Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/13-1.1.1.2-a+b-x-^m-c+d-x-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1917]. This is test number [13].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.0.1 (February 17, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.0.1.debian on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
5. Fricas 1.3.7 (June 30, 2021) based on based on ecl 21.2.1 on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
6. Giac/Xcas 1.9.0-7 (April 2022) on on Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem. Direct testing using C++ API.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 Ubuntu 20.04 Linux under window 10 WSL 2.0 subsystem via sagemath 9.6.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Mathics 4.0 via sagemath 9.6.

Maxima, Fricas, Mathics are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems. Mathics was called using its own interface in Sagemath as in this example

```
from sage.interfaces.mathics import mathics
res = mathics('Integrate[Sin[x]/(3 + Cos[x])^2,x]')
```

Sympy was called directly from Python. Giac was also called directly via its C++ interface.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1917)	0.00 (0)
Mathematica	100.00 (1917)	0.00 (0)
Fricas	83.62 (1603)	16.38 (314)
Maple	81.64 (1565)	18.36 (352)
Giac	72.30 (1386)	27.70 (531)
Maxima	69.27 (1328)	30.73 (589)
Mupad	64.74 (1241)	35.26 (676)
Sympy	62.55 (1199)	37.45 (718)
Mathics	62.44 (1197)	37.56 (720)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

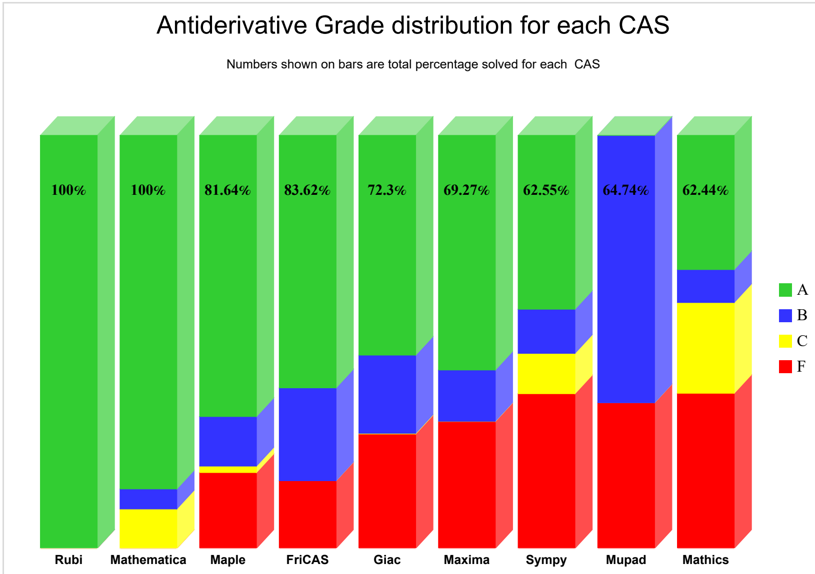
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

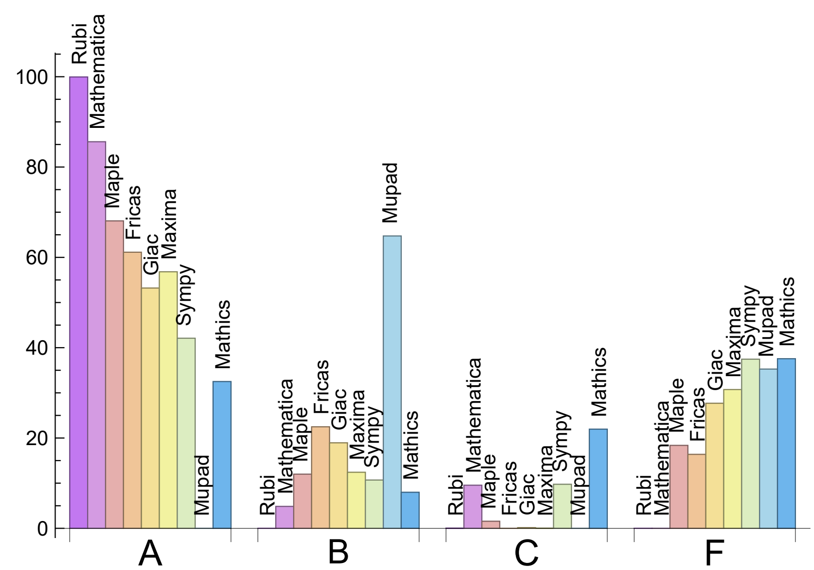
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.95	0.00	0.05	0.00
Mathematica	85.60	4.85	9.55	0.00
Maple	68.08	12.00	1.56	18.36
Fricas	61.14	22.48	0.00	16.38
Maxima	56.81	12.42	0.05	30.73
Giac	53.21	18.94	0.16	27.70
Sympy	42.10	10.69	9.75	37.45
Mathics	32.50	7.98	21.96	37.56
Mupad	N/A	64.74	0.00	35.26

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	352	100.00 %	0.00 %	0.00 %
Fricas	314	100.00 %	0.00 %	0.00 %
Giac	531	100.00 %	0.00 %	0.00 %
Maxima	589	78.10 %	0.00 %	21.90 %
Sympy	718	71.31 %	21.73 %	6.96 %
Mupad	676	100.00 %	0.00 %	0.00 %
Mathics	720	0.00 %	50.69 %	49.31 %

Table 1.4: Failure statistics for each CAS

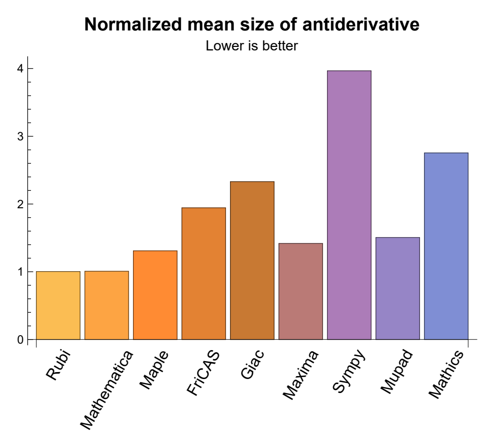
1.3 Performance

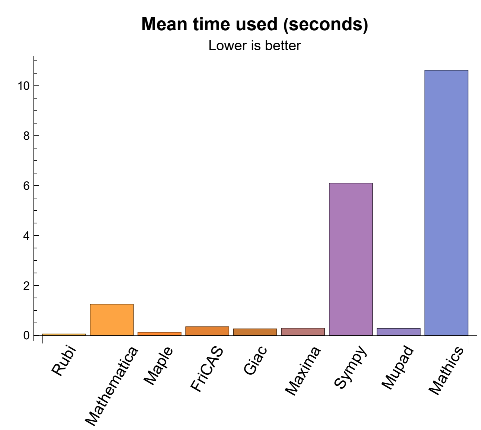
The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	108.09	1.00	66.00	1.00
Mathematica	1.25	80.02	1.01	56.00	0.90
Maple	0.13	100.32	1.31	57.00	0.94
Maxima	0.29	107.90	1.42	56.00	0.99
Fricas	0.34	200.82	1.94	77.00	1.34
Sympy	6.10	261.56	3.97	87.00	1.48
Giac	0.26	186.15	2.33	86.00	1.35
Mupad	0.28	123.92	1.51	52.00	0.97
Mathics	10.62	183.02	2.75	79.00	1.30

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

Mathics {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {1582, 1583, 1585, 1586, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1608, 1609, 1612, 1613, 1614, 1622, 1623, 1624, 1625, 1626, 1627}

Mathematica {}

Mathics {34, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 379, 380, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 396, 397, 398, 399, 400, 401, 403, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 438, 445, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 538, 539, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 562, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 612, 613, 614, 615, 619, 620, 621, 622, 626, 627, 628, 629, 632, 633, 634, 637, 638, 639, 640, 643, 644, 645, 646, 647, 648, 649, 661, 662, 663, 664, 665, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 698, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 897, 905, 913, 919, 989, 991, 1015, 1027, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1140, 1141, 1142, 1143, 1144, 1148, 1149, 1150, 1151, 1152, 1155, 1156, 1157, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1173, 1191, 1203, 1209, 1221, 1230, 1391, 1401, 1402, 1403, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1428, 1436, 1437, 1438, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1454, 1455, 1456, 1457, 1525, 1527, 1532, 1534, 1537, 1539, 1541, 1546, 1548, 1550, 1552, 1553, 1555, 1556, 1557, 1628, 1885, 1886, 1887, 1888, 1889}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi`, `Mathematica` and `Mathics`. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, Mathics and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

For Giac, the call `taille(anti_derivative,RAND_MAX);` is used to find leaf size.

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

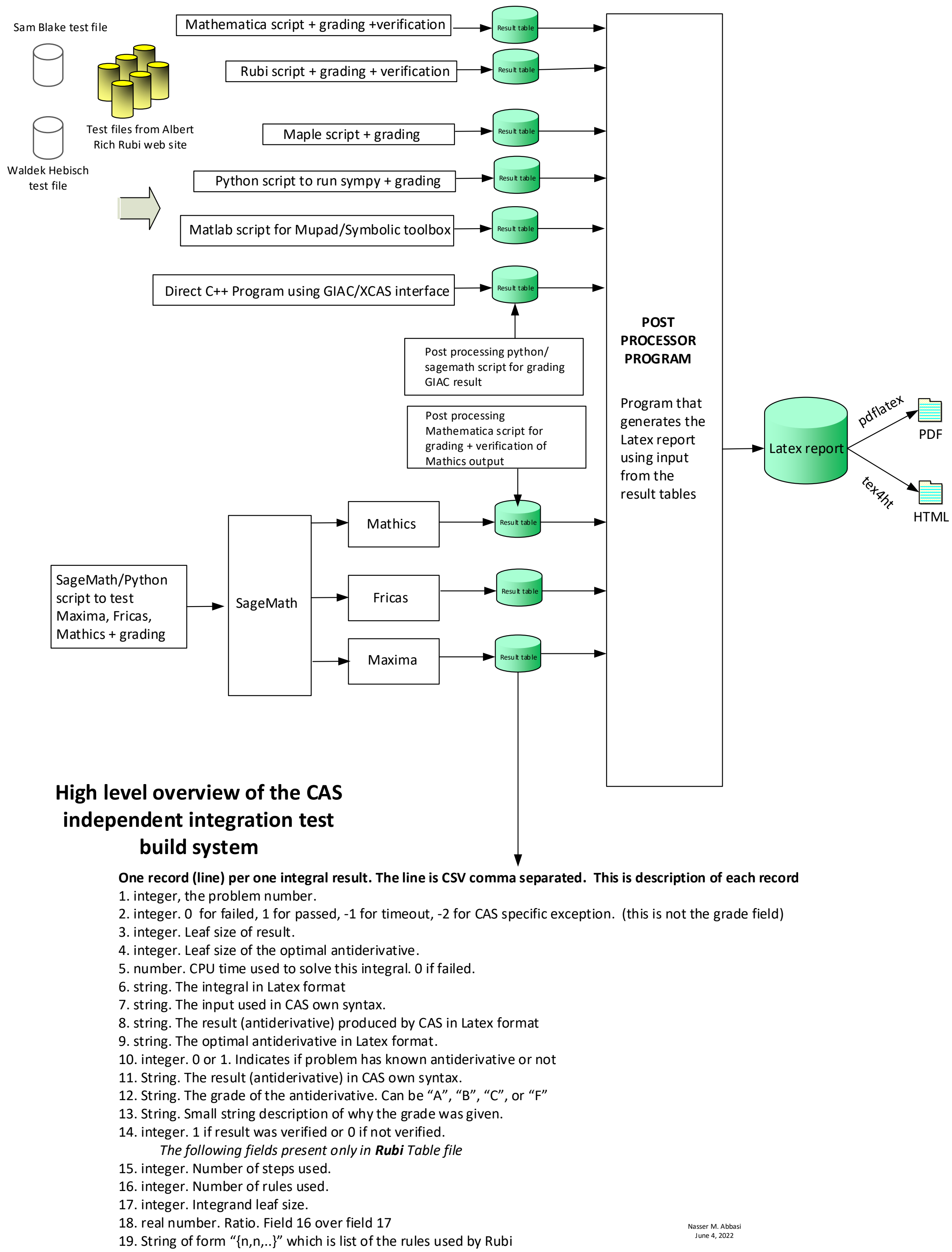
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 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621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 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1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 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B grade: { }

C grade: { 369 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 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1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1800, 1801, 1802, 1803, 1804, 1805, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1826, 1827, 1828, 1829, 1830, 1831, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 1847, 1848, 1849, 1850, 1851, 1852, 1853, 1854, 1855, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 648, 1110, 1154, 1155, 1156, 1157, 1162, 1236, 1246, 1258, 1259, 1268, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1284, 1285, 1288, 1289, 1291, 1292, 1293, 1294, 1295, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1362, 1363, 1365, 1366, 1525, 1527, 1532, 1534, 1537, 1539, 1540, 1543, 1545, 1546, 1548, 1550, 1555, 1628 }

C grade: { 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1189, 1190, 1191, 1192, 1193, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1582, 1583, 1584, 1585, 1586, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1680, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1700, 1701, 1702, 1703, 1704, 1712, 1713, 1714, 1715, 1716, 1717, 1723, 1724, 1725, 1726, 1727, 1728, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1780, 1799, 1806, 1825, 1832 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203,

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779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799,
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993, 994, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012,
1013, 1014, 1015, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032,
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1123, 1124, 1125, 1126, 1127, 1128, 1131, 1132, 1133, 1134, 1135, 1136, 1140, 1141, 1153, 1157, 1158,
1159, 1160, 1161, 1163, 1164, 1165, 1167, 1168, 1180, 1181, 1182, 1183, 1186, 1187, 1188, 1196, 1197,
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1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1469, 1470, 1471, 1472, 1473, 1474, 1481, 1482, 1483, 1484, 1492, 1493, 1494, 1495, 1497, 1498, 1499, 1500, 1501, 1506, 1507, 1508, 1509, 1510, 1511, 1517, 1518, 1519, 1520, 1521, 1530, 1541, 1544, 1578, 1579, 1580, 1581, 1590, 1591, 1592, 1593, 1604, 1605, 1606, 1607, 1618, 1619, 1620, 1621, 1682, 1683, 1684, 1685, 1696, 1697, 1698, 1699, 1708, 1709, 1710, 1711, 1719, 1720, 1721, 1722, 1775, 1776, 1777, 1778, 1782, 1783, 1784, 1785, 1801, 1802, 1803, 1804, 1808, 1809, 1810, 1811, 1827, 1828, 1829, 1830, 1834, 1835, 1836, 1837, 1844, 1848, 1854, 1855, 1860, 1861, 1868, 1869, 1874, 1875, 1881, 1882, 1883, 1884, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 199, 212, 213, 226, 227, 228, 243, 244, 494, 495, 502, 503, 510, 516, 517, 518, 572, 578, 584, 586, 593, 597, 602, 604, 611, 625, 632, 643, 648, 649, 699, 1016, 1017, 1018, 1034, 1066, 1067, 1068, 1069, 1070, 1078, 1079, 1081, 1082, 1083, 1084, 1091, 1098, 1099, 1100, 1101, 1105, 1109, 1110, 1118, 1119, 1129, 1130, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1154, 1155, 1156, 1162, 1166, 1169, 1225, 1236, 1237, 1246, 1247, 1254, 1258, 1259, 1260, 1261, 1262, 1268, 1269, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1343, 1352, 1362, 1363, 1364, 1365, 1366, 1367, 1424, 1434, 1453, 1466, 1467, 1468, 1477, 1478, 1479, 1480, 1488, 1489, 1490, 1491, 1496, 1516, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1542, 1543, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1846, 1847, 1852, 1853, 1859, 1870, 1871, 1876, 1877 }

C grade: { 721, 722, 726, 727, 1171, 1172, 1174, 1175, 1176, 1177, 1178, 1184, 1194, 1195, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1217, 1218, 1219, 1220, 1222, 1223, 1224, 1628 }

F grade: { 369, 579, 585, 598, 603, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 723, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1170, 1173, 1179, 1185, 1189, 1190, 1191, 1192, 1193, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1221, 1231, 1232, 1233, 1234, 1235, 1465, 1475, 1476, 1485, 1486, 1487, 1502, 1503, 1504, 1505, 1512, 1513, 1514, 1515, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1779, 1780, 1781, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1805, 1806, 1807, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1831, 1832, 1833, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 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B grade: { 1, 68, 73, 82, 83, 90, 105, 115, 116, 132, 133, 146, 147, 148, 186, 199, 212, 213, 214, 215, 216, 226, 227, 228, 229, 231, 232, 233, 243, 244, 489, 490, 491, 492, 505, 506, 507, 508, 521, 522, 523, 524, 533, 534, 535, 536, 545, 546, 547, 548, 557, 558, 559, 560, 561, 564, 570, 571, 572, 608, 609, 610, 611, 648, 830, 836, 855, 863, 880, 886, 887, 888, 917, 918, 919, 999, 1004, 1005, 1006, 1007, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1026, 1034, 1037, 1045, 1053, 1070, 1071, 1072, 1073, 1074, 1082, 1083, 1084, 1085, 1086, 1087, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1124, 1125, 1128, 1129, 1130, 1229, 1236, 1237, 1246, 1247, 1254, 1257, 1258, 1259, 1261, 1262, 1263, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1342, 1343, 1350, 1351, 1352, 1353, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1372, 1373, 1374, 1413, 1446, 1453, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1531, 1534, 1543, 1545, 1548, 1550, 1552, 1555, 1844, 1846, 1852 }

C grade: { 1027 }

F grade: { 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 754, 755, 760, 761, 762, 763, 769, 770, 771, 778, 779, 808, 809, 810, 811, 817, 818, 819, 824, 825, 826, 827, 856, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 992, 993, 994, 995, 996, 997, 998, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1231, 1232, 1233, 1234, 1235, 1381, 1382, 1383, 1384, 1385, 1386, 1393, 1394, 1395, 1396, 1397, 1398, 1405, 1406, 1407, 1408, 1409, 1410, 1419, 1420, 1421, 1422, 1423, 1430, 1431, 1432, 1433, 1440, 1441, 1442, 1443, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1551, 1554, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 204, 205, 206, 207, 208, 209, 210, 211, 222, 223, 230, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 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1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 37, 38, 42, 68, 73, 82, 83, 90, 105, 106, 115, 116, 132, 133, 134, 146, 147, 148, 186, 199, 201, 202, 203, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 243, 244, 295, 303, 314, 315, 316, 355, 356, 388, 410, 411, 418, 442, 586, 604, 625, 643, 648, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 697, 699, 700, 701, 999, 1004, 1005, 1006, 1007, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1026, 1034, 1037, 1045, 1068, 1069, 1070, 1082, 1083, 1084, 1098, 1099, 1100, 1101, 1109, 1110, 1119, 1129, 1130, 1156, 1162, 1170, 1236, 1237, 1246, 1247, 1254, 1257, 1258, 1259, 1261, 1262, 1263, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1342, 1343, 1344, 1345, 1350, 1351, 1352, 1353, 1354, 1355, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1408, 1409, 1410, 1420, 1421, 1422, 1423, 1424, 1431, 1432, 1433, 1434, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1453, 1458, 1459, 1465, 1466, 1467, 1468, 1469, 1470, 1476, 1477, 1478, 1479, 1480, 1481, 1486, 1487, 1488, 1489, 1490, 1491, 1496, 1498, 1499, 1500, 1501, 1502, 1505, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1527, 1529, 1531, 1534, 1536, 1537, 1538, 1539, 1540, 1543, 1545, 1548, 1550, 1551, 1553, 1554, 1555, 1557, 1558, 1576, 1577, 1578, 1579, 1580, 1581, 1587, 1588, 1589, 1591, 1592, 1593, 1601, 1602, 1603, 1605, 1606, 1607, 1615, 1617, 1619, 1620, 1621, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1693, 1694, 1695, 1697, 1698, 1699, 1705, 1706, 1707, 1709, 1710, 1711, 1717, 1718, 1720, 1721, 1722, 1730, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1809, 1810, 1811, 1824, 1825, 1826, 1828, 1829, 1830, 1831, 1832, 1833, 1835, 1836, 1837, 1846, 1847, 1852, 1853, 1859, 1860, 1869, 1870, 1871, 1875, 1876, 1877 }

C grade: { }

F grade: { 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1189, 1190, 1191, 1192, 1193, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1231, 1232, 1233, 1234, 1235, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1582, 1583, 1584, 1585, 1586, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1700, 1701, 1702, 1703, 1704, 1712, 1713, 1714, 1715, 1716, 1723, 1724, 1725, 1726, 1727, 1728, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 224, 225, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 327, 330, 333, 338, 339, 340, 341, 342, 346, 347, 349, 350, 352, 353, 354, 355, 359, 361, 363, 364, 370, 374, 381, 387, 388, 395, 402, 409, 415, 416, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 497, 498, 499, 500, 501, 503, 505, 506, 507, 508, 509, 511, 513, 514, 515, 516, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 560, 561, 562, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 625, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 666, 667, 668, 674, 675, 676, 677, 678, 679, 680, 681, 684, 685, 686, 687, 699, 700, 701, 702, 703, 732, 733, 734, 735, 741, 753, 754, 755, 756, 757, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 785, 786, 787, 788, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 812, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 825, 828, 829, 834, 835, 840, 841, 842, 843, 846, 847, 848, 849, 850, 851, 897, 905, 913, 919, 989, 991, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1015, 1022, 1023, 1024, 1025, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1038, 1039, 1040, 1042, 1043, 1044, 1046, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1069, 1070, 1076, 1080, 1081, 1093, 1094, 1095, 1104, 1105, 1106, 1107, 1108, 1111, 1112, 1116, 1117, 1118, 1120, 1126, 1127, 1130, 1131, 1132, 1156, 1160, 1161, 1162, 1163, 1173, 1191, 1203, 1209, 1221, 1229, 1230, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1248, 1249, 1251, 1252, 1253, 1255, 1264, 1265, 1266, 1267, 1335, 1336, 1337, 1338, 1339, 1343, 1344, 1345, 1346, 1347, 1348, 1354, 1355, 1356, 1357, 1376, 1377, 1378, 1379, 1380, 1381, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1456, 1457, 1525, 1530, 1532, 1544, 1546, 1552, 1553, 1555, 1556, 1557, 1642, 1846, 1847, 1848, 1852, 1853, 1854, 1855, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 55, 68, 73, 82, 83, 90, 91, 104, 105, 106, 115, 116, 131, 132, 133, 134, 146, 147, 148, 186, 187, 199, 201, 212, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 284, 285, 286, 288, 292, 293, 297, 334, 335, 336, 337, 343, 344, 345, 348, 351, 356, 357, 358, 371, 372, 373, 378, 379, 380, 385, 386, 392, 393, 394, 406, 407, 408, 413, 414, 494, 496, 510, 512, 575, 576, 582, 583, 584, 585, 586, 587, 588, 589, 614, 615, 621, 622, 623, 624, 626, 627, 628, 736, 737, 738, 808, 818, 826, 830, 836, 999, 1004, 1005, 1006, 1007, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1020, 1021, 1026, 1034, 1036, 1037, 1041, 1045, 1047, 1048, 1236, 1237, 1245, 1246, 1247, 1250, 1254, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1334, 1340, 1341, 1342, 1349, 1350, 1351, 1352, 1353, 1358, 1359, 1360, 1361, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1375, 1382, 1383, 1394, 1406, 1844, 1881 }

C grade: { 325, 326, 328, 329, 331, 332, 360, 362, 365, 367, 368, 369, 375, 376, 377, 382, 383, 384, 389, 390, 391, 396, 397, 398, 399, 400, 401, 403, 404, 405, 410, 411, 412, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 438, 502, 504, 518, 519, 520, 532, 544, 556, 568, 601, 602, 603, 604, 605, 606, 607, 634, 640, 641, 642, 643, 644, 645, 646, 661, 662, 663, 664, 665, 669, 670, 671, 672, 673, 698, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 742, 743, 744, 745, 746, 747, 748, 750, 751, 752, 1066, 1067, 1068, 1071, 1072, 1073, 1074, 1077, 1078, 1079, 1082, 1083, 1084, 1085, 1086, 1092, 1096, 1097, 1098, 1099, 1109, 1110, 1113, 1114, 1115, 1119, 1121, 1122, 1123, 1124, 1125, 1128, 1129, 1133, 1134, 1135, 1140, 1141, 1142, 1143, 1148, 1149, 1150, 1151, 1155, 1157, 1164, 1165, 1166, 1167, 1168, 1169, 1233, 1234, 1235, 1455, 1527, 1534, 1537, 1539, 1541, 1548, 1550, 1628, 1885, 1886, 1887, 1888, 1889 }

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2.1.7 Giac

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B grade: { 1, 37, 38, 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 284, 285, 286, 292, 293, 294, 295, 300, 301, 302, 303, 309, 310, 311, 312, 313, 314, 315, 316, 371, 372, 373, 378, 379, 380, 385, 386, 387, 388, 435, 442, 492, 494, 500, 501, 502, 508, 509, 510, 516, 517, 518, 523, 529, 535, 541, 545, 546, 547, 551, 552, 553, 557, 558, 559, 563, 564, 565, 580, 582, 583, 587, 594, 599, 601, 605, 607, 612, 619, 621, 622, 626, 628, 633, 634, 638, 639, 640, 644, 646, 649, 699, 700, 701, 702, 732, 733, 923, 924, 925, 930, 931, 938, 987, 989, 999, 1007, 1015, 1016, 1017, 1018, 1019, 1020, 1053, 1063, 1064, 1065, 1066, 1067, 1075, 1076, 1077, 1078, 1088, 1089, 1090, 1091, 1093, 1110, 1111, 1112, 1113, 1114, 1115, 1121, 1122, 1123, 1124, 1125, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1151, 1152, 1153, 1154, 1156, 1157, 1158, 1159, 1165, 1166, 1167, 1168, 1229, 1236, 1237, 1246, 1247, 1254, 1258, 1259, 1260, 1261, 1262, 1268, 1269, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1342, 1343, 1350, 1351, 1352, 1353, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1410, 1413, 1423, 1424, 1425, 1432, 1433, 1434, 1435, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1453, 1460, 1461, 1462, 1463, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1486, 1487, 1488, 1489, 1490, 1491, 1497, 1498, 1499, 1500, 1501, 1502, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1517, 1518, 1519, 1520, 1521, 1527, 1534, 1548, 1550, 1844, 1846, 1847, 1848, 1852, 1853, 1854 }

C grade: { 1027, 1541, 1542 }

F grade: { 368, 369, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 857, 858, 859, 865, 866, 867, 868, 875, 876, 881, 882, 883, 884, 889, 890, 891, 892, 898, 899, 900, 906, 907, 908, 914, 915, 916, 920, 921, 922, 927, 928, 929, 932, 935, 936, 937, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1231, 1232, 1233, 1234, 1235, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890 }

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1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1463, 1464, 1466, 1467, 1468, 1469, 1470, 1477, 1478, 1479, 1480, 1488, 1489, 1490, 1491, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1506, 1507, 1508, 1509, 1510, 1511, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 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C grade: { }

F grade: { 369, 489, 490, 493, 497, 498, 501, 505, 506, 509, 513, 514, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 577, 578, 579, 584, 585, 590, 591, 596, 597, 598, 602, 603, 608, 609, 616, 617, 618, 623, 624, 629, 630, 635, 636, 637, 641, 642, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 756, 757, 758, 761, 762, 764, 765, 766, 767, 768, 771, 772, 773, 774, 775, 776, 780, 788, 796, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 836, 837, 838, 839, 844, 845, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 898, 899, 900, 901, 902, 903, 904, 906, 907, 908, 909, 910, 911, 912, 914, 915, 916, 917, 918, 920, 921, 922, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1063, 1064, 1065, 1066, 1069, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1104, 1106, 1107, 1108, 1116, 1117, 1118, 1119, 1126, 1127, 1128, 1129, 1137, 1138, 1145, 1146, 1153, 1154, 1160, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1231, 1232, 1233, 1234, 1235, 1460, 1461, 1462, 1465, 1471, 1472, 1473, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1492, 1493, 1494, 1502, 1503, 1504, 1505, 1512, 1513, 1514, 1515, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1779, 1780, 1781, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1805, 1806, 1807, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.9 Mathics

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 219, 220, 221, 222, 223, 224, 225, 230, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 295, 296, 298, 303, 304, 305, 307, 308, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 347, 349, 350, 355, 358, 370, 374, 381, 388, 395, 402, 409, 416, 429, 430, 431, 432, 433, 434, 436, 437, 439, 440, 441, 443, 444, 446, 447, 448, 489, 490, 491, 492, 494, 495, 496, 505, 506, 507, 508, 509, 510, 511, 512, 521, 522, 523, 524, 525, 526, 533, 534, 535, 536, 537, 538, 545, 546, 547, 548, 549, 550, 557, 558, 559, 560, 561, 562, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 666, 667, 668, 756, 757, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 785, 786, 787, 788, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 812, 813, 814, 815, 816, 817, 818, 820, 821, 822, 823, 824, 825, 826, 828, 829, 830, 834, 835, 836, 840, 841, 842, 843, 846, 847, 848, 849, 850, 851, 999, 1000, 1001, 1002, 1003, 1007, 1008, 1009, 1010, 1011, 1021, 1022, 1023, 1024, 1025, 1026, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1255, 1256, 1257, 1260, 1261, 1262, 1264, 1265, 1266, 1267, 1270, 1271, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1355, 1356, 1357, 1358, 1368, 1369, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1392, 1393, 1404, 1405, 1418, 1419, 1424, 1425, 1426, 1427, 1429, 1430, 1434, 1435, 1439, 1440, 1525, 1530, 1532, 1544, 1844, 1881, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 68, 73, 83, 90, 105, 106, 115, 116, 132, 133, 134, 146, 147, 148, 199, 201, 212, 213, 216, 217, 218, 226, 227, 228, 229, 231, 232, 233, 234, 235, 243, 244, 284, 285, 286, 288, 291, 292, 293, 297, 299, 306, 334, 335, 336, 337, 343, 344, 345, 348, 351, 356, 357, 371, 372, 373, 379, 380, 385, 386, 392, 393, 394, 406, 407, 408, 413, 414, 493, 1004, 1005, 1006, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1020, 1236, 1246, 1254, 1258, 1259, 1263, 1268, 1269, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1342, 1350, 1351, 1352, 1353, 1354, 1359, 1360, 1361, 1365, 1366, 1367, 1370, 1371, 1372, 1373, 1374, 1382, 1383, 1384, 1387, 1388, 1389, 1390, 1394, 1395, 1399, 1400, 1406 }

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1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581,
1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598,
1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615,
1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633,
1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650,
1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667,
1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684,
1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701,
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1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752,
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1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803,
1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820,
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1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1855, 1856, 1857, 1858, 1859, 1860, 1861,
1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878,
1879, 1880, 1882, 1883, 1884, 1890 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.000	0.006	0.008	0.249	0.291	0.004	0.003	0.041	1.473

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.004	0.259	0.540	0.008	0.004	0.005	1.429

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.001	0.000	0.005	0.243	0.294	0.007	0.003	0.006	1.517

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00	1.00
time (sec)	N/A	0.000	0.000	0.007	0.250	0.297	0.007	0.007	0.004	1.517

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	3	3	5	5	3	3
N.S.	1	1.00	1.00	0.80	0.60	0.60	1.00	1.00	0.60	0.60
time (sec)	N/A	0.000	0.000	0.006	0.243	0.284	0.008	0.009	0.008	1.491

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.007	0.257	0.282	0.007	0.002	0.002	1.514

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.008	0.252	0.266	0.007	0.001	0.002	1.486

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.008	0.240	0.291	0.009	0.002	0.002	1.510

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	18	10	13	11	12
N.S.	1	1.00	1.00	0.86	0.79	1.29	0.71	0.93	0.79	0.86
time (sec)	N/A	0.005	0.000	0.022	0.237	0.295	0.024	0.003	0.003	1.563

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	6	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.86	0.71	0.71
time (sec)	N/A	0.001	0.000	0.011	0.238	0.293	0.025	0.002	0.118	1.509

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	6	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.86	0.71	0.71
time (sec)	N/A	0.000	0.000	0.005	0.259	0.288	0.024	0.002	0.022	1.515

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	6	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.86	0.71	0.71
time (sec)	N/A	0.000	0.000	0.005	0.238	0.286	0.024	0.002	0.010	1.512

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	6	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.86	0.71	0.71
time (sec)	N/A	0.000	0.000	0.007	0.269	0.271	0.009	0.002	0.012	1.518

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.003	0.250	0.289	0.007	0.000	0.002	1.427

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00	1.00
time (sec)	N/A	0.000	0.000	0.005	0.265	0.301	0.026	0.003	0.036	1.425

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	3	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	0.60	1.00	1.00
time (sec)	N/A	0.000	0.000	0.007	0.257	0.288	0.026	0.002	0.035	1.502

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.009	0.257	0.289	0.027	0.002	0.014	1.503

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.007	0.255	0.288	0.027	0.002	0.012	1.516

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.025	0.265	0.305	0.027	0.002	0.070	1.526

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	12	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.33	0.56	0.56
time (sec)	N/A	0.000	0.000	0.020	0.245	0.591	0.027	0.001	0.077	1.525

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	12	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.33	0.56	0.56
time (sec)	N/A	0.000	0.000	0.016	0.242	0.299	0.027	0.001	0.031	1.523

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	10	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.11	0.56	0.56
time (sec)	N/A	0.000	0.000	0.017	0.244	0.291	0.026	0.000	0.029	1.512

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.017	0.259	0.293	0.026	0.000	0.031	1.499

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	9	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	1.29	0.71	0.71
time (sec)	N/A	0.000	0.000	0.017	0.249	0.289	0.027	0.000	0.031	1.519

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	8	13	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.89	1.44	0.56	0.56
time (sec)	N/A	0.000	0.000	0.020	0.243	0.295	0.027	0.000	0.034	1.497

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	16	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.019	0.266	0.305	0.026	0.001	0.073	1.521

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	12	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.33	0.56	0.56
time (sec)	N/A	0.000	0.001	0.019	0.256	0.292	0.026	0.000	0.066	1.595

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	12	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.33	0.56	0.56
time (sec)	N/A	0.000	0.001	0.027	0.245	0.294	0.027	0.000	0.065	1.560

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	10	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.11	0.56	0.56
time (sec)	N/A	0.000	0.001	0.018	0.245	0.294	0.027	0.000	0.065	1.546

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	11	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	1.22	0.56	0.56
time (sec)	N/A	0.000	0.000	0.018	0.243	0.288	0.027	0.000	0.040	1.513

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.018	0.250	0.314	0.027	0.000	0.067	1.498

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	9	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	1.29	0.71	0.71
time (sec)	N/A	0.000	0.000	0.020	0.249	0.314	0.026	0.000	0.073	1.515

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	8	14	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.89	1.56	0.56	0.56
time (sec)	N/A	0.000	0.000	0.018	0.252	0.313	0.027	0.001	0.051	1.561

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	10	12	10	20	19
N.S.	1	1.00	1.00	1.09	1.00	0.91	1.09	0.91	1.82	1.73
time (sec)	N/A	0.001	0.001	0.007	0.244	0.314	0.028	0.000	0.345	1.583

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	17	16	12	17	14	12	0
N.S.	1	1.00	0.75	1.06	1.00	0.75	1.06	0.88	0.75	0.00
time (sec)	N/A	0.002	0.001	0.010	0.251	0.328	0.029	0.000	0.183	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	19	24	21	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.83	1.04	0.91	0.91
time (sec)	N/A	0.010	0.006	0.105	0.257	0.318	0.037	0.001	0.143	1.692

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	104	270	884	93	239
N.S.	1	1.00	1.00	0.87	0.83	4.52	11.74	38.43	4.04	10.39
time (sec)	N/A	0.009	0.022	0.093	0.247	0.306	1.640	0.007	0.183	3.217

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	59	156	357	45	142
N.S.	1	1.00	1.00	0.87	0.83	2.57	6.78	15.52	1.96	6.17
time (sec)	N/A	0.007	0.015	0.096	0.246	0.311	0.359	0.002	0.170	2.009

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	82	30	19	84
N.S.	1	1.00	1.00	0.87	0.83	0.83	3.57	1.30	0.83	3.65
time (sec)	N/A	0.007	0.012	0.092	0.246	0.308	0.124	0.000	0.075	1.795

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	19	31	19	19	43
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.48	0.90	0.90	2.05
time (sec)	N/A	0.007	0.010	0.092	0.256	0.308	0.678	0.000	0.106	2.170

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	34	58	21	19	71
N.S.	1	1.00	1.00	0.95	0.90	1.62	2.76	1.00	0.90	3.38
time (sec)	N/A	0.007	0.014	0.084	0.252	0.304	0.594	0.001	0.135	2.166

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	68	102	33	19	108
N.S.	1	1.00	1.00	0.87	0.83	2.96	4.43	1.43	0.83	4.70
time (sec)	N/A	0.007	0.013	0.081	0.248	0.306	1.463	0.002	0.179	3.189

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	17	13	12
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	1.00	0.76	0.71
time (sec)	N/A	0.007	0.001	0.011	0.246	0.301	0.027	0.000	0.020	1.624

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	17	13	12
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	1.00	0.76	0.71
time (sec)	N/A	0.005	0.001	0.013	0.261	0.309	0.027	0.000	0.019	1.632

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	17	13	12
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	1.00	0.76	0.71
time (sec)	N/A	0.004	0.001	0.010	0.258	0.268	0.027	0.000	0.019	1.608

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83	0.83
time (sec)	N/A	0.001	0.000	0.010	0.245	0.268	0.026	0.000	0.017	1.593

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00	1.00
time (sec)	N/A	0.002	0.001	0.012	0.246	0.323	0.040	0.000	0.017	1.588

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	7	11	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.64	1.00	1.00	1.00
time (sec)	N/A	0.004	0.002	0.012	0.257	0.300	0.049	0.000	0.033	1.631

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	14	11	11	12	15	11	12
N.S.	1	1.00	0.88	0.82	0.65	0.65	0.71	0.88	0.65	0.71
time (sec)	N/A	0.001	0.001	0.010	0.247	0.304	0.053	0.000	0.023	1.627

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	17	13	12
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	1.00	0.76	0.71
time (sec)	N/A	0.004	0.001	0.010	0.248	0.298	0.057	0.000	0.026	1.632

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	17	13	12
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	1.00	0.76	0.71
time (sec)	N/A	0.003	0.001	0.011	0.251	0.311	0.062	0.000	0.027	1.678

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	30	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	1.00	0.80	0.80
time (sec)	N/A	0.009	0.001	0.071	0.254	0.301	0.032	0.000	0.077	1.680

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	30	24	23
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	1.00	0.80	0.77
time (sec)	N/A	0.007	0.001	0.074	0.246	0.307	0.031	0.000	0.031	1.659

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	30	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	1.00	0.80	0.80
time (sec)	N/A	0.007	0.001	0.069	0.248	0.301	0.031	0.000	0.030	1.646

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	20	18
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43	1.29
time (sec)	N/A	0.001	0.001	0.087	0.247	0.300	0.030	0.000	0.029	1.733

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	20	23	20	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	0.91	1.05	0.91	0.91
time (sec)	N/A	0.005	0.001	0.072	0.255	0.298	0.048	0.000	0.029	1.745

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	17	20	20	22
N.S.	1	1.00	1.00	1.05	1.00	1.20	0.85	1.00	1.00	1.10
time (sec)	N/A	0.006	0.000	0.073	0.248	0.310	0.059	0.000	0.066	1.726

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	21	26	22	27	23	22
N.S.	1	1.00	1.00	0.96	0.88	1.08	0.92	1.12	0.96	0.92
time (sec)	N/A	0.006	0.002	0.078	0.246	0.305	0.075	0.000	0.044	1.800

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	26	25	22	22	24	27	22	23
N.S.	1	1.00	1.53	1.47	1.29	1.29	1.41	1.59	1.29	1.35
time (sec)	N/A	0.001	0.004	0.072	0.244	0.298	0.079	0.000	0.035	1.765

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	29	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.97	0.80	0.80
time (sec)	N/A	0.006	0.002	0.073	0.246	0.325	0.085	0.000	0.035	1.721

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	29	24	23
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.97	0.80	0.77
time (sec)	N/A	0.006	0.004	0.084	0.264	0.309	0.105	0.000	0.035	1.751

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	29	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.97	0.80	0.80
time (sec)	N/A	0.006	0.002	0.086	0.247	0.312	0.119	0.000	0.034	1.769

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	29	24	23
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.97	0.80	0.77
time (sec)	N/A	0.006	0.004	0.087	0.254	0.302	0.107	0.000	0.036	1.779

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	43	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	1.00	0.81	0.81
time (sec)	N/A	0.013	0.001	0.074	0.246	0.292	0.034	0.000	0.042	1.727

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	43	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	1.00	0.81	0.81
time (sec)	N/A	0.012	0.001	0.070	0.255	0.298	0.033	0.000	0.041	1.761

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	43	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	1.00	0.81	0.81
time (sec)	N/A	0.011	0.001	0.073	0.247	0.295	0.033	0.000	0.039	1.768

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	35	34	34	36	40	34	33
N.S.	1	1.00	1.33	1.17	1.13	1.13	1.20	1.33	1.13	1.10
time (sec)	N/A	0.006	0.001	0.080	0.246	0.303	0.033	0.000	0.040	1.717

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	12	31	32
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.21	2.29
time (sec)	N/A	0.001	0.001	0.070	0.246	0.309	0.032	0.000	0.041	1.710

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	34	36	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.97	1.03	0.89	0.89
time (sec)	N/A	0.007	0.002	0.079	0.257	0.307	0.067	0.000	0.035	1.727

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	31	34	32	34
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.91	1.00	0.94	1.00
time (sec)	N/A	0.009	0.003	0.084	0.245	0.321	0.063	0.000	0.035	1.729

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	30	37	32	36	32	31
N.S.	1	1.00	1.00	0.97	0.91	1.12	0.97	1.09	0.97	0.94
time (sec)	N/A	0.008	0.004	0.088	0.245	0.312	0.086	0.000	0.030	1.829

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	37	36	41	34	33
N.S.	1	1.00	1.00	0.92	0.92	1.00	0.97	1.11	0.92	0.89
time (sec)	N/A	0.009	0.003	0.086	0.243	0.298	0.101	0.000	0.071	1.878

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	39	36	33	33	36	39	33	34
N.S.	1	1.00	2.29	2.12	1.94	1.94	2.12	2.29	1.94	2.00
time (sec)	N/A	0.001	0.002	0.073	0.255	0.303	0.108	0.000	0.026	1.852

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	35	35	37	41	34	34
N.S.	1	1.00	1.14	1.00	0.97	0.97	1.03	1.14	0.94	0.94
time (sec)	N/A	0.004	0.004	0.073	0.247	0.308	0.117	0.000	0.027	1.860

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	41	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.95	0.81	0.81
time (sec)	N/A	0.009	0.002	0.082	0.243	0.303	0.125	0.000	0.025	1.864

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	41	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.95	0.81	0.81
time (sec)	N/A	0.009	0.002	0.088	0.250	0.294	0.134	0.000	0.026	1.861

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	66	56	57
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	1.00	0.85	0.86
time (sec)	N/A	0.023	0.002	0.090	0.267	0.448	0.036	0.000	0.025	1.844

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	69	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	1.00	0.83	0.83
time (sec)	N/A	0.019	0.002	0.073	0.250	0.300	0.036	0.000	0.023	1.840

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	69	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	1.00	0.83	0.83
time (sec)	N/A	0.017	0.002	0.082	0.247	0.308	0.036	0.000	0.024	1.839

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	57	56	56	63	66	56	57
N.S.	1	1.00	1.03	0.89	0.88	0.88	0.98	1.03	0.88	0.89
time (sec)	N/A	0.018	0.002	0.086	0.249	0.300	0.037	0.000	0.024	1.838

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	67	58	57	57	65	67	57	57
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.38	1.43	1.21	1.21
time (sec)	N/A	0.014	0.002	0.072	0.254	0.301	0.036	0.000	0.024	1.908

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	67	58	57	57	65	67	57	57
N.S.	1	1.00	2.23	1.93	1.90	1.90	2.17	2.23	1.90	1.90
time (sec)	N/A	0.006	0.001	0.095	0.253	0.291	0.036	0.000	0.023	1.935

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	53	53	60	12	53	54
N.S.	1	1.00	1.00	0.93	3.79	3.79	4.29	0.86	3.79	3.86
time (sec)	N/A	0.001	0.001	0.071	0.247	0.298	0.034	0.000	0.024	1.895

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	53	53	60	60	53	53
N.S.	1	1.00	1.00	0.92	0.90	0.90	1.02	1.02	0.90	0.90
time (sec)	N/A	0.012	0.002	0.085	0.245	0.313	0.063	0.000	0.029	1.908

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	54	59	56	58	54	57
N.S.	1	1.00	1.00	0.95	0.93	1.02	0.97	1.00	0.93	0.98
time (sec)	N/A	0.014	0.003	0.095	0.251	0.311	0.073	0.000	0.029	1.899

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	53	59	60	63	55	56
N.S.	1	1.00	1.00	0.92	0.88	0.98	1.00	1.05	0.92	0.93
time (sec)	N/A	0.014	0.003	0.075	0.255	0.318	0.095	0.000	0.029	1.991

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	55	59	60	64	55	58
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	1.07	0.92	0.97
time (sec)	N/A	0.014	0.003	0.098	0.252	0.311	0.117	0.000	0.039	2.055

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	54	59	58	62	54	53
N.S.	1	1.00	1.00	0.95	0.95	1.04	1.02	1.09	0.95	0.93
time (sec)	N/A	0.014	0.004	0.095	0.249	0.298	0.145	0.000	0.077	2.121

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	56	59	60	65	56	58
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.98	1.07	0.92	0.95
time (sec)	N/A	0.014	0.003	0.073	0.245	0.295	0.170	0.000	0.041	2.168

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	65	58	55	55	60	63	55	57
N.S.	1	1.00	3.82	3.41	3.24	3.24	3.53	3.71	3.24	3.35
time (sec)	N/A	0.001	0.003	0.076	0.258	0.306	0.180	0.000	0.039	2.068

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	67	58	57	57	61	65	57	57
N.S.	1	1.00	1.86	1.61	1.58	1.58	1.69	1.81	1.58	1.58
time (sec)	N/A	0.003	0.003	0.092	0.249	0.293	0.193	0.000	0.073	2.143

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	67	58	57	57	61	65	57	57
N.S.	1	1.00	1.20	1.04	1.02	1.02	1.09	1.16	1.02	1.02
time (sec)	N/A	0.007	0.003	0.080	0.249	0.300	0.207	0.000	0.039	2.157

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	65	56	57
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.97	0.84	0.85
time (sec)	N/A	0.015	0.004	0.089	0.249	0.303	0.221	0.000	0.084	2.192

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	61	65	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.88	0.94	0.83	0.83
time (sec)	N/A	0.015	0.003	0.088	0.256	0.300	0.231	0.000	0.082	2.169

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	61	65	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.88	0.94	0.83	0.83
time (sec)	N/A	0.015	0.003	0.084	0.257	0.297	0.247	0.000	0.042	2.091

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	65	56	57
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.97	0.84	0.85
time (sec)	N/A	0.015	0.003	0.088	0.246	0.302	0.257	0.000	0.040	2.103

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	65	56	57
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.97	0.84	0.85
time (sec)	N/A	0.015	0.003	0.090	0.248	0.297	0.264	0.000	0.038	2.163

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	94	95	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	1.00	0.83	0.83
time (sec)	N/A	0.033	0.002	0.091	0.256	0.307	0.040	0.000	0.153	1.979

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	92	95	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.97	1.00	0.83	0.83
time (sec)	N/A	0.028	0.002	0.084	0.245	0.317	0.040	0.000	0.074	2.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	94	95	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	1.00	0.83	0.83
time (sec)	N/A	0.025	0.002	0.089	0.245	0.293	0.040	0.000	0.068	1.991

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	79	78	78	90	92	78	79
N.S.	1	1.00	0.96	0.82	0.81	0.81	0.94	0.96	0.81	0.82
time (sec)	N/A	0.027	0.002	0.073	0.243	0.302	0.040	0.000	0.060	2.115

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	93	80	79	79	92	93	79	79
N.S.	1	1.00	1.15	0.99	0.98	0.98	1.14	1.15	0.98	0.98
time (sec)	N/A	0.023	0.002	0.072	0.247	0.305	0.039	0.000	0.062	2.039

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	80	79	79	92	93	79	79
N.S.	1	1.00	1.45	1.25	1.23	1.23	1.44	1.45	1.23	1.23
time (sec)	N/A	0.020	0.002	0.094	0.252	0.316	0.039	0.000	0.104	2.055

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	93	80	79	79	92	93	31	79
N.S.	1	1.00	1.98	1.70	1.68	1.68	1.96	1.98	0.66	1.68
time (sec)	N/A	0.016	0.002	0.076	0.244	0.298	0.039	0.000	0.121	2.003

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	91	80	79	79	90	91	25	79
N.S.	1	1.00	3.03	2.67	2.63	2.63	3.00	3.03	0.83	2.63
time (sec)	N/A	0.006	0.002	0.100	0.248	0.614	0.038	0.000	0.115	1.967

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	75	83	12	75	76
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.36	5.43
time (sec)	N/A	0.001	0.001	0.075	0.246	0.298	0.036	0.000	0.060	1.972

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	76	75	75	88	88	75	75
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	1.01	0.86	0.86
time (sec)	N/A	0.018	0.002	0.079	0.265	0.307	0.072	0.000	0.072	1.943

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	76	81	85	86	76	79
N.S.	1	1.00	1.00	0.90	0.88	0.94	0.99	1.00	0.88	0.92
time (sec)	N/A	0.021	0.003	0.079	0.246	0.299	0.081	0.000	0.055	2.003

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	75	81	85	87	77	78
N.S.	1	1.00	1.00	0.92	0.89	0.96	1.01	1.04	0.92	0.93
time (sec)	N/A	0.022	0.003	0.084	0.247	0.302	0.105	0.000	0.051	2.087

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	77	81	87	90	77	80
N.S.	1	1.00	1.00	0.90	0.90	0.94	1.01	1.05	0.90	0.93
time (sec)	N/A	0.021	0.003	0.076	0.245	0.303	0.127	0.000	0.051	2.201

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	77	81	85	89	77	81
N.S.	1	1.00	1.00	0.90	0.90	0.94	0.99	1.03	0.90	0.94
time (sec)	N/A	0.022	0.003	0.077	0.263	0.316	0.153	0.000	0.090	2.226

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	77	81	83	88	77	80
N.S.	1	1.00	1.00	0.92	0.92	0.96	0.99	1.05	0.92	0.95
time (sec)	N/A	0.021	0.003	0.079	0.270	0.301	0.193	0.000	0.106	2.284

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	76	81	82	86	81	80
N.S.	1	1.00	1.00	0.89	0.89	0.95	0.96	1.01	0.95	0.94
time (sec)	N/A	0.021	0.004	0.077	0.260	0.307	0.227	0.001	0.110	2.324

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	78	78	81	83	89	78	80
N.S.	1	1.00	1.00	0.88	0.88	0.91	0.93	1.00	0.88	0.90
time (sec)	N/A	0.021	0.003	0.091	0.246	0.299	0.257	0.000	0.068	2.381

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	87	80	77	77	83	87	77	79
N.S.	1	1.00	5.12	4.71	4.53	4.53	4.88	5.12	4.53	4.65
time (sec)	N/A	0.001	0.003	0.074	0.260	0.305	0.269	0.000	0.067	2.354

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	80	79	79	85	89	23	79
N.S.	1	1.00	2.53	2.22	2.19	2.19	2.36	2.47	0.64	2.19
time (sec)	N/A	0.004	0.003	0.073	0.248	0.294	0.286	0.000	0.093	2.295

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	93	80	79	79	85	89	79	79
N.S.	1	1.00	1.66	1.43	1.41	1.41	1.52	1.59	1.41	1.41
time (sec)	N/A	0.007	0.003	0.073	0.248	0.337	0.309	0.000	0.109	2.285

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	93	80	79	79	85	89	79	79
N.S.	1	1.00	1.22	1.05	1.04	1.04	1.12	1.17	1.04	1.04
time (sec)	N/A	0.012	0.003	0.079	0.250	0.298	0.326	0.001	0.106	2.278

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	93	80	79	79	85	89	79	79
N.S.	1	1.00	0.97	0.83	0.82	0.82	0.89	0.93	0.82	0.82
time (sec)	N/A	0.019	0.003	0.075	0.263	0.295	0.340	0.000	0.066	2.376

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	80	79	79	85	89	78	79
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.91	0.96	0.84	0.85
time (sec)	N/A	0.022	0.004	0.073	0.245	0.299	0.352	0.001	0.066	2.462

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	85	89	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.89	0.94	0.83	0.83
time (sec)	N/A	0.022	0.003	0.077	0.248	0.305	0.368	0.000	0.070	2.683

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	85	89	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.89	0.94	0.83	0.83
time (sec)	N/A	0.021	0.003	0.076	0.246	0.296	0.383	0.000	0.110	2.680

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	133	132	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	1.00	0.85	0.85
time (sec)	N/A	0.051	0.002	0.078	0.254	0.301	0.045	0.000	0.150	2.142

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	131	132	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	1.00	0.85	0.85
time (sec)	N/A	0.042	0.002	0.073	0.254	0.297	0.046	0.000	0.084	2.153

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	133	132	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	1.00	0.85	0.85
time (sec)	N/A	0.041	0.002	0.075	0.252	0.306	0.045	0.000	0.125	2.167

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	112	111	111	126	125	111	112
N.S.	1	1.00	0.85	0.76	0.76	0.76	0.86	0.85	0.76	0.76
time (sec)	N/A	0.043	0.002	0.091	0.258	0.305	0.045	0.000	0.087	2.298

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	130	113	112	112	131	130	112	112
N.S.	1	1.00	0.98	0.86	0.85	0.85	0.99	0.98	0.85	0.85
time (sec)	N/A	0.038	0.002	0.075	0.266	0.298	0.044	0.000	0.082	2.331

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	126	113	112	112	128	126	112	112
N.S.	1	1.00	1.12	1.01	1.00	1.00	1.14	1.12	1.00	1.00
time (sec)	N/A	0.034	0.002	0.075	0.255	0.304	0.045	0.000	0.121	2.273

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	132	113	112	112	133	132	112	112
N.S.	1	1.00	1.35	1.15	1.14	1.14	1.36	1.35	1.14	1.14
time (sec)	N/A	0.030	0.002	0.077	0.243	0.298	0.045	0.000	0.123	2.145

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	130	113	112	112	131	130	112	112
N.S.	1	1.00	1.60	1.40	1.38	1.38	1.62	1.60	1.38	1.38
time (sec)	N/A	0.027	0.002	0.080	0.248	0.307	0.045	0.000	0.120	2.195

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	128	113	112	112	129	128	112	112
N.S.	1	1.00	2.00	1.77	1.75	1.75	2.02	2.00	1.75	1.75
time (sec)	N/A	0.024	0.002	0.076	0.247	0.305	0.045	0.000	0.118	2.185

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	126	113	112	112	128	126	31	112
N.S.	1	1.00	2.68	2.40	2.38	2.38	2.72	2.68	0.66	2.38
time (sec)	N/A	0.020	0.002	0.075	0.245	0.298	0.043	0.000	0.070	2.207

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	128	113	112	112	129	128	25	112
N.S.	1	1.00	4.27	3.77	3.73	3.73	4.30	4.27	0.83	3.73
time (sec)	N/A	0.006	0.002	0.075	0.255	0.307	0.044	0.000	0.095	2.160

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	108	114	12	108	106
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.71	7.57
time (sec)	N/A	0.001	0.001	0.082	0.246	0.303	0.041	0.000	0.112	2.098

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	109	108	108	126	123	108	108
N.S.	1	1.00	1.00	0.89	0.89	0.89	1.03	1.01	0.89	0.89
time (sec)	N/A	0.029	0.003	0.118	0.248	0.303	0.087	0.000	0.079	2.213

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	109	114	117	115	109	112
N.S.	1	1.00	1.00	0.96	0.95	0.99	1.02	1.00	0.95	0.97
time (sec)	N/A	0.032	0.006	0.078	0.262	0.305	0.099	0.000	0.115	2.264

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	108	114	122	122	110	111
N.S.	1	1.00	1.00	0.92	0.91	0.96	1.03	1.03	0.92	0.93
time (sec)	N/A	0.031	0.003	0.096	0.243	0.303	0.121	0.001	0.071	2.352

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	108	114	119	119	110	111
N.S.	1	1.00	1.00	0.96	0.94	0.99	1.03	1.03	0.96	0.97
time (sec)	N/A	0.032	0.006	0.077	0.246	0.306	0.145	0.000	0.062	2.400

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	121	122	110	113
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.02	1.03	0.92	0.95
time (sec)	N/A	0.032	0.005	0.079	0.247	0.306	0.177	0.000	0.098	2.430

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	110	110	114	121	123	110	113
N.S.	1	1.00	1.00	0.94	0.94	0.97	1.03	1.05	0.94	0.97
time (sec)	N/A	0.032	0.006	0.078	0.259	0.303	0.217	0.000	0.099	2.441

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	122	126	110	113
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.03	1.06	0.92	0.95
time (sec)	N/A	0.032	0.003	0.077	0.248	0.302	0.244	0.000	0.055	2.584

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	110	114	119	123	110	113
N.S.	1	1.00	1.00	0.96	0.96	0.99	1.03	1.07	0.96	0.98
time (sec)	N/A	0.033	0.006	0.082	0.244	0.301	0.289	0.000	0.095	2.719

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	119	124	110	113
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.00	1.04	0.92	0.95
time (sec)	N/A	0.032	0.003	0.076	0.250	0.306	0.330	0.000	0.068	2.796

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	109	114	117	122	114	113
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	1.07	1.00	0.99
time (sec)	N/A	0.032	0.004	0.082	0.261	0.301	0.381	0.001	0.076	2.832

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	111	111	114	119	125	111	113
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.96	1.01	0.90	0.91
time (sec)	N/A	0.032	0.003	0.077	0.244	0.302	0.420	0.001	0.074	2.874

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	114	113	110	110	119	123	110	111
N.S.	1	1.00	6.71	6.65	6.47	6.47	7.00	7.24	6.47	6.53
time (sec)	N/A	0.001	0.006	0.080	0.244	0.294	0.442	0.000	0.134	2.793

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	128	113	112	112	121	125	23	112
N.S.	1	1.00	3.56	3.14	3.11	3.11	3.36	3.47	0.64	3.11
time (sec)	N/A	0.004	0.003	0.075	0.243	0.297	0.472	0.000	0.096	2.767

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	126	113	112	112	121	125	112	112
N.S.	1	1.00	2.25	2.02	2.00	2.00	2.16	2.23	2.00	2.00
time (sec)	N/A	0.008	0.006	0.075	0.255	0.303	0.493	0.001	0.132	2.922

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	128	113	112	112	121	125	112	112
N.S.	1	1.00	1.68	1.49	1.47	1.47	1.59	1.64	1.47	1.47
time (sec)	N/A	0.012	0.005	0.075	0.244	0.304	0.507	0.001	0.094	2.909

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	130	113	112	112	121	125	112	112
N.S.	1	1.00	1.35	1.18	1.17	1.17	1.26	1.30	1.17	1.17
time (sec)	N/A	0.019	0.006	0.078	0.246	0.302	0.562	0.000	0.130	2.881

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	132	113	112	112	121	125	112	112
N.S.	1	1.00	1.14	0.97	0.97	0.97	1.04	1.08	0.97	0.97
time (sec)	N/A	0.026	0.003	0.085	0.252	0.301	0.584	0.001	0.135	2.900

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	126	113	112	112	121	125	112	112
N.S.	1	1.00	0.93	0.83	0.82	0.82	0.89	0.92	0.82	0.82
time (sec)	N/A	0.035	0.006	0.090	0.245	0.291	0.593	0.001	0.134	2.937

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	113	112	112	121	125	112	112
N.S.	1	1.00	1.00	0.87	0.86	0.86	0.93	0.96	0.86	0.86
time (sec)	N/A	0.033	0.003	0.076	0.248	0.303	0.607	0.001	0.099	2.931

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	126	113	112	112	121	125	111	112
N.S.	1	1.00	1.00	0.90	0.89	0.89	0.96	0.99	0.88	0.89
time (sec)	N/A	0.032	0.005	0.076	0.244	0.304	0.633	0.001	0.137	2.999

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	12	12	13	11	11
N.S.	1	1.00	0.93	0.87	0.87	0.80	0.80	0.87	0.73	0.73
time (sec)	N/A	0.001	0.001	0.009	0.243	0.287	0.026	0.000	0.021	1.688

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	18	17	26	22	17	16	16
N.S.	1	1.00	0.95	0.90	0.85	1.30	1.10	0.85	0.80	0.80
time (sec)	N/A	0.003	0.001	0.008	0.254	0.302	0.031	0.000	0.075	1.758

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	63	64	63	61	75	62	61
N.S.	1	1.00	1.00	0.90	0.91	0.90	0.87	1.07	0.89	0.87
time (sec)	N/A	0.025	0.003	0.085	0.258	0.316	0.070	0.000	0.080	2.002

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	61	51	50
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	1.07	0.89	0.88
time (sec)	N/A	0.017	0.003	0.102	0.243	0.297	0.066	0.000	0.100	1.919

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	48	40	39
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	1.09	0.91	0.89
time (sec)	N/A	0.014	0.003	0.084	0.242	0.324	0.062	0.000	0.038	1.843

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	34	29	28
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	1.10	0.94	0.90
time (sec)	N/A	0.010	0.002	0.077	0.244	0.316	0.057	0.000	0.039	1.755

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18	17
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00	0.94
time (sec)	N/A	0.007	0.002	0.076	0.254	0.301	0.050	0.000	0.076	1.696

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	1.00
time (sec)	N/A	0.001	0.001	0.078	0.243	0.298	0.029	0.000	0.021	1.627

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	21	15	17
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.17	0.83	0.94
time (sec)	N/A	0.003	0.003	0.082	0.244	0.305	0.071	0.000	0.085	1.803

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	37	25	30
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.32	0.89	1.07
time (sec)	N/A	0.010	0.003	0.077	0.241	0.311	0.089	0.000	0.051	1.877

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	52	38	42
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.24	0.90	1.00
time (sec)	N/A	0.012	0.003	0.082	0.252	0.315	0.104	0.000	0.058	1.944

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	66	48	53
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.18	0.86	0.95
time (sec)	N/A	0.016	0.004	0.086	0.243	0.319	0.115	0.000	0.105	2.122

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	62	65	56	77	60	64
N.S.	1	1.00	1.00	0.93	0.91	0.96	0.82	1.13	0.88	0.94
time (sec)	N/A	0.018	0.003	0.088	0.244	0.302	0.131	0.001	0.065	2.204

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	78	82	96	78	92	83	98
N.S.	1	1.00	0.95	0.96	1.01	1.19	0.96	1.14	1.02	1.21
time (sec)	N/A	0.036	0.014	0.082	0.244	0.317	0.117	0.000	0.140	2.257

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	68	70	85	71	82	72	89
N.S.	1	1.00	0.92	0.94	0.97	1.18	0.99	1.14	1.00	1.24
time (sec)	N/A	0.030	0.010	0.084	0.251	0.304	0.112	0.000	0.070	2.215

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	67	62	72
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.16	1.07	1.24
time (sec)	N/A	0.024	0.011	0.082	0.246	0.307	0.104	0.000	0.067	2.129

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	46	47	62	44	53	50	54
N.S.	1	1.00	0.93	1.00	1.02	1.35	0.96	1.15	1.09	1.17
time (sec)	N/A	0.019	0.008	0.084	0.246	0.303	0.095	0.000	0.079	2.107

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	37	36	39
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.12	1.09	1.18
time (sec)	N/A	0.013	0.008	0.089	0.244	0.293	0.086	0.000	0.081	1.967

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	25	23	25
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.09	1.00	1.09
time (sec)	N/A	0.009	0.004	0.081	0.257	0.301	0.067	0.000	0.036	1.911

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	9	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	0.75	1.00	1.00
time (sec)	N/A	0.001	0.002	0.077	0.248	0.310	0.065	0.000	0.029	1.723

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	37	26	34
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.28	0.90	1.17
time (sec)	N/A	0.011	0.008	0.084	0.248	0.316	0.106	0.000	0.122	1.916

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	45	49
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.07	1.17
time (sec)	N/A	0.016	0.022	0.091	0.245	0.309	0.132	0.000	0.120	2.102

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	71	57	65
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.22	0.98	1.12
time (sec)	N/A	0.020	0.028	0.087	0.243	0.322	0.150	0.001	0.110	2.266

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	84	69	76
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.22	1.00	1.10
time (sec)	N/A	0.025	0.030	0.083	0.254	0.301	0.169	0.001	0.080	2.351

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	96	79	87
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.14	0.94	1.04
time (sec)	N/A	0.030	0.024	0.091	0.246	0.320	0.184	0.001	0.116	2.460

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	89	94	103	129	109	109	91	138
N.S.	1	1.00	0.90	0.95	1.04	1.30	1.10	1.10	0.92	1.39
time (sec)	N/A	0.049	0.015	0.095	0.249	0.300	0.182	0.001	0.234	2.494

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	83	91	117	92	92	78	126
N.S.	1	1.00	0.90	0.97	1.06	1.36	1.07	1.07	0.91	1.47
time (sec)	N/A	0.037	0.015	0.085	0.271	0.303	0.170	0.000	0.158	2.389

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	72	81	107	85	86	67	116
N.S.	1	1.00	0.87	0.94	1.05	1.39	1.10	1.12	0.87	1.51
time (sec)	N/A	0.031	0.012	0.085	0.268	0.311	0.164	0.001	0.124	2.330

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	62	69	95	70	70	54	107
N.S.	1	1.00	0.86	0.97	1.08	1.48	1.09	1.09	0.84	1.67
time (sec)	N/A	0.024	0.010	0.096	0.246	0.320	0.151	0.001	0.078	2.243

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	49	57	83	58	53	43	80
N.S.	1	1.00	0.80	0.98	1.14	1.66	1.16	1.06	0.86	1.60
time (sec)	N/A	0.019	0.023	0.085	0.248	0.296	0.138	0.001	0.146	2.147

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	40	48	61	46	41	46	57
N.S.	1	1.00	0.80	0.98	1.17	1.49	1.12	1.00	1.12	1.39
time (sec)	N/A	0.014	0.008	0.083	0.244	0.311	0.104	0.001	0.093	2.065

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	27	32	32	32	22	32	30
N.S.	1	1.00	1.18	1.59	1.88	1.88	1.88	1.29	1.88	1.76
time (sec)	N/A	0.001	0.003	0.085	0.260	0.300	0.096	0.000	0.072	1.847

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	24	26	12	26	23
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86	1.64
time (sec)	N/A	0.001	0.002	0.087	0.246	0.299	0.095	0.000	0.068	1.763

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	42	51	80	46	52	43	66
N.S.	1	1.00	0.86	0.98	1.19	1.86	1.07	1.21	1.00	1.53
time (sec)	N/A	0.015	0.016	0.095	0.246	0.318	0.154	0.001	0.100	2.202

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	56	69	109	66	73	63	83
N.S.	1	1.00	0.93	0.98	1.21	1.91	1.16	1.28	1.11	1.46
time (sec)	N/A	0.020	0.028	0.086	0.244	0.303	0.185	0.001	0.114	2.339

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	73	86	130	78	82	79	98
N.S.	1	1.00	0.89	0.96	1.13	1.71	1.03	1.08	1.04	1.29
time (sec)	N/A	0.027	0.026	0.090	0.270	0.309	0.197	0.001	0.119	2.498

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	84	97	141	92	100	91	109
N.S.	1	1.00	0.89	0.94	1.09	1.58	1.03	1.12	1.02	1.22
time (sec)	N/A	0.031	0.037	0.092	0.247	0.305	0.218	0.001	0.128	2.711

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	94	108	152	102	107	101	120
N.S.	1	1.00	0.93	0.97	1.11	1.57	1.05	1.10	1.04	1.24
time (sec)	N/A	0.037	0.030	0.085	0.255	0.313	0.233	0.001	0.092	2.851

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	109	125	162	131	120	103	177
N.S.	1	1.00	0.89	0.96	1.10	1.42	1.15	1.05	0.90	1.55
time (sec)	N/A	0.061	0.019	0.085	0.245	0.317	0.253	0.001	0.370	2.857

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	90	99	114	151	119	106	90	166
N.S.	1	1.00	0.86	0.94	1.09	1.44	1.13	1.01	0.86	1.58
time (sec)	N/A	0.050	0.016	0.097	0.266	0.300	0.242	0.001	0.221	2.776

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	87	102	139	107	96	79	154
N.S.	1	1.00	1.00	0.97	1.13	1.54	1.19	1.07	0.88	1.71
time (sec)	N/A	0.042	0.012	0.085	0.272	0.306	0.224	0.001	0.153	2.621

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	77	91	129	94	81	66	86
N.S.	1	1.00	0.84	0.95	1.12	1.59	1.16	1.00	0.81	1.06
time (sec)	N/A	0.034	0.014	0.084	0.245	0.315	0.210	0.001	0.125	2.537

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	64	79	116	82	65	55	74
N.S.	1	1.00	0.78	0.98	1.22	1.78	1.26	1.00	0.85	1.14
time (sec)	N/A	0.026	0.025	0.084	0.264	0.305	0.196	0.001	0.173	2.432

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	44	55	70	94	70	50	45	90
N.S.	1	1.00	0.76	0.95	1.21	1.62	1.21	0.86	0.78	1.55
time (sec)	N/A	0.021	0.009	0.081	0.269	0.294	0.150	0.001	0.070	2.320

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	41	54	54	56	34	56	52
N.S.	1	1.00	1.82	2.41	3.18	3.18	3.29	2.00	3.29	3.06
time (sec)	N/A	0.001	0.007	0.090	0.250	0.305	0.130	0.000	0.085	2.087

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	43	43	44	22	44	42
N.S.	1	1.00	0.67	0.90	1.43	1.43	1.47	0.73	1.47	1.40
time (sec)	N/A	0.010	0.003	0.079	0.243	0.304	0.124	0.000	0.072	2.005

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	35	37	12	37	34
N.S.	1	1.00	1.00	0.93	0.86	2.50	2.64	0.86	2.64	2.43
time (sec)	N/A	0.001	0.002	0.080	0.246	0.305	0.121	0.000	0.077	1.950

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	54	73	124	70	63	60	99
N.S.	1	1.00	0.84	0.95	1.28	2.18	1.23	1.11	1.05	1.74
time (sec)	N/A	0.020	0.019	0.085	0.296	0.306	0.194	0.001	0.128	2.516

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	69	91	153	90	85	85	116
N.S.	1	1.00	0.91	0.99	1.30	2.19	1.29	1.21	1.21	1.66
time (sec)	N/A	0.028	0.034	0.091	0.268	0.309	0.238	0.001	0.084	2.712

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	88	108	174	104	97	101	131
N.S.	1	1.00	0.85	0.95	1.16	1.87	1.12	1.04	1.09	1.41
time (sec)	N/A	0.033	0.032	0.085	0.267	0.312	0.250	0.001	0.136	2.906

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	88	99	117	183	114	105	113	142
N.S.	1	1.00	0.86	0.97	1.15	1.79	1.12	1.03	1.11	1.39
time (sec)	N/A	0.039	0.031	0.085	0.255	0.313	0.266	0.001	0.104	2.956

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	110	130	196	128	120	123	153
N.S.	1	1.00	0.86	0.94	1.11	1.68	1.09	1.03	1.05	1.31
time (sec)	N/A	0.049	0.037	0.087	0.269	0.311	0.278	0.001	0.173	3.115

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	144	180	250	190	139	126	283
N.S.	1	1.00	0.93	0.96	1.20	1.67	1.27	0.93	0.84	1.89
time (sec)	N/A	0.095	0.016	0.091	0.246	0.297	0.459	0.001	1.086	3.734

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	128	132	169	239	180	134	115	272
N.S.	1	1.00	0.92	0.95	1.22	1.72	1.29	0.96	0.83	1.96
time (sec)	N/A	0.077	0.018	0.105	0.265	0.311	0.436	0.001	0.553	3.613

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	122	157	228	165	114	102	152
N.S.	1	1.00	0.81	0.95	1.23	1.78	1.29	0.89	0.80	1.19
time (sec)	N/A	0.063	0.025	0.087	0.255	0.297	0.410	0.001	0.179	3.384

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	109	145	215	153	101	91	140
N.S.	1	1.00	0.88	0.92	1.23	1.82	1.30	0.86	0.77	1.19
time (sec)	N/A	0.052	0.017	0.087	0.263	0.314	0.377	0.001	0.336	3.371

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	100	136	193	141	83	81	189
N.S.	1	1.00	0.71	0.92	1.25	1.77	1.29	0.76	0.74	1.73
time (sec)	N/A	0.045	0.013	0.083	0.252	0.313	0.302	0.001	0.106	3.203

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	87	120	120	128	70	72	119
N.S.	1	1.00	3.76	5.12	7.06	7.06	7.53	4.12	4.24	7.00
time (sec)	N/A	0.001	0.006	0.082	0.250	0.305	0.279	0.001	0.122	2.715

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	53	72	109	109	116	58	22	108
N.S.	1	1.00	1.51	2.06	3.11	3.11	3.31	1.66	0.63	3.09
time (sec)	N/A	0.004	0.006	0.082	0.258	0.306	0.252	0.001	0.072	2.626

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	64	42	57	98	98	104	46	48	97
N.S.	1	1.23	0.81	1.10	1.88	1.88	2.00	0.88	0.92	1.87
time (sec)	N/A	0.022	0.006	0.084	0.246	0.311	0.234	0.001	0.068	2.473

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	42	87	87	92	34	31	86
N.S.	1	1.00	0.66	0.89	1.85	1.85	1.96	0.72	0.66	1.83
time (sec)	N/A	0.016	0.005	0.081	0.249	0.306	0.220	0.001	0.080	2.365

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	76	76	80	22	18	75
N.S.	1	1.00	0.67	0.90	2.53	2.53	2.67	0.73	0.60	2.50
time (sec)	N/A	0.010	0.004	0.082	0.241	0.302	0.209	0.001	0.100	2.429

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	68	73	12	70	67
N.S.	1	1.00	1.00	0.93	0.86	4.86	5.21	0.86	5.00	4.79
time (sec)	N/A	0.001	0.002	0.080	0.244	0.297	0.209	0.000	0.064	2.319

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	90	139	256	141	96	102	198
N.S.	1	1.00	0.82	0.91	1.40	2.59	1.42	0.97	1.03	2.00
time (sec)	N/A	0.036	0.034	0.090	0.248	0.313	0.326	0.001	0.455	3.202

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	108	157	285	162	121	151	215
N.S.	1	1.00	0.83	0.92	1.34	2.44	1.38	1.03	1.29	1.84
time (sec)	N/A	0.054	0.051	0.088	0.269	0.326	0.374	0.001	0.186	3.423

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	133	174	306	175	130	167	230
N.S.	1	1.00	0.78	0.92	1.21	2.12	1.22	0.90	1.16	1.60
time (sec)	N/A	0.065	0.038	0.086	0.261	0.308	0.391	0.001	0.209	3.526

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	144	185	317	187	148	179	241
N.S.	1	1.00	0.78	0.92	1.18	2.02	1.19	0.94	1.14	1.54
time (sec)	N/A	0.075	0.052	0.107	0.258	0.313	0.410	0.001	0.311	3.592

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	161	177	234	338	250	168	151	229
N.S.	1	1.00	0.87	0.95	1.26	1.82	1.34	0.90	0.81	1.23
time (sec)	N/A	0.125	0.025	0.086	0.280	0.303	0.706	0.001	0.979	4.291

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	167	223	327	236	147	138	218
N.S.	1	1.00	0.85	0.94	1.26	1.85	1.33	0.83	0.78	1.23
time (sec)	N/A	0.102	0.016	0.101	0.266	0.317	0.674	0.001	0.227	4.196

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	137	154	211	314	224	137	127	206
N.S.	1	1.00	0.86	0.97	1.33	1.97	1.41	0.86	0.80	1.30
time (sec)	N/A	0.085	0.018	0.097	0.275	0.312	0.621	0.001	0.943	4.070

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	111	145	202	292	212	116	117	288
N.S.	1	1.00	0.72	0.94	1.31	1.90	1.38	0.75	0.76	1.87
time (sec)	N/A	0.074	0.018	0.085	0.249	0.311	0.517	0.001	0.187	3.858

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	97	131	186	186	199	106	107	185
N.S.	1	1.00	5.71	7.71	10.94	10.94	11.71	6.24	6.29	10.88
time (sec)	N/A	0.001	0.011	0.082	0.273	0.310	0.477	0.001	0.141	3.324

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	86	117	175	175	187	94	22	174
N.S.	1	1.00	2.46	3.34	5.00	5.00	5.34	2.69	0.63	4.97
time (sec)	N/A	0.004	0.008	0.083	0.267	0.318	0.432	0.001	0.126	3.145

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	75	102	164	164	175	82	85	163
N.S.	1	1.00	1.44	1.96	3.15	3.15	3.37	1.58	1.63	3.13
time (sec)	N/A	0.007	0.009	0.090	0.259	0.298	0.401	0.001	0.140	3.215

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	86	153	153	163	70	71	152
N.S.	1	1.00	0.93	1.25	2.22	2.22	2.36	1.01	1.03	2.20
time (sec)	N/A	0.011	0.008	0.083	0.245	0.307	0.369	0.001	0.078	3.025

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	53	72	142	142	151	58	61	141
N.S.	1	1.00	0.65	0.89	1.75	1.75	1.86	0.72	0.75	1.74
time (sec)	N/A	0.028	0.008	0.085	0.269	0.313	0.349	0.001	0.077	2.895

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	42	57	131	131	139	46	48	130
N.S.	1	1.00	0.66	0.89	2.05	2.05	2.17	0.72	0.75	2.03
time (sec)	N/A	0.022	0.005	0.084	0.250	0.308	0.325	0.001	0.127	2.799

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	42	120	120	128	34	31	119
N.S.	1	1.00	0.66	0.89	2.55	2.55	2.72	0.72	0.66	2.53
time (sec)	N/A	0.016	0.006	0.095	0.253	0.310	0.312	0.001	0.150	2.747

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	109	109	116	22	18	108
N.S.	1	1.00	0.67	0.90	3.63	3.63	3.87	0.73	0.60	3.60
time (sec)	N/A	0.010	0.004	0.085	0.270	0.313	0.298	0.001	0.067	2.640

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	101	109	12	103	100
N.S.	1	1.00	1.00	0.93	0.86	7.21	7.79	0.86	7.36	7.14
time (sec)	N/A	0.001	0.002	0.089	0.263	0.297	0.307	0.000	0.141	2.567

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	127	126	205	388	212	129	145	297
N.S.	1	1.00	0.90	0.89	1.45	2.75	1.50	0.91	1.03	2.11
time (sec)	N/A	0.053	0.056	0.091	0.276	0.316	0.461	0.001	0.762	3.997

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	130	147	223	417	233	157	217	314
N.S.	1	1.00	0.82	0.93	1.41	2.64	1.47	0.99	1.37	1.99
time (sec)	N/A	0.088	0.072	0.087	0.257	0.317	0.528	0.001	0.395	4.167

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	145	178	240	438	246	163	233	329
N.S.	1	1.00	0.76	0.93	1.26	2.29	1.29	0.85	1.22	1.72
time (sec)	N/A	0.105	0.061	0.094	0.282	0.321	0.542	0.001	0.438	4.336

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	156	189	251	449	258	184	245	340
N.S.	1	1.00	0.79	0.95	1.27	2.27	1.30	0.93	1.24	1.72
time (sec)	N/A	0.115	0.070	0.136	0.276	0.323	0.559	0.001	0.586	4.319

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	132	132	136	143	147	132	135
N.S.	1	1.00	1.00	0.94	0.94	0.96	1.01	1.04	0.94	0.96
time (sec)	N/A	0.054	0.007	0.083	0.259	0.314	0.401	0.001	0.081	2.828

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	121	121	125	131	136	121	124
N.S.	1	1.00	1.00	0.92	0.92	0.95	0.99	1.03	0.92	0.94
time (sec)	N/A	0.047	0.004	0.087	0.268	0.316	0.388	0.001	0.092	2.740

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	109	114	117	122	114	113
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	1.07	1.00	0.99
time (sec)	N/A	0.038	0.004	0.077	0.245	0.313	0.387	0.000	0.002	2.683

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	100	100	103	107	113	100	102
N.S.	1	1.00	1.00	0.92	0.92	0.94	0.98	1.04	0.92	0.94
time (sec)	N/A	0.035	0.003	0.077	0.248	0.314	0.368	0.001	0.082	2.654

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	96	91	88	88	95	99	88	90
N.S.	1	1.00	5.65	5.35	5.18	5.18	5.59	5.82	5.18	5.29
time (sec)	N/A	0.001	0.006	0.084	0.273	0.301	0.325	0.000	0.090	2.284

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	80	79	79	85	89	23	79
N.S.	1	1.00	2.53	2.22	2.19	2.19	2.36	2.47	0.64	2.19
time (sec)	N/A	0.004	0.003	0.074	0.262	0.303	0.286	0.000	0.002	2.249

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	69	68	68	73	77	68	68
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.30	1.38	1.21	1.21
time (sec)	N/A	0.008	0.005	0.082	0.254	0.316	0.255	0.000	0.098	2.176

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	65	56	57
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.97	0.84	0.85
time (sec)	N/A	0.017	0.004	0.073	0.244	0.303	0.217	0.000	0.002	2.023

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	49	53	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.88	0.95	0.82	0.82
time (sec)	N/A	0.013	0.005	0.076	0.254	0.306	0.186	0.000	0.035	1.923

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	41	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.95	0.81	0.81
time (sec)	N/A	0.010	0.003	0.076	0.243	0.302	0.148	0.000	0.032	1.824

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	29	24	23
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.97	0.80	0.77
time (sec)	N/A	0.007	0.004	0.084	0.270	0.303	0.115	0.000	0.036	1.729

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	17	13	12
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	1.00	0.76	0.71
time (sec)	N/A	0.004	0.001	0.012	0.243	0.304	0.081	0.000	0.029	1.644

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.007	0.258	0.299	0.027	0.000	0.020	1.531

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	119	117	120	116	135	114	119
N.S.	1	1.00	1.00	0.89	0.87	0.90	0.87	1.01	0.85	0.89
time (sec)	N/A	0.040	0.004	0.089	0.256	0.316	0.194	0.001	0.127	2.661

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	134	135	141	163	139	155	135	142
N.S.	1	1.00	0.92	0.92	0.97	1.12	0.95	1.06	0.92	0.97
time (sec)	N/A	0.061	0.052	0.089	0.268	0.315	0.270	0.001	0.083	2.955

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	145	150	163	207	163	169	157	175
N.S.	1	1.00	0.89	0.92	1.00	1.27	1.00	1.04	0.96	1.07
time (sec)	N/A	0.075	0.059	0.089	0.268	0.306	0.340	0.001	0.228	3.197

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	18	10	11
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	1.06	0.59	0.65
time (sec)	N/A	0.002	0.002	0.086	0.250	0.304	0.054	0.000	0.170	1.605

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	18	10	11
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	1.06	0.59	0.65
time (sec)	N/A	0.002	0.002	0.091	0.242	0.314	0.053	0.000	0.143	1.612

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	18	21	20	27	18	19
N.S.	1	1.00	1.00	0.79	0.75	0.88	0.83	1.12	0.75	0.79
time (sec)	N/A	0.007	0.002	0.091	0.239	0.302	0.059	0.001	0.054	1.656

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	28	26	32	18	24
N.S.	1	1.00	1.00	0.77	0.74	0.90	0.84	1.03	0.58	0.77
time (sec)	N/A	0.007	0.002	0.092	0.241	0.309	0.062	0.000	0.037	1.678

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	28	33	31	38	24	29
N.S.	1	1.00	1.00	0.76	0.74	0.87	0.82	1.00	0.63	0.76
time (sec)	N/A	0.008	0.002	0.092	0.242	0.315	0.067	0.000	0.088	1.743

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	33	38	36	43	28	34
N.S.	1	1.00	1.00	0.76	0.73	0.84	0.80	0.96	0.62	0.76
time (sec)	N/A	0.009	0.002	0.096	0.282	0.314	0.069	0.000	0.040	1.760

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	23	22	32	19	29	20	26
N.S.	1	1.00	0.93	0.82	0.79	1.14	0.68	1.04	0.71	0.93
time (sec)	N/A	0.007	0.009	0.089	0.238	0.311	0.059	0.000	0.055	1.666

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	31	48	31	38	34	34
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.89	1.09	0.97	0.97
time (sec)	N/A	0.008	0.009	0.098	0.244	0.542	0.066	0.000	0.089	1.786

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	33	38	59	36	43	31	41
N.S.	1	1.00	0.86	0.79	0.90	1.40	0.86	1.02	0.74	0.98
time (sec)	N/A	0.010	0.008	0.099	0.248	0.310	0.072	0.000	0.042	1.829

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	43	64	41	50	37	46
N.S.	1	1.00	0.90	0.78	0.88	1.31	0.84	1.02	0.76	0.94
time (sec)	N/A	0.011	0.018	0.104	0.252	0.312	0.074	0.000	0.093	1.873

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	43	48	69	46	54	41	51
N.S.	1	1.00	1.00	0.77	0.86	1.23	0.82	0.96	0.73	0.91
time (sec)	N/A	0.014	0.006	0.111	0.240	0.310	0.077	0.000	0.095	1.905

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	30	50	27	35	29	39
N.S.	1	1.00	0.74	0.82	0.77	1.28	0.69	0.90	0.74	1.00
time (sec)	N/A	0.009	0.013	0.104	0.244	0.310	0.071	0.000	0.125	1.779

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	37	41	68	41	45	35	49
N.S.	1	1.00	0.85	0.80	0.89	1.48	0.89	0.98	0.76	1.07
time (sec)	N/A	0.011	0.013	0.122	0.255	0.310	0.078	0.000	0.092	1.861

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	42	48	79	46	53	41	56
N.S.	1	1.00	0.83	0.79	0.91	1.49	0.87	1.00	0.77	1.06
time (sec)	N/A	0.012	0.015	0.102	0.252	0.314	0.079	0.000	0.094	1.912

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	47	53	84	51	58	47	61
N.S.	1	1.00	0.82	0.78	0.88	1.40	0.85	0.97	0.78	1.02
time (sec)	N/A	0.015	0.011	0.103	0.237	0.313	0.086	0.000	0.046	1.948

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	58	89	56	61	51	66
N.S.	1	1.00	0.81	0.78	0.87	1.33	0.84	0.91	0.76	0.99
time (sec)	N/A	0.016	0.012	0.104	0.243	0.318	0.087	0.001	0.047	2.038

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	9	6	6	7	8	6	8
N.S.	1	1.00	1.25	1.12	0.75	0.75	0.88	1.00	0.75	1.00
time (sec)	N/A	0.001	0.001	0.082	0.245	0.299	0.027	0.000	0.151	1.547

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	11	6	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	1.10	0.60	0.80
time (sec)	N/A	0.001	0.001	0.083	0.279	0.311	0.029	0.000	0.077	1.566

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	10	14	18	10	12
N.S.	1	1.00	1.14	0.93	0.86	0.71	1.00	1.29	0.71	0.86
time (sec)	N/A	0.002	0.003	0.091	0.267	0.315	0.032	0.001	0.105	1.595

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	20	17	21	16	16
N.S.	1	1.00	1.00	0.85	0.80	1.00	0.85	1.05	0.80	0.80
time (sec)	N/A	0.007	0.003	0.089	0.299	0.299	0.035	0.000	0.108	1.641

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	19	23	14	18
N.S.	1	1.00	1.00	0.86	0.82	0.95	0.86	1.05	0.64	0.82
time (sec)	N/A	0.002	0.002	0.087	0.240	0.320	0.033	0.000	0.052	1.647

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	23	19	25	16	18
N.S.	1	1.00	1.00	0.86	0.82	1.05	0.86	1.14	0.73	0.82
time (sec)	N/A	0.002	0.002	0.091	0.278	0.315	0.033	0.000	0.057	1.665

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	18	24	19	28	16	17
N.S.	1	1.00	1.00	0.90	0.86	1.14	0.90	1.33	0.76	0.81
time (sec)	N/A	0.003	0.004	0.090	0.277	0.308	0.037	0.000	0.152	1.675

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	18	24	20	30	14	16
N.S.	1	1.00	1.10	0.95	0.90	1.20	1.00	1.50	0.70	0.80
time (sec)	N/A	0.003	0.006	0.088	0.240	0.321	0.039	0.000	0.175	1.631

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	12	9	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.09	0.82	1.00
time (sec)	N/A	0.002	0.002	0.094	0.239	0.306	0.057	0.000	0.097	1.577

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	8	12	9	13
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.67	1.00	0.75	1.08
time (sec)	N/A	0.002	0.002	0.088	0.257	0.299	0.058	0.000	0.036	1.571

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	21	14	23	16	19
N.S.	1	1.00	1.00	1.05	1.00	1.11	0.74	1.21	0.84	1.00
time (sec)	N/A	0.006	0.003	0.111	0.254	0.308	0.073	0.001	0.043	1.694

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	21	14	22	14	19
N.S.	1	1.00	1.00	1.00	0.94	1.17	0.78	1.22	0.78	1.06
time (sec)	N/A	0.007	0.002	0.110	0.270	0.577	0.080	0.000	0.029	1.701

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	15	10	28	20	14
N.S.	1	1.00	1.00	1.07	1.00	1.07	0.71	2.00	1.43	1.00
time (sec)	N/A	0.007	0.003	0.086	0.244	0.309	0.071	0.000	0.036	1.603

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	53	1742	192	56	316
N.S.	1	1.00	0.64	0.69	0.78	0.74	24.19	2.67	0.78	4.39
time (sec)	N/A	0.014	0.046	0.102	0.287	0.314	1.265	0.001	0.050	16.096

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	145	37	184
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	2.74	0.70	3.47
time (sec)	N/A	0.010	0.017	0.095	0.274	0.309	0.880	0.001	0.046	7.266

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	30	202	99	25	96
N.S.	1	1.00	1.00	0.76	0.76	0.88	5.94	2.91	0.74	2.82
time (sec)	N/A	0.006	0.014	0.080	0.292	0.305	0.580	0.001	0.028	3.456

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	21	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	1.31	0.75	0.75
time (sec)	N/A	0.001	0.006	0.081	0.279	0.294	0.029	0.000	0.021	1.585

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	73	68	42	27	74
N.S.	1	1.00	1.00	0.80	1.20	2.09	1.94	1.20	0.77	2.11
time (sec)	N/A	0.013	0.022	0.081	0.358	0.576	0.709	0.002	0.094	2.679

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	47	93	44	58	31	38
N.S.	1	1.00	1.00	0.95	1.21	2.38	1.13	1.49	0.79	0.97
time (sec)	N/A	0.008	0.045	0.085	0.405	0.310	0.929	0.002	0.053	2.692

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	54	88	119	97	89	48	100
N.S.	1	1.00	0.85	0.83	1.35	1.83	1.49	1.37	0.74	1.54
time (sec)	N/A	0.013	0.073	0.095	0.347	0.329	2.110	0.002	0.068	4.094

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	66	121	145	122	119	66	135
N.S.	1	1.00	0.77	0.76	1.39	1.67	1.40	1.37	0.76	1.55
time (sec)	N/A	0.018	0.099	0.092	0.382	0.316	5.282	0.002	0.107	7.082

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	64	1742	326	56	316
N.S.	1	1.00	0.64	0.69	0.78	0.89	24.19	4.53	0.78	4.39
time (sec)	N/A	0.013	0.022	0.085	0.248	0.709	1.368	0.002	0.048	16.413

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	53	733	257	37	197
N.S.	1	1.00	0.66	0.72	0.77	1.00	13.83	4.85	0.70	3.72
time (sec)	N/A	0.009	0.019	0.084	0.294	0.311	0.956	0.002	0.041	7.722

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	41	80	185	25	53
N.S.	1	1.00	0.71	0.76	0.76	1.21	2.35	5.44	0.74	1.56
time (sec)	N/A	0.006	0.017	0.090	0.279	0.301	0.154	0.001	0.026	1.995

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	113	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	7.06	0.75	0.75
time (sec)	N/A	0.001	0.007	0.099	0.266	0.298	0.029	0.001	0.016	1.572

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	52	88	71	63	37	62
N.S.	1	1.00	0.90	0.78	1.06	1.80	1.45	1.29	0.76	1.27
time (sec)	N/A	0.012	0.023	0.101	0.380	0.309	1.126	0.002	0.042	3.079

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	48	58	102	92	74	42	82
N.S.	1	1.00	0.88	0.94	1.14	2.00	1.80	1.45	0.82	1.61
time (sec)	N/A	0.011	0.052	0.104	0.359	0.304	1.344	0.003	0.100	3.413

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	52	86	124	76	86	46	61
N.S.	1	1.00	0.85	0.84	1.39	2.00	1.23	1.39	0.74	0.98
time (sec)	N/A	0.012	0.089	0.095	0.354	0.611	1.597	0.003	0.057	3.490

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	64	119	145	124	116	64	135
N.S.	1	1.00	0.80	0.76	1.42	1.73	1.48	1.38	0.76	1.61
time (sec)	N/A	0.016	0.104	0.095	0.373	0.318	3.699	0.003	0.102	5.744

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	75	146	481	56	88
N.S.	1	1.00	0.64	0.69	0.78	1.04	2.03	6.68	0.78	1.22
time (sec)	N/A	0.012	0.025	0.086	0.254	0.294	0.410	0.003	0.050	2.630

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	64	124	391	37	76
N.S.	1	1.00	0.66	0.72	0.77	1.21	2.34	7.38	0.70	1.43
time (sec)	N/A	0.009	0.022	0.090	0.257	0.310	0.347	0.002	0.044	2.385

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	52	102	297	25	64
N.S.	1	1.00	0.71	0.76	0.76	1.53	3.00	8.74	0.74	1.88
time (sec)	N/A	0.006	0.018	0.102	0.255	0.303	0.276	0.002	0.028	2.176

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	39	12	199	12	12
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	12.44	0.75	0.75
time (sec)	N/A	0.001	0.007	0.098	0.270	0.297	0.032	0.001	0.018	1.583

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	50	64	114	97	86	52	88
N.S.	1	1.00	0.86	0.77	0.98	1.75	1.49	1.32	0.80	1.35
time (sec)	N/A	0.014	0.043	0.086	0.360	0.308	2.366	0.003	0.053	4.297

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	62	71	126	99	100	58	85
N.S.	1	1.00	0.91	0.94	1.08	1.91	1.50	1.52	0.88	1.29
time (sec)	N/A	0.015	0.068	0.099	0.347	0.599	2.234	0.004	0.115	4.187

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	63	62	101	133	126	104	64	135
N.S.	1	1.00	0.81	0.79	1.29	1.71	1.62	1.33	0.82	1.73
time (sec)	N/A	0.015	0.090	0.095	0.347	0.307	2.523	0.004	0.053	4.573

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	64	115	146	104	110	64	84
N.S.	1	1.00	0.79	0.79	1.42	1.80	1.28	1.36	0.79	1.04
time (sec)	N/A	0.015	0.119	0.095	0.343	0.317	2.807	0.004	0.046	4.637

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	76	144	167	155	142	79	145
N.S.	1	1.00	0.76	0.74	1.40	1.62	1.50	1.38	0.77	1.41
time (sec)	N/A	0.022	0.133	0.102	0.364	0.588	7.653	0.005	0.111	9.712

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	90	97	116	141	279	1417	116	154
N.S.	1	1.00	0.62	0.66	0.79	0.97	1.91	9.71	0.79	1.05
time (sec)	N/A	0.029	0.042	0.087	0.252	0.304	1.673	0.006	0.036	4.274

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	85	101	130	257	1278	101	143
N.S.	1	1.00	0.62	0.67	0.80	1.02	2.02	10.06	0.80	1.13
time (sec)	N/A	0.026	0.037	0.085	0.269	0.312	1.496	0.006	0.031	4.050

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	68	74	86	119	235	1141	86	132
N.S.	1	1.00	0.62	0.67	0.78	1.08	2.14	10.37	0.78	1.20
time (sec)	N/A	0.021	0.030	0.089	0.265	0.306	1.339	0.005	0.027	3.782

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	62	71	108	212	1004	71	120
N.S.	1	1.00	0.63	0.68	0.78	1.19	2.33	11.03	0.78	1.32
time (sec)	N/A	0.018	0.028	0.087	0.264	0.315	1.202	0.005	0.025	3.553

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	97	190	866	56	109
N.S.	1	1.00	0.64	0.69	0.78	1.35	2.64	12.03	0.78	1.51
time (sec)	N/A	0.013	0.026	0.089	0.258	0.313	1.066	0.005	0.045	3.332

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	86	168	727	36	98
N.S.	1	1.00	0.66	0.72	0.77	1.62	3.17	13.72	0.68	1.85
time (sec)	N/A	0.010	0.023	0.098	0.263	0.302	0.948	0.004	0.040	3.138

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	74	146	586	25	86
N.S.	1	1.00	0.71	0.76	0.76	2.18	4.29	17.24	0.74	2.53
time (sec)	N/A	0.006	0.019	0.102	0.272	0.300	0.816	0.003	0.030	2.944

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	61	12	445	12	12
N.S.	1	1.00	1.00	0.81	0.75	3.81	0.75	27.81	0.75	0.75
time (sec)	N/A	0.001	0.008	0.095	0.253	0.297	0.039	0.003	0.018	1.600

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	74	88	158	148	132	76	132
N.S.	1	1.00	0.80	0.76	0.91	1.63	1.53	1.36	0.78	1.36
time (sec)	N/A	0.024	0.040	0.089	0.344	0.319	11.286	0.004	0.037	12.626

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	82	85	97	172	150	150	84	133
N.S.	1	1.00	0.84	0.87	0.99	1.76	1.53	1.53	0.86	1.36
time (sec)	N/A	0.026	0.075	0.097	0.362	0.329	10.913	0.005	0.043	12.331

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	86	87	131	180	184	154	117	155
N.S.	1	1.00	0.75	0.76	1.15	1.58	1.61	1.35	1.03	1.36
time (sec)	N/A	0.026	0.102	0.105	0.360	0.332	10.440	0.005	0.047	12.060

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	89	145	178	184	154	131	148
N.S.	1	1.00	0.75	0.78	1.27	1.56	1.61	1.35	1.15	1.30
time (sec)	N/A	0.026	0.122	0.100	0.344	0.311	9.889	0.006	0.121	11.717

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	86	155	177	182	152	94	185
N.S.	1	1.00	0.74	0.74	1.34	1.53	1.57	1.31	0.81	1.59
time (sec)	N/A	0.026	0.141	0.102	0.343	0.312	10.358	0.007	0.062	11.988

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	88	169	190	158	159	94	130
N.S.	1	1.00	0.72	0.74	1.42	1.60	1.33	1.34	0.79	1.09
time (sec)	N/A	0.027	0.174	0.125	0.349	0.328	11.040	0.006	0.119	12.499

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	100	198	211	209	190	109	181
N.S.	1	1.00	0.71	0.71	1.40	1.50	1.48	1.35	0.77	1.28
time (sec)	N/A	0.036	0.190	0.115	0.368	0.314	47.697	0.007	0.133	47.084

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	111	112	229	233	0	218	124	243
N.S.	1	1.00	0.68	0.69	1.40	1.43	0.00	1.34	0.76	1.49
time (sec)	N/A	0.047	0.212	0.107	0.353	0.319	0.000	0.007	0.126	275.322

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	31	78	148	43	31	153
N.S.	1	1.00	1.00	0.82	0.79	2.00	3.79	1.10	0.79	3.92
time (sec)	N/A	0.007	0.020	0.089	0.355	0.315	0.755	0.001	0.094	2.957

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	34	98	117	58	34	112
N.S.	1	1.00	1.00	0.98	0.81	2.33	2.79	1.38	0.81	2.67
time (sec)	N/A	0.007	0.036	0.098	0.345	0.321	0.950	0.002	0.097	2.894

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	59	83	124	207	90	54	206
N.S.	1	1.00	0.85	0.83	1.17	1.75	2.92	1.27	0.76	2.90
time (sec)	N/A	0.012	0.077	0.112	0.346	0.316	2.051	0.002	0.103	24.368

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	44	43	93	187	66	43	155
N.S.	1	1.00	0.87	0.80	0.78	1.69	3.40	1.20	0.78	2.82
time (sec)	N/A	0.011	0.032	0.100	0.348	0.330	1.206	0.002	0.041	3.359

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	54	47	105	197	75	47	194
N.S.	1	1.00	0.84	0.95	0.82	1.84	3.46	1.32	0.82	3.40
time (sec)	N/A	0.011	0.049	0.105	0.342	0.311	1.363	0.003	0.045	3.788

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	57	80	129	189	88	52	162
N.S.	1	1.00	0.82	0.84	1.18	1.90	2.78	1.29	0.76	2.38
time (sec)	N/A	0.011	0.081	0.110	0.356	0.317	1.573	0.003	0.097	3.620

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	58	57	119	240	92	57	207
N.S.	1	1.00	0.82	0.79	0.78	1.63	3.29	1.26	0.78	2.84
time (sec)	N/A	0.015	0.046	0.092	0.354	0.308	2.393	0.002	0.043	4.642

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	69	63	131	245	104	63	204
N.S.	1	1.00	0.86	0.93	0.85	1.77	3.31	1.41	0.85	2.76
time (sec)	N/A	0.015	0.060	0.107	0.355	0.309	2.303	0.004	0.103	4.504

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	70	97	139	267	106	69	266
N.S.	1	1.00	0.78	0.81	1.13	1.62	3.10	1.23	0.80	3.09
time (sec)	N/A	0.016	0.085	0.098	0.345	0.314	2.408	0.007	0.095	11.808

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	61	71	53	3755	108	71	494
N.S.	1	1.00	0.64	0.69	0.80	0.60	42.19	1.21	0.80	5.55
time (sec)	N/A	0.015	0.022	0.086	0.268	0.303	2.206	0.001	0.024	33.831

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	49	56	42	1640	82	56	308
N.S.	1	1.00	0.68	0.72	0.82	0.62	24.12	1.21	0.82	4.53
time (sec)	N/A	0.012	0.021	0.092	0.257	0.312	1.281	0.001	0.046	15.242

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	41	31	600	61	37	160
N.S.	1	1.00	0.69	0.73	0.80	0.61	11.76	1.20	0.73	3.14
time (sec)	N/A	0.009	0.018	0.088	0.282	0.298	0.851	0.001	0.038	7.001

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	26	26	19	162	36	25	73
N.S.	1	1.00	0.72	0.81	0.81	0.59	5.06	1.12	0.78	2.28
time (sec)	N/A	0.006	0.013	0.084	0.262	0.306	0.559	0.001	0.027	3.305

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	13	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.93	0.86	0.86
time (sec)	N/A	0.001	0.005	0.090	0.261	0.296	0.030	0.000	0.018	1.609

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	27	17	16
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	1.17	0.74	0.70
time (sec)	N/A	0.005	0.014	0.082	0.361	0.324	0.491	0.001	0.055	2.058

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	60	93	44	65	33	40
N.S.	1	1.00	1.00	0.98	1.46	2.27	1.07	1.59	0.80	0.98
time (sec)	N/A	0.008	0.044	0.111	0.345	0.330	1.170	0.001	0.111	3.010

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	66	92	123	102	97	51	102
N.S.	1	1.00	0.82	0.97	1.35	1.81	1.50	1.43	0.75	1.50
time (sec)	N/A	0.011	0.068	0.113	0.360	0.310	2.699	0.002	0.061	4.732

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	90	121	145	129	122	69	137
N.S.	1	1.00	0.74	1.00	1.34	1.61	1.43	1.36	0.77	1.52
time (sec)	N/A	0.018	0.072	0.095	0.360	0.522	7.925	0.001	0.052	9.905

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	62	71	63	3606	115	71	484
N.S.	1	1.00	0.67	0.73	0.84	0.74	42.42	1.35	0.84	5.69
time (sec)	N/A	0.016	0.024	0.094	0.259	0.296	2.196	0.002	0.028	32.007

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	49	56	51	1538	86	56	280
N.S.	1	1.00	0.68	0.74	0.85	0.77	23.30	1.30	0.85	4.24
time (sec)	N/A	0.013	0.020	0.112	0.257	0.297	1.342	0.003	0.048	14.563

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	38	41	40	534	64	35	145
N.S.	1	1.00	0.69	0.78	0.84	0.82	10.90	1.31	0.71	2.96
time (sec)	N/A	0.009	0.021	0.111	0.300	0.298	0.837	0.001	0.040	6.263

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	23	26	29	37	31	19	33
N.S.	1	1.00	0.70	0.77	0.87	0.97	1.23	1.03	0.63	1.10
time (sec)	N/A	0.006	0.012	0.114	0.256	0.313	0.292	0.001	0.087	1.909

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	15	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	1.07	0.86	0.86
time (sec)	N/A	0.001	0.008	0.115	0.261	0.304	0.030	0.001	0.019	1.626

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	45	110	146	47	30	78
N.S.	1	1.00	1.00	0.82	1.18	2.89	3.84	1.24	0.79	2.05
time (sec)	N/A	0.008	0.028	0.132	0.340	0.315	0.849	0.001	0.043	3.548

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	54	76	151	73	89	60	88
N.S.	1	1.00	0.86	0.95	1.33	2.65	1.28	1.56	1.05	1.54
time (sec)	N/A	0.012	0.060	0.103	0.368	0.318	1.847	0.002	0.122	3.709

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	68	108	189	107	113	90	103
N.S.	1	1.00	0.77	0.78	1.24	2.17	1.23	1.30	1.03	1.18
time (sec)	N/A	0.016	0.090	0.120	0.347	0.313	4.074	0.002	0.059	5.993

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	71	74	3456	109	68	454
N.S.	1	1.00	0.66	0.71	0.82	0.85	39.72	1.25	0.78	5.22
time (sec)	N/A	0.016	0.027	0.115	0.280	0.307	2.148	0.003	0.049	30.780

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	47	50	56	62	163	83	47	55
N.S.	1	1.00	0.69	0.74	0.82	0.91	2.40	1.22	0.69	0.81
time (sec)	N/A	0.013	0.024	0.098	0.285	0.313	0.412	0.002	0.043	3.186

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	41	52	121	53	35	45
N.S.	1	1.00	0.71	0.73	0.84	1.06	2.47	1.08	0.71	0.92
time (sec)	N/A	0.009	0.022	0.101	0.261	0.308	0.411	0.002	0.084	2.900

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	26	26	41	80	34	20	34
N.S.	1	1.00	0.75	0.81	0.81	1.28	2.50	1.06	0.62	1.06
time (sec)	N/A	0.006	0.014	0.093	0.247	0.303	0.395	0.002	0.033	2.464

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	31	14	23	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	1.44	0.75	0.75
time (sec)	N/A	0.001	0.007	0.092	0.253	0.305	0.040	0.001	0.018	1.614

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	43	53	177	697	65	42	205
N.S.	1	1.00	0.91	0.80	0.98	3.28	12.91	1.20	0.78	3.80
time (sec)	N/A	0.012	0.049	0.120	0.345	0.315	1.435	0.002	0.050	8.010

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	66	89	221	818	100	73	231
N.S.	1	1.00	0.85	0.89	1.20	2.99	11.05	1.35	0.99	3.12
time (sec)	N/A	0.016	0.075	0.105	0.355	0.317	2.862	0.002	0.109	10.337

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	78	81	123	255	464	134	105	172
N.S.	1	1.00	0.74	0.76	1.16	2.41	4.38	1.26	0.99	1.62
time (sec)	N/A	0.023	0.107	0.106	0.351	0.322	10.531	0.002	0.120	12.235

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	58	54	26	19	46
N.S.	1	1.00	1.00	0.80	0.76	2.32	2.16	1.04	0.76	1.84
time (sec)	N/A	0.006	0.015	0.095	0.339	0.320	0.509	0.001	0.048	2.390

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	44	46	97	121	63	36	117
N.S.	1	1.00	1.00	1.00	1.05	2.20	2.75	1.43	0.82	2.66
time (sec)	N/A	0.008	0.035	0.131	0.336	0.331	1.180	0.001	0.041	3.219

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	72	86	128	216	97	57	210
N.S.	1	1.00	0.81	0.97	1.16	1.73	2.92	1.31	0.77	2.84
time (sec)	N/A	0.012	0.059	0.094	0.351	0.317	2.571	0.001	0.045	95.098

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	124	437	48	34	311
N.S.	1	1.00	1.00	0.83	0.81	2.95	10.40	1.14	0.81	7.40
time (sec)	N/A	0.008	0.027	0.094	0.352	0.310	1.005	0.002	0.095	4.465

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	61	67	164	156	89	52	0
N.S.	1	1.00	0.82	0.98	1.08	2.65	2.52	1.44	0.84	0.00
time (sec)	N/A	0.011	0.057	0.103	0.368	0.313	1.814	0.002	0.063	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	77	104	198	226	115	101	0
N.S.	1	1.00	0.75	0.81	1.09	2.08	2.38	1.21	1.06	0.00
time (sec)	N/A	0.016	0.086	0.103	0.364	0.324	4.046	0.002	0.132	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	49	42	182	1950	68	48	1070
N.S.	1	1.00	0.87	0.82	0.70	3.03	32.50	1.13	0.80	17.83
time (sec)	N/A	0.012	0.052	0.097	0.379	0.322	104.416	0.002	0.092	9.946

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	67	74	82	226	0	101	70	1219
N.S.	1	1.00	0.83	0.91	1.01	2.79	0.00	1.25	0.86	15.05
time (sec)	N/A	0.016	0.076	0.128	0.356	0.317	0.000	0.003	0.118	12.203

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	82	90	121	260	1108	138	117	0
N.S.	1	1.00	0.71	0.78	1.04	2.24	9.55	1.19	1.01	0.00
time (sec)	N/A	0.022	0.116	0.117	0.346	0.328	13.203	0.004	0.071	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	78	0	11	69
N.S.	1	1.00	1.00	0.92	0.85	1.08	6.00	0.00	0.85	5.31
time (sec)	N/A	0.006	0.218	0.088	0.314	0.314	39.314	0.000	0.413	35.472

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	C	A	F	A	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	92	13	0	11	11	73	0	-1	64
N.S.	1	7.08	1.00	0.00	0.85	0.85	5.62	0.00	-0.08	4.92
time (sec)	N/A	0.030	0.011	0.039	0.302	0.314	2.796	0.000	0.000	4.381

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	27	17	16
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	1.17	0.74	0.70
time (sec)	N/A	0.006	0.002	0.107	0.350	0.315	0.493	0.001	0.002	1.991

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	53	1742	192	56	320
N.S.	1	1.00	0.64	0.69	0.78	0.74	24.19	2.67	0.78	4.44
time (sec)	N/A	0.012	0.021	0.099	0.257	0.309	1.314	0.002	0.052	15.916

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	147	37	197
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	2.77	0.70	3.72
time (sec)	N/A	0.009	0.018	0.095	0.272	0.300	0.889	0.001	0.038	7.133

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	30	202	98	25	96
N.S.	1	1.00	1.00	0.76	0.76	0.88	5.94	2.88	0.74	2.82
time (sec)	N/A	0.006	0.014	0.109	0.260	0.319	0.587	0.001	0.028	3.426

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	21	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	1.31	0.75	0.75
time (sec)	N/A	0.001	0.006	0.107	0.264	0.303	0.029	0.000	0.017	1.565

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	113	90	86	91	180	135	107	102
N.S.	1	1.00	1.24	0.99	0.95	1.00	1.98	1.48	1.18	1.12
time (sec)	N/A	0.044	0.066	0.099	0.356	0.309	1.069	0.005	0.121	3.716

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	119	95	93	139	643	164	117	110
N.S.	1	1.00	1.23	0.98	0.96	1.43	6.63	1.69	1.21	1.13
time (sec)	N/A	0.025	0.126	0.149	0.361	0.326	1.150	0.007	0.067	7.204

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	129	118	139	187	2266	199	196	136
N.S.	1	1.00	1.02	0.93	1.09	1.47	17.84	1.57	1.54	1.07
time (sec)	N/A	0.035	0.185	0.139	0.348	0.309	1.710	0.007	0.230	21.043

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	53	1742	217	56	0
N.S.	1	1.00	0.64	0.69	0.78	0.74	24.19	3.01	0.78	0.00
time (sec)	N/A	0.013	0.022	0.107	0.274	0.308	1.383	0.003	0.045	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	166	37	194
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	3.13	0.70	3.66
time (sec)	N/A	0.009	0.019	0.109	0.278	0.303	0.922	0.002	0.041	7.407

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	26	26	31	202	113	25	96
N.S.	1	1.00	1.03	0.76	0.76	0.91	5.94	3.32	0.74	2.82
time (sec)	N/A	0.006	0.015	0.092	0.264	0.299	0.612	0.002	0.028	3.521

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	23	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	1.44	0.75	0.75
time (sec)	N/A	0.001	0.007	0.089	0.269	0.304	0.029	0.000	0.018	1.616

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	114	90	85	110	182	147	117	102
N.S.	1	1.00	1.24	0.98	0.92	1.20	1.98	1.60	1.27	1.11
time (sec)	N/A	0.024	0.051	0.091	0.352	0.312	1.080	0.005	0.113	3.836

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	120	95	93	252	643	171	127	112
N.S.	1	1.00	1.28	1.01	0.99	2.68	6.84	1.82	1.35	1.19
time (sec)	N/A	0.026	0.116	0.100	0.357	0.317	1.171	0.007	0.114	7.319

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	147	118	139	350	2266	211	194	136
N.S.	1	1.00	1.16	0.93	1.09	2.76	17.84	1.66	1.53	1.07
time (sec)	N/A	0.034	0.188	0.119	0.353	0.317	1.735	0.007	0.329	21.454

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	64	1844	325	56	348
N.S.	1	1.00	0.64	0.69	0.78	0.89	25.61	4.51	0.78	4.83
time (sec)	N/A	0.013	0.023	0.107	0.258	0.300	1.485	0.004	0.047	17.176

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	53	733	256	37	207
N.S.	1	1.00	0.66	0.72	0.77	1.00	13.83	4.83	0.70	3.91
time (sec)	N/A	0.009	0.021	0.098	0.260	0.301	1.027	0.004	0.042	7.873

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	41	80	187	25	53
N.S.	1	1.00	0.71	0.76	0.76	1.21	2.35	5.50	0.74	1.56
time (sec)	N/A	0.006	0.016	0.108	0.274	0.316	0.225	0.003	0.027	2.067

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	112	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	7.00	0.75	0.75
time (sec)	N/A	0.001	0.007	0.107	0.263	0.306	0.029	0.002	0.018	1.590

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	102	96	98	209	158	123	118
N.S.	1	1.00	1.16	0.97	0.91	0.93	1.99	1.50	1.17	1.12
time (sec)	N/A	0.029	0.057	0.091	0.348	0.321	1.398	0.008	0.060	4.238

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	128	106	104	111	719	176	131	125
N.S.	1	1.00	1.20	0.99	0.97	1.04	6.72	1.64	1.22	1.17
time (sec)	N/A	0.030	0.150	0.151	0.360	0.312	1.508	0.009	0.073	8.332

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	136	110	136	162	2266	198	174	134
N.S.	1	1.00	1.10	0.89	1.10	1.31	18.27	1.60	1.40	1.08
time (sec)	N/A	0.032	0.202	0.141	0.359	0.314	1.600	0.010	0.122	21.907

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	42	1640	95	56	303
N.S.	1	1.00	0.64	0.69	0.78	0.58	22.78	1.32	0.78	4.21
time (sec)	N/A	0.013	0.021	0.109	0.264	0.301	1.291	0.003	0.043	15.338

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	31	600	69	37	171
N.S.	1	1.00	0.66	0.72	0.77	0.58	11.32	1.30	0.70	3.23
time (sec)	N/A	0.009	0.020	0.107	0.285	0.303	0.842	0.002	0.038	6.771

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	20	162	42	25	74
N.S.	1	1.00	0.71	0.76	0.76	0.59	4.76	1.24	0.74	2.18
time (sec)	N/A	0.006	0.013	0.122	0.275	0.318	0.566	0.001	0.029	3.186

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	17	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	1.06	0.75	0.75
time (sec)	N/A	0.001	0.006	0.108	0.273	0.300	0.028	0.001	0.016	1.568

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	95	75	76	213	155	137	99	82
N.S.	1	1.00	1.20	0.95	0.96	2.70	1.96	1.73	1.25	1.04
time (sec)	N/A	0.018	0.039	0.110	0.352	0.319	0.936	0.004	0.086	3.459

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	120	104	106	306	831	185	130	120
N.S.	1	1.00	1.20	1.04	1.06	3.06	8.31	1.85	1.30	1.20
time (sec)	N/A	0.024	0.127	0.115	0.344	0.324	1.228	0.005	0.137	9.775

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	149	130	142	296	2730	218	182	139
N.S.	1	1.00	1.15	1.00	1.09	2.28	21.00	1.68	1.40	1.07
time (sec)	N/A	0.032	0.121	0.112	0.373	0.330	2.141	0.006	0.226	26.899

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	48	58	64	44	4974	100	64	2502
N.S.	1	1.00	0.60	0.72	0.80	0.55	62.18	1.25	0.80	31.28
time (sec)	N/A	0.013	0.021	0.108	0.257	0.297	1.368	0.002	0.048	23.056

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	44	47	33	1326	73	43	837
N.S.	1	1.00	0.63	0.75	0.80	0.56	22.47	1.24	0.73	14.19
time (sec)	N/A	0.009	0.023	0.108	0.266	0.294	0.909	0.002	0.040	6.983

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	26	30	30	22	486	44	29	308
N.S.	1	1.00	0.68	0.79	0.79	0.58	12.79	1.16	0.76	8.11
time (sec)	N/A	0.006	0.013	0.100	0.267	0.300	0.622	0.001	0.029	3.959

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	18	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	1.00	0.78	0.78
time (sec)	N/A	0.001	0.006	0.106	0.265	0.309	0.030	0.001	0.020	1.574

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	100	83	86	285	160	152	117	91
N.S.	1	1.00	1.22	1.01	1.05	3.48	1.95	1.85	1.43	1.11
time (sec)	N/A	0.023	0.059	0.114	0.357	0.327	0.969	0.004	0.095	3.599

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	128	113	116	328	838	197	133	0
N.S.	1	1.00	1.24	1.10	1.13	3.18	8.14	1.91	1.29	0.00
time (sec)	N/A	0.024	0.105	0.121	0.344	0.325	1.259	0.005	0.175	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	160	141	159	374	2744	234	216	148
N.S.	1	1.00	1.18	1.04	1.17	2.75	20.18	1.72	1.59	1.09
time (sec)	N/A	0.031	0.106	0.105	0.356	0.324	2.173	0.006	0.220	28.131

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	50	56	42	1640	85	56	299
N.S.	1	1.00	0.66	0.71	0.80	0.60	23.43	1.21	0.80	4.27
time (sec)	N/A	0.013	0.022	0.111	0.273	0.294	1.299	0.002	0.045	15.194

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	41	31	600	60	37	162
N.S.	1	1.00	0.69	0.73	0.80	0.61	11.76	1.18	0.73	3.18
time (sec)	N/A	0.009	0.019	0.109	0.269	0.305	0.870	0.002	0.041	6.748

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	26	26	19	162	36	25	73
N.S.	1	1.00	0.72	0.81	0.81	0.59	5.06	1.12	0.78	2.28
time (sec)	N/A	0.006	0.012	0.108	0.257	0.304	0.579	0.002	0.029	3.192

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	13	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.93	0.86	0.86
time (sec)	N/A	0.001	0.005	0.115	0.261	0.300	0.029	0.001	0.017	1.578

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	76	77	115	150	129	95	82
N.S.	1	1.00	1.16	0.95	0.96	1.44	1.88	1.61	1.19	1.02
time (sec)	N/A	0.018	0.046	0.111	0.357	0.320	0.978	0.003	0.166	3.490

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	119	104	106	166	830	178	122	112
N.S.	1	1.00	1.21	1.06	1.08	1.69	8.47	1.82	1.24	1.14
time (sec)	N/A	0.024	0.114	0.133	0.346	0.317	1.258	0.004	0.130	9.159

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	149	130	142	162	2728	215	175	153
N.S.	1	1.00	1.15	1.00	1.09	1.25	20.98	1.65	1.35	1.18
time (sec)	N/A	0.034	0.116	0.148	0.353	0.313	2.205	0.004	0.131	25.885

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	49	56	52	1538	97	56	277
N.S.	1	1.00	0.66	0.70	0.80	0.74	21.97	1.39	0.80	3.96
time (sec)	N/A	0.013	0.022	0.123	0.266	0.304	1.377	0.002	0.053	14.772

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	38	41	40	534	67	35	144
N.S.	1	1.00	0.69	0.78	0.84	0.82	10.90	1.37	0.71	2.94
time (sec)	N/A	0.009	0.021	0.126	0.284	0.298	0.919	0.002	0.042	6.284

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	25	26	29	41	38	20	33
N.S.	1	1.00	0.72	0.78	0.81	0.91	1.28	1.19	0.62	1.03
time (sec)	N/A	0.006	0.013	0.115	0.266	0.306	0.312	0.001	0.030	1.856

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	15	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	1.07	0.86	0.86
time (sec)	N/A	0.001	0.008	0.113	0.264	0.300	0.030	0.001	0.021	1.566

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	111	95	88	285	184	156	114	131
N.S.	1	1.00	1.19	1.02	0.95	3.06	1.98	1.68	1.23	1.41
time (sec)	N/A	0.025	0.080	0.112	0.356	0.315	1.116	0.005	0.056	3.878

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	112	122	407	857	204	173	173
N.S.	1	1.00	1.09	0.99	1.08	3.60	7.58	1.81	1.53	1.53
time (sec)	N/A	0.032	0.189	0.112	0.340	0.318	1.609	0.007	0.068	9.855

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	142	126	158	407	2793	236	221	1120
N.S.	1	1.00	0.95	0.85	1.06	2.73	18.74	1.58	1.48	7.52
time (sec)	N/A	0.042	0.247	0.131	0.362	0.324	3.388	0.008	0.131	30.564

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	86	86	88	138	103	105	95
N.S.	1	1.00	1.37	1.21	1.21	1.24	1.94	1.45	1.48	1.34
time (sec)	N/A	0.024	0.050	0.120	0.343	0.325	1.049	0.002	0.101	3.701

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	101	90	90	92	136	107	108	98
N.S.	1	1.00	1.38	1.23	1.23	1.26	1.86	1.47	1.48	1.34
time (sec)	N/A	0.023	0.051	0.117	0.357	0.351	1.044	0.002	0.128	3.749

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	96	94	93	134	108	112	96
N.S.	1	1.00	1.38	1.30	1.27	1.26	1.81	1.46	1.51	1.30
time (sec)	N/A	0.022	0.052	0.099	0.361	0.310	1.100	0.002	0.108	3.746

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	100	98	97	139	112	115	93
N.S.	1	1.00	1.39	1.32	1.29	1.28	1.83	1.47	1.51	1.22
time (sec)	N/A	0.021	0.067	0.118	0.350	0.310	0.959	0.002	0.072	3.623

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	95	87	87	86	134	110	101	82
N.S.	1	1.00	1.32	1.21	1.21	1.19	1.86	1.53	1.40	1.14
time (sec)	N/A	0.018	0.046	0.118	0.357	0.322	0.953	0.003	0.141	3.473

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	99	91	91	90	136	114	104	89
N.S.	1	1.00	1.34	1.23	1.23	1.22	1.84	1.54	1.41	1.20
time (sec)	N/A	0.018	0.039	0.115	0.352	0.314	0.981	0.003	0.107	3.595

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	95	93	95	134	114	107	89
N.S.	1	1.00	1.38	1.28	1.26	1.28	1.81	1.54	1.45	1.20
time (sec)	N/A	0.017	0.039	0.119	0.340	0.317	0.979	0.003	0.156	3.571

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	99	97	99	133	118	110	82
N.S.	1	1.00	1.39	1.30	1.28	1.30	1.75	1.55	1.45	1.08
time (sec)	N/A	0.019	0.040	0.117	0.352	0.312	0.968	0.003	0.160	3.465

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	30	25	33	87	50	30	101
N.S.	1	1.00	0.88	1.20	1.00	1.32	3.48	2.00	1.20	4.04
time (sec)	N/A	0.006	0.021	0.012	0.261	0.312	0.130	0.001	0.313	1.700

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	27	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	1.29	0.62	0.62
time (sec)	N/A	0.003	0.009	0.047	0.263	0.295	0.245	0.000	0.091	1.759

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	27	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	1.29	0.62	0.62
time (sec)	N/A	0.003	0.009	0.047	0.258	0.307	0.135	0.000	0.027	1.646

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	25	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	1.19	0.62	0.62
time (sec)	N/A	0.003	0.008	0.047	0.269	0.299	0.726	0.000	0.025	2.127

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	17	20	12	12
N.S.	1	1.00	0.84	0.74	0.68	0.63	0.89	1.05	0.63	0.63
time (sec)	N/A	0.003	0.008	0.020	0.257	0.303	0.070	0.000	0.025	1.606

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	14	13	12	15	19	11	12
N.S.	1	1.00	0.76	0.82	0.76	0.71	0.88	1.12	0.65	0.71
time (sec)	N/A	0.003	0.010	0.026	0.257	0.301	0.159	0.001	0.028	1.609

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	14	11	11	19	21	13	13
N.S.	1	1.00	0.79	0.74	0.58	0.58	1.00	1.11	0.68	0.68
time (sec)	N/A	0.003	0.011	0.025	0.259	0.544	0.217	0.001	0.027	1.669

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	51	43	85	299	134	93	311
N.S.	1	1.00	0.88	1.19	1.00	1.98	6.95	3.12	2.16	7.23
time (sec)	N/A	0.011	0.037	0.087	0.260	0.322	0.193	0.002	0.416	1.977

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	45	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	1.25	0.67	0.67
time (sec)	N/A	0.005	0.013	0.087	0.265	0.320	0.331	0.001	0.103	1.852

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	45	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	1.25	0.67	0.67
time (sec)	N/A	0.005	0.012	0.088	0.257	0.301	0.187	0.001	0.035	1.743

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	1851	43	24	1012
N.S.	1	1.00	0.78	0.69	0.67	0.75	51.42	1.19	0.67	28.11
time (sec)	N/A	0.005	0.012	0.092	0.265	0.309	65.396	0.001	0.042	11.024

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	24	32	38	24	24
N.S.	1	1.00	0.82	0.74	0.71	0.71	0.94	1.12	0.71	0.71
time (sec)	N/A	0.005	0.012	0.089	0.258	0.307	0.098	0.001	0.037	1.673

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	23	31	35	24	21
N.S.	1	1.00	0.88	0.78	0.75	0.72	0.97	1.09	0.75	0.66
time (sec)	N/A	0.006	0.014	0.095	0.266	0.304	0.187	0.001	0.035	1.695

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	23	24	31	35	24	24
N.S.	1	1.00	0.81	0.78	0.72	0.75	0.97	1.09	0.75	0.75
time (sec)	N/A	0.005	0.016	0.089	0.268	0.313	0.226	0.001	0.030	1.716

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	61	157	663	255	167	662
N.S.	1	1.00	0.89	1.18	1.00	2.57	10.87	4.18	2.74	10.85
time (sec)	N/A	0.014	0.037	0.089	0.278	0.309	0.280	0.003	0.389	2.468

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	63	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	1.24	0.69	0.69
time (sec)	N/A	0.008	0.015	0.090	0.259	0.297	0.430	0.001	0.045	1.995

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	63	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	1.24	0.69	0.69
time (sec)	N/A	0.008	0.014	0.087	0.261	0.314	0.253	0.001	0.046	1.867

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	0	61	35	2556
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.00	1.20	0.69	50.12
time (sec)	N/A	0.008	0.014	0.087	0.249	0.300	0.000	0.001	0.042	27.146

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	36	35	35	46	56	35	35
N.S.	1	1.00	0.83	0.77	0.74	0.74	0.98	1.19	0.74	0.74
time (sec)	N/A	0.008	0.014	0.110	0.258	0.308	0.130	0.001	0.043	1.757

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	36	35	34	44	51	35	32
N.S.	1	1.00	0.87	0.80	0.78	0.76	0.98	1.13	0.78	0.71
time (sec)	N/A	0.008	0.017	0.096	0.260	0.300	0.222	0.001	0.047	1.762

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	36	34	34	46	51	35	32
N.S.	1	1.00	0.81	0.77	0.72	0.72	0.98	1.09	0.74	0.68
time (sec)	N/A	0.008	0.017	0.090	0.291	0.305	0.254	0.001	0.039	1.793

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	54	132	122	85	48	126
N.S.	1	1.00	0.90	0.79	0.79	1.94	1.79	1.25	0.71	1.85
time (sec)	N/A	0.023	0.045	0.105	0.353	0.334	2.998	0.001	0.057	4.382

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	42	103	107	65	37	115
N.S.	1	1.00	0.92	0.81	0.79	1.94	2.02	1.23	0.70	2.17
time (sec)	N/A	0.012	0.034	0.101	0.337	0.314	0.746	0.002	0.052	2.194

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	85	88	43	28	102
N.S.	1	1.00	1.00	0.80	0.78	2.12	2.20	1.08	0.70	2.55
time (sec)	N/A	0.009	0.020	0.102	0.351	0.312	0.355	0.001	0.041	1.833

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	26	19	92
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.90	0.66	3.17
time (sec)	N/A	0.006	0.014	0.098	0.351	0.313	0.470	0.001	0.044	1.966

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	93	85	45	28	100
N.S.	1	1.00	1.00	0.80	0.78	2.32	2.12	1.12	0.70	2.50
time (sec)	N/A	0.009	0.024	0.121	0.339	0.317	1.029	0.001	0.044	2.527

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	43	41	118	107	60	38	111
N.S.	1	1.00	0.91	0.81	0.77	2.23	2.02	1.13	0.72	2.09
time (sec)	N/A	0.012	0.036	0.107	0.339	0.313	3.568	0.001	0.102	3.532

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	52	144	126	77	49	126
N.S.	1	1.00	0.90	0.79	0.76	2.12	1.85	1.13	0.72	1.85
time (sec)	N/A	0.016	0.045	0.106	0.371	0.315	14.711	0.001	0.110	15.582

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	68	59	63	161	389	90	58	386
N.S.	1	1.00	0.97	0.84	0.90	2.30	5.56	1.29	0.83	5.51
time (sec)	N/A	0.016	0.074	0.118	0.361	0.309	14.134	0.002	0.114	15.364

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	47	49	134	332	66	46	332
N.S.	1	1.00	0.95	0.82	0.86	2.35	5.82	1.16	0.81	5.82
time (sec)	N/A	0.012	0.065	0.130	0.362	0.313	3.872	0.002	0.124	5.269

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	37	115	269	49	34	276
N.S.	1	1.00	1.00	0.80	0.80	2.50	5.85	1.07	0.74	6.00
time (sec)	N/A	0.009	0.050	0.102	0.347	0.630	1.697	0.002	0.040	3.107

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	116	277	48	33	281
N.S.	1	1.00	1.00	0.80	0.78	2.58	6.16	1.07	0.73	6.24
time (sec)	N/A	0.009	0.046	0.095	0.356	0.316	2.755	0.001	0.095	4.126

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	47	51	147	384	71	48	358
N.S.	1	1.00	0.96	0.84	0.91	2.62	6.86	1.27	0.86	6.39
time (sec)	N/A	0.012	0.066	0.112	0.342	0.315	7.381	0.001	0.124	8.477

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	58	64	184	452	81	58	419
N.S.	1	1.00	0.99	0.84	0.93	2.67	6.55	1.17	0.84	6.07
time (sec)	N/A	0.016	0.067	0.114	0.371	0.316	29.693	0.001	0.150	18.278

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	68	86	227	762	110	81	738
N.S.	1	1.00	0.85	0.72	0.91	2.39	8.02	1.16	0.85	7.77
time (sec)	N/A	0.022	0.111	0.132	0.344	0.311	93.791	0.002	0.122	92.821

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	56	73	200	683	85	69	665
N.S.	1	1.00	0.85	0.68	0.89	2.44	8.33	1.04	0.84	8.11
time (sec)	N/A	0.016	0.102	0.115	0.366	0.326	34.876	0.002	0.143	34.872

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	50	61	185	605	72	58	590
N.S.	1	1.00	0.84	0.71	0.87	2.64	8.64	1.03	0.83	8.43
time (sec)	N/A	0.013	0.100	0.099	0.351	0.313	16.536	0.002	0.131	17.350

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	52	64	186	627	68	56	612
N.S.	1	1.00	0.82	0.71	0.88	2.55	8.59	0.93	0.77	8.38
time (sec)	N/A	0.014	0.095	0.129	0.340	0.310	7.171	0.002	0.125	6.824

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	59	60	186	632	70	57	614
N.S.	1	1.00	0.84	0.84	0.86	2.66	9.03	1.00	0.81	8.77
time (sec)	N/A	0.014	0.066	0.108	0.334	0.324	11.763	0.001	0.126	12.604

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	56	73	214	779	89	70	706
N.S.	1	1.00	0.85	0.68	0.89	2.61	9.50	1.09	0.85	8.61
time (sec)	N/A	0.016	0.098	0.115	0.351	0.323	31.395	0.002	0.149	31.459

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	67	86	250	869	100	80	786
N.S.	1	1.00	0.85	0.71	0.91	2.63	9.15	1.05	0.84	8.27
time (sec)	N/A	0.021	0.099	0.134	0.357	0.324	81.483	0.002	0.155	43.447

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	70	131	117	90	51	122
N.S.	1	1.00	0.90	0.79	1.03	1.93	1.72	1.32	0.75	1.79
time (sec)	N/A	0.017	0.044	0.124	0.346	0.308	3.001	0.002	0.147	3.622

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	58	103	102	70	37	111
N.S.	1	1.00	0.92	0.81	1.09	1.94	1.92	1.32	0.70	2.09
time (sec)	N/A	0.013	0.036	0.119	0.351	0.623	0.741	0.001	0.114	2.227

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	47	83	83	47	28	98
N.S.	1	1.00	1.00	0.80	1.18	2.08	2.08	1.18	0.70	2.45
time (sec)	N/A	0.010	0.022	0.111	0.372	0.314	0.365	0.001	0.112	1.855

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	34	67	68	28	19	88
N.S.	1	1.00	1.00	0.66	1.17	2.31	2.34	0.97	0.66	3.03
time (sec)	N/A	0.007	0.014	0.095	0.365	0.315	0.465	0.001	0.125	1.970

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	47	91	76	48	28	96
N.S.	1	1.00	1.00	0.80	1.18	2.28	1.90	1.20	0.70	2.40
time (sec)	N/A	0.009	0.024	0.126	0.369	0.313	1.034	0.001	0.056	2.450

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	43	55	113	99	65	37	107
N.S.	1	1.00	0.91	0.81	1.04	2.13	1.87	1.23	0.70	2.02
time (sec)	N/A	0.012	0.041	0.103	0.355	0.682	3.493	0.001	0.122	3.598

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	68	143	117	81	48	122
N.S.	1	1.00	0.90	0.79	1.00	2.10	1.72	1.19	0.71	1.79
time (sec)	N/A	0.016	0.043	0.124	0.371	0.309	14.580	0.001	0.129	15.371

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	60	81	167	354	92	61	365
N.S.	1	1.00	1.00	0.86	1.16	2.39	5.06	1.31	0.87	5.21
time (sec)	N/A	0.016	0.072	0.116	0.359	0.305	13.907	0.002	0.071	15.180

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	48	68	138	301	69	47	314
N.S.	1	1.00	0.98	0.84	1.19	2.42	5.28	1.21	0.82	5.51
time (sec)	N/A	0.013	0.058	0.130	0.335	0.303	3.893	0.002	0.115	4.676

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	38	56	123	243	52	35	261
N.S.	1	1.00	1.04	0.81	1.19	2.62	5.17	1.11	0.74	5.55
time (sec)	N/A	0.010	0.052	0.123	0.397	0.303	1.748	0.002	0.112	3.222

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	56	122	252	53	34	266
N.S.	1	1.00	1.00	0.80	1.22	2.65	5.48	1.15	0.74	5.78
time (sec)	N/A	0.010	0.047	0.106	0.334	0.310	2.743	0.001	0.055	4.193

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	48	69	151	354	74	49	340
N.S.	1	1.00	0.96	0.84	1.21	2.65	6.21	1.30	0.86	5.96
time (sec)	N/A	0.013	0.059	0.130	0.356	0.313	7.223	0.001	0.073	7.247

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	59	82	187	416	86	60	398
N.S.	1	1.00	0.99	0.84	1.17	2.67	5.94	1.23	0.86	5.69
time (sec)	N/A	0.017	0.075	0.109	0.379	0.308	29.130	0.002	0.138	18.118

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	69	103	227	695	116	83	698
N.S.	1	1.00	0.85	0.71	1.06	2.34	7.16	1.20	0.86	7.20
time (sec)	N/A	0.022	0.112	0.113	0.357	0.321	94.025	0.002	0.141	95.955

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	57	90	199	624	92	69	629
N.S.	1	1.00	0.85	0.68	1.07	2.37	7.43	1.10	0.82	7.49
time (sec)	N/A	0.017	0.109	0.141	0.345	0.317	34.691	0.003	0.065	35.486

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	51	78	186	552	77	58	558
N.S.	1	1.00	0.83	0.71	1.08	2.58	7.67	1.07	0.81	7.75
time (sec)	N/A	0.015	0.098	0.103	0.344	0.313	16.384	0.003	0.143	17.386

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	53	80	183	575	74	57	580
N.S.	1	1.00	0.80	0.71	1.07	2.44	7.67	0.99	0.76	7.73
time (sec)	N/A	0.014	0.085	0.112	0.341	0.316	7.098	0.002	0.137	8.410

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	61	77	185	580	77	58	582
N.S.	1	1.00	0.83	0.85	1.07	2.57	8.06	1.07	0.81	8.08
time (sec)	N/A	0.015	0.076	0.115	0.349	0.324	11.865	0.001	0.135	9.954

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	57	90	213	716	93	69	670
N.S.	1	1.00	0.85	0.68	1.07	2.54	8.52	1.11	0.82	7.98
time (sec)	N/A	0.017	0.097	0.125	0.346	0.320	32.192	0.002	0.155	32.029

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	68	103	249	799	108	80	746
N.S.	1	1.00	0.85	0.70	1.06	2.57	8.24	1.11	0.82	7.69
time (sec)	N/A	0.022	0.102	0.133	0.340	0.324	85.475	0.002	0.165	44.150

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	88	128	178	162	153	151	-1	135
N.S.	1	1.00	0.72	1.05	1.46	1.33	1.25	1.24	-0.01	1.11
time (sec)	N/A	0.031	0.109	0.115	0.349	0.340	21.950	0.007	0.000	22.001

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	106	146	141	122	125	-1	125
N.S.	1	1.00	0.79	1.08	1.49	1.44	1.24	1.28	-0.01	1.28
time (sec)	N/A	0.021	0.076	0.122	0.366	0.332	5.353	0.006	0.000	7.245

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	84	108	114	97	92	52	90
N.S.	1	1.00	0.85	1.14	1.46	1.54	1.31	1.24	0.70	1.22
time (sec)	N/A	0.016	0.053	0.135	0.354	0.322	2.015	0.006	0.150	3.988

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	62	70	93	42	89	41	34
N.S.	1	1.00	1.07	1.41	1.59	2.11	0.95	2.02	0.93	0.77
time (sec)	N/A	0.012	0.045	0.120	0.361	0.311	0.937	10.401	0.683	2.625

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	61	54	89	68	96	-1	71
N.S.	1	1.00	1.04	1.36	1.20	1.98	1.51	2.13	-0.02	1.58
time (sec)	N/A	0.012	0.055	0.106	0.362	0.312	0.780	10.385	0.000	2.647

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	49	15	15	41	78	21	33
N.S.	1	1.00	1.00	2.33	0.71	0.71	1.95	3.71	1.00	1.57
time (sec)	N/A	0.001	0.018	0.119	0.282	0.324	0.793	0.007	0.237	2.543

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	71	31	34	65	113	32	44
N.S.	1	1.00	0.89	1.61	0.70	0.77	1.48	2.57	0.73	1.00
time (sec)	N/A	0.004	0.064	0.100	0.260	0.310	2.944	0.009	0.255	4.400

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	93	46	45	347	149	43	95
N.S.	1	1.00	0.75	1.37	0.68	0.66	5.10	2.19	0.63	1.40
time (sec)	N/A	0.007	0.069	0.113	0.263	0.305	9.619	0.010	0.264	11.732

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	135	170	164	323	158	-1	286
N.S.	1	1.00	0.72	1.06	1.34	1.29	2.54	1.24	-0.01	2.25
time (sec)	N/A	0.030	0.132	0.127	0.405	0.312	21.131	0.007	0.000	22.584

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	112	135	142	260	130	-1	252
N.S.	1	1.00	0.79	1.10	1.32	1.39	2.55	1.27	-0.01	2.47
time (sec)	N/A	0.022	0.110	0.128	0.361	0.313	5.338	0.007	0.000	12.847

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	89	95	118	207	98	58	192
N.S.	1	1.00	0.92	1.16	1.23	1.53	2.69	1.27	0.75	2.49
time (sec)	N/A	0.017	0.082	0.110	0.410	0.315	2.010	0.007	0.083	4.212

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	66	52	94	119	94	43	130
N.S.	1	1.00	1.15	1.43	1.13	2.04	2.59	2.04	0.93	2.83
time (sec)	N/A	0.012	0.055	0.119	0.381	0.317	0.956	10.598	0.593	2.953

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	53	66	35	91	148	102	-1	145
N.S.	1	1.00	1.13	1.40	0.74	1.94	3.15	2.17	-0.02	3.09
time (sec)	N/A	0.012	0.062	0.127	0.356	0.329	0.832	10.301	0.000	2.846

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	52	16	23	88	83	21	90
N.S.	1	1.00	1.00	2.36	0.73	1.05	4.00	3.77	0.95	4.09
time (sec)	N/A	0.001	0.060	0.111	0.294	0.298	0.821	0.008	0.243	2.649

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	75	33	34	241	120	32	229
N.S.	1	1.00	0.89	1.63	0.72	0.74	5.24	2.61	0.70	4.98
time (sec)	N/A	0.004	0.072	0.123	0.284	0.306	2.995	0.010	0.253	4.532

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	52	98	49	46	707	158	43	461
N.S.	1	1.00	0.73	1.38	0.69	0.65	9.96	2.23	0.61	6.49
time (sec)	N/A	0.008	0.078	0.102	0.281	0.312	11.983	0.011	0.269	45.141

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	76	121	163	140	117	140	-1	88
N.S.	1	1.00	0.70	1.12	1.51	1.30	1.08	1.30	-0.01	0.81
time (sec)	N/A	0.024	0.083	0.142	0.371	0.335	20.552	0.005	0.000	21.048

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	100	134	121	90	116	-1	81
N.S.	1	1.00	0.77	1.19	1.60	1.44	1.07	1.38	-0.01	0.96
time (sec)	N/A	0.015	0.071	0.104	0.353	0.319	4.859	0.005	0.000	6.588

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	79	98	101	71	86	46	66
N.S.	1	1.00	0.88	1.23	1.53	1.58	1.11	1.34	0.72	1.03
time (sec)	N/A	0.011	0.053	0.112	0.360	0.317	1.790	0.004	0.098	3.572

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	58	68	86	37	86	40	29
N.S.	1	1.00	1.15	1.45	1.70	2.15	0.92	2.15	1.00	0.72
time (sec)	N/A	0.005	0.043	0.126	0.345	0.313	0.842	1.141	0.617	2.527

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	49	54	87	48	96	-1	50
N.S.	1	1.00	1.15	1.20	1.32	2.12	1.17	2.34	-0.02	1.22
time (sec)	N/A	0.007	0.048	0.125	0.337	0.317	0.728	1.152	0.000	2.626

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	12	12	37	75	18	28
N.S.	1	1.00	1.00	1.50	0.67	0.67	2.06	4.17	1.00	1.56
time (sec)	N/A	0.001	0.019	0.119	0.277	0.311	0.774	0.008	0.208	2.530

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	43	26	25	56	102	26	35
N.S.	1	1.00	0.82	1.13	0.68	0.66	1.47	2.68	0.68	0.92
time (sec)	N/A	0.003	0.050	0.122	0.268	0.308	2.883	0.009	0.215	4.348

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	40	59	41	34	270	130	34	75
N.S.	1	1.00	0.68	1.00	0.69	0.58	4.58	2.20	0.58	1.27
time (sec)	N/A	0.006	0.056	0.129	0.257	0.305	9.443	0.009	0.224	11.364

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	82	128	147	141	250	147	-1	190
N.S.	1	1.00	0.73	1.14	1.31	1.26	2.23	1.31	-0.01	1.70
time (sec)	N/A	0.021	0.115	0.122	0.345	0.320	20.523	0.006	0.000	21.262

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	72	106	117	125	194	121	-1	0
N.S.	1	1.00	0.83	1.22	1.34	1.44	2.23	1.39	-0.01	0.00
time (sec)	N/A	0.015	0.092	0.123	0.347	0.320	4.949	0.005	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	85	81	107	155	92	53	0
N.S.	1	1.00	0.95	1.31	1.25	1.65	2.38	1.42	0.82	0.00
time (sec)	N/A	0.011	0.066	0.125	0.361	0.314	1.792	0.005	0.104	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	52	63	49	89	119	91	42	100
N.S.	1	1.00	1.27	1.54	1.20	2.17	2.90	2.22	1.02	2.44
time (sec)	N/A	0.006	0.049	0.109	0.360	0.313	0.860	1.138	0.562	2.653

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	64	35	90	122	102	-1	111
N.S.	1	1.00	1.26	1.52	0.83	2.14	2.90	2.43	-0.02	2.64
time (sec)	N/A	0.007	0.054	0.110	0.340	0.312	0.776	1.136	0.000	2.934

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	29	13	18	83	80	18	77
N.S.	1	1.00	1.00	1.53	0.68	0.95	4.37	4.21	0.95	4.05
time (sec)	N/A	0.001	0.050	0.141	0.275	0.316	0.805	0.008	0.224	2.592

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	46	28	25	194	109	26	117
N.S.	1	1.00	0.78	1.15	0.70	0.62	4.85	2.72	0.65	2.92
time (sec)	N/A	0.003	0.058	0.111	0.265	0.309	2.954	0.009	0.219	5.054

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	41	63	44	35	556	139	34	363
N.S.	1	1.00	0.66	1.02	0.71	0.56	8.97	2.24	0.55	5.85
time (sec)	N/A	0.007	0.064	0.133	0.275	0.302	11.453	0.010	0.224	11.215

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	99	144	212	184	178	334	-1	143
N.S.	1	1.00	0.69	1.01	1.48	1.29	1.24	2.34	-0.01	1.00
time (sec)	N/A	0.035	0.111	0.122	0.341	0.323	59.829	0.013	0.000	59.124

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	122	178	163	153	279	-1	133
N.S.	1	1.00	0.74	1.03	1.50	1.37	1.29	2.34	-0.01	1.12
time (sec)	N/A	0.027	0.083	0.105	0.354	0.328	12.472	0.014	0.000	13.845

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	76	97	144	140	124	378	-1	98
N.S.	1	1.00	0.80	1.02	1.52	1.47	1.31	3.98	-0.01	1.03
time (sec)	N/A	0.020	0.088	0.106	0.357	0.316	3.911	30.711	0.000	5.834

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	78	107	119	75	120	-1	51
N.S.	1	1.00	0.87	1.10	1.51	1.68	1.06	1.69	-0.01	0.72
time (sec)	N/A	0.016	0.072	0.123	0.331	0.325	1.728	10.362	0.000	3.514

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	71	84	109	92	126	-1	84
N.S.	1	1.00	0.86	1.13	1.33	1.73	1.46	2.00	-0.02	1.33
time (sec)	N/A	0.015	0.096	0.102	0.335	0.323	1.518	10.284	0.000	3.439

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	67	67	109	71	143	-1	76
N.S.	1	1.00	0.86	1.05	1.05	1.70	1.11	2.23	-0.02	1.19
time (sec)	N/A	0.015	0.096	0.106	0.373	0.311	1.778	10.355	0.000	3.705

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	104	152	207	185	376	348	-1	316
N.S.	1	1.00	0.70	1.02	1.39	1.24	2.52	2.34	-0.01	2.12
time (sec)	N/A	0.036	0.158	0.109	0.360	0.338	59.652	0.014	0.000	57.978

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	93	129	170	163	323	292	-1	282
N.S.	1	1.00	0.75	1.04	1.37	1.31	2.60	2.35	-0.01	2.27
time (sec)	N/A	0.027	0.125	0.108	0.341	0.319	12.226	0.013	0.000	24.029

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	104	133	141	264	406	-1	222
N.S.	1	1.00	0.83	1.05	1.34	1.42	2.67	4.10	-0.01	2.24
time (sec)	N/A	0.021	0.110	0.107	0.346	0.327	3.998	31.182	0.000	6.069

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	83	93	119	190	127	-1	162
N.S.	1	1.00	0.92	1.12	1.26	1.61	2.57	1.72	-0.01	2.19
time (sec)	N/A	0.016	0.089	0.105	0.341	0.328	1.780	10.343	0.000	3.824

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	74	68	109	197	135	-1	183
N.S.	1	1.00	0.92	1.12	1.03	1.65	2.98	2.05	-0.02	2.77
time (sec)	N/A	0.016	0.097	0.104	0.345	0.309	1.584	10.381	0.000	3.642

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	71	49	115	187	151	-1	175
N.S.	1	1.00	0.96	1.06	0.73	1.72	2.79	2.25	-0.01	2.61
time (sec)	N/A	0.015	0.105	0.105	0.344	0.320	1.923	10.430	0.000	3.926

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	84	135	194	156	136	308	-1	103
N.S.	1	1.00	0.67	1.07	1.54	1.24	1.08	2.44	-0.01	0.82
time (sec)	N/A	0.025	0.106	0.115	0.359	0.321	58.731	0.010	0.000	56.680

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	75	114	163	137	117	258	-1	89
N.S.	1	1.00	0.71	1.09	1.55	1.30	1.11	2.46	-0.01	0.85
time (sec)	N/A	0.018	0.076	0.111	0.340	0.311	11.517	0.009	0.000	12.886

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	87	132	124	92	361	-1	75
N.S.	1	1.00	0.79	1.06	1.61	1.51	1.12	4.40	-0.01	0.91
time (sec)	N/A	0.011	0.074	0.122	0.343	0.318	3.522	3.500	0.000	5.228

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	98	105	76	113	-1	54
N.S.	1	1.00	0.89	1.18	1.61	1.72	1.25	1.85	-0.02	0.89
time (sec)	N/A	0.008	0.064	0.116	0.347	0.326	1.587	1.174	0.000	3.322

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	55	81	99	73	118	-1	61
N.S.	1	1.00	0.88	0.95	1.40	1.71	1.26	2.03	-0.02	1.05
time (sec)	N/A	0.009	0.073	0.125	0.334	0.314	1.388	1.125	0.000	3.176

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	55	67	108	70	133	-1	74
N.S.	1	1.00	0.92	0.92	1.12	1.80	1.17	2.22	-0.02	1.23
time (sec)	N/A	0.010	0.083	0.129	0.358	0.336	1.666	1.128	0.000	3.581

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	90	143	179	157	289	322	-1	229
N.S.	1	1.00	0.69	1.09	1.37	1.20	2.21	2.46	-0.01	1.75
time (sec)	N/A	0.026	0.130	0.113	0.340	0.324	58.581	0.011	0.000	57.768

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	81	122	147	139	250	271	-1	0
N.S.	1	1.00	0.74	1.12	1.35	1.28	2.29	2.49	-0.01	0.00
time (sec)	N/A	0.020	0.104	0.109	0.349	0.318	11.510	0.011	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	94	115	125	197	389	-1	0
N.S.	1	1.00	0.85	1.12	1.37	1.49	2.35	4.63	-0.01	0.00
time (sec)	N/A	0.011	0.091	0.110	0.342	0.323	3.577	3.376	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	78	79	107	165	120	-1	125
N.S.	1	1.00	0.95	1.24	1.25	1.70	2.62	1.90	-0.02	1.98
time (sec)	N/A	0.009	0.074	0.112	0.344	0.319	1.588	1.122	0.000	3.407

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	70	63	101	158	128	-1	132
N.S.	1	1.00	0.97	1.17	1.05	1.68	2.63	2.13	-0.02	2.20
time (sec)	N/A	0.009	0.083	0.125	0.368	0.319	1.453	1.160	0.000	3.831

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	70	49	111	184	141	-1	163
N.S.	1	1.00	1.00	1.13	0.79	1.79	2.97	2.27	-0.02	2.63
time (sec)	N/A	0.010	0.088	0.125	0.346	0.317	1.806	1.134	0.000	3.709

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	110	160	244	206	207	546	-1	154
N.S.	1	1.00	0.67	0.98	1.49	1.26	1.26	3.33	-0.01	0.94
time (sec)	N/A	0.044	0.106	0.105	0.365	0.317	179.382	0.022	0.000	174.242

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	99	138	212	185	180	465	-1	144
N.S.	1	1.00	0.71	0.99	1.51	1.32	1.29	3.32	-0.01	1.03
time (sec)	N/A	0.033	0.118	0.119	0.388	0.317	33.750	0.019	0.000	34.078

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	87	113	176	162	155	589	-1	109
N.S.	1	1.00	0.75	0.97	1.52	1.40	1.34	5.08	-0.01	0.94
time (sec)	N/A	0.025	0.112	0.121	0.340	0.338	8.982	41.109	0.000	10.592

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	94	141	141	102	150	-1	62
N.S.	1	1.00	0.79	1.02	1.53	1.53	1.11	1.63	-0.01	0.67
time (sec)	N/A	0.019	0.091	0.104	0.340	0.322	4.040	10.430	0.000	5.632

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	84	125	137	126	155	-1	91
N.S.	1	1.00	0.82	0.94	1.40	1.54	1.42	1.74	-0.01	1.02
time (sec)	N/A	0.019	0.107	0.104	0.353	0.315	3.952	10.496	0.000	5.687

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	82	100	138	99	185	-1	99
N.S.	1	1.00	0.80	0.95	1.16	1.60	1.15	2.15	-0.01	1.15
time (sec)	N/A	0.019	0.121	0.109	0.337	0.317	3.746	10.495	0.000	5.521

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	114	169	242	208	435	568	-1	349
N.S.	1	1.00	0.67	0.99	1.42	1.22	2.54	3.32	-0.01	2.04
time (sec)	N/A	0.042	0.169	0.112	0.365	0.315	177.746	0.025	0.000	173.365

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	103	146	207	186	379	484	-1	315
N.S.	1	1.00	0.71	1.00	1.42	1.27	2.60	3.32	-0.01	2.16
time (sec)	N/A	0.034	0.157	0.141	0.360	0.338	33.664	0.022	0.000	49.673

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	121	168	164	326	629	-1	256
N.S.	1	1.00	0.77	1.00	1.39	1.36	2.69	5.20	-0.01	2.12
time (sec)	N/A	0.026	0.137	0.107	0.376	0.321	8.944	41.911	0.000	10.807

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	100	130	142	246	159	-1	195
N.S.	1	1.00	0.82	1.04	1.35	1.48	2.56	1.66	-0.01	2.03
time (sec)	N/A	0.020	0.112	0.104	0.363	0.307	4.026	10.418	0.000	5.992

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	88	112	137	267	167	-1	216
N.S.	1	1.00	0.85	0.95	1.20	1.47	2.87	1.80	-0.01	2.32
time (sec)	N/A	0.020	0.123	0.124	0.347	0.310	3.977	10.449	0.000	5.855

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	86	84	139	245	195	-1	224
N.S.	1	1.00	0.84	0.96	0.93	1.54	2.72	2.17	-0.01	2.49
time (sec)	N/A	0.019	0.135	0.120	0.365	0.316	3.826	10.488	0.000	5.730

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	92	147	223	172	158	500	-1	119
N.S.	1	1.00	0.64	1.02	1.55	1.19	1.10	3.47	-0.01	0.83
time (sec)	N/A	0.032	0.103	0.112	0.348	0.313	176.877	0.017	0.000	171.191

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	126	194	155	138	427	-1	97
N.S.	1	1.00	0.68	1.02	1.58	1.26	1.12	3.47	-0.01	0.79
time (sec)	N/A	0.020	0.108	0.110	0.348	0.311	32.986	0.015	0.000	32.895

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	76	99	161	140	119	559	-1	84
N.S.	1	1.00	0.75	0.97	1.58	1.37	1.17	5.48	-0.01	0.82
time (sec)	N/A	0.015	0.091	0.108	0.377	0.313	8.461	4.594	0.000	9.944

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	84	129	123	97	141	-1	63
N.S.	1	1.00	0.80	1.06	1.63	1.56	1.23	1.78	-0.01	0.80
time (sec)	N/A	0.012	0.077	0.112	0.352	0.326	3.733	1.147	0.000	5.303

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	62	63	113	116	94	145	-1	70
N.S.	1	1.00	0.78	0.80	1.43	1.47	1.19	1.84	-0.01	0.89
time (sec)	N/A	0.012	0.087	0.109	0.351	0.319	3.753	1.221	0.000	5.258

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	63	63	96	123	88	169	-1	92
N.S.	1	1.00	0.78	0.78	1.19	1.52	1.09	2.09	-0.01	1.14
time (sec)	N/A	0.013	0.097	0.139	0.347	0.316	3.476	1.234	0.000	5.286

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	98	157	209	173	0	522	-1	253
N.S.	1	1.00	0.65	1.05	1.39	1.15	0.00	3.48	-0.01	1.69
time (sec)	N/A	0.033	0.139	0.133	0.366	0.325	0.000	0.017	0.000	170.176

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	90	135	179	157	292	446	-1	0
N.S.	1	1.00	0.70	1.05	1.40	1.23	2.28	3.48	-0.01	0.00
time (sec)	N/A	0.021	0.129	0.125	0.345	0.328	32.953	0.017	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	82	107	145	141	253	599	-1	0
N.S.	1	1.00	0.77	1.01	1.37	1.33	2.39	5.65	-0.01	0.00
time (sec)	N/A	0.016	0.109	0.130	0.366	0.320	8.216	4.780	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	91	112	125	207	150	-1	149
N.S.	1	1.00	0.84	1.11	1.37	1.52	2.52	1.83	-0.01	1.82
time (sec)	N/A	0.013	0.094	0.139	0.356	0.317	3.687	1.222	0.000	5.395

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	78	96	117	201	157	-1	156
N.S.	1	1.00	0.83	0.95	1.17	1.43	2.45	1.91	-0.01	1.90
time (sec)	N/A	0.012	0.099	0.120	0.371	0.325	3.686	1.260	0.000	6.991

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	78	79	126	223	179	-1	200
N.S.	1	1.00	0.83	0.93	0.94	1.50	2.65	2.13	-0.01	2.38
time (sec)	N/A	0.013	0.108	0.148	0.357	0.311	3.623	1.192	0.000	5.462

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	109	146	140	128	124	-1	125
N.S.	1	1.00	0.75	1.08	1.45	1.39	1.27	1.23	-0.01	1.24
time (sec)	N/A	0.020	0.085	0.124	0.368	0.311	9.518	0.006	0.000	10.636

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	87	112	119	100	96	-1	93
N.S.	1	1.00	0.86	1.13	1.45	1.55	1.30	1.25	-0.01	1.21
time (sec)	N/A	0.016	0.066	0.104	0.341	0.322	2.811	0.005	0.000	4.645

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	65	73	91	44	61	44	38
N.S.	1	1.00	1.02	1.35	1.52	1.90	0.92	1.27	0.92	0.79
time (sec)	N/A	0.012	0.050	0.107	0.340	0.319	1.161	0.007	0.548	2.844

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	48	41	57	22	33	30	16
N.S.	1	1.00	1.07	1.71	1.46	2.04	0.79	1.18	1.07	0.57
time (sec)	N/A	0.010	0.029	0.115	0.357	0.314	0.511	0.003	0.030	2.009

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	19	37	15	20
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.00	1.95	0.79	1.05
time (sec)	N/A	0.001	0.016	0.120	0.249	0.305	0.484	0.004	0.349	1.999

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	33	31	23	42	72	25	33
N.S.	1	1.00	0.61	0.75	0.70	0.52	0.95	1.64	0.57	0.75
time (sec)	N/A	0.004	0.052	0.119	0.250	0.306	1.142	0.005	0.342	2.787

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	55	46	34	287	102	36	84
N.S.	1	1.00	0.59	0.81	0.68	0.50	4.22	1.50	0.53	1.24
time (sec)	N/A	0.007	0.063	0.117	0.250	0.309	4.085	0.007	0.353	6.956

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	51	77	61	45	488	131	47	117
N.S.	1	1.00	0.55	0.84	0.66	0.49	5.30	1.42	0.51	1.27
time (sec)	N/A	0.012	0.070	0.125	0.313	0.299	12.384	0.009	0.382	14.508

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	76	119	131	175	105	132	-1	93
N.S.	1	1.00	0.79	1.24	1.36	1.82	1.09	1.38	-0.01	0.97
time (sec)	N/A	0.022	0.107	0.129	0.351	0.318	5.743	0.008	0.000	7.152

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	106	92	145	71	97	-1	77
N.S.	1	1.00	0.84	1.56	1.35	2.13	1.04	1.43	-0.01	1.13
time (sec)	N/A	0.015	0.097	0.128	0.338	0.339	1.986	0.008	0.000	3.741

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	0	57	119	46	65	-1	38
N.S.	1	1.00	1.04	0.00	1.19	2.48	0.96	1.35	-0.02	0.79
time (sec)	N/A	0.013	0.064	0.028	0.333	0.318	0.862	0.008	0.000	2.580

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	22	17	53	22	20
N.S.	1	1.00	1.00	0.84	0.79	1.16	0.89	2.79	1.16	1.05
time (sec)	N/A	0.001	0.017	0.117	0.265	0.312	0.467	0.005	0.326	1.981

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	33	32	34	41	73	39	37
N.S.	1	1.00	0.64	0.85	0.82	0.87	1.05	1.87	1.00	0.95
time (sec)	N/A	0.004	0.056	0.144	0.258	0.305	0.882	0.005	0.386	2.621

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	38	55	50	49	219	152	46	73
N.S.	1	1.00	0.60	0.87	0.79	0.78	3.48	2.41	0.73	1.16
time (sec)	N/A	0.007	0.076	0.128	0.251	0.299	2.496	0.010	0.413	5.204

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	49	77	64	58	348	233	58	99
N.S.	1	1.00	0.56	0.89	0.74	0.67	4.00	2.68	0.67	1.14
time (sec)	N/A	0.012	0.088	0.129	0.257	0.312	8.267	0.016	0.426	10.438

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	147	109	214	396	138	-1	131
N.S.	1	1.00	0.79	1.62	1.20	2.35	4.35	1.52	-0.01	1.44
time (sec)	N/A	0.023	0.126	0.139	0.345	0.322	5.024	0.011	0.000	8.513

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	0	69	186	328	106	-1	102
N.S.	1	1.00	0.87	0.00	1.00	2.70	4.75	1.54	-0.01	1.48
time (sec)	N/A	0.015	0.117	0.026	0.348	0.322	2.148	0.011	0.000	5.873

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	54	15	33	42	42	36	26
N.S.	1	1.00	1.00	2.57	0.71	1.57	2.00	2.00	1.71	1.24
time (sec)	N/A	0.001	0.019	0.119	0.248	0.305	0.779	0.008	0.242	2.546

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	32	27	43	92	102	54	38
N.S.	1	1.00	0.67	0.74	0.63	1.00	2.14	2.37	1.26	0.88
time (sec)	N/A	0.004	0.061	0.118	0.260	0.308	1.084	0.009	0.400	3.435

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	40	54	46	58	153	115	71	58
N.S.	1	1.00	0.62	0.84	0.72	0.91	2.39	1.80	1.11	0.91
time (sec)	N/A	0.007	0.077	0.138	0.262	0.306	2.487	0.009	0.420	4.739

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	49	76	64	71	337	189	88	95
N.S.	1	1.00	0.58	0.90	0.76	0.85	4.01	2.25	1.05	1.13
time (sec)	N/A	0.011	0.085	0.131	0.295	0.316	4.656	0.013	0.471	7.833

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	82	116	135	141	270	131	-1	254
N.S.	1	1.00	0.78	1.10	1.29	1.34	2.57	1.25	-0.01	2.42
time (sec)	N/A	0.022	0.109	0.123	0.354	0.312	9.282	0.006	0.000	18.664

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	93	98	119	214	101	-1	195
N.S.	1	1.00	0.89	1.16	1.22	1.49	2.68	1.26	-0.01	2.44
time (sec)	N/A	0.016	0.089	0.127	0.340	0.317	2.792	0.007	0.000	5.143

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	70	56	93	121	67	47	135
N.S.	1	1.00	1.12	1.40	1.12	1.86	2.42	1.34	0.94	2.70
time (sec)	N/A	0.012	0.061	0.123	0.341	0.330	1.187	0.005	0.517	3.166

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	51	21	57	54	36	27	44
N.S.	1	1.00	1.21	1.76	0.72	1.97	1.86	1.24	0.93	1.52
time (sec)	N/A	0.010	0.040	0.106	0.369	0.315	0.537	0.003	0.031	2.198

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	46	40	16	55
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.30	2.00	0.80	2.75
time (sec)	N/A	0.001	0.017	0.128	0.247	0.307	0.510	0.005	0.401	2.193

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	28	35	32	22	177	76	26	174
N.S.	1	1.00	0.61	0.76	0.70	0.48	3.85	1.65	0.57	3.78
time (sec)	N/A	0.004	0.062	0.118	0.260	0.303	1.141	0.006	0.350	2.939

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	81	127	118	181	224	136	-1	201
N.S.	1	1.00	0.81	1.27	1.18	1.81	2.24	1.36	-0.01	2.01
time (sec)	N/A	0.021	0.141	0.128	0.337	0.322	5.726	0.011	0.000	7.525

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	114	75	152	155	103	-1	160
N.S.	1	1.00	0.90	1.61	1.06	2.14	2.18	1.45	-0.01	2.25
time (sec)	N/A	0.016	0.127	0.130	0.344	0.331	2.034	0.009	0.000	3.950

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	0	38	128	102	68	-1	89
N.S.	1	1.00	1.12	0.00	0.76	2.56	2.04	1.36	-0.02	1.78
time (sec)	N/A	0.012	0.093	0.031	0.345	0.327	0.929	0.008	0.000	2.719

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	25	44	59	24	55
N.S.	1	1.00	1.00	0.85	0.80	1.25	2.20	2.95	1.20	2.75
time (sec)	N/A	0.001	0.018	0.116	0.262	0.307	0.499	0.009	0.339	2.182

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	26	35	34	38	112	77	42	126
N.S.	1	1.00	0.63	0.85	0.83	0.93	2.73	1.88	1.02	3.07
time (sec)	N/A	0.004	0.071	0.132	0.253	0.545	0.918	0.005	0.400	2.730

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	58	52	51	452	162	48	311
N.S.	1	1.00	0.59	0.88	0.79	0.77	6.85	2.45	0.73	4.71
time (sec)	N/A	0.007	0.091	0.136	0.265	0.303	2.667	0.011	0.432	8.555

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	160	94	215	971	146	-1	523
N.S.	1	1.00	0.82	1.68	0.99	2.26	10.22	1.54	-0.01	5.51
time (sec)	N/A	0.021	0.160	0.122	0.342	0.325	5.426	0.013	0.000	11.137

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	52	188	833	110	-1	427
N.S.	1	1.00	0.92	0.00	0.72	2.61	11.57	1.53	-0.01	5.93
time (sec)	N/A	0.015	0.133	0.028	0.349	0.316	2.410	0.012	0.000	9.916

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	56	16	34	95	44	37	84
N.S.	1	1.00	1.00	2.55	0.73	1.55	4.32	2.00	1.68	3.82
time (sec)	N/A	0.001	0.020	0.107	0.248	0.306	0.809	0.010	0.252	2.684

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	34	30	44	197	109	56	157
N.S.	1	1.00	0.67	0.76	0.67	0.98	4.38	2.42	1.24	3.49
time (sec)	N/A	0.004	0.073	0.120	0.272	0.313	1.120	0.011	0.406	3.601

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	41	57	50	59	314	119	73	229
N.S.	1	1.00	0.61	0.85	0.75	0.88	4.69	1.78	1.09	3.42
time (sec)	N/A	0.007	0.090	0.155	0.258	0.318	2.563	0.008	0.441	5.418

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	80	68	70	688	200	92	447
N.S.	1	1.00	0.57	0.91	0.77	0.80	7.82	2.27	1.05	5.08
time (sec)	N/A	0.012	0.105	0.135	0.268	0.315	5.982	0.014	0.475	59.408

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	104	134	124	95	116	-1	75
N.S.	1	1.00	0.75	1.18	1.52	1.41	1.08	1.32	-0.01	0.85
time (sec)	N/A	0.014	0.073	0.133	0.337	0.319	8.849	0.005	0.000	10.115

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	83	102	105	75	90	-1	73
N.S.	1	1.00	0.85	1.24	1.52	1.57	1.12	1.34	-0.01	1.09
time (sec)	N/A	0.010	0.063	0.114	0.352	0.307	2.519	0.005	0.000	4.251

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	62	70	87	54	56	43	61
N.S.	1	1.00	1.14	1.44	1.63	2.02	1.26	1.30	1.00	1.42
time (sec)	N/A	0.007	0.045	0.123	0.347	0.317	1.018	0.004	0.585	2.786

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	46	41	55	24	32	30	17
N.S.	1	1.00	1.25	1.92	1.71	2.29	1.00	1.33	1.25	0.71
time (sec)	N/A	0.004	0.027	0.121	0.336	0.314	0.469	0.002	0.035	2.001

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	36	12	17
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	2.25	0.75	1.06
time (sec)	N/A	0.001	0.014	0.102	0.247	0.305	0.455	0.004	0.330	1.974

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	27	26	17	34	71	17	27
N.S.	1	1.00	0.61	0.71	0.68	0.45	0.89	1.87	0.45	0.71
time (sec)	N/A	0.003	0.044	0.136	0.258	0.315	1.041	0.005	0.316	2.769

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	32	43	41	26	224	97	26	67
N.S.	1	1.00	0.54	0.73	0.69	0.44	3.80	1.64	0.44	1.14
time (sec)	N/A	0.006	0.052	0.121	0.273	0.317	3.825	0.006	0.323	6.506

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	40	59	56	34	374	124	33	89
N.S.	1	1.00	0.50	0.74	0.70	0.42	4.68	1.55	0.41	1.11
time (sec)	N/A	0.009	0.056	0.128	0.255	0.318	11.819	0.013	0.333	13.573

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	65	63	119	152	80	123	-1	75
N.S.	1	1.00	0.76	0.73	1.38	1.77	0.93	1.43	-0.01	0.87
time (sec)	N/A	0.014	0.092	0.134	0.348	0.330	5.274	0.007	0.000	6.712

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	55	90	134	58	92	-1	63
N.S.	1	1.00	0.86	0.87	1.43	2.13	0.92	1.46	-0.02	1.00
time (sec)	N/A	0.010	0.078	0.126	0.339	0.321	1.821	0.007	0.000	3.427

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	50	48	57	117	41	64	-1	33
N.S.	1	1.00	1.14	1.09	1.30	2.66	0.93	1.45	-0.02	0.75
time (sec)	N/A	0.007	0.059	0.123	0.373	0.308	0.864	0.006	0.000	2.444

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	15	53	11	16
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.00	3.53	0.73	1.07
time (sec)	N/A	0.001	0.015	0.132	0.268	0.300	0.465	0.005	0.307	1.963

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	27	26	28	34	65	17	31
N.S.	1	1.00	0.66	0.84	0.81	0.88	1.06	2.03	0.53	0.97
time (sec)	N/A	0.003	0.048	0.136	0.271	0.313	0.861	0.005	0.348	2.540

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	32	43	41	39	170	138	37	57
N.S.	1	1.00	0.60	0.81	0.77	0.74	3.21	2.60	0.70	1.08
time (sec)	N/A	0.006	0.063	0.130	0.262	0.326	2.442	0.009	0.380	4.854

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	39	59	56	47	269	213	46	77
N.S.	1	1.00	0.53	0.80	0.76	0.64	3.64	2.88	0.62	1.04
time (sec)	N/A	0.009	0.069	0.138	0.276	0.306	8.060	0.015	0.426	9.766

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	63	105	186	308	121	-1	100
N.S.	1	1.00	0.77	0.73	1.22	2.16	3.58	1.41	-0.01	1.16
time (sec)	N/A	0.015	0.109	0.150	0.358	0.310	4.035	0.009	0.000	7.364

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	55	69	171	257	97	-1	86
N.S.	1	1.00	0.89	0.85	1.06	2.63	3.95	1.49	-0.02	1.32
time (sec)	N/A	0.011	0.099	0.128	0.337	0.320	1.899	0.008	0.000	5.010

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	46	12	27	27	40	12	12
N.S.	1	1.00	1.00	2.56	0.67	1.50	1.50	2.22	0.67	0.67
time (sec)	N/A	0.001	0.017	0.103	0.255	0.305	0.757	0.007	0.252	2.398

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	26	24	32	75	102	42	32
N.S.	1	1.00	0.62	0.70	0.65	0.86	2.03	2.76	1.14	0.86
time (sec)	N/A	0.003	0.051	0.123	0.284	0.313	1.035	0.011	0.357	3.328

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	32	42	40	45	117	96	57	48
N.S.	1	1.00	0.58	0.76	0.73	0.82	2.13	1.75	1.04	0.87
time (sec)	N/A	0.005	0.065	0.131	0.273	0.312	2.428	0.007	0.378	4.435

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	40	58	55	55	257	166	71	76
N.S.	1	1.00	0.56	0.82	0.77	0.77	3.62	2.34	1.00	1.07
time (sec)	N/A	0.006	0.067	0.131	0.285	0.308	4.576	0.012	0.418	7.240

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	111	117	125	204	123	-1	163
N.S.	1	1.00	0.79	1.22	1.29	1.37	2.24	1.35	-0.01	1.79
time (sec)	N/A	0.016	0.090	0.131	0.346	0.330	8.914	0.005	0.000	10.116

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	89	85	107	162	95	-1	0
N.S.	1	1.00	0.91	1.29	1.23	1.55	2.35	1.38	-0.01	0.00
time (sec)	N/A	0.011	0.078	0.110	0.351	0.448	2.554	0.005	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	67	52	90	119	62	46	0
N.S.	1	1.00	1.24	1.49	1.16	2.00	2.64	1.38	1.02	0.00
time (sec)	N/A	0.007	0.055	0.127	0.348	0.321	1.082	0.004	0.520	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	50	21	56	56	35	27	43
N.S.	1	1.00	1.46	2.08	0.88	2.33	2.33	1.46	1.12	1.79
time (sec)	N/A	0.005	0.037	0.121	0.342	0.319	0.524	0.002	0.032	2.141

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	41	39	13	45
N.S.	1	1.00	1.00	0.82	0.76	0.76	2.41	2.29	0.76	2.65
time (sec)	N/A	0.001	0.015	0.112	0.258	0.307	0.499	0.005	0.310	2.156

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	29	28	18	139	75	19	90
N.S.	1	1.00	0.62	0.72	0.70	0.45	3.48	1.88	0.48	2.25
time (sec)	N/A	0.003	0.055	0.115	0.266	0.315	1.127	0.005	0.292	3.301

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	81	101	155	172	128	-1	0
N.S.	1	1.00	0.80	0.91	1.13	1.74	1.93	1.44	-0.01	0.00
time (sec)	N/A	0.016	0.138	0.138	0.348	0.317	5.337	0.008	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	73	71	138	126	100	-1	0
N.S.	1	1.00	0.94	1.12	1.09	2.12	1.94	1.54	-0.02	0.00
time (sec)	N/A	0.011	0.114	0.138	0.353	0.323	1.827	0.008	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	67	38	122	90	70	-1	76
N.S.	1	1.00	1.24	1.49	0.84	2.71	2.00	1.56	-0.02	1.69
time (sec)	N/A	0.007	0.085	0.105	0.361	0.321	0.846	0.008	0.000	2.559

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	20	41	59	12	44
N.S.	1	1.00	1.00	0.81	0.75	1.25	2.56	3.69	0.75	2.75
time (sec)	N/A	0.001	0.018	0.110	0.266	0.306	0.495	0.006	0.299	2.077

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	28	28	29	90	72	27	81
N.S.	1	1.00	0.62	0.82	0.82	0.85	2.65	2.12	0.79	2.38
time (sec)	N/A	0.002	0.062	0.136	0.270	0.303	0.935	0.005	0.321	2.790

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	45	44	40	355	151	38	245
N.S.	1	1.00	0.59	0.80	0.79	0.71	6.34	2.70	0.68	4.38
time (sec)	N/A	0.006	0.080	0.138	0.304	0.313	2.626	0.011	0.362	4.903

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	81	86	187	751	129	-1	0
N.S.	1	1.00	0.81	0.91	0.97	2.10	8.44	1.45	-0.01	0.00
time (sec)	N/A	0.015	0.138	0.124	0.356	0.313	4.247	0.010	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	73	50	173	648	101	-1	0
N.S.	1	1.00	0.96	1.09	0.75	2.58	9.67	1.51	-0.01	0.00
time (sec)	N/A	0.011	0.119	0.126	0.343	0.324	2.067	0.010	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	49	13	28	63	42	13	51
N.S.	1	1.00	1.00	2.58	0.68	1.47	3.32	2.21	0.68	2.68
time (sec)	N/A	0.001	0.067	0.127	0.259	0.309	0.823	0.009	0.232	2.502

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	24	28	25	33	165	109	45	136
N.S.	1	1.00	0.62	0.72	0.64	0.85	4.23	2.79	1.15	3.49
time (sec)	N/A	0.003	0.067	0.114	0.268	0.319	1.114	0.009	0.357	3.482

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	33	45	42	46	245	100	59	182
N.S.	1	1.00	0.57	0.78	0.72	0.79	4.22	1.72	1.02	3.14
time (sec)	N/A	0.005	0.081	0.133	0.273	0.325	2.546	0.008	0.367	4.488

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	41	61	58	56	530	177	73	349
N.S.	1	1.00	0.55	0.81	0.77	0.75	7.07	2.36	0.97	4.65
time (sec)	N/A	0.007	0.094	0.117	0.259	0.319	5.784	0.015	0.438	7.276

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	35	41	37	27	54	29	31	54
N.S.	1	1.00	1.30	1.52	1.37	1.00	2.00	1.07	1.15	2.00
time (sec)	N/A	0.004	0.056	0.132	0.340	0.314	0.913	0.004	0.570	2.459

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	27	14	14	20	12	16	19
N.S.	1	1.00	4.75	3.38	1.75	1.75	2.50	1.50	2.00	2.38
time (sec)	N/A	0.003	0.025	0.125	0.347	0.308	0.461	0.001	0.051	1.937

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	48	21	57	42	35	23	35
N.S.	1	1.00	1.84	2.53	1.11	3.00	2.21	1.84	1.21	1.84
time (sec)	N/A	0.004	0.040	0.129	0.355	0.323	0.506	0.002	0.125	2.097

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	33	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	1.57	0.62	0.62
time (sec)	N/A	0.003	0.009	0.051	0.277	0.313	0.321	0.001	0.028	1.785

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	27	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	1.29	0.62	0.62
time (sec)	N/A	0.003	0.009	0.053	0.257	0.300	0.241	0.000	0.025	1.738

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	31	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	1.48	0.62	0.62
time (sec)	N/A	0.003	0.009	0.053	0.255	0.299	0.138	0.001	0.024	1.645

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	25	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	1.19	0.62	0.62
time (sec)	N/A	0.003	0.009	0.072	0.273	0.301	0.703	0.000	0.025	2.100

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	13	19	26	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.62	0.90	1.24	0.62	0.62
time (sec)	N/A	0.003	0.009	0.023	0.276	0.302	0.779	0.000	0.024	2.213

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	17	20	12	12
N.S.	1	1.00	0.84	0.74	0.68	0.63	0.89	1.05	0.63	0.63
time (sec)	N/A	0.003	0.009	0.026	0.275	0.309	0.528	0.000	0.023	1.910

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	12	17	23	13	12
N.S.	1	1.00	0.89	0.74	0.68	0.63	0.89	1.21	0.68	0.63
time (sec)	N/A	0.003	0.011	0.026	0.262	0.302	0.181	0.001	0.027	1.621

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	14	13	13	17	24	13	13
N.S.	1	1.00	0.79	0.74	0.68	0.68	0.89	1.26	0.68	0.68
time (sec)	N/A	0.003	0.011	0.027	0.266	0.300	0.196	0.001	0.026	1.615

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	55	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	1.53	0.67	0.67
time (sec)	N/A	0.005	0.013	0.111	0.260	0.568	0.485	0.002	0.045	1.987

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	45	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	1.25	0.67	0.67
time (sec)	N/A	0.005	0.013	0.107	0.259	0.292	0.385	0.001	0.038	1.886

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	34	51	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	0.94	1.42	0.67	0.67
time (sec)	N/A	0.005	0.013	0.092	0.270	0.302	0.232	0.001	0.038	1.760

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	2633	43	24	1235
N.S.	1	1.00	0.78	0.69	0.67	0.75	73.14	1.19	0.67	34.31
time (sec)	N/A	0.005	0.012	0.095	0.273	0.299	1.123	0.001	0.035	13.426

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	24	1765	48	24	965
N.S.	1	1.00	0.78	0.69	0.67	0.67	49.03	1.33	0.67	26.81
time (sec)	N/A	0.006	0.013	0.105	0.252	0.311	1.033	0.001	0.038	10.446

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	24	1741	38	24	965
N.S.	1	1.00	0.82	0.74	0.71	0.71	51.21	1.12	0.71	28.38
time (sec)	N/A	0.006	0.013	0.111	0.256	0.304	1.061	0.001	0.034	10.151

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	23	1826	39	24	968
N.S.	1	1.00	0.88	0.78	0.75	0.72	57.06	1.22	0.75	30.25
time (sec)	N/A	0.005	0.015	0.097	0.262	0.295	1.067	0.002	0.037	10.871

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	23	1957	40	24	968
N.S.	1	1.00	0.82	0.74	0.71	0.68	57.56	1.18	0.71	28.47
time (sec)	N/A	0.005	0.015	0.102	0.272	0.298	1.059	0.001	0.036	10.743

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	75	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	1.47	0.69	0.69
time (sec)	N/A	0.008	0.015	0.102	0.266	0.291	0.683	0.001	0.043	2.196

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	63	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	1.24	0.69	0.69
time (sec)	N/A	0.008	0.015	0.124	0.249	0.300	0.550	0.001	0.044	2.104

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	49	73	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.96	1.43	0.69	0.69
time (sec)	N/A	0.008	0.015	0.113	0.247	0.519	0.333	0.001	0.044	1.925

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	5012	61	35	2649
N.S.	1	1.00	0.76	0.71	0.69	0.75	98.27	1.20	0.69	51.94
time (sec)	N/A	0.008	0.014	0.094	0.273	0.303	1.621	0.001	0.048	27.183

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	35	6246	68	35	3003
N.S.	1	1.00	0.76	0.71	0.69	0.69	122.47	1.33	0.69	58.88
time (sec)	N/A	0.008	0.014	0.110	0.272	0.303	1.568	0.001	0.044	32.859

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	6667	56	35	3003
N.S.	1	1.00	0.80	0.73	0.71	0.71	136.06	1.14	0.71	61.29
time (sec)	N/A	0.008	0.014	0.092	0.256	0.348	1.583	0.001	0.044	33.407

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	4004	61	35	2278
N.S.	1	1.00	0.80	0.73	0.71	0.71	81.71	1.24	0.71	46.49
time (sec)	N/A	0.008	0.017	0.098	0.255	0.306	1.625	0.002	0.044	23.094

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	3964	58	35	2278
N.S.	1	1.00	0.80	0.73	0.71	0.71	80.90	1.18	0.71	46.49
time (sec)	N/A	0.008	0.017	0.111	0.269	0.314	1.630	0.001	0.043	23.093

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	140	124	130	147	180	214	151	173
N.S.	1	1.00	1.12	0.99	1.04	1.18	1.44	1.71	1.21	1.38
time (sec)	N/A	0.050	0.088	0.122	0.355	0.316	46.953	0.006	0.243	45.165

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	140	123	128	116	173	194	126	168
N.S.	1	1.00	1.14	1.00	1.04	0.94	1.41	1.58	1.02	1.37
time (sec)	N/A	0.041	0.073	0.173	0.347	0.317	21.264	0.005	0.068	10.514

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	127	112	114	128	162	187	130	160
N.S.	1	1.00	1.14	1.01	1.03	1.15	1.46	1.68	1.17	1.44
time (sec)	N/A	0.028	0.063	0.112	0.349	0.315	4.327	0.006	0.150	5.449

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	126	112	115	114	148	165	126	156
N.S.	1	1.00	1.16	1.03	1.06	1.05	1.36	1.51	1.16	1.43
time (sec)	N/A	0.028	0.060	0.146	0.399	0.317	2.332	0.005	0.074	3.655

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	103	96	103	313	141	171	120	147
N.S.	1	1.00	1.03	0.96	1.03	3.13	1.41	1.71	1.20	1.47
time (sec)	N/A	0.022	0.049	0.117	0.345	0.333	2.626	0.005	0.113	4.027

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	103	95	102	307	141	153	110	148
N.S.	1	1.00	1.03	0.95	1.02	3.07	1.41	1.53	1.10	1.48
time (sec)	N/A	0.021	0.053	0.098	0.340	0.320	6.548	0.002	0.206	5.407

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	127	112	111	113	151	183	124	157
N.S.	1	1.00	1.17	1.03	1.02	1.04	1.39	1.68	1.14	1.44
time (sec)	N/A	0.028	0.080	0.118	0.354	0.321	23.787	0.005	0.147	10.151

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	126	112	112	147	155	173	138	156
N.S.	1	1.00	1.14	1.01	1.01	1.32	1.40	1.56	1.24	1.41
time (sec)	N/A	0.028	0.080	0.172	0.353	0.306	41.418	0.003	0.072	41.431

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	142	124	133	162	0	217	150	566
N.S.	1	1.00	1.10	0.96	1.03	1.26	0.00	1.68	1.16	4.39
time (sec)	N/A	0.035	0.183	0.115	0.351	0.333	0.000	0.007	0.265	85.399

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	124	133	147	0	196	142	439
N.S.	1	1.00	1.14	0.99	1.06	1.18	0.00	1.57	1.14	3.51
time (sec)	N/A	0.034	0.178	0.258	0.352	0.333	0.000	0.006	0.152	66.084

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	133	118	120	394	527	198	142	506
N.S.	1	1.00	1.16	1.03	1.04	3.43	4.58	1.72	1.23	4.40
time (sec)	N/A	0.028	0.156	0.107	0.364	0.331	95.939	0.006	0.236	94.061

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	134	117	120	389	450	180	120	425
N.S.	1	1.00	1.15	1.00	1.03	3.32	3.85	1.54	1.03	3.63
time (sec)	N/A	0.028	0.151	0.102	0.351	0.326	60.304	0.006	0.063	59.099

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	116	127	396	544	200	144	515
N.S.	1	1.00	1.15	1.00	1.09	3.41	4.69	1.72	1.24	4.44
time (sec)	N/A	0.029	0.144	0.102	0.360	0.330	61.766	0.006	0.358	60.092

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	134	117	127	387	434	182	134	416
N.S.	1	1.00	1.19	1.04	1.12	3.42	3.84	1.61	1.19	3.68
time (sec)	N/A	0.028	0.142	0.113	0.349	0.323	109.515	0.004	0.223	39.785

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	142	124	132	156	0	215	151	616
N.S.	1	1.00	1.15	1.00	1.06	1.26	0.00	1.73	1.22	4.97
time (sec)	N/A	0.035	0.195	0.145	0.354	0.319	0.000	0.007	0.152	83.358

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	142	124	132	189	0	203	166	0
N.S.	1	1.00	1.11	0.97	1.03	1.48	0.00	1.59	1.30	0.00
time (sec)	N/A	0.034	0.201	0.223	0.344	0.323	0.000	0.004	0.165	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	130	143	506	0	221	165	0
N.S.	1	1.00	1.01	0.93	1.02	3.61	0.00	1.58	1.18	0.00
time (sec)	N/A	0.036	0.216	0.119	0.360	0.326	0.000	0.008	0.172	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	130	143	503	0	202	139	0
N.S.	1	1.00	1.01	0.93	1.02	3.59	0.00	1.44	0.99	0.00
time (sec)	N/A	0.035	0.208	0.118	0.357	0.321	0.000	0.007	0.066	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	133	132	153	508	0	220	172	0
N.S.	1	1.00	0.93	0.92	1.07	3.55	0.00	1.54	1.20	0.00
time (sec)	N/A	0.034	0.200	0.099	0.341	0.322	0.000	0.008	0.265	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	136	132	152	501	0	202	146	0
N.S.	1	1.00	0.95	0.92	1.06	3.50	0.00	1.41	1.02	0.00
time (sec)	N/A	0.034	0.193	0.106	0.345	0.342	0.000	0.007	0.235	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	139	151	510	0	221	167	0
N.S.	1	1.00	1.01	0.99	1.08	3.64	0.00	1.58	1.19	0.00
time (sec)	N/A	0.034	0.141	0.112	0.347	0.331	0.000	0.007	0.192	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	139	151	499	0	200	157	0
N.S.	1	1.00	1.01	0.99	1.08	3.56	0.00	1.43	1.12	0.00
time (sec)	N/A	0.037	0.138	0.126	0.367	0.339	0.000	0.004	0.241	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	153	133	154	211	0	235	174	0
N.S.	1	1.00	1.01	0.88	1.01	1.39	0.00	1.55	1.14	0.00
time (sec)	N/A	0.044	0.245	0.138	0.361	0.317	0.000	0.008	0.087	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	153	133	154	244	0	214	182	0
N.S.	1	1.00	1.01	0.88	1.01	1.61	0.00	1.41	1.20	0.00
time (sec)	N/A	0.045	0.244	0.158	0.353	0.313	0.000	0.005	0.173	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	60	61	82	243	80	46	98
N.S.	1	1.00	1.00	1.03	1.05	1.41	4.19	1.38	0.79	1.69
time (sec)	N/A	0.019	0.063	1.208	0.368	0.313	1.282	0.013	0.068	4.029

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	166	1534	187	1277	9996	2166	1274	9691
N.S.	1	1.00	0.89	8.20	1.00	6.83	53.45	11.58	6.81	51.82
time (sec)	N/A	0.060	0.182	0.107	0.268	0.328	1.227	0.016	1.370	8.775

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	118	156	133	665	4257	1119	683	4128
N.S.	1	1.00	0.89	1.17	1.00	5.00	32.01	8.41	5.14	31.04
time (sec)	N/A	0.034	0.075	0.105	0.260	0.322	0.707	0.007	0.777	5.366

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	61	157	663	255	167	662
N.S.	1	1.00	0.89	1.18	1.00	2.57	10.87	4.18	2.74	10.85
time (sec)	N/A	0.013	0.020	0.096	0.256	0.332	0.278	0.004	0.443	2.485

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	51	43	85	299	134	93	311
N.S.	1	1.00	0.88	1.19	1.00	1.98	6.95	3.12	2.16	7.23
time (sec)	N/A	0.009	0.017	0.117	0.272	0.321	0.194	0.002	0.370	1.983

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	30	25	33	87	50	30	101
N.S.	1	1.00	0.88	1.20	1.00	1.32	3.48	2.00	1.20	4.04
time (sec)	N/A	0.005	0.010	0.010	0.272	0.309	0.130	0.002	0.304	1.677

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	61	0	-1	24
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.10	0.00	-0.03	0.83
time (sec)	N/A	0.004	0.024	0.028	0.000	0.315	0.376	0.000	0.000	2.222

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	262	0	-1	57
N.S.	1	1.00	1.00	0.00	0.00	0.00	9.03	0.00	-0.03	1.97
time (sec)	N/A	0.004	0.019	0.032	0.000	0.310	0.503	0.000	0.000	4.007

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	717	0	-1	155
N.S.	1	1.00	1.00	0.00	0.00	0.00	24.72	0.00	-0.03	5.34
time (sec)	N/A	0.004	0.019	0.036	0.000	0.330	0.682	0.000	0.000	7.459

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	-0.02	0.71
time (sec)	N/A	0.008	0.092	0.026	0.000	0.313	6.457	0.000	0.000	6.867

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	-0.02	0.71
time (sec)	N/A	0.008	0.078	0.027	0.000	0.320	1.843	0.000	0.000	3.311

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	-0.02	0.71
time (sec)	N/A	0.008	0.056	0.025	0.000	0.313	0.810	0.000	0.000	2.463

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	-0.02	0.74
time (sec)	N/A	0.009	0.051	0.028	0.000	0.321	0.719	0.000	0.000	2.396

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	-0.02	0.74
time (sec)	N/A	0.008	0.076	0.029	0.000	0.327	0.928	0.000	0.000	2.568

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	36	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.02	0.71
time (sec)	N/A	0.009	0.077	0.026	0.000	0.322	1.927	0.000	0.000	3.342

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	-0.02	0.67
time (sec)	N/A	0.009	0.070	0.027	0.000	0.320	1.729	0.000	0.000	3.253

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	37	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	-0.02	0.69
time (sec)	N/A	0.009	0.065	0.031	0.000	0.317	1.240	0.000	0.000	2.830

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	-0.02	0.74
time (sec)	N/A	0.008	0.007	0.003	0.000	0.642	0.721	0.000	0.000	2.395

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	31	0	-1	28
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.65	0.00	-0.02	0.58
time (sec)	N/A	0.009	0.065	0.028	0.000	0.316	1.976	0.000	0.000	3.383

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	32	0	-1	32
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.65	0.00	-0.02	0.65
time (sec)	N/A	0.009	0.066	0.027	0.000	0.324	5.580	0.000	0.000	6.332

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	-1	34
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	-0.02	0.67
time (sec)	N/A	0.010	0.073	0.028	0.000	0.313	16.566	0.000	0.000	14.485

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	37	0	-1	32
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.19	0.00	-0.03	1.03
time (sec)	N/A	0.005	0.036	0.100	0.000	0.320	0.597	0.000	0.000	2.268

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	46	0	-1	33
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.48	0.00	-0.03	1.06
time (sec)	N/A	0.004	0.035	0.115	0.000	0.330	0.609	0.000	0.000	2.279

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	43	0	0	36	0	-1	30
N.S.	1	1.00	1.00	1.19	0.00	0.00	1.00	0.00	-0.03	0.83
time (sec)	N/A	0.007	0.034	0.130	0.000	0.313	0.608	0.000	0.000	2.255

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	30	0	0	41	0	-1	34
N.S.	1	1.00	0.96	0.60	0.00	0.00	0.82	0.00	-0.02	0.68
time (sec)	N/A	0.009	0.046	0.098	0.000	0.315	0.603	0.000	0.000	2.281

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	42	0	-1	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.88	0.00	-0.02	0.83
time (sec)	N/A	0.009	0.010	0.027	0.000	0.326	0.732	0.000	0.000	2.439

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	44	0	-1	32
N.S.	1	1.00	0.94	0.88	0.00	0.00	1.29	0.00	-0.03	0.94
time (sec)	N/A	0.005	0.005	0.105	0.000	0.316	0.591	0.000	0.000	2.282

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	53	0	-1	39
N.S.	1	1.00	0.94	0.88	0.00	0.00	1.56	0.00	-0.03	1.15
time (sec)	N/A	0.004	0.005	0.129	0.000	0.317	0.626	0.000	0.000	2.364

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	42	0	-1	36
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.86	0.00	-0.02	0.73
time (sec)	N/A	0.009	0.006	0.131	0.000	0.310	0.621	0.000	0.000	2.307

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	57	31	0	0	48	0	-1	33
N.S.	1	1.00	1.54	0.84	0.00	0.00	1.30	0.00	-0.03	0.89
time (sec)	N/A	0.004	0.010	0.119	0.000	0.309	0.583	0.000	0.000	2.296

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	0	26	0	-1	23
N.S.	1	1.00	1.00	0.88	0.00	0.00	1.00	0.00	-0.04	0.88
time (sec)	N/A	0.003	0.032	0.110	0.000	0.320	0.524	0.000	0.000	2.142

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	31	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.03	0.00	-0.03	0.87
time (sec)	N/A	0.004	0.103	0.032	0.000	0.323	0.549	0.000	0.000	2.185

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	34	0	-1	36
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.72	0.00	-0.02	0.77
time (sec)	N/A	0.008	0.021	0.040	0.000	0.318	1.728	0.000	0.000	3.371

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	0	0	37	0	-1	39
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.71	0.00	-0.02	0.75
time (sec)	N/A	0.011	0.007	0.055	0.000	0.743	1.516	0.000	0.000	3.163

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	126	101	143	1318	250	176	929
N.S.	1	1.00	0.81	1.52	1.22	1.72	15.88	3.01	2.12	11.19
time (sec)	N/A	0.022	0.055	0.107	0.271	0.331	0.632	0.002	0.534	7.731

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	154	192	496
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.57	3.20	8.27
time (sec)	N/A	0.014	0.035	0.114	0.283	0.321	0.408	0.002	0.557	4.324

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	83	94	213
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	2.13	2.41	5.46
time (sec)	N/A	0.009	0.025	0.122	0.260	0.309	0.249	0.002	0.378	2.401

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	16	18	0
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	0.89	1.00	0.00
time (sec)	N/A	0.002	0.016	0.112	0.259	0.313	0.029	0.000	0.198	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	B	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	83	0	-1	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.37	0.00	-0.03	1.14
time (sec)	N/A	0.005	0.021	0.028	0.000	0.315	0.644	0.000	0.000	2.866

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	B	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	354	0	-1	75
N.S.	1	1.00	1.00	0.00	0.00	0.00	10.11	0.00	-0.03	2.14
time (sec)	N/A	0.005	0.021	0.044	0.000	0.314	0.959	0.000	0.000	5.614

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	B	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	918	0	-1	150
N.S.	1	1.00	1.00	0.00	0.00	0.00	24.16	0.00	-0.03	3.95
time (sec)	N/A	0.006	0.021	0.030	0.000	0.326	1.990	0.000	0.000	11.581

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	64	77	0	104	0	0	136	0
N.S.	1	1.00	0.58	0.70	0.00	0.95	0.00	0.00	1.24	0.00
time (sec)	N/A	0.028	0.035	0.127	0.000	0.320	0.000	0.000	0.523	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	44	0	64	0	0	80	344
N.S.	1	1.00	0.61	0.69	0.00	1.00	0.00	0.00	1.25	5.38
time (sec)	N/A	0.007	0.029	0.115	0.000	0.319	0.000	0.000	0.447	190.943

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	29	0	33	85	0	29	120
N.S.	1	1.00	0.89	1.04	0.00	1.18	3.04	0.00	1.04	4.29
time (sec)	N/A	0.002	0.021	0.116	0.000	0.322	98.359	0.000	0.349	169.746

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	0	27	0	-1	30
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.69	0.00	-0.03	0.77
time (sec)	N/A	0.007	0.022	0.046	0.000	0.323	148.891	0.000	0.000	154.034

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	32	0	-1	36
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.71	0.00	-0.02	0.80
time (sec)	N/A	0.008	0.020	0.043	0.000	0.310	13.843	0.000	0.000	15.250

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	34	0	-1	36
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	-0.02	0.80
time (sec)	N/A	0.008	0.023	0.054	0.000	0.323	156.365	0.000	0.000	109.859

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	27	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.60	0.00	-0.02	0.58
time (sec)	N/A	0.007	0.079	0.027	0.000	0.328	77.311	0.000	0.000	39.614

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	27	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.60	0.00	-0.02	0.58
time (sec)	N/A	0.007	0.050	0.025	0.000	0.317	3.768	0.000	0.000	5.180

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	26	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.60	0.00	-0.02	0.60
time (sec)	N/A	0.007	0.054	0.026	0.000	0.317	2.351	0.000	0.000	3.867

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	29	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	-0.02	0.60
time (sec)	N/A	0.007	0.088	0.027	0.000	0.323	14.715	0.000	0.000	15.300

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	-1	26
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.58
time (sec)	N/A	0.007	0.068	0.024	0.000	0.331	0.000	0.000	0.000	172.893

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	0	0	37	0	-1	37
N.S.	1	1.00	0.89	0.91	0.00	0.00	1.06	0.00	-0.03	1.06
time (sec)	N/A	0.009	0.018	0.132	0.000	0.317	1.243	0.000	0.000	2.873

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	0	0	0	37	0	-1	36
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.92	0.00	-0.02	0.90
time (sec)	N/A	0.007	0.045	0.060	0.000	0.312	1.267	0.000	0.000	2.860

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	0	0	37	0	-1	39
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.71	0.00	-0.02	0.75
time (sec)	N/A	0.010	0.021	0.040	0.000	0.315	1.553	0.000	0.000	3.151

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	22	32	197	0	19	236
N.S.	1	1.00	1.00	1.05	1.16	1.68	10.37	0.00	1.00	12.42
time (sec)	N/A	0.002	0.027	0.113	0.330	0.322	128.509	0.000	0.504	202.575

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	64	328	0	86	302
N.S.	1	1.00	0.69	0.71	0.00	1.10	5.66	0.00	1.48	5.21
time (sec)	N/A	0.010	0.028	0.121	0.000	0.324	17.455	0.000	0.498	41.602

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	64	328	0	86	302
N.S.	1	1.00	0.69	0.71	0.00	1.10	5.66	0.00	1.48	5.21
time (sec)	N/A	0.009	0.003	0.120	0.000	0.316	17.516	0.000	0.002	18.004

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	33	22	29	25	-1	19
N.S.	1	1.00	0.69	0.60	0.94	0.63	0.83	0.71	-0.03	0.54
time (sec)	N/A	0.008	0.003	0.025	0.263	0.296	0.145	0.001	0.000	1.733

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	31	22	29	25	-1	19
N.S.	1	1.00	0.69	0.60	0.89	0.63	0.83	0.71	-0.03	0.54
time (sec)	N/A	0.007	0.003	0.024	0.259	0.304	0.120	0.000	0.000	1.722

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	28	22	29	25	-1	19
N.S.	1	1.00	0.69	0.60	0.80	0.63	0.83	0.71	-0.03	0.54
time (sec)	N/A	0.006	0.002	0.022	0.265	0.304	0.104	0.000	0.000	1.695

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	19	25	20	27	25	20	18
N.S.	1	1.00	0.67	0.58	0.76	0.61	0.82	0.76	0.61	0.55
time (sec)	N/A	0.006	0.002	0.021	0.264	0.301	0.089	0.000	0.542	1.657

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	17	0	16	22	19	14	14
N.S.	1	1.00	0.89	0.63	0.00	0.59	0.81	0.70	0.52	0.52
time (sec)	N/A	0.003	0.003	0.020	0.000	0.298	0.091	0.000	0.192	1.686

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	20	20	0	19	0	19	-1	0
N.S.	1	1.00	0.71	0.71	0.00	0.68	0.00	0.68	-0.04	0.00
time (sec)	N/A	0.003	0.003	0.024	0.000	0.314	0.000	0.001	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	20	21	0	20	0	21	-1	0
N.S.	1	1.00	0.62	0.66	0.00	0.62	0.00	0.66	-0.03	0.00
time (sec)	N/A	0.005	0.004	0.020	0.000	0.308	0.000	0.001	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	0	18	29	25	28	19
N.S.	1	1.00	0.85	0.73	0.00	0.69	1.12	0.96	1.08	0.73
time (sec)	N/A	0.003	0.003	0.020	0.000	0.306	0.202	0.001	0.144	1.739

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	33	24	29	26	-1	19
N.S.	1	1.00	0.65	0.57	0.89	0.65	0.78	0.70	-0.03	0.51
time (sec)	N/A	0.009	0.004	0.028	0.283	0.299	0.259	0.001	0.000	1.864

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	31	24	29	26	-1	19
N.S.	1	1.00	0.65	0.57	0.84	0.65	0.78	0.70	-0.03	0.51
time (sec)	N/A	0.009	0.004	0.025	0.275	0.296	0.224	0.001	0.000	1.833

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	28	24	29	26	-1	19
N.S.	1	1.00	0.65	0.57	0.76	0.65	0.78	0.70	-0.03	0.51
time (sec)	N/A	0.008	0.004	0.023	0.269	0.304	0.195	0.001	0.000	1.791

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	22	19	25	24	27	26	-1	18
N.S.	1	1.00	0.59	0.51	0.68	0.65	0.73	0.70	-0.03	0.49
time (sec)	N/A	0.007	0.004	0.023	0.256	0.301	0.165	0.000	0.000	1.760

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	18	22	24	24	26	-1	16
N.S.	1	1.00	0.68	0.49	0.59	0.65	0.65	0.70	-0.03	0.43
time (sec)	N/A	0.007	0.002	0.024	0.266	0.313	0.167	0.001	0.000	1.745

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	21	0	22	24	26	20	19
N.S.	1	1.00	0.66	0.60	0.00	0.63	0.69	0.74	0.57	0.54
time (sec)	N/A	0.006	0.002	0.021	0.000	0.296	0.173	0.000	0.267	1.755

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	20	0	18	26	20	14	17
N.S.	1	1.00	0.72	0.69	0.00	0.62	0.90	0.69	0.48	0.59
time (sec)	N/A	0.003	0.002	0.020	0.000	0.299	0.243	0.000	0.222	1.780

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	20	0	21	0	20	-1	0
N.S.	1	1.00	0.70	0.67	0.00	0.70	0.00	0.67	-0.03	0.00
time (sec)	N/A	0.004	0.004	0.023	0.000	0.325	0.000	0.001	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	33	28	29	31	-1	19
N.S.	1	1.00	0.59	0.51	0.80	0.68	0.71	0.76	-0.02	0.46
time (sec)	N/A	0.012	0.005	0.032	0.269	0.306	0.466	0.001	0.000	2.003

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	31	28	29	31	-1	19
N.S.	1	1.00	0.59	0.51	0.76	0.68	0.71	0.76	-0.02	0.46
time (sec)	N/A	0.010	0.005	0.031	0.253	0.301	0.411	0.000	0.000	1.963

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	28	28	29	31	-1	19
N.S.	1	1.00	0.59	0.51	0.68	0.68	0.71	0.76	-0.02	0.46
time (sec)	N/A	0.009	0.004	0.027	0.254	0.303	0.353	0.001	0.000	1.918

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	22	19	25	28	27	31	-1	18
N.S.	1	1.00	0.54	0.46	0.61	0.68	0.66	0.76	-0.02	0.44
time (sec)	N/A	0.008	0.004	0.026	0.262	0.313	0.303	0.001	0.000	1.856

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	22	28	24	31	-1	16
N.S.	1	1.00	0.61	0.44	0.54	0.68	0.59	0.76	-0.02	0.39
time (sec)	N/A	0.008	0.002	0.025	0.272	0.616	0.303	0.001	0.000	1.861

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	21	24	28	24	31	25	19
N.S.	1	1.00	0.56	0.51	0.59	0.68	0.59	0.76	0.61	0.46
time (sec)	N/A	0.007	0.002	0.023	0.284	0.293	0.318	0.001	0.284	1.858

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	0	28	27	31	25	19
N.S.	1	1.00	0.66	0.51	0.00	0.68	0.66	0.76	0.61	0.46
time (sec)	N/A	0.007	0.002	0.023	0.000	0.290	0.387	0.001	0.265	1.897

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	21	0	26	29	31	20	19
N.S.	1	1.00	0.64	0.54	0.00	0.67	0.74	0.79	0.51	0.49
time (sec)	N/A	0.006	0.002	0.021	0.000	0.297	0.382	0.001	0.260	1.902

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	33	25	29	25	-1	19
N.S.	1	1.00	0.69	0.60	0.94	0.71	0.83	0.71	-0.03	0.54
time (sec)	N/A	0.006	0.003	0.025	0.276	0.306	0.246	0.001	0.000	1.770

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	26	23	29	25	23	19
N.S.	1	1.00	0.69	0.60	0.74	0.66	0.83	0.71	0.66	0.54
time (sec)	N/A	0.006	0.003	0.021	0.256	0.299	0.225	0.001	0.251	1.741

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	20	22	19	27	21	19	17
N.S.	1	1.00	0.72	0.62	0.69	0.59	0.84	0.66	0.59	0.53
time (sec)	N/A	0.003	0.001	0.022	0.274	0.296	0.213	0.001	0.221	1.745

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	19	18	20	22	0	22	17	0
N.S.	1	1.00	0.66	0.62	0.69	0.76	0.00	0.76	0.59	0.00
time (sec)	N/A	0.003	0.002	0.030	0.265	0.299	0.000	0.001	0.513	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	18	17	23	0	24	22	0
N.S.	1	1.00	0.85	0.67	0.63	0.85	0.00	0.89	0.81	0.00
time (sec)	N/A	0.005	0.004	0.022	0.271	0.307	0.000	0.001	1.222	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	19	19	21	24	24	25	22
N.S.	1	1.00	0.88	0.73	0.73	0.81	0.92	0.92	0.96	0.85
time (sec)	N/A	0.003	0.004	0.022	0.255	0.300	0.237	0.001	0.157	1.755

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	22	21	19	23	29	26	26	22
N.S.	1	1.00	0.63	0.60	0.54	0.66	0.83	0.74	0.74	0.63
time (sec)	N/A	0.005	0.004	0.023	0.257	0.298	0.262	0.001	0.149	1.792

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	19	23	31	26	26	22
N.S.	1	1.00	0.69	0.60	0.54	0.66	0.89	0.74	0.74	0.63
time (sec)	N/A	0.005	0.008	0.025	0.256	0.299	0.284	0.002	0.149	1.818

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	32	19	27	23	-1	17
N.S.	1	1.00	0.61	0.53	0.84	0.50	0.71	0.61	-0.03	0.45
time (sec)	N/A	0.004	0.003	0.022	0.319	0.297	0.258	0.001	0.000	1.783

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	21	20	23	22	0	24	30	0
N.S.	1	1.00	0.60	0.57	0.66	0.63	0.00	0.69	0.86	0.00
time (sec)	N/A	0.004	0.003	0.025	0.267	0.318	0.000	0.001	0.320	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	21	21	23	0	26	28	0
N.S.	1	1.00	0.67	0.64	0.64	0.70	0.00	0.79	0.85	0.00
time (sec)	N/A	0.005	0.002	0.024	0.249	0.291	0.000	0.001	0.250	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	17	23	21	27	26	25	18
N.S.	1	1.00	0.76	0.59	0.79	0.72	0.93	0.90	0.86	0.62
time (sec)	N/A	0.003	0.002	0.022	0.259	0.293	0.243	0.001	0.149	1.765

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	19	23	26	28	26	16
N.S.	1	1.00	0.61	0.44	0.46	0.56	0.63	0.68	0.63	0.39
time (sec)	N/A	0.006	0.006	0.023	0.271	0.292	0.260	0.001	0.158	1.766

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	19	23	26	28	26	22
N.S.	1	1.00	0.66	0.51	0.46	0.56	0.63	0.68	0.63	0.54
time (sec)	N/A	0.006	0.005	0.027	0.269	0.285	0.280	0.002	0.151	1.781

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	22	21	19	23	29	28	26	22
N.S.	1	1.00	0.54	0.51	0.46	0.56	0.71	0.68	0.63	0.54
time (sec)	N/A	0.006	0.005	0.026	0.260	0.291	0.312	0.001	0.149	1.825

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	19	23	31	28	26	22
N.S.	1	1.00	0.59	0.51	0.46	0.56	0.76	0.68	0.63	0.54
time (sec)	N/A	0.006	0.004	0.028	0.271	0.293	0.351	0.001	0.151	1.854

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	21	24	23	0	31	-1	0
N.S.	1	1.00	0.67	0.64	0.73	0.70	0.00	0.94	-0.03	0.00
time (sec)	N/A	0.005	0.004	0.021	0.301	0.297	0.000	0.001	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	19	26	21	29	27	25	19
N.S.	1	1.00	0.83	0.66	0.90	0.72	1.00	0.93	0.86	0.66
time (sec)	N/A	0.004	0.004	0.021	0.255	0.293	0.323	0.001	0.153	1.846

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	23	23	31	29	26	19
N.S.	1	1.00	0.59	0.51	0.56	0.56	0.76	0.71	0.63	0.46
time (sec)	N/A	0.005	0.003	0.025	0.261	0.292	0.321	0.003	0.151	1.827

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	19	23	23	29	29	26	18
N.S.	1	1.00	0.66	0.46	0.56	0.56	0.71	0.71	0.63	0.44
time (sec)	N/A	0.005	0.002	0.024	0.268	0.289	0.309	0.002	0.160	1.827

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	18	19	23	26	29	26	16
N.S.	1	1.00	0.66	0.44	0.46	0.56	0.63	0.71	0.63	0.39
time (sec)	N/A	0.006	0.006	0.024	0.270	0.289	0.346	0.002	0.158	1.833

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	19	23	26	29	26	22
N.S.	1	1.00	0.66	0.51	0.46	0.56	0.63	0.71	0.63	0.54
time (sec)	N/A	0.006	0.005	0.029	0.273	0.295	0.386	0.001	0.162	1.901

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	22	21	19	23	29	29	26	22
N.S.	1	1.00	0.54	0.51	0.46	0.56	0.71	0.71	0.63	0.54
time (sec)	N/A	0.006	0.006	0.028	0.270	0.290	0.429	0.003	0.159	1.935

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	19	23	31	29	26	22
N.S.	1	1.00	0.59	0.51	0.46	0.56	0.76	0.71	0.63	0.54
time (sec)	N/A	0.006	0.005	0.028	0.275	0.289	0.459	0.002	0.162	1.970

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	54	33	49	39	-1	30
N.S.	1	1.00	0.61	0.56	0.95	0.58	0.86	0.68	-0.02	0.53
time (sec)	N/A	0.011	0.004	0.115	0.270	0.295	0.173	0.001	0.000	1.873

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	52	33	51	40	-1	31
N.S.	1	1.00	0.61	0.56	0.91	0.58	0.89	0.70	-0.02	0.54
time (sec)	N/A	0.010	0.004	0.129	0.267	0.289	0.151	0.001	0.000	1.845

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	49	33	49	39	-1	30
N.S.	1	1.00	0.61	0.56	0.86	0.58	0.86	0.68	-0.02	0.53
time (sec)	N/A	0.009	0.004	0.126	0.274	0.286	0.129	0.001	0.000	1.808

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	33	30	44	31	49	40	-1	29
N.S.	1	1.00	0.60	0.55	0.80	0.56	0.89	0.73	-0.02	0.53
time (sec)	N/A	0.009	0.005	0.112	0.273	0.291	0.112	0.000	0.000	1.801

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	23	0	27	41	32	-1	24
N.S.	1	1.00	0.96	0.88	0.00	1.04	1.58	1.23	-0.04	0.92
time (sec)	N/A	0.003	0.003	0.103	0.000	0.291	0.114	0.000	0.000	1.785

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	33	0	32	0	34	-1	0
N.S.	1	1.00	0.67	0.67	0.00	0.65	0.00	0.69	-0.02	0.00
time (sec)	N/A	0.007	0.006	0.120	0.000	0.292	0.000	0.001	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	31	32	0	31	0	32	-1	0
N.S.	1	1.00	0.63	0.65	0.00	0.63	0.00	0.65	-0.02	0.00
time (sec)	N/A	0.008	0.007	0.119	0.000	0.303	0.000	0.001	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	36	34	0	33	0	39	-1	0
N.S.	1	1.00	0.67	0.63	0.00	0.61	0.00	0.72	-0.02	0.00
time (sec)	N/A	0.009	0.007	0.102	0.000	0.294	0.000	0.001	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	54	36	49	40	-1	30
N.S.	1	1.00	0.58	0.53	0.90	0.60	0.82	0.67	-0.02	0.50
time (sec)	N/A	0.013	0.005	0.108	0.275	0.285	0.305	0.000	0.000	2.008

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	52	36	51	41	-1	31
N.S.	1	1.00	0.58	0.53	0.87	0.60	0.85	0.68	-0.02	0.52
time (sec)	N/A	0.013	0.006	0.105	0.264	0.292	0.265	0.000	0.000	1.981

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	49	36	49	40	-1	30
N.S.	1	1.00	0.58	0.53	0.82	0.60	0.82	0.67	-0.02	0.50
time (sec)	N/A	0.011	0.005	0.113	0.255	0.288	0.231	0.000	0.000	1.929

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	33	30	44	36	49	41	-1	29
N.S.	1	1.00	0.55	0.50	0.73	0.60	0.82	0.68	-0.02	0.48
time (sec)	N/A	0.011	0.005	0.125	0.257	0.285	0.197	0.000	0.000	1.870

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	36	29	40	36	44	40	-1	27
N.S.	1	1.00	0.60	0.48	0.67	0.60	0.73	0.67	-0.02	0.45
time (sec)	N/A	0.010	0.002	0.119	0.272	0.286	0.202	0.001	0.000	1.876

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	34	32	0	34	44	41	-1	29
N.S.	1	1.00	0.59	0.55	0.00	0.59	0.76	0.71	-0.02	0.50
time (sec)	N/A	0.010	0.002	0.109	0.000	0.288	0.206	0.000	0.000	1.878

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	23	0	30	41	33	-1	27
N.S.	1	1.00	0.96	0.85	0.00	1.11	1.52	1.22	-0.04	1.00
time (sec)	N/A	0.003	0.003	0.107	0.000	0.296	0.274	0.000	0.000	1.913

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	34	33	0	35	0	35	-1	0
N.S.	1	1.00	0.65	0.63	0.00	0.67	0.00	0.67	-0.02	0.00
time (sec)	N/A	0.007	0.005	0.105	0.000	0.296	0.000	0.001	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	49	42	49	48	-1	30
N.S.	1	1.00	0.53	0.48	0.74	0.64	0.74	0.73	-0.02	0.45
time (sec)	N/A	0.014	0.006	0.107	0.274	0.292	0.411	0.001	0.000	2.064

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	30	44	42	49	49	-1	29
N.S.	1	1.00	0.50	0.45	0.67	0.64	0.74	0.74	-0.02	0.44
time (sec)	N/A	0.013	0.006	0.103	0.260	0.292	0.355	0.001	0.000	2.001

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	36	29	40	42	44	48	-1	27
N.S.	1	1.00	0.55	0.44	0.61	0.64	0.67	0.73	-0.02	0.41
time (sec)	N/A	0.012	0.003	0.109	0.273	0.292	0.364	0.001	0.000	2.003

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	34	32	40	42	44	49	-1	29
N.S.	1	1.00	0.52	0.48	0.61	0.64	0.67	0.74	-0.02	0.44
time (sec)	N/A	0.011	0.004	0.131	0.267	0.286	0.365	0.001	0.000	2.023

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	0	42	44	48	-1	30
N.S.	1	1.00	0.58	0.48	0.00	0.64	0.67	0.73	-0.02	0.45
time (sec)	N/A	0.011	0.003	0.111	0.000	0.286	0.438	0.001	0.000	1.998

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	32	0	40	49	49	-1	31
N.S.	1	1.00	0.56	0.50	0.00	0.62	0.77	0.77	-0.02	0.48
time (sec)	N/A	0.011	0.002	0.121	0.000	0.288	0.433	0.001	0.000	2.029

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	0	36	46	42	-1	27
N.S.	1	1.00	0.90	0.79	0.00	1.24	1.59	1.45	-0.03	0.93
time (sec)	N/A	0.004	0.004	0.117	0.000	0.290	0.436	0.001	0.000	2.032

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	35	33	0	41	0	43	-1	0
N.S.	1	1.00	0.60	0.57	0.00	0.71	0.00	0.74	-0.02	0.00
time (sec)	N/A	0.008	0.006	0.122	0.000	0.295	0.000	0.001	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	54	36	49	37	-1	30
N.S.	1	1.00	0.61	0.56	0.95	0.63	0.86	0.65	-0.02	0.53
time (sec)	N/A	0.010	0.004	0.108	0.265	0.286	0.269	0.001	0.000	1.925

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	47	34	51	38	-1	31
N.S.	1	1.00	0.61	0.56	0.82	0.60	0.89	0.67	-0.02	0.54
time (sec)	N/A	0.009	0.004	0.104	0.261	0.288	0.253	0.001	0.000	1.877

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	42	30	46	31	-1	27
N.S.	1	1.00	1.00	0.88	1.75	1.25	1.92	1.29	-0.04	1.12
time (sec)	N/A	0.002	0.001	0.121	0.256	0.284	0.232	0.000	0.000	1.844

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	32	31	35	35	0	43	-1	0
N.S.	1	1.00	0.62	0.60	0.67	0.67	0.00	0.83	-0.02	0.00
time (sec)	N/A	0.007	0.003	0.121	0.270	0.299	0.000	0.001	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	29	35	34	0	36	-1	0
N.S.	1	1.00	0.72	0.62	0.74	0.72	0.00	0.77	-0.02	0.00
time (sec)	N/A	0.008	0.006	0.105	0.265	0.307	0.000	0.001	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	34	31	36	0	39	-1	0
N.S.	1	1.00	0.71	0.69	0.63	0.73	0.00	0.80	-0.02	0.00
time (sec)	N/A	0.008	0.006	0.102	0.259	0.302	0.000	0.002	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	33	30	33	32	42	36	33	29
N.S.	1	1.00	1.27	1.15	1.27	1.23	1.62	1.38	1.27	1.12
time (sec)	N/A	0.003	0.007	0.105	0.253	0.297	0.262	0.001	0.183	1.876

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	33	34	51	38	42	32
N.S.	1	1.00	0.61	0.56	0.58	0.60	0.89	0.67	0.74	0.56
time (sec)	N/A	0.008	0.005	0.102	0.264	0.293	0.290	0.001	0.187	1.956

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	23	52	30	46	33	-1	27
N.S.	1	1.00	0.96	0.85	1.93	1.11	1.70	1.22	-0.04	1.00
time (sec)	N/A	0.003	0.003	0.113	0.262	0.287	0.284	0.000	0.000	1.900

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	34	33	45	35	0	45	-1	0
N.S.	1	1.00	0.56	0.54	0.74	0.57	0.00	0.74	-0.02	0.00
time (sec)	N/A	0.008	0.005	0.122	0.293	0.292	0.000	0.001	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	32	42	34	0	38	-1	0
N.S.	1	1.00	0.59	0.57	0.75	0.61	0.00	0.68	-0.02	0.00
time (sec)	N/A	0.009	0.004	0.131	0.259	0.298	0.000	0.001	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	34	32	35	36	0	41	-1	0
N.S.	1	1.00	0.59	0.55	0.60	0.62	0.00	0.71	-0.02	0.00
time (sec)	N/A	0.009	0.003	0.106	0.292	0.295	0.000	0.002	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	36	27	37	32	42	38	33	27
N.S.	1	1.00	1.24	0.93	1.28	1.10	1.45	1.31	1.14	0.93
time (sec)	N/A	0.003	0.007	0.102	0.257	0.290	0.261	0.002	0.191	1.874

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	33	34	46	40	42	32
N.S.	1	1.00	0.58	0.48	0.50	0.52	0.70	0.61	0.64	0.48
time (sec)	N/A	0.010	0.006	0.116	0.264	0.294	0.278	0.001	0.193	1.904

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	32	33	34	46	40	42	29
N.S.	1	1.00	0.50	0.48	0.50	0.52	0.70	0.61	0.64	0.44
time (sec)	N/A	0.010	0.008	0.108	0.258	0.291	0.320	0.001	0.197	1.977

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	33	34	51	40	42	32
N.S.	1	1.00	0.53	0.48	0.50	0.52	0.77	0.61	0.64	0.48
time (sec)	N/A	0.010	0.006	0.122	0.262	0.289	0.353	0.002	0.175	2.017

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	32	45	34	0	47	-1	0
N.S.	1	1.00	0.59	0.57	0.80	0.61	0.00	0.84	-0.02	0.00
time (sec)	N/A	0.009	0.005	0.123	0.267	0.294	0.000	0.002	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	34	38	36	0	46	-1	0
N.S.	1	1.00	0.62	0.59	0.66	0.62	0.00	0.79	-0.02	0.00
time (sec)	N/A	0.009	0.006	0.109	0.281	0.302	0.000	0.002	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	30	44	32	48	39	33	30
N.S.	1	1.00	1.21	1.03	1.52	1.10	1.66	1.34	1.14	1.03
time (sec)	N/A	0.003	0.004	0.101	0.258	0.291	0.315	0.001	0.180	1.931

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	30	37	34	51	41	42	29
N.S.	1	1.00	0.58	0.45	0.56	0.52	0.77	0.62	0.64	0.44
time (sec)	N/A	0.009	0.003	0.128	0.273	0.288	0.316	0.002	0.175	1.901

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	29	37	34	46	41	42	27
N.S.	1	1.00	0.58	0.44	0.56	0.52	0.70	0.62	0.64	0.41
time (sec)	N/A	0.009	0.008	0.122	0.255	0.292	0.344	0.003	0.176	1.930

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	33	34	46	41	42	32
N.S.	1	1.00	0.58	0.48	0.50	0.52	0.70	0.62	0.64	0.48
time (sec)	N/A	0.009	0.006	0.114	0.255	0.293	0.384	0.002	0.180	2.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	32	33	34	46	41	42	29
N.S.	1	1.00	0.50	0.48	0.50	0.52	0.70	0.62	0.64	0.44
time (sec)	N/A	0.009	0.009	0.114	0.259	0.291	0.426	0.002	0.182	2.033

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	33	34	51	41	42	32
N.S.	1	1.00	0.53	0.48	0.50	0.52	0.77	0.62	0.64	0.48
time (sec)	N/A	0.010	0.006	0.103	0.272	0.293	0.466	0.002	0.180	2.074

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	63	128	62	0	88	-1	0
N.S.	1	1.00	0.62	0.62	1.25	0.61	0.00	0.86	-0.01	0.00
time (sec)	N/A	0.024	0.012	0.153	0.275	0.293	0.000	0.001	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	52	52	110	51	0	73	-1	0
N.S.	1	1.00	0.65	0.65	1.38	0.64	0.00	0.91	-0.01	0.00
time (sec)	N/A	0.018	0.010	0.140	0.271	0.301	0.000	0.001	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	40	91	39	0	59	-1	0
N.S.	1	1.00	0.69	0.69	1.57	0.67	0.00	1.02	-0.02	0.00
time (sec)	N/A	0.014	0.008	0.117	0.258	0.292	0.000	0.001	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	29	74	27	0	40	-1	0
N.S.	1	1.00	0.74	0.76	1.95	0.71	0.00	1.05	-0.03	0.00
time (sec)	N/A	0.009	0.005	0.131	0.283	0.291	0.000	0.001	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	21	0	20	0	28	-1	0
N.S.	1	1.00	0.95	0.95	0.00	0.91	0.00	1.27	-0.05	0.00
time (sec)	N/A	0.003	0.003	0.122	0.000	0.290	0.000	0.001	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	26	26	24	64	0	0	-1	0
N.S.	1	1.00	0.62	0.62	0.57	1.52	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.005	0.005	0.130	0.287	0.294	0.000	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	32	33	37	31	0	0	-1	0
N.S.	1	1.00	0.52	0.54	0.61	0.51	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.012	0.008	0.132	0.277	0.299	0.000	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	53	51	52	44	0	0	-1	0
N.S.	1	1.00	0.63	0.61	0.62	0.52	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.010	0.138	0.272	0.301	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	64	63	124	67	0	89	-1	0
N.S.	1	1.00	0.60	0.59	1.16	0.63	0.00	0.83	-0.01	0.00
time (sec)	N/A	0.023	0.009	0.130	0.278	0.290	0.000	0.001	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	53	52	109	55	0	74	-1	0
N.S.	1	1.00	0.63	0.62	1.30	0.65	0.00	0.88	-0.01	0.00
time (sec)	N/A	0.017	0.008	0.113	0.273	0.299	0.000	0.001	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	40	93	42	0	60	-1	0
N.S.	1	1.00	0.69	0.66	1.52	0.69	0.00	0.98	-0.02	0.00
time (sec)	N/A	0.013	0.004	0.111	0.279	0.292	0.000	0.001	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	29	75	29	0	41	-1	0
N.S.	1	1.00	0.75	0.72	1.88	0.72	0.00	1.02	-0.02	0.00
time (sec)	N/A	0.008	0.003	0.129	0.322	0.296	0.000	0.001	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	13	21	0	29	-1	0
N.S.	1	1.00	0.96	0.91	0.57	0.91	0.00	1.26	-0.04	0.00
time (sec)	N/A	0.002	0.003	0.128	0.259	0.294	0.000	0.001	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	26	24	66	0	0	-1	0
N.S.	1	1.00	0.61	0.59	0.55	1.50	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.005	0.005	0.112	0.258	0.298	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	34	33	37	33	0	0	-1	0
N.S.	1	1.00	0.53	0.52	0.58	0.52	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.011	0.008	0.112	0.273	0.292	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	53	51	52	47	0	0	-1	0
N.S.	1	1.00	0.60	0.58	0.59	0.53	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	0.009	0.138	0.294	0.301	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	65	62	66	59	0	0	-1	0
N.S.	1	1.00	0.58	0.55	0.59	0.53	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.017	0.133	0.265	0.299	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	76	74	146	91	0	123	-1	0
N.S.	1	1.00	0.54	0.52	1.03	0.64	0.00	0.87	-0.01	0.00
time (sec)	N/A	0.032	0.015	0.131	0.282	0.295	0.000	0.001	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	65	63	130	77	0	106	-1	0
N.S.	1	1.00	0.56	0.54	1.11	0.66	0.00	0.91	-0.01	0.00
time (sec)	N/A	0.024	0.004	0.147	0.289	0.293	0.000	0.001	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	54	52	114	63	0	88	-1	0
N.S.	1	1.00	0.59	0.57	1.24	0.68	0.00	0.96	-0.01	0.00
time (sec)	N/A	0.018	0.004	0.132	0.296	0.295	0.000	0.001	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	42	40	97	48	0	71	-1	0
N.S.	1	1.00	0.63	0.60	1.45	0.72	0.00	1.06	-0.01	0.00
time (sec)	N/A	0.013	0.004	0.166	0.277	0.292	0.000	0.001	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	29	77	33	0	49	-1	0
N.S.	1	1.00	0.68	0.66	1.75	0.75	0.00	1.11	-0.02	0.00
time (sec)	N/A	0.009	0.003	0.136	0.281	0.288	0.000	0.001	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	21	13	23	0	34	-1	0
N.S.	1	1.00	0.88	0.84	0.52	0.92	0.00	1.36	-0.04	0.00
time (sec)	N/A	0.003	0.003	0.134	0.265	0.295	0.000	0.002	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	28	26	24	70	0	0	-1	0
N.S.	1	1.00	0.58	0.54	0.50	1.46	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.006	0.006	0.140	0.256	0.300	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	34	33	37	37	0	0	-1	0
N.S.	1	1.00	0.49	0.47	0.53	0.53	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	0.008	0.132	0.262	0.298	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	50	142	54	0	86	-1	0
N.S.	1	1.00	0.61	0.60	1.71	0.65	0.00	1.04	-0.01	0.00
time (sec)	N/A	0.017	0.006	0.117	0.273	0.292	0.000	0.001	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	39	38	100	42	0	66	-1	0
N.S.	1	1.00	0.64	0.62	1.64	0.69	0.00	1.08	-0.02	0.00
time (sec)	N/A	0.012	0.008	0.112	0.296	0.293	0.000	0.001	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	27	27	64	30	0	43	-1	0
N.S.	1	1.00	0.69	0.69	1.64	0.77	0.00	1.10	-0.03	0.00
time (sec)	N/A	0.008	0.006	0.129	0.308	0.291	0.000	0.001	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	46	23	0	30	-1	0
N.S.	1	1.00	1.00	0.95	2.30	1.15	0.00	1.50	-0.05	0.00
time (sec)	N/A	0.002	0.002	0.131	0.275	0.300	0.000	0.001	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	24	35	70	0	0	-1	0
N.S.	1	1.00	0.66	0.63	0.92	1.84	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.004	0.003	0.112	0.272	0.303	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	36	30	37	34	0	0	-1	0
N.S.	1	1.00	0.67	0.56	0.69	0.63	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.007	0.132	0.272	0.291	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	52	51	55	47	0	0	-1	0
N.S.	1	1.00	0.68	0.66	0.71	0.61	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	0.007	0.138	0.278	0.300	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	63	62	69	58	0	0	-1	0
N.S.	1	1.00	0.63	0.62	0.69	0.58	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.020	0.008	0.135	0.268	0.296	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	53	52	162	54	0	88	-1	0
N.S.	1	1.00	0.56	0.55	1.71	0.57	0.00	0.93	-0.01	0.00
time (sec)	N/A	0.020	0.007	0.133	0.307	0.293	0.000	0.001	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	41	40	140	42	0	68	-1	0
N.S.	1	1.00	0.59	0.57	2.00	0.60	0.00	0.97	-0.01	0.00
time (sec)	N/A	0.014	0.006	0.133	0.303	0.290	0.000	0.001	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	29	29	116	30	0	45	-1	0
N.S.	1	1.00	0.64	0.64	2.58	0.67	0.00	1.00	-0.02	0.00
time (sec)	N/A	0.009	0.005	0.139	0.284	0.293	0.000	0.001	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	74	23	0	32	-1	0
N.S.	1	1.00	0.96	0.91	3.22	1.00	0.00	1.39	-0.04	0.00
time (sec)	N/A	0.003	0.003	0.129	0.290	0.290	0.000	0.001	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	26	35	70	0	0	-1	0
N.S.	1	1.00	0.61	0.59	0.80	1.59	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.005	0.005	0.131	0.267	0.299	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	35	33	51	34	0	0	-1	0
N.S.	1	1.00	0.56	0.52	0.81	0.54	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.012	0.005	0.114	0.274	0.296	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	51	49	65	47	0	0	-1	0
N.S.	1	1.00	0.57	0.55	0.73	0.53	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.005	0.112	0.268	0.300	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	66	59	69	58	0	0	-1	0
N.S.	1	1.00	0.57	0.51	0.60	0.50	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.010	0.140	0.271	0.296	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	81	88	135	83	0	101	-1	0
N.S.	1	1.00	0.76	0.83	1.27	0.78	0.00	0.95	-0.01	0.00
time (sec)	N/A	0.031	0.017	0.135	0.282	0.297	0.000	0.002	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	70	76	118	72	0	87	-1	0
N.S.	1	1.00	0.82	0.89	1.39	0.85	0.00	1.02	-0.01	0.00
time (sec)	N/A	0.024	0.015	0.115	0.291	0.298	0.000	0.001	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	62	96	57	0	64	-1	0
N.S.	1	1.00	0.82	0.95	1.48	0.88	0.00	0.98	-0.02	0.00
time (sec)	N/A	0.017	0.013	0.118	0.267	0.315	0.000	0.001	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	41	79	38	0	48	-1	0
N.S.	1	1.00	0.77	0.87	1.68	0.81	0.00	1.02	-0.02	0.00
time (sec)	N/A	0.012	0.009	0.135	0.269	0.299	0.000	0.001	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	23	16	23	32	26	22	39
N.S.	1	1.00	0.96	0.96	0.67	0.96	1.33	1.08	0.92	1.62
time (sec)	N/A	0.003	0.004	0.122	0.268	0.287	0.319	0.001	0.165	1.846

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	52	38	42	0	0	-1	0
N.S.	1	1.00	0.69	0.80	0.58	0.65	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.015	0.011	0.115	0.266	0.296	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	74	58	60	0	0	-1	0
N.S.	1	1.00	0.66	0.85	0.67	0.69	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.018	0.113	0.272	0.295	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	82	95	79	77	0	0	-1	0
N.S.	1	1.00	0.73	0.85	0.71	0.69	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	0.015	0.131	0.255	0.301	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	88	132	91	0	102	-1	0
N.S.	1	1.00	0.74	0.79	1.19	0.82	0.00	0.92	-0.01	0.00
time (sec)	N/A	0.027	0.014	0.155	0.273	0.298	0.000	0.001	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	76	115	79	0	88	-1	0
N.S.	1	1.00	0.80	0.85	1.29	0.89	0.00	0.99	-0.01	0.00
time (sec)	N/A	0.021	0.011	0.135	0.270	0.309	0.000	0.002	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	62	98	63	0	65	-1	0
N.S.	1	1.00	0.81	0.91	1.44	0.93	0.00	0.96	-0.01	0.00
time (sec)	N/A	0.015	0.005	0.121	0.285	0.299	0.000	0.001	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	41	80	43	0	49	-1	0
N.S.	1	1.00	0.78	0.84	1.63	0.88	0.00	1.00	-0.02	0.00
time (sec)	N/A	0.010	0.003	0.130	0.275	0.297	0.000	0.001	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	16	24	37	27	24	42
N.S.	1	1.00	0.96	0.92	0.64	0.96	1.48	1.08	0.96	1.68
time (sec)	N/A	0.003	0.004	0.115	0.273	0.293	0.638	0.001	0.148	2.232

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	52	38	47	0	0	-1	0
N.S.	1	1.00	0.68	0.76	0.56	0.69	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	0.010	0.125	0.260	0.293	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	74	58	65	0	0	-1	0
N.S.	1	1.00	0.65	0.81	0.64	0.71	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.015	0.132	0.269	0.304	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	95	79	82	0	0	-1	0
N.S.	1	1.00	0.70	0.81	0.68	0.70	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.014	0.137	0.270	0.297	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	80	86	168	85	0	114	-1	0
N.S.	1	1.00	0.75	0.80	1.57	0.79	0.00	1.07	-0.01	0.00
time (sec)	N/A	0.023	0.010	0.115	0.295	0.293	0.000	0.002	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	74	129	74	0	94	-1	0
N.S.	1	1.00	0.80	0.86	1.50	0.86	0.00	1.09	-0.01	0.00
time (sec)	N/A	0.019	0.008	0.122	0.271	0.296	0.000	0.002	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	60	88	59	0	67	-1	0
N.S.	1	1.00	0.81	0.94	1.38	0.92	0.00	1.05	-0.02	0.00
time (sec)	N/A	0.014	0.008	0.138	0.276	0.298	0.000	0.001	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	39	68	40	0	50	-1	0
N.S.	1	1.00	0.81	0.91	1.58	0.93	0.00	1.16	-0.02	0.00
time (sec)	N/A	0.010	0.006	0.130	0.259	0.294	0.000	0.001	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	21	25	68	26	25	83
N.S.	1	1.00	1.00	0.95	0.95	1.14	3.09	1.18	1.14	3.77
time (sec)	N/A	0.003	0.002	0.121	0.292	0.287	0.441	0.001	0.156	2.079

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	50	61	44	0	0	-1	0
N.S.	1	1.00	0.75	0.85	1.03	0.75	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.012	0.004	0.132	0.275	0.314	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	60	71	57	62	0	0	-1	0
N.S.	1	1.00	0.77	0.91	0.73	0.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.014	0.114	0.256	0.296	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	95	76	79	0	0	-1	0
N.S.	1	1.00	0.79	0.92	0.74	0.77	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.012	0.143	0.268	0.299	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	62	149	63	0	69	-1	0
N.S.	1	1.00	0.74	0.85	2.04	0.86	0.00	0.95	-0.01	0.00
time (sec)	N/A	0.016	0.010	0.130	0.287	0.294	0.000	0.001	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	41	108	44	0	52	-1	0
N.S.	1	1.00	0.76	0.84	2.20	0.90	0.00	1.06	-0.02	0.00
time (sec)	N/A	0.011	0.008	0.140	0.285	0.291	0.000	0.001	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	47	29	73	28	25	89
N.S.	1	1.00	0.96	0.92	1.88	1.16	2.92	1.12	1.00	3.56
time (sec)	N/A	0.003	0.004	0.137	0.283	0.286	0.569	0.001	0.167	2.219

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	52	82	48	0	0	-1	0
N.S.	1	1.00	0.68	0.76	1.21	0.71	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.009	0.116	0.275	0.294	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	59	74	79	66	0	0	-1	0
N.S.	1	1.00	0.66	0.82	0.88	0.73	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.007	0.126	0.273	0.300	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	93	98	83	0	0	-1	0
N.S.	1	1.00	0.68	0.79	0.83	0.70	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.023	0.006	0.135	0.273	0.301	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	97	136	116	153	0	327	214	0
N.S.	1	1.00	0.74	1.04	0.89	1.17	0.00	2.50	1.63	0.00
time (sec)	N/A	0.026	0.043	0.130	0.280	0.304	0.000	0.004	0.348	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	68	83	80	106	0	216	142	0
N.S.	1	1.00	0.71	0.86	0.83	1.10	0.00	2.25	1.48	0.00
time (sec)	N/A	0.020	0.031	0.112	0.261	0.300	0.000	0.003	0.252	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	46	51	63	0	129	85	0
N.S.	1	1.00	0.70	0.73	0.81	1.00	0.00	2.05	1.35	0.00
time (sec)	N/A	0.012	0.021	0.120	0.274	0.300	0.000	0.002	0.222	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	28	30	0	40	31	0
N.S.	1	1.00	0.97	0.97	0.93	1.00	0.00	1.33	1.03	0.00
time (sec)	N/A	0.005	0.010	0.130	0.263	0.297	0.000	0.002	0.228	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.009	0.033	0.000	0.299	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.010	0.026	0.000	0.303	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.012	0.026	0.000	0.305	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	132	199	157	233	0	464	307	0
N.S.	1	1.00	0.78	1.18	0.93	1.38	0.00	2.75	1.82	0.00
time (sec)	N/A	0.041	0.067	0.132	0.260	0.307	0.000	0.004	0.414	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	136	116	164	0	328	219	0
N.S.	1	1.00	0.73	1.01	0.86	1.21	0.00	2.43	1.62	0.00
time (sec)	N/A	0.031	0.033	0.140	0.271	0.305	0.000	0.004	0.320	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	83	80	113	0	0	146	0
N.S.	1	1.00	0.71	0.84	0.81	1.14	0.00	0.00	1.47	0.00
time (sec)	N/A	0.021	0.019	0.135	0.284	0.296	0.000	0.000	0.261	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	46	51	68	0	130	88	0
N.S.	1	1.00	0.71	0.71	0.78	1.05	0.00	2.00	1.35	0.00
time (sec)	N/A	0.014	0.007	0.114	0.271	0.298	0.000	0.002	0.231	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	28	33	0	41	45	0
N.S.	1	1.00	0.97	0.94	0.90	1.06	0.00	1.32	1.45	0.00
time (sec)	N/A	0.005	0.011	0.124	0.286	0.305	0.000	0.002	0.229	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.008	0.026	0.000	0.305	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.008	0.027	0.000	0.313	0.000	0.000	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.009	0.026	0.000	0.305	0.000	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	172	280	203	352	0	693	424	0
N.S.	1	1.00	0.79	1.29	0.94	1.62	0.00	3.19	1.95	0.00
time (sec)	N/A	0.054	0.085	0.145	0.291	0.304	0.000	0.006	0.496	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	133	199	157	265	0	0	319	0
N.S.	1	1.00	0.74	1.11	0.88	1.48	0.00	0.00	1.78	0.00
time (sec)	N/A	0.039	0.015	0.137	0.272	0.306	0.000	0.000	0.379	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	99	136	116	186	0	0	229	0
N.S.	1	1.00	0.69	0.95	0.81	1.30	0.00	0.00	1.60	0.00
time (sec)	N/A	0.030	0.012	0.137	0.268	0.298	0.000	0.000	0.317	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	83	80	127	0	0	154	0
N.S.	1	1.00	0.67	0.79	0.76	1.21	0.00	0.00	1.47	0.00
time (sec)	N/A	0.022	0.029	0.118	0.264	0.302	0.000	0.000	0.265	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	46	46	51	76	0	0	94	0
N.S.	1	1.00	0.67	0.67	0.74	1.10	0.00	0.00	1.36	0.00
time (sec)	N/A	0.014	0.007	0.156	0.263	0.302	0.000	0.000	0.239	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	29	28	37	0	0	49	0
N.S.	1	1.00	0.94	0.88	0.85	1.12	0.00	0.00	1.48	0.00
time (sec)	N/A	0.005	0.012	0.118	0.270	0.299	0.000	0.000	0.231	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.009	0.029	0.000	0.298	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.008	0.031	0.000	0.315	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	96	134	104	158	0	0	186	0
N.S.	1	1.00	0.78	1.09	0.85	1.28	0.00	0.00	1.51	0.00
time (sec)	N/A	0.025	0.023	0.143	0.263	0.312	0.000	0.000	0.369	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	81	83	110	0	0	121	0
N.S.	1	1.00	0.74	0.90	0.92	1.22	0.00	0.00	1.34	0.00
time (sec)	N/A	0.017	0.018	0.135	0.269	0.300	0.000	0.000	0.295	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	44	45	66	0	0	71	0
N.S.	1	1.00	0.73	0.75	0.76	1.12	0.00	0.00	1.20	0.00
time (sec)	N/A	0.012	0.014	0.136	0.273	0.303	0.000	0.000	0.277	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	31	33	0	0	36	0
N.S.	1	1.00	1.00	0.96	1.11	1.18	0.00	0.00	1.29	0.00
time (sec)	N/A	0.004	0.006	0.134	0.268	0.299	0.000	0.000	0.220	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.003	0.027	0.000	0.311	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	48	0	0	0	0	0	-1	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.008	0.029	0.000	0.309	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.009	0.030	0.000	0.303	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	136	104	168	0	0	201	0
N.S.	1	1.00	0.73	1.01	0.77	1.24	0.00	0.00	1.49	0.00
time (sec)	N/A	0.030	0.027	0.137	0.261	0.303	0.000	0.000	0.404	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	83	83	118	0	0	133	0
N.S.	1	1.00	0.70	0.84	0.84	1.19	0.00	0.00	1.34	0.00
time (sec)	N/A	0.021	0.022	0.131	0.275	0.296	0.000	0.000	0.313	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	46	45	72	0	0	80	0
N.S.	1	1.00	0.69	0.71	0.69	1.11	0.00	0.00	1.23	0.00
time (sec)	N/A	0.014	0.016	0.134	0.261	0.301	0.000	0.000	0.289	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	31	37	0	0	42	0
N.S.	1	1.00	0.97	0.94	1.00	1.19	0.00	0.00	1.35	0.00
time (sec)	N/A	0.005	0.010	0.138	0.261	0.301	0.000	0.000	0.229	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.006	0.029	0.000	0.298	0.000	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.005	0.027	0.000	0.302	0.000	0.000	0.000	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.003	0.024	0.000	0.301	0.000	0.000	0.000	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.026	0.026	0.000	0.320	0.000	0.000	0.000	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	99	136	104	168	0	0	201	0
N.S.	1	1.00	0.73	1.01	0.77	1.24	0.00	0.00	1.49	0.00
time (sec)	N/A	0.030	0.024	0.118	0.270	0.304	0.000	0.000	0.410	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	83	83	118	0	0	133	0
N.S.	1	1.00	0.71	0.84	0.84	1.19	0.00	0.00	1.34	0.00
time (sec)	N/A	0.022	0.020	0.145	0.261	0.300	0.000	0.000	0.355	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	46	45	72	0	0	80	0
N.S.	1	1.00	0.71	0.71	0.69	1.11	0.00	0.00	1.23	0.00
time (sec)	N/A	0.014	0.014	0.137	0.273	0.303	0.000	0.000	0.285	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	31	37	0	0	42	0
N.S.	1	1.00	1.00	0.94	1.00	1.19	0.00	0.00	1.35	0.00
time (sec)	N/A	0.005	0.009	0.128	0.267	0.302	0.000	0.000	0.230	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.007	0.028	0.000	0.299	0.000	0.000	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.007	0.026	0.000	0.305	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.007	0.027	0.000	0.306	0.000	0.000	0.000	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.003	0.024	0.000	0.298	0.000	0.000	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	38	40	39	58	0	0	44	0
N.S.	1	1.00	0.58	0.62	0.60	0.89	0.00	0.00	0.68	0.00
time (sec)	N/A	0.022	0.032	0.013	0.280	0.310	0.000	0.000	0.266	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	38	40	39	50	0	0	42	0
N.S.	1	1.00	0.62	0.66	0.64	0.82	0.00	0.00	0.69	0.00
time (sec)	N/A	0.019	0.029	0.011	0.300	0.300	0.000	0.000	0.237	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	40	39	44	0	0	39	0
N.S.	1	1.00	0.64	0.68	0.66	0.75	0.00	0.00	0.66	0.00
time (sec)	N/A	0.018	0.027	0.011	0.284	0.299	0.000	0.000	0.212	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	32	32	36	0	0	30	0
N.S.	1	1.00	0.69	0.67	0.67	0.75	0.00	0.00	0.62	0.00
time (sec)	N/A	0.015	0.024	0.012	0.273	0.301	0.000	0.000	0.211	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	38	40	39	53	0	0	48	0
N.S.	1	1.00	0.58	0.62	0.60	0.82	0.00	0.00	0.74	0.00
time (sec)	N/A	0.026	0.031	0.015	0.298	0.302	0.000	0.000	0.259	0.000

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	38	40	39	53	0	0	47	0
N.S.	1	1.00	0.57	0.60	0.58	0.79	0.00	0.00	0.70	0.00
time (sec)	N/A	0.025	0.032	0.014	0.274	0.301	0.000	0.000	0.275	0.000

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	48	95	64	123	0	0	127	0
N.S.	1	1.00	0.47	0.92	0.62	1.19	0.00	0.00	1.23	0.00
time (sec)	N/A	0.035	0.054	0.105	0.275	0.301	0.000	0.000	0.312	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	48	95	64	105	0	0	121	0
N.S.	1	1.00	0.49	0.98	0.66	1.08	0.00	0.00	1.25	0.00
time (sec)	N/A	0.032	0.049	0.108	0.273	0.307	0.000	0.000	0.279	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	95	64	94	0	0	116	0
N.S.	1	1.00	0.77	1.01	0.68	1.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.030	0.047	0.117	0.274	0.302	0.000	0.000	0.264	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	79	57	85	0	0	62	0
N.S.	1	1.00	0.77	0.98	0.70	1.05	0.00	0.00	0.77	0.00
time (sec)	N/A	0.026	0.043	0.131	0.287	0.301	0.000	0.000	0.258	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	62	83	59	92	0	0	66	0
N.S.	1	1.00	0.67	0.89	0.63	0.99	0.00	0.00	0.71	0.00
time (sec)	N/A	0.033	0.045	0.120	0.271	0.302	0.000	0.000	0.319	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	72	95	64	106	0	0	82	0
N.S.	1	1.00	0.69	0.90	0.61	1.01	0.00	0.00	0.78	0.00
time (sec)	N/A	0.038	0.049	0.112	0.280	0.308	0.000	0.000	0.339	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	0	0	0	0	0	-1	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.028	0.034	0.000	0.307	0.000	0.000	0.000	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0	-1	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.021	0.026	0.033	0.000	0.308	0.000	0.000	0.000	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.020	0.024	0.036	0.000	0.308	0.000	0.000	0.000	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.017	0.023	0.036	0.000	0.305	0.000	0.000	0.000	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	0	0	0	0	0	-1	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.026	0.037	0.000	0.309	0.000	0.000	0.000	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	0	0	0	0	0	-1	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.029	0.035	0.000	0.304	0.000	0.000	0.000	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	32	0	40	0	81	33	0
N.S.	1	1.00	0.97	0.97	0.00	1.21	0.00	2.45	1.00	0.00
time (sec)	N/A	0.008	0.041	0.142	0.000	0.308	0.000	0.009	0.266	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	33	0	40	0	0	34	0
N.S.	1	1.00	1.06	1.03	0.00	1.25	0.00	0.00	1.06	0.00
time (sec)	N/A	0.007	0.044	0.145	0.000	0.304	0.000	0.000	0.235	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	32	0	38	1409	81	33	1406
N.S.	1	1.00	0.97	0.97	0.00	1.15	42.70	2.45	1.00	42.61
time (sec)	N/A	0.008	0.035	0.142	0.000	0.304	147.575	0.008	0.217	141.623

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	31	0	36	0	0	32	0
N.S.	1	1.00	0.93	1.03	0.00	1.20	0.00	0.00	1.07	0.00
time (sec)	N/A	0.006	0.035	0.129	0.000	0.300	0.000	0.000	0.200	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	F	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	27	31	250	0	26	284
N.S.	1	1.00	1.00	0.96	1.04	1.19	9.62	0.00	1.00	10.92
time (sec)	N/A	0.004	0.031	0.125	0.265	0.310	18.541	0.000	0.262	19.399

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	38	0	37	0	0	32	0
N.S.	1	1.00	0.97	1.15	0.00	1.12	0.00	0.00	0.97	0.00
time (sec)	N/A	0.008	0.026	0.148	0.000	0.305	0.000	0.000	0.240	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	0	37	0	0	50	0
N.S.	1	1.00	0.91	0.91	0.00	1.06	0.00	0.00	1.43	0.00
time (sec)	N/A	0.008	0.029	0.149	0.000	0.301	0.000	0.000	0.248	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	33	0	37	0	0	51	0
N.S.	1	1.00	0.97	1.00	0.00	1.12	0.00	0.00	1.55	0.00
time (sec)	N/A	0.008	0.032	0.187	0.000	0.303	0.000	0.000	0.251	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	0	49	0	0	50	0
N.S.	1	1.00	1.00	1.03	0.00	1.29	0.00	0.00	1.32	0.00
time (sec)	N/A	0.008	0.049	0.161	0.000	0.308	0.000	0.000	0.338	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	0	57	0	0	39	0
N.S.	1	1.00	1.00	1.03	0.00	1.46	0.00	0.00	1.00	0.00
time (sec)	N/A	0.008	0.012	0.209	0.000	0.309	0.000	0.000	0.265	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.014	0.026	0.059	0.000	0.305	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	64	64	0	0	0	0	0	-1	0
N.S.	1	0.94	0.94	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	0.007	0.049	0.000	0.308	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	31	31	34	32	27	27
N.S.	1	1.00	1.00	0.94	1.82	1.82	2.00	1.88	1.59	1.59
time (sec)	N/A	0.003	0.001	0.134	0.260	0.291	0.057	0.001	0.050	1.660

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	18	20	20	20	22	16	16
N.S.	1	1.00	0.83	0.78	0.87	0.87	0.87	0.96	0.70	0.70
time (sec)	N/A	0.004	0.001	0.135	0.256	0.288	0.052	0.001	0.034	1.612

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00	1.00
time (sec)	N/A	0.001	0.000	0.149	0.248	0.288	0.048	0.001	0.010	1.538

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	19	17	13	17
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.46	1.31	1.00	1.31
time (sec)	N/A	0.002	0.002	0.149	0.271	0.298	0.050	0.001	0.048	1.628

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	19	19	19	15	15	15
N.S.	1	1.00	1.00	1.07	1.27	1.27	1.27	1.00	1.00	1.00
time (sec)	N/A	0.002	0.003	0.128	0.261	0.291	0.082	0.001	0.037	1.669

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	47	47	53	18	49	37
N.S.	1	1.00	1.00	0.94	2.76	2.76	3.12	1.06	2.88	2.18
time (sec)	N/A	0.002	0.003	0.146	0.257	0.289	0.142	0.001	0.153	1.941

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	61	61	68	18	63	48
N.S.	1	1.00	1.00	0.94	3.59	3.59	4.00	1.06	3.71	2.82
time (sec)	N/A	0.002	0.004	0.143	0.255	0.295	0.174	0.001	0.064	2.068

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	75	75	83	18	77	59
N.S.	1	1.00	1.00	0.94	4.41	4.41	4.88	1.06	4.53	3.47
time (sec)	N/A	0.002	0.004	0.137	0.257	0.291	0.208	0.001	0.054	2.222

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	35	35	34	36	27	27
N.S.	1	1.00	1.00	0.94	2.06	2.06	2.00	2.12	1.59	1.59
time (sec)	N/A	0.003	0.002	0.153	0.266	0.287	0.060	0.001	0.156	1.685

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	18	21	21	20	23	16	16
N.S.	1	1.00	0.83	0.78	0.91	0.91	0.87	1.00	0.70	0.70
time (sec)	N/A	0.004	0.001	0.129	0.267	0.291	0.055	0.001	0.030	1.599

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00	1.00
time (sec)	N/A	0.001	0.000	0.144	0.250	0.291	0.048	0.001	0.009	1.512

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	15	13	17
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.15	1.00	1.31
time (sec)	N/A	0.002	0.002	0.156	0.257	0.291	0.048	0.000	0.142	1.610

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	16	16	12	13	13	13
N.S.	1	1.00	1.00	1.08	1.23	1.23	0.92	1.00	1.00	1.00
time (sec)	N/A	0.002	0.003	0.131	0.255	0.291	0.075	0.001	0.040	1.612

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	36	36	44	12	38	34
N.S.	1	1.00	1.00	0.93	2.57	2.57	3.14	0.86	2.71	2.43
time (sec)	N/A	0.002	0.004	0.151	0.265	0.290	0.137	0.001	0.047	1.847

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	59	59	68	16	61	46
N.S.	1	1.00	1.00	0.93	3.93	3.93	4.53	1.07	4.07	3.07
time (sec)	N/A	0.002	0.004	0.132	0.254	0.289	0.176	0.001	0.050	2.061

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	75	75	83	18	77	59
N.S.	1	1.00	1.00	0.94	4.41	4.41	4.88	1.06	4.53	3.47
time (sec)	N/A	0.003	0.004	0.156	0.251	0.288	0.207	0.001	0.173	2.203

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	27	649	80	212	154	107	245
N.S.	1	1.00	1.04	1.12	27.04	3.33	8.83	6.42	4.46	10.21
time (sec)	N/A	0.006	0.026	0.168	0.289	0.302	0.653	0.006	0.326	2.268

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	114	113	113	124	119	113	90
N.S.	1	1.00	1.00	6.71	6.65	6.65	7.29	7.00	6.65	5.29
time (sec)	N/A	0.003	0.001	0.118	0.262	0.289	0.047	0.001	0.051	2.029

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	100	99	99	110	107	99	79
N.S.	1	1.00	1.00	5.88	5.82	5.82	6.47	6.29	5.82	4.65
time (sec)	N/A	0.003	0.001	0.140	0.272	0.286	0.043	0.000	0.041	1.939

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	72	71	71	78	73	71	66
N.S.	1	1.00	1.00	4.80	4.73	4.73	5.20	4.87	4.73	4.40
time (sec)	N/A	0.002	0.001	0.128	0.254	0.288	0.039	0.000	0.030	1.796

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	48	48	51	47	57	46
N.S.	1	1.00	1.00	0.94	2.82	2.82	3.00	2.76	3.35	2.71
time (sec)	N/A	0.003	0.001	0.150	0.277	0.290	0.049	0.000	0.025	1.755

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	37	37	46	40	43	35
N.S.	1	1.00	1.00	0.94	2.18	2.18	2.71	2.35	2.53	2.06
time (sec)	N/A	0.003	0.001	0.149	0.260	0.285	0.051	0.000	0.048	1.699

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	26	26	29	27	24	24
N.S.	1	1.00	1.00	0.94	1.53	1.53	1.71	1.59	1.41	1.41
time (sec)	N/A	0.003	0.001	0.151	0.279	0.293	0.051	0.001	0.036	1.635

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	15	15	15	15	17	13	13
N.S.	1	1.00	0.89	0.83	0.83	0.83	0.83	0.94	0.72	0.72
time (sec)	N/A	0.003	0.001	0.143	0.257	0.291	0.051	0.001	0.024	1.571

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	6	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.20	1.00	1.00
time (sec)	N/A	0.001	0.000	0.153	0.258	0.286	0.050	0.001	0.008	1.508

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	14	13	17
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.08	1.00	1.31
time (sec)	N/A	0.003	0.001	0.137	0.267	0.295	0.057	0.001	0.040	1.615

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	19	19	17	12	19	15
N.S.	1	1.00	1.00	1.07	1.27	1.27	1.13	0.80	1.27	1.00
time (sec)	N/A	0.003	0.002	0.148	0.265	0.288	0.097	0.001	0.046	1.650

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	33	33	36	15	35	26
N.S.	1	1.00	1.00	0.94	1.94	1.94	2.12	0.88	2.06	1.53
time (sec)	N/A	0.003	0.003	0.125	0.260	0.287	0.127	0.001	0.146	1.797

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	C	A	A	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	6	1	66	14	35	94
N.S.	1	1.00	1.00	0.82	0.21	0.04	2.36	0.50	1.25	3.36
time (sec)	N/A	0.002	0.004	0.158	0.342	0.300	0.589	0.001	0.222	2.388

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	45	44	44	44	49	44	36
N.S.	1	1.00	1.05	1.18	1.16	1.16	1.16	1.29	1.16	0.95
time (sec)	N/A	0.008	0.002	0.135	0.311	0.288	0.037	0.001	0.160	1.687

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	45	44	44	46	52	44	36
N.S.	1	1.00	1.11	1.18	1.16	1.16	1.21	1.37	1.16	0.95
time (sec)	N/A	0.013	0.002	0.106	0.263	0.285	0.035	0.000	0.048	1.683

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	19	18	18
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	1.06	1.00	1.00
time (sec)	N/A	0.003	0.001	0.037	0.276	0.286	0.029	0.001	0.023	1.564

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83	0.83
time (sec)	N/A	0.002	0.000	0.006	0.255	0.263	0.026	0.000	0.017	1.520

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	23	17	25	23	23
N.S.	1	1.00	1.00	0.96	1.04	1.00	0.74	1.09	1.00	1.00
time (sec)	N/A	0.009	0.003	0.112	0.257	0.290	0.068	0.000	0.046	1.651

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	31	37	39	29	33	37	34
N.S.	1	1.00	0.88	0.97	1.16	1.22	0.91	1.03	1.16	1.06
time (sec)	N/A	0.013	0.012	0.121	0.273	0.295	0.097	0.001	0.052	1.822

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	32	30	30	27	15	13	23
N.S.	1	1.00	1.00	2.46	2.31	2.31	2.08	1.15	1.00	1.77
time (sec)	N/A	0.002	0.006	0.138	0.255	0.291	0.120	0.001	0.148	1.763

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	33	54	54	56	25	54	44
N.S.	1	1.00	0.66	0.87	1.42	1.42	1.47	0.66	1.42	1.16
time (sec)	N/A	0.014	0.008	0.112	0.257	0.288	0.154	0.001	0.049	1.987

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	33	67	67	73	22	67	54
N.S.	1	1.00	0.63	0.87	1.76	1.76	1.92	0.58	1.76	1.42
time (sec)	N/A	0.014	0.007	0.128	0.261	0.289	0.194	0.001	0.167	2.103

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	33	84	84	88	27	82	68
N.S.	1	1.00	0.71	0.87	2.21	2.21	2.32	0.71	2.16	1.79
time (sec)	N/A	0.014	0.009	0.136	0.274	0.292	0.226	0.001	0.075	2.250

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	73	72	72	78	85	72	58
N.S.	1	1.00	1.19	1.28	1.26	1.26	1.37	1.49	1.26	1.02
time (sec)	N/A	0.021	0.003	0.168	0.254	0.288	0.040	0.000	0.032	1.803

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	34	34	36	39	31	31
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.95	1.03	0.82	0.82
time (sec)	N/A	0.011	0.001	0.129	0.280	0.291	0.037	0.000	0.040	1.644

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	37	36	36	39	44	36	34
N.S.	1	1.00	1.25	1.16	1.12	1.12	1.22	1.38	1.12	1.06
time (sec)	N/A	0.012	0.001	0.125	0.271	0.287	0.034	0.000	0.049	1.647

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	20	18
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43	1.29
time (sec)	N/A	0.001	0.001	0.125	0.290	0.283	0.030	0.000	0.027	1.578

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	31	35	34	31	50	34	33
N.S.	1	1.00	0.86	0.72	0.81	0.79	0.72	1.16	0.79	0.77
time (sec)	N/A	0.010	0.005	0.176	0.256	0.288	0.083	0.000	0.047	1.735

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	36	46	57	39	50	46	47
N.S.	1	1.00	0.85	0.88	1.12	1.39	0.95	1.22	1.12	1.15
time (sec)	N/A	0.016	0.019	0.155	0.255	0.294	0.105	0.001	0.149	1.890

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	33	48	61	69	54	48	59	61
N.S.	1	1.00	0.63	0.92	1.17	1.33	1.04	0.92	1.13	1.17
time (sec)	N/A	0.019	0.015	0.127	0.266	0.297	0.161	0.001	0.172	2.058

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	48	60	60	61	28	58	50
N.S.	1	1.00	1.11	1.71	2.14	2.14	2.18	1.00	2.07	1.79
time (sec)	N/A	0.003	0.012	0.144	0.263	0.288	0.179	0.001	0.046	2.039

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	48	78	78	85	33	76	65
N.S.	1	1.00	0.62	0.86	1.39	1.39	1.52	0.59	1.36	1.16
time (sec)	N/A	0.017	0.008	0.149	0.258	0.292	0.223	0.001	0.052	2.190

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	38	49	95	95	100	35	91	79
N.S.	1	1.00	0.67	0.86	1.67	1.67	1.75	0.61	1.60	1.39
time (sec)	N/A	0.018	0.012	0.146	0.273	0.293	0.253	0.001	0.187	2.324

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	49	108	108	117	35	104	89
N.S.	1	1.00	0.63	0.83	1.83	1.83	1.98	0.59	1.76	1.51
time (sec)	N/A	0.020	0.010	0.161	0.295	0.289	0.293	0.001	0.108	2.560

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	41	48	52	49	63	48	42
N.S.	1	1.00	0.69	0.67	0.79	0.85	0.80	1.03	0.79	0.69
time (sec)	N/A	0.015	0.004	0.152	0.282	0.287	0.096	0.001	0.049	1.786

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	30	34	38	34	48	32	30
N.S.	1	1.00	0.72	0.70	0.79	0.88	0.79	1.12	0.74	0.70
time (sec)	N/A	0.011	0.004	0.149	0.264	0.297	0.094	0.000	0.148	1.697

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	15	21	18	19
N.S.	1	1.00	1.00	1.06	1.00	1.11	0.83	1.17	1.00	1.06
time (sec)	N/A	0.007	0.002	0.137	0.258	0.295	0.061	0.001	0.039	1.607

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	1.00
time (sec)	N/A	0.001	0.001	0.129	0.268	0.284	0.029	0.000	0.019	1.535

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	35	37	28	22	37	17	31
N.S.	1	1.00	1.00	2.06	2.18	1.65	1.29	2.18	1.00	1.82
time (sec)	N/A	0.006	0.004	0.155	0.301	0.302	0.088	0.001	0.170	1.802

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	51	60	60	48	63	42	55
N.S.	1	1.00	1.26	1.21	1.43	1.43	1.14	1.50	1.00	1.31
time (sec)	N/A	0.021	0.011	0.146	0.253	0.298	0.146	0.001	0.071	2.120

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	67	82	98	71	74	64	83
N.S.	1	1.00	1.03	1.06	1.30	1.56	1.13	1.17	1.02	1.32
time (sec)	N/A	0.027	0.014	0.184	0.265	0.292	0.190	0.001	0.078	2.401

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	45	53	79	51	66	52	53
N.S.	1	1.00	0.85	0.83	0.98	1.46	0.94	1.22	0.96	0.98
time (sec)	N/A	0.022	0.013	0.153	0.261	0.290	0.119	0.001	0.055	1.888

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	34	40	61	36	46	39	42
N.S.	1	1.00	0.85	0.87	1.03	1.56	0.92	1.18	1.00	1.08
time (sec)	N/A	0.015	0.011	0.153	0.261	0.293	0.108	0.001	0.168	1.798

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	28	28	33	24	26	27	29
N.S.	1	1.00	0.85	1.04	1.04	1.22	0.89	0.96	1.00	1.07
time (sec)	N/A	0.010	0.007	0.145	0.268	0.293	0.093	0.001	0.043	1.739

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	9	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	0.75	1.00	1.00
time (sec)	N/A	0.001	0.001	0.128	0.270	0.285	0.065	0.000	0.024	1.601

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	50	55	51	44	56	37	53
N.S.	1	1.00	1.22	1.22	1.34	1.24	1.07	1.37	0.90	1.29
time (sec)	N/A	0.019	0.010	0.150	0.263	0.298	0.144	0.001	0.177	2.092

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	66	64	76	49	68	46	69
N.S.	1	1.00	1.61	1.43	1.39	1.65	1.07	1.48	1.00	1.50
time (sec)	N/A	0.011	0.015	0.165	0.271	0.300	0.135	0.001	0.181	2.169

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	82	108	146	104	93	86	117
N.S.	1	1.00	0.82	0.99	1.30	1.76	1.25	1.12	1.04	1.41
time (sec)	N/A	0.035	0.026	0.178	0.261	0.296	0.250	0.001	0.098	2.778

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	73	113	68	62	287	582	-1	196
N.S.	1	1.00	0.68	1.05	0.63	0.57	2.66	5.39	-0.01	1.81
time (sec)	N/A	0.016	0.086	0.164	0.348	0.306	96.653	0.037	0.000	70.670

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	99	54	57	252	429	-1	175
N.S.	1	1.00	0.77	1.12	0.61	0.65	2.86	4.88	-0.01	1.99
time (sec)	N/A	0.011	0.074	0.154	0.368	0.299	30.389	0.025	0.000	24.888

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	85	40	52	216	298	-1	154
N.S.	1	1.00	0.93	1.25	0.59	0.76	3.18	4.38	-0.01	2.26
time (sec)	N/A	0.007	0.072	0.163	0.361	0.296	9.329	0.019	0.000	9.831

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	52	71	28	47	167	189	-1	129
N.S.	1	1.00	1.08	1.48	0.58	0.98	3.48	3.94	-0.02	2.69
time (sec)	N/A	0.004	0.062	0.159	0.354	0.296	3.396	0.012	0.000	5.088

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	57	17	38	131	103	37	106
N.S.	1	1.00	1.32	2.04	0.61	1.36	4.68	3.68	1.32	3.79
time (sec)	N/A	0.003	0.040	0.155	0.355	0.298	1.614	0.006	0.205	3.422

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	34	42	14	37	99	35	14	90
N.S.	1	1.00	1.62	2.00	0.67	1.76	4.71	1.67	0.67	4.29
time (sec)	N/A	0.003	0.060	0.166	0.359	0.292	0.943	0.004	0.145	2.734

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	64	21	48	70	41	-1	63
N.S.	1	1.00	1.70	2.78	0.91	2.09	3.04	1.78	-0.04	2.74
time (sec)	N/A	0.003	0.035	0.186	0.346	0.293	0.764	0.005	0.000	2.478

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	30	38	33	60	39	34	52
N.S.	1	1.00	1.00	1.50	1.90	1.65	3.00	1.95	1.70	2.60
time (sec)	N/A	0.001	0.049	0.155	0.260	0.299	0.918	0.007	0.270	2.518

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	44	64	53	172	59	50	125
N.S.	1	1.00	0.56	1.07	1.56	1.29	4.20	1.44	1.22	3.05
time (sec)	N/A	0.003	0.043	0.149	0.255	0.298	4.765	0.011	0.244	5.657

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	58	95	70	566	77	64	344
N.S.	1	1.00	0.49	0.95	1.56	1.15	9.28	1.26	1.05	5.64
time (sec)	N/A	0.006	0.049	0.142	0.275	0.297	18.413	0.014	0.269	15.471

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	72	131	85	1561	96	80	867
N.S.	1	1.00	0.43	0.89	1.62	1.05	19.27	1.19	0.99	10.70
time (sec)	N/A	0.009	0.053	0.170	0.262	0.293	61.442	0.017	0.282	43.369

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	86	172	100	3648	119	94	1953
N.S.	1	1.00	0.40	0.85	1.70	0.99	36.12	1.18	0.93	19.34
time (sec)	N/A	0.013	0.060	0.158	0.262	0.296	178.230	0.023	0.292	112.433

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	78	127	66	67	0	647	-1	0
N.S.	1	1.00	0.72	1.17	0.61	0.61	0.00	5.94	-0.01	0.00
time (sec)	N/A	0.014	0.130	0.137	0.348	0.299	0.000	0.037	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	113	52	62	287	583	-1	196
N.S.	1	1.00	0.82	1.27	0.58	0.70	3.22	6.55	-0.01	2.20
time (sec)	N/A	0.010	0.113	0.161	0.350	0.295	114.357	0.035	0.000	98.884

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	99	40	57	248	343	-1	175
N.S.	1	1.00	0.99	1.43	0.58	0.83	3.59	4.97	-0.01	2.54
time (sec)	N/A	0.006	0.093	0.143	0.348	0.298	35.948	0.020	0.000	35.718

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	85	29	46	212	298	-1	154
N.S.	1	1.00	0.94	1.73	0.59	0.94	4.33	6.08	-0.02	3.14
time (sec)	N/A	0.005	0.065	0.141	0.364	0.300	11.655	0.019	0.000	13.502

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	54	71	28	47	163	124	-1	129
N.S.	1	1.00	1.12	1.48	0.58	0.98	3.40	2.58	-0.02	2.69
time (sec)	N/A	0.004	0.062	0.135	0.354	0.310	4.733	0.008	0.000	6.724

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	57	28	40	134	58	-1	112
N.S.	1	1.00	1.04	1.21	0.60	0.85	2.85	1.23	-0.02	2.38
time (sec)	N/A	0.005	0.049	0.159	0.359	0.299	2.356	0.006	0.000	4.365

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	72	42	52	99	61	-1	89
N.S.	1	1.00	1.00	1.76	1.02	1.27	2.41	1.49	-0.02	2.17
time (sec)	N/A	0.005	0.099	0.163	0.347	0.298	1.666	0.008	0.000	3.673

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	76	66	71	498	62	-1	266
N.S.	1	1.00	1.12	1.85	1.61	1.73	12.15	1.51	-0.02	6.49
time (sec)	N/A	0.004	0.059	0.164	0.364	0.297	2.185	0.008	0.000	6.392

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	57	94	52	87	53	50	75
N.S.	1	1.00	1.00	2.85	4.70	2.60	4.35	2.65	2.50	3.75
time (sec)	N/A	0.001	0.042	0.132	0.271	0.295	4.731	0.011	0.252	5.620

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	72	131	69	226	73	64	159
N.S.	1	1.00	0.56	1.76	3.20	1.68	5.51	1.78	1.56	3.88
time (sec)	N/A	0.003	0.050	0.138	0.284	0.301	18.369	0.014	0.268	14.750

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	86	172	86	675	91	80	410
N.S.	1	1.00	0.49	1.41	2.82	1.41	11.07	1.49	1.31	6.72
time (sec)	N/A	0.006	0.054	0.176	0.284	0.297	61.159	0.019	0.319	40.826

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	100	218	101	1751	111	94	975
N.S.	1	1.00	0.43	1.23	2.69	1.25	21.62	1.37	1.16	12.04
time (sec)	N/A	0.010	0.061	0.158	0.271	0.298	177.970	0.022	0.310	107.599

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	114	269	116	0	133	110	0
N.S.	1	1.00	0.40	1.13	2.66	1.15	0.00	1.32	1.09	0.00
time (sec)	N/A	0.015	0.067	0.140	0.278	0.306	0.000	0.028	0.328	0.000

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	155	78	77	0	1174	-1	0
N.S.	1	1.00	0.68	1.19	0.60	0.59	0.00	9.03	-0.01	0.00
time (sec)	N/A	0.019	0.167	0.139	0.346	0.293	0.000	0.061	0.000	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	141	64	72	0	954	-1	0
N.S.	1	1.00	0.75	1.28	0.58	0.65	0.00	8.67	-0.01	0.00
time (sec)	N/A	0.014	0.140	0.140	0.352	0.301	0.000	0.050	0.000	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	127	52	67	0	756	-1	0
N.S.	1	1.00	0.87	1.41	0.58	0.74	0.00	8.40	-0.01	0.00
time (sec)	N/A	0.009	0.119	0.162	0.350	0.302	0.000	0.040	0.000	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F(-1)	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	113	41	51	0	582	-1	196
N.S.	1	1.00	0.73	1.61	0.59	0.73	0.00	8.31	-0.01	2.80
time (sec)	N/A	0.008	0.080	0.152	0.405	0.305	0.000	0.033	0.000	194.105

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	99	40	57	245	255	-1	175
N.S.	1	1.00	0.99	1.43	0.58	0.83	3.55	3.70	-0.01	2.54
time (sec)	N/A	0.006	0.088	0.161	0.345	0.292	61.199	0.014	0.000	64.068

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	85	40	52	212	298	-1	154
N.S.	1	1.00	0.93	1.25	0.59	0.76	3.12	4.38	-0.01	2.26
time (sec)	N/A	0.007	0.064	0.136	0.342	0.295	21.502	0.020	0.000	22.849

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	71	42	47	170	78	-1	133
N.S.	1	1.00	0.84	1.06	0.63	0.70	2.54	1.16	-0.01	1.99
time (sec)	N/A	0.009	0.054	0.141	0.343	0.299	9.511	0.005	0.000	10.517

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	77	56	58	138	84	-1	110
N.S.	1	1.00	0.75	1.18	0.86	0.89	2.12	1.29	-0.02	1.69
time (sec)	N/A	0.008	0.066	0.184	0.384	0.297	6.184	0.006	0.000	7.253

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	84	99	75	575	82	-1	311
N.S.	1	1.00	0.81	1.33	1.57	1.19	9.13	1.30	-0.02	4.94
time (sec)	N/A	0.008	0.130	0.164	0.348	0.297	5.097	0.008	0.000	9.321

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	84	160	91	1606	83	-1	746
N.S.	1	1.00	0.81	1.33	2.54	1.44	25.49	1.32	-0.02	11.84
time (sec)	N/A	0.005	0.056	0.164	0.358	0.291	8.544	0.011	0.000	18.349

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	85	171	66	114	67	64	95
N.S.	1	1.00	1.00	4.25	8.55	3.30	5.70	3.35	3.20	4.75
time (sec)	N/A	0.001	0.045	0.138	0.264	0.301	19.710	0.014	0.279	14.128

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	100	218	83	280	87	80	191
N.S.	1	1.00	0.56	2.44	5.32	2.02	6.83	2.12	1.95	4.66
time (sec)	N/A	0.003	0.051	0.144	0.283	0.294	67.283	0.018	0.303	37.914

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	114	269	100	0	105	94	475
N.S.	1	1.00	0.49	1.87	4.41	1.64	0.00	1.72	1.54	7.79
time (sec)	N/A	0.006	0.057	0.138	0.260	0.297	0.000	0.023	0.310	103.284

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	128	325	115	0	124	110	0
N.S.	1	1.00	0.43	1.58	4.01	1.42	0.00	1.53	1.36	0.00
time (sec)	N/A	0.010	0.061	0.161	0.272	0.304	0.000	0.028	0.315	0.000

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	142	386	130	0	147	124	0
N.S.	1	1.00	0.40	1.41	3.82	1.29	0.00	1.46	1.23	0.00
time (sec)	N/A	0.014	0.071	0.160	0.262	0.296	0.000	0.034	0.352	0.000

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	F(-1)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	45	156	452	145	0	167	140	0
N.S.	1	1.00	0.37	1.29	3.74	1.20	0.00	1.38	1.16	0.00
time (sec)	N/A	0.019	0.076	0.176	0.274	0.296	0.000	0.043	0.370	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	98	42	55	88	71	-1	0
N.S.	1	1.00	1.00	1.53	0.66	0.86	1.38	1.11	-0.02	0.00
time (sec)	N/A	0.009	0.087	0.152	0.372	0.301	16.284	0.005	0.000	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	120	42	48	76	47	55	0
N.S.	1	1.00	1.11	1.94	0.68	0.77	1.23	0.76	0.89	0.00
time (sec)	N/A	0.024	0.163	0.155	0.343	0.299	3.114	0.007	0.151	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	85	56	52	197	107	-1	157
N.S.	1	1.00	0.72	0.98	0.64	0.60	2.26	1.23	-0.01	1.80
time (sec)	N/A	0.014	0.058	0.138	0.344	0.300	16.461	0.009	0.000	14.283

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	71	42	47	173	83	-1	133
N.S.	1	1.00	0.84	1.06	0.63	0.70	2.58	1.24	-0.01	1.99
time (sec)	N/A	0.008	0.057	0.162	0.361	0.305	4.536	0.007	0.000	5.903

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	57	28	40	138	61	-1	112
N.S.	1	1.00	1.04	1.21	0.60	0.85	2.94	1.30	-0.02	2.38
time (sec)	N/A	0.005	0.049	0.155	0.348	0.296	1.525	0.006	0.000	3.600

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	41	12	36	99	36	12	89
N.S.	1	1.00	1.60	2.05	0.60	1.80	4.95	1.80	0.60	4.45
time (sec)	N/A	0.003	0.057	0.159	0.354	0.299	0.795	0.004	0.119	2.955

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	B	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	14	27	2	22	39	19	22	35
N.S.	1	1.00	7.00	13.50	1.00	11.00	19.50	9.50	11.00	17.50
time (sec)	N/A	0.001	0.018	0.136	0.367	0.298	0.488	0.001	0.078	2.313

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	23	31	64	13	48
N.S.	1	1.00	1.00	0.82	0.94	1.35	1.82	3.76	0.76	2.82
time (sec)	N/A	0.003	0.014	0.132	0.347	0.303	0.494	0.002	0.283	2.664

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	30	38	39	128	149	43	106
N.S.	1	1.00	0.56	0.73	0.93	0.95	3.12	3.63	1.05	2.59
time (sec)	N/A	0.003	0.039	0.134	0.354	0.297	1.332	0.005	0.309	3.873

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	44	64	56	303	222	55	211
N.S.	1	1.00	0.49	0.72	1.05	0.92	4.97	3.64	0.90	3.46
time (sec)	N/A	0.006	0.045	0.145	0.349	0.296	5.540	0.007	0.324	8.258

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	58	95	71	542	293	67	350
N.S.	1	1.00	0.43	0.72	1.17	0.88	6.69	3.62	0.83	4.32
time (sec)	N/A	0.009	0.051	0.135	0.354	0.294	19.201	0.011	0.345	20.022

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	72	131	86	850	366	80	525
N.S.	1	1.00	0.40	0.71	1.30	0.85	8.42	3.62	0.79	5.20
time (sec)	N/A	0.013	0.055	0.158	0.351	0.297	57.544	0.015	0.363	49.641

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	56	84	70	65	206	108	-1	154
N.S.	1	1.00	0.66	0.99	0.82	0.76	2.42	1.27	-0.01	1.81
time (sec)	N/A	0.011	0.079	0.143	0.352	0.306	16.722	0.008	0.000	14.866

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	77	56	58	167	86	-1	129
N.S.	1	1.00	0.75	1.18	0.86	0.89	2.57	1.32	-0.02	1.98
time (sec)	N/A	0.008	0.065	0.161	0.347	0.294	4.888	0.006	0.000	6.235

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	43	71	41	53	131	61	-1	105
N.S.	1	1.00	1.05	1.73	1.00	1.29	3.20	1.49	-0.02	2.56
time (sec)	N/A	0.005	0.090	0.157	0.363	0.298	1.379	0.007	0.000	3.420

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	67	21	50	102	41	-1	77
N.S.	1	1.00	1.70	2.91	0.91	2.17	4.43	1.78	-0.04	3.35
time (sec)	N/A	0.003	0.033	0.136	0.343	0.303	0.737	0.005	0.000	2.774

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	23	31	21	14	39
N.S.	1	1.00	1.00	0.83	0.89	1.28	1.72	1.17	0.78	2.17
time (sec)	N/A	0.001	0.014	0.138	0.337	0.295	0.506	0.002	0.360	2.246

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	13	29	11	22	63	87	14	64
N.S.	1	1.00	0.72	1.61	0.61	1.22	3.50	4.83	0.78	3.56
time (sec)	N/A	0.001	0.027	0.154	0.267	0.296	0.973	0.003	0.305	3.010

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	44	40	54	160	171	42	133
N.S.	1	1.00	0.71	1.05	0.95	1.29	3.81	4.07	1.00	3.17
time (sec)	N/A	0.003	0.048	0.136	0.269	0.293	3.467	0.007	0.322	5.373

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	33	58	79	59	284	243	55	206
N.S.	1	1.00	0.53	0.94	1.27	0.95	4.58	3.92	0.89	3.32
time (sec)	N/A	0.006	0.053	0.162	0.255	0.300	13.056	0.009	0.337	13.122

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	40	72	134	86	425	315	68	290
N.S.	1	1.00	0.49	0.88	1.63	1.05	5.18	3.84	0.83	3.54
time (sec)	N/A	0.010	0.056	0.145	0.266	0.294	42.082	0.015	0.355	34.250

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	45	86	201	91	593	387	80	388
N.S.	1	1.00	0.44	0.84	1.97	0.89	5.81	3.79	0.78	3.80
time (sec)	N/A	0.013	0.062	0.139	0.270	0.291	117.565	0.021	0.363	88.366

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	89	125	85	248	127	-1	193
N.S.	1	1.00	0.59	0.86	1.21	0.83	2.41	1.23	-0.01	1.87
time (sec)	N/A	0.015	0.091	0.167	0.339	0.297	53.135	0.012	0.000	38.272

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	84	111	81	212	108	-1	172
N.S.	1	1.00	0.64	0.97	1.28	0.93	2.44	1.24	-0.01	1.98
time (sec)	N/A	0.012	0.078	0.161	0.344	0.299	16.625	0.010	0.000	14.597

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	79	98	75	162	86	-1	171
N.S.	1	1.00	0.81	1.25	1.56	1.19	2.57	1.37	-0.02	2.71
time (sec)	N/A	0.008	0.122	0.170	0.347	0.295	4.835	0.010	0.000	6.467

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	73	66	71	128	65	-1	137
N.S.	1	1.00	1.12	1.78	1.61	1.73	3.12	1.59	-0.02	3.34
time (sec)	N/A	0.004	0.052	0.163	0.356	0.296	2.057	0.008	0.000	4.146

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	30	38	37	66	41	32	66
N.S.	1	1.00	1.00	1.50	1.90	1.85	3.30	2.05	1.60	3.30
time (sec)	N/A	0.001	0.045	0.163	0.263	0.291	0.948	0.007	0.263	2.929

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	30	38	38	66	47	33	68
N.S.	1	1.00	0.56	0.73	0.93	0.93	1.61	1.15	0.80	1.66
time (sec)	N/A	0.003	0.037	0.156	0.345	0.296	1.371	0.004	0.310	3.421

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	30	43	38	49	167	111	48	133
N.S.	1	1.00	0.52	0.74	0.66	0.84	2.88	1.91	0.83	2.29
time (sec)	N/A	0.006	0.048	0.160	0.263	0.297	3.486	0.005	0.338	5.456

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	57	25	35	280	197	41	199
N.S.	1	1.00	0.56	1.33	0.58	0.81	6.51	4.58	0.95	4.63
time (sec)	N/A	0.003	0.040	0.154	0.272	0.290	6.713	0.007	0.365	8.559

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	40	72	52	84	425	269	75	285
N.S.	1	1.00	0.63	1.14	0.83	1.33	6.75	4.27	1.19	4.52
time (sec)	N/A	0.006	0.059	0.157	0.263	0.294	23.860	0.012	0.377	22.168

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	86	91	101	593	341	86	388
N.S.	1	1.00	0.54	1.04	1.10	1.22	7.14	4.11	1.04	4.67
time (sec)	N/A	0.010	0.065	0.164	0.271	0.297	71.344	0.017	0.413	56.745

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	50	100	146	114	0	414	99	495
N.S.	1	1.00	0.49	0.97	1.42	1.11	0.00	4.02	0.96	4.81
time (sec)	N/A	0.014	0.068	0.161	0.264	0.304	0.000	0.025	0.423	167.482

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	86	198	72	201	0	1165	-1	0
N.S.	1	1.00	0.68	1.57	0.57	1.60	0.00	9.25	-0.01	0.00
time (sec)	N/A	0.039	0.226	0.186	0.360	0.303	0.000	0.113	0.000	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	150	50	155	0	623	-1	0
N.S.	1	1.00	0.82	1.56	0.52	1.61	0.00	6.49	-0.01	0.00
time (sec)	N/A	0.026	0.162	0.178	0.334	0.307	0.000	0.065	0.000	0.000

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	102	28	127	0	231	59	0
N.S.	1	1.00	1.03	1.52	0.42	1.90	0.00	3.45	0.88	0.00
time (sec)	N/A	0.021	0.162	0.139	0.357	0.309	0.000	0.025	0.300	0.000

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	47	57	8	101	85	62	44	63
N.S.	1	1.00	1.09	1.33	0.19	2.35	1.98	1.44	1.02	1.47
time (sec)	N/A	0.017	0.044	0.137	0.355	0.304	13.199	0.009	0.175	15.525

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	47	21	39	82	131	23	63
N.S.	1	1.00	1.00	1.74	0.78	1.44	3.04	4.85	0.85	2.33
time (sec)	N/A	0.003	0.059	0.143	0.284	0.291	2.247	0.011	0.391	4.447

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	105	45	57	82	303	62	63
N.S.	1	1.00	0.69	1.72	0.74	0.93	1.34	4.97	1.02	1.03
time (sec)	N/A	0.007	0.072	0.141	0.277	0.293	8.481	0.035	0.415	9.619

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	49	163	67	74	85	459	50	63
N.S.	1	1.00	0.54	1.79	0.74	0.81	0.93	5.04	0.55	0.69
time (sec)	N/A	0.012	0.079	0.157	0.268	0.298	46.539	0.072	0.442	38.959

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	54	221	89	89	0	607	66	63
N.S.	1	1.00	0.45	1.83	0.74	0.74	0.00	5.02	0.55	0.52
time (sec)	N/A	0.020	0.086	0.137	0.281	0.302	0.000	0.180	0.478	188.872

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	103	242	89	232	0	951	-1	0
N.S.	1	1.00	0.76	1.79	0.66	1.72	0.00	7.04	-0.01	0.00
time (sec)	N/A	0.037	0.167	0.161	0.360	0.315	0.000	0.107	0.000	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	92	184	63	193	0	516	-1	0
N.S.	1	1.00	0.90	1.80	0.62	1.89	0.00	5.06	-0.01	0.00
time (sec)	N/A	0.025	0.123	0.172	0.364	0.307	0.000	0.058	0.000	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	78	126	39	159	0	199	72	0
N.S.	1	1.00	1.15	1.85	0.57	2.34	0.00	2.93	1.06	0.00
time (sec)	N/A	0.019	0.071	0.176	0.358	0.321	0.000	0.023	0.203	0.000

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	71	14	108	90	49	53	76
N.S.	1	1.00	1.26	1.87	0.37	2.84	2.37	1.29	1.39	2.00
time (sec)	N/A	0.014	0.044	0.189	0.345	0.307	13.413	0.009	0.177	15.774

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	59	25	45	94	112	26	79
N.S.	1	1.00	0.97	1.97	0.83	1.50	3.13	3.73	0.87	2.63
time (sec)	N/A	0.003	0.059	0.149	0.291	0.290	2.514	0.011	0.497	4.893

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	46	129	53	72	94	256	80	79
N.S.	1	1.00	0.69	1.93	0.79	1.07	1.40	3.82	1.19	1.18
time (sec)	N/A	0.007	0.071	0.145	0.269	0.297	9.477	0.030	0.584	10.811

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	202	79	98	97	389	111	79
N.S.	1	1.00	0.57	2.02	0.79	0.98	0.97	3.89	1.11	0.79
time (sec)	N/A	0.014	0.080	0.167	0.284	0.307	48.593	0.070	0.650	41.016

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	76	275	105	122	0	521	170	79
N.S.	1	1.00	0.57	2.07	0.79	0.92	0.00	3.92	1.28	0.59
time (sec)	N/A	0.022	0.091	0.151	0.266	0.332	0.000	0.148	0.714	190.343

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	193	134	46	65	0	690	-1	0
N.S.	1	1.00	1.93	1.34	0.46	0.65	0.00	6.90	-0.01	0.00
time (sec)	N/A	0.014	1.107	0.175	0.347	0.303	0.000	0.044	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	179	102	34	60	0	358	-1	0
N.S.	1	1.00	2.42	1.38	0.46	0.81	0.00	4.84	-0.01	0.00
time (sec)	N/A	0.008	0.809	0.144	0.386	0.290	0.000	0.024	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	108	70	22	52	187	125	44	129
N.S.	1	1.00	2.51	1.63	0.51	1.21	4.35	2.91	1.02	3.00
time (sec)	N/A	0.004	0.659	0.183	0.351	0.297	2.150	0.006	0.256	4.327

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	27	37	9	28	41	32	40	38
N.S.	1	1.00	2.08	2.85	0.69	2.15	3.15	2.46	3.08	2.92
time (sec)	N/A	0.002	0.043	0.161	0.346	0.298	1.060	0.002	0.051	2.798

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	59	34	12	26	156	112	24	90
N.S.	1	1.00	2.11	1.21	0.43	0.93	5.57	4.00	0.86	3.21
time (sec)	N/A	0.002	0.686	0.151	0.266	0.292	44.735	0.004	0.461	37.423

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-2)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	107	66	25	39	0	232	49	0
N.S.	1	1.00	1.88	1.16	0.44	0.68	0.00	4.07	0.86	0.00
time (sec)	N/A	0.004	0.803	0.148	0.259	0.293	0.000	0.008	0.311	0.000

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	149	98	37	49	0	339	66	0
N.S.	1	1.00	1.75	1.15	0.44	0.58	0.00	3.99	0.78	0.00
time (sec)	N/A	0.008	1.052	0.138	0.266	0.291	0.000	0.015	0.452	0.000

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	89	67	62	199	275	-1	144
N.S.	1	1.00	0.90	0.98	0.74	0.68	2.19	3.02	-0.01	1.58
time (sec)	N/A	0.017	0.084	0.164	0.379	0.298	12.089	0.015	0.000	13.978

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	72	61	38	52	124	93	41	100
N.S.	1	1.00	1.41	1.20	0.75	1.02	2.43	1.82	0.80	1.96
time (sec)	N/A	0.008	0.054	0.170	0.351	0.300	1.878	0.005	0.207	4.015

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	44	31	6	32	26	12	31	25
N.S.	1	1.00	5.50	3.88	0.75	4.00	3.25	1.50	3.88	3.12
time (sec)	N/A	0.003	0.028	0.167	0.340	0.297	0.788	0.002	0.178	2.560

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	21	30	30	29	100	71	32	78
N.S.	1	1.00	0.57	0.81	0.81	0.78	2.70	1.92	0.86	2.11
time (sec)	N/A	0.003	0.032	0.161	0.266	0.301	1.283	0.004	0.255	3.253

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	43	58	59	49	282	173	69	206
N.S.	1	1.00	0.54	0.73	0.75	0.62	3.57	2.19	0.87	2.61
time (sec)	N/A	0.010	0.043	0.167	0.271	0.307	7.052	0.007	0.370	8.753

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	16	30	12	22	71	87	22	66
N.S.	1	1.00	0.76	1.43	0.57	1.05	3.38	4.14	1.05	3.14
time (sec)	N/A	0.001	0.032	0.163	0.292	0.295	0.990	0.003	0.364	3.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	42	15	29	73	103	26	69
N.S.	1	1.00	0.79	1.75	0.62	1.21	3.04	4.29	1.08	2.88
time (sec)	N/A	0.002	0.040	0.149	0.258	0.292	2.396	0.004	0.463	4.266

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	21	30	12	22	92	92	22	75
N.S.	1	1.00	0.81	1.15	0.46	0.85	3.54	3.54	0.85	2.88
time (sec)	N/A	0.001	0.090	0.141	0.273	0.289	9.941	0.003	0.374	10.338

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	19	42	15	29	83	108	26	72
N.S.	1	1.00	0.66	1.45	0.52	1.00	2.86	3.72	0.90	2.48
time (sec)	N/A	0.002	0.155	0.179	0.289	0.292	12.053	0.003	0.321	12.461

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	76	39	108	88	47	56	76
N.S.	1	1.00	1.00	1.95	1.00	2.77	2.26	1.21	1.44	1.95
time (sec)	N/A	0.018	0.064	0.168	0.278	0.307	13.178	0.009	0.218	15.369

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	42	0	0	452	0	0	-1	0
N.S.	1	1.00	0.17	0.00	0.00	1.88	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.190	0.086	0.055	0.000	0.312	0.000	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	70	104	0	0	0	0	-1	0
N.S.	1	1.00	0.49	0.72	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.025	10.038	0.224	0.000	0.304	0.000	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	70	94	0	0	0	0	-1	0
N.S.	1	1.00	0.66	0.89	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.019	0.169	0.000	0.322	0.000	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	102	0	-1	74
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.44	0.00	-0.01	1.04
time (sec)	N/A	0.009	10.023	0.050	0.000	0.311	1.950	0.000	0.000	3.938

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	94	0	0	0	0	-1	0
N.S.	1	1.00	0.87	1.21	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	10.021	0.169	0.000	0.309	0.000	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0	-1	0
N.S.	1	1.00	0.85	1.28	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	10.026	0.184	0.000	0.305	0.000	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	113	0	0	0	0	-1	0
N.S.	1	1.00	0.61	0.98	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	10.031	0.175	0.000	0.303	0.000	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	70	114	0	0	0	0	-1	0
N.S.	1	1.00	0.47	0.77	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	10.024	0.201	0.000	0.310	0.000	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	111	479	0	194	0	0	-1	0
N.S.	1	1.00	0.43	1.87	0.00	0.76	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.125	0.346	1.432	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	77	0	0	221	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.95	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.098	0.148	0.050	0.000	0.317	0.000	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	0	31	0	0	38	0
N.S.	1	1.00	1.00	0.94	0.00	0.94	0.00	0.00	1.15	0.00
time (sec)	N/A	0.003	0.077	0.159	0.000	0.294	0.000	0.000	0.549	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	44	0	0	46	0
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	0.69	0.00
time (sec)	N/A	0.007	0.083	0.146	0.000	0.311	0.000	0.000	0.666	0.000

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	50	0	57	0	0	51	0
N.S.	1	1.00	0.52	0.50	0.00	0.57	0.00	0.00	0.51	0.00
time (sec)	N/A	0.012	0.089	0.162	0.000	0.304	0.000	0.000	0.752	0.000

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	57	55	0	68	0	0	57	0
N.S.	1	1.00	0.43	0.41	0.00	0.51	0.00	0.00	0.43	0.00
time (sec)	N/A	0.020	0.094	0.170	0.000	0.302	0.000	0.000	0.794	0.000

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	111	465	0	198	0	0	-1	0
N.S.	1	1.00	0.43	1.82	0.00	0.77	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.108	0.407	1.604	0.000	0.305	0.000	0.000	0.000	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	77	0	0	221	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.95	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.096	0.116	0.053	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	0	31	0	0	-1	0
N.S.	1	1.00	1.00	1.00	0.00	1.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.002	0.041	0.160	0.000	0.298	0.000	0.000	0.000	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	44	0	0	-1	0
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.007	0.076	0.151	0.000	0.299	0.000	0.000	0.000	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	50	0	57	0	0	-1	0
N.S.	1	1.00	0.52	0.50	0.00	0.57	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	0.079	0.159	0.000	0.298	0.000	0.000	0.000	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.021	0.013	0.000	0.304	0.000	0.000	0.000	0.000

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	10.022	0.009	0.000	0.301	0.000	0.000	0.000	0.000

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	68	0	0	0	100	0	-1	73
N.S.	1	1.00	1.58	0.00	0.00	0.00	2.33	0.00	-0.02	1.70
time (sec)	N/A	0.006	10.020	0.045	0.000	0.313	2.559	0.000	0.000	4.476

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	10.017	0.020	0.000	0.303	0.000	0.000	0.000	0.000

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.036	0.019	0.000	0.303	0.000	0.000	0.000	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	121	465	0	235	0	0	-1	0
N.S.	1	1.00	0.42	1.60	0.00	0.81	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.122	0.644	1.608	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	112	455	0	298	0	0	-1	0
N.S.	1	1.00	0.42	1.71	0.00	1.12	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.096	0.192	1.305	0.000	0.315	0.000	0.000	0.000	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	0	31	0	0	-1	0
N.S.	1	1.00	1.00	0.94	0.00	0.94	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.002	0.054	0.153	0.000	0.298	0.000	0.000	0.000	0.000

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	33	0	36	0	0	-1	0
N.S.	1	1.00	0.69	0.51	0.00	0.55	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.007	0.074	0.147	0.000	0.294	0.000	0.000	0.000	0.000

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	44	0	56	0	0	-1	0
N.S.	1	1.00	0.52	0.44	0.00	0.56	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	0.080	0.153	0.000	0.300	0.000	0.000	0.000	0.000

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.024	10.037	0.016	0.000	0.309	0.000	0.000	0.000	0.000

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.026	0.014	0.000	0.313	0.000	0.000	0.000	0.000

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	10.029	0.015	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	0	0	0	0	0	-1	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	10.015	0.017	0.000	0.302	0.000	0.000	0.000	0.000

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	95	0	-1	74
N.S.	1	1.00	0.86	0.00	0.00	0.00	1.17	0.00	-0.01	0.91
time (sec)	N/A	0.011	10.017	0.014	0.000	0.303	26.799	0.000	0.000	23.104

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.020	0.019	0.000	0.318	0.000	0.000	0.000	0.000

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F(-2)	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	70	0	0	0	0	0	-1	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.025	10.027	0.022	0.000	0.310	0.000	0.000	0.000	0.000

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	70	96	0	0	0	0	-1	0
N.S.	1	1.00	0.51	0.70	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.023	10.023	0.167	0.000	0.308	0.000	0.000	0.000	0.000

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	88	0	0	0	0	-1	0
N.S.	1	1.00	0.69	0.86	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.025	0.167	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	94	0	0	0	0	-1	0
N.S.	1	1.00	0.90	1.21	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	10.017	0.173	0.000	0.313	0.000	0.000	0.000	0.000

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	68	91	0	0	97	0	-1	72
N.S.	1	1.00	1.48	1.98	0.00	0.00	2.11	0.00	-0.02	1.57
time (sec)	N/A	0.006	10.018	0.199	0.000	0.304	6.481	0.000	0.000	7.735

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	107	0	0	0	0	-1	0
N.S.	1	1.00	0.85	1.30	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	10.021	0.179	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F(-2)	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	113	0	0	0	0	-1	0
N.S.	1	1.00	0.61	0.98	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	10.021	0.181	0.000	0.304	0.000	0.000	0.000	0.000

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	117	480	0	233	0	0	-1	0
N.S.	1	1.00	0.41	1.67	0.00	0.81	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.121	0.542	1.569	0.000	0.302	0.000	0.000	0.000	0.000

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	109	478	0	296	0	0	-1	0
N.S.	1	1.00	0.41	1.81	0.00	1.12	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.093	0.187	1.149	0.000	0.316	0.000	0.000	0.000	0.000

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	0	31	0	0	27	0
N.S.	1	1.00	1.00	1.00	0.00	1.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.002	0.045	0.184	0.000	0.296	0.000	0.000	1.161	0.000

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	33	0	36	0	0	40	0
N.S.	1	1.00	0.67	0.49	0.00	0.54	0.00	0.00	0.60	0.00
time (sec)	N/A	0.007	0.096	0.149	0.000	0.299	0.000	0.000	0.597	0.000

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	44	0	56	0	0	46	0
N.S.	1	1.00	0.52	0.44	0.00	0.56	0.00	0.00	0.46	0.00
time (sec)	N/A	0.013	0.104	0.150	0.000	0.296	0.000	0.000	0.764	0.000

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	70	101	0	0	0	0	-1	0
N.S.	1	1.00	0.50	0.72	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	10.032	0.177	0.000	0.311	0.000	0.000	0.000	0.000

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	107	0	0	0	0	-1	0
N.S.	1	1.00	0.61	0.93	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	10.024	0.211	0.000	0.308	0.000	0.000	0.000	0.000

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0	-1	0
N.S.	1	1.00	0.85	1.28	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.012	10.017	0.196	0.000	0.309	0.000	0.000	0.000	0.000

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	107	0	0	0	0	-1	0
N.S.	1	1.00	0.83	1.30	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	10.037	0.201	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	A	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	70	0	0	0	95	0	-1	74
N.S.	1	1.00	0.80	0.00	0.00	0.00	1.08	0.00	-0.01	0.84
time (sec)	N/A	0.012	10.024	0.067	0.000	0.304	125.792	0.000	0.000	94.497

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F(-2)	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	70	124	0	0	0	0	-1	0
N.S.	1	1.00	0.58	1.02	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.020	10.029	0.208	0.000	0.315	0.000	0.000	0.000	0.000

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	C	F(-2)	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	70	130	0	0	0	0	-1	0
N.S.	1	1.00	0.45	0.84	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.029	10.024	0.214	0.000	0.312	0.000	0.000	0.000	0.000

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	127	490	0	343	0	0	-1	0
N.S.	1	1.00	0.43	1.65	0.00	1.15	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.100	0.287	1.345	0.000	0.310	0.000	0.000	0.000	0.000

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	50	0	42	0	0	38	0
N.S.	1	1.00	1.00	1.52	0.00	1.27	0.00	0.00	1.15	0.00
time (sec)	N/A	0.003	0.079	0.147	0.000	0.297	0.000	0.000	0.546	0.000

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	44	0	0	38	0
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	0.57	0.00
time (sec)	N/A	0.007	0.097	0.167	0.000	0.292	0.000	0.000	0.633	0.000

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	44	0	56	0	0	45	0
N.S.	1	1.00	0.52	0.44	0.00	0.56	0.00	0.00	0.45	0.00
time (sec)	N/A	0.013	0.101	0.154	0.000	0.297	0.000	0.000	0.535	0.000

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	64	56	0	54	0	0	56	0
N.S.	1	1.00	0.48	0.42	0.00	0.41	0.00	0.00	0.42	0.00
time (sec)	N/A	0.022	0.106	0.165	0.000	0.311	0.000	0.000	0.687	0.000

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	103	167	128	819	282	133	693
N.S.	1	1.00	0.93	1.24	2.01	1.54	9.87	3.40	1.60	8.35
time (sec)	N/A	0.021	0.058	0.174	0.269	0.300	0.442	0.004	0.493	4.952

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	47	81	58	245	113	66	256
N.S.	1	1.00	0.81	0.89	1.53	1.09	4.62	2.13	1.25	4.83
time (sec)	N/A	0.013	0.044	0.123	0.269	0.310	0.277	0.004	0.319	2.803

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.011	0.039	0.058	0.000	0.299	0.000	0.000	0.000	0.000

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.033	0.059	0.000	0.304	0.000	0.000	0.000	0.000

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	53	0	0	0	124	0	-1	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	3.02	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.063	0.082	0.000	0.309	2.068	0.000	0.000	0.000

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	146	0	-1	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	2.56	0.00	-0.02	0.00
time (sec)	N/A	0.014	0.038	0.063	0.000	0.304	2.739	0.000	0.000	0.000

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	42	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.10	0.00	-0.05	0.00
time (sec)	N/A	0.005	0.051	0.057	0.000	0.299	2.284	0.000	0.000	0.000

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	84	97	96	96	100	109	88	85
N.S.	1	1.00	2.21	2.55	2.53	2.53	2.63	2.87	2.32	2.24
time (sec)	N/A	0.011	0.011	0.116	0.280	0.288	0.040	0.001	0.190	2.128

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	69	69	73	82	65	60
N.S.	1	1.00	1.76	1.92	1.82	1.82	1.92	2.16	1.71	1.58
time (sec)	N/A	0.010	0.008	0.117	0.263	0.289	0.037	0.001	0.159	2.011

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	49	48	48	49	57	47	43
N.S.	1	1.00	1.21	1.29	1.26	1.26	1.29	1.50	1.24	1.13
time (sec)	N/A	0.021	0.005	0.122	0.257	0.283	0.035	0.000	0.047	1.810

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	32	25	23
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	1.14	0.89	0.82
time (sec)	N/A	0.012	0.003	0.011	0.264	0.259	0.029	0.001	0.035	1.700

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	11	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.92	0.83	0.83
time (sec)	N/A	0.002	0.000	0.010	0.313	0.261	0.026	0.000	0.019	1.544

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	24	20	26	26	25
N.S.	1	1.00	1.00	1.04	1.00	0.96	0.80	1.04	1.04	1.00
time (sec)	N/A	0.013	0.006	0.124	0.271	0.285	0.084	0.000	0.049	1.709

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	33	35	39	27	33	31	32
N.S.	1	1.00	0.97	1.03	1.09	1.22	0.84	1.03	0.97	1.00
time (sec)	N/A	0.015	0.008	0.128	0.278	0.286	0.108	0.001	0.171	1.859

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	38	38	39	29	39	37
N.S.	1	1.00	0.93	1.25	1.36	1.36	1.39	1.04	1.39	1.32
time (sec)	N/A	0.004	0.008	0.117	0.271	0.298	0.151	0.001	0.159	1.857

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	50	50	53	30	52	48
N.S.	1	1.00	0.71	0.92	1.32	1.32	1.39	0.79	1.37	1.26
time (sec)	N/A	0.016	0.006	0.126	0.288	0.289	0.195	0.001	0.165	2.035

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	61	61	65	30	63	59
N.S.	1	1.00	0.71	0.92	1.61	1.61	1.71	0.79	1.66	1.55
time (sec)	N/A	0.015	0.006	0.123	0.280	0.284	0.248	0.001	0.041	2.123

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	148	163	156	156	168	186	144	139
N.S.	1	1.00	2.28	2.51	2.40	2.40	2.58	2.86	2.22	2.14
time (sec)	N/A	0.065	0.020	0.133	0.276	0.284	0.048	0.000	0.066	2.605

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	124	124	133	146	115	111
N.S.	1	1.00	1.88	1.92	1.91	1.91	2.05	2.25	1.77	1.71
time (sec)	N/A	0.047	0.010	0.132	0.261	0.293	0.044	0.000	0.050	2.322

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	79	87	81	81	87	101	74	72
N.S.	1	1.00	1.22	1.34	1.25	1.25	1.34	1.55	1.14	1.11
time (sec)	N/A	0.035	0.007	0.135	0.286	0.288	0.040	0.000	0.166	2.079

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	49	48	48	49	57	47	43
N.S.	1	1.00	1.24	1.29	1.26	1.26	1.29	1.50	1.24	1.13
time (sec)	N/A	0.022	0.006	0.115	0.280	0.285	0.034	0.000	0.044	1.777

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	20	18
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43	1.29
time (sec)	N/A	0.001	0.001	0.120	0.315	0.282	0.031	0.000	0.028	1.605

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	56	61	63	44	65	62	46
N.S.	1	1.00	0.88	1.14	1.24	1.29	0.90	1.33	1.27	0.94
time (sec)	N/A	0.014	0.012	0.168	0.264	0.294	0.133	0.000	0.190	1.950

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	63	67	92	60	69	71	67
N.S.	1	1.00	0.92	1.24	1.31	1.80	1.18	1.35	1.39	1.31
time (sec)	N/A	0.025	0.025	0.139	0.264	0.291	0.203	0.000	0.200	2.151

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	79	99	80	75	77	84
N.S.	1	1.00	0.83	1.17	1.34	1.68	1.36	1.27	1.31	1.42
time (sec)	N/A	0.026	0.017	0.143	0.281	0.288	0.267	0.001	0.197	2.400

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	70	84	84	88	67	80	80
N.S.	1	1.00	1.89	2.50	3.00	3.00	3.14	2.39	2.86	2.86
time (sec)	N/A	0.003	0.019	0.137	0.286	0.298	0.354	0.001	0.037	2.386

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	71	98	98	104	69	39	92
N.S.	1	1.00	0.86	1.09	1.51	1.51	1.60	1.06	0.60	1.42
time (sec)	N/A	0.024	0.015	0.140	0.275	0.293	0.441	0.001	0.193	2.649

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	71	109	109	116	69	107	103
N.S.	1	1.00	0.88	1.09	1.68	1.68	1.78	1.06	1.65	1.58
time (sec)	N/A	0.024	0.019	0.152	0.275	0.285	0.554	0.001	0.202	2.894

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	71	120	120	128	69	118	114
N.S.	1	1.00	0.89	1.09	1.85	1.85	1.97	1.06	1.82	1.75
time (sec)	N/A	0.024	0.013	0.162	0.277	0.282	0.668	0.001	0.089	3.086

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	235	281	277	277	308	335	261	252
N.S.	1	1.00	2.55	3.05	3.01	3.01	3.35	3.64	2.84	2.74
time (sec)	N/A	0.110	0.047	0.133	0.281	0.295	0.056	0.001	0.240	3.594

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	229	225	225	243	267	208	204
N.S.	1	1.00	2.36	2.49	2.45	2.45	2.64	2.90	2.26	2.22
time (sec)	N/A	0.084	0.022	0.132	0.262	0.291	0.052	0.001	0.214	3.178

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	161	177	167	167	190	212	152	153
N.S.	1	1.00	1.75	1.92	1.82	1.82	2.07	2.30	1.65	1.66
time (sec)	N/A	0.060	0.012	0.138	0.261	0.283	0.048	0.000	0.056	2.673

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	124	124	133	146	115	111
N.S.	1	1.00	1.88	1.92	1.91	1.91	2.05	2.25	1.77	1.71
time (sec)	N/A	0.046	0.009	0.139	0.270	0.291	0.043	0.000	0.046	2.347

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	69	69	73	82	65	60
N.S.	1	1.00	1.76	1.92	1.82	1.82	1.92	2.16	1.71	1.58
time (sec)	N/A	0.010	0.005	0.118	0.263	0.297	0.037	0.000	0.032	1.978

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	12	31	32
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.21	2.29
time (sec)	N/A	0.001	0.001	0.115	0.273	0.286	0.032	0.000	0.037	1.642

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	109	114	116	83	124	118	80
N.S.	1	1.00	1.01	1.49	1.56	1.59	1.14	1.70	1.62	1.10
time (sec)	N/A	0.020	0.020	0.140	0.261	0.290	0.182	0.000	0.201	2.344

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	109	118	173	102	126	123	108
N.S.	1	1.00	0.96	1.45	1.57	2.31	1.36	1.68	1.64	1.44
time (sec)	N/A	0.040	0.034	0.173	0.267	0.297	0.300	0.001	0.215	2.684

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	114	114	125	188	128	125	130	144
N.S.	1	1.00	1.46	1.46	1.60	2.41	1.64	1.60	1.67	1.85
time (sec)	N/A	0.037	0.029	0.142	0.282	0.305	0.504	0.001	0.821	3.075

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	120	142	176	148	128	138	155
N.S.	1	1.00	0.93	1.40	1.65	2.05	1.72	1.49	1.60	1.80
time (sec)	N/A	0.035	0.027	0.143	0.277	0.299	0.661	0.002	0.254	3.448

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	91	122	143	143	155	123	135	138
N.S.	1	1.00	3.25	4.36	5.11	5.11	5.54	4.39	4.82	4.93
time (sec)	N/A	0.002	0.020	0.145	0.271	0.286	0.914	0.001	0.072	3.494

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	97	121	160	160	172	126	39	151
N.S.	1	1.00	1.67	2.09	2.76	2.76	2.97	2.17	0.67	2.60
time (sec)	N/A	0.007	0.023	0.135	0.263	0.290	1.145	0.001	0.082	3.851

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	171	171	184	126	165	162
N.S.	1	1.00	1.05	1.33	1.86	1.86	2.00	1.37	1.79	1.76
time (sec)	N/A	0.037	0.025	0.136	0.336	0.294	1.502	0.001	0.222	4.171

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	182	182	196	126	176	173
N.S.	1	1.00	1.05	1.33	1.98	1.98	2.13	1.37	1.91	1.88
time (sec)	N/A	0.036	0.021	0.158	0.278	0.292	1.964	0.001	0.114	4.537

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	193	193	207	126	187	184
N.S.	1	1.00	1.05	1.33	2.10	2.10	2.25	1.37	2.03	2.00
time (sec)	N/A	0.036	0.023	0.138	0.275	0.289	3.124	0.001	0.232	5.011

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	993	1033	1023	1023	1163	1269	997	964
N.S.	1	1.00	4.96	5.16	5.12	5.12	5.82	6.34	4.98	4.82
time (sec)	N/A	0.462	0.091	0.137	0.285	0.304	0.111	0.001	0.554	9.565

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	897	925	921	921	1046	1140	892	862
N.S.	1	1.00	4.48	4.62	4.60	4.60	5.23	5.70	4.46	4.31
time (sec)	N/A	0.387	0.061	0.154	0.269	0.301	0.103	0.001	0.357	8.569

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	785	817	807	807	935	1024	781	755
N.S.	1	1.00	3.92	4.08	4.04	4.04	4.68	5.12	3.90	3.78
time (sec)	N/A	0.310	0.045	0.147	0.275	0.293	0.094	0.001	0.402	7.597

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	684	709	706	706	796	866	683	661
N.S.	1	1.00	3.95	4.10	4.08	4.08	4.60	5.01	3.95	3.82
time (sec)	N/A	0.300	0.046	0.135	0.276	0.288	0.086	0.000	0.256	6.856

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	574	601	594	594	673	734	570	553
N.S.	1	1.00	3.99	4.17	4.12	4.12	4.67	5.10	3.96	3.84
time (sec)	N/A	0.246	0.037	0.135	0.276	0.292	0.079	0.000	0.214	5.985

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	473	493	489	489	549	596	470	452
N.S.	1	1.00	3.97	4.14	4.11	4.11	4.61	5.01	3.95	3.80
time (sec)	N/A	0.192	0.032	0.136	0.270	0.294	0.071	0.001	0.313	5.169

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	360	385	376	376	427	464	356	343
N.S.	1	1.00	3.91	4.18	4.09	4.09	4.64	5.04	3.87	3.73
time (sec)	N/A	0.152	0.028	0.135	0.269	0.289	0.064	0.001	0.273	4.346

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	261	277	273	273	303	326	249	245
N.S.	1	1.00	4.02	4.26	4.20	4.20	4.66	5.02	3.83	3.77
time (sec)	N/A	0.108	0.018	0.132	0.271	0.283	0.057	0.001	0.107	3.550

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	151	169	163	163	178	189	143	140
N.S.	1	1.00	3.97	4.45	4.29	4.29	4.68	4.97	3.76	3.68
time (sec)	N/A	0.012	0.010	0.117	0.265	0.301	0.048	0.000	0.079	2.680

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	75	83	12	75	76
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.36	5.43
time (sec)	N/A	0.001	0.001	0.116	0.271	0.294	0.037	0.000	0.057	1.865

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	304	491	460	462	408	552	509	356
N.S.	1	1.00	1.80	2.91	2.72	2.73	2.41	3.27	3.01	2.11
time (sec)	N/A	0.048	0.093	0.143	0.276	0.320	0.492	0.001	0.217	5.100

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	388	479	467	632	428	530	841	404
N.S.	1	1.00	2.07	2.56	2.50	3.38	2.29	2.83	4.50	2.16
time (sec)	N/A	0.159	0.078	0.169	0.276	0.295	0.854	0.001	0.241	5.555

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	389	467	473	703	447	511	690	486
N.S.	1	1.00	2.10	2.52	2.56	3.80	2.42	2.76	3.73	2.63
time (sec)	N/A	0.147	0.082	0.146	0.272	0.291	1.734	0.001	0.266	6.861

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	199	459	484	739	474	502	559	550
N.S.	1	1.00	1.06	2.45	2.59	3.95	2.53	2.68	2.99	2.94
time (sec)	N/A	0.147	0.066	0.140	0.277	0.313	22.136	0.001	0.289	9.088

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	173	453	494	754	0	499	512	593
N.S.	1	1.00	0.93	2.42	2.64	4.03	0.00	2.67	2.74	3.17
time (sec)	N/A	0.132	0.070	0.145	0.300	0.298	0.000	0.001	0.772	19.493

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	389	451	504	732	0	493	508	612
N.S.	1	1.00	2.15	2.49	2.78	4.04	0.00	2.72	2.81	3.38
time (sec)	N/A	0.125	0.093	0.179	0.313	0.293	0.000	0.001	0.339	66.680

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	390	456	516	692	0	492	517	610
N.S.	1	1.00	2.10	2.45	2.77	3.72	0.00	2.65	2.78	3.28
time (sec)	N/A	0.113	0.126	0.147	0.315	0.292	0.000	0.001	0.369	236.591

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	308	462	534	624	0	494	461	0
N.S.	1	1.00	1.59	2.38	2.75	3.22	0.00	2.55	2.38	0.00
time (sec)	N/A	0.106	0.100	0.147	0.279	0.304	0.000	0.001	0.353	0.000

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	353	464	509	509	0	527	571	0
N.S.	1	1.00	12.61	16.57	18.18	18.18	0.00	18.82	20.39	0.00
time (sec)	N/A	0.003	0.075	0.141	0.286	0.298	0.000	0.001	0.169	0.000

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	367	464	548	548	0	534	39	0
N.S.	1	1.00	6.33	8.00	9.45	9.45	0.00	9.21	0.67	0.00
time (sec)	N/A	0.007	0.075	0.177	0.298	0.287	0.000	0.001	0.146	0.000

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	371	464	559	559	0	534	600	0
N.S.	1	1.00	4.17	5.21	6.28	6.28	0.00	6.00	6.74	0.00
time (sec)	N/A	0.015	0.076	0.143	0.281	0.292	0.000	0.001	0.446	0.000

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	369	464	570	570	0	534	548	0
N.S.	1	1.00	3.08	3.87	4.75	4.75	0.00	4.45	4.57	0.00
time (sec)	N/A	0.023	0.077	0.141	0.300	0.302	0.000	0.001	0.518	0.000

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	371	464	581	581	0	534	559	0
N.S.	1	1.00	2.46	3.07	3.85	3.85	0.00	3.54	3.70	0.00
time (sec)	N/A	0.034	0.075	0.145	0.297	0.300	0.000	0.001	0.229	0.000

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	369	463	592	592	0	534	570	0
N.S.	1	1.00	1.86	2.34	2.99	2.99	0.00	2.70	2.88	0.00
time (sec)	N/A	0.106	0.079	0.171	0.314	0.299	0.000	0.001	0.399	0.000

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	371	464	603	603	0	534	581	0
N.S.	1	1.00	1.86	2.32	3.02	3.02	0.00	2.67	2.90	0.00
time (sec)	N/A	0.098	0.077	0.161	0.290	0.299	0.000	0.001	1.238	0.000

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	371	464	614	614	0	534	592	0
N.S.	1	1.00	1.86	2.32	3.07	3.07	0.00	2.67	2.96	0.00
time (sec)	N/A	0.097	0.076	0.137	0.289	0.300	0.000	0.001	2.196	0.000

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	1817	1891	1877	1877	2088	2282	1847	1796
N.S.	1	1.00	6.61	6.88	6.83	6.83	7.59	8.30	6.72	6.53
time (sec)	N/A	1.010	0.175	0.138	0.297	0.313	0.171	0.002	0.984	15.989

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	1702	1741	1740	1740	1965	2150	1702	1650
N.S.	1	1.00	6.10	6.24	6.24	6.24	7.04	7.71	6.10	5.91
time (sec)	N/A	0.861	0.118	0.136	0.290	0.310	0.161	0.001	1.026	15.620

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	1539	1591	1581	1581	1775	1943	1549	1499
N.S.	1	1.00	5.52	5.70	5.67	5.67	6.36	6.96	5.55	5.37
time (sec)	N/A	0.756	0.097	0.134	0.281	0.306	0.163	0.001	0.689	13.771

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	1397	1441	1437	1437	1598	1744	1404	1358
N.S.	1	1.00	5.59	5.76	5.75	5.75	6.39	6.98	5.62	5.43
time (sec)	N/A	0.700	0.103	0.144	0.284	0.296	0.138	0.001	0.788	12.475

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	1241	1291	1283	1283	1428	1558	1253	1212
N.S.	1	1.00	5.52	5.74	5.70	5.70	6.35	6.92	5.57	5.39
time (sec)	N/A	0.608	0.090	0.154	0.285	0.292	0.127	0.002	0.707	11.470

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	1105	1141	1135	1135	1280	1396	1106	1070
N.S.	1	1.00	5.52	5.70	5.68	5.68	6.40	6.98	5.53	5.35
time (sec)	N/A	0.528	0.079	0.133	0.292	0.299	0.119	0.001	0.613	10.351

Problem 1305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	939	991	977	977	1088	1186	953	920
N.S.	1	1.00	5.52	5.83	5.75	5.75	6.40	6.98	5.61	5.41
time (sec)	N/A	0.458	0.070	0.131	0.280	0.296	0.110	0.000	0.532	9.107

Problem 1306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	811	841	835	835	940	1024	806	779
N.S.	1	1.00	5.55	5.76	5.72	5.72	6.44	7.01	5.52	5.34
time (sec)	N/A	0.370	0.053	0.135	0.268	0.300	0.102	0.000	0.341	7.881

Problem 1307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	660	691	686	686	748	811	664	637
N.S.	1	1.00	5.55	5.81	5.76	5.76	6.29	6.82	5.58	5.35
time (sec)	N/A	0.319	0.045	0.130	0.309	0.295	0.086	0.000	0.426	6.702

Problem 1308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	511	541	535	535	586	634	495	490
N.S.	1	1.00	5.55	5.88	5.82	5.82	6.37	6.89	5.38	5.33
time (sec)	N/A	0.250	0.036	0.231	0.267	0.293	0.079	0.001	0.231	5.603

Problem 1309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	358	391	384	384	415	445	348	342
N.S.	1	1.00	5.51	6.02	5.91	5.91	6.38	6.85	5.35	5.26
time (sec)	N/A	0.184	0.026	0.141	0.269	0.294	0.068	0.000	0.320	4.415

Problem 1310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	220	241	240	240	248	259	208	199
N.S.	1	1.00	5.79	6.34	6.32	6.32	6.53	6.82	5.47	5.24
time (sec)	N/A	0.012	0.016	0.118	0.276	0.290	0.056	0.001	0.129	3.233

Problem 1311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	108	114	12	108	106
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.71	7.57
time (sec)	N/A	0.001	0.001	0.113	0.273	0.291	0.041	0.000	0.080	2.012

Problem 1312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	591	2929	866	868	799	1041	979	708
N.S.	1	1.00	2.45	12.15	3.59	3.60	3.32	4.32	4.06	2.94
time (sec)	N/A	0.070	0.182	0.517	0.311	0.299	0.845	0.000	0.130	8.439

Problem 1313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	708	933	874	1124	816	998	3475	770
N.S.	1	1.00	2.74	3.62	3.39	4.36	3.16	3.87	13.47	2.98
time (sec)	N/A	0.325	0.144	0.151	0.278	0.314	1.509	0.001	0.352	9.257

Problem 1314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	708	914	881	1233	843	976	3299	889
N.S.	1	1.00	2.70	3.49	3.36	4.71	3.22	3.73	12.59	3.39
time (sec)	N/A	0.304	0.145	0.158	0.282	0.304	6.304	0.001	0.378	11.534

Problem 1315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	427	896	891	1316	0	957	2219	985
N.S.	1	1.00	1.66	3.47	3.45	5.10	0.00	3.71	8.60	3.82
time (sec)	N/A	0.303	0.114	0.168	0.298	0.306	0.000	0.001	0.385	28.968

Problem 1316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	359	881	903	1365	0	945	1494	1060
N.S.	1	1.00	1.37	3.36	3.45	5.21	0.00	3.61	5.70	4.05
time (sec)	N/A	0.293	0.122	0.153	0.293	0.298	0.000	0.001	0.381	196.771

Problem 1317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	305	870	912	1395	0	934	1141	0
N.S.	1	1.00	1.17	3.35	3.51	5.37	0.00	3.59	4.39	0.00
time (sec)	N/A	0.289	0.130	0.147	0.325	0.305	0.000	0.001	0.396	0.000

Problem 1318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	265	862	925	1386	0	928	997	0
N.S.	1	1.00	1.01	3.29	3.53	5.29	0.00	3.54	3.81	0.00
time (sec)	N/A	0.268	0.135	0.150	0.366	0.307	0.000	0.001	0.422	0.000

Problem 1319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	239	856	934	1362	0	924	950	0
N.S.	1	1.00	0.93	3.32	3.62	5.28	0.00	3.58	3.68	0.00
time (sec)	N/A	0.251	0.155	0.142	0.352	0.307	0.000	0.002	0.428	0.000

Problem 1320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	712	854	945	1296	0	922	946	0
N.S.	1	1.00	2.76	3.31	3.66	5.02	0.00	3.57	3.67	0.00
time (sec)	N/A	0.231	0.195	0.168	0.389	0.306	0.000	0.002	0.263	0.000

Problem 1321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	708	859	957	1216	0	924	955	0
N.S.	1	1.00	2.75	3.34	3.72	4.73	0.00	3.60	3.72	0.00
time (sec)	N/A	0.214	0.254	0.171	0.358	0.301	0.000	0.002	0.502	0.000

Problem 1322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	591	865	975	1107	0	926	866	0
N.S.	1	1.00	2.18	3.19	3.60	4.08	0.00	3.42	3.20	0.00
time (sec)	N/A	0.195	0.224	0.244	0.319	0.303	0.000	0.001	0.555	0.000

Problem 1323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	665	866	920	920	0	1019	1066	0
N.S.	1	1.00	23.75	30.93	32.86	32.86	0.00	36.39	38.07	0.00
time (sec)	N/A	0.003	0.171	0.137	0.336	0.309	0.000	0.001	0.458	0.000

Problem 1324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	684	867	986	986	0	1029	39	0
N.S.	1	1.00	11.79	14.95	17.00	17.00	0.00	17.74	0.67	0.00
time (sec)	N/A	0.007	0.168	0.141	0.320	0.301	0.000	0.002	0.394	0.000

Problem 1325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	690	867	997	997	0	1029	1098	0
N.S.	1	1.00	7.75	9.74	11.20	11.20	0.00	11.56	12.34	0.00
time (sec)	N/A	0.013	0.176	0.151	0.373	0.301	0.000	0.002	0.475	0.000

Problem 1326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	692	867	1008	1008	0	1029	1109	0
N.S.	1	1.00	5.77	7.22	8.40	8.40	0.00	8.58	9.24	0.00
time (sec)	N/A	0.022	0.173	0.148	0.328	0.297	0.000	0.001	1.295	0.000

Problem 1327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	690	867	1019	1019	0	1029	1120	0
N.S.	1	1.00	4.57	5.74	6.75	6.75	0.00	6.81	7.42	0.00
time (sec)	N/A	0.033	0.177	0.145	0.328	0.298	0.000	0.002	2.279	0.000

Problem 1328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	694	867	1030	1030	0	1029	1131	0
N.S.	1	1.00	3.81	4.76	5.66	5.66	0.00	5.65	6.21	0.00
time (sec)	N/A	0.046	0.173	0.209	0.341	0.312	0.000	0.002	0.584	0.000

Problem 1329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	690	867	1041	1041	0	1029	1142	0
N.S.	1	1.00	3.24	4.07	4.89	4.89	0.00	4.83	5.36	0.00
time (sec)	N/A	0.059	0.177	0.142	0.325	0.303	0.000	0.002	0.658	0.000

Problem 1330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	694	867	1052	1052	0	1029	1153	0
N.S.	1	1.00	2.84	3.55	4.31	4.31	0.00	4.22	4.73	0.00
time (sec)	N/A	0.074	0.174	0.163	0.335	0.306	0.000	0.002	12.020	0.000

Problem 1331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	692	866	1063	1063	0	1029	1164	0
N.S.	1	1.00	2.53	3.17	3.89	3.89	0.00	3.77	4.26	0.00
time (sec)	N/A	0.203	0.172	0.148	0.327	0.303	0.000	0.002	25.721	0.000

Problem 1332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	692	867	1074	1074	0	1029	1175	0
N.S.	1	1.00	2.48	3.11	3.85	3.85	0.00	3.69	4.21	0.00
time (sec)	N/A	0.179	0.181	0.160	0.338	0.305	0.000	0.002	0.799	0.000

Problem 1333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	692	867	1085	1085	0	1029	1186	0
N.S.	1	1.00	2.48	3.11	3.89	3.89	0.00	3.69	4.25	0.00
time (sec)	N/A	0.183	0.184	0.181	0.335	0.307	0.000	0.002	1.036	0.000

Problem 1334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	167	266	258	259	209	296	280	191
N.S.	1	1.00	1.37	2.18	2.11	2.12	1.71	2.43	2.30	1.57
time (sec)	N/A	0.039	0.046	0.141	0.270	0.293	0.302	0.000	0.074	3.698

Problem 1335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	115	189	177	179	136	198	189	127
N.S.	1	1.00	1.17	1.93	1.81	1.83	1.39	2.02	1.93	1.30
time (sec)	N/A	0.027	0.029	0.151	0.260	0.295	0.234	0.000	0.218	3.021

Problem 1336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	109	114	115	83	124	118	79
N.S.	1	1.00	1.00	1.47	1.54	1.55	1.12	1.68	1.59	1.07
time (sec)	N/A	0.021	0.021	0.137	0.273	0.290	0.181	0.000	0.065	2.485

Problem 1337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	56	60	62	44	65	62	46
N.S.	1	1.00	0.86	1.12	1.20	1.24	0.88	1.30	1.24	0.92
time (sec)	N/A	0.015	0.012	0.177	0.273	0.288	0.128	0.000	0.225	2.088

Problem 1338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	26	25	24
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.00	0.96	0.92
time (sec)	N/A	0.013	0.006	0.123	0.263	0.295	0.085	0.001	0.201	1.857

Problem 1339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	1.00
time (sec)	N/A	0.001	0.001	0.123	0.269	0.306	0.030	0.000	0.022	1.684

Problem 1340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	37	36	26	128	43	25	30
N.S.	1	1.00	0.72	1.03	1.00	0.72	3.56	1.19	0.69	0.83
time (sec)	N/A	0.006	0.009	0.156	0.282	0.295	0.188	0.001	0.258	3.591

Problem 1341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	92	93	233	101	46	93
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.77	0.81	1.63
time (sec)	N/A	0.023	0.018	0.164	0.315	0.297	0.401	0.001	0.142	4.806

Problem 1342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	202	242	381	172	182	227
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.10	2.22	2.77
time (sec)	N/A	0.035	0.042	0.172	0.285	0.301	0.620	0.001	0.161	6.843

Problem 1343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	228	259	264	373	231	290	327	227
N.S.	1	1.00	1.75	1.99	2.03	2.87	1.78	2.23	2.52	1.75
time (sec)	N/A	0.100	0.053	0.175	0.362	0.298	0.535	0.001	0.245	4.088

Problem 1344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	165	175	183	267	155	197	203	160
N.S.	1	1.00	1.59	1.68	1.76	2.57	1.49	1.89	1.95	1.54
time (sec)	N/A	0.074	0.041	0.144	0.262	0.295	0.414	0.001	0.073	3.316

Problem 1345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	114	108	117	172	102	126	123	107
N.S.	1	1.00	1.52	1.44	1.56	2.29	1.36	1.68	1.64	1.43
time (sec)	N/A	0.045	0.025	0.138	0.284	0.290	0.299	0.001	0.077	2.650

Problem 1346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	63	67	92	60	69	71	67
N.S.	1	1.00	0.92	1.24	1.31	1.80	1.18	1.35	1.39	1.31
time (sec)	N/A	0.028	0.027	0.138	0.269	0.298	0.204	0.001	0.236	2.281

Problem 1347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	33	34	37	27	33	32	32
N.S.	1	1.00	1.00	1.06	1.10	1.19	0.87	1.06	1.03	1.03
time (sec)	N/A	0.015	0.008	0.121	0.281	0.304	0.108	0.001	0.041	1.935

Problem 1348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	9	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	0.75	1.00	1.00
time (sec)	N/A	0.001	0.002	0.126	0.347	0.289	0.066	0.000	0.187	1.690

Problem 1349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	58	90	92	233	101	47	95
N.S.	1	1.00	0.95	1.04	1.61	1.64	4.16	1.80	0.84	1.70
time (sec)	N/A	0.023	0.019	0.158	0.295	0.289	0.399	0.001	0.292	5.092

Problem 1350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	66	82	208	241	406	165	74	248
N.S.	1	1.00	0.81	1.01	2.57	2.98	5.01	2.04	0.91	3.06
time (sec)	N/A	0.036	0.045	0.161	0.292	0.300	0.647	0.002	0.334	6.849

Problem 1351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	98	109	386	494	634	264	330	495
N.S.	1	1.00	0.90	1.00	3.54	4.53	5.82	2.42	3.03	4.54
time (sec)	N/A	0.058	0.050	0.165	0.312	0.301	1.041	0.002	0.396	9.777

Problem 1352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	303	351	364	548	340	384	441	380
N.S.	1	1.00	1.92	2.22	2.30	3.47	2.15	2.43	2.79	2.41
time (sec)	N/A	0.145	0.073	0.140	0.329	0.296	1.310	0.001	0.274	7.449

Problem 1353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	230	254	271	416	258	287	291	288
N.S.	1	1.00	1.73	1.91	2.04	3.13	1.94	2.16	2.19	2.17
time (sec)	N/A	0.086	0.049	0.140	0.266	0.293	0.995	0.001	0.099	4.768

Problem 1354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	167	172	191	291	185	198	196	210
N.S.	1	1.00	1.62	1.67	1.85	2.83	1.80	1.92	1.90	2.04
time (sec)	N/A	0.063	0.038	0.142	0.272	0.297	0.721	0.001	0.099	3.915

Problem 1355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	114	114	125	188	128	125	130	144
N.S.	1	1.00	1.46	1.46	1.60	2.41	1.64	1.60	1.67	1.85
time (sec)	N/A	0.042	0.028	0.147	0.276	0.294	0.495	0.001	0.109	3.195

Problem 1356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	69	80	100	80	75	77	84
N.S.	1	1.00	0.81	1.17	1.36	1.69	1.36	1.27	1.31	1.42
time (sec)	N/A	0.029	0.018	0.134	0.266	0.290	0.266	0.001	0.228	2.580

Problem 1357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	38	38	39	29	39	37
N.S.	1	1.00	0.93	1.25	1.36	1.36	1.39	1.04	1.39	1.32
time (sec)	N/A	0.003	0.007	0.121	0.280	0.283	0.152	0.001	0.029	2.013

Problem 1358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	24	26	12	26	23
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86	1.64
time (sec)	N/A	0.001	0.002	0.121	0.257	0.298	0.093	0.000	0.024	1.817

Problem 1359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	202	242	381	171	183	227
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.09	2.23	2.77
time (sec)	N/A	0.035	0.042	0.168	0.276	0.299	0.609	0.001	0.299	6.626

Problem 1360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	108	386	495	632	264	329	496
N.S.	1	1.00	0.88	0.98	3.51	4.50	5.75	2.40	2.99	4.51
time (sec)	N/A	0.051	0.074	0.174	0.272	0.300	0.992	0.002	0.395	9.910

Problem 1361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	128	140	594	760	881	357	542	783
N.S.	1	1.00	0.90	0.98	4.15	5.31	6.16	2.50	3.79	5.48
time (sec)	N/A	0.073	0.079	0.178	0.274	0.305	1.465	0.002	0.526	13.602

Problem 1362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	584	705	786	1093	0	764	784	0
N.S.	1	1.00	2.52	3.04	3.39	4.71	0.00	3.29	3.38	0.00
time (sec)	N/A	0.246	0.177	0.145	0.342	0.303	0.000	0.001	0.257	0.000

Problem 1363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	474	576	649	852	0	621	649	0
N.S.	1	1.00	2.27	2.76	3.11	4.08	0.00	2.97	3.11	0.00
time (sec)	N/A	0.188	0.128	0.144	0.299	0.304	0.000	0.001	0.430	0.000

Problem 1364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	308	462	535	625	0	494	460	0
N.S.	1	1.00	1.59	2.38	2.76	3.22	0.00	2.55	2.37	0.00
time (sec)	N/A	0.145	0.102	0.139	0.285	0.306	0.000	0.001	0.383	0.000

Problem 1365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	271	357	398	398	0	399	378	375
N.S.	1	1.00	9.68	12.75	14.21	14.21	0.00	14.25	13.50	13.39
time (sec)	N/A	0.003	0.057	0.136	0.275	0.291	0.000	0.001	0.146	193.321

Problem 1366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	205	265	326	326	0	294	39	307
N.S.	1	1.00	3.53	4.57	5.62	5.62	0.00	5.07	0.67	5.29
time (sec)	N/A	0.008	0.037	0.156	0.294	0.293	0.000	0.001	0.277	39.215

Problem 1367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	144	186	247	247	267	201	237	232
N.S.	1	1.00	1.62	2.09	2.78	2.78	3.00	2.26	2.66	2.61
time (sec)	N/A	0.014	0.032	0.141	0.269	0.304	108.564	0.001	0.110	9.403

Problem 1368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	94	122	182	182	196	126	176	173
N.S.	1	1.00	1.02	1.33	1.98	1.98	2.13	1.37	1.91	1.88
time (sec)	N/A	0.043	0.020	0.139	0.267	0.298	2.014	0.001	0.099	4.829

Problem 1369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	71	131	131	139	69	129	125
N.S.	1	1.00	0.85	1.09	2.02	2.02	2.14	1.06	1.98	1.92
time (sec)	N/A	0.028	0.017	0.141	0.282	0.295	0.816	0.001	0.086	3.571

Problem 1370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	94	94	100	30	96	92
N.S.	1	1.00	0.71	0.92	2.47	2.47	2.63	0.79	2.53	2.42
time (sec)	N/A	0.016	0.007	0.125	0.274	0.292	0.423	0.001	0.226	2.835

Problem 1371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	79	85	12	81	78
N.S.	1	1.00	1.00	0.93	0.86	5.64	6.07	0.86	5.79	5.57
time (sec)	N/A	0.001	0.003	0.125	0.285	0.299	0.239	0.001	0.223	2.456

Problem 1372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	196	192	1418	1589	1776	725	1299	1882
N.S.	1	1.00	0.97	0.95	7.02	7.87	8.79	3.59	6.43	9.32
time (sec)	N/A	0.120	0.061	0.198	0.357	0.331	7.988	0.012	0.873	27.841

Problem 1373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	213	223	1881	2264	2336	894	1738	2737
N.S.	1	1.00	0.92	0.97	8.14	9.80	10.11	3.87	7.52	11.85
time (sec)	N/A	0.190	0.151	0.201	0.460	0.353	25.483	0.008	1.386	42.523

Problem 1374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	254	265	2399	3016	0	1064	2224	2095
N.S.	1	1.00	0.92	0.96	8.69	10.93	0.00	3.86	8.06	7.59
time (sec)	N/A	0.250	0.125	0.240	0.513	0.375	0.000	0.009	1.911	65.952

Problem 1375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	217	121	259	338	314	1039	137	247
N.S.	1	1.00	1.39	0.78	1.66	2.17	2.01	6.66	0.88	1.58
time (sec)	N/A	0.042	0.112	0.138	0.268	0.304	2.415	0.007	0.081	7.075

Problem 1376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	154	100	181	245	223	748	112	172
N.S.	1	1.00	1.19	0.78	1.40	1.90	1.73	5.80	0.87	1.33
time (sec)	N/A	0.036	0.078	0.163	0.277	0.299	1.997	0.005	0.225	5.568

Problem 1377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	78	118	164	146	500	87	110
N.S.	1	1.00	1.02	0.78	1.18	1.64	1.46	5.00	0.87	1.10
time (sec)	N/A	0.024	0.063	0.151	0.288	0.300	1.545	0.004	0.065	4.378

Problem 1378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	56	68	99	85	300	68	63
N.S.	1	1.00	0.86	0.79	0.96	1.39	1.20	4.23	0.96	0.89
time (sec)	N/A	0.018	0.038	0.137	0.286	0.297	1.194	0.002	0.240	3.443

Problem 1379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	34	33	46	36	147	29	29
N.S.	1	1.00	0.71	0.81	0.79	1.10	0.86	3.50	0.69	0.69
time (sec)	N/A	0.011	0.021	0.123	0.268	0.308	0.915	0.001	0.043	2.742

Problem 1380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	21	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	1.31	0.75	0.75
time (sec)	N/A	0.001	0.007	0.119	0.261	0.298	0.028	0.000	0.021	1.618

Problem 1381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	61	0	143	61	73	50	48
N.S.	1	1.00	1.00	0.98	0.00	2.31	0.98	1.18	0.81	0.77
time (sec)	N/A	0.038	0.077	0.164	0.000	0.307	1.945	0.002	0.066	4.176

Problem 1382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	73	0	232	573	82	61	597
N.S.	1	1.00	0.99	1.04	0.00	3.31	8.19	1.17	0.87	8.53
time (sec)	N/A	0.021	0.176	0.157	0.000	0.316	21.974	0.003	0.242	22.411

Problem 1383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	99	106	0	456	1658	149	135	1307
N.S.	1	1.00	0.90	0.96	0.00	4.15	15.07	1.35	1.23	11.88
time (sec)	N/A	0.054	0.395	0.144	0.000	0.311	95.194	0.005	0.302	57.125

Problem 1384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	152	0	785	0	254	207	2591
N.S.	1	1.00	0.90	1.04	0.00	5.38	0.00	1.74	1.42	17.75
time (sec)	N/A	0.067	0.615	0.148	0.000	0.321	0.000	0.007	0.374	132.513

Problem 1385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	170	217	0	1176	0	388	297	0
N.S.	1	1.00	0.93	1.19	0.00	6.46	0.00	2.13	1.63	0.00
time (sec)	N/A	0.084	0.918	0.145	0.000	0.323	0.000	0.009	0.217	0.000

Problem 1386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	224	293	0	1673	0	551	401	0
N.S.	1	1.00	1.03	1.34	0.00	7.67	0.00	2.53	1.84	0.00
time (sec)	N/A	0.104	1.290	0.161	0.000	0.338	0.000	0.012	0.490	0.000

Problem 1387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	217	122	259	418	763	1798	137	452
N.S.	1	1.00	1.37	0.77	1.64	2.65	4.83	11.38	0.87	2.86
time (sec)	N/A	0.036	0.118	0.158	0.263	0.304	13.415	0.010	0.244	22.898

Problem 1388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	154	100	181	311	559	1320	112	331
N.S.	1	1.00	1.19	0.78	1.40	2.41	4.33	10.23	0.87	2.57
time (sec)	N/A	0.029	0.084	0.155	0.300	0.299	10.069	0.008	0.242	17.458

Problem 1389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	78	118	216	386	911	87	227
N.S.	1	1.00	1.02	0.78	1.18	2.16	3.86	9.11	0.87	2.27
time (sec)	N/A	0.022	0.064	0.169	0.272	0.299	7.304	0.006	0.249	12.953

Problem 1390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	56	68	137	240	571	68	142
N.S.	1	1.00	0.86	0.79	0.96	1.93	3.38	8.04	0.96	2.00
time (sec)	N/A	0.016	0.043	0.145	0.289	0.307	4.858	0.004	0.059	8.949

Problem 1391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	34	33	69	146	300	29	78
N.S.	1	1.00	0.71	0.81	0.79	1.64	3.48	7.14	0.69	1.86
time (sec)	N/A	0.011	0.024	0.130	0.263	0.304	0.166	0.002	0.215	2.328

Problem 1392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	113	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	7.06	0.75	0.75
time (sec)	N/A	0.001	0.007	0.135	0.278	0.301	0.029	0.001	0.017	1.626

Problem 1393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	99	0	188	82	133	93	86
N.S.	1	1.00	0.90	1.15	0.00	2.19	0.95	1.55	1.08	1.00
time (sec)	N/A	0.032	0.142	0.172	0.000	0.308	6.885	0.004	0.075	8.938

Problem 1394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	100	0	210	923	131	109	935
N.S.	1	1.00	0.98	1.18	0.00	2.47	10.86	1.54	1.28	11.00
time (sec)	N/A	0.027	0.228	0.220	0.000	0.307	67.898	0.007	0.105	77.123

Problem 1395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	97	0	383	0	137	135	2342
N.S.	1	1.00	0.90	0.97	0.00	3.83	0.00	1.37	1.35	23.42
time (sec)	N/A	0.031	0.338	0.158	0.000	0.322	0.000	0.007	0.284	246.497

Problem 1396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	126	0	666	0	228	209	0
N.S.	1	1.00	0.95	0.93	0.00	4.90	0.00	1.68	1.54	0.00
time (sec)	N/A	0.040	0.544	0.157	0.000	0.322	0.000	0.009	0.338	0.000

Problem 1397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	171	172	0	1043	0	360	296	0
N.S.	1	1.00	0.99	1.00	0.00	6.06	0.00	2.09	1.72	0.00
time (sec)	N/A	0.051	0.855	0.162	0.000	0.322	0.000	0.012	0.371	0.000

Problem 1398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	223	237	0	1492	0	523	398	0
N.S.	1	1.00	1.07	1.14	0.00	7.17	0.00	2.51	1.91	0.00
time (sec)	N/A	0.066	1.574	0.158	0.000	0.320	0.000	0.015	0.473	0.000

Problem 1399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	217	122	259	497	1292	2696	137	544
N.S.	1	1.00	1.37	0.77	1.64	3.15	8.18	17.06	0.87	3.44
time (sec)	N/A	0.036	0.123	0.150	0.285	0.297	21.930	0.017	0.267	38.502

Problem 1400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	154	100	181	377	960	2008	112	406
N.S.	1	1.00	1.19	0.78	1.40	2.92	7.44	15.57	0.87	3.15
time (sec)	N/A	0.029	0.090	0.149	0.261	0.300	16.794	0.012	0.234	27.917

Problem 1401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	78	118	268	549	1412	87	276
N.S.	1	1.00	1.02	0.78	1.18	2.68	5.49	14.12	0.87	2.76
time (sec)	N/A	0.023	0.072	0.145	0.281	0.291	0.454	0.009	0.076	4.598

Problem 1402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	56	68	174	355	911	68	183
N.S.	1	1.00	0.86	0.79	0.96	2.45	5.00	12.83	0.96	2.58
time (sec)	N/A	0.017	0.047	0.146	0.259	0.293	0.375	0.006	0.067	3.640

Problem 1403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	34	33	93	194	500	29	102
N.S.	1	1.00	0.71	0.81	0.79	2.21	4.62	11.90	0.69	2.43
time (sec)	N/A	0.010	0.027	0.133	0.277	0.298	0.298	0.005	0.045	2.907

Problem 1404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	39	12	199	12	12
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	12.44	0.75	0.75
time (sec)	N/A	0.001	0.008	0.135	0.264	0.288	0.031	0.002	0.021	1.747

Problem 1405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	161	0	290	121	221	130	140
N.S.	1	1.00	0.96	1.44	0.00	2.59	1.08	1.97	1.16	1.25
time (sec)	N/A	0.040	0.113	0.174	0.000	0.314	12.447	0.005	0.083	15.908

Problem 1406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	116	152	0	330	1312	216	161	1358
N.S.	1	1.00	1.05	1.38	0.00	3.00	11.93	1.96	1.46	12.35
time (sec)	N/A	0.037	0.290	0.188	0.000	0.309	120.019	0.006	0.124	134.052

Problem 1407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	138	0	344	0	213	199	0
N.S.	1	1.00	1.00	1.16	0.00	2.89	0.00	1.79	1.67	0.00
time (sec)	N/A	0.033	0.417	0.196	0.000	0.308	0.000	0.008	0.160	0.000

Problem 1408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	130	0	563	0	210	222	0
N.S.	1	1.00	0.94	1.03	0.00	4.47	0.00	1.67	1.76	0.00
time (sec)	N/A	0.033	0.527	0.165	0.000	0.313	0.000	0.010	0.365	0.000

Problem 1409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	172	159	0	894	0	332	309	0
N.S.	1	1.00	1.06	0.98	0.00	5.52	0.00	2.05	1.91	0.00
time (sec)	N/A	0.047	0.901	0.163	0.000	0.310	0.000	0.014	0.412	0.000

Problem 1410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	222	205	0	1337	0	494	411	0
N.S.	1	1.00	1.12	1.04	0.00	6.75	0.00	2.49	2.08	0.00
time (sec)	N/A	0.062	1.207	0.164	0.000	0.323	0.000	0.017	0.503	0.000

Problem 1411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	30	29	33	105	36	29	87
N.S.	1	1.00	1.00	0.86	0.83	0.94	3.00	1.03	0.83	2.49
time (sec)	N/A	0.007	0.041	0.168	0.351	0.296	0.749	0.001	0.062	2.830

Problem 1412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	40	43	46	168	56	45	165
N.S.	1	1.00	0.77	0.71	0.77	0.82	3.00	1.00	0.80	2.95
time (sec)	N/A	0.009	0.051	0.198	0.368	0.305	1.616	0.002	0.041	3.943

Problem 1413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	216	121	283	261	728	448	137	395
N.S.	1	1.00	1.40	0.79	1.84	1.69	4.73	2.91	0.89	2.56
time (sec)	N/A	0.034	0.103	0.148	0.269	0.294	36.744	0.003	0.069	45.966

Problem 1414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	153	99	204	182	532	314	112	291
N.S.	1	1.00	1.20	0.78	1.61	1.43	4.19	2.47	0.88	2.29
time (sec)	N/A	0.027	0.079	0.145	0.267	0.291	25.947	0.003	0.241	32.612

Problem 1415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	76	137	115	366	202	87	204
N.S.	1	1.00	1.05	0.79	1.43	1.20	3.81	2.10	0.91	2.12
time (sec)	N/A	0.022	0.054	0.147	0.275	0.297	16.892	0.002	0.264	21.994

Problem 1416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	55	82	64	231	116	68	134
N.S.	1	1.00	0.87	0.80	1.19	0.93	3.35	1.68	0.99	1.94
time (sec)	N/A	0.015	0.035	0.145	0.283	0.293	9.655	0.001	0.067	13.226

Problem 1417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	38	39	25	121	51	28	75
N.S.	1	1.00	0.72	0.95	0.98	0.62	3.02	1.28	0.70	1.88
time (sec)	N/A	0.009	0.024	0.131	0.265	0.289	2.114	0.001	0.048	4.624

Problem 1418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	13	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.93	0.86	0.86
time (sec)	N/A	0.001	0.006	0.128	0.264	0.292	0.029	0.000	0.021	1.642

Problem 1419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	37	0	119	44	46	38	49
N.S.	1	1.00	1.00	0.79	0.00	2.53	0.94	0.98	0.81	1.04
time (sec)	N/A	0.013	0.039	0.156	0.000	0.308	2.219	0.001	0.273	4.086

Problem 1420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	87	0	280	0	98	74	0
N.S.	1	1.00	0.99	1.14	0.00	3.68	0.00	1.29	0.97	0.00
time (sec)	N/A	0.020	0.172	0.160	0.000	0.311	0.000	0.002	0.094	0.000

Problem 1421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	138	0	549	0	178	142	0
N.S.	1	1.00	0.84	1.21	0.00	4.82	0.00	1.56	1.25	0.00
time (sec)	N/A	0.027	0.259	0.164	0.000	0.309	0.000	0.003	0.332	0.000

Problem 1422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	128	187	0	884	0	280	218	0
N.S.	1	1.00	0.87	1.27	0.00	6.01	0.00	1.90	1.48	0.00
time (sec)	N/A	0.036	0.317	0.161	0.000	0.309	0.000	0.004	0.395	0.000

Problem 1423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	166	236	0	1325	0	409	307	0
N.S.	1	1.00	0.92	1.31	0.00	7.36	0.00	2.27	1.71	0.00
time (sec)	N/A	0.047	0.518	0.156	0.000	0.323	0.000	0.005	0.455	0.000

Problem 1424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	214	324	267	271	243	465	192	192
N.S.	1	1.00	1.41	2.13	1.76	1.78	1.60	3.06	1.26	1.26
time (sec)	N/A	0.035	0.109	0.154	0.287	0.299	19.450	0.007	0.077	21.124

Problem 1425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	151	219	189	192	168	314	153	130
N.S.	1	1.00	1.23	1.78	1.54	1.56	1.37	2.55	1.24	1.06
time (sec)	N/A	0.029	0.077	0.155	0.278	0.295	14.338	0.005	0.056	14.839

Problem 1426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	136	125	124	109	192	114	82
N.S.	1	1.00	1.05	1.45	1.33	1.32	1.16	2.04	1.21	0.87
time (sec)	N/A	0.023	0.056	0.191	0.276	0.291	10.179	0.003	0.082	10.564

Problem 1427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	74	75	73	65	106	67	48
N.S.	1	1.00	0.88	1.10	1.12	1.09	0.97	1.58	1.00	0.72
time (sec)	N/A	0.016	0.042	0.174	0.270	0.295	6.288	0.002	0.264	7.244

Problem 1428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	33	37	35	60	42	25	46
N.S.	1	1.00	0.71	0.87	0.97	0.92	1.58	1.11	0.66	1.21
time (sec)	N/A	0.011	0.023	0.165	0.276	0.301	0.325	0.001	0.054	1.990

Problem 1429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	15	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	1.07	0.86	0.86
time (sec)	N/A	0.001	0.009	0.137	0.299	0.299	0.032	0.001	0.024	1.593

Problem 1430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	68	0	214	60	83	57	82
N.S.	1	1.00	1.00	0.99	0.00	3.10	0.87	1.20	0.83	1.19
time (sec)	N/A	0.021	0.094	0.187	0.000	0.301	5.295	0.002	0.270	6.269

Problem 1431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	90	100	0	423	0	169	123	0
N.S.	1	1.00	0.91	1.01	0.00	4.27	0.00	1.71	1.24	0.00
time (sec)	N/A	0.028	0.285	0.187	0.000	0.315	0.000	0.003	0.186	0.000

Problem 1432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	126	122	0	782	0	267	205	0
N.S.	1	1.00	0.90	0.87	0.00	5.59	0.00	1.91	1.46	0.00
time (sec)	N/A	0.035	0.501	0.198	0.000	0.324	0.000	0.005	0.444	0.000

Problem 1433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	170	156	0	1204	0	381	294	0
N.S.	1	1.00	0.98	0.90	0.00	6.96	0.00	2.20	1.70	0.00
time (sec)	N/A	0.046	0.656	0.189	0.000	0.317	0.000	0.007	0.541	0.000

Problem 1434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	217	294	265	283	196	421	229	152
N.S.	1	1.00	1.43	1.93	1.74	1.86	1.29	2.77	1.51	1.00
time (sec)	N/A	0.036	0.107	0.169	0.267	0.296	29.403	0.008	0.083	25.161

Problem 1435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	153	198	187	203	136	282	175	105
N.S.	1	1.00	1.22	1.58	1.50	1.62	1.09	2.26	1.40	0.84
time (sec)	N/A	0.029	0.088	0.164	0.283	0.305	22.292	0.006	0.302	19.426

Problem 1436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	122	122	136	461	172	128	136
N.S.	1	1.00	1.05	1.27	1.27	1.42	4.80	1.79	1.33	1.42
time (sec)	N/A	0.022	0.069	0.166	0.270	0.292	0.603	0.004	0.088	5.144

Problem 1437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	66	72	85	265	89	68	87
N.S.	1	1.00	0.93	0.99	1.07	1.27	3.96	1.33	1.01	1.30
time (sec)	N/A	0.017	0.047	0.163	0.285	0.298	0.603	0.003	0.072	3.782

Problem 1438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	28	46	124	42	29	47
N.S.	1	1.00	0.72	0.85	0.70	1.15	3.10	1.05	0.72	1.18
time (sec)	N/A	0.011	0.026	0.135	0.263	0.294	0.501	0.003	0.246	2.757

Problem 1439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	31	14	23	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	1.44	0.75	0.75
time (sec)	N/A	0.001	0.008	0.135	0.276	0.292	0.035	0.001	0.026	1.568

Problem 1440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	85	90	0	398	83	135	100	107
N.S.	1	1.00	0.91	0.97	0.00	4.28	0.89	1.45	1.08	1.15
time (sec)	N/A	0.029	0.172	0.168	0.000	0.312	7.040	0.003	0.333	7.508

Problem 1441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	125	121	0	782	0	241	161	0
N.S.	1	1.00	1.01	0.98	0.00	6.31	0.00	1.94	1.30	0.00
time (sec)	N/A	0.038	0.301	0.168	0.000	0.330	0.000	0.005	0.377	0.000

Problem 1442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	168	143	0	1226	0	344	243	0
N.S.	1	1.00	1.01	0.86	0.00	7.34	0.00	2.06	1.46	0.00
time (sec)	N/A	0.046	0.601	0.168	0.000	0.320	0.000	0.007	0.285	0.000

Problem 1443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	220	177	0	1840	0	469	334	0
N.S.	1	1.00	1.10	0.88	0.00	9.20	0.00	2.34	1.67	0.00
time (sec)	N/A	0.097	0.914	0.174	0.000	0.322	0.000	0.009	0.641	0.000

Problem 1444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	95	66	1080	17	81
N.S.	1	1.00	1.14	0.86	0.82	4.32	3.00	49.09	0.77	3.68
time (sec)	N/A	0.004	0.017	0.155	0.285	0.301	0.717	0.007	0.050	2.281

Problem 1445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	75	66	820	17	81
N.S.	1	1.00	1.14	0.86	0.82	3.41	3.00	37.27	0.77	3.68
time (sec)	N/A	0.004	0.012	0.174	0.265	0.287	0.661	0.005	0.034	2.132

Problem 1446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	374	67	88	608	17	119
N.S.	1	1.00	1.14	0.86	17.00	3.05	4.00	27.64	0.77	5.41
time (sec)	N/A	0.003	0.009	0.158	0.270	0.302	0.683	0.004	0.028	2.640

Problem 1447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	56	88	426	17	119
N.S.	1	1.00	1.14	0.86	0.82	2.55	4.00	19.36	0.77	5.41
time (sec)	N/A	0.004	0.014	0.160	0.273	0.296	0.599	0.003	0.028	2.671

Problem 1448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	45	88	273	17	119
N.S.	1	1.00	1.14	0.86	0.82	2.05	4.00	12.41	0.77	5.41
time (sec)	N/A	0.003	0.009	0.151	0.264	0.293	0.692	0.002	0.028	2.648

Problem 1449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	34	80	155	17	47
N.S.	1	1.00	1.14	0.86	0.82	1.55	3.64	7.05	0.77	2.14
time (sec)	N/A	0.003	0.009	0.178	0.265	0.290	0.891	0.003	0.028	2.490

Problem 1450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	19	18	23	53	67	17	38
N.S.	1	1.00	1.18	0.86	0.82	1.05	2.41	3.05	0.77	1.73
time (sec)	N/A	0.003	0.008	0.155	0.270	0.297	1.198	0.002	0.029	2.598

Problem 1451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	24	19	18	18	29	20	17	33
N.S.	1	1.00	1.20	0.95	0.90	0.90	1.45	1.00	0.85	1.65
time (sec)	N/A	0.003	0.005	0.168	0.277	0.295	1.802	0.002	0.030	2.906

Problem 1452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	24	19	18	29	41	22	17	40
N.S.	1	1.00	1.20	0.95	0.90	1.45	2.05	1.10	0.85	2.00
time (sec)	N/A	0.003	0.006	0.167	0.292	0.291	3.004	0.003	0.030	3.638

Problem 1453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	21	21	26	31	10	29
N.S.	1	1.00	1.00	1.57	1.50	1.50	1.86	2.21	0.71	2.07
time (sec)	N/A	0.003	0.012	0.164	0.285	0.292	0.355	0.001	0.054	1.909

Problem 1454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	19	18	18	61	27	15	49
N.S.	1	1.00	1.00	0.76	0.72	0.72	2.44	1.08	0.60	1.96
time (sec)	N/A	0.007	0.025	0.156	0.376	0.305	0.567	0.001	0.060	2.238

Problem 1455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	112	84	86	86	170	127	104	78
N.S.	1	1.00	1.33	1.00	1.02	1.02	2.02	1.51	1.24	0.93
time (sec)	N/A	0.034	0.099	3.086	0.352	0.302	1.436	0.004	0.068	3.552

Problem 1456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	20	19	19	112	114	14	67
N.S.	1	1.00	0.67	0.74	0.70	0.70	4.15	4.22	0.52	2.48
time (sec)	N/A	0.004	0.013	0.151	0.277	0.287	0.709	0.002	0.255	2.368

Problem 1457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	29	28	24	144	148	21	84
N.S.	1	1.00	0.61	0.76	0.74	0.63	3.79	3.89	0.55	2.21
time (sec)	N/A	0.006	0.015	0.173	0.263	0.288	0.972	0.001	0.046	2.584

Problem 1458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	154	161	0	570	0	280	204	0
N.S.	1	1.00	1.11	1.16	0.00	4.10	0.00	2.01	1.47	0.00
time (sec)	N/A	0.086	0.186	0.173	0.000	0.319	0.000	0.009	0.211	0.000

Problem 1459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	154	160	0	900	0	256	206	0
N.S.	1	1.00	1.10	1.14	0.00	6.43	0.00	1.83	1.47	0.00
time (sec)	N/A	0.056	0.152	0.166	0.000	0.319	0.000	0.007	0.369	0.000

Problem 1460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	166	239	0	702	0	1471	-1	0
N.S.	1	1.00	0.72	1.04	0.00	3.05	0.00	6.40	-0.00	0.00
time (sec)	N/A	0.115	0.407	0.165	0.000	0.336	0.000	0.104	0.000	0.000

Problem 1461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	144	206	0	540	0	972	-1	0
N.S.	1	1.00	0.75	1.07	0.00	2.81	0.00	5.06	-0.01	0.00
time (sec)	N/A	0.073	0.338	0.185	0.000	0.325	0.000	0.070	0.000	0.000

Problem 1462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	128	173	0	410	0	586	-1	0
N.S.	1	1.00	0.83	1.12	0.00	2.66	0.00	3.81	-0.01	0.00
time (sec)	N/A	0.053	0.242	0.153	0.000	0.316	0.000	0.043	0.000	0.000

Problem 1463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	95	140	0	300	0	299	88	0
N.S.	1	1.00	0.82	1.21	0.00	2.59	0.00	2.58	0.76	0.00
time (sec)	N/A	0.041	0.215	0.155	0.000	0.318	0.000	0.021	0.136	0.000

Problem 1464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	107	0	236	0	117	260	0
N.S.	1	1.00	1.03	1.49	0.00	3.28	0.00	1.62	3.61	0.00
time (sec)	N/A	0.027	0.200	0.163	0.000	0.315	0.000	0.010	4.005	0.000

Problem 1465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	0	241	0	127	-1	0
N.S.	1	1.00	1.00	0.00	0.00	3.65	0.00	1.92	-0.02	0.00
time (sec)	N/A	0.024	0.081	0.059	0.000	0.341	0.000	0.025	0.000	0.000

Problem 1466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	88	0	65	0	96	27	0
N.S.	1	1.00	1.00	2.75	0.00	2.03	0.00	3.00	0.84	0.00
time (sec)	N/A	0.002	0.031	0.156	0.000	0.341	0.000	0.039	0.717	0.000

Problem 1467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	128	0	175	0	195	127	0
N.S.	1	1.00	0.70	1.94	0.00	2.65	0.00	2.95	1.92	0.00
time (sec)	N/A	0.006	0.080	0.156	0.000	0.453	0.000	0.063	0.822	0.000

Problem 1468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	168	0	337	0	339	203	0
N.S.	1	1.00	0.76	1.66	0.00	3.34	0.00	3.36	2.01	0.00
time (sec)	N/A	0.013	0.090	0.157	0.000	0.779	0.000	0.106	0.966	0.000

Problem 1469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	208	0	532	0	527	292	0
N.S.	1	1.00	0.68	1.53	0.00	3.91	0.00	3.88	2.15	0.00
time (sec)	N/A	0.020	0.133	0.158	0.000	2.429	0.000	0.149	1.184	0.000

Problem 1470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	117	248	0	781	0	767	397	0
N.S.	1	1.00	0.68	1.45	0.00	4.57	0.00	4.49	2.32	0.00
time (sec)	N/A	0.030	0.142	0.156	0.000	5.205	0.000	0.197	1.432	0.000

Problem 1471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	166	239	0	702	0	2330	-1	0
N.S.	1	1.00	0.73	1.05	0.00	3.09	0.00	10.26	-0.00	0.00
time (sec)	N/A	0.084	0.499	0.157	0.000	0.334	0.000	0.164	0.000	0.000

Problem 1472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	144	206	0	534	0	1443	-1	0
N.S.	1	1.00	0.76	1.09	0.00	2.83	0.00	7.63	-0.01	0.00
time (sec)	N/A	0.065	0.378	0.160	0.000	0.326	0.000	0.108	0.000	0.000

Problem 1473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	173	0	410	0	768	-1	0
N.S.	1	1.00	0.84	1.15	0.00	2.72	0.00	5.09	-0.01	0.00
time (sec)	N/A	0.047	0.236	0.158	0.000	0.332	0.000	0.058	0.000	0.000

Problem 1474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	140	0	306	0	186	-1	0
N.S.	1	1.00	0.83	1.24	0.00	2.71	0.00	1.65	-0.01	0.00
time (sec)	N/A	0.035	0.166	0.156	0.000	0.315	0.000	0.015	0.000	0.000

Problem 1475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	89	0	0	311	0	198	-1	0
N.S.	1	1.00	0.91	0.00	0.00	3.17	0.00	2.02	-0.01	0.00
time (sec)	N/A	0.032	0.412	0.077	0.000	0.349	0.000	0.031	0.000	0.000

Problem 1476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	0	0	325	0	237	-1	0
N.S.	1	1.00	0.88	0.00	0.00	3.53	0.00	2.58	-0.01	0.00
time (sec)	N/A	0.028	0.140	0.066	0.000	0.387	0.000	0.049	0.000	0.000

Problem 1477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	161	0	104	0	149	27	0
N.S.	1	1.00	1.00	5.03	0.00	3.25	0.00	4.66	0.84	0.00
time (sec)	N/A	0.002	0.044	0.180	0.000	0.470	0.000	0.075	0.796	0.000

Problem 1478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	201	0	235	0	276	178	0
N.S.	1	1.00	0.70	3.05	0.00	3.56	0.00	4.18	2.70	0.00
time (sec)	N/A	0.006	0.093	0.165	0.000	0.818	0.000	0.117	0.926	0.000

Problem 1479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	241	0	426	0	448	268	0
N.S.	1	1.00	0.72	2.39	0.00	4.22	0.00	4.44	2.65	0.00
time (sec)	N/A	0.013	0.121	0.166	0.000	2.593	0.000	0.182	1.113	0.000

Problem 1480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	281	0	649	0	671	376	0
N.S.	1	1.00	0.70	2.07	0.00	4.77	0.00	4.93	2.76	0.00
time (sec)	N/A	0.020	0.147	0.166	0.000	5.500	0.000	0.249	1.333	0.000

Problem 1481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	272	0	882	0	4138	-1	0
N.S.	1	1.00	0.73	1.04	0.00	3.37	0.00	15.79	-0.00	0.00
time (sec)	N/A	0.109	0.609	0.165	0.000	0.345	0.000	0.304	0.000	0.000

Problem 1482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	169	239	0	702	0	2623	-1	0
N.S.	1	1.00	0.75	1.07	0.00	3.13	0.00	11.71	-0.00	0.00
time (sec)	N/A	0.086	0.480	0.167	0.000	0.332	0.000	0.192	0.000	0.000

Problem 1483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	144	206	0	540	0	1443	-1	0
N.S.	1	1.00	0.77	1.11	0.00	2.90	0.00	7.76	-0.01	0.00
time (sec)	N/A	0.061	0.331	0.164	0.000	0.323	0.000	0.107	0.000	0.000

Problem 1484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	124	173	0	412	0	263	-1	0
N.S.	1	1.00	0.84	1.17	0.00	2.78	0.00	1.78	-0.01	0.00
time (sec)	N/A	0.047	0.222	0.165	0.000	0.338	0.000	0.019	0.000	0.000

Problem 1485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	0	0	439	0	282	-1	0
N.S.	1	1.00	0.90	0.00	0.00	3.18	0.00	2.04	-0.01	0.00
time (sec)	N/A	0.046	0.250	0.057	0.000	0.383	0.000	0.043	0.000	0.000

Problem 1486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	121	0	0	475	0	352	-1	0
N.S.	1	1.00	0.95	0.00	0.00	3.71	0.00	2.75	-0.01	0.00
time (sec)	N/A	0.044	0.693	0.056	0.000	0.425	0.000	0.069	0.000	0.000

Problem 1487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	463	0	421	-1	0
N.S.	1	1.00	0.92	0.00	0.00	3.86	0.00	3.51	-0.01	0.00
time (sec)	N/A	0.037	0.130	0.057	0.000	0.588	0.000	0.103	0.000	0.000

Problem 1488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	234	0	138	0	199	27	0
N.S.	1	1.00	1.00	7.31	0.00	4.31	0.00	6.22	0.84	0.00
time (sec)	N/A	0.002	0.046	0.158	0.000	0.797	0.000	0.155	0.971	0.000

Problem 1489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	274	0	295	0	355	229	0
N.S.	1	1.00	0.70	4.15	0.00	4.47	0.00	5.38	3.47	0.00
time (sec)	N/A	0.007	0.098	0.158	0.000	2.699	0.000	0.242	1.140	0.000

Problem 1490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	314	0	513	0	561	333	0
N.S.	1	1.00	0.72	3.11	0.00	5.08	0.00	5.55	3.30	0.00
time (sec)	N/A	0.013	0.123	0.159	0.000	6.026	0.000	0.308	1.355	0.000

Problem 1491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F(-2)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	354	0	765	0	811	459	0
N.S.	1	1.00	0.70	2.60	0.00	5.62	0.00	5.96	3.38	0.00
time (sec)	N/A	0.021	0.150	0.166	0.000	12.236	0.000	0.485	1.615	0.000

Problem 1492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	164	206	0	542	0	353	-1	0
N.S.	1	1.00	0.90	1.13	0.00	2.96	0.00	1.93	-0.01	0.00
time (sec)	N/A	0.069	0.191	0.158	0.000	0.332	0.000	0.024	0.000	0.000

Problem 1493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	125	173	0	412	0	263	-1	0
N.S.	1	1.00	0.84	1.17	0.00	2.78	0.00	1.78	-0.01	0.00
time (sec)	N/A	0.050	0.190	0.155	0.000	0.327	0.000	0.019	0.000	0.000

Problem 1494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	140	0	306	0	186	-1	0
N.S.	1	1.00	0.83	1.24	0.00	2.71	0.00	1.65	-0.01	0.00
time (sec)	N/A	0.037	0.174	0.152	0.000	0.314	0.000	0.014	0.000	0.000

Problem 1495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	107	0	235	0	117	261	0
N.S.	1	1.00	1.01	1.47	0.00	3.22	0.00	1.60	3.58	0.00
time (sec)	N/A	0.024	0.193	0.156	0.000	0.314	0.000	0.010	3.803	0.000

Problem 1496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	76	0	178	0	63	45	0
N.S.	1	1.00	1.00	1.81	0.00	4.24	0.00	1.50	1.07	0.00
time (sec)	N/A	0.018	0.061	0.157	0.000	0.309	0.000	0.006	0.288	0.000

Problem 1497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	71	26	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	2.37	0.87	0.00
time (sec)	N/A	0.002	0.027	0.161	0.000	0.303	0.000	0.007	0.732	0.000

Problem 1498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	55	0	118	0	136	71	0
N.S.	1	1.00	0.70	0.83	0.00	1.79	0.00	2.06	1.08	0.00
time (sec)	N/A	0.006	0.071	0.164	0.000	0.335	0.000	0.014	0.894	0.000

Problem 1499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	95	0	251	0	254	133	0
N.S.	1	1.00	0.74	0.94	0.00	2.49	0.00	2.51	1.32	0.00
time (sec)	N/A	0.012	0.083	0.162	0.000	0.462	0.000	0.027	1.005	0.000

Problem 1500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	135	0	419	0	429	209	0
N.S.	1	1.00	0.68	0.99	0.00	3.08	0.00	3.15	1.54	0.00
time (sec)	N/A	0.020	0.116	0.197	0.000	0.729	0.000	0.046	1.192	0.000

Problem 1501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	115	175	0	638	0	663	303	0
N.S.	1	1.00	0.67	1.02	0.00	3.73	0.00	3.88	1.77	0.00
time (sec)	N/A	0.029	0.131	0.155	0.000	2.274	0.000	0.076	1.370	0.000

Problem 1502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	166	0	0	603	0	379	-1	0
N.S.	1	1.00	0.95	0.00	0.00	3.47	0.00	2.18	-0.01	0.00
time (sec)	N/A	0.062	0.290	0.057	0.000	0.450	0.000	0.053	0.000	0.000

Problem 1503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	0	0	441	0	282	-1	0
N.S.	1	1.00	0.90	0.00	0.00	3.20	0.00	2.04	-0.01	0.00
time (sec)	N/A	0.047	0.229	0.058	0.000	0.378	0.000	0.042	0.000	0.000

Problem 1504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	0	0	311	0	199	-1	0
N.S.	1	1.00	0.89	0.00	0.00	3.17	0.00	2.03	-0.01	0.00
time (sec)	N/A	0.032	0.380	0.062	0.000	0.346	0.000	0.030	0.000	0.000

Problem 1505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	0	241	0	127	-1	0
N.S.	1	1.00	1.00	0.00	0.00	3.65	0.00	1.92	-0.02	0.00
time (sec)	N/A	0.023	0.080	0.056	0.000	0.332	0.000	0.023	0.000	0.000

Problem 1506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	72	26	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	2.40	0.87	0.00
time (sec)	N/A	0.002	0.026	0.160	0.000	0.307	0.000	0.009	0.741	0.000

Problem 1507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	42	65	0	125	0	177	71	0
N.S.	1	1.00	0.68	1.05	0.00	2.02	0.00	2.85	1.15	0.00
time (sec)	N/A	0.006	0.084	0.162	0.000	0.338	0.000	0.021	0.858	0.000

Problem 1508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	71	105	0	273	0	433	141	0
N.S.	1	1.00	0.70	1.04	0.00	2.70	0.00	4.29	1.40	0.00
time (sec)	N/A	0.012	0.103	0.162	0.000	0.414	0.000	0.070	1.064	0.000

Problem 1509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	145	0	455	0	955	227	0
N.S.	1	1.00	0.68	1.07	0.00	3.35	0.00	7.02	1.67	0.00
time (sec)	N/A	0.020	0.115	0.160	0.000	0.601	0.000	0.178	1.312	0.000

Problem 1510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	117	185	0	689	0	1731	337	0
N.S.	1	1.00	0.68	1.08	0.00	4.03	0.00	10.12	1.97	0.00
time (sec)	N/A	0.030	0.137	0.158	0.000	1.499	0.000	0.394	1.500	0.000

Problem 1511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	139	225	0	955	0	2771	454	0
N.S.	1	1.00	0.67	1.09	0.00	4.64	0.00	13.45	2.20	0.00
time (sec)	N/A	0.042	0.134	0.161	0.000	3.591	0.000	0.769	1.959	0.000

Problem 1512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	165	0	0	879	0	613	-1	0
N.S.	1	1.00	0.81	0.00	0.00	4.31	0.00	3.00	-0.00	0.00
time (sec)	N/A	0.074	0.372	0.057	0.000	0.818	0.000	0.105	0.000	0.000

Problem 1513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	166	0	0	657	0	475	-1	0
N.S.	1	1.00	0.98	0.00	0.00	3.86	0.00	2.79	-0.01	0.00
time (sec)	N/A	0.056	0.254	0.055	0.000	0.573	0.000	0.085	0.000	0.000

Problem 1514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	123	0	0	475	0	353	-1	0
N.S.	1	1.00	0.96	0.00	0.00	3.71	0.00	2.76	-0.01	0.00
time (sec)	N/A	0.041	0.652	0.054	0.000	0.449	0.000	0.067	0.000	0.000

Problem 1515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	0	0	325	0	238	-1	0
N.S.	1	1.00	0.88	0.00	0.00	3.53	0.00	2.59	-0.01	0.00
time (sec)	N/A	0.028	0.140	0.056	0.000	0.404	0.000	0.049	0.000	0.000

Problem 1516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	88	0	65	0	97	130	0
N.S.	1	1.00	1.00	2.75	0.00	2.03	0.00	3.03	4.06	0.00
time (sec)	N/A	0.002	0.032	0.155	0.000	0.343	0.000	0.037	0.561	0.000

Problem 1517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	55	0	118	0	174	127	0
N.S.	1	1.00	0.70	0.83	0.00	1.79	0.00	2.64	1.92	0.00
time (sec)	N/A	0.007	0.071	0.158	0.000	0.366	0.000	0.015	0.897	0.000

Problem 1518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	95	0	273	0	459	132	0
N.S.	1	1.00	0.74	0.97	0.00	2.79	0.00	4.68	1.35	0.00
time (sec)	N/A	0.013	0.106	0.162	0.000	0.512	0.000	0.039	1.030	0.000

Problem 1519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	92	135	0	447	0	799	224	0
N.S.	1	1.00	0.68	1.00	0.00	3.31	0.00	5.92	1.66	0.00
time (sec)	N/A	0.021	0.126	0.155	0.000	0.663	0.000	0.126	1.289	0.000

Problem 1520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	117	175	0	715	0	1406	346	0
N.S.	1	1.00	0.68	1.02	0.00	4.16	0.00	8.17	2.01	0.00
time (sec)	N/A	0.031	0.138	0.160	0.000	1.954	0.000	0.337	1.531	0.000

Problem 1521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	139	215	0	999	0	2268	478	0
N.S.	1	1.00	0.67	1.04	0.00	4.83	0.00	10.96	2.31	0.00
time (sec)	N/A	0.043	0.129	0.178	0.000	4.344	0.000	0.735	1.908	0.000

Problem 1522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	26	86	48	31	0	27	50	0
N.S.	1	1.00	1.37	4.53	2.53	1.63	0.00	1.42	2.63	0.00
time (sec)	N/A	0.005	0.036	0.170	0.269	0.293	0.000	0.002	0.308	0.000

Problem 1523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	66	33	27	0	26	47	0
N.S.	1	1.00	1.32	3.47	1.74	1.42	0.00	1.37	2.47	0.00
time (sec)	N/A	0.004	0.035	0.169	0.263	0.294	0.000	0.002	0.339	0.000

Problem 1524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	66	33	27	0	26	43	0
N.S.	1	1.00	1.32	3.47	1.74	1.42	0.00	1.37	2.26	0.00
time (sec)	N/A	0.004	0.035	0.158	0.260	0.296	0.000	0.002	0.326	0.000

Problem 1525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	42	60	32	25	15	24	33	14
N.S.	1	1.00	2.47	3.53	1.88	1.47	0.88	1.41	1.94	0.82
time (sec)	N/A	0.003	0.034	0.142	0.273	0.295	0.680	0.002	0.309	2.378

Problem 1526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	64	33	27	0	26	44	0
N.S.	1	1.00	1.32	3.37	1.74	1.42	0.00	1.37	2.32	0.00
time (sec)	N/A	0.004	0.035	0.157	0.270	0.293	0.000	0.002	0.318	0.000

Problem 1527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	26	50	69
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.36	4.55	6.27
time (sec)	N/A	0.002	0.036	0.159	0.260	0.304	18.009	0.002	0.304	15.212

Problem 1528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	66	33	27	0	26	46	0
N.S.	1	1.00	1.32	3.47	1.74	1.42	0.00	1.37	2.42	0.00
time (sec)	N/A	0.004	0.034	0.161	0.262	0.292	0.000	0.002	0.292	0.000

Problem 1529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	25	66	33	28	0	26	47	0
N.S.	1	1.00	1.67	4.40	2.20	1.87	0.00	1.73	3.13	0.00
time (sec)	N/A	0.003	0.035	0.158	0.272	0.296	0.000	0.001	0.293	0.000

Problem 1530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00	1.00
time (sec)	N/A	0.001	0.001	0.140	0.278	0.290	0.031	0.000	0.257	1.636

Problem 1531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	25	66	33	28	0	26	43	0
N.S.	1	1.00	1.67	4.40	2.20	1.87	0.00	1.73	2.87	0.00
time (sec)	N/A	0.003	0.037	0.162	0.275	0.299	0.000	0.001	0.286	0.000

Problem 1532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	42	58	32	25	20	24	37	17
N.S.	1	1.00	2.21	3.05	1.68	1.32	1.05	1.26	1.95	0.89
time (sec)	N/A	0.004	0.004	0.143	0.256	0.292	0.686	0.001	0.280	2.343

Problem 1533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	65	30	28	0	26	44	0
N.S.	1	1.00	1.19	3.10	1.43	1.33	0.00	1.24	2.10	0.00
time (sec)	N/A	0.004	0.036	0.153	0.269	0.287	0.000	0.001	0.286	0.000

Problem 1534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	26	50	69
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.36	4.55	6.27
time (sec)	N/A	0.002	0.002	0.154	0.280	0.302	16.780	0.002	0.002	15.023

Problem 1535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	66	33	28	0	26	50	0
N.S.	1	1.00	1.19	3.14	1.57	1.33	0.00	1.24	2.38	0.00
time (sec)	N/A	0.006	0.036	0.155	0.273	0.297	0.000	0.001	0.283	0.000

Problem 1536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	65	21	44	0	23	44	0
N.S.	1	1.00	1.62	4.06	1.31	2.75	0.00	1.44	2.75	0.00
time (sec)	N/A	0.010	0.041	0.161	0.353	0.300	0.000	0.002	0.084	0.000

Problem 1537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	39	56	9	31	76	19	44	69
N.S.	1	1.00	3.55	5.09	0.82	2.82	6.91	1.73	4.00	6.27
time (sec)	N/A	0.003	0.033	0.161	0.348	0.288	17.166	0.002	0.078	15.093

Problem 1538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	66	19	43	0	23	40	0
N.S.	1	1.00	1.62	4.12	1.19	2.69	0.00	1.44	2.50	0.00
time (sec)	N/A	0.010	0.041	0.169	0.340	0.307	0.000	0.002	0.320	0.000

Problem 1539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	B	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	57	58	18	26	24	21	34	17
N.S.	1	1.00	5.70	5.80	1.80	2.60	2.40	2.10	3.40	1.70
time (sec)	N/A	0.007	0.014	0.148	0.365	0.295	0.827	0.001	0.293	2.322

Problem 1540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	59	66	21	44	0	16	41	0
N.S.	1	1.00	5.36	6.00	1.91	4.00	0.00	1.45	3.73	0.00
time (sec)	N/A	0.008	0.015	0.166	0.352	0.299	0.000	0.001	0.303	0.000

Problem 1541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	C	C	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	26	16	1	87	17	47	153
N.S.	1	1.00	0.97	0.90	0.55	0.03	3.00	0.59	1.62	5.28
time (sec)	N/A	0.003	0.006	0.241	0.255	0.291	1.140	0.001	0.069	3.367

Problem 1542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	C	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	66	21	44	0	28	47	0
N.S.	1	1.00	1.00	2.54	0.81	1.69	0.00	1.08	1.81	0.00
time (sec)	N/A	0.011	0.034	0.160	0.379	0.316	0.000	0.001	0.297	0.000

Problem 1543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	59	70	33	30	0	27	49	0
N.S.	1	1.00	3.69	4.38	2.06	1.88	0.00	1.69	3.06	0.00
time (sec)	N/A	0.004	0.040	0.159	0.273	0.298	0.000	0.001	0.312	0.000

Problem 1544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	11	11	8	11	11	11
N.S.	1	1.00	1.00	1.08	0.92	0.92	0.67	0.92	0.92	0.92
time (sec)	N/A	0.001	0.001	0.148	0.278	0.284	0.034	0.000	0.034	1.692

Problem 1545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	59	70	33	30	0	27	45	0
N.S.	1	1.00	3.69	4.38	2.06	1.88	0.00	1.69	2.81	0.00
time (sec)	N/A	0.004	0.040	0.162	0.280	0.291	0.000	0.002	0.311	0.000

Problem 1546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	48	64	32	27	51	25	39	43
N.S.	1	1.00	2.40	3.20	1.60	1.35	2.55	1.25	1.95	2.15
time (sec)	N/A	0.004	0.005	0.142	0.259	0.292	0.755	0.001	0.282	2.610

Problem 1547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	27	70	33	30	0	27	46	0
N.S.	1	1.00	1.23	3.18	1.50	1.36	0.00	1.23	2.09	0.00
time (sec)	N/A	0.006	0.037	0.158	0.275	0.292	0.000	0.002	0.285	0.000

Problem 1548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	27	61	26	28	78	27	52	71
N.S.	1	1.00	2.25	5.08	2.17	2.33	6.50	2.25	4.33	5.92
time (sec)	N/A	0.002	0.037	0.163	0.305	0.288	16.508	0.002	0.294	15.845

Problem 1549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	27	69	30	30	0	27	52	0
N.S.	1	1.00	1.23	3.14	1.36	1.36	0.00	1.23	2.36	0.00
time (sec)	N/A	0.005	0.037	0.171	0.254	0.291	0.000	0.002	0.292	0.000

Problem 1550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	C	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	26	40	69
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.36	3.64	6.27
time (sec)	N/A	0.002	0.035	0.168	0.252	0.292	17.289	0.001	0.324	16.029

Problem 1551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	100	0	175	0	57	66	0
N.S.	1	1.00	1.19	2.33	0.00	4.07	0.00	1.33	1.53	0.00
time (sec)	N/A	0.012	0.053	0.189	0.000	0.310	0.000	0.015	0.495	0.000

Problem 1552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	48	41	26	42	33	30	34
N.S.	1	1.00	1.36	2.18	1.86	1.18	1.91	1.50	1.36	1.55
time (sec)	N/A	0.003	0.032	0.162	0.379	0.294	0.547	0.004	0.439	2.160

Problem 1553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	57	28	46	56	46	43	43
N.S.	1	1.00	1.00	2.19	1.08	1.77	2.15	1.77	1.65	1.65
time (sec)	N/A	0.009	0.058	0.172	0.363	0.294	0.658	0.002	0.122	2.371

Problem 1554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	58	109	0	176	0	60	63	0
N.S.	1	1.00	1.38	2.60	0.00	4.19	0.00	1.43	1.50	0.00
time (sec)	N/A	0.013	0.064	0.183	0.000	0.313	0.000	0.015	0.515	0.000

Problem 1555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	B	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	38	27	14	14	24	15	16	23
N.S.	1	1.00	3.80	2.70	1.40	1.40	2.40	1.50	1.60	2.30
time (sec)	N/A	0.006	0.026	0.151	0.348	0.295	0.496	0.002	0.290	2.075

Problem 1556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	31	21	21	42	27	27	34
N.S.	1	1.00	1.90	1.55	1.05	1.05	2.10	1.35	1.35	1.70
time (sec)	N/A	0.005	0.050	0.151	0.337	0.291	0.546	0.002	0.295	2.198

Problem 1557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	39	11	44	56	35	40	43
N.S.	1	1.00	1.19	1.50	0.42	1.69	2.15	1.35	1.54	1.65
time (sec)	N/A	0.008	0.044	0.167	0.361	0.295	0.603	0.003	0.080	2.355

Problem 1558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F(-2)	B	F	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	84	0	185	0	66	44	0
N.S.	1	1.00	1.00	1.95	0.00	4.30	0.00	1.53	1.02	0.00
time (sec)	N/A	0.021	0.065	0.167	0.000	0.311	0.000	0.005	0.343	0.000

Problem 1559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.452	10.048	0.006	0.000	0.343	0.000	0.000	0.000	0.000

Problem 1560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.269	10.024	0.004	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.194	10.021	0.005	0.000	0.323	0.000	0.000	0.000	0.000

Problem 1562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.193	10.018	0.057	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.273	10.026	0.064	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.316	10.031	0.057	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.677	10.036	0.007	0.000	0.373	0.000	0.000	0.000	0.000

Problem 1566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	804	804	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.472	10.022	0.005	0.000	0.369	0.000	0.000	0.000	0.000

Problem 1567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	762	762	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.410	10.017	0.059	0.000	0.366	0.000	0.000	0.000	0.000

Problem 1568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.493	10.019	0.061	0.000	0.361	0.000	0.000	0.000	0.000

Problem 1569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	842	842	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.574	10.022	0.062	0.000	0.359	0.000	0.000	0.000	0.000

Problem 1570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.284	10.033	0.006	0.000	0.331	0.000	0.000	0.000	0.000

Problem 1571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.210	10.028	0.004	0.000	0.330	0.000	0.000	0.000	0.000

Problem 1572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.161	10.034	0.056	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.212	10.020	0.055	0.000	0.317	0.000	0.000	0.000	0.000

Problem 1574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.278	10.030	0.059	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	220	0	0	717	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	3.27	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.061	0.517	0.004	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	278	0	0	596	0	0	-1	0
N.S.	1	1.00	1.62	0.00	0.00	3.47	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.031	5.653	0.004	0.000	0.319	0.000	0.000	0.000	0.000

Problem 1577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	191	0	0	233	0	0	-1	0
N.S.	1	1.00	1.28	0.00	0.00	1.56	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.158	0.068	0.000	0.299	0.000	0.000	0.000	0.000

Problem 1578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	65	0	0	92	0
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	2.88	0.00
time (sec)	N/A	0.002	0.033	0.181	0.000	0.289	0.000	0.000	0.713	0.000

Problem 1579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	175	0	0	127	0
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	1.92	0.00
time (sec)	N/A	0.007	0.084	0.192	0.000	0.296	0.000	0.000	1.032	0.000

Problem 1580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	337	0	0	203	0
N.S.	1	1.00	0.72	1.04	0.00	3.34	0.00	0.00	2.01	0.00
time (sec)	N/A	0.013	0.104	0.167	0.000	0.305	0.000	0.000	1.017	0.000

Problem 1581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	533	0	0	293	0
N.S.	1	1.00	0.70	1.26	0.00	3.92	0.00	0.00	2.15	0.00
time (sec)	N/A	0.021	0.125	0.206	0.000	0.298	0.000	0.000	1.152	0.000

Problem 1582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.947	10.027	0.006	0.000	0.301	0.000	0.000	0.000	0.000

Problem 1583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.579	10.024	0.003	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	576	576	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.402	10.026	0.004	0.000	0.303	0.000	0.000	0.000	0.000

Problem 1585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.420	10.018	0.055	0.000	0.314	0.000	0.000	0.000	0.000

Problem 1586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.575	10.027	0.053	0.000	0.301	0.000	0.000	0.000	0.000

Problem 1587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	231	0	0	740	0	0	-1	0
N.S.	1	1.00	1.07	0.00	0.00	3.43	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.062	0.488	0.005	0.000	0.323	0.000	0.000	0.000	0.000

Problem 1588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	278	0	0	618	0	0	-1	0
N.S.	1	1.00	1.63	0.00	0.00	3.61	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.029	5.821	0.003	0.000	0.322	0.000	0.000	0.000	0.000

Problem 1589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	155	0	0	519	0	0	-1	0
N.S.	1	1.00	1.23	0.00	0.00	4.12	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.011	0.133	0.064	0.000	0.309	0.000	0.000	0.000	0.000

Problem 1590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	42	0	0	-1	0
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.002	0.031	0.184	0.000	0.287	0.000	0.000	0.000	0.000

Problem 1591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	-1	0
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.115	0.164	0.000	0.304	0.000	0.000	0.000	0.000

Problem 1592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	251	0	0	-1	0
N.S.	1	1.00	0.72	1.04	0.00	2.49	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.132	0.178	0.000	0.331	0.000	0.000	0.000	0.000

Problem 1593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	420	0	0	-1	0
N.S.	1	1.00	0.70	1.26	0.00	3.09	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.154	0.214	0.000	0.373	0.000	0.000	0.000	0.000

Problem 1594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1365	1365	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.918	10.042	0.053	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1330	1330	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.401	10.025	0.056	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1293	1293	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.118	10.026	0.053	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1257	1257	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.850	10.022	0.065	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1297	1297	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.103	10.024	0.057	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1335	1335	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.368	10.025	0.061	0.000	0.314	0.000	0.000	0.000	0.000

Problem 1600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1372	1372	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.670	10.019	0.061	0.000	0.316	0.000	0.000	0.000	0.000

Problem 1601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	231	0	0	741	0	0	-1	0
N.S.	1	1.00	1.07	0.00	0.00	3.43	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.055	0.325	0.006	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	278	0	0	619	0	0	-1	0
N.S.	1	1.00	1.64	0.00	0.00	3.66	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	6.548	0.005	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	155	0	0	521	0	0	-1	0
N.S.	1	1.00	1.23	0.00	0.00	4.13	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.010	0.121	0.061	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	26	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87	0.00
time (sec)	N/A	0.002	0.035	0.177	0.000	0.292	0.000	0.000	0.828	0.000

Problem 1605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	71	0
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.08	0.00
time (sec)	N/A	0.006	0.083	0.243	0.000	0.294	0.000	0.000	0.977	0.000

Problem 1606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	251	0	0	133	0
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	0.00	1.32	0.00
time (sec)	N/A	0.012	0.095	0.171	0.000	0.298	0.000	0.000	1.508	0.000

Problem 1607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	419	0	0	209	0
N.S.	1	1.00	0.70	1.26	0.00	3.08	0.00	0.00	1.54	0.00
time (sec)	N/A	0.020	0.124	0.188	0.000	0.296	0.000	0.000	1.273	0.000

Problem 1608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.752	10.033	0.007	0.000	0.300	0.000	0.000	0.000	0.000

Problem 1609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.577	10.034	0.008	0.000	0.302	0.000	0.000	0.000	0.000

Problem 1610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.385	10.029	0.004	0.000	0.302	0.000	0.000	0.000	0.000

Problem 1611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.308	10.023	0.059	0.000	0.300	0.000	0.000	0.000	0.000

Problem 1612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.397	10.024	0.062	0.000	0.305	0.000	0.000	0.000	0.000

Problem 1613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.588	10.023	0.070	0.000	0.314	0.000	0.000	0.000	0.000

Problem 1614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	656	656	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.739	10.033	0.066	0.000	0.309	0.000	0.000	0.000	0.000

Problem 1615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	267	0	0	423	0	0	-1	0
N.S.	1	1.00	1.11	0.00	0.00	1.76	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.075	0.484	0.009	0.000	0.308	0.000	0.000	0.000	0.000

Problem 1616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	245	0	0	306	0	0	-1	0
N.S.	1	1.00	1.26	0.00	0.00	1.57	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.048	10.089	0.007	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	191	0	0	233	0	0	-1	0
N.S.	1	1.00	1.28	0.00	0.00	1.56	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.179	0.056	0.000	0.302	0.000	0.000	0.000	0.000

Problem 1618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	-1	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.003	0.034	0.197	0.000	0.300	0.000	0.000	0.000	0.000

Problem 1619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	53	0	126	0	0	-1	0
N.S.	1	1.00	0.68	0.80	0.00	1.91	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.007	0.086	0.165	0.000	0.300	0.000	0.000	0.000	0.000

Problem 1620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	71	105	0	273	0	0	-1	0
N.S.	1	1.00	0.70	1.04	0.00	2.70	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.105	0.169	0.000	0.303	0.000	0.000	0.000	0.000

Problem 1621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	456	0	0	-1	0
N.S.	1	1.00	0.70	1.26	0.00	3.35	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.118	0.168	0.000	0.327	0.000	0.000	0.000	0.000

Problem 1622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1355	1355	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.726	10.046	0.060	0.000	0.322	0.000	0.000	0.000	0.000

Problem 1623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1317	1317	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.373	10.042	0.058	0.000	0.316	0.000	0.000	0.000	0.000

Problem 1624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1279	1279	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.061	10.035	0.055	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1298	1298	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.067	10.025	0.065	0.000	0.315	0.000	0.000	0.000	0.000

Problem 1626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1327	1327	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.397	10.040	0.062	0.000	0.312	0.000	0.000	0.000	0.000

Problem 1627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1370	1370	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.643	10.029	0.063	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	B	C	F	A	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	158	577	0	107	39	0	-1	25
N.S.	1	1.00	2.05	7.49	0.00	1.39	0.51	0.00	-0.01	0.32
time (sec)	N/A	0.011	0.184	0.436	0.000	0.302	1.715	0.000	0.000	3.361

Problem 1629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.148	10.052	0.006	0.000	0.313	0.000	0.000	0.000	0.000

Problem 1630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.066	10.033	0.004	0.000	0.308	0.000	0.000	0.000	0.000

Problem 1631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.047	10.018	0.006	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.045	10.025	0.056	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.065	10.041	0.053	0.000	0.314	0.000	0.000	0.000	0.000

Problem 1634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.088	10.023	0.055	0.000	0.311	0.000	0.000	0.000	0.000

Problem 1635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.255	10.051	0.007	0.000	0.324	0.000	0.000	0.000	0.000

Problem 1636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.177	10.036	0.004	0.000	0.319	0.000	0.000	0.000	0.000

Problem 1637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.158	10.034	0.007	0.000	0.330	0.000	0.000	0.000	0.000

Problem 1638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.152	10.025	0.060	0.000	0.327	0.000	0.000	0.000	0.000

Problem 1639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.165	10.053	0.058	0.000	0.322	0.000	0.000	0.000	0.000

Problem 1640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.196	10.041	0.054	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.125	10.058	0.008	0.000	0.322	0.000	0.000	0.000	0.000

Problem 1642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	A	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	73	0	0	0	218	0	-1	0
N.S.	1	1.00	0.40	0.00	0.00	0.00	1.20	0.00	-0.01	0.00
time (sec)	N/A	0.077	10.036	0.005	0.000	0.325	10.370	0.000	0.000	0.000

Problem 1643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.061	10.037	0.008	0.000	0.314	0.000	0.000	0.000	0.000

Problem 1644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	10.044	0.010	0.000	0.313	0.000	0.000	0.000	0.000

Problem 1645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.053	10.056	0.067	0.000	0.327	0.000	0.000	0.000	0.000

Problem 1646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.070	10.054	0.059	0.000	0.333	0.000	0.000	0.000	0.000

Problem 1647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.078	10.065	0.056	0.000	0.351	0.000	0.000	0.000	0.000

Problem 1648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.197	10.038	0.009	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.170	10.025	0.005	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.151	10.026	0.005	0.000	0.318	0.000	0.000	0.000	0.000

Problem 1651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.136	10.018	0.059	0.000	0.317	0.000	0.000	0.000	0.000

Problem 1652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.147	10.031	0.067	0.000	0.311	0.000	0.000	0.000	0.000

Problem 1653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.155	10.045	0.066	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.056	10.034	0.007	0.000	0.318	0.000	0.000	0.000	0.000

Problem 1655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.044	10.032	0.005	0.000	0.302	0.000	0.000	0.000	0.000

Problem 1656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.036	10.032	0.063	0.000	0.312	0.000	0.000	0.000	0.000

Problem 1657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.045	10.025	0.084	0.000	0.307	0.000	0.000	0.000	0.000

Problem 1658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.057	10.047	0.066	0.000	0.305	0.000	0.000	0.000	0.000

Problem 1659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.184	10.047	0.011	0.000	0.328	0.000	0.000	0.000	0.000

Problem 1660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.163	10.045	0.007	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.38	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.146	10.031	0.060	0.000	0.318	0.000	0.000	0.000	0.000

Problem 1662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.152	10.029	0.059	0.000	0.315	0.000	0.000	0.000	0.000

Problem 1663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.163	10.037	0.062	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.188	10.058	0.065	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.098	10.056	0.010	0.000	0.312	0.000	0.000	0.000	0.000

Problem 1666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	10.036	0.008	0.000	0.311	0.000	0.000	0.000	0.000

Problem 1667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.044	10.030	0.061	0.000	0.305	0.000	0.000	0.000	0.000

Problem 1668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.045	10.039	0.064	0.000	0.305	0.000	0.000	0.000	0.000

Problem 1669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.055	10.041	0.067	0.000	0.310	0.000	0.000	0.000	0.000

Problem 1670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.073	10.054	0.071	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.211	10.065	0.012	0.000	0.364	0.000	0.000	0.000	0.000

Problem 1672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.187	10.060	0.010	0.000	0.334	0.000	0.000	0.000	0.000

Problem 1673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.162	10.049	0.064	0.000	0.316	0.000	0.000	0.000	0.000

Problem 1674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.166	10.047	0.064	0.000	0.327	0.000	0.000	0.000	0.000

Problem 1675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.168	10.042	0.061	0.000	0.327	0.000	0.000	0.000	0.000

Problem 1676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.186	10.035	0.063	0.000	0.349	0.000	0.000	0.000	0.000

Problem 1677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.214	10.043	0.087	0.000	0.348	0.000	0.000	0.000	0.000

Problem 1678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	174	0	0	2151	0	0	-1	0
N.S.	1	1.00	0.85	0.00	0.00	10.49	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.099	0.505	0.006	0.000	0.421	0.000	0.000	0.000	0.000

Problem 1679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	0	0	1468	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	8.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.073	0.386	0.007	0.000	0.376	0.000	0.000	0.000	0.000

Problem 1680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	71	0	0	857	0	0	-1	0
N.S.	1	1.00	0.47	0.00	0.00	5.64	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.068	10.044	0.007	0.000	0.354	0.000	0.000	0.000	0.000

Problem 1681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	0	0	368	0	0	-1	0
N.S.	1	1.00	0.91	0.00	0.00	2.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.062	0.270	0.059	0.000	0.332	0.000	0.000	0.000	0.000

Problem 1682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	104	0	0	99	0
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	0.00	3.09	0.00
time (sec)	N/A	0.002	0.046	0.173	0.000	0.326	0.000	0.000	0.810	0.000

Problem 1683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	235	0	0	178	0
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	0.00	2.70	0.00
time (sec)	N/A	0.007	0.130	0.225	0.000	0.368	0.000	0.000	0.955	0.000

Problem 1684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	426	0	0	268	0
N.S.	1	1.00	0.72	1.04	0.00	4.22	0.00	0.00	2.65	0.00
time (sec)	N/A	0.013	0.157	0.218	0.000	0.382	0.000	0.000	1.126	0.000

Problem 1685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	649	0	0	376	0
N.S.	1	1.00	0.70	1.26	0.00	4.77	0.00	0.00	2.76	0.00
time (sec)	N/A	0.021	0.176	0.171	0.000	0.456	0.000	0.000	1.360	0.000

Problem 1686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.412	10.048	0.006	0.000	0.321	0.000	0.000	0.000	0.000

Problem 1687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.265	10.042	0.006	0.000	0.306	0.000	0.000	0.000	0.000

Problem 1688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.208	10.036	0.006	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.201	10.038	0.008	0.000	0.319	0.000	0.000	0.000	0.000

Problem 1690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.218	10.039	0.062	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.252	10.046	0.059	0.000	0.349	0.000	0.000	0.000	0.000

Problem 1692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.310	10.052	0.060	0.000	0.388	0.000	0.000	0.000	0.000

Problem 1693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	0	0	1468	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	8.79	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.071	0.341	0.006	0.000	0.379	0.000	0.000	0.000	0.000

Problem 1694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	129	0	0	814	0	0	-1	0
N.S.	1	1.00	1.02	0.00	0.00	6.41	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.050	6.506	0.003	0.000	0.337	0.000	0.000	0.000	0.000

Problem 1695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	0	0	234	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	2.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.040	0.084	0.065	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	42	0	0	-1	0
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.002	0.031	0.189	0.000	0.327	0.000	0.000	0.000	0.000

Problem 1697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	-1	0
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.007	0.114	0.218	0.000	0.425	0.000	0.000	0.000	0.000

Problem 1698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	252	0	0	-1	0
N.S.	1	1.00	0.72	1.04	0.00	2.50	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.133	0.209	0.000	0.647	0.000	0.000	0.000	0.000

Problem 1699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	419	0	0	-1	0
N.S.	1	1.00	0.70	1.26	0.00	3.08	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.156	0.210	0.000	1.260	0.000	0.000	0.000	0.000

Problem 1700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.556	10.031	0.059	0.000	0.343	0.000	0.000	0.000	0.000

Problem 1701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	705	705	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.439	10.033	0.057	0.000	0.338	0.000	0.000	0.000	0.000

Problem 1702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	688	688	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.362	10.023	0.061	0.000	0.337	0.000	0.000	0.000	0.000

Problem 1703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.443	10.028	0.064	0.000	0.355	0.000	0.000	0.000	0.000

Problem 1704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	760	760	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.531	10.032	0.067	0.000	0.334	0.000	0.000	0.000	0.000

Problem 1705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	0	0	1457	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	8.72	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.072	0.239	0.006	0.000	0.372	0.000	0.000	0.000	0.000

Problem 1706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	131	0	0	808	0	0	-1	0
N.S.	1	1.00	1.03	0.00	0.00	6.36	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.058	7.411	0.004	0.000	0.340	0.000	0.000	0.000	0.000

Problem 1707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	0	0	234	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	2.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.047	0.084	0.067	0.000	0.313	0.000	0.000	0.000	0.000

Problem 1708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	26	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87	0.00
time (sec)	N/A	0.002	0.037	0.185	0.000	0.293	0.000	0.000	0.706	0.000

Problem 1709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	71	0
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.08	0.00
time (sec)	N/A	0.007	0.085	0.187	0.000	0.292	0.000	0.000	0.868	0.000

Problem 1710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	251	0	0	133	0
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	0.00	1.32	0.00
time (sec)	N/A	0.013	0.100	0.204	0.000	0.303	0.000	0.000	1.020	0.000

Problem 1711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	171	0	419	0	0	209	0
N.S.	1	1.00	0.68	1.26	0.00	3.08	0.00	0.00	1.54	0.00
time (sec)	N/A	0.021	0.134	0.181	0.000	0.308	0.000	0.000	1.260	0.000

Problem 1712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.212	10.027	0.007	0.000	0.353	0.000	0.000	0.000	0.000

Problem 1713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.172	10.023	0.004	0.000	0.311	0.000	0.000	0.000	0.000

Problem 1714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.136	10.021	0.072	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.168	10.034	0.069	0.000	0.301	0.000	0.000	0.000	0.000

Problem 1716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.210	10.025	0.069	0.000	0.348	0.000	0.000	0.000	0.000

Problem 1717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	73	0	0	857	0	0	-1	0
N.S.	1	1.00	0.48	0.00	0.00	5.64	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.065	10.046	0.007	0.000	0.383	0.000	0.000	0.000	0.000

Problem 1718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	0	0	273	0	0	-1	0
N.S.	1	1.00	0.97	0.00	0.00	2.53	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.047	0.123	0.056	0.000	0.309	0.000	0.000	0.000	0.000

Problem 1719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	-1	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.003	0.037	0.212	0.000	0.300	0.000	0.000	0.000	0.000

Problem 1720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	53	0	126	0	0	-1	0
N.S.	1	1.00	0.68	0.80	0.00	1.91	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.007	0.087	0.175	0.000	0.295	0.000	0.000	0.000	0.000

Problem 1721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	273	0	0	-1	0
N.S.	1	1.00	0.72	1.04	0.00	2.70	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.107	0.193	0.000	0.422	0.000	0.000	0.000	0.000

Problem 1722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	457	0	0	-1	0
N.S.	1	1.00	0.70	1.26	0.00	3.36	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.021	0.121	0.211	0.000	0.618	0.000	0.000	0.000	0.000

Problem 1723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.631	10.056	0.069	0.000	0.340	0.000	0.000	0.000	0.000

Problem 1724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.539	10.043	0.069	0.000	0.369	0.000	0.000	0.000	0.000

Problem 1725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.455	10.036	0.061	0.000	0.352	0.000	0.000	0.000	0.000

Problem 1726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	719	719	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.447	10.035	0.067	0.000	0.355	0.000	0.000	0.000	0.000

Problem 1727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.546	10.038	0.071	0.000	0.345	0.000	0.000	0.000	0.000

Problem 1728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.641	10.035	0.072	0.000	0.348	0.000	0.000	0.000	0.000

Problem 1729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	156	0	0	247	0	0	-1	0
N.S.	1	1.00	0.56	0.00	0.00	0.89	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.219	0.196	0.068	0.000	0.336	0.000	0.000	0.000	0.000

Problem 1730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	96	0	0	448	0	0	-1	0
N.S.	1	1.00	0.50	0.00	0.00	2.32	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.096	0.103	0.059	0.000	0.334	0.000	0.000	0.000	0.000

Problem 1731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.019	10.024	0.008	0.000	1.718	0.000	0.000	0.000	0.000

Problem 1732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.029	0.006	0.000	1.740	0.000	0.000	0.000	0.000

Problem 1733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	10.026	0.062	0.000	1.680	0.000	0.000	0.000	0.000

Problem 1734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.028	0.061	0.000	1.735	0.000	0.000	0.000	0.000

Problem 1735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	10.025	0.064	0.000	1.591	0.000	0.000	0.000	0.000

Problem 1736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.403	10.066	0.006	0.000	0.382	0.000	0.000	0.000	0.000

Problem 1737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.249	10.023	0.006	0.000	0.359	0.000	0.000	0.000	0.000

Problem 1738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.210	10.021	0.005	0.000	0.346	0.000	0.000	0.000	0.000

Problem 1739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.173	10.027	0.006	0.000	0.345	0.000	0.000	0.000	0.000

Problem 1740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.173	10.034	0.060	0.000	0.340	0.000	0.000	0.000	0.000

Problem 1741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.206	10.049	0.059	0.000	0.364	0.000	0.000	0.000	0.000

Problem 1742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	896	896	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.777	10.032	0.006	0.000	0.421	0.000	0.000	0.000	0.000

Problem 1743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.573	10.026	0.005	0.000	0.425	0.000	0.000	0.000	0.000

Problem 1744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	817	817	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.495	10.040	0.006	0.000	0.423	0.000	0.000	0.000	0.000

Problem 1745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	798	798	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.462	10.028	0.061	0.000	0.429	0.000	0.000	0.000	0.000

Problem 1746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	854	854	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.552	10.047	0.058	0.000	0.421	0.000	0.000	0.000	0.000

Problem 1747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	896	896	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.635	10.037	0.054	0.000	0.437	0.000	0.000	0.000	0.000

Problem 1748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	890	890	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.649	10.026	0.007	0.000	0.469	0.000	0.000	0.000	0.000

Problem 1749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.534	10.022	0.006	0.000	0.420	0.000	0.000	0.000	0.000

Problem 1750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	820	820	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.464	10.022	0.004	0.000	0.432	0.000	0.000	0.000	0.000

Problem 1751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.399	10.024	0.059	0.000	0.408	0.000	0.000	0.000	0.000

Problem 1752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	813	813	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.459	10.037	0.060	0.000	0.426	0.000	0.000	0.000	0.000

Problem 1753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.519	10.057	0.064	0.000	0.409	0.000	0.000	0.000	0.000

Problem 1754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.236	10.034	0.008	0.000	0.336	0.000	0.000	0.000	0.000

Problem 1755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.199	10.033	0.007	0.000	0.337	0.000	0.000	0.000	0.000

Problem 1756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.169	10.032	0.005	0.000	0.361	0.000	0.000	0.000	0.000

Problem 1757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.137	10.025	0.055	0.000	0.341	0.000	0.000	0.000	0.000

Problem 1758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.162	10.019	0.056	0.000	0.358	0.000	0.000	0.000	0.000

Problem 1759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.205	10.040	0.059	0.000	0.351	0.000	0.000	0.000	0.000

Problem 1760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	880	880	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.649	10.054	0.008	0.000	0.456	0.000	0.000	0.000	0.000

Problem 1761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	844	844	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.561	10.043	0.007	0.000	0.437	0.000	0.000	0.000	0.000

Problem 1762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	806	806	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.479	10.041	0.057	0.000	0.461	0.000	0.000	0.000	0.000

Problem 1763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	817	817	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.484	10.037	0.060	0.000	0.451	0.000	0.000	0.000	0.000

Problem 1764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	844	844	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.552	10.039	0.059	0.000	0.445	0.000	0.000	0.000	0.000

Problem 1765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	893	893	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.616	10.054	0.062	0.000	0.429	0.000	0.000	0.000	0.000

Problem 1766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.060	0.005	0.000	0.329	0.000	0.000	0.000	0.000

Problem 1767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.038	0.004	0.000	0.356	0.000	0.000	0.000	0.000

Problem 1768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.032	0.003	0.000	0.329	0.000	0.000	0.000	0.000

Problem 1769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.034	0.003	0.000	0.376	0.000	0.000	0.000	0.000

Problem 1770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.055	0.056	0.000	0.342	0.000	0.000	0.000	0.000

Problem 1771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	81	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.033	0.063	0.000	0.329	0.000	0.000	0.000	0.000

Problem 1772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	267	0	0	5633	0	0	-1	0
N.S.	1	1.00	0.63	0.00	0.00	13.19	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.446	0.805	0.005	0.000	0.435	0.000	0.000	0.000	0.000

Problem 1773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	291	0	0	3025	0	0	-1	0
N.S.	1	1.00	0.77	0.00	0.00	8.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.357	10.209	0.005	0.000	0.447	0.000	0.000	0.000	0.000

Problem 1774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	239	0	0	663	0	0	-1	0
N.S.	1	1.00	0.72	0.00	0.00	2.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.341	0.249	0.054	0.000	0.357	0.000	0.000	0.000	0.000

Problem 1775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	65	0	0	130	0
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	4.06	0.00
time (sec)	N/A	0.003	0.033	0.171	0.000	0.300	0.000	0.000	0.565	0.000

Problem 1776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	175	0	0	137	0
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	2.08	0.00
time (sec)	N/A	0.007	0.119	0.199	0.000	0.328	0.000	0.000	0.750	0.000

Problem 1777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	338	0	0	213	0
N.S.	1	1.00	0.72	1.04	0.00	3.35	0.00	0.00	2.11	0.00
time (sec)	N/A	0.014	0.137	0.171	0.000	0.361	0.000	0.000	0.949	0.000

Problem 1778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	533	0	0	302	0
N.S.	1	1.00	0.70	1.26	0.00	3.92	0.00	0.00	2.22	0.00
time (sec)	N/A	0.025	0.157	0.210	0.000	0.341	0.000	0.000	1.147	0.000

Problem 1779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	267	0	0	5633	0	0	-1	0
N.S.	1	1.00	0.63	0.00	0.00	13.19	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.445	0.771	0.004	0.000	0.444	0.000	0.000	0.000	0.000

Problem 1780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	2997	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	7.93	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.398	10.035	0.003	0.000	0.400	0.000	0.000	0.000	0.000

Problem 1781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	244	0	0	755	0	0	-1	0
N.S.	1	1.00	0.73	0.00	0.00	2.26	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.386	0.302	0.053	0.000	0.409	0.000	0.000	0.000	0.000

Problem 1782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	65	0	0	130	0
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	4.06	0.00
time (sec)	N/A	0.003	0.041	0.175	0.000	0.340	0.000	0.000	0.587	0.000

Problem 1783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	175	0	0	137	0
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	2.08	0.00
time (sec)	N/A	0.007	0.127	0.177	0.000	0.331	0.000	0.000	0.744	0.000

Problem 1784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	338	0	0	214	0
N.S.	1	1.00	0.72	1.04	0.00	3.35	0.00	0.00	2.12	0.00
time (sec)	N/A	0.014	0.152	0.201	0.000	0.342	0.000	0.000	0.944	0.000

Problem 1785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	533	0	0	303	0
N.S.	1	1.00	0.70	1.26	0.00	3.92	0.00	0.00	2.23	0.00
time (sec)	N/A	0.023	0.171	0.198	0.000	0.349	0.000	0.000	1.158	0.000

Problem 1786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.054	0.063	0.000	0.575	0.000	0.000	0.000	0.000

Problem 1787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.026	0.053	0.000	0.606	0.000	0.000	0.000	0.000

Problem 1788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	10.048	0.058	0.000	0.592	0.000	0.000	0.000	0.000

Problem 1789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	10.044	0.055	0.000	0.570	0.000	0.000	0.000	0.000

Problem 1790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.052	0.056	0.000	0.631	0.000	0.000	0.000	0.000

Problem 1791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	0	0	0	0	0	-1	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.037	0.056	0.000	0.564	0.000	0.000	0.000	0.000

Problem 1792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.077	0.006	0.000	0.362	0.000	0.000	0.000	0.000

Problem 1793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.053	0.004	0.000	0.359	0.000	0.000	0.000	0.000

Problem 1794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.024	0.006	0.000	0.347	0.000	0.000	0.000	0.000

Problem 1795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.028	0.006	0.000	0.387	0.000	0.000	0.000	0.000

Problem 1796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.053	0.007	0.000	0.387	0.000	0.000	0.000	0.000

Problem 1797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	81	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.043	0.064	0.000	0.354	0.000	0.000	0.000	0.000

Problem 1798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	288	0	0	5633	0	0	-1	0
N.S.	1	1.00	0.68	0.00	0.00	13.29	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.400	0.680	0.007	0.000	0.455	0.000	0.000	0.000	0.000

Problem 1799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	73	0	0	3084	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	7.65	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.392	10.037	0.013	0.000	0.398	0.000	0.000	0.000	0.000

Problem 1800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	257	0	0	855	0	0	-1	0
N.S.	1	1.00	0.72	0.00	0.00	2.39	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.370	0.407	0.055	0.000	0.343	0.000	0.000	0.000	0.000

Problem 1801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	104	0	0	199	0
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	0.00	6.22	0.00
time (sec)	N/A	0.003	0.044	0.171	0.000	0.339	0.000	0.000	0.756	0.000

Problem 1802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	235	0	0	189	0
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	0.00	2.86	0.00
time (sec)	N/A	0.007	0.137	0.175	0.000	0.345	0.000	0.000	0.907	0.000

Problem 1803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	427	0	0	278	0
N.S.	1	1.00	0.72	1.04	0.00	4.23	0.00	0.00	2.75	0.00
time (sec)	N/A	0.014	0.162	0.201	0.000	0.360	0.000	0.000	1.144	0.000

Problem 1804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	649	0	0	385	0
N.S.	1	1.00	0.70	1.26	0.00	4.77	0.00	0.00	2.83	0.00
time (sec)	N/A	0.023	0.198	0.175	0.000	0.322	0.000	0.000	1.429	0.000

Problem 1805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	278	0	0	5633	0	0	-1	0
N.S.	1	1.00	0.66	0.00	0.00	13.29	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.453	0.699	0.006	0.000	0.413	0.000	0.000	0.000	0.000

Problem 1806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	3025	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	8.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.405	10.024	0.005	0.000	0.399	0.000	0.000	0.000	0.000

Problem 1807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	194	0	0	620	0	0	-1	0
N.S.	1	1.00	0.63	0.00	0.00	2.01	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.363	0.169	0.055	0.000	0.334	0.000	0.000	0.000	0.000

Problem 1808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	42	0	0	27	0
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	0.84	0.00
time (sec)	N/A	0.002	0.034	0.193	0.000	0.328	0.000	0.000	0.763	0.000

Problem 1809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	127	0
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.92	0.00
time (sec)	N/A	0.007	0.085	0.197	0.000	0.326	0.000	0.000	0.860	0.000

Problem 1810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	252	0	0	203	0
N.S.	1	1.00	0.72	1.04	0.00	2.50	0.00	0.00	2.01	0.00
time (sec)	N/A	0.014	0.106	0.205	0.000	0.326	0.000	0.000	1.033	0.000

Problem 1811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	420	0	0	292	0
N.S.	1	1.00	0.70	1.26	0.00	3.09	0.00	0.00	2.15	0.00
time (sec)	N/A	0.023	0.117	0.233	0.000	0.321	0.000	0.000	1.200	0.000

Problem 1812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.042	0.056	0.000	0.597	0.000	0.000	0.000	0.000

Problem 1813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.052	0.056	0.000	0.574	0.000	0.000	0.000	0.000

Problem 1814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.017	0.051	0.000	0.600	0.000	0.000	0.000	0.000

Problem 1815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.029	0.052	0.000	0.560	0.000	0.000	0.000	0.000

Problem 1816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.038	0.056	0.000	0.546	0.000	0.000	0.000	0.000

Problem 1817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	0	0	0	0	0	-1	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.047	0.060	0.000	0.555	0.000	0.000	0.000	0.000

Problem 1818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.044	0.008	0.000	0.368	0.000	0.000	0.000	0.000

Problem 1819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.030	0.006	0.000	0.332	0.000	0.000	0.000	0.000

Problem 1820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.027	0.003	0.000	0.318	0.000	0.000	0.000	0.000

Problem 1821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.021	0.054	0.000	0.350	0.000	0.000	0.000	0.000

Problem 1822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.045	0.058	0.000	0.328	0.000	0.000	0.000	0.000

Problem 1823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.029	0.064	0.000	0.354	0.000	0.000	0.000	0.000

Problem 1824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	278	0	0	5591	0	0	-1	0
N.S.	1	1.00	0.66	0.00	0.00	13.19	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.408	0.442	0.006	0.000	0.416	0.000	0.000	0.000	0.000

Problem 1825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	71	0	0	2997	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	7.93	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.353	10.022	0.005	0.000	0.415	0.000	0.000	0.000	0.000

Problem 1826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	194	0	0	620	0	0	-1	0
N.S.	1	1.00	0.63	0.00	0.00	2.01	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.318	0.160	0.056	0.000	0.360	0.000	0.000	0.000	0.000

Problem 1827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	-1	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.002	0.036	0.174	0.000	0.312	0.000	0.000	0.000	0.000

Problem 1828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	53	0	118	0	0	-1	0
N.S.	1	1.00	0.70	0.80	0.00	1.79	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.007	0.080	0.174	0.000	0.345	0.000	0.000	0.000	0.000

Problem 1829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	252	0	0	-1	0
N.S.	1	1.00	0.76	1.04	0.00	2.50	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.102	0.200	0.000	0.328	0.000	0.000	0.000	0.000

Problem 1830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	171	0	420	0	0	-1	0
N.S.	1	1.00	0.68	1.26	0.00	3.09	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.132	0.172	0.000	0.317	0.000	0.000	0.000	0.000

Problem 1831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	308	0	0	5690	0	0	-1	0
N.S.	1	1.00	0.69	0.00	0.00	12.67	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.472	0.765	0.009	0.000	0.422	0.000	0.000	0.000	0.000

Problem 1832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	C	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	71	0	0	3084	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	7.65	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.429	10.050	0.007	0.000	0.384	0.000	0.000	0.000	0.000

Problem 1833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	239	0	0	663	0	0	-1	0
N.S.	1	1.00	0.72	0.00	0.00	2.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.398	0.230	0.051	0.000	0.347	0.000	0.000	0.000	0.000

Problem 1834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	26	0
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87	0.00
time (sec)	N/A	0.003	0.037	0.174	0.000	0.311	0.000	0.000	0.680	0.000

Problem 1835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	53	0	126	0	0	72	0
N.S.	1	1.00	0.70	0.83	0.00	1.97	0.00	0.00	1.12	0.00
time (sec)	N/A	0.008	0.086	0.175	0.000	0.305	0.000	0.000	0.834	0.000

Problem 1836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	105	0	273	0	0	132	0
N.S.	1	1.00	0.74	1.07	0.00	2.79	0.00	0.00	1.35	0.00
time (sec)	N/A	0.016	0.105	0.171	0.000	0.322	0.000	0.000	0.960	0.000

Problem 1837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	95	171	0	457	0	0	209	0
N.S.	1	1.00	0.71	1.28	0.00	3.41	0.00	0.00	1.56	0.00
time (sec)	N/A	0.024	0.120	0.173	0.000	0.329	0.000	0.000	1.146	0.000

Problem 1838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	10.055	0.054	0.000	0.547	0.000	0.000	0.000	0.000

Problem 1839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.016	10.025	0.056	0.000	0.577	0.000	0.000	0.000	0.000

Problem 1840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.021	0.059	0.000	0.576	0.000	0.000	0.000	0.000

Problem 1841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.040	0.058	0.000	0.540	0.000	0.000	0.000	0.000

Problem 1842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.049	0.062	0.000	0.538	0.000	0.000	0.000	0.000

Problem 1843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	0	0	0	0	0	-1	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.015	10.035	0.056	0.000	0.561	0.000	0.000	0.000	0.000

Problem 1844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	106	17	20	27	11	11
N.S.	1	1.00	1.00	1.09	9.64	1.55	1.82	2.45	1.00	1.00
time (sec)	N/A	0.002	0.043	0.150	0.265	0.313	0.125	0.003	0.462	1.854

Problem 1845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	74	73	0	0	0	0	0	-1	0
N.S.	1	1.21	1.20	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.019	0.041	0.092	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	389	246	497	4058	920	478	2986
N.S.	1	1.00	0.85	3.54	2.24	4.52	36.89	8.36	4.35	27.15
time (sec)	N/A	0.043	0.092	0.182	0.294	0.326	2.650	0.004	0.942	15.827

Problem 1847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	138	235	1506	427	226	1256
N.S.	1	1.00	0.86	2.04	1.77	3.01	19.31	5.47	2.90	16.10
time (sec)	N/A	0.026	0.083	0.191	0.285	0.319	0.732	0.003	0.655	6.812

Problem 1848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	49	63	83	377	147	88	383
N.S.	1	1.00	0.89	1.07	1.37	1.80	8.20	3.20	1.91	8.33
time (sec)	N/A	0.014	0.045	0.146	0.273	0.328	0.436	0.003	0.484	3.043

Problem 1849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.040	0.055	0.000	0.318	0.000	0.000	0.000	0.000

Problem 1850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.042	0.061	0.000	0.322	0.000	0.000	0.000	0.000

Problem 1851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.010	0.043	0.087	0.000	0.332	0.000	0.000	0.000	0.000

Problem 1852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	B	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	386	246	496	4058	920	478	2986
N.S.	1	1.00	0.86	3.48	2.22	4.47	36.56	8.29	4.31	26.90
time (sec)	N/A	0.040	0.091	0.178	0.269	0.348	1.315	0.004	0.913	16.789

Problem 1853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	138	237	1506	427	226	1256
N.S.	1	1.00	0.86	2.04	1.77	3.04	19.31	5.47	2.90	16.10
time (sec)	N/A	0.024	0.075	0.188	0.289	0.325	0.810	0.003	0.618	6.897

Problem 1854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	46	63	83	377	147	88	383
N.S.	1	1.00	0.87	0.98	1.34	1.77	8.02	3.13	1.87	8.15
time (sec)	N/A	0.015	0.042	0.163	0.265	0.319	0.331	0.003	0.494	3.071

Problem 1855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	16	18	0
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	0.89	1.00	0.00
time (sec)	N/A	0.002	0.017	0.150	0.289	0.304	0.033	0.000	0.379	0.000

Problem 1856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.011	0.039	0.059	0.000	0.320	0.000	0.000	0.000	0.000

Problem 1857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	0	0	0	0	0	-1	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.009	0.041	0.064	0.000	0.323	0.000	0.000	0.000	0.000

Problem 1858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.012	0.044	0.081	0.000	0.315	0.000	0.000	0.000	0.000

Problem 1859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	112	322	0	512	0	0	528	0
N.S.	1	1.00	0.78	2.25	0.00	3.58	0.00	0.00	3.69	0.00
time (sec)	N/A	0.047	0.078	0.222	0.000	0.349	0.000	0.000	1.096	0.000

Problem 1860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	59	127	0	206	0	0	220	0
N.S.	1	1.00	0.69	1.48	0.00	2.40	0.00	0.00	2.56	0.00
time (sec)	N/A	0.010	0.051	0.189	0.000	0.333	0.000	0.000	0.767	0.000

Problem 1861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	0	60	0	0	102	0
N.S.	1	1.00	0.92	1.15	0.00	1.54	0.00	0.00	2.62	0.00
time (sec)	N/A	0.003	0.038	0.192	0.000	0.319	0.000	0.000	0.559	0.000

Problem 1862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	0	0	0	0	0	-1	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.017	0.042	0.084	0.000	0.330	0.000	0.000	0.000	0.000

Problem 1863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.035	0.078	0.000	0.323	0.000	0.000	0.000	0.000

Problem 1864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	89	0	0	0	0	0	-1	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.056	0.085	0.000	0.323	0.000	0.000	0.000	0.000

Problem 1865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	92	0	0	0	0	0	-1	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.017	0.057	0.080	0.000	0.344	0.000	0.000	0.000	0.000

Problem 1866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.024	0.036	0.073	0.000	0.326	0.000	0.000	0.000	0.000

Problem 1867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.043	0.108	0.000	0.325	0.000	0.000	0.000	0.000

Problem 1868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-2)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	41	0	59	0	0	97	0
N.S.	1	1.00	1.03	1.11	0.00	1.59	0.00	0.00	2.62	0.00
time (sec)	N/A	0.005	0.039	0.189	0.000	0.346	0.000	0.000	0.532	0.000

Problem 1869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	123	0	207	0	0	214	0
N.S.	1	1.00	0.75	1.54	0.00	2.59	0.00	0.00	2.68	0.00
time (sec)	N/A	0.016	0.050	0.188	0.000	0.318	0.000	0.000	0.736	0.000

Problem 1870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	113	318	0	509	0	0	525	0
N.S.	1	1.00	0.86	2.43	0.00	3.89	0.00	0.00	4.01	0.00
time (sec)	N/A	0.039	0.071	0.194	0.000	0.332	0.000	0.000	0.992	0.000

Problem 1871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	195	661	0	959	0	0	944	0
N.S.	1	1.00	1.05	3.55	0.00	5.16	0.00	0.00	5.08	0.00
time (sec)	N/A	0.067	0.091	0.211	0.000	0.334	0.000	0.000	1.642	0.000

Problem 1872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.018	0.005	0.004	0.000	0.319	0.000	0.000	0.000	0.000

Problem 1873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.020	0.046	0.087	0.000	0.328	0.000	0.000	0.000	0.000

Problem 1874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	42	0	58	0	0	98	0
N.S.	1	1.00	1.00	1.17	0.00	1.61	0.00	0.00	2.72	0.00
time (sec)	N/A	0.004	0.037	0.193	0.000	0.328	0.000	0.000	0.556	0.000

Problem 1875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	59	124	0	205	0	0	214	0
N.S.	1	1.00	0.75	1.57	0.00	2.59	0.00	0.00	2.71	0.00
time (sec)	N/A	0.014	0.050	0.233	0.000	0.320	0.000	0.000	0.745	0.000

Problem 1876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	112	319	0	507	0	0	528	0
N.S.	1	1.00	0.86	2.45	0.00	3.90	0.00	0.00	4.06	0.00
time (sec)	N/A	0.033	0.073	0.244	0.000	0.334	0.000	0.000	1.023	0.000

Problem 1877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	B	F	B	F(-1)	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	195	662	0	954	0	0	945	0
N.S.	1	1.00	1.05	3.58	0.00	5.16	0.00	0.00	5.11	0.00
time (sec)	N/A	0.049	0.091	0.233	0.000	0.341	0.000	0.000	1.609	0.000

Problem 1878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.066	0.084	0.000	0.333	0.000	0.000	0.000	0.000

Problem 1879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0	-1	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.022	0.059	0.076	0.000	0.314	0.000	0.000	0.000	0.000

Problem 1880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.004	0.004	0.000	0.310	0.000	0.000	0.000	0.000

Problem 1881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	92	93	233	101	46	93
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.77	0.81	1.63
time (sec)	N/A	0.025	0.016	0.193	0.267	0.312	0.480	0.001	0.439	4.683

Problem 1882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	54	57	0	85	0	0	81	0
N.S.	1	1.00	0.57	0.60	0.00	0.89	0.00	0.00	0.85	0.00
time (sec)	N/A	0.028	0.174	0.278	0.000	0.318	0.000	0.000	1.037	0.000

Problem 1883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F	F	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	46	66	0	84	0	0	119	0
N.S.	1	1.00	0.47	0.68	0.00	0.87	0.00	0.00	1.23	0.00
time (sec)	N/A	0.017	0.251	0.204	0.000	0.321	0.000	0.000	2.138	0.000

Problem 1884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	46	66	0	84	0	0	142	0
N.S.	1	1.00	0.47	0.68	0.00	0.87	0.00	0.00	1.46	0.00
time (sec)	N/A	0.015	0.122	0.208	0.000	0.359	0.000	0.000	0.850	0.000

Problem 1885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	29	0	-1	28
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.97	0.00	-0.03	0.93
time (sec)	N/A	0.006	0.057	0.059	0.000	0.316	1.305	0.000	0.000	2.782

Problem 1886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	31	0	-1	29
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.89	0.00	-0.03	0.83
time (sec)	N/A	0.005	0.058	0.051	0.000	0.336	1.285	0.000	0.000	2.783

Problem 1887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	37	0	-1	27
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.12	0.00	-0.03	0.82
time (sec)	N/A	0.006	0.073	0.051	0.000	0.363	77.086	0.000	0.000	52.351

Problem 1888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	42	0	-1	30
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.14	0.00	-0.03	0.81
time (sec)	N/A	0.005	0.074	0.050	0.000	0.323	76.091	0.000	0.000	52.445

Problem 1889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	C	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	42	0	-1	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.89	0.00	-0.02	0.87
time (sec)	N/A	0.012	0.028	0.057	0.000	0.311	19.800	0.000	0.000	16.337

Problem 1890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	F	F	F	F(-2)	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	88	0	0	0	0	0	-1	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.014	0.022	0.092	0.000	0.316	0.000	0.000	0.000	0.000

Problem 1891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	25	22	20
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.89	0.79	0.71
time (sec)	N/A	0.004	0.000	0.013	0.272	0.295	0.030	0.000	0.037	1.679

Problem 1892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	8	14	10	10
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	0.93	0.67	0.67
time (sec)	N/A	0.002	0.000	0.013	0.258	0.292	0.046	0.000	0.021	1.677

Problem 1893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	5	9	8	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.73	0.82
time (sec)	N/A	0.001	0.000	0.012	0.256	0.294	0.029	0.000	0.017	1.600

Problem 1894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	11	7	7
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.22	0.78	0.78
time (sec)	N/A	0.001	0.000	0.012	0.337	0.287	0.028	0.000	0.029	1.609

Problem 1895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	12	10	15	12	12
N.S.	1	1.00	1.00	1.07	0.86	0.86	0.71	1.07	0.86	0.86
time (sec)	N/A	0.002	0.000	0.038	0.273	0.292	0.035	0.000	0.020	1.635

Problem 1896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	15	10	10
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	1.36	0.91	0.91
time (sec)	N/A	0.001	0.000	0.011	0.257	0.289	0.028	0.000	0.019	1.654

Problem 1897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	8	17	8	8
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	1.13	0.53	0.53
time (sec)	N/A	0.001	0.000	0.013	0.266	0.291	0.050	0.000	0.021	1.614

Problem 1898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	15	19	13	13
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	1.06	0.72	0.72
time (sec)	N/A	0.002	0.000	0.014	0.299	0.292	0.028	0.000	0.024	1.636

Problem 1899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	12	21	15	14
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.60	1.05	0.75	0.70
time (sec)	N/A	0.002	0.000	0.007	0.289	0.299	0.027	0.000	0.026	1.654

Problem 1900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	17	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	1.06	0.75	0.75
time (sec)	N/A	0.001	0.000	0.013	0.268	0.321	0.030	0.000	0.023	1.634

Problem 1901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	12	25	13	11
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.92	1.92	1.00	0.85
time (sec)	N/A	0.001	0.000	0.012	0.260	0.298	0.028	0.000	0.028	1.619

Problem 1902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	27	20	23	20	20
N.S.	1	1.00	1.00	0.95	0.91	1.23	0.91	1.05	0.91	0.91
time (sec)	N/A	0.003	0.005	0.012	0.253	0.298	0.115	0.000	0.037	1.750

Problem 1903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	17	15	20	17	17
N.S.	1	1.00	1.00	0.77	0.73	0.77	0.68	0.91	0.77	0.77
time (sec)	N/A	0.002	0.001	0.014	0.281	0.292	0.044	0.000	0.029	1.704

Problem 1904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	14	14	11	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	0.93	0.93	0.73	0.87
time (sec)	N/A	0.002	0.002	0.008	0.281	0.295	0.049	0.000	0.029	1.659

Problem 1905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	11	7	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.10	0.70	1.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.012	0.258	0.294	0.045	0.000	0.033	1.630

Problem 1906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	10	16	12	12
N.S.	1	1.00	1.00	0.80	0.73	0.80	0.67	1.07	0.80	0.80
time (sec)	N/A	0.001	0.001	0.016	0.260	0.309	0.049	0.000	0.025	1.641

Problem 1907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	11	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	1.00	0.82	0.82
time (sec)	N/A	0.001	0.001	0.012	0.260	0.304	0.043	0.000	0.027	1.610

Problem 1908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	10	8	14	10	10
N.S.	1	1.00	1.00	0.92	0.85	0.77	0.62	1.08	0.77	0.77
time (sec)	N/A	0.001	0.001	0.010	0.274	0.293	0.053	0.000	0.031	1.630

Problem 1909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	17	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.31	0.85	0.85
time (sec)	N/A	0.002	0.001	0.010	0.264	0.309	0.041	0.000	0.026	1.629

Problem 1910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	12	20	11	11
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.71	1.18	0.65	0.65
time (sec)	N/A	0.002	0.001	0.039	0.255	0.312	0.031	0.000	0.030	1.683

Problem 1911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	20	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	1.54	0.62	0.69
time (sec)	N/A	0.001	0.001	0.169	0.281	0.301	0.038	0.000	0.024	1.602

Problem 1912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	10	9	10	12	21	10	10
N.S.	1	1.00	0.93	0.67	0.60	0.67	0.80	1.40	0.67	0.67
time (sec)	N/A	0.001	0.008	0.023	0.271	0.306	0.040	0.000	0.021	1.639

Problem 1913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	18	14	19	15	13
N.S.	1	1.00	1.00	0.93	0.87	1.20	0.93	1.27	1.00	0.87
time (sec)	N/A	0.002	0.014	0.046	0.261	0.294	0.032	0.000	0.291	1.639

Problem 1914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	12	11	10	14	22	12	10
N.S.	1	1.00	0.82	0.71	0.65	0.59	0.82	1.29	0.71	0.59
time (sec)	N/A	0.001	0.012	0.026	0.262	0.358	0.030	0.001	0.027	1.681

Problem 1915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	10	12	11	14	12	28	8	8
N.S.	1	1.00	0.67	0.80	0.73	0.93	0.80	1.87	0.53	0.53
time (sec)	N/A	0.001	0.008	0.044	0.276	0.318	0.031	0.001	0.027	1.620

Problem 1916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	16	14	19	28	15	16
N.S.	1	1.00	1.00	0.71	0.67	0.58	0.79	1.17	0.62	0.67
time (sec)	N/A	0.002	0.004	0.025	0.263	0.300	0.032	0.000	0.029	1.645

Problem 1917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	Mathics
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	15	19	20	31	17	15
N.S.	1	1.00	1.00	0.70	0.65	0.83	0.87	1.35	0.74	0.65
time (sec)	N/A	0.002	0.005	0.045	0.254	0.304	0.033	0.000	0.281	1.635

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1884] had the largest ratio of [51]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	1	1.000
2	A	1	1	1.00	1	1.000
3	A	1	1	1.00	1	1.000
4	A	1	1	1.00	1	1.000
5	A	1	1	1.00	3	0.333
6	A	1	1	1.00	1	1.000
7	A	1	1	1.00	1	1.000
8	A	1	1	1.00	3	0.333
9	A	1	1	1.00	13	0.077
10	A	1	1	1.00	3	0.333
11	A	1	1	1.00	3	0.333
12	A	1	1	1.00	3	0.333
13	A	1	1	1.00	1	1.000
14	A	1	1	1.00	1	1.000
15	A	1	1	1.00	3	0.333
16	A	1	1	1.00	3	0.333
17	A	1	1	1.00	3	0.333
18	A	1	1	1.00	3	0.333
19	A	1	1	1.00	3	0.333
20	A	1	1	1.00	5	0.200
21	A	1	1	1.00	5	0.200
22	A	1	1	1.00	5	0.200
23	A	1	1	1.00	5	0.200
24	A	1	1	1.00	5	0.200
25	A	1	1	1.00	5	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	1	1	1.00	5	0.200
27	A	1	1	1.00	5	0.200
28	A	1	1	1.00	5	0.200
29	A	1	1	1.00	5	0.200
30	A	1	1	1.00	5	0.200
31	A	1	1	1.00	5	0.200
32	A	1	1	1.00	5	0.200
33	A	1	1	1.00	5	0.200
34	A	1	1	1.00	3	0.333
35	A	1	1	1.00	5	0.200
36	A	2	2	1.00	17	0.118
37	A	2	2	1.00	13	0.154
38	A	2	2	1.00	13	0.154
39	A	2	2	1.00	13	0.154
40	A	2	2	1.00	13	0.154
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	13	0.154
43	A	2	1	1.00	9	0.111
44	A	2	1	1.00	9	0.111
45	A	2	1	1.00	7	0.143
46	A	1	0	1.00	5	0.000
47	A	2	1	1.00	9	0.111
48	A	2	1	1.00	9	0.111
49	A	1	1	1.00	9	0.111
50	A	2	1	1.00	9	0.111
51	A	2	1	1.00	9	0.111
52	A	2	1	1.00	11	0.091
53	A	2	1	1.00	11	0.091
54	A	2	1	1.00	9	0.111
55	A	1	1	1.00	7	0.143
56	A	2	1	1.00	11	0.091
57	A	2	1	1.00	11	0.091
58	A	2	1	1.00	11	0.091
59	A	1	1	1.00	11	0.091
60	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	1	1.00	11	0.091
62	A	2	1	1.00	11	0.091
63	A	2	1	1.00	11	0.091
64	A	2	1	1.00	11	0.091
65	A	2	1	1.00	11	0.091
66	A	2	1	1.00	11	0.091
67	A	2	1	1.00	9	0.111
68	A	1	1	1.00	7	0.143
69	A	2	1	1.00	11	0.091
70	A	2	1	1.00	11	0.091
71	A	2	1	1.00	11	0.091
72	A	2	1	1.00	11	0.091
73	A	1	1	1.00	11	0.091
74	A	2	2	1.00	11	0.182
75	A	2	1	1.00	11	0.091
76	A	2	1	1.00	11	0.091
77	A	2	1	1.00	11	0.091
78	A	2	1	1.00	11	0.091
79	A	2	1	1.00	11	0.091
80	A	2	1	1.00	11	0.091
81	A	2	1	1.00	11	0.091
82	A	2	1	1.00	9	0.111
83	A	1	1	1.00	7	0.143
84	A	2	1	1.00	11	0.091
85	A	2	1	1.00	11	0.091
86	A	2	1	1.00	11	0.091
87	A	2	1	1.00	11	0.091
88	A	2	1	1.00	11	0.091
89	A	2	1	1.00	11	0.091
90	A	1	1	1.00	11	0.091
91	A	2	2	1.00	11	0.182
92	A	3	2	1.00	11	0.182
93	A	2	1	1.00	11	0.091
94	A	2	1	1.00	11	0.091
95	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	1	1.00	11	0.091
97	A	2	1	1.00	11	0.091
98	A	2	1	1.00	11	0.091
99	A	2	1	1.00	11	0.091
100	A	2	1	1.00	11	0.091
101	A	2	1	1.00	11	0.091
102	A	2	1	1.00	11	0.091
103	A	2	1	1.00	11	0.091
104	A	2	1	1.00	11	0.091
105	A	2	1	1.00	9	0.111
106	A	1	1	1.00	7	0.143
107	A	2	1	1.00	11	0.091
108	A	2	1	1.00	11	0.091
109	A	2	1	1.00	11	0.091
110	A	2	1	1.00	11	0.091
111	A	2	1	1.00	11	0.091
112	A	2	1	1.00	11	0.091
113	A	2	1	1.00	11	0.091
114	A	2	1	1.00	11	0.091
115	A	1	1	1.00	11	0.091
116	A	2	2	1.00	11	0.182
117	A	3	2	1.00	11	0.182
118	A	4	2	1.00	11	0.182
119	A	5	2	1.00	11	0.182
120	A	2	1	1.00	11	0.091
121	A	2	1	1.00	11	0.091
122	A	2	1	1.00	11	0.091
123	A	2	1	1.00	11	0.091
124	A	2	1	1.00	11	0.091
125	A	2	1	1.00	11	0.091
126	A	2	1	1.00	11	0.091
127	A	2	1	1.00	11	0.091
128	A	2	1	1.00	11	0.091
129	A	2	1	1.00	11	0.091
130	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	1	1.00	11	0.091
132	A	2	1	1.00	11	0.091
133	A	2	1	1.00	9	0.111
134	A	1	1	1.00	7	0.143
135	A	2	1	1.00	11	0.091
136	A	2	1	1.00	11	0.091
137	A	2	1	1.00	11	0.091
138	A	2	1	1.00	11	0.091
139	A	2	1	1.00	11	0.091
140	A	2	1	1.00	11	0.091
141	A	2	1	1.00	11	0.091
142	A	2	1	1.00	11	0.091
143	A	2	1	1.00	11	0.091
144	A	2	1	1.00	11	0.091
145	A	2	1	1.00	11	0.091
146	A	1	1	1.00	11	0.091
147	A	2	2	1.00	11	0.182
148	A	3	2	1.00	11	0.182
149	A	4	2	1.00	11	0.182
150	A	5	2	1.00	11	0.182
151	A	6	2	1.00	11	0.182
152	A	7	2	1.00	11	0.182
153	A	2	1	1.00	11	0.091
154	A	2	1	1.00	11	0.091
155	A	1	1	1.00	7	0.143
156	A	1	1	1.00	12	0.083
157	A	2	1	1.00	11	0.091
158	A	2	1	1.00	11	0.091
159	A	2	1	1.00	11	0.091
160	A	2	1	1.00	11	0.091
161	A	2	1	1.00	9	0.111
162	A	1	1	1.00	7	0.143
163	A	3	3	1.00	11	0.273
164	A	2	1	1.00	11	0.091
165	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	1	1.00	11	0.091
167	A	2	1	1.00	11	0.091
168	A	2	1	1.00	11	0.091
169	A	2	1	1.00	11	0.091
170	A	2	1	1.00	11	0.091
171	A	2	1	1.00	11	0.091
172	A	2	1	1.00	11	0.091
173	A	2	1	1.00	9	0.111
174	A	1	1	1.00	7	0.143
175	A	2	1	1.00	11	0.091
176	A	2	1	1.00	11	0.091
177	A	2	1	1.00	11	0.091
178	A	2	1	1.00	11	0.091
179	A	2	1	1.00	11	0.091
180	A	2	1	1.00	11	0.091
181	A	2	1	1.00	11	0.091
182	A	2	1	1.00	11	0.091
183	A	2	1	1.00	11	0.091
184	A	2	1	1.00	11	0.091
185	A	2	1	1.00	11	0.091
186	A	1	1	1.00	9	0.111
187	A	1	1	1.00	7	0.143
188	A	2	1	1.00	11	0.091
189	A	2	1	1.00	11	0.091
190	A	2	1	1.00	11	0.091
191	A	2	1	1.00	11	0.091
192	A	2	1	1.00	11	0.091
193	A	2	1	1.00	11	0.091
194	A	2	1	1.00	11	0.091
195	A	2	1	1.00	11	0.091
196	A	2	1	1.00	11	0.091
197	A	2	1	1.00	11	0.091
198	A	2	1	1.00	11	0.091
199	A	1	1	1.00	11	0.091
200	A	2	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	1	1	1.00	7	0.143
202	A	2	1	1.00	11	0.091
203	A	2	1	1.00	11	0.091
204	A	2	1	1.00	11	0.091
205	A	2	1	1.00	11	0.091
206	A	2	1	1.00	11	0.091
207	A	2	1	1.00	11	0.091
208	A	2	1	1.00	11	0.091
209	A	2	1	1.00	11	0.091
210	A	2	1	1.00	11	0.091
211	A	2	1	1.00	11	0.091
212	A	1	1	1.00	11	0.091
213	A	2	2	1.00	11	0.182
214	A	2	1	1.23	11	0.091
215	A	2	1	1.00	11	0.091
216	A	2	1	1.00	9	0.111
217	A	1	1	1.00	7	0.143
218	A	2	1	1.00	11	0.091
219	A	2	1	1.00	11	0.091
220	A	2	1	1.00	11	0.091
221	A	2	1	1.00	11	0.091
222	A	2	1	1.00	11	0.091
223	A	2	1	1.00	11	0.091
224	A	2	1	1.00	11	0.091
225	A	2	1	1.00	11	0.091
226	A	1	1	1.00	11	0.091
227	A	2	2	1.00	11	0.182
228	A	3	2	1.00	11	0.182
229	A	4	2	1.00	11	0.182
230	A	2	1	1.00	11	0.091
231	A	2	1	1.00	11	0.091
232	A	2	1	1.00	11	0.091
233	A	2	1	1.00	9	0.111
234	A	1	1	1.00	7	0.143
235	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	1	1.00	11	0.091
237	A	2	1	1.00	11	0.091
238	A	2	1	1.00	11	0.091
239	A	2	1	1.00	11	0.091
240	A	2	1	1.00	11	0.091
241	A	2	1	1.00	11	0.091
242	A	2	1	1.00	11	0.091
243	A	1	1	1.00	11	0.091
244	A	2	2	1.00	11	0.182
245	A	3	2	1.00	11	0.182
246	A	2	1	1.00	11	0.091
247	A	2	1	1.00	11	0.091
248	A	2	1	1.00	11	0.091
249	A	2	1	1.00	11	0.091
250	A	2	1	1.00	9	0.111
251	A	1	1	1.00	3	0.333
252	A	2	1	1.00	11	0.091
253	A	2	1	1.00	11	0.091
254	A	2	1	1.00	11	0.091
255	A	3	3	1.00	11	0.273
256	A	3	3	1.00	11	0.273
257	A	2	1	1.00	11	0.091
258	A	2	1	1.00	11	0.091
259	A	2	1	1.00	11	0.091
260	A	2	1	1.00	11	0.091
261	A	2	1	1.00	11	0.091
262	A	2	1	1.00	11	0.091
263	A	2	1	1.00	11	0.091
264	A	2	1	1.00	11	0.091
265	A	2	1	1.00	11	0.091
266	A	2	1	1.00	11	0.091
267	A	2	1	1.00	11	0.091
268	A	2	1	1.00	11	0.091
269	A	2	1	1.00	11	0.091
270	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	1	1	1.00	7	0.143
272	A	1	1	1.00	7	0.143
273	A	1	1	1.00	11	0.091
274	A	1	1	1.00	13	0.077
275	A	1	1	1.00	15	0.067
276	A	1	1	1.00	15	0.067
277	A	1	1	1.00	15	0.067
278	A	1	1	1.00	15	0.067
279	A	3	3	1.00	11	0.273
280	A	3	3	1.00	11	0.273
281	A	2	1	1.00	11	0.091
282	A	2	1	1.00	11	0.091
283	A	3	1	1.00	17	0.059
284	A	2	1	1.00	13	0.077
285	A	2	1	1.00	13	0.077
286	A	2	1	1.00	11	0.091
287	A	1	1	1.00	9	0.111
288	A	3	3	1.00	13	0.231
289	A	3	3	1.00	13	0.231
290	A	4	4	1.00	13	0.308
291	A	5	4	1.00	13	0.308
292	A	2	1	1.00	13	0.077
293	A	2	1	1.00	13	0.077
294	A	2	1	1.00	11	0.091
295	A	1	1	1.00	9	0.111
296	A	4	3	1.00	13	0.231
297	A	4	4	1.00	13	0.308
298	A	4	3	1.00	13	0.231
299	A	5	4	1.00	13	0.308
300	A	2	1	1.00	13	0.077
301	A	2	1	1.00	13	0.077
302	A	2	1	1.00	11	0.091
303	A	1	1	1.00	9	0.111
304	A	5	3	1.00	13	0.231
305	A	5	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
306	A	5	4	1.00	13	0.308
307	A	5	3	1.00	13	0.231
308	A	6	4	1.00	13	0.308
309	A	2	1	1.00	13	0.077
310	A	2	1	1.00	13	0.077
311	A	2	1	1.00	13	0.077
312	A	2	1	1.00	13	0.077
313	A	2	1	1.00	13	0.077
314	A	2	1	1.00	13	0.077
315	A	2	1	1.00	11	0.091
316	A	1	1	1.00	9	0.111
317	A	7	3	1.00	13	0.231
318	A	7	4	1.00	13	0.308
319	A	7	4	1.00	13	0.308
320	A	7	4	1.00	13	0.308
321	A	7	4	1.00	13	0.308
322	A	7	3	1.00	13	0.231
323	A	8	4	1.00	13	0.308
324	A	9	4	1.00	13	0.308
325	A	3	3	1.00	15	0.200
326	A	3	3	1.00	15	0.200
327	A	4	4	1.00	15	0.267
328	A	4	3	1.00	15	0.200
329	A	4	4	1.00	15	0.267
330	A	4	3	1.00	15	0.200
331	A	5	3	1.00	15	0.200
332	A	5	4	1.00	15	0.267
333	A	5	4	1.00	15	0.267
334	A	2	1	1.00	13	0.077
335	A	2	1	1.00	13	0.077
336	A	2	1	1.00	13	0.077
337	A	2	1	1.00	11	0.091
338	A	1	1	1.00	9	0.111
339	A	2	2	1.00	13	0.154
340	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
341	A	4	3	1.00	13	0.231
342	A	5	3	1.00	13	0.231
343	A	2	1	1.00	13	0.077
344	A	2	1	1.00	13	0.077
345	A	2	1	1.00	13	0.077
346	A	2	1	1.00	11	0.091
347	A	1	1	1.00	9	0.111
348	A	3	3	1.00	13	0.231
349	A	4	4	1.00	13	0.308
350	A	5	4	1.00	13	0.308
351	A	2	1	1.00	13	0.077
352	A	2	1	1.00	13	0.077
353	A	2	1	1.00	13	0.077
354	A	2	1	1.00	11	0.091
355	A	1	1	1.00	9	0.111
356	A	4	3	1.00	13	0.231
357	A	5	4	1.00	13	0.308
358	A	6	4	1.00	13	0.308
359	A	2	2	1.00	15	0.133
360	A	3	3	1.00	15	0.200
361	A	4	3	1.00	15	0.200
362	A	3	3	1.00	15	0.200
363	A	4	4	1.00	15	0.267
364	A	5	4	1.00	15	0.267
365	A	4	3	1.00	15	0.200
366	A	5	4	1.00	15	0.267
367	A	6	4	1.00	15	0.267
368	A	2	2	1.00	31	0.065
369	C	5	2	7.08	34	0.059
370	A	3	3	1.00	29	0.103
371	A	2	1	1.00	13	0.077
372	A	2	1	1.00	13	0.077
373	A	2	1	1.00	11	0.091
374	A	1	1	1.00	9	0.111
375	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	5	5	1.00	13	0.385
377	A	6	6	1.00	13	0.462
378	A	2	1	1.00	13	0.077
379	A	2	1	1.00	13	0.077
380	A	2	1	1.00	11	0.091
381	A	1	1	1.00	9	0.111
382	A	5	5	1.00	13	0.385
383	A	5	5	1.00	13	0.385
384	A	6	6	1.00	13	0.462
385	A	2	1	1.00	13	0.077
386	A	2	1	1.00	13	0.077
387	A	2	1	1.00	11	0.091
388	A	1	1	1.00	9	0.111
389	A	6	5	1.00	13	0.385
390	A	6	6	1.00	13	0.462
391	A	6	5	1.00	13	0.385
392	A	2	1	1.00	13	0.077
393	A	2	1	1.00	13	0.077
394	A	2	1	1.00	11	0.091
395	A	1	1	1.00	9	0.111
396	A	4	4	1.00	13	0.308
397	A	5	5	1.00	13	0.385
398	A	6	5	1.00	13	0.385
399	A	2	1	1.00	15	0.067
400	A	2	1	1.00	15	0.067
401	A	2	1	1.00	13	0.077
402	A	1	1	1.00	11	0.091
403	A	4	4	1.00	15	0.267
404	A	5	5	1.00	15	0.333
405	A	6	5	1.00	15	0.333
406	A	2	1	1.00	13	0.077
407	A	2	1	1.00	13	0.077
408	A	2	1	1.00	11	0.091
409	A	1	1	1.00	9	0.111
410	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	5	5	1.00	13	0.385
412	A	6	5	1.00	13	0.385
413	A	2	1	1.00	13	0.077
414	A	2	1	1.00	13	0.077
415	A	2	1	1.00	11	0.091
416	A	1	1	1.00	9	0.111
417	A	5	5	1.00	13	0.385
418	A	6	6	1.00	13	0.462
419	A	7	6	1.00	13	0.462
420	A	4	4	1.00	17	0.235
421	A	4	4	1.00	18	0.222
422	A	4	4	1.00	19	0.210
423	A	4	4	1.00	20	0.200
424	A	4	4	1.00	17	0.235
425	A	4	4	1.00	18	0.222
426	A	4	4	1.00	19	0.210
427	A	4	4	1.00	20	0.200
428	A	2	1	1.00	9	0.111
429	A	2	1	1.00	11	0.091
430	A	2	1	1.00	11	0.091
431	A	2	1	1.00	11	0.091
432	A	2	1	1.00	11	0.091
433	A	2	1	1.00	11	0.091
434	A	2	1	1.00	11	0.091
435	A	2	1	1.00	11	0.091
436	A	2	1	1.00	13	0.077
437	A	2	1	1.00	13	0.077
438	A	2	1	1.00	13	0.077
439	A	2	1	1.00	13	0.077
440	A	2	1	1.00	13	0.077
441	A	2	1	1.00	13	0.077
442	A	2	1	1.00	11	0.091
443	A	2	1	1.00	13	0.077
444	A	2	1	1.00	13	0.077
445	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	2	1	1.00	13	0.077
447	A	2	1	1.00	13	0.077
448	A	2	1	1.00	13	0.077
449	A	5	3	1.00	13	0.231
450	A	4	3	1.00	13	0.231
451	A	3	3	1.00	13	0.231
452	A	2	2	1.00	13	0.154
453	A	3	3	1.00	13	0.231
454	A	4	3	1.00	13	0.231
455	A	5	3	1.00	13	0.231
456	A	5	4	1.00	13	0.308
457	A	4	4	1.00	13	0.308
458	A	3	3	1.00	13	0.231
459	A	3	3	1.00	13	0.231
460	A	4	4	1.00	13	0.308
461	A	5	4	1.00	13	0.308
462	A	6	4	1.00	13	0.308
463	A	5	4	1.00	13	0.308
464	A	4	3	1.00	13	0.231
465	A	4	4	1.00	13	0.308
466	A	4	3	1.00	13	0.231
467	A	5	4	1.00	13	0.308
468	A	6	4	1.00	13	0.308
469	A	5	3	1.00	15	0.200
470	A	4	3	1.00	15	0.200
471	A	3	3	1.00	15	0.200
472	A	2	2	1.00	15	0.133
473	A	3	3	1.00	15	0.200
474	A	4	3	1.00	15	0.200
475	A	5	3	1.00	15	0.200
476	A	5	4	1.00	15	0.267
477	A	4	4	1.00	15	0.267
478	A	3	3	1.00	15	0.200
479	A	3	3	1.00	15	0.200
480	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
481	A	5	4	1.00	15	0.267
482	A	6	4	1.00	15	0.267
483	A	5	4	1.00	15	0.267
484	A	4	3	1.00	15	0.200
485	A	4	4	1.00	15	0.267
486	A	4	3	1.00	15	0.200
487	A	5	4	1.00	15	0.267
488	A	6	4	1.00	15	0.267
489	A	7	4	1.00	15	0.267
490	A	6	4	1.00	15	0.267
491	A	5	4	1.00	15	0.267
492	A	4	4	1.00	15	0.267
493	A	4	4	1.00	15	0.267
494	A	1	1	1.00	15	0.067
495	A	2	2	1.00	15	0.133
496	A	3	2	1.00	15	0.133
497	A	7	4	1.00	16	0.250
498	A	6	4	1.00	16	0.250
499	A	5	4	1.00	16	0.250
500	A	4	4	1.00	16	0.250
501	A	4	4	1.00	16	0.250
502	A	1	1	1.00	16	0.062
503	A	2	2	1.00	16	0.125
504	A	3	2	1.00	16	0.125
505	A	6	3	1.00	15	0.200
506	A	5	3	1.00	15	0.200
507	A	4	3	1.00	15	0.200
508	A	3	3	1.00	15	0.200
509	A	3	3	1.00	15	0.200
510	A	1	1	1.00	15	0.067
511	A	2	2	1.00	15	0.133
512	A	3	2	1.00	15	0.133
513	A	6	3	1.00	16	0.188
514	A	5	3	1.00	16	0.188
515	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	3	3	1.00	16	0.188
517	A	3	3	1.00	16	0.188
518	A	1	1	1.00	16	0.062
519	A	2	2	1.00	16	0.125
520	A	3	2	1.00	16	0.125
521	A	8	4	1.00	15	0.267
522	A	7	4	1.00	15	0.267
523	A	6	4	1.00	15	0.267
524	A	5	4	1.00	15	0.267
525	A	5	5	1.00	15	0.333
526	A	5	4	1.00	15	0.267
527	A	8	4	1.00	16	0.250
528	A	7	4	1.00	16	0.250
529	A	6	4	1.00	16	0.250
530	A	5	4	1.00	16	0.250
531	A	5	5	1.00	16	0.312
532	A	5	4	1.00	16	0.250
533	A	7	3	1.00	15	0.200
534	A	6	3	1.00	15	0.200
535	A	5	3	1.00	15	0.200
536	A	4	3	1.00	15	0.200
537	A	4	4	1.00	15	0.267
538	A	4	3	1.00	15	0.200
539	A	7	3	1.00	16	0.188
540	A	6	3	1.00	16	0.188
541	A	5	3	1.00	16	0.188
542	A	4	3	1.00	16	0.188
543	A	4	4	1.00	16	0.250
544	A	4	3	1.00	16	0.188
545	A	9	4	1.00	15	0.267
546	A	8	4	1.00	15	0.267
547	A	7	4	1.00	15	0.267
548	A	6	4	1.00	15	0.267
549	A	6	5	1.00	15	0.333
550	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
551	A	9	4	1.00	16	0.250
552	A	8	4	1.00	16	0.250
553	A	7	4	1.00	16	0.250
554	A	6	4	1.00	16	0.250
555	A	6	5	1.00	16	0.312
556	A	6	5	1.00	16	0.312
557	A	8	3	1.00	15	0.200
558	A	7	3	1.00	15	0.200
559	A	6	3	1.00	15	0.200
560	A	5	3	1.00	15	0.200
561	A	5	4	1.00	15	0.267
562	A	5	4	1.00	15	0.267
563	A	8	3	1.00	16	0.188
564	A	7	3	1.00	16	0.188
565	A	6	3	1.00	16	0.188
566	A	5	3	1.00	16	0.188
567	A	5	4	1.00	16	0.250
568	A	5	4	1.00	16	0.250
569	A	6	4	1.00	15	0.267
570	A	5	4	1.00	15	0.267
571	A	4	4	1.00	15	0.267
572	A	3	3	1.00	15	0.200
573	A	1	1	1.00	15	0.067
574	A	2	2	1.00	15	0.133
575	A	3	2	1.00	15	0.133
576	A	4	2	1.00	15	0.133
577	A	6	5	1.00	15	0.333
578	A	5	5	1.00	15	0.333
579	A	4	4	1.00	15	0.267
580	A	1	1	1.00	15	0.067
581	A	2	2	1.00	15	0.133
582	A	3	2	1.00	15	0.133
583	A	4	2	1.00	15	0.133
584	A	6	5	1.00	15	0.333
585	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	1	1	1.00	15	0.067
587	A	2	2	1.00	15	0.133
588	A	3	2	1.00	15	0.133
589	A	4	2	1.00	15	0.133
590	A	6	4	1.00	16	0.250
591	A	5	4	1.00	16	0.250
592	A	4	4	1.00	16	0.250
593	A	3	3	1.00	16	0.188
594	A	1	1	1.00	16	0.062
595	A	2	2	1.00	16	0.125
596	A	6	5	1.00	16	0.312
597	A	5	5	1.00	16	0.312
598	A	4	4	1.00	16	0.250
599	A	1	1	1.00	16	0.062
600	A	2	2	1.00	16	0.125
601	A	3	2	1.00	16	0.125
602	A	6	5	1.00	16	0.312
603	A	5	4	1.00	16	0.250
604	A	1	1	1.00	16	0.062
605	A	2	2	1.00	16	0.125
606	A	3	2	1.00	16	0.125
607	A	4	2	1.00	16	0.125
608	A	5	3	1.00	15	0.200
609	A	4	3	1.00	15	0.200
610	A	3	3	1.00	15	0.200
611	A	2	2	1.00	15	0.133
612	A	1	1	1.00	15	0.067
613	A	2	2	1.00	15	0.133
614	A	3	2	1.00	15	0.133
615	A	4	2	1.00	15	0.133
616	A	5	4	1.00	15	0.267
617	A	4	4	1.00	15	0.267
618	A	3	3	1.00	15	0.200
619	A	1	1	1.00	15	0.067
620	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	3	2	1.00	15	0.133
622	A	4	2	1.00	15	0.133
623	A	5	4	1.00	15	0.267
624	A	4	3	1.00	15	0.200
625	A	1	1	1.00	15	0.067
626	A	2	2	1.00	15	0.133
627	A	3	2	1.00	15	0.133
628	A	4	2	1.00	15	0.133
629	A	5	3	1.00	16	0.188
630	A	4	3	1.00	16	0.188
631	A	3	3	1.00	16	0.188
632	A	2	2	1.00	16	0.125
633	A	1	1	1.00	16	0.062
634	A	2	2	1.00	16	0.125
635	A	5	4	1.00	16	0.250
636	A	4	4	1.00	16	0.250
637	A	3	3	1.00	16	0.188
638	A	1	1	1.00	16	0.062
639	A	2	2	1.00	16	0.125
640	A	3	2	1.00	16	0.125
641	A	5	4	1.00	16	0.250
642	A	4	3	1.00	16	0.188
643	A	1	1	1.00	16	0.062
644	A	2	2	1.00	16	0.125
645	A	3	2	1.00	16	0.125
646	A	4	2	1.00	16	0.125
647	A	4	4	1.00	15	0.267
648	A	3	3	1.00	15	0.200
649	A	2	2	1.00	16	0.125
650	A	2	1	1.00	11	0.091
651	A	2	1	1.00	11	0.091
652	A	2	1	1.00	11	0.091
653	A	2	1	1.00	11	0.091
654	A	2	1	1.00	11	0.091
655	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	2	1	1.00	11	0.091
657	A	2	1	1.00	11	0.091
658	A	2	1	1.00	13	0.077
659	A	2	1	1.00	13	0.077
660	A	2	1	1.00	13	0.077
661	A	2	1	1.00	13	0.077
662	A	2	1	1.00	13	0.077
663	A	2	1	1.00	13	0.077
664	A	2	1	1.00	13	0.077
665	A	2	1	1.00	13	0.077
666	A	2	1	1.00	13	0.077
667	A	2	1	1.00	13	0.077
668	A	2	1	1.00	13	0.077
669	A	2	1	1.00	13	0.077
670	A	2	1	1.00	13	0.077
671	A	2	1	1.00	13	0.077
672	A	2	1	1.00	13	0.077
673	A	2	1	1.00	13	0.077
674	A	6	5	1.00	13	0.385
675	A	6	5	1.00	13	0.385
676	A	5	5	1.00	13	0.385
677	A	5	5	1.00	13	0.385
678	A	4	4	1.00	13	0.308
679	A	4	4	1.00	13	0.308
680	A	5	5	1.00	13	0.385
681	A	5	5	1.00	13	0.385
682	A	6	6	1.00	13	0.462
683	A	6	6	1.00	13	0.462
684	A	5	5	1.00	13	0.385
685	A	5	5	1.00	13	0.385
686	A	5	5	1.00	13	0.385
687	A	5	5	1.00	13	0.385
688	A	6	6	1.00	13	0.462
689	A	6	6	1.00	13	0.462
690	A	6	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
691	A	6	5	1.00	13	0.385
692	A	6	6	1.00	13	0.462
693	A	6	6	1.00	13	0.462
694	A	6	5	1.00	13	0.385
695	A	6	5	1.00	13	0.385
696	A	7	6	1.00	13	0.462
697	A	7	6	1.00	13	0.462
698	A	5	5	1.00	15	0.333
699	A	2	1	1.00	11	0.091
700	A	2	1	1.00	11	0.091
701	A	2	1	1.00	11	0.091
702	A	2	1	1.00	11	0.091
703	A	2	1	1.00	9	0.111
704	A	1	1	1.00	11	0.091
705	A	1	1	1.00	11	0.091
706	A	1	1	1.00	11	0.091
707	A	2	2	1.00	13	0.154
708	A	2	2	1.00	13	0.154
709	A	2	2	1.00	13	0.154
710	A	2	2	1.00	13	0.154
711	A	2	2	1.00	13	0.154
712	A	2	2	1.00	13	0.154
713	A	2	2	1.00	15	0.133
714	A	2	2	1.00	15	0.133
715	A	2	2	1.00	13	0.154
716	A	2	2	1.00	15	0.133
717	A	2	2	1.00	15	0.133
718	A	2	2	1.00	15	0.133
719	A	1	1	1.00	13	0.077
720	A	1	1	1.00	13	0.077
721	A	1	1	1.00	13	0.077
722	A	3	3	1.00	13	0.231
723	A	2	2	1.00	15	0.133
724	A	1	1	1.00	15	0.067
725	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	3	3	1.00	15	0.200
727	A	1	1	1.00	15	0.067
728	A	1	1	1.00	13	0.077
729	A	1	1	1.00	14	0.071
730	A	2	2	1.00	11	0.182
731	A	2	2	1.00	13	0.154
732	A	2	1	1.00	11	0.091
733	A	2	1	1.00	11	0.091
734	A	2	1	1.00	9	0.111
735	A	1	1	1.00	7	0.143
736	A	1	1	1.00	11	0.091
737	A	1	1	1.00	11	0.091
738	A	1	1	1.00	11	0.091
739	A	3	2	1.00	15	0.133
740	A	2	2	1.00	15	0.133
741	A	1	1	1.00	15	0.067
742	A	2	2	1.00	15	0.133
743	A	2	2	1.00	13	0.154
744	A	2	2	1.00	15	0.133
745	A	2	2	1.00	13	0.154
746	A	2	2	1.00	13	0.154
747	A	2	2	1.00	13	0.154
748	A	2	2	1.00	13	0.154
749	A	2	2	1.00	13	0.154
750	A	1	1	1.00	13	0.077
751	A	1	1	1.00	15	0.067
752	A	2	2	1.00	13	0.154
753	A	1	1	1.00	17	0.059
754	A	2	2	1.00	15	0.133
755	A	2	2	1.00	19	0.105
756	A	3	2	1.00	18	0.111
757	A	3	2	1.00	18	0.111
758	A	3	2	1.00	16	0.125
759	A	3	2	1.00	15	0.133
760	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
761	A	3	2	1.00	18	0.111
762	A	3	2	1.00	18	0.111
763	A	2	2	1.00	18	0.111
764	A	3	2	1.00	18	0.111
765	A	3	2	1.00	18	0.111
766	A	3	2	1.00	16	0.125
767	A	3	2	1.00	15	0.133
768	A	3	2	1.00	18	0.111
769	A	3	2	1.00	18	0.111
770	A	2	1	1.00	18	0.056
771	A	3	2	1.00	18	0.111
772	A	3	2	1.00	18	0.111
773	A	3	2	1.00	18	0.111
774	A	3	2	1.00	16	0.125
775	A	3	2	1.00	15	0.133
776	A	3	2	1.00	18	0.111
777	A	3	2	1.00	18	0.111
778	A	3	2	1.00	18	0.111
779	A	3	2	1.00	18	0.111
780	A	3	2	1.00	18	0.111
781	A	3	2	1.00	18	0.111
782	A	2	1	1.00	16	0.062
783	A	3	2	1.00	15	0.133
784	A	3	2	1.00	18	0.111
785	A	2	2	1.00	18	0.111
786	A	3	2	1.00	18	0.111
787	A	3	2	1.00	18	0.111
788	A	2	1	1.00	18	0.056
789	A	3	2	1.00	18	0.111
790	A	3	2	1.00	16	0.125
791	A	2	2	1.00	15	0.133
792	A	3	2	1.00	18	0.111
793	A	3	2	1.00	18	0.111
794	A	3	2	1.00	18	0.111
795	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	3	2	1.00	18	0.111
797	A	2	2	1.00	18	0.111
798	A	3	2	1.00	16	0.125
799	A	3	2	1.00	15	0.133
800	A	3	2	1.00	18	0.111
801	A	3	2	1.00	18	0.111
802	A	3	2	1.00	18	0.111
803	A	3	2	1.00	18	0.111
804	A	3	2	1.00	20	0.100
805	A	3	2	1.00	20	0.100
806	A	3	2	1.00	18	0.111
807	A	3	2	1.00	17	0.118
808	A	2	2	1.00	20	0.100
809	A	3	2	1.00	20	0.100
810	A	3	2	1.00	20	0.100
811	A	3	2	1.00	20	0.100
812	A	3	2	1.00	20	0.100
813	A	3	2	1.00	20	0.100
814	A	3	2	1.00	18	0.111
815	A	3	2	1.00	17	0.118
816	A	3	2	1.00	20	0.100
817	A	3	2	1.00	20	0.100
818	A	2	2	1.00	20	0.100
819	A	3	2	1.00	20	0.100
820	A	3	2	1.00	18	0.111
821	A	3	2	1.00	17	0.118
822	A	3	2	1.00	20	0.100
823	A	3	2	1.00	20	0.100
824	A	3	2	1.00	20	0.100
825	A	3	2	1.00	20	0.100
826	A	2	2	1.00	20	0.100
827	A	3	2	1.00	20	0.100
828	A	3	2	1.00	20	0.100
829	A	3	2	1.00	20	0.100
830	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
831	A	3	2	1.00	17	0.118
832	A	3	2	1.00	20	0.100
833	A	3	2	1.00	20	0.100
834	A	2	2	1.00	20	0.100
835	A	3	2	1.00	20	0.100
836	A	2	2	1.00	20	0.100
837	A	3	2	1.00	20	0.100
838	A	3	2	1.00	18	0.111
839	A	3	2	1.00	17	0.118
840	A	2	2	1.00	20	0.100
841	A	3	2	1.00	20	0.100
842	A	3	2	1.00	20	0.100
843	A	3	2	1.00	20	0.100
844	A	3	2	1.00	20	0.100
845	A	3	2	1.00	20	0.100
846	A	2	2	1.00	18	0.111
847	A	3	2	1.00	17	0.118
848	A	3	2	1.00	20	0.100
849	A	3	2	1.00	20	0.100
850	A	3	2	1.00	20	0.100
851	A	3	2	1.00	20	0.100
852	A	3	2	1.00	20	0.100
853	A	3	2	1.00	20	0.100
854	A	3	2	1.00	18	0.111
855	A	3	2	1.00	17	0.118
856	A	2	2	1.00	20	0.100
857	A	4	4	1.00	20	0.200
858	A	3	2	1.00	20	0.100
859	A	3	2	1.00	20	0.100
860	A	3	2	1.00	18	0.111
861	A	3	2	1.00	17	0.118
862	A	3	2	1.00	20	0.100
863	A	3	2	1.00	20	0.100
864	A	2	2	1.00	20	0.100
865	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	3	2	1.00	20	0.100
867	A	3	2	1.00	20	0.100
868	A	3	2	1.00	20	0.100
869	A	3	2	1.00	17	0.118
870	A	3	2	1.00	20	0.100
871	A	3	2	1.00	20	0.100
872	A	3	2	1.00	20	0.100
873	A	3	2	1.00	20	0.100
874	A	2	2	1.00	20	0.100
875	A	4	4	1.00	20	0.200
876	A	3	2	1.00	20	0.100
877	A	3	2	1.00	20	0.100
878	A	3	2	1.00	20	0.100
879	A	3	2	1.00	20	0.100
880	A	2	2	1.00	18	0.111
881	A	4	4	1.00	17	0.235
882	A	3	2	1.00	20	0.100
883	A	3	2	1.00	20	0.100
884	A	3	2	1.00	20	0.100
885	A	3	2	1.00	20	0.100
886	A	3	2	1.00	20	0.100
887	A	3	2	1.00	20	0.100
888	A	2	2	1.00	20	0.100
889	A	4	4	1.00	20	0.200
890	A	3	2	1.00	18	0.111
891	A	3	2	1.00	17	0.118
892	A	3	2	1.00	20	0.100
893	A	3	2	1.00	20	0.100
894	A	3	2	1.00	20	0.100
895	A	3	2	1.00	18	0.111
896	A	3	2	1.00	17	0.118
897	A	2	2	1.00	20	0.100
898	A	3	2	1.00	20	0.100
899	A	3	2	1.00	20	0.100
900	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	3	2	1.00	18	0.111
902	A	3	2	1.00	17	0.118
903	A	3	2	1.00	20	0.100
904	A	3	2	1.00	20	0.100
905	A	2	2	1.00	20	0.100
906	A	3	2	1.00	20	0.100
907	A	3	2	1.00	20	0.100
908	A	3	2	1.00	20	0.100
909	A	3	2	1.00	20	0.100
910	A	3	2	1.00	20	0.100
911	A	3	2	1.00	20	0.100
912	A	3	2	1.00	20	0.100
913	A	2	2	1.00	18	0.111
914	A	3	2	1.00	17	0.118
915	A	3	2	1.00	20	0.100
916	A	3	2	1.00	20	0.100
917	A	3	2	1.00	20	0.100
918	A	3	2	1.00	20	0.100
919	A	2	2	1.00	20	0.100
920	A	3	2	1.00	20	0.100
921	A	3	2	1.00	18	0.111
922	A	3	2	1.00	17	0.118
923	A	3	2	1.00	20	0.100
924	A	3	2	1.00	18	0.111
925	A	3	2	1.00	17	0.118
926	A	2	2	1.00	20	0.100
927	A	2	2	1.00	20	0.100
928	A	2	2	1.00	20	0.100
929	A	2	2	1.00	20	0.100
930	A	3	2	1.00	18	0.111
931	A	3	2	1.00	17	0.118
932	A	3	2	1.00	20	0.100
933	A	3	2	1.00	20	0.100
934	A	2	2	1.00	20	0.100
935	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	2	2	1.00	20	0.100
937	A	2	2	1.00	20	0.100
938	A	3	2	1.00	17	0.118
939	A	3	2	1.00	20	0.100
940	A	3	2	1.00	20	0.100
941	A	3	2	1.00	20	0.100
942	A	3	2	1.00	20	0.100
943	A	2	2	1.00	20	0.100
944	A	2	2	1.00	20	0.100
945	A	2	2	1.00	20	0.100
946	A	3	2	1.00	20	0.100
947	A	3	2	1.00	20	0.100
948	A	3	2	1.00	20	0.100
949	A	2	2	1.00	18	0.111
950	A	2	2	1.00	17	0.118
951	A	2	2	1.00	20	0.100
952	A	2	2	1.00	20	0.100
953	A	3	2	1.00	20	0.100
954	A	3	2	1.00	20	0.100
955	A	3	2	1.00	20	0.100
956	A	2	2	1.00	20	0.100
957	A	2	2	1.00	20	0.100
958	A	2	2	1.00	18	0.111
959	A	2	2	1.00	17	0.118
960	A	2	2	1.00	20	0.100
961	A	3	2	1.00	20	0.100
962	A	3	2	1.00	20	0.100
963	A	3	2	1.00	20	0.100
964	A	2	2	1.00	20	0.100
965	A	2	2	1.00	20	0.100
966	A	2	2	1.00	20	0.100
967	A	2	2	1.00	20	0.100
968	A	2	2	1.00	18	0.111
969	A	4	3	1.00	20	0.150
970	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	4	3	1.00	20	0.150
972	A	4	3	1.00	20	0.150
973	A	4	3	1.00	20	0.150
974	A	4	3	1.00	20	0.150
975	A	4	3	1.00	22	0.136
976	A	4	3	1.00	22	0.136
977	A	4	3	1.00	22	0.136
978	A	4	3	1.00	22	0.136
979	A	4	3	1.00	22	0.136
980	A	4	3	1.00	22	0.136
981	A	4	4	1.00	22	0.182
982	A	4	4	1.00	22	0.182
983	A	4	4	1.00	22	0.182
984	A	4	4	1.00	22	0.182
985	A	4	4	1.00	22	0.182
986	A	4	4	1.00	22	0.182
987	A	2	2	1.00	22	0.091
988	A	2	2	1.00	22	0.091
989	A	2	2	1.00	20	0.100
990	A	2	2	1.00	19	0.105
991	A	2	2	1.00	22	0.091
992	A	2	2	1.00	20	0.100
993	A	2	2	1.00	22	0.091
994	A	2	2	1.00	22	0.091
995	A	2	2	1.00	25	0.080
996	A	3	3	1.00	27	0.111
997	A	3	3	1.00	18	0.167
998	A	4	4	0.94	20	0.200
999	A	2	2	1.00	20	0.100
1000	A	2	1	1.00	20	0.050
1001	A	2	2	1.00	20	0.100
1002	A	2	2	1.00	20	0.100
1003	A	2	2	1.00	18	0.111
1004	A	2	2	1.00	20	0.100
1005	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1006	A	2	2	1.00	20	0.100
1007	A	2	2	1.00	20	0.100
1008	A	2	1	1.00	20	0.050
1009	A	2	2	1.00	20	0.100
1010	A	2	2	1.00	20	0.100
1011	A	2	2	1.00	18	0.111
1012	A	2	2	1.00	20	0.100
1013	A	2	2	1.00	20	0.100
1014	A	2	2	1.00	20	0.100
1015	A	2	2	1.00	18	0.111
1016	A	2	2	1.00	18	0.111
1017	A	2	2	1.00	18	0.111
1018	A	2	2	1.00	16	0.125
1019	A	2	2	1.00	18	0.111
1020	A	2	2	1.00	18	0.111
1021	A	2	2	1.00	18	0.111
1022	A	2	1	1.00	18	0.056
1023	A	2	2	1.00	18	0.111
1024	A	2	2	1.00	18	0.111
1025	A	2	2	1.00	18	0.111
1026	A	2	2	1.00	18	0.111
1027	A	2	2	1.00	19	0.105
1028	A	2	1	1.00	17	0.059
1029	A	2	1	1.00	17	0.059
1030	A	2	1	1.00	15	0.067
1031	A	1	0	1.00	5	0.000
1032	A	2	1	1.00	17	0.059
1033	A	2	1	1.00	17	0.059
1034	A	1	1	1.00	17	0.059
1035	A	2	1	1.00	17	0.059
1036	A	2	1	1.00	17	0.059
1037	A	2	1	1.00	17	0.059
1038	A	2	1	1.00	19	0.053
1039	A	3	2	1.00	19	0.105
1040	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1041	A	1	1	1.00	7	0.143
1042	A	2	1	1.00	19	0.053
1043	A	2	1	1.00	19	0.053
1044	A	2	1	1.00	19	0.053
1045	A	1	1	1.00	19	0.053
1046	A	2	1	1.00	19	0.053
1047	A	2	1	1.00	19	0.053
1048	A	2	1	1.00	19	0.053
1049	A	2	1	1.00	19	0.053
1050	A	2	1	1.00	19	0.053
1051	A	2	1	1.00	17	0.059
1052	A	1	1	1.00	7	0.143
1053	A	2	2	1.00	19	0.105
1054	A	3	2	1.00	19	0.105
1055	A	3	2	1.00	19	0.105
1056	A	2	1	1.00	19	0.053
1057	A	2	1	1.00	19	0.053
1058	A	2	1	1.00	17	0.059
1059	A	1	1	1.00	7	0.143
1060	A	3	2	1.00	19	0.105
1061	A	3	3	1.00	19	0.158
1062	A	3	2	1.00	19	0.105
1063	A	7	4	1.00	17	0.235
1064	A	6	4	1.00	17	0.235
1065	A	5	4	1.00	17	0.235
1066	A	4	4	1.00	17	0.235
1067	A	3	3	1.00	17	0.176
1068	A	3	3	1.00	17	0.176
1069	A	3	3	1.00	17	0.176
1070	A	1	1	1.00	17	0.059
1071	A	2	2	1.00	17	0.118
1072	A	3	2	1.00	17	0.118
1073	A	4	2	1.00	17	0.118
1074	A	5	2	1.00	17	0.118
1075	A	7	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1076	A	6	4	1.00	17	0.235
1077	A	5	4	1.00	17	0.235
1078	A	4	3	1.00	17	0.176
1079	A	4	4	1.00	17	0.235
1080	A	4	3	1.00	17	0.176
1081	A	4	4	1.00	17	0.235
1082	A	4	3	1.00	17	0.176
1083	A	1	1	1.00	17	0.059
1084	A	2	2	1.00	17	0.118
1085	A	3	2	1.00	17	0.118
1086	A	4	2	1.00	17	0.118
1087	A	5	2	1.00	17	0.118
1088	A	8	4	1.00	17	0.235
1089	A	7	4	1.00	17	0.235
1090	A	6	4	1.00	17	0.235
1091	A	5	3	1.00	17	0.176
1092	A	5	4	1.00	17	0.235
1093	A	5	4	1.00	17	0.235
1094	A	5	3	1.00	17	0.176
1095	A	5	4	1.00	17	0.235
1096	A	5	4	1.00	17	0.235
1097	A	5	3	1.00	17	0.176
1098	A	1	1	1.00	17	0.059
1099	A	2	2	1.00	17	0.118
1100	A	3	2	1.00	17	0.118
1101	A	4	2	1.00	17	0.118
1102	A	5	2	1.00	17	0.118
1103	A	6	2	1.00	17	0.118
1104	A	4	3	1.00	20	0.150
1105	A	3	3	1.00	28	0.107
1106	A	6	3	1.00	17	0.176
1107	A	5	3	1.00	17	0.176
1108	A	4	3	1.00	17	0.176
1109	A	3	3	1.00	17	0.176
1110	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1111	A	1	1	1.00	17	0.059
1112	A	2	2	1.00	17	0.118
1113	A	3	2	1.00	17	0.118
1114	A	4	2	1.00	17	0.118
1115	A	5	2	1.00	17	0.118
1116	A	6	4	1.00	17	0.235
1117	A	5	4	1.00	17	0.235
1118	A	4	4	1.00	17	0.235
1119	A	3	3	1.00	17	0.176
1120	A	1	1	1.00	17	0.059
1121	A	1	1	1.00	17	0.059
1122	A	2	2	1.00	17	0.118
1123	A	3	2	1.00	17	0.118
1124	A	4	2	1.00	17	0.118
1125	A	5	2	1.00	17	0.118
1126	A	7	4	1.00	17	0.235
1127	A	6	4	1.00	17	0.235
1128	A	5	4	1.00	17	0.235
1129	A	4	3	1.00	17	0.176
1130	A	1	1	1.00	17	0.059
1131	A	2	2	1.00	17	0.118
1132	A	3	2	1.00	17	0.118
1133	A	2	2	1.00	17	0.118
1134	A	3	3	1.00	17	0.176
1135	A	4	3	1.00	17	0.176
1136	A	5	3	1.00	17	0.176
1137	A	6	4	1.00	20	0.200
1138	A	5	4	1.00	20	0.200
1139	A	4	4	1.00	20	0.200
1140	A	3	3	1.00	20	0.150
1141	A	1	1	1.00	20	0.050
1142	A	2	2	1.00	20	0.100
1143	A	3	2	1.00	20	0.100
1144	A	4	2	1.00	20	0.100
1145	A	6	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1146	A	5	4	1.00	23	0.174
1147	A	4	4	1.00	23	0.174
1148	A	3	3	1.00	23	0.130
1149	A	1	1	1.00	23	0.043
1150	A	2	2	1.00	23	0.087
1151	A	3	2	1.00	23	0.087
1152	A	4	2	1.00	23	0.087
1153	A	5	3	1.00	19	0.158
1154	A	4	3	1.00	19	0.158
1155	A	3	3	1.00	19	0.158
1156	A	2	2	1.00	19	0.105
1157	A	1	1	1.00	19	0.053
1158	A	2	2	1.00	19	0.105
1159	A	3	2	1.00	19	0.105
1160	A	7	4	1.00	17	0.235
1161	A	5	4	1.00	17	0.235
1162	A	3	3	1.00	17	0.176
1163	A	2	2	1.00	17	0.118
1164	A	4	2	1.00	17	0.118
1165	A	1	1	1.00	17	0.059
1166	A	1	1	1.00	20	0.050
1167	A	1	1	1.00	17	0.059
1168	A	1	1	1.00	20	0.050
1169	A	3	3	1.00	23	0.130
1170	A	11	8	1.00	20	0.400
1171	A	6	5	1.00	25	0.200
1172	A	5	5	1.00	25	0.200
1173	A	4	4	1.00	25	0.160
1174	A	4	4	1.00	25	0.160
1175	A	4	4	1.00	25	0.160
1176	A	5	5	1.00	25	0.200
1177	A	6	5	1.00	25	0.200
1178	A	12	9	1.00	25	0.360
1179	A	11	8	1.00	25	0.320
1180	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1181	A	2	2	1.00	25	0.080
1182	A	3	2	1.00	25	0.080
1183	A	4	2	1.00	25	0.080
1184	A	12	9	1.00	25	0.360
1185	A	11	8	1.00	25	0.320
1186	A	1	1	1.00	25	0.040
1187	A	2	2	1.00	25	0.080
1188	A	3	2	1.00	25	0.080
1189	A	5	4	1.00	25	0.160
1190	A	4	4	1.00	25	0.160
1191	A	3	3	1.00	25	0.120
1192	A	4	4	1.00	25	0.160
1193	A	5	4	1.00	25	0.160
1194	A	13	10	1.00	25	0.400
1195	A	12	9	1.00	25	0.360
1196	A	1	1	1.00	25	0.040
1197	A	2	2	1.00	25	0.080
1198	A	3	2	1.00	25	0.080
1199	A	6	5	1.00	25	0.200
1200	A	5	5	1.00	25	0.200
1201	A	4	4	1.00	25	0.160
1202	A	4	4	1.00	25	0.160
1203	A	4	4	1.00	25	0.160
1204	A	5	5	1.00	25	0.200
1205	A	6	5	1.00	25	0.200
1206	A	6	6	1.00	25	0.240
1207	A	5	5	1.00	25	0.200
1208	A	4	4	1.00	25	0.160
1209	A	3	3	1.00	25	0.120
1210	A	4	4	1.00	25	0.160
1211	A	5	4	1.00	25	0.160
1212	A	13	10	1.00	25	0.400
1213	A	12	9	1.00	25	0.360
1214	A	1	1	1.00	25	0.040
1215	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1216	A	3	2	1.00	25	0.080
1217	A	6	5	1.00	25	0.200
1218	A	5	5	1.00	25	0.200
1219	A	4	4	1.00	25	0.160
1220	A	5	5	1.00	25	0.200
1221	A	4	4	1.00	25	0.160
1222	A	5	5	1.00	25	0.200
1223	A	6	5	1.00	25	0.200
1224	A	13	9	1.00	25	0.360
1225	A	1	1	1.00	25	0.040
1226	A	2	2	1.00	25	0.080
1227	A	3	2	1.00	25	0.080
1228	A	4	2	1.00	25	0.080
1229	A	2	1	1.00	19	0.053
1230	A	2	1	1.00	17	0.059
1231	A	1	1	1.00	19	0.053
1232	A	1	1	1.00	19	0.053
1233	A	3	3	1.00	16	0.188
1234	A	3	3	1.00	19	0.158
1235	A	2	2	1.00	15	0.133
1236	A	2	1	1.00	13	0.077
1237	A	2	1	1.00	13	0.077
1238	A	2	1	1.00	13	0.077
1239	A	2	1	1.00	11	0.091
1240	A	1	0	1.00	5	0.000
1241	A	2	1	1.00	13	0.077
1242	A	2	1	1.00	13	0.077
1243	A	1	1	1.00	13	0.077
1244	A	2	1	1.00	13	0.077
1245	A	2	1	1.00	13	0.077
1246	A	2	1	1.00	15	0.067
1247	A	2	1	1.00	15	0.067
1248	A	2	1	1.00	15	0.067
1249	A	2	1	1.00	13	0.077
1250	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1251	A	2	1	1.00	15	0.067
1252	A	2	1	1.00	15	0.067
1253	A	2	1	1.00	15	0.067
1254	A	1	1	1.00	15	0.067
1255	A	2	1	1.00	15	0.067
1256	A	2	1	1.00	15	0.067
1257	A	2	1	1.00	15	0.067
1258	A	2	1	1.00	15	0.067
1259	A	2	1	1.00	15	0.067
1260	A	2	1	1.00	15	0.067
1261	A	2	1	1.00	15	0.067
1262	A	2	1	1.00	13	0.077
1263	A	1	1	1.00	7	0.143
1264	A	2	1	1.00	15	0.067
1265	A	2	1	1.00	15	0.067
1266	A	2	1	1.00	15	0.067
1267	A	2	1	1.00	15	0.067
1268	A	1	1	1.00	15	0.067
1269	A	2	2	1.00	15	0.133
1270	A	2	1	1.00	15	0.067
1271	A	2	1	1.00	15	0.067
1272	A	2	1	1.00	15	0.067
1273	A	2	1	1.00	15	0.067
1274	A	2	1	1.00	15	0.067
1275	A	2	1	1.00	15	0.067
1276	A	2	1	1.00	15	0.067
1277	A	2	1	1.00	15	0.067
1278	A	2	1	1.00	15	0.067
1279	A	2	1	1.00	15	0.067
1280	A	2	1	1.00	15	0.067
1281	A	2	1	1.00	13	0.077
1282	A	1	1	1.00	7	0.143
1283	A	2	1	1.00	15	0.067
1284	A	2	1	1.00	15	0.067
1285	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1286	A	2	1	1.00	15	0.067
1287	A	2	1	1.00	15	0.067
1288	A	2	1	1.00	15	0.067
1289	A	2	1	1.00	15	0.067
1290	A	2	1	1.00	15	0.067
1291	A	1	1	1.00	15	0.067
1292	A	2	2	1.00	15	0.133
1293	A	3	2	1.00	15	0.133
1294	A	4	2	1.00	15	0.133
1295	A	5	2	1.00	15	0.133
1296	A	2	1	1.00	15	0.067
1297	A	2	1	1.00	15	0.067
1298	A	2	1	1.00	15	0.067
1299	A	2	1	1.00	15	0.067
1300	A	2	1	1.00	15	0.067
1301	A	2	1	1.00	15	0.067
1302	A	2	1	1.00	15	0.067
1303	A	2	1	1.00	15	0.067
1304	A	2	1	1.00	15	0.067
1305	A	2	1	1.00	15	0.067
1306	A	2	1	1.00	15	0.067
1307	A	2	1	1.00	15	0.067
1308	A	2	1	1.00	15	0.067
1309	A	2	1	1.00	15	0.067
1310	A	2	1	1.00	13	0.077
1311	A	1	1	1.00	7	0.143
1312	A	2	1	1.00	15	0.067
1313	A	2	1	1.00	15	0.067
1314	A	2	1	1.00	15	0.067
1315	A	2	1	1.00	15	0.067
1316	A	2	1	1.00	15	0.067
1317	A	2	1	1.00	15	0.067
1318	A	2	1	1.00	15	0.067
1319	A	2	1	1.00	15	0.067
1320	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1321	A	2	1	1.00	15	0.067
1322	A	2	1	1.00	15	0.067
1323	A	1	1	1.00	15	0.067
1324	A	2	2	1.00	15	0.133
1325	A	3	2	1.00	15	0.133
1326	A	4	2	1.00	15	0.133
1327	A	5	2	1.00	15	0.133
1328	A	6	2	1.00	15	0.133
1329	A	7	2	1.00	15	0.133
1330	A	8	2	1.00	15	0.133
1331	A	2	1	1.00	15	0.067
1332	A	2	1	1.00	15	0.067
1333	A	2	1	1.00	15	0.067
1334	A	2	1	1.00	15	0.067
1335	A	2	1	1.00	15	0.067
1336	A	2	1	1.00	15	0.067
1337	A	2	1	1.00	15	0.067
1338	A	2	1	1.00	13	0.077
1339	A	1	1	1.00	7	0.143
1340	A	3	2	1.00	15	0.133
1341	A	2	1	1.00	15	0.067
1342	A	2	1	1.00	15	0.067
1343	A	2	1	1.00	15	0.067
1344	A	2	1	1.00	15	0.067
1345	A	2	1	1.00	15	0.067
1346	A	2	1	1.00	15	0.067
1347	A	2	1	1.00	13	0.077
1348	A	1	1	1.00	7	0.143
1349	A	2	1	1.00	15	0.067
1350	A	2	1	1.00	15	0.067
1351	A	2	1	1.00	15	0.067
1352	A	2	1	1.00	15	0.067
1353	A	2	1	1.00	15	0.067
1354	A	2	1	1.00	15	0.067
1355	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1356	A	2	1	1.00	15	0.067
1357	A	1	1	1.00	13	0.077
1358	A	1	1	1.00	7	0.143
1359	A	2	1	1.00	15	0.067
1360	A	2	1	1.00	15	0.067
1361	A	2	1	1.00	15	0.067
1362	A	2	1	1.00	15	0.067
1363	A	2	1	1.00	15	0.067
1364	A	2	1	1.00	15	0.067
1365	A	1	1	1.00	15	0.067
1366	A	2	2	1.00	15	0.133
1367	A	3	2	1.00	15	0.133
1368	A	2	1	1.00	15	0.067
1369	A	2	1	1.00	15	0.067
1370	A	2	1	1.00	13	0.077
1371	A	1	1	1.00	7	0.143
1372	A	2	1	1.00	15	0.067
1373	A	2	1	1.00	15	0.067
1374	A	2	1	1.00	15	0.067
1375	A	2	1	1.00	17	0.059
1376	A	2	1	1.00	17	0.059
1377	A	2	1	1.00	17	0.059
1378	A	2	1	1.00	17	0.059
1379	A	2	1	1.00	15	0.067
1380	A	1	1	1.00	9	0.111
1381	A	3	3	1.00	17	0.176
1382	A	3	3	1.00	17	0.176
1383	A	4	4	1.00	17	0.235
1384	A	5	4	1.00	17	0.235
1385	A	6	4	1.00	17	0.235
1386	A	7	4	1.00	17	0.235
1387	A	2	1	1.00	17	0.059
1388	A	2	1	1.00	17	0.059
1389	A	2	1	1.00	17	0.059
1390	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1391	A	2	1	1.00	15	0.067
1392	A	1	1	1.00	9	0.111
1393	A	4	3	1.00	17	0.176
1394	A	4	4	1.00	17	0.235
1395	A	4	3	1.00	17	0.176
1396	A	5	4	1.00	17	0.235
1397	A	6	4	1.00	17	0.235
1398	A	7	4	1.00	17	0.235
1399	A	2	1	1.00	17	0.059
1400	A	2	1	1.00	17	0.059
1401	A	2	1	1.00	17	0.059
1402	A	2	1	1.00	17	0.059
1403	A	2	1	1.00	15	0.067
1404	A	1	1	1.00	9	0.111
1405	A	5	3	1.00	17	0.176
1406	A	5	4	1.00	17	0.235
1407	A	5	4	1.00	17	0.235
1408	A	5	3	1.00	17	0.176
1409	A	6	4	1.00	17	0.235
1410	A	7	4	1.00	17	0.235
1411	A	3	3	1.00	13	0.231
1412	A	4	4	1.00	13	0.308
1413	A	2	1	1.00	17	0.059
1414	A	2	1	1.00	17	0.059
1415	A	2	1	1.00	17	0.059
1416	A	2	1	1.00	17	0.059
1417	A	2	1	1.00	15	0.067
1418	A	1	1	1.00	9	0.111
1419	A	2	2	1.00	17	0.118
1420	A	3	3	1.00	17	0.176
1421	A	4	3	1.00	17	0.176
1422	A	5	3	1.00	17	0.176
1423	A	6	3	1.00	17	0.176
1424	A	2	1	1.00	17	0.059
1425	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1426	A	2	1	1.00	17	0.059
1427	A	2	1	1.00	17	0.059
1428	A	2	1	1.00	15	0.067
1429	A	1	1	1.00	9	0.111
1430	A	3	3	1.00	17	0.176
1431	A	4	4	1.00	17	0.235
1432	A	5	4	1.00	17	0.235
1433	A	6	4	1.00	17	0.235
1434	A	2	1	1.00	17	0.059
1435	A	2	1	1.00	17	0.059
1436	A	2	1	1.00	17	0.059
1437	A	2	1	1.00	17	0.059
1438	A	2	1	1.00	15	0.067
1439	A	1	1	1.00	9	0.111
1440	A	4	3	1.00	17	0.176
1441	A	5	4	1.00	17	0.235
1442	A	6	4	1.00	17	0.235
1443	A	7	4	1.00	17	0.235
1444	A	2	2	1.00	20	0.100
1445	A	2	2	1.00	20	0.100
1446	A	2	2	1.00	20	0.100
1447	A	2	2	1.00	20	0.100
1448	A	2	2	1.00	20	0.100
1449	A	2	2	1.00	20	0.100
1450	A	2	2	1.00	20	0.100
1451	A	2	2	1.00	20	0.100
1452	A	2	2	1.00	20	0.100
1453	A	2	2	1.00	13	0.154
1454	A	2	2	1.00	17	0.118
1455	A	5	5	1.00	15	0.333
1456	A	2	1	1.00	13	0.077
1457	A	2	1	1.00	15	0.067
1458	A	4	4	1.00	17	0.235
1459	A	4	4	1.00	17	0.235
1460	A	8	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1461	A	7	4	1.00	19	0.210
1462	A	6	4	1.00	19	0.210
1463	A	5	4	1.00	19	0.210
1464	A	4	4	1.00	19	0.210
1465	A	4	4	1.00	19	0.210
1466	A	1	1	1.00	19	0.053
1467	A	2	2	1.00	19	0.105
1468	A	3	2	1.00	19	0.105
1469	A	4	2	1.00	19	0.105
1470	A	5	2	1.00	19	0.105
1471	A	8	4	1.00	19	0.210
1472	A	7	4	1.00	19	0.210
1473	A	6	4	1.00	19	0.210
1474	A	5	4	1.00	19	0.210
1475	A	5	5	1.00	19	0.263
1476	A	5	4	1.00	19	0.210
1477	A	1	1	1.00	19	0.053
1478	A	2	2	1.00	19	0.105
1479	A	3	2	1.00	19	0.105
1480	A	4	2	1.00	19	0.105
1481	A	9	4	1.00	19	0.210
1482	A	8	4	1.00	19	0.210
1483	A	7	4	1.00	19	0.210
1484	A	6	4	1.00	19	0.210
1485	A	6	5	1.00	19	0.263
1486	A	6	5	1.00	19	0.263
1487	A	6	4	1.00	19	0.210
1488	A	1	1	1.00	19	0.053
1489	A	2	2	1.00	19	0.105
1490	A	3	2	1.00	19	0.105
1491	A	4	2	1.00	19	0.105
1492	A	7	4	1.00	19	0.210
1493	A	6	4	1.00	19	0.210
1494	A	5	4	1.00	19	0.210
1495	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1496	A	3	3	1.00	19	0.158
1497	A	1	1	1.00	19	0.053
1498	A	2	2	1.00	19	0.105
1499	A	3	2	1.00	19	0.105
1500	A	4	2	1.00	19	0.105
1501	A	5	2	1.00	19	0.105
1502	A	7	5	1.00	19	0.263
1503	A	6	5	1.00	19	0.263
1504	A	5	5	1.00	19	0.263
1505	A	4	4	1.00	19	0.210
1506	A	1	1	1.00	19	0.053
1507	A	2	2	1.00	19	0.105
1508	A	3	2	1.00	19	0.105
1509	A	4	2	1.00	19	0.105
1510	A	5	2	1.00	19	0.105
1511	A	6	2	1.00	19	0.105
1512	A	8	5	1.00	19	0.263
1513	A	7	5	1.00	19	0.263
1514	A	6	5	1.00	19	0.263
1515	A	5	4	1.00	19	0.210
1516	A	1	1	1.00	19	0.053
1517	A	2	2	1.00	19	0.105
1518	A	3	2	1.00	19	0.105
1519	A	4	2	1.00	19	0.105
1520	A	5	2	1.00	19	0.105
1521	A	6	2	1.00	19	0.105
1522	A	2	2	1.00	20	0.100
1523	A	2	2	1.00	19	0.105
1524	A	2	2	1.00	19	0.105
1525	A	2	2	1.00	17	0.118
1526	A	2	2	1.00	19	0.105
1527	A	1	1	1.00	19	0.053
1528	A	2	2	1.00	19	0.105
1529	A	2	2	1.00	19	0.105
1530	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1531	A	2	2	1.00	19	0.105
1532	A	2	2	1.00	17	0.118
1533	A	2	2	1.00	19	0.105
1534	A	1	1	1.00	19	0.053
1535	A	2	2	1.00	19	0.105
1536	A	3	3	1.00	20	0.150
1537	A	2	2	1.00	20	0.100
1538	A	3	3	1.00	20	0.150
1539	A	3	3	1.00	18	0.167
1540	A	3	3	1.00	20	0.150
1541	A	2	2	1.00	20	0.100
1542	A	3	3	1.00	20	0.150
1543	A	2	2	1.00	21	0.095
1544	A	1	1	1.00	8	0.125
1545	A	2	2	1.00	21	0.095
1546	A	2	2	1.00	19	0.105
1547	A	2	2	1.00	21	0.095
1548	A	1	1	1.00	21	0.048
1549	A	2	2	1.00	21	0.095
1550	A	1	1	1.00	19	0.053
1551	A	2	2	1.00	29	0.069
1552	A	2	2	1.00	15	0.133
1553	A	2	2	1.00	19	0.105
1554	A	2	2	1.00	29	0.069
1555	A	3	3	1.00	15	0.200
1556	A	2	2	1.00	15	0.133
1557	A	2	2	1.00	19	0.105
1558	A	3	3	1.00	20	0.150
1559	A	5	3	1.00	19	0.158
1560	A	4	3	1.00	19	0.158
1561	A	3	3	1.00	19	0.158
1562	A	3	3	1.00	19	0.158
1563	A	4	4	1.00	19	0.210
1564	A	5	4	1.00	19	0.210
1565	A	6	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1566	A	5	5	1.00	19	0.263
1567	A	4	4	1.00	19	0.210
1568	A	5	5	1.00	19	0.263
1569	A	6	5	1.00	19	0.263
1570	A	4	3	1.00	19	0.158
1571	A	3	3	1.00	19	0.158
1572	A	2	2	1.00	19	0.105
1573	A	3	3	1.00	19	0.158
1574	A	4	3	1.00	19	0.158
1575	A	3	2	1.00	19	0.105
1576	A	2	2	1.00	19	0.105
1577	A	2	2	1.00	19	0.105
1578	A	1	1	1.00	19	0.053
1579	A	2	2	1.00	19	0.105
1580	A	3	2	1.00	19	0.105
1581	A	4	2	1.00	19	0.105
1582	A	6	4	1.00	19	0.210
1583	A	5	4	1.00	19	0.210
1584	A	4	4	1.00	19	0.210
1585	A	4	4	1.00	19	0.210
1586	A	5	5	1.00	19	0.263
1587	A	3	2	1.00	19	0.105
1588	A	2	2	1.00	19	0.105
1589	A	1	1	1.00	19	0.053
1590	A	1	1	1.00	19	0.053
1591	A	2	2	1.00	19	0.105
1592	A	3	2	1.00	19	0.105
1593	A	4	2	1.00	19	0.105
1594	A	8	6	1.00	19	0.316
1595	A	7	6	1.00	19	0.316
1596	A	6	6	1.00	19	0.316
1597	A	5	5	1.00	19	0.263
1598	A	6	6	1.00	19	0.316
1599	A	7	6	1.00	19	0.316
1600	A	8	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1601	A	3	2	1.00	19	0.105
1602	A	2	2	1.00	19	0.105
1603	A	1	1	1.00	19	0.053
1604	A	1	1	1.00	19	0.053
1605	A	2	2	1.00	19	0.105
1606	A	3	2	1.00	19	0.105
1607	A	4	2	1.00	19	0.105
1608	A	6	4	1.00	19	0.210
1609	A	5	4	1.00	19	0.210
1610	A	4	4	1.00	19	0.210
1611	A	3	3	1.00	19	0.158
1612	A	4	4	1.00	19	0.210
1613	A	5	4	1.00	19	0.210
1614	A	6	4	1.00	19	0.210
1615	A	4	3	1.00	19	0.158
1616	A	3	3	1.00	19	0.158
1617	A	2	2	1.00	19	0.105
1618	A	1	1	1.00	19	0.053
1619	A	2	2	1.00	19	0.105
1620	A	3	2	1.00	19	0.105
1621	A	4	2	1.00	19	0.105
1622	A	8	7	1.00	19	0.368
1623	A	7	7	1.00	19	0.368
1624	A	6	6	1.00	19	0.316
1625	A	6	6	1.00	19	0.316
1626	A	7	6	1.00	19	0.316
1627	A	8	6	1.00	19	0.316
1628	A	2	2	1.00	15	0.133
1629	A	6	4	1.00	19	0.210
1630	A	5	4	1.00	19	0.210
1631	A	4	4	1.00	19	0.210
1632	A	4	4	1.00	19	0.210
1633	A	5	5	1.00	19	0.263
1634	A	6	5	1.00	19	0.263
1635	A	10	8	1.00	19	0.421

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1636	A	9	8	1.00	19	0.421
1637	A	8	8	1.00	19	0.421
1638	A	8	8	1.00	19	0.421
1639	A	9	9	1.00	19	0.474
1640	A	10	9	1.00	19	0.474
1641	A	7	4	1.00	19	0.210
1642	A	6	4	1.00	19	0.210
1643	A	5	4	1.00	19	0.210
1644	A	5	5	1.00	19	0.263
1645	A	5	4	1.00	19	0.210
1646	A	6	5	1.00	19	0.263
1647	A	7	5	1.00	19	0.263
1648	A	10	8	1.00	19	0.421
1649	A	9	8	1.00	19	0.421
1650	A	8	8	1.00	19	0.421
1651	A	7	7	1.00	19	0.368
1652	A	8	8	1.00	19	0.421
1653	A	9	8	1.00	19	0.421
1654	A	5	4	1.00	19	0.210
1655	A	4	4	1.00	19	0.210
1656	A	3	3	1.00	19	0.158
1657	A	4	4	1.00	19	0.210
1658	A	5	4	1.00	19	0.210
1659	A	10	9	1.00	19	0.474
1660	A	9	9	1.00	19	0.474
1661	A	8	8	1.00	19	0.421
1662	A	8	8	1.00	19	0.421
1663	A	9	8	1.00	19	0.421
1664	A	10	8	1.00	19	0.421
1665	A	7	5	1.00	19	0.263
1666	A	5	5	1.00	19	0.263
1667	A	4	4	1.00	19	0.210
1668	A	4	4	1.00	19	0.210
1669	A	5	4	1.00	19	0.210
1670	A	6	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1671	A	11	9	1.00	19	0.474
1672	A	10	9	1.00	19	0.474
1673	A	9	8	1.00	19	0.421
1674	A	9	9	1.00	19	0.474
1675	A	9	8	1.00	19	0.421
1676	A	10	8	1.00	19	0.421
1677	A	11	8	1.00	19	0.421
1678	A	8	6	1.00	19	0.316
1679	A	7	6	1.00	19	0.316
1680	A	7	7	1.00	19	0.368
1681	A	7	6	1.00	19	0.316
1682	A	1	1	1.00	19	0.053
1683	A	2	2	1.00	19	0.105
1684	A	3	2	1.00	19	0.105
1685	A	4	2	1.00	19	0.105
1686	A	7	4	1.00	19	0.210
1687	A	6	4	1.00	19	0.210
1688	A	5	4	1.00	19	0.210
1689	A	5	5	1.00	19	0.263
1690	A	5	4	1.00	19	0.210
1691	A	6	5	1.00	19	0.263
1692	A	7	5	1.00	19	0.263
1693	A	7	6	1.00	19	0.316
1694	A	6	6	1.00	19	0.316
1695	A	5	5	1.00	19	0.263
1696	A	1	1	1.00	19	0.053
1697	A	2	2	1.00	19	0.105
1698	A	3	2	1.00	19	0.105
1699	A	4	2	1.00	19	0.105
1700	A	7	6	1.00	19	0.316
1701	A	6	6	1.00	19	0.316
1702	A	5	5	1.00	19	0.263
1703	A	6	6	1.00	19	0.316
1704	A	7	6	1.00	19	0.316
1705	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1706	A	6	6	1.00	19	0.316
1707	A	5	5	1.00	19	0.263
1708	A	1	1	1.00	19	0.053
1709	A	2	2	1.00	19	0.105
1710	A	3	2	1.00	19	0.105
1711	A	4	2	1.00	19	0.105
1712	A	5	4	1.00	19	0.210
1713	A	4	4	1.00	19	0.210
1714	A	3	3	1.00	19	0.158
1715	A	4	4	1.00	19	0.210
1716	A	5	4	1.00	19	0.210
1717	A	7	7	1.00	19	0.368
1718	A	6	6	1.00	19	0.316
1719	A	1	1	1.00	19	0.053
1720	A	2	2	1.00	19	0.105
1721	A	3	2	1.00	19	0.105
1722	A	4	2	1.00	19	0.105
1723	A	8	7	1.00	19	0.368
1724	A	7	7	1.00	19	0.368
1725	A	6	6	1.00	19	0.316
1726	A	6	6	1.00	19	0.316
1727	A	7	6	1.00	19	0.316
1728	A	8	6	1.00	19	0.316
1729	A	11	8	1.00	20	0.400
1730	A	11	8	1.00	20	0.400
1731	A	2	2	1.00	19	0.105
1732	A	2	2	1.00	19	0.105
1733	A	2	2	1.00	19	0.105
1734	A	2	2	1.00	19	0.105
1735	A	2	2	1.00	19	0.105
1736	A	6	3	1.00	19	0.158
1737	A	5	3	1.00	19	0.158
1738	A	4	3	1.00	19	0.158
1739	A	3	3	1.00	19	0.158
1740	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1741	A	4	4	1.00	19	0.210
1742	A	7	5	1.00	19	0.263
1743	A	6	5	1.00	19	0.263
1744	A	5	5	1.00	19	0.263
1745	A	5	5	1.00	19	0.263
1746	A	6	6	1.00	19	0.316
1747	A	7	6	1.00	19	0.316
1748	A	7	5	1.00	19	0.263
1749	A	6	5	1.00	19	0.263
1750	A	5	5	1.00	19	0.263
1751	A	4	4	1.00	19	0.210
1752	A	5	5	1.00	19	0.263
1753	A	6	5	1.00	19	0.263
1754	A	5	3	1.00	19	0.158
1755	A	4	3	1.00	19	0.158
1756	A	3	3	1.00	19	0.158
1757	A	2	2	1.00	19	0.105
1758	A	3	3	1.00	19	0.158
1759	A	4	3	1.00	19	0.158
1760	A	7	6	1.00	19	0.316
1761	A	6	6	1.00	19	0.316
1762	A	5	5	1.00	19	0.263
1763	A	5	5	1.00	19	0.263
1764	A	6	5	1.00	19	0.263
1765	A	7	5	1.00	19	0.263
1766	A	2	2	1.00	19	0.105
1767	A	2	2	1.00	19	0.105
1768	A	2	2	1.00	19	0.105
1769	A	2	2	1.00	19	0.105
1770	A	2	2	1.00	19	0.105
1771	A	2	2	1.00	19	0.105
1772	A	14	9	1.00	19	0.474
1773	A	13	9	1.00	19	0.474
1774	A	13	9	1.00	19	0.474
1775	A	1	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1776	A	2	2	1.00	19	0.105
1777	A	3	2	1.00	19	0.105
1778	A	4	2	1.00	19	0.105
1779	A	14	9	1.00	19	0.474
1780	A	13	9	1.00	19	0.474
1781	A	13	9	1.00	19	0.474
1782	A	1	1	1.00	19	0.053
1783	A	2	2	1.00	19	0.105
1784	A	3	2	1.00	19	0.105
1785	A	4	2	1.00	19	0.105
1786	A	2	2	1.00	19	0.105
1787	A	2	2	1.00	19	0.105
1788	A	2	2	1.00	19	0.105
1789	A	2	2	1.00	19	0.105
1790	A	2	2	1.00	19	0.105
1791	A	2	2	1.00	19	0.105
1792	A	2	2	1.00	19	0.105
1793	A	2	2	1.00	19	0.105
1794	A	2	2	1.00	19	0.105
1795	A	2	2	1.00	19	0.105
1796	A	2	2	1.00	19	0.105
1797	A	2	2	1.00	19	0.105
1798	A	14	9	1.00	19	0.474
1799	A	14	10	1.00	19	0.526
1800	A	14	9	1.00	19	0.474
1801	A	1	1	1.00	19	0.053
1802	A	2	2	1.00	19	0.105
1803	A	3	2	1.00	19	0.105
1804	A	4	2	1.00	19	0.105
1805	A	14	9	1.00	19	0.474
1806	A	13	9	1.00	19	0.474
1807	A	12	8	1.00	19	0.421
1808	A	1	1	1.00	19	0.053
1809	A	2	2	1.00	19	0.105
1810	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1811	A	4	2	1.00	19	0.105
1812	A	2	2	1.00	19	0.105
1813	A	2	2	1.00	19	0.105
1814	A	2	2	1.00	19	0.105
1815	A	2	2	1.00	19	0.105
1816	A	2	2	1.00	19	0.105
1817	A	2	2	1.00	19	0.105
1818	A	2	2	1.00	19	0.105
1819	A	2	2	1.00	19	0.105
1820	A	2	2	1.00	19	0.105
1821	A	2	2	1.00	19	0.105
1822	A	2	2	1.00	19	0.105
1823	A	2	2	1.00	19	0.105
1824	A	14	9	1.00	19	0.474
1825	A	13	9	1.00	19	0.474
1826	A	12	8	1.00	19	0.421
1827	A	1	1	1.00	19	0.053
1828	A	2	2	1.00	19	0.105
1829	A	3	2	1.00	19	0.105
1830	A	4	2	1.00	19	0.105
1831	A	15	10	1.00	19	0.526
1832	A	14	10	1.00	19	0.526
1833	A	13	9	1.00	19	0.474
1834	A	1	1	1.00	19	0.053
1835	A	2	2	1.00	19	0.105
1836	A	3	2	1.00	19	0.105
1837	A	4	2	1.00	19	0.105
1838	A	2	2	1.00	19	0.105
1839	A	2	2	1.00	19	0.105
1840	A	2	2	1.00	19	0.105
1841	A	2	2	1.00	19	0.105
1842	A	2	2	1.00	19	0.105
1843	A	2	2	1.00	19	0.105
1844	A	1	1	1.00	16	0.062
1845	A	2	2	1.21	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1846	A	2	1	1.00	15	0.067
1847	A	2	1	1.00	15	0.067
1848	A	2	1	1.00	13	0.077
1849	A	1	1	1.00	15	0.067
1850	A	1	1	1.00	15	0.067
1851	A	1	1	1.00	15	0.067
1852	A	2	1	1.00	15	0.067
1853	A	2	1	1.00	15	0.067
1854	A	2	1	1.00	13	0.077
1855	A	1	1	1.00	7	0.143
1856	A	1	1	1.00	15	0.067
1857	A	1	1	1.00	15	0.067
1858	A	1	1	1.00	15	0.067
1859	A	3	2	1.00	19	0.105
1860	A	2	2	1.00	19	0.105
1861	A	1	1	1.00	19	0.053
1862	A	2	2	1.00	19	0.105
1863	A	2	2	1.00	17	0.118
1864	A	2	2	1.00	19	0.105
1865	A	2	2	1.00	19	0.105
1866	A	2	2	1.00	17	0.118
1867	A	2	2	1.00	19	0.105
1868	A	1	1	1.00	19	0.053
1869	A	2	2	1.00	19	0.105
1870	A	3	2	1.00	19	0.105
1871	A	4	2	1.00	19	0.105
1872	A	2	2	1.00	17	0.118
1873	A	2	2	1.00	19	0.105
1874	A	1	1	1.00	19	0.053
1875	A	2	2	1.00	19	0.105
1876	A	3	2	1.00	19	0.105
1877	A	4	2	1.00	19	0.105
1878	A	2	2	1.00	21	0.095
1879	A	2	2	1.00	21	0.095
1880	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1881	A	3	2	1.00	24	0.083
1882	A	2	2	1.00	28	0.071
1883	A	2	2	1.00	44	0.045
1884	A	2	2	1.00	51	0.039
1885	A	1	1	1.00	15	0.067
1886	A	1	1	1.00	15	0.067
1887	A	1	1	1.00	15	0.067
1888	A	1	1	1.00	15	0.067
1889	A	1	1	1.00	17	0.059
1890	A	2	2	1.00	27	0.074
1891	A	1	0	1.00	15	0.000
1892	A	1	0	1.00	9	0.000
1893	A	1	0	1.00	5	0.000
1894	A	1	0	1.00	5	0.000
1895	A	1	0	1.00	9	0.000
1896	A	1	0	1.00	9	0.000
1897	A	1	0	1.00	15	0.000
1898	A	1	0	1.00	10	0.000
1899	A	1	0	1.00	10	0.000
1900	A	1	0	1.00	12	0.000
1901	A	1	0	1.00	15	0.000
1902	A	1	0	1.00	17	0.000
1903	A	1	0	1.00	8	0.000
1904	A	1	0	1.00	10	0.000
1905	A	1	0	1.00	11	0.000
1906	A	1	0	1.00	11	0.000
1907	A	1	0	1.00	6	0.000
1908	A	1	0	1.00	11	0.000
1909	A	1	0	1.00	10	0.000
1910	A	1	0	1.00	11	0.000
1911	A	1	0	1.00	7	0.000
1912	A	1	0	1.00	17	0.000
1913	A	1	0	1.00	18	0.000
1914	A	1	0	1.00	11	0.000
1915	A	1	0	1.00	15	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1916	A	1	0	1.00	18	0.000
1917	A	1	0	1.00	20	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int 0\,dx$

Optimal. Leaf size=1

$$0$$

```
[Out] 0
```

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$0$$

Antiderivative was successfully verified.

```
[In] Int[0,x]
```

```
[Out] 0
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 0\,dx = 0$$

Mathematica [A]
time = 0.01, size = 1, normalized size = 1.00

$$0$$

Antiderivative was successfully verified.

```
[In] Integrate[0,x]
```

```
[Out] 0
```

Mathics [A]
time = 1.47, size = 1, normalized size = 1.00

$$0$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[0,x]')
[Out] 0
```

Maple [A]
time = 0.01, size = 2, normalized size = 2.00

method	result	size
default	0	2
norman	0	2
meijerg	0	2
risch	0	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x,method=_RETURNVERBOSE)
[Out] 0
```

Maxima [B] Error detected during grading. Assigning place holder grade for now.
time = 0.25, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="maxima")
[Out] 0
```

Fricas [A]
time = 0.29, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="fricas")
[Out] 0
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x)
[Out] 0
```

Giac [**B**] N/A
time = 0.00, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x)
```

```
[Out] 0
```

Mupad [**B**]
time = 0.04, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
```

```
[Out] 0
```

3.2 $\int 1 \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$x$$

Antiderivative was successfully verified.

```
[In] Int[1,x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1 \, dx = x$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[1,x]
[Out] x
```

Mathics [A]
time = 1.43, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1,x]')
```

```
[Out] x
```

Maple [A]
time = 0.00, size = 2, normalized size = 2.00

method	result	size
default	x	2
norman	x	2
risch	x	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.26, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 0.54, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Giac [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Mupad [B]
time = 0.01, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

3.3 $\int 5\,dx$

Optimal. Leaf size=3

$$5x$$

[Out] 5*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$5x$$

Antiderivative was successfully verified.

[In] Int[5,x]

[Out] 5*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 5\,dx = 5x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$5x$$

Antiderivative was successfully verified.

[In] Integrate[5,x]

[Out] 5*x

Mathics [A]
time = 1.52, size = 3, normalized size = 1.00

$$5x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[5,x]')
```

```
[Out] 5 x
```

Maple [A]
time = 0.00, size = 4, normalized size = 1.33

method	result	size
default	$5x$	4
norman	$5x$	4
risch	$5x$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x
```

Maxima [A]
time = 0.24, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="maxima")
```

```
[Out] 5*x
```

Fricas [A]
time = 0.29, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="fricas")
```

```
[Out] 5*x
```

Sympy [A]
time = 0.01, size = 2, normalized size = 0.67

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x)
```

```
[Out] 5*x
```

Giac [A]
time = 0.00, size = 3, normalized size = 1.00

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x)
```

```
[Out] 5*x
```

Mupad [B]
time = 0.01, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x)
```

```
[Out] 5*x
```

3.4 $\int -2\,dx$

Optimal. Leaf size=3

$$-2x$$

[Out] -2*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$-2x$$

Antiderivative was successfully verified.

[In] Int[-2,x]

[Out] -2*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -2\,dx = -2x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Antiderivative was successfully verified.

[In] Integrate[-2,x]

[Out] -2*x

Mathics [A]
time = 1.52, size = 3, normalized size = 1.00

$$-2x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-2,x]')
```

```
[Out] -2 x
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size
default	$-2x$	4
norman	$-2x$	4
risch	$-2x$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x
```

Maxima [A]
time = 0.25, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="maxima")
```

```
[Out] -2*x
```

Fricas [A]
time = 0.30, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="fricas")
```

```
[Out] -2*x
```

Sympy [A]
time = 0.01, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x)
```

```
[Out] -2*x
```

Giac [A]
time = 0.01, size = 4, normalized size = 1.33

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x)
```

```
[Out] -2*x
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
```

```
[Out] -2*x
```

3.5 $\int -\frac{3}{2} \, dx$

Optimal. Leaf size=5

$$-\frac{3x}{2}$$

```
[Out] -3/2*x
```

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{8\}$

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-3/2,x]
[Out] (-3*x)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -\frac{3}{2} \, dx = -\frac{3x}{2}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/2,x]
[Out] (-3*x)/2
```

Mathics [A]
time = 1.49, size = 3, normalized size = 0.60

$$\frac{-3x}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-3/2,x]')
```

```
[Out] -3 x / 2
```

Maple [A]
time = 0.01, size = 4, normalized size = 0.80

method	result	size
default	$-\frac{3x}{2}$	4
norman	$-\frac{3x}{2}$	4
risch	$-\frac{3x}{2}$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x
```

Maxima [A]
time = 0.24, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="maxima")
```

```
[Out] -3/2*x
```

Fricas [A]
time = 0.28, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="fricas")
```

```
[Out] -3/2*x
```

Sympy [A]
time = 0.01, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x)
```

```
[Out] -3*x/2
```

Giac [A]
time = 0.01, size = 5, normalized size = 1.00

$$-\frac{3}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x)
```

```
[Out] -3/2*x
```

Mupad [B]
time = 0.01, size = 3, normalized size = 0.60

$$-\frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
```

```
[Out] -(3*x)/2
```

3.6 $\int \pi \, dx$

Optimal. Leaf size=3

$$\pi x$$

[Out] Pi*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$\pi x$$

Antiderivative was successfully verified.

[In] Int[Pi,x]

[Out] Pi*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \pi \, dx = \pi x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\pi x$$

Antiderivative was successfully verified.

[In] Integrate[Pi,x]

[Out] Pi*x

Mathics [A]
time = 1.51, size = 3, normalized size = 1.00

$$\text{Pi}x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Pi,x]')
```

```
[Out] Pi x
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size
default	πx	4
norman	πx	4
risch	πx	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x
```

Maxima [A]
time = 0.26, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="maxima")
```

```
[Out] pi*x
```

Fricas [A]
time = 0.28, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="fricas")
```

```
[Out] pi*x
```

Sympy [A]
time = 0.01, size = 2, normalized size = 0.67

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x)
```

```
[Out] pi*x
```

Giac [**A**]
time = 0.00, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x)
```

```
[Out] pi*x
```

Mupad [**B**]
time = 0.00, size = 3, normalized size = 1.00

$$\Pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x)
```

```
[Out] Pi*x
```

3.7 $\int a\,dx$

Optimal. Leaf size=3

$$ax$$

[Out] a*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {8}

$$ax$$

Antiderivative was successfully verified.

[In] Int[a,x]

[Out] a*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int a\,dx = ax$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$ax$$

Antiderivative was successfully verified.

[In] Integrate[a,x]

[Out] a*x

Mathics [A]
time = 1.49, size = 3, normalized size = 1.00

$$ax$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[a,x]')
```

```
[Out] a x
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size
default	ax	4
norman	ax	4
risch	ax	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x,method=_RETURNVERBOSE)
```

```
[Out] a*x
```

Maxima [A]
time = 0.25, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="maxima")
```

```
[Out] a*x
```

Fricas [A]
time = 0.27, size = 3, normalized size = 1.00

$$xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="fricas")
```

```
[Out] x*a
```

Sympy [A]
time = 0.01, size = 2, normalized size = 0.67

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x)
```

```
[Out] a*x
```

Giac [A]
time = 0.00, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x)
```

```
[Out] a*x
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x)
```

```
[Out] a*x
```

3.8 $\int 3a\,dx$

Optimal. Leaf size=4

$$3ax$$

```
[Out] 3*a*x
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{8\}$

$$3ax$$

Antiderivative was successfully verified.

```
[In] Int[3*a,x]
```

```
[Out] 3*a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 3a\,dx = 3ax$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$3ax$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a,x]
```

```
[Out] 3*a*x
```

Mathics [A]
time = 1.51, size = 4, normalized size = 1.00

$$3ax$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[3*a,x]')
```

```
[Out] 3 a x
```

Maple [A]
time = 0.01, size = 5, normalized size = 1.25

method	result	size
default	$3ax$	5
norman	$3ax$	5
risch	$3ax$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*x
```

Maxima [A]
time = 0.24, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="maxima")
```

```
[Out] 3*a*x
```

Fricas [A]
time = 0.29, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="fricas")
```

```
[Out] 3*a*x
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.75

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x)
```

```
[Out] 3*a*x
```

Giac [A]
time = 0.00, size = 4, normalized size = 1.00

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x)
```

```
[Out] 3*a*x
```

Mupad [B]
time = 0.00, size = 4, normalized size = 1.00

$$3\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a,x)
```

```
[Out] 3*a*x
```

3.9 $\int \frac{\pi}{\sqrt{16-e^2}}\,dx$

Optimal. Leaf size=14

$$\frac{\pi x}{\sqrt{16-e^2}}$$

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{8\}$

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Pi/Sqrt[16 - E^2],x]
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\pi}{\sqrt{16-e^2}}\,dx = \frac{\pi x}{\sqrt{16-e^2}}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi/Sqrt[16 - E^2],x]
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Mathics [A]
time = 1.56, size = 12, normalized size = 0.86

$$\frac{\text{Pi}x}{\sqrt{16-E^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Pi/Sqrt[16 - E^2],x]')
```

```
[Out] Pi x / Sqrt[16 - E ^ 2]
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.86

method	result	size
default	$\frac{\pi x}{\sqrt{16-\mathrm{e}^{2}}}$	12
norman	$-\frac{\pi \sqrt{16-\mathrm{e}^{2}}\, x}{-16+\mathrm{e}^{2}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16-exp(2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

Maxima [A]
time = 0.24, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="maxima")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

Fricas [A]
time = 0.29, size = 18, normalized size = 1.29

$$-\frac{\pi x\sqrt{-e^2+16}}{e^2-16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="fricas")
```

```
[Out] -pi*x*sqrt(-e^2 + 16)/(e^2 - 16)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.71

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))**(1/2),x)
```

```
[Out] pi*x/sqrt(16 - exp(2))
```

Giac [**A**]
time = 0.00, size = 13, normalized size = 0.93

$$\frac{\pi x}{\sqrt{-\mathrm{e}^{2} + 16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x)
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

Mupad [**B**]
time = 0.00, size = 11, normalized size = 0.79

$$\frac{\Pi\, x}{\sqrt{16 - \mathrm{e}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16 - exp(2))^(1/2),x)
```

```
[Out] (Pi*x)/(16 - exp(2))^(1/2)
```

3.10 $\int x^{100}\,dx$

Optimal. Leaf size=7

$$\frac{x^{101}}{101}$$

[Out] 1/101*x^101

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

[In] Int[x^100,x]

[Out] x^101/101

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{100}\,dx = \frac{x^{101}}{101}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

[In] Integrate[x^100,x]

[Out] x^101/101

Mathics [A]
time = 1.51, size = 5, normalized size = 0.71

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^100,x]')
```

```
[Out] x ^ 101 / 101
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^{101}}{101}$	6
default	$\frac{x^{101}}{101}$	6
risch	$\frac{x^{101}}{101}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x,method=_RETURNVERBOSE)
```

```
[Out] 1/101*x^101
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="maxima")
```

```
[Out] 1/101*x^101
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="fricas")
```

```
[Out] 1/101*x^101
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**100,x)
```

```
[Out] x**101/101
```

Giac [A]
time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x)
```

```
[Out] 1/101*x^101
```

Mupad [B]
time = 0.12, size = 5, normalized size = 0.71

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] x^101/101
```

3.11 $\int x^3 \, dx$

Optimal. Leaf size=7

$$\frac{x^4}{4}$$

```
[Out] 1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3,x]
```

```
[Out] x^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \, dx = \frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3,x]
```

```
[Out] x^4/4
```

Mathics [A]
time = 1.52, size = 5, normalized size = 0.71

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3,x]')
```

```
[Out] x ^ 4 / 4
```

Maple [A]
time = 0.00, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^4}{4}$	6
default	$\frac{x^4}{4}$	6
norman	$\frac{x^4}{4}$	6
risch	$\frac{x^4}{4}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4
```

Maxima [A]
time = 0.26, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3,x)
```

```
[Out] x**4/4
```

Giac [A]
time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x)
```

```
[Out] 1/4*x^4
```

Mupad [B]
time = 0.02, size = 5, normalized size = 0.71

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
```

```
[Out] x^4/4
```

3.12 $\int x^2\,dx$

Optimal. Leaf size=7

$$\frac{x^3}{3}$$

[Out] 1/3*x^3

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2,x]

[Out] x^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\,dx = \frac{x^3}{3}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2,x]

[Out] x^3/3

Mathics [A]
time = 1.51, size = 5, normalized size = 0.71

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2,x]')
```

```
[Out] x ^ 3 / 3
```

Maple [A]
time = 0.00, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^3}{3}$	6
default	$\frac{x^3}{3}$	6
norman	$\frac{x^3}{3}$	6
risch	$\frac{x^3}{3}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2,x)
```

```
[Out] x**3/3
```

Giac [A]
time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x)
```

```
[Out] 1/3*x^3
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
```

```
[Out] x^3/3
```

3.13 $\int x\,dx$

Optimal. Leaf size=7

$$\frac{x^2}{2}$$

```
[Out] 1/2*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{30\}$

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x,x]
```

```
[Out] x^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\,dx = \frac{x^2}{2}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x,x]
```

```
[Out] x^2/2
```

Mathics [A]
time = 1.52, size = 5, normalized size = 0.71

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1,x]')
```

```
[Out] x ^ 2 / 2
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^2}{2}$	6
default	$\frac{x^2}{2}$	6
norman	$\frac{x^2}{2}$	6
risch	$\frac{x^2}{2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2
```

Fricas [A]
time = 0.27, size = 5, normalized size = 0.71

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.43

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x)
```

```
[Out] x**2/2
```

Giac [A]
time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x)
```

```
[Out] 1/2*x^2
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x)
```

```
[Out] x^2/2
```

3.14 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$x$$

Antiderivative was successfully verified.

[In] Int[1,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[1,x]

[Out] x

Mathics [A]
time = 1.43, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0,x]')
```

```
[Out] x
```

Maple [A]
time = 0.00, size = 2, normalized size = 2.00

method	result	size
default	x	2
norman	x	2
risch	x	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.25, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 0.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Giac [**A**]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Mupad [**B**]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

3.15 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{29\}$

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Mathics [A]
time = 1.42, size = 2, normalized size = 1.00

$$\text{Log}\,[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^1,x]')
[Out] Log[x]
```

Maple [A]
time = 0.00, size = 3, normalized size = 1.50

method	result	size
default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x,method=_RETURNVERBOSE)
[Out] ln(x)
```

Maxima [A]
time = 0.26, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
[Out] log(x)
```

Fricas [A]
time = 0.30, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

Sympy [A]
time = 0.03, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
[Out] log(x)
```

Giac [**A**]
time = 0.00, size = 3, normalized size = 1.50

$$\ln |x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(abs(x))
```

Mupad [**B**]
time = 0.04, size = 2, normalized size = 1.00

$$\ln (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

3.16 $\int \frac{1}{x^2} \, dx$

Optimal. Leaf size=5

$$-\frac{1}{x}$$

```
[Out] -1/x
```

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2),x]
```

```
[Out] -x^(-1)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2} \, dx = -\frac{1}{x}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2),x]
```

```
[Out] -x^(-1)
```

Mathics [A]
time = 1.50, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^2,x]')
```

```
[Out] -1 / x
```

Maple [A]
time = 0.01, size = 6, normalized size = 1.20

method	result	size
gosper	$-\frac{1}{x}$	6
default	$-\frac{1}{x}$	6
norman	$-\frac{1}{x}$	6
risch	$-\frac{1}{x}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x
```

Maxima [A]
time = 0.26, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="maxima")
```

```
[Out] -1/x
```

Fricas [A]
time = 0.29, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="fricas")
```

```
[Out] -1/x
```

Sympy [A]
time = 0.03, size = 3, normalized size = 0.60

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2,x)
```

```
[Out] -1/x
```

Giac [A]
time = 0.00, size = 3, normalized size = 0.60

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x)
```

```
[Out] -1/x
```

Mupad [B]
time = 0.03, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x)
```

```
[Out] -1/x
```

3.17 $\int \frac{1}{x^3} dx$

Optimal. Leaf size=7

$$-\frac{1}{2x^2}$$

[Out] -1/2/x^2

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[x^(-3),x]

[Out] -1/2*1/x^2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3} dx = -\frac{1}{2x^2}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3),x]

[Out] -1/2*1/x^2

Mathics [A]
time = 1.50, size = 5, normalized size = 0.71

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^3,x]')
```

```
[Out] -1 / (2 x ^ 2)
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{2x^2}$	6
default	$-\frac{1}{2x^2}$	6
norman	$-\frac{1}{2x^2}$	6
risch	$-\frac{1}{2x^2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2
```

Maxima [A]
time = 0.26, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/x^2
```

Sympy [A]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3,x)
```

```
[Out] -1/(2*x**2)
```

Giac [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x)
```

```
[Out] -1/2/x^2
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
```

```
[Out] -1/(2*x^2)
```

3.18 $\int \frac{1}{x^4} dx$

Optimal. Leaf size=7

$$-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4),x]
```

```
[Out] -1/3*1/x^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4),x]
```

```
[Out] -1/3*1/x^3
```

Mathics [A]
time = 1.52, size = 5, normalized size = 0.71

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^4,x]')
```

```
[Out] -1 / (3 x ^ 3)
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{3x^3}$	6
default	$-\frac{1}{3x^3}$	6
norman	$-\frac{1}{3x^3}$	6
risch	$-\frac{1}{3x^3}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3
```

Maxima [A]
time = 0.26, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="maxima")
```

```
[Out] -1/3/x^3
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="fricas")
```

```
[Out] -1/3/x^3
```

Sympy [A]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4,x)
```

```
[Out] -1/(3*x**3)
```

Giac [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x)
```

```
[Out] -1/3/x^3
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
```

```
[Out] -1/(3*x^3)
```

3.19 $\int \frac{1}{x^{100}}\,dx$

Optimal. Leaf size=7

$$-\frac{1}{99x^{99}}$$

```
[Out] -1/99/x^99
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-100),x]
```

```
[Out] -1/99*1/x^99
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{100}}\,dx = -\frac{1}{99x^{99}}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-100),x]
```

```
[Out] -1/99*1/x^99
```

Mathics [A]
time = 1.53, size = 5, normalized size = 0.71

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^100,x]')
```

```
[Out] -1 / (99 x ^ 99)
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{99x^{99}}$	6
default	$-\frac{1}{99x^{99}}$	6
risch	$-\frac{1}{99x^{99}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x,method=_RETURNVERBOSE)
```

```
[Out] -1/99/x^99
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="maxima")
```

```
[Out] -1/99/x^99
```

Fricas [A]
time = 0.31, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="fricas")
```

```
[Out] -1/99/x^99
```

Sympy [A]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**100,x)
```

```
[Out] -1/(99*x**99)
```

Giac [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x)
```

```
[Out] -1/99/x^99
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/(99*x^99)
```

3.20 $\int x^{5/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{7/2}}{7}$$

```
[Out] 2/7*x^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2),x]
```

```
[Out] (2*x^(7/2))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\,dx = \frac{2x^{7/2}}{7}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2),x]
```

```
[Out] (2*x^(7/2))/7
```

Mathics [A]
time = 1.52, size = 5, normalized size = 0.56

$$\frac{2x^{\frac{7}{2}}}{7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2),x]')
```

```
[Out] 2 x ^ (7 / 2) / 7
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}}{7}$	6
derivativedivides	$\frac{2x^{\frac{7}{2}}}{7}$	6
default	$\frac{2x^{\frac{7}{2}}}{7}$	6
trager	$\frac{2x^{\frac{7}{2}}}{7}$	6
risch	$\frac{2x^{\frac{7}{2}}}{7}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^(7/2)
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2)
```

Fricas [A]
time = 0.59, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*x^(7/2)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2),x)
```

```
[Out] 2*x**(7/2)/7
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.33

$$\frac{2}{7}\sqrt{x}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x)
```

```
[Out] 2/7*x^(7/2)
```

Mupad [B]
time = 0.08, size = 5, normalized size = 0.56

$$\frac{2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x)
```

```
[Out] (2*x^(7/2))/7
```

3.21 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

[Out] 2/5*x^(5/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2),x]

[Out] (2*x^(5/2))/5

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2),x]

[Out] (2*x^(5/2))/5

Mathics [A]
time = 1.52, size = 5, normalized size = 0.56

$$\frac{2x^{\frac{5}{2}}}{5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2),x]')
```

```
[Out] 2 x ^ (5 / 2) / 5
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}}{5}$	6
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}$	6
default	$\frac{2x^{\frac{5}{2}}}{5}$	6
trager	$\frac{2x^{\frac{5}{2}}}{5}$	6
risch	$\frac{2x^{\frac{5}{2}}}{5}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.56

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

Fricas [A]
time = 0.30, size = 5, normalized size = 0.56

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.33

$$\frac{2}{5}\sqrt{x}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x)
```

```
[Out] 2/5*x^(5/2)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] (2*x^(5/2))/5
```

3.22 $\int \sqrt{x}\, dx$

Optimal. Leaf size=9

$$\frac{2x^{3/2}}{3}$$

[Out] 2/3*x^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x],x]

[Out] (2*x^(3/2))/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\, dx = \frac{2x^{3/2}}{3}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x],x]

[Out] (2*x^(3/2))/3

Mathics [A]
time = 1.51, size = 5, normalized size = 0.56

$$\frac{2x^{\frac{3}{2}}}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(1/2),x]')
```

```
[Out] 2 x ^ (3 / 2) / 3
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3}$	6
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}$	6
default	$\frac{2x^{\frac{3}{2}}}{3}$	6
trager	$\frac{2x^{\frac{3}{2}}}{3}$	6
risch	$\frac{2x^{\frac{3}{2}}}{3}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2),x)
[Out] 2*x**(3/2)/3
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.11

$$\frac{2}{3}\sqrt{x}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x)
[Out] 2/3*x^(3/2)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
[Out] (2*x^(3/2))/3
```

3.23 $\int \frac{1}{\sqrt{x}} dx$

Optimal. Leaf size=7

$$2\sqrt{x}$$

```
[Out] 2*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

Mathics [A]
time = 1.50, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(1/2),x]')
```

```
[Out] 2 Sqrt[x]
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$2\sqrt{x}$	6
derivativedivides	$2\sqrt{x}$	6
default	$2\sqrt{x}$	6
trager	$2\sqrt{x}$	6
risch	$2\sqrt{x}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)
```

Maxima [A]
time = 0.26, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2),x)
```

```
[Out] 2*sqrt(x)
```

Giac [A]
time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x)
```

```
[Out] 2*sqrt(x)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.71

$$2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

3.24 $\int \frac{1}{x^{3/2}} \, dx$

Optimal. Leaf size=7

$$-\frac{2}{\sqrt{x}}$$

[Out] -2/x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[x^(-3/2),x]

[Out] -2/Sqrt[x]

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}} \, dx = -\frac{2}{\sqrt{x}}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3/2),x]

[Out] -2/Sqrt[x]

Mathics [A]
time = 1.52, size = 5, normalized size = 0.71

$$\frac{-2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(3/2),x]')
```

```
[Out] -2 / Sqrt[x]
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{2}{\sqrt{x}}$	6
derivativedivides	$-\frac{2}{\sqrt{x}}$	6
default	$-\frac{2}{\sqrt{x}}$	6
trager	$-\frac{2}{\sqrt{x}}$	6
risch	$-\frac{2}{\sqrt{x}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/x^(1/2)
```

Maxima [A]
time = 0.25, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2/sqrt(x)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/sqrt(x)
```

Sympy [**A**]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2),x)
```

```
[Out] -2/sqrt(x)
```

Giac [**A**]
time = 0.00, size = 9, normalized size = 1.29

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x)
```

```
[Out] -2/sqrt(x)
```

Mupad [**B**]
time = 0.03, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
```

```
[Out] -2/x^(1/2)
```

3.25 $\int \frac{1}{x^{5/2}} dx$

Optimal. Leaf size=9

$$-\frac{2}{3x^{3/2}}$$

```
[Out] -2/3/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/2),x]
```

```
[Out] -2/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}} dx = -\frac{2}{3x^{3/2}}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/2),x]
```

```
[Out] -2/(3*x^(3/2))
```

Mathics [A]
time = 1.50, size = 5, normalized size = 0.56

$$\frac{-2}{3x^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(5/2),x]')
```

```
[Out] -2 / (3 x ^ (3 / 2))
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$-\frac{2}{3x^{\frac{3}{2}}}$	6
derivativedivides	$-\frac{2}{3x^{\frac{3}{2}}}$	6
default	$-\frac{2}{3x^{\frac{3}{2}}}$	6
trager	$-\frac{2}{3x^{\frac{3}{2}}}$	6
risch	$-\frac{2}{3x^{\frac{3}{2}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/x^(3/2)
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/x^(3/2)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3/x^(3/2)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.89

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2),x)
```

```
[Out] -2/(3*x**(3/2))
```

Giac [A]
time = 0.00, size = 13, normalized size = 1.44

$$-\frac{2}{3\sqrt{x}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x)
```

```
[Out] -2/3/x^(3/2)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x)
```

```
[Out] -2/(3*x^(3/2))
```

3.26 $\int x^{5/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{8/3}}{8}$$

[Out] 3/8*x^(8/3)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

[In] Int[x^(5/3),x]

[Out] (3*x^(8/3))/8

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/3}\,dx = \frac{3x^{8/3}}{8}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/3),x]

[Out] (3*x^(8/3))/8

Mathics [A]
time = 1.52, size = 5, normalized size = 0.56

$$\frac{3x^{\frac{8}{3}}}{8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/3),x]')
```

```
[Out] 3 x ^ (8 / 3) / 8
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}}{8}$	6
derivativedivides	$\frac{3x^{\frac{8}{3}}}{8}$	6
default	$\frac{3x^{\frac{8}{3}}}{8}$	6
trager	$\frac{3x^{\frac{8}{3}}}{8}$	6
risch	$\frac{3x^{\frac{8}{3}}}{8}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^(8/3)
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/8*x^(8/3)
```

Fricas [A]
time = 0.30, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/8*x^(8/3)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3),x)
```

```
[Out] 3*x**(8/3)/8
```

Giac [A]
time = 0.00, size = 16, normalized size = 1.78

$$\frac{3\left(x^{\frac{1}{3}}\right)^{2}x^{2}}{2\cdot 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x)
```

```
[Out] 3/8*x^(8/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
```

```
[Out] (3*x^(8/3))/8
```

3.27 $\int x^{4/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{7/3}}{7}$$

```
[Out] 3/7*x^(7/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3),x]
[Out] (3*x^(7/3))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{4/3}\,dx = \frac{3x^{7/3}}{7}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3),x]
[Out] (3*x^(7/3))/7
```

Mathics [A]
time = 1.60, size = 5, normalized size = 0.56

$$\frac{3x^{\frac{7}{3}}}{7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(4/3),x]')
```

```
[Out] 3 x ^ (7 / 3) / 7
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}}{7}$	6
derivativedivides	$\frac{3x^{\frac{7}{3}}}{7}$	6
default	$\frac{3x^{\frac{7}{3}}}{7}$	6
trager	$\frac{3x^{\frac{7}{3}}}{7}$	6
risch	$\frac{3x^{\frac{7}{3}}}{7}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*x^(7/3)
```

Maxima [A]
time = 0.26, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*x^(7/3)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*x^(7/3)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3),x)
```

```
[Out] 3*x**(7/3)/7
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.33

$$\frac{3}{7}x^{\frac{1}{3}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x)
```

```
[Out] 3/7*x^(7/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x)
```

```
[Out] (3*x^(7/3))/7
```

3.28 $\int x^{2/3} \, dx$

Optimal. Leaf size=9

$$\frac{3x^{5/3}}{5}$$

```
[Out] 3/5*x^(5/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{2/3} \, dx = \frac{3x^{5/3}}{5}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Mathics [A]
time = 1.56, size = 5, normalized size = 0.56

$$\frac{3x^{\frac{5}{3}}}{5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(2/3),x]')
```

```
[Out] 3 x ^ (5 / 3) / 5
```

Maple [A]
time = 0.03, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}}{5}$	6
derivativedivides	$\frac{3x^{\frac{5}{3}}}{5}$	6
default	$\frac{3x^{\frac{5}{3}}}{5}$	6
trager	$\frac{3x^{\frac{5}{3}}}{5}$	6
risch	$\frac{3x^{\frac{5}{3}}}{5}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*x^(5/3)
```

Maxima [A]
time = 0.25, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*x^(5/3)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*x^(5/3)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3),x)
```

```
[Out] 3*x**(5/3)/5
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.33

$$\frac{3}{5}\left(x^{\frac{1}{3}}\right)^{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x)
```

```
[Out] 3/5*x^(5/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
```

```
[Out] (3*x^(5/3))/5
```

3.29 $\int \sqrt[3]{x}\, dx$

Optimal. Leaf size=9

$$\frac{3x^{4/3}}{4}$$

```
[Out] 3/4*x^(4/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x}\, dx = \frac{3x^{4/3}}{4}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

Mathics [A]
time = 1.55, size = 5, normalized size = 0.56

$$\frac{3x^{\frac{4}{3}}}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(1/3),x]')
```

```
[Out] 3 x ^ (4 / 3) / 4
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}}{4}$	6
derivativedivides	$\frac{3x^{\frac{4}{3}}}{4}$	6
default	$\frac{3x^{\frac{4}{3}}}{4}$	6
trager	$\frac{3x^{\frac{4}{3}}}{4}$	6
risch	$\frac{3x^{\frac{4}{3}}}{4}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^(4/3)
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.56

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*x^(4/3)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3),x)
```

```
[Out] 3*x**(4/3)/4
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.11

$$\frac{3}{4}x^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x)
```

```
[Out] 3/4*x^(4/3)
```

Mupad [B]
time = 0.06, size = 5, normalized size = 0.56

$$\frac{3\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
```

```
[Out] (3*x^(4/3))/4
```

3.30 $\int \frac{1}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{2/3}}{2}$$

```
[Out] 3/2*x^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}}\,dx = \frac{3x^{2/3}}{2}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

Mathics [A]
time = 1.51, size = 5, normalized size = 0.56

$$\frac{3x^{\frac{2}{3}}}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(1/3),x]')
[Out] 3 x ^ (2 / 3) / 2
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{2}{3}}}{2}$	6
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2}$	6
default	$\frac{3x^{\frac{2}{3}}}{2}$	6
trager	$\frac{3x^{\frac{2}{3}}}{2}$	6
risch	$\frac{3x^{\frac{2}{3}}}{2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)
```

Maxima [A]
time = 0.24, size = 5, normalized size = 0.56

$$\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="maxima")
[Out] 3/2*x^(2/3)
```

Fricas [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3),x)
```

```
[Out] 3*x**(2/3)/2
```

Giac [A]
time = 0.00, size = 11, normalized size = 1.22

$$\frac{3}{2}\left(x^{\frac{1}{3}}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)
```

Mupad [B]
time = 0.04, size = 5, normalized size = 0.56

$$\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
```

```
[Out] (3*x^(2/3))/2
```

3.31 $\int \frac{1}{x^{2/3}} dx$

Optimal. Leaf size=7

$$3\sqrt[3]{x}$$

```
[Out] 3*x^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{2/3}} dx = 3\sqrt[3]{x}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

Mathics [A]
time = 1.50, size = 5, normalized size = 0.71

$$3x^{\frac{1}{3}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(2/3),x]')
```

```
[Out] 3 x ^ (1 / 3)
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$3x^{\frac{1}{3}}$	6
derivativedivides	$3x^{\frac{1}{3}}$	6
default	$3x^{\frac{1}{3}}$	6
trager	$3x^{\frac{1}{3}}$	6
risch	$3x^{\frac{1}{3}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)
```

Maxima [A]
time = 0.25, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3)
```

Fricas [A]
time = 0.31, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.71

$$3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3),x)
```

```
[Out] 3*x**(1/3)
```

Giac [A]
time = 0.00, size = 7, normalized size = 1.00

$$3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.71

$$3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

3.32 $\int \frac{1}{x^{4/3}} dx$

Optimal. Leaf size=7

$$-\frac{3}{\sqrt[3]{x}}$$

```
[Out] -3/x^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30}

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4/3),x]
[Out] -3/x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{4/3}} dx = -\frac{3}{\sqrt[3]{x}}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4/3),x]
[Out] -3/x^(1/3)
```

Mathics [A]
time = 1.52, size = 5, normalized size = 0.71

$$\frac{-3}{x^{\frac{1}{3}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(4/3),x]')
```

```
[Out] -3 / x ^ (1 / 3)
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{3}{x^{\frac{1}{3}}}$	6
derivativedivides	$-\frac{3}{x^{\frac{1}{3}}}$	6
default	$-\frac{3}{x^{\frac{1}{3}}}$	6
trager	$-\frac{3}{x^{\frac{1}{3}}}$	6
risch	$-\frac{3}{x^{\frac{1}{3}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/x^(1/3)
```

Maxima [A]
time = 0.25, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="maxima")
```

```
[Out] -3/x^(1/3)
```

Fricas [A]
time = 0.31, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="fricas")
```

```
[Out] -3/x^(1/3)
```

Sympy [**A**]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3),x)
[Out] -3/x**(1/3)
```

Giac [**A**]
time = 0.00, size = 9, normalized size = 1.29

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x)
[Out] -3/x^(1/3)
```

Mupad [**B**]
time = 0.07, size = 5, normalized size = 0.71

$$-\frac{3}{x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x)
[Out] -3/x^(1/3)
```

3.33 $\int \frac{1}{x^{5/3}} dx$

Optimal. Leaf size=9

$$-\frac{3}{2x^{2/3}}$$

[Out] -3/2/x^(2/3)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^(-5/3),x]

[Out] -3/(2*x^(2/3))

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/3}} dx = -\frac{3}{2x^{2/3}}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-5/3),x]

[Out] -3/(2*x^(2/3))

Mathics [A]
time = 1.56, size = 5, normalized size = 0.56

$$\frac{-3}{2x^{\frac{2}{3}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/x^(5/3),x]')
```

```
[Out] -3 / (2 x ^ (2 / 3))
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$-\frac{3}{2x^{\frac{2}{3}}}$	6
derivativedivides	$-\frac{3}{2x^{\frac{2}{3}}}$	6
default	$-\frac{3}{2x^{\frac{2}{3}}}$	6
trager	$-\frac{3}{2x^{\frac{2}{3}}}$	6
risch	$-\frac{3}{2x^{\frac{2}{3}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/x^(2/3)
```

Maxima [A]
time = 0.25, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="maxima")
```

```
[Out] -3/2/x^(2/3)
```

Fricas [A]
time = 0.31, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="fricas")
```

```
[Out] -3/2/x^(2/3)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.89

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3),x)
```

```
[Out] -3/(2*x**(2/3))
```

Giac [A]
time = 0.00, size = 14, normalized size = 1.56

$$-\frac{\frac{1}{2}\cdot 3}{\left(x^{\frac{1}{3}}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x)
```

```
[Out] -3/2/x^(2/3)
```

Mupad [B]
time = 0.05, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
```

```
[Out] -3/(2*x^(2/3))
```

3.34 $\int x^n\,dx$

Optimal. Leaf size=11

$$\frac{x^{1+n}}{1+n}$$

```
[Out] x^(1+n)/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n\,dx = \frac{x^{1+n}}{1+n}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.58, size = 19, normalized size = 1.73

$$\text{Piecewise}\left[\left\{\left\{\frac{x^{1+n}}{1+n}, n!=-1\right\}\right\}, \text{Log}\left[x\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^n,x]')
[Out] Piecewise[{{x ^ (1 + n) / (1 + n), n != -1}}, Log[x]]
```

Maple [A]
time = 0.01, size = 12, normalized size = 1.09

method	result	size
risch	$\frac{x\,x^{n}}{1+n}$	11
gosper	$\frac{x^{1+n}}{1+n}$	12
default	$\frac{x^{1+n}}{1+n}$	12
norman	$\frac{x\,{\mathrm e}^{n \ln\left(x\right)}}{1+n}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x,method=_RETURNVERBOSE)
[Out] x^(1+n)/(1+n)
```

Maxima [A]
time = 0.24, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")
[Out] x^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.31, size = 10, normalized size = 0.91

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
[Out] x*x^n/(n + 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for}\: n \neq -1 \\ \log{\left(x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
```

```
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Giac [A]
time = 0.00, size = 10, normalized size = 0.91

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x)
```

```
[Out] x^(n + 1)/(n + 1)
```

Mupad [B]
time = 0.35, size = 20, normalized size = 1.82

$$\begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
```

```
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

3.35 $\int (bx)^n \, dx$

Optimal. Leaf size=16

$$\frac{(bx)^{1+n}}{b(1+n)}$$

```
[Out] (b*x)^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {32}

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^n,x]
```

```
[Out] (b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^n \, dx = \frac{(bx)^{1+n}}{b(1+n)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.75

$$\frac{x(bx)^n}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^n,x]
```

```
[Out] (x*(b*x)^n)/(1 + n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(b*x)^n,x]')
[Out] cought exception:
```

Maple [A]
time = 0.01, size = 17, normalized size = 1.06

method	result	size
gosper	$\frac{x(bx)^n}{1+n}$	13
risch	$\frac{x(bx)^n}{1+n}$	13
norman	$\frac{x\,e^{n\ln(bx)}}{1+n}$	15
default	$\frac{(bx)^{1+n}}{b(1+n)}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x,method=_RETURNVERBOSE)
[Out] (b*x)^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.25, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="maxima")
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 0.33, size = 12, normalized size = 0.75

$$\frac{(bx)^n x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="fricas")
[Out] (b*x)^n*x/(n + 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 1.06

$$\frac{\begin{cases} \frac{(bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**n,x)
```

```
[Out] Piecewise(((b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(b*x), True))/b
```

Giac [A]
time = 0.00, size = 14, normalized size = 0.88

$$\frac{(bx)^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x)
```

```
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 0.18, size = 12, normalized size = 0.75

$$\frac{x\,(b\,x)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x)
```

```
[Out] (x*(b*x)^n)/(n + 1)
```

3.36 $\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

```
[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {33, 31}

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Mathics [A]
time = 1.69, size = 21, normalized size = 0.91

$$\frac{\text{Log}\left[ce + dex + \sqrt{-a}\right]}{de}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Sqrt[-a] + e*(c + d*x)),x]')
```

```
[Out] Log[c e + d e x + Sqrt[-a]] / (d e)
```

Maple [A]
time = 0.10, size = 22, normalized size = 0.96

method	result	size
default	$\frac{\ln\left(ce+dex+\sqrt{-a}\right)}{de}$	22
norman	$\frac{\ln\left(ce+dex+\sqrt{-a}\right)}{de}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(d*x+c)+(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e
```

Maxima [A]
time = 0.26, size = 21, normalized size = 0.91

$$\frac{e^{(-1)}\log\left((dx + c)e + \sqrt{-a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] e^(-1)*log((d*x + c)*e + sqrt(-a))/d
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.91

$$\frac{e^{(-1)}\log\left((dx + c)e + \sqrt{-a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] e^(-1)*log((d*x + c)*e + sqrt(-a))/d
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.83

$$\frac{\log\left(ce+dex+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)**(1/2)),x)
```

```
[Out] log(c*e + d*e*x + sqrt(-a))/(d*e)
```

Giac [A]
time = 0.00, size = 24, normalized size = 1.04

$$\frac{\ln\left|xed+\mathrm{e}c+\sqrt{-a}\right|}{\mathrm{e}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x)
```

```
[Out] e^(-1)*log(abs(d*x*e + c*e + sqrt(-a)))/d
```

Mupad [B]
time = 0.14, size = 21, normalized size = 0.91

$$\frac{\ln\left(\sqrt{-a}+c\,e+d\,e\,x\right)}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-a)^(1/2) + e*(c + d*x)),x)
```

```
[Out] log((-a)^(1/2) + c*e + d*e*x)/(d*e)
```

3.37 $\int (c+d(a+bx))^{5/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(c+d(a+bx))^{7/2}}{7bd}$$

```
[Out] 2/7*(c+d*(b*x+a))^(7/2)/b/d
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{5/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$\frac{2(c+ad+bdx)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(7/2))/(7*b*d)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.22, size = 239, normalized size = 10.39

$$\text{Piecewise}\left[\left\{\left\{c^{\frac{5}{2}}x, b{=}{=}0\&\&d{=}{=}0\right\}, \left\{x\,(ad+c)^{\frac{5}{2}}, b{=}{=}0\right\}, \left\{c^{\frac{5}{2}}x, d{=}{=}0\right\}\right\}, \frac{2a^3d^2\sqrt{ad+bdx+c}}{7b} + \frac{6a^2cd\sqrt{ad+bdx+c}}{7b} + \frac{6a^2d^2x\sqrt{ad+bdx+c}}{7} + \frac{6abd^2x^2\sqrt{ad+bdx+c}}{7} + \frac{6ac^2\sqrt{ad+bdx+c}}{7b} + \frac{12acdx\sqrt{ad+bdx+c}}{7} + \frac{2b^2d^2x^3\sqrt{ad+bdx+c}}{7} + \frac{6bcdx^2\sqrt{ad+bdx+c}}{7} + \frac{2c^3\sqrt{ad+bdx+c}}{7bd} + \frac{6c^2x\sqrt{ad+bdx+c}}{7}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*(a + b*x))^(5/2),x]')
```

```
[Out] Piecewise[{{c ^ (5 / 2) x, b == 0 && d == 0}, {x (a d + c) ^ (5 / 2), b ==
0}, {c ^ (5 / 2) x, d == 0}}, 2 a ^ 3 d ^ 2 Sqrt[a d + b d x + c] / (7 b) +
 6 a ^ 2 c d Sqrt[a d + b d x + c] / (7 b) + 6 a ^ 2 d ^ 2 x Sqrt[a d + b d
 x + c] / 7 + 6 a b d ^ 2 x ^ 2 Sqrt[a d + b d x + c] / 7 + 6 a c ^ 2 Sqrt[
a d + b d x + c] / (7 b) + 12 a c d x Sqrt[a d + b d x + c] / 7 + 2 b ^ 2 d
 ^ 2 x ^ 3 Sqrt[a d + b d x + c] / 7 + 6 b c d x ^ 2 Sqrt[a d + b d x + c]
/ 7 + 2 c ^ 3 Sqrt[a d + b d x + c] / (7 b d) + 6 c ^ 2 x Sqrt[a d + b d x
+ c] / 7]
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.87

method	result	si
gosper	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7bd}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7bd}$	20
default	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7bd}$	20
trager	$\frac{2\left(b^3d^3x^3+3a\,b^2d^3x^2+3a^2b\,d^3x+3b^2c\,d^2x^2+a^3d^3+6abc\,d^2x+3a^2c\,d^2+3b\,c^2dx+3a\,c^2d+c^3\right)\sqrt{bdx+ad+c}}{7bd}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(b*d*x+a*d+c)^(7/2)/b/d
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*((b*x + a)*d + c)^(7/2)/(b*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(19) = 38$.
time = 0.31, size = 104, normalized size = 4.52

$$\frac{2\left(b^{3}d^{3}x^{3}+a^{3}d^{3}+3\,a^{2}cd^{2}+3\,ac^{2}d+c^{3}+3\,(ab^{2}d^{3}+b^{2}cd^{2})x^{2}+3\,(a^{2}bd^{3}+2\,abcd^{2}+bc^{2}d)x\right)\sqrt{bdx+ad+c}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*d^3*x^3 + a^3*d^3 + 3*a^2*c*d^2 + 3*a*c^2*d + c^3 + 3*(a*b^2*d^3 +
 b^2*c*d^2)*x^2 + 3*(a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d)*x)*sqrt(b*d*x + a*d
 + c)/(b*d)
```

Sympy [**A**]
time = 1.64, size = 270, normalized size = 11.74

$$\begin{cases} c^{\frac{5}{2}}x & \text{for}\ b=0\wedge d=0 \\ x\,(ad+c)^{\frac{5}{2}} & \text{for}\ b=0 \\ c^{\frac{5}{2}}x & \text{for}\ d=0 \\ \frac{2a^{3}d^{2}\sqrt{ad+bdx+c}}{7b}+\frac{6a^{2}d^{2}x\sqrt{ad+bdx+c}}{7}+\frac{6a^{2}cd\sqrt{ad+bdx+c}}{7b}+\frac{6abd^{2}x^{2}\sqrt{ad+bdx+c}}{7}+\frac{12acdx\sqrt{ad+bdx+c}}{7}+\frac{6ac^{2}\sqrt{ad+bdx+c}}{7b}+\frac{2b^{2}d^{2}x^{3}\sqrt{ad+bdx+c}}{7}+\frac{6bcdx^{2}\sqrt{ad+bdx+c}}{7}+\frac{6c^{2}x\sqrt{ad+bdx+c}}{7}+\frac{2c^{3}\sqrt{ad+bdx+c}}{7bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((c**(5/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(5/2), Eq(b, 0))
, (c**(5/2)*x, Eq(d, 0)), (2*a**3*d**2*sqrt(a*d + b*d*x + c)/(7*b) + 6*a**2
*d**2*x*sqrt(a*d + b*d*x + c)/7 + 6*a**2*c*d*sqrt(a*d + b*d*x + c)/(7*b) +
6*a*b*d**2*x**2*sqrt(a*d + b*d*x + c)/7 + 12*a*c*d*x*sqrt(a*d + b*d*x + c)/
7 + 6*a*c**2*sqrt(a*d + b*d*x + c)/(7*b) + 2*b**2*d**2*x**3*sqrt(a*d + b*d*
x + c)/7 + 6*b*c*d*x**2*sqrt(a*d + b*d*x + c)/7 + 6*c**2*x*sqrt(a*d + b*d*x
 + c)/7 + 2*c**3*sqrt(a*d + b*d*x + c)/(7*b*d), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(19) = 38$.
time = 0.01, size = 884, normalized size = 38.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x)
```

```
[Out] 2/35*(35*(b*d*x + a*d + c)^(3/2)*a^2*d^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d
- (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a^2*d^2 - 21*(b*d*x
```

```
+ a*d + c)^(5/2)*a*d + 70*(b*d*x + a*d + c)^(3/2)*a*c*d - 70*(3*sqrt(b*d*x
+ a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*c*d
 + 5*(b*d*x + a*d + c)^(7/2) - 21*(b*d*x + a*d + c)^(5/2)*c + 35*(b*d*x + a
*d + c)^(3/2)*c^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/
2) + 3*sqrt(b*d*x + a*d + c)*c)*c^2 + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 -
 10*(b*d*x + a*d + c)^(3/2)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x
 + a*d + c)^(5/2) - 10*(b*d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)
*c^2)*a*d + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2
)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x + a*d + c)^(5/2) - 10*(b*
d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)*c^2)*c)/(b*d)
```

Mupad [B]
time = 0.18, size = 93, normalized size = 4.04

$$\frac{6\,x\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^2}{7}+\frac{2\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^3}{7\,b\,d}+\frac{2\,b^2\,d^2\,x^3\,\sqrt{c+d\,\left(a+b\,x\right)}}{7}+\frac{6\,b\,d\,x^2\,\sqrt{c+d\,\left(a+b\,x\right)}\,\left(c+a\,d\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(5/2),x)
```

```
[Out] (6*x*(c + d*(a + b*x))^(1/2)*(c + a*d)^2)/7 + (2*(c + d*(a + b*x))^(1/2)*(c
 + a*d)^3)/(7*b*d) + (2*b^2*d^2*x^3*(c + d*(a + b*x))^(1/2))/7 + (6*b*d*x^2
*(c + d*(a + b*x))^(1/2)*(c + a*d))/7
```

3.38 $\int (c+d(a+bx))^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(c+d(a+bx))^{5/2}}{5bd}$$

```
[Out] 2/5*(c+d*(b*x+a))^(5/2)/b/d
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(c+ad+bdx)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(5/2))/(5*b*d)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.01, size = 142, normalized size = 6.17

$$\text{Piecewise}\left[\left\{\left\{c^{\frac{3}{2}}x, d==0\&\&b==0||d==0\right\}, \left\{x\,(ad+c)^{\frac{3}{2}}, b==0\right\}\right\}, \frac{2a^2d\sqrt{ad+bdx+c}}{5b}+\frac{4ac\sqrt{ad+bdx+c}}{5b}+\frac{4adx\sqrt{ad+bdx+c}}{5}+\frac{2bdx^2\sqrt{ad+bdx+c}}{5}+\frac{2c^2\sqrt{ad+bdx+c}}{5bd}+\frac{4cx\sqrt{ad+bdx+c}}{5}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*(a + b*x))^(3/2),x]')
```

```
[Out] Piecewise[{{c ^ (3 / 2) x, d == 0 && b == 0 || d == 0}, {x (a d + c) ^ (3 /
 2), b == 0}}, 2 a ^ 2 d Sqrt[a d + b d x + c] / (5 b) + 4 a c Sqrt[a d + b
 d x + c] / (5 b) + 4 a d x Sqrt[a d + b d x + c] / 5 + 2 b d x ^ 2 Sqrt[a
d + b d x + c] / 5 + 2 c ^ 2 Sqrt[a d + b d x + c] / (5 b d) + 4 c x Sqrt[a
 d + b d x + c] / 5]
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.87

method	result	size
gosper	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5bd}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5bd}$	20
default	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5bd}$	20
trager	$\frac{2(b^2d^2x^2+2ab\,d^2x+a^2d^2+2bcdx+2acd+c^2)\sqrt{bdx+ad+c}}{5bd}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*d*x+a*d+c)^(5/2)/b/d
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*((b*x + a)*d + c)^(5/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.
time = 0.31, size = 59, normalized size = 2.57

$$\frac{2\left(b^{2} d^{2} x^{2} + a^{2} d^{2} + 2\, a c d + c^{2} + 2\left(a b d^{2} + b c d\right) x\right) \sqrt{b d x + a d + c}}{5\, b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*d^2*x^2 + a^2*d^2 + 2*a*c*d + c^2 + 2*(a*b*d^2 + b*c*d)*x)*sqrt(b*
d*x + a*d + c)/(b*d)
```

Sympy [A]
time = 0.36, size = 156, normalized size = 6.78

$$\begin{cases} c^{\frac{3}{2}} x & \text{for}\: b = 0 \wedge d = 0 \\ x \left(a d + c\right)^{\frac{3}{2}} & \text{for}\: b = 0 \\ c^{\frac{3}{2}} x & \text{for}\: d = 0 \\ \frac{2 a^{2} d \sqrt{a d + b d x + c}}{5 b} + \frac{4 a d x \sqrt{a d + b d x + c}}{5} + \frac{4 a c \sqrt{a d + b d x + c}}{5 b} + \frac{2 b d x^{2} \sqrt{a d + b d x + c}}{5} + \frac{4 c x \sqrt{a d + b d x + c}}{5} + \frac{2 c^{2} \sqrt{a d + b d x + c}}{5 b d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(3/2), Eq(b, 0))
, (c**(3/2)*x, Eq(d, 0)), (2*a**2*d*sqrt(a*d + b*d*x + c)/(5*b) + 4*a*d*x*s
qrt(a*d + b*d*x + c)/5 + 4*a*c*sqrt(a*d + b*d*x + c)/(5*b) + 2*b*d*x**2*sqr
t(a*d + b*d*x + c)/5 + 4*c*x*sqrt(a*d + b*d*x + c)/5 + 2*c**2*sqrt(a*d + b*
d*x + c)/(5*b*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(19) = 38$.
time = 0.00, size = 357, normalized size = 15.52

$$\frac{\frac{2 b^{2} d^{2} \left(\frac{1}{5} \sqrt{a d + b d x + c}\,(a d + b d x + c)^{2} - \frac{2}{3} \sqrt{a d + b d x + c}\,(a d + b d x + c) a d - \frac{2}{3} \sqrt{a d + b d x + c}\,(a d + b d x + c) c + \sqrt{a d + b d x + c}\, a^{2} d^{2} + 2 \sqrt{a d + b d x + c}\, a d c + \sqrt{a d + b d x + c}\, c^{2}\right)}{d^{2} b^{2}} + 2 a^{2} d^{2} \sqrt{a d + b d x + c} + \frac{4 a b d^{2} \left(\frac{1}{3} \sqrt{a d + b d x + c}\,(a d + b d x + c) - a d \sqrt{a d + b d x + c} - c \sqrt{a d + b d x + c}\right)}{b d} + 4 a c d \sqrt{a d + b d x + c} + \frac{4 b c d \left(\frac{1}{3} \sqrt{a d + b d x + c}\,(a d + b d x + c) - a d \sqrt{a d + b d x + c} - c \sqrt{a d + b d x + c}\right)}{b d} + 2 c^{2} \sqrt{a d + b d x + c}}{b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x)
```

```
[Out] 2/15*(30*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2)*a*d + 6
0*sqrt(b*d*x + a*d + c)*a*c*d - 10*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x +
a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*d + 3*(b*d*x + a*d + c)^(5/2)
 - 10*(b*d*x + a*d + c)^(3/2)*c + 30*sqrt(b*d*x + a*d + c)*c^2 - 10*(3*sqrt
(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c
)*c)/(b*d)
```

Mupad [B]
time = 0.17, size = 45, normalized size = 1.96

$$\sqrt{c+d\,(a+b\,x)}\,\left(x\,\left(\frac{4\,c}{5}+\frac{4\,a\,d}{5}\right)+\frac{2\,{\left(c+a\,d\right)}^2}{5\,b\,d}+\frac{2\,b\,d\,x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(3/2),x)
```

```
[Out] (c + d*(a + b*x))^(1/2)*(x*((4*c)/5 + (4*a*d)/5) + (2*(c + a*d)^2)/(5*b*d)
+ (2*b*d*x^2)/5)
```

3.39 $\int \sqrt{c+d(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{2(c+d(a+bx))^{3/2}}{3bd}$$

```
[Out] 2/3*(c+d*(b*x+a))^(3/2)/b/d
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+d(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{c+dx}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(c+ad+bdx)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + a*d + b*d*x)^(3/2))/(3*b*d)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 1.80, size = 84, normalized size = 3.65

$$\text{Piecewise}\left[\left\{\left\{\sqrt{c}\,x, d{=}{=}0\&\&b{=}{=}0||d{=}{=}0\right\}, \left\{x\sqrt{ad+c}\,, b{=}{=}0\right\}\right\}, \frac{2a\sqrt{ad+bdx+c}}{3b}+\frac{2c\sqrt{ad+bdx+c}}{3bd}+\frac{2x\sqrt{ad+bdx+c}}{3}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*(a + b*x))^(1/2),x]')
```

```
[Out] Piecewise[{{Sqrt[c] x, d == 0 && b == 0 || d == 0}, {x Sqrt[a d + c], b ==
0}}, 2 a Sqrt[a d + b d x + c] / (3 b) + 2 c Sqrt[a d + b d x + c] / (3 b d
) + 2 x Sqrt[a d + b d x + c] / 3]
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.87

method	result	size
gosper	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20
default	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20
trager	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*d*x+a*d+c)^(3/2)/b/d
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*((b*x + a)*d + c)^(3/2)/(b*d)
```

Fricas [A]
time = 0.31, size = 19, normalized size = 0.83

$$\frac{2\,(bdx+ad+c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

Sympy [A]
time = 0.12, size = 82, normalized size = 3.57

$$\begin{cases} \sqrt{c}\,x & \text{for } b=0 \wedge d=0 \\ x\sqrt{ad+c} & \text{for } b=0 \\ \sqrt{c}\,x & \text{for } d=0 \\ \frac{2a\sqrt{ad+bdx+c}}{3b}+\frac{2x\sqrt{ad+bdx+c}}{3}+\frac{2c\sqrt{ad+bdx+c}}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(1/2),x)
[Out] Piecewise((sqrt(c)*x, Eq(b, 0) & Eq(d, 0)), (x*sqrt(a*d + c), Eq(b, 0)), (s
qrt(c)*x, Eq(d, 0)), (2*a*sqrt(a*d + b*d*x + c)/(3*b) + 2*x*sqrt(a*d + b*d*
x + c)/3 + 2*c*sqrt(a*d + b*d*x + c)/(3*b*d), True))
```

Giac [A]
time = 0.00, size = 30, normalized size = 1.30

$$\frac{\sqrt{ad+bdx+c}\,(ad+bdx+c)}{\frac{3}{2}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x)
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

Mupad [B]
time = 0.08, size = 19, normalized size = 0.83

$$\frac{2\,(c+d\,(a+b\,x))^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(1/2),x)
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

3.40 $\int \frac{1}{\sqrt{c+d(a+bx)}} \, dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{c+d(a+bx)}}{bd}$$

```
[Out] 2*(c+d*(b*x+a))^(1/2)/b/d
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{c+d(a+bx)}} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}} \, dx, x, a+bx\right)}{b} \\ &= \frac{2\sqrt{c+d(a+bx)}}{bd} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{c+ad+bdx}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + a*d + b*d*x])/(b*d)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.17, size = 43, normalized size = 2.05

$$\text{Piecewise}\left[\left\{\left\{\frac{x}{\sqrt{ad+c}},b{=}{=}0\right\},\left\{\frac{x}{\sqrt{c}},d{=}{=}0\right\}\right\},\frac{2\sqrt{c+d\,(a+bx)}}{bd}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(c + d*(a + b*x))^(1/2),x]')
```

```
[Out] Piecewise[{{x / Sqrt[a d + c], b == 0}, {x / Sqrt[c], d == 0}}, 2 Sqrt[c +
d (a + b x)] / (b d)]
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.95

method	result	size
gosper	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20
derivativedivides	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20
default	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20
trager	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*d*x+a*d+c)^(1/2)/b/d
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.90

$$\frac{2\sqrt{(bx+a)d+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt((b*x + a)*d + c)/(b*d)
```

Fricas [A]
time = 0.31, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

Sympy [A]
time = 0.68, size = 31, normalized size = 1.48

$$\begin{cases} \frac{x}{\sqrt{ad+c}} & \text{for}\: b=0 \\ \frac{x}{\sqrt{c}} & \text{for}\: d=0 \\ \frac{2\sqrt{c+d\left(a+bx\right)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((x/sqrt(a*d + c), Eq(b, 0)), (x/sqrt(c), Eq(d, 0)), (2*sqrt(c + d
*(a + b*x))/(b*d), True))
```

Giac [A]
time = 0.00, size = 19, normalized size = 0.90

$$\frac{2\sqrt{ad+bdx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x)
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

Mupad [B]
time = 0.11, size = 19, normalized size = 0.90

$$\frac{2\sqrt{c+d\,\left(a+b\,x\right)}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(1/2),x)
```

```
[Out] (2*(c + d*(a + b*x))^(1/2))/(b*d)
```

3.41 $\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2}{bd\sqrt{c+d(a+bx)}}$$

[Out] -2/b/d/(c+d*(b*x+a))^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{bd\sqrt{c+d(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{c+ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + a*d + b*d*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.17, size = 71, normalized size = 3.38

$$\text{Piecewise}\left[\left\{\left\{\frac{x}{c^{\frac{3}{2}}},b{=}{=}0\&\&d{=}{=}0\right\},\left\{\frac{x}{(ad+c)^{\frac{3}{2}}},b{=}{=}0\right\},\left\{\frac{x}{c^{\frac{3}{2}}},d{=}{=}0\right\}\right\},\frac{-2\sqrt{ad+bdx+c}}{abd^2+b^2d^2x+bcd}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(c + d*(a + b*x))^(3/2),x]')
```

```
[Out] Piecewise[{{x / c ^ (3 / 2), b == 0 && d == 0}, {x / (a d + c) ^ (3 / 2), b
 == 0}, {x / c ^ (3 / 2), d == 0}}, -2 Sqrt[a d + b d x + c] / (a b d ^ 2 +
 b ^ 2 d ^ 2 x + b c d)]
```

Maple [A]
time = 0.08, size = 20, normalized size = 0.95

method	result	size
gosper	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20
derivativedivides	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20
default	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20
trager	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b*d*x+a*d+c)^(1/2)/b/d
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{(bx+a)d+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt((b*x + a)*d + c)*b*d)
```

Fricas [A]
time = 0.30, size = 34, normalized size = 1.62

$$-\frac{2\sqrt{bdx+ad+c}}{b^2d^2x+abd^2+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*d*x + a*d + c)/(b^2*d^2*x + a*b*d^2 + b*c*d)
```

Sympy [A]
time = 0.59, size = 58, normalized size = 2.76

$$\begin{cases} \frac{x}{c^{\frac{3}{2}}} & \text{for } b=0 \wedge d=0 \\ \frac{x}{(ad+c)^{\frac{3}{2}}} & \text{for } b=0 \\ \frac{x}{c^{\frac{3}{2}}} & \text{for } d=0 \\ -\frac{2\sqrt{ad+bdx+c}}{abd^2+b^2d^2x+bcd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((x/c**(3/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(3/2), Eq(b, 0))
, (x/c**(3/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(a*b*d**2 + b**2*d**2*x
 + b*c*d), True))
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{ad+bdx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x)
```

```
[Out] -2/(sqrt(b*d*x + a*d + c)*b*d)
```

Mupad [B]
time = 0.13, size = 19, normalized size = 0.90

$$-\frac{2}{b\,d\,\sqrt{c+d\,(a+b\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(3/2),x)
```

```
[Out] -2/(b*d*(c + d*(a + b*x))^(1/2))
```

3.42 $\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3bd(c+d(a+bx))^{3/2}}$$

```
[Out] -2/3/b/d/(c+d*(b*x+a))^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{3bd(c+d(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2}{3bd(c+ad+bdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + a*d + b*d*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.19, size = 108, normalized size = 4.70

$$\text{Piecewise}\left[\left\{\left\{\frac{x}{c^{\frac{5}{2}}},b{=}{=}0\&\&d{=}{=}0\right\},\left\{\frac{x}{(ad+c)^{\frac{5}{2}}},b{=}{=}0\right\},\left\{\frac{x}{c^{\frac{5}{2}}},d{=}{=}0\right\}\right\},\frac{-2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(c + d*(a + b*x))^(5/2),x]')
```

```
[Out] Piecewise[{{x / c ^ (5 / 2), b == 0 && d == 0}, {x / (a d + c) ^ (5 / 2), b
 == 0}, {x / c ^ (5 / 2), d == 0}}, -2 Sqrt[a d + b d x + c] / (3 a ^ 2 b d
 ^ 3 + 6 a b ^ 2 d ^ 3 x + 6 a b c d ^ 2 + 3 b ^ 3 d ^ 3 x ^ 2 + 6 b ^ 2 c
d ^ 2 x + 3 b c ^ 2 d)]
```

Maple [A]
time = 0.08, size = 20, normalized size = 0.87

method	result	size
gosper	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}bd}$	20
derivativedivides	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}bd}$	20
default	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}bd}$	20
trager	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}}bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(b*d*x+a*d+c)^(3/2)/b/d
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.83

$$-\frac{2}{3\left((bx+a)d+c\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/(((b*x + a)*d + c)^(3/2)*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.
time = 0.31, size = 68, normalized size = 2.96

$$-\frac{2\sqrt{bdx+ad+c}}{3\left(b^3d^3x^2+a^2bd^3+2\,abcd^2+bc^2d+2\left(ab^2d^3+b^2cd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*d*x + a*d + c)/(b^3*d^3*x^2 + a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d
 + 2*(a*b^2*d^3 + b^2*c*d^2)*x)
```

Sympy [A]
time = 1.46, size = 102, normalized size = 4.43

$$\begin{cases} \frac{x}{c^{\frac{5}{2}}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{\frac{5}{2}}} & \text{for}\: b = 0 \\ \frac{x}{c^{\frac{5}{2}}} & \text{for}\: d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(5/2),x)
[Out] Piecewise((x/c**(5/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(5/2), Eq(b, 0))
, (x/c**(5/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(3*a**2*b*d**3 + 6*a*b*
*2*d**3*x + 6*a*b*c*d**2 + 3*b**3*d**3*x**2 + 6*b**2*c*d**2*x + 3*b*c**2*d)
, True))
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.43

$$-\frac{2}{3bd\sqrt{ad+bdx+c}\,(ad+bdx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x)
[Out] -2/3/((b*d*x + a*d + c)^(3/2)*b*d)
```

Mupad [B]
time = 0.18, size = 19, normalized size = 0.83

$$-\frac{2}{3\,b\,d\,{\left(c+d\,\left(a+b\,x\right)\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(5/2),x)
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

3.43 $\int x^3(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)\,dx &= \int \left(ax^3+bx^4\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Mathics [A]
time = 1.62, size = 12, normalized size = 0.71

$$x^4\left(\frac{a}{4}+\frac{bx}{5}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(a + b*x),x]')
```

```
[Out] x ^ 4 (a / 4 + b x / 5)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A]
time = 0.30, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x)
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,\left(5\,a+4\,b\,x\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

3.44 $\int x^2(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)\,dx &= \int \left(ax^2+bx^3\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Mathics [A]
time = 1.63, size = 12, normalized size = 0.71

$$x^3\left(\frac{a}{3}+\frac{bx}{4}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x),x]')
```

```
[Out] x ^ 3 (a / 3 + b x / 4)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A]
time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a),x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x)
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x),x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

3.45 $\int x(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

[Out] 1/2*a*x^2+1/3*b*x^3

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {45}

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x),x]

[Out] (a*x^2)/2 + (b*x^3)/3

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)\,dx&=\int \left(ax+bx^2\right)\,dx\\&=\frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x),x]

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Mathics [A]
time = 1.61, size = 12, normalized size = 0.71

$$x^2\left(\frac{a}{2}+\frac{bx}{3}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x),x]')
```

```
[Out] x ^ 2 (a / 2 + b x / 3)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{3}x^3b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b + 1/2*x^2*a
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x)
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\,\left(3\,a+2\,b\,x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x),x)
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

3.46 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx = ax+\frac{bx^2}{2}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Mathics [A]
time = 1.59, size = 10, normalized size = 0.83

$$\frac{x\,(2a+bx)}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x),x]')
```

```
[Out] x (2 a + b x) / 2
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{1}{2}x^2b+ax$	11
default	$\frac{1}{2}x^2b+ax$	11
norman	$\frac{1}{2}x^2b+ax$	11
risch	$\frac{1}{2}x^2b+ax$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*b+a*x
```

Maxima [A]
time = 0.24, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.27, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [**A**]
time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [**B**]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

3.47 $\int \frac{a+bx}{x} \, dx$

Optimal. Leaf size=8

$$bx + a\log(x)$$

[Out] b*x+a*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$a\log(x) + bx$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x,x]

[Out] b*x + a*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x} \, dx &= \int \left(b+\frac{a}{x}\right) \, dx\\ &= bx + a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x,x]

[Out] b*x + a*Log[x]

Mathics [A]
time = 1.59, size = 8, normalized size = 1.00

$$a\text{Log}\,[x] + bx$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^1,x]')
```

```
[Out] a Log[x] + b x
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
default	$bx + a \ln(x)$	9
norman	$bx + a \ln(x)$	9
risch	$bx + a \ln(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] b*x+a*ln(x)
```

Maxima [A]
time = 0.25, size = 8, normalized size = 1.00

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="maxima")
```

```
[Out] b*x + a*log(x)
```

Fricas [A]
time = 0.32, size = 8, normalized size = 1.00

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [A]
time = 0.04, size = 7, normalized size = 0.88

$$a \log(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x)
```

```
[Out] a*log(x) + b*x
```

Giac [A]
time = 0.00, size = 9, normalized size = 1.12

$$xb + a \ln |x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x)
```

```
[Out] b*x + a*log(abs(x))
```

Mupad [B]
time = 0.02, size = 8, normalized size = 1.00

$$b\,x + a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x,x)
```

```
[Out] b*x + a*log(x)
```

3.48 $\int \frac{a+bx}{x^2} \, dx$

Optimal. Leaf size=11

$$-\frac{a}{x} + b\log(x)$$

[Out] -a/x+b*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^2,x]

[Out] -(a/x) + b*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^2} \, dx &= \int \left(\frac{a}{x^2} + \frac{b}{x}\right) \, dx \\ &= -\frac{a}{x} + b\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{a}{x} + b\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^2,x]

[Out] -(a/x) + b*Log[x]

Mathics [A]
time = 1.63, size = 11, normalized size = 1.00

$$-\frac{a}{x} + b\text{Log}[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^2,x]')
```

```
[Out] -a / x + b Log[x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 1.09

method	result	size
default	$-\frac{a}{x} + b\ln(x)$	12
norman	$-\frac{a}{x} + b\ln(x)$	12
risch	$-\frac{a}{x} + b\ln(x)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x+b*ln(x)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$b\log(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - a/x
```

Fricas [A]
time = 0.30, size = 13, normalized size = 1.18

$$\frac{bx\log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.64

$$-\frac{a}{x}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2,x)
```

```
[Out] -a/x + b*log(x)
```

Giac [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{a}{x}+b\ln\left|x\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x)
```

```
[Out] b*log(abs(x)) - a/x
```

Mupad [B]
time = 0.03, size = 11, normalized size = 1.00

$$b\,\ln\left(x\right)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^2,x)
```

```
[Out] b*log(x) - a/x
```

3.49 $\int \frac{a+bx}{x^3} \, dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^2}{2ax^2}$$

```
[Out] -1/2*(b*x+a)^2/a/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {37}

$$-\frac{(a+bx)^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^3,x]
```

```
[Out] -1/2*(a + b*x)^2/(a*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{x^3} \, dx = -\frac{(a+bx)^2}{2ax^2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 0.88

$$-\frac{a}{2x^2} - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^3,x]
```

```
[Out] -1/2*a/x^2 - b/x
```

Mathics [A]
time = 1.63, size = 12, normalized size = 0.71

$$\frac{-\frac{a}{2} - bx}{x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^3,x]')
```

```
[Out] (-a / 2 - b x) / x ^ 2
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2bx+a}{2x^2}$	12
norman	$\frac{-bx-\frac{a}{2}}{x^2}$	13
risch	$\frac{-bx-\frac{a}{2}}{x^2}$	13
default	$-\frac{b}{x} - \frac{a}{2x^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/x-1/2/x^2*a
```

Maxima [A]
time = 0.25, size = 11, normalized size = 0.65

$$-\frac{2\,bx + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Fricas [A]
time = 0.30, size = 11, normalized size = 0.65

$$-\frac{2\,bx + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.71

$$\frac{-a - 2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3,x)
```

```
[Out] (-a - 2*b*x)/(2*x**2)
```

Giac [A]
time = 0.00, size = 15, normalized size = 0.88

$$\frac{-2xb - a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x)
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.65

$$-\frac{a + 2\,b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^3,x)
```

```
[Out] -(a + 2*b*x)/(2*x^2)
```

3.50 $\int \frac{a+bx}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx\\ &= -\frac{a}{3x^3}-\frac{b}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2)
```

Mathics [A]
time = 1.63, size = 12, normalized size = 0.71

$$\frac{-\frac{a}{3} - \frac{bx}{2}}{x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^4,x]')
```

```
[Out] (-a / 3 - b x / 2) / x ^ 3
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^3}$	13
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^3}$	13
gosper	$-\frac{3bx+2a}{6x^3}$	14
default	$-\frac{a}{3x^3}-\frac{b}{2x^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/2/x^2*b
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.76

$$-\frac{3\,bx + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Fricas [A]
time = 0.30, size = 13, normalized size = 0.76

$$-\frac{3\,bx + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.82

$$\frac{-2a - 3bx}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4,x)
```

```
[Out] (-2*a - 3*b*x)/(6*x**3)
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{-3xb - 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x)
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,a+3\,b\,x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^4,x)
```

```
[Out] -(2*a + 3*b*x)/(6*x^3)
```

3.51 $\int \frac{a+bx}{x^5} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

```
[Out] -1/4*a/x^4-1/3*b/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^5} \, dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^4}\right) \, dx \\ &= -\frac{a}{4x^4}-\frac{b}{3x^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(3*x^3)
```

Mathics [A]
time = 1.68, size = 12, normalized size = 0.71

$$\frac{-\frac{a}{4}-\frac{bx}{3}}{x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^5,x]')
[Out] (-a / 4 - b x / 3) / x ^ 4
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^4}$	13
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^4}$	13
gosper	$-\frac{4bx+3a}{12x^4}$	14
default	$-\frac{a}{4x^4}-\frac{b}{3x^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4-1/3*b/x^3
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="maxima")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Fricas [A]
time = 0.31, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.82

$$\frac{-3a - 4bx}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**5,x)
```

```
[Out] (-3*a - 4*b*x)/(12*x**4)
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{-4xb - 3a}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x)
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{3\,a+4\,b\,x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^5,x)
```

```
[Out] -(3*a + 4*b*x)/(12*x^4)
```

3.52 $\int x^3(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^2\,dx&=\int\left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\&=\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Mathics [A]
time = 1.68, size = 24, normalized size = 0.80

$$\frac{x^4\left(15a^2+24abx+10b^2x^2\right)}{60}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(a + b*x)^2,x]')
[Out] x ^ 4 (15 a ^ 2 + 24 a b x + 10 b ^ 2 x ^ 2) / 60
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
default	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
norman	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Giac [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{1}{6}x^6b^2 + \frac{2}{5}x^5ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x)
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B]
time = 0.08, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

3.53 $\int x^2(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^2\,dx&=\int\left(a^2x^2+2abx^3+b^2x^4\right)\,dx\\&=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Mathics [A]
time = 1.66, size = 23, normalized size = 0.77

$$x^3\left(\frac{a^2}{3}+\frac{abx}{2}+\frac{b^2x^2}{5}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x)^2,x]')
```

```
[Out] x ^ 3 (a ^ 2 / 3 + a b x / 2 + b ^ 2 x ^ 2 / 5)
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
default	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
norman	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Giac [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{1}{5}x^5b^2 + \frac{1}{2}x^4ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x)
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

3.54 $\int x(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

```
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^2\,dx &= \int\left(a^2x+2abx^2+b^2x^3\right)\,dx\\ &= \frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Mathics [A]
time = 1.65, size = 24, normalized size = 0.80

$$\frac{x^2\left(6a^2+8abx+3b^2x^2\right)}{12}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x)^2,x]')
[Out] x ^ 2 (6 a ^ 2 + 8 a b x + 3 b ^ 2 x ^ 2) / 12
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
default	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
norman	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
risch	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.87

$$\frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2,x)
```

```
[Out] a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

Giac [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{1}{4}x^4b^2 + \frac{2}{3}x^3ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x)
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2} + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^4)/4 + (2*a*b*x^3)/3
```

3.55 $\int (a+bx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

```
[Out] 1/3*(b*x+a)^3/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2,x]
```

```
[Out] (a + b*x)^3/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2 \, dx = \frac{(a+bx)^3}{3b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2,x]
```

```
[Out] (a + b*x)^3/(3*b)
```

Mathics [A]
time = 1.73, size = 18, normalized size = 1.29

$$x\left(a^2 + abx + \frac{b^2x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^2,x]')
[Out] x (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3)
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
gosper	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
norman	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
risch	$\frac{b^2x^3}{3} + ab\,x^2 + a^2x + \frac{a^3}{3b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(b*x+a)^3/b
```

Maxima [A]
time = 0.25, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.03, size = 19, normalized size = 1.36

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x)
```

```
[Out] 1/3*(b*x + a)^3/b
```

Mupad [B]
time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
```

```
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

3.56 $\int \frac{(a+bx)^2}{x} \, dx$

Optimal. Leaf size=22

$$2abx + \frac{b^2x^2}{2} + a^2\log(x)$$

```
[Out] 2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x,x]
```

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x} \, dx &= \int \left(2ab + \frac{a^2}{x} + b^2x\right) dx \\ &= 2abx + \frac{b^2x^2}{2} + a^2\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$2abx + \frac{b^2x^2}{2} + a^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x,x]
```

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Mathics [A]
time = 1.74, size = 20, normalized size = 0.91

$$a^2\mathrm{Log}\left[x\right]+2abx+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^1,x]')
[Out] a ^ 2 Log[x] + 2 a b x + b ^ 2 x ^ 2 / 2
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.95

method	result	size
default	$2abx+\frac{x^2b^2}{2}+a^2\ln\left(x\right)$	21
norman	$2abx+\frac{x^2b^2}{2}+a^2\ln\left(x\right)$	21
risch	$2abx+\frac{x^2b^2}{2}+a^2\ln\left(x\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x,x,method=_RETURNVERBOSE)
[Out] 2*a*b*x+1/2*x^2*b^2+a^2*ln(x)
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2+2\,abx+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Fricas [A]
time = 0.30, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2+2\,abx+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.91

$$a^2 \log(x) + 2abx + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x,x)
[Out] a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.05

$$\frac{1}{2}x^2b^2 + 2xba + a^2 \ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x)
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.91

$$a^2 \ln(x) + \frac{b^2 x^2}{2} + 2 a b x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x,x)
[Out] a^2*log(x) + (b^2*x^2)/2 + 2*a*b*x
```

3.57 $\int \frac{(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{a^2}{x}+b^2x+2ab\log(x)$$

```
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2}\,dx &= \int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx\\ &= -\frac{a^2}{x}+b^2x+2ab\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{a^2}{x}+b^2x+2ab\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Mathics [A]
time = 1.73, size = 22, normalized size = 1.10

$$\frac{-a^2 + bx\,(2a\mathrm{Log}\,[x] + bx)}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^2,x]')
[Out] (-a ^ 2 + b x (2 a Log[x] + b x)) / x
```

Maple [A]
time = 0.07, size = 21, normalized size = 1.05

method	result	size
default	$-\frac{a^2}{x} + b^2x + 2ab\ln(x)$	21
risch	$-\frac{a^2}{x} + b^2x + 2ab\ln(x)$	21
norman	$\frac{x^2b^2-a^2}{x} + 2ab\ln(x)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

Maxima [A]
time = 0.25, size = 20, normalized size = 1.00

$$b^2x + 2\,ab\log(x) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="maxima")
[Out] b^2*x + 2*a*b*log(x) - a^2/x
```

Fricas [A]
time = 0.31, size = 24, normalized size = 1.20

$$\frac{b^2x^2 + 2\,abx\log(x) - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [A]
time = 0.06, size = 17, normalized size = 0.85

$$-\frac{a^2}{x} + 2ab\log(x) + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2,x)
[Out] -a**2/x + 2*a*b*log(x) + b**2*x
```

Giac [A]
time = 0.00, size = 20, normalized size = 1.00

$$xb^2 - \frac{a^2}{x} + 2ba\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x)
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

Mupad [B]
time = 0.07, size = 20, normalized size = 1.00

$$b^2\,x - \frac{a^2}{x} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^2,x)
[Out] b^2*x - a^2/x + 2*a*b*log(x)
```

3.58 $\int \frac{(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

```
[Out] -1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx\\ &= -\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Mathics [A]
time = 1.80, size = 22, normalized size = 0.92

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\text{Log}\left[x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^3,x]')
[Out] -a ^ 2 / (2 x ^ 2) - 2 a b / x + b ^ 2 Log[x]
```

Maple [A]
time = 0.08, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \ln\left(x\right)$	23
norman	$\frac{-\frac{1}{2}a^2 - 2abx}{x^2} + b^2 \ln\left(x\right)$	23
risch	$\frac{-\frac{1}{2}a^2 - 2abx}{x^2} + b^2 \ln\left(x\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Maxima [A]
time = 0.25, size = 21, normalized size = 0.88

$$b^2\log\left(x\right) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="maxima")
[Out] b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

Fricas [A]
time = 0.30, size = 26, normalized size = 1.08

$$\frac{2\,b^2x^2\log\left(x\right) - 4\,abx - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.92

$$b^2\log(x) + \frac{-a^2 - 4abx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.12

$$\frac{\frac{1}{2}\left(-4bax - a^2\right)}{x^2} + b^2\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x)
```

```
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.96

$$b^2\,\ln(x) - \frac{\frac{a^2}{2} + 2\,b\,x\,a}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^3,x)
```

```
[Out] b^2*log(x) - (a^2/2 + 2*a*b*x)/x^2
```

3.59 $\int \frac{(a+bx)^2}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^3}{3ax^3}$$

```
[Out] -1/3*(b*x+a)^3/a/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^3}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^4,x]
```

```
[Out] -1/3*(a + b*x)^3/(a*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4} dx = -\frac{(a+bx)^3}{3ax^3}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.53

$$-\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (a*b)/x^2 - b^2/x
```

Mathics [A]
time = 1.77, size = 23, normalized size = 1.35

$$\frac{-\frac{a^2}{3} - abx - b^2x^2}{x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^4,x]')
[Out] (-a ^ 2 / 3 - a b x - b ^ 2 x ^ 2) / x ^ 3
```

Maple [A]
time = 0.07, size = 25, normalized size = 1.47

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3x^3}$	23
norman	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{x^3}$	24
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{x^3}$	24
default	$-\frac{b^2}{x}-\frac{a^2}{3x^3}-\frac{ab}{x^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
[Out] -b^2/x-1/3*a^2/x^3-a*b/x^2
```

Maxima [A]
time = 0.24, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="maxima")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Fricas [A]
time = 0.30, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A]
time = 0.08, size = 24, normalized size = 1.41

$$\frac{-a^2 - 3abx - 3b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4,x)
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.59

$$\frac{-3x^2b^2 - 3xba - a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x)
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Mupad [B]
time = 0.04, size = 22, normalized size = 1.29

$$-\frac{\frac{a^2}{3} + a\,b\,x + b^2\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^4,x)
[Out] -(a^2/3 + b^2*x^2 + a*b*x)/x^3
```

3.60 $\int \frac{(a+bx)^2}{x^5} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

```
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^5} dx &= \int \left(\frac{a^2}{x^5} + \frac{2ab}{x^4} + \frac{b^2}{x^3}\right) dx \\ &= -\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Mathics [A]
time = 1.72, size = 24, normalized size = 0.80

$$\frac{-3a^2 - 8abx - 6b^2x^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^5,x]')
[Out] (-3 a ^ 2 - 8 a b x - 6 b ^ 2 x ^ 2) / (12 x ^ 4)
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^4}$	24
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^4}$	24
gosper	$-\frac{6x^2b^2+8abx+3a^2}{12x^4}$	25
default	$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="maxima")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Fricas [A]
time = 0.32, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A]
time = 0.08, size = 26, normalized size = 0.87

$$\frac{-3a^2 - 8abx - 6b^2x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**5,x)
```

```
[Out] (-3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.97

$$\frac{-6x^2b^2 - 8xba - 3a^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x)
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{4} + \frac{2\,a\,b\,x}{3} + \frac{b^2\,x^2}{2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^5,x)
```

```
[Out] -(a^2/4 + (b^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

3.61 $\int \frac{(a+bx)^2}{x^6} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

```
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^6} dx &= \int \left(\frac{a^2}{x^6} + \frac{2ab}{x^5} + \frac{b^2}{x^4}\right) dx \\ &= -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Mathics [A]
time = 1.75, size = 23, normalized size = 0.77

$$\frac{-\frac{a^2}{5} - \frac{abx}{2} - \frac{b^2x^2}{3}}{x^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^6,x]')
```

```
[Out] (-a ^ 2 / 5 - a b x / 2 - b ^ 2 x ^ 2 / 3) / x ^ 5
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{3}x^2b^2 - \frac{1}{2}abx - \frac{1}{5}a^2}{x^5}$	24
risch	$\frac{-\frac{1}{3}x^2b^2 - \frac{1}{2}abx - \frac{1}{5}a^2}{x^5}$	24
gosper	$-\frac{10x^2b^2+15abx+6a^2}{30x^5}$	25
default	$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Sympy [A]
time = 0.10, size = 26, normalized size = 0.87

$$\frac{-6a^2 - 15abx - 10b^2x^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**6,x)
```

```
[Out] (-6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.97

$$\frac{-10x^2b^2 - 15xba - 6a^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x)
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{5} + \frac{a\,b\,x}{2} + \frac{b^2\,x^2}{3}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^6,x)
```

```
[Out] -(a^2/5 + (b^2*x^2)/3 + (a*b*x)/2)/x^5
```

3.62 $\int \frac{(a+bx)^2}{x^7} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

```
[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right) dx\\ &= -\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Mathics [A]
time = 1.77, size = 24, normalized size = 0.80

$$\frac{-10a^2 - 24abx - 15b^2x^2}{60x^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^7,x]')
[Out] (-10 a ^ 2 - 24 a b x - 15 b ^ 2 x ^ 2) / (60 x ^ 6)
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{4}x^2b^2-\frac{2}{5}abx-\frac{1}{6}a^2}{x^6}$	24
risch	$\frac{-\frac{1}{4}x^2b^2-\frac{2}{5}abx-\frac{1}{6}a^2}{x^6}$	24
gosper	$-\frac{15x^2b^2+24abx+10a^2}{60x^6}$	25
default	$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="maxima")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Sympy [A]
time = 0.12, size = 26, normalized size = 0.87

$$\frac{-10a^2 - 24abx - 15b^2x^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**7,x)
```

```
[Out] (-10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.97

$$\frac{-15x^2b^2 - 24xba - 10a^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x)
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{6} + \frac{2\,a\,b\,x}{5} + \frac{b^2\,x^2}{4}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

3.63 $\int \frac{(a+bx)^2}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

```
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^8} dx &= \int \left(\frac{a^2}{x^8} + \frac{2ab}{x^7} + \frac{b^2}{x^6}\right) dx \\
&= -\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Mathics [A]
time = 1.78, size = 23, normalized size = 0.77

$$\frac{-\frac{a^2}{7}-\frac{abx}{3}-\frac{b^2x^2}{5}}{x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^8,x]')
[Out] (-a ^ 2 / 7 - a b x / 3 - b ^ 2 x ^ 2 / 5) / x ^ 7
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{5}x^2b^2-\frac{1}{3}abx-\frac{1}{7}a^2}{x^7}$	24
risch	$\frac{-\frac{1}{5}x^2b^2-\frac{1}{3}abx-\frac{1}{7}a^2}{x^7}$	24
gosper	$-\frac{21x^2b^2+35abx+15a^2}{105x^7}$	25
default	$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="maxima")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Sympy [A]
time = 0.11, size = 26, normalized size = 0.87

$$\frac{-15a^2 - 35abx - 21b^2x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**8,x)
```

```
[Out] (-15*a**2 - 35*a*b*x - 21*b**2*x**2)/(105*x**7)
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.97

$$\frac{-21x^2b^2 - 35xba - 15a^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x)
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{7} + \frac{a\,b\,x}{3} + \frac{b^2\,x^2}{5}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^2)/5 + (a*b*x)/3)/x^7
```

3.64 $\int x^4(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

```
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^3\,dx&=\int\left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)\,dx\\&=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Mathics [A]
time = 1.73, size = 35, normalized size = 0.81

$$\frac{x^5\left(56a^3+140a^2bx+120ab^2x^2+35b^3x^3\right)}{280}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4*(a + b*x)^3,x]')
[Out] x ^ 5 (56 a ^ 3 + 140 a ^ 2 b x + 120 a b ^ 2 x ^ 2 + 35 b ^ 3 x ^ 3) / 280
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
default	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
norman	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
risch	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Fricas [A]
time = 0.29, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Sympy [A]
time = 0.03, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5} + \frac{a^2bx^6}{2} + \frac{3ab^2x^7}{7} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**3,x)
```

```
[Out] a**3*x**5/5 + a**2*b*x**6/2 + 3*a*b**2*x**7/7 + b**3*x**8/8
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{1}{8}x^8b^3 + \frac{3}{7}x^7b^2a + \frac{1}{2}x^6ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x)
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5} + \frac{a^2\,b\,x^6}{2} + \frac{3\,a\,b^2\,x^7}{7} + \frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^8)/8 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7
```

3.65 $\int x^3(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

```
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^3\,dx &= \int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)\,dx\\ &= \frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Mathics [A]
time = 1.76, size = 35, normalized size = 0.81

$$\frac{x^4\left(35a^3+84a^2bx+70ab^2x^2+20b^3x^3\right)}{140}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(a + b*x)^3,x]')
[Out] x ^ 4 (35 a ^ 3 + 84 a ^ 2 b x + 70 a b ^ 2 x ^ 2 + 20 b ^ 3 x ^ 3) / 140
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
default	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
norman	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
risch	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Sympy [A]
time = 0.03, size = 37, normalized size = 0.86

$$\frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{ab^2x^6}{2} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**3,x)
```

```
[Out] a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{1}{7}x^7b^3 + \frac{1}{2}x^6b^2a + \frac{3}{5}x^5ba^2 + \frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x)
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^4}{4} + \frac{3\,a^2\,b\,x^5}{5} + \frac{a\,b^2\,x^6}{2} + \frac{b^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^7)/7 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2
```

3.66 $\int x^2(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

```
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^3\,dx &= \int \left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)\,dx\\ &= \frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Mathics [A]
time = 1.77, size = 35, normalized size = 0.81

$$\frac{x^3 \left(20a^3 + 45a^2bx + 36ab^2x^2 + 10b^3x^3\right)}{60}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x)^3,x]')
[Out] x ^ 3 (20 a ^ 3 + 45 a ^ 2 b x + 36 a b ^ 2 x ^ 2 + 10 b ^ 3 x ^ 3) / 60
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{3}a^3x^3 + \frac{3}{4}a^2b\,x^4 + \frac{3}{5}a\,b^2x^5 + \frac{1}{6}b^3x^6$	36
default	$\frac{1}{3}a^3x^3 + \frac{3}{4}a^2b\,x^4 + \frac{3}{5}a\,b^2x^5 + \frac{1}{6}b^3x^6$	36
norman	$\frac{1}{3}a^3x^3 + \frac{3}{4}a^2b\,x^4 + \frac{3}{5}a\,b^2x^5 + \frac{1}{6}b^3x^6$	36
risch	$\frac{1}{3}a^3x^3 + \frac{3}{4}a^2b\,x^4 + \frac{3}{5}a\,b^2x^5 + \frac{1}{6}b^3x^6$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6 + \frac{3}{5}\,ab^2x^5 + \frac{3}{4}\,a^2bx^4 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6 + \frac{3}{5}\,ab^2x^5 + \frac{3}{4}\,a^2bx^4 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^5}{5} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**3,x)
[Out] a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{1}{6}x^6b^3 + \frac{3}{5}x^5b^2a + \frac{3}{4}x^4ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x)
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3} + \frac{3\,a^2\,b\,x^4}{4} + \frac{3\,a\,b^2\,x^5}{5} + \frac{b^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^3,x)
[Out] (a^3*x^3)/3 + (b^3*x^6)/6 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5
```

3.67 $\int x(a+bx)^3\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}$$

```
[Out] -1/4*a*(b*x+a)^4/b^2+1/5*(b*x+a)^5/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = $\{45\}$

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^3,x]
```

```
[Out] -1/4*(a*(a + b*x)^4)/b^2 + (a + b*x)^5/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^3\,dx &= \int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx\\ &= -\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.33

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}ab^2x^4+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^3,x]
```

```
[Out] (a^3*x^2)/2 + a^2*b*x^3 + (3*a*b^2*x^4)/4 + (b^3*x^5)/5
```

Mathics [A]
time = 1.72, size = 33, normalized size = 1.10

$$x^2\left(\frac{a^3}{2}+a^2bx+\frac{3ab^2x^2}{4}+\frac{b^3x^3}{5}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x)^3,x]')
```

```
[Out] x ^ 2 (a ^ 3 / 2 + a ^ 2 b x + 3 a b ^ 2 x ^ 2 / 4 + b ^ 3 x ^ 3 / 5)
```

Maple [A]
time = 0.08, size = 35, normalized size = 1.17

method	result	size
gosper	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
default	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
norman	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
risch	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^3*x^5+3/4*a*b^2*x^4+a^2*b*x^3+1/2*a^3*x^2
```

Maxima [A]
time = 0.25, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Fricas [A]
time = 0.30, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [A]
time = 0.03, size = 36, normalized size = 1.20

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3ab^2x^4}{4}+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**3,x)
[Out] a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5
```

Giac [A]
time = 0.00, size = 40, normalized size = 1.33

$$\frac{1}{5}x^5b^3+\frac{3}{4}x^4b^2a+x^3ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x)
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Mupad [B]
time = 0.04, size = 34, normalized size = 1.13

$$\frac{a^3\,x^2}{2}+a^2\,b\,x^3+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^3,x)
[Out] (a^3*x^2)/2 + (b^3*x^5)/5 + a^2*b*x^3 + (3*a*b^2*x^4)/4
```

3.68 $\int (a+bx)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^4}{4b}$$

```
[Out] 1/4*(b*x+a)^4/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3,x]
```

```
[Out] (a + b*x)^4/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3\,dx = \frac{(a+bx)^4}{4b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3,x]
```

```
[Out] (a + b*x)^4/(4*b)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.
time = 1.71, size = 32, normalized size = 2.29

$$\frac{x\left(4a^3+6a^2bx+4ab^2x^2+b^3x^3\right)}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^3,x]')
```

```
[Out] x (4 a ^ 3 + 6 a ^ 2 b x + 4 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) / 4
```

Maple [A]
time = 0.07, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^4}{4b}$	13
gosper	$\frac{1}{4}b^3x^4 + a\,b^2x^3 + \frac{3}{2}a^2b\,x^2 + a^3x$	32
norman	$\frac{1}{4}b^3x^4 + a\,b^2x^3 + \frac{3}{2}a^2b\,x^2 + a^3x$	32
risch	$\frac{b^3x^4}{4} + a\,b^2x^3 + \frac{3a^2b\,x^2}{2} + a^3x + \frac{a^4}{4b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.25, size = 31, normalized size = 2.21

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.31, size = 31, normalized size = 2.21

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.
time = 0.03, size = 32, normalized size = 2.29

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(bx + a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x)
```

```
[Out] 1/4*(b*x + a)^4/b
```

Mupad [B]
time = 0.04, size = 31, normalized size = 2.21

$$a^3\,x + \frac{3\,a^2\,b\,x^2}{2} + a\,b^2\,x^3 + \frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3,x)
```

```
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

3.69 $\int \frac{(a+bx)^3}{x} dx$

Optimal. Leaf size=35

$$3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x)$$

```
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$a^3\log(x) + 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x} dx &= \int \left(3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2\right) dx \\ &= 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Mathics [A]
time = 1.73, size = 31, normalized size = 0.89

$$a^3\text{Log}\,[x] + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^1,x]')
```

```
[Out] a ^ 3 Log[x] + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 / 2 + b ^ 3 x ^ 3 / 3
```

Maple [A]
time = 0.08, size = 32, normalized size = 0.91

method	result	size
default	$3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln(x)$	32
norman	$3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln(x)$	32
risch	$3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln(x)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Maxima [A]
time = 0.26, size = 31, normalized size = 0.89

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Fricas [A]
time = 0.31, size = 31, normalized size = 0.89

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="fricas")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [**A**]
time = 0.07, size = 34, normalized size = 0.97

$$a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x,x)
[Out] a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3
```

Giac [**A**]
time = 0.00, size = 36, normalized size = 1.03

$$\frac{1}{3}x^3b^3 + \frac{3}{2}x^2b^2a + 3xba^2 + a^3\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x)
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

Mupad [**B**]
time = 0.03, size = 31, normalized size = 0.89

$$a^3\,\ln(x) + \frac{b^3\,x^3}{3} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x,x)
[Out] a^3*log(x) + (b^3*x^3)/3 + (3*a*b^2*x^2)/2 + 3*a^2*b*x
```

3.70 $\int \frac{(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)$$

```
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^2}\,dx &= \int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)dx\\ &= -\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Mathics [A]
time = 1.73, size = 34, normalized size = 1.00

$$\frac{-a^3 + \frac{bx\left(6a^2\mathrm{Log}[x]+6abx+b^2x^2\right)}{2}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^2,x]')
[Out] (-a ^ 3 + b x (6 a ^ 2 Log[x] + 6 a b x + b ^ 2 x ^ 2) / 2) / x
```

Maple [A]
time = 0.08, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{a^3}{x}+3a\,b^2x+\frac{b^3x^2}{2}+3a^2b\ln(x)$	33
risch	$-\frac{a^3}{x}+3a\,b^2x+\frac{b^3x^2}{2}+3a^2b\ln(x)$	33
norman	$\frac{-a^3+\frac{1}{2}b^3x^3+3a\,b^2x^2}{x}+3a^2b\ln(x)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Maxima [A]
time = 0.24, size = 32, normalized size = 0.94

$$\frac{1}{2}\,b^3x^2+3\,ab^2x+3\,a^2b\log(x)-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(x) - a^3/x
```

Fricas [A]
time = 0.32, size = 36, normalized size = 1.06

$$\frac{b^3x^3+6\,ab^2x^2+6\,a^2bx\log(x)-2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [A]
time = 0.06, size = 31, normalized size = 0.91

$$-\frac{a^3}{x} + 3a^2b\log{(x)} + 3ab^2x + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2
```

Giac [A]
time = 0.00, size = 34, normalized size = 1.00

$$\frac{1}{2}x^2b^3 + 3xb^2a - \frac{a^3}{x} + 3ba^2\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x)
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.94

$$\frac{b^3\,x^2}{2} - \frac{a^3}{x} + 3\,a^2\,b\,\ln{(x)} + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^2,x)
```

```
[Out] (b^3*x^2)/2 - a^3/x + 3*a^2*b*log(x) + 3*a*b^2*x
```

3.71 $\int \frac{(a+bx)^3}{x^3} dx$

Optimal. Leaf size=33

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)$$

```
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^3}\,dx &= \int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)dx\\ &= -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Mathics [A]
time = 1.83, size = 31, normalized size = 0.94

$$-\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\text{Log}\,[x] + b^3x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^3,x]')
[Out] -a ^ 3 / (2 x ^ 2) - 3 a ^ 2 b / x + 3 a b ^ 2 Log[x] + b ^ 3 x
```

Maple [A]
time = 0.09, size = 32, normalized size = 0.97

method	result	size
default	$-\frac{a^3}{2x^2} - \frac{3a^2b}{x} + b^3x + 3a\,b^2\ln(x)$	32
risch	$b^3x + \frac{-3a^2bx - \frac{1}{2}a^3}{x^2} + 3a\,b^2\ln(x)$	32
norman	$\frac{b^3x^3 - \frac{1}{2}a^3 - 3a^2bx}{x^2} + 3a\,b^2\ln(x)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Maxima [A]
time = 0.24, size = 30, normalized size = 0.91

$$b^3x + 3\,ab^2\log(x) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="maxima")
[Out] b^3*x + 3*a*b^2*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Fricas [A]
time = 0.31, size = 37, normalized size = 1.12

$$\frac{2\,b^3x^3 + 6\,ab^2x^2\log(x) - 6\,a^2bx - a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [A]
time = 0.09, size = 32, normalized size = 0.97

$$3ab^2\log(x) + b^3x + \frac{-a^3 - 6a^2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**3,x)
[Out] 3*a*b**2*log(x) + b**3*x + (-a**3 - 6*a**2*b*x)/(2*x**2)
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.09

$$xb^3 + \frac{\frac{1}{2}\left(-6ba^2x - a^3\right)}{x^2} + 3b^2a\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x)
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.97

$$b^3\,x - \frac{\frac{a^3}{2} + 3\,b\,x\,a^2}{x^2} + 3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^3,x)
[Out] b^3*x - (a^3/2 + 3*a^2*b*x)/x^2 + 3*a*b^2*log(x)
```

3.72 $\int \frac{(a+bx)^3}{x^4} dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

```
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^4}\,dx &= \int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Mathics [A]
time = 1.88, size = 33, normalized size = 0.89

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\mathrm{Log}\,[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^4,x]')
[Out] -a ^ 3 / (3 x ^ 3) - 3 a ^ 2 b / (2 x ^ 2) - 3 a b ^ 2 / x + b ^ 3 Log[x]
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.92

method	result	size
default	$-\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3a\,b^2}{x} + b^3\ln(x)$	34
norman	$\frac{-\frac{1}{3}a^3 - 3a\,b^2x^2 - \frac{3}{2}a^2bx}{x^3} + b^3\ln(x)$	34
risch	$\frac{-\frac{1}{3}a^3 - 3a\,b^2x^2 - \frac{3}{2}a^2bx}{x^3} + b^3\ln(x)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Maxima [A]
time = 0.24, size = 34, normalized size = 0.92

$$b^3\log(x) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="maxima")
[Out] b^3*log(x) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Fricas [A]
time = 0.30, size = 37, normalized size = 1.00

$$\frac{6\,b^3x^3\log(x) - 18\,ab^2x^2 - 9\,a^2bx - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [A]
time = 0.10, size = 36, normalized size = 0.97

$$b^3\log(x) + \frac{-2a^3 - 9a^2bx - 18ab^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**4,x)
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

Giac [A]
time = 0.00, size = 41, normalized size = 1.11

$$\frac{\frac{1}{6}\left(-18b^2ax^2 - 9ba^2x - 2a^3\right)}{x^3} + b^3\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x)
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Mupad [B]
time = 0.07, size = 34, normalized size = 0.92

$$b^3\ln(x) - \frac{\frac{a^3}{3} + \frac{3a^2bx}{2} + 3ab^2x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^4,x)
[Out] b^3*log(x) - (a^3/3 + 3*a*b^2*x^2 + (3*a^2*b*x)/2)/x^3
```

3.73 $\int \frac{(a+bx)^3}{x^5}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^4}{4ax^4}$$

```
[Out] -1/4*(b*x+a)^4/a/x^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^4}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^5,x]
```

```
[Out] -1/4*(a + b*x)^4/(a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^3}{x^5}\,dx = -\frac{(a+bx)^4}{4ax^4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.
time = 0.00, size = 39, normalized size = 2.29

$$-\frac{a^3}{4x^4}-\frac{a^2b}{x^3}-\frac{3ab^2}{2x^2}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (a^2*b)/x^3 - (3*a*b^2)/(2*x^2) - b^3/x
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.
time = 1.85, size = 34, normalized size = 2.00

$$\frac{-\frac{a^3}{4} - a^2bx - \frac{3ab^2x^2}{2} - b^3x^3}{x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^5,x]')
[Out] (-a ^ 3 / 4 - a ^ 2 b x - 3 a b ^ 2 x ^ 2 / 2 - b ^ 3 x ^ 3) / x ^ 4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.
time = 0.07, size = 36, normalized size = 2.12

method	result	size
gosper	$-\frac{4b^3x^3+6a\,b^2x^2+4a^2bx+a^3}{4x^4}$	34
norman	$\frac{-b^3x^3-\frac{3}{2}a\,b^2x^2-a^2bx-\frac{1}{4}a^3}{x^4}$	35
risch	$\frac{-b^3x^3-\frac{3}{2}a\,b^2x^2-a^2bx-\frac{1}{4}a^3}{x^4}$	35
default	$-\frac{b^3}{x}-\frac{a^2b}{x^3}-\frac{a^3}{4x^4}-\frac{3a\,b^2}{2x^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^5,x,method=_RETURNVERBOSE)
[Out] -b^3/x-a^2*b/x^3-1/4*a^3/x^4-3/2*a*b^2/x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.25, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="maxima")
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.30, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.11, size = 36, normalized size = 2.12

$$\frac{-a^3 - 4a^2bx - 6ab^2x^2 - 4b^3x^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**5,x)
```

```
[Out] (-a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.00, size = 39, normalized size = 2.29

$$\frac{-4x^3b^3 - 6x^2b^2a - 4xba^2 - a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x)
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Mupad [B]
time = 0.03, size = 33, normalized size = 1.94

$$-\frac{\frac{a^3}{4} + a^2\,b\,x + \frac{3\,a\,b^2\,x^2}{2} + b^3\,x^3}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^5,x)
```

```
[Out] -(a^3/4 + b^3*x^3 + (3*a*b^2*x^2)/2 + a^2*b*x)/x^4
```

3.74 $\int \frac{(a+bx)^3}{x^6} dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^4}{5ax^5}+\frac{b(a+bx)^4}{20a^2x^4}$$

```
[Out] -1/5*(b*x+a)^4/a/x^5+1/20*b*(b*x+a)^4/a^2/x^4
```

Rubi [A]

time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b(a+bx)^4}{20a^2x^4}-\frac{(a+bx)^4}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^6,x]
```

```
[Out] -1/5*(a + b*x)^4/(a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^6} dx &= -\frac{(a+bx)^4}{5ax^5}-\frac{b\int \frac{(a+bx)^3}{x^5} dx}{5a}\\ &= -\frac{(a+bx)^4}{5ax^5}+\frac{b(a+bx)^4}{20a^2x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.14

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{4x^4}-\frac{ab^2}{x^3}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^6,x]
[Out] -1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) - (a*b^2)/x^3 - b^3/(2*x^2)
```

Mathics [A]
time = 1.86, size = 34, normalized size = 0.94

$$\frac{-\frac{a^3}{5}-\frac{3a^2bx}{4}-ab^2x^2-\frac{b^3x^3}{2}}{x^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^6,x]')
[Out] (-a ^ 3 / 5 - 3 a ^ 2 b x / 4 - a b ^ 2 x ^ 2 - b ^ 3 x ^ 3 / 2) / x ^ 5
```

Maple [A]
time = 0.07, size = 36, normalized size = 1.00

method	result	size
norman	$\frac{-\frac{1}{2}b^3x^3-a\,b^2x^2-\frac{3}{4}a^2bx-\frac{1}{5}a^3}{x^5}$	35
risch	$\frac{-\frac{1}{2}b^3x^3-a\,b^2x^2-\frac{3}{4}a^2bx-\frac{1}{5}a^3}{x^5}$	35
gosper	$-\frac{10b^3x^3+20a\,b^2x^2+15a^2bx+4a^3}{20x^5}$	36
default	$-\frac{a\,b^2}{x^3}-\frac{3a^2b}{4x^4}-\frac{b^3}{2x^2}-\frac{a^3}{5x^5}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^6,x,method=_RETURNVERBOSE)
[Out] -a*b^2/x^3-3/4*a^2*b/x^4-1/2*b^3/x^2-1/5*a^3/x^5
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Fricas [A]
time = 0.31, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [A]
time = 0.12, size = 37, normalized size = 1.03

$$\frac{-4a^3-15a^2bx-20ab^2x^2-10b^3x^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**6,x)
```

```
[Out] (-4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

Giac [A]
time = 0.00, size = 41, normalized size = 1.14

$$\frac{-10x^3b^3-20x^2b^2a-15xba^2-4a^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x)
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.94

$$-\frac{\frac{a^3}{5}+\frac{3\,a^2\,b\,x}{4}+a\,b^2\,x^2+\frac{b^3\,x^3}{2}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^6,x)
```

```
[Out] -(a^3/5 + (b^3*x^3)/2 + a*b^2*x^2 + (3*a^2*b*x)/4)/x^5
```

3.75 $\int \frac{(a+bx)^3}{x^7} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

```
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^7}\,dx &= \int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Mathics [A]
time = 1.86, size = 35, normalized size = 0.81

$$\frac{-10a^3 - 36a^2bx - 45ab^2x^2 - 20b^3x^3}{60x^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^7,x]')
[Out] (-10 a ^ 3 - 36 a ^ 2 b x - 45 a b ^ 2 x ^ 2 - 20 b ^ 3 x ^ 3) / (60 x ^ 6)
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{3}b^3x^3-\frac{3}{4}a\,b^2x^2-\frac{3}{5}a^2bx-\frac{1}{6}a^3}{x^6}$	35
risch	$\frac{-\frac{1}{3}b^3x^3-\frac{3}{4}a\,b^2x^2-\frac{3}{5}a^2bx-\frac{1}{6}a^3}{x^6}$	35
gosper	$-\frac{20b^3x^3+45a\,b^2x^2+36a^2bx+10a^3}{60x^6}$	36
default	$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3a\,b^2}{4x^4}-\frac{b^3}{3x^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

Maxima [A]
time = 0.24, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3 + 45\,ab^2x^2 + 36\,a^2bx + 10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="maxima")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3 + 45\,ab^2x^2 + 36\,a^2bx + 10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="fricas")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Sympy [A]
time = 0.12, size = 37, normalized size = 0.86

$$\frac{-10a^3 - 36a^2bx - 45ab^2x^2 - 20b^3x^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**7,x)
[Out] (-10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.95

$$\frac{-20x^3b^3 - 45x^2b^2a - 36xba^2 - 10a^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x)
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{6} + \frac{3\,a^2\,b\,x}{5} + \frac{3\,a\,b^2\,x^2}{4} + \frac{b^3\,x^3}{3}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^7,x)
[Out] -(a^3/6 + (b^3*x^3)/3 + (3*a*b^2*x^2)/4 + (3*a^2*b*x)/5)/x^6
```

3.76 $\int \frac{(a+bx)^3}{x^8} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

```
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx\\ &= -\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Mathics [A]
time = 1.86, size = 35, normalized size = 0.81

$$\frac{-20a^3 - 70a^2bx - 84ab^2x^2 - 35b^3x^3}{140x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^8,x]')
[Out] (-20 a ^ 3 - 70 a ^ 2 b x - 84 a b ^ 2 x ^ 2 - 35 b ^ 3 x ^ 3) / (140 x ^ 7
)
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{4}b^3x^3-\frac{3}{5}a\,b^2x^2-\frac{1}{2}a^2bx-\frac{1}{7}a^3}{x^7}$	35
risch	$\frac{-\frac{1}{4}b^3x^3-\frac{3}{5}a\,b^2x^2-\frac{1}{2}a^2bx-\frac{1}{7}a^3}{x^7}$	35
gosper	$-\frac{35b^3x^3+84a\,b^2x^2+70a^2bx+20a^3}{140x^7}$	36
default	$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3a\,b^2}{5x^5}-\frac{b^3}{4x^4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3 + 84\,ab^2x^2 + 70\,a^2bx + 20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="maxima")
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Fricas [A]
time = 0.29, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3 + 84\,ab^2x^2 + 70\,a^2bx + 20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Sympy [A]
time = 0.13, size = 37, normalized size = 0.86

$$\frac{-20a^3 - 70a^2bx - 84ab^2x^2 - 35b^3x^3}{140x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**8,x)
```

```
[Out] (-20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.95

$$\frac{-35x^3b^3 - 84x^2b^2a - 70xba^2 - 20a^3}{140x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x)
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{7} + \frac{a^2\,b\,x}{2} + \frac{3\,a\,b^2\,x^2}{5} + \frac{b^3\,x^3}{4}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^8,x)
```

```
[Out] -(a^3/7 + (b^3*x^3)/4 + (3*a*b^2*x^2)/5 + (a^2*b*x)/2)/x^7
```

3.77 $\int x^6(a+bx)^5\,dx$

Optimal. Leaf size=66

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^5\,dx&=\int\left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)\,dx\\&=\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Mathics [A]
time = 1.84, size = 57, normalized size = 0.86

$$\frac{x^7\left(792a^5+3465a^4bx+6160a^3b^2x^2+5544a^2b^3x^3+2520ab^4x^4+462b^5x^5\right)}{5544}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6*(a + b*x)^5,x]')
```

```
[Out] x ^ 7 (792 a ^ 5 + 3465 a ^ 4 b x + 6160 a ^ 3 b ^ 2 x ^ 2 + 5544 a ^ 2 b ^
 3 x ^ 3 + 2520 a b ^ 4 x ^ 4 + 462 b ^ 5 x ^ 5) / 5544
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.86

method	result	size
gosper	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
default	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
norman	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
risch	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Fricas [A]
time = 0.45, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Sympy [A]
time = 0.04, size = 63, normalized size = 0.95

$$\frac{a^5x^7}{7} + \frac{5a^4bx^8}{8} + \frac{10a^3b^2x^9}{9} + a^2b^3x^{10} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**9/9 + a**2*b**3*x**10 + 5*a
*b**4*x**11/11 + b**5*x**12/12
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.00

$$\frac{1}{12}x^{12}b^5 + \frac{5}{11}x^{11}b^4a + x^{10}b^3a^2 + \frac{10}{9}x^9b^2a^3 + \frac{5}{8}x^8ba^4 + \frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x)
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Mupad [B]
time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^7}{7} + \frac{5\,a^4\,b\,x^8}{8} + \frac{10\,a^3\,b^2\,x^9}{9} + a^2\,b^3\,x^{10} + \frac{5\,a\,b^4\,x^{11}}{11} + \frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^12)/12 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^11)/11 + (10*a^3
*b^2*x^9)/9 + a^2*b^3*x^10
```

3.78 $\int x^5(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^5\,dx&=\int\left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)\,dx\\&=\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Mathics [A]
time = 1.84, size = 57, normalized size = 0.83

$$\frac{x^6\left(462a^5+1980a^4bx+3465a^3b^2x^2+3080a^2b^3x^3+1386ab^4x^4+252b^5x^5\right)}{2772}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5*(a + b*x)^5,x]')
```

```
[Out] x ^ 6 (462 a ^ 5 + 1980 a ^ 4 b x + 3465 a ^ 3 b ^ 2 x ^ 2 + 3080 a ^ 2 b ^
 3 x ^ 3 + 1386 a b ^ 4 x ^ 4 + 252 b ^ 5 x ^ 5) / 2772
```

Maple [A]
time = 0.07, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
default	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
norman	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
risch	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

Maxima [A]
time = 0.25, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Sympy [A]
time = 0.04, size = 65, normalized size = 0.94

$$\frac{a^5x^6}{6} + \frac{5a^4bx^7}{7} + \frac{5a^3b^2x^8}{4} + \frac{10a^2b^3x^9}{9} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**7/7 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**9/9 +
a*b**4*x**10/2 + b**5*x**11/11
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{1}{11}x^{11}b^5 + \frac{1}{2}x^{10}b^4a + \frac{10}{9}x^9b^3a^2 + \frac{5}{4}x^8b^2a^3 + \frac{5}{7}x^7ba^4 + \frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x)
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^6}{6} + \frac{5\,a^4\,b\,x^7}{7} + \frac{5\,a^3\,b^2\,x^8}{4} + \frac{10\,a^2\,b^3\,x^9}{9} + \frac{a\,b^4\,x^{10}}{2} + \frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^11)/11 + (5*a^4*b*x^7)/7 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^8)/4 + (10*a^2*b^3*x^9)/9
```

3.79 $\int x^4(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^5\,dx&=\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)\,dx\\&=\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Mathics [A]
time = 1.84, size = 57, normalized size = 0.83

$$\frac{x^5\left(252a^5+1050a^4bx+1800a^3b^2x^2+1575a^2b^3x^3+700ab^4x^4+126b^5x^5\right)}{1260}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4*(a + b*x)^5,x]')
```

```
[Out] x ^ 5 (252 a ^ 5 + 1050 a ^ 4 b x + 1800 a ^ 3 b ^ 2 x ^ 2 + 1575 a ^ 2 b ^
 3 x ^ 3 + 700 a b ^ 4 x ^ 4 + 126 b ^ 5 x ^ 5) / 1260
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
default	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
norman	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
risch	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

Maxima [A]
time = 0.25, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Fricas [A]
time = 0.31, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Sympy [A]
time = 0.04, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5} + \frac{5a^4bx^6}{6} + \frac{10a^3b^2x^7}{7} + \frac{5a^2b^3x^8}{4} + \frac{5ab^4x^9}{9} + \frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**7/7 + 5*a**2*b**3*x**8/4 +
5*a*b**4*x**9/9 + b**5*x**10/10
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{1}{10}x^{10}b^5 + \frac{5}{9}x^9b^4a + \frac{5}{4}x^8b^3a^2 + \frac{10}{7}x^7b^2a^3 + \frac{5}{6}x^6ba^4 + \frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x)
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5} + \frac{5\,a^4\,b\,x^6}{6} + \frac{10\,a^3\,b^2\,x^7}{7} + \frac{5\,a^2\,b^3\,x^8}{4} + \frac{5\,a\,b^4\,x^9}{9} + \frac{b^5\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^10)/10 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^9)/9 + (10*a^3*b
^2*x^7)/7 + (5*a^2*b^3*x^8)/4
```

3.80 $\int x^3(a+bx)^5\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}$$

```
[Out] -1/6*a^3*(b*x+a)^6/b^4+3/7*a^2*(b*x+a)^7/b^4-3/8*a*(b*x+a)^8/b^4+1/9*(b*x+a
)^9/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^5,x]
```

```
[Out] -1/6*(a^3*(a + b*x)^6)/b^4 + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^8
)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^5\,dx &= \int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.03

$$\frac{a^5x^4}{4}+a^4bx^5+\frac{5}{3}a^3b^2x^6+\frac{10}{7}a^2b^3x^7+\frac{5}{8}ab^4x^8+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^5,x]
```

```
[Out] (a^5*x^4)/4 + a^4*b*x^5 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^7)/7 + (5*a*b^4
*x^8)/8 + (b^5*x^9)/9
```

Mathics [A]
time = 1.84, size = 57, normalized size = 0.89

$$\frac{x^4\left(126a^5+504a^4bx+840a^3b^2x^2+720a^2b^3x^3+315ab^4x^4+56b^5x^5\right)}{504}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(a + b*x)^5,x]')
```

```
[Out] x ^ 4 (126 a ^ 5 + 504 a ^ 4 b x + 840 a ^ 3 b ^ 2 x ^ 2 + 720 a ^ 2 b ^ 3
x ^ 3 + 315 a b ^ 4 x ^ 4 + 56 b ^ 5 x ^ 5) / 504
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.89

method	result	size
gosper	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
default	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
norman	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57
risch	$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*x^9+5/8*a*b^4*x^8+10/7*a^2*b^3*x^7+5/3*a^3*b^2*x^6+a^4*b*x^5+1/4*a^
5*x^4
```

Maxima [A]
time = 0.25, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Fricas [A]
time = 0.30, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Sympy [A]
time = 0.04, size = 63, normalized size = 0.98

$$\frac{a^5x^4}{4} + a^4bx^5 + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^8}{8} + \frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**5,x)
```

```
[Out] a**5*x**4/4 + a**4*b*x**5 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**7/7 + 5*a*
b**4*x**8/8 + b**5*x**9/9
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.03

$$\frac{1}{9}x^9b^5 + \frac{5}{8}x^8b^4a + \frac{10}{7}x^7b^3a^2 + \frac{5}{3}x^6b^2a^3 + x^5ba^4 + \frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x)
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Mupad [B]
time = 0.02, size = 56, normalized size = 0.88

$$\frac{a^5\,x^4}{4} + a^4\,b\,x^5 + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{10\,a^2\,b^3\,x^7}{7} + \frac{5\,a\,b^4\,x^8}{8} + \frac{b^5\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^9)/9 + a^4*b*x^5 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x^6)/3
 + (10*a^2*b^3*x^7)/7
```

3.81 $\int x^2(a+bx)^5\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}$$

```
[Out] 1/6*a^2*(b*x+a)^6/b^3-2/7*a*(b*x+a)^7/b^3+1/8*(b*x+a)^8/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^5,x]
```

```
[Out] (a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^5\,dx&=\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx\\&=\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.43

$$\frac{a^5x^3}{3}+\frac{5}{4}a^4bx^4+2a^3b^2x^5+\frac{5}{3}a^2b^3x^6+\frac{5}{7}ab^4x^7+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^5,x]
```

```
[Out] (a^5*x^3)/3 + (5*a^4*b*x^4)/4 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^6)/3 + (5*a*b^
4*x^7)/7 + (b^5*x^8)/8
```

Mathics [A]
time = 1.91, size = 57, normalized size = 1.21

$$\frac{x^3\left(56a^5+210a^4bx+336a^3b^2x^2+280a^2b^3x^3+120ab^4x^4+21b^5x^5\right)}{168}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x)^5,x]')
```

```
[Out] x ^ 3 (56 a ^ 5 + 210 a ^ 4 b x + 336 a ^ 3 b ^ 2 x ^ 2 + 280 a ^ 2 b ^ 3 x
 ^ 3 + 120 a b ^ 4 x ^ 4 + 21 b ^ 5 x ^ 5) / 168
```

Maple [A]
time = 0.07, size = 58, normalized size = 1.23

method	result	size
gosper	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
default	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
norman	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
risch	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^5*x^8+5/7*a*b^4*x^7+5/3*a^2*b^3*x^6+2*a^3*b^2*x^5+5/4*a^4*b*x^4+1/3*a
^5*x^3
```

Maxima [A]
time = 0.25, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Fricas [A]
time = 0.30, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Sympy [A]
time = 0.04, size = 65, normalized size = 1.38

$$\frac{a^5x^3}{3}+\frac{5a^4bx^4}{4}+2a^3b^2x^5+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^7}{7}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**5,x)
```

```
[Out] a**5*x**3/3 + 5*a**4*b*x**4/4 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**6/3 + 5*a
*b**4*x**7/7 + b**5*x**8/8
```

Giac [A]
time = 0.00, size = 67, normalized size = 1.43

$$\frac{1}{8}x^8b^5+\frac{5}{7}x^7b^4a+\frac{5}{3}x^6b^3a^2+2x^5b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x)
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Mupad [B]
time = 0.02, size = 57, normalized size = 1.21

$$\frac{a^5\,x^3}{3}+\frac{5\,a^4\,b\,x^4}{4}+2\,a^3\,b^2\,x^5+\frac{5\,a^2\,b^3\,x^6}{3}+\frac{5\,a\,b^4\,x^7}{7}+\frac{b^5\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^8)/8 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^7)/7 + 2*a^3*b^2*x
^5 + (5*a^2*b^3*x^6)/3
```

3.82 $\int x(a+bx)^5\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}$$

[Out] -1/6*a*(b*x+a)^6/b^2+1/7*(b*x+a)^7/b^2

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x)^5,x]

[Out] -1/6*(a*(a + b*x)^6)/b^2 + (a + b*x)^7/(7*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^5\,dx &= \int \left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right)dx\\ &= -\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.
time = 0.00, size = 67, normalized size = 2.23

$$\frac{a^5x^2}{2}+\frac{5}{3}a^4bx^3+\frac{5}{2}a^3b^2x^4+2a^2b^3x^5+\frac{5}{6}ab^4x^6+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x)^5,x]

```
[Out] (a^5*x^2)/2 + (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^4)/2 + 2*a^2*b^3*x^5 + (5*a*b^
4*x^6)/6 + (b^5*x^7)/7
```

Mathics [**A**]
time = 1.93, size = 57, normalized size = 1.90

$$\frac{x^2 \left(21a^5 + 70a^4bx + 105a^3b^2x^2 + 84a^2b^3x^3 + 35ab^4x^4 + 6b^5x^5\right)}{42}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x)^5,x]')
```

```
[Out] x ^ 2 (21 a ^ 5 + 70 a ^ 4 b x + 105 a ^ 3 b ^ 2 x ^ 2 + 84 a ^ 2 b ^ 3 x ^
 3 + 35 a b ^ 4 x ^ 4 + 6 b ^ 5 x ^ 5) / 42
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.10, size = 58, normalized size = 1.93

method	result	size
gosper	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58
default	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58
norman	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58
risch	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^5*x^7+5/6*a*b^4*x^6+2*a^2*b^3*x^5+5/2*a^3*b^2*x^4+5/3*a^4*b*x^3+1/2*a
^5*x^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.25, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.29, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^{5}x^{7}+\frac{5}{6}\,ab^{4}x^{6}+2\,a^{2}b^{3}x^{5}+\frac{5}{2}\,a^{3}b^{2}x^{4}+\frac{5}{3}\,a^{4}bx^{3}+\frac{1}{2}\,a^{5}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(24) = 48$.
time = 0.04, size = 65, normalized size = 2.17

$$\frac{a^{5}x^{2}}{2}+\frac{5a^{4}bx^{3}}{3}+\frac{5a^{3}b^{2}x^{4}}{2}+2a^{2}b^{3}x^{5}+\frac{5ab^{4}x^{6}}{6}+\frac{b^{5}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**4/2 + 2*a**2*b**3*x**5 + 5*a
*b**4*x**6/6 + b**5*x**7/7
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.00, size = 67, normalized size = 2.23

$$\frac{1}{7}x^{7}b^{5}+\frac{5}{6}x^{6}b^{4}a+2x^{5}b^{3}a^{2}+\frac{5}{2}x^{4}b^{2}a^{3}+\frac{5}{3}x^{3}ba^{4}+\frac{1}{2}x^{2}a^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x)
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Mupad [B]
time = 0.02, size = 57, normalized size = 1.90

$$\frac{a^5\,x^2}{2}+\frac{5\,a^4\,b\,x^3}{3}+\frac{5\,a^3\,b^2\,x^4}{2}+2\,a^2\,b^3\,x^5+\frac{5\,a\,b^4\,x^6}{6}+\frac{b^5\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^7)/7 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^6)/6 + (5*a^3*b^2*
x^4)/2 + 2*a^2*b^3*x^5
```

3.83 $\int (a+bx)^5 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] 1/6*(b*x+a)^6/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^5 \, dx = \frac{(a+bx)^6}{6b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(14) = 28.
time = 1.89, size = 54, normalized size = 3.86

$$\frac{x\left(6a^5+15a^4bx+20a^3b^2x^2+15a^2b^3x^3+6ab^4x^4+b^5x^5\right)}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^5,x]')
```

```
[Out] x (6 a ^ 5 + 15 a ^ 4 b x + 20 a ^ 3 b ^ 2 x ^ 2 + 15 a ^ 2 b ^ 3 x ^ 3 + 6
 a b ^ 4 x ^ 4 + b ^ 5 x ^ 5) / 6
```

Maple [A]
time = 0.07, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^6}{6b}$	13
gosper	$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4b\,x^2 + a^5x$	54
norman	$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4b\,x^2 + a^5x$	54
risch	$\frac{b^5x^6}{6} + a\,b^4x^5 + \frac{5a^2b^3x^4}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^4b\,x^2}{2} + a^5x + \frac{a^6}{6b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x+a)^6/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.
time = 0.25, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.
time = 0.30, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(8) = 16$.
time = 0.03, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(bx+a)^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x)
```

```
[Out] 1/6*(b*x + a)^6/b
```

Mupad [B]
time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x + \frac{5\,a^4\,b\,x^2}{2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^4}{2} + a\,b^4\,x^5 + \frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

3.84 $\int \frac{(a+bx)^5}{x} \, dx$

Optimal. Leaf size=59

$$5a^4bx + 5a^3b^2x^2 + \frac{10}{3}a^2b^3x^3 + \frac{5}{4}ab^4x^4 + \frac{b^5x^5}{5} + a^5\log(x)$$

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$a^5\log(x) + 5a^4bx + 5a^3b^2x^2 + \frac{10}{3}a^2b^3x^3 + \frac{5}{4}ab^4x^4 + \frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x} \, dx &= \int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5x^4\right) dx \\ &= 5a^4bx + 5a^3b^2x^2 + \frac{10}{3}a^2b^3x^3 + \frac{5}{4}ab^4x^4 + \frac{b^5x^5}{5} + a^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 59, normalized size = 1.00

$$5a^4bx + 5a^3b^2x^2 + \frac{10}{3}a^2b^3x^3 + \frac{5}{4}ab^4x^4 + \frac{b^5x^5}{5} + a^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Mathics [A]
time = 1.91, size = 53, normalized size = 0.90

$$a^5\text{Log}\left[x\right]+5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5ab^4x^4}{4}+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^1,x]')
```

```
[Out] a ^ 5 Log[x] + 5 a ^ 4 b x + 5 a ^ 3 b ^ 2 x ^ 2 + 10 a ^ 2 b ^ 3 x ^ 3 / 3
 + 5 a b ^ 4 x ^ 4 / 4 + b ^ 5 x ^ 5 / 5
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.92

method	result	size
default	$5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$	54
norman	$5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$	54
risch	$5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

Maxima [A]
time = 0.25, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Fricas [A]
time = 0.31, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^{5}x^{5} + \frac{5}{4}\,ab^{4}x^{4} + \frac{10}{3}\,a^{2}b^{3}x^{3} + 5\,a^{3}b^{2}x^{2} + 5\,a^{4}bx + a^{5}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Sympy [A]
time = 0.06, size = 60, normalized size = 1.02

$$a^{5}\log\left(x\right) + 5a^{4}bx + 5a^{3}b^{2}x^{2} + \frac{10a^{2}b^{3}x^{3}}{3} + \frac{5ab^{4}x^{4}}{4} + \frac{b^{5}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x + 5*a**3*b**2*x**2 + 10*a**2*b**3*x**3/3 + 5*a*b**
4*x**4/4 + b**5*x**5/5
```

Giac [A]
time = 0.00, size = 60, normalized size = 1.02

$$\frac{1}{5}x^{5}b^{5} + \frac{5}{4}x^{4}b^{4}a + \frac{10}{3}x^{3}b^{3}a^{2} + 5x^{2}b^{2}a^{3} + 5xba^{4} + a^{5}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x)
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(abs(x))
```

Mupad [B]
time = 0.03, size = 53, normalized size = 0.90

$$a^{5}\,\ln\left(x\right) + \frac{b^{5}\,x^{5}}{5} + \frac{5\,a\,b^{4}\,x^{4}}{4} + 5\,a^{3}\,b^{2}\,x^{2} + \frac{10\,a^{2}\,b^{3}\,x^{3}}{3} + 5\,a^{4}\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^5)/5 + (5*a*b^4*x^4)/4 + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^
3)/3 + 5*a^4*b*x
```

3.85 $\int \frac{(a+bx)^5}{x^2} dx$

Optimal. Leaf size=58

$$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)$$

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{x}+5a^4b\log(x)+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^2}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx\\ &= -\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 1.00

$$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Mathics [A]
time = 1.90, size = 57, normalized size = 0.98

$$\frac{-a^5 + \frac{bx\left(60a^4\mathrm{Log}[x]+120a^3bx+60a^2b^2x^2+20ab^3x^3+3b^4x^4\right)}{12}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^2,x]')
```

```
[Out] (-a ^ 5 + b x (60 a ^ 4 Log[x] + 120 a ^ 3 b x + 60 a ^ 2 b ^ 2 x ^ 2 + 20
a b ^ 3 x ^ 3 + 3 b ^ 4 x ^ 4) / 12) / x
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.95

method	result	size
default	$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5a\,b^4x^3}{3}+\frac{b^5x^4}{4}+5a^4b\ln\left(x\right)$	55
risch	$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5a\,b^4x^3}{3}+\frac{b^5x^4}{4}+5a^4b\ln\left(x\right)$	55
norman	$\frac{-a^5+\frac{1}{4}b^5x^5+\frac{5}{3}a\,b^4x^4+5a^2b^3x^3+10a^3b^2x^2}{x}+5a^4b\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Maxima [A]
time = 0.25, size = 54, normalized size = 0.93

$$\frac{1}{4}\,b^5x^4+\frac{5}{3}\,ab^4x^3+5\,a^2b^3x^2+10\,a^3b^2x+5\,a^4b\log\left(x\right)-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(x)
 - a^5/x
```

Fricas [A]
time = 0.31, size = 59, normalized size = 1.02

$$\frac{3\,b^5x^5+20\,ab^4x^4+60\,a^2b^3x^3+120\,a^3b^2x^2+60\,a^4bx\log\left(x\right)-12\,a^5}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4*
b*x*log(x) - 12*a^5)/x
```

Sympy [A]
time = 0.07, size = 56, normalized size = 0.97

$$-\frac{a^5}{x} + 5a^4b\log(x) + 10a^3b^2x + 5a^2b^3x^2 + \frac{5ab^4x^3}{3} + \frac{b^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*log(x) + 10*a**3*b**2*x + 5*a**2*b**3*x**2 + 5*a*b**4*x*
*3/3 + b**5*x**4/4
```

Giac [A]
time = 0.00, size = 58, normalized size = 1.00

$$\frac{1}{4}x^4b^5 + \frac{5}{3}x^3b^4a + 5x^2b^3a^2 + 10xb^2a^3 - \frac{a^5}{x} + 5ba^4\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x)
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(ab
s(x)) - a^5/x
```

Mupad [B]
time = 0.03, size = 54, normalized size = 0.93

$$\frac{b^5\,x^4}{4} - \frac{a^5}{x} + 10\,a^3\,b^2\,x + \frac{5\,a\,b^4\,x^3}{3} + 5\,a^4\,b\,\ln(x) + 5\,a^2\,b^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^2,x)
```

```
[Out] (b^5*x^4)/4 - a^5/x + 10*a^3*b^2*x + (5*a*b^4*x^3)/3 + 5*a^4*b*log(x) + 5*a
^2*b^3*x^2
```

3.86 $\int \frac{(a+bx)^5}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)$$

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^3}\,dx &= \int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx\\ &= -\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Mathics [A]
time = 1.99, size = 56, normalized size = 0.93

$$\frac{-3a^4 (a + 10bx) + b^2x^2 (60a^3\text{Log}\,[x] + 60a^2bx + 15ab^2x^2 + 2b^3x^3)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^3,x]')
```

```
[Out] (-3 a ^ 4 (a + 10 b x) + b ^ 2 x ^ 2 (60 a ^ 3 Log[x] + 60 a ^ 2 b x + 15 a
 b ^ 2 x ^ 2 + 2 b ^ 3 x ^ 3)) / (6 x ^ 2)
```

Maple [A]
time = 0.08, size = 55, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{2x^2} - \frac{5a^4b}{x} + 10a^2b^3x + \frac{5a\,b^4x^2}{2} + \frac{b^5x^3}{3} + 10a^3b^2\ln(x)$	55
risch	$\frac{b^5x^3}{3} + \frac{5a\,b^4x^2}{2} + 10a^2b^3x + \frac{-5a^4bx-\frac{1}{2}a^5}{x^2} + 10a^3b^2\ln(x)$	55
norman	$\frac{-\frac{1}{2}a^5+\frac{1}{3}b^5x^5+\frac{5}{2}a\,b^4x^4+10a^2b^3x^3-5a^4bx}{x^2} + 10a^3b^2\ln(x)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.88

$$\frac{1}{3}\,b^5x^3 + \frac{5}{2}\,ab^4x^2 + 10\,a^2b^3x + 10\,a^3b^2\log(x) - \frac{10\,a^4bx + a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(x) - 1/2*(10*a^
4*b*x + a^5)/x^2
```

Fricas [A]
time = 0.32, size = 59, normalized size = 0.98

$$\frac{2\,b^{5}x^{5}+15\,ab^{4}x^{4}+60\,a^{2}b^{3}x^{3}+60\,a^{3}b^{2}x^{2}\log\left(x\right)-30\,a^{4}bx-3\,a^{5}}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 30
*a^4*b*x - 3*a^5)/x^2
```

Sympy [A]
time = 0.10, size = 60, normalized size = 1.00

$$10a^{3}b^{2}\log\left(x\right)+10a^{2}b^{3}x+\frac{5ab^{4}x^{2}}{2}+\frac{b^{5}x^{3}}{3}+\frac{-a^{5}-10a^{4}bx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**3,x)
```

```
[Out] 10*a**3*b**2*log(x) + 10*a**2*b**3*x + 5*a*b**4*x**2/2 + b**5*x**3/3 + (-a*
*5 - 10*a**4*b*x)/(2*x**2)
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.05

$$\frac{1}{3}x^{3}b^{5}+\frac{5}{2}x^{2}b^{4}a+10xb^{3}a^{2}+\frac{\frac{1}{2}\left(-10ba^{4}x-a^{5}\right)}{x^{2}}+10b^{2}a^{3}\ln\left|x\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x)
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(abs(x)) - 1/2*(
10*a^4*b*x + a^5)/x^2
```

Mupad [B]
time = 0.03, size = 55, normalized size = 0.92

$$\frac{b^{5}\,x^{3}}{3}-\frac{\frac{a^{5}}{2}+5\,b\,x\,a^{4}}{x^{2}}+10\,a^{2}\,b^{3}\,x+\frac{5\,a\,b^{4}\,x^{2}}{2}+10\,a^{3}\,b^{2}\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^3,x)
```

```
[Out] (b^5*x^3)/3 - (a^5/2 + 5*a^4*b*x)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + 10
*a^3*b^2*log(x)
```

3.87 $\int \frac{(a+bx)^5}{x^4} dx$

Optimal. Leaf size=60

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)$$

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^4}\,dx &= \int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Mathics [A]
time = 2.06, size = 58, normalized size = 0.97

$$\frac{-a^3\left(2a^2+15abx+60b^2x^2\right)+3b^3x^3\left(20a^2\text{Log}\left[x\right]+10abx+b^2x^2\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^4,x]')
```

```
[Out] (-a ^ 3 (2 a ^ 2 + 15 a b x + 60 b ^ 2 x ^ 2) + 3 b ^ 3 x ^ 3 (20 a ^ 2 Log
[x] + 10 a b x + b ^ 2 x ^ 2)) / (6 x ^ 3)
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5a\,b^4x+\frac{b^5x^2}{2}+10a^2b^3\ln\left(x\right)$	55
risch	$\frac{b^5x^2}{2}+5a\,b^4x+\frac{-10a^3b^2x^2-\frac{5}{2}a^4bx-\frac{1}{3}a^5}{x^3}+10a^2b^3\ln\left(x\right)$	55
norman	$\frac{-\frac{1}{3}a^5+\frac{1}{2}b^5x^5+5a\,b^4x^4-10a^3b^2x^2-\frac{5}{2}a^4bx}{x^3}+10a^2b^3\ln\left(x\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

Maxima [A]
time = 0.25, size = 55, normalized size = 0.92

$$\frac{1}{2}\,b^5x^2+5\,ab^4x+10\,a^2b^3\log\left(x\right)-\frac{60\,a^3b^2x^2+15\,a^4bx+2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(x) - 1/6*(60*a^3*b^2*x^2 + 15*a^4*
b*x + 2*a^5)/x^3
```

Fricas [A]
time = 0.31, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^5+30\,ab^4x^4+60\,a^2b^3x^3\log\left(x\right)-60\,a^3b^2x^2-15\,a^4bx-2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 15
*a^4*b*x - 2*a^5)/x^3
```

Sympy [A]
time = 0.12, size = 60, normalized size = 1.00

$$10a^2b^3\log\left(x\right)+5ab^4x+\frac{b^5x^2}{2}+\frac{-2a^5-15a^4bx-60a^3b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**4,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x + b**5*x**2/2 + (-2*a**5 - 15*a**4*b*x - 6
0*a**3*b**2*x**2)/(6*x**3)
```

Giac [A]
time = 0.00, size = 64, normalized size = 1.07

$$\frac{1}{2}x^2b^5+5xb^4a+\frac{\frac{1}{6}\left(-60b^2a^3x^2-15ba^4x-2a^5\right)}{x^3}+10b^3a^2\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x)
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(abs(x)) - 1/6*(60*a^3*b^2*x^2 + 15
*a^4*b*x + 2*a^5)/x^3
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.92

$$\frac{b^5\,x^2}{2}-\frac{\frac{a^5}{3}+\frac{5\,a^4\,b\,x}{2}+10\,a^3\,b^2\,x^2}{x^3}+10\,a^2\,b^3\,\ln\left(x\right)+5\,a\,b^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^4,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/3 + 10*a^3*b^2*x^2 + (5*a^4*b*x)/2)/x^3 + 10*a^2*b^3*log
(x) + 5*a*b^4*x
```

3.88 $\int \frac{(a+bx)^5}{x^5} dx$

Optimal. Leaf size=57

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)$$

```
[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^5}\,dx &= \int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Mathics [A]
time = 2.12, size = 53, normalized size = 0.93

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\text{Log}\,[x]+b^5x$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^5,x]')
```

```
[Out] -a ^ 5 / (4 x ^ 4) - 5 a ^ 4 b / (3 x ^ 3) - 5 a ^ 3 b ^ 2 / x ^ 2 - 10 a ^
 2 b ^ 3 / x + 5 a b ^ 4 Log[x] + b ^ 5 x
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.95

method	result	size
default	$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5a\,b^4\ln\left(x\right)$	54
risch	$b^5x+\frac{-10a^2b^3x^3-5a^3b^2x^2-\frac{5}{3}a^4bx-\frac{1}{4}a^5}{x^4}+5a\,b^4\ln\left(x\right)$	54
norman	$\frac{b^5x^5-\frac{1}{4}a^5-10a^2b^3x^3-5a^3b^2x^2-\frac{5}{3}a^4bx}{x^4}+5a\,b^4\ln\left(x\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Maxima [A]
time = 0.25, size = 54, normalized size = 0.95

$$b^5x+5\,ab^4\log\left(x\right)-\frac{120\,a^2b^3x^3+60\,a^3b^2x^2+20\,a^4bx+3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] b^5*x + 5*a*b^4*log(x) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a^4*b*
x + 3*a^5)/x^4
```

Fricas [A]
time = 0.30, size = 59, normalized size = 1.04

$$\frac{12\,b^5x^5+60\,ab^4x^4\log\left(x\right)-120\,a^2b^3x^3-60\,a^3b^2x^2-20\,a^4bx-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2 -
 20*a^4*b*x - 3*a^5)/x^4
```

Sympy [A]
time = 0.14, size = 58, normalized size = 1.02

$$5ab^4\log(x) + b^5x + \frac{-3a^5 - 20a^4bx - 60a^3b^2x^2 - 120a^2b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**5,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x + (-3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120
*a**2*b**3*x**3)/(12*x**4)
```

Giac [A]
time = 0.00, size = 62, normalized size = 1.09

$$xb^5 + \frac{\frac{1}{12}\left(-120b^3a^2x^3 - 60b^2a^3x^2 - 20ba^4x - 3a^5\right)}{x^4} + 5b^4a\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x)
```

```
[Out] b^5*x + 5*a*b^4*log(abs(x)) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a
^4*b*x + 3*a^5)/x^4
```

Mupad [B]
time = 0.08, size = 54, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{4} + \frac{5\,a^4\,b\,x}{3} + 5\,a^3\,b^2\,x^2 + 10\,a^2\,b^3\,x^3}{x^4} + 5\,a\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^5,x)
```

```
[Out] b^5*x - (a^5/4 + 5*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + (5*a^4*b*x)/3)/x^4 + 5*a*
b^4*log(x)
```

3.89 $\int \frac{(a+bx)^5}{x^6} dx$

Optimal. Leaf size=61

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^6}\,dx &= \int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Mathics [A]
time = 2.17, size = 58, normalized size = 0.95

$$\frac{-\frac{a(12a^4+75a^3bx+200a^2b^2x^2+300ab^3x^3+300b^4x^4)}{60}+b^5x^5\text{Log}\,[x]}{x^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^6,x]')
```

```
[Out] (-a (12 a ^ 4 + 75 a ^ 3 b x + 200 a ^ 2 b ^ 2 x ^ 2 + 300 a b ^ 3 x ^ 3 +
300 b ^ 4 x ^ 4) / 60 + b ^ 5 x ^ 5 Log[x]) / x ^ 5
```

Maple [A]
time = 0.07, size = 56, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5a\,b^4}{x}+b^5\ln(x)$	56
norman	$\frac{-\frac{1}{5}a^5-5a\,b^4x^4-5a^2b^3x^3-\frac{10}{3}a^3b^2x^2-\frac{5}{4}a^4bx}{x^5}+b^5\ln(x)$	56
risch	$\frac{-\frac{1}{5}a^5-5a\,b^4x^4-5a^2b^3x^3-\frac{10}{3}a^3b^2x^2-\frac{5}{4}a^4bx}{x^5}+b^5\ln(x)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

Maxima [A]
time = 0.24, size = 56, normalized size = 0.92

$$b^5\log(x)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 + 75*a
^4*b*x + 12*a^5)/x^5
```

Fricas [A]
time = 0.30, size = 59, normalized size = 0.97

$$\frac{60\, b^5 x^5 \log\left(x\right) - 300\, a b^4 x^4 - 300\, a^2 b^3 x^3 - 200\, a^3 b^2 x^2 - 75\, a^4 b x - 12\, a^5}{60\, x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^2
 - 75*a^4*b*x - 12*a^5)/x^5
```

Sympy [A]
time = 0.17, size = 60, normalized size = 0.98

$$b^5 \log\left(x\right) + \frac{-12a^5 - 75a^4bx - 200a^3b^2x^2 - 300a^2b^3x^3 - 300ab^4x^4}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**6,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a**2*b**3*
x**3 - 300*a*b**4*x**4)/(60*x**5)
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.07

$$\frac{\frac{1}{60}\left(-300b^4ax^4 - 300b^3a^2x^3 - 200b^2a^3x^2 - 75ba^4x - 12a^5\right)}{x^5} + b^5 \ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x)
```

```
[Out] b^5*log(abs(x)) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 +
 75*a^4*b*x + 12*a^5)/x^5
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.92

$$b^5\,\ln\left(x\right) - \frac{\frac{a^5}{5} + \frac{5\,a^4\,b\,x}{4} + \frac{10\,a^3\,b^2\,x^2}{3} + 5\,a^2\,b^3\,x^3 + 5\,a\,b^4\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^6,x)
```

```
[Out] b^5*log(x) - (a^5/5 + 5*a*b^4*x^4 + (10*a^3*b^2*x^2)/3 + 5*a^2*b^3*x^3 + (5
*a^4*b*x)/4)/x^5
```

3.90 $\int \frac{(a+bx)^5}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^6}{6ax^6}$$

```
[Out] -1/6*(b*x+a)^6/a/x^6
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^6}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^7,x]
```

```
[Out] -1/6*(a + b*x)^6/(a*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^7}\,dx = -\frac{(a+bx)^6}{6ax^6}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(17) = 34.
time = 0.00, size = 65, normalized size = 3.82

$$-\frac{a^5}{6x^6}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{2x^4}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{2x^2}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (a^4*b)/x^5 - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/(3*x^3) - (
5*a*b^4)/(2*x^2) - b^5/x
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 59 vs. $2(17) = 34$.
time = 2.07, size = 57, normalized size = 3.35

$$\frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^7,x]')
```

```
[Out] (-a ^ 5 - 6 a ^ 4 b x - 15 a ^ 3 b ^ 2 x ^ 2 - 20 a ^ 2 b ^ 3 x ^ 3 - 15 a
b ^ 4 x ^ 4 - 6 b ^ 5 x ^ 5) / (6 x ^ 6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(15) = 30$.
time = 0.08, size = 58, normalized size = 3.41

method	result	size
gosper	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6x^6}$	56
norman	$\frac{-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5}{x^6}$	57
risch	$\frac{-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5}{x^6}$	57
default	$-\frac{b^5}{x}-\frac{10a^2b^3}{3x^3}-\frac{5a^3b^2}{2x^4}-\frac{5a\,b^4}{2x^2}-\frac{a^4b}{x^5}-\frac{a^5}{6x^6}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -b^5/x-10/3*a^2*b^3/x^3-5/2*a^3*b^2/x^4-5/2*a*b^4/x^2-a^4*b/x^5-1/6*a^5/x^6
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.
time = 0.26, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.
time = 0.31, size = 55, normalized size = 3.24

$$-\frac{6\,b^{5}x^{5}+15\,ab^{4}x^{4}+20\,a^{2}b^{3}x^{3}+15\,a^{3}b^{2}x^{2}+6\,a^{4}bx+a^{5}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.
time = 0.18, size = 60, normalized size = 3.53

$$\frac{-a^{5}-6a^{4}bx-15a^{3}b^{2}x^{2}-20a^{2}b^{3}x^{3}-15ab^{4}x^{4}-6b^{5}x^{5}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.
time = 0.00, size = 63, normalized size = 3.71

$$\frac{-6x^{5}b^{5}-15x^{4}b^{4}a-20x^{3}b^{3}a^{2}-15x^{2}b^{2}a^{3}-6xba^{4}-a^{5}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x)
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Mupad [B]
time = 0.04, size = 55, normalized size = 3.24

$$-\frac{\frac{a^{5}}{6}+a^{4}\,b\,x+\frac{5\,a^{3}\,b^{2}\,x^{2}}{2}+\frac{10\,a^{2}\,b^{3}\,x^{3}}{3}+\frac{5\,a\,b^{4}\,x^{4}}{2}+b^{5}\,x^{5}}{x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^7,x)
```

```
[Out] -(a^5/6 + b^5*x^5 + (5*a*b^4*x^4)/2 + (5*a^3*b^2*x^2)/2 + (10*a^2*b^3*x^3)/
3 + a^4*b*x)/x^6
```

3.91 $\int \frac{(a+bx)^5}{x^8} dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^6}{7ax^7} + \frac{b(a+bx)^6}{42a^2x^6}$$

```
[Out] -1/7*(b*x+a)^6/a/x^7+1/42*b*(b*x+a)^6/a^2/x^6
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^8,x]
```

```
[Out] -1/7*(a + b*x)^6/(a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{x^8} dx &= -\frac{(a+bx)^6}{7ax^7} - \frac{b\int \frac{(a+bx)^5}{x^7} dx}{7a} \\
&= -\frac{(a+bx)^6}{7ax^7} + \frac{b(a+bx)^6}{42a^2x^6}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.86

$$-\frac{a^5}{7x^7}-\frac{5a^4b}{6x^6}-\frac{2a^3b^2}{x^5}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{3x^3}-\frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (5*a^4*b)/(6*x^6) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/(2*x^4) -
(5*a*b^4)/(3*x^3) - b^5/(2*x^2)
```

Mathics [A]
time = 2.14, size = 57, normalized size = 1.58

$$\frac{-6a^5-35a^4bx-84a^3b^2x^2-105a^2b^3x^3-70ab^4x^4-21b^5x^5}{42x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^8,x]')
```

```
[Out] (-6 a ^ 5 - 35 a ^ 4 b x - 84 a ^ 3 b ^ 2 x ^ 2 - 105 a ^ 2 b ^ 3 x ^ 3 - 7
0 a b ^ 4 x ^ 4 - 21 b ^ 5 x ^ 5) / (42 x ^ 7)
```

Maple [A]
time = 0.09, size = 58, normalized size = 1.61

method	result	size
norman	$\frac{-\frac{1}{2}b^5x^5-\frac{5}{3}a\,b^4x^4-\frac{5}{2}a^2b^3x^3-2a^3b^2x^2-\frac{5}{6}a^4bx-\frac{1}{7}a^5}{x^7}$	57
risch	$\frac{-\frac{1}{2}b^5x^5-\frac{5}{3}a\,b^4x^4-\frac{5}{2}a^2b^3x^3-2a^3b^2x^2-\frac{5}{6}a^4bx-\frac{1}{7}a^5}{x^7}$	57
gosper	$-\frac{21b^5x^5+70a\,b^4x^4+105a^2b^3x^3+84a^3b^2x^2+35a^4bx+6a^5}{42x^7}$	58
default	$-\frac{5a\,b^4}{3x^3}-\frac{5a^2b^3}{2x^4}-\frac{b^5}{2x^2}-\frac{2a^3b^2}{x^5}-\frac{5a^4b}{6x^6}-\frac{a^5}{7x^7}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3*a*b^4/x^3-5/2*a^2*b^3/x^4-1/2*b^5/x^2-2*a^3*b^2/x^5-5/6*a^4*b/x^6-1/7*
a^5/x^7
```

Maxima [A]
time = 0.25, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Fricas [A]
time = 0.29, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.
time = 0.19, size = 61, normalized size = 1.69

$$\frac{-6a^5-35a^4bx-84a^3b^2x^2-105a^2b^3x^3-70ab^4x^4-21b^5x^5}{42x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**8,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b**4
*x**4 - 21*b**5*x**5)/(42*x**7)
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.81

$$\frac{-21x^5b^5-70x^4b^4a-105x^3b^3a^2-84x^2b^2a^3-35xba^4-6a^5}{42x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x)
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Mupad [B]
time = 0.07, size = 57, normalized size = 1.58

$$-\frac{\frac{a^5}{7}+\frac{5\,a^4\,b\,x}{6}+2\,a^3\,b^2\,x^2+\frac{5\,a^2\,b^3\,x^3}{2}+\frac{5\,a\,b^4\,x^4}{3}+\frac{b^5\,x^5}{2}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^8,x)
```

```
[Out] -(a^5/7 + (b^5*x^5)/2 + (5*a*b^4*x^4)/3 + 2*a^3*b^2*x^2 + (5*a^2*b^3*x^3)/2
 + (5*a^4*b*x)/6)/x^7
```

3.92 $\int \frac{(a+bx)^5}{x^9} dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^6}{8ax^8}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{b^2(a+bx)^6}{168a^3x^6}$$

```
[Out] -1/8*(b*x+a)^6/a/x^8+1/28*b*(b*x+a)^6/a^2/x^7-1/168*b^2*(b*x+a)^6/a^3/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^9,x]
```

```
[Out] -1/8*(a + b*x)^6/(a*x^8) + (b*(a + b*x)^6)/(28*a^2*x^7) - (b^2*(a + b*x)^6)
/(168*a^3*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{(a+bx)^6}{8ax^8} - \frac{b\int \frac{(a+bx)^5}{x^8}\,dx}{4a}$$
$$= -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} + \frac{b^2\int \frac{(a+bx)^5}{x^7}\,dx}{28a^2}$$
$$= -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} - \frac{b^2(a+bx)^6}{168a^3x^6}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.20

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{7x^7} - \frac{5a^3b^2}{3x^6} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{4x^4} - \frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^9,x]
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(3*x^6) - (2*a^2*b^3)/x^5 -
(5*a*b^4)/(4*x^4) - b^5/(3*x^3)
```

Mathics [A]
time = 2.16, size = 57, normalized size = 1.02

$$\frac{-21a^5 - 120a^4bx - 280a^3b^2x^2 - 336a^2b^3x^3 - 210ab^4x^4 - 56b^5x^5}{168x^8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^9,x]')
[Out] (-21 a ^ 5 - 120 a ^ 4 b x - 280 a ^ 3 b ^ 2 x ^ 2 - 336 a ^ 2 b ^ 3 x ^ 3
- 210 a b ^ 4 x ^ 4 - 56 b ^ 5 x ^ 5) / (168 x ^ 8)
```

Maple [A]
time = 0.08, size = 58, normalized size = 1.04

method	result	size
norman	$\frac{-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5}{x^8}$	57
risch	$\frac{-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5}{x^8}$	57
gosper	$-\frac{56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5}{168x^8}$	58
default	$-\frac{b^5}{3x^3}-\frac{5a\,b^4}{4x^4}-\frac{a^5}{8x^8}-\frac{2a^2b^3}{x^5}-\frac{5a^3b^2}{3x^6}-\frac{5a^4b}{7x^7}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^5/x^3-5/4*a*b^4/x^4-1/8*a^5/x^8-2*a^2*b^3/x^5-5/3*a^3*b^2/x^6-5/7*a^
4*b/x^7
```

Maxima [A]
time = 0.25, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Fricas [A]
time = 0.30, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Sympy [A]
time = 0.21, size = 61, normalized size = 1.09

$$\frac{-21a^5-120a^4bx-280a^3b^2x^2-336a^2b^3x^3-210ab^4x^4-56b^5x^5}{168x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**9,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210*a*
b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.16

$$\frac{-56x^5b^5-210x^4b^4a-336x^3b^3a^2-280x^2b^2a^3-120xba^4-21a^5}{168x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x)
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Mupad [B]
time = 0.04, size = 57, normalized size = 1.02

$$-\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x}{7}+\frac{5\,a^3\,b^2\,x^2}{3}+2\,a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{4}+\frac{b^5\,x^5}{3}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^9,x)
```

```
[Out] -(a^5/8 + (b^5*x^5)/3 + (5*a*b^4*x^4)/4 + (5*a^3*b^2*x^2)/3 + 2*a^2*b^3*x^3
 + (5*a^4*b*x)/7)/x^8
```

3.93 $\int \frac{(a+bx)^5}{x^{10}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Mathics [A]
time = 2.19, size = 57, normalized size = 0.85

$$\frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^10,x]')
```

```
[Out] (-56 a ^ 5 - 315 a ^ 4 b x - 720 a ^ 3 b ^ 2 x ^ 2 - 840 a ^ 2 b ^ 3 x ^ 3
- 504 a b ^ 4 x ^ 4 - 126 b ^ 5 x ^ 5) / (504 x ^ 9)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
risch	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
gosper	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{a\,b^4}{x^5}-\frac{b^5}{4x^4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Maxima [A]
time = 0.25, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [A]
time = 0.22, size = 61, normalized size = 0.91

$$\frac{-56a^5-315a^4bx-720a^3b^2x^2-840a^2b^3x^3-504ab^4x^4-126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.97

$$\frac{-126x^5b^5-504x^4b^4a-840x^3b^3a^2-720x^2b^2a^3-315xba^4-56a^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x)
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Mupad [B]
time = 0.08, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9}+\frac{5\,a^4\,b\,x}{8}+\frac{10\,a^3\,b^2\,x^2}{7}+\frac{5\,a^2\,b^3\,x^3}{3}+a\,b^4\,x^4+\frac{b^5\,x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

3.94 $\int \frac{(a+bx)^5}{x^{11}} dx$

Optimal. Leaf size=69

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{11}}\,dx &= \int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx\\ &= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Mathics [A]
time = 2.17, size = 57, normalized size = 0.83

$$\frac{-126a^5 - 700a^4bx - 1575a^3b^2x^2 - 1800a^2b^3x^3 - 1050ab^4x^4 - 252b^5x^5}{1260x^{10}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^11,x]')
```

```
[Out] (-126 a ^ 5 - 700 a ^ 4 b x - 1575 a ^ 3 b ^ 2 x ^ 2 - 1800 a ^ 2 b ^ 3 x ^
 3 - 1050 a b ^ 4 x ^ 4 - 252 b ^ 5 x ^ 5) / (1260 x ^ 10)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5}{x^{10}}$	57
risch	$\frac{-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5}{x^{10}}$	57
gosper	$-\frac{252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5}{1260x^{10}}$	58
default	$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a\,b^4}{6x^6}-\frac{b^5}{5x^5}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5 + 1050\,ab^4x^4 + 1800\,a^2b^3x^3 + 1575\,a^3b^2x^2 + 700\,a^4bx + 126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.83

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Sympy [A]
time = 0.23, size = 61, normalized size = 0.88

$$\frac{-126a^{5}-700a^{4}bx-1575a^{3}b^{2}x^{2}-1800a^{2}b^{3}x^{3}-1050ab^{4}x^{4}-252b^{5}x^{5}}{1260x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**11,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 - 105
0*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.94

$$\frac{-252x^{5}b^{5}-1050x^{4}b^{4}a-1800x^{3}b^{3}a^{2}-1575x^{2}b^{2}a^{3}-700xba^{4}-126a^{5}}{1260x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x)
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Mupad [B]
time = 0.08, size = 57, normalized size = 0.83

$$-\frac{\frac{a^{5}}{10}+\frac{5\,a^{4}\,b\,x}{9}+\frac{5\,a^{3}\,b^{2}\,x^{2}}{4}+\frac{10\,a^{2}\,b^{3}\,x^{3}}{7}+\frac{5\,a\,b^{4}\,x^{4}}{6}+\frac{b^{5}\,x^{5}}{5}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^11,x)
```

```
[Out] -(a^5/10 + (b^5*x^5)/5 + (5*a*b^4*x^4)/6 + (5*a^3*b^2*x^2)/4 + (10*a^2*b^3*
x^3)/7 + (5*a^4*b*x)/9)/x^10
```

3.95 $\int \frac{(a+bx)^5}{x^{12}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{12}}\,dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{11}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^9}+\frac{5ab^4}{x^8}+\frac{b^5}{x^7}\right)dx\\ &= -\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Mathics [A]
time = 2.09, size = 57, normalized size = 0.83

$$\frac{-252a^5 - 1386a^4bx - 3080a^3b^2x^2 - 3465a^2b^3x^3 - 1980ab^4x^4 - 462b^5x^5}{2772x^{11}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^12,x]')
```

```
[Out] (-252 a ^ 5 - 1386 a ^ 4 b x - 3080 a ^ 3 b ^ 2 x ^ 2 - 3465 a ^ 2 b ^ 3 x
^ 3 - 1980 a b ^ 4 x ^ 4 - 462 b ^ 5 x ^ 5) / (2772 x ^ 11)
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5}{x^{11}}$	57
risch	$\frac{-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5}{x^{11}}$	57
gosper	$-\frac{462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5}{2772x^{11}}$	58
default	$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5a\,b^4}{7x^7}-\frac{b^5}{6x^6}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5 + 1980\,ab^4x^4 + 3465\,a^2b^3x^3 + 3080\,a^3b^2x^2 + 1386\,a^4bx + 252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Sympy [A]
time = 0.25, size = 61, normalized size = 0.88

$$\frac{-252a^5-1386a^4bx-3080a^3b^2x^2-3465a^2b^3x^3-1980ab^4x^4-462b^5x^5}{2772x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**12,x)
```

```
[Out] (-252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 - 19
80*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.94

$$\frac{-462x^5b^5-1980x^4b^4a-3465x^3b^3a^2-3080x^2b^2a^3-1386xba^4-252a^5}{2772x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x)
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Mupad [B]
time = 0.04, size = 57, normalized size = 0.83

$$-\frac{\frac{a^5}{11}+\frac{a^4\,b\,x}{2}+\frac{10\,a^3\,b^2\,x^2}{9}+\frac{5\,a^2\,b^3\,x^3}{4}+\frac{5\,a\,b^4\,x^4}{7}+\frac{b^5\,x^5}{6}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^12,x)
```

```
[Out] -(a^5/11 + (b^5*x^5)/6 + (5*a*b^4*x^4)/7 + (10*a^3*b^2*x^2)/9 + (5*a^2*b^3*
x^3)/4 + (a^4*b*x)/2)/x^11
```

3.96 $\int \frac{(a+bx)^5}{x^{13}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{13}}\,dx &= \int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Mathics [A]
time = 2.10, size = 57, normalized size = 0.85

$$\frac{-462a^5 - 2520a^4bx - 5544a^3b^2x^2 - 6160a^2b^3x^3 - 3465ab^4x^4 - 792b^5x^5}{5544x^{12}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^13,x]')
```

```
[Out] (-462 a ^ 5 - 2520 a ^ 4 b x - 5544 a ^ 3 b ^ 2 x ^ 2 - 6160 a ^ 2 b ^ 3 x
^ 3 - 3465 a b ^ 4 x ^ 4 - 792 b ^ 5 x ^ 5) / (5544 x ^ 12)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{7}b^5x^5-\frac{5}{8}a\,b^4x^4-\frac{10}{9}a^2b^3x^3-a^3b^2x^2-\frac{5}{11}a^4bx-\frac{1}{12}a^5}{x^{12}}$	57
risch	$\frac{-\frac{1}{7}b^5x^5-\frac{5}{8}a\,b^4x^4-\frac{10}{9}a^2b^3x^3-a^3b^2x^2-\frac{5}{11}a^4bx-\frac{1}{12}a^5}{x^{12}}$	57
gosper	$-\frac{792b^5x^5+3465a\,b^4x^4+6160a^2b^3x^3+5544a^3b^2x^2+2520a^4bx+462a^5}{5544x^{12}}$	58
default	$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a\,b^4}{8x^8}-\frac{b^5}{7x^7}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

Maxima [A]
time = 0.25, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.85

$$-\frac{792\, b^5 x^5 + 3465\, a b^4 x^4 + 6160\, a^2 b^3 x^3 + 5544\, a^3 b^2 x^2 + 2520\, a^4 b x + 462\, a^5}{5544\, x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Sympy [A]
time = 0.26, size = 61, normalized size = 0.91

$$\frac{-462a^5 - 2520a^4bx - 5544a^3b^2x^2 - 6160a^2b^3x^3 - 3465ab^4x^4 - 792b^5x^5}{5544x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**13,x)
```

```
[Out] (-462*a**5 - 2520*a**4*b*x - 5544*a**3*b**2*x**2 - 6160*a**2*b**3*x**3 - 34
65*a*b**4*x**4 - 792*b**5*x**5)/(5544*x**12)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.97

$$\frac{-792x^5b^5 - 3465x^4b^4a - 6160x^3b^3a^2 - 5544x^2b^2a^3 - 2520xba^4 - 462a^5}{5544x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x)
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{12} + \frac{5\, a^4\, b\, x}{11} + a^3\, b^2\, x^2 + \frac{10\, a^2\, b^3\, x^3}{9} + \frac{5\, a\, b^4\, x^4}{8} + \frac{b^5\, x^5}{7}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^5)/7 + (5*a*b^4*x^4)/8 + a^3*b^2*x^2 + (10*a^2*b^3*x^3)/9
 + (5*a^4*b*x)/11)/x^12
```

3.97 $\int \frac{(a+bx)^5}{x^{14}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{13}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^9}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Mathics [A]
time = 2.16, size = 57, normalized size = 0.85

$$\frac{-792a^5 - 4290a^4bx - 9360a^3b^2x^2 - 10296a^2b^3x^3 - 5720ab^4x^4 - 1287b^5x^5}{10296x^{13}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^14,x]')
```

```
[Out] (-792 a ^ 5 - 4290 a ^ 4 b x - 9360 a ^ 3 b ^ 2 x ^ 2 - 10296 a ^ 2 b ^ 3 x
 ^ 3 - 5720 a b ^ 4 x ^ 4 - 1287 b ^ 5 x ^ 5) / (10296 x ^ 13)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{8}b^5x^5-\frac{5}{9}a\,b^4x^4-a^2b^3x^3-\frac{10}{11}a^3b^2x^2-\frac{5}{12}a^4bx-\frac{1}{13}a^5}{x^{13}}$	57
risch	$\frac{-\frac{1}{8}b^5x^5-\frac{5}{9}a\,b^4x^4-a^2b^3x^3-\frac{10}{11}a^3b^2x^2-\frac{5}{12}a^4bx-\frac{1}{13}a^5}{x^{13}}$	57
gosper	$-\frac{1287b^5x^5+5720a\,b^4x^4+10296a^2b^3x^3+9360a^3b^2x^2+4290a^4bx+792a^5}{10296x^{13}}$	58
default	$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a\,b^4}{9x^9}-\frac{b^5}{8x^8}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

Maxima [A]
time = 0.25, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5 + 5720\,ab^4x^4 + 10296\,a^2b^3x^3 + 9360\,a^3b^2x^2 + 4290\,a^4bx + 792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Sympy [A]
time = 0.26, size = 61, normalized size = 0.91

$$\frac{-792a^5-4290a^4bx-9360a^3b^2x^2-10296a^2b^3x^3-5720ab^4x^4-1287b^5x^5}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**14,x)
```

```
[Out] (-792*a**5 - 4290*a**4*b*x - 9360*a**3*b**2*x**2 - 10296*a**2*b**3*x**3 - 5
720*a*b**4*x**4 - 1287*b**5*x**5)/(10296*x**13)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.97

$$\frac{-1287x^5b^5-5720x^4b^4a-10296x^3b^3a^2-9360x^2b^2a^3-4290xba^4-792a^5}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x)
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{13}+\frac{5\,a^4\,b\,x}{12}+\frac{10\,a^3\,b^2\,x^2}{11}+a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{9}+\frac{b^5\,x^5}{8}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^5)/8 + (5*a*b^4*x^4)/9 + (10*a^3*b^2*x^2)/11 + a^2*b^3*x^
3 + (5*a^4*b*x)/12)/x^13
```

3.98 $\int x^8(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^7\,dx &= \int \left(a^7x^8+7a^6bx^9+21a^5b^2x^{10}+35a^4b^3x^{11}+35a^3b^4x^{12}+21a^2b^5x^{13}+7ab^6x^{14}+b^7x^{15}\right) \\ &= \frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Mathics [A]
time = 1.98, size = 79, normalized size = 0.83

$$\frac{x^9 (11440a^7 + 72072a^6bx + 196560a^5b^2x^2 + 300300a^4b^3x^3 + 277200a^3b^4x^4 + 154440a^2b^5x^5 + 48048ab^6x^6 + 6435b^7x^7)}{102960}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^8*(a + b*x)^7,x]')
```

```
[Out] x ^ 9 (11440 a ^ 7 + 72072 a ^ 6 b x + 196560 a ^ 5 b ^ 2 x ^ 2 + 300300 a
^ 4 b ^ 3 x ^ 3 + 277200 a ^ 3 b ^ 4 x ^ 4 + 154440 a ^ 2 b ^ 5 x ^ 5 + 480
48 a b ^ 6 x ^ 6 + 6435 b ^ 7 x ^ 7) / 102960
```

Maple [A]
time = 0.09, size = 80, normalized size = 0.84

method	result	size
gosper	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$	80
default	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$	80
norman	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$	80
risch	$\frac{1}{9}a^7x^9+\frac{7}{10}a^6b\,x^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}a\,b^6x^{15}+\frac{1}{16}b^7x^{16}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Maxima [A]
time = 0.26, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Fricas [A]
time = 0.31, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Sympy [A]
time = 0.04, size = 94, normalized size = 0.99

$$\frac{a^7x^9}{9}+\frac{7a^6bx^{10}}{10}+\frac{21a^5b^2x^{11}}{11}+\frac{35a^4b^3x^{12}}{12}+\frac{35a^3b^4x^{13}}{13}+\frac{3a^2b^5x^{14}}{2}+\frac{7ab^6x^{15}}{15}+\frac{b^7x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**7,x)
```

```
[Out] a**7*x**9/9 + 7*a**6*b*x**10/10 + 21*a**5*b**2*x**11/11 + 35*a**4*b**3*x**1
2/12 + 35*a**3*b**4*x**13/13 + 3*a**2*b**5*x**14/2 + 7*a*b**6*x**15/15 + b*
*7*x**16/16
```

Giac [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{1}{16}x^{16}b^7+\frac{7}{15}x^{15}b^6a+\frac{3}{2}x^{14}b^5a^2+\frac{35}{13}x^{13}b^4a^3+\frac{35}{12}x^{12}b^3a^4+\frac{21}{11}x^{11}b^2a^5+\frac{7}{10}x^{10}ba^6+\frac{1}{9}x^9a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x)
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Mupad [B]
time = 0.15, size = 79, normalized size = 0.83

$$\frac{a^7\,x^9}{9}+\frac{7\,a^6\,b\,x^{10}}{10}+\frac{21\,a^5\,b^2\,x^{11}}{11}+\frac{35\,a^4\,b^3\,x^{12}}{12}+\frac{35\,a^3\,b^4\,x^{13}}{13}+\frac{3\,a^2\,b^5\,x^{14}}{2}+\frac{7\,a\,b^6\,x^{15}}{15}+\frac{b^7\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^7,x)
```

```
[Out] (a^7*x^9)/9 + (b^7*x^16)/16 + (7*a^6*b*x^10)/10 + (7*a*b^6*x^15)/15 + (21*a
^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*
x^14)/2
```

3.99 $\int x^7(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^7\,dx &= \int\left(a^7x^7+7a^6bx^8+21a^5b^2x^9+35a^4b^3x^{10}+35a^3b^4x^{11}+21a^2b^5x^{12}+7ab^6x^{13}+b^7x^{14}\right)\\ &= \frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Mathics [A]
time = 2.00, size = 79, normalized size = 0.83

$$\frac{x^8\left(6435a^7+40040a^6bx+108108a^5b^2x^2+163800a^4b^3x^3+150150a^3b^4x^4+83160a^2b^5x^5+25740ab^6x^6+3432b^7x^7\right)}{51480}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^7*(a + b*x)^7,x]')
```

```
[Out] x ^ 8 (6435 a ^ 7 + 40040 a ^ 6 b x + 108108 a ^ 5 b ^ 2 x ^ 2 + 163800 a ^
 4 b ^ 3 x ^ 3 + 150150 a ^ 3 b ^ 4 x ^ 4 + 83160 a ^ 2 b ^ 5 x ^ 5 + 25740
 a b ^ 6 x ^ 6 + 3432 b ^ 7 x ^ 7) / 51480
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.84

method	result	size
gosper	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80
default	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80
norman	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80
risch	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Maxima [A]
time = 0.24, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Fricas [A]
time = 0.32, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Sympy [A]
time = 0.04, size = 92, normalized size = 0.97

$$\frac{a^7x^8}{8}+\frac{7a^6bx^9}{9}+\frac{21a^5b^2x^{10}}{10}+\frac{35a^4b^3x^{11}}{11}+\frac{35a^3b^4x^{12}}{12}+\frac{21a^2b^5x^{13}}{13}+\frac{ab^6x^{14}}{2}+\frac{b^7x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**7,x)
```

```
[Out] a**7*x**8/8 + 7*a**6*b*x**9/9 + 21*a**5*b**2*x**10/10 + 35*a**4*b**3*x**11/
11 + 35*a**3*b**4*x**12/12 + 21*a**2*b**5*x**13/13 + a*b**6*x**14/2 + b**7*
x**15/15
```

Giac [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{1}{15}x^{15}b^7+\frac{1}{2}x^{14}b^6a+\frac{21}{13}x^{13}b^5a^2+\frac{35}{12}x^{12}b^4a^3+\frac{35}{11}x^{11}b^3a^4+\frac{21}{10}x^{10}b^2a^5+\frac{7}{9}x^9ba^6+\frac{1}{8}x^8a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x)
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^8}{8}+\frac{7\,a^6\,b\,x^9}{9}+\frac{21\,a^5\,b^2\,x^{10}}{10}+\frac{35\,a^4\,b^3\,x^{11}}{11}+\frac{35\,a^3\,b^4\,x^{12}}{12}+\frac{21\,a^2\,b^5\,x^{13}}{13}+\frac{a\,b^6\,x^{14}}{2}+\frac{b^7\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^7,x)
```

```
[Out] (a^7*x^8)/8 + (b^7*x^15)/15 + (7*a^6*b*x^9)/9 + (a*b^6*x^14)/2 + (21*a^5*b^
2*x^10)/10 + (35*a^4*b^3*x^11)/11 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13
)/13
```

3.100 $\int x^6(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^7\,dx&=\int\left(a^7x^6+7a^6bx^7+21a^5b^2x^8+35a^4b^3x^9+35a^3b^4x^{10}+21a^2b^5x^{11}+7ab^6x^{12}+b^7x^{13}\right)dx\\&=\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Mathics [A]
time = 1.99, size = 79, normalized size = 0.83

$$\frac{x^7\left(3432a^7+21021a^6bx+56056a^5b^2x^2+84084a^4b^3x^3+76440a^3b^4x^4+42042a^2b^5x^5+12936ab^6x^6+1716b^7x^7\right)}{24024}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6*(a + b*x)^7,x]')
```

```
[Out] x ^ 7 (3432 a ^ 7 + 21021 a ^ 6 b x + 56056 a ^ 5 b ^ 2 x ^ 2 + 84084 a ^ 4
 b ^ 3 x ^ 3 + 76440 a ^ 3 b ^ 4 x ^ 4 + 42042 a ^ 2 b ^ 5 x ^ 5 + 12936 a
b ^ 6 x ^ 6 + 1716 b ^ 7 x ^ 7) / 24024
```

Maple [A]
time = 0.09, size = 80, normalized size = 0.84

method	result	size
gosper	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
default	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
norman	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
risch	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Maxima [A]
time = 0.24, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Fricas [A]
time = 0.29, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Sympy [A]
time = 0.04, size = 94, normalized size = 0.99

$$\frac{a^7x^7}{7}+\frac{7a^6bx^8}{8}+\frac{7a^5b^2x^9}{3}+\frac{7a^4b^3x^{10}}{2}+\frac{35a^3b^4x^{11}}{11}+\frac{7a^2b^5x^{12}}{4}+\frac{7ab^6x^{13}}{13}+\frac{b^7x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**7,x)
```

```
[Out] a**7*x**7/7 + 7*a**6*b*x**8/8 + 7*a**5*b**2*x**9/3 + 7*a**4*b**3*x**10/2 +
35*a**3*b**4*x**11/11 + 7*a**2*b**5*x**12/4 + 7*a*b**6*x**13/13 + b**7*x**1
4/14
```

Giac [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{1}{14}x^{14}b^7+\frac{7}{13}x^{13}b^6a+\frac{7}{4}x^{12}b^5a^2+\frac{35}{11}x^{11}b^4a^3+\frac{7}{2}x^{10}b^3a^4+\frac{7}{3}x^9b^2a^5+\frac{7}{8}x^8ba^6+\frac{1}{7}x^7a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x)
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^7}{7}+\frac{7\,a^6\,b\,x^8}{8}+\frac{7\,a^5\,b^2\,x^9}{3}+\frac{7\,a^4\,b^3\,x^{10}}{2}+\frac{35\,a^3\,b^4\,x^{11}}{11}+\frac{7\,a^2\,b^5\,x^{12}}{4}+\frac{7\,a\,b^6\,x^{13}}{13}+\frac{b^7\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^7,x)
```

```
[Out] (a^7*x^7)/7 + (b^7*x^14)/14 + (7*a^6*b*x^8)/8 + (7*a*b^6*x^13)/13 + (7*a^5*
b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4
```

3.101 $\int x^5(a+bx)^7\,dx$

Optimal. Leaf size=96

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+bx)^{13}}{13b^6}$$

```
[Out] -1/8*a^5*(b*x+a)^8/b^6+5/9*a^4*(b*x+a)^9/b^6-a^3*(b*x+a)^10/b^6+10/11*a^2*(
b*x+a)^11/b^6-5/12*a*(b*x+a)^12/b^6+1/13*(b*x+a)^13/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^7,x]
```

```
[Out] -1/8*(a^5*(a + b*x)^8)/b^6 + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^1
0)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (a
+ b*x)^13/(13*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^7\,dx &= \int \left(-\frac{a^5(a+bx)^7}{b^5}+\frac{5a^4(a+bx)^8}{b^5}-\frac{10a^3(a+bx)^9}{b^5}+\frac{10a^2(a+bx)^{10}}{b^5}-\frac{5a(a+bx)^{11}}{b^5}\right.\\ &= -\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 92, normalized size = 0.96

$$\frac{a^7x^6}{6}+a^6bx^7+\frac{21}{8}a^5b^2x^8+\frac{35}{9}a^4b^3x^9+\frac{7}{2}a^3b^4x^{10}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{12}ab^6x^{12}+\frac{b^7x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^7,x]
```

```
[Out] (a^7*x^6)/6 + a^6*b*x^7 + (21*a^5*b^2*x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*
b^4*x^10)/2 + (21*a^2*b^5*x^11)/11 + (7*a*b^6*x^12)/12 + (b^7*x^13)/13
```

Mathics [A]
time = 2.11, size = 79, normalized size = 0.82

$$\frac{x^6 \left(1716a^7 + 10296a^6bx + 27027a^5b^2x^2 + 40040a^4b^3x^3 + 36036a^3b^4x^4 + 19656a^2b^5x^5 + 6006ab^6x^6 + 792b^7x^7\right)}{10296}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5*(a + b*x)^7,x]')
```

```
[Out] x ^ 6 (1716 a ^ 7 + 10296 a ^ 6 b x + 27027 a ^ 5 b ^ 2 x ^ 2 + 40040 a ^ 4
 b ^ 3 x ^ 3 + 36036 a ^ 3 b ^ 4 x ^ 4 + 19656 a ^ 2 b ^ 5 x ^ 5 + 6006 a b
 ^ 6 x ^ 6 + 792 b ^ 7 x ^ 7) / 10296
```

Maple [A]
time = 0.07, size = 79, normalized size = 0.82

method	result	size
gosper	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
default	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
norman	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
risch	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^7*x^13+7/12*a*b^6*x^12+21/11*a^2*b^5*x^11+7/2*a^3*b^4*x^10+35/9*a^4*
b^3*x^9+21/8*a^5*b^2*x^8+a^6*b*x^7+1/6*a^7*x^6
```

Maxima [A]
time = 0.24, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^7x^{13}+\frac{7}{12}\,ab^6x^{12}+\frac{21}{11}\,a^2b^5x^{11}+\frac{7}{2}\,a^3b^4x^{10}+\frac{35}{9}\,a^4b^3x^9+\frac{21}{8}\,a^5b^2x^8+a^6bx^7+\frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Fricas [A]
time = 0.30, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^{7}x^{13}+\frac{7}{12}\,ab^{6}x^{12}+\frac{21}{11}\,a^{2}b^{5}x^{11}+\frac{7}{2}\,a^{3}b^{4}x^{10}+\frac{35}{9}\,a^{4}b^{3}x^{9}+\frac{21}{8}\,a^{5}b^{2}x^{8}+a^{6}bx^{7}+\frac{1}{6}\,a^{7}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Sympy [A]
time = 0.04, size = 90, normalized size = 0.94

$$\frac{a^{7}x^{6}}{6}+a^{6}bx^{7}+\frac{21a^{5}b^{2}x^{8}}{8}+\frac{35a^{4}b^{3}x^{9}}{9}+\frac{7a^{3}b^{4}x^{10}}{2}+\frac{21a^{2}b^{5}x^{11}}{11}+\frac{7ab^{6}x^{12}}{12}+\frac{b^{7}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**7,x)
```

```
[Out] a**7*x**6/6 + a**6*b*x**7 + 21*a**5*b**2*x**8/8 + 35*a**4*b**3*x**9/9 + 7*a
**3*b**4*x**10/2 + 21*a**2*b**5*x**11/11 + 7*a*b**6*x**12/12 + b**7*x**13/1
3
```

Giac [A]
time = 0.00, size = 92, normalized size = 0.96

$$\frac{1}{13}x^{13}b^{7}+\frac{7}{12}x^{12}b^{6}a+\frac{21}{11}x^{11}b^{5}a^{2}+\frac{7}{2}x^{10}b^{4}a^{3}+\frac{35}{9}x^{9}b^{3}a^{4}+\frac{21}{8}x^{8}b^{2}a^{5}+x^{7}ba^{6}+\frac{1}{6}x^{6}a^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x)
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Mupad [B]
time = 0.06, size = 78, normalized size = 0.81

$$\frac{a^7\,x^6}{6}+a^6\,b\,x^7+\frac{21\,a^5\,b^2\,x^8}{8}+\frac{35\,a^4\,b^3\,x^9}{9}+\frac{7\,a^3\,b^4\,x^{10}}{2}+\frac{21\,a^2\,b^5\,x^{11}}{11}+\frac{7\,a\,b^6\,x^{12}}{12}+\frac{b^7\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^7,x)
```

```
[Out] (a^7*x^6)/6 + (b^7*x^13)/13 + a^6*b*x^7 + (7*a*b^6*x^12)/12 + (21*a^5*b^2*x
^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*b^4*x^10)/2 + (21*a^2*b^5*x^11)/11
```

3.102 $\int x^4(a+bx)^7\,dx$

Optimal. Leaf size=81

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}$$

```
[Out] 1/8*a^4*(b*x+a)^8/b^5-4/9*a^3*(b*x+a)^9/b^5+3/5*a^2*(b*x+a)^10/b^5-4/11*a*(
b*x+a)^11/b^5+1/12*(b*x+a)^12/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^7,x]
```

```
[Out] (a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)^
10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^7\,dx&=\int\left(\frac{a^4(a+bx)^7}{b^4}-\frac{4a^3(a+bx)^8}{b^4}+\frac{6a^2(a+bx)^9}{b^4}-\frac{4a(a+bx)^{10}}{b^4}+\frac{(a+bx)^{11}}{b^4}\right)dx\\&=\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.15

$$\frac{a^7x^5}{5}+\frac{7}{6}a^6bx^6+3a^5b^2x^7+\frac{35}{8}a^4b^3x^8+\frac{35}{9}a^3b^4x^9+\frac{21}{10}a^2b^5x^{10}+\frac{7}{11}ab^6x^{11}+\frac{b^7x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^7,x]
```

```
[Out] (a^7*x^5)/5 + (7*a^6*b*x^6)/6 + 3*a^5*b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^
3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10 + (7*a*b^6*x^11)/11 + (b^7*x^12)/12
```

Mathics [A]
time = 2.04, size = 79, normalized size = 0.98

$$\frac{x^5\left(792a^7+4620a^6bx+11880a^5b^2x^2+17325a^4b^3x^3+15400a^3b^4x^4+8316a^2b^5x^5+2520ab^6x^6+330b^7x^7\right)}{3960}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4*(a + b*x)^7,x]')
```

```
[Out] x ^ 5 (792 a ^ 7 + 4620 a ^ 6 b x + 11880 a ^ 5 b ^ 2 x ^ 2 + 17325 a ^ 4 b
 ^ 3 x ^ 3 + 15400 a ^ 3 b ^ 4 x ^ 4 + 8316 a ^ 2 b ^ 5 x ^ 5 + 2520 a b ^
6 x ^ 6 + 330 b ^ 7 x ^ 7) / 3960
```

Maple [A]
time = 0.07, size = 80, normalized size = 0.99

method	result	size
gosper	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
default	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
norman	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
risch	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^7*x^12+7/11*a*b^6*x^11+21/10*a^2*b^5*x^10+35/9*a^3*b^4*x^9+35/8*a^4*
b^3*x^8+3*a^5*b^2*x^7+7/6*a^6*b*x^6+1/5*a^7*x^5
```

Maxima [A]
time = 0.25, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Fricas [A]
time = 0.30, size = 79, normalized size = 0.98

$$\frac{1}{12}b^7x^{12} + \frac{7}{11}ab^6x^{11} + \frac{21}{10}a^2b^5x^{10} + \frac{35}{9}a^3b^4x^9 + \frac{35}{8}a^4b^3x^8 + 3a^5b^2x^7 + \frac{7}{6}a^6bx^6 + \frac{1}{5}a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Sympy [A]
time = 0.04, size = 92, normalized size = 1.14

$$\frac{a^7x^5}{5} + \frac{7a^6bx^6}{6} + 3a^5b^2x^7 + \frac{35a^4b^3x^8}{8} + \frac{35a^3b^4x^9}{9} + \frac{21a^2b^5x^{10}}{10} + \frac{7ab^6x^{11}}{11} + \frac{b^7x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**7,x)
```

```
[Out] a**7*x**5/5 + 7*a**6*b*x**6/6 + 3*a**5*b**2*x**7 + 35*a**4*b**3*x**8/8 + 35
*a**3*b**4*x**9/9 + 21*a**2*b**5*x**10/10 + 7*a*b**6*x**11/11 + b**7*x**12/
12
```

Giac [A]
time = 0.00, size = 93, normalized size = 1.15

$$\frac{1}{12}x^{12}b^7 + \frac{7}{11}x^{11}b^6a + \frac{21}{10}x^{10}b^5a^2 + \frac{35}{9}x^9b^4a^3 + \frac{35}{8}x^8b^3a^4 + 3x^7b^2a^5 + \frac{7}{6}x^6ba^6 + \frac{1}{5}x^5a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x)
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Mupad [B]
time = 0.06, size = 79, normalized size = 0.98

$$\frac{a^7\,x^5}{5} + \frac{7\,a^6\,b\,x^6}{6} + 3\,a^5\,b^2\,x^7 + \frac{35\,a^4\,b^3\,x^8}{8} + \frac{35\,a^3\,b^4\,x^9}{9} + \frac{21\,a^2\,b^5\,x^{10}}{10} + \frac{7\,a\,b^6\,x^{11}}{11} + \frac{b^7\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^7,x)
```

```
[Out] (a^7*x^5)/5 + (b^7*x^12)/12 + (7*a^6*b*x^6)/6 + (7*a*b^6*x^11)/11 + 3*a^5*b
^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10
```

3.103 $\int x^3(a+bx)^7\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}$$

```
[Out] -1/8*a^3*(b*x+a)^8/b^4+1/3*a^2*(b*x+a)^9/b^4-3/10*a*(b*x+a)^10/b^4+1/11*(b*
x+a)^11/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^7,x]
```

```
[Out] -1/8*(a^3*(a + b*x)^8)/b^4 + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10)
/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^7\,dx &= \int \left(-\frac{a^3(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^8}{b^3}-\frac{3a(a+bx)^9}{b^3}+\frac{(a+bx)^{10}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.45

$$\frac{a^7x^4}{4}+\frac{7}{5}a^6bx^5+\frac{7}{2}a^5b^2x^6+5a^4b^3x^7+\frac{35}{8}a^3b^4x^8+\frac{7}{3}a^2b^5x^9+\frac{7}{10}ab^6x^{10}+\frac{b^7x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x)^7,x]

[Out] (a^7*x^4)/4 + (7*a^6*b*x^5)/5 + (7*a^5*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3 *b^4*x^8)/8 + (7*a^2*b^5*x^9)/3 + (7*a*b^6*x^10)/10 + (b^7*x^11)/11

Mathics [A]
time = 2.06, size = 79, normalized size = 1.23

$$\frac{x^4\left(330a^7+1848a^6bx+4620a^5b^2x^2+6600a^4b^3x^3+5775a^3b^4x^4+3080a^2b^5x^5+924ab^6x^6+120b^7x^7\right)}{1320}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^3*(a + b*x)^7,x]')

[Out] x ^ 4 (330 a ^ 7 + 1848 a ^ 6 b x + 4620 a ^ 5 b ^ 2 x ^ 2 + 6600 a ^ 4 b ^ 3 x ^ 3 + 5775 a ^ 3 b ^ 4 x ^ 4 + 3080 a ^ 2 b ^ 5 x ^ 5 + 924 a b ^ 6 x ^ 6 + 120 b ^ 7 x ^ 7) / 1320

Maple [A]
time = 0.09, size = 80, normalized size = 1.25

method	result	size
gosper	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
default	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
norman	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
risch	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x+a)^7,x,method=_RETURNVERBOSE)

[Out] 1/11*b^7*x^11+7/10*a*b^6*x^10+7/3*a^2*b^5*x^9+35/8*a^3*b^4*x^8+5*a^4*b^3*x^7+7/2*a^5*b^2*x^6+7/5*a^6*b*x^5+1/4*a^7*x^4

Maxima [A]
time = 0.25, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x+a)^7,x, algorithm="maxima")

[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4

Fricas [A]
time = 0.32, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Sympy [A]
time = 0.04, size = 92, normalized size = 1.44

$$\frac{a^7x^4}{4}+\frac{7a^6bx^5}{5}+\frac{7a^5b^2x^6}{2}+5a^4b^3x^7+\frac{35a^3b^4x^8}{8}+\frac{7a^2b^5x^9}{3}+\frac{7ab^6x^{10}}{10}+\frac{b^7x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**7,x)
```

```
[Out] a**7*x**4/4 + 7*a**6*b*x**5/5 + 7*a**5*b**2*x**6/2 + 5*a**4*b**3*x**7 + 35*
a**3*b**4*x**8/8 + 7*a**2*b**5*x**9/3 + 7*a*b**6*x**10/10 + b**7*x**11/11
```

Giac [A]
time = 0.00, size = 93, normalized size = 1.45

$$\frac{1}{11}x^{11}b^7+\frac{7}{10}x^{10}b^6a+\frac{7}{3}x^9b^5a^2+\frac{35}{8}x^8b^4a^3+5x^7b^3a^4+\frac{7}{2}x^6b^2a^5+\frac{7}{5}x^5ba^6+\frac{1}{4}x^4a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x)
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Mupad [B]
time = 0.10, size = 79, normalized size = 1.23

$$\frac{a^7\,x^4}{4}+\frac{7\,a^6\,b\,x^5}{5}+\frac{7\,a^5\,b^2\,x^6}{2}+5\,a^4\,b^3\,x^7+\frac{35\,a^3\,b^4\,x^8}{8}+\frac{7\,a^2\,b^5\,x^9}{3}+\frac{7\,a\,b^6\,x^{10}}{10}+\frac{b^7\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^7,x)
```

```
[Out] (a^7*x^4)/4 + (b^7*x^11)/11 + (7*a^6*b*x^5)/5 + (7*a*b^6*x^10)/10 + (7*a^5*
b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3
```

3.104 $\int x^2(a+bx)^7\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}$$

```
[Out] 1/8*a^2*(b*x+a)^8/b^3-2/9*a*(b*x+a)^9/b^3+1/10*(b*x+a)^10/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^7,x]
```

```
[Out] (a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b^
3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^7\,dx&=\int\left(\frac{a^2(a+bx)^7}{b^2}-\frac{2a(a+bx)^8}{b^2}+\frac{(a+bx)^9}{b^2}\right)dx\\&=\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.98

$$\frac{a^7x^3}{3}+\frac{7}{4}a^6bx^4+\frac{21}{5}a^5b^2x^5+\frac{35}{6}a^4b^3x^6+5a^3b^4x^7+\frac{21}{8}a^2b^5x^8+\frac{7}{9}ab^6x^9+\frac{b^7x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^7,x]
```

```
[Out] (a^7*x^3)/3 + (7*a^6*b*x^4)/4 + (21*a^5*b^2*x^5)/5 + (35*a^4*b^3*x^6)/6 + 5
*a^3*b^4*x^7 + (21*a^2*b^5*x^8)/8 + (7*a*b^6*x^9)/9 + (b^7*x^10)/10
```

Mathics [A]
time = 2.00, size = 79, normalized size = 1.68

$$\frac{x^3 (120a^7 + 630a^6bx + 1512a^5b^2x^2 + 2100a^4b^3x^3 + 1800a^3b^4x^4 + 945a^2b^5x^5 + 280ab^6x^6 + 36b^7x^7)}{360}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x)^7,x]')
```

```
[Out] x ^ 3 (120 a ^ 7 + 630 a ^ 6 b x + 1512 a ^ 5 b ^ 2 x ^ 2 + 2100 a ^ 4 b ^
3 x ^ 3 + 1800 a ^ 3 b ^ 4 x ^ 4 + 945 a ^ 2 b ^ 5 x ^ 5 + 280 a b ^ 6 x ^
6 + 36 b ^ 7 x ^ 7) / 360
```

Maple [A]
time = 0.08, size = 80, normalized size = 1.70

method	result	size
gosper	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
default	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
norman	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80
risch	$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^7*x^10+7/9*a*b^6*x^9+21/8*a^2*b^5*x^8+5*a^3*b^4*x^7+35/6*a^4*b^3*x^6
+21/5*a^5*b^2*x^5+7/4*a^6*b*x^4+1/3*a^7*x^3
```

Maxima [A]
time = 0.24, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^7x^{10} + \frac{7}{9}\,ab^6x^9 + \frac{21}{8}\,a^2b^5x^8 + 5\,a^3b^4x^7 + \frac{35}{6}\,a^4b^3x^6 + \frac{21}{5}\,a^5b^2x^5 + \frac{7}{4}\,a^6bx^4 + \frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Fricas [A]
time = 0.30, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^{7}x^{10}+\frac{7}{9}\,ab^{6}x^{9}+\frac{21}{8}\,a^{2}b^{5}x^{8}+5\,a^{3}b^{4}x^{7}+\frac{35}{6}\,a^{4}b^{3}x^{6}+\frac{21}{5}\,a^{5}b^{2}x^{5}+\frac{7}{4}\,a^{6}bx^{4}+\frac{1}{3}\,a^{7}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 0.04, size = 92, normalized size = 1.96

$$\frac{a^{7}x^{3}}{3}+\frac{7a^{6}bx^{4}}{4}+\frac{21a^{5}b^{2}x^{5}}{5}+\frac{35a^{4}b^{3}x^{6}}{6}+5a^{3}b^{4}x^{7}+\frac{21a^{2}b^{5}x^{8}}{8}+\frac{7ab^{6}x^{9}}{9}+\frac{b^{7}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**7,x)
```

```
[Out] a**7*x**3/3 + 7*a**6*b*x**4/4 + 21*a**5*b**2*x**5/5 + 35*a**4*b**3*x**6/6 +
 5*a**3*b**4*x**7 + 21*a**2*b**5*x**8/8 + 7*a*b**6*x**9/9 + b**7*x**10/10
```

Giac [A]
time = 0.00, size = 93, normalized size = 1.98

$$\frac{1}{10}x^{10}b^{7}+\frac{7}{9}x^{9}b^{6}a+\frac{21}{8}x^{8}b^{5}a^{2}+5x^{7}b^{4}a^{3}+\frac{35}{6}x^{6}b^{3}a^{4}+\frac{21}{5}x^{5}b^{2}a^{5}+\frac{7}{4}x^{4}ba^{6}+\frac{1}{3}x^{3}a^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x)
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Mupad [B]
time = 0.12, size = 31, normalized size = 0.66

$$\frac{{\left(a+b\,x\right)}^8\,\left(8\,a^2-64\,a\,b\,x+288\,b^2\,x^2\right)}{2880\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^8*(8*a^2 + 288*b^2*x^2 - 64*a*b*x))/(2880*b^3)
```

3.105 $\int x(a+bx)^7\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}$$

```
[Out] -1/8*a*(b*x+a)^8/b^2+1/9*(b*x+a)^9/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^7,x]
```

```
[Out] -1/8*(a*(a + b*x)^8)/b^2 + (a + b*x)^9/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^7\,dx &= \int \left(-\frac{a(a+bx)^7}{b}+\frac{(a+bx)^8}{b}\right)dx\\ &= -\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(30) = 60.
time = 0.00, size = 91, normalized size = 3.03

$$\frac{a^7x^2}{2}+\frac{7}{3}a^6bx^3+\frac{21}{4}a^5b^2x^4+7a^4b^3x^5+\frac{35}{6}a^3b^4x^6+3a^2b^5x^7+\frac{7}{8}ab^6x^8+\frac{b^7x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^7,x]
```

```
[Out] (a^7*x^2)/2 + (7*a^6*b*x^3)/3 + (21*a^5*b^2*x^4)/4 + 7*a^4*b^3*x^5 + (35*a^
3*b^4*x^6)/6 + 3*a^2*b^5*x^7 + (7*a*b^6*x^8)/8 + (b^7*x^9)/9
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(30) = 60$.
time = 1.97, size = 79, normalized size = 2.63

$$\frac{x^2 (36a^7 + 168a^6bx + 378a^5b^2x^2 + 504a^4b^3x^3 + 420a^3b^4x^4 + 216a^2b^5x^5 + 63ab^6x^6 + 8b^7x^7)}{72}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x)^7,x]')
```

```
[Out] x ^ 2 (36 a ^ 7 + 168 a ^ 6 b x + 378 a ^ 5 b ^ 2 x ^ 2 + 504 a ^ 4 b ^ 3 x
 ^ 3 + 420 a ^ 3 b ^ 4 x ^ 4 + 216 a ^ 2 b ^ 5 x ^ 5 + 63 a b ^ 6 x ^ 6 + 8
 b ^ 7 x ^ 7) / 72
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.
time = 0.10, size = 80, normalized size = 2.67

method	result	size
gosper	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80
default	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80
norman	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80
risch	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^7*x^9+7/8*a*b^6*x^8+3*a^2*b^5*x^7+35/6*a^3*b^4*x^6+7*a^4*b^3*x^5+21/4
*a^5*b^2*x^4+7/3*a^6*b*x^3+1/2*a^7*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.
time = 0.25, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(26) = 52.
time = 0.61, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9+\frac{7}{8}\,ab^6x^8+3\,a^2b^5x^7+\frac{35}{6}\,a^3b^4x^6+7\,a^4b^3x^5+\frac{21}{4}\,a^5b^2x^4+\frac{7}{3}\,a^6bx^3+\frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(24) = 48.
time = 0.04, size = 90, normalized size = 3.00

$$\frac{a^7x^2}{2}+\frac{7a^6bx^3}{3}+\frac{21a^5b^2x^4}{4}+7a^4b^3x^5+\frac{35a^3b^4x^6}{6}+3a^2b^5x^7+\frac{7ab^6x^8}{8}+\frac{b^7x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**7,x)
```

```
[Out] a**7*x**2/2 + 7*a**6*b*x**3/3 + 21*a**5*b**2*x**4/4 + 7*a**4*b**3*x**5 + 35
*a**3*b**4*x**6/6 + 3*a**2*b**5*x**7 + 7*a*b**6*x**8/8 + b**7*x**9/9
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(26) = 52.
time = 0.00, size = 91, normalized size = 3.03

$$\frac{1}{9}x^9b^7+\frac{7}{8}x^8b^6a+3x^7b^5a^2+\frac{35}{6}x^6b^4a^3+7x^5b^3a^4+\frac{21}{4}x^4b^2a^5+\frac{7}{3}x^3ba^6+\frac{1}{2}x^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x)
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Mupad [B]
time = 0.12, size = 25, normalized size = 0.83

$$-\frac{2\,\left(\frac{a\,{\left(a+b\,x\right)}^8}{16}-\frac{{\left(a+b\,x\right)}^9}{18}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^7,x)
```

```
[Out] -(2*((a*(a + b*x)^8)/16 - (a + b*x)^9/18))/b^2
```

3.106 $\int (a+bx)^7 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^8}{8b}$$

```
[Out] 1/8*(b*x+a)^8/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^7 \, dx = \frac{(a+bx)^8}{8b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(14) = 28.
time = 1.97, size = 76, normalized size = 5.43

$$\frac{x\left(8a^7+28a^6bx+56a^5b^2x^2+70a^4b^3x^3+56a^3b^4x^4+28a^2b^5x^5+8ab^6x^6+b^7x^7\right)}{8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^7,x]')
```

```
[Out] x (8 a ^ 7 + 28 a ^ 6 b x + 56 a ^ 5 b ^ 2 x ^ 2 + 70 a ^ 4 b ^ 3 x ^ 3 + 5
6 a ^ 3 b ^ 4 x ^ 4 + 28 a ^ 2 b ^ 5 x ^ 5 + 8 a b ^ 6 x ^ 6 + b ^ 7 x ^ 7)
 / 8
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^8}{8b}$	13
gosper	$\frac{1}{8}b^7x^8 + a\,b^6x^7 + \frac{7}{2}a^2b^5x^6 + 7a^3b^4x^5 + \frac{35}{4}a^4b^3x^4 + 7a^5b^2x^3 + \frac{7}{2}a^6b\,x^2 + a^7x$	76
norman	$\frac{1}{8}b^7x^8 + a\,b^6x^7 + \frac{7}{2}a^2b^5x^6 + 7a^3b^4x^5 + \frac{35}{4}a^4b^3x^4 + 7a^5b^2x^3 + \frac{7}{2}a^6b\,x^2 + a^7x$	76
risch	$\frac{b^7x^8}{8} + a\,b^6x^7 + \frac{7a^2b^5x^6}{2} + 7a^3b^4x^5 + \frac{35a^4b^3x^4}{4} + 7a^5b^2x^3 + \frac{7a^6b\,x^2}{2} + a^7x + \frac{a^8}{8b}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(b*x+a)^8/b
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x + a)^8/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(12) = 24$.
time = 0.30, size = 75, normalized size = 5.36

$$\frac{1}{8}\,b^7x^8 + ab^6x^7 + \frac{7}{2}\,a^2b^5x^6 + 7\,a^3b^4x^5 + \frac{35}{4}\,a^4b^3x^4 + 7\,a^5b^2x^3 + \frac{7}{2}\,a^6bx^2 + a^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*b^7*x^8 + a*b^6*x^7 + 7/2*a^2*b^5*x^6 + 7*a^3*b^4*x^5 + 35/4*a^4*b^3*x^
4 + 7*a^5*b^2*x^3 + 7/2*a^6*b*x^2 + a^7*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.
time = 0.04, size = 83, normalized size = 5.93

$$a^7x + \frac{7a^6bx^2}{2} + 7a^5b^2x^3 + \frac{35a^4b^3x^4}{4} + 7a^3b^4x^5 + \frac{7a^2b^5x^6}{2} + ab^6x^7 + \frac{b^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7,x)
```

```
[Out] a**7*x + 7*a**6*b*x**2/2 + 7*a**5*b**2*x**3 + 35*a**4*b**3*x**4/4 + 7*a**3*
b**4*x**5 + 7*a**2*b**5*x**6/2 + a*b**6*x**7 + b**7*x**8/8
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(bx + a)^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x)
```

```
[Out] 1/8*(b*x + a)^8/b
```

Mupad [B]
time = 0.06, size = 75, normalized size = 5.36

$$a^7\,x + \frac{7\,a^6\,b\,x^2}{2} + 7\,a^5\,b^2\,x^3 + \frac{35\,a^4\,b^3\,x^4}{4} + 7\,a^3\,b^4\,x^5 + \frac{7\,a^2\,b^5\,x^6}{2} + a\,b^6\,x^7 + \frac{b^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7,x)
```

```
[Out] a^7*x + (b^7*x^8)/8 + (7*a^6*b*x^2)/2 + a*b^6*x^7 + 7*a^5*b^2*x^3 + (35*a^4
*b^3*x^4)/4 + 7*a^3*b^4*x^5 + (7*a^2*b^5*x^6)/2
```

3.107 $\int \frac{(a+bx)^7}{x}\,dx$

Optimal. Leaf size=87

$$7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)$$

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$a^7\log(x)+7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x}\,dx&=\int \left(7a^6b+\frac{a^7}{x}+21a^5b^2x+35a^4b^3x^2+35a^3b^4x^3+21a^2b^5x^4+7ab^6x^5+b^7x^6\right)dx\\&=7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 87, normalized size = 1.00

$$7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Mathics [A]
time = 1.94, size = 75, normalized size = 0.86

$$a^7\text{Log}\,[x] + 7a^6bx + \frac{21a^5b^2x^2}{2} + \frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4} + \frac{21a^2b^5x^5}{5} + \frac{7ab^6x^6}{6} + \frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^1,x]')
```

```
[Out] a ^ 7 Log[x] + 7 a ^ 6 b x + 21 a ^ 5 b ^ 2 x ^ 2 / 2 + 35 a ^ 4 b ^ 3 x ^
3 / 3 + 35 a ^ 3 b ^ 4 x ^ 4 / 4 + 21 a ^ 2 b ^ 5 x ^ 5 / 5 + 7 a b ^ 6 x ^
 6 / 6 + b ^ 7 x ^ 7 / 7
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.87

method	result	size
default	$7a^6bx + \frac{21a^5b^2x^2}{2} + \frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4} + \frac{21a^2b^5x^5}{5} + \frac{7a\,b^6x^6}{6} + \frac{b^7x^7}{7} + a^7\ln\left(x\right)$	76
norman	$7a^6bx + \frac{21a^5b^2x^2}{2} + \frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4} + \frac{21a^2b^5x^5}{5} + \frac{7a\,b^6x^6}{6} + \frac{b^7x^7}{7} + a^7\ln\left(x\right)$	76
risch	$7a^6bx + \frac{21a^5b^2x^2}{2} + \frac{35a^4b^3x^3}{3} + \frac{35a^3b^4x^4}{4} + \frac{21a^2b^5x^5}{5} + \frac{7a\,b^6x^6}{6} + \frac{b^7x^7}{7} + a^7\ln\left(x\right)$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x,x,method=_RETURNVERBOSE)
```

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Maxima [A]
time = 0.26, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^7x^7 + \frac{7}{6}\,ab^6x^6 + \frac{21}{5}\,a^2b^5x^5 + \frac{35}{4}\,a^3b^4x^4 + \frac{35}{3}\,a^4b^3x^3 + \frac{21}{2}\,a^5b^2x^2 + 7\,a^6bx + a^7\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="maxima")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Fricas [A]
time = 0.31, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="fricas")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Sympy [A]
time = 0.07, size = 88, normalized size = 1.01

$$a^{7}\log\left(x\right)+7a^{6}bx+\frac{21a^{5}b^{2}x^{2}}{2}+\frac{35a^{4}b^{3}x^{3}}{3}+\frac{35a^{3}b^{4}x^{4}}{4}+\frac{21a^{2}b^{5}x^{5}}{5}+\frac{7ab^{6}x^{6}}{6}+\frac{b^{7}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x,x)
```

```
[Out] a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**3/3 + 35*a
**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x**7/7
```

Giac [A]
time = 0.00, size = 88, normalized size = 1.01

$$\frac{1}{7}x^{7}b^{7}+\frac{7}{6}x^{6}b^{6}a+\frac{21}{5}x^{5}b^{5}a^{2}+\frac{35}{4}x^{4}b^{4}a^{3}+\frac{35}{3}x^{3}b^{3}a^{4}+\frac{21}{2}x^{2}b^{2}a^{5}+7xba^{6}+a^{7}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x)
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(abs(x))
```

Mupad [B]
time = 0.07, size = 75, normalized size = 0.86

$$a^{7}\,\ln\left(x\right)+\frac{b^{7}\,x^{7}}{7}+\frac{7\,a\,b^{6}\,x^{6}}{6}+\frac{21\,a^{5}\,b^{2}\,x^{2}}{2}+\frac{35\,a^{4}\,b^{3}\,x^{3}}{3}+\frac{35\,a^{3}\,b^{4}\,x^{4}}{4}+\frac{21\,a^{2}\,b^{5}\,x^{5}}{5}+7\,a^{6}\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x,x)
```

```
[Out] a^7*log(x) + (b^7*x^7)/7 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/2 + (35*a^4*b
^3*x^3)/3 + (35*a^3*b^4*x^4)/4 + (21*a^2*b^5*x^5)/5 + 7*a^6*b*x
```

3.108 $\int \frac{(a+bx)^7}{x^2} dx$

Optimal. Leaf size=86

$$-\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)$$

```
[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*
a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^7}{x}+7a^6b\log(x)+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^2}\,dx &= \int \left(21a^5b^2+\frac{a^7}{x^2}+\frac{7a^6b}{x}+35a^4b^3x+35a^3b^4x^2+21a^2b^5x^3+7ab^6x^4+b^7x^5\right)dx\\ &= -\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Mathics [A]
time = 2.00, size = 79, normalized size = 0.92

$$\frac{-a^7 + \frac{bx\left(420a^6\mathrm{Log}[x]+1260a^5bx+1050a^4b^2x^2+700a^3b^3x^3+315a^2b^4x^4+84ab^5x^5+10b^6x^6\right)}{60}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^2,x]')
```

```
[Out] (-a ^ 7 + b x (420 a ^ 6 Log[x] + 1260 a ^ 5 b x + 1050 a ^ 4 b ^ 2 x ^ 2 +
 700 a ^ 3 b ^ 3 x ^ 3 + 315 a ^ 2 b ^ 4 x ^ 4 + 84 a b ^ 5 x ^ 5 + 10 b ^
6 x ^ 6) / 60) / x
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.90

method	result	size
default	$-\frac{a^7}{x}+21a^5b^2x+\frac{35a^4b^3x^2}{2}+\frac{35a^3b^4x^3}{3}+\frac{21a^2b^5x^4}{4}+\frac{7a\,b^6x^5}{5}+\frac{b^7x^6}{6}+7a^6b\ln\left(x\right)$	77
risch	$-\frac{a^7}{x}+21a^5b^2x+\frac{35a^4b^3x^2}{2}+\frac{35a^3b^4x^3}{3}+\frac{21a^2b^5x^4}{4}+\frac{7a\,b^6x^5}{5}+\frac{b^7x^6}{6}+7a^6b\ln\left(x\right)$	77
norman	$\frac{-a^7+\frac{1}{6}b^7x^7+\frac{7}{5}a\,b^6x^6+\frac{21}{4}a^2b^5x^5+\frac{35}{3}a^3b^4x^4+\frac{35}{2}a^4b^3x^3+21a^5b^2x^2}{x}+7a^6b\ln\left(x\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*
a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Maxima [A]
time = 0.25, size = 76, normalized size = 0.88

$$\frac{1}{6}\,b^7x^6+\frac{7}{5}\,ab^6x^5+\frac{21}{4}\,a^2b^5x^4+\frac{35}{3}\,a^3b^4x^3+\frac{35}{2}\,a^4b^3x^2+21\,a^5b^2x+7\,a^6b\log\left(x\right)-\frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(x) - a^7/x
```

Fricas [A]
time = 0.30, size = 81, normalized size = 0.94

$$\frac{10\,b^{7}x^{7}+84\,ab^{6}x^{6}+315\,a^{2}b^{5}x^{5}+700\,a^{3}b^{4}x^{4}+1050\,a^{4}b^{3}x^{3}+1260\,a^{5}b^{2}x^{2}+420\,a^{6}bx\log\left(x\right)-60\,a^{7}}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*x^7 + 84*a*b^6*x^6 + 315*a^2*b^5*x^5 + 700*a^3*b^4*x^4 + 1050*
a^4*b^3*x^3 + 1260*a^5*b^2*x^2 + 420*a^6*b*x*log(x) - 60*a^7)/x
```

Sympy [A]
time = 0.08, size = 85, normalized size = 0.99

$$-\frac{a^{7}}{x}+7a^{6}b\log\left(x\right)+21a^{5}b^{2}x+\frac{35a^{4}b^{3}x^{2}}{2}+\frac{35a^{3}b^{4}x^{3}}{3}+\frac{21a^{2}b^{5}x^{4}}{4}+\frac{7ab^{6}x^{5}}{5}+\frac{b^{7}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**2,x)
```

```
[Out] -a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*a**3*
b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.00

$$\frac{1}{6}x^{6}b^{7}+\frac{7}{5}x^{5}b^{6}a+\frac{21}{4}x^{4}b^{5}a^{2}+\frac{35}{3}x^{3}b^{4}a^{3}+\frac{35}{2}x^{2}b^{3}a^{4}+21xb^{2}a^{5}-\frac{a^{7}}{x}+7ba^{6}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x)
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(abs(x)) - a^7/x
```

Mupad [B]
time = 0.05, size = 76, normalized size = 0.88

$$\frac{b^{7}\,x^{6}}{6}-\frac{a^{7}}{x}+21\,a^{5}\,b^{2}\,x+\frac{7\,a\,b^{6}\,x^{5}}{5}+7\,a^{6}\,b\,\ln\left(x\right)+\frac{35\,a^{4}\,b^{3}\,x^{2}}{2}+\frac{35\,a^{3}\,b^{4}\,x^{3}}{3}+\frac{21\,a^{2}\,b^{5}\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^2,x)
```

```
[Out] (b^7*x^6)/6 - a^7/x + 21*a^5*b^2*x + (7*a*b^6*x^5)/5 + 7*a^6*b*log(x) + (35
*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2*b^5*x^4)/4
```

3.109 $\int \frac{(a+bx)^7}{x^3} dx$

Optimal. Leaf size=84

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)$$

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^3,x]
```

```
[Out] -1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^3}\,dx &= \int \left(35a^4b^3+\frac{a^7}{x^3}+\frac{7a^6b}{x^2}+\frac{21a^5b^2}{x}+35a^3b^4x+21a^2b^5x^2+7ab^6x^3+b^7x^4\right)dx\\ &= -\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^3,x]
```

```
[Out] -1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Mathics [A]
time = 2.09, size = 78, normalized size = 0.93

$$\frac{-10a^6\left(a+14bx\right)+b^2x^2\left(420a^5\mathrm{Log}\left[x\right]+700a^4bx+350a^3b^2x^2+140a^2b^3x^3+35ab^4x^4+4b^5x^5\right)}{20x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^3,x]')
```

```
[Out] (-10 a ^ 6 (a + 14 b x) + b ^ 2 x ^ 2 (420 a ^ 5 Log[x] + 700 a ^ 4 b x + 3
50 a ^ 3 b ^ 2 x ^ 2 + 140 a ^ 2 b ^ 3 x ^ 3 + 35 a b ^ 4 x ^ 4 + 4 b ^ 5 x
 ^ 5)) / (20 x ^ 2)
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.92

method	result	size
default	$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35a^3b^4x^2}{2}+7a^2b^5x^3+\frac{7a\,b^6x^4}{4}+\frac{b^7x^5}{5}+21a^5b^2\ln\left(x\right)$	77
risch	$\frac{b^7x^5}{5}+\frac{7a\,b^6x^4}{4}+7a^2b^5x^3+\frac{35a^3b^4x^2}{2}+35a^4b^3x+\frac{-7a^6bx-\frac{1}{2}a^7}{x^2}+21a^5b^2\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{2}a^7+\frac{1}{5}b^7x^7+\frac{7}{4}a\,b^6x^6+7a^2b^5x^5+\frac{35}{2}a^3b^4x^4+35a^4b^3x^3-7a^6bx}{x^2}+21a^5b^2\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Maxima [A]
time = 0.25, size = 75, normalized size = 0.89

$$\frac{1}{5}\,b^7x^5+\frac{7}{4}\,ab^6x^4+7\,a^2b^5x^3+\frac{35}{2}\,a^3b^4x^2+35\,a^4b^3x+21\,a^5b^2\log\left(x\right)-\frac{14\,a^6bx+a^7}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(x) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Fricas [A]
time = 0.30, size = 81, normalized size = 0.96

$$\frac{4\,b^{7}x^{7}+35\,ab^{6}x^{6}+140\,a^{2}b^{5}x^{5}+350\,a^{3}b^{4}x^{4}+700\,a^{4}b^{3}x^{3}+420\,a^{5}b^{2}x^{2}\log\left(x\right)-140\,a^{6}bx-10\,a^{7}}{20\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 + 35*a*b^6*x^6 + 140*a^2*b^5*x^5 + 350*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 420*a^5*b^2*x^2*log(x) - 140*a^6*b*x - 10*a^7)/x^2
```

Sympy [A]
time = 0.10, size = 85, normalized size = 1.01

$$21a^{5}b^{2}\log\left(x\right)+35a^{4}b^{3}x+\frac{35a^{3}b^{4}x^{2}}{2}+7a^{2}b^{5}x^{3}+\frac{7ab^{6}x^{4}}{4}+\frac{b^{7}x^{5}}{5}+\frac{-a^{7}-14a^{6}bx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**3,x)
```

```
[Out] 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a**3*b**4*x**2/2 + 7*a**2*b**5*x*
*3 + 7*a*b**6*x**4/4 + b**7*x**5/5 + (-a**7 - 14*a**6*b*x)/(2*x**2)
```

Giac [A]
time = 0.00, size = 87, normalized size = 1.04

$$\frac{1}{5}x^{5}b^{7}+\frac{7}{4}x^{4}b^{6}a+7x^{3}b^{5}a^{2}+\frac{35}{2}x^{2}b^{4}a^{3}+35xb^{3}a^{4}+\frac{\frac{1}{2}\left(-14ba^{6}x-a^{7}\right)}{x^{2}}+21b^{2}a^{5}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x)
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(abs(x)) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.92

$$\frac{b^{7}\,x^{5}}{5}-\frac{\frac{a^{7}}{2}+7\,b\,x\,a^{6}}{x^{2}}+35\,a^{4}\,b^{3}\,x+\frac{7\,a\,b^{6}\,x^{4}}{4}+\frac{35\,a^{3}\,b^{4}\,x^{2}}{2}+7\,a^{2}\,b^{5}\,x^{3}+21\,a^{5}\,b^{2}\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^3,x)
```

```
[Out] (b^7*x^5)/5 - (a^7/2 + 7*a^6*b*x)/x^2 + 35*a^4*b^3*x + (7*a*b^6*x^4)/4 + (3
5*a^3*b^4*x^2)/2 + 7*a^2*b^5*x^3 + 21*a^5*b^2*log(x)
```

3.110 $\int \frac{(a+bx)^7}{x^4}\,dx$

Optimal. Leaf size=86

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)$$

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^4,x]
```

```
[Out] -1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^4}\,dx &= \int \left(35a^3b^4+\frac{a^7}{x^4}+\frac{7a^6b}{x^3}+\frac{21a^5b^2}{x^2}+\frac{35a^4b^3}{x}+21a^2b^5x+7ab^6x^2+b^7x^3\right)dx\\ &= -\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^4,x]
```

```
[Out] -1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Mathics [A]
time = 2.20, size = 80, normalized size = 0.93

$$\frac{-2a^5\left(2a^2+21abx+126b^2x^2\right)+b^3x^3\left(420a^4\mathrm{Log}\left[x\right]+420a^3bx+126a^2b^2x^2+28ab^3x^3+3b^4x^4\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^4,x]')
```

```
[Out] (-2 a ^ 5 (2 a ^ 2 + 21 a b x + 126 b ^ 2 x ^ 2) + b ^ 3 x ^ 3 (420 a ^ 4 L
og[x] + 420 a ^ 3 b x + 126 a ^ 2 b ^ 2 x ^ 2 + 28 a b ^ 3 x ^ 3 + 3 b ^ 4
x ^ 4)) / (12 x ^ 3)
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.90

method	result	size
default	$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21a^2b^5x^2}{2}+\frac{7a\,b^6x^3}{3}+\frac{b^7x^4}{4}+35a^4b^3\ln\left(x\right)$	77
risch	$\frac{b^7x^4}{4}+\frac{7a\,b^6x^3}{3}+\frac{21a^2b^5x^2}{2}+35a^3b^4x+\frac{-21a^5b^2x^2-\frac{7}{2}a^6bx-\frac{1}{3}a^7}{x^3}+35a^4b^3\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{3}a^7+\frac{1}{4}b^7x^7+\frac{7}{3}a\,b^6x^6+\frac{21}{2}a^2b^5x^5+35a^3b^4x^4-21a^5b^2x^2-\frac{7}{2}a^6bx}{x^3}+35a^4b^3\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Maxima [A]
time = 0.25, size = 77, normalized size = 0.90

$$\frac{1}{4}\,b^7x^4+\frac{7}{3}\,ab^6x^3+\frac{21}{2}\,a^2b^5x^2+35\,a^3b^4x+35\,a^4b^3\log\left(x\right)-\frac{126\,a^5b^2x^2+21\,a^6bx+2\,a^7}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(x) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Fricas [A]
time = 0.30, size = 81, normalized size = 0.94

$$\frac{3\,b^{7}x^{7}+28\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+420\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}\log\left(x\right)-252\,a^{5}b^{2}x^{2}-42\,a^{6}bx-4\,a^{7}}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 + 28*a*b^6*x^6 + 126*a^2*b^5*x^5 + 420*a^3*b^4*x^4 + 420*a^
4*b^3*x^3*log(x) - 252*a^5*b^2*x^2 - 42*a^6*b*x - 4*a^7)/x^3
```

Sympy [A]
time = 0.13, size = 87, normalized size = 1.01

$$35a^{4}b^{3}\log\left(x\right)+35a^{3}b^{4}x+\frac{21a^{2}b^{5}x^{2}}{2}+\frac{7ab^{6}x^{3}}{3}+\frac{b^{7}x^{4}}{4}+\frac{-2a^{7}-21a^{6}bx-126a^{5}b^{2}x^{2}}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**4,x)
```

```
[Out] 35*a**4*b**3*log(x) + 35*a**3*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/
3 + b**7*x**4/4 + (-2*a**7 - 21*a**6*b*x - 126*a**5*b**2*x**2)/(6*x**3)
```

Giac [A]
time = 0.00, size = 90, normalized size = 1.05

$$\frac{1}{4}x^{4}b^{7}+\frac{7}{3}x^{3}b^{6}a+\frac{21}{2}x^{2}b^{5}a^{2}+35xb^{4}a^{3}+\frac{\frac{1}{6}\left(-126b^{2}a^{5}x^{2}-21ba^{6}x-2a^{7}\right)}{x^{3}}+35b^{3}a^{4}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x)
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(abs(x)) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.90

$$\frac{b^{7}\,x^{4}}{4}-\frac{\frac{a^{7}}{3}+\frac{7\,a^{6}\,b\,x}{2}+21\,a^{5}\,b^{2}\,x^{2}}{x^{3}}+35\,a^{3}\,b^{4}\,x+\frac{7\,a\,b^{6}\,x^{3}}{3}+\frac{21\,a^{2}\,b^{5}\,x^{2}}{2}+35\,a^{4}\,b^{3}\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^4,x)
```

```
[Out] (b^7*x^4)/4 - (a^7/3 + 21*a^5*b^2*x^2 + (7*a^6*b*x)/2)/x^3 + 35*a^3*b^4*x +
 (7*a*b^6*x^3)/3 + (21*a^2*b^5*x^2)/2 + 35*a^4*b^3*log(x)
```

3.111 $\int \frac{(a+bx)^7}{x^5} dx$

Optimal. Leaf size=86

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)$$

```
[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a
*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+35a^3b^4\log(x)+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^5,x]
```

```
[Out] -1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^5}\,dx &= \int \left(21a^2b^5+\frac{a^7}{x^5}+\frac{7a^6b}{x^4}+\frac{21a^5b^2}{x^3}+\frac{35a^4b^3}{x^2}+\frac{35a^3b^4}{x}+7ab^6x+b^7x^2\right)dx\\ &= -\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^5,x]
```

```
[Out] -1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Mathics [A]
time = 2.23, size = 81, normalized size = 0.94

$$\frac{-a^4\left(3a^3+28a^2bx+126ab^2x^2+420b^3x^3\right)+2b^4x^4\left(210a^3\mathrm{Log}\left[x\right]+126a^2bx+21ab^2x^2+2b^3x^3\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^5,x]')
```

```
[Out] (-a ^ 4 (3 a ^ 3 + 28 a ^ 2 b x + 126 a b ^ 2 x ^ 2 + 420 b ^ 3 x ^ 3) + 2
b ^ 4 x ^ 4 (210 a ^ 3 Log[x] + 126 a ^ 2 b x + 21 a b ^ 2 x ^ 2 + 2 b ^ 3
x ^ 3)) / (12 x ^ 4)
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.90

method	result	size
default	$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7a\,b^6x^2}{2}+\frac{b^7x^3}{3}+35a^3b^4\ln\left(x\right)$	77
risch	$\frac{b^7x^3}{3}+\frac{7a\,b^6x^2}{2}+21a^2b^5x+\frac{-35a^4b^3x^3-\frac{21}{2}a^5b^2x^2-\frac{7}{3}a^6bx-\frac{1}{4}a^7}{x^4}+35a^3b^4\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{4}a^7+\frac{1}{3}b^7x^7+\frac{7}{2}a\,b^6x^6+21a^2b^5x^5-35a^4b^3x^3-\frac{21}{2}a^5b^2x^2-\frac{7}{3}a^6bx}{x^4}+35a^3b^4\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a
*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Maxima [A]
time = 0.26, size = 77, normalized size = 0.90

$$\frac{1}{3}\,b^7x^3+\frac{7}{2}\,ab^6x^2+21\,a^2b^5x+35\,a^3b^4\log\left(x\right)-\frac{420\,a^4b^3x^3+126\,a^5b^2x^2+28\,a^6bx+3\,a^7}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(x) - 1/12*(420*
a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Fricas [A]
time = 0.32, size = 81, normalized size = 0.94

$$\frac{4\,b^7x^7+42\,ab^6x^6+252\,a^2b^5x^5+420\,a^3b^4x^4\log\left(x\right)-420\,a^4b^3x^3-126\,a^5b^2x^2-28\,a^6bx-3\,a^7}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*x^7 + 42*a*b^6*x^6 + 252*a^2*b^5*x^5 + 420*a^3*b^4*x^4*log(x) -
 420*a^4*b^3*x^3 - 126*a^5*b^2*x^2 - 28*a^6*b*x - 3*a^7)/x^4
```

Sympy [A]
time = 0.15, size = 85, normalized size = 0.99

$$35a^3b^4\log\left(x\right)+21a^2b^5x+\frac{7ab^6x^2}{2}+\frac{b^7x^3}{3}+\frac{-3a^7-28a^6bx-126a^5b^2x^2-420a^4b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**5,x)
```

```
[Out] 35*a**3*b**4*log(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3 + (-3*
a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 420*a**4*b**3*x**3)/(12*x**4)
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.03

$$\frac{1}{3}x^3b^7+\frac{7}{2}x^2b^6a+21xb^5a^2+\frac{\frac{1}{12}\left(-420b^3a^4x^3-126b^2a^5x^2-28ba^6x-3a^7\right)}{x^4}+35b^4a^3\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x)
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(abs(x)) - 1/12*
(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Mupad [B]
time = 0.09, size = 77, normalized size = 0.90

$$\frac{b^7\,x^3}{3}-\frac{\frac{a^7}{4}+\frac{7\,a^6\,b\,x}{3}+\frac{21\,a^5\,b^2\,x^2}{2}+35\,a^4\,b^3\,x^3}{x^4}+21\,a^2\,b^5\,x+\frac{7\,a\,b^6\,x^2}{2}+35\,a^3\,b^4\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^5,x)
```

```
[Out] (b^7*x^3)/3 - (a^7/4 + (21*a^5*b^2*x^2)/2 + 35*a^4*b^3*x^3 + (7*a^6*b*x)/3)
/x^4 + 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + 35*a^3*b^4*log(x)
```

3.112 $\int \frac{(a+bx)^7}{x^6} dx$

Optimal. Leaf size=84

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)$$

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^6,x]
```

```
[Out] -1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^6}\,dx &= \int \left(7ab^6+\frac{a^7}{x^6}+\frac{7a^6b}{x^5}+\frac{21a^5b^2}{x^4}+\frac{35a^4b^3}{x^3}+\frac{35a^3b^4}{x^2}+\frac{21a^2b^5}{x}+b^7x\right)dx\\ &= -\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^6,x]
```

```
[Out] -1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Mathics [A]
time = 2.28, size = 80, normalized size = 0.95

$$\frac{-a^3\left(4a^4+35a^3bx+140a^2b^2x^2+350ab^3x^3+700b^4x^4\right)+10b^5x^5\left(42a^2\mathrm{Log}\left[x\right]+14abx+b^2x^2\right)}{20x^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^6,x]')
```

```
[Out] (-a ^ 3 (4 a ^ 4 + 35 a ^ 3 b x + 140 a ^ 2 b ^ 2 x ^ 2 + 350 a b ^ 3 x ^ 3
 + 700 b ^ 4 x ^ 4) + 10 b ^ 5 x ^ 5 (42 a ^ 2 Log[x] + 14 a b x + b ^ 2 x
^ 2)) / (20 x ^ 5)
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.92

method	result	size
default	$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7a\,b^6x+\frac{b^7x^2}{2}+21a^2b^5\ln\left(x\right)$	77
risch	$\frac{b^7x^2}{2}+7a\,b^6x+\frac{-35a^3b^4x^4-\frac{35}{2}a^4b^3x^3-7a^5b^2x^2-\frac{7}{4}a^6bx-\frac{1}{5}a^7}{x^5}+21a^2b^5\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{5}a^7+\frac{1}{2}b^7x^7+7a\,b^6x^6-35a^3b^4x^4-\frac{35}{2}a^4b^3x^3-7a^5b^2x^2-\frac{7}{4}a^6bx}{x^5}+21a^2b^5\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.92

$$\frac{1}{2}\,b^7x^2+7\,ab^6x+21\,a^2b^5\log\left(x\right)-\frac{700\,a^3b^4x^4+350\,a^4b^3x^3+140\,a^5b^2x^2+35\,a^6bx+4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(x) - 1/20*(700*a^3*b^4*x^4 + 350*a
^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Fricas [A]
time = 0.30, size = 81, normalized size = 0.96

$$\frac{10\,b^7x^7+140\,ab^6x^6+420\,a^2b^5x^5\log\left(x\right)-700\,a^3b^4x^4-350\,a^4b^3x^3-140\,a^5b^2x^2-35\,a^6bx-4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*x^7 + 140*a*b^6*x^6 + 420*a^2*b^5*x^5*log(x) - 700*a^3*b^4*x^4
 - 350*a^4*b^3*x^3 - 140*a^5*b^2*x^2 - 35*a^6*b*x - 4*a^7)/x^5
```

Sympy [A]
time = 0.19, size = 83, normalized size = 0.99

$$21a^2b^5\log\left(x\right)+7ab^6x+\frac{b^7x^2}{2}+\frac{-4a^7-35a^6bx-140a^5b^2x^2-350a^4b^3x^3-700a^3b^4x^4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**6,x)
```

```
[Out] 21*a**2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2 + (-4*a**7 - 35*a**6*b*x - 1
40*a**5*b**2*x**2 - 350*a**4*b**3*x**3 - 700*a**3*b**4*x**4)/(20*x**5)
```

Giac [A]
time = 0.00, size = 88, normalized size = 1.05

$$\frac{1}{2}x^2b^7+7xb^6a+\frac{\frac{1}{20}\left(-700b^4a^3x^4-350b^3a^4x^3-140b^2a^5x^2-35ba^6x-4a^7\right)}{x^5}+21b^5a^2\ln\left|x\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x)
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(abs(x)) - 1/20*(700*a^3*b^4*x^4 +
350*a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Mupad [B]
time = 0.11, size = 77, normalized size = 0.92

$$\frac{b^7\,x^2}{2}-\frac{\frac{a^7}{5}+\frac{7\,a^6\,b\,x}{4}+7\,a^5\,b^2\,x^2+\frac{35\,a^4\,b^3\,x^3}{2}+35\,a^3\,b^4\,x^4}{x^5}+21\,a^2\,b^5\,\ln\left(x\right)+7\,a\,b^6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^6,x)
```

```
[Out] (b^7*x^2)/2 - (a^7/5 + 7*a^5*b^2*x^2 + (35*a^4*b^3*x^3)/2 + 35*a^3*b^4*x^4
+ (7*a^6*b*x)/4)/x^5 + 21*a^2*b^5*log(x) + 7*a*b^6*x
```

3.113 $\int \frac{(a+bx)^7}{x^7} dx$

Optimal. Leaf size=85

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)$$

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^7,x]
```

```
[Out] -1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^7}\,dx &= \int \left(b^7+\frac{a^7}{x^7}+\frac{7a^6b}{x^6}+\frac{21a^5b^2}{x^5}+\frac{35a^4b^3}{x^4}+\frac{35a^3b^4}{x^3}+\frac{21a^2b^5}{x^2}+\frac{7ab^6}{x}\right)dx\\ &= -\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 85, normalized size = 1.00

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^7,x]
```

```
[Out] -1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Mathics [A]
time = 2.32, size = 80, normalized size = 0.94

$$\frac{-a^2\left(10a^5+84a^4bx+315a^3b^2x^2+700a^2b^3x^3+1050ab^4x^4+1260b^5x^5\right)+60b^6x^6\left(7a\mathrm{Log}\left[x\right]+bx\right)}{60x^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^7,x]')
```

```
[Out] (-a ^ 2 (10 a ^ 5 + 84 a ^ 4 b x + 315 a ^ 3 b ^ 2 x ^ 2 + 700 a ^ 2 b ^ 3
x ^ 3 + 1050 a b ^ 4 x ^ 4 + 1260 b ^ 5 x ^ 5) + 60 b ^ 6 x ^ 6 (7 a Log[x]
 + b x)) / (60 x ^ 6)
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.89

method	result	size
default	$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7a\,b^6\ln\left(x\right)$	76
risch	$b^7x+\frac{-21a^2b^5x^5-\frac{35}{2}a^3b^4x^4-\frac{35}{3}a^4b^3x^3-\frac{21}{4}a^5b^2x^2-\frac{7}{5}a^6bx-\frac{1}{6}a^7}{x^6}+7a\,b^6\ln\left(x\right)$	76
norman	$\frac{b^7x^7-\frac{1}{6}a^7-21a^2b^5x^5-\frac{35}{2}a^3b^4x^4-\frac{35}{3}a^4b^3x^3-\frac{21}{4}a^5b^2x^2-\frac{7}{5}a^6bx}{x^6}+7a\,b^6\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Maxima [A]
time = 0.26, size = 76, normalized size = 0.89

$$b^7x+7\,ab^6\log\left(x\right)-\frac{1260\,a^2b^5x^5+1050\,a^3b^4x^4+700\,a^4b^3x^3+315\,a^5b^2x^2+84\,a^6bx+10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="maxima")
```

```
[Out] b^7*x + 7*a*b^6*log(x) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Fricas [A]
time = 0.31, size = 81, normalized size = 0.95

$$\frac{60\,b^7x^7+420\,ab^6x^6\log\left(x\right)-1260\,a^2b^5x^5-1050\,a^3b^4x^4-700\,a^4b^3x^3-315\,a^5b^2x^2-84\,a^6bx-10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 420*a*b^6*x^6*log(x) - 1260*a^2*b^5*x^5 - 1050*a^3*b^4*x
^4 - 700*a^4*b^3*x^3 - 315*a^5*b^2*x^2 - 84*a^6*b*x - 10*a^7)/x^6
```

Sympy [A]
time = 0.23, size = 82, normalized size = 0.96

$$7ab^6\log\left(x\right)+b^7x+\frac{-10a^7-84a^6bx-315a^5b^2x^2-700a^4b^3x^3-1050a^3b^4x^4-1260a^2b^5x^5}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**7,x)
```

```
[Out] 7*a*b**6*log(x) + b**7*x + (-10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 - 7
00*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*x**6)
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.01

$$xb^7+\frac{\frac{1}{60}\left(-1260b^5a^2x^5-1050b^4a^3x^4-700b^3a^4x^3-315b^2a^5x^2-84ba^6x-10a^7\right)}{x^6}+7b^6a\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x)
```

```
[Out] b^7*x + 7*a*b^6*log(abs(x)) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 7
00*a^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Mupad [B]
time = 0.11, size = 81, normalized size = 0.95

$$-\frac{10\,a^7-60\,b^7\,x^7+315\,a^5\,b^2\,x^2+700\,a^4\,b^3\,x^3+1050\,a^3\,b^4\,x^4+1260\,a^2\,b^5\,x^5+84\,a^6\,b\,x-420\,a\,b^6\,x^6\,\ln\left(x\right)}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^7,x)
```

```
[Out] -(10*a^7 - 60*b^7*x^7 + 315*a^5*b^2*x^2 + 700*a^4*b^3*x^3 + 1050*a^3*b^4*x^
4 + 1260*a^2*b^5*x^5 + 84*a^6*b*x - 420*a*b^6*x^6*log(x))/(60*x^6)
```

3.114 $\int \frac{(a+bx)^7}{x^8}\,dx$

Optimal. Leaf size=89

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^8,x]
```

```
[Out] -1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^8}\,dx &= \int \left(\frac{a^7}{x^8}+\frac{7a^6b}{x^7}+\frac{21a^5b^2}{x^6}+\frac{35a^4b^3}{x^5}+\frac{35a^3b^4}{x^4}+\frac{21a^2b^5}{x^3}+\frac{7ab^6}{x^2}+\frac{b^7}{x}\right)dx\\ &= -\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 89, normalized size = 1.00

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^8,x]
```

```
[Out] -1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Mathics [**A**]
time = 2.38, size = 80, normalized size = 0.90

$$\frac{-\frac{a(60a^6+490a^5bx+1764a^4b^2x^2+3675a^3b^3x^3+4900a^2b^4x^4+4410ab^5x^5+2940b^6x^6)}{420}+b^7x^7\mathrm{Log}\left[x\right]}{x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^8,x]')
```

```
[Out] (-a (60 a ^ 6 + 490 a ^ 5 b x + 1764 a ^ 4 b ^ 2 x ^ 2 + 3675 a ^ 3 b ^ 3 x
 ^ 3 + 4900 a ^ 2 b ^ 4 x ^ 4 + 4410 a b ^ 5 x ^ 5 + 2940 b ^ 6 x ^ 6) / 42
0 + b ^ 7 x ^ 7 Log[x]) / x ^ 7
```

Maple [**A**]
time = 0.09, size = 78, normalized size = 0.88

method	result	size
default	$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a\,b^6}{x}+b^7\ln\left(x\right)$	78
norman	$\frac{-\frac{1}{7}a^7-7a\,b^6x^6-\frac{21}{2}a^2b^5x^5-\frac{35}{3}a^3b^4x^4-\frac{35}{4}a^4b^3x^3-\frac{21}{5}a^5b^2x^2-\frac{7}{6}a^6bx}{x^7}+b^7\ln\left(x\right)$	78
risch	$\frac{-\frac{1}{7}a^7-7a\,b^6x^6-\frac{21}{2}a^2b^5x^5-\frac{35}{3}a^3b^4x^4-\frac{35}{4}a^4b^3x^3-\frac{21}{5}a^5b^2x^2-\frac{7}{6}a^6bx}{x^7}+b^7\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Maxima [**A**]
time = 0.25, size = 78, normalized size = 0.88

$$b^7\log\left(x\right)-\frac{2940\,ab^6x^6+4410\,a^2b^5x^5+4900\,a^3b^4x^4+3675\,a^4b^3x^3+1764\,a^5b^2x^2+490\,a^6bx+60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="maxima")
```

```
[Out] b^7*log(x) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x^4 +
3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Fricas [A]
time = 0.30, size = 81, normalized size = 0.91

$$\frac{420\,b^7x^7\log\left(x\right)-2940\,ab^6x^6-4410\,a^2b^5x^5-4900\,a^3b^4x^4-3675\,a^4b^3x^3-1764\,a^5b^2x^2-490\,a^6bx-60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="fricas")
```

```
[Out] 1/420*(420*b^7*x^7*log(x) - 2940*a*b^6*x^6 - 4410*a^2*b^5*x^5 - 4900*a^3*b^
4*x^4 - 3675*a^4*b^3*x^3 - 1764*a^5*b^2*x^2 - 490*a^6*b*x - 60*a^7)/x^7
```

Sympy [A]
time = 0.26, size = 83, normalized size = 0.93

$$b^7\log\left(x\right)+\frac{-60a^7-490a^6bx-1764a^5b^2x^2-3675a^4b^3x^3-4900a^3b^4x^4-4410a^2b^5x^5-2940ab^6x^6}{420x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**8,x)
```

```
[Out] b**7*log(x) + (-60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 3675*a**4*b*
*3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6*x**6)/(42
0*x**7)
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.00

$$\frac{\frac{1}{420}\left(-2940b^6ax^6-4410b^5a^2x^5-4900b^4a^3x^4-3675b^3a^4x^3-1764b^2a^5x^2-490ba^6x-60a^7\right)}{x^7}+b^7\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x)
```

```
[Out] b^7*log(abs(x)) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x
^4 + 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Mupad [B]
time = 0.07, size = 78, normalized size = 0.88

$$b^7\,\ln\left(x\right)-\frac{\frac{a^7}{7}+\frac{7\,a^6\,b\,x}{6}+\frac{21\,a^5\,b^2\,x^2}{5}+\frac{35\,a^4\,b^3\,x^3}{4}+\frac{35\,a^3\,b^4\,x^4}{3}+\frac{21\,a^2\,b^5\,x^5}{2}+7\,a\,b^6\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^8,x)
```

```
[Out] b^7*log(x) - (a^7/7 + 7*a*b^6*x^6 + (21*a^5*b^2*x^2)/5 + (35*a^4*b^3*x^3)/4
 + (35*a^3*b^4*x^4)/3 + (21*a^2*b^5*x^5)/2 + (7*a^6*b*x)/6)/x^7
```

3.115 $\int \frac{(a+bx)^7}{x^9} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^8}{8ax^8}$$

```
[Out] -1/8*(b*x+a)^8/a/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^8}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^9,x]
```

```
[Out] -1/8*(a + b*x)^8/(a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^9} dx = -\frac{(a+bx)^8}{8ax^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(17) = 34.
time = 0.00, size = 87, normalized size = 5.12

$$-\frac{a^7}{8x^8} - \frac{a^6b}{x^7} - \frac{7a^5b^2}{2x^6} - \frac{7a^4b^3}{x^5} - \frac{35a^3b^4}{4x^4} - \frac{7a^2b^5}{x^3} - \frac{7ab^6}{2x^2} - \frac{b^7}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^9,x]
```

[Out] -1/8*a^7/x^8 - (a^6*b)/x^7 - (7*a^5*b^2)/(2*x^6) - (7*a^4*b^3)/x^5 - (35*a^3*b^4)/(4*x^4) - (7*a^2*b^5)/x^3 - (7*a*b^6)/(2*x^2) - b^7/x

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(17) = 34$.
time = 2.35, size = 79, normalized size = 4.65

$$\frac{-a^7 - 8a^6bx - 28a^5b^2x^2 - 56a^4b^3x^3 - 70a^3b^4x^4 - 56a^2b^5x^5 - 28ab^6x^6 - 8b^7x^7}{8x^8}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^7/x^9,x]')

[Out] (-a ^ 7 - 8 a ^ 6 b x - 28 a ^ 5 b ^ 2 x ^ 2 - 56 a ^ 4 b ^ 3 x ^ 3 - 70 a ^ 3 b ^ 4 x ^ 4 - 56 a ^ 2 b ^ 5 x ^ 5 - 28 a b ^ 6 x ^ 6 - 8 b ^ 7 x ^ 7) / (8 x ^ 8)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(15) = 30$.
time = 0.07, size = 80, normalized size = 4.71

method	result	size
gosper	$-\frac{8b^7x^7+28a\,b^6x^6+56a^2b^5x^5+70a^3b^4x^4+56a^4b^3x^3+28a^5b^2x^2+8a^6bx+a^7}{8x^8}$	78
norman	$\frac{-b^7x^7-\frac{7}{2}a\,b^6x^6-7a^2b^5x^5-\frac{35}{4}a^3b^4x^4-7a^4b^3x^3-\frac{7}{2}a^5b^2x^2-a^6bx-\frac{1}{8}a^7}{x^8}$	79
risch	$\frac{-b^7x^7-\frac{7}{2}a\,b^6x^6-7a^2b^5x^5-\frac{35}{4}a^3b^4x^4-7a^4b^3x^3-\frac{7}{2}a^5b^2x^2-a^6bx-\frac{1}{8}a^7}{x^8}$	79
default	$-\frac{b^7}{x}-\frac{7a^2b^5}{x^3}-\frac{35a^3b^4}{4x^4}-\frac{7a\,b^6}{2x^2}-\frac{a^7}{8x^8}-\frac{7a^4b^3}{x^5}-\frac{7a^5b^2}{2x^6}-\frac{a^6b}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^7/x^9,x,method=_RETURNVERBOSE)

[Out] -b^7/x-7*a^2*b^5/x^3-35/4*a^3*b^4/x^4-7/2*a*b^6/x^2-1/8*a^7/x^8-7*a^4*b^3/x^5-7/2*a^5*b^2/x^6-a^6*b/x^7

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.
time = 0.26, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^9,x, algorithm="maxima")

[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.
time = 0.30, size = 77, normalized size = 4.53

$$-\frac{8\,b^{7}x^{7}+28\,ab^{6}x^{6}+56\,a^{2}b^{5}x^{5}+70\,a^{3}b^{4}x^{4}+56\,a^{4}b^{3}x^{3}+28\,a^{5}b^{2}x^{2}+8\,a^{6}bx+a^{7}}{8\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(14) = 28$.
time = 0.27, size = 83, normalized size = 4.88

$$\frac{-a^{7}-8a^{6}bx-28a^{5}b^{2}x^{2}-56a^{4}b^{3}x^{3}-70a^{3}b^{4}x^{4}-56a^{2}b^{5}x^{5}-28ab^{6}x^{6}-8b^{7}x^{7}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**9,x)
```

```
[Out] (-a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b**4*
x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.
time = 0.00, size = 87, normalized size = 5.12

$$\frac{-8x^{7}b^{7}-28x^{6}b^{6}a-56x^{5}b^{5}a^{2}-70x^{4}b^{4}a^{3}-56x^{3}b^{3}a^{4}-28x^{2}b^{2}a^{5}-8xba^{6}-a^{7}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x)
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Mupad [B]
time = 0.07, size = 77, normalized size = 4.53

$$-\frac{\frac{a^{7}}{8}+a^{6}\,b\,x+\frac{7\,a^{5}\,b^{2}\,x^{2}}{2}+7\,a^{4}\,b^{3}\,x^{3}+\frac{35\,a^{3}\,b^{4}\,x^{4}}{4}+7\,a^{2}\,b^{5}\,x^{5}+\frac{7\,a\,b^{6}\,x^{6}}{2}+b^{7}\,x^{7}}{x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^9,x)
```

```
[Out] -(a^7/8 + b^7*x^7 + (7*a*b^6*x^6)/2 + (7*a^5*b^2*x^2)/2 + 7*a^4*b^3*x^3 + (
35*a^3*b^4*x^4)/4 + 7*a^2*b^5*x^5 + a^6*b*x)/x^8
```

3.116 $\int \frac{(a+bx)^7}{x^{10}} dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}$$

```
[Out] -1/9*(b*x+a)^8/a/x^9+1/72*b*(b*x+a)^8/a^2/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}}\,dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9}\,dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(36) = 72.
time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(36) = 72.
time = 2.29, size = 79, normalized size = 2.19

$$\frac{-8a^7-63a^6bx-216a^5b^2x^2-420a^4b^3x^3-504a^3b^4x^4-378a^2b^5x^5-168ab^6x^6-36b^7x^7}{72x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^10,x]')
```

```
[Out] (-8 a ^ 7 - 63 a ^ 6 b x - 216 a ^ 5 b ^ 2 x ^ 2 - 420 a ^ 4 b ^ 3 x ^ 3 -
504 a ^ 3 b ^ 4 x ^ 4 - 378 a ^ 2 b ^ 5 x ^ 5 - 168 a b ^ 6 x ^ 6 - 36 b ^
7 x ^ 7) / (72 x ^ 9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.07, size = 80, normalized size = 2.22

method	result	size
norman	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
risch	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
gosper	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80
default	$-\frac{7a\,b^6}{3x^3}-\frac{21a^2b^5}{4x^4}-\frac{a^7}{9x^9}-\frac{b^7}{2x^2}-\frac{7a^6b}{8x^8}-\frac{7a^3b^4}{x^5}-\frac{35a^4b^3}{6x^6}-\frac{3a^5b^2}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*a*b^6/x^3-21/4*a^2*b^5/x^4-1/9*a^7/x^9-1/2*b^7/x^2-7/8*a^6*b/x^8-7*a^3
*b^4/x^5-35/6*a^4*b^3/x^6-3*a^5*b^2/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.

time = 0.25, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.29, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.
time = 0.29, size = 85, normalized size = 2.36

$$\frac{-8a^{7}-63a^{6}bx-216a^{5}b^{2}x^{2}-420a^{4}b^{3}x^{3}-504a^{3}b^{4}x^{4}-378a^{2}b^{5}x^{5}-168ab^{6}x^{6}-36b^{7}x^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.00, size = 89, normalized size = 2.47

$$\frac{-36x^{7}b^{7}-168x^{6}b^{6}a-378x^{5}b^{5}a^{2}-504x^{4}b^{4}a^{3}-420x^{3}b^{3}a^{4}-216x^{2}b^{2}a^{5}-63xba^{6}-8a^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x)
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Mupad [B]
time = 0.09, size = 23, normalized size = 0.64

$$-\frac{(8\,a-b\,x)\,{(a+b\,x)}^8}{72\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

3.117 $\int \frac{(a+bx)^7}{x^{11}}\,dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^8}{10ax^{10}}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{b^2(a+bx)^8}{360a^3x^8}$$

```
[Out] -1/10*(b*x+a)^8/a/x^10+1/45*b*(b*x+a)^8/a^2/x^9-1/360*b^2*(b*x+a)^8/a^3/x^8
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^11,x]
```

```
[Out] -1/10*(a + b*x)^8/(a*x^10) + (b*(a + b*x)^8)/(45*a^2*x^9) - (b^2*(a + b*x)^
8)/(360*a^3*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{(a+bx)^8}{10ax^{10}} - \frac{b\int \frac{(a+bx)^7}{x^{10}}\,dx}{5a}$$
$$= -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} + \frac{b^2\int \frac{(a+bx)^7}{x^9}\,dx}{45a^2}$$
$$= -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} - \frac{b^2(a+bx)^8}{360a^3x^8}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.66

$$-\frac{a^7}{10x^{10}} - \frac{7a^6b}{9x^9} - \frac{21a^5b^2}{8x^8} - \frac{5a^4b^3}{x^7} - \frac{35a^3b^4}{6x^6} - \frac{21a^2b^5}{5x^5} - \frac{7ab^6}{4x^4} - \frac{b^7}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^11,x]
[Out] -1/10*a^7/x^10 - (7*a^6*b)/(9*x^9) - (21*a^5*b^2)/(8*x^8) - (5*a^4*b^3)/x^7
 - (35*a^3*b^4)/(6*x^6) - (21*a^2*b^5)/(5*x^5) - (7*a*b^6)/(4*x^4) - b^7/(3
*x^3)
```

Mathics [A]
time = 2.28, size = 79, normalized size = 1.41

$$\frac{-36a^7 - 280a^6bx - 945a^5b^2x^2 - 1800a^4b^3x^3 - 2100a^3b^4x^4 - 1512a^2b^5x^5 - 630ab^6x^6 - 120b^7x^7}{360x^{10}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^11,x]')
[Out] (-36 a ^ 7 - 280 a ^ 6 b x - 945 a ^ 5 b ^ 2 x ^ 2 - 1800 a ^ 4 b ^ 3 x ^ 3
 - 2100 a ^ 3 b ^ 4 x ^ 4 - 1512 a ^ 2 b ^ 5 x ^ 5 - 630 a b ^ 6 x ^ 6 - 12
0 b ^ 7 x ^ 7) / (360 x ^ 10)
```

Maple [A]
time = 0.07, size = 80, normalized size = 1.43

method	result	size
norman	$\frac{-\frac{1}{3}b^7x^7-\frac{7}{4}a\,b^6x^6-\frac{21}{5}a^2b^5x^5-\frac{35}{6}a^3b^4x^4-5a^4b^3x^3-\frac{21}{8}a^5b^2x^2-\frac{7}{9}a^6bx-\frac{1}{10}a^7}{x^{10}}$	79
risch	$\frac{-\frac{1}{3}b^7x^7-\frac{7}{4}a\,b^6x^6-\frac{21}{5}a^2b^5x^5-\frac{35}{6}a^3b^4x^4-5a^4b^3x^3-\frac{21}{8}a^5b^2x^2-\frac{7}{9}a^6bx-\frac{1}{10}a^7}{x^{10}}$	79
gosper	$-\frac{120b^7x^7+630a\,b^6x^6+1512a^2b^5x^5+2100a^3b^4x^4+1800a^4b^3x^3+945a^5b^2x^2+280a^6bx+36a^7}{360x^{10}}$	80

default	$-\frac{a^{7}}{10x^{10}}-\frac{b^{7}}{3x^{3}}-\frac{7a\,b^{6}}{4x^{4}}-\frac{7a^{6}b}{9x^{9}}-\frac{21a^{5}b^{2}}{8x^{8}}-\frac{21a^{2}b^{5}}{5x^{5}}-\frac{35a^{3}b^{4}}{6x^{6}}-\frac{5a^{4}b^{3}}{x^{7}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^7/x^10-1/3*b^7/x^3-7/4*a*b^6/x^4-7/9*a^6*b/x^9-21/8*a^5*b^2/x^8-21/
5*a^2*b^5/x^5-35/6*a^3*b^4/x^6-5*a^4*b^3/x^7
```

Maxima [A]
time = 0.25, size = 79, normalized size = 1.41

$$-\frac{120\,b^{7}x^{7}+630\,ab^{6}x^{6}+1512\,a^{2}b^{5}x^{5}+2100\,a^{3}b^{4}x^{4}+1800\,a^{4}b^{3}x^{3}+945\,a^{5}b^{2}x^{2}+280\,a^{6}bx+36\,a^{7}}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Fricas [A]
time = 0.34, size = 79, normalized size = 1.41

$$-\frac{120\,b^{7}x^{7}+630\,ab^{6}x^{6}+1512\,a^{2}b^{5}x^{5}+2100\,a^{3}b^{4}x^{4}+1800\,a^{4}b^{3}x^{3}+945\,a^{5}b^{2}x^{2}+280\,a^{6}bx+36\,a^{7}}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Sympy [A]
time = 0.31, size = 85, normalized size = 1.52

$$\frac{-36a^{7}-280a^{6}bx-945a^{5}b^{2}x^{2}-1800a^{4}b^{3}x^{3}-2100a^{3}b^{4}x^{4}-1512a^{2}b^{5}x^{5}-630ab^{6}x^{6}-120b^{7}x^{7}}{360x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**11,x)
```

```
[Out] (-36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 2100*
a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)/(36
0*x**10)
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.59

$$\frac{-120x^7b^7 - 630x^6b^6a - 1512x^5b^5a^2 - 2100x^4b^4a^3 - 1800x^3b^3a^4 - 945x^2b^2a^5 - 280xba^6 - 36a^7}{360x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x)
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Mupad [B]
time = 0.11, size = 79, normalized size = 1.41

$$-\frac{\frac{a^7}{10} + \frac{7\,a^6\,b\,x}{9} + \frac{21\,a^5\,b^2\,x^2}{8} + 5\,a^4\,b^3\,x^3 + \frac{35\,a^3\,b^4\,x^4}{6} + \frac{21\,a^2\,b^5\,x^5}{5} + \frac{7\,a\,b^6\,x^6}{4} + \frac{b^7\,x^7}{3}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^11,x)
```

```
[Out] -(a^7/10 + (b^7*x^7)/3 + (7*a*b^6*x^6)/4 + (21*a^5*b^2*x^2)/8 + 5*a^4*b^3*x
^3 + (35*a^3*b^4*x^4)/6 + (21*a^2*b^5*x^5)/5 + (7*a^6*b*x)/9)/x^10
```

3.118 $\int \frac{(a+bx)^7}{x^{12}}\,dx$

Optimal. Leaf size=76

$$-\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{b^3(a+bx)^8}{1320a^4x^8}$$

```
[Out] -1/11*(b*x+a)^8/a/x^11+3/110*b*(b*x+a)^8/a^2/x^10-1/165*b^2*(b*x+a)^8/a^3/x
^9+1/1320*b^3*(b*x+a)^8/a^4/x^8
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{b^3(a+bx)^8}{1320a^4x^8}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{(a+bx)^8}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^12,x]
```

```
[Out] -1/11*(a + b*x)^8/(a*x^11) + (3*b*(a + b*x)^8)/(110*a^2*x^10) - (b^2*(a + b
*x)^8)/(165*a^3*x^9) + (b^3*(a + b*x)^8)/(1320*a^4*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{(a+bx)^8}{11ax^{11}} - \frac{(3b)\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} + \frac{(3b^2)\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^2}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} - \frac{b^3\int \frac{(a+bx)^7}{x^9}\,dx}{165a^3}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} + \frac{b^3(a+bx)^8}{1320a^4x^8}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.22

$$-\frac{a^7}{11x^{11}} - \frac{7a^6b}{10x^{10}} - \frac{7a^5b^2}{3x^9} - \frac{35a^4b^3}{8x^8} - \frac{5a^3b^4}{x^7} - \frac{7a^2b^5}{2x^6} - \frac{7ab^6}{5x^5} - \frac{b^7}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^12,x]
```

```
[Out] -1/11*a^7/x^11 - (7*a^6*b)/(10*x^10) - (7*a^5*b^2)/(3*x^9) - (35*a^4*b^3)/(
8*x^8) - (5*a^3*b^4)/x^7 - (7*a^2*b^5)/(2*x^6) - (7*a*b^6)/(5*x^5) - b^7/(4
*x^4)
```

Mathics [A]
time = 2.28, size = 79, normalized size = 1.04

$$\frac{-120a^7 - 924a^6bx - 3080a^5b^2x^2 - 5775a^4b^3x^3 - 6600a^3b^4x^4 - 4620a^2b^5x^5 - 1848ab^6x^6 - 330b^7x^7}{1320x^{11}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^12,x]')
```

```
[Out] (-120 a ^ 7 - 924 a ^ 6 b x - 3080 a ^ 5 b ^ 2 x ^ 2 - 5775 a ^ 4 b ^ 3 x ^
 3 - 6600 a ^ 3 b ^ 4 x ^ 4 - 4620 a ^ 2 b ^ 5 x ^ 5 - 1848 a b ^ 6 x ^ 6 -
 330 b ^ 7 x ^ 7) / (1320 x ^ 11)
```

Maple [A]
time = 0.08, size = 80, normalized size = 1.05

method	result	size
norman	$\frac{-\frac{1}{4}b^7x^7-\frac{7}{5}a\,b^6x^6-\frac{7}{2}a^2b^5x^5-5a^3b^4x^4-\frac{35}{8}a^4b^3x^3-\frac{7}{3}a^5b^2x^2-\frac{7}{10}a^6bx-\frac{1}{11}a^7}{x^{11}}$	79
risch	$\frac{-\frac{1}{4}b^7x^7-\frac{7}{5}a\,b^6x^6-\frac{7}{2}a^2b^5x^5-5a^3b^4x^4-\frac{35}{8}a^4b^3x^3-\frac{7}{3}a^5b^2x^2-\frac{7}{10}a^6bx-\frac{1}{11}a^7}{x^{11}}$	79

gosper	$-\frac{330b^7x^7+1848a\,b^6x^6+4620a^2b^5x^5+6600a^3b^4x^4+5775a^4b^3x^3+3080a^5b^2x^2+924a^6bx+120a^7}{1320x^{11}}$	80
default	$-\frac{7a^6b}{10x^{10}}-\frac{a^7}{11x^{11}}-\frac{b^7}{4x^4}-\frac{7a^5b^2}{3x^9}-\frac{35a^4b^3}{8x^8}-\frac{7a\,b^6}{5x^5}-\frac{7a^2b^5}{2x^6}-\frac{5a^3b^4}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -7/10*a^6*b/x^10-1/11*a^7/x^11-1/4*b^7/x^4-7/3*a^5*b^2/x^9-35/8*a^4*b^3/x^8
-7/5*a*b^6/x^5-7/2*a^2*b^5/x^6-5*a^3*b^4/x^7
```

Maxima [A]
time = 0.25, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Fricas [A]
time = 0.30, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Sympy [A]
time = 0.33, size = 85, normalized size = 1.12

$$\frac{-120a^7-924a^6bx-3080a^5b^2x^2-5775a^4b^3x^3-6600a^3b^4x^4-4620a^2b^5x^5-1848ab^6x^6-330b^7x^7}{1320x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**12,x)
```

```
[Out] (-120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 - 660
0*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x**7)/
(1320*x**11)
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.17

$$\frac{-330x^7b^7 - 1848x^6b^6a - 4620x^5b^5a^2 - 6600x^4b^4a^3 - 5775x^3b^3a^4 - 3080x^2b^2a^5 - 924xba^6 - 120a^7}{1320x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x)
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Mupad [B]
time = 0.11, size = 79, normalized size = 1.04

$$-\frac{\frac{a^7}{11}+\frac{7\,a^6\,b\,x}{10}+\frac{7\,a^5\,b^2\,x^2}{3}+\frac{35\,a^4\,b^3\,x^3}{8}+5\,a^3\,b^4\,x^4+\frac{7\,a^2\,b^5\,x^5}{2}+\frac{7\,a\,b^6\,x^6}{5}+\frac{b^7\,x^7}{4}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^12,x)
```

```
[Out] -(a^7/11 + (b^7*x^7)/4 + (7*a*b^6*x^6)/5 + (7*a^5*b^2*x^2)/3 + (35*a^4*b^3*
x^3)/8 + 5*a^3*b^4*x^4 + (7*a^2*b^5*x^5)/2 + (7*a^6*b*x)/10)/x^11
```

3.119 $\int \frac{(a+bx)^7}{x^{13}} dx$

Optimal. Leaf size=96

$$-\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^4(a+bx)^8}{3960a^5x^8}$$

```
[Out] -1/12*(b*x+a)^8/a/x^12+1/33*b*(b*x+a)^8/a^2/x^11-1/110*b^2*(b*x+a)^8/a^3/x^
10+1/495*b^3*(b*x+a)^8/a^4/x^9-1/3960*b^4*(b*x+a)^8/a^5/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^13,x]
```

```
[Out] -1/12*(a + b*x)^8/(a*x^12) + (b*(a + b*x)^8)/(33*a^2*x^11) - (b^2*(a + b*x)
^8)/(110*a^3*x^10) + (b^3*(a + b*x)^8)/(495*a^4*x^9) - (b^4*(a + b*x)^8)/(3
960*a^5*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{(a+bx)^8}{12ax^{12}} - \frac{b\int \frac{(a+bx)^7}{x^{12}}\,dx}{3a}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} + \frac{b^2\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a^2}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} - \frac{b^3\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^3}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} + \frac{b^4\int \frac{(a+bx)^7}{x^9}\,dx}{495a^4}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} - \frac{b^4(a+bx)^8}{3960a^5x^8}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 0.97

$$-\frac{a^7}{12x^{12}} - \frac{7a^6b}{11x^{11}} - \frac{21a^5b^2}{10x^{10}} - \frac{35a^4b^3}{9x^9} - \frac{35a^3b^4}{8x^8} - \frac{3a^2b^5}{x^7} - \frac{7ab^6}{6x^6} - \frac{b^7}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^13,x]
```

```
[Out] -1/12*a^7/x^12 - (7*a^6*b)/(11*x^11) - (21*a^5*b^2)/(10*x^10) - (35*a^4*b^3
)/(9*x^9) - (35*a^3*b^4)/(8*x^8) - (3*a^2*b^5)/x^7 - (7*a*b^6)/(6*x^6) - b^
7/(5*x^5)
```

Mathics [A]
time = 2.38, size = 79, normalized size = 0.82

$$\frac{-330a^7 - 2520a^6bx - 8316a^5b^2x^2 - 15400a^4b^3x^3 - 17325a^3b^4x^4 - 11880a^2b^5x^5 - 4620ab^6x^6 - 792b^7x^7}{3960x^{12}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^13,x]')
```

```
[Out] (-330 a ^ 7 - 2520 a ^ 6 b x - 8316 a ^ 5 b ^ 2 x ^ 2 - 15400 a ^ 4 b ^ 3 x
 ^ 3 - 17325 a ^ 3 b ^ 4 x ^ 4 - 11880 a ^ 2 b ^ 5 x ^ 5 - 4620 a b ^ 6 x ^
 6 - 792 b ^ 7 x ^ 7) / (3960 x ^ 12)
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.83

method	result	size

norman	$\frac{-\frac{1}{5}b^7x^7-\frac{7}{6}a\,b^6x^6-3a^2b^5x^5-\frac{35}{8}a^3b^4x^4-\frac{35}{9}a^4b^3x^3-\frac{21}{10}a^5b^2x^2-\frac{7}{11}a^6bx-\frac{1}{12}a^7}{x^{12}}$	79
risch	$\frac{-\frac{1}{5}b^7x^7-\frac{7}{6}a\,b^6x^6-3a^2b^5x^5-\frac{35}{8}a^3b^4x^4-\frac{35}{9}a^4b^3x^3-\frac{21}{10}a^5b^2x^2-\frac{7}{11}a^6bx-\frac{1}{12}a^7}{x^{12}}$	79
gosper	$-\frac{792b^7x^7+4620a\,b^6x^6+11880a^2b^5x^5+17325a^3b^4x^4+15400a^4b^3x^3+8316a^5b^2x^2+2520a^6bx+330a^7}{3960x^{12}}$	80
default	$-\frac{21a^5b^2}{10x^{10}}-\frac{7a^6b}{11x^{11}}-\frac{a^7}{12x^{12}}-\frac{35a^4b^3}{9x^9}-\frac{35a^3b^4}{8x^8}-\frac{b^7}{5x^5}-\frac{7a\,b^6}{6x^6}-\frac{3a^2b^5}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -21/10*a^5*b^2/x^10-7/11*a^6*b/x^11-1/12*a^7/x^12-35/9*a^4*b^3/x^9-35/8*a^3
*b^4/x^8-1/5*b^7/x^5-7/6*a*b^6/x^6-3*a^2*b^5/x^7
```

Maxima [A]
time = 0.26, size = 79, normalized size = 0.82

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Fricas [A]
time = 0.29, size = 79, normalized size = 0.82

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Sympy [A]
time = 0.34, size = 85, normalized size = 0.89

$$\frac{-330a^7-2520a^6bx-8316a^5b^2x^2-15400a^4b^3x^3-17325a^3b^4x^4-11880a^2b^5x^5-4620ab^6x^6-792b^7x^7}{3960x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**13,x)
```

```
[Out] (-330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3 - 1
7325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**7*x*
*7)/(3960*x**12)
```

Giac [A]
time = 0.00, size = 89, normalized size = 0.93

$$\frac{-792x^7b^7-4620x^6b^6a-11880x^5b^5a^2-17325x^4b^4a^3-15400x^3b^3a^4-8316x^2b^2a^5-2520xba^6-330a^7}{3960x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x)
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.82

$$-\frac{\frac{a^7}{12}+\frac{7\,a^6\,b\,x}{11}+\frac{21\,a^5\,b^2\,x^2}{10}+\frac{35\,a^4\,b^3\,x^3}{9}+\frac{35\,a^3\,b^4\,x^4}{8}+3\,a^2\,b^5\,x^5+\frac{7\,a\,b^6\,x^6}{6}+\frac{b^7\,x^7}{5}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^13,x)
```

```
[Out] -(a^7/12 + (b^7*x^7)/5 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/10 + (35*a^4*b^
3*x^3)/9 + (35*a^3*b^4*x^4)/8 + 3*a^2*b^5*x^5 + (7*a^6*b*x)/11)/x^12
```

3.120 $\int \frac{(a+bx)^7}{x^{14}} dx$

Optimal. Leaf size=93

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

```
[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3
*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^14,x]
```

```
[Out] -1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{14}}\,dx &= \int \left(\frac{a^7}{x^{14}}+\frac{7a^6b}{x^{13}}+\frac{21a^5b^2}{x^{12}}+\frac{35a^4b^3}{x^{11}}+\frac{35a^3b^4}{x^{10}}+\frac{21a^2b^5}{x^9}+\frac{7ab^6}{x^8}+\frac{b^7}{x^7}\right)dx\\ &= -\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.00

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^14,x]
```

```
[Out] -1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Mathics [A]
time = 2.46, size = 79, normalized size = 0.85

$$\frac{-792a^7 - 6006a^6bx - 19656a^5b^2x^2 - 36036a^4b^3x^3 - 40040a^3b^4x^4 - 27027a^2b^5x^5 - 10296ab^6x^6 - 1716b^7x^7}{10296x^{13}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^14,x]')
```

```
[Out] (-792 a ^ 7 - 6006 a ^ 6 b x - 19656 a ^ 5 b ^ 2 x ^ 2 - 36036 a ^ 4 b ^ 3
x ^ 3 - 40040 a ^ 3 b ^ 4 x ^ 4 - 27027 a ^ 2 b ^ 5 x ^ 5 - 10296 a b ^ 6 x
 ^ 6 - 1716 b ^ 7 x ^ 7) / (10296 x ^ 13)
```

Maple [A]
time = 0.07, size = 80, normalized size = 0.86

method	result	size
norman	$\frac{-\frac{1}{6}b^7x^7 - a\,b^6x^6 - \frac{21}{8}a^2b^5x^5 - \frac{35}{9}a^3b^4x^4 - \frac{7}{2}a^4b^3x^3 - \frac{21}{11}a^5b^2x^2 - \frac{7}{12}a^6bx - \frac{1}{13}a^7}{x^{13}}$	79
risch	$\frac{-\frac{1}{6}b^7x^7 - a\,b^6x^6 - \frac{21}{8}a^2b^5x^5 - \frac{35}{9}a^3b^4x^4 - \frac{7}{2}a^4b^3x^3 - \frac{21}{11}a^5b^2x^2 - \frac{7}{12}a^6bx - \frac{1}{13}a^7}{x^{13}}$	79
gosper	$-\frac{1716b^7x^7 + 10296a\,b^6x^6 + 27027a^2b^5x^5 + 40040a^3b^4x^4 + 36036a^4b^3x^3 + 19656a^5b^2x^2 + 6006a^6bx + 792a^7}{10296x^{13}}$	80
default	$-\frac{a^7}{13x^{13}} - \frac{7a^6b}{12x^{12}} - \frac{21a^5b^2}{11x^{11}} - \frac{7a^4b^3}{2x^{10}} - \frac{35a^3b^4}{9x^9} - \frac{21a^2b^5}{8x^8} - \frac{a\,b^6}{x^7} - \frac{b^7}{6x^6}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3
*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

Maxima [A]
time = 0.24, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx + 792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Fricas [A]
time = 0.30, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7+10296\,ab^6x^6+27027\,a^2b^5x^5+40040\,a^3b^4x^4+36036\,a^4b^3x^3+19656\,a^5b^2x^2+6006\,a^6bx+792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Sympy [A]
time = 0.35, size = 85, normalized size = 0.91

$$\frac{-792a^7-6006a^6bx-19656a^5b^2x^2-36036a^4b^3x^3-40040a^3b^4x^4-27027a^2b^5x^5-10296ab^6x^6-1716b^7x^7}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**14,x)
```

```
[Out] (-792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3 -
40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*b**7
*x**7)/(10296*x**13)
```

Giac [A]
time = 0.00, size = 89, normalized size = 0.96

$$\frac{-1716x^7b^7-10296x^6b^6a-27027x^5b^5a^2-40040x^4b^4a^3-36036x^3b^3a^4-19656x^2b^2a^5-6006xba^6-792a^7}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x)
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Mupad [B]
time = 0.07, size = 78, normalized size = 0.84

$$-\frac{\frac{a^7}{13}+\frac{7\,a^6\,b\,x}{12}+\frac{21\,a^5\,b^2\,x^2}{11}+\frac{7\,a^4\,b^3\,x^3}{2}+\frac{35\,a^3\,b^4\,x^4}{9}+\frac{21\,a^2\,b^5\,x^5}{8}+a\,b^6\,x^6+\frac{b^7\,x^7}{6}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^14,x)
```

```
[Out] -(a^7/13 + (b^7*x^7)/6 + a*b^6*x^6 + (21*a^5*b^2*x^2)/11 + (7*a^4*b^3*x^3)/
2 + (35*a^3*b^4*x^4)/9 + (21*a^2*b^5*x^5)/8 + (7*a^6*b*x)/12)/x^13
```

3.121 $\int \frac{(a+bx)^7}{x^{15}} \, dx$

Optimal. Leaf size=95

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^15,x]
```

```
[Out] -1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{15}}\,dx &= \int \left(\frac{a^7}{x^{15}}+\frac{7a^6b}{x^{14}}+\frac{21a^5b^2}{x^{13}}+\frac{35a^4b^3}{x^{12}}+\frac{35a^3b^4}{x^{11}}+\frac{21a^2b^5}{x^{10}}+\frac{7ab^6}{x^9}+\frac{b^7}{x^8}\right)dx\\ &= -\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^15,x]
```

```
[Out] -1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Mathics [A]
time = 2.68, size = 79, normalized size = 0.83

$$\frac{-1716a^7 - 12936a^6bx - 42042a^5b^2x^2 - 76440a^4b^3x^3 - 84084a^3b^4x^4 - 56056a^2b^5x^5 - 21021ab^6x^6 - 3432b^7x^7}{24024x^{14}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^15,x]')
```

```
[Out] (-1716 a ^ 7 - 12936 a ^ 6 b x - 42042 a ^ 5 b ^ 2 x ^ 2 - 76440 a ^ 4 b ^
3 x ^ 3 - 84084 a ^ 3 b ^ 4 x ^ 4 - 56056 a ^ 2 b ^ 5 x ^ 5 - 21021 a b ^ 6
 x ^ 6 - 3432 b ^ 7 x ^ 7) / (24024 x ^ 14)
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{14}a^7-\frac{7}{13}a^6bx-\frac{7}{4}a^5b^2x^2-\frac{35}{11}a^4b^3x^3-\frac{7}{2}a^3b^4x^4-\frac{7}{3}a^2b^5x^5-\frac{7}{8}a\,b^6x^6-\frac{1}{7}b^7x^7}{x^{14}}$	79
risch	$\frac{-\frac{1}{14}a^7-\frac{7}{13}a^6bx-\frac{7}{4}a^5b^2x^2-\frac{35}{11}a^4b^3x^3-\frac{7}{2}a^3b^4x^4-\frac{7}{3}a^2b^5x^5-\frac{7}{8}a\,b^6x^6-\frac{1}{7}b^7x^7}{x^{14}}$	79
gosper	$-\frac{3432b^7x^7+21021a\,b^6x^6+56056a^2b^5x^5+84084a^3b^4x^4+76440a^4b^3x^3+42042a^5b^2x^2+12936a^6bx+1716a^7}{24024x^{14}}$	80
default	$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a\,b^6}{8x^8}-\frac{b^7}{7x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

Maxima [A]
time = 0.25, size = 79, normalized size = 0.83

$$-\frac{3432\,b^7x^7 + 21021\,ab^6x^6 + 56056\,a^2b^5x^5 + 84084\,a^3b^4x^4 + 76440\,a^4b^3x^3 + 42042\,a^5b^2x^2 + 12936\,a^6bx + 1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Fricas [A]

time = 0.31, size = 79, normalized size = 0.83

$$-\frac{3432\,b^7x^7+21021\,ab^6x^6+56056\,a^2b^5x^5+84084\,a^3b^4x^4+76440\,a^4b^3x^3+42042\,a^5b^2x^2+12936\,a^6bx+1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Sympy [A]

time = 0.37, size = 85, normalized size = 0.89

$$\frac{-1716a^7-12936a^6bx-42042a^5b^2x^2-76440a^4b^3x^3-84084a^3b^4x^4-56056a^2b^5x^5-21021ab^6x^6-3432b^7x^7}{24024x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**15,x)
```

```
[Out] (-1716*a**7 - 12936*a**6*b*x - 42042*a**5*b**2*x**2 - 76440*a**4*b**3*x**3
- 84084*a**3*b**4*x**4 - 56056*a**2*b**5*x**5 - 21021*a*b**6*x**6 - 3432*b*
*7*x**7)/(24024*x**14)
```

Giac [A]

time = 0.00, size = 89, normalized size = 0.94

$$\frac{-3432x^7b^7-21021x^6b^6a-56056x^5b^5a^2-84084x^4b^4a^3-76440x^3b^3a^4-42042x^2b^2a^5-12936xba^6-1716a^7}{24024x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x)
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Mupad [B]

time = 0.07, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{14}+\frac{7\,a^6\,b\,x}{13}+\frac{7\,a^5\,b^2\,x^2}{4}+\frac{35\,a^4\,b^3\,x^3}{11}+\frac{7\,a^3\,b^4\,x^4}{2}+\frac{7\,a^2\,b^5\,x^5}{3}+\frac{7\,a\,b^6\,x^6}{8}+\frac{b^7\,x^7}{7}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^15,x)
```

```
[Out] -(a^7/14 + (b^7*x^7)/7 + (7*a*b^6*x^6)/8 + (7*a^5*b^2*x^2)/4 + (35*a^4*b^3*
x^3)/11 + (7*a^3*b^4*x^4)/2 + (7*a^2*b^5*x^5)/3 + (7*a^6*b*x)/13)/x^14
```

3.122 $\int \frac{(a+bx)^7}{x^{16}} \, dx$

Optimal. Leaf size=95

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^16,x]
```

```
[Out] -1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{16}} \, dx &= \int \left(\frac{a^7}{x^{16}}+\frac{7a^6b}{x^{15}}+\frac{21a^5b^2}{x^{14}}+\frac{35a^4b^3}{x^{13}}+\frac{35a^3b^4}{x^{12}}+\frac{21a^2b^5}{x^{11}}+\frac{7ab^6}{x^{10}}+\frac{b^7}{x^9}\right) dx\\ &= -\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^16,x]
```

```
[Out] -1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Mathics [A]
time = 2.68, size = 79, normalized size = 0.83

$$\frac{-3432a^7 - 25740a^6bx - 83160a^5b^2x^2 - 150150a^4b^3x^3 - 163800a^3b^4x^4 - 108108a^2b^5x^5 - 40040ab^6x^6 - 6435b^7x^7}{51480x^{15}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^16,x]')
```

```
[Out] (-3432 a ^ 7 - 25740 a ^ 6 b x - 83160 a ^ 5 b ^ 2 x ^ 2 - 150150 a ^ 4 b ^
 3 x ^ 3 - 163800 a ^ 3 b ^ 4 x ^ 4 - 108108 a ^ 2 b ^ 5 x ^ 5 - 40040 a b
^ 6 x ^ 6 - 6435 b ^ 7 x ^ 7) / (51480 x ^ 15)
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{15}a^7-\frac{1}{2}a^6bx-\frac{21}{13}a^5b^2x^2-\frac{35}{12}a^4b^3x^3-\frac{35}{11}a^3b^4x^4-\frac{21}{10}a^2b^5x^5-\frac{7}{9}a\,b^6x^6-\frac{1}{8}b^7x^7}{x^{15}}$	79
risch	$\frac{-\frac{1}{15}a^7-\frac{1}{2}a^6bx-\frac{21}{13}a^5b^2x^2-\frac{35}{12}a^4b^3x^3-\frac{35}{11}a^3b^4x^4-\frac{21}{10}a^2b^5x^5-\frac{7}{9}a\,b^6x^6-\frac{1}{8}b^7x^7}{x^{15}}$	79
gosper	$-\frac{6435b^7x^7+40040a\,b^6x^6+108108a^2b^5x^5+163800a^3b^4x^4+150150a^4b^3x^3+83160a^5b^2x^2+25740a^6bx+3432a^7}{51480x^{15}}$	80
default	$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7a\,b^6}{9x^9}-\frac{b^7}{8x^8}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

Maxima [A]
time = 0.25, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7 + 40040\,ab^6x^6 + 108108\,a^2b^5x^5 + 163800\,a^3b^4x^4 + 150150\,a^4b^3x^3 + 83160\,a^5b^2x^2 + 25740\,a^6bx + 3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Fricas [A]

time = 0.30, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Sympy [A]

time = 0.38, size = 85, normalized size = 0.89

$$\frac{-3432a^7-25740a^6bx-83160a^5b^2x^2-150150a^4b^3x^3-163800a^3b^4x^4-108108a^2b^5x^5-40040ab^6x^6-6435b^7x^7}{51480x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**16,x)
```

```
[Out] (-3432*a**7 - 25740*a**6*b*x - 83160*a**5*b**2*x**2 - 150150*a**4*b**3*x**3
 - 163800*a**3*b**4*x**4 - 108108*a**2*b**5*x**5 - 40040*a*b**6*x**6 - 6435
*b**7*x**7)/(51480*x**15)
```

Giac [A]

time = 0.00, size = 89, normalized size = 0.94

$$\frac{-6435x^7b^7-40040x^6b^6a-108108x^5b^5a^2-163800x^4b^4a^3-150150x^3b^3a^4-83160x^2b^2a^5-25740xba^6-3432a^7}{51480x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x)
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Mupad [B]

time = 0.11, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{15}+\frac{a^6\,b\,x}{2}+\frac{21\,a^5\,b^2\,x^2}{13}+\frac{35\,a^4\,b^3\,x^3}{12}+\frac{35\,a^3\,b^4\,x^4}{11}+\frac{21\,a^2\,b^5\,x^5}{10}+\frac{7\,a\,b^6\,x^6}{9}+\frac{b^7\,x^7}{8}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^16,x)
```

```
[Out] -(a^7/15 + (b^7*x^7)/8 + (7*a*b^6*x^6)/9 + (21*a^5*b^2*x^2)/13 + (35*a^4*b^
3*x^3)/12 + (35*a^3*b^4*x^4)/11 + (21*a^2*b^5*x^5)/10 + (a^6*b*x)/2)/x^15
```

3.123 $\int x^{11}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab$$

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{11}(a+bx)^{10}\,dx &= \int \left(a^{10}x^{11}+10a^9bx^{12}+45a^8b^2x^{13}+120a^7b^3x^{14}+210a^6b^4x^{15}+252a^5b^5x^{16}+210a^4b^6\right.\\ &= \frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Mathics [A]
time = 2.14, size = 112, normalized size = 0.85

$$\frac{x^{12}\left(646646a^{10}+5969040a^{9}bx+24942060a^{8}b^{2}x^{2}+62078016a^{7}b^{3}x^{3}+101846745a^{6}b^{4}x^{4}+115026912a^{5}b^{5}x^{5}+90530440a^{4}b^{6}x^{6}+49008960a^{3}b^{7}x^{7}+17459442a^{2}b^{8}x^{8}+3695120ab^{9}x^{9}+352716b^{10}x^{10}\right)}{7759752}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^11*(a + b*x)^10,x]')
```

```
[Out] x ^ 12 (646646 a ^ 10 + 5969040 a ^ 9 b x + 24942060 a ^ 8 b ^ 2 x ^ 2 + 62
078016 a ^ 7 b ^ 3 x ^ 3 + 101846745 a ^ 6 b ^ 4 x ^ 4 + 115026912 a ^ 5 b
^ 5 x ^ 5 + 90530440 a ^ 4 b ^ 6 x ^ 6 + 49008960 a ^ 3 b ^ 7 x ^ 7 + 17459
442 a ^ 2 b ^ 8 x ^ 8 + 3695120 a b ^ 9 x ^ 9 + 352716 b ^ 10 x ^ 10) / 775
9752
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$
default	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$
norman	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$
risch	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Maxima [A]
time = 0.25, size = 112, normalized size = 0.85

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{14}+\frac{10}{13}\,a^{9}bx^{13}+\frac{1}{12}\,a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Fricas [A]
time = 0.30, size = 112, normalized size = 0.85

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{14}+\frac{10}{13}\,a^{9}bx^{13}+\frac{1}{12}\,a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Sympy [A]
time = 0.04, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{12}}{12}+\frac{10a^{9}bx^{13}}{13}+\frac{45a^{8}b^{2}x^{14}}{14}+8a^{7}b^{3}x^{15}+\frac{105a^{6}b^{4}x^{16}}{8}+\frac{252a^{5}b^{5}x^{17}}{17}+\frac{35a^{4}b^{6}x^{18}}{3}+\frac{120a^{3}b^{7}x^{19}}{19}+\frac{9a^{2}b^{8}x^{20}}{4}+\frac{10ab^{9}x^{21}}{21}+\frac{b^{10}x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x+a)**10,x)
```

```
[Out] a**10*x**12/12 + 10*a**9*b*x**13/13 + 45*a**8*b**2*x**14/14 + 8*a**7*b**3*x
**15 + 105*a**6*b**4*x**16/8 + 252*a**5*b**5*x**17/17 + 35*a**4*b**6*x**18/
3 + 120*a**3*b**7*x**19/19 + 9*a**2*b**8*x**20/4 + 10*a*b**9*x**21/21 + b**
10*x**22/22
```

Giac [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{1}{22}x^{22}b^{10}+\frac{10}{21}x^{21}b^{9}a+\frac{9}{4}x^{20}b^{8}a^{2}+\frac{120}{19}x^{19}b^{7}a^{3}+\frac{35}{3}x^{18}b^{6}a^{4}+\frac{252}{17}x^{17}b^{5}a^{5}+\frac{105}{8}x^{16}b^{4}a^{6}+8x^{15}b^{3}a^{7}+\frac{45}{14}x^{14}b^{2}a^{8}+\frac{10}{13}x^{13}ba^{9}+\frac{1}{12}x^{12}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x)
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Mupad [B]
time = 0.15, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{12}}{12}+\frac{10\,a^{9}\,b\,x^{13}}{13}+\frac{45\,a^{8}\,b^{2}\,x^{14}}{14}+8\,a^{7}\,b^{3}\,x^{15}+\frac{105\,a^{6}\,b^{4}\,x^{16}}{8}+\frac{252\,a^{5}\,b^{5}\,x^{17}}{17}+\frac{35\,a^{4}\,b^{6}\,x^{18}}{3}+\frac{120\,a^{3}\,b^{7}\,x^{19}}{19}+\frac{9\,a^{2}\,b^{8}\,x^{20}}{4}+\frac{10\,a\,b^{9}\,x^{21}}{21}+\frac{b^{10}\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x)^10,x)
```

```
[Out] (a^10*x^12)/12 + (b^10*x^22)/22 + (10*a^9*b*x^13)/13 + (10*a*b^9*x^21)/21 +
 (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^
5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)
/4
```

3.124 $\int x^{10}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x$$

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{10}(a+bx)^{10}\,dx &= \int \left(a^{10}x^{10}+10a^9bx^{11}+45a^8b^2x^{12}+120a^7b^3x^{13}+210a^6b^4x^{14}+252a^5b^5x^{15}+210a^4b^6\right.\\ &= \frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Mathics [A]
time = 2.15, size = 112, normalized size = 0.85

$$\frac{x^{11}\left(352716a^{10}+3233230a^9bx+13430340a^8b^2x^2+33256080a^7b^3x^3+54318264a^6b^4x^4+61108047a^5b^5x^5+47927880a^4b^6x^6+25865840a^3b^7x^7+9189180a^2b^8x^8+1939938ab^9x^9+184756b^{10}x^{10}\right)}{3879876}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^10*(a + b*x)^10,x]')
```

```
[Out] x ^ 11 (352716 a ^ 10 + 3233230 a ^ 9 b x + 13430340 a ^ 8 b ^ 2 x ^ 2 + 33
256080 a ^ 7 b ^ 3 x ^ 3 + 54318264 a ^ 6 b ^ 4 x ^ 4 + 61108047 a ^ 5 b ^
5 x ^ 5 + 47927880 a ^ 4 b ^ 6 x ^ 6 + 25865840 a ^ 3 b ^ 7 x ^ 7 + 9189180
 a ^ 2 b ^ 8 x ^ 8 + 1939938 a b ^ 9 x ^ 9 + 184756 b ^ 10 x ^ 10) / 387987
6
```

Maple [A]
time = 0.07, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}$
default	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}$
norman	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}$
risch	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Maxima [A]
time = 0.25, size = 112, normalized size = 0.85

$$\frac{1}{21}b^{10}x^{21}+\frac{1}{2}ab^9x^{20}+\frac{45}{19}a^2b^8x^{19}+\frac{20}{3}a^3b^7x^{18}+\frac{210}{17}a^4b^6x^{17}+\frac{63}{4}a^5b^5x^{16}+14a^6b^4x^{15}+\frac{60}{7}a^7b^3x^{14}+\frac{45}{13}a^8b^2x^{13}+\frac{5}{6}a^9bx^{12}+\frac{1}{11}a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="maxima")
```

[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 + 210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11

Fricas [A]
time = 0.30, size = 112, normalized size = 0.85

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}+\frac{5}{6}\,a^{9}bx^{12}+\frac{1}{11}\,a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(b*x+a)^10,x, algorithm="fricas")

[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 + 210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11

Sympy [A]
time = 0.05, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{11}}{11}+\frac{5a^{9}bx^{12}}{6}+\frac{45a^{8}b^{2}x^{13}}{13}+\frac{60a^{7}b^{3}x^{14}}{7}+14a^{6}b^{4}x^{15}+\frac{63a^{5}b^{5}x^{16}}{4}+\frac{210a^{4}b^{6}x^{17}}{17}+\frac{20a^{3}b^{7}x^{18}}{3}+\frac{45a^{2}b^{8}x^{19}}{19}+\frac{ab^{9}x^{20}}{2}+\frac{b^{10}x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10*(b*x+a)**10,x)

[Out] a**10*x**11/11 + 5*a**9*b*x**12/6 + 45*a**8*b**2*x**13/13 + 60*a**7*b**3*x**14/7 + 14*a**6*b**4*x**15 + 63*a**5*b**5*x**16/4 + 210*a**4*b**6*x**17/17 + 20*a**3*b**7*x**18/3 + 45*a**2*b**8*x**19/19 + a*b**9*x**20/2 + b**10*x**21/21

Giac [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{1}{21}x^{21}b^{10}+\frac{1}{2}x^{20}b^{9}a+\frac{45}{19}x^{19}b^{8}a^{2}+\frac{20}{3}x^{18}b^{7}a^{3}+\frac{210}{17}x^{17}b^{6}a^{4}+\frac{63}{4}x^{16}b^{5}a^{5}+14x^{15}b^{4}a^{6}+\frac{60}{7}x^{14}b^{3}a^{7}+\frac{45}{13}x^{13}b^{2}a^{8}+\frac{5}{6}x^{12}ba^{9}+\frac{1}{11}x^{11}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10*(b*x+a)^10,x)

[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 + 210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11

Mupad [B]
time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{11}}{11}+\frac{5\,a^{9}\,b\,x^{12}}{6}+\frac{45\,a^{8}\,b^{2}\,x^{13}}{13}+\frac{60\,a^{7}\,b^{3}\,x^{14}}{7}+14\,a^{6}\,b^{4}\,x^{15}+\frac{63\,a^{5}\,b^{5}\,x^{16}}{4}+\frac{210\,a^{4}\,b^{6}\,x^{17}}{17}+\frac{20\,a^{3}\,b^{7}\,x^{18}}{3}+\frac{45\,a^{2}\,b^{8}\,x^{19}}{19}+\frac{a\,b^{9}\,x^{20}}{2}+\frac{b^{10}\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b*x)^10,x)

[Out] (a^10*x^11)/11 + (b^10*x^21)/21 + (5*a^9*b*x^12)/6 + (a*b^9*x^20)/2 + (45*a
^8*b^2*x^13)/13 + (60*a^7*b^3*x^14)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)
/4 + (210*a^4*b^6*x^17)/17 + (20*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19
```

3.125 $\int x^9(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}a$$

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^9(a+bx)^{10}\,dx = \int \left(a^{10}x^9+10a^9bx^{10}+45a^8b^2x^{11}+120a^7b^3x^{12}+210a^6b^4x^{13}+252a^5b^5x^{14}+210a^4b^6x^{1}\right.$$

$$=\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Mathics [A]
time = 2.17, size = 112, normalized size = 0.85

$$\frac{x^{10}\left(184756a^{10}+1679600a^9bx+6928350a^8b^2x^2+17054400a^7b^3x^3+27713400a^6b^4x^4+31039008a^5b^5x^5+24249225a^4b^6x^6+13041600a^3b^7x^7+4618900a^2b^8x^8+972400ab^9x^9+92378b^{10}x^{10}\right)}{1847560}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^9*(a + b*x)^10,x]')
```

```
[Out] x ^ 10 (184756 a ^ 10 + 1679600 a ^ 9 b x + 6928350 a ^ 8 b ^ 2 x ^ 2 + 170
54400 a ^ 7 b ^ 3 x ^ 3 + 27713400 a ^ 6 b ^ 4 x ^ 4 + 31039008 a ^ 5 b ^ 5
 x ^ 5 + 24249225 a ^ 4 b ^ 6 x ^ 6 + 13041600 a ^ 3 b ^ 7 x ^ 7 + 4618900
a ^ 2 b ^ 8 x ^ 8 + 972400 a b ^ 9 x ^ 9 + 92378 b ^ 10 x ^ 10) / 1847560
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$
default	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$
norman	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$
risch	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Maxima [A]
time = 0.25, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20}+\frac{10}{19}\,ab^9x^{19}+\frac{5}{2}\,a^2b^8x^{18}+\frac{120}{17}\,a^3b^7x^{17}+\frac{105}{8}\,a^4b^6x^{16}+\frac{84}{5}\,a^5b^5x^{15}+15\,a^6b^4x^{14}+\frac{120}{13}\,a^7b^3x^{13}+\frac{15}{4}\,a^8b^2x^{12}+\frac{10}{11}\,a^9bx^{11}+\frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Fricas [A]
time = 0.31, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20}+\frac{10}{19}\,ab^{9}x^{19}+\frac{5}{2}\,a^{2}b^{8}x^{18}+\frac{120}{17}\,a^{3}b^{7}x^{17}+\frac{105}{8}\,a^{4}b^{6}x^{16}+\frac{84}{5}\,a^{5}b^{5}x^{15}+15\,a^{6}b^{4}x^{14}+\frac{120}{13}\,a^{7}b^{3}x^{13}+\frac{15}{4}\,a^{8}b^{2}x^{12}+\frac{10}{11}\,a^{9}bx^{11}+\frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Sympy [A]
time = 0.04, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{10}}{10}+\frac{10a^{9}bx^{11}}{11}+\frac{15a^{8}b^{2}x^{12}}{4}+\frac{120a^{7}b^{3}x^{13}}{13}+15a^{6}b^{4}x^{14}+\frac{84a^{5}b^{5}x^{15}}{5}+\frac{105a^{4}b^{6}x^{16}}{8}+\frac{120a^{3}b^{7}x^{17}}{17}+\frac{5a^{2}b^{8}x^{18}}{2}+\frac{10ab^{9}x^{19}}{19}+\frac{b^{10}x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x+a)**10,x)
```

```
[Out] a**10*x**10/10 + 10*a**9*b*x**11/11 + 15*a**8*b**2*x**12/4 + 120*a**7*b**3*
x**13/13 + 15*a**6*b**4*x**14 + 84*a**5*b**5*x**15/5 + 105*a**4*b**6*x**16/
8 + 120*a**3*b**7*x**17/17 + 5*a**2*b**8*x**18/2 + 10*a*b**9*x**19/19 + b**
10*x**20/20
```

Giac [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{1}{20}x^{20}b^{10}+\frac{10}{19}x^{19}b^{9}a+\frac{5}{2}x^{18}b^{8}a^{2}+\frac{120}{17}x^{17}b^{7}a^{3}+\frac{105}{8}x^{16}b^{6}a^{4}+\frac{84}{5}x^{15}b^{5}a^{5}+15x^{14}b^{4}a^{6}+\frac{120}{13}x^{13}b^{3}a^{7}+\frac{15}{4}x^{12}b^{2}a^{8}+\frac{10}{11}x^{11}ba^{9}+\frac{1}{10}x^{10}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x)
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Mupad [B]
time = 0.12, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{10}}{10}+\frac{10\,a^{9}\,b\,x^{11}}{11}+\frac{15\,a^{8}\,b^{2}\,x^{12}}{4}+\frac{120\,a^{7}\,b^{3}\,x^{13}}{13}+15\,a^{6}\,b^{4}\,x^{14}+\frac{84\,a^{5}\,b^{5}\,x^{15}}{5}+\frac{105\,a^{4}\,b^{6}\,x^{16}}{8}+\frac{120\,a^{3}\,b^{7}\,x^{17}}{17}+\frac{5\,a^{2}\,b^{8}\,x^{18}}{2}+\frac{10\,a\,b^{9}\,x^{19}}{19}+\frac{b^{10}\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x)^10,x)
```

```
[Out] (a^10*x^10)/10 + (b^10*x^20)/20 + (10*a^9*b*x^11)/11 + (10*a*b^9*x^19)/19 +
 (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^
5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)
/2
```

3.126 $\int x^8(a+bx)^{10}\,dx$

Optimal. Leaf size=147

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)}{17b^9}$$

```
[Out] 1/11*a^8*(b*x+a)^11/b^9-2/3*a^7*(b*x+a)^12/b^9+28/13*a^6*(b*x+a)^13/b^9-4*a
^5*(b*x+a)^14/b^9+14/3*a^4*(b*x+a)^15/b^9-7/2*a^3*(b*x+a)^16/b^9+28/17*a^2*
(b*x+a)^17/b^9-4/9*a*(b*x+a)^18/b^9+1/19*(b*x+a)^19/b^9
```

Rubi [A]
time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}+\frac{(a+bx)^{19}}{19b^9}-\frac{4a(a+bx)^{18}}{9b^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^10,x]
```

```
[Out] (a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a + b
*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^9)
 - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*(a
+ b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^{10}\,dx &= \int \left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+bx)^{1}}{b^8}\right.\\ &= \frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7}{}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 125, normalized size = 0.85

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}a^2b^8x^{17}+\frac{5}{9}ab^9x^{18}+\frac{b^{10}x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^10,x]
```

```
[Out] (a^10*x^9)/9 + a^9*b*x^10 + (45*a^8*b^2*x^11)/11 + 10*a^7*b^3*x^12 + (210*a
^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 +
 (45*a^2*b^8*x^17)/17 + (5*a*b^9*x^18)/9 + (b^10*x^19)/19
```

Mathics [A]
time = 2.30, size = 112, normalized size = 0.76

$$\frac{x^9\left(92378a^{10}+831402a^9bx+3401190a^8b^2x^2+8314020a^7b^3x^3+13430340a^6b^4x^4+14965236a^5b^5x^5+11639628a^4b^6x^6+6235515a^3b^7x^7+2200770a^2b^8x^8+461890ab^9x^9+43758b^{10}x^{10}\right)}{831402}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^8*(a + b*x)^10,x]')
```

```
[Out] x ^ 9 (92378 a ^ 10 + 831402 a ^ 9 b x + 3401190 a ^ 8 b ^ 2 x ^ 2 + 831402
0 a ^ 7 b ^ 3 x ^ 3 + 13430340 a ^ 6 b ^ 4 x ^ 4 + 14965236 a ^ 5 b ^ 5 x ^
 5 + 11639628 a ^ 4 b ^ 6 x ^ 6 + 6235515 a ^ 3 b ^ 7 x ^ 7 + 2200770 a ^ 2
 b ^ 8 x ^ 8 + 461890 a b ^ 9 x ^ 9 + 43758 b ^ 10 x ^ 10) / 831402
```

Maple [A]
time = 0.09, size = 112, normalized size = 0.76

method	result
gosper	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+$
default	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+$
norman	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+$
risch	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^10*x^9+a^9*b*x^10+45/11*a^8*b^2*x^11+10*a^7*b^3*x^12+210/13*a^6*b^4*x
^13+18*a^5*b^5*x^14+14*a^4*b^6*x^15+15/2*a^3*b^7*x^16+45/17*a^2*b^8*x^17+5/
9*a*b^9*x^18+1/19*b^10*x^19
```

Maxima [A]
time = 0.26, size = 111, normalized size = 0.76

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^9x^{18}+\frac{45}{17}\,a^2b^8x^{17}+\frac{15}{2}\,a^3b^7x^{16}+14\,a^4b^6x^{15}+18\,a^5b^5x^{14}+\frac{210}{13}\,a^6b^4x^{13}+10\,a^7b^3x^{12}+\frac{45}{11}\,a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}\,a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Fricas [A]
time = 0.30, size = 111, normalized size = 0.76

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^{9}x^{18}+\frac{45}{17}\,a^{2}b^{8}x^{17}+\frac{15}{2}\,a^{3}b^{7}x^{16}+14\,a^{4}b^{6}x^{15}+18\,a^{5}b^{5}x^{14}+\frac{210}{13}\,a^{6}b^{4}x^{13}+10\,a^{7}b^{3}x^{12}+\frac{45}{11}\,a^{8}b^{2}x^{11}+a^{9}bx^{10}+\frac{1}{9}\,a^{10}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Sympy [A]
time = 0.04, size = 126, normalized size = 0.86

$$\frac{a^{10}x^{9}}{9}+a^{9}bx^{10}+\frac{45a^{8}b^{2}x^{11}}{11}+10a^{7}b^{3}x^{12}+\frac{210a^{6}b^{4}x^{13}}{13}+18a^{5}b^{5}x^{14}+14a^{4}b^{6}x^{15}+\frac{15a^{3}b^{7}x^{16}}{2}+\frac{45a^{2}b^{8}x^{17}}{17}+\frac{5ab^{9}x^{18}}{9}+\frac{b^{10}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**10,x)
```

```
[Out] a**10*x**9/9 + a**9*b*x**10 + 45*a**8*b**2*x**11/11 + 10*a**7*b**3*x**12 +
210*a**6*b**4*x**13/13 + 18*a**5*b**5*x**14 + 14*a**4*b**6*x**15 + 15*a**3*
b**7*x**16/2 + 45*a**2*b**8*x**17/17 + 5*a*b**9*x**18/9 + b**10*x**19/19
```

Giac [A]
time = 0.00, size = 125, normalized size = 0.85

$$\frac{1}{19}x^{19}b^{10}+\frac{5}{9}x^{18}b^{9}a+\frac{45}{17}x^{17}b^{8}a^{2}+\frac{15}{2}x^{16}b^{7}a^{3}+14x^{15}b^{6}a^{4}+18x^{14}b^{5}a^{5}+\frac{210}{13}x^{13}b^{4}a^{6}+10x^{12}b^{3}a^{7}+\frac{45}{11}x^{11}b^{2}a^{8}+x^{10}ba^{9}+\frac{1}{9}x^{9}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x)
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Mupad [B]
time = 0.09, size = 111, normalized size = 0.76

$$\frac{a^{10}\,x^{9}}{9}+a^{9}\,b\,x^{10}+\frac{45\,a^{8}\,b^{2}\,x^{11}}{11}+10\,a^{7}\,b^{3}\,x^{12}+\frac{210\,a^{6}\,b^{4}\,x^{13}}{13}+18\,a^{5}\,b^{5}\,x^{14}+14\,a^{4}\,b^{6}\,x^{15}+\frac{15\,a^{3}\,b^{7}\,x^{16}}{2}+\frac{45\,a^{2}\,b^{8}\,x^{17}}{17}+\frac{5\,a\,b^{9}\,x^{18}}{9}+\frac{b^{10}\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^10,x)
```

```
[Out] (a^10*x^9)/9 + (b^10*x^19)/19 + a^9*b*x^10 + (5*a*b^9*x^18)/9 + (45*a^8*b^2
*x^11)/11 + 10*a^7*b^3*x^12 + (210*a^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*
a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 + (45*a^2*b^8*x^17)/17
```

3.127 $\int x^7(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a(a+b}{17b^8}$$

```
[Out] -1/11*a^7*(b*x+a)^11/b^8+7/12*a^6*(b*x+a)^12/b^8-21/13*a^5*(b*x+a)^13/b^8+5
/2*a^4*(b*x+a)^14/b^8-7/3*a^3*(b*x+a)^15/b^8+21/16*a^2*(b*x+a)^16/b^8-7/17*
a*(b*x+a)^17/b^8+1/18*(b*x+a)^18/b^8
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^10,x]
```

```
[Out] -1/11*(a^7*(a + b*x)^11)/b^8 + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a +
 b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/(3
*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (a +
 b*x)^18/(18*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^{10}\,dx &= \int\left(-\frac{a^7(a+bx)^{10}}{b^7}+\frac{7a^6(a+bx)^{11}}{b^7}-\frac{21a^5(a+bx)^{12}}{b^7}+\frac{35a^4(a+bx)^{13}}{b^7}-\frac{35a^3(a+b}{b^7}\right.\\ &= -\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 130, normalized size = 0.98

$$\frac{a^{10}x^8}{8}+\frac{10}{9}a^9bx^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}a^2b^8x^{16}+\frac{10}{17}ab^9x^{17}+\frac{b^{10}x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^10,x]
```

```
[Out] (a^10*x^8)/8 + (10*a^9*b*x^9)/9 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/1
1 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b
^7*x^15 + (45*a^2*b^8*x^16)/16 + (10*a*b^9*x^17)/17 + (b^10*x^18)/18
```

Mathics [A]
time = 2.33, size = 112, normalized size = 0.85

$$\frac{x^8\left(43758a^{10}+388960a^9bx+1575288a^8b^2x^2+3818880a^7b^3x^3+6126120a^6b^4x^4+6785856a^5b^5x^5+5250960a^4b^6x^6+2800512a^3b^7x^7+984555a^2b^8x^8+205920ab^9x^9+19448b^{10}x^{10}\right)}{350064}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^7*(a + b*x)^10,x]')
```

```
[Out] x ^ 8 (43758 a ^ 10 + 388960 a ^ 9 b x + 1575288 a ^ 8 b ^ 2 x ^ 2 + 381888
0 a ^ 7 b ^ 3 x ^ 3 + 6126120 a ^ 6 b ^ 4 x ^ 4 + 6785856 a ^ 5 b ^ 5 x ^ 5
 + 5250960 a ^ 4 b ^ 6 x ^ 6 + 2800512 a ^ 3 b ^ 7 x ^ 7 + 984555 a ^ 2 b ^
 8 x ^ 8 + 205920 a b ^ 9 x ^ 9 + 19448 b ^ 10 x ^ 10) / 350064
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}$
default	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}$
norman	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}$
risch	$\frac{1}{8}a^{10}x^8+\frac{10}{9}a^9b\,x^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^10*x^8+10/9*a^9*b*x^9+9/2*a^8*b^2*x^10+120/11*a^7*b^3*x^11+35/2*a^6*b
^4*x^12+252/13*a^5*b^5*x^13+15*a^4*b^6*x^14+8*a^3*b^7*x^15+45/16*a^2*b^8*x^
16+10/17*a*b^9*x^17+1/18*b^10*x^18
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.85

$$\frac{1}{18}\,b^{10}x^{18}+\frac{10}{17}\,ab^9x^{17}+\frac{45}{16}\,a^2b^8x^{16}+8\,a^3b^7x^{15}+15\,a^4b^6x^{14}+\frac{252}{13}\,a^5b^5x^{13}+\frac{35}{2}\,a^6b^4x^{12}+\frac{120}{11}\,a^7b^3x^{11}+\frac{9}{2}\,a^8b^2x^{10}+\frac{10}{9}\,a^9bx^9+\frac{1}{8}\,a^{10}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Fricas [A]
time = 0.30, size = 112, normalized size = 0.85

$$\frac{1}{18}\,b^{10}x^{18}+\frac{10}{17}\,ab^{9}x^{17}+\frac{45}{16}\,a^{2}b^{8}x^{16}+8\,a^{3}b^{7}x^{15}+15\,a^{4}b^{6}x^{14}+\frac{252}{13}\,a^{5}b^{5}x^{13}+\frac{35}{2}\,a^{6}b^{4}x^{12}+\frac{120}{11}\,a^{7}b^{3}x^{11}+\frac{9}{2}\,a^{8}b^{2}x^{10}+\frac{10}{9}\,a^{9}bx^{9}+\frac{1}{8}\,a^{10}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Sympy [A]
time = 0.04, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{8}}{8}+\frac{10a^{9}bx^{9}}{9}+\frac{9a^{8}b^{2}x^{10}}{2}+\frac{120a^{7}b^{3}x^{11}}{11}+\frac{35a^{6}b^{4}x^{12}}{2}+\frac{252a^{5}b^{5}x^{13}}{13}+15a^{4}b^{6}x^{14}+8a^{3}b^{7}x^{15}+\frac{45a^{2}b^{8}x^{16}}{16}+\frac{10ab^{9}x^{17}}{17}+\frac{b^{10}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**10,x)
```

```
[Out] a**10*x**8/8 + 10*a**9*b*x**9/9 + 9*a**8*b**2*x**10/2 + 120*a**7*b**3*x**11
/11 + 35*a**6*b**4*x**12/2 + 252*a**5*b**5*x**13/13 + 15*a**4*b**6*x**14 +
8*a**3*b**7*x**15 + 45*a**2*b**8*x**16/16 + 10*a*b**9*x**17/17 + b**10*x**1
8/18
```

Giac [A]
time = 0.00, size = 130, normalized size = 0.98

$$\frac{1}{18}x^{18}b^{10}+\frac{10}{17}x^{17}b^{9}a+\frac{45}{16}x^{16}b^{8}a^{2}+8x^{15}b^{7}a^{3}+15x^{14}b^{6}a^{4}+\frac{252}{13}x^{13}b^{5}a^{5}+\frac{35}{2}x^{12}b^{4}a^{6}+\frac{120}{11}x^{11}b^{3}a^{7}+\frac{9}{2}x^{10}b^{2}a^{8}+\frac{10}{9}x^{9}ba^{9}+\frac{1}{8}x^{8}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x)
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Mupad [B]
time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{8}}{8}+\frac{10\,a^{9}\,b\,x^{9}}{9}+\frac{9\,a^{8}\,b^{2}\,x^{10}}{2}+\frac{120\,a^{7}\,b^{3}\,x^{11}}{11}+\frac{35\,a^{6}\,b^{4}\,x^{12}}{2}+\frac{252\,a^{5}\,b^{5}\,x^{13}}{13}+15\,a^{4}\,b^{6}\,x^{14}+8\,a^{3}\,b^{7}\,x^{15}+\frac{45\,a^{2}\,b^{8}\,x^{16}}{16}+\frac{10\,a\,b^{9}\,x^{17}}{17}+\frac{b^{10}\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^10,x)
```

```
[Out] (a^10*x^8)/8 + (b^10*x^18)/18 + (10*a^9*b*x^9)/9 + (10*a*b^9*x^17)/17 + (9*
a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/11 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^
5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b^7*x^15 + (45*a^2*b^8*x^16)/16
```

3.128 $\int x^6(a+bx)^{10}\,dx$

Optimal. Leaf size=112

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a(a+bx)^{16}}{8b^7}+\frac{(a+bx)^{17}}{17b^7}$$

```
[Out] 1/11*a^6*(b*x+a)^11/b^7-1/2*a^5*(b*x+a)^12/b^7+15/13*a^4*(b*x+a)^13/b^7-10/
7*a^3*(b*x+a)^14/b^7+a^2*(b*x+a)^15/b^7-3/8*a*(b*x+a)^16/b^7+1/17*(b*x+a)^1
7/b^7
```

Rubi [A]
time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^10,x]
```

```
[Out] (a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*x
)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 - (
3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^{10}\,dx &= \int \left(\frac{a^6(a+bx)^{10}}{b^6}-\frac{6a^5(a+bx)^{11}}{b^6}+\frac{15a^4(a+bx)^{12}}{b^6}-\frac{20a^3(a+bx)^{13}}{b^6}+\frac{15a^2(a+bx)}{b^6}\right.\\ &= \frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-3a\end{aligned}$$

Mathematica [A]
time = 0.00, size = 126, normalized size = 1.12

$$\frac{a^{10}x^7}{7}+\frac{5}{4}a^9bx^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2b^8x^{15}+\frac{5}{8}ab^9x^{16}+\frac{b^{10}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^10,x]
```

```
[Out] (a^10*x^7)/7 + (5*a^9*b*x^8)/4 + 5*a^8*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6
*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)
/7 + 3*a^2*b^8*x^15 + (5*a*b^9*x^16)/8 + (b^10*x^17)/17
```

Mathics [A]
time = 2.27, size = 112, normalized size = 1.00

$$\frac{x^7\left(19448a^{10}+170170a^9bx+680680a^8b^2x^2+1633632a^7b^3x^3+2598960a^6b^4x^4+2858856a^5b^5x^5+2199120a^4b^6x^6+1166880a^3b^7x^7+408408a^2b^8x^8+85085ab^9x^9+8008b^{10}x^{10}\right)}{136136}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6*(a + b*x)^10,x]')
```

```
[Out] x ^ 7 (19448 a ^ 10 + 170170 a ^ 9 b x + 680680 a ^ 8 b ^ 2 x ^ 2 + 1633632
 a ^ 7 b ^ 3 x ^ 3 + 2598960 a ^ 6 b ^ 4 x ^ 4 + 2858856 a ^ 5 b ^ 5 x ^ 5
+ 2199120 a ^ 4 b ^ 6 x ^ 6 + 1166880 a ^ 3 b ^ 7 x ^ 7 + 408408 a ^ 2 b ^
8 x ^ 8 + 85085 a b ^ 9 x ^ 9 + 8008 b ^ 10 x ^ 10) / 136136
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.01

method	result
gosper	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2$
default	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2$
norman	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2$
risch	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^10*x^7+5/4*a^9*b*x^8+5*a^8*b^2*x^9+12*a^7*b^3*x^10+210/11*a^6*b^4*x^1
1+21*a^5*b^5*x^12+210/13*a^4*b^6*x^13+60/7*a^3*b^7*x^14+3*a^2*b^8*x^15+5/8*
a*b^9*x^16+1/17*b^10*x^17
```

Maxima [A]
time = 0.25, size = 112, normalized size = 1.00

$$\frac{1}{17}\,b^{10}x^{17}+\frac{5}{8}\,ab^9x^{16}+3\,a^2b^8x^{15}+\frac{60}{7}\,a^3b^7x^{14}+\frac{210}{13}\,a^4b^6x^{13}+21\,a^5b^5x^{12}+\frac{210}{11}\,a^6b^4x^{11}+12\,a^7b^3x^{10}+5\,a^8b^2x^9+\frac{5}{4}\,a^9bx^8+\frac{1}{7}\,a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Fricas [A]
time = 0.30, size = 112, normalized size = 1.00

$$\frac{1}{17}\,b^{10}x^{17}+\frac{5}{8}\,ab^{9}x^{16}+3\,a^{2}b^{8}x^{15}+\frac{60}{7}\,a^{3}b^{7}x^{14}+\frac{210}{13}\,a^{4}b^{6}x^{13}+21\,a^{5}b^{5}x^{12}+\frac{210}{11}\,a^{6}b^{4}x^{11}+12\,a^{7}b^{3}x^{10}+5\,a^{8}b^{2}x^{9}+\frac{5}{4}\,a^{9}bx^{8}+\frac{1}{7}\,a^{10}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Sympy [A]
time = 0.04, size = 128, normalized size = 1.14

$$\frac{a^{10}x^{7}}{7}+\frac{5a^{9}bx^{8}}{4}+5a^{8}b^{2}x^{9}+12a^{7}b^{3}x^{10}+\frac{210a^{6}b^{4}x^{11}}{11}+21a^{5}b^{5}x^{12}+\frac{210a^{4}b^{6}x^{13}}{13}+\frac{60a^{3}b^{7}x^{14}}{7}+3a^{2}b^{8}x^{15}+\frac{5ab^{9}x^{16}}{8}+\frac{b^{10}x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**10,x)
```

```
[Out] a**10*x**7/7 + 5*a**9*b*x**8/4 + 5*a**8*b**2*x**9 + 12*a**7*b**3*x**10 + 21
0*a**6*b**4*x**11/11 + 21*a**5*b**5*x**12 + 210*a**4*b**6*x**13/13 + 60*a**
3*b**7*x**14/7 + 3*a**2*b**8*x**15 + 5*a*b**9*x**16/8 + b**10*x**17/17
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.12

$$\frac{1}{17}x^{17}b^{10}+\frac{5}{8}x^{16}b^{9}a+3x^{15}b^{8}a^{2}+\frac{60}{7}x^{14}b^{7}a^{3}+\frac{210}{13}x^{13}b^{6}a^{4}+21x^{12}b^{5}a^{5}+\frac{210}{11}x^{11}b^{4}a^{6}+12x^{10}b^{3}a^{7}+5x^{9}b^{2}a^{8}+\frac{5}{4}x^{8}ba^{9}+\frac{1}{7}x^{7}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x)
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.00

$$\frac{a^{10}\,x^{7}}{7}+\frac{5\,a^{9}\,b\,x^{8}}{4}+5\,a^{8}\,b^{2}\,x^{9}+12\,a^{7}\,b^{3}\,x^{10}+\frac{210\,a^{6}\,b^{4}\,x^{11}}{11}+21\,a^{5}\,b^{5}\,x^{12}+\frac{210\,a^{4}\,b^{6}\,x^{13}}{13}+\frac{60\,a^{3}\,b^{7}\,x^{14}}{7}+3\,a^{2}\,b^{8}\,x^{15}+\frac{5\,a\,b^{9}\,x^{16}}{8}+\frac{b^{10}\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^10,x)
```

```
[Out] (a^10*x^7)/7 + (b^10*x^17)/17 + (5*a^9*b*x^8)/4 + (5*a*b^9*x^16)/8 + 5*a^8*
b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*
a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)/7 + 3*a^2*b^8*x^15
```

3.129 $\int x^5(a+bx)^{10}\,dx$

Optimal. Leaf size=98

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\frac{(a+bx)^{16}}{16b^6}$$

```
[Out] -1/11*a^5*(b*x+a)^11/b^6+5/12*a^4*(b*x+a)^12/b^6-10/13*a^3*(b*x+a)^13/b^6+5
/7*a^2*(b*x+a)^14/b^6-1/3*a*(b*x+a)^15/b^6+1/16*(b*x+a)^16/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^10,x]
```

```
[Out] -1/11*(a^5*(a + b*x)^11)/b^6 + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a +
 b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b^6
) + (a + b*x)^16/(16*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{10}\,dx &= \int \left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a+bx)^{14}}{b^5}\right.\\ &= -\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+(a\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.35

$$\frac{a^{10}x^6}{6}+\frac{10}{7}a^9bx^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+\frac{45}{14}a^2b^8x^{14}+\frac{2}{3}ab^9x^{15}+\frac{b^{10}x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^10,x]
```

```
[Out] (a^10*x^6)/6 + (10*a^9*b*x^7)/7 + (45*a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 +
 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/11 + (35*a^4*b^6*x^12)/2 + (120*a^3*b
^7*x^13)/13 + (45*a^2*b^8*x^14)/14 + (2*a*b^9*x^15)/3 + (b^10*x^16)/16
```

Mathics [A]
time = 2.14, size = 112, normalized size = 1.14

$$\frac{x^6\left(8008a^{10}+68640a^9bx+270270a^8b^2x^2+640640a^7b^3x^3+1009008a^6b^4x^4+1100736a^5b^5x^5+840840a^4b^6x^6+443520a^3b^7x^7+154440a^2b^8x^8+32032ab^9x^9+3003b^{10}x^{10}\right)}{48048}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5*(a + b*x)^10,x]')
```

```
[Out] x ^ 6 (8008 a ^ 10 + 68640 a ^ 9 b x + 270270 a ^ 8 b ^ 2 x ^ 2 + 640640 a
^ 7 b ^ 3 x ^ 3 + 1009008 a ^ 6 b ^ 4 x ^ 4 + 1100736 a ^ 5 b ^ 5 x ^ 5 + 8
40840 a ^ 4 b ^ 6 x ^ 6 + 443520 a ^ 3 b ^ 7 x ^ 7 + 154440 a ^ 2 b ^ 8 x ^
 8 + 32032 a b ^ 9 x ^ 9 + 3003 b ^ 10 x ^ 10) / 48048
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.15

method	result
gosper	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+$
default	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+$
norman	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+$
risch	$\frac{1}{6}a^{10}x^6+\frac{10}{7}a^9b\,x^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^10*x^6+10/7*a^9*b*x^7+45/8*a^8*b^2*x^8+40/3*a^7*b^3*x^9+21*a^6*b^4*x^
10+252/11*a^5*b^5*x^11+35/2*a^4*b^6*x^12+120/13*a^3*b^7*x^13+45/14*a^2*b^8*
x^14+2/3*a*b^9*x^15+1/16*b^10*x^16
```

Maxima [A]
time = 0.24, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^9x^{15}+\frac{45}{14}\,a^2b^8x^{14}+\frac{120}{13}\,a^3b^7x^{13}+\frac{35}{2}\,a^4b^6x^{12}+\frac{252}{11}\,a^5b^5x^{11}+21\,a^6b^4x^{10}+\frac{40}{3}\,a^7b^3x^9+\frac{45}{8}\,a^8b^2x^8+\frac{10}{7}\,a^9bx^7+\frac{1}{6}\,a^{10}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Fricas [A]
time = 0.30, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^{9}x^{15}+\frac{45}{14}\,a^{2}b^{8}x^{14}+\frac{120}{13}\,a^{3}b^{7}x^{13}+\frac{35}{2}\,a^{4}b^{6}x^{12}+\frac{252}{11}\,a^{5}b^{5}x^{11}+21\,a^{6}b^{4}x^{10}+\frac{40}{3}\,a^{7}b^{3}x^{9}+\frac{45}{8}\,a^{8}b^{2}x^{8}+\frac{10}{7}\,a^{9}bx^{7}+\frac{1}{6}\,a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Sympy [A]
time = 0.05, size = 133, normalized size = 1.36

$$\frac{a^{10}x^{6}}{6}+\frac{10a^{9}bx^{7}}{7}+\frac{45a^{8}b^{2}x^{8}}{8}+\frac{40a^{7}b^{3}x^{9}}{3}+21a^{6}b^{4}x^{10}+\frac{252a^{5}b^{5}x^{11}}{11}+\frac{35a^{4}b^{6}x^{12}}{2}+\frac{120a^{3}b^{7}x^{13}}{13}+\frac{45a^{2}b^{8}x^{14}}{14}+\frac{2ab^{9}x^{15}}{3}+\frac{b^{10}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**10,x)
```

```
[Out] a**10*x**6/6 + 10*a**9*b*x**7/7 + 45*a**8*b**2*x**8/8 + 40*a**7*b**3*x**9/3
 + 21*a**6*b**4*x**10 + 252*a**5*b**5*x**11/11 + 35*a**4*b**6*x**12/2 + 120
*a**3*b**7*x**13/13 + 45*a**2*b**8*x**14/14 + 2*a*b**9*x**15/3 + b**10*x**1
6/16
```

Giac [A]
time = 0.00, size = 132, normalized size = 1.35

$$\frac{1}{16}x^{16}b^{10}+\frac{2}{3}x^{15}b^{9}a+\frac{45}{14}x^{14}b^{8}a^{2}+\frac{120}{13}x^{13}b^{7}a^{3}+\frac{35}{2}x^{12}b^{6}a^{4}+\frac{252}{11}x^{11}b^{5}a^{5}+21x^{10}b^{4}a^{6}+\frac{40}{3}x^{9}b^{3}a^{7}+\frac{45}{8}x^{8}b^{2}a^{8}+\frac{10}{7}x^{7}ba^{9}+\frac{1}{6}x^{6}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x)
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.14

$$\frac{a^{10}\,x^{6}}{6}+\frac{10\,a^{9}\,b\,x^{7}}{7}+\frac{45\,a^{8}\,b^{2}\,x^{8}}{8}+\frac{40\,a^{7}\,b^{3}\,x^{9}}{3}+21\,a^{6}\,b^{4}\,x^{10}+\frac{252\,a^{5}\,b^{5}\,x^{11}}{11}+\frac{35\,a^{4}\,b^{6}\,x^{12}}{2}+\frac{120\,a^{3}\,b^{7}\,x^{13}}{13}+\frac{45\,a^{2}\,b^{8}\,x^{14}}{14}+\frac{2\,a\,b^{9}\,x^{15}}{3}+\frac{b^{10}\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^10,x)
```

```
[Out] (a^10*x^6)/6 + (b^10*x^16)/16 + (10*a^9*b*x^7)/7 + (2*a*b^9*x^15)/3 + (45*a
^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 + 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/1
1 + (35*a^4*b^6*x^12)/2 + (120*a^3*b^7*x^13)/13 + (45*a^2*b^8*x^14)/14
```

3.130 $\int x^4(a+bx)^{10}\,dx$

Optimal. Leaf size=81

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}$$

```
[Out] 1/11*a^4*(b*x+a)^11/b^5-1/3*a^3*(b*x+a)^12/b^5+6/13*a^2*(b*x+a)^13/b^5-2/7*
a*(b*x+a)^14/b^5+1/15*(b*x+a)^15/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^10,x]
```

```
[Out] (a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x)
^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{10}\,dx&=\int\left(\frac{a^4(a+bx)^{10}}{b^4}-\frac{4a^3(a+bx)^{11}}{b^4}+\frac{6a^2(a+bx)^{12}}{b^4}-\frac{4a(a+bx)^{13}}{b^4}+\frac{(a+bx)^{14}}{b^4}\right)dx\\&=\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 130, normalized size = 1.60

$$\frac{a^{10}x^5}{5}+\frac{5}{3}a^9bx^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a^2b^8x^{13}+\frac{5}{7}ab^9x^{14}+\frac{b^{10}x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^10,x]
```

```
[Out] (a^10*x^5)/5 + (5*a^9*b*x^6)/3 + (45*a^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*
a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*
x^12 + (45*a^2*b^8*x^13)/13 + (5*a*b^9*x^14)/7 + (b^10*x^15)/15
```

Mathics [A]
time = 2.20, size = 112, normalized size = 1.38

$$\frac{x^5\left(3003a^{10}+25025a^9bx+96525a^8b^2x^2+225225a^7b^3x^3+350350a^6b^4x^4+378378a^5b^5x^5+286650a^4b^6x^6+150150a^3b^7x^7+51975a^2b^8x^8+10725ab^9x^9+1001b^{10}x^{10}\right)}{15015}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4*(a + b*x)^10,x]')
```

```
[Out] x ^ 5 (3003 a ^ 10 + 25025 a ^ 9 b x + 96525 a ^ 8 b ^ 2 x ^ 2 + 225225 a ^
 7 b ^ 3 x ^ 3 + 350350 a ^ 6 b ^ 4 x ^ 4 + 378378 a ^ 5 b ^ 5 x ^ 5 + 2866
50 a ^ 4 b ^ 6 x ^ 6 + 150150 a ^ 3 b ^ 7 x ^ 7 + 51975 a ^ 2 b ^ 8 x ^ 8 +
 10725 a b ^ 9 x ^ 9 + 1001 b ^ 10 x ^ 10) / 15015
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.40

method	result
gosper	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}$
default	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}$
norman	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}$
risch	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^10*x^5+5/3*a^9*b*x^6+45/7*a^8*b^2*x^7+15*a^7*b^3*x^8+70/3*a^6*b^4*x^9
+126/5*a^5*b^5*x^10+210/11*a^4*b^6*x^11+10*a^3*b^7*x^12+45/13*a^2*b^8*x^13+
5/7*a*b^9*x^14+1/15*b^10*x^15
```

Maxima [A]
time = 0.25, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15}+\frac{5}{7}\,ab^9x^{14}+\frac{45}{13}\,a^2b^8x^{13}+10\,a^3b^7x^{12}+\frac{210}{11}\,a^4b^6x^{11}+\frac{126}{5}\,a^5b^5x^{10}+\frac{70}{3}\,a^6b^4x^9+15\,a^7b^3x^8+\frac{45}{7}\,a^8b^2x^7+\frac{5}{3}\,a^9bx^6+\frac{1}{5}\,a^{10}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Fricas [A]
time = 0.31, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15}+\frac{5}{7}\,ab^{9}x^{14}+\frac{45}{13}\,a^{2}b^{8}x^{13}+10\,a^{3}b^{7}x^{12}+\frac{210}{11}\,a^{4}b^{6}x^{11}+\frac{126}{5}\,a^{5}b^{5}x^{10}+\frac{70}{3}\,a^{6}b^{4}x^{9}+15\,a^{7}b^{3}x^{8}+\frac{45}{7}\,a^{8}b^{2}x^{7}+\frac{5}{3}\,a^{9}bx^{6}+\frac{1}{5}\,a^{10}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Sympy [A]
time = 0.04, size = 131, normalized size = 1.62

$$\frac{a^{10}x^{5}}{5}+\frac{5a^{9}bx^{6}}{3}+\frac{45a^{8}b^{2}x^{7}}{7}+15a^{7}b^{3}x^{8}+\frac{70a^{6}b^{4}x^{9}}{3}+\frac{126a^{5}b^{5}x^{10}}{5}+\frac{210a^{4}b^{6}x^{11}}{11}+10a^{3}b^{7}x^{12}+\frac{45a^{2}b^{8}x^{13}}{13}+\frac{5ab^{9}x^{14}}{7}+\frac{b^{10}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**10,x)
```

```
[Out] a**10*x**5/5 + 5*a**9*b*x**6/3 + 45*a**8*b**2*x**7/7 + 15*a**7*b**3*x**8 +
70*a**6*b**4*x**9/3 + 126*a**5*b**5*x**10/5 + 210*a**4*b**6*x**11/11 + 10*a
**3*b**7*x**12 + 45*a**2*b**8*x**13/13 + 5*a*b**9*x**14/7 + b**10*x**15/15
```

Giac [A]
time = 0.00, size = 130, normalized size = 1.60

$$\frac{1}{15}x^{15}b^{10}+\frac{5}{7}x^{14}b^{9}a+\frac{45}{13}x^{13}b^{8}a^{2}+10x^{12}b^{7}a^{3}+\frac{210}{11}x^{11}b^{6}a^{4}+\frac{126}{5}x^{10}b^{5}a^{5}+\frac{70}{3}x^{9}b^{4}a^{6}+15x^{8}b^{3}a^{7}+\frac{45}{7}x^{7}b^{2}a^{8}+\frac{5}{3}x^{6}ba^{9}+\frac{1}{5}x^{5}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x)
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.38

$$\frac{a^{10}\,x^{5}}{5}+\frac{5\,a^{9}\,b\,x^{6}}{3}+\frac{45\,a^{8}\,b^{2}\,x^{7}}{7}+15\,a^{7}\,b^{3}\,x^{8}+\frac{70\,a^{6}\,b^{4}\,x^{9}}{3}+\frac{126\,a^{5}\,b^{5}\,x^{10}}{5}+\frac{210\,a^{4}\,b^{6}\,x^{11}}{11}+10\,a^{3}\,b^{7}\,x^{12}+\frac{45\,a^{2}\,b^{8}\,x^{13}}{13}+\frac{5\,a\,b^{9}\,x^{14}}{7}+\frac{b^{10}\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^10,x)
```

```
[Out] (a^10*x^5)/5 + (b^10*x^15)/15 + (5*a^9*b*x^6)/3 + (5*a*b^9*x^14)/7 + (45*a^
8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 +
 (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*x^12 + (45*a^2*b^8*x^13)/13
```

3.131 $\int x^3(a+bx)^{10}\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}$$

```
[Out] -1/11*a^3*(b*x+a)^11/b^4+1/4*a^2*(b*x+a)^12/b^4-3/13*a*(b*x+a)^13/b^4+1/14*
(b*x+a)^14/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^10,x]
```

```
[Out] -1/11*(a^3*(a + b*x)^11)/b^4 + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)^
13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{10}\,dx&=\int\left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}+\frac{(a+bx)^{13}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 128, normalized size = 2.00

$$\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2b^8x^{12}+\frac{10}{13}ab^9x^{13}+\frac{b^{10}x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^10,x]
```

```
[Out] (a^10*x^4)/4 + 2*a^9*b*x^5 + (15*a^8*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (10
5*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11
 + (15*a^2*b^8*x^12)/4 + (10*a*b^9*x^13)/13 + (b^10*x^14)/14
```

Mathics [A]
time = 2.18, size = 112, normalized size = 1.75

$$\frac{x^4\left(1001a^{10}+8008a^9bx+30030a^8b^2x^2+68640a^7b^3x^3+105105a^6b^4x^4+112112a^5b^5x^5+84084a^4b^6x^6+43680a^3b^7x^7+15015a^2b^8x^8+3080ab^9x^9+286b^{10}x^{10}\right)}{4004}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(a + b*x)^10,x]')
```

```
[Out] x ^ 4 (1001 a ^ 10 + 8008 a ^ 9 b x + 30030 a ^ 8 b ^ 2 x ^ 2 + 68640 a ^ 7
 b ^ 3 x ^ 3 + 105105 a ^ 6 b ^ 4 x ^ 4 + 112112 a ^ 5 b ^ 5 x ^ 5 + 84084
a ^ 4 b ^ 6 x ^ 6 + 43680 a ^ 3 b ^ 7 x ^ 7 + 15015 a ^ 2 b ^ 8 x ^ 8 + 308
0 a b ^ 9 x ^ 9 + 286 b ^ 10 x ^ 10) / 4004
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.77

method	result
gosper	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2$
default	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2$
norman	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2$
risch	$\frac{1}{4}a^{10}x^4+2a^9b\,x^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^10*x^4+2*a^9*b*x^5+15/2*a^8*b^2*x^6+120/7*a^7*b^3*x^7+105/4*a^6*b^4*x
^8+28*a^5*b^5*x^9+21*a^4*b^6*x^10+120/11*a^3*b^7*x^11+15/4*a^2*b^8*x^12+10/
13*a*b^9*x^13+1/14*b^10*x^14
```

Maxima [A]
time = 0.25, size = 112, normalized size = 1.75

$$\frac{1}{14}\,b^{10}x^{14}+\frac{10}{13}\,ab^9x^{13}+\frac{15}{4}\,a^2b^8x^{12}+\frac{120}{11}\,a^3b^7x^{11}+21\,a^4b^6x^{10}+28\,a^5b^5x^9+\frac{105}{4}\,a^6b^4x^8+\frac{120}{7}\,a^7b^3x^7+\frac{15}{2}\,a^8b^2x^6+2\,a^9bx^5+\frac{1}{4}\,a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Fricas [A]
time = 0.31, size = 112, normalized size = 1.75

$$\frac{1}{14}\,b^{10}x^{14}+\frac{10}{13}\,ab^{9}x^{13}+\frac{15}{4}\,a^{2}b^{8}x^{12}+\frac{120}{11}\,a^{3}b^{7}x^{11}+21\,a^{4}b^{6}x^{10}+28\,a^{5}b^{5}x^{9}+\frac{105}{4}\,a^{6}b^{4}x^{8}+\frac{120}{7}\,a^{7}b^{3}x^{7}+\frac{15}{2}\,a^{8}b^{2}x^{6}+2\,a^{9}bx^{5}+\frac{1}{4}\,a^{10}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.
time = 0.05, size = 129, normalized size = 2.02

$$\frac{a^{10}x^{4}}{4}+2a^{9}bx^{5}+\frac{15a^{8}b^{2}x^{6}}{2}+\frac{120a^{7}b^{3}x^{7}}{7}+\frac{105a^{6}b^{4}x^{8}}{4}+28a^{5}b^{5}x^{9}+21a^{4}b^{6}x^{10}+\frac{120a^{3}b^{7}x^{11}}{11}+\frac{15a^{2}b^{8}x^{12}}{4}+\frac{10ab^{9}x^{13}}{13}+\frac{b^{10}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**10,x)
```

```
[Out] a**10*x**4/4 + 2*a**9*b*x**5 + 15*a**8*b**2*x**6/2 + 120*a**7*b**3*x**7/7 +
 105*a**6*b**4*x**8/4 + 28*a**5*b**5*x**9 + 21*a**4*b**6*x**10 + 120*a**3*b
**7*x**11/11 + 15*a**2*b**8*x**12/4 + 10*a*b**9*x**13/13 + b**10*x**14/14
```

Giac [A]
time = 0.00, size = 128, normalized size = 2.00

$$\frac{1}{14}x^{14}b^{10}+\frac{10}{13}x^{13}b^{9}a+\frac{15}{4}x^{12}b^{8}a^{2}+\frac{120}{11}x^{11}b^{7}a^{3}+21x^{10}b^{6}a^{4}+28x^{9}b^{5}a^{5}+\frac{105}{4}x^{8}b^{4}a^{6}+\frac{120}{7}x^{7}b^{3}a^{7}+\frac{15}{2}x^{6}b^{2}a^{8}+2x^{5}ba^{9}+\frac{1}{4}x^{4}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x)
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.75

$$\frac{a^{10}\,x^4}{4}+2\,a^9\,b\,x^5+\frac{15\,a^8\,b^2\,x^6}{2}+\frac{120\,a^7\,b^3\,x^7}{7}+\frac{105\,a^6\,b^4\,x^8}{4}+28\,a^5\,b^5\,x^9+21\,a^4\,b^6\,x^{10}+\frac{120\,a^3\,b^7\,x^{11}}{11}+\frac{15\,a^2\,b^8\,x^{12}}{4}+\frac{10\,a\,b^9\,x^{13}}{13}+\frac{b^{10}\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^10,x)
```

```
[Out] (a^10*x^4)/4 + (b^10*x^14)/14 + 2*a^9*b*x^5 + (10*a*b^9*x^13)/13 + (15*a^8*
b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (105*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 2
1*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11 + (15*a^2*b^8*x^12)/4
```

3.132 $\int x^2(a+bx)^{10}\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}$$

```
[Out] 1/11*a^2*(b*x+a)^11/b^3-1/6*a*(b*x+a)^12/b^3+1/13*(b*x+a)^13/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^10,x]
```

```
[Out] (a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*b
^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{10}\,dx&=\int\left(\frac{a^2(a+bx)^{10}}{b^2}-\frac{2a(a+bx)^{11}}{b^2}+\frac{(a+bx)^{12}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(47) = 94.
time = 0.00, size = 126, normalized size = 2.68

$$\frac{a^{10}x^3}{3}+\frac{5}{2}a^9bx^4+9a^8b^2x^5+20a^7b^3x^6+30a^6b^4x^7+\frac{63}{2}a^5b^5x^8+\frac{70}{3}a^4b^6x^9+12a^3b^7x^{10}+\frac{45}{11}a^2b^8x^{11}+\frac{5}{6}ab^9x^{12}+\frac{b^{10}x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^10,x]
```

```
[Out] (a^10*x^3)/3 + (5*a^9*b*x^4)/2 + 9*a^8*b^2*x^5 + 20*a^7*b^3*x^6 + 30*a^6*b^
4*x^7 + (63*a^5*b^5*x^8)/2 + (70*a^4*b^6*x^9)/3 + 12*a^3*b^7*x^10 + (45*a^2
*b^8*x^11)/11 + (5*a*b^9*x^12)/6 + (b^10*x^13)/13
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.
time = 2.21, size = 112, normalized size = 2.38

$$\frac{x^3 \left(286a^{10} + 2145a^9bx + 7722a^8b^2x^2 + 17160a^7b^3x^3 + 25740a^6b^4x^4 + 27027a^5b^5x^5 + 20020a^4b^6x^6 + 10296a^3b^7x^7 + 3510a^2b^8x^8 + 715ab^9x^9 + 66b^{10}x^{10}\right)}{858}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x)^10,x]')
```

```
[Out] x ^ 3 (286 a ^ 10 + 2145 a ^ 9 b x + 7722 a ^ 8 b ^ 2 x ^ 2 + 17160 a ^ 7 b
 ^ 3 x ^ 3 + 25740 a ^ 6 b ^ 4 x ^ 4 + 27027 a ^ 5 b ^ 5 x ^ 5 + 20020 a ^
4 b ^ 6 x ^ 6 + 10296 a ^ 3 b ^ 7 x ^ 7 + 3510 a ^ 2 b ^ 8 x ^ 8 + 715 a b
^ 9 x ^ 9 + 66 b ^ 10 x ^ 10) / 858
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.08, size = 113, normalized size = 2.40

method	result
gosper	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a$
default	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a$
norman	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a$
risch	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^10*x^13+5/6*a*b^9*x^12+45/11*a^2*b^8*x^11+12*a^3*b^7*x^10+70/3*a^4*b
^6*x^9+63/2*a^5*b^5*x^8+30*a^6*b^4*x^7+20*a^7*b^3*x^6+9*a^8*b^2*x^5+5/2*a^9
*b*x^4+1/3*a^10*x^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.24, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.30, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13}+\frac{5}{6}\,ab^{9}x^{12}+\frac{45}{11}\,a^{2}b^{8}x^{11}+12\,a^{3}b^{7}x^{10}+\frac{70}{3}\,a^{4}b^{6}x^{9}+\frac{63}{2}\,a^{5}b^{5}x^{8}+30\,a^{6}b^{4}x^{7}+20\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{5}+\frac{5}{2}\,a^{9}bx^{4}+\frac{1}{3}\,a^{10}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.
time = 0.04, size = 128, normalized size = 2.72

$$\frac{a^{10}x^{3}}{3}+\frac{5a^{9}bx^{4}}{2}+9a^{8}b^{2}x^{5}+20a^{7}b^{3}x^{6}+30a^{6}b^{4}x^{7}+\frac{63a^{5}b^{5}x^{8}}{2}+\frac{70a^{4}b^{6}x^{9}}{3}+12a^{3}b^{7}x^{10}+\frac{45a^{2}b^{8}x^{11}}{11}+\frac{5ab^{9}x^{12}}{6}+\frac{b^{10}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**10,x)
```

```
[Out] a**10*x**3/3 + 5*a**9*b*x**4/2 + 9*a**8*b**2*x**5 + 20*a**7*b**3*x**6 + 30*
a**6*b**4*x**7 + 63*a**5*b**5*x**8/2 + 70*a**4*b**6*x**9/3 + 12*a**3*b**7*x
**10 + 45*a**2*b**8*x**11/11 + 5*a*b**9*x**12/6 + b**10*x**13/13
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.00, size = 126, normalized size = 2.68

$$\frac{1}{13}x^{13}b^{10}+\frac{5}{6}x^{12}b^{9}a+\frac{45}{11}x^{11}b^{8}a^{2}+12x^{10}b^{7}a^{3}+\frac{70}{3}x^{9}b^{6}a^{4}+\frac{63}{2}x^{8}b^{5}a^{5}+30x^{7}b^{4}a^{6}+20x^{6}b^{3}a^{7}+9x^{5}b^{2}a^{8}+\frac{5}{2}x^{4}ba^{9}+\frac{1}{3}x^{3}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x)
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Mupad [B]
time = 0.07, size = 31, normalized size = 0.66

$$\frac{{\left(a+b\,x\right)}^{11}\,\left(8\,a^2-88\,a\,b\,x+528\,b^2\,x^2\right)}{6864\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^10,x)
[Out] ((a + b*x)^11*(8*a^2 + 528*b^2*x^2 - 88*a*b*x))/(6864*b^3)
```

3.133 $\int x(a+bx)^{10}\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}$$

```
[Out] -1/11*a*(b*x+a)^11/b^2+1/12*(b*x+a)^12/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^10,x]
```

```
[Out] -1/11*(a*(a + b*x)^11)/b^2 + (a + b*x)^12/(12*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{10}\,dx &= \int \left(-\frac{a(a+bx)^{10}}{b}+\frac{(a+bx)^{11}}{b}\right)dx\\ &= -\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(30) = 60.
time = 0.00, size = 128, normalized size = 4.27

$$\frac{a^{10}x^2}{2}+\frac{10}{3}a^9bx^3+\frac{45}{4}a^8b^2x^4+24a^7b^3x^5+35a^6b^4x^6+36a^5b^5x^7+\frac{105}{4}a^4b^6x^8+\frac{40}{3}a^3b^7x^9+\frac{9}{2}a^2b^8x^{10}+\frac{10}{11}ab^9x^{11}+\frac{b^{10}x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^10,x]
```

```
[Out] (a^10*x^2)/2 + (10*a^9*b*x^3)/3 + (45*a^8*b^2*x^4)/4 + 24*a^7*b^3*x^5 + 35*
a^6*b^4*x^6 + 36*a^5*b^5*x^7 + (105*a^4*b^6*x^8)/4 + (40*a^3*b^7*x^9)/3 + (
9*a^2*b^8*x^10)/2 + (10*a*b^9*x^11)/11 + (b^10*x^12)/12
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(30) = 60.
time = 2.16, size = 112, normalized size = 3.73

$$\frac{x^2\left(66a^{10}+440a^9bx+1485a^8b^2x^2+3168a^7b^3x^3+4620a^6b^4x^4+4752a^5b^5x^5+3465a^4b^6x^6+1760a^3b^7x^7+594a^2b^8x^8+120ab^9x^9+11b^{10}x^{10}\right)}{132}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x)^10,x]')
```

```
[Out] x ^ 2 (66 a ^ 10 + 440 a ^ 9 b x + 1485 a ^ 8 b ^ 2 x ^ 2 + 3168 a ^ 7 b ^
3 x ^ 3 + 4620 a ^ 6 b ^ 4 x ^ 4 + 4752 a ^ 5 b ^ 5 x ^ 5 + 3465 a ^ 4 b ^
6 x ^ 6 + 1760 a ^ 3 b ^ 7 x ^ 7 + 594 a ^ 2 b ^ 8 x ^ 8 + 120 a b ^ 9 x ^
9 + 11 b ^ 10 x ^ 10) / 132
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.
time = 0.08, size = 113, normalized size = 3.77

method	result
gosper	$\frac{1}{12}b^{10}x^{12}+\frac{10}{11}a\,b^9x^{11}+\frac{9}{2}a^2b^8x^{10}+\frac{40}{3}a^3b^7x^9+\frac{105}{4}a^4b^6x^8+36a^5b^5x^7+35a^6b^4x^6+24a^7b^3x^5+\frac{45}{4}$
default	$\frac{1}{12}b^{10}x^{12}+\frac{10}{11}a\,b^9x^{11}+\frac{9}{2}a^2b^8x^{10}+\frac{40}{3}a^3b^7x^9+\frac{105}{4}a^4b^6x^8+36a^5b^5x^7+35a^6b^4x^6+24a^7b^3x^5+\frac{45}{4}$
norman	$\frac{1}{12}b^{10}x^{12}+\frac{10}{11}a\,b^9x^{11}+\frac{9}{2}a^2b^8x^{10}+\frac{40}{3}a^3b^7x^9+\frac{105}{4}a^4b^6x^8+36a^5b^5x^7+35a^6b^4x^6+24a^7b^3x^5+\frac{45}{4}$
risch	$\frac{1}{12}b^{10}x^{12}+\frac{10}{11}a\,b^9x^{11}+\frac{9}{2}a^2b^8x^{10}+\frac{40}{3}a^3b^7x^9+\frac{105}{4}a^4b^6x^8+36a^5b^5x^7+35a^6b^4x^6+24a^7b^3x^5+\frac{45}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^10*x^12+10/11*a*b^9*x^11+9/2*a^2*b^8*x^10+40/3*a^3*b^7*x^9+105/4*a^4
*b^6*x^8+36*a^5*b^5*x^7+35*a^6*b^4*x^6+24*a^7*b^3*x^5+45/4*a^8*b^2*x^4+10/3
*a^9*b*x^3+1/2*a^10*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.
time = 0.26, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12}+\frac{10}{11}\,ab^9x^{11}+\frac{9}{2}\,a^2b^8x^{10}+\frac{40}{3}\,a^3b^7x^9+\frac{105}{4}\,a^4b^6x^8+36\,a^5b^5x^7+35\,a^6b^4x^6+24\,a^7b^3x^5+\frac{45}{4}\,a^8b^2x^4+\frac{10}{3}\,a^9bx^3+\frac{1}{2}\,a^{10}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.
time = 0.31, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^{9}x^{11} + \frac{9}{2}\,a^{2}b^{8}x^{10} + \frac{40}{3}\,a^{3}b^{7}x^{9} + \frac{105}{4}\,a^{4}b^{6}x^{8} + 36\,a^{5}b^{5}x^{7} + 35\,a^{6}b^{4}x^{6} + 24\,a^{7}b^{3}x^{5} + \frac{45}{4}\,a^{8}b^{2}x^{4} + \frac{10}{3}\,a^{9}bx^{3} + \frac{1}{2}\,a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(24) = 48$.
time = 0.04, size = 129, normalized size = 4.30

$$\frac{a^{10}x^{2}}{2} + \frac{10a^{9}bx^{3}}{3} + \frac{45a^{8}b^{2}x^{4}}{4} + 24a^{7}b^{3}x^{5} + 35a^{6}b^{4}x^{6} + 36a^{5}b^{5}x^{7} + \frac{105a^{4}b^{6}x^{8}}{4} + \frac{40a^{3}b^{7}x^{9}}{3} + \frac{9a^{2}b^{8}x^{10}}{2} + \frac{10ab^{9}x^{11}}{11} + \frac{b^{10}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**10,x)
```

```
[Out] a**10*x**2/2 + 10*a**9*b*x**3/3 + 45*a**8*b**2*x**4/4 + 24*a**7*b**3*x**5 +
 35*a**6*b**4*x**6 + 36*a**5*b**5*x**7 + 105*a**4*b**6*x**8/4 + 40*a**3*b**
7*x**9/3 + 9*a**2*b**8*x**10/2 + 10*a*b**9*x**11/11 + b**10*x**12/12
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.
time = 0.00, size = 128, normalized size = 4.27

$$\frac{1}{12}x^{12}b^{10} + \frac{10}{11}x^{11}b^{9}a + \frac{9}{2}x^{10}b^{8}a^{2} + \frac{40}{3}x^{9}b^{7}a^{3} + \frac{105}{4}x^{8}b^{6}a^{4} + 36x^{7}b^{5}a^{5} + 35x^{6}b^{4}a^{6} + 24x^{5}b^{3}a^{7} + \frac{45}{4}x^{4}b^{2}a^{8} + \frac{10}{3}x^{3}ba^{9} + \frac{1}{2}x^{2}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x)
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Mupad [B]
time = 0.09, size = 25, normalized size = 0.83

$$-\frac{2\,\left(\frac{a\,{\left(a+b\,x\right)}^{11}}{22}-\frac{{\left(a+b\,x\right)}^{12}}{24}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^10,x)
```

```
[Out] -(2*((a*(a + b*x)^11)/22 - (a + b*x)^12/24))/b^2
```

3.134 $\int (a+bx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

[Out] 1/11*(b*x+a)^11/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^10,x]

[Out] (a + b*x)^11/(11*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{10}\,dx = \frac{(a+bx)^{11}}{11b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^10,x]

[Out] (a + b*x)^11/(11*b)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(14) = 28.
time = 2.10, size = 106, normalized size = 7.57

$$x\left(a^{10}+5a^9bx+15a^8b^2x^2+30a^7b^3x^3+42a^6b^4x^4+42a^5b^5x^5+30a^4b^6x^6+15a^3b^7x^7+5a^2b^8x^8+ab^9x^9+\frac{b^{10}x^{10}}{11}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^10,x]')
```

```
[Out] x (a ^ 10 + 5 a ^ 9 b x + 15 a ^ 8 b ^ 2 x ^ 2 + 30 a ^ 7 b ^ 3 x ^ 3 + 42
a ^ 6 b ^ 4 x ^ 4 + 42 a ^ 5 b ^ 5 x ^ 5 + 30 a ^ 4 b ^ 6 x ^ 6 + 15 a ^ 3
b ^ 7 x ^ 7 + 5 a ^ 2 b ^ 8 x ^ 8 + a b ^ 9 x ^ 9 + b ^ 10 x ^ 10 / 11)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result
default	$\frac{(bx+a)^{11}}{11b}$
gosper	$\frac{1}{11}b^{10}x^{11}+a\,b^{9}x^{10}+5a^{2}b^{8}x^{9}+15a^{3}b^{7}x^{8}+30a^{4}b^{6}x^{7}+42a^{5}b^{5}x^{6}+42a^{6}b^{4}x^{5}+30a^{7}b^{3}x^{4}+15a^{8}b^{2}$
norman	$\frac{1}{11}b^{10}x^{11}+a\,b^{9}x^{10}+5a^{2}b^{8}x^{9}+15a^{3}b^{7}x^{8}+30a^{4}b^{6}x^{7}+42a^{5}b^{5}x^{6}+42a^{6}b^{4}x^{5}+30a^{7}b^{3}x^{4}+15a^{8}b^{2}$
risch	$\frac{b^{10}x^{11}}{11}+a\,b^{9}x^{10}+5a^{2}b^{8}x^{9}+15a^{3}b^{7}x^{8}+30a^{4}b^{6}x^{7}+42a^{5}b^{5}x^{6}+42a^{6}b^{4}x^{5}+30a^{7}b^{3}x^{4}+15a^{8}b^{2}x^{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*(b*x+a)^11/b
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.86

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(b*x + a)^11/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.
time = 0.30, size = 108, normalized size = 7.71

$$\frac{1}{11}\,b^{10}x^{11}+ab^{9}x^{10}+5\,a^{2}b^{8}x^{9}+15\,a^{3}b^{7}x^{8}+30\,a^{4}b^{6}x^{7}+42\,a^{5}b^{5}x^{6}+42\,a^{6}b^{4}x^{5}+30\,a^{7}b^{3}x^{4}+15\,a^{8}b^{2}x^{3}+5\,a^{9}bx^{2}+a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(8) = 16$.
time = 0.04, size = 114, normalized size = 8.14

$$a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(bx + a)^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x)
```

```
[Out] 1/11*(b*x + a)^11/b
```

Mupad [B]
time = 0.11, size = 108, normalized size = 7.71

$$a^{10}\,x + 5\,a^9\,b\,x^2 + 15\,a^8\,b^2\,x^3 + 30\,a^7\,b^3\,x^4 + 42\,a^6\,b^4\,x^5 + 42\,a^5\,b^5\,x^6 + 30\,a^4\,b^6\,x^7 + 15\,a^3\,b^7\,x^8 + 5\,a^2\,b^8\,x^9 + a\,b^9\,x^{10} + \frac{b^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

3.135 $\int \frac{(a+bx)^{10}}{x} dx$

Optimal. Leaf size=122

$$10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}+$$

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x}\,dx &= \int \left(10a^9b+\frac{a^{10}}{x}+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5+120a^3\right.\\ &= 10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 122, normalized size = 1.00

$$10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}+a^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Mathics [A]
time = 2.21, size = 108, normalized size = 0.89

$$a^{10}\text{Log}\,[x]+10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10ab^9x^9}{9}+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^1,x]')
```

```
[Out] a ^ 10 Log[x] + 10 a ^ 9 b x + 45 a ^ 8 b ^ 2 x ^ 2 / 2 + 40 a ^ 7 b ^ 3 x
^ 3 + 105 a ^ 6 b ^ 4 x ^ 4 / 2 + 252 a ^ 5 b ^ 5 x ^ 5 / 5 + 35 a ^ 4 b ^
6 x ^ 6 + 120 a ^ 3 b ^ 7 x ^ 7 / 7 + 45 a ^ 2 b ^ 8 x ^ 8 / 8 + 10 a b ^ 9
 x ^ 9 / 9 + b ^ 10 x ^ 10 / 10
```

Maple [A]
time = 0.12, size = 109, normalized size = 0.89

method	result
default	$10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10a\,b^9x^9}{9}+b$
norman	$10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10a\,b^9x^9}{9}+b$
risch	$10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10a\,b^9x^9}{9}+b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x,x,method=_RETURNVERBOSE)
```

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

Maxima [A]
time = 0.25, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^9x^9+\frac{45}{8}\,a^2b^8x^8+\frac{120}{7}\,a^3b^7x^7+35\,a^4b^6x^6+\frac{252}{5}\,a^5b^5x^5+\frac{105}{2}\,a^6b^4x^4+40\,a^7b^3x^3+\frac{45}{2}\,a^8b^2x^2+10\,a^9bx+a^{10}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Fricas [A]
time = 0.30, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^{9}x^{9}+\frac{45}{8}\,a^{2}b^{8}x^{8}+\frac{120}{7}\,a^{3}b^{7}x^{7}+35\,a^{4}b^{6}x^{6}+\frac{252}{5}\,a^{5}b^{5}x^{5}+\frac{105}{2}\,a^{6}b^{4}x^{4}+40\,a^{7}b^{3}x^{3}+\frac{45}{2}\,a^{8}b^{2}x^{2}+10\,a^{9}bx+a^{10}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Sympy [A]
time = 0.09, size = 126, normalized size = 1.03

$$a^{10}\log\left(x\right)+10a^{9}bx+\frac{45a^{8}b^{2}x^{2}}{2}+40a^{7}b^{3}x^{3}+\frac{105a^{6}b^{4}x^{4}}{2}+\frac{252a^{5}b^{5}x^{5}}{5}+35a^{4}b^{6}x^{6}+\frac{120a^{3}b^{7}x^{7}}{7}+\frac{45a^{2}b^{8}x^{8}}{8}+\frac{10ab^{9}x^{9}}{9}+\frac{b^{10}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x,x)
```

```
[Out] a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**3 + 105*
a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a**3*b**7
*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10
```

Giac [A]
time = 0.00, size = 123, normalized size = 1.01

$$\frac{1}{10}x^{10}b^{10}+\frac{10}{9}x^{9}b^{9}a+\frac{45}{8}x^{8}b^{8}a^{2}+\frac{120}{7}x^{7}b^{7}a^{3}+35x^{6}b^{6}a^{4}+\frac{252}{5}x^{5}b^{5}a^{5}+\frac{105}{2}x^{4}b^{4}a^{6}+40x^{3}b^{3}a^{7}+\frac{45}{2}x^{2}b^{2}a^{8}+10xba^{9}+a^{10}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x)
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(abs(x))
```

Mupad [B]
time = 0.08, size = 108, normalized size = 0.89

$$a^{10}\,\ln\left(x\right)+\frac{b^{10}\,x^{10}}{10}+\frac{10\,a\,b^9\,x^9}{9}+\frac{45\,a^8\,b^2\,x^2}{2}+40\,a^7\,b^3\,x^3+\frac{105\,a^6\,b^4\,x^4}{2}+\frac{252\,a^5\,b^5\,x^5}{5}+35\,a^4\,b^6\,x^6+\frac{120\,a^3\,b^7\,x^7}{7}+\frac{45\,a^2\,b^8\,x^8}{8}+10\,a^9\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x,x)
```

```
[Out] a^10*log(x) + (b^10*x^10)/10 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/2 + 40*a
^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (
120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)/8 + 10*a^9*b*x
```

3.136 $\int \frac{(a+bx)^{10}}{x^2} dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}+10a^9b\log(x$$

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^2}\,dx &= \int \left(45a^8b^2+\frac{a^{10}}{x^2}+\frac{10a^9b}{x}+120a^7b^3x+210a^6b^4x^2+252a^5b^5x^3+210a^4b^6x^4+120a^3b^7x^5\right.\\ &= -\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}+10a^9b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Mathics [A]
time = 2.26, size = 112, normalized size = 0.97

$$\frac{-a^{10} + \frac{bx\left(2520a^9\text{Log}[x]+11340a^8bx+15120a^7b^2x^2+17640a^6b^3x^3+15876a^5b^4x^4+10584a^4b^5x^5+5040a^3b^6x^6+1620a^2b^7x^7+315ab^8x^8+28b^9x^9\right)}{252}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^2,x]')
```

```
[Out] (-a ^ 10 + b x (2520 a ^ 9 Log[x] + 11340 a ^ 8 b x + 15120 a ^ 7 b ^ 2 x ^
 2 + 17640 a ^ 6 b ^ 3 x ^ 3 + 15876 a ^ 5 b ^ 4 x ^ 4 + 10584 a ^ 4 b ^ 5
x ^ 5 + 5040 a ^ 3 b ^ 6 x ^ 6 + 1620 a ^ 2 b ^ 7 x ^ 7 + 315 a b ^ 8 x ^ 8
 + 28 b ^ 9 x ^ 9) / 252) / x
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.96

method	result
default	$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5a\,b^9x^8}{4}+$
risch	$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5a\,b^9x^8}{4}+$
norman	$\frac{-a^{10}+\frac{1}{9}b^{10}x^{10}+\frac{5}{4}a\,b^9x^9+\frac{45}{7}a^2b^8x^8+20a^3b^7x^7+42a^4b^6x^6+63a^5b^5x^5+70a^6b^4x^4+60a^7b^3x^3+45a^8b^2x^2}{x}+10a^9b\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

Maxima [A]
time = 0.26, size = 109, normalized size = 0.95

$$\frac{1}{9}b^{10}x^9+\frac{5}{4}ab^9x^8+\frac{45}{7}a^2b^8x^7+20\,a^3b^7x^6+42\,a^4b^6x^5+63\,a^5b^5x^4+70\,a^6b^4x^3+60\,a^7b^3x^2+45\,a^8b^2x+10\,a^9b\log(x)-\frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(x) - a^10/x
```

Fricas [A]
time = 0.31, size = 114, normalized size = 0.99

$$\frac{28\,b^{10}x^{10}+315\,ab^{9}x^{9}+1620\,a^{2}b^{8}x^{8}+5040\,a^{3}b^{7}x^{7}+10584\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+17640\,a^{6}b^{4}x^{4}+15120\,a^{7}b^{3}x^{3}+11340\,a^{8}b^{2}x^{2}+2520\,a^{9}bx\log\left(x\right)-252\,a^{10}}{252\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*x^10 + 315*a*b^9*x^9 + 1620*a^2*b^8*x^8 + 5040*a^3*b^7*x^7 +
 10584*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 17640*a^6*b^4*x^4 + 15120*a^7*b^3*
x^3 + 11340*a^8*b^2*x^2 + 2520*a^9*b*x*log(x) - 252*a^10)/x
```

Sympy [A]
time = 0.10, size = 117, normalized size = 1.02

$$-\frac{a^{10}}{x}+10a^{9}b\log\left(x\right)+45a^{8}b^{2}x+60a^{7}b^{3}x^{2}+70a^{6}b^{4}x^{3}+63a^{5}b^{5}x^{4}+42a^{4}b^{6}x^{5}+20a^{3}b^{7}x^{6}+\frac{45a^{2}b^{8}x^{7}}{7}+\frac{5ab^{9}x^{8}}{4}+\frac{b^{10}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**2,x)
```

```
[Out] -a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b**3*x**2 + 70*a**6*
b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a**3*b**7*x**6 + 45*
a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9
```

Giac [A]
time = 0.00, size = 115, normalized size = 1.00

$$\frac{1}{9}x^{9}b^{10}+\frac{5}{4}x^{8}b^{9}a+\frac{45}{7}x^{7}b^{8}a^{2}+20x^{6}b^{7}a^{3}+42x^{5}b^{6}a^{4}+63x^{4}b^{5}a^{5}+70x^{3}b^{4}a^{6}+60x^{2}b^{3}a^{7}+45xb^{2}a^{8}-\frac{a^{10}}{x}+10ba^{9}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x)
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(abs(x)) - a^10/x
```

Mupad [B]
time = 0.12, size = 109, normalized size = 0.95

$$\frac{b^{10}\,x^9}{9}-\frac{a^{10}}{x}+45\,a^8\,b^2\,x+\frac{5\,a\,b^9\,x^8}{4}+10\,a^9\,b\,\ln\left(x\right)+60\,a^7\,b^3\,x^2+70\,a^6\,b^4\,x^3+63\,a^5\,b^5\,x^4+42\,a^4\,b^6\,x^5+20\,a^3\,b^7\,x^6+\frac{45\,a^2\,b^8\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^2,x)
```

```
[Out] (b^10*x^9)/9 - a^10/x + 45*a^8*b^2*x + (5*a*b^9*x^8)/4 + 10*a^9*b*log(x) +
60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4 + 42*a^4*b^6*x^5 + 20*a^3*
b^7*x^6 + (45*a^2*b^8*x^7)/7
```

3.137 $\int \frac{(a+bx)^{10}}{x^3} \, dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}+45a^8b^2$$

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^3} \, dx = \int \left(120a^7b^3+\frac{a^{10}}{x^3}+\frac{10a^9b}{x^2}+\frac{45a^8b^2}{x}+210a^6b^4x+252a^5b^5x^2+210a^4b^6x^3+120a^3b^7x^4\right.$$

$$= -\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6$$

Mathematica [A]
time = 0.00, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}+45a^8b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Mathics [A]
time = 2.35, size = 111, normalized size = 0.93

$$\frac{-28a^9(a+20bx)+b^2x^2(2520a^8\text{Log}[x]+6720a^7bx+5880a^6b^2x^2+4704a^5b^3x^3+2940a^4b^4x^4+1344a^3b^5x^5+420a^2b^6x^6+80ab^7x^7+7b^8x^8)}{56x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^3,x]')
```

```
[Out] (-28 a ^ 9 (a + 20 b x) + b ^ 2 x ^ 2 (2520 a ^ 8 Log[x] + 6720 a ^ 7 b x +
 5880 a ^ 6 b ^ 2 x ^ 2 + 4704 a ^ 5 b ^ 3 x ^ 3 + 2940 a ^ 4 b ^ 4 x ^ 4 +
 1344 a ^ 3 b ^ 5 x ^ 5 + 420 a ^ 2 b ^ 6 x ^ 6 + 80 a b ^ 7 x ^ 7 + 7 b ^
8 x ^ 8)) / (56 x ^ 2)
```

Maple [A]
time = 0.10, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105a^4b^6x^4}{2}+24a^3b^7x^5+\frac{15a^2b^8x^6}{2}+\frac{10a\,b^9x^7}{7}+\frac{b^{10}}{8}$
risch	$\frac{b^{10}x^8}{8}+\frac{10a\,b^9x^7}{7}+\frac{15a^2b^8x^6}{2}+24a^3b^7x^5+\frac{105a^4b^6x^4}{2}+84a^5b^5x^3+105a^6b^4x^2+120a^7b^3x+\frac{-10a^9bx-\frac{1}{2}a^1}{x^2}$
norman	$\frac{-\frac{1}{2}a^{10}+\frac{1}{8}b^{10}x^{10}+\frac{10}{7}a\,b^9x^9+\frac{15}{2}a^2b^8x^8+24a^3b^7x^7+\frac{105}{2}a^4b^6x^6+84a^5b^5x^5+105a^6b^4x^4+120a^7b^3x^3-10a^9bx}{x^2}+45a^8b^2\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

Maxima [A]
time = 0.24, size = 108, normalized size = 0.91

$$\frac{1}{8}b^{10}x^8+\frac{10}{7}ab^9x^7+\frac{15}{2}a^2b^8x^6+24a^3b^7x^5+\frac{105}{2}a^4b^6x^4+84a^5b^5x^3+105a^6b^4x^2+120a^7b^3x+45a^8b^2\log(x)-\frac{20a^9bx+a^{10}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(x) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Fricas [A]

time = 0.30, size = 114, normalized size = 0.96

$$\frac{7\,b^{10}x^{10}+80\,ab^9x^9+420\,a^2b^8x^8+1344\,a^3b^7x^7+2940\,a^4b^6x^6+4704\,a^5b^5x^5+5880\,a^6b^4x^4+6720\,a^7b^3x^3+2520\,a^8b^2x^2\log\left(x\right)-560\,a^9bx-28\,a^{10}}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*x^10 + 80*a*b^9*x^9 + 420*a^2*b^8*x^8 + 1344*a^3*b^7*x^7 + 294
0*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 5880*a^6*b^4*x^4 + 6720*a^7*b^3*x^3 + 25
20*a^8*b^2*x^2*log(x) - 560*a^9*b*x - 28*a^10)/x^2
```

Sympy [A]

time = 0.12, size = 122, normalized size = 1.03

$$45a^8b^2\log\left(x\right)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105a^4b^6x^4}{2}+24a^3b^7x^5+\frac{15a^2b^8x^6}{2}+\frac{10ab^9x^7}{7}+\frac{b^{10}x^8}{8}+\frac{-a^{10}-20a^9bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**3,x)
```

```
[Out] 45*a**8*b**2*log(x) + 120*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x
**3 + 105*a**4*b**6*x**4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a
*b**9*x**7/7 + b**10*x**8/8 + (-a**10 - 20*a**9*b*x)/(2*x**2)
```

Giac [A]

time = 0.00, size = 122, normalized size = 1.03

$$\frac{1}{8}x^8b^{10}+\frac{10}{7}x^7b^9a+\frac{15}{2}x^6b^8a^2+24x^5b^7a^3+\frac{105}{2}x^4b^6a^4+84x^3b^5a^5+105x^2b^4a^6+120xb^3a^7+\frac{\frac{1}{2}\left(-20ba^9x-a^{10}\right)}{x^2}+45b^2a^8\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x)
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(abs(x)) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Mupad [B]

time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^8}{8}-\frac{\frac{a^{10}}{2}+10\,b\,x\,a^9}{x^2}+120\,a^7\,b^3\,x+\frac{10\,a\,b^9\,x^7}{7}+105\,a^6\,b^4\,x^2+84\,a^5\,b^5\,x^3+\frac{105\,a^4\,b^6\,x^4}{2}+24\,a^3\,b^7\,x^5+\frac{15\,a^2\,b^8\,x^6}{2}+45\,a^8\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^3,x)
```

```
[Out] (b^10*x^8)/8 - (a^10/2 + 10*a^9*b*x)/x^2 + 120*a^7*b^3*x + (10*a*b^9*x^7)/7
 + 105*a^6*b^4*x^2 + 84*a^5*b^5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5
+ (15*a^2*b^8*x^6)/2 + 45*a^8*b^2*log(x)
```

3.138 $\int \frac{(a+bx)^{10}}{x^4} dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}+120a^7b^3\log(x)$$

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^4} dx = \int \left(210a^6b^4+\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+252a^5b^5x+210a^4b^6x^2+120a^3b^7x^3+\right.$$

$$=-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}+120a^7b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Mathics [A]
time = 2.40, size = 111, normalized size = 0.97

$$\frac{-7a^8\left(a^2+15abx+135b^2x^2\right)+b^3x^3\left(2520a^7\text{Log}\left[x\right]+4410a^6bx+2646a^5b^2x^2+1470a^4b^3x^3+630a^3b^4x^4+189a^2b^5x^5+35ab^6x^6+3b^7x^7\right)}{21x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^4,x]')
```

```
[Out] (-7 a ^ 8 (a ^ 2 + 15 a b x + 135 b ^ 2 x ^ 2) + b ^ 3 x ^ 3 (2520 a ^ 7 Lo
g[x] + 4410 a ^ 6 b x + 2646 a ^ 5 b ^ 2 x ^ 2 + 1470 a ^ 4 b ^ 3 x ^ 3 + 6
30 a ^ 3 b ^ 4 x ^ 4 + 189 a ^ 2 b ^ 5 x ^ 5 + 35 a b ^ 6 x ^ 6 + 3 b ^ 7 x
 ^ 7)) / (21 x ^ 3)
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.96

method	result
default	$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5a\,b^9x^6}{3}+\frac{b^{10}x^7}{7}$
risch	$\frac{b^{10}x^7}{7}+\frac{5a\,b^9x^6}{3}+9a^2b^8x^5+30a^3b^7x^4+70a^4b^6x^3+126a^5b^5x^2+210a^6b^4x+\frac{-45a^8b^2x^2-5a^9bx-\frac{1}{3}a^{10}}{x^3}$
norman	$\frac{-\frac{1}{3}a^{10}+\frac{1}{7}b^{10}x^{10}+\frac{5}{3}a\,b^9x^9+9a^2b^8x^8+30a^3b^7x^7+70a^4b^6x^6+126a^5b^5x^5+210a^6b^4x^4-45a^8b^2x^2-5a^9bx}{x^3}+120a^7b^3\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

Maxima [A]
time = 0.25, size = 108, normalized size = 0.94

$$\frac{1}{7}\,b^{10}x^7+\frac{5}{3}\,ab^9x^6+9\,a^2b^8x^5+30\,a^3b^7x^4+70\,a^4b^6x^3+126\,a^5b^5x^2+210\,a^6b^4x+120\,a^7b^3\log\left(x\right)-\frac{135\,a^8b^2x^2+15\,a^9bx+a^{10}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(x) - 1/3*(135*a^8*b
^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Fricas [A]
time = 0.31, size = 114, normalized size = 0.99

$$\frac{3\,b^{10}x^{10}+35\,ab^{9}x^{9}+189\,a^{2}b^{8}x^{8}+630\,a^{3}b^{7}x^{7}+1470\,a^{4}b^{6}x^{6}+2646\,a^{5}b^{5}x^{5}+4410\,a^{6}b^{4}x^{4}+2520\,a^{7}b^{3}x^{3}\log\left(x\right)-945\,a^{8}b^{2}x^{2}-105\,a^{9}bx-7\,a^{10}}{21\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*x^10 + 35*a*b^9*x^9 + 189*a^2*b^8*x^8 + 630*a^3*b^7*x^7 + 1470
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 4410*a^6*b^4*x^4 + 2520*a^7*b^3*x^3*log(x
) - 945*a^8*b^2*x^2 - 105*a^9*b*x - 7*a^10)/x^3
```

Sympy [A]
time = 0.14, size = 119, normalized size = 1.03

$$120a^{7}b^{3}\log\left(x\right)+210a^{6}b^{4}x+126a^{5}b^{5}x^{2}+70a^{4}b^{6}x^{3}+30a^{3}b^{7}x^{4}+9a^{2}b^{8}x^{5}+\frac{5ab^{9}x^{6}}{3}+\frac{b^{10}x^{7}}{7}+\frac{-a^{10}-15a^{9}bx-135a^{8}b^{2}x^{2}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**4,x)
```

```
[Out] 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b**6*
x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x**7/
7 + (-a**10 - 15*a**9*b*x - 135*a**8*b**2*x**2)/(3*x**3)
```

Giac [A]
time = 0.00, size = 119, normalized size = 1.03

$$\frac{1}{7}x^{7}b^{10}+\frac{5}{3}x^{6}b^{9}a+9x^{5}b^{8}a^{2}+30x^{4}b^{7}a^{3}+70x^{3}b^{6}a^{4}+126x^{2}b^{5}a^{5}+210xb^{4}a^{6}+\frac{\frac{1}{3}\left(-135b^{2}a^{8}x^{2}-15ba^{9}x-a^{10}\right)}{x^{3}}+120b^{3}a^{7}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x)
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(abs(x)) - 1/3*(135*
a^8*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Mupad [B]
time = 0.06, size = 110, normalized size = 0.96

$$\frac{b^{10}\,x^{7}}{7}-\frac{\frac{a^{10}}{3}+5\,a^{9}\,b\,x+45\,a^{8}\,b^{2}\,x^{2}}{x^{3}}+210\,a^{6}\,b^{4}\,x+\frac{5\,a\,b^{9}\,x^{6}}{3}+126\,a^{5}\,b^{5}\,x^{2}+70\,a^{4}\,b^{6}\,x^{3}+30\,a^{3}\,b^{7}\,x^{4}+9\,a^{2}\,b^{8}\,x^{5}+120\,a^{7}\,b^{3}\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^4,x)
```

```
[Out] (b^10*x^7)/7 - (a^10/3 + 45*a^8*b^2*x^2 + 5*a^9*b*x)/x^3 + 210*a^6*b^4*x +
(5*a*b^9*x^6)/3 + 126*a^5*b^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2
*b^8*x^5 + 120*a^7*b^3*log(x)
```

3.139 $\int \frac{(a+bx)^{10}}{x^5}\,dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}+210a^6b^4\log(x$$

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^5}\,dx &= \int \left(252a^5b^5+\frac{a^{10}}{x^5}+\frac{10a^9b}{x^4}+\frac{45a^8b^2}{x^3}+\frac{120a^7b^3}{x^2}+\frac{210a^6b^4}{x}+210a^4b^6x+120a^3b^7x^2+ \right.\\ &= -\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}+210a^6b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Mathics [A]
time = 2.43, size = 113, normalized size = 0.95

$$\frac{-a^7\left(3a^3+40a^2bx+270ab^2x^2+1440b^3x^3\right)+b^4x^4\left(2520a^6\text{Log}\,[x]+3024a^5bx+1260a^4b^2x^2+480a^3b^3x^3+135a^2b^4x^4+24ab^5x^5+2b^6x^6\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^5,x]')
```

```
[Out] (-a ^ 7 (3 a ^ 3 + 40 a ^ 2 b x + 270 a b ^ 2 x ^ 2 + 1440 b ^ 3 x ^ 3) + b
 ^ 4 x ^ 4 (2520 a ^ 6 Log[x] + 3024 a ^ 5 b x + 1260 a ^ 4 b ^ 2 x ^ 2 + 4
80 a ^ 3 b ^ 3 x ^ 3 + 135 a ^ 2 b ^ 4 x ^ 4 + 24 a b ^ 5 x ^ 5 + 2 b ^ 6 x
 ^ 6)) / (12 x ^ 4)
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45a^2b^8x^4}{4}+2a\,b^9x^5+\frac{b^{10}x^6}{6}+$
risch	$\frac{b^{10}x^6}{6}+2a\,b^9x^5+\frac{45a^2b^8x^4}{4}+40a^3b^7x^3+105a^4b^6x^2+252a^5b^5x+\frac{-120a^7b^3x^3-\frac{45}{2}a^8b^2x^2-\frac{10}{3}a^9bx-\frac{1}{4}a^{10}}{x^4}+2$
norman	$\frac{-\frac{1}{4}a^{10}+\frac{1}{6}b^{10}x^{10}+2a\,b^9x^9+\frac{45}{4}a^2b^8x^8+40a^3b^7x^7+105a^4b^6x^6+252a^5b^5x^5-120a^7b^3x^3-\frac{45}{2}a^8b^2x^2-\frac{10}{3}a^9bx}{x^4}+210a^6b^4\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

Maxima [A]
time = 0.25, size = 110, normalized size = 0.92

$$\frac{1}{6}\,b^{10}x^6+2\,ab^9x^5+\frac{45}{4}\,a^2b^8x^4+40\,a^3b^7x^3+105\,a^4b^6x^2+252\,a^5b^5x+210\,a^6b^4\log\left(x\right)-\frac{1440\,a^7b^3x^3+270\,a^8b^2x^2+40\,a^9bx+3\,a^{10}}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(x) - 1/12*(1440*a^7*b^3*x^3 + 270*a
^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Fricas [A]
time = 0.31, size = 114, normalized size = 0.96

$$\frac{2\,b^{10}x^{10}+24\,ab^{9}x^{9}+135\,a^{2}b^{8}x^{8}+480\,a^{3}b^{7}x^{7}+1260\,a^{4}b^{6}x^{6}+3024\,a^{5}b^{5}x^{5}+2520\,a^{6}b^{4}x^{4}\log\left(x\right)-1440\,a^{7}b^{3}x^{3}-270\,a^{8}b^{2}x^{2}-40\,a^{9}bx-3\,a^{10}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*x^10 + 24*a*b^9*x^9 + 135*a^2*b^8*x^8 + 480*a^3*b^7*x^7 + 1260
*a^4*b^6*x^6 + 3024*a^5*b^5*x^5 + 2520*a^6*b^4*x^4*log(x) - 1440*a^7*b^3*x^
3 - 270*a^8*b^2*x^2 - 40*a^9*b*x - 3*a^10)/x^4
```

Sympy [A]
time = 0.18, size = 121, normalized size = 1.02

$$210a^{6}b^{4}\log\left(x\right)+252a^{5}b^{5}x+105a^{4}b^{6}x^{2}+40a^{3}b^{7}x^{3}+\frac{45a^{2}b^{8}x^{4}}{4}+2ab^{9}x^{5}+\frac{b^{10}x^{6}}{6}+\frac{-3a^{10}-40a^{9}bx-270a^{8}b^{2}x^{2}-1440a^{7}b^{3}x^{3}}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**5,x)
```

```
[Out] 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105*a**4*b**6*x**2 + 40*a**3*b**7*
x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 + b**10*x**6/6 + (-3*a**10 - 40*
a**9*b*x - 270*a**8*b**2*x**2 - 1440*a**7*b**3*x**3)/(12*x**4)
```

Giac [A]
time = 0.00, size = 122, normalized size = 1.03

$$\frac{1}{6}x^{6}b^{10}+2x^{5}b^{9}a+\frac{45}{4}x^{4}b^{8}a^{2}+40x^{3}b^{7}a^{3}+105x^{2}b^{6}a^{4}+252xb^{5}a^{5}+\frac{\frac{1}{12}\left(-1440b^{3}a^{7}x^{3}-270b^{2}a^{8}x^{2}-40ba^{9}x-3a^{10}\right)}{x^{4}}+210b^{4}a^{6}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x)
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(abs(x)) - 1/12*(1440*a^7*b^3*x^3 +
270*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Mupad [B]
time = 0.10, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^6}{6}-\frac{\frac{a^{10}}{4}+\frac{10\,a^9\,b\,x}{3}+\frac{45\,a^8\,b^2\,x^2}{2}+120\,a^7\,b^3\,x^3}{x^4}+252\,a^5\,b^5\,x+2\,a\,b^9\,x^5+105\,a^4\,b^6\,x^2+40\,a^3\,b^7\,x^3+\frac{45\,a^2\,b^8\,x^4}{4}+210\,a^6\,b^4\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^5,x)
```

```
[Out] (b^10*x^6)/6 - (a^10/4 + (45*a^8*b^2*x^2)/2 + 120*a^7*b^3*x^3 + (10*a^9*b*x
)/3)/x^4 + 252*a^5*b^5*x + 2*a*b^9*x^5 + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 +
 (45*a^2*b^8*x^4)/4 + 210*a^6*b^4*log(x)
```

3.140 $\int \frac{(a+bx)^{10}}{x^6} dx$

Optimal. Leaf size=117

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}+252a^5b^5\log(x)$$

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^6}\,dx &= \int \left(210a^4b^6+\frac{a^{10}}{x^6}+\frac{10a^9b}{x^5}+\frac{45a^8b^2}{x^4}+\frac{120a^7b^3}{x^3}+\frac{210a^6b^4}{x^2}+\frac{252a^5b^5}{x}+120a^3b^7x+45a\right.\\ &= -\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}+252a^5b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Mathics [A]
time = 2.44, size = 113, normalized size = 0.97

$$\frac{-a^6\left(2a^4+25a^3bx+150a^2b^2x^2+600ab^3x^3+2100b^4x^4\right)+b^5x^5\left(2520a^5\mathrm{Log}\left[x\right]+2100a^4bx+600a^3b^2x^2+150a^2b^3x^3+25ab^4x^4+2b^5x^5\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^6,x]')
```

```
[Out] (-a ^ 6 (2 a ^ 4 + 25 a ^ 3 b x + 150 a ^ 2 b ^ 2 x ^ 2 + 600 a b ^ 3 x ^ 3
 + 2100 b ^ 4 x ^ 4) + b ^ 5 x ^ 5 (2520 a ^ 5 Log[x] + 2100 a ^ 4 b x + 60
0 a ^ 3 b ^ 2 x ^ 2 + 150 a ^ 2 b ^ 3 x ^ 3 + 25 a b ^ 4 x ^ 4 + 2 b ^ 5 x
^ 5)) / (10 x ^ 5)
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.94

method	result
default	$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5a\,b^9x^4}{2}+\frac{b^{10}x^5}{5}+252$
risch	$\frac{b^{10}x^5}{5}+\frac{5a\,b^9x^4}{2}+15a^2b^8x^3+60a^3b^7x^2+210a^4b^6x+\frac{-210a^6b^4x^4-60a^7b^3x^3-15a^8b^2x^2-\frac{5}{2}a^9bx-\frac{1}{5}a^{10}}{x^5}+252a$
norman	$\frac{-\frac{1}{5}a^{10}+\frac{1}{5}b^{10}x^{10}+\frac{5}{2}a\,b^9x^9+15a^2b^8x^8+60a^3b^7x^7+210a^4b^6x^6-210a^6b^4x^4-60a^7b^3x^3-15a^8b^2x^2-\frac{5}{2}a^9bx}{x^5}+252a^5b^5\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

Maxima [A]
time = 0.26, size = 110, normalized size = 0.94

$$\frac{1}{5}\,b^{10}x^5+\frac{5}{2}\,ab^9x^4+15\,a^2b^8x^3+60\,a^3b^7x^2+210\,a^4b^6x+252\,a^5b^5\log\left(x\right)-\frac{2100\,a^6b^4x^4+600\,a^7b^3x^3+150\,a^8b^2x^2+25\,a^9bx+2\,a^{10}}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(x) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 + 150*a
^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Fricas [A]
time = 0.30, size = 114, normalized size = 0.97

$$\frac{2\,b^{10}x^{10}+25\,ab^{9}x^{9}+150\,a^{2}b^{8}x^{8}+600\,a^{3}b^{7}x^{7}+2100\,a^{4}b^{6}x^{6}+2520\,a^{5}b^{5}x^{5}\log\left(x\right)-2100\,a^{6}b^{4}x^{4}-600\,a^{7}b^{3}x^{3}-150\,a^{8}b^{2}x^{2}-25\,a^{9}bx-2\,a^{10}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*x^10 + 25*a*b^9*x^9 + 150*a^2*b^8*x^8 + 600*a^3*b^7*x^7 + 2100
*a^4*b^6*x^6 + 2520*a^5*b^5*x^5*log(x) - 2100*a^6*b^4*x^4 - 600*a^7*b^3*x^3
 - 150*a^8*b^2*x^2 - 25*a^9*b*x - 2*a^10)/x^5
```

Sympy [A]
time = 0.22, size = 121, normalized size = 1.03

$$252a^{5}b^{5}\log\left(x\right)+210a^{4}b^{6}x+60a^{3}b^{7}x^{2}+15a^{2}b^{8}x^{3}+\frac{5ab^{9}x^{4}}{2}+\frac{b^{10}x^{5}}{5}+\frac{-2a^{10}-25a^{9}bx-150a^{8}b^{2}x^{2}-600a^{7}b^{3}x^{3}-2100a^{6}b^{4}x^{4}}{10x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**6,x)
```

```
[Out] 252*a**5*b**5*log(x) + 210*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x
**3 + 5*a*b**9*x**4/2 + b**10*x**5/5 + (-2*a**10 - 25*a**9*b*x - 150*a**8*b
**2*x**2 - 600*a**7*b**3*x**3 - 2100*a**6*b**4*x**4)/(10*x**5)
```

Giac [A]
time = 0.00, size = 123, normalized size = 1.05

$$\frac{1}{5}x^{5}b^{10}+\frac{5}{2}x^{4}b^{9}a+15x^{3}b^{8}a^{2}+60x^{2}b^{7}a^{3}+210xb^{6}a^{4}+\frac{\frac{1}{10}\left(-2100b^{4}a^{6}x^{4}-600b^{3}a^{7}x^{3}-150b^{2}a^{8}x^{2}-25ba^{9}x-2a^{10}\right)}{x^{5}}+252b^{5}a^{5}\ln\left|x\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x)
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(abs(x)) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 +
150*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Mupad [B]
time = 0.10, size = 110, normalized size = 0.94

$$\frac{b^{10}\,x^5}{5}-\frac{\frac{a^{10}}{5}+\frac{5\,a^9\,b\,x}{2}+15\,a^8\,b^2\,x^2+60\,a^7\,b^3\,x^3+210\,a^6\,b^4\,x^4}{x^5}+210\,a^4\,b^6\,x+\frac{5\,a\,b^9\,x^4}{2}+60\,a^3\,b^7\,x^2+15\,a^2\,b^8\,x^3+252\,a^5\,b^5\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^6,x)
```

```
[Out] (b^10*x^5)/5 - (a^10/5 + 15*a^8*b^2*x^2 + 60*a^7*b^3*x^3 + 210*a^6*b^4*x^4
+ (5*a^9*b*x)/2)/x^5 + 210*a^4*b^6*x + (5*a*b^9*x^4)/2 + 60*a^3*b^7*x^2 + 1
5*a^2*b^8*x^3 + 252*a^5*b^5*log(x)
```

3.141 $\int \frac{(a+bx)^{10}}{x^7} dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}+210a^4b^6\log(x)$$

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^7}\,dx &= \int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}+45a\right.\\ &= -\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab\end{aligned}$$

Mathematica [A]
time = 0.00, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}+210a^4b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Mathics [A]
time = 2.58, size = 113, normalized size = 0.95

$$\frac{-a^5\left(2a^5+24a^4bx+135a^3b^2x^2+480a^2b^3x^3+1260ab^4x^4+3024b^5x^5\right)+b^6x^6\left(2520a^4\mathrm{Log}\,[x]+1440a^3bx+270a^2b^2x^2+40ab^3x^3+3b^4x^4\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^7,x]')
```

```
[Out] (-a ^ 5 (2 a ^ 5 + 24 a ^ 4 b x + 135 a ^ 3 b ^ 2 x ^ 2 + 480 a ^ 2 b ^ 3 x
 ^ 3 + 1260 a b ^ 4 x ^ 4 + 3024 b ^ 5 x ^ 5) + b ^ 6 x ^ 6 (2520 a ^ 4 Log
[x] + 1440 a ^ 3 b x + 270 a ^ 2 b ^ 2 x ^ 2 + 40 a b ^ 3 x ^ 3 + 3 b ^ 4 x
 ^ 4)) / (12 x ^ 6)
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45a^2b^8x^2}{2}+\frac{10a\,b^9x^3}{3}+\frac{b^{10}x^4}{4}+210a^4b^6$
risch	$\frac{b^{10}x^4}{4}+\frac{10a\,b^9x^3}{3}+\frac{45a^2b^8x^2}{2}+120a^3b^7x+\frac{-252a^5b^5x^5-105a^6b^4x^4-40a^7b^3x^3-\frac{45}{4}a^8b^2x^2-2a^9bx-\frac{1}{6}a^{10}}{x^6}+210a^4b^6\ln$
norman	$\frac{-\frac{1}{6}a^{10}+\frac{1}{4}b^{10}x^{10}+\frac{10}{3}a\,b^9x^9+\frac{45}{2}a^2b^8x^8+120a^3b^7x^7-252a^5b^5x^5-105a^6b^4x^4-40a^7b^3x^3-\frac{45}{4}a^8b^2x^2-2a^9bx}{x^6}+210a^4b^6\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

Maxima [A]
time = 0.25, size = 110, normalized size = 0.92

$$\frac{1}{4}\,b^{10}x^4+\frac{10}{3}\,ab^9x^3+\frac{45}{2}\,a^2b^8x^2+120\,a^3b^7x+210\,a^4b^6\log\left(x\right)-\frac{3024\,a^5b^5x^5+1260\,a^6b^4x^4+480\,a^7b^3x^3+135\,a^8b^2x^2+24\,a^9bx+2\,a^{10}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(x) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x^3 +
135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Fricas [A]
time = 0.30, size = 114, normalized size = 0.96

$$\frac{3\,b^{10}x^{10}+40\,ab^{9}x^{9}+270\,a^{2}b^{8}x^{8}+1440\,a^{3}b^{7}x^{7}+2520\,a^{4}b^{6}x^{6}\log\left(x\right)-3024\,a^{5}b^{5}x^{5}-1260\,a^{6}b^{4}x^{4}-480\,a^{7}b^{3}x^{3}-135\,a^{8}b^{2}x^{2}-24\,a^{9}bx-2\,a^{10}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 + 40*a*b^9*x^9 + 270*a^2*b^8*x^8 + 1440*a^3*b^7*x^7 + 252
0*a^4*b^6*x^6*log(x) - 3024*a^5*b^5*x^5 - 1260*a^6*b^4*x^4 - 480*a^7*b^3*x^
3 - 135*a^8*b^2*x^2 - 24*a^9*b*x - 2*a^10)/x^6
```

Sympy [A]
time = 0.24, size = 122, normalized size = 1.03

$$210a^{4}b^{6}\log\left(x\right)+120a^{3}b^{7}x+\frac{45a^{2}b^{8}x^{2}}{2}+\frac{10ab^{9}x^{3}}{3}+\frac{b^{10}x^{4}}{4}+\frac{-2a^{10}-24a^{9}bx-135a^{8}b^{2}x^{2}-480a^{7}b^{3}x^{3}-1260a^{6}b^{4}x^{4}-3024a^{5}b^{5}x^{5}}{12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**7,x)
```

```
[Out] 210*a**4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x*
*3/3 + b**10*x**4/4 + (-2*a**10 - 24*a**9*b*x - 135*a**8*b**2*x**2 - 480*a*
*7*b**3*x**3 - 1260*a**6*b**4*x**4 - 3024*a**5*b**5*x**5)/(12*x**6)
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.06

$$\frac{1}{4}x^{4}b^{10}+\frac{10}{3}x^{3}b^{9}a+\frac{45}{2}x^{2}b^{8}a^{2}+120xb^{7}a^{3}+\frac{\frac{1}{12}\left(-3024b^{5}a^{5}x^{5}-1260b^{4}a^{6}x^{4}-480b^{3}a^{7}x^{3}-135b^{2}a^{8}x^{2}-24ba^{9}x-2a^{10}\right)}{x^{6}}+210b^{6}a^{4}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x)
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(abs(x)) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x
^3 + 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Mupad [B]
time = 0.05, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^4}{4}-\frac{\frac{a^{10}}{6}+2\,a^9\,b\,x+\frac{45\,a^8\,b^2\,x^2}{4}+40\,a^7\,b^3\,x^3+105\,a^6\,b^4\,x^4+252\,a^5\,b^5\,x^5}{x^6}+120\,a^3\,b^7\,x+\frac{10\,a\,b^9\,x^3}{3}+\frac{45\,a^2\,b^8\,x^2}{2}+210\,a^4\,b^6\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^7,x)
```

```
[Out] (b^10*x^4)/4 - (a^10/6 + (45*a^8*b^2*x^2)/4 + 40*a^7*b^3*x^3 + 105*a^6*b^4*
x^4 + 252*a^5*b^5*x^5 + 2*a^9*b*x)/x^6 + 120*a^3*b^7*x + (10*a*b^9*x^3)/3 +
 (45*a^2*b^8*x^2)/2 + 210*a^4*b^6*log(x)
```

3.142 $\int \frac{(a+bx)^{10}}{x^8} dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\log(x)$$

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^8}\,dx &= \int \left(45a^2b^8+\frac{a^{10}}{x^8}+\frac{10a^9b}{x^7}+\frac{45a^8b^2}{x^6}+\frac{120a^7b^3}{x^5}+\frac{210a^6b^4}{x^4}+\frac{252a^5b^5}{x^3}+\frac{210a^4b^6}{x^2}+\frac{120a^3}{x}\right.\\ &= -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Mathics [A]
time = 2.72, size = 113, normalized size = 0.98

$$\frac{-a^4\left(3a^6+35a^5bx+189a^4b^2x^2+630a^3b^3x^3+1470a^2b^4x^4+2646ab^5x^5+4410b^6x^6\right)+7b^7x^7\left(360a^3\text{Log}\left[x\right]+135a^2bx+15ab^2x^2+b^3x^3\right)}{21x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^8,x]')
```

```
[Out] (-a ^ 4 (3 a ^ 6 + 35 a ^ 5 b x + 189 a ^ 4 b ^ 2 x ^ 2 + 630 a ^ 3 b ^ 3 x
 ^ 3 + 1470 a ^ 2 b ^ 4 x ^ 4 + 2646 a b ^ 5 x ^ 5 + 4410 b ^ 6 x ^ 6) + 7
b ^ 7 x ^ 7 (360 a ^ 3 Log[x] + 135 a ^ 2 b x + 15 a b ^ 2 x ^ 2 + b ^ 3 x
^ 3)) / (21 x ^ 7)
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.96

method	result
default	$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5a\,b^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\ln$
risch	$\frac{b^{10}x^3}{3}+5a\,b^9x^2+45a^2b^8x+\frac{-210a^4b^6x^6-126a^5b^5x^5-70a^6b^4x^4-30a^7b^3x^3-9a^8b^2x^2-\frac{5}{3}a^9bx-\frac{1}{7}a^{10}}{x^7}+120a^3b^7\ln($
norman	$\frac{-\frac{1}{7}a^{10}+\frac{1}{3}b^{10}x^{10}+5a\,b^9x^9+45a^2b^8x^8-210a^4b^6x^6-126a^5b^5x^5-70a^6b^4x^4-30a^7b^3x^3-9a^8b^2x^2-\frac{5}{3}a^9bx}{x^7}+120a^3b^7\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

Maxima [A]
time = 0.24, size = 110, normalized size = 0.96

$$\frac{1}{3}\,b^{10}x^3+5\,ab^9x^2+45\,a^2b^8x+120\,a^3b^7\log\left(x\right)-\frac{4410\,a^4b^6x^6+2646\,a^5b^5x^5+1470\,a^6b^4x^4+630\,a^7b^3x^3+189\,a^8b^2x^2+35\,a^9bx+3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(x) - 1/21*(4410
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 + 189*
a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Fricas [A]

time = 0.30, size = 114, normalized size = 0.99

$$\frac{7\,b^{10}x^{10}+105\,ab^9x^9+945\,a^2b^8x^8+2520\,a^3b^7x^7\log\left(x\right)-4410\,a^4b^6x^6-2646\,a^5b^5x^5-1470\,a^6b^4x^4-630\,a^7b^3x^3-189\,a^8b^2x^2-35\,a^9bx-3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*x^10 + 105*a*b^9*x^9 + 945*a^2*b^8*x^8 + 2520*a^3*b^7*x^7*log(
x) - 4410*a^4*b^6*x^6 - 2646*a^5*b^5*x^5 - 1470*a^6*b^4*x^4 - 630*a^7*b^3*x
^3 - 189*a^8*b^2*x^2 - 35*a^9*b*x - 3*a^10)/x^7
```

Sympy [A]

time = 0.29, size = 119, normalized size = 1.03

$$120a^3b^7\log\left(x\right)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+\frac{-3a^{10}-35a^9bx-189a^8b^2x^2-630a^7b^3x^3-1470a^6b^4x^4-2646a^5b^5x^5-4410a^4b^6x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**8,x)
```

```
[Out] 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 + b**10*x**3/3 + (-3*
a**10 - 35*a**9*b*x - 189*a**8*b**2*x**2 - 630*a**7*b**3*x**3 - 1470*a**6*b
**4*x**4 - 2646*a**5*b**5*x**5 - 4410*a**4*b**6*x**6)/(21*x**7)
```

Giac [A]

time = 0.00, size = 123, normalized size = 1.07

$$\frac{1}{3}x^3b^{10}+5x^2b^9a+45xb^8a^2+\frac{\frac{1}{21}\left(-4410b^6a^4x^6-2646b^5a^5x^5-1470b^4a^6x^4-630b^3a^7x^3-189b^2a^8x^2-35ba^9x-3a^{10}\right)}{x^7}+120b^7a^3\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x)
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(abs(x)) - 1/21*
(4410*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 +
 189*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Mupad [B]

time = 0.10, size = 110, normalized size = 0.96

$$\frac{b^{10}\,x^3}{3}-\frac{\frac{a^{10}}{7}+\frac{5\,a^9\,b\,x}{3}+9\,a^8\,b^2\,x^2+30\,a^7\,b^3\,x^3+70\,a^6\,b^4\,x^4+126\,a^5\,b^5\,x^5+210\,a^4\,b^6\,x^6}{x^7}+45\,a^2\,b^8\,x+5\,a\,b^9\,x^2+120\,a^3\,b^7\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^8,x)
```

```
[Out] (b^10*x^3)/3 - (a^10/7 + 9*a^8*b^2*x^2 + 30*a^7*b^3*x^3 + 70*a^6*b^4*x^4 +
126*a^5*b^5*x^5 + 210*a^4*b^6*x^6 + (5*a^9*b*x)/3)/x^7 + 45*a^2*b^8*x + 5*a
*b^9*x^2 + 120*a^3*b^7*log(x)
```

3.143 $\int \frac{(a+bx)^{10}}{x^9}\,dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\frac{b^{10}x^2}{2}+45a^2b^8\log(x)$$

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^9}\,dx = \int \left(10ab^9+\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b^6}{x^3}+\frac{120a}{x^2}\right.$$
$$= -\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x$$

Mathematica [A]
time = 0.00, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\frac{b^{10}x^2}{2}+45a^2b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Mathics [A]
time = 2.80, size = 113, normalized size = 0.95

$$\frac{-a^3\left(7a^7+80a^6bx+420a^5b^2x^2+1344a^4b^3x^3+2940a^3b^4x^4+4704a^2b^5x^5+5880ab^6x^6+6720b^7x^7\right)+28b^8x^8\left(90a^2\text{Log}\left[x\right]+20abx+b^2x^2\right)}{56x^8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^9,x]')
```

```
[Out] (-a ^ 3 (7 a ^ 7 + 80 a ^ 6 b x + 420 a ^ 5 b ^ 2 x ^ 2 + 1344 a ^ 4 b ^ 3
x ^ 3 + 2940 a ^ 3 b ^ 4 x ^ 4 + 4704 a ^ 2 b ^ 5 x ^ 5 + 5880 a b ^ 6 x ^
6 + 6720 b ^ 7 x ^ 7) + 28 b ^ 8 x ^ 8 (90 a ^ 2 Log[x] + 20 a b x + b ^ 2
x ^ 2)) / (56 x ^ 8)
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10a\,b^9x+\frac{b^{10}x^2}{2}+45a^2b^8\ln(x$
risch	$\frac{b^{10}x^2}{2}+10a\,b^9x+\frac{-120a^3b^7x^7-105a^4b^6x^6-84a^5b^5x^5-\frac{105}{2}a^6b^4x^4-24a^7b^3x^3-\frac{15}{2}a^8b^2x^2-\frac{10}{7}a^9bx-\frac{1}{8}a^{10}}{x^8}+45a^2b^8\ln(x)$
norman	$\frac{-\frac{1}{8}a^{10}+\frac{1}{2}b^{10}x^{10}+10a\,b^9x^9-120a^3b^7x^7-105a^4b^6x^6-84a^5b^5x^5-\frac{105}{2}a^6b^4x^4-24a^7b^3x^3-\frac{15}{2}a^8b^2x^2-\frac{10}{7}a^9bx}{x^8}+45a^2b^8\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

Maxima [A]
time = 0.25, size = 110, normalized size = 0.92

$$\frac{1}{2}\,b^{10}x^2+10\,ab^9x+45\,a^2b^8\log\left(x\right)-\frac{6720\,a^3b^7x^7+5880\,a^4b^6x^6+4704\,a^5b^5x^5+2940\,a^6b^4x^4+1344\,a^7b^3x^3+420\,a^8b^2x^2+80\,a^9bx+7\,a^{10}}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(x) - 1/56*(6720*a^3*b^7*x^7 + 58
80*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^3 + 4
20*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Fricas [A]
time = 0.31, size = 114, normalized size = 0.96

$$\frac{28\,b^{10}x^{10}+560\,ab^{9}x^{9}+2520\,a^{2}b^{8}x^{8}\log\left(x\right)-6720\,a^{3}b^{7}x^{7}-5880\,a^{4}b^{6}x^{6}-4704\,a^{5}b^{5}x^{5}-2940\,a^{6}b^{4}x^{4}-1344\,a^{7}b^{3}x^{3}-420\,a^{8}b^{2}x^{2}-80\,a^{9}bx-7\,a^{10}}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*x^10 + 560*a*b^9*x^9 + 2520*a^2*b^8*x^8*log(x) - 6720*a^3*b^7
*x^7 - 5880*a^4*b^6*x^6 - 4704*a^5*b^5*x^5 - 2940*a^6*b^4*x^4 - 1344*a^7*b^
3*x^3 - 420*a^8*b^2*x^2 - 80*a^9*b*x - 7*a^10)/x^8
```

Sympy [A]
time = 0.33, size = 119, normalized size = 1.00

$$45a^{2}b^{8}\log\left(x\right)+10ab^{9}x+\frac{b^{10}x^{2}}{2}+\frac{-7a^{10}-80a^{9}bx-420a^{8}b^{2}x^{2}-1344a^{7}b^{3}x^{3}-2940a^{6}b^{4}x^{4}-4704a^{5}b^{5}x^{5}-5880a^{4}b^{6}x^{6}-6720a^{3}b^{7}x^{7}}{56x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**9,x)
```

```
[Out] 45*a**2*b**8*log(x) + 10*a*b**9*x + b**10*x**2/2 + (-7*a**10 - 80*a**9*b*x
- 420*a**8*b**2*x**2 - 1344*a**7*b**3*x**3 - 2940*a**6*b**4*x**4 - 4704*a**
5*b**5*x**5 - 5880*a**4*b**6*x**6 - 6720*a**3*b**7*x**7)/(56*x**8)
```

Giac [A]
time = 0.00, size = 124, normalized size = 1.04

$$\frac{1}{2}x^{2}b^{10}+10xb^{9}a+\frac{\frac{1}{56}\left(-6720b^{7}a^{3}x^{7}-5880b^{6}a^{4}x^{6}-4704b^{5}a^{5}x^{5}-2940b^{4}a^{6}x^{4}-1344b^{3}a^{7}x^{3}-420b^{2}a^{8}x^{2}-80ba^{9}x-7a^{10}\right)}{x^{8}}+45b^{8}a^{2}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x)
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(abs(x)) - 1/56*(6720*a^3*b^7*x^7
 + 5880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^
3 + 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Mupad [B]
time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^2}{2}-\frac{\frac{a^{10}}{8}+\frac{10\,a^9\,b\,x}{7}+\frac{15\,a^8\,b^2\,x^2}{2}+24\,a^7\,b^3\,x^3+\frac{105\,a^6\,b^4\,x^4}{2}+84\,a^5\,b^5\,x^5+105\,a^4\,b^6\,x^6+120\,a^3\,b^7\,x^7}{x^8}+45\,a^2\,b^8\,\ln\left(x\right)+10\,a\,b^9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^9,x)
```

```
[Out] (b^10*x^2)/2 - (a^10/8 + (15*a^8*b^2*x^2)/2 + 24*a^7*b^3*x^3 + (105*a^6*b^4
*x^4)/2 + 84*a^5*b^5*x^5 + 105*a^4*b^6*x^6 + 120*a^3*b^7*x^7 + (10*a^9*b*x)
/7)/x^8 + 45*a^2*b^8*log(x) + 10*a*b^9*x
```

3.144 $\int \frac{(a+bx)^{10}}{x^{10}} dx$

Optimal. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{10}} dx = \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}\right.$$
$$=-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}$$

Mathematica [A]
time = 0.00, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Mathics [A]
time = 2.83, size = 113, normalized size = 0.99

$$\frac{-a^2\left(28a^8+315a^7bx+1620a^6b^2x^2+5040a^5b^3x^3+10584a^4b^4x^4+15876a^3b^5x^5+17640a^2b^6x^6+15120ab^7x^7+11340b^8x^8\right)+252b^9x^9\left(10a\mathrm{Log}\left[x\right]+bx\right)}{252x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^10,x]')
```

```
[Out] (-a ^ 2 (28 a ^ 8 + 315 a ^ 7 b x + 1620 a ^ 6 b ^ 2 x ^ 2 + 5040 a ^ 5 b ^
 3 x ^ 3 + 10584 a ^ 4 b ^ 4 x ^ 4 + 15876 a ^ 3 b ^ 5 x ^ 5 + 17640 a ^ 2
b ^ 6 x ^ 6 + 15120 a b ^ 7 x ^ 7 + 11340 b ^ 8 x ^ 8) + 252 b ^ 9 x ^ 9 (1
0 a Log[x] + b x)) / (252 x ^ 9)
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.96

method	result	s
default	$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10a\,b^9\ln\left(x\right)$	1
risch	$b^{10}x+\frac{-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx-\frac{1}{9}a^{10}}{x^9}+10a\,b^9\ln\left(x\right)$	1
norman	$\frac{b^{10}x^{10}-\frac{1}{9}a^{10}-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx}{x^9}+10a\,b^9\ln\left(x\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Maxima [A]
time = 0.26, size = 109, normalized size = 0.96

$$b^{10}x+10\,ab^9\log\left(x\right)-\frac{11340\,a^2b^8x^8+15120\,a^3b^7x^7+17640\,a^4b^6x^6+15876\,a^5b^5x^5+10584\,a^6b^4x^4+5040\,a^7b^3x^3+1620\,a^8b^2x^2+315\,a^9bx+28\,a^{10}}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Fricas [A]

time = 0.30, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log\left(x\right)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A]

time = 0.38, size = 117, normalized size = 1.03

$$10ab^{9}\log\left(x\right)+b^{10}x+\frac{-28a^{10}-315a^{9}bx-1620a^{8}b^{2}x^{2}-5040a^{7}b^{3}x^{3}-10584a^{6}b^{4}x^{4}-15876a^{5}b^{5}x^{5}-17640a^{4}b^{6}x^{6}-15120a^{3}b^{7}x^{7}-11340a^{2}b^{8}x^{8}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

Giac [A]

time = 0.00, size = 122, normalized size = 1.07

$$xb^{10}+\frac{\frac{1}{252}\left(-11340b^{8}a^{2}x^{8}-15120b^{7}a^{3}x^{7}-17640b^{6}a^{4}x^{6}-15876b^{5}a^{5}x^{5}-10584b^{4}a^{6}x^{4}-5040b^{3}a^{7}x^{3}-1620b^{2}a^{8}x^{2}-315ba^{9}x-28a^{10}\right)}{x^{9}}+10b^{9}a\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x)
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Mupad [B]

time = 0.08, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9}-b^{10}\,x^{10}+\frac{45\,a^{8}\,b^{2}\,x^{2}}{7}+20\,a^{7}\,b^{3}\,x^{3}+42\,a^{6}\,b^{4}\,x^{4}+63\,a^{5}\,b^{5}\,x^{5}+70\,a^{4}\,b^{6}\,x^{6}+60\,a^{3}\,b^{7}\,x^{7}+45\,a^{2}\,b^{8}\,x^{8}+\frac{5\,a^{9}\,b\,x}{4}-10\,a\,b^{9}\,x^{9}\,\ln\left(x\right)}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

3.145 $\int \frac{(a+bx)^{10}}{x^{11}}\,dx$

Optimal. Leaf size=124

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

```
[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^
4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-1
0*a*b^9/x+b^10*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^11,x]
```

```
[Out] -1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{11}}\,dx &= \int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b^7}{x^4}+\frac{45\ldots}{\ldots}\right.\\ &= -\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2\ldots}{2x^2\ldots}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 124, normalized size = 1.00

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^11,x]
```

```
[Out] -1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Mathics [A]
time = 2.87, size = 113, normalized size = 0.91

$$\frac{-\frac{a(252a^9+2800a^8bx+14175a^7b^2x^2+43200a^6b^3x^3+88200a^5b^4x^4+127008a^4b^5x^5+132300a^3b^6x^6+100800a^2b^7x^7+56700ab^8x^8+25200b^9x^9)}{2520}+b^{10}x^{10}\text{Log}[x]}{x^{10}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^11,x]')
```

```
[Out] (-a (252 a ^ 9 + 2800 a ^ 8 b x + 14175 a ^ 7 b ^ 2 x ^ 2 + 43200 a ^ 6 b ^
 3 x ^ 3 + 88200 a ^ 5 b ^ 4 x ^ 4 + 127008 a ^ 4 b ^ 5 x ^ 5 + 132300 a ^
3 b ^ 6 x ^ 6 + 100800 a ^ 2 b ^ 7 x ^ 7 + 56700 a b ^ 8 x ^ 8 + 25200 b ^
9 x ^ 9) / 2520 + b ^ 10 x ^ 10 Log[x]) / x ^ 10
```

Maple [A]
time = 0.08, size = 111, normalized size = 0.90

method	result
default	$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a\,b^9}{x}+b^{10}\ln(x)$
norman	$\frac{-\frac{1}{10}a^{10}-10a\,b^9x^9-\frac{45}{2}a^2b^8x^8-40a^3b^7x^7-\frac{105}{2}a^4b^6x^6-\frac{252}{5}a^5b^5x^5-35a^6b^4x^4-\frac{120}{7}a^7b^3x^3-\frac{45}{8}a^8b^2x^2-\frac{10}{9}a^9bx}{x^{10}}+b^{10}\ln(x)$
risch	$\frac{-\frac{1}{10}a^{10}-10a\,b^9x^9-\frac{45}{2}a^2b^8x^8-40a^3b^7x^7-\frac{105}{2}a^4b^6x^6-\frac{252}{5}a^5b^5x^5-35a^6b^4x^4-\frac{120}{7}a^7b^3x^3-\frac{45}{8}a^8b^2x^2-\frac{10}{9}a^9bx}{x^{10}}+b^{10}\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^
4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-1
0*a*b^9/x+b^10*ln(x)
```

Maxima [A]
time = 0.24, size = 111, normalized size = 0.90

$$b^{10}\log(x)-\frac{25200\,ab^9x^9+56700\,a^2b^8x^8+100800\,a^3b^7x^7+132300\,a^4b^6x^6+127008\,a^5b^5x^5+88200\,a^6b^4x^4+43200\,a^7b^3x^3+14175\,a^8b^2x^2+2800\,a^9bx+252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="maxima")
```

```
[Out] b^10*log(x) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3*b^7*
x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43200*a
^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

Fricas [A]
time = 0.30, size = 114, normalized size = 0.92

$$\frac{2520\,b^{10}x^{10}\log\left(x\right)-25200\,ab^{9}x^{9}-56700\,a^{2}b^{8}x^{8}-100800\,a^{3}b^{7}x^{7}-132300\,a^{4}b^{6}x^{6}-127008\,a^{5}b^{5}x^{5}-88200\,a^{6}b^{4}x^{4}-43200\,a^{7}b^{3}x^{3}-14175\,a^{8}b^{2}x^{2}-2800\,a^{9}bx-252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^10*x^10*log(x) - 25200*a*b^9*x^9 - 56700*a^2*b^8*x^8 - 10080
0*a^3*b^7*x^7 - 132300*a^4*b^6*x^6 - 127008*a^5*b^5*x^5 - 88200*a^6*b^4*x^4
 - 43200*a^7*b^3*x^3 - 14175*a^8*b^2*x^2 - 2800*a^9*b*x - 252*a^10)/x^10
```

Sympy [A]
time = 0.42, size = 119, normalized size = 0.96

$$b^{10}\log\left(x\right)+\frac{-252a^{10}-2800a^{9}bx-14175a^{8}b^{2}x^{2}-43200a^{7}b^{3}x^{3}-88200a^{6}b^{4}x^{4}-127008a^{5}b^{5}x^{5}-132300a^{4}b^{6}x^{6}-100800a^{3}b^{7}x^{7}-56700a^{2}b^{8}x^{8}-25200ab^{9}x^{9}}{2520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**11,x)
```

```
[Out] b**10*log(x) + (-252*a**10 - 2800*a**9*b*x - 14175*a**8*b**2*x**2 - 43200*a
**7*b**3*x**3 - 88200*a**6*b**4*x**4 - 127008*a**5*b**5*x**5 - 132300*a**4*
b**6*x**6 - 100800*a**3*b**7*x**7 - 56700*a**2*b**8*x**8 - 25200*a*b**9*x**
9)/(2520*x**10)
```

Giac [A]
time = 0.00, size = 125, normalized size = 1.01

$$\frac{\frac{1}{2520}\left(-25200b^{9}ax^{9}-56700b^{8}a^{2}x^{8}-100800b^{7}a^{3}x^{7}-132300b^{6}a^{4}x^{6}-127008b^{5}a^{5}x^{5}-88200b^{4}a^{6}x^{4}-43200b^{3}a^{7}x^{3}-14175b^{2}a^{8}x^{2}-2800ba^{9}x-252a^{10}\right)}{x^{10}}+b^{10}\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x)
```

```
[Out] b^10*log(abs(x)) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3
*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43
200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

Mupad [B]
time = 0.07, size = 111, normalized size = 0.90

$$b^{10}\,\ln\left(x\right)-\frac{\frac{a^{10}}{10}+\frac{10\,a^9\,b\,x}{9}+\frac{45\,a^8\,b^2\,x^2}{8}+\frac{120\,a^7\,b^3\,x^3}{7}+35\,a^6\,b^4\,x^4+\frac{252\,a^5\,b^5\,x^5}{5}+\frac{105\,a^4\,b^6\,x^6}{2}+40\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{2}+10\,a\,b^9\,x^9}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^11,x)
```

```
[Out] b^10*log(x) - (a^10/10 + 10*a*b^9*x^9 + (45*a^8*b^2*x^2)/8 + (120*a^7*b^3*x
^3)/7 + 35*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/5 + (105*a^4*b^6*x^6)/2 + 40*a^3
*b^7*x^7 + (45*a^2*b^8*x^8)/2 + (10*a^9*b*x)/9)/x^10
```

3.146 $\int \frac{(a+bx)^{10}}{x^{12}}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

```
[Out] -1/11*(b*x+a)^11/a/x^11
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^12,x]
```

```
[Out] -1/11*(a + b*x)^11/(a*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{(a+bx)^{11}}{11ax^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(17) = 34.
time = 0.01, size = 114, normalized size = 6.71

$$-\frac{a^{10}}{11x^{11}}-\frac{a^9b}{x^{10}}-\frac{5a^8b^2}{x^9}-\frac{15a^7b^3}{x^8}-\frac{30a^6b^4}{x^7}-\frac{42a^5b^5}{x^6}-\frac{42a^4b^6}{x^5}-\frac{30a^3b^7}{x^4}-\frac{15a^2b^8}{x^3}-\frac{5ab^9}{x^2}-\frac{b^{10}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^12,x]
```

```
[Out] -1/11*a^10/x^11 - (a^9*b)/x^10 - (5*a^8*b^2)/x^9 - (15*a^7*b^3)/x^8 - (30*a
^6*b^4)/x^7 - (42*a^5*b^5)/x^6 - (42*a^4*b^6)/x^5 - (30*a^3*b^7)/x^4 - (15*
a^2*b^8)/x^3 - (5*a*b^9)/x^2 - b^10/x
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(17) = 34$.
time = 2.79, size = 111, normalized size = 6.53

$$\frac{-\frac{a^{10}}{11} - a^9bx - 5a^8b^2x^2 - 15a^7b^3x^3 - 30a^6b^4x^4 - 42a^5b^5x^5 - 42a^4b^6x^6 - 30a^3b^7x^7 - 15a^2b^8x^8 - 5ab^9x^9 - b^{10}x^{10}}{x^{11}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^12,x]')
```

```
[Out] (-a ^ 10 / 11 - a ^ 9 b x - 5 a ^ 8 b ^ 2 x ^ 2 - 15 a ^ 7 b ^ 3 x ^ 3 - 30
 a ^ 6 b ^ 4 x ^ 4 - 42 a ^ 5 b ^ 5 x ^ 5 - 42 a ^ 4 b ^ 6 x ^ 6 - 30 a ^ 3
 b ^ 7 x ^ 7 - 15 a ^ 2 b ^ 8 x ^ 8 - 5 a b ^ 9 x ^ 9 - b ^ 10 x ^ 10) / x
^ 11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(15) = 30$.
time = 0.08, size = 113, normalized size = 6.65

method	result	size
gosper	$-\frac{11b^{10}x^{10}+55a\,b^9x^9+165a^2b^8x^8+330a^3b^7x^7+462a^4b^6x^6+462a^5b^5x^5+330a^6b^4x^4+165a^7b^3x^3+55a^8b^2x^2+11a^9bx+a^{10}}{11x^{11}}$	111
norman	$\frac{-b^{10}x^{10}-5a\,b^9x^9-15a^2b^8x^8-30a^3b^7x^7-42a^4b^6x^6-42a^5b^5x^5-30a^6b^4x^4-15a^7b^3x^3-5a^8b^2x^2-a^9bx-\frac{1}{11}a^{10}}{x^{11}}$	112
risch	$\frac{-b^{10}x^{10}-5a\,b^9x^9-15a^2b^8x^8-30a^3b^7x^7-42a^4b^6x^6-42a^5b^5x^5-30a^6b^4x^4-15a^7b^3x^3-5a^8b^2x^2-a^9bx-\frac{1}{11}a^{10}}{x^{11}}$	112
default	$-\frac{a^9b}{x^{10}}-\frac{a^{10}}{11x^{11}}-\frac{b^{10}}{x}-\frac{15a^2b^8}{x^3}-\frac{30a^3b^7}{x^4}-\frac{5a^8b^2}{x^9}-\frac{5a\,b^9}{x^2}-\frac{15a^7b^3}{x^8}-\frac{42a^4b^6}{x^5}-\frac{42a^5b^5}{x^6}-\frac{30a^6b^4}{x^7}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -a^9*b/x^10-1/11*a^10/x^11-b^10/x-15*a^2*b^8/x^3-30*a^3*b^7/x^4-5*a^8*b^2/x
^9-5*a*b^9/x^2-15*a^7*b^3/x^8-42*a^4*b^6/x^5-42*a^5*b^5/x^6-30*a^6*b^4/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.
time = 0.24, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10} + 55\,ab^9x^9 + 165\,a^2b^8x^8 + 330\,a^3b^7x^7 + 462\,a^4b^6x^6 + 462\,a^5b^5x^5 + 330\,a^6b^4x^4 + 165\,a^7b^3x^3 + 55\,a^8b^2x^2 + 11\,a^9bx + a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.
time = 0.29, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^9x^9+165\,a^2b^8x^8+330\,a^3b^7x^7+462\,a^4b^6x^6+462\,a^5b^5x^5+330\,a^6b^4x^4+165\,a^7b^3x^3+55\,a^8b^2x^2+11\,a^9bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(14) = 28$.
time = 0.44, size = 119, normalized size = 7.00

$$\frac{-a^{10}-11a^9bx-55a^8b^2x^2-165a^7b^3x^3-330a^6b^4x^4-462a^5b^5x^5-462a^4b^6x^6-330a^3b^7x^7-165a^2b^8x^8-55ab^9x^9-11b^{10}x^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**12,x)
```

```
[Out] (-a**10 - 11*a**9*b*x - 55*a**8*b**2*x**2 - 165*a**7*b**3*x**3 - 330*a**6*b
**4*x**4 - 462*a**5*b**5*x**5 - 462*a**4*b**6*x**6 - 330*a**3*b**7*x**7 - 1
65*a**2*b**8*x**8 - 55*a*b**9*x**9 - 11*b**10*x**10)/(11*x**11)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.
time = 0.00, size = 123, normalized size = 7.24

$$\frac{-11x^{10}b^{10}-55x^9b^9a-165x^8b^8a^2-330x^7b^7a^3-462x^6b^6a^4-462x^5b^5a^5-330x^4b^4a^6-165x^3b^3a^7-55x^2b^2a^8-11xba^9-a^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x)
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Mupad [B]
time = 0.13, size = 110, normalized size = 6.47

$$-\frac{\frac{a^{10}}{11}+a^9\,b\,x+5\,a^8\,b^2\,x^2+15\,a^7\,b^3\,x^3+30\,a^6\,b^4\,x^4+42\,a^5\,b^5\,x^5+42\,a^4\,b^6\,x^6+30\,a^3\,b^7\,x^7+15\,a^2\,b^8\,x^8+5\,a\,b^9\,x^9+b^{10}\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^12,x)
```

```
[Out] -(a^10/11 + b^10*x^10 + 5*a*b^9*x^9 + 5*a^8*b^2*x^2 + 15*a^7*b^3*x^3 + 30*a
^6*b^4*x^4 + 42*a^5*b^5*x^5 + 42*a^4*b^6*x^6 + 30*a^3*b^7*x^7 + 15*a^2*b^8*
x^8 + a^9*b*x)/x^11
```

3.147 $\int \frac{(a+bx)^{10}}{x^{13}}\,dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^{11}}{12ax^{12}}+\frac{b(a+bx)^{11}}{132a^2x^{11}}$$

```
[Out] -1/12*(b*x+a)^11/a/x^12+1/132*b*(b*x+a)^11/a^2/x^11
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^13,x]
```

```
[Out] -1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{13}}\,dx &= -\frac{(a+bx)^{11}}{12ax^{12}}-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{12a}\\ &= -\frac{(a+bx)^{11}}{12ax^{12}}+\frac{b(a+bx)^{11}}{132a^2x^{11}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. $2(36) = 72$.
time = 0.00, size = 128, normalized size = 3.56

$$-\frac{a^{10}}{12x^{12}}-\frac{10a^9b}{11x^{11}}-\frac{9a^8b^2}{2x^{10}}-\frac{40a^7b^3}{3x^9}-\frac{105a^6b^4}{4x^8}-\frac{36a^5b^5}{x^7}-\frac{35a^4b^6}{x^6}-\frac{24a^3b^7}{x^5}-\frac{45a^2b^8}{4x^4}-\frac{10ab^9}{3x^3}-\frac{b^{10}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^13,x]
```

```
[Out] -1/12*a^10/x^12 - (10*a^9*b)/(11*x^11) - (9*a^8*b^2)/(2*x^10) - (40*a^7*b^3
)/(3*x^9) - (105*a^6*b^4)/(4*x^8) - (36*a^5*b^5)/x^7 - (35*a^4*b^6)/x^6 - (
24*a^3*b^7)/x^5 - (45*a^2*b^8)/(4*x^4) - (10*a*b^9)/(3*x^3) - b^10/(2*x^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(36) = 72$.
time = 2.77, size = 112, normalized size = 3.11

$$\frac{-11a^{10}-120a^9bx-594a^8b^2x^2-1760a^7b^3x^3-3465a^6b^4x^4-4752a^5b^5x^5-4620a^4b^6x^6-3168a^3b^7x^7-1485a^2b^8x^8-440ab^9x^9-66b^{10}x^{10}}{132x^{12}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^13,x]')
```

```
[Out] (-11 a ^ 10 - 120 a ^ 9 b x - 594 a ^ 8 b ^ 2 x ^ 2 - 1760 a ^ 7 b ^ 3 x ^
3 - 3465 a ^ 6 b ^ 4 x ^ 4 - 4752 a ^ 5 b ^ 5 x ^ 5 - 4620 a ^ 4 b ^ 6 x ^
6 - 3168 a ^ 3 b ^ 7 x ^ 7 - 1485 a ^ 2 b ^ 8 x ^ 8 - 440 a b ^ 9 x ^ 9 - 6
6 b ^ 10 x ^ 10) / (132 x ^ 12)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(32) = 64$.
time = 0.08, size = 113, normalized size = 3.14

method	result
norman	$\frac{-\frac{1}{2}b^{10}x^{10}-\frac{10}{3}a\,b^9x^9-\frac{45}{4}a^2b^8x^8-24a^3b^7x^7-35a^4b^6x^6-36a^5b^5x^5-\frac{105}{4}a^6b^4x^4-\frac{40}{3}a^7b^3x^3-\frac{9}{2}a^8b^2x^2-\frac{10}{11}a^9bx-\frac{1}{12}a^{10}}{x^{12}}$
risch	$\frac{-\frac{1}{2}b^{10}x^{10}-\frac{10}{3}a\,b^9x^9-\frac{45}{4}a^2b^8x^8-24a^3b^7x^7-35a^4b^6x^6-36a^5b^5x^5-\frac{105}{4}a^6b^4x^4-\frac{40}{3}a^7b^3x^3-\frac{9}{2}a^8b^2x^2-\frac{10}{11}a^9bx-\frac{1}{12}a^{10}}{x^{12}}$
gosper	$-\frac{66b^{10}x^{10}+440a\,b^9x^9+1485a^2b^8x^8+3168a^3b^7x^7+4620a^4b^6x^6+4752a^5b^5x^5+3465a^6b^4x^4+1760a^7b^3x^3+594a^8b^2x^2+120a^9bx+11a^1}{132x^{12}}$
default	$-\frac{9a^8b^2}{2x^{10}}-\frac{10a^9b}{11x^{11}}-\frac{10a\,b^9}{3x^3}-\frac{45a^2b^8}{4x^4}-\frac{a^{10}}{12x^{12}}-\frac{40a^7b^3}{3x^9}-\frac{b^{10}}{2x^2}-\frac{105a^6b^4}{4x^8}-\frac{24a^3b^7}{x^5}-\frac{35a^4b^6}{x^6}-\frac{36a^5b^5}{x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -9/2*a^8*b^2/x^10-10/11*a^9*b/x^11-10/3*a*b^9/x^3-45/4*a^2*b^8/x^4-1/12*a^1
0/x^12-40/3*a^7*b^3/x^9-1/2*b^10/x^2-105/4*a^6*b^4/x^8-24*a^3*b^7/x^5-35*a^
4*b^6/x^6-36*a^5*b^5/x^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(32) = 64$.
time = 0.24, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10} + 440\,ab^{9}x^{9} + 1485\,a^{2}b^{8}x^{8} + 3168\,a^{3}b^{7}x^{7} + 4620\,a^{4}b^{6}x^{6} + 4752\,a^{5}b^{5}x^{5} + 3465\,a^{6}b^{4}x^{4} + 1760\,a^{7}b^{3}x^{3} + 594\,a^{8}b^{2}x^{2} + 120\,a^{9}bx + 11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(32) = 64$.
time = 0.30, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10} + 440\,ab^{9}x^{9} + 1485\,a^{2}b^{8}x^{8} + 3168\,a^{3}b^{7}x^{7} + 4620\,a^{4}b^{6}x^{6} + 4752\,a^{5}b^{5}x^{5} + 3465\,a^{6}b^{4}x^{4} + 1760\,a^{7}b^{3}x^{3} + 594\,a^{8}b^{2}x^{2} + 120\,a^{9}bx + 11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(29) = 58$.
time = 0.47, size = 121, normalized size = 3.36

$$\frac{-11a^{10} - 120a^{9}bx - 594a^{8}b^{2}x^{2} - 1760a^{7}b^{3}x^{3} - 3465a^{6}b^{4}x^{4} - 4752a^{5}b^{5}x^{5} - 4620a^{4}b^{6}x^{6} - 3168a^{3}b^{7}x^{7} - 1485a^{2}b^{8}x^{8} - 440ab^{9}x^{9} - 66b^{10}x^{10}}{132x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**13,x)
```

```
[Out] (-11*a**10 - 120*a**9*b*x - 594*a**8*b**2*x**2 - 1760*a**7*b**3*x**3 - 3465
*a**6*b**4*x**4 - 4752*a**5*b**5*x**5 - 4620*a**4*b**6*x**6 - 3168*a**3*b**
7*x**7 - 1485*a**2*b**8*x**8 - 440*a*b**9*x**9 - 66*b**10*x**10)/(132*x**12
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(32) = 64$.
time = 0.00, size = 125, normalized size = 3.47

$$\frac{-66x^{10}b^{10} - 440x^{9}b^{9}a - 1485x^{8}b^{8}a^{2} - 3168x^{7}b^{7}a^{3} - 4620x^{6}b^{6}a^{4} - 4752x^{5}b^{5}a^{5} - 3465x^{4}b^{4}a^{6} - 1760x^{3}b^{3}a^{7} - 594x^{2}b^{2}a^{8} - 120xba^{9} - 11a^{10}}{132x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x)
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Mupad [B]
time = 0.10, size = 23, normalized size = 0.64

$$-\frac{(11\,a-b\,x)\,\left(a+b\,x\right)^{11}}{132\,a^2\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^13,x)
```

```
[Out] -((11*a - b*x)*(a + b*x)^11)/(132*a^2*x^12)
```

3.148 $\int \frac{(a+bx)^{10}}{x^{14}} dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^{11}}{13ax^{13}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}$$

```
[Out] -1/13*(b*x+a)^11/a/x^13+1/78*b*(b*x+a)^11/a^2/x^12-1/858*b^2*(b*x+a)^11/a^3
/x^11
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^14,x]
```

```
[Out] -1/13*(a + b*x)^11/(a*x^13) + (b*(a + b*x)^11)/(78*a^2*x^12) - (b^2*(a + b*
x)^11)/(858*a^3*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{(a+bx)^{11}}{13ax^{13}} - \frac{(2b)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{13a}$$
$$= -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{78a^2}$$
$$= -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} - \frac{b^2(a+bx)^{11}}{858a^3x^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(56) = 112$.
time = 0.01, size = 126, normalized size = 2.25

$$-\frac{a^{10}}{13x^{13}} - \frac{5a^9b}{6x^{12}} - \frac{45a^8b^2}{11x^{11}} - \frac{12a^7b^3}{x^{10}} - \frac{70a^6b^4}{3x^9} - \frac{63a^5b^5}{2x^8} - \frac{30a^4b^6}{x^7} - \frac{20a^3b^7}{x^6} - \frac{9a^2b^8}{x^5} - \frac{5ab^9}{2x^4} - \frac{b^{10}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^14,x]
```

```
[Out] -1/13*a^10/x^13 - (5*a^9*b)/(6*x^12) - (45*a^8*b^2)/(11*x^11) - (12*a^7*b^3
)/x^10 - (70*a^6*b^4)/(3*x^9) - (63*a^5*b^5)/(2*x^8) - (30*a^4*b^6)/x^7 - (
20*a^3*b^7)/x^6 - (9*a^2*b^8)/x^5 - (5*a*b^9)/(2*x^4) - b^10/(3*x^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(56) = 112$.
time = 2.92, size = 112, normalized size = 2.00

$$\frac{-66a^{10} - 715a^9bx - 3510a^8b^2x^2 - 10296a^7b^3x^3 - 20020a^6b^4x^4 - 27027a^5b^5x^5 - 25740a^4b^6x^6 - 17160a^3b^7x^7 - 7722a^2b^8x^8 - 2145ab^9x^9 - 286b^{10}x^{10}}{858x^{13}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^14,x]')
```

```
[Out] (-66 a ^ 10 - 715 a ^ 9 b x - 3510 a ^ 8 b ^ 2 x ^ 2 - 10296 a ^ 7 b ^ 3 x
^ 3 - 20020 a ^ 6 b ^ 4 x ^ 4 - 27027 a ^ 5 b ^ 5 x ^ 5 - 25740 a ^ 4 b ^ 6
 x ^ 6 - 17160 a ^ 3 b ^ 7 x ^ 7 - 7722 a ^ 2 b ^ 8 x ^ 8 - 2145 a b ^ 9 x
^ 9 - 286 b ^ 10 x ^ 10) / (858 x ^ 13)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.
time = 0.08, size = 113, normalized size = 2.02

method	result
norman	$\frac{-\frac{1}{3}b^{10}x^{10}-\frac{5}{2}a\,b^9x^9-9a^2b^8x^8-20a^3b^7x^7-30a^4b^6x^6-\frac{63}{2}a^5b^5x^5-\frac{70}{3}a^6b^4x^4-12a^7b^3x^3-\frac{45}{11}a^8b^2x^2-\frac{5}{6}a^9bx-\frac{1}{13}a^{10}}{x^{13}}$
risch	$\frac{-\frac{1}{3}b^{10}x^{10}-\frac{5}{2}a\,b^9x^9-9a^2b^8x^8-20a^3b^7x^7-30a^4b^6x^6-\frac{63}{2}a^5b^5x^5-\frac{70}{3}a^6b^4x^4-12a^7b^3x^3-\frac{45}{11}a^8b^2x^2-\frac{5}{6}a^9bx-\frac{1}{13}a^{10}}{x^{13}}$

gosper	$-\frac{286b^{10}x^{10}+2145a\,b^9x^9+7722a^2b^8x^8+17160a^3b^7x^7+25740a^4b^6x^6+27027a^5b^5x^5+20020a^6b^4x^4+10296a^7b^3x^3+3510a^8b^2x^2+715}{858x^{13}}$
default	$-\frac{12a^7b^3}{x^{10}}-\frac{45a^8b^2}{11x^{11}}-\frac{b^{10}}{3x^3}-\frac{5a\,b^9}{2x^4}-\frac{5a^9b}{6x^{12}}-\frac{70a^6b^4}{3x^9}-\frac{63a^5b^5}{2x^8}-\frac{9a^2b^8}{x^5}-\frac{a^{10}}{13x^{13}}-\frac{20a^3b^7}{x^6}-\frac{30a^4b^6}{x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -12*a^7*b^3/x^10-45/11*a^8*b^2/x^11-1/3*b^10/x^3-5/2*a*b^9/x^4-5/6*a^9*b/x^
12-70/3*a^6*b^4/x^9-63/2*a^5*b^5/x^8-9*a^2*b^8/x^5-1/13*a^10/x^13-20*a^3*b^
7/x^6-30*a^4*b^6/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.
time = 0.26, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^9x^9+7722\,a^2b^8x^8+17160\,a^3b^7x^7+25740\,a^4b^6x^6+27027\,a^5b^5x^5+20020\,a^6b^4x^4+10296\,a^7b^3x^3+3510\,a^8b^2x^2+715\,a^9bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.
time = 0.30, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^9x^9+7722\,a^2b^8x^8+17160\,a^3b^7x^7+25740\,a^4b^6x^6+27027\,a^5b^5x^5+20020\,a^6b^4x^4+10296\,a^7b^3x^3+3510\,a^8b^2x^2+715\,a^9bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(48) = 96$.
time = 0.49, size = 121, normalized size = 2.16

$$\frac{-66a^{10}-715a^9bx-3510a^8b^2x^2-10296a^7b^3x^3-20020a^6b^4x^4-27027a^5b^5x^5-25740a^4b^6x^6-17160a^3b^7x^7-7722a^2b^8x^8-2145ab^9x^9-286b^{10}x^{10}}{858x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**14,x)
```

```
[Out] (-66*a**10 - 715*a**9*b*x - 3510*a**8*b**2*x**2 - 10296*a**7*b**3*x**3 - 20
020*a**6*b**4*x**4 - 27027*a**5*b**5*x**5 - 25740*a**4*b**6*x**6 - 17160*a*
*3*b**7*x**7 - 7722*a**2*b**8*x**8 - 2145*a*b**9*x**9 - 286*b**10*x**10)/(8
58*x**13)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(50) = 100.
time = 0.00, size = 125, normalized size = 2.23

$$\frac{-286x^{10}b^{10} - 2145x^9b^9a - 7722x^8b^8a^2 - 17160x^7b^7a^3 - 25740x^6b^6a^4 - 27027x^5b^5a^5 - 20020x^4b^4a^6 - 10296x^3b^3a^7 - 3510x^2b^2a^8 - 715xba^9 - 66a^{10}}{858x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x)
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Mupad [B]
time = 0.13, size = 112, normalized size = 2.00

$$-\frac{\frac{a^{10}}{13} + \frac{5\,a^9\,b\,x}{6} + \frac{45\,a^8\,b^2\,x^2}{11} + 12\,a^7\,b^3\,x^3 + \frac{70\,a^6\,b^4\,x^4}{3} + \frac{63\,a^5\,b^5\,x^5}{2} + 30\,a^4\,b^6\,x^6 + 20\,a^3\,b^7\,x^7 + 9\,a^2\,b^8\,x^8 + \frac{5\,a\,b^9\,x^9}{2} + \frac{b^{10}\,x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^14,x)
```

```
[Out] -(a^10/13 + (b^10*x^10)/3 + (5*a*b^9*x^9)/2 + (45*a^8*b^2*x^2)/11 + 12*a^7*
b^3*x^3 + (70*a^6*b^4*x^4)/3 + (63*a^5*b^5*x^5)/2 + 30*a^4*b^6*x^6 + 20*a^3
*b^7*x^7 + 9*a^2*b^8*x^8 + (5*a^9*b*x)/6)/x^13
```

3.149 $\int \frac{(a+bx)^{10}}{x^{15}} dx$

Optimal. Leaf size=76

$$-\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}$$

```
[Out] -1/14*(b*x+a)^11/a/x^14+3/182*b*(b*x+a)^11/a^2/x^13-1/364*b^2*(b*x+a)^11/a^
3/x^12+1/4004*b^3*(b*x+a)^11/a^4/x^11
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^15,x]
```

```
[Out] -1/14*(a + b*x)^11/(a*x^14) + (3*b*(a + b*x)^11)/(182*a^2*x^13) - (b^2*(a +
 b*x)^11)/(364*a^3*x^12) + (b^3*(a + b*x)^11)/(4004*a^4*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = -\frac{(a+bx)^{11}}{14ax^{14}} - \frac{(3b)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{14a}$$
$$= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} + \frac{(3b^2)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{91a^2}$$
$$= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{364a^3}$$
$$= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} + \frac{b^3(a+bx)^{11}}{4004a^4x^{11}}$$

Mathematica [A]
time = 0.01, size = 128, normalized size = 1.68

$$-\frac{a^{10}}{14x^{14}} - \frac{10a^9b}{13x^{13}} - \frac{15a^8b^2}{4x^{12}} - \frac{120a^7b^3}{11x^{11}} - \frac{21a^6b^4}{x^{10}} - \frac{28a^5b^5}{x^9} - \frac{105a^4b^6}{4x^8} - \frac{120a^3b^7}{7x^7} - \frac{15a^2b^8}{2x^6} - \frac{2ab^9}{x^5} - \frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^15,x]
```

```
[Out] -1/14*a^10/x^14 - (10*a^9*b)/(13*x^13) - (15*a^8*b^2)/(4*x^12) - (120*a^7*b
^3)/(11*x^11) - (21*a^6*b^4)/x^10 - (28*a^5*b^5)/x^9 - (105*a^4*b^6)/(4*x^8
) - (120*a^3*b^7)/(7*x^7) - (15*a^2*b^8)/(2*x^6) - (2*a*b^9)/x^5 - b^10/(4*
x^4)
```

Mathics [A]
time = 2.91, size = 112, normalized size = 1.47

$$\frac{-286a^{10} - 3080a^9bx - 15015a^8b^2x^2 - 43680a^7b^3x^3 - 84084a^6b^4x^4 - 112112a^5b^5x^5 - 105105a^4b^6x^6 - 68640a^3b^7x^7 - 30030a^2b^8x^8 - 8008ab^9x^9 - 1001b^{10}x^{10}}{4004x^{14}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^15,x]')
```

```
[Out] (-286 a ^ 10 - 3080 a ^ 9 b x - 15015 a ^ 8 b ^ 2 x ^ 2 - 43680 a ^ 7 b ^ 3
 x ^ 3 - 84084 a ^ 6 b ^ 4 x ^ 4 - 112112 a ^ 5 b ^ 5 x ^ 5 - 105105 a ^ 4
b ^ 6 x ^ 6 - 68640 a ^ 3 b ^ 7 x ^ 7 - 30030 a ^ 2 b ^ 8 x ^ 8 - 8008 a b
^ 9 x ^ 9 - 1001 b ^ 10 x ^ 10) / (4004 x ^ 14)
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.49

method	result

norman	$\frac{-\frac{1}{14}a^{10}-\frac{10}{13}a^{9}bx-\frac{15}{4}a^{8}b^{2}x^{2}-\frac{120}{11}a^{7}b^{3}x^{3}-21a^{6}b^{4}x^{4}-28a^{5}b^{5}x^{5}-\frac{105}{4}a^{4}b^{6}x^{6}-\frac{120}{7}a^{3}b^{7}x^{7}-\frac{15}{2}a^{2}b^{8}x^{8}-2a\,b^{9}x^{9}-\frac{1}{4}b^{10}x^{10}}{x^{14}}$
risch	$\frac{-\frac{1}{14}a^{10}-\frac{10}{13}a^{9}bx-\frac{15}{4}a^{8}b^{2}x^{2}-\frac{120}{11}a^{7}b^{3}x^{3}-21a^{6}b^{4}x^{4}-28a^{5}b^{5}x^{5}-\frac{105}{4}a^{4}b^{6}x^{6}-\frac{120}{7}a^{3}b^{7}x^{7}-\frac{15}{2}a^{2}b^{8}x^{8}-2a\,b^{9}x^{9}-\frac{1}{4}b^{10}x^{10}}{x^{14}}$
gosper	$-\frac{1001b^{10}x^{10}+8008a\,b^{9}x^{9}+30030a^{2}b^{8}x^{8}+68640a^{3}b^{7}x^{7}+105105a^{4}b^{6}x^{6}+112112a^{5}b^{5}x^{5}+84084a^{6}b^{4}x^{4}+43680a^{7}b^{3}x^{3}+15015a^{8}b^{2}x^{2}}{4004x^{14}}$
default	$-\frac{21a^{6}b^{4}}{x^{10}}-\frac{120a^{7}b^{3}}{11x^{11}}-\frac{a^{10}}{14x^{14}}-\frac{b^{10}}{4x^{4}}-\frac{15a^{8}b^{2}}{4x^{12}}-\frac{28a^{5}b^{5}}{x^{9}}-\frac{105a^{4}b^{6}}{4x^{8}}-\frac{2a\,b^{9}}{x^{5}}-\frac{10a^{9}b}{13x^{13}}-\frac{15a^{2}b^{8}}{2x^{6}}-\frac{120a^{3}b^{7}}{7x^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -21*a^6*b^4/x^10-120/11*a^7*b^3/x^11-1/14*a^10/x^14-1/4*b^10/x^4-15/4*a^8*b
^2/x^12-28*a^5*b^5/x^9-105/4*a^4*b^6/x^8-2*a*b^9/x^5-10/13*a^9*b/x^13-15/2*
a^2*b^8/x^6-120/7*a^3*b^7/x^7
```

Maxima [A]
time = 0.24, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Fricas [A]
time = 0.30, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Sympy [A]
time = 0.51, size = 121, normalized size = 1.59

$$\frac{-286a^{10}-3080a^{9}bx-15015a^{8}b^{2}x^{2}-43680a^{7}b^{3}x^{3}-84084a^{6}b^{4}x^{4}-112112a^{5}b^{5}x^{5}-105105a^{4}b^{6}x^{6}-68640a^{3}b^{7}x^{7}-30030a^{2}b^{8}x^{8}-8008ab^{9}x^{9}-1001b^{10}x^{10}}{4004x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**15,x)
```

```
[Out] (-286*a**10 - 3080*a**9*b*x - 15015*a**8*b**2*x**2 - 43680*a**7*b**3*x**3 -
 84084*a**6*b**4*x**4 - 112112*a**5*b**5*x**5 - 105105*a**4*b**6*x**6 - 686
40*a**3*b**7*x**7 - 30030*a**2*b**8*x**8 - 8008*a*b**9*x**9 - 1001*b**10*x*
*10)/(4004*x**14)
```

Giac [A]
time = 0.00, size = 125, normalized size = 1.64

$$\frac{-1001x^{10}b^{10} - 8008x^9b^9a - 30030x^8b^8a^2 - 68640x^7b^7a^3 - 105105x^6b^6a^4 - 112112x^5b^5a^5 - 84084x^4b^4a^6 - 43680x^3b^3a^7 - 15015x^2b^2a^8 - 3080xba^9 - 286a^{10}}{4004x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x)
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Mupad [B]
time = 0.09, size = 112, normalized size = 1.47

$$-\frac{\frac{a^{10}}{14}+\frac{10\,a^9\,b\,x}{13}+\frac{15\,a^8\,b^2\,x^2}{4}+\frac{120\,a^7\,b^3\,x^3}{11}+21\,a^6\,b^4\,x^4+28\,a^5\,b^5\,x^5+\frac{105\,a^4\,b^6\,x^6}{4}+\frac{120\,a^3\,b^7\,x^7}{7}+\frac{15\,a^2\,b^8\,x^8}{2}+2\,a\,b^9\,x^9+\frac{b^{10}\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^15,x)
```

```
[Out] -(a^10/14 + (b^10*x^10)/4 + 2*a*b^9*x^9 + (15*a^8*b^2*x^2)/4 + (120*a^7*b^3
*x^3)/11 + 21*a^6*b^4*x^4 + 28*a^5*b^5*x^5 + (105*a^4*b^6*x^6)/4 + (120*a^3
*b^7*x^7)/7 + (15*a^2*b^8*x^8)/2 + (10*a^9*b*x)/13)/x^14
```

3.150 $\int \frac{(a+bx)^{10}}{x^{16}} dx$

Optimal. Leaf size=96

$$-\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}$$

```
[Out] -1/15*(b*x+a)^11/a/x^15+2/105*b*(b*x+a)^11/a^2/x^14-2/455*b^2*(b*x+a)^11/a^
3/x^13+1/1365*b^3*(b*x+a)^11/a^4/x^12-1/15015*b^4*(b*x+a)^11/a^5/x^11
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^16,x]
```

```
[Out] -1/15*(a + b*x)^11/(a*x^15) + (2*b*(a + b*x)^11)/(105*a^2*x^14) - (2*b^2*(a
 + b*x)^11)/(455*a^3*x^13) + (b^3*(a + b*x)^11)/(1365*a^4*x^12) - (b^4*(a +
 b*x)^11)/(15015*a^5*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx = -\frac{(a+bx)^{11}}{15ax^{15}} - \frac{(4b)\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{15a}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} + \frac{(2b^2)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{35a^2}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} - \frac{(4b^3)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{455a^3}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{1365a^4}$$
$$= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} - \frac{b^4(a+bx)^{11}}{15015a^5x^{11}}$$

Mathematica [A]
time = 0.01, size = 130, normalized size = 1.35

$$-\frac{a^{10}}{15x^{15}} - \frac{5a^9b}{7x^{14}} - \frac{45a^8b^2}{13x^{13}} - \frac{10a^7b^3}{x^{12}} - \frac{210a^6b^4}{11x^{11}} - \frac{126a^5b^5}{5x^{10}} - \frac{70a^4b^6}{3x^9} - \frac{15a^3b^7}{x^8} - \frac{45a^2b^8}{7x^7} - \frac{5ab^9}{3x^6} - \frac{b^{10}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^16,x]
```

```
[Out] -1/15*a^10/x^15 - (5*a^9*b)/(7*x^14) - (45*a^8*b^2)/(13*x^13) - (10*a^7*b^3
)/x^12 - (210*a^6*b^4)/(11*x^11) - (126*a^5*b^5)/(5*x^10) - (70*a^4*b^6)/(3
*x^9) - (15*a^3*b^7)/x^8 - (45*a^2*b^8)/(7*x^7) - (5*a*b^9)/(3*x^6) - b^10/
(5*x^5)
```

Mathics [A]
time = 2.88, size = 112, normalized size = 1.17

$$\frac{-1001a^{10} - 10725a^9bx - 51975a^8b^2x^2 - 150150a^7b^3x^3 - 286650a^6b^4x^4 - 378378a^5b^5x^5 - 350350a^4b^6x^6 - 225225a^3b^7x^7 - 96525a^2b^8x^8 - 25025ab^9x^9 - 3003b^{10}x^{10}}{15015x^{15}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^16,x]')
```

```
[Out] (-1001 a ^ 10 - 10725 a ^ 9 b x - 51975 a ^ 8 b ^ 2 x ^ 2 - 150150 a ^ 7 b
^ 3 x ^ 3 - 286650 a ^ 6 b ^ 4 x ^ 4 - 378378 a ^ 5 b ^ 5 x ^ 5 - 350350 a
^ 4 b ^ 6 x ^ 6 - 225225 a ^ 3 b ^ 7 x ^ 7 - 96525 a ^ 2 b ^ 8 x ^ 8 - 2502
5 a b ^ 9 x ^ 9 - 3003 b ^ 10 x ^ 10) / (15015 x ^ 15)
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.18

method	result
norman	$\frac{-\frac{1}{15}a^{10}-\frac{5}{7}a^9bx-\frac{45}{13}a^8b^2x^2-10a^7b^3x^3-\frac{210}{11}a^6b^4x^4-\frac{126}{5}a^5b^5x^5-\frac{70}{3}a^4b^6x^6-15a^3b^7x^7-\frac{45}{7}a^2b^8x^8-\frac{5}{3}a\,b^9x^9-\frac{1}{5}b^{10}x^{10}}{x^{15}}$
risch	$\frac{-\frac{1}{15}a^{10}-\frac{5}{7}a^9bx-\frac{45}{13}a^8b^2x^2-10a^7b^3x^3-\frac{210}{11}a^6b^4x^4-\frac{126}{5}a^5b^5x^5-\frac{70}{3}a^4b^6x^6-15a^3b^7x^7-\frac{45}{7}a^2b^8x^8-\frac{5}{3}a\,b^9x^9-\frac{1}{5}b^{10}x^{10}}{x^{15}}$
gosper	$-\frac{3003b^{10}x^{10}+25025a\,b^9x^9+96525a^2b^8x^8+225225a^3b^7x^7+350350a^4b^6x^6+378378a^5b^5x^5+286650a^6b^4x^4+150150a^7b^3x^3+51975a^8}{15015x^{15}}$
default	$-\frac{126a^5b^5}{5x^{10}}-\frac{210a^6b^4}{11x^{11}}-\frac{5a^9b}{7x^{14}}-\frac{10a^7b^3}{x^{12}}-\frac{70a^4b^6}{3x^9}-\frac{15a^3b^7}{x^8}-\frac{b^{10}}{5x^5}-\frac{45a^8b^2}{13x^{13}}-\frac{a^{10}}{15x^{15}}-\frac{5a\,b^9}{3x^6}-\frac{45a^2b^8}{7x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -126/5*a^5*b^5/x^10-210/11*a^6*b^4/x^11-5/7*a^9*b/x^14-10*a^7*b^3/x^12-70/3
*a^4*b^6/x^9-15*a^3*b^7/x^8-1/5*b^10/x^5-45/13*a^8*b^2/x^13-1/15*a^10/x^15-
5/3*a*b^9/x^6-45/7*a^2*b^8/x^7
```

Maxima [A]
time = 0.25, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^9x^9+96525\,a^2b^8x^8+225225\,a^3b^7x^7+350350\,a^4b^6x^6+378378\,a^5b^5x^5+286650\,a^6b^4x^4+150150\,a^7b^3x^3+51975\,a^8b^2x^2+10725\,a^9bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Fricas [A]
time = 0.30, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^9x^9+96525\,a^2b^8x^8+225225\,a^3b^7x^7+350350\,a^4b^6x^6+378378\,a^5b^5x^5+286650\,a^6b^4x^4+150150\,a^7b^3x^3+51975\,a^8b^2x^2+10725\,a^9bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Sympy [A]
time = 0.56, size = 121, normalized size = 1.26

$$\frac{-1001a^{10}-10725a^9bx-51975a^8b^2x^2-150150a^7b^3x^3-286650a^6b^4x^4-378378a^5b^5x^5-350350a^4b^6x^6-225225a^3b^7x^7-96525a^2b^8x^8-25025ab^9x^9-3003b^{10}x^{10}}{15015x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**16,x)
```

```
[Out] (-1001*a**10 - 10725*a**9*b*x - 51975*a**8*b**2*x**2 - 150150*a**7*b**3*x**
3 - 286650*a**6*b**4*x**4 - 378378*a**5*b**5*x**5 - 350350*a**4*b**6*x**6 -
 225225*a**3*b**7*x**7 - 96525*a**2*b**8*x**8 - 25025*a*b**9*x**9 - 3003*b*
*10*x**10)/(15015*x**15)
```

Giac [A]
time = 0.00, size = 125, normalized size = 1.30

$$\frac{-3003x^{10}b^{10} - 25025x^9b^9a - 96525x^8b^8a^2 - 225225x^7b^7a^3 - 350350x^6b^6a^4 - 378378x^5b^5a^5 - 286650x^4b^4a^6 - 150150x^3b^3a^7 - 51975x^2b^2a^8 - 10725xba^9 - 1001a^{10}}{15015x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x)
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Mupad [B]
time = 0.13, size = 112, normalized size = 1.17

$$-\frac{\frac{a^{10}}{15}+\frac{5\,a^9\,b\,x}{7}+\frac{45\,a^8\,b^2\,x^2}{13}+10\,a^7\,b^3\,x^3+\frac{210\,a^6\,b^4\,x^4}{11}+\frac{126\,a^5\,b^5\,x^5}{5}+\frac{70\,a^4\,b^6\,x^6}{3}+15\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{7}+\frac{5\,a\,b^9\,x^9}{3}+\frac{b^{10}\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^16,x)
```

```
[Out] -(a^10/15 + (b^10*x^10)/5 + (5*a*b^9*x^9)/3 + (45*a^8*b^2*x^2)/13 + 10*a^7*
b^3*x^3 + (210*a^6*b^4*x^4)/11 + (126*a^5*b^5*x^5)/5 + (70*a^4*b^6*x^6)/3 +
 15*a^3*b^7*x^7 + (45*a^2*b^8*x^8)/7 + (5*a^9*b*x)/7)/x^15
```

3.151 $\int \frac{(a+bx)^{10}}{x^{17}} dx$

Optimal. Leaf size=116

$$-\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}$$

```
[Out] -1/16*(b*x+a)^11/a/x^16+1/48*b*(b*x+a)^11/a^2/x^15-1/168*b^2*(b*x+a)^11/a^3
/x^14+1/728*b^3*(b*x+a)^11/a^4/x^13-1/4368*b^4*(b*x+a)^11/a^5/x^12+1/48048*
b^5*(b*x+a)^11/a^6/x^11
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^17,x]
```

```
[Out] -1/16*(a + b*x)^11/(a*x^16) + (b*(a + b*x)^11)/(48*a^2*x^15) - (b^2*(a + b*
x)^11)/(168*a^3*x^14) + (b^3*(a + b*x)^11)/(728*a^4*x^13) - (b^4*(a + b*x)^
11)/(4368*a^5*x^12) + (b^5*(a + b*x)^11)/(48048*a^6*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{17}}\,dx &= -\frac{(a+bx)^{11}}{16ax^{16}} - \frac{(5b)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{16a} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{12a^2} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{56a^3} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{364a^4} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} - \frac{b^5\int \frac{(a+bx)^{10}}{x^{12}}\,d}{4368a^5} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} + \frac{b^5(a+bx)^{11}}{48048a^6x^{11}}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.14

$$
-\frac{a^{10}}{16x^{16}} - \frac{2a^9b}{3x^{15}} - \frac{45a^8b^2}{14x^{14}} - \frac{120a^7b^3}{13x^{13}} - \frac{35a^6b^4}{2x^{12}} - \frac{252a^5b^5}{11x^{11}} - \frac{21a^4b^6}{x^{10}} - \frac{40a^3b^7}{3x^9} - \frac{45a^2b^8}{8x^8} - \frac{10ab^9}{7x^7} - \frac{b^{10}}{6x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^17,x]
```

```
[Out] -1/16*a^10/x^16 - (2*a^9*b)/(3*x^15) - (45*a^8*b^2)/(14*x^14) - (120*a^7*b^
3)/(13*x^13) - (35*a^6*b^4)/(2*x^12) - (252*a^5*b^5)/(11*x^11) - (21*a^4*b^
6)/x^10 - (40*a^3*b^7)/(3*x^9) - (45*a^2*b^8)/(8*x^8) - (10*a*b^9)/(7*x^7)
- b^10/(6*x^6)
```

Mathics [A]
time = 2.90, size = 112, normalized size = 0.97

$$
\frac{-3003a^{10} - 32032a^9bx - 154440a^8b^2x^2 - 443520a^7b^3x^3 - 840840a^6b^4x^4 - 1100736a^5b^5x^5 - 1009008a^4b^6x^6 - 640640a^3b^7x^7 - 270270a^2b^8x^8 - 68640ab^9x^9 - 8008b^{10}x^{10}}{48048x^{16}}
$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^17,x]')
```

```
[Out] (-3003 a ^ 10 - 32032 a ^ 9 b x - 154440 a ^ 8 b ^ 2 x ^ 2 - 443520 a ^ 7 b
 ^ 3 x ^ 3 - 840840 a ^ 6 b ^ 4 x ^ 4 - 1100736 a ^ 5 b ^ 5 x ^ 5 - 1009008
 a ^ 4 b ^ 6 x ^ 6 - 640640 a ^ 3 b ^ 7 x ^ 7 - 270270 a ^ 2 b ^ 8 x ^ 8 -
68640 a b ^ 9 x ^ 9 - 8008 b ^ 10 x ^ 10) / (48048 x ^ 16)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.97

method	result
norman	$\frac{-\frac{1}{16}a^{10}-\frac{2}{3}a^{9}bx-\frac{45}{14}a^{8}b^{2}x^{2}-\frac{120}{13}a^{7}b^{3}x^{3}-\frac{35}{2}a^{6}b^{4}x^{4}-\frac{252}{11}a^{5}b^{5}x^{5}-21a^{4}b^{6}x^{6}-\frac{40}{3}a^{3}b^{7}x^{7}-\frac{45}{8}a^{2}b^{8}x^{8}-\frac{10}{7}a\,b^{9}x^{9}-\frac{1}{6}b^{10}x^{10}}{x^{16}}$
risch	$\frac{-\frac{1}{16}a^{10}-\frac{2}{3}a^{9}bx-\frac{45}{14}a^{8}b^{2}x^{2}-\frac{120}{13}a^{7}b^{3}x^{3}-\frac{35}{2}a^{6}b^{4}x^{4}-\frac{252}{11}a^{5}b^{5}x^{5}-21a^{4}b^{6}x^{6}-\frac{40}{3}a^{3}b^{7}x^{7}-\frac{45}{8}a^{2}b^{8}x^{8}-\frac{10}{7}a\,b^{9}x^{9}-\frac{1}{6}b^{10}x^{10}}{x^{16}}$
gosper	$-\frac{8008b^{10}x^{10}+68640a\,b^{9}x^{9}+270270a^{2}b^{8}x^{8}+640640a^{3}b^{7}x^{7}+1009008a^{4}b^{6}x^{6}+1100736a^{5}b^{5}x^{5}+840840a^{6}b^{4}x^{4}+443520a^{7}b^{3}x^{3}+154}{48048x^{16}}$
default	$-\frac{21a^{4}b^{6}}{x^{10}}-\frac{252a^{5}b^{5}}{11x^{11}}-\frac{45a^{8}b^{2}}{14x^{14}}-\frac{35a^{6}b^{4}}{2x^{12}}-\frac{40a^{3}b^{7}}{3x^{9}}-\frac{45a^{2}b^{8}}{8x^{8}}-\frac{120a^{7}b^{3}}{13x^{13}}-\frac{2a^{9}b}{3x^{15}}-\frac{b^{10}}{6x^{6}}-\frac{10a\,b^{9}}{7x^{7}}-\frac{a^{10}}{16x^{16}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -21*a^4*b^6/x^10-252/11*a^5*b^5/x^11-45/14*a^8*b^2/x^14-35/2*a^6*b^4/x^12-4
0/3*a^3*b^7/x^9-45/8*a^2*b^8/x^8-120/13*a^7*b^3/x^13-2/3*a^9*b/x^15-1/6*b^1
0/x^6-10/7*a*b^9/x^7-1/16*a^10/x^16
```

Maxima [A]
time = 0.25, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^{9}x^{9}+270270\,a^{2}b^{8}x^{8}+640640\,a^{3}b^{7}x^{7}+1009008\,a^{4}b^{6}x^{6}+1100736\,a^{5}b^{5}x^{5}+840840\,a^{6}b^{4}x^{4}+443520\,a^{7}b^{3}x^{3}+154440\,a^{8}b^{2}x^{2}+32032\,a^{9}bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Fricas [A]
time = 0.30, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^{9}x^{9}+270270\,a^{2}b^{8}x^{8}+640640\,a^{3}b^{7}x^{7}+1009008\,a^{4}b^{6}x^{6}+1100736\,a^{5}b^{5}x^{5}+840840\,a^{6}b^{4}x^{4}+443520\,a^{7}b^{3}x^{3}+154440\,a^{8}b^{2}x^{2}+32032\,a^{9}bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Sympy [A]
time = 0.58, size = 121, normalized size = 1.04

$$\frac{-3003a^{10}-32032a^{9}bx-154440a^{8}b^{2}x^{2}-443520a^{7}b^{3}x^{3}-840840a^{6}b^{4}x^{4}-1100736a^{5}b^{5}x^{5}-1009008a^{4}b^{6}x^{6}-640640a^{3}b^{7}x^{7}-270270a^{2}b^{8}x^{8}-68640ab^{9}x^{9}-8008b^{10}x^{10}}{48048x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**17,x)
```

```
[Out] (-3003*a**10 - 32032*a**9*b*x - 154440*a**8*b**2*x**2 - 443520*a**7*b**3*x**3 - 840840*a**6*b**4*x**4 - 1100736*a**5*b**5*x**5 - 1009008*a**4*b**6*x**6 - 640640*a**3*b**7*x**7 - 270270*a**2*b**8*x**8 - 68640*a*b**9*x**9 - 8008*b**10*x**10)/(48048*x**16)
```

Giac [A]
time = 0.00, size = 125, normalized size = 1.08

$$\frac{-8008x^{10}b^{10} - 68640x^9b^9a - 270270x^8b^8a^2 - 640640x^7b^7a^3 - 1009008x^6b^6a^4 - 1100736x^5b^5a^5 - 840840x^4b^4a^6 - 443520x^3b^3a^7 - 154440x^2b^2a^8 - 32032xba^9 - 3003a^{10}}{48048x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x)
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4 + 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Mupad [B]
time = 0.13, size = 112, normalized size = 0.97

$$-\frac{\frac{a^{10}}{16} + \frac{2\,a^9\,b\,x}{3} + \frac{45\,a^8\,b^2\,x^2}{14} + \frac{120\,a^7\,b^3\,x^3}{13} + \frac{35\,a^6\,b^4\,x^4}{2} + \frac{252\,a^5\,b^5\,x^5}{11} + 21\,a^4\,b^6\,x^6 + \frac{40\,a^3\,b^7\,x^7}{3} + \frac{45\,a^2\,b^8\,x^8}{8} + \frac{10\,a\,b^9\,x^9}{7} + \frac{b^{10}\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^17,x)
```

```
[Out] -(a^10/16 + (b^10*x^10)/6 + (10*a*b^9*x^9)/7 + (45*a^8*b^2*x^2)/14 + (120*a^7*b^3*x^3)/13 + (35*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/11 + 21*a^4*b^6*x^6 + (40*a^3*b^7*x^7)/3 + (45*a^2*b^8*x^8)/8 + (2*a^9*b*x)/3)/x^16
```

3.152 $\int \frac{(a+bx)^{10}}{x^{18}} dx$

Optimal. Leaf size=136

$$-\frac{(a+bx)^{11}}{17ax^{17}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}$$

```
[Out] -1/17*(b*x+a)^11/a/x^17+3/136*b*(b*x+a)^11/a^2/x^16-1/136*b^2*(b*x+a)^11/a^
3/x^15+1/476*b^3*(b*x+a)^11/a^4/x^14-3/6188*b^4*(b*x+a)^11/a^5/x^13+1/12376
*b^5*(b*x+a)^11/a^6/x^12-1/136136*b^6*(b*x+a)^11/a^7/x^11
```

Rubi [A]
time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^18,x]
```

```
[Out] -1/17*(a + b*x)^11/(a*x^17) + (3*b*(a + b*x)^11)/(136*a^2*x^16) - (b^2*(a +
 b*x)^11)/(136*a^3*x^15) + (b^3*(a + b*x)^11)/(476*a^4*x^14) - (3*b^4*(a +
b*x)^11)/(6188*a^5*x^13) + (b^5*(a + b*x)^11)/(12376*a^6*x^12) - (b^6*(a +
b*x)^11)/(136136*a^7*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{(a+bx)^{11}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx)^{10}}{x^{17}}\,dx}{17a}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} + \frac{(15b^2)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{136a^2}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{34a^3}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} + \frac{(3b^4)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{476a^4}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} - \frac{(3b^5)\int \frac{(a+}{x}}{3094}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)}{12376a^6x}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)}{12376a^6x}$$

Mathematica [A]
time = 0.01, size = 126, normalized size = 0.93

$$-\frac{a^{10}}{17x^{17}} - \frac{5a^9b}{8x^{16}} - \frac{3a^8b^2}{x^{15}} - \frac{60a^7b^3}{7x^{14}} - \frac{210a^6b^4}{13x^{13}} - \frac{21a^5b^5}{x^{12}} - \frac{210a^4b^6}{11x^{11}} - \frac{12a^3b^7}{x^{10}} - \frac{5a^2b^8}{x^9} - \frac{5ab^9}{4x^8} - \frac{b^{10}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^18,x]
```

```
[Out] -1/17*a^10/x^17 - (5*a^9*b)/(8*x^16) - (3*a^8*b^2)/x^15 - (60*a^7*b^3)/(7*x
^14) - (210*a^6*b^4)/(13*x^13) - (21*a^5*b^5)/x^12 - (210*a^4*b^6)/(11*x^11
) - (12*a^3*b^7)/x^10 - (5*a^2*b^8)/x^9 - (5*a*b^9)/(4*x^8) - b^10/(7*x^7)
```

Mathics [A]
time = 2.94, size = 112, normalized size = 0.82

$$\frac{-8008a^{10} - 85085a^9bx - 408408a^8b^2x^2 - 1166880a^7b^3x^3 - 2199120a^6b^4x^4 - 2858856a^5b^5x^5 - 2598960a^4b^6x^6 - 1633632a^3b^7x^7 - 680680a^2b^8x^8 - 170170ab^9x^9 - 19448b^{10}x^{10}}{136136x^{17}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^18,x]')
```

```
[Out] (-8008 a ^ 10 - 85085 a ^ 9 b x - 408408 a ^ 8 b ^ 2 x ^ 2 - 1166880 a ^ 7
b ^ 3 x ^ 3 - 2199120 a ^ 6 b ^ 4 x ^ 4 - 2858856 a ^ 5 b ^ 5 x ^ 5 - 25989
60 a ^ 4 b ^ 6 x ^ 6 - 1633632 a ^ 3 b ^ 7 x ^ 7 - 680680 a ^ 2 b ^ 8 x ^ 8
 - 170170 a b ^ 9 x ^ 9 - 19448 b ^ 10 x ^ 10) / (136136 x ^ 17)
```

Maple [A]
time = 0.09, size = 113, normalized size = 0.83

method	result
norman	$\frac{-\frac{1}{17}a^{10}-\frac{5}{8}a^{9}bx-3a^{8}b^{2}x^{2}-\frac{60}{7}a^{7}b^{3}x^{3}-\frac{210}{13}a^{6}b^{4}x^{4}-21a^{5}b^{5}x^{5}-\frac{210}{11}a^{4}b^{6}x^{6}-12a^{3}b^{7}x^{7}-5a^{2}b^{8}x^{8}-\frac{5}{4}a\,b^{9}x^{9}-\frac{1}{7}b^{10}x^{10}}{x^{17}}$
risch	$\frac{-\frac{1}{17}a^{10}-\frac{5}{8}a^{9}bx-3a^{8}b^{2}x^{2}-\frac{60}{7}a^{7}b^{3}x^{3}-\frac{210}{13}a^{6}b^{4}x^{4}-21a^{5}b^{5}x^{5}-\frac{210}{11}a^{4}b^{6}x^{6}-12a^{3}b^{7}x^{7}-5a^{2}b^{8}x^{8}-\frac{5}{4}a\,b^{9}x^{9}-\frac{1}{7}b^{10}x^{10}}{x^{17}}$
gosper	$-\frac{19448b^{10}x^{10}+170170a\,b^{9}x^{9}+680680a^{2}b^{8}x^{8}+1633632a^{3}b^{7}x^{7}+2598960a^{4}b^{6}x^{6}+2858856a^{5}b^{5}x^{5}+2199120a^{6}b^{4}x^{4}+1166880a^{7}b^{3}x^{3}}{136136x^{17}}$
default	$-\frac{12a^{3}b^{7}}{x^{10}}-\frac{210a^{4}b^{6}}{11x^{11}}-\frac{60a^{7}b^{3}}{7x^{14}}-\frac{a^{10}}{17x^{17}}-\frac{21a^{5}b^{5}}{x^{12}}-\frac{5a^{2}b^{8}}{x^{9}}-\frac{5a\,b^{9}}{4x^{8}}-\frac{210a^{6}b^{4}}{13x^{13}}-\frac{3a^{8}b^{2}}{x^{15}}-\frac{b^{10}}{7x^{7}}-\frac{5a^{9}b}{8x^{16}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -12*a^3*b^7/x^10-210/11*a^4*b^6/x^11-60/7*a^7*b^3/x^14-1/17*a^10/x^17-21*a^
5*b^5/x^12-5*a^2*b^8/x^9-5/4*a*b^9/x^8-210/13*a^6*b^4/x^13-3*a^8*b^2/x^15-1
/7*b^10/x^7-5/8*a^9*b/x^16
```

Maxima [A]
time = 0.24, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^{9}x^{9}+680680\,a^{2}b^{8}x^{8}+1633632\,a^{3}b^{7}x^{7}+2598960\,a^{4}b^{6}x^{6}+2858856\,a^{5}b^{5}x^{5}+2199120\,a^{6}b^{4}x^{4}+1166880\,a^{7}b^{3}x^{3}+408408\,a^{8}b^{2}x^{2}+85085\,a^{9}bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Fricas [A]
time = 0.29, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^{9}x^{9}+680680\,a^{2}b^{8}x^{8}+1633632\,a^{3}b^{7}x^{7}+2598960\,a^{4}b^{6}x^{6}+2858856\,a^{5}b^{5}x^{5}+2199120\,a^{6}b^{4}x^{4}+1166880\,a^{7}b^{3}x^{3}+408408\,a^{8}b^{2}x^{2}+85085\,a^{9}bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Sympy [A]
time = 0.59, size = 121, normalized size = 0.89

$$\frac{-8008a^{10}-85085a^9bx-408408a^8b^2x^2-1166880a^7b^3x^3-2199120a^6b^4x^4-2858856a^5b^5x^5-2598960a^4b^6x^6-1633632a^3b^7x^7-680680a^2b^8x^8-170170ab^9x^9-19448b^{10}x^{10}}{136136x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**18,x)
```

```
[Out] (-8008*a**10 - 85085*a**9*b*x - 408408*a**8*b**2*x**2 - 1166880*a**7*b**3*x
**3 - 2199120*a**6*b**4*x**4 - 2858856*a**5*b**5*x**5 - 2598960*a**4*b**6*x
**6 - 1633632*a**3*b**7*x**7 - 680680*a**2*b**8*x**8 - 170170*a*b**9*x**9 -
 19448*b**10*x**10)/(136136*x**17)
```

Giac [A]
time = 0.00, size = 125, normalized size = 0.92

$$\frac{-19448x^{10}b^{10}-170170x^9b^9a-680680x^8b^8a^2-1633632x^7b^7a^3-2598960x^6b^6a^4-2858856x^5b^5a^5-2199120x^4b^4a^6-1166880x^3b^3a^7-408408x^2b^2a^8-85085xba^9-8008a^{10}}{136136x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x)
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Mupad [B]
time = 0.13, size = 112, normalized size = 0.82

$$-\frac{\frac{a^{10}}{17}+\frac{5\,a^9\,b\,x}{8}+3\,a^8\,b^2\,x^2+\frac{60\,a^7\,b^3\,x^3}{7}+\frac{210\,a^6\,b^4\,x^4}{13}+21\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{11}+12\,a^3\,b^7\,x^7+5\,a^2\,b^8\,x^8+\frac{5\,a\,b^9\,x^9}{4}+\frac{b^{10}\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^18,x)
```

```
[Out] -(a^10/17 + (b^10*x^10)/7 + (5*a*b^9*x^9)/4 + 3*a^8*b^2*x^2 + (60*a^7*b^3*x
^3)/7 + (210*a^6*b^4*x^4)/13 + 21*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/11 + 12*a
^3*b^7*x^7 + 5*a^2*b^8*x^8 + (5*a^9*b*x)/8)/x^17
```

3.153 $\int \frac{(a+bx)^{10}}{x^{19}} \, dx$

Optimal. Leaf size=130

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^19,x]
```

```
[Out] -1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{19}} \, dx &= \int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b^7}{x^{12}}+\frac{45}{x}\right.\\ &= -\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\end{aligned}$$

Mathematica [A]
time = 0.00, size = 130, normalized size = 1.00

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^19,x]
```

```
[Out] -1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Mathics [A]
time = 2.93, size = 112, normalized size = 0.86

$$\frac{-19448a^{10}-205920a^9bx-984555a^8b^2x^2-2800512a^7b^3x^3-5250960a^6b^4x^4-6785856a^5b^5x^5-6126120a^4b^6x^6-3818880a^3b^7x^7-1575288a^2b^8x^8-388960ab^9x^9-43758b^{10}x^{10}}{350064x^{18}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^19,x]')
```

```
[Out] (-19448 a ^ 10 - 205920 a ^ 9 b x - 984555 a ^ 8 b ^ 2 x ^ 2 - 2800512 a ^
7 b ^ 3 x ^ 3 - 5250960 a ^ 6 b ^ 4 x ^ 4 - 6785856 a ^ 5 b ^ 5 x ^ 5 - 612
6120 a ^ 4 b ^ 6 x ^ 6 - 3818880 a ^ 3 b ^ 7 x ^ 7 - 1575288 a ^ 2 b ^ 8 x
^ 8 - 388960 a b ^ 9 x ^ 9 - 43758 b ^ 10 x ^ 10) / (350064 x ^ 18)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.87

method	result
norman	$\frac{-\frac{1}{18}a^{10}-\frac{10}{17}a^9bx-\frac{45}{16}a^8b^2x^2-8a^7b^3x^3-15a^6b^4x^4-\frac{252}{13}a^5b^5x^5-\frac{35}{2}a^4b^6x^6-\frac{120}{11}a^3b^7x^7-\frac{9}{2}a^2b^8x^8-\frac{10}{9}ab^9x^9-\frac{1}{8}b^{10}x^{10}}{x^{18}}$
risch	$\frac{-\frac{1}{18}a^{10}-\frac{10}{17}a^9bx-\frac{45}{16}a^8b^2x^2-8a^7b^3x^3-15a^6b^4x^4-\frac{252}{13}a^5b^5x^5-\frac{35}{2}a^4b^6x^6-\frac{120}{11}a^3b^7x^7-\frac{9}{2}a^2b^8x^8-\frac{10}{9}ab^9x^9-\frac{1}{8}b^{10}x^{10}}{x^{18}}$
gosper	$-\frac{43758b^{10}x^{10}+388960ab^9x^9+1575288a^2b^8x^8+3818880a^3b^7x^7+6126120a^4b^6x^6+6785856a^5b^5x^5+5250960a^6b^4x^4+2800512a^7b^3x^3+}{350064x^{18}}$
default	$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

Maxima [A]
time = 0.25, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^9x^9+1575288\,a^2b^8x^8+3818880\,a^3b^7x^7+6126120\,a^4b^6x^6+6785856\,a^5b^5x^5+5250960\,a^6b^4x^4+2800512\,a^7b^3x^3+984555\,a^8b^2x^2+205920\,a^9bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10/x^19,x, algorithm="maxima")

[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18

Fricas [A]
time = 0.30, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10/x^19,x, algorithm="fricas")

[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18

Sympy [A]
time = 0.61, size = 121, normalized size = 0.93

$$\frac{-19448a^{10}-205920a^{9}bx-984555a^{8}b^{2}x^{2}-2800512a^{7}b^{3}x^{3}-5250960a^{6}b^{4}x^{4}-6785856a^{5}b^{5}x^{5}-6126120a^{4}b^{6}x^{6}-3818880a^{3}b^{7}x^{7}-1575288a^{2}b^{8}x^{8}-388960ab^{9}x^{9}-43758b^{10}x^{10}}{350064x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**10/x**19,x)

[Out] (-19448*a**10 - 205920*a**9*b*x - 984555*a**8*b**2*x**2 - 2800512*a**7*b**3
*x**3 - 5250960*a**6*b**4*x**4 - 6785856*a**5*b**5*x**5 - 6126120*a**4*b**6
*x**6 - 3818880*a**3*b**7*x**7 - 1575288*a**2*b**8*x**8 - 388960*a*b**9*x**
9 - 43758*b**10*x**10)/(350064*x**18)

Giac [A]
time = 0.00, size = 125, normalized size = 0.96

$$\frac{-43758x^{10}b^{10}-388960x^{9}b^{9}a-1575288x^{8}b^{8}a^{2}-3818880x^{7}b^{7}a^{3}-6126120x^{6}b^{6}a^{4}-6785856x^{5}b^{5}a^{5}-5250960x^{4}b^{4}a^{6}-2800512x^{3}b^{3}a^{7}-984555x^{2}b^{2}a^{8}-205920xba^{9}-19448a^{10}}{350064x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10/x^19,x)

[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18

Mupad [B]
time = 0.10, size = 112, normalized size = 0.86

$$-\frac{\frac{a^{10}}{18}+\frac{10\,a^9\,b\,x}{17}+\frac{45\,a^8\,b^2\,x^2}{16}+8\,a^7\,b^3\,x^3+15\,a^6\,b^4\,x^4+\frac{252\,a^5\,b^5\,x^5}{13}+\frac{35\,a^4\,b^6\,x^6}{2}+\frac{120\,a^3\,b^7\,x^7}{11}+\frac{9\,a^2\,b^8\,x^8}{2}+\frac{10\,a\,b^9\,x^9}{9}+\frac{b^{10}\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^19,x)
```

```
[Out] -(a^10/18 + (b^10*x^10)/8 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/16 + 8*a^7*
b^3*x^3 + 15*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/13 + (35*a^4*b^6*x^6)/2 + (120
*a^3*b^7*x^7)/11 + (9*a^2*b^8*x^8)/2 + (10*a^9*b*x)/17)/x^18
```

3.154 $\int \frac{(a+bx)^{10}}{x^{20}} dx$

Optimal. Leaf size=126

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^20,x]
```

```
[Out] -1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^{10}}{x^{20}}\,dx &= \int \left(\frac{a^{10}}{x^{20}}+\frac{10a^9b}{x^{19}}+\frac{45a^8b^2}{x^{18}}+\frac{120a^7b^3}{x^{17}}+\frac{210a^6b^4}{x^{16}}+\frac{252a^5b^5}{x^{15}}+\frac{210a^4b^6}{x^{14}}+\frac{120a^3b^7}{x^{13}}+\frac{45}{x} \right. \\ &= -\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}- \end{aligned}$$

Mathematica [A]
time = 0.00, size = 126, normalized size = 1.00

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^20,x]
```

```
[Out] -1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Mathics [A]
time = 3.00, size = 112, normalized size = 0.89

$$\frac{-43758a^{10} - 461890a^9bx - 2200770a^8b^2x^2 - 6235515a^7b^3x^3 - 11639628a^6b^4x^4 - 14965236a^5b^5x^5 - 13430340a^4b^6x^6 - 8314020a^3b^7x^7 - 3401190a^2b^8x^8 - 831402ab^9x^9 - 92378b^{10}x^{10}}{831402x^{19}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^20,x]')
```

```
[Out] (-43758 a ^ 10 - 461890 a ^ 9 b x - 2200770 a ^ 8 b ^ 2 x ^ 2 - 6235515 a ^
 7 b ^ 3 x ^ 3 - 11639628 a ^ 6 b ^ 4 x ^ 4 - 14965236 a ^ 5 b ^ 5 x ^ 5 -
13430340 a ^ 4 b ^ 6 x ^ 6 - 8314020 a ^ 3 b ^ 7 x ^ 7 - 3401190 a ^ 2 b ^
8 x ^ 8 - 831402 a b ^ 9 x ^ 9 - 92378 b ^ 10 x ^ 10) / (831402 x ^ 19)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.90

method	result
norman	$\frac{-\frac{1}{19}a^{10}-\frac{5}{9}a^9bx-\frac{45}{17}a^8b^2x^2-\frac{15}{2}a^7b^3x^3-14a^6b^4x^4-18a^5b^5x^5-\frac{210}{13}a^4b^6x^6-10a^3b^7x^7-\frac{45}{11}a^2b^8x^8-a\,b^9x^9-\frac{1}{9}b^{10}x^{10}}{x^{19}}$
risch	$\frac{-\frac{1}{19}a^{10}-\frac{5}{9}a^9bx-\frac{45}{17}a^8b^2x^2-\frac{15}{2}a^7b^3x^3-14a^6b^4x^4-18a^5b^5x^5-\frac{210}{13}a^4b^6x^6-10a^3b^7x^7-\frac{45}{11}a^2b^8x^8-a\,b^9x^9-\frac{1}{9}b^{10}x^{10}}{x^{19}}$
gosper	$-\frac{92378b^{10}x^{10}+831402a\,b^9x^9+3401190a^2b^8x^8+8314020a^3b^7x^7+13430340a^4b^6x^6+14965236a^5b^5x^5+11639628a^6b^4x^4+6235515a^7b^3}{831402x^{19}}$
default	$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{a\,b^9}{x^{10}}-\frac{b^{10}}{9x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

Maxima [A]
time = 0.24, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^9x^9 + 3401190\,a^2b^8x^8 + 8314020\,a^3b^7x^7 + 13430340\,a^4b^6x^6 + 14965236\,a^5b^5x^5 + 11639628\,a^6b^4x^4 + 6235515\,a^7b^3x^3 + 2200770\,a^8b^2x^2 + 461890\,a^9bx + 43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Fricas [A]
time = 0.30, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+11639628\,a^{6}b^{4}x^{4}+6235515\,a^{7}b^{3}x^{3}+2200770\,a^{8}b^{2}x^{2}+461890\,a^{9}bx+43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Sympy [A]
time = 0.63, size = 121, normalized size = 0.96

$$\frac{-43758a^{10}-461890a^{9}bx-2200770a^{8}b^{2}x^{2}-6235515a^{7}b^{3}x^{3}-11639628a^{6}b^{4}x^{4}-14965236a^{5}b^{5}x^{5}-13430340a^{4}b^{6}x^{6}-8314020a^{3}b^{7}x^{7}-3401190a^{2}b^{8}x^{8}-831402ab^{9}x^{9}-92378b^{10}x^{10}}{831402x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**20,x)
```

```
[Out] (-43758*a**10 - 461890*a**9*b*x - 2200770*a**8*b**2*x**2 - 6235515*a**7*b**
3*x**3 - 11639628*a**6*b**4*x**4 - 14965236*a**5*b**5*x**5 - 13430340*a**4*
b**6*x**6 - 8314020*a**3*b**7*x**7 - 3401190*a**2*b**8*x**8 - 831402*a*b**9
*x**9 - 92378*b**10*x**10)/(831402*x**19)
```

Giac [A]
time = 0.00, size = 125, normalized size = 0.99

$$\frac{-92378x^{10}b^{10}-831402x^{9}b^{9}a-3401190x^{8}b^{8}a^{2}-8314020x^{7}b^{7}a^{3}-13430340x^{6}b^{6}a^{4}-14965236x^{5}b^{5}a^{5}-11639628x^{4}b^{4}a^{6}-6235515x^{3}b^{3}a^{7}-2200770x^{2}b^{2}a^{8}-461890xba^{9}-43758a^{10}}{831402x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x)
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Mupad [B]
time = 0.14, size = 111, normalized size = 0.88

$$-\frac{\frac{a^{10}}{19}+\frac{5\,a^9\,b\,x}{9}+\frac{45\,a^8\,b^2\,x^2}{17}+\frac{15\,a^7\,b^3\,x^3}{2}+14\,a^6\,b^4\,x^4+18\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{13}+10\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{11}+a\,b^9\,x^9+\frac{b^{10}\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^20,x)
```

```
[Out] -(a^10/19 + (b^10*x^10)/9 + a*b^9*x^9 + (45*a^8*b^2*x^2)/17 + (15*a^7*b^3*x
^3)/2 + 14*a^6*b^4*x^4 + 18*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/13 + 10*a^3*b^7
*x^7 + (45*a^2*b^8*x^8)/11 + (5*a^9*b*x)/9)/x^19
```

3.155 $\int c(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^2}{2b}$$

[Out] 1/2*c*(b*x+a)^2/b

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{9\}$

$$\frac{c(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[c*(a + b*x),x]

[Out] (c*(a + b*x)^2)/(2*b)

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[a*((b + c*x)^2/(2*c)), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int c(a+bx)\,dx = \frac{c(a+bx)^2}{2b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 0.93

$$c\left(ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[c*(a + b*x),x]

[Out] c*(a*x + (b*x^2)/2)

Mathics [A]
time = 1.69, size = 11, normalized size = 0.73

$$\frac{cx\,(2a+bx)}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[c*(a + b*x),x]')
```

```
[Out] c x (2 a + b x) / 2
```

Maple [A]
time = 0.01, size = 13, normalized size = 0.87

method	result	size
gosper	$\frac{x(bx+2a)c}{2}$	12
default	$\left(\frac{1}{2}x^{2}b+ax\right)c$	13
norman	$acx+\frac{1}{2}bc\,x^{2}$	13
risch	$acx+\frac{1}{2}bc\,x^{2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*x^2*b+a*x)*c
```

Maxima [A]
time = 0.24, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^{2}+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

Fricas [A]
time = 0.29, size = 12, normalized size = 0.80

$$\frac{1}{2}x^{2}cb+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + x*c*a
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.80

$$acx+\frac{bcx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x)
```

```
[Out] a*c*x + b*c*x**2/2
```

Giac [A]
time = 0.00, size = 13, normalized size = 0.87

$$c\left(\frac{1}{2}bx^2+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.73

$$\frac{c\,x\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(a + b*x),x)
```

```
[Out] (c*x*(2*a + b*x))/2
```

3.156 $\int \frac{(c+d)(a+bx)}{e} \, dx$

Optimal. Leaf size=20

$$\frac{(c+d)(a+bx)^2}{2be}$$

[Out] 1/2*(c+d)*(b*x+a)^2/b/e

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {9}

$$\frac{(c+d)(a+bx)^2}{2be}$$

Antiderivative was successfully verified.

[In] Int[((c + d)*(a + b*x))/e,x]

[Out] ((c + d)*(a + b*x)^2)/(2*b*e)

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[a*((b + c*x)^2/(2*c)), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(c+d)(a+bx)}{e} \, dx = \frac{(c+d)(a+bx)^2}{2be}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.95

$$\frac{(c+d)\left(ax+\frac{bx^2}{2}\right)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[((c + d)*(a + b*x))/e,x]

[Out] ((c + d)*(a*x + (b*x^2)/2))/e

Mathics [A]
time = 1.76, size = 16, normalized size = 0.80

$$\frac{x\,(2a+bx)\,(c+d)}{2e}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c + d)*(a + b*x))/e,x]')
```

```
[Out] x (2 a + b x) (c + d) / (2 e)
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.90

method	result	size
gosper	$\frac{x(bx+2a)(c+d)}{2e}$	17
default	$\frac{\left(\frac{1}{2}x^2b+ax\right)(c+d)}{e}$	18
norman	$\frac{a(c+d)x}{e}+\frac{(c+d)b\,x^2}{2e}$	23
risch	$\frac{axc}{e}+\frac{axd}{e}+\frac{b\,x^2c}{2e}+\frac{b\,x^2d}{2e}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d)*(b*x+a)/e,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*x^2*b+a*x)*(c+d)/e
```

Maxima [A]
time = 0.25, size = 17, normalized size = 0.85

$$\frac{1}{2}\left(bx^2+2\,ax\right)(c+d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

Fricas [A]
time = 0.30, size = 26, normalized size = 1.30

$$\frac{1}{2}\left((bc+bd)x^2+2\,(ac+ad)x\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="fricas")
```

```
[Out] 1/2*((b*c + b*d)*x^2 + 2*(a*c + a*d)*x)*e^(-1)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 1.10

$$\frac{x^2\,(bc+bd)}{2e}+\frac{x\,(ac+ad)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x)
```

```
[Out] x**2*(b*c + b*d)/(2*e) + x*(a*c + a*d)/e
```

Giac [A]
time = 0.00, size = 17, normalized size = 0.85

$$(c+d)\,\mathrm{e}^{-1}\left(\frac{1}{2}bx^2+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

Mupad [B]
time = 0.07, size = 16, normalized size = 0.80

$$\frac{x\,\left(c+d\right)\,\left(2\,a+b\,x\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d)*(a + b*x))/e,x)
```

```
[Out] (x*(c + d)*(2*a + b*x))/(2*e)
```

3.157 $\int \frac{x^5}{a+bx} \, dx$

Optimal. Leaf size=70

$$\frac{a^4x}{b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^4}{4b^2} + \frac{x^5}{5b} - \frac{a^5\log(a+bx)}{b^6}$$

```
[Out] a^4*x/b^5-1/2*a^3*x^2/b^4+1/3*a^2*x^3/b^3-1/4*a*x^4/b^2+1/5*x^5/b-a^5*ln(b*
x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5\log(a+bx)}{b^6} + \frac{a^4x}{b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^4}{4b^2} + \frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx} \, dx &= \int \left(\frac{a^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a^5}{b^5(a+bx)}\right) dx \\ &= \frac{a^4x}{b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^4}{4b^2} + \frac{x^5}{5b} - \frac{a^5\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 70, normalized size = 1.00

$$\frac{a^4x}{b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^4}{4b^2} + \frac{x^5}{5b} - \frac{a^5\log(a+bx)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Mathics [A]
time = 2.00, size = 61, normalized size = 0.87

$$\frac{-a^5\text{Log}\left[a+bx\right]+a^4bx-\frac{a^3b^2x^2}{2}+\frac{a^2b^3x^3}{3}-\frac{ab^4x^4}{4}+\frac{b^5x^5}{5}}{b^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5/(a + b*x),x]')
```

```
[Out] (-a ^ 5 Log[a + b x] + a ^ 4 b x - a ^ 3 b ^ 2 x ^ 2 / 2 + a ^ 2 b ^ 3 x ^
3 / 3 - a b ^ 4 x ^ 4 / 4 + b ^ 5 x ^ 5 / 5) / b ^ 6
```

Maple [A]
time = 0.08, size = 63, normalized size = 0.90

method	result	size
default	$\frac{\frac{1}{5}b^4x^5-\frac{1}{4}a\,b^3x^4+\frac{1}{3}a^2b^2x^3-\frac{1}{2}a^3b\,x^2+a^4x}{b^5}-\frac{a^5\ln(bx+a)}{b^6}$	63
norman	$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{a\,x^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\ln(bx+a)}{b^6}$	63
risch	$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{a\,x^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\ln(bx+a)}{b^6}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*b^4*x^5-1/4*a*b^3*x^4+1/3*a^2*b^2*x^3-1/2*a^3*b*x^2+a^4*x)-a^5*l
n(b*x+a)/b^6
```

Maxima [A]
time = 0.26, size = 64, normalized size = 0.91

$$-\frac{a^5\log\left(bx+a\right)}{b^6}+\frac{12\,b^4x^5-15\,ab^3x^4+20\,a^2b^2x^3-30\,a^3bx^2+60\,a^4x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -a^5*log(b*x + a)/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x^3 -
30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Fricas [A]
time = 0.32, size = 63, normalized size = 0.90

$$\frac{12\,b^5x^5 - 15\,ab^4x^4 + 20\,a^2b^3x^3 - 30\,a^3b^2x^2 + 60\,a^4bx - 60\,a^5\log\left(bx + a\right)}{60\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*x^5 - 15*a*b^4*x^4 + 20*a^2*b^3*x^3 - 30*a^3*b^2*x^2 + 60*a^4*
b*x - 60*a^5*log(b*x + a))/b^6
```

Sympy [A]
time = 0.07, size = 61, normalized size = 0.87

$$-\frac{a^5\log\left(a + bx\right)}{b^6} + \frac{a^4x}{b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^4}{4b^2} + \frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a),x)
```

```
[Out] -a**5*log(a + b*x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b
**3) - a*x**4/(4*b**2) + x**5/(5*b)
```

Giac [A]
time = 0.00, size = 75, normalized size = 1.07

$$\frac{\frac{1}{5}x^5b^4 - \frac{1}{4}x^4b^3a + \frac{1}{3}x^3b^2a^2 - \frac{1}{2}x^2ba^3 + xa^4}{b^5} - \frac{a^5\ln|xb + a|}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x)
```

```
[Out] -a^5*log(abs(b*x + a))/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x
^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Mupad [B]
time = 0.08, size = 62, normalized size = 0.89

$$\frac{x^5}{5\,b} - \frac{a^5\,\ln\left(a + b\,x\right)}{b^6} - \frac{a\,x^4}{4\,b^2} + \frac{a^4\,x}{b^5} + \frac{a^2\,x^3}{3\,b^3} - \frac{a^3\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x),x)
```

```
[Out] x^5/(5*b) - (a^5*log(a + b*x))/b^6 - (a*x^4)/(4*b^2) + (a^4*x)/b^5 + (a^2*x
^3)/(3*b^3) - (a^3*x^2)/(2*b^4)
```

3.158 $\int \frac{x^4}{a+bx} dx$

Optimal. Leaf size=57

$$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b*x),x]

[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log[a + b*x])/b^5

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.00

$$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Mathics [A]
time = 1.92, size = 50, normalized size = 0.88

$$\frac{a^4\text{Log}\left[a+bx\right]-a^3bx+\frac{a^2b^2x^2}{2}-\frac{ab^3x^3}{3}+\frac{b^4x^4}{4}}{b^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4/(a + b*x),x]')
```

```
[Out] (a ^ 4 Log[a + b x] - a ^ 3 b x + a ^ 2 b ^ 2 x ^ 2 / 2 - a b ^ 3 x ^ 3 / 3
 + b ^ 4 x ^ 4 / 4) / b ^ 5
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{1}{3}a\,b^2x^3-\frac{1}{2}a^2b\,x^2+a^3x}{b^4}+\frac{a^4\ln(bx+a)}{b^5}$	52
norman	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52
risch	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.24, size = 52, normalized size = 0.91

$$\frac{a^4\log\left(bx+a\right)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

Fricas [A]
time = 0.30, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

Sympy [A]
time = 0.07, size = 49, normalized size = 0.86

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

Giac [A]
time = 0.00, size = 61, normalized size = 1.07

$$\frac{\frac{1}{4}x^4b^3-\frac{1}{3}x^3b^2a+\frac{1}{2}x^2ba^2-xa^3}{b^4}+\frac{a^4\ln|xb+a|}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x)
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

Mupad [B]
time = 0.10, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b}+\frac{a^4\,\ln(a+b\,x)}{b^5}-\frac{a\,x^3}{3\,b^2}-\frac{a^3\,x}{b^4}+\frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

3.159 $\int \frac{x^3}{a+bx}\,dx$

Optimal. Leaf size=44

$$\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}$$

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx}\,dx &= \int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Mathics [A]
time = 1.84, size = 39, normalized size = 0.89

$$\frac{-a^3\text{Log}\left[a+bx\right]+a^2bx-\frac{ab^2x^2}{2}+\frac{b^3x^3}{3}}{b^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(a + b*x),x]')
```

```
[Out] (-a ^ 3 Log[a + b x] + a ^ 2 b x - a b ^ 2 x ^ 2 / 2 + b ^ 3 x ^ 3 / 3) / b
 ^ 4
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.93

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x}{b^3}-\frac{a^3\ln(bx+a)}{b^4}$	41
norman	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
risch	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.24, size = 42, normalized size = 0.95

$$-\frac{a^3\log\left(bx+a\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Fricas [A]
time = 0.32, size = 41, normalized size = 0.93

$$\frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A]
time = 0.06, size = 37, normalized size = 0.84

$$-\frac{a^3 \log(a+bx)}{b^4}+\frac{a^2 x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Giac [A]
time = 0.00, size = 48, normalized size = 1.09

$$\frac{\frac{1}{3}x^3b^2-\frac{1}{2}x^2ba+xa^2}{b^3}-\frac{a^3 \ln|xb+a|}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x)
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [B]
time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,b}-\frac{a^3\,\ln(a+b\,x)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x),x)
```

```
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

3.160 $\int \frac{x^2}{a+bx} \, dx$

Optimal. Leaf size=31

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx}\,dx &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Mathics [A]
time = 1.75, size = 28, normalized size = 0.90

$$\frac{a^2\text{Log}\left[a+bx\right]-abx+\frac{b^2x^2}{2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(a + b*x),x]')
```

```
[Out] (a ^ 2 Log[a + b x] - a b x + b ^ 2 x ^ 2 / 2) / b ^ 3
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}x^2b+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
norman	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/2*x^2*b+a*x)+a^2*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.24, size = 29, normalized size = 0.94

$$\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A]
time = 0.32, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.84

$$\frac{a^2 \log{(a + bx)}}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [A]
time = 0.00, size = 34, normalized size = 1.10

$$\frac{\frac{1}{2}x^2b - xa}{b^2} + \frac{a^2 \ln{|xb + a|}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x)
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B]
time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln{(a + b\,x)} + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.161 $\int \frac{x}{a+bx} \, dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

[Out] x/b-a*ln(b*x+a)/b^2

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x),x]

[Out] x/b - (a*Log[a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a + bx} \, dx &= \int \left(\frac{1}{b} - \frac{a}{b(a + bx)} \right) \, dx \\ &= \frac{x}{b} - \frac{a \log(a + bx)}{b^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x),x]

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Mathics [A]
time = 1.70, size = 17, normalized size = 0.94

$$\frac{-a\text{Log}\left[a+bx\right]+bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1/(a + b*x),x]')
[Out] (-a Log[a + b x] + b x) / b ^ 2
```

Maple [A]
time = 0.08, size = 19, normalized size = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.25, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A]
time = 0.30, size = 17, normalized size = 0.94

$$\frac{bx-a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.78

$$-\frac{a\log{(a+bx)}}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
[Out] -a*log(a + b*x)/b**2 + x/b
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.06

$$\frac{x}{b}-\frac{a\ln|xb+a|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
[Out] x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B]
time = 0.08, size = 18, normalized size = 1.00

$$-\frac{a\,\ln{(a+b\,x)}-b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x),x)
[Out] -(a*log(a + b*x) - b*x)/b^2
```

3.162 $\int \frac{1}{a+bx} \, dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

[Out] ln(b*x+a)/b

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} \, dx = \frac{\log(a+bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Mathics [A]
time = 1.63, size = 10, normalized size = 1.00

$$\frac{\text{Log}\,[a+bx]}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x),x]')
```

```
[Out] Log[a + b x] / b
```

Maple [A]
time = 0.08, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A]
time = 0.24, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A]
time = 0.30, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.70

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\ln|xb + a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a + b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

3.163 $\int \frac{1}{x(a+bx)} dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

[Out] ln(x)/a-ln(b*x+a)/a

Rubi [A]

time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x)),x]

[Out] Log[x]/a - Log[a + b*x]/a

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)}\,dx &= \frac{\int \frac{1}{x}\,dx}{a} - \frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

Mathics [A]
time = 1.80, size = 17, normalized size = 0.94

$$\frac{\text{Log}\left[x\right] - \text{Log}\left[\frac{a}{b} + x\right]}{a}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)),x]')
[Out] (Log[x] - Log[a / b + x]) / a
```

Maple [A]
time = 0.08, size = 19, normalized size = 1.06

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
risch	$\frac{\ln(-x)}{a} - \frac{\ln(bx+a)}{a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A]
time = 0.24, size = 18, normalized size = 1.00

$$-\frac{\log\left(bx+a\right)}{a} + \frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A]
time = 0.31, size = 16, normalized size = 0.89

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A]
time = 0.07, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.17

$$\frac{\ln|x|}{a}-\frac{b\ln|xb+a|}{ba}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B]
time = 0.09, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

3.164 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Mathics [A]
time = 1.88, size = 30, normalized size = 1.07

$$\frac{-a - bx\left(\text{Log}\left[x\right] - \text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)),x]')
[Out] (-a - b x (Log[x] - Log[(a + b x) / b])) / (a ^ 2 x)
```

Maple [A]
time = 0.08, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A]
time = 0.24, size = 28, normalized size = 1.00

$$\frac{b\log\left(bx+a\right)}{a^2} - \frac{b\log\left(x\right)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A]
time = 0.31, size = 26, normalized size = 0.93

$$\frac{bx\log\left(bx+a\right) - bx\log\left(x\right) - a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A]
time = 0.09, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A]
time = 0.00, size = 37, normalized size = 1.32

$$\frac{b^2\ln|xb+a|}{ba^2}-\frac{b\ln|x|}{a^2}-\frac{a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x)
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.165 $\int \frac{1}{x^3(a+bx)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)}\,dx &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.00

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Mathics [A]
time = 1.94, size = 42, normalized size = 1.00

$$\frac{a(-a+2bx)+2b^2x^2\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)),x]')
[Out] (a (-a + 2 b x) + 2 b ^ 2 x ^ 2 (Log[x] - Log[(a + b x) / b])) / (2 a ^ 3 x
 ^ 2)
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a\,x^2}+\frac{b}{a^2x}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
norman	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(-x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.25, size = 40, normalized size = 0.95

$$-\frac{b^2\log(bx+a)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A]
time = 0.32, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log(bx+a)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A]
time = 0.10, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Giac [A]
time = 0.00, size = 52, normalized size = 1.24

$$-\frac{b^3\ln|xb+a|}{ba^3}+\frac{b^2\ln|x|}{a^3}+\frac{\frac{1}{2}\left(2bax-a^2\right)}{a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x)
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B]
time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.166 $\int \frac{1}{x^4(a+bx)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)}\,dx &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Mathics [A]
time = 2.12, size = 53, normalized size = 0.95

$$\frac{a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\left(\mathrm{Log}\left[\frac{a+bx}{b}\right]-\mathrm{Log}\left[x\right]\right)}{6a^4x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(a + b*x)),x]')
```

```
[Out] (a (-2 a ^ 2 + 3 a b x - 6 b ^ 2 x ^ 2) + 6 b ^ 3 x ^ 3 (Log[(a + b x) / b]
 - Log[x])) / (6 a ^ 4 x ^ 3)
```

Maple [A]
time = 0.09, size = 53, normalized size = 0.95

method	result	size
default	$-\frac{1}{3a\,x^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(bx+a)}{a^4}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{x^3}+\frac{b^3\ln(bx+a)}{a^4}-\frac{b^3\ln(x)}{a^4}$	53
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{x^3}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(-bx-a)}{a^4}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Maxima [A]
time = 0.24, size = 51, normalized size = 0.91

$$\frac{b^3\log\left(bx+a\right)}{a^4}-\frac{b^3\log\left(x\right)}{a^4}-\frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

Fricas [A]
time = 0.32, size = 54, normalized size = 0.96

$$\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

Sympy [A]
time = 0.11, size = 44, normalized size = 0.79

$$\frac{-2a^2 + 3abx - 6b^2x^2}{6a^3x^3} + \frac{b^3\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.18

$$\frac{b^4\ln|xb + a|}{ba^4} - \frac{b^3\ln|x|}{a^4} + \frac{\frac{1}{6}\left(-6b^2ax^2 + 3ba^2x - 2a^3\right)}{a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x)
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

Mupad [B]
time = 0.10, size = 48, normalized size = 0.86

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{a^3}{3} - \frac{a^2\,b\,x}{2} + a\,b^2\,x^2}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

3.167 $\int \frac{1}{x^5(a+bx)} dx$

Optimal. Leaf size=68

$$-\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}$$

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b^3}{a^4x}-\frac{b^2}{2a^3x^2}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x
])/a^5 - (b^4*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)}\,dx &= \int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.00

$$-\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x
])/a^5 - (b^4*Log[a + b*x])/a^5
```

Mathics [A]
time = 2.20, size = 64, normalized size = 0.94

$$\frac{a\left(-3a^3+4a^2bx-6ab^2x^2+12b^3x^3\right)+12b^4x^4\left(\operatorname{Log}\left[x\right]-\operatorname{Log}\left[\frac{a+bx}{b}\right]\right)}{12a^5x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(a + b*x)),x]')
```

```
[Out] (a (-3 a ^ 3 + 4 a ^ 2 b x - 6 a b ^ 2 x ^ 2 + 12 b ^ 3 x ^ 3) + 12 b ^ 4 x
 ^ 4 (Log[x] - Log[(a + b x) / b])) / (12 a ^ 5 x ^ 4)
```

Maple [A]
time = 0.09, size = 63, normalized size = 0.93

method	result	size
default	$-\frac{1}{4a\,x^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln(bx+a)}{a^5}$	63
norman	$\frac{\frac{b^3x^3}{a^4}-\frac{1}{4a}+\frac{bx}{3a^2}-\frac{b^2x^2}{2a^3}}{x^4}+\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln(bx+a)}{a^5}$	63
risch	$\frac{\frac{b^3x^3}{a^4}-\frac{1}{4a}+\frac{bx}{3a^2}-\frac{b^2x^2}{2a^3}}{x^4}-\frac{b^4\ln(bx+a)}{a^5}+\frac{b^4\ln(-x)}{a^5}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

Maxima [A]
time = 0.24, size = 62, normalized size = 0.91

$$-\frac{b^4\log\left(bx+a\right)}{a^5}+\frac{b^4\log\left(x\right)}{a^5}+\frac{12\,b^3x^3-6\,ab^2x^2+4\,a^2bx-3\,a^3}{12\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^4*log(b*x + a)/a^5 + b^4*log(x)/a^5 + 1/12*(12*b^3*x^3 - 6*a*b^2*x^2 + 4
*a^2*b*x - 3*a^3)/(a^4*x^4)
```

Fricas [A]
time = 0.30, size = 65, normalized size = 0.96

$$-\frac{12\,b^4x^4\log\left(bx+a\right)-12\,b^4x^4\log\left(x\right)-12\,ab^3x^3+6\,a^2b^2x^2-4\,a^3bx+3\,a^4}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^4*x^4*log(b*x + a) - 12*b^4*x^4*log(x) - 12*a*b^3*x^3 + 6*a^2*b
^2*x^2 - 4*a^3*b*x + 3*a^4)/(a^5*x^4)
```

Sympy [A]
time = 0.13, size = 56, normalized size = 0.82

$$\frac{-3a^3+4a^2bx-6ab^2x^2+12b^3x^3}{12a^4x^4}+\frac{b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x - 6*a*b**2*x**2 + 12*b**3*x**3)/(12*a**4*x**4) + b**4
*(log(x) - log(a/b + x))/a**5
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.13

$$-\frac{b^5\ln|xb+a|}{ba^5}+\frac{b^4\ln|x|}{a^5}+\frac{\frac{1}{12}\left(12b^3ax^3-6b^2a^2x^2+4ba^3x-3a^4\right)}{a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x)
```

```
[Out] -b^4*log(abs(b*x + a))/a^5 + b^4*log(abs(x))/a^5 + 1/12*(12*a*b^3*x^3 - 6*a
^2*b^2*x^2 + 4*a^3*b*x - 3*a^4)/(a^5*x^4)
```

Mupad [B]
time = 0.07, size = 60, normalized size = 0.88

$$-\frac{\frac{a^4}{4}-\frac{a^3\,b\,x}{3}+\frac{a^2\,b^2\,x^2}{2}-a\,b^3\,x^3}{a^5\,x^4}-\frac{2\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)),x)
```

```
[Out] - (a^4/4 - a*b^3*x^3 + (a^2*b^2*x^2)/2 - (a^3*b*x)/3)/(a^5*x^4) - (2*b^4*at
anh((2*b*x)/a + 1))/a^5
```

3.168 $\int \frac{x^6}{(a+bx)^2}\,dx$

Optimal. Leaf size=81

$$\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}$$

```
[Out] 5*a^4*x/b^6-2*a^3*x^2/b^5+a^2*x^3/b^4-1/2*a*x^4/b^3+1/5*x^5/b^2-a^6/b^7/(b*
x+a)-6*a^5*ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^2,x]
```

```
[Out] (5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5*
b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^2}\,dx &= \int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right)dx\\ &= \frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 0.95

$$\frac{50a^4bx-20a^3b^2x^2+10a^2b^3x^3-5ab^4x^4+2b^5x^5-\frac{10a^6}{a+bx}-60a^5\log(a+bx)}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^2,x]
```

```
[Out] (50*a^4*b*x - 20*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 5*a*b^4*x^4 + 2*b^5*x^5 - (
10*a^6)/(a + b*x) - 60*a^5*Log[a + b*x])/(10*b^7)
```

Mathics [**A**]
time = 2.26, size = 98, normalized size = 1.21

$$\frac{-6a^6\text{Log}\left[a+bx\right]-a^6-6a^5bx\text{Log}\left[a+bx\right]+5a^5bx+3a^4b^2x^2-a^3b^3x^3+\frac{a^2b^4x^4}{2}-\frac{3ab^5x^5}{10}+\frac{b^6x^6}{5}}{b^7\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6/(a + b*x)^2,x]')
```

```
[Out] (-6 a ^ 6 Log[a + b x] - a ^ 6 - 6 a ^ 5 b x Log[a + b x] + 5 a ^ 5 b x + 3
 a ^ 4 b ^ 2 x ^ 2 - a ^ 3 b ^ 3 x ^ 3 + a ^ 2 b ^ 4 x ^ 4 / 2 - 3 a b ^ 5
x ^ 5 / 10 + b ^ 6 x ^ 6 / 5) / (b ^ 7 (a + b x))
```

Maple [**A**]
time = 0.08, size = 78, normalized size = 0.96

method	result	size
default	$\frac{\frac{1}{5}b^4x^5-\frac{1}{2}a\,b^3x^4+a^2b^2x^3-2a^3b\,x^2+5a^4x}{b^6}-\frac{a^6}{b^7(bx+a)}-\frac{6a^5\ln(bx+a)}{b^7}$	78
risch	$\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a\,x^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(bx+a)}-\frac{6a^5\ln(bx+a)}{b^7}$	78
norman	$\frac{\frac{x^6}{5b}-\frac{3a\,x^5}{10b^2}-\frac{6a^6}{b^7}-\frac{a^3x^3}{b^4}+\frac{3a^4x^2}{b^5}+\frac{a^2x^4}{2b^3}}{bx+a}-\frac{6a^5\ln(bx+a)}{b^7}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/5*b^4*x^5-1/2*a*b^3*x^4+a^2*b^2*x^3-2*a^3*b*x^2+5*a^4*x)-a^6/b^7/(
b*x+a)-6*a^5*ln(b*x+a)/b^7
```

Maxima [**A**]
time = 0.24, size = 82, normalized size = 1.01

$$-\frac{a^6}{b^8x+ab^7}-\frac{6\,a^5\log\left(bx+a\right)}{b^7}+\frac{2\,b^4x^5-5\,ab^3x^4+10\,a^2b^2x^3-20\,a^3bx^2+50\,a^4x}{10\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^6/(b^8*x + a*b^7) - 6*a^5*log(b*x + a)/b^7 + 1/10*(2*b^4*x^5 - 5*a*b^3*x
^4 + 10*a^2*b^2*x^3 - 20*a^3*b*x^2 + 50*a^4*x)/b^6
```

Fricas [A]
time = 0.32, size = 96, normalized size = 1.19

$$\frac{2\,b^6x^6 - 3\,ab^5x^5 + 5\,a^2b^4x^4 - 10\,a^3b^3x^3 + 30\,a^4b^2x^2 + 50\,a^5bx - 10\,a^6 - 60\,(a^5bx + a^6)\log(bx + a)}{10\,(b^8x + ab^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^6*x^6 - 3*a*b^5*x^5 + 5*a^2*b^4*x^4 - 10*a^3*b^3*x^3 + 30*a^4*b^2
*x^2 + 50*a^5*b*x - 10*a^6 - 60*(a^5*b*x + a^6)*log(b*x + a))/(b^8*x + a*b^
7)
```

Sympy [A]
time = 0.12, size = 78, normalized size = 0.96

$$-\frac{a^6}{ab^7 + b^8x} - \frac{6a^5\log(a + bx)}{b^7} + \frac{5a^4x}{b^6} - \frac{2a^3x^2}{b^5} + \frac{a^2x^3}{b^4} - \frac{ax^4}{2b^3} + \frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**2,x)
```

```
[Out] -a**6/(a*b**7 + b**8*x) - 6*a**5*log(a + b*x)/b**7 + 5*a**4*x/b**6 - 2*a**3
*x**2/b**5 + a**2*x**3/b**4 - a*x**4/(2*b**3) + x**5/(5*b**2)
```

Giac [A]
time = 0.00, size = 92, normalized size = 1.14

$$\frac{\frac{1}{5}x^5b^8 - \frac{1}{2}x^4b^7a + x^3b^6a^2 - 2x^2b^5a^3 + 5xb^4a^4}{b^{10}} - \frac{a^6}{b^7\,(xb + a)} - \frac{6a^5\ln|xb + a|}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x)
```

```
[Out] -6*a^5*log(abs(b*x + a))/b^7 - a^6/((b*x + a)*b^7) + 1/10*(2*b^8*x^5 - 5*a*
b^7*x^4 + 10*a^2*b^6*x^3 - 20*a^3*b^5*x^2 + 50*a^4*b^4*x)/b^10
```

Mupad [B]
time = 0.14, size = 83, normalized size = 1.02

$$\frac{x^5}{5\,b^2} - \frac{6\,a^5\,\ln(a + b\,x)}{b^7} - \frac{a\,x^4}{2\,b^3} + \frac{5\,a^4\,x}{b^6} + \frac{a^2\,x^3}{b^4} - \frac{2\,a^3\,x^2}{b^5} - \frac{a^6}{b\,(x\,b^7 + a\,b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^2,x)
```

```
[Out] x^5/(5*b^2) - (6*a^5*log(a + b*x))/b^7 - (a*x^4)/(2*b^3) + (5*a^4*x)/b^6 +
(a^2*x^3)/b^4 - (2*a^3*x^2)/b^5 - a^6/(b*(a*b^6 + b^7*x))
```

3.169 $\int \frac{x^5}{(a+bx)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}$$

```
[Out] -4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a^
4*ln(b*x+a)/b^6
```

Rubi [A]

time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^2,x]
```

```
[Out] (-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a^
5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^2}\,dx &= \int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx\\ &= -\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 66, normalized size = 0.92

$$\frac{-48a^3bx+18a^2b^2x^2-8ab^3x^3+3b^4x^4+\frac{12a^5}{a+bx}+60a^4\log(a+bx)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^2,x]
```

```
[Out] (-48*a^3*b*x + 18*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4 + (12*a^5)/(a + b*x
) + 60*a^4*Log[a + b*x])/(12*b^6)
```

Mathics [A]
time = 2.22, size = 89, normalized size = 1.24

$$\frac{60a^4\text{Log}\left[a+bx\right](a+bx)+12a^5-48a^3bx(a+bx)+18a^2b^2x^2(a+bx)-8ab^3x^3(a+bx)+3b^4x^4(a+bx)}{12b^6(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5/(a + b*x)^2,x]')
```

```
[Out] (60 a ^ 4 Log[a + b x] (a + b x) + 12 a ^ 5 - 48 a ^ 3 b x (a + b x) + 18 a
 ^ 2 b ^ 2 x ^ 2 (a + b x) - 8 a b ^ 3 x ^ 3 (a + b x) + 3 b ^ 4 x ^ 4 (a +
 b x)) / (12 b ^ 6 (a + b x))
```

Maple [A]
time = 0.08, size = 68, normalized size = 0.94

method	result	size
risch	$-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2a\,x^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(bx+a)}+\frac{5a^4\ln(bx+a)}{b^6}$	67
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{2}{3}a\,b^2x^3-\frac{3}{2}a^2b\,x^2+4a^3x}{b^5}+\frac{a^5}{b^6(bx+a)}+\frac{5a^4\ln(bx+a)}{b^6}$	68
norman	$\frac{\frac{5a^5}{b^6}+\frac{x^5}{4b}-\frac{5a\,x^4}{12b^2}+\frac{5a^2x^3}{6b^3}-\frac{5a^3x^2}{2b^4}}{bx+a}+\frac{5a^4\ln(bx+a)}{b^6}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/4*b^3*x^4+2/3*a*b^2*x^3-3/2*a^2*b*x^2+4*a^3*x)+a^5/b^6/(b*x+a)+5
*a^4*ln(b*x+a)/b^6
```

Maxima [A]
time = 0.25, size = 70, normalized size = 0.97

$$\frac{a^5}{b^7x+ab^6}+\frac{5\,a^4\log\left(bx+a\right)}{b^6}+\frac{3\,b^3x^4-8\,ab^2x^3+18\,a^2bx^2-48\,a^3x}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^5/(b^7*x + a*b^6) + 5*a^4*log(b*x + a)/b^6 + 1/12*(3*b^3*x^4 - 8*a*b^2*x^
3 + 18*a^2*b*x^2 - 48*a^3*x)/b^5
```

Fricas [A]
time = 0.30, size = 85, normalized size = 1.18

$$\frac{3\,b^5x^5-5\,ab^4x^4+10\,a^2b^3x^3-30\,a^3b^2x^2-48\,a^4bx+12\,a^5+60\,(a^4bx+a^5)\log\left(bx+a\right)}{12\,(b^7x+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 - 5*a*b^4*x^4 + 10*a^2*b^3*x^3 - 30*a^3*b^2*x^2 - 48*a^4*b*
x + 12*a^5 + 60*(a^4*b*x + a^5)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A]
time = 0.11, size = 71, normalized size = 0.99

$$\frac{a^5}{ab^6+b^7x}+\frac{5a^4\log\left(a+bx\right)}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2,x)
```

```
[Out] a**5/(a*b**6 + b**7*x) + 5*a**4*log(a + b*x)/b**6 - 4*a**3*x/b**5 + 3*a**2*
x**2/(2*b**4) - 2*a*x**3/(3*b**3) + x**4/(4*b**2)
```

Giac [A]
time = 0.00, size = 82, normalized size = 1.14

$$\frac{\frac{1}{4}x^4b^6-\frac{2}{3}x^3b^5a+\frac{3}{2}x^2b^4a^2-4xb^3a^3}{b^8}+\frac{a^5}{b^6\left(xb+a\right)}+\frac{5a^4\ln\left|xb+a\right|}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x)
```

```
[Out] 5*a^4*log(abs(b*x + a))/b^6 + a^5/((b*x + a)*b^6) + 1/12*(3*b^6*x^4 - 8*a*b
^5*x^3 + 18*a^2*b^4*x^2 - 48*a^3*b^3*x)/b^8
```

Mupad [B]
time = 0.07, size = 72, normalized size = 1.00

$$\frac{x^4}{4\,b^2}+\frac{5\,a^4\,\ln\left(a+b\,x\right)}{b^6}-\frac{2\,a\,x^3}{3\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{3\,a^2\,x^2}{2\,b^4}+\frac{a^5}{b\,\left(x\,b^6+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^2,x)
```

```
[Out] x^4/(4*b^2) + (5*a^4*log(a + b*x))/b^6 - (2*a*x^3)/(3*b^3) - (4*a^3*x)/b^5
+ (3*a^2*x^2)/(2*b^4) + a^5/(b*(a*b^5 + b^6*x))
```

3.170 $\int \frac{x^4}{(a+bx)^2} dx$

Optimal. Leaf size=58

$$\frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2} - \frac{a^4}{b^5(a+bx)} - \frac{4a^3\log(a+bx)}{b^5}$$

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{b^5(a+bx)} - \frac{4a^3\log(a+bx)}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a+bx)^2} dx &= \int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right) dx \\
&= \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2} - \frac{a^4}{b^5(a+bx)} - \frac{4a^3\log(a+bx)}{b^5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.93

$$\frac{9a^2bx - 3ab^2x^2 + b^3x^3 - \frac{3a^4}{a+bx} - 12a^3\log(a+bx)}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

Mathics [A]
time = 2.13, size = 72, normalized size = 1.24

$$\frac{-4a^3\text{Log}\left[a+bx\right](a+bx)-a^4+3a^2bx\,(a+bx)-ab^2x^2\,(a+bx)+\frac{b^3x^3(a+bx)}{3}}{b^5\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4/(a + b*x)^2,x]')
```

```
[Out] (-4 a ^ 3 Log[a + b x] (a + b x) - a ^ 4 + 3 a ^ 2 b x (a + b x) - a b ^ 2
x ^ 2 (a + b x) + b ^ 3 x ^ 3 (a + b x) / 3) / (b ^ 5 (a + b x))
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^4}{3b}-\frac{2a\,x^3}{3b^2}-\frac{4a^4}{b^5}+\frac{2a^2x^2}{b^3}}{bx+a}-\frac{4a^3\ln(bx+a)}{b^5}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.25, size = 59, normalized size = 1.02

$$-\frac{a^4}{b^6x+ab^5}-\frac{4\,a^3\log\left(bx+a\right)}{b^5}+\frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

Fricas [A]
time = 0.31, size = 73, normalized size = 1.26

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\,(a^3bx + a^4)\log\left(bx + a\right)}{3\,(b^6x + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A]
time = 0.10, size = 54, normalized size = 0.93

$$-\frac{a^4}{ab^5 + b^6x} - \frac{4a^3\log\left(a + bx\right)}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

Giac [A]
time = 0.00, size = 67, normalized size = 1.16

$$\frac{\frac{1}{3}x^3b^4 - x^2b^3a + 3xb^2a^2}{b^6} - \frac{a^4}{b^5\,(xb + a)} - \frac{4a^3\ln|xb + a|}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x)
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

Mupad [B]
time = 0.07, size = 62, normalized size = 1.07

$$\frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a + b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5 + a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

3.171 $\int \frac{x^3}{(a+bx)^2} dx$

Optimal. Leaf size=46

$$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}$$

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^2} dx &= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx\\ &= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.93

$$\frac{-4abx+b^2x^2+\frac{2a^3}{a+bx}+6a^2\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*x)^2,x]

[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)

Mathics [A]
time = 2.11, size = 54, normalized size = 1.17

$$\frac{3a^2\text{Log}\left[a+bx\right](a+bx)+a^3-2abx(a+bx)+\frac{b^2x^2(a+bx)}{2}}{b^4(a+bx)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^3/(a + b*x)^2,x]')

[Out] (3 a ^ 2 Log[a + b x] (a + b x) + a ^ 3 - 2 a b x (a + b x) + b ^ 2 x ^ 2 (a + b x) / 2) / (b ^ 4 (a + b x))

Maple [A]
time = 0.08, size = 46, normalized size = 1.00

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	46
norman	$\frac{\frac{3a^3}{b^4}+\frac{x^3}{2b}-\frac{3a\,x^2}{2b^2}}{bx+a}+\frac{3a^2\ln(bx+a)}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/b^3*(-1/2*x^2*b+2*a*x)+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4

Maxima [A]
time = 0.25, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log\left(bx+a\right)}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x+a)^2,x, algorithm="maxima")

[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3

Fricas [A]
time = 0.30, size = 62, normalized size = 1.35

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [**A**]
time = 0.10, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4 + b^5x} + \frac{3a^2\log(a + bx)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

Giac [**A**]
time = 0.00, size = 53, normalized size = 1.15

$$\frac{\frac{1}{2}x^2b^2 - 2xba}{b^4} + \frac{a^3}{b^4(xb + a)} + \frac{3a^2\ln|xb + a|}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x)
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

Mupad [**B**]
time = 0.08, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,b^2} + \frac{3\,a^2\,\ln(a + b\,x)}{b^4} + \frac{a^3}{b\,(x\,b^4 + a\,b^3)} - \frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

3.172 $\int \frac{x^2}{(a+bx)^2}\,dx$

Optimal. Leaf size=33

$$\frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}$$

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^2}\,dx &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{bx-\frac{a^2}{a+bx}-2a\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^2,x]
```

```
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Mathics [A]
time = 1.97, size = 39, normalized size = 1.18

$$\frac{-2a\text{Log}\left[a+bx\right]\left(a+bx\right)-a^2+bx\left(a+bx\right)}{b^3\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(a + b*x)^2,x]')
```

```
[Out] (-2 a Log[a + b x] (a + b x) - a ^ 2 + b x (a + b x)) / (b ^ 3 (a + b x))
```

Maple [A]
time = 0.09, size = 34, normalized size = 1.03

method	result	size
default	$\frac{x}{b^{2}}-\frac{a^{2}}{b^{3}(bx+a)}-\frac{2a\ln(bx+a)}{b^{3}}$	34
risch	$\frac{x}{b^{2}}-\frac{a^{2}}{b^{3}(bx+a)}-\frac{2a\ln(bx+a)}{b^{3}}$	34
norman	$\frac{\frac{x^{2}}{b}-\frac{2a^{2}}{b^{3}}}{bx+a}-\frac{2a\ln(bx+a)}{b^{3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.24, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x+ab^3}+\frac{x}{b^2}-\frac{2\,a\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Fricas [A]
time = 0.29, size = 47, normalized size = 1.42

$$\frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A]
time = 0.09, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a\log(a + bx)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2,x)
```

```
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Giac [A]
time = 0.00, size = 37, normalized size = 1.12

$$\frac{x}{b^2} - \frac{a^2}{b^3(xb + a)} - \frac{2a\ln|xb + a|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x)
```

```
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

Mupad [B]
time = 0.08, size = 36, normalized size = 1.09

$$\frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln(a + b\,x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^2,x)
```

```
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

3.173 $\int \frac{x}{(a+bx)^2}\,dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = $\{45\}$

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^2}\,dx &= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx\\ &= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

Mathics [A]
time = 1.91, size = 25, normalized size = 1.09

$$\frac{a + \text{Log}\left[a + bx\right](a + bx)}{b^2 (a + bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1/(a + b*x)^2,x]')
[Out] (a + Log[a + b x] (a + b x)) / (b ^ 2 (a + b x))
```

Maple [A]
time = 0.08, size = 24, normalized size = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)} + \frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)} + \frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)} + \frac{\ln(bx+a)}{b^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Maxima [A]
time = 0.26, size = 26, normalized size = 1.13

$$\frac{a}{b^3x + ab^2} + \frac{\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Fricas [A]
time = 0.30, size = 28, normalized size = 1.22

$$\frac{(bx + a)\log(bx + a) + a}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.07, size = 20, normalized size = 0.87

$$\frac{a}{ab^2 + b^3x} + \frac{\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2,x)
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.09

$$\frac{a}{bb(xb + a)} + \frac{\ln|xb + a|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x)
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

Mupad [B]
time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln(a + b\,x)}{b^2} + \frac{a}{b^2\,(a + b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^2,x)
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

3.174 $\int \frac{1}{(a+bx)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b(a+bx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Mathics [A]
time = 1.72, size = 12, normalized size = 1.00

$$-\frac{1}{b\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^2,x]')
```

```
[Out] -1 / (b (a + b x))
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A]
time = 0.25, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A]
time = 0.31, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A]
time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{b\left(xb+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x)
```

```
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

3.175 $\int \frac{1}{x(a+bx)^2} \, dx$

Optimal. Leaf size=29

$$\frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}$$

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^2),x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^2} \, dx &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) \, dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}+\log(x)-\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^2),x]
```

[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2

Mathics [A]
time = 1.92, size = 34, normalized size = 1.17

$$\frac{a+(a+bx)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^2\left(a+bx\right)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^1*(a + b*x)^2),x]')

[Out] (a + (a + b x) (Log[x] - Log[(a + b x) / b])) / (a ^ 2 (a + b x))

Maple [A]
time = 0.08, size = 30, normalized size = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}+\frac{\ln(-x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2

Maxima [A]
time = 0.25, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log\left(bx+a\right)}{a^2}+\frac{\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^2,x, algorithm="maxima")

[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2

Fricas [A]
time = 0.32, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log\left(bx+a\right)-(bx+a)\log\left(x\right)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^2,x, algorithm="fricas")

```
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A]
time = 0.11, size = 22, normalized size = 0.76

$$\frac{1}{a^2 + abx} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2,x)
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Giac [A]
time = 0.00, size = 37, normalized size = 1.28

$$\frac{\ln|x|}{a^2} - \frac{b\ln|xb + a|}{ba^2} + \frac{a}{a^2(xb + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x)
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

Mupad [B]
time = 0.12, size = 26, normalized size = 0.90

$$\frac{1}{a^2 + b\,x\,a} - \frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^2),x)
[Out] 1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2
```

3.176 $\int \frac{1}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Mathics [A]
time = 2.10, size = 49, normalized size = 1.17

$$\frac{a\left(-a-2bx\right)-2bx\left(a+bx\right)\left(\operatorname{Log}\left[x\right]-\operatorname{Log}\left[\frac{a+bx}{b}\right]\right)}{a^3x\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^2),x]')
```

```
[Out] (a (-a - 2 b x) - 2 b x (a + b x) (Log[x] - Log[(a + b x) / b])) / (a ^ 3 x
 (a + b x))
```

Maple [A]
time = 0.09, size = 43, normalized size = 1.02

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b}{a^{2} \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	43
risch	$\frac{-\frac{2 b x}{a^{2}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(-b x -a \right)}{a^{3}}$	49
norman	$\frac{\frac{2 b^{2} x^{2}}{a^{3}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.25, size = 45, normalized size = 1.07

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A]
time = 0.31, size = 63, normalized size = 1.50

$$-\frac{2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(bx+a\right)+2\left(b^2x^2+abx\right)\log\left(x\right)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A]
time = 0.13, size = 37, normalized size = 0.88

$$\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Giac [A]
time = 0.00, size = 52, normalized size = 1.24

$$-\frac{2b\ln|x|}{a^3}+\frac{2b^2\ln|xb+a|}{ba^3}-\frac{2xb+a}{a^2\left(x^2b+xa\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x)
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

Mupad [B]
time = 0.12, size = 45, normalized size = 1.07

$$\frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3}-\frac{1}{a\,x\,\left(a+b\,x\right)}-\frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

3.177 $\int \frac{1}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}$$

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{46\}$

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.91

$$\frac{a\left(-\frac{a}{x^2}+\frac{4b}{x}+\frac{2b^2}{a+bx}\right)+6b^2\log(x)-6b^2\log(a+bx)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^2),x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

Mathics [A]
time = 2.27, size = 65, normalized size = 1.12

$$\frac{a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\left(a+bx\right)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{2a^4x^2\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^2),x]')
```

```
[Out] (a (-a ^ 2 + 3 a b x + 6 b ^ 2 x ^ 2) + 6 b ^ 2 x ^ 2 (a + b x) (Log[x] - L
og[(a + b x) / b])) / (2 a ^ 4 x ^ 2 (a + b x))
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	57
norman	$\frac{-\frac{3b^3x^3}{a^4}-\frac{1}{2a}+\frac{3bx}{2a^2}}{x^2(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	61
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)}-\frac{3b^2\ln(bx+a)}{a^4}+\frac{3b^2\ln(-x)}{a^4}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Maxima [A]
time = 0.24, size = 64, normalized size = 1.10

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3bx^3+a^4x^2\right)}-\frac{3\,b^2\log\left(bx+a\right)}{a^4}+\frac{3\,b^2\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Fricas [A]
time = 0.32, size = 86, normalized size = 1.48

$$\frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,(b^3x^3+ab^2x^2)\log{(bx+a)}+6\,(b^3x^3+ab^2x^2)\log{(x)}}{2\,(a^4bx^3+a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A]
time = 0.15, size = 54, normalized size = 0.93

$$\frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3}+\frac{3b^2\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

Giac [A]
time = 0.00, size = 71, normalized size = 1.22

$$-\frac{3b^3\ln|xb+a|}{ba^4}+\frac{3b^2\ln|x|}{a^4}+\frac{\frac{1}{2}\,(6b^2ax^2+3ba^2x-a^3)}{a^4x^2\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x)
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

Mupad [B]
time = 0.11, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2}-\frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^2),x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

3.178 $\int \frac{1}{x^4(a+bx)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4
*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.96

$$-\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)}+12b^3\log(x)-12b^3\log(a+bx)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Mathics [A]
time = 2.35, size = 76, normalized size = 1.10

$$\frac{a\left(-a^3+2a^2bx-6ab^2x^2-12b^3x^3\right)-12b^3x^3\left(a+bx\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{3a^5x^3\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(a + b*x)^2),x]')
```

```
[Out] (a (-a ^ 3 + 2 a ^ 2 b x - 6 a b ^ 2 x ^ 2 - 12 b ^ 3 x ^ 3) - 12 b ^ 3 x ^
 3 (a + b x) (Log[x] - Log[(a + b x) / b])) / (3 a ^ 5 x ^ 3 (a + b x))
```

Maple [A]
time = 0.08, size = 68, normalized size = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(-bx-a)}{a^5}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Maxima [A]
time = 0.25, size = 73, normalized size = 1.06

$$-\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\left(a^4bx^4+a^5x^3\right)}+\frac{4\,b^3\log\left(bx+a\right)}{a^5}-\frac{4\,b^3\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Fricas [A]
time = 0.30, size = 95, normalized size = 1.38

$$-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\,(b^4x^4+ab^3x^3)\log\left(bx+a\right)+12\,(b^4x^4+ab^3x^3)\log\left(x\right)}{3\,(a^5bx^4+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [A]
time = 0.17, size = 66, normalized size = 0.96

$$\frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Giac [A]
time = 0.00, size = 84, normalized size = 1.22

$$\frac{4b^4\ln|xb+a|}{ba^5}-\frac{4b^3\ln|x|}{a^5}+\frac{\frac{1}{3}\left(-12b^3ax^3-6b^2a^2x^2+2ba^3x-a^4\right)}{a^5x^3\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x)
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

Mupad [B]
time = 0.08, size = 69, normalized size = 1.00

$$\frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^2),x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

3.179 $\int \frac{1}{x^5(a+bx)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}$$

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^2),x]
```

```
[Out] -1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x
) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 79, normalized size = 0.94

$$\frac{\frac{a(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4)}{x^4(a+bx)}+60b^4\log(x)-60b^4\log(a+bx)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Mathics [A]
time = 2.46, size = 87, normalized size = 1.04

$$\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)+60b^4x^4\left(a+bx\right)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{12a^6x^4\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(a + b*x)^2),x]')
```

```
[Out] (a (-3 a ^ 4 + 5 a ^ 3 b x - 10 a ^ 2 b ^ 2 x ^ 2 + 30 a b ^ 3 x ^ 3 + 60 b
 ^ 4 x ^ 4) + 60 b ^ 4 x ^ 4 (a + b x) (Log[x] - Log[(a + b x) / b])) / (12
 a ^ 6 x ^ 4 (a + b x))
```

Maple [A]
time = 0.09, size = 79, normalized size = 0.94

method	result	size
default	$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	79
norman	$\frac{-\frac{5b^5x^5}{a^6}-\frac{1}{4a}+\frac{5bx}{12a^2}-\frac{5b^2x^2}{6a^3}+\frac{5b^3x^3}{2a^4}}{x^4(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	83
risch	$\frac{\frac{5b^4x^4}{a^5}+\frac{5b^3x^3}{2a^4}-\frac{5b^2x^2}{6a^3}+\frac{5bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)}-\frac{5b^4\ln(bx+a)}{a^6}+\frac{5b^4\ln(-x)}{a^6}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.25, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\left(a^5bx^5+a^6x^4\right)}-\frac{5\,b^4\log\left(bx+a\right)}{a^6}+\frac{5\,b^4\log\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Fricas [A]
time = 0.32, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\,(b^5x^5+ab^4x^4)\log(bx+a)+60\,(b^5x^5+ab^4x^4)\log(x)}{12\,(a^6bx^5+a^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

Sympy [A]
time = 0.18, size = 80, normalized size = 0.95

$$\frac{-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4}{12a^6x^4+12a^5bx^5}+\frac{5b^4\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Giac [A]
time = 0.00, size = 96, normalized size = 1.14

$$-\frac{5b^5\ln|xb+a|}{ba^6}+\frac{5b^4\ln|x|}{a^6}+\frac{\frac{1}{12}\left(60b^4ax^4+30b^3a^2x^3-10b^2a^3x^2+5ba^4x-3a^5\right)}{a^6x^4(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x)
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +
 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

Mupad [B]
time = 0.12, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4}-\frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

3.180 $\int \frac{x^7}{(a+bx)^3} \, dx$

Optimal. Leaf size=99

$$\frac{15a^4x}{b^7} - \frac{5a^3x^2}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^5}{5b^3} + \frac{a^7}{2b^8(a+bx)^2} - \frac{7a^6}{b^8(a+bx)} - \frac{21a^5\log(a+bx)}{b^8}$$

```
[Out] 15*a^4*x/b^7-5*a^3*x^2/b^6+2*a^2*x^3/b^5-3/4*a*x^4/b^4+1/5*x^5/b^3+1/2*a^7/
b^8/(b*x+a)^2-7*a^6/b^8/(b*x+a)-21*a^5*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^7}{2b^8(a+bx)^2} - \frac{7a^6}{b^8(a+bx)} - \frac{21a^5\log(a+bx)}{b^8} + \frac{15a^4x}{b^7} - \frac{5a^3x^2}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^3,x]
```

```
[Out] (15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x^
5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*Log
[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^3}\,dx &= \int \left(\frac{15a^4}{b^7} - \frac{10a^3x}{b^6} + \frac{6a^2x^2}{b^5} - \frac{3ax^3}{b^4} + \frac{x^4}{b^3} - \frac{a^7}{b^7(a+bx)^3} + \frac{7a^6}{b^7(a+bx)^2} - \frac{21a^5}{b^7(a+bx)}\right)dx\\ &= \frac{15a^4x}{b^7} - \frac{5a^3x^2}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^5}{5b^3} + \frac{a^7}{2b^8(a+bx)^2} - \frac{7a^6}{b^8(a+bx)} - \frac{21a^5\log(a+bx)}{b^8}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 89, normalized size = 0.90

$$\frac{300a^4bx - 100a^3b^2x^2 + 40a^2b^3x^3 - 15ab^4x^4 + 4b^5x^5 + \frac{10a^7}{(a+bx)^2} - \frac{140a^6}{a+bx} - 420a^5\log(a+bx)}{20b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^3,x]
```

```
[Out] (300*a^4*b*x - 100*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 15*a*b^4*x^4 + 4*b^5*x^5
+ (10*a^7)/(a + b*x)^2 - (140*a^6)/(a + b*x) - 420*a^5*Log[a + b*x])/(20*b^
8)
```

Mathics [A]
time = 2.49, size = 138, normalized size = 1.39

$$\frac{-420a^7\mathrm{Log}\,[a+bx]-130a^7-840a^6bx\mathrm{Log}\,[a+bx]+160a^6bx-420a^5b^2x^2\mathrm{Log}\,[a+bx]+500a^5b^2x^2+140a^4b^3x^3-35a^3b^4x^4+14a^2b^5x^5-7ab^6x^6+4b^7x^7}{20b^8\,(a^2+2abx+b^2x^2)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^7/(a + b*x)^3,x]')
```

```
[Out] (-420 a ^ 7 Log[a + b x] - 130 a ^ 7 - 840 a ^ 6 b x Log[a + b x] + 160 a ^
 6 b x - 420 a ^ 5 b ^ 2 x ^ 2 Log[a + b x] + 500 a ^ 5 b ^ 2 x ^ 2 + 140 a
 ^ 4 b ^ 3 x ^ 3 - 35 a ^ 3 b ^ 4 x ^ 4 + 14 a ^ 2 b ^ 5 x ^ 5 - 7 a b ^ 6
x ^ 6 + 4 b ^ 7 x ^ 7) / (20 b ^ 8 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.10, size = 94, normalized size = 0.95

method	result	size
risch	$\frac{x^5}{5b^3}-\frac{3a\,x^4}{4b^4}+\frac{2a^2x^3}{b^5}-\frac{5a^3x^2}{b^6}+\frac{15a^4x}{b^7}+\frac{-7a^6x-\frac{13a^7}{2b}}{b^7(bx+a)^2}-\frac{21a^5\ln(bx+a)}{b^8}$	90
norman	$\frac{\frac{x^7}{5b}-\frac{7a\,x^6}{20b^2}-\frac{63a^7}{2b^8}-\frac{7a^3x^4}{4b^4}+\frac{7a^4x^3}{b^5}+\frac{7a^2x^5}{10b^3}-\frac{42a^6x}{b^7}}{(bx+a)^2}-\frac{21a^5\ln(bx+a)}{b^8}$	92
default	$\frac{\frac{1}{5}b^4x^5-\frac{3}{4}a\,b^3x^4+2a^2b^2x^3-5a^3b\,x^2+15a^4x}{b^7}-\frac{7a^6}{b^8(bx+a)}+\frac{a^7}{2b^8(bx+a)^2}-\frac{21a^5\ln(bx+a)}{b^8}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/5*b^4*x^5-3/4*a*b^3*x^4+2*a^2*b^2*x^3-5*a^3*b*x^2+15*a^4*x)-7*a^6/
b^8/(b*x+a)+1/2*a^7/b^8/(b*x+a)^2-21*a^5*ln(b*x+a)/b^8
```

Maxima [A]
time = 0.25, size = 103, normalized size = 1.04

$$-\frac{14\,a^6bx+13\,a^7}{2\,(b^{10}x^2+2\,ab^9x+a^2b^8)}-\frac{21\,a^5\log\left(bx+a\right)}{b^8}+\frac{4\,b^4x^5-15\,ab^3x^4+40\,a^2b^2x^3-100\,a^3bx^2+300\,a^4x}{20\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(14*a^6*b*x + 13*a^7)/(b^10*x^2 + 2*a*b^9*x + a^2*b^8) - 21*a^5*log(b*
x + a)/b^8 + 1/20*(4*b^4*x^5 - 15*a*b^3*x^4 + 40*a^2*b^2*x^3 - 100*a^3*b*x^
2 + 300*a^4*x)/b^7
```

Fricas [A]
time = 0.30, size = 129, normalized size = 1.30

$$\frac{4\,b^7x^7 - 7\,ab^6x^6 + 14\,a^2b^5x^5 - 35\,a^3b^4x^4 + 140\,a^4b^3x^3 + 500\,a^5b^2x^2 + 160\,a^6bx - 130\,a^7 - 420\,(a^5b^2x^2 + 2\,a^6bx + a^7)\log{(bx + a)}}{20\,(b^{10}x^2 + 2\,ab^9x + a^2b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 - 7*a*b^6*x^6 + 14*a^2*b^5*x^5 - 35*a^3*b^4*x^4 + 140*a^4*b
^3*x^3 + 500*a^5*b^2*x^2 + 160*a^6*b*x - 130*a^7 - 420*(a^5*b^2*x^2 + 2*a^6
*b*x + a^7)*log(b*x + a))/(b^10*x^2 + 2*a*b^9*x + a^2*b^8)
```

Sympy [A]
time = 0.18, size = 109, normalized size = 1.10

$$-\frac{21a^5\log{(a + bx)}}{b^8} + \frac{15a^4x}{b^7} - \frac{5a^3x^2}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^4}{4b^4} + \frac{-13a^7 - 14a^6bx}{2a^2b^8 + 4ab^9x + 2b^{10}x^2} + \frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x)/b**8 + 15*a**4*x/b**7 - 5*a**3*x**2/b**6 + 2*a**2*x**
3/b**5 - 3*a*x**4/(4*b**4) + (-13*a**7 - 14*a**6*b*x)/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + x**5/(5*b**3)
```

Giac [A]
time = 0.00, size = 109, normalized size = 1.10

$$\frac{\frac{1}{5}x^5b^{12} - \frac{3}{4}x^4b^{11}a + 2x^3b^{10}a^2 - 5x^2b^9a^3 + 15xb^8a^4}{b^{15}} + \frac{\frac{1}{2}\left(-14ba^6x - 13a^7\right)}{b^8\left(xb + a\right)^2} - \frac{21a^5\ln|xb + a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x)
```

```
[Out] -21*a^5*log(abs(b*x + a))/b^8 - 1/2*(14*a^6*b*x + 13*a^7)/((b*x + a)^2*b^8)
 + 1/20*(4*b^12*x^5 - 15*a*b^11*x^4 + 40*a^2*b^10*x^3 - 100*a^3*b^9*x^2 + 3
00*a^4*b^8*x)/b^15
```

Mupad [B]
time = 0.23, size = 91, normalized size = 0.92

$$-\frac{\frac{7\,a\,{(a+b\,x)}^4}{4} - \frac{{(a+b\,x)}^5}{5} - 7\,a^2\,{(a + b\,x)}^3 + \frac{35\,a^3\,{(a+b\,x)}^2}{2} + \frac{7\,a^6}{a+b\,x} - \frac{a^7}{2\,{(a+b\,x)}^2} + 21\,a^5\,\ln{(a + b\,x)} - 35\,a^4\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^3,x)
```

```
[Out] -((7*a*(a + b*x)^4)/4 - (a + b*x)^5/5 - 7*a^2*(a + b*x)^3 + (35*a^3*(a + b*
x)^2)/2 + (7*a^6)/(a + b*x) - a^7/(2*(a + b*x)^2) + 21*a^5*log(a + b*x) - 3
5*a^4*b*x)/b^8
```

3.181 $\int \frac{x^6}{(a+bx)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}$$

```
[Out] -10*a^3*x/b^6+3*a^2*x^2/b^5-a*x^3/b^4+1/4*x^4/b^3-1/2*a^6/b^7/(b*x+a)^2+6*a
^5/b^7/(b*x+a)+15*a^4*ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^3,x]
```

```
[Out] (-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7*
(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^3}\,dx &= \int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx\\ &= -\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 0.90

$$\frac{-40a^3bx+12a^2b^2x^2-4ab^3x^3+b^4x^4-\frac{2a^6}{(a+bx)^2}+\frac{24a^5}{a+bx}+60a^4\log(a+bx)}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^3,x]
```

```
[Out] (-40*a^3*b*x + 12*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 - (2*a^6)/(a + b*x)^2
 + (24*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(4*b^7)
```

Mathics [A]
time = 2.39, size = 126, normalized size = 1.47

$$\frac{22a^6+60a^6\text{Log}\left[a+bx\right]-16a^5bx+120a^5bx\text{Log}\left[a+bx\right]-68a^4b^2x^2+60a^4b^2x^2\text{Log}\left[a+bx\right]-20a^3b^3x^3+5a^2b^4x^4-2ab^5x^5+b^6x^6}{4b^7\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6/(a + b*x)^3,x]')
```

```
[Out] (22 a ^ 6 + 60 a ^ 6 Log[a + b x] - 16 a ^ 5 b x + 120 a ^ 5 b x Log[a + b
x] - 68 a ^ 4 b ^ 2 x ^ 2 + 60 a ^ 4 b ^ 2 x ^ 2 Log[a + b x] - 20 a ^ 3 b
^ 3 x ^ 3 + 5 a ^ 2 b ^ 4 x ^ 4 - 2 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6) / (4 b ^ 7
 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.08, size = 83, normalized size = 0.97

method	result	size
risch	$\frac{x^4}{4b^3}-\frac{a\,x^3}{b^4}+\frac{3a^2x^2}{b^5}-\frac{10a^3x}{b^6}+\frac{6a^5x+\frac{11a^6}{2b}}{b^6\left(bx+a\right)^2}+\frac{15a^4\ln\left(bx+a\right)}{b^7}$	79
norman	$\frac{\frac{x^6}{4b}-\frac{a\,x^5}{2b^2}+\frac{45a^6}{2b^7}-\frac{5a^3x^3}{b^4}+\frac{5a^2x^4}{4b^3}+\frac{30a^5x}{b^6}}{\left(bx+a\right)^2}+\frac{15a^4\ln\left(bx+a\right)}{b^7}$	81
default	$-\frac{-\frac{1}{4}b^3x^4+a\,b^2x^3-3a^2b\,x^2+10a^3x}{b^6}+\frac{6a^5}{b^7\left(bx+a\right)}-\frac{a^6}{2b^7\left(bx+a\right)^2}+\frac{15a^4\ln\left(bx+a\right)}{b^7}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/4*b^3*x^4+a*b^2*x^3-3*a^2*b*x^2+10*a^3*x)+6*a^5/b^7/(b*x+a)-1/2*
a^6/b^7/(b*x+a)^2+15*a^4*ln(b*x+a)/b^7
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.06

$$\frac{12\,a^5bx+11\,a^6}{2\left(b^9x^2+2\,ab^8x+a^2b^7\right)}+\frac{15\,a^4\log\left(bx+a\right)}{b^7}+\frac{b^3x^4-4\,ab^2x^3+12\,a^2bx^2-40\,a^3x}{4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^5*b*x + 11*a^6)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7) + 15*a^4*log(b*x
+ a)/b^7 + 1/4*(b^3*x^4 - 4*a*b^2*x^3 + 12*a^2*b*x^2 - 40*a^3*x)/b^6
```

Fricas [A]
time = 0.30, size = 117, normalized size = 1.36

$$\frac{b^6x^6 - 2\,ab^5x^5 + 5\,a^2b^4x^4 - 20\,a^3b^3x^3 - 68\,a^4b^2x^2 - 16\,a^5bx + 22\,a^6 + 60\,(a^4b^2x^2 + 2\,a^5bx + a^6)\log{(bx + a)}}{4\,(b^9x^2 + 2\,ab^8x + a^2b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*x^6 - 2*a*b^5*x^5 + 5*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 68*a^4*b^2*x^
2 - 16*a^5*b*x + 22*a^6 + 60*(a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a))/
(b^9*x^2 + 2*a*b^8*x + a^2*b^7)
```

Sympy [A]
time = 0.17, size = 92, normalized size = 1.07

$$\frac{15a^4\log{(a + bx)}}{b^7} - \frac{10a^3x}{b^6} + \frac{3a^2x^2}{b^5} - \frac{ax^3}{b^4} + \frac{11a^6 + 12a^5bx}{2a^2b^7 + 4ab^8x + 2b^9x^2} + \frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x)/b**7 - 10*a**3*x/b**6 + 3*a**2*x**2/b**5 - a*x**3/b**4
 + (11*a**6 + 12*a**5*b*x)/(2*a**2*b**7 + 4*a*b**8*x + 2*b**9*x**2) + x**4/
(4*b**3)
```

Giac [A]
time = 0.00, size = 92, normalized size = 1.07

$$\frac{\frac{1}{4}x^4b^9 - x^3b^8a + 3x^2b^7a^2 - 10xb^6a^3}{b^{12}} + \frac{\frac{1}{2}\,(12ba^5x + 11a^6)}{b^7\,(xb + a)^2} + \frac{15a^4\ln{|xb + a|}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x)
```

```
[Out] 15*a^4*log(abs(b*x + a))/b^7 + 1/2*(12*a^5*b*x + 11*a^6)/((b*x + a)^2*b^7)
+ 1/4*(b^9*x^4 - 4*a*b^8*x^3 + 12*a^2*b^7*x^2 - 40*a^3*b^6*x)/b^12
```

Mupad [B]
time = 0.16, size = 78, normalized size = 0.91

$$\frac{\frac{(a+b\,x)^4}{4} - 2\,a\,(a + b\,x)^3 + \frac{15\,a^2\,(a+b\,x)^2}{2} + \frac{6\,a^5}{a+b\,x} - \frac{a^6}{2\,(a+b\,x)^2} + 15\,a^4\,\ln{(a + b\,x)} - 20\,a^3\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^4/4 - 2*a*(a + b*x)^3 + (15*a^2*(a + b*x)^2)/2 + (6*a^5)/(a + b*
x) - a^6/(2*(a + b*x)^2) + 15*a^4*log(a + b*x) - 20*a^3*b*x)/b^7
```

3.182 $\int \frac{x^5}{(a+bx)^3}\,dx$

Optimal. Leaf size=77

$$\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}+\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}$$

```
[Out] 6*a^2*x/b^5-3/2*a*x^2/b^4+1/3*x^3/b^3+1/2*a^5/b^6/(b*x+a)^2-5*a^4/b^6/(b*x+
a)-10*a^3*ln(b*x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^3,x]
```

```
[Out] (6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2) -
 (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^3}\,dx&=\int \left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx\\&=\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}+\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 0.87

$$\frac{36a^2bx-9ab^2x^2+2b^3x^3+\frac{3a^5}{(a+bx)^2}-\frac{30a^4}{a+bx}-60a^3\log(a+bx)}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^3,x]
```

```
[Out] (36*a^2*b*x - 9*a*b^2*x^2 + 2*b^3*x^3 + (3*a^5)/(a + b*x)^2 - (30*a^4)/(a +
 b*x) - 60*a^3*Log[a + b*x])/(6*b^6)
```

Mathics [A]
time = 2.33, size = 116, normalized size = 1.51

$$\frac{-60a^5\text{Log}\,[a+bx]-27a^5-120a^4bx\text{Log}\,[a+bx]+6a^4bx-60a^3b^2x^2\text{Log}\,[a+bx]+63a^3b^2x^2+20a^2b^3x^3-5ab^4x^4+2b^5x^5}{6b^6\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5/(a + b*x)^3,x]')
```

```
[Out] (-60 a ^ 5 Log[a + b x] - 27 a ^ 5 - 120 a ^ 4 b x Log[a + b x] + 6 a ^ 4 b
 x - 60 a ^ 3 b ^ 2 x ^ 2 Log[a + b x] + 63 a ^ 3 b ^ 2 x ^ 2 + 20 a ^ 2 b
^ 3 x ^ 3 - 5 a b ^ 4 x ^ 4 + 2 b ^ 5 x ^ 5) / (6 b ^ 6 (a ^ 2 + 2 a b x +
b ^ 2 x ^ 2))
```

Maple [A]
time = 0.08, size = 72, normalized size = 0.94

method	result	size
risch	$\frac{x^3}{3b^3}-\frac{3a\,x^2}{2b^4}+\frac{6a^2x}{b^5}+\frac{-5a^4x-\frac{9a^5}{2b}}{b^5(bx+a)^2}-\frac{10a^3\ln(bx+a)}{b^6}$	68
norman	$\frac{\frac{x^5}{3b}-\frac{5a\,x^4}{6b^2}+\frac{10a^2x^3}{3b^3}-\frac{15a^5}{b^6}-\frac{20a^4x}{b^5}}{(bx+a)^2}-\frac{10a^3\ln(bx+a)}{b^6}$	70
default	$\frac{\frac{1}{3}b^2x^3-\frac{3}{2}ab\,x^2+6a^2x}{b^5}-\frac{5a^4}{b^6(bx+a)}+\frac{a^5}{2b^6(bx+a)^2}-\frac{10a^3\ln(bx+a)}{b^6}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/3*b^2*x^3-3/2*a*b*x^2+6*a^2*x)-5*a^4/b^6/(b*x+a)+1/2*a^5/b^6/(b*x+
a)^2-10*a^3*ln(b*x+a)/b^6
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.05

$$-\frac{10\,a^4bx+9\,a^5}{2\,(b^8x^2+2\,ab^7x+a^2b^6)}-\frac{10\,a^3\log\left(bx+a\right)}{b^6}+\frac{2\,b^2x^3-9\,abx^2+36\,a^2x}{6\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*a^4*b*x + 9*a^5)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) - 10*a^3*log(b*x
+ a)/b^6 + 1/6*(2*b^2*x^3 - 9*a*b*x^2 + 36*a^2*x)/b^5
```

Fricas [A]
time = 0.31, size = 107, normalized size = 1.39

$$\frac{2\,b^5x^5-5\,ab^4x^4+20\,a^2b^3x^3+63\,a^3b^2x^2+6\,a^4bx-27\,a^5-60\,(a^3b^2x^2+2\,a^4bx+a^5)\log\left(bx+a\right)}{6\,(b^8x^2+2\,ab^7x+a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 - 5*a*b^4*x^4 + 20*a^2*b^3*x^3 + 63*a^3*b^2*x^2 + 6*a^4*b*x
- 27*a^5 - 60*(a^3*b^2*x^2 + 2*a^4*b*x + a^5)*log(b*x + a))/(b^8*x^2 + 2*a*
b^7*x + a^2*b^6)
```

Sympy [A]
time = 0.16, size = 85, normalized size = 1.10

$$-\frac{10a^3\log\left(a+bx\right)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{-9a^5-10a^4bx}{2a^2b^6+4ab^7x+2b^8x^2}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**3,x)
```

```
[Out] -10*a**3*log(a + b*x)/b**6 + 6*a**2*x/b**5 - 3*a*x**2/(2*b**4) + (-9*a**5 -
 10*a**4*b*x)/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) + x**3/(3*b**3)
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.12

$$\frac{\frac{1}{3}x^3b^6-\frac{3}{2}x^2b^5a+6xb^4a^2}{b^9}+\frac{\frac{1}{2}\left(-10ba^4x-9a^5\right)}{b^6\left(xb+a\right)^2}-\frac{10a^3\ln|xb+a|}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x)
```

```
[Out] -10*a^3*log(abs(b*x + a))/b^6 - 1/2*(10*a^4*b*x + 9*a^5)/((b*x + a)^2*b^6)
+ 1/6*(2*b^6*x^3 - 9*a*b^5*x^2 + 36*a^2*b^4*x)/b^9
```

Mupad [B]
time = 0.12, size = 67, normalized size = 0.87

$$-\frac{\frac{5\,a\,(a+b\,x)^2}{2}-\frac{(a+b\,x)^3}{3}+\frac{5\,a^4}{a+b\,x}-\frac{a^5}{2\,(a+b\,x)^2}+10\,a^3\,\ln\left(a+b\,x\right)-10\,a^2\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^3,x)
```

```
[Out] -((5*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (5*a^4)/(a + b*x) - a^5/(2*(a + b*x
)^2) + 10*a^3*log(a + b*x) - 10*a^2*b*x)/b^6
```

3.183 $\int \frac{x^4}{(a+bx)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}$$

```
[Out] -3*a*x/b^4+1/2*x^2/b^3-1/2*a^4/b^5/(b*x+a)^2+4*a^3/b^5/(b*x+a)+6*a^2*ln(b*x
+a)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^3,x]
```

```
[Out] (-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b*
x)) + (6*a^2*Log[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^3} \, dx &= \int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right) dx \\ &= -\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 0.86

$$\frac{-6abx+b^2x^2-\frac{a^4}{(a+bx)^2}+\frac{8a^3}{a+bx}+12a^2\log(a+bx)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^3,x]
```

```
[Out] (-6*a*b*x + b^2*x^2 - a^4/(a + b*x)^2 + (8*a^3)/(a + b*x) + 12*a^2*Log[a +
b*x])/(2*b^5)
```

Mathics [A]
time = 2.24, size = 107, normalized size = 1.67

$$\frac{12a^2\text{Log}\left[a+bx\right]\left(a^2+2abx+b^2x^2\right)+a^3\left(7a+8bx\right)-6abx\left(a^2+2abx+b^2x^2\right)+b^2x^2\left(a^2+2abx+b^2x^2\right)}{2b^5\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4/(a + b*x)^3,x]')
```

```
[Out] (12 a ^ 2 Log[a + b x] (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) + a ^ 3 (7 a + 8 b x
) - 6 a b x (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) + b ^ 2 x ^ 2 (a ^ 2 + 2 a b x
+ b ^ 2 x ^ 2)) / (2 b ^ 5 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.97

method	result	size
risch	$\frac{x^2}{2b^3}-\frac{3ax}{b^4}+\frac{4a^3x+\frac{7a^4}{2b}}{b^4(bx+a)^2}+\frac{6a^2\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{9a^4}{b^5}+\frac{x^4}{2b}-\frac{2a\,x^3}{b^2}+\frac{12a^3x}{b^4}}{(bx+a)^2}+\frac{6a^2\ln(bx+a)}{b^5}$	59
default	$-\frac{-\frac{1}{2}x^2b+3ax}{b^4}+\frac{4a^3}{b^5(bx+a)}-\frac{a^4}{2b^5(bx+a)^2}+\frac{6a^2\ln(bx+a)}{b^5}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/2*x^2*b+3*a*x)+4*a^3/b^5/(b*x+a)-1/2*a^4/b^5/(b*x+a)^2+6*a^2*ln(
b*x+a)/b^5
```

Maxima [A]
time = 0.25, size = 69, normalized size = 1.08

$$\frac{8\,a^3bx+7\,a^4}{2\,(b^7x^2+2\,ab^6x+a^2b^5)}+\frac{6\,a^2\log\left(bx+a\right)}{b^5}+\frac{bx^2-6\,ax}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*b*x + 7*a^4)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a
)/b^5 + 1/2*(b*x^2 - 6*a*x)/b^4
```

Fricas [A]
time = 0.32, size = 95, normalized size = 1.48

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\,(a^2b^2x^2 + 2\,a^3bx + a^4)\log(bx + a)}{2\,(b^7x^2 + 2\,ab^6x + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A]
time = 0.15, size = 70, normalized size = 1.09

$$\frac{6a^2\log(a + bx)}{b^5} - \frac{3ax}{b^4} + \frac{7a^4 + 8a^3bx}{2a^2b^5 + 4ab^6x + 2b^7x^2} + \frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**3,x)
```

```
[Out] 6*a**2*log(a + b*x)/b**5 - 3*a*x/b**4 + (7*a**4 + 8*a**3*b*x)/(2*a**2*b**5
+ 4*a*b**6*x + 2*b**7*x**2) + x**2/(2*b**3)
```

Giac [A]
time = 0.00, size = 70, normalized size = 1.09

$$\frac{\frac{1}{2}x^2b^3 - 3xb^2a}{b^6} + \frac{\frac{1}{2}\left(8ba^3x + 7a^4\right)}{b^5\left(xb + a\right)^2} + \frac{6a^2\ln|xb + a|}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x)
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

Mupad [B]
time = 0.08, size = 54, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^2}{2} + \frac{4\,a^3}{a+b\,x} - \frac{a^4}{2\,(a+b\,x)^2} + 6\,a^2\,\ln(a + b\,x) - 4\,a\,b\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^2/2 + (4*a^3)/(a + b*x) - a^4/(2*(a + b*x)^2) + 6*a^2*log(a + b*
x) - 4*a*b*x)/b^5
```

3.184 $\int \frac{x^3}{(a+bx)^3} dx$

Optimal. Leaf size=50

$$\frac{x}{b^3} + \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4}$$

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4} + \frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^3,x]
```

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^3} dx &= \int \left(\frac{1}{b^3} - \frac{a^3}{b^3(a+bx)^3} + \frac{3a^2}{b^3(a+bx)^2} - \frac{3a}{b^3(a+bx)}\right) dx \\ &= \frac{x}{b^3} + \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 0.80

$$-\frac{-2bx + \frac{a^2(5a+6bx)}{(a+bx)^2} + 6a\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^3,x]
```

```
[Out] -1/2*(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/b^4
```

Mathics [A]
time = 2.15, size = 80, normalized size = 1.60

$$\frac{-3a\mathrm{Log}\left[a+bx\right]\left(a^2+2abx+b^2x^2\right)-\frac{a^2(5a+6bx)}{2}+bx\left(a^2+2abx+b^2x^2\right)}{b^4\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(a + b*x)^3,x]')
```

```
[Out] (-3 a Log[a + b x] (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) - a ^ 2 (5 a + 6 b x) /
2 + b x (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)) / (b ^ 4 (a ^ 2 + 2 a b x + b ^ 2
x ^ 2))
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.98

method	result	size
risch	$\frac{x}{b^3}+\frac{-3a^2x-\frac{5a^3}{2b}}{b^3(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	45
norman	$\frac{\frac{x^3}{b}-\frac{9a^3}{2b^4}-\frac{6a^2x}{b^3}}{(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	47
default	$\frac{x}{b^3}+\frac{a^3}{2b^4(bx+a)^2}-\frac{3a^2}{b^4(bx+a)}-\frac{3a\ln(bx+a)}{b^4}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.25, size = 57, normalized size = 1.14

$$-\frac{6\,a^2bx+5\,a^3}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}+\frac{x}{b^3}-\frac{3\,a\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log(
b*x + a)/b^4
```

Fricas [A]
time = 0.30, size = 83, normalized size = 1.66

$$\frac{2 b^3 x^3 + 4 a b^2 x^2 - 4 a^2 b x - 5 a^3 - 6 (a b^2 x^2 + 2 a^2 b x + a^3) \log(b x + a)}{2 (b^6 x^2 + 2 a b^5 x + a^2 b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A]
time = 0.14, size = 58, normalized size = 1.16

$$-\frac{3a \log(a + bx)}{b^4} + \frac{-5a^3 - 6a^2 bx}{2a^2 b^4 + 4ab^5 x + 2b^6 x^2} + \frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**3,x)
[Out] -3*a*log(a + b*x)/b**4 + (-5*a**3 - 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + x/b**3
```

Giac [A]
time = 0.00, size = 53, normalized size = 1.06

$$\frac{x}{b^3} + \frac{\frac{1}{2} (-6ba^2 x - 5a^3)}{b^4 (xb + a)^2} - \frac{3a \ln |xb + a|}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x)
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

Mupad [B]
time = 0.15, size = 43, normalized size = 0.86

$$-\frac{3 a \ln(a + b x) - b x + \frac{3 a^2}{a + b x} - \frac{a^3}{2 (a + b x)^2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^3,x)
[Out] -(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2))/b^4
```

3.185 $\int \frac{x^2}{(a+bx)^3}\,dx$

Optimal. Leaf size=41

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^3,x]
```

```
[Out] -1/2*a^2/(b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^3}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx\\ &= -\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.80

$$\frac{\frac{a(3a+4bx)}{(a+bx)^2}+2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

Mathics [A]
time = 2.07, size = 57, normalized size = 1.39

$$\frac{\frac{a(3a+4bx)}{2}+\mathrm{Log}\left[a+bx\right]\left(a^2+2abx+b^2x^2\right)}{b^3\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(a + b*x)^3,x]')
```

```
[Out] (a (3 a + 4 b x) / 2 + Log[a + b x] (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)) / (b ^
 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.08, size = 40, normalized size = 0.98

method	result	size
norman	$\frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
risch	$\frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
default	$-\frac{a^2}{2b^3(bx+a)^2}+\frac{2a}{b^3(bx+a)}+\frac{\ln(bx+a)}{b^3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

Maxima [A]
time = 0.24, size = 48, normalized size = 1.17

$$\frac{4\,abx+3\,a^2}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}+\frac{\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Fricas [A]
time = 0.31, size = 61, normalized size = 1.49

$$\frac{4\,abx+3\,a^2+2\,(b^2x^2+2\,abx+a^2)\log\left(bx+a\right)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [A]
time = 0.10, size = 46, normalized size = 1.12

$$\frac{3a^2 + 4abx}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{\log(a + bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)/
b**3
```

Giac [A]
time = 0.00, size = 41, normalized size = 1.00

$$\frac{\frac{1}{2}\left(4ax + \frac{3a^2}{b}\right)}{b^2(xb + a)^2} + \frac{\ln|xb + a|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x)
```

```
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.09, size = 46, normalized size = 1.12

$$\frac{\ln(a + b\,x)}{b^3} + \frac{\frac{3\,a^2}{2\,b^3} + \frac{2\,a\,x}{b^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^3,x)
```

```
[Out] log(a + b*x)/b^3 + ((3*a^2)/(2*b^3) + (2*a*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*x
)
```

3.186 $\int \frac{x}{(a+bx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{x^2}{2a(a+bx)^2}$$

[Out] 1/2*x^2/a/(b*x+a)^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{37\}$

$$\frac{x^2}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x)^3,x]

[Out] x^2/(2*a*(a + b*x)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{(a+bx)^3}\,dx = \frac{x^2}{2a(a+bx)^2}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.18

$$-\frac{a+2bx}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x)^3,x]

[Out] -1/2*(a + 2*b*x)/(b^2*(a + b*x)^2)

Mathics [A]
time = 1.85, size = 30, normalized size = 1.76

$$\frac{-\frac{a}{2} - bx}{b^2 (a^2 + 2abx + b^2x^2)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1/(a + b*x)^3,x]')
```

```
[Out] (-a / 2 - b x) / (b ^ 2 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.08, size = 27, normalized size = 1.59

method	result	size
gosper	$-\frac{2bx+a}{2(bx+a)^2 b^2}$	19
norman	$\frac{-\frac{x}{b}-\frac{a}{2b^2}}{(bx+a)^2}$	22
risch	$\frac{-\frac{x}{b}-\frac{a}{2b^2}}{(bx+a)^2}$	22
default	$-\frac{1}{b^2(bx+a)}+\frac{a}{2b^2(bx+a)^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/(b*x+a)+1/2*a/b^2/(b*x+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.
time = 0.26, size = 32, normalized size = 1.88

$$-\frac{2\,bx + a}{2\,(b^4x^2 + 2\,ab^3x + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.
time = 0.30, size = 32, normalized size = 1.88

$$-\frac{2\,bx + a}{2\,(b^4x^2 + 2\,ab^3x + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.
time = 0.10, size = 32, normalized size = 1.88

$$\frac{-a - 2bx}{2a^2b^2 + 4ab^3x + 2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**3,x)
```

```
[Out] (-a - 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Giac [A]
time = 0.00, size = 22, normalized size = 1.29

$$\frac{-2xb - a}{2b^2\left(xb + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x)
```

```
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.07, size = 32, normalized size = 1.88

$$-\frac{\frac{a}{2\,b^2}+\frac{x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^3,x)
```

```
[Out] -(a/(2*b^2) + x/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.187 $\int \frac{1}{(a+bx)^3} dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

[Out] -1/2/b/(b*x+a)^2

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-3),x]

[Out] -1/2*1/(b*(a + b*x)^2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3} dx = -\frac{1}{2b(a+bx)^2}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-3),x]

[Out] -1/2*1/(b*(a + b*x)^2)

Mathics [A]
time = 1.76, size = 23, normalized size = 1.64

$$-\frac{1}{2b(a^2+2abx+b^2x^2)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^3,x]')
[Out] -1 / (2 b (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{2b(bx+a)^{2}}$	13
default	$-\frac{1}{2b(bx+a)^{2}}$	13
norman	$-\frac{1}{2b(bx+a)^{2}}$	13
risch	$-\frac{1}{2b(bx+a)^{2}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/(b*x+a)^2
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx+a)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2/((b*x + a)^2*b)
```

Fricas [A]
time = 0.30, size = 24, normalized size = 1.71

$$-\frac{1}{2\,(b^{3}x^{2}+2\,ab^{2}x+a^{2}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.10, size = 26, normalized size = 1.86

$$-\frac{1}{2a^{2}b+4ab^{2}x+2b^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3,x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{2b\left(xb+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x)
```

```
[Out] -1/2/((b*x + a)^2*b)
```

Mupad [B]
time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^3,x)
```

```
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

3.188 $\int \frac{1}{x(a+bx)^3} dx$

Optimal. Leaf size=43

$$\frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}$$

```
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a^2(a+bx)}+\frac{1}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^3),x]
```

```
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx\\ &= \frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.86

$$\frac{\frac{a(3a+2bx)}{(a+bx)^2}+2\log(x)-2\log(a+bx)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

Mathics [A]
time = 2.20, size = 66, normalized size = 1.53

$$\frac{\frac{a(3a+2bx)}{2} + (a^2 + 2abx + b^2x^2)\left(\text{Log}\,[x] - \text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^3\,(a^2 + 2abx + b^2x^2)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)^3),x]')
```

```
[Out] (a (3 a + 2 b x) / 2 + (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (Log[x] - Log[(a + b
 x) / b])) / (a ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.98

method	result	size
risch	$\frac{\frac{bx}{a^2}+\frac{3}{2a}}{(bx+a)^2} - \frac{\ln(bx+a)}{a^3} + \frac{\ln(-x)}{a^3}$	41
default	$\frac{1}{2a(bx+a)^2} + \frac{1}{a^2(bx+a)} + \frac{\ln(x)}{a^3} - \frac{\ln(bx+a)}{a^3}$	42
norman	$\frac{-\frac{2bx}{a^2}-\frac{3b^2x^2}{2a^3}}{(bx+a)^2} + \frac{\ln(x)}{a^3} - \frac{\ln(bx+a)}{a^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

Maxima [A]
time = 0.25, size = 51, normalized size = 1.19

$$\frac{2\,bx + 3\,a}{2\,(a^2b^2x^2 + 2\,a^3bx + a^4)} - \frac{\log\,(bx + a)}{a^3} + \frac{\log\,(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

Fricas [A]
time = 0.32, size = 80, normalized size = 1.86

$$\frac{2\,abx + 3\,a^2 - 2\,(b^2x^2 + 2\,abx + a^2)\log\,(bx + a) + 2\,(b^2x^2 + 2\,abx + a^2)\log\,(x)}{2\,(a^3b^2x^2 + 2\,a^4bx + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [A]
time = 0.15, size = 46, normalized size = 1.07

$$\frac{3a+2bx}{2a^4+4a^3bx+2a^2b^2x^2}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**3,x)
```

```
[Out] (3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
+ x))/a**3
```

Giac [A]
time = 0.00, size = 52, normalized size = 1.21

$$\frac{\ln|x|}{a^3}-\frac{b\ln|xb+a|}{ba^3}+\frac{\frac{1}{2}\left(2bax+3a^2\right)}{a^3\left(xb+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x)
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

Mupad [B]
time = 0.10, size = 43, normalized size = 1.00

$$\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,{\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^3),x)
```

```
[Out] (1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2)
```

3.189 $\int \frac{1}{x^2(a+bx)^3}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}$$

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{2b}{a^3(a+bx)}-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.93

$$-\frac{\frac{a(2a^2+9abx+6b^2x^2)}{x(a+bx)^2}+6b\log(x)-6b\log(a+bx)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b*
Log[a + b*x])/a^4
```

Mathics [A]
time = 2.34, size = 83, normalized size = 1.46

$$\frac{a\left(-2a^2-9abx-6b^2x^2\right)-6bx\left(a^2+2abx+b^2x^2\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{2a^4x\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^3),x]')
```

```
[Out] (a (-2 a ^ 2 - 9 a b x - 6 b ^ 2 x ^ 2) - 6 b x (a ^ 2 + 2 a b x + b ^ 2 x
^ 2) (Log[x] - Log[(a + b x) / b])) / (2 a ^ 4 x (a ^ 2 + 2 a b x + b ^ 2 x
 ^ 2))
```

Maple [A]
time = 0.09, size = 56, normalized size = 0.98

method	result	size
default	$-\frac{1}{a^3x}-\frac{b}{2a^2(bx+a)^2}-\frac{2b}{a^3(bx+a)}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(bx+a)}{a^4}$	56
risch	$\frac{-\frac{3b^2x^2}{a^3}-\frac{9bx}{2a^2}-\frac{1}{a}}{x(bx+a)^2}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(-bx-a)}{a^4}$	60
norman	$\frac{-\frac{1}{a}+\frac{6b^2x^2}{a^3}+\frac{9b^3x^3}{2a^4}}{x(bx+a)^2}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(bx+a)}{a^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

Maxima [A]
time = 0.24, size = 69, normalized size = 1.21

$$-\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Fricas [A]
time = 0.30, size = 109, normalized size = 1.91

$$-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log\left(bx+a\right)+6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log\left(x\right)}{2\,(a^4b^2x^3+2\,a^5bx^2+a^6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [A]
time = 0.19, size = 66, normalized size = 1.16

$$\frac{-2a^2-9abx-6b^2x^2}{2a^5x+4a^4bx^2+2a^3b^2x^3}+\frac{3b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x - 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*x
**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

Giac [A]
time = 0.00, size = 73, normalized size = 1.28

$$-\frac{3b\ln|x|}{a^4}+\frac{3b^2\ln|xb+a|}{ba^4}+\frac{\frac{1}{2}\left(-6b^2ax^2-9ba^2x-2a^3\right)}{a^4x\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x)
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

Mupad [B]
time = 0.11, size = 63, normalized size = 1.11

$$\frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{1}{a}+\frac{3\,b^2\,x^2}{a^3}+\frac{9\,b\,x}{2\,a^2}}{a^2\,x+2\,a\,b\,x^2+b^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^3),x)
```

```
[Out] (6*b*atanh((2*b*x)/a + 1))/a^4 - (1/a + (3*b^2*x^2)/a^3 + (9*b*x)/(2*a^2))/
(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

3.190 $\int \frac{1}{x^3(a+bx)^3} dx$

Optimal. Leaf size=76

$$-\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}$$

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b^2}{a^4(a+bx)}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*1/(a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(
a + b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.89

$$\frac{\frac{a(-a^3+4a^2bx+18ab^2x^2+12b^3x^3)}{x^2(a+bx)^2}+12b^2\log(x)-12b^2\log(a+bx)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12*
b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

Mathics [A]
time = 2.50, size = 98, normalized size = 1.29

$$\frac{a\left(-a^3+4a^2bx+18ab^2x^2+12b^3x^3\right)+12b^2x^2\left(a^2+2abx+b^2x^2\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{2a^5x^2\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^3),x]')
```

```
[Out] (a (-a ^ 3 + 4 a ^ 2 b x + 18 a b ^ 2 x ^ 2 + 12 b ^ 3 x ^ 3) + 12 b ^ 2 x
^ 2 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (Log[x] - Log[(a + b x) / b])) / (2 a ^
 5 x ^ 2 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.96

method	result	size
norman	$\frac{-\frac{9b^4x^4}{a^5}-\frac{1}{2a}+\frac{2bx}{a^2}-\frac{12b^3x^3}{a^4}}{x^2(bx+a)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$	72
default	$-\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(bx+a)^2}+\frac{3b^2}{a^4(bx+a)}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$	73
risch	$\frac{\frac{6b^3x^3}{a^4}+\frac{9b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^2}+\frac{6b^2\ln(-x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

Maxima [A]
time = 0.27, size = 86, normalized size = 1.13

$$\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}-\frac{6\,b^2\log\left(bx+a\right)}{a^5}+\frac{6\,b^2\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

Fricas [A]
time = 0.31, size = 130, normalized size = 1.71

$$\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log\left(bx+a\right)+12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log\left(x\right)}{2\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [A]
time = 0.20, size = 78, normalized size = 1.03

$$\frac{-a^3+4a^2bx+18ab^2x^2+12b^3x^3}{2a^6x^2+4a^5bx^3+2a^4b^2x^4}+\frac{6b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5*
b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

Giac [A]
time = 0.00, size = 82, normalized size = 1.08

$$\frac{6b^2\ln|x|}{a^5}-\frac{6b^3\ln|xb+a|}{ba^5}-\frac{-12x^3b^3-18x^2b^2a-4xba^2+a^3}{2a^4\left(x^2b+xa\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x)
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

Mupad [B]
time = 0.12, size = 79, normalized size = 1.04

$$\frac{\frac{9\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{6\,b^3\,x^3}{a^4}+\frac{2\,b\,x}{a^2}}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4}-\frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^3),x)
```

```
[Out] ((9*b^2*x^2)/a^3 - 1/(2*a) + (6*b^3*x^3)/a^4 + (2*b*x)/a^2)/(a^2*x^2 + b^2*
x^4 + 2*a*b*x^3) - (12*b^2*atanh((2*b*x)/a + 1))/a^5
```

3.191 $\int \frac{1}{x^4(a+bx)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}$$

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{46\}$

$$-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}-\frac{4b^3}{a^5(a+bx)}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/3*1/(a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b*
x)^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b*x
])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.89

$$-\frac{\frac{a(2a^4-5a^3bx+20a^2b^2x^2+90ab^3x^3+60b^4x^4)}{x^3(a+bx)^2}+60b^3\log(x)-60b^3\log(a+bx)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/6*((a*(2*a^4 - 5*a^3*b*x + 20*a^2*b^2*x^2 + 90*a*b^3*x^3 + 60*b^4*x^4))/
(x^3*(a + b*x)^2) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a^6
```

Mathics [A]
time = 2.71, size = 109, normalized size = 1.22

$$\frac{a\left(-2a^4+5a^3bx-20a^2b^2x^2-90ab^3x^3-60b^4x^4\right)-60b^3x^3\left(a^2+2abx+b^2x^2\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{6a^6x^3\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(a + b*x)^3),x]')
```

```
[Out] (a (-2 a ^ 4 + 5 a ^ 3 b x - 20 a ^ 2 b ^ 2 x ^ 2 - 90 a b ^ 3 x ^ 3 - 60 b
 ^ 4 x ^ 4) - 60 b ^ 3 x ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (Log[x] - Log[
(a + b x) / b])) / (6 a ^ 6 x ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.09, size = 84, normalized size = 0.94

method	result	size
norman	$\frac{\frac{15b^5x^5}{a^6}-\frac{1}{3a}+\frac{5bx}{6a^2}-\frac{10b^2x^2}{3a^3}+\frac{20b^4x^4}{a^5}}{x^3(bx+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(bx+a)}{a^6}$	83
default	$-\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(bx+a)^2}-\frac{4b^3}{a^5(bx+a)}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(bx+a)}{a^6}$	84
risch	$\frac{-\frac{10b^4x^4}{a^5}-\frac{15b^3x^3}{a^4}-\frac{10b^2x^2}{3a^3}+\frac{5bx}{6a^2}-\frac{1}{3a}}{x^3(bx+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(-bx-a)}{a^6}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.25, size = 97, normalized size = 1.09

$$-\frac{60\,b^4x^4+90\,ab^3x^3+20\,a^2b^2x^2-5\,a^3bx+2\,a^4}{6\,\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}+\frac{10\,b^3\log\left(bx+a\right)}{a^6}-\frac{10\,b^3\log\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(60*b^4*x^4 + 90*a*b^3*x^3 + 20*a^2*b^2*x^2 - 5*a^3*b*x + 2*a^4)/(a^5*
b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3) + 10*b^3*log(b*x + a)/a^6 - 10*b^3*log(x)/
a^6
```

Fricas [A]
time = 0.31, size = 141, normalized size = 1.58

$$-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5-60\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\log\left(bx+a\right)+60\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\log\left(x\right)}{6\,(a^6b^2x^5+2\,a^7bx^4+a^8x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*a*b^4*x^4 + 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5 -
60*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*log(b*x + a) + 60*(b^5*x^5 + 2*a*b
^4*x^4 + a^2*b^3*x^3)*log(x))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

Sympy [A]
time = 0.22, size = 92, normalized size = 1.03

$$\frac{-2a^4+5a^3bx-20a^2b^2x^2-90ab^3x^3-60b^4x^4}{6a^7x^3+12a^6bx^4+6a^5b^2x^5}+\frac{10b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x - 20*a**2*b**2*x**2 - 90*a*b**3*x**3 - 60*b**4*x**4)/
(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 10*b**3*(-log(x) + log(
a/b + x))/a**6
```

Giac [A]
time = 0.00, size = 100, normalized size = 1.12

$$\frac{10b^4\ln|xb+a|}{ba^6}-\frac{10b^3\ln|x|}{a^6}+\frac{\frac{1}{6}\left(-60b^4ax^4-90b^3a^2x^3-20b^2a^3x^2+5ba^4x-2a^5\right)}{a^6x^3\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x)
```

```
[Out] 10*b^3*log(abs(b*x + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/6*(60*a*b^4*x^4 +
 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5)/((b*x + a)^2*a^6*x^3)
```

Mupad [B]
time = 0.13, size = 91, normalized size = 1.02

$$\frac{20\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}-\frac{\frac{1}{3\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{15\,b^3\,x^3}{a^4}+\frac{10\,b^4\,x^4}{a^5}-\frac{5\,b\,x}{6\,a^2}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^3),x)
```

```
[Out] (20*b^3*atanh((2*b*x)/a + 1))/a^6 - (1/(3*a) + (10*b^2*x^2)/(3*a^3) + (15*b
^3*x^3)/a^4 + (10*b^4*x^4)/a^5 - (5*b*x)/(6*a^2))/(a^2*x^3 + b^2*x^5 + 2*a*
b*x^4)
```

3.192 $\int \frac{1}{x^5(a+bx)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}$$

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{5b^4}{a^6(a+bx)}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}-\frac{3b^2}{a^5x^2}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^3),x]
```

```
[Out] -1/4*1/(a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^4
/(2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (15*
b^4*Log[a + b*x])/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx)}\right) d\\ &= -\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(}{a^7}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 0.93

$$\frac{\frac{a\left(-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5\right)}{x^4(a+bx)^2}+60b^4\log(x)-60b^4\log(a+bx)}{4a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x - 5*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 90*a*b^4*x^4 + 60*
b^5*x^5))/(x^4*(a + b*x)^2) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(4*a^7)
```

Mathics [A]
time = 2.85, size = 120, normalized size = 1.24

$$\frac{a\left(-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5\right)+60b^4x^4\left(a^2+2abx+b^2x^2\right)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{4a^7x^4\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(a + b*x)^3),x]')
```

```
[Out] (a (-a ^ 5 + 2 a ^ 4 b x - 5 a ^ 3 b ^ 2 x ^ 2 + 20 a ^ 2 b ^ 3 x ^ 3 + 90
a b ^ 4 x ^ 4 + 60 b ^ 5 x ^ 5) + 60 b ^ 4 x ^ 4 (a ^ 2 + 2 a b x + b ^ 2 x
 ^ 2) (Log[x] - Log[(a + b x) / b])) / (4 a ^ 7 x ^ 4 (a ^ 2 + 2 a b x + b
^ 2 x ^ 2))
```

Maple [A]
time = 0.08, size = 94, normalized size = 0.97

method	result	size
default	$-\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(bx+a)^2}+\frac{5b^4}{a^6(bx+a)}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$	94
norman	$\frac{-\frac{1}{4a}+\frac{bx}{2a^2}-\frac{5b^2x^2}{4a^3}+\frac{5b^3x^3}{a^4}-\frac{30b^5x^5}{a^6}-\frac{45b^6x^6}{2a^7}}{x^4(bx+a)^2}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$	94
risch	$\frac{\frac{15b^5x^5}{a^6}+\frac{45b^4x^4}{2a^5}+\frac{5b^3x^3}{a^4}-\frac{5b^2x^2}{4a^3}+\frac{bx}{2a^2}-\frac{1}{4a}}{x^4(bx+a)^2}-\frac{15b^4\ln(bx+a)}{a^7}+\frac{15b^4\ln(-x)}{a^7}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Maxima [A]
time = 0.25, size = 108, normalized size = 1.11

$$\frac{60\,b^5x^5+90\,ab^4x^4+20\,a^2b^3x^3-5\,a^3b^2x^2+2\,a^4bx-a^5}{4\,(a^6b^2x^6+2\,a^7bx^5+a^8x^4)}-\frac{15\,b^4\log\left(bx+a\right)}{a^7}+\frac{15\,b^4\log\left(x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="maxima")
```

[Out] 1/4*(60*b^5*x^5 + 90*a*b^4*x^4 + 20*a^2*b^3*x^3 - 5*a^3*b^2*x^2 + 2*a^4*b*x - a^5)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4) - 15*b^4*log(b*x + a)/a^7 + 15*b^4*log(x)/a^7

Fricas [A]
time = 0.31, size = 152, normalized size = 1.57

$$\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6-60\,(b^6x^6+2\,ab^5x^5+a^2b^4x^4)\log(bx+a)+60\,(b^6x^6+2\,ab^5x^5+a^2b^4x^4)\log(x)}{4\,(a^7b^2x^6+2\,a^8bx^5+a^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="fricas")

[Out] 1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6 - 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(b*x + a) + 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(x))/(a^7*b^2*x^6 + 2*a^8*b*x^5 + a^9*x^4)

Sympy [A]
time = 0.23, size = 102, normalized size = 1.05

$$\frac{-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5}{4a^8x^4+8a^7bx^5+4a^6b^2x^6}+\frac{15b^4\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x+a)**3,x)

[Out] (-a**5 + 2*a**4*b*x - 5*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 90*a*b**4*x**4 + 60*b**5*x**5)/(4*a**8*x**4 + 8*a**7*b*x**5 + 4*a**6*b**2*x**6) + 15*b**4*(log(x) - log(a/b + x))/a**7

Giac [A]
time = 0.00, size = 107, normalized size = 1.10

$$-\frac{15b^5\ln|xb+a|}{ba^7}+\frac{15b^4\ln|x|}{a^7}+\frac{\frac{1}{4}\left(60b^5ax^5+90b^4a^2x^4+20b^3a^3x^3-5b^2a^4x^2+2ba^5x-a^6\right)}{a^7x^4\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x+a)^3,x)

[Out] -15*b^4*log(abs(b*x + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6)/((b*x + a)^2*a^7*x^4)

Mupad [B]
time = 0.09, size = 101, normalized size = 1.04

$$\frac{\frac{5\,b^3\,x^3}{a^4}-\frac{5\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{45\,b^4\,x^4}{2\,a^5}+\frac{15\,b^5\,x^5}{a^6}+\frac{b\,x}{2\,a^2}}{a^2\,x^4+2\,a\,b\,x^5+b^2\,x^6}-\frac{30\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^3),x)
```

```
[Out] ((5*b^3*x^3)/a^4 - (5*b^2*x^2)/(4*a^3) - 1/(4*a) + (45*b^4*x^4)/(2*a^5) + (
15*b^5*x^5)/a^6 + (b*x)/(2*a^2))/(a^2*x^4 + b^2*x^6 + 2*a*b*x^5) - (30*b^4*
atanh((2*b*x)/a + 1))/a^7
```

3.193 $\int \frac{x^8}{(a+bx)^4} dx$

Optimal. Leaf size=114

$$\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}$$

```
[Out] 35*a^4*x/b^8-10*a^3*x^2/b^7+10/3*a^2*x^3/b^6-a*x^4/b^5+1/5*x^5/b^4-1/3*a^8/
b^9/(b*x+a)^3+4*a^7/b^9/(b*x+a)^2-28*a^6/b^9/(b*x+a)-56*a^5*ln(b*x+a)/b^9
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$,
Rules used = {45}

$$-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^4,x]
```

```
[Out] (35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x^
5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6)/
(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^4}\,dx = \int \left(\frac{35a^4}{b^8}-\frac{20a^3x}{b^7}+\frac{10a^2x^2}{b^6}-\frac{4ax^3}{b^5}+\frac{x^4}{b^4}+\frac{a^8}{b^8(a+bx)^4}-\frac{8a^7}{b^8(a+bx)^3}+\frac{28a^6}{b^8(a+bx)^2}-\right.$$
$$=\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-56$$

Mathematica [A]
time = 0.02, size = 101, normalized size = 0.89

$$\frac{525a^4bx-150a^3b^2x^2+50a^2b^3x^3-15ab^4x^4+3b^5x^5-\frac{5a^8}{(a+bx)^3}+\frac{60a^7}{(a+bx)^2}-\frac{420a^6}{a+bx}-840a^5\log(a+bx)}{15b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^4,x]
```

```
[Out] (525*a^4*b*x - 150*a^3*b^2*x^2 + 50*a^2*b^3*x^3 - 15*a*b^4*x^4 + 3*b^5*x^5
- (5*a^8)/(a + b*x)^3 + (60*a^7)/(a + b*x)^2 - (420*a^6)/(a + b*x) - 840*a^
5*Log[a + b*x])/(15*b^9)
```

Mathics [A]
time = 2.86, size = 177, normalized size = 1.55

$$\frac{-840a^8\mathrm{Log}\left[a+bx\right]-365a^8-2520a^7bx\mathrm{Log}\left[a+bx\right]-255a^7bx-2520a^6b^2x^2\mathrm{Log}\left[a+bx\right]+1005a^6b^2x^2-840a^5b^3x^3\mathrm{Log}\left[a+bx\right]+1175a^5b^3x^3+210a^4b^4x^4-42a^3b^5x^5+14a^2b^6x^6-6ab^7x^7+3b^8x^8}{15b^9\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^8/(a + b*x)^4,x]')
```

```
[Out] (-840 a ^ 8 Log[a + b x] - 365 a ^ 8 - 2520 a ^ 7 b x Log[a + b x] - 255 a
^ 7 b x - 2520 a ^ 6 b ^ 2 x ^ 2 Log[a + b x] + 1005 a ^ 6 b ^ 2 x ^ 2 - 84
0 a ^ 5 b ^ 3 x ^ 3 Log[a + b x] + 1175 a ^ 5 b ^ 3 x ^ 3 + 210 a ^ 4 b ^ 4
 x ^ 4 - 42 a ^ 3 b ^ 5 x ^ 5 + 14 a ^ 2 b ^ 6 x ^ 6 - 6 a b ^ 7 x ^ 7 + 3
b ^ 8 x ^ 8) / (15 b ^ 9 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^
 3))
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.96

method	result	size
risch	$\frac{x^5}{5b^4}-\frac{a\,x^4}{b^5}+\frac{10a^2x^3}{3b^6}-\frac{10a^3x^2}{b^7}+\frac{35a^4x}{b^8}+\frac{-28a^6b\,x^2-52a^7x-\frac{73a^8}{3b}}{b^8\left(bx+a\right)^3}-\frac{56a^5\ln\left(bx+a\right)}{b^9}$	99
norman	$\frac{\frac{x^8}{5b}-\frac{2a\,x^7}{5b^2}-\frac{14a^3x^5}{5b^4}-\frac{308a^8}{3b^9}+\frac{14a^4x^4}{b^5}+\frac{14a^2x^6}{15b^3}-\frac{168a^6x^2}{b^7}-\frac{252a^7x}{b^8}}{\left(bx+a\right)^3}-\frac{56a^5\ln\left(bx+a\right)}{b^9}$	103
default	$\frac{\frac{1}{5}b^4x^5-a\,b^3x^4+\frac{10}{3}a^2b^2x^3-10a^3b\,x^2+35a^4x}{b^8}-\frac{28a^6}{b^9\left(bx+a\right)}+\frac{4a^7}{b^9\left(bx+a\right)^2}-\frac{56a^5\ln\left(bx+a\right)}{b^9}-\frac{a^8}{3b^9\left(bx+a\right)^3}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^8*(1/5*b^4*x^5-a*b^3*x^4+10/3*a^2*b^2*x^3-10*a^3*b*x^2+35*a^4*x)-28*a^6
/b^9/(b*x+a)+4*a^7/b^9/(b*x+a)^2-56*a^5*ln(b*x+a)/b^9-1/3*a^8/b^9/(b*x+a)^3
```

Maxima [A]
time = 0.24, size = 125, normalized size = 1.10

$$-\frac{84\,a^6b^2x^2+156\,a^7bx+73\,a^8}{3\left(b^{12}x^3+3\,ab^{11}x^2+3\,a^2b^{10}x+a^3b^9\right)}-\frac{56\,a^5\log\left(bx+a\right)}{b^9}+\frac{3\,b^4x^5-15\,ab^3x^4+50\,a^2b^2x^3-150\,a^3bx^2+525\,a^4x}{15\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)/(b^12*x^3 + 3*a*b^11*x^2 + 3*a
^2*b^10*x + a^3*b^9) - 56*a^5*log(b*x + a)/b^9 + 1/15*(3*b^4*x^5 - 15*a*b^3
*x^4 + 50*a^2*b^2*x^3 - 150*a^3*b*x^2 + 525*a^4*x)/b^8
```

Fricas [A]
time = 0.32, size = 162, normalized size = 1.42

$$\frac{3\,b^{8}x^{8}-6\,ab^{7}x^{7}+14\,a^{2}b^{6}x^{6}-42\,a^{3}b^{5}x^{5}+210\,a^{4}b^{4}x^{4}+1175\,a^{5}b^{3}x^{3}+1005\,a^{6}b^{2}x^{2}-255\,a^{7}bx-365\,a^{8}-840\,(a^{5}b^{3}x^{3}+3\,a^{6}b^{2}x^{2}+3\,a^{7}bx+a^{8})\log\left(bx+a\right)}{15\,(b^{12}x^{3}+3\,ab^{11}x^{2}+3\,a^{2}b^{10}x+a^{3}b^{9})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^8*x^8 - 6*a*b^7*x^7 + 14*a^2*b^6*x^6 - 42*a^3*b^5*x^5 + 210*a^4*b
^4*x^4 + 1175*a^5*b^3*x^3 + 1005*a^6*b^2*x^2 - 255*a^7*b*x - 365*a^8 - 840*
(a^5*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8)*log(b*x + a))/(b^12*x^3 + 3
*a*b^11*x^2 + 3*a^2*b^10*x + a^3*b^9)
```

Sympy [A]
time = 0.25, size = 131, normalized size = 1.15

$$-\frac{56a^{5}\log\left(a+bx\right)}{b^{9}}+\frac{35a^{4}x}{b^{8}}-\frac{10a^{3}x^{2}}{b^{7}}+\frac{10a^{2}x^{3}}{3b^{6}}-\frac{ax^{4}}{b^{5}}+\frac{-73a^{8}-156a^{7}bx-84a^{6}b^{2}x^{2}}{3a^{3}b^{9}+9a^{2}b^{10}x+9ab^{11}x^{2}+3b^{12}x^{3}}+\frac{x^{5}}{5b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**4,x)
```

```
[Out] -56*a**5*log(a + b*x)/b**9 + 35*a**4*x/b**8 - 10*a**3*x**2/b**7 + 10*a**2*x
**3/(3*b**6) - a*x**4/b**5 + (-73*a**8 - 156*a**7*b*x - 84*a**6*b**2*x**2)/
(3*a**3*b**9 + 9*a**2*b**10*x + 9*a*b**11*x**2 + 3*b**12*x**3) + x**5/(5*b*
*4)
```

Giac [A]
time = 0.00, size = 120, normalized size = 1.05

$$\frac{\frac{1}{5}x^{5}b^{16}-x^{4}b^{15}a+\frac{10}{3}x^{3}b^{14}a^{2}-10x^{2}b^{13}a^{3}+35xb^{12}a^{4}}{b^{20}}+\frac{\frac{1}{3}\left(-84b^{2}a^{6}x^{2}-156ba^{7}x-73a^{8}\right)}{b^{9}\left(xb+a\right)^{3}}-\frac{56a^{5}\ln\left|xb+a\right|}{b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x)
```

```
[Out] -56*a^5*log(abs(b*x + a))/b^9 - 1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)
/((b*x + a)^3*b^9) + 1/15*(3*b^16*x^5 - 15*a*b^15*x^4 + 50*a^2*b^14*x^3 - 1
50*a^3*b^13*x^2 + 525*a^4*b^12*x)/b^20
```

Mupad [B]
time = 0.37, size = 103, normalized size = 0.90

$$-\frac{2\,a\,{\left(a+b\,x\right)}^4-\frac{{\left(a+b\,x\right)}^5}{5}-\frac{28\,a^2\,{\left(a+b\,x\right)}^3}{3}+28\,a^3\,{\left(a+b\,x\right)}^2+\frac{28\,a^6}{a+b\,x}-\frac{4\,a^7}{{\left(a+b\,x\right)}^2}+\frac{a^8}{3\,{\left(a+b\,x\right)}^3}+56\,a^5\,\ln\left(a+b\,x\right)-70\,a^4\,b\,x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^4,x)
```

```
[Out] -(2*a*(a + b*x)^4 - (a + b*x)^5/5 - (28*a^2*(a + b*x)^3)/3 + 28*a^3*(a + b*
x)^2 + (28*a^6)/(a + b*x) - (4*a^7)/(a + b*x)^2 + a^8/(3*(a + b*x)^3) + 56*
a^5*log(a + b*x) - 70*a^4*b*x)/b^9
```

3.194 $\int \frac{x^7}{(a+bx)^4} \, dx$

Optimal. Leaf size=105

$$-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}$$

```
[Out] -20*a^3*x/b^7+5*a^2*x^2/b^6-4/3*a*x^3/b^5+1/4*x^4/b^4+1/3*a^7/b^8/(b*x+a)^3
-7/2*a^6/b^8/(b*x+a)^2+21*a^5/b^8/(b*x+a)+35*a^4*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^4,x]
```

```
[Out] (-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/(
3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x))
 + (35*a^4*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^4}\,dx &= \int \left(-\frac{20a^3}{b^7}+\frac{10a^2x}{b^6}-\frac{4ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^7}{b^7(a+bx)^4}+\frac{7a^6}{b^7(a+bx)^3}-\frac{21a^5}{b^7(a+bx)^2}+\frac{35a^4}{b^7(a+b\right.\\ &= -\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a}{b^8}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 90, normalized size = 0.86

$$\frac{-240a^3bx+60a^2b^2x^2-16ab^3x^3+3b^4x^4+\frac{4a^7}{(a+bx)^3}-\frac{42a^6}{(a+bx)^2}+\frac{252a^5}{a+bx}+420a^4\log(a+bx)}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^4,x]
```

```
[Out] (-240*a^3*b*x + 60*a^2*b^2*x^2 - 16*a*b^3*x^3 + 3*b^4*x^4 + (4*a^7)/(a + b*
x)^3 - (42*a^6)/(a + b*x)^2 + (252*a^5)/(a + b*x) + 420*a^4*Log[a + b*x])/(
12*b^8)
```

Mathics [A]
time = 2.78, size = 166, normalized size = 1.58

$$\frac{214a^7 + 420a^7\text{Log}\,[a + bx] + 222a^6bx + 1260a^6bx\text{Log}\,[a + bx] - 408a^5b^2x^2 + 1260a^5b^2x^2\text{Log}\,[a + bx] - 556a^4b^3x^3 + 420a^4b^3x^3\text{Log}\,[a + bx] - 105a^3b^4x^4 + 21a^2b^5x^5 - 7ab^6x^6 + 3b^7x^7}{12b^8\,(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^7/(a + b*x)^4,x]')
```

```
[Out] (214 a ^ 7 + 420 a ^ 7 Log[a + b x] + 222 a ^ 6 b x + 1260 a ^ 6 b x Log[a
+ b x] - 408 a ^ 5 b ^ 2 x ^ 2 + 1260 a ^ 5 b ^ 2 x ^ 2 Log[a + b x] - 556
a ^ 4 b ^ 3 x ^ 3 + 420 a ^ 4 b ^ 3 x ^ 3 Log[a + b x] - 105 a ^ 3 b ^ 4 x
^ 4 + 21 a ^ 2 b ^ 5 x ^ 5 - 7 a b ^ 6 x ^ 6 + 3 b ^ 7 x ^ 7) / (12 b ^ 8 (
a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.10, size = 99, normalized size = 0.94

method	result	size
risch	$\frac{x^4}{4b^4} - \frac{4a\,x^3}{3b^5} + \frac{5a^2x^2}{b^6} - \frac{20a^3x}{b^7} + \frac{21a^5b\,x^2 + \frac{77a^6x}{2} + \frac{107a^7}{6b}}{b^7\,(bx+a)^3} + \frac{35a^4\ln(bx+a)}{b^8}$	88
norman	$\frac{\frac{x^7}{4b} - \frac{7a\,x^6}{12b^2} - \frac{35a^3x^4}{4b^4} + \frac{385a^7}{6b^8} + \frac{7a^2x^5}{4b^3} + \frac{105a^5x^2}{b^6} + \frac{315a^6x}{2b^7}}{(bx+a)^3} + \frac{35a^4\ln(bx+a)}{b^8}$	92
default	$-\frac{-\frac{1}{4}b^3x^4 + \frac{4}{3}a\,b^2x^3 - 5a^2b\,x^2 + 20a^3x}{b^7} + \frac{21a^5}{b^8(bx+a)} - \frac{7a^6}{2b^8(bx+a)^2} + \frac{35a^4\ln(bx+a)}{b^8} + \frac{a^7}{3b^8(bx+a)^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^7*(-1/4*b^3*x^4+4/3*a*b^2*x^3-5*a^2*b*x^2+20*a^3*x)+21*a^5/b^8/(b*x+a)
-7/2*a^6/b^8/(b*x+a)^2+35*a^4*ln(b*x+a)/b^8+1/3*a^7/b^8/(b*x+a)^3
```

Maxima [A]
time = 0.27, size = 114, normalized size = 1.09

$$\frac{126\,a^5b^2x^2 + 231\,a^6bx + 107\,a^7}{6\,(b^{11}x^3 + 3\,ab^{10}x^2 + 3\,a^2b^9x + a^3b^8)} + \frac{35\,a^4\log\,(bx + a)}{b^8} + \frac{3\,b^3x^4 - 16\,ab^2x^3 + 60\,a^2bx^2 - 240\,a^3x}{12\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7)/(b^11*x^3 + 3*a*b^10*x^2 + 3*
a^2*b^9*x + a^3*b^8) + 35*a^4*log(b*x + a)/b^8 + 1/12*(3*b^3*x^4 - 16*a*b^2
*x^3 + 60*a^2*b*x^2 - 240*a^3*x)/b^7
```

Fricas [A]
time = 0.30, size = 151, normalized size = 1.44

$$\frac{3\,b^7x^7-7\,ab^6x^6+21\,a^2b^5x^5-105\,a^3b^4x^4-556\,a^4b^3x^3-408\,a^5b^2x^2+222\,a^6bx+214\,a^7+420\,(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7)\log{(bx+a)}}{12\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 - 7*a*b^6*x^6 + 21*a^2*b^5*x^5 - 105*a^3*b^4*x^4 - 556*a^4*
b^3*x^3 - 408*a^5*b^2*x^2 + 222*a^6*b*x + 214*a^7 + 420*(a^4*b^3*x^3 + 3*a^
5*b^2*x^2 + 3*a^6*b*x + a^7)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2
*b^9*x + a^3*b^8)
```

Sympy [A]
time = 0.24, size = 119, normalized size = 1.13

$$\frac{35a^4\log{(a+bx)}}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{107a^7+231a^6bx+126a^5b^2x^2}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3}+\frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**4,x)
```

```
[Out] 35*a**4*log(a + b*x)/b**8 - 20*a**3*x/b**7 + 5*a**2*x**2/b**6 - 4*a*x**3/(3
*b**5) + (107*a**7 + 231*a**6*b*x + 126*a**5*b**2*x**2)/(6*a**3*b**8 + 18*a
**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*x**3) + x**4/(4*b**4)
```

Giac [A]
time = 0.00, size = 106, normalized size = 1.01

$$\frac{\frac{1}{4}x^4b^{12}-\frac{4}{3}x^3b^{11}a+5x^2b^{10}a^2-20xb^9a^3}{b^{16}}+\frac{\frac{1}{6}\left(126b^2a^5x^2+231ba^6x+107a^7\right)}{b^8\left(xb+a\right)^3}+\frac{35a^4\ln|xb+a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x)
```

```
[Out] 35*a^4*log(abs(b*x + a))/b^8 + 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7
)/((b*x + a)^3*b^8) + 1/12*(3*b^12*x^4 - 16*a*b^11*x^3 + 60*a^2*b^10*x^2 -
240*a^3*b^9*x)/b^16
```

Mupad [B]
time = 0.22, size = 90, normalized size = 0.86

$$\frac{\frac{{\left(a+b\,x\right)}^4}{4}-\frac{7\,a\,{\left(a+b\,x\right)}^3}{3}+\frac{21\,a^2\,{\left(a+b\,x\right)}^2}{2}+\frac{21\,a^5}{a+b\,x}-\frac{7\,a^6}{2\,{\left(a+b\,x\right)}^2}+\frac{a^7}{3\,{\left(a+b\,x\right)}^3}+35\,a^4\,\ln\left(a+b\,x\right)-35\,a^3\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^4/4 - (7*a*(a + b*x)^3)/3 + (21*a^2*(a + b*x)^2)/2 + (21*a^5)/(a
 + b*x) - (7*a^6)/(2*(a + b*x)^2) + a^7/(3*(a + b*x)^3) + 35*a^4*log(a + b*
x) - 35*a^3*b*x)/b^8
```

3.195 $\int \frac{x^6}{(a+bx)^4}\,dx$

Optimal. Leaf size=90

$$\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}$$

```
[Out] 10*a^2*x/b^6-2*a*x^2/b^5+1/3*x^3/b^4-1/3*a^6/b^7/(b*x+a)^3+3*a^5/b^7/(b*x+a
)^2-15*a^4/b^7/(b*x+a)-20*a^3*ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^4}\,dx &= \int \left(\frac{10a^2}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{b^4}+\frac{a^6}{b^6(a+bx)^4}-\frac{6a^5}{b^6(a+bx)^3}+\frac{15a^4}{b^6(a+bx)^2}-\frac{20a^3}{b^6(a+bx)}\right)dx\\ &= \frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 90, normalized size = 1.00

$$\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Mathics [A]
time = 2.62, size = 154, normalized size = 1.71

$$\frac{-60a^6\text{Log}\left[a+bx\right]-37a^6-180a^5bx\text{Log}\left[a+bx\right]-51a^5bx-180a^4b^2x^2\text{Log}\left[a+bx\right]+39a^4b^2x^2-60a^3b^3x^3\text{Log}\left[a+bx\right]+73a^3b^3x^3+15a^2b^4x^4-3ab^5x^5+b^6x^6}{3b^7\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6/(a + b*x)^4,x]')
```

```
[Out] (-60 a ^ 6 Log[a + b x] - 37 a ^ 6 - 180 a ^ 5 b x Log[a + b x] - 51 a ^ 5
b x - 180 a ^ 4 b ^ 2 x ^ 2 Log[a + b x] + 39 a ^ 4 b ^ 2 x ^ 2 - 60 a ^ 3
b ^ 3 x ^ 3 Log[a + b x] + 73 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 - 3
a b ^ 5 x ^ 5 + b ^ 6 x ^ 6) / (3 b ^ 7 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x
^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 87, normalized size = 0.97

method	result	size
risch	$\frac{x^3}{3b^4}-\frac{2a\,x^2}{b^5}+\frac{10a^2x}{b^6}+\frac{-15a^4b\,x^2-27a^5x-\frac{37a^6}{3b}}{b^6(bx+a)^3}-\frac{20a^3\ln(bx+a)}{b^7}$	77
norman	$\frac{\frac{x^6}{3b}-\frac{a\,x^5}{b^2}-\frac{110a^6}{3b^7}+\frac{5a^2x^4}{b^3}-\frac{60a^4x^2}{b^5}-\frac{90a^5x}{b^6}}{(bx+a)^3}-\frac{20a^3\ln(bx+a)}{b^7}$	81
default	$\frac{\frac{1}{3}b^2x^3-2ab\,x^2+10a^2x}{b^6}-\frac{15a^4}{b^7(bx+a)}+\frac{3a^5}{b^7(bx+a)^2}-\frac{20a^3\ln(bx+a)}{b^7}-\frac{a^6}{3b^7(bx+a)^3}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/3*b^2*x^3-2*a*b*x^2+10*a^2*x)-15*a^4/b^7/(b*x+a)+3*a^5/b^7/(b*x+a)
^2-20*a^3*ln(b*x+a)/b^7-1/3*a^6/b^7/(b*x+a)^3
```

Maxima [A]
time = 0.27, size = 102, normalized size = 1.13

$$-\frac{45\,a^4b^2x^2+81\,a^5bx+37\,a^6}{3\,(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7)}-\frac{20\,a^3\log\left(bx+a\right)}{b^7}+\frac{b^2x^3-6\,abx^2+30\,a^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2
*b^8*x + a^3*b^7) - 20*a^3*log(b*x + a)/b^7 + 1/3*(b^2*x^3 - 6*a*b*x^2 + 30
*a^2*x)/b^6
```

Fricas [A]
time = 0.31, size = 139, normalized size = 1.54

$$\frac{b^6x^6 - 3\,ab^5x^5 + 15\,a^2b^4x^4 + 73\,a^3b^3x^3 + 39\,a^4b^2x^2 - 51\,a^5bx - 37\,a^6 - 60\,(a^3b^3x^3 + 3\,a^4b^2x^2 + 3\,a^5bx + a^6)\log{(bx + a)}}{3\,(b^{10}x^3 + 3\,ab^9x^2 + 3\,a^2b^8x + a^3b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*x^6 - 3*a*b^5*x^5 + 15*a^2*b^4*x^4 + 73*a^3*b^3*x^3 + 39*a^4*b^2*x
^2 - 51*a^5*b*x - 37*a^6 - 60*(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*x + a^
6)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

Sympy [A]
time = 0.22, size = 107, normalized size = 1.19

$$-\frac{20a^3\log{(a + bx)}}{b^7} + \frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{-37a^6 - 81a^5bx - 45a^4b^2x^2}{3a^3b^7 + 9a^2b^8x + 9ab^9x^2 + 3b^{10}x^3} + \frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**4,x)
```

```
[Out] -20*a**3*log(a + b*x)/b**7 + 10*a**2*x/b**6 - 2*a*x**2/b**5 + (-37*a**6 - 8
1*a**5*b*x - 45*a**4*b**2*x**2)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3) + x**3/(3*b**4)
```

Giac [A]
time = 0.00, size = 96, normalized size = 1.07

$$\frac{\frac{1}{3}x^3b^8 - 2x^2b^7a + 10xb^6a^2}{b^{12}} + \frac{\frac{1}{3}\left(-45b^2a^4x^2 - 81ba^5x - 37a^6\right)}{b^7\left(xb + a\right)^3} - \frac{20a^3\ln{|xb + a|}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x)
```

```
[Out] -20*a^3*log(abs(b*x + a))/b^7 - 1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/
((b*x + a)^3*b^7) + 1/3*(b^8*x^3 - 6*a*b^7*x^2 + 30*a^2*b^6*x)/b^12
```

Mupad [B]
time = 0.15, size = 79, normalized size = 0.88

$$-\frac{3\,a\,{\left(a+b\,x\right)}^2-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{15\,a^4}{a+b\,x}-\frac{3\,a^5}{{\left(a+b\,x\right)}^2}+\frac{a^6}{3\,{\left(a+b\,x\right)}^3}+20\,a^3\,\ln\left(a+b\,x\right)-15\,a^2\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^4,x)
```

```
[Out] -(3*a*(a + b*x)^2 - (a + b*x)^3/3 + (15*a^4)/(a + b*x) - (3*a^5)/(a + b*x)^
2 + a^6/(3*(a + b*x)^3) + 20*a^3*log(a + b*x) - 15*a^2*b*x)/b^7
```

3.196 $\int \frac{x^5}{(a+bx)^4} \, dx$

Optimal. Leaf size=81

$$-\frac{4ax}{b^5}+\frac{x^2}{2b^4}+\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}$$

```
[Out] -4*a*x/b^5+1/2*x^2/b^4+1/3*a^5/b^6/(b*x+a)^3-5/2*a^4/b^6/(b*x+a)^2+10*a^3/b
^6/(b*x+a)+10*a^2*ln(b*x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^4,x]
```

```
[Out] (-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a +
b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^4}\,dx &= \int \left(-\frac{4a}{b^5}+\frac{x}{b^4}-\frac{a^5}{b^5(a+bx)^4}+\frac{5a^4}{b^5(a+bx)^3}-\frac{10a^3}{b^5(a+bx)^2}+\frac{10a^2}{b^5(a+bx)}\right)dx\\ &= -\frac{4ax}{b^5}+\frac{x^2}{2b^4}+\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 0.84

$$\frac{-24abx+3b^2x^2+\frac{2a^5}{(a+bx)^3}-\frac{15a^4}{(a+bx)^2}+\frac{60a^3}{a+bx}+60a^2\log(a+bx)}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^4,x]
```

```
[Out] (-24*a*b*x + 3*b^2*x^2 + (2*a^5)/(a + b*x)^3 - (15*a^4)/(a + b*x)^2 + (60*a
^3)/(a + b*x) + 60*a^2*Log[a + b*x])/(6*b^6)
```

Mathics [A]
time = 2.54, size = 86, normalized size = 1.06

$$\frac{10a^2\mathrm{Log}\,[a+bx]}{b^6}+\frac{a^3\,(47a^2+105abx+60b^2x^2)}{6b^6\,(a^3+3a^2bx+3ab^2x^2+b^3x^3)}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5/(a + b*x)^4,x]')
```

```
[Out] 10 a ^ 2 Log[a + b x] / b ^ 6 + a ^ 3 (47 a ^ 2 + 105 a b x + 60 b ^ 2 x ^
2) / (6 b ^ 6 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) - 4 a
x / b ^ 5 + x ^ 2 / (2 b ^ 4)
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.95

method	result	size
risch	$\frac{x^2}{2b^4}-\frac{4ax}{b^5}+\frac{10a^3b\,x^2+\frac{35a^4x}{2}+\frac{47a^5}{6b}}{b^5(bx+a)^3}+\frac{10a^2\ln(bx+a)}{b^6}$	66
norman	$\frac{\frac{x^5}{2b}-\frac{5a\,x^4}{2b^2}+\frac{55a^5}{3b^6}+\frac{30a^3x^2}{b^4}+\frac{45a^4x}{b^5}}{(bx+a)^3}+\frac{10a^2\ln(bx+a)}{b^6}$	70
default	$-\frac{-\frac{1}{2}x^2b+4ax}{b^5}+\frac{10a^3}{b^6(bx+a)}-\frac{5a^4}{2b^6(bx+a)^2}+\frac{10a^2\ln(bx+a)}{b^6}+\frac{a^5}{3b^6(bx+a)^3}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/2*x^2*b+4*a*x)+10*a^3/b^6/(b*x+a)-5/2*a^4/b^6/(b*x+a)^2+10*a^2*l
n(b*x+a)/b^6+1/3*a^5/b^6/(b*x+a)^3
```

Maxima [A]
time = 0.24, size = 91, normalized size = 1.12

$$\frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\,(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6)}+\frac{10\,a^2\log\,(bx+a)}{b^6}+\frac{bx^2-8\,ax}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*a^3*b^2*x^2 + 105*a^4*b*x + 47*a^5)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*
b^7*x + a^3*b^6) + 10*a^2*log(b*x + a)/b^6 + 1/2*(b*x^2 - 8*a*x)/b^5
```

Fricas [A]

time = 0.32, size = 129, normalized size = 1.59

$$\frac{3\,b^5x^5 - 15\,ab^4x^4 - 63\,a^2b^3x^3 - 9\,a^3b^2x^2 + 81\,a^4bx + 47\,a^5 + 60\,(a^2b^3x^3 + 3\,a^3b^2x^2 + 3\,a^4bx + a^5)\log{(bx + a)}}{6\,(b^9x^3 + 3\,ab^8x^2 + 3\,a^2b^7x + a^3b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 - 15*a*b^4*x^4 - 63*a^2*b^3*x^3 - 9*a^3*b^2*x^2 + 81*a^4*b*x
 + 47*a^5 + 60*(a^2*b^3*x^3 + 3*a^3*b^2*x^2 + 3*a^4*b*x + a^5)*log(b*x + a)
)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

Sympy [A]

time = 0.21, size = 94, normalized size = 1.16

$$\frac{10a^2\log{(a + bx)}}{b^6} - \frac{4ax}{b^5} + \frac{47a^5 + 105a^4bx + 60a^3b^2x^2}{6a^3b^6 + 18a^2b^7x + 18ab^8x^2 + 6b^9x^3} + \frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**4,x)
```

```
[Out] 10*a**2*log(a + b*x)/b**6 - 4*a*x/b**5 + (47*a**5 + 105*a**4*b*x + 60*a**3*
b**2*x**2)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
x**2/(2*b**4)
```

Giac [A]

time = 0.00, size = 81, normalized size = 1.00

$$\frac{\frac{1}{2}x^2b^4 - 4xb^3a}{b^8} + \frac{\frac{1}{6}\left(60b^2a^3x^2 + 105ba^4x + 47a^5\right)}{b^6\left(xb + a\right)^3} + \frac{10a^2\ln{|xb + a|}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x)
```

```
[Out] 10*a^2*log(abs(b*x + a))/b^6 + 1/2*(b^4*x^2 - 8*a*b^3*x)/b^8 + 1/6*(60*a^3*
b^2*x^2 + 105*a^4*b*x + 47*a^5)/((b*x + a)^3*b^6)
```

Mupad [B]

time = 0.12, size = 66, normalized size = 0.81

$$\frac{\frac{(a+b\,x)^2}{2} + \frac{10\,a^3}{a+b\,x} - \frac{5\,a^4}{2\,(a+b\,x)^2} + \frac{a^5}{3\,(a+b\,x)^3} + 10\,a^2\,\ln{(a + b\,x)} - 5\,a\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^2/2 + (10*a^3)/(a + b*x) - (5*a^4)/(2*(a + b*x)^2) + a^5/(3*(a +
 b*x)^3) + 10*a^2*log(a + b*x) - 5*a*b*x)/b^6
```

3.197 $\int \frac{x^4}{(a+bx)^4} dx$

Optimal. Leaf size=65

$$\frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}$$

```
[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*
x+a)/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^4,x]
```

```
[Out] x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5*
(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^4}\,dx &= \int \left(\frac{1}{b^4}+\frac{a^4}{b^4(a+bx)^4}-\frac{4a^3}{b^4(a+bx)^3}+\frac{6a^2}{b^4(a+bx)^2}-\frac{4a}{b^4(a+bx)}\right)dx\\ &= \frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.78

$$-\frac{-3bx+\frac{a^2(13a^2+30abx+18b^2x^2)}{(a+bx)^3}+12a\log(a+bx)}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^4,x]
```

```
[Out] -1/3*(-3*b*x + (a^2*(13*a^2 + 30*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 12*a*Lo
g[a + b*x])/b^5
```

Mathics [A]
time = 2.43, size = 74, normalized size = 1.14

$$\frac{-4a\text{Log}\left[a+bx\right]}{b^5}-\frac{a^2\left(13a^2+30abx+18b^2x^2\right)}{3b^5\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4/(a + b*x)^4,x]')
```

```
[Out] -4 a Log[a + b x] / b ^ 5 - a ^ 2 (13 a ^ 2 + 30 a b x + 18 b ^ 2 x ^ 2) /
(3 b ^ 5 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) + x / b ^ 4
```

Maple [A]
time = 0.08, size = 64, normalized size = 0.98

method	result	size
risch	$\frac{x}{b^4}+\frac{-6a^2b\,x^2-10a^3x-\frac{13a^4}{3b}}{b^4(bx+a)^3}-\frac{4a\ln(bx+a)}{b^5}$	54
norman	$\frac{\frac{x^4}{b}-\frac{22a^4}{3b^5}-\frac{12a^2x^2}{b^3}-\frac{18a^3x}{b^4}}{(bx+a)^3}-\frac{4a\ln(bx+a)}{b^5}$	58
default	$\frac{x}{b^4}-\frac{a^4}{3b^5(bx+a)^3}+\frac{2a^3}{b^5(bx+a)^2}-\frac{6a^2}{b^5(bx+a)}-\frac{4a\ln(bx+a)}{b^5}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*
x+a)/b^5
```

Maxima [A]
time = 0.26, size = 79, normalized size = 1.22

$$-\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\left(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5\right)}+\frac{x}{b^4}-\frac{4\,a\log\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a^4)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*
b^6*x + a^3*b^5) + x/b^4 - 4*a*log(b*x + a)/b^5
```

Fricas [A]
time = 0.31, size = 116, normalized size = 1.78

$$\frac{3\,b^4x^4+9\,ab^3x^3-9\,a^2b^2x^2-27\,a^3bx-13\,a^4-12\,(ab^3x^3+3\,a^2b^2x^2+3\,a^3bx+a^4)\log{(bx+a)}}{3\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 9*a*b^3*x^3 - 9*a^2*b^2*x^2 - 27*a^3*b*x - 13*a^4 - 12*(a*
b^3*x^3 + 3*a^2*b^2*x^2 + 3*a^3*b*x + a^4)*log(b*x + a))/(b^8*x^3 + 3*a*b^7
*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [A]
time = 0.20, size = 82, normalized size = 1.26

$$-\frac{4a\log{(a+bx)}}{b^5}+\frac{-13a^4-30a^3bx-18a^2b^2x^2}{3a^3b^5+9a^2b^6x+9ab^7x^2+3b^8x^3}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**4,x)
```

```
[Out] -4*a*log(a + b*x)/b**5 + (-13*a**4 - 30*a**3*b*x - 18*a**2*b**2*x**2)/(3*a*
*3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + x/b**4
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.00

$$\frac{x}{b^4}+\frac{\frac{1}{3}\,(-18b^2a^2x^2-30ba^3x-13a^4)}{b^5\,(xb+a)^3}-\frac{4a\ln{|xb+a|}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x)
```

```
[Out] x/b^4 - 4*a*log(abs(b*x + a))/b^5 - 1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a
^4)/((b*x + a)^3*b^5)
```

Mupad [B]
time = 0.17, size = 55, normalized size = 0.85

$$-\frac{4\,a\,\ln{(a+b\,x)}-b\,x+\frac{6\,a^2}{a+b\,x}-\frac{2\,a^3}{(a+b\,x)^2}+\frac{a^4}{3\,(a+b\,x)^3}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^4,x)
```

```
[Out] -(4*a*log(a + b*x) - b*x + (6*a^2)/(a + b*x) - (2*a^3)/(a + b*x)^2 + a^4/(3
*(a + b*x)^3))/b^5
```

3.198 $\int \frac{x^3}{(a+bx)^4} \, dx$

Optimal. Leaf size=58

$$\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

```
[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^4,x]
```

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^4}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^4}+\frac{3a^2}{b^3(a+bx)^3}-\frac{3a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)dx\\ &= \frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.76

$$\frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3}+6\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^4,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4)
```

Mathics [A]
time = 2.32, size = 90, normalized size = 1.55

$$\frac{\frac{a(11a^2+27abx+18b^2x^2)}{6}+\mathrm{Log}\left[a+bx\right](a^3+3a^2bx+3ab^2x^2+b^3x^3)}{b^4\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(a + b*x)^4,x]')
```

```
[Out] (a (11 a ^ 2 + 27 a b x + 18 b ^ 2 x ^ 2) / 6 + Log[a + b x] (a ^ 3 + 3 a ^
 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) / (b ^ 4 (a ^ 3 + 3 a ^ 2 b x + 3
a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 55, normalized size = 0.95

method	result	size
norman	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
risch	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
default	$\frac{a^3}{3b^4(bx+a)^3}-\frac{3a^2}{2b^4(bx+a)^2}+\frac{3a}{b^4(bx+a)}+\frac{\ln(bx+a)}{b^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Maxima [A]
time = 0.27, size = 70, normalized size = 1.21

$$\frac{18\,ab^2x^2+27\,a^2bx+11\,a^3}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}+\frac{\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5
*x + a^3*b^4) + log(b*x + a)/b^4
```

Fricas [A]
time = 0.29, size = 94, normalized size = 1.62

$$\frac{18\,ab^{2}x^{2}+27\,a^{2}bx+11\,a^{3}+6\,(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3})\log\left(bx+a\right)}{6\,(b^{7}x^{3}+3\,ab^{6}x^{2}+3\,a^{2}b^{5}x+a^{3}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A]
time = 0.15, size = 70, normalized size = 1.21

$$\frac{11a^{3}+27a^{2}bx+18ab^{2}x^{2}}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}}+\frac{\log\left(a+bx\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

Giac [A]
time = 0.00, size = 50, normalized size = 0.86

$$\frac{\frac{1}{6}\left(18bax^{2}+27a^{2}x+\frac{11a^{3}}{b}\right)}{b^{3}\left(xb+a\right)^{3}}+\frac{\ln|xb+a|}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x)
```

```
[Out] log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^3
*b^3)
```

Mupad [B]
time = 0.07, size = 45, normalized size = 0.78

$$\frac{\ln\left(a+b\,x\right)+\frac{3\,a}{a+b\,x}-\frac{3\,a^{2}}{2\,{\left(a+b\,x\right)}^{2}}+\frac{a^{3}}{3\,{\left(a+b\,x\right)}^{3}}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^4,x)
```

```
[Out] (log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a + b*x
)^3))/b^4
```

3.199 $\int \frac{x^2}{(a+bx)^4} \, dx$

Optimal. Leaf size=17

$$\frac{x^3}{3a(a+bx)^3}$$

[Out] 1/3*x^3/a/(b*x+a)^3

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$\frac{x^3}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*x)^4,x]

[Out] x^3/(3*a*(a + b*x)^3)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx)^4} \, dx = \frac{x^3}{3a(a+bx)^3}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.82

$$-\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x)^4,x]

[Out] -1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 2.09, size = 52, normalized size = 3.06

$$\frac{-\frac{a^2}{3} - abx - b^2x^2}{b^3 (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(a + b*x)^4,x]')
```

```
[Out] (-a ^ 2 / 3 - a b x - b ^ 2 x ^ 2) / (b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^
2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.
time = 0.09, size = 41, normalized size = 2.41

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3(bx+a)^3b^3}$	30
norman	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
risch	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
default	$-\frac{1}{b^3(bx+a)}+\frac{a}{b^3(bx+a)^2}-\frac{a^2}{3b^3(bx+a)^3}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3/(b*x+a)+a/b^3/(b*x+a)^2-1/3/b^3*a^2/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.
time = 0.25, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,(b^6x^3 + 3\,ab^5x^2 + 3\,a^2b^4x + a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.

time = 0.31, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(12) = 24$.
time = 0.13, size = 56, normalized size = 3.29

$$\frac{-a^2-3abx-3b^2x^2}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**
2 + 3*b**6*x**3)
```

Giac [A]
time = 0.00, size = 34, normalized size = 2.00

$$\frac{-3x^2b^2-3xba-a^2}{3b^3\,(xb+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x)
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

Mupad [B]
time = 0.09, size = 56, normalized size = 3.29

$$-\frac{a^2+3\,a\,b\,x+3\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^4,x)
```

```
[Out] -(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5
*x^2)
```

3.200 $\int \frac{x}{(a+bx)^4}\,dx$

Optimal. Leaf size=30

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

```
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^4,x]
```

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^4}\,dx &= \int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right)dx\\ &= \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.67

$$-\frac{a+3bx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

Mathics [A]
time = 2.01, size = 42, normalized size = 1.40

$$\frac{-a-3bx}{6b^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1/(a + b*x)^4,x]')
```

```
[Out] (-a - 3 b x) / (6 b ^ 2 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^
3))
```

Maple [A]
time = 0.08, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{3bx+a}{6(bx+a)^3b^2}$	19
norman	$\frac{-\frac{x}{2b}-\frac{a}{6b^2}}{(bx+a)^3}$	22
risch	$\frac{-\frac{x}{2b}-\frac{a}{6b^2}}{(bx+a)^3}$	22
default	$\frac{a}{3b^2(bx+a)^3}-\frac{1}{2b^2(bx+a)^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Maxima [A]
time = 0.24, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Fricas [A]
time = 0.30, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Sympy [A]
time = 0.12, size = 44, normalized size = 1.47

$$\frac{-a-3bx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**4,x)
```

```
[Out] (-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

Giac [A]
time = 0.00, size = 22, normalized size = 0.73

$$\frac{-3xb-a}{6b^2\left(xb+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x)
```

```
[Out] -1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

Mupad [B]
time = 0.07, size = 44, normalized size = 1.47

$$-\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^4,x)
```

```
[Out] -(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.201 $\int \frac{1}{(a+bx)^4} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(a+bx)^3}$$

```
[Out] -1/3/b/(b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4),x]
```

```
[Out] -1/3*1/(b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4} \, dx = -\frac{1}{3b(a+bx)^3}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4),x]
```

```
[Out] -1/3*1/(b*(a + b*x)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.
time = 1.95, size = 34, normalized size = 2.43

$$-\frac{1}{3b\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^4,x]')
[Out] -1 / (3 b (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{3b(bx+a)^{3}}$	13
default	$-\frac{1}{3b(bx+a)^{3}}$	13
norman	$-\frac{1}{3b(bx+a)^{3}}$	13
risch	$-\frac{1}{3b(bx+a)^{3}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/3/b/(b*x+a)^3
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(bx+a)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="maxima")
[Out] -1/3/((b*x + a)^3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.
time = 0.31, size = 35, normalized size = 2.50

$$-\frac{1}{3\,(b^{4}x^{3}+3\,ab^{3}x^{2}+3\,a^{2}b^{2}x+a^{3}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.
time = 0.12, size = 37, normalized size = 2.64

$$-\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4,x)
```

```
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{3b\,(xb + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x)
```

```
[Out] -1/3/((b*x + a)^3*b)
```

Mupad [B]
time = 0.08, size = 37, normalized size = 2.64

$$-\frac{1}{3\,a^3\,b + 9\,a^2\,b^2\,x + 9\,a\,b^3\,x^2 + 3\,b^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^4,x)
```

```
[Out] -1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

3.202 $\int \frac{1}{x(a+bx)^4}\,dx$

Optimal. Leaf size=57

$$\frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}$$

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^4),x]
```

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x}-\frac{b}{a(a+bx)^4}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^4(a+bx)}\right)dx\\ &= \frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.84

$$\frac{\frac{a(11a^2+15abx+6b^2x^2)}{(a+bx)^3}+6\log(x)-6\log(a+bx)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^4),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*x
])/(6*a^4)
```

Mathics [A]
time = 2.52, size = 99, normalized size = 1.74

$$\frac{\frac{a(11a^2+15abx+6b^2x^2)}{6}+(a^3+3a^2bx+3ab^2x^2+b^3x^3)\left(\text{Log}\,[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^4\,(a^3+3a^2bx+3ab^2x^2+b^3x^3)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)^4),x]')
```

```
[Out] (a (11 a ^ 2 + 15 a b x + 6 b ^ 2 x ^ 2) / 6 + (a ^ 3 + 3 a ^ 2 b x + 3 a b
 ^ 2 x ^ 2 + b ^ 3 x ^ 3) (Log[x] - Log[(a + b x) / b])) / (a ^ 4 (a ^ 3 +
3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.95

method	result	size
risch	$\frac{\frac{b^2x^2}{a^3}+\frac{5bx}{2a^2}+\frac{11}{6a}}{(bx+a)^3}+\frac{\ln(-x)}{a^4}-\frac{\ln(bx+a)}{a^4}$	52
default	$\frac{1}{3a(bx+a)^3}+\frac{1}{2a^2(bx+a)^2}+\frac{1}{a^3(bx+a)}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$	54
norman	$\frac{-\frac{3bx}{a^2}-\frac{9b^2x^2}{2a^3}-\frac{11b^3x^3}{6a^4}}{(bx+a)^3}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Maxima [A]
time = 0.30, size = 73, normalized size = 1.28

$$\frac{6\,b^2x^2+15\,abx+11\,a^2}{6\,(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6)}-\frac{\log{(bx+a)}}{a^4}+\frac{\log{(x)}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*
x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.
time = 0.31, size = 124, normalized size = 2.18

$$\frac{6\,ab^2x^2 + 15\,a^2bx + 11\,a^3 - 6\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\log(bx + a) + 6\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\log(x)}{6\,(a^4b^3x^3 + 3\,a^5b^2x^2 + 3\,a^6bx + a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(x)
)/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

Sympy [A]
time = 0.19, size = 70, normalized size = 1.23

$$\frac{11a^2 + 15abx + 6b^2x^2}{6a^6 + 18a^5bx + 18a^4b^2x^2 + 6a^3b^3x^3} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**4,x)
```

```
[Out] (11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x**
2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.11

$$\frac{\ln|x|}{a^4} - \frac{b\ln|xb + a|}{ba^4} + \frac{\frac{1}{6}\,(6b^2ax^2 + 15ba^2x + 11a^3)}{a^4\,(xb + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x)
```

```
[Out] -log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
11*a^3)/((b*x + a)^3*a^4)
```

Mupad [B]
time = 0.13, size = 60, normalized size = 1.05

$$\frac{\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,{\left(a+b\,x\right)}^2}}{a}+\frac{1}{3\,a\,{\left(a+b\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^4),x)
```

```
[Out] ((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(3
*a*(a + b*x)^3)
```

3.203 $\int \frac{1}{x^2(a+bx)^4} dx$

Optimal. Leaf size=70

$$-\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}$$

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^2}-\frac{4b}{a^5x}+\frac{b^2}{a^2(a+bx)^4}+\frac{2b^2}{a^3(a+bx)^3}+\frac{3b^2}{a^4(a+bx)^2}+\frac{4b^2}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.91

$$-\frac{\frac{a(3a^3+22a^2bx+30ab^2x^2+12b^3x^3)}{x(a+bx)^3}+12b\log(x)-12b\log(a+bx)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
+ 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

Mathics [A]
time = 2.71, size = 116, normalized size = 1.66

$$\frac{a\left(-3a^3-22a^2bx-30ab^2x^2-12b^3x^3\right)-12bx\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{3a^5x\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^4),x]')
```

```
[Out] (a (-3 a ^ 3 - 22 a ^ 2 b x - 30 a b ^ 2 x ^ 2 - 12 b ^ 3 x ^ 3) - 12 b x (
a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (Log[x] - Log[(a + b x
) / b])) / (3 a ^ 5 x (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)
)
```

Maple [A]
time = 0.09, size = 69, normalized size = 0.99

method	result	size
default	$-\frac{1}{a^4x}-\frac{b}{3a^2(bx+a)^3}-\frac{b}{a^3(bx+a)^2}-\frac{3b}{a^4(bx+a)}-\frac{4b\ln(x)}{a^5}+\frac{4b\ln(bx+a)}{a^5}$	69
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{10b^2x^2}{a^3}-\frac{22bx}{3a^2}-\frac{1}{a}}{x(bx+a)^3}+\frac{4b\ln(-bx-a)}{a^5}-\frac{4b\ln(x)}{a^5}$	71
norman	$\frac{-\frac{1}{a}+\frac{12b^2x^2}{a^3}+\frac{18b^3x^3}{a^4}+\frac{22b^4x^4}{3a^5}}{x(bx+a)^3}-\frac{4b\ln(x)}{a^5}+\frac{4b\ln(bx+a)}{a^5}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.30

$$-\frac{12\,b^3x^3+30\,ab^2x^2+22\,a^2bx+3\,a^3}{3\left(a^4b^3x^4+3\,a^5b^2x^3+3\,a^6bx^2+a^7x\right)}+\frac{4\,b\log\left(bx+a\right)}{a^5}-\frac{4\,b\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5*
b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.
time = 0.31, size = 153, normalized size = 2.19

$$-\frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4-12\,(b^4x^4+3\,ab^3x^3+3\,a^2b^2x^2+a^3bx)\log\left(bx+a\right)+12\,(b^4x^4+3\,ab^3x^3+3\,a^2b^2x^2+a^3bx)\log\left(x\right)}{3\,(a^5b^3x^4+3\,a^6b^2x^3+3\,a^7bx^2+a^8x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3*
a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3*x
^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*
b*x^2 + a^8*x)
```

Sympy [A]
time = 0.24, size = 90, normalized size = 1.29

$$\frac{-3a^3-22a^2bx-30ab^2x^2-12b^3x^3}{3a^7x+9a^6bx^2+9a^5b^2x^3+3a^4b^3x^4}+\frac{4b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**4,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6*
b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b + x)
)/a**5
```

Giac [A]
time = 0.00, size = 85, normalized size = 1.21

$$-\frac{4b\ln|x|}{a^5}+\frac{4b^2\ln|xb+a|}{ba^5}+\frac{\frac{1}{3}\left(-12b^3ax^3-30b^2a^2x^2-22ba^3x-3a^4\right)}{a^5x\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x)
```

```
[Out] 4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a^
2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

Mupad [B]
time = 0.08, size = 85, normalized size = 1.21

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{a}+\frac{10\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}+\frac{22\,b\,x}{3\,a^2}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^4),x)
```

```
[Out] (8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
+ (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

3.204 $\int \frac{1}{x^3(a+bx)^4}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(a+bx)^3}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{6b^2}{a^5(a+bx)}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}$$

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{6b^2}{a^5(a+bx)}+\frac{4b}{a^5x}+\frac{3b^2}{2a^4(a+bx)^2}-\frac{1}{2a^4x^2}+\frac{b^2}{3a^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^4),x]
```

```
[Out] -1/2*1/(a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4
*(a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log
[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^3}-\frac{4b}{a^5x^2}+\frac{10b^2}{a^6x}-\frac{b^3}{a^3(a+bx)^4}-\frac{3b^3}{a^4(a+bx)^3}-\frac{6b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}\right) \\ &= -\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(a+bx)^3}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{6b^2}{a^5(a+bx)}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log}{a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.85

$$\frac{\frac{a\left(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4\right)}{x^2(a+bx)^3}+60b^2\log(x)-60b^2\log(a+bx)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/(
x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

Mathics [A]
time = 2.91, size = 131, normalized size = 1.41

$$\frac{a\left(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4\right)+60b^2x^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{6a^6x^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^4),x]')
```

```
[Out] (a (-3 a ^ 4 + 15 a ^ 3 b x + 110 a ^ 2 b ^ 2 x ^ 2 + 150 a b ^ 3 x ^ 3 + 6
0 b ^ 4 x ^ 4) + 60 b ^ 2 x ^ 2 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b
^ 3 x ^ 3) (Log[x] - Log[(a + b x) / b])) / (6 a ^ 6 x ^ 2 (a ^ 3 + 3 a ^ 2
 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 88, normalized size = 0.95

method	result	size
norman	$\frac{-\frac{1}{2a}+\frac{5bx}{2a^2}-\frac{30b^3x^3}{a^4}-\frac{45b^4x^4}{a^5}-\frac{55b^5x^5}{3a^6}}{x^2(bx+a)^3}+\frac{10b^2\ln(x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$	83
risch	$\frac{\frac{10b^4x^4}{a^5}+\frac{25b^3x^3}{a^4}+\frac{55b^2x^2}{3a^3}+\frac{5bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)^3}+\frac{10b^2\ln(-x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$	85
default	$-\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(bx+a)^3}+\frac{3b^2}{2a^4(bx+a)^2}+\frac{6b^2}{a^5(bx+a)}+\frac{10b^2\ln(x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.27, size = 108, normalized size = 1.16

$$\frac{60\,b^4x^4+150\,ab^3x^3+110\,a^2b^2x^2+15\,a^3bx-3\,a^4}{6\left(a^5b^3x^5+3\,a^6b^2x^4+3\,a^7bx^3+a^8x^2\right)}-\frac{10\,b^2\log\left(bx+a\right)}{a^6}+\frac{10\,b^2\log\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a^
5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/a^
6 + 10*b^2*log(x)/a^6
```

Fricas [A]
time = 0.31, size = 174, normalized size = 1.87

$$\frac{60\,ab^4x^4 + 150\,a^2b^3x^3 + 110\,a^3b^2x^2 + 15\,a^4bx - 3\,a^5 - 60\,(b^5x^5 + 3\,ab^4x^4 + 3\,a^2b^3x^3 + a^3b^2x^2)\log\left(bx + a\right) + 60\,(b^5x^5 + 3\,ab^4x^4 + 3\,a^2b^3x^3 + a^3b^2x^2)\log\left(x\right)}{6\,(a^6b^3x^5 + 3\,a^7b^2x^4 + 3\,a^8bx^3 + a^9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
- 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) + 6
0*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3*x^
5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

Sympy [A]
time = 0.25, size = 104, normalized size = 1.12

$$\frac{-3a^4 + 15a^3bx + 110a^2b^2x^2 + 150ab^3x^3 + 60b^4x^4}{6a^8x^2 + 18a^7bx^3 + 18a^6b^2x^4 + 6a^5b^3x^5} + \frac{10b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**4,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x**
4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5) +
10*b**2*(log(x) - log(a/b + x))/a**6
```

Giac [A]
time = 0.00, size = 97, normalized size = 1.04

$$\frac{10b^2\ln|x|}{a^6} - \frac{10b^3\ln|xb + a|}{ba^6} + \frac{\frac{1}{6}\left(60b^4ax^4 + 150b^3a^2x^3 + 110b^2a^3x^2 + 15ba^4x - 3a^5\right)}{a^6x^2\left(bx + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x)
```

```
[Out] -10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
+ 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^6*
x^2)
```

Mupad [B]
time = 0.14, size = 101, normalized size = 1.09

$$\frac{\frac{55\,b^2\,x^2}{3\,a^3} - \frac{1}{2\,a} + \frac{25\,b^3\,x^3}{a^4} + \frac{10\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{2\,a^2}}{a^3\,x^2 + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^4 + b^3\,x^5} - \frac{20\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^4),x)
```

```
[Out] ((55*b^2*x^2)/(3*a^3) - 1/(2*a) + (25*b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 + (5*
b*x)/(2*a^2))/(a^3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (20*b^2*ata
nh((2*b*x)/a + 1))/a^6
```

3.205 $\int \frac{1}{x^4(a+bx)^4}\,dx$

Optimal. Leaf size=102

$$-\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}$$

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}-\frac{10b^3}{a^6(a+bx)}-\frac{10b^2}{a^6x}-\frac{2b^3}{a^5(a+bx)^2}+\frac{2b}{a^5x^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/3*1/(a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*x
)^3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log[x
])/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^4}-\frac{4b}{a^5x^3}+\frac{10b^2}{a^6x^2}-\frac{20b^3}{a^7x}+\frac{b^4}{a^4(a+bx)^4}+\frac{4b^4}{a^5(a+bx)^3}+\frac{10b^4}{a^6(a+bx)^2}+\frac{20\cdots}{a^7(a\cdots}\right.\\ &= -\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\cdots\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.86

$$-\frac{\frac{a(a^5-3a^4bx+15a^3b^2x^2+110a^2b^3x^3+150ab^4x^4+60b^5x^5)}{x^3(a+bx)^3}+60b^3\log(x)-60b^3\log(a+bx)}{3a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(a^5 - 3*a^4*b*x + 15*a^3*b^2*x^2 + 110*a^2*b^3*x^3 + 150*a*b^4*x^
4 + 60*b^5*x^5))/(x^3*(a + b*x)^3) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a
^7
```

Mathics [A]
time = 2.96, size = 142, normalized size = 1.39

$$\frac{a\left(-a^5+3a^4bx-15a^3b^2x^2-110a^2b^3x^3-150ab^4x^4-60b^5x^5\right)-60b^3x^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{3a^7x^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(a + b*x)^4),x]')
```

```
[Out] (a (-a ^ 5 + 3 a ^ 4 b x - 15 a ^ 3 b ^ 2 x ^ 2 - 110 a ^ 2 b ^ 3 x ^ 3 - 1
50 a b ^ 4 x ^ 4 - 60 b ^ 5 x ^ 5) - 60 b ^ 3 x ^ 3 (a ^ 3 + 3 a ^ 2 b x +
3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (Log[x] - Log[(a + b x) / b])) / (3 a ^ 7 x
^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 99, normalized size = 0.97

method	result	size
norman	$\frac{\frac{bx}{a^2}-\frac{1}{3a}-\frac{5b^2x^2}{a^3}+\frac{60b^4x^4}{a^5}+\frac{90b^5x^5}{a^6}+\frac{110b^6x^6}{3a^7}}{x^3(bx+a)^3}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln(bx+a)}{a^7}$	93
risch	$\frac{-\frac{20b^5x^5}{a^6}-\frac{50b^4x^4}{a^5}-\frac{110b^3x^3}{3a^4}-\frac{5b^2x^2}{a^3}+\frac{bx}{a^2}-\frac{1}{3a}}{x^3(bx+a)^3}+\frac{20b^3\ln(-bx-a)}{a^7}-\frac{20b^3\ln(x)}{a^7}$	96
default	$-\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(bx+a)^3}-\frac{2b^3}{a^5(bx+a)^2}-\frac{10b^3}{a^6(bx+a)}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln(bx+a)}{a^7}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Maxima [A]
time = 0.25, size = 117, normalized size = 1.15

$$-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\,\left(a^6b^3x^6+3\,a^7b^2x^5+3\,a^8bx^4+a^9x^3\right)}+\frac{20\,b^3\log\left(bx+a\right)}{a^7}-\frac{20\,b^3\log\left(x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(60*b^5*x^5 + 150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4
*b*x + a^5)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*x^3) + 20*b^3*
log(b*x + a)/a^7 - 20*b^3*log(x)/a^7
```

Fricas [A]
time = 0.31, size = 183, normalized size = 1.79

$$-\frac{60\,ab^5x^5+150\,a^2b^4x^4+110\,a^3b^3x^3+15\,a^4b^2x^2-3\,a^5bx+a^6-60\,(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3)\log\left(bx+a\right)+60\,(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3)\log\left(x\right)}{3\,(a^7b^3x^6+3\,a^8b^2x^5+3\,a^9bx^4+a^{10}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(60*a*b^5*x^5 + 150*a^2*b^4*x^4 + 110*a^3*b^3*x^3 + 15*a^4*b^2*x^2 - 3
*a^5*b*x + a^6 - 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*l
og(b*x + a) + 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*log(
x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

Sympy [A]
time = 0.27, size = 114, normalized size = 1.12

$$\frac{-a^5+3a^4bx-15a^3b^2x^2-110a^2b^3x^3-150ab^4x^4-60b^5x^5}{3a^9x^3+9a^8bx^4+9a^7b^2x^5+3a^6b^3x^6}+\frac{20b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**4,x)
```

```
[Out] (-a**5 + 3*a**4*b*x - 15*a**3*b**2*x**2 - 110*a**2*b**3*x**3 - 150*a*b**4*x
**4 - 60*b**5*x**5)/(3*a**9*x**3 + 9*a**8*b*x**4 + 9*a**7*b**2*x**5 + 3*a**
6*b**3*x**6) + 20*b**3*(-log(x) + log(a/b + x))/a**7
```

Giac [A]
time = 0.00, size = 105, normalized size = 1.03

$$-\frac{20b^3\ln|x|}{a^7}+\frac{20b^4\ln|xb+a|}{ba^7}+\frac{-60x^5b^5-150x^4b^4a-110x^3b^3a^2-15x^2b^2a^3+3xba^4-a^5}{3a^6\left(x^2b+xa\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x)
```

```
[Out] 20*b^3*log(abs(b*x + a))/a^7 - 20*b^3*log(abs(x))/a^7 - 1/3*(60*b^5*x^5 + 1
50*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4*b*x + a^5)/((b*x^2
+ a*x)^3*a^6)
```

Mupad [B]
time = 0.10, size = 113, normalized size = 1.11

$$\frac{40\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}-\frac{\frac{1}{3\,a}+\frac{5\,b^2\,x^2}{a^3}+\frac{110\,b^3\,x^3}{3\,a^4}+\frac{50\,b^4\,x^4}{a^5}+\frac{20\,b^5\,x^5}{a^6}-\frac{b\,x}{a^2}}{a^3\,x^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^5+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^4),x)
```

```
[Out] (40*b^3*atanh((2*b*x)/a + 1))/a^7 - (1/(3*a) + (5*b^2*x^2)/a^3 + (110*b^3*x
^3)/(3*a^4) + (50*b^4*x^4)/a^5 + (20*b^5*x^5)/a^6 - (b*x)/a^2)/(a^3*x^3 + b
^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^5)
```

3.206 $\int \frac{1}{x^5(a+bx)^4}\,dx$

Optimal. Leaf size=117

$$-\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}$$

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{15b^4}{a^7(a+bx)}+\frac{20b^3}{a^7x}+\frac{5b^4}{2a^6(a+bx)^2}-\frac{5b^2}{a^6x^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^4),x]
```

```
[Out] -1/4*1/(a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x)
 + b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*(a
 + b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^5}-\frac{4b}{a^5x^4}+\frac{10b^2}{a^6x^3}-\frac{20b^3}{a^7x^2}+\frac{35b^4}{a^8x}-\frac{b^5}{a^5(a+bx)^4}-\frac{5b^5}{a^6(a+bx)^3}-\frac{15b^5}{a^7(a+bx)^2}\right.\\ &= -\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.86

$$\frac{\frac{a(-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6)}{x^4(a+bx)^3}+420b^4\log(x)-420b^4\log(a+bx)}{12a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^6 + 7*a^5*b*x - 21*a^4*b^2*x^2 + 105*a^3*b^3*x^3 + 770*a^2*b^4*x^
4 + 1050*a*b^5*x^5 + 420*b^6*x^6))/(x^4*(a + b*x)^3) + 420*b^4*Log[x] - 420
*b^4*Log[a + b*x])/(12*a^8)
```

Mathics [A]
time = 3.11, size = 153, normalized size = 1.31

$$\frac{a\left(-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6\right)+420b^4x^4\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(\mathrm{Log}\,[x]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{12a^8x^4\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(a + b*x)^4),x]')
```

```
[Out] (a (-3 a ^ 6 + 7 a ^ 5 b x - 21 a ^ 4 b ^ 2 x ^ 2 + 105 a ^ 3 b ^ 3 x ^ 3 +
 770 a ^ 2 b ^ 4 x ^ 4 + 1050 a b ^ 5 x ^ 5 + 420 b ^ 6 x ^ 6) + 420 b ^ 4
x ^ 4 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (Log[x] - Log[(
a + b x) / b])) / (12 a ^ 8 x ^ 4 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 +
b ^ 3 x ^ 3))
```

Maple [A]
time = 0.09, size = 110, normalized size = 0.94

method	result	size
norman	$\frac{-\frac{1}{4a}+\frac{7bx}{12a^2}-\frac{7b^2x^2}{4a^3}+\frac{35b^3x^3}{4a^4}-\frac{105b^5x^5}{a^6}-\frac{315b^6x^6}{2a^7}-\frac{385b^7x^7}{6a^8}}{x^4(bx+a)^3}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$	105
risch	$\frac{\frac{35b^6x^6}{a^7}+\frac{175b^5x^5}{2a^6}+\frac{385b^4x^4}{6a^5}+\frac{35b^3x^3}{4a^4}-\frac{7b^2x^2}{4a^3}+\frac{7bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)^3}+\frac{35b^4\ln(-x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$	107
default	$-\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(bx+a)^3}+\frac{5b^4}{2a^6(bx+a)^2}+\frac{15b^4}{a^7(bx+a)}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

Maxima [A]
time = 0.27, size = 130, normalized size = 1.11

$$\frac{420\,b^6x^6+1050\,ab^5x^5+770\,a^2b^4x^4+105\,a^3b^3x^3-21\,a^4b^2x^2+7\,a^5bx-3\,a^6}{12\left(a^7b^3x^7+3\,a^8b^2x^6+3\,a^9bx^5+a^{10}x^4\right)}-\frac{35\,b^4\log\left(bx+a\right)}{a^8}+\frac{35\,b^4\log\left(x\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(420*b^6*x^6 + 1050*a*b^5*x^5 + 770*a^2*b^4*x^4 + 105*a^3*b^3*x^3 - 21
*a^4*b^2*x^2 + 7*a^5*b*x - 3*a^6)/(a^7*b^3*x^7 + 3*a^8*b^2*x^6 + 3*a^9*b*x^
5 + a^10*x^4) - 35*b^4*log(b*x + a)/a^8 + 35*b^4*log(x)/a^8
```

Fricas [**A**]
time = 0.31, size = 196, normalized size = 1.68

$$\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7-420\,(b^7x^7+3\,ab^6x^6+3\,a^2b^5x^5+a^3b^4x^4)\log(bx+a)+420\,(b^7x^7+3\,ab^6x^6+3\,a^2b^5x^5+a^3b^4x^4)\log(x)}{12\,(a^8b^3x^7+3\,a^9b^2x^6+3\,a^{10}bx^5+a^{11}x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(420*a*b^6*x^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3
- 21*a^5*b^2*x^2 + 7*a^6*b*x - 3*a^7 - 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b
^5*x^5 + a^3*b^4*x^4)*log(b*x + a) + 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b^5
*x^5 + a^3*b^4*x^4)*log(x))/(a^8*b^3*x^7 + 3*a^9*b^2*x^6 + 3*a^10*b*x^5 + a
^11*x^4)
```

Sympy [**A**]
time = 0.28, size = 128, normalized size = 1.09

$$\frac{-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6}{12a^{10}x^4+36a^9bx^5+36a^8b^2x^6+12a^7b^3x^7}+\frac{35b^4\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**4,x)
```

```
[Out] (-3*a**6 + 7*a**5*b*x - 21*a**4*b**2*x**2 + 105*a**3*b**3*x**3 + 770*a**2*b
**4*x**4 + 1050*a*b**5*x**5 + 420*b**6*x**6)/(12*a**10*x**4 + 36*a**9*b*x**
5 + 36*a**8*b**2*x**6 + 12*a**7*b**3*x**7) + 35*b**4*(log(x) - log(a/b + x)
)/a**8
```

Giac [**A**]
time = 0.00, size = 120, normalized size = 1.03

$$-\frac{35b^5\ln|xb+a|}{ba^8}+\frac{35b^4\ln|x|}{a^8}+\frac{\frac{1}{12}\left(420b^6ax^6+1050b^5a^2x^5+770b^4a^3x^4+105b^3a^4x^3-21b^2a^5x^2+7ba^6x-3a^7\right)}{a^8x^4\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x)
```

```
[Out] -35*b^4*log(abs(b*x + a))/a^8 + 35*b^4*log(abs(x))/a^8 + 1/12*(420*a*b^6*x^
6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3 - 21*a^5*b^2*x^2 +
 7*a^6*b*x - 3*a^7)/((b*x + a)^3*a^8*x^4)
```

Mupad [B]

time = 0.17, size = 123, normalized size = 1.05

$$\frac{\frac{35\,b^3\,x^3}{4\,a^4}-\frac{7\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{385\,b^4\,x^4}{6\,a^5}+\frac{175\,b^5\,x^5}{2\,a^6}+\frac{35\,b^6\,x^6}{a^7}+\frac{7\,b\,x}{12\,a^2}}{a^3\,x^4+3\,a^2\,b\,x^5+3\,a\,b^2\,x^6+b^3\,x^7}-\frac{70\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^4),x)
```

```
[Out] ((35*b^3*x^3)/(4*a^4) - (7*b^2*x^2)/(4*a^3) - 1/(4*a) + (385*b^4*x^4)/(6*a^
5) + (175*b^5*x^5)/(2*a^6) + (35*b^6*x^6)/a^7 + (7*b*x)/(12*a^2))/(a^3*x^4
+ b^3*x^7 + 3*a^2*b*x^5 + 3*a*b^2*x^6) - (70*b^4*atanh((2*b*x)/a + 1))/a^8
```

3.207 $\int \frac{x^{10}}{(a+bx)^7}\,dx$

Optimal. Leaf size=150

$$-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\ldots$$

```
[Out] -84*a^3*x/b^10+14*a^2*x^2/b^9-7/3*a*x^3/b^8+1/4*x^4/b^7-1/6*a^10/b^11/(b*x+
a)^6+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4+40*a^7/b^11/(b*x+a)^3-105
*a^6/b^11/(b*x+a)^2+252*a^5/b^11/(b*x+a)+210*a^4*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\frac{252a^5}{b^{11}(a+bx)}+\frac{210a^4\log(a+bx)}{b^{11}}-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^7,x]
```

```
[Out] (-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^1
0/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a +
 b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) + (25
2*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a+bx)^7}\,dx &= \int \left(-\frac{84a^3}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{b^8}+\frac{x^3}{b^7}+\frac{a^{10}}{b^{10}(a+bx)^7}-\frac{10a^9}{b^{10}(a+bx)^6}+\frac{45a^8}{b^{10}(a+bx)^5}-\ldots\right.\\ &= -\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\ldots\end{aligned}$$

Mathematica [A]
time = 0.02, size = 139, normalized size = 0.93

$$\frac{2131a^{10}+10266a^9bx+18105a^8b^2x^2+11540a^7b^3x^3-3945a^6b^4x^4-9138a^5b^5x^5-4043a^4b^6x^6-360a^3b^7x^7+45a^2b^8x^8-10ab^9x^9+3b^{10}x^{10}+2520a^4(a+bx)^6\log(a+bx)}{12b^{11}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^7,x]
```

```
[Out] (2131*a^10 + 10266*a^9*b*x + 18105*a^8*b^2*x^2 + 11540*a^7*b^3*x^3 - 3945*a
^6*b^4*x^4 - 9138*a^5*b^5*x^5 - 4043*a^4*b^6*x^6 - 360*a^3*b^7*x^7 + 45*a^2
*b^8*x^8 - 10*a*b^9*x^9 + 3*b^10*x^10 + 2520*a^4*(a + b*x)^6*Log[a + b*x])/
(12*b^11*(a + b*x)^6)
```

Mathics [A]
time = 3.73, size = 283, normalized size = 1.89

$$\frac{2131a^{10}+2520a^{10}\mathrm{Log}\left[a+bx\right]+10266a^9bx+15120a^9bx\mathrm{Log}\left[a+bx\right]+18105a^8b^2x^2+37800a^8b^2x^2\mathrm{Log}\left[a+bx\right]+11540a^7b^3x^3+50400a^7b^3x^3\mathrm{Log}\left[a+bx\right]-3945a^6b^4x^4+37800a^6b^4x^4\mathrm{Log}\left[a+bx\right]-9138a^5b^5x^5+15120a^5b^5x^5\mathrm{Log}\left[a+bx\right]-4043a^4b^6x^6+2520a^4b^6x^6\mathrm{Log}\left[a+bx\right]-360a^3b^7x^7+45a^2b^8x^8-10ab^9x^9+3b^{10}x^{10}}{12b^{11}\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^10/(a + b*x)^7,x]')
```

```
[Out] (2131 a ^ 10 + 2520 a ^ 10 Log[a + b x] + 10266 a ^ 9 b x + 15120 a ^ 9 b x
 Log[a + b x] + 18105 a ^ 8 b ^ 2 x ^ 2 + 37800 a ^ 8 b ^ 2 x ^ 2 Log[a + b
 x] + 11540 a ^ 7 b ^ 3 x ^ 3 + 50400 a ^ 7 b ^ 3 x ^ 3 Log[a + b x] - 3945
 a ^ 6 b ^ 4 x ^ 4 + 37800 a ^ 6 b ^ 4 x ^ 4 Log[a + b x] - 9138 a ^ 5 b ^
5 x ^ 5 + 15120 a ^ 5 b ^ 5 x ^ 5 Log[a + b x] - 4043 a ^ 4 b ^ 6 x ^ 6 + 2
520 a ^ 4 b ^ 6 x ^ 6 Log[a + b x] - 360 a ^ 3 b ^ 7 x ^ 7 + 45 a ^ 2 b ^ 8
 x ^ 8 - 10 a b ^ 9 x ^ 9 + 3 b ^ 10 x ^ 10) / (12 b ^ 11 (a ^ 6 + 6 a ^ 5
b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 +
6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.09, size = 144, normalized size = 0.96

method	result
risch	$\frac{x^4}{4b^7}-\frac{7a\,x^3}{3b^8}+\frac{14a^2x^2}{b^9}-\frac{84a^3x}{b^{10}}+\frac{252a^5b^4x^5+1155a^6b^3x^4+2140a^7b^2x^3+\frac{7995a^8b\,x^2}{4}+\frac{1879a^9x}{2}+\frac{2131a^{10}}{12b}}{b^{10}(bx+a)^6}+\frac{210a^4\ln(bx+a)}{b^{11}}$
norman	$\frac{\frac{x^{10}}{4b}-\frac{5a\,x^9}{6b^2}-\frac{30a^3x^7}{b^4}+\frac{1029a^{10}}{2b^{11}}+\frac{15a^2x^8}{4b^3}+\frac{1260a^5x^5}{b^6}+\frac{4725a^6x^4}{b^7}+\frac{7700a^7x^3}{b^8}+\frac{13125a^8x^2}{2b^9}+\frac{2877a^9x}{b^{10}}}{(bx+a)^6}+\frac{210a^4\ln(bx+a)}{b^{11}}$
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{7}{3}a\,b^2x^3-14a^2b\,x^2+84a^3x}{b^{10}}+\frac{252a^5}{b^{11}(bx+a)}+\frac{2a^9}{b^{11}(bx+a)^5}-\frac{45a^8}{4b^{11}(bx+a)^4}-\frac{105a^6}{b^{11}(bx+a)^2}-\frac{a^{10}}{6b^{11}(bx+a)^6}+\frac{210a^4\ln}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^10*(-1/4*b^3*x^4+7/3*a*b^2*x^3-14*a^2*b*x^2+84*a^3*x)+252*a^5/b^11/(b*
x+a)+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4-105*a^6/b^11/(b*x+a)^2-1/
6*a^10/b^11/(b*x+a)^6+210*a^4*ln(b*x+a)/b^11+40*a^7/b^11/(b*x+a)^3
```

Maxima [A]
time = 0.25, size = 180, normalized size = 1.20

$$\frac{3024\,a^5b^5x^5+13860\,a^6b^4x^4+25680\,a^7b^3x^3+23985\,a^8b^2x^2+11274\,a^9bx+2131\,a^{10}}{12\left(b^{17}x^6+6\,ab^{16}x^5+15\,a^2b^{15}x^4+20\,a^3b^{14}x^3+15\,a^4b^{13}x^2+6\,a^5b^{12}x+a^6b^{11}\right)}+\frac{210\,a^4\log\left(bx+a\right)}{b^{11}}+\frac{3\,b^3x^4-28\,ab^2x^3+168\,a^2bx^2-1008\,a^3x}{12\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4 + 25680*a^7*b^3*x^3 + 23985*a^8*
b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15
*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 210*a
^4*log(b*x + a)/b^11 + 1/12*(3*b^3*x^4 - 28*a*b^2*x^3 + 168*a^2*b*x^2 - 100
8*a^3*x)/b^10
```

Fricas [A]
time = 0.30, size = 250, normalized size = 1.67

$$\frac{3\,b^{10}x^{10} - 10\,ab^{9}x^{9} + 45\,a^{2}b^{8}x^{8} - 360\,a^{3}b^{7}x^{7} - 4043\,a^{4}b^{6}x^{6} - 9138\,a^{5}b^{5}x^{5} - 3945\,a^{6}b^{4}x^{4} + 11540\,a^{7}b^{3}x^{3} + 18105\,a^{8}b^{2}x^{2} + 10266\,a^{9}bx + 2131\,a^{10} + 2520\,(a^{4}b^{6}x^{6} + 6\,a^{5}b^{5}x^{5} + 15\,a^{6}b^{4}x^{4} + 20\,a^{7}b^{3}x^{3} + 15\,a^{8}b^{2}x^{2} + 6\,a^{9}bx + a^{10})\log\left(bx + a\right)}{12\,(b^{17}x^{6} + 6\,ab^{16}x^{5} + 15\,a^{2}b^{15}x^{4} + 20\,a^{3}b^{14}x^{3} + 15\,a^{4}b^{13}x^{2} + 6\,a^{5}b^{12}x + a^{6}b^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 - 10*a*b^9*x^9 + 45*a^2*b^8*x^8 - 360*a^3*b^7*x^7 - 4043*
a^4*b^6*x^6 - 9138*a^5*b^5*x^5 - 3945*a^6*b^4*x^4 + 11540*a^7*b^3*x^3 + 181
05*a^8*b^2*x^2 + 10266*a^9*b*x + 2131*a^10 + 2520*(a^4*b^6*x^6 + 6*a^5*b^5*
x^5 + 15*a^6*b^4*x^4 + 20*a^7*b^3*x^3 + 15*a^8*b^2*x^2 + 6*a^9*b*x + a^10)*
log(b*x + a))/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3
+ 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11)
```

Sympy [A]
time = 0.46, size = 190, normalized size = 1.27

$$\frac{210a^{4}\log\left(a+bx\right)}{b^{11}} - \frac{84a^{3}x}{b^{10}} + \frac{14a^{2}x^{2}}{b^{9}} - \frac{7ax^{3}}{3b^{8}} + \frac{2131a^{10} + 11274a^{9}bx + 23985a^{8}b^{2}x^{2} + 25680a^{7}b^{3}x^{3} + 13860a^{6}b^{4}x^{4} + 3024a^{5}b^{5}x^{5}}{12a^{6}b^{11} + 72a^{5}b^{12}x + 180a^{4}b^{13}x^{2} + 240a^{3}b^{14}x^{3} + 180a^{2}b^{15}x^{4} + 72ab^{16}x^{5} + 12b^{17}x^{6}} + \frac{x^{4}}{4b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**7,x)
```

```
[Out] 210*a**4*log(a + b*x)/b**11 - 84*a**3*x/b**10 + 14*a**2*x**2/b**9 - 7*a*x**
3/(3*b**8) + (2131*a**10 + 11274*a**9*b*x + 23985*a**8*b**2*x**2 + 25680*a*
*7*b**3*x**3 + 13860*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*a**6*b**11 +
 72*a**5*b**12*x + 180*a**4*b**13*x**2 + 240*a**3*b**14*x**3 + 180*a**2*b**
15*x**4 + 72*a*b**16*x**5 + 12*b**17*x**6) + x**4/(4*b**7)
```

Giac [A]
time = 0.00, size = 139, normalized size = 0.93

$$\frac{\frac{1}{4}x^{4}b^{21} - \frac{7}{3}x^{3}b^{20}a + 14x^{2}b^{19}a^{2} - 84xb^{18}a^{3}}{b^{28}} + \frac{\frac{1}{12}\left(3024b^{5}a^{5}x^{5} + 13860b^{4}a^{6}x^{4} + 25680b^{3}a^{7}x^{3} + 23985b^{2}a^{8}x^{2} + 11274ba^{9}x + 2131a^{10}\right)}{b^{11}\left(xb+a\right)^{6}} + \frac{210a^{4}\ln\left|xb+a\right|}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x)
```

```
[Out] 210*a^4*log(abs(b*x + a))/b^11 + 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4
 + 25680*a^7*b^3*x^3 + 23985*a^8*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/((b*x
 + a)^6*b^11) + 1/12*(3*b^21*x^4 - 28*a*b^20*x^3 + 168*a^2*b^19*x^2 - 1008*
a^3*b^18*x)/b^28
```

Mupad [B]
time = 1.09, size = 126, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^4}{4}-\frac{10\,a\,(a+b\,x)^3}{3}+\frac{45\,a^2\,(a+b\,x)^2}{2}+\frac{252\,a^5}{a+b\,x}-\frac{105\,a^6}{(a+b\,x)^2}+\frac{40\,a^7}{(a+b\,x)^3}-\frac{45\,a^8}{4\,(a+b\,x)^4}+\frac{2\,a^9}{(a+b\,x)^5}-\frac{a^{10}}{6\,(a+b\,x)^6}+210\,a^4\,\ln\left(a+b\,x\right)-120\,a^3\,b\,x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^4/4 - (10*a*(a + b*x)^3)/3 + (45*a^2*(a + b*x)^2)/2 + (252*a^5)/
(a + b*x) - (105*a^6)/(a + b*x)^2 + (40*a^7)/(a + b*x)^3 - (45*a^8)/(4*(a +
 b*x)^4) + (2*a^9)/(a + b*x)^5 - a^10/(6*(a + b*x)^6) + 210*a^4*log(a + b*x
) - 120*a^3*b*x)/b^11
```

3.208 $\int \frac{x^9}{(a+bx)^7}\,dx$

Optimal. Leaf size=139

$$\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}$$

```
[Out] 28*a^2*x/b^9-7/2*a*x^2/b^8+1/3*x^3/b^7+1/6*a^9/b^10/(b*x+a)^6-9/5*a^8/b^10/
(b*x+a)^5+9*a^7/b^10/(b*x+a)^4-28*a^6/b^10/(b*x+a)^3+63*a^5/b^10/(b*x+a)^2-
126*a^4/b^10/(b*x+a)-84*a^3*ln(b*x+a)/b^10
```

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^7,x]
```

```
[Out] (28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6)
 - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(b^
10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x))
- (84*a^3*Log[a + b*x])/b^10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^7}\,dx &= \int \left(\frac{28a^2}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{b^7}-\frac{a^9}{b^9(a+bx)^7}+\frac{9a^8}{b^9(a+bx)^6}-\frac{36a^7}{b^9(a+bx)^5}+\frac{84a^6}{b^9(a+bx)^4}-\ldots\right.\\ &= \frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\ldots\end{aligned}$$

Mathematica [A]
time = 0.02, size = 128, normalized size = 0.92

$$-\frac{2509a^9+12534a^8bx+23775a^7b^2x^2+19100a^6b^3x^3+1725a^5b^4x^4-6870a^4b^5x^5-3665a^3b^6x^6-360a^2b^7x^7+45ab^8x^8-10b^9x^9+2520a^3(a+bx)^6\log(a+bx)}{30b^{10}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^7,x]
```

```
[Out] -1/30*(2509*a^9 + 12534*a^8*b*x + 23775*a^7*b^2*x^2 + 19100*a^6*b^3*x^3 + 1
725*a^5*b^4*x^4 - 6870*a^4*b^5*x^5 - 3665*a^3*b^6*x^6 - 360*a^2*b^7*x^7 + 4
5*a*b^8*x^8 - 10*b^9*x^9 + 2520*a^3*(a + b*x)^6*Log[a + b*x])/(b^10*(a + b*
x)^6)
```

Mathics [A]
time = 3.61, size = 272, normalized size = 1.96

$$\frac{-2520a^9\mathrm{Log}\left[a+bx\right]-2509a^9-15120a^8bx\mathrm{Log}\left[a+bx\right]-12534a^8bx-37800a^7b^2x^2\mathrm{Log}\left[a+bx\right]-23775a^7b^2x^2-50400a^6b^3x^3\mathrm{Log}\left[a+bx\right]-19100a^6b^3x^3-37800a^5b^4x^4\mathrm{Log}\left[a+bx\right]-1725a^5b^4x^4-15120a^4b^5x^5\mathrm{Log}\left[a+bx\right]+6870a^4b^5x^5-2520a^3b^6x^6\mathrm{Log}\left[a+bx\right]+3665a^3b^6x^6+360a^2b^7x^7-45ab^8x^8+10b^9x^9}{30b^{10}\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^9/(a + b*x)^7,x]')
```

```
[Out] (-2520 a ^ 9 Log[a + b x] - 2509 a ^ 9 - 15120 a ^ 8 b x Log[a + b x] - 125
34 a ^ 8 b x - 37800 a ^ 7 b ^ 2 x ^ 2 Log[a + b x] - 23775 a ^ 7 b ^ 2 x ^
 2 - 50400 a ^ 6 b ^ 3 x ^ 3 Log[a + b x] - 19100 a ^ 6 b ^ 3 x ^ 3 - 37800
 a ^ 5 b ^ 4 x ^ 4 Log[a + b x] - 1725 a ^ 5 b ^ 4 x ^ 4 - 15120 a ^ 4 b ^
5 x ^ 5 Log[a + b x] + 6870 a ^ 4 b ^ 5 x ^ 5 - 2520 a ^ 3 b ^ 6 x ^ 6 Log[
a + b x] + 3665 a ^ 3 b ^ 6 x ^ 6 + 360 a ^ 2 b ^ 7 x ^ 7 - 45 a b ^ 8 x ^
8 + 10 b ^ 9 x ^ 9) / (30 b ^ 10 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^
2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x
 ^ 6))
```

Maple [A]
time = 0.10, size = 132, normalized size = 0.95

method	result
risch	$\frac{x^{3}}{3 b^{7}}-\frac{7 a \,x^{2}}{2 b^{8}}+\frac{28 a^{2} x}{b^{9}}+\frac{-126 a^{4} b^{4} x^{5}-567 a^{5} b^{3} x^{4}-1036 a^{6} b^{2} x^{3}-957 a^{7} b \,x^{2}-\frac{2229 a^{8} x}{5}-\frac{2509 a^{9}}{30 b}}{b^{9} \left(b x +a \right)^{6}}-\frac{84 a^{3} \ln \left(b x +a \right)}{b^{10}}$
norman	$\frac{\frac{x^{9}}{3 b}-\frac{3 a \,x^{8}}{2 b^{2}}-\frac{1029 a^{9}}{5 b^{10}}+\frac{12 a^{2} x^{7}}{b^{3}}-\frac{504 a^{4} x^{5}}{b^{5}}-\frac{1890 a^{5} x^{4}}{b^{6}}-\frac{3080 a^{6} x^{3}}{b^{7}}-\frac{2625 a^{7} x^{2}}{b^{8}}-\frac{5754 a^{8} x}{5 b^{9}}}{\left(b x +a \right)^{6}}-\frac{84 a^{3} \ln \left(b x +a \right)}{b^{10}}$
default	$\frac{\frac{1}{3} b^{2} x^{3}-\frac{7}{2} a b \,x^{2}+28 a^{2} x}{b^{9}}-\frac{126 a^{4}}{b^{10} \left(b x +a \right)}+\frac{9 a^{7}}{b^{10} \left(b x +a \right)^{4}}+\frac{63 a^{5}}{b^{10} \left(b x +a \right)^{2}}+\frac{a^{9}}{6 b^{10} \left(b x +a \right)^{6}}-\frac{84 a^{3} \ln \left(b x +a \right)}{b^{10}}-\frac{9 a^{8}}{5 b^{10} \left(b x +a \right)^{5}}-\frac{2}{b^{10}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^9*(1/3*b^2*x^3-7/2*a*b*x^2+28*a^2*x)-126*a^4/b^10/(b*x+a)+9*a^7/b^10/(b
*x+a)^4+63*a^5/b^10/(b*x+a)^2+1/6*a^9/b^10/(b*x+a)^6-84*a^3*ln(b*x+a)/b^10-
9/5*a^8/b^10/(b*x+a)^5-28*a^6/b^10/(b*x+a)^3
```

Maxima [A]
time = 0.26, size = 169, normalized size = 1.22

$$-\frac{3780 \, a^{4} b^{5} x^{5} + 17010 \, a^{5} b^{4} x^{4} + 31080 \, a^{6} b^{3} x^{3} + 28710 \, a^{7} b^{2} x^{2} + 13374 \, a^{8} b x + 2509 \, a^{9}}{30 \, {\left(b^{16} x^{6} + 6 \, a b^{15} x^{5} + 15 \, a^{2} b^{14} x^{4} + 20 \, a^{3} b^{13} x^{3} + 15 \, a^{4} b^{12} x^{2} + 6 \, a^{5} b^{11} x + a^{6} b^{10}\right)}} - \frac{84 \, a^{3} \log\left(b x + a\right)}{b^{10}} + \frac{2 \, b^{2} x^{3} - 21 \, a b x^{2} + 168 \, a^{2} x}{6 \, b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4 + 31080*a^6*b^3*x^3 + 28710*a^7
*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/(b^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^14
*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 + 6*a^5*b^11*x + a^6*b^10) - 84*a^
3*log(b*x + a)/b^10 + 1/6*(2*b^2*x^3 - 21*a*b*x^2 + 168*a^2*x)/b^9
```

Fricas [A]
time = 0.31, size = 239, normalized size = 1.72

$$\frac{10\,b^9x^9 - 45\,ab^8x^8 + 360\,a^2b^7x^7 + 3665\,a^3b^6x^6 + 6870\,a^4b^5x^5 - 1725\,a^5b^4x^4 - 19100\,a^6b^3x^3 - 23775\,a^7b^2x^2 - 12534\,a^8bx - 2509\,a^9 - 2520\,(a^3b^6x^6 + 6\,a^4b^5x^5 + 15\,a^5b^4x^4 + 20\,a^6b^3x^3 + 15\,a^7b^2x^2 + 6\,a^8bx + a^9)\log(bx + a)}{30\,(b^{16}x^6 + 6\,ab^{15}x^5 + 15\,a^2b^{14}x^4 + 20\,a^3b^{13}x^3 + 15\,a^4b^{12}x^2 + 6\,a^5b^{11}x + a^6b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^9*x^9 - 45*a*b^8*x^8 + 360*a^2*b^7*x^7 + 3665*a^3*b^6*x^6 + 6870
*a^4*b^5*x^5 - 1725*a^5*b^4*x^4 - 19100*a^6*b^3*x^3 - 23775*a^7*b^2*x^2 - 1
2534*a^8*b*x - 2509*a^9 - 2520*(a^3*b^6*x^6 + 6*a^4*b^5*x^5 + 15*a^5*b^4*x^
4 + 20*a^6*b^3*x^3 + 15*a^7*b^2*x^2 + 6*a^8*b*x + a^9)*log(b*x + a))/(b^16*
x^6 + 6*a*b^15*x^5 + 15*a^2*b^14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 +
6*a^5*b^11*x + a^6*b^10)
```

Sympy [A]
time = 0.44, size = 180, normalized size = 1.29

$$-\frac{84a^3\log(a + bx)}{b^{10}} + \frac{28a^2x}{b^9} - \frac{7ax^2}{2b^8} + \frac{-2509a^9 - 13374a^8bx - 28710a^7b^2x^2 - 31080a^6b^3x^3 - 17010a^5b^4x^4 - 3780a^4b^5x^5}{30a^6b^{10} + 180a^5b^{11}x + 450a^4b^{12}x^2 + 600a^3b^{13}x^3 + 450a^2b^{14}x^4 + 180ab^{15}x^5 + 30b^{16}x^6} + \frac{x^3}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**7,x)
```

```
[Out] -84*a**3*log(a + b*x)/b**10 + 28*a**2*x/b**9 - 7*a*x**2/(2*b**8) + (-2509*a
**9 - 13374*a**8*b*x - 28710*a**7*b**2*x**2 - 31080*a**6*b**3*x**3 - 17010*
a**5*b**4*x**4 - 3780*a**4*b**5*x**5)/(30*a**6*b**10 + 180*a**5*b**11*x + 4
50*a**4*b**12*x**2 + 600*a**3*b**13*x**3 + 450*a**2*b**14*x**4 + 180*a*b**1
5*x**5 + 30*b**16*x**6) + x**3/(3*b**7)
```

Giac [A]
time = 0.00, size = 134, normalized size = 0.96

$$\frac{\frac{1}{3}x^3b^{14} - \frac{7}{2}x^2b^{13}a + 28xb^{12}a^2}{b^{21}} + \frac{\frac{1}{30}\left(-3780b^5a^4x^5 - 17010b^4a^5x^4 - 31080b^3a^6x^3 - 28710b^2a^7x^2 - 13374ba^8x - 2509a^9\right)}{b^{10}\left(xb + a\right)^6} - \frac{84a^3\ln|xb + a|}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x)
```

```
[Out] -84*a^3*log(abs(b*x + a))/b^10 - 1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4
 + 31080*a^6*b^3*x^3 + 28710*a^7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/((b*x
+ a)^6*b^10) + 1/6*(2*b^14*x^3 - 21*a*b^13*x^2 + 168*a^2*b^12*x)/b^21
```

Mupad [B]
time = 0.55, size = 115, normalized size = 0.83

$$-\frac{\frac{9\,a\,{\left(a+b\,x\right)}^2}{2}-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{126\,a^4}{a+b\,x}-\frac{63\,a^5}{{\left(a+b\,x\right)}^2}+\frac{28\,a^6}{{\left(a+b\,x\right)}^3}-\frac{9\,a^7}{{\left(a+b\,x\right)}^4}+\frac{9\,a^8}{5\,{\left(a+b\,x\right)}^5}-\frac{a^9}{6\,{\left(a+b\,x\right)}^6}+84\,a^3\,\ln\left(a+b\,x\right)-36\,a^2\,b\,x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^7,x)
```

```
[Out] -((9*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (126*a^4)/(a + b*x) - (63*a^5)/(a +
 b*x)^2 + (28*a^6)/(a + b*x)^3 - (9*a^7)/(a + b*x)^4 + (9*a^8)/(5*(a + b*x)
^5) - a^9/(6*(a + b*x)^6) + 84*a^3*log(a + b*x) - 36*a^2*b*x)/b^10
```

3.209 $\int \frac{x^8}{(a+bx)^7}\,dx$

Optimal. Leaf size=128

$$-\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a}{b^9}$$

```
[Out] -7*a*x/b^8+1/2*x^2/b^7-1/6*a^8/b^9/(b*x+a)^6+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4+56/3*a^5/b^9/(b*x+a)^3-35*a^4/b^9/(b*x+a)^2+56*a^3/b^9/(b*x+a)+
28*a^2*ln(b*x+a)/b^9
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^7,x]
```

```
[Out] (-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*a^
4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/b^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^7}\,dx = \int \left(-\frac{7a}{b^8}+\frac{x}{b^7}+\frac{a^8}{b^8(a+bx)^7}-\frac{8a^7}{b^8(a+bx)^6}+\frac{28a^6}{b^8(a+bx)^5}-\frac{56a^5}{b^8(a+bx)^4}+\frac{70a^4}{b^8(a+bx)^3}$$
$$= -\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 0.81

$$\frac{-210abx+15b^2x^2-\frac{5a^8}{(a+bx)^6}+\frac{48a^7}{(a+bx)^5}-\frac{210a^6}{(a+bx)^4}+\frac{560a^5}{(a+bx)^3}-\frac{1050a^4}{(a+bx)^2}+\frac{1680a^3}{a+bx}+840a^2\log(a+bx)}{30b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^7,x]
```

```
[Out] (-210*a*b*x + 15*b^2*x^2 - (5*a^8)/(a + b*x)^6 + (48*a^7)/(a + b*x)^5 - (21
0*a^6)/(a + b*x)^4 + (560*a^5)/(a + b*x)^3 - (1050*a^4)/(a + b*x)^2 + (1680
*a^3)/(a + b*x) + 840*a^2*Log[a + b*x])/(30*b^9)
```

Mathics [A]
time = 3.38, size = 152, normalized size = 1.19

$$\frac{28a^2\mathrm{Log}\left[a+bx\right]}{b^9}+\frac{a^3\left(1023a^5+5508a^4bx+11970a^3b^2x^2+13160a^2b^3x^3+7350ab^4x^4+1680b^5x^5\right)}{30b^9\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^8/(a + b*x)^7,x]')
```

```
[Out] 28 a ^ 2 Log[a + b x] / b ^ 9 + a ^ 3 (1023 a ^ 5 + 5508 a ^ 4 b x + 11970
a ^ 3 b ^ 2 x ^ 2 + 13160 a ^ 2 b ^ 3 x ^ 3 + 7350 a b ^ 4 x ^ 4 + 1680 b ^
 5 x ^ 5) / (30 b ^ 9 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^
3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6)) - 7
a x / b ^ 8 + x ^ 2 / (2 b ^ 7)
```

Maple [A]
time = 0.09, size = 122, normalized size = 0.95

method	result	si
risch	$\frac{x^2}{2b^7}-\frac{7ax}{b^8}+\frac{56a^3b^4x^5+245a^4b^3x^4+\frac{1316a^5b^2x^3}{3}+399a^6b\,x^2+\frac{918a^7x}{5}+\frac{341a^8}{10b}}{b^8(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}$	9
norman	$\frac{\frac{x^8}{2b}-\frac{4a\,x^7}{b^2}+\frac{343a^8}{5b^9}+\frac{168a^3x^5}{b^4}+\frac{630a^4x^4}{b^5}+\frac{3080a^5x^3}{3b^6}+\frac{875a^6x^2}{b^7}+\frac{1918a^7x}{5b^8}}{(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}$	1
default	$-\frac{-\frac{1}{2}x^2b+7ax}{b^8}+\frac{56a^3}{b^9(bx+a)}+\frac{8a^7}{5b^9(bx+a)^5}-\frac{7a^6}{b^9(bx+a)^4}-\frac{35a^4}{b^9(bx+a)^2}-\frac{a^8}{6b^9(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}+\frac{56a^5}{3b^9(bx+a)^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^8*(-1/2*x^2*b+7*a*x)+56*a^3/b^9/(b*x+a)+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4-35*a^4/b^9/(b*x+a)^2-1/6*a^8/b^9/(b*x+a)^6+28*a^2*ln(b*x+a)/b^9
+56/3*a^5/b^9/(b*x+a)^3
```

Maxima [A]
time = 0.25, size = 157, normalized size = 1.23

$$\frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\left(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^{11}x^2+6\,a^5b^{10}x+a^6b^9\right)}+\frac{28\,a^2\log\left(bx+a\right)}{b^9}+\frac{bx^2-14\,ax}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(1680*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b
^2*x^2 + 5508*a^7*b*x + 1023*a^8)/(b^15*x^6 + 6*a*b^14*x^5 + 15*a^2*b^13*x^
4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*b^9) + 28*a^2*lo
g(b*x + a)/b^9 + 1/2*(b*x^2 - 14*a*x)/b^8
```

Fricas [A]
time = 0.30, size = 228, normalized size = 1.78

$$\frac{15\,b^{8}x^{8}-120\,ab^{7}x^{7}-1035\,a^{2}b^{6}x^{6}-1170\,a^{3}b^{5}x^{5}+3375\,a^{4}b^{4}x^{4}+10100\,a^{5}b^{3}x^{3}+10725\,a^{6}b^{2}x^{2}+5298\,a^{7}bx+1023\,a^{8}+840\,\left(a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{5}+15\,a^{4}b^{4}x^{4}+20\,a^{5}b^{3}x^{3}+15\,a^{6}b^{2}x^{2}+6\,a^{7}bx+a^{8}\right)\log\left(bx+a\right)}{30\,\left(b^{15}x^{6}+6\,ab^{14}x^{5}+15\,a^{2}b^{13}x^{4}+20\,a^{3}b^{12}x^{3}+15\,a^{4}b^{11}x^{2}+6\,a^{5}b^{10}x+a^{6}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(15*b^8*x^8 - 120*a*b^7*x^7 - 1035*a^2*b^6*x^6 - 1170*a^3*b^5*x^5 + 33
75*a^4*b^4*x^4 + 10100*a^5*b^3*x^3 + 10725*a^6*b^2*x^2 + 5298*a^7*b*x + 102
3*a^8 + 840*(a^2*b^6*x^6 + 6*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 20*a^5*b^3*x^3
+ 15*a^6*b^2*x^2 + 6*a^7*b*x + a^8)*log(b*x + a))/(b^15*x^6 + 6*a*b^14*x^5
+ 15*a^2*b^13*x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*
b^9)
```

Sympy [A]
time = 0.41, size = 165, normalized size = 1.29

$$\frac{28a^{2}\log\left(a+bx\right)}{b^{9}}-\frac{7ax}{b^{8}}+\frac{1023a^{8}+5508a^{7}bx+11970a^{6}b^{2}x^{2}+13160a^{5}b^{3}x^{3}+7350a^{4}b^{4}x^{4}+1680a^{3}b^{5}x^{5}}{30a^{6}b^{9}+180a^{5}b^{10}x+450a^{4}b^{11}x^{2}+600a^{3}b^{12}x^{3}+450a^{2}b^{13}x^{4}+180ab^{14}x^{5}+30b^{15}x^{6}}+\frac{x^{2}}{2b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**7,x)
```

```
[Out] 28*a**2*log(a + b*x)/b**9 - 7*a*x/b**8 + (1023*a**8 + 5508*a**7*b*x + 11970
*a**6*b**2*x**2 + 13160*a**5*b**3*x**3 + 7350*a**4*b**4*x**4 + 1680*a**3*b*
*5*x**5)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*
b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + x**2
/(2*b**7)
```

Giac [A]
time = 0.00, size = 114, normalized size = 0.89

$$\frac{\frac{1}{2}x^{2}b^{7}-7xb^{6}a}{b^{14}}+\frac{\frac{1}{30}\left(1680b^{5}a^{3}x^{5}+7350b^{4}a^{4}x^{4}+13160b^{3}a^{5}x^{3}+11970b^{2}a^{6}x^{2}+5508ba^{7}x+1023a^{8}\right)}{b^{9}\left(xb+a\right)^{6}}+\frac{28a^{2}\ln\left|xb+a\right|}{b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x)
```

```
[Out] 28*a^2*log(abs(b*x + a))/b^9 + 1/2*(b^7*x^2 - 14*a*b^6*x)/b^14 + 1/30*(1680
*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b^2*x^2 + 5
508*a^7*b*x + 1023*a^8)/((b*x + a)^6*b^9)
```

Mupad [B]
time = 0.18, size = 102, normalized size = 0.80

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{56\,a^3}{a+b\,x}-\frac{35\,a^4}{(a+b\,x)^2}+\frac{56\,a^5}{3\,(a+b\,x)^3}-\frac{7\,a^6}{(a+b\,x)^4}+\frac{8\,a^7}{5\,(a+b\,x)^5}-\frac{a^8}{6\,(a+b\,x)^6}+28\,a^2\,\ln\left(a+b\,x\right)-8\,a\,b\,x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^2/2 + (56*a^3)/(a + b*x) - (35*a^4)/(a + b*x)^2 + (56*a^5)/(3*(a
 + b*x)^3) - (7*a^6)/(a + b*x)^4 + (8*a^7)/(5*(a + b*x)^5) - a^8/(6*(a + b*
x)^6) + 28*a^2*log(a + b*x) - 8*a*b*x)/b^9
```

3.210 $\int \frac{x^7}{(a+bx)^7} \, dx$

Optimal. Leaf size=118

$$\frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}$$

```
[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35
/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a
)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^7,x]
```

```
[Out] x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(4
*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b*x
)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^7} \, dx &= \int \left(\frac{1}{b^7}-\frac{a^7}{b^7(a+bx)^7}+\frac{7a^6}{b^7(a+bx)^6}-\frac{21a^5}{b^7(a+bx)^5}+\frac{35a^4}{b^7(a+bx)^4}-\frac{35a^3}{b^7(a+bx)^3}+\frac{21}{b^7(a-}\right.\\ &= \frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{2}{b^8(a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 0.88

$$-\frac{669a^7+3594a^6bx+7725a^5b^2x^2+8200a^4b^3x^3+4050a^3b^4x^4+360a^2b^5x^5-360ab^6x^6-60b^7x^7+420a(a+bx)^6\log(a+bx)}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b*x)^7,x]

[Out] -1/60*(669*a^7 + 3594*a^6*b*x + 7725*a^5*b^2*x^2 + 8200*a^4*b^3*x^3 + 4050*a^3*b^4*x^4 + 360*a^2*b^5*x^5 - 360*a*b^6*x^6 - 60*b^7*x^7 + 420*a*(a + b*x)^6*Log[a + b*x])/(b^8*(a + b*x)^6)

Mathics [A]
time = 3.37, size = 140, normalized size = 1.19

$$\frac{-7a\text{Log}\left[a+bx\right]}{b^8}-\frac{a^2\left(669a^5+3654a^4bx+8085a^3b^2x^2+9100a^2b^3x^3+5250ab^4x^4+1260b^5x^5\right)}{60b^8\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}+\frac{x}{b^7}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^7/(a + b*x)^7,x]')

[Out] -7 a Log[a + b x] / b ^ 8 - a ^ 2 (669 a ^ 5 + 3654 a ^ 4 b x + 8085 a ^ 3 b ^ 2 x ^ 2 + 9100 a ^ 2 b ^ 3 x ^ 3 + 5250 a b ^ 4 x ^ 4 + 1260 b ^ 5 x ^ 5) / (60 b ^ 8 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6)) + x / b ^ 7

Maple [A]
time = 0.09, size = 109, normalized size = 0.92

method	result	size
risch	$\frac{x}{b^7}+\frac{-21a^2b^4x^5-\frac{175a^3b^3x^4}{2}-\frac{455a^4b^2x^3}{3}-\frac{539a^5b\,x^2}{4}-\frac{609a^6x}{10}-\frac{223a^7}{20b}}{b^7\left(bx+a\right)^6}-\frac{7a\ln\left(bx+a\right)}{b^8}$	87
norman	$\frac{\frac{x^7}{b}-\frac{343a^7}{20b^8}-\frac{42a^2x^5}{b^3}-\frac{315a^3x^4}{2b^4}-\frac{770a^4x^3}{3b^5}-\frac{875a^5x^2}{4b^6}-\frac{959a^6x}{10b^7}}{\left(bx+a\right)^6}-\frac{7a\ln\left(bx+a\right)}{b^8}$	91
default	$\frac{x}{b^7}+\frac{a^7}{6b^8\left(bx+a\right)^6}-\frac{7a^6}{5b^8\left(bx+a\right)^5}+\frac{21a^5}{4b^8\left(bx+a\right)^4}-\frac{35a^4}{3b^8\left(bx+a\right)^3}+\frac{35a^3}{2b^8\left(bx+a\right)^2}-\frac{21a^2}{b^8\left(bx+a\right)}-\frac{7a\ln\left(bx+a\right)}{b^8}$	109

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(b*x+a)^7,x,method=_RETURNVERBOSE)

[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a)/b^8

Maxima [A]
time = 0.26, size = 145, normalized size = 1.23

$$-\frac{1260\,a^2b^5x^5+5250\,a^3b^4x^4+9100\,a^4b^3x^3+8085\,a^5b^2x^2+3654\,a^6bx+669\,a^7}{60\left(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+15\,a^4b^{10}x^2+6\,a^5b^9x+a^6b^8\right)}+\frac{x}{b^7}-\frac{7\,a\log\left(bx+a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^
2*x^2 + 3654*a^6*b*x + 669*a^7)/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4
+ 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8) + x/b^7 - 7*a*
log(b*x + a)/b^8
```

Fricas [A]
time = 0.31, size = 215, normalized size = 1.82

$$\frac{60\,b^{7}x^{7}+360\,ab^{6}x^{6}-360\,a^{2}b^{5}x^{5}-4050\,a^{3}b^{4}x^{4}-8200\,a^{4}b^{3}x^{3}-7725\,a^{5}b^{2}x^{2}-3594\,a^{6}bx-669\,a^{7}-420\,\left(ab^{6}x^{6}+6\,a^{2}b^{5}x^{5}+15\,a^{3}b^{4}x^{4}+20\,a^{4}b^{3}x^{3}+15\,a^{5}b^{2}x^{2}+6\,a^{6}bx+a^{7}\right)\log\left(bx+a\right)}{60\,\left(b^{14}x^{6}+6\,ab^{13}x^{5}+15\,a^{2}b^{12}x^{4}+20\,a^{3}b^{11}x^{3}+15\,a^{4}b^{10}x^{2}+6\,a^{5}b^{9}x+a^{6}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 360*a*b^6*x^6 - 360*a^2*b^5*x^5 - 4050*a^3*b^4*x^4 - 820
0*a^4*b^3*x^3 - 7725*a^5*b^2*x^2 - 3594*a^6*b*x - 669*a^7 - 420*(a*b^6*x^6
+ 6*a^2*b^5*x^5 + 15*a^3*b^4*x^4 + 20*a^4*b^3*x^3 + 15*a^5*b^2*x^2 + 6*a^6*
b*x + a^7)*log(b*x + a))/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^
3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8)
```

Sympy [A]
time = 0.38, size = 153, normalized size = 1.30

$$-\frac{7a\log\left(a+bx\right)}{b^{8}}+\frac{-669a^{7}-3654a^{6}bx-8085a^{5}b^{2}x^{2}-9100a^{4}b^{3}x^{3}-5250a^{3}b^{4}x^{4}-1260a^{2}b^{5}x^{5}}{60a^{6}b^{8}+360a^{5}b^{9}x+900a^{4}b^{10}x^{2}+1200a^{3}b^{11}x^{3}+900a^{2}b^{12}x^{4}+360ab^{13}x^{5}+60b^{14}x^{6}}+\frac{x}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**7,x)
```

```
[Out] -7*a*log(a + b*x)/b**8 + (-669*a**7 - 3654*a**6*b*x - 8085*a**5*b**2*x**2 -
 9100*a**4*b**3*x**3 - 5250*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*a**6*
b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a
**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) + x/b**7
```

Giac [A]
time = 0.00, size = 101, normalized size = 0.86

$$\frac{x}{b^{7}}+\frac{\frac{1}{60}\left(-1260b^{5}a^{2}x^{5}-5250b^{4}a^{3}x^{4}-9100b^{3}a^{4}x^{3}-8085b^{2}a^{5}x^{2}-3654ba^{6}x-669a^{7}\right)}{b^{8}\left(xb+a\right)^{6}}-\frac{7a\ln\left|xb+a\right|}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x)
```

```
[Out] x/b^7 - 7*a*log(abs(b*x + a))/b^8 - 1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x
^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^2*x^2 + 3654*a^6*b*x + 669*a^7)/((b*x +
a)^6*b^8)
```

Mupad [B]
time = 0.34, size = 91, normalized size = 0.77

$$-\frac{7\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{21\,a^2}{a+b\,x}-\frac{35\,a^3}{2\,{\left(a+b\,x\right)}^2}+\frac{35\,a^4}{3\,{\left(a+b\,x\right)}^3}-\frac{21\,a^5}{4\,{\left(a+b\,x\right)}^4}+\frac{7\,a^6}{5\,{\left(a+b\,x\right)}^5}-\frac{a^7}{6\,{\left(a+b\,x\right)}^6}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^7,x)
```

```
[Out] -(7*a*log(a + b*x) - b*x + (21*a^2)/(a + b*x) - (35*a^3)/(2*(a + b*x)^2) +
(35*a^4)/(3*(a + b*x)^3) - (21*a^5)/(4*(a + b*x)^4) + (7*a^6)/(5*(a + b*x)^
5) - a^7/(6*(a + b*x)^6))/b^8
```

3.211 $\int \frac{x^6}{(a+bx)^7}\,dx$

Optimal. Leaf size=109

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^7,x]
```

```
[Out] -1/6*a^6/(b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7*
(a + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2)
+ (6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^7}\,dx &= \int \left(\frac{a^6}{b^6(a+bx)^7}-\frac{6a^5}{b^6(a+bx)^6}+\frac{15a^4}{b^6(a+bx)^5}-\frac{20a^3}{b^6(a+bx)^4}+\frac{15a^2}{b^6(a+bx)^3}-\frac{6a}{b^6(a+bx)^2}\right.\\ &= -\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 0.71

$$\frac{\frac{a(147a^5+822a^4bx+1875a^3b^2x^2+2200a^2b^3x^3+1350ab^4x^4+360b^5x^5)}{(a+bx)^6}+60\log(a+bx)}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^7,x]
```

```
[Out] ((a*(147*a^5 + 822*a^4*b*x + 1875*a^3*b^2*x^2 + 2200*a^2*b^3*x^3 + 1350*a*b
^4*x^4 + 360*b^5*x^5))/(a + b*x)^6 + 60*Log[a + b*x])/(60*b^7)
```

Mathics [A]
time = 3.20, size = 189, normalized size = 1.73

$$\frac{\frac{a(147a^5+822a^4bx+1875a^3b^2x^2+2200a^2b^3x^3+1350ab^4x^4+360b^5x^5)}{60}+\text{Log}\left[a+bx\right]\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}{b^7\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6/(a + b*x)^7,x]')
```

```
[Out] (a (147 a ^ 5 + 822 a ^ 4 b x + 1875 a ^ 3 b ^ 2 x ^ 2 + 2200 a ^ 2 b ^ 3 x
 ^ 3 + 1350 a b ^ 4 x ^ 4 + 360 b ^ 5 x ^ 5) / 60 + Log[a + b x] (a ^ 6 + 6
 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x
 ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6)) / (b ^ 7 (a ^ 6 + 6 a ^ 5 b x + 15 a
 ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5
x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.08, size = 100, normalized size = 0.92

method	result	size
norman	$\frac{\frac{49a^6}{20b^7}+\frac{6a\,x^5}{b^2}+\frac{45a^2x^4}{2b^3}+\frac{125a^4x^2}{4b^5}+\frac{137a^5x}{10b^6}+\frac{110a^3x^3}{3b^4}}{(bx+a)^6}+\frac{\ln(bx+a)}{b^7}$	80
risch	$\frac{\frac{49a^6}{20b^7}+\frac{6a\,x^5}{b^2}+\frac{45a^2x^4}{2b^3}+\frac{125a^4x^2}{4b^5}+\frac{137a^5x}{10b^6}+\frac{110a^3x^3}{3b^4}}{(bx+a)^6}+\frac{\ln(bx+a)}{b^7}$	80
default	$-\frac{a^6}{6b^7(bx+a)^6}+\frac{6a^5}{5b^7(bx+a)^5}-\frac{15a^4}{4b^7(bx+a)^4}+\frac{20a^3}{3b^7(bx+a)^3}-\frac{15a^2}{2b^7(bx+a)^2}+\frac{6a}{b^7(bx+a)}+\frac{\ln(bx+a)}{b^7}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

Maxima [A]
time = 0.25, size = 136, normalized size = 1.25

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6}{60\,(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7)}+\frac{\log\left(bx+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6)/(b^13*x^6 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*
a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b^8*x + a^6*b^7) + log(b*x + a)/b^7
```

Fricas [A]
time = 0.31, size = 193, normalized size = 1.77

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6+60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\log\left(bx+a\right)}{60\,(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6 + 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20
*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a))/(b^13*x^6 +
6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b
^8*x + a^6*b^7)
```

Sympy [A]
time = 0.30, size = 141, normalized size = 1.29

$$\frac{147a^6+822a^5bx+1875a^4b^2x^2+2200a^3b^3x^3+1350a^2b^4x^4+360ab^5x^5}{60a^6b^7+360a^5b^8x+900a^4b^9x^2+1200a^3b^{10}x^3+900a^2b^{11}x^4+360ab^{12}x^5+60b^{13}x^6}+\frac{\log\left(a+bx\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**7,x)
```

```
[Out] (147*a**6 + 822*a**5*b*x + 1875*a**4*b**2*x**2 + 2200*a**3*b**3*x**3 + 1350
*a**2*b**4*x**4 + 360*a*b**5*x**5)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + log(a + b*x)/b**7
```

Giac [A]
time = 0.00, size = 83, normalized size = 0.76

$$\frac{\frac{1}{60}\left(360b^4ax^5+1350b^3a^2x^4+2200b^2a^3x^3+1875ba^4x^2+822a^5x+\frac{147a^6}{b}\right)}{b^6\left(xb+a\right)^6}+\frac{\ln|xb+a|}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x)
```

```
[Out] log(abs(b*x + a))/b^7 + 1/60*(360*a*b^4*x^5 + 1350*a^2*b^3*x^4 + 2200*a^3*b
^2*x^3 + 1875*a^4*b*x^2 + 822*a^5*x + 147*a^6/b)/((b*x + a)^6*b^6)
```

Mupad [B]
time = 0.11, size = 81, normalized size = 0.74

$$\frac{\ln(a+bx)+\frac{6a}{a+bx}-\frac{15a^2}{2(a+bx)^2}+\frac{20a^3}{3(a+bx)^3}-\frac{15a^4}{4(a+bx)^4}+\frac{6a^5}{5(a+bx)^5}-\frac{a^6}{6(a+bx)^6}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^7,x)
```

```
[Out] (log(a + b*x) + (6*a)/(a + b*x) - (15*a^2)/(2*(a + b*x)^2) + (20*a^3)/(3*(a
 + b*x)^3) - (15*a^4)/(4*(a + b*x)^4) + (6*a^5)/(5*(a + b*x)^5) - a^6/(6*(a
 + b*x)^6))/b^7
```

3.212 $\int \frac{x^5}{(a+bx)^7} dx$

Optimal. Leaf size=17

$$\frac{x^6}{6a(a+bx)^6}$$

```
[Out] 1/6*x^6/a/(b*x+a)^6
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$\frac{x^6}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^7,x]
```

```
[Out] x^6/(6*a*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^7} dx = \frac{x^6}{6a(a+bx)^6}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.
time = 0.01, size = 64, normalized size = 3.76

$$-\frac{a^5+6a^4bx+15a^3b^2x^2+20a^2b^3x^3+15ab^4x^4+6b^5x^5}{6b^6(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^7,x]
```

[Out] -1/6*(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15*a*b^4*x^4 + 6*b^5*x^5)/(b^6*(a + b*x)^6)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 121 vs. $2(17) = 34$.
time = 2.71, size = 119, normalized size = 7.00

$$\frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6b^6\left(a^6 + 6a^5bx + 15a^4b^2x^2 + 20a^3b^3x^3 + 15a^2b^4x^4 + 6ab^5x^5 + b^6x^6\right)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^5/(a + b*x)^7,x]')

[Out] (-a ^ 5 - 6 a ^ 4 b x - 15 a ^ 3 b ^ 2 x ^ 2 - 20 a ^ 2 b ^ 3 x ^ 3 - 15 a b ^ 4 x ^ 4 - 6 b ^ 5 x ^ 5) / (6 b ^ 6 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(15) = 30$.
time = 0.08, size = 87, normalized size = 5.12

method	result	size
gosper	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6(bx+a)^6b^6}$	63
norman	$\frac{-\frac{x^5}{b}-\frac{5a\,x^4}{2b^2}-\frac{10a^2x^3}{3b^3}-\frac{5a^3x^2}{2b^4}-\frac{a^4x}{b^5}-\frac{a^5}{6b^6}}{(bx+a)^6}$	66
risch	$\frac{-\frac{x^5}{b}-\frac{5a\,x^4}{2b^2}-\frac{10a^2x^3}{3b^3}-\frac{5a^3x^2}{2b^4}-\frac{a^4x}{b^5}-\frac{a^5}{6b^6}}{(bx+a)^6}$	66
default	$-\frac{1}{b^6(bx+a)}+\frac{5a^3}{2b^6(bx+a)^4}+\frac{5a}{2b^6(bx+a)^2}+\frac{a^5}{6b^6(bx+a)^6}-\frac{10a^2}{3b^6(bx+a)^3}-\frac{a^4}{b^6(bx+a)^5}$	87

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x+a)^7,x,method=_RETURNVERBOSE)

[Out] -1/b^6/(b*x+a)+5/2*a^3/b^6/(b*x+a)^4+5/2*a/b^6/(b*x+a)^2+1/6*a^5/b^6/(b*x+a)^6-10/3/b^6*a^2/(b*x+a)^3-a^4/b^6/(b*x+a)^5

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.
time = 0.25, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\left(b^{12}x^6 + 6\,ab^{11}x^5 + 15\,a^2b^{10}x^4 + 20\,a^3b^9x^3 + 15\,a^4b^8x^2 + 6\,a^5b^7x + a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x+a)^7,x, algorithm="maxima")

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.
time = 0.31, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,(b^{12}x^6+6\,ab^{11}x^5+15\,a^2b^{10}x^4+20\,a^3b^9x^3+15\,a^4b^8x^2+6\,a^5b^7x+a^6b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(12) = 24$.
time = 0.28, size = 128, normalized size = 7.53

$$\frac{-a^5-6a^4bx-15a^3b^2x^2-20a^2b^3x^3-15ab^4x^4-6b^5x^5}{6a^6b^6+36a^5b^7x+90a^4b^8x^2+120a^3b^9x^3+90a^2b^{10}x^4+36ab^{11}x^5+6b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*a**6*b**6 + 36*a**5*b**7*x + 90*a**4*b**8*x**2 + 120*a*
*3*b**9*x**3 + 90*a**2*b**10*x**4 + 36*a*b**11*x**5 + 6*b**12*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(15) = 30$.
time = 0.00, size = 70, normalized size = 4.12

$$\frac{-6x^5b^5-15x^4b^4a-20x^3b^3a^2-15x^2b^2a^3-6xba^4-a^5}{6b^6\,(xb+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x)
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/((b*x + a)^6*b^6)
```

Mupad [B]
time = 0.12, size = 72, normalized size = 4.24

$$\frac{\frac{5\,a}{2\,(a+b\,x)^2}-\frac{1}{a+b\,x}-\frac{10\,a^2}{3\,(a+b\,x)^3}+\frac{5\,a^3}{2\,(a+b\,x)^4}-\frac{a^4}{(a+b\,x)^5}+\frac{a^5}{6\,(a+b\,x)^6}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^7,x)
```

```
[Out] ((5*a)/(2*(a + b*x)^2) - 1/(a + b*x) - (10*a^2)/(3*(a + b*x)^3) + (5*a^3)/(
2*(a + b*x)^4) - a^4/(a + b*x)^5 + a^5/(6*(a + b*x)^6))/b^6
```

3.213 $\int \frac{x^4}{(a+bx)^7} dx$

Optimal. Leaf size=35

$$\frac{x^5}{6a(a+bx)^6}+\frac{x^5}{30a^2(a+bx)^5}$$

[Out] 1/6*x^5/a/(b*x+a)^6+1/30*x^5/a^2/(b*x+a)^5

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{x^5}{30a^2(a+bx)^5}+\frac{x^5}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b*x)^7,x]

[Out] x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^7}\,dx &= \frac{x^5}{6a(a+bx)^6}+\frac{\int \frac{x^4}{(a+bx)^6}\,dx}{6a}\\ &= \frac{x^5}{6a(a+bx)^6}+\frac{x^5}{30a^2(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.51

$$-\frac{a^4+6a^3bx+15a^2b^2x^2+20ab^3x^3+15b^4x^4}{30b^5(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^7,x]
```

```
[Out] -1/30*(a^4 + 6*a^3*b*x + 15*a^2*b^2*x^2 + 20*a*b^3*x^3 + 15*b^4*x^4)/(b^5*(
a + b*x)^6)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 110 vs. 2(35) = 70.
time = 2.63, size = 108, normalized size = 3.09

$$\frac{-a^4-6a^3bx-15a^2b^2x^2-20ab^3x^3-15b^4x^4}{30b^5\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4/(a + b*x)^7,x]')
```

```
[Out] (-a ^ 4 - 6 a ^ 3 b x - 15 a ^ 2 b ^ 2 x ^ 2 - 20 a b ^ 3 x ^ 3 - 15 b ^ 4
x ^ 4) / (30 b ^ 5 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b
 ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(31) = 62.
time = 0.08, size = 72, normalized size = 2.06

method	result	size
gosper	$-\frac{15b^4x^4+20a\,b^3x^3+15a^2b^2x^2+6a^3bx+a^4}{30(bx+a)^6b^5}$	52
norman	$\frac{-\frac{x^4}{2b}-\frac{2a\,x^3}{3b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3x}{5b^4}-\frac{a^4}{30b^5}}{(bx+a)^6}$	55
risch	$\frac{-\frac{x^4}{2b}-\frac{2a\,x^3}{3b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3x}{5b^4}-\frac{a^4}{30b^5}}{(bx+a)^6}$	55
default	$-\frac{3a^2}{2b^5(bx+a)^4}-\frac{1}{2b^5(bx+a)^2}-\frac{a^4}{6b^5(bx+a)^6}+\frac{4a^3}{5b^5(bx+a)^5}+\frac{4a}{3b^5(bx+a)^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/b^5*a^2/(b*x+a)^4-1/2/b^5/(b*x+a)^2-1/6*a^4/b^5/(b*x+a)^6+4/5*a^3/b^5/
(b*x+a)^5+4/3*a/b^5/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.
time = 0.26, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(b^{11}x^6 + 6\,ab^{10}x^5 + 15\,a^2b^9x^4 + 20\,a^3b^8x^3 + 15\,a^4b^7x^2 + 6\,a^5b^6x + a^6b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.
time = 0.31, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(b^{11}x^6 + 6\,ab^{10}x^5 + 15\,a^2b^9x^4 + 20\,a^3b^8x^3 + 15\,a^4b^7x^2 + 6\,a^5b^6x + a^6b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(27) = 54$.
time = 0.25, size = 116, normalized size = 3.31

$$\frac{-a^4 - 6a^3bx - 15a^2b^2x^2 - 20ab^3x^3 - 15b^4x^4}{30a^6b^5 + 180a^5b^6x + 450a^4b^7x^2 + 600a^3b^8x^3 + 450a^2b^9x^4 + 180ab^{10}x^5 + 30b^{11}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**7,x)
```

```
[Out] (-a**4 - 6*a**3*b*x - 15*a**2*b**2*x**2 - 20*a*b**3*x**3 - 15*b**4*x**4)/(3
0*a**6*b**5 + 180*a**5*b**6*x + 450*a**4*b**7*x**2 + 600*a**3*b**8*x**3 + 4
50*a**2*b**9*x**4 + 180*a*b**10*x**5 + 30*b**11*x**6)
```

Giac [A]
time = 0.00, size = 58, normalized size = 1.66

$$\frac{-15x^4b^4 - 20x^3b^3a - 15x^2b^2a^2 - 6xba^3 - a^4}{30b^5\,(xb + a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x)
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/((b*x
+ a)^6*b^5)
```

Mupad [B]
time = 0.07, size = 22, normalized size = 0.63

$$\frac{x^5\,\left(6\,a+b\,x\right)}{30\,a^2\,{\left(a+b\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^7,x)
```

```
[Out] (x^5*(6*a + b*x))/(30*a^2*(a + b*x)^6)
```

3.214 $\int \frac{x^3}{(a+bx)^7} \, dx$

Optimal. Leaf size=52

$$\frac{x^4}{6a(a+bx)^6}+\frac{x^4}{15a^2(a+bx)^5}+\frac{x^4}{60a^3(a+bx)^4}$$

```
[Out] 1/6*x^4/a/(b*x+a)^6+1/15*x^4/a^2/(b*x+a)^5+1/60*x^4/a^3/(b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^7,x]
```

```
[Out] a^3/(6*b^4*(a + b*x)^6) - (3*a^2)/(5*b^4*(a + b*x)^5) + (3*a)/(4*b^4*(a + b
*x)^4) - 1/(3*b^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^7} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^7}+\frac{3a^2}{b^3(a+bx)^6}-\frac{3a}{b^3(a+bx)^5}+\frac{1}{b^3(a+bx)^4}\right) \, dx\\ &= \frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.81

$$-\frac{a^3+6a^2bx+15ab^2x^2+20b^3x^3}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3 + 6*a^2*b*x + 15*a*b^2*x^2 + 20*b^3*x^3)/(b^4*(a + b*x)^6)
```

Mathics [A]
time = 2.47, size = 97, normalized size = 1.87

$$\frac{-a^3-6a^2bx-15ab^2x^2-20b^3x^3}{60b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(a + b*x)^7,x]')
```

```
[Out] (-a ^ 3 - 6 a ^ 2 b x - 15 a b ^ 2 x ^ 2 - 20 b ^ 3 x ^ 3) / (60 b ^ 4 (a ^
 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b
 ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.08, size = 57, normalized size = 1.10

method	result	size
gosper	$-\frac{20b^3x^3+15a\,b^2x^2+6a^2bx+a^3}{60\left(bx+a\right)^6b^4}$	41
norman	$\frac{-\frac{x^3}{3b}-\frac{a\,x^2}{4b^2}-\frac{a^2x}{10b^3}-\frac{a^3}{60b^4}}{\left(bx+a\right)^6}$	44
risch	$\frac{-\frac{x^3}{3b}-\frac{a\,x^2}{4b^2}-\frac{a^2x}{10b^3}-\frac{a^3}{60b^4}}{\left(bx+a\right)^6}$	44
default	$\frac{a^3}{6b^4\left(bx+a\right)^6}-\frac{3a^2}{5b^4\left(bx+a\right)^5}+\frac{3a}{4b^4\left(bx+a\right)^4}-\frac{1}{3b^4\left(bx+a\right)^3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^3/b^4/(b*x+a)^6-3/5*a^2/b^4/(b*x+a)^5+3/4*a/b^4/(b*x+a)^4-1/3/b^4/(b*
x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.
time = 0.25, size = 98, normalized size = 1.88

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.
time = 0.31, size = 98, normalized size = 1.88

$$-\frac{20\,b^3x^3 + 15\,ab^2x^2 + 6\,a^2bx + a^3}{60\,(b^{10}x^6 + 6\,ab^9x^5 + 15\,a^2b^8x^4 + 20\,a^3b^7x^3 + 15\,a^4b^6x^2 + 6\,a^5b^5x + a^6b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.
time = 0.23, size = 104, normalized size = 2.00

$$\frac{-a^3 - 6a^2bx - 15ab^2x^2 - 20b^3x^3}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**7,x)
```

```
[Out] (-a**3 - 6*a**2*b*x - 15*a*b**2*x**2 - 20*b**3*x**3)/(60*a**6*b**4 + 360*a*
*5*b**5*x + 900*a**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 +
 360*a*b**9*x**5 + 60*b**10*x**6)
```

Giac [A]
time = 0.00, size = 46, normalized size = 0.88

$$\frac{-20x^3b^3 - 15x^2b^2a - 6xba^2 - a^3}{60b^4\,(xb + a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x)
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/((b*x + a)^6*b^4)
```

Mupad [B]
time = 0.07, size = 48, normalized size = 0.92

$$\frac{\frac{3\,a}{4\,(a+b\,x)^4} - \frac{1}{3\,(a+b\,x)^3} - \frac{3\,a^2}{5\,(a+b\,x)^5} + \frac{a^3}{6\,(a+b\,x)^6}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^7,x)
```

```
[Out] ((3*a)/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5) + a^3/
(6*(a + b*x)^6))/b^4
```

3.215 $\int \frac{x^2}{(a+bx)^7} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^7,x]
```

```
[Out] -1/6*a^2/(b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x)
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^7} dx &= \int \left(\frac{a^2}{b^2(a+bx)^7}-\frac{2a}{b^2(a+bx)^6}+\frac{1}{b^2(a+bx)^5}\right) dx\\ &= -\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+6abx+15b^2x^2}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2 + 6*a*b*x + 15*b^2*x^2)/(b^3*(a + b*x)^6)
```

Mathics [A]
time = 2.37, size = 86, normalized size = 1.83

$$\frac{-a^2 - 6abx - 15b^2x^2}{60b^3\left(a^6 + 6a^5bx + 15a^4b^2x^2 + 20a^3b^3x^3 + 15a^2b^4x^4 + 6ab^5x^5 + b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(a + b*x)^7,x]')
```

```
[Out] (-a ^ 2 - 6 a b x - 15 b ^ 2 x ^ 2) / (60 b ^ 3 (a ^ 6 + 6 a ^ 5 b x + 15 a
 ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5
x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.08, size = 42, normalized size = 0.89

method	result	size
gosper	$-\frac{15x^2b^2+6abx+a^2}{60(bx+a)^6b^3}$	30
norman	$\frac{-\frac{x^2}{4b}-\frac{ax}{10b^2}-\frac{a^2}{60b^3}}{(bx+a)^6}$	33
risch	$\frac{-\frac{x^2}{4b}-\frac{ax}{10b^2}-\frac{a^2}{60b^3}}{(bx+a)^6}$	33
default	$-\frac{a^2}{6b^3(bx+a)^6}+\frac{2a}{5b^3(bx+a)^5}-\frac{1}{4b^3(bx+a)^4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.
time = 0.25, size = 87, normalized size = 1.85

$$-\frac{15\,b^2x^2 + 6\,abx + a^2}{60\left(b^9x^6 + 6\,ab^8x^5 + 15\,a^2b^7x^4 + 20\,a^3b^6x^3 + 15\,a^4b^5x^2 + 6\,a^5b^4x + a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.
time = 0.31, size = 87, normalized size = 1.85

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(42) = 84$.
time = 0.22, size = 92, normalized size = 1.96

$$\frac{-a^2-6abx-15b^2x^2}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**7,x)
```

```
[Out] (-a**2 - 6*a*b*x - 15*b**2*x**2)/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4
*b**5*x**2 + 1200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 6
0*b**9*x**6)
```

Giac [A]
time = 0.00, size = 34, normalized size = 0.72

$$\frac{-15x^2b^2-6xba-a^2}{60b^3\,(xb+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x)
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/((b*x + a)^6*b^3)
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.66

$$-\frac{8\,a^2+48\,a\,b\,x+120\,b^2\,x^2}{480\,b^3\,(a+b\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^7,x)
```

```
[Out] -(8*a^2 + 120*b^2*x^2 + 48*a*b*x)/(480*b^3*(a + b*x)^6)
```

3.216 $\int \frac{x}{(a+bx)^7} dx$

Optimal. Leaf size=30

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

```
[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^7,x]
```

```
[Out] a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(a+bx)^7} dx &= \int \left(-\frac{a}{b(a+bx)^7} + \frac{1}{b(a+bx)^6}\right) dx \\ &= \frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.67

$$-\frac{a+6bx}{30b^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^7,x]
```

`[Out] -1/30*(a + 6*b*x)/(b^2*(a + b*x)^6)`

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 77 vs. $2(30) = 60$.
time = 2.43, size = 75, normalized size = 2.50

$$\frac{-a - 6bx}{30b^2 (a^6 + 6a^5bx + 15a^4b^2x^2 + 20a^3b^3x^3 + 15a^2b^4x^4 + 6ab^5x^5 + b^6x^6)}$$

Antiderivative was successfully verified.

`[In] mathics('Integrate[x^1/(a + b*x)^7,x]')`

`[Out] (-a - 6 b x) / (30 b ^ 2 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))`

Maple [A]
time = 0.08, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{6bx+a}{30(bx+a)^6 b^2}$	19
norman	$\frac{-\frac{x}{5b}-\frac{a}{30b^2}}{(bx+a)^6}$	22
risch	$\frac{-\frac{x}{5b}-\frac{a}{30b^2}}{(bx+a)^6}$	22
default	$\frac{a}{6b^2(bx+a)^6}-\frac{1}{5b^2(bx+a)^5}$	27

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x/(b*x+a)^7,x,method=_RETURNVERBOSE)`

`[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5`

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.24, size = 76, normalized size = 2.53

$$-\frac{6\,bx + a}{30\,(b^8x^6 + 6\,ab^7x^5 + 15\,a^2b^6x^4 + 20\,a^3b^5x^3 + 15\,a^4b^4x^2 + 6\,a^5b^3x + a^6b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x/(b*x+a)^7,x, algorithm="maxima")`

`[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3 + 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)`

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.30, size = 76, normalized size = 2.53

$$-\frac{6\,bx + a}{30\,(b^8x^6 + 6\,ab^7x^5 + 15\,a^2b^6x^4 + 20\,a^3b^5x^3 + 15\,a^4b^4x^2 + 6\,a^5b^3x + a^6b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(26) = 52$.
time = 0.21, size = 80, normalized size = 2.67

$$\frac{-a - 6bx}{30a^6b^2 + 180a^5b^3x + 450a^4b^4x^2 + 600a^3b^5x^3 + 450a^2b^6x^4 + 180ab^7x^5 + 30b^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**7,x)
```

```
[Out] (-a - 6*b*x)/(30*a**6*b**2 + 180*a**5*b**3*x + 450*a**4*b**4*x**2 + 600*a**
3*b**5*x**3 + 450*a**2*b**6*x**4 + 180*a*b**7*x**5 + 30*b**8*x**6)
```

Giac [A]
time = 0.00, size = 22, normalized size = 0.73

$$\frac{-6xb - a}{30b^2\left(xb + a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x)
```

```
[Out] -1/30*(6*b*x + a)/((b*x + a)^6*b^2)
```

Mupad [B]
time = 0.10, size = 18, normalized size = 0.60

$$-\frac{a + 6\,b\,x}{30\,b^2\,{\left(a + b\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^7,x)
```

```
[Out] -(a + 6*b*x)/(30*b^2*(a + b*x)^6)
```

3.217 $\int \frac{1}{(a+bx)^7} dx$

Optimal. Leaf size=14

$$-\frac{1}{6b(a+bx)^6}$$

[Out] -1/6/b/(b*x+a)^6

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-7),x]

[Out] -1/6*1/(b*(a + b*x)^6)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^7} dx = -\frac{1}{6b(a+bx)^6}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-7),x]

[Out] -1/6*1/(b*(a + b*x)^6)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(14) = 28.
time = 2.32, size = 67, normalized size = 4.79

$$-\frac{1}{6b\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^7,x]')
```

```
[Out] -1 / (6 b (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^
3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{6b(bx+a)^{6}}$	13
default	$-\frac{1}{6b(bx+a)^{6}}$	13
norman	$-\frac{1}{6b(bx+a)^{6}}$	13
risch	$-\frac{1}{6b(bx+a)^{6}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b/(b*x+a)^6
```

Maxima [A]
time = 0.24, size = 12, normalized size = 0.86

$$-\frac{1}{6\,(bx+a)^{6}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.
time = 0.30, size = 68, normalized size = 4.86

$$-\frac{1}{6\,(b^{7}x^{6}+6\,ab^{6}x^{5}+15\,a^{2}b^{5}x^{4}+20\,a^{3}b^{4}x^{3}+15\,a^{4}b^{3}x^{2}+6\,a^{5}b^{2}x+a^{6}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b^4*x^3 + 15*a^4*b^3*
x^2 + 6*a^5*b^2*x + a^6*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.
time = 0.21, size = 73, normalized size = 5.21

$$-\frac{1}{6a^6b + 36a^5b^2x + 90a^4b^3x^2 + 120a^3b^4x^3 + 90a^2b^5x^4 + 36ab^6x^5 + 6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**7,x)
```

```
[Out] -1/(6*a**6*b + 36*a**5*b**2*x + 90*a**4*b**3*x**2 + 120*a**3*b**4*x**3 + 90
*a**2*b**5*x**4 + 36*a*b**6*x**5 + 6*b**7*x**6)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{6b\,(xb+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x)
```

```
[Out] -1/6/((b*x + a)^6*b)
```

Mupad [B]
time = 0.06, size = 70, normalized size = 5.00

$$-\frac{1}{6\,a^6\,b + 36\,a^5\,b^2\,x + 90\,a^4\,b^3\,x^2 + 120\,a^3\,b^4\,x^3 + 90\,a^2\,b^5\,x^4 + 36\,a\,b^6\,x^5 + 6\,b^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^7,x)
```

```
[Out] -1/(6*a^6*b + 6*b^7*x^6 + 36*a^5*b^2*x + 36*a*b^6*x^5 + 90*a^4*b^3*x^2 + 12
0*a^3*b^4*x^3 + 90*a^2*b^5*x^4)
```

3.218 $\int \frac{1}{x(a+bx)^7} \, dx$

Optimal. Leaf size=99

$$\frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx)}+\frac{\log(x)}{a^7}-\frac{\log(a+bx)}{a^7}$$

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^7),x]
```

```
[Out] 1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3*
a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7 -
 Log[a + b*x]/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^7} \, dx &= \int \left(\frac{1}{a^7 x}-\frac{b}{a(a+bx)^7}-\frac{b}{a^2(a+bx)^6}-\frac{b}{a^3(a+bx)^5}-\frac{b}{a^4(a+bx)^4}-\frac{b}{a^5(a+bx)^3}-\frac{b}{a^6(\right.\\ &= \frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 0.82

$$\frac{\frac{a(147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5)}{(a+bx)^6}+60\log(x)-60\log(a+bx)}{60a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^7),x]
```

```
[Out] ((a*(147*a^5 + 522*a^4*b*x + 855*a^3*b^2*x^2 + 740*a^2*b^3*x^3 + 330*a*b^4*
x^4 + 60*b^5*x^5))/(a + b*x)^6 + 60*Log[x] - 60*Log[a + b*x])/(60*a^7)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(99) = 198.
time = 3.20, size = 198, normalized size = 2.00

$$\frac{\frac{a(147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5)}{60}+(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^7\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)^7),x]')
```

```
[Out] (a (147 a ^ 5 + 522 a ^ 4 b x + 855 a ^ 3 b ^ 2 x ^ 2 + 740 a ^ 2 b ^ 3 x ^
 3 + 330 a b ^ 4 x ^ 4 + 60 b ^ 5 x ^ 5) / 60 + (a ^ 6 + 6 a ^ 5 b x + 15 a
 ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5
x ^ 5 + b ^ 6 x ^ 6) (Log[x] - Log[(a + b x) / b])) / (a ^ 7 (a ^ 6 + 6 a ^
 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4
 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [**A**]
time = 0.09, size = 90, normalized size = 0.91

method	result	size
risch	$\frac{\frac{b^5x^5}{a^6}+\frac{11b^4x^4}{2a^5}+\frac{37b^3x^3}{3a^4}+\frac{57b^2x^2}{4a^3}+\frac{87bx}{10a^2}+\frac{49}{20a}}{(bx+a)^6}-\frac{\ln(bx+a)}{a^7}+\frac{\ln(-x)}{a^7}$	85
default	$\frac{1}{6a(bx+a)^6}+\frac{1}{5a^2(bx+a)^5}+\frac{1}{4a^3(bx+a)^4}+\frac{1}{3a^4(bx+a)^3}+\frac{1}{2a^5(bx+a)^2}+\frac{1}{a^6(bx+a)}+\frac{\ln(x)}{a^7}-\frac{\ln(bx+a)}{a^7}$	90
norman	$\frac{-\frac{6bx}{a^2}-\frac{45b^2x^2}{2a^3}-\frac{110b^3x^3}{3a^4}-\frac{125b^4x^4}{4a^5}-\frac{137b^5x^5}{10a^6}-\frac{49b^6x^6}{20a^7}}{(bx+a)^6}+\frac{\ln(x)}{a^7}-\frac{\ln(bx+a)}{a^7}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

Maxima [**A**]
time = 0.25, size = 139, normalized size = 1.40

$$\frac{60\,b^5x^5+330\,ab^4x^4+740\,a^2b^3x^3+855\,a^3b^2x^2+522\,a^4bx+147\,a^5}{60\left(a^6b^6x^6+6\,a^7b^5x^5+15\,a^8b^4x^4+20\,a^9b^3x^3+15\,a^{10}b^2x^2+6\,a^{11}bx+a^{12}\right)}-\frac{\log\left(bx+a\right)}{a^7}+\frac{\log\left(x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(60*b^5*x^5 + 330*a*b^4*x^4 + 740*a^2*b^3*x^3 + 855*a^3*b^2*x^2 + 522*
a^4*b*x + 147*a^5)/(a^6*b^6*x^6 + 6*a^7*b^5*x^5 + 15*a^8*b^4*x^4 + 20*a^9*b
^3*x^3 + 15*a^10*b^2*x^2 + 6*a^11*b*x + a^12) - log(b*x + a)/a^7 + log(x)/a
^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(89) = 178$.
time = 0.31, size = 256, normalized size = 2.59

$$\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6-60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\log\left(bx+a\right)+60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\log\left(x\right)}{60\,(a^7b^6x^6+6\,a^8b^5x^5+15\,a^9b^4x^4+20\,a^{10}b^3x^3+15\,a^{11}b^2x^2+6\,a^{12}bx+a^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 +
522*a^5*b*x + 147*a^6 - 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3
*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a) + 60*(b^6*x^6 + 6
*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x +
 a^6)*log(x))/(a^7*b^6*x^6 + 6*a^8*b^5*x^5 + 15*a^9*b^4*x^4 + 20*a^10*b^3*x
^3 + 15*a^11*b^2*x^2 + 6*a^12*b*x + a^13)
```

Sympy [A]
time = 0.33, size = 141, normalized size = 1.42

$$\frac{147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5}{60a^{12}+360a^{11}bx+900a^{10}b^2x^2+1200a^9b^3x^3+900a^8b^4x^4+360a^7b^5x^5+60a^6b^6x^6}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**7,x)
```

```
[Out] (147*a**5 + 522*a**4*b*x + 855*a**3*b**2*x**2 + 740*a**2*b**3*x**3 + 330*a*
b**4*x**4 + 60*b**5*x**5)/(60*a**12 + 360*a**11*b*x + 900*a**10*b**2*x**2 +
 1200*a**9*b**3*x**3 + 900*a**8*b**4*x**4 + 360*a**7*b**5*x**5 + 60*a**6*b*
*6*x**6) + (log(x) - log(a/b + x))/a**7
```

Giac [A]
time = 0.00, size = 96, normalized size = 0.97

$$\frac{\ln|x|}{a^7}-\frac{b\ln|xb+a|}{ba^7}+\frac{\frac{1}{60}\left(60b^5ax^5+330b^4a^2x^4+740b^3a^3x^3+855b^2a^4x^2+522ba^5x+147a^6\right)}{a^7\left(xb+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x)
```

[Out] -log(abs(b*x + a))/a^7 + log(abs(x))/a^7 + 1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 + 522*a^5*b*x + 147*a^6)/((b*x + a)^6*a^7)

Mupad [B]
time = 0.45, size = 102, normalized size = 1.03

$$-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{15\,b^2\,x^2}{2\,{\left(a+b\,x\right)}^2}+\frac{20\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{15\,b^4\,x^4}{4\,{\left(a+b\,x\right)}^4}+\frac{6\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{b^6\,x^6}{6\,{\left(a+b\,x\right)}^6}+\frac{6\,b\,x}{a+b\,x}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x)^7),x)

[Out] -(log((a + b*x)/x) - (15*b^2*x^2)/(2*(a + b*x)^2) + (20*b^3*x^3)/(3*(a + b*x)^3) - (15*b^4*x^4)/(4*(a + b*x)^4) + (6*b^5*x^5)/(5*(a + b*x)^5) - (b^6*x^6)/(6*(a + b*x)^6) + (6*b*x)/(a + b*x))/a^7

3.219 $\int \frac{1}{x^2(a+bx)^7}\,dx$

Optimal. Leaf size=117

$$-\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-\frac{6b}{a^7(a+bx)}-\frac{7b\log(x)}{a^8}+\frac{7b}{}$$

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}-\frac{6b}{a^7(a+bx)}-\frac{1}{a^7x}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(4
*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2) -
 (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^7}\,dx=\int \left(\frac{1}{a^7x^2}-\frac{7b}{a^8x}+\frac{b^2}{a^2(a+bx)^7}+\frac{2b^2}{a^3(a+bx)^6}+\frac{3b^2}{a^4(a+bx)^5}+\frac{4b^2}{a^5(a+bx)^4}+\frac{5b^2}{a^6(a+b}\right.$$
$$=-\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.83

$$-\frac{\frac{a(60a^6+1029a^5bx+3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+2310ab^5x^5+420b^6x^6)}{x(a+bx)^6}+420b\log(x)-420b\log(a+bx)}{60a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -1/60*((a*(60*a^6 + 1029*a^5*b*x + 3654*a^4*b^2*x^2 + 5985*a^3*b^3*x^3 + 51
80*a^2*b^4*x^4 + 2310*a*b^5*x^5 + 420*b^6*x^6))/(x*(a + b*x)^6) + 420*b*Log
[x] - 420*b*Log[a + b*x])/a^8
```

Mathics [A]
time = 3.42, size = 215, normalized size = 1.84

$$\frac{a\left(-60a^6-1029a^5bx-3654a^4b^2x^2-5985a^3b^3x^3-5180a^2b^4x^4-2310ab^5x^5-420b^6x^6\right)-420bx\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{60a^8x\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^7),x]')
```

```
[Out] (a (-60 a ^ 6 - 1029 a ^ 5 b x - 3654 a ^ 4 b ^ 2 x ^ 2 - 5985 a ^ 3 b ^ 3
x ^ 3 - 5180 a ^ 2 b ^ 4 x ^ 4 - 2310 a b ^ 5 x ^ 5 - 420 b ^ 6 x ^ 6) - 42
0 b x (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 +
15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6) (Log[x] - Log[(a + b
x) / b])) / (60 a ^ 8 x (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a
^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.09, size = 108, normalized size = 0.92

method	result
risch	$\frac{-\frac{7b^6x^6}{a^7}-\frac{77b^5x^5}{2a^6}-\frac{259b^4x^4}{3a^5}-\frac{399b^3x^3}{4a^4}-\frac{609b^2x^2}{10a^3}-\frac{343bx}{20a^2}-\frac{1}{a}}{x(bx+a)^6}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(-bx-a)}{a^8}$
norman	$\frac{-\frac{1}{a}+\frac{42b^2x^2}{a^3}+\frac{315b^3x^3}{2a^4}+\frac{770b^4x^4}{3a^5}+\frac{875b^5x^5}{4a^6}+\frac{959b^6x^6}{10a^7}+\frac{343b^7x^7}{20a^8}}{x(bx+a)^6}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(bx+a)}{a^8}$
default	$-\frac{1}{a^7x}-\frac{b}{6a^2(bx+a)^6}-\frac{2b}{5a^3(bx+a)^5}-\frac{3b}{4a^4(bx+a)^4}-\frac{4b}{3a^5(bx+a)^3}-\frac{5b}{2a^6(bx+a)^2}-\frac{6b}{a^7(bx+a)}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(bx+a)}{a^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

Maxima [A]
time = 0.27, size = 157, normalized size = 1.34

$$-\frac{420\,b^6x^6+2310\,ab^5x^5+5180\,a^2b^4x^4+5985\,a^3b^3x^3+3654\,a^4b^2x^2+1029\,a^5bx+60\,a^6}{60\,(a^7b^6x^7+6\,a^8b^5x^6+15\,a^9b^4x^5+20\,a^{10}b^3x^4+15\,a^{11}b^2x^3+6\,a^{12}bx^2+a^{13}x)}+\frac{7\,b\log\left(bx+a\right)}{a^8}-\frac{7\,b\log\left(x\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(420*b^6*x^6 + 2310*a*b^5*x^5 + 5180*a^2*b^4*x^4 + 5985*a^3*b^3*x^3 +
 3654*a^4*b^2*x^2 + 1029*a^5*b*x + 60*a^6)/(a^7*b^6*x^7 + 6*a^8*b^5*x^6 + 1
5*a^9*b^4*x^5 + 20*a^10*b^3*x^4 + 15*a^11*b^2*x^3 + 6*a^12*b*x^2 + a^13*x)
+ 7*b*log(b*x + a)/a^8 - 7*b*log(x)/a^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(107) = 214$.
time = 0.33, size = 285, normalized size = 2.44

$$-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7-420\,(b^7x^7+6\,ab^6x^6+15\,a^2b^5x^5+20\,a^3b^4x^4+15\,a^4b^3x^3+6\,a^5b^2x^2+a^6bx)\log(bx+a)+420\,(b^7x^7+6\,ab^6x^6+15\,a^2b^5x^5+20\,a^3b^4x^4+15\,a^4b^3x^3+6\,a^5b^2x^2+a^6bx)\log(x)}{60\,(a^8b^6x^7+6\,a^9b^5x^6+15\,a^{10}b^4x^5+20\,a^{11}b^3x^4+15\,a^{12}b^2x^3+6\,a^{13}bx^2+a^{14}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(420*a*b^6*x^6 + 2310*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x
^3 + 3654*a^5*b^2*x^2 + 1029*a^6*b*x + 60*a^7 - 420*(b^7*x^7 + 6*a*b^6*x^6
+ 15*a^2*b^5*x^5 + 20*a^3*b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*
x)*log(b*x + a) + 420*(b^7*x^7 + 6*a*b^6*x^6 + 15*a^2*b^5*x^5 + 20*a^3*b^4*
x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*x)*log(x))/(a^8*b^6*x^7 + 6*a^
9*b^5*x^6 + 15*a^10*b^4*x^5 + 20*a^11*b^3*x^4 + 15*a^12*b^2*x^3 + 6*a^13*b*
x^2 + a^14*x)
```

Sympy [A]
time = 0.37, size = 162, normalized size = 1.38

$$\frac{-60a^6-1029a^5bx-3654a^4b^2x^2-5985a^3b^3x^3-5180a^2b^4x^4-2310ab^5x^5-420b^6x^6}{60a^{13}x+360a^{12}bx^2+900a^{11}b^2x^3+1200a^{10}b^3x^4+900a^9b^4x^5+360a^8b^5x^6+60a^7b^6x^7}+\frac{7b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**7,x)
```

```
[Out] (-60*a**6 - 1029*a**5*b*x - 3654*a**4*b**2*x**2 - 5985*a**3*b**3*x**3 - 518
0*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 420*b**6*x**6)/(60*a**13*x + 360*a**1
2*b*x**2 + 900*a**11*b**2*x**3 + 1200*a**10*b**3*x**4 + 900*a**9*b**4*x**5
+ 360*a**8*b**5*x**6 + 60*a**7*b**6*x**7) + 7*b*(-log(x) + log(a/b + x))/a*
*8
```

Giac [A]
time = 0.00, size = 121, normalized size = 1.03

$$-\frac{7b\ln|x|}{a^8}+\frac{7b^2\ln|xb+a|}{ba^8}+\frac{\frac{1}{60}\left(-420b^6ax^6-2310b^5a^2x^5-5180b^4a^3x^4-5985b^3a^4x^3-3654b^2a^5x^2-1029ba^6x-60a^7\right)}{a^8x\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x)
```

```
[Out] 7*b*log(abs(b*x + a))/a^8 - 7*b*log(abs(x))/a^8 - 1/60*(420*a*b^6*x^6 + 231
0*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x^3 + 3654*a^5*b^2*x^2 + 10
29*a^6*b*x + 60*a^7)/((b*x + a)^6*a^8*x)
```

Mupad [B]
time = 0.19, size = 151, normalized size = 1.29

$$\frac{14\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8}-\frac{\frac{1}{a}+\frac{609\,b^2\,x^2}{10\,a^3}+\frac{399\,b^3\,x^3}{4\,a^4}+\frac{259\,b^4\,x^4}{3\,a^5}+\frac{77\,b^5\,x^5}{2\,a^6}+\frac{7\,b^6\,x^6}{a^7}+\frac{343\,b\,x}{20\,a^2}}{a^6\,x+6\,a^5\,b\,x^2+15\,a^4\,b^2\,x^3+20\,a^3\,b^3\,x^4+15\,a^2\,b^4\,x^5+6\,a\,b^5\,x^6+b^6\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^7),x)
```

```
[Out] (14*b*atanh((2*b*x)/a + 1))/a^8 - (1/a + (609*b^2*x^2)/(10*a^3) + (399*b^3*
x^3)/(4*a^4) + (259*b^4*x^4)/(3*a^5) + (77*b^5*x^5)/(2*a^6) + (7*b^6*x^6)/a
^7 + (343*b*x)/(20*a^2))/(a^6*x + b^6*x^7 + 6*a^5*b*x^2 + 6*a*b^5*x^6 + 15*
a^4*b^2*x^3 + 20*a^3*b^3*x^4 + 15*a^2*b^4*x^5)
```

3.220 $\int \frac{1}{x^3(a+bx)^7}\,dx$

Optimal. Leaf size=144

$$-\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15b^2}{2a^7(a+bx)^2}+\frac{21b^2}{a^8(a+bx)}+\frac{28b^2 \log(x)}{a^9}$$

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{21b^2}{a^8(a+bx)}+\frac{7b}{a^8x}+\frac{15b^2}{2a^7(a+bx)^2}-\frac{1}{2a^7x^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^7),x]
```

```
[Out] -1/2*1/(a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^4
*(a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3)
+ (15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[x])
/a^9 - (28*b^2*Log[a + b*x])/a^9
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x^3}-\frac{7b}{a^8x^2}+\frac{28b^2}{a^9x}-\frac{b^3}{a^3(a+bx)^7}-\frac{3b^3}{a^4(a+bx)^6}-\frac{6b^3}{a^5(a+bx)^5}-\frac{10b^3}{a^6(a+bx)^4}-\right. \\ &= -\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15}{2a^7(a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 112, normalized size = 0.78

$$\frac{\frac{a(-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7)}{x^2(a+bx)^6}+840b^2\log(x)-840b^2\log(a+bx)}{30a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^7),x]
```

```
[Out] ((a*(-15*a^7 + 120*a^6*b*x + 2058*a^5*b^2*x^2 + 7308*a^4*b^3*x^3 + 11970*a^
3*b^4*x^4 + 10360*a^2*b^5*x^5 + 4620*a*b^6*x^6 + 840*b^7*x^7))/(x^2*(a + b*
x)^6) + 840*b^2*Log[x] - 840*b^2*Log[a + b*x])/(30*a^9)
```

Mathics [A]
time = 3.53, size = 230, normalized size = 1.60

$$\frac{a\left(-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7\right)+840b^2x^2\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{30a^9x^2\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^7),x]')
```

```
[Out] (a (-15 a ^ 7 + 120 a ^ 6 b x + 2058 a ^ 5 b ^ 2 x ^ 2 + 7308 a ^ 4 b ^ 3 x
 ^ 3 + 11970 a ^ 3 b ^ 4 x ^ 4 + 10360 a ^ 2 b ^ 5 x ^ 5 + 4620 a b ^ 6 x ^
 6 + 840 b ^ 7 x ^ 7) + 840 b ^ 2 x ^ 2 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^
 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 +
b ^ 6 x ^ 6) (Log[x] - Log[(a + b x) / b])) / (30 a ^ 9 x ^ 2 (a ^ 6 + 6 a
^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^
4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.09, size = 133, normalized size = 0.92

method	result
norman	$\frac{-\frac{1}{2a}+\frac{4bx}{a^2}-\frac{168b^3x^3}{a^4}-\frac{630b^4x^4}{a^5}-\frac{3080b^5x^5}{3a^6}-\frac{875b^6x^6}{a^7}-\frac{1918b^7x^7}{5a^8}-\frac{343b^8x^8}{5a^9}}{x^2(bx+a)^6}+\frac{28b^2\ln(x)}{a^9}-\frac{28b^2\ln(bx+a)}{a^9}$
risch	$\frac{\frac{28b^7x^7}{a^8}+\frac{154b^6x^6}{a^7}+\frac{1036b^5x^5}{3a^6}+\frac{399b^4x^4}{a^5}+\frac{1218b^3x^3}{5a^4}+\frac{343b^2x^2}{5a^3}+\frac{4bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^6}-\frac{28b^2\ln(bx+a)}{a^9}+\frac{28b^2\ln(-x)}{a^9}$
default	$-\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(bx+a)^6}+\frac{3b^2}{5a^4(bx+a)^5}+\frac{3b^2}{2a^5(bx+a)^4}+\frac{10b^2}{3a^6(bx+a)^3}+\frac{15b^2}{2a^7(bx+a)^2}+\frac{21b^2}{a^8(bx+a)}+\frac{28b^2\ln(x)}{a^9}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

Maxima [A]
time = 0.26, size = 174, normalized size = 1.21

$$\frac{840\,b^7x^7+4620\,ab^6x^6+10360\,a^2b^5x^5+11970\,a^3b^4x^4+7308\,a^4b^3x^3+2058\,a^5b^2x^2+120\,a^6bx-15\,a^7}{30\,(a^8b^6x^8+6\,a^9b^5x^7+15\,a^{10}b^4x^6+20\,a^{11}b^3x^5+15\,a^{12}b^2x^4+6\,a^{13}bx^3+a^{14}x^2)}-\frac{28\,b^2\log(bx+a)}{a^9}+\frac{28\,b^2\log(x)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(840*b^7*x^7 + 4620*a*b^6*x^6 + 10360*a^2*b^5*x^5 + 11970*a^3*b^4*x^4
+ 7308*a^4*b^3*x^3 + 2058*a^5*b^2*x^2 + 120*a^6*b*x - 15*a^7)/(a^8*b^6*x^8
+ 6*a^9*b^5*x^7 + 15*a^10*b^4*x^6 + 20*a^11*b^3*x^5 + 15*a^12*b^2*x^4 + 6*a
^13*b*x^3 + a^14*x^2) - 28*b^2*log(b*x + a)/a^9 + 28*b^2*log(x)/a^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(132) = 264$.
time = 0.31, size = 306, normalized size = 2.12

$$\frac{840\,ab^7x^7 + 4620\,a^2b^6x^6 + 10360\,a^3b^5x^5 + 11970\,a^4b^4x^4 + 7308\,a^5b^3x^3 + 2058\,a^6b^2x^2 + 120\,a^7bx - 15\,a^8 - 840\,(b^8x^8 + 6\,ab^7x^7 + 15\,a^2b^6x^6 + 20\,a^3b^5x^5 + 15\,a^4b^4x^4 + 6\,a^5b^3x^3 + a^6b^2x^2)\log(bx+a) + 840\,(b^8x^8 + 6\,ab^7x^7 + 15\,a^2b^6x^6 + 20\,a^3b^5x^5 + 15\,a^4b^4x^4 + 6\,a^5b^3x^3 + a^6b^2x^2)\log(x)}{30\,(a^9b^6x^8 + 6\,a^{10}b^5x^7 + 15\,a^{11}b^4x^6 + 20\,a^{12}b^3x^5 + 15\,a^{13}b^2x^4 + 6\,a^{14}bx^3 + a^{15}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*
x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8 - 840*(b^8
*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a
^5*b^3*x^3 + a^6*b^2*x^2)*log(b*x + a) + 840*(b^8*x^8 + 6*a*b^7*x^7 + 15*a^
2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*x^2)*
log(x))/(a^9*b^6*x^8 + 6*a^10*b^5*x^7 + 15*a^11*b^4*x^6 + 20*a^12*b^3*x^5 +
 15*a^13*b^2*x^4 + 6*a^14*b*x^3 + a^15*x^2)
```

Sympy [A]
time = 0.39, size = 175, normalized size = 1.22

$$\frac{-15a^7 + 120a^6bx + 2058a^5b^2x^2 + 7308a^4b^3x^3 + 11970a^3b^4x^4 + 10360a^2b^5x^5 + 4620ab^6x^6 + 840b^7x^7}{30a^{14}x^2 + 180a^{13}bx^3 + 450a^{12}b^2x^4 + 600a^{11}b^3x^5 + 450a^{10}b^4x^6 + 180a^9b^5x^7 + 30a^8b^6x^8} + \frac{28b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**7,x)
```

```
[Out] (-15*a**7 + 120*a**6*b*x + 2058*a**5*b**2*x**2 + 7308*a**4*b**3*x**3 + 1197
0*a**3*b**4*x**4 + 10360*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 840*b**7*x**7)
/(30*a**14*x**2 + 180*a**13*b*x**3 + 450*a**12*b**2*x**4 + 600*a**11*b**3*x
**5 + 450*a**10*b**4*x**6 + 180*a**9*b**5*x**7 + 30*a**8*b**6*x**8) + 28*b*
*2*(log(x) - log(a/b + x))/a**9
```

Giac [A]
time = 0.00, size = 130, normalized size = 0.90

$$\frac{28b^2\ln|x|}{a^9} - \frac{28b^3\ln|xb+a|}{ba^9} + \frac{\frac{1}{30}\left(840b^7ax^7 + 4620b^6a^2x^6 + 10360b^5a^3x^5 + 11970b^4a^4x^4 + 7308b^3a^5x^3 + 2058b^2a^6x^2 + 120ba^7x - 15a^8\right)}{a^9x^2\left(bx+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x)
```

```
[Out] -28*b^2*log(abs(b*x + a))/a^9 + 28*b^2*log(abs(x))/a^9 + 1/30*(840*a*b^7*x^
7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b^3
*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8)/((b*x + a)^6*a^9*x^2)
```

Mupad [B]
time = 0.21, size = 167, normalized size = 1.16

$$\frac{\frac{343\,b^2\,x^2}{5\,a^3}-\frac{1}{2\,a}+\frac{1218\,b^3\,x^3}{5\,a^4}+\frac{399\,b^4\,x^4}{a^5}+\frac{1036\,b^5\,x^5}{3\,a^6}+\frac{154\,b^6\,x^6}{a^7}+\frac{28\,b^7\,x^7}{a^8}+\frac{4\,b\,x}{a^2}}{a^6\,x^2+6\,a^5\,b\,x^3+15\,a^4\,b^2\,x^4+20\,a^3\,b^3\,x^5+15\,a^2\,b^4\,x^6+6\,a\,b^5\,x^7+b^6\,x^8}-\frac{56\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^7),x)
```

```
[Out] ((343*b^2*x^2)/(5*a^3) - 1/(2*a) + (1218*b^3*x^3)/(5*a^4) + (399*b^4*x^4)/a
^5 + (1036*b^5*x^5)/(3*a^6) + (154*b^6*x^6)/a^7 + (28*b^7*x^7)/a^8 + (4*b*x
)/a^2)/(a^6*x^2 + b^6*x^8 + 6*a^5*b*x^3 + 6*a*b^5*x^7 + 15*a^4*b^2*x^4 + 20
*a^3*b^3*x^5 + 15*a^2*b^4*x^6) - (56*b^2*atanh((2*b*x)/a + 1))/a^9
```

3.221 $\int \frac{1}{x^4(a+bx)^7}\,dx$

Optimal. Leaf size=157

$$-\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{35b^3}{2a^8(a+bx)^2}-\frac{56b^3}{a^9(a+bx}$$

```
[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(
b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^
2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Rubi [A]
time = 0.07, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}-\frac{56b^3}{a^9(a+bx)}-\frac{28b^2}{a^9x}-\frac{35b^3}{2a^8(a+bx)^2}+\frac{7b}{2a^8x^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{1}{3a^7x^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/3*1/(a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a + b
*x)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*b^
3)/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a +
b*x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x^4}-\frac{7b}{a^8x^3}+\frac{28b^2}{a^9x^2}-\frac{84b^3}{a^{10}x}+\frac{b^4}{a^4(a+bx)^7}+\frac{4b^4}{a^5(a+bx)^6}+\frac{10b^4}{a^6(a+bx)^5}+\frac{20b}{a^7(a+}\right.\\ &= -\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 123, normalized size = 0.78

$$-\frac{\frac{a(10a^8-45a^7bx+360a^6b^2x^2+6174a^5b^3x^3+21924a^4b^4x^4+35910a^3b^5x^5+31080a^2b^6x^6+13860ab^7x^7+2520b^8x^8)}{x^3(a+bx)^6}+2520b^3\log(x)-2520b^3\log(a+bx)}{30a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/30*((a*(10*a^8 - 45*a^7*b*x + 360*a^6*b^2*x^2 + 6174*a^5*b^3*x^3 + 21924
*a^4*b^4*x^4 + 35910*a^3*b^5*x^5 + 31080*a^2*b^6*x^6 + 13860*a*b^7*x^7 + 25
20*b^8*x^8))/(x^3*(a + b*x)^6) + 2520*b^3*Log[x] - 2520*b^3*Log[a + b*x])/a
^10
```

Mathics [A]
time = 3.59, size = 241, normalized size = 1.54

$$\frac{a\left(-10a^8+45a^7bx-360a^6b^2x^2-6174a^5b^3x^3-21924a^4b^4x^4-35910a^3b^5x^5-31080a^2b^6x^6-13860ab^7x^7-2520b^8x^8\right)-2520b^3x^3\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{30a^{10}x^3\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(a + b*x)^7),x]')
```

```
[Out] (a (-10 a ^ 8 + 45 a ^ 7 b x - 360 a ^ 6 b ^ 2 x ^ 2 - 6174 a ^ 5 b ^ 3 x ^
 3 - 21924 a ^ 4 b ^ 4 x ^ 4 - 35910 a ^ 3 b ^ 5 x ^ 5 - 31080 a ^ 2 b ^ 6
x ^ 6 - 13860 a b ^ 7 x ^ 7 - 2520 b ^ 8 x ^ 8) - 2520 b ^ 3 x ^ 3 (a ^ 6 +
 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4
 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6) (Log[x] - Log[(a + b x) / b])) / (3
0 a ^ 10 x ^ 3 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3
 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.11, size = 144, normalized size = 0.92

method	result
norman	$\frac{-\frac{1}{3a}+\frac{3bx}{2a^2}-\frac{12b^2x^2}{a^3}+\frac{504b^4x^4}{a^5}+\frac{1890b^5x^5}{a^6}+\frac{3080b^6x^6}{a^7}+\frac{2625b^7x^7}{a^8}+\frac{5754b^8x^8}{5a^9}+\frac{1029b^9x^9}{5a^{10}}}{x^3(bx+a)^6}-\frac{84b^3\ln(x)}{a^{10}}+\frac{84b^3\ln(bx+a)}{a^{10}}$
risch	$\frac{-\frac{84b^8x^8}{a^9}-\frac{462b^7x^7}{a^8}-\frac{1036b^6x^6}{a^7}-\frac{1197b^5x^5}{a^6}-\frac{3654b^4x^4}{5a^5}-\frac{1029b^3x^3}{5a^4}-\frac{12b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{3a}}{x^3(bx+a)^6}+\frac{84b^3\ln(-bx-a)}{a^{10}}-\frac{84b^3\ln(x)}{a^{10}}$
default	$-\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(bx+a)^6}-\frac{4b^3}{5a^5(bx+a)^5}-\frac{5b^3}{2a^6(bx+a)^4}-\frac{20b^3}{3a^7(bx+a)^3}-\frac{35b^3}{2a^8(bx+a)^2}-\frac{56b^3}{a^9(bx+a)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(
b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^
2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Maxima [A]
time = 0.26, size = 185, normalized size = 1.18

$$-\frac{2520\,b^8x^8+13860\,ab^7x^7+31080\,a^2b^6x^6+35910\,a^3b^5x^5+21924\,a^4b^4x^4+6174\,a^5b^3x^3+360\,a^6b^2x^2-45\,a^7bx+10\,a^8}{30\left(a^9b^6x^9+6\,a^{10}b^5x^8+15\,a^{11}b^4x^7+20\,a^{12}b^3x^6+15\,a^{13}b^2x^5+6\,a^{14}bx^4+a^{15}x^3\right)}+\frac{84\,b^3\log\left(bx+a\right)}{a^{10}}-\frac{84\,b^3\log\left(x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(2520*b^8*x^8 + 13860*a*b^7*x^7 + 31080*a^2*b^6*x^6 + 35910*a^3*b^5*x
^5 + 21924*a^4*b^4*x^4 + 6174*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 45*a^7*b*x +
10*a^8)/(a^9*b^6*x^9 + 6*a^10*b^5*x^8 + 15*a^11*b^4*x^7 + 20*a^12*b^3*x^6 +
 15*a^13*b^2*x^5 + 6*a^14*b*x^4 + a^15*x^3) + 84*b^3*log(b*x + a)/a^10 - 84
*b^3*log(x)/a^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(143) = 286$.
time = 0.31, size = 317, normalized size = 2.02

$$-\frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2-45\,a^8bx+10\,a^9-2520\,(b^9x^9+6\,ab^8x^8+15\,a^2b^7x^7+20\,a^3b^6x^6+15\,a^4b^5x^5+6\,a^5b^4x^4+a^6b^3x^3)\log(bx+a)+2520\,(b^9x^9+6\,ab^8x^8+15\,a^2b^7x^7+20\,a^3b^6x^6+15\,a^4b^5x^5+6\,a^5b^4x^4+a^6b^3x^3)\log(x)}{30\,(a^{10}b^6x^9+6\,a^{11}b^5x^8+15\,a^{12}b^4x^7+20\,a^{13}b^3x^6+15\,a^{14}b^2x^5+6\,a^{15}bx^4+a^{16}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b
^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*
x + 10*a^9 - 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6
+ 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(b*x + a) + 2520*(b^9*x^
9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6*a^5*
b^4*x^4 + a^6*b^3*x^3)*log(x))/(a^10*b^6*x^9 + 6*a^11*b^5*x^8 + 15*a^12*b^4
*x^7 + 20*a^13*b^3*x^6 + 15*a^14*b^2*x^5 + 6*a^15*b*x^4 + a^16*x^3)
```

Sympy [A]
time = 0.41, size = 187, normalized size = 1.19

$$\frac{-10a^8+45a^7bx-360a^6b^2x^2-6174a^5b^3x^3-21924a^4b^4x^4-35910a^3b^5x^5-31080a^2b^6x^6-13860ab^7x^7-2520b^8x^8}{30a^{15}x^3+180a^{14}bx^4+450a^{13}b^2x^5+600a^{12}b^3x^6+450a^{11}b^4x^7+180a^{10}b^5x^8+30a^9b^6x^9}+\frac{84b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**7,x)
```

```
[Out] (-10*a**8 + 45*a**7*b*x - 360*a**6*b**2*x**2 - 6174*a**5*b**3*x**3 - 21924*
a**4*b**4*x**4 - 35910*a**3*b**5*x**5 - 31080*a**2*b**6*x**6 - 13860*a*b**7
*x**7 - 2520*b**8*x**8)/(30*a**15*x**3 + 180*a**14*b*x**4 + 450*a**13*b**2*
x**5 + 600*a**12*b**3*x**6 + 450*a**11*b**4*x**7 + 180*a**10*b**5*x**8 + 30
*a**9*b**6*x**9) + 84*b**3*(-log(x) + log(a/b + x))/a**10
```

Giac [A]
time = 0.00, size = 148, normalized size = 0.94

$$-\frac{84b^3\ln|x|}{a^{10}}+\frac{84b^4\ln|xb+a|}{ba^{10}}+\frac{\frac{1}{30}\left(-2520b^8ax^8-13860b^7a^2x^7-31080b^6a^3x^6-35910b^5a^4x^5-21924b^4a^5x^4-6174b^3a^6x^3-360b^2a^7x^2+45ba^8x-10a^9\right)}{a^{10}x^3\,(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x)
```

```
[Out] 84*b^3*log(abs(b*x + a))/a^10 - 84*b^3*log(abs(x))/a^10 - 1/30*(2520*a*b^8*
x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a^5
*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9)/((b*x
+ a)^6*a^10*x^3)
```

Mupad [B]
time = 0.31, size = 179, normalized size = 1.14

$$\frac{168\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{10}}-\frac{\frac{1}{3\,a}+\frac{12\,b^2\,x^2}{a^3}+\frac{1029\,b^3\,x^3}{5\,a^4}+\frac{3654\,b^4\,x^4}{5\,a^5}+\frac{1197\,b^5\,x^5}{a^6}+\frac{1036\,b^6\,x^6}{a^7}+\frac{462\,b^7\,x^7}{a^8}+\frac{84\,b^8\,x^8}{a^9}-\frac{3\,b\,x}{2\,a^2}}{a^6\,x^3+6\,a^5\,b\,x^4+15\,a^4\,b^2\,x^5+20\,a^3\,b^3\,x^6+15\,a^2\,b^4\,x^7+6\,a\,b^5\,x^8+b^6\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^7),x)
```

```
[Out] (168*b^3*atanh((2*b*x)/a + 1))/a^10 - (1/(3*a) + (12*b^2*x^2)/a^3 + (1029*b
^3*x^3)/(5*a^4) + (3654*b^4*x^4)/(5*a^5) + (1197*b^5*x^5)/a^6 + (1036*b^6*x
^6)/a^7 + (462*b^7*x^7)/a^8 + (84*b^8*x^8)/a^9 - (3*b*x)/(2*a^2))/(a^6*x^3
+ b^6*x^9 + 6*a^5*b*x^4 + 6*a*b^5*x^8 + 15*a^4*b^2*x^5 + 20*a^3*b^3*x^6 + 1
5*a^2*b^4*x^7)
```

3.222 $\int \frac{x^{12}}{(a+bx)^{10}} dx$

Optimal. Leaf size=186

$$\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx}$$

```
[Out] 55*a^2*x/b^12-5*a*x^2/b^11+1/3*x^3/b^10-1/9*a^12/b^13/(b*x+a)^9+3/2*a^11/b^
13/(b*x+a)^8-66/7*a^10/b^13/(b*x+a)^7+110/3*a^9/b^13/(b*x+a)^6-99*a^8/b^13/
(b*x+a)^5+198*a^7/b^13/(b*x+a)^4-308*a^6/b^13/(b*x+a)^3+396*a^5/b^13/(b*x+a
)^2-495*a^4/b^13/(b*x+a)-220*a^3*ln(b*x+a)/b^13
```

Rubi [A]

time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{396a^5}{b^{13}(a+bx)^2}-\frac{495a^4}{b^{13}(a+bx)}-\frac{220a^3\log(a+bx)}{b^{13}}+\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x)^10,x]
```

```
[Out] (55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9)
 + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*a^
9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*(a
+ b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2) - (
495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx=\int \left(\frac{55a^2}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{b^{10}}+\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220a^9}{b^{12}(a+bx)^7}+\right.$$
$$=\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}$$

Mathematica [A]
time = 0.02, size = 161, normalized size = 0.87

$$-\frac{35201a^{12}+289089a^{11}bx+1031616a^{10}b^2x^2+2074464a^9b^3x^3+2529576a^8b^4x^4+1831032a^7b^5x^5+638568a^6b^6x^6-58968a^5b^7x^7-139482a^4b^8x^8-43218a^3b^9x^9-2772a^2b^{10}x^{10}+252ab^{11}x^{11}-42b^{12}x^{12}+27720a^3(a+bx)^9\log(a+bx)}{126b^{13}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x)^10,x]
```

```
[Out] -1/126*(35201*a^12 + 289089*a^11*b*x + 1031616*a^10*b^2*x^2 + 2074464*a^9*b
^3*x^3 + 2529576*a^8*b^4*x^4 + 1831032*a^7*b^5*x^5 + 638568*a^6*b^6*x^6 - 5
8968*a^5*b^7*x^7 - 139482*a^4*b^8*x^8 - 43218*a^3*b^9*x^9 - 2772*a^2*b^10*x
^10 + 252*a*b^11*x^11 - 42*b^12*x^12 + 27720*a^3*(a + b*x)^9*Log[a + b*x])/
(b^13*(a + b*x)^9)
```

Mathics [A]
time = 4.29, size = 229, normalized size = 1.23

$$\frac{-220a^3\text{Log}[a+bx]}{b^{13}}-\frac{a^4(35201a^8+296019a^7bx+1093356a^6b^2x^2+2318316a^5b^3x^3+3089394a^4b^4x^4+2652804a^3b^5x^5+1435896a^2b^6x^6+449064ab^7x^7+62370b^8x^8)}{126b^{13}(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9)}+\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^12/(a + b*x)^10,x]')
```

```
[Out] -220 a ^ 3 Log[a + b x] / b ^ 13 - a ^ 4 (35201 a ^ 8 + 296019 a ^ 7 b x +
1093356 a ^ 6 b ^ 2 x ^ 2 + 2318316 a ^ 5 b ^ 3 x ^ 3 + 3089394 a ^ 4 b ^ 4
 x ^ 4 + 2652804 a ^ 3 b ^ 5 x ^ 5 + 1435896 a ^ 2 b ^ 6 x ^ 6 + 449064 a b
 ^ 7 x ^ 7 + 62370 b ^ 8 x ^ 8) / (126 b ^ 13 (a ^ 9 + 9 a ^ 8 b x + 36 a ^
 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b
 ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8
+ b ^ 9 x ^ 9)) + 55 a ^ 2 x / b ^ 12 - 5 a x ^ 2 / b ^ 11 + x ^ 3 / (3 b ^
 10)
```

Maple [A]
time = 0.09, size = 177, normalized size = 0.95

method	result
risch	$\frac{x^3}{3b^{10}}-\frac{5a\,x^2}{b^{11}}+\frac{55a^2x}{b^{12}}+\frac{-495a^4b^7x^8-3564a^5b^6x^7-11396a^6b^5x^6-21054a^7b^4x^5-24519a^8b^3x^4-\frac{55198a^9b^2x^3}{3}-\frac{60742a^{10}b\,x^2}{7}-3}{b^{12}(bx+a)^9}$
norman	$\frac{\frac{x^{12}}{3b}-\frac{2a\,x^{11}}{b^2}-\frac{78419a^{12}}{126b^{13}}+\frac{22a^2x^{10}}{b^3}-\frac{1980a^4x^8}{b^5}-\frac{11880a^5x^7}{b^6}-\frac{33880a^6x^6}{b^7}-\frac{57750a^7x^5}{b^8}-\frac{63294a^8x^4}{b^9}-\frac{45276a^9x^3}{b^{10}}-\frac{143748a^{10}x^2}{7b^{11}}-\frac{75339a^{11}x}{14b^{12}}}{(bx+a)^9}$
default	$\frac{\frac{1}{3}b^2x^3-5ab\,x^2+55a^2x}{b^{12}}-\frac{495a^4}{b^{13}(bx+a)}+\frac{198a^7}{b^{13}(bx+a)^4}-\frac{66a^{10}}{7b^{13}(bx+a)^7}-\frac{a^{12}}{9b^{13}(bx+a)^9}+\frac{396a^5}{b^{13}(bx+a)^2}+\frac{110a^9}{3b^{13}(bx+a)^6}+\frac{3}{2b^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^12*(1/3*b^2*x^3-5*a*b*x^2+55*a^2*x)-495*a^4/b^13/(b*x+a)+198*a^7/b^13/(
b*x+a)^4-66/7*a^10/b^13/(b*x+a)^7-1/9*a^12/b^13/(b*x+a)^9+396*a^5/b^13/(b*x
+a)^2+110/3*a^9/b^13/(b*x+a)^6+3/2*a^11/b^13/(b*x+a)^8-99*a^8/b^13/(b*x+a)^
5-220*a^3*ln(b*x+a)/b^13-308*a^6/b^13/(b*x+a)^3
```

Maxima [A]
time = 0.28, size = 234, normalized size = 1.26

$$-\frac{62370\,a^{4}b^{8}x^{8}+449064\,a^{5}b^{7}x^{7}+1435896\,a^{6}b^{6}x^{6}+2652804\,a^{7}b^{5}x^{5}+3089394\,a^{8}b^{4}x^{4}+2318316\,a^{9}b^{3}x^{3}+1093356\,a^{10}b^{2}x^{2}+296019\,a^{11}bx+35201\,a^{12}}{126\,(b^{22}x^{9}+9\,ab^{21}x^{8}+36\,a^{2}b^{20}x^{7}+84\,a^{3}b^{19}x^{6}+126\,a^{4}b^{18}x^{5}+126\,a^{5}b^{17}x^{4}+84\,a^{6}b^{16}x^{3}+36\,a^{7}b^{15}x^{2}+9\,a^{8}b^{14}x+a^{9}b^{13})}-\frac{220\,a^{3}\log(bx+a)}{b^{13}}+\frac{b^{2}x^{3}-15\,abx^{2}+165\,a^{2}x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 2652
804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^10*
b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^2*b
^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^6*b^
16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13) - 220*a^3*log(b*x + a)/
b^13 + 1/3*(b^2*x^3 - 15*a*b*x^2 + 165*a^2*x)/b^12
```

Fricas [A]
time = 0.30, size = 338, normalized size = 1.82

$$\frac{42\,b^{12}x^{12}-252\,ab^{11}x^{11}+2772\,a^{2}b^{10}x^{10}+43218\,a^{3}b^{9}x^{9}+139482\,a^{4}b^{8}x^{8}+58968\,a^{5}b^{7}x^{7}-638568\,a^{6}b^{6}x^{6}-1831032\,a^{7}b^{5}x^{5}-2529576\,a^{8}b^{4}x^{4}-2074464\,a^{9}b^{3}x^{3}-1031616\,a^{10}b^{2}x^{2}-289089\,a^{11}bx-35201\,a^{12}-27720\,(a^{3}b^{9}x^{9}+9\,a^{4}b^{8}x^{8}+36\,a^{5}b^{7}x^{7}+84\,a^{6}b^{6}x^{6}+126\,a^{7}b^{5}x^{5}+126\,a^{8}b^{4}x^{4}+84\,a^{9}b^{3}x^{3}+36\,a^{10}b^{2}x^{2}+9\,a^{11}bx+a^{12})\log(bx+a)}{126\,(b^{22}x^{9}+9\,ab^{21}x^{8}+36\,a^{2}b^{20}x^{7}+84\,a^{3}b^{19}x^{6}+126\,a^{4}b^{18}x^{5}+126\,a^{5}b^{17}x^{4}+84\,a^{6}b^{16}x^{3}+36\,a^{7}b^{15}x^{2}+9\,a^{8}b^{14}x+a^{9}b^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 - 252*a*b^11*x^11 + 2772*a^2*b^10*x^10 + 43218*a^3*b^9*
x^9 + 139482*a^4*b^8*x^8 + 58968*a^5*b^7*x^7 - 638568*a^6*b^6*x^6 - 1831032
*a^7*b^5*x^5 - 2529576*a^8*b^4*x^4 - 2074464*a^9*b^3*x^3 - 1031616*a^10*b^2
*x^2 - 289089*a^11*b*x - 35201*a^12 - 27720*(a^3*b^9*x^9 + 9*a^4*b^8*x^8 +
36*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 126*a^7*b^5*x^5 + 126*a^8*b^4*x^4 + 84*a^
9*b^3*x^3 + 36*a^10*b^2*x^2 + 9*a^11*b*x + a^12)*log(b*x + a))/(b^22*x^9 +
9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a
^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13)
```

Sympy [A]
time = 0.71, size = 250, normalized size = 1.34

$$-\frac{220a^{3}\log(a+bx)}{b^{13}}+\frac{55a^{2}x}{b^{12}}-\frac{5ax^{2}}{b^{11}}+\frac{-35201a^{12}-296019a^{11}bx-1093356a^{10}b^{2}x^{2}-2318316a^{9}b^{3}x^{3}-3089394a^{8}b^{4}x^{4}-2652804a^{7}b^{5}x^{5}-1435896a^{6}b^{6}x^{6}-449064a^{5}b^{7}x^{7}-62370a^{4}b^{8}x^{8}}{126a^{9}b^{13}+1134a^{8}b^{14}x+4536a^{7}b^{15}x^{2}+10584a^{6}b^{16}x^{3}+15876a^{5}b^{17}x^{4}+15876a^{4}b^{18}x^{5}+10584a^{3}b^{19}x^{6}+4536a^{2}b^{20}x^{7}+1134ab^{21}x^{8}+126b^{22}x^{9}}+\frac{x^{3}}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x+a)**10,x)
```

```
[Out] -220*a**3*log(a + b*x)/b**13 + 55*a**2*x/b**12 - 5*a*x**2/b**11 + (-35201*a
**12 - 296019*a**11*b*x - 1093356*a**10*b**2*x**2 - 2318316*a**9*b**3*x**3
```

```
- 3089394*a**8*b**4*x**4 - 2652804*a**7*b**5*x**5 - 1435896*a**6*b**6*x**6
- 449064*a**5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*a**9*b**13 + 1134*a**8
*b**14*x + 4536*a**7*b**15*x**2 + 10584*a**6*b**16*x**3 + 15876*a**5*b**17*
x**4 + 15876*a**4*b**18*x**5 + 10584*a**3*b**19*x**6 + 4536*a**2*b**20*x**7
 + 1134*a*b**21*x**8 + 126*b**22*x**9) + x**3/(3*b**10)
```

Giac [A]
time = 0.00, size = 168, normalized size = 0.90

$$\frac{\frac{1}{3}x^3b^{20}-5x^2b^{19}a+55xb^{18}a^2}{b^{30}}+\frac{\frac{1}{126}\left(-62370b^8a^4x^8-449064b^7a^5x^7-1435896b^6a^6x^6-2652804b^5a^7x^5-3089394b^4a^8x^4-2318316b^3a^9x^3-1093356b^2a^{10}x^2-296019ba^{11}x-35201a^{12}\right)}{b^{13}\left(xb+a\right)^9}-\frac{220a^3\ln|xb+a|}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x)
```

```
[Out] -220*a^3*log(abs(b*x + a))/b^13 - 1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7
*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 23
18316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/((
b*x + a)^9*b^13) + 1/3*(b^20*x^3 - 15*a*b^19*x^2 + 165*a^2*b^18*x)/b^30
```

Mupad [B]
time = 0.98, size = 151, normalized size = 0.81

$$-\frac{6\,a\,{\left(a+b\,x\right)}^2-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{495\,a^4}{a+b\,x}-\frac{396\,a^5}{{\left(a+b\,x\right)}^2}+\frac{308\,a^6}{{\left(a+b\,x\right)}^3}-\frac{198\,a^7}{{\left(a+b\,x\right)}^4}+\frac{99\,a^8}{{\left(a+b\,x\right)}^5}-\frac{110\,a^9}{3\,{\left(a+b\,x\right)}^6}+\frac{66\,a^{10}}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^{11}}{2\,{\left(a+b\,x\right)}^8}+\frac{a^{12}}{9\,{\left(a+b\,x\right)}^9}+220\,a^3\,\ln\left(a+b\,x\right)-66\,a^2\,b\,x}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x)^10,x)
```

```
[Out] -(6*a*(a + b*x)^2 - (a + b*x)^3/3 + (495*a^4)/(a + b*x) - (396*a^5)/(a + b*
x)^2 + (308*a^6)/(a + b*x)^3 - (198*a^7)/(a + b*x)^4 + (99*a^8)/(a + b*x)^5
 - (110*a^9)/(3*(a + b*x)^6) + (66*a^10)/(7*(a + b*x)^7) - (3*a^11)/(2*(a +
 b*x)^8) + a^12/(9*(a + b*x)^9) + 220*a^3*log(a + b*x) - 66*a^2*b*x)/b^13
```

3.223 $\int \frac{x^{11}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=177

$$-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\ldots$$

```
[Out] -10*a*x/b^11+1/2*x^2/b^10+1/9*a^11/b^12/(b*x+a)^9-11/8*a^10/b^12/(b*x+a)^8+
55/7*a^9/b^12/(b*x+a)^7-55/2*a^8/b^12/(b*x+a)^6+66*a^7/b^12/(b*x+a)^5-231/2
*a^6/b^12/(b*x+a)^4+154*a^5/b^12/(b*x+a)^3-165*a^4/b^12/(b*x+a)^2+165*a^3/b
^12/(b*x+a)+55*a^2*ln(b*x+a)/b^12
```

Rubi [A]
time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165a^4}{b^{12}(a+bx)^2}+\frac{165a^3}{b^{12}(a+bx)}+\frac{55a^2\log(a+bx)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x)^10,x]
```

```
[Out] (-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b^
12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b*x
)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (154*
a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^12*(a
 + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx)^{10}}\,dx &= \int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\frac{330\ldots}{b^{11}(a+\ldots}\right.\\ &= -\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{6\ldots}{b^{12}(a\ldots}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 150, normalized size = 0.85

$$\frac{42131a^{11}+351459a^{10}bx+1281096a^9b^2x^2+2656584a^8b^3x^3+3402756a^7b^4x^4+2704212a^6b^5x^5+1220688a^5b^6x^6+190512a^4b^7x^7-77112a^3b^8x^8-36288a^2b^9x^9-2772ab^{10}x^{10}+252b^{11}x^{11}+27720a^2(a+bx)^9\log(a+bx)}{504b^{12}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x + 1281096*a^9*b^2*x^2 + 2656584*a^8*b^3*x^3 +
 3402756*a^7*b^4*x^4 + 2704212*a^6*b^5*x^5 + 1220688*a^5*b^6*x^6 + 190512*a
^4*b^7*x^7 - 77112*a^3*b^8*x^8 - 36288*a^2*b^9*x^9 - 2772*a*b^10*x^10 + 252
*b^11*x^11 + 27720*a^2*(a + b*x)^9*Log[a + b*x])/(504*b^12*(a + b*x)^9)
```

Mathics [A]
time = 4.20, size = 218, normalized size = 1.23

$$\frac{55a^2\text{Log}[a+bx]}{b^{12}}+\frac{a^3(42131a^8+356499a^7bx+1326204a^6b^2x^2+2835756a^5b^3x^3+3817044a^4b^4x^4+3318084a^3b^5x^5+1823976a^2b^6x^6+582120ab^7x^7+83160b^8x^8)}{504b^{12}(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9)}-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^11/(a + b*x)^10,x]')
```

```
[Out] 55 a ^ 2 Log[a + b x] / b ^ 12 + a ^ 3 (42131 a ^ 8 + 356499 a ^ 7 b x + 13
26204 a ^ 6 b ^ 2 x ^ 2 + 2835756 a ^ 5 b ^ 3 x ^ 3 + 3817044 a ^ 4 b ^ 4 x
 ^ 4 + 3318084 a ^ 3 b ^ 5 x ^ 5 + 1823976 a ^ 2 b ^ 6 x ^ 6 + 582120 a b ^
 7 x ^ 7 + 83160 b ^ 8 x ^ 8) / (504 b ^ 12 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7
 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^
 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 +
b ^ 9 x ^ 9)) - 10 a x / b ^ 11 + x ^ 2 / (2 b ^ 10)
```

Maple [A]
time = 0.10, size = 167, normalized size = 0.94

method	result
risch	$\frac{x^2}{2b^{10}}-\frac{10ax}{b^{11}}+\frac{165a^3b^7x^8+1155a^4b^6x^7+3619a^5b^5x^6+\frac{13167a^6b^4x^5}{2}+\frac{15147a^7b^3x^4}{2}+\frac{11253a^8b^2x^3}{2}+\frac{36839a^9b\,x^2}{14}+\frac{39611a^{10}x}{56}+\frac{42131\cdots}{504\cdots}}{b^{11}(bx+a)^9}$
norman	$\frac{\frac{x^{11}}{2b}-\frac{11a\,x^{10}}{2b^2}+\frac{78419a^{11}}{504b^{12}}+\frac{495a^3x^8}{b^4}+\frac{2970a^4x^7}{b^5}+\frac{8470a^5x^6}{b^6}+\frac{28875a^6x^5}{2b^7}+\frac{31647a^7x^4}{2b^8}+\frac{11319a^8x^3}{b^9}+\frac{35937a^9x^2}{7b^{10}}+\frac{75339a^{10}x}{56b^{11}}}{(bx+a)^9}+\frac{55a^2\ln(b\cdots}{b^{12}}$
default	$-\frac{-\frac{1}{2}x^2b+10ax}{b^{11}}+\frac{165a^3}{b^{12}(bx+a)}-\frac{231a^6}{2b^{12}(bx+a)^4}+\frac{a^{11}}{9b^{12}(bx+a)^9}+\frac{66a^7}{b^{12}(bx+a)^5}-\frac{165a^4}{b^{12}(bx+a)^2}-\frac{55a^8}{2b^{12}(bx+a)^6}+\frac{55a^2\ln(bx\cdots}{b^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^11*(-1/2*x^2*b+10*a*x)+165*a^3/b^12/(b*x+a)-231/2*a^6/b^12/(b*x+a)^4+1
/9*a^11/b^12/(b*x+a)^9+66*a^7/b^12/(b*x+a)^5-165*a^4/b^12/(b*x+a)^2-55/2*a^
```

```
8/b^12/(b*x+a)^6+55*a^2*ln(b*x+a)/b^12-11/8*a^10/b^12/(b*x+a)^8+154*a^5/b^1
2/(b*x+a)^3+55/7*a^9/b^12/(b*x+a)^7
```

Maxima [A]
time = 0.27, size = 223, normalized size = 1.26

$$\frac{83160\,a^{3}b^{8}x^{8}+582120\,a^{4}b^{7}x^{7}+1823976\,a^{5}b^{6}x^{6}+3318084\,a^{6}b^{5}x^{5}+3817044\,a^{7}b^{4}x^{4}+2835756\,a^{8}b^{3}x^{3}+1326204\,a^{9}b^{2}x^{2}+356499\,a^{10}bx+42131\,a^{11}}{504\,(b^{21}x^{9}+9\,ab^{20}x^{8}+36\,a^{2}b^{19}x^{7}+84\,a^{3}b^{18}x^{6}+126\,a^{4}b^{17}x^{5}+126\,a^{5}b^{16}x^{4}+84\,a^{6}b^{15}x^{3}+36\,a^{7}b^{14}x^{2}+9\,a^{8}b^{13}x+a^{9}b^{12})}+\frac{55\,a^{2}\log\left(bx+a\right)}{b^{12}}+\frac{bx^{2}-20\,ax}{2\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 33180
84*a^6*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^
2*x^2 + 356499*a^10*b*x + 42131*a^11)/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^2*b^1
9*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6*b^15
*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12) + 55*a^2*log(b*x + a)/b^1
2 + 1/2*(b*x^2 - 20*a*x)/b^11
```

Fricas [A]
time = 0.32, size = 327, normalized size = 1.85

$$\frac{252\,b^{11}x^{11}-2772\,ab^{10}x^{10}-36288\,a^{2}b^{9}x^{9}-77112\,a^{3}b^{8}x^{8}+190512\,a^{4}b^{7}x^{7}+1220688\,a^{5}b^{6}x^{6}+2704212\,a^{6}b^{5}x^{5}+3402756\,a^{7}b^{4}x^{4}+2656584\,a^{8}b^{3}x^{3}+1281096\,a^{9}b^{2}x^{2}+351459\,a^{10}bx+42131\,a^{11}+27720\,(a^{2}b^{9}x^{9}+9\,a^{3}b^{8}x^{8}+36\,a^{4}b^{7}x^{7}+84\,a^{5}b^{6}x^{6}+126\,a^{6}b^{5}x^{5}+126\,a^{7}b^{4}x^{4}+84\,a^{8}b^{3}x^{3}+36\,a^{9}b^{2}x^{2}+9\,a^{10}bx+a^{11})\log\left(bx+a\right)}{504\,(b^{21}x^{9}+9\,ab^{20}x^{8}+36\,a^{2}b^{19}x^{7}+84\,a^{3}b^{18}x^{6}+126\,a^{4}b^{17}x^{5}+126\,a^{5}b^{16}x^{4}+84\,a^{6}b^{15}x^{3}+36\,a^{7}b^{14}x^{2}+9\,a^{8}b^{13}x+a^{9}b^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 - 2772*a*b^10*x^10 - 36288*a^2*b^9*x^9 - 77112*a^3*b^8
*x^8 + 190512*a^4*b^7*x^7 + 1220688*a^5*b^6*x^6 + 2704212*a^6*b^5*x^5 + 340
2756*a^7*b^4*x^4 + 2656584*a^8*b^3*x^3 + 1281096*a^9*b^2*x^2 + 351459*a^10*
b*x + 42131*a^11 + 27720*(a^2*b^9*x^9 + 9*a^3*b^8*x^8 + 36*a^4*b^7*x^7 + 84
*a^5*b^6*x^6 + 126*a^6*b^5*x^5 + 126*a^7*b^4*x^4 + 84*a^8*b^3*x^3 + 36*a^9*
b^2*x^2 + 9*a^10*b*x + a^11)*log(b*x + a))/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^
2*b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6
*b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12)
```

Sympy [A]
time = 0.67, size = 236, normalized size = 1.33

$$\frac{55a^{2}\log\left(a+bx\right)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{42131a^{11}+356499a^{10}bx+1326204a^{9}b^{2}x^{2}+2835756a^{8}b^{3}x^{3}+3817044a^{7}b^{4}x^{4}+3318084a^{6}b^{5}x^{5}+1823976a^{5}b^{6}x^{6}+582120a^{4}b^{7}x^{7}+83160a^{3}b^{8}x^{8}}{504a^{9}b^{12}+4536a^{8}b^{13}x+18144a^{7}b^{14}x^{2}+42336a^{6}b^{15}x^{3}+63504a^{5}b^{16}x^{4}+63504a^{4}b^{17}x^{5}+42336a^{3}b^{18}x^{6}+18144a^{2}b^{19}x^{7}+4536ab^{20}x^{8}+504b^{21}x^{9}}+\frac{x^{2}}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x)/b**12 - 10*a*x/b**11 + (42131*a**11 + 356499*a**10*b*x
 + 1326204*a**9*b**2*x**2 + 2835756*a**8*b**3*x**3 + 3817044*a**7*b**4*x**4
 + 3318084*a**6*b**5*x**5 + 1823976*a**5*b**6*x**6 + 582120*a**4*b**7*x**7
```

```
+ 83160*a**3*b**8*x**8)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b*
*14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17
*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 +
 504*b**21*x**9) + x**2/(2*b**10)
```

Giac [A]
time = 0.00, size = 147, normalized size = 0.83

$$\frac{\frac{1}{2}x^2b^{10} - 10xb^9a}{b^{20}} + \frac{\frac{1}{504}\left(83160b^8a^3x^8 + 582120b^7a^4x^7 + 1823976b^6a^5x^6 + 3318084b^5a^6x^5 + 3817044b^4a^7x^4 + 2835756b^3a^8x^3 + 1326204b^2a^9x^2 + 356499ba^{10}x + 42131a^{11}\right)}{b^{12}(xb+a)^9} + \frac{55a^2\ln|xb+a|}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x)
```

```
[Out] 55*a^2*log(abs(b*x + a))/b^12 + 1/2*(b^10*x^2 - 20*a*b^9*x)/b^20 + 1/504*(8
3160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318084*a^6*b
^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^2*x^2 +
356499*a^10*b*x + 42131*a^11)/((b*x + a)^9*b^12)
```

Mupad [B]
time = 0.23, size = 138, normalized size = 0.78

$$\frac{\frac{(a+bx)^2}{2} + \frac{165a^3}{a+bx} - \frac{165a^4}{(a+bx)^2} + \frac{154a^5}{(a+bx)^3} - \frac{231a^6}{2(a+bx)^4} + \frac{66a^7}{(a+bx)^5} - \frac{55a^8}{2(a+bx)^6} + \frac{55a^9}{7(a+bx)^7} - \frac{11a^{10}}{8(a+bx)^8} + \frac{a^{11}}{9(a+bx)^9} + 55a^2\ln(a+bx) - 11abx}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^2/2 + (165*a^3)/(a + b*x) - (165*a^4)/(a + b*x)^2 + (154*a^5)/(a
 + b*x)^3 - (231*a^6)/(2*(a + b*x)^4) + (66*a^7)/(a + b*x)^5 - (55*a^8)/(2*
(a + b*x)^6) + (55*a^9)/(7*(a + b*x)^7) - (11*a^10)/(8*(a + b*x)^8) + a^11/
(9*(a + b*x)^9) + 55*a^2*log(a + b*x) - 11*a*b*x)/b^12
```

3.224 $\int \frac{x^{10}}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=159

$$\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+$$

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{60a^3}{b^{11}(a+bx)^2}-\frac{45a^2}{b^{11}(a+bx)}-\frac{10a\log(a+bx)}{b^{11}}+\frac{x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^10,x]
```

```
[Out] x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^8
)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a +
b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (60*a
^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x])/b^
11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a+bx)^{10}} \, dx &= \int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\right.\\ &= \frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 137, normalized size = 0.86

$$-\frac{4861a^{10}+41229a^9bx+153576a^8b^2x^2+328104a^7b^3x^3+439236a^6b^4x^4+375732a^5b^5x^5+197568a^4b^6x^6+54432a^3b^7x^7+2268a^2b^8x^8-2268ab^9x^9-252b^{10}x^{10}+2520a(a+bx)^9\log(a+bx)}{252b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10 + 41229*a^9*b*x + 153576*a^8*b^2*x^2 + 328104*a^7*b^3*x^3
 + 439236*a^6*b^4*x^4 + 375732*a^5*b^5*x^5 + 197568*a^4*b^6*x^6 + 54432*a^3
*b^7*x^7 + 2268*a^2*b^8*x^8 - 2268*a*b^9*x^9 - 252*b^10*x^10 + 2520*a*(a +
b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

Mathics [A]
time = 4.07, size = 206, normalized size = 1.30

$$\frac{-10a\mathrm{Log}\left[a+bx\right]}{b^{11}}-\frac{a^2\left(4861a^8+41481a^7bx+155844a^6b^2x^2+337176a^5b^3x^3+460404a^4b^4x^4+407484a^3b^5x^5+229320a^2b^6x^6+75600ab^7x^7+11340b^8x^8\right)}{252b^{11}\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}+\frac{x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^10/(a + b*x)^10,x]')
```

```
[Out] -10 a Log[a + b x] / b ^ 11 - a ^ 2 (4861 a ^ 8 + 41481 a ^ 7 b x + 155844
a ^ 6 b ^ 2 x ^ 2 + 337176 a ^ 5 b ^ 3 x ^ 3 + 460404 a ^ 4 b ^ 4 x ^ 4 + 4
07484 a ^ 3 b ^ 5 x ^ 5 + 229320 a ^ 2 b ^ 6 x ^ 6 + 75600 a b ^ 7 x ^ 7 +
11340 b ^ 8 x ^ 8) / (252 b ^ 11 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^
2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 +
84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9
)) + x / b ^ 10
```

Maple [A]
time = 0.10, size = 154, normalized size = 0.97

method	result
risch	$\frac{x}{b^{10}}+\frac{-45a^2b^7x^8-300a^3b^6x^7-910a^4b^5x^6-1617a^5b^4x^5-1827a^6b^3x^4-1338a^7b^2x^3-\frac{4329a^8b\,x^2}{7}-\frac{4609a^9x}{28}-\frac{4861a^{10}}{252b}}{b^{10}(bx+a)^9}-\frac{10a\ln(bx+}{b^{11}}$
norman	$\frac{\frac{x^{10}}{b}-\frac{7129a^{10}}{252b^{11}}-\frac{90a^2x^8}{b^3}-\frac{540a^3x^7}{b^4}-\frac{1540a^4x^6}{b^5}-\frac{2625a^5x^5}{b^6}-\frac{2877a^6x^4}{b^7}-\frac{2058a^7x^3}{b^8}-\frac{6534a^8x^2}{7b^9}-\frac{6849a^9x}{28b^{10}}}{(bx+a)^9}-\frac{10a\ln(bx+a)}{b^{11}}$
default	$\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(bx+a)^9}+\frac{5a^9}{4b^{11}(bx+a)^8}-\frac{45a^8}{7b^{11}(bx+a)^7}+\frac{20a^7}{b^{11}(bx+a)^6}-\frac{42a^6}{b^{11}(bx+a)^5}+\frac{63a^5}{b^{11}(bx+a)^4}-\frac{70a^4}{b^{11}(bx+a)^3}+\frac{60a}{b^{11}(bx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
```

```
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

Maxima [A]
time = 0.27, size = 211, normalized size = 1.33

$$-\frac{11340\,a^{2} b^{8} x^{8} + 75600\,a^{3} b^{7} x^{7} + 229320\,a^{4} b^{6} x^{6} + 407484\,a^{5} b^{5} x^{5} + 460404\,a^{6} b^{4} x^{4} + 337176\,a^{7} b^{3} x^{3} + 155844\,a^{8} b^{2} x^{2} + 41481\,a^{9} b x + 4861\,a^{10}}{252\,{\left(b^{20} x^{9} + 9\,a b^{19} x^{8} + 36\,a^{2} b^{18} x^{7} + 84\,a^{3} b^{17} x^{6} + 126\,a^{4} b^{16} x^{5} + 126\,a^{5} b^{15} x^{4} + 84\,a^{6} b^{14} x^{3} + 36\,a^{7} b^{13} x^{2} + 9\,a^{8} b^{12} x + a^{9} b^{11}\right)}} + \frac{x}{b^{10}} - \frac{10\,a \log\left(b x + a\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(11340*a^2*b^8*x^8 + 75600*a^3*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484
*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2
 + 41481*a^9*b*x + 4861*a^10)/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 +
84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 3
6*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + x/b^10 - 10*a*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(153) = 306.
time = 0.31, size = 314, normalized size = 1.97

$$\frac{252\,b^{10} x^{10} + 2268\,a b^{9} x^{9} - 2268\,a^{2} b^{8} x^{8} - 54432\,a^{3} b^{7} x^{7} - 197568\,a^{4} b^{6} x^{6} - 375732\,a^{5} b^{5} x^{5} - 439236\,a^{6} b^{4} x^{4} - 328104\,a^{7} b^{3} x^{3} - 153576\,a^{8} b^{2} x^{2} - 41229\,a^{9} b x - 4861\,a^{10} - 2520\,{\left(a b^{9} x^{9} + 9\,a^{2} b^{8} x^{8} + 36\,a^{3} b^{7} x^{7} + 84\,a^{4} b^{6} x^{6} + 126\,a^{5} b^{5} x^{5} + 126\,a^{6} b^{4} x^{4} + 84\,a^{7} b^{3} x^{3} + 36\,a^{8} b^{2} x^{2} + 9\,a^{9} b x + a^{10}\right)} \log\left(b x + a\right)}{252\,{\left(b^{20} x^{9} + 9\,a b^{19} x^{8} + 36\,a^{2} b^{18} x^{7} + 84\,a^{3} b^{17} x^{6} + 126\,a^{4} b^{16} x^{5} + 126\,a^{5} b^{15} x^{4} + 84\,a^{6} b^{14} x^{3} + 36\,a^{7} b^{13} x^{2} + 9\,a^{8} b^{12} x + a^{9} b^{11}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2268*a*b^9*x^9 - 2268*a^2*b^8*x^8 - 54432*a^3*b^7*x^
7 - 197568*a^4*b^6*x^6 - 375732*a^5*b^5*x^5 - 439236*a^6*b^4*x^4 - 328104*a
^7*b^3*x^3 - 153576*a^8*b^2*x^2 - 41229*a^9*b*x - 4861*a^10 - 2520*(a*b^9*x
^9 + 9*a^2*b^8*x^8 + 36*a^3*b^7*x^7 + 84*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 12
6*a^6*b^4*x^4 + 84*a^7*b^3*x^3 + 36*a^8*b^2*x^2 + 9*a^9*b*x + a^10)*log(b*x
 + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a
^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*
b^12*x + a^9*b^11)
```

Sympy [A]
time = 0.62, size = 224, normalized size = 1.41

$$-\frac{10a \log{\left(a + bx \right)}}{b^{11}} + \frac{-4861a^{10} - 41481a^{9}bx - 155844a^{8}b^{2}x^{2} - 337176a^{7}b^{3}x^{3} - 460404a^{6}b^{4}x^{4} - 407484a^{5}b^{5}x^{5} - 229320a^{4}b^{6}x^{6} - 75600a^{3}b^{7}x^{7} - 11340a^{2}b^{8}x^{8}}{252a^{9}b^{11} + 2268a^{8}b^{12}x + 9072a^{7}b^{13}x^{2} + 21168a^{6}b^{14}x^{3} + 31752a^{5}b^{15}x^{4} + 31752a^{4}b^{16}x^{5} + 21168a^{3}b^{17}x^{6} + 9072a^{2}b^{18}x^{7} + 2268ab^{19}x^{8} + 252b^{20}x^{9}} + \frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**10,x)
```

```
[Out] -10*a*log(a + b*x)/b**11 + (-4861*a**10 - 41481*a**9*b*x - 155844*a**8*b**2
*x**2 - 337176*a**7*b**3*x**3 - 460404*a**6*b**4*x**4 - 407484*a**5*b**5*x*
*5 - 229320*a**4*b**6*x**6 - 75600*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(
```

```
252*a**9*b**11 + 2268*a**8*b**12*x + 9072*a**7*b**13*x**2 + 21168*a**6*b**1
4*x**3 + 31752*a**5*b**15*x**4 + 31752*a**4*b**16*x**5 + 21168*a**3*b**17*x
**6 + 9072*a**2*b**18*x**7 + 2268*a*b**19*x**8 + 252*b**20*x**9) + x/b**10
```

Giac [A]
time = 0.00, size = 137, normalized size = 0.86

$$\frac{x}{b^{10}} + \frac{\frac{1}{252}\left(-11340b^8a^2x^8 - 75600b^7a^3x^7 - 229320b^6a^4x^6 - 407484b^5a^5x^5 - 460404b^4a^6x^4 - 337176b^3a^7x^3 - 155844b^2a^8x^2 - 41481ba^9x - 4861a^{10}\right)}{b^{11}\left(xb+a\right)^9} - \frac{10a\ln|xb+a|}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x)
```

```
[Out] x/b^10 - 10*a*log(abs(b*x + a))/b^11 - 1/252*(11340*a^2*b^8*x^8 + 75600*a^3
*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 3
37176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2 + 41481*a^9*b*x + 4861*a^10)/((b*x +
 a)^9*b^11)
```

Mupad [B]
time = 0.94, size = 127, normalized size = 0.80

$$-\frac{10\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{45\,a^2}{a+b\,x}-\frac{60\,a^3}{{\left(a+b\,x\right)}^2}+\frac{70\,a^4}{{\left(a+b\,x\right)}^3}-\frac{63\,a^5}{{\left(a+b\,x\right)}^4}+\frac{42\,a^6}{{\left(a+b\,x\right)}^5}-\frac{20\,a^7}{{\left(a+b\,x\right)}^6}+\frac{45\,a^8}{7\,{\left(a+b\,x\right)}^7}-\frac{5\,a^9}{4\,{\left(a+b\,x\right)}^8}+\frac{a^{10}}{9\,{\left(a+b\,x\right)}^9}}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^10,x)
```

```
[Out] -(10*a*log(a + b*x) - b*x + (45*a^2)/(a + b*x) - (60*a^3)/(a + b*x)^2 + (70
*a^4)/(a + b*x)^3 - (63*a^5)/(a + b*x)^4 + (42*a^6)/(a + b*x)^5 - (20*a^7)/
(a + b*x)^6 + (45*a^8)/(7*(a + b*x)^7) - (5*a^9)/(4*(a + b*x)^8) + a^10/(9*
(a + b*x)^9))/b^11
```

3.225 $\int \frac{x^9}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=154

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{}{b^1}$$

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a^2}{b^{10}(a+bx)^2}+\frac{9a}{b^{10}(a+bx)}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^10,x]
```

```
[Out] a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10*
(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^5)
 - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2)/(
b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a^4}{b^9(a+b}\right.\\ &= \frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63}{2b^{10}(a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 111, normalized size = 0.72

$$\frac{a\left(7129a^8+61641a^7bx+235224a^6b^2x^2+518616a^5b^3x^3+725004a^4b^4x^4+661500a^3b^5x^5+388080a^2b^6x^6+136080ab^7x^7+22680b^8x^8\right)}{2520b^{10}(a+bx)^9}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^10,x]
```

```
[Out] (a*(7129*a^8 + 61641*a^7*b*x + 235224*a^6*b^2*x^2 + 518616*a^5*b^3*x^3 + 72
5004*a^4*b^4*x^4 + 661500*a^3*b^5*x^5 + 388080*a^2*b^6*x^6 + 136080*a*b^7*x
^7 + 22680*b^8*x^8))/(2520*b^10*(a + b*x)^9) + Log[a + b*x]/b^10
```

Mathics [A]
time = 3.86, size = 288, normalized size = 1.87

$$\frac{\frac{a(7129a^8+61641a^7bx+235224a^6b^2x^2+518616a^5b^3x^3+725004a^4b^4x^4+661500a^3b^5x^5+388080a^2b^6x^6+136080ab^7x^7+22680b^8x^8)}{2520}+\mathrm{Log}\left[a+bx\right]\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}{b^{10}\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^9/(a + b*x)^10,x]')
```

```
[Out] (a (7129 a ^ 8 + 61641 a ^ 7 b x + 235224 a ^ 6 b ^ 2 x ^ 2 + 518616 a ^ 5
b ^ 3 x ^ 3 + 725004 a ^ 4 b ^ 4 x ^ 4 + 661500 a ^ 3 b ^ 5 x ^ 5 + 388080
a ^ 2 b ^ 6 x ^ 6 + 136080 a b ^ 7 x ^ 7 + 22680 b ^ 8 x ^ 8) / 2520 + Log[
a + b x] (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3
 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 3
6 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9)) / (b ^ 10 (a ^ 9 + 9
a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x
 ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7
+ 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [A]
time = 0.08, size = 145, normalized size = 0.94

method	result
norman	$\frac{\frac{7129a^9}{2520b^{10}}+\frac{9a\,x^8}{b^2}+\frac{54a^2x^7}{b^3}+\frac{154a^3x^6}{b^4}+\frac{525a^4x^5}{2b^5}+\frac{2877a^5x^4}{10b^6}+\frac{1029a^6x^3}{5b^7}+\frac{3267a^7x^2}{35b^8}+\frac{6849a^8x}{280b^9}}{(bx+a)^9}+\frac{\ln(bx+a)}{b^{10}}$
risch	$\frac{\frac{7129a^9}{2520b^{10}}+\frac{9a\,x^8}{b^2}+\frac{54a^2x^7}{b^3}+\frac{154a^3x^6}{b^4}+\frac{525a^4x^5}{2b^5}+\frac{2877a^5x^4}{10b^6}+\frac{1029a^6x^3}{5b^7}+\frac{3267a^7x^2}{35b^8}+\frac{6849a^8x}{280b^9}}{(bx+a)^9}+\frac{\ln(bx+a)}{b^{10}}$
default	$\frac{a^9}{9b^{10}(bx+a)^9}-\frac{9a^8}{8b^{10}(bx+a)^8}+\frac{36a^7}{7b^{10}(bx+a)^7}-\frac{14a^6}{b^{10}(bx+a)^6}+\frac{126a^5}{5b^{10}(bx+a)^5}-\frac{63a^4}{2b^{10}(bx+a)^4}+\frac{28a^3}{b^{10}(bx+a)^3}-\frac{18a^2}{b^{10}(bx+a)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

Maxima [A]
time = 0.25, size = 202, normalized size = 1.31

$$\frac{22680\,ab^8x^8+136080\,a^2b^7x^7+388080\,a^3b^6x^6+661500\,a^4b^5x^5+725004\,a^5b^4x^4+518616\,a^6b^3x^3+235224\,a^7b^2x^2+61641\,a^8bx+7129\,a^9}{2520\,(b^{19}x^9+9\,ab^{18}x^8+36\,a^2b^{17}x^7+84\,a^3b^{16}x^6+126\,a^4b^{15}x^5+126\,a^5b^{14}x^4+84\,a^6b^{13}x^3+36\,a^7b^{12}x^2+9\,a^8b^{11}x+a^9b^{10})}+\frac{\log(bx+a)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9)/(b^19*x^9 + 9*a*b^18*x^8 + 36*a^2*b^17*x^7 + 84
*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^6*b^13*x^3 + 36*
a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10) + log(b*x + a)/b^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.
time = 0.31, size = 292, normalized size = 1.90

$$\frac{22680\,ab^{8}x^{8} + 136080\,a^{2}b^{7}x^{7} + 388080\,a^{3}b^{6}x^{6} + 661500\,a^{4}b^{5}x^{5} + 725004\,a^{5}b^{4}x^{4} + 518616\,a^{6}b^{3}x^{3} + 235224\,a^{7}b^{2}x^{2} + 61641\,a^{8}bx + 7129\,a^{9} + 2520\,(b^{9}x^{9} + 9\,ab^{8}x^{8} + 36\,a^{2}b^{7}x^{7} + 84\,a^{3}b^{6}x^{6} + 126\,a^{4}b^{5}x^{5} + 126\,a^{5}b^{4}x^{4} + 84\,a^{6}b^{3}x^{3} + 36\,a^{7}b^{2}x^{2} + 9\,a^{8}bx + a^{9})\log\left(bx + a\right)}{2520\,(b^{19}x^{9} + 9\,ab^{18}x^{8} + 36\,a^{2}b^{17}x^{7} + 84\,a^{3}b^{16}x^{6} + 126\,a^{4}b^{15}x^{5} + 126\,a^{5}b^{14}x^{4} + 84\,a^{6}b^{13}x^{3} + 36\,a^{7}b^{12}x^{2} + 9\,a^{8}b^{11}x + a^{9}b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9 + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 +
 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a
^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a))/(b^19*x^9 + 9*a*b^18*x^8 + 36*a
^2*b^17*x^7 + 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^
6*b^13*x^3 + 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10)
```

Sympy [A]
time = 0.52, size = 212, normalized size = 1.38

$$\frac{7129a^{9} + 61641a^{8}bx + 235224a^{7}b^{2}x^{2} + 518616a^{6}b^{3}x^{3} + 725004a^{5}b^{4}x^{4} + 661500a^{4}b^{5}x^{5} + 388080a^{3}b^{6}x^{6} + 136080a^{2}b^{7}x^{7} + 22680ab^{8}x^{8}}{2520a^{9}b^{10} + 22680a^{8}b^{11}x + 90720a^{7}b^{12}x^{2} + 211680a^{6}b^{13}x^{3} + 317520a^{5}b^{14}x^{4} + 317520a^{4}b^{15}x^{5} + 211680a^{3}b^{16}x^{6} + 90720a^{2}b^{17}x^{7} + 22680ab^{18}x^{8} + 2520b^{19}x^{9}} + \frac{\log\left(a + bx\right)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x + 235224*a**7*b**2*x**2 + 518616*a**6*b**3*x**3
 + 725004*a**5*b**4*x**4 + 661500*a**4*b**5*x**5 + 388080*a**3*b**6*x**6 +
136080*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*a**9*b**10 + 22680*a**8*b*
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + log(a + b*x)/b**10
```

Giac [A]
time = 0.00, size = 116, normalized size = 0.75

$$\frac{\frac{1}{2520}\left(22680b^{7}ax^{8} + 136080b^{6}a^{2}x^{7} + 388080b^{5}a^{3}x^{6} + 661500b^{4}a^{4}x^{5} + 725004b^{3}a^{5}x^{4} + 518616b^{2}a^{6}x^{3} + 235224ba^{7}x^{2} + 61641a^{8}x + \frac{7129a^{9}}{b}\right)}{b^{9}\left(xb + a\right)^{9}} + \frac{\ln\left|xb + a\right|}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x)
```

```
[Out] log(abs(b*x + a))/b^10 + 1/2520*(22680*a*b^7*x^8 + 136080*a^2*b^6*x^7 + 388
080*a^3*b^5*x^6 + 661500*a^4*b^4*x^5 + 725004*a^5*b^3*x^4 + 518616*a^6*b^2*
x^3 + 235224*a^7*b*x^2 + 61641*a^8*x + 7129*a^9/b)/((b*x + a)^9*b^9)
```

Mupad [B]
time = 0.19, size = 117, normalized size = 0.76

$$\frac{\ln(a+b\,x)+\frac{9\,a}{a+b\,x}-\frac{18\,a^2}{(a+b\,x)^2}+\frac{28\,a^3}{(a+b\,x)^3}-\frac{63\,a^4}{2\,(a+b\,x)^4}+\frac{126\,a^5}{5\,(a+b\,x)^5}-\frac{14\,a^6}{(a+b\,x)^6}+\frac{36\,a^7}{7\,(a+b\,x)^7}-\frac{9\,a^8}{8\,(a+b\,x)^8}+\frac{a^9}{9\,(a+b\,x)^9}}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^10,x)
```

```
[Out] (log(a + b*x) + (9*a)/(a + b*x) - (18*a^2)/(a + b*x)^2 + (28*a^3)/(a + b*x)
^3 - (63*a^4)/(2*(a + b*x)^4) + (126*a^5)/(5*(a + b*x)^5) - (14*a^6)/(a + b
*x)^6 + (36*a^7)/(7*(a + b*x)^7) - (9*a^8)/(8*(a + b*x)^8) + a^9/(9*(a + b*
x)^9))/b^10
```

3.226 $\int \frac{x^8}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=17

$$\frac{x^9}{9a(a+bx)^9}$$

```
[Out] 1/9*x^9/a/(b*x+a)^9
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$\frac{x^9}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^10,x]
```

```
[Out] x^9/(9*a*(a + b*x)^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = \frac{x^9}{9a(a+bx)^9}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(17) = 34.
time = 0.01, size = 97, normalized size = 5.71

$$-\frac{a^8+9a^7bx+36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+36ab^7x^7+9b^8x^8}{9b^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^10,x]
```

```
[Out] -1/9*(a^8 + 9*a^7*b*x + 36*a^6*b^2*x^2 + 84*a^5*b^3*x^3 + 126*a^4*b^4*x^4 +
 126*a^3*b^5*x^5 + 84*a^2*b^6*x^6 + 36*a*b^7*x^7 + 9*b^8*x^8)/(b^9*(a + b*x
)^9)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 187 vs. $2(17) = 34$.
time = 3.32, size = 185, normalized size = 10.88

$$\frac{-a^8 - 9a^7bx - 36a^6b^2x^2 - 84a^5b^3x^3 - 126a^4b^4x^4 - 126a^3b^5x^5 - 84a^2b^6x^6 - 36ab^7x^7 - 9b^8x^8}{9b^9\left(a^9 + 9a^8bx + 36a^7b^2x^2 + 84a^6b^3x^3 + 126a^5b^4x^4 + 126a^4b^5x^5 + 84a^3b^6x^6 + 36a^2b^7x^7 + 9ab^8x^8 + b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^8/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 8 - 9 a ^ 7 b x - 36 a ^ 6 b ^ 2 x ^ 2 - 84 a ^ 5 b ^ 3 x ^ 3 - 126 a
 ^ 4 b ^ 4 x ^ 4 - 126 a ^ 3 b ^ 5 x ^ 5 - 84 a ^ 2 b ^ 6 x ^ 6 - 36 a b ^
7 x ^ 7 - 9 b ^ 8 x ^ 8) / (9 b ^ 9 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x
 ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5
 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x
^ 9))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(15) = 30$.
time = 0.08, size = 131, normalized size = 7.71

method	result
gosper	$-\frac{9x^8b^8+36a\,x^7b^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5x^3b^3+36a^6x^2b^2+9a^7xb+a^8}{9(bx+a)^9b^9}$
norman	$\frac{-\frac{x^8}{b}-\frac{4a\,x^7}{b^2}-\frac{28a^2x^6}{3b^3}-\frac{14a^3x^5}{b^4}-\frac{14a^4x^4}{b^5}-\frac{28a^5x^3}{3b^6}-\frac{4a^6x^2}{b^7}-\frac{a^7x}{b^8}-\frac{a^8}{9b^9}}{(bx+a)^9}$
risch	$\frac{-\frac{x^8}{b}-\frac{4a\,x^7}{b^2}-\frac{28a^2x^6}{3b^3}-\frac{14a^3x^5}{b^4}-\frac{14a^4x^4}{b^5}-\frac{28a^5x^3}{3b^6}-\frac{4a^6x^2}{b^7}-\frac{a^7x}{b^8}-\frac{a^8}{9b^9}}{(bx+a)^9}$
default	$-\frac{1}{b^9(bx+a)}-\frac{14a^4}{b^9(bx+a)^5}+\frac{14a^3}{b^9(bx+a)^4}-\frac{a^8}{9b^9(bx+a)^9}+\frac{4a}{b^9(bx+a)^2}+\frac{28a^5}{3b^9(bx+a)^6}-\frac{4a^6}{b^9(bx+a)^7}+\frac{a^7}{b^9(bx+a)^8}-\frac{28}{3b^9(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^9/(b*x+a)-14*a^4/b^9/(b*x+a)^5+14*a^3/b^9/(b*x+a)^4-1/9*a^8/b^9/(b*x+a
)^9+4/b^9*a/(b*x+a)^2+28/3*a^5/b^9/(b*x+a)^6-4*a^6/b^9/(b*x+a)^7+a^7/b^9/(b
*x+a)^8-28/3/b^9*a^2/(b*x+a)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(15) = 30$.
time = 0.27, size = 186, normalized size = 10.94

$$-\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx + a^8}{9\left(b^{18}x^9 + 9\,ab^{17}x^8 + 36\,a^2b^{16}x^7 + 84\,a^3b^{15}x^6 + 126\,a^4b^{14}x^5 + 126\,a^5b^{13}x^4 + 84\,a^6b^{12}x^3 + 36\,a^7b^{11}x^2 + 9\,a^8b^{10}x + a^9b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(15) = 30$.
time = 0.31, size = 186, normalized size = 10.94

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\,(b^{18}x^{9}+9\,ab^{17}x^{8}+36\,a^{2}b^{16}x^{7}+84\,a^{3}b^{15}x^{6}+126\,a^{4}b^{14}x^{5}+126\,a^{5}b^{13}x^{4}+84\,a^{6}b^{12}x^{3}+36\,a^{7}b^{11}x^{2}+9\,a^{8}b^{10}x+a^{9}b^{9})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(12) = 24$.
time = 0.48, size = 199, normalized size = 11.71

$$\frac{-a^{8}-9a^{7}bx-36a^{6}b^{2}x^{2}-84a^{5}b^{3}x^{3}-126a^{4}b^{4}x^{4}-126a^{3}b^{5}x^{5}-84a^{2}b^{6}x^{6}-36ab^{7}x^{7}-9b^{8}x^{8}}{9a^{9}b^{9}+81a^{8}b^{10}x+324a^{7}b^{11}x^{2}+756a^{6}b^{12}x^{3}+1134a^{5}b^{13}x^{4}+1134a^{4}b^{14}x^{5}+756a^{3}b^{15}x^{6}+324a^{2}b^{16}x^{7}+81ab^{17}x^{8}+9b^{18}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*a**9*b**9 + 81*a**8*b**10*x + 324*a**7*b**11*x**2 + 756*a**6*b**12*x
**3 + 1134*a**5*b**13*x**4 + 1134*a**4*b**14*x**5 + 756*a**3*b**15*x**6 + 3
24*a**2*b**16*x**7 + 81*a*b**17*x**8 + 9*b**18*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(15) = 30$.
time = 0.00, size = 106, normalized size = 6.24

$$\frac{-9x^{8}b^{8}-36x^{7}b^{7}a-84x^{6}b^{6}a^{2}-126x^{5}b^{5}a^{3}-126x^{4}b^{4}a^{4}-84x^{3}b^{3}a^{5}-36x^{2}b^{2}a^{6}-9xba^{7}-a^{8}}{9b^{9}\left(xb+a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x)
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/((b*x + a)^9*
b^9)
```

Mupad [B]
time = 0.14, size = 107, normalized size = 6.29

$$-\frac{\frac{1}{a+b\,x}-\frac{4\,a}{{\left(a+b\,x\right)}^2}+\frac{28\,a^2}{3\,{\left(a+b\,x\right)}^3}-\frac{14\,a^3}{{\left(a+b\,x\right)}^4}+\frac{14\,a^4}{{\left(a+b\,x\right)}^5}-\frac{28\,a^5}{3\,{\left(a+b\,x\right)}^6}+\frac{4\,a^6}{{\left(a+b\,x\right)}^7}-\frac{a^7}{{\left(a+b\,x\right)}^8}+\frac{a^8}{9\,{\left(a+b\,x\right)}^9}}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^10,x)
```

```
[Out] -(1/(a + b*x) - (4*a)/(a + b*x)^2 + (28*a^2)/(3*(a + b*x)^3) - (14*a^3)/(a
+ b*x)^4 + (14*a^4)/(a + b*x)^5 - (28*a^5)/(3*(a + b*x)^6) + (4*a^6)/(a + b
*x)^7 - a^7/(a + b*x)^8 + a^8/(9*(a + b*x)^9))/b^9
```

3.227 $\int \frac{x^7}{(a+bx)^{10}} dx$

Optimal. Leaf size=35

$$\frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}$$

```
[Out] 1/9*x^8/a/(b*x+a)^9+1/72*x^8/a^2/(b*x+a)^8
```

Rubi [A]

time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^10,x]
```

```
[Out] x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx)^{10}}\,dx &= \frac{x^8}{9a(a+bx)^9}+\frac{\int \frac{x^7}{(a+bx)^9}\,dx}{9a}\\
&= \frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(35) = 70.
time = 0.01, size = 86, normalized size = 2.46

$$-\frac{a^7+9a^6bx+36a^5b^2x^2+84a^4b^3x^3+126a^3b^4x^4+126a^2b^5x^5+84ab^6x^6+36b^7x^7}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7 + 9*a^6*b*x + 36*a^5*b^2*x^2 + 84*a^4*b^3*x^3 + 126*a^3*b^4*x^4
+ 126*a^2*b^5*x^5 + 84*a*b^6*x^6 + 36*b^7*x^7)/(b^8*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 176 vs. 2(35) = 70.
time = 3.15, size = 174, normalized size = 4.97

$$\frac{-a^7-9a^6bx-36a^5b^2x^2-84a^4b^3x^3-126a^3b^4x^4-126a^2b^5x^5-84ab^6x^6-36b^7x^7}{72b^8\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^7/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 7 - 9 a ^ 6 b x - 36 a ^ 5 b ^ 2 x ^ 2 - 84 a ^ 4 b ^ 3 x ^ 3 - 126 a
 ^ 3 b ^ 4 x ^ 4 - 126 a ^ 2 b ^ 5 x ^ 5 - 84 a b ^ 6 x ^ 6 - 36 b ^ 7 x ^
7) / (72 b ^ 8 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3
 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^
 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(31) = 62.
time = 0.08, size = 117, normalized size = 3.34

method	result	s
gosper	$-\frac{36b^7x^7+84a\,b^6x^6+126a^2b^5x^5+126a^3b^4x^4+84a^4b^3x^3+36a^5b^2x^2+9a^6bx+a^7}{72(bx+a)^9b^8}$	8
norman	$\frac{-\frac{x^7}{2b}-\frac{7a\,x^6}{6b^2}-\frac{7a^2x^5}{4b^3}-\frac{7a^3x^4}{4b^4}-\frac{7a^4x^3}{6b^5}-\frac{a^5x^2}{2b^6}-\frac{a^6x}{8b^7}-\frac{a^7}{72b^8}}{(bx+a)^9}$	8
risch	$\frac{-\frac{x^7}{2b}-\frac{7a\,x^6}{6b^2}-\frac{7a^2x^5}{4b^3}-\frac{7a^3x^4}{4b^4}-\frac{7a^4x^3}{6b^5}-\frac{a^5x^2}{2b^6}-\frac{a^6x}{8b^7}-\frac{a^7}{72b^8}}{(bx+a)^9}$	8
default	$-\frac{21a^2}{4b^8(bx+a)^4}+\frac{3a^5}{b^8(bx+a)^7}+\frac{a^7}{9b^8(bx+a)^9}-\frac{7a^6}{8b^8(bx+a)^8}-\frac{1}{2b^8(bx+a)^2}-\frac{35a^4}{6b^8(bx+a)^6}+\frac{7a^3}{b^8(bx+a)^5}+\frac{7a}{3b^8(bx+a)^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -21/4*a^2/b^8/(b*x+a)^4+3*a^5/b^8/(b*x+a)^7+1/9*a^7/b^8/(b*x+a)^9-7/8*a^6/b
^8/(b*x+a)^8-1/2/b^8/(b*x+a)^2-35/6*a^4/b^8/(b*x+a)^6+7*a^3/b^8/(b*x+a)^5+7
/3/b^8*a/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.
time = 0.27, size = 175, normalized size = 5.00

$$-\frac{36\,b^{7}x^{7}+84\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+126\,a^{3}b^{4}x^{4}+84\,a^{4}b^{3}x^{3}+36\,a^{5}b^{2}x^{2}+9\,a^{6}bx+a^{7}}{72\,\left(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a^{9}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.
time = 0.32, size = 175, normalized size = 5.00

$$-\frac{36\,b^{7}x^{7}+84\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+126\,a^{3}b^{4}x^{4}+84\,a^{4}b^{3}x^{3}+36\,a^{5}b^{2}x^{2}+9\,a^{6}bx+a^{7}}{72\,\left(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a^{9}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.
time = 0.43, size = 187, normalized size = 5.34

$$\frac{-a^{7}-9a^{6}bx-36a^{5}b^{2}x^{2}-84a^{4}b^{3}x^{3}-126a^{3}b^{4}x^{4}-126a^{2}b^{5}x^{5}-84ab^{6}x^{6}-36b^{7}x^{7}}{72a^{9}b^{8}+648a^{8}b^{9}x+2592a^{7}b^{10}x^{2}+6048a^{6}b^{11}x^{3}+9072a^{5}b^{12}x^{4}+9072a^{4}b^{13}x^{5}+6048a^{3}b^{14}x^{6}+2592a^{2}b^{15}x^{7}+648ab^{16}x^{8}+72b^{17}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x - 36*a**5*b**2*x**2 - 84*a**4*b**3*x**3 - 126*a**3*b**4
*x**4 - 126*a**2*b**5*x**5 - 84*a*b**6*x**6 - 36*b**7*x**7)/(72*a**9*b**8 +
 648*a**8*b**9*x + 2592*a**7*b**10*x**2 + 6048*a**6*b**11*x**3 + 9072*a**5*
```

```
b**12*x**4 + 9072*a**4*b**13*x**5 + 6048*a**3*b**14*x**6 + 2592*a**2*b**15*
x**7 + 648*a*b**16*x**8 + 72*b**17*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.
time = 0.00, size = 94, normalized size = 2.69

$$\frac{-36x^7b^7 - 84x^6b^6a - 126x^5b^5a^2 - 126x^4b^4a^3 - 84x^3b^3a^4 - 36x^2b^2a^5 - 9xba^6 - a^7}{72b^8 (xb + a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x)
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/((b*x + a)^9*b^8)
```

Mupad [B]
time = 0.13, size = 22, normalized size = 0.63

$$\frac{x^8\,\left(9\,a+b\,x\right)}{72\,a^2\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^10,x)
```

```
[Out] (x^8*(9*a + b*x))/(72*a^2*(a + b*x)^9)
```

3.228 $\int \frac{x^6}{(a+bx)^{10}} dx$

Optimal. Leaf size=52

$$\frac{x^7}{9a(a+bx)^9}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{252a^3(a+bx)^7}$$

```
[Out] 1/9*x^7/a/(b*x+a)^9+1/36*x^7/a^2/(b*x+a)^8+1/252*x^7/a^3/(b*x+a)^7
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{x^7}{252a^3(a+bx)^7}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^10,x]
```

```
[Out] x^7/(9*a*(a + b*x)^9) + x^7/(36*a^2*(a + b*x)^8) + x^7/(252*a^3*(a + b*x)^7
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{x^6}{(a+bx)^{10}} dx = \frac{x^7}{9a(a+bx)^9} + \frac{2\int \frac{x^6}{(a+bx)^9} dx}{9a}$$
$$= \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{\int \frac{x^6}{(a+bx)^8} dx}{36a^2}$$
$$= \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{252a^3(a+bx)^7}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.44

$$-\frac{a^6 + 9a^5bx + 36a^4b^2x^2 + 84a^3b^3x^3 + 126a^2b^4x^4 + 126ab^5x^5 + 84b^6x^6}{252b^7(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^6 + 9*a^5*b*x + 36*a^4*b^2*x^2 + 84*a^3*b^3*x^3 + 126*a^2*b^4*x^4
 + 126*a*b^5*x^5 + 84*b^6*x^6)/(b^7*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(52) = 104.
time = 3.21, size = 163, normalized size = 3.13

$$\frac{-a^6 - 9a^5bx - 36a^4b^2x^2 - 84a^3b^3x^3 - 126a^2b^4x^4 - 126ab^5x^5 - 84b^6x^6}{252b^7(a^9 + 9a^8bx + 36a^7b^2x^2 + 84a^6b^3x^3 + 126a^5b^4x^4 + 126a^4b^5x^5 + 84a^3b^6x^6 + 36a^2b^7x^7 + 9ab^8x^8 + b^9x^9)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^6/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 6 - 9 a ^ 5 b x - 36 a ^ 4 b ^ 2 x ^ 2 - 84 a ^ 3 b ^ 3 x ^ 3 - 126 a
 ^ 2 b ^ 4 x ^ 4 - 126 a b ^ 5 x ^ 5 - 84 b ^ 6 x ^ 6) / (252 b ^ 7 (a ^ 9
+ 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^
 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x
^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(46) = 92.
time = 0.09, size = 102, normalized size = 1.96

method	result	size
gosper	$-\frac{84x^6b^6+126a\,x^5b^5+126a^2x^4b^4+84a^3x^3b^3+36a^4x^2b^2+9a^5xb+a^6}{252(bx+a)^9b^7}$	74

norman	$\frac{-\frac{x^6}{3b}-\frac{a\,x^5}{2b^2}-\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{3b^4}-\frac{a^4x^2}{7b^5}-\frac{a^5x}{28b^6}-\frac{a^6}{252b^7}}{(bx+a)^9}$	77
risch	$\frac{-\frac{x^6}{3b}-\frac{a\,x^5}{2b^2}-\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{3b^4}-\frac{a^4x^2}{7b^5}-\frac{a^5x}{28b^6}-\frac{a^6}{252b^7}}{(bx+a)^9}$	77
default	$\frac{3a^5}{4b^7(bx+a)^8}+\frac{3a}{2b^7(bx+a)^4}-\frac{3a^2}{b^7(bx+a)^5}-\frac{a^6}{9b^7(bx+a)^9}+\frac{10a^3}{3b^7(bx+a)^6}-\frac{15a^4}{7b^7(bx+a)^7}-\frac{1}{3b^7(bx+a)^3}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*a^5/b^7/(b*x+a)^8+3/2*a/b^7/(b*x+a)^4-3/b^7*a^2/(b*x+a)^5-1/9*a^6/b^7/(
b*x+a)^9+10/3*a^3/b^7/(b*x+a)^6-15/7*a^4/b^7/(b*x+a)^7-1/3/b^7/(b*x+a)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(46) = 92$.
time = 0.26, size = 164, normalized size = 3.15

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(b^{16}x^9+9\,ab^{15}x^8+36\,a^2b^{14}x^7+84\,a^3b^{13}x^6+126\,a^4b^{12}x^5+126\,a^5b^{11}x^4+84\,a^6b^{10}x^3+36\,a^7b^9x^2+9\,a^8b^8x+a^9b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(46) = 92$.
time = 0.30, size = 164, normalized size = 3.15

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(b^{16}x^9+9\,ab^{15}x^8+36\,a^2b^{14}x^7+84\,a^3b^{13}x^6+126\,a^4b^{12}x^5+126\,a^5b^{11}x^4+84\,a^6b^{10}x^3+36\,a^7b^9x^2+9\,a^8b^8x+a^9b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(42) = 84$.
time = 0.40, size = 175, normalized size = 3.37

$$\frac{-a^6-9a^5bx-36a^4b^2x^2-84a^3b^3x^3-126a^2b^4x^4-126ab^5x^5-84b^6x^6}{252a^9b^7+2268a^8b^8x+9072a^7b^9x^2+21168a^6b^{10}x^3+31752a^5b^{11}x^4+31752a^4b^{12}x^5+21168a^3b^{13}x^6+9072a^2b^{14}x^7+2268ab^{15}x^8+252b^{16}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x - 36*a**4*b**2*x**2 - 84*a**3*b**3*x**3 - 126*a**2*b**4
*x**4 - 126*a*b**5*x**5 - 84*b**6*x**6)/(252*a**9*b**7 + 2268*a**8*b**8*x +
 9072*a**7*b**9*x**2 + 21168*a**6*b**10*x**3 + 31752*a**5*b**11*x**4 + 3175
2*a**4*b**12*x**5 + 21168*a**3*b**13*x**6 + 9072*a**2*b**14*x**7 + 2268*a*b
**15*x**8 + 252*b**16*x**9)
```

Giac [A]
time = 0.00, size = 82, normalized size = 1.58

$$\frac{-84x^6b^6 - 126x^5b^5a - 126x^4b^4a^2 - 84x^3b^3a^3 - 36x^2b^2a^4 - 9xba^5 - a^6}{252b^7\left(xb+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x)
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/((b*x + a)^9*b^7)
```

Mupad [B]
time = 0.14, size = 85, normalized size = 1.63

$$-\frac{\frac{1}{3\,{\left(a+b\,x\right)}^3}-\frac{3\,a}{2\,{\left(a+b\,x\right)}^4}+\frac{3\,a^2}{{\left(a+b\,x\right)}^5}-\frac{10\,a^3}{3\,{\left(a+b\,x\right)}^6}+\frac{15\,a^4}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^5}{4\,{\left(a+b\,x\right)}^8}+\frac{a^6}{9\,{\left(a+b\,x\right)}^9}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^10,x)
```

```
[Out] -(1/(3*(a + b*x)^3) - (3*a)/(2*(a + b*x)^4) + (3*a^2)/(a + b*x)^5 - (10*a^3
)/(3*(a + b*x)^6) + (15*a^4)/(7*(a + b*x)^7) - (3*a^5)/(4*(a + b*x)^8) + a^
6/(9*(a + b*x)^9))/b^7
```

3.229 $\int \frac{x^5}{(a+bx)^{10}} dx$

Optimal. Leaf size=69

$$\frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{504a^4(a+bx)^6}$$

```
[Out] 1/9*x^6/a/(b*x+a)^9+1/24*x^6/a^2/(b*x+a)^8+1/84*x^6/a^3/(b*x+a)^7+1/504*x^6
/a^4/(b*x+a)^6
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^10,x]
```

```
[Out] x^6/(9*a*(a + b*x)^9) + x^6/(24*a^2*(a + b*x)^8) + x^6/(84*a^3*(a + b*x)^7)
 + x^6/(504*a^4*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^{10}}\,dx &= \frac{x^6}{9a(a+bx)^9} + \frac{\int \frac{x^5}{(a+bx)^9}\,dx}{3a} \\ &= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{\int \frac{x^5}{(a+bx)^8}\,dx}{12a^2} \\ &= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{\int \frac{x^5}{(a+bx)^7}\,dx}{84a^3} \\ &= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{x^6}{504a^4(a+bx)^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.93

$$-\frac{a^5+9a^4bx+36a^3b^2x^2+84a^2b^3x^3+126ab^4x^4+126b^5x^5}{504b^6(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^5 + 9*a^4*b*x + 36*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 126*a*b^4*x^4 +
 126*b^5*x^5)/(b^6*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(69) = 138.
time = 3.03, size = 152, normalized size = 2.20

$$\frac{-a^5-9a^4bx-36a^3b^2x^2-84a^2b^3x^3-126ab^4x^4-126b^5x^5}{504b^6\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^5/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 5 - 9 a ^ 4 b x - 36 a ^ 3 b ^ 2 x ^ 2 - 84 a ^ 2 b ^ 3 x ^ 3 - 126 a
 b ^ 4 x ^ 4 - 126 b ^ 5 x ^ 5) / (504 b ^ 6 (a ^ 9 + 9 a ^ 8 b x + 36 a ^
7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b
^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 +
 b ^ 9 x ^ 9))
```

Maple [A]
time = 0.08, size = 86, normalized size = 1.25

method	result	size

gosper	$-\frac{126b^5x^5+126a\,b^4x^4+84a^2b^3x^3+36a^3b^2x^2+9a^4bx+a^5}{504(bx+a)^9b^6}$	63
norman	$\frac{-\frac{x^5}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^3}{6b^3}-\frac{a^3x^2}{14b^4}-\frac{a^4x}{56b^5}-\frac{a^5}{504b^6}}{(bx+a)^9}$	66
risch	$\frac{-\frac{x^5}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^3}{6b^3}-\frac{a^3x^2}{14b^4}-\frac{a^4x}{56b^5}-\frac{a^5}{504b^6}}{(bx+a)^9}$	66
default	$\frac{a}{b^6(bx+a)^5}-\frac{1}{4b^6(bx+a)^4}+\frac{a^5}{9b^6(bx+a)^9}-\frac{5a^2}{3b^6(bx+a)^6}+\frac{10a^3}{7b^6(bx+a)^7}-\frac{5a^4}{8b^6(bx+a)^8}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] a/b^6/(b*x+a)^5-1/4/b^6/(b*x+a)^4+1/9*a^5/b^6/(b*x+a)^9-5/3/b^6*a^2/(b*x+a)
^6+10/7*a^3/b^6/(b*x+a)^7-5/8*a^4/b^6/(b*x+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.
time = 0.25, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+9\,a^8b^7x+a^9b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.
time = 0.31, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+9\,a^8b^7x+a^9b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.
time = 0.37, size = 163, normalized size = 2.36

$$\frac{-a^5 - 9a^4bx - 36a^3b^2x^2 - 84a^2b^3x^3 - 126ab^4x^4 - 126b^5x^5}{504a^9b^6 + 4536a^8b^7x + 18144a^7b^8x^2 + 42336a^6b^9x^3 + 63504a^5b^{10}x^4 + 63504a^4b^{11}x^5 + 42336a^3b^{12}x^6 + 18144a^2b^{13}x^7 + 4536ab^{14}x^8 + 504b^{15}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**10,x)
```

```
[Out] (-a**5 - 9*a**4*b*x - 36*a**3*b**2*x**2 - 84*a**2*b**3*x**3 - 126*a*b**4*x*
*4 - 126*b**5*x**5)/(504*a**9*b**6 + 4536*a**8*b**7*x + 18144*a**7*b**8*x**
2 + 42336*a**6*b**9*x**3 + 63504*a**5*b**10*x**4 + 63504*a**4*b**11*x**5 +
42336*a**3*b**12*x**6 + 18144*a**2*b**13*x**7 + 4536*a*b**14*x**8 + 504*b**
15*x**9)
```

Giac [A]
time = 0.00, size = 70, normalized size = 1.01

$$\frac{-126x^5b^5 - 126x^4b^4a - 84x^3b^3a^2 - 36x^2b^2a^3 - 9xba^4 - a^5}{504b^6\,(xb+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x)
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/((b*x + a)^9*b^6)
```

Mupad [B]
time = 0.08, size = 71, normalized size = 1.03

$$\frac{\frac{a}{(a+b\,x)^5} - \frac{1}{4\,(a+b\,x)^4} - \frac{5\,a^2}{3\,(a+b\,x)^6} + \frac{10\,a^3}{7\,(a+b\,x)^7} - \frac{5\,a^4}{8\,(a+b\,x)^8} + \frac{a^5}{9\,(a+b\,x)^9}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^10,x)
```

```
[Out] (a/(a + b*x)^5 - 1/(4*(a + b*x)^4) - (5*a^2)/(3*(a + b*x)^6) + (10*a^3)/(7*
(a + b*x)^7) - (5*a^4)/(8*(a + b*x)^8) + a^5/(9*(a + b*x)^9))/b^6
```

3.230 $\int \frac{x^4}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=81

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^10,x]
```

```
[Out] -1/9*a^4/(b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a +
b*x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{10}} \, dx &= \int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right) dx\\ &= -\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.65

$$-\frac{a^4+9a^3bx+36a^2b^2x^2+84ab^3x^3+126b^4x^4}{630b^5(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^10,x]
```

```
[Out] -1/630*(a^4 + 9*a^3*b*x + 36*a^2*b^2*x^2 + 84*a*b^3*x^3 + 126*b^4*x^4)/(b^5
*(a + b*x)^9)
```

Mathics [A]
time = 2.89, size = 141, normalized size = 1.74

$$\frac{-a^4-9a^3bx-36a^2b^2x^2-84ab^3x^3-126b^4x^4}{630b^5\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^4/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 4 - 9 a ^ 3 b x - 36 a ^ 2 b ^ 2 x ^ 2 - 84 a b ^ 3 x ^ 3 - 126 b ^ 4
 x ^ 4) / (630 b ^ 5 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6
 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^
 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [A]
time = 0.08, size = 72, normalized size = 0.89

method	result	size
gosper	$-\frac{126b^4x^4+84a\,b^3x^3+36a^2b^2x^2+9a^3bx+a^4}{630(bx+a)^9b^5}$	52
norman	$\frac{-\frac{x^4}{5b}-\frac{2a\,x^3}{15b^2}-\frac{2a^2x^2}{35b^3}-\frac{a^3x}{70b^4}-\frac{a^4}{630b^5}}{(bx+a)^9}$	55
risch	$\frac{-\frac{x^4}{5b}-\frac{2a\,x^3}{15b^2}-\frac{2a^2x^2}{35b^3}-\frac{a^3x}{70b^4}-\frac{a^4}{630b^5}}{(bx+a)^9}$	55
default	$-\frac{a^4}{9b^5(bx+a)^9}+\frac{a^3}{2b^5(bx+a)^8}-\frac{6a^2}{7b^5(bx+a)^7}+\frac{2a}{3b^5(bx+a)^6}-\frac{1}{5b^5(bx+a)^5}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

Maxima [A]
time = 0.27, size = 142, normalized size = 1.75

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,\left(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+9\,a^8b^6x+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

Fricas [A]
time = 0.31, size = 142, normalized size = 1.75

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+9\,a^8b^6x+a^9b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(75) = 150.
time = 0.35, size = 151, normalized size = 1.86

$$\frac{-a^4-9a^3bx-36a^2b^2x^2-84ab^3x^3-126b^4x^4}{630a^9b^5+5670a^8b^6x+22680a^7b^7x^2+52920a^6b^8x^3+79380a^5b^9x^4+79380a^4b^{10}x^5+52920a^3b^{11}x^6+22680a^2b^{12}x^7+5670ab^{13}x^8+630b^{14}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x - 36*a**2*b**2*x**2 - 84*a*b**3*x**3 - 126*b**4*x**4)/(
630*a**9*b**5 + 5670*a**8*b**6*x + 22680*a**7*b**7*x**2 + 52920*a**6*b**8*x
**3 + 79380*a**5*b**9*x**4 + 79380*a**4*b**10*x**5 + 52920*a**3*b**11*x**6
+ 22680*a**2*b**12*x**7 + 5670*a*b**13*x**8 + 630*b**14*x**9)
```

Giac [A]
time = 0.00, size = 58, normalized size = 0.72

$$\frac{-126x^4b^4-84x^3b^3a-36x^2b^2a^2-9xba^3-a^4}{630b^5\,(xb+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x)
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/((b*
x + a)^9*b^5)
```

Mupad [B]
time = 0.08, size = 61, normalized size = 0.75

$$-\frac{\frac{1}{5\,(a+b\,x)^5}-\frac{2\,a}{3\,(a+b\,x)^6}+\frac{6\,a^2}{7\,(a+b\,x)^7}-\frac{a^3}{2\,(a+b\,x)^8}+\frac{a^4}{9\,(a+b\,x)^9}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^10,x)
```

```
[Out] -(1/(5*(a + b*x)^5) - (2*a)/(3*(a + b*x)^6) + (6*a^2)/(7*(a + b*x)^7) - a^3
/(2*(a + b*x)^8) + a^4/(9*(a + b*x)^9))/b^5
```

3.231 $\int \frac{x^3}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=64

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

```
[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*
x+a)^6
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^10,x]
```

```
[Out] a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a + b
*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right)dx\\ &= \frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.66

$$-\frac{a^3+9a^2bx+36ab^2x^2+84b^3x^3}{504b^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^3 + 9*a^2*b*x + 36*a*b^2*x^2 + 84*b^3*x^3)/(b^4*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(64) = 128.
time = 2.80, size = 130, normalized size = 2.03

$$\frac{-a^3 - 9a^2bx - 36ab^2x^2 - 84b^3x^3}{504b^4\left(a^9 + 9a^8bx + 36a^7b^2x^2 + 84a^6b^3x^3 + 126a^5b^4x^4 + 126a^4b^5x^5 + 84a^3b^6x^6 + 36a^2b^7x^7 + 9ab^8x^8 + b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 3 - 9 a ^ 2 b x - 36 a b ^ 2 x ^ 2 - 84 b ^ 3 x ^ 3) / (504 b ^ 4 (a
^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5
 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^
7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.89

method	result	size
gosper	$-\frac{84b^3x^3+36a\,b^2x^2+9a^2bx+a^3}{504(bx+a)^9b^4}$	41
norman	$\frac{-\frac{x^3}{6b}-\frac{a\,x^2}{14b^2}-\frac{a^2x}{56b^3}-\frac{a^3}{504b^4}}{(bx+a)^9}$	44
risch	$\frac{-\frac{x^3}{6b}-\frac{a\,x^2}{14b^2}-\frac{a^2x}{56b^3}-\frac{a^3}{504b^4}}{(bx+a)^9}$	44
default	$\frac{a^3}{9b^4(bx+a)^9}-\frac{3a^2}{8b^4(bx+a)^8}+\frac{3a}{7b^4(bx+a)^7}-\frac{1}{6b^4(bx+a)^6}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*
x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(56) = 112.
time = 0.25, size = 131, normalized size = 2.05

$$-\frac{84\,b^3x^3 + 36\,ab^2x^2 + 9\,a^2bx + a^3}{504\left(b^{13}x^9 + 9\,ab^{12}x^8 + 36\,a^2b^{11}x^7 + 84\,a^3b^{10}x^6 + 126\,a^4b^9x^5 + 126\,a^5b^8x^4 + 84\,a^6b^7x^3 + 36\,a^7b^6x^2 + 9\,a^8b^5x + a^9b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(56) = 112$.
time = 0.31, size = 131, normalized size = 2.05

$$-\frac{84\,b^{3}x^{3}+36\,ab^{2}x^{2}+9\,a^{2}bx+a^{3}}{504\,\left(b^{13}x^{9}+9\,ab^{12}x^{8}+36\,a^{2}b^{11}x^{7}+84\,a^{3}b^{10}x^{6}+126\,a^{4}b^{9}x^{5}+126\,a^{5}b^{8}x^{4}+84\,a^{6}b^{7}x^{3}+36\,a^{7}b^{6}x^{2}+9\,a^{8}b^{5}x+a^{9}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(60) = 120$.
time = 0.32, size = 139, normalized size = 2.17

$$\frac{-a^{3}-9a^{2}bx-36ab^{2}x^{2}-84b^{3}x^{3}}{504a^{9}b^{4}+4536a^{8}b^{5}x+18144a^{7}b^{6}x^{2}+42336a^{6}b^{7}x^{3}+63504a^{5}b^{8}x^{4}+63504a^{4}b^{9}x^{5}+42336a^{3}b^{10}x^{6}+18144a^{2}b^{11}x^{7}+4536ab^{12}x^{8}+504b^{13}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**10,x)
```

```
[Out] (-a**3 - 9*a**2*b*x - 36*a*b**2*x**2 - 84*b**3*x**3)/(504*a**9*b**4 + 4536*
a**8*b**5*x + 18144*a**7*b**6*x**2 + 42336*a**6*b**7*x**3 + 63504*a**5*b**8
*x**4 + 63504*a**4*b**9*x**5 + 42336*a**3*b**10*x**6 + 18144*a**2*b**11*x**
7 + 4536*a*b**12*x**8 + 504*b**13*x**9)
```

Giac [A]
time = 0.00, size = 46, normalized size = 0.72

$$\frac{-84x^{3}b^{3}-36x^{2}b^{2}a-9xba^{2}-a^{3}}{504b^{4}\left(xb+a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x)
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/((b*x + a)^9*b^4)
```

Mupad [B]
time = 0.13, size = 48, normalized size = 0.75

$$\frac{\frac{3\,a}{7\,{\left(a+b\,x\right)}^{7}}-\frac{1}{6\,{\left(a+b\,x\right)}^{6}}-\frac{3\,a^{2}}{8\,{\left(a+b\,x\right)}^{8}}+\frac{a^{3}}{9\,{\left(a+b\,x\right)}^{9}}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^10,x)
```

```
[Out] ((3*a)/(7*(a + b*x)^7) - 1/(6*(a + b*x)^6) - (3*a^2)/(8*(a + b*x)^8) + a^3/
(9*(a + b*x)^9))/b^4
```

3.232 $\int \frac{x^2}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=47

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

```
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^10,x]
```

```
[Out] -1/9*a^2/(b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a+bx)^{10}} \, dx &= \int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right) \, dx \\ &= -\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+9abx+36b^2x^2}{252b^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^2 + 9*a*b*x + 36*b^2*x^2)/(b^3*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 121 vs. $2(47) = 94$.
time = 2.75, size = 119, normalized size = 2.53

$$\frac{-a^2 - 9abx - 36b^2x^2}{252b^3\left(a^9 + 9a^8bx + 36a^7b^2x^2 + 84a^6b^3x^3 + 126a^5b^4x^4 + 126a^4b^5x^5 + 84a^3b^6x^6 + 36a^2b^7x^7 + 9ab^8x^8 + b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/(a + b*x)^10,x]')
```

```
[Out] (-a ^ 2 - 9 a b x - 36 b ^ 2 x ^ 2) / (252 b ^ 3 (a ^ 9 + 9 a ^ 8 b x + 36
a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^
4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^
 8 + b ^ 9 x ^ 9))
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.89

method	result	size
gosper	$-\frac{36x^2b^2+9abx+a^2}{252(bx+a)^9b^3}$	30
norman	$\frac{-\frac{x^2}{7b}-\frac{ax}{28b^2}-\frac{a^2}{252b^3}}{(bx+a)^9}$	33
risch	$\frac{-\frac{x^2}{7b}-\frac{ax}{28b^2}-\frac{a^2}{252b^3}}{(bx+a)^9}$	33
default	$-\frac{a^2}{9b^3(bx+a)^9}+\frac{a}{4b^3(bx+a)^8}-\frac{1}{7b^3(bx+a)^7}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.
time = 0.25, size = 120, normalized size = 2.55

$$-\frac{36\,b^2x^2 + 9\,abx + a^2}{252\,(b^{12}x^9 + 9\,ab^{11}x^8 + 36\,a^2b^{10}x^7 + 84\,a^3b^9x^6 + 126\,a^4b^8x^5 + 126\,a^5b^7x^4 + 84\,a^6b^6x^3 + 36\,a^7b^5x^2 + 9\,a^8b^4x + a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.
time = 0.31, size = 120, normalized size = 2.55

$$-\frac{36\,b^2x^2+9\,abx+a^2}{252\,(b^{12}x^9+9\,ab^{11}x^8+36\,a^2b^{10}x^7+84\,a^3b^9x^6+126\,a^4b^8x^5+126\,a^5b^7x^4+84\,a^6b^6x^3+36\,a^7b^5x^2+9\,a^8b^4x+a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(41) = 82$.
time = 0.31, size = 128, normalized size = 2.72

$$\frac{-a^2-9abx-36b^2x^2}{252a^9b^3+2268a^8b^4x+9072a^7b^5x^2+21168a^6b^6x^3+31752a^5b^7x^4+31752a^4b^8x^5+21168a^3b^9x^6+9072a^2b^{10}x^7+2268ab^{11}x^8+252b^{12}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x - 36*b**2*x**2)/(252*a**9*b**3 + 2268*a**8*b**4*x + 9072*a
**7*b**5*x**2 + 21168*a**6*b**6*x**3 + 31752*a**5*b**7*x**4 + 31752*a**4*b*
*8*x**5 + 21168*a**3*b**9*x**6 + 9072*a**2*b**10*x**7 + 2268*a*b**11*x**8 +
 252*b**12*x**9)
```

Giac [A]
time = 0.00, size = 34, normalized size = 0.72

$$\frac{-36x^2b^2-9xba-a^2}{252b^3\,(xb+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x)
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/((b*x + a)^9*b^3)
```

Mupad [B]
time = 0.15, size = 31, normalized size = 0.66

$$-\frac{8\,a^2+72\,a\,b\,x+288\,b^2\,x^2}{2016\,b^3\,(a+b\,x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^10,x)
```

```
[Out] -(8*a^2 + 288*b^2*x^2 + 72*a*b*x)/(2016*b^3*(a + b*x)^9)
```

3.233 $\int \frac{x}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=30

$$\frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8}$$

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^10,x]
```

```
[Out] a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{10}} \, dx &= \int \left(-\frac{a}{b(a+bx)^{10}} + \frac{1}{b(a+bx)^9}\right) dx \\ &= \frac{a}{9b^2(a+bx)^9} - \frac{1}{8b^2(a+bx)^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.67

$$-\frac{a+9bx}{72b^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^10,x]
```

```
[Out] -1/72*(a + 9*b*x)/(b^2*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 110 vs. $2(30) = 60$.
time = 2.64, size = 108, normalized size = 3.60

$$\frac{-a-9bx}{72b^2\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1/(a + b*x)^10,x]')
```

```
[Out] (-a - 9 b x) / (72 b ^ 2 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a
 ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3
 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [A]
time = 0.08, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{9bx+a}{72(bx+a)^9b^2}$	19
norman	$\frac{-\frac{x}{8b}-\frac{a}{72b^2}}{(bx+a)^9}$	22
risch	$\frac{-\frac{x}{8b}-\frac{a}{72b^2}}{(bx+a)^9}$	22
default	$\frac{a}{9b^2(bx+a)^9}-\frac{1}{8b^2(bx+a)^8}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.
time = 0.27, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,\left(b^{11}x^9+9\,ab^{10}x^8+36\,a^2b^9x^7+84\,a^3b^8x^6+126\,a^4b^7x^5+126\,a^5b^6x^4+84\,a^6b^5x^3+36\,a^7b^4x^2+9\,a^8b^3x+a^9b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.
time = 0.31, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,(b^{11}x^9+9\,ab^{10}x^8+36\,a^2b^9x^7+84\,a^3b^8x^6+126\,a^4b^7x^5+126\,a^5b^6x^4+84\,a^6b^5x^3+36\,a^7b^4x^2+9\,a^8b^3x+a^9b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(26) = 52$.
time = 0.30, size = 116, normalized size = 3.87

$$\frac{-a-9bx}{72a^9b^2+648a^8b^3x+2592a^7b^4x^2+6048a^6b^5x^3+9072a^5b^6x^4+9072a^4b^7x^5+6048a^3b^8x^6+2592a^2b^9x^7+648ab^{10}x^8+72b^{11}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**10,x)
```

```
[Out] (-a - 9*b*x)/(72*a**9*b**2 + 648*a**8*b**3*x + 2592*a**7*b**4*x**2 + 6048*a
**6*b**5*x**3 + 9072*a**5*b**6*x**4 + 9072*a**4*b**7*x**5 + 6048*a**3*b**8*
x**6 + 2592*a**2*b**9*x**7 + 648*a*b**10*x**8 + 72*b**11*x**9)
```

Giac [A]
time = 0.00, size = 22, normalized size = 0.73

$$\frac{-9xb-a}{72b^2\,(xb+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x)
```

```
[Out] -1/72*(9*b*x + a)/((b*x + a)^9*b^2)
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.60

$$-\frac{a+9\,b\,x}{72\,b^2\,(a+b\,x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^10,x)
```

```
[Out] -(a + 9*b*x)/(72*b^2*(a + b*x)^9)
```

3.234 $\int \frac{1}{(a+bx)^{10}} dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/9/b/(b*x+a)^9
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-10),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10}} dx = -\frac{1}{9b(a+bx)^9}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-10),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(14) = 28.
time = 2.57, size = 100, normalized size = 7.14

$$-\frac{1}{9b\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^10,x]')
```

```
[Out] -1 / (9 b (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^
3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 +
36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{9b(bx+a)^9}$	13
default	$-\frac{1}{9b(bx+a)^9}$	13
norman	$-\frac{1}{9b(bx+a)^9}$	13
risch	$-\frac{1}{9b(bx+a)^9}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(12) = 24.
time = 0.30, size = 101, normalized size = 7.21

$$-\frac{1}{9\,(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(12) = 24$.
time = 0.31, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9x^8 + 9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**10,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Giac [**A**]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{9b\,(xb+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x)
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Mupad [**B**]
time = 0.14, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x^7+81\,a\,b^9\,x^8+9\,b^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^10,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

3.235 $\int \frac{1}{x(a+bx)^{10}}\,dx$

Optimal. Leaf size=141

$$\frac{1}{9a(a+bx)^9}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{3a^7(a+bx)^3}+\frac{}{2a^8}$$

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^10),x]
```

```
[Out] 1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6*
a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a^7
*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10 - L
og[a + b*x]/a^10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{10}}\,dx = \int \left(\frac{1}{a^{10}x}-\frac{b}{a(a+bx)^{10}}-\frac{b}{a^2(a+bx)^9}-\frac{b}{a^3(a+bx)^8}-\frac{b}{a^4(a+bx)^7}-\frac{b}{a^5(a+bx)^6}-\right.$$
$$=\frac{1}{9a(a+bx)^9}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{4a^6(a+}$$

Mathematica [A]
time = 0.06, size = 127, normalized size = 0.90

$$\frac{280a^8+315a^7(a+bx)+360a^6(a+bx)^2+420a^5(a+bx)^3+504a^4(a+bx)^4+630a^3(a+bx)^5+840a^2(a+bx)^6+1260a(a+bx)^7+2520(a+bx)^8}{2520a^9(a+bx)^9}+\frac{\log(x)}{a^{10}}-\frac{\log(a+bx)}{a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^10),x]
```

```
[Out] (280*a^8 + 315*a^7*(a + b*x) + 360*a^6*(a + b*x)^2 + 420*a^5*(a + b*x)^3 +
504*a^4*(a + b*x)^4 + 630*a^3*(a + b*x)^5 + 840*a^2*(a + b*x)^6 + 1260*a*(a
 + b*x)^7 + 2520*(a + b*x)^8)/(2520*a^9*(a + b*x)^9) + Log[x]/a^10 - Log[a
+ b*x]/a^10
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 299 vs. $2(141) = 282$.
time = 4.00, size = 297, normalized size = 2.11

$$\frac{\frac{a(7129a^8+41481a^7bx+120564a^6b^2x^2+210756a^5b^3x^3+236754a^4b^4x^4+173250a^3b^5x^5+80220a^2b^6x^6+21420ab^7x^7+2520b^8x^8)}{2520}+(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{a^{10}\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)^10),x]')
```

```
[Out] (a (7129 a ^ 8 + 41481 a ^ 7 b x + 120564 a ^ 6 b ^ 2 x ^ 2 + 210756 a ^ 5
b ^ 3 x ^ 3 + 236754 a ^ 4 b ^ 4 x ^ 4 + 173250 a ^ 3 b ^ 5 x ^ 5 + 80220 a
 ^ 2 b ^ 6 x ^ 6 + 21420 a b ^ 7 x ^ 7 + 2520 b ^ 8 x ^ 8) / 2520 + (a ^ 9
+ 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^
 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x
^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9) (Log[x] - Log[(a + b x) / b])) / (a ^
10 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126
 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^
2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [**A**]
time = 0.09, size = 126, normalized size = 0.89

method	result
risch	$\frac{\frac{b^8x^8}{a^9}+\frac{17b^7x^7}{2a^8}+\frac{191b^6x^6}{6a^7}+\frac{275b^5x^5}{4a^6}+\frac{1879b^4x^4}{20a^5}+\frac{2509b^3x^3}{30a^4}+\frac{3349b^2x^2}{70a^3}+\frac{4609bx}{280a^2}+\frac{7129}{2520a}}{(bx+a)^9}-\frac{\ln(bx+a)}{a^{10}}+\frac{\ln(-x)}{a^{10}}$
norman	$\frac{-\frac{9bx}{a^2}-\frac{54b^2x^2}{a^3}-\frac{154b^3x^3}{a^4}-\frac{525b^4x^4}{2a^5}-\frac{2877b^5x^5}{10a^6}-\frac{1029b^6x^6}{5a^7}-\frac{3267b^7x^7}{35a^8}-\frac{6849b^8x^8}{280a^9}-\frac{7129b^9x^9}{2520a^{10}}}{(bx+a)^9}+\frac{\ln(x)}{a^{10}}-\frac{\ln(bx+a)}{a^{10}}$
default	$\frac{1}{9a(bx+a)^9}+\frac{1}{8a^2(bx+a)^8}+\frac{1}{7a^3(bx+a)^7}+\frac{1}{6a^4(bx+a)^6}+\frac{1}{5a^5(bx+a)^5}+\frac{1}{4a^6(bx+a)^4}+\frac{1}{3a^7(bx+a)^3}+\frac{1}{2a^8(bx+a)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

Maxima [**A**]
time = 0.28, size = 205, normalized size = 1.45

$$\frac{2520\,b^8x^8+21420\,ab^7x^7+80220\,a^2b^6x^6+173250\,a^3b^5x^5+236754\,a^4b^4x^4+210756\,a^5b^3x^3+120564\,a^6b^2x^2+41481\,a^7bx+7129\,a^8}{2520\,(a^9b^9x^9+9\,a^{10}b^8x^8+36\,a^{11}b^7x^7+84\,a^{12}b^6x^6+126\,a^{13}b^5x^5+126\,a^{14}b^4x^4+84\,a^{15}b^3x^3+36\,a^{16}b^2x^2+9\,a^{17}bx+a^{18})}-\frac{\log(bx+a)}{a^{10}}+\frac{\log(x)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(2520*b^8*x^8 + 21420*a*b^7*x^7 + 80220*a^2*b^6*x^6 + 173250*a^3*b^5
*x^5 + 236754*a^4*b^4*x^4 + 210756*a^5*b^3*x^3 + 120564*a^6*b^2*x^2 + 41481
*a^7*b*x + 7129*a^8)/(a^9*b^9*x^9 + 9*a^10*b^8*x^8 + 36*a^11*b^7*x^7 + 84*a
^12*b^6*x^6 + 126*a^13*b^5*x^5 + 126*a^14*b^4*x^4 + 84*a^15*b^3*x^3 + 36*a^
16*b^2*x^2 + 9*a^17*b*x + a^18) - log(b*x + a)/a^10 + log(x)/a^10
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(125) = 250$.
time = 0.32, size = 388, normalized size = 2.75

$$\frac{2520\,ab^8x^8 + 21420\,a^2b^7x^7 + 80220\,a^3b^6x^6 + 173250\,a^4b^5x^5 + 236754\,a^5b^4x^4 + 210756\,a^6b^3x^3 + 120564\,a^7b^2x^2 + 41481\,a^8bx + 7129\,a^9 - 2520\,(b^9x^9 + 9\,ab^8x^8 + 36\,a^2b^7x^7 + 84\,a^3b^6x^6 + 126\,a^4b^5x^5 + 126\,a^5b^4x^4 + 84\,a^6b^3x^3 + 36\,a^7b^2x^2 + 9\,a^8bx + a^9)\log(bx + a) + 2520\,(b^9x^9 + 9\,ab^8x^8 + 36\,a^2b^7x^7 + 84\,a^3b^6x^6 + 126\,a^4b^5x^5 + 126\,a^5b^4x^4 + 84\,a^6b^3x^3 + 36\,a^7b^2x^2 + 9\,a^8bx + a^9)\log(x)}{2520\,(a^{10}b^9x^9 + 9\,a^{11}b^8x^8 + 36\,a^{12}b^7x^7 + 84\,a^{13}b^6x^6 + 126\,a^{14}b^5x^5 + 126\,a^{15}b^4x^4 + 84\,a^{16}b^3x^3 + 36\,a^{17}b^2x^2 + 9\,a^{18}bx + a^{19})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*a*b^8*x^8 + 21420*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4
*b^5*x^5 + 236754*a^5*b^4*x^4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 4
1481*a^8*b*x + 7129*a^9 - 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 + 84
*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a^7*
b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a) + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*
a^2*b^7*x^7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b
^3*x^3 + 36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(x))/(a^10*b^9*x^9 + 9*a^11*b
^8*x^8 + 36*a^12*b^7*x^7 + 84*a^13*b^6*x^6 + 126*a^14*b^5*x^5 + 126*a^15*b^
4*x^4 + 84*a^16*b^3*x^3 + 36*a^17*b^2*x^2 + 9*a^18*b*x + a^19)
```

Sympy [**A**]
time = 0.46, size = 212, normalized size = 1.50

$$\frac{7129a^8 + 41481a^7bx + 120564a^6b^2x^2 + 210756a^5b^3x^3 + 236754a^4b^4x^4 + 173250a^3b^5x^5 + 80220a^2b^6x^6 + 21420ab^7x^7 + 2520b^8x^8}{2520a^{18} + 22680a^{17}bx + 90720a^{16}b^2x^2 + 211680a^{15}b^3x^3 + 317520a^{14}b^4x^4 + 317520a^{13}b^5x^5 + 211680a^{12}b^6x^6 + 90720a^{11}b^7x^7 + 22680a^{10}b^8x^8 + 2520a^9b^9x^9} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x + 120564*a**6*b**2*x**2 + 210756*a**5*b**3*x**3
 + 236754*a**4*b**4*x**4 + 173250*a**3*b**5*x**5 + 80220*a**2*b**6*x**6 + 2
1420*a*b**7*x**7 + 2520*b**8*x**8)/(2520*a**18 + 22680*a**17*b*x + 90720*a*
*16*b**2*x**2 + 211680*a**15*b**3*x**3 + 317520*a**14*b**4*x**4 + 317520*a*
*13*b**5*x**5 + 211680*a**12*b**6*x**6 + 90720*a**11*b**7*x**7 + 22680*a**1
0*b**8*x**8 + 2520*a**9*b**9*x**9) + (log(x) - log(a/b + x))/a**10
```

Giac [**A**]
time = 0.00, size = 129, normalized size = 0.91

$$\frac{\ln|x|}{a^{10}} - \frac{b\ln|xb + a|}{ba^{10}} + \frac{\frac{1}{2520}\left(2520b^8ax^8 + 21420b^7a^2x^7 + 80220b^6a^3x^6 + 173250b^5a^4x^5 + 236754b^4a^5x^4 + 210756b^3a^6x^3 + 120564b^2a^7x^2 + 41481ba^8x + 7129a^9\right)}{a^{10}\left(xb + a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x)
```

```
[Out] -log(abs(b*x + a))/a^10 + log(abs(x))/a^10 + 1/2520*(2520*a*b^8*x^8 + 21420
*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4*b^5*x^5 + 236754*a^5*b^4*x^4
+ 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 41481*a^8*b*x + 7129*a^9)/((b*x
 + a)^9*a^10)
```

Mupad [B]
time = 0.76, size = 145, normalized size = 1.03

$$\frac{1}{9\,a\,{\left(a+b\,x\right)}^9}-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{14\,b^2\,x^2}{{\left(a+b\,x\right)}^2}+\frac{56\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{35\,b^4\,x^4}{2\,{\left(a+b\,x\right)}^4}+\frac{56\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,b^6\,x^6}{3\,{\left(a+b\,x\right)}^6}+\frac{8\,b^7\,x^7}{7\,{\left(a+b\,x\right)}^7}-\frac{b^8\,x^8}{8\,{\left(a+b\,x\right)}^8}+\frac{8\,b\,x}{a+b\,x}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^10),x)
```

```
[Out] 1/(9*a*(a + b*x)^9) - (log((a + b*x)/x) - (14*b^2*x^2)/(a + b*x)^2 + (56*b^
3*x^3)/(3*(a + b*x)^3) - (35*b^4*x^4)/(2*(a + b*x)^4) + (56*b^5*x^5)/(5*(a
+ b*x)^5) - (14*b^6*x^6)/(3*(a + b*x)^6) + (8*b^7*x^7)/(7*(a + b*x)^7) - (b
^8*x^8)/(8*(a + b*x)^8) + (8*b*x)/(a + b*x))/a^10
```

3.236 $\int \frac{1}{x^2(a+bx)^{10}}\,dx$

Optimal. Leaf size=158

$$-\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}-\frac{3b}{2a^7(a+bx)^4}-\frac{7b}{3a^8(a+b}$$

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

Rubi [A]
time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{9b}{a^{10}(a+bx)}-\frac{1}{a^{10}x}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}-\frac{b}{9a^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a^
4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/(2
*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2) - (
9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\frac{}{a^6(a}\right.\\ &= -\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 130, normalized size = 0.82

$$-\frac{\frac{a(252a^9+7129a^8bx+41481a^7b^2x^2+120564a^6b^3x^3+210756a^5b^4x^4+236754a^4b^5x^5+173250a^3b^6x^6+80220a^2b^7x^7+21420ab^8x^8+2520b^9x^9)}{x(a+bx)^9}+2520b\log(x)-2520b\log(a+bx)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -1/252*((a*(252*a^9 + 7129*a^8*b*x + 41481*a^7*b^2*x^2 + 120564*a^6*b^3*x^3
 + 210756*a^5*b^4*x^4 + 236754*a^4*b^5*x^5 + 173250*a^3*b^6*x^6 + 80220*a^2
*b^7*x^7 + 21420*a*b^8*x^8 + 2520*b^9*x^9))/(x*(a + b*x)^9) + 2520*b*Log[x]
 - 2520*b*Log[a + b*x])/a^11
```

Mathics [A]
time = 4.17, size = 314, normalized size = 1.99

$$\frac{a\left(-252a^9-7129a^8bx-41481a^7b^2x^2-120564a^6b^3x^3-210756a^5b^4x^4-236754a^4b^5x^5-173250a^3b^6x^6-80220a^2b^7x^7-21420ab^8x^8-2520b^9x^9\right)-2520bx\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)\left(\mathrm{Log}\,[x]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{252a^{11}x\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^10),x]')
```

```
[Out] (a (-252 a ^ 9 - 7129 a ^ 8 b x - 41481 a ^ 7 b ^ 2 x ^ 2 - 120564 a ^ 6 b
^ 3 x ^ 3 - 210756 a ^ 5 b ^ 4 x ^ 4 - 236754 a ^ 4 b ^ 5 x ^ 5 - 173250 a
^ 3 b ^ 6 x ^ 6 - 80220 a ^ 2 b ^ 7 x ^ 7 - 21420 a b ^ 8 x ^ 8 - 2520 b ^
9 x ^ 9) - 2520 b x (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6
b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^
6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9) (Log[x] - L
og[(a + b x) / b])) / (252 a ^ 11 x (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x
 ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5
 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x
^ 9))
```

Maple [A]
time = 0.09, size = 147, normalized size = 0.93

method	result
risch	$\frac{-\frac{10b^9x^9}{a^{10}}-\frac{85b^8x^8}{a^9}-\frac{955b^7x^7}{3a^8}-\frac{1375b^6x^6}{2a^7}-\frac{1879b^5x^5}{2a^6}-\frac{2509b^4x^4}{3a^5}-\frac{3349b^3x^3}{7a^4}-\frac{4609b^2x^2}{28a^3}-\frac{7129bx}{252a^2}-\frac{1}{a}}{x(bx+a)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{10b\ln(-bx-a)}{a^{11}}$
norman	$\frac{-\frac{1}{a}+\frac{90b^2x^2}{a^3}+\frac{540b^3x^3}{a^4}+\frac{1540b^4x^4}{a^5}+\frac{2625b^5x^5}{a^6}+\frac{2877b^6x^6}{a^7}+\frac{2058b^7x^7}{a^8}+\frac{6534b^8x^8}{7a^9}+\frac{6849b^9x^9}{28a^{10}}+\frac{7129b^{10}x^{10}}{252a^{11}}}{x(bx+a)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{10b\ln(bx+a)}{a^{11}}$
default	$-\frac{1}{a^{10}x}-\frac{b}{9a^2(bx+a)^9}-\frac{b}{4a^3(bx+a)^8}-\frac{3b}{7a^4(bx+a)^7}-\frac{2b}{3a^5(bx+a)^6}-\frac{b}{a^6(bx+a)^5}-\frac{3b}{2a^7(bx+a)^4}-\frac{7b}{3a^8(bx+a)^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

Maxima [A]
time = 0.26, size = 223, normalized size = 1.41

$$-\frac{2520\,b^{9}x^{9}+21420\,ab^{8}x^{8}+80220\,a^{2}b^{7}x^{7}+173250\,a^{3}b^{6}x^{6}+236754\,a^{4}b^{5}x^{5}+210756\,a^{5}b^{4}x^{4}+120564\,a^{6}b^{3}x^{3}+41481\,a^{7}b^{2}x^{2}+7129\,a^{8}bx+252\,a^{9}}{252\,(a^{10}b^{9}x^{10}+9\,a^{11}b^{8}x^{9}+36\,a^{12}b^{7}x^{8}+84\,a^{13}b^{6}x^{7}+126\,a^{14}b^{5}x^{6}+126\,a^{15}b^{4}x^{5}+84\,a^{16}b^{3}x^{4}+36\,a^{17}b^{2}x^{3}+9\,a^{18}bx^{2}+a^{19}x)}+\frac{10\,b\log\left(bx+a\right)}{a^{11}}-\frac{10\,b\log\left(x\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 21420*a*b^8*x^8 + 80220*a^2*b^7*x^7 + 173250*a^3*b^6
*x^6 + 236754*a^4*b^5*x^5 + 210756*a^5*b^4*x^4 + 120564*a^6*b^3*x^3 + 41481
*a^7*b^2*x^2 + 7129*a^8*b*x + 252*a^9)/(a^10*b^9*x^10 + 9*a^11*b^8*x^9 + 36
*a^12*b^7*x^8 + 84*a^13*b^6*x^7 + 126*a^14*b^5*x^6 + 126*a^15*b^4*x^5 + 84*
a^16*b^3*x^4 + 36*a^17*b^2*x^3 + 9*a^18*b*x^2 + a^19*x) + 10*b*log(b*x + a)
/a^11 - 10*b*log(x)/a^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(146) = 292.
time = 0.32, size = 417, normalized size = 2.64

$$-\frac{2520\,ab^{9}x^{9}+21420\,a^{2}b^{8}x^{8}+80220\,a^{3}b^{7}x^{7}+173250\,a^{4}b^{6}x^{6}+236754\,a^{5}b^{5}x^{5}+210756\,a^{6}b^{4}x^{4}+120564\,a^{7}b^{3}x^{3}+41481\,a^{8}b^{2}x^{2}+7129\,a^{9}bx+252\,a^{10}-2520\,(b^{10}x^{10}+9\,ab^{9}x^{9}+36\,a^{2}b^{8}x^{8}+84\,a^{3}b^{7}x^{7}+126\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}+36\,a^{7}b^{3}x^{3}+9\,a^{8}b^{2}x^{2}+a^{9}bx)\log\left(bx+a\right)+2520\,(b^{10}x^{10}+9\,ab^{9}x^{9}+36\,a^{2}b^{8}x^{8}+84\,a^{3}b^{7}x^{7}+126\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}+36\,a^{7}b^{3}x^{3}+9\,a^{8}b^{2}x^{2}+a^{9}bx)\log\left(x\right)}{252\,(a^{11}b^{9}x^{10}+9\,a^{12}b^{8}x^{9}+36\,a^{13}b^{7}x^{8}+84\,a^{14}b^{6}x^{7}+126\,a^{15}b^{5}x^{6}+126\,a^{16}b^{4}x^{5}+84\,a^{17}b^{3}x^{4}+36\,a^{18}b^{2}x^{3}+9\,a^{19}bx^{2}+a^{20}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4
*b^6*x^6 + 236754*a^5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 4
1481*a^8*b^2*x^2 + 7129*a^9*b*x + 252*a^10 - 2520*(b^10*x^10 + 9*a*b^9*x^9
+ 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 84*
a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^9*b*x)*log(b*x + a) + 2520
*(b^10*x^10 + 9*a*b^9*x^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x
^6 + 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^
9*b*x)*log(x))/(a^11*b^9*x^10 + 9*a^12*b^8*x^9 + 36*a^13*b^7*x^8 + 84*a^14*
b^6*x^7 + 126*a^15*b^5*x^6 + 126*a^16*b^4*x^5 + 84*a^17*b^3*x^4 + 36*a^18*b
^2*x^3 + 9*a^19*b*x^2 + a^20*x)
```

Sympy [A]
time = 0.53, size = 233, normalized size = 1.47

$$\frac{-252a^{9}-7129a^{8}bx-41481a^{7}b^{2}x^{2}-120564a^{6}b^{3}x^{3}-210756a^{5}b^{4}x^{4}-236754a^{4}b^{5}x^{5}-173250a^{3}b^{6}x^{6}-80220a^{2}b^{7}x^{7}-21420ab^{8}x^{8}-2520b^{9}x^{9}}{252a^{19}x+2268a^{18}bx^{2}+9072a^{17}b^{2}x^{3}+21168a^{16}b^{3}x^{4}+31752a^{15}b^{4}x^{5}+31752a^{14}b^{5}x^{6}+21168a^{13}b^{6}x^{7}+9072a^{12}b^{7}x^{8}+2268a^{11}b^{8}x^{9}+252a^{10}b^{9}x^{10}}+\frac{10b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x - 41481*a**7*b**2*x**2 - 120564*a**6*b**3*x**3 -
 210756*a**5*b**4*x**4 - 236754*a**4*b**5*x**5 - 173250*a**3*b**6*x**6 - 80
```

```
220*a**2*b**7*x**7 - 21420*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**19*x + 226
8*a**18*b*x**2 + 9072*a**17*b**2*x**3 + 21168*a**16*b**3*x**4 + 31752*a**15
*b**4*x**5 + 31752*a**14*b**5*x**6 + 21168*a**13*b**6*x**7 + 9072*a**12*b**
7*x**8 + 2268*a**11*b**8*x**9 + 252*a**10*b**9*x**10) + 10*b*(-log(x) + log
(a/b + x))/a**11
```

Giac [A]
time = 0.00, size = 157, normalized size = 0.99

$$-\frac{10b\ln|x|}{a^{11}}+\frac{10b^2\ln|xb+a|}{ba^{11}}+\frac{\frac{1}{252}\left(-2520b^9ax^9-21420b^8a^2x^8-80220b^7a^3x^7-173250b^6a^4x^6-236754b^5a^5x^5-210756b^4a^6x^4-120564b^3a^7x^3-41481b^2a^8x^2-7129ba^9x-252a^{10}\right)}{a^{11}x\left(bx+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x)
```

```
[Out] 10*b*log(abs(b*x + a))/a^11 - 10*b*log(abs(x))/a^11 - 1/252*(2520*a*b^9*x^9
 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4*b^6*x^6 + 236754*a^5*
b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 41481*a^8*b^2*x^2 + 712
9*a^9*b*x + 252*a^10)/((b*x + a)^9*a^11*x)
```

Mupad [B]
time = 0.39, size = 217, normalized size = 1.37

$$\frac{20\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{a}+\frac{4609\,b^2\,x^2}{28\,a^3}+\frac{3349\,b^3\,x^3}{7\,a^4}+\frac{2509\,b^4\,x^4}{3\,a^5}+\frac{1879\,b^5\,x^5}{2\,a^6}+\frac{1375\,b^6\,x^6}{2\,a^7}+\frac{955\,b^7\,x^7}{3\,a^8}+\frac{85\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}+\frac{7129\,b\,x}{252\,a^2}}{a^9\,x+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^3+84\,a^6\,b^3\,x^4+126\,a^5\,b^4\,x^5+126\,a^4\,b^5\,x^6+84\,a^3\,b^6\,x^7+36\,a^2\,b^7\,x^8+9\,a\,b^8\,x^9+b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^10),x)
```

```
[Out] (20*b*atanh((2*b*x)/a + 1))/a^11 - (1/a + (4609*b^2*x^2)/(28*a^3) + (3349*b
^3*x^3)/(7*a^4) + (2509*b^4*x^4)/(3*a^5) + (1879*b^5*x^5)/(2*a^6) + (1375*b
^6*x^6)/(2*a^7) + (955*b^7*x^7)/(3*a^8) + (85*b^8*x^8)/a^9 + (10*b^9*x^9)/a
^10 + (7129*b*x)/(252*a^2))/(a^9*x + b^9*x^10 + 9*a^8*b*x^2 + 9*a*b^8*x^9 +
 36*a^7*b^2*x^3 + 84*a^6*b^3*x^4 + 126*a^5*b^4*x^5 + 126*a^4*b^5*x^6 + 84*a
^3*b^6*x^7 + 36*a^2*b^7*x^8)
```

3.237 $\int \frac{1}{x^3(a+bx)^{10}}\,dx$

Optimal. Leaf size=191

$$-\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(a+bx)^9}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{3b^2}{a^7(a+bx)^5}+\frac{21b^2}{4a^8(a+bx)^4}+3$$

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Rubi [A]
time = 0.10, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{55b^2\log(x)}{a^{12}}-\frac{55b^2\log(a+bx)}{a^{12}}+\frac{45b^2}{a^{11}(a+bx)}+\frac{10b}{a^{11}x}+\frac{18b^2}{a^{10}(a+bx)^2}-\frac{1}{2a^{10}x^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+bx)^5}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{b^2}{9a^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^10),x]
```

```
[Out] -1/2*1/(a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8*
a^4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)^6
) + (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/(3*
a^9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*x))
+ (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx = \int \left(\frac{1}{a^{10}x^3}-\frac{10b}{a^{11}x^2}+\frac{55b^2}{a^{12}x}-\frac{b^3}{a^3(a+bx)^{10}}-\frac{3b^3}{a^4(a+bx)^9}-\frac{6b^3}{a^5(a+bx)^8}-\frac{10b^3}{a^6(a+bx}\right.$$

$$= -\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(a+bx)^9}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{}{a^7}$$

Mathematica [A]

time = 0.06, size = 145, normalized size = 0.76

$$\frac{\frac{a(-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10})}{x^2(a+bx)^9}+27720b^2\log(x)-27720b^2\log(a+bx)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x + 78419*a^8*b^2*x^2 + 456291*a^7*b^3*x^3 + 13
26204*a^6*b^4*x^4 + 2318316*a^5*b^5*x^5 + 2604294*a^4*b^6*x^6 + 1905750*a^3
*b^7*x^7 + 882420*a^2*b^8*x^8 + 235620*a*b^9*x^9 + 27720*b^10*x^10))/(x^2*(
a + b*x)^9) + 27720*b^2*Log[x] - 27720*b^2*Log[a + b*x])/(504*a^12)
```

Mathics [A]

time = 4.34, size = 329, normalized size = 1.72

$$\frac{a(-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10})+27720b^2x^2(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9)\left(\mathrm{Log}[x]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{504a^{12}x^2(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^10),x]')
```

```
[Out] (a (-252 a ^ 10 + 2772 a ^ 9 b x + 78419 a ^ 8 b ^ 2 x ^ 2 + 456291 a ^ 7 b
 ^ 3 x ^ 3 + 1326204 a ^ 6 b ^ 4 x ^ 4 + 2318316 a ^ 5 b ^ 5 x ^ 5 + 260429
4 a ^ 4 b ^ 6 x ^ 6 + 1905750 a ^ 3 b ^ 7 x ^ 7 + 882420 a ^ 2 b ^ 8 x ^ 8
+ 235620 a b ^ 9 x ^ 9 + 27720 b ^ 10 x ^ 10) + 27720 b ^ 2 x ^ 2 (a ^ 9 +
9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4
 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^
7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9) (Log[x] - Log[(a + b x) / b])) / (504 a
^ 12 x ^ 2 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^ 3 x ^
 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x ^ 6 +
 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9))
```

Maple [A]

time = 0.09, size = 178, normalized size = 0.93

method	result
norman	$\frac{-\frac{1}{2a}+\frac{11bx}{2a^2}-\frac{495b^3x^3}{a^4}-\frac{2970b^4x^4}{a^5}-\frac{8470b^5x^5}{a^6}-\frac{28875b^6x^6}{2a^7}-\frac{31647b^7x^7}{2a^8}-\frac{11319b^8x^8}{a^9}-\frac{35937b^9x^9}{7a^{10}}-\frac{75339b^{10}x^{10}}{56a^{11}}-\frac{78419b^{11}x^{11}}{504a^{12}}}{x^2(bx+a)^9}+\frac{55b^2\ln(x)}{a^{12}}$
risch	$\frac{\frac{55b^{10}x^{10}}{a^{11}}+\frac{935b^9x^9}{2a^{10}}+\frac{105505b^8x^8}{6a^9}+\frac{151125b^7x^7}{4a^8}+\frac{20669b^6x^6}{4a^7}+\frac{275999b^5x^5}{6a^6}+\frac{36839b^4x^4}{14a^5}+\frac{50699b^3x^3}{56a^4}+\frac{78419b^2x^2}{504a^3}+\frac{11bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)^9}-\frac{55b^2\ln(bx+a)}{a^{12}}$
default	$-\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(bx+a)^9}+\frac{3b^2}{8a^4(bx+a)^8}+\frac{6b^2}{7a^5(bx+a)^7}+\frac{5b^2}{3a^6(bx+a)^6}+\frac{3b^2}{a^7(bx+a)^5}+\frac{21b^2}{4a^8(bx+a)^4}+\frac{28b^2}{3a^9(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Maxima [**A**]
time = 0.28, size = 240, normalized size = 1.26

$$\frac{27720\,b^{10}x^{10}+235620\,ab^{9}x^{9}+882420\,a^{2}b^{8}x^{8}+1905750\,a^{3}b^{7}x^{7}+2604294\,a^{4}b^{6}x^{6}+2318316\,a^{5}b^{5}x^{5}+1326204\,a^{6}b^{4}x^{4}+456291\,a^{7}b^{3}x^{3}+78419\,a^{8}b^{2}x^{2}+2772\,a^{9}bx-252\,a^{10}}{504\,(a^{11}b^{9}x^{11}+9\,a^{12}b^{8}x^{10}+36\,a^{13}b^{7}x^{9}+84\,a^{14}b^{6}x^{8}+126\,a^{15}b^{5}x^{7}+126\,a^{16}b^{4}x^{6}+84\,a^{17}b^{3}x^{5}+36\,a^{18}b^{2}x^{4}+9\,a^{19}bx^{3}+a^{20}x^{2})}-\frac{55\,b^{2}\log\left(bx+a\right)}{a^{12}}+\frac{55\,b^{2}\log\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(27720*b^10*x^10 + 235620*a*b^9*x^9 + 882420*a^2*b^8*x^8 + 1905750*a^
3*b^7*x^7 + 2604294*a^4*b^6*x^6 + 2318316*a^5*b^5*x^5 + 1326204*a^6*b^4*x^4
 + 456291*a^7*b^3*x^3 + 78419*a^8*b^2*x^2 + 2772*a^9*b*x - 252*a^10)/(a^11*
b^9*x^11 + 9*a^12*b^8*x^10 + 36*a^13*b^7*x^9 + 84*a^14*b^6*x^8 + 126*a^15*b
^5*x^7 + 126*a^16*b^4*x^6 + 84*a^17*b^3*x^5 + 36*a^18*b^2*x^4 + 9*a^19*b*x^
3 + a^20*x^2) - 55*b^2*log(b*x + a)/a^12 + 55*b^2*log(x)/a^12
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(177) = 354$.
time = 0.32, size = 438, normalized size = 2.29

$$\frac{27720\,ab^{10}x^{10}+235620\,a^{2}b^{9}x^{9}+882420\,a^{3}b^{8}x^{8}+1905750\,a^{4}b^{7}x^{7}+2604294\,a^{5}b^{6}x^{6}+2318316\,a^{6}b^{5}x^{5}+1326204\,a^{7}b^{4}x^{4}+456291\,a^{8}b^{3}x^{3}+78419\,a^{9}b^{2}x^{2}+2772\,a^{10}bx-252\,a^{11}-27720\,(b^{11}x^{11}+9\,ab^{10}x^{10}+36\,a^{2}b^{9}x^{9}+84\,a^{3}b^{8}x^{8}+126\,a^{4}b^{7}x^{7}+126\,a^{5}b^{6}x^{6}+84\,a^{6}b^{5}x^{5}+36\,a^{7}b^{4}x^{4}+9\,a^{8}b^{3}x^{3}+a^{9}b^{2}x^{2})\log\left(bx+a\right)+27720\,(b^{11}x^{11}+9\,ab^{10}x^{10}+36\,a^{2}b^{9}x^{9}+84\,a^{3}b^{8}x^{8}+126\,a^{4}b^{7}x^{7}+126\,a^{5}b^{6}x^{6}+84\,a^{6}b^{5}x^{5}+36\,a^{7}b^{4}x^{4}+9\,a^{8}b^{3}x^{3}+a^{9}b^{2}x^{2})\log\left(x\right)}{504\,(a^{12}b^{9}x^{11}+9\,a^{13}b^{8}x^{10}+36\,a^{14}b^{7}x^{9}+84\,a^{15}b^{6}x^{8}+126\,a^{16}b^{5}x^{7}+126\,a^{17}b^{4}x^{6}+84\,a^{18}b^{3}x^{5}+36\,a^{19}b^{2}x^{4}+9\,a^{20}bx^{3}+a^{21}x^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(27720*a*b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 190575
0*a^4*b^7*x^7 + 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4
*x^4 + 456291*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11 -
27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^
4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x
^3 + a^9*b^2*x^2)*log(b*x + a) + 27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*
b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x
^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x^3 + a^9*b^2*x^2)*log(x))/(a^12*b^9*x^11 +
 9*a^13*b^8*x^10 + 36*a^14*b^7*x^9 + 84*a^15*b^6*x^8 + 126*a^16*b^5*x^7 + 1
26*a^17*b^4*x^6 + 84*a^18*b^3*x^5 + 36*a^19*b^2*x^4 + 9*a^20*b*x^3 + a^21*x
^2)
```

Sympy [**A**]
time = 0.54, size = 246, normalized size = 1.29

$$\frac{-252a^{10}+2772a^{9}bx+78419a^{8}b^{2}x^{2}+456291a^{7}b^{3}x^{3}+1326204a^{6}b^{4}x^{4}+2318316a^{5}b^{5}x^{5}+2604294a^{4}b^{6}x^{6}+1905750a^{3}b^{7}x^{7}+882420a^{2}b^{8}x^{8}+235620ab^{9}x^{9}+27720b^{10}x^{10}}{504a^{20}x^{2}+4536a^{19}bx^{3}+18144a^{18}b^{2}x^{4}+42336a^{17}b^{3}x^{5}+63504a^{16}b^{4}x^{6}+63504a^{15}b^{5}x^{7}+42336a^{14}b^{6}x^{8}+18144a^{13}b^{7}x^{9}+4536a^{12}b^{8}x^{10}+504a^{11}b^{9}x^{11}}+\frac{55b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x + 78419*a**8*b**2*x**2 + 456291*a**7*b**3*x**3
+ 1326204*a**6*b**4*x**4 + 2318316*a**5*b**5*x**5 + 2604294*a**4*b**6*x**6
+ 1905750*a**3*b**7*x**7 + 882420*a**2*b**8*x**8 + 235620*a*b**9*x**9 + 277
20*b**10*x**10)/(504*a**20*x**2 + 4536*a**19*b*x**3 + 18144*a**18*b**2*x**4
 + 42336*a**17*b**3*x**5 + 63504*a**16*b**4*x**6 + 63504*a**15*b**5*x**7 +
42336*a**14*b**6*x**8 + 18144*a**13*b**7*x**9 + 4536*a**12*b**8*x**10 + 504
*a**11*b**9*x**11) + 55*b**2*(log(x) - log(a/b + x))/a**12
```

Giac [A]
time = 0.00, size = 163, normalized size = 0.85

$$\frac{55b^2\ln|x|}{a^{12}} - \frac{55b^3\ln|xb+a|}{ba^{12}} + \frac{\frac{1}{504}\left(27720b^{10}ax^{10} + 235620b^9a^2x^9 + 882420b^8a^3x^8 + 1905750b^7a^4x^7 + 2604294b^6a^5x^6 + 2318316b^5a^6x^5 + 1326204b^4a^7x^4 + 456291b^3a^8x^3 + 78419b^2a^9x^2 + 2772ba^{10}x - 252a^{11}\right)}{a^{12}x^2\left(bx+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x)
```

```
[Out] -55*b^2*log(abs(b*x + a))/a^12 + 55*b^2*log(abs(x))/a^12 + 1/504*(27720*a*b
^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 1905750*a^4*b^7*x^7 +
2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4*x^4 + 456291*a^
8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11)/((b*x + a)^9*a^12
*x^2)
```

Mupad [B]
time = 0.44, size = 233, normalized size = 1.22

$$\frac{\frac{78419\,b^2\,x^2}{504\,a^3} - \frac{1}{2\,a} + \frac{50699\,b^3\,x^3}{56\,a^4} + \frac{36839\,b^4\,x^4}{14\,a^5} + \frac{27599\,b^5\,x^5}{6\,a^6} + \frac{20669\,b^6\,x^6}{4\,a^7} + \frac{15125\,b^7\,x^7}{4\,a^8} + \frac{10505\,b^8\,x^8}{6\,a^9} + \frac{935\,b^9\,x^9}{2\,a^{10}} + \frac{55\,b^{10}\,x^{10}}{a^{11}} + \frac{11\,b\,x}{2\,a^2}}{a^9\,x^2 + 9\,a^8\,b\,x^3 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^5 + 126\,a^5\,b^4\,x^6 + 126\,a^4\,b^5\,x^7 + 84\,a^3\,b^6\,x^8 + 36\,a^2\,b^7\,x^9 + 9\,a\,b^8\,x^{10} + b^9\,x^{11}} - \frac{110\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^10),x)
```

```
[Out] ((78419*b^2*x^2)/(504*a^3) - 1/(2*a) + (50699*b^3*x^3)/(56*a^4) + (36839*b^
4*x^4)/(14*a^5) + (27599*b^5*x^5)/(6*a^6) + (20669*b^6*x^6)/(4*a^7) + (1512
5*b^7*x^7)/(4*a^8) + (10505*b^8*x^8)/(6*a^9) + (935*b^9*x^9)/(2*a^10) + (55
*b^10*x^10)/a^11 + (11*b*x)/(2*a^2))/(a^9*x^2 + b^9*x^11 + 9*a^8*b*x^3 + 9*
a*b^8*x^10 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^5 + 126*a^5*b^4*x^6 + 126*a^4*b^
5*x^7 + 84*a^3*b^6*x^8 + 36*a^2*b^7*x^9) - (110*b^2*atanh((2*b*x)/a + 1))/a
^12
```

3.238 $\int \frac{1}{x^4(a+bx)^{10}}\,dx$

Optimal. Leaf size=198

$$-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{7b^3}{a^8(a+bx)^5}-\frac{14b^3}{a^9(a+b$$

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Rubi [A]

time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}-\frac{165b^3}{a^{12}(a+bx)}-\frac{55b^2}{a^{12}x}-\frac{60b^3}{a^{11}(a+bx)^2}+\frac{5b}{a^{11}x^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{1}{3a^{10}x^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-\frac{b^3}{9a^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/3*1/(a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a +
b*x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^3)
/(3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*x)^
4) - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b^3)/
(a^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^4}-\frac{10b}{a^{11}x^3}+\frac{55b^2}{a^{12}x^2}-\frac{220b^3}{a^{13}x}+\frac{b^4}{a^4(a+bx)^{10}}+\frac{4b^4}{a^5(a+bx)^9}+\frac{10b^4}{a^6(a+bx)^8}+\right.\\ &= -\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+\end{aligned}$$

Mathematica [A]

time = 0.07, size = 156, normalized size = 0.79

$$-\frac{\frac{a(42a^{11}-252a^{10}bx+2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7+1905750a^3b^8x^8+882420a^2b^9x^9+235620ab^{10}x^{10}+27720b^{11}x^{11})}{x^3(a+bx)^9}+27720b^3\log(x)-27720b^3\log(a+bx)}{126a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/126*((a*(42*a^11 - 252*a^10*b*x + 2772*a^9*b^2*x^2 + 78419*a^8*b^3*x^3 +
 456291*a^7*b^4*x^4 + 1326204*a^6*b^5*x^5 + 2318316*a^5*b^6*x^6 + 2604294*a
^4*b^7*x^7 + 1905750*a^3*b^8*x^8 + 882420*a^2*b^9*x^9 + 235620*a*b^10*x^10
+ 27720*b^11*x^11))/(x^3*(a + b*x)^9) + 27720*b^3*Log[x] - 27720*b^3*Log[a
+ b*x])/a^13
```

Mathics [A]
time = 4.32, size = 340, normalized size = 1.72

$$\frac{a\left(-42a^{11}+252a^{10}bx-2772a^9b^2x^2-78419a^8b^3x^3-456291a^7b^4x^4-1326204a^6b^5x^5-2318316a^5b^6x^6-2604294a^4b^7x^7-1905750a^3b^8x^8-882420a^2b^9x^9-235620ab^{10}x^{10}-27720b^{11}x^{11}\right)-27720b^3x^3\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{126a^{13}x^3\left(a^9+9a^8bx+36a^7b^2x^2+84a^6b^3x^3+126a^5b^4x^4+126a^4b^5x^5+84a^3b^6x^6+36a^2b^7x^7+9ab^8x^8+b^9x^9\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(a + b*x)^10),x]')
```

```
[Out] (a (-42 a ^ 11 + 252 a ^ 10 b x - 2772 a ^ 9 b ^ 2 x ^ 2 - 78419 a ^ 8 b ^
3 x ^ 3 - 456291 a ^ 7 b ^ 4 x ^ 4 - 1326204 a ^ 6 b ^ 5 x ^ 5 - 2318316 a
^ 5 b ^ 6 x ^ 6 - 2604294 a ^ 4 b ^ 7 x ^ 7 - 1905750 a ^ 3 b ^ 8 x ^ 8 - 8
82420 a ^ 2 b ^ 9 x ^ 9 - 235620 a b ^ 10 x ^ 10 - 27720 b ^ 11 x ^ 11) - 2
7720 b ^ 3 x ^ 3 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2 x ^ 2 + 84 a ^ 6 b ^
 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 + 84 a ^ 3 b ^ 6 x
 ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x ^ 9) (Log[x] - Log[
(a + b x) / b])) / (126 a ^ 13 x ^ 3 (a ^ 9 + 9 a ^ 8 b x + 36 a ^ 7 b ^ 2
x ^ 2 + 84 a ^ 6 b ^ 3 x ^ 3 + 126 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^
5 + 84 a ^ 3 b ^ 6 x ^ 6 + 36 a ^ 2 b ^ 7 x ^ 7 + 9 a b ^ 8 x ^ 8 + b ^ 9 x
 ^ 9))
```

Maple [A]
time = 0.14, size = 189, normalized size = 0.95

method	result
norman	$\frac{-\frac{1}{3a}+\frac{2bx}{a^2}-\frac{22b^2x^2}{a^3}+\frac{1980b^4x^4}{a^5}+\frac{11880b^5x^5}{a^6}+\frac{33880b^6x^6}{a^7}+\frac{57750b^7x^7}{a^8}+\frac{63294b^8x^8}{a^9}+\frac{45276b^9x^9}{a^{10}}+\frac{143748b^{10}x^{10}}{7a^{11}}+\frac{75339b^{11}x^{11}}{14a^{12}}+\frac{78419b^{12}x^{12}}{126a^{13}}}{x^3(bx+a)^9}$
risch	$\frac{-\frac{220b^{11}x^{11}}{a^{12}}-\frac{1870b^{10}x^{10}}{a^{11}}-\frac{21010b^9x^9}{3a^{10}}-\frac{15125b^8x^8}{a^9}-\frac{20669b^7x^7}{a^8}-\frac{55198b^6x^6}{3a^7}-\frac{73678b^5x^5}{7a^6}-\frac{50699b^4x^4}{14a^5}-\frac{78419b^3x^3}{126a^4}-\frac{22b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{3a}}{x^3(bx+a)^9}-2$
default	$-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(bx+a)^9}-\frac{b^3}{2a^5(bx+a)^8}-\frac{10b^3}{7a^6(bx+a)^7}-\frac{10b^3}{3a^7(bx+a)^6}-\frac{7b^3}{a^8(bx+a)^5}-\frac{14b^3}{a^9(bx+a)^4}-a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
```

```
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Maxima [A]
time = 0.28, size = 251, normalized size = 1.27

$$-\frac{27720\,b^{11}x^{11}+235620\,ab^{10}x^{10}+882420\,a^2b^9x^9+1905750\,a^3b^8x^8+2604294\,a^4b^7x^7+2318316\,a^5b^6x^6+1326204\,a^6b^5x^5+456291\,a^7b^4x^4+78419\,a^8b^3x^3+2772\,a^9b^2x^2-252\,a^{10}bx+42\,a^{11}}{126\,(a^{12}b^9x^{12}+9\,a^{13}b^8x^{11}+36\,a^{14}b^7x^{10}+84\,a^{15}b^6x^9+126\,a^{16}b^5x^8+126\,a^{17}b^4x^7+84\,a^{18}b^3x^6+36\,a^{19}b^2x^5+9\,a^{20}bx^4+a^{21}x^3)}+\frac{220\,b^3\log(bx+a)}{a^{13}}-\frac{220\,b^3\log(x)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(27720*b^11*x^11 + 235620*a*b^10*x^10 + 882420*a^2*b^9*x^9 + 1905750
*a^3*b^8*x^8 + 2604294*a^4*b^7*x^7 + 2318316*a^5*b^6*x^6 + 1326204*a^6*b^5*
x^5 + 456291*a^7*b^4*x^4 + 78419*a^8*b^3*x^3 + 2772*a^9*b^2*x^2 - 252*a^10*
b*x + 42*a^11)/(a^12*b^9*x^12 + 9*a^13*b^8*x^11 + 36*a^14*b^7*x^10 + 84*a^1
5*b^6*x^9 + 126*a^16*b^5*x^8 + 126*a^17*b^4*x^7 + 84*a^18*b^3*x^6 + 36*a^19
*b^2*x^5 + 9*a^20*b*x^4 + a^21*x^3) + 220*b^3*log(b*x + a)/a^13 - 220*b^3*l
og(x)/a^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(188) = 376.
time = 0.32, size = 449, normalized size = 2.27

$$-\frac{27720\,ab^{11}x^{11}+235620\,a^2b^{10}x^{10}+882420\,a^3b^9x^9+1905750\,a^4b^8x^8+2604294\,a^5b^7x^7+2318316\,a^6b^6x^6+1326204\,a^7b^5x^5+456291\,a^8b^4x^4+78419\,a^9b^3x^3+2772\,a^{10}b^2x^2-252\,a^{11}bx+42\,a^{12}-27720\,(b^{12}x^{12}+9\,ab^{11}x^{11}+36\,a^2b^{10}x^{10}+84\,a^3b^9x^9+126\,a^4b^8x^8+126\,a^5b^7x^7+84\,a^6b^6x^6+36\,a^7b^5x^5+9\,a^8b^4x^4+a^9b^3x^3)\log(bx+a)+27720\,(b^{12}x^{12}+9\,ab^{11}x^{11}+36\,a^2b^{10}x^{10}+84\,a^3b^9x^9+126\,a^4b^8x^8+126\,a^5b^7x^7+84\,a^6b^6x^6+36\,a^7b^5x^5+9\,a^8b^4x^4+a^9b^3x^3)\log(x)}{126\,(a^{13}b^9x^{12}+9\,a^{14}b^8x^{11}+36\,a^{15}b^7x^{10}+84\,a^{16}b^6x^9+126\,a^{17}b^5x^8+126\,a^{18}b^4x^7+84\,a^{19}b^3x^6+36\,a^{20}b^2x^5+9\,a^{21}bx^4+a^{22}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/126*(27720*a*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 190
5750*a^4*b^8*x^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*
b^5*x^5 + 456291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*
a^11*b*x + 42*a^12 - 27720*(b^12*x^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^10 +
84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^
7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)*log(b*x + a) + 27720*(b^12*x^12 +
9*a*b^11*x^11 + 36*a^2*b^10*x^10 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a
^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)
*log(x))/(a^13*b^9*x^12 + 9*a^14*b^8*x^11 + 36*a^15*b^7*x^10 + 84*a^16*b^6*
x^9 + 126*a^17*b^5*x^8 + 126*a^18*b^4*x^7 + 84*a^19*b^3*x^6 + 36*a^20*b^2*x
^5 + 9*a^21*b*x^4 + a^22*x^3)
```

Sympy [A]
time = 0.56, size = 258, normalized size = 1.30

$$\frac{-42a^{11}+252a^{10}bx-2772a^9b^2x^2-78419a^8b^3x^3-456291a^7b^4x^4-1326204a^6b^5x^5-2318316a^5b^6x^6-2604294a^4b^7x^7-1905750a^3b^8x^8-882420a^2b^9x^9-235620ab^{10}x^{10}-27720b^{11}x^{11}}{126a^{21}x^3+1134a^{20}bx^4+4536a^{19}b^2x^5+10584a^{18}b^3x^6+15876a^{17}b^4x^7+15876a^{16}b^5x^8+10584a^{15}b^6x^9+4536a^{14}b^7x^{10}+1134a^{13}b^8x^{11}+126a^{12}b^9x^{12}}+\frac{220b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x - 2772*a**9*b**2*x**2 - 78419*a**8*b**3*x**3 - 4
56291*a**7*b**4*x**4 - 1326204*a**6*b**5*x**5 - 2318316*a**5*b**6*x**6 - 26
04294*a**4*b**7*x**7 - 1905750*a**3*b**8*x**8 - 882420*a**2*b**9*x**9 - 235
620*a*b**10*x**10 - 27720*b**11*x**11)/(126*a**21*x**3 + 1134*a**20*b*x**4
+ 4536*a**19*b**2*x**5 + 10584*a**18*b**3*x**6 + 15876*a**17*b**4*x**7 + 15
876*a**16*b**5*x**8 + 10584*a**15*b**6*x**9 + 4536*a**14*b**7*x**10 + 1134*
a**13*b**8*x**11 + 126*a**12*b**9*x**12) + 220*b**3*(-log(x) + log(a/b + x)
)/a**13
```

Giac [A]
time = 0.00, size = 184, normalized size = 0.93

$$-\frac{220b^3\ln|x|}{a^{13}}+\frac{220b^4\ln|xb+a|}{ba^{13}}+\frac{\frac{1}{126}\left(-27720b^{11}ax^{11}-235620b^{10}a^2x^{10}-882420b^9a^3x^9-1905750b^8a^4x^8-2604294b^7a^5x^7-2318316b^6a^6x^6-1326204b^5a^7x^5-456291b^4a^8x^4-78419b^3a^9x^3-2772b^2a^{10}x^2+252ba^{11}x-42a^{12}\right)}{a^{13}x^3\left(bx+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x)
```

```
[Out] 220*b^3*log(abs(b*x + a))/a^13 - 220*b^3*log(abs(x))/a^13 - 1/126*(27720*a*
b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 1905750*a^4*b^8*x^8
 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*b^5*x^5 + 456291
*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*a^11*b*x + 42*a^
12)/((b*x + a)^9*a^13*x^3)
```

Mupad [B]
time = 0.59, size = 245, normalized size = 1.24

$$\frac{440\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{13}}-\frac{\frac{1}{3\,a}+\frac{22\,b^2\,x^2}{a^3}+\frac{78419\,b^3\,x^3}{126\,a^4}+\frac{50699\,b^4\,x^4}{14\,a^5}+\frac{73678\,b^5\,x^5}{7\,a^6}+\frac{55198\,b^6\,x^6}{3\,a^7}+\frac{20669\,b^7\,x^7}{a^8}+\frac{15125\,b^8\,x^8}{a^9}+\frac{21010\,b^9\,x^9}{3\,a^{10}}+\frac{1870\,b^{10}\,x^{10}}{a^{11}}+\frac{220\,b^{11}\,x^{11}}{a^{12}}-\frac{2\,b\,x}{a^2}}{a^9\,x^3+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^5+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^7+126\,a^4\,b^5\,x^8+84\,a^3\,b^6\,x^9+36\,a^2\,b^7\,x^{10}+9\,a\,b^8\,x^{11}+b^9\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^10),x)
```

```
[Out] (440*b^3*atanh((2*b*x)/a + 1))/a^13 - (1/(3*a) + (22*b^2*x^2)/a^3 + (78419*
b^3*x^3)/(126*a^4) + (50699*b^4*x^4)/(14*a^5) + (73678*b^5*x^5)/(7*a^6) + (
55198*b^6*x^6)/(3*a^7) + (20669*b^7*x^7)/a^8 + (15125*b^8*x^8)/a^9 + (21010
*b^9*x^9)/(3*a^10) + (1870*b^10*x^10)/a^11 + (220*b^11*x^11)/a^12 - (2*b*x)
/a^2)/(a^9*x^3 + b^9*x^12 + 9*a^8*b*x^4 + 9*a*b^8*x^11 + 36*a^7*b^2*x^5 + 8
4*a^6*b^3*x^6 + 126*a^5*b^4*x^7 + 126*a^4*b^5*x^8 + 84*a^3*b^6*x^9 + 36*a^2
*b^7*x^10)
```

3.239 $\int \frac{(a+bx)^{12}}{x^{10}}\,dx$

Optimal. Leaf size=141

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x}{3}$$

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12/x^10,x]
```

```
[Out] -1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{12}}{x^{10}}\,dx &= \int \left(66a^2b^{10}+\frac{a^{12}}{x^{10}}+\frac{12a^{11}b}{x^9}+\frac{66a^{10}b^2}{x^8}+\frac{220a^9b^3}{x^7}+\frac{495a^8b^4}{x^6}+\frac{792a^7b^5}{x^5}+\frac{924a^6b^6}{x^4}+\cdots\right.\\ &= -\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a}{x}\cdots\end{aligned}$$

Mathematica [A]
time = 0.01, size = 141, normalized size = 1.00

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}+220a^3b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12/x^10,x]
```

```
[Out] -1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Mathics [A]
time = 2.83, size = 135, normalized size = 0.96

$$\frac{-a^4\left(14a^8+189a^7bx+1188a^6b^2x^2+4620a^5b^3x^3+12474a^4b^4x^4+24948a^3b^5x^5+38808a^2b^6x^6+49896ab^7x^7+62370b^8x^8\right)+42b^9x^9\left(660a^3\mathrm{Log}\left[x\right]+198a^2bx+18ab^2x^2+b^3x^3\right)}{126x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^12/x^10,x]')
```

```
[Out] (-a ^ 4 (14 a ^ 8 + 189 a ^ 7 b x + 1188 a ^ 6 b ^ 2 x ^ 2 + 4620 a ^ 5 b ^
 3 x ^ 3 + 12474 a ^ 4 b ^ 4 x ^ 4 + 24948 a ^ 3 b ^ 5 x ^ 5 + 38808 a ^ 2
b ^ 6 x ^ 6 + 49896 a b ^ 7 x ^ 7 + 62370 b ^ 8 x ^ 8) + 42 b ^ 9 x ^ 9 (66
0 a ^ 3 Log[x] + 198 a ^ 2 b x + 18 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) / (126 x
^ 9)
```

Maple [A]
time = 0.08, size = 132, normalized size = 0.94

method	result
default	$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6a\,b^{11}$
risch	$\frac{b^{12}x^3}{3}+6a\,b^{11}x^2+66a^2b^{10}x+\frac{-495a^4b^8x^8-396a^5b^7x^7-308a^6b^6x^6-198a^7b^5x^5-99a^8b^4x^4-\frac{110}{3}a^9b^3x^3-\frac{66}{7}a^{10}b^2x^2-\frac{3}{2}a^{11}}{x^9}$
norman	$\frac{-\frac{1}{9}a^{12}+\frac{1}{3}b^{12}x^{12}+6a\,b^{11}x^{11}+66a^2b^{10}x^{10}-495a^4b^8x^8-396a^5b^7x^7-308a^6b^6x^6-198a^7b^5x^5-99a^8b^4x^4-\frac{110}{3}a^9b^3x^3-\frac{66}{7}a^{10}b^2x^2-\frac{3}{2}a^{11}}{x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Maxima [A]
time = 0.26, size = 132, normalized size = 0.94

$$\frac{1}{3}\,b^{12}x^3+6\,ab^{11}x^2+66\,a^2b^{10}x+220\,a^3b^9\log\left(x\right)-\frac{62370\,a^4b^8x^8+49896\,a^5b^7x^7+38808\,a^6b^6x^6+24948\,a^7b^5x^5+12474\,a^8b^4x^4+4620\,a^9b^3x^3+1188\,a^{10}b^2x^2+189\,a^{11}bx+14\,a^{12}}{126\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(x) - 1/126*(6
2370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b^5*x^
5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^11*b*x
 + 14*a^12)/x^9
```

Fricas [A]
time = 0.31, size = 136, normalized size = 0.96

$$\frac{42\,b^{12}x^{12}+756\,ab^{11}x^{11}+8316\,a^{2}b^{10}x^{10}+27720\,a^{3}b^{9}x^{9}\log\left(x\right)-62370\,a^{4}b^{8}x^{8}-49896\,a^{5}b^{7}x^{7}-38808\,a^{6}b^{6}x^{6}-24948\,a^{7}b^{5}x^{5}-12474\,a^{8}b^{4}x^{4}-4620\,a^{9}b^{3}x^{3}-1188\,a^{10}b^{2}x^{2}-189\,a^{11}bx-14\,a^{12}}{126\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 + 756*a*b^11*x^11 + 8316*a^2*b^10*x^10 + 27720*a^3*b^9*
x^9*log(x) - 62370*a^4*b^8*x^8 - 49896*a^5*b^7*x^7 - 38808*a^6*b^6*x^6 - 24
948*a^7*b^5*x^5 - 12474*a^8*b^4*x^4 - 4620*a^9*b^3*x^3 - 1188*a^10*b^2*x^2
- 189*a^11*b*x - 14*a^12)/x^9
```

Sympy [A]
time = 0.40, size = 143, normalized size = 1.01

$$220a^{3}b^{9}\log\left(x\right)+66a^{2}b^{10}x+6ab^{11}x^{2}+\frac{b^{12}x^{3}}{3}+\frac{-14a^{12}-189a^{11}bx-1188a^{10}b^{2}x^{2}-4620a^{9}b^{3}x^{3}-12474a^{8}b^{4}x^{4}-24948a^{7}b^{5}x^{5}-38808a^{6}b^{6}x^{6}-49896a^{5}b^{7}x^{7}-62370a^{4}b^{8}x^{8}}{126x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12/x**10,x)
```

```
[Out] 220*a**3*b**9*log(x) + 66*a**2*b**10*x + 6*a*b**11*x**2 + b**12*x**3/3 + (-
14*a**12 - 189*a**11*b*x - 1188*a**10*b**2*x**2 - 4620*a**9*b**3*x**3 - 124
74*a**8*b**4*x**4 - 24948*a**7*b**5*x**5 - 38808*a**6*b**6*x**6 - 49896*a**
5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*x**9)
```

Giac [A]
time = 0.00, size = 147, normalized size = 1.04

$$\frac{1}{3}x^{3}b^{12}+6x^{2}b^{11}a+66xb^{10}a^{2}+\frac{\frac{1}{126}\left(-62370b^{8}a^{4}x^{8}-49896b^{7}a^{5}x^{7}-38808b^{6}a^{6}x^{6}-24948b^{5}a^{7}x^{5}-12474b^{4}a^{8}x^{4}-4620b^{3}a^{9}x^{3}-1188b^{2}a^{10}x^{2}-189ba^{11}x-14a^{12}\right)}{x^{9}}+220b^{9}a^{3}\ln\left|x\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x)
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(abs(x)) - 1/1
26*(62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b
^5*x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^1
1*b*x + 14*a^12)/x^9
```

Mupad [B]
time = 0.08, size = 132, normalized size = 0.94

$$\frac{b^{12}\,x^3}{3}-\frac{\frac{a^{12}}{9}+\frac{3\,a^{11}\,b\,x}{2}+\frac{66\,a^{10}\,b^2\,x^2}{7}+\frac{110\,a^9\,b^3\,x^3}{3}+99\,a^8\,b^4\,x^4+198\,a^7\,b^5\,x^5+308\,a^6\,b^6\,x^6+396\,a^5\,b^7\,x^7+495\,a^4\,b^8\,x^8}{x^9}+66\,a^2\,b^{10}\,x+6\,a\,b^{11}\,x^2+220\,a^3\,b^9\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12/x^10,x)
```

```
[Out] (b^12*x^3)/3 - (a^12/9 + (66*a^10*b^2*x^2)/7 + (110*a^9*b^3*x^3)/3 + 99*a^8
*b^4*x^4 + 198*a^7*b^5*x^5 + 308*a^6*b^6*x^6 + 396*a^5*b^7*x^7 + 495*a^4*b^
8*x^8 + (3*a^11*b*x)/2)/x^9 + 66*a^2*b^10*x + 6*a*b^11*x^2 + 220*a^3*b^9*lo
g(x)
```

3.240 $\int \frac{(a+bx)^{11}}{x^{10}}\,dx$

Optimal. Leaf size=132

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11ab^{10}x+\frac{b^{11}x^2}{2}+55a^2b^9$$

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11/x^10,x]
```

```
[Out] -1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx = \int \left(11ab^{10}+\frac{a^{11}}{x^{10}}+\frac{11a^{10}b}{x^9}+\frac{55a^9b^2}{x^8}+\frac{165a^8b^3}{x^7}+\frac{330a^7b^4}{x^6}+\frac{462a^6b^5}{x^5}+\frac{462a^5b^6}{x^4}+330\right.$$

$$= -\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3}{x}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11ab^{10}x+\frac{b^{11}x^2}{2}+55a^2b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11/x^10,x]
```

```
[Out] -1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Mathics [A]
time = 2.74, size = 124, normalized size = 0.94

$$\frac{-a^3\left(56a^8+693a^7bx+3960a^6b^2x^2+13860a^5b^3x^3+33264a^4b^4x^4+58212a^3b^5x^5+77616a^2b^6x^6+83160ab^7x^7+83160b^8x^8\right)+252b^9x^9\left(110a^2\mathrm{Log}\left[x\right]+22abx+b^2x^2\right)}{504x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^11/x^10,x]')
```

```
[Out] (-a ^ 3 (56 a ^ 8 + 693 a ^ 7 b x + 3960 a ^ 6 b ^ 2 x ^ 2 + 13860 a ^ 5 b
^ 3 x ^ 3 + 33264 a ^ 4 b ^ 4 x ^ 4 + 58212 a ^ 3 b ^ 5 x ^ 5 + 77616 a ^ 2
 b ^ 6 x ^ 6 + 83160 a b ^ 7 x ^ 7 + 83160 b ^ 8 x ^ 8) + 252 b ^ 9 x ^ 9 (
110 a ^ 2 Log[x] + 22 a b x + b ^ 2 x ^ 2)) / (504 x ^ 9)
```

Maple [A]
time = 0.09, size = 121, normalized size = 0.92

method	result
default	$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11a\,b^{10}x+\frac{b^{11}x^2}{2}+$
risch	$\frac{b^{11}x^2}{2}+11a\,b^{10}x+\frac{-165a^3b^8x^8-165a^4b^7x^7-154a^5b^6x^6-\frac{231}{2}a^6b^5x^5-66a^7b^4x^4-\frac{55}{2}a^8b^3x^3-\frac{55}{7}b^2a^9x^2-\frac{11}{8}a^{10}bx-\frac{1}{9}a^{11}}{x^9}+55$
norman	$\frac{-\frac{1}{9}a^{11}+\frac{1}{2}b^{11}x^{11}+11a\,b^{10}x^{10}-165a^3b^8x^8-165a^4b^7x^7-154a^5b^6x^6-\frac{231}{2}a^6b^5x^5-66a^7b^4x^4-\frac{55}{2}a^8b^3x^3-\frac{11}{8}a^{10}bx-\frac{55}{7}b^2a^9x^2}{x^9}+55a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Maxima [A]
time = 0.27, size = 121, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^2+11\,ab^{10}x+55\,a^2b^9\log\left(x\right)-\frac{83160\,a^3b^8x^8+83160\,a^4b^7x^7+77616\,a^5b^6x^6+58212\,a^6b^5x^5+33264\,a^7b^4x^4+13860\,a^8b^3x^3+3960\,a^9b^2x^2+693\,a^{10}bx+56\,a^{11}}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="maxima")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(x) - 1/504*(83160*a^3*b^8*x^8 +
 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7*b^4*
x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x^9
```

Fricas [A]
time = 0.32, size = 125, normalized size = 0.95

$$\frac{252 b^{11}x^{11} + 5544 ab^{10}x^{10} + 27720 a^2b^9x^9 \log(x) - 83160 a^3b^8x^8 - 83160 a^4b^7x^7 - 77616 a^5b^6x^6 - 58212 a^6b^5x^5 - 33264 a^7b^4x^4 - 13860 a^8b^3x^3 - 3960 a^9b^2x^2 - 693 a^{10}bx - 56 a^{11}}{504 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 + 5544*a*b^10*x^10 + 27720*a^2*b^9*x^9*log(x) - 83160*
a^3*b^8*x^8 - 83160*a^4*b^7*x^7 - 77616*a^5*b^6*x^6 - 58212*a^6*b^5*x^5 - 3
3264*a^7*b^4*x^4 - 13860*a^8*b^3*x^3 - 3960*a^9*b^2*x^2 - 693*a^10*b*x - 56
*a^11)/x^9
```

Sympy [A]
time = 0.39, size = 131, normalized size = 0.99

$$55a^2b^9 \log(x) + 11ab^{10}x + \frac{b^{11}x^2}{2} + \frac{-56a^{11} - 693a^{10}bx - 3960a^9b^2x^2 - 13860a^8b^3x^3 - 33264a^7b^4x^4 - 58212a^6b^5x^5 - 77616a^5b^6x^6 - 83160a^4b^7x^7 - 83160a^3b^8x^8}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11/x**10,x)
```

```
[Out] 55*a**2*b**9*log(x) + 11*a*b**10*x + b**11*x**2/2 + (-56*a**11 - 693*a**10*
b*x - 3960*a**9*b**2*x**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 - 5
8212*a**6*b**5*x**5 - 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 83160*a
**3*b**8*x**8)/(504*x**9)
```

Giac [A]
time = 0.00, size = 136, normalized size = 1.03

$$\frac{1}{2}x^2b^{11} + 11xb^{10}a + \frac{\frac{1}{504}\left(-83160b^8a^3x^8 - 83160b^7a^4x^7 - 77616b^6a^5x^6 - 58212b^5a^6x^5 - 33264b^4a^7x^4 - 13860b^3a^8x^3 - 3960b^2a^9x^2 - 693ba^{10}x - 56a^{11}\right)}{x^9} + 55b^9a^2 \ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x)
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(abs(x)) - 1/504*(83160*a^3*b^8*
x^8 + 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7
*b^4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x
^9
```

Mupad [B]
time = 0.09, size = 121, normalized size = 0.92

$$\frac{b^{11}x^2}{2} - \frac{\frac{a^{11}}{9} + \frac{11 a^{10} b x}{8} + \frac{55 a^9 b^2 x^2}{7} + \frac{55 a^8 b^3 x^3}{2} + 66 a^7 b^4 x^4 + \frac{231 a^6 b^5 x^5}{2} + 154 a^5 b^6 x^6 + 165 a^4 b^7 x^7 + 165 a^3 b^8 x^8}{x^9} + 55 a^2 b^9 \ln(x) + 11 a b^{10} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11/x^10,x)
```

```
[Out] (b^11*x^2)/2 - (a^11/9 + (55*a^9*b^2*x^2)/7 + (55*a^8*b^3*x^3)/2 + 66*a^7*b
^4*x^4 + (231*a^6*b^5*x^5)/2 + 154*a^5*b^6*x^6 + 165*a^4*b^7*x^7 + 165*a^3*
b^8*x^8 + (11*a^10*b*x)/8)/x^9 + 55*a^2*b^9*log(x) + 11*a*b^10*x
```

3.241 $\int \frac{(a+bx)^{10}}{x^{10}}\,dx$

Optimal. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}\right.$$
$$= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^1$$

Mathematica [A]
time = 0.00, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Mathics [A]
time = 2.68, size = 113, normalized size = 0.99

$$\frac{-a^2\left(28a^8+315a^7bx+1620a^6b^2x^2+5040a^5b^3x^3+10584a^4b^4x^4+15876a^3b^5x^5+17640a^2b^6x^6+15120ab^7x^7+11340b^8x^8\right)+252b^9x^9\left(10a\text{Log}\left[x\right]+bx\right)}{252x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10/x^10,x]')
```

```
[Out] (-a ^ 2 (28 a ^ 8 + 315 a ^ 7 b x + 1620 a ^ 6 b ^ 2 x ^ 2 + 5040 a ^ 5 b ^
 3 x ^ 3 + 10584 a ^ 4 b ^ 4 x ^ 4 + 15876 a ^ 3 b ^ 5 x ^ 5 + 17640 a ^ 2
b ^ 6 x ^ 6 + 15120 a b ^ 7 x ^ 7 + 11340 b ^ 8 x ^ 8) + 252 b ^ 9 x ^ 9 (1
0 a Log[x] + b x)) / (252 x ^ 9)
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.96

method	result	siz
default	$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10a\,b^9\ln\left(x\right)$	10
risch	$b^{10}x+\frac{-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx-\frac{1}{9}a^{10}}{x^9}+10a\,b^9\ln\left(x\right)$	10
norman	$\frac{b^{10}x^{10}-\frac{1}{9}a^{10}-45a^2b^8x^8-60a^3b^7x^7-70a^4b^6x^6-63a^5b^5x^5-42a^6b^4x^4-20a^7b^3x^3-\frac{45}{7}a^8b^2x^2-\frac{5}{4}a^9bx}{x^9}+10a\,b^9\ln\left(x\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Maxima [A]
time = 0.25, size = 109, normalized size = 0.96

$$b^{10}x+10\,ab^9\log\left(x\right)-\frac{11340\,a^2b^8x^8+15120\,a^3b^7x^7+17640\,a^4b^6x^6+15876\,a^5b^5x^5+10584\,a^6b^4x^4+5040\,a^7b^3x^3+1620\,a^8b^2x^2+315\,a^9bx+28\,a^{10}}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Fricas [A]

time = 0.31, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log\left(x\right)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A]

time = 0.39, size = 117, normalized size = 1.03

$$10ab^{9}\log\left(x\right)+b^{10}x+\frac{-28a^{10}-315a^{9}bx-1620a^{8}b^{2}x^{2}-5040a^{7}b^{3}x^{3}-10584a^{6}b^{4}x^{4}-15876a^{5}b^{5}x^{5}-17640a^{4}b^{6}x^{6}-15120a^{3}b^{7}x^{7}-11340a^{2}b^{8}x^{8}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

Giac [A]

time = 0.00, size = 122, normalized size = 1.07

$$xb^{10}+\frac{\frac{1}{252}\left(-11340b^{8}a^{2}x^{8}-15120b^{7}a^{3}x^{7}-17640b^{6}a^{4}x^{6}-15876b^{5}a^{5}x^{5}-10584b^{4}a^{6}x^{4}-5040b^{3}a^{7}x^{3}-1620b^{2}a^{8}x^{2}-315ba^{9}x-28a^{10}\right)}{x^{9}}+10b^{9}a\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x)
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Mupad [B]

time = 0.00, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9}-b^{10}\,x^{10}+\frac{45\,a^8\,b^2\,x^2}{7}+20\,a^7\,b^3\,x^3+42\,a^6\,b^4\,x^4+63\,a^5\,b^5\,x^5+70\,a^4\,b^6\,x^6+60\,a^3\,b^7\,x^7+45\,a^2\,b^8\,x^8+\frac{5\,a^9\,b\,x}{4}-10\,a\,b^9\,x^9\,\ln\left(x\right)}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

3.242 $\int \frac{(a+bx)^9}{x^{10}}\,dx$

Optimal. Leaf size=109

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/x^10,x]
```

```
[Out] -1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^9}{x^{10}}\,dx &= \int \left(\frac{a^9}{x^{10}}+\frac{9a^8b}{x^9}+\frac{36a^7b^2}{x^8}+\frac{84a^6b^3}{x^7}+\frac{126a^5b^4}{x^6}+\frac{126a^4b^5}{x^5}+\frac{84a^3b^6}{x^4}+\frac{36a^2b^7}{x^3}+\frac{9ab^8}{x^2}+\frac{b^9}{x}\right.\\ &= -\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log\end{aligned}$$

Mathematica [A]
time = 0.00, size = 109, normalized size = 1.00

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/x^10,x]
```

```
[Out] -1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Mathics [A]
time = 2.65, size = 102, normalized size = 0.94

$$\frac{-\frac{a(280a^8+2835a^7bx+12960a^6b^2x^2+35280a^5b^3x^3+63504a^4b^4x^4+79380a^3b^5x^5+70560a^2b^6x^6+45360ab^7x^7+22680b^8x^8)}{2520}+b^9x^9\text{Log}\,[x]}{x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^9/x^10,x]')
```

```
[Out] (-a (280 a ^ 8 + 2835 a ^ 7 b x + 12960 a ^ 6 b ^ 2 x ^ 2 + 35280 a ^ 5 b ^
 3 x ^ 3 + 63504 a ^ 4 b ^ 4 x ^ 4 + 79380 a ^ 3 b ^ 5 x ^ 5 + 70560 a ^ 2
b ^ 6 x ^ 6 + 45360 a b ^ 7 x ^ 7 + 22680 b ^ 8 x ^ 8) / 2520 + b ^ 9 x ^ 9
 Log[x]) / x ^ 9
```

Maple [A]
time = 0.08, size = 100, normalized size = 0.92

method	result	size
default	$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a\,b^8}{x}+b^9\ln\left(x\right)$	100
norman	$\frac{-\frac{1}{9}a^9-9a\,b^8x^8-18a^2b^7x^7-28a^3b^6x^6-\frac{63}{2}a^4b^5x^5-\frac{126}{5}a^5b^4x^4-14a^6b^3x^3-\frac{36}{7}a^7b^2x^2-\frac{9}{8}a^8bx}{x^9}+b^9\ln\left(x\right)$	100
risch	$\frac{-\frac{1}{9}a^9-9a\,b^8x^8-18a^2b^7x^7-28a^3b^6x^6-\frac{63}{2}a^4b^5x^5-\frac{126}{5}a^5b^4x^4-14a^6b^3x^3-\frac{36}{7}a^7b^2x^2-\frac{9}{8}a^8bx}{x^9}+b^9\ln\left(x\right)$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Maxima [A]
time = 0.25, size = 100, normalized size = 0.92

$$b^9\log\left(x\right)-\frac{22680\,ab^8x^8+45360\,a^2b^7x^7+70560\,a^3b^6x^6+79380\,a^4b^5x^5+63504\,a^5b^4x^4+35280\,a^6b^3x^3+12960\,a^7b^2x^2+2835\,a^8bx+280\,a^9}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="maxima")
```

```
[Out] b^9*log(x) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b^6*x^
6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*a^7*b
^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Fricas [A]
time = 0.31, size = 103, normalized size = 0.94

$$\frac{2520\,b^9x^9\log\left(x\right)-22680\,ab^8x^8-45360\,a^2b^7x^7-70560\,a^3b^6x^6-79380\,a^4b^5x^5-63504\,a^5b^4x^4-35280\,a^6b^3x^3-12960\,a^7b^2x^2-2835\,a^8bx-280\,a^9}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(x) - 22680*a*b^8*x^8 - 45360*a^2*b^7*x^7 - 70560*a
^3*b^6*x^6 - 79380*a^4*b^5*x^5 - 63504*a^5*b^4*x^4 - 35280*a^6*b^3*x^3 - 12
960*a^7*b^2*x^2 - 2835*a^8*b*x - 280*a^9)/x^9
```

Sympy [A]
time = 0.37, size = 107, normalized size = 0.98

$$b^9\log\left(x\right)+\frac{-280a^9-2835a^8bx-12960a^7b^2x^2-35280a^6b^3x^3-63504a^5b^4x^4-79380a^4b^5x^5-70560a^3b^6x^6-45360a^2b^7x^7-22680ab^8x^8}{2520x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/x**10,x)
```

```
[Out] b**9*log(x) + (-280*a**9 - 2835*a**8*b*x - 12960*a**7*b**2*x**2 - 35280*a**
6*b**3*x**3 - 63504*a**5*b**4*x**4 - 79380*a**4*b**5*x**5 - 70560*a**3*b**6
*x**6 - 45360*a**2*b**7*x**7 - 22680*a*b**8*x**8)/(2520*x**9)
```

Giac [A]
time = 0.00, size = 113, normalized size = 1.04

$$\frac{\frac{1}{2520}\left(-22680b^8ax^8-45360b^7a^2x^7-70560b^6a^3x^6-79380b^5a^4x^5-63504b^4a^5x^4-35280b^3a^6x^3-12960b^2a^7x^2-2835ba^8x-280a^9\right)}{x^9}+b^9\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x)
```

```
[Out] b^9*log(abs(x)) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b
^6*x^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*
a^7*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Mupad [B]
time = 0.08, size = 100, normalized size = 0.92

$$b^9\,\ln\left(x\right)-\frac{\frac{a^9}{9}+\frac{9\,a^8\,b\,x}{8}+\frac{36\,a^7\,b^2\,x^2}{7}+14\,a^6\,b^3\,x^3+\frac{126\,a^5\,b^4\,x^4}{5}+\frac{63\,a^4\,b^5\,x^5}{2}+28\,a^3\,b^6\,x^6+18\,a^2\,b^7\,x^7+9\,a\,b^8\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/x^10,x)
```

```
[Out] b^9*log(x) - (a^9/9 + 9*a*b^8*x^8 + (36*a^7*b^2*x^2)/7 + 14*a^6*b^3*x^3 + (
126*a^5*b^4*x^4)/5 + (63*a^4*b^5*x^5)/2 + 28*a^3*b^6*x^6 + 18*a^2*b^7*x^7 +
 (9*a^8*b*x)/8)/x^9
```

3.243 $\int \frac{(a+bx)^8}{x^{10}}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^9}{9ax^9}$$

```
[Out] -1/9*(b*x+a)^9/a/x^9
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$-\frac{(a+bx)^9}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/x^10,x]
```

```
[Out] -1/9*(a + b*x)^9/(a*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^8}{x^{10}}\,dx = -\frac{(a+bx)^9}{9ax^9}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(17) = 34.
time = 0.01, size = 96, normalized size = 5.65

$$-\frac{a^8}{9x^9}-\frac{a^7b}{x^8}-\frac{4a^6b^2}{x^7}-\frac{28a^5b^3}{3x^6}-\frac{14a^4b^4}{x^5}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{3x^3}-\frac{4ab^7}{x^2}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (a^7*b)/x^8 - (4*a^6*b^2)/x^7 - (28*a^5*b^3)/(3*x^6) - (14*a
^4*b^4)/x^5 - (14*a^3*b^5)/x^4 - (28*a^2*b^6)/(3*x^3) - (4*a*b^7)/x^2 - b^8
/x
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 92 vs. $2(17) = 34$.
time = 2.28, size = 90, normalized size = 5.29

$$\frac{-a^8 - 9a^7bx - 36a^6b^2x^2 - 84a^5b^3x^3 - 126a^4b^4x^4 - 126a^3b^5x^5 - 84a^2b^6x^6 - 36ab^7x^7 - 9b^8x^8}{9x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^8/x^10,x]')
```

```
[Out] (-a ^ 8 - 9 a ^ 7 b x - 36 a ^ 6 b ^ 2 x ^ 2 - 84 a ^ 5 b ^ 3 x ^ 3 - 126 a
 ^ 4 b ^ 4 x ^ 4 - 126 a ^ 3 b ^ 5 x ^ 5 - 84 a ^ 2 b ^ 6 x ^ 6 - 36 a b ^
7 x ^ 7 - 9 b ^ 8 x ^ 8) / (9 x ^ 9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(15) = 30$.
time = 0.08, size = 91, normalized size = 5.35

method	result	size
gosper	$-\frac{9b^8x^8+36a\,b^7x^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5x^3b^3+36a^6x^2b^2+9a^7xb+a^8}{9x^9}$	89
norman	$\frac{-b^8x^8-4a\,b^7x^7-\frac{28}{3}a^2x^6b^6-14a^3x^5b^5-14a^4x^4b^4-\frac{28}{3}a^5x^3b^3-4a^6x^2b^2-a^7xb-\frac{1}{9}a^8}{x^9}$	90
risch	$\frac{-b^8x^8-4a\,b^7x^7-\frac{28}{3}a^2x^6b^6-14a^3x^5b^5-14a^4x^4b^4-\frac{28}{3}a^5x^3b^3-4a^6x^2b^2-a^7xb-\frac{1}{9}a^8}{x^9}$	90
default	$-\frac{b^8}{x}-\frac{28b^6a^2}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{a^8}{9x^9}-\frac{4a\,b^7}{x^2}-\frac{a^7b}{x^8}-\frac{14a^4b^4}{x^5}-\frac{28a^5b^3}{3x^6}-\frac{4b^2a^6}{x^7}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -b^8/x-28/3*b^6*a^2/x^3-14*a^3*b^5/x^4-1/9*a^8/x^9-4*a*b^7/x^2-a^7*b/x^8-14
*a^4*b^4/x^5-28/3*a^5*b^3/x^6-4*b^2*a^6/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.
time = 0.27, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx + a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.

time = 0.30, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(14) = 28$.

time = 0.33, size = 95, normalized size = 5.59

$$\frac{-a^8-9a^7bx-36a^6b^2x^2-84a^5b^3x^3-126a^4b^4x^4-126a^3b^5x^5-84a^2b^6x^6-36ab^7x^7-9b^8x^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/x**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.

time = 0.00, size = 99, normalized size = 5.82

$$\frac{-9x^8b^8-36x^7b^7a-84x^6b^6a^2-126x^5b^5a^3-126x^4b^4a^4-84x^3b^3a^5-36x^2b^2a^6-9xba^7-a^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x)
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Mupad [B]

time = 0.09, size = 88, normalized size = 5.18

$$-\frac{\frac{a^8}{9}+a^7\,b\,x+4\,a^6\,b^2\,x^2+\frac{28\,a^5\,b^3\,x^3}{3}+14\,a^4\,b^4\,x^4+14\,a^3\,b^5\,x^5+\frac{28\,a^2\,b^6\,x^6}{3}+4\,a\,b^7\,x^7+b^8\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/x^10,x)
```

```
[Out] -(a^8/9 + b^8*x^8 + 4*a*b^7*x^7 + 4*a^6*b^2*x^2 + (28*a^5*b^3*x^3)/3 + 14*a
^4*b^4*x^4 + 14*a^3*b^5*x^5 + (28*a^2*b^6*x^6)/3 + a^7*b*x)/x^9
```

3.244 $\int \frac{(a+bx)^7}{x^{10}} dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}$$

```
[Out] -1/9*(b*x+a)^8/a/x^9+1/72*b*(b*x+a)^8/a^2/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}}\,dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9}\,dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. $2(36) = 72$.
time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.
time = 2.25, size = 79, normalized size = 2.19

$$\frac{-8a^7-63a^6bx-216a^5b^2x^2-420a^4b^3x^3-504a^3b^4x^4-378a^2b^5x^5-168ab^6x^6-36b^7x^7}{72x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7/x^10,x]')
```

```
[Out] (-8 a ^ 7 - 63 a ^ 6 b x - 216 a ^ 5 b ^ 2 x ^ 2 - 420 a ^ 4 b ^ 3 x ^ 3 -
504 a ^ 3 b ^ 4 x ^ 4 - 378 a ^ 2 b ^ 5 x ^ 5 - 168 a b ^ 6 x ^ 6 - 36 b ^
7 x ^ 7) / (72 x ^ 9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 0.07, size = 80, normalized size = 2.22

method	result	size
norman	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
risch	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
gosper	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80
default	$-\frac{7a\,b^6}{3x^3}-\frac{21a^2b^5}{4x^4}-\frac{a^7}{9x^9}-\frac{b^7}{2x^2}-\frac{7a^6b}{8x^8}-\frac{7a^3b^4}{x^5}-\frac{35a^4b^3}{6x^6}-\frac{3a^5b^2}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*a*b^6/x^3-21/4*a^2*b^5/x^4-1/9*a^7/x^9-1/2*b^7/x^2-7/8*a^6*b/x^8-7*a^3
*b^4/x^5-35/6*a^4*b^3/x^6-3*a^5*b^2/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

time = 0.26, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.30, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.
time = 0.29, size = 85, normalized size = 2.36

$$\frac{-8a^7-63a^6bx-216a^5b^2x^2-420a^4b^3x^3-504a^3b^4x^4-378a^2b^5x^5-168ab^6x^6-36b^7x^7}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.00, size = 89, normalized size = 2.47

$$\frac{-36x^7b^7-168x^6b^6a-378x^5b^5a^2-504x^4b^4a^3-420x^3b^3a^4-216x^2b^2a^5-63xba^6-8a^7}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x)
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Mupad [B]
time = 0.00, size = 23, normalized size = 0.64

$$-\frac{(8\,a-b\,x)\,{(a+b\,x)}^8}{72\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

3.245 $\int \frac{(a+bx)^6}{x^{10}} \, dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^7}{9ax^9}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{b^2(a+bx)^7}{252a^3x^7}$$

```
[Out] -1/9*(b*x+a)^7/a/x^9+1/36*b*(b*x+a)^7/a^2/x^8-1/252*b^2*(b*x+a)^7/a^3/x^7
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/x^10,x]
```

```
[Out] -1/9*(a + b*x)^7/(a*x^9) + (b*(a + b*x)^7)/(36*a^2*x^8) - (b^2*(a + b*x)^7)
/(252*a^3*x^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{(a+bx)^7}{9ax^9} - \frac{(2b)\int \frac{(a+bx)^6}{x^9}\,dx}{9a}$$
$$= -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} + \frac{b^2\int \frac{(a+bx)^6}{x^8}\,dx}{36a^2}$$
$$= -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} - \frac{b^2(a+bx)^7}{252a^3x^7}$$

Mathematica [A]
time = 0.01, size = 80, normalized size = 1.43

$$-\frac{a^6}{9x^9} - \frac{3a^5b}{4x^8} - \frac{15a^4b^2}{7x^7} - \frac{10a^3b^3}{3x^6} - \frac{3a^2b^4}{x^5} - \frac{3ab^5}{2x^4} - \frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (3*a^5*b)/(4*x^8) - (15*a^4*b^2)/(7*x^7) - (10*a^3*b^3)/(3*x
^6) - (3*a^2*b^4)/x^5 - (3*a*b^5)/(2*x^4) - b^6/(3*x^3)
```

Mathics [A]
time = 2.18, size = 68, normalized size = 1.21

$$\frac{-28a^6 - 189a^5bx - 540a^4b^2x^2 - 840a^3b^3x^3 - 756a^2b^4x^4 - 378ab^5x^5 - 84b^6x^6}{252x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^6/x^10,x]')
```

```
[Out] (-28 a ^ 6 - 189 a ^ 5 b x - 540 a ^ 4 b ^ 2 x ^ 2 - 840 a ^ 3 b ^ 3 x ^ 3
- 756 a ^ 2 b ^ 4 x ^ 4 - 378 a b ^ 5 x ^ 5 - 84 b ^ 6 x ^ 6) / (252 x ^ 9)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.23

method	result	size
norman	$\frac{-\frac{1}{3}x^6b^6-\frac{3}{2}a\,x^5b^5-3a^2x^4b^4-\frac{10}{3}a^3b^3x^3-\frac{15}{7}a^4x^2b^2-\frac{3}{4}a^5xb-\frac{1}{9}a^6}{x^9}$	68
risch	$\frac{-\frac{1}{3}x^6b^6-\frac{3}{2}a\,x^5b^5-3a^2x^4b^4-\frac{10}{3}a^3b^3x^3-\frac{15}{7}a^4x^2b^2-\frac{3}{4}a^5xb-\frac{1}{9}a^6}{x^9}$	68
gosper	$-\frac{84x^6b^6+378a\,x^5b^5+756a^2x^4b^4+840a^3b^3x^3+540a^4x^2b^2+189a^5xb+28a^6}{252x^9}$	69
default	$-\frac{b^6}{3x^3}-\frac{3a\,b^5}{2x^4}-\frac{a^6}{9x^9}-\frac{3a^5b}{4x^8}-\frac{3a^2b^4}{x^5}-\frac{10a^3b^3}{3x^6}-\frac{15a^4b^2}{7x^7}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^6/x^3-3/2*a*b^5/x^4-1/9*a^6/x^9-3/4*a^5*b/x^8-3*a^2*b^4/x^5-10/3*a^3
*b^3/x^6-15/7*a^4*b^2/x^7
```

Maxima [A]
time = 0.25, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Fricas [A]
time = 0.32, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Sympy [A]
time = 0.25, size = 73, normalized size = 1.30

$$\frac{-28a^6-189a^5bx-540a^4b^2x^2-840a^3b^3x^3-756a^2b^4x^4-378ab^5x^5-84b^6x^6}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/x**10,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756*a*
*2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.38

$$\frac{-84x^6b^6-378x^5b^5a-756x^4b^4a^2-840x^3b^3a^3-540x^2b^2a^4-189xba^5-28a^6}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x)
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Mupad [B]
time = 0.10, size = 68, normalized size = 1.21

$$-\frac{\frac{a^6}{9}+\frac{3\,a^5\,b\,x}{4}+\frac{15\,a^4\,b^2\,x^2}{7}+\frac{10\,a^3\,b^3\,x^3}{3}+3\,a^2\,b^4\,x^4+\frac{3\,a\,b^5\,x^5}{2}+\frac{b^6\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/x^10,x)
```

```
[Out] -(a^6/9 + (b^6*x^6)/3 + (3*a*b^5*x^5)/2 + (15*a^4*b^2*x^2)/7 + (10*a^3*b^3*
x^3)/3 + 3*a^2*b^4*x^4 + (3*a^5*b*x)/4)/x^9
```

3.246 $\int \frac{(a+bx)^5}{x^{10}} dx$

Optimal. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Mathics [A]
time = 2.02, size = 57, normalized size = 0.85

$$\frac{-56a^5-315a^4bx-720a^3b^2x^2-840a^2b^3x^3-504ab^4x^4-126b^5x^5}{504x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/x^10,x]')
```

```
[Out] (-56 a ^ 5 - 315 a ^ 4 b x - 720 a ^ 3 b ^ 2 x ^ 2 - 840 a ^ 2 b ^ 3 x ^ 3
- 504 a b ^ 4 x ^ 4 - 126 b ^ 5 x ^ 5) / (504 x ^ 9)
```

Maple [A]
time = 0.07, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
risch	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
gosper	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{a\,b^4}{x^5}-\frac{b^5}{4x^4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Maxima [A]
time = 0.24, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [A]
time = 0.22, size = 61, normalized size = 0.91

$$\frac{-56a^5-315a^4bx-720a^3b^2x^2-840a^2b^3x^3-504ab^4x^4-126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.97

$$\frac{-126x^5b^5-504x^4b^4a-840x^3b^3a^2-720x^2b^2a^3-315xba^4-56a^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x)
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Mupad [B]
time = 0.00, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9}+\frac{5\,a^4\,b\,x}{8}+\frac{10\,a^3\,b^2\,x^2}{7}+\frac{5\,a^2\,b^3\,x^3}{3}+a\,b^4\,x^4+\frac{b^5\,x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

3.247 $\int \frac{(a+bx)^4}{x^{10}} \, dx$

Optimal. Leaf size=56

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

```
[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{x^{10}}\,dx &= \int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^9}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^7}+\frac{b^4}{x^6}\right)dx\\ &= -\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Mathics [A]
time = 1.92, size = 46, normalized size = 0.82

$$\frac{-70a^4 - 315a^3bx - 540a^2b^2x^2 - 420ab^3x^3 - 126b^4x^4}{630x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/x^10,x]')
```

```
[Out] (-70 a ^ 4 - 315 a ^ 3 b x - 540 a ^ 2 b ^ 2 x ^ 2 - 420 a b ^ 3 x ^ 3 - 12
6 b ^ 4 x ^ 4) / (630 x ^ 9)
```

Maple [A]
time = 0.08, size = 47, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{5}b^4x^4-\frac{2}{3}a\,b^3x^3-\frac{6}{7}a^2b^2x^2-\frac{1}{2}a^3bx-\frac{1}{9}a^4}{x^9}$	46
risch	$\frac{-\frac{1}{5}b^4x^4-\frac{2}{3}a\,b^3x^3-\frac{6}{7}a^2b^2x^2-\frac{1}{2}a^3bx-\frac{1}{9}a^4}{x^9}$	46
gosper	$-\frac{126b^4x^4+420a\,b^3x^3+540a^2b^2x^2+315a^3bx+70a^4}{630x^9}$	47
default	$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2a\,b^3}{3x^6}-\frac{b^4}{5x^5}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

Maxima [A]
time = 0.25, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4 + 420\,ab^3x^3 + 540\,a^2b^2x^2 + 315\,a^3bx + 70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Fricas [A]
time = 0.31, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4 + 420\,ab^3x^3 + 540\,a^2b^2x^2 + 315\,a^3bx + 70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Sympy [A]
time = 0.19, size = 49, normalized size = 0.88

$$\frac{-70a^4 - 315a^3bx - 540a^2b^2x^2 - 420ab^3x^3 - 126b^4x^4}{630x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/x**10,x)
```

```
[Out] (-70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b**4*
x**4)/(630*x**9)
```

Giac [A]
time = 0.00, size = 53, normalized size = 0.95

$$\frac{-126x^4b^4 - 420x^3b^3a - 540x^2b^2a^2 - 315xba^3 - 70a^4}{630x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x)
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Mupad [B]
time = 0.03, size = 46, normalized size = 0.82

$$-\frac{\frac{a^4}{9} + \frac{a^3\,b\,x}{2} + \frac{6\,a^2\,b^2\,x^2}{7} + \frac{2\,a\,b^3\,x^3}{3} + \frac{b^4\,x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/x^10,x)
```

```
[Out] -(a^4/9 + (b^4*x^4)/5 + (2*a*b^3*x^3)/3 + (6*a^2*b^2*x^2)/7 + (a^3*b*x)/2)/
x^9
```

3.248 $\int \frac{(a+bx)^3}{x^{10}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

```
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{10}}\,dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right)dx\\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Mathics [A]
time = 1.82, size = 35, normalized size = 0.81

$$\frac{-56a^3 - 189a^2bx - 216ab^2x^2 - 84b^3x^3}{504x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^10,x]')
[Out] (-56 a ^ 3 - 189 a ^ 2 b x - 216 a b ^ 2 x ^ 2 - 84 b ^ 3 x ^ 3) / (504 x ^
 9)
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{6}b^3x^3-\frac{3}{7}a\,b^2x^2-\frac{3}{8}a^2bx-\frac{1}{9}a^3}{x^9}$	35
risch	$\frac{-\frac{1}{6}b^3x^3-\frac{3}{7}a\,b^2x^2-\frac{3}{8}a^2bx-\frac{1}{9}a^3}{x^9}$	35
gosper	$-\frac{84b^3x^3+216a\,b^2x^2+189a^2bx+56a^3}{504x^9}$	36
default	$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3a\,b^2}{7x^7}-\frac{b^3}{6x^6}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

Maxima [A]
time = 0.24, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3 + 216\,ab^2x^2 + 189\,a^2bx + 56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="maxima")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3 + 216\,ab^2x^2 + 189\,a^2bx + 56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Sympy [A]
time = 0.15, size = 37, normalized size = 0.86

$$\frac{-56a^3 - 189a^2bx - 216ab^2x^2 - 84b^3x^3}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**10,x)
```

```
[Out] (-56*a**3 - 189*a**2*b*x - 216*a*b**2*x**2 - 84*b**3*x**3)/(504*x**9)
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.95

$$\frac{-84x^3b^3 - 216x^2b^2a - 189xba^2 - 56a^3}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x)
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{9} + \frac{3\,a^2\,b\,x}{8} + \frac{3\,a\,b^2\,x^2}{7} + \frac{b^3\,x^3}{6}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^10,x)
```

```
[Out] -(a^3/9 + (b^3*x^3)/6 + (3*a*b^2*x^2)/7 + (3*a^2*b*x)/8)/x^9
```

3.249 $\int \frac{(a+bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

```
[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Mathics [A]
time = 1.73, size = 23, normalized size = 0.77

$$\frac{-\frac{a^2}{9} - \frac{abx}{4} - \frac{b^2x^2}{7}}{x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^10,x]')
[Out] (-a ^ 2 / 9 - a b x / 4 - b ^ 2 x ^ 2 / 7) / x ^ 9
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{7}x^2b^2-\frac{1}{4}abx-\frac{1}{9}a^2}{x^9}$	24
risch	$\frac{-\frac{1}{7}x^2b^2-\frac{1}{4}abx-\frac{1}{9}a^2}{x^9}$	24
gosper	$-\frac{36x^2b^2+63abx+28a^2}{252x^9}$	25
default	$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2 + 63\,abx + 28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="maxima")
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2 + 63\,abx + 28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Sympy [A]
time = 0.12, size = 26, normalized size = 0.87

$$\frac{-28a^2 - 63abx - 36b^2x^2}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**10,x)
```

```
[Out] (-28*a**2 - 63*a*b*x - 36*b**2*x**2)/(252*x**9)
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.97

$$\frac{-36x^2b^2 - 63xba - 28a^2}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x)
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{9} + \frac{a\,b\,x}{4} + \frac{b^2\,x^2}{7}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^10,x)
```

```
[Out] -(a^2/9 + (b^2*x^2)/7 + (a*b*x)/4)/x^9
```

3.250 $\int \frac{a+bx}{x^{10}}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

```
[Out] -1/9*a/x^9-1/8*b/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^10,x]
```

```
[Out] -1/9*a/x^9 - b/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{10}}\,dx &= \int \left(\frac{a}{x^{10}}+\frac{b}{x^9}\right)dx\\ &= -\frac{a}{9x^9}-\frac{b}{8x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^10,x]
```

```
[Out] -1/9*a/x^9 - b/(8*x^8)
```

Mathics [**A**]
time = 1.64, size = 12, normalized size = 0.71

$$\frac{-\frac{a}{9} - \frac{bx}{8}}{x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1/x^10,x]')
```

```
[Out] (-a / 9 - b x / 8) / x ^ 9
```

Maple [**A**]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{bx}{8}-\frac{a}{9}}{x^9}$	13
risch	$\frac{-\frac{bx}{8}-\frac{a}{9}}{x^9}$	13
gosper	$-\frac{9bx+8a}{72x^9}$	14
default	$-\frac{a}{9x^9}-\frac{b}{8x^8}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a/x^9-1/8*b/x^8
```

Maxima [**A**]
time = 0.24, size = 13, normalized size = 0.76

$$-\frac{9\,bx + 8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Fricas [**A**]
time = 0.30, size = 13, normalized size = 0.76

$$-\frac{9\,bx + 8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Sympy [A]
time = 0.08, size = 14, normalized size = 0.82

$$\frac{-8a - 9bx}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**10,x)
```

```
[Out] (-8*a - 9*b*x)/(72*x**9)
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{-9xb - 8a}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x)
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{8\,a+9\,b\,x}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^10,x)
```

```
[Out] -(8*a + 9*b*x)/(72*x^9)
```

3.251 $\int \frac{1}{x^{10}} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{9x^9}$$

[Out] -1/9/x^9

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

[In] Int[x^(-10),x]

[Out] -1/9*1/x^9

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9x^9}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-10),x]

[Out] -1/9*1/x^9

Mathics [A]
time = 1.53, size = 5, normalized size = 0.71

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/x^10,x]')
```

```
[Out] -1 / (9 x ^ 9)
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{9x^9}$	6
default	$-\frac{1}{9x^9}$	6
norman	$-\frac{1}{9x^9}$	6
risch	$-\frac{1}{9x^9}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x^9
```

Maxima [A]
time = 0.26, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="maxima")
```

```
[Out] -1/9/x^9
```

Fricas [A]
time = 0.30, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="fricas")
```

```
[Out] -1/9/x^9
```

Sympy [A]
time = 0.03, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10,x)
```

```
[Out] -1/(9*x**9)
```

Giac [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x)
```

```
[Out] -1/9/x^9
```

Mupad [B]
time = 0.02, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/(9*x^9)
```

3.252 $\int \frac{1}{x^{10}(a+bx)}\,dx$

Optimal. Leaf size=134

$$-\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}$$

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

Rubi [A]
time = 0.04, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5
*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*
x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)}\,dx &= \int \left(\frac{1}{ax^{10}}-\frac{b}{a^2x^9}+\frac{b^2}{a^3x^8}-\frac{b^3}{a^4x^7}+\frac{b^4}{a^5x^6}-\frac{b^5}{a^6x^5}+\frac{b^6}{a^7x^4}-\frac{b^7}{a^8x^3}+\frac{b^8}{a^9x^2}-\frac{b^9}{a^{10}x}+\frac{}{a^{10}(}\right.\\ &= -\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(}{a^{10}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 134, normalized size = 1.00

$$-\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5
*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*
x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Mathics [A]
time = 2.66, size = 119, normalized size = 0.89

$$\frac{a\left(-280a^8+315a^7bx-360a^6b^2x^2+420a^5b^3x^3-504a^4b^4x^4+630a^3b^5x^5-840a^2b^6x^6+1260ab^7x^7-2520b^8x^8\right)+2520b^9x^9\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}\left[x\right]\right)}{2520a^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^1*x^10),x]')
```

```
[Out] (a (-280 a ^ 8 + 315 a ^ 7 b x - 360 a ^ 6 b ^ 2 x ^ 2 + 420 a ^ 5 b ^ 3 x
^ 3 - 504 a ^ 4 b ^ 4 x ^ 4 + 630 a ^ 3 b ^ 5 x ^ 5 - 840 a ^ 2 b ^ 6 x ^ 6
 + 1260 a b ^ 7 x ^ 7 - 2520 b ^ 8 x ^ 8) + 2520 b ^ 9 x ^ 9 (Log[(a + b x)
 / b] - Log[x])) / (2520 a ^ 10 x ^ 9)
```

Maple [A]
time = 0.09, size = 119, normalized size = 0.89

method	result	size
default	$-\frac{1}{9a\,x^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(bx+a)}{a^{10}}$	119
norman	$\frac{-\frac{1}{9a}+\frac{bx}{8a^2}-\frac{b^2x^2}{7a^3}+\frac{b^3x^3}{6a^4}-\frac{b^4x^4}{5a^5}+\frac{b^5x^5}{4a^6}-\frac{b^6x^6}{3a^7}+\frac{b^7x^7}{2a^8}-\frac{b^8x^8}{a^9}}{x^9}+\frac{b^9\ln(bx+a)}{a^{10}}-\frac{b^9\ln(x)}{a^{10}}$	119
risch	$\frac{-\frac{1}{9a}+\frac{bx}{8a^2}-\frac{b^2x^2}{7a^3}+\frac{b^3x^3}{6a^4}-\frac{b^4x^4}{5a^5}+\frac{b^5x^5}{4a^6}-\frac{b^6x^6}{3a^7}+\frac{b^7x^7}{2a^8}-\frac{b^8x^8}{a^9}}{x^9}-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(-bx-a)}{a^{10}}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

Maxima [A]
time = 0.26, size = 117, normalized size = 0.87

$$\frac{b^9\log\left(bx+a\right)}{a^{10}}-\frac{b^9\log\left(x\right)}{a^{10}}-\frac{2520\,b^8x^8-1260\,ab^7x^7+840\,a^2b^6x^6-630\,a^3b^5x^5+504\,a^4b^4x^4-420\,a^5b^3x^3+360\,a^6b^2x^2-315\,a^7bx+280\,a^8}{2520\,a^9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="maxima")
```

[Out] b^9*log(b*x + a)/a^10 - b^9*log(x)/a^10 - 1/2520*(2520*b^8*x^8 - 1260*a*b^7*x^7 + 840*a^2*b^6*x^6 - 630*a^3*b^5*x^5 + 504*a^4*b^4*x^4 - 420*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 315*a^7*b*x + 280*a^8)/(a^9*x^9)

Fricas [A]

time = 0.32, size = 120, normalized size = 0.90

$$\frac{2520\,b^{9}x^{9}\log\left(bx+a\right)-2520\,b^{9}x^{9}\log\left(x\right)-2520\,ab^{8}x^{8}+1260\,a^{2}b^{7}x^{7}-840\,a^{3}b^{6}x^{6}+630\,a^{4}b^{5}x^{5}-504\,a^{5}b^{4}x^{4}+420\,a^{6}b^{3}x^{3}-360\,a^{7}b^{2}x^{2}+315\,a^{8}bx-280\,a^{9}}{2520\,a^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a),x, algorithm="fricas")

[Out] 1/2520*(2520*b^9*x^9*log(b*x + a) - 2520*b^9*x^9*log(x) - 2520*a*b^8*x^8 + 1260*a^2*b^7*x^7 - 840*a^3*b^6*x^6 + 630*a^4*b^5*x^5 - 504*a^5*b^4*x^4 + 420*a^6*b^3*x^3 - 360*a^7*b^2*x^2 + 315*a^8*b*x - 280*a^9)/(a^10*x^9)

Sympy [A]

time = 0.19, size = 116, normalized size = 0.87

$$\frac{-280a^{8}+315a^{7}bx-360a^{6}b^{2}x^{2}+420a^{5}b^{3}x^{3}-504a^{4}b^{4}x^{4}+630a^{3}b^{5}x^{5}-840a^{2}b^{6}x^{6}+1260ab^{7}x^{7}-2520b^{8}x^{8}}{2520a^{9}x^{9}}+\frac{b^{9}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**10/(b*x+a),x)

[Out] (-280*a**8 + 315*a**7*b*x - 360*a**6*b**2*x**2 + 420*a**5*b**3*x**3 - 504*a**4*b**4*x**4 + 630*a**3*b**5*x**5 - 840*a**2*b**6*x**6 + 1260*a*b**7*x**7 - 2520*b**8*x**8)/(2520*a**9*x**9) + b**9*(-log(x) + log(a/b + x))/a**10

Giac [A]

time = 0.00, size = 135, normalized size = 1.01

$$\frac{b^{10}\ln|xb+a|}{ba^{10}}-\frac{b^{9}\ln|x|}{a^{10}}+\frac{\frac{1}{2520}\left(-2520b^{8}ax^{8}+1260b^{7}a^{2}x^{7}-840b^{6}a^{3}x^{6}+630b^{5}a^{4}x^{5}-504b^{4}a^{5}x^{4}+420b^{3}a^{6}x^{3}-360b^{2}a^{7}x^{2}+315ba^{8}x-280a^{9}\right)}{a^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a),x)

[Out] b^9*log(abs(b*x + a))/a^10 - b^9*log(abs(x))/a^10 - 1/2520*(2520*a*b^8*x^8 - 1260*a^2*b^7*x^7 + 840*a^3*b^6*x^6 - 630*a^4*b^5*x^5 + 504*a^5*b^4*x^4 - 420*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 315*a^8*b*x + 280*a^9)/(a^10*x^9)

Mupad [B]

time = 0.13, size = 114, normalized size = 0.85

$$-\frac{280\,a^{9}+2520\,a\,b^{8}\,x^{8}-5040\,b^{9}\,x^{9}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)+360\,a^{7}\,b^{2}\,x^{2}-420\,a^{6}\,b^{3}\,x^{3}+504\,a^{5}\,b^{4}\,x^{4}-630\,a^{4}\,b^{5}\,x^{5}+840\,a^{3}\,b^{6}\,x^{6}-1260\,a^{2}\,b^{7}\,x^{7}-315\,a^{8}\,b\,x}{2520\,a^{10}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^10*(a + b*x)),x)

[Out] -(280*a^9 + 2520*a*b^8*x^8 - 5040*b^9*x^9*atanh((2*b*x)/a + 1) + 360*a^7*b^2*x^2 - 420*a^6*b^3*x^3 + 504*a^5*b^4*x^4 - 630*a^4*b^5*x^5 + 840*a^3*b^6*x^6 - 1260*a^2*b^7*x^7 - 315*a^8*b*x)/(2520*a^10*x^9)

3.253 $\int \frac{1}{x^{10}(a+bx)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+bx)}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a}{a^{11}}$$

```
[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*
b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*
b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Rubi [A]
time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}+\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/9*1/(a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^6
) - b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(a^
9*x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11 + (
10*b^9*Log[a + b*x])/a^11
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^{10}}-\frac{2b}{a^3x^9}+\frac{3b^2}{a^4x^8}-\frac{4b^3}{a^5x^7}+\frac{5b^4}{a^6x^6}-\frac{6b^5}{a^7x^5}+\frac{7b^6}{a^8x^4}-\frac{8b^7}{a^9x^3}+\frac{9b^8}{a^{10}x^2}-\frac{10b^9}{a^{11}x}+\right.\\ &= -\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b}{a^{10}(a}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 134, normalized size = 0.92

$$-\frac{\frac{a(28a^9-35a^8bx+45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7+1260ab^8x^8+2520b^9x^9)}{x^9(a+bx)}+2520b^9\log(x)-2520b^9\log(a+bx)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/252*((a*(28*a^9 - 35*a^8*b*x + 45*a^7*b^2*x^2 - 60*a^6*b^3*x^3 + 84*a^5*
b^4*x^4 - 126*a^4*b^5*x^5 + 210*a^3*b^6*x^6 - 420*a^2*b^7*x^7 + 1260*a*b^8*
x^8 + 2520*b^9*x^9))/(x^9*(a + b*x)) + 2520*b^9*Log[x] - 2520*b^9*Log[a + b
*x])/a^11
```

Mathics [A]
time = 2.96, size = 142, normalized size = 0.97

$$\frac{a\left(-28a^9+35a^8bx-45a^7b^2x^2+60a^6b^3x^3-84a^5b^4x^4+126a^4b^5x^5-210a^3b^6x^6+420a^2b^7x^7-1260ab^8x^8-2520b^9x^9\right)-2520b^9x^9\left(a+bx\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{252a^{11}x^9\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^2*x^10),x]')
```

```
[Out] (a (-28 a ^ 9 + 35 a ^ 8 b x - 45 a ^ 7 b ^ 2 x ^ 2 + 60 a ^ 6 b ^ 3 x ^ 3
- 84 a ^ 5 b ^ 4 x ^ 4 + 126 a ^ 4 b ^ 5 x ^ 5 - 210 a ^ 3 b ^ 6 x ^ 6 + 42
0 a ^ 2 b ^ 7 x ^ 7 - 1260 a b ^ 8 x ^ 8 - 2520 b ^ 9 x ^ 9) - 2520 b ^ 9 x
 ^ 9 (a + b x) (Log[x] - Log[(a + b x) / b])) / (252 a ^ 11 x ^ 9 (a + b x)
)
```

Maple [A]
time = 0.09, size = 135, normalized size = 0.92

method	result
default	$-\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(bx+a)}-\frac{10b^9\ln(x)}{a^{11}}+$ $10b^9\ln($
norman	$\frac{\frac{10b^{10}x^{10}}{a^{11}}-\frac{1}{9a}+\frac{5bx}{36a^2}-\frac{5b^2x^2}{28a^3}+\frac{5b^3x^3}{21a^4}-\frac{b^4x^4}{3a^5}+\frac{b^5x^5}{2a^6}-\frac{5b^6x^6}{6a^7}+\frac{5b^7x^7}{3a^8}-\frac{5b^8x^8}{a^9}}{x^9(bx+a)}-\frac{10b^9\ln(x)}{a^{11}}+\frac{10b^9\ln(bx+a)}{a^{11}}$
risch	$\frac{-\frac{10b^9x^9}{a^{10}}-\frac{5b^8x^8}{a^9}+\frac{5b^7x^7}{3a^8}-\frac{5b^6x^6}{6a^7}+\frac{b^5x^5}{2a^6}-\frac{b^4x^4}{3a^5}+\frac{5b^3x^3}{21a^4}-\frac{5b^2x^2}{28a^3}+\frac{5bx}{36a^2}-\frac{1}{9a}}{x^9(bx+a)}+\frac{10b^9\ln(-bx-a)}{a^{11}}-\frac{10b^9\ln(x)}{a^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*
b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*
b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Maxima [A]
time = 0.27, size = 141, normalized size = 0.97

$$-\frac{2520\,b^9x^9+1260\,ab^8x^8-420\,a^2b^7x^7+210\,a^3b^6x^6-126\,a^4b^5x^5+84\,a^5b^4x^4-60\,a^6b^3x^3+45\,a^7b^2x^2-35\,a^8bx+28\,a^9}{252\left(a^{10}bx^{10}+a^{11}x^9\right)}+\frac{10\,b^9\log\left(bx+a\right)}{a^{11}}-\frac{10\,b^9\log\left(x\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 1260*a*b^8*x^8 - 420*a^2*b^7*x^7 + 210*a^3*b^6*x^6 -
 126*a^4*b^5*x^5 + 84*a^5*b^4*x^4 - 60*a^6*b^3*x^3 + 45*a^7*b^2*x^2 - 35*a^
8*b*x + 28*a^9)/(a^10*b*x^10 + a^11*x^9) + 10*b^9*log(b*x + a)/a^11 - 10*b^
9*log(x)/a^11
```

Fricas [A]
time = 0.32, size = 163, normalized size = 1.12

$$-\frac{2520\,ab^9x^9+1260\,a^2b^8x^8-420\,a^3b^7x^7+210\,a^4b^6x^6-126\,a^5b^5x^5+84\,a^6b^4x^4-60\,a^7b^3x^3+45\,a^8b^2x^2-35\,a^9bx+28\,a^{10}-2520\,(b^{10}x^{10}+ab^9x^9)\log\left(bx+a\right)+2520\,(b^{10}x^{10}+ab^9x^9)\log\left(x\right)}{252\,(a^{11}bx^{10}+a^{12}x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*x
^6 - 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 - 3
5*a^9*b*x + 28*a^10 - 2520*(b^10*x^10 + a*b^9*x^9)*log(b*x + a) + 2520*(b^1
0*x^10 + a*b^9*x^9)*log(x))/(a^11*b*x^10 + a^12*x^9)
```

Sympy [A]
time = 0.27, size = 139, normalized size = 0.95

$$\frac{-28a^9+35a^8bx-45a^7b^2x^2+60a^6b^3x^3-84a^5b^4x^4+126a^4b^5x^5-210a^3b^6x^6+420a^2b^7x^7-1260ab^8x^8-2520b^9x^9}{252a^{11}x^9+252a^{10}bx^{10}}+\frac{10b^9\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**2,x)
```

```
[Out] (-28*a**9 + 35*a**8*b*x - 45*a**7*b**2*x**2 + 60*a**6*b**3*x**3 - 84*a**5*b
**4*x**4 + 126*a**4*b**5*x**5 - 210*a**3*b**6*x**6 + 420*a**2*b**7*x**7 - 1
260*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**11*x**9 + 252*a**10*b*x**10) + 10
*b**9*(-log(x) + log(a/b + x))/a**11
```

Giac [A]
time = 0.00, size = 155, normalized size = 1.06

$$\frac{10b^{10}\ln\left|xb+a\right|}{ba^{11}}-\frac{10b^9\ln\left|x\right|}{a^{11}}+\frac{\frac{1}{252}\left(-2520b^9ax^9-1260b^8a^2x^8+420b^7a^3x^7-210b^6a^4x^6+126b^5a^5x^5-84b^4a^6x^4+60b^3a^7x^3-45b^2a^8x^2+35ba^9x-28a^{10}\right)}{a^{11}x^9\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x)
```

```
[Out] 10*b^9*log(abs(b*x + a))/a^11 - 10*b^9*log(abs(x))/a^11 - 1/252*(2520*a*b^9
*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*x^6 - 126*a^5*b^5*x
^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 - 35*a^9*b*x + 28*a^1
0)/((b*x + a)*a^11*x^9)
```

Mupad [B]
time = 0.08, size = 135, normalized size = 0.92

$$\frac{20\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{9\,a}+\frac{5\,b^2\,x^2}{28\,a^3}-\frac{5\,b^3\,x^3}{21\,a^4}+\frac{b^4\,x^4}{3\,a^5}-\frac{b^5\,x^5}{2\,a^6}+\frac{5\,b^6\,x^6}{6\,a^7}-\frac{5\,b^7\,x^7}{3\,a^8}+\frac{5\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}-\frac{5\,b\,x}{36\,a^2}}{b\,x^{10}+a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^2),x)
```

```
[Out] (20*b^9*atanh((2*b*x)/a + 1))/a^11 - (1/(9*a) + (5*b^2*x^2)/(28*a^3) - (5*b
^3*x^3)/(21*a^4) + (b^4*x^4)/(3*a^5) - (b^5*x^5)/(2*a^6) + (5*b^6*x^6)/(6*a
^7) - (5*b^7*x^7)/(3*a^8) + (5*b^8*x^8)/a^9 + (10*b^9*x^9)/a^10 - (5*b*x)/(
36*a^2))/(a*x^9 + b*x^10)
```

3.254 $\int \frac{1}{x^{10}(a+bx)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{10b^9}{a^{11}(a+bx)}-\frac{55b^9\,\mathrm{lo}}{a^{12}}$$

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

Rubi [A]

time = 0.07, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx)}{a^{12}}-\frac{10b^9}{a^{11}(a+bx)}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}+\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}+\frac{3b}{8a^4x^8}-\frac{1}{9a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/9*1/(a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^6
*x^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) + (
18*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*b^9
)/(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^{10}}-\frac{3b}{a^4x^9}+\frac{6b^2}{a^5x^8}-\frac{10b^3}{a^6x^7}+\frac{15b^4}{a^7x^6}-\frac{21b^5}{a^8x^5}+\frac{28b^6}{a^9x^4}-\frac{36b^7}{a^{10}x^3}+\frac{45b^8}{a^{11}x^2}-\frac{55b^9}{a^{12}x}+\right.\\ &= -\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{}{2a^{10}(}\end{aligned}$$

Mathematica [A]

time = 0.06, size = 145, normalized size = 0.89

$$-\frac{\frac{a(56a^{10}-77a^9bx+110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8+41580ab^9x^9+27720b^{10}x^{10})}{x^9(a+bx)^2}+27720b^9\log(x)-27720b^9\log(a+bx)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/504*((a*(56*a^10 - 77*a^9*b*x + 110*a^8*b^2*x^2 - 165*a^7*b^3*x^3 + 264*
a^6*b^4*x^4 - 462*a^5*b^5*x^5 + 924*a^4*b^6*x^6 - 2310*a^3*b^7*x^7 + 9240*a
^2*b^8*x^8 + 41580*a*b^9*x^9 + 27720*b^10*x^10))/(x^9*(a + b*x)^2) + 27720*
b^9*Log[x] - 27720*b^9*Log[a + b*x])/a^12
```

Mathics [A]
time = 3.20, size = 175, normalized size = 1.07

$$\frac{a\left(-56a^{10}+77a^{9}bx-110a^{8}b^{2}x^{2}+165a^{7}b^{3}x^{3}-264a^{6}b^{4}x^{4}+462a^{5}b^{5}x^{5}-924a^{4}b^{6}x^{6}+2310a^{3}b^{7}x^{7}-9240a^{2}b^{8}x^{8}-41580ab^{9}x^{9}-27720b^{10}x^{10}\right)-27720b^{9}x^{9}\left(a^{2}+2abx+b^{2}x^{2}\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)}{504a^{12}x^{9}\left(a^{2}+2abx+b^{2}x^{2}\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^3*x^10),x]')
```

```
[Out] (a (-56 a ^ 10 + 77 a ^ 9 b x - 110 a ^ 8 b ^ 2 x ^ 2 + 165 a ^ 7 b ^ 3 x ^
 3 - 264 a ^ 6 b ^ 4 x ^ 4 + 462 a ^ 5 b ^ 5 x ^ 5 - 924 a ^ 4 b ^ 6 x ^ 6
+ 2310 a ^ 3 b ^ 7 x ^ 7 - 9240 a ^ 2 b ^ 8 x ^ 8 - 41580 a b ^ 9 x ^ 9 - 2
7720 b ^ 10 x ^ 10) - 27720 b ^ 9 x ^ 9 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (Lo
g[x] - Log[(a + b x) / b])) / (504 a ^ 12 x ^ 9 (a ^ 2 + 2 a b x + b ^ 2 x
^ 2))
```

Maple [A]
time = 0.09, size = 150, normalized size = 0.92

method	result
norman	$\frac{-\frac{1}{9a}+\frac{11bx}{72a^{2}}-\frac{55b^{2}x^{2}}{252a^{3}}+\frac{55b^{3}x^{3}}{168a^{4}}-\frac{11b^{4}x^{4}}{21a^{5}}+\frac{11b^{5}x^{5}}{12a^{6}}-\frac{11b^{6}x^{6}}{6a^{7}}+\frac{55b^{7}x^{7}}{12a^{8}}-\frac{55b^{8}x^{8}}{3a^{9}}+\frac{110b^{10}x^{10}}{a^{11}}+\frac{165b^{11}x^{11}}{2a^{12}}}{x^{9}\left(bx+a\right)^{2}}-\frac{55b^{9}\ln\left(x\right)}{a^{12}}+\frac{55b^{9}\ln\left(bx+a\right)}{a^{12}}$
default	$-\frac{1}{9a^{3}x^{9}}+\frac{3b}{8a^{4}x^{8}}-\frac{6b^{2}}{7a^{5}x^{7}}+\frac{5b^{3}}{3a^{6}x^{6}}-\frac{3b^{4}}{a^{7}x^{5}}+\frac{21b^{5}}{4a^{8}x^{4}}-\frac{28b^{6}}{3a^{9}x^{3}}+\frac{18b^{7}}{a^{10}x^{2}}-\frac{45b^{8}}{a^{11}x}-\frac{b^{9}}{2a^{10}\left(bx+a\right)^{2}}-\frac{10b^{9}}{a^{11}\left(bx+a\right)}-$
risch	$\frac{-\frac{55b^{10}x^{10}}{a^{11}}-\frac{165b^{9}x^{9}}{2a^{10}}-\frac{55b^{8}x^{8}}{3a^{9}}+\frac{55b^{7}x^{7}}{12a^{8}}-\frac{11b^{6}x^{6}}{6a^{7}}+\frac{11b^{5}x^{5}}{12a^{6}}-\frac{11b^{4}x^{4}}{21a^{5}}+\frac{55b^{3}x^{3}}{168a^{4}}-\frac{55b^{2}x^{2}}{252a^{3}}+\frac{11bx}{72a^{2}}-\frac{1}{9a}}{x^{9}\left(bx+a\right)^{2}}+\frac{55b^{9}\ln\left(-bx-a\right)}{a^{12}}-\frac{55b^{9}\ln\left(x\right)}{a^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

Maxima [A]
time = 0.27, size = 163, normalized size = 1.00

$$-\frac{27720\,b^{10}x^{10}+41580\,ab^{9}x^{9}+9240\,a^{2}b^{8}x^{8}-2310\,a^{3}b^{7}x^{7}+924\,a^{4}b^{6}x^{6}-462\,a^{5}b^{5}x^{5}+264\,a^{6}b^{4}x^{4}-165\,a^{7}b^{3}x^{3}+110\,a^{8}b^{2}x^{2}-77\,a^{9}bx+56\,a^{10}}{504\,\left(a^{11}b^{2}x^{11}+2\,a^{12}bx^{10}+a^{13}x^{9}\right)}+\frac{55\,b^{9}\log\left(bx+a\right)}{a^{12}}-\frac{55\,b^{9}\log\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/504*(27720*b^10*x^10 + 41580*a*b^9*x^9 + 9240*a^2*b^8*x^8 - 2310*a^3*b^7
*x^7 + 924*a^4*b^6*x^6 - 462*a^5*b^5*x^5 + 264*a^6*b^4*x^4 - 165*a^7*b^3*x^
3 + 110*a^8*b^2*x^2 - 77*a^9*b*x + 56*a^10)/(a^11*b^2*x^11 + 2*a^12*b*x^10
+ a^13*x^9) + 55*b^9*log(b*x + a)/a^12 - 55*b^9*log(x)/a^12
```

Fricas [A]
time = 0.31, size = 207, normalized size = 1.27

$$-\frac{27720\,ab^{10}x^{10}+41580\,a^{2}b^{9}x^{9}+9240\,a^{3}b^{8}x^{8}-2310\,a^{4}b^{7}x^{7}+924\,a^{5}b^{6}x^{6}-462\,a^{6}b^{5}x^{5}+264\,a^{7}b^{4}x^{4}-165\,a^{8}b^{3}x^{3}+110\,a^{9}b^{2}x^{2}-77\,a^{10}bx+56\,a^{11}-27720\,(b^{11}x^{11}+2\,ab^{10}x^{10}+a^{2}b^{9}x^{9})\log(bx+a)+27720\,(b^{11}x^{11}+2\,ab^{10}x^{10}+a^{2}b^{9}x^{9})\log(x)}{504\,(a^{12}b^{2}x^{11}+2\,a^{13}bx^{10}+a^{14}x^{9})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/504*(27720*a*b^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4
*b^7*x^7 + 924*a^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^
3*x^3 + 110*a^9*b^2*x^2 - 77*a^10*b*x + 56*a^11 - 27720*(b^11*x^11 + 2*a*b^
10*x^10 + a^2*b^9*x^9)*log(b*x + a) + 27720*(b^11*x^11 + 2*a*b^10*x^10 + a^
2*b^9*x^9)*log(x))/(a^12*b^2*x^11 + 2*a^13*b*x^10 + a^14*x^9)
```

Sympy [A]
time = 0.34, size = 163, normalized size = 1.00

$$\frac{-56a^{10}+77a^{9}bx-110a^{8}b^{2}x^{2}+165a^{7}b^{3}x^{3}-264a^{6}b^{4}x^{4}+462a^{5}b^{5}x^{5}-924a^{4}b^{6}x^{6}+2310a^{3}b^{7}x^{7}-9240a^{2}b^{8}x^{8}-41580ab^{9}x^{9}-27720b^{10}x^{10}}{504a^{13}x^{9}+1008a^{12}bx^{10}+504a^{11}b^{2}x^{11}}+\frac{55b^{9}\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**3,x)
```

```
[Out] (-56*a**10 + 77*a**9*b*x - 110*a**8*b**2*x**2 + 165*a**7*b**3*x**3 - 264*a*
*6*b**4*x**4 + 462*a**5*b**5*x**5 - 924*a**4*b**6*x**6 + 2310*a**3*b**7*x**
7 - 9240*a**2*b**8*x**8 - 41580*a*b**9*x**9 - 27720*b**10*x**10)/(504*a**13
*x**9 + 1008*a**12*b*x**10 + 504*a**11*b**2*x**11) + 55*b**9*(-log(x) + log
(a/b + x))/a**12
```

Giac [A]
time = 0.00, size = 169, normalized size = 1.04

$$\frac{55b^{10}\ln|xb+a|}{ba^{12}}-\frac{55b^{9}\ln|x|}{a^{12}}+\frac{\frac{1}{504}\left(-27720b^{10}ax^{10}-41580b^{9}a^{2}x^{9}-9240b^{8}a^{3}x^{8}+2310b^{7}a^{4}x^{7}-924b^{6}a^{5}x^{6}+462b^{5}a^{6}x^{5}-264b^{4}a^{7}x^{4}+165b^{3}a^{8}x^{3}-110b^{2}a^{9}x^{2}+77ba^{10}x-56a^{11}\right)}{a^{12}x^{9}(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x)
```

```
[Out] 55*b^9*log(abs(b*x + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/504*(27720*a*b^
10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4*b^7*x^7 + 924*a^5
```

```
*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^3*x^3 + 110*a^9*b^
2*x^2 - 77*a^10*b*x + 56*a^11)/((b*x + a)^2*a^12*x^9)
```

Mupad [B]
time = 0.23, size = 157, normalized size = 0.96

$$\frac{110\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}-\frac{\frac{1}{9\,a}+\frac{55\,b^2\,x^2}{252\,a^3}-\frac{55\,b^3\,x^3}{168\,a^4}+\frac{11\,b^4\,x^4}{21\,a^5}-\frac{11\,b^5\,x^5}{12\,a^6}+\frac{11\,b^6\,x^6}{6\,a^7}-\frac{55\,b^7\,x^7}{12\,a^8}+\frac{55\,b^8\,x^8}{3\,a^9}+\frac{165\,b^9\,x^9}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}-\frac{11\,b\,x}{72\,a^2}}{a^2\,x^9+2\,a\,b\,x^{10}+b^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^3),x)
```

```
[Out] (110*b^9*atanh((2*b*x)/a + 1))/a^12 - (1/(9*a) + (55*b^2*x^2)/(252*a^3) - (
55*b^3*x^3)/(168*a^4) + (11*b^4*x^4)/(21*a^5) - (11*b^5*x^5)/(12*a^6) + (11
*b^6*x^6)/(6*a^7) - (55*b^7*x^7)/(12*a^8) + (55*b^8*x^8)/(3*a^9) + (165*b^9
*x^9)/(2*a^10) + (55*b^10*x^10)/a^11 - (11*b*x)/(72*a^2))/(a^2*x^9 + b^2*x^
11 + 2*a*b*x^10)
```

3.255 $\int \frac{1}{x(2+3x)}\,dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2}-\frac{1}{2}\log(2+3x)$$

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(2+3x)}\,dx &= \frac{1}{2}\int \frac{1}{x}\,dx-\frac{3}{2}\int \frac{1}{2+3x}\,dx\\ &= \frac{\log(x)}{2}-\frac{1}{2}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Mathics [A]
time = 1.60, size = 11, normalized size = 0.65

$$-\frac{\text{Log}\left[\frac{2}{3}+x\right]}{2}+\frac{\text{Log}\left[x\right]}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(2 + 3*x)),x]')
```

```
[Out] -Log[2 / 3 + x] / 2 + Log[x] / 2
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}$	14
norman	$\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}$	14
risch	$\frac{\ln(x)}{2}-\frac{\ln(2+3x)}{2}$	14
meijerg	$\frac{\ln(x)}{2}-\frac{\ln(2)}{2}+\frac{\ln(3)}{2}-\frac{\ln\left(1+\frac{3x}{2}\right)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x)
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.76

$$-\frac{1}{2}\log\left(3\,x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Fricas [A]
time = 0.30, size = 13, normalized size = 0.76

$$-\frac{1}{2}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="fricas")
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.71

$$\frac{\log\left(x\right)}{2}-\frac{\log\left(x+\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x)
[Out] log(x)/2 - log(x + 2/3)/2
```

Giac [A]
time = 0.00, size = 18, normalized size = 1.06

$$\frac{\ln|x|}{2}-\frac{\ln|3x+2|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x)
[Out] -1/2*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.17, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{2}{x}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + 2)),x)
[Out] -log(2/x + 3)/2
```

3.256 $\int \frac{1}{x(4+6x)}\,dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)$$

```
[Out] 1/4*ln(x)-1/4*ln(2+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)}\,dx &= \frac{1}{4}\int \frac{1}{x}\,dx - \frac{3}{2}\int \frac{1}{4+6x}\,dx\\ &= \frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)),x]
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Mathics [A]
time = 1.61, size = 11, normalized size = 0.65

$$-\frac{\mathrm{Log}\left[\frac{2}{3}+x\right]}{4} + \frac{\mathrm{Log}\left[x\right]}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(4 + 6*x)),x]')
[Out] -Log[2 / 3 + x] / 4 + Log[x] / 4
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
norman	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
risch	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
meijerg	$\frac{\ln(x)}{4}-\frac{\ln(2)}{4}+\frac{\ln(3)}{4}-\frac{\ln\left(1+\frac{3x}{2}\right)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x)-1/4*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 13, normalized size = 0.76

$$-\frac{1}{4}\log\left(3\,x+2\right)+\frac{1}{4}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

Fricas [A]
time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{4}\,\log\left(3\,x+2\right)+\frac{1}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.71

$$\frac{\log\left(x\right)}{4}-\frac{\log\left(x+\frac{2}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x)
```

```
[Out] log(x)/4 - log(x + 2/3)/4
```

Giac [A]
time = 0.00, size = 18, normalized size = 1.06

$$\frac{\ln|x|}{4}-\frac{\ln|3x+2|}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x)
```

```
[Out] -1/4*log(abs(3*x + 2)) + 1/4*log(abs(x))
```

Mupad [B]
time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{4}{x}+6\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)),x)
```

```
[Out] -log(4/x + 6)/4
```

3.257 $\int \frac{1}{x^2(4+6x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)$$

[Out] -1/4/x-3/8*ln(x)+3/8*ln(2+3*x)

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{46\}$

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(4 + 6*x)),x]

[Out] -1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)}\,dx &= \int \left(\frac{1}{4x^2}-\frac{3}{8x}+\frac{9}{8(2+3x)}\right)dx\\ &= -\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(4 + 6*x)),x]

```
[Out] -1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Mathics [A]
time = 1.66, size = 19, normalized size = 0.79

$$\frac{-2 + 3x\left(\text{Log}\left[\frac{2}{3} + x\right] - \text{Log}\left[x\right]\right)}{8x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(4 + 6*x)),x]')
[Out] (-2 + 3 x (Log[2 / 3 + x] - Log[x])) / (8 x)
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.79

method	result	size
default	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
norman	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
risch	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
meijerg	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2)}{8}-\frac{3\ln(3)}{8}+\frac{3\ln\left(1+\frac{3x}{2}\right)}{8}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x),x,method=_RETURNVERBOSE)
[Out] -1/4/x-3/8*ln(x)+3/8*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 18, normalized size = 0.75

$$-\frac{1}{4\,x} + \frac{3}{8}\,\log\left(3\,x + 2\right) - \frac{3}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="maxima")
[Out] -1/4/x + 3/8*log(3*x + 2) - 3/8*log(x)
```

Fricas [A]
time = 0.30, size = 21, normalized size = 0.88

$$\frac{3\,x\log\left(3\,x + 2\right) - 3\,x\log\left(x\right) - 2}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*x*log(3*x + 2) - 3*x*log(x) - 2)/x
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.83

$$-\frac{3\log(x)}{8}+\frac{3\log\left(x+\frac{2}{3}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x),x)
```

```
[Out] -3*log(x)/8 + 3*log(x + 2/3)/8 - 1/(4*x)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.12

$$\frac{3}{8}\ln|3x+2|-\frac{3}{8}\ln|x|-\frac{\frac{1}{8}\cdot 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x)
```

```
[Out] -1/4/x + 3/8*log(abs(3*x + 2)) - 3/8*log(abs(x))
```

Mupad [B]
time = 0.05, size = 18, normalized size = 0.75

$$-\frac{3\ln\left(\frac{x}{6\,x+4}\right)}{8}-\frac{1}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)),x)
```

```
[Out] - (3*log(x/(6*x + 4)))/8 - 1/(4*x)
```

3.258 $\int \frac{1}{x^3(4+6x)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)$$

```
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)),x]
```

```
[Out] -1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)}\,dx &= \int \left(\frac{1}{4x^3}-\frac{3}{8x^2}+\frac{9}{16x}-\frac{27}{16(2+3x)}\right)dx\\ &= -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)),x]
```

```
[Out] -1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Mathics [A]
time = 1.68, size = 24, normalized size = 0.77

$$\frac{-2+6x+9x^2\left(\mathrm{Log}\,[x]-\mathrm{Log}\left[\frac{2}{3}+x\right]\right)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(4 + 6*x)),x]')
[Out] (-2 + 6 x + 9 x ^ 2 (Log[x] - Log[2 / 3 + x])) / (16 x ^ 2)
```

Maple [A]
time = 0.09, size = 24, normalized size = 0.77

method	result	size
norman	$\frac{-\frac{1}{8}+\frac{3x}{8}}{x^2}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	23
risch	$\frac{-\frac{1}{8}+\frac{3x}{8}}{x^2}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	23
default	$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	24
meijerg	$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\ln(x)}{16}-\frac{9\ln(2)}{16}+\frac{9\ln(3)}{16}-\frac{9\ln\left(1+\frac{3x}{2}\right)}{16}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x),x,method=_RETURNVERBOSE)
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 23, normalized size = 0.74

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log\left(3\,x+2\right)+\frac{9}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="maxima")
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(3*x + 2) + 9/16*log(x)
```

Fricas [A]
time = 0.31, size = 28, normalized size = 0.90

$$-\frac{9\,x^2\log\left(3\,x+2\right)-9\,x^2\log\left(x\right)-6\,x+2}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/16*(9*x^2*log(3*x + 2) - 9*x^2*log(x) - 6*x + 2)/x^2
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.84

$$\frac{9\log{(x)}}{16}-\frac{9\log{\left(x+\frac{2}{3}\right)}}{16}+\frac{3x-1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x),x)
```

```
[Out] 9*log(x)/16 - 9*log(x + 2/3)/16 + (3*x - 1)/(8*x**2)
```

Giac [A]
time = 0.00, size = 32, normalized size = 1.03

$$-\frac{9}{16}\ln|3x+2|+\frac{9}{16}\ln|x|+\frac{\frac{1}{32}(12x-4)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x)
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(abs(3*x + 2)) + 9/16*log(abs(x))
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.58

$$\frac{\frac{3\,x}{8}-\frac{1}{8}}{x^2}-\frac{9\,\mathrm{atanh}\left(3\,x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)),x)
```

```
[Out] ((3*x)/8 - 1/8)/x^2 - (9*atanh(3*x + 1))/8
```

3.259 $\int \frac{1}{x^4(4+6x)}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

```
[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)}\,dx &= \int \left(\frac{1}{4x^4}-\frac{3}{8x^3}+\frac{9}{16x^2}-\frac{27}{32x}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.00

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Mathics [A]
time = 1.74, size = 29, normalized size = 0.76

$$\frac{-8+18x-54x^2+81x^3\left(\text{Log}\left[\frac{2}{3}+x\right]-\text{Log}\left[x\right]\right)}{96x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(4 + 6*x)),x]')
[Out] (-8 + 18 x - 54 x ^ 2 + 81 x ^ 3 (Log[2 / 3 + x] - Log[x])) / (96 x ^ 3)
```

Maple [A]
time = 0.09, size = 29, normalized size = 0.76

method	result	size
norman	$\frac{-\frac{1}{12}+\frac{3}{16}x-\frac{9}{16}x^2}{x^3}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	28
risch	$\frac{-\frac{1}{12}+\frac{3}{16}x-\frac{9}{16}x^2}{x^3}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	28
default	$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	29
meijerg	$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\ln(x)}{32}+\frac{27\ln(2)}{32}-\frac{27\ln(3)}{32}+\frac{27\ln(1+\frac{3x}{2})}{32}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x),x,method=_RETURNVERBOSE)
[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 28, normalized size = 0.74

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="maxima")
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(3*x + 2) - 27/32*log(x)
```

Fricas [A]
time = 0.31, size = 33, normalized size = 0.87

$$\frac{81\,x^3\log\left(3\,x+2\right)-81\,x^3\log\left(x\right)-54\,x^2+18\,x-8}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="fricas")
[Out] 1/96*(81*x^3*log(3*x + 2) - 81*x^3*log(x) - 54*x^2 + 18*x - 8)/x^3
```

Sympy [A]
time = 0.07, size = 31, normalized size = 0.82

$$-\frac{27\log\left(x\right)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-27x^{2}+9x-4}{48x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x),x)
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-27*x**2 + 9*x - 4)/(48*x**3)
```

Giac [A]
time = 0.00, size = 38, normalized size = 1.00

$$\frac{27}{32}\ln\left|3x+2\right|-\frac{27}{32}\ln\left|x\right|+\frac{\frac{1}{192}\left(-108x^{2}+36x-16\right)}{x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x)
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(abs(3*x + 2)) - 27/32*log(abs(x))
```

Mupad [B]
time = 0.09, size = 24, normalized size = 0.63

$$\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{16}-\frac{\frac{9\,x^{2}}{16}-\frac{3\,x}{16}+\frac{1}{12}}{x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)),x)
[Out] (27*atanh(3*x + 1))/16 - ((9*x^2)/16 - (3*x)/16 + 1/12)/x^3
```

3.260 $\int \frac{1}{x^5(4+6x)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)$$

```
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)}\,dx &= \int \left(\frac{1}{4x^5}-\frac{3}{8x^4}+\frac{9}{16x^3}-\frac{27}{32x^2}+\frac{81}{64x}-\frac{243}{64(2+3x)}\right)dx\\ &= -\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.00

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Mathics [A]
time = 1.76, size = 34, normalized size = 0.76

$$\frac{-4+8x-18x^2+54x^3+81x^4\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{2}{3}+x\right]\right)}{64x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(4 + 6*x)),x]')
```

```
[Out] (-4 + 8 x - 18 x ^ 2 + 54 x ^ 3 + 81 x ^ 4 (Log[x] - Log[2 / 3 + x])) / (64
 x ^ 4)
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.76

method	result	size
norman	$\frac{-\frac{1}{16}+\frac{1}{8}x-\frac{9}{32}x^2+\frac{27}{32}x^3}{x^4}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	33
risch	$\frac{-\frac{1}{16}+\frac{1}{8}x-\frac{9}{32}x^2+\frac{27}{32}x^3}{x^4}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	33
default	$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	34
meijerg	$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}-\frac{81\ln(2)}{64}+\frac{81\ln(3)}{64}-\frac{81\ln(1+\frac{3x}{2})}{64}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(2+3*x)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.73

$$\frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4}-\frac{81}{64}\,\log\left(3\,x+2\right)+\frac{81}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(3*x + 2) + 81/64*log(x)
```

Fricas [A]
time = 0.31, size = 38, normalized size = 0.84

$$-\frac{81\,x^4\log\left(3\,x+2\right)-81\,x^4\log\left(x\right)-54\,x^3+18\,x^2-8\,x+4}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/64*(81*x^4*log(3*x + 2) - 81*x^4*log(x) - 54*x^3 + 18*x^2 - 8*x + 4)/x^4
```

Sympy [A]
time = 0.07, size = 36, normalized size = 0.80

$$\frac{81\log{(x)}}{64}-\frac{81\log{\left(x+\frac{2}{3}\right)}}{64}+\frac{27x^3-9x^2+4x-2}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x),x)
```

```
[Out] 81*log(x)/64 - 81*log(x + 2/3)/64 + (27*x**3 - 9*x**2 + 4*x - 2)/(32*x**4)
```

Giac [A]
time = 0.00, size = 43, normalized size = 0.96

$$-\frac{81}{64}\ln|3x+2|+\frac{81}{64}\ln|x|+\frac{\frac{1}{256}\left(216x^3-72x^2+32x-16\right)}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x)
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(abs(3*x + 2)) + 81/64*log(a
bs(x))
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.62

$$\frac{\frac{27\,x^3}{32}-\frac{9\,x^2}{32}+\frac{x}{8}-\frac{1}{16}}{x^4}-\frac{81\,\mathrm{atanh}\left(3\,x+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)),x)
```

```
[Out] (x/8 - (9*x^2)/32 + (27*x^3)/32 - 1/16)/x^4 - (81*atanh(3*x + 1))/32
```

3.261 $\int \frac{1}{x(4+6x)^2} dx$

Optimal. Leaf size=28

$$\frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)$$

```
[Out] 1/8/(2+3*x)+1/16*ln(x)-1/16*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{46\}$

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^2),x]
```

```
[Out] 1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^2}\,dx &= \int \left(\frac{1}{16x}-\frac{3}{8(2+3x)^2}-\frac{3}{16(2+3x)}\right) dx\\ &= \frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{16}\left(\frac{2}{2+3x}+\log(-6x)-\log(4+6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^2),x]
[Out] (2/(2 + 3*x) + Log[-6*x] - Log[4 + 6*x])/16
```

Mathics [A]
time = 1.67, size = 26, normalized size = 0.93

$$\frac{2+(2+3x)\left(\text{Log}\left[x\right]-\text{Log}\left[\frac{2}{3}+x\right]\right)}{32+48x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(4 + 6*x)^2),x]')
[Out] (2 + (2 + 3 x) (Log[x] - Log[2 / 3 + x])) / (16 (2 + 3 x))
```

Maple [A]
time = 0.09, size = 23, normalized size = 0.82

method	result	size
risch	$\frac{1}{16+24x}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	21
default	$\frac{1}{16+24x}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	23
norman	$-\frac{3x}{16(2+3x)}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	24
meijerg	$\frac{1}{16}+\frac{\ln(x)}{16}-\frac{\ln(2)}{16}+\frac{\ln(3)}{16}-\frac{3x}{16(2+3x)}-\frac{\ln(1+\frac{3x}{2})}{16}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8/(2+3*x)+1/16*ln(x)-1/16*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 22, normalized size = 0.79

$$\frac{1}{8\,(3\,x+2)}-\frac{1}{16}\,\log\left(3\,x+2\right)+\frac{1}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="maxima")
[Out] 1/8/(3*x + 2) - 1/16*log(3*x + 2) + 1/16*log(x)
```

Fricas [A]
time = 0.31, size = 32, normalized size = 1.14

$$-\frac{(3\,x+2)\log\left(3\,x+2\right)-(3\,x+2)\log\left(x\right)-2}{16\,(3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*((3*x + 2)*log(3*x + 2) - (3*x + 2)*log(x) - 2)/(3*x + 2)
```

Sympy [A]
time = 0.06, size = 19, normalized size = 0.68

$$\frac{\log(x)}{16} - \frac{\log\left(x + \frac{2}{3}\right)}{16} + \frac{1}{24x + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**2,x)
```

```
[Out] log(x)/16 - log(x + 2/3)/16 + 1/(24*x + 16)
```

Giac [A]
time = 0.00, size = 29, normalized size = 1.04

$$\frac{\ln|x|}{16} - \frac{\ln|3x + 2|}{16} + \frac{\frac{1}{16} \cdot 2}{3x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x)
```

```
[Out] 1/8/(3*x + 2) - 1/16*log(abs(3*x + 2)) + 1/16*log(abs(x))
```

Mupad [B]
time = 0.06, size = 20, normalized size = 0.71

$$\frac{1}{8\,(3\,x+2)} - \frac{\ln\left(\frac{6\,x+4}{x}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^2),x)
```

```
[Out] 1/(8*(3*x + 2)) - log((6*x + 4)/x)/16
```

3.262 $\int \frac{1}{x^2(4+6x)^2} dx$

Optimal. Leaf size=35

$$-\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)$$

```
[Out] -1/16/x-3/16/(2+3*x)-3/16*ln(x)+3/16*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] -1/16*1/x - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^2}-\frac{3}{16x}+\frac{9}{16(2+3x)^2}+\frac{9}{16(2+3x)}\right)dx\\ &= -\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{1}{x}-\frac{3}{2+3x}-3\log(x)+3\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^2),x]
[Out] (-x^(-1) - 3/(2 + 3*x) - 3*Log[x] + 3*Log[2 + 3*x])/16
```

Mathics [A]
time = 1.79, size = 34, normalized size = 0.97

$$\frac{-2-6x+3x\,(2+3x)\left(\mathrm{Log}\left[\frac{2}{3}+x\right]-\mathrm{Log}\,[x]\right)}{16x\,(2+3x)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(4 + 6*x)^2),x]')
[Out] (-2 - 6 x + 3 x (2 + 3 x) (Log[2 / 3 + x] - Log[x])) / (16 x (2 + 3 x))
```

Maple [A]
time = 0.10, size = 28, normalized size = 0.80

method	result	size
default	$-\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	28
risch	$\frac{-\frac{3x}{8}-\frac{1}{8}}{x(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	31
norman	$\frac{-\frac{1}{8}+\frac{9x^2}{16}}{x(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	32
meijerg	$-\frac{1}{16x}-\frac{3}{32}-\frac{3\ln(x)}{16}+\frac{3\ln(2)}{16}-\frac{3\ln(3)}{16}+\frac{27x}{64(\frac{9x}{2}+3)}+\frac{3\ln(1+\frac{3x}{2})}{16}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/16/x-3/16/(2+3*x)-3/16*ln(x)+3/16*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 31, normalized size = 0.89

$$-\frac{3\,x+1}{8\,(3\,x^2+2\,x)}+\frac{3}{16}\,\log\left(3\,x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="maxima")
[Out] -1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(3*x + 2) - 3/16*log(x)
```

Fricas [A]
time = 0.54, size = 48, normalized size = 1.37

$$\frac{3\,(3\,x^2+2\,x)\log\left(3\,x+2\right)-3\,(3\,x^2+2\,x)\log\left(x\right)-6\,x-2}{16\,(3\,x^2+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(3*x^2 + 2*x)*log(3*x + 2) - 3*(3*x^2 + 2*x)*log(x) - 6*x - 2)/(3*x
^2 + 2*x)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 0.89

$$\frac{-3x-1}{24x^2+16x}-\frac{3\log{(x)}}{16}+\frac{3\log{\left(x+\frac{2}{3}\right)}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**2,x)
```

```
[Out] (-3*x - 1)/(24*x**2 + 16*x) - 3*log(x)/16 + 3*log(x + 2/3)/16
```

Giac [A]
time = 0.00, size = 38, normalized size = 1.09

$$-\frac{3}{16}\ln|x|+\frac{3}{16}\ln|3x+2|-\frac{3x+1}{8(3x^2+2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x)
```

```
[Out] -1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(abs(3*x + 2)) - 3/16*log(abs(x))
```

Mupad [B]
time = 0.09, size = 34, normalized size = 0.97

$$\frac{3\ln\left(\frac{6\,x+4}{x}\right)}{16}-\frac{3}{4\,\left(6\,x+4\right)}-\frac{1}{4\,x\,\left(6\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^2),x)
```

```
[Out] (3*log((6*x + 4)/x))/16 - 3/(4*(6*x + 4)) - 1/(4*x*(6*x + 4))
```

3.263 $\int \frac{1}{x^3(4+6x)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)$$

```
[Out] -1/32/x^2+3/16/x+9/32/(2+3*x)+27/64*ln(x)-27/64*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] -1/32*1/x^2 + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3*
x])/64
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^3}-\frac{3}{16x^2}+\frac{27}{64x}-\frac{27}{32(2+3x)^2}-\frac{81}{64(2+3x)}\right)dx\\ &= -\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.86

$$\frac{1}{64}\left(-\frac{2}{x^2}+\frac{12}{x}+\frac{18}{2+3x}+27\log(x)-27\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^2),x]
[Out] (-2/x^2 + 12/x + 18/(2 + 3*x) + 27*Log[x] - 27*Log[2 + 3*x])/64
```

Mathics [A]
time = 1.83, size = 41, normalized size = 0.98

$$\frac{-4+18x+27x^2(2+3x)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{2}{3}+x\right]\right)+54x^2}{64x^2(2+3x)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(4 + 6*x)^2),x]')
[Out] (-4 + 18 x + 27 x ^ 2 (2 + 3 x) (Log[x] - Log[2 / 3 + x]) + 54 x ^ 2) / (64
 x ^ 2 (2 + 3 x))
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.79

method	result	size
default	$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	33
norman	$\frac{-\frac{1}{16}-\frac{81}{64}x^3+\frac{9}{32}x}{x^2(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	35
risch	$\frac{\frac{27}{32}x^2+\frac{9}{32}x-\frac{1}{16}}{x^2(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	36
meijerg	$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{64}+\frac{27\ln(x)}{64}-\frac{27\ln(2)}{64}+\frac{27\ln(3)}{64}-\frac{27x}{32(4+6x)}-\frac{27\ln(1+\frac{3x}{2})}{64}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/32/x^2+3/16/x+9/32/(2+3*x)+27/64*ln(x)-27/64*ln(2+3*x)
```

Maxima [A]
time = 0.25, size = 38, normalized size = 0.90

$$\frac{27\,x^2+9\,x-2}{32\,(3\,x^3+2\,x^2)}-\frac{27}{64}\,\log\left(3\,x+2\right)+\frac{27}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="maxima")
[Out] 1/32*(27*x^2 + 9*x - 2)/(3*x^3 + 2*x^2) - 27/64*log(3*x + 2) + 27/64*log(x)
```

Fricas [A]
time = 0.31, size = 59, normalized size = 1.40

$$\frac{54\,x^2-27\,(3\,x^3+2\,x^2)\log\left(3\,x+2\right)+27\,(3\,x^3+2\,x^2)\log\left(x\right)+18\,x-4}{64\,(3\,x^3+2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(54*x^2 - 27*(3*x^3 + 2*x^2)*log(3*x + 2) + 27*(3*x^3 + 2*x^2)*log(x)
+ 18*x - 4)/(3*x^3 + 2*x^2)
```

Sympy [A]
time = 0.07, size = 36, normalized size = 0.86

$$\frac{27\log\left(x\right)}{64}-\frac{27\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^{2}+9x-2}{96x^{3}+64x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**2,x)
```

```
[Out] 27*log(x)/64 - 27*log(x + 2/3)/64 + (27*x**2 + 9*x - 2)/(96*x**3 + 64*x**2)
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.02

$$-\frac{27}{64}\ln|3x+2|+\frac{27}{64}\ln|x|+\frac{\frac{1}{128}\left(108x^{2}+36x-8\right)}{x^{2}\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x)
```

```
[Out] 1/32*(27*x^2 + 9*x - 2)/((3*x + 2)*x^2) - 27/64*log(abs(3*x + 2)) + 27/64*l
og(abs(x))
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.74

$$\frac{\frac{9\,x^2}{32}+\frac{3\,x}{32}-\frac{1}{48}}{x^3+\frac{2\,x^2}{3}}-\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^2),x)
```

```
[Out] ((3*x)/32 + (9*x^2)/32 - 1/48)/((2*x^2)/3 + x^3) - (27*atanh(3*x + 1))/32
```

3.264 $\int \frac{1}{x^4(4+6x)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

```
[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(2+3*x)-27/32*ln(x)+27/32*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] -1/48*1/x^3 + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32 +
 (27*Log[2 + 3*x])/32
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^4}-\frac{3}{16x^3}+\frac{27}{64x^2}-\frac{27}{32x}+\frac{81}{64(2+3x)^2}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.90

$$\frac{1}{192}\left(-\frac{4\left(2-6x+27x^2+81x^3\right)}{x^3(2+3x)}-162\log(x)+162\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] ((-4*(2 - 6*x + 27*x^2 + 81*x^3))/(x^3*(2 + 3*x)) - 162*Log[x] + 162*Log[2
+ 3*x])/192
```

Mathics [A]
time = 1.87, size = 46, normalized size = 0.94

$$\frac{-4+12x-54x^2-162x^3+81x^3(2+3x)\left(\text{Log}\left[\frac{2}{3}+x\right]-\text{Log}[x]\right)}{96x^3(2+3x)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(4 + 6*x)^2),x]')
```

```
[Out] (-4 + 12 x - 54 x ^ 2 - 162 x ^ 3 + 81 x ^ 3 (2 + 3 x) (Log[2 / 3 + x] - Lo
g[x])) / (96 x ^ 3 (2 + 3 x))
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.78

method	result	size
default	$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	38
norman	$\frac{-\frac{1}{24}+\frac{81}{32}x^4+\frac{1}{8}x-\frac{9}{16}x^2}{x^3(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	40
risch	$\frac{-\frac{27}{16}x^3-\frac{9}{16}x^2+\frac{1}{8}x-\frac{1}{24}}{x^3(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	41
meijerg	$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{128}-\frac{27\ln(x)}{32}+\frac{27\ln(2)}{32}-\frac{27\ln(3)}{32}+\frac{405x}{256\left(5+\frac{15x}{2}\right)}+\frac{27\ln\left(1+\frac{3x}{2}\right)}{32}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(2+3*x)-27/32*ln(x)+27/32*ln(2+3*x)
```

Maxima [A]
time = 0.25, size = 43, normalized size = 0.88

$$-\frac{81\,x^3+27\,x^2-6\,x+2}{48\,(3\,x^4+2\,x^3)}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(81*x^3 + 27*x^2 - 6*x + 2)/(3*x^4 + 2*x^3) + 27/32*log(3*x + 2) - 27
/32*log(x)
```

Fricas [A]
time = 0.31, size = 64, normalized size = 1.31

$$-\frac{162\,x^3+54\,x^2-81\,(3\,x^4+2\,x^3)\log(3\,x+2)+81\,(3\,x^4+2\,x^3)\log(x)-12\,x+4}{96\,(3\,x^4+2\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(162*x^3 + 54*x^2 - 81*(3*x^4 + 2*x^3)*log(3*x + 2) + 81*(3*x^4 + 2*x
^3)*log(x) - 12*x + 4)/(3*x^4 + 2*x^3)
```

Sympy [A]
time = 0.07, size = 41, normalized size = 0.84

$$-\frac{27\log(x)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-81x^3-27x^2+6x-2}{144x^4+96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**2,x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-81*x**3 - 27*x**2 + 6*x - 2)/(144*x*
*4 + 96*x**3)
```

Giac [A]
time = 0.00, size = 50, normalized size = 1.02

$$\frac{27}{32}\ln|3x+2|-\frac{27}{32}\ln|x|+\frac{\frac{1}{192}\left(-324x^3-108x^2+24x-8\right)}{x^3\,(3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x)
```

```
[Out] -1/48*(81*x^3 + 27*x^2 - 6*x + 2)/((3*x + 2)*x^3) + 27/32*log(abs(3*x + 2))
 - 27/32*log(abs(x))
```

Mupad [B]
time = 0.09, size = 37, normalized size = 0.76

$$\frac{27\,\mathrm{atanh}\,(3\,x+1)}{16}-\frac{\frac{9\,x^3}{16}+\frac{3\,x^2}{16}-\frac{x}{24}+\frac{1}{72}}{x^4+\frac{2\,x^3}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^2),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((3*x^2)/16 - x/24 + (9*x^3)/16 + 1/72)/((2*x^3)/3
 + x^4)
```

3.265 $\int \frac{1}{x^5(4+6x)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)$$

[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(2+3*x)+405/256*ln(x)-405/256*ln(2+3*x)

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(4 + 6*x)^2),x]

[Out] -1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) + (405*Log[x])/256 - (405*Log[2 + 3*x])/256

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^5}-\frac{3}{16x^4}+\frac{27}{64x^3}-\frac{27}{32x^2}+\frac{405}{256x}-\frac{243}{128(2+3x)^2}-\frac{1215}{256(2+3x)}\right)dx\\ &= -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Mathics [A]
time = 1.91, size = 51, normalized size = 0.91

$$\frac{-8+20x-60x^2+270x^3+405x^4\left(2+3x\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{2}{3}+x\right]\right)+810x^4}{256x^4\left(2+3x\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(4 + 6*x)^2),x]')
```

```
[Out] (-8 + 20 x - 60 x ^ 2 + 270 x ^ 3 + 405 x ^ 4 (2 + 3 x) (Log[x] - Log[2 / 3
 + x]) + 810 x ^ 4) / (256 x ^ 4 (2 + 3 x))
```

Maple [A]
time = 0.11, size = 43, normalized size = 0.77

method	result	size
default	$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	43
norman	$\frac{-\frac{1}{32}-\frac{1215}{256}x^5+\frac{5}{64}x-\frac{15}{64}x^2+\frac{135}{128}x^3}{x^4(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	45
risch	$\frac{\frac{405}{128}x^4+\frac{135}{128}x^3-\frac{15}{64}x^2+\frac{5}{64}x-\frac{1}{32}}{x^4(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	46
meijerg	$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{256}+\frac{405\ln(x)}{256}-\frac{405\ln(2)}{256}+\frac{405\ln(3)}{256}-\frac{729x}{256(9x+6)}-\frac{405\ln\left(1+\frac{3x}{2}\right)}{256}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(2+3*x)+405/256*ln(x)-405/256*
ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 48, normalized size = 0.86

$$\frac{405\,x^4+135\,x^3-30\,x^2+10\,x-4}{128\,(3\,x^5+2\,x^4)}-\frac{405}{256}\,\log\left(3\,x+2\right)+\frac{405}{256}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/(3*x^5 + 2*x^4) - 405/256*log
(3*x + 2) + 405/256*log(x)
```

Fricas [A]
time = 0.31, size = 69, normalized size = 1.23

$$\frac{810\,x^4+270\,x^3-60\,x^2-405\,(3\,x^5+2\,x^4)\log\left(3\,x+2\right)+405\,(3\,x^5+2\,x^4)\log\left(x\right)+20\,x-8}{256\,(3\,x^5+2\,x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/256*(810*x^4 + 270*x^3 - 60*x^2 - 405*(3*x^5 + 2*x^4)*log(3*x + 2) + 405*
(3*x^5 + 2*x^4)*log(x) + 20*x - 8)/(3*x^5 + 2*x^4)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 0.82

$$\frac{405\log\left(x\right)}{256}-\frac{405\log\left(x+\frac{2}{3}\right)}{256}+\frac{405x^4+135x^3-30x^2+10x-4}{384x^5+256x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**2,x)
```

```
[Out] 405*log(x)/256 - 405*log(x + 2/3)/256 + (405*x**4 + 135*x**3 - 30*x**2 + 10
*x - 4)/(384*x**5 + 256*x**4)
```

Giac [A]
time = 0.00, size = 54, normalized size = 0.96

$$-\frac{405}{256}\ln|3x+2|+\frac{405}{256}\ln|x|+\frac{\frac{1}{1024}\left(3240x^4+1080x^3-240x^2+80x-32\right)}{x^4\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x)
```

```
[Out] 1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/((3*x + 2)*x^4) - 405/256*log
(abs(3*x + 2)) + 405/256*log(abs(x))
```

Mupad [B]
time = 0.09, size = 41, normalized size = 0.73

$$\frac{\frac{135\,x^4}{128}+\frac{45\,x^3}{128}-\frac{5\,x^2}{64}+\frac{5\,x}{192}-\frac{1}{96}}{x^5+\frac{2\,x^4}{3}}-\frac{405\,\mathrm{atanh}\left(3\,x+1\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^2),x)
```

```
[Out] ((5*x)/192 - (5*x^2)/64 + (45*x^3)/128 + (135*x^4)/128 - 1/96)/((2*x^4)/3 +
 x^5) - (405*atanh(3*x + 1))/128
```

3.266 $\int \frac{1}{x(4+6x)^3}\,dx$

Optimal. Leaf size=39

$$\frac{1}{32(2+3x)^2}+\frac{1}{32(2+3x)}+\frac{\log(x)}{64}-\frac{1}{64}\log(2+3x)$$

```
[Out] 1/32/(2+3*x)^2+1/32/(2+3*x)+1/64*ln(x)-1/64*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^3),x]
```

```
[Out] 1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^3}\,dx &= \int \left(\frac{1}{64x}-\frac{3}{16(2+3x)^3}-\frac{3}{32(2+3x)^2}-\frac{3}{64(2+3x)}\right)dx\\ &= \frac{1}{32(2+3x)^2}+\frac{1}{32(2+3x)}+\frac{\log(x)}{64}-\frac{1}{64}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.74

$$\frac{1}{64}\left(\frac{6(1+x)}{(2+3x)^2}+\log(-6x)-\log(4+6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^3),x]
[Out] ((6*(1 + x))/(2 + 3*x)^2 + Log[-6*x] - Log[4 + 6*x])/64
```

Mathics [A]
time = 1.78, size = 39, normalized size = 1.00

$$\frac{6+6x+(4+12x+9x^2)\left(\text{Log}[x]-\text{Log}\left[\frac{2}{3}+x\right]\right)}{256+768x+576x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^1*(4 + 6*x)^3),x]')
[Out] (6 + 6 x + (4 + 12 x + 9 x ^ 2) (Log[x] - Log[2 / 3 + x])) / (64 (4 + 12 x
+ 9 x ^ 2))
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.82

method	result	size
risch	$\frac{\frac{3x}{32}+\frac{3}{32}}{(2+3x)^2}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	28
norman	$\frac{-\frac{3}{16}x-\frac{27}{128}x^2}{(2+3x)^2}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	31
default	$\frac{1}{32(2+3x)^2}+\frac{1}{64+96x}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	32
meijerg	$\frac{3}{128}+\frac{\ln(x)}{64}-\frac{\ln(2)}{64}+\frac{\ln(3)}{64}-\frac{3x\left(\frac{9x}{2}+4\right)}{256\left(1+\frac{3x}{2}\right)^2}-\frac{\ln\left(1+\frac{3x}{2}\right)}{64}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/32/(2+3*x)^2+1/32/(2+3*x)+1/64*ln(x)-1/64*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 30, normalized size = 0.77

$$\frac{3(x+1)}{32(9x^2+12x+4)}-\frac{1}{64}\log(3x+2)+\frac{1}{64}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="maxima")
[Out] 3/32*(x + 1)/(9*x^2 + 12*x + 4) - 1/64*log(3*x + 2) + 1/64*log(x)
```

Fricas [A]
time = 0.31, size = 50, normalized size = 1.28

$$-\frac{(9x^2+12x+4)\log(3x+2)-(9x^2+12x+4)\log(x)-6x-6}{64(9x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/64*((9*x^2 + 12*x + 4)*log(3*x + 2) - (9*x^2 + 12*x + 4)*log(x) - 6*x -
6)/(9*x^2 + 12*x + 4)
```

Sympy [A]
time = 0.07, size = 27, normalized size = 0.69

$$\frac{3x+3}{288x^2+384x+128}+\frac{\log\left(x\right)}{64}-\frac{\log\left(x+\frac{2}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**3,x)
```

```
[Out] (3*x + 3)/(288*x**2 + 384*x + 128) + log(x)/64 - log(x + 2/3)/64
```

Giac [A]
time = 0.00, size = 35, normalized size = 0.90

$$\frac{\ln|x|}{64}-\frac{\ln|3x+2|}{64}+\frac{\frac{1}{128}\left(12x+12\right)}{\left(3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x)
```

```
[Out] 3/32*(x + 1)/(3*x + 2)^2 - 1/64*log(abs(3*x + 2)) + 1/64*log(abs(x))
```

Mupad [B]
time = 0.13, size = 29, normalized size = 0.74

$$\frac{1}{32\,\left(3\,x+2\right)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{64}+\frac{1}{8\,{\left(6\,x+4\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^3),x)
```

```
[Out] 1/(32*(3*x + 2)) - log((6*x + 4)/x)/64 + 1/(8*(6*x + 4)^2)
```

3.267 $\int \frac{1}{x^2(4+6x)^3} dx$

Optimal. Leaf size=46

$$-\frac{1}{64x} - \frac{3}{64(2+3x)^2} - \frac{3}{32(2+3x)} - \frac{9\log(x)}{128} + \frac{9}{128}\log(2+3x)$$

```
[Out] -1/64/x-3/64/(2+3*x)^2-3/32/(2+3*x)-9/128*ln(x)+9/128*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{1}{64x} - \frac{3}{32(3x+2)} - \frac{3}{64(3x+2)^2} - \frac{9\log(x)}{128} + \frac{9}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] -1/64*1/x - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Log
[2 + 3*x])/128
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^2} - \frac{9}{128x} + \frac{9}{32(2+3x)^3} + \frac{9}{32(2+3x)^2} + \frac{27}{128(2+3x)}\right) dx \\ &= -\frac{1}{64x} - \frac{3}{64(2+3x)^2} - \frac{3}{32(2+3x)} - \frac{9\log(x)}{128} + \frac{9}{128}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.85

$$\frac{1}{128}\left(-\frac{2\left(4+27x+27x^2\right)}{x(2+3x)^2} - 9\log(x) + 9\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^3),x]
[Out] ((-2*(4 + 27*x + 27*x^2))/(x*(2 + 3*x)^2) - 9*Log[x] + 9*Log[2 + 3*x])/128
```

Mathics [A]
time = 1.86, size = 49, normalized size = 1.07

$$\frac{-8-54x+9x\left(4+12x+9x^2\right)\left(\operatorname{Log}\left[\frac{2}{3}+x\right]-\operatorname{Log}[x]\right)-54x^2}{128x\left(4+12x+9x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(4 + 6*x)^3),x]')
[Out] (-8 - 54 x + 9 x (4 + 12 x + 9 x ^ 2) (Log[2 / 3 + x] - Log[x]) - 54 x ^ 2)
 / (128 x (4 + 12 x + 9 x ^ 2))
```

Maple [A]
time = 0.12, size = 37, normalized size = 0.80

method	result	size
risch	$\frac{-\frac{27}{64}x^2-\frac{27}{64}x-\frac{1}{16}}{x(2+3x)^2}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	36
default	$-\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	37
norman	$\frac{-\frac{1}{16}+\frac{27}{32}x^2+\frac{243}{256}x^3}{x(2+3x)^2}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	37
meijerg	$-\frac{1}{64x}-\frac{15}{256}-\frac{9\ln(x)}{128}+\frac{9\ln(2)}{128}-\frac{9\ln(3)}{128}+\frac{9x\left(\frac{15x}{2}+6\right)}{512\left(1+\frac{3x}{2}\right)^2}+\frac{9\ln\left(1+\frac{3x}{2}\right)}{128}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/64/x-3/64/(2+3*x)^2-3/32/(2+3*x)-9/128*ln(x)+9/128*ln(2+3*x)
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.89

$$-\frac{27\,x^2+27\,x+4}{64\left(9\,x^3+12\,x^2+4\,x\right)}+\frac{9}{128}\,\log\left(3\,x+2\right)-\frac{9}{128}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="maxima")
[Out] -1/64*(27*x^2 + 27*x + 4)/(9*x^3 + 12*x^2 + 4*x) + 9/128*log(3*x + 2) - 9/1
28*log(x)
```

Fricas [A]
time = 0.31, size = 68, normalized size = 1.48

$$-\frac{54\,x^{2}-9\,(9\,x^{3}+12\,x^{2}+4\,x)\log\left(3\,x+2\right)+9\,(9\,x^{3}+12\,x^{2}+4\,x)\log\left(x\right)+54\,x+8}{128\,(9\,x^{3}+12\,x^{2}+4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(54*x^2 - 9*(9*x^3 + 12*x^2 + 4*x)*log(3*x + 2) + 9*(9*x^3 + 12*x^2
+ 4*x)*log(x) + 54*x + 8)/(9*x^3 + 12*x^2 + 4*x)
```

Sympy [A]
time = 0.08, size = 41, normalized size = 0.89

$$\frac{-27x^{2}-27x-4}{576x^{3}+768x^{2}+256x}-\frac{9\log\left(x\right)}{128}+\frac{9\log\left(x+\frac{2}{3}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**3,x)
```

```
[Out] (-27*x**2 - 27*x - 4)/(576*x**3 + 768*x**2 + 256*x) - 9*log(x)/128 + 9*log(
x + 2/3)/128
```

Giac [A]
time = 0.00, size = 45, normalized size = 0.98

$$-\frac{9}{128}\ln|x|+\frac{9}{128}\ln|3x+2|+\frac{\frac{1}{256}\left(-108x^{2}-108x-16\right)}{x\left(3x+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x)
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/((3*x + 2)^2*x) + 9/128*log(abs(3*x + 2)) - 9/128
*log(abs(x))
```

Mupad [B]
time = 0.09, size = 35, normalized size = 0.76

$$\frac{9\,\mathrm{atanh}\left(3\,x+1\right)}{64}-\frac{\frac{3\,x^{2}}{64}+\frac{3\,x}{64}+\frac{1}{144}}{x^{3}+\frac{4\,x^{2}}{3}+\frac{4\,x}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^3),x)
```

```
[Out] (9*atanh(3*x + 1))/64 - ((3*x)/64 + (3*x^2)/64 + 1/144)/((4*x)/9 + (4*x^2)/
3 + x^3)
```

3.268 $\int \frac{1}{x^3(4+6x)^3} \, dx$

Optimal. Leaf size=53

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)$$

```
[Out] -1/128/x^2+9/128/x+9/128/(2+3*x)^2+27/128/(2+3*x)+27/128*ln(x)-27/128*ln(2+
3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] -1/128*1/x^2 + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*L
og[x])/128 - (27*Log[2 + 3*x])/128
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^3}-\frac{9}{128x^2}+\frac{27}{128x}-\frac{27}{64(2+3x)^3}-\frac{81}{128(2+3x)^2}-\frac{81}{128(2+3x)}\right)dx\\ &= -\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.83

$$\frac{1}{128}\left(\frac{2\left(-2+12x+81x^2+81x^3\right)}{x^2(2+3x)^2}+27\log(x)-27\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-2 + 12*x + 81*x^2 + 81*x^3))/(x^2*(2 + 3*x)^2) + 27*Log[x] - 27*Log[2
 + 3*x])/128
```

Mathics [A]
time = 1.91, size = 56, normalized size = 1.06

$$\frac{-4+24x+27x^2\left(4+12x+9x^2\right)\left(\mathrm{Log}\,[x]-\mathrm{Log}\left[\frac{2}{3}+x\right]\right)+162x^2+162x^3}{128x^2\left(4+12x+9x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^3*(4 + 6*x)^3),x]')
```

```
[Out] (-4 + 24 x + 27 x ^ 2 (4 + 12 x + 9 x ^ 2) (Log[x] - Log[2 / 3 + x]) + 162
x ^ 2 + 162 x ^ 3) / (128 x ^ 2 (4 + 12 x + 9 x ^ 2))
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.79

method	result	size
norman	$\frac{-\frac{1}{32}-\frac{81}{32}x^3-\frac{729}{256}x^4+\frac{3}{16}x}{x^2(2+3x)^2}+\frac{27\ln(x)}{128}-\frac{27\ln(2+3x)}{128}$	40
risch	$\frac{\frac{81}{64}x^3+\frac{81}{64}x^2+\frac{3}{16}x-\frac{1}{32}}{x^2(2+3x)^2}+\frac{27\ln(x)}{128}-\frac{27\ln(2+3x)}{128}$	41
default	$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\ln(x)}{128}-\frac{27\ln(2+3x)}{128}$	42
meijerg	$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{63}{512}+\frac{27\ln(x)}{128}-\frac{27\ln(2)}{128}+\frac{27\ln(3)}{128}-\frac{27x\left(\frac{21x}{2}+8\right)}{1024\left(1+\frac{3x}{2}\right)^2}-\frac{27\ln\left(1+\frac{3x}{2}\right)}{128}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/x^2+9/128/x+9/128/(2+3*x)^2+27/128/(2+3*x)+27/128*ln(x)-27/128*ln(2+
3*x)
```

Maxima [A]
time = 0.25, size = 48, normalized size = 0.91

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\left(9\,x^4+12\,x^3+4\,x^2\right)}-\frac{27}{128}\,\log\left(3\,x+2\right)+\frac{27}{128}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(9*x^4 + 12*x^3 + 4*x^2) - 27/128*log(3*x
 + 2) + 27/128*log(x)
```

Fricas [A]
time = 0.31, size = 79, normalized size = 1.49

$$\frac{162\,x^3+162\,x^2-27\,(9\,x^4+12\,x^3+4\,x^2)\log\left(3\,x+2\right)+27\,(9\,x^4+12\,x^3+4\,x^2)\log\left(x\right)+24\,x-4}{128\,(9\,x^4+12\,x^3+4\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(162*x^3 + 162*x^2 - 27*(9*x^4 + 12*x^3 + 4*x^2)*log(3*x + 2) + 27*(9
*x^4 + 12*x^3 + 4*x^2)*log(x) + 24*x - 4)/(9*x^4 + 12*x^3 + 4*x^2)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 0.87

$$\frac{27\log\left(x\right)}{128}-\frac{27\log\left(x+\frac{2}{3}\right)}{128}+\frac{81x^3+81x^2+12x-2}{576x^4+768x^3+256x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**3,x)
```

```
[Out] 27*log(x)/128 - 27*log(x + 2/3)/128 + (81*x**3 + 81*x**2 + 12*x - 2)/(576*x
**4 + 768*x**3 + 256*x**2)
```

Giac [A]
time = 0.00, size = 53, normalized size = 1.00

$$\frac{27}{128}\ln|x|-\frac{27}{128}\ln|3x+2|-\frac{-81x^3-81x^2-12x+2}{64\left(3x^2+2x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x)
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(3*x^2 + 2*x)^2 - 27/128*log(abs(3*x + 2)
) + 27/128*log(abs(x))
```

Mupad [B]
time = 0.09, size = 41, normalized size = 0.77

$$\frac{\frac{9\,x^3}{64}+\frac{9\,x^2}{64}+\frac{x}{48}-\frac{1}{288}}{x^4+\frac{4\,x^3}{3}+\frac{4\,x^2}{9}}-\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^3),x)
```

```
[Out] (x/48 + (9*x^2)/64 + (9*x^3)/64 - 1/288)/((4*x^2)/9 + (4*x^3)/3 + x^4) - (2
7*atanh(3*x + 1))/64
```

3.269 $\int \frac{1}{x^4(4+6x)^3}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3x)$$

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(2+3*x)^2-27/64/(2+3*x)-135/256*ln(x)+
135/256*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] -1/192*1/x^3 + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
+ 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^4}-\frac{9}{128x^3}+\frac{27}{128x^2}-\frac{135}{256x}+\frac{81}{128(2+3x)^3}+\frac{81}{64(2+3x)^2}+\frac{405}{256(2+3x)}\right) d\\ &= -\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.82

$$\frac{1}{768}\left(-\frac{2\left(8-30x+180x^2+1215x^3+1215x^4\right)}{x^3(2+3x)^2}-405\log(x)+405\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(8 - 30*x + 180*x^2 + 1215*x^3 + 1215*x^4))/(x^3*(2 + 3*x)^2) - 405*Lo
g[x] + 405*Log[2 + 3*x])/768
```

Mathics [A]
time = 1.95, size = 61, normalized size = 1.02

$$\frac{-16+60x-360x^2-2430x^3+405x^3\left(4+12x+9x^2\right)\left(\text{Log}\left[\frac{2}{3}+x\right]-\text{Log}\left[x\right]\right)-2430x^4}{768x^3\left(4+12x+9x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^4*(4 + 6*x)^3),x]')
```

```
[Out] (-16 + 60 x - 360 x ^ 2 - 2430 x ^ 3 + 405 x ^ 3 (4 + 12 x + 9 x ^ 2) (Log[
2 / 3 + x] - Log[x]) - 2430 x ^ 4) / (768 x ^ 3 (4 + 12 x + 9 x ^ 2))
```

Maple [A]
time = 0.10, size = 47, normalized size = 0.78

method	result	size
norman	$\frac{-\frac{1}{48}+\frac{405}{64}x^4+\frac{3645}{512}x^5+\frac{5}{64}x-\frac{15}{32}x^2}{x^3(2+3x)^2}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$	45
risch	$\frac{-\frac{405}{128}x^4-\frac{405}{128}x^3-\frac{15}{32}x^2+\frac{5}{64}x-\frac{1}{48}}{x^3(2+3x)^2}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$	46
default	$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$	47
meijerg	$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{243}{1024}-\frac{135\ln(x)}{256}+\frac{135\ln(2)}{256}-\frac{135\ln(3)}{256}+\frac{81x\left(\frac{27x}{2}+10\right)}{2048\left(1+\frac{3x}{2}\right)^2}+\frac{135\ln\left(1+\frac{3x}{2}\right)}{256}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(2+3*x)^2-27/64/(2+3*x)-135/256*ln(x)+
135/256*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 53, normalized size = 0.88

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(9\,x^5+12\,x^4+4\,x^3)}+\frac{135}{256}\,\log\left(3\,x+2\right)-\frac{135}{256}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/(9*x^5 + 12*x^4 + 4*x^3)
+ 135/256*log(3*x + 2) - 135/256*log(x)
```

Fricas [A]
time = 0.31, size = 84, normalized size = 1.40

$$-\frac{2430\,x^4+2430\,x^3+360\,x^2-405\,(9\,x^5+12\,x^4+4\,x^3)\log{(3\,x+2)}+405\,(9\,x^5+12\,x^4+4\,x^3)\log{(x)}-60\,x+16}{768\,(9\,x^5+12\,x^4+4\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/768*(2430*x^4 + 2430*x^3 + 360*x^2 - 405*(9*x^5 + 12*x^4 + 4*x^3)*log(3*
x + 2) + 405*(9*x^5 + 12*x^4 + 4*x^3)*log(x) - 60*x + 16)/(9*x^5 + 12*x^4 +
 4*x^3)
```

Sympy [A]
time = 0.09, size = 51, normalized size = 0.85

$$-\frac{135\log{(x)}}{256}+\frac{135\log{\left(x+\frac{2}{3}\right)}}{256}+\frac{-1215x^4-1215x^3-180x^2+30x-8}{3456x^5+4608x^4+1536x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**3,x)
```

```
[Out] -135*log(x)/256 + 135*log(x + 2/3)/256 + (-1215*x**4 - 1215*x**3 - 180*x**2
 + 30*x - 8)/(3456*x**5 + 4608*x**4 + 1536*x**3)
```

Giac [A]
time = 0.00, size = 58, normalized size = 0.97

$$\frac{135}{256}\ln|3x+2|-\frac{135}{256}\ln|x|+\frac{\frac{1}{768}\left(-2430x^4-2430x^3-360x^2+60x-16\right)}{x^3\left(3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x)
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/((3*x + 2)^2*x^3) + 135/2
56*log(abs(3*x + 2)) - 135/256*log(abs(x))
```

Mupad [B]
time = 0.05, size = 47, normalized size = 0.78

$$\frac{135\,\mathrm{atanh}\left(3\,x+1\right)}{128}-\frac{\frac{45\,x^4}{128}+\frac{45\,x^3}{128}+\frac{5\,x^2}{96}-\frac{5\,x}{576}+\frac{1}{432}}{x^5+\frac{4\,x^4}{3}+\frac{4\,x^3}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^3),x)
```

```
[Out] (135*atanh(3*x + 1))/128 - ((5*x^2)/96 - (5*x)/576 + (45*x^3)/128 + (45*x^4
)/128 + 1/432)/((4*x^3)/9 + (4*x^4)/3 + x^5)
```

3.270 $\int \frac{1}{x^5(4+6x)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{1215\log(2+3x)}{1024}$$

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(2+3*x)^2+405/512/(2+3*x)+
1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] -1/256*1/x^4 + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x)
^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^5}-\frac{9}{128x^4}+\frac{27}{128x^3}-\frac{135}{256x^2}+\frac{1215}{1024x}-\frac{243}{256(2+3x)^3}-\frac{1215}{512(2+3x)^2}-\frac{3}{1024}\right.\\ &= -\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{121}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.81

$$\frac{\frac{2(-8+24x-90x^2+540x^3+3645x^4+3645x^5)}{x^4(2+3x)^2}+1215\log(x)-1215\log(2+3x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-8 + 24*x - 90*x^2 + 540*x^3 + 3645*x^4 + 3645*x^5))/(x^4*(2 + 3*x)^2)
 + 1215*Log[x] - 1215*Log[2 + 3*x])/1024
```

Mathics [A]
time = 2.04, size = 66, normalized size = 0.99

$$\frac{-16+48x-180x^2+1080x^3+1215x^4\left(4+12x+9x^2\right)\left(\mathrm{Log}\left[x\right]-\mathrm{Log}\left[\frac{2}{3}+x\right]\right)+7290x^4+7290x^5}{1024x^4\left(4+12x+9x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^5*(4 + 6*x)^3),x]')
```

```
[Out] (-16 + 48 x - 180 x ^ 2 + 1080 x ^ 3 + 1215 x ^ 4 (4 + 12 x + 9 x ^ 2) (Log
[x] - Log[2 / 3 + x]) + 7290 x ^ 4 + 7290 x ^ 5) / (1024 x ^ 4 (4 + 12 x +
9 x ^ 2))
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.78

method	result
norman	$\frac{-\frac{1}{64}-\frac{3645}{256}x^5-\frac{32805}{2048}x^6+\frac{3}{64}x-\frac{45}{256}x^2+\frac{135}{128}x^3}{x^4(2+3x)^2}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
risch	$\frac{\frac{3645}{512}x^5+\frac{3645}{512}x^4+\frac{135}{128}x^3-\frac{45}{256}x^2+\frac{3}{64}x-\frac{1}{64}}{x^4(2+3x)^2}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
default	$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
meijerg	$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{891}{2048}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2)}{1024}+\frac{1215\ln(3)}{1024}-\frac{243x\left(\frac{33x}{2}+12\right)}{4096\left(1+\frac{3x}{2}\right)^2}-\frac{1215\ln\left(1+\frac{3x}{2}\right.}{1024}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(2+3*x)^2+405/512/(2+3*x)+
1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Maxima [A]
time = 0.24, size = 58, normalized size = 0.87

$$\frac{3645\,x^5+3645\,x^4+540\,x^3-90\,x^2+24\,x-8}{512\,\left(9\,x^6+12\,x^5+4\,x^4\right)}-\frac{1215}{1024}\,\log\left(3\,x+2\right)+\frac{1215}{1024}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/(9*x^6 + 12*x^5 +
 4*x^4) - 1215/1024*log(3*x + 2) + 1215/1024*log(x)
```

Fricas [A]
time = 0.32, size = 89, normalized size = 1.33

$$\frac{7290\,x^5+7290\,x^4+1080\,x^3-180\,x^2-1215\,(9\,x^6+12\,x^5+4\,x^4)\log(3\,x+2)+1215\,(9\,x^6+12\,x^5+4\,x^4)\log(x)+48\,x-16}{1024\,(9\,x^6+12\,x^5+4\,x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(7290*x^5 + 7290*x^4 + 1080*x^3 - 180*x^2 - 1215*(9*x^6 + 12*x^5 + 4
*x^4)*log(3*x + 2) + 1215*(9*x^6 + 12*x^5 + 4*x^4)*log(x) + 48*x - 16)/(9*x
^6 + 12*x^5 + 4*x^4)
```

Sympy [A]
time = 0.09, size = 56, normalized size = 0.84

$$\frac{1215\log(x)}{1024}-\frac{1215\log\left(x+\frac{2}{3}\right)}{1024}+\frac{3645x^5+3645x^4+540x^3-90x^2+24x-8}{4608x^6+6144x^5+2048x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**3,x)
```

```
[Out] 1215*log(x)/1024 - 1215*log(x + 2/3)/1024 + (3645*x**5 + 3645*x**4 + 540*x*
*3 - 90*x**2 + 24*x - 8)/(4608*x**6 + 6144*x**5 + 2048*x**4)
```

Giac [A]
time = 0.00, size = 61, normalized size = 0.91

$$-\frac{1215}{1024}\ln|3x+2|+\frac{1215}{1024}\ln|x|+\frac{\frac{1}{4096}(29160x^5+29160x^4+4320x^3-720x^2+192x-64)}{x^4\,(3x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x)
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/((3*x + 2)^2*x^4)
 - 1215/1024*log(abs(3*x + 2)) + 1215/1024*log(abs(x))
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.76

$$\frac{\frac{405\,x^5}{512}+\frac{405\,x^4}{512}+\frac{15\,x^3}{128}-\frac{5\,x^2}{256}+\frac{x}{192}-\frac{1}{576}}{x^6+\frac{4\,x^5}{3}+\frac{4\,x^4}{9}}-\frac{1215\,\mathrm{atanh}\,(3\,x+1)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^3),x)
```

```
[Out] (x/192 - (5*x^2)/256 + (15*x^3)/128 + (405*x^4)/512 + (405*x^5)/512 - 1/576
)/((4*x^4)/9 + (4*x^5)/3 + x^6) - (1215*atanh(3*x + 1))/512
```

3.271 $\int \frac{1}{2+2x}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2}\log(1+x)$$

```
[Out] 1/2*ln(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)^(-1),x]
```

```
[Out] Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+2x}\,dx = \frac{1}{2}\log(1+x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.25

$$\frac{1}{2}\log(2+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)^(-1),x]
```

```
[Out] Log[2 + 2*x]/2
```

Mathics [A]
time = 1.55, size = 8, normalized size = 1.00

$$\frac{\text{Log}\,[2+2x]}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(2 + 2*x),x]')
```

```
[Out] Log[2 + 2 x] / 2
```

Maple [A]
time = 0.08, size = 9, normalized size = 1.12

method	result	size
meijerg	$\frac{\ln(1+x)}{2}$	7
risch	$\frac{\ln(1+x)}{2}$	7
default	$\frac{\ln(2+2x)}{2}$	9
norman	$\frac{\ln(2+2x)}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(2+2*x)
```

Maxima [A]
time = 0.24, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1)
```

Fricas [A]
time = 0.30, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.88

$$\frac{\log\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x)
```

```
[Out] log(2*x + 2)/2
```

Giac [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\ln|x+1|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x)
```

```
[Out] 1/2*log(abs(x + 1))
```

Mupad [B]
time = 0.15, size = 6, normalized size = 0.75

$$\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 2),x)
```

```
[Out] log(x + 1)/2
```

3.272 $\int \frac{1}{4-6x}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{6}\log(2-3x)$$

```
[Out] -1/6*ln(2-3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$-\frac{1}{6}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*x)^(-1),x]
```

```
[Out] -1/6*Log[2 - 3*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{4-6x}\,dx = -\frac{1}{6}\log(2-3x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{6}\log(4-6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*x)^(-1),x]
```

```
[Out] -1/6*Log[4 - 6*x]
```

Mathics [A]
time = 1.57, size = 8, normalized size = 0.80

$$-\frac{\text{Log}\left[-4+6x\right]}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(4 - 6*x),x]')
[Out] -Log[-4 + 6 x] / 6
```

Maple [A]
time = 0.08, size = 9, normalized size = 0.90

method	result	size
default	$-\frac{\ln(4-6x)}{6}$	9
norman	$-\frac{\ln(6x-4)}{6}$	9
meijerg	$-\frac{\ln\left(1-\frac{3x}{2}\right)}{6}$	9
risch	$-\frac{\ln(-2+3x)}{6}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*x),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(4-6*x)
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.80

$$-\frac{1}{6}\log\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="maxima")
[Out] -1/6*log(3*x - 2)
```

Fricas [A]
time = 0.31, size = 8, normalized size = 0.80

$$-\frac{1}{6}\log\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="fricas")
[Out] -1/6*log(3*x - 2)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.80

$$-\frac{\log\left(6x-4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x)
```

```
[Out] -log(6*x - 4)/6
```

Giac [A]
time = 0.00, size = 11, normalized size = 1.10

$$-\frac{\ln|3x-2|}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x)
```

```
[Out] -1/6*log(abs(3*x - 2))
```

Mupad [B]
time = 0.08, size = 6, normalized size = 0.60

$$-\frac{\ln\left(x-\frac{2}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(6*x - 4),x)
```

```
[Out] -log(x - 2/3)/6
```

3.273 $\int \frac{1}{a+\sqrt{a}\,x}\,dx$

Optimal. Leaf size=14

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

```
[Out] ln(x+a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{31\}$

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] Log[Sqrt[a] + x]/Sqrt[a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{a}\,x}\,dx=\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.14

$$\frac{\log\left(a+\sqrt{a}\,x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[a]*x]/Sqrt[a]
```

Mathics [A]
time = 1.60, size = 12, normalized size = 0.86

$$\frac{\text{Log}\left[a+\sqrt{a}\,x\right]}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + Sqrt[a]*x),x]')
[Out] Log[a + Sqrt[a] x] / Sqrt[a]
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
default	$\frac{\ln\left(a+x\sqrt{a}\right)}{\sqrt{a}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*a^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(a+x*a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{\log\left(\sqrt{a}\,x+a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

Fricas [A]
time = 0.32, size = 10, normalized size = 0.71

$$\frac{\log\left(x+\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="fricas")
[Out] log(x + sqrt(a))/sqrt(a)
```

Sympy [**A**]
time = 0.03, size = 14, normalized size = 1.00

$$\frac{\log\left(\sqrt{a}\,x+a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a**(1/2)),x)
```

```
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

Giac [**A**]
time = 0.00, size = 18, normalized size = 1.29

$$\frac{\ln\left|\sqrt{a}\,x+a\right|}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x)
```

```
[Out] log(abs(sqrt(a)*x + a))/sqrt(a)
```

Mupad [**B**]
time = 0.11, size = 10, normalized size = 0.71

$$\frac{\ln\left(x+\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a^(1/2)*x),x)
```

```
[Out] log(x + a^(1/2))/a^(1/2)
```

3.274 $\int \frac{1}{a+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

```
[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{31\}$

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{-a}\,x}\,dx=\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Mathics [A]
time = 1.64, size = 16, normalized size = 0.80

$$\frac{\text{Log}\left[a+x\sqrt{-a}\right]}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + Sqrt[-a]*x),x]')
```

```
[Out] Log[a + x Sqrt[-a]] / Sqrt[-a]
```

Maple [A]
time = 0.09, size = 17, normalized size = 0.85

method	result	size
default	$\frac{\ln\left(a+x\sqrt{-a}\right)}{\sqrt{-a}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.30, size = 16, normalized size = 0.80

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + a)/sqrt(-a)
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.00

$$-\frac{\sqrt{-a}\,\log\left(x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(x - sqrt(-a))/a
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.85

$$\frac{\log\left(a + x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)**(1/2)),x)
```

```
[Out] log(a + x*sqrt(-a))/sqrt(-a)
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.05

$$-\frac{\sqrt{-a}\,\ln\left|x - \sqrt{-a}\right|}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x)
```

```
[Out] -sqrt(-a)*log(abs(x - sqrt(-a)))/a
```

Mupad [B]
time = 0.11, size = 16, normalized size = 0.80

$$\frac{\ln\left(x - \sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - (-a)^(1/2))/(-a)^(1/2)
```

3.275 $\int \frac{1}{a^2+\sqrt{-a}\, x} \, dx$

Optimal. Leaf size=22

$$\frac{\log \left(a^2+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

```
[Out] ln(a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {31}

$$\frac{\log \left(a^2+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^2+\sqrt{-a}\, x} \, dx = \frac{\log \left(a^2+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log \left(a^2+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Mathics [A]
time = 1.65, size = 18, normalized size = 0.82

$$\frac{\text{Log}\left[a^2 + x\sqrt{-a}\right]}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a^2 + Sqrt[-a]*x),x]')
[Out] Log[a ^ 2 + x Sqrt[-a]] / Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.86

method	result	size
default	$\frac{\ln\left(a^2+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.24, size = 18, normalized size = 0.82

$$\frac{\log\left(a^2 + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(a^2 + sqrt(-a)*x)/sqrt(-a)
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.95

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(-sqrt(-a)*a + x)/a
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.86

$$\frac{\log\left(a^2 + x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2 + x*sqrt(-a))/sqrt(-a)
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.05

$$-\frac{\sqrt{-a}\,\ln\left|x - a\sqrt{-a}\right|}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x)
```

```
[Out] -sqrt(-a)*log(abs(-sqrt(-a)*a + x))/a
```

Mupad [B]
time = 0.05, size = 14, normalized size = 0.64

$$\frac{\ln\left(x + (-a)^{3/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + (-a)^(3/2))/(-a)^(1/2)
```

3.276 $\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=22

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

```
[Out] ln(a^3+x*(-a)^(1/2))/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{31\}$

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx=\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Mathics [A]
time = 1.66, size = 18, normalized size = 0.82

$$\frac{\text{Log}\left[a^3 + x\sqrt{-a}\right]}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a^3 + Sqrt[-a]*x),x]')
```

```
[Out] Log[a ^ 3 + x Sqrt[-a]] / Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.86

method	result	size
default	$\frac{\ln\left(a^3+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a^3+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{\log\left(a^3 + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a^3 + sqrt(-a)*x)/sqrt(-a)
```

Fricas [A]
time = 0.32, size = 23, normalized size = 1.05

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a^2 + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(-sqrt(-a)*a^2 + x)/a
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.86

$$\frac{\log\left(a^3 + x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x*(-a)**(1/2)),x)
```

```
[Out] log(a**3 + x*sqrt(-a))/sqrt(-a)
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.14

$$-\frac{\sqrt{-a}\,\ln\left|x - a^2\sqrt{-a}\right|}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x)
```

```
[Out] -sqrt(-a)*log(abs(-sqrt(-a)*a^2 + x))/a
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.73

$$\frac{\ln\left(x - (-a)^{5/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + (-a)^(1/2)*x),x)
```

```
[Out] log(x - (-a)^(5/2))/(-a)^(1/2)
```

3.277 $\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

```
[Out] ln(1-(-a)^(3/2)*x)/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 - (-a)^(3/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx=\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{\log\left(1+\sqrt{-a}\,ax\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + Sqrt[-a]*a*x]/Sqrt[-a]
```

Mathics [A]
time = 1.67, size = 17, normalized size = 0.81

$$\frac{\text{Log}\left[1-x\left(-a\right)^{\frac{3}{2}}\right]}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(1/a + Sqrt[-a]*x),x]')
[Out] Log[1 - x (-a) ^ (3 / 2)] / Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.90

method	result	size
default	$\frac{\ln\left(\frac{1}{a}+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(1/a+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.86

$$\frac{\log\left(\sqrt{-a}\,x+\frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(-a)*x + 1/a)/sqrt(-a)
```

Fricas [A]
time = 0.31, size = 24, normalized size = 1.14

$$-\frac{\sqrt{-a}\,\log\left(a^{2}x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(a^2*x - sqrt(-a))/a
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.90

$$\frac{\log \left(ax\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)**(1/2)),x)
[Out] log(a*x*sqrt(-a) + 1)/sqrt(-a)
```

Giac [A]
time = 0.00, size = 28, normalized size = 1.33

$$-\frac{a\sqrt{-a}\,\ln\left|a^2x-\sqrt{-a}\right|}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x)
[Out] -sqrt(-a)*log(abs(a^2*x - sqrt(-a)))/a
```

Mupad [B]
time = 0.15, size = 16, normalized size = 0.76

$$\frac{\ln\left(x-\frac{1}{(-a)^{3/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a + (-a)^(1/2)*x),x)
[Out] log(x - 1/(-a)^(3/2))/(-a)^(1/2)
```

3.278 $\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

```
[Out] ln(1+(-a)^(5/2)*x)/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {31}

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-2) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + (-a)^(5/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx = \frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.10

$$\frac{\log\left(\frac{1}{a^2}+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-2) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^(-2) + Sqrt[-a]*x]/Sqrt[-a]
```

Mathics [A]
time = 1.63, size = 16, normalized size = 0.80

$$\frac{\operatorname{Log}\left[1+x(-a)^{\frac{5}{2}}\right]}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(1/a^2 + Sqrt[-a]*x),x]')
```

```
[Out] Log[1 + x (-a) ^ (5 / 2)] / Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.95

method	result	size
default	$\frac{\ln\left(\frac{1}{a^2}+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1/a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.24, size = 18, normalized size = 0.90

$$\frac{\log\left(\sqrt{-a}\,x+\frac{1}{a^2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a^2)/sqrt(-a)
```

Fricas [A]
time = 0.32, size = 24, normalized size = 1.20

$$-\frac{\sqrt{-a}\,\log\left(a^3x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^3*x - sqrt(-a))/a
```

Sympy [A]
time = 0.04, size = 20, normalized size = 1.00

$$\frac{\log\left(a^{2}x\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2*x*sqrt(-a) + 1)/sqrt(-a)
```

Giac [A]
time = 0.00, size = 30, normalized size = 1.50

$$-\frac{a^{2}\sqrt{-a}\ln\left|a^{3}x-\sqrt{-a}\right|}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x)
```

```
[Out] -sqrt(-a)*log(abs(a^3*x - sqrt(-a)))/a
```

Mupad [B]
time = 0.18, size = 14, normalized size = 0.70

$$\frac{\ln\left(x+\frac{1}{{(-a)}^{5/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + 1/(-a)^(5/2))/(-a)^(1/2)
```

3.279 $\int \frac{1}{x(1+bx)} dx$

Optimal. Leaf size=11

$$\log(x) - \log(1 + bx)$$

```
[Out] ln(x)-ln(b*x+1)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\log(x) - \log(bx + 1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x)),x]
```

```
[Out] Log[x] - Log[1 + b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+bx)}\,dx &= -\left(b\int \frac{1}{1+bx}\,dx\right) + \int \frac{1}{x}\,dx\\ &= \log(x) - \log(1+bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\log(x) - \log(1 + bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x)),x]
```

```
[Out] Log[x] - Log[1 + b*x]
```

Mathics [A]
time = 1.58, size = 11, normalized size = 1.00

$$\mathrm{Log}\,[x] - \mathrm{Log}\left[\frac{1}{b} + x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x*(1 + b*x)),x]')
```

```
[Out] Log[x] - Log[1 / b + x]
```

Maple [A]
time = 0.09, size = 12, normalized size = 1.09

method	result	size
default	$\ln(x) - \ln(bx + 1)$	12
norman	$\ln(x) - \ln(bx + 1)$	12
meijerg	$\ln(x) + \ln(b) - \ln(bx + 1)$	14
risch	$\ln(-x) - \ln(bx + 1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-ln(b*x+1)
```

Maxima [A]
time = 0.24, size = 11, normalized size = 1.00

$$-\log(bx + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="maxima")
```

```
[Out] -log(b*x + 1) + log(x)
```

Fricas [A]
time = 0.31, size = 11, normalized size = 1.00

$$-\log(bx + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="fricas")
```

```
[Out] -log(b*x + 1) + log(x)
```

Sympy [A]
time = 0.06, size = 8, normalized size = 0.73

$$\log(x) - \log\left(x + \frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x)
```

```
[Out] log(x) - log(x + 1/b)
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.09

$$\ln|x| - \ln|xb + 1|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x)
```

```
[Out] -log(abs(b*x + 1)) + log(abs(x))
```

Mupad [B]
time = 0.10, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}\left(2\,b\,x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + 1)),x)
```

```
[Out] -2*atanh(2*b*x + 1)
```

3.280 $\int \frac{1}{x(-1+bx)}\,dx$

Optimal. Leaf size=12

$$-\log(x)+\log(1-bx)$$

```
[Out] -ln(x)+ln(-b*x+1)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {36, 29, 31}

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(-1+bx)}\,dx &= b\int \frac{1}{-1+bx}\,dx-\int \frac{1}{x}\,dx\\ &= -\log(x)+\log(1-bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\log(x)+\log(1-bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

Mathics [A]
time = 1.57, size = 13, normalized size = 1.08

$$\text{Log}\left[-\frac{1}{b}+x\right]-\text{Log}\left[x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x*(-1 + b*x)),x]')
```

```
[Out] Log[-1 / b + x] - Log[x]
```

Maple [A]
time = 0.09, size = 12, normalized size = 1.00

method	result	size
default	$-\ln(x)+\ln(bx-1)$	12
norman	$-\ln(x)+\ln(bx-1)$	12
risch	$-\ln(x)+\ln(-bx+1)$	13
meijerg	$-\ln(x)-\ln(-b)+\ln(-bx+1)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(b*x-1)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.92

$$\log(bx-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="maxima")
```

```
[Out] log(b*x - 1) - log(x)
```

Fricas [A]
time = 0.30, size = 11, normalized size = 0.92

$$\log(bx-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="fricas")
```

```
[Out] log(b*x - 1) - log(x)
```

Sympy [A]
time = 0.06, size = 8, normalized size = 0.67

$$-\log(x) + \log\left(x - \frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x)
```

```
[Out] -log(x) + log(x - 1/b)
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\ln|x| + \ln|xb - 1|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x)
```

```
[Out] log(abs(b*x - 1)) - log(abs(x))
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.75

$$-2\,\mathrm{atanh}\left(2\,b\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - 1)),x)
```

```
[Out] -2*atanh(2*b*x - 1)
```

3.281 $\int \frac{1}{x^2(1+bx)}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{x} - b\log(x) + b\log(1+bx)$$

```
[Out] -1/x-b*ln(x)+b*ln(b*x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-b\log(x) + b\log(bx+1) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(1+bx)}\,dx &= \int \left(\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{1+bx}\right) dx\\ &= -\frac{1}{x} - b\log(x) + b\log(1+bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{x} - b\log(x) + b\log(1+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Mathics [A]
time = 1.69, size = 19, normalized size = 1.00

$$-b\text{Log}\left[x\right]+b\text{Log}\left[\frac{1}{b}+x\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(x^2*(1 + b*x)),x]')
[Out] -b Log[x] + b Log[1 / b + x] - 1 / x
```

Maple [A]
time = 0.11, size = 20, normalized size = 1.05

method	result	size
default	$-\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx+1\right)$	20
norman	$-\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx+1\right)$	20
risch	$-\frac{1}{x}+b\ln\left(-bx-1\right)-b\ln\left(x\right)$	21
meijerg	$b\left(-\frac{1}{xb}-\ln\left(x\right)-\ln\left(b\right)+\ln\left(bx+1\right)\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+1),x,method=_RETURNVERBOSE)
[Out] -1/x-b*ln(x)+b*ln(b*x+1)
```

Maxima [A]
time = 0.25, size = 19, normalized size = 1.00

$$b\log\left(bx+1\right)-b\log\left(x\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="maxima")
[Out] b*log(b*x + 1) - b*log(x) - 1/x
```

Fricas [A]
time = 0.31, size = 21, normalized size = 1.11

$$\frac{bx\log\left(bx+1\right)-bx\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + 1) - b*x*log(x) - 1)/x
```

Sympy [A]
time = 0.07, size = 14, normalized size = 0.74

$$b\left(-\log\left(x\right)+\log\left(x+\frac{1}{b}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+1),x)
```

```
[Out] b*(-log(x) + log(x + 1/b)) - 1/x
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.21

$$\frac{b^2\ln|xb+1|}{b}-b\ln|x|-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x)
```

```
[Out] b*log(abs(b*x + 1)) - b*log(abs(x)) - 1/x
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.84

$$2\,b\,\mathrm{atanh}\left(2\,b\,x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)),x)
```

```
[Out] 2*b*atanh(2*b*x + 1) - 1/x
```

3.282 $\int \frac{1}{x^2(-1+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{x} - b\log(x) + b\log(1 - bx)$$

```
[Out] 1/x-b*ln(x)+b*ln(-b*x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(-1+bx)}\,dx &= \int \left(-\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{-1+bx}\right) dx \\
&= \frac{1}{x} - b\log(x) + b\log(1 - bx)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{x} - b\log(x) + b\log(1 - bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-1 + b*x)),x]
```

[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]

Mathics [A]
time = 1.70, size = 19, normalized size = 1.06

$$-b\text{Log}\,[x] + b\text{Log}\left[-\frac{1}{b} + x\right] + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[1/(x^2*(-1 + b*x)),x]')

[Out] -b Log[x] + b Log[-1 / b + x] + 1 / x

Maple [A]
time = 0.11, size = 18, normalized size = 1.00

method	result	size
default	$\frac{1}{x} - b\ln\left(x\right) + b\ln\left(bx-1\right)$	18
norman	$\frac{1}{x} - b\ln\left(x\right) + b\ln\left(bx-1\right)$	18
risch	$\frac{1}{x} - b\ln\left(x\right) + b\ln\left(-bx+1\right)$	19
meijerg	$b\left(\frac{1}{xb} - \ln\left(x\right) - \ln\left(-b\right) + \ln\left(-bx+1\right)\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x-1),x,method=_RETURNVERBOSE)

[Out] 1/x-b*ln(x)+b*ln(b*x-1)

Maxima [A]
time = 0.27, size = 17, normalized size = 0.94

$$b\log\left(bx-1\right) - b\log\left(x\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x-1),x, algorithm="maxima")

[Out] b*log(b*x - 1) - b*log(x) + 1/x

Fricas [A]
time = 0.58, size = 21, normalized size = 1.17

$$\frac{bx\log\left(bx-1\right) - bx\log\left(x\right) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x - 1) - b*x*log(x) + 1)/x
```

Sympy [A]
time = 0.08, size = 14, normalized size = 0.78

$$b\left(-\log\left(x\right)+\log\left(x-\frac{1}{b}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-1),x)
```

```
[Out] b*(-log(x) + log(x - 1/b)) + 1/x
```

Giac [A]
time = 0.00, size = 22, normalized size = 1.22

$$\frac{b^2\ln|xb-1|}{b}-b\ln|x|+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x)
```

```
[Out] b*log(abs(b*x - 1)) - b*log(abs(x)) + 1/x
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.78

$$\frac{1}{x}-2\,b\,\mathrm{atanh}\left(2\,b\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - 1)),x)
```

```
[Out] 1/x - 2*b*atanh(2*b*x - 1)
```

3.283 $\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx$

Optimal. Leaf size=14

$$-\frac{1}{x}+b\log(1+bx)$$

```
[Out] -1/x+b*ln(b*x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {46}

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx &= b\log(x)+\int \frac{1}{x^2(1+bx)}\,dx\\ &= b\log(x)+\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right) dx\\ &= -\frac{1}{x}+b\log(1+bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{x}+b\log(1+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Mathics [A]
time = 1.60, size = 14, normalized size = 1.00

$$b\text{Log}\left[1+bx\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[b/x + 1/(x^2*(1 + b*x)),x]')
```

```
[Out] b Log[1 + b x] - 1 / x
```

Maple [A]
time = 0.09, size = 15, normalized size = 1.07

method	result	size
default	$-\frac{1}{x}+b\ln\left(bx+1\right)$	15
norman	$-\frac{1}{x}+b\ln\left(bx+1\right)$	15
risch	$-\frac{1}{x}+b\ln\left(-bx-1\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/x+1/x^2/(b*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+b*ln(b*x+1)
```

Maxima [A]
time = 0.24, size = 14, normalized size = 1.00

$$b\log\left(bx+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="maxima")
```

```
[Out] b*log(b*x + 1) - 1/x
```

Fricas [A]
time = 0.31, size = 15, normalized size = 1.07

$$\frac{bx\log\left(bx+1\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + 1) - 1)/x
```

Sympy [A]
time = 0.07, size = 10, normalized size = 0.71

$$b\log(bx+1)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x**2/(b*x+1),x)
[Out] b*log(b*x + 1) - 1/x
```

Giac [A]
time = 0.00, size = 28, normalized size = 2.00

$$\frac{b^2\ln|xb+1|}{b}-b\ln|x|-\frac{1}{x}+b\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x)
[Out] b*log(abs(b*x + 1)) - 1/x
```

Mupad [B]
time = 0.04, size = 20, normalized size = 1.43

$$b\,\ln(x)+2\,b\,\mathrm{atanh}\left(2\,b\,x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)) + b/x,x)
[Out] b*log(x) + 2*b*atanh(2*b*x + 1) - 1/x
```

3.284 $\int x^3\sqrt{a+bx}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}$$

```
[Out] -2/3*a^3*(b*x+a)^(3/2)/b^4+6/5*a^2*(b*x+a)^(5/2)/b^4-6/7*a*(b*x+a)^(7/2)/b^
4+2/9*(b*x+a)^(9/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*(
a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx}\,dx&=\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(315
*b^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(72) = 144$.
time = 16.10, size = 316, normalized size = 4.39

$$\frac{2\sqrt{a}\left(16a^{10}\left(1-\sqrt{\frac{a+bx}{a}}\right)+8a^9bx\left(12-11\sqrt{\frac{a+bx}{a}}\right)+6a^8b^2x^2\left(40-33\sqrt{\frac{a+bx}{a}}\right)+a^7b^3x^3\left(320-231\sqrt{\frac{a+bx}{a}}\right)-105a^6b^4x^4\sqrt{\frac{a+bx}{a}}+240a^6b^4x^4+189a^4b^5x^5\left(a+3bx\right)\sqrt{\frac{a+bx}{a}}+96a^5b^5x^5+16a^4b^6x^6+9a^2b^7x^7\left(83a+61bx\right)\sqrt{\frac{a+bx}{a}}+215ab^9x^9\sqrt{\frac{a+bx}{a}}+35b^{10}x^{10}\sqrt{\frac{a+bx}{a}}\right)}{315b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (16 a ^ 10 (1 - Sqrt[(a + b x) / a]) + 8 a ^ 9 b x (12 - 11 Sqrt[
(a + b x) / a]) + 6 a ^ 8 b ^ 2 x ^ 2 (40 - 33 Sqrt[(a + b x) / a]) + a ^ 7
 b ^ 3 x ^ 3 (320 - 231 Sqrt[(a + b x) / a]) - 105 a ^ 6 b ^ 4 x ^ 4 Sqrt[(
a + b x) / a] + 240 a ^ 6 b ^ 4 x ^ 4 + 189 a ^ 4 b ^ 5 x ^ 5 (a + 3 b x) S
qrt[(a + b x) / a] + 96 a ^ 5 b ^ 5 x ^ 5 + 16 a ^ 4 b ^ 6 x ^ 6 + 9 a ^ 2
b ^ 7 x ^ 7 (83 a + 61 b x) Sqrt[(a + b x) / a] + 215 a b ^ 9 x ^ 9 Sqrt[(a
 + b x) / a] + 35 b ^ 10 x ^ 10 Sqrt[(a + b x) / a]) / (315 b ^ 4 (a ^ 6 +
6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4
x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)}{315b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{6a(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{6a(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}}{b^4}$	50
trager	$-\frac{2\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)\sqrt{bx+a}}{315b^4}$	54
risch	$-\frac{2\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)\sqrt{bx+a}}{315b^4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b*x+a)^(5/2)-1/3*a^3*
(b*x+a)^(3/2))
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^4}-\frac{6\,(bx+a)^{\frac{7}{2}}a}{7\,b^4}+\frac{6\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^4}-\frac{2\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^4 - 6/7*(b*x + a)^(7/2)*a/b^4 + 6/5*(b*x + a)^(5/2)*a
^2/b^4 - 2/3*(b*x + a)^(3/2)*a^3/b^4
```

Fricas [A]
time = 0.31, size = 53, normalized size = 0.74

$$\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(68) = 136.
time = 1.27, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(49/2)*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a*
*18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b*
*9*x**5 + 315*a**14*b**10*x**6) + 32*a**(49/2)/(315*a**20*b**4 + 1890*a**19
*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**
4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 176*a**(47/2)*b*x*sqrt(1
 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300
*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*
b**10*x**6) + 192*a**(47/2)*b*x/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) - 396*a**(45/2)*b**2*x**2*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 480*a**(45/2)*b**2*x**2/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**
18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**
```

```
9*x**5 + 315*a**14*b**10*x**6) - 462*a**(43/2)*b**3*x**3*sqrt(1 + b*x/a)/(3
15*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*
x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
+ 640*a**(43/2)*b**3*x**3/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*
b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x
**5 + 315*a**14*b**10*x**6) - 210*a**(41/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*
a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**
3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 4
80*a**(41/2)*b**4*x**4/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**
6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5
 + 315*a**14*b**10*x**6) + 378*a**(39/2)*b**5*x**5*sqrt(1 + b*x/a)/(315*a**
20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 +
 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 192*
a**(39/2)*b**5*x**5/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x
**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 +
315*a**14*b**10*x**6) + 1134*a**(37/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**20
*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4
725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 32*a**
(37/2)*b**6*x**6/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2
 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315
*a**14*b**10*x**6) + 1494*a**(35/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**20*b*
*4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725
*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 1098*a**(
33/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) + 430*a**(31/2)*b**9*x**9*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 70*a**(29/2)*b**10*x**10*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*
b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4
 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.

time = 0.00, size = 192, normalized size = 2.67

$$\frac{\frac{2b\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3+\sqrt{a+bx}\,a^4\right)}{b^4}+\frac{2a\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
 - 35*sqrt(b*x + a)*a^3)*a/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*
a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a
^4)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^4) - (2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a +
 b*x)^(5/2))/(5*b^4) - (6*a*(a + b*x)^(7/2))/(7*b^4)
```

3.285 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx&=\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 200 vs. $2(53) = 106$.
time = 7.27, size = 184, normalized size = 3.47

$$\frac{2\sqrt{a}\left(8a^6\left(-1+\sqrt{\frac{a+bx}{a}}\right)+4a^5bx\left(-6+5\sqrt{\frac{a+bx}{a}}\right)+3a^4b^2x^2\left(-8+5\sqrt{\frac{a+bx}{a}}\right)+10a^2b^3x^3\left(2a+5bx\right)\sqrt{\frac{a+bx}{a}}-8a^3b^3x^3+48ab^5x^5\sqrt{\frac{a+bx}{a}}+15b^6x^6\sqrt{\frac{a+bx}{a}}\right)}{105b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (8 a ^ 6 (-1 + Sqrt[(a + b x) / a]) + 4 a ^ 5 b x (-6 + 5 Sqrt[(a
 + b x) / a]) + 3 a ^ 4 b ^ 2 x ^ 2 (-8 + 5 Sqrt[(a + b x) / a]) + 10 a ^ 2
 b ^ 3 x ^ 3 (2 a + 5 b x) Sqrt[(a + b x) / a] - 8 a ^ 3 b ^ 3 x ^ 3 + 48 a
 b ^ 5 x ^ 5 Sqrt[(a + b x) / a] + 15 b ^ 6 x ^ 6 Sqrt[(a + b x) / a]) / (1
05 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}\left(15x^2b^2-12abx+8a^2\right)}{105b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{b^3}$	38
trager	$\frac{2\left(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3\right)\sqrt{bx+a}}{105b^3}$	43
risch	$\frac{2\left(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3\right)\sqrt{bx+a}}{105b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

Fricas [A]
time = 0.31, size = 42, normalized size = 0.79

$$\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(49) = 98.
time = 0.88, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(41) = 82.
time = 0.00, size = 145, normalized size = 2.74

$$\frac{\frac{2b\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}+\frac{2a\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.70

$$\frac{30\,{\left(a+b\,x\right)}^{7/2}-84\,a\,{\left(a+b\,x\right)}^{5/2}+70\,a^2\,{\left(a+b\,x\right)}^{3/2}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/2),x)
```

```
[Out] (30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b
^3)
```

3.286 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx &= \int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{2\sqrt{a+bx}\,\left(-2a^2+abx+3b^2x^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(34) = 68$.
time = 3.46, size = 96, normalized size = 2.82

$$\frac{2\sqrt{a}\left(2a^3\left(1-\sqrt{\frac{a+bx}{a}}\right)+a^2bx\left(2-\sqrt{\frac{a+bx}{a}}\right)+4ab^2x^2\sqrt{\frac{a+bx}{a}}+3b^3x^3\sqrt{\frac{a+bx}{a}}\right)}{15b^2\left(a+bx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1*Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (2 a ^ 3 (1 - Sqrt[(a + b x) / a]) + a ^ 2 b x (2 - Sqrt[(a + b x
) / a]) + 4 a b ^ 2 x ^ 2 Sqrt[(a + b x) / a] + 3 b ^ 3 x ^ 3 Sqrt[(a + b x
) / a]) / (15 b ^ 2 (a + b x))
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
trager	$-\frac{2(-3x^2b^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32
risch	$-\frac{2(-3x^2b^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.76

$$\frac{2\left(bx+a\right)^{\frac{5}{2}}}{5\,b^2}-\frac{2\left(bx+a\right)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Fricas [A]
time = 0.31, size = 30, normalized size = 0.88

$$\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.
time = 0.58, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.00, size = 99, normalized size = 2.91

$$\frac{\frac{2b\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}+\frac{2a\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx)-a\sqrt{a+bx}\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10\,a\,{(a+b\,x)}^{3/2}-6\,{(a+b\,x)}^{5/2}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/2),x)
```

```
[Out] -(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

3.287 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Mathics [A]
time = 1.58, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{3}{2}}}{3b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*Sqrt[a + b*x],x]')
```

```
[Out] 2 (a + b x) ^ (3 / 2) / (3 b)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
trager	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
risch	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A]
time = 0.29, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.31

$$\frac{\sqrt{a+bx}\,\,(a+bx)}{\frac{3}{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x)
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

3.288 $\int \frac{\sqrt{a+bx}}{x} \, dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+bx} + a\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2\sqrt{a+bx} + \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= 2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 94 vs. 2(35) = 70.
time = 2.68, size = 74, normalized size = 2.11

$$\frac{2\left(-\sqrt{a}\,\mathrm{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right](a+bx) + a\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx}{bx}} + b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{a+bx}{bx}}\right)}{a+bx}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^1,x]')
```

```
[Out] 2 (-Sqrt[a] ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) + a Sqrt[b] Sqrt
[x] Sqrt[(a + b x) / (b x)] + b ^ (3 / 2) x ^ (3 / 2) Sqrt[(a + b x) / (b x
)]) / (a + b x)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.80

method	result	size
derivativedivides	$-2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28

default	$-2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\;+2\sqrt{bx+a}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.36, size = 42, normalized size = 1.20

$$\sqrt{a}\,\log\left(\frac{\sqrt{bx+a}\;-\sqrt{a}}{\sqrt{bx+a}\;+\sqrt{a}}\right)+2\,\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

Fricas [A]
time = 0.58, size = 73, normalized size = 2.09

$$\left[\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}\;+2\,a}{x}\right)+2\,\sqrt{bx+a}\,,2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+2\,\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.
time = 0.71, size = 68, normalized size = 1.94

$$-2\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 0.00, size = 42, normalized size = 1.20

$$2\sqrt{a+bx}+\frac{4a\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x)
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

Mupad [B]
time = 0.09, size = 27, normalized size = 0.77

$$2\sqrt{a+b\,x}-2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x,x)
```

```
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

3.289 $\int \frac{\sqrt{a+bx}}{x^2} dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^2}\,dx &= -\frac{\sqrt{a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= -\frac{\sqrt{a+bx}}{x}+\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= -\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 39, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Mathics [A]
time = 2.69, size = 38, normalized size = 0.97

$$-\frac{b\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}}-\frac{\sqrt{b}\sqrt{1+\frac{a}{bx}}}{\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^2,x]')
```

```
[Out] -b ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a] - Sqrt[b] Sqrt[1 + a / (b
 x)] / Sqrt[x]
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.95

method	result	size
risch	$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$	32

derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37
default	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.40, size = 47, normalized size = 1.21

$$\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

Fricas [A]
time = 0.31, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2\,ax},\ \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A]
time = 0.93, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

Giac [A]
time = 0.00, size = 58, normalized size = 1.49

$$\frac{\frac{\sqrt{a+bx}\,b^2}{-a-bx+a}+\frac{2b^2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x)
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+b\,x}}{x}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.290 $\int \frac{\sqrt{a+bx}}{x^3} \, dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x+a)^(1/2)/x^2-1/4*b*
(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x]/x^2 - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^3}\,dx &= -\frac{\sqrt{a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a}\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.85

$$-\frac{\sqrt{a+bx}\,(2a+bx)}{4ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^3,x]
[Out] -1/4*(Sqrt[a + b*x]*(2*a + b*x))/(a*x^2) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*a^(3/2))
```

Mathics [A]
time = 4.09, size = 100, normalized size = 1.54

$$\frac{\frac{-2a^{\frac{7}{2}}x(a+bx)}{b}-3a^{\frac{5}{2}}x^2(a+bx)-a^{\frac{3}{2}}bx^3(a+bx)+ab^{\frac{5}{2}}x^{\frac{9}{2}}\mathrm{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}}{4a^{\frac{5}{2}}\sqrt{b}\,x^{\frac{9}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^3,x]')
```

```
[Out] (-2 a ^ (7 / 2) x (a + b x) / b - 3 a ^ (5 / 2) x ^ 2 (a + b x) - a ^ (3 /
2) b x ^ 3 (a + b x) + a b ^ (5 / 2) x ^ (9 / 2) ArcSinh[Sqrt[a] / (Sqrt[b]
 Sqrt[x])] ((a + b x) / (b x)) ^ (3 / 2)) / (4 a ^ (5 / 2) Sqrt[b] x ^ (9 /
 2) ((a + b x) / (b x)) ^ (3 / 2))
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.83

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(bx+2a)}{4x^{2}a}+\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	44
derivativedivides	$2b^{2}\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54
default	$2b^{2}\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-(1/8/a*(b*x+a)^(3/2)+1/8*(b*x+a)^(1/2))/b^2/x^2+1/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.35, size = 88, normalized size = 1.35

$$-\frac{b^{2}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{3}{2}}b^{2}+\sqrt{bx+a}\,ab^{2}}{4\left((bx+a)^{2}a-2\,(bx+a)a^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) -
 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a - 2*(b*x +
a)*a^2 + a^3)
```

Fricas [A]
time = 0.33, size = 119, normalized size = 1.83

$$\left[\frac{\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(abx+2\,a^{2})\sqrt{bx+a}}{8\,a^{2}x^{2}},\,-\frac{\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(abx+2\,a^{2})\sqrt{bx+a}}{4\,a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [A]
time = 2.11, size = 97, normalized size = 1.49

$$-\frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(b
*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)/(
sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.37

$$\frac{-\frac{\sqrt{a+bx}\,(a+bx)b^3+\sqrt{a+bx}\,ab^3}{4a(a+bx-a)^2}-\frac{b^3\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2a\cdot 2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x)
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.07, size = 48, normalized size = 0.74

$$\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{{\left(a+b\,x\right)}^{3/2}}{4\,a\,x^2}-\frac{\sqrt{a+b\,x}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^3,x)
```

```
[Out] (b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x)^(3/2)/(4*a*x^2
) - (a + b*x)^(1/2)/(4*x^2)
```

3.291 $\int \frac{\sqrt{a+bx}}{x^4}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)-1/3*(b*x+a)^(1/2)/x^3-1/12*
b*(b*x+a)^(1/2)/a/x^2+1/8*b^2*(b*x+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 214}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x]/x^3 - (b*Sqrt[a + b*x])/(12*a*x^2) + (b^2*Sqrt[a + b*x])
/(8*a^2*x) - (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^4}\,dx &= -\frac{\sqrt{a+bx}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}\,dx \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^2} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a^2} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 0.77

$$
-\frac{\sqrt{a+bx}\,(8a^2+2abx-3b^2x^2)}{24a^2x^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2*x^2))/(a^2*x^3) - (b^3*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(87) = 174.
time = 7.08, size = 135, normalized size = 1.55

$$
-\frac{ab^{\frac{3}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}(a+bx)^2}-\frac{b^3\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{8a^{\frac{5}{2}}}-\frac{5b^{\frac{5}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{12\sqrt{x}\,(a+bx)^2}+\frac{b^{\frac{7}{2}}\sqrt{x}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{24a\,(a+bx)^2}+\frac{b^{\frac{9}{2}}x^{\frac{3}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{8a^2\,(a+bx)^2}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^4,x]')
```

```
[Out] -a b ^ (3 / 2) (1 + a / (b x)) ^ (3 / 2) / (3 x ^ (3 / 2) (a + b x) ^ 2) -
b ^ 3 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (8 a ^ (5 / 2)) - 5 b ^ (5 / 2
) (1 + a / (b x)) ^ (3 / 2) / (12 Sqrt[x] (a + b x) ^ 2) + b ^ (7 / 2) Sqrt
[x] (1 + a / (b x)) ^ (3 / 2) / (24 a (a + b x) ^ 2) + b ^ (9 / 2) x ^ (3 /
 2) (1 + a / (b x)) ^ (3 / 2) / (8 a ^ 2 (a + b x) ^ 2)
```

Maple [**A**]
time = 0.09, size = 66, normalized size = 0.76

method	result	size
risch	$-\frac{\sqrt{bx+a}\,\left(-3x^{2}b^{2}+2abx+8a^{2}\right)}{24x^{3}a^{2}}-\frac{b^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$	56
derivativedivides	$2b^{3}\left(-\frac{-\frac{(bx+a)^{\frac{5}{2}}}{16a^{2}}+\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{\sqrt{bx+a}}{16}}{b^{3}x^{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	66
default	$2b^{3}\left(-\frac{-\frac{(bx+a)^{\frac{5}{2}}}{16a^{2}}+\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{\sqrt{bx+a}}{16}}{b^{3}x^{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-(-1/16/a^2*(b*x+a)^(5/2)+1/6/a*(b*x+a)^(3/2)+1/16*(b*x+a)^(1/2))/b^
3/x^3-1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2))
```

Maxima [**A**]
time = 0.38, size = 121, normalized size = 1.39

$$\frac{b^{3}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}b^{3}-8\,(bx+a)^{\frac{3}{2}}ab^{3}-3\,\sqrt{bx+a}\,a^{2}b^{3}}{24\,\left((bx+a)^{3}a^{2}-3\,(bx+a)^{2}a^{3}+3\,(bx+a)a^{4}-a^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
 1/24*(3*(b*x + a)^(5/2)*b^3 - 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a^
2*b^3)/((b*x + a)^3*a^2 - 3*(b*x + a)^2*a^3 + 3*(b*x + a)*a^4 - a^5)
```

Fricas [A]
time = 0.32, size = 145, normalized size = 1.67

$$\left[\frac{3\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(3\,ab^2x^2-2\,a^2bx-8\,a^3)\sqrt{bx+a}}{48\,a^3x^3},\ \frac{3\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,ab^2x^2-2\,a^2bx-8\,a^3)\sqrt{bx+a}}{24\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [A]
time = 5.28, size = 122, normalized size = 1.40

$$-\frac{a}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{5}{2}}}{8a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**4,x)
```

```
[Out] -a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*sqrt(b)/(12*x**(5/2)*sqrt(a/(
b*x) + 1)) + b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + b**(5/2)/(8*a**2*
sqrt(x)*sqrt(a/(b*x) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a**(5
/2))
```

Giac [A]
time = 0.00, size = 119, normalized size = 1.37

$$\frac{\frac{3\sqrt{a+bx}\,(a+bx)^2b^4-8\sqrt{a+bx}\,(a+bx)ab^4-3\sqrt{a+bx}\,a^2b^4}{24a^2(a+bx-a)^3}+\frac{b^4\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4a^2\cdot 2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x)
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
/b
```

Mupad [B]
time = 0.11, size = 66, normalized size = 0.76

$$\frac{(a+b\,x)^{5/2}}{8\,a^2\,x^3}-\frac{(a+b\,x)^{3/2}}{3\,a\,x^3}-\frac{\sqrt{a+b\,x}}{8\,x^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\;1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^4,x)
```

```
[Out] (a + b*x)^(5/2)/(8*a^2*x^3) - (a + b*x)^(3/2)/(3*a*x^3) - (a + b*x)^(1/2)/(
8*x^3) + (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(5/2))
```

3.292 $\int x^3(a+bx)^{3/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}$$

```
[Out] -2/5*a^3*(b*x+a)^(5/2)/b^4+6/7*a^2*(b*x+a)^(7/2)/b^4-2/3*a*(b*x+a)^(9/2)/b^
4+2/11*(b*x+a)^(11/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*(
a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{3/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{5/2}\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(11
55*b^4)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(72) = 144$.
time = 16.41, size = 316, normalized size = 4.39

$$\frac{2\sqrt{a}\left(16a^{11}\left(1-\sqrt{\frac{a+bx}{a}}\right)+8a^{10}bx\left(12-11\sqrt{\frac{a+bx}{a}}\right)+6a^9b^2x^2\left(40-33\sqrt{\frac{a+bx}{a}}\right)+a^8b^3x^3\left(320-231\sqrt{\frac{a+bx}{a}}\right)+240a^7b^4x^4+924a^5b^5x^5\left(a+3bx\right)\sqrt{\frac{a+bx}{a}}+96a^6b^5x^5+16a^5b^6x^6+66a^3b^7x^7\left(67a+64bx\right)\sqrt{\frac{a+bx}{a}}+2420a^2b^9x^9\sqrt{\frac{a+bx}{a}}+770ab^{10}x^{10}\sqrt{\frac{a+bx}{a}}+105b^{11}x^{11}\sqrt{\frac{a+bx}{a}}\right)}{1155b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(a + b*x)^(3/2),x]')
```

```
[Out] 2 Sqrt[a] (16 a ^ 11 (1 - Sqrt[(a + b x) / a]) + 8 a ^ 10 b x (12 - 11 Sqrt
[(a + b x) / a]) + 6 a ^ 9 b ^ 2 x ^ 2 (40 - 33 Sqrt[(a + b x) / a]) + a ^
8 b ^ 3 x ^ 3 (320 - 231 Sqrt[(a + b x) / a]) + 240 a ^ 7 b ^ 4 x ^ 4 + 924
 a ^ 5 b ^ 5 x ^ 5 (a + 3 b x) Sqrt[(a + b x) / a] + 96 a ^ 6 b ^ 5 x ^ 5 +
 16 a ^ 5 b ^ 6 x ^ 6 + 66 a ^ 3 b ^ 7 x ^ 7 (67 a + 64 b x) Sqrt[(a + b x)
 / a] + 2420 a ^ 2 b ^ 9 x ^ 9 Sqrt[(a + b x) / a] + 770 a b ^ 10 x ^ 10 Sq
rt[(a + b x) / a] + 105 b ^ 11 x ^ 11 Sqrt[(a + b x) / a]) / (1155 b ^ 4 (a
 ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2
 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [**A**]
time = 0.08, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)}{1155b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{2a(bx+a)^{\frac{9}{2}}}{3}+\frac{6a^2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^3(bx+a)^{\frac{5}{2}}}{5}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{2a(bx+a)^{\frac{9}{2}}}{3}+\frac{6a^2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^3(bx+a)^{\frac{5}{2}}}{5}}{b^4}$	50
trager	$-\frac{2\left(-105b^5x^5-140a\,b^4x^4-5a^2b^3x^3+6a^3b^2x^2-8a^4bx+16a^5\right)\sqrt{bx+a}}{1155b^4}$	65
risch	$-\frac{2\left(-105b^5x^5-140a\,b^4x^4-5a^2b^3x^3+6a^3b^2x^2-8a^4bx+16a^5\right)\sqrt{bx+a}}{1155b^4}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/11*(b*x+a)^(11/2)-1/3*a*(b*x+a)^(9/2)+3/7*a^2*(b*x+a)^(7/2)-1/5*a^
3*(b*x+a)^(5/2))
```

Maxima [A]
time = 0.25, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^4}-\frac{2\,(bx+a)^{\frac{9}{2}}a}{3\,b^4}+\frac{6\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^4}-\frac{2\,(bx+a)^{\frac{5}{2}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^4 - 2/3*(b*x + a)^(9/2)*a/b^4 + 6/7*(b*x + a)^(7/2)
*a^2/b^4 - 2/5*(b*x + a)^(5/2)*a^3/b^4
```

Fricas [A]
time = 0.71, size = 64, normalized size = 0.89

$$\frac{2\,(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(68) = 136.
time = 1.37, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(3/2),x)
```

```
[Out] -32*a**(51/2)*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*
a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**1
5*b**9*x**5 + 1155*a**14*b**10*x**6) + 32*a**(51/2)/(1155*a**20*b**4 + 6930
*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16
*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 176*a**(49/2)*
b*x*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6
*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**
5 + 1155*a**14*b**10*x**6) + 192*a**(49/2)*b*x/(1155*a**20*b**4 + 6930*a**1
9*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8
*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 396*a**(47/2)*b**2*
x**2*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**
6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x*
*5 + 1155*a**14*b**10*x**6) + 480*a**(47/2)*b**2*x**2/(1155*a**20*b**4 + 69
30*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**
```

```
16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 462*a**(45/2
)*b**3*x**3*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a*
*18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*
b**9*x**5 + 1155*a**14*b**10*x**6) + 640*a**(45/2)*b**3*x**3/(1155*a**20*b*
*4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17
325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 480*a
**(43/2)*b**4*x**4/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*
x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5
 + 1155*a**14*b**10*x**6) + 1848*a**(41/2)*b**5*x**5*sqrt(1 + b*x/a)/(1155*
a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x
**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
 + 192*a**(41/2)*b**5*x**5/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**
18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b
**9*x**5 + 1155*a**14*b**10*x**6) + 5544*a**(39/2)*b**6*x**6*sqrt(1 + b*x/a
)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**1
7*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**
10*x**6) + 32*a**(39/2)*b**6*x**6/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 8844*a**(37/2)*b**7*x**7*sqrt(1
+ b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 231
00*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a*
*14*b**10*x**6) + 8448*a**(35/2)*b**8*x**8*sqrt(1 + b*x/a)/(1155*a**20*b**4
 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1732
5*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 4840*a*
*(33/2)*b**9*x**9*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 1540*a**(31/2)*b**10*x**10*sqrt(
1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 2
3100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*
a**14*b**10*x**6) + 210*a**(29/2)*b**11*x**11*sqrt(1 + b*x/a)/(1155*a**20*b
**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1
7325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(56) = 112.

time = 0.00, size = 326, normalized size = 4.53

$$\frac{\frac{2b^2\left(\frac{1}{11}\sqrt{a+bx}\,(a+bx)^5-\frac{5}{9}\sqrt{a+bx}\,(a+bx)^4a+\frac{10}{7}\sqrt{a+bx}\,(a+bx)^3a^2-2\sqrt{a+bx}\,(a+bx)^2a^3+\frac{5}{3}\sqrt{a+bx}\,(a+bx)a^4-\sqrt{a+bx}\,a^5\right)}{b^5}+\frac{4ab\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3+\sqrt{a+bx}\,a^4\right)}{b^4}+\frac{2a^2\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x)
```

```
[Out] 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a
^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)
^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x
```

```
 + a)*a^4)*a/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*
x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 69
3*sqrt(b*x + a)*a^5)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b^4) - (2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a
 + b*x)^(7/2))/(7*b^4) - (2*a*(a + b*x)^(9/2))/(3*b^4)
```

3.293 $\int x^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}$$

```
[Out] 2/5*a^2*(b*x+a)^(5/2)/b^3-4/7*a*(b*x+a)^(7/2)/b^3+2/9*(b*x+a)^(9/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + b
*x)^(9/2))/(9*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{3/2}\,dx &= \int \left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{5/2}\left(8a^2-20abx+35b^2x^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 213 vs. $2(53) = 106$.
time = 7.72, size = 197, normalized size = 3.72

$$\frac{2\sqrt{a}\left(8a^7\left(-1+\sqrt{\frac{a+bx}{a}}\right)+4a^6bx\left(-6+5\sqrt{\frac{a+bx}{a}}\right)+3a^5b^2x^2\left(-8+5\sqrt{\frac{a+bx}{a}}\right)+5ab^3x^3\left(11a^3+38a^2bx+31b^3x^3\right)\sqrt{\frac{a+bx}{a}}-8a^4b^3x^3+258a^2b^5x^5\sqrt{\frac{a+bx}{a}}+35b^7x^7\sqrt{\frac{a+bx}{a}}\right)}{315b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(a + b*x)^(3/2),x]')
```

```
[Out] 2 Sqrt[a] (8 a ^ 7 (-1 + Sqrt[(a + b x) / a]) + 4 a ^ 6 b x (-6 + 5 Sqrt[(a
 + b x) / a]) + 3 a ^ 5 b ^ 2 x ^ 2 (-8 + 5 Sqrt[(a + b x) / a]) + 5 a b ^
3 x ^ 3 (11 a ^ 3 + 38 a ^ 2 b x + 31 b ^ 3 x ^ 3) Sqrt[(a + b x) / a] - 8
a ^ 4 b ^ 3 x ^ 3 + 258 a ^ 2 b ^ 5 x ^ 5 Sqrt[(a + b x) / a] + 35 b ^ 7 x
^ 7 Sqrt[(a + b x) / a]) / (315 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^
2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.08, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}\left(35x^2b^2-20abx+8a^2\right)}{315b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{4a(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{4a(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}}{b^3}$	38
trager	$\frac{2\left(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4\right)\sqrt{bx+a}}{315b^3}$	54
risch	$\frac{2\left(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4\right)\sqrt{bx+a}}{315b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/9*(b*x+a)^(9/2)-2/7*a*(b*x+a)^(7/2)+1/5*a^2*(b*x+a)^(5/2))
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^3}-\frac{4\,(bx+a)^{\frac{7}{2}}a}{7\,b^3}+\frac{2\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^3 - 4/7*(b*x + a)^(7/2)*a/b^3 + 2/5*(b*x + a)^(5/2)*a
^2/b^3
```

Fricas [A]
time = 0.31, size = 53, normalized size = 1.00

$$\frac{2\left(35\,b^{4}x^{4}+50\,ab^{3}x^{3}+3\,a^{2}b^{2}x^{2}-4\,a^{3}bx+8\,a^{4}\right)\sqrt{bx+a}}{315\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(49) = 98$.
time = 0.96, size = 733, normalized size = 13.83

$$\frac{16a^{\frac{25}{2}}\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{25}{2}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{40a^{\frac{23}{2}}bx\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{23}{2}}bx}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{30a^{\frac{21}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{21}{2}}b^2x^2}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{110a^{\frac{19}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{19}{2}}b^3x^3}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{380a^{\frac{17}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{516a^{\frac{15}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{310a^{\frac{13}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{70a^{\frac{11}{2}}b^7x^7\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(3/2),x)
```

```
[Out] 16*a**(25/2)*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b*
*5*x**2 + 315*a**5*b**6*x**3) - 16*a**(25/2)/(315*a**8*b**3 + 945*a**7*b**4
*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 40*a**(23/2)*b*x*sqrt(1 + b
*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6
*x**3) - 48*a**(23/2)*b*x/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*
x**2 + 315*a**5*b**6*x**3) + 30*a**(21/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a*
*8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 48*a
**(21/2)*b**2*x**2/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 +
315*a**5*b**6*x**3) + 110*a**(19/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**8*b**
3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 16*a**(19/
2)*b**3*x**3/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a*
*5*b**6*x**3) + 380*a**(17/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*a**8*b**3 + 94
5*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 516*a**(15/2)*b*
*5*x**5*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x*
*2 + 315*a**5*b**6*x**3) + 310*a**(13/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**
8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 70*a*
*(11/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a*
*6*b**5*x**2 + 315*a**5*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(41) = 82$.

time = 0.00, size = 257, normalized size = 4.85

$$\frac{\frac{2b^2\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3+\sqrt{a+bx}\,a^4\right)}{b^4}+\frac{4ab\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}+\frac{2a^2\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x)
```

```
[Out] 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2
)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(
7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x +
 a)*a^4)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{70\,{\left(a+b\,x\right)}^{9/2}-180\,a\,{\left(a+b\,x\right)}^{7/2}+126\,a^2\,{\left(a+b\,x\right)}^{5/2}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(3/2),x)
```

```
[Out] (70*(a + b*x)^(9/2) - 180*a*(a + b*x)^(7/2) + 126*a^2*(a + b*x)^(5/2))/(315
*b^3)
```

3.294 $\int x(a+bx)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}$$

```
[Out] -2/5*a*(b*x+a)^(5/2)/b^2+2/7*(b*x+a)^(7/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{3/2}\,dx &= \int \left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{5/2}(-2a+5bx)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(3/2),x]
```

[Out] (2*(a + b*x)^(5/2)*(-2*a + 5*b*x))/(35*b^2)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 1.99, size = 53, normalized size = 1.56

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2a^3+a^2bx+b^2x^2\left(8a+5bx\right)\right)\sqrt{a+bx}}{35b^2},b!{=}0\right\}\right\},\frac{a^{\frac{3}{2}}x^2}{2}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^1*(a + b*x)^(3/2),x]')

[Out] Piecewise[{{2 (-2 a ^ 3 + a ^ 2 b x + b ^ 2 x ^ 2 (8 a + 5 b x)) Sqrt[a + b x] / (35 b ^ 2), b != 0}}, a ^ (3 / 2) x ^ 2 / 2]

Maple [A]
time = 0.09, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}}{b^2}$	26
trager	$-\frac{2(-5b^3x^3-8a\,b^2x^2-a^2bx+2a^3)\sqrt{bx+a}}{35b^2}$	43
risch	$-\frac{2(-5b^3x^3-8a\,b^2x^2-a^2bx+2a^3)\sqrt{bx+a}}{35b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/b^2*(1/7*(b*x+a)^(7/2)-1/5*a*(b*x+a)^(5/2))

Maxima [A]
time = 0.28, size = 26, normalized size = 0.76

$$\frac{2\left(bx+a\right)^{\frac{7}{2}}}{7\,b^2}-\frac{2\left(bx+a\right)^{\frac{5}{2}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^(3/2),x, algorithm="maxima")

[Out] 2/7*(b*x + a)^(7/2)/b^2 - 2/5*(b*x + a)^(5/2)*a/b^2

Fricas [A]
time = 0.30, size = 41, normalized size = 1.21

$$\frac{2\,(5\,b^3x^3 + 8\,ab^2x^2 + a^2bx - 2\,a^3)\sqrt{bx + a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Sympy [A]
time = 0.15, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{4a^3\sqrt{a+bx}}{35b^2} + \frac{2a^2x\sqrt{a+bx}}{35b} + \frac{16ax^2\sqrt{a+bx}}{35} + \frac{2bx^3\sqrt{a+bx}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**3*sqrt(a + b*x)/(35*b**2) + 2*a**2*x*sqrt(a + b*x)/(35*b)
+ 16*a*x**2*sqrt(a + b*x)/35 + 2*b*x**3*sqrt(a + b*x)/7, Ne(b, 0)), (a**(3/
2)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(26) = 52.
time = 0.00, size = 185, normalized size = 5.44

$$\frac{\frac{2b^2\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3 - \frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a + \sqrt{a+bx}\,(a+bx)a^2 - \sqrt{a+bx}\,a^3\right)}{b^3} + \frac{4ab\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2 - \frac{2}{3}\sqrt{a+bx}\,(a+bx)a + \sqrt{a+bx}\,a^2\right)}{b^2} + \frac{2a^2\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx) - a\sqrt{a+bx}\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x)
```

```
[Out] 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a/b + 3*(5*(b*x + a)^(7/2
) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b
)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{14\,a\,{\left(a+b\,x\right)}^{5/2}-10\,{\left(a+b\,x\right)}^{7/2}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(3/2),x)
```

```
[Out] -(14*a*(a + b*x)^(5/2) - 10*(a + b*x)^(7/2))/(35*b^2)
```

3.295 $\int (a+bx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{5/2}}{5b}$$

[Out] 2/5*(b*x+a)^(5/2)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2),x]

[Out] (2*(a + b*x)^(5/2))/(5*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{3/2}\,dx = \frac{2(a+bx)^{5/2}}{5b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(3/2),x]

[Out] (2*(a + b*x)^(5/2))/(5*b)

Mathics [A]
time = 1.57, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{5}{2}}}{5b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^(3/2),x]')
```

```
[Out] 2 (a + b x) ^ (5 / 2) / (5 b)
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
default	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
trager	$\frac{2(x^2b^2+2abx+a^2)\sqrt{bx+a}}{5b}$	29
risch	$\frac{2(x^2b^2+2abx+a^2)\sqrt{bx+a}}{5b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*x+a)^(5/2)/b
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.30, size = 28, normalized size = 1.75

$$\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2),x)
```

```
[Out] 2*(a + b*x)**(5/2)/(5*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.
time = 0.00, size = 113, normalized size = 7.06

$$\frac{\frac{2b^2\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}+4a\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx)-a\sqrt{a+bx}\right)+2a^2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x)
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 30*sqrt(b*x + a)*a^2 + 10*
((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

3.296 $\int \frac{(a+bx)^{3/2}}{x}\,dx$

Optimal. Leaf size=49

$$2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x,x]
```

```
[Out] 2*a*Sqrt[a + b*x] + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(a+bx)^{3/2}+a\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+a^2\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.90

$$\frac{2}{3}\sqrt{a+bx}\,(4a+bx)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Mathics [A]
time = 3.08, size = 62, normalized size = 1.27

$$\frac{\sqrt{a}\left(-6a\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+3a\text{Log}\left[\frac{bx}{a}\right]+8a\sqrt{\frac{a+bx}{a}}+2bx\sqrt{\frac{a+bx}{a}}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/x^1,x]')
```

```
[Out] Sqrt[a] (-6 a Log[1 + Sqrt[(a + b x) / a]] + 3 a Log[b x / a] + 8 a Sqrt[(a
 + b x) / a] + 2 b x Sqrt[(a + b x) / a]) / 3
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.78

method	result	size

derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a\sqrt{bx+a}$	38
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a\sqrt{bx+a}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

Maxima [A]
time = 0.38, size = 52, normalized size = 1.06

$$a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}(bx+a)^{\frac{3}{2}}+2\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(b*x
 + a)^(3/2) + 2*sqrt(b*x + a)*a
```

Fricas [A]
time = 0.31, size = 88, normalized size = 1.80

$$\left[a^{\frac{3}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{3}(bx+4a)\sqrt{bx+a}\,,2\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{3}(bx+4a)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

Sympy [A]
time = 1.13, size = 71, normalized size = 1.45

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{3}+a^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,bx\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x/a)/3 + a**(3/2)*log(b*x/a) - 2*a**(3/2)*log(sqrt(1
+ b*x/a) + 1) + 2*sqrt(a)*b*x*sqrt(1 + b*x/a)/3
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.29

$$\frac{2}{3}\sqrt{a+bx}\,(a+bx)+2\sqrt{a+bx}\,a+\frac{4a^2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x)
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sqr
t(b*x + a)*a
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.76

$$2\,a\,\sqrt{a+b\,x}+\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3}-2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x,x)
```

```
[Out] 2*a*(a + b*x)^(1/2) + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*atanh((a + b*x)^(1/
2)/a^(1/2))
```

3.297 $\int \frac{(a+bx)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=51

$$3b\sqrt{a+bx} - \frac{(a+bx)^{3/2}}{x} - 3\sqrt{a}\, b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -(b*x+a)^(3/2)/x-3*b*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+3*b*(b*x+a)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-\frac{(a+bx)^{3/2}}{x} + 3b\sqrt{a+bx} - 3\sqrt{a}\, b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[a + b*x] - (a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^2}\,dx &= -\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+(3a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.88

$$-\frac{(a-2bx)\sqrt{a+bx}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] -(((a - 2*b*x)*Sqrt[a + b*x])/x) - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(51) = 102.
time = 3.41, size = 82, normalized size = 1.61

$$-\frac{a^3}{b^{\frac{3}{2}}x^{\frac{5}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}+\frac{3a\sqrt{b}}{\sqrt{x}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}-3\sqrt{a}\,b\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/x^2,x]')
```

```
[Out] -a ^ 3 / (b ^ (3 / 2) x ^ (5 / 2) (1 + a / (b x)) ^ (3 / 2)) + 3 a Sqrt[b]
/ (Sqrt[x] (1 + a / (b x)) ^ (3 / 2)) - 3 Sqrt[a] b ArcSinh[Sqrt[a] / (Sqrt
[b] Sqrt[x])] + 2 b ^ (3 / 2) Sqrt[x] / (1 + a / (b x)) ^ (3 / 2)
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.94

method	result	size
risch	$-\frac{a\sqrt{bx+a}}{x}+\frac{b\left(4\sqrt{bx+a}-6\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\right)}{2}$	45
derivativedivides	$2b\left(\sqrt{bx+a}-a\left(\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	48
default	$2b\left(\sqrt{bx+a}-a\left(\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*((b*x+a)^(1/2)-a*(1/2*(b*x+a)^(1/2)/b/x+3/2*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(1/2)))
```

Maxima [A]
time = 0.36, size = 58, normalized size = 1.14

$$\frac{3}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b-\frac{\sqrt{bx+a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(a)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*
sqrt(b*x + a)*b - sqrt(b*x + a)*a/x
```

Fricas [A]
time = 0.30, size = 102, normalized size = 2.00

$$\left[\frac{3\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(2\,bx-a)\sqrt{bx+a}}{2\,x},\ \frac{3\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2\,bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.
time = 1.34, size = 92, normalized size = 1.80

$$-3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{a\sqrt{b}}{\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**2,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt
(a/(b*x) + 1)) + a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)
/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 0.00, size = 74, normalized size = 1.45

$$\frac{2\sqrt{a+bx}\,b^2+\frac{\sqrt{a+bx}\,b^2a}{-a-bx+a}+\frac{6b^2a\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x)
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*b^2 - sq
rt(b*x + a)*a*b/x)/b
```

Mupad [B]
time = 0.10, size = 42, normalized size = 0.82

$$2\,b\,\sqrt{a+b\,x}-3\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)-\frac{a\,\sqrt{a+b\,x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^2,x)
```

```
[Out] 2*b*(a + b*x)^(1/2) - 3*a^(1/2)*b*atanh((a + b*x)^(1/2)/a^(1/2)) - (a*(a +
b*x)^(1/2))/x
```

3.298 $\int \frac{(a+bx)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=62

$$-\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/2*(b*x+a)^(3/2)/x^2-3/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-3/4*b
*(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b\sqrt{a+bx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[a + b*x])/(4*x) - (a + b*x)^(3/2)/(2*x^2) - (3*b^2*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^3}\,dx &= -\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\ &= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 53, normalized size = 0.85

$$-\frac{\sqrt{a+bx}\,(2a+5bx)}{4x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b*x]*(2*a + 5*b*x))/x^2 - (3*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*Sqrt[a])
```

Mathics [**A**]
time = 3.49, size = 61, normalized size = 0.98

$$-\frac{a\sqrt{b}\sqrt{1+\frac{a}{bx}}}{2x^{\frac{3}{2}}}-\frac{3b^2\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{4\sqrt{a}}-\frac{5b^{\frac{3}{2}}\sqrt{1+\frac{a}{bx}}}{4\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/x^3,x]')
```

```
[Out] -a Sqrt[b] Sqrt[1 + a / (b x)] / (2 x ^ (3 / 2)) - 3 b ^ 2 ArcSinh[Sqrt[a]
/ (Sqrt[b] Sqrt[x])] / (4 Sqrt[a]) - 5 b ^ (3 / 2) Sqrt[1 + a / (b x)] / (4
 Sqrt[x])
```

Maple [**A**]
time = 0.10, size = 52, normalized size = 0.84

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(5bx+2a)}{4x^{2}}-\frac{3b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4\sqrt{a}}$	42
derivativedivides	$2b^{2}\left(-\frac{\frac{5(bx+a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx+a}}{8}}{b^{2}x^{2}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	52
default	$2b^{2}\left(-\frac{\frac{5(bx+a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx+a}}{8}}{b^{2}x^{2}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-(5/8*(b*x+a)^(3/2)-3/8*a*(b*x+a)^(1/2))/b^2/x^2-3/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.35, size = 86, normalized size = 1.39

$$\frac{3\,b^{2}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^{2}-3\,\sqrt{bx+a}\,ab^{2}}{4\,\left((bx+a)^{2}-2\,(bx+a)a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
1/4*(5*(b*x + a)^(3/2)*b^2 - 3*sqrt(b*x + a)*a*b^2)/((b*x + a)^2 - 2*(b*x +
 a)*a + a^2)
```

Fricas [A]
time = 0.61, size = 124, normalized size = 2.00

$$\left[\frac{3\,\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(5\,abx+2\,a^{2})\sqrt{bx+a}}{8\,ax^{2}},\frac{3\,\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(5\,abx+2\,a^{2})\sqrt{bx+a}}{4\,ax^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

Sympy [A]
time = 1.60, size = 76, normalized size = 1.23

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(a/(b*x) + 1)/(4
*sqrt(x)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.39

$$\frac{-\frac{5\sqrt{a+bx}\,(a+bx)b^3-3\sqrt{a+bx}\,b^3a}{4(a+bx-a)^2}+\frac{3b^3\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x)
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3
 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.06, size = 46, normalized size = 0.74

$$\frac{3\,a\,\sqrt{a+b\,x}}{4\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^3,x)
```

```
[Out] (3*a*(a + b*x)^(1/2))/(4*x^2) - (3*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a
^(1/2)) - (5*(a + b*x)^(3/2))/(4*x^2)
```

3.299 $\int \frac{(a+bx)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=84

$$-\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

```
[Out] -1/3*(b*x+a)^(3/2)/x^3+1/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*b
*(b*x+a)^(1/2)/x^2-1/8*b^2*(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b\sqrt{a+bx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] -1/4*(b*Sqrt[a + b*x])/x^2 - (b^2*Sqrt[a + b*x])/(8*a*x) - (a + b*x)^(3/2)/
(3*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^4}\,dx &= -\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}\,dx\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{8}b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a}\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a}\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 0.80

$$-\frac{\sqrt{a+bx}\,(8a^2+14abx+3b^2x^2)}{24ax^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^4,x]
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^2))/(a*x^3) + (b^3*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(84) = 168.
time = 5.74, size = 135, normalized size = 1.61

$$-\frac{a^2b^{\frac{3}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}\left(a+bx\right)^2}-\frac{11ab^{\frac{5}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{12\sqrt{x}\left(a+bx\right)^2}+\frac{b^3\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{8a^{\frac{3}{2}}}-\frac{17b^{\frac{7}{2}}\sqrt{x}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{24\left(a+bx\right)^2}-\frac{b^{\frac{9}{2}}x^{\frac{3}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{8a\left(a+bx\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/x^4,x]')
```

```
[Out] -a ^ 2 b ^ (3 / 2) (1 + a / (b x)) ^ (3 / 2) / (3 x ^ (3 / 2) (a + b x) ^ 2
) - 11 a b ^ (5 / 2) (1 + a / (b x)) ^ (3 / 2) / (12 Sqrt[x] (a + b x) ^ 2)
 + b ^ 3 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (8 a ^ (3 / 2)) - 17 b ^ (7
 / 2) Sqrt[x] (1 + a / (b x)) ^ (3 / 2) / (24 (a + b x) ^ 2) - b ^ (9 / 2)
x ^ (3 / 2) (1 + a / (b x)) ^ (3 / 2) / (8 a (a + b x) ^ 2)
```

Maple [**A**]
time = 0.10, size = 64, normalized size = 0.76

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(3x^2b^2+14abx+8a^2)}{24x^3a}+\frac{b^3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}$	56
derivativedivides	$2b^3\left(-\frac{\frac{(bx+a)^{\frac{5}{2}}}{16a}+\frac{(bx+a)^{\frac{3}{2}}}{6}-\frac{a\sqrt{bx+a}}{16}}{b^3x^3}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	64
default	$2b^3\left(-\frac{\frac{(bx+a)^{\frac{5}{2}}}{16a}+\frac{(bx+a)^{\frac{3}{2}}}{6}-\frac{a\sqrt{bx+a}}{16}}{b^3x^3}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-(1/16/a*(b*x+a)^(5/2)+1/6*(b*x+a)^(3/2)-1/16*a*(b*x+a)^(1/2))/b^3/x
^3+1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [**A**]
time = 0.37, size = 119, normalized size = 1.42

$$-\frac{b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}b^3+8\,(bx+a)^{\frac{3}{2}}ab^3-3\,\sqrt{bx+a}\,a^2b^3}{24\,\left((bx+a)^3a-3\,(bx+a)^2a^2+3\,(bx+a)a^3-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
- 1/24*(3*(b*x + a)^(5/2)*b^3 + 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a
^2*b^3)/((b*x + a)^3*a - 3*(b*x + a)^2*a^2 + 3*(b*x + a)*a^3 - a^4)
```

Fricas [A]
time = 0.32, size = 145, normalized size = 1.73

$$\left[\frac{3\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,a^2x^3},-\frac{3\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{24\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

Sympy [A]
time = 3.70, size = 124, normalized size = 1.48

$$-\frac{a^2}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{17b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**4,x)
```

```
[Out] -a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*a*sqrt(b)/(12*x**(5/2)*sq
rt(a/(b*x) + 1)) - 17*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - b**(5/2)/(
8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a
**(3/2))
```

Giac [A]
time = 0.00, size = 116, normalized size = 1.38

$$\frac{\frac{-3\sqrt{a+bx}\,(a+bx)^2b^4-8\sqrt{a+bx}\,(a+bx)b^4a+3\sqrt{a+bx}\,b^4a^2}{24a(a+bx-a)^3}-\frac{b^4\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4a\cdot 2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x)
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x + a)^(5/
2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a*b^3*x^3))/b
```

Mupad [B]
time = 0.10, size = 64, normalized size = 0.76

$$\frac{a\sqrt{a+b\,x}}{8\,x^3}-\frac{(a+b\,x)^{5/2}}{8\,a\,x^3}-\frac{(a+b\,x)^{3/2}}{3\,x^3}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^4,x)
```

```
[Out] (a*(a + b*x)^(1/2))/(8*x^3) - (a + b*x)^(5/2)/(8*a*x^3) - (b^3*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(3/2)) - (a + b*x)^(3/2)/(3*x^3)
```

3.300 $\int x^3(a+bx)^{5/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}$$

```
[Out] -2/7*a^3*(b*x+a)^(7/2)/b^4+2/3*a^2*(b*x+a)^(9/2)/b^4-6/11*a*(b*x+a)^(11/2)/
b^4+2/13*(b*x+a)^(13/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*(
a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{5/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{7/2}\left(-16a^3+56a^2bx-126ab^2x^2+231b^3x^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(3
003*b^4)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.63, size = 88, normalized size = 1.22

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-16a^6+8a^5bx-6a^4b^2x^2+5a^3b^3x^3+7b^4x^4\left(53a^2+81abx+33b^2x^2\right)\right)\sqrt{a+bx}}{3003b^4},b!=0\right\}\right\},\frac{a^{\frac{5}{2}}x^4}{4}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(a + b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (-16 a ^ 6 + 8 a ^ 5 b x - 6 a ^ 4 b ^ 2 x ^ 2 + 5 a ^ 3 b ^
3 x ^ 3 + 7 b ^ 4 x ^ 4 (53 a ^ 2 + 81 a b x + 33 b ^ 2 x ^ 2)) Sqrt[a + b
x] / (3003 b ^ 4), b != 0}}, a ^ (5 / 2) x ^ 4 / 4]
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-231b^3x^3+126a\,b^2x^2-56a^2bx+16a^3\right)}{3003b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{6a(bx+a)^{\frac{11}{2}}}{11}+\frac{2a^2(bx+a)^{\frac{9}{2}}}{3}-\frac{2a^3(bx+a)^{\frac{7}{2}}}{7}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{6a(bx+a)^{\frac{11}{2}}}{11}+\frac{2a^2(bx+a)^{\frac{9}{2}}}{3}-\frac{2a^3(bx+a)^{\frac{7}{2}}}{7}}{b^4}$	50
trager	$-\frac{2(-231x^6b^6-567a\,x^5b^5-371a^2x^4b^4-5a^3b^3x^3+6a^4x^2b^2-8a^5xb+16a^6)\sqrt{bx+a}}{3003b^4}$	76
risch	$-\frac{2(-231x^6b^6-567a\,x^5b^5-371a^2x^4b^4-5a^3b^3x^3+6a^4x^2b^2-8a^5xb+16a^6)\sqrt{bx+a}}{3003b^4}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/13*(b*x+a)^(13/2)-3/11*a*(b*x+a)^(11/2)+1/3*a^2*(b*x+a)^(9/2)-1/7*
a^3*(b*x+a)^(7/2))
```

Maxima [A]
time = 0.25, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^4}-\frac{6\,(bx+a)^{\frac{11}{2}}a}{11\,b^4}+\frac{2\,(bx+a)^{\frac{9}{2}}a^2}{3\,b^4}-\frac{2\,(bx+a)^{\frac{7}{2}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^4 - 6/11*(b*x + a)^(11/2)*a/b^4 + 2/3*(b*x + a)^(9/
2)*a^2/b^4 - 2/7*(b*x + a)^(7/2)*a^3/b^4
```

Fricas [A]
time = 0.29, size = 75, normalized size = 1.04

$$\frac{2\left(231\,b^{6}x^{6}+567\,ab^{5}x^{5}+371\,a^{2}b^{4}x^{4}+5\,a^{3}b^{3}x^{3}-6\,a^{4}b^{2}x^{2}+8\,a^{5}bx-16\,a^{6}\right)\sqrt{bx+a}}{3003\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Sympy [A]
time = 0.41, size = 146, normalized size = 2.03

$$\begin{cases} -\frac{32a^{6}\sqrt{a+bx}}{3003b^{4}}+\frac{16a^{5}x\sqrt{a+bx}}{3003b^{3}}-\frac{4a^{4}x^{2}\sqrt{a+bx}}{1001b^{2}}+\frac{10a^{3}x^{3}\sqrt{a+bx}}{3003b}+\frac{106a^{2}x^{4}\sqrt{a+bx}}{429}+\frac{54abx^{5}\sqrt{a+bx}}{143}+\frac{2b^{2}x^{6}\sqrt{a+bx}}{13} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**6*sqrt(a + b*x)/(3003*b**4) + 16*a**5*x*sqrt(a + b*x)/(30
03*b**3) - 4*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*a**3*x**3*sqrt(a + b*
x)/(3003*b) + 106*a**2*x**4*sqrt(a + b*x)/429 + 54*a*b*x**5*sqrt(a + b*x)/1
43 + 2*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(56) = 112.
time = 0.00, size = 481, normalized size = 6.68

$$\frac{\frac{2b^{3}\left(\frac{1}{13}\sqrt{a+bx}\,(a+bx)^{6}-\frac{6}{11}\sqrt{a+bx}\,(a+bx)^{5}a+\frac{5}{3}\sqrt{a+bx}\,(a+bx)^{4}a^{2}-\frac{20}{7}\sqrt{a+bx}\,(a+bx)^{3}a^{3}+3\sqrt{a+bx}\,(a+bx)^{2}a^{4}-2\sqrt{a+bx}\,(a+bx)a^{5}+\sqrt{a+bx}\,a^{6}\right)}{b^{6}}+\frac{6ab^{2}\left(\frac{1}{11}\sqrt{a+bx}\,(a+bx)^{5}-\frac{5}{9}\sqrt{a+bx}\,(a+bx)^{4}a+\frac{10}{7}\sqrt{a+bx}\,(a+bx)^{3}a^{2}-2\sqrt{a+bx}\,(a+bx)^{2}a^{3}+\frac{5}{3}\sqrt{a+bx}\,(a+bx)a^{4}-\sqrt{a+bx}\,a^{5}\right)}{b^{5}}+\frac{6a^{2}b\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^{4}-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^{3}a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^{2}a^{2}-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^{3}+\sqrt{a+bx}\,a^{4}\right)}{b^{4}}+\frac{2a^{3}\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^{3}-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^{2}a+\sqrt{a+bx}\,(a+bx)a^{2}-\sqrt{a+bx}\,a^{3}\right)}{b^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x)
```

```
[Out] 2/15015*(429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)
*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*a^2/b^3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
```

```
4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)
^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x
 + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(13/2))/(13*b^4) - (2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a
 + b*x)^(9/2))/(3*b^4) - (6*a*(a + b*x)^(11/2))/(11*b^4)
```

3.301 $\int x^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}$$

```
[Out] 2/7*a^2*(b*x+a)^(7/2)/b^3-4/9*a*(b*x+a)^(9/2)/b^3+2/11*(b*x+a)^(11/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a + b
*x)^(11/2))/(11*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{5/2}\,dx &= \int \left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{7/2}\left(8a^2-28abx+63b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.38, size = 76, normalized size = 1.43

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(8a^5-4a^4bx+3a^3b^2x^2+b^3x^3\left(113a^2+161abx+63b^2x^2\right)\right)\sqrt{a+bx}}{693b^3},b!=0\right\}\right\},\frac{a^{\frac{5}{2}}x^3}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(a + b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (8 a ^ 5 - 4 a ^ 4 b x + 3 a ^ 3 b ^ 2 x ^ 2 + b ^ 3 x ^ 3 (1
13 a ^ 2 + 161 a b x + 63 b ^ 2 x ^ 2)) Sqrt[a + b x] / (693 b ^ 3), b != 0
}}, a ^ (5 / 2) x ^ 3 / 3]
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}\left(63x^2b^2-28abx+8a^2\right)}{693b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{4a(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^2(bx+a)^{\frac{7}{2}}}{7}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{4a(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^2(bx+a)^{\frac{7}{2}}}{7}}{b^3}$	38
trager	$\frac{2\left(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5\right)\sqrt{bx+a}}{693b^3}$	65
risch	$\frac{2\left(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5\right)\sqrt{bx+a}}{693b^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/11*(b*x+a)^(11/2)-2/9*a*(b*x+a)^(9/2)+1/7*a^2*(b*x+a)^(7/2))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^3}-\frac{4\,(bx+a)^{\frac{9}{2}}a}{9\,b^3}+\frac{2\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^3 - 4/9*(b*x + a)^(9/2)*a/b^3 + 2/7*(b*x + a)^(7/2)
*a^2/b^3
```

Fricas [A]
time = 0.31, size = 64, normalized size = 1.21

$$\frac{2\,(63\,b^5x^5 + 161\,ab^4x^4 + 113\,a^2b^3x^3 + 3\,a^3b^2x^2 - 4\,a^4bx + 8\,a^5)\sqrt{bx + a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Sympy [A]
time = 0.35, size = 124, normalized size = 2.34

$$\begin{cases} \frac{16a^5\sqrt{a+bx}}{693b^3} - \frac{8a^4x\sqrt{a+bx}}{693b^2} + \frac{2a^3x^2\sqrt{a+bx}}{231b} + \frac{226a^2x^3\sqrt{a+bx}}{693} + \frac{46abx^4\sqrt{a+bx}}{99} + \frac{2b^2x^5\sqrt{a+bx}}{11} & \text{for}\: b \neq 0 \\ \frac{a^{\frac{5}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**5*sqrt(a + b*x)/(693*b**3) - 8*a**4*x*sqrt(a + b*x)/(693*b
**2) + 2*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*a**2*x**3*sqrt(a + b*x)/693
+ 46*a*b*x**4*sqrt(a + b*x)/99 + 2*b**2*x**5*sqrt(a + b*x)/11, Ne(b, 0)), (
a**(5/2)*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(41) = 82.
time = 0.00, size = 391, normalized size = 7.38

$$\frac{\frac{2b^3\left(\frac{1}{11}\sqrt{a+bx}(a+bx)^5 - \frac{5}{9}\sqrt{a+bx}(a+bx)^4a + \frac{10}{7}\sqrt{a+bx}(a+bx)^3a^2 - 2\sqrt{a+bx}(a+bx)^2a^3 + \frac{5}{3}\sqrt{a+bx}(a+bx)a^4 - \sqrt{a+bx}a^5\right)}{b^5} + \frac{6ab^2\left(\frac{1}{9}\sqrt{a+bx}(a+bx)^4 - \frac{4}{7}\sqrt{a+bx}(a+bx)^3a + \frac{6}{5}\sqrt{a+bx}(a+bx)^2a^2 - \frac{4}{3}\sqrt{a+bx}(a+bx)a^3 + \sqrt{a+bx}a^4\right)}{b^4} + \frac{6a^2b\left(\frac{1}{7}\sqrt{a+bx}(a+bx)^3 - \frac{3}{5}\sqrt{a+bx}(a+bx)^2a + \sqrt{a+bx}(a+bx)a^2 - \sqrt{a+bx}a^3\right)}{b^3} + \frac{2a^3\left(\frac{1}{5}\sqrt{a+bx}(a+bx)^2 - \frac{2}{3}\sqrt{a+bx}(a+bx)a + \sqrt{a+bx}a^2\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x)
```

```
[Out] 2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*a^3/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*(35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a/b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{126\,{\left(a+b\,x\right)}^{11/2}-308\,a\,{\left(a+b\,x\right)}^{9/2}+198\,a^2\,{\left(a+b\,x\right)}^{7/2}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(5/2),x)
[Out] (126*(a + b*x)^(11/2) - 308*a*(a + b*x)^(9/2) + 198*a^2*(a + b*x)^(7/2))/(6
93*b^3)
```

3.302 $\int x(a+bx)^{5/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}$$

```
[Out] -2/7*a*(b*x+a)^(7/2)/b^2+2/9*(b*x+a)^(9/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{5/2}\,dx&=\int\left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}(-2a+7bx)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-2*a + 7*b*x))/(63*b^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.18, size = 64, normalized size = 1.88

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2a^4+a^3bx+b^2x^2\left(15a^2+19abx+7b^2x^2\right)\right)\sqrt{a+bx}}{63b^2}, b!{=}0\right\}\right\}, \frac{a^{\frac{5}{2}}x^2}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1*(a + b*x)^(5/2),x]')
[Out] Piecewise[{{2 (-2 a ^ 4 + a ^ 3 b x + b ^ 2 x ^ 2 (15 a ^ 2 + 19 a b x + 7
b ^ 2 x ^ 2)) Sqrt[a + b x] / (63 b ^ 2), b != 0}}, a ^ (5 / 2) x ^ 2 / 2]
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bx+2a)}{63b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(bx+a)^{\frac{7}{2}}}{7}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(bx+a)^{\frac{7}{2}}}{7}}{b^2}$	26
trager	$-\frac{2\left(-7b^4x^4-19a\,b^3x^3-15a^2b^2x^2-a^3bx+2a^4\right)\sqrt{bx+a}}{63b^2}$	54
risch	$-\frac{2\left(-7b^4x^4-19a\,b^3x^3-15a^2b^2x^2-a^3bx+2a^4\right)\sqrt{bx+a}}{63b^2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/9*(b*x+a)^(9/2)-1/7*a*(b*x+a)^(7/2))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.76

$$\frac{2\left(bx+a\right)^{\frac{9}{2}}}{9\,b^2}-\frac{2\left(bx+a\right)^{\frac{7}{2}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="maxima")
[Out] 2/9*(b*x + a)^(9/2)/b^2 - 2/7*(b*x + a)^(7/2)*a/b^2
```

Fricas [A]
time = 0.30, size = 52, normalized size = 1.53

$$\frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

Sympy [A]
time = 0.28, size = 102, normalized size = 3.00

$$\begin{cases} -\frac{4a^4\sqrt{a+bx}}{63b^2}+\frac{2a^3x\sqrt{a+bx}}{63b}+\frac{10a^2x^2\sqrt{a+bx}}{21}+\frac{38abx^3\sqrt{a+bx}}{63}+\frac{2b^2x^4\sqrt{a+bx}}{9} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a**4*sqrt(a + b*x)/(63*b**2) + 2*a**3*x*sqrt(a + b*x)/(63*b)
+ 10*a**2*x**2*sqrt(a + b*x)/21 + 38*a*b*x**3*sqrt(a + b*x)/63 + 2*b**2*x**
4*sqrt(a + b*x)/9, Ne(b, 0)), (a**(5/2)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(26) = 52.
time = 0.00, size = 297, normalized size = 8.74

$$\frac{\frac{2b^3\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3+\sqrt{a+bx}\,a^4\right)}{b^4}+\frac{6ab^2\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}+\frac{6a^2b\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}+\frac{2a^3\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx)-a\sqrt{a+bx}\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x)
```

```
[Out] 2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2/b + 27*(5*(b*x + a)^
(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^
3)*a/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{18\,a\,{\left(a+b\,x\right)}^{7/2}-14\,{\left(a+b\,x\right)}^{9/2}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^(7/2) - 14*(a + b*x)^(9/2))/(63*b^2)
```

3.303 $\int (a+bx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{7/2}}{7b}$$

```
[Out] 2/7*(b*x+a)^(7/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2),x]
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{5/2}\,dx = \frac{2(a+bx)^{7/2}}{7b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2),x]
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Mathics [A]
time = 1.58, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{7}{2}}}{7b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^(5/2),x]')
[Out] 2 (a + b x) ^ (7 / 2) / (7 b)
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
default	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
trager	$\frac{2(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)\sqrt{bx+a}}{7b}$	40
risch	$\frac{2(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)\sqrt{bx+a}}{7b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(b*x+a)^(7/2)/b
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="maxima")
[Out] 2/7*(b*x + a)^(7/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.
time = 0.30, size = 39, normalized size = 2.44

$$\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="fricas")
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2),x)
```

```
[Out] 2*(a + b*x)**(7/2)/(7*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(12) = 24.
time = 0.00, size = 199, normalized size = 12.44

$$\frac{\frac{2b^3\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}+\frac{6ab^2\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}+6a^2\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx)-a\sqrt{a+bx}\right)+2a^3\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x)
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 3
5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2 + 7*(3*(b*x + a)^(5/2) - 10*(b*
x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

3.304 $\int \frac{(a+bx)^{5/2}}{x} dx$

Optimal. Leaf size=65

$$2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} - 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[a + b*x] + (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 - 2*a
^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(a+bx)^{5/2}+a\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+a^2\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+a^3\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+\frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b} \\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.86

$$\frac{2}{15}\sqrt{a+bx}\,\left(23a^2+11abx+3b^2x^2\right)-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(23*a^2 + 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]]
```

Mathics [A]
time = 4.30, size = 88, normalized size = 1.35

$$\frac{\sqrt{a}\left(-30a^2\mathrm{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+15a^2\mathrm{Log}\left[\frac{bx}{a}\right]+46a^2\sqrt{\frac{a+bx}{a}}+22abx\sqrt{\frac{a+bx}{a}}+6b^2x^2\sqrt{\frac{a+bx}{a}}\right)}{15}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/x^1,x]')
```

```
[Out] Sqrt[a] (-30 a ^ 2 Log[1 + Sqrt[(a + b x) / a]] + 15 a ^ 2 Log[b x / a] + 4
6 a ^ 2 Sqrt[(a + b x) / a] + 22 a b x Sqrt[(a + b x) / a] + 6 b ^ 2 x ^ 2
Sqrt[(a + b x) / a]) / 15
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.77

method	result	size
derivativedivides	$\frac{2a(bx+a)^{\frac{3}{2}}}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^2\sqrt{bx+a}$	50
default	$\frac{2a(bx+a)^{\frac{3}{2}}}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^2\sqrt{bx+a}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.36, size = 64, normalized size = 0.98

$$a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}(bx+a)^{\frac{5}{2}}+\frac{2}{3}(bx+a)^{\frac{3}{2}}a+2\sqrt{bx+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] a^(5/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/5*(b*x
 + a)^(5/2) + 2/3*(b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

Fricas [A]
time = 0.31, size = 114, normalized size = 1.75

$$\left[a^{\frac{5}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a},\,2\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

Sympy [A]
time = 2.37, size = 97, normalized size = 1.49

$$\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{15}+a^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{22a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}\,b^2x^2\sqrt{1+\frac{bx}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x,x)
```

```
[Out] 46*a**(5/2)*sqrt(1 + b*x/a)/15 + a**(5/2)*log(b*x/a) - 2*a**(5/2)*log(sqrt(
1 + b*x/a) + 1) + 22*a**(3/2)*b*x*sqrt(1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*
sqrt(1 + b*x/a)/5
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.32

$$\frac{2}{5}\sqrt{a+bx}\,(a+bx)^2+\frac{2}{3}\sqrt{a+bx}\,(a+bx)\,a+2\sqrt{a+bx}\,a^2+\frac{4a^3\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x)
```

```
[Out] 2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/5*(b*x + a)^(5/2) + 2/3*(
b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.80

$$\frac{2\,a\,(a+b\,x)^{3/2}}{3}+\frac{2\,(a+b\,x)^{5/2}}{5}+2\,a^2\,\sqrt{a+b\,x}\,+a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 + 2*a^2*(a + b*x)^(1/2) + a
^(5/2)*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

3.305 $\int \frac{(a+bx)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=66

$$5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 5/3*b*(b*x+a)^(3/2)-(b*x+a)^(5/2)/x-5*a^(3/2)*b*arctanh((b*x+a)^(1/2)/a^(1/
2))+5*a*b*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] 5*a*b*Sqrt[a + b*x] + (5*b*(a + b*x)^(3/2))/3 - (a + b*x)^(5/2)/x - 5*a^(3/
2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/2}}{x^2}\,dx &= -\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(a+bx)^{3/2}}{x}\,dx\\ &= \frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5ab)\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.91

$$\frac{\sqrt{a+bx}\,\left(-3a^2+14abx+2b^2x^2\right)}{3x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a^2 + 14*a*b*x + 2*b^2*x^2))/(3*x) - 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]]
```

Mathics [A]
time = 4.19, size = 85, normalized size = 1.29

$$\frac{\sqrt{a}\left(-a^2\sqrt{\frac{a+bx}{a}}+\frac{bx\left(-30a\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+15a\text{Log}\left[\frac{bx}{a}\right]+28a\sqrt{\frac{a+bx}{a}}+4bx\sqrt{\frac{a+bx}{a}}\right)}{6}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/x^2,x]')
```

```
[Out] Sqrt[a] (-a ^ 2 Sqrt[(a + b x) / a] + b x (-30 a Log[1 + Sqrt[(a + b x) / a
]] + 15 a Log[b x / a] + 28 a Sqrt[(a + b x) / a] + 4 b x Sqrt[(a + b x) /
a]) / 6) / x
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.94

method	result	size
risch	$-\frac{a^{2} \sqrt{b x +a}}{x}+\frac{b \left(\frac{4 \left(b x +a \right)^{\frac{3}{2}}}{3}+8 a \sqrt{b x +a}-10 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)\right)}{2}$	57
derivativedivides	$2 b \left(\frac{\left(b x +a \right)^{\frac{3}{2}}}{3}+2 a \sqrt{b x +a}-a^{2} \left(\frac{\sqrt{b x +a}}{2 b x}+\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{2 \sqrt{a}}\right)\right)$	62
default	$2 b \left(\frac{\left(b x +a \right)^{\frac{3}{2}}}{3}+2 a \sqrt{b x +a}-a^{2} \left(\frac{\sqrt{b x +a}}{2 b x}+\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{2 \sqrt{a}}\right)\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/3*(b*x+a)^(3/2)+2*a*(b*x+a)^(1/2)-a^2*(1/2*(b*x+a)^(1/2)/b/x+5/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.35, size = 71, normalized size = 1.08

$$\frac{5}{2} a^{\frac{3}{2}} b \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3} (bx+a)^{\frac{3}{2}} b+4 \sqrt{bx+a}\, ab-\frac{\sqrt{bx+a}\, a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/2*a^(3/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
3*(b*x + a)^(3/2)*b + 4*sqrt(b*x + a)*a*b - sqrt(b*x + a)*a^2/x
```

Fricas [A]
time = 0.60, size = 126, normalized size = 1.91

$$\left[\frac{15\, a^{\frac{3}{2}} bx \log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\, a}{x}\right)+2\, (2\, b^{2} x^{2}+14\, abx-3\, a^{2})\sqrt{bx+a}}{6\, x},\ \frac{15\, \sqrt{-a}\, abx \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2\, b^{2} x^{2}+14\, abx-3\, a^{2})\sqrt{bx+a}}{3\, x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

Sympy [A]
time = 2.23, size = 99, normalized size = 1.50

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}-5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,b^2x\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x/a)/x + 14*a**(3/2)*b*sqrt(1 + b*x/a)/3 + 5*a**(3/2)*
b*log(b*x/a)/2 - 5*a**(3/2)*b*log(sqrt(1 + b*x/a) + 1) + 2*sqrt(a)*b**2*x*s
qrt(1 + b*x/a)/3
```

Giac [A]
time = 0.00, size = 100, normalized size = 1.52

$$\frac{\frac{2}{3}\sqrt{a+bx}\,(a+bx)\,b^2+4\sqrt{a+bx}\,b^2a+\frac{\sqrt{a+bx}\,b^2a^2}{-a-bx+a}+\frac{10b^2a^2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x)
```

```
[Out] 1/3*(15*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*(b*x + a)^(3/2)
*b^2 + 12*sqrt(b*x + a)*a*b^2 - 3*sqrt(b*x + a)*a^2*b/x)/b
```

Mupad [B]
time = 0.11, size = 58, normalized size = 0.88

$$\frac{2\,b\,{\left(a+b\,x\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{a+b\,x}}{x}+4\,a\,b\,\sqrt{a+b\,x}+a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(3/2))/3 - (a^2*(a + b*x)^(1/2))/x + a^(3/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*5i + 4*a*b*(a + b*x)^(1/2)
```

3.306 $\int \frac{(a+bx)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -5/4*b*(b*x+a)^(3/2)/x-1/2*(b*x+a)^(5/2)/x^2-15/4*b^2*arctanh((b*x+a)^(1/2)
/a^(1/2))*a^(1/2)+15/4*b^2*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{2x^2}-\frac{5b(a+bx)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x])/4 - (5*b*(a + b*x)^(3/2))/(4*x) - (a + b*x)^(5/2)/(2
*x^2) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^3}\,dx &= -\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(a+bx)^{3/2}}{x^2}\,dx \\
&= -\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(15ab)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+ \right. \\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.81

$$\frac{1}{4}\left(\frac{\sqrt{a+bx}\,\left(-2a^2-9abx+8b^2x^2\right)}{x^2}-15\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] ((Sqrt[a + b*x]*(-2*a^2 - 9*a*b*x + 8*b^2*x^2))/x^2 - 15*Sqrt[a]*b^2*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/4
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(78) = 156.
time = 4.57, size = 135, normalized size = 1.73

$$\frac{b^{\frac{3}{2}}\left(-2a^3x^2\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}-15\sqrt{a}\,\sqrt{b}\,x^{\frac{5}{2}}\mathrm{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right](a+bx)^2-11a^2bx^3\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}-ab^2x^4\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}+8b^3x^5\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}\right)}{4x^{\frac{5}{2}}(a+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/x^3,x]')
```

```
[Out] b ^ (3 / 2) (-2 a ^ 3 x ^ 2 ((a + b x) / (b x)) ^ (3 / 2) - 15 Sqrt[a] Sqrt
[b] x ^ (5 / 2) ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) ^ 2 - 11 a ^
 2 b x ^ 3 ((a + b x) / (b x)) ^ (3 / 2) - a b ^ 2 x ^ 4 ((a + b x) / (b x)
) ^ (3 / 2) + 8 b ^ 3 x ^ 5 ((a + b x) / (b x)) ^ (3 / 2)) / (4 x ^ (5 / 2)
 (a + b x) ^ 2)
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.79

method	result	size
risch	$-\frac{a\sqrt{bx+a}\,(9bx+2a)}{4x^{2}}+\frac{b^{2}\left(16\sqrt{bx+a}-30\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\right)}{8}$	55
derivativedivides	$2b^{2}\left(\sqrt{bx+a}-a\left(\frac{\frac{9(bx+a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	62
default	$2b^{2}\left(\sqrt{bx+a}-a\left(\frac{\frac{9(bx+a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx+a}}{8}}{b^{2}x^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((b*x+a)^(1/2)-a*((9/8*(b*x+a)^(3/2)-7/8*a*(b*x+a)^(1/2))/b^2/x^2+15/
8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.35, size = 101, normalized size = 1.29

$$\frac{15}{8}\sqrt{a}\,b^{2}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^{2}-\frac{9\,(bx+a)^{\frac{3}{2}}ab^{2}-7\sqrt{bx+a}\,a^{2}b^{2}}{4\left((bx+a)^{2}-2\,(bx+a)a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 15/8*sqrt(a)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2*sqrt(b*x + a)*b^2 - 1/4*(9*(b*x + a)^(3/2)*a*b^2 - 7*sqrt(b*x + a)*a^2*b
^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

Fricas [A]
time = 0.31, size = 133, normalized size = 1.71

$$\left[\frac{15\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(8\,b^{2}x^{2}-9\,abx-2\,a^{2})\sqrt{bx+a}}{8\,x^{2}},\ \frac{15\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(8\,b^{2}x^{2}-9\,abx-2\,a^{2})\sqrt{bx+a}}{4\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

Sympy [A]
time = 2.52, size = 126, normalized size = 1.62

$$-\frac{15\sqrt{a}\,b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4}-\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**3,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**3/(2*sqrt(b)*x**(5
/2)*sqrt(a/(b*x) + 1)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - a
*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(a/(b*x) +
 1)
```

Giac [A]
time = 0.00, size = 104, normalized size = 1.33

$$\frac{2\sqrt{a+bx}\,b^3-\frac{9\sqrt{a+bx}\,(a+bx)b^3a-7\sqrt{a+bx}\,b^3a^2}{4(a+bx-a)^2}+\frac{15b^3a\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x)
```

```
[Out] 1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x + a)*b^3
 - (9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x + a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.05, size = 64, normalized size = 0.82

$$2\,b^2\,\sqrt{a+b\,x}+\frac{7\,a^2\,\sqrt{a+b\,x}}{4\,x^2}-\frac{9\,a\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}+\frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,15{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(a + b*x)^(1/2) + (7*a^2*(a + b*x)^(1/2))/(4*x^2) + (a^(1/2)*b^2*atan
(((a + b*x)^(1/2)*1i)/a^(1/2))*15i)/4 - (9*a*(a + b*x)^(3/2))/(4*x^2)
```

3.307 $\int \frac{(a+bx)^{5/2}}{x^4} dx$

Optimal. Leaf size=81

$$-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

```
[Out] -5/12*b*(b*x+a)^(3/2)/x^2-1/3*(b*x+a)^(5/2)/x^3-5/8*b^3*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(1/2)-5/8*b^2*(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b(a+bx)^{3/2}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(8*x) - (5*b*(a + b*x)^(3/2))/(12*x^2) - (a + b*x)^(
5/2)/(3*x^3) - (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^4}\,dx &= -\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{6}(5b)\int \frac{(a+bx)^{3/2}}{x^3}\,dx \\
&= -\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^2}\,dx \\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{16}\left(5b^3\right)\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a
\end{aligned}
$$

$$
= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}
$$

Mathematica [**A**]
time = 0.12, size = 64, normalized size = 0.79

$$
-\frac{\sqrt{a+bx}\,\left(8a^2+26abx+33b^2x^2\right)}{24x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 + 26*a*b*x + 33*b^2*x^2))/x^3 - (5*b^3*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Mathics [**A**]
time = 4.64, size = 84, normalized size = 1.04

$$
-\frac{a^2\sqrt{b}\,\sqrt{1+\frac{a}{bx}}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{1+\frac{a}{bx}}}{12x^{\frac{3}{2}}}-\frac{5b^3\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right]}{8\sqrt{a}}-\frac{11b^{\frac{5}{2}}\sqrt{1+\frac{a}{bx}}}{8\sqrt{x}}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/x^4,x]')
```

```
[Out] -a ^ 2 Sqrt[b] Sqrt[1 + a / (b x)] / (3 x ^ (5 / 2)) - 13 a b ^ (3 / 2) Sqr
t[1 + a / (b x)] / (12 x ^ (3 / 2)) - 5 b ^ 3 ArcSinh[Sqrt[a] / (Sqrt[b] Sq
rt[x])] / (8 Sqrt[a]) - 11 b ^ (5 / 2) Sqrt[1 + a / (b x)] / (8 Sqrt[x])
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.79

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(33x^2b^2+26abx+8a^2)}{24x^3}-\frac{5b^3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}$	53
derivativedivides	$2b^3\left(-\frac{\frac{11(bx+a)^{\frac{5}{2}}}{16}-\frac{5a(bx+a)^{\frac{3}{2}}}{6}+\frac{5a^2\sqrt{bx+a}}{16}}{b^3x^3}-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}\right)$	64
default	$2b^3\left(-\frac{\frac{11(bx+a)^{\frac{5}{2}}}{16}-\frac{5a(bx+a)^{\frac{3}{2}}}{6}+\frac{5a^2\sqrt{bx+a}}{16}}{b^3x^3}-\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-(11/16*(b*x+a)^(5/2)-5/6*a*(b*x+a)^(3/2)+5/16*a^2*(b*x+a)^(1/2))/b^
3/x^3-5/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.34, size = 115, normalized size = 1.42

$$\frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,\sqrt{a}}-\frac{33\,(bx+a)^{\frac{5}{2}}b^3-40\,(bx+a)^{\frac{3}{2}}ab^3+15\,\sqrt{bx+a}\,a^2b^3}{24\,\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
 1/24*(33*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 15*sqrt(b*x + a)
*a^2*b^3)/((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

Fricas [A]
time = 0.32, size = 146, normalized size = 1.80

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,ax^3},\ \frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

Sympy [A]
time = 2.81, size = 104, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{12x^{\frac{3}{2}}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{8\sqrt{x}}-\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x**(5/2)) - 13*a*b**(3/2)*sqrt(a/(b*x) +
 1)/(12*x**(3/2)) - 11*b**(5/2)*sqrt(a/(b*x) + 1)/(8*sqrt(x)) - 5*b**3*asin
h(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a))
```

Giac [A]
time = 0.00, size = 110, normalized size = 1.36

$$\frac{\frac{-33\sqrt{a+bx}\,(a+bx)^2b^4+40\sqrt{a+bx}\,(a+bx)b^4a-15\sqrt{a+bx}\,b^4a^2}{24(a+bx-a)^3}+\frac{5b^4\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{8\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x)
```

```
[Out] 1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (33*(b*x + a)^(5/2)*
b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a^2*b^4)/(b^3*x^3))/b
```

Mupad [B]
time = 0.05, size = 64, normalized size = 0.79

$$\frac{5\,a\,{\left(a+b\,x\right)}^{3/2}}{3\,x^3}-\frac{5\,a^2\,\sqrt{a+b\,x}}{8\,x^3}-\frac{11\,{\left(a+b\,x\right)}^{5/2}}{8\,x^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^4,x)
```

```
[Out] (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(1/2)) - (5*a^2*(a + b*x)^
(1/2))/(8*x^3) - (11*(a + b*x)^(5/2))/(8*x^3) + (5*a*(a + b*x)^(3/2))/(3*x^
3)
```

3.308 $\int \frac{(a+bx)^{5/2}}{x^5} dx$

Optimal. Leaf size=103

$$-\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

```
[Out] -5/24*b*(b*x+a)^(3/2)/x^3-1/4*(b*x+a)^(5/2)/x^4+5/64*b^4*arctanh((b*x+a)^(1
/2)/a^(1/2))/a^(3/2)-5/32*b^2*(b*x+a)^(1/2)/x^2-5/64*b^3*(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 214}

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{5b(a+bx)^{3/2}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(32*x^2) - (5*b^3*Sqrt[a + b*x])/(64*a*x) - (5*b*(a
+ b*x)^(3/2))/(24*x^3) - (a + b*x)^(5/2)/(4*x^4) + (5*b^4*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^5}\,dx &= -\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{8}(5b)\int \frac{(a+bx)^{3/2}}{x^4}\,dx \\
&= -\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^3}\,dx \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{64}\left(5b^3\right)\int \frac{1}{x^2\sqrt{a+bx}}\,dx \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^4\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{128a} \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^3\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{64a} \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{64a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 78, normalized size = 0.76

$$-\frac{\sqrt{a+bx}\,\left(48a^3+136a^2bx+118ab^2x^2+15b^3x^3\right)}{192ax^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^5,x]
[Out] -1/192*(Sqrt[a + b*x]*(48*a^3 + 136*a^2*b*x + 118*a*b^2*x^2 + 15*b^3*x^3))/
(a*x^4) + (5*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(3/2))
```

Mathics [A]
time = 9.71, size = 145, normalized size = 1.41

$$\frac{\frac{-48a^{\frac{11}{2}}x^6(a+bx)^2}{b^2}-\frac{184a^{\frac{9}{2}}x^7(a+bx)^2}{b}-254a^{\frac{7}{2}}x^8\,(a+bx)^2-133a^{\frac{5}{2}}bx^9\,(a+bx)^2-15a^{\frac{3}{2}}b^2x^{10}\,(a+bx)^2+15ab^{\frac{9}{2}}x^{\frac{25}{2}}\operatorname{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right]\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}}{192a^{\frac{5}{2}}\sqrt{b}\,x^{\frac{25}{2}}\,\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/x^5,x]')
```

```
[Out] (-48 a ^ (11 / 2) x ^ 6 (a + b x) ^ 2 / b ^ 2 - 184 a ^ (9 / 2) x ^ 7 (a +
b x) ^ 2 / b - 254 a ^ (7 / 2) x ^ 8 (a + b x) ^ 2 - 133 a ^ (5 / 2) b x ^
9 (a + b x) ^ 2 - 15 a ^ (3 / 2) b ^ 2 x ^ 10 (a + b x) ^ 2 + 15 a b ^ (9 /
 2) x ^ (25 / 2) ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] ((a + b x) / (b x)) ^
 (5 / 2)) / (192 a ^ (5 / 2) Sqrt[b] x ^ (25 / 2) ((a + b x) / (b x)) ^ (5
/ 2))
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.74

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(15b^3x^3+118a\,b^2x^2+136a^2bx+48a^3)}{192x^4a}+\frac{5b^4\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64a^{\frac{3}{2}}}$	67
derivativedivides	$2b^4\left(-\frac{\frac{5(bx+a)^{\frac{7}{2}}}{128a}+\frac{73(bx+a)^{\frac{5}{2}}}{384}-\frac{55a(bx+a)^{\frac{3}{2}}}{384}+\frac{5a^2\sqrt{bx+a}}{128}}{b^4x^4}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}\right)$	76
default	$2b^4\left(-\frac{\frac{5(bx+a)^{\frac{7}{2}}}{128a}+\frac{73(bx+a)^{\frac{5}{2}}}{384}-\frac{55a(bx+a)^{\frac{3}{2}}}{384}+\frac{5a^2\sqrt{bx+a}}{128}}{b^4x^4}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}\right)$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4*(-(5/128/a*(b*x+a)^(7/2)+73/384*(b*x+a)^(5/2)-55/384*a*(b*x+a)^(3/2)+
5/128*a^2*(b*x+a)^(1/2))/b^4/x^4+5/128*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/
2))
```

Maxima [A]
time = 0.36, size = 144, normalized size = 1.40

$$-\frac{5\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}}-\frac{15\,(bx+a)^{\frac{7}{2}}b^4+73\,(bx+a)^{\frac{5}{2}}ab^4-55\,(bx+a)^{\frac{3}{2}}a^2b^4+15\,\sqrt{bx+a}\,a^3b^4}{192\,\left((bx+a)^4a-4\,(bx+a)^3a^2+6\,(bx+a)^2a^3-4\,(bx+a)a^4+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -5/128*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
 - 1/192*(15*(b*x + a)^(7/2)*b^4 + 73*(b*x + a)^(5/2)*a*b^4 - 55*(b*x + a)^
```

```
(3/2)*a^2*b^4 + 15*sqrt(b*x + a)*a^3*b^4)/((b*x + a)^4*a - 4*(b*x + a)^3*a^
2 + 6*(b*x + a)^2*a^3 - 4*(b*x + a)*a^4 + a^5)
```

Fricas [A]
time = 0.59, size = 167, normalized size = 1.62

$$\left[\frac{15\,\sqrt{a}\,b^4x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{384\,a^2x^4},-\frac{15\,\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{192\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

Sympy [A]
time = 7.65, size = 155, normalized size = 1.50

$$-\frac{a^3}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{127ab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{133b^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{7}{2}}}{64a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**5,x)
```

```
[Out] -a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*a**2*sqrt(b)/(24*x**(7/2)
*sqrt(a/(b*x) + 1)) - 127*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)) - 133*
b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(7/2)/(64*a*sqrt(x)*sqrt(a
/(b*x) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(3/2))
```

Giac [A]
time = 0.00, size = 142, normalized size = 1.38

$$\frac{\frac{-15\sqrt{a+bx}\,(a+bx)^3b^5-73\sqrt{a+bx}\,(a+bx)^2b^5a+55\sqrt{a+bx}\,(a+bx)b^5a^2-15\sqrt{a+bx}\,b^5a^3}{192a(a+bx-a)^4}-\frac{5b^5\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{32a\cdot 2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x)
```

```
[Out] -1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x + a)^
(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)^(3/2)*a^2*b^5 + 15*sqrt
(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

Mupad [B]
time = 0.11, size = 79, normalized size = 0.77

$$\frac{55\,a\,{\left(a+b\,x\right)}^{3/2}}{192\,x^4}-\frac{5\,a^2\,\sqrt{a+b\,x}}{64\,x^4}-\frac{5\,{\left(a+b\,x\right)}^{7/2}}{64\,a\,x^4}-\frac{73\,{\left(a+b\,x\right)}^{5/2}}{192\,x^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{64\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^5,x)
```

```
[Out] (55*a*(a + b*x)^(3/2))/(192*x^4) - (5*a^2*(a + b*x)^(1/2))/(64*x^4) - (5*(a
 + b*x)^(7/2))/(64*a*x^4) - (b^4*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(64
*a^(3/2)) - (73*(a + b*x)^(5/2))/(192*x^4)
```

3.309 $\int x^7(a+bx)^{9/2}\,dx$

Optimal. Leaf size=146

$$-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{21/2}}{b^8}$$

```
[Out] -2/11*a^7*(b*x+a)^(11/2)/b^8+14/13*a^6*(b*x+a)^(13/2)/b^8-14/5*a^5*(b*x+a)^
(15/2)/b^8+70/17*a^4*(b*x+a)^(17/2)/b^8-70/19*a^3*(b*x+a)^(19/2)/b^8+2*a^2*
(b*x+a)^(21/2)/b^8-14/23*a*(b*x+a)^(23/2)/b^8+2/25*(b*x+a)^(25/2)/b^8
```

Rubi [A]
time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{21/2}}{b^8}+\frac{2(a+bx)^{25/2}}{25b^8}-\frac{14a(a+bx)^{23/2}}{23b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) - (
14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (70
*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(a +
 b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-35a\right.\\ &= -\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-70a^3\end{aligned}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-2048a^7+11264a^6bx-36608a^5b^2x^2+91520a^4b^3x^3-194480a^3b^4x^4+369512a^2b^5x^5-646646ab^6x^6+1062347b^7x^7\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2048*a^7 + 11264*a^6*b*x - 36608*a^5*b^2*x^2 + 91520*
a^4*b^3*x^3 - 194480*a^3*b^4*x^4 + 369512*a^2*b^5*x^5 - 646646*a*b^6*x^6 +
1062347*b^7*x^7))/(26558675*b^8)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.27, size = 154, normalized size = 1.05

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2048a^{12}+1024a^{11}bx-768a^{10}b^2x^2+640a^9b^3x^3-560a^8b^4x^4+504a^7b^5x^5-462a^6b^6x^6+429a^5b^7x^7+143b^8x^8\left(11605a^4+40630a^3bx+54264a^2b^2x^2+32623ab^3x^3+7429b^4x^4\right)\right)\sqrt{a+bx}}{26558675b^8},b!=0\right\}\right\},\frac{a^{\frac{9}{2}}x^8}{8}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^7*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (-2048 a ^ 12 + 1024 a ^ 11 b x - 768 a ^ 10 b ^ 2 x ^ 2 + 64
0 a ^ 9 b ^ 3 x ^ 3 - 560 a ^ 8 b ^ 4 x ^ 4 + 504 a ^ 7 b ^ 5 x ^ 5 - 462 a
 ^ 6 b ^ 6 x ^ 6 + 429 a ^ 5 b ^ 7 x ^ 7 + 143 b ^ 8 x ^ 8 (11605 a ^ 4 + 4
0630 a ^ 3 b x + 54264 a ^ 2 b ^ 2 x ^ 2 + 32623 a b ^ 3 x ^ 3 + 7429 b ^ 4
 x ^ 4)) Sqrt[a + b x] / (26558675 b ^ 8), b != 0}}, a ^ (9 / 2) x ^ 8 / 8]
```

Maple [A]
time = 0.09, size = 97, normalized size = 0.66

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-1062347b^7x^7+646646a\,b^6x^6-369512a^2b^5x^5+194480a^3b^4x^4-91520a^4b^3x^3+36608a^5b^2x^2-11264a^6bx+\right.}{26558675b^8}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{25}{2}}}{25}-\frac{14a(bx+a)^{\frac{23}{2}}}{23}+2a^2(bx+a)^{\frac{21}{2}}-\frac{70a^3(bx+a)^{\frac{19}{2}}}{19}+\frac{70a^4(bx+a)^{\frac{17}{2}}}{17}-\frac{14a^5(bx+a)^{\frac{15}{2}}}{5}+\frac{14a^6(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^7(bx+a)^{\frac{11}{2}}}{11}}{b^8}$
default	$\frac{\frac{2(bx+a)^{\frac{25}{2}}}{25}-\frac{14a(bx+a)^{\frac{23}{2}}}{23}+2a^2(bx+a)^{\frac{21}{2}}-\frac{70a^3(bx+a)^{\frac{19}{2}}}{19}+\frac{70a^4(bx+a)^{\frac{17}{2}}}{17}-\frac{14a^5(bx+a)^{\frac{15}{2}}}{5}+\frac{14a^6(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^7(bx+a)^{\frac{11}{2}}}{11}}{b^8}$
trager	$-\frac{2\left(-1062347b^{12}x^{12}-4665089a\,b^{11}x^{11}-7759752a^2b^{10}x^{10}-5810090a^3b^9x^9-1659515a^4b^8x^8-429a^5b^7x^7+462a^6b^6x^6-\right.}{26558675b^8}$
risch	$-\frac{2\left(-1062347b^{12}x^{12}-4665089a\,b^{11}x^{11}-7759752a^2b^{10}x^{10}-5810090a^3b^9x^9-1659515a^4b^8x^8-429a^5b^7x^7+462a^6b^6x^6-\right.}{26558675b^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^8*(1/25*(b*x+a)^(25/2)-7/23*a*(b*x+a)^(23/2)+a^2*(b*x+a)^(21/2)-35/19*a
^3*(b*x+a)^(19/2)+35/17*a^4*(b*x+a)^(17/2)-7/5*a^5*(b*x+a)^(15/2)+7/13*a^6*
(b*x+a)^(13/2)-1/11*a^7*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.25, size = 116, normalized size = 0.79

$$\frac{2\,(bx+a)^{\frac{25}{2}}}{25\,b^8}-\frac{14\,(bx+a)^{\frac{23}{2}}a}{23\,b^8}+\frac{2\,(bx+a)^{\frac{21}{2}}a^2}{b^8}-\frac{70\,(bx+a)^{\frac{19}{2}}a^3}{19\,b^8}+\frac{70\,(bx+a)^{\frac{17}{2}}a^4}{17\,b^8}-\frac{14\,(bx+a)^{\frac{15}{2}}a^5}{5\,b^8}+\frac{14\,(bx+a)^{\frac{13}{2}}a^6}{13\,b^8}-\frac{2\,(bx+a)^{\frac{11}{2}}a^7}{11\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/25*(b*x + a)^(25/2)/b^8 - 14/23*(b*x + a)^(23/2)*a/b^8 + 2*(b*x + a)^(21/
2)*a^2/b^8 - 70/19*(b*x + a)^(19/2)*a^3/b^8 + 70/17*(b*x + a)^(17/2)*a^4/b^
8 - 14/5*(b*x + a)^(15/2)*a^5/b^8 + 14/13*(b*x + a)^(13/2)*a^6/b^8 - 2/11*(
b*x + a)^(11/2)*a^7/b^8
```

Fricas [**A**]
time = 0.30, size = 141, normalized size = 0.97

$$\frac{2\,(1062347\,b^{12}x^{12} + 4665089\,ab^{11}x^{11} + 7759752\,a^{2}b^{10}x^{10} + 5810090\,a^{3}b^{9}x^{9} + 1659515\,a^{4}b^{8}x^{8} + 429\,a^{5}b^{7}x^{7} - 462\,a^{6}b^{6}x^{6} + 504\,a^{7}b^{5}x^{5} - 560\,a^{8}b^{4}x^{4} + 640\,a^{9}b^{3}x^{3} - 768\,a^{10}b^{2}x^{2} + 1024\,a^{11}bx - 2048\,a^{12})\sqrt{bx+a}}{26558675\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/26558675*(1062347*b^12*x^12 + 4665089*a*b^11*x^11 + 7759752*a^2*b^10*x^10
 + 5810090*a^3*b^9*x^9 + 1659515*a^4*b^8*x^8 + 429*a^5*b^7*x^7 - 462*a^6*b^
6*x^6 + 504*a^7*b^5*x^5 - 560*a^8*b^4*x^4 + 640*a^9*b^3*x^3 - 768*a^10*b^2*
x^2 + 1024*a^11*b*x - 2048*a^12)*sqrt(b*x + a)/b^8
```

Sympy [**A**]
time = 1.67, size = 279, normalized size = 1.91

$$\begin{cases} - \frac{4096 a^{12} \sqrt{a + b x}}{26558675 b^{8}} + \frac{2048 a^{11} x \sqrt{a + b x}}{26558675 b^{7}} - \frac{1536 a^{10} x^{2} \sqrt{a + b x}}{26558675 b^{6}} + \frac{256 a^{9} x^{3} \sqrt{a + b x}}{5311735 b^{5}} - \frac{224 a^{8} x^{4} \sqrt{a + b x}}{5311735 b^{4}} + \frac{1008 a^{7} x^{5} \sqrt{a + b x}}{26558675 b^{3}} - \frac{84 a^{6} x^{6} \sqrt{a + b x}}{2414425 b^{2}} + \frac{6 a^{5} x^{7} \sqrt{a + b x}}{185725 b} + \frac{4642 a^{4} x^{8} \sqrt{a + b x}}{37145} + \frac{956 a^{3} b x^{9} \sqrt{a + b x}}{2185} + \frac{336 a^{2} b^{2} x^{10} \sqrt{a + b x}}{575} + \frac{202 a b^{3} x^{11} \sqrt{a + b x}}{575} + \frac{2 b^{4} x^{12} \sqrt{a + b x}}{25} & \text{for}\: b \neq 0 \\ \frac{a^{\frac{9}{2}} x^{8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4096*a**12*sqrt(a + b*x)/(26558675*b**8) + 2048*a**11*x*sqrt(a
+ b*x)/(26558675*b**7) - 1536*a**10*x**2*sqrt(a + b*x)/(26558675*b**6) + 25
6*a**9*x**3*sqrt(a + b*x)/(5311735*b**5) - 224*a**8*x**4*sqrt(a + b*x)/(531
1735*b**4) + 1008*a**7*x**5*sqrt(a + b*x)/(26558675*b**3) - 84*a**6*x**6*sq
rt(a + b*x)/(2414425*b**2) + 6*a**5*x**7*sqrt(a + b*x)/(185725*b) + 4642*a*
*4*x**8*sqrt(a + b*x)/37145 + 956*a**3*b*x**9*sqrt(a + b*x)/2185 + 336*a**2
*b**2*x**10*sqrt(a + b*x)/575 + 202*a*b**3*x**11*sqrt(a + b*x)/575 + 2*b**4
*x**12*sqrt(a + b*x)/25, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(116) = 232.
time = 0.01, size = 1417, normalized size = 9.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x)
```

```
[Out] 2/1673196525*(260015*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a^5/b^7 + 76475*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^4/b^7 + 72450*(12
155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*
a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230
*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2
)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a^3/b^7 + 17
250*(46189*(b*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)
^(17/2)*a^2 - 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4
- 22221108*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(
b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*
a^9 + 969969*sqrt(b*x + a)*a^10)*a^2/b^7 + 4125*(88179*(b*x + a)^(23/2) - 1
062347*(b*x + a)^(21/2)*a + 5870865*(b*x + a)^(19/2)*a^2 - 19684665*(b*x +
a)^(17/2)*a^3 + 44618574*(b*x + a)^(15/2)*a^4 - 72076158*(b*x + a)^(13/2)*a
^5 + 85180914*(b*x + a)^(11/2)*a^6 - 74364290*(b*x + a)^(9/2)*a^7 + 4780561
5*(b*x + a)^(7/2)*a^8 - 22309287*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3
/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)*a/b^7 + 99*(676039*(b*x + a)^(25/2)
- 8817900*(b*x + a)^(23/2)*a + 53117350*(b*x + a)^(21/2)*a^2 - 195695500*(b
*x + a)^(19/2)*a^3 + 492116625*(b*x + a)^(17/2)*a^4 - 892371480*(b*x + a)^(
15/2)*a^5 + 1201269300*(b*x + a)^(13/2)*a^6 - 1216870200*(b*x + a)^(11/2)*a
^7 + 929553625*(b*x + a)^(9/2)*a^8 - 531173500*(b*x + a)^(7/2)*a^9 + 223092
870*(b*x + a)^(5/2)*a^10 - 67603900*(b*x + a)^(3/2)*a^11 + 16900975*sqrt(b*
x + a)*a^12)/b^7)/b
```

Mupad [B]
time = 0.04, size = 116, normalized size = 0.79

$$\frac{2\,{\left(a+b\,x\right)}^{25/2}}{25\,b^8}-\frac{2\,a^7\,{\left(a+b\,x\right)}^{11/2}}{11\,b^8}+\frac{14\,a^6\,{\left(a+b\,x\right)}^{13/2}}{13\,b^8}-\frac{14\,a^5\,{\left(a+b\,x\right)}^{15/2}}{5\,b^8}+\frac{70\,a^4\,{\left(a+b\,x\right)}^{17/2}}{17\,b^8}-\frac{70\,a^3\,{\left(a+b\,x\right)}^{19/2}}{19\,b^8}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{21/2}}{b^8}-\frac{14\,a\,{\left(a+b\,x\right)}^{23/2}}{23\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(25/2))/(25*b^8) - (2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6
*(a + b*x)^(13/2))/(13*b^8) - (14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(
a + b*x)^(17/2))/(17*b^8) - (70*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a
+ b*x)^(21/2))/b^8 - (14*a*(a + b*x)^(23/2))/(23*b^8)
```

3.310 $\int x^6(a+bx)^{9/2}\,dx$

Optimal. Leaf size=127

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}+$$

```
[Out] 2/11*a^6*(b*x+a)^(11/2)/b^7-12/13*a^5*(b*x+a)^(13/2)/b^7+2*a^4*(b*x+a)^(15/
2)/b^7-40/17*a^3*(b*x+a)^(17/2)/b^7+30/19*a^2*(b*x+a)^(19/2)/b^7-4/7*a*(b*x
+a)^(21/2)/b^7+2/23*(b*x+a)^(23/2)/b^7
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (2
*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2*(
a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x)^(
23/2))/(23*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^{9/2}\,dx&=\int\left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+15a^2(\right.\\&=\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a-}{19}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(1024a^6-5632a^5bx+18304a^4b^2x^2-45760a^3b^3x^3+97240a^2b^4x^4-184756ab^5x^5+323323b^6x^6\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(1024*a^6 - 5632*a^5*b*x + 18304*a^4*b^2*x^2 - 45760*a^
3*b^3*x^3 + 97240*a^2*b^4*x^4 - 184756*a*b^5*x^5 + 323323*b^6*x^6))/(743642
9*b^7)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.05, size = 143, normalized size = 1.13

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(1024a^{11}-512a^{10}bx+384a^{9}b^{2}x^{2}-320a^{8}b^{3}x^{3}+280a^{7}b^{4}x^{4}-252a^{6}b^{5}x^{5}+231a^{5}b^{6}x^{6}+143b^{7}x^{7}\left(3713a^{4}+12770a^{3}bx+16830a^{2}b^{2}x^{2}+10013ab^{3}x^{3}+2261b^{4}x^{4}\right)\right)\sqrt{a+bx}}{7436429b^{7}},b!=0\right\}\right\},\frac{a^{\frac{9}{2}}x^{7}}{7}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^6*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (1024 a ^ 11 - 512 a ^ 10 b x + 384 a ^ 9 b ^ 2 x ^ 2 - 320 a
 ^ 8 b ^ 3 x ^ 3 + 280 a ^ 7 b ^ 4 x ^ 4 - 252 a ^ 6 b ^ 5 x ^ 5 + 231 a ^
5 b ^ 6 x ^ 6 + 143 b ^ 7 x ^ 7 (3713 a ^ 4 + 12770 a ^ 3 b x + 16830 a ^ 2
 b ^ 2 x ^ 2 + 10013 a b ^ 3 x ^ 3 + 2261 b ^ 4 x ^ 4)) Sqrt[a + b x] / (74
36429 b ^ 7), b != 0}}, a ^ (9 / 2) x ^ 7 / 7]
```

Maple [A]
time = 0.08, size = 85, normalized size = 0.67

method	result
gosper	$\frac{2(bx+a)^{\frac{11}{2}}\left(323323x^{6}b^{6}-184756a\,x^{5}b^{5}+97240a^{2}x^{4}b^{4}-45760a^{3}b^{3}x^{3}+18304a^{4}x^{2}b^{2}-5632a^{5}xb+1024a^{6}\right)}{7436429b^{7}}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{23}{2}}}{23}-\frac{4a(bx+a)^{\frac{21}{2}}}{7}+\frac{30a^{2}(bx+a)^{\frac{19}{2}}}{19}-\frac{40a^{3}(bx+a)^{\frac{17}{2}}}{17}+2a^{4}(bx+a)^{\frac{15}{2}}-\frac{12a^{5}(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^{6}(bx+a)^{\frac{11}{2}}}{11}}{b^{7}}$
default	$\frac{\frac{2(bx+a)^{\frac{23}{2}}}{23}-\frac{4a(bx+a)^{\frac{21}{2}}}{7}+\frac{30a^{2}(bx+a)^{\frac{19}{2}}}{19}-\frac{40a^{3}(bx+a)^{\frac{17}{2}}}{17}+2a^{4}(bx+a)^{\frac{15}{2}}-\frac{12a^{5}(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^{6}(bx+a)^{\frac{11}{2}}}{11}}{b^{7}}$
trager	$\frac{2(323323b^{11}x^{11}+1431859a\,b^{10}x^{10}+2406690a^{2}b^{9}x^{9}+1826110a^{3}b^{8}x^{8}+530959a^{4}b^{7}x^{7}+231a^{5}b^{6}x^{6}-252a^{6}b^{5}x^{5}+280a^{7}b^{4}x}{7436429b^{7}}$
risch	$\frac{2(323323b^{11}x^{11}+1431859a\,b^{10}x^{10}+2406690a^{2}b^{9}x^{9}+1826110a^{3}b^{8}x^{8}+530959a^{4}b^{7}x^{7}+231a^{5}b^{6}x^{6}-252a^{6}b^{5}x^{5}+280a^{7}b^{4}x}{7436429b^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^7*(1/23*(b*x+a)^(23/2)-2/7*a*(b*x+a)^(21/2)+15/19*a^2*(b*x+a)^(19/2)-20
/17*a^3*(b*x+a)^(17/2)+a^4*(b*x+a)^(15/2)-6/13*a^5*(b*x+a)^(13/2)+1/11*a^6*
(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 101, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{23}{2}}}{23\,b^7}-\frac{4\,(bx+a)^{\frac{21}{2}}a}{7\,b^7}+\frac{30\,(bx+a)^{\frac{19}{2}}a^2}{19\,b^7}-\frac{40\,(bx+a)^{\frac{17}{2}}a^3}{17\,b^7}+\frac{2\,(bx+a)^{\frac{15}{2}}a^4}{b^7}-\frac{12\,(bx+a)^{\frac{13}{2}}a^5}{13\,b^7}+\frac{2\,(bx+a)^{\frac{11}{2}}a^6}{11\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/23*(b*x + a)^(23/2)/b^7 - 4/7*(b*x + a)^(21/2)*a/b^7 + 30/19*(b*x + a)^(1
9/2)*a^2/b^7 - 40/17*(b*x + a)^(17/2)*a^3/b^7 + 2*(b*x + a)^(15/2)*a^4/b^7
- 12/13*(b*x + a)^(13/2)*a^5/b^7 + 2/11*(b*x + a)^(11/2)*a^6/b^7
```

Fricas [A]
time = 0.31, size = 130, normalized size = 1.02

$$\frac{2\,(323323\,b^{11}x^{11}+1431859\,ab^{10}x^{10}+2406690\,a^{2}b^{9}x^{9}+1826110\,a^{3}b^{8}x^{8}+530959\,a^{4}b^{7}x^{7}+231\,a^{5}b^{6}x^{6}-252\,a^{6}b^{5}x^{5}+280\,a^{7}b^{4}x^{4}-320\,a^{8}b^{3}x^{3}+384\,a^{9}b^{2}x^{2}-512\,a^{10}bx+1024\,a^{11})\sqrt{bx+a}}{7436429\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/7436429*(323323*b^11*x^11 + 1431859*a*b^10*x^10 + 2406690*a^2*b^9*x^9 + 1
826110*a^3*b^8*x^8 + 530959*a^4*b^7*x^7 + 231*a^5*b^6*x^6 - 252*a^6*b^5*x^5
 + 280*a^7*b^4*x^4 - 320*a^8*b^3*x^3 + 384*a^9*b^2*x^2 - 512*a^10*b*x + 102
4*a^11)*sqrt(b*x + a)/b^7
```

Sympy [A]
time = 1.50, size = 257, normalized size = 2.02

$$\begin{cases} \frac{2048a^{11}\sqrt{a+bx}}{7436429b^{7}}-\frac{1024a^{10}x\sqrt{a+bx}}{7436429b^{6}}+\frac{768a^{9}x^{2}\sqrt{a+bx}}{7436429b^{5}}-\frac{640a^{8}x^{3}\sqrt{a+bx}}{7436429b^{4}}+\frac{80a^{7}x^{4}\sqrt{a+bx}}{1062347b^{3}}-\frac{72a^{6}x^{5}\sqrt{a+bx}}{1062347b^{2}}+\frac{6a^{5}x^{6}\sqrt{a+bx}}{96577b}+\frac{7426a^{4}x^{7}\sqrt{a+bx}}{52003}+\frac{25540a^{3}bx^{8}\sqrt{a+bx}}{52003}+\frac{1980a^{2}b^{2}x^{9}\sqrt{a+bx}}{3059}+\frac{62ab^{3}x^{10}\sqrt{a+bx}}{161}+\frac{2b^{4}x^{11}\sqrt{a+bx}}{23} & \text{for}\ b\neq 0\\ \frac{a^{\frac{9}{2}}x^{7}}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((2048*a**11*sqrt(a + b*x)/(7436429*b**7) - 1024*a**10*x*sqrt(a +
b*x)/(7436429*b**6) + 768*a**9*x**2*sqrt(a + b*x)/(7436429*b**5) - 640*a**8
*x**3*sqrt(a + b*x)/(7436429*b**4) + 80*a**7*x**4*sqrt(a + b*x)/(1062347*b*
*3) - 72*a**6*x**5*sqrt(a + b*x)/(1062347*b**2) + 6*a**5*x**6*sqrt(a + b*x)
/(96577*b) + 7426*a**4*x**7*sqrt(a + b*x)/52003 + 25540*a**3*b*x**8*sqrt(a
+ b*x)/52003 + 1980*a**2*b**2*x**9*sqrt(a + b*x)/3059 + 62*a*b**3*x**10*sqr
t(a + b*x)/161 + 2*b**4*x**11*sqrt(a + b*x)/23, Ne(b, 0)), (a**(9/2)*x**7/7
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(101) = 202.
time = 0.01, size = 1278, normalized size = 10.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x)
```

[Out] 2/66927861*(22287*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b *x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6 006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^5/b^6 + 52003*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025 *(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^ 5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^4/b^6 + 6118*(643 5*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 1 09395*sqrt(b*x + a)*a^8)*a^3/b^6 + 2898*(12155*(b*x + a)^(19/2) - 122265*(b *x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a ^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a ^8 - 230945*sqrt(b*x + a)*a^9)*a^2/b^6 + 345*(46189*(b*x + a)^(21/2) - 5105 10*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x + a)^(1 5/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^10)*a /b^6 + 33*(88179*(b*x + a)^(23/2) - 1062347*(b*x + a)^(21/2)*a + 5870865*(b *x + a)^(19/2)*a^2 - 19684665*(b*x + a)^(17/2)*a^3 + 44618574*(b*x + a)^(15 /2)*a^4 - 72076158*(b*x + a)^(13/2)*a^5 + 85180914*(b*x + a)^(11/2)*a^6 - 7 4364290*(b*x + a)^(9/2)*a^7 + 47805615*(b*x + a)^(7/2)*a^8 - 22309287*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3/2)*a^10 - 2028117*sqrt(b*x + a)*a^11) /b^6)/b

Mupad [B]
time = 0.03, size = 101, normalized size = 0.80

$$\frac{2\,{\left(a+b\,x\right)}^{23/2}}{23\,b^7}+\frac{2\,a^6\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7}-\frac{12\,a^5\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{15/2}}{b^7}-\frac{40\,a^3\,{\left(a+b\,x\right)}^{17/2}}{17\,b^7}+\frac{30\,a^2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^7}-\frac{4\,a\,{\left(a+b\,x\right)}^{21/2}}{7\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(a + b*x)^(9/2),x)

[Out] (2*(a + b*x)^(23/2))/(23*b^7) + (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5 *(a + b*x)^(13/2))/(13*b^7) + (2*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b *x)^(17/2))/(17*b^7) + (30*a^2*(a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^ (21/2))/(7*b^7)

3.311 $\int x^5(a+bx)^{9/2}\,dx$

Optimal. Leaf size=110

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}+\frac{2(a+bx)^{21/2}}{21b^6}$$

```
[Out] -2/11*a^5*(b*x+a)^(11/2)/b^6+10/13*a^4*(b*x+a)^(13/2)/b^6-4/3*a^3*(b*x+a)^(
15/2)/b^6+20/17*a^2*(b*x+a)^(17/2)/b^6-10/19*a*(b*x+a)^(19/2)/b^6+2/21*(b*x
+a)^(21/2)/b^6
```

Rubi [A]
time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (
4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*
a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5(a+bx)^{9/2}\,dx=\int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\frac{5a(\ldots}{}\right.$$
$$=-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a\ldots}{1\ldots}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-256a^5+1408a^4bx-4576a^3b^2x^2+11440a^2b^3x^3-24310ab^4x^4+46189b^5x^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-256*a^5 + 1408*a^4*b*x - 4576*a^3*b^2*x^2 + 11440*a^2
*b^3*x^3 - 24310*a*b^4*x^4 + 46189*b^5*x^5))/(969969*b^6)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.78, size = 132, normalized size = 1.20

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-256a^{10}+128a^9bx-96a^8b^2x^2+80a^7b^3x^3-70a^6b^4x^4+63a^5b^5x^5+11b^6x^6\left(7343a^4+24674a^3bx+31980a^2b^2x^2+18785ab^3x^3+4199b^4x^4\right)\right)\sqrt{a+bx}}{969969b^6},b!=0\right\}\right\},\frac{a^{\frac{9}{2}}x^6}{6}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^5*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (-256 a ^ 10 + 128 a ^ 9 b x - 96 a ^ 8 b ^ 2 x ^ 2 + 80 a ^
7 b ^ 3 x ^ 3 - 70 a ^ 6 b ^ 4 x ^ 4 + 63 a ^ 5 b ^ 5 x ^ 5 + 11 b ^ 6 x ^
6 (7343 a ^ 4 + 24674 a ^ 3 b x + 31980 a ^ 2 b ^ 2 x ^ 2 + 18785 a b ^ 3 x
 ^ 3 + 4199 b ^ 4 x ^ 4)) Sqrt[a + b x] / (969969 b ^ 6), b != 0}}, a ^ (9
/ 2) x ^ 6 / 6]
```

Maple [A]
time = 0.09, size = 74, normalized size = 0.67

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-46189b^5x^5+24310a\,b^4x^4-11440a^2b^3x^3+4576a^3b^2x^2-1408a^4bx+256a^5\right)}{969969b^6}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{21}{2}}}{21}-\frac{10a(bx+a)^{\frac{19}{2}}}{19}+\frac{20a^2(bx+a)^{\frac{17}{2}}}{17}-\frac{4a^3(bx+a)^{\frac{15}{2}}}{3}+\frac{10a^4(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^5(bx+a)^{\frac{11}{2}}}{11}}{b^6}$
default	$\frac{\frac{2(bx+a)^{\frac{21}{2}}}{21}-\frac{10a(bx+a)^{\frac{19}{2}}}{19}+\frac{20a^2(bx+a)^{\frac{17}{2}}}{17}-\frac{4a^3(bx+a)^{\frac{15}{2}}}{3}+\frac{10a^4(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^5(bx+a)^{\frac{11}{2}}}{11}}{b^6}$
trager	$-\frac{2(-46189b^{10}x^{10}-206635a\,b^9x^9-351780a^2b^8x^8-271414a^3b^7x^7-80773a^4b^6x^6-63a^5b^5x^5+70a^6b^4x^4-80a^7b^3x^3+96a^8}{969969b^6}$
risch	$-\frac{2(-46189b^{10}x^{10}-206635a\,b^9x^9-351780a^2b^8x^8-271414a^3b^7x^7-80773a^4b^6x^6-63a^5b^5x^5+70a^6b^4x^4-80a^7b^3x^3+96a^8}{969969b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^6*(1/21*(b*x+a)^(21/2)-5/19*a*(b*x+a)^(19/2)+10/17*a^2*(b*x+a)^(17/2)-2
/3*a^3*(b*x+a)^(15/2)+5/13*a^4*(b*x+a)^(13/2)-1/11*a^5*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 86, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{21}{2}}}{21\,b^6}-\frac{10\,(bx+a)^{\frac{19}{2}}a}{19\,b^6}+\frac{20\,(bx+a)^{\frac{17}{2}}a^2}{17\,b^6}-\frac{4\,(bx+a)^{\frac{15}{2}}a^3}{3\,b^6}+\frac{10\,(bx+a)^{\frac{13}{2}}a^4}{13\,b^6}-\frac{2\,(bx+a)^{\frac{11}{2}}a^5}{11\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x + a)^(21/2)/b^6 - 10/19*(b*x + a)^(19/2)*a/b^6 + 20/17*(b*x + a)^
(17/2)*a^2/b^6 - 4/3*(b*x + a)^(15/2)*a^3/b^6 + 10/13*(b*x + a)^(13/2)*a^4/
b^6 - 2/11*(b*x + a)^(11/2)*a^5/b^6
```

Fricas [A]
time = 0.31, size = 119, normalized size = 1.08

$$\frac{2\,(46189\,b^{10}x^{10}+206635\,ab^{9}x^{9}+351780\,a^{2}b^{8}x^{8}+271414\,a^{3}b^{7}x^{7}+80773\,a^{4}b^{6}x^{6}+63\,a^{5}b^{5}x^{5}-70\,a^{6}b^{4}x^{4}+80\,a^{7}b^{3}x^{3}-96\,a^{8}b^{2}x^{2}+128\,a^{9}bx-256\,a^{10})\sqrt{bx+a}}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(46189*b^10*x^10 + 206635*a*b^9*x^9 + 351780*a^2*b^8*x^8 + 271414*
a^3*b^7*x^7 + 80773*a^4*b^6*x^6 + 63*a^5*b^5*x^5 - 70*a^6*b^4*x^4 + 80*a^7*
b^3*x^3 - 96*a^8*b^2*x^2 + 128*a^9*b*x - 256*a^10)*sqrt(b*x + a)/b^6
```

Sympy [A]
time = 1.34, size = 235, normalized size = 2.14

$$\begin{cases} -\frac{512a^{10}\sqrt{a+bx}}{969969b^{6}}+\frac{256a^{9}x\sqrt{a+bx}}{969969b^{5}}-\frac{64a^{8}x^{2}\sqrt{a+bx}}{323323b^{4}}+\frac{160a^{7}x^{3}\sqrt{a+bx}}{969969b^{3}}-\frac{20a^{6}x^{4}\sqrt{a+bx}}{138567b^{2}}+\frac{6a^{5}x^{5}\sqrt{a+bx}}{46189b}+\frac{2098a^{4}x^{6}\sqrt{a+bx}}{12597}+\frac{3796a^{3}bx^{7}\sqrt{a+bx}}{6783}+\frac{1640a^{2}b^{2}x^{8}\sqrt{a+bx}}{2261}+\frac{170ab^{3}x^{9}\sqrt{a+bx}}{399}+\frac{2b^{4}x^{10}\sqrt{a+bx}}{21} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{9}{2}}x^{6}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-512*a**10*sqrt(a + b*x)/(969969*b**6) + 256*a**9*x*sqrt(a + b*x
)/(969969*b**5) - 64*a**8*x**2*sqrt(a + b*x)/(323323*b**4) + 160*a**7*x**3*
sqrt(a + b*x)/(969969*b**3) - 20*a**6*x**4*sqrt(a + b*x)/(138567*b**2) + 6*
a**5*x**5*sqrt(a + b*x)/(46189*b) + 2098*a**4*x**6*sqrt(a + b*x)/12597 + 37
96*a**3*b*x**7*sqrt(a + b*x)/6783 + 1640*a**2*b**2*x**8*sqrt(a + b*x)/2261
+ 170*a*b**3*x**9*sqrt(a + b*x)/399 + 2*b**4*x**10*sqrt(a + b*x)/21, Ne(b,
0)), (a**(9/2)*x**6/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(86) = 172.
time = 0.00, size = 1141, normalized size = 10.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x)
```

```
[Out] 2/2909907*(4199*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
```

```
t(b*x + a)*a^5)*a^5/b^5 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2
)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^
(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^4/b^5 + 45
22*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)
*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^3/b^5
 + 266*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)
^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 87
5160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3
/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2/b^5 + 63*(12155*(b*x + a)^(19/2) -
122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)
^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 +
2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a
)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a/b^5 + 3*(46189*(b*x + a)^(21/2) -
 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x +
a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a
^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*
(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^
10)/b^5)/b
```

Mupad [B]
time = 0.03, size = 86, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{21/2}}{21\,b^6}-\frac{2\,a^5\,{\left(a+b\,x\right)}^{11/2}}{11\,b^6}+\frac{10\,a^4\,{\left(a+b\,x\right)}^{13/2}}{13\,b^6}-\frac{4\,a^3\,{\left(a+b\,x\right)}^{15/2}}{3\,b^6}+\frac{20\,a^2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^6}-\frac{10\,a\,{\left(a+b\,x\right)}^{19/2}}{19\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(21/2))/(21*b^6) - (2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4
*(a + b*x)^(13/2))/(13*b^6) - (4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a
 + b*x)^(17/2))/(17*b^6) - (10*a*(a + b*x)^(19/2))/(19*b^6)
```

3.312 $\int x^4(a+bx)^{9/2}\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19/2}}{19b^5}$$

```
[Out] 2/11*a^4*(b*x+a)^(11/2)/b^5-8/13*a^3*(b*x+a)^(13/2)/b^5+4/5*a^2*(b*x+a)^(15
/2)/b^5-8/17*a*(b*x+a)^(17/2)/b^5+2/19*(b*x+a)^(19/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4*
a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a + b
*x)^(19/2))/(19*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{9/2}\,dx&=\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a+bx)^{1}}{b^4}\right.\\&=\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19}}{19b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.63

$$\frac{2(a+bx)^{11/2}\left(128a^4-704a^3bx+2288a^2b^2x^2-5720ab^3x^3+12155b^4x^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(128*a^4 - 704*a^3*b*x + 2288*a^2*b^2*x^2 - 5720*a*b^3*
x^3 + 12155*b^4*x^4))/(230945*b^5)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.55, size = 120, normalized size = 1.32

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(128a^9-64a^8bx+48a^7b^2x^2-40a^6b^3x^3+35a^5b^4x^4+b^5x^5\left(23063a^4+75086a^3bx+95238a^2b^2x^2+55055ab^3x^3+12155b^4x^4\right)\right)\sqrt{a+bx}}{230945b^5},b!=0\right\}\right\},\frac{a^{\frac{9}{2}}x^5}{5}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (128 a ^ 9 - 64 a ^ 8 b x + 48 a ^ 7 b ^ 2 x ^ 2 - 40 a ^ 6 b
 ^ 3 x ^ 3 + 35 a ^ 5 b ^ 4 x ^ 4 + b ^ 5 x ^ 5 (23063 a ^ 4 + 75086 a ^ 3
b x + 95238 a ^ 2 b ^ 2 x ^ 2 + 55055 a b ^ 3 x ^ 3 + 12155 b ^ 4 x ^ 4)) S
qrt[a + b x] / (230945 b ^ 5), b != 0}}, a ^ (9 / 2) x ^ 5 / 5]
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.68

method	result
gosper	$\frac{2(bx+a)^{\frac{11}{2}}\left(12155b^4x^4-5720a\,b^3x^3+2288a^2b^2x^2-704a^3bx+128a^4\right)}{230945b^5}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{19}{2}}}{19}-\frac{8a(bx+a)^{\frac{17}{2}}}{17}+\frac{4a^2(bx+a)^{\frac{15}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^4(bx+a)^{\frac{11}{2}}}{11}}{b^5}$
default	$\frac{\frac{2(bx+a)^{\frac{19}{2}}}{19}-\frac{8a(bx+a)^{\frac{17}{2}}}{17}+\frac{4a^2(bx+a)^{\frac{15}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^4(bx+a)^{\frac{11}{2}}}{11}}{b^5}$
trager	$\frac{2(12155b^9x^9+55055a\,b^8x^8+95238a^2b^7x^7+75086a^3b^6x^6+23063a^4b^5x^5+35a^5b^4x^4-40a^6b^3x^3+48a^7b^2x^2-64a^8bx+128a^9}{230945b^5}$
risch	$\frac{2(12155b^9x^9+55055a\,b^8x^8+95238a^2b^7x^7+75086a^3b^6x^6+23063a^4b^5x^5+35a^5b^4x^4-40a^6b^3x^3+48a^7b^2x^2-64a^8bx+128a^9}{230945b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/19*(b*x+a)^(19/2)-4/17*a*(b*x+a)^(17/2)+2/5*a^2*(b*x+a)^(15/2)-4/1
3*a^3*(b*x+a)^(13/2)+1/11*a^4*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.26, size = 71, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{19}{2}}}{19\,b^5}-\frac{8\,(bx+a)^{\frac{17}{2}}\,a}{17\,b^5}+\frac{4\,(bx+a)^{\frac{15}{2}}\,a^2}{5\,b^5}-\frac{8\,(bx+a)^{\frac{13}{2}}\,a^3}{13\,b^5}+\frac{2\,(bx+a)^{\frac{11}{2}}\,a^4}{11\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/19*(b*x + a)^(19/2)/b^5 - 8/17*(b*x + a)^(17/2)*a/b^5 + 4/5*(b*x + a)^(15
/2)*a^2/b^5 - 8/13*(b*x + a)^(13/2)*a^3/b^5 + 2/11*(b*x + a)^(11/2)*a^4/b^5
```

Fricas [A]
time = 0.32, size = 108, normalized size = 1.19

$$\frac{2\,(12155\,b^9x^9+55055\,ab^8x^8+95238\,a^2b^7x^7+75086\,a^3b^6x^6+23063\,a^4b^5x^5+35\,a^5b^4x^4-40\,a^6b^3x^3+48\,a^7b^2x^2-64\,a^8bx+128\,a^9)\sqrt{bx+a}}{230945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/230945*(12155*b^9*x^9 + 55055*a*b^8*x^8 + 95238*a^2*b^7*x^7 + 75086*a^3*b
^6*x^6 + 23063*a^4*b^5*x^5 + 35*a^5*b^4*x^4 - 40*a^6*b^3*x^3 + 48*a^7*b^2*x
^2 - 64*a^8*b*x + 128*a^9)*sqrt(b*x + a)/b^5
```

Sympy [A]
time = 1.20, size = 212, normalized size = 2.33

$$\begin{cases} \frac{256a^9\sqrt{a+bx}}{230945b^5} - \frac{128a^8x\sqrt{a+bx}}{230945b^4} + \frac{96a^7x^2\sqrt{a+bx}}{230945b^3} - \frac{16a^6x^3\sqrt{a+bx}}{46189b^2} + \frac{14a^5x^4\sqrt{a+bx}}{46189b} + \frac{46126a^4x^5\sqrt{a+bx}}{230945} + \frac{13652a^3bx^6\sqrt{a+bx}}{20995} + \frac{1332a^2b^2x^7\sqrt{a+bx}}{1615} + \frac{154ab^3x^8\sqrt{a+bx}}{323} + \frac{2b^4x^9\sqrt{a+bx}}{19} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((256*a**9*sqrt(a + b*x)/(230945*b**5) - 128*a**8*x*sqrt(a + b*x)/
(230945*b**4) + 96*a**7*x**2*sqrt(a + b*x)/(230945*b**3) - 16*a**6*x**3*sqr
t(a + b*x)/(46189*b**2) + 14*a**5*x**4*sqrt(a + b*x)/(46189*b) + 46126*a**4
*x**5*sqrt(a + b*x)/230945 + 13652*a**3*b*x**6*sqrt(a + b*x)/20995 + 1332*a
**2*b**2*x**7*sqrt(a + b*x)/1615 + 154*a*b**3*x**8*sqrt(a + b*x)/323 + 2*b*
*4*x**9*sqrt(a + b*x)/19, Ne(b, 0)), (a**(9/2)*x**5/5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(71) = 142.
time = 0.00, size = 1004, normalized size = 11.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x)
```

```
[Out] 2/14549535*(46189*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^5/b^4 + 1
04975*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
```

```
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)
*a^5)*a^4/b^4 + 48450*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 500
5*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3/b^4 + 22610*(429*
(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 2
5025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2
)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^2/b^4 + 665*(
6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*
a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*
x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7
+ 109395*sqrt(b*x + a)*a^8)*a/b^4 + 63*(12155*(b*x + a)^(19/2) - 122265*(b*
x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^
3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b
*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^
8 - 230945*sqrt(b*x + a)*a^9)/b^4)/b
```

Mupad [B]
time = 0.02, size = 71, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^5}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{11/2}}{11\,b^5}-\frac{8\,a^3\,{\left(a+b\,x\right)}^{13/2}}{13\,b^5}+\frac{4\,a^2\,{\left(a+b\,x\right)}^{15/2}}{5\,b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{17/2}}{17\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(19/2))/(19*b^5) + (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*
(a + b*x)^(13/2))/(13*b^5) + (4*a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b
*x)^(17/2))/(17*b^5)
```

3.313 $\int x^3(a+bx)^{9/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}$$

```
[Out] -2/11*a^3*(b*x+a)^(11/2)/b^4+6/13*a^2*(b*x+a)^(13/2)/b^4-2/5*a*(b*x+a)^(15/2)/b^4+2/17*(b*x+a)^(17/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (2*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right)dx\\ &= -\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{11/2}\left(-16a^3+88a^2bx-286ab^2x^2+715b^3x^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-16*a^3 + 88*a^2*b*x - 286*a*b^2*x^2 + 715*b^3*x^3))/(
12155*b^4)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.33, size = 109, normalized size = 1.51

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-16a^8+8a^7bx-6a^6b^2x^2+5a^5b^3x^3+b^4x^4\left(1515a^4+4714a^3bx+5808a^2b^2x^2+3289ab^3x^3+715b^4x^4\right)\right)\sqrt{a+bx}}{12155b^4},b!=0\right\}\right\},\frac{a^{\frac{9}{2}}x^4}{4}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (-16 a ^ 8 + 8 a ^ 7 b x - 6 a ^ 6 b ^ 2 x ^ 2 + 5 a ^ 5 b ^
3 x ^ 3 + b ^ 4 x ^ 4 (1515 a ^ 4 + 4714 a ^ 3 b x + 5808 a ^ 2 b ^ 2 x ^ 2
 + 3289 a b ^ 3 x ^ 3 + 715 b ^ 4 x ^ 4)) Sqrt[a + b x] / (12155 b ^ 4), b
!= 0}}, a ^ (9 / 2) x ^ 4 / 4]
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.69

method	result	s
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-715b^3x^3+286a\,b^2x^2-88a^2bx+16a^3\right)}{12155b^4}$	[illegible]
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{17}{2}}}{17}-\frac{2a(bx+a)^{\frac{15}{2}}}{5}+\frac{6a^2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^3(bx+a)^{\frac{11}{2}}}{11}}{b^4}$	[illegible]
default	$\frac{\frac{2(bx+a)^{\frac{17}{2}}}{17}-\frac{2a(bx+a)^{\frac{15}{2}}}{5}+\frac{6a^2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^3(bx+a)^{\frac{11}{2}}}{11}}{b^4}$	[illegible]
trager	$-\frac{2\left(-715b^8x^8-3289a\,b^7x^7-5808a^2x^6b^6-4714a^3x^5b^5-1515a^4x^4b^4-5a^5x^3b^3+6a^6x^2b^2-8a^7xb+16a^8\right)\sqrt{bx+a}}{12155b^4}$	[illegible]
risch	$-\frac{2\left(-715b^8x^8-3289a\,b^7x^7-5808a^2x^6b^6-4714a^3x^5b^5-1515a^4x^4b^4-5a^5x^3b^3+6a^6x^2b^2-8a^7xb+16a^8\right)\sqrt{bx+a}}{12155b^4}$	[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/17*(b*x+a)^(17/2)-1/5*a*(b*x+a)^(15/2)+3/13*a^2*(b*x+a)^(13/2)-1/1
1*a^3*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{17}{2}}}{17\,b^4}-\frac{2\,(bx+a)^{\frac{15}{2}}\,a}{5\,b^4}+\frac{6\,(bx+a)^{\frac{13}{2}}\,a^2}{13\,b^4}-\frac{2\,(bx+a)^{\frac{11}{2}}\,a^3}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/17*(b*x + a)^(17/2)/b^4 - 2/5*(b*x + a)^(15/2)*a/b^4 + 6/13*(b*x + a)^(13
/2)*a^2/b^4 - 2/11*(b*x + a)^(11/2)*a^3/b^4
```

Fricas [A]
time = 0.31, size = 97, normalized size = 1.35

$$\frac{2\,(715\,b^8x^8+3289\,ab^7x^7+5808\,a^2b^6x^6+4714\,a^3b^5x^5+1515\,a^4b^4x^4+5\,a^5b^3x^3-6\,a^6b^2x^2+8\,a^7bx-16\,a^8)\sqrt{bx+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/12155*(715*b^8*x^8 + 3289*a*b^7*x^7 + 5808*a^2*b^6*x^6 + 4714*a^3*b^5*x^5
 + 1515*a^4*b^4*x^4 + 5*a^5*b^3*x^3 - 6*a^6*b^2*x^2 + 8*a^7*b*x - 16*a^8)*s
qrt(b*x + a)/b^4
```

Sympy [A]
time = 1.07, size = 190, normalized size = 2.64

$$\begin{cases} -\frac{32a^8\sqrt{a+bx}}{12155b^4}+\frac{16a^7x\sqrt{a+bx}}{12155b^3}-\frac{12a^6x^2\sqrt{a+bx}}{12155b^2}+\frac{2a^5x^3\sqrt{a+bx}}{2431b}+\frac{606a^4x^4\sqrt{a+bx}}{2431}+\frac{9428a^3bx^5\sqrt{a+bx}}{12155}+\frac{1056a^2b^2x^6\sqrt{a+bx}}{1105}+\frac{46ab^3x^7\sqrt{a+bx}}{85}+\frac{2b^4x^8\sqrt{a+bx}}{17} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-32*a**8*sqrt(a + b*x)/(12155*b**4) + 16*a**7*x*sqrt(a + b*x)/(1
2155*b**3) - 12*a**6*x**2*sqrt(a + b*x)/(12155*b**2) + 2*a**5*x**3*sqrt(a +
 b*x)/(2431*b) + 606*a**4*x**4*sqrt(a + b*x)/2431 + 9428*a**3*b*x**5*sqrt(a
 + b*x)/12155 + 1056*a**2*b**2*x**6*sqrt(a + b*x)/1105 + 46*a*b**3*x**7*sqr
t(a + b*x)/85 + 2*b**4*x**8*sqrt(a + b*x)/17, Ne(b, 0)), (a**(9/2)*x**4/4,
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(56) = 112.
time = 0.00, size = 866, normalized size = 12.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x)
```

```
[Out] 2/765765*(21879*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^5/b^3 + 12155*(35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
```

```
sqrt(b*x + a)*a^4)*a^4/b^3 + 11050*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/
2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^
(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3/b^3 + 2550*(231*(b*x + a)^(13/2) - 1
638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^
3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a
)*a^6)*a^2/b^3 + 595*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a/b^3 + 7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 23
5620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^
(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 29172
0*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)/b^3)/b
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{15/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(17/2))/(17*b^4) - (2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*
(a + b*x)^(13/2))/(13*b^4) - (2*a*(a + b*x)^(15/2))/(5*b^4)
```

3.314 $\int x^2(a+bx)^{9/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}$$

```
[Out] 2/11*a^2*(b*x+a)^(11/2)/b^3-4/13*a*(b*x+a)^(13/2)/b^3+2/15*(b*x+a)^(15/2)/b
^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(a
 + b*x)^(15/2))/(15*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{9/2}\,dx&=\int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{11/2}\left(8a^2-44abx+143b^2x^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(8*a^2 - 44*a*b*x + 143*b^2*x^2))/(2145*b^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.14, size = 98, normalized size = 1.85

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(8a^7-4a^6bx+3a^5b^2x^2+b^3x^3\left(355a^4+1030a^3bx+1218a^2b^2x^2+671ab^3x^3+143b^4x^4\right)\right)\sqrt{a+bx}}{2145b^3},b!{=}0\right\}\right\},\frac{a^{\frac{9}{2}}x^3}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (8 a ^ 7 - 4 a ^ 6 b x + 3 a ^ 5 b ^ 2 x ^ 2 + b ^ 3 x ^ 3 (3
55 a ^ 4 + 1030 a ^ 3 b x + 1218 a ^ 2 b ^ 2 x ^ 2 + 671 a b ^ 3 x ^ 3 + 14
3 b ^ 4 x ^ 4)) Sqrt[a + b x] / (2145 b ^ 3), b != 0}}, a ^ (9 / 2) x ^ 3 /
 3]
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{11}{2}}\left(143x^2b^2-44abx+8a^2\right)}{2145b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{15}{2}}}{15}-\frac{4a(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^2(bx+a)^{\frac{11}{2}}}{11}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{15}{2}}}{15}-\frac{4a(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^2(bx+a)^{\frac{11}{2}}}{11}}{b^3}$	38
trager	$\frac{2\left(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+8a^7\right)\sqrt{bx+a}}{2145b^3}$	87
risch	$\frac{2\left(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+8a^7\right)\sqrt{bx+a}}{2145b^3}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/15*(b*x+a)^(15/2)-2/13*a*(b*x+a)^(13/2)+1/11*a^2*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{15}{2}}}{15\,b^3}-\frac{4\,(bx+a)^{\frac{13}{2}}\,a}{13\,b^3}+\frac{2\,(bx+a)^{\frac{11}{2}}\,a^2}{11\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x + a)^(15/2)/b^3 - 4/13*(b*x + a)^(13/2)*a/b^3 + 2/11*(b*x + a)^(1
1/2)*a^2/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.
time = 0.30, size = 86, normalized size = 1.62

$$\frac{2\,(143\,b^7x^7 + 671\,ab^6x^6 + 1218\,a^2b^5x^5 + 1030\,a^3b^4x^4 + 355\,a^4b^3x^3 + 3\,a^5b^2x^2 - 4\,a^6bx + 8\,a^7)\sqrt{bx+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/2145*(143*b^7*x^7 + 671*a*b^6*x^6 + 1218*a^2*b^5*x^5 + 1030*a^3*b^4*x^4 +
 355*a^4*b^3*x^3 + 3*a^5*b^2*x^2 - 4*a^6*b*x + 8*a^7)*sqrt(b*x + a)/b^3
```

Sympy [A]
time = 0.95, size = 168, normalized size = 3.17

$$\begin{cases} \frac{16a^7\sqrt{a+bx}}{2145b^3} - \frac{8a^6x\sqrt{a+bx}}{2145b^2} + \frac{2a^5x^2\sqrt{a+bx}}{715b} + \frac{142a^4x^3\sqrt{a+bx}}{429} + \frac{412a^3bx^4\sqrt{a+bx}}{429} + \frac{812a^2b^2x^5\sqrt{a+bx}}{715} + \frac{122ab^3x^6\sqrt{a+bx}}{195} + \frac{2b^4x^7\sqrt{a+bx}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((16*a**7*sqrt(a + b*x)/(2145*b**3) - 8*a**6*x*sqrt(a + b*x)/(2145
*b**2) + 2*a**5*x**2*sqrt(a + b*x)/(715*b) + 142*a**4*x**3*sqrt(a + b*x)/42
9 + 412*a**3*b*x**4*sqrt(a + b*x)/429 + 812*a**2*b**2*x**5*sqrt(a + b*x)/71
5 + 122*a*b**3*x**6*sqrt(a + b*x)/195 + 2*b**4*x**7*sqrt(a + b*x)/15, Ne(b,
 0)), (a**(9/2)*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(41) = 82$.
time = 0.00, size = 727, normalized size = 13.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x)
```

```
[Out] 2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*
a^2)*a^5/b^2 + 6435*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^4/b^2 + 1430*(35*(b*x + a)^(9/2) - 18
0*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 3
15*sqrt(b*x + a)*a^4)*a^3/b^2 + 650*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9
```

```
/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)
^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2/b^2 + 75*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*a/b^2 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*
x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 -
 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)
*a^7)/b^2)/b
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.68

$$\frac{\frac{2\,{\left(a+b\,x\right)}^{15/2}}{15}-\frac{4\,a\,{\left(a+b\,x\right)}^{13/2}}{13}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{11/2}}{11}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(9/2),x)
```

```
[Out] ((2*(a + b*x)^(15/2))/15 - (4*a*(a + b*x)^(13/2))/13 + (2*a^2*(a + b*x)^(11
/2))/11)/b^3
```

3.315 $\int x(a+bx)^{9/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}$$

```
[Out] -2/11*a*(b*x+a)^(11/2)/b^2+2/13*(b*x+a)^(13/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{9/2}\,dx &= \int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}(-2a+11bx)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(9/2),x]
```

[Out] (2*(a + b*x)^(11/2)*(-2*a + 11*b*x))/(143*b^2)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.94, size = 86, normalized size = 2.53

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2a^6+a^5bx+b^2x^2\left(35a^4+90a^3bx+100a^2b^2x^2+53ab^3x^3+11b^4x^4\right)\right)\sqrt{a+bx}}{143b^2},b!=0\right\}\right\},\frac{a^{\frac{9}{2}}x^2}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1*(a + b*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (-2 a ^ 6 + a ^ 5 b x + b ^ 2 x ^ 2 (35 a ^ 4 + 90 a ^ 3 b x
+ 100 a ^ 2 b ^ 2 x ^ 2 + 53 a b ^ 3 x ^ 3 + 11 b ^ 4 x ^ 4)) Sqrt[a + b x]
 / (143 b ^ 2), b != 0}}, a ^ (9 / 2) x ^ 2 / 2]
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}(-11bx+2a)}{143b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a(bx+a)^{\frac{11}{2}}}{11}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a(bx+a)^{\frac{11}{2}}}{11}}{b^2}$	26
trager	$-\frac{2\left(-11x^6b^6-53a\,x^5b^5-100a^2x^4b^4-90a^3b^3x^3-35a^4x^2b^2-a^5xb+2a^6\right)\sqrt{bx+a}}{143b^2}$	76
risch	$-\frac{2\left(-11x^6b^6-53a\,x^5b^5-100a^2x^4b^4-90a^3b^3x^3-35a^4x^2b^2-a^5xb+2a^6\right)\sqrt{bx+a}}{143b^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/13*(b*x+a)^(13/2)-1/11*a*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^2}-\frac{2\,(bx+a)^{\frac{11}{2}}\,a}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^2 - 2/11*(b*x + a)^(11/2)*a/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.
time = 0.30, size = 74, normalized size = 2.18

$$\frac{2\,(11\,b^6x^6 + 53\,ab^5x^5 + 100\,a^2b^4x^4 + 90\,a^3b^3x^3 + 35\,a^4b^2x^2 + a^5bx - 2\,a^6)\sqrt{bx+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*b^6*x^6 + 53*a*b^5*x^5 + 100*a^2*b^4*x^4 + 90*a^3*b^3*x^3 + 35*a^
4*b^2*x^2 + a^5*b*x - 2*a^6)*sqrt(b*x + a)/b^2
```

Sympy [A]
time = 0.82, size = 146, normalized size = 4.29

$$\begin{cases} -\frac{4a^6\sqrt{a+bx}}{143b^2} + \frac{2a^5x\sqrt{a+bx}}{143b} + \frac{70a^4x^2\sqrt{a+bx}}{143} + \frac{180a^3bx^3\sqrt{a+bx}}{143} + \frac{200a^2b^2x^4\sqrt{a+bx}}{143} + \frac{106ab^3x^5\sqrt{a+bx}}{143} + \frac{2b^4x^6\sqrt{a+bx}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4*a**6*sqrt(a + b*x)/(143*b**2) + 2*a**5*x*sqrt(a + b*x)/(143*b
) + 70*a**4*x**2*sqrt(a + b*x)/143 + 180*a**3*b*x**3*sqrt(a + b*x)/143 + 20
0*a**2*b**2*x**4*sqrt(a + b*x)/143 + 106*a*b**3*x**5*sqrt(a + b*x)/143 + 2*
b**4*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(9/2)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(26) = 52$.
time = 0.00, size = 586, normalized size = 17.24

$$\frac{\frac{2b^5\left(\frac{1}{13}\sqrt{a+bx}\,(a+bx)^6 - \frac{6}{11}\sqrt{a+bx}\,(a+bx)^5a + \frac{5}{3}\sqrt{a+bx}\,(a+bx)^4a^2 - \frac{20}{7}\sqrt{a+bx}\,(a+bx)^3a^3 + 3\sqrt{a+bx}\,(a+bx)^2a^4 - 2\sqrt{a+bx}\,(a+bx)a^5 + \sqrt{a+bx}\,a^6\right)}{b^6} + \frac{10ab^4\left(\frac{1}{11}\sqrt{a+bx}\,(a+bx)^5 - \frac{5}{9}\sqrt{a+bx}\,(a+bx)^4a + \frac{10}{7}\sqrt{a+bx}\,(a+bx)^3a^2 - 2\sqrt{a+bx}\,(a+bx)^2a^3 + \frac{5}{3}\sqrt{a+bx}\,(a+bx)a^4 - \sqrt{a+bx}\,a^5\right)}{b^5} + \frac{20a^2b^3\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4 - \frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a + \frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2 - \frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3 + \sqrt{a+bx}\,a^4\right)}{b^4} + \frac{20a^3b^2\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3 - \frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a + \sqrt{a+bx}\,(a+bx)a^2 - \sqrt{a+bx}\,a^3\right)}{b^3} + \frac{10a^4b\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2 - \frac{2}{3}\sqrt{a+bx}\,(a+bx)a + \sqrt{a+bx}\,a^2\right)}{b^2} + \frac{2a^5\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx) - a\sqrt{a+bx}\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x)
```

```
[Out] 2/9009*(3003*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^5/b + 3003*(3*(b*x + a
)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^4/b + 2574*(5*(b*x
 + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x +
 a)*a^3)*a^3/b + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x
 + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b +
65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 -
 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^
5)*a/b + 3*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)
^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*
x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{26\,a\,{\left(a+b\,x\right)}^{11/2}-22\,{\left(a+b\,x\right)}^{13/2}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(9/2),x)
[Out] -(26*a*(a + b*x)^(11/2) - 22*(a + b*x)^(13/2))/(143*b^2)
```

3.316 $\int (a+bx)^{9/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{11/2}}{11b}$$

[Out] 2/11*(b*x+a)^(11/2)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(9/2),x]

[Out] (2*(a + b*x)^(11/2))/(11*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{9/2}\,dx = \frac{2(a+bx)^{11/2}}{11b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(9/2),x]

[Out] (2*(a + b*x)^(11/2))/(11*b)

Mathics [A]
time = 1.60, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{11}{2}}}{11b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0*(a + b*x)^(9/2),x]')
```

```
[Out] 2 (a + b x) ^ (11 / 2) / (11 b)
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
default	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
trager	$\frac{2\left(b^{5} x^{5}+5 a\, b^{4} x^{4}+10 a^{2} b^{3} x^{3}+10 a^{3} b^{2} x^{2}+5 a^{4} b x +a^{5}\right) \sqrt{b x +a}}{11 b}$	62
risch	$\frac{2\left(b^{5} x^{5}+5 a\, b^{4} x^{4}+10 a^{2} b^{3} x^{3}+10 a^{3} b^{2} x^{2}+5 a^{4} b x +a^{5}\right) \sqrt{b x +a}}{11 b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*(b*x+a)^(11/2)/b
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.
time = 0.30, size = 61, normalized size = 3.81

$$\frac{2\,\left(b^{5}x^{5}+5\,ab^{4}x^{4}+10\,a^{2}b^{3}x^{3}+10\,a^{3}b^{2}x^{2}+5\,a^{4}bx+a^{5}\right)\sqrt{bx+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2),x)
```

```
[Out] 2*(a + b*x)**(11/2)/(11*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(12) = 24.
time = 0.00, size = 445, normalized size = 27.81

$$\frac{\frac{2b^5\left(\frac{1}{11}\sqrt{a+bx}\,(a+bx)^5-\frac{5}{9}\sqrt{a+bx}\,(a+bx)^4a+\frac{10}{7}\sqrt{a+bx}\,(a+bx)^3a^2-2\sqrt{a+bx}\,(a+bx)^2a^3+\frac{5}{3}\sqrt{a+bx}\,(a+bx)a^4-\sqrt{a+bx}\,a^5\right)}{b^5}+\frac{10ab^4\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3+\sqrt{a+bx}\,a^4\right)}{b^4}+\frac{20a^2b^3\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{b^3}+\frac{20a^3b^2\left(\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2-\frac{2}{3}\sqrt{a+bx}\,(a+bx)a+\sqrt{a+bx}\,a^2\right)}{b^2}+10a^4\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx)-a\sqrt{a+bx}\right)+2a^5\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x)
```

```
[Out] 2/693*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 + 1155*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a^4 + 462*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)
*a + 15*sqrt(b*x + a)*a^2)*a^3 + 198*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2 + 11*(35*(b*x + a)
^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3
/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

3.317 $\int \frac{(a+bx)^{9/2}}{x} dx$

Optimal. Leaf size=97

$$2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x,x]
```

```
[Out] 2*a^4*Sqrt[a + b*x] + (2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5
 + (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 - 2*a^(9/2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x}\,dx &= \frac{2}{9}(a+bx)^{9/2} + a\int \frac{(a+bx)^{7/2}}{x}\,dx \\
&= \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^2\int \frac{(a+bx)^{5/2}}{x}\,dx \\
&= \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^3\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^4\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^5 \\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + \frac{(2a}{} \\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} - 2a^9
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.80

$$
\frac{2}{315}\sqrt{a+bx}\,\left(563a^4+506a^3bx+408a^2b^2x^2+185ab^3x^3+35b^4x^4\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(563*a^4 + 506*a^3*b*x + 408*a^2*b^2*x^2 + 185*a*b^3*x^3 +
 35*b^4*x^4))/315 - 2*a^(9/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Mathics [A]
time = 12.63, size = 132, normalized size = 1.36

$$
\frac{\sqrt{a}\left(-630a^4\mathrm{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+315a^4\mathrm{Log}\left[\frac{bx}{a}\right]+1126a^4\sqrt{\frac{a+bx}{a}}+1012a^3bx\sqrt{\frac{a+bx}{a}}+816a^2b^2x^2\sqrt{\frac{a+bx}{a}}+370ab^3x^3\sqrt{\frac{a+bx}{a}}+70b^4x^4\sqrt{\frac{a+bx}{a}}\right)}{315}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^1,x]')
```

```
[Out] Sqrt[a] (-630 a ^ 4 Log[1 + Sqrt[(a + b x) / a]] + 315 a ^ 4 Log[b x / a] +
 1126 a ^ 4 Sqrt[(a + b x) / a] + 1012 a ^ 3 b x Sqrt[(a + b x) / a] + 816
```

```
a ^ 2 b ^ 2 x ^ 2 Sqrt[(a + b x) / a] + 370 a b ^ 3 x ^ 3 Sqrt[(a + b x) /
a] + 70 b ^ 4 x ^ 4 Sqrt[(a + b x) / a]) / 315
```

Maple [A]
time = 0.09, size = 74, normalized size = 0.76

method	result
derivativedivides	$\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}+\frac{2a(bx+a)^{\frac{7}{2}}}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}-2a^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^4\sqrt{bx+a}$
default	$\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}+\frac{2a(bx+a)^{\frac{7}{2}}}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}-2a^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^4\sqrt{bx+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 88, normalized size = 0.91

$$a^{\frac{9}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{9}(bx+a)^{\frac{9}{2}}+\frac{2}{7}(bx+a)^{\frac{7}{2}}a+\frac{2}{5}(bx+a)^{\frac{5}{2}}a^2+\frac{2}{3}(bx+a)^{\frac{3}{2}}a^3+2\sqrt{bx+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="maxima")
[Out] a^(9/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/9*(b*x
 + a)^(9/2) + 2/7*(b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x +
a)^(3/2)*a^3 + 2*sqrt(b*x + a)*a^4
```

Fricas [A]
time = 0.32, size = 158, normalized size = 1.63

$$\left[a^{\frac{9}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a},\ 2\sqrt{-a}\,a^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="fricas")
[Out] [a^(9/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/315*(35*b^4*x^4 +
 185*a*b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a), 2*
sqrt(-a)*a^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/315*(35*b^4*x^4 + 185*a*b
^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a)]
```

Sympy [A]
time = 11.29, size = 148, normalized size = 1.53

$$\frac{1126a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{315}+a^{\frac{9}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{1012a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{315}+\frac{272a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105}+\frac{74a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{63}+\frac{2\sqrt{a}\,b^4x^4\sqrt{1+\frac{bx}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x,x)
```

```
[Out] 1126*a**(9/2)*sqrt(1 + b*x/a)/315 + a**(9/2)*log(b*x/a) - 2*a**(9/2)*log(sq
rt(1 + b*x/a) + 1) + 1012*a**(7/2)*b*x*sqrt(1 + b*x/a)/315 + 272*a**(5/2)*b
**2*x**2*sqrt(1 + b*x/a)/105 + 74*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/63 + 2
*sqrt(a)*b**4*x**4*sqrt(1 + b*x/a)/9
```

Giac [A]
time = 0.00, size = 132, normalized size = 1.36

$$\frac{2}{9}\sqrt{a+bx}\,(a+bx)^4+\frac{2}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{2}{5}\sqrt{a+bx}\,(a+bx)^2a^2+\frac{2}{3}\sqrt{a+bx}\,(a+bx)a^3+2\sqrt{a+bx}\,a^4+\frac{4a^5\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x)
```

```
[Out] 2*a^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x + a)^(9/2) + 2/7*(
b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x + a)^(3/2)*a^3 + 2*sq
rt(b*x + a)*a^4
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.78

$$\frac{2\,a\,{\left(a+b\,x\right)}^{7/2}}{7}+\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9}+2\,a^4\,\sqrt{a+b\,x}+\frac{2\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5}+a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 + 2*a^4*(a + b*x)^(1/2) + (
2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5 + a^(9/2)*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*2i
```

3.318 $\int \frac{(a+bx)^{9/2}}{x^2}\,dx$

Optimal. Leaf size=98

$$9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 3*a^2*b*(b*x+a)^(3/2)+9/5*a*b*(b*x+a)^(5/2)+9/7*b*(b*x+a)^(7/2)-(b*x+a)^(9/
2)/x-9*a^(7/2)*b*arctanh((b*x+a)^(1/2)/a^(1/2))+9*a^3*b*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] 9*a^3*b*Sqrt[a + b*x] + 3*a^2*b*(a + b*x)^(3/2) + (9*a*b*(a + b*x)^(5/2))/5
 + (9*b*(a + b*x)^(7/2))/7 - (a + b*x)^(9/2)/x - 9*a^(7/2)*b*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^2}\,dx = -\frac{(a+bx)^{9/2}}{x} + \frac{1}{2}(9b)\int \frac{(a+bx)^{7/2}}{x}\,dx$$
$$= \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}(9ab)\int \frac{(a+bx)^{5/2}}{x}\,dx$$
$$= \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}\left(9a^2b\right)\int \frac{(a+bx)^{3/2}}{x}\,dx$$
$$= 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}\left(9a^3b\right)\int \frac{\sqrt{a+bx}}{x}$$
$$= 9a^3b\sqrt{a+bx} + 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}($$
$$= 9a^3b\sqrt{a+bx} + 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + (9a$$
$$= 9a^3b\sqrt{a+bx} + 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} - 9a^7$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\left(-35a^4+388a^3bx+156a^2b^2x^2+58ab^3x^3+10b^4x^4\right)}{35x} - 9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-35*a^4 + 388*a^3*b*x + 156*a^2*b^2*x^2 + 58*a*b^3*x^3 + 10
*b^4*x^4))/(35*x) - 9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Mathics [A]

time = 12.33, size = 133, normalized size = 1.36

$$\frac{\sqrt{a}\left(-a^4\sqrt{\frac{a+bx}{a}}+\frac{bx\left(-630a^3\mathrm{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+315a^3\mathrm{Log}\left[\frac{bx}{a}\right]+776a^3\sqrt{\frac{a+bx}{a}}+312a^2bx\sqrt{\frac{a+bx}{a}}+116ab^2x^2\sqrt{\frac{a+bx}{a}}+20b^3x^3\sqrt{\frac{a+bx}{a}}\right)}{70}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^2,x]')
```

```
[Out] Sqrt[a] (-a ^ 4 Sqrt[(a + b x) / a] + b x (-630 a ^ 3 Log[1 + Sqrt[(a + b x
) / a]] + 315 a ^ 3 Log[b x / a] + 776 a ^ 3 Sqrt[(a + b x) / a] + 312 a ^
2 b x Sqrt[(a + b x) / a] + 116 a b ^ 2 x ^ 2 Sqrt[(a + b x) / a] + 20 b ^
3 x ^ 3 Sqrt[(a + b x) / a]) / 70) / x
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.87

method	result
risch	$-\frac{a^4\sqrt{bx+a}}{x}+\frac{b\left(\frac{4(bx+a)^{\frac{7}{2}}}{7}+\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^2(bx+a)^{\frac{3}{2}}+16a^3\sqrt{bx+a}-18a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{2}$
derivativedivides	$2b\left(\frac{(bx+a)^{\frac{7}{2}}}{7}+\frac{2a(bx+a)^{\frac{5}{2}}}{5}+a^2(bx+a)^{\frac{3}{2}}+4a^3\sqrt{bx+a}-a^4\left(\frac{\sqrt{bx+a}}{2bx}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{2\sqrt{a}}\right.\right.$
default	$2b\left(\frac{(bx+a)^{\frac{7}{2}}}{7}+\frac{2a(bx+a)^{\frac{5}{2}}}{5}+a^2(bx+a)^{\frac{3}{2}}+4a^3\sqrt{bx+a}-a^4\left(\frac{\sqrt{bx+a}}{2bx}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{2\sqrt{a}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/7*(b*x+a)^(7/2)+2/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)+4*a^3*(b*x+a)^
(1/2)-a^4*(1/2*(b*x+a)^(1/2)/b/x+9/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
))
```

Maxima [A]
time = 0.36, size = 97, normalized size = 0.99

$$\frac{9}{2}a^{\frac{7}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{7}(bx+a)^{\frac{7}{2}}b+\frac{4}{5}(bx+a)^{\frac{5}{2}}ab+2(bx+a)^{\frac{3}{2}}a^2b+8\sqrt{bx+a}\,a^3b-\frac{\sqrt{bx+a}\,a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/2*a^(7/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
7*(b*x + a)^(7/2)*b + 4/5*(b*x + a)^(5/2)*a*b + 2*(b*x + a)^(3/2)*a^2*b + 8
*sqrt(b*x + a)*a^3*b - sqrt(b*x + a)*a^4/x
```

Fricas [A]
time = 0.33, size = 172, normalized size = 1.76

$$\left[\frac{315 \, a^{\frac{7}{2}} b x \log\left(\frac{b x - 2 \, \sqrt{b x + a} \sqrt{a} + 2 \, a}{x}\right) + 2 \, {\left(10 \, b^{4} x^{4} + 58 \, a b^{3} x^{3} + 156 \, a^{2} b^{2} x^{2} + 388 \, a^{3} b x - 35 \, a^{4}\right)} \sqrt{b x + a}}{70 \, x}, \frac{315 \, \sqrt{-a} a^{3} b x \arctan\left(\frac{\sqrt{b x + a} \sqrt{-a}}{a}\right) + {\left(10 \, b^{4} x^{4} + 58 \, a b^{3} x^{3} + 156 \, a^{2} b^{2} x^{2} + 388 \, a^{3} b x - 35 \, a^{4}\right)} \sqrt{b x + a}}{35 \, x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/70*(315*a^(7/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(10
*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x
+ a))/x, 1/35*(315*sqrt(-a)*a^3*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (10*
b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x +
 a))/x]
```

Sympy [A]
time = 10.91, size = 150, normalized size = 1.53

$$-\frac{a^{\frac{9}{2}} \sqrt{1 + \frac{b x}{a}}}{x} + \frac{388 a^{\frac{7}{2}} b \sqrt{1 + \frac{b x}{a}}}{35} + \frac{9 a^{\frac{7}{2}} b \log{\left(\frac{b x}{a} \right)}}{2} - 9 a^{\frac{7}{2}} b \log{\left(\sqrt{1 + \frac{b x}{a}} + 1 \right)} + \frac{156 a^{\frac{5}{2}} b^{2} x \sqrt{1 + \frac{b x}{a}}}{35} + \frac{58 a^{\frac{3}{2}} b^{3} x^{2} \sqrt{1 + \frac{b x}{a}}}{35} + \frac{2 \sqrt{a} b^{4} x^{3} \sqrt{1 + \frac{b x}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x/a)/x + 388*a**(7/2)*b*sqrt(1 + b*x/a)/35 + 9*a**(7/2
)*b*log(b*x/a)/2 - 9*a**(7/2)*b*log(sqrt(1 + b*x/a) + 1) + 156*a**(5/2)*b**
2*x*sqrt(1 + b*x/a)/35 + 58*a**(3/2)*b**3*x**2*sqrt(1 + b*x/a)/35 + 2*sqrt(
a)*b**4*x**3*sqrt(1 + b*x/a)/7
```

Giac [A]
time = 0.00, size = 150, normalized size = 1.53

$$\frac{\frac{2}{7} \sqrt{a + b x} \left(a + b x\right)^{3} b^{2} + \frac{4}{5} \sqrt{a + b x} \left(a + b x\right)^{2} b^{2} a + 2 \sqrt{a + b x} \left(a + b x\right) b^{2} a^{2} + 8 \sqrt{a + b x} b^{2} a^{3} + \frac{\sqrt{a + b x} b^{2} a^{4}}{-a - b x + a} + \frac{18 b^{2} a^{4} \arctan\left(\frac{\sqrt{a + b x}}{\sqrt{-a}}\right)}{2 \sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x)
```

```
[Out] 1/35*(315*a^4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 10*(b*x + a)^(7
/2)*b^2 + 28*(b*x + a)^(5/2)*a*b^2 + 70*(b*x + a)^(3/2)*a^2*b^2 + 280*sqrt(
b*x + a)*a^3*b^2 - 35*sqrt(b*x + a)*a^4*b/x)/b
```

Mupad [B]
time = 0.04, size = 84, normalized size = 0.86

$$\frac{2\,b\,{\left(a+b\,x\right)}^{7/2}}{7}-\frac{a^4\,\sqrt{a+b\,x}}{x}+\frac{4\,a\,b\,{\left(a+b\,x\right)}^{5/2}}{5}+8\,a^3\,b\,\sqrt{a+b\,x}+2\,a^2\,b\,{\left(a+b\,x\right)}^{3/2}+a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,9{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(7/2))/7 - (a^4*(a + b*x)^(1/2))/x + a^(7/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*9i + (4*a*b*(a + b*x)^(5/2))/5 + 8*a^3*b*(a + b*x)^(1
/2) + 2*a^2*b*(a + b*x)^(3/2)
```

3.319 $\int \frac{(a+bx)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=114

$$\frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 21/4*a*b^2*(b*x+a)^(3/2)+63/20*b^2*(b*x+a)^(5/2)-9/4*b*(b*x+a)^(7/2)/x-1/2*
(b*x+a)^(9/2)/x^2-63/4*a^(5/2)*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))+63/4*a^2*
b^2*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x])/4 + (21*a*b^2*(a + b*x)^(3/2))/4 + (63*b^2*(a +
b*x)^(5/2))/20 - (9*b*(a + b*x)^(7/2))/(4*x) - (a + b*x)^(9/2)/(2*x^2) - (6
3*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^3}\,dx &= -\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{4}(9b)\int \frac{(a+bx)^{7/2}}{x^2}\,dx\\ &= -\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x}\,dx\\ &= \frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63ab^2\right)\int \frac{(a+bx)^{3/2}}{x}\,dx\\ &= \frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63a^2b^2\right)\int \frac{\sqrt{a\cdots}}{\cdots}\\ &= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\cdots\\ &= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\cdots\\ &= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}-\cdots\end{aligned}$$

Mathematica [A]
time = 0.10, size = 86, normalized size = 0.75

$$\frac{\sqrt{a+bx}\,\left(-10a^4-85a^3bx+288a^2b^2x^2+56ab^3x^3+8b^4x^4\right)}{20x^2}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x]*(-10*a^4 - 85*a^3*b*x + 288*a^2*b^2*x^2 + 56*a*b^3*x^3 + 8*b
^4*x^4))/(20*x^2) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Mathics [A]
time = 12.06, size = 155, normalized size = 1.36

$$\frac{b^{\frac{3}{2}}\left(-10a^5x^2\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}-315a^{\frac{5}{2}}\sqrt{b}\,x^{\frac{5}{2}}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right](a+bx)^2-95a^4bx^3\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}+203a^3b^2x^4\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}+8b^3x^5\left(43a^2+8abx+b^2x^2\right)\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}\right)}{20x^{\frac{5}{2}}\left(a+bx\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^3,x]')
```

```
[Out] b ^ (3 / 2) (-10 a ^ 5 x ^ 2 ((a + b x) / (b x)) ^ (3 / 2) - 315 a ^ (5 / 2
) Sqrt[b] x ^ (5 / 2) ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) ^ 2 -
95 a ^ 4 b x ^ 3 ((a + b x) / (b x)) ^ (3 / 2) + 203 a ^ 3 b ^ 2 x ^ 4 ((a
+ b x) / (b x)) ^ (3 / 2) + 8 b ^ 3 x ^ 5 (43 a ^ 2 + 8 a b x + b ^ 2 x ^ 2
) ((a + b x) / (b x)) ^ (3 / 2)) / (20 x ^ (5 / 2) (a + b x) ^ 2)
```

Maple [A]
time = 0.10, size = 87, normalized size = 0.76

method	result
risch	$-\frac{a^3\sqrt{bx+a}\,(17bx+2a)}{4x^2}+\frac{b^2\left(\frac{16(bx+a)^{\frac{5}{2}}}{5}+16a(bx+a)^{\frac{3}{2}}+96a^2\sqrt{bx+a}-126a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{8}$
derivativedivides	$2b^2\left(\frac{(bx+a)^{\frac{5}{2}}}{5}+a\left(bx+a\right)^{\frac{3}{2}}+6a^2\sqrt{bx+a}-a^3\left(\frac{\frac{17(bx+a)^{\frac{3}{2}}}{8}-\frac{15a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{63\operatorname{arctanh}\left(\sqrt{\ldots}\right.}{8\sqrt{\ldots}}\right.\right.$
default	$2b^2\left(\frac{(bx+a)^{\frac{5}{2}}}{5}+a\left(bx+a\right)^{\frac{3}{2}}+6a^2\sqrt{bx+a}-a^3\left(\frac{\frac{17(bx+a)^{\frac{3}{2}}}{8}-\frac{15a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{63\operatorname{arctanh}\left(\sqrt{\ldots}\right.}{8\sqrt{\ldots}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/5*(b*x+a)^(5/2)+a*(b*x+a)^(3/2)+6*a^2*(b*x+a)^(1/2)-a^3*((17/8*(b*
x+a)^(3/2)-15/8*a*(b*x+a)^(1/2))/b^2/x^2+63/8*arctanh((b*x+a)^(1/2)/a^(1/2)
)/a^(1/2)))
```

Maxima [A]
time = 0.36, size = 131, normalized size = 1.15

$$\frac{63}{8}\,a^{\frac{5}{2}}b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}\left(bx+a\right)^{\frac{5}{2}}b^2+2\left(bx+a\right)^{\frac{3}{2}}ab^2+12\sqrt{bx+a}\,a^2b^2-\frac{17\left(bx+a\right)^{\frac{3}{2}}a^3b^2-15\sqrt{bx+a}\,a^4b^2}{4\left(\left(bx+a\right)^2-2\left(bx+a\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] 63/8*a^(5/2)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2/5*(b*x + a)^(5/2)*b^2 + 2*(b*x + a)^(3/2)*a*b^2 + 12*sqrt(b*x + a)*a^2*b
^2 - 1/4*(17*(b*x + a)^(3/2)*a^3*b^2 - 15*sqrt(b*x + a)*a^4*b^2)/((b*x + a)
^2 - 2*(b*x + a)*a + a^2)
```

Fricas [A]
time = 0.33, size = 180, normalized size = 1.58

$$\left[\frac{315\, a^{\frac{5}{2}} b^{2} x^{2} \log\left(\frac{b x - 2 \, \sqrt{b x + a} \sqrt{a} + 2 \, a}{x}\right) + 2 \, {\left(8 \, b^{4} x^{4} + 56 \, a b^{3} x^{3} + 288 \, a^{2} b^{2} x^{2} - 85 \, a^{3} b x - 10 \, a^{4}\right)} \sqrt{b x + a}}{40 \, x^{2}}, \frac{315 \, \sqrt{-a} a^{2} b^{2} x^{2} \arctan\left(\frac{\sqrt{b x + a} \sqrt{-a}}{a}\right) + {\left(8 \, b^{4} x^{4} + 56 \, a b^{3} x^{3} + 288 \, a^{2} b^{2} x^{2} - 85 \, a^{3} b x - 10 \, a^{4}\right)} \sqrt{b x + a}}{20 \, x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*a^(5/2)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt(b*
x + a))/x^2, 1/20*(315*sqrt(-a)*a^2*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt
(b*x + a))/x^2]
```

Sympy [A]
time = 10.44, size = 184, normalized size = 1.61

$$- \frac{63 a^{\frac{5}{2}} b^{2} \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right)}}{4} - \frac{a^{5}}{2 \sqrt{b} x^{\frac{5}{2}} \sqrt{\frac{a}{b x} + 1}} - \frac{19 a^{4} \sqrt{b}}{4 x^{\frac{3}{2}} \sqrt{\frac{a}{b x} + 1}} + \frac{203 a^{3} b^{\frac{3}{2}}}{20 \sqrt{x} \sqrt{\frac{a}{b x} + 1}} + \frac{86 a^{2} b^{\frac{5}{2}} \sqrt{x}}{5 \sqrt{\frac{a}{b x} + 1}} + \frac{16 a b^{\frac{7}{2}} x^{\frac{3}{2}}}{5 \sqrt{\frac{a}{b x} + 1}} + \frac{2 b^{\frac{9}{2}} x^{\frac{5}{2}}}{5 \sqrt{\frac{a}{b x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**3,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**5/(2*sqrt(b)*x**(
5/2)*sqrt(a/(b*x) + 1)) - 19*a**4*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) +
203*a**3*b**(3/2)/(20*sqrt(x)*sqrt(a/(b*x) + 1)) + 86*a**2*b**(5/2)*sqrt(x)
/(5*sqrt(a/(b*x) + 1)) + 16*a*b**(7/2)*x**(3/2)/(5*sqrt(a/(b*x) + 1)) + 2*b
**(9/2)*x**(5/2)/(5*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.00, size = 154, normalized size = 1.35

$$\frac{\frac{2}{5} \sqrt{a + b x} (a + b x)^{2} b^{3} + 2 \sqrt{a + b x} (a + b x) b^{3} a + 12 \sqrt{a + b x} b^{3} a^{2} - \frac{17 \sqrt{a + b x} (a + b x) b^{3} a^{3} - 15 \sqrt{a + b x} b^{3} a^{4}}{4 (a + b x - a)^{2}} + \frac{63 b^{3} a^{3} \arctan\left(\frac{\sqrt{a + b x}}{\sqrt{-a}}\right)}{4 \sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x)
```

```
[Out] 1/20*(315*a^3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*(b*x + a)^(5/
2)*b^3 + 40*(b*x + a)^(3/2)*a*b^3 + 240*sqrt(b*x + a)*a^2*b^3 - 5*(17*(b*x
+ a)^(3/2)*a^3*b^3 - 15*sqrt(b*x + a)*a^4*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.05, size = 117, normalized size = 1.03

$$\frac{2\,b^2\,{\left(a+b\,x\right)}^{5/2}}{5}+\frac{\frac{15\,a^4\,b^2\,\sqrt{a+b\,x}}{4}-\frac{17\,a^3\,b^2\,{\left(a+b\,x\right)}^{3/2}}{4}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}+12\,a^2\,b^2\,\sqrt{a+b\,x}+2\,a\,b^2\,{\left(a+b\,x\right)}^{3/2}+\frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^3,x)
```

```
[Out] (2*b^2*(a + b*x)^(5/2))/5 + ((15*a^4*b^2*(a + b*x)^(1/2))/4 - (17*a^3*b^2*(
a + b*x)^(3/2))/4)/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + 12*a^2*b^2*(a + b*
x)^(1/2) + (a^(5/2)*b^2*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/4 + 2*a*b^2
*(a + b*x)^(3/2)
```

3.320 $\int \frac{(a+bx)^{9/2}}{x^4} dx$

Optimal. Leaf size=114

$$\frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.$$

```
[Out] 35/8*b^3*(b*x+a)^(3/2)-21/8*b^2*(b*x+a)^(5/2)/x-3/4*b*(b*x+a)^(7/2)/x^2-1/3
*(b*x+a)^(9/2)/x^3-105/8*a^(3/2)*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))+105/8*a
*b^3*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x])/8 + (35*b^3*(a + b*x)^(3/2))/8 - (21*b^2*(a + b*x
)^(5/2))/(8*x) - (3*b*(a + b*x)^(7/2))/(4*x^2) - (a + b*x)^(9/2)/(3*x^3) -
(105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^4}\,dx &= -\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{2}(3b)\int \frac{(a+bx)^{7/2}}{x^3}\,dx\\
&= -\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{8}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^2}\,dx\\
&= -\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= \frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105ab^3\right)\int \frac{\sqrt{a}}{}\\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}-\end{aligned}$$

Mathematica [A]
time = 0.12, size = 85, normalized size = 0.75

$$\frac{1}{24}\left(\frac{\sqrt{a+bx}\,\left(-8a^4-50a^3bx-165a^2b^2x^2+208ab^3x^3+16b^4x^4\right)}{x^3}-315a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^4,x]
[Out] ((Sqrt[a + b*x]*(-8*a^4 - 50*a^3*b*x - 165*a^2*b^2*x^2 + 208*a*b^3*x^3 + 16
*b^4*x^4))/x^3 - 315*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/24
```

Mathics [A]
time = 11.72, size = 148, normalized size = 1.30

$$\frac{-8a^5x^{\frac{5}{2}}(a+bx)^2-58a^4bx^{\frac{7}{2}}(a+bx)^2-215a^3b^2x^{\frac{9}{2}}(a+bx)^2+43a^2b^3x^{\frac{11}{2}}(a+bx)^2+16b^4x^{\frac{13}{2}}(14a+bx)(a+bx)^2-315a^{\frac{3}{2}}b^{\frac{11}{2}}x^8\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}}{24b^{\frac{5}{2}}x^8\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^4,x]')
```

```
[Out] (-8 a ^ 5 x ^ (5 / 2) (a + b x) ^ 2 - 58 a ^ 4 b x ^ (7 / 2) (a + b x) ^ 2
- 215 a ^ 3 b ^ 2 x ^ (9 / 2) (a + b x) ^ 2 + 43 a ^ 2 b ^ 3 x ^ (11 / 2) (
a + b x) ^ 2 + 16 b ^ 4 x ^ (13 / 2) (14 a + b x) (a + b x) ^ 2 - 315 a ^ (
3 / 2) b ^ (11 / 2) x ^ 8 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] ((a + b x) /
 (b x)) ^ (5 / 2)) / (24 b ^ (5 / 2) x ^ 8 ((a + b x) / (b x)) ^ (5 / 2))
```

Maple [A]
time = 0.10, size = 89, normalized size = 0.78

method	result
risch	$-\frac{a^{2} \sqrt{b x+a}\, \left(165 x^{2} b^{2}+50 a b x+8 a^{2}\right)}{24 x^{3}}+\frac{b^{3} \left(\frac{32 \left(b x+a\right)^{\frac{3}{2}}}{3}+128 a \sqrt{b x+a}-210 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)\right)}{16}$
derivativedivides	$2 b^{3} \left(\frac{\left(b x+a\right)^{\frac{3}{2}}}{3}+4 a \sqrt{b x+a}-a^{2} \left(-\frac{-\frac{55 \left(b x+a\right)^{\frac{5}{2}}}{16}+\frac{35 a \left(b x+a\right)^{\frac{3}{2}}}{6}-\frac{41 a^{2} \sqrt{b x+a}}{16}}{b^{3} x^{3}}+\frac{105 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{16 \sqrt{a}}\right)\right)$
default	$2 b^{3} \left(\frac{\left(b x+a\right)^{\frac{3}{2}}}{3}+4 a \sqrt{b x+a}-a^{2} \left(-\frac{-\frac{55 \left(b x+a\right)^{\frac{5}{2}}}{16}+\frac{35 a \left(b x+a\right)^{\frac{3}{2}}}{6}-\frac{41 a^{2} \sqrt{b x+a}}{16}}{b^{3} x^{3}}+\frac{105 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{16 \sqrt{a}}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(1/3*(b*x+a)^(3/2)+4*a*(b*x+a)^(1/2)-a^2*(-(-55/16*(b*x+a)^(5/2)+35/6
*a*(b*x+a)^(3/2)-41/16*a^2*(b*x+a)^(1/2))/b^3/x^3+105/16*arctanh((b*x+a)^(1
/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.34, size = 145, normalized size = 1.27

$$\frac{105}{16}\, a^{\frac{3}{2}} b^{3} \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}} b^{3}+8\,\sqrt{bx+a}\,ab^{3}-\frac{165\,(bx+a)^{\frac{5}{2}} a^{2} b^{3}-280\,(bx+a)^{\frac{3}{2}} a^{3} b^{3}+123\,\sqrt{bx+a}\,a^{4} b^{3}}{24\,\left((bx+a)^{3}-3\,(bx+a)^{2} a+3\,(bx+a)a^{2}-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 105/16*a^(3/2)*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
 + 2/3*(b*x + a)^(3/2)*b^3 + 8*sqrt(b*x + a)*a*b^3 - 1/24*(165*(b*x + a)^(5
/2)*a^2*b^3 - 280*(b*x + a)^(3/2)*a^3*b^3 + 123*sqrt(b*x + a)*a^4*b^3)/((b*
x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

Fricas [A]
time = 0.31, size = 178, normalized size = 1.56

$$\left[\frac{315\,a^{\frac{3}{2}}b^{3}x^{3}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(16\,b^{4}x^{4}+208\,ab^{3}x^{3}-165\,a^{2}b^{2}x^{2}-50\,a^{3}bx-8\,a^{4})\sqrt{bx+a}}{48\,x^{3}},\ \frac{315\,\sqrt{-a}\,ab^{3}x^{3}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(16\,b^{4}x^{4}+208\,ab^{3}x^{3}-165\,a^{2}b^{2}x^{2}-50\,a^{3}bx-8\,a^{4})\sqrt{bx+a}}{24\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(315*a^(3/2)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt(b
*x + a))/x^3, 1/24*(315*sqrt(-a)*a*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a)
 + (16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt
(b*x + a))/x^3]
```

Sympy [A]
time = 9.89, size = 184, normalized size = 1.61

$$-\frac{105a^{\frac{3}{2}}b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{8}-\frac{a^{5}}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{29a^{4}\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{215a^{3}b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{43a^{2}b^{\frac{5}{2}}}{24\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{28ab^{\frac{7}{2}}\sqrt{x}}{3\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**4,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/8 - a**5/(3*sqrt(b)*x**
(7/2)*sqrt(a/(b*x) + 1)) - 29*a**4*sqrt(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1))
- 215*a**3*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) + 43*a**2*b**(5/2)/(24*
sqrt(x)*sqrt(a/(b*x) + 1)) + 28*a*b**(7/2)*sqrt(x)/(3*sqrt(a/(b*x) + 1)) +
2*b**(9/2)*x**(3/2)/(3*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.01, size = 154, normalized size = 1.35

$$\frac{\frac{2}{3}\sqrt{a+bx}\,(a+bx)\,b^{4}+8\sqrt{a+bx}\,b^{4}a+\frac{-165\sqrt{a+bx}\,(a+bx)^{2}b^{4}a^{2}+280\sqrt{a+bx}\,(a+bx)b^{4}a^{3}-123\sqrt{a+bx}\,b^{4}a^{4}}{24(a+bx-a)^{3}}+\frac{105b^{4}a^{2}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{8\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x)
```

```
[Out] 1/24*(315*a^2*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 16*(b*x + a)^(3
/2)*b^4 + 192*sqrt(b*x + a)*a*b^4 - (165*(b*x + a)^(5/2)*a^2*b^4 - 280*(b*x
 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x + a)*a^4*b^4)/(b^3*x^3))/b
```

Mupad [B]
time = 0.12, size = 131, normalized size = 1.15

$$\frac{2\,b^{3}\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{\frac{41\,a^{4}\,b^{3}\,\sqrt{a+b\,x}}{8}-\frac{35\,a^{3}\,b^{3}\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{55\,a^{2}\,b^{3}\,{\left(a+b\,x\right)}^{5/2}}{8}}{3\,a\,{\left(a+b\,x\right)}^{2}-3\,a^{2}\,\left(a+b\,x\right)-{\left(a+b\,x\right)}^{3}+a^{3}}+8\,a\,b^{3}\,\sqrt{a+b\,x}+\frac{a^{3/2}\,b^{3}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,105{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^4,x)
```

```
[Out] (2*b^3*(a + b*x)^(3/2))/3 + ((41*a^4*b^3*(a + b*x)^(1/2))/8 - (35*a^3*b^3*(
a + b*x)^(3/2))/3 + (55*a^2*b^3*(a + b*x)^(5/2))/8)/(3*a*(a + b*x)^2 - 3*a^
2*(a + b*x) - (a + b*x)^3 + a^3) + (a^(3/2)*b^3*atan(((a + b*x)^(1/2)*1i)/a
^(1/2))*105i)/8 + 8*a*b^3*(a + b*x)^(1/2)
```

3.321 $\int \frac{(a+bx)^{9/2}}{x^5}\,dx$

Optimal. Leaf size=116

$$\frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -105/64*b^3*(b*x+a)^(3/2)/x-21/32*b^2*(b*x+a)^(5/2)/x^2-3/8*b*(b*x+a)^(7/2)
/x^3-1/4*(b*x+a)^(9/2)/x^4-315/64*b^4*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2
)+315/64*b^4*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$\frac{315}{64}b^4\sqrt{a+bx}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{(a+bx)^{9/2}}{4x^4}-\frac{3b(a+bx)^{7/2}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x])/64 - (105*b^3*(a + b*x)^(3/2))/(64*x) - (21*b^2*(a
+ b*x)^(5/2))/(32*x^2) - (3*b*(a + b*x)^(7/2))/(8*x^3) - (a + b*x)^(9/2)/(4
*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^5}\,dx &= -\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{8}(9b)\int \frac{(a+bx)^{7/2}}{x^4}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{16}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^3}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{64}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x^2}\,dx \\
&= -\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{128}\left(315b^4\right)\int \frac{\sqrt{}}{} \\
&= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+ \\
&= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+ \\
&= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}-
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 86, normalized size = 0.74

$$\frac{1}{64}\left(-\frac{\sqrt{a+bx}\,\left(16a^4+88a^3bx+210a^2b^2x^2+325ab^3x^3-128b^4x^4\right)}{x^4}-315\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (-((Sqrt[a + b*x]*(16*a^4 + 88*a^3*b*x + 210*a^2*b^2*x^2 + 325*a*b^3*x^3 -
128*b^4*x^4))/x^4) - 315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Mathics [A]
time = 11.99, size = 185, normalized size = 1.59

$$\frac{b^{\frac{5}{2}}\left(-16a^5x^8\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}-104a^4bx^9\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}-315\sqrt{a}\,b^{\frac{3}{2}}x^{\frac{19}{2}}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right](a+bx)^3-298a^3b^2x^{10}\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}-535a^2b^3x^{11}\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}-197ab^4x^{12}\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}+128b^5x^{13}\left(\frac{a+bx}{bx}\right)^{\frac{5}{2}}\right)}{64x^{\frac{19}{2}}(a+bx)^3}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^5,x]')
```

```
[Out] b ^ (5 / 2) (-16 a ^ 5 x ^ 8 ((a + b x) / (b x)) ^ (5 / 2) - 104 a ^ 4 b x
^ 9 ((a + b x) / (b x)) ^ (5 / 2) - 315 Sqrt[a] b ^ (3 / 2) x ^ (19 / 2) Ar
cSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) ^ 3 - 298 a ^ 3 b ^ 2 x ^ 10 (
(a + b x) / (b x)) ^ (5 / 2) - 535 a ^ 2 b ^ 3 x ^ 11 ((a + b x) / (b x)) ^
 (5 / 2) - 197 a b ^ 4 x ^ 12 ((a + b x) / (b x)) ^ (5 / 2) + 128 b ^ 5 x ^
 13 ((a + b x) / (b x)) ^ (5 / 2)) / (64 x ^ (19 / 2) (a + b x) ^ 3)
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.74

method	result
risch	$-\frac{a\sqrt{bx+a}\,(325b^3x^3+210a\,b^2x^2+88a^2bx+16a^3)}{64x^4}+\frac{b^4\left(256\sqrt{bx+a}-630\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\right)}{128}$
derivativedivides	$2b^4\left(\sqrt{bx+a}-a\left(\frac{\frac{325(bx+a)^{\frac{7}{2}}}{128}-\frac{765a(bx+a)^{\frac{5}{2}}}{128}+\frac{643a^2(bx+a)^{\frac{3}{2}}}{128}-\frac{187a^3\sqrt{bx+a}}{128}}{b^4x^4}+\frac{315\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{128\sqrt{a}}\right.\right.$
default	$2b^4\left(\sqrt{bx+a}-a\left(\frac{\frac{325(bx+a)^{\frac{7}{2}}}{128}-\frac{765a(bx+a)^{\frac{5}{2}}}{128}+\frac{643a^2(bx+a)^{\frac{3}{2}}}{128}-\frac{187a^3\sqrt{bx+a}}{128}}{b^4x^4}+\frac{315\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{128\sqrt{a}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4*((b*x+a)^(1/2)-a*((325/128*(b*x+a)^(7/2)-765/128*a*(b*x+a)^(5/2)+643/
128*a^2*(b*x+a)^(3/2)-187/128*a^3*(b*x+a)^(1/2))/b^4/x^4+315/128*arctanh((b
*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.34, size = 155, normalized size = 1.34

$$\frac{315}{128}\sqrt{a}\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^4-\frac{325\,(bx+a)^{\frac{7}{2}}ab^4-765\,(bx+a)^{\frac{5}{2}}a^2b^4+643\,(bx+a)^{\frac{3}{2}}a^3b^4-187\sqrt{bx+a}\,a^4b^4}{64\left((bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2-4\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))
) + 2*sqrt(b*x + a)*b^4 - 1/64*(325*(b*x + a)^(7/2)*a*b^4 - 765*(b*x + a)^(
5/2)*a^2*b^4 + 643*(b*x + a)^(3/2)*a^3*b^4 - 187*sqrt(b*x + a)*a^4*b^4)/((b
*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2 - 4*(b*x + a)*a^3 + a^4)
```

Fricas [A]
time = 0.31, size = 177, normalized size = 1.53

$$\left[\frac{315\,\sqrt{a}\,b^4x^4\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4)\sqrt{bx+a}}{128\,x^4},\frac{315\,\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4)\sqrt{bx+a}}{64\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(315*sqrt(a)*b^4*x^4*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*sqr
t(b*x + a))/x^4, 1/64*(315*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*s
qrt(b*x + a))/x^4]
```

Sympy [A]
time = 10.36, size = 182, normalized size = 1.57

$$-\frac{315\sqrt{a}\,b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{64}-\frac{a^5}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{13a^4\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{149a^3b^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{535a^2b^{\frac{5}{2}}}{64x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{197ab^{\frac{7}{2}}}{64\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**5,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/64 - a**5/(4*sqrt(b)*x**
(9/2)*sqrt(a/(b*x) + 1)) - 13*a**4*sqrt(b)/(8*x**(7/2)*sqrt(a/(b*x) + 1)) -
 149*a**3*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) - 535*a**2*b**(5/2)/(64*
x**(3/2)*sqrt(a/(b*x) + 1)) - 197*a*b**(7/2)/(64*sqrt(x)*sqrt(a/(b*x) + 1))
 + 2*b**(9/2)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 0.01, size = 152, normalized size = 1.31

$$\frac{2\sqrt{a+bx}\,b^5+\frac{-325\sqrt{a+bx}\,(a+bx)^3b^5a+765\sqrt{a+bx}\,(a+bx)^2b^5a^2-643\sqrt{a+bx}\,(a+bx)b^5a^3+187\sqrt{a+bx}\,b^5a^4}{64(a+bx-a)^4}+\frac{315b^5a\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{64\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x)
```

```
[Out] 1/64*(315*a*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x + a)
*b^5 - (325*(b*x + a)^(7/2)*a*b^5 - 765*(b*x + a)^(5/2)*a^2*b^5 + 643*(b*x
+ a)^(3/2)*a^3*b^5 - 187*sqrt(b*x + a)*a^4*b^5)/(b^4*x^4))/b
```

Mupad [B]
time = 0.06, size = 94, normalized size = 0.81

$$2\,b^4\,\sqrt{a+b\,x}+\frac{187\,a^4\,\sqrt{a+b\,x}}{64\,x^4}-\frac{643\,a^3\,{\left(a+b\,x\right)}^{3/2}}{64\,x^4}+\frac{765\,a^2\,{\left(a+b\,x\right)}^{5/2}}{64\,x^4}-\frac{325\,a\,{\left(a+b\,x\right)}^{7/2}}{64\,x^4}+\frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,315{}\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*(a + b*x)^(1/2) + (187*a^4*(a + b*x)^(1/2))/(64*x^4) - (643*a^3*(a +
b*x)^(3/2))/(64*x^4) + (765*a^2*(a + b*x)^(5/2))/(64*x^4) + (a^(1/2)*b^4*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*315i)/64 - (325*a*(a + b*x)^(7/2))/(64*x^4
)
```

3.322 $\int \frac{(a+bx)^{9/2}}{x^6}\,dx$

Optimal. Leaf size=119

$$-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}$$

```
[Out] -21/64*b^3*(b*x+a)^(3/2)/x^2-21/80*b^2*(b*x+a)^(5/2)/x^3-9/40*b*(b*x+a)^(7/
2)/x^4-1/5*(b*x+a)^(9/2)/x^5-63/128*b^5*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1
/2)-63/128*b^4*(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{9b(a+bx)^{7/2}}{40x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x])/(128*x) - (21*b^3*(a + b*x)^(3/2))/(64*x^2) - (21*b
^2*(a + b*x)^(5/2))/(80*x^3) - (9*b*(a + b*x)^(7/2))/(40*x^4) - (a + b*x)^(
9/2)/(5*x^5) - (63*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = -\frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{10}(9b)\int \frac{(a+bx)^{7/2}}{x^5}\,dx$$

$$= -\frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{80}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x^4}\,dx$$

$$= -\frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{32}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$

$$= -\frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{128}\left(63b^4\right)\int \frac{\sqrt{}}{}$$

$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} +$$

$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} +$$

$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} -$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.72

$$\frac{1}{640}\left(-\frac{\sqrt{a+bx}\,\left(128a^4+656a^3bx+1368a^2b^2x^2+1490ab^3x^3+965b^4x^4\right)}{x^5}-\frac{315b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-((Sqrt[a + b*x]*(128*a^4 + 656*a^3*b*x + 1368*a^2*b^2*x^2 + 1490*a*b^3*x^
3 + 965*b^4*x^4))/x^5) - (315*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/
640
```

Mathics [A]
time = 12.50, size = 130, normalized size = 1.09

$$-\frac{a^4\sqrt{b}\sqrt{1+\frac{a}{bx}}}{5x^{\frac{9}{2}}}-\frac{41a^3b^{\frac{3}{2}}\sqrt{1+\frac{a}{bx}}}{40x^{\frac{7}{2}}}-\frac{171a^2b^{\frac{5}{2}}\sqrt{1+\frac{a}{bx}}}{80x^{\frac{5}{2}}}-\frac{149ab^{\frac{7}{2}}\sqrt{1+\frac{a}{bx}}}{64x^{\frac{3}{2}}}-\frac{63b^5\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{128\sqrt{a}}-\frac{193b^{\frac{9}{2}}\sqrt{1+\frac{a}{bx}}}{128\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^6,x]')
```

```
[Out] -a ^ 4 Sqrt[b] Sqrt[1 + a / (b x)] / (5 x ^ (9 / 2)) - 41 a ^ 3 b ^ (3 / 2)
 Sqrt[1 + a / (b x)] / (40 x ^ (7 / 2)) - 171 a ^ 2 b ^ (5 / 2) Sqrt[1 + a
/ (b x)] / (80 x ^ (5 / 2)) - 149 a b ^ (7 / 2) Sqrt[1 + a / (b x)] / (64 x
 ^ (3 / 2)) - 63 b ^ 5 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (128 Sqrt[a])
 - 193 b ^ (9 / 2) Sqrt[1 + a / (b x)] / (128 Sqrt[x])
```

Maple [A]
time = 0.12, size = 88, normalized size = 0.74

method	result
risch	$-\frac{\sqrt{bx+a}\,(965b^4x^4+1490a\,b^3x^3+1368a^2b^2x^2+656a^3bx+128a^4)}{640x^5}-\frac{63b^5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}$
derivativedivides	$2b^5\left(-\frac{\frac{193(bx+a)^{\frac{9}{2}}}{256}-\frac{237a(bx+a)^{\frac{7}{2}}}{128}+\frac{21a^2(bx+a)^{\frac{5}{2}}}{10}-\frac{147a^3(bx+a)^{\frac{3}{2}}}{128}+\frac{63a^4\sqrt{bx+a}}{256}}{b^5x^5}-\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}\right)$
default	$2b^5\left(-\frac{\frac{193(bx+a)^{\frac{9}{2}}}{256}-\frac{237a(bx+a)^{\frac{7}{2}}}{128}+\frac{21a^2(bx+a)^{\frac{5}{2}}}{10}-\frac{147a^3(bx+a)^{\frac{3}{2}}}{128}+\frac{63a^4\sqrt{bx+a}}{256}}{b^5x^5}-\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^5*(-(193/256*(b*x+a)^(9/2)-237/128*a*(b*x+a)^(7/2)+21/10*a^2*(b*x+a)^(5
/2)-147/128*a^3*(b*x+a)^(3/2)+63/256*a^4*(b*x+a)^(1/2))/b^5/x^5-63/256*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.35, size = 169, normalized size = 1.42

$$\frac{63\,b^5\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{256\,\sqrt{a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^5-2370\,(bx+a)^{\frac{7}{2}}ab^5+2688\,(bx+a)^{\frac{5}{2}}a^2b^5-1470\,(bx+a)^{\frac{3}{2}}a^3b^5+315\,\sqrt{bx+a}\,a^4b^5}{640\,\left((bx+a)^5-5\,(bx+a)^4a+10\,(bx+a)^3a^2-10\,(bx+a)^2a^3+5\,(bx+a)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 63/256*b^5*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
 - 1/640*(965*(b*x + a)^(9/2)*b^5 - 2370*(b*x + a)^(7/2)*a*b^5 + 2688*(b*x
+ a)^(5/2)*a^2*b^5 - 1470*(b*x + a)^(3/2)*a^3*b^5 + 315*sqrt(b*x + a)*a^4*b
^5)/((b*x + a)^5 - 5*(b*x + a)^4*a + 10*(b*x + a)^3*a^2 - 10*(b*x + a)^2*a^
3 + 5*(b*x + a)*a^4 - a^5)
```

Fricas [A]
time = 0.33, size = 190, normalized size = 1.60

$$\left[\frac{315\,\sqrt{a}\,b^5x^5\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(965\,ab^4x^4+1490\,a^2b^3x^3+1368\,a^3b^2x^2+656\,a^4bx+128\,a^5)\sqrt{bx+a}}{1280\,ax^5},\ \frac{315\,\sqrt{-a}\,b^5x^5\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(965\,ab^4x^4+1490\,a^2b^3x^3+1368\,a^3b^2x^2+656\,a^4bx+128\,a^5)\sqrt{bx+a}}{640\,ax^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(315*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
 2*(965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 656*a^4*b*x + 128
*a^5)*sqrt(b*x + a))/(a*x^5), 1/640*(315*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x +
 a)*sqrt(-a)/a) - (965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 65
6*a^4*b*x + 128*a^5)*sqrt(b*x + a))/(a*x^5)]
```

Sympy [A]
time = 11.04, size = 158, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{5x^{\frac{9}{2}}}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{40x^{\frac{7}{2}}}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{80x^{\frac{5}{2}}}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{64x^{\frac{3}{2}}}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{128\sqrt{x}}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**6,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**(9/2)) - 41*a**3*b**(3/2)*sqrt(a/(b*x
) + 1)/(40*x**(7/2)) - 171*a**2*b**(5/2)*sqrt(a/(b*x) + 1)/(80*x**(5/2)) -
149*a*b**(7/2)*sqrt(a/(b*x) + 1)/(64*x**(3/2)) - 193*b**(9/2)*sqrt(a/(b*x)
+ 1)/(128*sqrt(x)) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*sqrt(a))
```

Giac [A]
time = 0.01, size = 159, normalized size = 1.34

$$\frac{-\frac{965\sqrt{a+bx}\,(a+bx)^4b^6-2370\sqrt{a+bx}\,(a+bx)^3b^6a+2688\sqrt{a+bx}\,(a+bx)^2b^6a^2-1470\sqrt{a+bx}\,(a+bx)b^6a^3+315\sqrt{a+bx}\,b^6a^4}{640(a+bx-a)^5}+\frac{63b^6\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{128\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x)
```

```
[Out] 1/640*(315*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (965*(b*x + a)^(9/
2)*b^6 - 2370*(b*x + a)^(7/2)*a*b^6 + 2688*(b*x + a)^(5/2)*a^2*b^6 - 1470*(
b*x + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x + a)*a^4*b^6)/(b^5*x^5))/b
```

Mupad [B]
time = 0.12, size = 94, normalized size = 0.79

$$\frac{147\,a^3\,(a+b\,x)^{3/2}}{64\,x^5}-\frac{63\,a^4\,\sqrt{a+b\,x}}{128\,x^5}-\frac{193\,(a+b\,x)^{9/2}}{128\,x^5}-\frac{21\,a^2\,(a+b\,x)^{5/2}}{5\,x^5}+\frac{237\,a\,(a+b\,x)^{7/2}}{64\,x^5}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{128\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^6,x)
```

```
[Out] (147*a^3*(a + b*x)^(3/2))/(64*x^5) - (63*a^4*(a + b*x)^(1/2))/(128*x^5) - (
193*(a + b*x)^(9/2))/(128*x^5) - (21*a^2*(a + b*x)^(5/2))/(5*x^5) + (b^5*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/(128*a^(1/2)) + (237*a*(a + b*x)^(7/2
))/(64*x^5)
```

3.323 $\int \frac{(a+bx)^{9/2}}{x^7}\,dx$

Optimal. Leaf size=141

$$-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{21b^6\tanh^{-1}}{5}$$

```
[Out] -7/64*b^3*(b*x+a)^(3/2)/x^3-21/160*b^2*(b*x+a)^(5/2)/x^4-3/20*b*(b*x+a)^(7/
2)/x^5-1/6*(b*x+a)^(9/2)/x^6+21/512*b^6*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3
/2)-21/256*b^4*(b*x+a)^(1/2)/x^2-21/512*b^5*(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{(a+bx)^{9/2}}{6x^6}-\frac{3b(a+bx)^{7/2}}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x])/(256*x^2) - (21*b^5*Sqrt[a + b*x])/(512*a*x) - (7*b
^3*(a + b*x)^(3/2))/(64*x^3) - (21*b^2*(a + b*x)^(5/2))/(160*x^4) - (3*b*(a
 + b*x)^(7/2))/(20*x^5) - (a + b*x)^(9/2)/(6*x^6) + (21*b^6*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^7}\,dx = -\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{4}(3b)\int \frac{(a+bx)^{7/2}}{x^6}\,dx$$

$$= -\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{40}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^5}\,dx$$

$$= -\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{64}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^4}\,dx$$

$$= -\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{128}\left(21b^4\right)\int \frac{\sqrt{a+}}{x}$$

$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{51}$$

$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-$$

$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-$$

$$= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-$$

Mathematica [A]
time = 0.19, size = 100, normalized size = 0.71

$$-\frac{\sqrt{a+bx}\,\left(1280a^5+6272a^4bx+12144a^3b^2x^2+11432a^2b^3x^3+4910ab^4x^4+315b^5x^5\right)}{7680ax^6}+\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^7,x]
```

[Out] -1/7680*(Sqrt[a + b*x]*(1280*a^5 + 6272*a^4*b*x + 12144*a^3*b^2*x^2 + 11432 *a^2*b^3*x^3 + 4910*a*b^4*x^4 + 315*b^5*x^5))/(a*x^6) + (21*b^6*ArcTanh[Sqr t[a + b*x]/Sqrt[a]])/(512*a^(3/2))

Mathics [A]
time = 47.08, size = 181, normalized size = 1.28

$$\frac{\frac{-1280a^{\frac{15}{2}}x^{15}(a+bx)^3}{b^3}-\frac{7552a^{\frac{13}{2}}x^{16}(a+bx)^3}{b^2}-\frac{18416a^{\frac{11}{2}}x^{17}(a+bx)^3}{b}-23576a^{\frac{9}{2}}x^{18}(a+bx)^3-16342a^{\frac{7}{2}}bx^{19}(a+bx)^3-5225a^{\frac{5}{2}}b^2x^{20}(a+bx)^3-315a^{\frac{3}{2}}b^3x^{21}(a+bx)^3+315ab^{\frac{13}{2}}x^{\frac{49}{2}}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}}{7680a^{\frac{5}{2}}\sqrt{b}\,x^{\frac{49}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^7,x]')
```

```
[Out] (-1280 a ^ (15 / 2) x ^ 15 (a + b x) ^ 3 / b ^ 3 - 7552 a ^ (13 / 2) x ^ 16
 (a + b x) ^ 3 / b ^ 2 - 18416 a ^ (11 / 2) x ^ 17 (a + b x) ^ 3 / b - 2357
6 a ^ (9 / 2) x ^ 18 (a + b x) ^ 3 - 16342 a ^ (7 / 2) b x ^ 19 (a + b x) ^
 3 - 5225 a ^ (5 / 2) b ^ 2 x ^ 20 (a + b x) ^ 3 - 315 a ^ (3 / 2) b ^ 3 x
^ 21 (a + b x) ^ 3 + 315 a b ^ (13 / 2) x ^ (49 / 2) ArcSinh[Sqrt[a] / (Sqr
t[b] Sqrt[x])] ((a + b x) / (b x)) ^ (7 / 2)) / (7680 a ^ (5 / 2) Sqrt[b] x
 ^ (49 / 2) ((a + b x) / (b x)) ^ (7 / 2))
```

Maple [A]
time = 0.12, size = 100, normalized size = 0.71

method	result
risch	$-\frac{\sqrt{bx+a}\,(315b^5x^5+4910a\,b^4x^4+11432a^2b^3x^3+12144a^3b^2x^2+6272a^4bx+1280a^5)}{7680x^6a}+\frac{21b^6\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{512a^{\frac{3}{2}}}$
derivativedivides	$2b^6\left(-\frac{\frac{21(bx+a)^{\frac{11}{2}}}{1024a}+\frac{667(bx+a)^{\frac{9}{2}}}{3072}-\frac{843a(bx+a)^{\frac{7}{2}}}{2560}+\frac{693a^2(bx+a)^{\frac{5}{2}}}{2560}-\frac{119a^3(bx+a)^{\frac{3}{2}}}{1024}+\frac{21a^4\sqrt{bx+a}}{1024}}{b^6x^6}+\frac{21\operatorname{arctanh}(\ldots}{10\ldots}\right.$
default	$2b^6\left(-\frac{\frac{21(bx+a)^{\frac{11}{2}}}{1024a}+\frac{667(bx+a)^{\frac{9}{2}}}{3072}-\frac{843a(bx+a)^{\frac{7}{2}}}{2560}+\frac{693a^2(bx+a)^{\frac{5}{2}}}{2560}-\frac{119a^3(bx+a)^{\frac{3}{2}}}{1024}+\frac{21a^4\sqrt{bx+a}}{1024}}{b^6x^6}+\frac{21\operatorname{arctanh}(\ldots}{10\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^6*(-(21/1024/a*(b*x+a)^(11/2)+667/3072*(b*x+a)^(9/2)-843/2560*a*(b*x+a)
^(7/2)+693/2560*a^2*(b*x+a)^(5/2)-119/1024*a^3*(b*x+a)^(3/2)+21/1024*a^4*(b
*x+a)^(1/2))/b^6/x^6+21/1024*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.37, size = 198, normalized size = 1.40

$$-\frac{21\,b^6\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{1024\,a^{\frac{3}{2}}}-\frac{315\,(bx+a)^{\frac{11}{2}}b^6+3335\,(bx+a)^{\frac{9}{2}}ab^6-5058\,(bx+a)^{\frac{7}{2}}a^2b^6+4158\,(bx+a)^{\frac{5}{2}}a^3b^6-1785\,(bx+a)^{\frac{3}{2}}a^4b^6+315\,\sqrt{bx+a}\,a^5b^6}{7680\,\left((bx+a)^6a-6\,(bx+a)^5a^2+15\,(bx+a)^4a^3-20\,(bx+a)^3a^4+15\,(bx+a)^2a^5-6\,(bx+a)a^6+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -21/1024*b^6*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/
2) - 1/7680*(315*(b*x + a)^(11/2)*b^6 + 3335*(b*x + a)^(9/2)*a*b^6 - 5058*(
b*x + a)^(7/2)*a^2*b^6 + 4158*(b*x + a)^(5/2)*a^3*b^6 - 1785*(b*x + a)^(3/2
)*a^4*b^6 + 315*sqrt(b*x + a)*a^5*b^6)/((b*x + a)^6*a - 6*(b*x + a)^5*a^2 +
 15*(b*x + a)^4*a^3 - 20*(b*x + a)^3*a^4 + 15*(b*x + a)^2*a^5 - 6*(b*x + a)
*a^6 + a^7)
```

Fricas [A]
time = 0.31, size = 211, normalized size = 1.50

$$\left[\frac{315\,\sqrt{a}\,b^6x^6\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+1280\,a^6)\sqrt{bx+a}}{15360\,a^2x^6},\ -\frac{315\,\sqrt{-a}\,b^6x^6\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+1280\,a^6)\sqrt{bx+a}}{7680\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/15360*(315*sqrt(a)*b^6*x^6*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
- 2*(315*a*b^5*x^5 + 4910*a^2*b^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x
^2 + 6272*a^5*b*x + 1280*a^6)*sqrt(b*x + a))/(a^2*x^6), -1/7680*(315*sqrt(-
a)*b^6*x^6*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^5*x^5 + 4910*a^2*b^4
*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x^2 + 6272*a^5*b*x + 1280*a^6)*sqr
t(b*x + a))/(a^2*x^6)]
```

Sympy [A]
time = 47.70, size = 209, normalized size = 1.48

$$-\frac{a^5}{6\sqrt{b}\,x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{59a^4\sqrt{b}}{60x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{480x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{960x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{8171ab^{\frac{7}{2}}}{3840x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1045b^{\frac{9}{2}}}{1536x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{21b^{\frac{11}{2}}}{512a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{21b^6\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{512a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**7,x)
```

```
[Out] -a**5/(6*sqrt(b)*x**(13/2)*sqrt(a/(b*x) + 1)) - 59*a**4*sqrt(b)/(60*x**(11/
2)*sqrt(a/(b*x) + 1)) - 1151*a**3*b**(3/2)/(480*x**(9/2)*sqrt(a/(b*x) + 1))
 - 2947*a**2*b**(5/2)/(960*x**(7/2)*sqrt(a/(b*x) + 1)) - 8171*a*b**(7/2)/(3
840*x**(5/2)*sqrt(a/(b*x) + 1)) - 1045*b**(9/2)/(1536*x**(3/2)*sqrt(a/(b*x)
 + 1)) - 21*b**(11/2)/(512*a*sqrt(x)*sqrt(a/(b*x) + 1)) + 21*b**6*asinh(sqr
t(a)/(sqrt(b)*sqrt(x)))/(512*a**(3/2))
```

Giac [A]
time = 0.01, size = 190, normalized size = 1.35

$$-\frac{\frac{315\sqrt{a+bx}\,(a+bx)^5b^7+3335\sqrt{a+bx}\,(a+bx)^4b^7a-5058\sqrt{a+bx}\,(a+bx)^3b^7a^2+4158\sqrt{a+bx}\,(a+bx)^2b^7a^3-1785\sqrt{a+bx}\,(a+bx)b^7a^4+315\sqrt{a+bx}\,b^7a^5}{7680a(a+bx-a)^6}-\frac{21b^7\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{256a\cdot 2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x)
```

```
[Out] -1/7680*(315*b^7*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x +
a)^(11/2)*b^7 + 3335*(b*x + a)^(9/2)*a*b^7 - 5058*(b*x + a)^(7/2)*a^2*b^7 +
 4158*(b*x + a)^(5/2)*a^3*b^7 - 1785*(b*x + a)^(3/2)*a^4*b^7 + 315*sqrt(b*x
 + a)*a^5*b^7)/(a*b^6*x^6))/b
```

Mupad [B]
time = 0.13, size = 109, normalized size = 0.77

$$\frac{119\,a^3\,{\left(a+b\,x\right)}^{3/2}}{512\,x^6}-\frac{21\,a^4\,\sqrt{a+b\,x}}{512\,x^6}-\frac{667\,{\left(a+b\,x\right)}^{9/2}}{1536\,x^6}-\frac{693\,a^2\,{\left(a+b\,x\right)}^{5/2}}{1280\,x^6}-\frac{21\,{\left(a+b\,x\right)}^{11/2}}{512\,a\,x^6}+\frac{843\,a\,{\left(a+b\,x\right)}^{7/2}}{1280\,x^6}-\frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,21{}\mathrm{i}}{512\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^7,x)
```

```
[Out] (119*a^3*(a + b*x)^(3/2))/(512*x^6) - (21*a^4*(a + b*x)^(1/2))/(512*x^6) -
(667*(a + b*x)^(9/2))/(1536*x^6) - (693*a^2*(a + b*x)^(5/2))/(1280*x^6) - (
21*(a + b*x)^(11/2))/(512*a*x^6) - (b^6*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*
21i)/(512*a^(3/2)) + (843*a*(a + b*x)^(7/2))/(1280*x^6)
```

3.324 $\int \frac{(a+bx)^{9/2}}{x^8} dx$

Optimal. Leaf size=163

$$-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}-$$

```
[Out] -3/64*b^3*(b*x+a)^(3/2)/x^4-3/40*b^2*(b*x+a)^(5/2)/x^5-3/28*b*(b*x+a)^(7/2)
/x^6-1/7*(b*x+a)^(9/2)/x^7-9/1024*b^7*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2
)-3/128*b^4*(b*x+a)^(1/2)/x^3-3/512*b^5*(b*x+a)^(1/2)/a/x^2+9/1024*b^6*(b*x
+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.05, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{(a+bx)^{9/2}}{7x^7}-\frac{3b(a+bx)^{7/2}}{28x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x])/(128*x^3) - (3*b^5*Sqrt[a + b*x])/(512*a*x^2) + (9*b
^6*Sqrt[a + b*x])/(1024*a^2*x) - (3*b^3*(a + b*x)^(3/2))/(64*x^4) - (3*b^2*
(a + b*x)^(5/2))/(40*x^5) - (3*b*(a + b*x)^(7/2))/(28*x^6) - (a + b*x)^(9/2
)/(7*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^8}\,dx &= -\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{14}(9b)\int \frac{(a+bx)^{7/2}}{x^7}\,dx\\
&= -\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{8}\left(3b^2\right)\int \frac{(a+bx)^{5/2}}{x^6}\,dx\\
&= -\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{16}\left(3b^3\right)\int \frac{(a+bx)^{3/2}}{x^5}\,dx\\
&= -\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{128}\left(9b^4\right)\int \frac{\sqrt{a+}}{x^4}\\
&= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{25}\\
&= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(}{}\\
&= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b}{}\\
&= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b}{}\\
&= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b}{}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 111, normalized size = 0.68

$$-\frac{\sqrt{a+bx}\,\left(5120a^6+24320a^5bx+44928a^4b^2x^2+39056a^3b^3x^3+14168a^2b^4x^4+210ab^5x^5-315b^6x^6\right)}{35840a^2x^7}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] -1/35840*(Sqrt[a + b*x]*(5120*a^6 + 24320*a^5*b*x + 44928*a^4*b^2*x^2 + 390
56*a^3*b^3*x^3 + 14168*a^2*b^4*x^4 + 210*a*b^5*x^5 - 315*b^6*x^6))/(a^2*x^7
) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Mathics [A]
time = 275.32, size = 243, normalized size = 1.49

$$\frac{b^{\frac{7}{2}}\left(-5120a^{\frac{21}{2}}x^{\frac{49}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}-29440a^{\frac{19}{2}}bx^{\frac{51}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}-69248a^{\frac{17}{2}}b^2x^{\frac{53}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}-83984a^{\frac{15}{2}}b^3x^{\frac{55}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}-315a^3b^{\frac{7}{2}}x^{28}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right](a+bx)^4-53224a^{\frac{13}{2}}b^4x^{\frac{57}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}-14378a^{\frac{11}{2}}b^5x^{\frac{59}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}+105a^{\frac{9}{2}}b^6x^{\frac{61}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}+315a^{\frac{7}{2}}b^7x^{\frac{63}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{7}{2}}\right)}{35840a^{\frac{11}{2}}x^{28}(a+bx)^4}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/x^8,x]')
```

```
[Out] b ^ (7 / 2) (-5120 a ^ (21 / 2) x ^ (49 / 2) ((a + b x) / (b x)) ^ (7 / 2)
- 29440 a ^ (19 / 2) b x ^ (51 / 2) ((a + b x) / (b x)) ^ (7 / 2) - 69248 a
 ^ (17 / 2) b ^ 2 x ^ (53 / 2) ((a + b x) / (b x)) ^ (7 / 2) - 83984 a ^ (1
5 / 2) b ^ 3 x ^ (55 / 2) ((a + b x) / (b x)) ^ (7 / 2) - 315 a ^ 3 b ^ (7
/ 2) x ^ 28 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) ^ 4 - 53224 a ^
(13 / 2) b ^ 4 x ^ (57 / 2) ((a + b x) / (b x)) ^ (7 / 2) - 14378 a ^ (11 /
 2) b ^ 5 x ^ (59 / 2) ((a + b x) / (b x)) ^ (7 / 2) + 105 a ^ (9 / 2) b ^
6 x ^ (61 / 2) ((a + b x) / (b x)) ^ (7 / 2) + 315 a ^ (7 / 2) b ^ 7 x ^ (6
3 / 2) ((a + b x) / (b x)) ^ (7 / 2)) / (35840 a ^ (11 / 2) x ^ 28 (a + b x
) ^ 4)
```

Maple [A]
time = 0.11, size = 112, normalized size = 0.69

method	result
risch	$-\frac{\sqrt{bx+a}\,(-315x^6b^6+210a\,x^5b^5+14168a^2x^4b^4+39056a^3b^3x^3+44928a^4x^2b^2+24320a^5xb+5120a^6)}{35840x^7a^2}-\frac{9b^7\operatorname{arctanh}}{\ldots}$
derivativedivides	$2b^7\left(-\frac{-\frac{9(bx+a)^{\frac{13}{2}}}{2048a^2}+\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{1199(bx+a)^{\frac{9}{2}}}{10240}-\frac{9a(bx+a)^{\frac{7}{2}}}{70}+\frac{849a^2(bx+a)^{\frac{5}{2}}}{10240}-\frac{15a^3(bx+a)^{\frac{3}{2}}}{512}+\frac{9a^4\sqrt{bx+a}}{2048}}{b^7x^7}-\frac{9a}{\ldots}\right.$
default	$2b^7\left(-\frac{-\frac{9(bx+a)^{\frac{13}{2}}}{2048a^2}+\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{1199(bx+a)^{\frac{9}{2}}}{10240}-\frac{9a(bx+a)^{\frac{7}{2}}}{70}+\frac{849a^2(bx+a)^{\frac{5}{2}}}{10240}-\frac{15a^3(bx+a)^{\frac{3}{2}}}{512}+\frac{9a^4\sqrt{bx+a}}{2048}}{b^7x^7}-\frac{9a}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^7*(-(-9/2048/a^2*(b*x+a)^(13/2)+15/512/a*(b*x+a)^(11/2)+1199/10240*(b*x
+a)^(9/2)-9/70*a*(b*x+a)^(7/2)+849/10240*a^2*(b*x+a)^(5/2)-15/512*a^3*(b*x+
a)^(3/2)+9/2048*a^4*(b*x+a)^(1/2))/b^7/x^7-9/2048*arctanh((b*x+a)^(1/2)/a^(
1/2))/a^(5/2))
```

Maxima [A]
time = 0.35, size = 229, normalized size = 1.40

$$\frac{9\,b^7\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2048\,a^{\frac{5}{2}}}+\frac{315\,(bx+a)^{\frac{13}{2}}b^7-2100\,(bx+a)^{\frac{11}{2}}ab^7-8393\,(bx+a)^{\frac{9}{2}}a^2b^7+9216\,(bx+a)^{\frac{7}{2}}a^3b^7-5943\,(bx+a)^{\frac{5}{2}}a^4b^7+2100\,(bx+a)^{\frac{3}{2}}a^5b^7-315\,\sqrt{bx+a}\,a^6b^7}{35840\,\left((bx+a)^7a^2-7\,(bx+a)^6a^3+21\,(bx+a)^5a^4-35\,(bx+a)^4a^5+35\,(bx+a)^3a^6-21\,(bx+a)^2a^7+7\,(bx+a)a^8-a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2048*b^7*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
 + 1/35840*(315*(b*x + a)^(13/2)*b^7 - 2100*(b*x + a)^(11/2)*a*b^7 - 8393*(
b*x + a)^(9/2)*a^2*b^7 + 9216*(b*x + a)^(7/2)*a^3*b^7 - 5943*(b*x + a)^(5/2
)*a^4*b^7 + 2100*(b*x + a)^(3/2)*a^5*b^7 - 315*sqrt(b*x + a)*a^6*b^7)/((b*x
 + a)^7*a^2 - 7*(b*x + a)^6*a^3 + 21*(b*x + a)^5*a^4 - 35*(b*x + a)^4*a^5 +
 35*(b*x + a)^3*a^6 - 21*(b*x + a)^2*a^7 + 7*(b*x + a)*a^8 - a^9)
```

Fricas [A]
time = 0.32, size = 233, normalized size = 1.43

$$\left[\frac{315\,\sqrt{a}\,b^7x^7\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(315\,ab^6x^6-210\,a^2b^5x^5-14168\,a^3b^4x^4-39056\,a^4b^3x^3-44928\,a^5b^2x^2-24320\,a^6bx-5120\,a^7)\sqrt{bx+a}}{71680\,a^3x^7},\ \frac{315\,\sqrt{-a}\,b^7x^7\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(315\,ab^6x^6-210\,a^2b^5x^5-14168\,a^3b^4x^4-39056\,a^4b^3x^3-44928\,a^5b^2x^2-24320\,a^6bx-5120\,a^7)\sqrt{bx+a}}{35840\,a^3x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/71680*(315*sqrt(a)*b^7*x^7*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(315*a*b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^
3 - 44928*a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7),
 1/35840*(315*sqrt(-a)*b^7*x^7*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^
6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^3 - 44928*a^5
*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 218, normalized size = 1.34

$$\frac{\frac{315\sqrt{a+bx}\,(a+bx)^6b^8-2100\sqrt{a+bx}\,(a+bx)^5b^8a-8393\sqrt{a+bx}\,(a+bx)^4b^8a^2+9216\sqrt{a+bx}\,(a+bx)^3b^8a^3-5943\sqrt{a+bx}\,(a+bx)^2b^8a^4+2100\sqrt{a+bx}\,(a+bx)b^8a^5-315\sqrt{a+bx}\,b^8a^6}{35840a^2(a+bx-a)^7}+\frac{9b^8\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{512a^2\cdot 2\sqrt{-a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x)
```

```
[Out] 1/35840*(315*b^8*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*x
+ a)^(13/2)*b^8 - 2100*(b*x + a)^(11/2)*a*b^8 - 8393*(b*x + a)^(9/2)*a^2*b^
8 + 9216*(b*x + a)^(7/2)*a^3*b^8 - 5943*(b*x + a)^(5/2)*a^4*b^8 + 2100*(b*x
 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x + a)*a^6*b^8)/(a^2*b^7*x^7))/b
```

Mupad [B]
time = 0.13, size = 124, normalized size = 0.76

$$\frac{15\,a^3\,{\left(a+b\,x\right)}^{3/2}}{256\,x^7}-\frac{9\,a^4\,\sqrt{a+b\,x}}{1024\,x^7}-\frac{1199\,{\left(a+b\,x\right)}^{9/2}}{5120\,x^7}-\frac{849\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5120\,x^7}-\frac{15\,{\left(a+b\,x\right)}^{11/2}}{256\,a\,x^7}+\frac{9\,{\left(a+b\,x\right)}^{13/2}}{1024\,a^2\,x^7}+\frac{9\,a\,{\left(a+b\,x\right)}^{7/2}}{35\,x^7}+\frac{b^7\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,9{}\mathrm{i}}{1024\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^8,x)
```

```
[Out] (15*a^3*(a + b*x)^(3/2))/(256*x^7) - (9*a^4*(a + b*x)^(1/2))/(1024*x^7) - (
1199*(a + b*x)^(9/2))/(5120*x^7) - (849*a^2*(a + b*x)^(5/2))/(5120*x^7) - (
15*(a + b*x)^(11/2))/(256*a*x^7) + (9*(a + b*x)^(13/2))/(1024*a^2*x^7) + (b
^7*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*9i)/(1024*a^(5/2)) + (9*a*(a + b*x)^(
7/2))/(35*x^7)
```

3.325 $\int \frac{\sqrt{-a+bx}}{x}\,dx$

Optimal. Leaf size=39

$$2\sqrt{-a+bx} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 65, 211}

$$2\sqrt{bx-a} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = 2\sqrt{-a+bx} - a\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= 2\sqrt{-a+bx} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}$$
$$= 2\sqrt{-a+bx} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

<u>Mathematica</u> [A]
time = 0.02, size = 39, normalized size = 1.00

$$2\sqrt{-a+bx} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

<u>Mathics</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.96, size = 153, normalized size = 3.92

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\left(-\sqrt{a}\,\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right](a-bx)+a\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx}{bx}}-b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{a-bx}{bx}}\right)}{a-bx}, \text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\}, 2\sqrt{a}\,\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]-\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{1-\frac{a}{bx}}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[-a + b*x]/x,x]')
```

```
[Out] Piecewise[{{2 I (-Sqrt[a] ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a - b x) +
a Sqrt[b] Sqrt[x] Sqrt[(a - b x) / (b x)] - b ^ (3 / 2) x ^ (3 / 2) Sqrt[(a
 - b x) / (b x)]) / (a - b x), Abs[a / (b x)] > 1}}, 2 Sqrt[a] ArcSin[Sqrt[
a] / (Sqrt[b] Sqrt[x])] - 2 a / (Sqrt[b] Sqrt[x] Sqrt[1 - a / (b x)]) + 2 S
qrt[b] Sqrt[x] / Sqrt[1 - a / (b x)]]
```

<u>Maple</u> [A]
time = 0.09, size = 32, normalized size = 0.82

method	result	size

derivativedivides	$-2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx-a}$	32
default	$-2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx-a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

Maxima [A]
time = 0.35, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

Fricas [A]
time = 0.32, size = 78, normalized size = 2.00

$$\left[\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a},\,-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(b*x - a),
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.76, size = 148, normalized size = 3.79

$$\begin{cases} -2i\sqrt{a}\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2ia}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2i\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ 2\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x,x)
```

```
[Out] Piecewise((-2*I*sqrt(a)*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*I*a/(sqrt(b)*s
qrt(x)*sqrt(a/(b*x) - 1)) - 2*I*sqrt(b)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b
*x)) > 1), (2*sqrt(a)*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - 2*a/(sqrt(b)*sqrt(x
)*sqrt(-a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Giac [**A**]
time = 0.00, size = 43, normalized size = 1.10

$$2\sqrt{-a+bx}-\frac{4a\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

Mupad [**B**]
time = 0.09, size = 31, normalized size = 0.79

$$2\,\sqrt{b\,x-a}-2\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x,x)
```

```
[Out] 2*(b*x - a)^(1/2) - 2*a^(1/2)*atan((b*x - a)^(1/2)/a^(1/2))
```

3.326 $\int \frac{\sqrt{-a+bx}}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{-a+bx}}{x}+\frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-(b*x-a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 65, 211}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = -\frac{\sqrt{-a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= -\frac{\sqrt{-a+bx}}{x} + \text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)$$
$$= -\frac{\sqrt{-a+bx}}{x} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.00

$$-\frac{\sqrt{-a+bx}}{x} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.89, size = 112, normalized size = 2.67

$$\text{Piecewise}\left[\left\{\left\{\frac{Ib\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}} - \frac{I\sqrt{b}\sqrt{-1+\frac{a}{bx}}}{\sqrt{x}}, \text{Abs}\left[\frac{a}{bx}\right] > 1\right\}\right\}, \frac{a}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}} - \frac{b\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}} - \frac{\sqrt{b}}{\sqrt{x}\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[-a + b*x]/x^2,x]')
```

```
[Out] Piecewise[{{I b ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a] - I Sqrt[b]
Sqrt[-1 + a / (b x)] / Sqrt[x], Abs[a / (b x)] > 1}}, a / (Sqrt[b] x ^ (3 /
 2) Sqrt[1 - a / (b x)]) - b ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a]
- Sqrt[b] / (Sqrt[x] Sqrt[1 - a / (b x)])]
```

Maple [A]
time = 0.10, size = 41, normalized size = 0.98

method	result	size

risch	$\frac{-bx+a}{x\sqrt{bx-a}}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	40
derivativedivides	$2b\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	41
default	$2b\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x-a)^(1/2)/b/x+1/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.35, size = 34, normalized size = 0.81

$$\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="maxima")
[Out] b*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)/x
```

Fricas [A]
time = 0.32, size = 98, normalized size = 2.33

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}\,a}{2\,ax}, \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="fricas")
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(
b*x - a)*a)/(a*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x -
a)*a)/(a*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 117, normalized size = 2.79

$$\begin{cases} -\frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{\sqrt{x}}+\frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{a}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**2,x)
```

```
[Out] Piecewise((-I*sqrt(b)*sqrt(a/(b*x) - 1)/sqrt(x) + I*b*acosh(sqrt(a)/(sqrt(b
)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1), (a/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x)
+ 1)) - sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(b)*sqrt
(x)))/sqrt(a), True))
```

Giac [A]
time = 0.00, size = 58, normalized size = 1.38

$$\frac{-\frac{\sqrt{-a+bx}\,b^2}{-a+bx+a}+\frac{2b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2\sqrt{a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x)
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)*b/x)/b
```

Mupad [B]
time = 0.10, size = 34, normalized size = 0.81

$$\frac{b\operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{b\,x-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^2,x)
```

```
[Out] (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2) - (b*x - a)^(1/2)/x
```

3.327 $\int \frac{\sqrt{-a+bx}}{x^3} \, dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x-a)^(1/2)/x^2+1/4*b*(
b*x-a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 44, 65, 211}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{bx-a}}{2x^2}+\frac{b\sqrt{bx-a}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -1/2*Sqrt[-a + b*x]/x^2 + (b*Sqrt[-a + b*x])/(4*a*x) + (b^2*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+bx}}{x^3}\,dx &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{-a+bx}}\,dx\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a}\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.85

$$-\frac{(2a-bx)\sqrt{-a+bx}}{4ax^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -1/4*((2*a - b*x)*Sqrt[-a + b*x])/(a*x^2) + (b^2*ArcTan[Sqrt[-a + b*x]/Sqrt
[a]])/(4*a^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 24.37, size = 206, normalized size = 2.90

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-2a^{\frac{7}{2}}x\,(a-bx)+3a^{\frac{5}{2}}bx^2\,(a-bx)-a^{\frac{3}{2}}b^2x^3\,(a-bx)+ab^{\frac{7}{2}}x^{\frac{9}{2}}\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}\right)}{4a^{\frac{5}{2}}b^{\frac{3}{2}}x^{\frac{9}{2}}\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{1-\frac{a}{bx}}}+\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{1-\frac{a}{bx}}}-\frac{b^2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{4a^{\frac{3}{2}}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[-a + b*x]/x^3,x]')
```

```
[Out] Piecewise[{{I / 4 (-2 a ^ (7 / 2) x (a - b x) + 3 a ^ (5 / 2) b x ^ 2 (a -
b x) - a ^ (3 / 2) b ^ 2 x ^ 3 (a - b x) + a b ^ (7 / 2) x ^ (9 / 2) ArcCos
h[Sqrt[a] / (Sqrt[b] Sqrt[x])] ((a - b x) / (b x)) ^ (3 / 2)) / (a ^ (5 / 2
) b ^ (3 / 2) x ^ (9 / 2) ((a - b x) / (b x)) ^ (3 / 2)), Abs[a / (b x)] >
1}}, a / (2 Sqrt[b] x ^ (5 / 2) Sqrt[1 - a / (b x)]) + b ^ (3 / 2) / (4 a S
qrt[x] Sqrt[1 - a / (b x)]) - b ^ 2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (
4 a ^ (3 / 2)) - 3 Sqrt[b] / (4 x ^ (3 / 2) Sqrt[1 - a / (b x)])]
```

Maple [A]
time = 0.11, size = 59, normalized size = 0.83

method	result	size
risch	$\frac{(-bx+a)(-bx+2a)}{4x^2\sqrt{bx-a}\,a}+\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	55
derivativedivides	$2b^2\left(\frac{\frac{(bx-a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx-a}}{8}}{b^2x^2}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	59
default	$2b^2\left(\frac{\frac{(bx-a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx-a}}{8}}{b^2x^2}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((1/8/a*(b*x-a)^(3/2)-1/8*(b*x-a)^(1/2))/b^2/x^2+1/8*arctan((b*x-a)^(
1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.35, size = 83, normalized size = 1.17

$$\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}+\frac{(bx-a)^{\frac{3}{2}}b^2-\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2a+2\,(bx-a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + 1/4*((b*x - a)^(3/2)*b^2 -
sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a + 2*(b*x - a)*a^2 + a^3)
```

Fricas [A]
time = 0.32, size = 124, normalized size = 1.75

$$\left[-\frac{\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(abx-2\,a^2)\sqrt{bx-a}}{8\,a^2x^2},\ \frac{\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(abx-2\,a^2)\sqrt{bx-a}}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(
a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2), 1/4*(sqrt(a)*b^2*x^2*arctan(sqrt(b
*x - a)/sqrt(a)) + (a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2)]
```

Sympy [A]
time = 2.05, size = 207, normalized size = 2.92

$$\begin{cases} -\frac{ia}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{ib^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**3,x)
```

```
[Out] Piecewise((-I*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(4*x**
(3/2)*sqrt(a/(b*x) - 1)) - I*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) - 1)) + I*b
**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), Abs(a/(b*x)) > 1), (a/(2
*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x)
 + 1)) + b**(3/2)/(4*a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b**2*asin(sqrt(a)/(sqr
t(b)*sqrt(x)))/(4*a**(3/2)), True))
```

Giac [A]
time = 0.00, size = 90, normalized size = 1.27

$$\frac{\frac{\sqrt{-a+bx}\,(-a+bx)b^3-\sqrt{-a+bx}\,ab^3}{4a(-a+bx+a)^2}+\frac{b^3\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2a\cdot 2\sqrt{a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x)
```

```
[Out] 1/4*(b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + ((b*x - a)^(3/2)*b^3 - sqr
t(b*x - a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.10, size = 54, normalized size = 0.76

$$\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{\sqrt{b\,x-a}}{4\,x^2}+\frac{{\left(b\,x-a\right)}^{3/2}}{4\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^3,x)
```

```
[Out] (b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (b*x - a)^(1/2)/(4*x^2) +
 (b*x - a)^(3/2)/(4*a*x^2)
```

3.328 $\int \frac{(-a+bx)^{3/2}}{x} dx$

Optimal. Leaf size=55

$$-2a\sqrt{-a+bx}+\frac{2}{3}(-a+bx)^{3/2}+2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 65, 211}

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x,x]
```

```
[Out] -2*a*Sqrt[-a + b*x] + (2*(-a + b*x)^(3/2))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(-a+bx)^{3/2} - a\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + a^2\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b} \\
&= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + 2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.87

$$
\frac{2}{3}(-4a+bx)\sqrt{-a+bx} + 2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x,x]
```

```
[Out] (2*(-4*a + b*x)*Sqrt[-a + b*x])/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.36, size = 155, normalized size = 2.82

$$
\mathrm{Piecewise}\left[\left\{\left\{\frac{\sqrt{a}\left(-8a\sqrt{\frac{-a+bx}{a}} - 6a\mathrm{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right] - 3Ia\mathrm{Log}\left[\frac{bx}{a}\right] + 6Ia\mathrm{Log}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right] + 2bx\sqrt{\frac{-a+bx}{a}}\right)}{3}, \mathrm{Abs}\left[\frac{bx}{a}\right] > 1\right\}\right\}, \frac{-8Ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3} - Ia^{\frac{3}{2}}\mathrm{Log}\left[\frac{bx}{a}\right] + I2a^{\frac{3}{2}}\mathrm{Log}\left[1+\sqrt{1-\frac{bx}{a}}\right] + \frac{I2\sqrt{a}\,bx\sqrt{1-\frac{bx}{a}}}{3}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-a + b*x)^(3/2)/x,x]')
```

```
[Out] Piecewise[{{Sqrt[a] (-8 a Sqrt[(-a + b x) / a] - 6 a ArcSin[Sqrt[a] / (Sqrt
[b] Sqrt[x])] - 3 I a Log[b x / a] + 6 I a Log[Sqrt[b] Sqrt[x] / Sqrt[a]] +
 2 b x Sqrt[(-a + b x) / a]) / 3, Abs[b x / a] > 1}}, -8 I a ^ (3 / 2) Sqrt
[1 - b x / a] / 3 - I a ^ (3 / 2) Log[b x / a] + I 2 a ^ (3 / 2) Log[1 + Sq
rt[1 - b x / a]] + I 2 Sqrt[a] b x Sqrt[1 - b x / a] / 3]
```

Maple [A]
time = 0.10, size = 44, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2(bx-a)^{\frac{3}{2}}}{3}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}$	44
default	$\frac{2(bx-a)^{\frac{3}{2}}}{3}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

Maxima [A]
time = 0.35, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

Fricas [A]
time = 0.33, size = 93, normalized size = 1.69

$$\left[\sqrt{-a}\,a\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}\,-2\,a}{x}\right)+\frac{2}{3}\,\sqrt{bx-a}\,(bx-4\,a),2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,\sqrt{bx-a}\,(bx-4\,a)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/3*sqrt(b*x -
a)*(b*x - 4*a), 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*sqrt(b*x - a)
*(b*x - 4*a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.21, size = 187, normalized size = 3.40

$$\begin{cases} -\frac{8a^{\frac{3}{2}}\sqrt{-1+\frac{bx}{a}}}{3}-ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)+2ia^{\frac{3}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-2a^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2\sqrt{a}\,bx\sqrt{-1+\frac{bx}{a}}}{3} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{8ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3}-ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)+2ia^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}\,bx\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x,x)
```

```
[Out] Piecewise((-8*a**(3/2)*sqrt(-1 + b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**
(3/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) - 2*a**(3/2)*asin(sqrt(a)/(sqrt(b)*sqrt(
x))) + 2*sqrt(a)*b*x*sqrt(-1 + b*x/a)/3, Abs(b*x/a) > 1), (-8*I*a**(3/2)*sq
rt(1 - b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**(3/2)*log(sqrt(1 - b*x/a)
+ 1) + 2*I*sqrt(a)*b*x*sqrt(1 - b*x/a)/3, True))
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.20

$$\frac{2}{3}\sqrt{-a+bx}\,(-a+bx)-2\sqrt{-a+bx}\,a+\frac{4a^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x)
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.78

$$2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-2\,a\,\sqrt{b\,x-a}+\frac{2\,{\left(b\,x-a\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x,x)
```

```
[Out] 2*a^(3/2)*atan((b*x - a)^(1/2)/a^(1/2)) - 2*a*(b*x - a)^(1/2) + (2*(b*x - a
)^(3/2))/3
```

3.329 $\int \frac{(-a+bx)^{3/2}}{x^2} dx$

Optimal. Leaf size=57

$$3b\sqrt{-a+bx} - \frac{(-a+bx)^{3/2}}{x} - 3\sqrt{a}\, b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -(b*x-a)^(3/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+3*b*(b*x-a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 211}

$$-\frac{(bx-a)^{3/2}}{x} + 3b\sqrt{bx-a} - 3\sqrt{a}\, b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[-a + b*x] - (-a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^2}\,dx &= -\frac{(-a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-(3a)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right) \\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.84

$$\frac{\sqrt{-a+bx}\,(a+2bx)}{x}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[-a + b*x]*(a + 2*b*x))/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.79, size = 194, normalized size = 3.40

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(a^2(a-bx)+abx(a-bx)-2b^2x^2(a-bx)-3\sqrt{a}\,b^{\frac{5}{2}}x^{\frac{5}{2}}\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}\right)}{b^{\frac{3}{2}}x^{\frac{5}{2}}\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}-\frac{a\sqrt{b}}{\sqrt{x}\sqrt{1-\frac{a}{bx}}}+3\sqrt{a}\,b\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-a + b*x)^(3/2)/x^2,x]')
```

```
[Out] Piecewise[{{I (a ^ 2 (a - b x) + a b x (a - b x) - 2 b ^ 2 x ^ 2 (a - b x)
- 3 Sqrt[a] b ^ (5 / 2) x ^ (5 / 2) ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] ((
a - b x) / (b x)) ^ (3 / 2)) / (b ^ (3 / 2) x ^ (5 / 2) ((a - b x) / (b x))
 ^ (3 / 2)), Abs[a / (b x)] > 1}}, -a ^ 2 / (Sqrt[b] x ^ (3 / 2) Sqrt[1 - a
 / (b x)]) - a Sqrt[b] / (Sqrt[x] Sqrt[1 - a / (b x)]) + 3 Sqrt[a] b ArcSin
[Sqrt[a] / (Sqrt[b] Sqrt[x])] + 2 b ^ (3 / 2) Sqrt[x] / Sqrt[1 - a / (b x)]
]
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.95

method	result	size
derivativedivides	$2b\left(\sqrt{bx-a}-a\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	54
default	$2b\left(\sqrt{bx-a}-a\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	54
risch	$-\frac{a(-bx+a)}{x\sqrt{bx-a}}-3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2b\sqrt{bx-a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*((b*x-a)^(1/2)-a*(-1/2*(b*x-a)^(1/2)/b/x+3/2*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(1/2)))
```

Maxima [A]
time = 0.34, size = 47, normalized size = 0.82

$$-3\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\,b+\frac{\sqrt{bx-a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3*sqrt(a)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b + sqrt(b*x -
 a)*a/x
```

Fricas [A]
time = 0.31, size = 105, normalized size = 1.84

$$\left[\frac{3\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\,(2\,bx+a)\sqrt{bx-a}}{2\,x},-\frac{3\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(2\,bx+a)\sqrt{bx-a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(2*b
*x + a)*sqrt(b*x - a))/x, -(3*sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - (
2*b*x + a)*sqrt(b*x - a))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.36, size = 197, normalized size = 3.46

$$\begin{cases} -3i\sqrt{a}\,b\,\text{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{ia^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{ia\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ 3\sqrt{a}\,b\,\text{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{a\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**2,x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + I*a**2/(sqrt(b
)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) - 1)) - 2
*I*b**(3/2)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (3*sqrt(a)*b*asin
(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) -
a*sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)/sqrt(-a/(b*x) +
 1), True))
```

Giac [A]
time = 0.00, size = 75, normalized size = 1.32

$$\frac{2\sqrt{-a+bx}\,b^2+\frac{\sqrt{-a+bx}\,b^2a}{-a+bx+a}-\frac{6b^2a\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2\sqrt{a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x)
```

```
[Out] -(3*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) - 2*sqrt(b*x - a)*b^2 - sqrt(
b*x - a)*a*b/x)/b
```

Mupad [B]
time = 0.04, size = 47, normalized size = 0.82

$$2\,b\,\sqrt{b\,x-a}+\frac{a\,\sqrt{b\,x-a}}{x}-3\,\sqrt{a}\,b\,\text{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^2,x)
```

```
[Out] 2*b*(b*x - a)^(1/2) + (a*(b*x - a)^(1/2))/x - 3*a^(1/2)*b*atan((b*x - a)^(1
/2)/a^(1/2))
```

3.330 $\int \frac{(-a+bx)^{3/2}}{x^3} dx$

Optimal. Leaf size=68

$$-\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/2*(b*x-a)^(3/2)/x^2+3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-3/4*b*
(b*x-a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {43, 65, 211}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(bx-a)^{3/2}}{2x^2}-\frac{3b\sqrt{bx-a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[-a + b*x])/(4*x) - (-a + b*x)^(3/2)/(2*x^2) + (3*b^2*ArcTan[Sqrt
[-a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^3}\,dx &= -\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{-a+bx}}{x^2}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.82

$$
\frac{1}{4}\left(\frac{(2a-5bx)\sqrt{-a+bx}}{x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] (((2*a - 5*b*x)*Sqrt[-a + b*x])/x^2 + (3*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
)/Sqrt[a])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.62, size = 162, normalized size = 2.38

$$
\text{Piecewise}\left[\left\{\left\{\frac{Ia\sqrt{b}\sqrt{-1+\frac{a}{bx}}}{2x^{\frac{3}{2}}}+\frac{3Ib^2\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{4\sqrt{a}}-\frac{5Ib^{\frac{3}{2}}\sqrt{-1+\frac{a}{bx}}}{4\sqrt{x}},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},-\frac{a^2}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{1-\frac{a}{bx}}}+\frac{7a\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}-\frac{3b^2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{4\sqrt{a}}-\frac{5b^{\frac{3}{2}}}{4\sqrt{x}\sqrt{1-\frac{a}{bx}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-a + b*x)^(3/2)/x^3,x]')
```

```
[Out] Piecewise[{{I / 2 a Sqrt[b] Sqrt[-1 + a / (b x)] / x ^ (3 / 2) + 3 I / 4 b
^ 2 ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a] - 5 I / 4 b ^ (3 / 2) Sq
rt[-1 + a / (b x)] / Sqrt[x], Abs[a / (b x)] > 1}}, -a ^ 2 / (2 Sqrt[b] x ^
 (5 / 2) Sqrt[1 - a / (b x)]) + 7 a Sqrt[b] / (4 x ^ (3 / 2) Sqrt[1 - a / (
b x)]) - 3 b ^ 2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (4 Sqrt[a]) - 5 b ^
(3 / 2) / (4 Sqrt[x] Sqrt[1 - a / (b x)])]
```

Maple [A]
time = 0.11, size = 57, normalized size = 0.84

method	result	size
risch	$-\frac{(-bx+a)(-5bx+2a)}{4x^2\sqrt{bx-a}}+\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}$	52
derivativedivides	$2b^2\left(\frac{-\frac{5(bx-a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	57
default	$2b^2\left(\frac{-\frac{5(bx-a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((-5/8*(b*x-a)^(3/2)-3/8*a*(b*x-a)^(1/2))/b^2/x^2+3/8*arctan((b*x-a)^
(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.36, size = 80, normalized size = 1.18

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^2+3\,\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2+2\,(bx-a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - 1/4*(5*(b*x - a)^(3/2)*b^2
+ 3*sqrt(b*x - a)*a*b^2)/((b*x - a)^2 + 2*(b*x - a)*a + a^2)
```

Fricas [A]
time = 0.32, size = 129, normalized size = 1.90

$$\left[-\frac{3\,\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,(5\,abx-2\,a^2)\sqrt{bx-a}}{8\,ax^2},\frac{3\,\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(5\,abx-2\,a^2)\sqrt{bx-a}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(sq
rt(b*x - a)/sqrt(a)) - (5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2)]
```

Sympy [A]
time = 1.57, size = 189, normalized size = 2.78

$$\begin{cases} \frac{ia\sqrt{b}\sqrt{\frac{a}{bx}-1}}{2x^{\frac{3}{2}}} - \frac{5ib^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{4\sqrt{x}} + \frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{a^2}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{7a\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{5b^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**3,x)
```

```
[Out] Piecewise((I*a*sqrt(b)*sqrt(a/(b*x) - 1)/(2*x**(3/2)) - 5*I*b**(3/2)*sqrt(a
/(b*x) - 1)/(4*sqrt(x)) + 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt
(a)), Abs(a/(b*x)) > 1), (-a**2/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 7
*a*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) - 5*b**(3/2)/(4*sqrt(x)*sqrt(-a/
(b*x) + 1)) - 3*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), True))
```

Giac [A]
time = 0.00, size = 88, normalized size = 1.29

$$\frac{\frac{-5\sqrt{-a+bx}\,(-a+bx)b^3-3\sqrt{-a+bx}\,b^3a}{4(-a+bx+a)^2} + \frac{3b^3\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x)
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - (5*(b*x - a)^(3/2)*b^3 +
 3*sqrt(b*x - a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.10, size = 52, normalized size = 0.76

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}} - \frac{5\,{\left(b\,x-a\right)}^{3/2}}{4\,x^2} - \frac{3\,a\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^3,x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(1/2)) - (5*(b*x - a)^(3/2))/(4*
x^2) - (3*a*(b*x - a)^(1/2))/(4*x^2)
```

3.331 $\int \frac{(-a+bx)^{5/2}}{x}\,dx$

Optimal. Leaf size=73

$$2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 65, 211}

$$-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^2\sqrt{bx-a}-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[-a + b*x] - (2*a*(-a + b*x)^(3/2))/3 + (2*(-a + b*x)^(5/2))/5 -
2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(-a+bx)^{5/2} - a\int \frac{(-a+bx)^{3/2}}{x}\,dx \\
&= -\frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} + a^2\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - a^3\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a}\right.}{b} \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.82

$$
\frac{2}{15}\sqrt{-a+bx}\,\left(23a^2 - 11abx + 3b^2x^2\right) - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*x]*(23*a^2 - 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.64, size = 207, normalized size = 2.84

$$
\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{a}\left(-30Ia^2\text{Log}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right] + 15Ia^2\text{Log}\left[\frac{bx}{a}\right] + 30a^2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right] + 46a^2\sqrt{\frac{-a+bx}{a}} - 22abx\sqrt{\frac{-a+bx}{a}} + 6b^2x^2\sqrt{\frac{-a+bx}{a}}\right)}{15}, \text{Abs}\left[\frac{bx}{a}\right] > 1\right\}\right\}, -2Ia^{\frac{5}{2}}\text{Log}\left[1+\sqrt{1-\frac{bx}{a}}\right] + \frac{I46a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15} + Ia^{\frac{5}{2}}\text{Log}\left[\frac{bx}{a}\right] - \frac{22Ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15} + \frac{I2\sqrt{a}\,b^2x^2\sqrt{1-\frac{bx}{a}}}{5}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-a + b*x)^(5/2)/x,x]')
```

```
[Out] Piecewise[{{Sqrt[a] (-30 I a ^ 2 Log[Sqrt[b] Sqrt[x] / Sqrt[a]] + 15 I a ^
2 Log[b x / a] + 30 a ^ 2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] + 46 a ^ 2 Sq
rt[(-a + b x) / a] - 22 a b x Sqrt[(-a + b x) / a] + 6 b ^ 2 x ^ 2 Sqrt[(-a
 + b x) / a]) / 15, Abs[b x / a] > 1}}, -2 I a ^ (5 / 2) Log[1 + Sqrt[1 - b
 x / a]] + I 46 a ^ (5 / 2) Sqrt[1 - b x / a] / 15 + I a ^ (5 / 2) Log[b x
```

```
/ a] - 22 I a ^ (3 / 2) b x Sqrt[1 - b x / a] / 15 + I 2 Sqrt[a] b ^ 2 x ^
2 Sqrt[1 - b x / a] / 5]
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2a(bx-a)^{\frac{3}{2}}}{3}+\frac{2(bx-a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^{2}\sqrt{bx-a}$	58
default	$-\frac{2a(bx-a)^{\frac{3}{2}}}{3}+\frac{2(bx-a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^{2}\sqrt{bx-a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x,x,method=_RETURNVERBOSE)
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

Maxima [A]
time = 0.35, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="maxima")
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

Fricas [A]
time = 0.31, size = 119, normalized size = 1.63

$$\left[\sqrt{-a}\,a^{2}\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+\frac{2}{15}\,(3\,b^{2}x^{2}-11\,abx+23\,a^{2})\sqrt{bx-a}\,,-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{15}\,(3\,b^{2}x^{2}-11\,abx+23\,a^{2})\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="fricas")
[Out] [sqrt(-a)*a^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/15*(3*b^2*x
^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a), -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt
(a)) + 2/15*(3*b^2*x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.39, size = 240, normalized size = 3.29

$$\begin{cases} \frac{46a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+2a^{\frac{5}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{22a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}\,b^{2}x^{2}\sqrt{-1+\frac{bx}{a}}}{5} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{46ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)-\frac{22ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15}+\frac{2i\sqrt{a}\,b^{2}x^{2}\sqrt{1-\frac{bx}{a}}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x,x)
```

```
[Out] Piecewise((46*a**(5/2)*sqrt(-1 + b*x/a)/15 + I*a**(5/2)*log(b*x/a) - 2*I*a*
*(5/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) + 2*a**(5/2)*asin(sqrt(a)/(sqrt(b)*sqrt
(x))) - 22*a**(3/2)*b*x*sqrt(-1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*sqrt(-1 +
 b*x/a)/5, Abs(b*x/a) > 1), (46*I*a**(5/2)*sqrt(1 - b*x/a)/15 + I*a**(5/2)*
log(b*x/a) - 2*I*a**(5/2)*log(sqrt(1 - b*x/a) + 1) - 22*I*a**(3/2)*b*x*sqrt
(1 - b*x/a)/15 + 2*I*sqrt(a)*b**2*x**2*sqrt(1 - b*x/a)/5, True))
```

Giac [A]
time = 0.00, size = 92, normalized size = 1.26

$$\frac{2}{5}\sqrt{-a+bx}\,(-a+bx)^2-\frac{2}{3}\sqrt{-a+bx}\,(-a+bx)\,a+2\sqrt{-a+bx}\,a^2-\frac{4a^3\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x)
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

Mupad [B]
time = 0.04, size = 57, normalized size = 0.78

$$\frac{2\,{\left(b\,x-a\right)}^{5/2}}{5}-\frac{2\,a\,{\left(b\,x-a\right)}^{3/2}}{3}-2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)+2\,a^2\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x,x)
```

```
[Out] (2*(b*x - a)^(5/2))/5 - (2*a*(b*x - a)^(3/2))/3 - 2*a^(5/2)*atan((b*x - a)^
(1/2)/a^(1/2)) + 2*a^2*(b*x - a)^(1/2)
```

3.332 $\int \frac{(-a+bx)^{5/2}}{x^2} \, dx$

Optimal. Leaf size=74

$$-5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] 5/3*b*(b*x-a)^(3/2)-(b*x-a)^(5/2)/x+5*a^(3/2)*b*arctan((b*x-a)^(1/2)/a^(1/2
))-5*a*b*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 211}

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -5*a*b*Sqrt[-a + b*x] + (5*b*(-a + b*x)^(3/2))/3 - (-a + b*x)^(5/2)/x + 5*a
^(3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x^2}\,dx &= -\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(-a+bx)^{3/2}}{x}\,dx\\
&= \frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}-\frac{1}{2}(5ab)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{}\right.\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.86

$$-\frac{\sqrt{-a+bx}\,\left(3a^2+14abx-2b^2x^2\right)}{3x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -1/3*(Sqrt[-a + b*x]*(3*a^2 + 14*a*b*x - 2*b^2*x^2))/x + 5*a^(3/2)*b*ArcTan
[Sqrt[-a + b*x]/Sqrt[a]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.50, size = 204, normalized size = 2.76

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{a}\left(-a^2\sqrt{\frac{-a+bx}{a}}+\frac{bx\left(-30a\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]-28a\sqrt{\frac{-a+bx}{a}}-15Ia\text{Log}\left[\frac{bx}{a}\right]+30Ia\text{Log}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]+4bx\sqrt{\frac{-a+bx}{a}}\right)}{6}\right)}{x},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},-\frac{Ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x}-\frac{14Ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3}-\frac{5Ia^{\frac{3}{2}}b\text{Log}\left[\frac{bx}{a}\right]}{2}+I5a^{\frac{3}{2}}b\text{Log}\left[1+\sqrt{1-\frac{bx}{a}}\right]+\frac{I2\sqrt{a}\,b^2x\sqrt{1-\frac{bx}{a}}}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-a + b*x)^(5/2)/x^2,x]')
```

```
[Out] Piecewise[{{Sqrt[a] (-a ^ 2 Sqrt[(-a + b x) / a] + b x (-30 a ArcSin[Sqrt[a
] / (Sqrt[b] Sqrt[x])] - 28 a Sqrt[(-a + b x) / a] - 15 I a Log[b x / a] +
30 I a Log[Sqrt[b] Sqrt[x] / Sqrt[a]] + 4 b x Sqrt[(-a + b x) / a]) / 6) /
x, Abs[b x / a] > 1}}, -I a ^ (5 / 2) Sqrt[1 - b x / a] / x - 14 I a ^ (3 /
 2) b Sqrt[1 - b x / a] / 3 - 5 I a ^ (3 / 2) b Log[b x / a] / 2 + I 5 a ^
(3 / 2) b Log[1 + Sqrt[1 - b x / a]] + I 2 Sqrt[a] b ^ 2 x Sqrt[1 - b x / a
] / 3]
```

Maple [A]
time = 0.11, size = 69, normalized size = 0.93

method	result	size
derivativedivides	$2b\left(\frac{(bx-a)^{\frac{3}{2}}}{3}-2a\sqrt{bx-a}+a^{2}\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	69
default	$2b\left(\frac{(bx-a)^{\frac{3}{2}}}{3}-2a\sqrt{bx-a}+a^{2}\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	69
risch	$\frac{a^{2}(-bx+a)}{x\sqrt{bx-a}}+\frac{2b(bx-a)^{\frac{3}{2}}}{3}-4ab\sqrt{bx-a}+5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/3*(b*x-a)^(3/2)-2*a*(b*x-a)^(1/2)+a^2*(-1/2*(b*x-a)^(1/2)/b/x+5/2*ar
ctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.36, size = 63, normalized size = 0.85

$$5\,a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}b-4\,\sqrt{bx-a}\,ab-\frac{\sqrt{bx-a}\,a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5*a^(3/2)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2)*b - 4*sqrt(
b*x - a)*a*b - sqrt(b*x - a)*a^2/x
```

Fricas [A]
time = 0.31, size = 131, normalized size = 1.77

$$\left[\frac{15\sqrt{-a}\,abx\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{6x},\frac{15a^{\frac{3}{2}}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*sqrt(-a)*a*b*x*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(
2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x, 1/3*(15*a^(3/2)*b*x*arctan(
sqrt(b*x - a)/sqrt(a)) + (2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.30, size = 245, normalized size = 3.31

$$\begin{cases} -\frac{a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{x}-\frac{14a^{\frac{3}{2}}b\sqrt{-1+\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-5a^{\frac{3}{2}}b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2\sqrt{a}\,b^2x\sqrt{-1+\frac{bx}{a}}}{3} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x}-\frac{14ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}\,b^2x\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**2,x)
```

```
[Out] Piecewise((-a**(5/2)*sqrt(-1 + b*x/a)/x - 14*a**(3/2)*b*sqrt(-1 + b*x/a)/3
- 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqrt(b)*sqrt(x)/sqrt(a))
 - 5*a**(3/2)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*sqrt(a)*b**2*x*sqrt(-1
+ b*x/a)/3, Abs(b*x/a) > 1), (-I*a**(5/2)*sqrt(1 - b*x/a)/x - 14*I*a**(3/2)
*b*sqrt(1 - b*x/a)/3 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqr
t(1 - b*x/a) + 1) + 2*I*sqrt(a)*b**2*x*sqrt(1 - b*x/a)/3, True))
```

Giac [A]
time = 0.00, size = 104, normalized size = 1.41

$$\frac{\frac{2}{3}\sqrt{-a+bx}\,(-a+bx)\,b^2-4\sqrt{-a+bx}\,b^2a-\frac{\sqrt{-a+bx}\,b^2a^2}{-a+bx+a}+\frac{10b^2a^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2\sqrt{a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x)
```

```
[Out] 1/3*(15*a^(3/2)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*(b*x - a)^(3/2)*b^2 -
 12*sqrt(b*x - a)*a*b^2 - 3*sqrt(b*x - a)*a^2*b/x)/b
```

Mupad [B]
time = 0.10, size = 63, normalized size = 0.85

$$\frac{2\,b\,{\left(b\,x-a\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{b\,x-a}}{x}+5\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-4\,a\,b\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^2,x)
```

```
[Out] (2*b*(b*x - a)^(3/2))/3 - (a^2*(b*x - a)^(1/2))/x + 5*a^(3/2)*b*atan((b*x -
 a)^(1/2)/a^(1/2)) - 4*a*b*(b*x - a)^(1/2)
```

3.333 $\int \frac{(-a+bx)^{5/2}}{x^3} dx$

Optimal. Leaf size=86

$$\frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -5/4*b*(b*x-a)^(3/2)/x-1/2*(b*x-a)^(5/2)/x^2-15/4*b^2*arctan((b*x-a)^(1/2)/
a^(1/2))*a^(1/2)+15/4*b^2*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 211}

$$\frac{15}{4}b^2\sqrt{bx-a}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{2x^2}-\frac{5b(bx-a)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[-a + b*x])/4 - (5*b*(-a + b*x)^(3/2))/(4*x) - (-a + b*x)^(5/2)
/(2*x^2) - (15*Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{5/2}}{x^3}\,dx &= -\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(-a+bx)^{3/2}}{x^2}\,dx\\ &= -\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{-a+bx}}{x}\,dx\\ &= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\ &= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{4}(15ab)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x\right.\\ &= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.78

$$\frac{1}{4}\left(\frac{\sqrt{-a+bx}\,\left(-2a^2+9abx+8b^2x^2\right)}{x^2}-15\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] ((Sqrt[-a + b*x]*(-2*a^2 + 9*a*b*x + 8*b^2*x^2))/x^2 - 15*Sqrt[a]*b^2*ArcTa
n[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 11.81, size = 266, normalized size = 3.09

$$\text{Piecewise}\left[\left\{\left\{\frac{Ib^{\frac{3}{2}}\left(-2a^3x^2\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}-15\sqrt{a}\,\sqrt{b}\,x^{\frac{5}{2}}\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right](a-bx)^2+11a^2bx^3\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}-ab^2x^4\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}-8b^3x^5\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}\right)}{4x^{\frac{5}{2}}(a-bx)^2},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{1-\frac{a}{bx}}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}+\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\,\sqrt{1-\frac{a}{bx}}}+\frac{15\sqrt{a}\,b^2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right]}{4}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-a + b*x)^(5/2)/x^3,x]')
```

```
[Out] Piecewise[{{I / 4 b ^ (3 / 2) (-2 a ^ 3 x ^ 2 ((a - b x) / (b x)) ^ (3 / 2)
 - 15 Sqrt[a] Sqrt[b] x ^ (5 / 2) ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a -
 b x) ^ 2 + 11 a ^ 2 b x ^ 3 ((a - b x) / (b x)) ^ (3 / 2) - a b ^ 2 x ^ 4
((a - b x) / (b x)) ^ (3 / 2) - 8 b ^ 3 x ^ 5 ((a - b x) / (b x)) ^ (3 / 2)
) / (x ^ (5 / 2) (a - b x) ^ 2), Abs[a / (b x)] > 1}}, a ^ 3 / (2 Sqrt[b] x
 ^ (5 / 2) Sqrt[1 - a / (b x)]) - 11 a ^ 2 Sqrt[b] / (4 x ^ (3 / 2) Sqrt[1
- a / (b x)]) + a b ^ (3 / 2) / (4 Sqrt[x] Sqrt[1 - a / (b x)]) + 15 Sqrt[a
] b ^ 2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] / 4 + 2 b ^ (5 / 2) Sqrt[x] / S
qrt[1 - a / (b x)]]
```

Maple [A]
time = 0.10, size = 70, normalized size = 0.81

method	result	size
risch	$\frac{a(-bx+a)(-9bx+2a)}{4x^{2}\sqrt{bx-a}}-\frac{15b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}}{4}+2b^{2}\sqrt{bx-a}$	67
derivativedivides	$2b^{2}\left(\sqrt{bx-a}-a\left(\frac{-\frac{9(bx-a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	70
default	$2b^{2}\left(\sqrt{bx-a}-a\left(\frac{-\frac{9(bx-a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((b*x-a)^(1/2)-a*((-9/8*(b*x-a)^(3/2)-7/8*a*(b*x-a)^(1/2))/b^2/x^2+15
/8*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.35, size = 97, normalized size = 1.13

$$-\frac{15}{4}\sqrt{a}\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\,b^{2}+\frac{9\,(bx-a)^{\frac{3}{2}}ab^{2}+7\sqrt{bx-a}\,a^{2}b^{2}}{4\left((bx-a)^{2}+2\,(bx-a)a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -15/4*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b^2 + 1/4
*(9*(b*x - a)^(3/2)*a*b^2 + 7*sqrt(b*x - a)*a^2*b^2)/((b*x - a)^2 + 2*(b*x
- a)*a + a^2)
```

Fricas [A]
time = 0.31, size = 139, normalized size = 1.62

$$\left[\frac{15\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\,(8\,b^2x^2+9\,abx-2\,a^2)\sqrt{bx-a}}{8\,x^2},-\frac{15\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(8\,b^2x^2+9\,abx-2\,a^2)\sqrt{bx-a}}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a))/x^2, -1/4*(15*sqrt(a)*b^2*x^2
*arctan(sqrt(b*x - a)/sqrt(a)) - (8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a
))/x^2]
```

Sympy [A]
time = 2.41, size = 267, normalized size = 3.10

$$\begin{cases}-\frac{15i\sqrt{a}\,b^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{ia^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{11ia^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{iab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1\\ \frac{15\sqrt{a}\,b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}+\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**3,x)
```

```
[Out] Piecewise((-15*I*sqrt(a)*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - I*a**3/(
2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 11*I*a**2*sqrt(b)/(4*x**(3/2)*sqrt(
a/(b*x) - 1)) - I*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*b**(5/2)*s
qrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (15*sqrt(a)*b**2*asin(sqrt(a)/
(sqrt(b)*sqrt(x)))/4 + a**3/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 11*a*
*2*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) + a*b**(3/2)/(4*sqrt(x)*sqrt(-a/
(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Giac [A]
time = 0.01, size = 106, normalized size = 1.23

$$\frac{2\sqrt{-a+bx}\,b^3+\frac{9\sqrt{-a+bx}\,(-a+bx)b^3a+7\sqrt{-a+bx}\,b^3a^2}{4(-a+bx+a)^2}-\frac{15b^3a\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x)
```

```
[Out] -1/4*(15*sqrt(a)*b^3*arctan(sqrt(b*x - a)/sqrt(a)) - 8*sqrt(b*x - a)*b^3 -
(9*(b*x - a)^(3/2)*a*b^3 + 7*sqrt(b*x - a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.09, size = 69, normalized size = 0.80

$$2\,b^2\,\sqrt{b\,x-a}-\frac{15\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4}+\frac{9\,a\,{\left(b\,x-a\right)}^{3/2}}{4\,x^2}+\frac{7\,a^2\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(b*x - a)^(1/2) - (15*a^(1/2)*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/4 +
(9*a*(b*x - a)^(3/2))/(4*x^2) + (7*a^2*(b*x - a)^(1/2))/(4*x^2)
```

3.334 $\int \frac{x^4}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}+\frac{2(a+bx)^{9/2}}{9b^5}$$

```
[Out] -8/3*a^3*(b*x+a)^(3/2)/b^5+12/5*a^2*(b*x+a)^(5/2)/b^5-8/7*a*(b*x+a)^(7/2)/b
^5+2/9*(b*x+a)^(9/2)/b^5+2*a^4*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a +
b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))/(
9*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^4}{b^4\sqrt{a+bx}}-\frac{4a^3\sqrt{a+bx}}{b^4}+\frac{6a^2(a+bx)^{3/2}}{b^4}-\frac{4a(a+bx)^{5/2}}{b^4}+\frac{(a+bx)^{7/2}}{b^4}\right)dx\\ &= \frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}+\frac{2(a+bx)^{9/2}}{9b^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\,(128a^4-64a^3bx+48a^2b^2x^2-40ab^3x^3+35b^4x^4)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4 - 64*a^3*b*x + 48*a^2*b^2*x^2 - 40*a*b^3*x^3 + 35
*b^4*x^4))/(315*b^5)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 526 vs. $2(89) = 178$.
time = 33.83, size = 494, normalized size = 5.55

$$\frac{2\sqrt{a}\left(128a^{14}\left(-1+\sqrt{\frac{a+bx}{a}}\right)+64a^{13}bx\left(-20+19\sqrt{\frac{a+bx}{a}}\right)+16a^{12}b^2x^2\left(-360+323\sqrt{\frac{a+bx}{a}}\right)+40a^{11}b^3x^3\left(-384+323\sqrt{\frac{a+bx}{a}}\right)+5a^{10}b^4x^4\left(-5376+4199\sqrt{\frac{a+bx}{a}}\right)+a^2b^5x^5\left(23126a^7\sqrt{\frac{a+bx}{a}}-128a^2b^5x^5+4198a^2b^5x^5\sqrt{\frac{a+bx}{a}}+2816ab^6x^6\sqrt{\frac{a+bx}{a}}+1223b^7x^7\sqrt{\frac{a+bx}{a}}\right)-32256a^9b^5x^5+3a^7b^6x^6(5869a+3272bx)\sqrt{\frac{a+bx}{a}}-26880a^8b^6x^6+30a^6b^7x^7\left(-512a+181bx\sqrt{\frac{a+bx}{a}}\right)-5760a^6b^8x^8+10ab^9x^9\left(-128a^4+458a^4\sqrt{\frac{a+bx}{a}}+31b^4x^4\sqrt{\frac{a+bx}{a}}\right)+35b^{14}x^{14}\sqrt{\frac{a+bx}{a}}\right)}{315b^5\left(a^{10}+10a^9bx+45a^8b^2x^2+120a^7b^3x^3+210a^6b^4x^4+252a^5b^5x^5+210a^4b^6x^6+120a^3b^7x^7+45a^2b^8x^8+10ab^9x^9+b^{10}x^{10}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (128 a ^ 14 (-1 + Sqrt[(a + b x) / a]) + 64 a ^ 13 b x (-20 + 19
Sqrt[(a + b x) / a]) + 16 a ^ 12 b ^ 2 x ^ 2 (-360 + 323 Sqrt[(a + b x) / a
]) + 40 a ^ 11 b ^ 3 x ^ 3 (-384 + 323 Sqrt[(a + b x) / a]) + 5 a ^ 10 b ^
4 x ^ 4 (-5376 + 4199 Sqrt[(a + b x) / a]) + a ^ 2 b ^ 5 x ^ 5 (23126 a ^ 7
 Sqrt[(a + b x) / a] - 128 a ^ 2 b ^ 5 x ^ 5 + 4198 a ^ 2 b ^ 5 x ^ 5 Sqrt[
(a + b x) / a] + 2816 a b ^ 6 x ^ 6 Sqrt[(a + b x) / a] + 1223 b ^ 7 x ^ 7
Sqrt[(a + b x) / a]) - 32256 a ^ 9 b ^ 5 x ^ 5 + 3 a ^ 7 b ^ 6 x ^ 6 (5869
a + 3272 b x) Sqrt[(a + b x) / a] - 26880 a ^ 8 b ^ 6 x ^ 6 + 30 a ^ 6 b ^
7 x ^ 7 (-512 a + 181 b x Sqrt[(a + b x) / a]) - 5760 a ^ 6 b ^ 8 x ^ 8 + 1
0 a b ^ 9 x ^ 9 (-128 a ^ 4 + 458 a ^ 4 Sqrt[(a + b x) / a] + 31 b ^ 4 x ^
4 Sqrt[(a + b x) / a]) + 35 b ^ 14 x ^ 14 Sqrt[(a + b x) / a]) / (315 b ^ 5
 (a ^ 10 + 10 a ^ 9 b x + 45 a ^ 8 b ^ 2 x ^ 2 + 120 a ^ 7 b ^ 3 x ^ 3 + 21
0 a ^ 6 b ^ 4 x ^ 4 + 252 a ^ 5 b ^ 5 x ^ 5 + 210 a ^ 4 b ^ 6 x ^ 6 + 120 a
 ^ 3 b ^ 7 x ^ 7 + 45 a ^ 2 b ^ 8 x ^ 8 + 10 a b ^ 9 x ^ 9 + b ^ 10 x ^ 10)
)
```

Maple [**A**]
time = 0.09, size = 61, normalized size = 0.69

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,\left(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4\right)}{315b^5}$	54
trager	$\frac{2\sqrt{bx+a}\,\left(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4\right)}{315b^5}$	54
risch	$\frac{2\sqrt{bx+a}\,\left(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4\right)}{315b^5}$	54
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{8a(bx+a)^{\frac{7}{2}}}{7}+\frac{12a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{3}{2}}}{3}+2a^4\sqrt{bx+a}}{b^5}$	61
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{8a(bx+a)^{\frac{7}{2}}}{7}+\frac{12a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{3}{2}}}{3}+2a^4\sqrt{bx+a}}{b^5}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/b^5*(1/9*(b*x+a)^(9/2)-4/7*a*(b*x+a)^(7/2)+6/5*a^2*(b*x+a)^(5/2)-4/3*a^3*(b*x+a)^(3/2)+a^4*(b*x+a)^(1/2))

Maxima [**A**]
time = 0.27, size = 71, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^5}-\frac{8\,(bx+a)^{\frac{7}{2}}a}{7\,b^5}+\frac{12\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^5}+\frac{2\,\sqrt{bx+a}\,a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] 2/9*(b*x + a)^(9/2)/b^5 - 8/7*(b*x + a)^(7/2)*a/b^5 + 12/5*(b*x + a)^(5/2)*a^2/b^5 - 8/3*(b*x + a)^(3/2)*a^3/b^5 + 2*sqrt(b*x + a)*a^4/b^5

Fricas [**A**]
time = 0.30, size = 53, normalized size = 0.60

$$\frac{2\,(35\,b^4x^4-40\,ab^3x^3+48\,a^2b^2x^2-64\,a^3bx+128\,a^4)\sqrt{bx+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*sqrt(b*x + a)/b^5

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3755 vs. $2(85) = 170$.
time = 2.21, size = 3755, normalized size = 42.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x+a)**(1/2),x)

[Out] 256*a**(89/2)*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(89/2)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 2432*a**(87/2)*b*x*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*

```
x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x
**9 + 315*a**30*b**15*x**10) - 2560*a**(87/2)*b*x/(315*a**40*b**5 + 3150*a*
*39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b*
*9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**
12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*
x**10) + 10336*a**(85/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a
**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b
**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b*
*12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15
*x**10) - 11520*a**(85/2)*b**2*x**2/(315*a**40*b**5 + 3150*a**39*b**6*x + 1
4175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 7938
0*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 1417
5*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 25840
*a**(83/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x +
14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 793
80*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 141
75*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 3072
0*a**(83/2)*b**3*x**3/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**
7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*
x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*
x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 41990*a**(81/2)*b**
4*x**4*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b*
*7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10
*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13
*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 53760*a**(81/2)*b*
*4*x**4/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800
*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a
**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a*
*31*b**14*x**9 + 315*a**30*b**15*x**10) + 46252*a**(79/2)*b**5*x**5*sqrt(1
+ b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 3780
0*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*
a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a
**31*b**14*x**9 + 315*a**30*b**15*x**10) - 64512*a**(79/2)*b**5*x**5/(315*a
**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x*
*3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**
6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9
 + 315*a**30*b**15*x**10) + 35214*a**(77/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*
a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x
**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x*
*6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**
9 + 315*a**30*b**15*x**10) - 53760*a**(77/2)*b**6*x**6/(315*a**40*b**5 + 31
50*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**
36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**3
3*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b
**15*x**10) + 19632*a**(75/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3
```

150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a*
*36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**
33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*
b**15*x**10) - 30720*a**(75/2)*b**7*x**7/(315*a**40*b**5 + 3150*a**39*b**6*
x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 +
 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 +
 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) +
10860*a**(73/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6
*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4
+ 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7
+ 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) -
 11520*a**(73/2)*b**8*x**8/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
10*x5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
13*x8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 9160*a**(71/2)
*b**9*x**9*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
10*x5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
13*x8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 2560*a**(71/2)
*b**9*x**9/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37
800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 6615
0*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150
*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 8396*a**(69/2)*b**10*x**10*sqr
t(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 +
37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66
150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 31
50*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(69/2)*b**10*x**10/(3
15*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**
8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11
*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*
x**9 + 315*a**30*b**15*x**10) + 5632*a**(67/2)*b**11*x**11*sqrt(1 + b*x/a)/
(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b
8*x3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**
11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**1
4*x**9 + 315*a**30*b**15*x**10) + 2446*a**(65/2)*b**12*x**12*sqrt(1 + b*x/a
)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37
*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b
11*x6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b*
*14*x**9 + 315*a**30*b**15*x**10) + 620*a**(63/2)*b**13*x**13*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
14*x9 + 315*a**30*b**15*x**10) + 70*a**(61/2)*b**14*x**14*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*

```
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10)
```

Giac [A]
time = 0.00, size = 108, normalized size = 1.21

$$\frac{2\left(\frac{1}{9}\sqrt{a+bx}\,(a+bx)^4-\frac{4}{7}\sqrt{a+bx}\,(a+bx)^3a+\frac{6}{5}\sqrt{a+bx}\,(a+bx)^2a^2-\frac{4}{3}\sqrt{a+bx}\,(a+bx)a^3+\sqrt{a+bx}\,a^4\right)}{bb^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

Mupad [B]
time = 0.02, size = 71, normalized size = 0.80

$$\frac{2\,(a+b\,x)^{9/2}}{9\,b^5}+\frac{2\,a^4\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a^3\,(a+b\,x)^{3/2}}{3\,b^5}+\frac{12\,a^2\,(a+b\,x)^{5/2}}{5\,b^5}-\frac{8\,a\,(a+b\,x)^{7/2}}{7\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^5) + (2*a^4*(a + b*x)^(1/2))/b^5 - (8*a^3*(a + b*x
)^(3/2))/(3*b^5) + (12*a^2*(a + b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))
/(7*b^5)
```

3.335 $\int \frac{x^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}$$

```
[Out] 2*a^2*(b*x+a)^(3/2)/b^4-6/5*a*(b*x+a)^(5/2)/b^4+2/7*(b*x+a)^(7/2)/b^4-2*a^3
*(b*x+a)^(1/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^(
5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx\\ &= -\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.68

$$\frac{2\sqrt{a+bx}\,\left(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 330 vs. $2(68) = 136$.
time = 15.24, size = 308, normalized size = 4.53

$$\frac{2\sqrt{a}\left(16a^9\left(1-\sqrt{\frac{a+bx}{a}}\right)+8a^8bx\left(12-11\sqrt{\frac{a+bx}{a}}\right)+6a^7b^2x^2\left(40-33\sqrt{\frac{a+bx}{a}}\right)+a^6b^3x^3\left(320-231\sqrt{\frac{a+bx}{a}}\right)+5b^4x^4\left(48a^5+b^5x^5\sqrt{\frac{a+bx}{a}}\right)-140a^5b^4x^4\sqrt{\frac{a+bx}{a}}+ab^5x^5\left(96a^3+16a^2bx+47ab^2x^2\sqrt{\frac{a+bx}{a}}+24b^3x^3\sqrt{\frac{a+bx}{a}}\right)+21a^3b^5x^5(-a+2bx)\sqrt{\frac{a+bx}{a}}\right)}{35b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (16 a ^ 9 (1 - Sqrt[(a + b x) / a]) + 8 a ^ 8 b x (12 - 11 Sqrt[(
a + b x) / a]) + 6 a ^ 7 b ^ 2 x ^ 2 (40 - 33 Sqrt[(a + b x) / a]) + a ^ 6
b ^ 3 x ^ 3 (320 - 231 Sqrt[(a + b x) / a]) + 5 b ^ 4 x ^ 4 (48 a ^ 5 + b ^
 5 x ^ 5 Sqrt[(a + b x) / a]) - 140 a ^ 5 b ^ 4 x ^ 4 Sqrt[(a + b x) / a] +
 a b ^ 5 x ^ 5 (96 a ^ 3 + 16 a ^ 2 b x + 47 a b ^ 2 x ^ 2 Sqrt[(a + b x) /
 a] + 24 b ^ 3 x ^ 3 Sqrt[(a + b x) / a]) + 21 a ^ 3 b ^ 5 x ^ 5 (-a + 2 b
x) Sqrt[(a + b x) / a]) / (35 b ^ 4 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x
 ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^
6 x ^ 6))
```

Maple [**A**]
time = 0.09, size = 49, normalized size = 0.72

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)}{35b^4}$	43
trager	$-\frac{2\sqrt{bx+a}\,\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)}{35b^4}$	43
risch	$-\frac{2\sqrt{bx+a}\,\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)}{35b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{6a(bx+a)^{\frac{5}{2}}}{5}+2a^2(bx+a)^{\frac{3}{2}}-2a^3\sqrt{bx+a}}{b^4}$	49
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{6a(bx+a)^{\frac{5}{2}}}{5}+2a^2(bx+a)^{\frac{3}{2}}-2a^3\sqrt{bx+a}}{b^4}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/7*(b*x+a)^(7/2)-3/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)-a^3*(b*x+a)^
(1/2))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^4}-\frac{6\,(bx+a)^{\frac{5}{2}}a}{5\,b^4}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{b^4}-\frac{2\,\sqrt{bx+a}\,a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^4 - 6/5*(b*x + a)^(5/2)*a/b^4 + 2*(b*x + a)^(3/2)*a^2
/b^4 - 2*sqrt(b*x + a)*a^3/b^4
```

Fricas [A]
time = 0.31, size = 42, normalized size = 0.62

$$\frac{2\,(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3)\sqrt{bx+a}}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(65) = 130$.
time = 1.28, size = 1640, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(47/2)*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18
*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**
5 + 35*a**14*b**10*x**6) + 32*a**(47/2)/(35*a**20*b**4 + 210*a**19*b**5*x +
 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**1
5*b**9*x**5 + 35*a**14*b**10*x**6) - 176*a**(45/2)*b*x*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(45/2)*b*x/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a*
*17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*
x**6) - 396*a**(43/2)*b**2*x**2*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*
b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 +
210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 480*a**(43/2)*b**2*x**2/(35*a*
*20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 5
25*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 462*a**(4
1/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
```

```
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6) + 640*a**(41/2)*b**3*x**3/(35*a**20*b**4 + 210*a*
*19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**
4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 280*a**(39/2)*b**4*x**4*sq
rt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700
*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**
10*x**6) + 480*a**(39/2)*b**4*x**4/(35*a**20*b**4 + 210*a**19*b**5*x + 525*
a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**
9*x**5 + 35*a**14*b**10*x**6) - 42*a**(37/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(37/2)*b**5*x**5/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 +
700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*
b**10*x**6) + 84*a**(35/2)*b**6*x**6*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a
**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x*
*4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 32*a**(35/2)*b**6*x**6/(3
5*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3
 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 94*a*
*(33/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a
**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9
*x**5 + 35*a**14*b**10*x**6) + 48*a**(31/2)*b**8*x**8*sqrt(1 + b*x/a)/(35*a
**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 10*a**(2
9/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6)
```

Giac [A]
time = 0.00, size = 82, normalized size = 1.21

$$\frac{2\left(\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3-\frac{3}{5}\sqrt{a+bx}\,(a+bx)^2a+\sqrt{a+bx}\,(a+bx)a^2-\sqrt{a+bx}\,a^3\right)}{bb^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x)
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.82

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}-\frac{2\,a^3\,\sqrt{a+b\,x}}{b^4}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{3/2}}{b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^4) - (2*a^3*(a + b*x)^(1/2))/b^4 + (2*a^2*(a + b*x
)^(3/2))/b^4 - (6*a*(a + b*x)^(5/2))/(5*b^4)
```

3.336 $\int \frac{x^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}$$

```
[Out] -4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3+2*a^2*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\,\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 174 vs. 2(51) = 102.
time = 7.00, size = 160, normalized size = 3.14

$$\frac{2\sqrt{a}\left(8a^5\left(-1+\sqrt{\frac{a+bx}{a}}\right)+4a^4bx\left(-6+5\sqrt{\frac{a+bx}{a}}\right)+3a^3b^2x^2\left(-8+5\sqrt{\frac{a+bx}{a}}\right)+\frac{5b^3x^3(a+bx)^2}{\sqrt{\frac{a+bx}{a}}}-8a^2b^3x^3+3b^5x^5\sqrt{\frac{a+bx}{a}}\right)}{15b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (8 a ^ 5 (-1 + Sqrt[(a + b x) / a]) + 4 a ^ 4 b x (-6 + 5 Sqrt[(a
 + b x) / a]) + 3 a ^ 3 b ^ 2 x ^ 2 (-8 + 5 Sqrt[(a + b x) / a]) + 5 b ^ 3
x ^ 3 (a + b x) ^ 2 / Sqrt[(a + b x) / a] - 8 a ^ 2 b ^ 3 x ^ 3 + 3 b ^ 5 x
 ^ 5 Sqrt[(a + b x) / a]) / (15 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^
2 + b ^ 3 x ^ 3))
```

Maple [**A**]
time = 0.09, size = 37, normalized size = 0.73

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,\left(3x^2b^2-4abx+8a^2\right)}{15b^3}$	32
trager	$\frac{2\sqrt{bx+a}\,\left(3x^2b^2-4abx+8a^2\right)}{15b^3}$	32
risch	$\frac{2\sqrt{bx+a}\,\left(3x^2b^2-4abx+8a^2\right)}{15b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*(b*x+a)^(5/2)-2/3*a*(b*x+a)^(3/2)+a^2*(b*x+a)^(1/2))
```

Maxima [**A**]
time = 0.28, size = 41, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

Fricas [A]
time = 0.30, size = 31, normalized size = 0.61

$$\frac{2\,(3\,b^2x^2 - 4\,abx + 8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(48) = 96.
time = 0.85, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Giac [A]
time = 0.00, size = 61, normalized size = 1.20

$$\frac{2\left(\frac{1}{5}\sqrt{a+bx}\,\left(a+bx\right)^2 - \frac{2}{3}\sqrt{a+bx}\,\left(a+bx\right)a + \sqrt{a+bx}\,a^2\right)}{bb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,{\left(a+b\,x\right)}^{5/2}-20\,a\,{\left(a+b\,x\right)}^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/2),x)
```

```
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

3.337 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$-\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b^2-2*a*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\ &= -\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.72

$$\frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 83 vs. $2(32) = 64$.
time = 3.31, size = 73, normalized size = 2.28

$$\frac{2\sqrt{a}\left(2a^2\left(1-\sqrt{\frac{a+bx}{a}}\right)+abx\left(2-\sqrt{\frac{a+bx}{a}}\right)+b^2x^2\sqrt{\frac{a+bx}{a}}\right)}{3b^2\left(a+bx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1/Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a] (2 a ^ 2 (1 - Sqrt[(a + b x) / a]) + a b x (2 - Sqrt[(a + b x) / a]) + b ^ 2 x ^ 2 Sqrt[(a + b x) / a]) / (3 b ^ 2 (a + b x))
```

Maple [**A**]
time = 0.08, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
risch	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/3*(b*x+a)^(3/2)-a*(b*x+a)^(1/2))
```

Maxima [**A**]
time = 0.26, size = 26, normalized size = 0.81

$$\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,b^2}-\frac{2\,\sqrt{bx+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Fricas [A]
time = 0.31, size = 19, normalized size = 0.59

$$\frac{2\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.
time = 0.56, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x}+\frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.12

$$\frac{2\left(\frac{1}{3}\sqrt{a+bx}\,(a+bx)-a\sqrt{a+bx}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x)
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6\,a\,\sqrt{a+b\,x}-2\,{\left(a+b\,x\right)}^{3/2}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/2),x)
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

3.338 $\int \frac{1}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

```
[Out] 2*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Mathics [A]
time = 1.61, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/Sqrt[a + b*x],x]')
```

```
[Out] 2 Sqrt[a + b x] / b
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{2\sqrt{bx+a}}{b}$	13
derivativedivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

Giac [A]
time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x)
```

```
[Out] 2*sqrt(b*x + a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\,\sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2))/b
```

3.339 $\int \frac{1}{x\sqrt{a+bx}} \, dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}} \, dx &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} \, dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Mathics [A]
time = 2.06, size = 16, normalized size = 0.70

$$\frac{-2\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^1*Sqrt[a + b*x]),x]')
[Out] -2 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a]
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.36, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.32, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A]
time = 0.49, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.17

$$\frac{2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x)
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.74

$$-\frac{2\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.340 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{a+bx}}{ax}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}$$
$$= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a}$$
$$= -\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Mathics [A]
time = 3.01, size = 40, normalized size = 0.98

$$-\frac{\sqrt{b}\sqrt{1+\frac{a}{bx}}}{a\sqrt{x}} + \frac{b\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{a^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*Sqrt[a + b*x]),x]')
```

```
[Out] -Sqrt[b] Sqrt[1 + a / (b x)] / (a Sqrt[x]) + b ArcSinh[Sqrt[a] / (Sqrt[b] S
qrt[x])] / a ^ (3 / 2)
```

Maple [A]
time = 0.11, size = 40, normalized size = 0.98

method	result	size
risch	$\frac{b\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{bx+a}}{ax}$	34

derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.34, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a}\,b}{(bx+a)a-a^2}-\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

Fricas [A]
time = 0.33, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,a}{2\,a^2x},-\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A]
time = 1.17, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.59

$$\frac{2\left(-\frac{\sqrt{a+bx}\,b^2}{2a(a+bx-a)}-\frac{b^2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{a\cdot 2\sqrt{-a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

Mupad [B]
time = 0.11, size = 33, normalized size = 0.80

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\,x}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

3.341 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

```
[Out] -3/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)-1/2*(b*x+a)^(1/2)/a/x^2+3/4
*b*(b*x+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {44, 65, 214}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] -1/2*Sqrt[a + b*x]/(a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{2ax^2}-\frac{(3b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a}\\
&= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}+\frac{(3b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^2}\\
&= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}+\frac{(3b)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a^2}\\
&= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 56, normalized size = 0.82

$$
\frac{\sqrt{a+bx}\,(-2a+3bx)}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(-2*a + 3*b*x))/(4*a^2*x^2) - (3*b^2*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(4*a^(5/2))
```

Mathics [**A**]
time = 4.73, size = 102, normalized size = 1.50

$$
\frac{\frac{-2a^{\frac{11}{2}}x(a+bx)}{b}+a^{\frac{9}{2}}x^2\,(a+bx)+3a^{\frac{7}{2}}bx^3\,(a+bx)-3a^3b^{\frac{5}{2}}x^{\frac{9}{2}}\mathrm{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right]\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}}{4a^{\frac{11}{2}}\sqrt{b}\,x^{\frac{9}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*Sqrt[a + b*x]),x]')
```

```
[Out] (-2 a ^ (11 / 2) x (a + b x) / b + a ^ (9 / 2) x ^ 2 (a + b x) + 3 a ^ (7 /
 2) b x ^ 3 (a + b x) - 3 a ^ 3 b ^ (5 / 2) x ^ (9 / 2) ArcSinh[Sqrt[a] / (
Sqrt[b] Sqrt[x])] ((a + b x) / (b x)) ^ (3 / 2)) / (4 a ^ (11 / 2) Sqrt[b]
x ^ (9 / 2) ((a + b x) / (b x)) ^ (3 / 2))
```

Maple [A]
time = 0.11, size = 66, normalized size = 0.97

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3bx+2a)}{4a^2x^2}-\frac{3b^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	45
derivativedivides	$2b^2\left(-\frac{\sqrt{bx+a}}{4a\,b^2x^2}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66
default	$2b^2\left(-\frac{\sqrt{bx+a}}{4a\,b^2x^2}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/4*(b*x+a)^(1/2)/a/b^2/x^2-3/4/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)))
```

Maxima [A]
time = 0.36, size = 92, normalized size = 1.35

$$\frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{3}{2}}b^2-5\,\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
1/4*(3*(b*x + a)^(3/2)*b^2 - 5*sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a^2 - 2*(b
*x + a)*a^3 + a^4)
```

Fricas [A]
time = 0.31, size = 123, normalized size = 1.81

$$\left[\frac{3\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(3\,abx-2\,a^2)\sqrt{bx+a}}{8\,a^3x^2},\frac{3\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx-2\,a^2)\sqrt{bx+a}}{4\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [A]
time = 2.70, size = 102, normalized size = 1.50

$$-\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(b
*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

Giac [A]
time = 0.00, size = 97, normalized size = 1.43

$$\frac{2\left(-\frac{-3\sqrt{a+bx}\,(a+bx)b^3+5\sqrt{a+bx}\,ab^3}{8a^2(a+bx-a)^2}+\frac{3b^3\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4a^2\cdot 2\sqrt{-a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x)
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.06, size = 51, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{3/2}}{4\,a^2\,x^2}-\frac{5\,\sqrt{a+b\,x}}{4\,a\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/2)),x)
```

```
[Out] (3*(a + b*x)^(3/2))/(4*a^2*x^2) - (5*(a + b*x)^(1/2))/(4*a*x^2) - (3*b^2*at
anh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

3.342 $\int \frac{1}{x^4\sqrt{a+bx}}\,dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

```
[Out] 5/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)-1/3*(b*x+a)^(1/2)/a/x^3+5/12
*b*(b*x+a)^(1/2)/a^2/x^2-5/8*b^2*(b*x+a)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 214}

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{\sqrt{a+bx}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -1/3*Sqrt[a + b*x]/(a*x^3) + (5*b*Sqrt[a + b*x])/(12*a^2*x^2) - (5*b^2*Sqrt
[a + b*x])/(8*a^3*x) + (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{3ax^3}-\frac{(5b)\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{6a}\\
&= -\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}+\frac{(5b^2)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a^2}\\
&= -\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{5b^2\sqrt{a+bx}}{8a^3x}-\frac{(5b^3)\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^3}\\
&= -\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{5b^2\sqrt{a+bx}}{8a^3x}-\frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a^3}\\
&= -\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 67, normalized size = 0.74

$$-\frac{\sqrt{a+bx}\,(8a^2-10abx+15b^2x^2)}{24a^3x^3}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 - 10*a*b*x + 15*b^2*x^2))/(a^3*x^3) + (5*b^3*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Mathics [A]
time = 9.90, size = 137, normalized size = 1.52

$$\frac{5b^3\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right]}{8a^{\frac{7}{2}}}-\frac{b^{\frac{3}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}\left(a+bx\right)^2}+\frac{b^{\frac{5}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{12a\sqrt{x}\left(a+bx\right)^2}-\frac{5b^{\frac{7}{2}}\sqrt{x}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{24a^2\left(a+bx\right)^2}-\frac{5b^{\frac{9}{2}}x^{\frac{3}{2}}\left(1+\frac{a}{bx}\right)^{\frac{3}{2}}}{8a^3\left(a+bx\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^4*Sqrt[a + b*x]),x]')
```

```
[Out] 5 b ^ 3 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (8 a ^ (7 / 2)) - b ^ (3 / 2
) (1 + a / (b x)) ^ (3 / 2) / (3 x ^ (3 / 2) (a + b x) ^ 2) + b ^ (5 / 2) (
1 + a / (b x)) ^ (3 / 2) / (12 a Sqrt[x] (a + b x) ^ 2) - 5 b ^ (7 / 2) Sqr
```

```
t[x] (1 + a / (b x)) ^ (3 / 2) / (24 a ^ 2 (a + b x) ^ 2) - 5 b ^ (9 / 2) x
 ^ (3 / 2) (1 + a / (b x)) ^ (3 / 2) / (8 a ^ 3 (a + b x) ^ 2)
```

Maple [A]
time = 0.10, size = 90, normalized size = 1.00

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(15x^2b^2-10abx+8a^2)}{24a^3x^3}+\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{7}{2}}}$	56
derivativedivides	$2b^3\left(-\frac{\sqrt{bx+a}}{6a\,b^3x^3}+\frac{\frac{5\sqrt{bx+a}}{24a\,b^2x^2}+\frac{5\left(-\frac{3\sqrt{bx+a}}{8abx}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}}{a}\right)$	90
default	$2b^3\left(-\frac{\sqrt{bx+a}}{6a\,b^3x^3}+\frac{\frac{5\sqrt{bx+a}}{24a\,b^2x^2}+\frac{5\left(-\frac{3\sqrt{bx+a}}{8abx}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}}{a}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-1/6*(b*x+a)^(1/2)/a/b^3/x^3+5/6/a*(1/4*(b*x+a)^(1/2)/a/b^2/x^2+3/4/
a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))))
```

Maxima [A]
time = 0.36, size = 121, normalized size = 1.34

$$-\frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{5}{2}}b^3-40\,(bx+a)^{\frac{3}{2}}ab^3+33\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3a^3-3\,(bx+a)^2a^4+3\,(bx+a)a^5-a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
- 1/24*(15*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 33*sqrt(b*x + a
)*a^2*b^3)/((b*x + a)^3*a^3 - 3*(b*x + a)^2*a^4 + 3*(b*x + a)*a^5 - a^6)
```

Fricas [A]
time = 0.52, size = 145, normalized size = 1.61

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,a^4x^3},-\frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^2x^2-10\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,a^4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

Sympy [A]
time = 7.93, size = 129, normalized size = 1.43

$$-\frac{1}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{3}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{5}{2}}}{8a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**(1/2),x)
```

```
[Out] -1/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(12*a*x**(5/2)*sqrt(a/(
b*x) + 1)) - 5*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(5/2)/(
8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/(8*a**(7/2))
```

Giac [A]
time = 0.00, size = 122, normalized size = 1.36

$$\frac{2\left(\frac{-15\sqrt{a+bx}\,(a+bx)^2b^4+40\sqrt{a+bx}\,(a+bx)ab^4-33\sqrt{a+bx}\,a^2b^4}{48a^3(a+bx-a)^3}-\frac{5b^4\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{8a^3\cdot 2\sqrt{-a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x)
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.77

$$\frac{5\,{\left(a+b\,x\right)}^{3/2}}{3\,a^2\,x^3}-\frac{11\,\sqrt{a+b\,x}}{8\,a\,x^3}-\frac{5\,{\left(a+b\,x\right)}^{5/2}}{8\,a^3\,x^3}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^(1/2)),x)
```

```
[Out] (5*(a + b*x)^(3/2))/(3*a^2*x^3) - (11*(a + b*x)^(1/2))/(8*a*x^3) - (5*(a +
b*x)^(5/2))/(8*a^3*x^3) - (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^
(7/2))
```

3.343 $\int \frac{x^4}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

```
[Out] 4*a^2*(b*x+a)^(3/2)/b^5-8/5*a*(b*x+a)^(5/2)/b^5+2/7*(b*x+a)^(7/2)/b^5-2*a^4
/b^5/(b*x+a)^(1/2)-8*a^3*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x)
^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{3/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.67

$$\frac{2\left(-128a^4-64a^3bx+16a^2b^2x^2-8ab^3x^3+5b^4x^4\right)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-128*a^4 - 64*a^3*b*x + 16*a^2*b^2*x^2 - 8*a*b^3*x^3 + 5*b^4*x^4))/(35*
b^5*Sqrt[a + b*x])
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 516 vs. 2(85) = 170.
time = 32.01, size = 484, normalized size = 5.69

$$\frac{2\sqrt{a}\left(128a^{13}\left(1-\sqrt{\frac{a+bx}{a}}\right)+64a^{12}bx\left(20-19\sqrt{\frac{a+bx}{a}}\right)+16a^{11}b^2x^2\left(360-323\sqrt{\frac{a+bx}{a}}\right)+40a^{10}b^3x^3\left(384-323\sqrt{\frac{a+bx}{a}}\right)+5a^9b^4x^4\left(5376-4199\sqrt{\frac{a+bx}{a}}\right)+ab^5x^5\left(-23091a^7\sqrt{\frac{a+bx}{a}}-17292a^6bx\sqrt{\frac{a+bx}{a}}-8556a^5b^2x^2\sqrt{\frac{a+bx}{a}}+128a^2b^5x^5+212a^2b^5x^5\sqrt{\frac{a+bx}{a}}+124ab^6x^6\sqrt{\frac{a+bx}{a}}+37b^7x^7\sqrt{\frac{a+bx}{a}}\right)+32256a^8b^5x^5+26880a^7b^6x^6+5b^7x^7\left(3072a^6-498a^5bx\sqrt{\frac{a+bx}{a}}+1152a^5bx-34a^4b^2x^2\sqrt{\frac{a+bx}{a}}+256a^4b^2x^2+b^6x^6\sqrt{\frac{a+bx}{a}}\right)\right)}{35b^5\left(a^{10}+10a^9bx+45a^8b^2x^2+120a^7b^3x^3+210a^6b^4x^4+252a^5b^5x^5+210a^4b^6x^6+120a^3b^7x^7+45a^2b^8x^8+10ab^9x^9+b^{10}x^{10}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/(a + b*x)^(3/2),x]')
```

```
[Out] 2 Sqrt[a] (128 a ^ 13 (1 - Sqrt[(a + b x) / a]) + 64 a ^ 12 b x (20 - 19 Sq
rt[(a + b x) / a]) + 16 a ^ 11 b ^ 2 x ^ 2 (360 - 323 Sqrt[(a + b x) / a])
+ 40 a ^ 10 b ^ 3 x ^ 3 (384 - 323 Sqrt[(a + b x) / a]) + 5 a ^ 9 b ^ 4 x ^
 4 (5376 - 4199 Sqrt[(a + b x) / a]) + a b ^ 5 x ^ 5 (-23091 a ^ 7 Sqrt[(a
+ b x) / a] - 17292 a ^ 6 b x Sqrt[(a + b x) / a] - 8556 a ^ 5 b ^ 2 x ^ 2
Sqrt[(a + b x) / a] + 128 a ^ 2 b ^ 5 x ^ 5 + 212 a ^ 2 b ^ 5 x ^ 5 Sqrt[(a
 + b x) / a] + 124 a b ^ 6 x ^ 6 Sqrt[(a + b x) / a] + 37 b ^ 7 x ^ 7 Sqrt[
(a + b x) / a]) + 32256 a ^ 8 b ^ 5 x ^ 5 + 26880 a ^ 7 b ^ 6 x ^ 6 + 5 b ^
 7 x ^ 7 (3072 a ^ 6 - 498 a ^ 5 b x Sqrt[(a + b x) / a] + 1152 a ^ 5 b x -
 34 a ^ 4 b ^ 2 x ^ 2 Sqrt[(a + b x) / a] + 256 a ^ 4 b ^ 2 x ^ 2 + b ^ 6 x
 ^ 6 Sqrt[(a + b x) / a])) / (35 b ^ 5 (a ^ 10 + 10 a ^ 9 b x + 45 a ^ 8 b
^ 2 x ^ 2 + 120 a ^ 7 b ^ 3 x ^ 3 + 210 a ^ 6 b ^ 4 x ^ 4 + 252 a ^ 5 b ^ 5
 x ^ 5 + 210 a ^ 4 b ^ 6 x ^ 6 + 120 a ^ 3 b ^ 7 x ^ 7 + 45 a ^ 2 b ^ 8 x ^
 8 + 10 a b ^ 9 x ^ 9 + b ^ 10 x ^ 10))
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.73

method	result	size
gosper	$-\frac{2\left(-5b^4x^4+8a\,b^3x^3-16a^2b^2x^2+64a^3bx+128a^4\right)}{35\sqrt{bx+a}\;b^5}$	54
trager	$-\frac{2\left(-5b^4x^4+8a\,b^3x^3-16a^2b^2x^2+64a^3bx+128a^4\right)}{35\sqrt{bx+a}\;b^5}$	54
risch	$-\frac{2\left(-5b^3x^3+13a\,b^2x^2-29a^2bx+93a^3\right)\sqrt{bx+a}}{35b^5}-\frac{2a^4}{b^5\sqrt{bx+a}}$	59
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^2(bx+a)^{\frac{3}{2}}-8a^3\sqrt{bx+a}-\frac{2a^4}{\sqrt{bx+a}}}{b^5}$	62
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^2(bx+a)^{\frac{3}{2}}-8a^3\sqrt{bx+a}-\frac{2a^4}{\sqrt{bx+a}}}{b^5}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/7*(b*x+a)^(7/2)-4/5*a*(b*x+a)^(5/2)+2*a^2*(b*x+a)^(3/2)-4*a^3*(b*x
+a)^(1/2)-a^4/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.26, size = 71, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^5}-\frac{8\,(bx+a)^{\frac{5}{2}}a}{5\,b^5}+\frac{4\,(bx+a)^{\frac{3}{2}}a^2}{b^5}-\frac{8\,\sqrt{bx+a}\,a^3}{b^5}-\frac{2\,a^4}{\sqrt{bx+a}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^5 - 8/5*(b*x + a)^(5/2)*a/b^5 + 4*(b*x + a)^(3/2)*a^2
/b^5 - 8*sqrt(b*x + a)*a^3/b^5 - 2*a^4/(sqrt(b*x + a)*b^5)
```

Fricas [A]
time = 0.30, size = 63, normalized size = 0.74

$$\frac{2\,(5\,b^4x^4-8\,ab^3x^3+16\,a^2b^2x^2-64\,a^3bx-128\,a^4)\sqrt{bx+a}}{35\,(b^6x+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3606 vs. 2(82) = 164.
time = 2.20, size = 3606, normalized size = 42.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(3/2),x)
```

```
[Out] -256*a**(87/2)*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**
38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**
10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*
x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(87/2)/(35*a**
40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 +
7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200
```

*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30
*b**15*x**10) - 2432*a**(85/2)*b*x*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**
39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x
4 + 8820*a35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 2
560*a**(85/2)*b*x/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2
+ 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 735
0*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a*
*31*b**14*x**9 + 35*a**30*b**15*x**10) - 10336*a**(83/2)*b**2*x**2*sqrt(1 +
 b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**
37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b*
*11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*
x**9 + 35*a**30*b**15*x**10) + 11520*a**(83/2)*b**2*x**2/(35*a**40*b**5 + 3
50*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*
b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**1
2*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**1
0) - 25840*a**(81/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b
6*x + 1575*a38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4
+ 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1
575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 30720
*a**(81/2)*b**3*x**3/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x*
*2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 +
7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350
*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 41990*a**(79/2)*b**4*x**4*sqrt(
1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*
a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34
*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**
14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(79/2)*b**4*x**4/(35*a**40*b**5
+ 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**
36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b
12*x7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x
10) - 46182*a(77/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**3
9*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x*
*4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
+ 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 64
512*a**(77/2)*b**5*x**5/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7
*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5
 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 +
350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 34584*a**(75/2)*b**6*x**6*sq
rt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 42
00*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a*
*34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*
b**14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(75/2)*b**6*x**6/(35*a**40*b*
*5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*
a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**3

```
3*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**1
5*x**10) - 17112*a**(73/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 30720*a**(73/2)*b**7*x**7/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b
**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x
**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 4980*a**(71/2)*b**8*x**8*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 11520*a**(71/2)*b**8*x**8/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) - 340*a**(69/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 2560*a**(69/2)*b**9*x**9/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b*
*7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x*
*5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
+ 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 424*a**(67/2)*b**10*x**10*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(67/2)*b**10*x**10/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) + 248*a**(65/2)*b**11*x**11*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350
*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b*
*9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*
x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
 + 74*a**(63/2)*b**12*x**12*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6
*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8
820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575
*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 10*a**(6
1/2)*b**13*x**13*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a
**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b
**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**1
3*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
```

Giac [A]

time = 0.00, size = 115, normalized size = 1.35

$$2\left(\frac{\frac{1}{7}\sqrt{a+bx}\,(a+bx)^3\,b^{30}-\frac{4}{5}\sqrt{a+bx}\,(a+bx)^2\,ab^{30}+2\sqrt{a+bx}\,(a+bx)\,a^2b^{30}-4\sqrt{a+bx}\,a^3b^{30}}{b^{35}}-\frac{a^4}{b^5\sqrt{a+bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x)
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.84

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}-\frac{8\,a^3\,\sqrt{a+b\,x}}{b^5}+\frac{4\,a^2\,{\left(a+b\,x\right)}^{3/2}}{b^5}-\frac{2\,a^4}{b^5\,\sqrt{a+b\,x}}-\frac{8\,a\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^5) - (8*a^3*(a + b*x)^(1/2))/b^5 + (4*a^2*(a + b*x
)^(3/2))/b^5 - (2*a^4)/(b^5*(a + b*x)^(1/2)) - (8*a*(a + b*x)^(5/2))/(5*b^5
)
```

3.344 $\int \frac{x^3}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=66

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

```
[Out] -2*a*(b*x+a)^(3/2)/b^4+2/5*(b*x+a)^(5/2)/b^4+2*a^3/b^4/(b*x+a)^(1/2)+6*a^2*
(b*x+a)^(1/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(3
/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx\\ &= \frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.68

$$\frac{2\left(16a^3+8a^2bx-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[a + b*x])
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 302 vs. 2(66) = 132.
time = 14.56, size = 280, normalized size = 4.24

$$\frac{2\sqrt{a}\left(16a^8\left(-1+\sqrt{\frac{a+bx}{a}}\right)+8a^7bx\left(-12+11\sqrt{\frac{a+bx}{a}}\right)+6a^6b^2x^2\left(-40+33\sqrt{\frac{a+bx}{a}}\right)+a^5b^3x^3\left(-320+231\sqrt{\frac{a+bx}{a}}\right)+5a^4b^4x^4\left(-48+29\sqrt{\frac{a+bx}{a}}\right)+b^5x^5\left(-96a^3+46a^3\sqrt{\frac{a+bx}{a}}-16a^2bx+8a^2bx\sqrt{\frac{a+bx}{a}}+3ab^2x^2\sqrt{\frac{a+bx}{a}}+b^3x^3\sqrt{\frac{a+bx}{a}}\right)\right)}{5b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(a + b*x)^(3/2),x]')
```

```
[Out] 2 Sqrt[a] (16 a ^ 8 (-1 + Sqrt[(a + b x) / a]) + 8 a ^ 7 b x (-12 + 11 Sqrt
[(a + b x) / a]) + 6 a ^ 6 b ^ 2 x ^ 2 (-40 + 33 Sqrt[(a + b x) / a]) + a ^
 5 b ^ 3 x ^ 3 (-320 + 231 Sqrt[(a + b x) / a]) + 5 a ^ 4 b ^ 4 x ^ 4 (-48
+ 29 Sqrt[(a + b x) / a]) + b ^ 5 x ^ 5 (-96 a ^ 3 + 46 a ^ 3 Sqrt[(a + b x
) / a] - 16 a ^ 2 b x + 8 a ^ 2 b x Sqrt[(a + b x) / a] + 3 a b ^ 2 x ^ 2 S
qrt[(a + b x) / a] + b ^ 3 x ^ 3 Sqrt[(a + b x) / a])) / (5 b ^ 4 (a ^ 6 +
6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4
x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.11, size = 49, normalized size = 0.74

method	result	size
gosper	$\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{b^4\sqrt{bx+a}}$	42
trager	$\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{b^4\sqrt{bx+a}}$	42
risch	$\frac{2(x^2b^2-3abx+11a^2)\sqrt{bx+a}}{5b^4}+\frac{2a^3}{b^4\sqrt{bx+a}}$	47
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a(bx+a)^{\frac{3}{2}}+6a^2\sqrt{bx+a}+\frac{2a^3}{\sqrt{bx+a}}}{b^4}$	49
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a(bx+a)^{\frac{3}{2}}+6a^2\sqrt{bx+a}+\frac{2a^3}{\sqrt{bx+a}}}{b^4}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/5*(b*x+a)^(5/2)-a*(b*x+a)^(3/2)+3*a^2*(b*x+a)^(1/2)+a^3/(b*x+a)^(1
/2))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^4}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{b^4}+\frac{6\,\sqrt{bx+a}\,a^2}{b^4}+\frac{2\,a^3}{\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^4 - 2*(b*x + a)^(3/2)*a/b^4 + 6*sqrt(b*x + a)*a^2/b^4
 + 2*a^3/(sqrt(b*x + a)*b^4)
```

Fricas [A]
time = 0.30, size = 51, normalized size = 0.77

$$\frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)\,\sqrt{bx+a}}{5\,(b^5x+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(63) = 126$.
time = 1.34, size = 1538, normalized size = 23.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(3/2),x)
```

```
[Out] 32*a**(45/2)*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**
6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*
a**14*b**10*x**6) - 32*a**(45/2)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18
*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5
+ 5*a**14*b**10*x**6) + 176*a**(43/2)*b*x*sqrt(1 + b*x/a)/(5*a**20*b**4 + 3
0*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x
**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 192*a**(43/2)*b*x/(5*a**20
*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**
16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 396*a**(41/2)*b**
2*x**2*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) - 480*a**(41/2)*b**2*x**2/(5*a**20*b**4 + 30*a**19*b**5*x + 75*
a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*
```

```
x**5 + 5*a**14*b**10*x**6) + 462*a**(39/2)*b**3*x**3*sqrt(1 + b*x/a)/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 640*a**(39/2)*b
**3*x**3/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b
**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) +
290*a**(37/2)*b**4*x**4*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 7
5*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**
9*x**5 + 5*a**14*b**10*x**6) - 480*a**(37/2)*b**4*x**4/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 92*a**(35/2)*b**5*x**5*sqrt(1
 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*
b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) -
 192*a**(35/2)*b**5*x**5/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x*
*2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**1
4*b**10*x**6) + 16*a**(33/2)*b**6*x**6*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 32*a**(33/2)*b**6*x**6/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 6*a**(31/2)*b**
7*x**7*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) + 2*a**(29/2)*b**8*x**8*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**1
9*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 +
30*a**15*b**9*x**5 + 5*a**14*b**10*x**6)
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.30

$$2\left(\frac{\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2b^{16}-\sqrt{a+bx}\,(a+bx)\,ab^{16}+3\sqrt{a+bx}\,a^2b^{16}}{b^{20}}+\frac{a^3}{b^4\sqrt{a+bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x)
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.85

$$\frac{2\,(a+b\,x)^{5/2}}{5\,b^4}+\frac{6\,a^2\,\sqrt{a+b\,x}}{b^4}+\frac{2\,a^3}{b^4\,\sqrt{a+b\,x}}-\frac{2\,a\,(a+b\,x)^{3/2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(3/2),x)
[Out] (2*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(1/2))/b^4 + (2*a^3)/(b^4*(a
 + b*x)^(1/2)) - (2*a*(a + b*x)^(3/2))/b^4
```

3.345 $\int \frac{x^2}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{b^3\sqrt{a+bx}} - \frac{4a\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{3/2}}{3b^3}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b^3-2*a^2/b^3/(b*x+a)^(1/2)-4*a*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^2}{b^3\sqrt{a+bx}} - \frac{4a\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2)
)/(3*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{3/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{3/2}} - \frac{2a}{b^2\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b^2}\right) dx \\ &= -\frac{2a^2}{b^3\sqrt{a+bx}} - \frac{4a\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.69

$$\frac{2\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[a + b*x])
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(49) = 98.
time = 6.26, size = 145, normalized size = 2.96

$$\frac{2\sqrt{a}\left(8a^4\left(1-\sqrt{\frac{a+bx}{a}}\right)+4a^3bx\left(6-5\sqrt{\frac{a+bx}{a}}\right)+3a^2b^2x^2\left(8-5\sqrt{\frac{a+bx}{a}}\right)+b^3x^3\left(-2a\sqrt{\frac{a+bx}{a}}+8a+bx\sqrt{\frac{a+bx}{a}}\right)\right)}{3b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(a + b*x)^(3/2),x]')
```

```
[Out] 2 Sqrt[a] (8 a ^ 4 (1 - Sqrt[(a + b x) / a]) + 4 a ^ 3 b x (6 - 5 Sqrt[(a +
 b x) / a]) + 3 a ^ 2 b ^ 2 x ^ 2 (8 - 5 Sqrt[(a + b x) / a]) + b ^ 3 x ^ 3
 (-2 a Sqrt[(a + b x) / a] + 8 a + b x Sqrt[(a + b x) / a])) / (3 b ^ 3 (a
^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.78

method	result	size
gosper	$-\frac{2\left(-x^2b^2+4abx+8a^2\right)}{3\sqrt{bx+a}\,b^3}$	32
trager	$-\frac{2\left(-x^2b^2+4abx+8a^2\right)}{3\sqrt{bx+a}\,b^3}$	32
risch	$-\frac{2(-bx+5a)\sqrt{bx+a}}{3b^3}-\frac{2a^2}{b^3\sqrt{bx+a}}$	37
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-4a\sqrt{bx+a}-\frac{2a^2}{\sqrt{bx+a}}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-4a\sqrt{bx+a}-\frac{2a^2}{\sqrt{bx+a}}}{b^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/3*(b*x+a)^(3/2)-2*a*(b*x+a)^(1/2)-a^2/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 41, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^3}-\frac{4\,\sqrt{bx+a}\,a}{b^3}-\frac{2\,a^2}{\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^3 - 4*sqrt(b*x + a)*a/b^3 - 2*a^2/(sqrt(b*x + a)*b^3)
```

Fricas [A]
time = 0.30, size = 40, normalized size = 0.82

$$\frac{2\left(b^{2}x^{2}-4\,abx-8\,a^{2}\right)\sqrt{bx+a}}{3\left(b^{4}x+ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(46) = 92$.
time = 0.84, size = 534, normalized size = 10.90

$$-\frac{16a^{\frac{19}{2}}\sqrt{1+\frac{bx}{a}}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}+\frac{16a^{\frac{19}{2}}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}-\frac{40a^{\frac{17}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}+\frac{48a^{\frac{17}{2}}bx}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}-\frac{30a^{\frac{15}{2}}b^{2}x^{2}\sqrt{1+\frac{bx}{a}}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}+\frac{48a^{\frac{15}{2}}b^{2}x^{2}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}-\frac{4a^{\frac{13}{2}}b^{3}x^{3}\sqrt{1+\frac{bx}{a}}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}+\frac{16a^{\frac{13}{2}}b^{3}x^{3}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}+\frac{2a^{\frac{11}{2}}b^{4}x^{4}\sqrt{1+\frac{bx}{a}}}{3a^{8}b^{3}+9a^{7}b^{4}x+9a^{6}b^{5}x^{2}+3a^{5}b^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(3/2),x)
```

```
[Out] -16*a**(19/2)*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**
*2 + 3*a**5*b**6*x**3) + 16*a**(19/2)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6
*b**5*x**2 + 3*a**5*b**6*x**3) - 40*a**(17/2)*b*x*sqrt(1 + b*x/a)/(3*a**8*b
**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(17/2)*b
*x/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 30
*a**(15/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*
b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(15/2)*b**2*x**2/(3*a**8*b**3 + 9*a**
7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 4*a**(13/2)*b**3*x**3*sqr
t(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*
x**3) + 16*a**(13/2)*b**3*x**3/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x
**2 + 3*a**5*b**6*x**3) + 2*a**(11/2)*b**4*x**4*sqrt(1 + b*x/a)/(3*a**8*b**
3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
```

Giac [A]
time = 0.00, size = 64, normalized size = 1.31

$$2\left(\frac{\frac{1}{3}\sqrt{a+bx}\,\left(a+bx\right)b^{6}-2\sqrt{a+bx}\,ab^{6}}{b^{9}}-\frac{a^{2}}{b^{3}\sqrt{a+bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x)
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$-\frac{12\,a\,\left(a+b\,x\right)-2\,{\left(a+b\,x\right)}^2+6\,a^2}{3\,b^3\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(3/2),x)
```

```
[Out] -(12*a*(a + b*x) - 2*(a + b*x)^2 + 6*a^2)/(3*b^3*(a + b*x)^(1/2))
```

3.346 $\int \frac{x}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=30

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

[Out] 2*a/b^2/(b*x+a)^(1/2)+2*(b*x+a)^(1/2)/b^2

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x)^(3/2),x]

[Out] (2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx\\ &= \frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.70

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 1.91, size = 33, normalized size = 1.10

$$\text{Piecewise}\left[\left\{\left\{\frac{2\,(2a+bx)}{b^2\sqrt{a+bx}},b!{=}0\right\}\right\},\frac{x^2}{2a^{\frac{3}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1/(a + b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{2 (2 a + b x) / (b ^ 2 Sqrt[a + b x]), b != 0}}, x ^ 2 / (2 a ^
 (3 / 2))]
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.77

method	result	size
gosper	$\frac{2bx+4a}{b^2\sqrt{bx+a}}$	20
trager	$\frac{2bx+4a}{b^2\sqrt{bx+a}}$	20
derivativedivides	$\frac{2\sqrt{bx+a}+\frac{2a}{\sqrt{bx+a}}}{b^2}$	23
default	$\frac{2\sqrt{bx+a}+\frac{2a}{\sqrt{bx+a}}}{b^2}$	23
risch	$\frac{2a}{b^2\sqrt{bx+a}}+\frac{2\sqrt{bx+a}}{b^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*((b*x+a)^(1/2)+a/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.87

$$\frac{2\,\sqrt{bx+a}}{b^2}+\frac{2\,a}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^2 + 2*a/(sqrt(b*x + a)*b^2)
```

Fricas [A]
time = 0.31, size = 29, normalized size = 0.97

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.29, size = 37, normalized size = 1.23

$$\begin{cases} \frac{4a}{b^2\sqrt{a+bx}} + \frac{2x}{b\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(3/2),x)
[Out] Piecewise((4*a/(b**2*sqrt(a + b*x)) + 2*x/(b*sqrt(a + b*x)), Ne(b, 0)), (x*
*2/(2*a**(3/2)), True))
```

Giac [A]
time = 0.00, size = 31, normalized size = 1.03

$$\frac{2\left(\frac{\sqrt{a+bx}}{b} + \frac{a}{b\sqrt{a+bx}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x)
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

Mupad [B]
time = 0.09, size = 19, normalized size = 0.63

$$\frac{4\,a+2\,b\,x}{b^2\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(3/2),x)
[Out] (4*a + 2*b*x)/(b^2*(a + b*x)^(1/2))
```

3.347 $\int \frac{1}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=14

$$-\frac{2}{b\sqrt{a+bx}}$$

[Out] -2/b/(b*x+a)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-3/2),x]

[Out] -2/(b*Sqrt[a + b*x])

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}} dx = -\frac{2}{b\sqrt{a+bx}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-3/2),x]

[Out] -2/(b*Sqrt[a + b*x])

Mathics [A]
time = 1.63, size = 12, normalized size = 0.86

$$\frac{-2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^(3/2),x]')
```

```
[Out] -2 / (b Sqrt[a + b x])
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{2}{b\sqrt{bx+a}}$	13
derivativedivides	$-\frac{2}{b\sqrt{bx+a}}$	13
default	$-\frac{2}{b\sqrt{bx+a}}$	13
trager	$-\frac{2}{b\sqrt{bx+a}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.86

$$-\frac{2}{b\sqrt{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2),x)
```

```
[Out] -2/(b*sqrt(a + b*x))
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.07

$$-\frac{2}{b\sqrt{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x)
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{b\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(3/2),x)
```

```
[Out] -2/(b*(a + b*x)^(1/2))
```

3.348 $\int \frac{1}{x(a+bx)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 214}

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{a+bx}} + \frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{a\sqrt{a+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{ab}$$
$$= \frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [**A**]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(38) = 76.
time = 3.55, size = 78, normalized size = 2.05

$$\frac{a\left(\text{Log}\left[\frac{bx}{a}\right] - 2\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right] + 2\sqrt{\frac{a+bx}{a}}\right) + bx\left(\text{Log}\left[\frac{bx}{a}\right] - 2\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)}{a^{\frac{3}{2}}(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)^(3/2)),x]')
```

```
[Out] (a (Log[b x / a] - 2 Log[1 + Sqrt[(a + b x) / a]] + 2 Sqrt[(a + b x) / a])
+ b x (Log[b x / a] - 2 Log[1 + Sqrt[(a + b x) / a]])) / (a ^ (3 / 2) (a +
b x))
```

Maple [**A**]
time = 0.13, size = 31, normalized size = 0.82

method	result	size

derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{bx+a}}$	31
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{bx+a}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 45, normalized size = 1.18

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) + 2/(sqrt(
b*x + a)*a)
```

Fricas [A]
time = 0.32, size = 110, normalized size = 2.89

$$\left[\frac{(bx+a)\sqrt{a}\,\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a}{a^2bx+a^3},\ \frac{2\left((bx+a)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.
time = 0.85, size = 146, normalized size = 3.84

$$\frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/2
) + a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*
x) + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt(1
+ b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
```

Giac [**A**]
time = 0.00, size = 47, normalized size = 1.24

$$2\left(\frac{1}{a\sqrt{a+bx}}+\frac{2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{a\cdot 2\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x)
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

Mupad [**B**]
time = 0.04, size = 30, normalized size = 0.79

$$\frac{2}{a\,\sqrt{a+b\,x}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(3/2)),x)
```

```
[Out] 2/(a*(a + b*x)^(1/2)) - (2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2)
```

3.349 $\int \frac{1}{x^2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 3*b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)-3*b/a^2/(b*x+a)^(1/2)-1/a/x/(b*x
+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 214}

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (-3*b)/(a^2*Sqrt[a + b*x]) - 1/(a*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{3/2}}\,dx &= \frac{2}{ax\sqrt{a+bx}}+\frac{3\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}-\frac{(3b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^2}\\ &= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}-\frac{3\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^2}\\ &= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.86

$$\frac{-a-3bx}{a^2x\sqrt{a+bx}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(3/2)),x]
[Out] (-a - 3*b*x)/(a^2*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a
^(5/2)
```

Mathics [A]
time = 3.71, size = 88, normalized size = 1.54

$$\frac{\sqrt{b}\left(-a^{\frac{9}{2}}\sqrt{x}\sqrt{\frac{a+bx}{bx}}+3a^3\sqrt{b}\,x\mathrm{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right](a+bx)-3a^{\frac{7}{2}}bx^{\frac{3}{2}}\sqrt{\frac{a+bx}{bx}}\right)}{a^{\frac{11}{2}}x\,(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^(3/2)),x]')
```

```
[Out] Sqrt[b] (-a ^ (9 / 2) Sqrt[x] Sqrt[(a + b x) / (b x)] + 3 a ^ 3 Sqrt[b] x A
rcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) - 3 a ^ (7 / 2) b x ^ (3 / 2)
 Sqrt[(a + b x) / (b x)]) / (a ^ (11 / 2) x (a + b x))
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.95

method	result	size
risch	$-\frac{\sqrt{bx+a}}{a^{2}x}-\frac{b\left(\frac{4}{\sqrt{bx+a}}-\frac{6\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{2a^{2}}$	50
derivativedivides	$2b\left(-\frac{1}{a^{2}\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}\right)$	54
default	$2b\left(-\frac{1}{a^{2}\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/a^2/(b*x+a)^(1/2)+1/a^2*(-1/2*(b*x+a)^(1/2)/b/x+3/2*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.37, size = 76, normalized size = 1.33

$$-\frac{3\,(bx+a)b-2\,ab}{(bx+a)^{\frac{3}{2}}a^{2}-\sqrt{bx+a}\,a^{3}}-\frac{3\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x + a)*b - 2*a*b)/((b*x + a)^(3/2)*a^2 - sqrt(b*x + a)*a^3) - 3/2*b*
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
```

Fricas [A]
time = 0.32, size = 151, normalized size = 2.65

$$\left[\frac{3\,(b^2x^2+abx)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(3\,abx+a^2)\sqrt{bx+a}}{2\,(a^3bx^2+a^4x)},-\frac{3\,(b^2x^2+abx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx+a^2)\sqrt{bx+a}}{a^3bx^2+a^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [A]
time = 1.85, size = 73, normalized size = 1.28

$$-\frac{1}{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{3b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a/
(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.56

$$2\left(-\frac{3\,(a+bx)\,b-2ba}{2a^2\left(\sqrt{a+bx}\,(a+bx)-\sqrt{a+bx}\,a\right)}-\frac{3b\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2a^2\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x)
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

Mupad [B]
time = 0.12, size = 60, normalized size = 1.05

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,b}{a}-\frac{3\,b\,(a+b\,x)}{a^2}}{a\,\sqrt{a+b\,x}-{(a+b\,x)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(3/2)),x)
[Out] (3*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*b)/a - (3*b*(a + b*x))/a
^2)/(a*(a + b*x)^(1/2) - (a + b*x)^(3/2))
```

3.350 $\int \frac{1}{x^3(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{15b^2}{4a^3\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)+15/4*b^2/a^3/(b*x+a)^(1/2)
-1/2/a/x^2/(b*x+a)^(1/2)+5/4*b/a^2/x/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 214}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15b^2}{4a^3\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (15*b^2)/(4*a^3*Sqrt[a + b*x]) - 1/(2*a*x^2*Sqrt[a + b*x]) + (5*b)/(4*a^2*x
*Sqrt[a + b*x]) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx)^{3/2}}\,dx &= \frac{2}{ax^2\sqrt{a+bx}}+\frac{5\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{a}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}-\frac{(15b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^2}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{(15b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^3}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{(15b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a^3}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.77

$$\frac{-2a^2+5abx+15b^2x^2}{4a^3x^2\sqrt{a+bx}}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(3/2)),x]
[Out] (-2*a^2 + 5*a*b*x + 15*b^2*x^2)/(4*a^3*x^2*Sqrt[a + b*x]) - (15*b^2*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Mathics [A]

time = 5.99, size = 103, normalized size = 1.18

$$\frac{\frac{-2a^{\frac{17}{2}}x(a+bx)}{b}+5a^{\frac{15}{2}}x^2(a+bx)+15a^{\frac{13}{2}}bx^3(a+bx)-15a^6b^{\frac{5}{2}}x^{\frac{9}{2}}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}}{4a^{\frac{19}{2}}\sqrt{b}\,x^{\frac{9}{2}}\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^(3/2)),x]')
```

```
[Out] (-2 a ^ (17 / 2) x (a + b x) / b + 5 a ^ (15 / 2) x ^ 2 (a + b x) + 15 a ^
(13 / 2) b x ^ 3 (a + b x) - 15 a ^ 6 b ^ (5 / 2) x ^ (9 / 2) ArcSinh[Sqrt[
a] / (Sqrt[b] Sqrt[x])] ((a + b x) / (b x)) ^ (3 / 2)) / (4 a ^ (19 / 2) Sq
rt[b] x ^ (9 / 2) ((a + b x) / (b x)) ^ (3 / 2))
```

Maple [A]
time = 0.12, size = 68, normalized size = 0.78

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-7bx+2a)}{4a^3x^2}+\frac{b^2\left(\frac{16}{\sqrt{bx+a}}-\frac{30\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{8a^3}$	60
derivativedivides	$2b^2\left(\frac{1}{a^3\sqrt{bx+a}}-\frac{\frac{-\frac{7(bx+a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}\right)$	68
default	$2b^2\left(\frac{1}{a^3\sqrt{bx+a}}-\frac{\frac{-\frac{7(bx+a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}\right)$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/a^3/(b*x+a)^(1/2)-1/a^3*((-7/8*(b*x+a)^(3/2)+9/8*a*(b*x+a)^(1/2))/
b^2/x^2+15/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]

time = 0.35, size = 108, normalized size = 1.24

$$\frac{15\,{\left(bx+a\right)}^{2}b^{2}-25\,{\left(bx+a\right)}ab^{2}+8\,a^{2}b^{2}}{4\,{\left({\left(bx+a\right)}^{\frac{5}{2}}a^{3}-2\,{\left(bx+a\right)}^{\frac{3}{2}}a^{4}+\sqrt{bx+a}\,a^{5}\right)}}+\frac{15\,b^{2}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(b*x + a)^2*b^2 - 25*(b*x + a)*a*b^2 + 8*a^2*b^2)/((b*x + a)^(5/2)*
a^3 - 2*(b*x + a)^(3/2)*a^4 + sqrt(b*x + a)*a^5) + 15/8*b^2*log((sqrt(b*x +
 a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
```

Fricas [A]
time = 0.31, size = 189, normalized size = 2.17

$$\left[\frac{15\,{\left(b^{3}x^{3}+ab^{2}x^{2}\right)}\sqrt{a}\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\,{\left(15\,ab^{2}x^{2}+5\,a^{2}bx-2\,a^{3}\right)}\sqrt{bx+a}}{8\,{\left(a^{4}bx^{3}+a^{5}x^{2}\right)}},\ \frac{15\,{\left(b^{3}x^{3}+ab^{2}x^{2}\right)}\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+{\left(15\,ab^{2}x^{2}+5\,a^{2}bx-2\,a^{3}\right)}\sqrt{bx+a}}{4\,{\left(a^{4}bx^{3}+a^{5}x^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

Sympy [A]
time = 4.07, size = 107, normalized size = 1.23

$$-\frac{1}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{15b^{\frac{3}{2}}}{4a^{3}\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{15b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*sq
rt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))
```

Giac [A]
time = 0.00, size = 113, normalized size = 1.30

$$2\left(\frac{b^{2}}{a^{3}\sqrt{a+bx}}-\frac{-7\sqrt{a+bx}\,(a+bx)\,b^{2}+9\sqrt{a+bx}\,b^{2}a}{8a^{3}\,(a+bx-a)^{2}}+\frac{15b^{2}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4a^{3}\cdot 2\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x)
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B]
time = 0.06, size = 90, normalized size = 1.03

$$\frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,(a+b\,x)^2}{4\,a^3}-\frac{25\,b^2\,(a+b\,x)}{4\,a^2}}{(a+b\,x)^{5/2}-2\,a\,(a+b\,x)^{3/2}+a^2\,\sqrt{a+b\,x}}-\frac{15\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(3/2)),x)
```

```
[Out] ((2*b^2)/a + (15*b^2*(a + b*x)^2)/(4*a^3) - (25*b^2*(a + b*x))/(4*a^2))/((a
 + b*x)^(5/2) - 2*a*(a + b*x)^(3/2) + a^2*(a + b*x)^(1/2)) - (15*b^2*atanh(
(a + b*x)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

3.351 $\int \frac{x^4}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

```
[Out] -2/3*a^4/b^5/(b*x+a)^(3/2)-8/3*a*(b*x+a)^(3/2)/b^5+2/5*(b*x+a)^(5/2)/b^5+8*
a^3/b^5/(b*x+a)^(1/2)+12*a^2*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*Sq
rt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5*b
^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.66

$$\frac{2\left(128a^4+192a^3bx+48a^2b^2x^2-8ab^3x^3+3b^4x^4\right)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (2*(128*a^4 + 192*a^3*b*x + 48*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4))/(15*
b^5*(a + b*x)^(3/2))
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 484 vs. $2(87) = 174$.
time = 30.78, size = 454, normalized size = 5.22

$$\frac{2\sqrt{a}\left(128a^{12}\left(-1+\sqrt{\frac{a+bx}{a}}\right)+64a^{11}bx\left(-20+19\sqrt{\frac{a+bx}{a}}\right)+16a^{10}b^2x^2\left(-360+323\sqrt{\frac{a+bx}{a}}\right)+40a^9b^3x^3\left(-384+323\sqrt{\frac{a+bx}{a}}\right)+5a^8b^4x^4\left(-5376+4199\sqrt{\frac{a+bx}{a}}\right)+3b^5x^5\left(-10752a^7+b^7x^7\sqrt{\frac{a+bx}{a}}\right)+4ab^5x^5\left(5774a^6\sqrt{\frac{a+bx}{a}}+4333a^5bx\sqrt{\frac{a+bx}{a}}+2174a^4b^2x^2\sqrt{\frac{a+bx}{a}}-32ab^5x^5+17ab^5x^5\sqrt{\frac{a+bx}{a}}+4b^6x^6\sqrt{\frac{a+bx}{a}}\right)-1920a^4b^6x^6\left(14a^2+8abx+3b^2x^2\right)+10a^3b^8x^8\left(277a\sqrt{\frac{a+bx}{a}}-128bx+52bx\sqrt{\frac{a+bx}{a}}\right)\right)}{15b^5\left(a^{10}+10a^9bx+45a^8b^2x^2+120a^7b^3x^3+210a^6b^4x^4+252a^5b^5x^5+210a^4b^6x^6+120a^3b^7x^7+45a^2b^8x^8+10ab^9x^9+b^{10}x^{10}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/(a + b*x)^(5/2),x]')
```

```
[Out] 2 Sqrt[a] (128 a ^ 12 (-1 + Sqrt[(a + b x) / a]) + 64 a ^ 11 b x (-20 + 19
Sqrt[(a + b x) / a]) + 16 a ^ 10 b ^ 2 x ^ 2 (-360 + 323 Sqrt[(a + b x) / a
]) + 40 a ^ 9 b ^ 3 x ^ 3 (-384 + 323 Sqrt[(a + b x) / a]) + 5 a ^ 8 b ^ 4
x ^ 4 (-5376 + 4199 Sqrt[(a + b x) / a]) + 3 b ^ 5 x ^ 5 (-10752 a ^ 7 + b
^ 7 x ^ 7 Sqrt[(a + b x) / a]) + 4 a b ^ 5 x ^ 5 (5774 a ^ 6 Sqrt[(a + b x)
 / a] + 4333 a ^ 5 b x Sqrt[(a + b x) / a] + 2174 a ^ 4 b ^ 2 x ^ 2 Sqrt[(a
 + b x) / a] - 32 a b ^ 5 x ^ 5 + 17 a b ^ 5 x ^ 5 Sqrt[(a + b x) / a] + 4
b ^ 6 x ^ 6 Sqrt[(a + b x) / a]) - 1920 a ^ 4 b ^ 6 x ^ 6 (14 a ^ 2 + 8 a b
 x + 3 b ^ 2 x ^ 2) + 10 a ^ 3 b ^ 8 x ^ 8 (277 a Sqrt[(a + b x) / a] - 128
 b x + 52 b x Sqrt[(a + b x) / a])) / (15 b ^ 5 (a ^ 10 + 10 a ^ 9 b x + 45
 a ^ 8 b ^ 2 x ^ 2 + 120 a ^ 7 b ^ 3 x ^ 3 + 210 a ^ 6 b ^ 4 x ^ 4 + 252 a
^ 5 b ^ 5 x ^ 5 + 210 a ^ 4 b ^ 6 x ^ 6 + 120 a ^ 3 b ^ 7 x ^ 7 + 45 a ^ 2
b ^ 8 x ^ 8 + 10 a b ^ 9 x ^ 9 + b ^ 10 x ^ 10))
```

Maple [A]
time = 0.12, size = 62, normalized size = 0.71

method	result	size
gosper	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,b^3x^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}\,b^5}$	54
trager	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,b^3x^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}\,b^5}$	54
risch	$\frac{2(3x^2b^2-14abx+73a^2)\sqrt{bx+a}}{15b^5}+\frac{2a^3(12bx+11a)}{3b^5(bx+a)^{\frac{3}{2}}}$	56
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}+12a^2\sqrt{bx+a}-\frac{2a^4}{3(bx+a)^{\frac{3}{2}}}+\frac{8a^3}{\sqrt{bx+a}}}{b^5}$	62
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}+12a^2\sqrt{bx+a}-\frac{2a^4}{3(bx+a)^{\frac{3}{2}}}+\frac{8a^3}{\sqrt{bx+a}}}{b^5}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/5*(b*x+a)^(5/2)-4/3*a*(b*x+a)^(3/2)+6*a^2*(b*x+a)^(1/2)-1/3*a^4/(b
*x+a)^(3/2)+4*a^3/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a}{3\,b^5}+\frac{12\,\sqrt{bx+a}\,a^2}{b^5}+\frac{8\,a^3}{\sqrt{bx+a}\,b^5}-\frac{2\,a^4}{3\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^5 - 8/3*(b*x + a)^(3/2)*a/b^5 + 12*sqrt(b*x + a)*a^2/
b^5 + 8*a^3/(sqrt(b*x + a)*b^5) - 2/3*a^4/((b*x + a)^(3/2)*b^5)
```

Fricas [A]
time = 0.31, size = 74, normalized size = 0.85

$$\frac{2\,(3\,b^4x^4-8\,ab^3x^3+48\,a^2b^2x^2+192\,a^3bx+128\,a^4)\sqrt{bx+a}}{15\,(b^7x^2+2\,ab^6x+a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3456 vs. 2(83) = 166.
time = 2.15, size = 3456, normalized size = 39.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(5/2),x)
```

```
[Out] 256*a**(85/2)*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(85/2)/(15*a**40*
b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150
*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**
33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**1
5*x**10) + 2432*a**(83/2)*b*x*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
```

3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**
(83/2)*b*x/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a
37*b8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*
b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14
*x**9 + 15*a**30*b**15*x**10) + 10336*a**(81/2)*b**2*x**2*sqrt(1 + b*x/a)/(
15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x*
*3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 +
1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a
30*b15*x**10) - 11520*a**(81/2)*b**2*x**2/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 25840*a*
*(79/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a
38*b7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b
10*x5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13
*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(79/2)*b**3
*x**3/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*
b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11
*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9
+ 15*a**30*b**15*x**10) + 41990*a**(77/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a*
*40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 +
3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800
*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*
b**15*x**10) - 53760*a**(77/2)*b**4*x**4/(15*a**40*b**5 + 150*a**39*b**6*x
+ 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*
a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**3
2*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 46192*a**(75/
2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*
b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*
x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8
+ 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 64512*a**(75/2)*b**5*x**5
/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*
x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6
+ 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15
*a**30*b**15*x**10) + 34664*a**(73/2)*b**6*x**6*sqrt(1 + b*x/a)/(15*a**40*b
5 + 150*a39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*
a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**3
3*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15
*x**10) - 53760*a**(73/2)*b**6*x**6/(15*a**40*b**5 + 150*a**39*b**6*x + 675
*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35
*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**
13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 17392*a**(71/2)*b*
*7*x**7*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*
x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5

```
+ 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 15
0*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(71/2)*b**7*x**7/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10) + 5540*a**(69/2)*b**8*x**8*sqrt(1 + b*x/a)/(15*a**40*b**5 +
150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*
b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**1
2*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10
) - 11520*a**(69/2)*b**8*x**8/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 1040*a**(67/2)*b**9*x**9
*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 +
1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*
a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31
*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**(67/2)*b**9*x**9/(15*a**40*b*
*5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a
**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33
*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*
x**10) + 136*a**(65/2)*b**10*x**10*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**
39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x*
*4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7
+ 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256
*a**(65/2)*b**10*x**10/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x
**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 +
 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150
*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 32*a**(63/2)*b**11*x**11*sqrt(1
 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a*
*37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b
**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*
x**9 + 15*a**30*b**15*x**10) + 6*a**(61/2)*b**12*x**12*sqrt(1 + b*x/a)/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10)
```

Giac [A]

time = 0.00, size = 109, normalized size = 1.25

$$2\left(\frac{\frac{1}{5}\sqrt{a+bx}\,(a+bx)^2b^{20}-\frac{4}{3}\sqrt{a+bx}\,(a+bx)ab^{20}+6\sqrt{a+bx}\,a^2b^{20}}{b^{25}}+\frac{12\,(a+bx)\,a^3-a^4}{3b^5\sqrt{a+bx}\,(a+bx)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x)
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

Mupad [B]
time = 0.05, size = 68, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5}+\frac{8\,a^3\,\left(a+b\,x\right)-\frac{2\,a^4}{3}}{b^5\,{\left(a+b\,x\right)}^{3/2}}+\frac{12\,a^2\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{3/2}}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^5) + (8*a^3*(a + b*x) - (2*a^4)/3)/(b^5*(a + b*x)^
(3/2)) + (12*a^2*(a + b*x)^(1/2))/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5)
```

3.352 $\int \frac{x^3}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=68

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

```
[Out] 2/3*a^3/b^4/(b*x+a)^(3/2)+2/3*(b*x+a)^(3/2)/b^4-6*a^2/b^4/(b*x+a)^(1/2)-6*a
*(b*x+a)^(1/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[a
 + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{5/2}} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{5/2}}+\frac{3a^2}{b^3(a+bx)^{3/2}}-\frac{3a}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^3}\right) dx\\ &= \frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.69

$$\frac{2\left(a^3-9a^2(a+bx)-9a(a+bx)^2+(a+bx)^3\right)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(a^3 - 9*a^2*(a + b*x) - 9*a*(a + b*x)^2 + (a + b*x)^3))/(3*b^4*(a + b*x
)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.19, size = 55, normalized size = 0.81

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-16a^3-24a^2bx-6ab^2x^2+b^3x^3\right)}{3b^4\left(a+bx\right)^{\frac{3}{2}}},b!{=}0\right\}\right\},\frac{x^4}{4a^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(a + b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (-16 a ^ 3 - 24 a ^ 2 b x - 6 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) /
(3 b ^ 4 (a + b x) ^ (3 / 2)), b != 0}}, x ^ 4 / (4 a ^ (5 / 2))]
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.74

method	result	size
trager	$-\frac{2(bx+2a)\left(-x^2b^2+8abx+8a^2\right)}{3b^4(bx+a)^{\frac{3}{2}}}$	39
gosper	$-\frac{2\left(-b^3x^3+6a\,b^2x^2+24a^2bx+16a^3\right)}{3(bx+a)^{\frac{3}{2}}b^4}$	43
risch	$-\frac{2(-bx+8a)\sqrt{bx+a}}{3b^4}-\frac{2a^2(9bx+8a)}{3b^4(bx+a)^{\frac{3}{2}}}$	45
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-6a\sqrt{bx+a}+\frac{2a^3}{3(bx+a)^{\frac{3}{2}}}-\frac{6a^2}{\sqrt{bx+a}}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-6a\sqrt{bx+a}+\frac{2a^3}{3(bx+a)^{\frac{3}{2}}}-\frac{6a^2}{\sqrt{bx+a}}}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/3*(b*x+a)^(3/2)-3*a*(b*x+a)^(1/2)+1/3*a^3/(b*x+a)^(3/2)-3*a^2/(b*x
+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^4}-\frac{6\,\sqrt{bx+a}\,a}{b^4}-\frac{6\,a^2}{\sqrt{bx+a}\,b^4}+\frac{2\,a^3}{3\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^4 - 6*sqrt(b*x + a)*a/b^4 - 6*a^2/(sqrt(b*x + a)*b^4)
 + 2/3*a^3/((b*x + a)^(3/2)*b^4)
```

Fricas [**A**]
time = 0.31, size = 62, normalized size = 0.91

$$\frac{2\left(b^{3}x^{3}-6\,ab^{2}x^{2}-24\,a^{2}bx-16\,a^{3}\right)\sqrt{bx+a}}{3\left(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

Sympy [**A**]
time = 0.41, size = 163, normalized size = 2.40

$$\begin{cases} -\frac{32a^{3}}{3ab^{4}\sqrt{a+bx}+3b^{5}x\sqrt{a+bx}}-\frac{48a^{2}bx}{3ab^{4}\sqrt{a+bx}+3b^{5}x\sqrt{a+bx}}-\frac{12ab^{2}x^{2}}{3ab^{4}\sqrt{a+bx}+3b^{5}x\sqrt{a+bx}}+\frac{2b^{3}x^{3}}{3ab^{4}\sqrt{a+bx}+3b^{5}x\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**3/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48*
a**2*b*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*a*b**2*x**2
/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*b**3*x**3/(3*a*b**4*
sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), (x**4/(4*a**(5/2)), Tru
e))
```

Giac [**A**]
time = 0.00, size = 83, normalized size = 1.22

$$2\left(\frac{\frac{1}{3}\sqrt{a+bx}\,\left(a+bx\right)b^{8}-3\sqrt{a+bx}\,ab^{8}}{b^{12}}+\frac{-9\left(a+bx\right)a^{2}+a^{3}}{3b^{4}\sqrt{a+bx}\,\left(a+bx\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

Mupad [B]
time = 0.04, size = 47, normalized size = 0.69

$$-\frac{18\,a\,{\left(a+b\,x\right)}^2+18\,a^2\,\left(a+b\,x\right)-2\,{\left(a+b\,x\right)}^3-2\,a^3}{3\,b^4\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^2 + 18*a^2*(a + b*x) - 2*(a + b*x)^3 - 2*a^3)/(3*b^4*(a +
b*x)^(3/2))
```

3.353 $\int \frac{x^2}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

```
[Out] -2/3*a^2/b^3/(b*x+a)^(3/2)+4*a/b^3/(b*x+a)^(1/2)+2*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a +
b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx\\ &= -\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.71

$$\frac{2\left(8a^2+12abx+3b^2x^2\right)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.90, size = 45, normalized size = 0.92

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(8a^2+12abx+3b^2x^2\right)}{3b^3\left(a+bx\right)^{\frac{3}{2}}},b!{=}0\right\}\right\},\frac{x^3}{3a^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(a + b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (8 a ^ 2 + 12 a b x + 3 b ^ 2 x ^ 2) / (3 b ^ 3 (a + b x) ^ (
3 / 2)), b != 0}}, x ^ 3 / (3 a ^ (5 / 2))]
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.73

method	result	size
gosper	$\frac{2x^2b^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
trager	$\frac{2x^2b^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
risch	$\frac{2\sqrt{bx+a}}{b^3}+\frac{2a(6bx+5a)}{3b^3(bx+a)^{\frac{3}{2}}}$	35
derivativedivides	$\frac{2\sqrt{bx+a}+\frac{4a}{\sqrt{bx+a}}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}}}{b^3}$	36
default	$\frac{2\sqrt{bx+a}+\frac{4a}{\sqrt{bx+a}}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}}}{b^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*((b*x+a)^(1/2)+2*a/(b*x+a)^(1/2)-1/3*a^2/(b*x+a)^(3/2))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.84

$$\frac{2\sqrt{bx+a}}{b^3}+\frac{4\,a}{\sqrt{bx+a}\,b^3}-\frac{2\,a^2}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 4*a/(sqrt(b*x + a)*b^3) - 2/3*a^2/((b*x + a)^(3/2)*b^
3)
```

Fricas [A]
time = 0.31, size = 52, normalized size = 1.06

$$\frac{2\,(3\,b^2x^2 + 12\,abx + 8\,a^2)\,\sqrt{bx+a}}{3\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

Sympy [A]
time = 0.41, size = 121, normalized size = 2.47

$$\begin{cases} \frac{16a^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{24abx}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{6b^2x^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} & \text{for}\ b \neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**2/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*a
*b*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*b**2*x**2/(3*a*b
**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), (x**3/(3*a**(5/2)),
 True))
```

Giac [A]
time = 0.00, size = 53, normalized size = 1.08

$$2\left(\frac{\sqrt{a+bx}}{b^3} + \frac{6\,(a+bx)\,a - a^2}{3b^3\sqrt{a+bx}\,\,(a+bx)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

Mupad [B]
time = 0.08, size = 35, normalized size = 0.71

$$\frac{6\,{\left(a+b\,x\right)}^2+12\,a\,\left(a+b\,x\right)-2\,a^2}{3\,b^3\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(5/2),x)
[Out] (6*(a + b*x)^2 + 12*a*(a + b*x) - 2*a^2)/(3*b^3*(a + b*x)^(3/2))
```

3.354 $\int \frac{x}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=32

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

```
[Out] 2/3*a/b^2/(b*x+a)^(3/2)-2/b^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(5/2),x]
```

```
[Out] (2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{5/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx\\ &= \frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.46, size = 34, normalized size = 1.06

$$\text{Piecewise}\left[\left\{\left\{\frac{2(-2a-3bx)}{3b^2(a+bx)^{\frac{3}{2}}}, b!{=}0\right\}\right\}, \frac{x^2}{2a^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1/(a + b*x)^(5/2),x]')
[Out] Piecewise[{{2 (-2 a - 3 b x) / (3 b ^ 2 (a + b x) ^ (3 / 2)), b != 0}}, x ^
 2 / (2 a ^ (5 / 2))]
```

Maple [A]
time = 0.09, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}}b^2}$	21
trager	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}}b^2}$	21
derivativedivides	$\frac{-\frac{2}{\sqrt{bx+a}}+\frac{2a}{3(bx+a)^{\frac{3}{2}}}}{b^2}$	26
default	$\frac{-\frac{2}{\sqrt{bx+a}}+\frac{2a}{3(bx+a)^{\frac{3}{2}}}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(-1/(b*x+a)^(1/2)+1/3*a/(b*x+a)^(3/2))
```

Maxima [A]
time = 0.25, size = 26, normalized size = 0.81

$$-\frac{2}{\sqrt{bx+a}\,b^2}+\frac{2\,a}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b^2) + 2/3*a/((b*x + a)^(3/2)*b^2)
```

Fricas [A]
time = 0.30, size = 41, normalized size = 1.28

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A]
time = 0.39, size = 80, normalized size = 2.50

$$\begin{cases} -\frac{4a}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} - \frac{6bx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(5/2),x)
[Out] Piecewise((-4*a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*b*x/(
3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), (x**2/(2*a**(5
/2)), True))
```

Giac [A]
time = 0.00, size = 34, normalized size = 1.06

$$\frac{2\,(-3\,(a+bx)+a)}{b\cdot 3b\sqrt{a+bx}\,\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x)
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.62

$$-\frac{4\,a+6\,b\,x}{3\,b^2\,{(a+b\,x)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(5/2),x)
[Out] -(4*a + 6*b*x)/(3*b^2*(a + b*x)^(3/2))
```

3.355 $\int \frac{1}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=16

$$-\frac{2}{3b(a+bx)^{3/2}}$$

```
[Out] -2/3/b/(b*x+a)^(3/2)
```

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5/2),x]
```

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}} dx = -\frac{2}{3b(a+bx)^{3/2}}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5/2),x]
```

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Mathics [A]

time = 1.61, size = 12, normalized size = 0.75

$$\frac{-2}{3b\,(a+bx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^0/(a + b*x)^(5/2),x]')
```

```
[Out] -2 / (3 b (a + b x) ^ (3 / 2))
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
default	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
trager	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b/(b*x+a)^(3/2)
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.31, size = 31, normalized size = 1.94

$$-\frac{2\,\sqrt{bx+a}}{3\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.88

$$-\frac{2}{3b\left(a+bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2),x)
[Out] -2/(3*b*(a + b*x)**(3/2))
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.44

$$-\frac{2}{3b\sqrt{a+bx}\,(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x)
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$-\frac{2}{3\,b\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(5/2),x)
[Out] -2/(3*b*(a + b*x)^(3/2))
```

3.356 $\int \frac{1}{x(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x+a)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {53, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*x)^(5/2)),x]

[Out] 2/(3*a*(a + b*x)^(3/2)) + 2/(a^2*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(5/2)

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(a+bx)^{5/2}}\,dx &= \frac{2}{3a(a+bx)^{3/2}}+\frac{\int \frac{1}{x(a+bx)^{3/2}}\,dx}{a}\\
&= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a^2}\\
&= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^2b}\\
&= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.91

$$\frac{2(a+3(a+bx))}{3a^2(a+bx)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(5/2)),x]
[Out] (2*(a + 3*(a + b*x)))/(3*a^2*(a + b*x)^(3/2)) - (2*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/a^(5/2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 223 vs. 2(54) = 108.
time = 8.01, size = 205, normalized size = 3.80

$$\frac{3a^3\left(\text{Log}\left[\frac{bx}{a}\right]-2\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)+8a^3\sqrt{\frac{a+bx}{a}}+a^2bx\left(-18\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+9\text{Log}\left[\frac{bx}{a}\right]+14\sqrt{\frac{a+bx}{a}}\right)+3ab^2x^2\left(-6\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]+2\sqrt{\frac{a+bx}{a}}+3\text{Log}\left[\frac{bx}{a}\right]\right)+3b^3x^3\left(\text{Log}\left[\frac{bx}{a}\right]-2\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)}{3a^{\frac{5}{2}}\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^1*(a + b*x)^(5/2)),x]')
[Out] (3 a ^ 3 (Log[b x / a] - 2 Log[1 + Sqrt[(a + b x) / a]]) + 8 a ^ 3 Sqrt[(a
+ b x) / a] + a ^ 2 b x (-18 Log[1 + Sqrt[(a + b x) / a]] + 9 Log[b x / a]
+ 14 Sqrt[(a + b x) / a]) + 3 a b ^ 2 x ^ 2 (-6 Log[1 + Sqrt[(a + b x) / a]
```

```
] + 2 Sqrt[(a + b x) / a] + 3 Log[b x / a]) + 3 b ^ 3 x ^ 3 (Log[b x / a] -
 2 Log[1 + Sqrt[(a + b x) / a]])) / (3 a ^ (5 / 2) (a ^ 3 + 3 a ^ 2 b x + 3
 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2}{3a(bx+a)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{bx+a}}$	43
default	$\frac{2}{3a(bx+a)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{bx+a}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x+a)^
(1/2)
```

Maxima [A]
time = 0.35, size = 53, normalized size = 0.98

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) + 2/3*(3*b
*x + 4*a)/((b*x + a)^(3/2)*a^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(42) = 84.
time = 0.31, size = 177, normalized size = 3.28

$$\left[\frac{3\,(b^{2}x^{2}+2\,abx+a^{2})\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx+4\,a^{2})\sqrt{bx+a}}{3\,(a^{3}b^{2}x^{2}+2\,a^{4}bx+a^{5})},\frac{2\left(3\,(b^{2}x^{2}+2\,abx+a^{2})\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx+4\,a^{2})\sqrt{bx+a}\right)}{3\,(a^{3}b^{2}x^{2}+2\,a^{4}bx+a^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
```

```
a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(48) = 96$.
time = 1.44, size = 697, normalized size = 12.91

$$\frac{8a^7\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{3a^7\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{6a^7\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{14a^6bx\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{9a^6bx\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{18a^6bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{6a^5b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{9a^5b^2x^2\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{18a^5b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{3a^4b^3x^3\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{6a^4b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x*
*2 + 3*a**(13/2)*b**3*x**3) + 3*a**7*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*
b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**7*log(sqrt(1 +
b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(
13/2)*b**3*x**3) + 14*a**6*b*x*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b
*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*a**6*b*x*log(b*x/a)
/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) - 18*a**6*b*x*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(1
 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 9*a**5*b**2*x**2*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**5*b**2*x**2*log(s
qrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 3*a**4*b**3*x**3*log(b*x/a)/(3*a**(19/2) + 9*a**
(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x
**3*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3)
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.20

$$2\left(\frac{3(a+bx)+a}{3a^2\sqrt{a+bx}\,(a+bx)}+\frac{\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{a^2\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x)
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

Mupad [B]

time = 0.05, size = 42, normalized size = 0.78

$$\frac{\frac{2(a+bx)}{a^2}+\frac{2}{3a}}{(a+bx)^{3/2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*(a + b*x))/a^2 + 2/(3*a))/(a + b*x)^(3/2) - (2*atanh((a + b*x)^(1/2)/a^
(1/2)))/a^(5/2)
```

3.357 $\int \frac{1}{x^2(a+bx)^{5/2}} dx$

Optimal. Leaf size=74

$$-\frac{5b}{3a^2(a+bx)^{3/2}}-\frac{1}{ax(a+bx)^{3/2}}-\frac{5b}{a^3\sqrt{a+bx}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3*b/a^2/(b*x+a)^(3/2)-1/a/x/(b*x+a)^(3/2)+5*b*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(7/2)-5*b/a^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 214}

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{a+bx}}-\frac{5b}{3a^2(a+bx)^{3/2}}-\frac{1}{ax(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-5*b)/(3*a^2*(a + b*x)^(3/2)) - 1/(a*x*(a + b*x)^(3/2)) - (5*b)/(a^3*Sqrt[
a + b*x]) + (5*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(a+bx)^{5/2}}\,dx &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{5\int \frac{1}{x^2(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a^2}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{(5b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^3}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{5\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^3}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.85

$$\frac{-3a^2-20abx-15b^2x^2}{3a^3x(a+bx)^{3/2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-3*a^2 - 20*a*b*x - 15*b^2*x^2)/(3*a^3*x*(a + b*x)^(3/2)) + (5*b*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 251 vs. $2(74) = 148$.
time = 10.34, size = 231, normalized size = 3.12

$$\frac{-6a^4\sqrt{\frac{a+bx}{a}}+a^3bx\left(-46\sqrt{\frac{a+bx}{a}}-15\text{Log}\left[\frac{bx}{a}\right]+30\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)+5a^2b^2x^2\left(-14\sqrt{\frac{a+bx}{a}}-9\text{Log}\left[\frac{bx}{a}\right]+18\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)+15ab^3x^3\left(-3\text{Log}\left[\frac{bx}{a}\right]-2\sqrt{\frac{a+bx}{a}}+6\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)+15b^4x^4\left(-\text{Log}\left[\frac{bx}{a}\right]+2\text{Log}\left[1+\sqrt{\frac{a+bx}{a}}\right]\right)}{6a^{\frac{7}{2}}x\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^(5/2)),x]')
```

```
[Out] (-6 a ^ 4 Sqrt[(a + b x) / a] + a ^ 3 b x (-46 Sqrt[(a + b x) / a] - 15 Log
[b x / a] + 30 Log[1 + Sqrt[(a + b x) / a]]) + 5 a ^ 2 b ^ 2 x ^ 2 (-14 Sqr
t[(a + b x) / a] - 9 Log[b x / a] + 18 Log[1 + Sqrt[(a + b x) / a]]) + 15 a
 b ^ 3 x ^ 3 (-3 Log[b x / a] - 2 Sqrt[(a + b x) / a] + 6 Log[1 + Sqrt[(a +
 b x) / a]]) + 15 b ^ 4 x ^ 4 (-Log[b x / a] + 2 Log[1 + Sqrt[(a + b x) / a
]])) / (6 a ^ (7 / 2) x (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^
3))
```

Maple [A]
time = 0.10, size = 66, normalized size = 0.89

method	result	size
risch	$-\frac{\sqrt{bx+a}}{a^{3}x}-\frac{b\left(-\frac{10\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{8}{\sqrt{bx+a}}+\frac{4a}{3(bx+a)^{\frac{3}{2}}}\right)}{2a^{3}}$	60
derivativedivides	$2b\left(-\frac{1}{3a^{2}(bx+a)^{\frac{3}{2}}}-\frac{2}{a^{3}\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}\right)$	66
default	$2b\left(-\frac{1}{3a^{2}(bx+a)^{\frac{3}{2}}}-\frac{2}{a^{3}\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/3/a^2/(b*x+a)^(3/2)-2/a^3/(b*x+a)^(1/2)+1/a^3*(-1/2*(b*x+a)^(1/2)/b
/x+5/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.35, size = 89, normalized size = 1.20

$$-\frac{15\,(bx+a)^{2}b-10\,(bx+a)ab-2\,a^{2}b}{3\left((bx+a)^{\frac{5}{2}}a^{3}-(bx+a)^{\frac{3}{2}}a^{4}\right)}-\frac{5\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(15*(b*x + a)^2*b - 10*(b*x + a)*a*b - 2*a^2*b)/((b*x + a)^(5/2)*a^3 -
 (b*x + a)^(3/2)*a^4) - 5/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a)
+ sqrt(a)))/a^(7/2)
```

Fricas [A]
time = 0.32, size = 221, normalized size = 2.99

$$\left[\frac{15\,(b^3x^3 + 2\,ab^2x^2 + a^2bx)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right) - 2\,(15\,ab^2x^2 + 20\,a^2bx + 3\,a^3)\sqrt{bx+a}}{6\,(a^4b^2x^3 + 2\,a^5bx^2 + a^6x)}, -\frac{15\,(b^3x^3 + 2\,ab^2x^2 + a^2bx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right) + (15\,ab^2x^2 + 20\,a^2bx + 3\,a^3)\sqrt{bx+a}}{3\,(a^4b^2x^3 + 2\,a^5bx^2 + a^6x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(68) = 136.
time = 2.86, size = 818, normalized size = 11.05

$$-\frac{6a^{17}\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{46a^{16}bx\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{15a^{16}bx\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} + \frac{30a^{16}bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{70a^{15}b^2x^2\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{45a^{15}b^2x^2\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} + \frac{90a^{15}b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{30a^{14}b^3x^3\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{45a^{14}b^3x^3\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} + \frac{90a^{14}b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} - \frac{15a^{13}b^4x^4\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4} + \frac{30a^{13}b^4x^4\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x + 18a^{\frac{37}{2}}bx^2 + 18a^{\frac{35}{2}}b^2x^3 + 6a^{\frac{33}{2}}b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a**(
39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x
**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 + b*x/
a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a
**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*a
**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log(sqrt(1 + b*x
/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*
a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6*a**(39/2)*x +
18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*
a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35
/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*log(sqrt(1 + b*
```

```
x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6
*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(6*a**(39/2)*x + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**1
3*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
```

Giac [A]
time = 0.00, size = 100, normalized size = 1.35

$$2\left(-\frac{\sqrt{a+bx}\,b}{2a^3\,(a+bx-a)}-\frac{6\,(a+bx)\,b+ba}{3a^3\sqrt{a+bx}\,(a+bx)}-\frac{5b\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{2a^3\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x)
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

Mupad [B]
time = 0.11, size = 73, normalized size = 0.99

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2\,b}{3\,a}+\frac{10\,b\,(a+b\,x)}{3\,a^2}-\frac{5\,b\,(a+b\,x)^2}{a^3}}{a\,(a+b\,x)^{3/2}-(a+b\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(7/2) - ((2*b)/(3*a) + (10*b*(a + b*
x))/(3*a^2) - (5*b*(a + b*x)^2)/a^3)/(a*(a + b*x)^(3/2) - (a + b*x)^(5/2))
```

3.358 $\int \frac{1}{x^3(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=106

$$\frac{35b^2}{12a^3(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}+\frac{35b^2}{4a^4\sqrt{a+bx}}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] 35/12*b^2/a^3/(b*x+a)^(3/2)-1/2/a/x^2/(b*x+a)^(3/2)+7/4*b/a^2/x/(b*x+a)^(3/
2)-35/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(9/2)+35/4*b^2/a^4/(b*x+a)^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {44, 53, 65, 214}

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{a+bx}}+\frac{35b^2}{12a^3(a+bx)^{3/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (35*b^2)/(12*a^3*(a + b*x)^(3/2)) - 1/(2*a*x^2*(a + b*x)^(3/2)) + (7*b)/(4*
a^2*x*(a + b*x)^(3/2)) + (35*b^2)/(4*a^4*Sqrt[a + b*x]) - (35*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx)^{5/2}}\,dx &= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{7\int \frac{1}{x^3(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}+\frac{35\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{3a^2}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}-\frac{(35b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^3}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{(35b^2)\int \frac{1}{x\sqrt{a+}}}{8a^4}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{(35b)\mathrm{Subst}\left(\int\right.}{}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}-\frac{35b^2\tanh^{-1}\left(\right.}{4a^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 78, normalized size = 0.74

$$\frac{-6a^3+21a^2bx+140ab^2x^2+105b^3x^3}{12a^4x^2(a+bx)^{3/2}}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(5/2)),x]
[Out] (-6*a^3 + 21*a^2*b*x + 140*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^2*(a + b*x)^(
3/2)) - (35*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Mathics [A]
time = 12.23, size = 172, normalized size = 1.62

$$\frac{b^{\frac{3}{2}}\left(-6a^{17}x^2\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}-105a^{\frac{29}{2}}\sqrt{b}\,x^{\frac{5}{2}}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right](a+bx)^2+21a^{16}bx^3\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}-105a^{\frac{27}{2}}b^{\frac{3}{2}}x^{\frac{7}{2}}\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right](a+bx)^2+140a^{15}b^2x^4\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}+105a^{14}b^3x^5\left(\frac{a+bx}{bx}\right)^{\frac{3}{2}}\right)}{12a^{18}x^{\frac{5}{2}}(a+bx)^3}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^(5/2)),x]')
```

```
[Out] b ^ (3 / 2) (-6 a ^ 17 x ^ 2 ((a + b x) / (b x)) ^ (3 / 2) - 105 a ^ (29 /
2) Sqrt[b] x ^ (5 / 2) ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) ^ 2 +
 21 a ^ 16 b x ^ 3 ((a + b x) / (b x)) ^ (3 / 2) - 105 a ^ (27 / 2) b ^ (3
/ 2) x ^ (7 / 2) ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] (a + b x) ^ 2 + 140 a
 ^ 15 b ^ 2 x ^ 4 ((a + b x) / (b x)) ^ (3 / 2) + 105 a ^ 14 b ^ 3 x ^ 5 ((
a + b x) / (b x)) ^ (3 / 2)) / (12 a ^ 18 x ^ (5 / 2) (a + b x) ^ 3)
```

Maple [A]
time = 0.11, size = 81, normalized size = 0.76

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-11bx+2a)}{4a^4x^2}+\frac{b^2\left(-\frac{70\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{48}{\sqrt{bx+a}}+\frac{16a}{3(bx+a)^{\frac{3}{2}}}\right)}{8a^4}$	70
derivativedivides	$2b^2\left(-\frac{\frac{-\frac{11(bx+a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{35\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}+\frac{3}{a^4\sqrt{bx+a}}+\frac{1}{3a^3(bx+a)^{\frac{3}{2}}}\right)$	81
default	$2b^2\left(-\frac{\frac{-\frac{11(bx+a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{35\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}+\frac{3}{a^4\sqrt{bx+a}}+\frac{1}{3a^3(bx+a)^{\frac{3}{2}}}\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/a^4*((-11/8*(b*x+a)^(3/2)+13/8*a*(b*x+a)^(1/2))/b^2/x^2+35/8*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))+3/a^4/(b*x+a)^(1/2)+1/3/a^3/(b*x+a)^(3/
2))
```

Maxima [A]
time = 0.35, size = 123, normalized size = 1.16

$$\frac{105\,(bx+a)^3b^2-175\,(bx+a)^2ab^2+56\,(bx+a)a^2b^2+8\,a^3b^2}{12\left((bx+a)^{\frac{7}{2}}a^4-2\,(bx+a)^{\frac{5}{2}}a^5+(bx+a)^{\frac{3}{2}}a^6\right)}+\frac{35\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x + a)^3*b^2 - 175*(b*x + a)^2*a*b^2 + 56*(b*x + a)*a^2*b^2 +
8*a^3*b^2)/((b*x + a)^(7/2)*a^4 - 2*(b*x + a)^(5/2)*a^5 + (b*x + a)^(3/2)*a
^6) + 35/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(
9/2)
```

Fricas [A]
time = 0.32, size = 255, normalized size = 2.41

$$\left[\frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4)\sqrt{bx+a}}{24\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)},\ \frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4)\sqrt{bx+a}}{12\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(99) = 198$.
time = 10.53, size = 464, normalized size = 4.38

$$-\frac{6a^{\frac{89}{2}}b^{75}x^{75}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{21a^{\frac{87}{2}}b^{76}x^{76}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{140a^{\frac{85}{2}}b^{77}x^{77}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{105a^{\frac{83}{2}}b^{78}x^{78}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{105a^{42}b^{\frac{155}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{105a^{41}b^{\frac{157}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) +
 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(87/2)*
b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(
91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/
(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(15
```

```
3/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12*a**(93/2
)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) + 1)*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a
/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a
**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**
(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.00, size = 134, normalized size = 1.26

$$2\left(-\frac{-11\sqrt{a+bx}\,(a+bx)\,b^2+13\sqrt{a+bx}\,b^2a}{8a^4\,(a+bx-a)^2}+\frac{9\,(a+bx)\,b^2+b^2a}{3a^4\sqrt{a+bx}\,(a+bx)}+\frac{35b^2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{4a^4\cdot 2\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x)
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

Mupad [B]
time = 0.12, size = 105, normalized size = 0.99

$$\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,(a+b\,x)^2}{12\,a^3}+\frac{35\,b^2\,(a+b\,x)^3}{4\,a^4}+\frac{14\,b^2\,(a+b\,x)}{3\,a^2}}{(a+b\,x)^{7/2}-2\,a\,(a+b\,x)^{5/2}+a^2\,(a+b\,x)^{3/2}}-\frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*b^2)/(3*a) - (175*b^2*(a + b*x)^2)/(12*a^3) + (35*b^2*(a + b*x)^3)/(4*a
^4) + (14*b^2*(a + b*x))/(3*a^2))/((a + b*x)^(7/2) - 2*a*(a + b*x)^(5/2) +
a^2*(a + b*x)^(3/2)) - (35*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(9/2))
```

3.359 $\int \frac{1}{x\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {65, 211}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.39, size = 46, normalized size = 1.84

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^1*Sqrt[-a + b*x]),x]')
[Out] Piecewise[{{2 I ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a], Abs[a / (b
x)] > 1}}, -2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a]]
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
default	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.34, size = 19, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

Fricas [A]
time = 0.32, size = 58, normalized size = 2.32

$$\left[-\frac{\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)}{a}, \frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x)/a, 2*arctan(sqrt(b
*x - a)/sqrt(a))/sqrt(a)]
```

Sympy [A]
time = 0.51, size = 54, normalized size = 2.16

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1),
 (-2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), True))
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.04

$$\frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x)
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/2)),x)
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2)
```

3.360 $\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{-a+bx}}{ax}+\frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)+(b*x-a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {44, 65, 211}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}}{ax} + \frac{b\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.00

$$\frac{\sqrt{-a+bx}}{ax} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.22, size = 117, normalized size = 2.66

$$\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{b}\sqrt{-1+\frac{a}{bx}}}{a\sqrt{x}} + \frac{Ib\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{a^{\frac{3}{2}}}, \text{Abs}\left[\frac{a}{bx}\right] > 1\right\}\right\}, -\frac{b\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{a^{\frac{3}{2}}} - \frac{1}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}} + \frac{\sqrt{b}}{a\sqrt{x}\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*Sqrt[-a + b*x]),x]')
```

```
[Out] Piecewise[{{I Sqrt[b] Sqrt[-1 + a / (b x)] / (a Sqrt[x]) + I b ArcCosh[Sqrt
[a] / (Sqrt[b] Sqrt[x])] / a ^ (3 / 2), Abs[a / (b x)] > 1}}, -b ArcSin[Sqr
t[a] / (Sqrt[b] Sqrt[x])] / a ^ (3 / 2) - 1 / (Sqrt[b] x ^ (3 / 2) Sqrt[1 -
 a / (b x)]) + Sqrt[b] / (a Sqrt[x] Sqrt[1 - a / (b x)])]
```

Maple [A]
time = 0.13, size = 44, normalized size = 1.00

method	result	size

derivativedivides	$2b\left(\frac{\sqrt{bx-a}}{2abx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	44
default	$2b\left(\frac{\sqrt{bx-a}}{2abx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	44
risch	$-\frac{-bx+a}{ax\sqrt{bx-a}}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/2*(b*x-a)^(1/2)/a/b/x+1/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.34, size = 46, normalized size = 1.05

$$\frac{\sqrt{bx-a}\,b}{(bx-a)a+a^2}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x - a)*b/((b*x - a)*a + a^2) + b*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/
2)
```

Fricas [A]
time = 0.33, size = 97, normalized size = 2.20

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\sqrt{bx-a}\,a}{2a^2x},\ \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*sqrt(
b*x - a)*a)/(a^2*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) + sqrt(b*x
- a)*a)/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.18, size = 121, normalized size = 2.75

$$\begin{cases} \frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a\sqrt{x}} + \frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{1}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{\sqrt{b}}{a\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(b)*sqrt(a/(b*x) - 1)/(a*sqrt(x)) + I*b*acosh(sqrt(a)/(sqr
t(b)*sqrt(x)))/a**(3/2), Abs(a/(b*x)) > 1), (-1/(sqrt(b)*x**(3/2)*sqrt(-a/(
b*x) + 1)) + sqrt(b)/(a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(
b)*sqrt(x)))/a**(3/2), True))
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.43

$$\frac{2\left(\frac{\sqrt{-a+bx}\,b^2}{2a(-a+bx+a)} + \frac{b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a\cdot 2\sqrt{a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x)
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + sqrt(b*x - a)*b/(a*x))/b
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.82

$$\frac{\sqrt{b\,x-a}}{a\,x} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/2)),x)
```

```
[Out] (b*x - a)^(1/2)/(a*x) + (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2)
```

3.361 $\int \frac{1}{x^3\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

```
[Out] 3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(b*x-a)^(1/2)/a/x^2+3/4*b
*(b*x-a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {44, 65, 211}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{bx-a}}{4a^2x}+\frac{\sqrt{bx-a}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(2*a*x^2) + (3*b*Sqrt[-a + b*x])/(4*a^2*x) + (3*b^2*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{-a+bx}}\,dx &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{(3b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{(3b^2)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^2}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{(3b)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a^2}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 0.81

$$\frac{\sqrt{-a+bx}\,(2a+3bx)}{4a^2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (Sqrt[-a + b*x]*(2*a + 3*b*x))/(4*a^2*x^2) + (3*b^2*ArcTan[Sqrt[-a + b*x]/S
qrt[a]])/(4*a^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 95.10, size = 210, normalized size = 2.84

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(2a^{\frac{11}{2}}x\,(a-bx)+a^{\frac{9}{2}}bx^2\,(a-bx)-3a^{\frac{7}{2}}b^2x^3\,(a-bx)+3a^3b^{\frac{7}{2}}x^{\frac{9}{2}}\text{ArcCosh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}\right)}{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^{\frac{9}{2}}\left(\frac{a-bx}{bx}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-3b^2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{4a^{\frac{5}{2}}}-\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{1-\frac{a}{bx}}}-\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*Sqrt[-a + b*x]),x]')
```

```
[Out] Piecewise[{{I / 4 (2 a ^ (11 / 2) x (a - b x) + a ^ (9 / 2) b x ^ 2 (a - b
x) - 3 a ^ (7 / 2) b ^ 2 x ^ 3 (a - b x) + 3 a ^ 3 b ^ (7 / 2) x ^ (9 / 2)
ArcCosh[Sqrt[a] / (Sqrt[b] Sqrt[x])] ((a - b x) / (b x)) ^ (3 / 2)) / (a ^
(11 / 2) b ^ (3 / 2) x ^ (9 / 2) ((a - b x) / (b x)) ^ (3 / 2)), Abs[a / (b
 x)] > 1}}, -3 b ^ 2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] / (4 a ^ (5 / 2))
- 1 / (2 Sqrt[b] x ^ (5 / 2) Sqrt[1 - a / (b x)]) - Sqrt[b] / (4 a x ^ (3 /
```

```
2) Sqrt[1 - a / (b x)]) + 3 b ^ (3 / 2) / (4 a ^ 2 Sqrt[x] Sqrt[1 - a / (b
x)])]
```

Maple [A]
time = 0.09, size = 72, normalized size = 0.97

method	result	size
risch	$-\frac{(-bx+a)(3bx+2a)}{4a^2x^2\sqrt{bx-a}}+\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	55
derivativedivides	$2b^2\left(\frac{\sqrt{bx-a}}{4a\,b^2x^2}+\frac{\frac{3\sqrt{bx-a}}{8abx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}}{a}\right)$	72
default	$2b^2\left(\frac{\sqrt{bx-a}}{4a\,b^2x^2}+\frac{\frac{3\sqrt{bx-a}}{8abx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}}{a}\right)$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*b^2*(1/4*(b*x-a)^(1/2)/a/b^2/x^2+3/4/a*(1/2*(b*x-a)^(1/2)/a/b/x+1/2*arcta
n((b*x-a)^(1/2)/a^(1/2))/a^(3/2)))
```

Maxima [A]
time = 0.35, size = 86, normalized size = 1.16

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}+\frac{3\,(bx-a)^{\frac{3}{2}}b^2+5\,\sqrt{bx-a}\,ab^2}{4\,\left((bx-a)^2a^2+2\,(bx-a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="maxima")
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 1/4*(3*(b*x - a)^(3/2)*b^2
+ 5*sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a^4)
```

Fricas [A]
time = 0.32, size = 128, normalized size = 1.73

$$\left[-\frac{3\,\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(3\,abx+2\,a^2)\sqrt{bx-a}}{8\,a^3x^2},\frac{3\,\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(3\,abx+2\,a^2)\sqrt{bx-a}}{4\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2
*(3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(
sqrt(b*x - a)/sqrt(a)) + (3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2)]
```

Sympy [A]
time = 2.57, size = 216, normalized size = 2.92

$$\begin{cases} \frac{i}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{i\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{3ib^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}} + \frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(4*a*x**(3/
2)*sqrt(a/(b*x) - 1)) - 3*I*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) - 1)) + 3
*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), Abs(a/(b*x)) > 1), (
-1/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - sqrt(b)/(4*a*x**(3/2)*sqrt(-a/
(b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) - 3*b**2*asin(
sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), True))
```

Giac [A]
time = 0.00, size = 97, normalized size = 1.31

$$\frac{2\left(\frac{3\sqrt{-a+bx}\,(-a+bx)b^3+5\sqrt{-a+bx}\,ab^3}{8a^2(-a+bx+a)^2} + \frac{3b^3\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^2\cdot 2\sqrt{a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x)
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + (3*(b*x - a)^(3/2)*b^3 +
 5*sqrt(b*x - a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.05, size = 57, normalized size = 0.77

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{5/2}} + \frac{5\,\sqrt{b\,x-a}}{4\,a\,x^2} + \frac{3\,{\left(b\,x-a\right)}^{3/2}}{4\,a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/2)),x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*(b*x - a)^(1/2))/(4*
a*x^2) + (3*(b*x - a)^(3/2))/(4*a^2*x^2)
```

3.362 $\int \frac{1}{x(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2}{a\sqrt{-a+bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 65, 211}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{-a+bx}} - \frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{ab}$$
$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2}{a\sqrt{-a+bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(3/2)),x]
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.47, size = 311, normalized size = 7.40

$$\text{Piecewise}\left[\left\{\left\{\frac{a\left(-2I\text{Log}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]+I\text{Log}\left[\frac{bx}{a}\right]+2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]+2\sqrt{\frac{-a+bx}{a}}\right)+bx\left(-2\text{ArcSin}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]-I\text{Log}\left[\frac{bx}{a}\right]+2I\text{Log}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\right)}{a^{\frac{3}{2}}(a-bx)}, \text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\}, \frac{-2Ia^3\sqrt{1-\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{Ia^3\text{Log}\left[\frac{bx}{a}\right]}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{I2a^3\text{Log}\left[1+\sqrt{1-\frac{bx}{a}}\right]}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{\text{Pi}a^3}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} - \frac{2Ia^2bx\text{Log}\left[1+\sqrt{1-\frac{bx}{a}}\right]}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{Ia^2bx\text{Log}\left[\frac{bx}{a}\right]}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} + \frac{\text{Pi}a^2bx}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^1*(-a + b*x)^(3/2)),x]')
[Out] Piecewise[{{(a (-2 I Log[Sqrt[b] Sqrt[x] / Sqrt[a]] + I Log[b x / a] + 2 Ar
cSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] + 2 Sqrt[(-a + b x) / a]) + b x (-2 ArcSi
n[Sqrt[a] / (Sqrt[b] Sqrt[x])] - I Log[b x / a] + 2 I Log[Sqrt[b] Sqrt[x] /
 Sqrt[a]])) / (a ^ (3 / 2) (a - b x)), Abs[b x / a] > 1}}, -2 I a ^ 3 Sqrt[
1 - b x / a] / (-a ^ (9 / 2) + a ^ (7 / 2) b x) - I a ^ 3 Log[b x / a] / (-
a ^ (9 / 2) + a ^ (7 / 2) b x) + I 2 a ^ 3 Log[1 + Sqrt[1 - b x / a]] / (-a
 ^ (9 / 2) + a ^ (7 / 2) b x) - Pi a ^ 3 / (-a ^ (9 / 2) + a ^ (7 / 2) b x)
 - 2 I a ^ 2 b x Log[1 + Sqrt[1 - b x / a]] / (-a ^ (9 / 2) + a ^ (7 / 2) b
 x) + I a ^ 2 b x Log[b x / a] / (-a ^ (9 / 2) + a ^ (7 / 2) b x) + Pi a ^
2 b x / (-a ^ (9 / 2) + a ^ (7 / 2) b x)]
```

Maple [A]
time = 0.09, size = 35, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{a\sqrt{bx-a}}$	35
default	$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{a\sqrt{bx-a}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

Maxima [A]
time = 0.35, size = 34, normalized size = 0.81

$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{\sqrt{bx-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

Fricas [A]
time = 0.31, size = 124, normalized size = 2.95

$$\left[-\frac{(bx-a)\sqrt{-a}\,\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}\,a}{a^2bx-a^3},\,-\frac{2\left((bx-a)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a\right)}{a^2bx-a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="fricas")
[Out] [-((b*x - a)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqr
t(b*x - a)*a)/(a^2*b*x - a^3), -2*((b*x - a)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + sqrt(b*x - a)*a)/(a^2*b*x - a^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.00, size = 437, normalized size = 10.40

$$\begin{cases} -\frac{2a^3\sqrt{-1+\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{2ia^3\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2ia^2bx\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{2a^2bx\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2ia^3\sqrt{1-\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{2ia^3\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{\pi a^3}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2ia^2bx\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{\pi a^2bx}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(-1 + b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*log
(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(b)*sqrt(x)/sqrt(a))/
(-a**(9/2) + a**(7/2)*b*x) - 2*a**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9
/2) + a**(7/2)*b*x) + I*a**2*b*x*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*
I*a**2*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-a**(9/2) + a**(7/2)*b*x) + 2*a**2
*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9/2) + a**(7/2)*b*x), Abs(b*x/a)
 > 1), (-2*I*a**3*sqrt(1 - b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*log(b
*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(1 - b*x/a) + 1)/(-a**(
9/2) + a**(7/2)*b*x) - pi*a**3/(-a**(9/2) + a**(7/2)*b*x) + I*a**2*b*x*log(
b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*I*a**2*b*x*log(sqrt(1 - b*x/a) + 1)/(
-a**(9/2) + a**(7/2)*b*x) + pi*a**2*b*x/(-a**(9/2) + a**(7/2)*b*x), True))
```

Giac [**A**]
time = 0.00, size = 48, normalized size = 1.14

$$-2\left(\frac{1}{a\sqrt{-a+bx}}+\frac{2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a\cdot 2\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x)
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

Mupad [**B**]
time = 0.10, size = 34, normalized size = 0.81

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\,\sqrt{b\,x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(3/2)),x)
```

```
[Out] - (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(b*x - a)^(1/2))
```

3.363 $\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{3b}{a^2\sqrt{-a+bx}}+\frac{1}{ax\sqrt{-a+bx}}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)-3*b/a^2/(b*x-a)^(1/2)+1/a/x/(b*x
-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {44, 53, 65, 211}

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{bx-a}}+\frac{1}{ax\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] (-3*b)/(a^2*Sqrt[-a + b*x]) + 1/(a*x*Sqrt[-a + b*x]) - (3*b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a}\\ &= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{(3b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^2}\\ &= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{3\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^2}\\ &= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.82

$$\frac{a-3bx}{a^2x\sqrt{-a+bx}}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(3/2)),x]
[Out] (a - 3*b*x)/(a^2*x*Sqrt[-a + b*x]) - (3*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a
^(5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*(-a + b*x)^(3/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.10, size = 61, normalized size = 0.98

method	result	size
risch	$\frac{-bx+a}{a^{2}x\sqrt{bx-a}}-\frac{2b}{a^{2}\sqrt{bx-a}}-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	59
derivativedivides	$2b\left(-\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}-\frac{1}{a^{2}\sqrt{bx-a}}\right)$	61
default	$2b\left(-\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}-\frac{1}{a^{2}\sqrt{bx-a}}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/a^2*(1/2*(b*x-a)^(1/2)/b/x+3/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2
))-1/a^2/(b*x-a)^(1/2))
```

Maxima [A]
time = 0.37, size = 67, normalized size = 1.08

$$-\frac{3\,(bx-a)b+2\,ab}{(bx-a)^{\frac{3}{2}}a^{2}+\sqrt{bx-a}\,a^{3}}-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x - a)*b + 2*a*b)/((b*x - a)^(3/2)*a^2 + sqrt(b*x - a)*a^3) - 3*b*ar
ctan(sqrt(b*x - a)/sqrt(a))/a^(5/2)
```

Fricas [A]
time = 0.31, size = 164, normalized size = 2.65

$$\left[-\frac{3\,(b^{2}x^{2}-abx)\sqrt{-a}\,\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}\,-2\,a}{x}\right)+2\,(3\,abx-a^{2})\sqrt{bx-a}}{2\,(a^{3}bx^{2}-a^{4}x)},-\frac{3\,(b^{2}x^{2}-abx)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(3\,abx-a^{2})\sqrt{bx-a}}{a^{3}bx^{2}-a^{4}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^2*x^2 - a*b*x)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2
*a)/x) + 2*(3*a*b*x - a^2)*sqrt(b*x - a))/(a^3*b*x^2 - a^4*x), -(3*(b^2*x^2
 - a*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (3*a*b*x - a^2)*sqrt(b*x
- a))/(a^3*b*x^2 - a^4*x)]
```

Sympy [A]
time = 1.81, size = 156, normalized size = 2.52

$$\begin{cases} -\frac{i}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{3i\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{3ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ \frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{3b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(a**2*sq
rt(x)*sqrt(a/(b*x) - 1)) - 3*I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2),
 Abs(a/(b*x)) > 1), (1/(a*sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/
(a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) + 3*b*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/a**
(5/2), True))
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.44

$$-2\left(\frac{3(-a+bx)b+2ba}{2a^2\left(\sqrt{-a+bx}(-a+bx)+\sqrt{-a+bx}\,a\right)} + \frac{3b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2a^2\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x)
```

```
[Out] -3*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) - (3*(b*x - a)*b + 2*a*b)/(((b*x
 - a)^(3/2) + sqrt(b*x - a)*a)*a^2)
```

Mupad [B]
time = 0.06, size = 52, normalized size = 0.84

$$\frac{1}{a\,x\,\sqrt{b\,x-a}} - \frac{3\,b}{a^2\,\sqrt{b\,x-a}} - \frac{3\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(3/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(1/2)) - (3*b)/(a^2*(b*x - a)^(1/2)) - (3*b*atan((b*x - a)
^(1/2)/a^(1/2)))/a^(5/2)
```

3.364 $\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{15b^2}{4a^3\sqrt{-a+bx}}+\frac{1}{2ax^2\sqrt{-a+bx}}+\frac{5b}{4a^2x\sqrt{-a+bx}}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)-15/4*b^2/a^3/(b*x-a)^(1/2)+
1/2/a/x^2/(b*x-a)^(1/2)+5/4*b/a^2/x/(b*x-a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 211}

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15b^2}{4a^3\sqrt{bx-a}}+\frac{5b}{4a^2x\sqrt{bx-a}}+\frac{1}{2ax^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] (-15*b^2)/(4*a^3*Sqrt[-a + b*x]) + 1/(2*a*x^2*Sqrt[-a + b*x]) + (5*b)/(4*a^
2*x*Sqrt[-a + b*x]) - (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{a}\\
&= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{(15b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^2}\\
&= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{15b\sqrt{-a+bx}}{4a^3x}-\frac{(15b^2)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^3}\\
&= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{15b\sqrt{-a+bx}}{4a^3x}-\frac{(15b)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-\right.}{4a^3}\\
&= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{15b\sqrt{-a+bx}}{4a^3x}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.75

$$\frac{2a^2+5abx-15b^2x^2}{4a^3x^2\sqrt{-a+bx}}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(3/2)),x]
[Out] (2*a^2 + 5*a*b*x - 15*b^2*x^2)/(4*a^3*x^2*Sqrt[-a + b*x]) - (15*b^2*ArcTan[
Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^3*(-a + b*x)^(3/2)),x]')

[Out] Timed out

Maple [A]
time = 0.10, size = 77, normalized size = 0.81

method	result	size
risch	$\frac{(-bx+a)(7bx+2a)}{4a^3x^2\sqrt{bx-a}}-\frac{2b^2}{a^3\sqrt{bx-a}}-\frac{15b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$	72
derivativedivides	$2b^2\left(-\frac{\frac{\frac{7(bx-a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}-\frac{1}{a^3\sqrt{bx-a}}\right)$	77
default	$2b^2\left(-\frac{\frac{\frac{7(bx-a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}-\frac{1}{a^3\sqrt{bx-a}}\right)$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*b^2*(-1/a^3*((7/8*(b*x-a)^(3/2)+9/8*a*(b*x-a)^(1/2))/b^2/x^2+15/8*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2))-1/a^3/(b*x-a)^(1/2))

Maxima [A]
time = 0.36, size = 104, normalized size = 1.09

$$-\frac{15\,(bx-a)^2b^2+25\,(bx-a)ab^2+8\,a^2b^2}{4\left((bx-a)^{\frac{5}{2}}a^3+2\,(bx-a)^{\frac{3}{2}}a^4+\sqrt{bx-a}\,a^5\right)}-\frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="maxima")

[Out] -1/4*(15*(b*x - a)^2*b^2 + 25*(b*x - a)*a*b^2 + 8*a^2*b^2)/((b*x - a)^(5/2)*a^3 + 2*(b*x - a)^(3/2)*a^4 + sqrt(b*x - a)*a^5) - 15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2)

Fricas [A]
time = 0.32, size = 198, normalized size = 2.08

$$\left[-\frac{15\,(b^3x^3 - ab^2x^2)\sqrt{-a}\,\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right) + 2\,(15\,ab^2x^2 - 5\,a^2bx - 2\,a^3)\sqrt{bx-a}}{8\,(a^4bx^3 - a^5x^2)}, -\frac{15\,(b^3x^3 - ab^2x^2)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + (15\,ab^2x^2 - 5\,a^2bx - 2\,a^3)\sqrt{bx-a}}{4\,(a^4bx^3 - a^5x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a
) - 2*a)/x) + 2*(15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^
3 - a^5*x^2), -1/4*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + (15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^3 - a^
5*x^2)]
```

Sympy [A]
time = 4.05, size = 226, normalized size = 2.38

$$\begin{cases} -\frac{i}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{5i\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{15ib^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{15ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{1}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{15b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 5*I*sqrt(b)/(4*a**
2*x**(3/2)*sqrt(a/(b*x) - 1)) + 15*I*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x)
- 1)) - 15*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), Abs(a/(b*x
)) > 1), (1/(2*a*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x
**(3/2)*sqrt(-a/(b*x) + 1)) - 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(-a/(b*x) + 1
)) + 15*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), True))
```

Giac [A]
time = 0.00, size = 115, normalized size = 1.21

$$-2\left(\frac{b^2}{a^3\sqrt{-a+bx}} + \frac{7\sqrt{-a+bx}\,(-a+bx)\,b^2 + 9\sqrt{-a+bx}\,b^2a}{8a^3\,(-a+bx+a)^2} + \frac{15b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^3\cdot 2\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x)
```

```
[Out] -15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) - 2*b^2/(sqrt(b*x - a)*a^3)
 - 1/4*(7*(b*x - a)^(3/2)*b^2 + 9*sqrt(b*x - a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B]
time = 0.13, size = 101, normalized size = 1.06

$$-\frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,{\left(a-b\,x\right)}^2}{4\,a^3}-\frac{25\,b^2\,\left(a-b\,x\right)}{4\,a^2}}{2\,a\,{\left(b\,x-a\right)}^{3/2}+{\left(b\,x-a\right)}^{5/2}+a^2\,\sqrt{b\,x-a}}-\frac{15\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(3/2)),x)
```

```
[Out] - ((2*b^2)/a + (15*b^2*(a - b*x)^2)/(4*a^3) - (25*b^2*(a - b*x))/(4*a^2))/(
2*a*(b*x - a)^(3/2) + (b*x - a)^(5/2) + a^2*(b*x - a)^(1/2)) - (15*b^2*atan
((b*x - a)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

3.365 $\int \frac{1}{x(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 65, 211}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{bx-a}}-\frac{2}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*(-a + b*x)^(3/2)) + 2/(a^2*Sqrt[-a + b*x]) + (2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(-a+bx)^{5/2}}\,dx &= -\frac{2}{3a(-a+bx)^{3/2}}-\frac{\int \frac{1}{x(-a+bx)^{3/2}}\,dx}{a}\\
&= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a^2}\\
&= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^2b}\\
&= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.87

$$-\frac{8a-6bx}{3a^2(-a+bx)^{3/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] -1/3*(8*a - 6*b*x)/(a^2*(-a + b*x)^(3/2)) + (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a
]])/a^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 9.95, size = 1070, normalized size = 17.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^1*(-a + b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{(-8 a ^ 3 Sqrt[(-a + b x) / a] + 3 a ^ 3 (-2 ArcSin[Sqrt[a] / (
Sqrt[b] Sqrt[x])] - I Log[b x / a] + 2 I Log[Sqrt[b] Sqrt[x] / Sqrt[a]]) +
a ^ 2 b x (-18 I Log[Sqrt[b] Sqrt[x] / Sqrt[a]] + 9 I Log[b x / a] + 14 Sqr
t[(-a + b x) / a] + 18 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])]) + 3 a b ^ 2 x ^
```

```
2 (-6 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] - 2 Sqrt[(-a + b x) / a] - 3 I L
og[b x / a] + 6 I Log[Sqrt[b] Sqrt[x] / Sqrt[a]]) + 3 b ^ 3 x ^ 3 (-2 I Log
[Sqrt[b] Sqrt[x] / Sqrt[a]] + I Log[b x / a] + 2 ArcSin[Sqrt[a] / (Sqrt[b]
Sqrt[x])])) / (3 a ^ (5 / 2) (a ^ 3 - 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 - b ^ 3
 x ^ 3)), Abs[b x / a] > 1}}, -6 I a ^ 7 Log[1 + Sqrt[1 - b x / a]] / (-3 a
 ^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 /
 2) b ^ 3 x ^ 3) + I 3 a ^ 7 Log[b x / a] / (-3 a ^ (19 / 2) + 9 a ^ (17 /
2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) + I 8 a ^
 7 Sqrt[1 - b x / a] / (-3 a ^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 /
2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) + 3 Pi a ^ 7 / (-3 a ^ (19 / 2
) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3
x ^ 3) - 14 I a ^ 6 b x Sqrt[1 - b x / a] / (-3 a ^ (19 / 2) + 9 a ^ (17 /
2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) - 9 I a ^
 6 b x Log[b x / a] / (-3 a ^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2
) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) + I 18 a ^ 6 b x Log[1 + Sqrt[1
 - b x / a]] / (-3 a ^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2
 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) - 9 Pi a ^ 6 b x / (-3 a ^ (19 / 2) +
9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^
3) - 18 I a ^ 5 b ^ 2 x ^ 2 Log[1 + Sqrt[1 - b x / a]] / (-3 a ^ (19 / 2) +
 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^
 3) + I 6 a ^ 5 b ^ 2 x ^ 2 Sqrt[1 - b x / a] / (-3 a ^ (19 / 2) + 9 a ^ (1
7 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) + I 9
 a ^ 5 b ^ 2 x ^ 2 Log[b x / a] / (-3 a ^ (19 / 2) + 9 a ^ (17 / 2) b x - 9
 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3) + 9 Pi a ^ 5 b ^ 2
x ^ 2 / (-3 a ^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2
+ 3 a ^ (13 / 2) b ^ 3 x ^ 3) - 3 I a ^ 4 b ^ 3 x ^ 3 Log[b x / a] / (-3 a
^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 /
2) b ^ 3 x ^ 3) + I 6 a ^ 4 b ^ 3 x ^ 3 Log[1 + Sqrt[1 - b x / a]] / (-3 a
^ (19 / 2) + 9 a ^ (17 / 2) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 /
2) b ^ 3 x ^ 3) - 3 Pi a ^ 4 b ^ 3 x ^ 3 / (-3 a ^ (19 / 2) + 9 a ^ (17 / 2
) b x - 9 a ^ (15 / 2) b ^ 2 x ^ 2 + 3 a ^ (13 / 2) b ^ 3 x ^ 3)]
```

Maple [**A**]
time = 0.10, size = 49, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2}{3a(bx-a)^{\frac{3}{2}}}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{bx-a}}$	49
default	$-\frac{2}{3a(bx-a)^{\frac{3}{2}}}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{bx-a}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

Maxima [A]
time = 0.38, size = 42, normalized size = 0.70

$$\frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

Fricas [A]
time = 0.32, size = 182, normalized size = 3.03

$$\left[-\frac{3\,(b^{2}x^{2}-2\,abx+a^{2})\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(3\,abx-4\,a^{2})\sqrt{bx-a}}{3\,(a^{3}b^{2}x^{2}-2\,a^{4}bx+a^{5})},\ \frac{2\left(3\,(b^{2}x^{2}-2\,abx+a^{2})\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(3\,abx-4\,a^{2})\sqrt{bx-a}\right)}{3\,(a^{3}b^{2}x^{2}-2\,a^{4}bx+a^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt
(-a) - 2*a)/x) - 2*(3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*
x + a^5), 2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqr
t(a)) + (3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*x + a^5)]
```

Sympy [C] Result contains complex when optimal does not.
time = 104.42, size = 1950, normalized size = 32.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(
15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(b*x/a)/(-3*a**(19/2
) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*I*
a**7*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(1
5/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**7*asin(sqrt(a)/(sqrt(b)*sqrt
(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)
*b**3*x**3) - 14*a**6*b*x*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x
- 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log(b*x/a)/
(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
```

```
x**3) + 18*I*a**6*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17
/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**6*b*x*asin
(sqrt(a)/(sqrt(b)*sqrt(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(-1 + b*x/a)/(-3*a
**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3)
 + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15
/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*a**5*b**2*x**2*log(sqrt(b)*sq
rt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*
a**(13/2)*b**3*x**3) + 18*a**5*b**2*x**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-
3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x*
*3) - 3*I*a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**
(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**4*b**3*x**3*log(sqrt(b)*
sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 +
3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(
-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x
**3), Abs(b*x/a) > 1), (8*I*a**7*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2
)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(b*x/a
)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**
3*x**3) - 6*I*a**7*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x
 - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*pi*a**7/(-3*a**(19/2)
 + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 14*I*
a**6*b*x*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2
*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log(b*x/a)/(-3*a**(19/2) + 9*
a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 18*I*a**6*
b*x*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*
b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*pi*a**6*b*x/(-3*a**(19/2) + 9*a**(17
/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**5*b**2*x*
*2*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2) + 9*
a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*a**5*
b**2*x**2*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(
15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*pi*a**5*b**2*x**2/(-3*a**(19/2
) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 3*I*
a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**
2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**4*b**3*x**3*log(sqrt(1 - b*x/a) +
1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b*
*3*x**3) - 3*pi*a**4*b**3*x**3/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2
)*b**2*x**2 + 3*a**(13/2)*b**3*x**3), True))
```

Giac [**A**]
time = 0.00, size = 68, normalized size = 1.13

$$2\left(\frac{3\left(-a+bx\right)-a}{3a^{2}\sqrt{-a+bx}\,\left(-a+bx\right)}+\frac{\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{2}\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x)
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

Mupad [B]
time = 0.09, size = 48, normalized size = 0.80

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,\left(a-b\,x\right)}{a^2}+\frac{2}{3\,a}}{{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(5/2)),x)
```

```
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a - b*x))/a^2 + 2/(3*a))/(
b*x - a)^(3/2)
```

3.366 $\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{5b}{3a^2(-a+bx)^{3/2}}+\frac{1}{ax(-a+bx)^{3/2}}+\frac{5b}{a^3\sqrt{-a+bx}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3*b/a^2/(b*x-a)^(3/2)+1/a/x/(b*x-a)^(3/2)+5*b*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(7/2)+5*b/a^3/(b*x-a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 211}

$$\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{bx-a}}-\frac{5b}{3a^2(bx-a)^{3/2}}+\frac{1}{ax(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (-5*b)/(3*a^2*(-a + b*x)^(3/2)) + 1/(a*x*(-a + b*x)^(3/2)) + (5*b)/(a^3*Sqr
t[-a + b*x]) + (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx = -\frac{2}{3ax(-a+bx)^{3/2}} - \frac{5\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx}{3a}$$

$$= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a^2}$$

$$= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{(5b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^3}$$

$$= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\cdot}\right.}{a^3}$$

$$= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 0.83

$$\frac{3a^2-20abx+15b^2x^2}{3a^3x(-a+bx)^{3/2}} + \frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (3*a^2 - 20*a*b*x + 15*b^2*x^2)/(3*a^3*x*(-a + b*x)^(3/2)) + (5*b*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 12.20, size = 1219, normalized size = 15.05

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*(-a + b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{(6 a ^ 4 Sqrt[(-a + b x) / a] + a ^ 3 b x (-46 Sqrt[(-a + b x)
/ a] - 30 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])] - 15 I Log[b x / a] + 30 I Lo
g[Sqrt[b] Sqrt[x] / Sqrt[a]]) + 5 a ^ 2 b ^ 2 x ^ 2 (-18 I Log[Sqrt[b] Sqrt
[x] / Sqrt[a]] + 9 I Log[b x / a] + 14 Sqrt[(-a + b x) / a] + 18 ArcSin[Sqr
t[a] / (Sqrt[b] Sqrt[x])]) + 15 a b ^ 3 x ^ 3 (-6 ArcSin[Sqrt[a] / (Sqrt[b]
 Sqrt[x])] - 2 Sqrt[(-a + b x) / a] - 3 I Log[b x / a] + 6 I Log[Sqrt[b] Sq
rt[x] / Sqrt[a]]) + 15 b ^ 4 x ^ 4 (-2 I Log[Sqrt[b] Sqrt[x] / Sqrt[a]] + I
 Log[b x / a] + 2 ArcSin[Sqrt[a] / (Sqrt[b] Sqrt[x])])) / (6 a ^ (7 / 2) x
(a ^ 3 - 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 - b ^ 3 x ^ 3)), Abs[b x / a] > 1}},
 -6 I a ^ 17 Sqrt[1 - b x / a] / (-6 a ^ (39 / 2) x + 18 a ^ (37 / 2) b x ^
 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^ 4) - 30 I a ^ 1
6 b x Log[1 + Sqrt[1 - b x / a]] / (-6 a ^ (39 / 2) x + 18 a ^ (37 / 2) b x
 ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^ 4) + I 15 a ^
 16 b x Log[b x / a] / (-6 a ^ (39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a
^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^ 4) + I 46 a ^ 16 b x Sqrt
[1 - b x / a] / (-6 a ^ (39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 /
 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^ 4) + 15 Pi a ^ 16 b x / (-6 a ^
(39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^
(33 / 2) b ^ 3 x ^ 4) - 70 I a ^ 15 b ^ 2 x ^ 2 Sqrt[1 - b x / a] / (-6 a ^
 (39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^
 (33 / 2) b ^ 3 x ^ 4) - 45 I a ^ 15 b ^ 2 x ^ 2 Log[b x / a] / (-6 a ^ (39
 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33
 / 2) b ^ 3 x ^ 4) + I 90 a ^ 15 b ^ 2 x ^ 2 Log[1 + Sqrt[1 - b x / a]] / (
-6 a ^ (39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 +
 6 a ^ (33 / 2) b ^ 3 x ^ 4) - 45 Pi a ^ 15 b ^ 2 x ^ 2 / (-6 a ^ (39 / 2)
x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2)
b ^ 3 x ^ 4) - 90 I a ^ 14 b ^ 3 x ^ 3 Log[1 + Sqrt[1 - b x / a]] / (-6 a ^
 (39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^
 (33 / 2) b ^ 3 x ^ 4) + I 30 a ^ 14 b ^ 3 x ^ 3 Sqrt[1 - b x / a] / (-6 a
^ (39 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a
^ (33 / 2) b ^ 3 x ^ 4) + I 45 a ^ 14 b ^ 3 x ^ 3 Log[b x / a] / (-6 a ^ (3
9 / 2) x + 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (3
3 / 2) b ^ 3 x ^ 4) + 45 Pi a ^ 14 b ^ 3 x ^ 3 / (-6 a ^ (39 / 2) x + 18 a
^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^
 4) - 15 I a ^ 13 b ^ 4 x ^ 4 Log[b x / a] / (-6 a ^ (39 / 2) x + 18 a ^ (3
7 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^ 4)
+ I 30 a ^ 13 b ^ 4 x ^ 4 Log[1 + Sqrt[1 - b x / a]] / (-6 a ^ (39 / 2) x +
 18 a ^ (37 / 2) b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^
 3 x ^ 4) - 15 Pi a ^ 13 b ^ 4 x ^ 4 / (-6 a ^ (39 / 2) x + 18 a ^ (37 / 2)
 b x ^ 2 - 18 a ^ (35 / 2) b ^ 2 x ^ 3 + 6 a ^ (33 / 2) b ^ 3 x ^ 4)]
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.91

method	result	size
derivativedivides	$2b\left(\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}-\frac{1}{3a^{2}(bx-a)^{\frac{3}{2}}}+\frac{2}{a^{3}\sqrt{bx-a}}\right)$	74
default	$2b\left(\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}-\frac{1}{3a^{2}(bx-a)^{\frac{3}{2}}}+\frac{2}{a^{3}\sqrt{bx-a}}\right)$	74
risch	$-\frac{-bx+a}{a^{3}x\sqrt{bx-a}}+\frac{5b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{4b}{a^{3}\sqrt{bx-a}}-\frac{2b}{3a^{2}(bx-a)^{\frac{3}{2}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/a^3*(1/2*(b*x-a)^(1/2)/b/x+5/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
)-1/3/a^2/(b*x-a)^(3/2)+2/a^3/(b*x-a)^(1/2))
```

Maxima [A]
time = 0.36, size = 82, normalized size = 1.01

$$\frac{15\,(bx-a)^{2}b+10\,(bx-a)ab-2\,a^{2}b}{3\,\left((bx-a)^{\frac{5}{2}}a^{3}+(bx-a)^{\frac{3}{2}}a^{4}\right)}+\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(b*x - a)^2*b + 10*(b*x - a)*a*b - 2*a^2*b)/((b*x - a)^(5/2)*a^3 +
(b*x - a)^(3/2)*a^4) + 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2)
```

Fricas [A]
time = 0.32, size = 226, normalized size = 2.79

$$\left[-\frac{15\,(b^{3}x^{3}-2\,ab^{2}x^{2}+a^{2}bx)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(15\,ab^{2}x^{2}-20\,a^{2}bx+3\,a^{3})\sqrt{bx-a}}{6\,(a^{4}b^{2}x^{3}-2\,a^{5}bx^{2}+a^{6}x)},\ \frac{15\,(b^{3}x^{3}-2\,ab^{2}x^{2}+a^{2}bx)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(15\,ab^{2}x^{2}-20\,a^{2}bx+3\,a^{3})\sqrt{bx-a}}{3\,(a^{4}b^{2}x^{3}-2\,a^{5}bx^{2}+a^{6}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^3*x^3 - 2*a*b^2*x^2 + a^2*b*x)*sqrt(-a)*log((b*x - 2*sqrt(b*x
- a)*sqrt(-a) - 2*a)/x) - 2*(15*a*b^2*x^2 - 20*a^2*b*x + 3*a^3)*sqrt(b*x -
a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x), 1/3*(15*(b^3*x^3 - 2*a*b^2*x^2 + a
^2*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (15*a*b^2*x^2 - 20*a^2*b*x
+ 3*a^3)*sqrt(b*x - a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 101, normalized size = 1.25

$$2\left(\frac{\sqrt{-a+bx}\,b}{2a^3\left(-a+bx+a\right)}+\frac{6\left(-a+bx\right)b-ba}{3a^3\sqrt{-a+bx}\,\left(-a+bx\right)}+\frac{5b\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{2a^3\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x)
```

```
[Out] 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) + 2/3*(6*(b*x - a)*b - a*b)/((b*x
 - a)^(3/2)*a^3) + sqrt(b*x - a)/(a^3*x)
```

Mupad [B]
time = 0.12, size = 70, normalized size = 0.86

$$\frac{1}{a\,x\,{\left(b\,x-a\right)}^{3/2}}-\frac{20\,b}{3\,a^2\,{\left(b\,x-a\right)}^{3/2}}+\frac{5\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5\,b^2\,x}{a^3\,{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(5/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(3/2)) - (20*b)/(3*a^2*(b*x - a)^(3/2)) + (5*b*atan((b*x -
 a)^(1/2)/a^(1/2)))/a^(7/2) + (5*b^2*x)/(a^3*(b*x - a)^(3/2))
```

3.367 $\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{35b^2}{12a^3(-a+bx)^{3/2}}+\frac{1}{2ax^2(-a+bx)^{3/2}}+\frac{7b}{4a^2x(-a+bx)^{3/2}}+\frac{35b^2}{4a^4\sqrt{-a+bx}}+\frac{35b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] -35/12*b^2/a^3/(b*x-a)^(3/2)+1/2/a/x^2/(b*x-a)^(3/2)+7/4*b/a^2/x/(b*x-a)^(3
/2)+35/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(9/2)+35/4*b^2/a^4/(b*x-a)^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 211}

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{bx-a}}-\frac{35b^2}{12a^3(bx-a)^{3/2}}+\frac{7b}{4a^2x(bx-a)^{3/2}}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (-35*b^2)/(12*a^3*(-a + b*x)^(3/2)) + 1/(2*a*x^2*(-a + b*x)^(3/2)) + (7*b)/
(4*a^2*x*(-a + b*x)^(3/2)) + (35*b^2)/(4*a^4*Sqrt[-a + b*x]) + (35*b^2*ArcT
an[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx &= -\frac{2}{3ax^2(-a+bx)^{3/2}}-\frac{7\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx}{3a}\\ &= -\frac{2}{3ax^2(-a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{-a+bx}}+\frac{35\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{3a^2}\\ &= -\frac{2}{3ax^2(-a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{-a+bx}}+\frac{35\sqrt{-a+bx}}{6a^3x^2}+\frac{(35b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^3}\\ &= -\frac{2}{3ax^2(-a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{-a+bx}}+\frac{35\sqrt{-a+bx}}{6a^3x^2}+\frac{35b\sqrt{-a+bx}}{4a^4x}+\frac{(35b^2)}{}\\ &= -\frac{2}{3ax^2(-a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{-a+bx}}+\frac{35\sqrt{-a+bx}}{6a^3x^2}+\frac{35b\sqrt{-a+bx}}{4a^4x}+\frac{(35b)}{}\\ &= -\frac{2}{3ax^2(-a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{-a+bx}}+\frac{35\sqrt{-a+bx}}{6a^3x^2}+\frac{35b\sqrt{-a+bx}}{4a^4x}+\frac{35b^2\,\mathrm{t}}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.71

$$\frac{6a^3+21a^2bx-140ab^2x^2+105b^3x^3}{12a^4x^2(-a+bx)^{3/2}}+\frac{35b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (6*a^3 + 21*a^2*b*x - 140*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^2*(-a + b*x)^(
3/2)) + (35*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*(-a + b*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.12, size = 90, normalized size = 0.78

method	result	size
risch	$-\frac{(-bx+a)(11bx+2a)}{4a^4x^2\sqrt{bx-a}}+\frac{35b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{9}{2}}}+\frac{6b^2}{a^4\sqrt{bx-a}}-\frac{2b^2}{3a^3(bx-a)^{\frac{3}{2}}}$	89
derivativedivides	$2b^2\left(-\frac{1}{3a^3(bx-a)^{\frac{3}{2}}}+\frac{3}{a^4\sqrt{bx-a}}+\frac{\frac{\frac{11(bx-a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{35\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}\right)$	90
default	$2b^2\left(-\frac{1}{3a^3(bx-a)^{\frac{3}{2}}}+\frac{3}{a^4\sqrt{bx-a}}+\frac{\frac{\frac{11(bx-a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{35\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/3/a^3/(b*x-a)^(3/2)+3/a^4/(b*x-a)^(1/2)+1/a^4*((11/8*(b*x-a)^(3/2
)+13/8*a*(b*x-a)^(1/2))/b^2/x^2+35/8*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2))
)
```

Maxima [A]
time = 0.35, size = 121, normalized size = 1.04

$$\frac{105\,(bx-a)^3b^2+175\,(bx-a)^2ab^2+56\,(bx-a)a^2b^2-8\,a^3b^2}{12\left((bx-a)^{\frac{7}{2}}a^4+2\,(bx-a)^{\frac{5}{2}}a^5+(bx-a)^{\frac{3}{2}}a^6\right)}+\frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x - a)^3*b^2 + 175*(b*x - a)^2*a*b^2 + 56*(b*x - a)*a^2*b^2 -
8*a^3*b^2)/((b*x - a)^(7/2)*a^4 + 2*(b*x - a)^(5/2)*a^5 + (b*x - a)^(3/2)*a
^6) + 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2)
```

Fricas [A]
time = 0.33, size = 260, normalized size = 2.24

$$\left[-\frac{105\,(b^4x^4-2\,ab^3x^3+a^2b^2x^2)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4)\sqrt{bx-a}}{24\,(a^5b^2x^4-2\,a^6bx^3+a^7x^2)},\ \frac{105\,(b^4x^4-2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4)\sqrt{bx-a}}{12\,(a^5b^2x^4-2\,a^6bx^3+a^7x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*log((b*x - 2*sqr
t(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3
*b*x + 6*a^4)*sqrt(b*x - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
05*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(
a)) + (105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3*b*x + 6*a^4)*sqrt(b*x - a))
/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.20, size = 1108, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((12*I*a**(89/2)*b**75*x**75/(24*a**(93/2)*b**(151/2)*x**(155/2)*s
qrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) +
42*I*a**(87/2)*b**76*x**76/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x)
 - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 280*I*a**(8
5/2)*b**77*x**77/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24
*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**(83/2)*b**78
*x**78/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)
*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**42*b**(155/2)*x**(155/
2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151
/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(
a/(b*x) - 1)) - 105*pi*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) - 1)/(24*a*
*(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x
**(157/2)*sqrt(a/(b*x) - 1)) - 210*I*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*
x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1))
 + 105*pi*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) - 1)/(24*a**(93/2)*b**(1
51/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqr
t(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-6*a**(89/2)*b**75*x**75/(12*a**(93/2)
```

```
*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(-a/(b*x) + 1)) - 21*a**(87/2)*b**76*x**76/(12*a**(93/2)*b**(151/2)
*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a
/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/(12*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1
)) - 105*a**(83/2)*b**78*x**78/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/
(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) - 105*a
**42*b**(155/2)*x**(155/2)*sqrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b*
*(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) + 105*a**41*b**(157/2)*x**(157/2)*s
qrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*
x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/
(b*x) + 1)), True))
```

Giac [A]
time = 0.00, size = 138, normalized size = 1.19

$$2\left(\frac{11\sqrt{-a+bx}\,(-a+bx)\,b^2+13\sqrt{-a+bx}\,b^2a}{8a^4\,(-a+bx+a)^2}+\frac{9\,(-a+bx)\,b^2-b^2a}{3a^4\sqrt{-a+bx}\,(-a+bx)}+\frac{35b^2\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^4\cdot 2\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x)
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2) + 2/3*(9*(b*x - a)*b^2 - a*b
^2)/((b*x - a)^(3/2)*a^4) + 1/4*(11*(b*x - a)^(3/2)*b^2 + 13*sqrt(b*x - a)*
a*b^2)/(a^4*b^2*x^2)
```

Mupad [B]
time = 0.07, size = 117, normalized size = 1.01

$$\frac{35\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{9/2}}-\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,{\left(a-b\,x\right)}^2}{12\,a^3}+\frac{35\,b^2\,{\left(a-b\,x\right)}^3}{4\,a^4}+\frac{14\,b^2\,\left(a-b\,x\right)}{3\,a^2}}{2\,a\,{\left(b\,x-a\right)}^{5/2}+{\left(b\,x-a\right)}^{7/2}+a^2\,{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(5/2)),x)
```

```
[Out] (35*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(9/2)) - ((2*b^2)/(3*a) - (175*
b^2*(a - b*x)^2)/(12*a^3) + (35*b^2*(a - b*x)^3)/(4*a^4) + (14*b^2*(a - b*x
))/(3*a^2))/(2*a*(b*x - a)^(5/2) + (b*x - a)^(7/2) + a^2*(b*x - a)^(3/2))
```

3.368 $\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

[Out] x^m/(b*x+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {12, 75}

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]

[Out] x^m/Sqrt[a + b*x]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{(a+bx)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 35.47, size = 69, normalized size = 5.31

$$\frac{a\,(1+m)\,x^m\text{hyper}\left[\left\{\frac{3}{2},m\right\},\{1+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]+\frac{b(-1+2m)x^{1+m}\text{hyper}\left[\left\{\frac{3}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{2}}{a^{\frac{3}{2}}\,(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)
),x]')
```

```
[Out] (a (1 + m) x ^ m hyper[{3 / 2, m}, {1 + m}, b x exp_polar[I Pi] / a] + b (-
1 + 2 m) x ^ (1 + m) hyper[{3 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a
] / 2) / (a ^ (3 / 2) (1 + m))
```

Maple [A]
time = 0.09, size = 12, normalized size = 0.92

method	result	size
gosper	$\frac{x^m}{\sqrt{bx+a}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x^m/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.31, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="max
ima")
```

```
[Out] x^m/sqrt(b*x + a)
```

Fricas [A]
time = 0.31, size = 14, normalized size = 1.08

$$\frac{x x^{m-1}}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="fri
cas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x + a)
```

Sympy [C] Result contains complex when optimal does not.
time = 39.31, size = 78, normalized size = 6.00

$$\frac{mx^{m}\Gamma(m)\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}+\frac{bxx^{m}(2m-1)\,\Gamma(m+1)\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((3/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) + b*x*x**m*(2*m - 1)*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,)
, b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.41, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x*(2*m - 1)))/(2*(a + b*x)^(3/2)),x)
```

```
[Out] x^m/(a + b*x)^(1/2)
```

3.369 $\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

[Out] x^m/(b*x+a)^(1/2)

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 7.08, number of steps used = 5, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {69, 67}

$$\frac{x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{\sqrt{a+bx}}-\frac{2mx^m\sqrt{a+bx}\,\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]

[Out] (x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/((-((b*x)/a))^m*Sqrt[a + b*x]) - (2*m*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(-((b*x)/a))^m)

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)dx = -\left(\frac{1}{2}b\int \frac{x^m}{(a+bx)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx$$

$$= -\left(\frac{1}{2}\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}\,dx\right)-\frac{\left(bmx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int$$

$$= \frac{x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{\sqrt{a+bx}}-\frac{2mx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2}{a}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.38, size = 64, normalized size = 4.92

$$\frac{a\left(1+m\right)x^m\text{hyper}\left[\left\{\frac{1}{2},m\right\},\left\{1+m\right\},\frac{bx\text{exp\_polar}\left[I\text{Pi}\right]}{a}\right]-\frac{bx^{1+m}\text{hyper}\left[\left\{\frac{3}{2},1+m\right\},\left\{2+m\right\},\frac{bx\text{exp\_polar}\left[I\text{Pi}\right]}{a}\right]}{2}}{a^{\frac{3}{2}}\left(1+m\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[-((b*x^m)/(2*(a + b*x)^(3/2))) + (m*x^(-1 + m))/Sqrt[a +
 b*x],x]')
```

```
[Out] (a (1 + m) x ^ m hyper[{1 / 2, m}, {1 + m}, b x exp_polar[I Pi] / a] - b x
^ (1 + m) hyper[{3 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / 2) / (a
 ^ (3 / 2) (1 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int -\frac{b\,x^m}{2\left(bx+a\right)^{\frac{3}{2}}}+\frac{m\,x^{-1+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)

[Out] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)

Maxima [**A**]
time = 0.30, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] x^m/sqrt(b*x + a)

Fricas [**A**]
time = 0.31, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] x^m/sqrt(b*x + a)

Sympy [**C**] Result contains complex when optimal does not.
time = 2.80, size = 73, normalized size = 5.62

$$\frac{mx^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}-\frac{bxx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/2*b*x**m/(b*x+a)**(3/2)+m*x**(-1+m)/(b*x+a)**(1/2),x)

[Out] m*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*gamma(m + 1)) - b*x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x}}-\frac{b\,x^m}{2\,{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)),x)
```

```
[Out] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)), x)
```

3.370 $\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {7, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Mathics [A]
time = 1.99, size = 16, normalized size = 0.70

$$\frac{-2\text{ArcSinh}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]}{\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^((1 - n)/2 + (1/2)*(-3 + n))/Sqrt[a + b*x],x]')
[Out] -2 ArcSinh[Sqrt[a] / (Sqrt[b] Sqrt[x])] / Sqrt[a]
```

Maple [A]
time = 0.11, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.35, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.31, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A]
time = 0.49, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.17

$$\frac{2\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x)
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 0.00, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.371 $\int x^3 \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}$$

```
[Out] -3/4*a^3*(b*x+a)^(4/3)/b^4+9/7*a^2*(b*x+a)^(7/3)/b^4-9/10*a*(b*x+a)^(10/3)/
b^4+3/13*(b*x+a)^(13/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*(
a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx\\&=-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{4/3}\left(-81a^3+108a^2bx-126ab^2x^2+140b^3x^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-81*a^3 + 108*a^2*b*x - 126*a*b^2*x^2 + 140*b^3*x^3))/(
1820*b^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 342 vs. $2(72) = 144$.
time = 15.92, size = 320, normalized size = 4.44

$$\frac{3a^{\frac{1}{3}}\left(81a^{10}\left(1-\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+27a^9bx\left(18-17\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+9a^8b^2x^2\left(135-119\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^7b^3x^3\left(1620-1309\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+7a^4b^4x^4\left(-103a^2+87abx+313b^2x^2\right)\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+1215a^6b^4x^4+6a^2b^5x^5\left(81a^3+361b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^3b^6x^6\left(81a+2929bx\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+854ab^9x^9\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+140b^{10}x^{10}\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{1820b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(a + b*x)^(1/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (81 a ^ 10 (1 - ((a + b x) / a) ^ (1 / 3)) + 27 a ^ 9 b x (18
 - 17 ((a + b x) / a) ^ (1 / 3)) + 9 a ^ 8 b ^ 2 x ^ 2 (135 - 119 ((a + b x
) / a) ^ (1 / 3)) + a ^ 7 b ^ 3 x ^ 3 (1620 - 1309 ((a + b x) / a) ^ (1 / 3
)) + 7 a ^ 4 b ^ 4 x ^ 4 (-103 a ^ 2 + 87 a b x + 313 b ^ 2 x ^ 2) ((a + b
x) / a) ^ (1 / 3) + 1215 a ^ 6 b ^ 4 x ^ 4 + 6 a ^ 2 b ^ 5 x ^ 5 (81 a ^ 3
+ 361 b ^ 3 x ^ 3 ((a + b x) / a) ^ (1 / 3)) + a ^ 3 b ^ 6 x ^ 6 (81 a + 29
29 b x ((a + b x) / a) ^ (1 / 3)) + 854 a b ^ 9 x ^ 9 ((a + b x) / a) ^ (1
/ 3) + 140 b ^ 10 x ^ 10 ((a + b x) / a) ^ (1 / 3)) / (1820 b ^ 4 (a ^ 6 +
6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4
x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{4}{3}}\left(-140b^3x^3+126a\,b^2x^2-108a^2bx+81a^3\right)}{1820b^4}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{9a(bx+a)^{\frac{10}{3}}}{10}+\frac{9a^2(bx+a)^{\frac{7}{3}}}{7}-\frac{3a^3(bx+a)^{\frac{4}{3}}}{4}}{b^4}$	50
default	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{9a(bx+a)^{\frac{10}{3}}}{10}+\frac{9a^2(bx+a)^{\frac{7}{3}}}{7}-\frac{3a^3(bx+a)^{\frac{4}{3}}}{4}}{b^4}$	50
trager	$-\frac{3\left(-140b^4x^4-14a\,b^3x^3+18a^2b^2x^2-27a^3bx+81a^4\right)(bx+a)^{\frac{1}{3}}}{1820b^4}$	54
risch	$-\frac{3\left(-140b^4x^4-14a\,b^3x^3+18a^2b^2x^2-27a^3bx+81a^4\right)(bx+a)^{\frac{1}{3}}}{1820b^4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/13*(b*x+a)^(13/3)-3/10*a*(b*x+a)^(10/3)+3/7*a^2*(b*x+a)^(7/3)-1/4*
a^3*(b*x+a)^(4/3))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.78

$$\frac{3 (bx + a)^{\frac{13}{3}}}{13 b^4} - \frac{9 (bx + a)^{\frac{10}{3}} a}{10 b^4} + \frac{9 (bx + a)^{\frac{7}{3}} a^2}{7 b^4} - \frac{3 (bx + a)^{\frac{4}{3}} a^3}{4 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^4 - 9/10*(b*x + a)^(10/3)*a/b^4 + 9/7*(b*x + a)^(7/
3)*a^2/b^4 - 3/4*(b*x + a)^(4/3)*a^3/b^4
```

Fricas [A]
time = 0.31, size = 53, normalized size = 0.74

$$\frac{3 (140 b^4 x^4 + 14 a b^3 x^3 - 18 a^2 b^2 x^2 + 27 a^3 b x - 81 a^4)(bx + a)^{\frac{1}{3}}}{1820 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^4*x^4 + 14*a*b^3*x^3 - 18*a^2*b^2*x^2 + 27*a^3*b*x - 81*a^4)*
(b*x + a)^(1/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(68) = 136.
time = 1.31, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 2
7300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 1092
0*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(73/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 1377*
a**(70/3)*b*x*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27
300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920
*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1458*a**(70/3)*b*x/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 3
213*a**(67/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b
**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x*
*4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 3645*a**(67/3)*b**2*x
**2/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a
```

```
**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14
*b**10*x**6) - 3927*a**(64/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 4860*
a**(64/3)*b**3*x**3/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**
6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x
**5 + 1820*a**14*b**10*x**6) - 2163*a**(61/3)*b**4*x**4*(1 + b*x/a)**(1/3)/
(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17
*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**
10*x**6) + 3645*a**(61/3)*b**4*x**4/(1820*a**20*b**4 + 10920*a**19*b**5*x +
 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10
920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1827*a**(58/3)*b**5*x**5*(1
+ b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x*
*2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5
+ 1820*a**14*b**10*x**6) + 1458*a**(58/3)*b**5*x**5/(1820*a**20*b**4 + 1092
0*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**1
6*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6573*a**(55/
3)*b**6*x**6*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 273
00*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*
a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(55/3)*b**6*x**6/(1820*a*
*20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x*
*3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
 + 8787*a**(52/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**
19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**
8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6498*a**(49/3)*b*
*8*x**8*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a*
*18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15
*b**9*x**5 + 1820*a**14*b**10*x**6) + 2562*a**(46/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 3640
0*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a*
*14*b**10*x**6) + 420*a**(43/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(56) = 112.

time = 0.00, size = 192, normalized size = 2.67

$$\frac{\frac{3b\left(\frac{1}{13}(a+bx)^{\frac{1}{3}}(a+bx)^4-\frac{2}{5}(a+bx)^{\frac{1}{3}}(a+bx)^3a+\frac{6}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a^2-(a+bx)^{\frac{1}{3}}(a+bx)a^3+(a+bx)^{\frac{1}{3}}a^4\right)}{b^4}+\frac{3a\left(\frac{1}{10}(a+bx)^{\frac{1}{3}}(a+bx)^3-\frac{3}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a+\frac{3}{4}(a+bx)^{\frac{1}{3}}(a+bx)a^2-(a+bx)^{\frac{1}{3}}a^3\right)}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x)
```

```
[Out] 3/1820*(13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^3 + 4*(35*(b*x + a)^(13/3) - 182*(b*x
```

```
+ a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*(b*
x + a)^(1/3)*a^4)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{13/3}}{13\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{4/3}}{4\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(13/3))/(13*b^4) - (3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a
 + b*x)^(7/3))/(7*b^4) - (9*a*(a + b*x)^(10/3))/(10*b^4)
```

3.372 $\int x^2\sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}$$

```
[Out] 3/4*a^2*(b*x+a)^(4/3)/b^3-6/7*a*(b*x+a)^(7/3)/b^3+3/10*(b*x+a)^(10/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a + b
*x)^(10/3))/(10*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[3]{a+bx}\,dx&=\int\left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{4/3}\left(9a^2-12abx+14b^2x^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(9*a^2 - 12*a*b*x + 14*b^2*x^2))/(140*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 215 vs. $2(53) = 106$.
time = 7.13, size = 197, normalized size = 3.72

$$\frac{3a^{\frac{1}{3}}\left(9a^6\left(-1+\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+3a^5bx\left(-9+8\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^4b^2x^2\left(-27+20\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)-9a^3b^3x^3+20a^3b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+45a^2b^4x^4\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+44ab^5x^5\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+14b^6x^6\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{140b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(a + b*x)^(1/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (9 a ^ 6 (-1 + ((a + b x) / a) ^ (1 / 3)) + 3 a ^ 5 b x (-9 +
 8 ((a + b x) / a) ^ (1 / 3)) + a ^ 4 b ^ 2 x ^ 2 (-27 + 20 ((a + b x) / a)
 ^ (1 / 3)) - 9 a ^ 3 b ^ 3 x ^ 3 + 20 a ^ 3 b ^ 3 x ^ 3 ((a + b x) / a) ^
(1 / 3) + 45 a ^ 2 b ^ 4 x ^ 4 ((a + b x) / a) ^ (1 / 3) + 44 a b ^ 5 x ^ 5
 ((a + b x) / a) ^ (1 / 3) + 14 b ^ 6 x ^ 6 ((a + b x) / a) ^ (1 / 3)) / (1
40 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{4}{3}}\left(14x^2b^2-12abx+9a^2\right)}{140b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{6a(bx+a)^{\frac{7}{3}}}{7}+\frac{3a^2(bx+a)^{\frac{4}{3}}}{4}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{6a(bx+a)^{\frac{7}{3}}}{7}+\frac{3a^2(bx+a)^{\frac{4}{3}}}{4}}{b^3}$	38
trager	$\frac{3\left(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3\right)(bx+a)^{\frac{1}{3}}}{140b^3}$	43
risch	$\frac{3\left(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3\right)(bx+a)^{\frac{1}{3}}}{140b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/10*(b*x+a)^(10/3)-2/7*a*(b*x+a)^(7/3)+1/4*a^2*(b*x+a)^(4/3))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^3}-\frac{6\,(bx+a)^{\frac{7}{3}}a}{7\,b^3}+\frac{3\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^3 - 6/7*(b*x + a)^(7/3)*a/b^3 + 3/4*(b*x + a)^(4/3)
*a^2/b^3
```

Fricas [A]
time = 0.30, size = 42, normalized size = 0.79

$$\frac{3\,(14\,b^3x^3+2\,ab^2x^2-3\,a^2bx+9\,a^3)(bx+a)^{\frac{1}{3}}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 + 2*a*b^2*x^2 - 3*a^2*b*x + 9*a^3)*(b*x + a)^(1/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.
time = 0.89, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{27a^{\frac{34}{3}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{72a^{\frac{31}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{81a^{\frac{31}{3}}bx}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{60a^{\frac{28}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{81a^{\frac{28}{3}}b^2x^2}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{60a^{\frac{25}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{27a^{\frac{25}{3}}b^3x^3}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{135a^{\frac{22}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{132a^{\frac{19}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{42a^{\frac{16}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) - 27*a**(34/3)/(140*a**8*b**3 + 420*a**7*b
**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 72*a**(31/3)*b*x*(1 + b*
x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**
5*b**6*x**3) - 81*a**(31/3)*b*x/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(28/3)*b**2*x**2*(1 + b*x/a)**(1/3
)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**
3) - 81*a**(28/3)*b**2*x**2/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**
5*x**2 + 140*a**5*b**6*x**3) + 60*a**(25/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1
40*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) -
 27*a**(25/3)*b**3*x**3/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x*
*2 + 140*a**5*b**6*x**3) + 135*a**(22/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(140*
a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 13
2*a**(19/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x +
 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 42*a**(16/3)*b**6*x**6*(1 + b*x
/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5
*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 0.00, size = 147, normalized size = 2.77

$$\frac{\frac{3b\left(\frac{1}{10}(a+bx)^{\frac{1}{3}}(a+bx)^3-\frac{3}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a+\frac{3}{4}(a+bx)^{\frac{1}{3}}(a+bx)a^2-(a+bx)^{\frac{1}{3}}a^3\right)}{b^3}+\frac{3a\left(\frac{1}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2-\frac{1}{2}(a+bx)^{\frac{1}{3}}(a+bx)a+(a+bx)^{\frac{1}{3}}a^2\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x)
```

```
[Out] 3/140*(10*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a/b^2 + (14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)
*a^2 - 140*(b*x + a)^(1/3)*a^3)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{42\,{\left(a+b\,x\right)}^{10/3}-120\,a\,{\left(a+b\,x\right)}^{7/3}+105\,a^2\,{\left(a+b\,x\right)}^{4/3}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/3),x)
```

```
[Out] (42*(a + b*x)^(10/3) - 120*a*(a + b*x)^(7/3) + 105*a^2*(a + b*x)^(4/3))/(14
0*b^3)
```

3.373 $\int x\sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}$$

```
[Out] -3/4*a*(b*x+a)^(4/3)/b^2+3/7*(b*x+a)^(7/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(1/3),x]
[Out] (-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}\,\left(-3a^2+abx+4b^2x^2\right)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-3*a^2 + a*b*x + 4*b^2*x^2))/(28*b^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(34) = 68$.
time = 3.43, size = 96, normalized size = 2.82

$$\frac{3a^{\frac{1}{3}}\left(3a^3\left(1-\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^2bx\left(3-2\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+5ab^2x^2\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+4b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{28b^2\left(a+bx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*(a + b*x)^(1/3),x]')
[Out] 3 a ^ (1 / 3) (3 a ^ 3 (1 - ((a + b x) / a) ^ (1 / 3)) + a ^ 2 b x (3 - 2 (
(a + b x) / a) ^ (1 / 3)) + 5 a b ^ 2 x ^ 2 ((a + b x) / a) ^ (1 / 3) + 4 b
 ^ 3 x ^ 3 ((a + b x) / a) ^ (1 / 3)) / (28 b ^ 2 (a + b x))
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{4}{3}}(-4bx+3a)}{28b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{4}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{4}}{b^2}$	26
trager	$-\frac{3(-4x^2b^2-abx+3a^2)(bx+a)^{\frac{1}{3}}}{28b^2}$	32
risch	$-\frac{3(-4x^2b^2-abx+3a^2)(bx+a)^{\frac{1}{3}}}{28b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/7*(b*x+a)^(7/3)-1/4*a*(b*x+a)^(4/3))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.76

$$\frac{3\left(bx+a\right)^{\frac{7}{3}}}{7\,b^2}-\frac{3\left(bx+a\right)^{\frac{4}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/7*(b*x + a)^(7/3)/b^2 - 3/4*(b*x + a)^(4/3)*a/b^2
```

Fricas [A]
time = 0.32, size = 30, normalized size = 0.88

$$\frac{3\left(4\,b^{2}x^{2}+abx-3\,a^{2}\right)(bx+a)^{\frac{1}{3}}}{28\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^2*x^2 + a*b*x - 3*a^2)*(b*x + a)^(1/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(31) = 62.
time = 0.59, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx}{a}}}{28a^{2}b^{2}+28ab^{3}x}+\frac{9a^{\frac{13}{3}}}{28a^{2}b^{2}+28ab^{3}x}-\frac{6a^{\frac{10}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{28a^{2}b^{2}+28ab^{3}x}+\frac{9a^{\frac{10}{3}}bx}{28a^{2}b^{2}+28ab^{3}x}+\frac{15a^{\frac{7}{3}}b^{2}x^{2}\sqrt[3]{1+\frac{bx}{a}}}{28a^{2}b^{2}+28ab^{3}x}+\frac{12a^{\frac{4}{3}}b^{3}x^{3}\sqrt[3]{1+\frac{bx}{a}}}{28a^{2}b^{2}+28ab^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 9*a**(13/3)/
(28*a**2*b**2 + 28*a*b**3*x) - 6*a**(10/3)*b*x*(1 + b*x/a)**(1/3)/(28*a**2*
b**2 + 28*a*b**3*x) + 9*a**(10/3)*b*x/(28*a**2*b**2 + 28*a*b**3*x) + 15*a**
(7/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 12*a**(4/
3)*b**3*x**3*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(26) = 52.
time = 0.00, size = 98, normalized size = 2.88

$$\frac{\frac{3b\left(\frac{1}{7}(a+bx)^{\frac{1}{3}}(a+bx)^{2}-\frac{1}{2}(a+bx)^{\frac{1}{3}}(a+bx)a+(a+bx)^{\frac{1}{3}}a^{2}\right)}{b^{2}}+\frac{3a\left(\frac{1}{4}(a+bx)^{\frac{1}{3}}(a+bx)-a(a+bx)^{\frac{1}{3}}\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x)
```

```
[Out] 3/28*(7*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a/b + 2*(2*(b*x + a)^(7/3)
- 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{21\,a\,{\left(a+b\,x\right)}^{4/3}-12\,{\left(a+b\,x\right)}^{7/3}}{28\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/3),x)
```

```
[Out] -(21*a*(a + b*x)^(4/3) - 12*(a + b*x)^(7/3))/(28*b^2)
```

3.374 $\int \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{4/3}}{4b}$$

[Out] 3/4*(b*x+a)^(4/3)/b

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(1/3),x]

[Out] (3*(a + b*x)^(4/3))/(4*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\,dx = \frac{3(a+bx)^{4/3}}{4b}$$

Mathematica [A]

time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(1/3),x]

[Out] (3*(a + b*x)^(4/3))/(4*b)

Mathics [A]

time = 1.56, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{4}{3}}}{4b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(1/3),x]')
[Out] 3 (a + b x) ^ (4 / 3) / (4 b)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
default	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
trager	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
risch	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/4*(b*x+a)^(4/3)/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x + a)^(4/3)/b
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x + a)^(4/3)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{3\left(a+bx\right)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3),x)
```

```
[Out] 3*(a + b*x)**(4/3)/(4*b)
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.31

$$\frac{\left(bx+a\right)^{\frac{1}{3}}\left(bx+a\right)}{\frac{4}{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x)
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\left(a+b\,x\right)^{4/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

3.375 $\int \frac{\sqrt[3]{a+bx}}{x}\,dx$

Optimal. Leaf size=91

$$3\sqrt[3]{a+bx}-\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 3*(b*x+a)^(1/3)-1/2*a^(1/3)*ln(x)+3/2*a^(1/3)*ln(a^(1/3)-(b*x+a)^(1/3))-a^(
1/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 59, 631, 210, 31}

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx}}{x}\,dx &= 3\sqrt[3]{a+bx}+a\int \frac{1}{x(a+bx)^{2/3}}\,dx\\ &= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\,\log(x)-\frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{1}{2}\left(3a^{2/3}\right)\\ &= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.\\ &= 3\sqrt[3]{a+bx}-\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a}\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 113, normalized size = 1.24

$$3\sqrt[3]{a+bx}-\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}\sqrt[3]{a}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3)
)/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.72, size = 102, normalized size = 1.12

$$-a^{\frac{1}{3}}\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]+a^{\frac{1}{3}}\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]+3b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)/x,x]')
```

```
[Out] -1 ^ (1 / 3) a ^ (1 / 3) Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_pola
r[2 I / 3 Pi] / a ^ (1 / 3)] + -1 ^ (2 / 3) a ^ (1 / 3) Log[1 - b ^ (1 / 3)
 (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] + a ^ (1 / 3) L
og[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] + 3 b ^ (1 / 3) (a
/ b + x) ^ (1 / 3)
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.99

method	result
derivativedivides	$3\left(bx+a\right)^{\frac{1}{3}}+3\left(\frac{\ln\left(\left(bx+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left(\left(bx+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(bx+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2\left(bx+a\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$
default	$3\left(bx+a\right)^{\frac{1}{3}}+3\left(\frac{\ln\left(\left(bx+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left(\left(bx+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(bx+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2\left(bx+a\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(b*x+a)^(1/3)+3*(1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(2/3)*ln((b*
x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))*a
```

Maxima [A]
time = 0.36, size = 86, normalized size = 0.95

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\left(bx+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left(\left(bx+a\right)^{\frac{2}{3}}+\left(bx+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left(\left(bx+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(bx+a\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Fricas [A]
time = 0.31, size = 91, normalized size = 1.00

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.07, size = 180, normalized size = 1.98

$$\frac{4\sqrt[3]{a}\,\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma(
7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pol
ar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)
/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Giac [A]
time = 0.00, size = 135, normalized size = 1.48

$$-\frac{1}{2}a^{\frac{1}{3}}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}\,(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)+\frac{3aa^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}+3\,(a+bx)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x)
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

Mupad [B]
time = 0.12, size = 107, normalized size = 1.18

$$a^{1/3}\,\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-9\,a^{4/3}\right)+3\,{\left(a+b\,x\right)}^{1/3}+\frac{a^{1/3}\,\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{1/3}\,\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}+\frac{9\,a^{4/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x,x)
```

```
[Out] a^(1/3)*log(9*a*(a + b*x)^(1/3) - 9*a^(4/3)) + 3*(a + b*x)^(1/3) + (a^(1/3)
*log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/2 - (a^(1/3)*log(9*a*(a + b*x)^(1/3) + (9*a^(4/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/2
```

3.376 $\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt[3]{a+bx}}{x}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

```
[Out] -(b*x+a)^(1/3)/x-1/6*b*ln(x)/a^(2/3)+1/2*b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3
)-1/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 59, 631, 210, 31}

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx &= -\frac{\sqrt[3]{a+bx}}{x}+\frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\log(x)}{6a^{2/3}}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,d\right.}{2\sqrt[3]{a}}\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}+\frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right.}{a^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 119, normalized size = 1.23

$$-\frac{6a^{2/3}\sqrt[3]{a+bx}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -1/6*(6*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1
/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(2/3)*x)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.20, size = 110, normalized size = 1.13

$$\frac{-3a^{\frac{2}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}} - bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right] + bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right] + bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3a^{\frac{2}{3}}x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)/x^2,x]')
```

```
[Out] (-3 a ^ (2 / 3) b ^ (1 / 3) (a / b + x) ^ (1 / 3) - -1 ^ (1 / 3) b x Log[1
- b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)] +
-1 ^ (2 / 3) b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I /
3 Pi] / a ^ (1 / 3)] + b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^
(1 / 3)]) / (3 a ^ (2 / 3) x)
```

Maple [A]
time = 0.15, size = 95, normalized size = 0.98

method	result
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{9a^{\frac{2}{3}}}\right.$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{9a^{\frac{2}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/3*(b*x+a)^(1/3)/b/x+1/9/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/18/a^(2
/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/9/a^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Maxima [A]
time = 0.36, size = 93, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}}+\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/6*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2
/3) + 1/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - (b*x + a)^(1/3)/x
```

Fricas [A]
time = 0.33, size = 139, normalized size = 1.43

$$-\frac{2\sqrt{3}\,(a^2)^{\frac{1}{6}}abx\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^2}\right)+(a^2)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\,(a^2)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+6\,(bx+a)^{\frac{1}{3}}a^2}{6\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*b*x*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/
3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b
*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2
/3)*b*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*
x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.15, size = 643, normalized size = 6.63

$$\frac{4a^{\frac{7}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{12a^2b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**2,x)
```

```
[Out] 4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*
gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*
pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2
*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(
4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
```

```
))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(
7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b
+ x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a
**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*e
xp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*
gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi
/3)*gamma(7/3))
```

Giac [A]
time = 0.01, size = 164, normalized size = 1.69

$$\frac{-\frac{\frac{1}{6}b^2\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^2+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{\left(a^{\frac{1}{3}}\right)^2}-\frac{b^2\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\left(a^{\frac{1}{3}}\right)^2}+\frac{b^2a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}-\frac{(a+bx)^{\frac{1}{3}}b^2}{a+bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x)
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(2/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1
/3)*b/x)/b
```

Mupad [B]
time = 0.07, size = 117, normalized size = 1.21

$$\frac{b\ln\left(3\,b\,(a+b\,x)^{1/3}-3\,a^{1/3}\,b\right)}{3\,a^{2/3}}-\frac{(a+b\,x)^{1/3}}{x}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{2}+3\,b\,(a+b\,x)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{2/3}}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{2}+3\,b\,(a+b\,x)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^2,x)
```

```
[Out] (b*log(3*b*(a + b*x)^(1/3) - 3*a^(1/3)*b))/(3*a^(2/3)) - (a + b*x)^(1/3)/x
- (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)^(1/3))*(b - 3^(1/2)
*b*1i))/(6*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)
^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(2/3))
```

3.377 $\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}$$

```
[Out] -1/2*(b*x+a)^(1/3)/x^2-1/6*b*(b*x+a)^(1/3)/a/x+1/18*b^2*ln(x)/a^(5/3)-1/6*b
^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/3)+1/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^
(1/3))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 59, 631, 210, 31}

$$\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x)^(1/3)/x^2 - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx &= -\frac{\sqrt[3]{a+bx}}{2x^2}+\frac{1}{6}b\int \frac{1}{x^2(a+bx)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}-\frac{b^2\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a}\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}+\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\text{Subst}
\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3
\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a\right.}{6a^{5/3}}
\end{aligned}$$

Mathematica [A]

time = 0.19, size = 129, normalized size = 1.02

$$\frac{-\frac{3a^{2/3}\sqrt[3]{a+bx}\,(3a+bx)}{x^2}+2\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] ((-3*a^(2/3)*(a + b*x)^(1/3)*(3*a + b*x))/x^2 + 2*Sqrt[3]*b^2*ArcTan[(1 + (
2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b^2*Log[a^(1/3) - (a + b*x)^(1/3)]
 + b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(5/3
))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 21.04, size = 136, normalized size = 1.07

$$-\frac{b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{6ax}-\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{9a^{\frac{5}{3}}}-\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{5}{3}}}+\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{5}{3}}}-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)/x^3,x]')
```

```
[Out] -b ^ (4 / 3) (a / b + x) ^ (1 / 3) / (6 a x) - -1 ^ (2 / 3) b ^ 2 Log[1 - b
 ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (9
a ^ (5 / 3)) - b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3
)] / (9 a ^ (5 / 3)) + -1 ^ (1 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^
 (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (5 / 3)) - b ^ (1 /
3) (a / b + x) ^ (1 / 3) / (2 x ^ 2)
```

Maple [A]
time = 0.14, size = 118, normalized size = 0.93

method	result
derivativedivides	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{3a^{\frac{2}{3}}}}{9a}\right.$

default	$3b^2\left(-\frac{-\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{2}{3}}}}{9a}\right.$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-(1/18/a*(b*x+a)^(4/3)+1/9*(b*x+a)^(1/3))/b^2/x^2-1/9/a*(1/3/a^(2/3)
*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/
3)+a^(2/3))-1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)
+1))))
```

Maxima [A]
time = 0.35, size = 139, normalized size = 1.09

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{4}{3}}b^2+2\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(5/3) - 1/6*((b*x + a)^(
4/3)*b^2 + 2*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3)
```

Fricas [A]
time = 0.31, size = 187, normalized size = 1.47

$$\frac{2\sqrt{3}\,ab^2x^2\sqrt{-(-a^2)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\,(-a^2)^{\frac{1}{3}}a-2\sqrt{3}\,(-a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{3a^2}\right)+(-a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a-(-a^2)^{\frac{1}{3}}a+(-a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\,(-a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-(-a^2)^{\frac{2}{3}}\right)-3\,(a^2bx+3\,a^3)(bx+a)^{\frac{1}{3}}}{18\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) +
 (-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)
*(b*x + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(
2/3)) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.71, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**3,x)
```

```
[Out] -4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a*
*7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)
+ 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)
*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(
a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi
/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(
7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x
)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3
)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*
(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*p
i/3)*gamma(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/
3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)*
*2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma
(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*e
xp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*e
xp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3
)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*g
amma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/
b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
```

```
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
) + 4*a**(7/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**
6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*
pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 4*
a**(7/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3)
 - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2
*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7
/3)) - 12*a**5*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**4*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(
2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**3*b**(13/3)*(a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
)
```

Giac [A]
time = 0.01, size = 199, normalized size = 1.57

$$\frac{\frac{a^{\frac{1}{3}}b^{3}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{18a^{2}}+\frac{\frac{1}{3}a^{\frac{1}{3}}b^{3}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{2}}-\frac{b^{3}a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3\cdot 3a^{2}}-\frac{\frac{1}{6}\left((a+bx)^{\frac{1}{3}}(a+bx)b^{3}+2(a+bx)^{\frac{1}{3}}ab^{3}\right)}{a(a+bx-a)^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x)
```

```
[Out] 1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a)^(
4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.23, size = 196, normalized size = 1.54

$$\frac{b^{2}\ln\left(\frac{b^{2}}{(-a)^{2/3}}-\frac{b^{2}\,(a+b\,x)^{1/3}}{a}\right)}{9\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}}{2\,(-a)^{2/3}}+\frac{b^{2}\,(a+b\,x)^{1/3}}{a}\right)\,\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)}{18\,(-a)^{5/3}}-\frac{\frac{b^{2}\,(a+b\,x)^{1/3}}{3}+\frac{b^{2}\,(a+b\,x)^{4/3}}{6\,a}}{(a+b\,x)^{2}-2\,a\,(a+b\,x)+a^{2}}+\frac{b^{2}\ln\left(\frac{b^{2}\,(a+b\,x)^{1/3}}{a}-\frac{b^{2}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{2/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^3,x)
```

```
[Out] (b^2*log(b^2/(-a)^(2/3) - (b^2*(a + b*x)^(1/3))/a))/(9*(-a)^(5/3)) - (log((
3^(1/2)*b^2*1i + b^2)/(2*(-a)^(2/3)) + (b^2*(a + b*x)^(1/3))/a)*(3^(1/2)*b^
2*1i + b^2))/(18*(-a)^(5/3)) - ((b^2*(a + b*x)^(1/3))/3 + (b^2*(a + b*x)^(4
/3))/(6*a))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + (b^2*log((b^2*(a + b*x)^(
1/3))/a - (b^2*((3^(1/2)*1i)/2 - 1/2))/(-a)^(2/3))*((3^(1/2)*1i)/2 - 1/2))/
(9*(-a)^(5/3))
```

3.378 $\int x^3(a+bx)^{2/3}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}$$

```
[Out] -3/5*a^3*(b*x+a)^(5/3)/b^4+9/8*a^2*(b*x+a)^(8/3)/b^4-9/11*a*(b*x+a)^(11/3)/
b^4+3/14*(b*x+a)^(14/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*(
a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{2/3}\,dx&=\int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx\\&=-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{5/3}\left(-81a^3+135a^2bx-180ab^2x^2+220b^3x^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-81*a^3 + 135*a^2*b*x - 180*a*b^2*x^2 + 220*b^3*x^3))/(
3080*b^4)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(a + b*x)^(2/3),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{5}{3}}\left(-220b^{3}x^{3}+180a\,b^{2}x^{2}-135a^{2}bx+81a^{3}\right)}{3080b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{14}{3}}}{14}-\frac{9a(bx+a)^{\frac{11}{3}}}{11}+\frac{9a^{2}(bx+a)^{\frac{8}{3}}}{8}-\frac{3a^{3}(bx+a)^{\frac{5}{3}}}{5}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{14}{3}}}{14}-\frac{9a(bx+a)^{\frac{11}{3}}}{11}+\frac{9a^{2}(bx+a)^{\frac{8}{3}}}{8}-\frac{3a^{3}(bx+a)^{\frac{5}{3}}}{5}}{b^{4}}$	50
trager	$-\frac{3\left(-220b^{4}x^{4}-40a\,b^{3}x^{3}+45a^{2}b^{2}x^{2}-54a^{3}bx+81a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080b^{4}}$	54
risch	$-\frac{3\left(-220b^{4}x^{4}-40a\,b^{3}x^{3}+45a^{2}b^{2}x^{2}-54a^{3}bx+81a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080b^{4}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/14*(b*x+a)^(14/3)-3/11*a*(b*x+a)^(11/3)+3/8*a^2*(b*x+a)^(8/3)-1/5*
a^3*(b*x+a)^(5/3))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{14}{3}}}{14\,b^4}-\frac{9\,(bx+a)^{\frac{11}{3}}a}{11\,b^4}+\frac{9\,(bx+a)^{\frac{8}{3}}a^2}{8\,b^4}-\frac{3\,(bx+a)^{\frac{5}{3}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x + a)^(14/3)/b^4 - 9/11*(b*x + a)^(11/3)*a/b^4 + 9/8*(b*x + a)^(8/
3)*a^2/b^4 - 3/5*(b*x + a)^(5/3)*a^3/b^4
```

Fricas [A]
time = 0.31, size = 53, normalized size = 0.74

$$\frac{3\,(220\,b^4x^4 + 40\,ab^3x^3 - 45\,a^2b^2x^2 + 54\,a^3bx - 81\,a^4)(bx + a)^{\frac{2}{3}}}{3080\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^4*x^4 + 40*a*b^3*x^3 - 45*a^2*b^2*x^2 + 54*a^3*b*x - 81*a^4)*
(b*x + a)^(2/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(68) = 136.
time = 1.38, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 4
6200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 1848
0*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(74/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 1296*
a**(71/3)*b*x*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 1458*a**(71/3)*b*x/(3080*a**20*
b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 +
 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 2
808*a**(68/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b
**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x*
*4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 3645*a**(68/3)*b**2*x
**2/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a
**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14
*b**10*x**6) - 3120*a**(65/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4860*
a**(65/3)*b**3*x**3/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**
6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x
**5 + 3080*a**14*b**10*x**6) - 1050*a**(62/3)*b**4*x**4*(1 + b*x/a)**(2/3)/
(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17
*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**
```

```
10*x**6) + 3645*a**(62/3)*b**4*x**4/(3080*a**20*b**4 + 18480*a**19*b**5*x +
 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18
480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4032*a**(59/3)*b**5*x**5*(1
+ b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x*
*2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5
+ 3080*a**14*b**10*x**6) + 1458*a**(59/3)*b**5*x**5/(3080*a**20*b**4 + 1848
0*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**1
6*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 11004*a**(56
/3)*b**6*x**6*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(56/3)*b**6*x**6/(3080*a
**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x
**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6
) + 14352*a**(53/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a
**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b
**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 10485*a**(50/3)
*b**8*x**8*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200
*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a*
*15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4080*a**(47/3)*b**9*x**9*(1 + b*x/
a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 6
1600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080
*a**14*b**10*x**6) + 660*a**(44/3)*b**10*x**10*(1 + b*x/a)**(2/3)/(3080*a**
20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**
3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(56) = 112.
time = 0.00, size = 217, normalized size = 3.01

$$\frac{\frac{3b\left(\frac{1}{14}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{4}-\frac{4}{11}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{3}a+\frac{3}{4}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{2}a^{2}-\frac{4}{5}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)a^{3}+\frac{1}{2}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}a^{4}\right)}{b^{4}}+\frac{3a\left(\frac{1}{11}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{3}-\frac{3}{8}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{2}a+\frac{3}{5}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)a^{2}-\frac{1}{2}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}a^{3}\right)}{b^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x)
```

```
[Out] 3/3080*(7*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3
)*a^2 - 220*(b*x + a)^(2/3)*a^3)*a/b^3 + 2*(110*(b*x + a)^(14/3) - 560*(b*x
 + a)^(11/3)*a + 1155*(b*x + a)^(8/3)*a^2 - 1232*(b*x + a)^(5/3)*a^3 + 770*
(b*x + a)^(2/3)*a^4)/b^3)/b
```

Mupad [**B**]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{14/3}}{14\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(2/3),x)
[Out] (3*(a + b*x)^(14/3))/(14*b^4) - (3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a
 + b*x)^(8/3))/(8*b^4) - (9*a*(a + b*x)^(11/3))/(11*b^4)
```

3.379 $\int x^2(a+bx)^{2/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}$$

```
[Out] 3/5*a^2*(b*x+a)^(5/3)/b^3-3/4*a*(b*x+a)^(8/3)/b^3+3/11*(b*x+a)^(11/3)/b^3
```

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a + b
*x)^(11/3))/(11*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{2/3}\,dx&=\int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}\end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{5/3}\left(9a^2-15abx+20b^2x^2\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(9*a^2 - 15*a*b*x + 20*b^2*x^2))/(220*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 212 vs. $2(53) = 106$.
time = 7.41, size = 194, normalized size = 3.66

$$\frac{3a^{\frac{2}{3}}\left(9a^6\left(-1+\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+3a^5bx\left(-9+7\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+a^4b^2x^2\left(-27+14\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+3a^2b^3x^3\left(-3a+23bx\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+26a^3b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+65ab^5x^5\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+20b^6x^6\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)}{220b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(a + b*x)^(2/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (9 a ^ 6 (-1 + ((a + b x) / a) ^ (2 / 3)) + 3 a ^ 5 b x (-9 +
 7 ((a + b x) / a) ^ (2 / 3)) + a ^ 4 b ^ 2 x ^ 2 (-27 + 14 ((a + b x) / a)
 ^ (2 / 3)) + 3 a ^ 2 b ^ 3 x ^ 3 (-3 a + 23 b x ((a + b x) / a) ^ (2 / 3))
 + 26 a ^ 3 b ^ 3 x ^ 3 ((a + b x) / a) ^ (2 / 3) + 65 a b ^ 5 x ^ 5 ((a +
b x) / a) ^ (2 / 3) + 20 b ^ 6 x ^ 6 ((a + b x) / a) ^ (2 / 3)) / (220 b ^
3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{5}{3}}\left(20x^2b^2-15abx+9a^2\right)}{220b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{3a(bx+a)^{\frac{8}{3}}}{4}+\frac{3a^2(bx+a)^{\frac{5}{3}}}{5}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{3a(bx+a)^{\frac{8}{3}}}{4}+\frac{3a^2(bx+a)^{\frac{5}{3}}}{5}}{b^3}$	38
trager	$\frac{3\left(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3\right)(bx+a)^{\frac{2}{3}}}{220b^3}$	43
risch	$\frac{3\left(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3\right)(bx+a)^{\frac{2}{3}}}{220b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/11*(b*x+a)^(11/3)-1/4*a*(b*x+a)^(8/3)+1/5*a^2*(b*x+a)^(5/3))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^3}-\frac{3\,(bx+a)^{\frac{8}{3}}a}{4\,b^3}+\frac{3\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^3 - 3/4*(b*x + a)^(8/3)*a/b^3 + 3/5*(b*x + a)^(5/3)
*a^2/b^3
```

Fricas [A]
time = 0.30, size = 42, normalized size = 0.79

$$\frac{3\,(20\,b^3x^3+5\,ab^2x^2-6\,a^2bx+9\,a^3)(bx+a)^{\frac{2}{3}}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/220*(20*b^3*x^3 + 5*a*b^2*x^2 - 6*a^2*b*x + 9*a^3)*(b*x + a)^(2/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(49) = 98.
time = 0.92, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{35}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{63a^{\frac{32}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{32}{3}}bx}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{42a^{\frac{29}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{29}{3}}b^2x^2}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{78a^{\frac{26}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{26}{3}}b^3x^3}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{207a^{\frac{23}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{195a^{\frac{20}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{60a^{\frac{17}{3}}b^6x^6\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) - 27*a**(35/3)/(220*a**8*b**3 + 660*a**7*b
**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 63*a**(32/3)*b*x*(1 + b*
x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**
5*b**6*x**3) - 81*a**(32/3)*b*x/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) + 42*a**(29/3)*b**2*x**2*(1 + b*x/a)**(2/3
)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**
3) - 81*a**(29/3)*b**2*x**2/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**
5*x**2 + 220*a**5*b**6*x**3) + 78*a**(26/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(2
20*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) -
 27*a**(26/3)*b**3*x**3/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x*
*2 + 220*a**5*b**6*x**3) + 207*a**(23/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(220*
a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 19
5*a**(20/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x +
 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 60*a**(17/3)*b**6*x**6*(1 + b*x
/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5
*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(41) = 82.
time = 0.00, size = 166, normalized size = 3.13

$$\frac{\frac{3b\left(\frac{1}{11}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)^3-\frac{3}{8}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)^2a+\frac{3}{5}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)a^2-\frac{1}{2}\left((a+bx)^{\frac{1}{3}}\right)^2a^3\right)}{b^3}+\frac{3a\left(\frac{1}{8}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)^2-\frac{2}{5}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)a+\frac{1}{2}\left((a+bx)^{\frac{1}{3}}\right)^2a^2\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x)
```

```
[Out] 3/440*(11*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^
2)*a/b^2 + (40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/
3)*a^2 - 220*(b*x + a)^(2/3)*a^3)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{60\,{\left(a+b\,x\right)}^{11/3}-165\,a\,{\left(a+b\,x\right)}^{8/3}+132\,a^2\,{\left(a+b\,x\right)}^{5/3}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(2/3),x)
```

```
[Out] (60*(a + b*x)^(11/3) - 165*a*(a + b*x)^(8/3) + 132*a^2*(a + b*x)^(5/3))/(22
0*b^3)
```

3.380 $\int x(a+bx)^{2/3}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}$$

```
[Out] -3/5*a*(b*x+a)^(5/3)/b^2+3/8*(b*x+a)^(8/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{2/3}\,dx &= \int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.03

$$\frac{3(a+bx)^{2/3}\left(-3a^2+2abx+5b^2x^2\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-3*a^2 + 2*a*b*x + 5*b^2*x^2))/(40*b^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. $2(34) = 68$.
time = 3.52, size = 96, normalized size = 2.82

$$\frac{3a^{\frac{2}{3}}\left(3a^3\left(1-\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+a^2bx\left(3-\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+7ab^2x^2\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+5b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)}{40b^2\left(a+bx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*(a + b*x)^(2/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (3 a ^ 3 (1 - ((a + b x) / a) ^ (2 / 3)) + a ^ 2 b x (3 - ((a
 + b x) / a) ^ (2 / 3)) + 7 a b ^ 2 x ^ 2 ((a + b x) / a) ^ (2 / 3) + 5 b ^
 3 x ^ 3 ((a + b x) / a) ^ (2 / 3)) / (40 b ^ 2 (a + b x))
```

Maple [A]
time = 0.09, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{5}{3}}(-5bx+3a)}{40b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{3a(bx+a)^{\frac{5}{3}}}{5}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{3a(bx+a)^{\frac{5}{3}}}{5}}{b^2}$	26
trager	$-\frac{3(-5x^2b^2-2abx+3a^2)(bx+a)^{\frac{2}{3}}}{40b^2}$	32
risch	$-\frac{3(-5x^2b^2-2abx+3a^2)(bx+a)^{\frac{2}{3}}}{40b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/8*(b*x+a)^(8/3)-1/5*a*(b*x+a)^(5/3))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^2}-\frac{3\,(bx+a)^{\frac{5}{3}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^2 - 3/5*(b*x + a)^(5/3)*a/b^2
```

Fricas [A]
time = 0.30, size = 31, normalized size = 0.91

$$\frac{3\left(5\,b^{2}x^{2}+2\,abx-3\,a^{2}\right)\left(bx+a\right)^{\frac{2}{3}}}{40\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 2*a*b*x - 3*a^2)*(b*x + a)^(2/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(31) = 62.
time = 0.61, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{14}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{2}b^{2}+40ab^{3}x}+\frac{9a^{\frac{14}{3}}}{40a^{2}b^{2}+40ab^{3}x}-\frac{3a^{\frac{11}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{2}b^{2}+40ab^{3}x}+\frac{9a^{\frac{11}{3}}bx}{40a^{2}b^{2}+40ab^{3}x}+\frac{21a^{\frac{8}{3}}b^{2}x^{2}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{2}b^{2}+40ab^{3}x}+\frac{15a^{\frac{5}{3}}b^{3}x^{3}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{2}b^{2}+40ab^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(14/3)*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 9*a**(14/3)/
(40*a**2*b**2 + 40*a*b**3*x) - 3*a**(11/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**2*
b**2 + 40*a*b**3*x) + 9*a**(11/3)*b*x/(40*a**2*b**2 + 40*a*b**3*x) + 21*a**
(8/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 15*a**(5/
3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.
time = 0.00, size = 113, normalized size = 3.32

$$\frac{\frac{3b\left(\frac{1}{8}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{2}-\frac{2}{5}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)a+\frac{1}{2}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}a^{2}\right)}{b^{2}}+\frac{3a\left(\frac{1}{5}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)-\frac{1}{2}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}a\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x)
```

```
[Out] 3/40*(4*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)*a/b + (5*(b*x + a)^(8/3)
- 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{24\,a\,{\left(a+b\,x\right)}^{5/3}-15\,{\left(a+b\,x\right)}^{8/3}}{40\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(2/3),x)
```

```
[Out] -(24*a*(a + b*x)^(5/3) - 15*(a + b*x)^(8/3))/(40*b^2)
```

3.381 $\int (a+bx)^{2/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{5/3}}{5b}$$

```
[Out] 3/5*(b*x+a)^(5/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3),x]
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3),x]
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Mathics [A]
time = 1.62, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{5}{3}}}{5b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(2/3),x]')
[Out] 3 (a + b x) ^ (5 / 3) / (5 b)
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
default	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
trager	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
risch	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/5*(b*x+a)^(5/3)/b
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="maxima")
[Out] 3/5*(b*x + a)^(5/3)/b
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="fricas")
[Out] 3/5*(b*x + a)^(5/3)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{3(a+bx)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3),x)
```

```
[Out] 3*(a + b*x)**(5/3)/(5*b)
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.44

$$\frac{\left((bx+a)^{\frac{1}{3}}\right)^{2}(bx+a)}{\frac{5}{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x)
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{5/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

3.382 $\int \frac{(a+bx)^{2/3}}{x} dx$

Optimal. Leaf size=92

$$\frac{3}{2}(a+bx)^{2/3}+\sqrt{3}\, a^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a}\, +2\sqrt[3]{a+bx}}{\sqrt{3}\, \sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}\, -\sqrt[3]{a+bx}\right)$$

```
[Out] 3/2*(b*x+a)^(2/3)-1/2*a^(2/3)*ln(x)+3/2*a^(2/3)*ln(a^(1/3)-(b*x+a)^(1/3))+a
^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 57, 631, 210, 31}

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}\, -\sqrt[3]{a+bx}\right)+\sqrt{3}\, a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}\, +\sqrt[3]{a}}{\sqrt{3}\, \sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3)
)/(Sqrt[3]*a^(1/3))] - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3) - (a + b
*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \frac{3}{2}(a+bx)^{2/3} + a\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$

$$= \frac{3}{2}(a+bx)^{2/3} - \frac{1}{2}a^{2/3}\log(x) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) + \frac{1}{2}(3a)$$

$$= \frac{3}{2}(a+bx)^{2/3} - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-$$

$$= \frac{3}{2}(a+bx)^{2/3} + \sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-$$

Mathematica [A]
time = 0.05, size = 114, normalized size = 1.24

$$\frac{3}{2}(a+bx)^{2/3} + \sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \frac{1}{2}a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(
1/3))/Sqrt[3]] + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(2/3)*Log[a^(2
/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.84, size = 102, normalized size = 1.11

$$-a^{\frac{2}{3}}\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]+a^{\frac{2}{3}}\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]+\frac{3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)/x,x]')
```

```
[Out] -1 ^ (1 / 3) a ^ (2 / 3) Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_pola
r[4 I / 3 Pi] / a ^ (1 / 3)] + -1 ^ (2 / 3) a ^ (2 / 3) Log[1 - b ^ (1 / 3)
 (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)] + a ^ (2 / 3) L
og[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] + 3 b ^ (2 / 3) (a
/ b + x) ^ (2 / 3) / 2
```

Maple [A]
time = 0.09, size = 90, normalized size = 0.98

method	result
derivativedivides	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}\right.$
default	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(b*x+a)^(2/3)+3*(1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((
b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*
3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))*a
```

Maxima [A]
time = 0.35, size = 85, normalized size = 0.92

$$\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/2*(b*x + a)^(2/3)
```

Fricas [A]
time = 0.31, size = 110, normalized size = 1.20

$$\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{2}\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)+\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+\frac{3}{2}(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(a^2)^(1/3)*(b*x + a)
^(1/3))/a) - 1/2*(a^2)^(1/3)*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^
(2/3)*(b*x + a)^(1/3)) + (a^2)^(1/3)*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) +
 3/2*(b*x + a)^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 182, normalized size = 1.98

$$\frac{5a^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x,x)
```

```
[Out] 5*a**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(3*gamma(
8/3)) + 5*a**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(5/3)/(3*gamma(8/3)) + 5*a**(2/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)
/(3*gamma(8/3)) + 5*b**(2/3)*(a/b + x)**(2/3)*gamma(5/3)/(2*gamma(8/3))
```

Giac [A]
time = 0.00, size = 147, normalized size = 1.60

$$-\frac{1}{2}\left(a^{\frac{1}{3}}\right)^{2}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)+\sqrt{3}\left(a^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)+\frac{3a^{\frac{1}{3}}aa^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}+\frac{3}{2}\left((a+bx)^{\frac{1}{3}}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x)
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/2*(b*x + a)^(2/3)
```

Mupad [B]
time = 0.11, size = 117, normalized size = 1.27

$$\frac{3\,{\left(a+b\,x\right)}^{2/3}}{2}+a^{2/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{7/3}\right)+\frac{a^{2/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{7/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{2/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{7/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x)^(2/3))/2 + a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - 9*a^(7/3)) + (a
^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/
2)*1i - 1))/2 - (a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i
 + 1)^2)/4)*(3^(1/2)*1i + 1))/2
```

3.383 $\int \frac{(a+bx)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{(a+bx)^{2/3}}{x}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

```
[Out] -(b*x+a)^(2/3)/x-1/3*b*ln(x)/a^(1/3)+b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(1/3)+2/
3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 57, 631, 210, 31}

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3) - (a
+ b*x)^(1/3)])/a^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = -\frac{(a+bx)^{2/3}}{x}+\frac{1}{3}(2b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$

$$= -\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{b\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}-\frac{(2b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{}}{}\right.}{\sqrt[3]{a}}$$

$$= -\frac{(a+bx)^{2/3}}{x}+\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

Mathematica [A]
time = 0.12, size = 120, normalized size = 1.28

$$\frac{-3\sqrt[3]{a}\,(a+bx)^{2/3}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-bx\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3\sqrt[3]{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] (-3*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3))
/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a^(2/3)
 + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(1/3)*x)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.32, size = 112, normalized size = 1.19

$$\frac{-3a^{\frac{1}{3}}b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}-bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]-bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]+2bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{3a^{\frac{1}{3}}x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)/x^2,x]')
```

```
[Out] (-3 a ^ (1 / 3) b ^ (2 / 3) (a / b + x) ^ (2 / 3) - 2 -1 ^ (1 / 3) b x Log[
1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)]
+ 2 -1 ^ (2 / 3) b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2
I / 3 Pi] / a ^ (1 / 3)] + 2 b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3)
/ a ^ (1 / 3)]) / (3 a ^ (1 / 3) x)
```

Maple [A]
time = 0.10, size = 95, normalized size = 1.01

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{x}+\frac{2b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{2b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{3a^{\frac{1}{3}}}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3bx}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right.}{3}\right.}{9a^{\frac{1}{3}}}\right.$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3bx}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right.}{3}\right.}{9a^{\frac{1}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/3*(b*x+a)^(2/3)/b/x+2/9/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/9/a^(1/
3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+2/9*3^(1/2)/a^(1/3)*arct
an(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))
```

Maxima [A]
time = 0.36, size = 93, normalized size = 0.99

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{1}{3}}}+\frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/
3) + 2/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(1/3) - (b*x + a)^(2/3)/x
```

Fricas [A]
time = 0.32, size = 252, normalized size = 2.68

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(bx+a)^{\frac{2}{3}}a}{3\,ax},\ \frac{6\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}bx\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(bx+a)^{\frac{2}{3}}a}{3\,ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^
(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3
*(b*x + a)^(2/3)*a)/(a*x), 1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/3)*(2
*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b
*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1
/3)) - 3*(b*x + a)^(2/3)*a)/(a*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.17, size = 643, normalized size = 6.84

$$\frac{10a^{\frac{8}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{15a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**2,x)
```

```
[Out] 10*a**(8/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gam
ma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)
```

```
*gamma(8/3)) + 10*a**(8/3)*b*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3
) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*ex
p(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 10*
a**(5/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**
(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*
exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 1
0*a**(5/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(8/3)) + 15*a**2*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*
pi/3)*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(
2*I*pi/3)*gamma(8/3))
```

Giac [A]
time = 0.01, size = 171, normalized size = 1.82

$$\frac{-\frac{\frac{1}{3} b^2 \ln\left(\left((a+bx)^{\frac{1}{3}}\right)^2 + a^{\frac{1}{3}} (a+bx)^{\frac{1}{3}} + a^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} + \frac{2\left(a^{\frac{1}{3}}\right)^2 b^2 \arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}} + \frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\, a^{\frac{1}{3}}}\right)}{\sqrt{3}\, a} + \frac{2a^{\frac{1}{3}} b^2 a^{\frac{1}{3}} \ln\left|(a+bx)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|}{3a} + \frac{\left((a+bx)^{\frac{1}{3}}\right)^2 b^2}{-a-bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x)
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(1/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x + a)^(2/
3)*b/x)/b
```

Mupad [B]
time = 0.11, size = 127, normalized size = 1.35

$$\frac{2\,b\,\ln\left(4\,a^{1/3}\,b^2-4\,b^2\,{\left(a+b\,x\right)}^{1/3}\right)}{3\,a^{1/3}}-\frac{{\left(a+b\,x\right)}^{2/3}}{x}-\frac{\ln\left(a^{1/3}\,{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,b^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{1/3}}-\frac{\ln\left(a^{1/3}\,{\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,b^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^2,x)
```

```
[Out] (2*b*log(4*a^(1/3)*b^2 - 4*b^2*(a + b*x)^(1/3)))/(3*a^(1/3)) - (a + b*x)^(2
/3)/x - (log(a^(1/3)*(b - 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*x)^(1/3))*(b - 3^(
1/2)*b*1i))/(3*a^(1/3)) - (log(a^(1/3)*(b + 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*
x)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(1/3))
```

3.384 $\int \frac{(a+bx)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}$$

```
[Out] -1/2*(b*x+a)^(2/3)/x^2-1/3*b*(b*x+a)^(2/3)/a/x+1/18*b^2*ln(x)/a^(4/3)-1/6*b
^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)-1/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^
(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 57, 631, 210, 31}

$$\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x)^(2/3)/x^2 - (b*(a + b*x)^(2/3))/(3*a*x) - (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b^2*Log[
x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{2/3}}{x^3}\,dx &= -\frac{(a+bx)^{2/3}}{2x^2}+\frac{1}{3}b\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a}\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}+\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\text{S}}{}\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}+\frac{b^2\text{Subst}\left(\int \frac{}{-3}\right.}{}\\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\right.}{6a^{4/}}
\end{aligned}
$$

Mathematica [A]

time = 0.19, size = 147, normalized size = 1.16

$$-\frac{(a+bx)^{2/3}(a+2(a+bx))}{6ax^2}-\frac{b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{4/3}}+\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(a + 2*(a + b*x)))/(a*x^2) - (b^2*ArcTan[1/Sqrt[3] +
(2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) - (b^2*Log[a^(1
/3) - (a + b*x)^(1/3)])/(9*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^
(1/3) + (a + b*x)^(2/3)])/(18*a^(4/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 21.45, size = 136, normalized size = 1.07

$$-\frac{b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}{3ax}-\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{9a^{\frac{4}{3}}}-\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{4}{3}}}+\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{4}{3}}}-\frac{b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)/x^3,x]')
```

```
[Out] -b ^ (5 / 3) (a / b + x) ^ (2 / 3) / (3 a x) - -1 ^ (2 / 3) b ^ 2 Log[1 - b
 ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)] / (9
a ^ (4 / 3)) - b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3
)] / (9 a ^ (4 / 3)) + -1 ^ (1 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^
 (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (4 / 3)) - b ^ (2 /
3) (a / b + x) ^ (2 / 3) / (2 x ^ 2)
```

Maple [A]
time = 0.12, size = 118, normalized size = 0.93

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(2bx+3a)}{6x^2a}-\frac{b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{4}{3}}}+\frac{b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{4}{3}}}-\frac{b^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx}{}\right.}{3}\right.}{9a^{\frac{4}{3}}}$

derivativedivides	$3b^2\left(-\frac{-\frac{(bx+a)^{\frac{5}{3}}}{9a}+\frac{(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{9a}\right.$
default	$3b^2\left(-\frac{-\frac{(bx+a)^{\frac{5}{3}}}{9a}+\frac{(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{9a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-(1/9/a*(b*x+a)^(5/3)+1/18*(b*x+a)^(2/3))/b^2/x^2-1/9/a*(1/3/a^(1/3)
*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/
3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)
+1))))
```

Maxima [A]
time = 0.35, size = 139, normalized size = 1.09

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{4}{3}}}-\frac{2\,(bx+a)^{\frac{5}{3}}b^2+(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(4/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(4/3) - 1/6*(2*(b*x + a
)^(5/3)*b^2 + (b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3
)
```

Fricas [A]
time = 0.32, size = 350, normalized size = 2.76

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x}\right)+(-a)^{\frac{2}{3}}\,b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\,b^2x^2\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\,(2\,abx+3\,a^2)(bx+a)^{\frac{2}{3}}}{18\,a^2x^2},\ -\frac{6\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}\,b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2\,(-a)^{\frac{2}{3}}\,b^2x^2\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\,(2\,abx+3\,a^2)(bx+a)^{\frac{2}{3}}}{18\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2
*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b^2*x^2*log((
b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b^
2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x + 3*a^2)*(b*x + a)^(2/
3))/(a^2*x^2), -1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-
a)^(2/3)*b^2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(2*a*b*x + 3*a^2)*(b
*x + a)^(2/3))/(a^2*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.74, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**3,x)
```

```
[Out] -10*a**(17/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(2*I
*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(
2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3
*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a**7
*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) +
 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(2*I*pi
```

```
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2*I
*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**
5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*ex
p(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(2*I*
pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2
*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a
**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*
exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**5*b**(8/3)*(a/b + x)**(2/3)*exp(
2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b +
x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamm
a(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**4*b**(
11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamm
a(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)
*gamma(8/3)) + 30*a**3*b**(14/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(5/3)/
(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamm
a(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3))
```

Giac [**A**]
time = 0.01, size = 211, normalized size = 1.66

$$\frac{\frac{\left(a^{\frac{1}{3}}\right)^{2} b^{3} \ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}} (a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}} a^{\frac{1}{3}}\right)}{18a^{2}}-\frac{\frac{1}{3} b^{3} \arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\, a^{\frac{1}{3}}}\right)}{\sqrt{3}\, a a^{\frac{1}{3}}}-\frac{a^{\frac{1}{3}} b^{3} a^{\frac{1}{3}} \ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3\cdot 3a^{2}}+\frac{\frac{1}{6}\left(-2\left((a+bx)^{\frac{1}{3}}\right)^{2} (a+bx) b^{3}-\left((a+bx)^{\frac{1}{3}}\right)^{2} a b^{3}\right)}{a(a+bx-a)^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x)
```

```
[Out] -1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) - b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x + a
)^(5/3)*b^3 + (b*x + a)^(2/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.33, size = 194, normalized size = 1.53

$$\frac{(-1)^{1/3}\,b^2\,\ln\left((a+b\,x)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{9\,a^{4/3}}-\frac{\frac{b^2\,(a+b\,x)^{2/3}}{6}+\frac{b^2\,(a+b\,x)^{5/3}}{3\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2}+\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}}-\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^3,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x)^(1/3) - (-1)^(2/3)*a^(1/3)))/(9*a^(4/3)) - ((
b^2*(a + b*x)^(2/3))/6 + (b^2*(a + b*x)^(5/3))/(3*a))/((a + b*x)^2 - 2*a*(a
 + b*x) + a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^
(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(9
*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^(2/3)
*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(4
/3))
```

3.385 $\int x^3(a+bx)^{4/3}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}$$

```
[Out] -3/7*a^3*(b*x+a)^(7/3)/b^4+9/10*a^2*(b*x+a)^(10/3)/b^4-9/13*a*(b*x+a)^(13/3
)/b^4+3/16*(b*x+a)^(16/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*a
*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{4/3}\,dx &= \int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{7/3}\left(-81a^3+189a^2bx-315ab^2x^2+455b^3x^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-81*a^3 + 189*a^2*b*x - 315*a*b^2*x^2 + 455*b^3*x^3))/(
7280*b^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 372 vs. $2(72) = 144$.
time = 17.18, size = 348, normalized size = 4.83

$$\frac{3a^{\frac{1}{3}}\left(81a^{11}\left(1-\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+27a^{10}bx\left(18-17\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+9a^{9}b^{2}x^{2}\left(135-119\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^{8}b^{3}x^{3}\left(1620-1309\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+14a^{5}b^{4}x^{4}\left(-19a^{2}+271abx+839b^{2}x^{2}\right)\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+1215a^{7}b^{4}x^{4}+2a^{4}b^{5}x^{5}\left(243a^{2}+9427b^{2}x^{2}\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^{3}b^{6}x^{6}\left(81a^{2}+18091b^{2}x^{2}\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+10409a^{2}b^{9}x^{9}\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+3325ab^{10}x^{10}\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+455b^{11}x^{11}\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{7280b^{4}\left(a^{6}+6a^{5}bx+15a^{4}b^{2}x^{2}+20a^{3}b^{3}x^{3}+15a^{2}b^{4}x^{4}+6ab^{5}x^{5}+b^{6}x^{6}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(a + b*x)^(4/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (81 a ^ 11 (1 - ((a + b x) / a) ^ (1 / 3)) + 27 a ^ 10 b x (1
8 - 17 ((a + b x) / a) ^ (1 / 3)) + 9 a ^ 9 b ^ 2 x ^ 2 (135 - 119 ((a + b
x) / a) ^ (1 / 3)) + a ^ 8 b ^ 3 x ^ 3 (1620 - 1309 ((a + b x) / a) ^ (1 /
3)) + 14 a ^ 5 b ^ 4 x ^ 4 (-19 a ^ 2 + 271 a b x + 839 b ^ 2 x ^ 2) ((a +
b x) / a) ^ (1 / 3) + 1215 a ^ 7 b ^ 4 x ^ 4 + 2 a ^ 4 b ^ 5 x ^ 5 (243 a ^
 2 + 9427 b ^ 2 x ^ 2 ((a + b x) / a) ^ (1 / 3)) + a ^ 3 b ^ 6 x ^ 6 (81 a
^ 2 + 18091 b ^ 2 x ^ 2 ((a + b x) / a) ^ (1 / 3)) + 10409 a ^ 2 b ^ 9 x ^
9 ((a + b x) / a) ^ (1 / 3) + 3325 a b ^ 10 x ^ 10 ((a + b x) / a) ^ (1 / 3
) + 455 b ^ 11 x ^ 11 ((a + b x) / a) ^ (1 / 3)) / (7280 b ^ 4 (a ^ 6 + 6 a
 ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^
 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{7}{3}}\left(-455b^{3}x^{3}+315a\,b^{2}x^{2}-189a^{2}bx+81a^{3}\right)}{7280b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{16}{3}}}{16}-\frac{9a(bx+a)^{\frac{13}{3}}}{13}+\frac{9a^{2}(bx+a)^{\frac{10}{3}}}{10}-\frac{3a^{3}(bx+a)^{\frac{7}{3}}}{7}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{16}{3}}}{16}-\frac{9a(bx+a)^{\frac{13}{3}}}{13}+\frac{9a^{2}(bx+a)^{\frac{10}{3}}}{10}-\frac{3a^{3}(bx+a)^{\frac{7}{3}}}{7}}{b^{4}}$	50
trager	$-\frac{3\left(-455b^{5}x^{5}-595a\,b^{4}x^{4}-14a^{2}b^{3}x^{3}+18a^{3}b^{2}x^{2}-27a^{4}bx+81a^{5}\right)(bx+a)^{\frac{1}{3}}}{7280b^{4}}$	65
risch	$-\frac{3\left(-455b^{5}x^{5}-595a\,b^{4}x^{4}-14a^{2}b^{3}x^{3}+18a^{3}b^{2}x^{2}-27a^{4}bx+81a^{5}\right)(bx+a)^{\frac{1}{3}}}{7280b^{4}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/16*(b*x+a)^(16/3)-3/13*a*(b*x+a)^(13/3)+3/10*a^2*(b*x+a)^(10/3)-1/
7*a^3*(b*x+a)^(7/3))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{16}{3}}}{16\,b^4}-\frac{9\,(bx+a)^{\frac{13}{3}}a}{13\,b^4}+\frac{9\,(bx+a)^{\frac{10}{3}}a^2}{10\,b^4}-\frac{3\,(bx+a)^{\frac{7}{3}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x + a)^(16/3)/b^4 - 9/13*(b*x + a)^(13/3)*a/b^4 + 9/10*(b*x + a)^(1
0/3)*a^2/b^4 - 3/7*(b*x + a)^(7/3)*a^3/b^4
```

Fricas [A]
time = 0.30, size = 64, normalized size = 0.89

$$\frac{3\,(455\,b^5x^5+595\,ab^4x^4+14\,a^2b^3x^3-18\,a^3b^2x^2+27\,a^4bx-81\,a^5)(bx+a)^{\frac{1}{3}}}{7280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^5*x^5 + 595*a*b^4*x^4 + 14*a^2*b^3*x^3 - 18*a^3*b^2*x^2 + 27*
a^4*b*x - 81*a^5)*(b*x + a)^(1/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1844 vs. 2(68) = 136.
time = 1.48, size = 1844, normalized size = 25.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(4/3),x)
```

```
[Out] -243*a**(76/3)*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 1
09200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 4
3680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 243*a**(76/3)/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) -
 1377*a**(73/3)*b*x*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*
x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**
4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(73/3)*b*x/(72
80*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*
b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**
10*x**6) - 3213*a**(70/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 4
3680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 10920
0*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a
**(70/3)*b**2*x**2/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**
```

```
6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9
*x**5 + 7280*a**14*b**10*x**6) - 3927*a**(67/3)*b**3*x**3*(1 + b*x/a)**(1/3
)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a
**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**1
4*b**10*x**6) + 4860*a**(67/3)*b**3*x**3/(7280*a**20*b**4 + 43680*a**19*b**
5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x
**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) - 798*a**(64/3)*b**4*x
**4*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18
*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*
b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a**(64/3)*b**4*x**4/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) +
 11382*a**(61/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**1
9*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b
**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(61/3)*
b**5*x**5/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 +
145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7
280*a**14*b**10*x**6) + 35238*a**(58/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(7280*
a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**
7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*
x**6) + 243*a**(58/3)*b**6*x**6/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109
200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 436
80*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 56562*a**(55/3)*b**7*x**7*(1
+ b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x
**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x*
*5 + 7280*a**14*b**10*x**6) + 54273*a**(52/3)*b**8*x**8*(1 + b*x/a)**(1/3)/
(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**
17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*
b**10*x**6) + 31227*a**(49/3)*b**9*x**9*(1 + b*x/a)**(1/3)/(7280*a**20*b**4
 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 1
09200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 99
75*a**(46/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*
b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**
8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1365*a**(43/3)*b*
*11*x**11*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200
*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*
a**15*b**9*x**5 + 7280*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(56) = 112$.

time = 0.00, size = 325, normalized size = 4.51

$$\frac{\frac{3b^2\left(\frac{1}{16}(a+bx)^{\frac{1}{3}}(a+bx)^5-\frac{5}{13}(a+bx)^{\frac{1}{3}}(a+bx)^4a+(a+bx)^{\frac{1}{3}}(a+bx)^3a^2-\frac{10}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a^3+\frac{5}{4}(a+bx)^{\frac{1}{3}}(a+bx)a^4-(a+bx)^{\frac{1}{3}}a^5\right)}{b^5}+\frac{6ab\left(\frac{1}{13}(a+bx)^{\frac{1}{3}}(a+bx)^4-\frac{2}{5}(a+bx)^{\frac{1}{3}}(a+bx)^3a+\frac{6}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a^2-(a+bx)^{\frac{1}{3}}(a+bx)a^3+(a+bx)^{\frac{1}{3}}a^4\right)}{b^4}+\frac{3a^2\left(\frac{1}{10}(a+bx)^{\frac{1}{3}}(a+bx)^3-\frac{3}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a+\frac{3}{4}(a+bx)^{\frac{1}{3}}(a+bx)a^2-(a+bx)^{\frac{1}{3}}a^3\right)}{b^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x)
```

```
[Out] 3/7280*(52*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a^2/b^3 + 32*(35*(b*x + a)^(13/3) - 182*(b
*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*
(b*x + a)^(1/3)*a^4)*a/b^3 + 5*(91*(b*x + a)^(16/3) - 560*(b*x + a)^(13/3)*
a + 1456*(b*x + a)^(10/3)*a^2 - 2080*(b*x + a)^(7/3)*a^3 + 1820*(b*x + a)^(
4/3)*a^4 - 1456*(b*x + a)^(1/3)*a^5)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{16/3}}{16\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{13/3}}{13\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(16/3))/(16*b^4) - (3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a
 + b*x)^(10/3))/(10*b^4) - (9*a*(a + b*x)^(13/3))/(13*b^4)
```

3.386 $\int x^2(a+bx)^{4/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}$$

```
[Out] 3/7*a^2*(b*x+a)^(7/3)/b^3-3/5*a*(b*x+a)^(10/3)/b^3+3/13*(b*x+a)^(13/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a +
b*x)^(13/3))/(13*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{4/3}\,dx&=\int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{7/3}\left(9a^2-21abx+35b^2x^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(9*a^2 - 21*a*b*x + 35*b^2*x^2))/(455*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 225 vs. $2(53) = 106$.
time = 7.87, size = 207, normalized size = 3.91

$$\frac{3a^{\frac{1}{3}}\left(9a^7\left(-1+\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+3a^6bx\left(-9+8\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^5b^2x^2\left(-27+20\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^2b^3x^3\left(-9a^2+254b^2x^2\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+5a^3b^3x^3(11a+37bx)\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+154ab^6x^6\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+35b^7x^7\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{455b^3(a^3+3a^2bx+3ab^2x^2+b^3x^3)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(a + b*x)^(4/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (9 a ^ 7 (-1 + ((a + b x) / a) ^ (1 / 3)) + 3 a ^ 6 b x (-9 +
 8 ((a + b x) / a) ^ (1 / 3)) + a ^ 5 b ^ 2 x ^ 2 (-27 + 20 ((a + b x) / a)
 ^ (1 / 3)) + a ^ 2 b ^ 3 x ^ 3 (-9 a ^ 2 + 254 b ^ 2 x ^ 2 ((a + b x) / a)
 ^ (1 / 3)) + 5 a ^ 3 b ^ 3 x ^ 3 (11 a + 37 b x) ((a + b x) / a) ^ (1 / 3)
 + 154 a b ^ 6 x ^ 6 ((a + b x) / a) ^ (1 / 3) + 35 b ^ 7 x ^ 7 ((a + b x)
/ a) ^ (1 / 3)) / (455 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3
 x ^ 3))
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{7}{3}}(35x^2b^2-21abx+9a^2)}{455b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{3a(bx+a)^{\frac{10}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{7}{3}}}{7}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{3a(bx+a)^{\frac{10}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{7}{3}}}{7}}{b^3}$	38
trager	$\frac{3(35b^4x^4+49a\,b^3x^3+2a^2b^2x^2-3a^3bx+9a^4)(bx+a)^{\frac{1}{3}}}{455b^3}$	54
risch	$\frac{3(35b^4x^4+49a\,b^3x^3+2a^2b^2x^2-3a^3bx+9a^4)(bx+a)^{\frac{1}{3}}}{455b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/13*(b*x+a)^(13/3)-1/5*a*(b*x+a)^(10/3)+1/7*a^2*(b*x+a)^(7/3))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.77

$$\frac{3(bx+a)^{\frac{13}{3}}}{13\,b^3}-\frac{3(bx+a)^{\frac{10}{3}}a}{5\,b^3}+\frac{3(bx+a)^{\frac{7}{3}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^3 - 3/5*(b*x + a)^(10/3)*a/b^3 + 3/7*(b*x + a)^(7/3
)*a^2/b^3
```

Fricas [A]
time = 0.30, size = 53, normalized size = 1.00

$$\frac{3\,(35\,b^4x^4+49\,ab^3x^3+2\,a^2b^2x^2-3\,a^3bx+9\,a^4)(bx+a)^{\frac{1}{3}}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^4*x^4 + 49*a*b^3*x^3 + 2*a^2*b^2*x^2 - 3*a^3*b*x + 9*a^4)*(b*x
+ a)^(1/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(49) = 98$.
time = 1.03, size = 733, normalized size = 13.83

$$\frac{27a^{\frac{37}{3}}\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{37}{3}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{72a^{\frac{34}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{81a^{\frac{34}{3}}bx}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{60a^{\frac{31}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{81a^{\frac{31}{3}}b^2x^2}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{165a^{\frac{28}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{28}{3}}b^3x^3}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{555a^{\frac{25}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{762a^{\frac{22}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{462a^{\frac{19}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{105a^{\frac{16}{3}}b^7x^7\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(4/3),x)
```

```
[Out] 27*a**(37/3)*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a*
*6*b**5*x**2 + 455*a**5*b**6*x**3) - 27*a**(37/3)/(455*a**8*b**3 + 1365*a**
7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 72*a**(34/3)*b*x*(1
+ b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 4
55*a**5*b**6*x**3) - 81*a**(34/3)*b*x/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 60*a**(31/3)*b**2*x**2*(1 + b*x/
a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**
5*b**6*x**3) - 81*a**(31/3)*b**2*x**2/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 165*a**(28/3)*b**3*x**3*(1 + b*x
/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a*
*5*b**6*x**3) - 27*a**(28/3)*b**3*x**3/(455*a**8*b**3 + 1365*a**7*b**4*x +
1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 555*a**(25/3)*b**4*x**4*(1 + b*
x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a
**5*b**6*x**3) + 762*a**(22/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(455*a**8*b**3
+ 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 462*a**(19
/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a
**6*b**5*x**2 + 455*a**5*b**6*x**3) + 105*a**(16/3)*b**7*x**7*(1 + b*x/a)**
(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b*
*6*x**3)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(41) = 82$.
time = 0.00, size = 256, normalized size = 4.83

$$\frac{\frac{3b^2\left(\frac{1}{13}(a+bx)^{\frac{1}{3}}(a+bx)^4-\frac{2}{5}(a+bx)^{\frac{1}{3}}(a+bx)^3a+\frac{6}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a^2-(a+bx)^{\frac{1}{3}}(a+bx)a^3+(a+bx)^{\frac{1}{3}}a^4\right)}{b^4}+\frac{6ab\left(\frac{1}{10}(a+bx)^{\frac{1}{3}}(a+bx)^3-\frac{3}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a+\frac{3}{4}(a+bx)^{\frac{1}{3}}(a+bx)a^2-(a+bx)^{\frac{1}{3}}a^3\right)}{b^3}+\frac{3a^2\left(\frac{1}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2-\frac{1}{2}(a+bx)^{\frac{1}{3}}(a+bx)a+(a+bx)^{\frac{1}{3}}a^2\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x)
```

```
[Out] 3/910*(65*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a^2/b^2 + 13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^
(4/3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^2 + 2*(35*(b*x + a)^(13/3) - 182*(
b*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455
*(b*x + a)^(1/3)*a^4)/b^2)/b
```

Mupad [**B**]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{105\,{\left(a+b\,x\right)}^{13/3}-273\,a\,{\left(a+b\,x\right)}^{10/3}+195\,a^2\,{\left(a+b\,x\right)}^{7/3}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(4/3),x)
```

```
[Out] (105*(a + b*x)^(13/3) - 273*a*(a + b*x)^(10/3) + 195*a^2*(a + b*x)^(7/3))/(
455*b^3)
```

3.387 $\int x(a+bx)^{4/3}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}$$

```
[Out] -3/7*a*(b*x+a)^(7/3)/b^2+3/10*(b*x+a)^(10/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{4/3}\,dx &= \int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}(-3a+7bx)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-3*a + 7*b*x))/(70*b^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.07, size = 53, normalized size = 1.56

$$\text{Piecewise}\left[\left\{\left\{\frac{3\left(-3a^3+a^2bx+b^2x^2\left(11a+7bx\right)\right)\left(a+bx\right)^{\frac{1}{3}}}{70b^2},b!{=}0\right\}\right\},\frac{a^{\frac{4}{3}}x^2}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x*(a + b*x)^(4/3),x]')
```

```
[Out] Piecewise[{{3 (-3 a ^ 3 + a ^ 2 b x + b ^ 2 x ^ 2 (11 a + 7 b x)) (a + b x)
 ^ (1 / 3) / (70 b ^ 2), b != 0}}, a ^ (4 / 3) x ^ 2 / 2]
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{7}{3}}(-7bx+3a)}{70b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{3a(bx+a)^{\frac{7}{3}}}{7}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{3a(bx+a)^{\frac{7}{3}}}{7}}{b^2}$	26
trager	$-\frac{3(-7b^3x^3-11a\,b^2x^2-a^2bx+3a^3)(bx+a)^{\frac{1}{3}}}{70b^2}$	43
risch	$-\frac{3(-7b^3x^3-11a\,b^2x^2-a^2bx+3a^3)(bx+a)^{\frac{1}{3}}}{70b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/10*(b*x+a)^(10/3)-1/7*a*(b*x+a)^(7/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{3\left(bx+a\right)^{\frac{10}{3}}}{10\,b^2}-\frac{3\left(bx+a\right)^{\frac{7}{3}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^2 - 3/7*(b*x + a)^(7/3)*a/b^2
```

Fricas [A]
time = 0.32, size = 41, normalized size = 1.21

$$\frac{3\left(7\,b^3x^3+11\,ab^2x^2+a^2bx-3\,a^3\right)(bx+a)^{\frac{1}{3}}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/70*(7*b^3*x^3 + 11*a*b^2*x^2 + a^2*b*x - 3*a^3)*(b*x + a)^(1/3)/b^2
```

Sympy [A]
time = 0.23, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{9a^3\sqrt[3]{a+bx}}{70b^2}+\frac{3a^2x\sqrt[3]{a+bx}}{70b}+\frac{33ax^2\sqrt[3]{a+bx}}{70}+\frac{3bx^3\sqrt[3]{a+bx}}{10} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x)**(1/3)/(70*b**2) + 3*a**2*x*(a + b*x)**(1/3)/(
70*b) + 33*a*x**2*(a + b*x)**(1/3)/70 + 3*b*x**3*(a + b*x)**(1/3)/10, Ne(b,
 0)), (a**(4/3)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(26) = 52.
time = 0.00, size = 187, normalized size = 5.50

$$\frac{\frac{3b^2\left(\frac{1}{10}(a+bx)^{\frac{1}{3}}(a+bx)^3-\frac{3}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2a+\frac{3}{4}(a+bx)^{\frac{1}{3}}(a+bx)a^2-(a+bx)^{\frac{1}{3}}a^3\right)}{b^3}+\frac{6ab\left(\frac{1}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2-\frac{1}{2}(a+bx)^{\frac{1}{3}}(a+bx)a+(a+bx)^{\frac{1}{3}}a^2\right)}{b^2}+\frac{3a^2\left(\frac{1}{4}(a+bx)^{\frac{1}{3}}(a+bx)-a(a+bx)^{\frac{1}{3}}\right)}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x)
```

```
[Out] 3/140*(35*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a^2/b + 20*(2*(b*x + a)^(
7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)*a/b + (14*(b*x + a)^(1
0/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2 - 140*(b*x + a)^(1/3)
*a^3)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{30\,a\,{\left(a+b\,x\right)}^{7/3}-21\,{\left(a+b\,x\right)}^{10/3}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(4/3),x)
```

```
[Out] -(30*a*(a + b*x)^(7/3) - 21*(a + b*x)^(10/3))/(70*b^2)
```

3.388 $\int (a+bx)^{4/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{7/3}}{7b}$$

[Out] 3/7*(b*x+a)^(7/3)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(4/3),x]

[Out] (3*(a + b*x)^(7/3))/(7*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{4/3}\,dx = \frac{3(a+bx)^{7/3}}{7b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(4/3),x]

[Out] (3*(a + b*x)^(7/3))/(7*b)

Mathics [A]
time = 1.59, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{7}{3}}}{7b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(4/3),x]')
[Out] 3 (a + b x) ^ (7 / 3) / (7 b)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
default	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
trager	$\frac{3(x^2b^2+2abx+a^2)(bx+a)^{\frac{1}{3}}}{7b}$	29
risch	$\frac{3(x^2b^2+2abx+a^2)(bx+a)^{\frac{1}{3}}}{7b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/7*(b*x+a)^(7/3)/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="maxima")
[Out] 3/7*(b*x + a)^(7/3)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.31, size = 28, normalized size = 1.75

$$\frac{3\,(b^2x^2+2\,abx+a^2)(bx+a)^{\frac{1}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="fricas")
[Out] 3/7*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{3(a+bx)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3),x)
```

```
[Out] 3*(a + b*x)**(7/3)/(7*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.
time = 0.00, size = 112, normalized size = 7.00

$$\frac{6a\left(\frac{1}{4}(a+bx)^{\frac{1}{3}}(a+bx)-a(a+bx)^{\frac{1}{3}}\right)+\frac{3b^2\left(\frac{1}{7}(a+bx)^{\frac{1}{3}}(a+bx)^2-\frac{1}{2}(a+bx)^{\frac{1}{3}}(a+bx)a+(a+bx)^{\frac{1}{3}}a^2\right)}{b^2}+3a^2(a+bx)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x)
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 28*(b*x + a)^(1/3)*a^2 + 7*
((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{7/3}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

3.389 $\int \frac{(a+bx)^{4/3}}{x}\,dx$

Optimal. Leaf size=105

$$3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 3*a*(b*x+a)^(1/3)+3/4*(b*x+a)^(4/3)-1/2*a^(4/3)*ln(x)+3/2*a^(4/3)*ln(a^(1/3
)-(b*x+a)^(1/3))-a^(4/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/
2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 59, 631, 210, 31}

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x,x]
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 - Sqrt[3]*a^(4/3)*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] - (a^(4/3)*Log[x])/2 + (3*a^(4/3
)*Log[a^(1/3) - (a + b*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{x}\,dx &= \frac{3}{4}(a+bx)^{4/3}+a\int \frac{\sqrt[3]{a+bx}}{x}\,dx\\
&= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}+a^2\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{2}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+\right.\\
&= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(3a^{4/3}\right)\text{Su}\\
&= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 122, normalized size = 1.16

$$\frac{3}{4}\sqrt[3]{a+bx}\,(5a+bx)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x,x]
```

```
[Out] (3*(a + b*x)^(1/3)*(5*a + b*x))/4 - Sqrt[3]*a^(4/3)*ArcTan[(1 + (2*(a + b*x
)^(1/3))/a^(1/3))/Sqrt[3]] + a^(4/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(4
/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.24, size = 118, normalized size = 1.12

$$-a^{\frac{4}{3}}\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]+a^{\frac{4}{3}}\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]+\frac{15ab^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{4}+\frac{3b^{\frac{4}{3}}x\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{4}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)/x,x]')
```

```
[Out] -1 ^ (1 / 3) a ^ (4 / 3) Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_pola
r[2 I / 3 Pi] / a ^ (1 / 3)] + -1 ^ (2 / 3) a ^ (4 / 3) Log[1 - b ^ (1 / 3)
 (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] + a ^ (4 / 3) L
og[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] + 15 a b ^ (1 / 3)
(a / b + x) ^ (1 / 3) / 4 + 3 b ^ (4 / 3) x (a / b + x) ^ (1 / 3) / 4
```

Maple [A]

time = 0.09, size = 102, normalized size = 0.97

method	result
derivativedivides	$\frac{3(bx+a)^{\frac{4}{3}}}{4}+3a\left(bx+a\right)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}}{}\right.}{3}\right.$
default	$\frac{3(bx+a)^{\frac{4}{3}}}{4}+3a\left(bx+a\right)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}}{}\right.}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(b*x+a)^(4/3)+3*a*(b*x+a)^(1/3)+3*(1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3)
)-1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3/a^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))*a^2
```

Maxima [A]

time = 0.35, size = 96, normalized size = 0.91

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+a)^{\frac{4}{3}}+3\,(bx+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + a)^(4/3) + 3*(b*x + a)^(1
/3)*a
```

Fricas [A]
time = 0.32, size = 98, normalized size = 0.93

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+5\,a)(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + 5*a)*(b*x + a)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.40, size = 209, normalized size = 1.99

$$\frac{7a^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{7}{3}\right)}{\Gamma\left(\frac{10}{3}\right)}+\frac{7b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}{4\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x,x)
```

```
[Out] 7*a**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(3*gamma(
10/3)) + 7*a**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(3*gamma(10/3)) + 7*a**(4/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/
3)/(3*gamma(10/3)) + 7*a*b**(1/3)*(a/b + x)**(1/3)*gamma(7/3)/gamma(10/3) +
 7*b**(4/3)*(a/b + x)**(4/3)*gamma(7/3)/(4*gamma(10/3))
```

Giac [A]
time = 0.01, size = 158, normalized size = 1.50

$$\frac{3}{4}\,(a+bx)^{\frac{1}{3}}\,(a+bx)+3\,(a+bx)^{\frac{1}{3}}\,a-\frac{1}{2}aa^{\frac{1}{3}}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}\,(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)-\sqrt{3}\,aa^{\frac{1}{3}}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)+\frac{3a^{2}a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x)
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/4*(b*x + a)^(4/3) + 3*(b*x +
a)^(1/3)*a
```

Mupad [B]
time = 0.06, size = 123, normalized size = 1.17

$$3\,a\,{\left(a+b\,x\right)}^{1/3}+\frac{3\,{\left(a+b\,x\right)}^{4/3}}{4}+a^{4/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{7/3}\right)+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-9\,a^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+9\,a^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x,x)
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 + a^(4/3)*log(9*a^2*(a + b*x)^(
1/3) - 9*a^(7/3)) + (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i - 1))/2 - 9*a^2*(a
+ b*x)^(1/3))*(3^(1/2)*1i - 1))/2 - (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i + 1
))/2 + 9*a^2*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/2
```

3.390 $\int \frac{(a+bx)^{4/3}}{x^2}\,dx$

Optimal. Leaf size=107

$$4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 4*b*(b*x+a)^(1/3)-(b*x+a)^(4/3)/x-2/3*a^(1/3)*b*ln(x)+2*a^(1/3)*b*ln(a^(1/3
)-(b*x+a)^(1/3))-4/3*a^(1/3)*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)
*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 59, 631, 210, 31}

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] 4*b*(a + b*x)^(1/3) - (a + b*x)^(4/3)/x - (4*a^(1/3)*b*ArcTan[(a^(1/3) + 2*
(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log[x])/3 + 2*a
^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{4/3}}{x^2}\,dx &= -\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4b)\int \frac{\sqrt[3]{a+bx}}{x}\,dx \\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4ab)\int \frac{1}{x(a+bx)^{2/3}}\,dx \\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)-\left(2\sqrt[3]{a}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right. \\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(4\sqrt[3]{a}\,b\right)\text{S} \\
&= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 128, normalized size = 1.20

$$\frac{1}{3}\left(-\frac{3(a-3bx)\sqrt[3]{a+bx}}{x}-4\sqrt{3}\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-2\sqrt[3]{a}\,b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] ((-3*(a - 3*b*x)*(a + b*x)^(1/3))/x - 4*Sqrt[3]*a^(1/3)*b*ArcTan[(1 + (2*(a
 + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3
)] - 2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/
3
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 8.33, size = 125, normalized size = 1.17

$$-\frac{ab^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{x}-\frac{a^{\frac{1}{3}}b\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3}-\frac{a^{\frac{1}{3}}b\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3}+\frac{4a^{\frac{1}{3}}b\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{3}+3b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)/x^2,x]')
```

```
[Out] -a b ^ (1 / 3) (a / b + x) ^ (1 / 3) / x - 4 -1 ^ (1 / 3) a ^ (1 / 3) b Log
[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)]
 / 3 + 4 -1 ^ (2 / 3) a ^ (1 / 3) b Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 /
3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / 3 + 4 a ^ (1 / 3) b Log[1 - b ^ (
1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] / 3 + 3 b ^ (4 / 3) (a / b + x)
 ^ (1 / 3)
```

Maple [A]
time = 0.15, size = 106, normalized size = 0.99

method	result
derivativedivides	$3b\left((bx+a)^{\frac{1}{3}}-a\left(\frac{(bx+a)^{\frac{1}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{4\sqrt{3}\ \text{arcta}}{}\right.\right.$

default	$3b\left((bx+a)^{\frac{1}{3}}-a\left(\frac{(bx+a)^{\frac{1}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{4\sqrt{3}\arctan}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*((b*x+a)^(1/3)-a*(1/3*(b*x+a)^(1/3)/b/x-4/9/a^(2/3)*ln((b*x+a)^(1/3)-a^
(1/3))+2/9/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+4/9/a^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.36, size = 104, normalized size = 0.97

$$-\frac{4}{3}\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{2}{3}a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{4}{3}a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3(bx+a)^{\frac{1}{3}}b-\frac{(bx+a)^{\frac{1}{3}}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(
1/3)) - 2/3*a^(1/3)*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/
3)) + 4/3*a^(1/3)*b*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)*b -
(b*x + a)^(1/3)*a/x
```

Fricas [A]
time = 0.31, size = 111, normalized size = 1.04

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+2a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3(3bx-a)(bx+a)^{\frac{1}{3}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + 2*a^(1/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3
) + a^(2/3)) - 4*a^(1/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3*(3*b*x - a)
*(b*x + a)^(1/3))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 1.51, size = 719, normalized size = 6.72

$$\frac{28a^{\frac{10}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{84a^3b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{63a^2b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**2,x)
```

```
[Out] 28*a**(10/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*ga
mma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/
3)*gamma(10/3)) + 28*a**(10/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2
*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(10/3)*b*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*
a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/
3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**
(1/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*log(1 - b**(
1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(
2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(7/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) -
9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 84*a**3*b**(4/3)*(a/b + x)*
*(1/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*
b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 63*a**2*b**(7/3)*(a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3))
```

Giac [A]
time = 0.01, size = 176, normalized size = 1.64

$$\frac{3\left(a+bx\right)^{\frac{1}{3}}b^{2}-\frac{\left(a+bx\right)^{\frac{1}{3}}ab^{2}}{a+bx-a}-\frac{2}{3}a^{\frac{1}{3}}b^{2}\ln\left(\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}\left(a+bx\right)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)-\frac{4a^{\frac{1}{3}}b^{2}\arctan\left(\frac{2\left(\left(a+bx\right)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}}+\frac{4ab^{2}a^{\frac{1}{3}}\ln\left|\left(a+bx\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x)
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3)
)/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x + a)^(1/3) - a^(1/3))) - 9*(b*x + a)^
(1/3)*b^2 + 3*(b*x + a)^(1/3)*a*b/x)/b
```

Mupad [B]
time = 0.07, size = 131, normalized size = 1.22

$$3\,b\,{\left(a+b\,x\right)}^{1/3}+\frac{4\,a^{1/3}\,b\,\ln\left(12\,a^{4/3}\,b-12\,a\,b\,{\left(a+b\,x\right)}^{1/3}\right)}{3}-\frac{a\,{\left(a+b\,x\right)}^{1/3}}{x}+\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,{\left(a+b\,x\right)}^{1/3}-6\,a^{4/3}\,b\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{3}-\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,{\left(a+b\,x\right)}^{1/3}+6\,a^{4/3}\,b\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^2,x)
```

```
[Out] 3*b*(a + b*x)^(1/3) + (4*a^(1/3)*b*log(12*a^(4/3)*b - 12*a*b*(a + b*x)^(1/3
)))/3 - (a*(a + b*x)^(1/3))/x + (2*a^(1/3)*b*log(12*a*b*(a + b*x)^(1/3) - 6
*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (2*a^(1/3)*b*log(12*a*b*
(a + b*x)^(1/3) + 6*a^(4/3)*b*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/3
```

3.391 $\int \frac{(a+bx)^{4/3}}{x^3} \, dx$

Optimal. Leaf size=124

$$-\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}$$

```
[Out] -2/3*b*(b*x+a)^(1/3)/x-1/2*(b*x+a)^(4/3)/x^2-1/9*b^2*ln(x)/a^(2/3)+1/3*b^2*
ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3)-2/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/
3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 59, 631, 210, 31}

$$-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-2*b*(a + b*x)^(1/3))/(3*x) - (a + b*x)^(4/3)/(2*x^2) - (2*b^2*ArcTan[(a^(
1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^2*Lo
g[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{x^3}\,dx &= -\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{3}(2b)\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{9}\left(2b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{b^2\text{Sul}}{}\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{}{}\right.}{}\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\right.}{3a^{2/}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 136, normalized size = 1.10

$$\frac{1}{18}\left(-\frac{3\sqrt[3]{a+bx}\,(3a+7bx)}{x^2}-\frac{4\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{2/3}}-\frac{2b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{a^{2/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] ((-3*(a + b*x)^(1/3)*(3*a + 7*b*x))/x^2 - (4*Sqrt[3]*b^2*ArcTan[(1 + (2*(a
+ b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*b^2*Log[a^(1/3) - (a + b*x)^(
1/3)])/a^(2/3) - (2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(
2/3)])/a^(2/3))/18
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 21.91, size = 134, normalized size = 1.08

$$-\frac{ab^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{2x^2}-\frac{b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{2}{3}}}-\frac{b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{2}{3}}}+\frac{2b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{9a^{\frac{2}{3}}}-\frac{7b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{6x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)/x^3,x]')
```

```
[Out] -a b ^ (1 / 3) (a / b + x) ^ (1 / 3) / (2 x ^ 2) - 2 -1 ^ (1 / 3) b ^ 2 Log
[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)]
 / (9 a ^ (2 / 3)) + 2 -1 ^ (2 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^
 (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (2 / 3)) + 2 b ^ 2 L
og[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] / (9 a ^ (2 / 3)) -
 7 b ^ (4 / 3) (a / b + x) ^ (1 / 3) / (6 x)
```

Maple [A]

time = 0.14, size = 110, normalized size = 0.89

method	result
derivativedivides	$3b^2\left(-\frac{\frac{7(bx+a)^{\frac{4}{3}}}{18}-\frac{2a(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{2\sqrt{3}\,\arctan\left(\right.}{2}\right.$
default	$3b^2\left(-\frac{\frac{7(bx+a)^{\frac{4}{3}}}{18}-\frac{2a(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{2\sqrt{3}\,\arctan\left(\right.}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-(7/18*(b*x+a)^(4/3)-2/9*a*(b*x+a)^(1/3))/b^2/x^2+2/27/a^(2/3)*ln((b
*x+a)^(1/3)-a^(1/3))-1/27/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^
(2/3))-2/27/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
)
```

Maxima [A]
time = 0.36, size = 136, normalized size = 1.10

$$-\frac{2\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}}-\frac{b^{2}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{2}{3}}}+\frac{2\,b^{2}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{2}{3}}}-\frac{7\,(bx+a)^{\frac{4}{3}}b^{2}-4\,(bx+a)^{\frac{1}{3}}ab^{2}}{6\left((bx+a)^{2}-2\,(bx+a)a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - 1/6*(7*(b*x + a)
^(4/3)*b^2 - 4*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

Fricas [A]
time = 0.31, size = 162, normalized size = 1.31

$$-\frac{4\sqrt{3}\,(a^{2})^{\frac{1}{6}}ab^{2}x^{2}\arctan\left(\frac{(a^{2})^{\frac{1}{6}}\left(\sqrt{3}\,(a^{2})^{\frac{1}{3}}a+2\sqrt{3}\,(a^{2})^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^{2}}\right)+2\,(a^{2})^{\frac{2}{3}}b^{2}x^{2}\log\left((bx+a)^{\frac{2}{3}}a+(a^{2})^{\frac{1}{3}}a+(a^{2})^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-4\,(a^{2})^{\frac{2}{3}}b^{2}x^{2}\log\left((bx+a)^{\frac{1}{3}}a-(a^{2})^{\frac{2}{3}}\right)+3\,(7\,a^{2}bx+3\,a^{3})(bx+a)^{\frac{1}{3}}}{18\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 2*(a^2)^(2/3)*b^2
*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) -
 4*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x
+ 3*a^3)*(b*x + a)^(1/3))/(a^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.60, size = 2266, normalized size = 18.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**3,x)
```

```
[Out] 28*a**(19/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)
```

```
/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*ga
mma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*
b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3
)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a
**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b +
 x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma
(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**
3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(54
*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*exp(
-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*
gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*
I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*
(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a/b
 + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*ga
mma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*
pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) -
54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a
/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(
a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/
3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(10/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*g
amma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(
10/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)
*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a*
*(10/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3)
 - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)
**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(10/3)) + 84*a**6*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(7/3)/(54*
a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(1
```

```
0/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*
(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 231*a**5*b**(10/3)*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b
*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*p
i/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 1
47*a**4*b**(13/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*
I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**
3*exp(2*I*pi/3)*gamma(10/3))
```

Giac [A]
time = 0.01, size = 198, normalized size = 1.60

$$\frac{-\frac{\frac{1}{9}b^3\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^2+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{\left(a^{\frac{1}{3}}\right)^2}-\frac{\frac{1}{3}\cdot 2b^3\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\left(a^{\frac{1}{3}}\right)^2}+\frac{2b^3a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3\cdot 3a}+\frac{\frac{1}{6}\left(-7(a+bx)^{\frac{1}{3}}(a+bx)b^3+4(a+bx)^{\frac{1}{3}}ab^3\right)}{(a+bx-a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x)
```

```
[Out] -1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*x +
 a)^(4/3)*b^3 - 4*(b*x + a)^(1/3)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.12, size = 174, normalized size = 1.40

$$\frac{2\,b^2\,\ln\left(2\,b^2\,{\left(a+b\,x\right)}^{1/3}-2\,a^{1/3}\,b^2\right)}{9\,a^{2/3}}-\frac{\frac{7\,b^2\,{\left(a+b\,x\right)}^{4/3}}{6}-\frac{2\,a\,b^2\,{\left(a+b\,x\right)}^{1/3}}{3}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}-\frac{\ln\left(2\,b^2\,{\left(a+b\,x\right)}^{1/3}+a^{1/3}\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,a^{2/3}}+\frac{b^2\,\ln\left(2\,b^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{1/3}\,b^2\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^3,x)
```

```
[Out] (2*b^2*log(2*b^2*(a + b*x)^(1/3) - 2*a^(1/3)*b^2))/(9*a^(2/3)) - ((7*b^2*(a
 + b*x)^(4/3))/6 - (2*a*b^2*(a + b*x)^(1/3))/3)/((a + b*x)^2 - 2*a*(a + b*x
) + a^2) - (log(2*b^2*(a + b*x)^(1/3) + a^(1/3)*(3^(1/2)*b^2*1i + b^2))*(3^
(1/2)*b^2*1i + b^2))/(9*a^(2/3)) + (b^2*log(2*b^2*(a + b*x)^(1/3) - 9*a^(1/
3)*b^2*((3^(1/2)*1i)/9 - 1/9))*((3^(1/2)*1i)/9 - 1/9))/a^(2/3)
```

3.392 $\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}$$

```
[Out] -3/2*a^3*(b*x+a)^(2/3)/b^4+9/5*a^2*(b*x+a)^(5/3)/b^4-9/8*a*(b*x+a)^(8/3)/b^
4+3/11*(b*x+a)^(11/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*(
a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{2/3}\left(-81a^3+54a^2bx-45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-81*a^3 + 54*a^2*b*x - 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 323 vs. 2(72) = 144.
time = 15.34, size = 303, normalized size = 4.21

$$\frac{3a^{\frac{2}{3}}\left(81a^9\left(1-\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+54a^8bx\left(9-8\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+9a^7b^2x^2\left(135-104\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+20a^6b^3x^3\left(81-52\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+15ab^4x^4\left(81a^4+13b^4x^4\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)-570a^5b^4x^4\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+8a^2b^5x^5\left(3a^2+46abx+48b^2x^2\right)\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+486a^4b^5x^5+81a^3b^6x^6+40b^9x^9\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)}{440b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(a + b*x)^(1/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (81 a ^ 9 (1 - ((a + b x) / a) ^ (2 / 3)) + 54 a ^ 8 b x (9 -
 8 ((a + b x) / a) ^ (2 / 3)) + 9 a ^ 7 b ^ 2 x ^ 2 (135 - 104 ((a + b x) /
 a) ^ (2 / 3)) + 20 a ^ 6 b ^ 3 x ^ 3 (81 - 52 ((a + b x) / a) ^ (2 / 3)) +
 15 a b ^ 4 x ^ 4 (81 a ^ 4 + 13 b ^ 4 x ^ 4 ((a + b x) / a) ^ (2 / 3)) - 5
70 a ^ 5 b ^ 4 x ^ 4 ((a + b x) / a) ^ (2 / 3) + 8 a ^ 2 b ^ 5 x ^ 5 (3 a ^
 2 + 46 a b x + 48 b ^ 2 x ^ 2) ((a + b x) / a) ^ (2 / 3) + 486 a ^ 4 b ^ 5
 x ^ 5 + 81 a ^ 3 b ^ 6 x ^ 6 + 40 b ^ 9 x ^ 9 ((a + b x) / a) ^ (2 / 3)) /
 (440 b ^ 4 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x
^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$	43
trager	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$	43
risch	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{9a(bx+a)^{\frac{8}{3}}}{8}+\frac{9a^2(bx+a)^{\frac{5}{3}}}{5}-\frac{3a^3(bx+a)^{\frac{2}{3}}}{2}}{b^4}$	50
default	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{9a(bx+a)^{\frac{8}{3}}}{8}+\frac{9a^2(bx+a)^{\frac{5}{3}}}{5}-\frac{3a^3(bx+a)^{\frac{2}{3}}}{2}}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/11*(b*x+a)^(11/3)-3/8*a*(b*x+a)^(8/3)+3/5*a^2*(b*x+a)^(5/3)-1/2*a^
3*(b*x+a)^(2/3))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^4}-\frac{9\,(bx+a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^4}-\frac{3\,(bx+a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^4 - 9/8*(b*x + a)^(8/3)*a/b^4 + 9/5*(b*x + a)^(5/3)
*a^2/b^4 - 3/2*(b*x + a)^(2/3)*a^3/b^4
```

Fricas [A]
time = 0.30, size = 42, normalized size = 0.58

$$\frac{3\,(40\,b^3x^3-45\,ab^2x^2+54\,a^2bx-81\,a^3)(bx+a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 - 45*a*b^2*x^2 + 54*a^2*b*x - 81*a^3)*(b*x + a)^(2/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(68) = 136$.
time = 1.29, size = 1640, normalized size = 22.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 660
0*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**1
5*b**9*x**5 + 440*a**14*b**10*x**6) + 243*a**(71/3)/(440*a**20*b**4 + 2640*
a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**
8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1296*a**(68/3)*b*x*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(68/3)*b*x/(440*a**20*b**4 + 2640*a**19*b**5
*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2
640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 2808*a**(65/3)*b**2*x**2*(1 +
 b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 +
 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a
**14*b**10*x**6) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4 + 2640*a**19*b*
*5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 +
 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 3120*a**(62/3)*b**3*x**3*(1
```

```
 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2
 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440
*a**14*b**10*x**6) + 4860*a**(62/3)*b**3*x**3/(440*a**20*b**4 + 2640*a**19*
b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4
 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1710*a**(59/3)*b**4*x**4*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 72*a**(56/3)*b**5*x**5*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1104*a**(53/3)*b**6*x**
6*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*
x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 +
 440*a**14*b**10*x**6) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4 + 2640*a**
19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x
**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1152*a**(50/3)*b**7*x*
*7*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6
*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5
+ 440*a**14*b**10*x**6) + 585*a**(47/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(440*a
**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3
 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 12
0*a**(44/3)*b**9*x**9*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*
x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 26
40*a**15*b**9*x**5 + 440*a**14*b**10*x**6)
```

Giac [A]
time = 0.00, size = 95, normalized size = 1.32

$$\frac{3\left(\frac{1}{11}\left((a+bx)^{\frac{1}{3}}\right)^{2}(a+bx)^{3}-\frac{3}{8}\left((a+bx)^{\frac{1}{3}}\right)^{2}(a+bx)^{2}a+\frac{3}{5}\left((a+bx)^{\frac{1}{3}}\right)^{2}(a+bx)a^{2}-\frac{1}{2}\left((a+bx)^{\frac{1}{3}}\right)^{2}a^{3}\right)}{bb^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x)
```

```
[Out] 3/440*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a^
2 - 220*(b*x + a)^(2/3)*a^3)/b^4
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{2/3}}{2\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(11/3))/(11*b^4) - (3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a
 + b*x)^(5/3))/(5*b^4) - (9*a*(a + b*x)^(8/3))/(8*b^4)
```

3.393 $\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{8/3}}{8b^3}$$

```
[Out] 3/2*a^2*(b*x+a)^(2/3)/b^3-6/5*a*(b*x+a)^(5/3)/b^3+3/8*(b*x+a)^(8/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a + b
*x)^(8/3))/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{2/3}}{2b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{2/3}\left(9a^2-6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 187 vs. $2(53) = 106$.
time = 6.77, size = 171, normalized size = 3.23

$$\frac{3a^{\frac{2}{3}}\left(9a^5\left(-1+\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+3a^4bx\left(-9+7\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+a^3b^2x^2\left(-27+14\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+9ab^3x^3\left(-a+bx\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+6a^2b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+5b^5x^5\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)}{40b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(a + b*x)^(1/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (9 a ^ 5 (-1 + ((a + b x) / a) ^ (2 / 3)) + 3 a ^ 4 b x (-9 +
 7 ((a + b x) / a) ^ (2 / 3)) + a ^ 3 b ^ 2 x ^ 2 (-27 + 14 ((a + b x) / a)
 ^ (2 / 3)) + 9 a b ^ 3 x ^ 3 (-a + b x ((a + b x) / a) ^ (2 / 3)) + 6 a ^
2 b ^ 3 x ^ 3 ((a + b x) / a) ^ (2 / 3) + 5 b ^ 5 x ^ 5 ((a + b x) / a) ^ (
2 / 3)) / (40 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{2}{3}}\left(5x^2b^2-6abx+9a^2\right)}{40b^3}$	32
trager	$\frac{3(bx+a)^{\frac{2}{3}}\left(5x^2b^2-6abx+9a^2\right)}{40b^3}$	32
risch	$\frac{3(bx+a)^{\frac{2}{3}}\left(5x^2b^2-6abx+9a^2\right)}{40b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{6a(bx+a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{2}{3}}}{2}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{6a(bx+a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{2}{3}}}{2}}{b^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/8*(b*x+a)^(8/3)-2/5*a*(b*x+a)^(5/3)+1/2*a^2*(b*x+a)^(2/3))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.77

$$\frac{3(bx+a)^{\frac{8}{3}}}{8b^3}-\frac{6(bx+a)^{\frac{5}{3}}a}{5b^3}+\frac{3(bx+a)^{\frac{2}{3}}a^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^3 - 6/5*(b*x + a)^(5/3)*a/b^3 + 3/2*(b*x + a)^(2/3)*a
^2/b^3
```

Fricas [A]
time = 0.30, size = 31, normalized size = 0.58

$$\frac{3\,(5\,b^2x^2-6\,abx+9\,a^2)(bx+a)^{\frac{2}{3}}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 - 6*a*b*x + 9*a^2)*(b*x + a)^(2/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(49) = 98$.
time = 0.84, size = 600, normalized size = 11.32

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{32}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{63a^{\frac{29}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{29}{3}}bx}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{42a^{\frac{26}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{26}{3}}b^2x^2}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{18a^{\frac{23}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{23}{3}}b^3x^3}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{27a^{\frac{20}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{15a^{\frac{17}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*
b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(32/3)/(40*a**8*b**3 + 120*a**7*b**4
*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 + b*x/a)
**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6
*x**3) - 81*a**(29/3)*b*x/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x
**2 + 40*a**5*b**6*x**3) + 42*a**(26/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a*
*8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a*
*(26/3)*b**2*x**2/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40
*a**5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**8*b**3
+ 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*
b**3*x**3/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b*
*6*x**3) + 27*a**(20/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a*
*7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b**5*x**
5*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3)
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.30

$$\frac{3\left(\frac{1}{8}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)^2-\frac{2}{5}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)\,a+\frac{1}{2}\left((a+bx)^{\frac{1}{3}}\right)^2a^2\right)}{bb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x)
```

```
[Out] 3/40*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b^
3
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{15\,{\left(a+b\,x\right)}^{8/3}-48\,a\,{\left(a+b\,x\right)}^{5/3}+60\,a^2\,{\left(a+b\,x\right)}^{2/3}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/3),x)
```

```
[Out] (15*(a + b*x)^(8/3) - 48*a*(a + b*x)^(5/3) + 60*a^2*(a + b*x)^(2/3))/(40*b^
3)
```

3.394 $\int \frac{x}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{2/3}}{2b^2}+\frac{3(a+bx)^{5/3}}{5b^2}$$

```
[Out] -3/2*a*(b*x+a)^(2/3)/b^2+3/5*(b*x+a)^(5/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3(a+bx)^{5/3}}{5b^2}-\frac{3a(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{2/3}}{2b^2}+\frac{3(a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}(-3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(1/3),x]
```

[Out] (3*(a + b*x)^(2/3)*(-3*a + 2*b*x))/(10*b^2)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(34) = 68.
time = 3.19, size = 74, normalized size = 2.18

$$\frac{3a^{\frac{2}{3}}\left(3a^2\left(1-\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+abx\left(3-\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+2b^2x^2\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)}{10b^2\left(a+bx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/(a + b*x)^(1/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (3 a ^ 2 (1 - ((a + b x) / a) ^ (2 / 3)) + a b x (3 - ((a + b
 x) / a) ^ (2 / 3)) + 2 b ^ 2 x ^ 2 ((a + b x) / a) ^ (2 / 3)) / (10 b ^ 2
(a + b x))
```

Maple [A]
time = 0.12, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
trager	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
risch	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-\frac{3a(bx+a)^{\frac{2}{3}}}{2}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-\frac{3a(bx+a)^{\frac{2}{3}}}{2}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/5*(b*x+a)^(5/3)-1/2*a*(b*x+a)^(2/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^2}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^2 - 3/2*(b*x + a)^(2/3)*a/b^2
```

Fricas [A]
time = 0.32, size = 20, normalized size = 0.59

$$\frac{3\,(2\,bx-3\,a)(bx+a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x - 3*a)*(b*x + a)^(2/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(31) = 62$.
time = 0.57, size = 162, normalized size = 4.76

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{11}{3}}}{10a^2b^2+10ab^3x}-\frac{3a^{\frac{8}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{8}{3}}bx}{10a^2b^2+10ab^3x}+\frac{6a^{\frac{5}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x) + 9*a**(11/3)/
(10*a**2*b**2 + 10*a*b**3*x) - 3*a**(8/3)*b*x*(1 + b*x/a)**(2/3)/(10*a**2*b
**2 + 10*a*b**3*x) + 9*a**(8/3)*b*x/(10*a**2*b**2 + 10*a*b**3*x) + 6*a**(5/
3)*b**2*x**2*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x)
```

Giac [A]
time = 0.00, size = 42, normalized size = 1.24

$$\frac{3\left(\frac{1}{5}\left((a+bx)^{\frac{1}{3}}\right)^2(a+bx)-\frac{1}{2}\left((a+bx)^{\frac{1}{3}}\right)^2a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x)
```

```
[Out] 3/10*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)/b^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{15\,a\,{\left(a+b\,x\right)}^{2/3}-6\,{\left(a+b\,x\right)}^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/3),x)
```

```
[Out] -(15*a*(a + b*x)^(2/3) - 6*(a + b*x)^(5/3))/(10*b^2)
```

3.395 $\int \frac{1}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{2/3}}{2b}$$

```
[Out] 3/2*(b*x+a)^(2/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Mathics [A]
time = 1.57, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{2}{3}}}{2b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-1/3),x]')
```

```
[Out] 3 (a + b x) ^ (2 / 3) / (2 b)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
default	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
trager	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
risch	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(b*x+a)^(2/3)/b
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3),x)
```

```
[Out] 3*(a + b*x)**(2/3)/(2*b)
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.06

$$\frac{3\left((a+bx)^{\frac{1}{3}}\right)^{2}}{b\cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x)
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

3.396 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

```
[Out] -1/2*ln(x)/a^(1/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(1/3)+arctan(1/3*(a^(1/3
)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A]

time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {57, 631, 210, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = -\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \right.}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [A]
time = 0.04, size = 95, normalized size = 1.20

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3
) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2
/3)])/(2*a^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.46, size = 82, normalized size = 1.04

$$\frac{\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]+\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]-\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{a^{\frac{1}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a + b*x)^(1/3)),x]')
```

```
[Out] (Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] - -1 ^ (1 / 3) Lo
g[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)
] + -1 ^ (2 / 3) Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I /
3 Pi] / a ^ (1 / 3)]) / a ^ (1 / 3)
```

Maple [A]
time = 0.11, size = 75, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75
default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^
(1/3)+1))
```

Maxima [A]
time = 0.35, size = 76, normalized size = 0.96

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(1/3)
```

Fricas [A]
time = 0.32, size = 213, normalized size = 2.70

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\,\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a^
(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a
^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*a^(2
/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*log
((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x
+ a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.94, size = 155, normalized size = 1.96

$$\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/3),x)
```

```
[Out] 2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(
5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a**(1
/3)*gamma(5/3))
```

Giac [A]
time = 0.00, size = 137, normalized size = 1.73

$$3\left(-\frac{\left(a^{\frac{1}{3}}\right)^{2}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a}+\frac{\left(a^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a}+\frac{a^{\frac{1}{3}}a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x)
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B]
time = 0.09, size = 99, normalized size = 1.25

$$\frac{\ln\left(9\,{\left(a+b\,x\right)}^{1/3}-9\,a^{1/3}\right)}{a^{1/3}}+\frac{\ln\left(9\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{1/3}}-\frac{\ln\left(9\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(1/3) + (log(9*(a + b*x)^(1/3) - (9*a^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a^(1/3)) - (log(9*(a + b*
x)^(1/3) - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a^(1/3))
```

3.397 $\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=100

$$-\frac{(a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

```
[Out] -(b*x+a)^(2/3)/a/x+1/6*b*ln(x)/a^(4/3)-1/2*b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4
/3)-1/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(
1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 57, 631, 210, 31}

$$\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{ax} - \frac{b\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a}$$

$$= -\frac{(a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x\cdots}\right.}{2\cdots}$$

$$= -\frac{(a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a\cdots}}{\sqrt[3]{\cdots}}\right.}{a^{4/3}}$$

$$= -\frac{(a+bx)^{2/3}}{ax} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

Mathematica [A]
time = 0.13, size = 120, normalized size = 1.20

$$-\frac{6\sqrt[3]{a}\,(a+bx)^{2/3}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-bx\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*(6*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1
/3))/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(4/3)*x)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 9.77, size = 120, normalized size = 1.20

$$-\frac{(a+bx)^2}{ab^{\frac{4}{3}}x\left(\frac{a}{b}+x\right)^{\frac{4}{3}}}-\frac{b\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{3a^{\frac{4}{3}}}-\frac{b\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3a^{\frac{4}{3}}}+\frac{b\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3a^{\frac{4}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^(1/3)),x]')
```

```
[Out] -(a + b x) ^ 2 / (a b ^ (4 / 3) x (a / b + x) ^ (4 / 3)) - -1 ^ (2 / 3) b L
og[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3
)] / (3 a ^ (4 / 3)) - b Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1
 / 3)] / (3 a ^ (4 / 3)) + -1 ^ (1 / 3) b Log[1 - b ^ (1 / 3) (a / b + x) ^
 (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (3 a ^ (4 / 3))
```

Maple [A]
time = 0.12, size = 104, normalized size = 1.04

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{ax}-\frac{b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right.}{3a^{\frac{4}{3}}}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/3*(b*x+a)^(2/3)/a/b/x+1/3/a*(-1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))
+1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3*3^(1/2)/a^
(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.34, size = 106, normalized size = 1.06

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}b}{(bx+a)a-a^2}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}}-\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - (b*x + a)^(2/3)*b/((b*x + a)*a - a^2) + 1/6*b*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x + a)^(1/3) - a
^(1/3))/a^(4/3)
```

Fricas [A]
time = 0.32, size = 306, normalized size = 3.06

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6\,(bx+a)^{\frac{2}{3}}a}{6\,a^2x},\ -\frac{6\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2\,(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+6\,(bx+a)^{\frac{2}{3}}a}{6\,a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*x
 + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x + a)^
(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqrt(1/3
)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3
))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x + a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3) + (-a)^
(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.23, size = 831, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*p
i/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a
/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x
)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**
3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7
/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a*
*(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/
3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3
)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*
I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +
 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3))
```

Giac [A]
time = 0.00, size = 185, normalized size = 1.85

$$\frac{3\left(\frac{\left(a^{\frac{1}{3}}\right)^{2}b^{2}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{18a^{2}}-\frac{\frac{1}{3}b^{2}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,aa^{\frac{1}{3}}}-\frac{a^{\frac{1}{3}}b^{2}a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3\cdot 3a^{2}}-\frac{\frac{1}{3}\left((a+bx)^{\frac{1}{3}}\right)^{2}b^{2}}{a(a+bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x)
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2
/3)*b/(a*x))/b
```

Mupad [B]

time = 0.14, size = 130, normalized size = 1.30

$$-\frac{(a+b\,x)^{2/3}}{a\,x}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}-\frac{b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b - 3^(
1/2)*b*1i))/(6*a^(4/3)) - (a + b*x)^(2/3)/(a*x) + (log((b + 3^(1/2)*b*1i)^2
/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3)) -
 (b*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(4/3))
```

3.398 $\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=130

$$-\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}+\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}$$

```
[Out] -1/2*(b*x+a)^(2/3)/a/x^2+2/3*b*(b*x+a)^(2/3)/a^2/x-1/9*b^2*ln(x)/a^(7/3)+1/
3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(7/3)+2/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+
a)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 57, 631, 210, 31}

$$-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{(a+bx)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x)^(2/3)/(a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) -
 (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{2ax^2}-\frac{(2b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}+\frac{(2b^2)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^2}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{b^2\text{S}}{}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}-\frac{(2b^2)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}+\frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}\right.}{3a}$$

Mathematica [A]
time = 0.12, size = 149, normalized size = 1.15

$$-\frac{(a+bx)^{2/3}(7a-4(a+bx))}{6a^2x^2}+\frac{2b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{7/3}}-\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(7*a - 4*(a + b*x)))/(a^2*x^2) + (2*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(7/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 26.90, size = 139, normalized size = 1.07

$$-\frac{b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}{2ax^2}+\frac{2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}{3a^2x}-\frac{b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{7}{3}}}-\frac{b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{7}{3}}}+\frac{2b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{9a^{\frac{7}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^(1/3)),x]')
```

```
[Out] -b ^ (2 / 3) (a / b + x) ^ (2 / 3) / (2 a x ^ 2) + 2 b ^ (5 / 3) (a / b + x
) ^ (2 / 3) / (3 a ^ 2 x) - 2 -1 ^ (1 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b
 + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (7 / 3)) + 2
-1 ^ (2 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I
/ 3 Pi] / a ^ (1 / 3)] / (9 a ^ (7 / 3)) + 2 b ^ 2 Log[1 - b ^ (1 / 3) (a /
 b + x) ^ (1 / 3) / a ^ (1 / 3)] / (9 a ^ (7 / 3))
```

Maple [A]
time = 0.11, size = 130, normalized size = 1.00

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(-4bx+3a)}{6a^2x^2}+\frac{2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{\ }\right.}{9a^{\frac{7}{3}}}$

derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$
default	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-1/6/a*(b*x+a)^(2/3)/b^2/x^2-2/3/a*(-1/3*(b*x+a)^(2/3)/a/b/x+1/3/a*(
-1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)
*(b*x+a)^(1/3)+a^(2/3))-1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(
b*x+a)^(1/3)+1)))))
```

Maxima [A]

time = 0.37, size = 142, normalized size = 1.09

$$\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{4(bx+a)^{\frac{5}{3}}b^2-7(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a^2-2(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3) + 1/6*(4*(b*x + a)^
(5/3)*b^2 - 7*(b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 + a
^4)
```

Fricas [A]
time = 0.33, size = 296, normalized size = 2.28

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3(4abx-3a^2)(bx+a)^{\frac{2}{3}}}{18a^3x^2}, \frac{12\sqrt{\frac{1}{3}}a^{\frac{2}{3}}b^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3(4abx-3a^2)(bx+a)^{\frac{2}{3}}}{18a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(
b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*
(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3)
 - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt
(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^
2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.14, size = 2730, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/3),x)
```

```
[Out] 4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**
4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b**(1
0/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b
```

```
**(10/3)*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 8
1*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(1
3/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(
a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*
a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)
*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**
(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(7/3)*
exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/
3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3)
- 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(
10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b +
 x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(
7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*
b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3
)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(2
*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*ex
p(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a*
*7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*p
i/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3
)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(
7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I
*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(5/3)*b**(19/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3
)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3)
 - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/
3)*b**(19/3)*(a/b + x)**(13/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a*
*(5/3)*b**(19/3)*(a/b + x)**(13/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
```

```
lar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a
**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 21*a**4*b**4*(a
/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*
a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(
a/b + x)**3*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**2*b**6*
(a/b + x)**4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 2
7*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A]
time = 0.01, size = 218, normalized size = 1.68

$$\frac{3\left(-\frac{\frac{1}{27}b^{3}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{a^{2}a^{\frac{1}{3}}}+\frac{\frac{1}{9}\cdot 2\left(a^{\frac{1}{3}}\right)^{2}b^{3}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{3}}+\frac{2a^{\frac{1}{3}}b^{3}a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{9\cdot 3a^{3}}+\frac{\frac{1}{18}\left(4\left((a+bx)^{\frac{1}{3}}\right)^{2}(a+bx)b^{3}-7\left((a+bx)^{\frac{1}{3}}\right)^{2}ab^{3}\right)}{a^{2}(a+bx-a)^{2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x)
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x +
a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.23, size = 182, normalized size = 1.40

$$\frac{2\,b^2\,\ln\left({\left(a+b\,x\right)}^{1/3}-a^{1/3}\right)}{9\,a^{7/3}}-\frac{\frac{7\,b^2\,{\left(a+b\,x\right)}^{2/3}}{6\,a}-\frac{2\,b^2\,{\left(a+b\,x\right)}^{5/3}}{3\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}-\frac{\ln\left(\frac{4\,b^4\,{\left(a+b\,x\right)}^{1/3}}{9\,a^4}-\frac{{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}^2}{9\,a^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,a^{7/3}}+\frac{b^2\,\ln\left(\frac{4\,b^4\,{\left(a+b\,x\right)}^{1/3}}{9\,a^4}-\frac{9\,b^4\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2}{a^{11/3}}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/3)),x)
```

```
[Out] (2*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(7/3)) - ((7*b^2*(a + b*x)^(2/3
))/(6*a) - (2*b^2*(a + b*x)^(5/3))/(3*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) - (log((4*b^4*(a + b*x)^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(9*a
```

```
^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*a^(7/3)) + (b^2*log((4*b^4*(a + b*x)^(
1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1i)/9 - 1/9)^2)/a^(11/3))*((3^(1/2)*1i)/9
- 1/9))/a^(7/3)
```

3.399 $\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}$$

```
[Out] 3/2*a^3*(b*x-a)^(2/3)/b^4+9/5*a^2*(b*x-a)^(5/3)/b^4+9/8*a*(b*x-a)^(8/3)/b^4
+3/11*(b*x-a)^(11/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a*
(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^3}{b^3\sqrt[3]{-a+bx}}+\frac{3a^2(-a+bx)^{2/3}}{b^3}+\frac{3a(-a+bx)^{5/3}}{b^3}+\frac{(-a+bx)^{8/3}}{b^3}\right)dx\\ &= \frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.60

$$\frac{3(-a+bx)^{2/3}\left(81a^3+54a^2bx+45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(81*a^3 + 54*a^2*b*x + 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 23.06, size = 2502, normalized size = 31.28

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(-a + b*x)^(1/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (2 / 3) (81 a ^ 9 (-1 ^ (2 / 3) + ((-a + b x) / a) ^ (2 /
 3)) + 54 a ^ 8 b x (9 -1 ^ (2 / 3) - 8 ((-a + b x) / a) ^ (2 / 3)) + 9 a ^
 7 b ^ 2 x ^ 2 (-135 -1 ^ (2 / 3) + 104 ((-a + b x) / a) ^ (2 / 3)) + 20 a
^ 6 b ^ 3 x ^ 3 (81 -1 ^ (2 / 3) - 52 ((-a + b x) / a) ^ (2 / 3)) - 15 a b
^ 4 x ^ 4 (81 -1 ^ (2 / 3) a ^ 4 + 13 b ^ 4 x ^ 4 ((-a + b x) / a) ^ (2 / 3
)) + 570 a ^ 5 b ^ 4 x ^ 4 ((-a + b x) / a) ^ (2 / 3) + 8 a ^ 2 b ^ 5 x ^ 5
 (3 a ^ 2 - 46 a b x + 48 b ^ 2 x ^ 2) ((-a + b x) / a) ^ (2 / 3) + 486 -1
^ (2 / 3) a ^ 4 b ^ 5 x ^ 5 - 81 -1 ^ (2 / 3) a ^ 3 b ^ 6 x ^ 6 + 40 b ^ 9
x ^ 9 ((-a + b x) / a) ^ (2 / 3)) / (440 b ^ 4 (a ^ 6 - 6 a ^ 5 b x + 15 a
^ 4 b ^ 2 x ^ 2 - 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 - 6 a b ^ 5 x
 ^ 5 + b ^ 6 x ^ 6)), Abs[b x / a] > 1}}, -243 a ^ (71 / 3) (1 - b x / a) ^
 (2 / 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b
^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17
 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3)
 a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 243 a ^ (71
 / 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5
 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b
^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a
^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) - 1458 a ^ (68 /
 3) b x / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^
 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17
b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3)
a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 1296 a ^ (68
 / 3) b x (1 - b x / a) ^ (2 / 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640
 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8
800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x
^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b
^ 10 x ^ 6) - 2808 a ^ (65 / 3) b ^ 2 x ^ 2 (1 - b x / a) ^ (2 / 3) / (440
E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^
 (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6
600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x
```

^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 3645 a ^ (65 / 3) b ^ 2 x ^
2 / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x
+ 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7
x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 1
5 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) - 4860 a ^ (62 / 3)
b ^ 3 x ^ 3 / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 1
9 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a
^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi
/ 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 3120 a
^ (62 / 3) b ^ 3 x ^ 3 (1 - b x / a) ^ (2 / 3) / (440 E ^ (I Pi / 3) a ^ 20
b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b
^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a
^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi /
3) a ^ 14 b ^ 10 x ^ 6) - 1710 a ^ (59 / 3) b ^ 4 x ^ 4 (1 - b x / a) ^ (2
/ 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5
x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b
^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a
^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 3645 a ^ (59 /
3) b ^ 4 x ^ 4 / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a
^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3)
a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I
Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) - 1458
a ^ (56 / 3) b ^ 5 x ^ 5 / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I
Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^
(I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 26
40 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^
6) - 72 a ^ (56 / 3) b ^ 5 x ^ 5 (1 - b x / a) ^ (2 / 3) / (440 E ^ (I Pi
/ 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3
) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I
Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440
E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 243 a ^ (53 / 3) b ^ 6 x ^ 6 / (440 E
^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^
(I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 66
00 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^
5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 1104 a ^ (53 / 3) b ^ 6 x ^
6 (1 - b x / a) ^ (2 / 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I
Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^
(I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 26
40 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^
6) - 1152 a ^ (50 / 3) b ^ 7 x ^ 7 (1 - b x / a) ^ (2 / 3) / (440 E ^ (I P
i / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi /
3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^
(I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 44
0 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) + 585 a ^ (47 / 3) b ^ 8 x ^ 8 (1 - b
x / a) ^ (2 / 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ 4 - 2640 E ^ (I Pi / 3)

```
a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 18 b ^ 6 x ^ 2 - 8800 E ^ (I Pi /
3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (
I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ 10 x ^ 6) - 12
0 a ^ (44 / 3) b ^ 9 x ^ 9 (1 - b x / a) ^ (2 / 3) / (440 E ^ (I Pi / 3) a
^ 20 b ^ 4 - 2640 E ^ (I Pi / 3) a ^ 19 b ^ 5 x + 6600 E ^ (I Pi / 3) a ^ 1
8 b ^ 6 x ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ 7 x ^ 3 + 6600 E ^ (I Pi / 3
) a ^ 16 b ^ 8 x ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ 9 x ^ 5 + 440 E ^ (I
Pi / 3) a ^ 14 b ^ 10 x ^ 6)]
```

Maple [A]
time = 0.11, size = 58, normalized size = 0.72

method	result	size
gosper	$\frac{3\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)\left(bx-a\right)^{\frac{2}{3}}}{440b^{4}}$	45
trager	$\frac{3\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)\left(bx-a\right)^{\frac{2}{3}}}{440b^{4}}$	45
risch	$-\frac{3\left(-bx+a\right)\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)}{440b^{4}\left(bx-a\right)^{\frac{1}{3}}}$	51
derivativedivides	$\frac{\frac{3\left(bx-a\right)^{\frac{11}{3}}}{11}+\frac{9a\left(bx-a\right)^{\frac{8}{3}}}{8}+\frac{9a^{2}\left(bx-a\right)^{\frac{5}{3}}}{5}+\frac{3a^{3}\left(bx-a\right)^{\frac{2}{3}}}{2}}{b^{4}}$	58
default	$\frac{\frac{3\left(bx-a\right)^{\frac{11}{3}}}{11}+\frac{9a\left(bx-a\right)^{\frac{8}{3}}}{8}+\frac{9a^{2}\left(bx-a\right)^{\frac{5}{3}}}{5}+\frac{3a^{3}\left(bx-a\right)^{\frac{2}{3}}}{2}}{b^{4}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/11*(b*x-a)^(11/3)+3/8*a*(b*x-a)^(8/3)+3/5*a^2*(b*x-a)^(5/3)+1/2*a^
3*(b*x-a)^(2/3))
```

Maxima [A]
time = 0.26, size = 64, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{11}{3}}}{11\,b^4}+\frac{9\,(bx-a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx-a)^{\frac{5}{3}}a^2}{5\,b^4}+\frac{3\,(bx-a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x - a)^(11/3)/b^4 + 9/8*(b*x - a)^(8/3)*a/b^4 + 9/5*(b*x - a)^(5/3)
*a^2/b^4 + 3/2*(b*x - a)^(2/3)*a^3/b^4
```

Fricas [A]
time = 0.30, size = 44, normalized size = 0.55

$$\frac{3\,(40\,b^3x^3+45\,ab^2x^2+54\,a^2bx+81\,a^3)(bx-a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 45*a*b^2*x^2 + 54*a^2*b*x + 81*a^3)*(b*x - a)^(2/3)/b^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.37, size = 4974, normalized size = 62.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*ex
p(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3
) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2
640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a
**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600
*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**1
6*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**1
0*x**6*exp(I*pi/3)) - 1296*a**(68/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(4
40*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6
*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4
*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(
I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5
*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*ex
p(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*p
i/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2808*a**(65/3)*b**2*x**2*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
 + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b
**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(
I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/
3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) -
 3120*a**(62/3)*b**3*x**3*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/
3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) -
2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860
*a**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*
pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3)
 + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 44
0*a**14*b**10*x**6*exp(I*pi/3)) + 1710*a**(59/3)*b**4*x**4*(-1 + b*x/a)**(2
/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
 + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 66
00*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**
```

14*b**10*x**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I
*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640
*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 72*a**(5
6/3)*b**5*x**5*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3)
- 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a
17*b7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*
b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*
b**5*x**5/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 660
0*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**
16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**
10*x**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*x**6*(-1 + b*x/a)**(2/3)*exp(I*p
i/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**
18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b*
*8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x*
*6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4*exp(I*pi/3) - 264
0*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*
b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*
x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*
x**7*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**
19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*
x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*
exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*x**8*(
-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 120*a**(44/3)*b**9*x**9*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)), Abs(b*x/a) > 1), (-243*a**(71/3)*(1 -
b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640
*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b
7*x3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x
5*exp(I*pi/3) + 440*a14*b**10*x**6*exp(I*pi/3)) + 1296*a**(68/3)*b*x*(1
- b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3
) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6
600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a*
*14*b**10*x**6*exp(I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3
) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800
*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**1
5*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 2808*a**(65/3

)*b**2*x**2*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3120*a**(62/3)*b**3*x**3*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860*a**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*x**4*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 72*a**(56/3)*b**5*x**5*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1104*a**(53/3)*b**6*x**6*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1152*a**(50/3)*b**7*x**7*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 585*a**(47/3)*b**8*x**8*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*x**9*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)

```
+ 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 660
0*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**1
4*b**10*x**6*exp(I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 100, normalized size = 1.25

$$\frac{3\left(\frac{1}{11}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}\left(-a+bx\right)^{3}+\frac{3}{8}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}\left(-a+bx\right)^{2}a+\frac{3}{5}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}\left(-a+bx\right)a^{2}+\frac{1}{2}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}a^{3}\right)}{bb^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x)
```

```
[Out] 3/440*(40*(b*x - a)^(11/3) + 165*(b*x - a)^(8/3)*a + 264*(b*x - a)^(5/3)*a^
2 + 220*(b*x - a)^(2/3)*a^3)/b^4
```

Mupad [B]
time = 0.05, size = 64, normalized size = 0.80

$$\frac{3\,{\left(b\,x-a\right)}^{11/3}}{11\,b^4}+\frac{9\,a\,{\left(b\,x-a\right)}^{8/3}}{8\,b^4}+\frac{3\,a^3\,{\left(b\,x-a\right)}^{2/3}}{2\,b^4}+\frac{9\,a^2\,{\left(b\,x-a\right)}^{5/3}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x - a)^(1/3),x)
```

```
[Out] (3*(b*x - a)^(11/3))/(11*b^4) + (9*a*(b*x - a)^(8/3))/(8*b^4) + (3*a^3*(b*x
 - a)^(2/3))/(2*b^4) + (9*a^2*(b*x - a)^(5/3))/(5*b^4)
```

3.400 $\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}$$

```
[Out] 3/2*a^2*(b*x-a)^(2/3)/b^3+6/5*a*(b*x-a)^(5/3)/b^3+3/8*(b*x-a)^(8/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
+ b*x)^(8/3))/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{-a+bx}}+\frac{2a(-a+bx)^{2/3}}{b^2}+\frac{(-a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.63

$$\frac{3(-a+bx)^{2/3}\left(9a^2+6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(9*a^2 + 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 6.98, size = 837, normalized size = 14.19

$$\text{Piecewise}\left[\left\{\left\{\frac{3a^{\frac{2}{3}}\left(9a^5\left(-1^{\frac{2}{3}}+\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)+3a^4bx\left(9-1^{\frac{2}{3}}-7\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)+a^3b^2x^2\left(-27-1^{\frac{2}{3}}+14\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)+9ab^3x^3\left(-1^{\frac{2}{3}}a+bx\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)-6a^2b^3x^3\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}-5b^5x^5\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)}{40b^3\left(a^3-3a^2bx+3ab^2x^2-b^3x^3\right)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-27E^{\frac{I2\pi}{3}}a^{\frac{32}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}+\frac{27E^{\frac{I2\pi}{3}}a^{\frac{32}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}-\frac{63E^{\frac{I2\pi}{3}}a^{\frac{29}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}+\frac{81E^{\frac{I2\pi}{3}}a^{\frac{29}{3}}bx}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}-\frac{81E^{\frac{I2\pi}{3}}a^{\frac{26}{3}}b^2x^2}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}+\frac{42E^{\frac{I2\pi}{3}}a^{\frac{26}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}-\frac{18E^{\frac{I2\pi}{3}}a^{\frac{23}{3}}b^3x^3\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}+\frac{27E^{\frac{I2\pi}{3}}a^{\frac{23}{3}}b^3x^3}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}+\frac{27E^{\frac{I2\pi}{3}}a^{\frac{20}{3}}b^4x^4\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}-\frac{15E^{\frac{I2\pi}{3}}a^{\frac{17}{3}}b^5x^5\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3-120a^7b^4x+120a^6b^5x^2-40a^5b^6x^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(-a + b*x)^(1/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (2 / 3) (9 a ^ 5 (-1 ^ (2 / 3) + ((-a + b x) / a) ^ (2 /
3)) + 3 a ^ 4 b x (9 -1 ^ (2 / 3) - 7 ((-a + b x) / a) ^ (2 / 3)) + a ^ 3 b
 ^ 2 x ^ 2 (-27 -1 ^ (2 / 3) + 14 ((-a + b x) / a) ^ (2 / 3)) + 9 a b ^ 3 x
 ^ 3 (-1 ^ (2 / 3) a + b x ((-a + b x) / a) ^ (2 / 3)) - 6 a ^ 2 b ^ 3 x ^
3 ((-a + b x) / a) ^ (2 / 3) - 5 b ^ 5 x ^ 5 ((-a + b x) / a) ^ (2 / 3)) /
(40 b ^ 3 (a ^ 3 - 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 - b ^ 3 x ^ 3)), Abs[b x /
 a] > 1}}, -27 E ^ (I 2 Pi / 3) a ^ (32 / 3) / (40 a ^ 8 b ^ 3 - 120 a ^ 7
b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^ 5 b ^ 6 x ^ 3) + 27 E ^ (I 2 Pi /
3) a ^ (32 / 3) (1 - b x / a) ^ (2 / 3) / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4
 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^ 5 b ^ 6 x ^ 3) - 63 E ^ (I 2 Pi / 3) a
^ (29 / 3) b x (1 - b x / a) ^ (2 / 3) / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4
x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^ 5 b ^ 6 x ^ 3) + 81 E ^ (I 2 Pi / 3) a ^
 (29 / 3) b x / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2
 - 40 a ^ 5 b ^ 6 x ^ 3) - 81 E ^ (I 2 Pi / 3) a ^ (26 / 3) b ^ 2 x ^ 2 / (
40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^ 5 b ^ 6
 x ^ 3) + 42 E ^ (I 2 Pi / 3) a ^ (26 / 3) b ^ 2 x ^ 2 (1 - b x / a) ^ (2 /
 3) / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^
5 b ^ 6 x ^ 3) - 18 E ^ (I 2 Pi / 3) a ^ (23 / 3) b ^ 3 x ^ 3 (1 - b x / a)
 ^ (2 / 3) / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 -
40 a ^ 5 b ^ 6 x ^ 3) + 27 E ^ (I 2 Pi / 3) a ^ (23 / 3) b ^ 3 x ^ 3 / (40
a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^ 5 b ^ 6 x
^ 3) + 27 E ^ (I 2 Pi / 3) a ^ (20 / 3) b ^ 4 x ^ 4 (1 - b x / a) ^ (2 / 3)
 / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40 a ^ 5 b
 ^ 6 x ^ 3) - 15 E ^ (I 2 Pi / 3) a ^ (17 / 3) b ^ 5 x ^ 5 (1 - b x / a) ^
(2 / 3) / (40 a ^ 8 b ^ 3 - 120 a ^ 7 b ^ 4 x + 120 a ^ 6 b ^ 5 x ^ 2 - 40
a ^ 5 b ^ 6 x ^ 3)]
```

Maple [A]
time = 0.11, size = 44, normalized size = 0.75

method	result	size

gosper	$\frac{3(5x^2b^2+6abx+9a^2)(bx-a)^{\frac{2}{3}}}{40b^3}$	34
trager	$\frac{3(5x^2b^2+6abx+9a^2)(bx-a)^{\frac{2}{3}}}{40b^3}$	34
risch	$-\frac{3(-bx+a)(5x^2b^2+6abx+9a^2)}{40b^3(bx-a)^{\frac{1}{3}}}$	40
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{8}{3}}}{8}+\frac{6a(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx-a)^{\frac{2}{3}}}{2}}{b^3}$	44
default	$\frac{\frac{3(bx-a)^{\frac{8}{3}}}{8}+\frac{6a(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx-a)^{\frac{2}{3}}}{2}}{b^3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/8*(b*x-a)^(8/3)+2/5*a*(b*x-a)^(5/3)+1/2*a^2*(b*x-a)^(2/3))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.80

$$\frac{3(bx-a)^{\frac{8}{3}}}{8b^3}+\frac{6(bx-a)^{\frac{5}{3}}a}{5b^3}+\frac{3(bx-a)^{\frac{2}{3}}a^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x - a)^(8/3)/b^3 + 6/5*(b*x - a)^(5/3)*a/b^3 + 3/2*(b*x - a)^(2/3)*a
^2/b^3
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.56

$$\frac{3(5b^2x^2+6abx+9a^2)(bx-a)^{\frac{2}{3}}}{40b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 6*a*b*x + 9*a^2)*(b*x - a)^(2/3)/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.91, size = 1326, normalized size = 22.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4
*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(
-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) +
 63*a**(29/3)*b*x*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 12
0*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40
*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42
*a**(26/3)*b**2*x**2*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x -
 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*p
i/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x
**3) + 18*a**(23/3)*b**3*x**3*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7
*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*b**3*x**3*
exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a*
*5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b
**5*x**5*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b*
*5*x**2 + 40*a**5*b**6*x**3), Abs(b*x/a) > 1), (-27*a**(32/3)*(1 - b*x/a)**
(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*
b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 - b*
x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*
x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42*a**(26/3)*b
**2*x**2*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x
- 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*
pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*
x**3) + 18*a**(23/3)*b**3*x**3*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b
**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23
/3)*b**3*x**3*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**
5*x**2 + 40*a**5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(1 - b*x/a)**(2/3)*exp
(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*
b**6*x**3) + 15*a**(17/3)*b**5*x**5*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a
**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3), True)
)
```

Giac [A]
time = 0.00, size = 73, normalized size = 1.24

$$\frac{3\left(\frac{1}{8}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}\left(-a+bx\right)^{2}+\frac{2}{5}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}\left(-a+bx\right)a+\frac{1}{2}\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}a^{2}\right)}{bb^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x)
```

```
[Out] 3/40*(5*(b*x - a)^(8/3) + 16*(b*x - a)^(5/3)*a + 20*(b*x - a)^(2/3)*a^2)/b^
3
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.73

$$\frac{48\,a\,{\left(b\,x-a\right)}^{5/3}+15\,{\left(b\,x-a\right)}^{8/3}+60\,a^2\,{\left(b\,x-a\right)}^{2/3}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x - a)^(1/3),x)
```

```
[Out] (48*a*(b*x - a)^(5/3) + 15*(b*x - a)^(8/3) + 60*a^2*(b*x - a)^(2/3))/(40*b^
3)
```

3.401 $\int \frac{x}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=38

$$\frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}$$

```
[Out] 3/2*a*(b*x-a)^(2/3)/b^2+3/5*(b*x-a)^(5/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a}{b\sqrt[3]{-a+bx}}+\frac{(-a+bx)^{2/3}}{b}\right)dx\\ &= \frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.68

$$\frac{3(-a+bx)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.96, size = 308, normalized size = 8.11

$$\text{Piecewise}\left[\left\{\left\{\frac{3a^{\frac{2}{3}}\left(3a^2\left(-1^{\frac{2}{3}}+\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)+abx\left(3-1^{\frac{2}{3}}-\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)-2b^2x^2\left(\frac{-a+bx}{a}\right)^{\frac{2}{3}}\right)}{10b^2\left(a-bx\right)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{10E^{\frac{I\text{Pi}}{3}}a^2b^2-10E^{\frac{I\text{Pi}}{3}}ab^3x}+\frac{9a^{\frac{11}{3}}}{10E^{\frac{I\text{Pi}}{3}}a^2b^2-10E^{\frac{I\text{Pi}}{3}}ab^3x}-\frac{9a^{\frac{8}{3}}bx}{10E^{\frac{I\text{Pi}}{3}}a^2b^2-10E^{\frac{I\text{Pi}}{3}}ab^3x}+\frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{10E^{\frac{I\text{Pi}}{3}}a^2b^2-10E^{\frac{I\text{Pi}}{3}}ab^3x}+\frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{10E^{\frac{I\text{Pi}}{3}}a^2b^2-10E^{\frac{I\text{Pi}}{3}}ab^3x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/(-a + b*x)^(1/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (2 / 3) (3 a ^ 2 (-1 ^ (2 / 3) + ((-a + b x) / a) ^ (2 /
3)) + a b x (3 -1 ^ (2 / 3) - ((-a + b x) / a) ^ (2 / 3)) - 2 b ^ 2 x ^ 2 (
(-a + b x) / a) ^ (2 / 3)) / (10 b ^ 2 (a - b x)), Abs[b x / a] > 1}}, -9 a
 ^ (11 / 3) (1 - b x / a) ^ (2 / 3) / (10 E ^ (I Pi / 3) a ^ 2 b ^ 2 - 10 E
 ^ (I Pi / 3) a b ^ 3 x) + 9 a ^ (11 / 3) / (10 E ^ (I Pi / 3) a ^ 2 b ^ 2
- 10 E ^ (I Pi / 3) a b ^ 3 x) - 9 a ^ (8 / 3) b x / (10 E ^ (I Pi / 3) a ^
 2 b ^ 2 - 10 E ^ (I Pi / 3) a b ^ 3 x) + 3 a ^ (8 / 3) b x (1 - b x / a) ^
 (2 / 3) / (10 E ^ (I Pi / 3) a ^ 2 b ^ 2 - 10 E ^ (I Pi / 3) a b ^ 3 x) +
6 a ^ (5 / 3) b ^ 2 x ^ 2 (1 - b x / a) ^ (2 / 3) / (10 E ^ (I Pi / 3) a ^
2 b ^ 2 - 10 E ^ (I Pi / 3) a b ^ 3 x)]
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.79

method	result	size
gosper	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^2}$	23
trager	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^2}$	23
risch	$-\frac{3(-bx+a)(2bx+3a)}{10b^2(bx-a)^{\frac{1}{3}}}$	29
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{5}{3}}}{5}+\frac{3a(bx-a)^{\frac{2}{3}}}{2}}{b^2}$	30
default	$\frac{\frac{3(bx-a)^{\frac{5}{3}}}{5}+\frac{3a(bx-a)^{\frac{2}{3}}}{2}}{b^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/5*(b*x-a)^(5/3)+1/2*a*(b*x-a)^(2/3))
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.79

$$\frac{3\,(bx-a)^{\frac{5}{3}}}{5\,b^2}+\frac{3\,(bx-a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x - a)^(5/3)/b^2 + 3/2*(b*x - a)^(2/3)*a/b^2
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.58

$$\frac{3\,(2\,bx+3\,a)(bx-a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x + 3*a)*(b*x - a)^(2/3)/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 486, normalized size = 12.79

$$\begin{cases} -\frac{9a^{\frac{11}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{3a^{\frac{8}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{6a^{\frac{5}{3}}b^2x^2\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} + \frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} - \frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-9*a**(11/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(
I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*pi/3)
 + 10*a*b**3*x*exp(I*pi/3)) + 3*a**(8/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3
)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-
10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 6*a**(5/3)*b**2*x**2*
(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*ex
p(I*pi/3)), Abs(b*x/a) > 1), (9*a**(11/3)*(1 - b*x/a)**(2/3)/(-10*a**2*b**2
*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*
pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 3*a**(8/3)*b*x*(1 - b*x/a)**(2/3)/(-10*a
**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-10*a**2*
b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 6*a**(5/3)*b**2*x**2*(1 - b*x
/a)**(2/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 44, normalized size = 1.16

$$\frac{3\left(\frac{1}{5}\left((-a+bx)^{\frac{1}{3}}\right)^2(-a+bx)+\frac{1}{2}\left((-a+bx)^{\frac{1}{3}}\right)^2a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x)
```

```
[Out] 3/10*(2*(b*x - a)^(5/3) + 5*(b*x - a)^(2/3)*a)/b^2
```

Mupad [B]
time = 0.03, size = 29, normalized size = 0.76

$$\frac{15\,a\,{\left(b\,x-a\right)}^{2/3}+6\,{\left(b\,x-a\right)}^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x - a)^(1/3),x)
```

```
[Out] (15*a*(b*x - a)^(2/3) + 6*(b*x - a)^(5/3))/(10*b^2)
```

3.402 $\int \frac{1}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=18

$$\frac{3(-a+bx)^{2/3}}{2b}$$

```
[Out] 3/2*(b*x-a)^(2/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {32}

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(-1/3),x]
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx = \frac{3(-a+bx)^{2/3}}{2b}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{3(-a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(-1/3),x]
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Mathics [A]
time = 1.57, size = 14, normalized size = 0.78

$$\frac{3\left(-a+bx\right)^{\frac{2}{3}}}{2b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(-a + b*x)^(-1/3),x]')
```

```
[Out] 3 (-a + b x) ^ (2 / 3) / (2 b)
```

Maple [A]
time = 0.11, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
derivativedivides	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
default	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
trager	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
risch	$-\frac{3(-bx+a)}{2b(bx-a)^{\frac{1}{3}}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(b*x-a)^(2/3)/b
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.78

$$\frac{3\,\left(bx-a\right)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

Fricas [A]
time = 0.31, size = 14, normalized size = 0.78

$$\frac{3\,\left(bx-a\right)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.67

$$\frac{3\left(-a+bx\right)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**(1/3),x)
```

```
[Out] 3*(-a + b*x)**(2/3)/(2*b)
```

Giac [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{3\left(\left(-a+bx\right)^{\frac{1}{3}}\right)^{2}}{b\cdot 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x)
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.78

$$\frac{3\left(b\,x-a\right)^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x - a)^(1/3),x)
```

```
[Out] (3*(b*x - a)^(2/3))/(2*b)
```

3.403 $\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

```
[Out] 1/2*ln(x)/a^(1/3)-3/2*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)-arctan(1/3*(a^(1/3)
-2*(b*x-a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {58, 631, 210, 31}

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3
)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{-a+bx}\right) - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x,\right.}{2\sqrt[3]{a}}$$

$$= \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [A]
time = 0.06, size = 100, normalized size = 1.22

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) + (-a + b*x)^(1/3)] + Log[a^(2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*
x)^(2/3)])/(2*a^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.60, size = 91, normalized size = 1.11

$$\frac{-\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(-\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right] - \text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(-\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[I\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{a^{\frac{1}{3}}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*(-a + b*x)^(1/3)),x]')

[Out] (-1 ^ (2 / 3) Log[1 - b ^ (1 / 3) (-a / b + x) ^ (1 / 3) exp_polar[5 I / 3 Pi] / a ^ (1 / 3)] - Log[1 - b ^ (1 / 3) (-a / b + x) ^ (1 / 3) exp_polar[I Pi] / a ^ (1 / 3)] + -1 ^ (1 / 3) Log[1 - b ^ (1 / 3) (-a / b + x) ^ (1 / 3) exp_polar[I / 3 Pi] / a ^ (1 / 3)]) / a ^ (1 / 3)

Maple [A]
time = 0.11, size = 83, normalized size = 1.01

method	result	size
derivativedivides	$-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}$	83
default	$-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}$	83

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)

[Out] -ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/2/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(1/3)-1))

Maxima [A]
time = 0.36, size = 86, normalized size = 1.05

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}+\frac{\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="maxima")

[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(1/3) + 1/2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - log((b*x - a)^(1/3) + a^(1/3))/a^(1/3)

Fricas [A]
time = 0.33, size = 285, normalized size = 3.48

$$\left[\frac{\sqrt{3}\,a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\,\sqrt{3}\,a\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt((-a)^(1/3)/a)*log((2*b*x + sqrt(3)*(2*(b*x - a)^(2/3)*
(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x
- a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a
)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)^(1/3) - (-a)^
(1/3)))/a, 1/2*(2*sqrt(3)*a*sqrt(-(-a)^(1/3)/a)*arctan(1/3*sqrt(3)*(2*(b*x
- a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*log((b*x - a)^(2
/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)
^(1/3) - (-a)^(1/3)))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.97, size = 160, normalized size = 1.95

$$-\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/3),x)
```

```
[Out] -2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(
1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*log(1 - b**(1/3)*(-a/b + x)**(
1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*exp(2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(3*a**(1/3)*gamma(5/3))
```

Giac [A]
time = 0.00, size = 152, normalized size = 1.85

$$3\left(\frac{\left((-a)^{\frac{1}{3}}\right)^{2}\ln\left(\left((-a+bx)^{\frac{1}{3}}\right)^{2}+(-a)^{\frac{1}{3}}(-a+bx)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\right)}{6a}-\frac{\left((-a)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left((-a+bx)^{\frac{1}{3}}+\frac{(-a)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,(-a)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a}-\frac{(-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\ln\left|(-a+bx)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x)
```

```
[Out] -sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/a + 1/2*(-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3
) + (-a)^(2/3))/a - (-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)^(1/3)))/a
```

Mupad [B]
time = 0.09, size = 117, normalized size = 1.43

$$\frac{\ln\left(9\,(b\,x-a)^{1/3}-9\,(-a)^{1/3}\right)}{(-a)^{1/3}}+\frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}-\frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/3)),x)
```

```
[Out] log(9*(b*x - a)^(1/3) - 9*(-a)^(1/3))/(-a)^(1/3) + (log(9*(b*x - a)^(1/3) -
 (9*(-a)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-a)^(1/3)) - (l
og(9*(b*x - a)^(1/3) - (9*(-a)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1
))/(2*(-a)^(1/3))
```

3.404 $\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=103

$$\frac{(-a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}$$

```
[Out] (b*x-a)^(2/3)/a/x+1/6*b*ln(x)/a^(4/3)-1/2*b*ln(a^(1/3)+(b*x-a)^(1/3))/a^(4/
3)-1/3*b*arctan(1/3*(a^(1/3)-2*(b*x-a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {44, 58, 631, 210, 31}

$$\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2a^{4/3}}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) + (-
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = \frac{(-a+bx)^{2/3}}{ax}+\frac{b\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{3a}$$

$$= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx,x,\sqrt[3]{-a+bx}\right)}{2a^{4/3}}+\frac{b\text{Subst}\left(\int \frac{}{a^{2/3}-}\right.}{}$$

$$= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}+\frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\right.}{a^{4/3}}$$

$$= \frac{(-a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}$$

Mathematica [A]
time = 0.11, size = 128, normalized size = 1.24

$$\frac{6\sqrt[3]{a}\,(-a+bx)^{2/3}-2\sqrt{3}\,bx\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)+bx\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{6a^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(1/3)),x]
```

[Out] (6*a^(1/3)*(-a + b*x)^(2/3) - 2*Sqrt[3]*b*x*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) + (-a + b*x)^(1/3)] + b*x*Log[a^(2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(6*a^(4/3)*x)

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^2*(-a + b*x)^(1/3)),x]')

[Out] Exception raised: AttributeError >> 'SympyExpression' object has no attribute 'expr'

Maple [A]
time = 0.12, size = 113, normalized size = 1.10

method	result
risch	$-\frac{-bx+a}{ax(bx-a)^{\frac{1}{3}}}-\frac{b\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{4}{3}}}$
derivativedivides	$3b\left(\frac{(bx-a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(\frac{(bx-a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)

[Out] 3*b*(1/3*(b*x-a)^(2/3)/a/b/x+1/3/a*(-1/3*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/6/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(1/3)-1))))

Maxima [A]
time = 0.34, size = 116, normalized size = 1.13

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}}+\frac{(bx-a)^{\frac{2}{3}}b}{(bx-a)a+a^2}+\frac{b\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}}-\frac{b\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(
4/3) + (b*x - a)^(2/3)*b/((b*x - a)*a + a^2) + 1/6*b*log((b*x - a)^(2/3) -
(b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x - a)^(1/3) + a^
(1/3))/a^(4/3)
```

Fricas [A]
time = 0.33, size = 328, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6\,(bx-a)^{\frac{2}{3}}a}{6a^2x},\ \frac{6\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6\,(bx-a)^{\frac{2}{3}}a}{6a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2*(b*x
 - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*b*x*log((b*x - a)^
(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x - a)^(1/3) - (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x), 1/6*(6*sqrt(1/3)
*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3)
)*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*b*x*log((b*x - a)^(2/3) + (b*x - a)^(1/
3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*x - a)^(1/3) - (-a)^(
1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.26, size = 838, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*e
xp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**
(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/
```

```
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)*
*(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3
))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) +
 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b
**(10/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*p
i/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) -
2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b
 + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(
2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2
/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 6*a*b**3*(-a/b + x)**2
*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A]
time = 0.00, size = 197, normalized size = 1.91

$$\frac{3\left(-\frac{\frac{1}{18}b^{2}\ln\left(\left((-a+bx)^{\frac{1}{3}}\right)^{2}+(-a)^{\frac{1}{3}}(-a+bx)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\right)}{a(-a)^{\frac{1}{3}}}+\frac{\frac{1}{3}b^{2}\arctan\left(\frac{2\left((-a+bx)^{\frac{1}{3}}+\frac{(-a)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,(-a)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a(-a)^{\frac{1}{3}}}-\frac{(-a)^{\frac{1}{3}}b^{2}(-a)^{\frac{1}{3}}\ln\left|(-a+bx)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|}{3\cdot 3a^{2}}+\frac{\frac{1}{3}\left((-a+bx)^{\frac{1}{3}}\right)^{2}b^{2}}{a(-a+bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x)
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a)
^(1/3))/((-a)^(1/3)*a) - b^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/
3) + (-a)^(2/3))/((-a)^(1/3)*a) - 2*(-a)^(2/3)*b^2*log(abs((b*x - a)^(1/3)
- (-a)^(1/3)))/a^2 + 6*(b*x - a)^(2/3)*b/(a*x))/b
```

Mupad [B]
time = 0.18, size = 133, normalized size = 1.29

$$\frac{(b\,x-a)^{2/3}}{a\,x}-\frac{b\,\ln\left((b\,x-a)^{1/3}+a^{1/3}\right)}{3\,a^{4/3}}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/3)),x)
```

```
[Out] (b*x - a)^(2/3)/(a*x) - (b*log((b*x - a)^(1/3) + a^(1/3)))/(3*a^(4/3)) + (l
og((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x - a)^(1/3))/a^2)*(b - 3^(1/
2)*b*1i))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x -
 a)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3))
```

3.405 $\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=136

$$\frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}}$$

```
[Out] 1/2*(b*x-a)^(2/3)/a/x^2+2/3*b*(b*x-a)^(2/3)/a^2/x+1/9*b^2*ln(x)/a^(7/3)-1/3
*b^2*ln(a^(1/3)+(b*x-a)^(1/3))/a^(7/3)-2/9*b^2*arctan(1/3*(a^(1/3)-2*(b*x-a
)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {44, 58, 631, 210, 31}

$$\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{3a^{7/3}}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b(bx-a)^{2/3}}{3a^2x}+\frac{(bx-a)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(2*a*x^2) + (2*b*(-a + b*x)^(2/3))/(3*a^2*x) - (2*b^2*ArcT
an[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) +
 (b^2*Log[x])/(9*a^(7/3)) - (b^2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(7/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx &= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{(2b)\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx}{3a}\\
&= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}+\frac{(2b^2)\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{9a^2}\\
&= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx,x,\sqrt[3]{-a+bx}\right)}{3a^{7/3}}\\
&= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}}+\frac{(2b^2)\,\text{Sul}}{}\\
&= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}-\frac{2b^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 160, normalized size = 1.18

$$\frac{(-a+bx)^{2/3}(7a+4(-a+bx))}{6a^2x^2}-\frac{2b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{2b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{9a^{7/3}}+\frac{b^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] ((-a + b*x)^(2/3)*(7*a + 4*(-a + b*x)))/(6*a^2*x^2) - (2*b^2*ArcTan[1/Sqrt[
3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (2*b^2*
Log[a^(1/3) + (-a + b*x)^(1/3)])/(9*a^(7/3)) + (b^2*Log[a^(2/3) - a^(1/3)*(
-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(9*a^(7/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 28.13, size = 148, normalized size = 1.09

$$\frac{b^{\frac{2}{3}}\left(-\frac{a}{b}+x\right)^{\frac{2}{3}}}{2ax^2}+\frac{2b^{\frac{5}{3}}\left(-\frac{a}{b}+x\right)^{\frac{2}{3}}}{3a^2x}-\frac{2b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(-\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[I\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{7}{3}}}-\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(-\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{7}{3}}}-\frac{b^2\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(-\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{5I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{7}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*(-a + b*x)^(1/3)),x]')
```

```
[Out] b ^ (2 / 3) (-a / b + x) ^ (2 / 3) / (2 a x ^ 2) + 2 b ^ (5 / 3) (-a / b +
x) ^ (2 / 3) / (3 a ^ 2 x) - 2 -1 ^ (2 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (-a /
 b + x) ^ (1 / 3) exp_polar[5 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (7 / 3)) -
2 b ^ 2 Log[1 - b ^ (1 / 3) (-a / b + x) ^ (1 / 3) exp_polar[I Pi] / a ^ (1
 / 3)] / (9 a ^ (7 / 3)) + 2 -1 ^ (1 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (-a / b
 + x) ^ (1 / 3) exp_polar[I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (7 / 3))
```

Maple [A]
time = 0.10, size = 141, normalized size = 1.04

method	result
risch	$-\frac{(-bx+a)(4bx+3a)}{6a^2x^2(bx-a)^{\frac{1}{3}}}-\frac{2b^2\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{b^2\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(}{}\right.}{9a^{\frac{7}{3}}}$
derivativedivides	$3b^2\left(\frac{(bx-a)^{\frac{2}{3}}}{6a\,b^2x^2}+\frac{\frac{2(bx-a)^{\frac{2}{3}}}{9abx}+\frac{2\left(-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx-a}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}\right.}{9a}}{a}\right.$

default	$3b^2 \left(\frac{(bx-a)^{\frac{2}{3}}}{6a\,b^2x^2} + \frac{\frac{2(bx-a)^{\frac{2}{3}}}{9abx} + \frac{2\left(-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}} + \frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}\right.}{9a}}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(1/6/a*(b*x-a)^(2/3)/b^2/x^2+2/3/a*(1/3*(b*x-a)^(2/3)/a/b/x+1/3/a*(-1
/3*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/6/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(
b*x-a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*
x-a)^(1/3)-1)))))
```

Maxima [A]

time = 0.36, size = 159, normalized size = 1.17

$$\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}} + \frac{b^2\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}} - \frac{2\,b^2\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}} + \frac{4\,(bx-a)^{\frac{5}{3}}b^2+7\,(bx-a)^{\frac{2}{3}}ab^2}{6\left((bx-a)^2a^2+2\,(bx-a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a
^(7/3) + 1/9*b^2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) - 2/9*b^2*log((b*x - a)^(1/3) + a^(1/3))/a^(7/3) + 1/6*(4*(b*x - a)^
(5/3)*b^2 + 7*(b*x - a)^(2/3)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a
^4)
```

Fricas [A]

time = 0.32, size = 374, normalized size = 2.75

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+2\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\,(4\,abx+3\,a^2)(bx-a)^{\frac{2}{3}}}{18\,a^3x^2}, \frac{12\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+2\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\,(4\,abx+3\,a^2)(bx-a)^{\frac{2}{3}}}{18\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2
*(b*x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + 2*(-a)^(2/3)*b^2*x^2*log
((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(-a)^(2/3)*
b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2)*(b*x - a)^(
2/3))/(a^3*x^2), 1/18*(12*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sq
rt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 2*(-a)^(2/3
)*b^2*x^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) -
4*(-a)^(2/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2
)*(b*x - a)^(2/3))/(a^3*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.17, size = 2744, normalized size = 20.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/3),x)
```

```
[Out] -4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)
*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)
*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3
) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(
14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)
**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I
*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamm
a(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4
*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/
b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b +
x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp
(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
 - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b*
```

```
*(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*log(1 - b**(1/
3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)
**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp
(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*log(1
 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3
)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/
3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*log(
1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a
**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*
(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**
(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(
2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/
(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(
7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b +
 x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)
*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(-2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81
*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/
3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b +
x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 21*a**4*b**4*(-a/b + x)**2*exp(2*I*p
i/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3
) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b
**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-
a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(-a/b + x)**3*exp
(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*
a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13
/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**2*b**6*(-a/b + x)*
*4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
```

```
 + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*
b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A]
time = 0.01, size = 234, normalized size = 1.72

$$\frac{3\left(-\frac{\frac{1}{27}b^3\ln\left(\left((-a+bx)^{\frac{1}{3}}\right)^2+(-a)^{\frac{1}{3}}(-a+bx)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\right)}{a^2(-a)^{\frac{1}{3}}}+\frac{\frac{1}{9}\cdot 2b^3\arctan\left(\frac{2\left((-a+bx)^{\frac{1}{3}}+\frac{(-a)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,(-a)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2(-a)^{\frac{1}{3}}}-\frac{2(-a)^{\frac{1}{3}}b^3(-a)^{\frac{1}{3}}\ln\left|(-a+bx)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|}{9\cdot 3a^3}+\frac{\frac{1}{18}\left(4\left((-a+bx)^{\frac{1}{3}}\right)^2(-a+bx)b^3+7\left((-a+bx)^{\frac{1}{3}}\right)^2ab^3\right)}{a^2(-a+bx+a)^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x)
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/((-a)^(1/3)*a^2) - 2*b^3*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3))/((-a)^(1/3)*a^2) - 4*(-a)^(2/3)*b^3*log(abs((b*x - a)
^(1/3) - (-a)^(1/3)))/a^3 + 3*(4*(b*x - a)^(5/3)*b^3 + 7*(b*x - a)^(2/3)*a*
b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.22, size = 216, normalized size = 1.59

$$\frac{\frac{7\,b^2\,{\left(b\,x-a\right)}^{2/3}}{6\,a}+\frac{2\,b^2\,{\left(b\,x-a\right)}^{5/3}}{3\,a^2}}{{\left(a-b\,x\right)}^2-2\,a\,\left(a-b\,x\right)+a^2}-\frac{\ln\left(\frac{4\,b^4\,{\left(b\,x-a\right)}^{1/3}}{9\,a^4}-\frac{{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}^2}{9\,{\left(-a\right)}^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,{\left(-a\right)}^{7/3}}+\frac{2\,b^2\,\ln\left(\frac{4\,b^4\,{\left(b\,x-a\right)}^{1/3}}{9\,a^4}-\frac{4\,b^4}{9\,{\left(-a\right)}^{11/3}}\right)}{9\,{\left(-a\right)}^{7/3}}+\frac{b^2\,\ln\left(\frac{4\,b^4\,{\left(b\,x-a\right)}^{1/3}}{9\,a^4}-\frac{9\,b^4\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2}{{\left(-a\right)}^{11/3}}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{{\left(-a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/3)),x)
```

```
[Out] ((7*b^2*(b*x - a)^(2/3))/(6*a) + (2*b^2*(b*x - a)^(5/3))/(3*a^2))/((a - b*x
)^2 - 2*a*(a - b*x) + a^2) - (log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (3^(1/2
)*b^2*1i + b^2)^2/(9*(-a)^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*(-a)^(7/3)) +
 (2*b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (4*b^4)/(9*(-a)^(11/3))))/(9*
(-a)^(7/3)) + (b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1
i)/9 - 1/9)^2)/(-a)^(11/3))*((3^(1/2)*1i)/9 - 1/9))/(-a)^(7/3)
```

3.406 $\int \frac{x^3}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=70

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}$$

```
[Out] -3*a^3*(b*x+a)^(1/3)/b^4+9/4*a^2*(b*x+a)^(4/3)/b^4-9/7*a*(b*x+a)^(7/3)/b^4+
3/10*(b*x+a)^(10/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a +
b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx\\ &= -\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx}\,\left(-81a^3+27a^2bx-18ab^2x^2+14b^3x^3\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-81*a^3 + 27*a^2*b*x - 18*a*b^2*x^2 + 14*b^3*x^3))/(140
*b^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 319 vs. $2(70) = 140$.
time = 15.19, size = 299, normalized size = 4.27

$$\frac{3a^{\frac{1}{3}}\left(81a^9\left(1-\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+27a^8bx\left(18-17\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+9a^7b^2x^2\left(135-119\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^6b^3x^3\left(1620-1309\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+7a^3b^4x^4\left(-123a^2-33abx+13b^2x^2\right)\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+1215a^5b^4x^4+6ab^5x^5\left(81a^3+11b^3x^3\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+3a^2b^6x^6\left(27a+43bx\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+14b^9x^9\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{140b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(a + b*x)^(2/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (81 a ^ 9 (1 - ((a + b x) / a) ^ (1 / 3)) + 27 a ^ 8 b x (18
- 17 ((a + b x) / a) ^ (1 / 3)) + 9 a ^ 7 b ^ 2 x ^ 2 (135 - 119 ((a + b x)
 / a) ^ (1 / 3)) + a ^ 6 b ^ 3 x ^ 3 (1620 - 1309 ((a + b x) / a) ^ (1 / 3)
) + 7 a ^ 3 b ^ 4 x ^ 4 (-123 a ^ 2 - 33 a b x + 13 b ^ 2 x ^ 2) ((a + b x)
 / a) ^ (1 / 3) + 1215 a ^ 5 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 (81 a ^ 3 + 11 b
 ^ 3 x ^ 3 ((a + b x) / a) ^ (1 / 3)) + 3 a ^ 2 b ^ 6 x ^ 6 (27 a + 43 b x
((a + b x) / a) ^ (1 / 3)) + 14 b ^ 9 x ^ 9 ((a + b x) / a) ^ (1 / 3)) / (1
40 b ^ 4 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3
 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.71

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^3x^3+18a\,b^2x^2-27a^2bx+81a^3\right)}{140b^4}$	43
trager	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^3x^3+18a\,b^2x^2-27a^2bx+81a^3\right)}{140b^4}$	43
risch	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^3x^3+18a\,b^2x^2-27a^2bx+81a^3\right)}{140b^4}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{9a(bx+a)^{\frac{7}{3}}}{7}+\frac{9a^2(bx+a)^{\frac{4}{3}}}{4}-3a^3(bx+a)^{\frac{1}{3}}}{b^4}$	50
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{9a(bx+a)^{\frac{7}{3}}}{7}+\frac{9a^2(bx+a)^{\frac{4}{3}}}{4}-3a^3(bx+a)^{\frac{1}{3}}}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/10*(b*x+a)^(10/3)-3/7*a*(b*x+a)^(7/3)+3/4*a^2*(b*x+a)^(4/3)-a^3*(b
*x+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^4}-\frac{9\,(bx+a)^{\frac{7}{3}}a}{7\,b^4}+\frac{9\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^4}-\frac{3\,(bx+a)^{\frac{1}{3}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^4 - 9/7*(b*x + a)^(7/3)*a/b^4 + 9/4*(b*x + a)^(4/3)
*a^2/b^4 - 3*(b*x + a)^(1/3)*a^3/b^4
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.60

$$\frac{3\,(14\,b^3x^3-18\,ab^2x^2+27\,a^2bx-81\,a^3)(bx+a)^{\frac{1}{3}}}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 - 18*a*b^2*x^2 + 27*a^2*b*x - 81*a^3)*(b*x + a)^(1/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. 2(66) = 132.
time = 1.30, size = 1640, normalized size = 23.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) + 243*a**(70/3)/(140*a**20*b**4 + 840*a**
19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x
**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 1377*a**(67/3)*b*x*(1 +
 b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 +
2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**
14*b**10*x**6) + 1458*a**(67/3)*b*x/(140*a**20*b**4 + 840*a**19*b**5*x + 21
00*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**1
5*b**9*x**5 + 140*a**14*b**10*x**6) - 3213*a**(64/3)*b**2*x**2*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6) + 3645*a**(64/3)*b**2*x**2/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) - 3927*a**(61/3)*b**3*x**3*(1 + b*x/a)**(
```

```
1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17
*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x
**6) + 4860*a**(61/3)*b**3*x**3/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a
**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b*
*9*x**5 + 140*a**14*b**10*x**6) - 2583*a**(58/3)*b**4*x**4*(1 + b*x/a)**(1/
3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b
**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**
6) + 3645*a**(58/3)*b**4*x**4/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**
18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9
*x**5 + 140*a**14*b**10*x**6) - 693*a**(55/3)*b**5*x**5*(1 + b*x/a)**(1/3)/
(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7
*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6)
+ 1458*a**(55/3)*b**5*x**5/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*
b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x*
*5 + 140*a**14*b**10*x**6) + 273*a**(52/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(14
0*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x*
*3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 2
43*a**(52/3)*b**6*x**6/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6
*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
 140*a**14*b**10*x**6) + 387*a**(49/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(140*a*
*20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 +
 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 198*a
**(46/3)*b**8*x**8*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x +
2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a*
*15*b**9*x**5 + 140*a**14*b**10*x**6) + 42*a**(43/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6)
```

Giac [A]
time = 0.00, size = 85, normalized size = 1.21

$$\frac{3\left(\frac{1}{10}\left(a+bx\right)^{\frac{1}{3}}\left(a+bx\right)^{3}-\frac{3}{7}\left(a+bx\right)^{\frac{1}{3}}\left(a+bx\right)^{2}a+\frac{3}{4}\left(a+bx\right)^{\frac{1}{3}}\left(a+bx\right)a^{2}-\left(a+bx\right)^{\frac{1}{3}}a^{3}\right)}{bb^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x)
```

```
[Out] 3/140*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2
 - 140*(b*x + a)^(1/3)*a^3)/b^4
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{1/3}}{b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{4/3}}{4\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(10/3))/(10*b^4) - (3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b
*x)^(4/3))/(4*b^4) - (9*a*(a + b*x)^(7/3))/(7*b^4)
```

3.407 $\int \frac{x^2}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=51

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}$$

```
[Out] 3*a^2*(b*x+a)^(1/3)/b^3-3/2*a*(b*x+a)^(4/3)/b^3+3/7*(b*x+a)^(7/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)^
(7/3))/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{2/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx\\ &= \frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.69

$$\frac{3\sqrt[3]{a+bx}\,\left(9a^2-3abx+2b^2x^2\right)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 176 vs. 2(51) = 102.
time = 6.75, size = 162, normalized size = 3.18

$$\frac{3a^{\frac{1}{3}}\left(9a^5\left(-1+\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+3a^4bx\left(-9+8\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+a^3b^2x^2\left(-27+20\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+2b^3x^3\left(3a^2+b^2x^2\right)\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}+3ab^3x^3\left(-3a+bx\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)\right)}{14b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(a + b*x)^(2/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (9 a ^ 5 (-1 + ((a + b x) / a) ^ (1 / 3)) + 3 a ^ 4 b x (-9 +
 8 ((a + b x) / a) ^ (1 / 3)) + a ^ 3 b ^ 2 x ^ 2 (-27 + 20 ((a + b x) / a)
 ^ (1 / 3)) + 2 b ^ 3 x ^ 3 (3 a ^ 2 + b ^ 2 x ^ 2) ((a + b x) / a) ^ (1 /
3) + 3 a b ^ 3 x ^ 3 (-3 a + b x ((a + b x) / a) ^ (1 / 3))) / (14 b ^ 3 (a
 ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.73

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}\left(2x^2b^2-3abx+9a^2\right)}{14b^3}$	32
trager	$\frac{3(bx+a)^{\frac{1}{3}}\left(2x^2b^2-3abx+9a^2\right)}{14b^3}$	32
risch	$\frac{3(bx+a)^{\frac{1}{3}}\left(2x^2b^2-3abx+9a^2\right)}{14b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{2}+3a^2(bx+a)^{\frac{1}{3}}}{b^3}$	37
default	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{2}+3a^2(bx+a)^{\frac{1}{3}}}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/7*(b*x+a)^(7/3)-1/2*a*(b*x+a)^(4/3)+a^2*(b*x+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^3}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{2\,b^3}+\frac{3\,(bx+a)^{\frac{1}{3}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^3 - 3/2*(b*x + a)^(4/3)*a/b^3 + 3*(b*x + a)^(1/3)*a^2
/b^3
```

Fricas [**A**]
time = 0.31, size = 31, normalized size = 0.61

$$\frac{3\,(2\,b^2x^2-3\,abx+9\,a^2)(bx+a)^{\frac{1}{3}}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 - 3*a*b*x + 9*a^2)*(b*x + a)^(1/3)/b^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(48) = 96.
time = 0.87, size = 600, normalized size = 11.76

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{31}{3}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{72a^{\frac{28}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{28}{3}}bx}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{60a^{\frac{25}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{25}{3}}b^2x^2}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{18a^{\frac{22}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{22}{3}}b^3x^3}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{9a^{\frac{19}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{6a^{\frac{16}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b*
*5*x**2 + 14*a**5*b**6*x**3) - 27*a**(31/3)/(14*a**8*b**3 + 42*a**7*b**4*x
+ 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 72*a**(28/3)*b*x*(1 + b*x/a)**(1
/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
 - 81*a**(28/3)*b*x/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14
*a**5*b**6*x**3) + 60*a**(25/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(14*a**8*b**3
+ 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 81*a**(25/3)*b*
*2*x**2/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x
**3) + 18*a**(22/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b*
*4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(22/3)*b**3*x**3/(14*
a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 9*a**
(19/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**
6*b**5*x**2 + 14*a**5*b**6*x**3) + 6*a**(16/3)*b**5*x**5*(1 + b*x/a)**(1/3)
/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
```

Giac [**A**]
time = 0.00, size = 60, normalized size = 1.18

$$\frac{3\left(\frac{1}{7}\,(a+bx)^{\frac{1}{3}}\,(a+bx)^2-\frac{1}{2}\,(a+bx)^{\frac{1}{3}}\,(a+bx)\,a+(a+bx)^{\frac{1}{3}}\,a^2\right)}{bb^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x)
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b^3
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,{\left(a+b\,x\right)}^{7/3}-21\,a\,{\left(a+b\,x\right)}^{4/3}+42\,a^2\,{\left(a+b\,x\right)}^{1/3}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(2/3),x)
```

```
[Out] (6*(a + b*x)^(7/3) - 21*a*(a + b*x)^(4/3) + 42*a^2*(a + b*x)^(1/3))/(14*b^3
)
```

3.408 $\int \frac{x}{(a+bx)^{2/3}} \, dx$

Optimal. Leaf size=32

$$-\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}$$

```
[Out] -3*a*(b*x+a)^(1/3)/b^2+3/4*(b*x+a)^(4/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{2/3}} \, dx &= \int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right) dx\\ &= -\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(-3a+bx)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(2/3),x]
```

```
[Out] (3*(-3*a + b*x)*(a + b*x)^(1/3))/(4*b^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 83 vs. $2(32) = 64$.
time = 3.19, size = 73, normalized size = 2.28

$$\frac{3a^{\frac{1}{3}}\left(3a^2\left(1-\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+abx\left(3-2\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)+b^2x^2\left(\frac{a+bx}{a}\right)^{\frac{1}{3}}\right)}{4b^2\left(a+bx\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/(a + b*x)^(2/3),x]')
```

```
[Out] 3 a ^ (1 / 3) (3 a ^ 2 (1 - ((a + b x) / a) ^ (1 / 3)) + a b x (3 - 2 ((a +
 b x) / a) ^ (1 / 3)) + b ^ 2 x ^ 2 ((a + b x) / a) ^ (1 / 3)) / (4 b ^ 2 (
a + b x))
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^2}$	21
trager	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^2}$	21
risch	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{4}{3}}}{4}-3a(bx+a)^{\frac{1}{3}}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{4}{3}}}{4}-3a(bx+a)^{\frac{1}{3}}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/4*(b*x+a)^(4/3)-a*(b*x+a)^(1/3))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.81

$$\frac{3\left(bx+a\right)^{\frac{4}{3}}}{4\,b^2}-\frac{3\left(bx+a\right)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b^2 - 3*(b*x + a)^(1/3)*a/b^2
```

Fricas [A]
time = 0.30, size = 19, normalized size = 0.59

$$\frac{3\,(bx+a)^{\frac{1}{3}}(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(1/3)*(b*x - 3*a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.
time = 0.58, size = 162, normalized size = 5.06

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{10}{3}}}{4a^2b^2+4ab^3x}-\frac{6a^{\frac{7}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{7}{3}}bx}{4a^2b^2+4ab^3x}+\frac{3a^{\frac{4}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x) + 9*a**(10/3)/(4
*a**2*b**2 + 4*a*b**3*x) - 6*a**(7/3)*b*x*(1 + b*x/a)**(1/3)/(4*a**2*b**2 +
 4*a*b**3*x) + 9*a**(7/3)*b*x/(4*a**2*b**2 + 4*a*b**3*x) + 3*a**(4/3)*b**2*
x**2*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x)
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.12

$$\frac{3\left(\frac{1}{4}\,(a+bx)^{\frac{1}{3}}\,(a+bx)-a\,(a+bx)^{\frac{1}{3}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x)
```

```
[Out] 3/4*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)/b^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.78

$$-\frac{12\,a\,(a+b\,x)^{1/3}-3\,(a+b\,x)^{4/3}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(2/3),x)
```

```
[Out] -(12*a*(a + b*x)^(1/3) - 3*(a + b*x)^(4/3))/(4*b^2)
```

3.409 $\int \frac{1}{(a+bx)^{2/3}} dx$

Optimal. Leaf size=14

$$\frac{3\sqrt[3]{a+bx}}{b}$$

[Out] 3*(b*x+a)^(1/3)/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-2/3),x]

[Out] (3*(a + b*x)^(1/3))/b

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}}\, dx = \frac{3\sqrt[3]{a+bx}}{b}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-2/3),x]

[Out] (3*(a + b*x)^(1/3))/b

Mathics [A]
time = 1.58, size = 12, normalized size = 0.86

$$\frac{3\,(a+bx)^{\frac{1}{3}}}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-2/3),x]')
```

```
[Out] 3 (a + b x) ^ (1 / 3) / b
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
default	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
trager	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
risch	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*(b*x+a)^(1/3)/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.86

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3),x)
```

```
[Out] 3*(a + b*x)**(1/3)/b
```

Giac [A]
time = 0.00, size = 13, normalized size = 0.93

$$\frac{3\,(a+bx)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x)
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{3\,{\left(a+b\,x\right)}^{1/3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

3.410 $\int \frac{1}{x(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3)-arctan(1/3*(a^(1/3
)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {59, 631, 210, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/3)
) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{2/3}}\,dx &= -\frac{\log(x)}{2a^{2/3}}-\frac{3\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{3\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+\right.}{2\sqrt[3]{a}}\\ &= -\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}+\frac{3\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}\\ &= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 1.16

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(2/3)),x]
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*
x)^(2/3)])/a^(2/3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.49, size = 82, normalized size = 1.02

$$\frac{\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]+\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]-\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{a^{\frac{2}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a + b*x)^(2/3)),x]')
```

```
[Out] (Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] - -1 ^ (1 / 3) Lo
g[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)
] + -1 ^ (2 / 3) Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I /
3 Pi] / a ^ (1 / 3)]) / a ^ (2 / 3)
```

Maple [A]
time = 0.11, size = 76, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}$	76
default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))-1/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a
)^(1/3)+1))
```

Maxima [A]
time = 0.36, size = 77, normalized size = 0.96

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g((b*x + a)^(1/3) - a^(1/3))/a^(2/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(57) = 114.
time = 0.32, size = 115, normalized size = 1.44

$$-\frac{2\sqrt{3}\,(a^2)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\,(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^2}\right)+(a^2)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\,(a^2)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*log((b*x + a)^(2/3)*a
+ (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*log((b*x + a
)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.98, size = 150, normalized size = 1.88

$$\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(2/3),x)
```

```
[Out] log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/
3)) + exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/
a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b**(1/
3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*ga
mma(4/3))
```

Giac [A]
time = 0.00, size = 129, normalized size = 1.61

$$3\left(-\frac{a^{\frac{1}{3}}\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a}-\frac{a^{\frac{1}{3}}\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a}+\frac{a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x)
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3)
```

Mupad [B]
time = 0.17, size = 95, normalized size = 1.19

$$\frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-9\,(a+b\,x)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+9\,(a+b\,x)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x)^(2/3)),x)

[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(2/3) + (log((9*a^(1/3)*(3^(1/2)*1i - 1))/2 - 9*(a + b*x)^(1/3))*(3^(1/2)*1i - 1))/(2*a^(2/3)) - (log((9*a^(1/3)*(3^(1/2)*1i + 1))/2 + 9*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/(2*a^(2/3))

3.411 $\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=98

$$-\frac{\sqrt[3]{a+bx}}{ax}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}$$

```
[Out] -(b*x+a)^(1/3)/a/x+1/3*b*ln(x)/a^(5/3)-b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/3)+
2/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 59, 631, 210, 31}

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/3) -
 (a + b*x)^(1/3)])/a^(5/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx}}{ax} - \frac{(2b)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{3a}$$

$$= -\frac{\sqrt[3]{a+bx}}{ax} + \frac{b\log(x)}{3a^{5/3}} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{5/3}} + \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\ldots}\right.}{a\ldots}$$

$$= -\frac{\sqrt[3]{a+bx}}{ax} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\ldots\right.}{a^{5/3}}$$

$$= -\frac{\sqrt[3]{a+bx}}{ax} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}$$

Mathematica [A]
time = 0.11, size = 119, normalized size = 1.21

$$\frac{-3a^{2/3}\sqrt[3]{a+bx}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{5/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(2/3)),x]
```

[Out] (-3*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(5/3)*x)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 9.16, size = 112, normalized size = 1.14

$$\frac{-3a^{\frac{2}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}-2bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]-bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{2I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]-bx\mathrm{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{4I}{3}\mathrm{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3a^{\frac{5}{3}}x}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^2*(a + b*x)^(2/3)),x]')

[Out] (-3 a ^ (2 / 3) b ^ (1 / 3) (a / b + x) ^ (1 / 3) - 2 -1 ^ (2 / 3) b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] - 2 b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] + 2 -1 ^ (1 / 3) b x Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)]) / (3 a ^ (5 / 3) x)

Maple [A]
time = 0.13, size = 104, normalized size = 1.06

method	result
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}}{a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)

```
[Out] 3*b*(-1/3*(b*x+a)^(1/3)/a/b/x+2/3/a*(-1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))
+1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.35, size = 106, normalized size = 1.08

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}b}{(bx+a)a-a^2}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{5}{3}}}-\frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
5/3) - (b*x + a)^(1/3)*b/((b*x + a)*a - a^2) + 1/3*b*log((b*x + a)^(2/3) +
(b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/3*b*log((b*x + a)^(1/3) - a^
(1/3))/a^(5/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(75) = 150$.
time = 0.32, size = 166, normalized size = 1.69

$$\frac{2\sqrt{3}\,abx\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3\,a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3\,(bx+a)^{\frac{1}{3}}a^2}{3\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*a*b*x*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*
a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^
2)^(2/3)*b*x*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x + a
)^(1/3)) - 2*(-a^2)^(2/3)*b*x*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)) - 3*(b*
x + a)^(1/3)*a^2)/(a^3*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.26, size = 830, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(2/3),x)
```

```
[Out] -2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*p
i/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)
) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
```

```
exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*g
amma(4/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b*
*(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x
)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3
*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b
+ x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3
)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5
/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b +
 x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*g
amma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 3*
a*b**2*(a/b + x)*exp(2*I*pi/3)*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)
*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*
gamma(4/3))
```

Giac [A]
time = 0.00, size = 178, normalized size = 1.82

$$\frac{3\left(\frac{a^{\frac{1}{3}}b^2\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^2+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{9a^2}+\frac{\frac{1}{3}\cdot 2a^{\frac{1}{3}}b^2\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2}-\frac{2b^2a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3\cdot 3a^2}-\frac{\frac{1}{3}(a+bx)^{\frac{1}{3}}b^2}{a(a+bx-a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x)
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(5/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x + a)^(1/
3)*b/(a*x))/b
```

Mupad [B]
time = 0.13, size = 122, normalized size = 1.24

$$-\frac{(a+b\,x)^{1/3}}{a\,x}+\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}}-\frac{2\,b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/a^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b - 3^(1/2)
*b*1i))/(3*a^(5/3)) - (a + b*x)^(1/3)/(a*x) + (log((3*(b + 3^(1/2)*b*1i))/a
^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(3*a^(5/3)) - (2*b*lo
g((a + b*x)^(1/3) - a^(1/3)))/(3*a^(5/3))
```

3.412 $\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}$$

```
[Out] -1/2*(b*x+a)^(1/3)/a/x^2+5/6*b*(b*x+a)^(1/3)/a^2/x-5/18*b^2*ln(x)/a^(8/3)+5
/6*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(8/3)-5/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x
+a)^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 59, 631, 210, 31}

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -1/2*(a + b*x)^(1/3)/(a*x^2) + (5*b*(a + b*x)^(1/3))/(6*a^2*x) - (5*b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) -
 (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^
(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{2ax^2}-\frac{(5b)\int \frac{1}{x^2(a+bx)^{2/3}}\,dx}{6a}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}+\frac{(5b^2)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a^2}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{8/3}} -\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}+\frac{(5b^2)\,\text{Subst}}{}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}\right.}{6}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 149, normalized size = 1.15

$$-\frac{\sqrt[3]{a+bx}\,(8a-5(a+bx))}{6a^2x^2}-\frac{5b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{8/3}}-\frac{5b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x)^(2/3)),x]

[Out] -1/6*((a + b*x)^(1/3)*(8*a - 5*(a + b*x)))/(a^2*x^2) - (5*b^2*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(8/3)) - (5*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(8/3))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 25.89, size = 153, normalized size = 1.18

$$\frac{b^{\frac{1}{3}}}{3ax\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}+\frac{5b^{\frac{4}{3}}}{6a^2\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}-\frac{b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{8}{3}}}-\frac{b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{9a^{\frac{8}{3}}}+\frac{5b^2\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{9a^{\frac{8}{3}}}-\frac{1}{2b^{\frac{2}{3}}x^2\left(\frac{a}{b}+x\right)^{\frac{2}{3}}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^3*(a + b*x)^(2/3)),x]')

[Out] b ^ (1 / 3) / (3 a x (a / b + x) ^ (2 / 3)) + 5 b ^ (4 / 3) / (6 a ^ 2 (a / b + x) ^ (2 / 3)) - 5 -1 ^ (1 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (8 / 3)) + 5 -1 ^ (2 / 3) b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (9 a ^ (8 / 3)) + 5 b ^ 2 Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] / (9 a ^ (8 / 3)) - 1 / (2 b ^ (2 / 3) x ^ 2 (a / b + x) ^ (2 / 3))

Maple [A]
time = 0.15, size = 130, normalized size = 1.00

method	result
derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{1}{3}}}{6a\,b^2x^2}-\frac{5\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\sqrt{3}\left(\frac{2(b \cdots\right)\right)}{9a^{\frac{2}{3}}}}{a}\right)}{6a}\right.$

default	$3b^2$	$\left(-\frac{(bx+a)^{\frac{1}{3}}}{6a\,b^2x^2}-\frac{5\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2}{}\right.}{9a^{\frac{2}{3}}}}{a}\right.}{6a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-1/6/a*(b*x+a)^(1/3)/b^2/x^2-5/6/a*(-1/3*(b*x+a)^(1/3)/a/b/x+2/3/a*(
-1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)
*(b*x+a)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(
b*x+a)^(1/3)+1)))))
```

Maxima [A]
time = 0.35, size = 142, normalized size = 1.09

$$-\frac{5\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{8}{3}}}-\frac{5\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{8}{3}}}+\frac{5\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{8}{3}}}+\frac{5\,(bx+a)^{\frac{4}{3}}b^2-8\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -5/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(8/3) - 5/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(8/3) + 5/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(8/3) + 1/6*(5*(b*x + a
)^(4/3)*b^2 - 8*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 +
 a^4)
```

Fricas [A]
time = 0.31, size = 162, normalized size = 1.25

$$-\frac{10\sqrt{3}\,(a^2)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^2}\right)+5\,(a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-10\,(a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)-3\,(5\,a^2bx-3\,a^3)(bx+a)^{\frac{1}{3}}}{18\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="fricas")

[Out] -1/18*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 5*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 10*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^2*b*x - 3*a^3)*(b*x + a)^(1/3))/(a^4*x^2)

Sympy [C] Result contains complex when optimal does not.
time = 2.20, size = 2728, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x+a)**(2/3),x)

[Out] 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3

```
)*exp(2*I*pi/3)*gamma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(2*
I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**
(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b +
x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp
(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*g
amma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b
+ x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*
gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
+ 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(
a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/
3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi
/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/
3)) - 10*a**(4/3)*b**(17/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)
*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) -
 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3
)*b**(17/3)*(a/b + x)**(11/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi
/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3
) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b
**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3)*b**(17/3
)*(a/b + x)**(11/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 24*a**4*b**3*(a
/b + x)*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 39*a**3*b**4*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*
I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma
(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a*
*4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 15*a**2*b**5*(a/
b + x)**3*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
```

Giac [A]

time = 0.00, size = 215, normalized size = 1.65

$$\frac{3\left(-\frac{\frac{1}{54}\cdot 5b^3\ln\left(\left((a+bx)^{\frac{1}{3}}\right)^2+a^{\frac{1}{3}}(a+bx)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{a^2\left(a^{\frac{1}{3}}\right)^2}-\frac{\frac{1}{9}\cdot 5b^3\arctan\left(\frac{2\left((a+bx)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2\left(a^{\frac{1}{3}}\right)^2}+\frac{5b^3a^{\frac{1}{3}}\ln\left|(a+bx)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{9\cdot 3a^3}-\frac{\frac{1}{18}\left(-5(a+bx)^{\frac{1}{3}}(a+bx)b^3+8(a+bx)^{\frac{1}{3}}ab^3\right)}{a^2(a+bx-a)^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x)
```

```
[Out] -1/18*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(8/3) + 5*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(8/3) - 10*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(b*x
 + a)^(4/3)*b^3 - 8*(b*x + a)^(1/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.13, size = 175, normalized size = 1.35

$$\frac{5\,b^2\,\ln\left({\left(a+b\,x\right)}^{1/3}-a^{1/3}\right)}{9\,a^{8/3}}-\frac{\frac{4\,b^2\,{\left(a+b\,x\right)}^{1/3}}{3\,a}-\frac{5\,b^2\,{\left(a+b\,x\right)}^{4/3}}{6\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}+\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,{\left(a+b\,x\right)}^{1/3}}{a^2}-\frac{5\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}}-\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,{\left(a+b\,x\right)}^{1/3}}{a^2}+\frac{5\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(2/3)),x)
```

```
[Out] (5*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(8/3)) - ((4*b^2*(a + b*x)^(1/3
))/(3*a) - (5*b^2*(a + b*x)^(4/3))/(6*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) + (5*b^2*log((5*b^2*(a + b*x)^(1/3))/a^2 - (5*b^2*((3^(1/2)*1i)/2 - 1/
2))/a^(5/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) - (5*b^2*log((5*b^2*(a + b
*x)^(1/3))/a^2 + (5*b^2*((3^(1/2)*1i)/2 + 1/2))/a^(5/3))*((3^(1/2)*1i)/2 +
1/2))/(9*a^(8/3))
```

3.413 $\int \frac{x^3}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=70

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

```
[Out] 3*a^3/b^4/(b*x+a)^(1/3)+9/2*a^2*(b*x+a)^(2/3)/b^4-9/5*a*(b*x+a)^(5/3)/b^4+3
/8*(b*x+a)^(8/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a +
 b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{4/3}}\,dx&=\int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx\\&=\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.66

$$\frac{3\left(81a^3+27a^2bx-9ab^2x^2+5b^3x^3\right)}{40b^4\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x - 9*a*b^2*x^2 + 5*b^3*x^3))/(40*b^4*(a + b*x)^(1/3)
)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 295 vs. $2(70) = 140$.
time = 14.77, size = 277, normalized size = 3.96

$$\frac{3a^{\frac{2}{3}}\left(81a^8\left(-1+\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+54a^7bx\left(-9+8\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+9a^6b^2x^2\left(-135+104\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+20a^5b^3x^3\left(-81+52\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+5b^4x^4\left(-243a^4+b^4x^4\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+610a^4b^4x^4\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+16ab^5x^5\left(11a^2+2abx+b^2x^2\right)\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}-486a^3b^5x^5-81a^2b^6x^6\right)}{40b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(a + b*x)^(4/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (81 a ^ 8 (-1 + ((a + b x) / a) ^ (2 / 3)) + 54 a ^ 7 b x (-9
 + 8 ((a + b x) / a) ^ (2 / 3)) + 9 a ^ 6 b ^ 2 x ^ 2 (-135 + 104 ((a + b x
) / a) ^ (2 / 3)) + 20 a ^ 5 b ^ 3 x ^ 3 (-81 + 52 ((a + b x) / a) ^ (2 / 3
)) + 5 b ^ 4 x ^ 4 (-243 a ^ 4 + b ^ 4 x ^ 4 ((a + b x) / a) ^ (2 / 3)) + 6
10 a ^ 4 b ^ 4 x ^ 4 ((a + b x) / a) ^ (2 / 3) + 16 a b ^ 5 x ^ 5 (11 a ^ 2
 + 2 a b x + b ^ 2 x ^ 2) ((a + b x) / a) ^ (2 / 3) - 486 a ^ 3 b ^ 5 x ^ 5
 - 81 a ^ 2 b ^ 6 x ^ 6) / (40 b ^ 4 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2
x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^
 6 x ^ 6))
```

Maple [A]
time = 0.12, size = 49, normalized size = 0.70

method	result	size
gosper	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
trager	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
risch	$\frac{3\left(5x^2b^2-14abx+41a^2\right)(bx+a)^{\frac{2}{3}}}{40b^4}+\frac{3a^3}{b^4(bx+a)^{\frac{1}{3}}}$	48
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{9a(bx+a)^{\frac{5}{3}}}{5}+\frac{9a^2(bx+a)^{\frac{2}{3}}}{2}+\frac{3a^3}{(bx+a)^{\frac{1}{3}}}}{b^4}$	49
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{9a(bx+a)^{\frac{5}{3}}}{5}+\frac{9a^2(bx+a)^{\frac{2}{3}}}{2}+\frac{3a^3}{(bx+a)^{\frac{1}{3}}}}{b^4}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/8*(b*x+a)^(8/3)-3/5*a*(b*x+a)^(5/3)+3/2*a^2*(b*x+a)^(2/3)+a^3/(b*x
+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^4}-\frac{9\,(bx+a)^{\frac{5}{3}}a}{5\,b^4}+\frac{9\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^4}+\frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^4 - 9/5*(b*x + a)^(5/3)*a/b^4 + 9/2*(b*x + a)^(2/3)*a
^2/b^4 + 3*a^3/((b*x + a)^(1/3)*b^4)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 0.74

$$\frac{3\,(5\,b^3x^3-9\,ab^2x^2+27\,a^2bx+81\,a^3)(bx+a)^{\frac{2}{3}}}{40\,(b^5x+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 - 9*a*b^2*x^2 + 27*a^2*b*x + 81*a^3)*(b*x + a)^(2/3)/(b^5*x
 + a*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(66) = 132$.
time = 1.38, size = 1538, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a*
*18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*
x**5 + 40*a**14*b**10*x**6) - 243*a**(68/3)/(40*a**20*b**4 + 240*a**19*b**5
*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*
a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1296*a**(65/3)*b*x*(1 + b*x/a)**(2
/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**
7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) -
 1458*a**(65/3)*b*x/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2
 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**
14*b**10*x**6) + 2808*a**(62/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**20*b**4
 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16
*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(62/3)*b*
*2*x**2/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17
```

```
*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**
6) + 3120*a**(59/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19
*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 +
 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 4860*a**(59/3)*b**3*x**3/(40*
a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1830*a*
*(56/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 60
0*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b
**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(56/3)*b**4*x**4/(40*a**20*b**4 +
 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b
**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 528*a**(53/3)*b**5*
x**5*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*
x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40
*a**14*b**10*x**6) - 1458*a**(53/3)*b**5*x**5/(40*a**20*b**4 + 240*a**19*b*
*5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 24
0*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 96*a**(50/3)*b**6*x**6*(1 + b*x/
a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**
17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x
**6) - 243*a**(50/3)*b**6*x**6/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**1
8*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x*
*5 + 40*a**14*b**10*x**6) + 48*a**(47/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(40*a
**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 15*a**(4
4/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a
**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9
*x**5 + 40*a**14*b**10*x**6)
```

Giac [A]
time = 0.00, size = 97, normalized size = 1.39

$$3\left(\frac{\frac{1}{8}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)^{2}b^{28}-\frac{3}{5}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}\left(a+bx\right)ab^{28}+\frac{3}{2}\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}a^{2}b^{28}}{b^{32}}+\frac{a^{3}}{b^{4}\left(a+bx\right)^{\frac{1}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x)
[Out] 3*a^3/((b*x + a)^(1/3)*b^4) + 3/40*(5*(b*x + a)^(8/3)*b^28 - 24*(b*x + a)^(
5/3)*a*b^28 + 60*(b*x + a)^(2/3)*a^2*b^28)/b^32
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{2/3}}{2\,b^4}+\frac{3\,a^3}{b^4\,{\left(a+b\,x\right)}^{1/3}}-\frac{9\,a\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(8/3))/(8*b^4) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) + (3*a^3)/(b^
4*(a + b*x)^(1/3)) - (9*a*(a + b*x)^(5/3))/(5*b^4)
```

3.414 $\int \frac{x^2}{(a+bx)^{4/3}} dx$

Optimal. Leaf size=49

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

```
[Out] -3*a^2/b^3/(b*x+a)^(1/3)-3*a*(b*x+a)^(2/3)/b^3+3/5*(b*x+a)^(5/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^(
5/3))/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{4/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx\\ &= -\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.69

$$\frac{3\left(-9a^2-3abx+b^2x^2\right)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (3*(-9*a^2 - 3*a*b*x + b^2*x^2))/(5*b^3*(a + b*x)^(1/3))
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(49) = 98$.
time = 6.28, size = 144, normalized size = 2.94

$$\frac{3a^{\frac{2}{3}}\left(9a^4\left(1-\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+3a^3bx\left(9-7\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+a^2b^2x^2\left(27-14\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)+b^3x^3\left(-a\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}+9a+bx\left(\frac{a+bx}{a}\right)^{\frac{2}{3}}\right)\right)}{5b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(a + b*x)^(4/3),x]')
```

```
[Out] 3 a ^ (2 / 3) (9 a ^ 4 (1 - ((a + b x) / a) ^ (2 / 3)) + 3 a ^ 3 b x (9 - 7
 ((a + b x) / a) ^ (2 / 3)) + a ^ 2 b ^ 2 x ^ 2 (27 - 14 ((a + b x) / a) ^
(2 / 3)) + b ^ 3 x ^ 3 (-a ((a + b x) / a) ^ (2 / 3) + 9 a + b x ((a + b x)
 / a) ^ (2 / 3))) / (5 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3
 x ^ 3))
```

Maple [A]
time = 0.13, size = 38, normalized size = 0.78

method	result	size
gosper	$-\frac{3\left(-x^{2} b^{2}+3 a b x+9 a^{2}\right)}{5\left(b x+a\right)^{\frac{1}{3}} b^{3}}$	32
trager	$-\frac{3\left(-x^{2} b^{2}+3 a b x+9 a^{2}\right)}{5\left(b x+a\right)^{\frac{1}{3}} b^{3}}$	32
risch	$-\frac{3\left(-b x+4 a\right)\left(b x+a\right)^{\frac{2}{3}}}{5 b^{3}}-\frac{3 a^{2}}{b^{3}\left(b x+a\right)^{\frac{1}{3}}}$	37
derivativedivides	$\frac{\frac{3\left(b x+a\right)^{\frac{5}{3}}}{5}-3 a\left(b x+a\right)^{\frac{2}{3}}-\frac{3 a^{2}}{\left(b x+a\right)^{\frac{1}{3}}}}{b^{3}}$	38
default	$\frac{\frac{3\left(b x+a\right)^{\frac{5}{3}}}{5}-3 a\left(b x+a\right)^{\frac{2}{3}}-\frac{3 a^{2}}{\left(b x+a\right)^{\frac{1}{3}}}}{b^{3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/5*(b*x+a)^(5/3)-a*(b*x+a)^(2/3)-a^2/(b*x+a)^(1/3))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.84

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^3}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{b^3}-\frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^3 - 3*(b*x + a)^(2/3)*a/b^3 - 3*a^2/((b*x + a)^(1/3)*
b^3)
```

Fricas [**A**]
time = 0.30, size = 40, normalized size = 0.82

$$\frac{3\left(b^{2} x^{2}-3\,a b x-9\,a^{2}\right)(bx+a)^{\frac{2}{3}}}{5\left(b^{4} x+a b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b^2*x^2 - 3*a*b*x - 9*a^2)*(b*x + a)^(2/3)/(b^4*x + a*b^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(46) = 92$.
time = 0.92, size = 534, normalized size = 10.90

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}+\frac{27a^{\frac{29}{3}}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}-\frac{63a^{\frac{26}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}+\frac{81a^{\frac{26}{3}}bx}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}-\frac{42a^{\frac{23}{3}}b^{2}x^{2}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}+\frac{81a^{\frac{23}{3}}b^{2}x^{2}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}-\frac{3a^{\frac{20}{3}}b^{3}x^{3}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}+\frac{27a^{\frac{20}{3}}b^{3}x^{3}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}+\frac{3a^{\frac{17}{3}}b^{4}x^{4}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3}+15a^{7}b^{4}x+15a^{6}b^{5}x^{2}+5a^{5}b^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b*
*5*x**2 + 5*a**5*b**6*x**3) + 27*a**(29/3)/(5*a**8*b**3 + 15*a**7*b**4*x +
15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 63*a**(26/3)*b*x*(1 + b*x/a)**(2/3)
/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81
*a**(26/3)*b*x/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b
**6*x**3) - 42*a**(23/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**
7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81*a**(23/3)*b**2*x**2/(
5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 3*a*
*(20/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**
6*b**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(20/3)*b**3*x**3/(5*a**8*b**3 + 15*
a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 3*a**(17/3)*b**4*x**4
*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a
**5*b**6*x**3)
```

Giac [**A**]
time = 0.00, size = 67, normalized size = 1.37

$$3\left(\frac{\frac{1}{5}\left((a+bx)^{\frac{1}{3}}\right)^{2}(a+bx)\,b^{12}-\left((a+bx)^{\frac{1}{3}}\right)^{2}ab^{12}}{b^{15}}-\frac{a^{2}}{b^{3}\,(a+bx)^{\frac{1}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3*a^2/((b*x + a)^(1/3)*b^3) + 3/5*((b*x + a)^(5/3)*b^12 - 5*(b*x + a)^(2/3
)*a*b^12)/b^15
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$-\frac{15\,a\,\left(a+b\,x\right)-3\,{\left(a+b\,x\right)}^2+15\,a^2}{5\,b^3\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x) - 3*(a + b*x)^2 + 15*a^2)/(5*b^3*(a + b*x)^(1/3))
```

3.415 $\int \frac{x}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=32

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

```
[Out] 3*a/b^2/(b*x+a)^(1/3)+3/2*(b*x+a)^(2/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx\\ &= \frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 1.86, size = 33, normalized size = 1.03

$$\text{Piecewise}\left[\left\{\left\{\frac{3\,(3a+bx)}{2b^2\,(a+bx)^{\frac{1}{3}}}, b!{=}0\right\}\right\}, \frac{x^2}{2a^{\frac{4}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/(a + b*x)^(4/3),x]')
```

```
[Out] Piecewise[{{3 (3 a + b x) / (2 b ^ 2 (a + b x) ^ (1 / 3)), b != 0}}, x ^ 2
/ (2 a ^ (4 / 3))]
```

Maple [A]
time = 0.12, size = 25, normalized size = 0.78

method	result	size
gosper	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}}\,b^2}$	20
trager	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}}\,b^2}$	20
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{2}{3}}}{2}+\frac{3a}{(bx+a)^{\frac{1}{3}}}}{b^2}$	25
default	$\frac{\frac{3(bx+a)^{\frac{2}{3}}}{2}+\frac{3a}{(bx+a)^{\frac{1}{3}}}}{b^2}$	25
risch	$\frac{3a}{b^2(bx+a)^{\frac{1}{3}}}+\frac{3(bx+a)^{\frac{2}{3}}}{2b^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/2*(b*x+a)^(2/3)+a/(b*x+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b^2}+\frac{3\,a}{(bx+a)^{\frac{1}{3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b^2 + 3*a/((b*x + a)^(1/3)*b^2)
```

Fricas [A]
time = 0.31, size = 29, normalized size = 0.91

$$\frac{3\,(bx+3\,a)(bx+a)^{\frac{2}{3}}}{2\,(b^{3}x+ab^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + 3*a)*(b*x + a)^(2/3)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.31, size = 41, normalized size = 1.28

$$\begin{cases} \frac{9a}{2b^{2}\sqrt[3]{a+bx}}+\frac{3x}{2b\sqrt[3]{a+bx}} & \text{for}\ b\neq 0\\ \frac{x^{2}}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(2*b**2*(a + b*x)**(1/3)) + 3*x/(2*b*(a + b*x)**(1/3)), Ne(b
, 0)), (x**2/(2*a**(4/3)), True))
```

Giac [A]
time = 0.00, size = 38, normalized size = 1.19

$$\frac{3\left(\frac{\left((a+bx)^{\frac{1}{3}}\right)^{2}b}{2b^{2}}+\frac{a}{b(a+bx)^{\frac{1}{3}}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x)
```

```
[Out] 3/2*((b*x + a)^(2/3)/b + 2*a/((b*x + a)^(1/3)*b))/b
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.62

$$\frac{9\,a+3\,b\,x}{2\,b^{2}\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(4/3),x)
```

```
[Out] (9*a + 3*b*x)/(2*b^2*(a + b*x)^(1/3))
```

3.416 $\int \frac{1}{(a+bx)^{4/3}} dx$

Optimal. Leaf size=14

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

[Out] -3/b/(b*x+a)^(1/3)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-4/3),x]

[Out] -3/(b*(a + b*x)^(1/3))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}} dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-4/3),x]

[Out] -3/(b*(a + b*x)^(1/3))

Mathics [A]
time = 1.57, size = 12, normalized size = 0.86

$$\frac{-3}{b(a+bx)^{\frac{1}{3}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-4/3),x]')
[Out] -3 / (b (a + b x) ^ (1 / 3))
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
derivativedivides	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
default	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
trager	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] -3/b/(b*x+a)^(1/3)
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="maxima")
[Out] -3/((b*x + a)^(1/3)*b)
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.43

$$-\frac{3\,(bx+a)^{\frac{2}{3}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)/(b^2*x + a*b)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.86

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3),x)
```

```
[Out] -3/(b*(a + b*x)**(1/3))
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.07

$$-\frac{3}{b\,(a+bx)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x)
```

```
[Out] -3/((b*x + a)^(1/3)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{3}{b\,(a+b\,x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(4/3),x)
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

3.417 $\int \frac{1}{x(a+bx)^{4/3}} dx$

Optimal. Leaf size=93

$$\frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

```
[Out] 3/a/(b*x+a)^(1/3)-1/2*ln(x)/a^(4/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)+a
rctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(4/3)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {53, 57, 631, 210, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt
[3]*a^(1/3))])/a^(4/3) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a + b*x)^(1
/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{4/3}}\,dx = \frac{3}{a\sqrt[3]{a+bx}} + \frac{\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{a\sqrt[3]{a+bx}} - \frac{\log(x)}{2a^{4/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{4/3}} + \frac{3\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+\ldots}\right.}{2a\ldots}$$

$$= \frac{3}{a\sqrt[3]{a+bx}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+\ldots}}{\sqrt[3]{a}}\ldots\right.}{a^{4/3}}$$

$$= \frac{3}{a\sqrt[3]{a+bx}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - \frac{\log(x)}{2a^{4/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

Mathematica [A]
time = 0.08, size = 111, normalized size = 1.19

$$\frac{\frac{6\sqrt[3]{a}}{\sqrt[3]{a+bx}} + 2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x)^(4/3)),x]

[Out] ((6*a^(1/3))/(a + b*x)^(1/3) + 2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(2*a^(4/3))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 3.88, size = 131, normalized size = 1.41

$$-\frac{\text{Gamma}\left[-\frac{1}{3}\right]\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]}{3a^{\frac{4}{3}}\text{Gamma}\left[\frac{2}{3}\right]}-\frac{\text{Gamma}\left[-\frac{1}{3}\right]\text{Log}\left[1-\frac{b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]}{3a^{\frac{4}{3}}\text{Gamma}\left[\frac{2}{3}\right]}-\frac{\text{Gamma}\left[-\frac{1}{3}\right]}{ab^{\frac{1}{3}}\text{Gamma}\left[\frac{2}{3}\right]\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x*(a + b*x)^(4/3)),x]')

[Out] -1 ^ (2 / 3) Gamma[-1 / 3] Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a ^ (1 / 3)] / (3 a ^ (4 / 3) Gamma[2 / 3]) - Gamma[-1 / 3] Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) / a ^ (1 / 3)] / (3 a ^ (4 / 3) Gamma[2 / 3]) + -1 ^ (1 / 3) Gamma[-1 / 3] Log[1 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a ^ (1 / 3)] / (3 a ^ (4 / 3) Gamma[2 / 3]) - Gamma[-1 / 3] / (a b ^ (1 / 3) Gamma[2 / 3] (a / b + x) ^ (1 / 3))

Maple [A]

time = 0.11, size = 95, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}}{a}+\frac{3}{a(bx+a)^{\frac{1}{3}}}$	95
default	$\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}}{a}+\frac{3}{a(bx+a)^{\frac{1}{3}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)

[Out] 3/a*(1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))+3/a/(b*x+a)^(1/3)

Maxima [A]
time = 0.36, size = 88, normalized size = 0.95

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

Fricas [A]
time = 0.31, size = 285, normalized size = 3.06

$$\left[\frac{\sqrt{3}\,(abx+a^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6\,(bx+a)^{\frac{2}{3}}a}{2\,(a^2bx+a^3)},\ -\frac{(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}\,(abx+a^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-6\,(bx+a)^{\frac{2}{3}}a}{2\,(a^2bx+a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(a*b*x + a^2)*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - (b*x + a)*a^(2/3)*log((b*x + a)^(2/3) + (b*x
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x + a)*a^(2/3)*log((b*x + a)^(1/3) -
a^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*b*x + a^3), -1/2*((b*x + a)*a^(2/3)*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x + a)*a^(2/3
)*log((b*x + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x + a^2)*arctan(1/3*sqrt(
3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x + a)^(2/3)*a)/(a
^2*b*x + a^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.12, size = 184, normalized size = 1.98

$$-\frac{\Gamma\left(-\frac{1}{3}\right)}{a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(4/3),x)
```

```
[Out] -gamma(-1/3)/(a*b**(1/3)*(a/b + x)**(1/3)*gamma(2/3)) - log(1 - b**(1/3)*(a
/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(2*I*pi/3
```

```
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/
3)/(3*a**(4/3)*gamma(2/3)) - exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3))
```

Giac [A]
time = 0.00, size = 156, normalized size = 1.68

$$3\left(-\frac{\left(a^{\frac{1}{3}}\right)^{2}\ln\left(\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}\left(a+bx\right)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{2}}+\frac{\left(a^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(\left(a+bx\right)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{2}}+\frac{a^{\frac{1}{3}}a^{\frac{1}{3}}\ln\left|\left(a+bx\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3a^{2}}+\frac{1}{a\left(a+bx\right)^{\frac{1}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x)
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

Mupad [B]
time = 0.06, size = 114, normalized size = 1.23

$$\frac{\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-9\,a^{4/3}\right)}{a^{4/3}}+\frac{3}{a\,{\left(a+b\,x\right)}^{1/3}}+\frac{\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{4/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{4/3}}-\frac{\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{4/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(4/3)),x)
```

```
[Out] log(9*a*(a + b*x)^(1/3) - 9*a^(4/3))/a^(4/3) + 3/(a*(a + b*x)^(1/3)) + (log
(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/
(2*a^(4/3)) - (log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i + 1)^2)/4)*
(3^(1/2)*1i + 1))/(2*a^(4/3))
```

3.418 $\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=113

$$-\frac{4b}{a^2\sqrt[3]{a+bx}}-\frac{1}{ax\sqrt[3]{a+bx}}-\frac{4b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}$$

```
[Out] -4*b/a^2/(b*x+a)^(1/3)-1/a/x/(b*x+a)^(1/3)+2/3*b*ln(x)/a^(7/3)-2*b*ln(a^(1/
3)-(b*x+a)^(1/3))/a^(7/3)-4/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3
)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 57, 631, 210, 31}

$$\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{4b}{a^2\sqrt[3]{a+bx}}-\frac{1}{ax\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] (-4*b)/(a^2*(a + b*x)^(1/3)) - 1/(a*x*(a + b*x)^(1/3)) - (4*b*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b*Log[x]
)/(3*a^(7/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \frac{3}{ax\sqrt[3]{a+bx}} + \frac{4\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{(4b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a^2}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{7/3}} - \cdots$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}} + \frac{(4b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\cdots\right.}{a^{7/\cdots}}$$

Mathematica [A]
time = 0.19, size = 123, normalized size = 1.09

$$\frac{-\frac{3\sqrt[3]{a}(a+4bx)}{x\sqrt[3]{a+bx}}-4\sqrt{3}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] ((-3*a^(1/3)*(a + 4*b*x))/(x*(a + b*x)^(1/3)) - 4*Sqrt[3]*b*ArcTan[(1 + (2*
(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 4*b*Log[a^(1/3) - (a + b*x)^(1/3)] + 2
*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(7/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 9.85, size = 173, normalized size = 1.53

$$\frac{\text{Gamma}\left[-\frac{1}{3}\right]\left(3a^{\frac{4}{3}}+12a^{\frac{1}{3}}bx-b^{\frac{4}{3}}x\text{Log}\left[-\frac{-a^{\frac{1}{3}}+b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]\left(\frac{a}{b}+x\right)^{\frac{1}{3}}-b^{\frac{4}{3}}x\text{Log}\left[-\frac{-a^{\frac{1}{3}}+b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a^{\frac{1}{3}}}\right]\left(\frac{a}{b}+x\right)^{\frac{1}{3}}+4b^{\frac{4}{3}}x\text{Log}\left[-\frac{-a^{\frac{1}{3}}+b^{\frac{1}{3}}\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\right]\left(\frac{a}{b}+x\right)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}b^{\frac{1}{3}}x\text{Gamma}\left[\frac{2}{3}\right]\left(\frac{a}{b}+x\right)^{\frac{1}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*(a + b*x)^(4/3)),x]')
```

```
[Out] Gamma[-1 / 3] (3 a ^ (4 / 3) + 12 a ^ (1 / 3) b x - 4 -1 ^ (1 / 3) b ^ (4 /
 3) x Log[-(-a ^ (1 / 3) + b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I
/ 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 4 -1 ^ (2 / 3) b ^ (4 / 3)
x Log[-(-a ^ (1 / 3) + b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3
Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 4 b ^ (4 / 3) x Log[-(-a ^ (1 /
 3) + b ^ (1 / 3) (a / b + x) ^ (1 / 3)) / a ^ (1 / 3)] (a / b + x) ^ (1 /
3)) / (9 a ^ (7 / 3) b ^ (1 / 3) x Gamma[2 / 3] (a / b + x) ^ (1 / 3))
```

Maple [A]
time = 0.11, size = 112, normalized size = 0.99

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{a^{2}x}-\frac{3b}{a^{2}(bx+a)^{\frac{1}{3}}}-\frac{4b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{4b\sqrt{3}\,\arctan\left(\sqrt{\ldots}\right.}{3a\ldots}$

derivativedivides	$3b$	$\left(-\frac{1}{a^{2}(bx+a)^{\frac{1}{3}}}+\frac{-\frac{(bx+a)^{\frac{2}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+\cdots}{a^{\frac{1}{3}}}\right.}{3}\right.}{9a^{\frac{1}{3}}}}{a^{2}}\right.$
default	$3b$	$\left(-\frac{1}{a^{2}(bx+a)^{\frac{1}{3}}}+\frac{-\frac{(bx+a)^{\frac{2}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+\cdots}{a^{\frac{1}{3}}}\right.}{3}\right.}{9a^{\frac{1}{3}}}}{a^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/a^2/(b*x+a)^(1/3)+1/a^2*(-1/3*(b*x+a)^(2/3)/b/x-4/9/a^(1/3)*ln((b*x
+a)^(1/3)-a^(1/3))+2/9/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/
3))-4/9*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.34, size = 122, normalized size = 1.08

$$-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{7}{3}}}-\frac{4(bx+a)b-3ab}{(bx+a)^{\frac{4}{3}}a^{2}-(bx+a)^{\frac{1}{3}}a^{3}}+\frac{2b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{4b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) - (4*(b*x + a)*b - 3*a*b)/((b*x + a)^(4/3)*a^2 - (b*x + a)^(1/3)*a^3)
 + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7/3) -
 4/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(88) = 176.
time = 0.32, size = 407, normalized size = 3.60

$$\left[\frac{6\sqrt{\frac{1}{3}}(ab^{2}x^{2}+a^{2}bx)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+2(b^{2}x^{2}+abx)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4(b^{2}x^{2}+abx)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3(4abx+a^{2})(bx+a)^{\frac{2}{3}}}{3(a^{3}bx^{2}+a^{4}x)},-\frac{12\sqrt{\frac{1}{3}}(ab^{2}x^{2}+a^{2}bx)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2(b^{2}x^{2}+abx)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+4(b^{2}x^{2}+abx)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3(4abx+a^{2})(bx+a)^{\frac{2}{3}}}{3(a^{3}bx^{2}+a^{4}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*s
qrt(1/3)*(2*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*
sqrt((-a)^(1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + 2*(b^2*x^2 +
a*b*x)*(-a)^(2/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(
2/3)) - 4*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) -
3*(4*a*b*x + a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x), -1/3*(12*sqrt(1/3)*
(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log(
(b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^2 + a
*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x + a^2)*(b*x
 + a)^(2/3))/(a^3*b*x^2 + a^4*x)]
```

Sympy [**C**] Result contains complex when optimal does not.
time = 1.61, size = 857, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(4/3),x)
```

```
[Out] -9*a**(4/3)*b**(2/3)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*g
amma(2/3)) + 12*a**(1/3)*b**(5/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a
**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)
**(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*
pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a
/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*log(1 - b**(1/3)*(a/b + x)*
*(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi
/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3)) +
4*b**2*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/
3)) + 4*b**2*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)
*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3))
```

Giac [**A**]

time = 0.01, size = 204, normalized size = 1.81

$$3\left(\frac{2\left(a^{\frac{1}{3}}\right)^{2}b\ln\left(\left(\left(a+bx\right)^{\frac{1}{3}}\right)^{2}+a^{\frac{1}{3}}\left(a+bx\right)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{9a^{3}}-\frac{\frac{1}{3}\cdot 4b\arctan\left(\frac{2\left(\left(a+bx\right)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{2}a^{\frac{1}{3}}}-\frac{4a^{\frac{1}{3}}ba^{\frac{1}{3}}\ln\left|\left(a+bx\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{3\cdot 3a^{3}}+\frac{\frac{1}{3}\left(-4\left(a+bx\right)b+3ba\right)}{a^{2}\left(\left(a+bx\right)^{\frac{1}{3}}\left(a+bx\right)-\left(a+bx\right)^{\frac{1}{3}}a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x)
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7
/3) - 4/3*b*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) - (4*(b*x + a)*b -
3*a*b)/(((b*x + a)^(4/3) - (b*x + a)^(1/3)*a)*a^2)
```

Mupad [B]
time = 0.07, size = 173, normalized size = 1.53

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\,\left(a+b\,x\right)}{a^2}}{a\,\left(a+b\,x\right)^{1/3}-\left(a+b\,x\right)^{4/3}}+\frac{\ln\left(a^{7/3}\,\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,\left(a+b\,x\right)^{1/3}\right)\,\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\,\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,\left(a+b\,x\right)^{1/3}\right)\,\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}-\frac{4\,b\,\ln\left(16\,a^{7/3}\,b^2-16\,a^2\,b^2\,\left(a+b\,x\right)^{1/3}\right)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(2*b - 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a + b*x)^(1/3))*(2*b - 3^
(1/2)*b*2i))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x))/a^2)/(a*(a + b*x)^(1/
3) - (a + b*x)^(4/3)) + (log(a^(7/3)*(2*b + 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a
 + b*x)^(1/3))*(2*b + 3^(1/2)*b*2i))/(3*a^(7/3)) - (4*b*log(16*a^(7/3)*b^2
- 16*a^2*b^2*(a + b*x)^(1/3)))/(3*a^(7/3))
```

3.419 $\int \frac{1}{x^3(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$\frac{14b^2}{3a^3\sqrt[3]{a+bx}}-\frac{1}{2ax^2\sqrt[3]{a+bx}}+\frac{7b}{6a^2x\sqrt[3]{a+bx}}+\frac{14b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\ldots\right.}{3a^{10}\ldots}$$

```
[Out] 14/3*b^2/a^3/(b*x+a)^(1/3)-1/2/a/x^2/(b*x+a)^(1/3)+7/6*b/a^2/x/(b*x+a)^(1/3
)-7/9*b^2*ln(x)/a^(10/3)+7/3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(10/3)+14/9*b^
2*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {44, 53, 57, 631, 210, 31}

$$-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{14b^2}{3a^3\sqrt[3]{a+bx}}+\frac{7b}{6a^2x\sqrt[3]{a+bx}}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] (14*b^2)/(3*a^3*(a + b*x)^(1/3)) - 1/(2*a*x^2*(a + b*x)^(1/3)) + (7*b)/(6*a
^2*x*(a + b*x)^(1/3)) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) + (7*b^2*L
og[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{3}{ax^2\sqrt[3]{a+bx}} + \frac{7\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} - \frac{(14b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a^2}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{(14b^2)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^3}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{(7b^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\right.}{3a^{10/}}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{3a^{10/3}}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{7b^2}{9}$$

Mathematica [A]
time = 0.25, size = 142, normalized size = 0.95

$$\frac{\frac{3\sqrt[3]{a}\,(-3a^2+7abx+28b^2x^2)}{x^2\sqrt[3]{a+bx}} + 28\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - 14b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] ((3*a^(1/3)*(-3*a^2 + 7*a*b*x + 28*b^2*x^2))/(x^2*(a + b*x)^(1/3)) + 28*Sqr
t[3]*b^2*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^2*Log[a^(
1/3) - (a + b*x)^(1/3)] - 14*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a
 + b*x)^(2/3)])/(18*a^(10/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 30.56, size = 1120, normalized size = 7.52

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*(a + b*x)^(4/3)),x]')

[Out] -Gamma[-1 / 3] (-9 a ^ (55 / 3) b - 33 a ^ (52 / 3) b ^ 2 x + 75 a ^ (49 /
3) b ^ 3 x ^ 2 - 28 -1 ^ (1 / 3) a ^ 16 b ^ (10 / 3) x ^ 2 Log[(a ^ (1 / 3)
 - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi]) / a ^ (1 / 3)]
(a / b + x) ^ (1 / 3) + 28 -1 ^ (2 / 3) a ^ 16 b ^ (10 / 3) x ^ 2 Log[(a ^
(1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi]) / a ^ (1
 / 3)] (a / b + x) ^ (1 / 3) + 28 a ^ 16 b ^ (10 / 3) x ^ 2 Log[(a ^ (1 / 3
) - b ^ (1 / 3) (a / b + x) ^ (1 / 3)) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3)
 + 639 a ^ (46 / 3) b ^ 4 x ^ 3 - 168 -1 ^ (1 / 3) a ^ 15 b ^ (13 / 3) x ^
3 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi
]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 168 -1 ^ (2 / 3) a ^ 15 b ^ (13 /
 3) x ^ 3 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2
I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 168 a ^ 15 b ^ (13 / 3) x
 ^ 3 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3)) / a ^ (1 / 3)] (
a / b + x) ^ (1 / 3) + 1545 a ^ (43 / 3) b ^ 5 x ^ 4 - 420 -1 ^ (1 / 3) a ^
 14 b ^ (16 / 3) x ^ 4 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3)
 exp_polar[4 I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 420 -1 ^ (2
/ 3) a ^ 14 b ^ (16 / 3) x ^ 4 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^
 (1 / 3) exp_polar[2 I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 420
a ^ 14 b ^ (16 / 3) x ^ 4 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 /
 3)) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 1941 a ^ (40 / 3) b ^ 6 x ^ 5 -
 560 -1 ^ (1 / 3) a ^ 13 b ^ (19 / 3) x ^ 5 Log[(a ^ (1 / 3) - b ^ (1 / 3)
(a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (
1 / 3) + 560 -1 ^ (2 / 3) a ^ 13 b ^ (19 / 3) x ^ 5 Log[(a ^ (1 / 3) - b ^
(1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi]) / a ^ (1 / 3)] (a / b
+ x) ^ (1 / 3) + 560 a ^ 13 b ^ (19 / 3) x ^ 5 Log[(a ^ (1 / 3) - b ^ (1 /
3) (a / b + x) ^ (1 / 3)) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 1377 a ^ (
37 / 3) b ^ 7 x ^ 6 - 420 -1 ^ (1 / 3) a ^ 12 b ^ (22 / 3) x ^ 6 Log[(a ^ (
1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4 I / 3 Pi]) / a ^ (1
/ 3)] (a / b + x) ^ (1 / 3) + 420 -1 ^ (2 / 3) a ^ 12 b ^ (22 / 3) x ^ 6 Lo
g[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[2 I / 3 Pi]) /
 a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 420 a ^ 12 b ^ (22 / 3) x ^ 6 Log[(a
^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3)) / a ^ (1 / 3)] (a / b + x) ^
 (1 / 3) + 525 a ^ (34 / 3) b ^ 8 x ^ 7 - 168 -1 ^ (1 / 3) a ^ 11 b ^ (25 /
 3) x ^ 7 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_polar[4
I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 168 -1 ^ (2 / 3) a ^ 11 b
 ^ (25 / 3) x ^ 7 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3) exp_
polar[2 I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 168 a ^ 11 b ^ (2
5 / 3) x ^ 7 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3)) / a ^ (1
 / 3)] (a / b + x) ^ (1 / 3) + 84 a ^ (31 / 3) b ^ 9 x ^ 8 - 28 -1 ^ (1 / 3
) a ^ 10 b ^ (28 / 3) x ^ 8 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1
 / 3) exp_polar[4 I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) + 28 -1 ^
 (2 / 3) a ^ 10 b ^ (28 / 3) x ^ 8 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b +
x) ^ (1 / 3) exp_polar[2 I / 3 Pi]) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3) +
```

28 a ^ 10 b ^ (28 / 3) x ^ 8 Log[(a ^ (1 / 3) - b ^ (1 / 3) (a / b + x) ^ (1 / 3)) / a ^ (1 / 3)] (a / b + x) ^ (1 / 3)) / (54 a ^ (40 / 3) b ^ (22 / 3) x ^ 2 Gamma[2 / 3] (a / b + x) ^ (19 / 3))

Maple [A]
time = 0.13, size = 126, normalized size = 0.85

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(-10bx+3a)}{6a^3x^2}+\frac{3b^2}{a^3(bx+a)^{\frac{1}{3}}}+\frac{14b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2}{}$
derivativedivides	$3b^2\left(\frac{1}{a^3(bx+a)^{\frac{1}{3}}}-\frac{\frac{-\frac{5(bx+a)^{\frac{5}{3}}}{9}+\frac{13a(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{1}{3}}}+\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{1}{3}}}-\frac{14\sqrt{3}\ \mathrm{arc}}{}}{a^3}\right.$
default	$3b^2\left(\frac{1}{a^3(bx+a)^{\frac{1}{3}}}-\frac{\frac{-\frac{5(bx+a)^{\frac{5}{3}}}{9}+\frac{13a(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{1}{3}}}+\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{1}{3}}}-\frac{14\sqrt{3}\ \mathrm{arc}}{}}{a^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(1/a^3/(b*x+a)^(1/3)-1/a^3*((-5/9*(b*x+a)^(5/3)+13/18*a*(b*x+a)^(2/3)
)/b^2/x^2-14/27/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))+7/27/a^(1/3)*ln((b*x+a)^(
2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-14/27*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2
)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.36, size = 158, normalized size = 1.06

$$\frac{14\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}+\frac{28\,(bx+a)^2b^2-49\,(bx+a)ab^2+18\,a^2b^2}{6\left((bx+a)^{\frac{7}{3}}a^3-2\,(bx+a)^{\frac{4}{3}}a^4+(bx+a)^{\frac{1}{3}}a^5\right)}-\frac{7\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{10}{3}}}+\frac{14\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) + 1/6*(28*(b*x + a)^2*b^2 - 49*(b*x + a)*a*b^2 + 18*a^2*b^2)/((b*x
 + a)^(7/3)*a^3 - 2*(b*x + a)^(4/3)*a^4 + (b*x + a)^(1/3)*a^5) - 7/9*b^2*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(10/3) + 14/9*b^2*
log((b*x + a)^(1/3) - a^(1/3))/a^(10/3)
```

Fricas [A]
time = 0.32, size = 407, normalized size = 2.73

$$\left[\frac{42\,\sqrt{\frac{1}{3}}\,(ab^3x^3 + a^2b^2x^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx + 3\,\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}} - (bx+a)^{\frac{1}{3}}a - a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}} - 3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}} + 3\,a}{x}\right) - 14\,(b^3x^3 + ab^2x^2)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}} + (bx+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + 28\,(b^3x^3 + ab^2x^2)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) + 3\,(28\,ab^2x^2 + 7\,a^2bx - 3\,a^3)(bx+a)^{\frac{2}{3}}}{18\,(a^4bx^3 + a^5x^2)}, -\frac{14\,(b^3x^3 + ab^2x^2)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}} + (bx+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) - 28\,(b^3x^3 + ab^2x^2)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) - \frac{84\,\sqrt{\frac{1}{3}}\,(ab^3x^3 + a^2b^2x^2)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - 3\,(28\,ab^2x^2 + 7\,a^2bx - 3\,a^3)(bx+a)^{\frac{2}{3}}}{18\,(a^4bx^3 + a^5x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/18*(42*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-1/a^(2/3))*log((2*b*x +
 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt
(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 14*(b^3*x^3 + a*b^2*x^
2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 28*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(28*a*b^2*x^
2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2), -1/18*(14*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
 a^(2/3)) - 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3))
 - 84*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x
+ a)^(2/3))/(a^4*b*x^3 + a^5*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.39, size = 2793, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(4/3),x)
```

```
[Out] 54*a**(13/3)*b**(5/3)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/
3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 201*a*
*(10/3)*b**(8/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 2
31*a**(7/3)*b**(11/3)*(a/b + x)**2*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
```

/3)) - 84*a**(4/3)*b**(14/3)*(a/b + x)**3*exp(2*I*pi/3)*gamma(-1/3)/(-54*a*
(22/3)(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*ex
p(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(2/3)) + 28*a**4*b**2*(a/b + x)**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma
(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a
(13/3)*b3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a
/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3
)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) -
162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)
*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a/b + x)*
*(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(
19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)
*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*b**3*(a/b + x)**(4/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b +
 x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) -
84*a**3*b**3*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1
/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3)
 + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*
b**3*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)
/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(
2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(
16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a
/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a
(22/3)*(a/b + x)(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
 x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)
*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2
*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(

```
2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a*
*(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b +
 x)**(10/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma
(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(1
9/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
+ x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*
exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(
1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-5
4*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/
b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3
)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi
/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1
/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3
) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3))
```

Giac [A]
time = 0.01, size = 236, normalized size = 1.58

$$3\left(\frac{b^2}{a^3\left(a+bx\right)^{\frac{1}{3}}}-\frac{\frac{1}{18}\left(-10\left(\left(a+bx\right)^{\frac{1}{3}}\right)^2\left(a+bx\right)b^2+13\left(\left(a+bx\right)^{\frac{1}{3}}\right)^2b^2a\right)}{a^3\left(a+bx-a\right)^2}-\frac{\frac{1}{27}\cdot 7b^2\ln\left(\left(\left(a+bx\right)^{\frac{1}{3}}\right)^2+a^{\frac{1}{3}}\left(a+bx\right)^{\frac{1}{3}}+a^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{a^3a^{\frac{1}{3}}}+\frac{\frac{1}{9}\cdot 14\left(a^{\frac{1}{3}}\right)^2b^2\arctan\left(\frac{2\left(\left(a+bx\right)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\,a^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^4}+\frac{14a^{\frac{1}{3}}b^2a^{\frac{1}{3}}\ln\left|\left(a+bx\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|}{9\cdot 3a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x)
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) - 7/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(10/3) + 14/9*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(10/3) + 3*b^2/(
(b*x + a)^(1/3)*a^3) + 1/6*(10*(b*x + a)^(5/3)*b^2 - 13*(b*x + a)^(2/3)*a*b
^2)/(a^3*b^2*x^2)
```

Mupad [B]
time = 0.13, size = 221, normalized size = 1.48

$$\frac{\frac{3\,b^2}{a}+\frac{14\,b^2\,{\left(a+b\,x\right)}^2}{3\,a^3}-\frac{49\,b^2\,\left(a+b\,x\right)}{6\,a^2}}{{\left(a+b\,x\right)}^{7/3}-2\,a\,{\left(a+b\,x\right)}^{4/3}+a^2\,{\left(a+b\,x\right)}^{1/3}}+\frac{\ln\left(588\,a^3\,b^4\,{\left(a+b\,x\right)}^{1/3}-3\,a^{10/3}\,{\left(-7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}^2\right)\,\left(-7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}{9\,a^{10/3}}-\frac{\ln\left(588\,a^3\,b^4\,{\left(a+b\,x\right)}^{1/3}-3\,a^{10/3}\,{\left(7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}^2\right)\,\left(7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}{9\,a^{10/3}}+\frac{14\,b^2\,\ln\left(588\,a^3\,b^4\,{\left(a+b\,x\right)}^{1/3}-588\,a^{10/3}\,b^4\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(4/3)),x)
```

```
[Out] ((3*b^2)/a + (14*b^2*(a + b*x)^2)/(3*a^3) - (49*b^2*(a + b*x))/(6*a^2))/((a
 + b*x)^(7/3) - 2*a*(a + b*x)^(4/3) + a^2*(a + b*x)^(1/3)) + (log(588*a^3*b
^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^(1/2)*b^2*7i - 7*b^2)^2)*(3^(1/2)*b^2*7i
 - 7*b^2))/(9*a^(10/3)) - (log(588*a^3*b^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^
(1/2)*b^2*7i + 7*b^2)^2)*(3^(1/2)*b^2*7i + 7*b^2))/(9*a^(10/3)) + (14*b^2*l
og(588*a^3*b^4*(a + b*x)^(1/3) - 588*a^(10/3)*b^4))/(9*a^(10/3))
```

3.420 $\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

```
[Out] -1/2*ln(x)/a+3/2*ln(a-(b^3*x+a^3)^(1/3))/a+arctan(1/3*(a+2*(b^3*x+a^3)^(1/3
))/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 631, 210, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx &= -\frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3+b^3x}\right)-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3+b^3x}\right)}{2a}\\
&= -\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a}\\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 1.37

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)+2\log\left(a-\sqrt[3]{a^3+b^3x}\right)-\log\left(a^2+a\sqrt[3]{a^3+b^3x}+(a^3+b^3x)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] + 2*Log[a - (a^3
 + b^3*x)^(1/3)] - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/3)])/
(2*a)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.70, size = 95, normalized size = 1.34

$$\frac{\text{Gamma}\left[-\frac{1}{3}\right]\left(-\text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]-\text{Log}\left[1-\frac{a\text{exp\_polar}\left[2I\text{Pi}\right]}{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]\right)}{3a\text{Gamma}\left[\frac{2}{3}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]')
```

```
[Out] Gamma[-1 / 3] (-1 ^ (2 / 3) Log[1 - a exp_polar[4 I / 3 Pi] / (b (a ^ 3 / b
 ^ 3 + x) ^ (1 / 3))] - Log[1 - a exp_polar[2 I Pi] / (b (a ^ 3 / b ^ 3 + x
) ^ (1 / 3))] + -1 ^ (1 / 3) Log[1 - a exp_polar[2 I / 3 Pi] / (b (a ^ 3 /
b ^ 3 + x) ^ (1 / 3))]) / (3 a Gamma[2 / 3])
```

Maple [A]
time = 0.12, size = 86, normalized size = 1.21

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\, \arctan\left(\frac{\left(a+2\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a}\right)}{a}$	86
default	$\frac{\ln\left(a-\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\, \arctan\left(\frac{\left(a+2\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a}\right)}{a}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(b^3*x+a^3)^(1/3))/a+1/a*(-1/2*ln(a^2+a*(b^3*x+a^3)^(1/3)+(b^3*x+a^3)^
(2/3))+3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.34, size = 86, normalized size = 1.21

$$\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(a+2\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{3 a}\right)}{a}-\frac{\log\left(a^{2}+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}} a+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)}{2 a}+\frac{\log\left(-a+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(-a + (b^3*x + a^3)^(1
/3))/a
```

Fricas [A]
time = 0.32, size = 88, normalized size = 1.24

$$\frac{2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\, a+2 \sqrt{3}\, \left(b^{3} x+a^{3}\right)^{\frac{1}{3}}}{3 a}\right)-\log\left(a^{2}+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}} a+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)+2 \log\left(-a+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) -
log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) + 2*log(-a + (b^3*x
+ a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 1.05, size = 138, normalized size = 1.94

$$\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(1/3),x)
```

```
[Out] exp(I*pi/3)*log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamm
a(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(2*I
*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.00, size = 103, normalized size = 1.45

$$3\left(\frac{\ln\left|(a^3+b^3x)^{\frac{1}{3}}-a\right|}{3a}-\frac{\ln\left(\left((a^3+b^3x)^{\frac{1}{3}}\right)^2+(a^3+b^3x)^{\frac{1}{3}}a+a^2\right)}{6a}+\frac{\arctan\left(\frac{a+2(a^3+b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x)
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(abs(-a + (b^3*x + a^3
)^(1/3)))/a
```

Mupad [B]
time = 0.10, size = 105, normalized size = 1.48

$$\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-9\,a\right)}{a}+\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}-\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(1/3)),x)
```

```
[Out] log(9*(b^3*x + a^3)^(1/3) - 9*a)/a + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(b^3*x + a^3)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

3.421 $\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

```
[Out] -1/2*ln(x)/a+3/2*ln(a-(-b^3*x+a^3)^(1/3))/a+arctan(1/3*(a+2*(-b^3*x+a^3)^(1
/3))/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {57, 631, 210, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = -\frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-\right.}}{2a}$$

$$= -\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 1.38

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)+2\log\left(a-\sqrt[3]{a^3-b^3x}\right)-\log\left(a^2+a\sqrt[3]{a^3-b^3x}+(a^3-b^3x)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] + 2*Log[a - (a^3
 - b^3*x)^(1/3)] - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/3)])/
(2*a)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.75, size = 98, normalized size = 1.34

$$\frac{\text{Gamma}\left[-\frac{1}{3}\right]\left(-\text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{I}{3}\text{Pi}\right]}{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]-\text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{5I}{3}\text{Pi}\right]}{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]\right)}{3a\text{Gamma}\left[\frac{2}{3}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]')
```

```
[Out] Gamma[-1 / 3] (-1 ^ (2 / 3) Log[1 - a exp_polar[I Pi] / (b (-a ^ 3 / b ^ 3
+ x) ^ (1 / 3))] - Log[1 - a exp_polar[5 I / 3 Pi] / (b (-a ^ 3 / b ^ 3 + x
) ^ (1 / 3))] + -1 ^ (1 / 3) Log[1 - a exp_polar[I / 3 Pi] / (b (-a ^ 3 / b
 ^ 3 + x) ^ (1 / 3))]) / (3 a Gamma[2 / 3])
```

Maple [A]
time = 0.12, size = 90, normalized size = 1.23

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	90
default	$\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(-b^3*x+a^3)^(1/3))/a+1/a*(-1/2*ln(a^2+a*(-b^3*x+a^3)^(1/3)+(-b^3*x+a^
3)^(2/3))+3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.36, size = 90, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(-a + (-b^3*x + a^3
)^(1/3))/a
```

Fricas [A]
time = 0.35, size = 92, normalized size = 1.26

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a) -
 log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) + 2*log(-a + (-b^
3*x + a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 1.04, size = 136, normalized size = 1.86

$$-\frac{e^{-\frac{2i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(1/3),x)
```

```
[Out] -exp(-2*I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*g
amma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(I*pi)/(b*(-a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(5*I
*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.00, size = 107, normalized size = 1.47

$$3\left(\frac{\ln\left|(a^3-b^3x)^{\frac{1}{3}}-a\right|}{3a}-\frac{\ln\left(\left((a^3-b^3x)^{\frac{1}{3}}\right)^2+(a^3-b^3x)^{\frac{1}{3}}a+a^2\right)}{6a}+\frac{\arctan\left(\frac{a+2(a^3-b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x)
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(abs(-a + (-b^3*x +
 a^3)^(1/3)))/a
```

Mupad [B]
time = 0.13, size = 108, normalized size = 1.48

$$\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-9a\right)}{a}+\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a}-\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(1/3)),x)
```

```
[Out] log(9*(a^3 - b^3*x)^(1/3) - 9*a)/a + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(a^3 - b^3*x)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

3.422 $\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

```
[Out] 1/2*ln(x)/a-3/2*ln(a+(b^3*x-a^3)^(1/3))/a-arctan(1/3*(a-2*(b^3*x-a^3)^(1/3)
)/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {58, 631, 210, 31}

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\log(x)}{2a} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3+b^3x}\right) - \frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-\right.}{2a}$$

$$= \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a} + \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.05, size = 102, normalized size = 1.38

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right) - 2\log\left(a+\sqrt[3]{-a^3+b^3x}\right) + \log\left(a^2-a\sqrt[3]{-a^3+b^3x}+(-a^3+b^3x)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a + (-
a^3 + b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x)^(2/
3)])/(2*a)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.75, size = 96, normalized size = 1.30

$$\frac{\text{Gamma}\left[-\frac{1}{3}\right]\left(\text{Log}\left[1-\frac{a\text{exp\_polar}[I\text{Pi}]}{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right] + \text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{5I}{3}\text{Pi}\right]}{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right] - \text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{I}{3}\text{Pi}\right]}{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]\right)}{3a\text{Gamma}\left[\frac{2}{3}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]')
```

```
[Out] Gamma[-1 / 3] (Log[1 - a exp_polar[I Pi] / (b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3
))] - -1 ^ (1 / 3) Log[1 - a exp_polar[5 I / 3 Pi] / (b (-a ^ 3 / b ^ 3 + x
) ^ (1 / 3))] + -1 ^ (2 / 3) Log[1 - a exp_polar[I / 3 Pi] / (b (-a ^ 3 / b
 ^ 3 + x) ^ (1 / 3))]) / (3 a Gamma[2 / 3])
```

Maple [A]
time = 0.10, size = 96, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{\ln\left(a+\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{\frac{\ln\left(a^{2}-a\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3} x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\, \arctan\left(\frac{\left(-a+2\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a}\right)}{a}$	96
default	$-\frac{\ln\left(a+\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{\frac{\ln\left(a^{2}-a\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}+\left(b^{3} x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\, \arctan\left(\frac{\left(-a+2\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a}\right)}{a}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a+(b^3*x-a^3)^(1/3))/a+1/a*(1/2*ln(a^2-a*(b^3*x-a^3)^(1/3)+(b^3*x-a^3)^
(2/3))+3^(1/2)*arctan(1/3*(-a+2*(b^3*x-a^3)^(1/3))*3^(1/2)/a))
```

Maxima [A]
time = 0.36, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)}{3 a}\right)}{a}+\frac{\log\left(a^{2}-\left(b^{3} x-a^{3}\right)^{\frac{1}{3}} a+\left(b^{3} x-a^{3}\right)^{\frac{2}{3}}\right)}{2 a}-\frac{\log\left(a+\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(a + (b^3*x - a^3)^(1
/3))/a
```

Fricas [A]
time = 0.31, size = 93, normalized size = 1.26

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,a-2\sqrt{3}\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}}{3 a}\right)+\log\left(a^{2}-\left(b^{3} x-a^{3}\right)^{\frac{1}{3}} a+\left(b^{3} x-a^{3}\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(b^{3} x-a^{3}\right)^{\frac{1}{3}}\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) +
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) - 2*log(a + (b^3*x
- a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 1.10, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gam
ma(-1/3)/(3*a*gamma(2/3)) + log(-a*exp_polar(I*pi)/(b*(-a**3/b**3 + x)**(1/
3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(5*I*pi
/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.00, size = 108, normalized size = 1.46

$$3\left(-\frac{\ln\left|(-a^3+b^3x)^{\frac{1}{3}}+a\right|}{3a}+\frac{\ln\left(\left((-a^3+b^3x)^{\frac{1}{3}}\right)^2-(-a^3+b^3x)^{\frac{1}{3}}a+a^2\right)}{6a}+\frac{\arctan\left(\frac{-a+2(-a^3+b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x)
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(abs(a + (b^3*x - a^3
)^(1/3)))/a
```

Mupad [B]
time = 0.11, size = 112, normalized size = 1.51

$$-\frac{\ln\left(9\,a+9\,\left(b^3\,x-a^3\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\,\left(b^3\,x-a^3\right)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}+\frac{\ln\left(\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\,\left(b^3\,x-a^3\right)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1))/
(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1
i - 1))/(2*a) - log(9*a + 9*(b^3*x - a^3)^(1/3))/a
```

3.423 $\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

```
[Out] 1/2*ln(x)/a-3/2*ln(a+(-b^3*x-a^3)^(1/3))/a-arctan(1/3*(a-2*(-b^3*x-a^3)^(1/
3))/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {58, 631, 210, 31}

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\log(x)}{2a} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3-b^3x}\right) - \frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-\right.}{2a}$$

$$= \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a} + \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.07, size = 106, normalized size = 1.39

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right) - 2\log\left(a+\sqrt[3]{-a^3-b^3x}\right) + \log\left(a^2-a\sqrt[3]{-a^3-b^3x}+(-a^3-b^3x)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a + (-
a^3 - b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x)^(2/
3)])/(2*a)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.62, size = 93, normalized size = 1.22

$$\frac{\text{Gamma}\left[-\frac{1}{3}\right]\left(\text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]+\text{Log}\left[1-\frac{a\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]-\text{Log}\left[1-\frac{a\text{exp\_polar}\left[2I\text{Pi}\right]}{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}\right]\right)}{3a\text{Gamma}\left[\frac{2}{3}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]')
```

[Out] Gamma[-1 / 3] (Log[1 - a exp_polar[2 I / 3 Pi] / (b (a ^ 3 / b ^ 3 + x) ^ (1 / 3))] - -1 ^ (1 / 3) Log[1 - a exp_polar[4 I / 3 Pi] / (b (a ^ 3 / b ^ 3 + x) ^ (1 / 3))] + -1 ^ (2 / 3) Log[1 - a exp_polar[2 I Pi] / (b (a ^ 3 / b ^ 3 + x) ^ (1 / 3))]) / (3 a Gamma[2 / 3])

Maple [A]
time = 0.12, size = 100, normalized size = 1.32

method	result	size
derivativedivides	$\frac{\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$	100
default	$\frac{\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$	100

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-b^3*x-a^3)^(1/3),x,method=_RETURNVERBOSE)

[Out] 1/a*(1/2*ln(a^2-a*(-b^3*x-a^3)^(1/3)+(-b^3*x-a^3)^(2/3))+3^(1/2)*arctan(1/3 *(-a+2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a))-ln(a+(-b^3*x-a^3)^(1/3))/a

Maxima [A]
time = 0.35, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}+\frac{\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2\,a}-\frac{\log\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="maxima")

[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(a + (-b^3*x - a^3)^(1/3))/a

Fricas [A]
time = 0.31, size = 97, normalized size = 1.28

$$\frac{2\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,a-2\sqrt{3}\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)-2\,\log\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="fricas")

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
+ log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) - 2*log(a + (-b^
3*x - a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 139, normalized size = 1.83

$$\frac{\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(1/3),x)
```

```
[Out] log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a
*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3/b**3 + x)**(
1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) + exp(2*I*pi/3)*log(-a*exp_polar(2*
I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.00, size = 112, normalized size = 1.47

$$3\left(-\frac{\ln\left|(-a^3-b^3x)^{\frac{1}{3}}+a\right|}{3a}+\frac{\ln\left(\left((-a^3-b^3x)^{\frac{1}{3}}\right)^2-(-a^3-b^3x)^{\frac{1}{3}}a+a^2\right)}{6a}+\frac{\arctan\left(\frac{-a+2(-a^3-b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x)
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(abs(a + (-b^3*x -
 a^3)^(1/3)))/a
```

Mupad [B]
time = 0.07, size = 115, normalized size = 1.51

$$-\frac{\ln\left(9\,a+9\,{\left(-a^3-x\,b^3\right)}^{1/3}\right)}{a}-\frac{\ln\left(\frac{9\,a\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}+9\,{\left(-a^3-x\,b^3\right)}^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}+\frac{\ln\left(\frac{9\,a\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}+9\,{\left(-a^3-x\,b^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1)
)/(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/
2)*1i - 1))/(2*a) - log(9*a + 9*(- b^3*x - a^3)^(1/3))/a
```

3.424 $\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}\,a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a-(b^3*x+a^3)^(1/3))/a^2-arctan(1/3*(a+2*(b^3*x+a^3)^
(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {59, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}\,a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx &= -\frac{\log(x)}{2a^2}-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{3\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3+b^3x}\right)}{2a}\\ &= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a^2}\\ &= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 95, normalized size = 1.32

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}\,a}\right)-2\log\left(a-\sqrt[3]{a^3+b^3x}\right)+\log\left(a^2+a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 + b^3*x)^(1/3)] + Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/
3)])/a^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.47, size = 82, normalized size = 1.14

$$\frac{\text{Log}\left[1-\frac{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}{a}\right]+\text{Log}\left[1-\frac{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a}\right]-\text{Log}\left[1-\frac{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a}\right]}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]')
```

```
[Out] (Log[1 - b (a ^ 3 / b ^ 3 + x) ^ (1 / 3) / a] - -1 ^ (1 / 3) Log[1 - b (a ^
 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a] + -1 ^ (2 / 3) Log[1 -
 b (a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a]) / a ^ 2
```

Maple [A]
time = 0.12, size = 87, normalized size = 1.21

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}+\frac{-\frac{\ln\left(a^{2}+a\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a}\right)}{a^{2}}$	87
default	$\frac{\ln\left(a-\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}+\frac{-\frac{\ln\left(a^{2}+a\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a}\right)}{a^{2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(b^3*x+a^3)^(1/3))/a^2+1/a^2*(-1/2*ln(a^2+a*(b^3*x+a^3)^(1/3)+(b^3*x+a
^3)^(2/3))-3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.36, size = 87, normalized size = 1.21

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(a+2\,\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}} a+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(-a+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(-a + (b^3*x + a^
3)^(1/3))/a^2
```

Fricas [A]
time = 0.32, size = 86, normalized size = 1.19

$$-\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^{2}+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}} a+\left(b^{3} x+a^{3}\right)^{\frac{2}{3}}\right)-2\,\log\left(-a+\left(b^{3} x+a^{3}\right)^{\frac{1}{3}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) +
 log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) - 2*log(-a + (b^3*x
 + a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 134, normalized size = 1.86

$$\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(-2
*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/a)*gamma(1/3)
/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_p
olar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 0.00, size = 110, normalized size = 1.53

$$3\left(\frac{\ln\left|(a^3+b^3x)^{\frac{1}{3}}-a\right|}{3a^2}-\frac{\ln\left(\left((a^3+b^3x)^{\frac{1}{3}}\right)^2+(a^3+b^3x)^{\frac{1}{3}}\,a+a^2\right)}{6a^2}-\frac{\arctan\left(\frac{a+2(a^3+b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{a\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x)
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (b^3*x
+ a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.14, size = 101, normalized size = 1.40

$$\frac{\ln\left(9\,a-9\,{\left(a^3+x\,b^3\right)}^{1/3}\right)}{a^2}+\frac{\ln\left(9\,{\left(a^3+x\,b^3\right)}^{1/3}-\frac{9\,a\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\,{\left(a^3+x\,b^3\right)}^{1/3}+\frac{9\,a\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(2/3)),x)
```

```
[Out] log(9*a - 9*(b^3*x + a^3)^(1/3))/a^2 + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x + a^3)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.425 $\int \frac{1}{x(a^3-b^3x)^{2/3}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a-(-b^3*x+a^3)^(1/3))/a^2-arctan(1/3*(a+2*(-b^3*x+a^3
)^(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {59, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx &= -\frac{\log(x)}{2a^2}-\frac{3\mathrm{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{3\mathrm{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a}\\ &= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}+\frac{3\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a^2}\\ &= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 1.34

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)-2\log\left(a-\sqrt[3]{a^3-b^3x}\right)+\log\left(a^2+a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 - b^3*x)^(1/3)] + Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/
3)])/a^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.59, size = 89, normalized size = 1.20

$$\frac{\mathrm{Log}\left[1-\frac{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{I}{3}\mathrm{Pi}\right]}{a}\right]+\mathrm{Log}\left[1-\frac{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[I\mathrm{Pi}\right]}{a}\right]-\mathrm{Log}\left[1-\frac{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\mathrm{exp\_polar}\left[\frac{5I}{3}\mathrm{Pi}\right]}{a}\right]}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]')
```

```
[Out] (Log[1 - b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[I / 3 Pi] / a] - -1 ^ (
1 / 3) Log[1 - b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[I Pi] / a] + -1 ^
 (2 / 3) Log[1 - b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[5 I / 3 Pi] / a
]) / a ^ 2
```

Maple [A]
time = 0.12, size = 91, normalized size = 1.23

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}+\frac{-\frac{\ln\left(a^2+a\left(-b^3x+a^3\right)^{\frac{1}{3}}+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}$	91
default	$\frac{\ln\left(a-\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}+\frac{-\frac{\ln\left(a^2+a\left(-b^3x+a^3\right)^{\frac{1}{3}}+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(-b^3*x+a^3)^(1/3))/a^2+1/a^2*(-1/2*ln(a^2+a*(-b^3*x+a^3)^(1/3)+(-b^3*
x+a^3)^(2/3))-3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.35, size = 91, normalized size = 1.23

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(-a + (-b^3*x
+ a^3)^(1/3))/a^2
```

Fricas [A]
time = 0.31, size = 90, normalized size = 1.22

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a)
+ log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) - 2*log(-a + (-b
^3*x + a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.98, size = 136, normalized size = 1.84

$$\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(1/3)/(3*a**2*g
amma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 0.00, size = 114, normalized size = 1.54

$$3\left(\frac{\ln\left|(a^3-b^3x)^{\frac{1}{3}}-a\right|}{3a^2}-\frac{\ln\left(\left((a^3-b^3x)^{\frac{1}{3}}\right)^2+(a^3-b^3x)^{\frac{1}{3}}a+a^2\right)}{6a^2}-\frac{\arctan\left(\frac{a+2(a^3-b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{a\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x)
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (-b^
3*x + a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.11, size = 104, normalized size = 1.41

$$\frac{\ln\left(9\,a-9\,(a^3-b^3\,x)^{1/3}\right)}{a^2}+\frac{\ln\left(9\,(a^3-b^3\,x)^{1/3}-\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\,(a^3-b^3\,x)^{1/3}+\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(2/3)),x)
```

```
[Out] log(9*a - 9*(a^3 - b^3*x)^(1/3))/a^2 + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(a^3 - b^3*x)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.426 $\int \frac{1}{x(-a^3+b^3x)^{2/3}} dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a+(b^3*x-a^3)^(1/3))/a^2-arctan(1/3*(a-2*(b^3*x-a^3)^
(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]

time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {60, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3\right.}}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right.}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

Mathematica [A]
time = 0.04, size = 102, normalized size = 1.38

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)-2\log\left(a+\sqrt[3]{-a^3+b^3x}\right)+\log\left(a^2-a\sqrt[3]{-a^3+b^3x}+\left(-a^3+b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
+ (-a^3 + b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x)
^(2/3)])/a^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.57, size = 89, normalized size = 1.20

$$\frac{\text{Log}\left[1-\frac{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}[I\text{Pi}]}{a}\right]+\text{Log}\left[1-\frac{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{5I}{3}\text{Pi}\right]}{a}\right]-\text{Log}\left[1-\frac{b\left(-\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{I}{3}\text{Pi}\right]}{a}\right]}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]')
```

[Out] (Log[1 - b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[I Pi] / a] - -1 ^ (1 / 3) Log[1 - b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[5 I / 3 Pi] / a] + -1 ^ (2 / 3) Log[1 - b (-a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[I / 3 Pi] / a]) / a ^ 2

Maple [A]
time = 0.12, size = 95, normalized size = 1.28

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a^2-a\left(b^3x-a^3\right)^{\frac{1}{3}}+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$	95
default	$\frac{-\frac{\ln\left(a^2-a\left(b^3x-a^3\right)^{\frac{1}{3}}+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^3*x-a^3)^(2/3),x,method=_RETURNVERBOSE)

[Out] 1/a^2*(-1/2*ln(a^2-a*(b^3*x-a^3)^(1/3)+(b^3*x-a^3)^(2/3))+3^(1/2)*arctan(1/3*(-a+2*(b^3*x-a^3)^(1/3))*3^(1/2)/a))+ln(a+(b^3*x-a^3)^(1/3))/a^2

Maxima [A]
time = 0.34, size = 93, normalized size = 1.26

$$\frac{\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="maxima")

[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(a + (b^3*x - a^3)^(1/3))/a^2

Fricas [A]
time = 0.32, size = 95, normalized size = 1.28

$$\frac{2\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3\,a}\right)-\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)+2\,\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="fricas")

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) -
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) + 2*log(a + (b^3*x
- a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.98, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(2/3),x)
```

```
[Out] -exp(-I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(
1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 0.00, size = 114, normalized size = 1.54

$$3\left(\frac{\ln\left|(-a^3+b^3x)^{\frac{1}{3}}+a\right|}{3a^2}-\frac{\ln\left(\left((-a^3+b^3x)^{\frac{1}{3}}\right)^2-(-a^3+b^3x)^{\frac{1}{3}}\,a+a^2\right)}{6a^2}+\frac{\arctan\left(\frac{-a+2(-a^3+b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{a\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x)
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (b^3*x -
 a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.16, size = 107, normalized size = 1.45

$$\frac{\ln\left(9\,a+9\,(b^3\,x-a^3)^{1/3}\right)}{a^2}+\frac{\ln\left(9\,(b^3\,x-a^3)^{1/3}+\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\,(b^3\,x-a^3)^{1/3}-\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(b^3*x - a^3)^(1/3))/a^2 + (log(9*(b^3*x - a^3)^(1/3) + (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x - a^3)^(1/3) -
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.427 $\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}\,a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a+(-b^3*x-a^3)^(1/3))/a^2-arctan(1/3*(a-2*(-b^3*x-a^3
)^(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}\,a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3\right.}}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right.}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

Mathematica [A]
time = 0.04, size = 106, normalized size = 1.39

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)-2\log\left(a+\sqrt[3]{-a^3-b^3x}\right)+\log\left(a^2-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
+ (-a^3 - b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x)
^(2/3)])/a^2
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.47, size = 82, normalized size = 1.08

$$\frac{\text{Log}\left[1-\frac{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}}{a}\right]+\text{Log}\left[1-\frac{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{4I}{3}\text{Pi}\right]}{a}\right]-\text{Log}\left[1-\frac{b\left(\frac{a^3}{b^3}+x\right)^{\frac{1}{3}}\text{exp\_polar}\left[\frac{2I}{3}\text{Pi}\right]}{a}\right]}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]')
```

```
[Out] (Log[1 - b (a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[4 I / 3 Pi] / a] - -1 ^
(1 / 3) Log[1 - b (a ^ 3 / b ^ 3 + x) ^ (1 / 3) / a] + -1 ^ (2 / 3) Log[1 -
 b (a ^ 3 / b ^ 3 + x) ^ (1 / 3) exp_polar[2 I / 3 Pi] / a]) / a ^ 2
```

Maple [A]
time = 0.12, size = 99, normalized size = 1.30

method	result	siz
derivativedivides	$\frac{-\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	99
default	$\frac{-\frac{\ln\left(a^{2}-a\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*ln(a^2-a*(-b^3*x-a^3)^(1/3)+(-b^3*x-a^3)^(2/3))+3^(1/2)*arctan(
1/3*(-a+2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a))+ln(a+(-b^3*x-a^3)^(1/3))/a^2
```

Maxima [A]
time = 0.35, size = 97, normalized size = 1.28

$$\frac{\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(a + (-b^3*x -
 a^3)^(1/3))/a^2
```

Fricas [A]
time = 0.31, size = 99, normalized size = 1.30

$$\frac{2\,\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,a-2\,\sqrt{3}\,\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)-\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)+2\,\log\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
- log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) + 2*log(a + (-b^
3*x - a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.97, size = 133, normalized size = 1.75

$$\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(2/3),x)
```

```
[Out] exp(-2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma
(4/3)) - exp(-I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(a**3/b**3 + x)**(1/3)*exp_po
lar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 0.00, size = 118, normalized size = 1.55

$$3\left(\frac{\ln\left|(-a^3-b^3x)^{\frac{1}{3}}+a\right|}{3a^2}-\frac{\ln\left(\left((-a^3-b^3x)^{\frac{1}{3}}\right)^2-(-a^3-b^3x)^{\frac{1}{3}}a+a^2\right)}{6a^2}+\frac{\arctan\left(\frac{-a+2(-a^3-b^3x)^{\frac{1}{3}}}{a\sqrt{3}}\right)}{a\sqrt{3}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x)
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (-b^3
*x - a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.16, size = 110, normalized size = 1.45

$$\frac{\ln\left(9\,a+9\,(-a^3-x\,b^3)^{1/3}\right)}{a^2}+\frac{\ln\left(9\,(-a^3-x\,b^3)^{1/3}+\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\,(-a^3-x\,b^3)^{1/3}-\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(- b^3*x - a^3)^(1/3))/a^2 + (log(9*(- b^3*x - a^3)^(1/3) + (9*
a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(- b^3*x - a^3)^(
1/3) - (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.428 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}$$

[Out] a*x^(1+m)/(1+m)+b*x^(2+m)/(2+m)

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a + b*x),x]

[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx &= \int \left(ax^m+bx^{1+m}\right)\,dx\\ &= \frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.88

$$x^{1+m}\left(\frac{a}{1+m}+\frac{bx}{2+m}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x),x]

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.70, size = 101, normalized size = 4.04

$$\text{Piecewise}\left[\left\{\left\{-\frac{a}{x}+b\text{Log}[x], m==-2\right\}, \{a\text{Log}[x]+bx, m==-1\}\right\}, \frac{2axx^m}{2+3m+m^2}+\frac{amxx^m}{2+3m+m^2}+\frac{bx^2x^m}{2+3m+m^2}+\frac{bmx^2x^m}{2+3m+m^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(a + b*x),x]')
[Out] Piecewise[{{-a / x + b Log[x], m == -2}, {a Log[x] + b x, m == -1}}, 2 a x
x ^ m / (2 + 3 m + m ^ 2) + a m x x ^ m / (2 + 3 m + m ^ 2) + b x ^ 2 x ^ m
 / (2 + 3 m + m ^ 2) + b m x ^ 2 x ^ m / (2 + 3 m + m ^ 2)]
```

Maple [A]
time = 0.01, size = 30, normalized size = 1.20

method	result	size
norman	$\frac{ax\,e^{m\ln(x)}}{1+m}+\frac{b\,x^2e^{m\ln(x)}}{2+m}$	30
risch	$\frac{x(bmx+am+bx+2a)x^m}{(2+m)(1+m)}$	30
gosper	$\frac{x^{1+m}(bmx+am+bx+2a)}{(2+m)(1+m)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x,method=_RETURNVERBOSE)
[Out] a/(1+m)*x*exp(m*ln(x))+b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.31, size = 33, normalized size = 1.32

$$\frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [A]
time = 0.13, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x} + b\log\left(x\right) & \text{for}\: m = -2 \\ a\log\left(x\right) + bx & \text{for}\: m = -1 \\ \frac{amxx^{m}}{m^{2}+3m+2} + \frac{2axx^{m}}{m^{2}+3m+2} + \frac{bmx^{2}x^{m}}{m^{2}+3m+2} + \frac{bx^{2}x^{m}}{m^{2}+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

Giac [A]
time = 0.00, size = 50, normalized size = 2.00

$$\frac{amx\mathrm{e}^{m\ln x} + 2ax\mathrm{e}^{m\ln x} + bmx^2\mathrm{e}^{m\ln x} + bx^2\mathrm{e}^{m\ln x}}{m^2 + 3m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x)
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [B]
time = 0.31, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,\left(2\,a+a\,m+b\,x+b\,m\,x\right)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
```

```
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

3.429 $\int x^{5/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

```
[Out] 2/7*a*x^(7/2)+2/9*b*x^(9/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x),x]
[Out] (2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)\,dx &= \int \left(ax^{5/2}+bx^{7/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{63}x^{7/2}(9a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x),x]
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

Mathics [A]
time = 1.76, size = 13, normalized size = 0.62

$$\frac{2x^{\frac{7}{2}}\,(9a+7bx)}{63}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)*(a + b*x),x]')
```

```
[Out] 2 x ^ (7 / 2) (9 a + 7 b x) / 63
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
trager	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
risch	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*a*x^(7/2)+2/9*b*x^(9/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Fricas [A]
time = 0.29, size = 18, normalized size = 0.86

$$\frac{2}{63}\left(7\,bx^4+9\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

Sympy [A]
time = 0.24, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(9/2)/9
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.29

$$\frac{2}{9}b\sqrt{x}\,x^4+\frac{2}{7}a\sqrt{x}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x)
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Mupad [B]
time = 0.09, size = 13, normalized size = 0.62

$$\frac{2\,x^{7/2}\,\left(9\,a+7\,b\,x\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x),x)
```

```
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

3.430 $\int x^{3/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

```
[Out] 2/5*a*x^(5/2)+2/7*b*x^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x),x]
[Out] (2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)\,dx&=\int\left(ax^{3/2}+bx^{5/2}\right)\,dx\\&=\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{35}x^{5/2}(7a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x),x]
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

Mathics [A]
time = 1.65, size = 13, normalized size = 0.62

$$\frac{2x^{\frac{5}{2}}\left(7a+5bx\right)}{35}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)*(a + b*x),x]')
[Out] 2 x ^ (5 / 2) (7 a + 5 b x) / 35
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(5bx+7a\right)}{35}$	14
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
trager	$\frac{2x^{\frac{5}{2}}\left(5bx+7a\right)}{35}$	14
risch	$\frac{2x^{\frac{5}{2}}\left(5bx+7a\right)}{35}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2/5*a*x^(5/2)+2/7*b*x^(7/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Fricas [A]
time = 0.31, size = 18, normalized size = 0.86

$$\frac{2}{35}\left(5\,bx^3+7\,ax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

Sympy [A]
time = 0.13, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(7/2)/7
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.29

$$\frac{2}{7}b\sqrt{x}\,x^3 + \frac{2}{5}a\sqrt{x}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x)
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.62

$$\frac{2\,x^{5/2}\,\left(7\,a+5\,b\,x\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x),x)
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

3.431 $\int \sqrt{x}\,(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

```
[Out] 2/3*a*x^(3/2)+2/5*b*x^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)\,dx &= \int \left(a\sqrt{x}+bx^{3/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{15}x^{3/2}(5a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

Mathics [A]
time = 2.13, size = 13, normalized size = 0.62

$$\frac{2x^{\frac{3}{2}}\left(5a+3bx\right)}{15}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]*(a + b*x),x]')
```

```
[Out] 2 x ^ (3 / 2) (5 a + 3 b x) / 15
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
trager	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
risch	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*x^(3/2)+2/5*b*x^(5/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Fricas [A]
time = 0.30, size = 16, normalized size = 0.76

$$\frac{2}{15}\,\left(3\,bx^{2}+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

Sympy [A]
time = 0.73, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(5/2)/5
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.19

$$\frac{2}{5}b\sqrt{x}\,x^2 + \frac{2}{3}a\sqrt{x}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x)
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{2\,x^{3/2}\,\left(5\,a+3\,b\,x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x),x)
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

3.432 $\int \frac{a+bx}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

```
[Out] 2/3*b*x^(3/2)+2*a*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+b\sqrt{x}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{x}\,(3a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/3
```

Mathics [A]
time = 1.61, size = 12, normalized size = 0.63

$$\frac{2\sqrt{x}\ (3a+bx)}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/Sqrt[x],x]')
```

```
[Out] 2 Sqrt[x] (3 a + b x) / 3
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
gosper	$\frac{2\sqrt{x}\ (bx+3a)}{3}$	13
trager	$\left(\frac{2bx}{3}+2a\right)\sqrt{x}$	13
risch	$\frac{2\sqrt{x}\ (bx+3a)}{3}$	13
derivativedivides	$\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}$	14
default	$\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b*x^(3/2)+2*a*x^(1/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.63

$$\frac{2}{3}\,(bx+3\,a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

Sympy [A]
time = 0.07, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/2),x)
[Out] 2*a*sqrt(x) + 2*b*x**(3/2)/3
```

Giac [A]
time = 0.00, size = 20, normalized size = 1.05

$$\frac{2}{3}b\sqrt{x}\,x + 2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x)
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.63

$$\frac{2\,\sqrt{x}\,\left(3\,a+b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/2),x)
[Out] (2*x^(1/2)*(3*a + b*x))/3
```

3.433 $\int \frac{a+bx}{x^{3/2}}\,dx$

Optimal. Leaf size=17

$$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

```
[Out] -2*a/x^(1/2)+2*b*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 0.76

$$-\frac{2(a-bx)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*(a - b*x))/Sqrt[x]
```

Mathics [A]
time = 1.61, size = 12, normalized size = 0.71

$$\frac{2(-a+bx)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^(3/2),x]')
[Out] 2 (-a + b x) / Sqrt[x]
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
trager	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
risch	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
derivativedivides	$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$	14
default	$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*a/x^(1/2)+2*b*x^(1/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="maxima")
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.71

$$\frac{2(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/sqrt(x)
```

Sympy [A]
time = 0.16, size = 15, normalized size = 0.88

$$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*sqrt(x)
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.12

$$2\sqrt{x}\,b-\frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x)
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.65

$$-\frac{2\,\left(a-b\,x\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(3/2),x)
```

```
[Out] -(2*(a - b*x))/x^(1/2)
```

3.434 $\int \frac{a+bx}{x^{5/2}} dx$

Optimal. Leaf size=19

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

```
[Out] -2/3*a/x^(3/2)-2*b/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{3/2}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

Mathics [A]
time = 1.67, size = 13, normalized size = 0.68

$$\frac{2(-a-3bx)}{3x^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^(5/2),x]')
```

```
[Out] 2 (-a - 3 b x) / (3 x ^ (3 / 2))
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
trager	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
risch	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$	14
default	$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a/x^(3/2)-2*b/x^(1/2)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.58

$$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Fricas [A]
time = 0.54, size = 11, normalized size = 0.58

$$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Sympy [A]
time = 0.22, size = 19, normalized size = 1.00

$$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) - 2*b/sqrt(x)
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.11

$$\frac{-6xb-2a}{3\sqrt{x}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x)
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$-\frac{2\,a+6\,b\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/2),x)
```

```
[Out] -(2*a + 6*b*x)/(3*x^(3/2))
```

3.435 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(2+m)/(2+m)+b^2*x^(3+m)/(3+m)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx &= \int \left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)\,dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.88

$$x^{1+m}\left(\frac{a^2}{1+m}+\frac{2abx}{2+m}+\frac{b^2x^2}{3+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.98, size = 311, normalized size = 7.23

$$\text{Piecewise}\left[\left\{\left\{-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\text{Log}\left[x\right],m==-3\right\},\left\{\frac{-a^2+bx\left(2a\text{Log}\left[x\right]+bx\right)}{x},m==-2\right\},\left\{a^2\text{Log}\left[x\right]+2abx+\frac{b^2x^2}{2},m==-1\right\}\right\},\frac{6a^2xx^m}{6+11m+6m^2+m^3}+\frac{5a^2mxx^m}{6+11m+6m^2+m^3}+\frac{a^2m^2xx^m}{6+11m+6m^2+m^3}+\frac{6abx^2x^m}{6+11m+6m^2+m^3}+\frac{8abmx^2x^m}{6+11m+6m^2+m^3}+\frac{2abm^2x^2x^m}{6+11m+6m^2+m^3}+\frac{2b^2x^3x^m}{6+11m+6m^2+m^3}+\frac{3b^2mx^3x^m}{6+11m+6m^2+m^3}+\frac{b^2m^2x^3x^m}{6+11m+6m^2+m^3}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^m*(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{-a ^ 2 / (2 x ^ 2) - 2 a b / x + b ^ 2 Log[x], m == -3}, {(-a ^
 2 + b x (2 a Log[x] + b x)) / x, m == -2}, {a ^ 2 Log[x] + 2 a b x + b ^ 2
 x ^ 2 / 2, m == -1}}, 6 a ^ 2 x x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + 5 a
 ^ 2 m x x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + a ^ 2 m ^ 2 x x ^ m / (6 +
11 m + 6 m ^ 2 + m ^ 3) + 6 a b x ^ 2 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3)
+ 8 a b m x ^ 2 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + 2 a b m ^ 2 x ^ 2 x
^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + 2 b ^ 2 x ^ 3 x ^ m / (6 + 11 m + 6 m
^ 2 + m ^ 3) + 3 b ^ 2 m x ^ 3 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + b ^ 2
 m ^ 2 x ^ 3 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3)]
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.19

method	result	size
norman	$\frac{a^2 x\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b^2 x^3 \mathrm{e}^{m\ln(x)}}{3+m}+\frac{2ab\,x^2 \mathrm{e}^{m\ln(x)}}{2+m}$	51
risch	$\frac{x\left(b^2 m^2 x^2+2ab\,m^2 x+3m\,x^2 b^2+a^2 m^2+8mxab+2x^2 b^2+5m\,a^2+6abx+6a^2\right)x^m}{(3+m)(2+m)(1+m)}$	86
gosper	$\frac{x^{1+m}\left(b^2 m^2 x^2+2ab\,m^2 x+3m\,x^2 b^2+a^2 m^2+8mxab+2x^2 b^2+5m\,a^2+6abx+6a^2\right)}{(3+m)(2+m)(1+m)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(1+m)*x*exp(m*ln(x))+b^2/(3+m)*x^3*exp(m*ln(x))+2*a*b/(2+m)*x^2*exp(m*l
n(x))
```

Maxima [A]
time = 0.26, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+3}}{m+3}+\frac{2\,abx^{m+2}}{m+2}+\frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)

Fricas [A]
time = 0.32, size = 85, normalized size = 1.98

$$\frac{((b^2m^2 + 3\,b^2m + 2\,b^2)x^3 + 2\,(abm^2 + 4\,abm + 3\,ab)x^2 + (a^2m^2 + 5\,a^2m + 6\,a^2)x)x^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")

[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)

Sympy [A]
time = 0.19, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x) & \text{for } m = -3 \\ -\frac{a^2}{x} + 2ab\log(x) + b^2x & \text{for } m = -2 \\ a^2\log(x) + 2abx + \frac{b^2x^2}{2} & \text{for } m = -1 \\ \frac{a^2m^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2mxx^m}{m^3+6m^2+11m+6} + \frac{6a^2xx^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2m^2x^3x^m}{m^3+6m^2+11m+6} + \frac{3b^2mx^3x^m}{m^3+6m^2+11m+6} + \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(b*x+a)**2,x)

[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.
time = 0.00, size = 134, normalized size = 3.12

$$\frac{a^2m^2xe^{m\ln x} + 5a^2mxe^{m\ln x} + 6a^2xe^{m\ln x} + 2abm^2x^2e^{m\ln x} + 8abmx^2e^{m\ln x} + 6abx^2e^{m\ln x} + b^2m^2x^3e^{m\ln x} + 3b^2mx^3e^{m\ln x} + 2b^2x^3e^{m\ln x}}{m^3 + 6m^2 + 11m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x+a)^2,x)

[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m)/(m^3 + 6*m^2 + 11*m + 6)

Mupad [B]
time = 0.42, size = 93, normalized size = 2.16

$$x^m\,\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

3.436 $\int x^{5/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/9*a*b*x^(9/2)+2/11*b^2*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^2\,dx&=\int\left(a^2x^{5/2}+2abx^{7/2}+b^2x^{9/2}\right)dx\\&=\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{693}x^{7/2}\left(99a^2+154abx+63b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*(99*a^2 + 154*a*b*x + 63*b^2*x^2))/693
```

Mathics [A]
time = 1.85, size = 24, normalized size = 0.67

$$\frac{2x^{\frac{7}{2}}\left(99a^2+154abx+63b^2x^2\right)}{693}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)*(a + b*x)^2,x]')
```

```
[Out] 2 x ^ (7 / 2) (99 a ^ 2 + 154 a b x + 63 b ^ 2 x ^ 2) / 693
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(63x^{2}b^{2}+154abx+99a^{2}\right)}{693}$	25
derivativedivides	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$	25
default	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^{2}x^{\frac{11}{2}}}{11}$	25
trager	$\frac{2x^{\frac{7}{2}}\left(63x^{2}b^{2}+154abx+99a^{2}\right)}{693}$	25
risch	$\frac{2x^{\frac{7}{2}}\left(63x^{2}b^{2}+154abx+99a^{2}\right)}{693}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*a^2*x^(7/2)+4/9*a*b*x^(9/2)+2/11*b^2*x^(11/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Fricas [A]
time = 0.32, size = 29, normalized size = 0.81

$$\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

Sympy [A]
time = 0.33, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(11/2)/11
```

Giac [A]
time = 0.00, size = 45, normalized size = 1.25

$$\frac{2}{11}b^2\sqrt{x}\,x^5 + \frac{4}{9}ab\sqrt{x}\,x^4 + \frac{2}{7}a^2\sqrt{x}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x)
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Mupad [B]
time = 0.10, size = 24, normalized size = 0.67

$$\frac{2\,x^{7/2}\,\left(99\,a^2+154\,a\,b\,x+63\,b^2\,x^2\right)}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(7/2)*(99*a^2 + 63*b^2*x^2 + 154*a*b*x))/693
```

3.437 $\int x^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/7*a*b*x^(7/2)+2/9*b^2*x^(9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^2,x]
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^2\,dx&=\int\left(a^2x^{3/2}+2abx^{5/2}+b^2x^{7/2}\right)\,dx\\&=\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{315}x^{5/2}\left(63a^2+90abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^2,x]
[Out] (2*x^(5/2)*(63*a^2 + 90*a*b*x + 35*b^2*x^2))/315
```

Mathics [A]
time = 1.74, size = 24, normalized size = 0.67

$$\frac{2x^{\frac{5}{2}}\left(63a^2+90abx+35b^2x^2\right)}{315}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)*(a + b*x)^2,x]')
```

```
[Out] 2 x ^ (5 / 2) (63 a ^ 2 + 90 a b x + 35 b ^ 2 x ^ 2) / 315
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(35x^2b^2+90abx+63a^2\right)}{315}$	25
derivativedivides	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
default	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
trager	$\frac{2x^{\frac{5}{2}}\left(35x^2b^2+90abx+63a^2\right)}{315}$	25
risch	$\frac{2x^{\frac{5}{2}}\left(35x^2b^2+90abx+63a^2\right)}{315}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^2*x^(5/2)+4/7*a*b*x^(7/2)+2/9*b^2*x^(9/2)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Fricas [A]
time = 0.30, size = 29, normalized size = 0.81

$$\frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

Sympy [A]
time = 0.19, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(9/2)/9
```

Giac [A]
time = 0.00, size = 45, normalized size = 1.25

$$\frac{2}{9}b^2\sqrt{x}\,x^4 + \frac{4}{7}ab\sqrt{x}\,x^3 + \frac{2}{5}a^2\sqrt{x}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x)
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{5/2}\,\left(63\,a^2+90\,a\,b\,x+35\,b^2\,x^2\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(5/2)*(63*a^2 + 35*b^2*x^2 + 90*a*b*x))/315
```

3.438 $\int \sqrt{x}\,(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*b^2*x^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{3/2}+b^2x^{5/2}\right)\,dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{105}x^{3/2}\left(35a^2+42abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*x^(3/2)*(35*a^2 + 42*a*b*x + 15*b^2*x^2))/105
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 11.02, size = 1012, normalized size = 28.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{2 Sqrt[a] (8 I a ^ 3 + 35 a ^ 2 b x Sqrt[b x / a] + 42 a b ^ 2
x ^ 2 Sqrt[b x / a] + 15 b ^ 3 x ^ 3 Sqrt[b x / a]) / (105 b ^ (3 / 2)), Ab
s[(a + b x) / a] > 1}}, -16 I a ^ (23 / 2) Sqrt[1 - b (a / b + x) / a] / (1
05 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 / 2) (a / b + x) + 315 a ^ 6 b ^ (7
 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b + x) ^ 3) + I 16 a ^ (
23 / 2) / (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 / 2) (a / b + x) + 315
a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b + x) ^ 3)
- 48 I a ^ (21 / 2) b (a / b + x) / (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^
(5 / 2) (a / b + x) + 315 a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^
 (9 / 2) (a / b + x) ^ 3) + I 40 a ^ (21 / 2) b (a / b + x) Sqrt[1 - b (a /
 b + x) / a] / (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 / 2) (a / b + x) +
 315 a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b + x)
^ 3) - 30 I a ^ (19 / 2) b ^ 2 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 2
/ (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 / 2) (a / b + x) + 315 a ^ 6 b
^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b + x) ^ 3) + I 48 a
 ^ (19 / 2) b ^ 2 (a / b + x) ^ 2 / (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^
(5 / 2) (a / b + x) + 315 a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^
 (9 / 2) (a / b + x) ^ 3) - 16 I a ^ (17 / 2) b ^ 3 (a / b + x) ^ 3 / (105
a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 / 2) (a / b + x) + 315 a ^ 6 b ^ (7 /
2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b + x) ^ 3) + I 40 a ^ (17
/ 2) b ^ 3 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 3 / (105 a ^ 8 b ^ (3
/ 2) - 315 a ^ 7 b ^ (5 / 2) (a / b + x) + 315 a ^ 6 b ^ (7 / 2) (a / b + x
) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b + x) ^ 3) - 100 I a ^ (15 / 2) b ^ 4 S
qrt[1 - b (a / b + x) / a] (a / b + x) ^ 4 / (105 a ^ 8 b ^ (3 / 2) - 315 a
 ^ 7 b ^ (5 / 2) (a / b + x) + 315 a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105
a ^ 5 b ^ (9 / 2) (a / b + x) ^ 3) + I 96 a ^ (13 / 2) b ^ 5 Sqrt[1 - b (a
/ b + x) / a] (a / b + x) ^ 5 / (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 /
 2) (a / b + x) + 315 a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9
/ 2) (a / b + x) ^ 3) - 30 I a ^ (11 / 2) b ^ 6 Sqrt[1 - b (a / b + x) / a]
 (a / b + x) ^ 6 / (105 a ^ 8 b ^ (3 / 2) - 315 a ^ 7 b ^ (5 / 2) (a / b +
x) + 315 a ^ 6 b ^ (7 / 2) (a / b + x) ^ 2 - 105 a ^ 5 b ^ (9 / 2) (a / b +
 x) ^ 3)]
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size

gosper	$\frac{2x^{\frac{3}{2}}(15x^2b^2+42abx+35a^2)}{105}$	25
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{7}{2}}}{7}$	25
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{7}{2}}}{7}$	25
trager	$\frac{2x^{\frac{3}{2}}(15x^2b^2+42abx+35a^2)}{105}$	25
risch	$\frac{2x^{\frac{3}{2}}(15x^2b^2+42abx+35a^2)}{105}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*b^2*x^(7/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Fricas [A]
time = 0.31, size = 27, normalized size = 0.75

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

Sympy [C] Result contains complex when optimal does not.
time = 65.40, size = 1851, normalized size = 51.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*x**(1/2),x)
[Out] Piecewise((16*a**(23/2)*sqrt(-1 + b*(a/b + x)/a)/(-105*a**8*b**(3/2) + 315*
a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2
```

)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a
/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3)
- 40*a**(21/2)*b*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)/(-105*a**8*b**(3/2) + 3
15*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(
9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**8*b**(3/2) + 315*a
7*b(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)
*(a/b + x)**3) + 30*a**(19/2)*b**2*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**2/(-
105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b +
x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)*b**2*(a/b + x)**2/
(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b
+ x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*a**(17/2)*b**3*sqrt(-1 + b*(
a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x)
- 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 16*I*a
(17/2)*b3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x
) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 100*
a**(15/2)*b**4*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**4/(-105*a**8*b**(3/2) +
315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**
(9/2)*(a/b + x)**3) - 96*a**(13/2)*b**5*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)*
*5/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a
/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 30*a**(11/2)*b**6*sqrt(-1 +
b*(a/b + x)/a)*(a/b + x)**6/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b +
x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3), Abs(
b*(a/b + x)/a) > 1), (16*I*a**(23/2)*sqrt(1 - b*(a/b + x)/a)/(-105*a**8*b**
(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*
a**5*b**(9/2)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315*a**7
*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a
/b + x)**3) - 40*I*a**(21/2)*b*sqrt(1 - b*(a/b + x)/a)*(a/b + x)/(-105*a**8
*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 +
105*a**5*b**(9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**8*b**
(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*
a**5*b**(9/2)*(a/b + x)**3) + 30*I*a**(19/2)*b**2*sqrt(1 - b*(a/b + x)/a)*(
a/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b*
(7/2)(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)*b**2
*(a/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*
b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*I*a**(17/2)*b*
*3*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(
5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b +
x)**3) + 16*I*a**(17/2)*b**3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b*
(5/2)(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b
+ x)**3) + 100*I*a**(15/2)*b**4*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**4/(-105*
a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**
2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 96*I*a**(13/2)*b**5*sqrt(1 - b*(a/b +
 x)/a)*(a/b + x)**5/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315
*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 30*I*a**(11
/2)*b**6*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**6/(-105*a**8*b**(3/2) + 315*a**

```
7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(
a/b + x)**3), True))
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.19

$$\frac{2}{7}b^2\sqrt{x}\,x^3 + \frac{4}{5}ab\sqrt{x}\,x^2 + \frac{2}{3}a^2\sqrt{x}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x)
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{3/2}\,\left(35\,a^2+42\,a\,b\,x+15\,b^2\,x^2\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2)*(35*a^2 + 15*b^2*x^2 + 42*a*b*x))/105
```

3.439 $\int \frac{(a+bx)^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

[Out] 4/3*a*b*x^(3/2)+2/5*b^2*x^(5/2)+2*a^2*x^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/Sqrt[x],x]

[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2ab\sqrt{x}+b^2x^{3/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$\frac{2}{15}\sqrt{x}\,\left(15a^2+10abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2/Sqrt[x],x]

```
[Out] (2*Sqrt[x]*(15*a^2 + 10*a*b*x + 3*b^2*x^2))/15
```

Mathics [A]
time = 1.67, size = 24, normalized size = 0.71

$$\frac{2\sqrt{x}\ (15a^2+10abx+3b^2x^2)}{15}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/Sqrt[x],x]')
```

```
[Out] 2 Sqrt[x] (15 a ^ 2 + 10 a b x + 3 b ^ 2 x ^ 2) / 15
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.74

method	result	size
trager	$\left(\frac{2}{5}x^2b^2+\frac{4}{3}abx+2a^2\right)\sqrt{x}$	24
gosper	$\frac{2\sqrt{x}\ (3x^2b^2+10abx+15a^2)}{15}$	25
derivativedivides	$\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}+2a^2\sqrt{x}$	25
default	$\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}+2a^2\sqrt{x}$	25
risch	$\frac{2\sqrt{x}\ (3x^2b^2+10abx+15a^2)}{15}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*a*b*x^(3/2)+2/5*b^2*x^(5/2)+2*a^2*x^(1/2)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.71

$$\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

Sympy [A]
time = 0.10, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(5/2)/5
```

Giac [A]
time = 0.00, size = 38, normalized size = 1.12

$$\frac{2}{5}b^2\sqrt{x}\,x^2 + \frac{4}{3}ab\sqrt{x}\,x + 2a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x)
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.71

$$\frac{2\,\sqrt{x}\,\left(15\,a^2+10\,a\,b\,x+3\,b^2\,x^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(15*a^2 + 3*b^2*x^2 + 10*a*b*x))/15
```

3.440 $\int \frac{(a+bx)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

```
[Out] 2/3*b^2*x^(3/2)-2*a^2/x^(1/2)+4*a*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+\frac{2ab}{\sqrt{x}}+b^2\sqrt{x}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.88

$$-\frac{2\left(3a^2-6abx-b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (-2*(3*a^2 - 6*a*b*x - b^2*x^2))/(3*Sqrt[x])
```

Mathics [A]
time = 1.69, size = 21, normalized size = 0.66

$$\frac{2\left(-3a^2+bx\left(6a+bx\right)\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^(3/2),x]')
```

```
[Out] 2 (-3 a ^ 2 + b x (6 a + b x)) / (3 Sqrt[x])
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.78

method	result	size
gosper	$-\frac{2\left(-x^{2} b^{2}-6abx+3a^{2}\right)}{3\sqrt{x}}$	25
derivativedivides	$\frac{2b^{2} x^{\frac{3}{2}}}{3}-\frac{2a^{2}}{\sqrt{x}}+4ab\sqrt{x}$	25
default	$\frac{2b^{2} x^{\frac{3}{2}}}{3}-\frac{2a^{2}}{\sqrt{x}}+4ab\sqrt{x}$	25
trager	$-\frac{2\left(-x^{2} b^{2}-6abx+3a^{2}\right)}{3\sqrt{x}}$	25
risch	$-\frac{2\left(-x^{2} b^{2}-6abx+3a^{2}\right)}{3\sqrt{x}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b^2*x^(3/2)-2*a^2/x^(1/2)+4*a*b*x^(1/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.72

$$\frac{2\,(b^2x^2+6\,abx-3\,a^2)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

Sympy [A]
time = 0.19, size = 31, normalized size = 0.97

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(3/2),x)
[Out] -2*a**2/sqrt(x) + 4*a*b*sqrt(x) + 2*b**2*x**(3/2)/3
```

Giac [A]
time = 0.00, size = 35, normalized size = 1.09

$$\frac{2}{3}\sqrt{x}\,xb^2+4\sqrt{x}\,ba-\frac{2a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x)
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.75

$$\frac{-6\,a^2+12\,a\,b\,x+2\,b^2\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(3/2),x)
[Out] (2*b^2*x^2 - 6*a^2 + 12*a*b*x)/(3*x^(1/2))
```

3.441 $\int \frac{(a+bx)^2}{x^{5/2}} dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

```
[Out] -2/3*a^2/x^(3/2)-4*a*b/x^(1/2)+2*b^2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/2}} dx &= \int \left(\frac{a^2}{x^{5/2}} + \frac{2ab}{x^{3/2}} + \frac{b^2}{\sqrt{x}}\right) dx\\ &= -\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.81

$$-\frac{2\left(a^2+6abx-3b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*(a^2 + 6*a*b*x - 3*b^2*x^2))/(3*x^(3/2))
```

Mathics [A]
time = 1.72, size = 24, normalized size = 0.75

$$\frac{2\left(-a^2-6abx+3b^2x^2\right)}{3x^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/x^(5/2),x]')
[Out] 2 (-a ^ 2 - 6 a b x + 3 b ^ 2 x ^ 2) / (3 x ^ (3 / 2))
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.78

method	result	size
gosper	$-\frac{2\left(-3x^2b^2+6abx+a^2\right)}{3x^{\frac{3}{2}}}$	23
trager	$-\frac{2\left(-3x^2b^2+6abx+a^2\right)}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{2\left(-3x^2b^2+6abx+a^2\right)}{3x^{\frac{3}{2}}}$	23
derivativedivides	$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$	25
default	$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a^2/x^(3/2)-4*a*b/x^(1/2)+2*b^2*x^(1/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.72

$$2\,b^2\sqrt{x}-\frac{2\left(6\,abx+a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.75

$$\frac{2\left(3\,b^2x^2-6\,abx-a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

Sympy [A]
time = 0.23, size = 31, normalized size = 0.97

$$-\frac{2a^2}{3x^{\frac{3}{2}}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) - 4*a*b/sqrt(x) + 2*b**2*sqrt(x)
```

Giac [A]
time = 0.00, size = 35, normalized size = 1.09

$$2\sqrt{x}\,b^2 + \frac{-12xba - 2a^2}{3\sqrt{x}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x)
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.75

$$-\frac{2\,a^2 + 12\,a\,b\,x - 6\,b^2\,x^2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/2),x)
```

```
[Out] -(2*a^2 - 6*b^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

3.442 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(2+m)/(2+m)+3*a*b^2*x^(3+m)/(3+m)+b^3*x^(4+m)/(
4+m)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx&=\int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)\,dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 0.89

$$x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx}{2+m}+\frac{3ab^2x^2}{3+m}+\frac{b^3x^3}{4+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.47, size = 662, normalized size = 10.85

$$\text{Piecewise}\left[\left\{\left\{-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\text{Log}[x], m==-4\right\}, \left\{-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\text{Log}[x]+b^3x, m==-3\right\}, \left\{\frac{-a^3+\frac{bx(6a^2\text{Log}[x]+6abx+b^2x^2)}{2}}{x}, m==-2\right\}, \left\{a^3\text{Log}[x]+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3}, m==-1\right\}\right\}, \frac{24a^3xx^m}{24+50m+35m^2+10m^3+m^4}+\frac{26a^3mxx^m}{24+50m+35m^2+10m^3+m^4}+\frac{9a^3m^2xx^m}{24+50m+35m^2+10m^3+m^4}+\frac{a^3m^3xx^m}{24+50m+35m^2+10m^3+m^4}+\frac{36a^2bx^2x^m}{24+50m+35m^2+10m^3+m^4}+\frac{57a^2bmx^2x^m}{24+50m+35m^2+10m^3+m^4}+\cdots\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^m*(a + b*x)^3,x]')
```

```
[Out] Piecewise[{{-a ^ 3 / (3 x ^ 3) - 3 a ^ 2 b / (2 x ^ 2) - 3 a b ^ 2 / x + b
^ 3 Log[x], m == -4}, {-a ^ 3 / (2 x ^ 2) - 3 a ^ 2 b / x + 3 a b ^ 2 Log[x
] + b ^ 3 x, m == -3}, {(-a ^ 3 + b x (6 a ^ 2 Log[x] + 6 a b x + b ^ 2 x ^
 2) / 2) / x, m == -2}, {a ^ 3 Log[x] + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 / 2 +
 b ^ 3 x ^ 3 / 3, m == -1}}, 24 a ^ 3 x x ^ m / (24 + 50 m + 35 m ^ 2 + 10
m ^ 3 + m ^ 4) + 26 a ^ 3 m x x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m
^ 4) + 9 a ^ 3 m ^ 2 x x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) +
a ^ 3 m ^ 3 x x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 36 a ^ 2
b x ^ 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 57 a ^ 2 b m x
^ 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 24 a ^ 2 b m ^ 2 x
^ 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 3 a ^ 2 b m ^ 3 x ^
 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 24 a b ^ 2 x ^ 3 x ^
 m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 42 a b ^ 2 m x ^ 3 x ^ m /
 (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 21 a b ^ 2 m ^ 2 x ^ 3 x ^ m /
 (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 3 a b ^ 2 m ^ 3 x ^ 3 x ^ m /
(24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 6 b ^ 3 x ^ 4 x ^ m / (24 + 50
m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 11 b ^ 3 m x ^ 4 x ^ m / (24 + 50 m + 35
 m ^ 2 + 10 m ^ 3 + m ^ 4) + 6 b ^ 3 m ^ 2 x ^ 4 x ^ m / (24 + 50 m + 35 m
^ 2 + 10 m ^ 3 + m ^ 4) + b ^ 3 m ^ 3 x ^ 4 x ^ m / (24 + 50 m + 35 m ^ 2 +
 10 m ^ 3 + m ^ 4)]
```

Maple [A]
time = 0.09, size = 72, normalized size = 1.18

method	result
norman	$\frac{a^3 x\,e^{m\ln(x)}}{1+m}+\frac{b^3 x^4 e^{m\ln(x)}}{4+m}+\frac{3a\,b^2 x^3 e^{m\ln(x)}}{3+m}+\frac{3a^2 b\,x^2 e^{m\ln(x)}}{2+m}$
risch	$\frac{x\left(b^3m^3x^3+3a\,b^2m^3x^2+6b^3m^2x^3+3a^2b\,m^3x+21a\,b^2m^2x^2+11m\,x^3b^3+a^3m^3+24a^2b\,m^2x+42m\,x^2a\,b^2+6b^3x^3+9a^3m^2+57mx\,a^2b+\right.}{(4+m)(3+m)(2+m)(1+m)}$
gosper	$\frac{x^{1+m}\left(b^3m^3x^3+3a\,b^2m^3x^2+6b^3m^2x^3+3a^2b\,m^3x+21a\,b^2m^2x^2+11m\,x^3b^3+a^3m^3+24a^2b\,m^2x+42m\,x^2a\,b^2+6b^3x^3+9a^3m^2+57mx\right.}{(4+m)(3+m)(2+m)(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/(1+m)*x*exp(m*ln(x))+b^3/(4+m)*x^4*exp(m*ln(x))+3*a*b^2/(3+m)*x^3*exp(m
*ln(x))+3*a^2*b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+4}}{m+4}+\frac{3\,ab^2x^{m+3}}{m+3}+\frac{3\,a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.
time = 0.31, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b\right)x^2+\left(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3\right)x\right)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

Sympy [A]
time = 0.28, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log{(x)} & \text{for}\: m=-4 \\ -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log{(x)}+b^3x & \text{for}\: m=-3 \\ -\frac{a^3}{x}+3a^2b\log{(x)}+3ab^2x+\frac{b^3x^2}{2} & \text{for}\: m=-2 \\ a^3\log{(x)}+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3} & \text{for}\: m=-1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
```

```
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(61) = 122$.

time = 0.00, size = 255, normalized size = 4.18

$$\frac{a^3m^3xe^{m\ln x} + 9a^3m^2xe^{m\ln x} + 26a^3mxe^{m\ln x} + 24a^3xe^{m\ln x} + 3a^2bm^3x^2e^{m\ln x} + 24a^2bm^2x^2e^{m\ln x} + 57a^2bmx^2e^{m\ln x} + 36a^2bx^2e^{m\ln x} + 3ab^2m^3x^3e^{m\ln x} + 21ab^2m^2x^3e^{m\ln x} + 42ab^2mx^3e^{m\ln x} + 24ab^2x^3e^{m\ln x} + b^3m^3x^4e^{m\ln x} + 6b^3m^2x^4e^{m\ln x} + 11b^3mx^4e^{m\ln x} + 6b^3x^4e^{m\ln x}}{m^4 + 10m^3 + 35m^2 + 50m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x)
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B]

time = 0.39, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

3.443 $\int x^{5/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

```
[Out] 2/7*a^3*x^(7/2)+2/3*a^2*b*x^(9/2)+6/11*a*b^2*x^(11/2)+2/13*b^3*x^(13/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(13/2))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^3\,dx&=\int \left(a^3x^{5/2}+3a^2bx^{7/2}+3ab^2x^{9/2}+b^3x^{11/2}\right)\,dx\\&=\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{2x^{7/2}\left(429a^3+1001a^2bx+819ab^2x^2+231b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)*(a + b*x)^3,x]

[Out] (2*x^(7/2)*(429*a^3 + 1001*a^2*b*x + 819*a*b^2*x^2 + 231*b^3*x^3))/3003

Mathics [A]
time = 1.99, size = 35, normalized size = 0.69

$$\frac{2x^{\frac{7}{2}}\left(429a^3+1001a^2bx+819ab^2x^2+231b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^(5/2)*(a + b*x)^3,x]')

[Out] 2 x ^ (7 / 2) (429 a ^ 3 + 1001 a ^ 2 b x + 819 a b ^ 2 x ^ 2 + 231 b ^ 3 x ^ 3) / 3003

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3\right)}{3003}$	36
derivativedivides	$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{13}{2}}}{13}$	36
default	$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{13}{2}}}{13}$	36
trager	$\frac{2x^{\frac{7}{2}}\left(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3\right)}{3003}$	36
risch	$\frac{2x^{\frac{7}{2}}\left(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3\right)}{3003}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 2/7*a^3*x^(7/2)+2/3*a^2*b*x^(9/2)+6/11*a*b^2*x^(11/2)+2/13*b^3*x^(13/2)

Maxima [A]
time = 0.26, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="maxima")

[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/2)

Fricas [A]
time = 0.30, size = 40, normalized size = 0.78

$$\frac{2}{3003}\left(231\, b^3 x^6 + 819\, a b^2 x^5 + 1001\, a^2 b x^4 + 429\, a^3 x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

Sympy [A]
time = 0.43, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(13/2)/13
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.24

$$\frac{2}{13}b^3\sqrt{x}\,x^6 + \frac{6}{11}ab^2\sqrt{x}\,x^5 + \frac{6}{9}a^2b\sqrt{x}\,x^4 + \frac{2}{7}a^3\sqrt{x}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x)
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7} + \frac{2\,b^3\,x^{13/2}}{13} + \frac{2\,a^2\,b\,x^{9/2}}{3} + \frac{6\,a\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(13/2))/13 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(11/2))/11
```

3.444 $\int x^{3/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

```
[Out] 2/5*a^3*x^(5/2)+6/7*a^2*b*x^(7/2)+2/3*a*b^2*x^(9/2)+2/11*b^3*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(1
1/2))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{5/2}+3ab^2x^{7/2}+b^3x^{9/2}\right)\,dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{2x^{5/2}\left(231a^3+495a^2bx+385ab^2x^2+105b^3x^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*x^(5/2)*(231*a^3 + 495*a^2*b*x + 385*a*b^2*x^2 + 105*b^3*x^3))/1155
```

Mathics [A]
time = 1.87, size = 35, normalized size = 0.69

$$\frac{2x^{\frac{5}{2}}\left(231a^3+495a^2bx+385ab^2x^2+105b^3x^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)*(a + b*x)^3,x]')
```

```
[Out] 2 x ^ (5 / 2) (231 a ^ 3 + 495 a ^ 2 b x + 385 a b ^ 2 x ^ 2 + 105 b ^ 3 x
^ 3) / 1155
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)}{1155}$	36
derivativedivides	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2b\,x^{\frac{7}{2}}}{7}+\frac{2a\,b^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
default	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2b\,x^{\frac{7}{2}}}{7}+\frac{2a\,b^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
trager	$\frac{2x^{\frac{5}{2}}\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)}{1155}$	36
risch	$\frac{2x^{\frac{5}{2}}\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)}{1155}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^3*x^(5/2)+6/7*a^2*b*x^(7/2)+2/3*a*b^2*x^(9/2)+2/11*b^3*x^(11/2)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

Fricas [A]
time = 0.31, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(105\,b^3x^5+385\,ab^2x^4+495\,a^2bx^3+231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

Sympy [A]
time = 0.25, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 6*a**2*b*x**(7/2)/7 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(
11/2)/11
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.24

$$\frac{2}{11}b^3\sqrt{x}\,x^5+\frac{6}{9}ab^2\sqrt{x}\,x^4+\frac{6}{7}a^2b\sqrt{x}\,x^3+\frac{2}{5}a^3\sqrt{x}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x)
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{11/2}}{11}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{2\,a\,b^2\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(11/2))/11 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^
(9/2))/3
```

3.445 $\int \sqrt{x}\,(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/5*a^2*b*x^(5/2)+6/7*a*b^2*x^(7/2)+2/9*b^3*x^(9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(9
/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{3/2}+3ab^2x^{5/2}+b^3x^{7/2}\right)\,dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{2}{315}x^{3/2}\left(105a^3+189a^2bx+135ab^2x^2+35b^3x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(a + b*x)^3,x]

[Out] (2*x^(3/2)*(105*a^3 + 189*a^2*b*x + 135*a*b^2*x^2 + 35*b^3*x^3))/315

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 27.15, size = 2556, normalized size = 50.12

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[x]*(a + b*x)^3,x]')

[Out] Piecewise[{{2 Sqrt[a] (16 I a ^ 4 + 105 a ^ 3 b x Sqrt[b x / a] + 189 a ^ 2 b ^ 2 x ^ 2 Sqrt[b x / a] + 135 a b ^ 3 x ^ 3 Sqrt[b x / a] + 35 b ^ 4 x ^ 4 Sqrt[b x / a]) / (315 b ^ (3 / 2)), Abs[(a + b x) / a] > 1}}, -32 I a ^ (49 / 2) Sqrt[1 - b (a / b + x) / a] / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 1 9 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 32 a ^ (49 / 2) / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 192 I a ^ (47 / 2) b (a / b + x) / (315 a ^ 20 b ^ (3 / 2) - 1 890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 176 a ^ (47 / 2) b (a / b + x) Sqrt[1 - b (a / b + x) / a] / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 396 I a ^ (45 / 2) b ^ 2 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 2 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 480 a ^ (45 / 2) b ^ 2 (a / b + x) ^ 2 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 640 I a ^ (43 / 2) b ^ 3 (a / b + x) ^ 3 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 462 a ^ (43 / 2) b ^ 3 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 3 / (

315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 210 I a ^ (41 / 2) b ^ 4 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 4 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 480 a ^ (41 / 2) b ^ 4 (a / b + x) ^ 4 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 378 I a ^ (39 / 2) b ^ 5 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 5 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 192 I a ^ (39 / 2) b ^ 5 (a / b + x) ^ 5 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 32 a ^ (37 / 2) b ^ 6 (a / b + x) ^ 6 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 1134 a ^ (37 / 2) b ^ 6 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 6 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 1494 I a ^ (35 / 2) b ^ 7 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 7 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 1098 a ^ (33 / 2) b ^ 8 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 8 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) - 430 I a ^ (31 / 2) b ^ 9 Sqrt[1 - b (a / b + x) / a] (a / b + x) ^ 9 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 / 2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6) + I 70 a ^ (29 / 2) b ^ 10 Sqrt[1 - b (a / b + x) /

```
a] (a / b + x) ^ 10 / (315 a ^ 20 b ^ (3 / 2) - 1890 a ^ 19 b ^ (5 / 2) (a
 / b + x) + 4725 a ^ 18 b ^ (7 / 2) (a / b + x) ^ 2 - 6300 a ^ 17 b ^ (9 /
2) (a / b + x) ^ 3 + 4725 a ^ 16 b ^ (11 / 2) (a / b + x) ^ 4 - 1890 a ^ 15
 b ^ (13 / 2) (a / b + x) ^ 5 + 315 a ^ 14 b ^ (15 / 2) (a / b + x) ^ 6)]
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
derivativedivides	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
default	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
trager	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
risch	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a^3*x^(3/2)+6/5*a^2*b*x^(5/2)+6/7*a*b^2*x^(7/2)+2/9*b^3*x^(9/2)
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^{3}x^{\frac{9}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+\frac{6}{5}\,a^{2}bx^{\frac{5}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="maxima")
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 0.75

$$\frac{2}{315}\left(35\,b^{3}x^{4}+135\,ab^{2}x^{3}+189\,a^{2}bx^{2}+105\,a^{3}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="fricas")
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 61, normalized size = 1.20

$$\frac{2}{9}b^3\sqrt{x}\,x^4+\frac{6}{7}ab^2\sqrt{x}\,x^3+\frac{6}{5}a^2b\sqrt{x}\,x^2+\frac{2}{3}a^3\sqrt{x}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x)
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{9/2}}{9}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{6\,a\,b^2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(9/2))/9 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7
/2))/7
```

3.446 $\int \frac{(a+bx)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=47

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

```
[Out] 2*a^2*b*x^(3/2)+6/5*a*b^2*x^(5/2)+2/7*b^3*x^(7/2)+2*a^3*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2b\sqrt{x}+3ab^2x^{3/2}+b^3x^{5/2}\right)dx\\ &= 2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.83

$$\frac{2}{35}\sqrt{x}\left(35a^3+35a^2bx+21ab^2x^2+5b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[x],x]
[Out] (2*Sqrt[x]*(35*a^3 + 35*a^2*b*x + 21*a*b^2*x^2 + 5*b^3*x^3))/35
```

Mathics [A]
time = 1.76, size = 35, normalized size = 0.74

$$\frac{2\sqrt{x}\ (35a^3+35a^2bx+21ab^2x^2+5b^3x^3)}{35}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/Sqrt[x],x]')
[Out] 2 Sqrt[x] (35 a ^ 3 + 35 a ^ 2 b x + 21 a b ^ 2 x ^ 2 + 5 b ^ 3 x ^ 3) / 35
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.77

method	result	size
trager	$\left(\frac{2}{7}b^3x^3+\frac{6}{5}a\,b^2x^2+2a^2bx+2a^3\right)\sqrt{x}$	35
gosper	$\frac{2\sqrt{x}\ (5b^3x^3+21a\,b^2x^2+35a^2bx+35a^3)}{35}$	36
derivativedivides	$2a^2b\,x^{\frac{3}{2}}+\frac{6a\,b^2x^{\frac{5}{2}}}{5}+\frac{2b^3x^{\frac{7}{2}}}{7}+2a^3\sqrt{x}$	36
default	$2a^2b\,x^{\frac{3}{2}}+\frac{6a\,b^2x^{\frac{5}{2}}}{5}+\frac{2b^3x^{\frac{7}{2}}}{7}+2a^3\sqrt{x}$	36
risch	$\frac{2\sqrt{x}\ (5b^3x^3+21a\,b^2x^2+35a^2bx+35a^3)}{35}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a^2*b*x^(3/2)+6/5*a*b^2*x^(5/2)+2/7*b^3*x^(7/2)+2*a^3*x^(1/2)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.74

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="maxima")
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Fricas [A]
time = 0.31, size = 35, normalized size = 0.74

$$\frac{2}{35}\left(5\,b^3x^3+21\,ab^2x^2+35\,a^2bx+35\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

Sympy [A]
time = 0.13, size = 46, normalized size = 0.98

$$2a^3\sqrt{x} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(7/2)/
7
```

Giac [A]
time = 0.00, size = 56, normalized size = 1.19

$$\frac{2}{7}b^3\sqrt{x}\,x^3 + \frac{6}{5}ab^2\sqrt{x}\,x^2 + \frac{6}{3}a^2b\sqrt{x}\,x + 2a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x)
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.74

$$2\,a^3\,\sqrt{x} + \frac{2\,b^3\,x^{7/2}}{7} + 2\,a^2\,b\,x^{3/2} + \frac{6\,a\,b^2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/2),x)
```

```
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(7/2))/7 + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5
```

3.447 $\int \frac{(a+bx)^3}{x^{3/2}} dx$

Optimal. Leaf size=45

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

```
[Out] 2*a*b^2*x^(3/2)+2/5*b^3*x^(5/2)-2*a^3/x^(1/2)+6*a^2*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2\sqrt{x}+b^3x^{3/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.87

$$-\frac{2\left(5a^3-15a^2bx-5ab^2x^2-b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (-2*(5*a^3 - 15*a^2*b*x - 5*a*b^2*x^2 - b^3*x^3))/(5*Sqrt[x])
```

Mathics [A]
time = 1.76, size = 32, normalized size = 0.71

$$\frac{2\left(-5a^3+bx\left(15a^2+5abx+b^2x^2\right)\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^(3/2),x]')
```

```
[Out] 2 (-5 a ^ 3 + b x (15 a ^ 2 + 5 a b x + b ^ 2 x ^ 2)) / (5 Sqrt[x])
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.80

method	result	size
gosper	$-\frac{2\left(-b^3x^3-5a\,b^2x^2-15a^2bx+5a^3\right)}{5\sqrt{x}}$	36
derivativedivides	$2a\,b^2x^{\frac{3}{2}}+\frac{2b^3x^{\frac{5}{2}}}{5}-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}$	36
default	$2a\,b^2x^{\frac{3}{2}}+\frac{2b^3x^{\frac{5}{2}}}{5}-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}$	36
trager	$-\frac{2\left(-b^3x^3-5a\,b^2x^2-15a^2bx+5a^3\right)}{5\sqrt{x}}$	36
risch	$-\frac{2\left(-b^3x^3-5a\,b^2x^2-15a^2bx+5a^3\right)}{5\sqrt{x}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b^2*x^(3/2)+2/5*b^3*x^(5/2)-2*a^3/x^(1/2)+6*a^2*b*x^(1/2)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^3x^{\frac{5}{2}}+2\,ab^2x^{\frac{3}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Fricas [A]
time = 0.30, size = 34, normalized size = 0.76

$$\frac{2\left(b^{3}x^{3}+5\,ab^{2}x^{2}+15\,a^{2}bx-5\,a^{3}\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

Sympy [A]
time = 0.22, size = 44, normalized size = 0.98

$$-\frac{2a^{3}}{\sqrt{x}}+6a^{2}b\sqrt{x}+2ab^{2}x^{\frac{3}{2}}+\frac{2b^{3}x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(3/2),x)
[Out] -2*a**3/sqrt(x) + 6*a**2*b*sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(5/2)/5
```

Giac [A]
time = 0.00, size = 51, normalized size = 1.13

$$\frac{2}{5}\sqrt{x}\,x^{2}b^{3}+2\sqrt{x}\,xb^{2}a+6\sqrt{x}\,ba^{2}-\frac{2a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x)
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.78

$$\frac{2\,b^{3}\,x^{5/2}}{5}-\frac{2\,a^{3}}{\sqrt{x}}+6\,a^{2}\,b\,\sqrt{x}+2\,a\,b^{2}\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(3/2),x)
[Out] (2*b^3*x^(5/2))/5 - (2*a^3)/x^(1/2) + 6*a^2*b*x^(1/2) + 2*a*b^2*x^(3/2)
```

3.448 $\int \frac{(a+bx)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

```
[Out] -2/3*a^3/x^(3/2)+2/3*b^3*x^(3/2)-6*a^2*b/x^(1/2)+6*a*b^2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2)
)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+b^3\sqrt{x}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.81

$$\frac{2\left(-a^3-9a^2bx+9ab^2x^2+b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/2),x]
```

```
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

Mathics [A]
time = 1.79, size = 32, normalized size = 0.68

$$\frac{2\left(-a^3-9a^2bx+b^2x^2\left(9a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/x^(5/2),x]')
```

```
[Out] 2 (-a ^ 3 - 9 a ^ 2 b x + b ^ 2 x ^ 2 (9 a + b x)) / (3 x ^ (3 / 2))
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.77

method	result	size
gosper	$-\frac{2\left(-b^{3} x^{3}-9 a\, b^{2} x^{2}+9 a^{2} b x+a^{3}\right)}{3 x^{\frac{3}{2}}}$	34
trager	$-\frac{2\left(-b^{3} x^{3}-9 a\, b^{2} x^{2}+9 a^{2} b x+a^{3}\right)}{3 x^{\frac{3}{2}}}$	34
risch	$-\frac{2\left(-b^{3} x^{3}-9 a\, b^{2} x^{2}+9 a^{2} b x+a^{3}\right)}{3 x^{\frac{3}{2}}}$	34
derivativedivides	$-\frac{2 a^{3}}{3 x^{\frac{3}{2}}}+\frac{2 b^{3} x^{\frac{3}{2}}}{3}-\frac{6 a^{2} b}{\sqrt{x}}+6 a\, b^{2} \sqrt{x}$	36
default	$-\frac{2 a^{3}}{3 x^{\frac{3}{2}}}+\frac{2 b^{3} x^{\frac{3}{2}}}{3}-\frac{6 a^{2} b}{\sqrt{x}}+6 a\, b^{2} \sqrt{x}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^3/x^(3/2)+2/3*b^3*x^(3/2)-6*a^2*b/x^(1/2)+6*a*b^2*x^(1/2)
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}}+6\,ab^2\sqrt{x}-\frac{2\left(9\,a^2bx+a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Fricas [A]
time = 0.30, size = 34, normalized size = 0.72

$$\frac{2\left(b^{3} x^{3}+9\,a b^{2} x^{2}-9\,a^{2} b x-a^{3}\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

Sympy [A]
time = 0.25, size = 46, normalized size = 0.98

$$-\frac{2a^{3}}{3x^{\frac{3}{2}}}-\frac{6a^{2}b}{\sqrt{x}}+6ab^{2}\sqrt{x}+\frac{2b^{3}x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) - 6*a**2*b/sqrt(x) + 6*a*b**2*sqrt(x) + 2*b**3*x**(3/2
)/3
```

Giac [A]
time = 0.00, size = 51, normalized size = 1.09

$$\frac{2}{3}\sqrt{x}\,xb^{3}+6\sqrt{x}\,b^{2}a+\frac{-18xba^{2}-2a^{3}}{3\sqrt{x}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x)
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.74

$$-\frac{2\,a^{3}+18\,a^{2}\,b\,x-18\,a\,b^{2}\,x^{2}-2\,b^{3}\,x^{3}}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/2),x)
```

```
[Out] -(2*a^3 - 2*b^3*x^3 - 18*a*b^2*x^2 + 18*a^2*b*x)/(3*x^(3/2))
```

3.449 $\int \frac{x^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=68

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] -2/3*a*x^(3/2)/b^2+2/5*x^(5/2)/b-2*a^(5/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/
b^(7/2)+2*a^2*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 211}

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{a+bx}\,dx &= \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx}\,dx}{b} \\
&= -\frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{a+bx}\,dx}{b^2} \\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{a^3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b^3} \\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.90

$$
\frac{2\sqrt{x}\,(15a^2 - 5abx + 3b^2x^2)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.38, size = 126, normalized size = 1.85

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{5}{2}}\right], a{=}{=}0\&\&b{=}{=}0\right\}, \left\{\frac{2x^{\frac{7}{2}}}{7a}, b{=}{=}0\right\}, \left\{\frac{2x^{\frac{5}{2}}}{5b}, a{=}{=}0\right\}\right\}, -\frac{a^3\text{Log}\left[\sqrt{x} - \sqrt{-\frac{a}{b}}\right]}{b^4\sqrt{-\frac{a}{b}}} + \frac{a^3\text{Log}\left[\sqrt{x} + \sqrt{-\frac{a}{b}}\right]}{b^4\sqrt{-\frac{a}{b}}} + \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{2x^{\frac{5}{2}}}{5b}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (5 / 2)], a == 0 && b == 0}, {2 x ^ (7 / 2
) / (7 a), b == 0}, {2 x ^ (5 / 2) / (5 b), a == 0}}, -a ^ 3 Log[Sqrt[x] -
Sqrt[-a / b]] / (b ^ 4 Sqrt[-a / b]) + a ^ 3 Log[Sqrt[x] + Sqrt[-a / b]] /
```

```
(b ^ 4 Sqrt[-a / b]) + 2 a ^ 2 Sqrt[x] / b ^ 3 - 2 a x ^ (3 / 2) / (3 b ^ 2
) + 2 x ^ (5 / 2) / (5 b)]
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.79

method	result	size
risch	$\frac{2(3x^2b^2-5abx+15a^2)\sqrt{x}}{15b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}-\frac{2abx^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}-\frac{2abx^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)-1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.35, size = 54, normalized size = 0.79

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}-5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Fricas [A]
time = 0.33, size = 132, normalized size = 1.94

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\,-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [A]
time = 3.00, size = 122, normalized size = 1.79

$$\begin{cases} \tilde{\infty} x^{\frac{5}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a = 0 \\ -\frac{a^3 \log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{b^4 \sqrt{-\frac{a}{b}}} + \frac{a^3 \log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{b^4 \sqrt{-\frac{a}{b}}} + \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0))
, (2*x**(5/2)/(5*b), Eq(a, 0)), (-a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt
(-a/b)) + a**3*log(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*a**2*sqrt(x)
/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Giac [A]
time = 0.00, size = 85, normalized size = 1.25

$$2\left(\frac{\frac{1}{5}\sqrt{x}\,x^2b^4 - \frac{1}{3}\sqrt{x}\,xb^3a + \sqrt{x}\,b^2a^2}{b^5} - \frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2b^3\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x)
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B]
time = 0.06, size = 48, normalized size = 0.71

$$\frac{2\,x^{5/2}}{5\,b} - \frac{2\,a\,x^{3/2}}{3\,b^2} + \frac{2\,a^2\,\sqrt{x}}{b^3} - \frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 - (2*a^(5/2
)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

3.450 $\int \frac{x^{3/2}}{a+bx} \, dx$

Optimal. Leaf size=53

$$-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b+2*a^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)-2*a*x^(1/2)
/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 211}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{a+bx}\,dx &= \frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}\,dx}{b}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{a^2\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.92

$$
\frac{2\sqrt{x}\,(-3a+bx)}{3b^2}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.19, size = 115, normalized size = 2.17

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{3}{2}}\right],a{=}{=}0\&\&b{=}{=}0\right\},\left\{\frac{2x^{\frac{5}{2}}}{5a},b{=}{=}0\right\},\left\{\frac{2x^{\frac{3}{2}}}{3b},a{=}{=}0\right\}\right\},-\frac{a^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{b^3\sqrt{-\frac{a}{b}}}+\frac{a^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{b^3\sqrt{-\frac{a}{b}}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (3 / 2)], a == 0 && b == 0}, {2 x ^ (5 / 2
) / (5 a), b == 0}, {2 x ^ (3 / 2) / (3 b), a == 0}}, -a ^ 2 Log[Sqrt[x] +
Sqrt[-a / b]] / (b ^ 3 Sqrt[-a / b]) + a ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] /
(b ^ 3 Sqrt[-a / b]) - 2 a Sqrt[x] / b ^ 2 + 2 x ^ (3 / 2) / (3 b)]
```

Maple [A]
time = 0.10, size = 43, normalized size = 0.81

method	result	size
risch	$-\frac{2(-bx+3a)\sqrt{x}}{3b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	42
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2*(-1/3*b*x^(3/2)+a*x^(1/2))+2*a^2/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a
*b)^(1/2))
```

Maxima [A]
time = 0.34, size = 42, normalized size = 0.79

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

Fricas [A]
time = 0.31, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [A]
time = 0.75, size = 107, normalized size = 2.02

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ \frac{a^2 \log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}} - \frac{a^2 \log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}} - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a), Eq(b, 0))
, (2*x**(3/2)/(3*b), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(b**3*sqrt(
-a/b)) - a**2*log(sqrt(x) + sqrt(-a/b))/(b**3*sqrt(-a/b)) - 2*a*sqrt(x)/b**
2 + 2*x**(3/2)/(3*b), True))
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.23

$$2\left(\frac{\frac{1}{3}\sqrt{x}\,xb^2 - \sqrt{x}\,ba}{b^3} + \frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2b^2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x)
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b} - \frac{2\,a\,\sqrt{x}}{b^2} + \frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) - (2*a*x^(1/2))/b^2 + (2*a^(3/2)*atan((b^(1/2)*x^(1/2))/a
^(1/2)))/b^(5/2)
```

3.451 $\int \frac{\sqrt{x}}{a+bx}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)+2*x^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {52, 65, 211}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{a+bx}\,dx &= \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b} \\
&= \frac{2\sqrt{x}}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$
\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.83, size = 102, normalized size = 2.55

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\sqrt{x}\right], a==0\&\&b==0\right\}, \left\{\frac{2\sqrt{x}}{b}, a==0\right\}, \left\{\frac{2x^{\frac{3}{2}}}{3a}, b==0\right\}\right\}, -\frac{a\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{b^2\sqrt{-\frac{a}{b}}} + \frac{a\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{b^2\sqrt{-\frac{a}{b}}} + \frac{2\sqrt{x}}{b}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {2 Sqrt[x] / b, a
 == 0}, {2 x ^ (3 / 2) / (3 a), b == 0}}, -a Log[Sqrt[x] - Sqrt[-a / b]] /
(b ^ 2 Sqrt[-a / b]) + a Log[Sqrt[x] + Sqrt[-a / b]] / (b ^ 2 Sqrt[-a / b])
 + 2 Sqrt[x] / b]
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.80

method	result	size

derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.35, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Fricas [A]
time = 0.31, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b},-\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [A]
time = 0.35, size = 88, normalized size = 2.20

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)),
 (2*sqrt(x)/b, Eq(a, 0)), (-a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b)) +
 a*log(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*sqrt(x)/b, True))
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.08

$$2\left(\frac{\sqrt{x}}{b} - \frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x)
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.70

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\ \mathrm{atan}\left(\frac{\sqrt{b}\ \sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

3.452 $\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] 2*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 211}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx &= 2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.97, size = 92, normalized size = 3.17

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{\sqrt{x}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{b\sqrt{x}}, a==0\right\}, \left\{\frac{2\sqrt{x}}{a}, b==0\right\}\right\}, -\frac{\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{b\sqrt{-\frac{a}{b}}} + \frac{\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{b\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a + b*x)),x]')
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[x]], a == 0 && b == 0}, {-2 / (b Sqrt
[x]), a == 0}, {2 Sqrt[x] / a, b == 0}}, -Log[Sqrt[x] + Sqrt[-a / b]] / (b
Sqrt[-a / b]) + Log[Sqrt[x] - Sqrt[-a / b]] / (b Sqrt[-a / b])]
```

Maple [A]
time = 0.10, size = 19, normalized size = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.35, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="maxima")
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.31, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [A]
time = 0.47, size = 73, normalized size = 2.52

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\: a = 0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x**(1/2),x)
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a, Eq(b, 0)), (-2/
(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - sqrt(-a/b))/(b*sqrt(-a/b)) - log(sqr
t(x) + sqrt(-a/b))/(b*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.90

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x)
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 0.04, size = 19, normalized size = 0.66

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)),x)
```

```
[Out] (2*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.453 $\int \frac{1}{x^{3/2}(a+bx)} \, dx$

Optimal. Leaf size=40

$$-\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)-2/a/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 211}

$$-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{b\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [**A**]
time = 0.02, size = 40, normalized size = 1.00

$$-\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.53, size = 100, normalized size = 2.50

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{3bx^{\frac{3}{2}}}, a==0\right\}, \left\{\frac{-2}{a\sqrt{x}}, b==0\right\}\right\}, -\frac{\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{a\sqrt{-\frac{a}{b}}} + \frac{\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{a\sqrt{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (3 / 2)], a == 0 && b == 0}, {-2 / (3
b x ^ (3 / 2)), a == 0}, {-2 / (a Sqrt[x]), b == 0}}, -Log[Sqrt[x] - Sqrt[-
a / b]] / (a Sqrt[-a / b]) + Log[Sqrt[x] + Sqrt[-a / b]] / (a Sqrt[-a / b])
 - 2 / (a Sqrt[x])]
```

Maple [**A**]
time = 0.12, size = 32, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{2}{a\sqrt{x}}$	32
default	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{2}{a\sqrt{x}}$	32
risch	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{2}{a\sqrt{x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))-2/a/x^(1/2)
```

Maxima [A]
time = 0.34, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Fricas [A]
time = 0.32, size = 93, normalized size = 2.32

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\sqrt{x}}{ax},\frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

Sympy [A]
time = 1.03, size = 85, normalized size = 2.12

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{\log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}} + \frac{\log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - sqrt(-a/b))/(a*sqrt(-a/b)) +
 log(sqrt(x) + sqrt(-a/b))/(a*sqrt(-a/b)) - 2/(a*sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 45, normalized size = 1.12

$$2\left(-\frac{1}{a\sqrt{x}} - \frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a \cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x)
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.70

$$-\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\ \mathrm{atan}\left(\frac{\sqrt{b}\ \sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)),x)
```

```
[Out] - 2/(a*x^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

3.454 $\int \frac{1}{x^{5/2}(a+bx)} dx$

Optimal. Leaf size=53

$$-\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -2/3/a/x^(3/2)+2*b^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)+2*b/a^2/x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 211}

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)}\,dx &= -\frac{2}{3ax^{3/2}}-\frac{b\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^2}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.91

$$-\frac{2(a-3bx)}{3a^2x^{3/2}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] (-2*(a - 3*b*x))/(3*a^2*x^(3/2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.53, size = 111, normalized size = 2.09

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{5bx^{\frac{5}{2}}},a==0\right\},\left\{\frac{-2}{3ax^{\frac{3}{2}}},b==0\right\}\right\},\frac{-2}{3ax^{\frac{3}{2}}}-\frac{b\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{a^2\sqrt{-\frac{a}{b}}}+\frac{b\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{a^2\sqrt{-\frac{a}{b}}}+\frac{2b}{a^2\sqrt{x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 2)], a == 0 && b == 0}, {-2 / (5
b x ^ (5 / 2)), a == 0}, {-2 / (3 a x ^ (3 / 2)), b == 0}}, -2 / (3 a x ^ (
```

```
3 / 2)) - b Log[Sqrt[x] + Sqrt[-a / b]] / (a ^ 2 Sqrt[-a / b]) + b Log[Sqrt
[x] - Sqrt[-a / b]] / (a ^ 2 Sqrt[-a / b]) + 2 b / (a ^ 2 Sqrt[x])]
```

Maple [A]
time = 0.11, size = 43, normalized size = 0.81

method	result	size
risch	$-\frac{2(-3bx+a)}{3a^2x^{\frac{3}{2}}}+\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	40
derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	43
default	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)+2*b/a^2/x^(1/2)+2*b^2/a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)
^(1/2))
```

Maxima [A]
time = 0.34, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

Fricas [A]
time = 0.31, size = 118, normalized size = 2.23

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\,-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

Sympy [A]
time = 3.57, size = 107, normalized size = 2.02

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} + \frac{b\log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}} - \frac{b\log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + b*log(sqrt(x) - sqrt
(-a/b))/(a**2*sqrt(-a/b)) - b*log(sqrt(x) + sqrt(-a/b))/(a**2*sqrt(-a/b)) +
 2*b/(a**2*sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 60, normalized size = 1.13

$$2\left(\frac{3xb - a}{3a^2\sqrt{x}\,x} + \frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a^2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x)
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

Mupad [B]
time = 0.10, size = 38, normalized size = 0.72

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2}{3\,a} - \frac{2\,b\,x}{a^2}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/(3*a) - (2*b*x)/a^
2)/x^(3/2)
```

3.455 $\int \frac{1}{x^{7/2}(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)-2*b^(5/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/
a^(7/2)-2*b^2/a^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 211}

$$-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) - (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)}\,dx &= -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{5/2}(a+bx)}\,dx}{a} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} + \frac{b^2\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a^2} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{b^3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.90

$$-\frac{2\left(3a^2-5abx+15b^2x^2\right)}{15a^3x^{5/2}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] (-2*(3*a^2 - 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.58, size = 126, normalized size = 1.85

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{7}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{7bx^{\frac{7}{2}}}, a==0\right\}, \left\{\frac{-2}{5ax^{\frac{5}{2}}}, b==0\right\}\right\}, \frac{-2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} - \frac{b^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{a^3\sqrt{-\frac{a}{b}}} + \frac{b^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{a^3\sqrt{-\frac{a}{b}}} - \frac{2b^2}{a^3\sqrt{x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(7/2)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 2)], a == 0 && b == 0}, {-2 / (7
b x ^ (7 / 2)), a == 0}, {-2 / (5 a x ^ (5 / 2)), b == 0}}, -2 / (5 a x ^ (
5 / 2)) + 2 b / (3 a ^ 2 x ^ (3 / 2)) - b ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] /
 (a ^ 3 Sqrt[-a / b]) + b ^ 2 Log[Sqrt[x] + Sqrt[-a / b]] / (a ^ 3 Sqrt[-a
/ b]) - 2 b ^ 2 / (a ^ 3 Sqrt[x])]
```

Maple [A]
time = 0.11, size = 54, normalized size = 0.79

method	result	size
risch	$-\frac{2(15x^2b^2-5abx+3a^2)}{15a^3x^{\frac{5}{2}}}-\frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	53
derivativedivides	$-\frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}-\frac{2}{5a\,x^{\frac{5}{2}}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}$	54
default	$-\frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}-\frac{2}{5a\,x^{\frac{5}{2}}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))-2/5/a/x^(5/2)-2*b^2/a^
3/x^(1/2)+2/3*b/a^2/x^(3/2)
```

Maxima [A]
time = 0.37, size = 52, normalized size = 0.76

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\,(15\,b^2x^2-5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

Fricas [A]
time = 0.32, size = 144, normalized size = 2.12

$$\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\ \frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [A]
time = 14.71, size = 126, normalized size = 1.85

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\ a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} - \frac{b^2\log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}} + \frac{b^2\log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}} - \frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2)
) - b**2*log(sqrt(x) - sqrt(-a/b))/(a**3*sqrt(-a/b)) + b**2*log(sqrt(x) + s
qrt(-a/b))/(a**3*sqrt(-a/b)) - 2*b**2/(a**3*sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.13

$$2\left(-\frac{15x^2b^2 - 5xba + 3a^2}{15a^3\sqrt{x}\,x^2} - \frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a^3\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x)
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B]
time = 0.11, size = 49, normalized size = 0.72

$$-\frac{\frac{2}{5\,a} + \frac{2\,b^2\,x^2}{a^3} - \frac{2\,b\,x}{3\,a^2}}{x^{5/2}} - \frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)),x)
```

```
[Out] - (2/(5*a) + (2*b^2*x^2)/a^3 - (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atan((
b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

3.456 $\int \frac{x^{5/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] 5/3*x^(3/2)/b^2-x^(5/2)/b/(b*x+a)+5*a^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))
/b^(7/2)-5*a*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^2}\,dx &= -\frac{x^{5/2}}{b(a+bx)}+\frac{5\int \frac{x^{3/2}}{a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}-\frac{(5a)\int \frac{\sqrt{x}}{a+bx}\,dx}{2b^2}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{(5a^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 0.97

$$
\frac{\sqrt{x}\,\left(-15a^2-10abx+2b^2x^2\right)}{3b^3(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(-15*a^2 - 10*a*b*x + 2*b^2*x^2))/(3*b^3*(a + b*x)) + (5*a^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.36, size = 386, normalized size = 5.51

$$
\mathrm{Piecewise}\left[\left\{\left\{\mathrm{DirectedInfinity}\left[x^{\frac{3}{2}}\right],a==0\&\&b==0\right\},\left\{\frac{2x^{\frac{7}{2}}}{7a^2},b==0\right\},\left\{\frac{2x^{\frac{3}{2}}}{3b^2},a==0\right\}\right\},\frac{-15a^3\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}+\frac{15a^3\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}-\frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}-\frac{15a^2bx\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}+\frac{15a^2bx\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}-\frac{20ab^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}+\frac{4b^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}}+6b^5x\sqrt{-\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^(5/2)/(a + b*x)^2,x]')

[Out] Piecewise[{{DirectedInfinity[x ^ (3 / 2)], a == 0 && b == 0}, {2 x ^ (7 / 2) / (7 a ^ 2), b == 0}, {2 x ^ (3 / 2) / (3 b ^ 2), a == 0}}, -15 a ^ 3 Log [Sqrt[x] + Sqrt[-a / b]] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b]) + 15 a ^ 3 Log[Sqrt[x] - Sqrt[-a / b]] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b]) - 30 a ^ 2 b Sqrt[x] Sqrt[-a / b] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b]) - 15 a ^ 2 b x Log[Sqrt[x] + Sqrt[-a / b]] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b]) + 15 a ^ 2 b x Log[Sqrt[x] - Sqrt[-a / b]] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b]) - 20 a b ^ 2 x ^ (3 / 2) Sqrt[-a / b] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b]) + 4 b ^ 3 x ^ (5 / 2) Sqrt[-a / b] / (6 a b ^ 4 Sqrt[-a / b] + 6 b ^ 5 x Sqrt[-a / b])]

Maple [A]
time = 0.12, size = 59, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	59
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	59
risch	$-\frac{2(-bx+6a)\sqrt{x}}{3b^{3}}-\frac{a^{2}\sqrt{x}}{b^{3}(bx+a)}+\frac{5a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -2/b^3*(-1/3*b*x^(3/2)+2*a*x^(1/2))+2/b^3*a^2*(-1/2*x^(1/2)/(b*x+a)+5/2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))

Maxima [A]
time = 0.36, size = 63, normalized size = 0.90

$$-\frac{a^{2}\sqrt{x}}{b^{4}x+ab^{3}}+\frac{5\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{2\left(bx^{\frac{3}{2}}-6\,a\sqrt{x}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x + a*b^3) + 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^3) + 2/3*(b*x^(3/2) - 6*a*sqrt(x))/b^3
```

Fricas [A]
time = 0.31, size = 161, normalized size = 2.30

$$\left[\frac{15\,(abx+a^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x+ab^3)},\ \frac{15\,(abx+a^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)+(2\,b^2x^2-10\,abx-15\,a^2)\sqrt{x}}{3\,(b^4x+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [A]
time = 14.13, size = 389, normalized size = 5.56

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 a^{2}} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b^{2}} & \text{for}\: a = 0 \\ \frac{15 a^{3} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} - \frac{15 a^{3} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} - \frac{30 a^{2} b \sqrt{x} \sqrt{- \frac{a}{b}}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} + \frac{15 a^{2} b x \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} - \frac{15 a^{2} b x \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} - \frac{20 a b^{2} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} + \frac{4 b^{3} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}}{6 a b^{4} \sqrt{- \frac{a}{b}} + 6 b^{5} x \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (15*a**3*log(sqrt(x) - sqrt(-a/b))/(6
*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**3*log(sqrt(x) + sqrt(-a/b
))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 30*a**2*b*sqrt(x)*sqrt(-a/
b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 15*a**2*b*x*log(sqrt(x) -
sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**2*b*x*log(s
qrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 20*a*b**
2*x**(3/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 4*b**3*
x**(5/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 90, normalized size = 1.29

$$2\left(\frac{\frac{1}{3}\sqrt{x}\,xb^4-2\sqrt{x}\,b^3a}{b^6}-\frac{\sqrt{x}\,a^2}{2b^3\,(xb+a)}+\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2b^3\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x)
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

Mupad [B]
time = 0.11, size = 58, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3\,b^2}-\frac{4\,a\,\sqrt{x}}{b^3}-\frac{a^2\,\sqrt{x}}{x\,b^4+a\,b^3}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*a*x^(1/2))/b^3 - (a^2*x^(1/2))/(a*b^3 + b^4*x) + (
5*a^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

3.457 $\int \frac{x^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] -x^(3/2)/b/(b*x+a)-3*arctan(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)+3*x^(1
/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^2}\,dx &= -\frac{x^{3/2}}{b(a+bx)} + \frac{3\int \frac{\sqrt{x}}{a+bx}\,dx}{2b} \\
&= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{(3a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b^2} \\
&= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{(3a)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.95

$$
\frac{\sqrt{x}\,(3a+2bx)}{b^2(a+bx)} - \frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^2,x]
[Out] (Sqrt[x]*(3*a + 2*b*x))/(b^2*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.27, size = 332, normalized size = 5.82

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\sqrt{x}\right], a==0\&\&b==0\right\}, \left\{\frac{2x^{\frac{5}{2}}}{5a^2}, b==0\right\}, \left\{\frac{2\sqrt{x}}{b^2}, a==0\right\}\right\}, \frac{-3a^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{3a^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{3abx\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{3abx\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{4b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {2 x ^ (5 / 2) /
(5 a ^ 2), b == 0}, {2 Sqrt[x] / b ^ 2, a == 0}}, -3 a ^ 2 Log[Sqrt[x] - Sq
rt[-a / b]] / (2 a b ^ 3 Sqrt[-a / b] + 2 b ^ 4 x Sqrt[-a / b]) + 3 a ^ 2 L
og[Sqrt[x] + Sqrt[-a / b]] / (2 a b ^ 3 Sqrt[-a / b] + 2 b ^ 4 x Sqrt[-a /
b]) + 6 a b Sqrt[x] Sqrt[-a / b] / (2 a b ^ 3 Sqrt[-a / b] + 2 b ^ 4 x Sqrt
[-a / b]) - 3 a b x Log[Sqrt[x] - Sqrt[-a / b]] / (2 a b ^ 3 Sqrt[-a / b] +
 2 b ^ 4 x Sqrt[-a / b]) + 3 a b x Log[Sqrt[x] + Sqrt[-a / b]] / (2 a b ^ 3
 Sqrt[-a / b] + 2 b ^ 4 x Sqrt[-a / b]) + 4 b ^ 2 x ^ (3 / 2) Sqrt[-a / b]
/ (2 a b ^ 3 Sqrt[-a / b] + 2 b ^ 4 x Sqrt[-a / b])]
```

Maple [A]
time = 0.13, size = 47, normalized size = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	47
default	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	47
risch	$\frac{2\sqrt{x}}{b^2}+\frac{a\sqrt{x}}{b^2(bx+a)}-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^2-2*a/b^2*(-1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/2
)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.36, size = 49, normalized size = 0.86

$$\frac{a\sqrt{x}}{b^3x+ab^2}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*sqrt(x)/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + 2*sqrt(x)/b^2
```

Fricas [A]
time = 0.31, size = 134, normalized size = 2.35

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,(b^3x+ab^2)},\ -\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [A]
time = 3.87, size = 332, normalized size = 5.82

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b^2} & \text{for } a=0 \\ -\frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{6ab\sqrt{x}\,\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}-\frac{3abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{3abx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}}+\frac{4b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (-3*a**2*log(sqrt(x) - sqrt(-a/b))/(2*a*b**
3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a**2*log(sqrt(x) + sqrt(-a/b))/(2*a
*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 6*a*b*sqrt(x)*sqrt(-a/b)/(2*a*b**
3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*a*b*x*log(sqrt(x) - sqrt(-a/b))/(2*
a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a*b*x*log(sqrt(x) + sqrt(-a/b)
)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 4*b**2*x**(3/2)*sqrt(-a/b)/
(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.16

$$2\left(\frac{\sqrt{x}}{b^2}+\frac{\sqrt{x}\,a}{2b^2\,(xb+a)}-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2b^2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x)
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

Mupad [B]
time = 0.12, size = 46, normalized size = 0.81

$$\frac{2\,\sqrt{x}}{b^2}+\frac{a\,\sqrt{x}}{x\,b^3+a\,b^2}-\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 + b^3*x) - (3*a^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/b^(5/2)
```

3.458 $\int \frac{\sqrt{x}}{(a+bx)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{x}}{b(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

[Out] arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-x^(1/2)/b/(b*x+a)

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 211}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b*x)^2,x]

[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2))

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.00

$$-\frac{\sqrt{x}}{b(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.11, size = 276, normalized size = 6.00

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{\sqrt{x}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{b^2\sqrt{x}}, a==0\right\}, \left\{\frac{2x^{\frac{3}{2}}}{3a^2}, b==0\right\}\right\}, -\frac{a\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} + \frac{a\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} - \frac{bx\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} + \frac{bx\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[x]], a == 0 && b == 0}, {-2 / (b ^ 2
Sqrt[x]), a == 0}, {2 x ^ (3 / 2) / (3 a ^ 2), b == 0}}, -a Log[Sqrt[x] + S
qrt[-a / b]] / (2 a b ^ 2 Sqrt[-a / b] + 2 b ^ 3 x Sqrt[-a / b]) + a Log[Sq
rt[x] - Sqrt[-a / b]] / (2 a b ^ 2 Sqrt[-a / b] + 2 b ^ 3 x Sqrt[-a / b]) -
 2 b Sqrt[x] Sqrt[-a / b] / (2 a b ^ 2 Sqrt[-a / b] + 2 b ^ 3 x Sqrt[-a / b
]) - b x Log[Sqrt[x] + Sqrt[-a / b]] / (2 a b ^ 2 Sqrt[-a / b] + 2 b ^ 3 x
Sqrt[-a / b]) + b x Log[Sqrt[x] - Sqrt[-a / b]] / (2 a b ^ 2 Sqrt[-a / b] +
 2 b ^ 3 x Sqrt[-a / b])]
```

Maple [A]
time = 0.10, size = 37, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{b(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37
default	$-\frac{\sqrt{x}}{b(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -x^(1/2)/b/(b*x+a)+1/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.35, size = 37, normalized size = 0.80

$$-\frac{\sqrt{x}}{b^2x+ab}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.63, size = 115, normalized size = 2.50

$$\left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\,(ab^3x+a^2b^2)},-\frac{ab\sqrt{x}+\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [A]
time = 1.70, size = 269, normalized size = 5.85

$$\begin{cases} \frac{\infty}{\sqrt{x}} & \text{for}\ a=0\wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for}\ b=0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for}\ a=0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}-\frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}+\frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}-\frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*s
qrt(-a/b) + 2*b**3*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*sq
rt(-a/b) + 2*b**3*x*sqrt(-a/b)) - 2*b*sqrt(x)*sqrt(-a/b)/(2*a*b**2*sqrt(-a/
b) + 2*b**3*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*sqrt(-a
/b) + 2*b**3*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*sqrt(-
a/b) + 2*b**3*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 49, normalized size = 1.07

$$2\left(-\frac{\sqrt{x}}{2b\,(xb+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x)
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

Mupad [B]
time = 0.04, size = 34, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^2,x)
```

```
[Out] atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2)) - x^(1/2)/(b*(a + b*x))
```

3.459 $\int \frac{1}{\sqrt{x}(a+bx)^2} dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x}}{a(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[Out] arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)+x^(1/2)/a/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 211}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.00

$$\frac{\sqrt{x}}{a(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.13, size = 281, normalized size = 6.24

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{2\sqrt{x}}{a^2}, b==0\right\}, \left\{\frac{-2}{3b^2x^{\frac{3}{2}}}, a==0\right\}\right\}, -\frac{a\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{a\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{bx\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{bx\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (3 / 2)], a == 0 && b == 0}, {2 Sqrt[x
] / a ^ 2, b == 0}, {-2 / (3 b ^ 2 x ^ (3 / 2)), a == 0}}, -a Log[Sqrt[x] +
 Sqrt[-a / b]] / (2 a ^ 2 b Sqrt[-a / b] + 2 a b ^ 2 x Sqrt[-a / b]) + a Lo
g[Sqrt[x] - Sqrt[-a / b]] / (2 a ^ 2 b Sqrt[-a / b] + 2 a b ^ 2 x Sqrt[-a /
 b]) + 2 b Sqrt[x] Sqrt[-a / b] / (2 a ^ 2 b Sqrt[-a / b] + 2 a b ^ 2 x Sqr
t[-a / b]) - b x Log[Sqrt[x] + Sqrt[-a / b]] / (2 a ^ 2 b Sqrt[-a / b] + 2
a b ^ 2 x Sqrt[-a / b]) + b x Log[Sqrt[x] - Sqrt[-a / b]] / (2 a ^ 2 b Sqrt
[-a / b] + 2 a b ^ 2 x Sqrt[-a / b])]
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36
default	$\frac{\sqrt{x}}{a(bx+a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.36, size = 35, normalized size = 0.78

$$\frac{\sqrt{x}}{abx+a^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="maxima")
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.32, size = 116, normalized size = 2.58

$$\left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\,(a^2b^2x+a^3b)},\ \frac{ab\sqrt{x}-\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [A]
time = 2.75, size = 277, normalized size = 6.16

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a=0\wedge b=0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for}\ a=0 \\ \frac{2\sqrt{x}}{a^2} & \text{for}\ b=0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}}+\frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}}+\frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}}-\frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*sq
rt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b*s
qrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) + 2*b*sqrt(x)*sqrt(-a/b)/(2*a**2*b*sqrt(
-a/b) + 2*a*b**2*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*sq
rt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b
*sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 48, normalized size = 1.07

$$2\left(\frac{\sqrt{x}}{2a\left(xb+a\right)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x)
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

Mupad [B]
time = 0.09, size = 33, normalized size = 0.73

$$\frac{\sqrt{x}}{a\,\left(a+b\,x\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^2),x)
```

```
[Out] x^(1/2)/(a*(a + b*x)) + atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

3.460 $\int \frac{1}{x^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*arctan(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)-3/a^2/x^(1/2)+1/a/(b*x+a
)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(a+bx)^2}\,dx &= \frac{1}{a\sqrt{x}\,(a+bx)}+\frac{3\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^2}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.96

$$\frac{-2a-3bx}{a^2\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a - 3*b*x)/(a^2*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 8.48, size = 358, normalized size = 6.39

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{5b^2x^{\frac{5}{2}}},a==0\right\},\left\{\frac{-2}{a^2\sqrt{x}},b==0\right\}\right\},\frac{-4a\sqrt{-\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}-\frac{3a\sqrt{x}\,\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}+\frac{3a\sqrt{x}\,\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}-\frac{6bx\sqrt{-\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}-\frac{3bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}+\frac{3bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 2)], a == 0 && b == 0}, {-2 / (5
b ^ 2 x ^ (5 / 2)), a == 0}, {-2 / (a ^ 2 Sqrt[x]), b == 0}}, -4 a Sqrt[-a
/ b] / (2 a ^ 3 Sqrt[x] Sqrt[-a / b] + 2 a ^ 2 b x ^ (3 / 2) Sqrt[-a / b])
- 3 a Sqrt[x] Log[Sqrt[x] - Sqrt[-a / b]] / (2 a ^ 3 Sqrt[x] Sqrt[-a / b] +
 2 a ^ 2 b x ^ (3 / 2) Sqrt[-a / b]) + 3 a Sqrt[x] Log[Sqrt[x] + Sqrt[-a /
b]] / (2 a ^ 3 Sqrt[x] Sqrt[-a / b] + 2 a ^ 2 b x ^ (3 / 2) Sqrt[-a / b]) -
 6 b x Sqrt[-a / b] / (2 a ^ 3 Sqrt[x] Sqrt[-a / b] + 2 a ^ 2 b x ^ (3 / 2)
 Sqrt[-a / b]) - 3 b x ^ (3 / 2) Log[Sqrt[x] - Sqrt[-a / b]] / (2 a ^ 3 Sqr
t[x] Sqrt[-a / b] + 2 a ^ 2 b x ^ (3 / 2) Sqrt[-a / b]) + 3 b x ^ (3 / 2) L
og[Sqrt[x] + Sqrt[-a / b]] / (2 a ^ 3 Sqrt[x] Sqrt[-a / b] + 2 a ^ 2 b x ^
(3 / 2) Sqrt[-a / b])]
```

Maple [A]
time = 0.11, size = 47, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2b\left(\frac{\sqrt{x}}{2bx+2a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{2}{a^2\sqrt{x}}$	47
default	$-\frac{2b\left(\frac{\sqrt{x}}{2bx+2a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{2}{a^2\sqrt{x}}$	47
risch	$-\frac{2}{a^2\sqrt{x}}-\frac{b\sqrt{x}}{a^2(bx+a)}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a^2*(1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
)-2/a^2/x^(1/2)
```

Maxima [A]
time = 0.34, size = 51, normalized size = 0.91

$$-\frac{3\,bx+2\,a}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*a^2)
```

Fricas [**A**]
time = 0.32, size = 147, normalized size = 2.62

$$\left[\frac{3\,(bx^2+ax)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\,(a^2bx^2+a^3x)},\ \frac{3\,(bx^2+ax)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx+2\,a)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

Sympy [**A**]
time = 7.38, size = 384, normalized size = 6.86

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{5 b^{2} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{3 a \sqrt{x} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{- \frac{a}{b}} + 2 a^{2} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}} + \frac{3 a \sqrt{x} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{- \frac{a}{b}} + 2 a^{2} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}} - \frac{4 a \sqrt{- \frac{a}{b}}}{2 a^{3} \sqrt{x} \sqrt{- \frac{a}{b}} + 2 a^{2} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}} - \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{- \frac{a}{b}} + 2 a^{2} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}} + \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{- \frac{a}{b}} + 2 a^{2} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}} - \frac{6 b x \sqrt{- \frac{a}{b}}}{2 a^{3} \sqrt{x} \sqrt{- \frac{a}{b}} + 2 a^{2} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)
), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-3*a*sqrt(x)*log(sqrt(x) - sqrt(-a/b)
)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) + 3*a*sqrt(x)*
log(sqrt(x) + sqrt(-a/b))/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sq
rt(-a/b)) - 4*a*sqrt(-a/b)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*s
qrt(-a/b)) - 3*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(2*a**3*sqrt(x)*sqrt(-a
/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) + 3*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b)
)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) - 6*b*x*sqrt(-
a/b)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)), True))
```

Giac [**A**]

time = 0.00, size = 71, normalized size = 1.27

$$2\left(\frac{-3xb-2a}{2a^{2}\left(\sqrt{x}\,xb+\sqrt{x}\,a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a^{2}\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x)
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

Mupad [B]
time = 0.12, size = 48, normalized size = 0.86

$$-\frac{\frac{2}{a}+\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}\,+b\,x^{3/2}}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^2),x)
```

```
[Out] - (2/a + (3*b*x)/a^2)/(a*x^(1/2) + b*x^(3/2)) - (3*b^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/a^(5/2)
```

3.461 $\int \frac{1}{x^{5/2}(a+bx)^2} dx$

Optimal. Leaf size=69

$$-\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3/a^2/x^(3/2)+1/a/x^(3/2)/(b*x+a)+5*b^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/
2))/a^(7/2)+5*b/a^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) + (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a + b*x)) + (5*b^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a+bx)}+\frac{5\int \frac{1}{x^{5/2}(a+bx)}\,dx}{2a}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}-\frac{(5b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a^2}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{(5b^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^3}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 0.99

$$\frac{-2a^2+10abx+15b^2x^2}{3a^3x^{3/2}(a+bx)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a^2 + 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a + b*x)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 18.28, size = 419, normalized size = 6.07

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{7}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{7b^2x^{\frac{7}{2}}},a==0\right\},\left\{\frac{-2}{3a^2x^{\frac{3}{2}}},b==0\right\}\right\},\frac{-4a^2\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{20abx\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{15abx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{15abx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{30b^2x^2\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{15b^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{15b^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^(5/2)*(a + b*x)^2),x]')

[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 2)], a == 0 && b == 0}, {-2 / (7 b ^ 2 x ^ (7 / 2)), a == 0}, {-2 / (3 a ^ 2 x ^ (3 / 2)), b == 0}}, -4 a ^ 2 Sqrt[-a / b] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b]) + 20 a b x Sqrt[-a / b] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b]) - 15 a b x ^ (3 / 2) Log[Sqrt[x] + Sqrt[-a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b]) + 15 a b x ^ (3 / 2) Log[Sqrt[x] - Sqrt[-a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b]) + 30 b ^ 2 x ^ 2 Sqrt[-a / b] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b]) - 15 b ^ 2 x ^ (5 / 2) Log[Sqrt[x] + Sqrt[-a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b]) + 15 b ^ 2 x ^ (5 / 2) Log[Sqrt[x] - Sqrt[-a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[-a / b] + 6 a ^ 3 b x ^ (5 / 2) Sqrt[-a / b])]

Maple [A]
time = 0.11, size = 58, normalized size = 0.84

method	result	size
risch	$-\frac{2(-6bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{b^2\sqrt{x}}{a^3(bx+a)}+\frac{5b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	57
derivativedivides	$-\frac{2}{3a^2x^{\frac{3}{2}}}+\frac{4b}{a^3\sqrt{x}}+\frac{2b^2\left(\frac{\sqrt{x}}{2bx+2a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	58
default	$-\frac{2}{3a^2x^{\frac{3}{2}}}+\frac{4b}{a^3\sqrt{x}}+\frac{2b^2\left(\frac{\sqrt{x}}{2bx+2a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -2/3/a^2/x^(3/2)+4*b/a^3/x^(1/2)+2*b^2/a^3*(1/2*x^(1/2)/(b*x+a)+5/2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))

Maxima [A]

time = 0.37, size = 64, normalized size = 0.93

$$\frac{15\,b^{2}x^{2}+10\,abx-2\,a^{2}}{3\,\left(a^{3}bx^{\frac{5}{2}}+a^{4}x^{\frac{3}{2}}\right)}+\frac{5\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.32, size = 184, normalized size = 2.67

$$\left[\frac{15\,(b^{2}x^{3}+abx^{2})\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,b^{2}x^{2}+10\,abx-2\,a^{2})\sqrt{x}}{6\,(a^{3}bx^{3}+a^{4}x^{2})},\,-\frac{15\,(b^{2}x^{3}+abx^{2})\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^{2}x^{2}+10\,abx-2\,a^{2})\sqrt{x}}{3\,(a^{3}bx^{3}+a^{4}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [A]
time = 29.69, size = 452, normalized size = 6.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{7b^{2}x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{3a^{2}x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{4a^{2}\sqrt{-\frac{a}{b}}}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{20abx\sqrt{-\frac{a}{b}}}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{15b^{2}x^{\frac{5}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{15b^{2}x^{\frac{5}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{30b^{2}x^{2}\sqrt{-\frac{a}{b}}}{6a^{4}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^{3}bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*a**2*sqrt(-a/b)/(6*a**4*x**(3/2
)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*a*b*x**(3/2)*log(sqrt(x)
- sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) -
 15*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*
a**3*b*x**(5/2)*sqrt(-a/b)) + 20*a*b*x*sqrt(-a/b)/(6*a**4*x**(3/2)*sqrt(-a/
```

```
b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*b**2*x**(5/2)*log(sqrt(x) - sqrt(-a
/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 15*b**2*
x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x
**(5/2)*sqrt(-a/b)) + 30*b**2*x**2*sqrt(-a/b)/(6*a**4*x**(3/2)*sqrt(-a/b) +
 6*a**3*b*x**(5/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 81, normalized size = 1.17

$$2\left(\frac{\sqrt{x}\,b^2}{2a^3\,(xb+a)}+\frac{6xb-a}{3a^3\sqrt{x}\,x}+\frac{5b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a^3\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x)
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

Mupad [B]
time = 0.15, size = 58, normalized size = 0.84

$$\frac{\frac{5\,b^2\,x^2}{a^3}-\frac{2}{3\,a}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}+b\,x^{5/2}}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^2),x)
```

```
[Out] ((5*b^2*x^2)/a^3 - 2/(3*a) + (10*b*x)/(3*a^2))/(a*x^(3/2) + b*x^(5/2)) + (5
*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

3.462 $\int \frac{x^{7/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=95

$$-\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

```
[Out] 35/12*x^(3/2)/b^3-1/2*x^(7/2)/b/(b*x+a)^2-7/4*x^(5/2)/b^2/(b*x+a)+35/4*a^(3
/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)-35/4*a*x^(1/2)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{35a\sqrt{x}}{4b^4}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a + b*x)^2)
 - (7*x^(5/2))/(4*b^2*(a + b*x)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{(a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a+bx)^2}+\frac{7\int \frac{x^{5/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35\int \frac{x^{3/2}}{a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{(35a)\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^3}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{(35a^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{(35a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 0.85

$$\frac{\sqrt{x}\,\left(-105a^3-175a^2bx-56ab^2x^2+8b^3x^3\right)}{12b^4(a+bx)^2}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(-105*a^3 - 175*a^2*b*x - 56*a*b^2*x^2 + 8*b^3*x^3))/(12*b^4*(a +
b*x)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 92.82, size = 738, normalized size = 7.77

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{3}{2}}\right],a==0\&\&b==0\right\},\left\{\frac{2x^{\frac{3}{2}}}{3b^3},a==0\right\},\left\{\frac{2x^{\frac{9}{2}}}{9a^3},b==0\right\}\right\},\frac{-105a^4\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}+\frac{105a^4\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}-\frac{210a^3b\sqrt{x}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}-\frac{210a^3bx\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}+\frac{210a^3bx\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}-\frac{350a^2b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}-\frac{105a^2b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}+\frac{105a^2b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}-\frac{112ab^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}+\frac{16b^4x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(7/2)/(a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (3 / 2)], a == 0 && b == 0}, {2 x ^ (3 / 2
) / (3 b ^ 3), a == 0}, {2 x ^ (9 / 2) / (9 a ^ 3), b == 0}}, -105 a ^ 4 Lo
g[Sqrt[x] + Sqrt[-a / b]] / (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqr
t[-a / b] + 24 b ^ 7 x ^ 2 Sqrt[-a / b]) + 105 a ^ 4 Log[Sqrt[x] - Sqrt[-a
/ b]] / (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqrt[-a / b] + 24 b ^ 7
 x ^ 2 Sqrt[-a / b]) - 210 a ^ 3 b Sqrt[x] Sqrt[-a / b] / (24 a ^ 2 b ^ 5 S
qrt[-a / b] + 48 a b ^ 6 x Sqrt[-a / b] + 24 b ^ 7 x ^ 2 Sqrt[-a / b]) - 21
0 a ^ 3 b x Log[Sqrt[x] + Sqrt[-a / b]] / (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48
 a b ^ 6 x Sqrt[-a / b] + 24 b ^ 7 x ^ 2 Sqrt[-a / b]) + 210 a ^ 3 b x Log[
Sqrt[x] - Sqrt[-a / b]] / (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqrt[
-a / b] + 24 b ^ 7 x ^ 2 Sqrt[-a / b]) - 350 a ^ 2 b ^ 2 x ^ (3 / 2) Sqrt[-
a / b] / (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqrt[-a / b] + 24 b ^
7 x ^ 2 Sqrt[-a / b]) - 105 a ^ 2 b ^ 2 x ^ 2 Log[Sqrt[x] + Sqrt[-a / b]] /
 (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqrt[-a / b] + 24 b ^ 7 x ^ 2
Sqrt[-a / b]) + 105 a ^ 2 b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] / (24 a ^
 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqrt[-a / b] + 24 b ^ 7 x ^ 2 Sqrt[-a
/ b]) - 112 a b ^ 3 x ^ (5 / 2) Sqrt[-a / b] / (24 a ^ 2 b ^ 5 Sqrt[-a / b]
 + 48 a b ^ 6 x Sqrt[-a / b] + 24 b ^ 7 x ^ 2 Sqrt[-a / b]) + 16 b ^ 4 x ^
(7 / 2) Sqrt[-a / b] / (24 a ^ 2 b ^ 5 Sqrt[-a / b] + 48 a b ^ 6 x Sqrt[-a
/ b] + 24 b ^ 7 x ^ 2 Sqrt[-a / b])]
```

Maple [A]
time = 0.13, size = 68, normalized size = 0.72

method	result	size
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}\right)}{b^{4}}+\frac{2a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{\left(bx+a\right)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$	68
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}\right)}{b^{4}}+\frac{2a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{\left(bx+a\right)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$	68
risch	$-\frac{2\left(-bx+9a\right)\sqrt{x}}{3b^{4}}-\frac{13a^{2}x^{\frac{3}{2}}}{4b^{3}\left(bx+a\right)^{2}}-\frac{11a^{3}\sqrt{x}}{4b^{4}\left(bx+a\right)^{2}}+\frac{35a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{4}\sqrt{ab}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*(-1/3*b*x^(3/2)+3*a*x^(1/2))+2/b^4*a^2*((-13/8*b*x^(3/2)-11/8*a*x^(1
/2))/(b*x+a)^2+35/8/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.34, size = 86, normalized size = 0.91

$$-\frac{13\,a^{2}bx^{\frac{3}{2}}+11\,a^{3}\sqrt{x}}{4\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})}+\frac{35\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^{4}}+\frac{2\,\left(bx^{\frac{3}{2}}-9\,a\sqrt{x}\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) + 11*a^3*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(b*x^(3/2) - 9*a
*sqrt(x))/b^4
```

Fricas [A]
time = 0.31, size = 227, normalized size = 2.39

$$\left[\frac{105\,(ab^{2}x^{2}+2\,a^{2}bx+a^{3})\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^{3}x^{3}-56\,ab^{2}x^{2}-175\,a^{2}bx-105\,a^{3})\sqrt{x}}{24\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})},\ \frac{105\,(ab^{2}x^{2}+2\,a^{2}bx+a^{3})\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(8\,b^{3}x^{3}-56\,ab^{2}x^{2}-175\,a^{2}bx-105\,a^{3})\sqrt{x}}{12\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [A]
time = 93.79, size = 762, normalized size = 8.02

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\: a=0\wedge b=0 \\ \frac{2x^{\frac{9}{2}}}{9a^{3}} & \text{for}\: b=0 \\ \frac{2x^{\frac{3}{2}}}{3b^{3}} & \text{for}\: a=0 \\ \frac{105a^{4}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}-\frac{105a^{4}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}-\frac{210a^{3}b\sqrt{x}\sqrt{-\frac{a}{b}}}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}+\frac{210a^{3}bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}-\frac{210a^{3}bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}-\frac{350a^{2}b^{2}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}+\frac{105a^{2}b^{2}x^{2}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}-\frac{105a^{2}b^{2}x^{2}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}-\frac{112ab^{3}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}}+\frac{16b^{4}x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}}{24a^{2}b^{5}\sqrt{-\frac{a}{b}}+48ab^{6}x\sqrt{-\frac{a}{b}}+24b^{7}x^{2}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a**3), Eq(b,
0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (105*a**4*log(sqrt(x) - sqrt(-a/b))/(
24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b))
 - 105*a**4*log(sqrt(x) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*
x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*a**3*b*sqrt(x)*sqrt(-a/b)/(24
*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) +
 210*a**3*b*x*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**
6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*a**3*b*x*log(sqrt(x) + sqrt
(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sq
rt(-a/b)) - 350*a**2*b**2*x**(3/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(-a/b) + 48
*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 105*a**2*b**2*x**2*log(sq
rt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*
b**7*x**2*sqrt(-a/b)) - 105*a**2*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(24*a*
*2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 11
2*a*b**3*x**(5/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a
/b) + 24*b**7*x**2*sqrt(-a/b)) + 16*b**4*x**(7/2)*sqrt(-a/b)/(24*a**2*b**5*
sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 110, normalized size = 1.16

$$2\left(\frac{\frac{1}{3}\sqrt{x}\,xb^6-3\sqrt{x}\,b^5a}{b^9}+\frac{-13\sqrt{x}\,xba^2-11\sqrt{x}\,a^3}{8b^4\,(xb+a)^2}+\frac{35a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^4\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x)
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

Mupad [B]
time = 0.12, size = 81, normalized size = 0.85

$$\frac{2\,x^{3/2}}{3\,b^3}-\frac{\frac{11\,a^3\,\sqrt{x}}{4}+\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4+2\,a\,b^5\,x+b^6\,x^2}-\frac{6\,a\,\sqrt{x}}{b^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x)^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) - ((11*a^3*x^(1/2))/4 + (13*a^2*b*x^(3/2))/4)/(a^2*b^4
+ b^6*x^2 + 2*a*b^5*x) - (6*a*x^(1/2))/b^4 + (35*a^(3/2)*atan((b^(1/2)*x^(1
/2))/a^(1/2)))/(4*b^(9/2))
```

3.463 $\int \frac{x^{5/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=82

$$\frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b(a+bx)^2} - \frac{5x^{3/2}}{4b^2(a+bx)} - \frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

```
[Out] -1/2*x^(5/2)/b/(b*x+a)^2-5/4*x^(3/2)/b^2/(b*x+a)-15/4*arctan(b^(1/2)*x^(1/2
)/a^(1/2))*a^(1/2)/b^(7/2)+15/4*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 211}

$$-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}} - \frac{5x^{3/2}}{4b^2(a+bx)} - \frac{x^{5/2}}{2b(a+bx)^2} + \frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a+bx)^2}+\frac{5\int \frac{x^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}+\frac{15\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 0.85

$$\frac{\sqrt{x}\,(15a^2+25abx+8b^2x^2)}{4b^3(a+bx)^2}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(15*a^2 + 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a + b*x)^2) - (15*Sqrt[a]
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 34.87, size = 665, normalized size = 8.11

$$\mathrm{Piecewise}\left[\left\{\left\{\mathrm{DirectedInfinity}\left[\sqrt{x}\right],a==0\&\&b==0\right\},\left\{\frac{2x^{\frac{7}{2}}}{7a^3},b==0\right\},\left\{\frac{2\sqrt{x}}{b^3},a==0\right\}\right\},\frac{-15a^3\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{15a^3\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}-\frac{30a^2bx\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{30a^2bx\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{50ab^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}-\frac{15ab^2x^2\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{15ab^2x^2\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{16b^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^(5/2)/(a + b*x)^3,x]')

[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {2 x ^ (7 / 2) / (7 a ^ 3), b == 0}, {2 Sqrt[x] / b ^ 3, a == 0}}, -15 a ^ 3 Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) + 15 a ^ 3 Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) + 30 a ^ 2 b Sqrt[x] Sqrt[-a / b] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) - 30 a ^ 2 b x Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) + 30 a ^ 2 b x Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) + 50 a b ^ 2 x ^ (3 / 2) Sqrt[-a / b] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) - 15 a b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) + 15 a b ^ 2 x ^ 2 Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b]) + 16 b ^ 3 x ^ (5 / 2) Sqrt[-a / b] / (8 a ^ 2 b ^ 4 Sqrt[-a / b] + 16 a b ^ 5 x Sqrt[-a / b] + 8 b ^ 6 x ^ 2 Sqrt[-a / b])]

Maple [A]
time = 0.12, size = 56, normalized size = 0.68

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^{3}}-\frac{2a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{3}}$	56
default	$\frac{2\sqrt{x}}{b^{3}}-\frac{2a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{3}}$	56
risch	$\frac{2\sqrt{x}}{b^{3}}+\frac{9a\,x^{\frac{3}{2}}}{4b^{2}(bx+a)^{2}}+\frac{7a^{2}\sqrt{x}}{4b^{3}(bx+a)^{2}}-\frac{15a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{3}\sqrt{ab}}$	66

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 2*x^(1/2)/b^3-2/b^3*a*((-9/8*b*x^(3/2)-7/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))

Maxima [A]
time = 0.37, size = 73, normalized size = 0.89

$$\frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3}+\frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 15/4*
a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3
```

Fricas [A]
time = 0.33, size = 200, normalized size = 2.44

$$\left[\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)},-\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [A]
time = 34.88, size = 683, normalized size = 8.33

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a=0\wedge b=0 \\ \frac{2\sqrt{x}}{b^3} & \text{for}\: a=0 \\ \frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\: b=0 \\ -\frac{15a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{15a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}-\frac{30a^2bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{30a^2bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{50ab^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}-\frac{15ab^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{15ab^2x^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}}+\frac{16b^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**3, Eq(a, 0)), (
2*x**(7/2)/(7*a**3), Eq(b, 0)), (-15*a**3*log(sqrt(x) - sqrt(-a/b))/(8*a**2
*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15*a*
*3*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/
b) + 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**4*sq
rt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 30*a**2*b*x*l
```

```
og(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) +
 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 50*a*b**
2*x**(3/2)*sqrt(-a/b)/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*
b**6*x**2*sqrt(-a/b)) - 15*a*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15*a*b**
2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt
(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 16*b**3*x**(5/2)*sqrt(-a/b)/(8*a**2*b**4
*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 85, normalized size = 1.04

$$2\left(\frac{\sqrt{x}}{b^3}+\frac{9\sqrt{x}\,xba+7\sqrt{x}\,a^2}{8b^3\,(xb+a)^2}-\frac{15a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^3\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x)
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

Mupad [B]
time = 0.14, size = 69, normalized size = 0.84

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

3.464 $\int \frac{x^{3/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=70

$$-\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

```
[Out] -1/2*x^(3/2)/b/(b*x+a)^2+3/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)/a^(1/2
)-3/4*x^(1/2)/b^2/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {43, 65, 211}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{3\sqrt{x}}{4b^2(a+bx)}-\frac{x^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a+bx)^2}+\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^2}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.84

$$
-\frac{\sqrt{x}\,(3a+5bx)}{4b^2(a+bx)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(3*a + 5*b*x))/(b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 17.35, size = 590, normalized size = 8.43

$$
\mathrm{Piecewise}\left[\left\{\left\{\mathrm{DirectedInfinity}\left[\frac{1}{\sqrt{x}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{b^3\sqrt{x}},a==0\right\},\left\{\frac{2x^{\frac{5}{2}}}{5a^3},b==0\right\}\right\},\frac{-3a^2\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}+\frac{3a^2\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}-\frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}-\frac{6abx\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}+\frac{6abx\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}-\frac{10b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}-\frac{3b^2x^2\mathrm{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}+\frac{3b^2x^2\mathrm{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[x]], a == 0 && b == 0}, {-2 / (b ^ 3
Sqrt[x]), a == 0}, {2 x ^ (5 / 2) / (5 a ^ 3), b == 0}}, -3 a ^ 2 Log[Sqrt[
x] + Sqrt[-a / b]] / (8 a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a b ^ 4 x Sqrt[-a / b
] + 8 b ^ 5 x ^ 2 Sqrt[-a / b]) + 3 a ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] / (8
a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a b ^ 4 x Sqrt[-a / b] + 8 b ^ 5 x ^ 2 Sqrt[-
a / b]) - 6 a b Sqrt[x] Sqrt[-a / b] / (8 a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a b
```

```
 ^ 4 x Sqrt[-a / b] + 8 b ^ 5 x ^ 2 Sqrt[-a / b]) - 6 a b x Log[Sqrt[x] + S
qrt[-a / b]] / (8 a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a b ^ 4 x Sqrt[-a / b] + 8
b ^ 5 x ^ 2 Sqrt[-a / b]) + 6 a b x Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 2
b ^ 3 Sqrt[-a / b] + 16 a b ^ 4 x Sqrt[-a / b] + 8 b ^ 5 x ^ 2 Sqrt[-a / b]
) - 10 b ^ 2 x ^ (3 / 2) Sqrt[-a / b] / (8 a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a
b ^ 4 x Sqrt[-a / b] + 8 b ^ 5 x ^ 2 Sqrt[-a / b]) - 3 b ^ 2 x ^ 2 Log[Sqrt
[x] + Sqrt[-a / b]] / (8 a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a b ^ 4 x Sqrt[-a /
b] + 8 b ^ 5 x ^ 2 Sqrt[-a / b]) + 3 b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[-a / b]
] / (8 a ^ 2 b ^ 3 Sqrt[-a / b] + 16 a b ^ 4 x Sqrt[-a / b] + 8 b ^ 5 x ^ 2
 Sqrt[-a / b])]
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	50
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.35, size = 61, normalized size = 0.87

$$-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.31, size = 185, normalized size = 2.64

$$\left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)+2\,(5\,ab^2x+3\,a^2b)\sqrt{x}}{8\,(ab^5x^2+2\,a^2b^4x+a^3b^3)},-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5\,ab^2x+3\,a^2b)\sqrt{x}}{4\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

Sympy [A]
time = 16.54, size = 605, normalized size = 8.64

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{5}{2}}}{5 a^{3}} & \text{for}\: b = 0 \\ - \frac{2}{b^{3} \sqrt{x}} & \text{for}\: a = 0 \\ \frac{3 a^{2} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} - \frac{3 a^{2} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} - \frac{6 a b \sqrt{x} \sqrt{- \frac{a}{b}}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} + \frac{6 a b x \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} - \frac{6 a b x \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} - \frac{10 b^{2} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} + \frac{3 b^{2} x^{2} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} - \frac{3 b^{2} x^{2} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{- \frac{a}{b}} + 16 a b^{4} x \sqrt{- \frac{a}{b}} + 8 b^{5} x^{2} \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b, 0
)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a**
2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*a*
*2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/
b) + 8*b**5*x**2*sqrt(-a/b)) - 6*a*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**3*sqrt(-
a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) + 6*a*b*x*log(sqrt(
x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*
x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b
) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 10*b**2*x**(3/2)*sqr
t(-a/b)/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt
(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 1
6*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*b**2*x**2*log(sqrt(x) +
 sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2
*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 72, normalized size = 1.03

$$2 \left(\frac{-5 \sqrt{x}\, x b - 3 \sqrt{x}\, a}{8 b^{2} \left(x b + a\right)^{2}} + \frac{3 \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{4 b^{2} \cdot 2 \sqrt{a b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x)
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.13, size = 58, normalized size = 0.83

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}-\frac{\frac{5\,x^{3/2}}{4\,b}+\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2)) - ((5*x^(3/2))/(4*b
) + (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.465 $\int \frac{\sqrt{x}}{(a+bx)^3} \, dx$

Optimal. Leaf size=73

$$-\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[Out] 1/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/2*x^(1/2)/b/(b*x+a)^2
+1/4*x^(1/2)/a/b/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 211}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*
Sqrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.82

$$
-\frac{\sqrt{x}\,(a-bx)}{4ab(a+bx)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(a - b*x))/(a*b*(a + b*x)^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]]/(4*a^(3/2)*b^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.82, size = 612, normalized size = 8.38

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{3b^3x^{\frac{3}{2}}},a==0\right\},\left\{\frac{2x^{\frac{3}{2}}}{3a^3},b==0\right\}\right\},-\frac{a^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}+\frac{a^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}-\frac{2ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}-\frac{2abx\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}+\frac{2abx\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}+\frac{2b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}-\frac{b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}+\frac{b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^3b^2\sqrt{-\frac{a}{b}}+16a^2b^3x\sqrt{-\frac{a}{b}}+8ab^4x^2\sqrt{-\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (3 / 2)], a == 0 && b == 0}, {-2 / (3
b ^ 3 x ^ (3 / 2)), a == 0}, {2 x ^ (3 / 2) / (3 a ^ 3), b == 0}}, -a ^ 2 L
og[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 3 b ^ 2 Sqrt[-a / b] + 16 a ^ 2 b ^ 3 x
 Sqrt[-a / b] + 8 a b ^ 4 x ^ 2 Sqrt[-a / b]) + a ^ 2 Log[Sqrt[x] - Sqrt[-a
 / b]] / (8 a ^ 3 b ^ 2 Sqrt[-a / b] + 16 a ^ 2 b ^ 3 x Sqrt[-a / b] + 8 a
b ^ 4 x ^ 2 Sqrt[-a / b]) - 2 a b Sqrt[x] Sqrt[-a / b] / (8 a ^ 3 b ^ 2 Sqr
t[-a / b] + 16 a ^ 2 b ^ 3 x Sqrt[-a / b] + 8 a b ^ 4 x ^ 2 Sqrt[-a / b]) -
 2 a b x Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 3 b ^ 2 Sqrt[-a / b] + 16 a ^
 2 b ^ 3 x Sqrt[-a / b] + 8 a b ^ 4 x ^ 2 Sqrt[-a / b]) + 2 a b x Log[Sqrt[
x] - Sqrt[-a / b]] / (8 a ^ 3 b ^ 2 Sqrt[-a / b] + 16 a ^ 2 b ^ 3 x Sqrt[-a
 / b] + 8 a b ^ 4 x ^ 2 Sqrt[-a / b]) + 2 b ^ 2 x ^ (3 / 2) Sqrt[-a / b] /
(8 a ^ 3 b ^ 2 Sqrt[-a / b] + 16 a ^ 2 b ^ 3 x Sqrt[-a / b] + 8 a b ^ 4 x ^
 2 Sqrt[-a / b]) - b ^ 2 x ^ 2 Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 3 b ^ 2
 Sqrt[-a / b] + 16 a ^ 2 b ^ 3 x Sqrt[-a / b] + 8 a b ^ 4 x ^ 2 Sqrt[-a / b
]) + b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 3 b ^ 2 Sqrt[-a / b]
+ 16 a ^ 2 b ^ 3 x Sqrt[-a / b] + 8 a b ^ 4 x ^ 2 Sqrt[-a / b])]
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	52
default	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/8/a*x^(3/2)-1/8*x^(1/2)/b)/(b*x+a)^2+1/4/a/b/(a*b)^(1/2)*arctan(b*x^(1
/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.34, size = 64, normalized size = 0.88

$$\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\,(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.31, size = 186, normalized size = 2.55

$$\left[-\frac{\left(b^{2} x^{2}+2 \, a b x+a^{2}\right) \sqrt{-a b} \log \left(\frac{b x-a-2 \, \sqrt{-a b} \sqrt{x}}{b x+a}\right)-2 \, \left(a b^{2} x-a^{2} b\right) \sqrt{x}}{8 \, \left(a^{2} b^{4} x^{2}+2 \, a^{3} b^{3} x+a^{4} b^{2}\right)}, -\frac{\left(b^{2} x^{2}+2 \, a b x+a^{2}\right) \sqrt{a b} \arctan \left(\frac{\sqrt{a b}}{b \sqrt{x}}\right)-\left(a b^{2} x-a^{2} b\right) \sqrt{x}}{4 \, \left(a^{2} b^{4} x^{2}+2 \, a^{3} b^{3} x+a^{4} b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

Sympy [A]
time = 7.17, size = 627, normalized size = 8.59

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 a^{3}} & \text{for}\: b = 0 \\ - \frac{2}{3 b^{3} x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{a^{2} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} - \frac{a^{2} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} - \frac{2 a b \sqrt{x} \sqrt{- \frac{a}{b}}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} + \frac{2 a b x \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} - \frac{2 a b x \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} + \frac{2 b^{2} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} + \frac{b^{2} x^{2} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} - \frac{b^{2} x^{2} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{- \frac{a}{b}} + 16 a^{2} b^{3} x \sqrt{- \frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(8*a
**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b))
 - a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*
sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*sqrt(x)*sqrt(-a/b)/(8*a**3*b
**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + 2*
a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sq
rt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b
)) + 2*b**2*x**(3/2)*sqrt(-a/b)/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sq
rt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + b**2*x**2*log(sqrt(x) - sqrt(-a/b))/
(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a
/b)) - b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**
2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 68, normalized size = 0.93

$$2 \left(\frac{\sqrt{x}\, x b-\sqrt{x}\, a}{8 b a \left(x b+a\right)^{2}}+\frac{\arctan \left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{4 b a \cdot 2 \sqrt{a b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x)
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

Mupad [B]
time = 0.13, size = 56, normalized size = 0.77

$$\frac{\frac{x^{3/2}}{4\,a}-\frac{\sqrt{x}}{4\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^3,x)
```

```
[Out] (x^(3/2)/(4*a) - x^(1/2)/(4*b))/(a^2 + b^2*x^2 + 2*a*b*x) + atan((b^(1/2)*x
^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2))
```

3.466 $\int \frac{1}{\sqrt{x}\,(a+bx)^3}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

```
[Out] 3/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/2*x^(1/2)/a/(b*x+a)^2
+3/4*x^(1/2)/a^2/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 211}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\,(a+bx)^3}\,dx &= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx}{4a}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.84

$$
\frac{\sqrt{x}\,(5a+3bx)}{4a^2(a+bx)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (Sqrt[x]*(5*a + 3*b*x))/(4*a^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 12.60, size = 614, normalized size = 8.77

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{5b^3x^{\frac{5}{2}}},a==0\right\},\left\{\frac{2\sqrt{x}}{a^3},b==0\right\}\right\},\frac{-3a^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{3a^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{10ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}-\frac{6abx\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{6abx\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{6b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}-\frac{3b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{3b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a + b*x)^3),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 2)], a == 0 && b == 0}, {-2 / (5
b ^ 3 x ^ (5 / 2)), a == 0}, {2 Sqrt[x] / a ^ 3, b == 0}}, -3 a ^ 2 Log[Sqr
t[x] + Sqrt[-a / b]] / (8 a ^ 4 b Sqrt[-a / b] + 16 a ^ 3 b ^ 2 x Sqrt[-a /
 b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[-a / b]) + 3 a ^ 2 Log[Sqrt[x] - Sqrt[-a / b
]] / (8 a ^ 4 b Sqrt[-a / b] + 16 a ^ 3 b ^ 2 x Sqrt[-a / b] + 8 a ^ 2 b ^
3 x ^ 2 Sqrt[-a / b]) + 10 a b Sqrt[x] Sqrt[-a / b] / (8 a ^ 4 b Sqrt[-a /
```

```
b] + 16 a ^ 3 b ^ 2 x Sqrt[-a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[-a / b]) - 6
a b x Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 4 b Sqrt[-a / b] + 16 a ^ 3 b ^
2 x Sqrt[-a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[-a / b]) + 6 a b x Log[Sqrt[x]
- Sqrt[-a / b]] / (8 a ^ 4 b Sqrt[-a / b] + 16 a ^ 3 b ^ 2 x Sqrt[-a / b] +
 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[-a / b]) + 6 b ^ 2 x ^ (3 / 2) Sqrt[-a / b] / (8
a ^ 4 b Sqrt[-a / b] + 16 a ^ 3 b ^ 2 x Sqrt[-a / b] + 8 a ^ 2 b ^ 3 x ^ 2
Sqrt[-a / b]) - 3 b ^ 2 x ^ 2 Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 4 b Sqrt
[-a / b] + 16 a ^ 3 b ^ 2 x Sqrt[-a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[-a / b]
) + 3 b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 4 b Sqrt[-a / b] + 1
6 a ^ 3 b ^ 2 x Sqrt[-a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[-a / b])]
```

Maple [A]
time = 0.11, size = 59, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2a(bx+a)^{2}}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59
default	$\frac{\sqrt{x}}{2a(bx+a)^{2}}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)/a/(b*x+a)^2+3/2/a*(1/2*x^(1/2)/a/(b*x+a)+1/2/a/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.33, size = 60, normalized size = 0.86

$$\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.32, size = 186, normalized size = 2.66

$$\left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\,(3\,ab^2x+5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2+2\,a^4b^2x+a^5b)},-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(3\,ab^2x+5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="fricas")

[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]

Sympy [A]
time = 11.76, size = 632, normalized size = 9.03

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^3} & \text{for}\: b = 0 \\ -\frac{2}{5b^3x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{10ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{6abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} - \frac{6abx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{6b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} + \frac{3b^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} - \frac{3b^2x^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**3/x**(1/2),x)

[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0)),
(-2/(5*b**3*x**(5/2)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a**4
*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) -
3*a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt
(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 10*a*b*sqrt(x)*sqrt(-a/b)/(8*a**4*b
*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 6*
a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(
-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b
)) + 6*b**2*x**(3/2)*sqrt(-a/b)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(
-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b)
)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(
-a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a*
*3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)), True))

Giac [A]
time = 0.00, size = 70, normalized size = 1.00

$$2\left(\frac{3\sqrt{x}\,xb+5\sqrt{x}\,a}{8a^2\left(xb+a\right)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^2\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^3/x^(1/2),x)

[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)

Mupad [B]
time = 0.13, size = 57, normalized size = 0.81

$$\frac{\frac{5\sqrt{x}}{4a}+\frac{3bx^{3/2}}{4a^2}}{a^2+2abx+b^2x^2}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^3),x)
```

```
[Out] ((5*x^(1/2))/(4*a) + (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (3*
atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

3.467 $\int \frac{1}{x^{3/2}(a+bx)^3} dx$

Optimal. Leaf size=82

$$-\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)-15/4/a^3/x^(1/2)+1/2/
a/(b*x+a)^2/x^(1/2)+5/4/a^2/(b*x+a)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {44, 53, 65, 211}

$$-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15}{4a^3\sqrt{x}}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] -15/(4*a^3*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x)^2) + 5/(4*a^2*Sqrt[x]*(a + b
*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}(a+bx)^3}\,dx &= \frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5\int \frac{1}{x^{3/2}(a+bx)^2}\,dx}{4a}\\
&= \frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{15\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^2}\\
&= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{(15b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^3}\\
&= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{(15b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^3}\\
&= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 0.85

$$
\frac{-8a^2-25abx-15b^2x^2}{4a^3\sqrt{x}\,(a+bx)^2}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] (-8*a^2 - 25*a*b*x - 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a + b*x)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 31.46, size = 706, normalized size = 8.61

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{7b^3x^{\frac{7}{2}}},a==0\right\},\left\{\frac{-2}{a^3\sqrt{x}},b==0\right\}\right\},\frac{-16a^2\sqrt{-\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{15a^2\sqrt{x}\,\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{15a^2\sqrt{x}\,\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{50abx\sqrt{-\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{30abx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{30abx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{30b^2x^2\sqrt{-\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}-\frac{15b^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}+\frac{15b^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{-\frac{a}{b}}+16a^4bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a + b*x)^3),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 2)], a == 0 && b == 0}, {-2 / (7
b ^ 3 x ^ (7 / 2)), a == 0}, {-2 / (a ^ 3 Sqrt[x]), b == 0}}, -16 a ^ 2 Sqr
t[-a / b] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a ^ 4 b x ^ (3 / 2) Sqrt[-a
/ b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) - 15 a ^ 2 Sqrt[x] Log[Sqrt[
x] - Sqrt[-a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a ^ 4 b x ^ (3 / 2)
 Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) + 15 a ^ 2 Sqrt[x]
Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a ^ 4 b x
^ (3 / 2) Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) - 50 a b x
 Sqrt[-a / b] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a ^ 4 b x ^ (3 / 2) Sqrt
[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) - 30 a b x ^ (3 / 2) Log
[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a ^ 4 b x ^ (
3 / 2) Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) + 30 a b x ^
(3 / 2) Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a
^ 4 b x ^ (3 / 2) Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) -
30 b ^ 2 x ^ 2 Sqrt[-a / b] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] + 16 a ^ 4 b x
^ (3 / 2) Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b]) - 15 b ^ 2
 x ^ (5 / 2) Log[Sqrt[x] - Sqrt[-a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[-a / b] +
16 a ^ 4 b x ^ (3 / 2) Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[-a / b
]) + 15 b ^ 2 x ^ (5 / 2) Log[Sqrt[x] + Sqrt[-a / b]] / (8 a ^ 5 Sqrt[x] Sq
rt[-a / b] + 16 a ^ 4 b x ^ (3 / 2) Sqrt[-a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2
) Sqrt[-a / b])]
```

Maple [A]
time = 0.12, size = 56, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{2}{a^{3}\sqrt{x}}-\frac{2b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	56
default	$-\frac{2}{a^{3}\sqrt{x}}-\frac{2b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	56
risch	$-\frac{2}{a^{3}\sqrt{x}}-\frac{7b^{2}x^{\frac{3}{2}}}{4a^{3}(bx+a)^{2}}-\frac{9b\sqrt{x}}{4a^{2}(bx+a)^{2}}-\frac{15b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^{3}\sqrt{ab}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3/x^(1/2)-2/a^3*b*((7/8*b*x^(3/2)+9/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)^
(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.35, size = 73, normalized size = 0.89

$$-\frac{15\,b^{2}x^{2}+25\,abx+8\,a^{2}}{4\,\left(a^{3}b^{2}x^{\frac{5}{2}}+2\,a^{4}bx^{\frac{3}{2}}+a^{5}\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.32, size = 214, normalized size = 2.61

$$\left[\frac{15\,\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,\left(15\,b^{2}x^{2}+25\,abx+8\,a^{2}\right)\sqrt{x}}{8\,\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)},\ \frac{15\,\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^{2}x^{2}+25\,abx+8\,a^{2}\right)\sqrt{x}}{4\,\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [A]
time = 31.39, size = 779, normalized size = 9.50

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{a^{3}\sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{7b^{3}x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ - \frac{15a^{2}\sqrt{x}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{15a^{2}\sqrt{x}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{16a^{2}\sqrt{-\frac{a}{b}}}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{50abx\sqrt{-\frac{a}{b}}}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{15b^{2}x^{\frac{5}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{15b^{2}x^{\frac{5}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{30b^{2}x^{2}\sqrt{-\frac{a}{b}}}{8a^{5}\sqrt{x}\sqrt{-\frac{a}{b}}+16a^{4}bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+8a^{3}b^{2}x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**3*sqrt(x)), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-15*a**2*sqrt(x)*log(sqrt(x) - sqrt(-
```

```
a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b
**2*x**(5/2)*sqrt(-a/b)) + 15*a**2*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(8*a**
5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)
*sqrt(-a/b)) - 16*a**2*sqrt(-a/b)/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x*
*(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 30*a*b*x**(3/2)*log(
sqrt(x) - sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(
-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) + 30*a*b*x**(3/2)*log(sqrt(x) + sq
rt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a*
*3*b**2*x**(5/2)*sqrt(-a/b)) - 50*a*b*x*sqrt(-a/b)/(8*a**5*sqrt(x)*sqrt(-a/
b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 15*
b**2*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**
4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) + 15*b**2*x**(5/
2)*log(sqrt(x) + sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2
)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 30*b**2*x**2*sqrt(-a/b)/(
8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**
(5/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.09

$$2\left(-\frac{1}{a^3\sqrt{x}}+\frac{-7\sqrt{x}\,xb^2-9\sqrt{x}\,ba}{8a^3\,(xb+a)^2}-\frac{15b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^3\cdot 2\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x)
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

Mupad [B]
time = 0.15, size = 70, normalized size = 0.85

$$-\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}+\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}+2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^3),x)
```

```
[Out] - (2/a + (15*b^2*x^2)/(4*a^3) + (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2
) + 2*a*b*x^(3/2)) - (15*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2
))
```

3.468 $\int \frac{1}{x^{5/2}(a+bx)^3} dx$

Optimal. Leaf size=95

$$-\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] -35/12/a^3/x^(3/2)+1/2/a/x^(3/2)/(b*x+a)^2+7/4/a^2/x^(3/2)/(b*x+a)+35/4*b^(
3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)+35/4*b/a^4/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] -35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) + 1/(2*a*x^(3/2)*(a + b*x)^2)
 + 7/(4*a^2*x^(3/2)*(a + b*x)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)^3}\,dx &= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}\,dx}{4a}\\
&= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35\int \frac{1}{x^{5/2}(a+bx)}\,dx}{8a^2}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}-\frac{(35b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^3}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{(35b^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^4}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{(35b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,d\right.}{4a^4}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right.}{4a^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.85

$$\frac{-8a^3+56a^2bx+175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a+bx)^2}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] (-8*a^3 + 56*a^2*b*x + 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a + b*
x)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 43.45, size = 786, normalized size = 8.27

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{9}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{3a^3x^{\frac{3}{2}}}, b==0\right\}, \left\{\frac{-2}{9b^3x^{\frac{9}{2}}}, a==0\right\}\right\}, \frac{-16a^3\sqrt{-\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} + \frac{112a^2bx\sqrt{-\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} - \frac{105a^2bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} + \frac{105a^2bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} + \frac{350ab^2x^2\sqrt{-\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} - \frac{210ab^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} + \frac{210ab^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} + \frac{210b^3x^3\sqrt{-\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} - \frac{105b^3x^{\frac{7}{2}}\text{Log}\left[\sqrt{x}+\sqrt{-\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}} + \frac{105b^3x^{\frac{7}{2}}\text{Log}\left[\sqrt{x}-\sqrt{-\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+48a^5bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(a + b*x)^3),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (9 / 2)], a == 0 && b == 0}, {-2 / (3
a ^ 3 x ^ (3 / 2)), b == 0}, {-2 / (9 b ^ 3 x ^ (9 / 2)), a == 0}}, -16 a ^
 3 Sqrt[-a / b] / (24 a ^ 6 x ^ (3 / 2) Sqrt[-a / b] + 48 a ^ 5 b x ^ (5 /
2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b]) + 112 a ^ 2 b x
Sqrt[-a / b] / (24 a ^ 6 x ^ (3 / 2) Sqrt[-a / b] + 48 a ^ 5 b x ^ (5 / 2)
Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b]) - 105 a ^ 2 b x ^ (
3 / 2) Log[Sqrt[x] + Sqrt[-a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[-a / b] + 4
8 a ^ 5 b x ^ (5 / 2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b
]) + 105 a ^ 2 b x ^ (3 / 2) Log[Sqrt[x] - Sqrt[-a / b]] / (24 a ^ 6 x ^ (3
 / 2) Sqrt[-a / b] + 48 a ^ 5 b x ^ (5 / 2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x
 ^ (7 / 2) Sqrt[-a / b]) + 350 a b ^ 2 x ^ 2 Sqrt[-a / b] / (24 a ^ 6 x ^ (
3 / 2) Sqrt[-a / b] + 48 a ^ 5 b x ^ (5 / 2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2
x ^ (7 / 2) Sqrt[-a / b]) - 210 a b ^ 2 x ^ (5 / 2) Log[Sqrt[x] + Sqrt[-a /
 b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[-a / b] + 48 a ^ 5 b x ^ (5 / 2) Sqrt[-a
/ b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b]) + 210 a b ^ 2 x ^ (5 / 2) L
og[Sqrt[x] - Sqrt[-a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[-a / b] + 48 a ^ 5
b x ^ (5 / 2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b]) + 210
 b ^ 3 x ^ 3 Sqrt[-a / b] / (24 a ^ 6 x ^ (3 / 2) Sqrt[-a / b] + 48 a ^ 5 b
 x ^ (5 / 2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b]) - 105
b ^ 3 x ^ (7 / 2) Log[Sqrt[x] + Sqrt[-a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[
-a / b] + 48 a ^ 5 b x ^ (5 / 2) Sqrt[-a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2)
Sqrt[-a / b]) + 105 b ^ 3 x ^ (7 / 2) Log[Sqrt[x] - Sqrt[-a / b]] / (24 a ^
 6 x ^ (3 / 2) Sqrt[-a / b] + 48 a ^ 5 b x ^ (5 / 2) Sqrt[-a / b] + 24 a ^
4 b ^ 2 x ^ (7 / 2) Sqrt[-a / b])]
```

Maple [A]
time = 0.13, size = 67, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(bx+a)^2}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	67

default	$-\frac{2}{3a^{3}x^{\frac{3}{2}}}+\frac{6b}{a^{4}\sqrt{x}}+\frac{2b^{2}\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	67
risch	$-\frac{2(-9bx+a)}{3a^{4}x^{\frac{3}{2}}}+\frac{11b^{3}x^{\frac{3}{2}}}{4a^{4}(bx+a)^{2}}+\frac{13b^{2}\sqrt{x}}{4a^{3}(bx+a)^{2}}+\frac{35b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^{4}\sqrt{ab}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)+2/a^4*b^2*((11/8*b*x^(3/2)+13/8*a*x^(1/2))
/(b*x+a)^2+35/8/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.36, size = 86, normalized size = 0.91

$$\frac{105\,b^{3}x^{3}+175\,ab^{2}x^{2}+56\,a^{2}bx-8\,a^{3}}{12\left(a^{4}b^{2}x^{\frac{7}{2}}+2\,a^{5}bx^{\frac{5}{2}}+a^{6}x^{\frac{3}{2}}\right)}+\frac{35\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

Fricas [A]
time = 0.32, size = 250, normalized size = 2.63

$$\left[\frac{105\,(b^{3}x^{4}+2\,ab^{2}x^{3}+a^{2}bx^{2})\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(105\,b^{3}x^{3}+175\,ab^{2}x^{2}+56\,a^{2}bx-8\,a^{3})\sqrt{x}}{24\,(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2})},-\frac{105\,(b^{3}x^{4}+2\,ab^{2}x^{3}+a^{2}bx^{2})\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(105\,b^{3}x^{3}+175\,ab^{2}x^{2}+56\,a^{2}bx-8\,a^{3})\sqrt{x}}{12\,(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))
```

```
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [A]
time = 81.48, size = 869, normalized size = 9.15

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 a^{3} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{16 a^{3} \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} - \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{112 a^{2} b x \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} - \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{350 a b^{2} x^{2} \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} - \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{210 b^{3} x^{3} \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**3*x**(3/2)), Eq(b,
 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-16*a**3*sqrt(-a/b)/(24*a**6*x**(3
/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt
(-a/b)) + 105*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/2)*s
qrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b
)) - 105*a**2*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-
a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) +
112*a**2*b*x*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*s
qrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 210*a*b**2*x**(5/2)*log(sqr
t(x) - sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-
a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) - 210*a*b**2*x**(5/2)*log(sqrt(x)
+ sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b)
+ 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 350*a*b**2*x**2*sqrt(-a/b)/(24*a**6*x
**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*
sqrt(-a/b)) + 105*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/2)
*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a
/b)) - 105*b**3*x**(7/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-
a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) +
210*b**3*x**3*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*
sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 0.00, size = 100, normalized size = 1.05

$$2 \left(\frac{11 \sqrt{x}\, x b^{3} + 13 \sqrt{x}\, b^{2} a}{8 a^{4} \left(x b + a\right)^{2}} + \frac{9 x b - a}{3 a^{4} \sqrt{x}\, x} + \frac{35 b^{2} \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{4 a^{4} \cdot 2 \sqrt{a b}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x)
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

Mupad [B]
time = 0.16, size = 80, normalized size = 0.84

$$\frac{\frac{175\,b^2\,x^2}{12\,a^3}-\frac{2}{3\,a}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}+2\,a\,b\,x^{5/2}}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^3),x)
```

```
[Out] ((175*b^2*x^2)/(12*a^3) - 2/(3*a) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) + 2*a*b*x^(5/2)) + (35*b^(3/2)*atan((b^(1/2)*x
^(1/2))/a^(1/2)))/(4*a^(9/2))
```

3.469 $\int \frac{x^{5/2}}{-a+bx}\,dx$

Optimal. Leaf size=68

$$\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] 2/3*a*x^(3/2)/b^2+2/5*x^(5/2)/b-2*a^(5/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/
b^(7/2)+2*a^2*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 65, 214}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 + (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{-a+bx}\,dx &= \frac{2x^{5/2}}{5b}+\frac{a\int \frac{x^{3/2}}{-a+bx}\,dx}{b}\\
&= \frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{a^2\int \frac{\sqrt{x}}{-a+bx}\,dx}{b^2}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{a^3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.90

$$
\frac{2\sqrt{x}\,(15a^2+5abx+3b^2x^2)}{15b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.62, size = 122, normalized size = 1.79

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{5}{2}}\right],a==0\&\&b==0\right\},\left\{\frac{-2x^{\frac{7}{2}}}{7a},b==0\right\},\left\{\frac{2x^{\frac{5}{2}}}{5b},a==0\right\}\right\},-\frac{a^3\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{b^4\sqrt{\frac{a}{b}}}+\frac{a^3\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{b^4\sqrt{\frac{a}{b}}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(-a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (5 / 2)], a == 0 && b == 0}, {-2 x ^ (7 /
2) / (7 a), b == 0}, {2 x ^ (5 / 2) / (5 b), a == 0}}, -a ^ 3 Log[Sqrt[x] +
 Sqrt[a / b]] / (b ^ 4 Sqrt[a / b]) + a ^ 3 Log[Sqrt[x] - Sqrt[a / b]] / (b
```

```
^ 4 Sqrt[a / b]) + 2 a ^ 2 Sqrt[x] / b ^ 3 + 2 a x ^ (3 / 2) / (3 b ^ 2) +
2 x ^ (5 / 2) / (5 b)]
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.79

method	result	size
risch	$\frac{2(3x^2b^2+5abx+15a^2)\sqrt{x}}{15b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)+1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.35, size = 70, normalized size = 1.03

$$\frac{a^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}+5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) +
2/15*(3*b^2*x^(5/2) + 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Fricas [A]
time = 0.31, size = 131, normalized size = 1.93

$$\left[\frac{15\,a^2\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(3\,b^2x^2+5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\ \frac{2\left(15\,a^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(3\,b^2x^2+5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) +
2*(3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3, 2/15*(15*a^2*sqrt(-a/b)*arct
an(b*sqrt(x)*sqrt(-a/b)/a) + (3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [A]
time = 3.00, size = 117, normalized size = 1.72

$$\begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a = 0 \\ \frac{a^3 \log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{b^4\sqrt{\frac{a}{b}}} - \frac{a^3 \log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{b^4\sqrt{\frac{a}{b}}} + \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(7/2)/(7*a), Eq(b, 0)
), (2*x**(5/2)/(5*b), Eq(a, 0)), (a**3*log(sqrt(x) - sqrt(a/b))/(b**4*sqrt(
a/b)) - a**3*log(sqrt(x) + sqrt(a/b))/(b**4*sqrt(a/b)) + 2*a**2*sqrt(x)/b**
3 + 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Giac [A]
time = 0.00, size = 90, normalized size = 1.32

$$-2\left(\frac{-\frac{1}{5}\sqrt{x}\,x^2b^4 - \frac{1}{3}\sqrt{x}\,xb^3a - \sqrt{x}\,b^2a^2}{b^5} - \frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2b^3\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x)
```

```
[Out] 2*a^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2/15*(3*b^4*x^(5/2) +
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B]
time = 0.15, size = 51, normalized size = 0.75

$$\frac{2\,x^{5/2}}{5\,b} + \frac{2\,a\,x^{3/2}}{3\,b^2} + \frac{2\,a^2\,\sqrt{x}}{b^3} + \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) + (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 + (a^(5/2)*
atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*2i)/b^(7/2)
```

3.470 $\int \frac{x^{3/2}}{-a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b-2*a^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)+2*a*x^(1/2
)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 65, 214}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{-a+bx}\,dx &= \frac{2x^{3/2}}{3b}+\frac{a\int \frac{\sqrt{x}}{-a+bx}\,dx}{b}\\
&= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{a^2\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b^2}\\
&= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.92

$$\frac{2\sqrt{x}\,(3a+bx)}{3b^2}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.23, size = 111, normalized size = 2.09

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{3}{2}}\right],a==0\&\&b==0\right\},\left\{\frac{-2x^{\frac{5}{2}}}{5a},b==0\right\},\left\{\frac{2x^{\frac{3}{2}}}{3b},a==0\right\}\right\},-\frac{a^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{b^3\sqrt{\frac{a}{b}}}+\frac{a^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{b^3\sqrt{\frac{a}{b}}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(-a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (3 / 2)], a == 0 && b == 0}, {-2 x ^ (5 /
2) / (5 a), b == 0}, {2 x ^ (3 / 2) / (3 b), a == 0}}, -a ^ 2 Log[Sqrt[x] +
 Sqrt[a / b]] / (b ^ 3 Sqrt[a / b]) + a ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (b
 ^ 3 Sqrt[a / b]) + 2 a Sqrt[x] / b ^ 2 + 2 x ^ (3 / 2) / (3 b)]
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.81

method	result	size
risch	$\frac{2(bx+3a)\sqrt{x}}{3b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	41
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/3*b*x^(3/2)+a*x^(1/2))-2*a^2/b^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*
b)^(1/2))
```

Maxima [A]
time = 0.35, size = 58, normalized size = 1.09

$$\frac{a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}+3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2) +
2/3*(b*x^(3/2) + 3*a*sqrt(x))/b^2
```

Fricas [A]
time = 0.62, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(bx+3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(bx+3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(b
*x + 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a)
+ (b*x + 3*a)*sqrt(x))/b^2]
```

Sympy [A]
time = 0.74, size = 102, normalized size = 1.92

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{5}{2}}}{5a} & \text{for } b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ \frac{a^2 \log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{b^3 \sqrt{\frac{a}{b}}} - \frac{a^2 \log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{b^3 \sqrt{\frac{a}{b}}} + \frac{2a\sqrt{x}}{b^2} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a), Eq(b, 0)
), (2*x**(3/2)/(3*b), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(a/b))/(b**3*sqrt(
a/b)) - a**2*log(sqrt(x) + sqrt(a/b))/(b**3*sqrt(a/b)) + 2*a*sqrt(x)/b**2 +
 2*x**(3/2)/(3*b), True))
```

Giac [A]
time = 0.00, size = 70, normalized size = 1.32

$$-2\left(\frac{-\frac{1}{3}\sqrt{x}\,xb^2 - \sqrt{x}\,ba}{b^3} - \frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2b^2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x)
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) + 2/3*(b^2*x^(3/2) + 3*
a*b*sqrt(x))/b^3
```

Mupad [B]
time = 0.11, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b} + \frac{2\,a\,\sqrt{x}}{b^2} - \frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) + (2*a*x^(1/2))/b^2 - (2*a^(3/2)*atanh((b^(1/2)*x^(1/2))/
a^(1/2)))/b^(5/2)
```

3.471 $\int \frac{\sqrt{x}}{-a+bx} dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)+2*x^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 65, 214}

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{2\sqrt{x}}{b} + \frac{a\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} + \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.86, size = 98, normalized size = 2.45

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\sqrt{x}\right], a==0\&\&b==0\right\}, \left\{\frac{-2x^{\frac{3}{2}}}{3a}, b==0\right\}, \left\{\frac{2\sqrt{x}}{b}, a==0\right\}\right\}, -\frac{a\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{b^2\sqrt{\frac{a}{b}}} + \frac{a\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{b^2\sqrt{\frac{a}{b}}} + \frac{2\sqrt{x}}{b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(-a + b*x),x]')
[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {-2 x ^ (3 / 2) /
 (3 a), b == 0}, {2 Sqrt[x] / b, a == 0}}, -a Log[Sqrt[x] + Sqrt[a / b]] /
(b ^ 2 Sqrt[a / b]) + a Log[Sqrt[x] - Sqrt[a / b]] / (b ^ 2 Sqrt[a / b]) +
2 Sqrt[x] / b]
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.80

method	result	size

derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.37, size = 47, normalized size = 1.18

$$\frac{a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b) + 2*sq
rt(x)/b
```

Fricas [A]
time = 0.31, size = 83, normalized size = 2.08

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\sqrt{x}}{b},\ \frac{2\left(\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*sqrt(x))/b
, 2*(sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + sqrt(x))/b]
```

Sympy [A]
time = 0.37, size = 83, normalized size = 2.08

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^2\sqrt{\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^2\sqrt{\frac{a}{b}}} + \frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(3/2)/(3*a), Eq(b, 0))
, (2*sqrt(x)/b, Eq(a, 0)), (a*log(sqrt(x) - sqrt(a/b))/(b**2*sqrt(a/b)) - a
*log(sqrt(x) + sqrt(a/b))/(b**2*sqrt(a/b)) + 2*sqrt(x)/b, True))
```

Giac [A]
time = 0.00, size = 47, normalized size = 1.18

$$-2\left(-\frac{\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{b\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x)
```

```
[Out] 2*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) + 2*sqrt(x)/b
```

Mupad [B]
time = 0.11, size = 28, normalized size = 0.70

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\ \mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

3.472 $\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

[Out] -2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(-a + b*x)),x]

[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx &= 2\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.97, size = 88, normalized size = 3.03

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{\sqrt{x}}\right],a{=}{=}0\&\&b{=}{=}0\right\},\left\{\frac{-2}{b\sqrt{x}},a{=}{=}0\right\},\left\{\frac{-2\sqrt{x}}{a},b{=}{=}0\right\}\right\},-\frac{\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{b\sqrt{\frac{a}{b}}}+\frac{\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{b\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(-a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[x]], a == 0 && b == 0}, {-2 / (b Sqrt
[x]), a == 0}, {-2 Sqrt[x] / a, b == 0}}, -Log[Sqrt[x] + Sqrt[a / b]] / (b
Sqrt[a / b]) + Log[Sqrt[x] - Sqrt[a / b]] / (b Sqrt[a / b])]
```

Maple [A]
time = 0.10, size = 19, normalized size = 0.66

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.37, size = 34, normalized size = 1.17

$$\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/sqrt(a*b)
```

Fricas [A]
time = 0.31, size = 67, normalized size = 2.31

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{ab}, \frac{2\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a))/(a*b), 2*sqrt(-a*
b)*arctan(sqrt(-a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [A]
time = 0.47, size = 68, normalized size = 2.34

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\ a = 0 \\ -\frac{2\sqrt{x}}{a} & \text{for}\ b = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{b\sqrt{\frac{a}{b}}} - \frac{\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{b\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
-2*sqrt(x)/a, Eq(b, 0)), (log(sqrt(x) - sqrt(a/b))/(b*sqrt(a/b)) - log(sqrt
(x) + sqrt(a/b))/(b*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 28, normalized size = 0.97

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x)
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B]
time = 0.13, size = 19, normalized size = 0.66

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)),x)
```

```
[Out] -(2*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.473 $\int \frac{1}{x^{3/2}(-a+bx)}\,dx$

Optimal. Leaf size=40

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] -2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)+2/a/x^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 65, 214}

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(-a + b*x)),x]

[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\sqrt{x}} + \frac{b\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a}$$
$$= \frac{2}{a\sqrt{x}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)),x]
```

```
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.45, size = 96, normalized size = 2.40

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{2}{a\sqrt{x}}, b==0\right\}, \left\{\frac{-2}{3bx^{\frac{3}{2}}}, a==0\right\}\right\}, -\frac{\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{a\sqrt{\frac{a}{b}}} + \frac{\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{a\sqrt{\frac{a}{b}}} + \frac{2}{a\sqrt{x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(-a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (3 / 2)], a == 0 && b == 0}, {2 / (a S
qrt[x]), b == 0}, {-2 / (3 b x ^ (3 / 2)), a == 0}}, -Log[Sqrt[x] + Sqrt[a
/ b]] / (a Sqrt[a / b]) + Log[Sqrt[x] - Sqrt[a / b]] / (a Sqrt[a / b]) + 2
/ (a Sqrt[x])]
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.80

method	result	size

derivativedivides	$\frac{2}{a\sqrt{x}}-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
default	$\frac{2}{a\sqrt{x}}-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
risch	$\frac{2}{a\sqrt{x}}-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a),x,method=_RETURNVERBOSE)
[Out] 2/a/x^(1/2)-2*b/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.37, size = 47, normalized size = 1.18

$$\frac{b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="maxima")
[Out] b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a) + 2/(a
*sqrt(x))
```

Fricas [A]
time = 0.31, size = 91, normalized size = 2.28

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\sqrt{x}}{ax},\ \frac{2\left(x\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*sqrt(x))
/(a*x), 2*(x*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + sqrt(x))/(a*x)]
```

Sympy [A]
time = 1.03, size = 76, normalized size = 1.90

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 b x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{2}{a \sqrt{x}} & \text{for}\: b = 0 \\ \frac{\log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{a \sqrt{\frac{a}{b}}} - \frac{\log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{a \sqrt{\frac{a}{b}}} + \frac{2}{a \sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (2/(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(a/b))/(a*sqrt(a/b)) - log
(sqrt(x) + sqrt(a/b))/(a*sqrt(a/b)) + 2/(a*sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 48, normalized size = 1.20

$$-2\left(-\frac{1}{a\sqrt{x}}-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{a\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x)
```

```
[Out] 2*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) + 2/(a*sqrt(x))
```

Mupad [B]
time = 0.06, size = 28, normalized size = 0.70

$$\frac{2}{a\,\sqrt{x}}-\frac{2\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)),x)
```

```
[Out] 2/(a*x^(1/2)) - (2*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

3.474 $\int \frac{1}{x^{5/2}(-a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 2/3/a/x^(3/2)-2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)+2*b/a^2/x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 65, 214}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] 2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(-a+bx)}\,dx &= \frac{2}{3ax^{3/2}}+\frac{b\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a^2}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.91

$$\frac{2(a+3bx)}{3a^2x^{3/2}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)),x]
[Out] (2*(a + 3*b*x))/(3*a^2*x^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.60, size = 107, normalized size = 2.02

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{5bx^{\frac{5}{2}}},a==0\right\},\left\{\frac{2}{3ax^{\frac{3}{2}}},b==0\right\}\right\},\frac{2}{3ax^{\frac{3}{2}}}-\frac{b\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{a^2\sqrt{\frac{a}{b}}}+\frac{b\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{a^2\sqrt{\frac{a}{b}}}+\frac{2b}{a^2\sqrt{x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(-a + b*x)),x]')
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 2)], a == 0 && b == 0}, {-2 / (5
b x ^ (5 / 2)), a == 0}, {2 / (3 a x ^ (3 / 2)), b == 0}}, 2 / (3 a x ^ (3
```

```
/ 2)) - b Log[Sqrt[x] + Sqrt[a / b]] / (a ^ 2 Sqrt[a / b]) + b Log[Sqrt[x]
- Sqrt[a / b]] / (a ^ 2 Sqrt[a / b]) + 2 b / (a ^ 2 Sqrt[x])]
```

Maple [A]
time = 0.10, size = 43, normalized size = 0.81

method	result	size
risch	$\frac{2bx+\frac{2a}{3}}{a^2x^{\frac{3}{2}}}-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	40
derivativedivides	$-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}+\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}$	43
default	$-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}+\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2/a^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))+2/3/a/x^(3/2)+2*b/a^2
/x^(1/2)
```

Maxima [A]
time = 0.36, size = 55, normalized size = 1.04

$$\frac{b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2) +
2/3*(3*b*x + a)/(a^2*x^(3/2))
```

Fricas [A]
time = 0.68, size = 113, normalized size = 2.13

$$\left[\frac{3\,bx^2\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(3\,bx+a)\sqrt{x}}{3\,a^2x^2},\frac{2\left(3\,bx^2\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx+a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) +
2*(3*b*x + a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(-b/a)*arctan(a*sqrt(-b/
a)/(b*sqrt(x))) + (3*b*x + a)*sqrt(x))/(a^2*x^2)]
```

Sympy [A]
time = 3.49, size = 99, normalized size = 1.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} + \frac{b\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{a^2\sqrt{\frac{a}{b}}} - \frac{b\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{a^2\sqrt{\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (2/(3*a*x**(3/2)), Eq(b, 0)), (2/(3*a*x**(3/2)) + b*log(sqrt(x) - sqrt(a
/b))/(a**2*sqrt(a/b)) - b*log(sqrt(x) + sqrt(a/b))/(a**2*sqrt(a/b)) + 2*b/(
a**2*sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.23

$$-2\left(\frac{-3xb-a}{3a^2\sqrt{x}\,x} - \frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2a^2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x)
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 2/3*(3*b*x + a)/(a^2*
x^(3/2))
```

Mupad [B]
time = 0.12, size = 37, normalized size = 0.70

$$\frac{\frac{2}{3\,a}+\frac{2\,b\,x}{a^2}}{x^{3/2}}-\frac{2\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)),x)
```

```
[Out] (2/(3*a) + (2*b*x)/a^2)/x^(3/2) - (2*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2
)))/a^(5/2)
```

3.475 $\int \frac{1}{x^{7/2}(-a+bx)}\,dx$

Optimal. Leaf size=68

$$\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)-2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/
a^(7/2)+2*b^2/a^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 65, 214}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] 2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) + (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}(-a+bx)}\,dx &= \frac{2}{5ax^{5/2}}+\frac{b\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{a}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{b^2\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a^2}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{b^3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a^3}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{(2b^3)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.90

$$\frac{2\left(3a^2+5abx+15b^2x^2\right)}{15a^3x^{5/2}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] (2*(3*a^2 + 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.37, size = 122, normalized size = 1.79

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{7}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{7bx^{\frac{7}{2}}},a==0\right\},\left\{\frac{2}{5ax^{\frac{5}{2}}},b==0\right\}\right\},\frac{2}{5ax^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{b^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{a^3\sqrt{\frac{a}{b}}}+\frac{b^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{a^3\sqrt{\frac{a}{b}}}+\frac{2b^2}{a^3\sqrt{x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(7/2)*(-a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 2)], a == 0 && b == 0}, {-2 / (7
b x ^ (7 / 2)), a == 0}, {2 / (5 a x ^ (5 / 2)), b == 0}}, 2 / (5 a x ^ (5
/ 2)) + 2 b / (3 a ^ 2 x ^ (3 / 2)) - b ^ 2 Log[Sqrt[x] + Sqrt[a / b]] / (a
 ^ 3 Sqrt[a / b]) + b ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (a ^ 3 Sqrt[a / b])
+ 2 b ^ 2 / (a ^ 3 Sqrt[x])]
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.79

method	result	size
risch	$\frac{2x^2b^2+\frac{2}{3}abx+\frac{2}{5}a^2}{a^3x^{\frac{5}{2}}}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	53
derivativedivides	$-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}+\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}$	54
default	$-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}+\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^3/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))+2/5/a/x^(5/2)+2/3*b/a
^2/x^(3/2)+2*b^2/a^3/x^(1/2)
```

Maxima [A]
time = 0.37, size = 68, normalized size = 1.00

$$\frac{b^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{2\,(15\,b^2x^2+5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3) +
2/15*(15*b^2*x^2 + 5*a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Fricas [A]
time = 0.31, size = 143, normalized size = 2.10

$$\left[\frac{15\,b^2x^3\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(15\,b^2x^2+5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\ \frac{2\left(15\,b^2x^3\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(15\,b^2x^2+5\,abx+3\,a^2)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)
) + 2*(15*b^2*x^2 + 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3*s
qrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 + 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [A]
time = 14.58, size = 117, normalized size = 1.72

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} + \frac{b^2 \log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{a^3\sqrt{\frac{a}{b}}} - \frac{b^2 \log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{a^3\sqrt{\frac{a}{b}}} + \frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (2/(5*a*x**(5/2)), Eq(b, 0)), (2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2))
+ b**2*log(sqrt(x) - sqrt(a/b))/(a**3*sqrt(a/b)) - b**2*log(sqrt(x) + sqrt(
a/b))/(a**3*sqrt(a/b)) + 2*b**2/(a**3*sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 81, normalized size = 1.19

$$-2\left(\frac{-15x^2b^2 - 5xba - 3a^2}{15a^3\sqrt{x}\,x^2} - \frac{2b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2a^3\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x)
```

```
[Out] 2*b^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/15*(15*b^2*x^2 + 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B]
time = 0.13, size = 48, normalized size = 0.71

$$\frac{\frac{2}{5\,a} + \frac{2\,b^2\,x^2}{a^3} + \frac{2\,b\,x}{3\,a^2}}{x^{5/2}} - \frac{2\,b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(7/2)*(a - b*x)),x)
```

```
[Out] (2/(5*a) + (2*b^2*x^2)/a^3 + (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atanh((b
^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

3.476 $\int \frac{x^{5/2}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] 5/3*x^(3/2)/b^2+x^(5/2)/b/(-b*x+a)-5*a^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2
))/b^(7/2)+5*a*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) + x^(5/2)/(b*(a - b*x)) - (5*a^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^2}\,dx &= \frac{x^{5/2}}{b(a-bx)}+\frac{5\int \frac{x^{3/2}}{-a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b^2}\\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b^3}\\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 70, normalized size = 1.00

$$\frac{\sqrt{x}\,\left(-15a^2+10abx+2b^2x^2\right)}{3b^3(-a+bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^2,x]
[Out] (Sqrt[x]*(-15*a^2 + 10*a*b*x + 2*b^2*x^2))/(3*b^3*(-a + b*x)) - (5*a^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.18, size = 365, normalized size = 5.21

$$\mathrm{Piecewise}\left[\left\{\left\{\mathrm{DirectedInfinity}\left[x^{\frac{3}{2}}\right],a==0\&\&b==0\right\},\left\{\frac{2x^{\frac{3}{2}}}{3b^2},a==0\right\},\left\{\frac{2x^{\frac{7}{2}}}{7a^2},b==0\right\}\right\},\frac{-15a^3\mathrm{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}+\frac{15a^3\mathrm{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}+\frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}-\frac{15a^2bx\mathrm{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}+\frac{15a^2bx\mathrm{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}-\frac{20ab^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}-\frac{4b^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{6ab^4\sqrt{\frac{a}{b}}-6b^5x\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(-a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (3 / 2)], a == 0 && b == 0}, {2 x ^ (3 / 2
) / (3 b ^ 2), a == 0}, {2 x ^ (7 / 2) / (7 a ^ 2), b == 0}}, -15 a ^ 3 Log
[Sqrt[x] + Sqrt[a / b]] / (6 a b ^ 4 Sqrt[a / b] - 6 b ^ 5 x Sqrt[a / b]) +
 15 a ^ 3 Log[Sqrt[x] - Sqrt[a / b]] / (6 a b ^ 4 Sqrt[a / b] - 6 b ^ 5 x S
qrt[a / b]) + 30 a ^ 2 b Sqrt[x] Sqrt[a / b] / (6 a b ^ 4 Sqrt[a / b] - 6 b
 ^ 5 x Sqrt[a / b]) - 15 a ^ 2 b x Log[Sqrt[x] - Sqrt[a / b]] / (6 a b ^ 4
Sqrt[a / b] - 6 b ^ 5 x Sqrt[a / b]) + 15 a ^ 2 b x Log[Sqrt[x] + Sqrt[a /
b]] / (6 a b ^ 4 Sqrt[a / b] - 6 b ^ 5 x Sqrt[a / b]) - 20 a b ^ 2 x ^ (3 /
 2) Sqrt[a / b] / (6 a b ^ 4 Sqrt[a / b] - 6 b ^ 5 x Sqrt[a / b]) - 4 b ^ 3
 x ^ (5 / 2) Sqrt[a / b] / (6 a b ^ 4 Sqrt[a / b] - 6 b ^ 5 x Sqrt[a / b])]
```

Maple [A]
time = 0.12, size = 60, normalized size = 0.86

method	result	size
risch	$\frac{2(bx+6a)\sqrt{x}}{3b^{3}}+\frac{a^{2}\left(-\frac{\sqrt{x}}{bx-a}-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^{3}}$	57
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^{3}}-\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	60
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^{3}}-\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/3*b*x^(3/2)+2*a*x^(1/2))-2/b^3*a^2*(-1/2*x^(1/2)/(-b*x+a)+5/2/(a*b
)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.36, size = 81, normalized size = 1.16

$$-\frac{a^{2}\sqrt{x}}{b^{4}x-ab^{3}}+\frac{5\,a^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}+\frac{2\,\left(bx^{\frac{3}{2}}+6\,a\sqrt{x}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x - a*b^3) + 5/2*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(
x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2/3*(b*x^(3/2) + 6*a*sqrt(x))/b^3
```

Fricas [A]
time = 0.31, size = 167, normalized size = 2.39

$$\left[\frac{15\,(abx-a^2)\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,b^2x^2+10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x-ab^3)},\ \frac{15\,(abx-a^2)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+(2\,b^2x^2+10\,abx-15\,a^2)\sqrt{x}}{3\,(b^4x-ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x - a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x
 - a)) + 2*(2*b^2*x^2 + 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3), 1/3*(1
5*(a*b*x - a^2)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b^2*x^2 + 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [A]
time = 13.91, size = 354, normalized size = 5.06

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b^{2}} & \text{for}\: a = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 a^{2}} & \text{for}\: b = 0 \\ - \frac{15 a^{3} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} + \frac{15 a^{3} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} - \frac{30 a^{2} b \sqrt{x} \sqrt{\frac{a}{b}}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} + \frac{15 a^{2} b x \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} - \frac{15 a^{2} b x \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} + \frac{20 a b^{2} x^{\frac{3}{2}} \sqrt{\frac{a}{b}}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} + \frac{4 b^{3} x^{\frac{5}{2}} \sqrt{\frac{a}{b}}}{- 6 a b^{4} \sqrt{\frac{a}{b}} + 6 b^{5} x \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**2), Eq(a,
0)), (2*x**(7/2)/(7*a**2), Eq(b, 0)), (-15*a**3*log(sqrt(x) - sqrt(a/b))/(-
6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 15*a**3*log(sqrt(x) + sqrt(a/b))
/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) - 30*a**2*b*sqrt(x)*sqrt(a/b)/(
-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 15*a**2*b*x*log(sqrt(x) - sqrt(
a/b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) - 15*a**2*b*x*log(sqrt(x)
+ sqrt(a/b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 20*a*b**2*x**(3/2
)*sqrt(a/b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 4*b**3*x**(5/2)*sq
rt(a/b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 92, normalized size = 1.31

$$2\left(\frac{\frac{1}{3}\sqrt{x}\,xb^4+2\sqrt{x}\,b^3a}{b^6}-\frac{\sqrt{x}\,a^2}{2b^3\,(xb-a)}+\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2b^3\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x)
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) - a^2*sqrt(x)/((b*x - a
)*b^3) + 2/3*(b^4*x^(3/2) + 6*a*b^3*sqrt(x))/b^6
```

Mupad [B]
time = 0.07, size = 61, normalized size = 0.87

$$\frac{2\,x^{3/2}}{3\,b^2}+\frac{4\,a\,\sqrt{x}}{b^3}+\frac{a^2\,\sqrt{x}}{a\,b^3-b^4\,x}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) + (4*a*x^(1/2))/b^3 + (a^2*x^(1/2))/(a*b^3 - b^4*x) + (
a^(3/2)*atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*5i)/b^(7/2)
```

3.477 $\int \frac{x^{3/2}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] x^(3/2)/b/(-b*x+a)-3*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)+3*x^(
1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {43, 52, 65, 214}

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 + x^(3/2)/(b*(a - b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(-a+bx)^2}\,dx &= \frac{x^{3/2}}{b(a-bx)} + \frac{3\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b} \\
&= \frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} + \frac{(3a)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b^2} \\
&= \frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} + \frac{(3a)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} - \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.98

$$\frac{\sqrt{x}\,(-3a+2bx)}{b^2(-a+bx)} - \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^2,x]
[Out] (Sqrt[x]*(-3*a + 2*b*x))/(b^2*(-a + b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/b^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.68, size = 314, normalized size = 5.51

$$\text{Piecewise}\left[\left\{\{\text{DirectedInfinity}\left[\sqrt{x}\right], a==0\&\&b==0\}, \left\{\frac{2\sqrt{x}}{b^2}, a==0\right\}, \left\{\frac{2x^{\frac{5}{2}}}{5a^2}, b==0\right\}\right\}, \frac{-3a^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2ab^3\sqrt{\frac{a}{b}}-2b^4x\sqrt{\frac{a}{b}}} + \frac{3a^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2ab^3\sqrt{\frac{a}{b}}-2b^4x\sqrt{\frac{a}{b}}} + \frac{6ab\sqrt{x}\,\sqrt{\frac{a}{b}}}{2ab^3\sqrt{\frac{a}{b}}-2b^4x\sqrt{\frac{a}{b}}} - \frac{3abx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2ab^3\sqrt{\frac{a}{b}}-2b^4x\sqrt{\frac{a}{b}}} + \frac{3abx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2ab^3\sqrt{\frac{a}{b}}-2b^4x\sqrt{\frac{a}{b}}} - \frac{4b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{2ab^3\sqrt{\frac{a}{b}}-2b^4x\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(-a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {2 Sqrt[x] / b ^
2, a == 0}, {2 x ^ (5 / 2) / (5 a ^ 2), b == 0}}, -3 a ^ 2 Log[Sqrt[x] + Sq
rt[a / b]] / (2 a b ^ 3 Sqrt[a / b] - 2 b ^ 4 x Sqrt[a / b]) + 3 a ^ 2 Log[
Sqrt[x] - Sqrt[a / b]] / (2 a b ^ 3 Sqrt[a / b] - 2 b ^ 4 x Sqrt[a / b]) +
6 a b Sqrt[x] Sqrt[a / b] / (2 a b ^ 3 Sqrt[a / b] - 2 b ^ 4 x Sqrt[a / b])
 - 3 a b x Log[Sqrt[x] - Sqrt[a / b]] / (2 a b ^ 3 Sqrt[a / b] - 2 b ^ 4 x
Sqrt[a / b]) + 3 a b x Log[Sqrt[x] + Sqrt[a / b]] / (2 a b ^ 3 Sqrt[a / b]
- 2 b ^ 4 x Sqrt[a / b]) - 4 b ^ 2 x ^ (3 / 2) Sqrt[a / b] / (2 a b ^ 3 Sqr
t[a / b] - 2 b ^ 4 x Sqrt[a / b])]
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.84

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	48
default	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	48
risch	$\frac{2\sqrt{x}}{b^2}+\frac{a\left(-\frac{\sqrt{x}}{bx-a}-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^2-2*a/b^2*(-1/2*x^(1/2)/(-b*x+a)+3/2/(a*b)^(1/2)*arctanh(b*x^(1
/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.33, size = 68, normalized size = 1.19

$$-\frac{a\sqrt{x}}{b^3x-ab^2}+\frac{3\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

[Out] -a*sqrt(x)/(b^3*x - a*b^2) + 3/2*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2) + 2*sqrt(x)/b^2

Fricas [A]
time = 0.30, size = 138, normalized size = 2.42

$$\left[\frac{3\,(bx-a)\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,bx-3\,a)\sqrt{x}}{2\,(b^3x-ab^2)},\ \frac{3\,(bx-a)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(2\,bx-3\,a)\sqrt{x}}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="fricas")

[Out] [1/2*(3*(b*x - a)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2), (3*(b*x - a)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2)]

Sympy [A]
time = 3.89, size = 301, normalized size = 5.28

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\ a=0\wedge b=0 \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \text{for}\ b=0 \\ \frac{2\sqrt{x}}{b^2} & \text{for}\ a=0 \\ -\frac{3a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}}+\frac{3a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}}-\frac{6ab\sqrt{x}\sqrt{\frac{a}{b}}}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}}+\frac{3abx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}}-\frac{3abx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}}+\frac{4b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{-2ab^3\sqrt{\frac{a}{b}}+2b^4x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(b*x-a)**2,x)

[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0)), (2*sqrt(x)/b**2, Eq(a, 0)), (-3*a**2*log(sqrt(x) - sqrt(a/b))/(-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 3*a**2*log(sqrt(x) + sqrt(a/b))/(-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) - 6*a*b*sqrt(x)*sqrt(a/b)/(-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 3*a*b*x*log(sqrt(x) - sqrt(a/b))/(-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) - 3*a*b*x*log(sqrt(x) + sqrt(a/b))/(-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 4*b**2*x**(3/2)*sqrt(a/b)/(-2*a*b**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)), True))

Giac [A]
time = 0.00, size = 69, normalized size = 1.21

$$2\left(\frac{\sqrt{x}}{b^2}-\frac{\sqrt{x}\,a}{2b^2\,(xb-a)}+\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2b^2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x)
```

```
[Out] 3*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - a*sqrt(x)/((b*x - a)*b^
2) + 2*sqrt(x)/b^2
```

Mupad [B]
time = 0.11, size = 47, normalized size = 0.82

$$\frac{2\sqrt{x}}{b^2}+\frac{a\sqrt{x}}{a\,b^2-b^3\,x}-\frac{3\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 - b^3*x) - (3*a^(1/2)*atanh((b^(1/2)*x
^(1/2))/a^(1/2)))/b^(5/2)
```

3.478 $\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] -arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)+x^(1/2)/b/(-b*x+a)
```

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 65, 214}

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2)
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx &= \frac{\sqrt{x}}{b(a-bx)}+\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b}\\ &= \frac{\sqrt{x}}{b(a-bx)}+\frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}\\ &= \frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.04

$$-\frac{\sqrt{x}}{b(-a+bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(-a + b*x))) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(
3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.22, size = 261, normalized size = 5.55

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{\sqrt{x}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{b^2\sqrt{x}}, a==0\right\}, \left\{\frac{2x^{\frac{3}{2}}}{3a^2}, b==0\right\}\right\}, -\frac{a\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2ab^2\sqrt{\frac{a}{b}}-2b^3x\sqrt{\frac{a}{b}}}+\frac{a\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2ab^2\sqrt{\frac{a}{b}}-2b^3x\sqrt{\frac{a}{b}}}+\frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{2ab^2\sqrt{\frac{a}{b}}-2b^3x\sqrt{\frac{a}{b}}}-\frac{bx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2ab^2\sqrt{\frac{a}{b}}-2b^3x\sqrt{\frac{a}{b}}}+\frac{bx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2ab^2\sqrt{\frac{a}{b}}-2b^3x\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(-a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[x]], a == 0 && b == 0}, {-2 / (b ^ 2
Sqrt[x]), a == 0}, {2 x ^ (3 / 2) / (3 a ^ 2), b == 0}}, -a Log[Sqrt[x] + S
qrt[a / b]] / (2 a b ^ 2 Sqrt[a / b] - 2 b ^ 3 x Sqrt[a / b]) + a Log[Sqrt[
x] - Sqrt[a / b]] / (2 a b ^ 2 Sqrt[a / b] - 2 b ^ 3 x Sqrt[a / b]) + 2 b S
qrt[x] Sqrt[a / b] / (2 a b ^ 2 Sqrt[a / b] - 2 b ^ 3 x Sqrt[a / b]) - b x
Log[Sqrt[x] - Sqrt[a / b]] / (2 a b ^ 2 Sqrt[a / b] - 2 b ^ 3 x Sqrt[a / b]
) + b x Log[Sqrt[x] + Sqrt[a / b]] / (2 a b ^ 2 Sqrt[a / b] - 2 b ^ 3 x Sqr
t[a / b])]
```

Maple [A]
time = 0.12, size = 38, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\sqrt{x}}{b(-bx+a)}-\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	38
default	$\frac{\sqrt{x}}{b(-bx+a)}-\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/b/(-b*x+a)-1/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.40, size = 56, normalized size = 1.19

$$-\frac{\sqrt{x}}{b^2x-ab}+\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x - a*b) + 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.30, size = 123, normalized size = 2.62

$$\left[-\frac{2\,ab\sqrt{x}-\sqrt{ab}\,(bx-a)\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\,(ab^3x-a^2b^2)},-\frac{ab\sqrt{x}-\sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab^3x-a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a - 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a*b^3*x - a^2*b^2), -(a*b*sqrt(x) - sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a*b^3*x - a^2*b^2)]
```

Sympy [A]

time = 1.75, size = 243, normalized size = 5.17

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b=0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a=0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-a*log(sqrt(x) - sqrt(a/b))/(-2*a*b**2*
sqrt(a/b) + 2*b**3*x*sqrt(a/b)) + a*log(sqrt(x) + sqrt(a/b))/(-2*a*b**2*sqr
t(a/b) + 2*b**3*x*sqrt(a/b)) - 2*b*sqrt(x)*sqrt(a/b)/(-2*a*b**2*sqrt(a/b) +
 2*b**3*x*sqrt(a/b)) + b*x*log(sqrt(x) - sqrt(a/b))/(-2*a*b**2*sqrt(a/b) +
2*b**3*x*sqrt(a/b)) - b*x*log(sqrt(x) + sqrt(a/b))/(-2*a*b**2*sqrt(a/b) + 2
*b**3*x*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 52, normalized size = 1.11

$$2\left(-\frac{\sqrt{x}}{2b(xb-a)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{b\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x)
```

```
[Out] arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) - sqrt(x)/((b*x - a)*b)
```

Mupad [B]
time = 0.11, size = 35, normalized size = 0.74

$$\frac{\sqrt{x}}{b\,(a-b\,x)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^2,x)
```

```
[Out] x^(1/2)/(b*(a - b*x)) - atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2))
```

3.479 $\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{x}}{a(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[Out] arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)+x^(1/2)/a/(-b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {44, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a-bx)} - \frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} - \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [**A**]
time = 0.05, size = 46, normalized size = 1.00

$$\frac{\sqrt{x}}{a(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.19, size = 266, normalized size = 5.78

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{2\sqrt{x}}{a^2}, b==0\right\}, \left\{\frac{-2}{3b^2x^{\frac{3}{2}}}, a==0\right\}\right\}, -\frac{a\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2a^2b\sqrt{\frac{a}{b}}-2ab^2x\sqrt{\frac{a}{b}}} + \frac{a\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2a^2b\sqrt{\frac{a}{b}}-2ab^2x\sqrt{\frac{a}{b}}} + \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{2a^2b\sqrt{\frac{a}{b}}-2ab^2x\sqrt{\frac{a}{b}}} - \frac{bx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2a^2b\sqrt{\frac{a}{b}}-2ab^2x\sqrt{\frac{a}{b}}} + \frac{bx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2a^2b\sqrt{\frac{a}{b}}-2ab^2x\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (3 / 2)], a == 0 && b == 0}, {2 Sqrt[x
] / a ^ 2, b == 0}, {-2 / (3 b ^ 2 x ^ (3 / 2)), a == 0}}, -a Log[Sqrt[x] -
 Sqrt[a / b]] / (2 a ^ 2 b Sqrt[a / b] - 2 a b ^ 2 x Sqrt[a / b]) + a Log[S
qrt[x] + Sqrt[a / b]] / (2 a ^ 2 b Sqrt[a / b] - 2 a b ^ 2 x Sqrt[a / b]) +
 2 b Sqrt[x] Sqrt[a / b] / (2 a ^ 2 b Sqrt[a / b] - 2 a b ^ 2 x Sqrt[a / b]
) - b x Log[Sqrt[x] + Sqrt[a / b]] / (2 a ^ 2 b Sqrt[a / b] - 2 a b ^ 2 x S
qrt[a / b]) + b x Log[Sqrt[x] - Sqrt[a / b]] / (2 a ^ 2 b Sqrt[a / b] - 2 a
 b ^ 2 x Sqrt[a / b])]
```

Maple [**A**]
time = 0.11, size = 37, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37
default	$\frac{\sqrt{x}}{a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/a/(-b*x+a)+1/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.33, size = 56, normalized size = 1.22

$$-\frac{\sqrt{x}}{abx-a^2}-\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(a*b*x - a^2) - 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.31, size = 122, normalized size = 2.65

$$\left[-\frac{2\,ab\sqrt{x}-\sqrt{ab}\,(bx-a)\log\left(\frac{bx+a+2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\,(a^2b^2x-a^3b)},-\frac{ab\sqrt{x}+\sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{a^2b^2x-a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a + 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a^2*b^2*x - a^3*b), -(a*b*sqrt(x) + sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a^2*b^2*x - a^3*b)]
```

Sympy [A]

time = 2.74, size = 252, normalized size = 5.48

$$\begin{cases} \frac{\infty}{x^{\frac{3}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for}\ b = 0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for}\ a = 0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(a/b))/(-2*a**2*b*sq
rt(a/b) + 2*a*b**2*x*sqrt(a/b)) - a*log(sqrt(x) + sqrt(a/b))/(-2*a**2*b*sqr
t(a/b) + 2*a*b**2*x*sqrt(a/b)) - 2*b*sqrt(x)*sqrt(a/b)/(-2*a**2*b*sqrt(a/b)
 + 2*a*b**2*x*sqrt(a/b)) - b*x*log(sqrt(x) - sqrt(a/b))/(-2*a**2*b*sqrt(a/b
) + 2*a*b**2*x*sqrt(a/b)) + b*x*log(sqrt(x) + sqrt(a/b))/(-2*a**2*b*sqrt(a/
b) + 2*a*b**2*x*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 53, normalized size = 1.15

$$2\left(-\frac{\sqrt{x}}{2a(xb-a)} - \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{a\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x)
```

```
[Out] -arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) - sqrt(x)/((b*x - a)*a)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}}{a\,(a-b\,x)} + \frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^2),x)
```

```
[Out] x^(1/2)/(a*(a - b*x)) + atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

3.480 $\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 3*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)-3/a^2/x^(1/2)+1/a/(-b*x+
a)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 214}

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx &= \frac{1}{a\sqrt{x}\,(a-bx)}-\frac{3\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^2}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}-\frac{(3b)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.96

$$\frac{-2a+3bx}{a^2\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a + 3*b*x)/(a^2*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/a^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.25, size = 340, normalized size = 5.96

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right],a==0\&\&b==0\right\},\left\{\frac{-2}{a^2\sqrt{x}},b==0\right\},\left\{\frac{-2}{5b^2x^{\frac{5}{2}}},a==0\right\}\right\},\frac{-4a\sqrt{\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}-\frac{3a\sqrt{x}\,\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}+\frac{3a\sqrt{x}\,\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}+\frac{6bx\sqrt{\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}-\frac{3bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}+\frac{3bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{2a^3\sqrt{x}\sqrt{\frac{a}{b}}-2a^2bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(-a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 2)], a == 0 && b == 0}, {-2 / (a
^ 2 Sqrt[x]), b == 0}, {-2 / (5 b ^ 2 x ^ (5 / 2)), a == 0}}, -4 a Sqrt[a /
 b] / (2 a ^ 3 Sqrt[x] Sqrt[a / b] - 2 a ^ 2 b x ^ (3 / 2) Sqrt[a / b]) - 3
 a Sqrt[x] Log[Sqrt[x] - Sqrt[a / b]] / (2 a ^ 3 Sqrt[x] Sqrt[a / b] - 2 a
^ 2 b x ^ (3 / 2) Sqrt[a / b]) + 3 a Sqrt[x] Log[Sqrt[x] + Sqrt[a / b]] / (
2 a ^ 3 Sqrt[x] Sqrt[a / b] - 2 a ^ 2 b x ^ (3 / 2) Sqrt[a / b]) + 6 b x Sq
rt[a / b] / (2 a ^ 3 Sqrt[x] Sqrt[a / b] - 2 a ^ 2 b x ^ (3 / 2) Sqrt[a / b
]) - 3 b x ^ (3 / 2) Log[Sqrt[x] + Sqrt[a / b]] / (2 a ^ 3 Sqrt[x] Sqrt[a /
 b] - 2 a ^ 2 b x ^ (3 / 2) Sqrt[a / b]) + 3 b x ^ (3 / 2) Log[Sqrt[x] - Sq
rt[a / b]] / (2 a ^ 3 Sqrt[x] Sqrt[a / b] - 2 a ^ 2 b x ^ (3 / 2) Sqrt[a /
b])]
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2}{a^{2} \sqrt{x}}+\frac{2 b \left(\frac{\sqrt{x}}{-2 b x +2 a}+\frac{3 \,\mathrm{arctanh}\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{2}}$	48
default	$-\frac{2}{a^{2} \sqrt{x}}+\frac{2 b \left(\frac{\sqrt{x}}{-2 b x +2 a}+\frac{3 \,\mathrm{arctanh}\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{2}}$	48
risch	$-\frac{2}{a^{2} \sqrt{x}}-\frac{b \left(\frac{\sqrt{x}}{b x -a}-\frac{3 \,\mathrm{arctanh}\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{\sqrt{a b}}\right)}{a^{2}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2/x^(1/2)+2*b/a^2*(1/2*x^(1/2)/(-b*x+a)+3/2/(a*b)^(1/2)*arctanh(b*x^(1
/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.36, size = 69, normalized size = 1.21

$$-\frac{3 \, b x - 2 \, a}{a^{2} b x^{\frac{3}{2}} - a^{3} \sqrt{x}} - \frac{3 \, b \log\left(\frac{b \sqrt{x} - \sqrt{a b}}{b \sqrt{x} + \sqrt{a b}}\right)}{2 \, \sqrt{a b} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x - 2*a)/(a^2*b*x^(3/2) - a^3*sqrt(x)) - 3/2*b*log((b*sqrt(x) - sqrt(
a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.31, size = 151, normalized size = 2.65

$$\left[\frac{3\,(bx^2-ax)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(3\,bx-2\,a)\sqrt{x}}{2\,(a^2bx^2-a^3x)},\ -\frac{3\,(bx^2-ax)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx-2\,a)\sqrt{x}}{a^2bx^2-a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 - a*x)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x
- a)) - 2*(3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 - a^3*x), -(3*(b*x^2 - a*x)*sqr
t(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (3*b*x - 2*a)*sqrt(x))/(a^2*b*x^
2 - a^3*x)]
```

Sympy [A]
time = 7.22, size = 354, normalized size = 6.21

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b^{2} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{3 a \sqrt{x} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} + \frac{3 a \sqrt{x} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} - \frac{4 a \sqrt{\frac{a}{b}}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} + \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} - \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} + \frac{6 b x \sqrt{\frac{a}{b}}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-3*a*sqrt(x)*log(sqrt(x) - sqrt(a/b))
/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)) + 3*a*sqrt(x)*log
(sqrt(x) + sqrt(a/b))/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/
b)) - 4*a*sqrt(a/b)/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)
) + 3*b*x**(3/2)*log(sqrt(x) - sqrt(a/b))/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**
2*b*x**(3/2)*sqrt(a/b)) - 3*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(2*a**3*sqr
t(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)) + 6*b*x*sqrt(a/b)/(2*a**3*sqr
t(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 74, normalized size = 1.30

$$2\left(-\frac{3xb-2a}{2a^2\left(\sqrt{x}\,xb-\sqrt{x}\,a\right)}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2a^2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x)
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) - (3*b*x - 2*a)/((b*x^(3
/2) - a*sqrt(x))*a^2)
```

Mupad [B]
time = 0.07, size = 49, normalized size = 0.86

$$\frac{3\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2}{a}-\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}-b\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^2),x)
```

```
[Out] (3*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/a - (3*b*x)/a^2)/
(a*x^(1/2) - b*x^(3/2))
```

3.481 $\int \frac{1}{x^{5/2}(-a+bx)^2} \, dx$

Optimal. Leaf size=70

$$-\frac{5}{3a^2x^{3/2}}-\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a-bx)}+\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3/a^2/x^(3/2)+1/a/x^(3/2)/(-b*x+a)+5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(
1/2))/a^(7/2)-5*b/a^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 214}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) - (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a - b*x)) + (5*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a-bx)} - \frac{5\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{2a} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a^2} \\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^3} \\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.99

$$\frac{-2a^2-10abx+15b^2x^2}{3a^3x^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a^2 - 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a - b*x)) + (5*b^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 18.12, size = 398, normalized size = 5.69

$$\mathrm{Piecewise}\left[\left\{\left\{\mathrm{DirectedInfinity}\left[\frac{1}{x^{\frac{7}{2}}}\right], a{=}{=}0\&\&b{=}{=}0\right\}, \left\{\frac{-2}{3a^2x^{\frac{3}{2}}}, b{=}{=}0\right\}, \left\{\frac{-2}{7b^2x^{\frac{7}{2}}}, a{=}{=}0\right\}\right\}, \frac{-4a^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{20abx\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15abx^{\frac{3}{2}}\mathrm{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15abx^{\frac{3}{2}}\mathrm{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30b^2x^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15b^2x^{\frac{5}{2}}\mathrm{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15b^2x^{\frac{5}{2}}\mathrm{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(-a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 2)], a == 0 && b == 0}, {-2 / (3
a ^ 2 x ^ (3 / 2)), b == 0}, {-2 / (7 b ^ 2 x ^ (7 / 2)), a == 0}}, -4 a ^
2 Sqrt[a / b] / (6 a ^ 4 x ^ (3 / 2) Sqrt[a / b] - 6 a ^ 3 b x ^ (5 / 2) Sq
rt[a / b]) - 20 a b x Sqrt[a / b] / (6 a ^ 4 x ^ (3 / 2) Sqrt[a / b] - 6 a
^ 3 b x ^ (5 / 2) Sqrt[a / b]) - 15 a b x ^ (3 / 2) Log[Sqrt[x] - Sqrt[a /
b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[a / b] - 6 a ^ 3 b x ^ (5 / 2) Sqrt[a / b])
 + 15 a b x ^ (3 / 2) Log[Sqrt[x] + Sqrt[a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqr
t[a / b] - 6 a ^ 3 b x ^ (5 / 2) Sqrt[a / b]) + 30 b ^ 2 x ^ 2 Sqrt[a / b]
/ (6 a ^ 4 x ^ (3 / 2) Sqrt[a / b] - 6 a ^ 3 b x ^ (5 / 2) Sqrt[a / b]) - 1
5 b ^ 2 x ^ (5 / 2) Log[Sqrt[x] + Sqrt[a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[
a / b] - 6 a ^ 3 b x ^ (5 / 2) Sqrt[a / b]) + 15 b ^ 2 x ^ (5 / 2) Log[Sqrt
[x] - Sqrt[a / b]] / (6 a ^ 4 x ^ (3 / 2) Sqrt[a / b] - 6 a ^ 3 b x ^ (5 /
2) Sqrt[a / b])]
```

Maple [A]
time = 0.11, size = 59, normalized size = 0.84

method	result	size
risch	$-\frac{2(6bx+a)}{3a^3x^{\frac{3}{2}}}-\frac{b^2\left(\frac{\sqrt{x}}{bx-a}-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^3}$	56
derivativedivides	$\frac{2b^2\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{2}{3a^2x^{\frac{3}{2}}}-\frac{4b}{a^3\sqrt{x}}$	59
default	$\frac{2b^2\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{2}{3a^2x^{\frac{3}{2}}}-\frac{4b}{a^3\sqrt{x}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/a^3*(1/2*x^(1/2)/(-b*x+a)+5/2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/
2)))-2/3/a^2/x^(3/2)-4*b/a^3/x^(1/2)
```

Maxima [A]

time = 0.38, size = 82, normalized size = 1.17

$$-\frac{15\,b^2x^2-10\,abx-2\,a^2}{3\left(a^3bx^{\frac{5}{2}}-a^4x^{\frac{3}{2}}\right)}-\frac{5\,b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*b^2*x^2 - 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) - a^4*x^(3/2)) - 5/2*b^
2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.31, size = 187, normalized size = 2.67

$$\left[\frac{15\,(b^2x^3-abx^2)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(15\,b^2x^2-10\,abx-2\,a^2)\sqrt{x}}{6\,(a^3bx^3-a^4x^2)},\ -\frac{15\,(b^2x^3-abx^2)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(15\,b^2x^2-10\,abx-2\,a^2)\sqrt{x}}{3\,(a^3bx^3-a^4x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 - a*b*x^2)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a
)/(b*x - a)) - 2*(15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*
x^2), -1/3*(15*(b^2*x^3 - a*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x
))) + (15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*x^2)]
```

Sympy [A]
time = 29.13, size = 416, normalized size = 5.94

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{7b^2x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{3a^2x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{4a^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}-\frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}+\frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}-\frac{20abx\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}+\frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}-\frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}}+\frac{30b^2x^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*a**2*sqrt(a/b)/(6*a**4*x**(3/2)
*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) - 15*a*b*x**(3/2)*log(sqrt(x) - s
qrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) + 15*a*
b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x
```

```
**(5/2)*sqrt(a/b)) - 20*a*b*x*sqrt(a/b)/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3
*b*x**(5/2)*sqrt(a/b)) + 15*b**2*x**(5/2)*log(sqrt(x) - sqrt(a/b))/(6*a**4*
x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) - 15*b**2*x**(5/2)*log(sq
rt(x) + sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)
) + 30*b**2*x**2*sqrt(a/b)/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*s
qrt(a/b)), True))
```

Giac [**A**]
time = 0.00, size = 86, normalized size = 1.23

$$2\left(-\frac{\sqrt{x}\,b^2}{2a^3\,(xb-a)}-\frac{6xb+a}{3a^3\sqrt{x}\,x}-\frac{5b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{2a^3\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x)
```

```
[Out] -5*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) - b^2*sqrt(x)/((b*x -
a)*a^3) - 2/3*(6*b*x + a)/(a^3*x^(3/2))
```

Mupad [**B**]
time = 0.14, size = 60, normalized size = 0.86

$$\frac{5\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2}{3\,a}-\frac{5\,b^2\,x^2}{a^3}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}-b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^2),x)
```

```
[Out] (5*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2) - (2/(3*a) - (5*b^2*x^
2)/a^3 + (10*b*x)/(3*a^2))/(a*x^(3/2) - b*x^(5/2))
```

3.482 $\int \frac{x^{7/2}}{(-a+bx)^3} dx$

Optimal. Leaf size=97

$$\frac{35a\sqrt{x}}{4b^4} + \frac{35x^{3/2}}{12b^3} - \frac{x^{7/2}}{2b(a-bx)^2} + \frac{7x^{5/2}}{4b^2(a-bx)} - \frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

```
[Out] 35/12*x^(3/2)/b^3-1/2*x^(7/2)/b/(-b*x+a)^2+7/4*x^(5/2)/b^2/(-b*x+a)-35/4*a^
(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)+35/4*a*x^(1/2)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}} + \frac{35a\sqrt{x}}{4b^4} + \frac{7x^{5/2}}{4b^2(a-bx)} - \frac{x^{7/2}}{2b(a-bx)^2} + \frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a - b*x)^2)
+ (7*x^(5/2))/(4*b^2*(a - b*x)) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{(-a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7\int \frac{x^{5/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35\int \frac{x^{3/2}}{-a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^3}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 82, normalized size = 0.85

$$
\frac{\sqrt{x}\,\left(105a^3-175a^2bx+56ab^2x^2+8b^3x^3\right)}{12b^4(a-bx)^2}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(105*a^3 - 175*a^2*b*x + 56*a*b^2*x^2 + 8*b^3*x^3))/(12*b^4*(a - b
*x)^2) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 95.95, size = 698, normalized size = 7.20

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{3}{2}}\right], a==0\&\&b==0\right\}, \left\{\frac{2x^{\frac{3}{2}}}{3b^3}, a==0\right\}, \left\{\frac{-2x^{\frac{9}{2}}}{9a^3}, b==0\right\}\right\}, \frac{-105a^4\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{105a^4\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3b\sqrt{x}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{210a^3bx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3bx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{350a^2b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{105a^2b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{105a^2b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{112ab^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{16b^4x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(7/2)/(-a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (3 / 2)], a == 0 && b == 0}, {2 x ^ (3 / 2
) / (3 b ^ 3), a == 0}, {-2 x ^ (9 / 2) / (9 a ^ 3), b == 0}}, -105 a ^ 4 L
og[Sqrt[x] + Sqrt[a / b]] / (24 a ^ 2 b ^ 5 Sqrt[a / b] - 48 a b ^ 6 x Sqrt
[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b]) + 105 a ^ 4 Log[Sqrt[x] - Sqrt[a / b]
] / (24 a ^ 2 b ^ 5 Sqrt[a / b] - 48 a b ^ 6 x Sqrt[a / b] + 24 b ^ 7 x ^ 2
 Sqrt[a / b]) + 210 a ^ 3 b Sqrt[x] Sqrt[a / b] / (24 a ^ 2 b ^ 5 Sqrt[a /
b] - 48 a b ^ 6 x Sqrt[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b]) - 210 a ^ 3 b x
 Log[Sqrt[x] - Sqrt[a / b]] / (24 a ^ 2 b ^ 5 Sqrt[a / b] - 48 a b ^ 6 x Sq
rt[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b]) + 210 a ^ 3 b x Log[Sqrt[x] + Sqrt[
a / b]] / (24 a ^ 2 b ^ 5 Sqrt[a / b] - 48 a b ^ 6 x Sqrt[a / b] + 24 b ^ 7
 x ^ 2 Sqrt[a / b]) - 350 a ^ 2 b ^ 2 x ^ (3 / 2) Sqrt[a / b] / (24 a ^ 2 b
 ^ 5 Sqrt[a / b] - 48 a b ^ 6 x Sqrt[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b]) -
 105 a ^ 2 b ^ 2 x ^ 2 Log[Sqrt[x] + Sqrt[a / b]] / (24 a ^ 2 b ^ 5 Sqrt[a
/ b] - 48 a b ^ 6 x Sqrt[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b]) + 105 a ^ 2 b
 ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (24 a ^ 2 b ^ 5 Sqrt[a / b] - 48 a
b ^ 6 x Sqrt[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b]) + 112 a b ^ 3 x ^ (5 / 2)
 Sqrt[a / b] / (24 a ^ 2 b ^ 5 Sqrt[a / b] - 48 a b ^ 6 x Sqrt[a / b] + 24
b ^ 7 x ^ 2 Sqrt[a / b]) + 16 b ^ 4 x ^ (7 / 2) Sqrt[a / b] / (24 a ^ 2 b ^
 5 Sqrt[a / b] - 48 a b ^ 6 x Sqrt[a / b] + 24 b ^ 7 x ^ 2 Sqrt[a / b])]
```

Maple [A]
time = 0.11, size = 69, normalized size = 0.71

method	result	size
risch	$\frac{2(bx+9a)\sqrt{x}}{3b^4}+\frac{a^2\left(\frac{-\frac{13bx^{\frac{3}{2}}}{4}+\frac{11a\sqrt{x}}{4}}{(bx-a)^2}-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^4}$	67
derivativedivides	$\frac{\frac{2bx^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}-\frac{2a^2\left(\frac{\frac{13bx^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	69
default	$\frac{\frac{2bx^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}-\frac{2a^2\left(\frac{\frac{13bx^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/3*b*x^(3/2)+3*a*x^(1/2))-2/b^4*a^2*((13/8*b*x^(3/2)-11/8*a*x^(1/2)
)/(-b*x+a)^2+35/8/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.36, size = 103, normalized size = 1.06

$$-\frac{13\,a^2bx^{\frac{3}{2}}-11\,a^3\sqrt{x}}{4\,(b^6x^2-2\,ab^5x+a^2b^4)}+\frac{35\,a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}+\frac{2\left(bx^{\frac{3}{2}}+9\,a\sqrt{x}\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) - 11*a^3*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4) +
35/8*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^
4) + 2/3*(b*x^(3/2) + 9*a*sqrt(x))/b^4
```

Fricas [A]
time = 0.32, size = 227, normalized size = 2.34

$$\left[\frac{105\,(ab^2x^2-2\,a^2bx+a^3)\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(8\,b^3x^3+56\,ab^2x^2-175\,a^2bx+105\,a^3)\sqrt{x}}{24\,(b^6x^2-2\,ab^5x+a^2b^4)},\ \frac{105\,(ab^2x^2-2\,a^2bx+a^3)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+(8\,b^3x^3+56\,ab^2x^2-175\,a^2bx+105\,a^3)\sqrt{x}}{12\,(b^6x^2-2\,ab^5x+a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*s
qrt(a/b) + a)/(b*x - a)) + 2*(8*b^3*x^3 + 56*a*b^2*x^2 - 175*a^2*b*x + 105*
a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^3*x^3 + 56*a*b
^2*x^2 - 175*a^2*b*x + 105*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4)]
```

Sympy [A]
time = 94.02, size = 695, normalized size = 7.16

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{9}{2}}}{9a^3} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^3} & \text{for}\: a = 0 \\ \frac{105a^4\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{105a^4\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3b\sqrt{x}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{210a^3bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{350a^2b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{105a^2b^2x^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{105a^2b^2x^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{112ab^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{16b^4x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(9/2)/(9*a**3), Eq(b,
 0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (105*a**4*log(sqrt(x) - sqrt(a/b))/(
24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) -
105*a**4*log(sqrt(x) + sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqr
t(a/b) + 24*b**7*x**2*sqrt(a/b)) + 210*a**3*b*sqrt(x)*sqrt(a/b)/(24*a**2*b*
*5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 210*a**3*b
*x*log(sqrt(x) - sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b)
 + 24*b**7*x**2*sqrt(a/b)) + 210*a**3*b*x*log(sqrt(x) + sqrt(a/b))/(24*a**2
*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 350*a**
2*b**2*x**(3/2)*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) +
 24*b**7*x**2*sqrt(a/b)) + 105*a**2*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(24*
a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 105
*a**2*b**2*x**2*log(sqrt(x) + sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**
6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 112*a*b**3*x**(5/2)*sqrt(a/b)/(24
*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 16
*b**4*x**(7/2)*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) +
24*b**7*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 116, normalized size = 1.20

$$-2\left(\frac{-\frac{1}{3}\sqrt{x}\,xb^6-3\sqrt{x}\,b^5a}{b^9}-\frac{-13\sqrt{x}\,xba^2+11\sqrt{x}\,a^3}{8b^4\,(xb-a)^2}-\frac{35a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4b^4\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x)
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^4) - 1/4*(13*a^2*b*x^(3
/2) - 11*a^3*sqrt(x))/((b*x - a)^2*b^4) + 2/3*(b^6*x^(3/2) + 9*a*b^5*sqrt(x
))/b^9
```

Mupad [B]
time = 0.14, size = 83, normalized size = 0.86

$$\frac{\frac{11\,a^3\,\sqrt{x}}{4}-\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4-2\,a\,b^5\,x+b^6\,x^2}+\frac{2\,x^{3/2}}{3\,b^3}+\frac{6\,a\,\sqrt{x}}{b^4}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,35\mathrm{i}}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(7/2)/(a - b*x)^3,x)
```

```
[Out] ((11*a^3*x^(1/2))/4 - (13*a^2*b*x^(3/2))/4)/(a^2*b^4 + b^6*x^2 - 2*a*b^5*x)
 + (2*x^(3/2))/(3*b^3) + (6*a*x^(1/2))/b^4 + (a^(3/2)*atan((b^(1/2)*x^(1/2)
*1i)/a^(1/2))*35i)/(4*b^(9/2))
```

3.483 $\int \frac{x^{5/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

```
[Out] -1/2*x^(5/2)/b/(-b*x+a)^2+5/4*x^(3/2)/b^2/(-b*x+a)-15/4*arctanh(b^(1/2)*x^(
1/2)/a^(1/2))*a^(1/2)/b^(7/2)+15/4*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a - b*x)^2) + (5*x^(3/2))/(4*b^2*(a -
b*x)) - (15*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5\int \frac{x^{3/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{15\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 71, normalized size = 0.85

$$\frac{\sqrt{x}\,(15a^2-25abx+8b^2x^2)}{4b^3(a-bx)^2}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(15*a^2 - 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a - b*x)^2) - (15*Sqrt[a]
*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 35.49, size = 629, normalized size = 7.49

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\sqrt{x}\right],a==0\&\&b==0\right\},\left\{\frac{-2x^{\frac{5}{2}}}{7a^3},b==0\right\},\left\{\frac{2\sqrt{x}}{b^3},a==0\right\}\right\},\frac{-15a^3\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}+\frac{15a^3\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}+\frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}-\frac{30a^2bx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}+\frac{30a^2bx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}-\frac{50ab^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}-\frac{15ab^2x^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}+\frac{15ab^2x^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}+\frac{16b^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(-a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[Sqrt[x]], a == 0 && b == 0}, {-2 x ^ (7 / 2) /
 (7 a ^ 3), b == 0}, {2 Sqrt[x] / b ^ 3, a == 0}}, -15 a ^ 3 Log[Sqrt[x] +
Sqrt[a / b]] / (8 a ^ 2 b ^ 4 Sqrt[a / b] - 16 a b ^ 5 x Sqrt[a / b] + 8 b
^ 6 x ^ 2 Sqrt[a / b]) + 15 a ^ 3 Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 2 b ^
 4 Sqrt[a / b] - 16 a b ^ 5 x Sqrt[a / b] + 8 b ^ 6 x ^ 2 Sqrt[a / b]) + 30
 a ^ 2 b Sqrt[x] Sqrt[a / b] / (8 a ^ 2 b ^ 4 Sqrt[a / b] - 16 a b ^ 5 x Sq
rt[a / b] + 8 b ^ 6 x ^ 2 Sqrt[a / b]) - 30 a ^ 2 b x Log[Sqrt[x] - Sqrt[a
/ b]] / (8 a ^ 2 b ^ 4 Sqrt[a / b] - 16 a b ^ 5 x Sqrt[a / b] + 8 b ^ 6 x ^
 2 Sqrt[a / b]) + 30 a ^ 2 b x Log[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 2 b ^ 4
Sqrt[a / b] - 16 a b ^ 5 x Sqrt[a / b] + 8 b ^ 6 x ^ 2 Sqrt[a / b]) - 50 a
b ^ 2 x ^ (3 / 2) Sqrt[a / b] / (8 a ^ 2 b ^ 4 Sqrt[a / b] - 16 a b ^ 5 x S
qrt[a / b] + 8 b ^ 6 x ^ 2 Sqrt[a / b]) - 15 a b ^ 2 x ^ 2 Log[Sqrt[x] + Sq
rt[a / b]] / (8 a ^ 2 b ^ 4 Sqrt[a / b] - 16 a b ^ 5 x Sqrt[a / b] + 8 b ^
6 x ^ 2 Sqrt[a / b]) + 15 a b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^
 2 b ^ 4 Sqrt[a / b] - 16 a b ^ 5 x Sqrt[a / b] + 8 b ^ 6 x ^ 2 Sqrt[a / b]
) + 16 b ^ 3 x ^ (5 / 2) Sqrt[a / b] / (8 a ^ 2 b ^ 4 Sqrt[a / b] - 16 a b
^ 5 x Sqrt[a / b] + 8 b ^ 6 x ^ 2 Sqrt[a / b])]
```

Maple [A]
time = 0.14, size = 57, normalized size = 0.68

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	57
default	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	57
risch	$\frac{2\sqrt{x}}{b^3}+\frac{a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{4}+\frac{7a\sqrt{x}}{4}}{(bx-a)^2}-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^3-2/b^3*a*((9/8*b*x^(3/2)-7/8*a*x^(1/2))/(-b*x+a)^2+15/8/(a*b)^
(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.35, size = 90, normalized size = 1.07

$$-\frac{9\,abx^{\frac{3}{2}}-7\,a^2\sqrt{x}}{4\,(b^5x^2-2\,ab^4x+a^2b^3)}+\frac{15\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/(b^5*x^2 - 2*a*b^4*x + a^2*b^3) + 15/8
*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2
*sqrt(x)/b^3
```

Fricas [A]
time = 0.32, size = 199, normalized size = 2.37

$$\left[\frac{15\,(b^2x^2-2\,abx+a^2)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(8\,b^2x^2-25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2-2\,ab^4x+a^2b^3)},\ \frac{15\,(b^2x^2-2\,abx+a^2)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(8\,b^2x^2-25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2-2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/
b) + a)/(b*x - a)) + 2*(8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2 -
2*a*b^4*x + a^2*b^3), 1/4*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a/b)*arctan(b
*sqrt(x)*sqrt(-a/b)/a) + (8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
- 2*a*b^4*x + a^2*b^3)]
```

Sympy [A]
time = 34.69, size = 624, normalized size = 7.43

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^3} & \text{for}\: a = 0 \\ \frac{15a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{15a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{30a^2bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{30a^2bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{50ab^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{15ab^2x^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{15ab^2x^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{16b^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(7/2)/(7*a**3), Eq(b,
0)), (2*sqrt(x)/b**3, Eq(a, 0)), (15*a**3*log(sqrt(x) - sqrt(a/b))/(8*a**2*
b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 15*a**3*l
og(sqrt(x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*
b**6*x**2*sqrt(a/b)) + 30*a**2*b*sqrt(x)*sqrt(a/b)/(8*a**2*b**4*sqrt(a/b) -
```

```
 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 30*a**2*b*x*log(sqrt(x) -
 sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sq
rt(a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16
*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 50*a*b**2*x**(3/2)*sqrt(a/b)
/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) +
15*a*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*
x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 15*a*b**2*x**2*log(sqrt(x) + sqrt(a/
b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b))
 + 16*b**3*x**(5/2)*sqrt(a/b)/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b
) + 8*b**6*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 92, normalized size = 1.10

$$-2\left(-\frac{\sqrt{x}}{b^3}-\frac{-9\sqrt{x}\,xba+7\sqrt{x}\,a^2}{8b^3\left(xb-a\right)^2}-\frac{15a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4b^3\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 15/4*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2*sqrt(x)/b^3 - 1/4*
(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/((b*x - a)^2*b^3)
```

Mupad [B]
time = 0.06, size = 69, normalized size = 0.82

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4}-\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3-2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 - (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 - 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

3.484 $\int \frac{x^{3/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

```
[Out] -1/2*x^(3/2)/b/(-b*x+a)^2-3/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)/a^(1
/2)+3/4*x^(1/2)/b^2/(-b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {43, 65, 214}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{x^{3/2}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a - b*x)^2) + (3*Sqrt[x])/(4*b^2*(a - b*x)) - (3*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(-a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^2}\\
&= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\
&= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.83

$$
\frac{\sqrt{x}\,(3a-5bx)}{4b^2(a-bx)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(3*a - 5*b*x))/(4*b^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 17.39, size = 558, normalized size = 7.75

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{\sqrt{x}}\right],a==0\&\&b==0\right\},\left\{\frac{-2x^{\frac{5}{2}}}{5a^3},b==0\right\},\left\{\frac{-2}{b^3\sqrt{x}},a==0\right\}\right\},\frac{-3a^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}+\frac{3a^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}+\frac{6ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}-\frac{6abx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}+\frac{6abx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}-\frac{10b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}-\frac{3b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}+\frac{3b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^2b^3\sqrt{\frac{a}{b}}-16ab^4x\sqrt{\frac{a}{b}}+8b^5x^2\sqrt{\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(-a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[x]], a == 0 && b == 0}, {-2 x ^ (5 /
2) / (5 a ^ 3), b == 0}, {-2 / (b ^ 3 Sqrt[x]), a == 0}}, -3 a ^ 2 Log[Sqrt
[x] + Sqrt[a / b]] / (8 a ^ 2 b ^ 3 Sqrt[a / b] - 16 a b ^ 4 x Sqrt[a / b]
+ 8 b ^ 5 x ^ 2 Sqrt[a / b]) + 3 a ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^
2 b ^ 3 Sqrt[a / b] - 16 a b ^ 4 x Sqrt[a / b] + 8 b ^ 5 x ^ 2 Sqrt[a / b])
 + 6 a b Sqrt[x] Sqrt[a / b] / (8 a ^ 2 b ^ 3 Sqrt[a / b] - 16 a b ^ 4 x Sq
```

```
rt[a / b] + 8 b ^ 5 x ^ 2 Sqrt[a / b]) - 6 a b x Log[Sqrt[x] - Sqrt[a / b]]
 / (8 a ^ 2 b ^ 3 Sqrt[a / b] - 16 a b ^ 4 x Sqrt[a / b] + 8 b ^ 5 x ^ 2 Sq
rt[a / b]) + 6 a b x Log[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 2 b ^ 3 Sqrt[a / b
] - 16 a b ^ 4 x Sqrt[a / b] + 8 b ^ 5 x ^ 2 Sqrt[a / b]) - 10 b ^ 2 x ^ (3
 / 2) Sqrt[a / b] / (8 a ^ 2 b ^ 3 Sqrt[a / b] - 16 a b ^ 4 x Sqrt[a / b] +
 8 b ^ 5 x ^ 2 Sqrt[a / b]) - 3 b ^ 2 x ^ 2 Log[Sqrt[x] + Sqrt[a / b]] / (8
 a ^ 2 b ^ 3 Sqrt[a / b] - 16 a b ^ 4 x Sqrt[a / b] + 8 b ^ 5 x ^ 2 Sqrt[a
/ b]) + 3 b ^ 2 x ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 2 b ^ 3 Sqrt[a /
b] - 16 a b ^ 4 x Sqrt[a / b] + 8 b ^ 5 x ^ 2 Sqrt[a / b])]
```

Maple [A]
time = 0.10, size = 51, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2\left(\frac{5x^{\frac{3}{2}}}{8b}-\frac{3a\sqrt{x}}{8b^{2}}\right)}{\left(-bx+a\right)^{2}}-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}\sqrt{ab}}$	51
default	$-\frac{2\left(\frac{5x^{\frac{3}{2}}}{8b}-\frac{3a\sqrt{x}}{8b^{2}}\right)}{\left(-bx+a\right)^{2}}-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{2}\sqrt{ab}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(-b*x+a)^2-3/4/b^2/(a*b)^(1/2)*arctanh
(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.34, size = 78, normalized size = 1.08

$$-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\,(b^4x^2-2\,ab^3x+a^2b^2)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) - 3*a*sqrt(x))/(b^4*x^2 - 2*a*b^3*x + a^2*b^2) + 3/8*log(
(b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.31, size = 186, normalized size = 2.58

$$\left[\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)-2\,(5\,ab^2x-3\,a^2b)\sqrt{x}}{8\,(ab^5x^2-2\,a^2b^4x+a^3b^3)},\ \frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)-(5\,ab^2x-3\,a^2b)\sqrt{x}}{4\,(ab^5x^2-2\,a^2b^4x+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) - 2*(5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x
 + a^3*b^3), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) - (5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x + a^
3*b^3)]
```

Sympy [A]
time = 16.38, size = 552, normalized size = 7.67

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{b^{3} \sqrt{x}} & \text{for}\: a = 0 \\ - \frac{2 x^{\frac{5}{2}}}{5 a^{3}} & \text{for}\: b = 0 \\ \frac{3 a^{2} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} - \frac{3 a^{2} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} + \frac{6 a b \sqrt{x} \sqrt{\frac{a}{b}}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} - \frac{6 a b x \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} + \frac{6 a b x \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} - \frac{10 b^{2} x^{\frac{3}{2}} \sqrt{\frac{a}{b}}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} + \frac{3 b^{2} x^{2} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} - \frac{3 b^{2} x^{2} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{2} b^{3} \sqrt{\frac{a}{b}} - 16 a b^{4} x \sqrt{\frac{a}{b}} + 8 b^{5} x^{2} \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**3*sqrt(x)), Eq(a, 0))
, (-2*x**(5/2)/(5*a**3), Eq(b, 0)), (3*a**2*log(sqrt(x) - sqrt(a/b))/(8*a**
2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 3*a**2*
log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8
*b**5*x**2*sqrt(a/b)) + 6*a*b*sqrt(x)*sqrt(a/b)/(8*a**2*b**3*sqrt(a/b) - 16
*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 6*a*b*x*log(sqrt(x) - sqrt(a
/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)
) + 6*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*s
qrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 10*b**2*x**(3/2)*sqrt(a/b)/(8*a**2*b**3
*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) + 3*b**2*x**2*l
og(sqrt(x) - sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*
b**5*x**2*sqrt(a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sq
rt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.07

$$-2\left(-\frac{-5\sqrt{x}\,xb+3\sqrt{x}\,a}{8b^2\,(xb-a)^2}-\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4b^2\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - 1/4*(5*b*x^(3/2) - 3*a*
sqrt(x))/((b*x - a)^2*b^2)
```

Mupad [B]
time = 0.14, size = 58, normalized size = 0.81

$$-\frac{\frac{5\,x^{3/2}}{4\,b}-\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x)^3,x)
```

```
[Out] - ((5*x^(3/2))/(4*b) - (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2))
```

3.485 $\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[Out] 1/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/2*x^(1/2)/b/(-b*x+a)
^2+1/4*x^(1/2)/a/b/(-b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {43, 44, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a - b*x)^2) + Sqrt[x]/(4*a*b*(a - b*x)) + ArcTanh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.80

$$
-\frac{\sqrt{x}\,(a+bx)}{4ab(a-bx)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(a + b*x))/(a*b*(a - b*x)^2) + ArcTanh[(Sqrt[b]*Sqrt[x]])/Sqrt
[a]]/(4*a^(3/2)*b^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 8.41, size = 580, normalized size = 7.73

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{3}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{3b^3x^{\frac{3}{2}}}, a==0\right\}, \left\{\frac{-2x^{\frac{3}{2}}}{3a^3}, b==0\right\}\right\}, -\frac{a^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}+\frac{a^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}-\frac{2ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}-\frac{2abx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}+\frac{2abx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}-\frac{2b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}-\frac{b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}+\frac{b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^3b^2\sqrt{\frac{a}{b}}-16a^2b^3x\sqrt{\frac{a}{b}}+8ab^4x^2\sqrt{\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(-a + b*x)^3,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (3 / 2)], a == 0 && b == 0}, {-2 / (3
b ^ 3 x ^ (3 / 2)), a == 0}, {-2 x ^ (3 / 2) / (3 a ^ 3), b == 0}}, -a ^ 2
Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 3 b ^ 2 Sqrt[a / b] - 16 a ^ 2 b ^ 3 x
Sqrt[a / b] + 8 a b ^ 4 x ^ 2 Sqrt[a / b]) + a ^ 2 Log[Sqrt[x] + Sqrt[a / b
]] / (8 a ^ 3 b ^ 2 Sqrt[a / b] - 16 a ^ 2 b ^ 3 x Sqrt[a / b] + 8 a b ^ 4
x ^ 2 Sqrt[a / b]) - 2 a b Sqrt[x] Sqrt[a / b] / (8 a ^ 3 b ^ 2 Sqrt[a / b]
 - 16 a ^ 2 b ^ 3 x Sqrt[a / b] + 8 a b ^ 4 x ^ 2 Sqrt[a / b]) - 2 a b x Lo
g[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 3 b ^ 2 Sqrt[a / b] - 16 a ^ 2 b ^ 3 x Sq
rt[a / b] + 8 a b ^ 4 x ^ 2 Sqrt[a / b]) + 2 a b x Log[Sqrt[x] - Sqrt[a / b
]] / (8 a ^ 3 b ^ 2 Sqrt[a / b] - 16 a ^ 2 b ^ 3 x Sqrt[a / b] + 8 a b ^ 4
x ^ 2 Sqrt[a / b]) - 2 b ^ 2 x ^ (3 / 2) Sqrt[a / b] / (8 a ^ 3 b ^ 2 Sqrt[
a / b] - 16 a ^ 2 b ^ 3 x Sqrt[a / b] + 8 a b ^ 4 x ^ 2 Sqrt[a / b]) - b ^
2 x ^ 2 Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 3 b ^ 2 Sqrt[a / b] - 16 a ^ 2
b ^ 3 x Sqrt[a / b] + 8 a b ^ 4 x ^ 2 Sqrt[a / b]) + b ^ 2 x ^ 2 Log[Sqrt[x
] + Sqrt[a / b]] / (8 a ^ 3 b ^ 2 Sqrt[a / b] - 16 a ^ 2 b ^ 3 x Sqrt[a / b
] + 8 a b ^ 4 x ^ 2 Sqrt[a / b])]
```

Maple [A]
time = 0.11, size = 53, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2\left(\frac{x^{\frac{3}{2}}}{8a}+\frac{\sqrt{x}}{8b}\right)}{(-bx+a)^{2}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	53
default	$-\frac{2\left(\frac{x^{\frac{3}{2}}}{8a}+\frac{\sqrt{x}}{8b}\right)}{(-bx+a)^{2}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/8/a*x^(3/2)+1/8*x^(1/2)/b)/(-b*x+a)^2+1/4/a/b/(a*b)^(1/2)*arctanh(b*x
^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.34, size = 80, normalized size = 1.07

$$-\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\left(ab^{3}x^{2}-2\,a^{2}b^{2}x+a^{3}b\right)}-\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^(3/2) + a*sqrt(x))/(a*b^3*x^2 - 2*a^2*b^2*x + a^3*b) - 1/8*log((b
*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.32, size = 183, normalized size = 2.44

$$\left[\frac{\left(b^{2} x^{2} - 2\,abx + a^{2}\right)\sqrt{ab}\,\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right) - 2\left(ab^{2} x + a^{2} b\right)\sqrt{x}}{8\left(a^{2} b^{4} x^{2} - 2\,a^{3} b^{3} x + a^{4} b^{2}\right)}, -\frac{\left(b^{2} x^{2} - 2\,abx + a^{2}\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right) + \left(ab^{2} x + a^{2} b\right)\sqrt{x}}{4\left(a^{2} b^{4} x^{2} - 2\,a^{3} b^{3} x + a^{4} b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a + 2*sqrt(a*b)*sqrt(x
))/(b*x - a)) - 2*(a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a
^4*b^2), -1/4*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b*sq
rt(x))) + (a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a^4*b^2)]
```

Sympy [A]
time = 7.10, size = 575, normalized size = 7.67

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 b^{3} x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ - \frac{2 x^{\frac{3}{2}}}{3 a^{3}} & \text{for}\: b = 0 \\ - \frac{a^{2} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} + \frac{a^{2} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} - \frac{2 a b \sqrt{x} \sqrt{\frac{a}{b}}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} + \frac{2 a b x \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} - \frac{2 a b x \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} - \frac{2 b^{2} x^{\frac{3}{2}} \sqrt{\frac{a}{b}}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} - \frac{b^{2} x^{2} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} + \frac{b^{2} x^{2} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{3} b^{2} \sqrt{\frac{a}{b}} - 16 a^{2} b^{3} x \sqrt{\frac{a}{b}} + 8 a b^{4} x^{2} \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**3*x**(3/2)), Eq(a,
 0)), (-2*x**(3/2)/(3*a**3), Eq(b, 0)), (-a**2*log(sqrt(x) - sqrt(a/b))/(8*
a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) +
 a**2*log(sqrt(x) + sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt
(a/b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*a*b*sqrt(x)*sqrt(a/b)/(8*a**3*b**2*sqr
t(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) + 2*a*b*x*log(
sqrt(x) - sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*
a*b**4*x**2*sqrt(a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**3*b**2*sqrt
(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*b**2*x**(3/
2)*sqrt(a/b)/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x
**2*sqrt(a/b)) - b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**3*b**2*sqrt(a/b)
- 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) + b**2*x**2*log(sqrt(
x) + sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**
4*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 74, normalized size = 0.99

$$-2\left(-\frac{-\sqrt{x}\,xb - \sqrt{x}\,a}{8ba\left(xb-a\right)^{2}} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4ba\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x)
```

```
[Out] -1/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a*b) - 1/4*(b*x^(3/2) + a*sqr
t(x))/((b*x - a)^2*a*b)
```

Mupad [B]
time = 0.14, size = 57, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}-\frac{\frac{x^{3/2}}{4\,a}+\frac{\sqrt{x}}{4\,b}}{a^2-2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x)^3,x)
```

```
[Out] atanh((b^(1/2)*x^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(3/2)/(4*a) + x^(
1/2)/(4*b))/(a^2 + b^2*x^2 - 2*a*b*x)
```

3.486 $\int \frac{1}{\sqrt{x}\,(-a+bx)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{x}}{2a(a-bx)^2}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

```
[Out] -3/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/2*x^(1/2)/a/(-b*x+a
)^2-3/4*x^(1/2)/a^2/(-b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {44, 65, 214}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] -1/2*Sqrt[x]/(a*(a - b*x)^2) - (3*Sqrt[x])/(4*a^2*(a - b*x)) - (3*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\,(-a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx}{4a} \\
&= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^2} \\
&= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^2} \\
&= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.83

$$
\frac{\sqrt{x}\,(-5a+3bx)}{4a^2(a-bx)^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] (Sqrt[x]*(-5*a + 3*b*x))/(4*a^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 9.95, size = 582, normalized size = 8.08

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{5b^3x^{\frac{5}{2}}}, a==0\right\}, \left\{\frac{-2\sqrt{x}}{a^3}, b==0\right\}\right\}, \frac{-3a^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{3a^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{10ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{6abx\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{6abx\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{6b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{3b^2x^2\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{3b^2x^2\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 2)], a == 0 && b == 0}, {-2 / (5
b ^ 3 x ^ (5 / 2)), a == 0}, {-2 Sqrt[x] / a ^ 3, b == 0}}, -3 a ^ 2 Log[Sq
rt[x] + Sqrt[a / b]] / (8 a ^ 4 b Sqrt[a / b] - 16 a ^ 3 b ^ 2 x Sqrt[a / b
] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[a / b]) + 3 a ^ 2 Log[Sqrt[x] - Sqrt[a / b]] /
 (8 a ^ 4 b Sqrt[a / b] - 16 a ^ 3 b ^ 2 x Sqrt[a / b] + 8 a ^ 2 b ^ 3 x ^
2 Sqrt[a / b]) - 10 a b Sqrt[x] Sqrt[a / b] / (8 a ^ 4 b Sqrt[a / b] - 16 a
```

```
 ^ 3 b ^ 2 x Sqrt[a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[a / b]) - 6 a b x Log[S
qrt[x] - Sqrt[a / b]] / (8 a ^ 4 b Sqrt[a / b] - 16 a ^ 3 b ^ 2 x Sqrt[a /
b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[a / b]) + 6 a b x Log[Sqrt[x] + Sqrt[a / b]]
/ (8 a ^ 4 b Sqrt[a / b] - 16 a ^ 3 b ^ 2 x Sqrt[a / b] + 8 a ^ 2 b ^ 3 x ^
 2 Sqrt[a / b]) + 6 b ^ 2 x ^ (3 / 2) Sqrt[a / b] / (8 a ^ 4 b Sqrt[a / b]
- 16 a ^ 3 b ^ 2 x Sqrt[a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[a / b]) - 3 b ^ 2
 x ^ 2 Log[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 4 b Sqrt[a / b] - 16 a ^ 3 b ^ 2
 x Sqrt[a / b] + 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[a / b]) + 3 b ^ 2 x ^ 2 Log[Sqrt[
x] - Sqrt[a / b]] / (8 a ^ 4 b Sqrt[a / b] - 16 a ^ 3 b ^ 2 x Sqrt[a / b] +
 8 a ^ 2 b ^ 3 x ^ 2 Sqrt[a / b])]
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2a(-bx+a)^{2}}-\frac{3\left(\frac{\sqrt{x}}{2a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{2a}$	61
default	$-\frac{\sqrt{x}}{2a(-bx+a)^{2}}-\frac{3\left(\frac{\sqrt{x}}{2a(-bx+a)}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{2a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/a/(-b*x+a)^2-3/2/a*(1/2*x^(1/2)/a/(-b*x+a)+1/2/a/(a*b)^(1/2)*a
rctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.35, size = 77, normalized size = 1.07

$$\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(a^2b^2x^2-2\,a^3bx+a^4)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) - 5*a*sqrt(x))/(a^2*b^2*x^2 - 2*a^3*b*x + a^4) + 3/8*log((
b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.32, size = 185, normalized size = 2.57

$$\left[\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)+2\,(3\,ab^2x-5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2-2\,a^4b^2x+a^5b)},\ \frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+(3\,ab^2x-5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2-2\,a^4b^2x+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) + 2*(3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2
*x + a^5*b), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) + (3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*x +
a^5*b)]
```

Sympy [A]
time = 11.86, size = 580, normalized size = 8.06

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2 \sqrt{x}}{a^{3}} & \text{for}\: b = 0 \\ - \frac{2}{5 b^{3} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{3 a^{2} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} - \frac{3 a^{2} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} - \frac{10 a b \sqrt{x} \sqrt{\frac{a}{b}}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} - \frac{6 a b x \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} + \frac{6 a b x \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} + \frac{6 b^{2} x^{\frac{3}{2}} \sqrt{\frac{a}{b}}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} + \frac{3 b^{2} x^{2} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} - \frac{3 b^{2} x^{2} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{8 a^{4} b \sqrt{\frac{a}{b}} - 16 a^{3} b^{2} x \sqrt{\frac{a}{b}} + 8 a^{2} b^{3} x^{2} \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*sqrt(x)/a**3, Eq(b, 0)),
 (-2/(5*b**3*x**(5/2)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(a/b))/(8*a**4
*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 3*a
**2*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b)
 + 8*a**2*b**3*x**2*sqrt(a/b)) - 10*a*b*sqrt(x)*sqrt(a/b)/(8*a**4*b*sqrt(a/
b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 6*a*b*x*log(s
qrt(x) - sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2
*b**3*x**2*sqrt(a/b)) + 6*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b
) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) + 6*b**2*x**(3/2
)*sqrt(a/b)/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x*
*2*sqrt(a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**4*b*sqrt(a/b) -
16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 3*b**2*x**2*log(sq
rt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*
b**3*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.07

$$-2\left(-\frac{3\sqrt{x}\,xb-5\sqrt{x}\,a}{8a^2\,(xb-a)^2}-\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4a^2\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x)
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 1/4*(3*b*x^(3/2) - 5*a*
sqrt(x))/((b*x - a)^2*a^2)
```

Mupad [B]
time = 0.13, size = 58, normalized size = 0.81

$$-\frac{\frac{5\sqrt{x}}{4a}-\frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)^3),x)
```

```
[Out] - ((5*x^(1/2))/(4*a) - (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

3.487 $\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)+15/4/a^3/x^(1/2)-1/2
/a/(-b*x+a)^2/x^(1/2)-5/4/a^2/(-b*x+a)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 214}

$$-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] 15/(4*a^3*Sqrt[x]) - 1/(2*a*Sqrt[x]*(a - b*x)^2) - 5/(4*a^2*Sqrt[x]*(a - b*
x)) - (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx &= -\frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx}{4a} \\
&= -\frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{15\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^2} \\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{(15b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^3} \\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{(15b)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^3} \\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} - \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.85

$$\frac{8a^2-25abx+15b^2x^2}{4a^3\sqrt{x}\,(a-bx)^2} - \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^3),x]
[Out] (8*a^2 - 25*a*b*x + 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a - b*x)^2) - (15*Sqrt[b]*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 32.03, size = 670, normalized size = 7.98

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{7b^3x^{\frac{7}{2}}}, a==0\right\}, \left\{\frac{2}{a^3\sqrt{x}}, b==0\right\}\right\}, \frac{16a^2\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15a^2\sqrt{x}\,\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15a^2\sqrt{x}\,\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{50abx\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{30abx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30abx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30b^2x^2\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15b^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15b^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^(3/2)*(-a + b*x)^3),x]')

[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 2)], a == 0 && b == 0}, {-2 / (7 b ^ 3 x ^ (7 / 2)), a == 0}, {2 / (a ^ 3 Sqrt[x]), b == 0}}, 16 a ^ 2 Sqrt[a / b] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) - 15 a ^ 2 Sqrt[x] Log[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) + 15 a ^ 2 Sqrt[x] Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) - 50 a b x Sqrt[a / b] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) - 30 a b x ^ (3 / 2) Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) + 30 a b x ^ (3 / 2) Log[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) + 30 b ^ 2 x ^ 2 Sqrt[a / b] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) - 15 b ^ 2 x ^ (5 / 2) Log[Sqrt[x] + Sqrt[a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b]) + 15 b ^ 2 x ^ (5 / 2) Log[Sqrt[x] - Sqrt[a / b]] / (8 a ^ 5 Sqrt[x] Sqrt[a / b] - 16 a ^ 4 b x ^ (3 / 2) Sqrt[a / b] + 8 a ^ 3 b ^ 2 x ^ (5 / 2) Sqrt[a / b])]

Maple [A]
time = 0.12, size = 57, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{2b\left(\frac{-\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(-bx+a)^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}+\frac{2}{a^{3}\sqrt{x}}$	57
default	$-\frac{2b\left(\frac{-\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(-bx+a)^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}+\frac{2}{a^{3}\sqrt{x}}$	57
risch	$\frac{2}{a^{3}\sqrt{x}}+\frac{b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{4}-\frac{9a\sqrt{x}}{4}}{(bx-a)^{2}}-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3*b*((-7/8*b*x^(3/2)+9/8*a*x^(1/2))/(-b*x+a)^2+15/8/(a*b)^(1/2)*arctan
h(b*x^(1/2)/(a*b)^(1/2)))+2/a^3/x^(1/2)
```

Maxima [A]
time = 0.35, size = 90, normalized size = 1.07

$$\frac{15\,b^2x^2 - 25\,abx + 8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}} - 2\,a^4bx^{\frac{3}{2}} + a^5\sqrt{x}\right)} + \frac{15\,b\log\left(\frac{b\sqrt{x} - \sqrt{ab}}{b\sqrt{x} + \sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(15*b^2*x^2 - 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) - 2*a^4*b*x^(3/2) + a^
5*sqrt(x)) + 15/8*b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(s
qrt(a*b)*a^3)
```

Fricas [A]
time = 0.32, size = 213, normalized size = 2.54

$$\left[\frac{15\,(b^2x^3 - 2\,abx^2 + a^2x)\sqrt{\frac{b}{a}}\log\left(\frac{bx - 2\,a\sqrt{x}\sqrt{\frac{b}{a}} + a}{bx - a}\right) + 2\,(15\,b^2x^2 - 25\,abx + 8\,a^2)\sqrt{x}}{8\,(a^3b^2x^3 - 2\,a^4bx^2 + a^5x)}, \frac{15\,(b^2x^3 - 2\,abx^2 + a^2x)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right) + (15\,b^2x^2 - 25\,abx + 8\,a^2)\sqrt{x}}{4\,(a^3b^2x^3 - 2\,a^4bx^2 + a^5x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqr
t(b/a) + a)/(b*x - a)) + 2*(15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*b^
2*x^3 - 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(-b
/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 - 2*a^4*b*x^2 + a^5*x)]
```

Sympy [A]
time = 32.19, size = 716, normalized size = 8.52

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{a^3\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{2}{7b^3x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{15a^2\sqrt{x}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15a^2\sqrt{x}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{16a^2\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{50abx\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30b^2x^2\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}} - 16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}} + 8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2/(a**3*sqrt(x)), Eq(b, 0))
, (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (15*a**2*sqrt(x)*log(sqrt(x) - sqrt(a/b
```

```
))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x
**(5/2)*sqrt(a/b)) - 15*a**2*sqrt(x)*log(sqrt(x) + sqrt(a/b))/(8*a**5*sqrt(
x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b
)) + 16*a**2*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(
a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) - 30*a*b*x**(3/2)*log(sqrt(x) - sqrt
(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b*
*2*x**(5/2)*sqrt(a/b)) + 30*a*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(8*a**5*s
qrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt
(a/b)) - 50*a*b*x*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*
sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) + 15*b**2*x**(5/2)*log(sqrt(x)
- sqrt(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a
**3*b**2*x**(5/2)*sqrt(a/b)) - 15*b**2*x**(5/2)*log(sqrt(x) + sqrt(a/b))/(8
*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/
2)*sqrt(a/b)) + 30*b**2*x**2*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*
b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)), True))
```

Giac [A]
time = 0.00, size = 93, normalized size = 1.11

$$-2\left(-\frac{1}{a^3\sqrt{x}}-\frac{7\sqrt{x}\,xb^2-9\sqrt{x}\,ba}{8a^3\,(xb-a)^2}-\frac{15b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4a^3\cdot 2\sqrt{-ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 15/4*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/(a^3*sqrt(x)) + 1/
4*(7*b^2*x^(3/2) - 9*a*b*sqrt(x))/((b*x - a)^2*a^3)
```

Mupad [B]
time = 0.15, size = 69, normalized size = 0.82

$$\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}-\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}-2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)^3),x)
```

```
[Out] (2/a + (15*b^2*x^2)/(4*a^3) - (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2)
- 2*a*b*x^(3/2)) - (15*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2)
)
```

3.488 $\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx$

Optimal. Leaf size=97

$$\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] 35/12/a^3/x^(3/2)-1/2/a/x^(3/2)/(-b*x+a)^2-7/4/a^2/x^(3/2)/(-b*x+a)-35/4*b^
(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)+35/4*b/a^4/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {44, 53, 65, 214}

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] 35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) - 1/(2*a*x^(3/2)*(a - b*x)^2)
- 7/(4*a^2*x^(3/2)*(a - b*x)) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx &= -\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx}{4a}\\ &= -\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{35\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{8a^2}\\ &= \frac{35}{12a^3x^{3/2}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{(35b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^3}\\ &= \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{(35b^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,d}{8a^4}\\ &= \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{(35b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx}\right.}{4a^4}\\ &= \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt{a}}\right.}{4a^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 0.85

$$\frac{8a^3+56a^2bx-175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a-bx)^2}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^3),x]
[Out] (8*a^3 + 56*a^2*b*x - 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a - b*x
)^2) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 44.15, size = 746, normalized size = 7.69

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{9}{2}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-2}{9b^3x^{\frac{9}{2}}}, a==0\right\}, \left\{\frac{2}{3a^3x^{\frac{3}{2}}}, b==0\right\}\right\}, \frac{16a^3\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}+\frac{112a^2bx\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}-\frac{105a^2bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}+\frac{105a^2bx^{\frac{3}{2}}\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}-\frac{350ab^2x^2\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}-\frac{210ab^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}+\frac{210ab^2x^{\frac{5}{2}}\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}+\frac{210b^3x^3\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}-\frac{105b^3x^{\frac{7}{2}}\text{Log}\left[\sqrt{x}+\sqrt{\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}+\frac{105b^3x^{\frac{7}{2}}\text{Log}\left[\sqrt{x}-\sqrt{\frac{a}{b}}\right]}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}}-48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}+24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(-a + b*x)^3),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (9 / 2)], a == 0 && b == 0}, {-2 / (9
b ^ 3 x ^ (9 / 2)), a == 0}, {2 / (3 a ^ 3 x ^ (3 / 2)), b == 0}}, 16 a ^ 3
 Sqrt[a / b] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b x ^ (5 / 2) S
qrt[a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b]) + 112 a ^ 2 b x Sqrt[a
 / b] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b x ^ (5 / 2) Sqrt[a /
 b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b]) - 105 a ^ 2 b x ^ (3 / 2) Log
[Sqrt[x] + Sqrt[a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b x
^ (5 / 2) Sqrt[a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b]) + 105 a ^ 2
 b x ^ (3 / 2) Log[Sqrt[x] - Sqrt[a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a /
b] - 48 a ^ 5 b x ^ (5 / 2) Sqrt[a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a
 / b]) - 350 a b ^ 2 x ^ 2 Sqrt[a / b] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b]
- 48 a ^ 5 b x ^ (5 / 2) Sqrt[a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a /
b]) - 210 a b ^ 2 x ^ (5 / 2) Log[Sqrt[x] - Sqrt[a / b]] / (24 a ^ 6 x ^ (3
 / 2) Sqrt[a / b] - 48 a ^ 5 b x ^ (5 / 2) Sqrt[a / b] + 24 a ^ 4 b ^ 2 x ^
 (7 / 2) Sqrt[a / b]) + 210 a b ^ 2 x ^ (5 / 2) Log[Sqrt[x] + Sqrt[a / b]]
/ (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b x ^ (5 / 2) Sqrt[a / b] +
24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b]) + 210 b ^ 3 x ^ 3 Sqrt[a / b] / (24
 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b x ^ (5 / 2) Sqrt[a / b] + 24 a
^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b]) - 105 b ^ 3 x ^ (7 / 2) Log[Sqrt[x] + Sq
rt[a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b x ^ (5 / 2) Sqr
t[a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b]) + 105 b ^ 3 x ^ (7 / 2)
Log[Sqrt[x] - Sqrt[a / b]] / (24 a ^ 6 x ^ (3 / 2) Sqrt[a / b] - 48 a ^ 5 b
 x ^ (5 / 2) Sqrt[a / b] + 24 a ^ 4 b ^ 2 x ^ (7 / 2) Sqrt[a / b])]
```

Maple [A]
time = 0.13, size = 68, normalized size = 0.70

method	result	size
risch	$\frac{6bx+\frac{2a}{3}}{a^4x^{\frac{3}{2}}}+\frac{b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{4}-\frac{13a\sqrt{x}}{4}}{(bx-a)^2}-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^4}$	66

derivativedivides	$\frac{2}{3a^{3}x^{\frac{3}{2}}}+\frac{6b}{a^{4}\sqrt{x}}-\frac{2b^{2}\left(\frac{-\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{\left(-bx+a\right)^{2}}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	68
default	$\frac{2}{3a^{3}x^{\frac{3}{2}}}+\frac{6b}{a^{4}\sqrt{x}}-\frac{2b^{2}\left(\frac{-\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{\left(-bx+a\right)^{2}}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)-2/a^4*b^2*((-11/8*b*x^(3/2)+13/8*a*x^(1/2))
/(-b*x+a)^2+35/8/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.34, size = 103, normalized size = 1.06

$$\frac{105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}}{12\left(a^{4}b^{2}x^{\frac{7}{2}}-2\,a^{5}bx^{\frac{5}{2}}+a^{6}x^{\frac{3}{2}}\right)}+\frac{35\,b^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)/(a^4*b^2*x^(7/2) -
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/8*b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sq
rt(x) + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.32, size = 249, normalized size = 2.57

$$\left[\frac{105\left(b^{3}x^{4}-2\,ab^{2}x^{3}+a^{2}bx^{2}\right)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}\right)\sqrt{x}}{24\left(a^{4}b^{2}x^{4}-2\,a^{5}bx^{3}+a^{6}x^{2}\right)},\ \frac{105\left(b^{3}x^{4}-2\,ab^{2}x^{3}+a^{2}bx^{2}\right)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}\right)\sqrt{x}}{12\left(a^{4}b^{2}x^{4}-2\,a^{5}bx^{3}+a^{6}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*log((b*x - 2*a*sqr
t(x)*sqrt(b/a) + a)/(b*x - a)) + 2*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*
x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(b^3*x
```

```
^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) +
 (105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 -
 2*a^5*b*x^3 + a^6*x^2)]
```

<u>Sympy</u> [A]
time = 85.48, size = 799, normalized size = 8.24

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ \frac{2}{3 a^{3} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ \frac{16 a^{3} \sqrt{\frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} + \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} - \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} + \frac{112 a^{2} b x \sqrt{\frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} - \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} + \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} - \frac{350 a b^{2} x^{2} \sqrt{\frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} + \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} - \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} + \frac{210 b^{3} x^{3} \sqrt{\frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{\frac{a}{b}} - 48 a^{5} b x^{\frac{5}{2}} \sqrt{\frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b**3*x**(9/2)), Eq(a,
 0)), (2/(3*a**3*x**(3/2)), Eq(b, 0)), (16*a**3*sqrt(a/b)/(24*a**6*x**(3/2)
*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)
) + 105*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b
) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) - 105*a
**2*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a*
*5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 112*a**2*b*x*s
qrt(a/b)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a*
*4*b**2*x**(7/2)*sqrt(a/b)) - 210*a*b**2*x**(5/2)*log(sqrt(x) - sqrt(a/b))/
(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x
**(7/2)*sqrt(a/b)) + 210*a*b**2*x**(5/2)*log(sqrt(x) + sqrt(a/b))/(24*a**6*
x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*s
qrt(a/b)) - 350*a*b**2*x**2*sqrt(a/b)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5
*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 105*b**3*x**(7/2
)*log(sqrt(x) - sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)
*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) - 105*b**3*x**(7/2)*log(sqrt(
x) + sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b)
+ 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 210*b**3*x**3*sqrt(a/b)/(24*a**6*x**(3
/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a
/b)), True))
```

<u>Giac</u> [A]
time = 0.00, size = 108, normalized size = 1.11

$$-2 \left(-\frac{11 \sqrt{x}\, x b^{3}-13 \sqrt{x}\, b^{2} a}{8 a^{4} \left(x b-a\right)^{2}}+\frac{-9 x b-a}{3 a^{4} \sqrt{x}\, x}-\frac{35 b^{2} \arctan \left(\frac{b \sqrt{x}}{\sqrt{-a b}}\right)}{4 a^{4} \cdot 2 \sqrt{-a b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^4) + 2/3*(9*b*x + a)/(a
^4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) - 13*a*b^2*sqrt(x))/((b*x - a)^2*a^4)
```

Mupad [B]
time = 0.17, size = 80, normalized size = 0.82

$$\frac{\frac{2}{3\,a}-\frac{175\,b^2\,x^2}{12\,a^3}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}-2\,a\,b\,x^{5/2}}-\frac{35\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)^3),x)
```

```
[Out] (2/(3*a) - (175*b^2*x^2)/(12*a^3) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) - 2*a*b*x^(5/2)) - (35*b^(3/2)*atanh((b^(1/2)*
x^(1/2))/a^(1/2)))/(4*a^(9/2))
```

3.489 $\int x^{5/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=122

$$\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

```
[Out] -5/64*a^4*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-5/96*a^2*x^(3/2)*(
b*x+a)^(1/2)/b^2+1/24*a*x^(5/2)*(b*x+a)^(1/2)/b+1/4*x^(7/2)*(b*x+a)^(1/2)+5
/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a + b*x])/(96*
b^2) + (a*x^(5/2)*Sqrt[a + b*x])/(24*b) + (x^(7/2)*Sqrt[a + b*x])/4 - (5*a^
4*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$
$$= \frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^2)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{48b}$$
$$= -\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{(5a^3)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b^2}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\int}{}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\,\mathrm{Su}}{}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\,\mathrm{Su}}{}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\,\mathrm{tanl}}{}$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 0.72

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(15a^3-10a^2bx+8ab^2x^2+48b^3x^3)+15a^4\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a + b*x],x]
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3*
x^3) + 15*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(192*b^(7/2))
```

Mathics [A]
time = 22.00, size = 135, normalized size = 1.11

$$\frac{-15a^{\frac{9}{2}}b^6\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{5}{2}}+15a^2b^{\frac{13}{2}}\sqrt{x}\,(a+bx)^2+5ab^{\frac{15}{2}}x^{\frac{3}{2}}(a+bx)^2-2b^{\frac{17}{2}}x^{\frac{5}{2}}(a+bx)^2+\frac{56b^{\frac{19}{2}}x^{\frac{7}{2}}(a+bx)^2}{a}+\frac{48b^{\frac{21}{2}}x^{\frac{9}{2}}(a+bx)^2}{a^2}}{192\sqrt{a}\,b^{\frac{19}{2}}\left(\frac{a+bx}{a}\right)^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*Sqrt[a + b*x],x]')
```

```
[Out] (-15 a ^ (9 / 2) b ^ 6 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^
 (5 / 2) + 15 a ^ 2 b ^ (13 / 2) Sqrt[x] (a + b x) ^ 2 + 5 a b ^ (15 / 2) x
 ^ (3 / 2) (a + b x) ^ 2 - 2 b ^ (17 / 2) x ^ (5 / 2) (a + b x) ^ 2 + 56 b
^ (19 / 2) x ^ (7 / 2) (a + b x) ^ 2 / a + 48 b ^ (21 / 2) x ^ (9 / 2) (a +
 b x) ^ 2 / a ^ 2) / (192 Sqrt[a] b ^ (19 / 2) ((a + b x) / a) ^ (5 / 2))
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.05

method	result
risch	$\frac{(48b^3x^3+8ab^2x^2-10a^2bx+15a^3)\sqrt{x}\,\sqrt{bx+a}}{192b^3}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{128b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+a)^(3/2)-5/8*a/b*(1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/2*a/b*(1
/2/b*x^(1/2)*(b*x+a)^(3/2)-1/4*a/b*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))
^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1
/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(88) = 176$.
time = 0.35, size = 178, normalized size = 1.46

$$\frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}+\frac{73\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{55\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^7-\frac{4\,(bx+a)b^6}{x}+\frac{6\,(bx+a)^2b^5}{x^2}-\frac{4\,(bx+a)^3b^4}{x^3}+\frac{(bx+a)^4b^3}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) + 73*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) - 55*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^7 - 4*(b*x + a)*b^6/x + 6*(b*x + a)^2*b^5/x^2 - 4*(b*x +
a)^3*b^4/x^3 + (b*x + a)^4*b^3/x^4)
```

Fricas [A]
time = 0.34, size = 162, normalized size = 1.33

$$\left[\frac{15\,a^4\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{384\,b^4},\ \frac{15\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{192\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^4, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^4]
```

Sympy [A]
time = 21.95, size = 153, normalized size = 1.25

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*a**(5/2)*x**(3/2)/(192*b**
2*sqrt(1 + b*x/a)) - a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*sqrt(a)*x
```

```
**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b*
*(7/2)) + b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 151, normalized size = 1.24

$$2\left(2\left(\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\,\sqrt{x}}{b^6}+\frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{10a^4\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{256b^3\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x)
[Out] 1/192*(2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x
) + 5/64*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(1/2),x)
[Out] int(x^(5/2)*(a + b*x)^(1/2), x)
```

3.490 $\int x^{3/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)+1/12*a*x^(3/2)*(b*x+
a)^(1/2)/b+1/3*x^(5/2)*(b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {52, 65, 223, 212}

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] -1/8*(a^2*Sqrt[x]*Sqrt[a + b*x])/b^2 + (a*x^(3/2)*Sqrt[a + b*x])/(12*b) + (
x^(5/2)*Sqrt[a + b*x])/3 + (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
8*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}\sqrt{a+bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}-\frac{a^2\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\int\frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b^2}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,\right.}{8b^2}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{}}{\sqrt{a-}}\right.}{8b^2}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.79

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{a+bx}\,(-3a^2+2abx+8b^2x^2)-3a^3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a + b*x],x]
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) - 3*a^3*Log[-
(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(24*b^(5/2))
```

Mathics [A]
time = 7.24, size = 125, normalized size = 1.28

$$\frac{a^{\frac{3}{2}}\left(3a^{\frac{3}{2}}b^3\mathrm{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](a+bx)^2-3a^3b^{\frac{7}{2}}\sqrt{x}\,\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}-a^2b^{\frac{9}{2}}x^{\frac{3}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+10ab^{\frac{11}{2}}x^{\frac{5}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+8b^{\frac{13}{2}}x^{\frac{7}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}\right)}{24b^{\frac{11}{2}}(a+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*Sqrt[a + b*x],x]')
```

```
[Out] a ^ (3 / 2) (3 a ^ (3 / 2) b ^ 3 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] (a + b
x) ^ 2 - 3 a ^ 3 b ^ (7 / 2) Sqrt[x] ((a + b x) / a) ^ (3 / 2) - a ^ 2 b ^
(9 / 2) x ^ (3 / 2) ((a + b x) / a) ^ (3 / 2) + 10 a b ^ (11 / 2) x ^ (5 /
2) ((a + b x) / a) ^ (3 / 2) + 8 b ^ (13 / 2) x ^ (7 / 2) ((a + b x) / a) ^
 (3 / 2)) / (24 b ^ (11 / 2) (a + b x) ^ 2)
```

Maple [A]
time = 0.12, size = 106, normalized size = 1.08

method	result	size
risch	$-\frac{\left(-8x^{2}b^{2}-2abx+3a^{2}\right)\sqrt{x}\,\sqrt{bx+a}}{24b^{2}}+\frac{a^{3}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\left(bx+a\right)}}{16b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\left(bx+a\right)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/2*a/b*(1/2/b*x^(1/2)*(b*x+a)^(3/2)-1/4*a/b*(x
^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*
a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(70) = 140.
time = 0.37, size = 146, normalized size = 1.49

$$-\frac{a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{5}{2}}}-\frac{\frac{3\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{8\,(bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{5}-\frac{3\,(bx+a)b^{4}}{x}+\frac{3\,(bx+a)^{2}b^{3}}{x^{2}}-\frac{(bx+a)^{3}b^{2}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(5/2) - 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) + 8*(b*x + a)^(3/2
)*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x + a)*b^4/x +
 3*(b*x + a)^2*b^3/x^2 - (b*x + a)^3*b^2/x^3)
```

Fricas [A]
time = 0.33, size = 141, normalized size = 1.44

$$\left[\frac{3\,a^{3}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^{3}},-\frac{3\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A]
time = 5.35, size = 122, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}+\frac{a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(1/2),x)
```

```
[Out] -a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - a**(3/2)*x**(3/2)/(24*b*sqrt(1
 + b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(5/2)) + b*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 125, normalized size = 1.28

$$2\left(2\left(\left(\left(\frac{\frac{1}{576}\cdot 48b^{4}\sqrt{x}\sqrt{x}}{b^{4}}+\frac{\frac{1}{576}\cdot 12b^{3}a}{b^{4}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{576}\cdot 18b^{2}a^{2}}{b^{4}}\right)\sqrt{x}\sqrt{a+bx}-\frac{2a^{3}\ln\left|\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right|}{32b^{2}\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/24*sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 1/8*a^3*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(1/2),x)
[Out] int(x^(3/2)*(a + b*x)^(1/2), x)
```

3.491 $\int \sqrt{x}\,\sqrt{a+bx}\,dx$

Optimal. Leaf size=74

$$\frac{a\sqrt{x}\,\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}$$

```
[Out] -1/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)+1/2*x^(3/2)*(b*x+a)
^(1/2)+1/4*a*x^(1/2)*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\,\sqrt{a+bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (a*Sqrt[x]*Sqrt[a + b*x])/(4*b) + (x^(3/2)*Sqrt[a + b*x])/2 - (a^2*ArcTanh[
(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,\sqrt{a+bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{a\sqrt{x}\,\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b}\\
&= \frac{a\sqrt{x}\,\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{4b}\\
&= \frac{a\sqrt{x}\,\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b}\\
&= \frac{a\sqrt{x}\,\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 63, normalized size = 0.85

$$\frac{\sqrt{x}\,\sqrt{a+bx}\,(a+2bx)}{4b}+\frac{a^2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a + b*x],x]
[Out] (Sqrt[x]*Sqrt[a + b*x]*(a + 2*b*x))/(4*b) + (a^2*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/(4*b^(3/2))
```

Mathics [**A**]
time = 3.99, size = 90, normalized size = 1.22

$$\frac{-a^{\frac{5}{2}}b\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+ab^{\frac{3}{2}}\sqrt{x}\,(a+bx)+3b^{\frac{5}{2}}x^{\frac{3}{2}}(a+bx)+\frac{2b^{\frac{7}{2}}x^{\frac{5}{2}}(a+bx)}{a}}{4\sqrt{a}\,b^{\frac{5}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*Sqrt[a + b*x],x]')
```

```
[Out] (-a ^ (5 / 2) b ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^ (3 / 2
) + a b ^ (3 / 2) Sqrt[x] (a + b x) + 3 b ^ (5 / 2) x ^ (3 / 2) (a + b x) +
 2 b ^ (7 / 2) x ^ (5 / 2) (a + b x) / a) / (4 Sqrt[a] b ^ (5 / 2) ((a + b
x) / a) ^ (3 / 2))
```

Maple [A]
time = 0.14, size = 84, normalized size = 1.14

method	result	size
risch	$\frac{(2bx+a)\sqrt{x}\,\sqrt{bx+a}}{4b}-\frac{a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\left(bx+a\right)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	74
default	$\frac{\sqrt{x}\,\left(bx+a\right)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\left(bx+a\right)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*x^(1/2)*(b*x+a)^(3/2)-1/4*a/b*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a)
)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(
1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(52) = 104.
time = 0.35, size = 108, normalized size = 1.46

$$\frac{a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}+\frac{(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx+a)b^{2}}{x}+\frac{(bx+a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqr
t(x)))/b^(3/2) + 1/4*(sqrt(b*x + a)*a^2*b/sqrt(x) + (b*x + a)^(3/2)*a^2/x^(
3/2))/(b^3 - 2*(b*x + a)*b^2/x + (b*x + a)^2*b/x^2)
```

Fricas [A]
time = 0.32, size = 114, normalized size = 1.54

$$\left[\frac{a^2\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^2x+ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b^2},\ \frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(2\,b^2x+ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A]
time = 2.01, size = 97, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)/(4*b*sqrt(1 + b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 + b*x
/a)) - a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) + b*x**(5/2)/(2*sqr
t(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 92, normalized size = 1.24

$$2\left(2\left(\frac{\frac{1}{32}\cdot 4b^2\sqrt{x}\,\sqrt{x}}{b^2}+\frac{\frac{1}{32}\cdot 2ba}{b^2}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{2a^2\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{16b\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/4*sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + 1/4*a^2*log(abs(-sqrt(b)*sqrt(x) +
sqrt(b*x + a)))/b^(3/2)
```

Mupad [B]
time = 0.15, size = 52, normalized size = 0.70

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)\,\sqrt{a+b\,x}-\frac{a^2\,\ln\left(a+2\,b\,x+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{a+b\,x}\right)}{8\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + a/(4*b))*(a + b*x)^(1/2) - (a^2*log(a + 2*b*x + 2*b^(1/2)*x^
(1/2)*(a + b*x)^(1/2)))/(8*b^(3/2))
```

3.492 $\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=44

$$\sqrt{x}\,\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

```
[Out] a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)+x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$\sqrt{x}\,\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b
]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx &= \sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}a\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\ &= \sqrt{x}\,\sqrt{a+bx}+a\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\,\sqrt{a+bx}+a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= \sqrt{x}\,\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.07

$$\sqrt{x}\,\sqrt{a+bx}-\frac{a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[a + b*x] - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

Mathics [A]
time = 2.62, size = 34, normalized size = 0.77

$$\frac{a\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{\sqrt{b}}+\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/Sqrt[x],x]')
```

[Out] a ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] / Sqrt[b] + Sqrt[a] Sqrt[x] Sqrt[1 + b x / a]

Maple [A]
time = 0.12, size = 62, normalized size = 1.41

method	result	size
default	$\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}$	62
risch	$\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.36, size = 70, normalized size = 1.59

$$-\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,\sqrt{b}}-\frac{\sqrt{bx+a}\,a}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")

[Out] -1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/sqrt(b) - sqrt(b*x + a)*a/((b - (b*x + a)/x)*sqrt(x))

Fricas [A]
time = 0.31, size = 93, normalized size = 2.11

$$\left[\frac{a\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

Sympy [A]
time = 0.94, size = 42, normalized size = 0.95

$$\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(1/2),x)
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.
time = 10.40, size = 89, normalized size = 2.02

$$\frac{b^2\left(\frac{\frac{1}{2}\cdot 2\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{b}-\frac{2a\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{2\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x)
[Out] -(a*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt(b) - sq
rt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*b/abs(b)
```

Mupad [B]
time = 0.68, size = 41, normalized size = 0.93

$$\sqrt{x}\,\sqrt{a+b\,x}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(1/2),x)
[Out] x^(1/2)*(a + b*x)^(1/2) + (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2) - a
^(1/2))))/b^(1/2)
```

3.493 $\int \frac{\sqrt{a+bx}}{x^{3/2}} \, dx$

Optimal. Leaf size=45

$$-\frac{2\sqrt{a+bx}}{\sqrt{x}}+2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))*b^(1/2)-2*(b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b
*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+(2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+(2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 1.04

$$-\frac{2\sqrt{a+bx}}{\sqrt{x}}-2\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(3/2),x]
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*
x]]
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.
time = 2.65, size = 71, normalized size = 1.58

$$\frac{2\sqrt{a}\left(-a\sqrt{\frac{a+bx}{a}}+\frac{\sqrt{b}\,\sqrt{x}\,\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]_{(a+bx)}}{\sqrt{a}}-bx\sqrt{\frac{a+bx}{a}}\right)}{\sqrt{x}\,(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^(3/2),x]')
```

```
[Out] 2 Sqrt[a] (-a Sqrt[(a + b x) / a] + Sqrt[b] Sqrt[x] ArcSinh[Sqrt[b] Sqrt[x]
 / Sqrt[a]] (a + b x) / Sqrt[a] - b x Sqrt[(a + b x) / a]) / (Sqrt[x] (a +
b x))
```

Maple [A]
time = 0.11, size = 61, normalized size = 1.36

method	result	size
risch	$-\frac{2\sqrt{bx+a}}{\sqrt{x}}+\frac{\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x+a)^(1/2)/x^(1/2)+b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*
(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.36, size = 54, normalized size = 1.20

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)/sqrt(x)
```

Fricas [A]
time = 0.31, size = 89, normalized size = 1.98

$$\left[\frac{\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 0.78, size = 68, normalized size = 1.51

$$-\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt
(a)) - 2*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 10.38, size = 96, normalized size = 2.13

$$\frac{bb^2\left(-\frac{2\sqrt{a+bx}\sqrt{-ab+b(a+bx)}}{-ab+b(a+bx)}-\frac{2\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] -2*b^2*(log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt(b)
+ sqrt(b*x + a)/sqrt((b*x + a)*b - a*b))/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/x^(3/2), x)
```

3.494 $\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

```
[Out] -2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A]

time = 0.02, size = 21, normalized size = 1.00

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Mathics [A]
time = 2.54, size = 33, normalized size = 1.57

$$\frac{2\sqrt{b}\,(-a-bx)\sqrt{\frac{a+bx}{bx}}}{3ax}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^(5/2),x]')
```

```
[Out] 2 Sqrt[b] (-a - b x) Sqrt[(a + b x) / (b x)] / (3 a x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.
time = 0.12, size = 49, normalized size = 2.33

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
risch	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
default	$-\frac{\sqrt{bx+a}}{x^{\frac{3}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1/2)/x^(3/2)-1/2*a*(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1
/2)/a^2/x^(1/2))
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A]
time = 0.32, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 0.79, size = 41, normalized size = 1.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.01, size = 78, normalized size = 3.71

$$-\frac{\frac{1}{9}\cdot 3\cdot 2b^2b^3\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{|b|\,ba\,(-ab+b\,(a+bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x)
```

```
[Out] -2/3*(b*x + a)^(3/2)*b^4/(((b*x + a)*b - a*b)^(3/2)*a*abs(b))
```

Mupad [B]
time = 0.24, size = 21, normalized size = 1.00

$$-\frac{\left(\frac{2\,b\,x}{3\,a}+\frac{2}{3}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(5/2),x)
```

```
[Out] -(((2*b*x)/(3*a) + 2/3)*(a + b*x)^(1/2))/x^(3/2)
```

3.495 $\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}$$

```
[Out] -2/5*(b*x+a)^(3/2)/a/x^(5/2)+4/15*b*(b*x+a)^(3/2)/a^2/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}-\frac{(2b)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 39, normalized size = 0.89

$$-\frac{2\sqrt{a+bx}\,(3a^2+abx-2b^2x^2)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*a^2 + a*b*x - 2*b^2*x^2))/(15*a^2*x^(5/2))
```

Mathics [**A**]
time = 4.40, size = 44, normalized size = 1.00

$$\frac{2\sqrt{b}\,(-3a^2-abx+2b^2x^2)\sqrt{\frac{a+bx}{bx}}}{15a^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^(7/2),x]')
```

```
[Out] 2 Sqrt[b] (-3 a ^ 2 - a b x + 2 b ^ 2 x ^ 2) Sqrt[(a + b x) / (b x)] / (15
a ^ 2 x ^ 2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.10, size = 71, normalized size = 1.61

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-2bx+3a)}{15x^{\frac{5}{2}}a^2}$	24
risch	$-\frac{2\sqrt{bx+a}\,(-2x^2b^2+abx+3a^2)}{15x^{\frac{5}{2}}a^2}$	34
default	$-\frac{\sqrt{bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{4}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x+a)^(1/2)/x^(5/2)-1/4*a*(-2/5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*(-2/
3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)))
```

Maxima [A]
time = 0.26, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b*x + a)^(3/2)*b/x^(3/2) - 3*(b*x + a)^(5/2)/x^(5/2))/a^2
```

Fricas [A]
time = 0.31, size = 34, normalized size = 0.77

$$\frac{2\,(2\,b^2x^2-abx-3\,a^2)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Sympy [A]
time = 2.94, size = 65, normalized size = 1.48

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a*
x) + 4*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2)
```

Giac [A]
time = 0.01, size = 113, normalized size = 2.57

$$\frac{2b^2\left(\frac{\frac{1}{225}\cdot 30b^5\sqrt{a+bx}\,\sqrt{a+bx}}{a^2}-\frac{\frac{1}{225}\cdot 75b^5a}{a^2}\right)\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{|b|\,b\,(-ab+b\,(a+bx))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x)
```

```
[Out] 2/15*(2*(b*x + a)*b^5/a^2 - 5*b^5/a)*(b*x + a)^(3/2)*b/(((b*x + a)*b - a*b)
^(5/2)*abs(b))
```

Mupad [B]
time = 0.25, size = 32, normalized size = 0.73

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,b\,x}{15\,a}-\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(7/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((2*b*x)/(15*a) - (4*b^2*x^2)/(15*a^2) + 2/5))/x^(5/2)
```

3.496 $\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}$$

```
[Out] -2/7*(b*x+a)^(3/2)/a/x^(7/2)+8/35*b*(b*x+a)^(3/2)/a^2/x^(5/2)-16/105*b^2*(b
*x+a)^(3/2)/a^3/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) + (8*b*(a + b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a + b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} - \frac{(4b)\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx}{7a}$$
$$= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} + \frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}} + \frac{(8b^2)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{35a^2}$$
$$= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} + \frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.75

$$-\frac{2\sqrt{a+bx}\,(15a^3+3a^2bx-4ab^2x^2+8b^3x^3)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

Mathics [A]
time = 11.73, size = 95, normalized size = 1.40

$$\frac{2\sqrt{b}\,(-15a^5-33a^4bx-17a^3b^2x^2-3a^2b^3x^3-12ab^4x^4-8b^5x^5)\sqrt{\frac{a+bx}{bx}}}{105a^3x^3\,(a^2+2abx+b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a + b*x]/x^(9/2),x]')
```

```
[Out] 2 Sqrt[b] (-15 a ^ 5 - 33 a ^ 4 b x - 17 a ^ 3 b ^ 2 x ^ 2 - 3 a ^ 2 b ^ 3
x ^ 3 - 12 a b ^ 4 x ^ 4 - 8 b ^ 5 x ^ 5) Sqrt[(a + b x) / (b x)] / (105 a
^ 3 x ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.11, size = 93, normalized size = 1.37

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(8x^2b^2-12abx+15a^2)}{105x^{\frac{7}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}\,(8b^3x^3-4a\,b^2x^2+3a^2bx+15a^3)}{105x^{\frac{7}{2}}a^3}$	46

default	$-\frac{\sqrt{bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x+a)^(1/2)/x^(7/2)-1/6*a*(-2/7*(b*x+a)^(1/2)/a/x^(7/2)-6/7*b/a*(-2/
5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+
a)^(1/2)/a^2/x^(1/2))))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{35\,(bx+a)^{\frac{3}{2}}\,b^{2}}{x^{\frac{3}{2}}}-\frac{42\,(bx+a)^{\frac{5}{2}}\,b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(b*x + a)^(3/2)*b^2/x^(3/2) - 42*(b*x + a)^(5/2)*b/x^(5/2) + 15*
(b*x + a)^(7/2)/x^(7/2))/a^3
```

Fricas [A]
time = 0.30, size = 45, normalized size = 0.66

$$-\frac{2\,(8\,b^{3}x^{3}-4\,ab^{2}x^{2}+3\,a^{2}bx+15\,a^{3})\sqrt{bx+a}}{105\,a^{3}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.
time = 9.62, size = 347, normalized size = 5.10

$$-\frac{30a^{5}b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}+210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{66a^{4}b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}+210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{34a^{3}b^{\frac{13}{2}}x^{2}\sqrt{\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}+210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{6a^{2}b^{\frac{15}{2}}x^{3}\sqrt{\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}+210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{24ab^{\frac{17}{2}}x^{4}\sqrt{\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}+210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{16b^{\frac{19}{2}}x^{5}\sqrt{\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}+210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] -30*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*x**
2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b*x) +
1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 16*b**(
19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105
*a**3*b**6*x**5)
```

Giac [A]
time = 0.01, size = 149, normalized size = 2.19

$$\frac{2b^2\left(\left(\left(-\frac{\frac{1}{3675}\cdot 280b^7\sqrt{a+bx}\,\sqrt{a+bx}}{a^3}+\frac{\frac{1}{3675}\cdot 980b^7a}{a^3}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{\frac{1}{3675}\cdot 1225b^7a^2}{a^3}\right)\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{|b|\,b\,(-ab+b\,(a+bx))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(35*b^7/a + 4*(2*(b*x + a)*b^7/a^3 - 7*b^7/a^2)*(b*x + a))*(b*x + a)
^(3/2)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.26, size = 43, normalized size = 0.63

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{16\,b^3\,x^3}{105\,a^3}-\frac{8\,b^2\,x^2}{105\,a^2}+\frac{2\,b\,x}{35\,a}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(9/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((16*b^3*x^3)/(105*a^3) - (8*b^2*x^2)/(105*a^2) + (2*b*x)
/(35*a) + 2/7))/x^(7/2)
```

3.497 $\int x^{5/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=127

$$-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}$$

```
[Out] 5/64*a^4*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)-5/96*a^2*x^(3/2)*(-
b*x+a)^(1/2)/b^2-1/24*a*x^(5/2)*(-b*x+a)^(1/2)/b+1/4*x^(7/2)*(-b*x+a)^(1/2)
-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a - b*x])/(96
*b^2) - (a*x^(5/2)*Sqrt[a - b*x])/(24*b) + (x^(7/2)*Sqrt[a - b*x])/4 + (5*a
^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a-bx}\,dx=\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$

$$=-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^2)\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{48b}$$

$$=-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^3)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b^2}$$

$$=-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^4)}{}$$

$$=-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^4)}{}$$

$$=-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^4)}{}$$

$$=-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{5a^4\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.72

$$\frac{1}{192}\left(\frac{\sqrt{x}\sqrt{a-bx}\,(-15a^3-10a^2bx-8ab^2x^2+48b^3x^3)}{b^3}+\frac{15a^4\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-15*a^3 - 10*a^2*b*x - 8*a*b^2*x^2 + 48*b^3*x^3))/
b^3 + (15*a^4*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2))/192
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 22.58, size = 286, normalized size = 2.25

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-15a^{\frac{13}{2}}b^{6}\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{5}{2}}+15a^{4}b^{\frac{13}{2}}\sqrt{x}\,(-a+bx)^{2}-5a^{3}b^{\frac{15}{2}}x^{\frac{3}{2}}(-a+bx)^{2}-2a^{2}b^{\frac{17}{2}}x^{\frac{5}{2}}(-a+bx)^{2}-56ab^{\frac{19}{2}}x^{\frac{7}{2}}(-a+bx)^{2}+48b^{\frac{21}{2}}x^{\frac{9}{2}}(-a+bx)^{2}\right)}{192a^{\frac{5}{2}}b^{\frac{19}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{5}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{5a^{4}\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{64b^{\frac{7}{2}}}-\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}-\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*Sqrt[a - b*x],x]')
```

```
[Out] Piecewise[{{I / 192 (-15 a ^ (13 / 2) b ^ 6 ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[
a]] ((-a + b x) / a) ^ (5 / 2) + 15 a ^ 4 b ^ (13 / 2) Sqrt[x] (-a + b x) ^
 2 - 5 a ^ 3 b ^ (15 / 2) x ^ (3 / 2) (-a + b x) ^ 2 - 2 a ^ 2 b ^ (17 / 2)
 x ^ (5 / 2) (-a + b x) ^ 2 - 56 a b ^ (19 / 2) x ^ (7 / 2) (-a + b x) ^ 2
+ 48 b ^ (21 / 2) x ^ (9 / 2) (-a + b x) ^ 2) / (a ^ (5 / 2) b ^ (19 / 2) (
(-a + b x) / a) ^ (5 / 2)), Abs[b x / a] > 1}}, 5 a ^ 4 ArcSin[Sqrt[b] Sqrt
[x] / Sqrt[a]] / (64 b ^ (7 / 2)) - 5 a ^ (7 / 2) Sqrt[x] / (64 b ^ 3 Sqrt[
1 - b x / a]) + 5 a ^ (5 / 2) x ^ (3 / 2) / (192 b ^ 2 Sqrt[1 - b x / a]) +
 a ^ (3 / 2) x ^ (5 / 2) / (96 b Sqrt[1 - b x / a]) + 7 Sqrt[a] x ^ (7 / 2)
 / (24 Sqrt[1 - b x / a]) - b x ^ (9 / 2) / (4 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.13, size = 135, normalized size = 1.06

method	result
risch	$-\frac{\left(-48 b^{3} x^{3}+8 a\,b^{2} x^{2}+10 a^{2} b x+15 a^{3}\right) \sqrt{x}\,\sqrt{-b x+a}}{192 b^{3}}+\frac{5 a^{4} \arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right) \sqrt{x\left(-b x+a\right)}}{128 b^{\frac{7}{2}} \sqrt{x}\,\sqrt{-b x+a}}$

default	$-\frac{x^{\frac{5}{2}}(-bx+a)^{\frac{3}{2}}}{4b}+\frac{5a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3b}+\frac{a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,(}{\sqrt{-x^2}}\right.}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right.}{4b}\right.}{2b}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+a)^(3/2)+5/8*a/b*(-1/3/b*x^(3/2)*(-b*x+a)^(3/2)+1/2*a/
b*(-1/2/b*x^(1/2)*(-b*x+a)^(3/2)+1/4*a/b*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(
-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-
b*x^2+a*x)^(1/2)))))
```

Maxima [A]

time = 0.40, size = 170, normalized size = 1.34

$$-\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{7}{2}}}+\frac{\frac{15\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}-\frac{73\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{55\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^7-\frac{4\,(bx-a)b^6}{x}+\frac{6\,(bx-a)^2b^5}{x^2}-\frac{4\,(bx-a)^3b^4}{x^3}+\frac{(bx-a)^4b^3}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) - 73*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 55*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^7 - 4*(b*x
- a)*b^6/x + 6*(b*x - a)^2*b^5/x^2 - 4*(b*x - a)^3*b^4/x^3 + (b*x - a)^4*b^
3/x^4)
```

Fricas [A]

time = 0.31, size = 164, normalized size = 1.29

$$\left[-\frac{15\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^4},-\frac{15\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*s
qrt(x))/b^4, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))
) - (48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^4]
```

Sympy [A]
time = 21.13, size = 323, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} + \frac{ibx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}} + \frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} - \frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(5/2)*x
**(3/2)/(192*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(5/2)/(96*b*sqrt(-1 + b
*x/a)) - 7*I*sqrt(a)*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqrt(b
)*sqrt(x)/sqrt(a))/(64*b**(7/2)) + I*b*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a)
), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 - b*x/a)) + 5*a**(
5/2)*x**(3/2)/(192*b**2*sqrt(1 - b*x/a)) + a**(3/2)*x**(5/2)/(96*b*sqrt(1 -
 b*x/a)) + 7*sqrt(a)*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(7/2)) - b*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True
))
```

Giac [A]
time = 0.01, size = 158, normalized size = 1.24

$$2\left(2\left(\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\sqrt{x}}{b^6} - \frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\sqrt{a-bx} - \frac{10a^4\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{256b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x)
```

```
[Out] 1/192*(2*(4*(6*x - a/b)*x - 5*a^2/b^2)*x - 15*a^3/b^3)*sqrt(-b*x + a)*sqrt(
x) - 5/64*a^4*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\sqrt{a-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(1/2),x)
[Out] int(x^(5/2)*(a - b*x)^(1/2), x)
```

3.498 $\int x^{3/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=102

$$-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)-1/12*a*x^(3/2)*(-b*x
+a)^(1/2)/b+1/3*x^(5/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] -1/8*(a^2*Sqrt[x]*Sqrt[a - b*x])/b^2 - (a*x^(3/2)*Sqrt[a - b*x])/(12*b) + (
x^(5/2)*Sqrt[a - b*x])/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8
*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}\sqrt{a-bx}\,dx=\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$
$$=-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^2\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\int\frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\text{Subst}\left(\int\frac{1}{\sqrt{a-bx^2}}\,dx,\right.}{8b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\text{Subst}\left(\int\frac{1}{1+bx^2}\,dx,x,\frac{}{\sqrt{a}}\right.}{8b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 0.79

$$\frac{1}{24}\left(\frac{\sqrt{x}\sqrt{a-bx}\,(-3a^2-2abx+8b^2x^2)}{b^2}-\frac{3a^3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-3*a^2 - 2*a*b*x + 8*b^2*x^2))/b^2 - (3*a^3*Log[-(
Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(5/2))/24
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 12.85, size = 252, normalized size = 2.47

$$\text{Piecewise}\left[\left\{\left\{\frac{Ia^{\frac{3}{2}}\left(-3a^{\frac{3}{2}}b^3\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right](-a+bx)^2+3a^3b^{\frac{7}{2}}\sqrt{x}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-a^2b^{\frac{9}{2}}x^{\frac{3}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-10ab^{\frac{11}{2}}x^{\frac{5}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+8b^{\frac{13}{2}}x^{\frac{7}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}\right)}{24b^{\frac{11}{2}}(-a+bx)^2},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{a^3\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{8b^{\frac{5}{2}}}-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1-\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}-\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*Sqrt[a - b*x],x]')
```

```
[Out] Piecewise[{{I / 24 a ^ (3 / 2) (-3 a ^ (3 / 2) b ^ 3 ArcCosh[Sqrt[b] Sqrt[x
] / Sqrt[a]] (-a + b x) ^ 2 + 3 a ^ 3 b ^ (7 / 2) Sqrt[x] ((-a + b x) / a)
^ (3 / 2) - a ^ 2 b ^ (9 / 2) x ^ (3 / 2) ((-a + b x) / a) ^ (3 / 2) - 10 a
 b ^ (11 / 2) x ^ (5 / 2) ((-a + b x) / a) ^ (3 / 2) + 8 b ^ (13 / 2) x ^ (
7 / 2) ((-a + b x) / a) ^ (3 / 2)) / (b ^ (11 / 2) (-a + b x) ^ 2), Abs[b x
 / a] > 1}}, a ^ 3 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (8 b ^ (5 / 2)) - a
^ (5 / 2) Sqrt[x] / (8 b ^ 2 Sqrt[1 - b x / a]) + a ^ (3 / 2) x ^ (3 / 2) /
 (24 b Sqrt[1 - b x / a]) + 5 Sqrt[a] x ^ (5 / 2) / (12 Sqrt[1 - b x / a])
- b x ^ (7 / 2) / (3 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.13, size = 112, normalized size = 1.10

method	result	siz
risch	$-\frac{(-8x^2b^2+2abx+3a^2)\sqrt{x}\,\sqrt{-bx+a}}{24b^2}+\frac{a^3\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)\sqrt{x(-bx+a)}}{16b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	91
default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3b}+\frac{a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+a)^(3/2)+1/2*a/b*(-1/2/b*x^(1/2)*(-b*x+a)^(3/2)+1/4*a/
b*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b
^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]

time = 0.36, size = 135, normalized size = 1.32

$$-\frac{a^3 \arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{3\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}-\frac{8\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^5-\frac{3\,(bx-a)b^4}{x}+\frac{3\,(bx-a)^2b^3}{x^2}-\frac{(bx-a)^3b^2}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) - 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x - a)*b^4/x + 3*(b*x - a)^2*b^3/x^2 - (b*x -
 a)^3*b^2/x^3)
```

Fricas [A]
time = 0.31, size = 142, normalized size = 1.39

$$\left[-\frac{3\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\,(8\,b^3x^2-2\,ab^2x-3\,a^2b)\sqrt{-bx+a}\sqrt{x}}{48\,b^3},-\frac{3\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-(8\,b^3x^2-2\,ab^2x-3\,a^2b)\sqrt{-bx+a}\sqrt{x}}{24\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
- 2*(8*b^3*x^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/24*(3
*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 2*a*b^
2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [A]
time = 5.34, size = 260, normalized size = 2.55

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{-1+\frac{bx}{a}}}-\frac{5i\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{ibx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1-\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(3/
2)/(24*b*sqrt(-1 + b*x/a)) - 5*I*sqrt(a)*x**(5/2)/(12*sqrt(-1 + b*x/a)) - I
*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + I*b*x**(7/2)/(3*sqrt(a)
```

```
*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 - b*
x/a)) + a**(3/2)*x**(3/2)/(24*b*sqrt(1 - b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*s
qrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) - b*x**(7
/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 130, normalized size = 1.27

$$2\left(2\left(\left(\left(\frac{\frac{1}{576}\cdot 48b^4\sqrt{x}\,\sqrt{x}}{b^4}-\frac{\frac{1}{576}\cdot 12b^3a}{b^4}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{576}\cdot 18b^2a^2}{b^4}\right)\sqrt{x}\,\sqrt{a-bx}-\frac{2a^3\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{32b^2\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x)
```

```
[Out] 1/24*sqrt(-b*x + a)*(2*(4*x - a/b)*x - 3*a^2/b^2)*sqrt(x) - 1/8*a^3*log(abs
(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a-b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(1/2), x)
```

3.499 $\int \sqrt{x}\sqrt{a-bx}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}$$

```
[Out] 1/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)+1/2*x^(3/2)*(-b*x+a)
^(1/2)-1/4*a*x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] -1/4*(a*Sqrt[x]*Sqrt[a - b*x])/b + (x^(3/2)*Sqrt[a - b*x])/2 + (a^2*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{a-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 0.92

$$\frac{\sqrt{x}\sqrt{a-bx}(-a+2bx)}{4b}+\frac{a^2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{4(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a - b*x],x]
[Out] (Sqrt[x]*Sqrt[a - b*x]*(-a + 2*b*x))/(4*b) + (a^2*Log[-(Sqrt[-b]*Sqrt[x]) +
 Sqrt[a - b*x]])/(4*(-b)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.21, size = 192, normalized size = 2.49

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-a^{\frac{7}{2}}b\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+a^2b^{\frac{3}{2}}\sqrt{x}\,(-a+bx)+3ab^{\frac{5}{2}}x^{\frac{3}{2}}\,(a-bx)+2b^{\frac{7}{2}}x^{\frac{5}{2}}\,(-a+bx)\right)}{4a^{\frac{3}{2}}b^{\frac{5}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{a^2\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{4b^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1-\frac{bx}{a}}}+\frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}-\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*Sqrt[a - b*x],x]')
```

```
[Out] Piecewise[{{I / 4 (-a ^ (7 / 2) b ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((-a +
 b x) / a) ^ (3 / 2) + a ^ 2 b ^ (3 / 2) Sqrt[x] (-a + b x) + 3 a b ^ (5 /
2) x ^ (3 / 2) (a - b x) + 2 b ^ (7 / 2) x ^ (5 / 2) (-a + b x)) / (a ^ (3
/ 2) b ^ (5 / 2) ((-a + b x) / a) ^ (3 / 2)), Abs[b x / a] > 1}}, a ^ 2 Arc
Sin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (4 b ^ (3 / 2)) - a ^ (3 / 2) Sqrt[x] / (4
 b Sqrt[1 - b x / a]) + 3 Sqrt[a] x ^ (3 / 2) / (4 Sqrt[1 - b x / a]) - b x
 ^ (5 / 2) / (2 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.11, size = 89, normalized size = 1.16

method	result	size
risch	$-\frac{(-2bx+a)\sqrt{x}\,\sqrt{-bx+a}}{4b}+\frac{a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\left(-bx+a\right)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	78
default	$-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*x^(1/2)*(-b*x+a)^(3/2)+1/4*a/b*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*
x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x
^2+a*x)^(1/2)))
```

Maxima [A]
time = 0.41, size = 95, normalized size = 1.23

$$-\frac{a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}-\frac{(-bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx-a)b^{2}}{x}+\frac{(bx-a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/4*(sqrt(-b*x
+ a)*a^2*b/sqrt(x) - (-b*x + a)^(3/2)*a^2/x^(3/2))/(b^3 - 2*(b*x - a)*b^2/x
 + (b*x - a)^2*b/x^2)
```

Fricas [A]
time = 0.31, size = 118, normalized size = 1.53

$$\left[-\frac{a^{2} \sqrt{-b} \log \left(-2 \, b x + 2 \, \sqrt{-b x + a} \sqrt{-b} \sqrt{x} + a\right) - 2 \, {\left(2 \, b^{2} x - a b\right)} \sqrt{-b x + a} \sqrt{x}}{8 \, b^{2}}, -\frac{a^{2} \sqrt{b} \arctan \left(\frac{\sqrt{-b x + a}}{\sqrt{b} \sqrt{x}}\right) - {\left(2 \, b^{2} x - a b\right)} \sqrt{-b x + a} \sqrt{x}}{4 \, b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2
*(2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/4*(a^2*sqrt(b)*arctan(sqrt
(-b*x + a)/(sqrt(b)*sqrt(x))) - (2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2
]
```

Sympy [A]
time = 2.01, size = 207, normalized size = 2.69

$$\begin{cases} \frac{i a^{\frac{3}{2}} \sqrt{x}}{4 b \sqrt{-1 + \frac{b x}{a}}} - \frac{3 i \sqrt{a} x^{\frac{3}{2}}}{4 \sqrt{-1 + \frac{b x}{a}}} - \frac{i a^{2} \operatorname{acosh}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{4 b^{\frac{3}{2}}} + \frac{i b x^{\frac{5}{2}}}{2 \sqrt{a} \sqrt{-1 + \frac{b x}{a}}} & \text{for}\: \left|{\frac{b x}{a}}\right| > 1 \\- \frac{a^{\frac{3}{2}} \sqrt{x}}{4 b \sqrt{1 - \frac{b x}{a}}} + \frac{3 \sqrt{a} x^{\frac{3}{2}}}{4 \sqrt{1 - \frac{b x}{a}}} + \frac{a^{2} \operatorname{asin}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{4 b^{\frac{3}{2}}} - \frac{b x^{\frac{5}{2}}}{2 \sqrt{a} \sqrt{1 - \frac{b x}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(3/2)*sqrt(x)/(4*b*sqrt(-1 + b*x/a)) - 3*I*sqrt(a)*x**(3/2)
/(4*sqrt(-1 + b*x/a)) - I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2))
+ I*b*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(3/2)*sq
rt(x)/(4*b*sqrt(1 - b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 - b*x/a)) + a**2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) - b*x**(5/2)/(2*sqrt(a)*sqrt(1
- b*x/a)), True))
```

Giac [A]
time = 0.01, size = 98, normalized size = 1.27

$$2 \left(2 \left(\frac{\frac{1}{32} \cdot 4 b^{2} \sqrt{x} \sqrt{x}}{b^{2}} - \frac{\frac{1}{32} \cdot 2 b a}{b^{2}}\right) \sqrt{x} \sqrt{a - b x} - \frac{2 a^{2} \ln \left|\sqrt{a - b x} - \sqrt{-b} \sqrt{x}\right|}{16 b \sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x)
```

```
[Out] 1/4*sqrt(-b*x + a)*(2*x - a/b)*sqrt(x) - 1/4*a^2*log(abs(-sqrt(-b)*sqrt(x)
+ sqrt(-b*x + a)))/(sqrt(-b)*b)
```

Mupad [B]
time = 0.08, size = 58, normalized size = 0.75

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{a}{4\,b}\right)\,\sqrt{a-b\,x}-\frac{a^2\,\ln\left(a-2\,b\,x+2\,\sqrt{-b}\,\sqrt{x}\,\sqrt{a-b\,x}\right)}{8\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - a/(4*b))*(a - b*x)^(1/2) - (a^2*log(a - 2*b*x + 2*(-b)^(1/2)
*x^(1/2)*(a - b*x)^(1/2)))/(8*(-b)^(3/2))
```

3.500 $\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=46

$$\sqrt{x}\,\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

```
[Out] a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)+x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\sqrt{x}\,\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}a\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \sqrt{x}\,\sqrt{a-bx}+a\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}\,\sqrt{a-bx}+a\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \sqrt{x}\,\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.15

$$\sqrt{x}\,\sqrt{a-bx}+\frac{ab\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*b*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)
^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.95, size = 130, normalized size = 2.83

$$\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{a}\left(-\sqrt{a}\,\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](-a+bx)-a\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{-a+bx}{a}}+b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{-a+bx}{a}}\right)}{\sqrt{b}\,(-a+bx)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{a\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{\sqrt{b}}+\sqrt{a}\,\sqrt{x}\,\sqrt{1-\frac{bx}{a}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a - b*x]/Sqrt[x],x]')
```

```
[Out] Piecewise[{{I Sqrt[a] (-Sqrt[a] ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] (-a + b
x) - a Sqrt[b] Sqrt[x] Sqrt[(-a + b x) / a] + b ^ (3 / 2) x ^ (3 / 2) Sqrt[
(-a + b x) / a]) / (Sqrt[b] (-a + b x)), Abs[b x / a] > 1}}, a ArcSin[Sqrt[
b] Sqrt[x] / Sqrt[a]] / Sqrt[b] + Sqrt[a] Sqrt[x] Sqrt[1 - b x / a]]
```

Maple [A]
time = 0.12, size = 66, normalized size = 1.43

method	result	size
default	$\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}$	66
risch	$\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1
/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))
```

Maxima [A]
time = 0.38, size = 52, normalized size = 1.13

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\sqrt{-bx+a}\,a}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + sqrt(-b*x + a)*a/((b
- (b*x - a)/x)*sqrt(x))
```

Fricas [A]
time = 0.32, size = 94, normalized size = 2.04

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*s
qrt(-b*x + a)*b*sqrt(x))/b, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt
(x))) - sqrt(-b*x + a)*b*sqrt(x))/b]
```

Sympy [A]
time = 0.96, size = 119, normalized size = 2.59

$$\begin{cases} -\frac{i\sqrt{a}\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}} - \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{ibx^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \sqrt{a}\sqrt{x}\sqrt{1-\frac{bx}{a}} + \frac{a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(1/2),x)
[Out] Piecewise((-I*sqrt(a)*sqrt(x)/sqrt(-1 + b*x/a) - I*a*acosh(sqrt(b)*sqrt(x)/
sqrt(a))/sqrt(b) + I*b*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (sqrt(a)*sqrt(x)*sqrt(1 - b*x/a) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.
time = 10.60, size = 94, normalized size = 2.04

$$\frac{b^2\left(\frac{\frac{1}{2}\cdot 2\sqrt{a-bx}\sqrt{ab-b(a-bx)}}{b} + \frac{2a\ln\left|\sqrt{ab-b(a-bx)}-\sqrt{-b}\sqrt{a-bx}\right|}{2\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x)
[Out] (a*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) +
sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*b/abs(b)
```

Mupad [B]
time = 0.59, size = 43, normalized size = 0.93

$$\sqrt{x}\sqrt{a-b\,x} + \frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(1/2),x)
[Out] x^(1/2)*(a - b*x)^(1/2) + (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^
(1/2))))/b^(1/2)
```

3.501 $\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

[Out] -2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))*b^(1/2)-2*(-b*x+a)^(1/2)/x^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 65, 223, 209}

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b*x]/x^(3/2),x]

[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.13

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a -
b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.85, size = 145, normalized size = 3.09

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\left(a^{\frac{3}{2}}\sqrt{\frac{-a+bx}{a}}+\sqrt{b}\,\sqrt{x}\,\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](-a+bx)-\sqrt{a}\,bx\sqrt{\frac{-a+bx}{a}}\right)}{\sqrt{x}\,(-a+bx)}, \text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\}, \frac{-2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-2\sqrt{b}\,\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]+\frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a - b*x]/x^(3/2),x]')
```

```
[Out] Piecewise[{{2 I (a ^ (3 / 2) Sqrt[(-a + b x) / a] + Sqrt[b] Sqrt[x] ArcCosh
[Sqrt[b] Sqrt[x] / Sqrt[a]] (-a + b x) - Sqrt[a] b x Sqrt[(-a + b x) / a])
/ (Sqrt[x] (-a + b x)), Abs[b x / a] > 1}}, -2 Sqrt[a] / (Sqrt[x] Sqrt[1 -
b x / a]) - 2 Sqrt[b] ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] + 2 b Sqrt[x] / (Sq
rt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.13, size = 66, normalized size = 1.40

method	result	size
risch	$-\frac{2\sqrt{-bx+a}}{\sqrt{x}}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\left(-bx+a\right)}}{\sqrt{x}\,\sqrt{-bx+a}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-b*x+a)^(1/2)/x^(1/2)-b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(
1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.36, size = 35, normalized size = 0.74

$$2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\sqrt{-bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)/sqrt(
x)
```

Fricas [A]
time = 0.33, size = 91, normalized size = 1.94

$$\left[\frac{\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,\sqrt{x}}{x},\ \frac{2\,\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*sqrt(-
b*x + a)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))
- sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 0.83, size = 148, normalized size = 3.15

$$\begin{cases} \frac{2i\sqrt{a}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} + 2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2ib\sqrt{x}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} - 2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(a)/(sqrt(x)*sqrt(-1 + b*x/a)) + 2*I*sqrt(b)*acosh(sqrt(
b)*sqrt(x)/sqrt(a)) - 2*I*b*sqrt(x)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a)
> 1), (-2*sqrt(a)/(sqrt(x)*sqrt(1 - b*x/a)) - 2*sqrt(b)*asin(sqrt(b)*sqrt(x
)/sqrt(a)) + 2*b*sqrt(x)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 10.30, size = 102, normalized size = 2.17

$$-\frac{bb^2\left(\frac{2\sqrt{a-bx}\sqrt{ab-b(a-bx)}}{ab-b(a-bx)} + \frac{2\ln\left|\sqrt{ab-b(a-bx)}-\sqrt{-b}\sqrt{a-bx}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] -2*b^2*(log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-
b) + sqrt(-b*x + a)/sqrt((b*x - a)*b + a*b))/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(1/2)/x^(3/2), x)
```

3.502 $\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

```
[Out] -2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx=-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A]
time = 0.06, size = 22, normalized size = 1.00

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.65, size = 90, normalized size = 4.09

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\,(-a+bx)\sqrt{\frac{a-bx}{bx}}}{3ax},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{I2b^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}{3a}-\frac{2I\sqrt{b}\sqrt{1-\frac{a}{bx}}}{3x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a - b*x]/x^(5/2),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-a + b x) Sqrt[(a - b x) / (b x)] / (3 a x), Abs[a /
 (b x)] > 1}}, I 2 b ^ (3 / 2) Sqrt[1 - a / (b x)] / (3 a) - 2 I Sqrt[b] Sq
rt[1 - a / (b x)] / (3 x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.
time = 0.11, size = 52, normalized size = 2.36

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
risch	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
default	$-\frac{\sqrt{-bx+a}}{x^{\frac{3}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+a)^(1/2)/x^(3/2)-1/2*a*(-2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)
^(1/2)/a^2/x^(1/2))
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.73

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A]
time = 0.30, size = 23, normalized size = 1.05

$$\frac{2\,(bx-a)\sqrt{-bx+a}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)/(a*x^(3/2))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 88, normalized size = 4.00

$$\begin{cases} -\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{3x}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{2i\sqrt{b}\,\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{2ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(5/2),x)
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 2*b**(3/2)*sqrt(a/(b*x) - 1
)/(3*a), Abs(a/(b*x)) > 1), (-2*I*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 2*I*b*
*(3/2)*sqrt(-a/(b*x) + 1)/(3*a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(16) = 32.
time = 0.01, size = 83, normalized size = 3.77

$$-\frac{\frac{1}{9}\cdot 3\cdot 2b^2b^3\sqrt{a-bx}\,\sqrt{a-bx}\,\sqrt{a-bx}\,\sqrt{ab-b\,(a-bx)}}{|b|\,ba\,(ab-b\,(a-bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x)
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)*b^4/(((b*x - a)*b + a*b)^(3/2)*a*abs(b))
```

Mupad [B]
time = 0.24, size = 21, normalized size = 0.95

$$\frac{\left(\frac{2\,b\,x}{3\,a}-\frac{2}{3}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(5/2),x)
[Out] (((2*b*x)/(3*a) - 2/3)*(a - b*x)^(1/2))/x^(3/2)
```

3.503 $\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}$$

```
[Out] -2/5*(-b*x+a)^(3/2)/a/x^(5/2)-4/15*b*(-b*x+a)^(3/2)/a^2/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}+\frac{(2b)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.89

$$-\frac{2\sqrt{a-bx}\,(3a^2-abx-2b^2x^2)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(3*a^2 - a*b*x - 2*b^2*x^2))/(15*a^2*x^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.53, size = 229, normalized size = 4.98

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\,(-3a^2+abx+2b^2x^2)\sqrt{\frac{a-bx}{bx}}}{15a^2x^2},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-6Ia^3b^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}{x\,(15a^3bx-15a^2b^2x^2)}+\frac{I8a^2b^{\frac{5}{2}}\sqrt{1-\frac{a}{bx}}}{15a^3bx-15a^2b^2x^2}+\frac{I2ab^{\frac{7}{2}}x\sqrt{1-\frac{a}{bx}}}{15a^3bx-15a^2b^2x^2}-\frac{4Ib^{\frac{9}{2}}x^2\sqrt{1-\frac{a}{bx}}}{15a^3bx-15a^2b^2x^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a - b*x]/x^(7/2),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-3 a ^ 2 + a b x + 2 b ^ 2 x ^ 2) Sqrt[(a - b x) / (
b x)] / (15 a ^ 2 x ^ 2), Abs[a / (b x)] > 1}}, -6 I a ^ 3 b ^ (3 / 2) Sqrt
[1 - a / (b x)] / (x (15 a ^ 3 b x - 15 a ^ 2 b ^ 2 x ^ 2)) + I 8 a ^ 2 b ^
 (5 / 2) Sqrt[1 - a / (b x)] / (15 a ^ 3 b x - 15 a ^ 2 b ^ 2 x ^ 2) + I 2
a b ^ (7 / 2) x Sqrt[1 - a / (b x)] / (15 a ^ 3 b x - 15 a ^ 2 b ^ 2 x ^ 2)
 - 4 I b ^ (9 / 2) x ^ 2 Sqrt[1 - a / (b x)] / (15 a ^ 3 b x - 15 a ^ 2 b ^
 2 x ^ 2)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.12, size = 75, normalized size = 1.63

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}(2bx+3a)}{15x^{\frac{5}{2}}a^2}$	25
risch	$-\frac{2\sqrt{-bx+a}\,(-2x^2b^2-abx+3a^2)}{15x^{\frac{5}{2}}a^2}$	36
default	$-\frac{\sqrt{-bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-b*x+a)^(1/2)/x^(5/2)-1/4*a*(-2/5/a/x^(5/2)*(-b*x+a)^(1/2)+4/5*b/a*(-
2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)))
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.72

$$-\frac{2\left(\frac{5(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(-b*x + a)^(5/2)/x^(5/2))/a^2
```

Fricas [A]
time = 0.31, size = 34, normalized size = 0.74

$$\frac{2(2b^2x^2+abx-3a^2)\sqrt{-bx+a}}{15a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 + a*b*x - 3*a^2)*sqrt(-b*x + a)/(a^2*x^(5/2))
```

Sympy [A]
time = 3.00, size = 241, normalized size = 5.24

$$\begin{cases}-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{5x^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}{15a^2} & \text{for } \left|\frac{a}{bx}\right|>1\\ \frac{6ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{x(-15a^3bx+15a^2b^2x^2)}-\frac{8ia^2b^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}-\frac{2iab^{\frac{7}{2}}x\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}+\frac{4ib^{\frac{9}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(5*x**2) + 2*b**(3/2)*sqrt(a/(b*x)
- 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) - 1)/(15*a**2), Abs(a/(b*x)) > 1),
(6*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1)/(x*(-15*a**3*b*x + 15*a**2*b**2*x**2)
) - 8*I*a**2*b**(5/2)*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2)
 - 2*I*a*b**(7/2)*x*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2) +
 4*I*b**(9/2)*x**2*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2), T
rue))
```

Giac [A]
time = 0.01, size = 120, normalized size = 2.61

$$\frac{2b^2\left(\frac{\frac{1}{225}\cdot 30b^5\sqrt{a-bx}\sqrt{a-bx}}{a^2}-\frac{\frac{1}{225}\cdot 75b^5a}{a^2}\right)\sqrt{a-bx}\sqrt{a-bx}\sqrt{a-bx}\sqrt{ab-b(a-bx)}}{|b|\,b\,(ab-b(a-bx))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x)
```

```
[Out] 2/15*(2*(b*x - a)*b^5/a^2 + 5*b^5/a)*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a)
*b + a*b)^(5/2)*abs(b))
```

Mupad [B]
time = 0.25, size = 32, normalized size = 0.70

$$\frac{\sqrt{a-b\,x}\,\left(\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2\,b\,x}{15\,a}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(7/2),x)
```

```
[Out] ((a - b*x)^(1/2)*((4*b^2*x^2)/(15*a^2) + (2*b*x)/(15*a) - 2/5))/x^(5/2)
```

3.504 $\int \frac{\sqrt{a-bx}}{x^{9/2}} \, dx$

Optimal. Leaf size=71

$$-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}$$

```
[Out] -2/7*(-b*x+a)^(3/2)/a/x^(7/2)-8/35*b*(-b*x+a)^(3/2)/a^2/x^(5/2)-16/105*b^2*
(-b*x+a)^(3/2)/a^3/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) - (8*b*(a - b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a - b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} + \frac{(4b)\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx}{7a}$$
$$= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} + \frac{(8b^2)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{35a^2}$$
$$= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}$$

Mathematica [**A**]
time = 0.08, size = 52, normalized size = 0.73

$$-\frac{2\sqrt{a-bx}\,(15a^3-3a^2bx-4ab^2x^2-8b^3x^3)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(15*a^3 - 3*a^2*b*x - 4*a*b^2*x^2 - 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 45.14, size = 461, normalized size = 6.49

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\,(-15a^5+33a^4bx-17a^3b^2x^2+3a^2b^3x^3-12ab^4x^4+8b^5x^5)\sqrt{\frac{a-bx}{bx}}}{105a^3x^3(a^2-2abx+b^2x^2)}, \text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\}, \frac{-30Ia^5b^{\frac{9}{2}}\sqrt{1-\frac{a}{bx}}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} + \frac{I66a^4b^{\frac{11}{2}}x\sqrt{1-\frac{a}{bx}}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} - \frac{34Ia^3b^{\frac{13}{2}}x^2\sqrt{1-\frac{a}{bx}}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} + \frac{I6a^2b^{\frac{15}{2}}x^3\sqrt{1-\frac{a}{bx}}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} - \frac{24Iab^{\frac{17}{2}}x^4\sqrt{1-\frac{a}{bx}}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5} + \frac{I16b^{\frac{19}{2}}x^5\sqrt{1-\frac{a}{bx}}}{105a^5b^4x^3-210a^4b^5x^4+105a^3b^6x^5}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[a - b*x]/x^(9/2),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-15 a ^ 5 + 33 a ^ 4 b x - 17 a ^ 3 b ^ 2 x ^ 2 + 3
a ^ 2 b ^ 3 x ^ 3 - 12 a b ^ 4 x ^ 4 + 8 b ^ 5 x ^ 5) Sqrt[(a - b x) / (b x
)] / (105 a ^ 3 x ^ 3 (a ^ 2 - 2 a b x + b ^ 2 x ^ 2)), Abs[a / (b x)] > 1}
}, -30 I a ^ 5 b ^ (9 / 2) Sqrt[1 - a / (b x)] / (105 a ^ 5 b ^ 4 x ^ 3 - 2
10 a ^ 4 b ^ 5 x ^ 4 + 105 a ^ 3 b ^ 6 x ^ 5) + I 66 a ^ 4 b ^ (11 / 2) x S
qrt[1 - a / (b x)] / (105 a ^ 5 b ^ 4 x ^ 3 - 210 a ^ 4 b ^ 5 x ^ 4 + 105 a
 ^ 3 b ^ 6 x ^ 5) - 34 I a ^ 3 b ^ (13 / 2) x ^ 2 Sqrt[1 - a / (b x)] / (10
5 a ^ 5 b ^ 4 x ^ 3 - 210 a ^ 4 b ^ 5 x ^ 4 + 105 a ^ 3 b ^ 6 x ^ 5) + I 6
a ^ 2 b ^ (15 / 2) x ^ 3 Sqrt[1 - a / (b x)] / (105 a ^ 5 b ^ 4 x ^ 3 - 210
 a ^ 4 b ^ 5 x ^ 4 + 105 a ^ 3 b ^ 6 x ^ 5) - 24 I a b ^ (17 / 2) x ^ 4 Sqr
t[1 - a / (b x)] / (105 a ^ 5 b ^ 4 x ^ 3 - 210 a ^ 4 b ^ 5 x ^ 4 + 105 a ^
 3 b ^ 6 x ^ 5) + I 16 b ^ (19 / 2) x ^ 5 Sqrt[1 - a / (b x)] / (105 a ^ 5
b ^ 4 x ^ 3 - 210 a ^ 4 b ^ 5 x ^ 4 + 105 a ^ 3 b ^ 6 x ^ 5)]
```

Maple [A]
time = 0.10, size = 98, normalized size = 1.38

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}(8x^2b^2+12abx+15a^2)}{105x^{\frac{7}{2}}a^3}$	36
risch	$-\frac{2\sqrt{-bx+a}\,(-8b^3x^3-4a\,b^2x^2-3a^2bx+15a^3)}{105x^{\frac{7}{2}}a^3}$	47
default	$-\frac{\sqrt{-bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{7a\,x^{\frac{7}{2}}}+\frac{6b\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b*x+a)^(1/2)/x^(7/2)-1/6*a*(-2/7/a/x^(7/2)*(-b*x+a)^(1/2)+6/7*b/a*(-
2/5/a/x^(5/2)*(-b*x+a)^(1/2)+4/5*b/a*(-2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(
-b*x+a)^(1/2)/a^2/x^(1/2))))
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.69

$$-\frac{2\left(\frac{35\,(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{42\,(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 42*(-b*x + a)^(5/2)*b/x^(5/2) + 1
5*(-b*x + a)^(7/2)/x^(7/2))/a^3
```

Fricas [A]
time = 0.31, size = 46, normalized size = 0.65

$$\frac{2\,(8\,b^3x^3+4\,ab^2x^2+3\,a^2bx-15\,a^3)\sqrt{-bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

[Out] 2/105*(8*b^3*x^3 + 4*a*b^2*x^2 + 3*a^2*b*x - 15*a^3)*sqrt(-b*x + a)/(a^3*x^(7/2))

Sympy [C] Result contains complex when optimal does not.
time = 11.98, size = 707, normalized size = 9.96

$$\begin{cases} - \frac{30 a^{5} b^{\frac{9}{2}} \sqrt{\frac{a}{b x} - 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} + \frac{66 a^{4} b^{\frac{11}{2}} x \sqrt{\frac{a}{b x} - 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} - \frac{34 a^{3} b^{\frac{13}{2}} x^{2} \sqrt{\frac{a}{b x} - 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} + \frac{6 a^{2} b^{\frac{15}{2}} x^{3} \sqrt{\frac{a}{b x} - 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} - \frac{24 a b^{\frac{17}{2}} x^{4} \sqrt{\frac{a}{b x} - 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} + \frac{16 b^{\frac{19}{2}} x^{5} \sqrt{\frac{a}{b x} - 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} & \text{for}\: \left|{\frac{a}{b x}}\right| > 1 \\- \frac{30 i a^{5} b^{\frac{9}{2}} \sqrt{- \frac{a}{b x} + 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} + \frac{66 i a^{4} b^{\frac{11}{2}} x \sqrt{- \frac{a}{b x} + 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} - \frac{34 i a^{3} b^{\frac{13}{2}} x^{2} \sqrt{- \frac{a}{b x} + 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} + \frac{6 i a^{2} b^{\frac{15}{2}} x^{3} \sqrt{- \frac{a}{b x} + 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} - \frac{24 i a b^{\frac{17}{2}} x^{4} \sqrt{- \frac{a}{b x} + 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} + \frac{16 i b^{\frac{19}{2}} x^{5} \sqrt{- \frac{a}{b x} + 1}}{105 a^{5} b^{4} x^{3} - 210 a^{4} b^{5} x^{4} + 105 a^{3} b^{6} x^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)**(1/2)/x**(9/2),x)

[Out] Piecewise((-30*a**5*b**(9/2)*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 66*a**4*b**(11/2)*x*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 16*b**(19/2)*x**5*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5), Abs(a/(b*x)) > 1), (-30*I*a**5*b**(9/2)*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 66*I*a**4*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*I*a**3*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 6*I*a**2*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*I*a*b**(17/2)*x**4*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 16*I*b**(19/2)*x**5*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5), True))

Giac [A]
time = 0.01, size = 158, normalized size = 2.23

$$\frac{2 b^{2} \left(\left(-\frac{\frac{1}{3675} \cdot 280 b^{7} \sqrt{a-bx}\,\sqrt{a-bx}}{a^{3}} + \frac{\frac{1}{3675} \cdot 980 b^{7} a}{a^{3}} \right) \sqrt{a-bx}\,\sqrt{a-bx} - \frac{\frac{1}{3675} \cdot 1225 b^{7} a^{2}}{a^{3}} \right) \sqrt{a-bx}\,\sqrt{a-bx}\,\sqrt{a-bx}\,\sqrt{ab-b\left(a-bx\right)}}{|b|\, b \left(ab-b\left(a-bx\right)\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)^(1/2)/x^(9/2),x)

[Out] 2/105*(35*b^7/a + 4*(2*(b*x - a)*b^7/a^3 + 7*b^7/a^2)*(b*x - a))*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))

Mupad [B]
time = 0.27, size = 43, normalized size = 0.61

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,b^2\,x^2}{105\,a^2}+\frac{16\,b^3\,x^3}{105\,a^3}+\frac{2\,b\,x}{35\,a}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((a - b*x)^(1/2)*((8*b^2*x^2)/(105*a^2) + (16*b^3*x^3)/(105*a^3) + (2*b*x)/
(35*a) - 2/7))/x^(7/2)
```

3.505 $\int x^{5/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=108

$$\frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] -5/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/24*x^(3/2)*(b*x+2)^(1/2
)/b^2+1/12*x^(5/2)*(b*x+2)^(1/2)/b+1/4*x^(7/2)*(b*x+2)^(1/2)+5/8*x^(1/2)*(b
*x+2)^(1/2)/b^3
```

Rubi [A]

time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(24*b^2) + (x
^(5/2)*Sqrt[2 + b*x])/(12*b) + (x^(7/2)*Sqrt[2 + b*x])/4 - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{2+bx} + \frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{12b}$$
$$= -\frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b^2}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\int \frac{1}{\sqrt{x}\sqrt{2+}}}{8b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{4b^{7/2}}$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.70

$$\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2+6b^3x^3)}{24b^3} + \frac{5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5*
Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(7/2))
```

Mathics [A]
time = 21.05, size = 88, normalized size = 0.81

$$\frac{-5\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{4b^{\frac{7}{2}}} + \frac{5\sqrt{x}}{4b^3\sqrt{2+bx}} + \frac{5x^{\frac{3}{2}}}{24b^2\sqrt{2+bx}} - \frac{x^{\frac{5}{2}}}{24b\sqrt{2+bx}} + \frac{7x^{\frac{7}{2}}}{12\sqrt{2+bx}} + \frac{bx^{\frac{9}{2}}}{4\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)*Sqrt[2 + b*x],x]')
```

```
[Out] -5 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / (4 b ^ (7 / 2)) + 5 Sqrt[x] / (4
b ^ 3 Sqrt[2 + b x]) + 5 x ^ (3 / 2) / (24 b ^ 2 Sqrt[2 + b x]) - x ^ (5 /
```

2) / (24 b Sqrt[2 + b x]) + 7 x ^ (7 / 2) / (12 Sqrt[2 + b x]) + b x ^ (9 / 2) / (4 Sqrt[2 + b x])

Maple [A]
time = 0.14, size = 121, normalized size = 1.12

method	result	size
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(42b^{3}x^{3}+14x^{2}b^{2}-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{1344}+\frac{5\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{32}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	71
risch	$\frac{\left(6b^{3}x^{3}+2x^{2}b^{2}-5bx+15\right)\sqrt{x}\,\sqrt{bx+2}}{24b^{3}}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)\sqrt{x\left(bx+2\right)}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	85
default	$\frac{x^{\frac{5}{2}}\left(bx+2\right)^{\frac{3}{2}}}{4b}-\frac{5\left(\frac{x^{\frac{3}{2}}\left(bx+2\right)^{\frac{3}{2}}}{3b}-\frac{\frac{\sqrt{x}\,\left(bx+2\right)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\left(bx+2\right)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}\right)}{4b}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+2)^(3/2)-5/4/b*(1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/b*(1/2/b*x
^(1/2)*(b*x+2)^(3/2)-1/2/b*(x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)
^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(75) = 150$.
time = 0.37, size = 163, normalized size = 1.51

$$\frac{\frac{15\sqrt{bx+2}\,b^{3}}{\sqrt{x}}+\frac{73\left(bx+2\right)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}-\frac{55\left(bx+2\right)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\left(bx+2\right)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^{7}-\frac{4\left(bx+2\right)b^{6}}{x}+\frac{6\left(bx+2\right)^{2}b^{5}}{x^{2}}-\frac{4\left(bx+2\right)^{3}b^{4}}{x^{3}}+\frac{\left(bx+2\right)^{4}b^{3}}{x^{4}}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) + 73*(b*x + 2)^(3/2)*b^2/x^(3/2) - 55*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x + 2)*b^
```

```
6/x + 6*(b*x + 2)^2*b^5/x^2 - 4*(b*x + 2)^3*b^4/x^3 + (b*x + 2)^4*b^3/x^4)
+ 5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(
x)))/b^(7/2)
```

Fricas [A]
time = 0.33, size = 140, normalized size = 1.30

$$\left[\frac{\left(6\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^{4}},\ \frac{\left(6\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*
sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/24*((6*b^4*x^3
 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

Sympy [A]
time = 20.55, size = 117, normalized size = 1.08

$$\frac{bx^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{24b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^{2}\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b^{3}\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(9/2)/(4*sqrt(b*x + 2)) + 7*x**(7/2)/(12*sqrt(b*x + 2)) - x**(5/2)/(24
*b*sqrt(b*x + 2)) + 5*x**(3/2)/(24*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(4*b**3*
sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2))
```

Giac [A]
time = 0.00, size = 140, normalized size = 1.30

$$2\left(2\left(\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^{6}\sqrt{x}\sqrt{x}}{b^{6}}+\frac{\frac{1}{2880}\cdot 60b^{5}}{b^{6}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^{4}}{b^{6}}\right)\sqrt{x}\sqrt{x}+\frac{\frac{1}{2880}\cdot 450b^{3}}{b^{6}}\right)\sqrt{x}\sqrt{bx+2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{8b^{3}\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/24*((2*(3*x + 1/b)*x - 5/b^2)*x + 15/b^3)*sqrt(b*x + 2)*sqrt(x) + 5/4*log
(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{b\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(1/2),x)
[Out] int(x^(5/2)*(b*x + 2)^(1/2), x)
```

3.506 $\int x^{3/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+1/6*x^(3/2)*(b*x+2)^(1/2)/b+1/
3*x^(5/2)*(b*x+2)^(1/2)-1/2*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\,\sqrt{bx+2}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 + b*x])/b^2 + (x^(3/2)*Sqrt[2 + b*x])/(6*b) + (x^(5/2)
*Sqrt[2 + b*x])/3 + ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2+bx}\,dx = \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{1}{3}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} - \frac{\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A]
time = 0.07, size = 65, normalized size = 0.77

$$\frac{\sqrt{x}\sqrt{2+bx}\,(-3+bx+2b^2x^2)}{6b^2} - \frac{\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 2*b^2*x^2))/(6*b^2) - Log[-(Sqrt[b]*Sqrt
[x]) + Sqrt[2 + b*x]]/b^(5/2)
```

Mathics [A]
time = 6.59, size = 81, normalized size = 0.96

$$\frac{6b^3\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](2+bx)^{\frac{3}{2}} - 6b^{\frac{7}{2}}\sqrt{x}\,(2+bx) - b^{\frac{9}{2}}x^{\frac{3}{2}}(2+bx) + b^{\frac{11}{2}}x^{\frac{5}{2}}(2+bx)(5+2bx)}{6b^{\frac{11}{2}}(2+bx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)*Sqrt[2 + b*x],x]')
```

```
[Out] (6 b ^ 3 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) ^ (3 / 2) - 6 b ^ (
7 / 2) Sqrt[x] (2 + b x) - b ^ (9 / 2) x ^ (3 / 2) (2 + b x) + b ^ (11 / 2)
 x ^ (5 / 2) (2 + b x) (5 + 2 b x)) / (6 b ^ (11 / 2) (2 + b x) ^ (3 / 2))
```

Maple [A]
time = 0.10, size = 100, normalized size = 1.19

method	result	size
meijerg	$-\frac{4\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-10x^{2}b^{2}-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{120}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	63
risch	$\frac{(2x^{2}b^{2}+bx-3)\sqrt{x}\,\sqrt{bx+2}}{6b^{2}}+\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	76
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/b*(1/2/b*x^(1/2)*(b*x+2)^(3/2)-1/2/b*(x^(1/2)
*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(
b*x^2+2*x)^(1/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(59) = 118$.
time = 0.35, size = 134, normalized size = 1.60

$$-\frac{\frac{3\sqrt{bx+2}\,b^{2}}{\sqrt{x}}+\frac{8\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{5}-\frac{3\,(bx+2)b^{4}}{x}+\frac{3\,(bx+2)^{2}b^{3}}{x^{2}}-\frac{(bx+2)^{3}b^{2}}{x^{3}}\right)}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) + 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x +
2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x + 2)*b^4/x + 3*(b*x + 2)^2*b^3/x^2 - (b*x +
 2)^3*b^2/x^3) - 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt
(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 121, normalized size = 1.44

$$\left[\frac{(2\,b^{3}x^{2}+b^{2}x-3\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^{3}},\ \frac{(2\,b^{3}x^{2}+b^{2}x-3\,b)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x +
 sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 + b^2*x - 3*b)*sq
rt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))
)/b^3]
```

Sympy [A]
time = 4.86, size = 90, normalized size = 1.07

$$\frac{bx^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{5x^{\frac{5}{2}}}{6\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{6b\sqrt{bx+2}}-\frac{\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(7/2)/(3*sqrt(b*x + 2)) + 5*x**(5/2)/(6*sqrt(b*x + 2)) - x**(3/2)/(6*b
*sqrt(b*x + 2)) - sqrt(x)/(b**2*sqrt(b*x + 2)) + asinh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/b**(5/2)
```

Giac [A]
time = 0.00, size = 116, normalized size = 1.38

$$2\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 12b^4\sqrt{x}\sqrt{x}}{b^4}+\frac{\frac{1}{144}\cdot 6b^3}{b^4}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{144}\cdot 18b^2}{b^4}\right)\sqrt{x}\sqrt{bx+2}-\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{2b^2\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/6*sqrt(b*x + 2)*((2*x + 1/b)*x - 3/b^2)*sqrt(x) - log(-sqrt(b)*sqrt(x) +
sqrt(b*x + 2))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{b\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(1/2), x)
```

3.507 $\int \sqrt{x}\,\sqrt{2+bx}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] -arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*x^(3/2)*(b*x+2)^(1/2)+1/2
*x^(1/2)*(b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 - ArcSinh[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.88

$$\frac{\sqrt{x}(1+bx)\sqrt{2+bx}}{2b} + \frac{\log\left(-\sqrt{b}\sqrt{x} + \sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(1 + b*x)*Sqrt[2 + b*x])/(2*b) + Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 +
 b*x]]/b^(3/2)
```

Mathics [A]
time = 3.57, size = 66, normalized size = 1.03

$$\frac{-b\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](2+bx) + b^{\frac{3}{2}}\sqrt{x}\sqrt{2+bx} + \frac{b^{\frac{5}{2}}x^{\frac{3}{2}}(3+bx)\sqrt{2+bx}}{2}}{b^{\frac{5}{2}}(2+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]*Sqrt[2 + b*x],x]')
```

```
[Out] (-b ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + b ^ (3 / 2) Sqrt[x] Sq
rt[2 + b x] + b ^ (5 / 2) x ^ (3 / 2) (3 + b x) Sqrt[2 + b x] / 2) / (b ^ (
5 / 2) (2 + b x))
```

Maple [A]
time = 0.11, size = 79, normalized size = 1.23

method	result	size
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(3bx+3)\sqrt{\frac{bx}{2}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	55
risch	$\frac{(bx+1)\sqrt{x}\,\sqrt{bx+2}}{2b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	68
default	$\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*x^(1/2)*(b*x+2)^(3/2)-1/2/b*(x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/
(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.
time = 0.36, size = 98, normalized size = 1.53

$$\frac{\frac{\sqrt{bx+2}\,b}{\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3-\frac{2\,(bx+2)b^2}{x}+\frac{(bx+2)^2b}{x^2}}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b*x + 2)*b/sqrt(x) + (b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x + 2)*b^2/
x + (b*x + 2)^2*b/x^2) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b
) + sqrt(b*x + 2)/sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.32, size = 101, normalized size = 1.58

$$\left[\frac{(b^2x+b)\sqrt{bx+2}\,\sqrt{x}+\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b^2},\ \frac{(b^2x+b)\sqrt{bx+2}\,\sqrt{x}+2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*s
qrt(b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + 2*sqrt(-
b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A]
time = 1.79, size = 71, normalized size = 1.11

$$\frac{bx^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{3x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(5/2)/(2*sqrt(b*x + 2)) + 3*x**(3/2)/(2*sqrt(b*x + 2)) + sqrt(x)/(b*sq
rt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.34

$$2\left(2\left(\frac{\frac{1}{16}\cdot 2b^2\sqrt{x}\sqrt{x}}{b^2}+\frac{\frac{1}{16}\cdot 2b}{b^2}\right)\sqrt{x}\sqrt{bx+2}+\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{2b\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/2*sqrt(b*x + 2)*(x + 1/b)*sqrt(x) + log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))
/b^(3/2)
```

Mupad [B]
time = 0.10, size = 46, normalized size = 0.72

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{1}{2\,b}\right)\,\sqrt{b\,x+2}-\frac{\ln\left(b\,x+\sqrt{b}\,\sqrt{x}\,\sqrt{b\,x+2}+1\right)}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + 1/(2*b))*(b*x + 2)^(1/2) - log(b*x + b^(1/2)*x^(1/2)*(b*x +
2)^(1/2) + 1)/(2*b^(3/2))
```

3.508 $\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=40

$$\sqrt{x}\,\sqrt{2+bx}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {52, 56, 221}

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{2+bx} + \int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= \sqrt{x}\,\sqrt{2+bx} + 2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{2+bx} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.15

$$\sqrt{x}\,\sqrt{2+bx} - \frac{2\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 + b*x] - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Mathics [A]
time = 2.53, size = 29, normalized size = 0.72

$$\frac{2\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{\sqrt{b}} + \sqrt{x}\,\sqrt{2+bx}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[2 + b*x]/Sqrt[x],x]')
[Out] 2 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / Sqrt[b] + Sqrt[x] Sqrt[2 + b x]
```

Maple [A]
time = 0.13, size = 58, normalized size = 1.45

method	result	size
meijerg	$-\frac{-\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\sqrt{\frac{bx}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{\pi}}$	49

default	$\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	58
risch	$\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^
(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(29) = 58$.
time = 0.34, size = 68, normalized size = 1.70

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}-\frac{2\sqrt{bx+2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b) - 2*sqrt(b*x + 2)/((b - (b*x + 2)/x)*sqrt(x))
```

Fricas [A]
time = 0.31, size = 86, normalized size = 2.15

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x}+\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{b},\ \frac{\sqrt{bx+2}\,b\sqrt{x}-2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b, (sqrt(b*x + 2)*b*sqrt(x) - 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-
b)/(b*sqrt(x))))/b]
```

Sympy [A]
time = 0.84, size = 37, normalized size = 0.92

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(1/2),x)
[Out] sqrt(x)*sqrt(b*x + 2) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.
time = 1.14, size = 86, normalized size = 2.15

$$\frac{b^2\left(\frac{\frac{1}{2}\cdot 2\sqrt{bx+2}\,\sqrt{b\,(bx+2)-2b}}{b}-\frac{2\ln\left|\sqrt{b\,(bx+2)-2b}-\sqrt{b}\,\sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x)
[Out] -b*(2*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b) -
sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b)/abs(b)
```

Mupad [B]
time = 0.62, size = 40, normalized size = 1.00

$$\sqrt{x}\,\sqrt{b\,x+2}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(1/2),x)
[Out] x^(1/2)*(b*x + 2)^(1/2) - (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(
1/2))))/b^(1/2)
```

3.509 $\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2\sqrt{2+bx}}{\sqrt{x}}+2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 56, 221}

$$2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx = -\frac{2\sqrt{2+bx}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= -\frac{2\sqrt{2+bx}}{\sqrt{x}} + (2b)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2\sqrt{2+bx}}{\sqrt{x}} + 2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [**A**]
time = 0.05, size = 47, normalized size = 1.15

$$-\frac{2\sqrt{2+bx}}{\sqrt{x}} - 2\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*
x]]
```

Mathics [**A**]
time = 2.63, size = 50, normalized size = 1.22

$$\sqrt{b}\left(-2\sqrt{\frac{2+bx}{bx}} - \text{Log}\left[\frac{1}{bx}\right] + 2\text{Log}\left[1+\sqrt{\frac{2+bx}{bx}}\right]\right)$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 + b*x]/x^(3/2),x]')
```

```
[Out] Sqrt[b] (-2 Sqrt[(2 + b x) / (b x)] - Log[1 / (b x)] + 2 Log[1 + Sqrt[(2 +
b x) / (b x)]])
```

Maple [**A**]
time = 0.12, size = 49, normalized size = 1.20

method	result	size
meijerg	$-\frac{\sqrt{b}\left(\frac{4\sqrt{\pi}\,\sqrt{2}\,\sqrt{\frac{bx}{2}+1}}{\sqrt{x}\,\sqrt{b}} - 4\sqrt{\pi}\,\text{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	49

risch	$-\frac{2\sqrt{bx+2}}{\sqrt{x}}+\frac{\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^(1/2)/Pi^(1/2)*(4*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(1/2*b*x+1)^(1/2)
-4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.34, size = 54, normalized size = 1.32

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.32, size = 87, normalized size = 2.12

$$\left[\frac{\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-2\sqrt{bx+2}\,\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b*x + 2)*
sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + sqr
t(b*x + 2)*sqrt(x))/x]
```

Sympy [A]
time = 0.73, size = 48, normalized size = 1.17

$$-2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}-\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2\sqrt{b}\,\log\left(\sqrt{1+\frac{2}{bx}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(1 + 2/(b*x)) - sqrt(b)*log(1/(b*x)) + 2*sqrt(b)*log(sqrt(1
+ 2/(b*x)) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.
time = 1.15, size = 96, normalized size = 2.34

$$\frac{bb^2\left(-\frac{2\sqrt{bx+2}\sqrt{b(bx+2)-2b}}{b(bx+2)-2b}-\frac{2\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] -2*b^2*(log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b)
+ sqrt(b*x + 2)/sqrt((b*x + 2)*b - 2*b))/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x+2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(1/2)/x^(3/2), x)
```

3.510 $\int \frac{\sqrt{2+bx}}{x^{5/2}} \, dx$

Optimal. Leaf size=18

$$-\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

```
[Out] -1/3*(b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{5/2}} \, dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$-\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

Mathics [A]
time = 2.53, size = 28, normalized size = 1.56

$$\frac{\sqrt{b}\,(-2-bx)\sqrt{\frac{2+bx}{bx}}}{3x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 + b*x]/x^(5/2),x]')
```

```
[Out] Sqrt[b] (-2 - b x) Sqrt[(2 + b x) / (b x)] / (3 x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.
time = 0.12, size = 27, normalized size = 1.50

method	result	size
gosper	$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	13
meijerg	$-\frac{2\sqrt{2}\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	17
risch	$-\frac{x^{2}b^{2}+4bx+4}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	26
default	$-\frac{2\sqrt{bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{bx+2}}{3\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x+2)^(1/2)/x^(3/2)-1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.31, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.
time = 0.77, size = 37, normalized size = 2.06

$$-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(5/2),x)
```

```
[Out] -b**(3/2)*sqrt(1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.
time = 0.01, size = 75, normalized size = 4.17

$$-\frac{3\cdot 2b^2b^3\sqrt{bx+2}\sqrt{bx+2}\sqrt{bx+2}\sqrt{b(bx+2)-2b}}{|b|\,b\cdot 18\,(b\,(bx+2)-2b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x)
```

```
[Out] -1/3*(b*x + 2)^(3/2)*b^4/(((b*x + 2)*b - 2*b)^(3/2)*abs(b))
```

Mupad [B]
time = 0.21, size = 18, normalized size = 1.00

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}+\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(5/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/3 + 2/3))/x^(3/2)
```

3.511 $\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}$$

```
[Out] -1/5*(b*x+2)^(3/2)/x^(5/2)+1/15*b*(b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {47, 37}

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(7/2),x]
```

```
[Out] -1/5*(2 + b*x)^(3/2)/x^(5/2) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}-\frac{1}{5}b\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 0.82

$$\frac{\sqrt{2+bx}\,\left(-6-bx+b^2x^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(7/2),x]
[Out] (Sqrt[2 + b*x]*(-6 - b*x + b^2*x^2))/(15*x^(5/2))
```

Mathics [A]
time = 4.35, size = 35, normalized size = 0.92

$$\frac{\sqrt{b}\,\left(-6-bx+b^2x^2\right)\sqrt{\frac{2+bx}{bx}}}{15x^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 + b*x]/x^(7/2),x]')
[Out] Sqrt[b] (-6 - b x + b ^ 2 x ^ 2) Sqrt[(2 + b x) / (b x)] / (15 x ^ 2)
```

Maple [A]
time = 0.12, size = 43, normalized size = 1.13

method	result	size
gosper	$\frac{(bx+2)^{\frac{3}{2}}(bx-3)}{15x^{\frac{5}{2}}}$	18
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{6}x^2b^2+\frac{1}{6}bx+1\right)\sqrt{\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
risch	$\frac{b^3x^3+x^2b^2-8bx-12}{15x^{\frac{5}{2}}\sqrt{bx+2}}$	33
default	$-\frac{2\sqrt{bx+2}}{5x^{\frac{5}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(b*x+2)^(1/2)/x^(5/2)+1/5*b*(-1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^
(1/2)/x^(1/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.68

$$\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.31, size = 25, normalized size = 0.66

$$\frac{(b^2x^2 - bx - 6)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
[Out] 1/15*(b^2*x^2 - b*x - 6)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [A]
time = 2.88, size = 56, normalized size = 1.47

$$\frac{b^{\frac{5}{2}}\sqrt{1+\frac{2}{bx}}}{15} - \frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{15x} - \frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(7/2),x)
[Out] b**(5/2)*sqrt(1 + 2/(b*x))/15 - b**(3/2)*sqrt(1 + 2/(b*x))/(15*x) - 2*sqrt(
b)*sqrt(1 + 2/(b*x))/(5*x**2)
```

Giac [A]
time = 0.01, size = 102, normalized size = 2.68

$$\frac{2b^2\left(\frac{15}{450}b^5\sqrt{bx+2}\sqrt{bx+2} - \frac{75}{450}b^5\right)\sqrt{bx+2}\sqrt{bx+2}\sqrt{bx+2}\sqrt{b(bx+2)-2b}}{|b|\,b\,(b\,(bx+2)-2b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x)
[Out] 1/15*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(5/2)*a
bs(b))
```

Mupad [B]
time = 0.22, size = 26, normalized size = 0.68

$$-\frac{\sqrt{b\,x+2}\,\left(-\frac{b^2\,x^2}{15}+\frac{b\,x}{15}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(7/2),x)
[Out] -((b*x + 2)^(1/2)*((b*x)/15 - (b^2*x^2)/15 + 2/5))/x^(5/2)
```

3.512 $\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{(2+bx)^{3/2}}{7x^{7/2}}+\frac{2b(2+bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}$$

```
[Out] -1/7*(b*x+2)^(3/2)/x^(7/2)+2/35*b*(b*x+2)^(3/2)/x^(5/2)-2/105*b^2*(b*x+2)^(
3/2)/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -1/7*(2 + b*x)^(3/2)/x^(7/2) + (2*b*(2 + b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 + b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx = -\frac{(2+bx)^{3/2}}{7x^{7/2}} - \frac{1}{7}(2b)\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$$
$$= -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} + \frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx$$
$$= -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.68

$$\frac{\sqrt{2+bx}\,\left(-30-3bx+2b^2x^2-2b^3x^3\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-30 - 3*b*x + 2*b^2*x^2 - 2*b^3*x^3))/(105*x^(7/2))
```

Mathics [A]
time = 11.36, size = 75, normalized size = 1.27

$$\frac{\sqrt{b}\,\left(-120-132bx-34b^2x^2-3b^3x^3-6b^4x^4-2b^5x^5\right)\sqrt{\frac{2+bx}{bx}}}{105x^3\left(4+4bx+b^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 + b*x]/x^(9/2),x]')
```

```
[Out] Sqrt[b] (-120 - 132 b x - 34 b ^ 2 x ^ 2 - 3 b ^ 3 x ^ 3 - 6 b ^ 4 x ^ 4 -
2 b ^ 5 x ^ 5) Sqrt[(2 + b x) / (b x)] / (105 x ^ 3 (4 + 4 b x + b ^ 2 x ^
2))
```

Maple [A]
time = 0.13, size = 59, normalized size = 1.00

method	result	size
gosper	$-\frac{(bx+2)^{\frac{3}{2}}\left(2x^2b^2-6bx+15\right)}{105x^{\frac{7}{2}}}$	27
meijerg	$-\frac{2\sqrt{2}\,\left(\frac{1}{15}b^3x^3-\frac{1}{15}x^2b^2+\frac{1}{10}bx+1\right)\sqrt{\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39

risch	$-\frac{2b^4x^4+2b^3x^3-x^2b^2+36bx+60}{105x^{\frac{7}{2}}\sqrt{bx+2}}$	43
default	$-\frac{2\sqrt{bx+2}}{7x^{\frac{7}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x+2)^(1/2)/x^(7/2)+1/7*b*(-1/5*(b*x+2)^(1/2)/x^(5/2)-2/5*b*(-1/3*(b
*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.69

$$-\frac{(bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b*x + 2)^(3/2)*b^2/x^(3/2) + 1/10*(b*x + 2)^(5/2)*b/x^(5/2) - 1/28*(
b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A]
time = 0.31, size = 34, normalized size = 0.58

$$-\frac{(2\,b^3x^3-2\,b^2x^2+3\,bx+30)\sqrt{bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x + 30)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(53) = 106$.
time = 9.44, size = 270, normalized size = 4.58

$$-\frac{2b^{\frac{19}{2}}x^5\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{3b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{132b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{120b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] -2*b**(19/2)*x**5*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b*
*4*x**3) - 6*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**
4 + 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 42
0*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(105*b**
6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 132*b**(11/2)*x*sqrt(1 + 2/(b*x))
/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(1 + 2/
(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3)
```

Giac [A]
time = 0.01, size = 130, normalized size = 2.20

$$\frac{2b^2\left(\left(\left(-\frac{70}{7350}b^7\sqrt{bx+2}\sqrt{bx+2}+\frac{490}{7350}b^7\right)\sqrt{bx+2}\sqrt{bx+2}-\frac{1225}{7350}b^7\right)\sqrt{bx+2}\sqrt{bx+2}\sqrt{bx+2}\sqrt{b\left(bx+2\right)-2b}}{|b|\,b\left(b\left(bx+2\right)-2b\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x)
```

```
[Out] -1/105*(35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2)^(3/2)*b/(((
b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 34, normalized size = 0.58

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{105}-\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(9/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/35 - (2*b^2*x^2)/105 + (2*b^3*x^3)/105 + 2/7))/x^(
7/2)
```

3.513 $\int x^{5/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=112

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] 5/4*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/24*x^(3/2)*(-b*x+2)^(1/2)
/b^2-1/12*x^(5/2)*(-b*x+2)^(1/2)/b+1/4*x^(7/2)*(-b*x+2)^(1/2)-5/8*x^(1/2)*(
-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(24*b^2) - (
x^(5/2)*Sqrt[2 - b*x])/(12*b) + (x^(7/2)*Sqrt[2 - b*x])/4 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}\sqrt{2-bx}\,dx &= \frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{1}{4}\int\frac{x^{5/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int\frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{12b}\\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int\frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int\frac{1}{\sqrt{x}\sqrt{2-}}}{8b^3}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\mathrm{Subst}\left(\int\frac{}{\sqrt{}}\right.}{}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{4b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-15-5bx-2b^2x^2+6b^3x^3)}{24b^3}+\frac{5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 21.26, size = 190, normalized size = 1.70

$$\mathrm{Piecewise}\left[\left\{\left\{\frac{-5I\,\mathrm{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{4b^{\frac{7}{2}}}+\frac{5I\sqrt{x}}{4b^3\sqrt{-2+bx}}-\frac{5Ix^{\frac{3}{2}}}{24b^2\sqrt{-2+bx}}-\frac{Ix^{\frac{5}{2}}}{24b\sqrt{-2+bx}}-\frac{7Ix^{\frac{7}{2}}}{12\sqrt{-2+bx}}+\frac{Ibx^{\frac{9}{2}}}{4\sqrt{-2+bx}},\mathrm{Abs}[bx]>2\right\}\right\},\frac{5\,\mathrm{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{4b^{\frac{7}{2}}}-\frac{5\sqrt{x}}{4b^3\sqrt{2-bx}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{2-bx}}+\frac{x^{\frac{5}{2}}}{24b\sqrt{2-bx}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{2-bx}}-\frac{bx^{\frac{9}{2}}}{4\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*Sqrt[2 - b*x],x]')
```

```
[Out] Piecewise[{{-5 I / 4 ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (7 / 2) + 5
 I / 4 Sqrt[x] / (b ^ 3 Sqrt[-2 + b x]) - 5 I / 24 x ^ (3 / 2) / (b ^ 2 Sqr
```

```
t[-2 + b x]) - I / 24 x ^ (5 / 2) / (b Sqrt[-2 + b x]) - 7 I / 12 x ^ (7 /
2) / Sqrt[-2 + b x] + I / 4 b x ^ (9 / 2) / Sqrt[-2 + b x], Abs[b x] > 2}},
 5 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / (4 b ^ (7 / 2)) - 5 Sqrt[x] / (4 b
 ^ 3 Sqrt[2 - b x]) + 5 x ^ (3 / 2) / (24 b ^ 2 Sqrt[2 - b x]) + x ^ (5 / 2
) / (24 b Sqrt[2 - b x]) + 7 x ^ (7 / 2) / (12 Sqrt[2 - b x]) - b x ^ (9 /
2) / (4 Sqrt[2 - b x])]
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.14

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(-42b^3x^3+14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{168b^3}-\frac{5\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{7}{2}}}}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{(6b^3x^3-2x^2b^2-5bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{24b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\arctan\left(\frac{\sqrt{b}\,(x-\frac{1}{b})}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+2)^{\frac{3}{2}}}{4b}+\frac{-\frac{5x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{12b}+\frac{5\left(-\frac{\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\,\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,(x-\frac{1}{b})}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}\right)}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+2)^(3/2)+5/4/b*(-1/3/b*x^(3/2)*(-b*x+2)^(3/2)+1/b*(-1/
2/b*x^(1/2)*(-b*x+2)^(3/2)+1/2/b*(x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)
/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))
))
```

Maxima [A]
time = 0.34, size = 147, normalized size = 1.31

$$\frac{\frac{15\sqrt{-bx+2}\,b^3}{\sqrt{x}}-\frac{73(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4(bx-2)b^6}{x}+\frac{6(bx-2)^2b^5}{x^2}-\frac{4(bx-2)^3b^4}{x^3}+\frac{(bx-2)^4b^3}{x^4}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) - 73*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 55*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x - 2
)*b^6/x + 6*(b*x - 2)^2*b^5/x^2 - 4*(b*x - 2)^3*b^4/x^3 + (b*x - 2)^4*b^3/x
^4) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 141, normalized size = 1.26

$$\left[\frac{(6\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{24\,b^4},\frac{(6\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 15
*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, 1/24*((6*b^
4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arc
tan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

Sympy [A]
time = 20.52, size = 250, normalized size = 2.23

$$\begin{cases} \frac{ibx^{\frac{9}{2}}}{4\sqrt{bx-2}}-\frac{7ix^{\frac{7}{2}}}{12\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{24b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{24b^2\sqrt{bx-2}}+\frac{5i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } |bx|>2 \\ -\frac{bx^{\frac{9}{2}}}{4\sqrt{-bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{24b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{-bx+2}}-\frac{5\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{5\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(9/2)/(4*sqrt(b*x - 2)) - 7*I*x**(7/2)/(12*sqrt(b*x - 2))
 - I*x**(5/2)/(24*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(24*b**2*sqrt(b*x - 2)) +
 5*I*sqrt(x)/(4*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(4*b**(7/2)), Abs(b*x) > 2), (-b*x**(9/2)/(4*sqrt(-b*x + 2)) + 7*x**(7/2)/(
12*sqrt(-b*x + 2)) + x**(5/2)/(24*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(24*b**2*s
qrt(-b*x + 2)) - 5*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)
*sqrt(x)/2)/(4*b**(7/2)), True))
```

Giac [A]
time = 0.01, size = 147, normalized size = 1.31

$$2\left(2\left(\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^6\sqrt{x}\,\sqrt{x}}{b^6}-\frac{\frac{1}{2880}\cdot 60b^5}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^4}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 450b^3}{b^6}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{5\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^3\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x)
```

```
[Out] 1/24*((2*(3*x - 1/b)*x - 5/b^2)*x - 15/b^3)*sqrt(-b*x + 2)*sqrt(x) - 5/4*lo
g(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\sqrt{2-b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(1/2), x)
```

3.514 $\int x^{3/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-1/6*x^(3/2)*(-b*x+2)^(1/2)/b+1/
3*x^(5/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {52, 56, 222}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\,\sqrt{2-bx}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x])/b^2 - (x^(3/2)*Sqrt[2 - b*x])/(6*b) + (x^(5/2)
*Sqrt[2 - b*x])/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2-bx}\,dx = \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{1}{3}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right.}{b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.83

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-3-bx+2b^2x^2)}{6b^2} + \frac{b\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-3 - b*x + 2*b^2*x^2))/(6*b^2) + (b*Log[-(Sqrt[-b]*
Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*Sqrt[2 - b*x],x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.12, size = 106, normalized size = 1.22

method	result	size

meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(-10x^2b^2+5bx+15\right)\sqrt{-\frac{bx}{2}+1}}{30b^2}-\frac{\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}}}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$	81
default	$-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{3b}+\frac{-\frac{\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\,\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}$	106
risch	$-\frac{(2x^2b^2-bx-3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+2)^(3/2)+1/b*(-1/2/b*x^(1/2)*(-b*x+2)^(3/2)+1/2/b*(x^(
1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arcta
n(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))))
```

Maxima [A]
time = 0.35, size = 117, normalized size = 1.34

$$\frac{\frac{3\sqrt{-bx+2}\,b^2}{\sqrt{x}}-\frac{8(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5-\frac{3(bx-2)b^4}{x}+\frac{3(bx-2)^2b^3}{x^2}-\frac{(bx-2)^3b^2}{x^3}\right)}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) - 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x - 2)*b^4/x + 3*(b*x - 2)^2*b^3/x^2 - (b*x
 - 2)^3*b^2/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 125, normalized size = 1.44

$$\left[\frac{(2\,b^3x^2-b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}-3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^3},\ \frac{(2\,b^3x^2-b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}-6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x) - 3*sqrt(-b)*log(-b*
x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 - b^2*x - 3*
b)*sqrt(-b*x + 2)*sqrt(x) - 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x
))))/b^3]
```

Sympy [A]
time = 4.95, size = 194, normalized size = 2.23

$$\begin{cases} \frac{ibx^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{5ix^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{6b\sqrt{bx-2}} + \frac{i\sqrt{x}}{b^2\sqrt{bx-2}} - \frac{i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ -\frac{bx^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{5x^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{6b\sqrt{-bx+2}} - \frac{\sqrt{x}}{b^2\sqrt{-bx+2}} + \frac{\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(7/2)/(3*sqrt(b*x - 2)) - 5*I*x**(5/2)/(6*sqrt(b*x - 2))
- I*x**(3/2)/(6*b*sqrt(b*x - 2)) + I*sqrt(x)/(b**2*sqrt(b*x - 2)) - I*acosh
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x) > 2), (-b*x**(7/2)/(3*sqrt(-
b*x + 2)) + 5*x**(5/2)/(6*sqrt(-b*x + 2)) + x**(3/2)/(6*b*sqrt(-b*x + 2)) -
 sqrt(x)/(b**2*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2),
True))
```

Giac [A]
time = 0.00, size = 121, normalized size = 1.39

$$2\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 12b^4\sqrt{x}\sqrt{x}}{b^4} - \frac{\frac{1}{144}\cdot 6b^3}{b^4}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{144}\cdot 18b^2}{b^4}\right)\sqrt{x}\sqrt{-bx+2} - \frac{\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{2b^2\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x)
```

```
[Out] 1/6*sqrt(-b*x + 2)*((2*x - 1/b)*x - 3/b^2)*sqrt(x) - log(-sqrt(-b)*sqrt(x)
+ sqrt(-b*x + 2))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\sqrt{2-b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(1/2), x)
```

3.515 $\int \sqrt{x}\,\sqrt{2-bx}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*
x^(1/2)*(-b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x])/b + (x^(3/2)*Sqrt[2 - b*x])/2 + ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}\sqrt{2-bx}\,dx = \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [**A**]
time = 0.07, size = 62, normalized size = 0.95

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-1+bx)}{2b} + \frac{\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-1 + b*x))/(2*b) + Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2
 - b*x]]/(-b)^(3/2)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*Sqrt[2 - b*x],x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.12, size = 85, normalized size = 1.31

method	result	size

meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}(-3bx+3)\sqrt{-\frac{bx}{2}+1}}{6b}-\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}}}{\sqrt{-b}\,\sqrt{\pi}\,b}$	73
default	$-\frac{\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\,\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}$	85
risch	$-\frac{(bx-1)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*x^(1/2)*(-b*x+2)^(3/2)+1/2/b*(x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1
/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2
)))
```

Maxima [A]
time = 0.36, size = 81, normalized size = 1.25

$$\frac{\frac{\sqrt{-bx+2}\,b}{\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{3}-\frac{2\,(bx-2)b^{2}}{x}+\frac{(bx-2)^{2}b}{x^{2}}}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(-b*x + 2)*b/sqrt(x) - (-b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x - 2)*b^
2/x + (b*x - 2)^2*b/x^2) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.31, size = 107, normalized size = 1.65

$$\left[\frac{(b^{2}x-b)\sqrt{-bx+2}\,\sqrt{x}-\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b^{2}},\ \frac{(b^{2}x-b)\sqrt{-bx+2}\,\sqrt{x}-2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x +
2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - 2*
sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [A]
time = 1.79, size = 155, normalized size = 2.38

$$\begin{cases} \frac{ibx^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{3ix^{\frac{3}{2}}}{2\sqrt{bx-2}} + \frac{i\sqrt{x}}{b\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx| > 2 \\ -\frac{bx^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{3x^{\frac{3}{2}}}{2\sqrt{-bx+2}} - \frac{\sqrt{x}}{b\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(5/2)/(2*sqrt(b*x - 2)) - 3*I*x**(3/2)/(2*sqrt(b*x - 2))
+ I*sqrt(x)/(b*sqrt(b*x - 2)) - I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
, Abs(b*x) > 2), (-b*x**(5/2)/(2*sqrt(-b*x + 2)) + 3*x**(3/2)/(2*sqrt(-b*x
+ 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/
2), True))
```

Giac [A]
time = 0.00, size = 92, normalized size = 1.42

$$2\left(2\left(\frac{\frac{1}{16}\cdot 2b^2\sqrt{x}\sqrt{x}}{b^2} - \frac{\frac{1}{16}\cdot 2b}{b^2}\right)\sqrt{x}\sqrt{-bx+2} - \frac{\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{2b\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x)
```

```
[Out] 1/2*sqrt(-b*x + 2)*(x - 1/b)*sqrt(x) - log(-sqrt(-b)*sqrt(x) + sqrt(-b*x +
2))/(sqrt(-b)*b)
```

Mupad [B]
time = 0.10, size = 53, normalized size = 0.82

$$\sqrt{x}\,\left(\frac{x}{2} - \frac{1}{2\,b}\right)\,\sqrt{2-b\,x} - \frac{\ln\left(\sqrt{-b}\,\sqrt{x}\,\sqrt{2-b\,x} - b\,x + 1\right)}{2\,{(-b)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(2*b))*(2 - b*x)^(1/2) - log((-b)^(1/2)*x^(1/2)*(2 - b*x)^
(1/2) - b*x + 1)/(2*(-b)^(3/2))
```

3.516 $\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

[Out] 2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+x^(1/2)*(-b*x+2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 - b*x]/Sqrt[x],x]

[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{2-bx} + \int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx$$
$$= \sqrt{x}\,\sqrt{2-bx} + 2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{2-bx} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 1.27

$$\sqrt{x}\,\sqrt{2-bx} - \frac{2\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{2-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] - (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/Sqrt[-
b]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.65, size = 100, normalized size = 2.44

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(\sqrt{b}\,\sqrt{x}\,(-2+bx) - 2\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]\sqrt{-2+bx}\right)}{\sqrt{b}\,\sqrt{-2+bx}}, \text{Abs}\,[bx] > 2\right\}\right\}, \frac{2\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{\sqrt{b}} + \frac{2\sqrt{x}}{\sqrt{2-bx}} - \frac{bx^{\frac{3}{2}}}{\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 - b*x]/Sqrt[x],x]')
```

```
[Out] Piecewise[{{I (Sqrt[b] Sqrt[x] (-2 + b x) - 2 ArcCosh[Sqrt[2] Sqrt[b] Sqrt[
x] / 2] Sqrt[-2 + b x]) / (Sqrt[b] Sqrt[-2 + b x]), Abs[b x] > 2}}, 2 ArcSi
n[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / Sqrt[b] + 2 Sqrt[x] / Sqrt[2 - b x] - b x
^ (3 / 2) / Sqrt[2 - b x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(30) = 60$.
time = 0.11, size = 63, normalized size = 1.54

method	result	size

default	$\sqrt{x}\,\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	63
meijerg	$\frac{\sqrt{-b}\,\left(-\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{-b}\,\sqrt{-\frac{bx}{2}+1}-\frac{2\sqrt{\pi}\,\sqrt{-b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}\right)}{\sqrt{\pi}\,b}$	63
risch	$-\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*ar
ctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.36, size = 49, normalized size = 1.20

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{2\sqrt{-bx+2}}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 2*sqrt(-b*x + 2)/((b
 - (b*x - 2)/x)*sqrt(x))
```

Fricas [A]
time = 0.31, size = 89, normalized size = 2.17

$$\left[\frac{\sqrt{-bx+2}\,b\sqrt{x}-\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b},\ \frac{\sqrt{-bx+2}\,b\sqrt{x}-2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b*x + 2)*b*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sq
rt(x) + 1))/b, (sqrt(-b*x + 2)*b*sqrt(x) - 2*sqrt(b)*arctan(sqrt(-b*x + 2)/
(sqrt(b)*sqrt(x))))/b]
```

Sympy [A]
time = 0.86, size = 119, normalized size = 2.90

$$\begin{cases} \frac{ibx^{\frac{3}{2}}}{\sqrt{bx-2}} - \frac{2i\sqrt{x}}{\sqrt{bx-2}} - \frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx| > 2 \\ -\frac{bx^{\frac{3}{2}}}{\sqrt{-bx+2}} + \frac{2\sqrt{x}}{\sqrt{-bx+2}} + \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(3/2)/sqrt(b*x - 2) - 2*I*sqrt(x)/sqrt(b*x - 2) - 2*I*aco
sh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x) > 2), (-b*x**(3/2)/sqrt(-b*
x + 2) + 2*sqrt(x)/sqrt(-b*x + 2) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(
b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.
time = 1.14, size = 91, normalized size = 2.22

$$\frac{b^2\left(\frac{\frac{1}{2}\cdot 2\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{b} + \frac{2\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] b*(2*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqrt(-b)
+ sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b)/abs(b)
```

Mupad [B]
time = 0.56, size = 42, normalized size = 1.02

$$\sqrt{x}\,\sqrt{2-b\,x} - \frac{4\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(2 - b*x)^(1/2) - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1
/2))))/b^(1/2)
```

3.517 $\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {49, 56, 222}

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\ &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right) \\ &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 1.26

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 -
b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.93, size = 111, normalized size = 2.64

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\left(\sqrt{b}\,\sqrt{x}\,\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](-2+bx) - bx\sqrt{-2+bx} + 2\sqrt{-2+bx}\right)}{\sqrt{x}\,(-2+bx)}, \text{Abs}[bx] > 2\right\}\right\}, -2\sqrt{b}\,\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right] - \frac{4}{\sqrt{x}\,\sqrt{2-bx}} + \frac{2b\sqrt{x}}{\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 - b*x]/x^(3/2),x]')
```

```
[Out] Piecewise[{{2 I (Sqrt[b] Sqrt[x] ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (-2 +
 b x) - b x Sqrt[-2 + b x] + 2 Sqrt[-2 + b x]) / (Sqrt[x] (-2 + b x)), Abs[
b x] > 2}}, -2 Sqrt[b] ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] - 4 / (Sqrt[x] S
qrt[2 - b x]) + 2 b Sqrt[x] / Sqrt[2 - b x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.
time = 0.11, size = 64, normalized size = 1.52

method	result	size

meijerg	$\frac{(-b)^{\frac{3}{2}}\left(\frac{4\sqrt{\pi}\,\sqrt{2}\,\sqrt{-\frac{bx}{2}+1}}{\sqrt{x}\,\sqrt{-b}}+\frac{4\sqrt{\pi}\,\sqrt{b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{-b}}\right)}{2\sqrt{\pi}\,b}$	64
risch	$\frac{2(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b)^(3/2)/Pi^(1/2)/b*(4*Pi^(1/2)/x^(1/2)*2^(1/2)/(-b)^(1/2)*(-1/2*b*x+
1)^(1/2)+4*Pi^(1/2)/(-b)^(1/2)*b^(1/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.34, size = 35, normalized size = 0.83

$$2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + 2)/sqrt(
x)
```

Fricas [A]
time = 0.31, size = 90, normalized size = 2.14

$$\left[\frac{\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-2\sqrt{-bx+2}\,\sqrt{x}}{x},\ \frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - 2*sqrt(-b*x
+ 2)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - sq
rt(-b*x + 2)*sqrt(x))/x]
```

Sympy [**A**]
time = 0.78, size = 122, normalized size = 2.90

$$\begin{cases} 2i\sqrt{b}\ \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) - \frac{2ib\sqrt{x}}{\sqrt{bx-2}} + \frac{4i}{\sqrt{x}\sqrt{bx-2}} & \text{for } |bx| > 2 \\ -2\sqrt{b}\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) + \frac{2b\sqrt{x}}{\sqrt{-bx+2}} - \frac{4}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - 2*I*b*sqrt(x)/sqr
t(b*x - 2) + 4*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x) > 2), (-2*sqrt(b)*asin(s
qrt(2)*sqrt(b)*sqrt(x)/2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 4/(sqrt(x)*sqrt(-b
*x + 2)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(31) = 62$.
time = 1.14, size = 102, normalized size = 2.43

$$-\frac{bb^2\left(\frac{2\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{-b(-bx+2)+2b} + \frac{2\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] -2*b^2*(log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqrt(-
b) + sqrt(-b*x + 2)/sqrt((b*x - 2)*b + 2*b))/abs(b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(1/2)/x^(3/2), x)
```

3.518 $\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

```
[Out] -1/3*(-b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(5/2),x]
[Out] -1/3*(2 - b*x)^(3/2)/x^(3/2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx = -\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 1.00

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(5/2),x]
[Out] -1/3*(2 - b*x)^(3/2)/x^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.59, size = 77, normalized size = 4.05

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,(-2+bx)\sqrt{\frac{2-bx}{bx}}}{3x},\frac{1}{\text{Abs}\,[bx]}>\frac{1}{2}\right\}\right\},\frac{Ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}-\frac{2I\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 - b*x]/x^(5/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-2 + b x) Sqrt[(2 - b x) / (b x)] / (3 x), 1 / Abs[b x
] > 1 / 2}}, I b ^ (3 / 2) Sqrt[1 - 2 / (b x)] / 3 - 2 I Sqrt[b] Sqrt[1 - 2
 / (b x)] / (3 x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.14, size = 29, normalized size = 1.53

method	result	size
gosper	$-\frac{(-bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	14
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	17
default	$-\frac{2\sqrt{-bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{-bx+2}}{3\sqrt{x}}$	29
risch	$-\frac{\sqrt{(-bx+2)\,x}\,(x^{2}b^{2}-4bx+4)}{3x^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{-x\,(bx-2)}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-b*x+2)^(1/2)/x^(3/2)+1/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 18, normalized size = 0.95

$$\frac{(bx-2)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)/x^(3/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.80, size = 83, normalized size = 4.37

$$\begin{cases} \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2} \\ \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}-\frac{2i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(5/2),x)
[Out] Piecewise((b**(3/2)*sqrt(-1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 1/Abs(b*x) > 1/2), (I*b**(3/2)*sqrt(1 - 2/(b*x))/3 - 2*I*sqrt(b)*sqrt(1
 - 2/(b*x))/(3*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.
time = 0.01, size = 80, normalized size = 4.21

$$-\frac{3\cdot 2b^2b^3\sqrt{-bx+2}\,\sqrt{-bx+2}\,\sqrt{-bx+2}\,\sqrt{-b\,(-bx+2)+2b}}{|b|\,b\cdot 18\,(-b\,(-bx+2)+2b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x)
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)*b^4/(((b*x - 2)*b + 2*b)^(3/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 18, normalized size = 0.95

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}-\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(5/2),x)
[Out] ((2 - b*x)^(1/2)*((b*x)/3 - 2/3))/x^(3/2)
```

3.519 $\int \frac{\sqrt{2-bx}}{x^{7/2}} \, dx$

Optimal. Leaf size=40

$$-\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}$$

```
[Out] -1/5*(-b*x+2)^(3/2)/x^(5/2)-1/15*b*(-b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -1/5*(2 - b*x)^(3/2)/x^(5/2) - (b*(2 - b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{7/2}} \, dx &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}+\frac{1}{5}b\int \frac{\sqrt{2-bx}}{x^{5/2}} \, dx\\ &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.78

$$\frac{\sqrt{2-bx}\,\left(-6+bx+b^2x^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-6 + b*x + b^2*x^2))/(15*x^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.05, size = 117, normalized size = 2.92

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,\left(12-8bx+b^2x^2\left(-1+bx\right)\right)\sqrt{\frac{2-bx}{bx}}}{15x^2\left(-2+bx\right)},\frac{1}{\text{Abs}\left[bx\right]}>\frac{1}{2}\right\}\right\},\frac{Ib^{\frac{5}{2}}\sqrt{1-\frac{2}{bx}}}{15}-\frac{2I\sqrt{b}\sqrt{1-\frac{2}{bx}}}{5x^2}+\frac{Ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{15x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 - b*x]/x^(7/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (12 - 8 b x + b ^ 2 x ^ 2 (-1 + b x)) Sqrt[(2 - b x) /
(b x)] / (15 x ^ 2 (-2 + b x)), 1 / Abs[b x] > 1 / 2}}, I b ^ (5 / 2) Sqrt[
1 - 2 / (b x)] / 15 - 2 I Sqrt[b] Sqrt[1 - 2 / (b x)] / (5 x ^ 2) + I b ^ (
3 / 2) Sqrt[1 - 2 / (b x)] / (15 x)]
```

Maple [A]
time = 0.11, size = 46, normalized size = 1.15

method	result	size
gosper	$-\frac{(bx+3)(-bx+2)^{\frac{3}{2}}}{15x^{\frac{5}{2}}}$	19
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{6}x^2b^2-\frac{1}{6}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
default	$-\frac{2\sqrt{-bx+2}}{5x^{\frac{5}{2}}}-\frac{b\left(-\frac{\sqrt{-bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{-bx+2}}{3\sqrt{x}}\right)}{5}$	46
risch	$-\frac{\sqrt{(-bx+2)\,x}\,\left(b^3x^3-x^2b^2-8bx+12\right)}{15x^{\frac{5}{2}}\sqrt{-bx+2}\,\sqrt{-x\,(bx-2)}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-b*x+2)^(1/2)/x^(5/2)-1/5*b*(-1/3*(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+
2)^(1/2)/x^(1/2))
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.70

$$-\frac{(-bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] -1/6*(-b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(-b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.31, size = 25, normalized size = 0.62

$$\frac{(b^2x^2+bx-6)\sqrt{-bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
[Out] 1/15*(b^2*x^2 + b*x - 6)*sqrt(-b*x + 2)/x^(5/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.95, size = 194, normalized size = 4.85

$$\begin{cases}\frac{b^{\frac{9}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{15b^2x^2-30bx}-\frac{b^{\frac{7}{2}}x\sqrt{-1+\frac{2}{bx}}}{15b^2x^2-30bx}-\frac{8b^{\frac{5}{2}}\sqrt{-1+\frac{2}{bx}}}{15b^2x^2-30bx}+\frac{12b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{x(15b^2x^2-30bx)} & \text{for } \frac{1}{|bx|}>\frac{1}{2}\\ \frac{ib^{\frac{5}{2}}\sqrt{1-\frac{2}{bx}}}{15}+\frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{15x}-\frac{2i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{5x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(7/2),x)
[Out] Piecewise((b**(9/2)*x**2*sqrt(-1 + 2/(b*x))/(15*b**2*x**2 - 30*b*x) - b**(7
/2)*x*sqrt(-1 + 2/(b*x))/(15*b**2*x**2 - 30*b*x) - 8*b**(5/2)*sqrt(-1 + 2/(
b*x))/(15*b**2*x**2 - 30*b*x) + 12*b**(3/2)*sqrt(-1 + 2/(b*x))/(x*(15*b**2*
x**2 - 30*b*x)), 1/Abs(b*x) > 1/2), (I*b**(5/2)*sqrt(1 - 2/(b*x))/15 + I*b*
*(3/2)*sqrt(1 - 2/(b*x))/(15*x) - 2*I*sqrt(b)*sqrt(1 - 2/(b*x))/(5*x**2), T
rue))
```

Giac [A]
time = 0.01, size = 109, normalized size = 2.72

$$\frac{2b^2\left(\frac{15}{450}b^5\sqrt{-bx+2}\sqrt{-bx+2}-\frac{75}{450}b^5\right)\sqrt{-bx+2}\sqrt{-bx+2}\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{|b|\,b\,(-b(-bx+2)+2b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x)
```

```
[Out] 1/15*((b*x - 2)*b^5 + 5*b^5)*(b*x - 2)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b
)^(5/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 26, normalized size = 0.65

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b^2\,x^2}{15}+\frac{b\,x}{15}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(7/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/15 + (b^2*x^2)/15 - 2/5))/x^(5/2)
```

3.520 $\int \frac{\sqrt{2-bx}}{x^{9/2}} \, dx$

Optimal. Leaf size=62

$$-\frac{(2-bx)^{3/2}}{7x^{7/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}$$

```
[Out] -1/7*(-b*x+2)^(3/2)/x^(7/2)-2/35*b*(-b*x+2)^(3/2)/x^(5/2)-2/105*b^2*(-b*x+2
)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{47, 37\}$

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -1/7*(2 - b*x)^(3/2)/x^(7/2) - (2*b*(2 - b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 - b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx &= -\frac{(2-bx)^{3/2}}{7x^{7/2}} + \frac{1}{7}(2b)\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx \\ &= -\frac{(2-bx)^{3/2}}{7x^{7/2}} - \frac{2b(2-bx)^{3/2}}{35x^{5/2}} + \frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx \\ &= -\frac{(2-bx)^{3/2}}{7x^{7/2}} - \frac{2b(2-bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.66

$$\frac{\sqrt{2-bx}\,\left(-30+3bx+2b^2x^2+2b^3x^3\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-30 + 3*b*x + 2*b^2*x^2 + 2*b^3*x^3))/(105*x^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 11.22, size = 363, normalized size = 5.85

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,\left(-120+132bx-34b^2x^2+3b^3x^3\,(1-2bx)+2b^5x^5\right)\sqrt{\frac{2-bx}{bx}}}{105x^3\,(4-4bx+b^2x^2)},\frac{1}{\text{Abs}\,[bx]}>\frac{1}{2}\right\}\right\},\frac{-120Ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{420b^4x^3-420b^5x^4+105b^6x^5}+\frac{I132b^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{420b^4x^3-420b^5x^4+105b^6x^5}-\frac{34Ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{420b^4x^3-420b^5x^4+105b^6x^5}+\frac{I3b^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{420b^4x^3-420b^5x^4+105b^6x^5}-\frac{6Ib^{\frac{17}{2}}x^4\sqrt{1-\frac{2}{bx}}}{420b^4x^3-420b^5x^4+105b^6x^5}+\frac{I2b^{\frac{19}{2}}x^5\sqrt{1-\frac{2}{bx}}}{420b^4x^3-420b^5x^4+105b^6x^5}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[2 - b*x]/x^(9/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-120 + 132 b x - 34 b ^ 2 x ^ 2 + 3 b ^ 3 x ^ 3 (1 - 2
 b x) + 2 b ^ 5 x ^ 5) Sqrt[(2 - b x) / (b x)] / (105 x ^ 3 (4 - 4 b x + b
^ 2 x ^ 2)), 1 / Abs[b x] > 1 / 2}}, -120 I b ^ (9 / 2) Sqrt[1 - 2 / (b x)]
 / (420 b ^ 4 x ^ 3 - 420 b ^ 5 x ^ 4 + 105 b ^ 6 x ^ 5) + I 132 b ^ (11 /
2) x Sqrt[1 - 2 / (b x)] / (420 b ^ 4 x ^ 3 - 420 b ^ 5 x ^ 4 + 105 b ^ 6 x
 ^ 5) - 34 I b ^ (13 / 2) x ^ 2 Sqrt[1 - 2 / (b x)] / (420 b ^ 4 x ^ 3 - 42
0 b ^ 5 x ^ 4 + 105 b ^ 6 x ^ 5) + I 3 b ^ (15 / 2) x ^ 3 Sqrt[1 - 2 / (b x
)] / (420 b ^ 4 x ^ 3 - 420 b ^ 5 x ^ 4 + 105 b ^ 6 x ^ 5) - 6 I b ^ (17 /
2) x ^ 4 Sqrt[1 - 2 / (b x)] / (420 b ^ 4 x ^ 3 - 420 b ^ 5 x ^ 4 + 105 b ^
 6 x ^ 5) + I 2 b ^ (19 / 2) x ^ 5 Sqrt[1 - 2 / (b x)] / (420 b ^ 4 x ^ 3 -
 420 b ^ 5 x ^ 4 + 105 b ^ 6 x ^ 5)]
```

Maple [A]
time = 0.13, size = 63, normalized size = 1.02

method	result	size
gosper	$-\frac{(2x^{2}b^{2}+6bx+15)(-bx+2)^{\frac{3}{2}}}{105x^{\frac{7}{2}}}$	28
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{15}b^{3}x^{3}-\frac{1}{15}x^{2}b^{2}-\frac{1}{10}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
default	$-\frac{2\sqrt{-bx+2}}{7x^{\frac{7}{2}}}-\frac{b\left(-\frac{\sqrt{-bx+2}}{5x^{\frac{5}{2}}}+\frac{2b\left(-\frac{\sqrt{-bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{-bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	63
risch	$-\frac{\sqrt{(-bx+2)\,x}\,(2b^{4}x^{4}-2b^{3}x^{3}-x^{2}b^{2}-36bx+60)}{105x^{\frac{7}{2}}\sqrt{-bx+2}\,\sqrt{-x\,(bx-2)}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(-b*x+2)^(1/2)/x^(7/2)-1/7*b*(-1/5*(-b*x+2)^(1/2)/x^(5/2)+2/5*b*(-1/3*
(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+2)^(1/2)/x^(1/2)))
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.71

$$-\frac{(-bx+2)^{\frac{3}{2}}b^{2}}{12\,x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(-bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 1/10*(-b*x + 2)^(5/2)*b/x^(5/2) - 1/28
*(-b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.56

$$\frac{(2\,b^{3}x^{3}+2\,b^{2}x^{2}+3\,bx-30)\sqrt{-bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*b^3*x^3 + 2*b^2*x^2 + 3*b*x - 30)*sqrt(-b*x + 2)/x^(7/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.45, size = 556, normalized size = 8.97

$$\begin{cases} \frac{2b^{\frac{19}{2}}x^5\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} - \frac{6b^{\frac{17}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} + \frac{3b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} - \frac{34b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} + \frac{132b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} - \frac{120b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ \frac{2ib^{\frac{19}{2}}x^5\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} - \frac{6ib^{\frac{17}{2}}x^4\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} + \frac{3ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} - \frac{34ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} + \frac{132ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} - \frac{120ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((2*b**(19/2)*x**5*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 6*b**(17/2)*x**4*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 3*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 132*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3), 1/Abs(b*x) > 1/2), (2*I*b**(19/2)*x**5*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 6*I*b**(17/2)*x**4*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 3*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 34*I*b**(13/2)*x**2*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 132*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 120*I*b**(9/2)*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3), True))
```

Giac [A]
time = 0.01, size = 139, normalized size = 2.24

$$\frac{2b^2\left(\left(\left(-\frac{70}{7350}b^7\sqrt{-bx+2}\sqrt{-bx+2}+\frac{490}{7350}b^7\right)\sqrt{-bx+2}\sqrt{-bx+2}-\frac{1225}{7350}b^7\right)\sqrt{-bx+2}\sqrt{-bx+2}\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{|b|\,b\,(-b(-bx+2)+2b)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x)
```

```
[Out] 1/105*(35*b^7 + 2*((b*x - 2)*b^7 + 7*b^7)*(b*x - 2))*(b*x - 2)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 34, normalized size = 0.55

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,b^3\,x^3}{105}+\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/35 + (2*b^2*x^2)/105 + (2*b^3*x^3)/105 - 2/7))/x^(7/2)
```

3.521 $\int x^{5/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=143

$$\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right)}{128b^{7/\ldots}}$$

```
[Out] 1/5*x^(7/2)*(b*x+a)^(3/2)-3/128*a^5*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/
b^(7/2)-1/64*a^3*x^(3/2)*(b*x+a)^(1/2)/b^2+1/80*a^2*x^(5/2)*(b*x+a)^(1/2)/b
+3/40*a*x^(7/2)*(b*x+a)^(1/2)+3/128*a^4*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a + b*x])/(64*b
^2) + (a^2*x^(5/2)*Sqrt[a + b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a + b*x])/40 +
 (x^(7/2)*(a + b*x)^(3/2))/5 - (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(128*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{10}(3a)\int x^{5/2}\sqrt{a+bx}\,dx$$

$$= \frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{80}\left(3a^2\right)\int\frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{a^3\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{32b}$$

$$= -\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{(3a^4}{}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}$$

Mathematica [A]
time = 0.11, size = 99, normalized size = 0.69

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4)+15a^5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{640b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*a
*b^3*x^3 + 128*b^4*x^4) + 15*a^5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(
640*b^(7/2))
```

Mathics [A]
time = 59.12, size = 143, normalized size = 1.00

$$\frac{-15a^{\frac{15}{2}}b^6\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{5}{2}}+15a^5b^{\frac{13}{2}}\sqrt{x}\,(a+bx)^2+5a^4b^{\frac{15}{2}}x^{\frac{3}{2}}(a+bx)^2-2a^3b^{\frac{17}{2}}x^{\frac{5}{2}}(a+bx)^2+8ab^{\frac{19}{2}}x^{\frac{7}{2}}(23a+38bx)(a+bx)^2+128b^{\frac{23}{2}}x^{\frac{11}{2}}(a+bx)^2}{640a^{\frac{5}{2}}b^{\frac{19}{2}}\left(\frac{a+bx}{a}\right)^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*(a + b*x)^(3/2),x]')
```

```
[Out] (-15 a ^ (15 / 2) b ^ 6 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a)
^ (5 / 2) + 15 a ^ 5 b ^ (13 / 2) Sqrt[x] (a + b x) ^ 2 + 5 a ^ 4 b ^ (15 /
 2) x ^ (3 / 2) (a + b x) ^ 2 - 2 a ^ 3 b ^ (17 / 2) x ^ (5 / 2) (a + b x)
^ 2 + 8 a b ^ (19 / 2) x ^ (7 / 2) (23 a + 38 b x) (a + b x) ^ 2 + 128 b ^
(23 / 2) x ^ (11 / 2) (a + b x) ^ 2) / (640 a ^ (5 / 2) b ^ (19 / 2) ((a +
b x) / a) ^ (5 / 2))
```

Maple [A]
time = 0.12, size = 144, normalized size = 1.01

method	result
risch	$\frac{(128b^4x^4+176a\,b^3x^3+8a^2b^2x^2-10a^3bx+15a^4)\sqrt{x}\,\sqrt{bx+a}}{640b^3}-\frac{3a^5\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{256b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$

default	$\frac{x^{\frac{5}{2}}(bx+a)^{\frac{5}{2}}}{5b} - \frac{a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4b} - \frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b} - \frac{a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2} + \frac{3a\left(\sqrt{x}\,\sqrt{bx+a} + \frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{\cdots}\right.}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{\cdots}}\right.}{4}\right.}{6b}\right.}{8b}\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+a)^(5/2)-1/2*a/b*(1/4/b*x^(3/2)*(b*x+a)^(5/2)-3/8*a/b*(1
/3/b*x^(1/2)*(b*x+a)^(5/2)-1/6*a/b*(1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2
)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x
)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(103) = 206$.

time = 0.34, size = 212, normalized size = 1.48

$$\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^5b^4}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}-\frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8-\frac{5\,(bx+a)b^7}{x}+\frac{10\,(bx+a)^2b^6}{x^2}-\frac{10\,(bx+a)^3b^5}{x^3}+\frac{5\,(bx+a)^4b^4}{x^4}-\frac{(bx+a)^5b^3}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^(
3/2)*a^5*b^3/x^(3/2) - 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^(
7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x + a)*b^7
/x + 10*(b*x + a)^2*b^6/x^2 - 10*(b*x + a)^3*b^5/x^3 + 5*(b*x + a)^4*b^4/x^
4 - (b*x + a)^5*b^3/x^5)
```

Fricas [A]
time = 0.32, size = 184, normalized size = 1.29

$$\left[\frac{15\,a^5\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b)\sqrt{bx+a}\,\sqrt{x}}{1280\,b^4},\ \frac{15\,a^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b)\sqrt{bx+a}\,\sqrt{x}}{640\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)*s
qrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt
(-b)/(b*sqrt(x))) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b
^2*x + 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A]
time = 59.83, size = 178, normalized size = 1.24

$$\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}-\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(3/2),x)
```

```
[Out] 3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + a**(7/2)*x**(3/2)/(128*b**2
*sqrt(1 + b*x/a)) - a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*a**(3/2)
*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*x/a))
```

```
 - 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(5
*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 334, normalized size = 2.34

$$2b\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{1612800}\cdot 80640b^8\sqrt{x}\,\sqrt{x}}{b^8}+\frac{\frac{1}{1612800}\cdot 10080b^7a}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 11760b^6a^2}{b^8}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{1612800}\cdot 14700b^5a^3}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 22050b^4a^4}{b^8}\right)\sqrt{x}\,\sqrt{a+bx}-\frac{14a^5\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{512b^4\sqrt{b}}\right)+2a\left(2\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\,\sqrt{x}}{b^6}+\frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{10a^4\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{256b^3\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x)
```

```
[Out] 1/192*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(
x) + 15*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*a + 1/1920*
((2*(4*(6*(8*x + a/b)*x - 7*a^2/b^2)*x + 35*a^3/b^3)*x - 105*a^4/b^4)*sqrt(
b*x + a)*sqrt(x) - 105*a^5*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/
2))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(3/2), x)
```

3.522 $\int x^{3/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=119

$$-\frac{3a^3\sqrt{x}\,\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(b*x+a)^(3/2)+3/64*a^4*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b
^(5/2)+1/32*a^2*x^(3/2)*(b*x+a)^(1/2)/b+1/8*a*x^(5/2)*(b*x+a)^(1/2)-3/64*a^
3*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\,\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^2) + (a^2*x^(3/2)*Sqrt[a + b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a + b*x])/8 + (x^(5/2)*(a + b*x)^(3/2))/4 + (3*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}(a+bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{1}{8}(3a)\int x^{3/2}\sqrt{a+bx}\,dx$$

$$= \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{1}{16}a^2\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} - \frac{(3a^3)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b}$$

$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4}{}$$

$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4}{}$$

$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4}{}$$

$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{3a^4 t}{}$$

Mathematica [A]
time = 0.08, size = 88, normalized size = 0.74

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3) - 3a^4\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3*
x^3) - 3*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64*b^(5/2))
```

Mathics [A]
time = 13.84, size = 133, normalized size = 1.12

$$\frac{a^{\frac{3}{2}}\left(3a^{\frac{5}{2}}b^3\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](a+bx)^2-3a^4b^{\frac{7}{2}}\sqrt{x}\,\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}-a^3b^{\frac{9}{2}}x^{\frac{3}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+2ab^{\frac{11}{2}}x^{\frac{5}{2}}(13a+20bx)\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+16b^{\frac{15}{2}}x^{\frac{9}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}\right)}{64b^{\frac{11}{2}}(a+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(a + b*x)^(3/2),x]')
```

```
[Out] a ^ (3 / 2) (3 a ^ (5 / 2) b ^ 3 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] (a + b
x) ^ 2 - 3 a ^ 4 b ^ (7 / 2) Sqrt[x] ((a + b x) / a) ^ (3 / 2) - a ^ 3 b ^
(9 / 2) x ^ (3 / 2) ((a + b x) / a) ^ (3 / 2) + 2 a b ^ (11 / 2) x ^ (5 / 2
) (13 a + 20 b x) ((a + b x) / a) ^ (3 / 2) + 16 b ^ (15 / 2) x ^ (9 / 2) (
(a + b x) / a) ^ (3 / 2)) / (64 b ^ (11 / 2) (a + b x) ^ 2)
```

Maple [A]
time = 0.10, size = 122, normalized size = 1.03

method	result
risch	$-\frac{(-16b^3x^3-24a\,b^2x^2-2a^2bx+3a^3)\sqrt{x}\,\sqrt{bx+a}}{64b^2}+\frac{3a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{128b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b}-\frac{a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+a)^(5/2)-3/8*a/b*(1/3/b*x^(1/2)*(b*x+a)^(5/2)-1/6*a/b*(1
/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/
2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2))
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(85) = 170.
time = 0.35, size = 178, normalized size = 1.50

$$-\frac{3\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{5}{2}}}-\frac{\frac{3\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{11\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx+a)b^5}{x}+\frac{6\,(bx+a)^2b^4}{x^2}-\frac{4\,(bx+a)^3b^3}{x^3}+\frac{(bx+a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/64*(3*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 11*(b*x + a)^(3
/2)*a^4*b^2/x^(3/2) - 11*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 3*(b*x + a)^(7/2)*
a^4/x^(7/2))/(b^6 - 4*(b*x + a)*b^5/x + 6*(b*x + a)^2*b^4/x^2 - 4*(b*x + a)
^3*b^3/x^3 + (b*x + a)^4*b^2/x^4)
```

Fricas [A]
time = 0.33, size = 163, normalized size = 1.37

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{128\,b^3},-\frac{3\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{64\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

Sympy [A]
time = 12.47, size = 153, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}}+\frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - a**(5/2)*x**(3/2)/(64*b*sqr
t(1 + b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*sqrt(a)*b*x**
```

```
(7/2)/(8*sqrt(1 + b*x/a)) + 3*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(5
/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 279, normalized size = 2.34

$$2b\left(2\left(\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\,\sqrt{x}}{b^6}+\frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{10a^4\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{256b^3\sqrt{b}}\right)\right)+2a\left(2\left(\left(\left(\frac{\frac{1}{576}\cdot 48b^4\sqrt{x}\,\sqrt{x}}{b^4}+\frac{\frac{1}{576}\cdot 12b^3a}{b^4}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{576}\cdot 18b^2a^2}{b^4}\right)\sqrt{x}\,\sqrt{a+bx}-\frac{2a^3\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{32b^2\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x)
```

```
[Out] 1/24*(sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-
sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))*a + 1/192*((2*(4*(6*x + a/b)*x -
 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 15*a^4*log(abs(-sqrt(b)
*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(3/2), x)
```

3.523 $\int \sqrt{x}\,(a+bx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)-1/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^
(3/2)+1/4*a*x^(3/2)*(b*x+a)^(1/2)+1/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b) + (a*x^(3/2)*Sqrt[a + b*x])/4 + (x^(3/2)*
(a + b*x)^(3/2))/3 - (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8*b^(3
/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(a+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\,\sqrt{a+bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b}\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,d\right.}{8b}\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\right.}{8b}\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.80

$$\frac{\sqrt{x}\,\sqrt{a+bx}\,(3a^2+14abx+8b^2x^2)}{24b}+\frac{a^3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(3/2),x]
[Out] (Sqrt[x]*Sqrt[a + b*x]*(3*a^2 + 14*a*b*x + 8*b^2*x^2))/(24*b) + (a^3*Log[-(
Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(3/2))
```

Mathics [A]
time = 5.83, size = 98, normalized size = 1.03

$$\frac{-3a^{\frac{9}{2}}b\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+3a^3b^{\frac{3}{2}}\sqrt{x}\,(a+bx)+ab^{\frac{5}{2}}x^{\frac{3}{2}}(a+bx)(17a+22bx)+8b^{\frac{9}{2}}x^{\frac{7}{2}}(a+bx)}{24a^{\frac{3}{2}}b^{\frac{5}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[x]*(a + b*x)^(3/2),x]')

[Out] (-3 a ^ (9 / 2) b ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^ (3 / 2) + 3 a ^ 3 b ^ (3 / 2) Sqrt[x] (a + b x) + a b ^ (5 / 2) x ^ (3 / 2) (a + b x) (17 a + 22 b x) + 8 b ^ (9 / 2) x ^ (7 / 2) (a + b x)) / (24 a ^ (3 / 2) b ^ (5 / 2) ((a + b x) / a) ^ (3 / 2))

Maple [A]
time = 0.11, size = 97, normalized size = 1.02

method	result	size
risch	$\frac{\left(8x^2b^2+14abx+3a^2\right)\sqrt{x}\,\sqrt{bx+a}}{24b}-\frac{a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\left(bx+a\right)}}{16b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2}+\frac{a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\left(bx+a\right)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4}\right)}{2}$	97

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)+1/2*a*(1/2*x^(3/2)*(b*x+a)^(1/2)+1/4*a*(x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(67) = 134.
time = 0.36, size = 144, normalized size = 1.52

$$\frac{a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\frac{3\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{8\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^4-\frac{3\,(bx+a)b^3}{x}+\frac{3\,(bx+a)^2b^2}{x^2}-\frac{(bx+a)^3b}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")

```
[Out] 1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(3/2) + 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 8*(b*x + a)^(3/2)
*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x + a)*b^3/x +
3*(b*x + a)^2*b^2/x^2 - (b*x + a)^3*b/x^3)
```

Fricas [A]
time = 0.32, size = 140, normalized size = 1.47

$$\left[\frac{3\,a^{3}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}+14\,ab^{2}x+3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^{2}},\ \frac{3\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^{3}x^{2}+14\,ab^{2}x+3\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A]
time = 3.91, size = 124, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^{2}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 +
b*x/a)) + 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 + b*x/a)) - a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(67) = 134.
time = 30.71, size = 378, normalized size = 3.98

$$\frac{\frac{2b^{2}|b|\left(2\left(\left(\left(\frac{\frac{1}{2304}\cdot 192b^{5}\sqrt{a+bx}\,\sqrt{a+bx}}{b^{7}}-\frac{\frac{1}{2304}\cdot 624b^{5}a}{b^{7}}\right)\sqrt{a+bx}\,\sqrt{a+bx}+\frac{\frac{1}{2304}\cdot 792b^{5}a^{2}}{b^{7}}\right)\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}+\frac{10a^{3}\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{32b\sqrt{b}}\right)\right)}{b^{2}}+\frac{4ab|b|\left(2\left(\frac{1}{8}\sqrt{a+bx}\,\sqrt{a+bx}-\frac{10}{32}a\right)\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}-\frac{6a^{2}b\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{16\sqrt{b}}\right)}{b^{2}b}+\frac{2a^{2}|b|\left(\frac{1}{2}\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}+\frac{2ab\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{4\sqrt{b}}\right)}{b^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/24*((15*a^3*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/b^
(3/2) + sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2
```

```
 - 13*a/b^2) + 33*a^2/b^2))*abs(b) + 24*(a*sqrt(b)*log(abs(-sqrt(b*x + a)*s
qrt(b) + sqrt((b*x + a)*b - a*b))) + sqrt((b*x + a)*b - a*b)*sqrt(b*x + a))
*a^2*abs(b)/b^2 - 12*(3*a^2*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((
b*x + a)*b - a*b))) - sqrt((b*x + a)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*
a*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\, (a+b\,x)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(3/2), x)
```

3.524 $\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=71

$$\frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}$$

```
[Out] 3/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)+1/2*(b*x+a)^(3/2)*x^
(1/2)+3/4*a*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a + b*x])/4 + (Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a^2*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 0.87

$$\frac{1}{4}\sqrt{x}\,\sqrt{a+bx}\,(5a+2bx)-\frac{3a^2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[x],x]
[Out] (Sqrt[x]*Sqrt[a + b*x]*(5*a + 2*b*x))/4 - (3*a^2*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/(4*Sqrt[b])
```

Mathics [A]
time = 3.51, size = 51, normalized size = 0.72

$$\frac{3a^2\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]+\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,(5a+2bx)\sqrt{\frac{a+bx}{a}}}{4\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/Sqrt[x],x]')
```

```
[Out] (3 a ^ 2 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] + Sqrt[a] Sqrt[b] Sqrt[x] (5 a
+ 2 b x) Sqrt[(a + b x) / a]) / (4 Sqrt[b])
```

Maple [A]
time = 0.12, size = 78, normalized size = 1.10

method	result	size
risch	$\frac{(2bx+5a)\sqrt{x}\,\sqrt{bx+a}}{4}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\,(bx+a)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	73
default	$\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1
/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(49) = 98.
time = 0.33, size = 107, normalized size = 1.51

$$-\frac{3\,a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,\sqrt{b}}-\frac{\frac{3\,\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{2}-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/sqrt(b) - 1/4*(3*sqrt(b*x + a)*a^2*b/sqrt(x) - 5*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x + a)^2/x^2)
```

Fricas [A]
time = 0.33, size = 119, normalized size = 1.68

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x+5\,ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b},\,-\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{2}x+5\,ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

Sympy [A]
time = 1.73, size = 75, normalized size = 1.06

$$\frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{4}+\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 5*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a)/4 + sqrt(a)*b*x**(3/2)*sqrt(1 + b*x/a)/2
 + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b))
```

Giac [A]
time = 10.36, size = 120, normalized size = 1.69

$$\frac{b^2\left(2\left(\frac{\frac{1}{8}\cdot 2\sqrt{a+bx}\sqrt{a+bx}}{b}+\frac{\frac{1}{8}\cdot 3a}{b}\right)\sqrt{a+bx}\sqrt{-ab+b(a+bx)}-\frac{6a^2\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{8\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] -1/4*(3*a^2*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt
(b) - sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b + 3*a/b))*b/abs(
b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(1/2), x)
```

3.525 $\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=63

$$3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 3*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))*b^(1/2)-2*(b*x+a)^(3/2)/x^(1/2)+
3*b*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\,\sqrt{a+bx}+3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[a + b*x] - (2*(a + b*x)^(3/2))/Sqrt[x] + 3*a*Sqrt[b]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx \\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx \\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3ab)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right) \\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right) \\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.86

$$
\frac{(-2a+bx)\sqrt{a+bx}}{\sqrt{x}}-3a\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-2*a + b*x)*Sqrt[a + b*x])/Sqrt[x] - 3*a*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]]
```

Mathics [A]
time = 3.44, size = 84, normalized size = 1.33

$$\frac{-2a\left(a+bx\right)+3a^{\frac{3}{2}}\sqrt{b}\,\sqrt{x}\,\mathrm{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}-bx\left(a+bx\right)+\frac{b^2x^2(a+bx)}{a}}{\sqrt{a}\,\sqrt{x}\,\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/x^(3/2),x]')
```

```
[Out] (-2 a (a + b x) + 3 a ^ (3 / 2) Sqrt[b] Sqrt[x] ArcSinh[Sqrt[b] Sqrt[x] / S
qrt[a]] ((a + b x) / a) ^ (3 / 2) - b x (a + b x) + b ^ 2 x ^ 2 (a + b x) /
 a) / (Sqrt[a] Sqrt[x] ((a + b x) / a) ^ (3 / 2))
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.13

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-bx+2a)}{\sqrt{x}}+\frac{3a\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1/2)*(-b*x+2*a)/x^(1/2)+3/2*a*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x
^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 84, normalized size = 1.33

$$-\frac{3}{2}\,a\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*a*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 2*sqrt(b*x + a)*a/sqrt(x) - sqrt(b*x + a)*a*b/((b - (b*x +
a)/x)*sqrt(x))
```

Fricas [A]
time = 0.32, size = 109, normalized size = 1.73

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{2\,x},-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

Sympy [A]
time = 1.52, size = 92, normalized size = 1.46

$$-\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}}+3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] -2*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) - sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a) +
 3*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1
 + b*x/a))
```

Giac [A]
time = 10.28, size = 126, normalized size = 2.00

$$\frac{bb^2\left(\frac{2\left(\frac{1}{2}\sqrt{a+bx}\,\sqrt{a+bx}-\frac{3}{2}a\right)\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{-ab+b(a+bx)}-\frac{6a\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{2\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] -(3*a*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqrt(b) -
sqrt(b*x + a)*(b*x - 2*a)/sqrt((b*x + a)*b - a*b))*b^2/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(3/2), x)
```

3.526 $\int \frac{(a+bx)^{3/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=64

$$-\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] -2/3*(b*x+a)^(3/2)/x^(3/2)+2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
-2*b*(b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[a + b*x])/Sqrt[x] - (2*(a + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 55, normalized size = 0.86

$$-\frac{2\sqrt{a+bx}\,(a+4bx)}{3x^{3/2}}-2b^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(5/2),x]
[Out] (-2*Sqrt[a + b*x]*(a + 4*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]]
```

Mathics [**A**]
time = 3.71, size = 76, normalized size = 1.19

$$\frac{\sqrt{b}\left(-2a\sqrt{\frac{a+bx}{bx}}+bx\left(-8\sqrt{\frac{a+bx}{bx}}-3\text{Log}\left[\frac{a}{bx}\right]+6\text{Log}\left[1+\sqrt{\frac{a+bx}{bx}}\right]\right)\right)}{3x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/x^(5/2),x]')
```

```
[Out] Sqrt[b] (-2 a Sqrt[(a + b x) / (b x)] + b x (-8 Sqrt[(a + b x) / (b x)] - 3
 Log[a / (b x)] + 6 Log[1 + Sqrt[(a + b x) / (b x)]])) / (3 x)
```

Maple [A]
time = 0.11, size = 67, normalized size = 1.05

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(4bx+a)}{3x^{\frac{3}{2}}}+\frac{b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(4*b*x+a)/x^(3/2)+b^(3/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+
a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.37, size = 67, normalized size = 1.05

$$-b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)*b/sqrt(x) - 2/3*(b*x + a)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.31, size = 109, normalized size = 1.70

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{3\,x^2},-\frac{2\left(3\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A]
time = 1.78, size = 71, normalized size = 1.11

$$-\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-b^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] -2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 8*b**(3/2)*sqrt(a/(b*x) + 1)/3 - b**
(3/2)*log(a/(b*x)) + 2*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)
```

Giac [A]
time = 10.36, size = 143, normalized size = 2.23

$$\frac{b^2\left(\frac{2\left(-\frac{\frac{1}{9}\cdot 12b^3a\sqrt{a+bx}\sqrt{a+bx}}{a}+\frac{\frac{1}{9}\cdot 9b^3a^2}{a}\right)\sqrt{a+bx}\sqrt{-ab+b(a+bx)}}{(-ab+b(a+bx))^2}-\frac{2b^2\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] -2/3*(3*b^(3/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))
+ (4*(b*x + a)*b^3 - 3*a*b^3)*sqrt(b*x + a)/((b*x + a)*b - a*b)^(3/2))*b/ab
s(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(5/2), x)
```

3.527 $\int x^{5/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=149

$$-\frac{3a^4\sqrt{x}\,\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}$$

```
[Out] 1/5*x^(7/2)*(-b*x+a)^(3/2)+3/128*a^5*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))
/b^(7/2)-1/64*a^3*x^(3/2)*(-b*x+a)^(1/2)/b^2-1/80*a^2*x^(5/2)*(-b*x+a)^(1/2
)/b+3/40*a*x^(7/2)*(-b*x+a)^(1/2)-3/128*a^4*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{3a^4\sqrt{x}\,\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b^2) - (a^2*x^(5/2)*Sqrt[a - b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a - b*x])/40
+ (x^(7/2)*(a - b*x)^(3/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/(128*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{1}{10}(3a)\int x^{5/2}\sqrt{a-bx}\,dx$$
$$= \frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{1}{80}\left(3a^2\right)\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{a^3\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{32b}$$
$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{(3a^4}{}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7}$$

Mathematica [A]
time = 0.16, size = 104, normalized size = 0.70

$$\frac{1}{640}\left(-\frac{\sqrt{x}\sqrt{a-bx}\,\left(15a^4+10a^3bx+8a^2b^2x^2-176ab^3x^3+128b^4x^4\right)}{b^3}+\frac{15a^5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-((Sqrt[x]*Sqrt[a - b*x]*(15*a^4 + 10*a^3*b*x + 8*a^2*b^2*x^2 - 176*a*b^3*
x^3 + 128*b^4*x^4))/b^3) + (15*a^5*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
)/(-b)^(7/2))/640
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 57.98, size = 316, normalized size = 2.12

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-15a^{\frac{15}{2}}b^{6}\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{5}{2}}+15a^{5}b^{\frac{13}{2}}\sqrt{x}\left(-a+bx\right)^{2}-5a^{4}b^{\frac{15}{2}}x^{\frac{3}{2}}\left(-a+bx\right)^{2}-2a^{3}b^{\frac{17}{2}}x^{\frac{5}{2}}\left(-a+bx\right)^{2}+8ab^{\frac{19}{2}}x^{\frac{7}{2}}\left(-23a+38bx\right)\left(-a+bx\right)^{2}-128b^{\frac{23}{2}}x^{\frac{11}{2}}\left(-a+bx\right)^{2}\right)}{640a^{\frac{5}{2}}b^{\frac{19}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{5}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{3a^{5}\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{128b^{\frac{7}{2}}}-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}-\frac{19\sqrt{a}bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{b^{2}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*(a - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 640 (-15 a ^ (15 / 2) b ^ 6 ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[
a]] ((-a + b x) / a) ^ (5 / 2) + 15 a ^ 5 b ^ (13 / 2) Sqrt[x] (-a + b x) ^
 2 - 5 a ^ 4 b ^ (15 / 2) x ^ (3 / 2) (-a + b x) ^ 2 - 2 a ^ 3 b ^ (17 / 2)
 x ^ (5 / 2) (-a + b x) ^ 2 + 8 a b ^ (19 / 2) x ^ (7 / 2) (-23 a + 38 b x)
 (-a + b x) ^ 2 - 128 b ^ (23 / 2) x ^ (11 / 2) (-a + b x) ^ 2) / (a ^ (5 /
 2) b ^ (19 / 2) ((-a + b x) / a) ^ (5 / 2)), Abs[b x / a] > 1}}, 3 a ^ 5 A
rcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (128 b ^ (7 / 2)) - 3 a ^ (9 / 2) Sqrt[x
] / (128 b ^ 3 Sqrt[1 - b x / a]) + a ^ (7 / 2) x ^ (3 / 2) / (128 b ^ 2 Sq
rt[1 - b x / a]) + a ^ (5 / 2) x ^ (5 / 2) / (320 b Sqrt[1 - b x / a]) + 23
 a ^ (3 / 2) x ^ (7 / 2) / (80 Sqrt[1 - b x / a]) - 19 Sqrt[a] b x ^ (9 / 2
) / (40 Sqrt[1 - b x / a]) + b ^ 2 x ^ (11 / 2) / (5 Sqrt[a] Sqrt[1 - b x /
 a])]
```

Maple [A]
time = 0.11, size = 152, normalized size = 1.02

method	result
risch	$-\frac{\left(128 b^{4} x^{4}-176 a\,b^{3} x^{3}+8 a^{2} b^{2} x^{2}+10 a^{3} b x+15 a^{4}\right) \sqrt{x}\,\sqrt{-b x+a}}{640 b^{3}}+\frac{3 a^{5} \arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right) \sqrt{x \left(-b x+a\right)}}{256 b^{\frac{7}{2}} \sqrt{x}\,\sqrt{-b x+a}}$

default	$-\frac{x^{\frac{5}{2}}(-bx+a)^{\frac{5}{2}}}{5b}+\frac{a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3b}+\frac{a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a}}{2\sqrt{-b}}\right.}{4}\right.}{6b}\right.}{8b}\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/5/b*x^(5/2)*(-b*x+a)^(5/2)+1/2*a/b*(-1/4/b*x^(3/2)*(-b*x+a)^(5/2)+3/8*a/b*(-1/3/b*x^(1/2)*(-b*x+a)^(5/2)+1/6*a/b*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))))

Maxima [A]

time = 0.36, size = 207, normalized size = 1.39

$$-\frac{3\,a^5\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^5b^4}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}-\frac{128\,(-bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8-\frac{5\,(bx-a)b^7}{x}+\frac{10\,(bx-a)^2b^6}{x^2}-\frac{10\,(bx-a)^3b^5}{x^3}+\frac{5\,(bx-a)^4b^4}{x^4}-\frac{(bx-a)^5b^3}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) - 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x - a)*b^7/x + 10*(b*x - a)^2*b^6/x^2
 - 10*(b*x - a)^3*b^5/x^3 + 5*(b*x - a)^4*b^4/x^4 - (b*x - a)^5*b^3/x^5)
```

Fricas [A]
time = 0.34, size = 185, normalized size = 1.24

$$\left[-\frac{15\,a^5\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+10\,a^3b^2x+15\,a^4b)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^4},\ -\frac{15\,a^5\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+10\,a^3b^2x+15\,a^4b)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a^3*b^2*x + 15*a^4
*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) + (128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a
^3*b^2*x + 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [A]
time = 59.65, size = 376, normalized size = 2.52

$$\begin{cases}\frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{-1+\frac{bx}{a}}}-\frac{23ia^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}+\frac{19i\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}-\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}-\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**3*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**(5/2)/(320*b*sqrt(-1 + b
*x/a)) - 23*I*a**(3/2)*x**(7/2)/(80*sqrt(-1 + b*x/a)) + 19*I*sqrt(a)*b*x**(
9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b
```

```
**(7/2)) - I*b**2*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1),
(-3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(128*b*
*2*sqrt(1 - b*x/a)) + a**(5/2)*x**(5/2)/(320*b*sqrt(1 - b*x/a)) + 23*a**(3/
2)*x**(7/2)/(80*sqrt(1 - b*x/a)) - 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 - b*x/a
)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(
5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 348, normalized size = 2.34

$$-2b\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{1612800}\cdot 80640b^8\sqrt{x}\,\sqrt{x}}{b^8}-\frac{\frac{1}{1612800}\cdot 10080b^7a}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 11760b^6a^2}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 14700b^5a^3}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 22050b^4a^4}{b^8}\right)\sqrt{x}\,\sqrt{a-bx}-\frac{14a^5\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{512b^4\sqrt{-b}}\right)+2a\left(2\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\,\sqrt{x}}{b^6}-\frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\,\sqrt{a-bx}-\frac{10a^4\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{256b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x)
```

```
[Out] 1/192*((2*(4*(6*x - a/b)*x - 5*a^2/b^2)*x - 15*a^3/b^3)*sqrt(-b*x + a)*sqrt
(x) - 15*a^4*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3))*a
 - 1/1920*((2*(4*(6*(8*x - a/b)*x - 7*a^2/b^2)*x - 35*a^3/b^3)*x - 105*a^4/
b^4)*sqrt(-b*x + a)*sqrt(x) - 105*a^5*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x
 + a)))/(sqrt(-b)*b^4))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a-b\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(3/2), x)
```

3.528 $\int x^{3/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{3a^3\sqrt{x}\,\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(-b*x+a)^(3/2)+3/64*a^4*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/
b^(5/2)-1/32*a^2*x^(3/2)*(-b*x+a)^(1/2)/b+1/8*a*x^(5/2)*(-b*x+a)^(1/2)-3/64
*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\,\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^2) - (a^2*x^(3/2)*Sqrt[a - b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a - b*x])/8 + (x^(5/2)*(a - b*x)^(3/2))/4 + (3*a^4*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a-bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{8}(3a)\int x^{3/2}\sqrt{a-bx}\,dx\\ &= \frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{16}a^2\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx\\ &= -\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a^3)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a^4}{\quad}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a^4}{\quad}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a^4}{\quad}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{3a^4}{\quad}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.75

$$\frac{1}{64}\left(-\frac{\sqrt{x}\sqrt{a-bx}\,(3a^3+2a^2bx-24ab^2x^2+16b^3x^3)}{b^2}-\frac{3a^4\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-((Sqrt[x]*Sqrt[a - b*x]*(3*a^3 + 2*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/
b^2) - (3*a^4*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(5/2))/64
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 24.03, size = 282, normalized size = 2.27

$$\text{Piecewise}\left[\left\{\left\{\frac{Ia^{\frac{3}{2}}\left(-3a^{\frac{5}{2}}b^3\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right](-a+bx)^2+3a^4b^{\frac{7}{2}}\sqrt{x}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-a^3b^{\frac{9}{2}}x^{\frac{3}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+2ab^{\frac{11}{2}}x^{\frac{5}{2}}(-13a+20bx)\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-16b^{\frac{15}{2}}x^{\frac{9}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}\right)}{64b^{\frac{11}{2}}(-a+bx)^2},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{3a^4\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{64b^{\frac{5}{2}}}-\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(a - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 64 a ^ (3 / 2) (-3 a ^ (5 / 2) b ^ 3 ArcCosh[Sqrt[b] Sqrt[x
] / Sqrt[a]] (-a + b x) ^ 2 + 3 a ^ 4 b ^ (7 / 2) Sqrt[x] ((-a + b x) / a)
^ (3 / 2) - a ^ 3 b ^ (9 / 2) x ^ (3 / 2) ((-a + b x) / a) ^ (3 / 2) + 2 a
b ^ (11 / 2) x ^ (5 / 2) (-13 a + 20 b x) ((-a + b x) / a) ^ (3 / 2) - 16 b
 ^ (15 / 2) x ^ (9 / 2) ((-a + b x) / a) ^ (3 / 2)) / (b ^ (11 / 2) (-a + b
 x) ^ 2), Abs[b x / a] > 1}}, 3 a ^ 4 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (
64 b ^ (5 / 2)) - 3 a ^ (7 / 2) Sqrt[x] / (64 b ^ 2 Sqrt[1 - b x / a]) + a
^ (5 / 2) x ^ (3 / 2) / (64 b Sqrt[1 - b x / a]) + 13 a ^ (3 / 2) x ^ (5 /
2) / (32 Sqrt[1 - b x / a]) - 5 Sqrt[a] b x ^ (7 / 2) / (8 Sqrt[1 - b x / a
]) + b ^ 2 x ^ (9 / 2) / (4 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.11, size = 129, normalized size = 1.04

method	result
risch	$-\frac{(16b^3x^3-24a\,b^2x^2+2a^2bx+3a^3)\sqrt{x}\,\sqrt{-bx+a}}{64b^2}+\frac{3a^4\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{128b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3b}+\frac{a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\ldots}{\sqrt{-x\ldots}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{\ldots}}\right)}{4}\right)}{6b}\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+a)^(5/2)+3/8*a/b*(-1/3/b*x^(1/2)*(-b*x+a)^(5/2)+1/6*a/
b*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+
a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2
+a*x)^(1/2)))))
```

Maxima [A]
time = 0.34, size = 170, normalized size = 1.37

$$-\frac{3\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{5}{2}}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{11\,(-bx+a)^{\frac{3}{2}}\,a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+a)^{\frac{5}{2}}\,a^4b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+a)^{\frac{7}{2}}\,a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx-a)b^5}{x}+\frac{6\,(bx-a)^2b^4}{x^2}-\frac{4\,(bx-a)^3b^3}{x^3}+\frac{(bx-a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/64*(3*sqrt(-
b*x + a)*a^4*b^3/sqrt(x) + 11*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 11*(-b*x +
 a)^(5/2)*a^4*b/x^(5/2) - 3*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^6 - 4*(b*x - a
)*b^5/x + 6*(b*x - a)^2*b^4/x^2 - 4*(b*x - a)^3*b^3/x^3 + (b*x - a)^4*b^2/x
^4)
```

Fricas [A]
time = 0.32, size = 163, normalized size = 1.31

$$\left[-\frac{3\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{128\,b^3},\ -\frac{3\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{64\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(3*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^3, -1/64*(3*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) +
(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqrt(x))
/b^3]
```

Sympy [A]

time = 12.23, size = 323, normalized size = 2.60

$$\begin{cases} \frac{3ia^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{-1+\frac{bx}{a}}} - \frac{13ia^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{-1+\frac{bx}{a}}} + \frac{5i\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}} - \frac{ib^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}} + \frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}} - \frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}} + \frac{3a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}} + \frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(7/2)*sqrt(x)/(64*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**
(3/2)/(64*b*sqrt(-1 + b*x/a)) - 13*I*a**(3/2)*x**(5/2)/(32*sqrt(-1 + b*x/a)
) + 5*I*sqrt(a)*b*x**(7/2)/(8*sqrt(-1 + b*x/a)) - 3*I*a**4*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(5/2)) - I*b**2*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a))
, Abs(b*x/a) > 1), (-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 - b*x/a)) + a**(5/2
)*x**(3/2)/(64*b*sqrt(1 - b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 - b*x/a
)) - 5*sqrt(a)*b*x**(7/2)/(8*sqrt(1 - b*x/a)) + 3*a**4*asin(sqrt(b)*sqrt(x)
/sqrt(a))/(64*b**(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 292, normalized size = 2.35

$$-2b\left(2\left(\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\sqrt{x}}{b^6} - \frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\sqrt{a-bx} - \frac{10a^4\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{256b^3\sqrt{-b}}\right)\right) + 2a\left(2\left(\left(\left(\frac{\frac{1}{576}\cdot 48b^4\sqrt{x}\sqrt{x}}{b^4} - \frac{\frac{1}{576}\cdot 12b^3a}{b^4}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{576}\cdot 18b^2a^2}{b^4}\right)\sqrt{x}\sqrt{a-bx} - \frac{2a^3\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{32b^2\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x)
```

```
[Out] 1/24*(sqrt(-b*x + a)*(2*(4*x - a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(
-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^2))*a - 1/192*((2*(4*(6*x
- a/b)*x - 5*a^2/b^2)*x - 15*a^3/b^3)*sqrt(-b*x + a)*sqrt(x) - 15*a^4*log(a
bs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(3/2), x)
```

3.529 $\int \sqrt{x}\,(a-bx)^{3/2}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b
^(3/2)+1/4*a*x^(3/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] -1/8*(a^2*Sqrt[x]*Sqrt[a - b*x])/b + (a*x^(3/2)*Sqrt[a - b*x])/4 + (x^(3/2)
*(a - b*x)^(3/2))/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*b^(3
/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\,(a-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\,\sqrt{a-bx}\,dx \\
&= \frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{16b} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\right.}{8b} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x\right.}{8b} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.83

$$
-\frac{\sqrt{x}\,\sqrt{a-bx}\,(3a^2-14abx+8b^2x^2)}{24b}+\frac{a^3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{8(-b)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] -1/24*(Sqrt[x]*Sqrt[a - b*x]*(3*a^2 - 14*a*b*x + 8*b^2*x^2))/b + (a^3*Log[-
(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*(-b)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.07, size = 222, normalized size = 2.24

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(-3a^{\frac{9}{2}}b\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+3a^3b^{\frac{3}{2}}\sqrt{x}\,(-a+bx)+ab^{\frac{5}{2}}x^{\frac{3}{2}}(-17a+22bx)(-a+bx)+8b^{\frac{9}{2}}x^{\frac{7}{2}}(a-bx)\right)}{24a^{\frac{3}{2}}b^{\frac{5}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{a^3\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{8b^{\frac{3}{2}}}-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}}-\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1-\frac{bx}{a}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*(a - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 24 (-3 a ^ (9 / 2) b ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((-
a + b x) / a) ^ (3 / 2) + 3 a ^ 3 b ^ (3 / 2) Sqrt[x] (-a + b x) + a b ^ (5
 / 2) x ^ (3 / 2) (-17 a + 22 b x) (-a + b x) + 8 b ^ (9 / 2) x ^ (7 / 2) (
a - b x)) / (a ^ (3 / 2) b ^ (5 / 2) ((-a + b x) / a) ^ (3 / 2)), Abs[b x /
 a] > 1}}, a ^ 3 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (8 b ^ (3 / 2)) - a ^
(5 / 2) Sqrt[x] / (8 b Sqrt[1 - b x / a]) + 17 a ^ (3 / 2) x ^ (3 / 2) / (2
4 Sqrt[1 - b x / a]) - 11 Sqrt[a] b x ^ (5 / 2) / (12 Sqrt[1 - b x / a]) +
b ^ 2 x ^ (7 / 2) / (3 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.11, size = 104, normalized size = 1.05

method	result	siz
risch	$-\frac{\left(8 x^{2} b^{2}-14 a b x+3 a^{2}\right) \sqrt{x}\, \sqrt{-b x+a}}{24 b}+\frac{a^{3} \arctan\left(\frac{\sqrt{b}\, \left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right) \sqrt{x \left(-b x+a\right)}}{16 b^{\frac{3}{2}} \sqrt{x}\, \sqrt{-b x+a}}$	91
default	$\frac{x^{\frac{3}{2}} \left(-b x+a\right)^{\frac{3}{2}}}{3}+\frac{a \left(\frac{x^{\frac{3}{2}} \sqrt{-b x+a}}{2}+\frac{a \left(-\frac{\sqrt{x}\, \sqrt{-b x+a}}{b}+\frac{a \sqrt{x \left(-b x+a\right)}\, \arctan\left(\frac{\sqrt{b}\, \left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right)}{2 b^{\frac{3}{2}} \sqrt{x}\, \sqrt{-b x+a}}\right)}{4}\right)}{2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/2*a*(1/2*x^(3/2)*(-b*x+a)^(1/2)+1/4*a*(-x^(1/2
)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*
arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.35, size = 133, normalized size = 1.34

$$-\frac{a^{3} \arctan\left(\frac{\sqrt{-b x+a}}{\sqrt{b} \sqrt{x}}\right)}{8 \, b^{\frac{3}{2}}}+\frac{\frac{3 \, \sqrt{-b x+a} a^{3} b^{2}}{\sqrt{x}}+\frac{8 \, {\left(-b x+a\right)}^{\frac{3}{2}} a^{3} b}{x^{\frac{3}{2}}}-\frac{3 \, {\left(-b x+a\right)}^{\frac{5}{2}} a^{3}}{x^{\frac{5}{2}}}}{24 \, {\left(b^{4}-\frac{3 \, {\left(b x-a\right)} b^{3}}{x}+\frac{3 \, {\left(b x-a\right)}^{2} b^{2}}{x^{2}}-\frac{{\left(b x-a\right)}^{3} b}{x^{3}}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")

[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/24*(3*sqrt(-b*x + a)*a^3*b^2/sqrt(x) + 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x - a)*b^3/x + 3*(b*x - a)^2*b^2/x^2 - (b*x - a)^3*b/x^3)

Fricas [A]
time = 0.33, size = 141, normalized size = 1.42

$$\left[-\frac{3\,a^{3}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}-14\,ab^{2}x+3\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^{2}},-\frac{3\,a^{3}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(8\,b^{3}x^{2}-14\,ab^{2}x+3\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")

[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*(8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/24*(3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2]

Sympy [A]
time = 4.00, size = 264, normalized size = 2.67

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{-1+\frac{bx}{a}}}-\frac{17ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}+\frac{11i\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}-\frac{ib^{2}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}}-\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^{2}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)**(3/2)*x**(1/2),x)

[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b*sqrt(-1 + b*x/a)) - 17*I*a**(3/2)*x**(3/2)/(24*sqrt(-1 + b*x/a)) + 11*I*sqrt(a)*b*x**(5/2)/(12*sqrt(-1 + b*x/a)) - I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) - I*b**2*x**(7/2)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b*sqrt(1 - b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 - b*x/a)) - 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(71) = 142.
time = 31.18, size = 406, normalized size = 4.10

$$-\frac{-\frac{2b^{2}|b|\left(2\left(\left(\left(\frac{\frac{1}{2304}\cdot 192b^{5}\sqrt{a-bx}\,\sqrt{a-bx}}{b^{7}}-\frac{\frac{1}{2304}\cdot 624b^{5}a}{b^{7}}\right)\sqrt{a-bx}\,\sqrt{a-bx}+\frac{\frac{1}{2304}\cdot 792b^{5}a^{2}}{b^{7}}\right)\sqrt{a-bx}\,\sqrt{ab-b\left(a-bx\right)}-\frac{10a^{3}\ln\left|\sqrt{ab-b\left(a-bx\right)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{32b\sqrt{-b}}\right)\right)}{b^{2}}-\frac{4ab|b|\left(2\left(\frac{1}{8}\sqrt{a-bx}\,\sqrt{a-bx}-\frac{10}{32}a\right)\sqrt{a-bx}\,\sqrt{ab-b\left(a-bx\right)}+\frac{6a^{2}b\ln\left|\sqrt{ab-b\left(a-bx\right)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{16\sqrt{-b}}\right)}{b^{2}b}-\frac{2a^{2}|b|\left(\frac{1}{2}\sqrt{a-bx}\,\sqrt{ab-b\left(a-bx\right)}-\frac{2ab\ln\left|\sqrt{ab-b\left(a-bx\right)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{4\sqrt{-b}}\right)}{b^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)^(3/2)*x^(1/2),x)

[Out] 1/24*((15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/(sqrt(-b)*b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*(b*x - a)/b^2 + 13*a/b^2) + 33*a^2/b^2))*abs(b) + 24*(a*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a))*a^2*abs(b)/b^2 - 12*(3*a^2*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) - sqrt((b*x - a)*b + a*b)*(2*b*x + 3*a)*sqrt(-b*x + a))*a*abs(b)/b^2)/b

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a-b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(a - b*x)^(3/2),x)

[Out] int(x^(1/2)*(a - b*x)^(3/2), x)

3.530 $\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=74

$$\frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}$$

```
[Out] 3/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)+1/2*(-b*x+a)^(3/2)*x
^(1/2)+3/4*a*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a - b*x])/4 + (Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a^2*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 68, normalized size = 0.92

$$-\frac{1}{4}\sqrt{x}\,\sqrt{a-bx}\,(-5a+2bx)-\frac{3a^2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{4\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/Sqrt[x],x]
[Out] -1/4*(Sqrt[x]*Sqrt[a - b*x]*(-5*a + 2*b*x)) - (3*a^2*Log[-(Sqrt[-b]*Sqrt[x]
) + Sqrt[a - b*x]])/(4*Sqrt[-b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.82, size = 162, normalized size = 2.19

$$\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{a}\left(-3a^{\frac{3}{2}}\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](-a+bx)+a\sqrt{b}\,\sqrt{x}\,(-5a+7bx)\sqrt{\frac{-a+bx}{a}}-2b^{\frac{5}{2}}x^{\frac{5}{2}}\sqrt{\frac{-a+bx}{a}}\right)}{4\sqrt{b}\,(-a+bx)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{3a^2\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{4\sqrt{b}}+\frac{5a^{\frac{3}{2}}\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{4}-\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - b*x)^(3/2)/Sqrt[x],x]')
```

```
[Out] Piecewise[{{I / 4 Sqrt[a] (-3 a ^ (3 / 2) ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]
] (-a + b x) + a Sqrt[b] Sqrt[x] (-5 a + 7 b x) Sqrt[(-a + b x) / a] - 2 b
^ (5 / 2) x ^ (5 / 2) Sqrt[(-a + b x) / a]) / (Sqrt[b] (-a + b x)), Abs[b x
 / a] > 1}}, 3 a ^ 2 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (4 Sqrt[b]) + 5 a
^ (3 / 2) Sqrt[x] Sqrt[1 - b x / a] / 4 - Sqrt[a] b x ^ (3 / 2) Sqrt[1 - b
x / a] / 2]
```

Maple [A]
time = 0.10, size = 83, normalized size = 1.12

method	result	size
risch	$\frac{(-2bx+5a)\sqrt{x}\,\sqrt{-bx+a}}{4}+\frac{3a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\left(-bx+a\right)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{-bx+a}}$	77
default	$\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))
^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*
x)^(1/2)))
```

Maxima [A]
time = 0.34, size = 93, normalized size = 1.26

$$-\frac{3\,a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,\sqrt{b}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{2}-\frac{2\,(bx-a)b}{x}+\frac{(bx-a)^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/4*(3*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 5*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^2 - 2*(b*x - a)*b
/x + (b*x - a)^2/x^2)
```

Fricas [A]
time = 0.33, size = 119, normalized size = 1.61

$$\left[-\frac{3\,a^{2}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x-5\,ab)\sqrt{-bx+a}\,\sqrt{x}}{8\,b},-\frac{3\,a^{2}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^{2}x-5\,ab)\sqrt{-bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(b)*arctan(
sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x
))/b]
```

Sympy [A]
time = 1.78, size = 190, normalized size = 2.57

$$\begin{cases} -\frac{5ia^{\frac{3}{2}}\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{7i\sqrt{a}\,bx^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{2}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}-\frac{ib^{2}x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{5a^{\frac{3}{2}}\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{4}-\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2}+\frac{3a^{2}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-5*I*a**(3/2)*sqrt(x)/(4*sqrt(-1 + b*x/a)) + 7*I*sqrt(a)*b*x**(3
/2)/(4*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(
b)) - I*b**2*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (5*a**
(3/2)*sqrt(x)*sqrt(1 - b*x/a)/4 - sqrt(a)*b*x**(3/2)*sqrt(1 - b*x/a)/2 + 3*
a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b)), True))
```

Giac [A]
time = 10.34, size = 127, normalized size = 1.72

$$\frac{b^{2}\left(2\left(\frac{\frac{1}{8}\cdot 2\sqrt{a-bx}\,\sqrt{a-bx}}{b}+\frac{\frac{1}{8}\cdot 3a}{b}\right)\sqrt{a-bx}\,\sqrt{ab-b\,(a-bx)}+\frac{6a^{2}\ln\left|\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{8\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/4*(3*a^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqr
t(-b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b - 3*a/b))*b/a
bs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(1/2), x)
```

3.531 $\int \frac{(a-bx)^{3/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=66

$$-3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -3*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))*b^(1/2)-2*(-b*x+a)^(3/2)/x^(1/2
)-3*b*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{a-bx}-3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[a - b*x] - (2*(a - b*x)^(3/2))/Sqrt[x] - 3*a*Sqrt[b]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-\frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3ab)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3ab)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 61, normalized size = 0.92

$$\frac{(-2a-bx)\sqrt{a-bx}}{\sqrt{x}}-3a\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-2*a - b*x)*Sqrt[a - b*x])/Sqrt[x] - 3*a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x])
 + Sqrt[a - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.64, size = 183, normalized size = 2.77

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(2a^2(-a+bx)+3a^{\frac{5}{2}}\sqrt{b}\sqrt{x}\,\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-abx(-a+bx)+b^2x^2(a-bx)\right)}{a^{\frac{3}{2}}\sqrt{x}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-3a\sqrt{b}\,\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]+\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}}+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - b*x)^(3/2)/x^(3/2),x]')
```

```
[Out] Piecewise[{{I (2 a ^ 2 (-a + b x) + 3 a ^ (5 / 2) Sqrt[b] Sqrt[x] ArcCosh[S
qrt[b] Sqrt[x] / Sqrt[a]] ((-a + b x) / a) ^ (3 / 2) - a b x (-a + b x) + b
 ^ 2 x ^ 2 (a - b x)) / (a ^ (3 / 2) Sqrt[x] ((-a + b x) / a) ^ (3 / 2)), A
bs[b x / a] > 1}}, -2 a ^ (3 / 2) / (Sqrt[x] Sqrt[1 - b x / a]) - 3 a Sqrt[
b] ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] + Sqrt[a] b Sqrt[x] / Sqrt[1 - b x / a
] + b ^ 2 x ^ (3 / 2) / (Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.10, size = 74, normalized size = 1.12

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(bx+2a)}{\sqrt{x}}-\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{2\sqrt{x}\,\sqrt{-bx+a}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+a)^(1/2)*(b*x+2*a)/x^(1/2)-3/2*a*b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/
(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.35, size = 68, normalized size = 1.03

$$3\,a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{-bx+a}\,ab}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 3*a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)*a/s
qrt(x) - sqrt(-b*x + a)*a*b/((b - (b*x - a)/x)*sqrt(x))
```

Fricas [A]
time = 0.31, size = 109, normalized size = 1.65

$$\left[\frac{3\,a\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{2\,x},\frac{3\,a\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
2*(b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x, (3*a*sqrt(b)*x*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) - (b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 1.58, size = 197, normalized size = 2.98

$$\begin{cases} \frac{2ia^{\frac{3}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,b\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}+3ia\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{ib^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}}-3a\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(3/2)/(sqrt(x)*sqrt(-1 + b*x/a)) - I*sqrt(a)*b*sqrt(x)/sq
rt(-1 + b*x/a) + 3*I*a*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a)) - I*b**2*x**(
3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(3/2)/(sqrt(x)*sqr
t(1 - b*x/a)) + sqrt(a)*b*sqrt(x)/sqrt(1 - b*x/a) - 3*a*sqrt(b)*asin(sqrt(b
)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 10.38, size = 135, normalized size = 2.05

$$-\frac{bb^2\left(\frac{2\left(-\frac{1}{2}\sqrt{a-bx}\,\sqrt{a-bx}+\frac{3}{2}a\right)\sqrt{a-bx}\,\sqrt{ab-b\,(a-bx)}}{ab-b(a-bx)}+\frac{6a\ln\left|\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{2\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] -(3*a*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b)
 + (b*x + 2*a)*sqrt(-b*x + a)/sqrt((b*x - a)*b + a*b))*b^2/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(3/2),x)
[Out] int((a - b*x)^(3/2)/x^(3/2), x)
```

3.532 $\int \frac{(a-bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=67

$$\frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -2/3*(-b*x+a)^(3/2)/x^(3/2)+2*b^(3/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2)
)+2*b*(-b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 65, 223, 209}

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right) - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[a - b*x])/Sqrt[x] - (2*(a - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.96

$$
\frac{2\sqrt{a-bx}\,(-a+4bx)}{3x^{3/2}} + 2\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(5/2),x]
[Out] (2*Sqrt[a - b*x]*(-a + 4*b*x))/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[a - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.93, size = 175, normalized size = 2.61

$$
\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\left(-2a\sqrt{\frac{a-bx}{bx}}+bx\left(-6I\text{Log}\left[\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right]+3I\text{Log}\left[\frac{a}{bx}\right]+6\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]+8\sqrt{\frac{a-bx}{bx}}\right)\right)}{3x}, \text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\}, \frac{-2Ia\sqrt{b}\,\sqrt{1-\frac{a}{bx}}}{3x} - 2Ib^{\frac{3}{2}}\text{Log}\left[1+\sqrt{1-\frac{a}{bx}}\right] + \frac{I8b^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}{3} + Ib^{\frac{3}{2}}\text{Log}\left[\frac{a}{bx}\right]\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - b*x)^(3/2)/x^(5/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-2 a Sqrt[(a - b x) / (b x)] + b x (-6 I Log[Sqrt[a] /
 (Sqrt[b] Sqrt[x])] + 3 I Log[a / (b x)] + 6 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[
a]] + 8 Sqrt[(a - b x) / (b x)])) / (3 x), Abs[a / (b x)] > 1}}, -2 I a Sqr
t[b] Sqrt[1 - a / (b x)] / (3 x) - 2 I b ^ (3 / 2) Log[1 + Sqrt[1 - a / (b
x)]] + I 8 b ^ (3 / 2) Sqrt[1 - a / (b x)] / 3 + I b ^ (3 / 2) Log[a / (b x
)]]
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.06

method	result	size
risch	$-\frac{2\sqrt{-bx+a}\,(-4bx+a)}{3x^{\frac{3}{2}}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\,(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-b*x+a)^(1/2)*(-4*b*x+a)/x^(3/2)+b^(3/2)*arctan(b^(1/2)*(x-1/2*a/b)/(
-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 49, normalized size = 0.73

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+a}\,b}{\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + a)*b/sq
rt(x) - 2/3*(-b*x + a)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 115, normalized size = 1.72

$$\left[\frac{3\,\sqrt{-b}\,bx^{2}\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^{2}},-\frac{2\left(3\,b^{\frac{3}{2}}x^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqr
t(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.92, size = 187, normalized size = 2.79

$$\begin{cases} -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x} + \frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3} - 2ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right) + 2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2ia\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x} + \frac{8ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3} + ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 8*b**(3/2)*sqrt(a/(b*x) -
 1)/3 - 2*I*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + I*b**(3/2)*log(a/(b*x
)) + 2*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)), Abs(a/(b*x)) > 1), (-2*I*a*s
qrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 8*I*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + I*b**
(3/2)*log(a/(b*x)) - 2*I*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1), True))
```

Giac [A]
time = 10.43, size = 151, normalized size = 2.25

$$\frac{b^2\left(\frac{2\left(-\frac{\frac{1}{9}\cdot 12b^3a\sqrt{a-bx}\sqrt{a-bx}}{a}+\frac{\frac{1}{9}\cdot 9b^3a^2}{a}\right)\sqrt{a-bx}\sqrt{ab-b(a-bx)}}{(ab-b(a-bx))^2}+\frac{2b^2\ln\left|\sqrt{ab-b(a-bx)}-\sqrt{-b}\sqrt{a-bx}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] 2/3*(3*b^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqr
t(-b) + (4*(b*x - a)*b^3 + 3*a*b^3)*sqrt(-b*x + a)/((b*x - a)*b + a*b)^(3/2
))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(5/2), x)
```

3.533 $\int x^{5/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{3\sqrt{x}\,\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] 1/5*x^(7/2)*(b*x+2)^(3/2)-3/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-
1/8*x^(3/2)*(b*x+2)^(1/2)/b^2+1/20*x^(5/2)*(b*x+2)^(1/2)/b+3/20*x^(7/2)*(b*
x+2)^(1/2)+3/8*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{8b^3}-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 + b*x])/(8*b^2) + (x^(5
/2)*Sqrt[2 + b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 + b*x])/20 + (x^(7/2)*(2 + b*
x)^(3/2))/5 - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(2+bx)^{3/2} + \frac{3}{5}\int x^{5/2}\sqrt{2+bx}\,dx$$
$$= \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} + \frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} - \frac{\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{4b}$$
$$= -\frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2+b}}}{8b^2}$$
$$= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)$$
$$= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)$$
$$= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.67

$$\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2+22b^3x^3+8b^4x^4)}{40b^3} + \frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 22*b^3*x^3 + 8*b^4*x^4))/(
40*b^3) + (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(7/2))
```

Mathics [A]
time = 56.68, size = 103, normalized size = 0.82

$$\frac{-3\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{4b^{\frac{7}{2}}} + \frac{3\sqrt{x}}{4b^3\sqrt{2+bx}} + \frac{x^{\frac{3}{2}}}{8b^2\sqrt{2+bx}} - \frac{x^{\frac{5}{2}}}{40b\sqrt{2+bx}} + \frac{23x^{\frac{7}{2}}}{20\sqrt{2+bx}} + \frac{19bx^{\frac{9}{2}}}{20\sqrt{2+bx}} + \frac{b^2x^{\frac{11}{2}}}{5\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)*(2 + b*x)^(3/2),x]')
```

```
[Out] -3 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / (4 b ^ (7 / 2)) + 3 Sqrt[x] / (4
b ^ 3 Sqrt[2 + b x]) + x ^ (3 / 2) / (8 b ^ 2 Sqrt[2 + b x]) - x ^ (5 / 2)
/ (40 b Sqrt[2 + b x]) + 23 x ^ (7 / 2) / (20 Sqrt[2 + b x]) + 19 b x ^ (9
/ 2) / (20 Sqrt[2 + b x]) + b ^ 2 x ^ (11 / 2) / (5 Sqrt[2 + b x])
```

Maple [A]
time = 0.12, size = 135, normalized size = 1.07

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(56b^4x^4+154b^3x^3+14x^2b^2-35bx+105)\sqrt{\frac{bx}{2}+1}}{280}-\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}}{b^{\frac{7}{2}}\sqrt{\pi}}$
risch	$\frac{(8b^4x^4+22b^3x^3+2x^2b^2-5bx+15)\sqrt{x}\,\sqrt{bx+2}}{40b^3}-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{5}{2}}}{5b}-\frac{\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3b}-\frac{\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2}\right.}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}\right.}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+2)^(5/2)-1/b*(1/4/b*x^(3/2)*(b*x+2)^(5/2)-3/4/b*(1/3/b*x
^(1/2)*(b*x+2)^(5/2)-1/3/b*(1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(
1/2)+3/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+
2*x)^(1/2))/b^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(87) = 174$.
time = 0.36, size = 194, normalized size = 1.54

$$\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70\,(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128\,(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5\,(bx+2)b^7}{x}+\frac{10\,(bx+2)^2b^6}{x^2}-\frac{10\,(bx+2)^3b^5}{x^3}+\frac{5\,(bx+2)^4b^4}{x^4}-\frac{(bx+2)^5b^3}{x^5}\right)}+\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) - 128*(
b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(9
/2)/x^(9/2))/(b^8 - 5*(b*x + 2)*b^7/x + 10*(b*x + 2)^2*b^6/x^2 - 10*(b*x +
2)^3*b^5/x^3 + 5*(b*x + 2)^4*b^4/x^4 - (b*x + 2)^5*b^3/x^5) + 3/8*log(-(sqr
t(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 156, normalized size = 1.24

$$\left[\frac{(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{40\,b^4},\frac{(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*
sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/4
0*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt
(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

Sympy [A]
time = 58.73, size = 136, normalized size = 1.08

$$\frac{b^2x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{19bx^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{40b\sqrt{bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{bx+2}}+\frac{3\sqrt{x}}{4b^3\sqrt{bx+2}}-\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(11/2)/(5*sqrt(b*x + 2)) + 19*b*x**(9/2)/(20*sqrt(b*x + 2)) + 23*x*
*(7/2)/(20*sqrt(b*x + 2)) - x**(5/2)/(40*b*sqrt(b*x + 2)) + x**(3/2)/(8*b**
2*sqrt(b*x + 2)) + 3*sqrt(x)/(4*b**3*sqrt(b*x + 2)) - 3*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2))
```

Giac [A]
time = 0.01, size = 308, normalized size = 2.44

$$2b\left(2\left(\left(\left(\left(\frac{\frac{1}{100800}\cdot 5040b^8\sqrt{x}\,\sqrt{x}}{b^8}+\frac{\frac{1}{100800}\cdot 1260b^7}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 2940b^6}{b^8}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{100800}\cdot 7350b^5}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 22050b^4}{b^8}\right)\sqrt{x}\,\sqrt{bx+2}-\frac{7\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^4\sqrt{b}}\right)+4\left(2\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^6\sqrt{x}\,\sqrt{x}}{b^6}+\frac{\frac{1}{2880}\cdot 60b^5}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^4}{b^6}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{2880}\cdot 450b^3}{b^6}\right)\sqrt{x}\,\sqrt{bx+2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^3\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/120*(((2*(3*(4*x + 1/b)*x - 7/b^2)*x + 35/b^3)*x - 105/b^4)*sqrt(b*x + 2)
*sqrt(x) - 210*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(9/2))*b + 1/12*((2*
(3*x + 1/b)*x - 5/b^2)*x + 15/b^3)*sqrt(b*x + 2)*sqrt(x) + 5/2*log(-sqrt(b)
*sqrt(x) + sqrt(b*x + 2))/b^(7/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(b\,x+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(3/2), x)
```

3.534 $\int x^{3/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=105

$$-\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(b*x+2)^(3/2)+3/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+
1/8*x^(3/2)*(b*x+2)^(1/2)/b+1/4*x^(5/2)*(b*x+2)^(1/2)-3/8*x^(1/2)*(b*x+2)^(
1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (3*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\int \frac{}{\sqrt{x}\cdots}}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\mathrm{Subst}\Big(\cdots}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\sinh^{-1}\cdots}{4\cdots}$$

Mathematica [A]
time = 0.08, size = 75, normalized size = 0.71

$$\frac{\sqrt{x}\sqrt{2+bx}\,(-3+bx+6b^2x^2+2b^3x^3)}{8b^2} - \frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 6*b^2*x^2 + 2*b^3*x^3))/(8*b^2) - (3*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(5/2))
```

Mathics [A]
time = 12.89, size = 89, normalized size = 0.85

$$\frac{6b^3\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](2+bx)^{\frac{3}{2}} - 6b^{\frac{7}{2}}\sqrt{x}\,(2+bx) - b^{\frac{9}{2}}x^{\frac{3}{2}}(2+bx) + b^{\frac{11}{2}}x^{\frac{5}{2}}(2+bx)\,(13+10bx+2b^2x^2)}{8b^{\frac{11}{2}}(2+bx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)*(2 + b*x)^(3/2),x]')
```

```
[Out] (6 b ^ 3 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) ^ (3 / 2) - 6 b ^ (
7 / 2) Sqrt[x] (2 + b x) - b ^ (9 / 2) x ^ (3 / 2) (2 + b x) + b ^ (11 / 2)
```

```
x ^ (5 / 2) (2 + b x) (13 + 10 b x + 2 b ^ 2 x ^ 2)) / (8 b ^ (11 / 2) (2
+ b x) ^ (3 / 2))
```

Maple [A]
time = 0.11, size = 114, normalized size = 1.09

method	result	siz
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-10b^{3}x^{3}-30x^{2}b^{2}-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{40}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}}{b^{\frac{5}{2}}\sqrt{\pi}}$	71
risch	$\frac{\left(2b^{3}x^{3}+6x^{2}b^{2}+bx-3\right)\sqrt{x}\,\sqrt{bx+2}}{8b^{2}}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)\sqrt{x\left(bx+2\right)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	84
default	$\frac{x^{\frac{3}{2}}\left(bx+2\right)^{\frac{5}{2}}}{4b}-\frac{3\left(\frac{\sqrt{x}\,\left(bx+2\right)^{\frac{5}{2}}}{3b}-\frac{\frac{\left(bx+2\right)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x\left(bx+2\right)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}\right)}{4b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+2)^(5/2)-3/4/b*(1/3/b*x^(1/2)*(b*x+2)^(5/2)-1/3/b*(1/2*(
b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(1/2)+3/2*(x*(b*x+2))^(1/2)/(b*x+2
)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(72) = 144.
time = 0.34, size = 163, normalized size = 1.55

$$-\frac{\frac{3\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{11\left(bx+2\right)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}-\frac{11\left(bx+2\right)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{3\left(bx+2\right)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^{6}-\frac{4\left(bx+2\right)b^{5}}{x}+\frac{6\left(bx+2\right)^{2}b^{4}}{x^{2}}-\frac{4\left(bx+2\right)^{3}b^{3}}{x^{3}}+\frac{\left(bx+2\right)^{4}b^{2}}{x^{4}}\right)}-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*sqrt(b*x + 2)*b^3/sqrt(x) - 11*(b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(b*
x + 2)^(5/2)*b/x^(5/2) + 3*(b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x + 2)*b^5/
x + 6*(b*x + 2)^2*b^4/x^2 - 4*(b*x + 2)^3*b^3/x^3 + (b*x + 2)^4*b^2/x^4) -
```

```
3/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(5/2)
```

Fricas [A]
time = 0.31, size = 137, normalized size = 1.30

$$\left[\frac{(2\,b^4x^3+6\,b^3x^2+b^2x-3\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{8\,b^3}, \frac{(2\,b^4x^3+6\,b^3x^2+b^2x-3\,b)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*b^4*x^3 + 6*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(
b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/8*((2*b^4*x^3 + 6*b
^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A]
time = 11.52, size = 117, normalized size = 1.11

$$\frac{b^2x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{5bx^{\frac{7}{2}}}{4\sqrt{bx+2}}+\frac{13x^{\frac{5}{2}}}{8\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^2\sqrt{bx+2}}+\frac{3\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(9/2)/(4*sqrt(b*x + 2)) + 5*b*x**(7/2)/(4*sqrt(b*x + 2)) + 13*x**(5
/2)/(8*sqrt(b*x + 2)) - x**(3/2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sq
rt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Giac [A]
time = 0.01, size = 258, normalized size = 2.46

$$2b\left(2\left(\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^6\sqrt{x}\,\sqrt{x}}{b^6}+\frac{\frac{1}{2880}\cdot 60b^5}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^4}{b^6}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{2880}\cdot 450b^3}{b^6}\right)\sqrt{x}\,\sqrt{bx+2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^3\sqrt{b}}\right)\right)+4\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 12b^4\sqrt{x}\,\sqrt{x}}{b^4}+\frac{\frac{1}{144}\cdot 6b^3}{b^4}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{144}\cdot 18b^2}{b^4}\right)\sqrt{x}\,\sqrt{bx+2}-\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{2b^2\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/24*(((2*(3*x + 1/b)*x - 5/b^2)*x + 15/b^3)*sqrt(b*x + 2)*sqrt(x) + 30*log
(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(7/2))*b + 1/3*sqrt(b*x + 2)*((2*x + 1
/b)*x - 3/b^2)*sqrt(x) - 2*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(b\,x+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(3/2),x)
[Out] int(x^(3/2)*(b*x + 2)^(3/2), x)
```

3.535 $\int \sqrt{x}\,(2+bx)^{3/2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)-arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*
x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b
*x)^(3/2))/3 - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\,(2+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2+bx)^{3/2} + \int \sqrt{x}\,\sqrt{2+bx}\,dx \\
&= \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx \\
&= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b} \\
&= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{\right.}{b} \\
&= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 65, normalized size = 0.79

$$
\frac{\sqrt{x}\,\sqrt{2+bx}\,\left(3+7bx+2b^2x^2\right)}{6b} + \frac{\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(3 + 7*b*x + 2*b^2*x^2))/(6*b) + Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[2 + b*x]]/b^(3/2)
```

Mathics [**A**]
time = 5.23, size = 75, normalized size = 0.91

$$
\frac{-b\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](2+bx) + b^{\frac{3}{2}}\sqrt{x}\,\sqrt{2+bx} + \frac{b^{\frac{5}{2}}x^{\frac{3}{2}}\left(17+11bx+2b^2x^2\right)\sqrt{2+bx}}{6}}{b^{\frac{5}{2}}(2+bx)}
$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]')
```

```
[Out] (-b ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + b ^ (3 / 2) Sqrt[x] Sq
rt[2 + b x] + b ^ (5 / 2) x ^ (3 / 2) (17 + 11 b x + 2 b ^ 2 x ^ 2) Sqrt[2
+ b x] / 6) / (b ^ (5 / 2) (2 + b x))
```

Maple [**A**]
time = 0.12, size = 87, normalized size = 1.06

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(2x^2b^2+7bx+3)\sqrt{\frac{bx}{2}+1}}{6}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2+7bx+3)\sqrt{x}\,\sqrt{bx+2}}{6b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	77
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3}+\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)+1/2*x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/
2)/b-1/2/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)
+(b*x^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.
time = 0.34, size = 132, normalized size = 1.61

$$\frac{\frac{3\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{8\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^4-\frac{3\,(bx+2)b^3}{x}+\frac{3\,(bx+2)^2b^2}{x^2}-\frac{(bx+2)^3b}{x^3}\right)}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) - 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x + 2
)^(5/2)/x^(5/2))/(b^4 - 3*(b*x + 2)*b^3/x + 3*(b*x + 2)^2*b^2/x^2 - (b*x +
2)^3*b/x^3) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*
x + 2)/sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.32, size = 124, normalized size = 1.51

$$\left[\frac{(2\,b^3x^2+7\,b^2x+3\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^2},\frac{(2\,b^3x^2+7\,b^2x+3\,b)\sqrt{bx+2}\,\sqrt{x}+6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 7*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x
 - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/6*((2*b^3*x^2 + 7*b^2*x + 3*b
)*sqrt(b*x + 2)*sqrt(x) + 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))))/b^2]
```

Sympy [A]
time = 3.52, size = 92, normalized size = 1.12

$$\frac{b^2x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{11bx^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{17x^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] b**2*x**(7/2)/(3*sqrt(b*x + 2)) + 11*b*x**(5/2)/(6*sqrt(b*x + 2)) + 17*x**(
3/2)/(6*sqrt(b*x + 2)) + sqrt(x)/(b*sqrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*
sqrt(x)/2)/b**(3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(57) = 114.
time = 3.50, size = 361, normalized size = 4.40

$$\frac{\frac{2b^2|b|\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 12b^5\sqrt{bx+2}\sqrt{bx+2}}{b^7}-\frac{\frac{1}{144}\cdot 78b^5}{b^7}\right)\sqrt{bx+2}\sqrt{bx+2}+\frac{\frac{1}{144}\cdot 198b^5}{b^7}\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}+\frac{5\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{2b\sqrt{b}}\right)\right)}{b^2}+\frac{8b|b|\left(2\left(\frac{1}{8}\sqrt{bx+2}\sqrt{bx+2}-\frac{5}{8}\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}-\frac{6b\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{4\sqrt{b}}\right)}{b^2b}+\frac{8|b|\left(\frac{1}{2}\sqrt{bx+2}\sqrt{b(bx+2)-2b}+\frac{2b\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{2\sqrt{b}}\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] 1/6*((sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)*(2*(b*x + 2)/b^2 - 1
3/b^2) + 33/b^2) + 30*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2
*b)))/b^(3/2))*abs(b) + 12*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3)
 - 6*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*ab
s(b)/b^2 + 24*(2*sqrt(b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b
- 2*b))) + sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2))*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,{\left(b\,x+2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(3/2), x)
```

3.536 $\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$\frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 3*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+1/2*(b*x+2)^(3/2)*x^(1/2)+3/
2*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{1}{2}\sqrt{x}\,(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/2 + (Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*ArcSinh[(Sqr
t[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+3\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.89

$$\frac{1}{2}\sqrt{x}\,\sqrt{2+bx}\,(5+bx)-\frac{3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(5 + b*x))/2 - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 +
b*x]])/Sqrt[b]
```

Mathics [A]
time = 3.32, size = 54, normalized size = 0.89

$$\frac{\sqrt{b}\,\sqrt{x}\,(10+7bx+b^2x^2)+6\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]\sqrt{2+bx}}{2\sqrt{b}\,\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]')
```

```
[Out] (Sqrt[b] Sqrt[x] (10 + 7 b x + b ^ 2 x ^ 2) + 6 ArcSinh[Sqrt[2] Sqrt[b] Sqr
t[x] / 2] Sqrt[2 + b x]) / (2 Sqrt[b] Sqrt[2 + b x])
```

Maple [A]
time = 0.12, size = 72, normalized size = 1.18

method	result	size

meijerg	$\frac{4\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\left(\frac{bx}{8}+\frac{5}{8}\right)\sqrt{\frac{bx}{2}+1}+3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{\pi}}$	54
risch	$\frac{(bx+5)\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	65
default	$\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(1/2)+3/2*(x*(b*x+2))^(1/2)/(
b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(42) = 84.
time = 0.35, size = 98, normalized size = 1.61

$$-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\sqrt{b}}-\frac{\frac{3\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{5(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^2-\frac{2(bx+2)b}{x}+\frac{(bx+2)^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - (3*sqrt(b*x + 2)*b/sqrt(x) - 5*(b*x + 2)^(3/2)/x^(3/2))/(b^2
- 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2)
```

Fricas [A]
time = 0.33, size = 105, normalized size = 1.72

$$\left[\frac{(b^2x+5b)\sqrt{bx+2}\,\sqrt{x}+3\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2b},\ \frac{(b^2x+5b)\sqrt{bx+2}\,\sqrt{x}-6\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b, 1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) - 6*sq
rt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b]
```

Sympy [A]
time = 1.59, size = 76, normalized size = 1.25

$$\frac{b^2x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{7bx^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{5\sqrt{x}}{\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] b**2*x**(5/2)/(2*sqrt(b*x + 2)) + 7*b*x**(3/2)/(2*sqrt(b*x + 2)) + 5*sqrt(x
)/sqrt(b*x + 2) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [A]
time = 1.17, size = 113, normalized size = 1.85

$$\frac{b^2\left(2\left(\frac{\frac{1}{4}\sqrt{bx+2}\sqrt{bx+2}}{b}+\frac{\frac{1}{4}\cdot 3}{b}\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}-\frac{3\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)/b + 3/b) - 6*log(abs(
-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(1/2), x)
```

3.537 $\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=58

$$3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 6*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(b*x+2)^(3/2)/x^(1/2)+3*b*
x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\,\sqrt{bx+2}+6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[2 + b*x] - (2*(2 + b*x)^(3/2))/Sqrt[x] + 6*Sqrt[b]*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\
&= 3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\
&= 3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+(6b)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.88

$$\frac{(-4+bx)\sqrt{2+bx}}{\sqrt{x}}-6\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(3/2),x]
[Out] ((-4 + b*x)*Sqrt[2 + b*x])/Sqrt[x] - 6*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[2 + b*x]]
```

Mathics [A]
time = 3.18, size = 61, normalized size = 1.05

$$\frac{6\sqrt{b}\,\sqrt{x}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](2+bx)+bx\,(-2+bx)\,\sqrt{2+bx}-8\sqrt{2+bx}}{\sqrt{x}\,(2+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2 + b*x)^(3/2)/x^(3/2),x]')
```

```
[Out] (6 Sqrt[b] Sqrt[x] ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + b x (-2
 + b x) Sqrt[2 + b x] - 8 Sqrt[2 + b x]) / (Sqrt[x] (2 + b x))
```

Maple [A]
time = 0.12, size = 55, normalized size = 0.95

method	result	size
meijerg	$\frac{3\sqrt{b}\left(-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{bx}{4}+1\right)\sqrt{\frac{bx}{2}+1}}{3\sqrt{x}\,\sqrt{b}}+4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	55
risch	$\frac{x^2b^2-2bx-8}{\sqrt{x}\,\sqrt{bx+2}}+\frac{3\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*b^(1/2)/Pi^(1/2)*(-8/3*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(-1/4*b*x+1)*(1
/2*b*x+1)^(1/2)+4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.33, size = 81, normalized size = 1.40

$$-3\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+2}}{\sqrt{x}}-\frac{2\sqrt{bx+2}\,b}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 4*sqrt(b*x + 2)/sqrt(x) - 2*sqrt(b*x + 2)*b/((b - (b*x + 2)/x)*
sqrt(x))
```

Fricas [A]
time = 0.31, size = 99, normalized size = 1.71

$$\left[\frac{3\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x},\ -\frac{6\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + sqrt(b*x + 2)*
(b*x - 4)*sqrt(x))/x, -(6*sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))) - sqrt(b*x + 2)*(b*x - 4)*sqrt(x))/x]
```

Sympy [A]
time = 1.39, size = 73, normalized size = 1.26

$$6\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{bx+2}}-\frac{2b\sqrt{x}}{\sqrt{bx+2}}-\frac{8}{\sqrt{x}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] 6*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/sqrt(b*x + 2) -
2*b*sqrt(x)/sqrt(b*x + 2) - 8/(sqrt(x)*sqrt(b*x + 2))
```

Giac [A]
time = 1.12, size = 118, normalized size = 2.03

$$\frac{bb^2\left(\frac{2\left(\frac{1}{2}\sqrt{bx+2}\sqrt{bx+2}-3\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}}{b(bx+2)-2b}-\frac{6\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (sqrt(b*x + 2)*(b*x - 4)/sqrt((b*x + 2)*b - 2*b) - 6*log(abs(-sqrt(b*x + 2)
*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b))*b^2/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(3/2), x)
```

3.538 $\int \frac{(2+bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=60

$$-\frac{2b\sqrt{2+bx}}{\sqrt{x}}-\frac{2(2+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(b*x+2)^(3/2)/x^(3/2)+2*b^(3/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))-2
*b*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 56, 221}

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[2 + b*x])/Sqrt[x] - (2*(2 + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(2+bx)^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx \\
&= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx \\
&= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + (2b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.92

$$
-\frac{4\sqrt{2+bx}\,(1+2bx)}{3x^{3/2}} - 2b^{3/2}\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-4*Sqrt[2 + b*x]*(1 + 2*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[2 + b*x]]
```

Mathics [A]
time = 3.58, size = 74, normalized size = 1.23

$$
\frac{\sqrt{b}\left(bx\left(-8\sqrt{\frac{2+bx}{bx}} - 3\text{Log}\left[\frac{1}{bx}\right] + 6\text{Log}\left[1+\sqrt{\frac{2+bx}{bx}}\right]\right) - 4\sqrt{\frac{2+bx}{bx}}\right)}{3x}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 + b*x)^(3/2)/x^(5/2),x]')
```

```
[Out] Sqrt[b] (b x (-8 Sqrt[(2 + b x) / (b x)] - 3 Log[1 / (b x)] + 6 Log[1 + Sqr
t[(2 + b x) / (b x)]]) - 4 Sqrt[(2 + b x) / (b x)]) / (3 x)
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.92

method	result	size

meijerg	$\frac{3b^{\frac{3}{2}}\left(-\frac{16\sqrt{\pi}\,\sqrt{2}\,(2bx+1)\sqrt{\frac{bx}{2}+1}}{9x^{\frac{3}{2}}b^{\frac{3}{2}}}+\frac{8\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3}\right)}{4\sqrt{\pi}}$	55
risch	$-\frac{4(2x^2b^2+5bx+2)}{3x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^{\frac{3}{2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*b^(3/2)/Pi^(1/2)*(-16/9*Pi^(1/2)/x^(3/2)*2^(1/2)/b^(3/2)*(2*b*x+1)*(1/2
*b*x+1)^(1/2)+8/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.36, size = 67, normalized size = 1.12

$$-b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)*b/sqrt(x) - 2/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.34, size = 108, normalized size = 1.80

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-4\,(2\,bx+1)\sqrt{bx+2}\,\sqrt{x}}{3\,x^2},\ -\frac{2\left(3\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,bx+1)\sqrt{bx+2}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 4*(2*b*x
 + 1)*sqrt(b*x + 2)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + 2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2]
```

Sympy [A]
time = 1.67, size = 70, normalized size = 1.17

$$-\frac{8b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{4\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(5/2),x)
[Out] -8*b**(3/2)*sqrt(1 + 2/(b*x))/3 - b**(3/2)*log(1/(b*x)) + 2*b**(3/2)*log(sq
rt(1 + 2/(b*x)) + 1) - 4*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [A]
time = 1.13, size = 133, normalized size = 2.22

$$\frac{b^2\left(\frac{2\left(-\frac{12}{9}b^3\sqrt{bx+2}\sqrt{bx+2}+\frac{18}{9}b^3\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}}{(b(bx+2)-2b)^2}-\frac{2b^2\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x)
[Out] -2/3*(3*b^(3/2)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))
+ 2*(2*(b*x + 2)*b^3 - 3*b^3)*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(3/2))*b/ab
s(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(5/2),x)
[Out] int((b*x + 2)^(3/2)/x^(5/2), x)
```

3.539 $\int x^{5/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] 1/5*x^(7/2)*(-b*x+2)^(3/2)+3/4*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-
1/8*x^(3/2)*(-b*x+2)^(1/2)/b^2-1/20*x^(5/2)*(-b*x+2)^(1/2)/b+3/20*x^(7/2)*(
-b*x+2)^(1/2)-3/8*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 - b*x])/(8*b^2) - (x^(
5/2)*Sqrt[2 - b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 - b*x])/20 + (x^(7/2)*(2 - b
*x)^(3/2))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(2-bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{5}\int x^{5/2}\sqrt{2-bx}\,dx \\
&= \frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{4b} \\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2-}}}{8b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-b \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-b \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-b
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 90, normalized size = 0.69

$$
-\frac{\sqrt{x}\sqrt{2-bx}\,(15+5bx+2b^2x^2-22b^3x^3+8b^4x^4)}{40b^3}+\frac{3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/40*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2 - 22*b^3*x^3 + 8*b^4*x
^4))/b^3 + (3*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 57.77, size = 229, normalized size = 1.75

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(-30b^6\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](-2+bx)^2+30b^{\frac{13}{2}}\sqrt{x}(-2+bx)^{\frac{3}{2}}-5b^{\frac{15}{2}}x^{\frac{3}{2}}(-2+bx)^{\frac{3}{2}}-b^{\frac{17}{2}}x^{\frac{5}{2}}(-2+bx)^{\frac{3}{2}}+2b^{\frac{19}{2}}x^{\frac{7}{2}}(-23+19bx-4b^2x^2)(-2+bx)^{\frac{3}{2}}\right)}{40b^{\frac{19}{2}}(-2+bx)^2},\text{Abs}[bx]>2\right\}\right\},\frac{3\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{4b^{\frac{7}{2}}}-\frac{3\sqrt{x}}{4b^3\sqrt{2-bx}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{2-bx}}+\frac{x^{\frac{5}{2}}}{40b\sqrt{2-bx}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{2-bx}}-\frac{19bx^{\frac{9}{2}}}{20\sqrt{2-bx}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{2-bx}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*(2 - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 40 (-30 b ^ 6 ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (-2 + b
x) ^ 2 + 30 b ^ (13 / 2) Sqrt[x] (-2 + b x) ^ (3 / 2) - 5 b ^ (15 / 2) x ^
(3 / 2) (-2 + b x) ^ (3 / 2) - b ^ (17 / 2) x ^ (5 / 2) (-2 + b x) ^ (3 / 2
) + 2 b ^ (19 / 2) x ^ (7 / 2) (-23 + 19 b x - 4 b ^ 2 x ^ 2) (-2 + b x) ^
(3 / 2)) / (b ^ (19 / 2) (-2 + b x) ^ 2), Abs[b x] > 2}}, 3 ArcSin[Sqrt[2]
Sqrt[b] Sqrt[x] / 2] / (4 b ^ (7 / 2)) - 3 Sqrt[x] / (4 b ^ 3 Sqrt[2 - b x]
) + x ^ (3 / 2) / (8 b ^ 2 Sqrt[2 - b x]) + x ^ (5 / 2) / (40 b Sqrt[2 - b
x]) + 23 x ^ (7 / 2) / (20 Sqrt[2 - b x]) - 19 b x ^ (9 / 2) / (20 Sqrt[2 -
 b x]) + b ^ 2 x ^ (11 / 2) / (5 Sqrt[2 - b x])]
```

Maple [A]

time = 0.11, size = 143, normalized size = 1.09

method	result
meijerg	$-\frac{24\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(56b^4x^4-154b^3x^3+14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{6720b^3}+\frac{\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{32b^{\frac{7}{2}}}\right)}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$\frac{(8b^4x^4-22b^3x^3+2x^2b^2+5bx+15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{40b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+2)^{\frac{5}{2}}}{5b}+\frac{-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}}{4b}+\frac{-\frac{\sqrt{x}\,(-bx+2)^{\frac{5}{2}}}{4b}+\frac{\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\cdots\right)}{2\sqrt{-bx+2}\,\sqrt{\cdots}}}{4b}}{b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+2)^(5/2)+1/b*(-1/4/b*x^(3/2)*(-b*x+2)^(5/2)+3/4/b*(-1/
3/b*x^(1/2)*(-b*x+2)^(5/2)+1/3/b*(1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-
b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b
^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))))
```

Maxima [A]

time = 0.34, size = 179, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^4}{\sqrt{x}}+\frac{70\,(-bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128\,(-bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}-\frac{70\,(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5\,(bx-2)b^7}{x}+\frac{10\,(bx-2)^2b^6}{x^2}-\frac{10\,(bx-2)^3b^5}{x^3}+\frac{5\,(bx-2)^4b^4}{x^4}-\frac{(bx-2)^5b^3}{x^5}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) - 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^8 - 5*(b*x - 2)*b^7/x + 10*(b*x - 2)^2*b^6/x^2 - 10*(b
*x - 2)^3*b^5/x^3 + 5*(b*x - 2)^4*b^4/x^4 - (b*x - 2)^5*b^3/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 157, normalized size = 1.20

$$\left[-\frac{\left(8\,b^5x^4-22\,b^4x^3+2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{40\,b^4}, -\frac{\left(8\,b^5x^4-22\,b^4x^3+2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{40\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2
)*sqrt(x) + 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
4, -1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x +
 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

Sympy [A]
time = 58.58, size = 289, normalized size = 2.21

$$\begin{cases} -\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{bx-2}}+\frac{19ibx^{\frac{9}{2}}}{20\sqrt{bx-2}}-\frac{23ix^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{40b\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b^2\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{3i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } |bx|>2 \\ \frac{b^2x^{\frac{11}{2}}}{5\sqrt{-bx+2}}-\frac{19bx^{\frac{9}{2}}}{20\sqrt{-bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{40b\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{3\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(11/2)/(5*sqrt(b*x - 2)) + 19*I*b*x**(9/2)/(20*sqrt(b
*x - 2)) - 23*I*x**(7/2)/(20*sqrt(b*x - 2)) - I*x**(5/2)/(40*b*sqrt(b*x - 2
)) - I*x**(3/2)/(8*b**2*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**3*sqrt(b*x - 2))
 - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2)), Abs(b*x) > 2), (b**2*
x**(11/2)/(5*sqrt(-b*x + 2)) - 19*b*x**(9/2)/(20*sqrt(-b*x + 2)) + 23*x**(7
/2)/(20*sqrt(-b*x + 2)) + x**(5/2)/(40*b*sqrt(-b*x + 2)) + x**(3/2)/(8*b**2
*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

Giac [A]
time = 0.01, size = 322, normalized size = 2.46

$$-2b\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{100800}\cdot 5040b^8\sqrt{x}\,\sqrt{x}}{b^8}-\frac{\frac{1}{100800}\cdot 1260b^7}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 2940b^6}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 7350b^5}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 22050b^4}{b^8}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{7\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^4\sqrt{-b}}\right)+4\left(2\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^6\sqrt{x}\,\sqrt{x}}{b^6}-\frac{\frac{1}{2880}\cdot 60b^5}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^4}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 450b^3}{b^6}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{5\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/120*(((2*(3*(4*x - 1/b)*x - 7/b^2)*x - 35/b^3)*x - 105/b^4)*sqrt(-b*x +
2)*sqrt(x) - 210*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^4))*b
+ 1/12*((2*(3*x - 1/b)*x - 5/b^2)*x - 15/b^3)*sqrt(-b*x + 2)*sqrt(x) - 5/2*
log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(2-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(3/2), x)
```

3.540 $\int x^{3/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(-b*x+2)^(3/2)+3/4*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-
1/8*x^(3/2)*(-b*x+2)^(1/2)/b+1/4*x^(5/2)*(-b*x+2)^(1/2)-3/8*x^(1/2)*(-b*x+2
)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (3*ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(2-bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx\\
&= \frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\int \frac{}{\sqrt{x}\,\cdots}}{}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\text{Subst}\big(\cdots}{}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\sin^{-1}\big(\cdots}{4\cdots}
\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 81, normalized size = 0.74

$$-\frac{\sqrt{x}\sqrt{2-bx}\,(3+bx-6b^2x^2+2b^3x^3)}{8b^2}-\frac{3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/8*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x - 6*b^2*x^2 + 2*b^3*x^3))/b^2 - (3*Log
[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(5/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(2 - b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.11, size = 122, normalized size = 1.12

method	result
meijerg	$-\frac{12\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(10b^3x^3-30x^2b^2+5bx+15\right)\sqrt{-\frac{bx}{2}+1}}{480b^2}+\frac{\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{16b^{\frac{5}{2}}}\right)}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$
risch	$\frac{(2b^3x^3-6x^2b^2+bx+3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{8b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}}{4b}+\frac{-\frac{\sqrt{x}\,(-bx+2)^{\frac{5}{2}}}{4b}+\frac{\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+2)^(5/2)+3/4/b*(-1/3/b*x^(1/2)*(-b*x+2)^(5/2)+1/3/b*(1
/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2)
/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))
)
```

Maxima [A]
time = 0.35, size = 147, normalized size = 1.35

$$\frac{\frac{3\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{11\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^6-\frac{4\,(bx-2)b^5}{x}+\frac{6\,(bx-2)^2b^4}{x^2}-\frac{4\,(bx-2)^3b^3}{x^3}+\frac{(bx-2)^4b^2}{x^4}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*sqrt(-b*x + 2)*b^3/sqrt(x) + 11*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(-
b*x + 2)^(5/2)*b/x^(5/2) - 3*(-b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x - 2)*b
^5/x + 6*(b*x - 2)^2*b^4/x^2 - 4*(b*x - 2)^3*b^3/x^3 + (b*x - 2)^4*b^2/x^4)
 - 3/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 139, normalized size = 1.28

$$\left[-\frac{\left(2\,b^4x^3-6\,b^3x^2+b^2x+3\,b\right)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{8\,b^3},-\frac{\left(2\,b^4x^3-6\,b^3x^2+b^2x+3\,b\right)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*b^4*x^3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqr
t(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/8*((2*b^4*x^
3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt
(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [**A**]
time = 11.51, size = 250, normalized size = 2.29

$$\begin{cases} -\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{bx-2}} + \frac{5ibx^{\frac{7}{2}}}{4\sqrt{bx-2}} - \frac{13ix^{\frac{5}{2}}}{8\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ \frac{b^2x^{\frac{9}{2}}}{4\sqrt{-bx+2}} - \frac{5bx^{\frac{7}{2}}}{4\sqrt{-bx+2}} + \frac{13x^{\frac{5}{2}}}{8\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(9/2)/(4*sqrt(b*x - 2)) + 5*I*b*x**(7/2)/(4*sqrt(b*x
- 2)) - 13*I*x**(5/2)/(8*sqrt(b*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) +
3*I*sqrt(x)/(4*b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(
4*b**(5/2)), Abs(b*x) > 2), (b**2*x**(9/2)/(4*sqrt(-b*x + 2)) - 5*b*x**(7/2
)/(4*sqrt(-b*x + 2)) + 13*x**(5/2)/(8*sqrt(-b*x + 2)) + x**(3/2)/(8*b*sqrt(
-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqr
t(x)/2)/(4*b**(5/2)), True))
```

Giac [**A**]
time = 0.01, size = 271, normalized size = 2.49

$$-2b\left(2\left(\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^6\sqrt{x}\sqrt{x}}{b^6} - \frac{\frac{1}{2880}\cdot 60b^5}{b^6}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{2880}\cdot 150b^4}{b^6}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{2880}\cdot 450b^3}{b^6}\right)\sqrt{x}\sqrt{-bx+2} - \frac{5\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{8b^3\sqrt{-b}}\right)\right) + 4\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 12b^4\sqrt{x}\sqrt{x}}{b^4} - \frac{\frac{1}{144}\cdot 6b^3}{b^4}\right)\sqrt{x}\sqrt{x} - \frac{\frac{1}{144}\cdot 18b^2}{b^4}\right)\sqrt{x}\sqrt{-bx+2} - \frac{\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{2b^2\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/24*(((2*(3*x - 1/b)*x - 5/b^2)*x - 15/b^3)*sqrt(-b*x + 2)*sqrt(x) - 30*l
og(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3))*b + 1/3*sqrt(-b*x +
2)*((2*x - 1/b)*x - 3/b^2)*sqrt(x) - 2*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x +
2))/(sqrt(-b)*b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(2-b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(3/2),x)
[Out] int(x^(3/2)*(2 - b*x)^(3/2), x)
```

3.541 $\int \sqrt{x}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*
x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x])/b + (x^(3/2)*Sqrt[2 - b*x])/2 + (x^(3/2)*(2 -
b*x)^(3/2))/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(2-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2-bx)^{3/2}+\int \sqrt{x}\,\sqrt{2-bx}\,dx\\ &= \frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b}\\ &= -\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\right.}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.85

$$-\frac{\sqrt{x}\,\sqrt{2-bx}\,(3-7bx+2b^2x^2)}{6b}+\frac{\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(3 - 7*b*x + 2*b^2*x^2))/b + Log[-(Sqrt[-b]*Sqr
t[x]) + Sqrt[2 - b*x]]/(-b)^(3/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*(2 - b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.11, size = 94, normalized size = 1.12

method	result	size

meijerg	$-\frac{6\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}(2x^2b^2-7bx+3)\sqrt{-\frac{bx}{2}+1}}{36b}+\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{6b^{\frac{3}{2}}}\right)}{\sqrt{-b}\,\sqrt{\pi}\,b}$	81
default	$\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{3}+\frac{x^{\frac{3}{2}}\sqrt{-bx+2}}{2}-\frac{\sqrt{x}\,\sqrt{-bx+2}}{2b}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	94
risch	$\frac{(2x^2b^2-7bx+3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^
(1/2)/b+1/2/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2
)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.34, size = 115, normalized size = 1.37

$$\frac{\frac{3\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{8(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^4-\frac{3(bx-2)b^3}{x}+\frac{3(bx-2)^2b^2}{x^2}-\frac{(bx-2)^3b}{x^3}\right)}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) + 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^4 - 3*(b*x - 2)*b^3/x + 3*(b*x - 2)^2*b^2/x^2 - (b*x
 - 2)^3*b/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.32, size = 125, normalized size = 1.49

$$\left[-\frac{(2\,b^3x^2-7\,b^2x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^2},-\frac{(2\,b^3x^2-7\,b^2x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 - 7*b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(
-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, -1/6*((2*b^3*x^2 - 7*b^2*
x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x))))/b^2]
```

Sympy [A]
time = 3.58, size = 197, normalized size = 2.35

$$\begin{cases} -\frac{ib^2x^{\frac{7}{2}}}{3\sqrt{bx-2}} + \frac{11ibx^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{17ix^{\frac{3}{2}}}{6\sqrt{bx-2}} + \frac{i\sqrt{x}}{b\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx| > 2 \\ \frac{b^2x^{\frac{7}{2}}}{3\sqrt{-bx+2}} - \frac{11bx^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{17x^{\frac{3}{2}}}{6\sqrt{-bx+2}} - \frac{\sqrt{x}}{b\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(7/2)/(3*sqrt(b*x - 2)) + 11*I*b*x**(5/2)/(6*sqrt(b*x
 - 2)) - 17*I*x**(3/2)/(6*sqrt(b*x - 2)) + I*sqrt(x)/(b*sqrt(b*x - 2)) - I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x) > 2), (b**2*x**(7/2)/(3
*sqrt(-b*x + 2)) - 11*b*x**(5/2)/(6*sqrt(-b*x + 2)) + 17*x**(3/2)/(6*sqrt(-
b*x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(3/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(59) = 118.
time = 3.38, size = 389, normalized size = 4.63

$$\frac{-\frac{2b^2|b|\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 12b^5\sqrt{-bx+2}\sqrt{-bx+2}}{b^7} - \frac{\frac{1}{144}\cdot 78b^5}{b^7}\right)\sqrt{-bx+2}\sqrt{-bx+2} + \frac{\frac{1}{144}\cdot 198b^5}{b^7}\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}\right) - \frac{5\ln\left|\sqrt{-b(-bx+2)+2b} - \sqrt{-b}\sqrt{-bx+2}\right|}{2b\sqrt{-b}}\right)}{b^2} - \frac{8b|b|\left(2\left(\frac{1}{8}\sqrt{-bx+2}\sqrt{-bx+2} - \frac{5}{8}\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b} + \frac{6b\ln\left|\sqrt{-b(-bx+2)+2b} - \sqrt{-b}\sqrt{-bx+2}\right|}{4\sqrt{-b}}\right)}{b^2b} - \frac{8|b|\left(\frac{1}{2}\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b} - \frac{2b\ln\left|\sqrt{-b(-bx+2)+2b} - \sqrt{-b}\sqrt{-bx+2}\right|}{2\sqrt{-b}}\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] -1/6*((sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)*(2*(b*x - 2)/b^2 +
 13/b^2) + 33/b^2) - 30*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b
 + 2*b)))/(sqrt(-b)*b))*abs(b) - 12*(sqrt((b*x - 2)*b + 2*b)*(b*x + 3)*sqrt
(-b*x + 2) - 6*b*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)
))/sqrt(-b))*abs(b)/b^2 - 24*(2*b*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((
b*x - 2)*b + 2*b)))/sqrt(-b) - sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2))*abs(
b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,{\left(2-b\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(3/2), x)
```

3.542 $\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=63

$$\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 3*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+1/2*(-b*x+2)^(3/2)*x^(1/2)+3/
2*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 - b*x])/2 + (Sqrt[x]*(2 - b*x)^(3/2))/2 + (3*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+3\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.95

$$
-\frac{1}{2}\sqrt{x}\,\sqrt{2-bx}\,(-5+bx)-\frac{3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(-5 + b*x)) - (3*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt
[2 - b*x]])/Sqrt[-b]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.41, size = 125, normalized size = 1.98

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(\sqrt{b}\,\sqrt{x}\,(-10+7bx-b^2x^2)-6\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]\sqrt{-2+bx}\right)}{2\sqrt{b}\,\sqrt{-2+bx}},\text{Abs}\,[bx]>2\right\}\right\},\frac{3\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{\sqrt{b}}+\frac{5\sqrt{x}}{\sqrt{2-bx}}-\frac{7bx^{\frac{3}{2}}}{2\sqrt{2-bx}}+\frac{b^2x^{\frac{5}{2}}}{2\sqrt{2-bx}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - b*x)^(3/2)/Sqrt[x],x]')
```

```
[Out] Piecewise[{{I / 2 (Sqrt[b] Sqrt[x] (-10 + 7 b x - b ^ 2 x ^ 2) - 6 ArcCosh[
Sqrt[2] Sqrt[b] Sqrt[x] / 2] Sqrt[-2 + b x]) / (Sqrt[b] Sqrt[-2 + b x]), Ab
s[b x] > 2}}, 3 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / Sqrt[b] + 5 Sqrt[x] /
 Sqrt[2 - b x] - 7 b x ^ (3 / 2) / (2 Sqrt[2 - b x]) + b ^ 2 x ^ (5 / 2) /
(2 Sqrt[2 - b x])]
```

Maple [A]
time = 0.11, size = 78, normalized size = 1.24

method	result	size
meijerg	$-\frac{3\sqrt{-b}\left(\frac{4\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{-b}\,\left(-\frac{bx}{8}+\frac{5}{8}\right)\sqrt{-\frac{bx}{2}+1}}{3}+\frac{\sqrt{\pi}\,\sqrt{-b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}\right)}{\sqrt{\pi}\,b}$	69
default	$\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	78
risch	$\frac{(bx-5)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2
)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.34, size = 79, normalized size = 1.25

$$-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{3\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{5\,(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{2}-\frac{2\,(bx-2)b}{x}+\frac{(bx-2)^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + (3*sqrt(-b*x + 2)*b/s
qrt(x) + 5*(-b*x + 2)^(3/2)/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)^2/x
^2)
```

Fricas [A]
time = 0.32, size = 107, normalized size = 1.70

$$\left[-\frac{(b^{2}x-5\,b)\sqrt{-bx+2}\,\sqrt{x}+3\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b},-\frac{(b^{2}x-5\,b)\sqrt{-bx+2}\,\sqrt{x}+6\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, -1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x
) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b]
```

Sympy [A]
time = 1.59, size = 165, normalized size = 2.62

$$\begin{cases} -\frac{ib^2x^{\frac{5}{2}}}{2\sqrt{bx-2}} + \frac{7ibx^{\frac{3}{2}}}{2\sqrt{bx-2}} - \frac{5i\sqrt{x}}{\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx|>2 \\ \frac{b^2x^{\frac{5}{2}}}{2\sqrt{-bx+2}} - \frac{7bx^{\frac{3}{2}}}{2\sqrt{-bx+2}} + \frac{5\sqrt{x}}{\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(5/2)/(2*sqrt(b*x - 2)) + 7*I*b*x**(3/2)/(2*sqrt(b*x
- 2)) - 5*I*sqrt(x)/sqrt(b*x - 2) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sq
rt(b), Abs(b*x) > 2), (b**2*x**(5/2)/(2*sqrt(-b*x + 2)) - 7*b*x**(3/2)/(2*s
qrt(-b*x + 2)) + 5*sqrt(x)/sqrt(-b*x + 2) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x)/
2)/sqrt(b), True))
```

Giac [A]
time = 1.12, size = 120, normalized size = 1.90

$$\frac{b^2\left(2\left(\frac{\frac{1}{4}\sqrt{-bx+2}\sqrt{-bx+2}}{b}+\frac{\frac{1}{4}\cdot 3}{b}\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}+\frac{3\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] -1/2*(sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)/b - 3/b) - 6*log(ab
s(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqrt(-b))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(1/2), x)
```

3.543 $\int \frac{(2-bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=60

$$-3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -6*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(-b*x+2)^(3/2)/x^(1/2)-3*b
*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{2-bx}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[2 - b*x] - (2*(2 - b*x)^(3/2))/Sqrt[x] - 6*Sqrt[b]*ArcSin
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\
&= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(6b)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 58, normalized size = 0.97

$$\frac{(-4-bx)\sqrt{2-bx}}{\sqrt{x}}-6\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(3/2),x]
[Out] ((-4 - b*x)*Sqrt[2 - b*x])/Sqrt[x] - 6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + S
qrt[2 - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.83, size = 132, normalized size = 2.20

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(6\sqrt{b}\,\sqrt{x}\,\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](-2+bx)-bx\,(2+bx)\,\sqrt{-2+bx}+8\sqrt{-2+bx}\right)}{\sqrt{x}\,(-2+bx)},\text{Abs}\,[bx]>2\right\}\right\},-6\sqrt{b}\,\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]-\frac{8}{\sqrt{x}\,\sqrt{2-bx}}+\frac{2b\sqrt{x}}{\sqrt{2-bx}}+\frac{b^2x^{\frac{3}{2}}}{\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - b*x)^(3/2)/x^(3/2),x]')
[Out] Piecewise[{{I (6 Sqrt[b] Sqrt[x] ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (-2 +
 b x) - b x (2 + b x) Sqrt[-2 + b x] + 8 Sqrt[-2 + b x]) / (Sqrt[x] (-2 + b
```

```
x)), Abs[b x] > 2}}, -6 Sqrt[b] ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] - 8 /
(Sqrt[x] Sqrt[2 - b x]) + 2 b Sqrt[x] / Sqrt[2 - b x] + b ^ 2 x ^ (3 / 2) /
Sqrt[2 - b x]]
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.17

method	result	size
meijerg	$-\frac{3(-b)^{\frac{3}{2}}\left(-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(\frac{bx}{4}+1\right)\sqrt{-\frac{bx}{2}+1}}{3\sqrt{x}\,\sqrt{-b}}-\frac{4\sqrt{\pi}\,\sqrt{b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{-b}}\right)}{2\sqrt{\pi}\,b}$	70
risch	$\frac{(x^2b^2+2bx-8)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}-\frac{3\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*(-b)^(3/2)/Pi^(1/2)/b*(-8/3*Pi^(1/2)/x^(1/2)*2^(1/2)/(-b)^(1/2)*(1/4*b
*x+1)*(-1/2*b*x+1)^(1/2)-4*Pi^(1/2)/(-b)^(1/2)*b^(1/2)*arcsin(1/2*b^(1/2)*x
^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.37, size = 63, normalized size = 1.05

$$6\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{4\sqrt{-bx+2}}{\sqrt{x}}-\frac{2\sqrt{-bx+2}\,b}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 4*sqrt(-b*x + 2)/sqrt(
x) - 2*sqrt(-b*x + 2)*b/((b - (b*x - 2)/x)*sqrt(x))
```

Fricas [A]
time = 0.32, size = 101, normalized size = 1.68

$$\left[\frac{3\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(bx+4)\sqrt{-bx+2}\,\sqrt{x}}{x},\frac{6\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+4)\sqrt{-bx+2}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (b*x + 4)*
sqrt(-b*x + 2)*sqrt(x))/x, (6*sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt
(x))) - (b*x + 4)*sqrt(-b*x + 2)*sqrt(x))/x]
```

Sympy [A]
time = 1.45, size = 158, normalized size = 2.63

$$\begin{cases} 6i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) - \frac{ib^2x^{\frac{3}{2}}}{\sqrt{bx-2}} - \frac{2ib\sqrt{x}}{\sqrt{bx-2}} + \frac{8i}{\sqrt{x}\sqrt{bx-2}} & \text{for}\ |bx| > 2 \\ -6\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) + \frac{b^2x^{\frac{3}{2}}}{\sqrt{-bx+2}} + \frac{2b\sqrt{x}}{\sqrt{-bx+2}} - \frac{8}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((6*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - I*b**2*x**(3/2)/s
qrt(b*x - 2) - 2*I*b*sqrt(x)/sqrt(b*x - 2) + 8*I/(sqrt(x)*sqrt(b*x - 2)), A
bs(b*x) > 2), (-6*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/s
qrt(-b*x + 2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 8/(sqrt(x)*sqrt(-b*x + 2)), Tr
ue))
```

Giac [A]
time = 1.16, size = 128, normalized size = 2.13

$$-\frac{bb^2\left(\frac{2\left(-\frac{1}{2}\sqrt{-bx+2}\sqrt{-bx+2}+3\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{-b(-bx+2)+2b} + \frac{6\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] -((b*x + 4)*sqrt(-b*x + 2)/sqrt((b*x - 2)*b + 2*b) + 6*log(abs(-sqrt(-b*x +
 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqrt(-b))*b^2/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(3/2), x)
```

3.544 $\int \frac{(2-bx)^{3/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=62

$$\frac{2b\sqrt{2-bx}}{\sqrt{x}}-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(-b*x+2)^(3/2)/x^(3/2)+2*b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))+2
*b*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[2 - b*x])/Sqrt[x] - (2*(2 - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(2-bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2(2-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx$$
$$= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx$$
$$= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 1.00

$$\frac{4\sqrt{2-bx}\,(-1+2bx)}{3x^{3/2}} + 2\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (4*Sqrt[2 - b*x]*(-1 + 2*b*x))/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[2 - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.71, size = 163, normalized size = 2.63

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\left(bx\left(-6I\text{Log}\left[\frac{1}{\sqrt{b}\sqrt{x}}\right]+3I\text{Log}\left[\frac{1}{bx}\right]+6\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]+8\sqrt{\frac{2-bx}{bx}}\right)-4\sqrt{\frac{2-bx}{bx}}\right)}{3x},\frac{1}{\text{Abs}[bx]}>\frac{1}{2}\right\}\right\},-2Ib^{\frac{3}{2}}\text{Log}\left[1+\sqrt{1-\frac{2}{bx}}\right]+\frac{I8b^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+Ib^{\frac{3}{2}}\text{Log}\left[\frac{1}{bx}\right]-\frac{4I\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - b*x)^(3/2)/x^(5/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (b x (-6 I Log[1 / (Sqrt[b] Sqrt[x])] + 3 I Log[1 / (b
x)] + 6 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] + 8 Sqrt[(2 - b x) / (b x)]) -
4 Sqrt[(2 - b x) / (b x)]) / (3 x), 1 / Abs[b x] > 1 / 2}}, -2 I b ^ (3 / 2
) Log[1 + Sqrt[1 - 2 / (b x)]] + I 8 b ^ (3 / 2) Sqrt[1 - 2 / (b x)] / 3 +
I b ^ (3 / 2) Log[1 / (b x)] - 4 I Sqrt[b] Sqrt[1 - 2 / (b x)] / (3 x)]
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.13

method	result	size
meijerg	$-\frac{3(-b)^{\frac{5}{2}}\left(-\frac{16\sqrt{\pi}\,\sqrt{2}\,(-2bx+1)\sqrt{-\frac{bx}{2}+1}}{9x^{\frac{3}{2}}(-b)^{\frac{3}{2}}}+\frac{8\sqrt{\pi}\,b^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3(-b)^{\frac{3}{2}}}\right)}{4\sqrt{\pi}\,b}$	70
risch	$-\frac{4(2x^2b^2-5bx+2)\sqrt{(-bx+2)\,x}}{3x^{\frac{3}{2}}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*(-b)^(5/2)/Pi^(1/2)/b*(-16/9*Pi^(1/2)/x^(3/2)*2^(1/2)/(-b)^(3/2)*(-2*b
*x+1)*(-1/2*b*x+1)^(1/2)+8/3*Pi^(1/2)/(-b)^(3/2)*b^(3/2)*arcsin(1/2*b^(1/2)
*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.35, size = 49, normalized size = 0.79

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+2}\,b}{\sqrt{x}}-\frac{2\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + 2)*b/sq
rt(x) - 2/3*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 111, normalized size = 1.79

$$\left[\frac{3\,\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}\,+1\right)+4\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2},\,-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 4*
(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqrt(-b
*x + 2)/(sqrt(b)*sqrt(x))) - 2*(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.81, size = 184, normalized size = 2.97

$$\begin{cases} \frac{8b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} + ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right) + 2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) - \frac{4\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{1}{|bx|} > \frac{1}{2} \\ \frac{8ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} + ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right) - \frac{4i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((8*b**(3/2)*sqrt(-1 + 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*
b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 2*b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2) - 4*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x), 1/Abs(b*x) > 1/2), (8*I*b**(3/2)*
sqrt(1 - 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*b**(3/2)*log(sqrt(1 - 2
/(b*x)) + 1) - 4*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [A]
time = 1.13, size = 141, normalized size = 2.27

$$\frac{b^2\left(\frac{2\left(-\frac{12}{9}b^3\sqrt{-bx+2}\sqrt{-bx+2}+\frac{18}{9}b^3\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{(-b(-bx+2)+2b)^2} + \frac{2b^2\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] 2/3*(3*b^2*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqr
t(-b) + 2*(2*(b*x - 2)*b^3 + 3*b^3)*sqrt(-b*x + 2)/((b*x - 2)*b + 2*b)^(3/2
))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(5/2), x)
```

3.545 $\int x^{5/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=164

$$\frac{5a^5\sqrt{x}\,\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx$$

```
[Out] 1/12*a*x^(7/2)*(b*x+a)^(3/2)+1/6*x^(7/2)*(b*x+a)^(5/2)-5/512*a^6*arctanh(b^
(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-5/768*a^4*x^(3/2)*(b*x+a)^(1/2)/b^2+1/
192*a^3*x^(5/2)*(b*x+a)^(1/2)/b+1/32*a^2*x^(7/2)*(b*x+a)^(1/2)+5/512*a^5*x^
(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{5a^5\sqrt{x}\,\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^5*Sqrt[x]*Sqrt[a + b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a + b*x])/(76
8*b^2) + (a^3*x^(5/2)*Sqrt[a + b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a + b*x])/
32 + (a*x^(7/2)*(a + b*x)^(3/2))/12 + (x^(7/2)*(a + b*x)^(5/2))/6 - (5*a^6*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(512*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{12}(5a)\int x^{5/2}(a+bx)^{3/2}\,dx$$

$$= \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{8}a^2\int x^{5/2}\sqrt{a+bx}\,dx$$

$$= \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} -$$

$$= -\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} +$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} +$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} +$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} +$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} +$$

Mathematica [A]
time = 0.11, size = 110, normalized size = 0.67

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(15a^5-10a^4bx+8a^3b^2x^2+432a^2b^3x^3+640ab^4x^4+256b^5x^5)+15a^6\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*a
^2*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) + 15*a^6*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]])/(1536*b^(7/2))
```

Mathics [A]
time = 174.24, size = 154, normalized size = 0.94

$$\frac{-15a^{\frac{17}{2}}b^{6}\mathrm{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{5}{2}}+15a^{6}b^{\frac{13}{2}}\sqrt{x}\,(a+bx)^{2}+5a^{5}b^{\frac{15}{2}}x^{\frac{3}{2}}(a+bx)^{2}-2a^{4}b^{\frac{17}{2}}x^{\frac{5}{2}}(a+bx)^{2}+8ab^{\frac{19}{2}}x^{\frac{7}{2}}(55a^{2}+134abx+112b^{2}x^{2})(a+bx)^{2}+256b^{\frac{25}{2}}x^{\frac{13}{2}}(a+bx)^{2}}{1536a^{\frac{5}{2}}b^{\frac{19}{2}}\left(\frac{a+bx}{a}\right)^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*(a + b*x)^(5/2),x]')
```

```
[Out] (-15 a ^ (17 / 2) b ^ 6 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a)
^ (5 / 2) + 15 a ^ 6 b ^ (13 / 2) Sqrt[x] (a + b x) ^ 2 + 5 a ^ 5 b ^ (15 /
 2) x ^ (3 / 2) (a + b x) ^ 2 - 2 a ^ 4 b ^ (17 / 2) x ^ (5 / 2) (a + b x)
^ 2 + 8 a b ^ (19 / 2) x ^ (7 / 2) (55 a ^ 2 + 134 a b x + 112 b ^ 2 x ^ 2)
 (a + b x) ^ 2 + 256 b ^ (25 / 2) x ^ (13 / 2) (a + b x) ^ 2) / (1536 a ^ (
5 / 2) b ^ (19 / 2) ((a + b x) / a) ^ (5 / 2))
```

Maple [A]
time = 0.10, size = 160, normalized size = 0.98

method	result
risch	$\frac{(256b^{5}x^{5}+640a\,b^{4}x^{4}+432a^{2}b^{3}x^{3}+8a^{3}b^{2}x^{2}-10a^{4}bx+15a^{5})\sqrt{x}\,\sqrt{bx+a}}{1536b^{3}}-\frac{5a^{6}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\,(bx+a}}{1024b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$

$$5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{7}{2}}}{5b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{7}{2}}}{4b}-\frac{a\left(\frac{(bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx}{}\right.}{}\right.}{6}\right.}{8b}\right.}{10b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+a)^(7/2)-5/12*a/b*(1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/10*a/b*
(1/4/b*x^(1/2)*(b*x+a)^(7/2)-1/8*a/b*(1/3*(b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*
(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/
(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(118) = 236$.
time = 0.37, size = 244, normalized size = 1.49

$$\frac{5\,a^{6}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{\frac{15\sqrt{bx+a}\,a^{6}b^{5}}{\sqrt{x}}-\frac{85\,(bx+a)^{\frac{3}{2}}a^{6}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(bx+a)^{\frac{5}{2}}a^{6}b^{3}}{x^{\frac{5}{2}}}+\frac{198\,(bx+a)^{\frac{7}{2}}a^{6}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(bx+a)^{\frac{9}{2}}a^{6}b}{x^{\frac{9}{2}}}+\frac{15\,(bx+a)^{\frac{11}{2}}a^{6}}{x^{\frac{11}{2}}}}{1536\left(b^{9}-\frac{6\,(bx+a)b^{8}}{x}+\frac{15\,(bx+a)^{2}b^{7}}{x^{2}}-\frac{20\,(bx+a)^{3}b^{6}}{x^{3}}+\frac{15\,(bx+a)^{4}b^{5}}{x^{4}}-\frac{6\,(bx+a)^{5}b^{4}}{x^{5}}+\frac{(bx+a)^{6}b^{3}}{x^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/1024*a^6*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(7/2) + 1/1536*(15*sqrt(b*x + a)*a^6*b^5/sqrt(x) - 85*(b*x + a)
^(3/2)*a^6*b^4/x^(3/2) + 198*(b*x + a)^(5/2)*a^6*b^3/x^(5/2) + 198*(b*x + a
)^(7/2)*a^6*b^2/x^(7/2) - 85*(b*x + a)^(9/2)*a^6*b/x^(9/2) + 15*(b*x + a)^(
11/2)*a^6/x^(11/2))/(b^9 - 6*(b*x + a)*b^8/x + 15*(b*x + a)^2*b^7/x^2 - 20*
(b*x + a)^3*b^6/x^3 + 15*(b*x + a)^4*b^5/x^4 - 6*(b*x + a)^5*b^4/x^5 + (b*x
 + a)^6*b^3/x^6)
```

Fricas [A]
time = 0.32, size = 206, normalized size = 1.26

$$\left[\frac{15\,a^{6}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(256\,b^{6}x^{5}+640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}+8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x+15\,a^{5}b)\sqrt{bx+a}\,\sqrt{x}}{3072\,b^{4}},\ \frac{15\,a^{6}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(256\,b^{6}x^{5}+640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}+8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x+15\,a^{5}b)\sqrt{bx+a}\,\sqrt{x}}{1536\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*a^6*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b
^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan
(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a
^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x)
)/b^4]
```

Sympy [A]
time = 179.38, size = 207, normalized size = 1.26

$$\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1+\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,b^{2}x^{\frac{11}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{5a^{6}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}+\frac{b^{3}x^{\frac{13}{2}}}{6\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(5/2),x)
```

```
[Out] 5*a**(11/2)*sqrt(x)/(512*b**3*sqrt(1 + b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*
b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(5/2)/(768*b*sqrt(1 + b*x/a)) + 55*a**(
5/2)*x**(7/2)/(192*sqrt(1 + b*x/a)) + 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 + b
*x/a)) + 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 + b*x/a)) - 5*a**6*asinh(sqrt(
b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 + b*x
/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(118) = 236.
time = 0.02, size = 546, normalized size = 3.33

$$2b^{2}\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{174182400}\cdot 72576000b^{10}\sqrt{x}\,\sqrt{x}}{b^{10}}+\frac{\frac{1}{174182400}\cdot 725760b^{9}a}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{174182400}\cdot 816480b^{8}a^{2}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{174182400}\cdot 952560b^{7}a^{3}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{174182400}\cdot 1190700b^{6}a^{4}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{174182400}\cdot 1786050b^{5}a^{5}}{b^{10}}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{42a^{6}\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{2048b^{5}\sqrt{b}}\right)+4ab\left(2\left(\left(\left(\left(\frac{\frac{1}{1612800}\cdot 806400b^{8}\sqrt{x}\,\sqrt{x}}{b^{8}}+\frac{\frac{1}{1612800}\cdot 10080b^{7}a}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 11760b^{6}a^{2}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{1612800}\cdot 14700b^{5}a^{3}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 22050b^{4}a^{4}}{b^{8}}\right)\sqrt{x}\,\sqrt{a+bx}-\frac{14a^{5}\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{512b^{4}\sqrt{b}}\right)+2a^{2}\left(2\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^{6}\sqrt{x}\,\sqrt{x}}{b^{6}}+\frac{\frac{1}{23040}\cdot 240b^{5}a}{b^{6}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^{4}a^{2}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{23040}\cdot 450b^{3}a^{3}}{b^{6}}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{10a^{4}\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{256b^{3}\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/192*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(
x) + 15*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*a^2 + 1/960
*((2*(4*(6*(8*x + a/b)*x - 7*a^2/b^2)*x + 35*a^3/b^3)*x - 105*a^4/b^4)*sqrt
(b*x + a)*sqrt(x) - 105*a^5*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9
/2))*a*b + 1/7680*((2*(4*(2*(8*(10*x + a/b)*x - 9*a^2/b^2)*x + 21*a^3/b^3)*
x - 105*a^4/b^4)*x + 315*a^5/b^5)*sqrt(b*x + a)*sqrt(x) + 315*a^6*log(abs(-
sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(11/2))*b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(5/2), x)
```

3.546 $\int x^{3/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{3a^5\tanh^{-1}\Big(}{128b}$$

```
[Out] 1/8*a*x^(5/2)*(b*x+a)^(3/2)+1/5*x^(5/2)*(b*x+a)^(5/2)+3/128*a^5*arctanh(b^(
1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)+1/64*a^3*x^(3/2)*(b*x+a)^(1/2)/b+1/16*a
^2*x^(5/2)*(b*x+a)^(1/2)-3/128*a^4*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^2) + (a^3*x^(3/2)*Sqrt[a + b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a + b*x])/16 + (a*x^(5/2)*(a + b*x)^(3/2))/8 + (x^(5
/2)*(a + b*x)^(5/2))/5 + (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
128*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}\,dx\\ &= \frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a+bx}\,dx\\ &= \frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{32}a^3\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\ &= \frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}-\frac{(3}{}\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\end{aligned}$$

Mathematica [A]
time = 0.12, size = 99, normalized size = 0.71

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(-15a^4+10a^3bx+248a^2b^2x^2+336ab^3x^3+128b^4x^4)-15a^5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 33
6*a*b^3*x^3 + 128*b^4*x^4) - 15*a^5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
)/(640*b^(5/2))
```

Mathics [A]
time = 34.08, size = 144, normalized size = 1.03

$$\frac{a^{\frac{3}{2}}\left(15a^{\frac{7}{2}}b^3\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](a+bx)^2-15a^5b^{\frac{7}{2}}\sqrt{x}\,\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}-5a^4b^{\frac{9}{2}}x^{\frac{3}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+2ab^{\frac{11}{2}}x^{\frac{5}{2}}\left(129a^2+292abx+232b^2x^2\right)\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+128b^{\frac{17}{2}}x^{\frac{11}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}\right)}{640b^{\frac{11}{2}}\,(a+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(a + b*x)^(5/2),x]')
```

```
[Out] a ^ (3 / 2) (15 a ^ (7 / 2) b ^ 3 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] (a + b
 x) ^ 2 - 15 a ^ 5 b ^ (7 / 2) Sqrt[x] ((a + b x) / a) ^ (3 / 2) - 5 a ^ 4
b ^ (9 / 2) x ^ (3 / 2) ((a + b x) / a) ^ (3 / 2) + 2 a b ^ (11 / 2) x ^ (5
 / 2) (129 a ^ 2 + 292 a b x + 232 b ^ 2 x ^ 2) ((a + b x) / a) ^ (3 / 2) +
 128 b ^ (17 / 2) x ^ (11 / 2) ((a + b x) / a) ^ (3 / 2)) / (640 b ^ (11 /
2) (a + b x) ^ 2)
```

Maple [A]
time = 0.12, size = 138, normalized size = 0.99

method	result
risch	$-\frac{\left(-128b^4x^4-336a\,b^3x^3-248a^2b^2x^2-10a^3bx+15a^4\right)\sqrt{x}\,\sqrt{bx+a}}{640b^2}+\frac{3a^5\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{256b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$

default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{7}{2}}}{5b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{7}{2}}}{4b}-\frac{a\left(\frac{(bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}\cdots\right)}{2\sqrt{bx+a}\,\sqrt{\cdots}}\right)}{4}\right)}{6}\right)}{8b}\right)}{10b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/10*a/b*(1/4/b*x^(1/2)*(b*x+a)^(7/2)-1/8*a/b*(
1/3*(b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(
b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b
^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(100) = 200$.

time = 0.39, size = 212, normalized size = 1.51

$$-\frac{3\,a^{5}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{5}{2}}}-\frac{\frac{15\,\sqrt{bx+a}\,a^{5}b^{4}}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}\,a^{5}b^{3}}{x^{\frac{3}{2}}}+\frac{128\,(bx+a)^{\frac{5}{2}}\,a^{5}b^{2}}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}\,a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}\,a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{7}-\frac{5\,(bx+a)b^{6}}{x}+\frac{10\,(bx+a)^{2}b^{5}}{x^{2}}-\frac{10\,(bx+a)^{3}b^{4}}{x^{3}}+\frac{5\,(bx+a)^{4}b^{3}}{x^{4}}-\frac{(bx+a)^{5}b^{2}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^
(3/2)*a^5*b^3/x^(3/2) + 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^
(7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x + a)*b^
6/x + 10*(b*x + a)^2*b^5/x^2 - 10*(b*x + a)^3*b^4/x^3 + 5*(b*x + a)^4*b^3/x
^4 - (b*x + a)^5*b^2/x^5)
```

Fricas [A]
time = 0.32, size = 185, normalized size = 1.32

$$\left[\frac{15\,a^{5}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(128\,b^{5}x^{4}+336\,ab^{4}x^{3}+248\,a^{2}b^{3}x^{2}+10\,a^{3}b^{2}x-15\,a^{4}b)\sqrt{bx+a}\,\sqrt{x}}{1280\,b^{3}},\ -\frac{15\,a^{5}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(128\,b^{5}x^{4}+336\,ab^{4}x^{3}+248\,a^{2}b^{3}x^{2}+10\,a^{3}b^{2}x-15\,a^{4}b)\sqrt{bx+a}\,\sqrt{x}}{640\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)
*sqrt(b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*
a^3*b^2*x - 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A]
time = 33.75, size = 180, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1+\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1+\frac{bx}{a}}}+\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}+\frac{3a^{5}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(3/2)/(128*b*s
qrt(1 + b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 + b*x/a)) + 73*a**(3/2)
*b*x**(7/2)/(80*sqrt(1 + b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1 + b*
```

```
x/a)) + 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) + b**3*x**(11/
2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(100) = 200.
time = 0.02, size = 465, normalized size = 3.32

$$2b^2\left(2\left(\left(\left(\left(\frac{\frac{1}{1612800}\cdot 80640b^8\sqrt{x}\,\sqrt{x}}{b^8}+\frac{\frac{1}{1612800}\cdot 10080b^7a}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 11760b^6a^2}{b^8}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{1612800}\cdot 14700b^5a^3}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{1612800}\cdot 22050b^4a^4}{b^8}\right)\sqrt{x}\,\sqrt{a+bx}-\frac{14a^5\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{512b^4\sqrt{b}}\right)+4ab\left(2\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\,\sqrt{x}}{b^6}+\frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{10a^4\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{256b^3\sqrt{b}}\right)+2a^2\left(2\left(\left(\frac{\frac{1}{576}\cdot 48b^4\sqrt{x}\,\sqrt{x}}{b^4}+\frac{\frac{1}{576}\cdot 12b^3a}{b^4}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{576}\cdot 18b^2a^2}{b^4}\right)\sqrt{x}\,\sqrt{a+bx}-\frac{2a^3\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{32b^2\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/24*(sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-
sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))*a^2 + 1/96*((2*(4*(6*x + a/b)*x
- 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 15*a^4*log(abs(-sqrt(b
)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*a*b + 1/1920*((2*(4*(6*(8*x + a/b)*x -
 7*a^2/b^2)*x + 35*a^3/b^3)*x - 105*a^4/b^4)*sqrt(b*x + a)*sqrt(x) - 105*a^
5*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2))*b^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(5/2), x)
```

3.547 $\int \sqrt{x}\,(a+bx)^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}$$

```
[Out] 5/24*a*x^(3/2)*(b*x+a)^(3/2)+1/4*x^(3/2)*(b*x+a)^(5/2)-5/64*a^4*arctanh(b^(
1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)+5/32*a^2*x^(3/2)*(b*x+a)^(1/2)+5/64*a^3
*x^(1/2)*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a + b*x])/32 + (
5*a*x^(3/2)*(a + b*x)^(3/2))/24 + (x^(3/2)*(a + b*x)^(5/2))/4 - (5*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{x}\,(a+bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}\,(a+bx)^{3/2}\,dx$$

$$= \frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{16}\left(5a^2\right)\int \sqrt{x}\,\sqrt{a+bx}\,dx$$

$$= \frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}$$

$$= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-$$

$$= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-$$

$$= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-$$

$$= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-$$

Mathematica [A]
time = 0.11, size = 87, normalized size = 0.75

$$\frac{\sqrt{x}\,\sqrt{a+bx}\,\left(15a^3+118a^2bx+136ab^2x^2+48b^3x^3\right)}{192b}+\frac{5a^4\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*b^3*x^3))
/(192*b) + (5*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64*b^(3/2))
```

Mathics [A]
time = 10.59, size = 109, normalized size = 0.94

$$\frac{-15a^{\frac{11}{2}}b\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+15a^4b^{\frac{3}{2}}\sqrt{x}\,(a+bx)+ab^{\frac{5}{2}}x^{\frac{3}{2}}(a+bx)\left(133a^2+254abx+184b^2x^2\right)+48b^{\frac{11}{2}}x^{\frac{9}{2}}(a+bx)}{192a^{\frac{3}{2}}b^{\frac{5}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*(a + b*x)^(5/2),x]')
```

```
[Out] (-15 a ^ (11 / 2) b ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^ (3
 / 2) + 15 a ^ 4 b ^ (3 / 2) Sqrt[x] (a + b x) + a b ^ (5 / 2) x ^ (3 / 2)
(a + b x) (133 a ^ 2 + 254 a b x + 184 b ^ 2 x ^ 2) + 48 b ^ (11 / 2) x ^ (
9 / 2) (a + b x)) / (192 a ^ (3 / 2) b ^ (5 / 2) ((a + b x) / a) ^ (3 / 2))
```

Maple [A]
time = 0.12, size = 113, normalized size = 0.97

method	result
risch	$\frac{\left(48b^3x^3+136a\,b^2x^2+118a^2bx+15a^3\right)\sqrt{x}\,\sqrt{bx+a}}{192b}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{128b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4}+\frac{5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2}+\frac{a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4}\right)}{2}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(b*x+a)^(5/2)+5/8*a*(1/3*x^(3/2)*(b*x+a)^(3/2)+1/2*a*(1/2*x^(3/
2)*(b*x+a)^(1/2)+1/4*a*(x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(
1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.
time = 0.34, size = 176, normalized size = 1.52

$$\frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{3}{2}}}+\frac{\frac{15\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{55\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx+a)b^4}{x}+\frac{6\,(bx+a)^2b^3}{x^2}-\frac{4\,(bx+a)^3b^2}{x^3}+\frac{(bx+a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(3/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 55*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) + 73*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^5 - 4*(b*x + a)*b^4/x + 6*(b*x + a)^2*b^3/x^2 - 4*(b*x +
a)^3*b^2/x^3 + (b*x + a)^4*b/x^4)
```

Fricas [A]
time = 0.34, size = 162, normalized size = 1.40

$$\left[\frac{15\,a^4\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{384\,b^2},\ \frac{15\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{192\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

Sympy [A]
time = 8.98, size = 155, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqrt
(1 + b*x/a)) + 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*sqrt(a)*b*
```

```
*2*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(6
4*b**(3/2)) + b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(82) = 164.
time = 41.11, size = 589, normalized size = 5.08

$$\frac{\frac{2b^3|b|\left(2\left(\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^{11}\sqrt{a+bx}\sqrt{a+bx}}{b^{14}}-\frac{\frac{1}{23040}\cdot 6000b^{11}a}{b^{14}}\right)\sqrt{a+bx}\sqrt{a+bx}+\frac{\frac{1}{23040}\cdot 9780b^{11}a^2}{b^{14}}\right)\sqrt{a+bx}\sqrt{a+bx}-\frac{\frac{1}{23040}\cdot 8370b^{11}a^3}{b^{14}}\right)\sqrt{a+bx}\sqrt{-ab+b(a+bx)}-\frac{70a^4\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{256b^2\sqrt{b}}\right)}{b^2}+\frac{6ab^2|b|\left(2\left(\left(\left(\frac{\frac{1}{2304}\cdot 192b^5\sqrt{a+bx}\sqrt{a+bx}}{b^7}-\frac{\frac{1}{2304}\cdot 624b^5a}{b^7}\right)\sqrt{a+bx}\sqrt{a+bx}+\frac{\frac{1}{2304}\cdot 792b^5a^2}{b^7}\right)\sqrt{a+bx}\sqrt{-ab+b(a+bx)}+\frac{10a^3\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{32b\sqrt{b}}\right)\right)}{b^2}+\frac{6a^2b|b|\left(2\left(\left(\frac{1}{8}\sqrt{a+bx}\sqrt{a+bx}-\frac{10}{32}a\right)\sqrt{a+bx}\sqrt{-ab+b(a+bx)}-\frac{6a^2b\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{16\sqrt{b}}\right)\right)}{b^2b}+\frac{2a^3|b|\left(\frac{1}{2}\sqrt{a+bx}\sqrt{-ab+b(a+bx)}+\frac{2ab\ln\left|\sqrt{-ab+b(a+bx)}-\sqrt{b}\sqrt{a+bx}\right|}{4\sqrt{b}}\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/192*(24*(15*a^3*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))
)/b^(3/2) + sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)
/b^2 - 13*a/b^2) + 33*a^2/b^2))*a*abs(b) + 192*(a*sqrt(b)*log(abs(-sqrt(b*x
 + a)*sqrt(b) + sqrt((b*x + a)*b - a*b))) + sqrt((b*x + a)*b - a*b)*sqrt(b*
x + a))*a^3*abs(b)/b^2 - (105*a^4*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*
x + a)*b - a*b)))/b^(5/2) - (2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 - 25
*a/b^3) + 163*a^2/b^3) - 279*a^3/b^3)*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)
)*b*abs(b) - 144*(3*a^2*sqrt(b)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x
+ a)*b - a*b))) - sqrt((b*x + a)*b - a*b)*(2*b*x - 3*a)*sqrt(b*x + a))*a^2*
abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(5/2), x)
```

3.548 $\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=92

$$\frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

```
[Out] 5/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)+5/12*a*(b*x+a)^(3/2)
*x^(1/2)+1/3*(b*x+a)^(5/2)*x^(1/2)+5/8*a^2*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/8 + (5*a*Sqrt[x]*(a + b*x)^(3/2))/12 + (Sqrt[
x]*(a + b*x)^(5/2))/3 + (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8
*Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{6}(5a)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+}}\right.$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx}\right.$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.79

$$\frac{1}{24}\sqrt{x}\,\sqrt{a+bx}\,\left(33a^2+26abx+8b^2x^2\right) - \frac{5a^3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[x],x]
[Out] (Sqrt[x]*Sqrt[a + b*x]*(33*a^2 + 26*a*b*x + 8*b^2*x^2))/24 - (5*a^3*Log[-(S
qrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*Sqrt[b])
```

Mathics [A]
time = 5.63, size = 62, normalized size = 0.67

$$\frac{15a^3\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right] + \sqrt{a}\,\sqrt{b}\,\sqrt{x}\,\left(33a^2+26abx+8b^2x^2\right)\sqrt{\frac{a+bx}{a}}}{24\sqrt{b}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(5/2)/Sqrt[x],x]')

[Out] (15 a ^ 3 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] + Sqrt[a] Sqrt[b] Sqrt[x] (33 a ^ 2 + 26 a b x + 8 b ^ 2 x ^ 2) Sqrt[(a + b x) / a]) / (24 Sqrt[b])

Maple [A]
time = 0.10, size = 94, normalized size = 1.02

method	result	size
risch	$\frac{(8x^2b^2+26abx+33a^2)\sqrt{x}\,\sqrt{bx+a}}{24}+\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{16\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	84
default	$\frac{(bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}$	94

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.
time = 0.34, size = 141, normalized size = 1.53

$$-\frac{5\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,\sqrt{b}}-\frac{\frac{15\,\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{33\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3-\frac{3\,(bx+a)b^2}{x}+\frac{3\,(bx+a)^2b}{x^2}-\frac{(bx+a)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")

[Out] -5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/sqrt(b) - 1/24*(15*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3

```
/2)*a^3*b/x^(3/2) + 33*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x + a)*b^2/
x + 3*(b*x + a)^2*b/x^2 - (b*x + a)^3/x^3)
```

Fricas [A]
time = 0.32, size = 141, normalized size = 1.53

$$\left[\frac{15\,a^{3}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b},-\frac{15\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [A]
time = 4.04, size = 102, normalized size = 1.11

$$\frac{11a^{\frac{5}{2}}\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{8}+\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^{2}x^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 11*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a)/8 + 13*a**(3/2)*b*x**(3/2)*sqrt(1 + b*x
/a)/12 + sqrt(a)*b**2*x**(5/2)*sqrt(1 + b*x/a)/3 + 5*a**3*asinh(sqrt(b)*sqr
t(x)/sqrt(a))/(8*sqrt(b))
```

Giac [A]
time = 10.43, size = 150, normalized size = 1.63

$$\frac{b^{2}\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 24\sqrt{a+bx}\,\sqrt{a+bx}}{b}+\frac{\frac{1}{144}\cdot 30a}{b}\right)\sqrt{a+bx}\,\sqrt{a+bx}+\frac{\frac{1}{144}\cdot 45a^{2}}{b}\right)\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}-\frac{10a^{3}\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{16\sqrt{b}}\right)\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] -1/24*(15*a^3*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sq
rt(b) - sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b +
 5*a/b) + 15*a^2/b))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{\sqrt{x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(1/2),x)
[Out] int((a + b*x)^(5/2)/x^(1/2), x)
```

3.549 $\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{15}{4}ab\sqrt{x}\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 15/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))*b^(1/2)-2*(b*x+a)^(5/2)/x^(
1/2)+5/2*b*(b*x+a)^(3/2)*x^(1/2)+15/4*a*b*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {49, 52, 65, 223, 212}

$$\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*a*b*Sqrt[x]*Sqrt[a + b*x])/4 + (5*b*Sqrt[x]*(a + b*x)^(3/2))/2 - (2*(a
+ b*x)^(5/2))/Sqrt[x] + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]])/4
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{\sqrt{x}} + (5b)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$$
$$= \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{4}(15ab)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,d$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\right.$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right.$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.82

$$\frac{\sqrt{a+bx}\,\left(-8a^2+9abx+2b^2x^2\right)}{4\sqrt{x}} - \frac{15}{4}a^2\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(3/2),x]
```

[Out] (Sqrt[a + b*x]*(-8*a^2 + 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/4

Mathics [A]
time = 5.69, size = 91, normalized size = 1.02

$$\frac{-8a^3(a+bx)+15a^{\frac{7}{2}}\sqrt{b}\sqrt{x}\,\mathrm{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+abx(a+11bx)(a+bx)+2b^3x^3(a+bx)}{4a^{\frac{3}{2}}\sqrt{x}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(5/2)/x^(3/2),x]')

[Out] (-8 a ^ 3 (a + b x) + 15 a ^ (7 / 2) Sqrt[b] Sqrt[x] ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^ (3 / 2) + a b x (a + 11 b x) (a + b x) + 2 b ^ 3 x ^ 3 (a + b x)) / (4 a ^ (3 / 2) Sqrt[x] ((a + b x) / a) ^ (3 / 2))

Maple [A]
time = 0.10, size = 84, normalized size = 0.94

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-2x^2b^2-9abx+8a^2)}{4\sqrt{x}}+\frac{15a^2\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{8\sqrt{x}\sqrt{bx+a}}$	84

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4*(b*x+a)^(1/2)*(-2*b^2*x^2-9*a*b*x+8*a^2)/x^(1/2)+15/8*a^2*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)

Maxima [A]
time = 0.35, size = 125, normalized size = 1.40

$$-\frac{15}{8}a^2\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\sqrt{bx+a}\,a^2b^2}{\sqrt{x}}-\frac{9(bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2(bx+a)b}{x}+\frac{(bx+a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")

[Out] -15/8*a^2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x))) - 2*sqrt(b*x + a)*a^2/sqrt(x) - 1/4*(7*sqrt(b*x + a)*a^2*b

```
^2/sqrt(x) - 9*(b*x + a)^(3/2)*a^2*b/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x
 + a)^2/x^2)
```

Fricas [A]
time = 0.31, size = 137, normalized size = 1.54

$$\left[\frac{15\,a^{2}\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x^{2}+9\,abx-8\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{8\,x},\ -\frac{15\,a^{2}\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{2}x^{2}+9\,abx-8\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 3.95, size = 126, normalized size = 1.42

$$-\frac{2a^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^{2}x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}+\frac{15a^{2}\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{b^{3}x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] -2*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + a**(3/2)*b*sqrt(x)/(4*sqrt(1 + b*x/
a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sqrt(1 + b*x/a)) + 15*a**2*sqrt(b)*asinh(
sqrt(b)*sqrt(x)/sqrt(a))/4 + b**3*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 10.50, size = 155, normalized size = 1.74

$$\frac{bb^{2}\left(\frac{2\left(\left(\frac{1}{4}\sqrt{a+bx}\,\sqrt{a+bx}+\frac{5}{8}a\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{15}{8}a^{2}\right)\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{-ab+b(a+bx)}-\frac{30a^{2}\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{8\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] -1/4*(15*a^2*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)))/sqr
t(b) - ((2*b*x + 7*a)*(b*x + a) - 15*a^2)*sqrt(b*x + a)/sqrt((b*x + a)*b -
a*b))*b^2/abs(b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(3/2),x)
[Out] int((a + b*x)^(5/2)/x^(3/2), x)
```

3.550 $\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=86

$$5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] -2/3*(b*x+a)^(5/2)/x^(3/2)+5*a*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2
))-10/3*b*(b*x+a)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)+5b^2\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a + b*x] - (10*b*(a + b*x)^(3/2))/(3*Sqrt[x]) - (2*(a +
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx$$

$$= -\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$

$$= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx$$

$$= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\right.$$

$$= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{}{\sqrt{a}}\right.$$

$$= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.80

$$\frac{\sqrt{a+bx}\,\left(-2a^2-14abx+3b^2x^2\right)}{3x^{3/2}}-5ab^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-2*a^2 - 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 5*a*b^(3/2)*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

Mathics [A]
time = 5.52, size = 99, normalized size = 1.15

$$\frac{\sqrt{b}\left(-4a^2\sqrt{\frac{a+bx}{bx}}+bx\left(-28a\sqrt{\frac{a+bx}{bx}}-15a\text{Log}\left[\frac{a}{bx}\right]+30a\text{Log}\left[1+\sqrt{\frac{a+bx}{bx}}\right]+6bx\sqrt{\frac{a+bx}{bx}}\right)\right)}{6x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/x^(5/2),x]')
```

```
[Out] Sqrt[b] (-4 a ^ 2 Sqrt[(a + b x) / (b x)] + b x (-28 a Sqrt[(a + b x) / (b
x)] - 15 a Log[a / (b x)] + 30 a Log[1 + Sqrt[(a + b x) / (b x)]] + 6 b x S
qrt[(a + b x) / (b x)])) / (6 x)
```

Maple [A]
time = 0.11, size = 82, normalized size = 0.95

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3x^2b^2+14abx+2a^2)}{3x^{\frac{3}{2}}}+\frac{5a\,b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x+a)^(1/2)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*ln((1/
2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 100, normalized size = 1.16

$$-\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+a}\,ab}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab^2}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5/2*a*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 4*sqrt(b*x + a)*a*b/sqrt(x) - sqrt(b*x + a)*a*b^2/((b - (b*
x + a)/x)*sqrt(x)) - 2/3*(b*x + a)^(3/2)*a/x^(3/2)
```

Fricas [A]
time = 0.32, size = 138, normalized size = 1.60

$$\left[\frac{15\,ab^{\frac{3}{2}}x^{2}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(3\,b^{2}x^{2}-14\,abx-2\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{6\,x^{2}},\,-\frac{15\,a\sqrt{-b}\,bx^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(3\,b^{2}x^{2}-14\,abx-2\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A]
time = 3.75, size = 99, normalized size = 1.15

$$-\frac{2a^{2}\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] -2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 14*a*b**(3/2)*sqrt(a/(b*x) + 1)/3
 - 5*a*b**(3/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1) +
b**(5/2)*x*sqrt(a/(b*x) + 1)
```

Giac [A]
time = 10.50, size = 185, normalized size = 2.15

$$\frac{b^{2}\left(\frac{2\left(\left(\left(\frac{\frac{1}{18}\cdot 9b^{4}a\sqrt{a+bx}\,\sqrt{a+bx}}{ba}-\frac{\frac{1}{18}\cdot 60b^{4}a^{2}}{ba}\right)\sqrt{a+bx}\,\sqrt{a+bx}+\frac{\frac{1}{18}\cdot 45b^{4}a^{3}}{ba}\right)\sqrt{a+bx}\,\sqrt{-ab+b\,(a+bx)}}{(-ab+b(a+bx))^{2}}-\frac{10ab^{2}\ln\left|\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right|}{2\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(15*a*b^(3/2)*log(abs(-sqrt(b*x + a)*sqrt(b) + sqrt((b*x + a)*b - a*b)
)) - (15*a^2*b^3 + (3*(b*x + a)*b^3 - 20*a*b^3)*(b*x + a))*sqrt(b*x + a)/((
b*x + a)*b - a*b)^(3/2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{x^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(5/2),x)
[Out] int((a + b*x)^(5/2)/x^(5/2), x)
```

3.551 $\int x^{5/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=171

$$-\frac{5a^5\sqrt{x}\,\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-$$

```
[Out] 1/12*a*x^(7/2)*(-b*x+a)^(3/2)+1/6*x^(7/2)*(-b*x+a)^(5/2)+5/512*a^6*arctan(b
^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)-5/768*a^4*x^(3/2)*(-b*x+a)^(1/2)/b^2
-1/192*a^3*x^(5/2)*(-b*x+a)^(1/2)/b+1/32*a^2*x^(7/2)*(-b*x+a)^(1/2)-5/512*a
^5*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{5a^5\sqrt{x}\,\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^5*Sqrt[x]*Sqrt[a - b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a - b*x])/(7
68*b^2) - (a^3*x^(5/2)*Sqrt[a - b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a - b*x])
/32 + (a*x^(7/2)*(a - b*x)^(3/2))/12 + (x^(7/2)*(a - b*x)^(5/2))/6 + (5*a^6
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(512*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a-bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{12}(5a)\int x^{5/2}(a-bx)^{3/2}\,dx$$

$$= \frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{8}a^2\int x^{5/2}\sqrt{a-bx}\,dx$$

$$= \frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+$$

$$= -\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+$$

Mathematica [A]
time = 0.17, size = 114, normalized size = 0.67

$$\frac{\frac{\sqrt{x}\sqrt{a-bx}\,(-15a^5-10a^4bx-8a^3b^2x^2+432a^2b^3x^3-640ab^4x^4+256b^5x^5)}{b^3}+\frac{15a^6\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}}{1536}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)*(a - b*x)^(5/2),x]

[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-15*a^5 - 10*a^4*b*x - 8*a^3*b^2*x^2 + 432*a^2*b^3*x^3 - 640*a*b^4*x^4 + 256*b^5*x^5))/b^3 + (15*a^6*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2))/1536

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 173.37, size = 349, normalized size = 2.04

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-15a^{\frac{17}{2}}b^6\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{5}{2}}+15a^6b^{\frac{13}{2}}\sqrt{x}\,(-a+bx)^2-5a^5b^{\frac{15}{2}}x^{\frac{3}{2}}(-a+bx)^2-2a^4b^{\frac{17}{2}}x^{\frac{5}{2}}(-a+bx)^2+8ab^{\frac{19}{2}}x^{\frac{7}{2}}\left(-55a^2+134abx-112b^2x^2\right)(-a+bx)^2+256b^{\frac{25}{2}}x^{\frac{13}{2}}(-a+bx)^2\right)}{1536a^{\frac{5}{2}}b^{\frac{19}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{5}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{5a^6\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{512b^{\frac{7}{2}}}-\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}}-\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}}+\frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}}-\frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^(5/2)*(a - b*x)^(5/2),x]')

[Out] Piecewise[{{I / 1536 (-15 a ^ (17 / 2) b ^ 6 ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((-a + b x) / a) ^ (5 / 2) + 15 a ^ 6 b ^ (13 / 2) Sqrt[x] (-a + b x) ^ 2 - 5 a ^ 5 b ^ (15 / 2) x ^ (3 / 2) (-a + b x) ^ 2 - 2 a ^ 4 b ^ (17 / 2) x ^ (5 / 2) (-a + b x) ^ 2 + 8 a b ^ (19 / 2) x ^ (7 / 2) (-55 a ^ 2 + 13 4 a b x - 112 b ^ 2 x ^ 2) (-a + b x) ^ 2 + 256 b ^ (25 / 2) x ^ (13 / 2) (-a + b x) ^ 2) / (a ^ (5 / 2) b ^ (19 / 2) ((-a + b x) / a) ^ (5 / 2)), Abs [b x / a] > 1}}, 5 a ^ 6 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (512 b ^ (7 / 2)) - 5 a ^ (11 / 2) Sqrt[x] / (512 b ^ 3 Sqrt[1 - b x / a]) + 5 a ^ (9 / 2) x ^ (3 / 2) / (1536 b ^ 2 Sqrt[1 - b x / a]) + a ^ (7 / 2) x ^ (5 / 2) / (768 b Sqrt[1 - b x / a]) + 55 a ^ (5 / 2) x ^ (7 / 2) / (192 Sqrt[1 - b x / a]) - 67 a ^ (3 / 2) b x ^ (9 / 2) / (96 Sqrt[1 - b x / a]) + 7 Sqrt[a] b ^ 2 x ^ (11 / 2) / (12 Sqrt[1 - b x / a]) - b ^ 3 x ^ (13 / 2) / (6 Sqrt[a] Sqrt[1 - b x / a])]

Maple [A]
time = 0.11, size = 169, normalized size = 0.99

method	result
risch	$-\frac{\left(-256b^5x^5+640a\,b^4x^4-432a^2b^3x^3+8a^3b^2x^2+10a^4bx+15a^5\right)\sqrt{x}\,\sqrt{-bx+a}}{1536b^3}+\frac{5a^6\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-}{1024b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+a}}$

$$3a\left(\sqrt{x}\,\sqrt{-bx+\right.$$

$$5a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{\quad}{\quad}\right.$$

$$a\left(\frac{(-bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{\quad}{\quad}\right.$$

$$3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{7}{2}}}{4b}+\frac{\quad}{8b}\right.$$

$$5a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}}{5b}+\frac{\quad}{10b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+a)^(7/2)+5/12*a/b*(-1/5/b*x^(3/2)*(-b*x+a)^(7/2)+3/10*
a/b*(-1/4/b*x^(1/2)*(-b*x+a)^(7/2)+1/8*a/b*(1/3*(-b*x+a)^(5/2)*x^(1/2)+5/6*
a*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+
a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2
+a*x)^(1/2))))))
```

Maxima [A]
time = 0.37, size = 242, normalized size = 1.42

$$-\frac{5\,a^6 \arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{512\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^6b^5}{\sqrt{x}}+\frac{85\,(-bx+a)^{\frac{3}{2}}a^6b^4}{x^{\frac{3}{2}}}+\frac{198\,(-bx+a)^{\frac{5}{2}}a^6b^3}{x^{\frac{5}{2}}}-\frac{198\,(-bx+a)^{\frac{7}{2}}a^6b^2}{x^{\frac{7}{2}}}-\frac{85\,(-bx+a)^{\frac{9}{2}}a^6b}{x^{\frac{9}{2}}}-\frac{15\,(-bx+a)^{\frac{11}{2}}a^6}{x^{\frac{11}{2}}}}{1536\left(b^9-\frac{6\,(bx-a)b^8}{x}+\frac{15\,(bx-a)^2b^7}{x^2}-\frac{20\,(bx-a)^3b^6}{x^3}+\frac{15\,(bx-a)^4b^5}{x^4}-\frac{6\,(bx-a)^5b^4}{x^5}+\frac{(bx-a)^6b^3}{x^6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -5/512*a^6*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/1536*(15*sq
rt(-b*x + a)*a^6*b^5/sqrt(x) + 85*(-b*x + a)^(3/2)*a^6*b^4/x^(3/2) + 198*(-
b*x + a)^(5/2)*a^6*b^3/x^(5/2) - 198*(-b*x + a)^(7/2)*a^6*b^2/x^(7/2) - 85*
(-b*x + a)^(9/2)*a^6*b/x^(9/2) - 15*(-b*x + a)^(11/2)*a^6/x^(11/2))/(b^9 -
6*(b*x - a)*b^8/x + 15*(b*x - a)^2*b^7/x^2 - 20*(b*x - a)^3*b^6/x^3 + 15*(b
*x - a)^4*b^5/x^4 - 6*(b*x - a)^5*b^4/x^5 + (b*x - a)^6*b^3/x^6)
```

Fricas [A]
time = 0.32, size = 208, normalized size = 1.22

$$\left[-\frac{15\,a^6\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b)\sqrt{-bx+a}\,\sqrt{x}}{3072\,b^4},-\frac{15\,a^6\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b)\sqrt{-bx+a}\,\sqrt{x}}{1536\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(15*a^6*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(256*b^6*x^5 - 640*a*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*
a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/1536*(15*a^6*sqrt(b)*
arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (256*b^6*x^5 - 640*a*b^5*x^4 + 4
32*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sq
rt(x))/b^4]
```

Sympy [A]
time = 177.75, size = 435, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{-1+\frac{bx}{a}}} - \frac{55ia^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{67ia^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^6\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} + \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}} + \frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{5a^6\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} - \frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((5*I*a**(11/2)*sqrt(x)/(512*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(9/2)
*x**(3/2)/(1536*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x**(5/2)/(768*b*sqrt(-1
 + b*x/a)) - 55*I*a**(5/2)*x**(7/2)/(192*sqrt(-1 + b*x/a)) + 67*I*a**(3/2)*
b*x**(9/2)/(96*sqrt(-1 + b*x/a)) - 7*I*sqrt(a)*b**2*x**(11/2)/(12*sqrt(-1 +
 b*x/a)) - 5*I*a**6*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + I*b**3*
x**(13/2)/(6*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(11/2)*sqrt
(x)/(512*b**3*sqrt(1 - b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*b**2*sqrt(1 - b*
x/a)) + a**(7/2)*x**(5/2)/(768*b*sqrt(1 - b*x/a)) + 55*a**(5/2)*x**(7/2)/(1
92*sqrt(1 - b*x/a)) - 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 - b*x/a)) + 7*sqrt(
a)*b**2*x**(11/2)/(12*sqrt(1 - b*x/a)) + 5*a**6*asin(sqrt(b)*sqrt(x)/sqrt(a
))/(512*b**(7/2)) - b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(125) = 250.
time = 0.02, size = 568, normalized size = 3.32

$$2b^2\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{174182400}\cdot 7257600b^{10}\sqrt{x}\sqrt{x}}{b^{10}}-\frac{\frac{1}{174182400}\cdot 725760b^9a}{b^{10}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{174182400}\cdot 816480b^8a^2}{b^{10}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{174182400}\cdot 952560b^7a^3}{b^{10}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{174182400}\cdot 1190700b^6a^4}{b^{10}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{174182400}\cdot 1786050b^5a^5}{b^{10}}\right)\sqrt{x}\sqrt{a-bx}-\frac{42a^6\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{2048b^5\sqrt{-b}}\right)-4ab\left(2\left(\left(\left(\left(\frac{\frac{1}{1612800}\cdot 80640b^8\sqrt{x}\sqrt{x}}{b^8}-\frac{\frac{1}{1612800}\cdot 10080b^7a}{b^8}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{1612800}\cdot 11760b^6a^2}{b^8}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{1612800}\cdot 14700b^5a^3}{b^8}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{1612800}\cdot 22050b^4a^4}{b^8}\right)\sqrt{x}\sqrt{a-bx}-\frac{14a^5\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{512b^4\sqrt{-b}}\right)+2a^2\left(2\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^6\sqrt{x}\sqrt{x}}{b^6}-\frac{\frac{1}{23040}\cdot 240b^5a}{b^6}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^4a^2}{b^6}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{23040}\cdot 450b^3a^3}{b^6}\right)\sqrt{x}\sqrt{a-bx}-\frac{10a^4\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{256b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x)
```

```
[Out] 1/192*((2*(4*(6*x - a/b)*x - 5*a^2/b^2)*x - 15*a^3/b^3)*sqrt(-b*x + a)*sqrt
(x) - 15*a^4*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3))*a
^2 - 1/960*((2*(4*(6*(8*x - a/b)*x - 7*a^2/b^2)*x - 35*a^3/b^3)*x - 105*a^4
/b^4)*sqrt(-b*x + a)*sqrt(x) - 105*a^5*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*
x + a)))/(sqrt(-b)*b^4))*a*b + 1/7680*((2*(4*(2*(8*(10*x - a/b)*x - 9*a^2/b
^2)*x - 21*a^3/b^3)*x - 105*a^4/b^4)*x - 315*a^5/b^5)*sqrt(-b*x + a)*sqrt(x
) - 315*a^6*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^5))*b^
2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a-b\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(5/2),x)
[Out] int(x^(5/2)*(a - b*x)^(5/2), x)
```

3.552 $\int x^{3/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{3a^4\sqrt{x}\,\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{3a^5\tan^{-1}\left(\ldots\right.}{128\ldots}$$

```
[Out] 1/8*a*x^(5/2)*(-b*x+a)^(3/2)+1/5*x^(5/2)*(-b*x+a)^(5/2)+3/128*a^5*arctan(b^
(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)-1/64*a^3*x^(3/2)*(-b*x+a)^(1/2)/b+1/1
6*a^2*x^(5/2)*(-b*x+a)^(1/2)-3/128*a^4*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\,\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^2) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a - b*x])/16 + (a*x^(5/2)*(a - b*x)^(3/2))/8 + (x^(5
/2)*(a - b*x)^(5/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(1
28*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(a-bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a-bx)^{3/2}\,dx\\
&= \frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a-bx}\,dx\\
&= \frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{32}a^3\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{(3}{}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 103, normalized size = 0.71

$$\frac{1}{640}\left(\frac{\sqrt{x}\sqrt{a-bx}\left(-15a^4-10a^3bx+248a^2b^2x^2-336ab^3x^3+128b^4x^4\right)}{b^2}-\frac{15a^5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-15*a^4 - 10*a^3*b*x + 248*a^2*b^2*x^2 - 336*a*b^3
*x^3 + 128*b^4*x^4))/b^2 - (15*a^5*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
)/(-b)^(5/2))/640
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 49.67, size = 315, normalized size = 2.16

$$\text{Piecewise}\left[\left\{\left\{\frac{Ia^{\frac{3}{2}}\left(-15a^{\frac{7}{2}}b^3\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right](-a+bx)^2+15a^5b^{\frac{7}{2}}\sqrt{x}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-5a^4b^{\frac{9}{2}}x^{\frac{3}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+2ab^{\frac{11}{2}}x^{\frac{5}{2}}\left(-129a^2+292abx-232b^2x^2\right)\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+128b^{\frac{17}{2}}x^{\frac{11}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}\right)}{640b^{\frac{11}{2}}(-a+bx)^2},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{3a^5\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{128b^{\frac{5}{2}}}-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}b^2x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}-\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(a - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 640 a ^ (3 / 2) (-15 a ^ (7 / 2) b ^ 3 ArcCosh[Sqrt[b] Sqrt
[x] / Sqrt[a]] (-a + b x) ^ 2 + 15 a ^ 5 b ^ (7 / 2) Sqrt[x] ((-a + b x) /
a) ^ (3 / 2) - 5 a ^ 4 b ^ (9 / 2) x ^ (3 / 2) ((-a + b x) / a) ^ (3 / 2) +
 2 a b ^ (11 / 2) x ^ (5 / 2) (-129 a ^ 2 + 292 a b x - 232 b ^ 2 x ^ 2) ((
-a + b x) / a) ^ (3 / 2) + 128 b ^ (17 / 2) x ^ (11 / 2) ((-a + b x) / a) ^
 (3 / 2)) / (b ^ (11 / 2) (-a + b x) ^ 2), Abs[b x / a] > 1}}, 3 a ^ 5 ArcS
in[Sqrt[b] Sqrt[x] / Sqrt[a]] / (128 b ^ (5 / 2)) - 3 a ^ (9 / 2) Sqrt[x] /
 (128 b ^ 2 Sqrt[1 - b x / a]) + a ^ (7 / 2) x ^ (3 / 2) / (128 b Sqrt[1 -
b x / a]) + 129 a ^ (5 / 2) x ^ (5 / 2) / (320 Sqrt[1 - b x / a]) - 73 a ^
(3 / 2) b x ^ (7 / 2) / (80 Sqrt[1 - b x / a]) + 29 Sqrt[a] b ^ 2 x ^ (9 /
2) / (40 Sqrt[1 - b x / a]) - b ^ 3 x ^ (11 / 2) / (5 Sqrt[a] Sqrt[1 - b x
/ a])]
```

Maple [A]
time = 0.14, size = 146, normalized size = 1.00

method	result
risch	$-\frac{\left(-128b^4x^4+336a\,b^3x^3-248a^2b^2x^2+10a^3bx+15a^4\right)\sqrt{x}\,\sqrt{-bx+a}}{640b^2}+\frac{3a^5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\left(-bx+\right.}}{256b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$

default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}}{5b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{7}{2}}}{4b}+\frac{a\left(\frac{(-bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+}}{2\sqrt{-}}\right.}{4}\right.}{6}\right.}{8b}\right.}{10b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/5/b*x^(3/2)*(-b*x+a)^(7/2)+3/10*a/b*(-1/4/b*x^(1/2)*(-b*x+a)^(7/2)+1/8*a /b*(1/3*(-b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2) *arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))))

Maxima [A]

time = 0.36, size = 207, normalized size = 1.42

$$-\frac{3\,a^{5}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{5}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{5}b^{4}}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}a^{5}b^{3}}{x^{\frac{3}{2}}}+\frac{128\,(-bx+a)^{\frac{5}{2}}a^{5}b^{2}}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{7}-\frac{5\,(bx-a)b^{6}}{x}+\frac{10\,(bx-a)^{2}b^{5}}{x^{2}}-\frac{10\,(bx-a)^{3}b^{4}}{x^{3}}+\frac{5\,(bx-a)^{4}b^{3}}{x^{4}}-\frac{(bx-a)^{5}b^{2}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) + 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x - a)*b^6/x + 10*(b*x - a)^2*b^5/x^2
 - 10*(b*x - a)^3*b^4/x^3 + 5*(b*x - a)^4*b^3/x^4 - (b*x - a)^5*b^2/x^5)
```

Fricas [A]
time = 0.34, size = 186, normalized size = 1.27

$$\left[-\frac{15\,a^{5}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(128\,b^{5}x^{4}-336\,ab^{4}x^{3}+248\,a^{2}b^{3}x^{2}-10\,a^{3}b^{2}x-15\,a^{4}b)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^{3}},\ -\frac{15\,a^{5}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(128\,b^{5}x^{4}-336\,ab^{4}x^{3}+248\,a^{2}b^{3}x^{2}-10\,a^{3}b^{2}x-15\,a^{4}b)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 - 10*a^3*b^2*x - 15*a
^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x
+ a)/(sqrt(b)*sqrt(x))) - (128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 -
10*a^3*b^2*x - 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [A]
time = 33.66, size = 379, normalized size = 2.60

$$\begin{cases}\frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{-1+\frac{bx}{a}}}-\frac{129ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{-1+\frac{bx}{a}}}+\frac{73ia^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}-\frac{29i\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{5}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{ib^{3}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^{5}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b*sqrt(-1 + b*x/a)) - 129*I*a**(5/2)*x**(5/2)/(320*sqrt(-1 + b*
x/a)) + 73*I*a**(3/2)*b*x**(7/2)/(80*sqrt(-1 + b*x/a)) - 29*I*sqrt(a)*b**2*
x**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(1
```

```
28*b**(5/2)) + I*b**3*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(12
8*b*sqrt(1 - b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 - b*x/a)) - 73*a**
(3/2)*b*x**(7/2)/(80*sqrt(1 - b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1
 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) - b**3*x**
(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(106) = 212.
time = 0.02, size = 484, normalized size = 3.32

$$2b^{2}\left(2\left(\left(\left(\left(\frac{\frac{1}{1612800}\cdot 80640b^{8}\sqrt{x}\sqrt{x}}{b^{8}}-\frac{\frac{1}{1612800}\cdot 10080b^{7}a}{b^{8}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{1612800}\cdot 11760b^{6}a^{2}}{b^{8}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{1612800}\cdot 14700b^{5}a^{3}}{b^{8}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{1612800}\cdot 22050b^{4}a^{4}}{b^{8}}\right)\sqrt{x}\sqrt{a-bx}-\frac{14a^{5}\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{512b^{4}\sqrt{-b}}\right)-4ab\left(2\left(\left(\left(\frac{\frac{1}{23040}\cdot 1440b^{6}\sqrt{x}\sqrt{x}}{b^{6}}-\frac{\frac{1}{23040}\cdot 240b^{5}a}{b^{6}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{23040}\cdot 300b^{4}a^{2}}{b^{6}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{23040}\cdot 450b^{3}a^{3}}{b^{6}}\right)\sqrt{x}\sqrt{a-bx}-\frac{10a^{4}\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{256b^{3}\sqrt{-b}}\right)+2a^{2}\left(2\left(\left(\frac{\frac{1}{576}\cdot 48b^{4}\sqrt{x}\sqrt{x}}{b^{4}}-\frac{\frac{1}{576}\cdot 12b^{3}a}{b^{4}}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{576}\cdot 18b^{2}a^{2}}{b^{4}}\right)\sqrt{x}\sqrt{a-bx}-\frac{2a^{3}\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{32b^{2}\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x)
```

```
[Out] 1/24*(sqrt(-b*x + a)*(2*(4*x - a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(
-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^2))*a^2 - 1/96*((2*(4*(6*x
 - a/b)*x - 5*a^2/b^2)*x - 15*a^3/b^3)*sqrt(-b*x + a)*sqrt(x) - 15*a^4*log(
abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3))*a*b + 1/1920*((2*(
4*(6*(8*x - a/b)*x - 7*a^2/b^2)*x - 35*a^3/b^3)*x - 105*a^4/b^4)*sqrt(-b*x
+ a)*sqrt(x) - 105*a^5*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-
b)*b^4))*b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(a-b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(5/2), x)
```

3.553 $\int \sqrt{x}\,(a-bx)^{5/2}\,dx$

Optimal. Leaf size=121

$$-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}$$

```
[Out] 5/24*a*x^(3/2)*(-b*x+a)^(3/2)+1/4*x^(3/2)*(-b*x+a)^(5/2)+5/64*a^4*arctan(b^
(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)+5/32*a^2*x^(3/2)*(-b*x+a)^(1/2)-5/64*
a^3*x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a - b*x])/32 +
(5*a*x^(3/2)*(a - b*x)^(3/2))/24 + (x^(3/2)*(a - b*x)^(5/2))/4 + (5*a^4*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{x}\,(a-bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{1}{8}(5a)\int \sqrt{x}\,(a-bx)^{3/2}\,dx$$
$$= \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{1}{16}\left(5a^2\right)\int \sqrt{x}\,\sqrt{a-bx}\,dx$$
$$= \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}$$
$$= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} +$$
$$= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} +$$
$$= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} +$$
$$= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} +$$

Mathematica [**A**]
time = 0.14, size = 93, normalized size = 0.77

$$\frac{\sqrt{x}\,\sqrt{a-bx}\,\left(-15a^3+118a^2bx-136ab^2x^2+48b^3x^3\right)}{192b} + \frac{5a^4\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{64(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(5/2),x]

[Out] (Sqrt[x]*Sqrt[a - b*x]*(-15*a^3 + 118*a^2*b*x - 136*a*b^2*x^2 + 48*b^3*x^3)
)/(192*b) + (5*a^4*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(64*(-b)^(3/2)
)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 10.81, size = 256, normalized size = 2.12

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-15a^{\frac{11}{2}}b\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+15a^4b^{\frac{3}{2}}\sqrt{x}\,(-a+bx)+ab^{\frac{5}{2}}x^{\frac{3}{2}}(-a+bx)\left(-133a^2+254abx-184b^2x^2\right)+48b^{\frac{11}{2}}x^{\frac{9}{2}}(-a+bx)\right)}{192a^{\frac{3}{2}}b^{\frac{5}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{5a^4\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{64b^{\frac{3}{2}}}-\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}}-\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}-\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*(a - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 192 (-15 a ^ (11 / 2) b ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]]
((-a + b x) / a) ^ (3 / 2) + 15 a ^ 4 b ^ (3 / 2) Sqrt[x] (-a + b x) + a b
^ (5 / 2) x ^ (3 / 2) (-a + b x) (-133 a ^ 2 + 254 a b x - 184 b ^ 2 x ^ 2)
 + 48 b ^ (11 / 2) x ^ (9 / 2) (-a + b x)) / (a ^ (3 / 2) b ^ (5 / 2) ((-a
+ b x) / a) ^ (3 / 2)), Abs[b x / a] > 1}}, 5 a ^ 4 ArcSin[Sqrt[b] Sqrt[x]
/ Sqrt[a]] / (64 b ^ (3 / 2)) - 5 a ^ (7 / 2) Sqrt[x] / (64 b Sqrt[1 - b x
/ a]) + 133 a ^ (5 / 2) x ^ (3 / 2) / (192 Sqrt[1 - b x / a]) - 127 a ^ (3
/ 2) b x ^ (5 / 2) / (96 Sqrt[1 - b x / a]) + 23 Sqrt[a] b ^ 2 x ^ (7 / 2)
/ (24 Sqrt[1 - b x / a]) - b ^ 3 x ^ (9 / 2) / (4 Sqrt[a] Sqrt[1 - b x / a]
)]
```

Maple [A]

time = 0.11, size = 121, normalized size = 1.00

method	result
risch	$-\frac{\left(-48b^3x^3+136a\,b^2x^2-118a^2bx+15a^3\right)\sqrt{x}\,\sqrt{-bx+a}}{192b}+\frac{5a^4\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{128b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4}+\frac{5a\left(\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2}+\frac{a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,(x \ldots}{\sqrt{-x^2b \ldots}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4}\right)}{2}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(-b*x+a)^(5/2)+5/8*a*(1/3*x^(3/2)*(-b*x+a)^(3/2)+1/2*a*(1/2*x^(
3/2)*(-b*x+a)^(1/2)+1/4*a*(-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x
+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(
1/2)))))
```

Maxima [A]
time = 0.38, size = 168, normalized size = 1.39

$$-\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{3}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{55\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx-a)b^4}{x}+\frac{6\,(bx-a)^2b^3}{x^2}-\frac{4\,(bx-a)^3b^2}{x^3}+\frac{(bx-a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) + 55*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) + 73*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^5 - 4*(b*x
- a)*b^4/x + 6*(b*x - a)^2*b^3/x^2 - 4*(b*x - a)^3*b^2/x^3 + (b*x - a)^4*b/
x^4)
```

Fricas [A]
time = 0.32, size = 164, normalized size = 1.36

$$\left[-\frac{15\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^2},\,-\frac{15\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a
)*sqrt(x))/b^2, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(
x))) - (48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x +
a)*sqrt(x))/b^2]
```

Sympy [A]

time = 8.94, size = 326, normalized size = 2.69

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{-1+\frac{bx}{a}}} - \frac{133ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{127ia^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{23i\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} + \frac{ib^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}} + \frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}} + \frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} - \frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b*sqrt(-1 + b*x/a)) - 133*I*a**(5/2)*x**(3/2)/(192*sqrt(-1 + b*x/a)) + 127*I*a**(3/2)*b*x**(5/2)/(96*sqrt(-1 + b*x/a)) - 23*I*sqrt(a)*b**2*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) + I*b**3*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 - b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqrt(1 - b*x/a)) - 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 - b*x/a)) + 23*sqrt(a)*b**2*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) - b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(87) = 174.
time = 41.91, size = 629, normalized size = 5.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/192*(24*(15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/(sqrt(-b)*b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*(b*x - a)/b^2 + 13*a/b^2) + 33*a^2/b^2))*a*abs(b) + 192*(a*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) - sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a))*a^3*abs(b)/b^2 - (105*a^4*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/(sqrt(-b)*b^2) - (2*(b*x - a)*(4*(b*x - a)*(6*(b*x - a)/b^3 + 25*a/b^3) + 163*a^2/b^3) + 279*a^3/b^3)*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a))*b*abs(b) - 144*(3*a^2*b*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/sqrt(-b) - sqrt((b*x - a)*b + a*b)*(2*b*x + 3*a)*sqrt(-b*x + a))*a^2*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a-b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a - b*x)^(5/2), x)
```

3.554 $\int \frac{(a-bx)^{5/2}}{\sqrt{x}} dx$

Optimal. Leaf size=96

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}$$

```
[Out] 5/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)+5/12*a*(-b*x+a)^(3/2
)*x^(1/2)+1/3*(-b*x+a)^(5/2)*x^(1/2)+5/8*a^2*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a - b*x])/8 + (5*a*Sqrt[x]*(a - b*x)^(3/2))/12 + (Sqrt[
x]*(a - b*x)^(5/2))/3 + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*
Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{6}(5a)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx \\
&= \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}} \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-}} \right. \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1+bx} \right. \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 79, normalized size = 0.82

$$\frac{1}{24}\sqrt{x}\,\sqrt{a-bx}\,\left(33a^2-26abx+8b^2x^2\right)-\frac{5a^3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{8\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/Sqrt[x],x]
[Out] (Sqrt[x]*Sqrt[a - b*x]*(33*a^2 - 26*a*b*x + 8*b^2*x^2))/24 - (5*a^3*Log[-(S
qrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*Sqrt[-b])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.99, size = 195, normalized size = 2.03

$$\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{a}\left(-15a^{\frac{5}{2}}\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](-a+bx)+a\sqrt{b}\,\sqrt{x}\,(-33a^2+59abx-34b^2x^2)\sqrt{\frac{-a+bx}{a}}+8b^{\frac{7}{2}}x^{\frac{7}{2}}\sqrt{\frac{-a+bx}{a}}\right)}{24\sqrt{b}\,(-a+bx)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{5a^3\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{8\sqrt{b}}+\frac{11a^{\frac{5}{2}}\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{8}-\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^2x^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - b*x)^(5/2)/Sqrt[x],x]')
```

```
[Out] Piecewise[{{I / 24 Sqrt[a] (-15 a ^ (5 / 2) ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[
a]] (-a + b x) + a Sqrt[b] Sqrt[x] (-33 a ^ 2 + 59 a b x - 34 b ^ 2 x ^ 2)
Sqrt[(-a + b x) / a] + 8 b ^ (7 / 2) x ^ (7 / 2) Sqrt[(-a + b x) / a]) / (S
qrt[b] (-a + b x)), Abs[b x / a] > 1}}, 5 a ^ 3 ArcSin[Sqrt[b] Sqrt[x] / Sq
rt[a]] / (8 Sqrt[b]) + 11 a ^ (5 / 2) Sqrt[x] Sqrt[1 - b x / a] / 8 - 13 a
^ (3 / 2) b x ^ (3 / 2) Sqrt[1 - b x / a] / 12 + Sqrt[a] b ^ 2 x ^ (5 / 2)
Sqrt[1 - b x / a] / 3]
```

Maple [A]
time = 0.10, size = 100, normalized size = 1.04

method	result	s
risch	$\frac{(8x^2b^2-26abx+33a^2)\sqrt{x}\,\sqrt{-bx+a}}{24}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{16\sqrt{b}\,\sqrt{x}\,\sqrt{-bx+a}}$	8
default	$\frac{(-bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)
*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arc
tan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.36, size = 130, normalized size = 1.35

$$-\frac{5\,a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,\sqrt{b}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{33\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3-\frac{3\,(bx-a)b^2}{x}+\frac{3\,(bx-a)^2b}{x^2}-\frac{(bx-a)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/24*(15*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 33*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x - a)*b^2/x + 3*(b*x - a)^2*b/x^2 - (b*x
- a)^3/x^3)
```

Fricas [A]
time = 0.31, size = 142, normalized size = 1.48

$$\left[-\frac{15\,a^{3}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(8\,b^{3}x^{2}-26\,ab^{2}x+33\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b},-\frac{15\,a^{3}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(8\,b^{3}x^{2}-26\,ab^{2}x+33\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 - 2*(8*b^3*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/24*(
15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 26*a
*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b]
```

Sympy [A]
time = 4.03, size = 246, normalized size = 2.56

$$\begin{cases} -\frac{11ia^{\frac{5}{2}}\sqrt{x}}{8\sqrt{-1+\frac{bx}{a}}}+\frac{59ia^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{17i\sqrt{a}\,b^{2}x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{ib^{3}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{11a^{\frac{5}{2}}\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{8}-\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^{2}x^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{3}+\frac{5a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-11*I*a**(5/2)*sqrt(x)/(8*sqrt(-1 + b*x/a)) + 59*I*a**(3/2)*b*x*
*(3/2)/(24*sqrt(-1 + b*x/a)) - 17*I*sqrt(a)*b**2*x**(5/2)/(12*sqrt(-1 + b*x
/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + I*b**3*x**(7/2
)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (11*a**(5/2)*sqrt(x)*sqrt(
1 - b*x/a)/8 - 13*a**(3/2)*b*x**(3/2)*sqrt(1 - b*x/a)/12 + sqrt(a)*b**2*x**
(5/2)*sqrt(1 - b*x/a)/3 + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)),
 True))
```

Giac [A]
time = 10.42, size = 159, normalized size = 1.66

$$\frac{b^{2}\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 24\sqrt{a-bx}\,\sqrt{a-bx}}{b}+\frac{\frac{1}{144}\cdot 30a}{b}\right)\sqrt{a-bx}\,\sqrt{a-bx}+\frac{\frac{1}{144}\cdot 45a^{2}}{b}\right)\sqrt{a-bx}\,\sqrt{ab-b\,(a-bx)}+\frac{10a^{3}\ln\left|\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{16\sqrt{-b}}\right)\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/24*(15*a^3*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/s
qrt(-b) + sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)*(4*(b*x - a)/
b - 5*a/b) + 15*a^2/b))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(1/2), x)
```

3.555 $\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -15/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))*b^(1/2)-2*(-b*x+a)^(5/2)/x
^(1/2)-5/2*b*(-b*x+a)^(3/2)*x^(1/2)-15/4*a*b*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*a*b*Sqrt[x]*Sqrt[a - b*x])/4 - (5*b*Sqrt[x]*(a - b*x)^(3/2))/2 - (2*(a
 - b*x)^(5/2))/Sqrt[x] - (15*a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a -
b*x]])/4
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= -\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}(15ab)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-}}\right.\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1+b}\right.\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 79, normalized size = 0.85

$$\frac{\sqrt{a-bx}\,\left(-8a^2-9abx+2b^2x^2\right)}{4\sqrt{x}}-\frac{15}{4}a^2\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-8*a^2 - 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[-
b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.86, size = 216, normalized size = 2.32

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(8a^3(-a+bx)+15a^{\frac{7}{2}}\sqrt{b}\sqrt{x}\,\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+abx(a-11bx)(-a+bx)+2b^3x^3(-a+bx)\right)}{4a^{\frac{3}{2}}\sqrt{x}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-\frac{15a^2\sqrt{b}\,\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{4}-\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - b*x)^(5/2)/x^(3/2),x]')
```

```
[Out] Piecewise[{{I / 4 (8 a ^ 3 (-a + b x) + 15 a ^ (7 / 2) Sqrt[b] Sqrt[x] ArcC
osh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((-a + b x) / a) ^ (3 / 2) + a b x (a - 11 b
 x) (-a + b x) + 2 b ^ 3 x ^ 3 (-a + b x)) / (a ^ (3 / 2) Sqrt[x] ((-a + b
x) / a) ^ (3 / 2)), Abs[b x / a] > 1}}, -2 a ^ (5 / 2) / (Sqrt[x] Sqrt[1 -
b x / a]) - 15 a ^ 2 Sqrt[b] ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / 4 - a ^ (3
 / 2) b Sqrt[x] / (4 Sqrt[1 - b x / a]) + 11 Sqrt[a] b ^ 2 x ^ (3 / 2) / (4
 Sqrt[1 - b x / a]) - b ^ 3 x ^ (5 / 2) / (2 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.12, size = 88, normalized size = 0.95

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(-2x^2b^2+9abx+8a^2)}{4\sqrt{x}}-\frac{15a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)\sqrt{x(-bx+a)}}{8\sqrt{x}\,\sqrt{-bx+a}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-b*x+a)^(1/2)*(-2*b^2*x^2+9*a*b*x+8*a^2)/x^(1/2)-15/8*a^2*b^(1/2)*arc
tan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*
x+a)^(1/2)
```

Maxima [A]
time = 0.35, size = 112, normalized size = 1.20

$$\frac{15}{4}\,a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\sqrt{-bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\sqrt{-bx+a}\,a^2b^2}{\sqrt{x}}+\frac{9(-bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2(bx-a)b}{x}+\frac{(bx-a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 15/4*a^2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a
)*a^2/sqrt(x) - 1/4*(7*sqrt(-b*x + a)*a^2*b^2/sqrt(x) + 9*(-b*x + a)^(3/2)*
a^2*b/x^(3/2))/(b^2 - 2*(b*x - a)*b/x + (b*x - a)^2/x^2)
```

Fricas [A]
time = 0.31, size = 137, normalized size = 1.47

$$\left[\frac{15\,a^{2}\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x^{2}-9\,abx-8\,a^{2})\sqrt{-bx+a}\,\sqrt{x}}{8\,x},\ \frac{15\,a^{2}\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^{2}x^{2}-9\,abx-8\,a^{2})\sqrt{-bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x, 1/4*(15*a^2*s
qrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x^2 - 9*a*b*x -
8*a^2)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 3.98, size = 267, normalized size = 2.87

$$\begin{cases} \frac{2ia^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{-1+\frac{bx}{a}}}+\frac{ia^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{11i\sqrt{a}\,b^{2}x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{15ia^{2}\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{ib^{3}x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^{2}x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}-\frac{15a^{2}\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}-\frac{b^{3}x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(5/2)/(sqrt(x)*sqrt(-1 + b*x/a)) + I*a**(3/2)*b*sqrt(x)/(
4*sqrt(-1 + b*x/a)) - 11*I*sqrt(a)*b**2*x**(3/2)/(4*sqrt(-1 + b*x/a)) + 15*
I*a**2*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/4 + I*b**3*x**(5/2)/(2*sqrt(a
)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(5/2)/(sqrt(x)*sqrt(1 - b*x/a)
) - a**(3/2)*b*sqrt(x)/(4*sqrt(1 - b*x/a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sq
rt(1 - b*x/a)) - 15*a**2*sqrt(b)*asin(sqrt(b)*sqrt(x)/sqrt(a))/4 - b**3*x**
(5/2)/(2*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 10.45, size = 167, normalized size = 1.80

$$-\frac{bb^{2}\left(\frac{2\left(\left(-\frac{1}{4}\sqrt{a-bx}\,\sqrt{a-bx}-\frac{5}{8}a\right)\sqrt{a-bx}\,\sqrt{a-bx}+\frac{15}{8}a^{2}\right)\sqrt{a-bx}\,\sqrt{ab-b\,(a-bx)}}{ab-b(a-bx)}+\frac{30a^{2}\ln\left|\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{8\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] -1/4*(15*a^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/s
qrt(-b) - ((2*b*x - 7*a)*(b*x - a) - 15*a^2)*sqrt(-b*x + a)/sqrt((b*x - a)*
b + a*b))*b^2/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(3/2), x)
```

3.556 $\int \frac{(a-bx)^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=90

$$5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -2/3*(-b*x+a)^(5/2)/x^(3/2)+5*a*b^(3/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/
2))+10/3*b*(-b*x+a)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)+5b^2\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a - b*x] + (10*b*(a - b*x)^(3/2))/(3*Sqrt[x]) - (2*(a -
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x
]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx \\
&= \frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\right. \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\right. \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 76, normalized size = 0.84

$$\frac{\sqrt{a-bx}\,\left(-2a^2+14abx+3b^2x^2\right)}{3x^{3/2}}+5a\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-2*a^2 + 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 5*a*Sqrt[-b]*
b*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.73, size = 224, normalized size = 2.49

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\left(-4a^2\sqrt{\frac{a-bx}{bx}}+bx\left(-30Ia\text{Log}\left[\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right]+15Ia\text{Log}\left[\frac{a}{bx}\right]+28a\sqrt{\frac{a-bx}{bx}}+30a\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]+6bx\sqrt{\frac{a-bx}{bx}}\right)\right)}{6x},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2Ia^2\sqrt{b}\sqrt{1-\frac{a}{bx}}}{3x}-5Iab^{\frac{3}{2}}\text{Log}\left[1+\sqrt{1-\frac{a}{bx}}\right]+\frac{I5ab^{\frac{3}{2}}\text{Log}\left[\frac{a}{bx}\right]}{2}+\frac{I14ab^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}{3}+Ib^{\frac{5}{2}}x\sqrt{1-\frac{a}{bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - b*x)^(5/2)/x^(5/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-4 a ^ 2 Sqrt[(a - b x) / (b x)] + b x (-30 I a Log[Sq
rt[a] / (Sqrt[b] Sqrt[x])] + 15 I a Log[a / (b x)] + 28 a Sqrt[(a - b x) /
(b x)] + 30 a ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] + 6 b x Sqrt[(a - b x) / (b
 x)])) / (6 x), Abs[a / (b x)] > 1}}, -2 I a ^ 2 Sqrt[b] Sqrt[1 - a / (b x)
] / (3 x) - 5 I a b ^ (3 / 2) Log[1 + Sqrt[1 - a / (b x)]] + I 5 a b ^ (3 /
 2) Log[a / (b x)] / 2 + I 14 a b ^ (3 / 2) Sqrt[1 - a / (b x)] / 3 + I b ^
 (5 / 2) x Sqrt[1 - a / (b x)]]
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.96

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,\left(-3x^2b^2-14abx+2a^2\right)}{3x^{\frac{3}{2}}}+\frac{5a\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\left(-bx+a\right)}}{2\sqrt{x}\,\sqrt{-bx+a}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b*x+a)^(1/2)*(-3*b^2*x^2-14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*arcta
n(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+
a)^(1/2)
```

Maxima [A]
time = 0.37, size = 84, normalized size = 0.93

$$-5\,ab^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{4\,\sqrt{-bx+a}\,ab}{\sqrt{x}}+\frac{\sqrt{-bx+a}\,ab^2}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*a*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 4*sqrt(-b*x + a)*a*
b/sqrt(x) + sqrt(-b*x + a)*a*b^2/((b - (b*x - a)/x)*sqrt(x)) - 2/3*(-b*x +
a)^(3/2)*a/x^(3/2)
```

Fricas [A]
time = 0.32, size = 139, normalized size = 1.54

$$\left[\frac{15\,a\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(3\,b^2x^2+14\,abx-2\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{6\,x^2}, -\frac{15\,ab^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(3\,b^2x^2+14\,abx-2\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(3*b^2*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2, -1/3*(15
*a*b^(3/2)*x^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 14*a
*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.83, size = 245, normalized size = 2.72

$$\begin{cases} -\frac{2a^2\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{3x}+\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-5iab^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}-1} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia^2\sqrt{b}\,\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{14iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}-5iab^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right)+ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 14*a*b**(3/2)*sqrt(a/(
b*x) - 1)/3 - 5*I*a*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + 5*I*a*b**(3/2
)*log(a/(b*x))/2 + 5*a*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)) + b**(5/2)*x*
sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (-2*I*a**2*sqrt(b)*sqrt(-a/(b*x) + 1)
/(3*x) + 14*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + 5*I*a*b**(3/2)*log(a/(b*x))
/2 - 5*I*a*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1) + I*b**(5/2)*x*sqrt(-a/(b*x
) + 1), True))
```

Giac [A]
time = 10.49, size = 195, normalized size = 2.17

$$\frac{b^2\left(\frac{2\left(\left(\frac{\frac{1}{18}\cdot 9b^4a\sqrt{a-bx}\,\sqrt{a-bx}}{ba}-\frac{\frac{1}{18}\cdot 60b^4a^2}{ba}\right)\sqrt{a-bx}\,\sqrt{a-bx}+\frac{\frac{1}{18}\cdot 45b^4a^3}{ba}\right)\sqrt{a-bx}\,\sqrt{ab-b\,(a-bx)}}{(ab-b(a-bx))^2}+\frac{10ab^2\ln\left|\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right|}{2\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] 1/3*(15*a*b^2*log(abs(-sqrt(-b*x + a)*sqrt(-b) + sqrt((b*x - a)*b + a*b)))/
sqrt(-b) + (15*a^2*b^3 + (3*(b*x - a)*b^3 + 20*a*b^3)*(b*x - a))*sqrt(-b*x
+ a)/((b*x - a)*b + a*b)^(3/2))*b/abs(b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(5/2), x)
```

3.557 $\int x^{5/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=144

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}-\frac{5\sinh^{-1}}{}$$

```
[Out] 1/6*x^(7/2)*(b*x+2)^(3/2)+1/6*x^(7/2)*(b*x+2)^(5/2)-5/8*arcsinh(1/2*b^(1/2)
*x^(1/2)*2^(1/2))/b^(7/2)-5/48*x^(3/2)*(b*x+2)^(1/2)/b^2+1/24*x^(5/2)*(b*x+
2)^(1/2)/b+1/8*x^(7/2)*(b*x+2)^(1/2)+5/16*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]

time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{16b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(48*b^2) + (
x^(5/2)*Sqrt[2 + b*x])/(24*b) + (x^(7/2)*Sqrt[2 + b*x])/8 + (x^(7/2)*(2 + b
*x)^(3/2))/6 + (x^(7/2)*(2 + b*x)^(5/2))/6 - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(2+bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{5}{6}\int x^{5/2}(2+bx)^{3/2}\,dx \\
&= \frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{1}{2}\int x^{5/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}+\frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}-\frac{5\int \frac{x^{3/}}{\sqrt{2+}}}{24b} \\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+ \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 92, normalized size = 0.64

$$
\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2+54b^3x^3+40b^4x^4+8b^5x^5)}{48b^3}+\frac{5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{8b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 54*b^3*x^3 + 40*b^4*x^4 +
8*b^5*x^5))/(48*b^3) + (5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(8*b^(7/
2))
```

Mathics [A]
time = 171.19, size = 119, normalized size = 0.83

$$
\frac{-30b^6\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](2+bx)^2+30b^{\frac{13}{2}}\sqrt{x}\,(2+bx)^{\frac{3}{2}}+5b^{\frac{15}{2}}x^{\frac{3}{2}}(2+bx)^{\frac{3}{2}}-b^{\frac{17}{2}}x^{\frac{5}{2}}(2+bx)^{\frac{3}{2}}+2b^{\frac{19}{2}}x^{\frac{7}{2}}(55+67bx+28b^2x^2+4b^3x^3)(2+bx)^{\frac{3}{2}}}{48b^{\frac{19}{2}}(2+bx)^2}
$$

Antiderivative was successfully verified.

[In] mathics('Integrate[x^(5/2)*(2 + b*x)^(5/2),x]')

[Out] (-30 b ^ 6 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) ^ 2 + 30 b ^ (13 / 2) Sqrt[x] (2 + b x) ^ (3 / 2) + 5 b ^ (15 / 2) x ^ (3 / 2) (2 + b x) ^ (3 / 2) - b ^ (17 / 2) x ^ (5 / 2) (2 + b x) ^ (3 / 2) + 2 b ^ (19 / 2) x ^ (7 / 2) (55 + 67 b x + 28 b ^ 2 x ^ 2 + 4 b ^ 3 x ^ 3) (2 + b x) ^ (3 / 2)) / (48 b ^ (19 / 2) (2 + b x) ^ 2)

Maple [A]
time = 0.11, size = 147, normalized size = 1.02

method	result
meijerg	$-\frac{120\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(56b^5x^5+280b^4x^4+378b^3x^3+14x^2b^2-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{40320}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{192}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$
risch	$\frac{\left(8b^5x^5+40b^4x^4+54b^3x^3+2x^2b^2-5bx+15\right)\sqrt{x}\,\sqrt{bx+2}}{48b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\left(bx+2\right)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{7}{2}}}{6b}-\frac{5\left(\frac{x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}}{5b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x\,(bx+2)}\,\ln\left(\ldots\right)}{2\sqrt{bx+2}\ldots}}{4b}\right)}{5b}\right)}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/6/b*x^(5/2)*(b*x+2)^(7/2)-5/6/b*(1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/5/b*(1/4/b *x^(1/2)*(b*x+2)^(7/2)-1/4/b*(1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x ^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/2)+5/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2))))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(99) = 198.

time = 0.35, size = 223, normalized size = 1.55

$$\frac{\frac{15\sqrt{bx+2}\,b^5}{\sqrt{x}} - \frac{85\,(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{198\,(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}} + \frac{198\,(bx+2)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}} - \frac{85\,(bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}} + \frac{15\,(bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^9 - \frac{6\,(bx+2)b^8}{x} + \frac{15\,(bx+2)^2b^7}{x^2} - \frac{20\,(bx+2)^3b^6}{x^3} + \frac{15\,(bx+2)^4b^5}{x^4} - \frac{6\,(bx+2)^5b^4}{x^5} + \frac{(bx+2)^6b^3}{x^6}\right)} + \frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(b*x + 2)*b^5/sqrt(x) - 85*(b*x + 2)^(3/2)*b^4/x^(3/2) + 198*(
b*x + 2)^(5/2)*b^3/x^(5/2) + 198*(b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(b*x + 2)
^(9/2)*b/x^(9/2) + 15*(b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x + 2)*b^8/x +
 15*(b*x + 2)^2*b^7/x^2 - 20*(b*x + 2)^3*b^6/x^3 + 15*(b*x + 2)^4*b^5/x^4 -
 6*(b*x + 2)^5*b^4/x^5 + (b*x + 2)^6*b^3/x^6) + 5/16*log(-(sqrt(b) - sqrt(b
*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.31, size = 172, normalized size = 1.19

$$\left[\frac{\left(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{48\,b^4}, \frac{\left(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{48\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*s
qrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) +
 1))/b^4, 1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x
+ 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))))/b^4]
```

Sympy [A]
time = 176.88, size = 158, normalized size = 1.10

$$\frac{b^3x^{\frac{13}{2}}}{6\sqrt{bx+2}} + \frac{7b^2x^{\frac{11}{2}}}{6\sqrt{bx+2}} + \frac{67bx^{\frac{9}{2}}}{24\sqrt{bx+2}} + \frac{55x^{\frac{7}{2}}}{24\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{48b\sqrt{bx+2}} + \frac{5x^{\frac{3}{2}}}{48b^2\sqrt{bx+2}} + \frac{5\sqrt{x}}{8b^3\sqrt{bx+2}} - \frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(13/2)/(6*sqrt(b*x + 2)) + 7*b**2*x**(11/2)/(6*sqrt(b*x + 2)) + 67*
b*x**(9/2)/(24*sqrt(b*x + 2)) + 55*x**(7/2)/(24*sqrt(b*x + 2)) - x**(5/2)/(
48*b*sqrt(b*x + 2)) + 5*x**(3/2)/(48*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(8*b**
3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(99) = 198.
time = 0.02, size = 500, normalized size = 3.47

$$2b^{2}\left(2\left(\left(\left(\left(\left(\left(\frac{\frac{1}{5443200}\cdot 226800b^{10}\sqrt{x}\,\sqrt{x}}{b^{10}}+\frac{\frac{1}{5443200}\cdot 45360b^{9}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{5443200}\cdot 102060b^{8}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{5443200}\cdot 238140b^{7}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{5443200}\cdot 595350b^{6}}{b^{10}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{5443200}\cdot 1786050b^{5}}{b^{10}}\right)\sqrt{x}\,\sqrt{bx+2}+\frac{21\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{16b^{5}\sqrt{b}}\right)+8b\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{100800}\cdot 5040b^{8}\sqrt{x}\,\sqrt{x}}{b^{8}}+\frac{\frac{1}{100800}\cdot 1260b^{7}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 2940b^{6}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{100800}\cdot 7350b^{5}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 22050b^{4}}{b^{8}}\right)\sqrt{x}\,\sqrt{bx+2}-\frac{7\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^{4}\sqrt{b}}\right)+8\left(2\left(\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^{6}\sqrt{x}\,\sqrt{x}}{b^{6}}+\frac{\frac{1}{2880}\cdot 60b^{5}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^{4}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{2880}\cdot 450b^{3}}{b^{6}}\right)\sqrt{x}\,\sqrt{bx+2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^{3}\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/240*(((2*((4*(5*x + 1/b)*x - 9/b^2)*x + 21/b^3)*x - 105/b^4)*x + 315/b^5)
*sqrt(b*x + 2)*sqrt(x) + 630*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(11/2)
)*b^2 + 1/30*(((2*(3*(4*x + 1/b)*x - 7/b^2)*x + 35/b^3)*x - 105/b^4)*sqrt(b
*x + 2)*sqrt(x) - 210*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(9/2))*b + 1/
6*((2*(3*x + 1/b)*x - 5/b^2)*x + 15/b^3)*sqrt(b*x + 2)*sqrt(x) + 5*log(-sqr
t(b)*sqrt(x) + sqrt(b*x + 2))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(b\,x+2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(5/2), x)
```

3.558 $\int x^{3/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{3\sqrt{x}\,\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2+bx)^{5/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(b*x+2)^(3/2)+1/5*x^(5/2)*(b*x+2)^(5/2)+3/4*arcsinh(1/2*b^(1/2)
*x^(1/2)*2^(1/2))/b^(5/2)+1/8*x^(3/2)*(b*x+2)^(1/2)/b+1/4*x^(5/2)*(b*x+2)^(
1/2)-3/8*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]

time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2
))/5 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \int x^{3/2}(2+bx)^{3/2}\,dx \\
&= \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+b}}}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+ \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+ \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.68

$$
\frac{\sqrt{x}\sqrt{2+bx}\left(-15+5bx+62b^2x^2+42b^3x^3+8b^4x^4\right)}{40b^2} - \frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-15 + 5*b*x + 62*b^2*x^2 + 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(5/2))
```

Mathics [A]
time = 32.89, size = 97, normalized size = 0.79

$$
\frac{30b^3\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](2+bx)^{\frac{3}{2}} - 30b^{\frac{7}{2}}\sqrt{x}\,(2+bx) - 5b^{\frac{9}{2}}x^{\frac{3}{2}}(2+bx) + b^{\frac{11}{2}}x^{\frac{5}{2}}(2+bx)\left(129+146bx+58b^2x^2+8b^3x^3\right)}{40b^{\frac{11}{2}}(2+bx)^{\frac{3}{2}}}
$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)*(2 + b*x)^(5/2),x]')
```

```
[Out] (30 b ^ 3 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) ^ (3 / 2) - 30 b ^
 (7 / 2) Sqrt[x] (2 + b x) - 5 b ^ (9 / 2) x ^ (3 / 2) (2 + b x) + b ^ (11
/ 2) x ^ (5 / 2) (2 + b x) (129 + 146 b x + 58 b ^ 2 x ^ 2 + 8 b ^ 3 x ^ 3)
) / (40 b ^ (11 / 2) (2 + b x) ^ (3 / 2))
```

Maple [A]
time = 0.11, size = 126, normalized size = 1.02

method	result
meijerg	$-\frac{60\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-8b^4x^4-42b^3x^3-62x^2b^2-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{2400}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{80}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$
risch	$\frac{\left(8b^4x^4+42b^3x^3+62x^2b^2+5bx-15\right)\sqrt{x}\,\sqrt{bx+2}}{40b^2}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}}{5b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b}\right.}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}\right.}{5b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/5/b*(1/4/b*x^(1/2)*(b*x+2)^(7/2)-1/4/b*(1/3*(
b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/2)+5/
2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1
/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(84) = 168.
time = 0.35, size = 194, normalized size = 1.58

$$-\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70\,(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{128\,(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7-\frac{5\,(bx+2)b^6}{x}+\frac{10\,(bx+2)^2b^5}{x^2}-\frac{10\,(bx+2)^3b^4}{x^3}+\frac{5\,(bx+2)^4b^3}{x^4}-\frac{(bx+2)^5b^2}{x^5}\right)}-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) + 128*
(b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(
9/2)/x^(9/2))/(b^7 - 5*(b*x + 2)*b^6/x + 10*(b*x + 2)^2*b^5/x^2 - 10*(b*x +
 2)^3*b^4/x^3 + 5*(b*x + 2)^4*b^3/x^4 - (b*x + 2)^5*b^2/x^5) - 3/8*log(-(sq
rt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.31, size = 155, normalized size = 1.26

$$\left[\frac{\left(8\,b^{5}x^{4}+42\,b^{4}x^{3}+62\,b^{3}x^{2}+5\,b^{2}x-15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{40\,b^{3}}, \frac{\left(8\,b^{5}x^{4}+42\,b^{4}x^{3}+62\,b^{3}x^{2}+5\,b^{2}x-15\,b\right)\sqrt{bx+2}\,\sqrt{x}-30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)
*sqrt(x) + 15*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/
40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)*sq
rt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A]
time = 32.99, size = 138, normalized size = 1.12

$$\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{29b^{2}x^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{73bx^{\frac{7}{2}}}{20\sqrt{bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(11/2)/(5*sqrt(b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(b*x + 2)) + 73
*b*x**(7/2)/(20*sqrt(b*x + 2)) + 129*x**(5/2)/(40*sqrt(b*x + 2)) - x**(3/2)
/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*s
qrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(84) = 168.
time = 0.02, size = 427, normalized size = 3.47

$$2b^{2}\left(2\left(\left(\left(\left(\frac{\frac{1}{100800}\cdot5040b^{8}\sqrt{x}\,\sqrt{x}}{b^{8}}+\frac{\frac{1}{100800}\cdot1260b^{7}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot2940b^{6}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{100800}\cdot7350b^{5}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot22050b^{4}}{b^{8}}\right)\sqrt{x}\,\sqrt{bx+2}-\frac{7\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^{4}\sqrt{b}}\right)+8b\left(2\left(\left(\left(\frac{\frac{1}{2880}\cdot180b^{6}\sqrt{x}\,\sqrt{x}}{b^{6}}+\frac{\frac{1}{2880}\cdot60b^{5}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot150b^{4}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{2880}\cdot450b^{3}}{b^{6}}\right)\sqrt{x}\,\sqrt{bx+2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{8b^{3}\sqrt{b}}\right)+8\left(2\left(\left(\frac{\frac{1}{144}\cdot12b^{4}\sqrt{x}\,\sqrt{x}}{b^{4}}+\frac{\frac{1}{144}\cdot6b^{3}}{b^{4}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{144}\cdot18b^{2}}{b^{4}}\right)\sqrt{x}\,\sqrt{bx+2}-\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{2b^{2}\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/120*(((2*(3*(4*x + 1/b)*x - 7/b^2)*x + 35/b^3)*x - 105/b^4)*sqrt(b*x + 2)
*sqrt(x) - 210*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(9/2))*b^2 + 1/6*(((
2*(3*x + 1/b)*x - 5/b^2)*x + 15/b^3)*sqrt(b*x + 2)*sqrt(x) + 30*log(-sqrt(b
```

```
)*sqrt(x) + sqrt(b*x + 2))/b^(7/2))*b + 2/3*sqrt(b*x + 2)*((2*x + 1/b)*x -
3/b^2)*sqrt(x) - 4*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(b\,x+2)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(5/2), x)
```

3.559 $\int \sqrt{x}\,(2+bx)^{5/2}\,dx$

Optimal. Leaf size=102

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

```
[Out] 5/12*x^(3/2)*(b*x+2)^(3/2)+1/4*x^(3/2)*(b*x+2)^(5/2)-5/4*arcsinh(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(3/2)+5/8*x^(3/2)*(b*x+2)^(1/2)+5/8*x^(1/2)*(b*x+2)^(1
/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 + b*x])/8 + (5*x^(3/2)*
(2 + b*x)^(3/2))/12 + (x^(3/2)*(2 + b*x)^(5/2))/4 - (5*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(2+bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\,(2+bx)^{3/2}\,dx \\
&= \frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\,\sqrt{2+bx}\,dx \\
&= \frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx \\
&= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\int \cdots}{\cdots} \\
&= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\text{Subst}\cdots}{\cdots} \\
&= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\sinh^{-1}\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.75

$$\frac{\sqrt{x}\,\sqrt{2+bx}\,\left(15+59bx+34b^2x^2+6b^3x^3\right)}{24b}+\frac{5\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 + 59*b*x + 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5*
Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(3/2))
```

Mathics [A]
time = 9.94, size = 84, normalized size = 0.82

$$\frac{-30b\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](2+bx)+30b^{\frac{3}{2}}\sqrt{x}\,\sqrt{2+bx}+b^{\frac{5}{2}}x^{\frac{3}{2}}\left(133+127bx+46b^2x^2+6b^3x^3\right)\sqrt{2+bx}}{24b^{\frac{5}{2}}(2+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]')
```

```
[Out] (-30 b ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + 30 b ^ (3 / 2) Sqrt
[x] Sqrt[2 + b x] + b ^ (5 / 2) x ^ (3 / 2) (133 + 127 b x + 46 b ^ 2 x ^ 2
 + 6 b ^ 3 x ^ 3) Sqrt[2 + b x]) / (24 b ^ (5 / 2) (2 + b x))
```

Maple [A]
time = 0.11, size = 99, normalized size = 0.97

method	result
meijerg	$-\frac{30\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(6b^3x^3+34x^2b^2+59bx+15)\sqrt{\frac{bx}{2}+1}}{720}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{24}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$
risch	$\frac{(6b^3x^3+34x^2b^2+59bx+15)\sqrt{x}\,\sqrt{bx+2}}{24b}-\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{8b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4}+\frac{5x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{12}+\frac{5x^{\frac{3}{2}}\sqrt{bx+2}}{8}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{8b}-\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{8b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(b*x+2)^(5/2)+5/12*x^(3/2)*(b*x+2)^(3/2)+5/8*x^(3/2)*(b*x+2)^(1
/2)+5/8*x^(1/2)*(b*x+2)^(1/2)/b-5/8/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(69) = 138$.
time = 0.38, size = 161, normalized size = 1.58

$$\frac{\frac{15\sqrt{bx+2}\,b^3}{\sqrt{x}}-\frac{55(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{73(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^5-\frac{4(bx+2)b^4}{x}+\frac{6(bx+2)^2b^3}{x^2}-\frac{4(bx+2)^3b^2}{x^3}+\frac{(bx+2)^4b}{x^4}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) - 55*(b*x + 2)^(3/2)*b^2/x^(3/2) + 73*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x + 2)*b^
4/x + 6*(b*x + 2)^2*b^3/x^2 - 4*(b*x + 2)^3*b^2/x^3 + (b*x + 2)^4*b/x^4) +
5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(3/2)
```

Fricas [A]
time = 0.31, size = 140, normalized size = 1.37

$$\left[\frac{(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^2},\frac{(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 1
5*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/24*((6*b^4*x
^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arct
an(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A]
time = 8.46, size = 119, normalized size = 1.17

$$\frac{b^3x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{23b^2x^{\frac{7}{2}}}{12\sqrt{bx+2}}+\frac{127bx^{\frac{5}{2}}}{24\sqrt{bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] b**3*x**(9/2)/(4*sqrt(b*x + 2)) + 23*b**2*x**(7/2)/(12*sqrt(b*x + 2)) + 127
*b*x**(5/2)/(24*sqrt(b*x + 2)) + 133*x**(3/2)/(24*sqrt(b*x + 2)) + 5*sqrt(x
)/(4*b*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(69) = 138.
time = 4.59, size = 559, normalized size = 5.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/24*((((b*x + 2)*(2*(b*x + 2)*(3*(b*x + 2)/b^3 - 25/b^3) + 163/b^3) - 279/
b^3)*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2) - 210*log(abs(-sqrt(b*x + 2)*sqr
t(b) + sqrt((b*x + 2)*b - 2*b)))/b^(5/2))*b*abs(b) + 24*(sqrt((b*x + 2)*b -
 2*b)*sqrt(b*x + 2)*((b*x + 2)*(2*(b*x + 2)/b^2 - 13/b^2) + 33/b^2) + 30*lo
g(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/b^(3/2))*abs(b) +
144*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*(b*x - 3) - 6*sqrt(b)*log(abs(-s
qrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))))*abs(b)/b^2 + 192*(2*sqrt(
b)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b))) + sqrt((b*x +
 2)*b - 2*b)*sqrt(b*x + 2))*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(b\,x+2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(5/2),x)
[Out] int(x^(1/2)*(b*x + 2)^(5/2), x)
```

3.560 $\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=79

$$\frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 5*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+1/
3*(b*x+2)^(5/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\frac{1}{3}\sqrt{x}\,(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}\,(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*Sqrt[x]*(2 + b*x)^(3/2))/6 + (Sqrt[x]*(2 +
 b*x)^(5/2))/3 + (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{3}\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx \\
&= \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx \\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx \\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+5\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\surd\right. \\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 63, normalized size = 0.80

$$\frac{1}{6}\sqrt{x}\,\sqrt{2+bx}\,\left(33+13bx+2b^2x^2\right)-\frac{5\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(33 + 13*b*x + 2*b^2*x^2))/6 - (5*Log[-(Sqrt[b]*Sqrt
[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Mathics [**A**]
time = 5.30, size = 63, normalized size = 0.80

$$\frac{\sqrt{b}\,\sqrt{x}\,\left(66+59bx+17b^2x^2+2b^3x^3\right)+30\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]\sqrt{2+bx}}{6\sqrt{b}\,\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]')
```

```
[Out] (Sqrt[b] Sqrt[x] (66 + 59 b x + 17 b ^ 2 x ^ 2 + 2 b ^ 3 x ^ 3) + 30 ArcSin
h[Sqrt[2] Sqrt[b] Sqrt[x] / 2] Sqrt[2 + b x]) / (6 Sqrt[b] Sqrt[2 + b x])
```

Maple [**A**]
time = 0.11, size = 84, normalized size = 1.06

method	result	size
meijerg	$-\frac{15\left(-\frac{8\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\left(\frac{1}{24}x^2b^2+\frac{13}{48}bx+\frac{11}{16}\right)\sqrt{\frac{bx}{2}+1}}{15}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3}\right)}{\sqrt{b}\,\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2+13bx+33)\sqrt{x}\,\sqrt{bx+2}}{6}+\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	74
default	$\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/
2)+5/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*
x)^(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(54) = 108$.
time = 0.35, size = 129, normalized size = 1.63

$$-\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\sqrt{b}}-\frac{\frac{15\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{40(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^3-\frac{3(bx+2)b^2}{x}+\frac{3(bx+2)^2b}{x^2}-\frac{(bx+2)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - 1/3*(15*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3
/2) + 33*(b*x + 2)^(5/2)/x^(5/2))/(b^3 - 3*(b*x + 2)*b^2/x + 3*(b*x + 2)^2*
b/x^2 - (b*x + 2)^3/x^3)
```

Fricas [A]
time = 0.33, size = 123, normalized size = 1.56

$$\left[\frac{(2b^3x^2+13b^2x+33b)\sqrt{bx+2}\,\sqrt{x}+15\sqrt{b}\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6b},\ \frac{(2b^3x^2+13b^2x+33b)\sqrt{bx+2}\,\sqrt{x}-30\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 13*b^2*x + 33*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(
b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 + 13*b^2*x + 3
3*b)*sqrt(b*x + 2)*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b]
```

Sympy [A]
time = 3.73, size = 97, normalized size = 1.23

$$\frac{b^3x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{17b^2x^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{59bx^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{11\sqrt{x}}{\sqrt{bx+2}}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b**3*x**(7/2)/(3*sqrt(b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(b*x + 2)) + 59*b
*x**(3/2)/(6*sqrt(b*x + 2)) + 11*sqrt(x)/sqrt(b*x + 2) + 5*asinh(sqrt(2)*sq
rt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [A]
time = 1.15, size = 141, normalized size = 1.78

$$\frac{b^2\left(2\left(\left(\left(\frac{\frac{1}{36}\cdot 6\sqrt{bx+2}\sqrt{bx+2}}{b}+\frac{\frac{1}{36}\cdot 15}{b}\right)\sqrt{bx+2}\sqrt{bx+2}+\frac{\frac{1}{36}\cdot 45}{b}\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}-\frac{5\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{\sqrt{b}}\right)\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/6*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)*(2*(b*x + 2)/b + 5/b)
 + 15/b) - 30*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sq
rt(b))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x+2\right)}^{5/2}}{\sqrt{x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(1/2), x)
```

3.561 $\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 15*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(b*x+2)^(5/2)/x^(1/2)+5/2
*b*(b*x+2)^(3/2)*x^(1/2)+15/2*b*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\,\sqrt{bx+2}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*b*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*b*Sqrt[x]*(2 + b*x)^(3/2))/2 - (2*(2 +
b*x)^(5/2))/Sqrt[x] + 15*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{\sqrt{x}}+(5b)\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+\frac{1}{2}(15b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\
&= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+\frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\
&= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+(15b)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x\right.\\
&= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 0.78

$$\frac{\sqrt{2+bx}\,\left(-16+9bx+b^2x^2\right)}{2\sqrt{x}}-15\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(3/2),x]
[Out] (Sqrt[2 + b*x]*(-16 + 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[b]*Log[-(Sqrt
[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

Mathics [A]
time = 5.26, size = 70, normalized size = 0.89

$$\frac{30\sqrt{b}\,\sqrt{x}\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](2+bx)+bx\left(2+11bx+b^2x^2\right)\sqrt{2+bx}-32\sqrt{2+bx}}{2\sqrt{x}\,(2+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(2 + b*x)^(5/2)/x^(3/2),x]')
```

```
[Out] (30 Sqrt[b] Sqrt[x] ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + b x (2
 + 11 b x + b ^ 2 x ^ 2) Sqrt[2 + b x] - 32 Sqrt[2 + b x]) / (2 Sqrt[x] (2
+ b x))
```

Maple [A]
time = 0.11, size = 63, normalized size = 0.80

method	result	size
meijerg	$-\frac{15\sqrt{b}\left(\frac{16\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{1}{16}x^2b^2-\frac{9}{16}bx+1\right)\sqrt{\frac{bx}{2}+1}}{15\sqrt{x}\,\sqrt{b}}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	63
risch	$\frac{b^3x^3+11x^2b^2+2bx-32}{2\sqrt{x}\,\sqrt{bx+2}}+\frac{15\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2\sqrt{x}\,\sqrt{bx+2}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/2*b^(1/2)/Pi^(1/2)*(16/15*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(-1/16*x^2*b
^2-9/16*b*x+1)*(1/2*b*x+1)^(1/2)-2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(
1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(56) = 112.
time = 0.35, size = 113, normalized size = 1.43

$$-\frac{15}{2}\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{\frac{7\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{9\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2-\frac{2\,(bx+2)b}{x}+\frac{(bx+2)^2}{x^2}}-\frac{8\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x +
2)/sqrt(x))) - (7*sqrt(b*x + 2)*b^2/sqrt(x) - 9*(b*x + 2)^(3/2)*b/x^(3/2))/
(b^2 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2) - 8*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.32, size = 116, normalized size = 1.47

$$\left[\frac{15\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(b^2x^2+9\,bx-16)\sqrt{bx+2}\,\sqrt{x}}{2\,x},-\frac{30\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-(b^2x^2+9\,bx-16)\sqrt{bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x^2
+ 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/x, -1/2*(30*sqrt(-b)*x*arctan(sqrt(b*x
 + 2)*sqrt(-b)/(b*sqrt(x))) - (b^2*x^2 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))
/x]
```

Sympy [A]
time = 3.75, size = 94, normalized size = 1.19

$$15\sqrt{b}\ \operatorname{asinh}\left(\frac{\sqrt{2}\ \sqrt{b}\ \sqrt{x}}{2}\right)+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{b\sqrt{x}}{\sqrt{bx+2}}-\frac{16}{\sqrt{x}\ \sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] 15*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**3*x**(5/2)/(2*sqrt(b*x + 2
)) + 11*b**2*x**(3/2)/(2*sqrt(b*x + 2)) + b*sqrt(x)/sqrt(b*x + 2) - 16/(sqr
t(x)*sqrt(b*x + 2))
```

Giac [A]
time = 1.22, size = 145, normalized size = 1.84

$$\frac{bb^2\left(\frac{2\left(\left(\frac{1}{4}\sqrt{bx+2}\ \sqrt{bx+2}+\frac{5}{4}\right)\sqrt{bx+2}\ \sqrt{bx+2}-\frac{15}{2}\right)\sqrt{bx+2}\ \sqrt{b\left(bx+2\right)-2b}}{b(bx+2)-2b}-\frac{15\ln\left|\sqrt{b\left(bx+2\right)-2b}-\sqrt{b}\ \sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] 1/2*(((b*x + 7)*(b*x + 2) - 30)*sqrt(b*x + 2)/sqrt((b*x + 2)*b - 2*b) - 30*
log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))/sqrt(b))*b^2/abs
(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x+2\right)}^{5/2}}{x^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(3/2), x)
```

3.562 $\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=81

$$5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(b*x+2)^(5/2)/x^(3/2)+10*b^(3/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))-
10/3*b*(b*x+2)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\,\sqrt{bx+2}-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 + b*x] - (10*b*(2 + b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 +
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx\\
&= -\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\right.\\
&= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.78

$$\frac{\sqrt{2+bx}\,\left(-8-28bx+3b^2x^2\right)}{3x^{3/2}}-10b^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(5/2),x]
[Out] (Sqrt[2 + b*x]*(-8 - 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 10*b^(3/2)*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

Mathics [A]
time = 5.29, size = 92, normalized size = 1.14

$$\frac{\sqrt{b}\left(bx\left(3bx\sqrt{\frac{2+bx}{bx}}-28\sqrt{\frac{2+bx}{bx}}-15\text{Log}\left[\frac{1}{bx}\right]+30\text{Log}\left[1+\sqrt{\frac{2+bx}{bx}}\right]\right)-8\sqrt{\frac{2+bx}{bx}}\right)}{3x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 + b*x)^(5/2)/x^(5/2),x]')
```

```
[Out] Sqrt[b] (b x (3 b x Sqrt[(2 + b x) / (b x)] - 28 Sqrt[(2 + b x) / (b x)] -
15 Log[1 / (b x)] + 30 Log[1 + Sqrt[(2 + b x) / (b x)]]) - 8 Sqrt[(2 + b x)
 / (b x)]) / (3 x)
```

Maple [A]
time = 0.14, size = 63, normalized size = 0.78

method	result	size
meijerg	$-\frac{15 b^{\frac{3}{2}} \left(\frac{32 \sqrt{\pi}\, \sqrt{2}\, \left(-\frac{3}{8} x^{2} b^{2}+\frac{7}{2} b x+1\right) \sqrt{\frac{b x}{2}+1}}{45 x^{\frac{3}{2}} b^{\frac{3}{2}}}-\frac{8 \sqrt{\pi}\, \mathrm{arcsinh}\left(\frac{\sqrt{b}\, \sqrt{x}\, \sqrt{2}}{2}\right)}{3}\right)}{4 \sqrt{\pi}}$	63
risch	$\frac{3 b^{3} x^{3}-22 x^{2} b^{2}-64 b x-16}{3 x^{\frac{3}{2}} \sqrt{b x+2}}+\frac{5 b^{\frac{3}{2}} \ln\left(\frac{b x+1}{\sqrt{b}}+\sqrt{x^{2} b+2 x}\right) \sqrt{x \left(b x+2\right)}}{\sqrt{x}\, \sqrt{b x+2}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/4*b^(3/2)/Pi^(1/2)*(32/45*Pi^(1/2)/x^(3/2)*2^(1/2)/b^(3/2)*(-3/8*x^2*b^
2+7/2*b*x+1)*(1/2*b*x+1)^(1/2)-8/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(
1/2)))
```

Maxima [A]
time = 0.35, size = 96, normalized size = 1.19

$$-5\, b^{\frac{3}{2}} \log\left(-\frac{\sqrt{b} - \frac{\sqrt{b x + 2}}{\sqrt{x}}}{\sqrt{b} + \frac{\sqrt{b x + 2}}{\sqrt{x}}}\right) - \frac{8 \sqrt{b x + 2}\, b}{\sqrt{x}} - \frac{2 \sqrt{b x + 2}\, b^{2}}{\left(b - \frac{b x + 2}{x}\right) \sqrt{x}} - \frac{4 \left(b x + 2\right)^{\frac{3}{2}}}{3 \, x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 8*sqrt(b*x + 2)*b/sqrt(x) - 2*sqrt(b*x + 2)*b^2/((b - (b*x + 2)
/x)*sqrt(x)) - 4/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 123, normalized size = 1.52

$$\left[\frac{15 \, b^{\frac{3}{2}} x^{2} \log\left(b x + \sqrt{b x + 2}\, \sqrt{b}\, \sqrt{x} + 1\right) + \left(3 \, b^{2} x^{2} - 28 \, b x - 8\right) \sqrt{b x + 2}\, \sqrt{x}}{3 \, x^{2}}, -\frac{30 \, \sqrt{-b}\, b x^{2} \arctan\left(\frac{\sqrt{b x + 2}\, \sqrt{-b}}{b \sqrt{x}}\right) - \left(3 \, b^{2} x^{2} - 28 \, b x - 8\right) \sqrt{b x + 2}\, \sqrt{x}}{3 \, x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (3*b^2*
x^2 - 28*b*x - 8)*sqrt(b*x + 2)*sqrt(x))/x^2, -1/3*(30*sqrt(-b)*b*x^2*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 28*b*x - 8)*sqrt(b*x +
 2)*sqrt(x))/x^2]
```

Sympy [A]
time = 3.48, size = 88, normalized size = 1.09

$$b^{\frac{5}{2}}x\sqrt{1+\frac{2}{bx}}-\frac{28b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-5b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+10b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{8\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] b**(5/2)*x*sqrt(1 + 2/(b*x)) - 28*b**(3/2)*sqrt(1 + 2/(b*x))/3 - 5*b**(3/2)
*log(1/(b*x)) + 10*b**(3/2)*log(sqrt(1 + 2/(b*x)) + 1) - 8*sqrt(b)*sqrt(1 +
 2/(b*x))/(3*x)
```

Giac [A]
time = 1.23, size = 169, normalized size = 2.09

$$\frac{b^2\left(\frac{2\left(\left(\frac{\frac{1}{18}\cdot 9b^4\sqrt{bx+2}\sqrt{bx+2}}{b}-\frac{\frac{1}{18}\cdot 120b^4}{b}\right)\sqrt{bx+2}\sqrt{bx+2}+\frac{\frac{1}{18}\cdot 180b^4}{b}\right)\sqrt{bx+2}\sqrt{b(bx+2)-2b}}{(b(bx+2)-2b)^2}-\frac{10b^2\ln\left|\sqrt{b(bx+2)-2b}-\sqrt{b}\sqrt{bx+2}\right|}{\sqrt{b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(30*b^(3/2)*log(abs(-sqrt(b*x + 2)*sqrt(b) + sqrt((b*x + 2)*b - 2*b)))
 - (60*b^3 + (3*(b*x + 2)*b^3 - 40*b^3)*(b*x + 2))*sqrt(b*x + 2)/((b*x + 2)
*b - 2*b)^(3/2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(5/2), x)
```

3.563 $\int x^{5/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5\sin\ldots}{\ldots}$$

```
[Out] 1/6*x^(7/2)*(-b*x+2)^(3/2)+1/6*x^(7/2)*(-b*x+2)^(5/2)+5/8*arcsin(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(7/2)-5/48*x^(3/2)*(-b*x+2)^(1/2)/b^2-1/24*x^(5/2)*(-b
*x+2)^(1/2)/b+1/8*x^(7/2)*(-b*x+2)^(1/2)-5/16*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]

time = 0.03, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(48*b^2) -
(x^(5/2)*Sqrt[2 - b*x])/(24*b) + (x^(7/2)*Sqrt[2 - b*x])/8 + (x^(7/2)*(2 -
b*x)^(3/2))/6 + (x^(7/2)*(2 - b*x)^(5/2))/6 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}(2-bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5}{6}\int x^{5/2}(2-bx)^{3/2}\,dx\\
&= \frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{2}\int x^{5/2}\sqrt{2-bx}\,dx\\
&= \frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{8}\int\frac{x^{5/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5\int\frac{x^{3/}}{\sqrt{2-}}}{24}\\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-b\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-b\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-b
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 98, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-15-5bx-2b^2x^2+54b^3x^3-40b^4x^4+8b^5x^5)}{48b^3}+\frac{5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{8(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 54*b^3*x^3 - 40*b^4*x^4 +
 8*b^5*x^5))/(48*b^3) + (5*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(8*(-b
)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 170.18, size = 253, normalized size = 1.69

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-30b^6\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](-2+bx)^2+30b^{\frac{13}{2}}\sqrt{x}(-2+bx)^{\frac{3}{2}}-5b^{\frac{15}{2}}x^{\frac{3}{2}}(-2+bx)^{\frac{3}{2}}-b^{\frac{17}{2}}x^{\frac{5}{2}}(-2+bx)^{\frac{3}{2}}+2b^{\frac{19}{2}}x^{\frac{7}{2}}(-55+67bx-28b^2x^2+4b^3x^3)(-2+bx)^{\frac{3}{2}}\right)}{48b^{\frac{19}{2}}(-2+bx)^2},\text{Abs}[bx]>2\right\}\right\},\frac{5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{8b^{\frac{7}{2}}}-\frac{5\sqrt{x}}{8b^3\sqrt{2-bx}}+\frac{5x^{\frac{3}{2}}}{48b^2\sqrt{2-bx}}+\frac{x^{\frac{5}{2}}}{48b\sqrt{2-bx}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{2-bx}}-\frac{67bx^{\frac{9}{2}}}{24\sqrt{2-bx}}+\frac{7b^2x^{\frac{11}{2}}}{6\sqrt{2-bx}}-\frac{b^3x^{\frac{13}{2}}}{6\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)*(2 - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 48 (-30 b ^ 6 ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (-2 + b
x) ^ 2 + 30 b ^ (13 / 2) Sqrt[x] (-2 + b x) ^ (3 / 2) - 5 b ^ (15 / 2) x ^
(3 / 2) (-2 + b x) ^ (3 / 2) - b ^ (17 / 2) x ^ (5 / 2) (-2 + b x) ^ (3 / 2
) + 2 b ^ (19 / 2) x ^ (7 / 2) (-55 + 67 b x - 28 b ^ 2 x ^ 2 + 4 b ^ 3 x ^
 3) (-2 + b x) ^ (3 / 2)) / (b ^ (19 / 2) (-2 + b x) ^ 2), Abs[b x] > 2}},
5 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / (8 b ^ (7 / 2)) - 5 Sqrt[x] / (8 b
^ 3 Sqrt[2 - b x]) + 5 x ^ (3 / 2) / (48 b ^ 2 Sqrt[2 - b x]) + x ^ (5 / 2)
 / (48 b Sqrt[2 - b x]) + 55 x ^ (7 / 2) / (24 Sqrt[2 - b x]) - 67 b x ^ (9
 / 2) / (24 Sqrt[2 - b x]) + 7 b ^ 2 x ^ (11 / 2) / (6 Sqrt[2 - b x]) - b ^
 3 x ^ (13 / 2) / (6 Sqrt[2 - b x])]
```

Maple [A]

time = 0.13, size = 157, normalized size = 1.05

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(-56b^5x^5+280b^4x^4-378b^3x^3+14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{336b^3}-\frac{5\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{8b^{\frac{7}{2}}}}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{\left(8b^5x^5-40b^4x^4+54b^3x^3-2x^2b^2-5bx-15\right)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{48b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2 \cdots}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+2)^{\frac{7}{2}}}{6b}+\frac{-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{7}{2}}}{6b}+\frac{5\left(-\frac{3\sqrt{x}\,(-bx+2)^{\frac{7}{2}}}{20b}+\frac{3\left(\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{5\sqrt{(\cdots}}{\cdots}\right)}{20b}\cdots\right)}{6b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+2)^(7/2)+5/6/b*(-1/5/b*x^(3/2)*(-b*x+2)^(7/2)+3/5/b*(-
1/4/b*x^(1/2)*(-b*x+2)^(7/2)+1/4/b*(1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)
^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(
1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))))
```

Maxima [A]
time = 0.37, size = 209, normalized size = 1.39

$$\frac{\frac{15\sqrt{-bx+2}\,b^5}{\sqrt{x}}+\frac{85\,(-bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{198\,(-bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}-\frac{198\,(-bx+2)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}}-\frac{85\,(-bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}}-\frac{15\,(-bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^9-\frac{6\,(bx-2)b^8}{x}+\frac{15\,(bx-2)^2b^7}{x^2}-\frac{20\,(bx-2)^3b^6}{x^3}+\frac{15\,(bx-2)^4b^5}{x^4}-\frac{6\,(bx-2)^5b^4}{x^5}+\frac{(bx-2)^6b^3}{x^6}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(-b*x + 2)*b^5/sqrt(x) + 85*(-b*x + 2)^(3/2)*b^4/x^(3/2) + 198
*(-b*x + 2)^(5/2)*b^3/x^(5/2) - 198*(-b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(-b*x
 + 2)^(9/2)*b/x^(9/2) - 15*(-b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x - 2)*b
^8/x + 15*(b*x - 2)^2*b^7/x^2 - 20*(b*x - 2)^3*b^6/x^3 + 15*(b*x - 2)^4*b^5
/x^4 - 6*(b*x - 2)^5*b^4/x^5 + (b*x - 2)^6*b^3/x^6) - 5/8*arctan(sqrt(-b*x
+ 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.33, size = 173, normalized size = 1.15

$$\left[\frac{(8\,b^6x^5-40\,b^5x^4+54\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{48\,b^4},\ \frac{(8\,b^6x^5-40\,b^5x^4+54\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{48\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*s
qrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt
(x) + 1))/b^4, 1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b
^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))))/b^4]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(105) = 210.
time = 0.02, size = 522, normalized size = 3.48

$$2b^2\left(2\left(\left(\left(\left(\left(\frac{\frac{1}{5443200}\cdot 226800b^{10}\sqrt{x}\,\sqrt{x}}{b^{10}}-\frac{\frac{1}{5443200}\cdot 45360b^9}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{5443200}\cdot 102060b^8}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{5443200}\cdot 238140b^7}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{5443200}\cdot 595350b^6}{b^{10}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{5443200}\cdot 1786050b^5}{b^{10}}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{21\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{16b^5\sqrt{-b}}\right)-8b\left(2\left(\left(\left(\left(\frac{\frac{1}{100800}\cdot 5040b^8\sqrt{x}\,\sqrt{x}}{b^8}-\frac{\frac{1}{100800}\cdot 1260b^7}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 2940b^6}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 7350b^5}{b^8}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 22050b^4}{b^8}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{7\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^4\sqrt{-b}}\right)+8\left(2\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^6\sqrt{x}\,\sqrt{x}}{b^6}-\frac{\frac{1}{2880}\cdot 60b^5}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^4}{b^6}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 450b^3}{b^6}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{5\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x)
```

```
[Out] 1/240*(((2*((4*(5*x - 1/b)*x - 9/b^2)*x - 21/b^3)*x - 105/b^4)*x - 315/b^5)
*sqrt(-b*x + 2)*sqrt(x) - 630*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt
(-b)*b^5))*b^2 - 1/30*(((2*(3*(4*x - 1/b)*x - 7/b^2)*x - 35/b^3)*x - 105/b^
4)*sqrt(-b*x + 2)*sqrt(x) - 210*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sq
rt(-b)*b^4))*b + 1/6*((2*(3*x - 1/b)*x - 5/b^2)*x - 15/b^3)*sqrt(-b*x + 2)*
sqrt(x) - 5*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2} (2 - b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(5/2), x)
```

3.564 $\int x^{3/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(-b*x+2)^(3/2)+1/5*x^(5/2)*(-b*x+2)^(5/2)+3/4*arcsin(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(5/2)-1/8*x^(3/2)*(-b*x+2)^(1/2)/b+1/4*x^(5/2)*(-b*x+2
)^(1/2)-3/8*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (x^(5/2)*(2 - b*x)^(5/2
))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2-bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2-bx)^{5/2}+\int x^{3/2}(2-bx)^{3/2}\,dx\\
&= \frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx\\
&= \frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{3\int\frac{\sqrt{}}{\sqrt{2}}}{8}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-
\end{aligned}
$$

Mathematica [**A**]
time = 0.13, size = 90, normalized size = 0.70

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-15-5bx+62b^2x^2-42b^3x^3+8b^4x^4)}{40b^2}-\frac{3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x + 62*b^2*x^2 - 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) - (3*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(5/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(2 - b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.12, size = 135, normalized size = 1.05

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(-8b^4x^4+42b^3x^3-62x^2b^2+5bx+15\right)\sqrt{-\frac{bx}{2}+1}}{40b^2}-\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{5}{2}}}}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{(8b^4x^4-42b^3x^3+62x^2b^2-5bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{40b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{7}{2}}}{5b}+\frac{-\frac{3\sqrt{x}\,(-bx+2)^{\frac{7}{2}}}{20b}+\frac{3\left(\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{5\sqrt{(-bx+2)\,x}\,\text{arcta}}{2\sqrt{-bx+2}}\right.}{20b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+2)^(7/2)+3/5/b*(-1/4/b*x^(1/2)*(-b*x+2)^(7/2)+1/4/b*(1
/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^(
1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(
x-1/b)/(-b*x^2+2*x)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(89) = 178.
time = 0.34, size = 179, normalized size = 1.40

$$\frac{\frac{15\sqrt{-bx+2}\,b^4}{\sqrt{x}}+\frac{70\,(-bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{128\,(-bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}-\frac{70\,(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7-\frac{5\,(bx-2)b^6}{x}+\frac{10\,(bx-2)^2b^5}{x^2}-\frac{10\,(bx-2)^3b^4}{x^3}+\frac{5\,(bx-2)^4b^3}{x^4}-\frac{(bx-2)^5b^2}{x^5}\right)}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) + 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^7 - 5*(b*x - 2)*b^6/x + 10*(b*x - 2)^2*b^5/x^2 - 10*(b
*x - 2)^3*b^4/x^3 + 5*(b*x - 2)^4*b^3/x^4 - (b*x - 2)^5*b^2/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.33, size = 157, normalized size = 1.23

$$\left[\frac{\left(8\,b^{5}x^{4}-42\,b^{4}x^{3}+62\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{40\,b^{3}}, \frac{\left(8\,b^{5}x^{4}-42\,b^{4}x^{3}+62\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{40\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2
)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
3, 1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x +
 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [A]
time = 32.95, size = 292, normalized size = 2.28

$$\begin{cases} \frac{ib^{3}x^{\frac{11}{2}}}{5\sqrt{bx-2}}-\frac{29ib^{2}x^{\frac{9}{2}}}{20\sqrt{bx-2}}+\frac{73ibx^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{129ix^{\frac{5}{2}}}{40\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^{2}\sqrt{bx-2}}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for}\ \left|bx\right|>2 \\ -\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{-bx+2}}+\frac{29b^{2}x^{\frac{9}{2}}}{20\sqrt{-bx+2}}-\frac{73bx^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(11/2)/(5*sqrt(b*x - 2)) - 29*I*b**2*x**(9/2)/(20*sqrt
(b*x - 2)) + 73*I*b*x**(7/2)/(20*sqrt(b*x - 2)) - 129*I*x**(5/2)/(40*sqrt(b
*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**2*sqrt(b*x -
2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), Abs(b*x) > 2), (-b
**3*x**(11/2)/(5*sqrt(-b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(-b*x + 2)) - 7
3*b*x**(7/2)/(20*sqrt(-b*x + 2)) + 129*x**(5/2)/(40*sqrt(-b*x + 2)) + x**(3
/2)/(8*b*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(89) = 178.
time = 0.02, size = 446, normalized size = 3.48

$$2b^{2}\left(2\left(\left(\left(\left(\frac{\frac{1}{100800}\cdot 5040b^{8}\sqrt{x}\,\sqrt{x}}{b^{8}}-\frac{\frac{1}{100800}\cdot 1260b^{7}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 2940b^{6}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 7350b^{5}}{b^{8}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{100800}\cdot 22050b^{4}}{b^{8}}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{7\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^{4}\sqrt{-b}}\right)-8b\left(2\left(\left(\left(\frac{\frac{1}{2880}\cdot 180b^{6}\sqrt{x}\,\sqrt{x}}{b^{6}}-\frac{\frac{1}{2880}\cdot 60b^{5}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 150b^{4}}{b^{6}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{2880}\cdot 450b^{3}}{b^{6}}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{5\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{8b^{3}\sqrt{-b}}\right)+8\left(2\left(\left(\frac{\frac{1}{144}\cdot 12b^{4}\sqrt{x}\,\sqrt{x}}{b^{4}}-\frac{\frac{1}{144}\cdot 6b^{3}}{b^{4}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{144}\cdot 18b^{2}}{b^{4}}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{2b^{2}\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x)
```

```
[Out] 1/120*(((2*(3*(4*x - 1/b)*x - 7/b^2)*x - 35/b^3)*x - 105/b^4)*sqrt(-b*x + 2
)*sqrt(x) - 210*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^4))*b^2
```

```
 - 1/6*(((2*(3*x - 1/b)*x - 5/b^2)*x - 15/b^3)*sqrt(-b*x + 2)*sqrt(x) - 30*
log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3))*b + 2/3*sqrt(-b*x +
 2)*((2*x - 1/b)*x - 3/b^2)*sqrt(x) - 4*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x +
 2))/(sqrt(-b)*b^2)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(5/2), x)
```

3.565 $\int \sqrt{x}\,(2-bx)^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

```
[Out] 5/12*x^(3/2)*(-b*x+2)^(3/2)+1/4*x^(3/2)*(-b*x+2)^(5/2)+5/4*arcsin(1/2*b^(1/
2)*x^(1/2)*2^(1/2))/b^(3/2)+5/8*x^(3/2)*(-b*x+2)^(1/2)-5/8*x^(1/2)*(-b*x+2)
^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 - b*x])/8 + (5*x^(3/2)
*(2 - b*x)^(3/2))/12 + (x^(3/2)*(2 - b*x)^(5/2))/4 + (5*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}(2-bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{4}\int \sqrt{x}(2-bx)^{3/2}\,dx$$
$$=\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\sqrt{2-bx}\,dx$$
$$=\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$$
$$=-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\int \frac{}{\sqrt{}}}{}$$
$$=-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\text{Subs}}{}$$
$$=-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\sin^{-}}{}$$

Mathematica [**A**]
time = 0.11, size = 82, normalized size = 0.77

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-15+59bx-34b^2x^2+6b^3x^3)}{24b}+\frac{5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 + 59*b*x - 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(3/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]*(2 - b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.13, size = 107, normalized size = 1.01

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}\left(-6b^3x^3+34x^2b^2-59bx+15\right)\sqrt{-\frac{bx}{2}+1}}{24b}-\frac{5\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{3}{2}}}}{\sqrt{-b}\,\sqrt{\pi}\,b}$
default	$\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}}{4}+\frac{5x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{12}+\frac{5x^{\frac{3}{2}}\sqrt{-bx+2}}{8}-\frac{5\sqrt{x}\,\sqrt{-bx+2}}{8b}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,(}{\sqrt{-x^2b}}\right.}{8b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$
risch	$-\frac{(6b^3x^3-34x^2b^2+59bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{24b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{8b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(-b*x+2)^(5/2)+5/12*x^(3/2)*(-b*x+2)^(3/2)+5/8*x^(3/2)*(-b*x+2)
^(1/2)-5/8*x^(1/2)*(-b*x+2)^(1/2)/b+5/8/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)
^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]

time = 0.37, size = 145, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{55\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{73\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^5-\frac{4\,(bx-2)b^4}{x}+\frac{6\,(bx-2)^2b^3}{x^2}-\frac{4\,(bx-2)^3b^2}{x^3}+\frac{(bx-2)^4b}{x^4}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) + 55*(-b*x + 2)^(3/2)*b^2/x^(3/2) + 73*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x - 2
)*b^4/x + 6*(b*x - 2)^2*b^3/x^2 - 4*(b*x - 2)^3*b^2/x^3 + (b*x - 2)^4*b/x^4
) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

Fricas [A]

time = 0.32, size = 141, normalized size = 1.33

$$\left[\frac{(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{24\,b^2},\ \frac{(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) -
15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/24*((6*
b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)
*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [A]
time = 8.22, size = 253, normalized size = 2.39

$$\begin{cases} \frac{ib^{3}x^{\frac{9}{2}}}{4\sqrt{bx-2}} - \frac{23ib^{2}x^{\frac{7}{2}}}{12\sqrt{bx-2}} + \frac{127ibx^{\frac{5}{2}}}{24\sqrt{bx-2}} - \frac{133ix^{\frac{3}{2}}}{24\sqrt{bx-2}} + \frac{5i\sqrt{x}}{4b\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{for}\ |bx| > 2 \\ -\frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{-bx+2}} + \frac{23b^{2}x^{\frac{7}{2}}}{12\sqrt{-bx+2}} - \frac{127bx^{\frac{5}{2}}}{24\sqrt{-bx+2}} + \frac{133x^{\frac{3}{2}}}{24\sqrt{-bx+2}} - \frac{5\sqrt{x}}{4b\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(9/2)/(4*sqrt(b*x - 2)) - 23*I*b**2*x**(7/2)/(12*sqrt(
b*x - 2)) + 127*I*b*x**(5/2)/(24*sqrt(b*x - 2)) - 133*I*x**(3/2)/(24*sqrt(b
*x - 2)) + 5*I*sqrt(x)/(4*b*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/(4*b**(3/2)), Abs(b*x) > 2), (-b**3*x**(9/2)/(4*sqrt(-b*x + 2)) + 23
*b**2*x**(7/2)/(12*sqrt(-b*x + 2)) - 127*b*x**(5/2)/(24*sqrt(-b*x + 2)) + 1
33*x**(3/2)/(24*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b*sqrt(-b*x + 2)) + 5*asin(s
qrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(73) = 146.
time = 4.78, size = 599, normalized size = 5.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/24*((((b*x - 2)*(2*(b*x - 2)*(3*(b*x - 2)/b^3 + 25/b^3) + 163/b^3) + 279/
b^3)*sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2) - 210*log(abs(-sqrt(-b*x + 2)*s
qrt(-b) + sqrt((b*x - 2)*b + 2*b)))/(sqrt(-b)*b^2))*b*abs(b) - 24*(sqrt((b*
x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)*(2*(b*x - 2)/b^2 + 13/b^2) + 33/b
^2) - 30*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/(sqrt
(-b)*b))*abs(b) + 144*(sqrt((b*x - 2)*b + 2*b)*(b*x + 3)*sqrt(-b*x + 2) - 6
*b*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqrt(-b))*a
bs(b)/b^2 + 192*(2*b*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b +
2*b)))/sqrt(-b) - sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2))*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(2-b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(5/2),x)
[Out] int(x^(1/2)*(2 - b*x)^(5/2), x)
```

3.566 $\int \frac{(2-bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=82

$$\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 5*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+1/
3*(-b*x+2)^(5/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 - b*x])/2 + (5*Sqrt[x]*(2 - b*x)^(3/2))/6 + (Sqrt[x]*(2 -
 b*x)^(5/2))/3 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2-bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{3}\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+5\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\right.\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.84

$$
\frac{1}{6}\sqrt{x}\,\sqrt{2-bx}\,\left(33-13bx+2b^2x^2\right)-\frac{5\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(33 - 13*b*x + 2*b^2*x^2))/6 - (5*Log[-(Sqrt[-b]*Sqr
t[x]) + Sqrt[2 - b*x]])/Sqrt[-b]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.39, size = 149, normalized size = 1.82

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(\sqrt{b}\,\sqrt{x}\,\left(-66+59bx-17b^2x^2+2b^3x^3\right)-30\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]\sqrt{-2+bx}\right)}{6\sqrt{b}\,\sqrt{-2+bx}},\text{Abs}[bx]>2\right\}\right\},\frac{5\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{\sqrt{b}}+\frac{11\sqrt{x}}{\sqrt{2-bx}}-\frac{59bx^{\frac{3}{2}}}{6\sqrt{2-bx}}+\frac{17b^2x^{\frac{5}{2}}}{6\sqrt{2-bx}}-\frac{b^3x^{\frac{7}{2}}}{3\sqrt{2-bx}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - b*x)^(5/2)/Sqrt[x],x]')
```

```
[Out] Piecewise[{{I / 6 (Sqrt[b] Sqrt[x] (-66 + 59 b x - 17 b ^ 2 x ^ 2 + 2 b ^ 3
 x ^ 3) - 30 ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] Sqrt[-2 + b x]) / (Sqrt[b
] Sqrt[-2 + b x]), Abs[b x] > 2}}, 5 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] /
Sqrt[b] + 11 Sqrt[x] / Sqrt[2 - b x] - 59 b x ^ (3 / 2) / (6 Sqrt[2 - b x])
 + 17 b ^ 2 x ^ (5 / 2) / (6 Sqrt[2 - b x]) - b ^ 3 x ^ (7 / 2) / (3 Sqrt[2
 - b x])]
```

Maple [A]
time = 0.14, size = 91, normalized size = 1.11

method	result	size
meijerg	$\frac{15\sqrt{-b}\left(-\frac{8\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{-b}\left(\frac{1}{24}x^2b^2-\frac{13}{48}bx+\frac{11}{16}\right)\sqrt{-\frac{bx}{2}+1}}{15}-\frac{\sqrt{\pi}\sqrt{-b}\arcsin\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{3\sqrt{b}}\right)}{\sqrt{\pi}\, b}$	78
default	$\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\sqrt{-bx+2}}{2}+\frac{5\sqrt{(-bx+2)x}\arctan\left(\frac{\sqrt{b}\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\sqrt{x}\sqrt{b}}$	91
risch	$-\frac{(2x^2b^2-13bx+33)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)x}}{6\sqrt{-x(bx-2)}\sqrt{-bx+2}}+\frac{5\sqrt{(-bx+2)x}\arctan\left(\frac{\sqrt{b}\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\sqrt{x}\sqrt{b}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^
(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*
(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.36, size = 112, normalized size = 1.37

$$-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{15\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{40(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^3-\frac{3(bx-2)b^2}{x}+\frac{3(bx-2)^2b}{x^2}-\frac{(bx-2)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/3*(15*sqrt(-b*x + 2
)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 33*(-b*x + 2)^(5/2)/x^(5/2)
)/(b^3 - 3*(b*x - 2)*b^2/x + 3*(b*x - 2)^2*b/x^2 - (b*x - 2)^3/x^3)
```

Fricas [A]
time = 0.32, size = 125, normalized size = 1.52

$$\left[\frac{(2b^3x^2-13b^2x+33b)\sqrt{-bx+2}\sqrt{x}-15\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6b},\frac{(2b^3x^2-13b^2x+33b)\sqrt{-bx+2}\sqrt{x}-30\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{6b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 13*b^2*x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 - 13*b^2*
x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b
)*sqrt(x))))/b]
```

Sympy [A]
time = 3.69, size = 207, normalized size = 2.52

$$\begin{cases} \frac{ib^3x^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{17ib^2x^{\frac{5}{2}}}{6\sqrt{bx-2}} + \frac{59ibx^{\frac{3}{2}}}{6\sqrt{bx-2}} - \frac{11i\sqrt{x}}{\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx| > 2 \\ -\frac{b^3x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{17b^2x^{\frac{5}{2}}}{6\sqrt{-bx+2}} - \frac{59bx^{\frac{3}{2}}}{6\sqrt{-bx+2}} + \frac{11\sqrt{x}}{\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(7/2)/(3*sqrt(b*x - 2)) - 17*I*b**2*x**(5/2)/(6*sqrt(b
*x - 2)) + 59*I*b*x**(3/2)/(6*sqrt(b*x - 2)) - 11*I*sqrt(x)/sqrt(b*x - 2) -
 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x) > 2), (-b**3*x**(7/
2)/(3*sqrt(-b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(-b*x + 2)) - 59*b*x**(3/2)
/(6*sqrt(-b*x + 2)) + 11*sqrt(x)/sqrt(-b*x + 2) + 5*asin(sqrt(2)*sqrt(b)*sq
rt(x)/2)/sqrt(b), True))
```

Giac [A]
time = 1.22, size = 150, normalized size = 1.83

$$\frac{b^2\left(2\left(\left(\left(\frac{\frac{1}{36}\cdot 6\sqrt{-bx+2}\sqrt{-bx+2}}{b}+\frac{\frac{1}{36}\cdot 15}{b}\right)\sqrt{-bx+2}\sqrt{-bx+2}+\frac{\frac{1}{36}\cdot 45}{b}\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}+\frac{5\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/6*(sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)*(2*(b*x - 2)/b - 5/b
) + 15/b) + 30*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))
/sqrt(-b))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(1/2), x)
```

3.567 $\int \frac{(2-bx)^{5/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=82

$$-\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -15*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(-b*x+2)^(5/2)/x^(1/2)-5/
2*b*(-b*x+2)^(3/2)*x^(1/2)-15/2*b*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*b*Sqrt[x]*Sqrt[2 - b*x])/2 - (5*b*Sqrt[x]*(2 - b*x)^(3/2))/2 - (2*(2 -
 b*x)^(5/2))/Sqrt[x] - 15*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx\\ &= -\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{1}{2}(15b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-(15b)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,d\right.\\ &= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.83

$$\frac{\sqrt{2-bx}\,\left(-16-9bx+b^2x^2\right)}{2\sqrt{x}}-15\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(3/2),x]
[Out] (Sqrt[2 - b*x]*(-16 - 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[-b]*Log[-(Sqr
t[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.99, size = 156, normalized size = 1.90

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(30\sqrt{b}\,\sqrt{x}\,\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](-2+bx)+bx\left(2-11bx+b^2x^2\right)\sqrt{-2+bx}+32\sqrt{-2+bx}\right)}{2\sqrt{x}\,(-2+bx)},\text{Abs}[bx]>2\right\}\right\},-15\sqrt{b}\,\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]-\frac{16}{\sqrt{x}\,\sqrt{2-bx}}-\frac{b\sqrt{x}}{\sqrt{2-bx}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{2-bx}}-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - b*x)^(5/2)/x^(3/2),x]')
```

```
[Out] Piecewise[{{I / 2 (30 Sqrt[b] Sqrt[x] ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2]
(-2 + b x) + b x (2 - 11 b x + b ^ 2 x ^ 2) Sqrt[-2 + b x] + 32 Sqrt[-2 + b
 x]) / (Sqrt[x] (-2 + b x)), Abs[b x] > 2}}, -15 Sqrt[b] ArcSin[Sqrt[2] Sqr
t[b] Sqrt[x] / 2] - 16 / (Sqrt[x] Sqrt[2 - b x]) - b Sqrt[x] / Sqrt[2 - b x
] + 11 b ^ 2 x ^ (3 / 2) / (2 Sqrt[2 - b x]) - b ^ 3 x ^ (5 / 2) / (2 Sqrt[
2 - b x])]
```

Maple [A]
time = 0.12, size = 78, normalized size = 0.95

method	result	size
meijerg	$\frac{15(-b)^{\frac{3}{2}}\left(\frac{16\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{1}{16}x^2b^2+\frac{9}{16}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{15\sqrt{x}\,\sqrt{-b}}+\frac{2\sqrt{\pi}\,\sqrt{b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{-b}}\right)}{2\sqrt{\pi}\,b}$	78
risch	$-\frac{(b^3x^3-11x^2b^2+2bx+32)\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}-\frac{15\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{2\sqrt{x}\,\sqrt{-bx+2}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15/2*(-b)^(3/2)/Pi^(1/2)/b*(16/15*Pi^(1/2)/x^(1/2)*2^(1/2)/(-b)^(1/2)*(-1/1
6*x^2*b^2+9/16*b*x+1)*(-1/2*b*x+1)^(1/2)+2*Pi^(1/2)/(-b)^(1/2)*b^(1/2)*arcs
in(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.37, size = 96, normalized size = 1.17

$$15\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{\frac{7\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{9\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2-\frac{2\,(bx-2)b}{x}+\frac{(bx-2)^2}{x^2}}-\frac{8\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 15*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (7*sqrt(-b*x + 2)*b^2
/sqrt(x) + 9*(-b*x + 2)^(3/2)*b/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)
^2/x^2) - 8*sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.33, size = 117, normalized size = 1.43

$$\left[\frac{15\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+(b^2x^2-9\,bx-16)\sqrt{-bx+2}\,\sqrt{x}}{2\,x},\ \frac{30\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(b^2x^2-9\,bx-16)\sqrt{-bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (b^2*
x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x))/x, 1/2*(30*sqrt(b)*x*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x
))/x]
```

Sympy [A]
time = 3.69, size = 201, normalized size = 2.45

$$\begin{cases} 15i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{ib^3x^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{11ib^2x^{\frac{3}{2}}}{2\sqrt{bx-2}}+\frac{ib\sqrt{x}}{\sqrt{bx-2}}+\frac{16i}{\sqrt{x}\sqrt{bx-2}} & \text{for } |bx|>2 \\ -15\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{-bx+2}}-\frac{b\sqrt{x}}{\sqrt{-bx+2}}-\frac{16}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((15*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) + I*b**3*x**(5/2)/
(2*sqrt(b*x - 2)) - 11*I*b**2*x**(3/2)/(2*sqrt(b*x - 2)) + I*b*sqrt(x)/sqrt
(b*x - 2) + 16*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x) > 2), (-15*sqrt(b)*asin(
sqrt(2)*sqrt(b)*sqrt(x)/2) - b**3*x**(5/2)/(2*sqrt(-b*x + 2)) + 11*b**2*x**
(3/2)/(2*sqrt(-b*x + 2)) - b*sqrt(x)/sqrt(-b*x + 2) - 16/(sqrt(x)*sqrt(-b*x
 + 2)), True))
```

Giac [A]
time = 1.26, size = 157, normalized size = 1.91

$$-\frac{bb^2\left(\frac{2\left(\left(-\frac{5}{4}-\frac{1}{4}\sqrt{-bx+2}\sqrt{-bx+2}\right)\sqrt{-bx+2}\sqrt{-bx+2}+\frac{15}{2}\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{-b(-bx+2)+2b}+\frac{15\ln\left|\sqrt{-b(-bx+2)+2b}-\sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] 1/2*(((b*x - 2)*(b*x - 7) - 30)*sqrt(-b*x + 2)/sqrt((b*x - 2)*b + 2*b) - 30
*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sqrt(-b))*b^2
/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(3/2), x)
```

3.568 $\int \frac{(2-bx)^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=84

$$5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(-b*x+2)^(5/2)/x^(3/2)+10*b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))+
10/3*b*(-b*x+2)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 - b*x] + (10*b*(2 - b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 -
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx \\
&= \frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\
&= 5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\
&= 5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x\right. \\
&= 5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.83

$$\frac{\sqrt{2-bx}\,\left(-8+28bx+3b^2x^2\right)}{3x^{3/2}}+10\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-8 + 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 10*Sqrt[-b]*b*Log[-
(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.46, size = 200, normalized size = 2.38

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\left(bx\left(3bx\sqrt{\frac{2-bx}{bx}}-30I\text{Log}\left[\frac{1}{\sqrt{b}\,\sqrt{x}}\right]+15I\text{Log}\left[\frac{1}{bx}\right]+28\sqrt{\frac{2-bx}{bx}}+30\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]\right)-8\sqrt{\frac{2-bx}{bx}}\right)}{3x},\frac{1}{\text{Abs}\left[bx\right]}>\frac{1}{2}\right\}\right\},-10Ib^{\frac{3}{2}}\text{Log}\left[1+\sqrt{1-\frac{2}{bx}}\right]+I5b^{\frac{3}{2}}\text{Log}\left[\frac{1}{bx}\right]+\frac{I28b^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}-\frac{8I\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x}+Ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 - b*x)^(5/2)/x^(5/2),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (b x (3 b x Sqrt[(2 - b x) / (b x)] - 30 I Log[1 / (Sqr
t[b] Sqrt[x])] + 15 I Log[1 / (b x)] + 28 Sqrt[(2 - b x) / (b x)] + 30 ArcS
in[Sqrt[2] Sqrt[b] Sqrt[x] / 2]) - 8 Sqrt[(2 - b x) / (b x)]) / (3 x), 1 /
Abs[b x] > 1 / 2}}, -10 I b ^ (3 / 2) Log[1 + Sqrt[1 - 2 / (b x)]] + I 5 b
^ (3 / 2) Log[1 / (b x)] + I 28 b ^ (3 / 2) Sqrt[1 - 2 / (b x)] / 3 - 8 I S
qrt[b] Sqrt[1 - 2 / (b x)] / (3 x) + I b ^ (5 / 2) x Sqrt[1 - 2 / (b x)]]
```

Maple [A]
time = 0.15, size = 78, normalized size = 0.93

method	result	size
meijerg	$\frac{15(-b)^{\frac{5}{2}}\left(\frac{32\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{3}{8}x^2b^2-\frac{7}{2}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{45x^{\frac{3}{2}}(-b)^{\frac{3}{2}}}-\frac{8\sqrt{\pi}\,b^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3(-b)^{\frac{3}{2}}}\right)}{4\sqrt{\pi}\,b}$	78
risch	$-\frac{(3b^3x^3+22x^2b^2-64bx+16)\sqrt{(-bx+2)x}}{3x^{\frac{3}{2}}\sqrt{-x(bx-2)}\,\sqrt{-bx+2}}+\frac{5b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)x}}{\sqrt{x}\,\sqrt{-bx+2}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15/4*(-b)^(5/2)/Pi^(1/2)/b*(32/45*Pi^(1/2)/x^(3/2)*2^(1/2)/(-b)^(3/2)*(-3/8
*x^2*b^2-7/2*b*x+1)*(-1/2*b*x+1)^(1/2)-8/3*Pi^(1/2)/(-b)^(3/2)*b^(3/2)*arcs
in(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.36, size = 79, normalized size = 0.94

$$-10\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{8\,\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{2\,\sqrt{-bx+2}\,b^2}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}-\frac{4\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -10*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 8*sqrt(-b*x + 2)*b/s
qrt(x) + 2*sqrt(-b*x + 2)*b^2/((b - (b*x - 2)/x)*sqrt(x)) - 4/3*(-b*x + 2)^
(3/2)/x^(3/2)
```

Fricas [A]
time = 0.31, size = 126, normalized size = 1.50

$$\left[\frac{15\,\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+(3\,b^2x^2+28\,bx-8)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2},-\frac{30\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-(3\,b^2x^2+28\,bx-8)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (
3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*x + 2)*sqrt(x))/x^2, -1/3*(30*b^(3/2)*x^2*a
rctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*
x + 2)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.62, size = 223, normalized size = 2.65

$$\begin{cases} b^{\frac{5}{2}} x \sqrt{-1 + \frac{2}{bx}} + \frac{28 b^{\frac{3}{2}} \sqrt{-1 + \frac{2}{bx}}}{3} + 5 i b^{\frac{3}{2}} \log\left(\frac{1}{bx}\right) - 10 i b^{\frac{3}{2}} \log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right) + 10 b^{\frac{3}{2}} \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) - \frac{8\sqrt{b}\sqrt{-1 + \frac{2}{bx}}}{3x} & \text{for}\ \frac{1}{|bx|} > \frac{1}{2} \\ i b^{\frac{5}{2}} x \sqrt{1 - \frac{2}{bx}} + \frac{28 i b^{\frac{3}{2}} \sqrt{1 - \frac{2}{bx}}}{3} + 5 i b^{\frac{3}{2}} \log\left(\frac{1}{bx}\right) - 10 i b^{\frac{3}{2}} \log\left(\sqrt{1 - \frac{2}{bx}} + 1\right) - \frac{8 i \sqrt{b}\sqrt{1 - \frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(5/2)*x*sqrt(-1 + 2/(b*x)) + 28*b**(3/2)*sqrt(-1 + 2/(b*x))/3
 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 10*
b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) - 8*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 1/Abs(b*x) > 1/2), (I*b**(5/2)*x*sqrt(1 - 2/(b*x)) + 28*I*b**(3/2)*sqrt
(1 - 2/(b*x))/3 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(sqrt(1 - 2/
(b*x)) + 1) - 8*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [A]
time = 1.19, size = 179, normalized size = 2.13

$$\frac{b^2 \left(\frac{2\left(\left(\frac{\frac{1}{18}\cdot 9b^4\sqrt{-bx+2}\sqrt{-bx+2}}{b} - \frac{\frac{1}{18}\cdot 120b^4}{b}\right)\sqrt{-bx+2}\sqrt{-bx+2} + \frac{\frac{1}{18}\cdot 180b^4}{b}\right)\sqrt{-bx+2}\sqrt{-b(-bx+2)+2b}}{(-b(-bx+2)+2b)^2} + \frac{10b^2 \ln\left|\sqrt{-b(-bx+2)+2b} - \sqrt{-b}\sqrt{-bx+2}\right|}{\sqrt{-b}} \right)}{|b|\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] 1/3*(30*b^2*log(abs(-sqrt(-b*x + 2)*sqrt(-b) + sqrt((b*x - 2)*b + 2*b)))/sq
rt(-b) + (60*b^3 + (3*(b*x - 2)*b^3 + 40*b^3)*(b*x - 2))*sqrt(-b*x + 2)/((b
*x - 2)*b + 2*b)^(3/2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2 - b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(5/2), x)
```

3.569 $\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=101

$$\frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

```
[Out] -5/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-5/12*a*x^(3/2)*(b*x
+a)^(1/2)/b^2+1/3*x^(5/2)*(b*x+a)^(1/2)/b+5/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a + b*x])/(12*b^2
) + (x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx &= \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{6b} \\
&= -\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} + \frac{(5a^2)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b^2} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a^3)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b^3} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x \ldots\right)}{8b^3} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a^3)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{\ldots}}{\sqrt{a \ldots}}\right)}{8b^3} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.75

$$\frac{\sqrt{x}\,\sqrt{a+bx}\,(15a^2-10abx+8b^2x^2)}{24b^3} + \frac{5a^3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2))/(24*b^3) + (5*a^3*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(7/2))
```

Mathics [A]
time = 10.64, size = 125, normalized size = 1.24

$$\frac{a^{\frac{3}{2}}\left(-15a^{\frac{3}{2}}b^{6}\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right](a+bx)^{2}+15a^{3}b^{\frac{13}{2}}\sqrt{x}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+5a^{2}b^{\frac{15}{2}}x^{\frac{3}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}-2ab^{\frac{17}{2}}x^{\frac{5}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+8b^{\frac{19}{2}}x^{\frac{7}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}\right)}{24b^{\frac{19}{2}}(a+bx)^{2}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/Sqrt[a + b*x],x]')
```

```
[Out] a ^ (3 / 2) (-15 a ^ (3 / 2) b ^ 6 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] (a +
b x) ^ 2 + 15 a ^ 3 b ^ (13 / 2) Sqrt[x] ((a + b x) / a) ^ (3 / 2) + 5 a ^
2 b ^ (15 / 2) x ^ (3 / 2) ((a + b x) / a) ^ (3 / 2) - 2 a b ^ (17 / 2) x ^
 (5 / 2) ((a + b x) / a) ^ (3 / 2) + 8 b ^ (19 / 2) x ^ (7 / 2) ((a + b x)
/ a) ^ (3 / 2)) / (24 b ^ (19 / 2) (a + b x) ^ 2)
```

Maple [A]
time = 0.12, size = 109, normalized size = 1.08

method	result	size
risch	$\frac{(8x^{2}b^{2}-10abx+15a^{2})\sqrt{x}\,\sqrt{bx+a}}{24b^{3}}-\frac{5a^{3}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4b}\right)}{6b}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(5/2)*(b*x+a)^(1/2)/b-5/6*a/b*(1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a/b*(x
^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2
)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.37, size = 146, normalized size = 1.45

$$\frac{5\,a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}-\frac{\frac{33\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}+\frac{15\,(bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{6}-\frac{3\,(bx+a)b^{5}}{x}+\frac{3\,(bx+a)^{2}b^{4}}{x^{2}}-\frac{(bx+a)^{3}b^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(7/2) - 1/24*(33*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3/
2)*a^3*b/x^(3/2) + 15*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x + a)*b^5/x
 + 3*(b*x + a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3)
```

Fricas [A]
time = 0.31, size = 140, normalized size = 1.39

$$\left[\frac{15\,a^{3}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^{4}},\frac{15\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A]
time = 9.52, size = 128, normalized size = 1.27

$$\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}}-\frac{5a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*
sqrt(1 + b*x/a)) - sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*a**3*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 124, normalized size = 1.23

$$2\left(2\left(\left(\left(\frac{\frac{1}{288}\cdot 24b^{4}\sqrt{x}\,\sqrt{x}}{b^{5}}-\frac{\frac{1}{288}\cdot 30b^{3}a}{b^{5}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{288}\cdot 45b^{2}a^{2}}{b^{5}}\right)\sqrt{x}\,\sqrt{a+bx}+\frac{10a^{3}\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{32b^{3}\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(1/2), x)
```

3.570 $\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=77

$$-\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)+1/2*x^(3/2)*(b*x+a)^
(1/2)/b-3/4*a*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^2) + (x^(3/2)*Sqrt[a + b*x])/(2*b) + (3*a
^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = \frac{x^{3/2}\sqrt{a+bx}}{2b} - \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.86

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(-3a+2bx) - 3a^2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b*x],x]
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a + 2*b*x) - 3*a^2*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[a + b*x]])/(4*b^(5/2))
```

Mathics [A]
time = 4.65, size = 93, normalized size = 1.21

$$\frac{3a^{\frac{5}{2}}b^3\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}} - 3ab^{\frac{7}{2}}\sqrt{x}\,(a+bx) - b^{\frac{9}{2}}x^{\frac{3}{2}}(a+bx) + \frac{2b^{\frac{11}{2}}x^{\frac{5}{2}}(a+bx)}{a}}{4\sqrt{a}\,b^{\frac{11}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/Sqrt[a + b*x],x]')
```

```
[Out] (3 a ^ (5 / 2) b ^ 3 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^ (
3 / 2) - 3 a b ^ (7 / 2) Sqrt[x] (a + b x) - b ^ (9 / 2) x ^ (3 / 2) (a + b
 x) + 2 b ^ (11 / 2) x ^ (5 / 2) (a + b x) / a) / (4 Sqrt[a] b ^ (11 / 2) (
(a + b x) / a) ^ (3 / 2))
```

Maple [A]
time = 0.10, size = 87, normalized size = 1.13

method	result	size
risch	$-\frac{(-2bx+3a)\sqrt{x}\,\sqrt{bx+a}}{4b^2}+\frac{3a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$	76
default	$\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4b}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a/b*(x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*
(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^
(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.
time = 0.34, size = 112, normalized size = 1.45

$$-\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{5\sqrt{bx+a}\,a^2b}{\sqrt{x}}-\frac{3\,(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^4-\frac{2\,(bx+a)b^3}{x}+\frac{(bx+a)^2b^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(5/2) + 1/4*(5*sqrt(b*x + a)*a^2*b/sqrt(x) - 3*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2)
```

Fricas [A]
time = 0.32, size = 119, normalized size = 1.55

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,(2\,b^{2}x-3\,ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b^{3}},\,-\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{2}x-3\,ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

Sympy [A]
time = 2.81, size = 100, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - sqrt(a)*x**(3/2)/(4*b*sqrt(1
 + b*x/a)) + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/
(2*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.00, size = 96, normalized size = 1.25

$$2\left(2\left(\frac{\frac{1}{16}\cdot 2b^{2}\sqrt{x}\,\sqrt{x}}{b^{3}}-\frac{\frac{1}{16}\cdot 3ba}{b^{3}}\right)\sqrt{x}\,\sqrt{a+bx}\,-\frac{6a^{2}\ln\left|\sqrt{a+bx}\,-\sqrt{b}\,\sqrt{x}\right|}{16b^{2}\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(1/2),x)
[Out] int(x^(3/2)/(a + b*x)^(1/2), x)
```

3.571 $\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

[Out] -a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)+x^(1/2)*(b*x+a)^(1/2)/b

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[a + b*x],x]

[Out] (Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx &= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b} \\
&= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b} \\
&= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b} \\
&= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.02

$$\frac{\sqrt{x}\sqrt{a+bx}}{b} + \frac{a\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a + b*x],x]
[Out] (Sqrt[x]*Sqrt[a + b*x])/b + (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(
3/2)
```

Mathics [A]
time = 2.84, size = 38, normalized size = 0.79

$$-\frac{a\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{b^{\frac{3}{2}}} + \frac{\sqrt{a}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/Sqrt[a + b*x],x]')
```

```
[Out] -a ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] / b ^ (3 / 2) + Sqrt[a] Sqrt[x] Sqrt[
1 + b x / a] / b
```

Maple [A]
time = 0.11, size = 65, normalized size = 1.35

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\left(bx+a\right)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	65
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\left(bx+a\right)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/
2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.
time = 0.34, size = 73, normalized size = 1.52

$$\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}-\frac{\sqrt{bx+a}\,a}{\left(b^{2}-\frac{(bx+a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(
x)))/b^(3/2) - sqrt(b*x + a)*a/((b^2 - (b*x + a)*b/x)*sqrt(x))
```

Fricas [A]
time = 0.32, size = 91, normalized size = 1.90

$$\left[\frac{a\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b^{2}},\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}}{b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

Sympy [A]
time = 1.16, size = 44, normalized size = 0.92

$$\frac{\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a)/b - a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/
2)
```

Giac [A]
time = 0.01, size = 61, normalized size = 1.27

$$2\left(\frac{\frac{1}{4}\cdot 2\sqrt{x}\,\sqrt{a+bx}}{b}+\frac{2a\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{4b\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

Mupad [B]
time = 0.55, size = 44, normalized size = 0.92

$$\frac{\sqrt{x}\,\sqrt{a+b\,x}}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (x^(1/2)*(a + b*x)^(1/2))/b - (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2)
 - a^(1/2))))/b^(3/2)
```

3.572 $\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx = 2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

<u>**Mathematica**</u> [**A**]
time = 0.03, size = 30, normalized size = 1.07

$$-\frac{2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

<u>**Mathics**</u> [**A**]
time = 2.01, size = 16, normalized size = 0.57

$$\frac{2\mathrm{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]')
[Out] 2 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] / Sqrt[b]
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.
time = 0.12, size = 48, normalized size = 1.71

method	result	size
default	$\frac{\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{\sqrt{x}\,\sqrt{bx+a}\,\sqrt{b}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^
(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.
time = 0.36, size = 41, normalized size = 1.46

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
sqrt(b)
```

Fricas [A]
time = 0.31, size = 57, normalized size = 2.04

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [A]
time = 0.51, size = 22, normalized size = 0.79

$$\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.18

$$-\frac{2\ln\left|\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right|}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

Mupad [B]
time = 0.03, size = 30, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

3.573 $\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=19

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

```
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Mathics [A]
time = 2.00, size = 20, normalized size = 1.05

$$\frac{-2\sqrt{b}\,\sqrt{1+\frac{a}{bx}}}{a}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]')
```

```
[Out] -2 Sqrt[b] Sqrt[1 + a / (b x)] / a
```

Maple [A]
time = 0.12, size = 16, normalized size = 0.84

method	result	size
gosper	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
default	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
risch	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

Maxima [A]
time = 0.25, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Fricas [A]
time = 0.30, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Sympy [A]
time = 0.48, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(1/2),x)
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

Giac [A]
time = 0.00, size = 37, normalized size = 1.95

$$\frac{8\sqrt{b}}{2\left(\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x)
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

Mupad [B]
time = 0.35, size = 15, normalized size = 0.79

$$-\frac{2\,\sqrt{a+b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(1/2)),x)
[Out] -(2*(a + b*x)^(1/2))/(a*x^(1/2))
```

3.574 $\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\sqrt{a+bx}}{3ax^{3/2}}+\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}$$

```
[Out] -2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}+\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 27, normalized size = 0.61

$$-\frac{2(a-2bx)\sqrt{a+bx}}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]
[Out] (-2*(a - 2*b*x)*Sqrt[a + b*x])/(3*a^2*x^(3/2))
```

Mathics [A]
time = 2.79, size = 33, normalized size = 0.75

$$\frac{2\sqrt{b}\,(-a+2bx)\sqrt{\frac{a+bx}{bx}}}{3a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]')
[Out] 2 Sqrt[b] (-a + 2 b x) Sqrt[(a + b x) / (b x)] / (3 a ^ 2 x)
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.75

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^2}$	22
risch	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^2}$	22
default	$-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Maxima [A]
time = 0.25, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{3\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^2
```

Fricas [A]
time = 0.31, size = 23, normalized size = 0.52

$$\frac{2\,(2\,bx - a)\sqrt{bx + a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [A]
time = 1.14, size = 42, normalized size = 0.95

$$-\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx} + 1}}{3ax} + \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx} + 1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(1/2),x)
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**2
)
```

Giac [A]
time = 0.00, size = 72, normalized size = 1.64

$$-\frac{32\sqrt{b}\,b\left(-3\left(\sqrt{a + bx} - \sqrt{b}\,\sqrt{x}\right)^2 + a\right)}{2\cdot 6\left(\left(\sqrt{a + bx} - \sqrt{b}\,\sqrt{x}\right)^2 - a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x)
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

Mupad [B]
time = 0.34, size = 25, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a} - \frac{4\,b\,x}{3\,a^2}\right)\,\sqrt{a + b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(1/2)),x)
[Out] -((2/(3*a) - (4*b*x)/(3*a^2))*(a + b*x)^(1/2))/x^(3/2)
```

3.575 $\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}$$

```
[Out] -2/5*(b*x+a)^(1/2)/a/x^(5/2)+8/15*b*(b*x+a)^(1/2)/a^2/x^(3/2)-16/15*b^2*(b*
x+a)^(1/2)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(5*a*x^(5/2)) + (8*b*Sqrt[a + b*x])/(15*a^2*x^(3/2)) - (
16*b^2*Sqrt[a + b*x])/(15*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{5ax^{5/2}} - \frac{(4b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a}$$
$$= -\frac{2\sqrt{a+bx}}{5ax^{5/2}} + \frac{8b\sqrt{a+bx}}{15a^2x^{3/2}} + \frac{(8b^2)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{15a^2}$$
$$= -\frac{2\sqrt{a+bx}}{5ax^{5/2}} + \frac{8b\sqrt{a+bx}}{15a^2x^{3/2}} - \frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}$$

Mathematica [**A**]
time = 0.06, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\,(3a^2-4abx+8b^2x^2)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

Mathics [**A**]
time = 6.96, size = 84, normalized size = 1.24

$$\frac{2\sqrt{b}\,(-3a^4-2a^3bx-3a^2b^2x^2-12ab^3x^3-8b^4x^4)\sqrt{\frac{a+bx}{bx}}}{15a^3x^2\,(a^2+2abx+b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]')
```

```
[Out] 2 Sqrt[b] (-3 a ^ 4 - 2 a ^ 3 b x - 3 a ^ 2 b ^ 2 x ^ 2 - 12 a b ^ 3 x ^ 3
- 8 b ^ 4 x ^ 4) Sqrt[(a + b x) / (b x)] / (15 a ^ 3 x ^ 2 (a ^ 2 + 2 a b x
 + b ^ 2 x ^ 2))
```

Maple [**A**]
time = 0.12, size = 55, normalized size = 0.81

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(8x^2b^2-4abx+3a^2)}{15x^{\frac{5}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}\,(8x^2b^2-4abx+3a^2)}{15x^{\frac{5}{2}}a^3}$	35

default	$-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{5a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b
*x+a)^(1/2)/a^2/x^(1/2))
```

Maxima [A]
time = 0.25, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{15\sqrt{bx+a}\,b^{2}}{\sqrt{x}}-\frac{10\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*sqrt(b*x + a)*b^2/sqrt(x) - 10*(b*x + a)^(3/2)*b/x^(3/2) + 3*(b*x
 + a)^(5/2)/x^(5/2))/a^3
```

Fricas [A]
time = 0.31, size = 34, normalized size = 0.50

$$-\frac{2\,(8\,b^{2}x^{2}-4\,abx+3\,a^{2})\sqrt{bx+a}}{15\,a^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(63) = 126$.
time = 4.08, size = 287, normalized size = 4.22

$$-\frac{6a^{4}b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^{5}b^{4}x^{2}+30a^{4}b^{5}x^{3}+15a^{3}b^{6}x^{4}}-\frac{4a^{3}b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^{5}b^{4}x^{2}+30a^{4}b^{5}x^{3}+15a^{3}b^{6}x^{4}}-\frac{6a^{2}b^{\frac{13}{2}}x^{2}\sqrt{\frac{a}{bx}+1}}{15a^{5}b^{4}x^{2}+30a^{4}b^{5}x^{3}+15a^{3}b^{6}x^{4}}-\frac{24ab^{\frac{15}{2}}x^{3}\sqrt{\frac{a}{bx}+1}}{15a^{5}b^{4}x^{2}+30a^{4}b^{5}x^{3}+15a^{3}b^{6}x^{4}}-\frac{16b^{\frac{17}{2}}x^{4}\sqrt{\frac{a}{bx}+1}}{15a^{5}b^{4}x^{2}+30a^{4}b^{5}x^{3}+15a^{3}b^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 +
 15*a**3*b**6*x**4) - 4*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
```

```
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2)*x**2*sqrt(a/
(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 24
*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3
+ 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

Giac [A]
time = 0.01, size = 102, normalized size = 1.50

$$-\frac{128\sqrt{b}\,b^2\left(-10\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^4+5\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2a-a^2\right)}{60\left(\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x)
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

Mupad [B]
time = 0.35, size = 36, normalized size = 0.53

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a}+\frac{16\,b^2\,x^2}{15\,a^3}-\frac{8\,b\,x}{15\,a^2}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(1/2)),x)
[Out] -((a + b*x)^(1/2)*(2/(5*a) + (16*b^2*x^2)/(15*a^3) - (8*b*x)/(15*a^2)))/x^(
5/2)
```

3.576 $\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\sqrt{a+bx}}{7ax^{7/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}$$

```
[Out] -2/7*(b*x+a)^(1/2)/a/x^(7/2)+12/35*b*(b*x+a)^(1/2)/a^2/x^(5/2)-16/35*b^2*(b
*x+a)^(1/2)/a^3/x^(3/2)+32/35*b^3*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(7*a*x^(7/2)) + (12*b*Sqrt[a + b*x])/(35*a^2*x^(5/2)) -
(16*b^2*Sqrt[a + b*x])/(35*a^3*x^(3/2)) + (32*b^3*Sqrt[a + b*x])/(35*a^4*Sq
rt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{7ax^{7/2}} - \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{7a}$$
$$= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} + \frac{(24b^2)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{35a^2}$$
$$= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} - \frac{(16b^3)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{35a^3}$$
$$= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} + \frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.55

$$-\frac{2\sqrt{a+bx}\,(5a^3-6a^2bx+8ab^2x^2-16b^3x^3)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

Mathics [A]
time = 14.51, size = 117, normalized size = 1.27

$$\frac{2\sqrt{b}\,(-5a^6-9a^5bx-5a^4b^2x^2+5a^3b^3x^3+30a^2b^4x^4+40ab^5x^5+16b^6x^6)\sqrt{\frac{a+bx}{bx}}}{35a^4x^3\,(a^3+3a^2bx+3ab^2x^2+b^3x^3)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]')
```

```
[Out] 2 Sqrt[b] (-5 a ^ 6 - 9 a ^ 5 b x - 5 a ^ 4 b ^ 2 x ^ 2 + 5 a ^ 3 b ^ 3 x ^
 3 + 30 a ^ 2 b ^ 4 x ^ 4 + 40 a b ^ 5 x ^ 5 + 16 b ^ 6 x ^ 6) Sqrt[(a + b
x) / (b x)] / (35 a ^ 4 x ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^
3 x ^ 3))
```

Maple [A]
time = 0.12, size = 77, normalized size = 0.84

method	result	size

gosper	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
risch	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
default	$-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x+a)^(1/2)/a/x^(7/2)-6/7*b/a*(-2/5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*
(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)))
```

Maxima [A]
time = 0.31, size = 61, normalized size = 0.66

$$\frac{2\left(\frac{35\sqrt{bx+a}\,b^3}{\sqrt{x}}-\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{21\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(b*x + a)*b^3/sqrt(x) - 35*(b*x + a)^(3/2)*b^2/x^(3/2) + 21*(b
*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^4
```

Fricas [A]
time = 0.30, size = 45, normalized size = 0.49

$$\frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(87) = 174$.
time = 12.38, size = 488, normalized size = 5.30

$$-\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{18a^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{10a^4b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{10a^3b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{60a^2b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{80ab^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{32b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x*
*4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(21/2)*x*sqrt(a
/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5
+ 35*a**4*b**12*x**6) - 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(35*a**7*b
**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
+ 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b*
*10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*a**2*b**(27/2)*x*
*4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b*
*11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(35*
a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*
x**6) + 32*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b
**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
```

Giac [A]
time = 0.01, size = 131, normalized size = 1.42

$$-\frac{512\sqrt{b}\,b^3\left(-35\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^6+21\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^4a-7\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2a^2+a^3\right)}{280\left(\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

Mupad [B]
time = 0.38, size = 47, normalized size = 0.51

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{7\,a}+\frac{16\,b^2\,x^2}{35\,a^3}-\frac{32\,b^3\,x^3}{35\,a^4}-\frac{12\,b\,x}{35\,a^2}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(7*a) + (16*b^2*x^2)/(35*a^3) - (32*b^3*x^3)/(35*a^4)
- (12*b*x)/(35*a^2)))/x^(7/2)
```

3.577 $\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}$$

```
[Out] 15/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-2*x^(5/2)/b/(b*x+a)
^(1/2)+5/2*x^(3/2)*(b*x+a)^(1/2)/b^2-15/4*a*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]

time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {49, 52, 65, 223, 212}

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[a + b*x]) - (15*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^3) + (5*
x^(3/2)*Sqrt[a + b*x])/(2*b^2) + (15*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/(4*b^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{b} \\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b^2} \\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{(15a^2)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b^3} \\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x\right.}{4b^3} \\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{}}{\sqrt{a}}\right.}{4b^3} \\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.11, size = 76, normalized size = 0.79

$$\frac{\sqrt{x}\,(-15a^2-5abx+2b^2x^2)}{4b^3\sqrt{a+bx}}-\frac{15a^2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(-15*a^2 - 5*a*b*x + 2*b^2*x^2))/(4*b^3*Sqrt[a + b*x]) - (15*a^2*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(7/2))
```

Mathics [A]
time = 7.15, size = 93, normalized size = 0.97

$$\frac{15a^{\frac{5}{2}}b^6\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}-15ab^{\frac{13}{2}}\sqrt{x}\,(a+bx)-5b^{\frac{15}{2}}x^{\frac{3}{2}}(a+bx)+\frac{2b^{\frac{17}{2}}x^{\frac{5}{2}}(a+bx)}{a}}{4\sqrt{a}\,b^{\frac{19}{2}}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(a + b*x)^(3/2),x]')
```

```
[Out] (15 a ^ (5 / 2) b ^ 6 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^
(3 / 2) - 15 a b ^ (13 / 2) Sqrt[x] (a + b x) - 5 b ^ (15 / 2) x ^ (3 / 2)
(a + b x) + 2 b ^ (17 / 2) x ^ (5 / 2) (a + b x) / a) / (4 Sqrt[a] b ^ (19
/ 2) ((a + b x) / a) ^ (3 / 2))
```

Maple [A]
time = 0.13, size = 119, normalized size = 1.24

method	result
risch	$-\frac{(-2bx+7a)\sqrt{x}\,\sqrt{bx+a}}{4b^3}+\frac{\left(\frac{15a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{8b^{\frac{7}{2}}}-\frac{2a^2\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{b^4\left(x+\frac{a}{b}\right)}\right)\sqrt{x\,(bx+a}}{\sqrt{x}\,\sqrt{bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*b*x+7*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+(15/8/b^(7/2)*a^2*ln((1/2*a+b*x
)/b^(1/2)+(b*x^2+a*x)^(1/2))-2/b^4*a^2/(x+a/b)*((x+a/b)^2*b-a*(x+a/b))^(1/2
))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.35, size = 131, normalized size = 1.36

$$-\frac{8\,a^2b^2 - \frac{25\,(bx+a)a^2b}{x} + \frac{15\,(bx+a)^2a^2}{x^2}}{4\left(\frac{\sqrt{bx+a}\,b^5}{\sqrt{x}} - \frac{2\,(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} - \frac{15\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*b^2 - 25*(b*x + a)*a^2*b/x + 15*(b*x + a)^2*a^2/x^2)/(sqrt(b*x
+ a)*b^5/sqrt(x) - 2*(b*x + a)^(3/2)*b^4/x^(3/2) + (b*x + a)^(5/2)*b^3/x^(5
/2)) - 15/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
+ a)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 175, normalized size = 1.82

$$\left[\frac{15\,(a^2bx+a^3)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^3x^2-5\,ab^2x-15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{8\,(b^5x+ab^4)}, -\frac{15\,(a^2bx+a^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^3x^2-5\,ab^2x-15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{4\,(b^5x+ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

Sympy [A]
time = 5.74, size = 105, normalized size = 1.09

$$-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}} - \frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}} + \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2*
sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) + x*
*(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 132, normalized size = 1.38

$$2\left(\frac{2\left(\left(\left(\frac{\frac{1}{16}\cdot 2b^4\sqrt{x}\sqrt{x}}{b^5}-\frac{\frac{1}{16}\cdot 5b^3a}{b^5}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{16}\cdot 15b^2a^2}{b^5}\right)\sqrt{x}\sqrt{a+bx}\right)}{a+bx}-\frac{30a^2\ln\left|\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right|}{16b^3\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] 1/4*(x*(2*x/b - 5*a/b^2) - 15*a^2/b^3)*sqrt(x)/sqrt(b*x + a) - 15/4*a^2*log
(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(3/2), x)
```

3.578 $\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

```
[Out] -3*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)-2*x^(3/2)/b/(b*x+a)^(1/
2)+3*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*Sqrt[x]*Sqrt[a + b*x])/b^2 - (3*a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 57, normalized size = 0.84

$$\frac{\sqrt{x}\,(3a+bx)}{b^2\sqrt{a+bx}}+\frac{3a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(3*a + b*x))/(b^2*Sqrt[a + b*x]) + (3*a*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/b^(5/2)
```

Mathics [**A**]
time = 3.74, size = 77, normalized size = 1.13

$$\frac{\sqrt{a}\left(-3\sqrt{a}\,b^3\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](a+bx)+3ab^{\frac{7}{2}}\sqrt{x}\,\sqrt{\frac{a+bx}{a}}+b^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{a+bx}{a}}\right)}{b^{\frac{11}{2}}\,(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a + b*x)^(3/2),x]')
```

```
[Out] Sqrt[a] (-3 Sqrt[a] b ^ 3 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] (a + b x) + 3
a b ^ (7 / 2) Sqrt[x] Sqrt[(a + b x) / a] + b ^ (9 / 2) x ^ (3 / 2) Sqrt[(a
 + b x) / a]) / (b ^ (11 / 2) (a + b x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(52) = 104$.
time = 0.13, size = 106, normalized size = 1.56

method	result	size
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b^2}+\frac{\left(-\frac{3a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{5}{2}}}+\frac{2a\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{b^3\left(x+\frac{a}{b}\right)}\right)\sqrt{x\,(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))+2*a/b^3/(x+a/b)*((x+a/b)^2*b-a*(x+a/b))^(1/2))*(x*(b*x+a))^(1/2)/x
^(1/2)/(b*x+a)^(1/2)
```

Maxima [**A**]
time = 0.34, size = 92, normalized size = 1.35

$$\frac{2\,ab-\frac{3\,(bx+a)a}{x}}{\frac{\sqrt{bx+a}\,b^3}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x + a)*a/x)/(sqrt(b*x + a)*b^3/sqrt(x) - (b*x + a)^(3/2)*b^2/
x^(3/2)) + 3/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
 + a)/sqrt(x)))/b^(5/2)
```

Fricas [**A**]
time = 0.34, size = 145, normalized size = 2.13

$$\left[\frac{3\,(abx+a^2)\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(b^2x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{2\,(b^4x+ab^3)}, \frac{3\,(abx+a^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(b^2x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{b^4x+ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [**A**]
time = 1.99, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\,\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sqrt(a
))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))
```

Giac [**A**]
time = 0.01, size = 97, normalized size = 1.43

$$2\left(\frac{2\left(\frac{\frac{1}{4}b^2\sqrt{x}\,\sqrt{x}}{b^3}+\frac{\frac{1}{4}\cdot 3ba}{b^3}\right)\sqrt{x}\,\sqrt{a+bx}}{a+bx}+\frac{6a\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{4b^2\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] sqrt(x)*(x/b + 3*a/b^2)/sqrt(b*x + a) + 3*a*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(5/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(3/2), x)
```

3.579 $\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)-2*x^(1/2)/b/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{b}\\ &= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}\\ &= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.04

$$-\frac{2\sqrt{x}}{b\sqrt{a+bx}}-\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
)/b^(3/2)
```

Mathics [A]
time = 2.58, size = 38, normalized size = 0.79

$$\frac{2\mathrm{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[Sqrt[x]/(a + b*x)^(3/2),x]')

[Out] 2 ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] / b ^ (3 / 2) - 2 Sqrt[x] / (Sqrt[a] b Sqrt[1 + b x / a])

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x+a)^(3/2),x)

[Out] int(x^(1/2)/(b*x+a)^(3/2),x)

Maxima [A]
time = 0.33, size = 57, normalized size = 1.19

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+a}\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")

[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(3/2) - 2*sqrt(x)/(sqrt(b*x + a)*b)

Fricas [A]
time = 0.32, size = 119, normalized size = 2.48

$$\left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,b\sqrt{x}}{b^3x+ab^2}, -\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")

[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]

Sympy [A]
time = 0.86, size = 46, normalized size = 0.96

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 + b
*x/a))
```

Giac [A]
time = 0.01, size = 65, normalized size = 1.35

$$2\left(-\frac{\frac{1}{2}\cdot 2\sqrt{x}\,\sqrt{a+bx}}{b\,(a+bx)}-\frac{\ln\left|\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right|}{b\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) - 2*sqrt(x)/(sqrt(b*x
 + a)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a + b*x)^(3/2), x)
```

3.580 $\int \frac{1}{\sqrt{x}\,(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

```
[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Mathics [A]
time = 1.98, size = 20, normalized size = 1.05

$$\frac{2}{a\sqrt{b}\sqrt{1+\frac{a}{bx}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]')
[Out] 2 / (a Sqrt[b] Sqrt[1 + a / (b x)])
```

Maple [A]
time = 0.12, size = 16, normalized size = 0.84

method	result	size
gosper	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16
default	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.26, size = 15, normalized size = 0.79

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

Fricas [A]
time = 0.31, size = 22, normalized size = 1.16

$$\frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)

Sympy [A]
time = 0.47, size = 17, normalized size = 0.89

$$\frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(3/2)/x**(1/2),x)

[Out] 2/(a*sqrt(b)*sqrt(a/(b*x) + 1))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(15) = 30.
time = 0.00, size = 53, normalized size = 2.79

$$\frac{8b\sqrt{b}}{2\,|b|\left(\left(\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right)^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x)

[Out] 4*b^(3/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*abs(
b))

Mupad [B]
time = 0.33, size = 22, normalized size = 1.16

$$\frac{2\,\sqrt{x}\,\sqrt{a+b\,x}}{a^2+b\,x\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(a + b*x)^(3/2)),x)

[Out] (2*x^(1/2)*(a + b*x)^(1/2))/(a^2 + a*b*x)

3.581 $\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{a\sqrt{x}\,\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

[Out] 2/a/x^(1/2)/(b*x+a)^(1/2)-4*(b*x+a)^(1/2)/a^2/x^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{2}{a\sqrt{x}\,\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(a + b*x)^(3/2)),x]

[Out] 2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{x}\,\sqrt{a+bx}}+\frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{x}\,\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

Mathics [A]
time = 2.62, size = 37, normalized size = 0.95

$$\frac{2\sqrt{b}\,(-a-2bx)\sqrt{\frac{a+bx}{bx}}}{a^2\,(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]')
[Out] 2 Sqrt[b] (-a - 2 b x) Sqrt[(a + b x) / (b x)] / (a ^ 2 (a + b x))
```

Maple [A]
time = 0.14, size = 33, normalized size = 0.85

method	result	size
gosper	$-\frac{2(2bx+a)}{\sqrt{x}\,\sqrt{bx+a}\,a^2}$	22
default	$-\frac{2}{a\sqrt{x}\,\sqrt{bx+a}}-\frac{4b\sqrt{x}}{a^2\sqrt{bx+a}}$	33
risch	$-\frac{2\sqrt{bx+a}}{a^2\sqrt{x}}-\frac{2b\sqrt{x}}{a^2\sqrt{bx+a}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/a/x^(1/2)/(b*x+a)^(1/2)-4*b/a^2*x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.26, size = 32, normalized size = 0.82

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}\,a^2}-\frac{2\,\sqrt{bx+a}}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) - 2*sqrt(b*x + a)/(a^2*sqrt(x))
```

Fricas [A]
time = 0.30, size = 34, normalized size = 0.87

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [A]
time = 0.88, size = 41, normalized size = 1.05

$$-\frac{2}{a\sqrt{b}\,x\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.00, size = 73, normalized size = 1.87

$$2\left(-\frac{\frac{1}{2}\cdot 2b\sqrt{x}\,\sqrt{a+bx}}{a^2\,(a+bx)}+\frac{4\sqrt{b}}{2a\left(\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2-a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) + 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x
+ a))^2 - a)*a)
```

Mupad [B]
time = 0.39, size = 39, normalized size = 1.00

$$-\frac{2\,a\,\sqrt{a+b\,x}+4\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(a^3+b\,x\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a + b*x)^(1/2) + 4*b*x*(a + b*x)^(1/2))/(x^(1/2)*(a^3 + a^2*b*x))
```

3.582 $\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{ax^{3/2}\sqrt{a+bx}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/a/x^(3/2)/(b*x+a)^(1/2)-8/3*(b*x+a)^(1/2)/a^2/x^(3/2)+16/3*b*(b*x+a)^(1/2
)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a + b*x]) - (8*Sqrt[a + b*x])/(3*a^2*x^(3/2)) + (16*b*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a+bx}} + \frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} - \frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} + \frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}$$

Mathematica [A]
time = 0.08, size = 38, normalized size = 0.60

$$-\frac{2(a^2-4abx-8b^2x^2)}{3a^3x^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

Mathics [A]
time = 5.20, size = 73, normalized size = 1.16

$$\frac{2\sqrt{b}\,(-a^3+3a^2bx+12ab^2x^2+8b^3x^3)\sqrt{\frac{a+bx}{bx}}}{3a^3x\,(a^2+2abx+b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]')
[Out] 2 Sqrt[b] (-a ^ 3 + 3 a ^ 2 b x + 12 a b ^ 2 x ^ 2 + 8 b ^ 3 x ^ 3) Sqrt[(a
 + b x) / (b x)] / (3 a ^ 3 x (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.87

method	result	size
gosper	$-\frac{2(-8x^2b^2-4abx+a^2)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^3}$	33
risch	$-\frac{2\sqrt{bx+a}\,(-5bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{2b^2\sqrt{x}}{a^3\sqrt{bx+a}}$	41

default	$-\frac{2}{3 a x^{\frac{3}{2}} \sqrt{b x + a}} - \frac{4 b \left(-\frac{2}{a \sqrt{x}\, \sqrt{b x + a}} - \frac{4 b \sqrt{x}}{a^{2} \sqrt{b x + a}}\right)}{3 a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(b*x+a)^(1/2)-4/3*b/a*(-2/a/x^(1/2)/(b*x+a)^(1/2)-4*b/a^2*x^
(1/2)/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.25, size = 50, normalized size = 0.79

$$\frac{2 b^{2} \sqrt{x}}{\sqrt{b x + a}\, a^{3}} + \frac{2 \left(\frac{6 \sqrt{b x + a}\, b}{\sqrt{x}} - \frac{(b x + a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3 a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) + 2/3*(6*sqrt(b*x + a)*b/sqrt(x) - (b*x +
 a)^(3/2)/x^(3/2))/a^3
```

Fricas [A]
time = 0.30, size = 49, normalized size = 0.78

$$\frac{2 \left(8 b^{2} x^{2} + 4 a b x - a^{2}\right) \sqrt{b x + a}\, \sqrt{x}}{3 \left(a^{3} b x^{3} + a^{4} x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(58) = 116$.
time = 2.50, size = 219, normalized size = 3.48

$$-\frac{2 a^{3} b^{\frac{9}{2}} \sqrt{\frac{a}{b x} + 1}}{3 a^{5} b^{4} x + 6 a^{4} b^{5} x^{2} + 3 a^{3} b^{6} x^{3}} + \frac{6 a^{2} b^{\frac{11}{2}} x \sqrt{\frac{a}{b x} + 1}}{3 a^{5} b^{4} x + 6 a^{4} b^{5} x^{2} + 3 a^{3} b^{6} x^{3}} + \frac{24 a b^{\frac{13}{2}} x^{2} \sqrt{\frac{a}{b x} + 1}}{3 a^{5} b^{4} x + 6 a^{4} b^{5} x^{2} + 3 a^{3} b^{6} x^{3}} + \frac{16 b^{\frac{15}{2}} x^{3} \sqrt{\frac{a}{b x} + 1}}{3 a^{5} b^{4} x + 6 a^{4} b^{5} x^{2} + 3 a^{3} b^{6} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a*
*3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**
```

```
4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*
a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x**3*sqrt
(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(47) = 94$.

time = 0.01, size = 152, normalized size = 2.41

$$2\left(\frac{\frac{1}{2}\cdot 2b^2\sqrt{x}\,\sqrt{a+bx}}{a^3\,(a+bx)}+\frac{2\left(-3b\sqrt{b}\,\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^4+12b\sqrt{b}\,\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2a-5b\sqrt{b}\,a^2\right)}{3a^2\left(\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)^2-a\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) - 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(
b*x + a))^4 - 12*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 5*a^2*b^(3
/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^2)
```

Mupad [B]

time = 0.41, size = 46, normalized size = 0.73

$$\frac{\sqrt{a+b\,x}\,\left(\frac{8\,x}{3\,a^2}-\frac{2}{3\,a\,b}+\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(3/2)),x)
```

```
[Out] ((a + b*x)^(1/2)*((8*x)/(3*a^2) - 2/(3*a*b) + (16*b*x^2)/(3*a^3)))/(x^(5/2)
 + (a*x^(3/2))/b)
```

3.583 $\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}$$

```
[Out] 2/a/x^(5/2)/(b*x+a)^(1/2)-12/5*(b*x+a)^(1/2)/a^2/x^(5/2)+16/5*b*(b*x+a)^(1/
2)/a^3/x^(3/2)-32/5*b^2*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(5/2)*Sqrt[a + b*x]) - (12*Sqrt[a + b*x])/(5*a^2*x^(5/2)) + (16*b*Sq
rt[a + b*x])/(5*a^3*x^(3/2)) - (32*b^2*Sqrt[a + b*x])/(5*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{5/2}\sqrt{a+bx}} + \frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} - \frac{(24b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a^2}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} + \frac{(16b^2)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{5a^3}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} - \frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.56

$$-\frac{2\left(a^3-2a^2bx+8ab^2x^2+16b^3x^3\right)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

Mathics [A]
time = 10.44, size = 99, normalized size = 1.14

$$\frac{2\sqrt{b}\left(-a^5-5a^3b^2x^2-30a^2b^3x^3-40ab^4x^4-16b^5x^5\right)\sqrt{\frac{a+bx}{bx}}}{5a^4x^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]')
```

```
[Out] 2 Sqrt[b] (-a ^ 5 - 5 a ^ 3 b ^ 2 x ^ 2 - 30 a ^ 2 b ^ 3 x ^ 3 - 40 a b ^ 4
 x ^ 4 - 16 b ^ 5 x ^ 5) Sqrt[(a + b x) / (b x)] / (5 a ^ 4 x ^ 2 (a ^ 3 +
3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.13, size = 77, normalized size = 0.89

method	result	size

gosper	$-\frac{2\left(16 b^{3} x^{3}+8 a \,b^{2} x^{2}-2 a^{2} b x +a^{3}\right)}{5 x^{\frac{5}{2}} \sqrt{b x +a}\, a^{4}}$	44
risch	$-\frac{2 \sqrt{b x +a}\, \left(11 x^{2} b^{2}-3 a b x +a^{2}\right)}{5 a^{4} x^{\frac{5}{2}}}-\frac{2 b^{3} \sqrt{x}}{a^{4} \sqrt{b x +a}}$	52
default	$-\frac{2}{5 a \,x^{\frac{5}{2}} \sqrt{b x +a}}-\frac{6 b \left(-\frac{2}{3 a \,x^{\frac{3}{2}} \sqrt{b x +a}}-\frac{4 b \left(-\frac{2}{a \sqrt{x}\, \sqrt{b x +a}}-\frac{4 b \sqrt{x}}{a^{2} \sqrt{b x +a}}\right)}{3 a}\right)}{5 a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/a/x^(5/2)/(b*x+a)^(1/2)-6/5*b/a*(-2/3/a/x^(3/2)/(b*x+a)^(1/2)-4/3*b/a*
(-2/a/x^(1/2)/(b*x+a)^(1/2)-4*b/a^2*x^(1/2)/(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.26, size = 64, normalized size = 0.74

$$-\frac{2 \, b^{3} \sqrt{x}}{\sqrt{b x + a} a^{4}} - \frac{2 \, {\left(\frac{15 \, \sqrt{b x + a} b^{2}}{\sqrt{x}} - \frac{5 \, {\left(b x + a\right)}^{\frac{3}{2}} b}{x^{\frac{3}{2}}} + \frac{{\left(b x + a\right)}^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}}{5 \, a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) - 2/5*(15*sqrt(b*x + a)*b^2/sqrt(x) - 5*
(b*x + a)^(3/2)*b/x^(3/2) + (b*x + a)^(5/2)/x^(5/2))/a^4
```

Fricas [A]
time = 0.31, size = 58, normalized size = 0.67

$$-\frac{2 \, {\left(16 \, b^{3} x^{3} + 8 \, a b^{2} x^{2} - 2 \, a^{2} b x + a^{3}\right)} \sqrt{b x + a} \sqrt{x}}{5 \, {\left(a^{4} b x^{4} + a^{5} x^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(82) = 164$.
time = 8.27, size = 348, normalized size = 4.00

$$-\frac{2 a^{5} b^{\frac{19}{2}} \sqrt{\frac{a}{b x} + 1}}{5 a^{7} b^{9} x^{2} + 15 a^{6} b^{10} x^{3} + 15 a^{5} b^{11} x^{4} + 5 a^{4} b^{12} x^{5}} - \frac{10 a^{3} b^{\frac{23}{2}} x^{2} \sqrt{\frac{a}{b x} + 1}}{5 a^{7} b^{9} x^{2} + 15 a^{6} b^{10} x^{3} + 15 a^{5} b^{11} x^{4} + 5 a^{4} b^{12} x^{5}} - \frac{60 a^{2} b^{\frac{25}{2}} x^{3} \sqrt{\frac{a}{b x} + 1}}{5 a^{7} b^{9} x^{2} + 15 a^{6} b^{10} x^{3} + 15 a^{5} b^{11} x^{4} + 5 a^{4} b^{12} x^{5}} - \frac{80 a b^{\frac{27}{2}} x^{4} \sqrt{\frac{a}{b x} + 1}}{5 a^{7} b^{9} x^{2} + 15 a^{6} b^{10} x^{3} + 15 a^{5} b^{11} x^{4} + 5 a^{4} b^{12} x^{5}} - \frac{32 b^{\frac{29}{2}} x^{5} \sqrt{\frac{a}{b x} + 1}}{5 a^{7} b^{9} x^{2} + 15 a^{6} b^{10} x^{3} + 15 a^{5} b^{11} x^{4} + 5 a^{4} b^{12} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3
+ 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqrt(a/(
b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a
**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**
2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 80*a*b**
(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a
**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(
5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x
**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(65) = 130.
time = 0.02, size = 233, normalized size = 2.68

$$2\left(-\frac{\frac{1}{2}\cdot 2b^{3}\sqrt{x}\sqrt{a+bx}}{a^{4}(a+bx)}+\frac{2\left(5b^{2}\sqrt{b}\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^{8}-30b^{2}\sqrt{b}\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^{6}a+80b^{2}\sqrt{b}\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^{4}a^{2}-50b^{2}\sqrt{b}\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^{2}a^{3}+11b^{2}\sqrt{b}\,a^{4}\right)}{5a^{3}\left(\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^{2}-a\right)^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) + 4/5*(5*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^8 - 30*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 80*a^2*b^
(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 50*a^3*b^(5/2)*(sqrt(b)*sqrt(x)
 - sqrt(b*x + a))^2 + 11*a^4*b^(5/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2
 - a)^5*a^3)
```

Mupad [B]
time = 0.43, size = 58, normalized size = 0.67

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a\,b}-\frac{4\,x}{5\,a^2}+\frac{16\,b\,x^2}{5\,a^3}+\frac{32\,b^2\,x^3}{5\,a^4}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(3/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(5*a*b) - (4*x)/(5*a^2) + (16*b*x^2)/(5*a^3) + (32*b^2
*x^3)/(5*a^4)))/(x^(7/2) + (a*x^(5/2))/b)
```

3.584 $\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}$$

```
[Out] -2/3*x^(5/2)/b/(b*x+a)^(3/2)-5*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(
7/2)-10/3*x^(3/2)/b^2/(b*x+a)^(1/2)+5*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a + b*x]) + (
5*Sqrt[x]*Sqrt[a + b*x])/b^3 - (5*a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx = -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b^2}$$

$$= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{(5a)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{2b^3}$$

$$= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{(5a)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{\ }\right.}{b^3}$$

$$= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{(5a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+b}}\right.}{b^3}$$

$$= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}$$

Mathematica [A]

time = 0.13, size = 72, normalized size = 0.79

$$\frac{\sqrt{x}\,(15a^2+20abx+3b^2x^2)}{3b^3(a+bx)^{3/2}}+\frac{5a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*(15*a^2 + 20*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2)) + (5*a*Lo
g[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(7/2)
```

Mathics [A]
time = 8.51, size = 131, normalized size = 1.44

$$\frac{-15a^{\frac{7}{2}}b^{\frac{17}{2}}\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+15a^2b^9\sqrt{x}\,(a+bx)-15a^{\frac{5}{2}}b^{\frac{19}{2}}x\text{ArcSinh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}+20ab^{10}x^{\frac{3}{2}}(a+bx)+3b^{11}x^{\frac{5}{2}}(a+bx)}{3a^{\frac{3}{2}}b^{12}\left(\frac{a+bx}{a}\right)^{\frac{3}{2}}(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(a + b*x)^(5/2),x]')
```

```
[Out] (-15 a ^ (7 / 2) b ^ (17 / 2) ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x)
 / a) ^ (3 / 2) + 15 a ^ 2 b ^ 9 Sqrt[x] (a + b x) - 15 a ^ (5 / 2) b ^ (19
 / 2) x ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((a + b x) / a) ^ (3 / 2) + 20 a
 b ^ 10 x ^ (3 / 2) (a + b x) + 3 b ^ 11 x ^ (5 / 2) (a + b x)) / (3 a ^ (3
 / 2) b ^ 12 ((a + b x) / a) ^ (3 / 2) (a + b x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(67) = 134$.
time = 0.14, size = 147, normalized size = 1.62

method	result
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b^3}+\frac{\left(-\frac{5a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{7}{2}}}-\frac{2a^2\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{3b^5\left(x+\frac{a}{b}\right)^2}+\frac{14a\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x\right.}}{3b^4\left(x+\frac{a}{b}\right)}\right.}{\sqrt{x}\,\sqrt{bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^3+(-5/2/b^(7/2)*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))-2/3/b^5*a^2/(x+a/b)^2*((x+a/b)^2*b-a*(x+a/b))^(1/2)+14/3/b^4*a/(x+
a/b)*((x+a/b)^2*b-a*(x+a/b))^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 109, normalized size = 1.20

$$\frac{2\,ab^2+\frac{10\,(bx+a)ab}{x}-\frac{15\,(bx+a)^2a}{x^2}}{3\left(\frac{(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{5\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x + a)*a*b/x - 15*(b*x + a)^2*a/x^2)/((b*x + a)^(3/2)*
b^4/x^(3/2) - (b*x + a)^(5/2)*b^3/x^(5/2)) + 5/2*a*log(-(sqrt(b) - sqrt(b*x
 + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 214, normalized size = 2.35

$$\left[\frac{15\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(3\,b^3x^2+20\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{6\,(b^6x^2+2\,ab^5x+a^2b^4)},\ \frac{15\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(3\,b^3x^2+20\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{3\,(b^6x^2+2\,ab^5x+a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(85) = 170$.
time = 5.02, size = 396, normalized size = 4.35

$$-\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a)
)/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*
x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 + b*x/a)*a
sinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x
```

```
/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 15*a**40*b**(45/2)
*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(5
3/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**
(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 +
 b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1
+ b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 138, normalized size = 1.52

$$2\left(\frac{2\left(\left(\left(\frac{\frac{1}{36}\cdot 9b^4a\sqrt{x}\sqrt{x}}{b^5a}+\frac{\frac{1}{36}\cdot 60b^3a^2}{b^5a}\right)\sqrt{x}\sqrt{x}+\frac{\frac{1}{36}\cdot 45b^2a^3}{b^5a}\right)\sqrt{x}\sqrt{a+bx}}{(a+bx)^2}+\frac{10a\ln\left|\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right|}{4b^3\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x)
[Out] 1/3*(x*(3*x/b + 20*a/b^2) + 15*a^2/b^3)*sqrt(x)/(b*x + a)^(3/2) + 5*a*log(a
bs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(5/2),x)
[Out] int(x^(5/2)/(a + b*x)^(5/2), x)
```

3.585 $\int \frac{x^{3/2}}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=69

$$-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

```
[Out] -2/3*x^(3/2)/b/(b*x+a)^(3/2)+2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/
2)-2*x^(1/2)/b^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a + b*x]) + (2*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}+\frac{\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{b^2}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 60, normalized size = 0.87

$$-\frac{2\sqrt{x}\,(3a+4bx)}{3b^2(a+bx)^{3/2}}-\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[x]*(3*a + 4*b*x))/(3*b^2*(a + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]])/b^(5/2)
```

Mathics [A]

time = 5.87, size = 102, normalized size = 1.48

$$\frac{2\left(3ab^{\frac{9}{2}}\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\sqrt{\frac{a+bx}{a}}-3\sqrt{a}\,b^5\sqrt{x}+3b^{\frac{11}{2}}x\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\sqrt{\frac{a+bx}{a}}-\frac{4b^6x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^7\sqrt{\frac{a+bx}{a}}\,(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a + b*x)^(5/2),x]')
```

```
[Out] 2 (3 a b ^ (9 / 2) ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[a]] Sqrt[(a + b x) / a] -
 3 Sqrt[a] b ^ 5 Sqrt[x] + 3 b ^ (11 / 2) x ArcSinh[Sqrt[b] Sqrt[x] / Sqrt[
a]] Sqrt[(a + b x) / a] - 4 b ^ 6 x ^ (3 / 2) / Sqrt[a]) / (3 b ^ 7 Sqrt[(a
 + b x) / a] (a + b x))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x+a)^(5/2),x)
```

Maxima [A]
time = 0.35, size = 69, normalized size = 1.00

$$-\frac{2\left(b+\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}b^2}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 186, normalized size = 2.70

$$\left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,(4\,b^2x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{3\,(b^5x^2+2\,ab^4x+a^2b^3)},\,-\frac{2\left(3\,(b^2x^2+2\,abx+a^2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(4\,b^2x+3\,ab)\sqrt{bx+a}\,\sqrt{x}\right)}{3\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(63) = 126$.

time = 2.15, size = 328, normalized size = 4.75

$$\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] 6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/
(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x*
*(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a)*asin
h(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a)
 + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(23/2)*x**
14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)
*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2
)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/
a))
```

Giac [A]

time = 0.01, size = 106, normalized size = 1.54

$$2\left(\frac{2\left(-\frac{\frac{1}{18}\cdot 12b^2a\sqrt{x}\sqrt{x}}{b^3a}-\frac{\frac{1}{18}\cdot 9ba^2}{b^3a}\right)\sqrt{x}\sqrt{a+bx}}{\left(a+bx\right)^2}-\frac{\ln\left|\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right|}{b^2\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*sqrt(x)*(4*x/b + 3*a/b^2)/(b*x + a)^(3/2) - 2*log(abs(-sqrt(b)*sqrt(x)
 + sqrt(b*x + a)))/b^(5/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(5/2),x)
[Out] int(x^(3/2)/(a + b*x)^(5/2), x)
```

3.586 $\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

```
[Out] 2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx = \frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Mathics [A]
time = 2.55, size = 26, normalized size = 1.24

$$\frac{2x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{\frac{a+bx}{a}}\,(a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(a + b*x)^(5/2),x]')
```

```
[Out] 2 x ^ (3 / 2) / (3 a ^ (3 / 2) Sqrt[(a + b x) / a] (a + b x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.12, size = 54, normalized size = 2.57

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3a(bx+a)^{\frac{3}{2}}}$	16
default	$-\frac{\sqrt{x}}{b(bx+a)^{\frac{3}{2}}}+\frac{a\left(\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{2b}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*x^(1/2)/(b*x+a)^(3/2)+1/2*a/b*(2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)
/a^2/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.25, size = 15, normalized size = 0.71

$$\frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.30, size = 33, normalized size = 1.57

$$\frac{2\sqrt{bx+a}\,x^{\frac{3}{2}}}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.
time = 0.78, size = 42, normalized size = 2.00

$$\frac{2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] 2*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
```

Giac [A]
time = 0.01, size = 42, normalized size = 2.00

$$\frac{\frac{1}{18}\cdot 12b\sqrt{x}\,\sqrt{x}\,\sqrt{x}\,\sqrt{a+bx}}{ba\left(a+bx\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

Mupad [B]
time = 0.24, size = 36, normalized size = 1.71

$$\frac{2\,x^{3/2}\,\sqrt{a+b\,x}}{3\,\left(a^3+2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*x^(3/2)*(a + b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 + 2*a^2*b*x))
```

3.587 $\int \frac{1}{\sqrt{x}\,(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}$$

```
[Out] 2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {47, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx)^{5/2}}\,dx&=\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}\,(a+bx)^{3/2}}\,dx}{3a}\\&=\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

Mathics [A]
time = 3.44, size = 38, normalized size = 0.88

$$\frac{2\sqrt{b}\,x\,(3a+2bx)\sqrt{\frac{a+bx}{bx}}}{3a^2\,(a+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]')
```

```
[Out] 2 Sqrt[b] x (3 a + 2 b x) Sqrt[(a + b x) / (b x)] / (3 a ^ 2 (a + b x) ^ 2)
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.74

method	result	size
gosper	$\frac{2\sqrt{x}\,(2bx+3a)}{3(bx+a)^{\frac{3}{2}}a^2}$	24
default	$\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{bx+a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.26, size = 27, normalized size = 0.63

$$-\frac{2\left(b-\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b - 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*a^2)
```

Fricas [A]
time = 0.31, size = 43, normalized size = 1.00

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.
time = 1.08, size = 92, normalized size = 2.14

$$\frac{6a}{3a^3\sqrt{b}\,\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}+\frac{4bx}{3a^3\sqrt{b}\,\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)
) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x
) + 1))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(31) = 62.
time = 0.01, size = 102, normalized size = 2.37

$$-\frac{32b\sqrt{b}\,b\left(-3\left(\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right)^2-ab\right)}{2\cdot 6\,|b|\left(\left(\sqrt{-ab+b\,(a+bx)}-\sqrt{b}\,\sqrt{a+bx}\right)^2+ab\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(5/2)/(
((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*abs(b))
```

Mupad [B]
time = 0.40, size = 54, normalized size = 1.26

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a+b\,x}+4\,b\,x^{3/2}\,\sqrt{a+b\,x}}{3\,a^4+6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a + b*x)^(1/2) + 4*b*x^(3/2)*(a + b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 + 6*a^3*b*x)
```

3.588 $\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=64

$$\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/3/a/(b*x+a)^(3/2)/x^(1/2)+8/3/a^2/x^(1/2)/(b*x+a)^(1/2)-16/3*(b*x+a)^(1/2
)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a + b*x]) - (16*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}} + \frac{8\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.62

$$-\frac{2\,(3a^2+12abx+8b^2x^2)}{3a^3\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

Mathics [A]
time = 4.74, size = 58, normalized size = 0.91

$$\frac{\sqrt{b}\,\left(-2a^2-8abx-\frac{16b^2x^2}{3}\right)\sqrt{\frac{a+bx}{bx}}}{a^3\,(a^2+2abx+b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]')
[Out] Sqrt[b] (-2 a ^ 2 - 8 a b x - 16 b ^ 2 x ^ 2 / 3) Sqrt[(a + b x) / (b x)] /
 (a ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.14, size = 54, normalized size = 0.84

method	result	size
gosper	$-\frac{2(8x^2b^2+12abx+3a^2)}{3\sqrt{x}\,(bx+a)^{\frac{3}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}}{a^3\sqrt{x}} - \frac{2b(5bx+6a)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}a^3}$	41

default	$-\frac{2}{a(bx+a)^{\frac{3}{2}}\sqrt{x}}-\frac{4b\left(\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/(b*x+a)^(3/2)/x^(1/2)-4*b/a*(2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a
^2/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 0.72

$$\frac{2\left(b^{2}-\frac{6\,(bx+a)b}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^{3}}-\frac{2\,\sqrt{bx+a}}{a^{3}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x + a)*b/x)*x^(3/2)/((b*x + a)^(3/2)*a^3) - 2*sqrt(b*x + a)
/(a^3*sqrt(x))
```

Fricas [A]
time = 0.31, size = 58, normalized size = 0.91

$$-\frac{2\,(8\,b^{2}x^{2}+12\,abx+3\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{3\,(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(58) = 116$.
time = 2.49, size = 153, normalized size = 2.39

$$-\frac{6a^{2}b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x+3a^{3}b^{6}x^{2}}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x+3a^{3}b^{6}x^{2}}-\frac{16b^{\frac{13}{2}}x^{2}\sqrt{\frac{a}{bx}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x+3a^{3}b^{6}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b*
*6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x + 3*a**3*b**6*x**2)
```

Giac [A]
time = 0.01, size = 115, normalized size = 1.80

$$2\left(\frac{2\left(-\frac{\frac{1}{18}\cdot 15b^3a^2\sqrt{x}\sqrt{x}}{ba^5}-\frac{\frac{1}{18}\cdot 18b^2a^3}{ba^5}\right)\sqrt{x}\sqrt{a+bx}}{(a+bx)^2}+\frac{4\sqrt{b}}{2a^2\left(\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^2-a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*sqrt(x)*(5*b^2*x/a^3 + 6*b/a^2)/(b*x + a)^(3/2) + 4*sqrt(b)/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)*a^2)
```

Mupad [B]
time = 0.42, size = 71, normalized size = 1.11

$$-\frac{6\,a^2\,\sqrt{a+b\,x}+16\,b^2\,x^2\,\sqrt{a+b\,x}+24\,a\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b+3\,x\,a^3\,b^2\right)+3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(6*a^2*(a + b*x)^(1/2) + 16*b^2*x^2*(a + b*x)^(1/2) + 24*a*b*x*(a + b*x)^(
1/2))/(x^(1/2)*(x*(6*a^4*b + 3*a^3*b^2*x) + 3*a^5))
```

3.589 $\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}$$

```
[Out] 2/3/a/x^(3/2)/(b*x+a)^(3/2)+4/a^2/x^(3/2)/(b*x+a)^(1/2)-16/3*(b*x+a)^(1/2)/
a^3/x^(3/2)+32/3*b*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a + b*x]) - (16*Sqrt[
a + b*x])/(3*a^3*x^(3/2)) + (32*b*Sqrt[a + b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3x^{3/2}} - \frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^3}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3x^{3/2}} + \frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}$$

Mathematica [A]
time = 0.08, size = 49, normalized size = 0.58

$$-\frac{2\left(a^3-6a^2bx-24ab^2x^2-16b^3x^3\right)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 - 6*a^2*b*x - 24*a*b^2*x^2 - 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

Mathics [A]
time = 7.83, size = 95, normalized size = 1.13

$$\frac{2\sqrt{b}\,\left(-a^4+5a^3bx+30a^2b^2x^2+40ab^3x^3+16b^4x^4\right)\sqrt{\frac{a+bx}{bx}}}{3a^4x\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]')
```

```
[Out] 2 Sqrt[b] (-a ^ 4 + 5 a ^ 3 b x + 30 a ^ 2 b ^ 2 x ^ 2 + 40 a b ^ 3 x ^ 3 +
 16 b ^ 4 x ^ 4) Sqrt[(a + b x) / (b x)] / (3 a ^ 4 x (a ^ 3 + 3 a ^ 2 b x
+ 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.13, size = 76, normalized size = 0.90

method	result	size

gosper	$-\frac{2\left(-16b^{3}x^{3}-24a\,b^{2}x^{2}-6a^{2}bx+a^{3}\right)}{3x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}a^{4}}$	44
risch	$-\frac{2\sqrt{bx+a}\,\left(-8bx+a\right)}{3a^{4}x^{\frac{3}{2}}}+\frac{2b^{2}\left(8bx+9a\right)\sqrt{x}}{3\left(bx+a\right)^{\frac{3}{2}}a^{4}}$	49
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\left(bx+a\right)^{\frac{3}{2}}}-\frac{2b\left(-\frac{2}{a\left(bx+a\right)^{\frac{3}{2}}\sqrt{x}}-\frac{4b\left(\frac{2\sqrt{x}}{3a\left(bx+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{a}\right)}{a}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(b*x+a)^(3/2)-2*b/a*(-2/a/(b*x+a)^(3/2)/x^(1/2)-4*b/a*(2/3*x
^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.76

$$\frac{2\left(\frac{9\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{\left(bx+a\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^{4}}-\frac{2\left(b^{3}-\frac{9\left(bx+a\right)b^{2}}{x}\right)x^{\frac{3}{2}}}{3\left(bx+a\right)^{\frac{3}{2}}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(9*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^4 - 2/3*(b^3 -
9*(b*x + a)*b^2/x)*x^(3/2)/((b*x + a)^(3/2)*a^4)
```

Fricas [A]
time = 0.32, size = 71, normalized size = 0.85

$$\frac{2\left(16\,b^{3}x^{3}+24\,ab^{2}x^{2}+6\,a^{2}bx-a^{3}\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(78) = 156$.
time = 4.66, size = 337, normalized size = 4.01

$$-\frac{2a^{4}b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^{7}b^{9}x+9a^{6}b^{10}x^{2}+9a^{5}b^{11}x^{3}+3a^{4}b^{12}x^{4}}+\frac{10a^{3}b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^{7}b^{9}x+9a^{6}b^{10}x^{2}+9a^{5}b^{11}x^{3}+3a^{4}b^{12}x^{4}}+\frac{60a^{2}b^{\frac{23}{2}}x^{2}\sqrt{\frac{a}{bx}+1}}{3a^{7}b^{9}x+9a^{6}b^{10}x^{2}+9a^{5}b^{11}x^{3}+3a^{4}b^{12}x^{4}}+\frac{80ab^{\frac{25}{2}}x^{3}\sqrt{\frac{a}{bx}+1}}{3a^{7}b^{9}x+9a^{6}b^{10}x^{2}+9a^{5}b^{11}x^{3}+3a^{4}b^{12}x^{4}}+\frac{32b^{\frac{27}{2}}x^{4}\sqrt{\frac{a}{bx}+1}}{3a^{7}b^{9}x+9a^{6}b^{10}x^{2}+9a^{5}b^{11}x^{3}+3a^{4}b^{12}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*
a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x) + 1
)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**
4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10
*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**3*sqrt(a
/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4
*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*
b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)
```

Giac [A]
time = 0.01, size = 189, normalized size = 2.25

$$2\left(\frac{2\left(\frac{\frac{1}{18}\cdot 24b^4a^3\sqrt{x}\sqrt{x}}{ba^7}+\frac{\frac{1}{18}\cdot 27b^3a^4}{ba^7}\right)\sqrt{x}\sqrt{a+bx}}{(a+bx)^2}+\frac{2\left(-6b\sqrt{b}\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^4+18b\sqrt{b}\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^2a-8b\sqrt{b}a^2\right)}{3a^3\left(\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)^2-a\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3*sqrt(x)*(8*b^3*x/a^4 + 9*b^2/a^3)/(b*x + a)^(3/2) - 8/3*(3*b^(3/2)*(sqr
t(b)*sqrt(x) - sqrt(b*x + a))^4 - 9*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x +
 a))^2 + 4*a^2*b^(3/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^3)
```

Mupad [B]
time = 0.47, size = 88, normalized size = 1.05

$$\frac{32\,b^3\,x^3\,\sqrt{a+b\,x}-2\,a^3\,\sqrt{a+b\,x}+12\,a^2\,b\,x\,\sqrt{a+b\,x}+48\,a\,b^2\,x^2\,\sqrt{a+b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b+3\,x\,a^4\,b^2\right)+3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (32*b^3*x^3*(a + b*x)^(1/2) - 2*a^3*(a + b*x)^(1/2) + 12*a^2*b*x*(a + b*x)^
(1/2) + 48*a*b^2*x^2*(a + b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b + 3*a^4*b^2*x) +
 3*a^6))
```

3.590 $\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=105

$$-\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}$$

```
[Out] 5/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)-5/12*a*x^(3/2)*(-b*x
+a)^(1/2)/b^2-1/3*x^(5/2)*(-b*x+a)^(1/2)/b-5/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b
^3
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}-\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (-5*a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a - b*x])/(12*b^
2) - (x^(5/2)*Sqrt[a - b*x])/(3*b) + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
 - b*x]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{6b}\\
&= -\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a^2)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b^2}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a^3)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,\ldots\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\ldots\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.78

$$-\frac{\sqrt{x}\sqrt{a-bx}\,(15a^2+10abx+8b^2x^2)}{24b^3}+\frac{5a^3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{8(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] -1/24*(Sqrt[x]*Sqrt[a - b*x]*(15*a^2 + 10*a*b*x + 8*b^2*x^2))/b^3 + (5*a^3*
Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*(-b)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 18.66, size = 254, normalized size = 2.42

$$\text{Piecewise}\left[\left\{\left\{\frac{Ia^{\frac{3}{2}}\left(-15a^{\frac{3}{2}}b^{6}\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right](-a+bx)^{2}+15a^{3}b^{\frac{13}{2}}\sqrt{x}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-5a^{2}b^{\frac{15}{2}}x^{\frac{3}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-2ab^{\frac{17}{2}}x^{\frac{5}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-8b^{\frac{19}{2}}x^{\frac{7}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}\right)}{24b^{\frac{19}{2}}(-a+bx)^{2}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{5a^{3}\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{8b^{\frac{7}{2}}}-\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/Sqrt[a - b*x],x]')
```

```
[Out] Piecewise[{{I / 24 a ^ (3 / 2) (-15 a ^ (3 / 2) b ^ 6 ArcCosh[Sqrt[b] Sqrt[
x] / Sqrt[a]] (-a + b x) ^ 2 + 15 a ^ 3 b ^ (13 / 2) Sqrt[x] ((-a + b x) /
a) ^ (3 / 2) - 5 a ^ 2 b ^ (15 / 2) x ^ (3 / 2) ((-a + b x) / a) ^ (3 / 2)
- 2 a b ^ (17 / 2) x ^ (5 / 2) ((-a + b x) / a) ^ (3 / 2) - 8 b ^ (19 / 2)
x ^ (7 / 2) ((-a + b x) / a) ^ (3 / 2)) / (b ^ (19 / 2) (-a + b x) ^ 2), Ab
s[b x / a] > 1}}, 5 a ^ 3 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (8 b ^ (7 / 2
)) - 5 a ^ (5 / 2) Sqrt[x] / (8 b ^ 3 Sqrt[1 - b x / a]) + 5 a ^ (3 / 2) x
^ (3 / 2) / (24 b ^ 2 Sqrt[1 - b x / a]) + Sqrt[a] x ^ (5 / 2) / (12 b Sqrt
[1 - b x / a]) + x ^ (7 / 2) / (3 Sqrt[a] Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.12, size = 116, normalized size = 1.10

method	result
risch	$-\frac{(8x^2b^2+10abx+15a^2)\sqrt{x}\,\sqrt{-bx+a}}{24b^3}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$-\frac{x^{\frac{5}{2}}\sqrt{-bx+a}}{3b}+\frac{5a\left(-\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2b}+\frac{3a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,(x-\frac{a}{2b})}{\sqrt{-x^2b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4b}\right)}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^(5/2)*(-b*x+a)^(1/2)/b+5/6*a/b*(-1/2*x^(3/2)*(-b*x+a)^(1/2)/b+3/4*a/
b*(-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x
+a)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.35, size = 135, normalized size = 1.29

$$-\frac{5\,a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{6}-\frac{3\,(bx-a)b^{5}}{x}+\frac{3\,(bx-a)^{2}b^{4}}{x^{2}}-\frac{(bx-a)^{3}b^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) - 1/24*(33*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 15*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x - a)*b^5/x + 3*(b*x - a)^2*b^4/x^2 - (b*
x - a)^3*b^3/x^3)
```

Fricas [A]
time = 0.31, size = 141, normalized size = 1.34

$$\left[-\frac{15\,a^{3}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(8\,b^{3}x^{2}+10\,ab^{2}x+15\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^{4}},-\frac{15\,a^{3}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(8\,b^{3}x^{2}+10\,ab^{2}x+15\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(8*b^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/24
*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 + 10
*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [A]
time = 9.28, size = 270, normalized size = 2.57

$$\begin{cases}\frac{5ia^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}-\frac{ix^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}}+\frac{5a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(5/2)*sqrt(x)/(8*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(3/2)*x*
*(3/2)/(24*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(5/2)/(12*b*sqrt(-1 + b*x/
```

```
a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) - I*x**(7/2)/(3*
sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(5/2)*sqrt(x)/(8*b**3*sq
rt(1 - b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x*
*(5/2)/(12*b*sqrt(1 - b*x/a)) + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**
(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 131, normalized size = 1.25

$$2\left(2\left(\left(\left(-\frac{\frac{1}{288}\cdot 24b^4\sqrt{x}\,\sqrt{x}}{b^5}-\frac{\frac{1}{288}\cdot 30b^3a}{b^5}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{288}\cdot 45b^2a^2}{b^5}\right)\sqrt{x}\,\sqrt{a-bx}-\frac{10a^3\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{32b^3\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/24*sqrt(-b*x + a)*(2*x*(4*x/b + 5*a/b^2) + 15*a^2/b^3)*sqrt(x) - 5/8*a^3
*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(1/2), x)
```

3.591 $\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)-1/2*x^(3/2)*(-b*x+a)
^(1/2)/b-3/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^2) - (x^(3/2)*Sqrt[a - b*x])/(2*b) + (3*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b}\\
&= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a^2)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{8b^2}\\
&= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^2}\\
&= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^2}\\
&= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.89

$$
-\frac{\sqrt{x}\,\sqrt{a-bx}\,(3a+2bx)}{4b^2}-\frac{3a^2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{4(-b)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a - b*x],x]
[Out] -1/4*(Sqrt[x]*Sqrt[a - b*x]*(3*a + 2*b*x))/b^2 - (3*a^2*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[a - b*x]])/(4*(-b)^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.14, size = 195, normalized size = 2.44

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(-3a^{\frac{7}{2}}b^3\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}+3a^2b^{\frac{7}{2}}\sqrt{x}\,(-a+bx)+ab^{\frac{9}{2}}x^{\frac{3}{2}}(a-bx)+2b^{\frac{11}{2}}x^{\frac{5}{2}}(a-bx)\right)}{4a^{\frac{3}{2}}b^{\frac{11}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{3a^2\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{4b^{\frac{5}{2}}}-\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1-\frac{bx}{a}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1-\frac{bx}{a}}}\right]
$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^(3/2)/Sqrt[a - b*x],x]')

[Out] Piecewise[{{I / 4 (-3 a ^ (7 / 2) b ^ 3 ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] ((-a + b x) / a) ^ (3 / 2) + 3 a ^ 2 b ^ (7 / 2) Sqrt[x] (-a + b x) + a b ^ (9 / 2) x ^ (3 / 2) (a - b x) + 2 b ^ (11 / 2) x ^ (5 / 2) (a - b x)) / (a ^ (3 / 2) b ^ (11 / 2) ((-a + b x) / a) ^ (3 / 2)), Abs[b x / a] > 1}}, 3 a ^ 2 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (4 b ^ (5 / 2)) - 3 a ^ (3 / 2) Sqrt[x] / (4 b ^ 2 Sqrt[1 - b x / a]) + Sqrt[a] x ^ (3 / 2) / (4 b Sqrt[1 - b x / a]) + x ^ (5 / 2) / (2 Sqrt[a] Sqrt[1 - b x / a])]

Maple [A]
time = 0.13, size = 93, normalized size = 1.16

method	result	size
risch	$-\frac{(2bx+3a)\sqrt{x}\,\sqrt{-bx+a}}{4b^2}+\frac{3a^2\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	80
default	$-\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2b}+\frac{3a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4b}$	93

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*x^(3/2)*(-b*x+a)^(1/2)/b+3/4*a/b*(-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2)))

Maxima [A]
time = 0.34, size = 98, normalized size = 1.22

$$-\frac{3\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}-\frac{\frac{5\sqrt{-bx+a}\,a^2b}{\sqrt{x}}+\frac{3\,(-bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^4-\frac{2\,(bx-a)b^3}{x}+\frac{(bx-a)^2b^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")

[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) - 1/4*(5*sqrt(-b*x + a)*a^2*b/sqrt(x) + 3*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^4 - 2*(b*x - a)*b^3/x + (b*x - a)^2*b^2/x^2)

Fricas [A]
time = 0.32, size = 119, normalized size = 1.49

$$\left[-\frac{3\,a^{2}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x+3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{8\,b^{3}},-\frac{3\,a^{2}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^{2}x+3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{4\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(b)*arcta
n(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt
(x))/b^3]
```

Sympy [A]
time = 2.79, size = 214, normalized size = 2.68

$$\begin{cases} \frac{3ia^{\frac{3}{2}}\sqrt{x}}{4b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{2}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}-\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1-\frac{bx}{a}}}+\frac{3a^{2}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((3*I*a**(3/2)*sqrt(x)/(4*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(3
/2)/(4*b*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**
(5/2)) - I*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-3*a**(
3/2)*sqrt(x)/(4*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**(3/2)/(4*b*sqrt(1 - b*x/
a)) + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/(2*sqrt(
a)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 101, normalized size = 1.26

$$2\left(2\left(-\frac{\frac{1}{16}\cdot 2b^{2}\sqrt{x}\,\sqrt{x}}{b^{3}}-\frac{\frac{1}{16}\cdot 3ba}{b^{3}}\right)\sqrt{x}\,\sqrt{a-bx}-\frac{6a^{2}\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{16b^{2}\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/4*sqrt(-b*x + a)*sqrt(x)*(2*x/b + 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(-b)*s
qrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(1/2), x)
```

3.592 $\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

```
[Out] a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)-x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/
b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b}\\ &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.12

$$-\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a - b*x],x]
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])
/(-b)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.17, size = 135, normalized size = 2.70

$$\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{a}\left(-\sqrt{a}\,b\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right](-a+bx)+ab^{\frac{3}{2}}\sqrt{x}\,\sqrt{\frac{-a+bx}{a}}-b^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{\frac{-a+bx}{a}}\right)}{b^{\frac{5}{2}}(-a+bx)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{a\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{b^{\frac{3}{2}}}-\frac{\sqrt{a}\,\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/Sqrt[a - b*x],x]')
```

```
[Out] Piecewise[{{I Sqrt[a] (-Sqrt[a] b ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] (-a +
b x) + a b ^ (3 / 2) Sqrt[x] Sqrt[(-a + b x) / a] - b ^ (5 / 2) x ^ (3 / 2)
 Sqrt[(-a + b x) / a]) / (b ^ (5 / 2) (-a + b x)), Abs[b x / a] > 1}}, a Ar
cSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / b ^ (3 / 2) - Sqrt[a] Sqrt[x] Sqrt[1 - b
x / a] / b]
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.40

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	70
risch	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)
^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))
```

Maxima [A]
time = 0.34, size = 56, normalized size = 1.12

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{\sqrt{-bx+a}\,a}{\left(b^{2}-\frac{(bx-a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) - sqrt(-b*x + a)*a/((b^
2 - (b*x - a)*b/x)*sqrt(x))
```

Fricas [A]
time = 0.33, size = 93, normalized size = 1.86

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)+2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b^{2}},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\sqrt{-bx+a}\,b\sqrt{x}}{b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*s
qrt(-b*x + a)*b*sqrt(x))/b^2, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sq
rt(x))) + sqrt(-b*x + a)*b*sqrt(x))/b^2]
```

Sympy [A]
time = 1.19, size = 121, normalized size = 2.42

$$\begin{cases} \frac{i\sqrt{a}\sqrt{x}}{b\sqrt{-1+\frac{bx}{a}}} - \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{ix^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{\sqrt{a}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{b} + \frac{a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(a)*sqrt(x)/(b*sqrt(-1 + b*x/a)) - I*a*acosh(sqrt(b)*sqrt(
x)/sqrt(a))/b**(3/2) - I*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (-sqrt(a)*sqrt(x)*sqrt(1 - b*x/a)/b + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/b
**(3/2), True))
```

Giac [A]
time = 0.00, size = 67, normalized size = 1.34

$$2\left(-\frac{\frac{1}{4}\cdot 2\sqrt{x}\sqrt{a-bx}}{b} - \frac{2a\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{4b\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -a*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b) - sqrt(-b*x +
a)*sqrt(x)/b
```

Mupad [B]
time = 0.52, size = 47, normalized size = 0.94

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{b^{3/2}} - \frac{\sqrt{x}\,\sqrt{a-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(1/2),x)
```

```
[Out] (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^(1/2))))/b^(3/2) - (x^(1/2
)*(a - b*x)^(1/2))/b
```

3.593 $\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {65, 223, 209}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Mathematica [**A**]
time = 0.04, size = 35, normalized size = 1.21

$$-\frac{2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/Sqrt[-b]
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.20, size = 44, normalized size = 1.52

$$\text{Piecewise}\left[\left\{\left\{\frac{-2I\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{\sqrt{b}}, \text{Abs}\left[\frac{bx}{a}\right] > 1\right\}\right\}, \frac{2\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{\sqrt{b}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]')
```

```
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] / Sqrt[b], Abs[b x / a]
 > 1}}, 2 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / Sqrt[b]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.
time = 0.11, size = 51, normalized size = 1.76

method	result	size

default	$\frac{\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{\sqrt{x}\,\sqrt{-bx+a}\,\sqrt{b}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b
)/(-b*x^2+a*x)^(1/2))
```

Maxima [A]
time = 0.37, size = 21, normalized size = 0.72

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Fricas [A]
time = 0.31, size = 57, normalized size = 1.97

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)}{b},\,-\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)/b, -2*arctan
(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [A]
time = 0.54, size = 54, normalized size = 1.86

$$\begin{cases} -\frac{2i\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{2\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), Abs(b*x/a) > 1), (2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), True))
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.24

$$-\frac{2 \ln \left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -2*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/sqrt(-b)
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.93

$$-\frac{4 \operatorname{atan}\left(\frac{\sqrt{a-b\,x}-\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a - b*x)^(1/2) - a^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

3.594 $\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

```
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.19, size = 55, normalized size = 2.75

$$\text{Piecewise}\left[\left\{\left\{\frac{-2\sqrt{b}\sqrt{-1+\frac{a}{bx}}}{a}, \text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\}, \frac{-2I\sqrt{b}\sqrt{1-\frac{a}{bx}}}{a}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]')
[Out] Piecewise[{{-2 Sqrt[b] Sqrt[-1 + a / (b x)] / a, Abs[a / (b x)] > 1}}, -2 I
 Sqrt[b] Sqrt[1 - a / (b x)] / a]
```

Maple [A]
time = 0.13, size = 17, normalized size = 0.85

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
default	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
risch	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Maxima [A]
time = 0.25, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

Fricas [A]
time = 0.31, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

Sympy [A]
time = 0.51, size = 46, normalized size = 2.30

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(1/2),x)
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/a, Abs(a/(b*x)) > 1), (-2*I*sqrt(b)
*sqrt(-a/(b*x) + 1)/a, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.
time = 0.00, size = 40, normalized size = 2.00

$$\frac{8\sqrt{-b}}{2\left(\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x)
[Out] 4*sqrt(-b)/((sqrt(-b)*sqrt(x) - sqrt(-b*x + a))^2 - a)
```

Mupad [B]
time = 0.40, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{a-b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(1/2)),x)
[Out] -(2*(a - b*x)^(1/2))/(a*x^(1/2))
```

3.595 $\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{a-bx}}{3ax^{3/2}}-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}$$

```
[Out] -2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}+\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}\,(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.94, size = 174, normalized size = 3.78

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\,(-a-2bx)\sqrt{\frac{a-bx}{bx}}}{3a^2x},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2Ia^2b^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}{3a^3bx-3a^2b^2x^2}-\frac{2Iab^{\frac{5}{2}}x\sqrt{1-\frac{a}{bx}}}{3a^3bx-3a^2b^2x^2}+\frac{I4b^{\frac{7}{2}}x^2\sqrt{1-\frac{a}{bx}}}{3a^3bx-3a^2b^2x^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-a - 2 b x) Sqrt[(a - b x) / (b x)] / (3 a ^ 2 x), A
bs[a / (b x)] > 1}}, -2 I a ^ 2 b ^ (3 / 2) Sqrt[1 - a / (b x)] / (3 a ^ 3
b x - 3 a ^ 2 b ^ 2 x ^ 2) - 2 I a b ^ (5 / 2) x Sqrt[1 - a / (b x)] / (3 a
 ^ 3 b x - 3 a ^ 2 b ^ 2 x ^ 2) + I 4 b ^ (7 / 2) x ^ 2 Sqrt[1 - a / (b x)]
 / (3 a ^ 3 b x - 3 a ^ 2 b ^ 2 x ^ 2)]
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.76

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^2}$	23
risch	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^2}$	23
default	$-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Maxima [A]
time = 0.26, size = 32, normalized size = 0.70

$$-\frac{2\left(\frac{3\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^2
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.48

$$-\frac{2\,(2\,bx+a)\sqrt{-bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x + a)*sqrt(-b*x + a)/(a^2*x^(3/2))
```

Sympy [A]
time = 1.14, size = 177, normalized size = 3.85

$$\begin{cases} -\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{3ax}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{2ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2}+\frac{2iab^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2}-\frac{4ib^{\frac{7}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*a*x) - 4*b**(3/2)*sqrt(a/(b*x) -
 1)/(3*a**2), Abs(a/(b*x)) > 1), (2*I*a**2*b**(3/2)*sqrt(-a/(b*x) + 1)/(-3*
a**3*b*x + 3*a**2*b**2*x**2) + 2*I*a*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-3*a**3
*b*x + 3*a**2*b**2*x**2) - 4*I*b**(7/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**3*b*
x + 3*a**2*b**2*x**2), True))
```

Giac [A]
time = 0.01, size = 76, normalized size = 1.65

$$\frac{32\sqrt{-b}\,b\left(-3\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^2+a\right)}{2\cdot 6\left(\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -8/3*(3*(sqrt(-b)*sqrt(x) - sqrt(-b*x + a))^2 - a)*sqrt(-b)*b/((sqrt(-b)*sq
rt(x) - sqrt(-b*x + a))^2 - a)^3
```

Mupad [B]
time = 0.35, size = 26, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}+\frac{4\,b\,x}{3\,a^2}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) + (4*b*x)/(3*a^2))*(a - b*x)^(1/2))/x^(3/2)
```

3.596 $\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}$$

```
[Out] -15/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)+2*x^(5/2)/b/(-b*x+
a)^(1/2)+5/2*x^(3/2)*(-b*x+a)^(1/2)/b^2+15/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[a - b*x]) + (15*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^3) + (5*x
^(3/2)*Sqrt[a - b*x])/(2*b^2) - (15*a^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]])/(4*b^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \frac{2x^{5/2}}{b\sqrt{a-bx}} - \frac{5\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{b}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b^2}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a^2)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b^3}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x,\right.}{4b^3}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{\;}}{\sqrt{a-}}\right.}{4b^3}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}$$

Mathematica [A]

time = 0.14, size = 81, normalized size = 0.81

$$\frac{1}{4}\left(\frac{\sqrt{x}\,(15a^2-5abx-2b^2x^2)}{b^3\sqrt{a-bx}}-\frac{15a^2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] ((Sqrt[x]*(15*a^2 - 5*a*b*x - 2*b^2*x^2))/(b^3*Sqrt[a - b*x]) - (15*a^2*Log
[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2))/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.52, size = 201, normalized size = 2.01

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(15a^{\frac{7}{2}}b^6\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}-15a^2b^{\frac{13}{2}}\sqrt{x}\,(-a+bx)+5ab^{\frac{15}{2}}x^{\frac{3}{2}}(-a+bx)+2b^{\frac{17}{2}}x^{\frac{5}{2}}(-a+bx)\right)}{4a^{\frac{3}{2}}b^{\frac{19}{2}}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-15a^2\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]}{4b^{\frac{7}{2}}}+\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1-\frac{bx}{a}}}-\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(a - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 4 (15 a ^ (7 / 2) b ^ 6 ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]]
((-a + b x) / a) ^ (3 / 2) - 15 a ^ 2 b ^ (13 / 2) Sqrt[x] (-a + b x) + 5 a
 b ^ (15 / 2) x ^ (3 / 2) (-a + b x) + 2 b ^ (17 / 2) x ^ (5 / 2) (-a + b x
)) / (a ^ (3 / 2) b ^ (19 / 2) ((-a + b x) / a) ^ (3 / 2)), Abs[b x / a] >
1}}, -15 a ^ 2 ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / (4 b ^ (7 / 2)) + 15 a ^
 (3 / 2) Sqrt[x] / (4 b ^ 3 Sqrt[1 - b x / a]) - 5 Sqrt[a] x ^ (3 / 2) / (4
 b ^ 2 Sqrt[1 - b x / a]) - x ^ (5 / 2) / (2 Sqrt[a] b Sqrt[1 - b x / a])]
```

Maple [A]
time = 0.13, size = 127, normalized size = 1.27

method	result
risch	$\frac{(2bx+7a)\sqrt{x}\,\sqrt{-bx+a}}{4b^3}+\frac{\left(-\frac{15a^2\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{8b^{\frac{7}{2}}}-\frac{2a^2\sqrt{-\left(-\frac{a}{b}+x\right)^2b-a\left(-\frac{a}{b}+x\right)}}{b^4\left(-\frac{a}{b}+x\right)}\right)\sqrt{x\,(-}}{\sqrt{x}\,\sqrt{-bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*b*x+7*a)/b^3*x^(1/2)*(-b*x+a)^(1/2)+(-15/8/b^(7/2)*a^2*arctan(b^(1/2
)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))-2/b^4*a^2/(-a/b+x)*(-(-a/b+x)^2*b-a*(-a/b
+x))^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 118, normalized size = 1.18

$$\frac{8\,a^2b^2 - \frac{25\,(bx-a)a^2b}{x} + \frac{15\,(bx-a)^2a^2}{x^2}}{4\left(\frac{\sqrt{-bx+a}\,b^5}{\sqrt{x}} + \frac{2\,(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}} + \frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)} + \frac{15\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*b^2 - 25*(b*x - a)*a^2*b/x + 15*(b*x - a)^2*a^2/x^2)/(sqrt(-b*x
+ a)*b^5/sqrt(x) + 2*(-b*x + a)^(3/2)*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^
(5/2)) + 15/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 181, normalized size = 1.81

$$\left[-\frac{15\,(a^2bx-a^3)\sqrt{-b}\,\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(2\,b^3x^2+5\,ab^2x-15\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{8\,(b^5x-ab^4)}, \frac{15\,(a^2bx-a^3)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^3x^2+5\,ab^2x-15\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{4\,(b^5x-ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(a^2*b*x - a^3)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*s
qrt(x) + a) - 2*(2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/
(b^5*x - a*b^4), 1/4*(15*(a^2*b*x - a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqr
t(b)*sqrt(x))) + (2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))
/(b^5*x - a*b^4)]
```

Sympy [A]
time = 5.73, size = 224, normalized size = 2.24

$$\begin{cases} -\frac{15ia^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{-1+\frac{bx}{a}}} + \frac{5i\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{-1+\frac{bx}{a}}} + \frac{15ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1-\frac{bx}{a}}} - \frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1-\frac{bx}{a}}} - \frac{15a^2\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} - \frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-15*I*a**(3/2)*sqrt(x)/(4*b**3*sqrt(-1 + b*x/a)) + 5*I*sqrt(a)*x
**(3/2)/(4*b**2*sqrt(-1 + b*x/a)) + 15*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a)
)/(4*b**(7/2)) + I*x**(5/2)/(2*sqrt(a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 - b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**
2*sqrt(1 - b*x/a)) - 15*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) - x
**(5/2)/(2*sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 136, normalized size = 1.36

$$2\left(\frac{2\left(\left(\left(-\frac{\frac{1}{16}\cdot 2b^4\sqrt{x}\,\sqrt{x}}{b^5}-\frac{\frac{1}{16}\cdot 5b^3a}{b^5}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{16}\cdot 15b^2a^2}{b^5}\right)\sqrt{x}\,\sqrt{a-bx}}{a-bx}+\frac{30a^2\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{16b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] 1/4*sqrt(-b*x + a)*(x*(2*x/b + 5*a/b^2) - 15*a^2/b^3)*sqrt(x)/(b*x - a) + 1
5/4*a^2*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{{\left(a-b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(3/2), x)
```

3.597 $\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

```
[Out] -3*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)+2*x^(3/2)/b/(-b*x+a)^(1
/2)+3*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[a - b*x]) + (3*Sqrt[x]*Sqrt[a - b*x])/b^2 - (3*a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{a-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b} \\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{(3a)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{(3a)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{3a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 64, normalized size = 0.90

$$
\frac{\sqrt{x}\,(3a-bx)}{b^2\sqrt{a-bx}} + \frac{3a\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(3*a - b*x))/(b^2*Sqrt[a - b*x]) + (3*a*Log[-(Sqrt[-b]*Sqrt[x]) +
Sqrt[a - b*x]])/(-b)^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.95, size = 160, normalized size = 2.25

$$\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{a}\left(3\sqrt{a}\,b^3\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right](-a+bx)-3ab^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{-a+bx}{a}}+b^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{-a+bx}{a}}\right)}{b^{\frac{11}{2}}(-a+bx)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-3a\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{b^{\frac{5}{2}}}+\frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}}-\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{I Sqrt[a] (3 Sqrt[a] b ^ 3 ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] (
-a + b x) - 3 a b ^ (7 / 2) Sqrt[x] Sqrt[(-a + b x) / a] + b ^ (9 / 2) x ^
(3 / 2) Sqrt[(-a + b x) / a]) / (b ^ (11 / 2) (-a + b x)), Abs[b x / a] > 1
}}, -3 a ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] / b ^ (5 / 2) + 3 Sqrt[a] Sqrt[x
] / (b ^ 2 Sqrt[1 - b x / a]) - x ^ (3 / 2) / (Sqrt[a] b Sqrt[1 - b x / a])
]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.
time = 0.13, size = 114, normalized size = 1.61

method	result
risch	$\frac{\sqrt{x}\,\sqrt{-bx+a}}{b^2}+\frac{\left(-\frac{3a\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2b^{\frac{5}{2}}}-\frac{2a\sqrt{-\left(-\frac{a}{b}+x\right)^2b-a\left(-\frac{a}{b}+x\right)}}{b^3\left(-\frac{a}{b}+x\right)}\right)\sqrt{x\,(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x
^2+a*x)^(1/2))-2*a/b^3/(-a/b+x)*(-(-a/b+x)^2*b-a*(-a/b+x))^(1/2))*(x*(-b*x+
a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.34, size = 75, normalized size = 1.06

$$\frac{2\,ab-\frac{3\,(bx-a)a}{x}}{\frac{\sqrt{-bx+a}\,b^3}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{3\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x - a)*a/x)/(sqrt(-b*x + a)*b^3/sqrt(x) + (-b*x + a)^(3/2)*b^
2/x^(3/2)) + 3*a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.33, size = 152, normalized size = 2.14

$$\left[-\frac{3\,(abx-a^2)\sqrt{-b}\,\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(b^2x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{2\,(b^4x-ab^3)},\ \frac{3\,(abx-a^2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(b^2x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{b^4x-ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(a*b*x - a^2)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt
(x) + a) - 2*(b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3), (3*(a
*b*x - a^2)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (b^2*x - 3*a
*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [A]
time = 2.03, size = 155, normalized size = 2.18

$$\begin{cases} -\frac{3i\sqrt{a}\,\sqrt{x}}{b^2\sqrt{-1+\frac{bx}{a}}}+\frac{3ia\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{ix^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{3\sqrt{a}\,\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}}-\frac{3a\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}-\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*sqrt(x)/(b**2*sqrt(-1 + b*x/a)) + 3*I*a*acosh(sqrt(
b)*sqrt(x)/sqrt(a))/b**(5/2) + I*x**(3/2)/(sqrt(a)*b*sqrt(-1 + b*x/a)), Abs
(b*x/a) > 1), (3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 - b*x/a)) - 3*a*asin(sqrt(b)*
sqrt(x)/sqrt(a))/b**(5/2) - x**(3/2)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 103, normalized size = 1.45

$$2\left(\frac{2\left(-\frac{\frac{1}{4}b^2\sqrt{x}\,\sqrt{x}}{b^3}+\frac{\frac{1}{4}\cdot 3ba}{b^3}\right)\sqrt{x}\,\sqrt{a-bx}}{a-bx}+\frac{6a\ln\left|\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right|}{4b^2\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] sqrt(-b*x + a)*sqrt(x)*(x/b - 3*a/b^2)/(b*x - a) + 3*a*log(abs(-sqrt(-b)*sq
rt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(3/2), x)
```

3.598 $\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)+2*x^(1/2)/b/(-b*x+a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 65, 223, 209}

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b}\\ &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}\\ &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 56, normalized size = 1.12

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(3/2),x]
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
)/(-b)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.72, size = 89, normalized size = 1.78

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{b^{\frac{3}{2}}}-\frac{2I\sqrt{x}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-2\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(a - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{2 I ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]] / b ^ (3 / 2) - 2 I Sqrt
[x] / (Sqrt[a] b Sqrt[-1 + b x / a]), Abs[b x / a] > 1}}, -2 ArcSin[Sqrt[b]
 Sqrt[x] / Sqrt[a]] / b ^ (3 / 2) + 2 Sqrt[x] / (Sqrt[a] b Sqrt[1 - b x / a
])]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(-bx+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

Maxima [A]
time = 0.35, size = 38, normalized size = 0.76

$$\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\,\sqrt{x}}{\sqrt{-bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 a)*b)
```

Fricas [A]
time = 0.33, size = 128, normalized size = 2.56

$$\left[-\frac{(bx-a)\sqrt{-b}\,\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\sqrt{-bx+a}\,b\sqrt{x}}{b^3x-ab^2},\ \frac{2\left((bx-a)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}\right)}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2), 2*((b*x - a)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2
)]
```

Sympy [A]
time = 0.93, size = 102, normalized size = 2.04

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{2i\sqrt{x}}{\sqrt{a}\, b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2 \operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} + \frac{2\sqrt{x}}{\sqrt{a}\, b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*I*sqrt(x)/(sqrt(
a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*asin(sqrt(b)*sqrt(x)/sqrt(a))/
b**(3/2) + 2*sqrt(x)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 68, normalized size = 1.36

$$2\left(\frac{\frac{1}{2}\cdot 2\sqrt{x}\sqrt{a-bx}}{b\,(a-bx)} + \frac{\ln\left|\sqrt{a-bx} - \sqrt{-b}\sqrt{x}\right|}{b\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x)
```

```
[Out] 2*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b) - 2*sqrt(-b*x +
 a)*sqrt(x)/((b*x - a)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a - b*x)^(3/2), x)
```

3.599 $\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

```
[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.18, size = 55, normalized size = 2.75

$$\text{Piecewise}\left[\left\{\left\{\frac{2}{a\sqrt{b}\sqrt{-1+\frac{a}{bx}}},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2I}{a\sqrt{b}\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]')
```

```
[Out] Piecewise[{{2 / (a Sqrt[b] Sqrt[-1 + a / (b x)]), Abs[a / (b x)] > 1}}, -2
I / (a Sqrt[b] Sqrt[1 - a / (b x)])]
```

Maple [A]
time = 0.12, size = 17, normalized size = 0.85

method	result	size
gosper	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17
default	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.80

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(-b*x + a)*a)
```

Fricas [A]
time = 0.31, size = 25, normalized size = 1.25

$$-\frac{2\sqrt{-bx+a}\sqrt{x}}{abx-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)*sqrt(x)/(a*b*x - a^2)
```

Sympy [A]
time = 0.50, size = 44, normalized size = 2.20

$$\begin{cases} \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i}{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((2/(a*sqrt(b)*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-2*I/(a*sqr
t(b)*sqrt(-a/(b*x) + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(16) = 32.
time = 0.01, size = 59, normalized size = 2.95

$$-\frac{8b\sqrt{-b}}{2|b|\left(\left(\sqrt{ab-b(a-bx)}-\sqrt{-b}\sqrt{a-bx}\right)^2-ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
)*abs(b))
```

Mupad [B]
time = 0.34, size = 24, normalized size = 1.20

$$\frac{2\sqrt{x}\sqrt{a-b\,x}}{a^2-a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a - b*x)^(1/2))/(a^2 - a*b*x)
```

3.600 $\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

```
[Out] 2/a/x^(1/2)/(-b*x+a)^(1/2)-4*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{x}\sqrt{a-bx}}+\frac{2\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 26, normalized size = 0.63

$$-\frac{2(a-2bx)}{a^2\sqrt{x}\,\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a - 2*b*x))/(a^2*Sqrt[x]*Sqrt[a - b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.73, size = 126, normalized size = 3.07

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\,(-a+2bx)\sqrt{\frac{a-bx}{bx}}}{a^2\,(a-bx)},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2Iab^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}}{a^3b-a^2b^2x}+\frac{I4b^{\frac{5}{2}}x\sqrt{1-\frac{a}{bx}}}{a^3b-a^2b^2x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-a + 2 b x) Sqrt[(a - b x) / (b x)] / (a ^ 2 (a - b
x)), Abs[a / (b x)] > 1}}, -2 I a b ^ (3 / 2) Sqrt[1 - a / (b x)] / (a ^ 3
b - a ^ 2 b ^ 2 x) + I 4 b ^ (5 / 2) x Sqrt[1 - a / (b x)] / (a ^ 3 b - a ^
 2 b ^ 2 x)]
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.85

method	result	size
gosper	$-\frac{2(-2bx+a)}{\sqrt{x}\,\sqrt{-bx+a}\,a^2}$	23
default	$-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^2\sqrt{-bx+a}}$	35
risch	$-\frac{2\sqrt{-bx+a}}{a^2\sqrt{x}}+\frac{2b\sqrt{x}}{a^2\sqrt{-bx+a}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/x^(1/2)/(-b*x+a)^(1/2)+4*b/a^2*x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.25, size = 34, normalized size = 0.83

$$\frac{2\,b\sqrt{x}}{\sqrt{-bx+a}\,a^2}-\frac{2\,\sqrt{-bx+a}}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x)/(sqrt(-b*x + a)*a^2) - 2*sqrt(-b*x + a)/(a^2*sqrt(x))
```

Fricas [A]
time = 0.55, size = 38, normalized size = 0.93

$$-\frac{2\,(2\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{a^2bx^2-a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x - a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b*x^2 - a^3*x)
```

Sympy [A]
time = 0.92, size = 112, normalized size = 2.73

$$\begin{cases} -\frac{2}{a\sqrt{b}\,x\sqrt{\frac{a}{bx}-1}}+\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{2iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-a^3b+a^2b^2x}-\frac{4ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-a^3b+a^2b^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2/(a*sqrt(b)*x*sqrt(a/(b*x) - 1)) + 4*sqrt(b)/(a**2*sqrt(a/(b*x
) - 1)), Abs(a/(b*x)) > 1), (2*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/(-a**3*b + a
**2*b**2*x) - 4*I*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-a**3*b + a**2*b**2*x), Tr
ue))
```

Giac [A]
time = 0.00, size = 77, normalized size = 1.88

$$2\left(\frac{\frac{1}{2}\cdot 2b\sqrt{x}\,\sqrt{a-bx}}{a^2\,(a-bx)}+\frac{4\sqrt{-b}}{2a\left(\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^2-a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2*sqrt(-b*x + a)*b*sqrt(x)/((b*x - a)*a^2) + 4*sqrt(-b)/(((sqrt(-b)*sqrt(x
) - sqrt(-b*x + a))^2 - a)*a)
```

Mupad [B]
time = 0.40, size = 42, normalized size = 1.02

$$-\frac{2\,a\,\sqrt{a-b\,x}\,-4\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(a^3-a^2\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a - b*x)^(1/2) - 4*b*x*(a - b*x)^(1/2))/(x^(1/2)*(a^3 - a^2*b*x))
```

3.601 $\int \frac{1}{x^{5/2}(a-bx)^{3/2}} dx$

Optimal. Leaf size=66

$$\frac{2}{ax^{3/2}\sqrt{a-bx}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} - \frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/a/x^(3/2)/(-b*x+a)^(1/2)-8/3*(-b*x+a)^(1/2)/a^2/x^(3/2)-16/3*b*(-b*x+a)^(
1/2)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} + \frac{2}{ax^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a - b*x]) - (8*Sqrt[a - b*x])/(3*a^2*x^(3/2)) - (16*b*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx &= \frac{2}{ax^{3/2}\sqrt{a-bx}}+\frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a}\\ &= \frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}+\frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}\\ &= \frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 39, normalized size = 0.59

$$-\frac{2\left(a^2+4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a^2 + 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a - b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 8.56, size = 311, normalized size = 4.71

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\left(-a^3-3a^2bx+12ab^2x^2-8b^3x^3\right)\sqrt{\frac{a-bx}{bx}}}{3a^3x\left(a^2-2abx+b^2x^2\right)},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2Ia^3b^{\frac{9}{2}}\sqrt{1-\frac{a}{bx}}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{6Ia^2b^{\frac{11}{2}}x\sqrt{1-\frac{a}{bx}}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}+\frac{I24ab^{\frac{13}{2}}x^2\sqrt{1-\frac{a}{bx}}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{16Ib^{\frac{15}{2}}x^3\sqrt{1-\frac{a}{bx}}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-a ^ 3 - 3 a ^ 2 b x + 12 a b ^ 2 x ^ 2 - 8 b ^ 3 x
^ 3) Sqrt[(a - b x) / (b x)] / (3 a ^ 3 x (a ^ 2 - 2 a b x + b ^ 2 x ^ 2)),
 Abs[a / (b x)] > 1}}, -2 I a ^ 3 b ^ (9 / 2) Sqrt[1 - a / (b x)] / (3 a ^
5 b ^ 4 x - 6 a ^ 4 b ^ 5 x ^ 2 + 3 a ^ 3 b ^ 6 x ^ 3) - 6 I a ^ 2 b ^ (11
/ 2) x Sqrt[1 - a / (b x)] / (3 a ^ 5 b ^ 4 x - 6 a ^ 4 b ^ 5 x ^ 2 + 3 a ^
 3 b ^ 6 x ^ 3) + I 24 a b ^ (13 / 2) x ^ 2 Sqrt[1 - a / (b x)] / (3 a ^ 5
b ^ 4 x - 6 a ^ 4 b ^ 5 x ^ 2 + 3 a ^ 3 b ^ 6 x ^ 3) - 16 I b ^ (15 / 2) x
^ 3 Sqrt[1 - a / (b x)] / (3 a ^ 5 b ^ 4 x - 6 a ^ 4 b ^ 5 x ^ 2 + 3 a ^ 3
b ^ 6 x ^ 3)]
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.88

method	result	size
gosper	$-\frac{2(-8x^2b^2+4abx+a^2)}{3x^{\frac{3}{2}}\sqrt{-bx+a}\, a^3}$	34
risch	$-\frac{2\sqrt{-bx+a}\,(5bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{2b^2\sqrt{x}}{a^3\sqrt{-bx+a}}$	43
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{-bx+a}}+\frac{4b\left(-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^2\sqrt{-bx+a}}\right)}{3a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(-b*x+a)^(1/2)+4/3*b/a*(-2/a/x^(1/2)/(-b*x+a)^(1/2)+4*b/a^2*
x^(1/2)/(-b*x+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.79

$$\frac{2\,b^2\sqrt{x}}{\sqrt{-bx+a}\;a^3}-\frac{2\left(\frac{6\sqrt{-bx+a}\;b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(-b*x + a)*a^3) - 2/3*(6*sqrt(-b*x + a)*b/sqrt(x) + (-b*
x + a)^(3/2)/x^(3/2))/a^3
```

Fricas [A]
time = 0.30, size = 51, normalized size = 0.77

$$-\frac{2\,(8\,b^2x^2-4\,abx-a^2)\sqrt{-bx+a}\;\sqrt{x}}{3\,(a^3bx^3-a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 4*a*b*x - a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b*x^3 - a^4*x^
2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.67, size = 452, normalized size = 6.85

$$\begin{cases} -\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{6ia^2b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}+\frac{24iab^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{16ib^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*
x**2 + 3*a**3*b**6*x**3) - 6*a**2*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**
4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*
x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 16*b**(15/2
)*x**3*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x*
*3), Abs(a/(b*x)) > 1), (-2*I*a**3*b**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4
*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 6*I*a**2*b**(11/2)*x*sqrt(-a/(b
*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*I*a*b**
(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b
**6*x**3) - 16*I*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*
b**5*x**2 + 3*a**3*b**6*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(50) = 100$.
time = 0.01, size = 162, normalized size = 2.45

$$2\left(\frac{\frac{1}{2}\cdot 2b^{2}\sqrt{x}\,\sqrt{a-bx}}{a^{3}\,(a-bx)}+\frac{2\left(3b\sqrt{-b}\,\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^{4}-12b\sqrt{-b}\,\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^{2}a+5b\sqrt{-b}\,a^{2}\right)}{3a^{2}\left(\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^{2}-a\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2*sqrt(-b*x + a)*b^2*sqrt(x)/((b*x - a)*a^3) + 4/3*(3*sqrt(-b)*b*(sqrt(-b)
*sqrt(x) - sqrt(-b*x + a))^4 - 12*a*sqrt(-b)*b*(sqrt(-b)*sqrt(x) - sqrt(-b*
x + a))^2 + 5*a^2*sqrt(-b)*b)/(((sqrt(-b)*sqrt(x) - sqrt(-b*x + a))^2 - a)^
3*a^2)
```

Mupad [B]
time = 0.43, size = 48, normalized size = 0.73

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,x}{3\,a^2}+\frac{2}{3\,a\,b}-\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}-\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(3/2)),x)
```

```
[Out] ((a - b*x)^(1/2)*((8*x)/(3*a^2) + 2/(3*a*b) - (16*b*x^2)/(3*a^3)))/(x^(5/2)
 - (a*x^(3/2))/b)
```

3.602 $\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}$$

```
[Out] 2/3*x^(5/2)/b/(-b*x+a)^(3/2)+5*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(
7/2)-10/3*x^(3/2)/b^2/(-b*x+a)^(1/2)-5*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {49, 52, 65, 223, 209}

$$\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a - b*x]) - (5
*Sqrt[x]*Sqrt[a - b*x])/b^3 + (5*a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx}{3b}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{(5a)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{(5a)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{(5a)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 78, normalized size = 0.82

$$-\frac{\sqrt{x}\ (15a^2-20abx+3b^2x^2)}{3b^3(a-bx)^{3/2}}+\frac{5a\log\left(-\sqrt{-b}\ \sqrt{x}\ +\sqrt{a-bx}\ \right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[x]*(15*a^2 - 20*a*b*x + 3*b^2*x^2))/(b^3*(a - b*x)^(3/2)) + (5*a
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 11.14, size = 523, normalized size = 5.51

$$\text{Piecewise}\left[\left\{\left\{\frac{15a^{\frac{7}{2}}b^{\frac{17}{2}}\left(\text{Pi}-2I\,\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\right)\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}(a-bx)^4+30Ia^2b^9\sqrt{x}(-a+bx)^5+15a^{\frac{5}{2}}b^{\frac{19}{2}}x\left(-\text{Pi}+2I\,\text{ArcCosh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\right)\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}(a-bx)^4-40Iab^{10}x^{\frac{3}{2}}(-a+bx)^5+6Ib^{11}x^{\frac{5}{2}}(-a+bx)^5}{6a^{\frac{3}{2}}b^{12}\left(\frac{-a+bx}{a}\right)^{\frac{3}{2}}(a-bx)^5},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\sqrt{1-\frac{bx}{a}}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right]\sqrt{1-\frac{bx}{a}}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(a - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{(15 a ^ (7 / 2) b ^ (17 / 2) (Pi - 2 I ArcCosh[Sqrt[b] Sqrt[x]
/ Sqrt[a]]) ((-a + b x) / a) ^ (3 / 2) (a - b x) ^ 4 + 30 I a ^ 2 b ^ 9 Sqr
t[x] (-a + b x) ^ 5 + 15 a ^ (5 / 2) b ^ (19 / 2) x (-Pi + 2 I ArcCosh[Sqrt
[b] Sqrt[x] / Sqrt[a]]) ((-a + b x) / a) ^ (3 / 2) (a - b x) ^ 4 - 40 I a b
 ^ 10 x ^ (3 / 2) (-a + b x) ^ 5 + 6 I b ^ 11 x ^ (5 / 2) (-a + b x) ^ 5) /
 (6 a ^ (3 / 2) b ^ 12 ((-a + b x) / a) ^ (3 / 2) (a - b x) ^ 5), Abs[b x /
 a] > 1}}, 15 a ^ (81 / 2) b ^ 22 x ^ (51 / 2) ArcSin[Sqrt[b] Sqrt[x] / Sqr
t[a]] Sqrt[1 - b x / a] / (3 a ^ (79 / 2) b ^ (51 / 2) x ^ (51 / 2) Sqrt[1
- b x / a] - 3 a ^ (77 / 2) b ^ (53 / 2) x ^ (53 / 2) Sqrt[1 - b x / a]) -
15 a ^ 40 b ^ (45 / 2) x ^ 26 / (3 a ^ (79 / 2) b ^ (51 / 2) x ^ (51 / 2) S
qrt[1 - b x / a] - 3 a ^ (77 / 2) b ^ (53 / 2) x ^ (53 / 2) Sqrt[1 - b x /
a]) - 15 a ^ (79 / 2) b ^ 23 x ^ (53 / 2) ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]]
 Sqrt[1 - b x / a] / (3 a ^ (79 / 2) b ^ (51 / 2) x ^ (51 / 2) Sqrt[1 - b x
 / a] - 3 a ^ (77 / 2) b ^ (53 / 2) x ^ (53 / 2) Sqrt[1 - b x / a]) + 20 a
^ 39 b ^ (47 / 2) x ^ 27 / (3 a ^ (79 / 2) b ^ (51 / 2) x ^ (51 / 2) Sqrt[1
 - b x / a] - 3 a ^ (77 / 2) b ^ (53 / 2) x ^ (53 / 2) Sqrt[1 - b x / a]) -
 3 a ^ 38 b ^ (49 / 2) x ^ 28 / (3 a ^ (79 / 2) b ^ (51 / 2) x ^ (51 / 2) S
qrt[1 - b x / a] - 3 a ^ (77 / 2) b ^ (53 / 2) x ^ (53 / 2) Sqrt[1 - b x /
a])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(71) = 142$.
time = 0.12, size = 160, normalized size = 1.68

method	result
risch	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b^{3}}+\frac{\left(\frac{5a\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2b^{\frac{7}{2}}}+\frac{2a^{2}\sqrt{-\left(-\frac{a}{b}+x\right)^{2}b-a\left(-\frac{a}{b}+x\right)}}{3b^{5}\left(-\frac{a}{b}+x\right)^{2}}+\frac{14a\sqrt{-\left(-\frac{a}{b}+\right.}}{3b}\right.}{\sqrt{x}\,\sqrt{-bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b^3+(5/2/b^(7/2)*a*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x
^2+a*x)^(1/2))+2/3/b^5*a^2/(-a/b+x)^2*(-(-a/b+x)^2*b-a*(-a/b+x))^(1/2)+14/3
/b^4*a/(-a/b+x)*(-(-a/b+x)^2*b-a*(-a/b+x))^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2
)/(-b*x+a)^(1/2)
```

Maxima [A]

time = 0.34, size = 94, normalized size = 0.99

$$\frac{2\,ab^{2}+\frac{10\,(bx-a)ab}{x}-\frac{15\,(bx-a)^{2}a}{x^{2}}}{3\left(\frac{(-bx+a)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}\right)}-\frac{5\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x - a)*a*b/x - 15*(b*x - a)^2*a/x^2)/((-b*x + a)^(3/2)
*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^(5/2)) - 5*a*arctan(sqrt(-b*x + a)/(s
qrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]

time = 0.33, size = 215, normalized size = 2.26

$$\left[-\frac{15\,(ab^{2}x^{2}-2\,a^{2}bx+a^{3})\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(3\,b^{3}x^{2}-20\,ab^{2}x+15\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{6\,(b^{6}x^{2}-2\,ab^{5}x+a^{2}b^{4})},-\frac{15\,(ab^{2}x^{2}-2\,a^{2}bx+a^{3})\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(3\,b^{3}x^{2}-20\,ab^{2}x+15\,a^{2}b)\sqrt{-bx+a}\,\sqrt{x}}{3\,(b^{6}x^{2}-2\,ab^{5}x+a^{2}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x +
 a)*sqrt(-b)*sqrt(x) + a) + 2*(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x
 + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), -1/3*(15*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 -
```

```
 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*
b^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 5.43, size = 971, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-30*I*a**(81/2)*b**22*x**(51/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*s
qrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(7
7/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 15*pi*a**(81/2)*b**22*x**(51/2
)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a*
*(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 30*I*a**(79/2)*b**23*x**(53
/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*
x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/
a)) - 15*pi*a**(79/2)*b**23*x**(53/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/
2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b
*x/a)) + 30*I*a**40*b**(45/2)*x**26/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-
1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) - 40*I*a**39
*b**(47/2)*x**27/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(
77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 6*I*a**38*b**(49/2)*x**28/(6*
a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(
53/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (15*a**(81/2)*b**22*x**(51/2)*sqr
t(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)
*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 15*a*
*(79/2)*b**23*x**(53/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a*
*(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/
2)*sqrt(1 - b*x/a)) - 15*a**40*b**(45/2)*x**26/(3*a**(79/2)*b**(51/2)*x**(5
1/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) + 2
0*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) -
3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 3*a**38*b**(49/2)*x**28/
(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x*
*(53/2)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 146, normalized size = 1.54

$$2\left(\frac{2\left(\left(\left(-\frac{\frac{1}{36}\cdot 9b^4a\sqrt{x}\sqrt{x}}{b^5a}+\frac{\frac{1}{36}\cdot 60b^3a^2}{b^5a}\right)\sqrt{x}\sqrt{x}-\frac{\frac{1}{36}\cdot 45b^2a^3}{b^5a}\right)\sqrt{x}\sqrt{a-bx}\right)}{(a-bx)^2}-\frac{10a\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{4b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -1/3*sqrt(-b*x + a)*(x*(3*x/b - 20*a/b^2) + 15*a^2/b^3)*sqrt(x)/(b*x - a)^2
 - 5*a*log(abs(-sqrt(-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(5/2), x)
```

3.603 $\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b/(-b*x+a)^(3/2)+2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/
2)-2*x^(1/2)/b^2/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 65, 223, 209}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a - b*x]) + (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx}{b}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 66, normalized size = 0.92

$$\frac{2\sqrt{x}\,(-3a+4bx)}{3b^2(a-bx)^{3/2}} - \frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(5/2),x]
[Out] (2*Sqrt[x]*(-3*a + 4*b*x))/(3*b^2*(a - b*x)^(3/2)) - (2*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[a - b*x]])/(-b)^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 9.92, size = 427, normalized size = 5.93

$$\text{Piecewise}\left[\left\{\left\{\frac{ab^{\frac{9}{2}}\left(-\text{Pi}+2I\,\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\right)(-a+bx)^4+2Ia^{\frac{3}{2}}b^5\sqrt{x}\,\sqrt{\frac{-a+bx}{a}}\,(a-bx)^3+b^{\frac{11}{2}}x\left(\text{Pi}-2I\,\text{ArcCosh}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\right)(-a+bx)^4-\frac{8I\sqrt{a}\,b^6x^{\frac{3}{2}}\sqrt{\frac{-a+bx}{a}}\,(a-bx)^3}{3}}{b^7(-a+bx)^5},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\sqrt{1-\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\text{ArcSin}\left[\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right]\sqrt{1-\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}+\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(a - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{(a b ^ (9 / 2) (-Pi + 2 I ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]]) (
-a + b x) ^ 4 + 2 I a ^ (3 / 2) b ^ 5 Sqrt[x] Sqrt[(-a + b x) / a] (a - b x
) ^ 3 + b ^ (11 / 2) x (Pi - 2 I ArcCosh[Sqrt[b] Sqrt[x] / Sqrt[a]]) (-a +
b x) ^ 4 - 8 I / 3 Sqrt[a] b ^ 6 x ^ (3 / 2) Sqrt[(-a + b x) / a] (a - b x)
 ^ 3) / (b ^ 7 (-a + b x) ^ 5), Abs[b x / a] > 1}}, 6 a ^ (39 / 2) b ^ 11 x
 ^ (27 / 2) ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] Sqrt[1 - b x / a] / (3 a ^ (3
9 / 2) b ^ (27 / 2) x ^ (27 / 2) Sqrt[1 - b x / a] - 3 a ^ (37 / 2) b ^ (29
 / 2) x ^ (29 / 2) Sqrt[1 - b x / a]) - 6 a ^ 19 b ^ (23 / 2) x ^ 14 / (3 a
 ^ (39 / 2) b ^ (27 / 2) x ^ (27 / 2) Sqrt[1 - b x / a] - 3 a ^ (37 / 2) b
^ (29 / 2) x ^ (29 / 2) Sqrt[1 - b x / a]) - 6 a ^ (37 / 2) b ^ 12 x ^ (29
/ 2) ArcSin[Sqrt[b] Sqrt[x] / Sqrt[a]] Sqrt[1 - b x / a] / (3 a ^ (39 / 2)
b ^ (27 / 2) x ^ (27 / 2) Sqrt[1 - b x / a] - 3 a ^ (37 / 2) b ^ (29 / 2) x
 ^ (29 / 2) Sqrt[1 - b x / a]) + 8 a ^ 18 b ^ (25 / 2) x ^ 15 / (3 a ^ (39
/ 2) b ^ (27 / 2) x ^ (27 / 2) Sqrt[1 - b x / a] - 3 a ^ (37 / 2) b ^ (29 /
 2) x ^ (29 / 2) Sqrt[1 - b x / a])]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(-bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

Maxima [A]
time = 0.35, size = 52, normalized size = 0.72

$$\frac{2\left(b+\frac{3(bx-a)}{x}\right)x^{\frac{3}{2}}}{3(-bx+a)^{\frac{3}{2}}b^2}-\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 188, normalized size = 2.61

$$\left[-\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(4\,b^2x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{3\,(b^5x^2-2\,ab^4x+a^2b^3)},\,-\frac{2\left(3\,(b^2x^2-2\,abx+a^2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(4\,b^2x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\,(b^5x^2-2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*s
qrt(-b)*sqrt(x) + a) - 2*(4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^5*x^2
 - 2*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x)
)/(b^5*x^2 - 2*a*b^4*x + a^2*b^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.41, size = 833, normalized size = 11.57

$$\begin{cases} -\frac{6ia^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{3\pi a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{3\pi a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{8ia^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}} & \text{for}\: \left|\frac{bx}{a}\right|>1 \\ \frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}+\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*I*a**(39/2)*b**11*x**(27/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37
/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 3*pi*a**(39/2)*b**11*x**(27/2)*
sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(
37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 6*I*a**(37/2)*b**12*x**(29/2)
*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**
(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a))
 - 3*pi*a**(37/2)*b**12*x**(29/2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x
**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a
)) + 6*I*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b
*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) - 8*I*a**18*b**(2
5/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (6*a**(39/2)*b**11*
x**(27/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27
/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*
x/a)) - 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sq
rt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(2
9/2)*x**(29/2)*sqrt(1 - b*x/a)) - 6*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(
```

```
27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 -
b*x/a)) + 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 -
 b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 110, normalized size = 1.53

$$2\left(\frac{2\left(\frac{\frac{1}{18}\cdot 12b^2a\sqrt{x}\sqrt{x}}{b^3a}-\frac{\frac{1}{18}\cdot 9ba^2}{b^3a}\right)\sqrt{x}\sqrt{a-bx}}{(a-bx)^2}-\frac{\ln\left|\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right|}{b^2\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x)
[Out] 2/3*sqrt(-b*x + a)*sqrt(x)*(4*x/b - 3*a/b^2)/(b*x - a)^2 - 2*log(abs(-sqrt(
-b)*sqrt(x) + sqrt(-b*x + a)))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a-b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(5/2),x)
[Out] int(x^(3/2)/(a - b*x)^(5/2), x)
```

3.604 $\int \frac{\sqrt{x}}{(a-bx)^{5/2}} dx$

Optimal. Leaf size=22

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

```
[Out] 2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}} dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.68, size = 84, normalized size = 3.82

$$\text{Piecewise}\left[\left\{\left\{\frac{2Ix^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{\frac{-a+bx}{a}}\,(-a+bx)},\text{Abs}\left[\frac{bx}{a}\right]>1\right\}\right\},\frac{-2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(a - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 I / 3 x ^ (3 / 2) / (a ^ (3 / 2) Sqrt[(-a + b x) / a] (-a + b
 x)), Abs[b x / a] > 1}}, -2 x ^ (3 / 2) / (-3 a ^ (5 / 2) Sqrt[1 - b x / a
] + 3 a ^ (3 / 2) b x Sqrt[1 - b x / a])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(16) = 32$.
time = 0.11, size = 56, normalized size = 2.55

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3a(-bx+a)^{\frac{3}{2}}}$	17
default	$\frac{\sqrt{x}}{b(-bx+a)^{\frac{3}{2}}}-\frac{a\left(\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{-bx+a}}\right)}{2b}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*x^(1/2)/(-b*x+a)^(3/2)-1/2*a/b*(2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2
)/a^2/(-b*x+a)^(1/2))
```

Maxima [A]
time = 0.25, size = 16, normalized size = 0.73

$$\frac{2\,x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((-b*x + a)^(3/2)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 0.31, size = 34, normalized size = 1.55

$$\frac{2\sqrt{-bx+a}\,x^{\frac{3}{2}}}{3\,(ab^2x^2-2\,a^2bx+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-b*x + a)*x^(3/2)/(a*b^2*x^2 - 2*a^2*b*x + a^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.81, size = 95, normalized size = 4.32

$$\begin{cases} \frac{2ix^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(5/2),x)
[Out] Piecewise((2*I*x**(3/2)/(-3*a**(5/2)*sqrt(-1 + b*x/a) + 3*a**(3/2)*b*x*sqrt
(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*x**(3/2)/(-3*a**(5/2)*sqrt(1 - b*x/a) +
 3*a**(3/2)*b*x*sqrt(1 - b*x/a)), True))
```

Giac [A]
time = 0.01, size = 44, normalized size = 2.00

$$\frac{\frac{1}{18}\cdot 12b\sqrt{x}\sqrt{x}\sqrt{x}\sqrt{a-bx}}{ba\,(a-bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x)
[Out] 2/3*sqrt(-b*x + a)*x^(3/2)/((b*x - a)^2*a)
```

Mupad [B]
time = 0.25, size = 37, normalized size = 1.68

$$\frac{2\,x^{3/2}\,\sqrt{a-b\,x}}{3\,\left(a^3-2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(5/2),x)
[Out] (2*x^(3/2)*(a - b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 - 2*a^2*b*x))
```

3.605 $\int \frac{1}{\sqrt{x}\,(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}$$

```
[Out] 2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a-bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a-2bx)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
[Out] (2*Sqrt[x]*(3*a - 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.60, size = 157, normalized size = 3.49

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\,x\,(3a-2bx)\sqrt{\frac{a-bx}{bx}}}{3a^2\,(a-bx)^2},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-6Iab}{3a^3b^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}-3a^2b^{\frac{5}{2}}x\sqrt{1-\frac{a}{bx}}}+\frac{I4b^2x}{3a^3b^{\frac{3}{2}}\sqrt{1-\frac{a}{bx}}-3a^2b^{\frac{5}{2}}x\sqrt{1-\frac{a}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]')
[Out] Piecewise[{{2 Sqrt[b] x (3 a - 2 b x) Sqrt[(a - b x) / (b x)] / (3 a ^ 2 (a
 - b x) ^ 2), Abs[a / (b x)] > 1}}, -6 I a b / (3 a ^ 3 b ^ (3 / 2) Sqrt[1
- a / (b x)] - 3 a ^ 2 b ^ (5 / 2) x Sqrt[1 - a / (b x)]) + I 4 b ^ 2 x / (
3 a ^ 3 b ^ (3 / 2) Sqrt[1 - a / (b x)] - 3 a ^ 2 b ^ (5 / 2) x Sqrt[1 - a
/ (b x)])]
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.76

method	result	size
gosper	$\frac{2\sqrt{x}\,(-2bx+3a)}{3(-bx+a)^{\frac{3}{2}}a^2}$	25
default	$\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{-bx+a}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.67

$$\frac{2\left(b-\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b - 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*a^2)
```

Fricas [A]
time = 0.31, size = 44, normalized size = 0.98

$$-\frac{2\,(2\,bx-3\,a)\sqrt{-bx+a}\,\sqrt{x}}{3\,(a^2b^2x^2-2\,a^3bx+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x - 3*a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b^2*x^2 - 2*a^3*b*x + a^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.12, size = 197, normalized size = 4.38

$$\begin{cases} -\frac{6a}{-3a^3\sqrt{b}\,\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}}+\frac{4bx}{-3a^3\sqrt{b}\,\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{6iab}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}-\frac{4ib^2x}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-6*a/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)*x*sqrt
(a/(b*x) - 1)) + 4*b*x/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)
*x*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (6*I*a*b/(-3*a**3*b**(3/2)*sqrt(-
a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)) - 4*I*b**2*x/(-3*a**3*
b**(3/2)*sqrt(-a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.
time = 0.01, size = 109, normalized size = 2.42

$$-\frac{32b\sqrt{-b}\,b\left(-3\left(\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right)^2+ab\right)}{2\cdot 6\,|b|\left(\left(\sqrt{ab-b\,(a-bx)}-\sqrt{-b}\,\sqrt{a-bx}\right)^2-ab\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*abs(
b))
```

Mupad [B]
time = 0.41, size = 56, normalized size = 1.24

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a-b\,x}-4\,b\,x^{3/2}\,\sqrt{a-b\,x}}{3\,a^4-6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a - b*x)^(1/2) - 4*b*x^(3/2)*(a - b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 - 6*a^3*b*x)
```

3.606 $\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/3/a/(-b*x+a)^(3/2)/x^(1/2)+8/3/a^2/x^(1/2)/(-b*x+a)^(1/2)-16/3*(-b*x+a)^(
1/2)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a - b*x]) - (16*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{4\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}} + \frac{8\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}$$
$$= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 41, normalized size = 0.61

$$-\frac{2\left(3a^2-12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.42, size = 229, normalized size = 3.42

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\left(-2a^2+8abx-\frac{16b^2x^2}{3}\right)\sqrt{\frac{a-bx}{bx}}}{a^3\left(a^2-2abx+b^2x^2\right)},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-6Ia^2b^{\frac{9}{2}}\sqrt{1-\frac{a}{bx}}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{I24ab^{\frac{11}{2}}x\sqrt{1-\frac{a}{bx}}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16Ib^{\frac{13}{2}}x^2\sqrt{1-\frac{a}{bx}}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-2 a ^ 2 + 8 a b x - 16 b ^ 2 x ^ 2 / 3) Sqrt[(a - b x
) / (b x)] / (a ^ 3 (a ^ 2 - 2 a b x + b ^ 2 x ^ 2)), Abs[a / (b x)] > 1}},
 -6 I a ^ 2 b ^ (9 / 2) Sqrt[1 - a / (b x)] / (3 a ^ 5 b ^ 4 - 6 a ^ 4 b ^
5 x + 3 a ^ 3 b ^ 6 x ^ 2) + I 24 a b ^ (11 / 2) x Sqrt[1 - a / (b x)] / (3
 a ^ 5 b ^ 4 - 6 a ^ 4 b ^ 5 x + 3 a ^ 3 b ^ 6 x ^ 2) - 16 I b ^ (13 / 2) x
 ^ 2 Sqrt[1 - a / (b x)] / (3 a ^ 5 b ^ 4 - 6 a ^ 4 b ^ 5 x + 3 a ^ 3 b ^ 6
 x ^ 2)]
```

Maple [A]
time = 0.16, size = 57, normalized size = 0.85

method	result	size

gosper	$-\frac{2(8x^2b^2-12abx+3a^2)}{3\sqrt{x}\,(-bx+a)^{\frac{3}{2}}a^3}$	36
risch	$-\frac{2\sqrt{-bx+a}}{a^3\sqrt{x}}+\frac{2b(-5bx+6a)\sqrt{x}}{3(-bx+a)^{\frac{3}{2}}a^3}$	43
default	$-\frac{2}{a(-bx+a)^{\frac{3}{2}}\sqrt{x}}+\frac{4b\left(\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{-bx+a}}\right)}{a}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/(-b*x+a)^(3/2)/x^(1/2)+4*b/a*(2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)
/a^2/(-b*x+a)^(1/2))
```

Maxima [A]
time = 0.26, size = 50, normalized size = 0.75

$$\frac{2\left(b^2-\frac{6\,(bx-a)b}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^3}-\frac{2\,\sqrt{-bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x - a)*b/x)*x^(3/2)/((-b*x + a)^(3/2)*a^3) - 2*sqrt(-b*x +
a)/(a^3*sqrt(x))
```

Fricas [A]
time = 0.32, size = 59, normalized size = 0.88

$$-\frac{2\,(8\,b^2x^2-12\,abx+3\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{3\,(a^3b^2x^3-2\,a^4bx^2+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 12*a*b*x + 3*a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b^2*x^3 - 2
*a^4*b*x^2 + a^5*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.56, size = 314, normalized size = 4.69

$$\begin{cases} -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{6ia^2b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{24iab^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16ib^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*a**2*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) + 24*a*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a
**4*b**5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(3*a**
5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), Abs(a/(b*x)) > 1), (-6*I*a**2*b
**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2)
 + 24*I*a*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a
**3*b**6*x**2) - 16*I*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a*
*4*b**5*x + 3*a**3*b**6*x**2), True))
```

Giac [A]
time = 0.01, size = 119, normalized size = 1.78

$$2\left(\frac{2\left(-\frac{\frac{1}{18}\cdot 15b^3a^2\sqrt{x}\,\sqrt{x}}{ba^5}+\frac{\frac{1}{18}\cdot 18b^2a^3}{ba^5}\right)\sqrt{x}\,\sqrt{a-bx}}{(a-bx)^2}+\frac{4\sqrt{-b}}{2a^2\left(\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)^2-a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*sqrt(-b*x + a)*sqrt(x)*(5*b^2*x/a^3 - 6*b/a^2)/(b*x - a)^2 + 4*sqrt(-b
)/(((sqrt(-b)*sqrt(x) - sqrt(-b*x + a))^2 - a)*a^2)
```

Mupad [B]
time = 0.44, size = 73, normalized size = 1.09

$$\frac{6\,a^2\,\sqrt{a-b\,x}+16\,b^2\,x^2\,\sqrt{a-b\,x}-24\,a\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b-3\,a^3\,b^2\,x\right)-3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a^2*(a - b*x)^(1/2) + 16*b^2*x^2*(a - b*x)^(1/2) - 24*a*b*x*(a - b*x)^(1
/2))/(x^(1/2)*(x*(6*a^4*b - 3*a^3*b^2*x) - 3*a^5))
```

3.607 $\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}$$

```
[Out] 2/3/a/x^(3/2)/(-b*x+a)^(3/2)+4/a^2/x^(3/2)/(-b*x+a)^(1/2)-16/3*(-b*x+a)^(1/
2)/a^3/x^(3/2)-32/3*b*(-b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a - b*x]) - (16*Sqrt[
a - b*x])/(3*a^3*x^(3/2)) - (32*b*Sqrt[a - b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{2\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3x^{3/2}} + \frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^3}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3x^{3/2}} - \frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}$$

Mathematica [A]
time = 0.11, size = 50, normalized size = 0.57

$$-\frac{2\left(a^3+6a^2bx-24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 59.41, size = 447, normalized size = 5.08

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{b}\left(-a^4-5a^3bx+30a^2b^2x^2-40ab^3x^3+16b^4x^4\right)\sqrt{\frac{a-bx}{bx}}}{3a^4x\left(a^3-3a^2bx+3ab^2x^2-b^3x^3\right)},\text{Abs}\left[\frac{a}{bx}\right]>1\right\}\right\},\frac{-2Ia^4b^{\frac{19}{2}}\sqrt{1-\frac{a}{bx}}}{3a^7b^9x-9a^6b^{10}x^2+9a^5b^{11}x^3-3a^4b^{12}x^4}-\frac{10Ia^3b^{\frac{21}{2}}x\sqrt{1-\frac{a}{bx}}}{3a^7b^9x-9a^6b^{10}x^2+9a^5b^{11}x^3-3a^4b^{12}x^4}+\frac{I60a^2b^{\frac{23}{2}}x^2\sqrt{1-\frac{a}{bx}}}{3a^7b^9x-9a^6b^{10}x^2+9a^5b^{11}x^3-3a^4b^{12}x^4}-\frac{80Iab^{\frac{25}{2}}x^3\sqrt{1-\frac{a}{bx}}}{3a^7b^9x-9a^6b^{10}x^2+9a^5b^{11}x^3-3a^4b^{12}x^4}+\frac{I32b^{\frac{27}{2}}x^4\sqrt{1-\frac{a}{bx}}}{3a^7b^9x-9a^6b^{10}x^2+9a^5b^{11}x^3-3a^4b^{12}x^4}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{2 Sqrt[b] (-a ^ 4 - 5 a ^ 3 b x + 30 a ^ 2 b ^ 2 x ^ 2 - 40 a b
 ^ 3 x ^ 3 + 16 b ^ 4 x ^ 4) Sqrt[(a - b x) / (b x)] / (3 a ^ 4 x (a ^ 3 -
3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 - b ^ 3 x ^ 3)), Abs[a / (b x)] > 1}}, -2 I a
 ^ 4 b ^ (19 / 2) Sqrt[1 - a / (b x)] / (3 a ^ 7 b ^ 9 x - 9 a ^ 6 b ^ 10 x
 ^ 2 + 9 a ^ 5 b ^ 11 x ^ 3 - 3 a ^ 4 b ^ 12 x ^ 4) - 10 I a ^ 3 b ^ (21 /
2) x Sqrt[1 - a / (b x)] / (3 a ^ 7 b ^ 9 x - 9 a ^ 6 b ^ 10 x ^ 2 + 9 a ^
5 b ^ 11 x ^ 3 - 3 a ^ 4 b ^ 12 x ^ 4) + I 60 a ^ 2 b ^ (23 / 2) x ^ 2 Sqrt
[1 - a / (b x)] / (3 a ^ 7 b ^ 9 x - 9 a ^ 6 b ^ 10 x ^ 2 + 9 a ^ 5 b ^ 11
x ^ 3 - 3 a ^ 4 b ^ 12 x ^ 4) - 80 I a b ^ (25 / 2) x ^ 3 Sqrt[1 - a / (b x
)] / (3 a ^ 7 b ^ 9 x - 9 a ^ 6 b ^ 10 x ^ 2 + 9 a ^ 5 b ^ 11 x ^ 3 - 3 a ^
```

```
4 b ^ 12 x ^ 4) + I 32 b ^ (27 / 2) x ^ 4 Sqrt[1 - a / (b x)] / (3 a ^ 7 b
 ^ 9 x - 9 a ^ 6 b ^ 10 x ^ 2 + 9 a ^ 5 b ^ 11 x ^ 3 - 3 a ^ 4 b ^ 12 x ^ 4
)]
```

Maple [A]
time = 0.14, size = 80, normalized size = 0.91

method	result	size
gosper	$-\frac{2\left(16 b^{3} x^{3}-24 a\, b^{2} x^{2}+6 a^{2} b x +a^{3}\right)}{3 x^{\frac{3}{2}} \left(-b x +a \right)^{\frac{3}{2}} a^{4}}$	45
risch	$-\frac{2 \sqrt{-b x +a}\, \left(8 b x +a \right)}{3 a^{4} x^{\frac{3}{2}}}+\frac{2 b^{2} \left(-8 b x +9 a \right) \sqrt{x}}{3 \left(-b x +a \right)^{\frac{3}{2}} a^{4}}$	51
default	$-\frac{2}{3 a\, x^{\frac{3}{2}} \left(-b x +a \right)^{\frac{3}{2}}}+\frac{2 b \left(-\frac{2}{a \left(-b x +a \right)^{\frac{3}{2}} \sqrt{x}}+\frac{4 b \left(\frac{2 \sqrt{x}}{3 a \left(-b x +a \right)^{\frac{3}{2}}}+\frac{4 \sqrt{x}}{3 a^{2} \sqrt{-b x +a}}\right)}{a}\right)}{a}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(-b*x+a)^(3/2)+2*b/a*(-2/a/(-b*x+a)^(3/2)/x^(1/2)+4*b/a*(2/3
*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)))
```

Maple [A] is followed by

Maxima [A]
time = 0.27, size = 68, normalized size = 0.77

$$-\frac{2\left(\frac{9\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}+\frac{2\left(b^3-\frac{9\,(bx-a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(9*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^4 + 2/3*(b^3
 - 9*(b*x - a)*b^2/x)*x^(3/2)/((-b*x + a)^(3/2)*a^4)
```

Fricas [A]
time = 0.32, size = 70, normalized size = 0.80

$$-\frac{2\,(16\,b^3x^3-24\,ab^2x^2+6\,a^2bx+a^3)\sqrt{-bx+a}\,\sqrt{x}}{3\,(a^4b^2x^4-2\,a^5bx^3+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*b^3*x^3 - 24*a*b^2*x^2 + 6*a^2*b*x + a^3)*sqrt(-b*x + a)*sqrt(x)/(
a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.98, size = 688, normalized size = 7.82

$$\begin{cases} \frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^4b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10ia^3b^{\frac{21}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60ia^2b^{\frac{23}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80iab^{\frac{25}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{32ib^{\frac{27}{2}}x^4\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*a**4*b**(19/2)*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**1
0*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(
a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a*
*4*b**12*x**4) - 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x +
 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2
)*x**3*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11
*x**3 + 3*a**4*b**12*x**4) - 32*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-3*a**7*b
**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4), Abs(a/(
b*x)) > 1), (2*I*a**4*b**(19/2)*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6
*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*I*a**3*b**(21/2)*
x*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**
3 + 3*a**4*b**12*x**4) - 60*I*a**2*b**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a*
*7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80
*I*a*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2
- 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) - 32*I*b**(27/2)*x**4*sqrt(-a/(b*x
) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**
12*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(66) = 132.
time = 0.01, size = 200, normalized size = 2.27

$$2\left(\frac{2\left(-\frac{\frac{1}{18}\cdot 24b^4a^3\sqrt{x}\sqrt{x}}{ba^7}+\frac{\frac{1}{18}\cdot 27b^3a^4}{ba^7}\right)\sqrt{x}\sqrt{a-bx}}{(a-bx)^2}+\frac{2\left(6b\sqrt{-b}\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)^4-18b\sqrt{-b}\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)^2a+8b\sqrt{-b}a^2\right)}{3a^3\left(\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)^2-a\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*sqrt(-b*x + a)*sqrt(x)*(8*b^3*x/a^4 - 9*b^2/a^3)/(b*x - a)^2 + 8/3*(3*
sqrt(-b)*b*(sqrt(-b)*sqrt(x) - sqrt(-b*x + a))^4 - 9*a*sqrt(-b)*b*(sqrt(-b)
*sqrt(x) - sqrt(-b*x + a))^2 + 4*a^2*sqrt(-b)*b)/(((sqrt(-b)*sqrt(x) - sqrt
(-b*x + a))^2 - a)^3*a^3)
```

Mupad [B]
time = 0.47, size = 92, normalized size = 1.05

$$\frac{2\,a^3\,\sqrt{a-b\,x}+32\,b^3\,x^3\,\sqrt{a-b\,x}+12\,a^2\,b\,x\,\sqrt{a-b\,x}-48\,a\,b^2\,x^2\,\sqrt{a-b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b-3\,a^4\,b^2\,x\right)-3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (2*a^3*(a - b*x)^(1/2) + 32*b^3*x^3*(a - b*x)^(1/2) + 12*a^2*b*x*(a - b*x)^
(1/2) - 48*a*b^2*x^2*(a - b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b - 3*a^4*b^2*x) -
 3*a^6))
```

3.608 $\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=88

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] -5*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/6*x^(3/2)*(b*x+2)^(1/2)/b
^2+1/3*x^(5/2)*(b*x+2)^(1/2)/b+5/2*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(6*b^2) + (x^
(5/2)*Sqrt[2 + b*x])/(3*b) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx &= \frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{3b}\\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^3}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 66, normalized size = 0.75

$$\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2)}{6b^3}+\frac{5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2))/(6*b^3) + (5*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

Mathics [A]
time = 10.12, size = 75, normalized size = 0.85

$$\frac{-5\mathrm{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b^{\frac{7}{2}}}+\frac{5\sqrt{x}}{b^3\sqrt{2+bx}}+\frac{5x^{\frac{3}{2}}}{6b^2\sqrt{2+bx}}-\frac{x^{\frac{5}{2}}}{6b\sqrt{2+bx}}+\frac{x^{\frac{7}{2}}}{3\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)/Sqrt[2 + b*x],x]')
```

```
[Out] -5 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (7 / 2) + 5 Sqrt[x] / (b ^ 3
Sqrt[2 + b x]) + 5 x ^ (3 / 2) / (6 b ^ 2 Sqrt[2 + b x]) - x ^ (5 / 2) / (6
 b Sqrt[2 + b x]) + x ^ (7 / 2) / (3 Sqrt[2 + b x])
```

Maple [A]
time = 0.13, size = 104, normalized size = 1.18

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(14x^2b^2-35bx+105)\sqrt{\frac{bx}{2}+1}}{42}-5\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2-5bx+15)\sqrt{x}\,\sqrt{bx+2}}{6b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x(bx+2)}}{2b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	77
default	$\frac{x^{\frac{5}{2}}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}\right)}{2b}\right)}{3b}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(5/2)*(b*x+2)^(1/2)/b-5/3/b*(1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2/b*(x^(1/
2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*
x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(63) = 126$.
time = 0.34, size = 134, normalized size = 1.52

$$-\frac{\frac{33\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{40(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{15(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^6-\frac{3(bx+2)b^5}{x}+\frac{3(bx+2)^2b^4}{x^2}-\frac{(bx+2)^3b^3}{x^3}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3/2) + 15*(b*x
 + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x + 2)*b^5/x + 3*(b*x + 2)^2*b^4/x^2 - (b*
x + 2)^3*b^3/x^3) + 5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + s
qrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 124, normalized size = 1.41

$$\left[\frac{\left(2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^{4}},\frac{\left(2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] [1/6*((2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b
*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/6*((2*b^3*x^2 - 5*b^2*x + 1
5*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b^4]
```

Sympy [A]
time = 8.85, size = 95, normalized size = 1.08

$$\frac{x^{\frac{7}{2}}}{3\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{6b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{6b^{2}\sqrt{bx+2}}+\frac{5\sqrt{x}}{b^{3}\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(1/2),x)
[Out] x**(7/2)/(3*sqrt(b*x + 2)) - x**(5/2)/(6*b*sqrt(b*x + 2)) + 5*x**(3/2)/(6*b
**2*sqrt(b*x + 2)) + 5*sqrt(x)/(b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(7/2)
```

Giac [A]
time = 0.00, size = 116, normalized size = 1.32

$$2\left(2\left(\left(\left(\frac{\frac{1}{72}\cdot 6b^{4}\sqrt{x}\,\sqrt{x}}{b^{5}}-\frac{\frac{1}{72}\cdot 15b^{3}}{b^{5}}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{72}\cdot 45b^{2}}{b^{5}}\right)\sqrt{x}\,\sqrt{bx+2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{2b^{3}\sqrt{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x)
[Out] 1/6*sqrt(b*x + 2)*(x*(2*x/b - 5/b^2) + 15/b^3)*sqrt(x) + 5*log(-sqrt(b)*sqr
t(x) + sqrt(b*x + 2))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(1/2),x)
[Out] int(x^(5/2)/(b*x + 2)^(1/2), x)
```

3.609 $\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=67

$$-\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{2b}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] 3*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+1/2*x^(3/2)*(b*x+2)^(1/2)/b-
3/2*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{bx+2}}{2b^2}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(2*b) + (3*Arc
Sinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{x^{3/2}\sqrt{2+bx}}{2b} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^2}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [**A**]
time = 0.06, size = 57, normalized size = 0.85

$$\frac{\sqrt{x}\,(-3+bx)\sqrt{2+bx}}{2b^2} - \frac{3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(-3 + b*x)*Sqrt[2 + b*x])/(2*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sq
rt[2 + b*x]])/b^(5/2)
```

Mathics [**A**]
time = 4.25, size = 73, normalized size = 1.09

$$\frac{3\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{b^{\frac{5}{2}}} - \frac{6\sqrt{x}}{b^2\,(2+bx)^{\frac{3}{2}}} - \frac{4x^{\frac{3}{2}}}{b\,(2+bx)^{\frac{3}{2}}} + \frac{x^{\frac{5}{2}}}{2\,(2+bx)^{\frac{3}{2}}} + \frac{bx^{\frac{7}{2}}}{2\,(2+bx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)/Sqrt[2 + b*x],x]')
```

```
[Out] 3 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (5 / 2) - 6 Sqrt[x] / (b ^ 2 (
2 + b x) ^ (3 / 2)) - 4 x ^ (3 / 2) / (b (2 + b x) ^ (3 / 2)) + x ^ (5 / 2)
 / (2 (2 + b x) ^ (3 / 2)) + b x ^ (7 / 2) / (2 (2 + b x) ^ (3 / 2))
```

Maple [**A**]
time = 0.11, size = 83, normalized size = 1.24

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(-5bx+15)\sqrt{\frac{bx}{2}+1}}{10}+3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55
risch	$\frac{(bx-3)\sqrt{x}\,\sqrt{bx+2}}{2b^2}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	68
default	$\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}\right)}{2b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2/b*(x^(1/2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*
x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(48) = 96$.
time = 0.35, size = 102, normalized size = 1.52

$$\frac{\frac{5\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{3\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^4-\frac{2\,(bx+2)b^3}{x}+\frac{(bx+2)^2b^2}{x^2}}-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (5*sqrt(b*x + 2)*b/sqrt(x) - 3*(b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x + 2)*
b^3/x + (b*x + 2)^2*b^2/x^2) - 3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(
sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.31, size = 105, normalized size = 1.57

$$\left[\frac{(b^2x-3\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b^3},\frac{(b^2x-3\,b)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*
sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A]
time = 2.52, size = 75, normalized size = 1.12

$$\frac{x^{\frac{5}{2}}}{2\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{2b\sqrt{bx+2}}-\frac{3\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(5/2)/(2*sqrt(b*x + 2)) - x**(3/2)/(2*b*sqrt(b*x + 2)) - 3*sqrt(x)/(b**2
*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Giac [A]
time = 0.00, size = 90, normalized size = 1.34

$$2\left(2\left(\frac{\frac{1}{8}b^2\sqrt{x}\sqrt{x}}{b^3}-\frac{\frac{1}{8}\cdot 3b}{b^3}\right)\sqrt{x}\sqrt{bx+2}-\frac{3\ln\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{2b^2\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/2*sqrt(b*x + 2)*sqrt(x)*(x/b - 3/b^2) - 3*log(-sqrt(b)*sqrt(x) + sqrt(b*x
 + 2))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(1/2), x)
```

3.610 $\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] -2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+x^(1/2)*(b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b}$$
$$= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.14

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{b} + \frac{2\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b + (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(
3/2)
```

Mathics [A]
time = 2.79, size = 61, normalized size = 1.42

$$\frac{-2b\mathrm{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](2+bx) + 2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{2+bx} + b^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{2+bx}}{b^{\frac{5}{2}}(2+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]/Sqrt[2 + b*x],x]')
```

```
[Out] (-2 b ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + 2 b ^ (3 / 2) Sqrt[x
] Sqrt[2 + b x] + b ^ (5 / 2) x ^ (3 / 2) Sqrt[2 + b x]) / (b ^ (5 / 2) (2
+ b x))
```

Maple [A]
time = 0.12, size = 62, normalized size = 1.44

method	result	size

meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\sqrt{\frac{bx}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	49
default	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	62
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*l
n((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.35, size = 70, normalized size = 1.63

$$\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{bx+2}}{\left(b^2-\frac{(bx+2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b
^(3/2) - 2*sqrt(b*x + 2)/((b^2 - (b*x + 2)*b/x)*sqrt(x))
```

Fricas [A]
time = 0.32, size = 87, normalized size = 2.02

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x}+\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{b^2}, \frac{\sqrt{bx+2}\,b\sqrt{x}+2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b^2, (sqrt(b*x + 2)*b*sqrt(x) + 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt
(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A]
time = 1.02, size = 54, normalized size = 1.26

$$\frac{x^{\frac{3}{2}}}{\sqrt{bx+2}}+\frac{2\sqrt{x}}{b\sqrt{bx+2}}-\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(3/2)/sqrt(b*x + 2) + 2*sqrt(x)/(b*sqrt(b*x + 2)) - 2*asinh(sqrt(2)*sqrt
(b)*sqrt(x)/2)/b**(3/2)
```

Giac [A]
time = 0.00, size = 56, normalized size = 1.30

$$2\left(\frac{\frac{1}{4}\cdot 2\sqrt{x}\sqrt{bx+2}}{b}+\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] sqrt(b*x + 2)*sqrt(x)/b + 2*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(3/2)
```

Mupad [B]
time = 0.59, size = 43, normalized size = 1.00

$$\frac{4\operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{b\,x+2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(1/2),x)
```

```
[Out] (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(1/2))))/b^(3/2) + (x^(1/2)
*(b*x + 2)^(1/2))/b
```

3.611 $\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {56, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.25

$$-\frac{2\log\left(-\sqrt{b}\,\sqrt{x}\,+\sqrt{2+bx}\,\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Mathics [A]
time = 2.00, size = 17, normalized size = 0.71

$$\frac{2\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]')
```

```
[Out] 2 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / Sqrt[b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.
time = 0.12, size = 46, normalized size = 1.92

method	result	size
meijerg	$\frac{2\,\mathrm{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}$	18
default	$\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2
))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.

time = 0.34, size = 41, normalized size = 1.71

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b)
```

Fricas [A]
time = 0.31, size = 55, normalized size = 2.29

$$\left[\frac{\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{\sqrt{b}},-\frac{2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1)/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [A]
time = 0.47, size = 24, normalized size = 1.00

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [A]
time = 0.00, size = 32, normalized size = 1.33

$$-\frac{2\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/sqrt(b)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 1.25

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{b\,x+2}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (b*x + 2)^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

3.612 $\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{2+bx}}{\sqrt{x}}$$

```
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{\sqrt{2+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Mathics [A]
time = 1.97, size = 17, normalized size = 1.06

$$-\sqrt{b}\sqrt{1+\frac{2}{bx}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]')
```

```
[Out] -Sqrt[b] Sqrt[1 + 2 / (b x)]
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
default	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
risch	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
meijerg	$-\frac{\sqrt{2}\sqrt{\frac{bx}{2}+1}}{\sqrt{x}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.31, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x + 2)/sqrt(x)
```

Sympy [A]
time = 0.45, size = 15, normalized size = 0.94

$$-\sqrt{b}\sqrt{1+\frac{2}{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(1 + 2/(b*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 0.00, size = 36, normalized size = 2.25

$$\frac{8\sqrt{b}}{2\left(\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 2)
```

Mupad [B]
time = 0.33, size = 12, normalized size = 0.75

$$-\frac{\sqrt{b\,x+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(b*x + 2)^(1/2)/x^(1/2)
```

3.613 $\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] -1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/3*Sqrt[2 + b*x]/x^(3/2) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.61

$$\frac{(-1+bx)\sqrt{2+bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

Mathics [A]
time = 2.77, size = 27, normalized size = 0.71

$$\frac{\sqrt{b}\,(-1+bx)\sqrt{\frac{2+bx}{bx}}}{3x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]')
```

```
[Out] Sqrt[b] (-1 + b x) Sqrt[(2 + b x) / (b x)] / (3 x)
```

Maple [A]
time = 0.14, size = 27, normalized size = 0.71

method	result	size
gosper	$\frac{\sqrt{bx+2}\,(bx-1)}{3x^{\frac{3}{2}}}$	18
meijerg	$-\frac{\sqrt{2}\,(-bx+1)\sqrt{\frac{bx}{2}+1}}{3x^{\frac{3}{2}}}$	23
risch	$\frac{x^2b^2+bx-2}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	25
default	$-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.68

$$\frac{\sqrt{bx+2}\,b}{2\,\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/6*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 17, normalized size = 0.45

$$\frac{\sqrt{bx+2}\,(bx-1)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x + 2)*(b*x - 1)/x^(3/2)
```

Sympy [A]
time = 1.04, size = 34, normalized size = 0.89

$$\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(1/2),x)
[Out] b**(3/2)*sqrt(1 + 2/(b*x))/3 - sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [A]
time = 0.00, size = 71, normalized size = 1.87

$$-\frac{32\sqrt{b}\,b\left(-3\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^{2}+2\right)}{2\cdot 6\left(\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^{2}-2\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x)
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 2)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + 2))^2 - 2)^3
```

Mupad [B]
time = 0.32, size = 17, normalized size = 0.45

$$\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}-\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(1/2)),x)
[Out] ((b*x + 2)^(1/2)*((b*x)/3 - 1/3))/x^(3/2)
```

3.614 $\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}$$

```
[Out] -1/5*(b*x+2)^(1/2)/x^(5/2)+2/15*b*(b*x+2)^(1/2)/x^(3/2)-2/15*b^2*(b*x+2)^(1
/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/5*Sqrt[2 + b*x]/x^(5/2) + (2*b*Sqrt[2 + b*x])/(15*x^(3/2)) - (2*b^2*Sqrt
[2 + b*x])/(15*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{5x^{5/2}} - \frac{1}{5}(2b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$$
$$= -\frac{\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{15x^{3/2}} + \frac{1}{15}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$$
$$= -\frac{\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{15x^{3/2}} - \frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.54

$$\frac{\sqrt{2+bx}\,\left(-3+2bx-2b^2x^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*(-3 + 2*b*x - 2*b^2*x^2))/(15*x^(5/2))
```

Mathics [A]
time = 6.51, size = 67, normalized size = 1.14

$$\frac{\sqrt{b}\,\left(-12-4bx-3b^2x^2-6b^3x^3-2b^4x^4\right)\sqrt{\frac{2+bx}{bx}}}{15x^2\left(4+4bx+b^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]')
[Out] Sqrt[b] (-12 - 4 b x - 3 b ^ 2 x ^ 2 - 6 b ^ 3 x ^ 3 - 2 b ^ 4 x ^ 4) Sqrt[
(2 + b x) / (b x)] / (15 x ^ 2 (4 + 4 b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.73

method	result	size
gosper	$-\frac{\sqrt{bx+2}\,\left(2x^2b^2-2bx+3\right)}{15x^{\frac{5}{2}}}$	27
meijerg	$-\frac{\sqrt{2}\,\left(\frac{2}{3}x^2b^2-\frac{2}{3}bx+1\right)\sqrt{\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
risch	$-\frac{2b^3x^3+2x^2b^2-bx+6}{15x^{\frac{5}{2}}\sqrt{bx+2}}$	35

default	$-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(b*x+2)^(1/2)/x^(5/2)-2/5*b*(-1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^
(1/2)/x^(1/2))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.69

$$-\frac{\sqrt{bx+2}\,b^2}{4\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{20\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/sqrt(x) + 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/20*(b*x
+ 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.32, size = 26, normalized size = 0.44

$$-\frac{(2\,b^2x^2-2\,bx+3)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*b^2*x^2 - 2*b*x + 3)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(53) = 106$.
time = 3.83, size = 224, normalized size = 3.80

$$-\frac{2b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{6b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{3b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{4b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{12b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 60*b**4*
x**2) - 6*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 6
0*b**4*x**2) - 3*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x
```

```
**3 + 60*b**4*x**2) - 4*b**(11/2)*x*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b*
*5*x**3 + 60*b**4*x**2) - 12*b**(9/2)*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*
b**5*x**3 + 60*b**4*x**2)
```

Giac [A]
time = 0.01, size = 97, normalized size = 1.64

$$-\frac{128\sqrt{b}\,b^2\left(-5\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^4+5\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2-2\right)}{30\left(\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2-2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x)
[Out] 64/15*(5*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^4 - 5*(sqrt(b)*sqrt(x) - sqrt(b*
x + 2))^2 + 2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 2)^5
```

Mupad [B]
time = 0.32, size = 26, normalized size = 0.44

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^2\,x^2}{15}-\frac{2\,b\,x}{15}+\frac{1}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(1/2)),x)
[Out] -((b*x + 2)^(1/2)*((2*b^2*x^2)/15 - (2*b*x)/15 + 1/5))/x^(5/2)
```

3.615 $\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}+\frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}$$

```
[Out] -1/7*(b*x+2)^(1/2)/x^(7/2)+3/35*b*(b*x+2)^(1/2)/x^(5/2)-2/35*b^2*(b*x+2)^(1
/2)/x^(3/2)+2/35*b^3*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {47, 37}

$$\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/7*Sqrt[2 + b*x]/x^(7/2) + (3*b*Sqrt[2 + b*x])/(35*x^(5/2)) - (2*b^2*Sqrt
[2 + b*x])/(35*x^(3/2)) + (2*b^3*Sqrt[2 + b*x])/(35*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{7x^{7/2}} - \frac{1}{7}(3b)\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx \\ &= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} + \frac{1}{35}\left(6b^2\right)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\ &= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} - \frac{2b^2\sqrt{2+bx}}{35x^{3/2}} - \frac{1}{35}\left(2b^3\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\ &= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} - \frac{2b^2\sqrt{2+bx}}{35x^{3/2}} + \frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.50

$$\frac{\sqrt{2+bx}\,\left(-5+3bx-2b^2x^2+2b^3x^3\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

Mathics [A]
time = 13.57, size = 89, normalized size = 1.11

$$\frac{\sqrt{b}\,\left(-40-36bx-10b^2x^2+5b^3x^3\left(1+3bx+2b^2x^2\right)+2b^6x^6\right)\sqrt{\frac{2+bx}{bx}}}{35x^3\left(8+12bx+6b^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]')
[Out] Sqrt[b] (-40 - 36 b x - 10 b ^ 2 x ^ 2 + 5 b ^ 3 x ^ 3 (1 + 3 b x + 2 b ^ 2
 x ^ 2) + 2 b ^ 6 x ^ 6) Sqrt[(2 + b x) / (b x)] / (35 x ^ 3 (8 + 12 b x +
6 b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.74

method	result	size
gosper	$\frac{\sqrt{bx+2}\,\left(2b^3x^3-2x^2b^2+3bx-5\right)}{35x^{\frac{7}{2}}}$	35

meijerg	$-\frac{\sqrt{2}\,\left(-\frac{2}{5}b^3x^3+\frac{2}{5}x^2b^2-\frac{3}{5}bx+1\right)\sqrt{\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
risch	$\frac{2b^4x^4+2b^3x^3-x^2b^2+bx-10}{35x^{\frac{7}{2}}\sqrt{bx+2}}$	42
default	$-\frac{\sqrt{bx+2}}{7x^{\frac{7}{2}}}-\frac{3b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*(b*x+2)^(1/2)/x^(7/2)-3/7*b*(-1/5*(b*x+2)^(1/2)/x^(5/2)-2/5*b*(-1/3*(b
*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.70

$$\frac{\sqrt{bx+2}\,b^3}{8\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^2}{8\,x^{\frac{3}{2}}}+\frac{3\,(bx+2)^{\frac{5}{2}}b}{40\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{56\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x + 2)*b^3/sqrt(x) - 1/8*(b*x + 2)^(3/2)*b^2/x^(3/2) + 3/40*(b*x
 + 2)^(5/2)*b/x^(5/2) - 1/56*(b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A]
time = 0.32, size = 34, normalized size = 0.42

$$\frac{(2\,b^3x^3-2\,b^2x^2+3\,bx-5)\sqrt{bx+2}}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x - 5)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(73) = 146.
time = 11.82, size = 374, normalized size = 4.68

$$\frac{2b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{10b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{15b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{5b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{10b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{36b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{40b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b*
*10*x**4 + 280*b**9*x**3) + 10*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(35*b**12*x
**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) + 15*b**(27/2)*x**4*
sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b*
*9*x**3) + 5*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x*
*5 + 420*b**10*x**4 + 280*b**9*x**3) - 10*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/
(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) - 36*b**(
21/2)*x*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4
+ 280*b**9*x**3) - 40*b**(19/2)*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**1
1*x**5 + 420*b**10*x**4 + 280*b**9*x**3)
```

Giac [A]
time = 0.01, size = 124, normalized size = 1.55

$$-\frac{512\sqrt{b}\,b^{3}\left(-35\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^{6}+42\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^{4}-28\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^{2}+8\right)}{280\left(\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^{2}-2\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x)
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^6 - 42*(sqrt(b)*sqrt(x) - sqrt(
b*x + 2))^4 + 28*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 8)*b^(7/2)/((sqrt(b)
*sqrt(x) - sqrt(b*x + 2))^2 - 2)^7
```

Mupad [B]
time = 0.33, size = 33, normalized size = 0.41

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{35}-\frac{2\,b^2\,x^2}{35}+\frac{3\,b\,x}{35}-\frac{1}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((3*b*x)/35 - (2*b^2*x^2)/35 + (2*b^3*x^3)/35 - 1/7))/x^(7
/2)
```

3.616 $\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] 15*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-2*x^(5/2)/b/(b*x+2)^(1/2)+5
/2*x^(3/2)*(b*x+2)^(1/2)/b^2-15/2*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 52, 56, 221}

$$\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{15\sqrt{x}\,\sqrt{bx+2}}{2b^3}+\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[2 + b*x]) - (15*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) + (5*x^
(3/2)*Sqrt[2 + b*x])/(2*b^2) + (15*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}-\frac{15\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 65, normalized size = 0.76

$$\frac{\sqrt{x}\,\left(-30-5bx+b^2x^2\right)}{2b^3\sqrt{2+bx}}-\frac{15\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(3/2),x]
[Out] (Sqrt[x]*(-30 - 5*b*x + b^2*x^2))/(2*b^3*Sqrt[2 + b*x]) - (15*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

Mathics [A]
time = 6.71, size = 75, normalized size = 0.87

$$\frac{30b^6\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right](2+bx)^{\frac{3}{2}}-30b^{\frac{13}{2}}\sqrt{x}\,(2+bx)-5b^{\frac{15}{2}}x^{\frac{3}{2}}(2+bx)+b^{\frac{17}{2}}x^{\frac{5}{2}}(2+bx)}{2b^{\frac{19}{2}}(2+bx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)/(2 + b*x)^(3/2),x]')
```

```
[Out] (30 b ^ 6 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) ^ (3 / 2) - 30 b ^
 (13 / 2) Sqrt[x] (2 + b x) - 5 b ^ (15 / 2) x ^ (3 / 2) (2 + b x) + b ^ (1
7 / 2) x ^ (5 / 2) (2 + b x)) / (2 b ^ (19 / 2) (2 + b x) ^ (3 / 2))
```

Maple [A]
time = 0.13, size = 63, normalized size = 0.73

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}\left(-\frac{7}{2}x^2b^2+\frac{35}{2}bx+105\right)}{14\sqrt{\frac{bx}{2}+1}}+15\sqrt{\pi}\,\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{(bx-7)\sqrt{x}\sqrt{bx+2}}{2b^3}+\frac{\left(\frac{15\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2b^{\frac{7}{2}}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{b^4\left(x+\frac{2}{b}\right)}\right)\sqrt{x(bx+2)}}{\sqrt{x}\sqrt{bx+2}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/b^(7/2)/Pi^(1/2)*(-1/112*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-7/2*x^2*b^2+3
5/2*b*x+105)/(1/2*b*x+1)^(1/2)+15/8*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^
(1/2)))
```

Maxima [A]
time = 0.35, size = 119, normalized size = 1.38

$$-\frac{8\,b^2-\frac{25\,(bx+2)b}{x}+\frac{15\,(bx+2)^2}{x^2}}{\frac{\sqrt{bx+2}\,b^5}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}}-\frac{15\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(8*b^2 - 25*(b*x + 2)*b/x + 15*(b*x + 2)^2/x^2)/(sqrt(b*x + 2)*b^5/sqrt(x)
 - 2*(b*x + 2)^(3/2)*b^4/x^(3/2) + (b*x + 2)^(5/2)*b^3/x^(5/2)) - 15/2*log(
-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/
2)
```

Fricas [A]
time = 0.33, size = 152, normalized size = 1.77

$$\left[\frac{15\,(bx+2)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(b^3x^2-5\,b^2x-30\,b)\sqrt{bx+2}\,\sqrt{x}}{2\,(b^5x+2\,b^4)},-\frac{30\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-(b^3x^2-5\,b^2x-30\,b)\sqrt{bx+2}\,\sqrt{x}}{2\,(b^5x+2\,b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*(b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (
b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4), -1/2*(30*
(b*x + 2)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (b^3*x^2 -
5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4)]
```

Sympy [A]
time = 5.27, size = 80, normalized size = 0.93

$$\frac{x^{\frac{5}{2}}}{2b\sqrt{bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^2\sqrt{bx+2}}-\frac{15\sqrt{x}}{b^3\sqrt{bx+2}}+\frac{15\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(5/2)/(2*b*sqrt(b*x + 2)) - 5*x**(3/2)/(2*b**2*sqrt(b*x + 2)) - 15*sqrt(
x)/(b**3*sqrt(b*x + 2)) + 15*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2)
```

Giac [A]
time = 0.01, size = 123, normalized size = 1.43

$$2\left(\frac{2\left(\left(\frac{\frac{1}{8}b^4\sqrt{x}\,\sqrt{x}}{b^5}-\frac{\frac{1}{8}\cdot 5b^3}{b^5}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{8}\cdot 30b^2}{b^5}\right)\sqrt{x}\,\sqrt{bx+2}}{bx+2}-\frac{15\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{2b^3\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/2*(x*(x/b - 5/b^2) - 30/b^3)*sqrt(x)/sqrt(b*x + 2) - 15*log(-sqrt(b)*sqrt
(x) + sqrt(b*x + 2))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(3/2), x)
```

3.617 $\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] -6*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-2*x^(3/2)/b/(b*x+2)^(1/2)+3
*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{b^2}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*Sqrt[x]*Sqrt[2 + b*x])/b^2 - (6*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 0.86

$$\frac{\sqrt{x}\,(6+bx)}{b^2\sqrt{2+bx}}+\frac{6\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(6 + b*x))/(b^2*Sqrt[2 + b*x]) + (6*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[
2 + b*x]])/b^(5/2)
```

Mathics [A]
time = 3.43, size = 63, normalized size = 1.00

$$\frac{-6b^3\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right](2+bx)+6b^{\frac{7}{2}}\sqrt{x}\,\sqrt{2+bx}+b^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{2+bx}}{b^{\frac{11}{2}}\,(2+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)/(2 + b*x)^(3/2),x]')
```

```
[Out] (-6 b ^ 3 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] (2 + b x) + 6 b ^ (7 / 2) Sq
rt[x] Sqrt[2 + b x] + b ^ (9 / 2) x ^ (3 / 2) Sqrt[2 + b x]) / (b ^ (11 / 2
) (2 + b x))
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.87

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(\frac{5bx}{2}+15\right)}{5\sqrt{\frac{bx}{2}+1}}-6\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b^{2}}+\frac{\left(-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{\left(x+\frac{2}{b}\right)^{2}b-2x-\frac{4}{b}}}{b^{3}\left(x+\frac{2}{b}\right)}\right)\sqrt{x\left(bx+2\right)}}{\sqrt{x}\,\sqrt{bx+2}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b^(5/2)/Pi^(1/2)*(1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(5/2*b*x+15)/(1/2
*b*x+1)^(1/2)-3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.34, size = 90, normalized size = 1.43

$$\frac{2\left(2\,b-\frac{3\,(bx+2)}{x}\right)}{\frac{\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x + 2)/x)/(sqrt(b*x + 2)*b^3/sqrt(x) - (b*x + 2)^(3/2)*b^2/x^
(3/2)) + 3*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 134, normalized size = 2.13

$$\left[\frac{3\,(bx+2)\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(b^2x+6\,b)\sqrt{bx+2}\,\sqrt{x}}{b^4x+2\,b^3},\frac{6\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+(b^2x+6\,b)\sqrt{bx+2}\,\sqrt{x}}{b^4x+2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*(b*x + 2)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x
 + 6*b)*sqrt(b*x + 2)*sqrt(x))/(b^4*x + 2*b^3), (6*(b*x + 2)*sqrt(-b)*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 6*b)*sqrt(b*x + 2)*sqrt(x)
)/(b^4*x + 2*b^3)]
```

Sympy [A]
time = 1.82, size = 58, normalized size = 0.92

$$\frac{x^{\frac{3}{2}}}{b\sqrt{bx+2}}+\frac{6\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{6\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(3/2)/(b*sqrt(b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(b*x + 2)) - 6*asinh(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Giac [A]
time = 0.01, size = 92, normalized size = 1.46

$$2\left(\frac{2\left(\frac{\frac{1}{4}b^2\sqrt{x}\,\sqrt{x}}{b^3}+\frac{\frac{1}{4}\cdot 6b}{b^3}\right)\sqrt{x}\,\sqrt{bx+2}}{bx+2}+\frac{3\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^2\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x)
```

```
[Out] sqrt(x)*(x/b + 6/b^2)/sqrt(b*x + 2) + 6*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2
))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{{(b\,x+2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(3/2), x)
```

3.618 $\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\sqrt{x}}{b\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)-2*x^(1/2)/b/(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 56, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.14

$$-\frac{2\sqrt{x}}{b\sqrt{2+bx}} - \frac{2\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
)/b^(3/2)
```

Mathics [A]
time = 2.44, size = 33, normalized size = 0.75

$$\frac{2\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{b^{\frac{3}{2}}} - \frac{2\sqrt{x}}{b\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]')
[Out] 2 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (3 / 2) - 2 Sqrt[x] / (b Sqrt[
2 + b x])
```

Maple [A]
time = 0.12, size = 48, normalized size = 1.09

method	result	size

meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}}{\sqrt{\frac{bx}{2}+1}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^(3/2)/Pi^(1/2)*(-1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)/(1/2*b*x+1)^(1/2)
+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.37, size = 57, normalized size = 1.30

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + 2)*b)
```

Fricas [A]
time = 0.31, size = 117, normalized size = 2.66

$$\left[\frac{(bx+2)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-2\sqrt{bx+2}\,b\sqrt{x}}{b^3x+2\,b^2},\,-\frac{2\left((bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\,b\sqrt{x}\right)}{b^3x+2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b
*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2), -2*((b*x + 2)*sqrt(-b)*arctan(sqrt(b*x
+ 2)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2)]
```

Sympy [A]
time = 0.86, size = 41, normalized size = 0.93

$$-\frac{2\sqrt{x}}{b\sqrt{bx+2}}+\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*sqrt(x)/(b*sqrt(b*x + 2)) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Giac [A]
time = 0.01, size = 64, normalized size = 1.45

$$2\left(-\frac{\frac{1}{2}\cdot 2\sqrt{x}\,\sqrt{bx+2}}{b\,(bx+2)}-\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x)
```

```
[Out] -2*log(-sqrt(b)*sqrt(x) + sqrt(b*x + 2))/b^(3/2) - 2*sqrt(x)/(sqrt(b*x + 2)
*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x + 2)^(3/2), x)
```

3.619 $\int \frac{1}{\sqrt{x}\,(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{x}}{\sqrt{2+bx}}$$

[Out] x^(1/2)/(b*x+2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]

[Out] Sqrt[x]/Sqrt[2 + b*x]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2+bx}}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]

[Out] Sqrt[x]/Sqrt[2 + b*x]

Mathics [A]
time = 1.96, size = 16, normalized size = 1.07

$$\frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]')
```

```
[Out] 1 / (Sqrt[b] Sqrt[1 + 2 / (b x)])
```

Maple [A]
time = 0.13, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
default	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
meijerg	$\frac{\sqrt{x}\,\sqrt{2}}{2\sqrt{\frac{bx}{2}+1}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

Fricas [A]
time = 0.30, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

Sympy [A]
time = 0.47, size = 15, normalized size = 1.00

$$\frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] 1/(sqrt(b)*sqrt(1 + 2/(b*x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.
time = 0.00, size = 53, normalized size = 3.53

$$\frac{8b\sqrt{b}}{2\,|b|\left(\left(\sqrt{b\,(bx+2)-2b}-\sqrt{b}\,\sqrt{bx+2}\right)^2+2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(
b))
```

Mupad [B]
time = 0.31, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] x^(1/2)/(b*x + 2)^(1/2)
```

3.620 $\int \frac{1}{x^{3/2}(2+bx)^{3/2}} dx$

Optimal. Leaf size=32

$$\frac{1}{\sqrt{x}\sqrt{2+bx}}-\frac{\sqrt{2+bx}}{\sqrt{x}}$$

```
[Out] 1/x^(1/2)/(b*x+2)^(1/2)-(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{1}{\sqrt{x}\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\sqrt{2+bx}}+\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{\sqrt{x}\sqrt{2+bx}}-\frac{\sqrt{2+bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 0.66

$$\frac{-1-bx}{\sqrt{x}\,\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

Mathics [A]
time = 2.54, size = 31, normalized size = 0.97

$$\frac{\sqrt{b}\,(-1-bx)\sqrt{\frac{2+bx}{bx}}}{2+bx}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]')
[Out] Sqrt[b] (-1 - b x) Sqrt[(2 + b x) / (b x)] / (2 + b x)
```

Maple [A]
time = 0.14, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{bx+1}{\sqrt{x}\,\sqrt{bx+2}}$	18
meijerg	$-\frac{\sqrt{2}\,(bx+1)}{2\sqrt{x}\,\sqrt{\frac{bx}{2}+1}}$	22
default	$-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}$	27
risch	$-\frac{\sqrt{bx+2}}{2\sqrt{x}}-\frac{b\sqrt{x}}{2\sqrt{bx+2}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/x^(1/2)/(b*x+2)^(1/2)-b*x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.81

$$-\frac{b\sqrt{x}}{2\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b*sqrt(x)/sqrt(b*x + 2) - 1/2*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.31, size = 28, normalized size = 0.88

$$-\frac{\sqrt{bx+2}\,(bx+1)\sqrt{x}}{bx^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(b*x + 2)*(b*x + 1)*sqrt(x)/(b*x^2 + 2*x)
```

Sympy [A]
time = 0.86, size = 34, normalized size = 1.06

$$-\frac{\sqrt{b}}{\sqrt{1+\frac{2}{bx}}}-\frac{1}{\sqrt{b}\,x\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(3/2),x)
[Out] -sqrt(b)/sqrt(1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(1 + 2/(b*x)))
```

Giac [A]
time = 0.00, size = 65, normalized size = 2.03

$$2\left(-\frac{2b\sqrt{x}\,\sqrt{bx+2}}{8\,(bx+2)}+\frac{2\sqrt{b}}{2\left(\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x)
[Out] -1/2*b*sqrt(x)/sqrt(b*x + 2) + 2*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + 2))
^2 - 2)
```

Mupad [B]
time = 0.35, size = 17, normalized size = 0.53

$$-\frac{b\,x+1}{\sqrt{x}\,\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(3/2)),x)
[Out] -(b*x + 1)/(x^(1/2)*(b*x + 2)^(1/2))
```

3.621 $\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] 1/x^(3/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(3/2)+2/3*b*(b*x+2)^(1/2)/x^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x])/(3*x^(3/2)) + (2*b*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2+bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\ &= \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} - \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\ &= \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} + \frac{2b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 0.60

$$\frac{-1+2bx+2b^2x^2}{3x^{3/2}\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

Mathics [A]
time = 4.85, size = 57, normalized size = 1.08

$$\frac{\sqrt{b}\,\left(-2+3bx\,(1+2bx)+2b^3x^3\right)\sqrt{\frac{2+bx}{bx}}}{3x\,(4+4bx+b^2x^2)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]')
```

```
[Out] Sqrt[b] (-2 + 3 b x (1 + 2 b x) + 2 b ^ 3 x ^ 3) Sqrt[(2 + b x) / (b x)] /
(3 x (4 + 4 b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.13, size = 43, normalized size = 0.81

method	result	size
gosper	$\frac{2x^2b^2+2bx-1}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	27
meijerg	$-\frac{\sqrt{2}\,(-2x^2b^2-2bx+1)}{6x^{\frac{3}{2}}\sqrt{\frac{bx}{2}+1}}$	31

default	$-\frac{1}{3x^{\frac{3}{2}}\sqrt{bx+2}}-\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}\right)}{3}$	43
risch	$\frac{5x^{2}b^{2}+8bx-4}{12x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^{2}\sqrt{x}}{4\sqrt{bx+2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)/(b*x+2)^(1/2)-2/3*b*(-1/x^(1/2)/(b*x+2)^(1/2)-b*x^(1/2)/(b*x+2
)^(1/2))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.77

$$\frac{b^2\sqrt{x}}{4\sqrt{bx+2}}+\frac{\sqrt{bx+2}\,b}{2\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(b*x + 2) + 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/12*(b*x + 2
)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.33, size = 39, normalized size = 0.74

$$\frac{(2\,b^2x^2+2\,bx-1)\sqrt{bx+2}\,\sqrt{x}}{3\,(bx^3+2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*x^2 + 2*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^3 + 2*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(49) = 98$.
time = 2.44, size = 170, normalized size = 3.21

$$\frac{2b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{3b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] 2*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
 + 6*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) + 3*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) - 2*b**(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.
time = 0.01, size = 138, normalized size = 2.60

$$2\left(\frac{2b^2\sqrt{x}\,\sqrt{bx+2}}{16\,(bx+2)}+\frac{2\left(-3b\sqrt{b}\,\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^4+24b\sqrt{b}\,\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2-20b\sqrt{b}\right)}{12\left(\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2-2\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(b*x + 2) - 1/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x
+ 2))^4 - 24*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 + 20*b^(3/2))/((sq
rt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 2)^3
```

Mupad [B]
time = 0.38, size = 37, normalized size = 0.70

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,x}{3}+\frac{2\,b\,x^2}{3}-\frac{1}{3\,b}\right)}{x^{5/2}+\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((2*x)/3 + (2*b*x^2)/3 - 1/(3*b)))/(x^(5/2) + (2*x^(3/2))/
b)
```

3.622 $\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}$$

```
[Out] 1/x^(5/2)/(b*x+2)^(1/2)-3/5*(b*x+2)^(1/2)/x^(5/2)+2/5*b*(b*x+2)^(1/2)/x^(3/
2)-2/5*b^2*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(5/2)*Sqrt[2 + b*x]) - (3*Sqrt[2 + b*x])/(5*x^(5/2)) + (2*b*Sqrt[2 + b
*x])/(5*x^(3/2)) - (2*b^2*Sqrt[2 + b*x])/(5*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{5/2}\sqrt{2+bx}} + 3\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx \\
&= \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} - \frac{1}{5}(6b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\
&= \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{5x^{3/2}} + \frac{1}{5}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\
&= \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{5x^{3/2}} - \frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 39, normalized size = 0.53

$$\frac{-1+bx-2b^2x^2-2b^3x^3}{5x^{5/2}\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

Mathics [**A**]
time = 9.77, size = 77, normalized size = 1.04

$$\frac{\sqrt{b}\,\left(-4+5b^2x^2\left(-1-3bx-2b^2x^2\right)-2b^5x^5\right)\sqrt{\frac{2+bx}{bx}}}{5x^2\left(8+12bx+6b^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]')
```

```
[Out] Sqrt[b] (-4 + 5 b ^ 2 x ^ 2 (-1 - 3 b x - 2 b ^ 2 x ^ 2) - 2 b ^ 5 x ^ 5) S
qrt[(2 + b x) / (b x)] / (5 x ^ 2 (8 + 12 b x + 6 b ^ 2 x ^ 2 + b ^ 3 x ^ 3
))
```

Maple [**A**]
time = 0.14, size = 59, normalized size = 0.80

method	result	size
gosper	$-\frac{2b^3x^3+2x^2b^2-bx+1}{5x^{\frac{5}{2}}\sqrt{bx+2}}$	35

meijerg	$-\frac{\sqrt{2}\,(2b^3x^3+2x^2b^2-bx+1)}{10x^{\frac{5}{2}}\sqrt{\frac{bx}{2}+1}}$	39
risch	$-\frac{11b^3x^3+16x^2b^2-8bx+8}{40x^{\frac{5}{2}}\sqrt{bx+2}}-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}$	51
default	$-\frac{1}{5x^{\frac{5}{2}}\sqrt{bx+2}}-\frac{3b\left(-\frac{1}{3x^{\frac{3}{2}}\sqrt{bx+2}}-\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}\right)}{3}\right)}{5}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^(5/2)/(b*x+2)^(1/2)-3/5*b*(-1/3/x^(3/2)/(b*x+2)^(1/2)-2/3*b*(-1/x^(1
/2)/(b*x+2)^(1/2)-b*x^(1/2)/(b*x+2)^(1/2)))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.76

$$-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}-\frac{3\sqrt{bx+2}\,b^2}{8\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{8x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{40x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*sqrt(x)/sqrt(b*x + 2) - 3/8*sqrt(b*x + 2)*b^2/sqrt(x) + 1/8*(b*x +
 2)^(3/2)*b/x^(3/2) - 1/40*(b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.31, size = 47, normalized size = 0.64

$$-\frac{(2b^3x^3+2b^2x^2-bx+1)\sqrt{bx+2}\,\sqrt{x}}{5(bx^4+2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*b^3*x^3 + 2*b^2*x^2 - b*x + 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^4 + 2*x^3
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(70) = 140.
time = 8.06, size = 269, normalized size = 3.64

$$-\frac{2b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{10b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{15b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{5b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{4b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**1
0*x**3 + 40*b**9*x**2) - 10*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(5*b**12*x**5
+ 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 15*b**(25/2)*x**3*sqrt(1
+ 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) -
5*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10
*x**3 + 40*b**9*x**2) - 4*b**(19/2)*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b*
*11*x**4 + 60*b**10*x**3 + 40*b**9*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(52) = 104.
time = 0.02, size = 213, normalized size = 2.88

$$2\left(-\frac{2b^3\sqrt{x}\,\sqrt{bx+2}}{32\,(bx+2)}+\frac{2\left(5b^2\sqrt{b}\,\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^8-60b^2\sqrt{b}\,\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^6+320b^2\sqrt{b}\,\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^4-400b^2\sqrt{b}\,\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2+176b^2\sqrt{b}\right)}{40\left(\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)^2-2\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x)
```

```
[Out] -1/8*b^3*sqrt(x)/sqrt(b*x + 2) + 1/10*(5*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*
x + 2))^8 - 60*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^6 + 320*b^(5/2)*(s
qrt(b)*sqrt(x) - sqrt(b*x + 2))^4 - 400*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x
 + 2))^2 + 176*b^(5/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 2)^5
```

Mupad [B]
time = 0.43, size = 46, normalized size = 0.62

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b\,x^2}{5}-\frac{x}{5}+\frac{1}{5\,b}+\frac{2\,b^2\,x^3}{5}\right)}{x^{7/2}+\frac{2\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] -((b*x + 2)^(1/2)*((2*b*x^2)/5 - x/5 + 1/(5*b) + (2*b^2*x^3)/5))/(x^(7/2) +
 (2*x^(5/2))/b)
```

3.623 $\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] -2/3*x^(5/2)/b/(b*x+2)^(3/2)-10*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2
)-10/3*x^(3/2)/b^2/(b*x+2)^(1/2)+5*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 + b*x]) + (
5*Sqrt[x]*Sqrt[2 + b*x])/b^3 - (10*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b^2}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\sqrt{2+bx}}{b^3}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^3}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\sqrt{2+bx}}{b^3}-\frac{10\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\sqrt{2+bx}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 0.77

$$\frac{\sqrt{x}\,(60+40bx+3b^2x^2)}{3b^3(2+bx)^{3/2}}+\frac{10\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*(60 + 40*b*x + 3*b^2*x^2))/(3*b^3*(2 + b*x)^(3/2)) + (10*Log[-(Sqr
t[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

Mathics [A]
time = 7.36, size = 100, normalized size = 1.16

$$\frac{20\sqrt{x}}{b^3 (2+bx)^{\frac{3}{2}}} - \frac{10x\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{2b^{\frac{5}{2}} + b^{\frac{7}{2}}x} + \frac{40x^{\frac{3}{2}}}{3b^2 (2+bx)^{\frac{3}{2}}} + \frac{x^{\frac{5}{2}}}{b (2+bx)^{\frac{3}{2}}} - \frac{20\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{2b^{\frac{7}{2}} + b^{\frac{9}{2}}x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/2)/(2 + b*x)^(5/2),x]')
```

```
[Out] 20 Sqrt[x] / (b ^ 3 (2 + b x) ^ (3 / 2)) - 10 x ArcSinh[Sqrt[2] Sqrt[b] Sqr
t[x] / 2] / (2 b ^ (5 / 2) + b ^ (7 / 2) x) + 40 x ^ (3 / 2) / (3 b ^ 2 (2
+ b x) ^ (3 / 2)) + x ^ (5 / 2) / (b (2 + b x) ^ (3 / 2)) - 20 ArcSinh[Sqrt
[2] Sqrt[b] Sqrt[x] / 2] / (2 b ^ (7 / 2) + b ^ (9 / 2) x)
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.73

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(\frac{21}{4}x^2b^2+70bx+105\right)}{21\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}-10\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b^3}+\frac{\left(-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{7}{2}}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{3b^5\left(x+\frac{2}{b}\right)^2}+\frac{28\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{3b^4\left(x+\frac{2}{b}\right)}\right)\cdots}{\sqrt{x}\,\sqrt{bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3/b^(7/2)/Pi^(1/2)*(1/56*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(21/4*x^2*b^2+7
0*b*x+105)/(1/2*b*x+1)^(3/2)-15/4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1
/2)))
```

Maxima [A]
time = 0.36, size = 105, normalized size = 1.22

$$\frac{2\left(2\,b^2+\frac{10\,(bx+2)b}{x}-\frac{15\,(bx+2)^2}{x^2}\right)}{3\left(\frac{(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x + 2)*b/x - 15*(b*x + 2)^2/x^2)/((b*x + 2)^(3/2)*b^4/x^
(3/2) - (b*x + 2)^(5/2)*b^3/x^(5/2)) + 5*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt
(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.31, size = 186, normalized size = 2.16

$$\left[\frac{15\,(b^2x^2+4\,bx+4)\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(3\,b^3x^2+40\,b^2x+60\,b)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^6x^2+4\,b^5x+4\,b^4)}, \frac{30\,(b^2x^2+4\,bx+4)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+(3\,b^3x^2+40\,b^2x+60\,b)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^6x^2+4\,b^5x+4\,b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt
(x) + 1) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt(x))/(b^6*x^2 +
4*b^5*x + 4*b^4), 1/3*(30*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt
(x))/(b^6*x^2 + 4*b^5*x + 4*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.
time = 4.03, size = 308, normalized size = 3.58

$$\frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{30b^{10}x^{\frac{27}{2}}\sqrt{bx+2}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{bx+2}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25
/2)*sqrt(b*x + 2)) + 40*b**(21/2)*x**14/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2
) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) + 60*b**(19/2)*x**13/(3*b**(27/2)*
x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 30*b**10*x
**(27/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27
/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 60*b**9*x**(25/2
)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqr
t(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2))
```

Giac [A]
time = 0.01, size = 121, normalized size = 1.41

$$2\left(\frac{2\left(\left(\frac{\frac{1}{36}\cdot 9b^4\sqrt{x}\,\sqrt{x}}{b^5}+\frac{\frac{1}{36}\cdot 120b^3}{b^5}\right)\sqrt{x}\,\sqrt{x}+\frac{\frac{1}{36}\cdot 180b^2}{b^5}\right)\sqrt{x}\,\sqrt{bx+2}}{(bx+2)^2}+\frac{5\ln\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^3\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*(x*(3*x/b + 40/b^2) + 60/b^3)*sqrt(x)/(b*x + 2)^(3/2) + 10*log(-sqrt(b)
*sqrt(x) + sqrt(b*x + 2))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(5/2), x)
```

3.624 $\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] -2/3*x^(3/2)/b/(b*x+2)^(3/2)+2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)
-2*x^(1/2)/b^2/(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 56, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 + b*x]) + (2*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx &= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}+\frac{\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx}{b}\\ &= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2}\\ &= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 58, normalized size = 0.89

$$-\frac{4\sqrt{x}\,(3+2bx)}{3b^2(2+bx)^{3/2}}-\frac{2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*Sqrt[x]*(3 + 2*b*x))/(3*b^2*(2 + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[2 + b*x]])/b^(5/2)
```

Mathics [A]
time = 5.01, size = 86, normalized size = 1.32

$$\frac{-4\sqrt{x}}{b^2\,(2+bx)^{\frac{3}{2}}}+\frac{2x\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{2b^{\frac{3}{2}}+b^{\frac{5}{2}}x}-\frac{8x^{\frac{3}{2}}}{3b\,(2+bx)^{\frac{3}{2}}}+\frac{4\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{2b^{\frac{5}{2}}+b^{\frac{7}{2}}x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(3/2)/(2 + b*x)^(5/2),x]')
```

```
[Out] -4 Sqrt[x] / (b ^ 2 (2 + b x) ^ (3 / 2)) + 2 x ArcSinh[Sqrt[2] Sqrt[b] Sqrt
[x] / 2] / (2 b ^ (3 / 2) + b ^ (5 / 2) x) - 8 x ^ (3 / 2) / (3 b (2 + b x)
 ^ (3 / 2)) + 4 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / (2 b ^ (5 / 2) + b ^
 (7 / 2) x)
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.85

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(10bx+15)}{15\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3/b^(5/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(10*b*x+15)/(1
/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.34, size = 69, normalized size = 1.06

$$-\frac{2\left(b+\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}b^{2}}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + 2)/x)*x^(3/2)/((b*x + 2)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 171, normalized size = 2.63

$$\left[\frac{3\,(b^{2}x^{2}+4\,bx+4)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-4\,(2\,b^{2}x+3\,b)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^{5}x^{2}+4\,b^{4}x+4\,b^{3})},\,-\frac{2\left(3\,(b^{2}x^{2}+4\,bx+4)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,b^{2}x+3\,b)\sqrt{bx+2}\,\sqrt{x}\right)}{3\,(b^{5}x^{2}+4\,b^{4}x+4\,b^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(
x) + 1) - 4*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x + 4*b
^3), -2/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))) + 2*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x +
 4*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(61) = 122$.
time = 1.90, size = 257, normalized size = 3.95

$$-\frac{8b^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}-\frac{12b^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{6b^5x^{\frac{15}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{12b^4x^{\frac{13}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -8*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x**(13
/2)*sqrt(b*x + 2)) - 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2)
+ 6*b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 6*b**5*x**(15/2)*sqrt(b*x + 2)*asi
nh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(
13/2)*x**(13/2)*sqrt(b*x + 2)) + 12*b**4*x**(13/2)*sqrt(b*x + 2)*asinh(sqrt
(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x
**(13/2)*sqrt(b*x + 2))
```

Giac [A]
time = 0.01, size = 97, normalized size = 1.49

$$2\left(\frac{2\left(-\frac{\frac{1}{18}\cdot 12b^2\sqrt{x}\sqrt{x}}{b^3}-\frac{\frac{1}{18}\cdot 18b}{b^3}\right)\sqrt{x}\sqrt{bx+2}}{(bx+2)^2}-\frac{\ln\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^2\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] -4/3*sqrt(x)*(2*x/b + 3/b^2)/(b*x + 2)^(3/2) - 2*log(-sqrt(b)*sqrt(x) + sqr
t(b*x + 2))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(5/2), x)
```

3.625 $\int \frac{\sqrt{x}}{(2+bx)^{5/2}} dx$

Optimal. Leaf size=18

$$\frac{x^{3/2}}{3(2+bx)^{3/2}}$$

```
[Out] 1/3*x^(3/2)/(b*x+2)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}} dx = \frac{x^{3/2}}{3(2+bx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{x^{3/2}}{3(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Mathics [A]
time = 2.40, size = 12, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{3\,(2+bx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]')
[Out] x ^ (3 / 2) / (3 (2 + b x) ^ (3 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(12) = 24.
time = 0.10, size = 46, normalized size = 2.56

method	result	size
gosper	$\frac{x^{\frac{3}{2}}}{3(bx+2)^{\frac{3}{2}}}$	13
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}}{12\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	17
default	$-\frac{\sqrt{x}}{b(bx+2)^{\frac{3}{2}}}+\frac{\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}}{b}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/b*x^(1/2)/(b*x+2)^(3/2)+1/b*(1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+
2)^(1/2))
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")
[Out] 1/3*x^(3/2)/(b*x + 2)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 0.30, size = 27, normalized size = 1.50

$$\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{3\,(b^2x^2+4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x + 2)*x^(3/2)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [A]
time = 0.76, size = 27, normalized size = 1.50

$$\frac{x^{\frac{3}{2}}}{3bx\sqrt{bx+2}+6\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(5/2),x)
```

```
[Out] x**(3/2)/(3*b*x*sqrt(b*x + 2) + 6*sqrt(b*x + 2))
```

Giac [A]
time = 0.01, size = 40, normalized size = 2.22

$$\frac{\frac{1}{36}\cdot 12b\sqrt{x}\,\sqrt{x}\,\sqrt{x}\,\sqrt{bx+2}}{b\,(bx+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(b*x + 2)^(3/2)
```

Mupad [B]
time = 0.25, size = 12, normalized size = 0.67

$$\frac{x^{3/2}}{3\,(b\,x+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(5/2),x)
```

```
[Out] x^(3/2)/(3*(b*x + 2)^(3/2))
```

3.626 $\int \frac{1}{\sqrt{x}\,(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}$$

```
[Out] 1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(2+bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}\,(2+bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}(3+bx)}{3(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

Mathics [A]
time = 3.33, size = 32, normalized size = 0.86

$$\frac{\sqrt{b}\,x\,(3+bx)\sqrt{\frac{2+bx}{bx}}}{3\,(2+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]')
[Out] Sqrt[b] x (3 + b x) Sqrt[(2 + b x) / (b x)] / (3 (2 + b x) ^ 2)
```

Maple [A]
time = 0.12, size = 26, normalized size = 0.70

method	result	size
gosper	$\frac{\sqrt{x}\,(bx+3)}{3(bx+2)^{\frac{3}{2}}}$	18
meijerg	$\frac{\sqrt{x}\,\sqrt{2}\,(bx+3)}{12\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	22
default	$\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.65

$$-\frac{\left(b-\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{6\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(b - 3*(b*x + 2)/x)*x^(3/2)/(b*x + 2)^(3/2)
```

Fricas [A]
time = 0.31, size = 32, normalized size = 0.86

$$\frac{(bx+3)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^2x^2+4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(29) = 58$.
time = 1.04, size = 75, normalized size = 2.03

$$\frac{bx}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}+\frac{3}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b*x/(3*b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x))) + 3/(3*b
**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.
time = 0.01, size = 102, normalized size = 2.76

$$-\frac{32b\sqrt{b}\,b\left(-3\left(\sqrt{b\,(bx+2)-2b}-\sqrt{b}\,\sqrt{bx+2}\right)^2-2b\right)}{2\cdot 6\,|b|\left(\left(\sqrt{b\,(bx+2)-2b}-\sqrt{b}\,\sqrt{bx+2}\right)^2+2b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 8/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(5/2)/(
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

Mupad [B]
time = 0.36, size = 42, normalized size = 1.14

$$\frac{3\,\sqrt{x}\,\,\sqrt{b\,x+2}\,+b\,x^{3/2}\,\sqrt{b\,x+2}}{3\,b^2\,x^2+12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(b*x + 2)^(1/2) + b*x^(3/2)*(b*x + 2)^(1/2))/(12*b*x + 3*b^2*x^2
 + 12)
```

3.627 $\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\, dx$

Optimal. Leaf size=55

$$\frac{1}{3\sqrt{x}\,(2+bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] 1/3/(b*x+2)^(3/2)/x^(1/2)+2/3/x^(1/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(1/
2)
```

Rubi [A]
time = 0.00, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{bx+2}}+\frac{1}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 + b*x]) - (2*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx = \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx$$
$$= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2+bx}} + \frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3\sqrt{x}}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 0.58

$$\frac{-3-6bx-2b^2x^2}{3\sqrt{x}\,(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

Mathics [A]
time = 4.44, size = 48, normalized size = 0.87

$$\frac{\sqrt{b}\,(-3-6bx-2b^2x^2)\sqrt{\frac{2+bx}{bx}}}{12+12bx+3b^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]')
```

```
[Out] Sqrt[b] (-3 - 6 b x - 2 b ^ 2 x ^ 2) Sqrt[(2 + b x) / (b x)] / (3 (4 + 4 b
x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.76

method	result	size
gosper	$-\frac{2x^2b^2+6bx+3}{3\sqrt{x}\,(bx+2)^{\frac{3}{2}}}$	27
meijerg	$-\frac{\sqrt{2}\,(2x^2b^2+6bx+3)}{12\sqrt{x}\,\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	31
risch	$-\frac{\sqrt{bx+2}}{4\sqrt{x}}-\frac{b(5bx+12)\sqrt{x}}{12(bx+2)^{\frac{3}{2}}}$	33

default	$-\frac{1}{(bx+2)^{\frac{3}{2}}\sqrt{x}}-2b\left(\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*x+2)^(3/2)/x^(1/2)-2*b*(1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)
^(1/2))
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.73

$$\frac{\left(b^2-\frac{6\,(bx+2)b}{x}\right)x^{\frac{3}{2}}}{12\,(bx+2)^{\frac{3}{2}}}-\frac{\sqrt{bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x + 2)*b/x)*x^(3/2)/(b*x + 2)^(3/2) - 1/4*sqrt(b*x + 2)/sq
rt(x)
```

Fricas [A]
time = 0.31, size = 45, normalized size = 0.82

$$-\frac{(2\,b^2x^2+6\,bx+3)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^2x^3+4\,bx^2+4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 + 6*b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^3 + 4*b*x^2 + 4*x
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(49) = 98.
time = 2.43, size = 117, normalized size = 2.13

$$-\frac{2b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{6b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{3b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -2*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 6
*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 3*b**(
9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4)
```

Giac [A]
time = 0.01, size = 96, normalized size = 1.75

$$2\left(\frac{2\left(-\frac{\frac{1}{576}\cdot 60b^3\sqrt{x}\sqrt{x}}{b}-\frac{\frac{1}{576}\cdot 144b^2}{b}\right)\sqrt{x}\sqrt{bx+2}}{(bx+2)^2}+\frac{2\sqrt{b}}{4\left(\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)^2-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] -1/12*(5*b^2*x + 12*b)*sqrt(x)/(b*x + 2)^(3/2) + sqrt(b)/((sqrt(b)*sqrt(x)
- sqrt(b*x + 2))^2 - 2)
```

Mupad [B]
time = 0.38, size = 57, normalized size = 1.04

$$-\frac{3\sqrt{b\,x+2}+6\,b\,x\,\sqrt{b\,x+2}+2\,b^2\,x^2\,\sqrt{b\,x+2}}{\sqrt{x}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] -(3*(b*x + 2)^(1/2) + 6*b*x*(b*x + 2)^(1/2) + 2*b^2*x^2*(b*x + 2)^(1/2))/(x
^(1/2)*(x*(12*b + 3*b^2*x) + 12))
```

3.628 $\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] 1/3/x^(3/2)/(b*x+2)^(3/2)+1/x^(3/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(3/2)
+2/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x
])/(3*x^(3/2)) + (2*b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx = \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx$$
$$= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2+bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} - \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} + \frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.56

$$\frac{-1+3bx+6b^2x^2+2b^3x^3}{3x^{3/2}(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

Mathics [A]
time = 7.24, size = 76, normalized size = 1.07

$$\frac{\sqrt{b}\,\left(-2+bx\left(5+10b^2x^2+2b^3x^3\right)+15b^2x^2\right)\sqrt{\frac{2+bx}{bx}}}{3x\left(8+12bx+6b^2x^2+b^3x^3\right)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]')
```

```
[Out] Sqrt[b] (-2 + b x (5 + 10 b ^ 2 x ^ 2 + 2 b ^ 3 x ^ 3) + 15 b ^ 2 x ^ 2) Sq
rt[(2 + b x) / (b x)] / (3 x (8 + 12 b x + 6 b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.13, size = 58, normalized size = 0.82

method	result	size
gosper	$\frac{2b^3x^3+6x^2b^2+3bx-1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}$	35
meijerg	$-\frac{\sqrt{2}\,(-2b^3x^3-6x^2b^2-3bx+1)}{12x^{\frac{3}{2}}\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	39

risch	$\frac{4x^2b^2+7bx-2}{12x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^2(4bx+9)\sqrt{x}}{12(bx+2)^{\frac{3}{2}}}$	49
default	$-\frac{1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}-b\left(-\frac{1}{(bx+2)^{\frac{3}{2}}\sqrt{x}}-2b\left(\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}\right)\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)/(b*x+2)^(3/2)-b*(-1/(b*x+2)^(3/2)/x^(1/2)-2*b*(1/3*x^(1/2)/(b*
x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.77

$$\frac{3\sqrt{bx+2}\,b}{8\sqrt{x}}-\frac{\left(b^3-\frac{9\,(bx+2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(bx+2)^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(b*x + 2)*b/sqrt(x) - 1/24*(b^3 - 9*(b*x + 2)*b^2/x)*x^(3/2)/(b*x +
 2)^(3/2) - 1/24*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.31, size = 55, normalized size = 0.77

$$\frac{(2\,b^3x^3+6\,b^2x^2+3\,bx-1)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^2x^4+4\,bx^3+4\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^3*x^3 + 6*b^2*x^2 + 3*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^4 + 4*
b*x^3 + 4*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(66) = 132.
time = 4.58, size = 257, normalized size = 3.62

$$\frac{2b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{10b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{15b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{5b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}-\frac{2b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 2*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10
*x**2 + 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18
*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2*sqrt(1 + 2/(b*
x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 5*b**(21/2
)*x*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b*
*9*x) - 2*b**(19/2)*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b*
*10*x**2 + 24*b**9*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.

time = 0.01, size = 166, normalized size = 2.34

$$2\left(\frac{2\left(\frac{\frac{1}{2304}\cdot 192b^4\sqrt{x}\sqrt{x}}{b}+\frac{\frac{1}{2304}\cdot 432b^3}{b}\right)\sqrt{x}\sqrt{bx+2}}{(bx+2)^2}+\frac{2\left(-3b\sqrt{b}\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)^4+18b\sqrt{b}\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)^2-16b\sqrt{b}\right)}{12\left(\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)^2-2\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/12*(4*b^3*x + 9*b^2)*sqrt(x)/(b*x + 2)^(3/2) - 1/3*(3*b^(3/2)*(sqrt(b)*sq
rt(x) - sqrt(b*x + 2))^4 - 18*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 +
 16*b^(3/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + 2))^2 - 2)^3
```

Mupad [B]

time = 0.42, size = 71, normalized size = 1.00

$$\frac{3\,b\,x\,\sqrt{b\,x+2}-\sqrt{b\,x+2}+6\,b^2\,x^2\,\sqrt{b\,x+2}+2\,b^3\,x^3\,\sqrt{b\,x+2}}{x^{3/2}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*b*x*(b*x + 2)^(1/2) - (b*x + 2)^(1/2) + 6*b^2*x^2*(b*x + 2)^(1/2) + 2*b^
3*x^3*(b*x + 2)^(1/2))/(x^(3/2)*(x*(12*b + 3*b^2*x) + 12))
```

3.629 $\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=91

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] 5*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/6*x^(3/2)*(-b*x+2)^(1/2)/b^
2-1/3*x^(5/2)*(-b*x+2)^(1/2)/b-5/2*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(6*b^2) - (x
^(5/2)*Sqrt[2 - b*x])/(3*b) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx &= -\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{3b}\\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}\\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^3}\\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.79

$$-\frac{\sqrt{x}\,\sqrt{2-bx}\,(15+5bx+2b^2x^2)}{6b^3}+\frac{5\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2))/b^3 + (5*Log[-(Sqrt[-
b]*Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 10.12, size = 163, normalized size = 1.79

$$\text{Piecewise}\left[\left\{\left\{\frac{-5I\,\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b^{\frac{7}{2}}}+\frac{5I\sqrt{x}}{b^3\sqrt{-2+bx}}-\frac{5Ix^{\frac{3}{2}}}{6b^2\sqrt{-2+bx}}-\frac{Ix^{\frac{5}{2}}}{6b\sqrt{-2+bx}}-\frac{Ix^{\frac{7}{2}}}{3\sqrt{-2+bx}},\text{Abs}[bx]>2\right\}\right\},\frac{5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b^{\frac{7}{2}}}-\frac{5\sqrt{x}}{b^3\sqrt{2-bx}}+\frac{5x^{\frac{3}{2}}}{6b^2\sqrt{2-bx}}+\frac{x^{\frac{5}{2}}}{6b\sqrt{2-bx}}+\frac{x^{\frac{7}{2}}}{3\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/Sqrt[2 - b*x],x]')
```

```
[Out] Piecewise[{{-5 I ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (7 / 2) + 5 I S
qrt[x] / (b ^ 3 Sqrt[-2 + b x]) - 5 I / 6 x ^ (3 / 2) / (b ^ 2 Sqrt[-2 + b
x]) - I / 6 x ^ (5 / 2) / (b Sqrt[-2 + b x]) - I / 3 x ^ (7 / 2) / Sqrt[-2
```

```
+ b x], Abs[b x] > 2}}, 5 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (7 / 2)
 - 5 Sqrt[x] / (b ^ 3 Sqrt[2 - b x]) + 5 x ^ (3 / 2) / (6 b ^ 2 Sqrt[2 - b
x]) + x ^ (5 / 2) / (6 b Sqrt[2 - b x]) + x ^ (7 / 2) / (3 Sqrt[2 - b x])]
```

Maple [A]
time = 0.13, size = 111, normalized size = 1.22

method	result
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(14x^{2}b^{2}+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{336b^{3}}+\frac{5\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{8b^{\frac{7}{2}}}\right)}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$\frac{\left(2x^{2}b^{2}+5bx+15\right)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6b^{3}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}\sqrt{-bx+2}}{3b}+\frac{-\frac{5x^{\frac{3}{2}}\sqrt{-bx+2}}{6b}+\frac{5\left(-\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2b}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}\right)}{3b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^(5/2)*(-b*x+2)^(1/2)/b+5/3/b*(-1/2*x^(3/2)*(-b*x+2)^(1/2)/b+3/2/b*(-
x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2
)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))))
```

Maxima [A]
time = 0.35, size = 117, normalized size = 1.29

$$-\frac{\frac{33\sqrt{-bx+2}\,b^{2}}{\sqrt{x}}+\frac{40\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{6}-\frac{3\,(bx-2)b^{5}}{x}+\frac{3\,(bx-2)^{2}b^{4}}{x^{2}}-\frac{(bx-2)^{3}b^{3}}{x^{3}}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(-b*x + 2)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 15*(-
b*x + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x - 2)*b^5/x + 3*(b*x - 2)^2*b^4/x^2 -
(b*x - 2)^3*b^3/x^3) - 5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.33, size = 125, normalized size = 1.37

$$\left[-\frac{(2\,b^3x^2+5\,b^2x+15\,b)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^4},-\frac{(2\,b^3x^2+5\,b^2x+15\,b)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, -1/6*((2*b^3*x^2 + 5*b^
2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt
(b)*sqrt(x))))/b^4]
```

Sympy [A]
time = 8.91, size = 204, normalized size = 2.24

$$\begin{cases} -\frac{ix^{\frac{7}{2}}}{3\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{6b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{6b^2\sqrt{bx-2}}+\frac{5i\sqrt{x}}{b^3\sqrt{bx-2}}-\frac{5i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for}\ |bx|>2 \\ \frac{x^{\frac{7}{2}}}{3\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{6b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{6b^2\sqrt{-bx+2}}-\frac{5\sqrt{x}}{b^3\sqrt{-bx+2}}+\frac{5\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(7/2)/(3*sqrt(b*x - 2)) - I*x**(5/2)/(6*b*sqrt(b*x - 2)) -
 5*I*x**(3/2)/(6*b**2*sqrt(b*x - 2)) + 5*I*sqrt(x)/(b**3*sqrt(b*x - 2)) - 5
*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), Abs(b*x) > 2), (x**(7/2)/(3*s
qrt(-b*x + 2)) + x**(5/2)/(6*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(6*b**2*sqrt(-b
*x + 2)) - 5*sqrt(x)/(b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2)/b**(7/2), True))
```

Giac [A]
time = 0.00, size = 123, normalized size = 1.35

$$2\left(2\left(\left(\left(-\frac{\frac{1}{72}\cdot 6b^4\sqrt{x}\,\sqrt{x}}{b^5}-\frac{\frac{1}{72}\cdot 15b^3}{b^5}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{72}\cdot 45b^2}{b^5}\right)\sqrt{x}\,\sqrt{-bx+2}-\frac{5\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{2b^3\sqrt{-b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/6*sqrt(-b*x + 2)*(x*(2*x/b + 5/b^2) + 15/b^3)*sqrt(x) - 5*log(-sqrt(-b)*
sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(1/2),x)
[Out] int(x^(5/2)/(2 - b*x)^(1/2), x)
```

3.630 $\int \frac{x^{3/2}}{\sqrt{2-bx}} \, dx$

Optimal. Leaf size=69

$$-\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] 3*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-1/2*x^(3/2)*(-b*x+2)^(1/2)/b-
3/2*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]

time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(2*b) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx &= -\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}\\
&= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^2}\\
&= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 63, normalized size = 0.91

$$
-\frac{\sqrt{x}\,\sqrt{2-bx}\,(3+bx)}{2b^2}-\frac{3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x))/b^2 - (3*Log[-(Sqrt[-b]*Sqrt[x]) + S
qrt[2 - b*x]])/(-b)^(5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/Sqrt[2 - b*x],x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.11, size = 89, normalized size = 1.29

method	result	size

meijerg	$-\frac{4\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}(5bx+15)\sqrt{-\frac{bx}{2}+1}}{40b^{2}}+\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{5}{2}}}\right)}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$	73
default	$-\frac{x^{\frac{3}{2}}\sqrt{-bx+2}}{2b}+\frac{-\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2b}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}}{b}$	89
risch	$\frac{(bx+3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2b^{2}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)*(-b*x+2)^(1/2)/b+3/2/b*(-x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((
-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)
^(1/2)))
```

Maxima [A]
time = 0.35, size = 85, normalized size = 1.23

$$-\frac{\frac{5\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{3\,(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{4}-\frac{2\,(bx-2)b^{3}}{x}+\frac{(bx-2)^{2}b^{2}}{x^{2}}}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(5*sqrt(-b*x + 2)*b/sqrt(x) + 3*(-b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x -
2)*b^3/x + (b*x - 2)^2*b^2/x^2) - 3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))
)/b^(5/2)
```

Fricas [A]
time = 0.45, size = 107, normalized size = 1.55

$$\left[-\frac{(b^{2}x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b^{3}},-\frac{(b^{2}x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt
(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [A]
time = 2.55, size = 162, normalized size = 2.35

$$\begin{cases} -\frac{ix^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{2b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ \frac{x^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{2b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(5/2)/(2*sqrt(b*x - 2)) - I*x**(3/2)/(2*b*sqrt(b*x - 2)) +
 3*I*sqrt(x)/(b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(5/2), Abs(b*x) > 2), (x**(5/2)/(2*sqrt(-b*x + 2)) + x**(3/2)/(2*b*sqrt(-b
*x + 2)) - 3*sqrt(x)/(b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2)/b**(5/2), True))
```

Giac [A]
time = 0.00, size = 95, normalized size = 1.38

$$2\left(2\left(-\frac{\frac{1}{8}b^2\sqrt{x}\sqrt{x}}{b^3} - \frac{\frac{1}{8}\cdot 3b}{b^3}\right)\sqrt{x}\sqrt{-bx+2} - \frac{3\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{2b^2\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/2*sqrt(-b*x + 2)*sqrt(x)*(x/b + 3/b^2) - 3*log(-sqrt(-b)*sqrt(x) + sqrt(
-b*x + 2))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(1/2), x)
```

3.631 $\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{b}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)-x^(1/2)*(-b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx = -\frac{\sqrt{x}\,\sqrt{2-bx}}{b} + \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b}$$
$$= -\frac{\sqrt{x}\,\sqrt{2-bx}}{b} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\,\sqrt{2-bx}}{b} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [**A**]
time = 0.05, size = 56, normalized size = 1.24

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{b} + \frac{2\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])
/(-b)^(3/2)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/Sqrt[2 - b*x],x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.13, size = 67, normalized size = 1.49

method	result	size

meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}\sqrt{-\frac{bx}{2}+1}}{2b}+\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}}\right)}{\sqrt{-b}\,\sqrt{\pi}\,b}$	66
default	$-\frac{\sqrt{x}\,\sqrt{-bx+2}}{b}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	67
risch	$\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/
2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.35, size = 52, normalized size = 1.16

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{-bx+2}}{\left(b^{2}-\frac{(bx-2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) - 2*sqrt(-b*x + 2)/((b^
2 - (b*x - 2)*b/x)*sqrt(x))
```

Fricas [A]
time = 0.32, size = 90, normalized size = 2.00

$$\left[-\frac{\sqrt{-bx+2}\,b\sqrt{x}+\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b^{2}},-\frac{\sqrt{-bx+2}\,b\sqrt{x}+2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-b*x + 2)*b*sqrt(x) + sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*s
qrt(x) + 1))/b^2, -(sqrt(-b*x + 2)*b*sqrt(x) + 2*sqrt(b)*arctan(sqrt(-b*x +
 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [A]
time = 1.08, size = 119, normalized size = 2.64

$$\begin{cases} -\frac{ix^{\frac{3}{2}}}{\sqrt{bx-2}} + \frac{2i\sqrt{x}}{b\sqrt{bx-2}} - \frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx| > 2 \\ \frac{x^{\frac{3}{2}}}{\sqrt{-bx+2}} - \frac{2\sqrt{x}}{b\sqrt{-bx+2}} + \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(3/2)/sqrt(b*x - 2) + 2*I*sqrt(x)/(b*sqrt(b*x - 2)) - 2*I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x) > 2), (x**(3/2)/sqrt(-b
*x + 2) - 2*sqrt(x)/(b*sqrt(-b*x + 2)) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
b**(3/2), True))
```

Giac [A]
time = 0.00, size = 62, normalized size = 1.38

$$2\left(-\frac{\frac{1}{4}\cdot 2\sqrt{x}\sqrt{-bx+2}}{b} - \frac{\ln\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)}{b\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -sqrt(-b*x + 2)*sqrt(x)/b - 2*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt
(-b)*b)
```

Mupad [B]
time = 0.52, size = 46, normalized size = 1.02

$$-\frac{4\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{b^{3/2}} - \frac{\sqrt{x}\,\sqrt{2-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(1/2),x)
```

```
[Out] - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1/2))))/b^(3/2) - (x^(1/2
)*(2 - b*x)^(1/2))/b
```

3.632 $\int \frac{1}{\sqrt{x}\sqrt{2-bx}} dx$

Optimal. Leaf size=24

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

[Out] 2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {56, 222}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[2 - b*x]),x]

[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{2-bx}} dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} dx, x, \sqrt{x}\right) \\
&= \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.46

$$-\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/Sqrt[-b]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.14, size = 43, normalized size = 1.79

$$\text{Piecewise}\left[\left\{\left\{\frac{-2I\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{\sqrt{b}},\text{Abs}\left[bx\right]>2\right\}\right\},\frac{2\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{\sqrt{b}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[2 - b*x]),x]')
```

```
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / Sqrt[b], Abs[b x] >
 2}}, 2 ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / Sqrt[b]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.
time = 0.12, size = 50, normalized size = 2.08

method	result	size
meijerg	$\frac{2\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}$	18
default	$\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-
b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.34, size = 21, normalized size = 0.88

$$-\frac{2 \arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Fricas [A]
time = 0.32, size = 56, normalized size = 2.33

$$\left[-\frac{\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan(sqr
t(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [A]
time = 0.52, size = 56, normalized size = 2.33

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx|>2 \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+2)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x) > 2), (2
*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), True))
```

Giac [A]
time = 0.00, size = 35, normalized size = 1.46

$$-\frac{2\ln\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -2*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/sqrt(-b)
```

Mupad [B]
time = 0.03, size = 27, normalized size = 1.12

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{2-b\,x}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (2 - b*x)^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

3.633 $\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

```
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {37}

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx = -\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.16, size = 45, normalized size = 2.65

$$\text{Piecewise}\left[\left\{\left\{-\sqrt{b}\sqrt{-1+\frac{2}{bx}},\frac{1}{\text{Abs}\,[bx]}>\frac{1}{2}\right\}\right\},-I\sqrt{b}\sqrt{1-\frac{2}{bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*Sqrt[2 - b*x]),x]')
```

```
[Out] Piecewise[{{-Sqrt[b] Sqrt[-1 + 2 / (b x)], 1 / Abs[b x] > 1 / 2}}, -I Sqrt[
b] Sqrt[1 - 2 / (b x)]]
```

Maple [A]
time = 0.11, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$	14
default	$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$	14
meijerg	$-\frac{\sqrt{2}\,\sqrt{-\frac{bx}{2}+1}}{\sqrt{x}}$	17
risch	$\frac{(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.31, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

Sympy [A]
time = 0.50, size = 41, normalized size = 2.41

$$\begin{cases} -\sqrt{b}\sqrt{-1+\frac{2}{bx}} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ -i\sqrt{b}\sqrt{1-\frac{2}{bx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(1/2),x)
[Out] Piecewise((-sqrt(b)*sqrt(-1 + 2/(b*x)), 1/Abs(b*x) > 1/2), (-I*sqrt(b)*sqrt
(1 - 2/(b*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.
time = 0.00, size = 39, normalized size = 2.29

$$\frac{8\sqrt{-b}}{2\left(\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x)
[Out] 4*sqrt(-b)/((sqrt(-b)*sqrt(x) - sqrt(-b*x + 2))^2 - 2)
```

Mupad [B]
time = 0.31, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2-b\,x}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(1/2)),x)
[Out] -(2 - b*x)^(1/2)/x^(1/2)
```

3.634 $\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] -1/3*(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 - b*x]),x]
[Out] -1/3*Sqrt[2 - b*x]/x^(3/2) - (b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx &= -\frac{\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 0.62

$$\frac{(-1-bx)\sqrt{2-bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] ((-1 - b*x)*Sqrt[2 - b*x])/(3*x^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.30, size = 90, normalized size = 2.25

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\ (2+bx(1-bx))\sqrt{\frac{2-bx}{bx}}}{3x(-2+bx)}, \frac{1}{\text{Abs}[bx]}>\frac{1}{2}\right\}\right\}, -\frac{Ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}-\frac{I\sqrt{b}\ \sqrt{1-\frac{2}{bx}}}{3x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*Sqrt[2 - b*x]),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (2 + b x (1 - b x)) Sqrt[(2 - b x) / (b x)] / (3 x (-2
+ b x)), 1 / Abs[b x] > 1 / 2}}, -I b ^ (3 / 2) Sqrt[1 - 2 / (b x)] / 3 - I
 Sqrt[b] Sqrt[1 - 2 / (b x)] / (3 x)]
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.72

method	result	size
gosper	$-\frac{(bx+1)\sqrt{-bx+2}}{3x^{\frac{3}{2}}}$	19
meijerg	$-\frac{\sqrt{2}\ (bx+1)\sqrt{-\frac{bx}{2}+1}}{3x^{\frac{3}{2}}}$	22
default	$-\frac{\sqrt{-bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{-bx+2}}{3\sqrt{x}}$	29
risch	$\frac{\sqrt{(-bx+2)\,x}\ (x^{2}b^{2}-bx-2)}{3x^{\frac{3}{2}}\sqrt{-bx+2}\ \sqrt{-x\,(bx-2)}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.70

$$-\frac{\sqrt{-bx+2}\,b}{2\,\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/6*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 18, normalized size = 0.45

$$-\frac{(bx+1)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x + 1)*sqrt(-b*x + 2)/x^(3/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.13, size = 139, normalized size = 3.48

$$\begin{cases} -\frac{b^{\frac{7}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^2x^2-6bx}+\frac{b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^2x^2-6bx}+\frac{2b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^2x^2-6bx} & \text{for } \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}-\frac{i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise(((-b**(7/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**2*x**2 - 6*b*x) + b**(5/
2)*x*sqrt(-1 + 2/(b*x))/(3*b**2*x**2 - 6*b*x) + 2*b**(3/2)*sqrt(-1 + 2/(b*x
))/(3*b**2*x**2 - 6*b*x), 1/Abs(b*x) > 1/2), (-I*b**(3/2)*sqrt(1 - 2/(b*x))
/3 - I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.
time = 0.00, size = 75, normalized size = 1.88

$$\frac{32\sqrt{-b}\,b\left(-3\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)^2+2\right)}{2\cdot 6\left(\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)^2-2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -8/3*(3*(sqrt(-b)*sqrt(x) - sqrt(-b*x + 2))^2 - 2)*sqrt(-b)*b/((sqrt(-b)*sq
rt(x) - sqrt(-b*x + 2))^2 - 2)^3
```

Mupad [B]
time = 0.29, size = 19, normalized size = 0.48

$$-\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}+\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -((2 - b*x)^(1/2)*((b*x)/3 + 1/3))/x^(3/2)
```

3.635 $\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2x^{5/2}}{b\sqrt{2-bx}}+\frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] -15*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)+2*x^(5/2)/b/(-b*x+2)^(1/2)+
5/2*x^(3/2)*(-b*x+2)^(1/2)/b^2+15/2*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[2 - b*x]) + (15*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) + (5*x^(
3/2)*Sqrt[2 - b*x])/(2*b^2) - (15*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{2-bx}} - \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{b}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 71, normalized size = 0.80

$$
-\frac{\sqrt{x}\,(-30+5bx+b^2x^2)}{2b^3\sqrt{2-bx}} - \frac{15\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(3/2),x]
[Out] -1/2*(Sqrt[x]*(-30 + 5*b*x + b^2*x^2))/(b^3*Sqrt[2 - b*x]) - (15*Log[-(Sqrt
[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(2 - b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 81, normalized size = 0.91

method	result
meijerg	$-\frac{8\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(-\frac{7}{2}x^{2}b^{2}-\frac{35}{2}bx+105\right)}{112b^{3}\sqrt{-\frac{bx}{2}+1}}-\frac{15\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{8b^{\frac{7}{2}}}\right)}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{(bx+7)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2b^{3}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}-\frac{\left(\frac{15\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{2b^{\frac{7}{2}}}+\frac{8\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{b^{4}\left(x-\frac{2}{b}\right)}\right)\sqrt{(-bx}}{\sqrt{x}\,\sqrt{-bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/(-b)^(5/2)/Pi^(1/2)/b*(1/112*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(7/2)*(-7/2*x
^2*b^2-35/2*b*x+105)/b^3/(-1/2*b*x+1)^(1/2)-15/8*Pi^(1/2)*(-b)^(7/2)/b^(7/2
)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.35, size = 101, normalized size = 1.13

$$\frac{8\,b^{2}-\frac{25\,(bx-2)b}{x}+\frac{15\,(bx-2)^{2}}{x^{2}}}{\frac{\sqrt{-bx+2}\,b^{5}}{\sqrt{x}}+\frac{2\,(-bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}}+\frac{15\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] (8*b^2 - 25*(b*x - 2)*b/x + 15*(b*x - 2)^2/x^2)/(sqrt(-b*x + 2)*b^5/sqrt(x)
 + 2*(-b*x + 2)^(3/2)*b^4/x^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) + 15*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.32, size = 155, normalized size = 1.74

$$\left[-\frac{15\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(b^3x^2+5\,b^2x-30\,b)\sqrt{-bx+2}\,\sqrt{x}}{2\,(b^5x-2\,b^4)},\ \frac{30\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(b^3x^2+5\,b^2x-30\,b)\sqrt{-bx+2}\,\sqrt{x}}{2\,(b^5x-2\,b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(15*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1
) - (b^3*x^2 + 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4), 1/2
*(30*(b*x - 2)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^3*x^2
+ 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4)]
```

Sympy [A]
time = 5.34, size = 172, normalized size = 1.93

$$\begin{cases} \frac{i x^{\frac{5}{2}}}{2 b \sqrt{b x - 2}} + \frac{5 i x^{\frac{3}{2}}}{2 b^{2} \sqrt{b x - 2}} - \frac{15 i \sqrt{x}}{b^{3} \sqrt{b x - 2}} + \frac{15 i \operatorname{acosh}{\left(\frac{\sqrt{2} \sqrt{b} \sqrt{x}}{2} \right)}}{b^{\frac{7}{2}}} & \text{for}\: \left|{b x}\right| > 2 \\- \frac{x^{\frac{5}{2}}}{2 b \sqrt{- b x + 2}} - \frac{5 x^{\frac{3}{2}}}{2 b^{2} \sqrt{- b x + 2}} + \frac{15 \sqrt{x}}{b^{3} \sqrt{- b x + 2}} - \frac{15 \operatorname{asin}{\left(\frac{\sqrt{2} \sqrt{b} \sqrt{x}}{2} \right)}}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(5/2)/(2*b*sqrt(b*x - 2)) + 5*I*x**(3/2)/(2*b**2*sqrt(b*x -
 2)) - 15*I*sqrt(x)/(b**3*sqrt(b*x - 2)) + 15*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), Abs(b*x) > 2), (-x**(5/2)/(2*b*sqrt(-b*x + 2)) - 5*x**(3/2)
/(2*b**2*sqrt(-b*x + 2)) + 15*sqrt(x)/(b**3*sqrt(-b*x + 2)) - 15*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(7/2), True))
```

Giac [A]
time = 0.01, size = 128, normalized size = 1.44

$$-2\left(\frac{2\left(\left(\left(\frac{\frac{1}{8}b^4\sqrt{x}\,\sqrt{x}}{b^5}+\frac{\frac{1}{8}\cdot 5b^3}{b^5}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{8}\cdot 30b^2}{b^5}\right)\sqrt{x}\,\sqrt{-bx+2}}{-bx+2}-\frac{15\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{2b^3\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] 1/2*sqrt(-b*x + 2)*(x*(x/b + 5/b^2) - 30/b^3)*sqrt(x)/(b*x - 2) + 15*log(-s
qrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(3/2), x)
```

3.636 $\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2x^{3/2}}{b\sqrt{2-bx}}+\frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] -6*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+2*x^(3/2)/b/(-b*x+2)^(1/2)+3
*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[2 - b*x]) + (3*Sqrt[x]*Sqrt[2 - b*x])/b^2 - (6*ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{2-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b} \\
&= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2} - \frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2} - \frac{6\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2} - \frac{6\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.94

$$\frac{\sqrt{x}\,(6-bx)}{b^2\sqrt{2-bx}} + \frac{6\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(3/2),x]
[Out] (Sqrt[x]*(6 - b*x))/(b^2*Sqrt[2 - b*x]) + (6*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt
[2 - b*x]])/(-b)^(5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(2 - b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 73, normalized size = 1.12

method	result
meijerg	$-\frac{4\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(-\frac{5bx}{2}+15\right)}{20b^{2}\sqrt{-\frac{bx}{2}+1}}-\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2b^{\frac{5}{2}}}\right)}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{b^{2}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}-\frac{\left(\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{b^{3}\left(x-\frac{2}{b}\right)}\right)\sqrt{(-bx+}}{\sqrt{x}\,\sqrt{-bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(-b)^(3/2)/Pi^(1/2)/b*(1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-5/2*b*
x+15)/b^2/(-1/2*b*x+1)^(1/2)-3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*b^(
1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.35, size = 71, normalized size = 1.09

$$\frac{2\left(2\,b-\frac{3\,(bx-2)}{x}\right)}{\frac{\sqrt{-bx+2}\,b^{3}}{\sqrt{x}}+\frac{(-bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{6\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x - 2)/x)/(sqrt(-b*x + 2)*b^3/sqrt(x) + (-b*x + 2)^(3/2)*b^2/
x^(3/2)) + 6*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 138, normalized size = 2.12

$$\left[-\frac{3\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(b^{2}x-6\,b)\sqrt{-bx+2}\,\sqrt{x}}{b^{4}x-2\,b^{3}},\ \frac{6\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(b^{2}x-6\,b)\sqrt{-bx+2}\,\sqrt{x}}{b^{4}x-2\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(3*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (
b^2*x - 6*b)*sqrt(-b*x + 2)*sqrt(x))/(b^4*x - 2*b^3), (6*(b*x - 2)*sqrt(b)*
arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x - 6*b)*sqrt(-b*x + 2)*sqr
t(x))/(b^4*x - 2*b^3)]
```

Sympy [A]
time = 1.83, size = 126, normalized size = 1.94

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{b\sqrt{bx-2}} - \frac{6i\sqrt{x}}{b^2\sqrt{bx-2}} + \frac{6i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ -\frac{x^{\frac{3}{2}}}{b\sqrt{-bx+2}} + \frac{6\sqrt{x}}{b^2\sqrt{-bx+2}} - \frac{6\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(b*sqrt(b*x - 2)) - 6*I*sqrt(x)/(b**2*sqrt(b*x - 2))
+ 6*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x) > 2), (-x**(3/2)/
(b*sqrt(-b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(-b*x + 2)) - 6*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(5/2), True))
```

Giac [A]
time = 0.01, size = 100, normalized size = 1.54

$$-2\left(\frac{2\left(\frac{\frac{1}{4}b^2\sqrt{x}\sqrt{x}}{b^3} - \frac{\frac{1}{4}\cdot 6b}{b^3}\right)\sqrt{x}\sqrt{-bx+2}}{-bx+2} - \frac{3\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{b^2\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] sqrt(-b*x + 2)*sqrt(x)*(x/b - 6/b^2)/(b*x - 2) + 6*log(-sqrt(-b)*sqrt(x) +
sqrt(-b*x + 2))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{{(2-b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(3/2),x)
[Out] int(x^(3/2)/(2 - b*x)^(3/2), x)
```

3.637 $\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] -2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+2*x^(1/2)/b/(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 1.24

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
)/(-b)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.56, size = 76, normalized size = 1.69

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b^{\frac{3}{2}}} - \frac{2I\sqrt{x}}{b\sqrt{-2+bx}}, \text{Abs}\left[bx\right]>2\right\}\right\}, \frac{-2\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b^{\frac{3}{2}}} + \frac{2\sqrt{x}}{b\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(2 - b*x)^(3/2),x]')
```

```
[Out] Piecewise[{{2 I ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b ^ (3 / 2) - 2 I Sq
rt[x] / (b Sqrt[-2 + b x]), Abs[b x] > 2}}, -2 ArcSin[Sqrt[2] Sqrt[b] Sqrt[
x] / 2] / b ^ (3 / 2) + 2 Sqrt[x] / (b Sqrt[2 - b x])]
```

Maple [A]
time = 0.10, size = 67, normalized size = 1.49

method	result	size

meijerg	$-\frac{2\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}}{2b\sqrt{-\frac{bx}{2}+1}}-\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}}\right)}{\sqrt{-b}\,\sqrt{\pi}\,b}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-b)^(1/2)/Pi^(1/2)/b*(1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(3/2)/b/(-1/2*b
*x+1)^(1/2)-Pi^(1/2)*(-b)^(3/2)/b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))
)
```

Maxima [A]
time = 0.36, size = 38, normalized size = 0.84

$$\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{-bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 2)*b)
```

Fricas [A]
time = 0.32, size = 122, normalized size = 2.71

$$\left[-\frac{(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+2\sqrt{-bx+2}\,b\sqrt{x}}{b^3x-2b^2},\ \frac{2\left((bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+2}\,b\sqrt{x}\right)}{b^3x-2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 2*s
qrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2), 2*((b*x - 2)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2)]
```

Sympy [A]

time = 0.85, size = 90, normalized size = 2.00

$$\begin{cases} -\frac{2i\sqrt{x}}{b\sqrt{bx-2}} + \frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx| > 2 \\ \frac{2\sqrt{x}}{b\sqrt{-bx+2}} - \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*I*sqrt(x)/(b*sqrt(b*x - 2)) + 2*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(3/2), Abs(b*x) > 2), (2*sqrt(x)/(b*sqrt(-b*x + 2)) - 2*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(3/2), True))
```

Giac [A]
time = 0.01, size = 70, normalized size = 1.56

$$-2\left(-\frac{\frac{1}{2}\cdot 2\sqrt{x}\sqrt{-bx+2}}{b\left(-bx+2\right)} - \frac{\ln\left(\sqrt{-bx+2} - \sqrt{-b}\sqrt{x}\right)}{b\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x)
```

```
[Out] 2*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b) - 2*sqrt(-b*x + 2)*s
qrt(x)/((b*x - 2)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\left(2-b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(2 - b*x)^(3/2), x)
```

3.638 $\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

[Out] x^(1/2)/(-b*x+2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]

[Out] Sqrt[x]/Sqrt[2 - b*x]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2-bx}}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]

[Out] Sqrt[x]/Sqrt[2 - b*x]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.08, size = 44, normalized size = 2.75

$$\text{Piecewise}\left[\left\{\left\{\frac{1}{\sqrt{b}\sqrt{-1+\frac{2}{bx}}}, \frac{1}{\text{Abs}\,[bx]} > \frac{1}{2}\right\}\right\}, -\frac{I}{\sqrt{b}\sqrt{1-\frac{2}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]')
```

```
[Out] Piecewise[{{1 / (Sqrt[b] Sqrt[-1 + 2 / (b x)]), 1 / Abs[b x] > 1 / 2}}, -I
/ (Sqrt[b] Sqrt[1 - 2 / (b x)])]
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{\sqrt{x}}{\sqrt{-bx+2}}$	13
default	$\frac{\sqrt{x}}{\sqrt{-bx+2}}$	13
meijerg	$\frac{\sqrt{x}\,\sqrt{2}}{2\sqrt{-\frac{bx}{2}+1}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(-b*x + 2)
```

Fricas [A]
time = 0.31, size = 20, normalized size = 1.25

$$-\frac{\sqrt{-bx+2}\,\sqrt{x}}{bx-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)
```

Sympy [A]
time = 0.49, size = 41, normalized size = 2.56

$$\begin{cases} \frac{1}{\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{i}{\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((1/(sqrt(b)*sqrt(-1 + 2/(b*x))), 1/Abs(b*x) > 1/2), (-I/(sqrt(b)*
sqrt(1 - 2/(b*x))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(12) = 24$.
time = 0.01, size = 59, normalized size = 3.69

$$-\frac{8b\sqrt{-b}}{2\,|b|\left(\left(\sqrt{-b\,(-bx+2)+2b}-\sqrt{-b}\,\sqrt{-bx+2}\right)^{2}-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b))
```

Mupad [B]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] x^(1/2)/(2 - b*x)^(1/2)
```

3.639 $\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

```
[Out] 1/x^(1/2)/(-b*x+2)^(1/2)-(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {47, 37}

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 - b*x]) - Sqrt[2 - b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\sqrt{2-bx}}+\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 21, normalized size = 0.62

$$\frac{-1+bx}{\sqrt{x}\,\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x)/(Sqrt[x]*Sqrt[2 - b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.79, size = 81, normalized size = 2.38

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,(1-bx)\sqrt{\frac{2-bx}{bx}}}{-2+bx},\frac{1}{\text{Abs}\,[bx]}>\frac{1}{2}\right\}\right\},-\frac{I\sqrt{b}}{\sqrt{1-\frac{2}{bx}}}+\frac{I}{\sqrt{b}\,x\sqrt{1-\frac{2}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(2 - b*x)^(3/2)),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (1 - b x) Sqrt[(2 - b x) / (b x)] / (-2 + b x), 1 / Abs
[b x] > 1 / 2}}, -I Sqrt[b] / Sqrt[1 - 2 / (b x)] + I / (Sqrt[b] x Sqrt[1 -
 2 / (b x)])]
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.82

method	result	size
gosper	$\frac{bx-1}{\sqrt{x}\,\sqrt{-bx+2}}$	18
meijerg	$-\frac{\sqrt{2}\,(-bx+1)}{2\sqrt{x}\,\sqrt{-\frac{bx}{2}+1}}$	23
default	$-\frac{1}{\sqrt{x}\,\sqrt{-bx+2}}+\frac{b\sqrt{x}}{\sqrt{-bx+2}}$	28
risch	$\frac{(bx-2)\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}+\frac{b\sqrt{x}\,\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^(1/2)/(-b*x+2)^(1/2)+b*x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.82

$$\frac{b\sqrt{x}}{2\sqrt{-bx+2}} - \frac{\sqrt{-bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.30, size = 29, normalized size = 0.85

$$-\frac{(bx-1)\sqrt{-bx+2}\sqrt{x}}{bx^2-2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^2 - 2*x)
```

Sympy [A]
time = 0.94, size = 90, normalized size = 2.65

$$\begin{cases} -\frac{b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}}{b^2x-2b} + \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{b^2x-2b} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{i\sqrt{b}}{\sqrt{1-\frac{2}{bx}}} + \frac{i}{\sqrt{b}\,x\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-b**(5/2)*x*sqrt(-1 + 2/(b*x))/(b**2*x - 2*b) + b**(3/2)*sqrt(-1
 + 2/(b*x))/(b**2*x - 2*b), 1/Abs(b*x) > 1/2), (-I*sqrt(b)/sqrt(1 - 2/(b*x)
) + I/(sqrt(b)*x*sqrt(1 - 2/(b*x))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(26) = 52$.
time = 0.00, size = 72, normalized size = 2.12

$$-2\left(-\frac{2b\sqrt{x}\sqrt{-bx+2}}{8(-bx+2)} - \frac{2\sqrt{-b}}{2\left(\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)^2-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/2*sqrt(-b*x + 2)*b*sqrt(x)/(b*x - 2) + 2*sqrt(-b)/((sqrt(-b)*sqrt(x) - s
qrt(-b*x + 2))^2 - 2)
```

Mupad [B]
time = 0.32, size = 27, normalized size = 0.79

$$\frac{b\,\sqrt{x}}{\sqrt{2-b\,x}}-\frac{1}{\sqrt{x}\,\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] (b*x^(1/2))/(2 - b*x)^(1/2) - 1/(x^(1/2)*(2 - b*x)^(1/2))
```

3.640 $\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/x^(3/2)/(-b*x+2)^(1/2)-2/3*(-b*x+2)^(1/2)/x^(3/2)-2/3*b*(-b*x+2)^(1/2)/x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x])/(3*x^(3/2)) - (2*b*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx = \frac{1}{x^{3/2}\sqrt{2-bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx$$
$$= \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$$
$$= \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} - \frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 0.59

$$\frac{-1-2bx+2b^2x^2}{3x^{3/2}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 - 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 - b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.90, size = 245, normalized size = 4.38

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,(-2+3bx(-1+2bx)-2b^3x^3)\sqrt{\frac{2-bx}{bx}}}{3x(4-4bx+b^2x^2)},\frac{1}{\text{Abs}[bx]}>\frac{1}{2}\right\}\right\},\frac{-2Ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{12b^4x-12b^5x^2+3b^6x^3}-\frac{3Ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{12b^4x-12b^5x^2+3b^6x^3}+\frac{I6b^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{12b^4x-12b^5x^2+3b^6x^3}-\frac{2Ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{12b^4x-12b^5x^2+3b^6x^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(2 - b*x)^(3/2)),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-2 + 3 b x (-1 + 2 b x) - 2 b ^ 3 x ^ 3) Sqrt[(2 - b x
) / (b x)] / (3 x (4 - 4 b x + b ^ 2 x ^ 2)), 1 / Abs[b x] > 1 / 2}}, -2 I
b ^ (9 / 2) Sqrt[1 - 2 / (b x)] / (12 b ^ 4 x - 12 b ^ 5 x ^ 2 + 3 b ^ 6 x
^ 3) - 3 I b ^ (11 / 2) x Sqrt[1 - 2 / (b x)] / (12 b ^ 4 x - 12 b ^ 5 x ^
2 + 3 b ^ 6 x ^ 3) + I 6 b ^ (13 / 2) x ^ 2 Sqrt[1 - 2 / (b x)] / (12 b ^ 4
 x - 12 b ^ 5 x ^ 2 + 3 b ^ 6 x ^ 3) - 2 I b ^ (15 / 2) x ^ 3 Sqrt[1 - 2 /
(b x)] / (12 b ^ 4 x - 12 b ^ 5 x ^ 2 + 3 b ^ 6 x ^ 3)]
```

Maple [A]
time = 0.14, size = 45, normalized size = 0.80

method	result	size

gosper	$\frac{2x^2b^2-2bx-1}{3x^{\frac{3}{2}}\sqrt{-bx+2}}$	28
meijerg	$-\frac{\sqrt{2}\,(-2x^2b^2+2bx+1)}{6x^{\frac{3}{2}}\sqrt{-\frac{bx}{2}+1}}$	31
default	$-\frac{1}{3x^{\frac{3}{2}}\sqrt{-bx+2}}+\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{-bx+2}}+\frac{b\sqrt{x}}{\sqrt{-bx+2}}\right)}{3}$	45
risch	$\frac{(5x^2b^2-8bx-4)\sqrt{(-bx+2)\,x}}{12x^{\frac{3}{2}}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{b^2\sqrt{x}\,\sqrt{(-bx+2)\,x}}{4\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)/(-b*x+2)^(1/2)+2/3*b*(-1/x^(1/2)/(-b*x+2)^(1/2)+b*x^(1/2)/(-b*
x+2)^(1/2))
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.79

$$\frac{b^2\sqrt{x}}{4\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}\,b}{2\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/12*(-b*x
+ 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.31, size = 40, normalized size = 0.71

$$-\frac{(2\,b^2x^2-2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}}{3\,(bx^3-2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^3 - 2*x^2)
```

Sympy [C] Result contains complex when optimal does not.

time = 2.63, size = 355, normalized size = 6.34

$$\begin{cases} -\frac{2b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} + \frac{6b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} - \frac{3b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} - \frac{2b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{2ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} + \frac{6ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} - \frac{3ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} - \frac{2ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*x**2
 + 12*b**4*x) + 6*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*
x**2 + 12*b**4*x) - 3*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5
*x**2 + 12*b**4*x) - 2*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*x
**2 + 12*b**4*x), 1/Abs(b*x) > 1/2), (-2*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))
/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) + 6*I*b**(13/2)*x**2*sqrt(1 - 2/(
b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) - 3*I*b**(11/2)*x*sqrt(1 - 2
/(b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) - 2*I*b**(9/2)*sqrt(1 - 2/
(b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(40) = 80.

time = 0.01, size = 151, normalized size = 2.70

$$-2\left(-\frac{2b^2\sqrt{x}\,\sqrt{-bx+2}}{16\,(-bx+2)} + \frac{2\left(-3b\sqrt{-b}\,\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)^4 + 24b\sqrt{-b}\,\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)^2 - 20b\sqrt{-b}\right)}{12\left(\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)^2-2\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/4*sqrt(-b*x + 2)*b^2*sqrt(x)/(b*x - 2) + 1/3*(3*sqrt(-b)*b*(sqrt(-b)*sqr
t(x) - sqrt(-b*x + 2))^4 - 24*sqrt(-b)*b*(sqrt(-b)*sqrt(x) - sqrt(-b*x + 2)
)^2 + 20*sqrt(-b)*b)/((sqrt(-b)*sqrt(x) - sqrt(-b*x + 2))^2 - 2)^3
```

Mupad [B]

time = 0.36, size = 38, normalized size = 0.68

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,x}{3}-\frac{2\,b\,x^2}{3}+\frac{1}{3\,b}\right)}{x^{5/2}-\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] ((2 - b*x)^(1/2)*((2*x)/3 - (2*b*x^2)/3 + 1/(3*b)))/(x^(5/2) - (2*x^(3/2))/
b)
```

3.641 $\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] 2/3*x^(5/2)/b/(-b*x+2)^(3/2)+10*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)
-10/3*x^(3/2)/b^2/(-b*x+2)^(1/2)-5*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 52, 56, 222}

$$\frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(2 - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 - b*x]) - (5
*Sqrt[x]*Sqrt[2 - b*x])/b^3 + (10*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{5\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx}{3b} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b^2} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b^3} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{10\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 72, normalized size = 0.81

$$-\frac{\sqrt{x}\,(60-40bx+3b^2x^2)}{3b^3(2-bx)^{3/2}} + \frac{10\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[x]*(60 - 40*b*x + 3*b^2*x^2))/(b^3*(2 - b*x)^(3/2)) + (10*Log[-(
Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/2)/(2 - b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.12, size = 81, normalized size = 0.91

method	result
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(\frac{21}{4}x^2b^2-70bx+105\right)}{56b^3\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}+\frac{15\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{7}{2}}}\right)}{3(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\left(\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{b^{\frac{7}{2}}}+\frac{28\sqrt{-\left(x-\frac{2}{b}\right)^2b-2x+\frac{4}{b}}}{3b^4\left(x-\frac{2}{b}\right)}+\frac{8\sqrt{-(x-\cdots}}{3b\cdots}\right.}{\sqrt{x}\,\sqrt{-bx+2}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3/(-b)^(5/2)/Pi^(1/2)/b*(-1/56*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(7/2)*(21/4
*x^2*b^2-70*b*x+105)/b^3/(-1/2*b*x+1)^(3/2)+15/4*Pi^(1/2)*(-b)^(7/2)/b^(7/2
)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.36, size = 86, normalized size = 0.97

$$\frac{2\left(2\,b^2+\frac{10\,(bx-2)b}{x}-\frac{15\,(bx-2)^2}{x^2}\right)}{3\left(\frac{(-bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{10\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x - 2)*b/x - 15*(b*x - 2)^2/x^2)/((-b*x + 2)^(3/2)*b^4/x
^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) - 10*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.31, size = 187, normalized size = 2.10

$$\left[-\frac{15\,(b^2x^2-4\,bx+4)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+(3\,b^3x^2-40\,b^2x+60\,b)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^6x^2-4\,b^5x+4\,b^4)},\,-\frac{30\,(b^2x^2-4\,bx+4)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(3\,b^3x^2-40\,b^2x+60\,b)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^6x^2-4\,b^5x+4\,b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(15*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)
*sqrt(x) + 1) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)*sqrt(x))/(b^6*
x^2 - 4*b^5*x + 4*b^4), -1/3*(30*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)
*sqrt(x))/(b^6*x^2 - 4*b^5*x + 4*b^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 4.25, size = 751, normalized size = 8.44

$$\begin{cases} -\frac{3ib^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} + \frac{40ib^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} - \frac{60ib^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} - \frac{30ib^{10}x^{\frac{27}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} + \frac{15\pi b^{10}x^{\frac{27}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} + \frac{60ib^{9}x^{\frac{25}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} - \frac{30\pi b^{9}x^{\frac{25}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} & \text{for}\: |bx| > 2 \\ \frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} - \frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} + \frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} + \frac{30b^{10}x^{\frac{27}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} - \frac{60b^{9}x^{\frac{25}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-3*I*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b*
*(25/2)*x**(25/2)*sqrt(b*x - 2)) + 40*I*b**(21/2)*x**14/(3*b**(27/2)*x**(27
/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 60*I*b**(19/2)*x
**13/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x
- 2)) - 30*I*b**10*x**(27/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)
/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)
) + 15*pi*b**10*x**(27/2)*sqrt(b*x - 2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2
) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) + 60*I*b**9*x**(25/2)*sqrt(b*x - 2
)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6
*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 30*pi*b**9*x**(25/2)*sqrt(b*x - 2)/(3
*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)),
Abs(b*x) > 2), (3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6
*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) - 40*b**(21/2)*x**14/(3*b**(27/2)*x**(
27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) + 60*b**(19/2)
*x**13/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-
b*x + 2)) + 30*b**10*x**(27/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x
+ 2)) - 60*b**9*x**(25/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3
*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2))
, True))
```

Giac [A]
time = 0.01, size = 129, normalized size = 1.45

$$2\left(\frac{2\left(\left(\left(-\frac{\frac{1}{36}\cdot 9b^{4}\sqrt{x}\,\sqrt{x}}{b^{5}}+\frac{\frac{1}{36}\cdot 120b^{3}}{b^{5}}\right)\sqrt{x}\,\sqrt{x}-\frac{\frac{1}{36}\cdot 180b^{2}}{b^{5}}\right)\sqrt{x}\,\sqrt{-bx+2}}{(-bx+2)^{2}}-\frac{5\ln\left(\sqrt{-bx+2}-\sqrt{-b}\,\sqrt{x}\right)}{b^{3}\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*sqrt(-b*x + 2)*(x*(3*x/b - 40/b^2) + 60/b^3)*sqrt(x)/(b*x - 2)^2 - 10*
log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(5/2), x)
```

3.642 $\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b/(-b*x+2)^(3/2)+2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-
2*x^(1/2)/b^2/(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(2 - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 - b*x]) + (2*Ar
cSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx}{b} \\
&= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2} \\
&= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 0.96

$$\frac{4\sqrt{x}\,(-3+2bx)}{3b^2(2-bx)^{3/2}} - \frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[x]*(-3 + 2*b*x))/(3*b^2*(2 - b*x)^(3/2)) - (2*Log[-(Sqrt[-b]*Sqrt[x
]) + Sqrt[2 - b*x]])/(-b)^(5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)/(2 - b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.13, size = 73, normalized size = 1.09

method	result	size

meijerg	$-\frac{4\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}(-10bx+15)}{20b^2\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}+\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2b^{\frac{5}{2}}}\right)}{3(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/(-b)^(3/2)/Pi^(1/2)/b*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-10*
b*x+15)/b^2/(-1/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*b
^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.34, size = 50, normalized size = 0.75

$$\frac{2\left(b+\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+2)^{\frac{3}{2}}b^2}-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - 2)/x)*x^(3/2)/((-b*x + 2)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.32, size = 173, normalized size = 2.58

$$\left[-\frac{3\,(b^2x^2-4\,bx+4)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-4\,(2\,b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^5x^2-4\,b^4x+4\,b^3)},-\frac{2\left(3\,(b^2x^2-4\,bx+4)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\,(2\,b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\,(b^5x^2-4\,b^4x+4\,b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*
sqrt(x) + 1) - 4*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4*x
 + 4*b^3), -2/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))) - 2*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4
*x + 4*b^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.07, size = 648, normalized size = 9.67

$$\begin{cases} \frac{8ib^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{12ib^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6ib^5x^{\frac{15}{2}}\sqrt{bx-2}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{3\pi b^5x^{\frac{15}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{12ib^4x^{\frac{13}{2}}\sqrt{bx-2}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6\pi b^4x^{\frac{13}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} & \text{for}\ |bx|>2 \\ -\frac{8b^{\frac{11}{2}}x^8}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{12b^{\frac{9}{2}}x^7}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{6b^5x^{\frac{15}{2}}\sqrt{-bx+2}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}-\frac{12b^4x^{\frac{13}{2}}\sqrt{-bx+2}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((8*I*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(
13/2)*x**(13/2)*sqrt(b*x - 2)) - 12*I*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*
sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) - 6*I*b**5*x**(15/2)*s
qrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b
*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) + 3*pi*b**5*x**(15/2)*sqrt(b
*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b
*x - 2)) + 12*I*b**4*x**(13/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x -
2)) - 6*pi*b**4*x**(13/2)*sqrt(b*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2
) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)), Abs(b*x) > 2), (-8*b**(11/2)*x**8
/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*sqrt(-b*x +
2)) + 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*
x**(13/2)*sqrt(-b*x + 2)) + 6*b**5*x**(15/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sq
rt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13
/2)*sqrt(-b*x + 2)) - 12*b**4*x**(13/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)
*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*s
qrt(-b*x + 2)), True))
```

Giac [A]
time = 0.01, size = 101, normalized size = 1.51

$$2\left(\frac{2\left(\frac{\frac{1}{18}\cdot 12b^2\sqrt{x}\sqrt{x}}{b^3}-\frac{\frac{1}{18}\cdot 18b}{b^3}\right)\sqrt{x}\sqrt{-bx+2}}{(-bx+2)^2}-\frac{\ln\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)}{b^2\sqrt{-b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 4/3*sqrt(-b*x + 2)*sqrt(x)*(2*x/b - 3/b^2)/(b*x - 2)^2 - 2*log(-sqrt(-b)*sq
rt(x) + sqrt(-b*x + 2))/(sqrt(-b)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(5/2), x)
```

3.643 $\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

```
[Out] 1/3*x^(3/2)/(-b*x+2)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx = \frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Mathematica [A]
time = 0.07, size = 19, normalized size = 1.00

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.50, size = 51, normalized size = 2.68

$$\text{Piecewise}\left[\left\{\left\{\frac{Ix^{\frac{3}{2}}}{3\left(-2+bx\right)^{\frac{3}{2}}},\text{Abs}\left[bx\right]>2\right\}\right\},-\frac{x^{\frac{3}{2}}}{3bx\sqrt{2-bx}-6\sqrt{2-bx}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/(2 - b*x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 3 x ^ (3 / 2) / (-2 + b x) ^ (3 / 2), Abs[b x] > 2}}, -x ^
(3 / 2) / (3 b x Sqrt[2 - b x] - 6 Sqrt[2 - b x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(13) = 26.
time = 0.13, size = 49, normalized size = 2.58

method	result	size
gosper	$\frac{x^{\frac{3}{2}}}{3(-bx+2)^{\frac{3}{2}}}$	14
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}}{12\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	17
default	$\frac{\sqrt{x}}{b(-bx+2)^{\frac{3}{2}}}-\frac{\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}}{b}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*x^(1/2)/(-b*x+2)^(3/2)-1/b*(1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*
x+2)^(1/2))
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.68

$$\frac{x^{\frac{3}{2}}}{3\left(-bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)/(-b*x + 2)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.31, size = 28, normalized size = 1.47

$$\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{3\,(b^2x^2-4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-b*x + 2)*x^(3/2)/(b^2*x^2 - 4*b*x + 4)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 63, normalized size = 3.32

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{3bx\sqrt{bx-2}-6\sqrt{bx-2}} & \text{for } |bx|>2 \\ -\frac{x^{\frac{3}{2}}}{3bx\sqrt{-bx+2}-6\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(3*b*x*sqrt(b*x - 2) - 6*sqrt(b*x - 2)), Abs(b*x) > 2
), (-x**(3/2)/(3*b*x*sqrt(-b*x + 2) - 6*sqrt(-b*x + 2)), True))
```

Giac [A]
time = 0.01, size = 42, normalized size = 2.21

$$\frac{\frac{1}{36}\cdot 12b\sqrt{x}\,\sqrt{x}\,\sqrt{x}\,\sqrt{-bx+2}}{b\,(-bx+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/3*sqrt(-b*x + 2)*x^(3/2)/(b*x - 2)^2
```

Mupad [B]
time = 0.23, size = 13, normalized size = 0.68

$$\frac{x^{3/2}}{3\,(2-b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(5/2),x)
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

3.644 $\int \frac{1}{\sqrt{x}\,(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}$$

```
[Out] 1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x+2)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
[Out] Sqrt[x]/(3*(2 - b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(2-bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 24, normalized size = 0.62

$$-\frac{\sqrt{x}\,(-3+bx)}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(Sqrt[x]*(-3 + b*x))/(2 - b*x)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.48, size = 136, normalized size = 3.49

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,x\,(3-bx)\sqrt{\frac{2-bx}{bx}}}{3\,(4-4bx+b^2x^2)},\frac{1}{\text{Abs}\,[bx]}>\frac{1}{2}\right\}\right\},-\frac{Ibx}{-6\sqrt{b}\sqrt{1-\frac{2}{bx}}+3b^{\frac{3}{2}}x\sqrt{1-\frac{2}{bx}}}+\frac{I3}{-6\sqrt{b}\sqrt{1-\frac{2}{bx}}+3b^{\frac{3}{2}}x\sqrt{1-\frac{2}{bx}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{Sqrt[b] x (3 - b x) Sqrt[(2 - b x) / (b x)] / (3 (4 - 4 b x + b
 ^ 2 x ^ 2)), 1 / Abs[b x] > 1 / 2}}, -I b x / (-6 Sqrt[b] Sqrt[1 - 2 / (b
x)] + 3 b ^ (3 / 2) x Sqrt[1 - 2 / (b x)]) + I 3 / (-6 Sqrt[b] Sqrt[1 - 2 /
 (b x)] + 3 b ^ (3 / 2) x Sqrt[1 - 2 / (b x)])]
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.72

method	result	size
gosper	$-\frac{\sqrt{x}\,(bx-3)}{3(-bx+2)^{\frac{3}{2}}}$	19
meijerg	$\frac{\sqrt{x}\,\sqrt{2}\,(-bx+3)}{12\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	23
default	$\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.64

$$\frac{\left(b-\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{6\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
[Out] 1/6*(b - 3*(b*x - 2)/x)*x^(3/2)/(-b*x + 2)^(3/2)
```

Fricas [A]
time = 0.32, size = 33, normalized size = 0.85

$$-\frac{(bx-3)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^2x^2-4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
[Out] -1/3*(b*x - 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^2 - 4*b*x + 4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.11, size = 165, normalized size = 4.23

$$\begin{cases} \frac{b^2x}{3b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}-6b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}-\frac{3b}{3b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}-6b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{ibx}{3b^{\frac{3}{2}}x\sqrt{1-\frac{2}{bx}}-6\sqrt{b}\sqrt{1-\frac{2}{bx}}}+\frac{3i}{3b^{\frac{3}{2}}x\sqrt{1-\frac{2}{bx}}-6\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(5/2)/x**(1/2),x)
[Out] Piecewise((b**2*x/(3*b**(5/2)*x*sqrt(-1 + 2/(b*x)) - 6*b**(3/2)*sqrt(-1 + 2
/(b*x))) - 3*b/(3*b**(5/2)*x*sqrt(-1 + 2/(b*x)) - 6*b**(3/2)*sqrt(-1 + 2/(b
*x))), 1/Abs(b*x) > 1/2), (-I*b*x/(3*b**(3/2)*x*sqrt(1 - 2/(b*x)) - 6*sqrt(
b)*sqrt(1 - 2/(b*x))) + 3*I/(3*b**(3/2)*x*sqrt(1 - 2/(b*x)) - 6*sqrt(b)*sqr
t(1 - 2/(b*x))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(27) = 54.
time = 0.01, size = 109, normalized size = 2.79

$$-\frac{32b\sqrt{-b}\,b\left(-3\left(\sqrt{-b\,(-bx+2)+2b}-\sqrt{-b}\,\sqrt{-bx+2}\right)^2+2b\right)}{2\cdot 6\,|b|\left(\left(\sqrt{-b\,(-bx+2)+2b}-\sqrt{-b}\,\sqrt{-bx+2}\right)^2-2b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 8/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(
b))
```

Mupad [B]
time = 0.36, size = 45, normalized size = 1.15

$$\frac{3\,\sqrt{x}\,\sqrt{2-b\,x}-b\,x^{3/2}\,\sqrt{2-b\,x}}{3\,b^2\,x^2-12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(2 - b*x)^(1/2) - b*x^(3/2)*(2 - b*x)^(1/2))/(3*b^2*x^2 - 12*b*x
 + 12)
```

3.645 $\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/3/(-b*x+2)^(3/2)/x^(1/2)+2/3/x^(1/2)/(-b*x+2)^(1/2)-2/3*(-b*x+2)^(1/2)/x^
(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 - b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 - b*x]) - (2*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx = \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}} + \frac{2}{3}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx$$
$$= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2-bx}} + \frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$$
$$= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3\sqrt{x}}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 0.57

$$-\frac{3-6bx+2b^2x^2}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(3 - 6*b*x + 2*b^2*x^2)/(Sqrt[x]*(2 - b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.49, size = 182, normalized size = 3.14

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,(-3+6bx-2b^2x^2)\sqrt{\frac{2-bx}{bx}}}{3\,(4-4bx+b^2x^2)},\frac{1}{\text{Abs}\,[bx]}>\frac{1}{2}\right\}\right\},\frac{-3Ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{12b^4-12b^5x+3b^6x^2}+\frac{I6b^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{12b^4-12b^5x+3b^6x^2}-\frac{2Ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{12b^4-12b^5x+3b^6x^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(3/2)*(2 - b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (-3 + 6 b x - 2 b ^ 2 x ^ 2) Sqrt[(2 - b x) / (b x)] /
(3 (4 - 4 b x + b ^ 2 x ^ 2)), 1 / Abs[b x] > 1 / 2}}, -3 I b ^ (9 / 2) Sqr
t[1 - 2 / (b x)] / (12 b ^ 4 - 12 b ^ 5 x + 3 b ^ 6 x ^ 2) + I 6 b ^ (11 /
2) x Sqrt[1 - 2 / (b x)] / (12 b ^ 4 - 12 b ^ 5 x + 3 b ^ 6 x ^ 2) - 2 I b
^ (13 / 2) x ^ 2 Sqrt[1 - 2 / (b x)] / (12 b ^ 4 - 12 b ^ 5 x + 3 b ^ 6 x ^
 2)]
```

Maple [A]
time = 0.13, size = 45, normalized size = 0.78

method	result	size

gosper	$-\frac{2x^2b^2-6bx+3}{3\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}$	28
meijerg	$-\frac{\sqrt{2}\,(2x^2b^2-6bx+3)}{12\sqrt{x}\,\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	31
default	$-\frac{1}{(-bx+2)^{\frac{3}{2}}\sqrt{x}}+2b\left(\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}\right)$	45
risch	$\frac{(bx-2)\sqrt{(-bx+2)\,x}}{4\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}+\frac{b(5bx-12)\sqrt{x}\,\sqrt{(-bx+2)\,x}}{12\sqrt{-x\,(bx-2)}\,(bx-2)\sqrt{-bx+2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-b*x+2)^(3/2)/x^(1/2)+2*b*(1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x
+2)^(1/2))
```

Maxima [A]
time = 0.27, size = 42, normalized size = 0.72

$$\frac{\left(b^2-\frac{6\,(bx-2)b}{x}\right)x^{\frac{3}{2}}}{12\,(-bx+2)^{\frac{3}{2}}}-\frac{\sqrt{-bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x - 2)*b/x)*x^(3/2)/(-b*x + 2)^(3/2) - 1/4*sqrt(-b*x + 2)/
sqrt(x)
```

Fricas [A]
time = 0.32, size = 46, normalized size = 0.79

$$-\frac{(2\,b^2x^2-6\,bx+3)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^2x^3-4\,bx^2+4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 6*b*x + 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^3 - 4*b*x^2 + 4*
x)
```

Sympy [C] Result contains complex when optimal does not.

time = 2.55, size = 245, normalized size = 4.22

$$\begin{cases} -\frac{2b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{2ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4) + 6*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b
**4) - 3*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b**4), 1
/Abs(b*x) > 1/2), (-2*I*b**(13/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*
b**5*x + 12*b**4) + 6*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**
5*x + 12*b**4) - 3*I*b**(9/2)*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4), True))
```

Giac [A]
time = 0.01, size = 100, normalized size = 1.72

$$2\left(\frac{2\left(-\frac{\frac{1}{576}\cdot 60b^3\sqrt{x}\sqrt{x}}{b} + \frac{\frac{1}{576}\cdot 144b^2}{b}\right)\sqrt{x}\sqrt{-bx+2}}{(-bx+2)^2} + \frac{2\sqrt{-b}}{4\left(\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)^2-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/12*(5*b^2*x - 12*b)*sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)^2 + sqrt(-b)/((sqrt
(-b)*sqrt(x) - sqrt(-b*x + 2))^2 - 2)
```

Mupad [B]
time = 0.37, size = 59, normalized size = 1.02

$$\frac{3\sqrt{2-b\,x}-6\,b\,x\,\sqrt{2-b\,x}+2\,b^2\,x^2\,\sqrt{2-b\,x}}{\sqrt{x}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*(2 - b*x)^(1/2) - 6*b*x*(2 - b*x)^(1/2) + 2*b^2*x^2*(2 - b*x)^(1/2))/(x^
(1/2)*(x*(12*b - 3*b^2*x) - 12))
```

3.646 $\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/3/x^(3/2)/(-b*x+2)^(3/2)+1/x^(3/2)/(-b*x+2)^(1/2)-2/3*(-b*x+2)^(1/2)/x^(3
/2)-2/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {47, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 - b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x
])/(3*x^(3/2)) - (2*b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx \\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx \\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx \\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} - \frac{2b\sqrt{2-bx}}{3\sqrt{x}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 41, normalized size = 0.55

$$
-\frac{1+3bx-6b^2x^2+2b^3x^3}{3x^{3/2}(2-bx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(1 + 3*b*x - 6*b^2*x^2 + 2*b^3*x^3)/(x^(3/2)*(2 - b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 7.28, size = 349, normalized size = 4.65

$$
\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{b}\,(2+bx(5+10b^2x^2-2b^3x^3)-15b^2x^2)\sqrt{\frac{2-bx}{bx}}}{3x(-8+12bx-6b^2x^2+b^3x^3)}, \frac{1}{\text{Abs}[bx]}>\frac{1}{2}\right\}\right\}, \frac{I2b^{\frac{19}{2}}\sqrt{1-\frac{2}{bx}}}{-24b^9x+36b^{10}x^2-18b^{11}x^3+3b^{12}x^4} + \frac{I5b^{\frac{21}{2}}x\sqrt{1-\frac{2}{bx}}}{-24b^9x+36b^{10}x^2-18b^{11}x^3+3b^{12}x^4} - \frac{15Ib^{\frac{23}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-24b^9x+36b^{10}x^2-18b^{11}x^3+3b^{12}x^4} + \frac{I10b^{\frac{25}{2}}x^3\sqrt{1-\frac{2}{bx}}}{-24b^9x+36b^{10}x^2-18b^{11}x^3+3b^{12}x^4} - \frac{2Ib^{\frac{27}{2}}x^4\sqrt{1-\frac{2}{bx}}}{-24b^9x+36b^{10}x^2-18b^{11}x^3+3b^{12}x^4}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/2)*(2 - b*x)^(5/2)),x]')
```

```
[Out] Piecewise[{{Sqrt[b] (2 + b x (5 + 10 b ^ 2 x ^ 2 - 2 b ^ 3 x ^ 3) - 15 b ^
2 x ^ 2) Sqrt[(2 - b x) / (b x)] / (3 x (-8 + 12 b x - 6 b ^ 2 x ^ 2 + b ^
3 x ^ 3)), 1 / Abs[b x] > 1 / 2}}, I 2 b ^ (19 / 2) Sqrt[1 - 2 / (b x)] / (
-24 b ^ 9 x + 36 b ^ 10 x ^ 2 - 18 b ^ 11 x ^ 3 + 3 b ^ 12 x ^ 4) + I 5 b ^
 (21 / 2) x Sqrt[1 - 2 / (b x)] / (-24 b ^ 9 x + 36 b ^ 10 x ^ 2 - 18 b ^ 1
1 x ^ 3 + 3 b ^ 12 x ^ 4) - 15 I b ^ (23 / 2) x ^ 2 Sqrt[1 - 2 / (b x)] / (
-24 b ^ 9 x + 36 b ^ 10 x ^ 2 - 18 b ^ 11 x ^ 3 + 3 b ^ 12 x ^ 4) + I 10 b
^ (25 / 2) x ^ 3 Sqrt[1 - 2 / (b x)] / (-24 b ^ 9 x + 36 b ^ 10 x ^ 2 - 18
b ^ 11 x ^ 3 + 3 b ^ 12 x ^ 4) - 2 I b ^ (27 / 2) x ^ 4 Sqrt[1 - 2 / (b x)]
 / (-24 b ^ 9 x + 36 b ^ 10 x ^ 2 - 18 b ^ 11 x ^ 3 + 3 b ^ 12 x ^ 4)]
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.81

method	result	size
gosper	$-\frac{2b^{3}x^{3}-6x^{2}b^{2}+3bx+1}{3x^{\frac{3}{2}}\left(-bx+2\right)^{\frac{3}{2}}}$	36
meijerg	$-\frac{\sqrt{2}\,\left(2b^{3}x^{3}-6x^{2}b^{2}+3bx+1\right)}{12x^{\frac{3}{2}}\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	39
default	$-\frac{1}{3x^{\frac{3}{2}}\left(-bx+2\right)^{\frac{3}{2}}}+b\left(-\frac{1}{\left(-bx+2\right)^{\frac{3}{2}}\sqrt{x}}+2b\left(\frac{\sqrt{x}}{3\left(-bx+2\right)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}\right)\right)$	61
risch	$\frac{\left(4x^{2}b^{2}-7bx-2\right)\sqrt{\left(-bx+2\right)x}}{12x^{\frac{3}{2}}\sqrt{-x\left(bx-2\right)}\,\sqrt{-bx+2}}+\frac{b^{2}\left(4bx-9\right)\sqrt{x}\,\sqrt{\left(-bx+2\right)x}}{12\sqrt{-x\left(bx-2\right)}\,\left(bx-2\right)\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)/(-b*x+2)^(3/2)+b*(-1/(-b*x+2)^(3/2)/x^(1/2)+2*b*(1/3*x^(1/2)/(
-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x+2)^(1/2)))
```

Maxima [A]
time = 0.26, size = 58, normalized size = 0.77

$$-\frac{3\sqrt{-bx+2}\,b}{8\sqrt{x}}+\frac{\left(b^{3}-\frac{9\,(bx-2)b^{2}}{x}\right)x^{\frac{3}{2}}}{24\left(-bx+2\right)^{\frac{3}{2}}}-\frac{\left(-bx+2\right)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*sqrt(-b*x + 2)*b/sqrt(x) + 1/24*(b^3 - 9*(b*x - 2)*b^2/x)*x^(3/2)/(-b*
x + 2)^(3/2) - 1/24*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.32, size = 56, normalized size = 0.75

$$-\frac{\left(2\,b^{3}x^{3}-6\,b^{2}x^{2}+3\,bx+1\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^{2}x^{4}-4\,bx^{3}+4\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^3*x^3 - 6*b^2*x^2 + 3*b*x + 1)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^4 -
4*b*x^3 + 4*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.78, size = 530, normalized size = 7.07

$$\begin{cases} -\frac{2b^{\frac{27}{2}}x^{4}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{10b^{\frac{25}{2}}x^{3}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}-\frac{15b^{\frac{23}{2}}x^{2}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{5b^{\frac{21}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{2b^{\frac{19}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{2ib^{\frac{27}{2}}x^{4}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{10ib^{\frac{25}{2}}x^{3}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}-\frac{15ib^{\frac{23}{2}}x^{2}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{5ib^{\frac{21}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{2ib^{\frac{19}{2}}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(27/2)*x**4*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x*
*3 + 36*b**10*x**2 - 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(-1 + 2/(b*x))/(3*b
**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) - 15*b**(23/2)*x**2*
sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*
x) + 5*b**(21/2)*x*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b*
*10*x**2 - 24*b**9*x) + 2*b**(19/2)*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b
**11*x**3 + 36*b**10*x**2 - 24*b**9*x), 1/Abs(b*x) > 1/2), (-2*I*b**(27/2)*
x**4*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b
**9*x) + 10*I*b**(25/2)*x**3*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**
3 + 36*b**10*x**2 - 24*b**9*x) - 15*I*b**(23/2)*x**2*sqrt(1 - 2/(b*x))/(3*b
**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) + 5*I*b**(21/2)*x*sq
rt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x)
+ 2*I*b**(19/2)*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*
x**2 - 24*b**9*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(53) = 106.
time = 0.02, size = 177, normalized size = 2.36

$$2\left(\frac{2\left(-\frac{\frac{1}{2304}\cdot 192b^4\sqrt{x}\sqrt{x}}{b}+\frac{\frac{1}{2304}\cdot 432b^3}{b}\right)\sqrt{x}\sqrt{-bx+2}}{(-bx+2)^2}+\frac{2\left(3b\sqrt{-b}\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)^4-18b\sqrt{-b}\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)^2+16b\sqrt{-b}\right)}{12\left(\left(\sqrt{-bx+2}-\sqrt{-b}\sqrt{x}\right)^2-2\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/12*(4*b^3*x - 9*b^2)*sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)^2 + 1/3*(3*sqrt(-b
)*b*(sqrt(-b)*sqrt(x) - sqrt(-b*x + 2))^4 - 18*sqrt(-b)*b*(sqrt(-b)*sqrt(x)
 - sqrt(-b*x + 2))^2 + 16*sqrt(-b)*b)/((sqrt(-b)*sqrt(x) - sqrt(-b*x + 2))^
2 - 2)^3
```

Mupad [B]
time = 0.44, size = 73, normalized size = 0.97

$$\frac{\sqrt{2-b\,x}+3\,b\,x\,\sqrt{2-b\,x}-6\,b^2\,x^2\,\sqrt{2-b\,x}+2\,b^3\,x^3\,\sqrt{2-b\,x}}{x^{3/2}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] ((2 - b*x)^(1/2) + 3*b*x*(2 - b*x)^(1/2) - 6*b^2*x^2*(2 - b*x)^(1/2) + 2*b^
3*x^3*(2 - b*x)^(1/2))/(x^(3/2)*(x*(12*b - 3*b^2*x) - 12))
```

3.647 $\int \frac{\sqrt{x}}{\sqrt{1-x}} \, dx$

Optimal. Leaf size=27

$$-\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

[Out] 1/2*arcsin(-1+2*x)-(1-x)^(1/2)*x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 55, 633, 222}

$$-\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[1 - x],x]

[Out] -(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx &= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\ &= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 1.30

$$-\sqrt{-((-1+x)x)}+2\tan^{-1}\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 - x],x]
[Out] -Sqrt[-((-1 + x)*x)] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.46, size = 54, normalized size = 2.00

$$\text{Piecewise}\left[\left\{\left\{I\left(-\sqrt{x}\,\sqrt{-1+x}-\text{ArcCosh}\left[\sqrt{x}\right]\right),\text{Abs}\left[x\right]>1\right\}\right\},-\frac{\sqrt{x}}{\sqrt{1-x}}+\frac{x^{\frac{3}{2}}}{\sqrt{1-x}}+\text{ArcSin}\left[\sqrt{x}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[x]/Sqrt[1 - x],x]')
[Out] Piecewise[{{I (-Sqrt[x] Sqrt[-1 + x] - ArcCosh[Sqrt[x]]), Abs[x] > 1}}, -Sq
rt[x] / Sqrt[1 - x] + x ^ (3 / 2) / Sqrt[1 - x] + ArcSin[Sqrt[x]]]
```

Maple [A]
time = 0.13, size = 41, normalized size = 1.52

method	result	size
meijerg	$\frac{i\left(i\sqrt{\pi}\,\sqrt{x}\,\sqrt{1-x}-i\sqrt{\pi}\,\arcsin\left(\sqrt{x}\right)\right)}{\sqrt{\pi}}$	34

default	$-\sqrt{1-x}\,\sqrt{x}+\frac{\sqrt{x(1-x)}\,\arcsin(2x-1)}{2\sqrt{x}\,\sqrt{1-x}}$	41
risch	$\frac{\sqrt{x}\,(-1+x)\sqrt{x(1-x)}}{\sqrt{-x(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{x(1-x)}\,\arcsin(2x-1)}{2\sqrt{x}\,\sqrt{1-x}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(1-x)^(1/2)*x^(1/2)+1/2*(x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(2*x-1)
```

Maxima [A]
time = 0.34, size = 37, normalized size = 1.37

$$\frac{\sqrt{-x+1}}{\sqrt{x}\left(\frac{x-1}{x}-1\right)}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] sqrt(-x + 1)/(sqrt(x)*((x - 1)/x - 1)) - arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [A]
time = 0.31, size = 27, normalized size = 1.00

$$-\sqrt{x}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A]
time = 0.91, size = 54, normalized size = 2.00

$$\begin{cases}-i\sqrt{x}\,\sqrt{x-1}-i\operatorname{acosh}\left(\sqrt{x}\right) & \text{for } |x|>1\\ \frac{x^{\frac{3}{2}}}{\sqrt{1-x}}-\frac{\sqrt{x}}{\sqrt{1-x}}+\operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1-x)**(1/2),x)
[Out] Piecewise((-I*sqrt(x)*sqrt(x - 1) - I*acosh(sqrt(x)), Abs(x) > 1), (x**(3/2
)/sqrt(1 - x) - sqrt(x)/sqrt(1 - x) + asin(sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 29, normalized size = 1.07

$$2\left(-\frac{1}{2}\sqrt{x}\,\sqrt{-x+1}+\frac{\arcsin\left(\sqrt{x}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -sqrt(x)*sqrt(-x + 1) + arcsin(sqrt(x))
```

Mupad [B]
time = 0.57, size = 31, normalized size = 1.15

$$2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-\sqrt{x}\,\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1 - x)^(1/2),x)
```

```
[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - x^(1/2)*(1 - x)^(1/2)
```

3.648 $\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] arcsin(-1+2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 633, 222}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-x}\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\
&= -\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(8) = 16$.
time = 0.03, size = 38, normalized size = 4.75

$$\frac{2\sqrt{-1+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-1+x}}\right)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[-1 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-1 + x]])/Sqrt[-((-1 + x)*x)]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.94, size = 19, normalized size = 2.38

$$\text{Piecewise}\left[\left\{\left\{-2I\text{ArcCosh}\left[\sqrt{x}\right],\text{Abs}\left[x\right]>1\right\}\right\},2\text{ArcSin}\left[\sqrt{x}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[1 - x]),x]')
```

```
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[x]], Abs[x] > 1}}, 2 ArcSin[Sqrt[x]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.
time = 0.12, size = 27, normalized size = 3.38

method	result	size
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{x\left(1-x\right)}\;\arcsin(2x-1)}{\sqrt{x}\;\sqrt{1-x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(2*x-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.35, size = 14, normalized size = 1.75

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.31, size = 14, normalized size = 1.75

$$-2 \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A]
time = 0.46, size = 20, normalized size = 2.50

$$\begin{cases} -2i \operatorname{acosh}\left(\sqrt{x}\right) & \text{for } |x| > 1 \\ 2 \operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x)), Abs(x) > 1), (2*asin(sqrt(x)), True))
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.50

$$-2 \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x)
```

```
[Out] -2*arcsin(sqrt(-x + 1))
```

Mupad [B]
time = 0.05, size = 16, normalized size = 2.00

$$-4 \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] -4*atan(((1 - x)^(1/2) - 1)/x^(1/2))
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sin^{-1}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$$

```
[Out] 2*arcsin(b^(1/2)*x^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {56, 222}

$$\frac{2\sin^{-1}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,\sqrt{1-bx}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.84

$$-\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{1-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[1 - b*x]])/Sqrt[-b]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.10, size = 35, normalized size = 1.84

$$\text{Piecewise}\left[\left\{\left\{\frac{-2I\text{ArcCosh}\left[\sqrt{b}\sqrt{x}\right]}{\sqrt{b}}, \text{Abs}\left[bx\right]>1\right\}\right\}, \frac{2\text{ArcSin}\left[\sqrt{b}\sqrt{x}\right]}{\sqrt{b}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[1 - b*x]),x]')
```

```
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[b] Sqrt[x]] / Sqrt[b], Abs[b x] > 1}}, 2 ArcS
in[Sqrt[b] Sqrt[x]] / Sqrt[b]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.
time = 0.13, size = 48, normalized size = 2.53

method	result	size
meijerg	$\frac{2\arcsin\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$	14
default	$\frac{\sqrt{x\left(-bx+1\right)}\arctan\left(\frac{\sqrt{b}\left(x-\frac{1}{2b}\right)}{\sqrt{-x^{2}b+x}}\right)}{\sqrt{x}\sqrt{-bx+1}\sqrt{b}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(-b*x+1))^(1/2)/x^(1/2)/(-b*x+1)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b)/
(-b*x^2+x)^(1/2))
```

Maxima [A]
time = 0.35, size = 21, normalized size = 1.11

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Fricas [A]
time = 0.32, size = 57, normalized size = 3.00

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+1}\,\sqrt{-b}\,\sqrt{x}\,+1\right)}{b},-\frac{2\,\arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + 1)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan
(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [A]
time = 0.51, size = 42, normalized size = 2.21

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ |bx|>1 \\ \frac{2\operatorname{asin}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+1)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x))/sqrt(b), Abs(b*x) > 1), (2*asin(sqrt
(b)*sqrt(x))/sqrt(b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.
time = 0.00, size = 35, normalized size = 1.84

$$-\frac{2\ln\left(\sqrt{-bx+1}\,-\sqrt{-b}\,\sqrt{x}\,\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x)
[Out] -2*log(-sqrt(-b)*sqrt(x) + sqrt(-b*x + 1))/sqrt(-b)
```

Mupad [B]
time = 0.13, size = 23, normalized size = 1.21

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{1-b\,x}-1}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - b*x)^(1/2)),x)
[Out] -(4*atan(((1 - b*x)^(1/2) - 1)/(b^(1/2)*x^(1/2))))/b^(1/2)
```

3.650 $\int x^{5/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

[Out] 3/8*a*x^(8/3)+3/11*b*x^(11/3)

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

Antiderivative was successfully verified.

[In] Int[x^(5/3)*(a + b*x),x]

[Out] (3*a*x^(8/3))/8 + (3*b*x^(11/3))/11

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)\,dx &= \int \left(ax^{5/3}+bx^{8/3}\right)\,dx\\ &= \frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{88}x^{8/3}(11a+8bx)$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/3)*(a + b*x),x]

[Out] (3*x^(8/3)*(11*a + 8*b*x))/88

Mathics [A]
time = 1.78, size = 13, normalized size = 0.62

$$\frac{3x^{\frac{8}{3}}\left(11a+8bx\right)}{88}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/3)*(a + b*x),x]')
```

```
[Out] 3 x ^ (8 / 3) (11 a + 8 b x) / 88
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}\left(8bx+11a\right)}{88}$	14
derivativedivides	$\frac{3a\,x^{\frac{8}{3}}}{8}+\frac{3b\,x^{\frac{11}{3}}}{11}$	14
default	$\frac{3a\,x^{\frac{8}{3}}}{8}+\frac{3b\,x^{\frac{11}{3}}}{11}$	14
trager	$\frac{3x^{\frac{8}{3}}\left(8bx+11a\right)}{88}$	14
risch	$\frac{3x^{\frac{8}{3}}\left(8bx+11a\right)}{88}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a*x^(8/3)+3/11*b*x^(11/3)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Fricas [A]
time = 0.31, size = 18, normalized size = 0.86

$$\frac{3}{88}\left(8\,bx^{3}+11\,ax^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/88*(8*b*x^3 + 11*a*x^2)*x^(2/3)
```

Sympy [A]
time = 0.32, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{8}{3}}}{8} + \frac{3bx^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(8/3)/8 + 3*b*x**(11/3)/11
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.57

$$\frac{3}{11}b\left(x^{\frac{1}{3}}\right)^2 x^3 + \frac{3a\left(x^{\frac{1}{3}}\right)^2 x^2}{2\cdot 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x)
```

```
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.62

$$\frac{3\,x^{8/3}\,\left(11\,a+8\,b\,x\right)}{88}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x),x)
```

```
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

3.651 $\int x^{4/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

```
[Out] 3/7*a*x^(7/3)+3/10*b*x^(10/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)\,dx&=\int\left(ax^{4/3}+bx^{7/3}\right)\,dx\\&=\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{70}x^{7/3}(10a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

Mathics [A]
time = 1.74, size = 13, normalized size = 0.62

$$\frac{3x^{\frac{7}{3}}\,(10a+7bx)}{70}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(4/3)*(a + b*x),x]')
```

```
[Out] 3 x ^ (7 / 3) (10 a + 7 b x) / 70
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
derivativedivides	$\frac{3a\,x^{\frac{7}{3}}}{7}+\frac{3b\,x^{\frac{10}{3}}}{10}$	14
default	$\frac{3a\,x^{\frac{7}{3}}}{7}+\frac{3b\,x^{\frac{10}{3}}}{10}$	14
trager	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
risch	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*a*x^(7/3)+3/10*b*x^(10/3)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.62

$$\frac{3}{10}\,bx^{\frac{10}{3}}+\frac{3}{7}\,ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Fricas [A]
time = 0.30, size = 18, normalized size = 0.86

$$\frac{3}{70}\left(7\,bx^3+10\,ax^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/70*(7*b*x^3 + 10*a*x^2)*x^(1/3)
```

Sympy [A]
time = 0.24, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{7}{3}}}{7}+\frac{3bx^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(7/3)/7 + 3*b*x**(10/3)/10
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.29

$$\frac{3}{10}bx^{\frac{1}{3}}x^3+\frac{3}{7}ax^{\frac{1}{3}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x)
```

```
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{7/3}\,\left(10\,a+7\,b\,x\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x),x)
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

3.652 $\int x^{2/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

```
[Out] 3/5*a*x^(5/3)+3/8*b*x^(8/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)\,dx&=\int\left(ax^{2/3}+bx^{5/3}\right)\,dx\\&=\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{40}x^{5/3}(8a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

Mathics [A]
time = 1.64, size = 13, normalized size = 0.62

$$\frac{3x^{\frac{5}{3}}(8a + 5bx)}{40}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(2/3)*(a + b*x),x]')
```

```
[Out] 3 x ^ (5 / 3) (8 a + 5 b x) / 40
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
derivativedivides	$\frac{3a\,x^{\frac{5}{3}}}{5}+\frac{3b\,x^{\frac{8}{3}}}{8}$	14
default	$\frac{3a\,x^{\frac{5}{3}}}{5}+\frac{3b\,x^{\frac{8}{3}}}{8}$	14
trager	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
risch	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*a*x^(5/3)+3/8*b*x^(8/3)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.62

$$\frac{3}{8}\,bx^{\frac{8}{3}} + \frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Fricas [A]
time = 0.30, size = 16, normalized size = 0.76

$$\frac{3}{40}\,(5\,bx^2 + 8\,ax)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b*x^2 + 8*a*x)*x^(2/3)
```

Sympy [A]
time = 0.14, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{5}{3}}}{5}+\frac{3bx^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(5/3)/5 + 3*b*x**(8/3)/8
```

Giac [A]
time = 0.00, size = 31, normalized size = 1.48

$$\frac{3b\left(x^{\frac{1}{3}}\right)^{2}x^{2}}{2\cdot 4}+\frac{3}{5}a\left(x^{\frac{1}{3}}\right)^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x)
```

```
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{5/3}\,\left(8\,a+5\,b\,x\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x),x)
```

```
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

3.653 $\int \sqrt[3]{x}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

```
[Out] 3/4*a*x^(4/3)+3/7*b*x^(7/3)
```

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)\,dx &= \int \left(a\sqrt[3]{x}+bx^{4/3}\right)\,dx\\ &= \frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{28}x^{4/3}(7a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

Mathics [A]
time = 2.10, size = 13, normalized size = 0.62

$$\frac{3x^{\frac{4}{3}}\left(7a+4bx\right)}{28}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(1/3)*(a + b*x),x]')
[Out] 3 x ^ (4 / 3) (7 a + 4 b x) / 28
```

Maple [A]
time = 0.07, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
derivativedivides	$\frac{3a\,x^{\frac{4}{3}}}{4}+\frac{3b\,x^{\frac{7}{3}}}{7}$	14
default	$\frac{3a\,x^{\frac{4}{3}}}{4}+\frac{3b\,x^{\frac{7}{3}}}{7}$	14
trager	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
risch	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 3/4*a*x^(4/3)+3/7*b*x^(7/3)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{3}{7}\,bx^{\frac{7}{3}}+\frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Fricas [A]
time = 0.30, size = 16, normalized size = 0.76

$$\frac{3}{28}\,\left(4\,bx^{2}+7\,ax\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b*x^2 + 7*a*x)*x^(1/3)
```

Sympy [A]
time = 0.70, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{4}{3}}}{4}+\frac{3bx^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(4/3)/4 + 3*b*x**(7/3)/7
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.19

$$\frac{3}{7}bx^{\frac{1}{3}}x^2+\frac{3}{4}ax^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x)
```

```
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{4/3}\,\left(7\,a+4\,b\,x\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x),x)
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

3.654 $\int \frac{a+bx}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=21

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

```
[Out] 3/2*a*x^(2/3)+3/5*b*x^(5/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a}{\sqrt[3]{x}}+bx^{2/3}\right)\,dx\\ &= \frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{10}x^{2/3}(5a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

Mathics [A]
time = 2.21, size = 13, normalized size = 0.62

$$\frac{3x^{\frac{2}{3}}\left(5a+2bx\right)}{10}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^(1/3),x]')
```

```
[Out] 3 x ^ (2 / 3) (5 a + 2 b x) / 10
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
trager	$\left(\frac{3bx}{5}+\frac{3a}{2}\right)x^{\frac{2}{3}}$	13
gosper	$\frac{3x^{\frac{2}{3}}\left(2bx+5a\right)}{10}$	14
derivativedivides	$\frac{3a\,x^{\frac{2}{3}}}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	14
default	$\frac{3a\,x^{\frac{2}{3}}}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	14
risch	$\frac{3x^{\frac{2}{3}}\left(2bx+5a\right)}{10}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*a*x^(2/3)+3/5*b*x^(5/3)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Fricas [A]
time = 0.30, size = 13, normalized size = 0.62

$$\frac{3}{10}\,(2\,bx+5\,a)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x + 5*a)*x^(2/3)
```

Sympy [A]
time = 0.78, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{2}{3}}}{2} + \frac{3bx^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/3),x)
```

```
[Out] 3*a*x**(2/3)/2 + 3*b*x**(5/3)/5
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.24

$$\frac{3}{5}b\left(x^{\frac{1}{3}}\right)^{2}x + \frac{3}{2}a\left(x^{\frac{1}{3}}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x)
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{2/3}\,\left(5\,a+2\,b\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

3.655 $\int \frac{a+bx}{x^{2/3}} dx$

Optimal. Leaf size=19

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

[Out] 3*a*x^(1/3)+3/4*b*x^(4/3)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^(2/3),x]

[Out] 3*a*x^(1/3) + (3*b*x^(4/3))/4

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{2/3}}\,dx &= \int \left(\frac{a}{x^{2/3}}+b\sqrt[3]{x}\right)dx\\ &= 3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{3}{4}\sqrt[3]{x}\,(4a+bx)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^(2/3),x]

[Out] (3*x^(1/3)*(4*a + b*x))/4

Mathics [A]
time = 1.91, size = 12, normalized size = 0.63

$$\frac{3x^{\frac{1}{3}}(4a+bx)}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^(2/3),x]')
```

```
[Out] 3 x ^ (1 / 3) (4 a + b x) / 4
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
trager	$\left(\frac{3bx}{4}+3a\right)x^{\frac{1}{3}}$	13
risch	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
derivativedivides	$3a\,x^{\frac{1}{3}}+\frac{3b\,x^{\frac{4}{3}}}{4}$	14
default	$3a\,x^{\frac{1}{3}}+\frac{3b\,x^{\frac{4}{3}}}{4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*x^(1/3)+3/4*b*x^(4/3)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.68

$$\frac{3}{4}\,bx^{\frac{4}{3}}+3\,ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Fricas [A]
time = 0.31, size = 12, normalized size = 0.63

$$\frac{3}{4}\,(bx+4\,a)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + 4*a)*x^(1/3)
```

Sympy [A]
time = 0.53, size = 17, normalized size = 0.89

$$3a\sqrt[3]{x} + \frac{3bx^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(2/3),x)
```

```
[Out] 3*a*x**(1/3) + 3*b*x**(4/3)/4
```

Giac [A]
time = 0.00, size = 20, normalized size = 1.05

$$\frac{3}{4}bx^{\frac{1}{3}}x + 3ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x)
```

```
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.63

$$\frac{3\,x^{1/3}\,\left(4\,a+b\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(2/3),x)
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

3.656 $\int \frac{a+bx}{x^{4/3}}\,dx$

Optimal. Leaf size=19

$$-\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}$$

[Out] -3*a/x^(1/3)+3/2*b*x^(2/3)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^(4/3),x]

[Out] (-3*a)/x^(1/3) + (3*b*x^(2/3))/2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{4/3}}\,dx &= \int \left(\frac{a}{x^{4/3}}+\frac{b}{\sqrt[3]{x}}\right)dx\\ &= -\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.89

$$-\frac{3(2a-bx)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^(4/3),x]

```
[Out] (-3*(2*a - b*x))/(2*x^(1/3))
```

Mathics [A]
time = 1.62, size = 12, normalized size = 0.63

$$\frac{3(-2a+bx)}{2x^{\frac{1}{3}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^(4/3),x]')
[Out] 3 (-2 a + b x) / (2 x ^ (1 / 3))
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
derivativedivides	$-\frac{3a}{x^{\frac{1}{3}}}+\frac{3bx^{\frac{2}{3}}}{2}$	14
default	$-\frac{3a}{x^{\frac{1}{3}}}+\frac{3bx^{\frac{2}{3}}}{2}$	14
trager	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
risch	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(4/3),x,method=_RETURNVERBOSE)
[Out] -3*a/x^(1/3)+3/2*b*x^(2/3)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.68

$$\frac{3}{2}bx^{\frac{2}{3}}-\frac{3a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="maxima")
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.63

$$\frac{3(bx-2a)}{2x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x - 2*a)/x^(1/3)
```

Sympy [A]
time = 0.18, size = 17, normalized size = 0.89

$$-\frac{3a}{\sqrt[3]{x}}+\frac{3bx^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(4/3),x)
```

```
[Out] -3*a/x**(1/3) + 3*b*x**(2/3)/2
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.21

$$\frac{3}{2}\left(x^{\frac{1}{3}}\right)^{2}b-\frac{3a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x)
```

```
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$-\frac{6\,a-3\,b\,x}{2\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(4/3),x)
```

```
[Out] -(6*a - 3*b*x)/(2*x^(1/3))
```

3.657 $\int \frac{a+bx}{x^{5/3}}\,dx$

Optimal. Leaf size=19

$$-\frac{3a}{2x^{2/3}}+3b\sqrt[3]{x}$$

```
[Out] -3/2*a/x^(2/3)+3*b*x^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$3b\sqrt[3]{x}-\frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/3),x]
```

```
[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/3}}\,dx &= \int \left(\frac{a}{x^{5/3}}+\frac{b}{x^{2/3}}\right)dx\\ &= -\frac{3a}{2x^{2/3}}+3b\sqrt[3]{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.79

$$-\frac{3(a-2bx)}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/3),x]
```

```
[Out] (-3*(a - 2*b*x))/(2*x^(2/3))
```

Mathics [A]
time = 1.61, size = 13, normalized size = 0.68

$$\frac{3(-a+2bx)}{2x^{\frac{2}{3}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/x^(5/3),x]')
[Out] 3 (-a + 2 b x) / (2 x ^ (2 / 3))
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
trager	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
risch	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
derivativedivides	$-\frac{3a}{2x^{\frac{2}{3}}}+3b\,x^{\frac{1}{3}}$	14
default	$-\frac{3a}{2x^{\frac{2}{3}}}+3b\,x^{\frac{1}{3}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/3),x,method=_RETURNVERBOSE)
[Out] -3/2*a/x^(2/3)+3*b*x^(1/3)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}}-\frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="maxima")
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Fricas [A]
time = 0.30, size = 13, normalized size = 0.68

$$\frac{3\,(2\,bx-a)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/2*(2*b*x - a)/x^(2/3)
```

Sympy [A]
time = 0.20, size = 17, normalized size = 0.89

$$-\frac{3a}{2x^{\frac{2}{3}}}+3b\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/3),x)
```

```
[Out] -3*a/(2*x**(2/3)) + 3*b*x**(1/3)
```

Giac [A]
time = 0.00, size = 24, normalized size = 1.26

$$3x^{\frac{1}{3}}b-\frac{\frac{1}{2}\cdot 3a}{\left(x^{\frac{1}{3}}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x)
```

```
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$-\frac{3\,a-6\,b\,x}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/3),x)
```

```
[Out] -(3*a - 6*b*x)/(2*x^(2/3))
```

3.658 $\int x^{5/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

```
[Out] 3/8*a^2*x^(8/3)+6/11*a*b*x^(11/3)+3/14*b^2*x^(14/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^2,x]
[Out] (3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^2\,dx&=\int\left(a^2x^{5/3}+2abx^{8/3}+b^2x^{11/3}\right)\,dx\\&=\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^2,x]
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

Mathics [A]
time = 1.99, size = 24, normalized size = 0.67

$$\frac{3x^{\frac{8}{3}}\left(77a^2+112abx+44b^2x^2\right)}{616}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/3)*(a + b*x)^2,x]')
[Out] 3 x ^ (8 / 3) (77 a ^ 2 + 112 a b x + 44 b ^ 2 x ^ 2) / 616
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}\left(44x^2b^2+112abx+77a^2\right)}{616}$	25
derivativedivides	$\frac{3a^2x^{\frac{8}{3}}}{8}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{3b^2x^{\frac{14}{3}}}{14}$	25
default	$\frac{3a^2x^{\frac{8}{3}}}{8}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{3b^2x^{\frac{14}{3}}}{14}$	25
trager	$\frac{3x^{\frac{8}{3}}\left(44x^2b^2+112abx+77a^2\right)}{616}$	25
risch	$\frac{3x^{\frac{8}{3}}\left(44x^2b^2+112abx+77a^2\right)}{616}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/8*a^2*x^(8/3)+6/11*a*b*x^(11/3)+3/14*b^2*x^(14/3)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^2x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^2x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Fricas [A]
time = 0.57, size = 29, normalized size = 0.81

$$\frac{3}{616}\left(44\,b^2x^4+112\,abx^3+77\,a^2x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/616*(44*b^2*x^4 + 112*a*b*x^3 + 77*a^2*x^2)*x^(2/3)
```

Sympy [A]
time = 0.48, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{8}{3}}}{8}+\frac{6abx^{\frac{11}{3}}}{11}+\frac{3b^2x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(8/3)/8 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(14/3)/14
```

Giac [A]
time = 0.00, size = 55, normalized size = 1.53

$$\frac{3b^2\left(x^{\frac{1}{3}}\right)^2x^4}{2\cdot 7}+\frac{6}{11}ab\left(x^{\frac{1}{3}}\right)^2x^3+\frac{3a^2\left(x^{\frac{1}{3}}\right)^2x^2}{2\cdot 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x)
```

```
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{8/3}\,\left(77\,a^2+112\,a\,b\,x+44\,b^2\,x^2\right)}{616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(8/3)*(77*a^2 + 44*b^2*x^2 + 112*a*b*x))/616
```

3.659 $\int x^{4/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

```
[Out] 3/7*a^2*x^(7/3)+3/5*a*b*x^(10/3)+3/13*b^2*x^(13/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^2\,dx &= \int \left(a^2x^{4/3}+2abx^{7/3}+b^2x^{10/3}\right)\,dx\\ &= \frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

Mathics [A]
time = 1.89, size = 24, normalized size = 0.67

$$\frac{3x^{\frac{7}{3}}\left(65a^2+91abx+35b^2x^2\right)}{455}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(4/3)*(a + b*x)^2,x]')
```

```
[Out] 3 x ^ (7 / 3) (65 a ^ 2 + 91 a b x + 35 b ^ 2 x ^ 2) / 455
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}\left(35x^2b^2+91abx+65a^2\right)}{455}$	25
derivativedivides	$\frac{3a^2x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{13}{3}}}{13}$	25
default	$\frac{3a^2x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{13}{3}}}{13}$	25
trager	$\frac{3x^{\frac{7}{3}}\left(35x^2b^2+91abx+65a^2\right)}{455}$	25
risch	$\frac{3x^{\frac{7}{3}}\left(35x^2b^2+91abx+65a^2\right)}{455}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*a^2*x^(7/3)+3/5*a*b*x^(10/3)+3/13*b^2*x^(13/3)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^2x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Fricas [A]
time = 0.29, size = 29, normalized size = 0.81

$$\frac{3}{455}\left(35\,b^2x^4+91\,abx^3+65\,a^2x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^2*x^4 + 91*a*b*x^3 + 65*a^2*x^2)*x^(1/3)
```

Sympy [A]
time = 0.39, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{7}{3}}}{7} + \frac{3abx^{\frac{10}{3}}}{5} + \frac{3b^2x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(7/3)/7 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(13/3)/13
```

Giac [A]
time = 0.00, size = 45, normalized size = 1.25

$$\frac{3}{13}b^2x^{\frac{1}{3}}x^4 + \frac{6}{10}abx^{\frac{1}{3}}x^3 + \frac{3}{7}a^2x^{\frac{1}{3}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x)
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{7/3}\,\left(65\,a^2+91\,a\,b\,x+35\,b^2\,x^2\right)}{455}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(7/3)*(65*a^2 + 35*b^2*x^2 + 91*a*b*x))/455
```

3.660 $\int x^{2/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

```
[Out] 3/5*a^2*x^(5/3)+3/4*a*b*x^(8/3)+3/11*b^2*x^(11/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^2\,dx&=\int\left(a^2x^{2/3}+2abx^{5/3}+b^2x^{8/3}\right)\,dx\\&=\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{220}x^{5/3}\left(44a^2+55abx+20b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*(44*a^2 + 55*a*b*x + 20*b^2*x^2))/220
```

Mathics [A]
time = 1.76, size = 24, normalized size = 0.67

$$\frac{3x^{\frac{5}{3}}\left(44a^2+55abx+20b^2x^2\right)}{220}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(2/3)*(a + b*x)^2,x]')
[Out] 3 x ^ (5 / 3) (44 a ^ 2 + 55 a b x + 20 b ^ 2 x ^ 2) / 220
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}\left(20x^2b^2+55abx+44a^2\right)}{220}$	25
derivativedivides	$\frac{3a^2x^{\frac{5}{3}}}{5}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{11}{3}}}{11}$	25
default	$\frac{3a^2x^{\frac{5}{3}}}{5}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{11}{3}}}{11}$	25
trager	$\frac{3x^{\frac{5}{3}}\left(20x^2b^2+55abx+44a^2\right)}{220}$	25
risch	$\frac{3x^{\frac{5}{3}}\left(20x^2b^2+55abx+44a^2\right)}{220}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/5*a^2*x^(5/3)+3/4*a*b*x^(8/3)+3/11*b^2*x^(11/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Fricas [A]
time = 0.30, size = 27, normalized size = 0.75

$$\frac{3}{220}\left(20\,b^2x^3+55\,abx^2+44\,a^2x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/220*(20*b^2*x^3 + 55*a*b*x^2 + 44*a^2*x)*x^(2/3)
```

Sympy [A]
time = 0.23, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{5}{3}}}{5}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(5/3)/5 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(11/3)/11
```

Giac [A]
time = 0.00, size = 51, normalized size = 1.42

$$\frac{3}{11}b^2\left(x^{\frac{1}{3}}\right)^2x^3+\frac{6ab\left(x^{\frac{1}{3}}\right)^2x^2}{2\cdot 4}+\frac{3}{5}a^2\left(x^{\frac{1}{3}}\right)^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x)
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{5/3}\,\left(44\,a^2+55\,a\,b\,x+20\,b^2\,x^2\right)}{220}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(5/3)*(44*a^2 + 20*b^2*x^2 + 55*a*b*x))/220
```

3.661 $\int \sqrt[3]{x}\,(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

```
[Out] 3/4*a^2*x^(4/3)+6/7*a*b*x^(7/3)+3/10*b^2*x^(10/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}\,(a+bx)^2\,dx &= \int \left(a^2\sqrt[3]{x}+2abx^{4/3}+b^2x^{7/3}\right)\,dx\\ &= \frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{140}x^{4/3}\left(35a^2+40abx+14b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*(35*a^2 + 40*a*b*x + 14*b^2*x^2))/140
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 13.43, size = 1235, normalized size = 34.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1/3)*(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{3 a ^ (1 / 3) (9 -1 ^ (1 / 3) a ^ 3 + 35 a ^ 2 b x (b x / a) ^
(1 / 3) + 40 a b ^ 2 x ^ 2 (b x / a) ^ (1 / 3) + 14 b ^ 3 x ^ 3 (b x / a) ^
 (1 / 3)) / (140 b ^ (4 / 3)), Abs[(a + b x) / a] > 1}}, -27 a ^ (34 / 3) /
 (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4 / 3) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (
7 / 3) (a / b + x) + 420 E ^ (I 2 Pi / 3) a ^ 6 b ^ (10 / 3) (a / b + x) ^
2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (13 / 3) (a / b + x) ^ 3) + 27 a ^ (34 /
 3) (1 - b (a / b + x) / a) ^ (1 / 3) / (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4
/ 3) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E ^ (I 2 Pi
 / 3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (
13 / 3) (a / b + x) ^ 3) - 72 a ^ (31 / 3) b (a / b + x) (1 - b (a / b + x)
 / a) ^ (1 / 3) / (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4 / 3) - 420 E ^ (I 2 Pi
 / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E ^ (I 2 Pi / 3) a ^ 6 b ^ (10 /
3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (13 / 3) (a / b + x) ^
3) + 81 a ^ (31 / 3) b (a / b + x) / (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4 / 3
) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E ^ (I 2 Pi /
3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (13
/ 3) (a / b + x) ^ 3) - 81 a ^ (28 / 3) b ^ 2 (a / b + x) ^ 2 / (140 E ^ (I
 2 Pi / 3) a ^ 8 b ^ (4 / 3) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a /
b + x) + 420 E ^ (I 2 Pi / 3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^
(I 2 Pi / 3) a ^ 5 b ^ (13 / 3) (a / b + x) ^ 3) + 60 a ^ (28 / 3) b ^ 2 (1
 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 2 / (140 E ^ (I 2 Pi / 3) a ^
 8 b ^ (4 / 3) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E
 ^ (I 2 Pi / 3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a
 ^ 5 b ^ (13 / 3) (a / b + x) ^ 3) - 60 a ^ (25 / 3) b ^ 3 (1 - b (a / b +
x) / a) ^ (1 / 3) (a / b + x) ^ 3 / (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4 / 3)
 - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E ^ (I 2 Pi / 3
) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (13 /
 3) (a / b + x) ^ 3) + 27 a ^ (25 / 3) b ^ 3 (a / b + x) ^ 3 / (140 E ^ (I
2 Pi / 3) a ^ 8 b ^ (4 / 3) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b
 + x) + 420 E ^ (I 2 Pi / 3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (
I 2 Pi / 3) a ^ 5 b ^ (13 / 3) (a / b + x) ^ 3) + 135 a ^ (22 / 3) b ^ 4 (1
 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 4 / (140 E ^ (I 2 Pi / 3) a ^
 8 b ^ (4 / 3) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E
 ^ (I 2 Pi / 3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a
 ^ 5 b ^ (13 / 3) (a / b + x) ^ 3) - 132 a ^ (19 / 3) b ^ 5 (1 - b (a / b +
 x) / a) ^ (1 / 3) (a / b + x) ^ 5 / (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4 / 3
) - 420 E ^ (I 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E ^ (I 2 Pi /
```

```
3) a ^ 6 b ^ (10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (13
/ 3) (a / b + x) ^ 3) + 42 a ^ (16 / 3) b ^ 6 (1 - b (a / b + x) / a) ^ (1
/ 3) (a / b + x) ^ 6 / (140 E ^ (I 2 Pi / 3) a ^ 8 b ^ (4 / 3) - 420 E ^ (I
 2 Pi / 3) a ^ 7 b ^ (7 / 3) (a / b + x) + 420 E ^ (I 2 Pi / 3) a ^ 6 b ^ (
10 / 3) (a / b + x) ^ 2 - 140 E ^ (I 2 Pi / 3) a ^ 5 b ^ (13 / 3) (a / b +
x) ^ 3)]
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}\left(14x^2b^2+40abx+35a^2\right)}{140}$	25
derivativedivides	$\frac{3a^2x^{\frac{4}{3}}}{4}+\frac{6ab\,x^{\frac{7}{3}}}{7}+\frac{3b^2x^{\frac{10}{3}}}{10}$	25
default	$\frac{3a^2x^{\frac{4}{3}}}{4}+\frac{6ab\,x^{\frac{7}{3}}}{7}+\frac{3b^2x^{\frac{10}{3}}}{10}$	25
trager	$\frac{3x^{\frac{4}{3}}\left(14x^2b^2+40abx+35a^2\right)}{140}$	25
risch	$\frac{3x^{\frac{4}{3}}\left(14x^2b^2+40abx+35a^2\right)}{140}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/4*a^2*x^(4/3)+6/7*a*b*x^(7/3)+3/10*b^2*x^(10/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^2x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Fricas [A]
time = 0.30, size = 27, normalized size = 0.75

$$\frac{3}{140}\left(14\,b^2x^3+40\,abx^2+35\,a^2x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/140*(14*b^2*x^3 + 40*a*b*x^2 + 35*a^2*x)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.12, size = 2633, normalized size = 73.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**2,x)
```

```
[Out] Piecewise((27*a**(34/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(-140*a**
8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*
a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3
*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7
*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I
*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 72*a**(31/3)*b*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2
*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(
a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3))
- 81*a**(31/3)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(28/3)*b**2*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp
(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)
*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)
) + 81*a**(28/3)*b**2*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*
a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp
(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(25/3)*
b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-140*a**8*b**(
4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b
**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2
*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3
) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x
)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 135*a
**(22/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(2*I*pi/3)/(-140*
a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 4
20*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)
**3*exp(2*I*pi/3)) - 132*a**(19/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b +
x)**5*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(
a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 14
0*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 42*a**(16/3)*b**6*(-1 + b*(a
/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi
/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b +
 x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), Abs(
b*(a/b + x)/a) > 1), (-27*a**(34/3)*(1 - b*(a/b + x)/a)**(1/3)/(-140*a**8*b
**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**
6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*ex
p(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b*
```

```
*(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi
/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 72*a**(31/3)*b*(1 -
b*(a/b + x)/a)**(1/3)*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**
7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*
I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(31/3)*b*(
a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*ex
p(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(
13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(28/3)*b**2*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a
/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140
*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 81*a**(28/3)*b**2*(a/b + x)**
2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*p
i/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(25/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(
a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)
*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b
**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140
*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) -
420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x
)**3*exp(2*I*pi/3)) - 135*a**(22/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b +
x)**4/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2
*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/
3)*(a/b + x)**3*exp(2*I*pi/3)) + 132*a**(19/3)*b**5*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)**5/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b
 + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a
**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 42*a**(16/3)*b**6*(1 - b*(a/b +
 x)/a)**(1/3)*(a/b + x)**6/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.19

$$\frac{3}{10}b^2x^{\frac{1}{3}}x^3+\frac{6}{7}abx^{\frac{1}{3}}x^2+\frac{3}{4}a^2x^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x)
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{4/3}\,\left(35\,a^2+40\,a\,b\,x+14\,b^2\,x^2\right)}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(4/3)*(35*a^2 + 14*b^2*x^2 + 40*a*b*x))/140
```

3.662 $\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=36

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

```
[Out] 3/2*a^2*x^(2/3)+6/5*a*b*x^(5/3)+3/8*b^2*x^(8/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^2}{\sqrt[3]{x}}+2abx^{2/3}+b^2x^{5/3}\right)\,dx\\ &= \frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{40}x^{2/3}\left(20a^2+16abx+5b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(1/3),x]
```

[Out] (3*x^(2/3)*(20*a^2 + 16*a*b*x + 5*b^2*x^2))/40

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 10.45, size = 965, normalized size = 26.81

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^2/x^(1/3),x]')

[Out] Piecewise[{{3 a ^ (2 / 3) (-9 -1 ^ (2 / 3) a ^ 2 + 20 a ^ 2 (b x / a) ^ (2 / 3) + 16 a b x (b x / a) ^ (2 / 3) + 5 b ^ 2 x ^ 2 (b x / a) ^ (2 / 3)) / (40 b ^ (2 / 3)), Abs[(a + b x) / a] > 1}}, -27 E ^ (I 2 Pi / 3) a ^ (32 / 3) / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) + 27 E ^ (I 2 Pi / 3) a ^ (32 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) - 63 E ^ (I 2 Pi / 3) a ^ (29 / 3) b (a / b + x) (1 - b (a / b + x) / a) ^ (2 / 3) / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) + 81 E ^ (I 2 Pi / 3) a ^ (29 / 3) b (a / b + x) / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) - 81 E ^ (I 2 Pi / 3) a ^ (26 / 3) b ^ 2 (a / b + x) ^ 2 / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) + 42 E ^ (I 2 Pi / 3) a ^ (26 / 3) b ^ 2 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 2 / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) - 18 E ^ (I 2 Pi / 3) a ^ (23 / 3) b ^ 3 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 3 / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) + 27 E ^ (I 2 Pi / 3) a ^ (23 / 3) b ^ 3 (a / b + x) ^ 3 / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) + 27 E ^ (I 2 Pi / 3) a ^ (20 / 3) b ^ 4 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 4 / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3) - 15 E ^ (I 2 Pi / 3) a ^ (17 / 3) b ^ 5 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 5 / (40 a ^ 8 b ^ (2 / 3) - 120 a ^ 7 b ^ (5 / 3) (a / b + x) + 120 a ^ 6 b ^ (8 / 3) (a / b + x) ^ 2 - 40 a ^ 5 b ^ (11 / 3) (a / b + x) ^ 3)]

Maple [A]
time = 0.10, size = 25, normalized size = 0.69

---

method	result	size
trager	$\left(\frac{3}{8}x^{2}b^{2}+\frac{6}{5}abx+\frac{3}{2}a^{2}\right)x^{\frac{2}{3}}$	24
gosper	$\frac{3x^{\frac{2}{3}}\left(5x^{2}b^{2}+16abx+20a^{2}\right)}{40}$	25
derivativedivides	$\frac{3a^{2}x^{\frac{2}{3}}}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+\frac{3b^{2}x^{\frac{8}{3}}}{8}$	25
default	$\frac{3a^{2}x^{\frac{2}{3}}}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+\frac{3b^{2}x^{\frac{8}{3}}}{8}$	25
risch	$\frac{3x^{\frac{2}{3}}\left(5x^{2}b^{2}+16abx+20a^{2}\right)}{40}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*a^2*x^(2/3)+6/5*a*b*x^(5/3)+3/8*b^2*x^(8/3)
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^{2}x^{\frac{8}{3}}+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{2}\,a^{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.67

$$\frac{3}{40}\left(5\,b^{2}x^{2}+16\,abx+20\,a^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 16*a*b*x + 20*a^2)*x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.03, size = 1765, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*(a/b + x)/a)**(2/3)/(-40*a**8*b**(2/3) + 1
20*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(1
```

```
1/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a*
*7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*
(a/b + x)**3) + 63*a**(29/3)*b*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)
/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b
+ x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(-1 + b*(a/b
+ x)/a)**(2/3)*(a/b + x)**2/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 81*a
**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(
5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b +
x)**3) + 18*a**(23/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-40*a*
*8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
+ 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b + x)**3*exp(2*I*
pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*
(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(20/3)*b**4*(-1 + b*
(a/b + x)/a)**(2/3)*(a/b + x)**4/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 15*a**(17/3)*b**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5/(-40*a**8*b**(2
/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**
5*b**(11/3)*(a/b + x)**3), Abs(b*(a/b + x)/a) > 1), (-27*a**(32/3)*(1 - b*(
a/b + x)/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 63*a
**(29/3)*b*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(-40*a**8*b**
(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a
**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(1 - b*(a/b + x)/a)*
*(2/3)*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a
/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3)
+ 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**
7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(
a/b + x)**3) + 18*a**(23/3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*ex
p(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**
(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a
/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) -
 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(
20/3)*b**4*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(-40*a**8*
b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 4
0*a**5*b**(11/3)*(a/b + x)**3) + 15*a**(17/3)*b**5*(1 - b*(a/b + x)/a)**(2/
3)*(a/b + x)**5*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b +
 x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3), Tru
e))
```

Giac [A]
time = 0.00, size = 48, normalized size = 1.33

$$\frac{3b^2\left(x^{\frac{1}{3}}\right)^2x^2}{2\cdot 4}+\frac{6}{5}ab\left(x^{\frac{1}{3}}\right)^2x+\frac{3}{2}a^2\left(x^{\frac{1}{3}}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x)
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{2/3}\,\left(20\,a^2+16\,a\,b\,x+5\,b^2\,x^2\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(20*a^2 + 5*b^2*x^2 + 16*a*b*x))/40
```

3.663 $\int \frac{(a+bx)^2}{x^{2/3}} \, dx$

Optimal. Leaf size=34

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

```
[Out] 3*a^2*x^(1/3)+3/2*a*b*x^(4/3)+3/7*b^2*x^(7/3)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(2/3),x]
```

```
[Out] 3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^{2/3}} \, dx &= \int \left(\frac{a^2}{x^{2/3}}+2ab\sqrt[3]{x}+b^2x^{4/3}\right) dx \\ &= 3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$\frac{3}{14}\sqrt[3]{x}\left(14a^2+7abx+2b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(14*a^2 + 7*a*b*x + 2*b^2*x^2))/14
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 10.15, size = 965, normalized size = 28.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/x^(2/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (1 / 3) (-9 -1 ^ (1 / 3) a ^ 2 + 14 a ^ 2 (b x / a) ^ (1
/ 3) + 7 a b x (b x / a) ^ (1 / 3) + 2 b ^ 2 x ^ 2 (b x / a) ^ (1 / 3)) / (
14 b ^ (1 / 3)), Abs[(a + b x) / a] > 1}}, -27 E ^ (I Pi / 3) a ^ (31 / 3)
/ (14 a ^ 8 b ^ (1 / 3) - 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42 a ^ 6 b ^ (
7 / 3) (a / b + x) ^ 2 - 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3) + 27 E ^ (I
 Pi / 3) a ^ (31 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) / (14 a ^ 8 b ^ (1
/ 3) - 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b + x)
^ 2 - 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3) - 72 E ^ (I Pi / 3) a ^ (28 /
3) b (a / b + x) (1 - b (a / b + x) / a) ^ (1 / 3) / (14 a ^ 8 b ^ (1 / 3)
- 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b + x) ^ 2 -
 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3) + 81 E ^ (I Pi / 3) a ^ (28 / 3) b
(a / b + x) / (14 a ^ 8 b ^ (1 / 3) - 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42
 a ^ 6 b ^ (7 / 3) (a / b + x) ^ 2 - 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3)
 - 81 E ^ (I Pi / 3) a ^ (25 / 3) b ^ 2 (a / b + x) ^ 2 / (14 a ^ 8 b ^ (1
/ 3) - 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b + x)
^ 2 - 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3) + 60 E ^ (I Pi / 3) a ^ (25 /
3) b ^ 2 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 2 / (14 a ^ 8 b ^
(1 / 3) - 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b +
x) ^ 2 - 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3) - 18 E ^ (I Pi / 3) a ^ (22
 / 3) b ^ 3 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 3 / (14 a ^ 8 b
 ^ (1 / 3) - 42 a ^ 7 b ^ (4 / 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b
 + x) ^ 2 - 14 a ^ 5 b ^ (10 / 3) (a / b + x) ^ 3) + 27 E ^ (I Pi / 3) a ^
(22 / 3) b ^ 3 (a / b + x) ^ 3 / (14 a ^ 8 b ^ (1 / 3) - 42 a ^ 7 b ^ (4 /
3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b + x) ^ 2 - 14 a ^ 5 b ^ (10 /
3) (a / b + x) ^ 3) + 9 E ^ (I Pi / 3) a ^ (19 / 3) b ^ 4 (1 - b (a / b + x
) / a) ^ (1 / 3) (a / b + x) ^ 4 / (14 a ^ 8 b ^ (1 / 3) - 42 a ^ 7 b ^ (4
/ 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b + x) ^ 2 - 14 a ^ 5 b ^ (10
/ 3) (a / b + x) ^ 3) - 6 E ^ (I Pi / 3) a ^ (16 / 3) b ^ 5 (1 - b (a / b +
 x) / a) ^ (1 / 3) (a / b + x) ^ 5 / (14 a ^ 8 b ^ (1 / 3) - 42 a ^ 7 b ^ (
4 / 3) (a / b + x) + 42 a ^ 6 b ^ (7 / 3) (a / b + x) ^ 2 - 14 a ^ 5 b ^ (1
0 / 3) (a / b + x) ^ 3)]
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.74

method	result	size
trager	$\left(\frac{3}{7}x^2b^2+\frac{3}{2}abx+3a^2\right)x^{\frac{1}{3}}$	24
gosper	$\frac{3x^{\frac{1}{3}}(2x^2b^2+7abx+14a^2)}{14}$	25
derivativedivides	$3a^2x^{\frac{1}{3}}+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^2x^{\frac{7}{3}}}{7}$	25
default	$3a^2x^{\frac{1}{3}}+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^2x^{\frac{7}{3}}}{7}$	25
risch	$\frac{3x^{\frac{1}{3}}(2x^2b^2+7abx+14a^2)}{14}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*x^(1/3)+3/2*a*b*x^(4/3)+3/7*b^2*x^(7/3)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.71

$$\frac{3}{14}\left(2\,b^2x^2+7\,abx+14\,a^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 + 7*a*b*x + 14*a^2)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 1741, normalized size = 51.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(2/3),x)
```

```
[Out] Piecewise((-27*a**(31/3)*(-1 + b*(a/b + x)/a)**(1/3)/(-14*a**8*b**(1/3) + 4
2*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/
```

3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b*
(4/3)(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b +
x)**3) + 72*a**(28/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(-14*a**8*b*
*(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a*
*5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8
*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14
*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(-1 + b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b
(7/3)*(a/b + x)2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*
(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22
/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-14*a**8*b**(1/3) + 42*a
7*b(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*
(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/
3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b
(10/3)*(a/b + x)3) - 9*a**(19/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**4/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*
(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(-1 + b*(
a/b + x)/a)**(1/3)*(a/b + x)**5/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b
+ x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3), Abs
(b*(a/b + x)/a) > 1), (-27*a**(31/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)
/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b +
x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**
8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 1
4*a**5*b**(10/3)*(a/b + x)**3) + 72*a**(28/3)*b*(1 - b*(a/b + x)/a)**(1/3)*
(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*
a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)
b(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25
/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(
1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5
*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(-14*
a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2
+ 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22/3)*b**3*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b +
x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 27*
a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/
3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)*
*3) - 9*a**(19/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*pi/3)/
(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x
)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(1 - b*(a/b + x)/
a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a
/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3),
True))

Giac [A]
time = 0.00, size = 38, normalized size = 1.12

$$\frac{3}{7}b^2x^{\frac{1}{3}}x^2 + \frac{6}{4}abx^{\frac{1}{3}}x + 3a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x)
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.71

$$\frac{3\,x^{1/3}\,\left(14\,a^2+7\,a\,b\,x+2\,b^2\,x^2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(2/3),x)
```

```
[Out] (3*x^(1/3)*(14*a^2 + 2*b^2*x^2 + 7*a*b*x))/14
```

3.664 $\int \frac{(a+bx)^2}{x^{4/3}} dx$

Optimal. Leaf size=32

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

```
[Out] -3*a^2/x^(1/3)+3*a*b*x^(2/3)+3/5*b^2*x^(5/3)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(4/3),x]
```

```
[Out] (-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{4/3}}\,dx &= \int \left(\frac{a^2}{x^{4/3}}+\frac{2ab}{\sqrt[3]{x}}+b^2x^{2/3}\right)dx\\ &= -\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.88

$$-\frac{3\left(5a^2-5abx-b^2x^2\right)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(4/3),x]
```

```
[Out] (-3*(5*a^2 - 5*a*b*x - b^2*x^2))/(5*x^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 10.87, size = 968, normalized size = 30.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/x^(4/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (2 / 3) (-5 a ^ 2 (b x / a) ^ (2 / 3) - 9 -1 ^ (2 / 3) a
b x + 5 a b x (b x / a) ^ (2 / 3) + b ^ 2 x ^ 2 (b x / a) ^ (2 / 3)) / (5 b
 ^ (2 / 3) x), Abs[(a + b x) / a] > 1}}, -27 a ^ (29 / 3) b ^ (1 / 3) (1 -
b (a / b + x) / a) ^ (2 / 3) / (5 E ^ (I Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3)
a ^ 7 b (a / b + x) + 15 E ^ (I Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^
 (I Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 27 a ^ (29 / 3) b ^ (1 / 3) / (5
 E ^ (I Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15 E ^ (I P
i / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3 (a / b +
x) ^ 3) - 81 a ^ (26 / 3) b ^ (4 / 3) (a / b + x) / (5 E ^ (I Pi / 3) a ^ 8
 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15 E ^ (I Pi / 3) a ^ 6 b ^ 2 (a
 / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 63 a ^ (26
/ 3) b ^ (4 / 3) (a / b + x) (1 - b (a / b + x) / a) ^ (2 / 3) / (5 E ^ (I
Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15 E ^ (I Pi / 3) a
 ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3)
- 42 a ^ (23 / 3) b ^ (7 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x)
 ^ 2 / (5 E ^ (I Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15
 E ^ (I Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3
(a / b + x) ^ 3) + 81 a ^ (23 / 3) b ^ (7 / 3) (a / b + x) ^ 2 / (5 E ^ (I
Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15 E ^ (I Pi / 3) a
 ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3)
- 27 a ^ (20 / 3) b ^ (10 / 3) (a / b + x) ^ 3 / (5 E ^ (I Pi / 3) a ^ 8 -
15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15 E ^ (I Pi / 3) a ^ 6 b ^ 2 (a /
b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 3 a ^ (20 / 3)
 b ^ (10 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 3 / (5 E ^ (I
 Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 15 E ^ (I Pi / 3)
a ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3)
 + 3 a ^ (17 / 3) b ^ (13 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x
) ^ 4 / (5 E ^ (I Pi / 3) a ^ 8 - 15 E ^ (I Pi / 3) a ^ 7 b (a / b + x) + 1
5 E ^ (I Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 5 E ^ (I Pi / 3) a ^ 5 b ^ 3
 (a / b + x) ^ 3)]
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.78

method	result	size
gosper	$-\frac{3(-x^2b^2-5abx+5a^2)}{5x^{\frac{1}{3}}}$	25
derivativedivides	$-\frac{3a^2}{x^{\frac{1}{3}}}+3ab\,x^{\frac{2}{3}}+\frac{3b^2x^{\frac{5}{3}}}{5}$	25
default	$-\frac{3a^2}{x^{\frac{1}{3}}}+3ab\,x^{\frac{2}{3}}+\frac{3b^2x^{\frac{5}{3}}}{5}$	25
trager	$-\frac{3(-x^2b^2-5abx+5a^2)}{5x^{\frac{1}{3}}}$	25
risch	$-\frac{3(-x^2b^2-5abx+5a^2)}{5x^{\frac{1}{3}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*a^2/x^(1/3)+3*a*b*x^(2/3)+3/5*b^2*x^(5/3)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.72

$$\frac{3\,(b^2x^2+5\,abx-5\,a^2)}{5\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b^2*x^2 + 5*a*b*x - 5*a^2)/x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.07, size = 1826, normalized size = 57.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(4/3),x)
```

```
[Out] Piecewise((-27*a**(29/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(
-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
 x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/3)*b
**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b*
*2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 63*a*
*(26/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(-5*a**8
*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*
exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)
*(a/b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6
*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 42
*a**(23/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(I*pi/3)/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b*
*(7/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3)
- 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi
/3)) + 3*a**(20/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I
*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**
2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**
(20/3)*b**(10/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*ex
p(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**
3*exp(I*pi/3)) + 3*a**(17/3)*b**(13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x
)**4*exp(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 1
5*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)
), Abs(b*(a/b + x)/a) > 1), (27*a**(29/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/
3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a
/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/
3)*b**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**
6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 6
3*a**(26/3)*b**(4/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)/(-5*a**8*exp(I*pi
/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/
3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)*(a/b + x
)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/
b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 42*a**(23/3
)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 1
5*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*
a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b**(7/3)*(a/b + x)**2/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(20/3)*b**
(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a*
*7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5
*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**(20/3)*b**(10/3)*(a/b + x)**3/(-5*a
**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)*
*2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(17/3)*b**(13
/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-5*a**8*exp(I*pi/3) + 15*a**7*
b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b*
```

```
*3*(a/b + x)**3*exp(I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 39, normalized size = 1.22

$$\frac{3}{5}\left(x^{\frac{1}{3}}\right)^{2}xb^{2}+3\left(x^{\frac{1}{3}}\right)^{2}ba-\frac{3a^{2}}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x)
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.75

$$\frac{-15\,a^2+15\,a\,b\,x+3\,b^2\,x^2}{5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(4/3),x)
[Out] (3*b^2*x^2 - 15*a^2 + 15*a*b*x)/(5*x^(1/3))
```

3.665 $\int \frac{(a+bx)^2}{x^{5/3}} \, dx$

Optimal. Leaf size=34

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

```
[Out] -3/2*a^2/x^(2/3)+6*a*b*x^(1/3)+3/4*b^2*x^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/3),x]
```

```
[Out] (-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^{5/3}} \, dx &= \int \left(\frac{a^2}{x^{5/3}}+\frac{2ab}{x^{2/3}}+b^2\sqrt[3]{x}\right) dx \\ &= -\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$-\frac{3\left(2a^2-8abx-b^2x^2\right)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/3),x]
```

```
[Out] (-3*(2*a^2 - 8*a*b*x - b^2*x^2))/(4*x^(2/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 10.74, size = 968, normalized size = 28.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/x^(5/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (1 / 3) (-2 a ^ 2 (b x / a) ^ (1 / 3) - 9 -1 ^ (1 / 3) a
b x + 8 a b x (b x / a) ^ (1 / 3) + b ^ 2 x ^ 2 (b x / a) ^ (1 / 3)) / (4 b
 ^ (1 / 3) x), Abs[(a + b x) / a] > 1}}, -27 a ^ (28 / 3) b ^ (2 / 3) (1 -
b (a / b + x) / a) ^ (1 / 3) / (4 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi /
 3) a ^ 7 b (a / b + x) + 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 -
 4 E ^ (I 2 Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 27 a ^ (28 / 3) b ^ (2 /
 3) / (4 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi / 3) a ^ 7 b (a / b + x) +
 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 4 E ^ (I 2 Pi / 3) a ^ 5
 b ^ 3 (a / b + x) ^ 3) - 81 a ^ (25 / 3) b ^ (5 / 3) (a / b + x) / (4 E ^
(I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi / 3) a ^ 7 b (a / b + x) + 12 E ^ (I 2
Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 4 E ^ (I 2 Pi / 3) a ^ 5 b ^ 3 (a / b
 + x) ^ 3) + 72 a ^ (25 / 3) b ^ (5 / 3) (a / b + x) (1 - b (a / b + x) / a
) ^ (1 / 3) / (4 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi / 3) a ^ 7 b (a /
b + x) + 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 4 E ^ (I 2 Pi /
3) a ^ 5 b ^ 3 (a / b + x) ^ 3) - 60 a ^ (22 / 3) b ^ (8 / 3) (1 - b (a / b
 + x) / a) ^ (1 / 3) (a / b + x) ^ 2 / (4 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (
I 2 Pi / 3) a ^ 7 b (a / b + x) + 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2 (a / b +
x) ^ 2 - 4 E ^ (I 2 Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 81 a ^ (22 / 3)
b ^ (8 / 3) (a / b + x) ^ 2 / (4 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi /
3) a ^ 7 b (a / b + x) + 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 -
4 E ^ (I 2 Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) - 27 a ^ (19 / 3) b ^ (11 /
 3) (a / b + x) ^ 3 / (4 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi / 3) a ^ 7
 b (a / b + x) + 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 4 E ^ (I
 2 Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 12 a ^ (19 / 3) b ^ (11 / 3) (1 -
 b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 3 / (4 E ^ (I 2 Pi / 3) a ^ 8 -
 12 E ^ (I 2 Pi / 3) a ^ 7 b (a / b + x) + 12 E ^ (I 2 Pi / 3) a ^ 6 b ^ 2
(a / b + x) ^ 2 - 4 E ^ (I 2 Pi / 3) a ^ 5 b ^ 3 (a / b + x) ^ 3) + 3 a ^ (
16 / 3) b ^ (14 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 4 / (4
 E ^ (I 2 Pi / 3) a ^ 8 - 12 E ^ (I 2 Pi / 3) a ^ 7 b (a / b + x) + 12 E ^
(I 2 Pi / 3) a ^ 6 b ^ 2 (a / b + x) ^ 2 - 4 E ^ (I 2 Pi / 3) a ^ 5 b ^ 3 (
a / b + x) ^ 3)]
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.74

method	result	size
gosper	$-\frac{3(-x^2b^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
derivativedivides	$-\frac{3a^2}{2x^{\frac{2}{3}}}+6ab\,x^{\frac{1}{3}}+\frac{3b^2x^{\frac{4}{3}}}{4}$	25
default	$-\frac{3a^2}{2x^{\frac{2}{3}}}+6ab\,x^{\frac{1}{3}}+\frac{3b^2x^{\frac{4}{3}}}{4}$	25
trager	$-\frac{3(-x^2b^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
risch	$-\frac{3(-x^2b^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a^2/x^(2/3)+6*a*b*x^(1/3)+3/4*b^2*x^(4/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}}+6\,abx^{\frac{1}{3}}-\frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.68

$$\frac{3\,(b^2x^2+8\,abx-2\,a^2)}{4\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b^2*x^2 + 8*a*b*x - 2*a^2)/x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 1957, normalized size = 57.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/3),x)
```

[Out] Piecewise((-27*a**(28/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)
/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*
(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 27*a
(28/3)*b(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3
) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(
2*I*pi/3)) + 72*a**(25/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*ex
p(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12
*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi
/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(
a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b
3*(a/b + x)3*exp(2*I*pi/3)) - 60*a**(22/3)*b**(8/3)*(-1 + b*(a/b + x)/a
)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b
 + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3
*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*
exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)*
*2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 12*a**(19/3)*b
(11/3)*(-1 + b*(a/b + x)/a)(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-4*a**8*ex
p(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2
*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**
(11/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi
/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*ex
p(2*I*pi/3)) + 3*a**(16/3)*b**(14/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*
*4*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
 - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2
*I*pi/3)), Abs(b*(a/b + x)/a) > 1), (27*a**(28/3)*b**(2/3)*(1 - b*(a/b + x)
/a)**(1/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*
a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/
3)) - 27*a**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*ex
p(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
x)**3*exp(2*I*pi/3)) - 72*a**(25/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/
b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6
*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3))
+ 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b +
 x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(22/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
- 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*
I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a
7*b*(a/b + x)*exp(2*I*pi/3) - 12*a6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4
*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 12*a**(19/3)*b**(11/3)*(1 - b*(a/b
 + x)/a)**(1/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*e
xp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
 x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**(11/3)*(a/b + x)**3/(-4*a**8*exp(2*
I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp
(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 3*a**(16/3)*b**(14/3

```
)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(-4*a**8*exp(2*I*pi/3) + 12*a**7*
b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**
5*b**3*(a/b + x)**3*exp(2*I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 40, normalized size = 1.18

$$\frac{3}{4}x^{\frac{1}{3}}xb^2 + 6x^{\frac{1}{3}}ba - \frac{\frac{1}{2} \cdot 3a^2}{\left(x^{\frac{1}{3}}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x)
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.71

$$\frac{-6\,a^2 + 24\,a\,b\,x + 3\,b^2\,x^2}{4\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/3),x)
[Out] (3*b^2*x^2 - 6*a^2 + 24*a*b*x)/(4*x^(2/3))
```

3.666 $\int x^{5/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

```
[Out] 3/8*a^3*x^(8/3)+9/11*a^2*b*x^(11/3)+9/14*a*b^2*x^(14/3)+3/17*b^3*x^(17/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3*
x^(17/3))/17
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^3\,dx &= \int \left(a^3x^{5/3}+3a^2bx^{8/3}+3ab^2x^{11/3}+b^3x^{14/3}\right)\,dx\\ &= \frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{8/3}\left(1309a^3+2856a^2bx+2244ab^2x^2+616b^3x^3\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*(1309*a^3 + 2856*a^2*b*x + 2244*a*b^2*x^2 + 616*b^3*x^3))/10472
```

Mathics [A]
time = 2.20, size = 35, normalized size = 0.69

$$\frac{3x^{\frac{8}{3}}\left(1309a^3+2856a^2bx+2244ab^2x^2+616b^3x^3\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/3)*(a + b*x)^3,x]')
```

```
[Out] 3 x ^ (8 / 3) (1309 a ^ 3 + 2856 a ^ 2 b x + 2244 a b ^ 2 x ^ 2 + 616 b ^ 3
 x ^ 3) / 10472
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}\left(616b^3x^3+2244a\,b^2x^2+2856a^2bx+1309a^3\right)}{10472}$	36
derivativedivides	$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2b\,x^{\frac{11}{3}}}{11}+\frac{9a\,b^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$	36
default	$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2b\,x^{\frac{11}{3}}}{11}+\frac{9a\,b^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$	36
trager	$\frac{3x^{\frac{8}{3}}\left(616b^3x^3+2244a\,b^2x^2+2856a^2bx+1309a^3\right)}{10472}$	36
risch	$\frac{3x^{\frac{8}{3}}\left(616b^3x^3+2244a\,b^2x^2+2856a^2bx+1309a^3\right)}{10472}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^3*x^(8/3)+9/11*a^2*b*x^(11/3)+9/14*a*b^2*x^(14/3)+3/17*b^3*x^(17/3)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

Fricas [A]
time = 0.29, size = 40, normalized size = 0.78

$$\frac{3}{10472}\left(616\,b^3x^5+2244\,ab^2x^4+2856\,a^2bx^3+1309\,a^3x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/10472*(616*b^3*x^5 + 2244*a*b^2*x^4 + 2856*a^2*b*x^3 + 1309*a^3*x^2)*x^(2
/3)
```

Sympy [A]
time = 0.68, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(8/3)/8 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(14/3)/14 + 3*b**3*
x**(17/3)/17
```

Giac [A]
time = 0.00, size = 75, normalized size = 1.47

$$\frac{3}{17}b^3\left(x^{\frac{1}{3}}\right)^2x^5+\frac{9ab^2\left(x^{\frac{1}{3}}\right)^2x^4}{2\cdot 7}+\frac{9}{11}a^2b\left(x^{\frac{1}{3}}\right)^2x^3+\frac{3a^3\left(x^{\frac{1}{3}}\right)^2x^2}{2\cdot 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x)
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{8/3}}{8}+\frac{3\,b^3\,x^{17/3}}{17}+\frac{9\,a^2\,b\,x^{11/3}}{11}+\frac{9\,a\,b^2\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(8/3))/8 + (3*b^3*x^(17/3))/17 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*
x^(14/3))/14
```

3.667 $\int x^{4/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

```
[Out] 3/7*a^3*x^(7/3)+9/10*a^2*b*x^(10/3)+9/13*a*b^2*x^(13/3)+3/16*b^3*x^(16/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3*
x^(16/3))/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^3\,dx&=\int\left(a^3x^{4/3}+3a^2bx^{7/3}+3ab^2x^{10/3}+b^3x^{13/3}\right)dx\\&=\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{7/3}\left(1040a^3+2184a^2bx+1680ab^2x^2+455b^3x^3\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*(1040*a^3 + 2184*a^2*b*x + 1680*a*b^2*x^2 + 455*b^3*x^3))/7280
```

Mathics [A]
time = 2.10, size = 35, normalized size = 0.69

$$\frac{3x^{\frac{7}{3}}\left(1040a^3+2184a^2bx+1680ab^2x^2+455b^3x^3\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(4/3)*(a + b*x)^3,x]')
```

```
[Out] 3 x ^ (7 / 3) (1040 a ^ 3 + 2184 a ^ 2 b x + 1680 a b ^ 2 x ^ 2 + 455 b ^ 3
 x ^ 3) / 7280
```

Maple [A]
time = 0.12, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}\left(455b^3x^3+1680a\,b^2x^2+2184a^2bx+1040a^3\right)}{7280}$	36
derivativedivides	$\frac{3a^3x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^2x^{\frac{13}{3}}}{13}+\frac{3b^3x^{\frac{16}{3}}}{16}$	36
default	$\frac{3a^3x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^2x^{\frac{13}{3}}}{13}+\frac{3b^3x^{\frac{16}{3}}}{16}$	36
trager	$\frac{3x^{\frac{7}{3}}\left(455b^3x^3+1680a\,b^2x^2+2184a^2bx+1040a^3\right)}{7280}$	36
risch	$\frac{3x^{\frac{7}{3}}\left(455b^3x^3+1680a\,b^2x^2+2184a^2bx+1040a^3\right)}{7280}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*a^3*x^(7/3)+9/10*a^2*b*x^(10/3)+9/13*a*b^2*x^(13/3)+3/16*b^3*x^(16/3)
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^3x^{\frac{16}{3}}+\frac{9}{13}\,ab^2x^{\frac{13}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

Fricas [A]
time = 0.30, size = 40, normalized size = 0.78

$$\frac{3}{7280}\left(455\,b^{3}x^{5}+1680\,ab^{2}x^{4}+2184\,a^{2}bx^{3}+1040\,a^{3}x^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^3*x^5 + 1680*a*b^2*x^4 + 2184*a^2*b*x^3 + 1040*a^3*x^2)*x^(1/
3)
```

Sympy [A]
time = 0.55, size = 49, normalized size = 0.96

$$\frac{3a^{3}x^{\frac{7}{3}}}{7}+\frac{9a^{2}bx^{\frac{10}{3}}}{10}+\frac{9ab^{2}x^{\frac{13}{3}}}{13}+\frac{3b^{3}x^{\frac{16}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(7/3)/7 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(13/3)/13 + 3*b**3*
x**(16/3)/16
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.24

$$\frac{3}{16}b^{3}x^{\frac{1}{3}}x^{5}+\frac{9}{13}ab^{2}x^{\frac{1}{3}}x^{4}+\frac{9}{10}a^{2}bx^{\frac{1}{3}}x^{3}+\frac{3}{7}a^{3}x^{\frac{1}{3}}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x)
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^{3}\,x^{7/3}}{7}+\frac{3\,b^{3}\,x^{16/3}}{16}+\frac{9\,a^{2}\,b\,x^{10/3}}{10}+\frac{9\,a\,b^{2}\,x^{13/3}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(7/3))/7 + (3*b^3*x^(16/3))/16 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*
x^(13/3))/13
```

3.668 $\int x^{2/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

```
[Out] 3/5*a^3*x^(5/3)+9/8*a^2*b*x^(8/3)+9/11*a*b^2*x^(11/3)+3/14*b^3*x^(14/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x^
(14/3))/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^3\,dx&=\int\left(a^3x^{2/3}+3a^2bx^{5/3}+3ab^2x^{8/3}+b^3x^{11/3}\right)\,dx\\&=\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{5/3}\left(616a^3+1155a^2bx+840ab^2x^2+220b^3x^3\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*(616*a^3 + 1155*a^2*b*x + 840*a*b^2*x^2 + 220*b^3*x^3))/3080
```

Mathics [A]
time = 1.93, size = 35, normalized size = 0.69

$$\frac{3x^{\frac{5}{3}}\left(616a^3+1155a^2bx+840ab^2x^2+220b^3x^3\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(2/3)*(a + b*x)^3,x]')
```

```
[Out] 3 x ^ (5 / 3) (616 a ^ 3 + 1155 a ^ 2 b x + 840 a b ^ 2 x ^ 2 + 220 b ^ 3 x
 ^ 3) / 3080
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}\left(220b^3x^3+840a\,b^2x^2+1155a^2bx+616a^3\right)}{3080}$	36
derivativedivides	$\frac{3a^3x^{\frac{5}{3}}}{5}+\frac{9a^2b\,x^{\frac{8}{3}}}{8}+\frac{9a\,b^2x^{\frac{11}{3}}}{11}+\frac{3b^3x^{\frac{14}{3}}}{14}$	36
default	$\frac{3a^3x^{\frac{5}{3}}}{5}+\frac{9a^2b\,x^{\frac{8}{3}}}{8}+\frac{9a\,b^2x^{\frac{11}{3}}}{11}+\frac{3b^3x^{\frac{14}{3}}}{14}$	36
trager	$\frac{3x^{\frac{5}{3}}\left(220b^3x^3+840a\,b^2x^2+1155a^2bx+616a^3\right)}{3080}$	36
risch	$\frac{3x^{\frac{5}{3}}\left(220b^3x^3+840a\,b^2x^2+1155a^2bx+616a^3\right)}{3080}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*a^3*x^(5/3)+9/8*a^2*b*x^(8/3)+9/11*a*b^2*x^(11/3)+3/14*b^3*x^(14/3)
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

Fricas [A]
time = 0.52, size = 38, normalized size = 0.75

$$\frac{3}{3080}\left(220\,b^3x^4+840\,ab^2x^3+1155\,a^2bx^2+616\,a^3x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^3*x^4 + 840*a*b^2*x^3 + 1155*a^2*b*x^2 + 616*a^3*x)*x^(2/3)
```

Sympy [A]
time = 0.33, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{5}{3}}}{5}+\frac{9a^2bx^{\frac{8}{3}}}{8}+\frac{9ab^2x^{\frac{11}{3}}}{11}+\frac{3b^3x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(5/3)/5 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(11/3)/11 + 3*b**3*x*
*(14/3)/14
```

Giac [A]
time = 0.00, size = 73, normalized size = 1.43

$$\frac{3b^3\left(x^{\frac{1}{3}}\right)^2x^4}{2\cdot 7}+\frac{9}{11}ab^2\left(x^{\frac{1}{3}}\right)^2x^3+\frac{9a^2b\left(x^{\frac{1}{3}}\right)^2x^2}{2\cdot 4}+\frac{3}{5}a^3\left(x^{\frac{1}{3}}\right)^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x)
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{5/3}}{5}+\frac{3\,b^3\,x^{14/3}}{14}+\frac{9\,a^2\,b\,x^{8/3}}{8}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(5/3))/5 + (3*b^3*x^(14/3))/14 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^
(11/3))/11
```

3.669 $\int \sqrt[3]{x}\,(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

```
[Out] 3/4*a^3*x^(4/3)+9/7*a^2*b*x^(7/3)+9/10*a*b^2*x^(10/3)+3/13*b^3*x^(13/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x^
(13/3))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}\,(a+bx)^3\,dx &= \int \left(a^3\sqrt[3]{x}+3a^2bx^{4/3}+3ab^2x^{7/3}+b^3x^{10/3}\right)dx\\ &= \frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{4/3}\left(455a^3+780a^2bx+546ab^2x^2+140b^3x^3\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*(455*a^3 + 780*a^2*b*x + 546*a*b^2*x^2 + 140*b^3*x^3))/1820
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 27.18, size = 2649, normalized size = 51.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1/3)*(a + b*x)^3,x]')
```

```
[Out] Piecewise[{{3 a ^ (1 / 3) (81 -1 ^ (1 / 3) a ^ 4 + 455 a ^ 3 b x (b x / a)
^ (1 / 3) + 780 a ^ 2 b ^ 2 x ^ 2 (b x / a) ^ (1 / 3) + 546 a b ^ 3 x ^ 3 (
b x / a) ^ (1 / 3) + 140 b ^ 4 x ^ 4 (b x / a) ^ (1 / 3)) / (1820 b ^ (4 /
3)), Abs[(a + b x) / a] > 1}}, -243 E ^ (I Pi / 3) a ^ (73 / 3) (1 - b (a /
 b + x) / a) ^ (1 / 3) / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3
) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b
^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10
920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b +
 x) ^ 6) + 243 E ^ (I Pi / 3) a ^ (73 / 3) / (1820 a ^ 20 b ^ (4 / 3) - 109
20 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^
 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3)
(a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b
 ^ (22 / 3) (a / b + x) ^ 6) - 1458 E ^ (I Pi / 3) a ^ (70 / 3) b (a / b +
x) / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 2730
0 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b +
x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 /
 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 1377 E ^
(I Pi / 3) a ^ (70 / 3) b (a / b + x) (1 - b (a / b + x) / a) ^ (1 / 3) / (
1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^
18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3
 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a
 / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) - 3213 E ^ (I Pi
/ 3) a ^ (67 / 3) b ^ 2 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 2 /
 (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a
^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^
 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3)
(a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 3645 E ^ (I P
i / 3) a ^ (67 / 3) b ^ 2 (a / b + x) ^ 2 / (1820 a ^ 20 b ^ (4 / 3) - 1092
0 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^
2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (
a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b
^ (22 / 3) (a / b + x) ^ 6) - 4860 E ^ (I Pi / 3) a ^ (64 / 3) b ^ 3 (a / b
 + x) ^ 3 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x)
 + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a
```

/ b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 3927 E ^ (I Pi / 3) a ^ (64 / 3) b ^ 3 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 3 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) - 2163 E ^ (I Pi / 3) a ^ (61 / 3) b ^ 4 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 4 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 3645 E ^ (I Pi / 3) a ^ (61 / 3) b ^ 4 (a / b + x) ^ 4 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) - 1827 E ^ (I Pi / 3) a ^ (58 / 3) b ^ 5 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 5 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) - 1458 E ^ (I Pi / 3) a ^ (58 / 3) b ^ 5 (a / b + x) ^ 5 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 243 E ^ (I Pi / 3) a ^ (55 / 3) b ^ 6 (a / b + x) ^ 6 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 6573 E ^ (I Pi / 3) a ^ (55 / 3) b ^ 6 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 6 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) - 8787 E ^ (I Pi / 3) a ^ (52 / 3) b ^ 7 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 7 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 6498 E ^ (I Pi / 3) a ^ (49 / 3) b ^ 8 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 8 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 / 3)

```
 (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) - 2562 E ^ (I
Pi / 3) a ^ (46 / 3) b ^ 9 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^
9 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27300
 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b + x
) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19 /
3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6) + 420 E ^ (I
 Pi / 3) a ^ (43 / 3) b ^ 10 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x)
^ 10 / (1820 a ^ 20 b ^ (4 / 3) - 10920 a ^ 19 b ^ (7 / 3) (a / b + x) + 27
300 a ^ 18 b ^ (10 / 3) (a / b + x) ^ 2 - 36400 a ^ 17 b ^ (13 / 3) (a / b
+ x) ^ 3 + 27300 a ^ 16 b ^ (16 / 3) (a / b + x) ^ 4 - 10920 a ^ 15 b ^ (19
 / 3) (a / b + x) ^ 5 + 1820 a ^ 14 b ^ (22 / 3) (a / b + x) ^ 6)]
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{4}{3}}}{4}+\frac{9a^{2}b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^{2}x^{\frac{10}{3}}}{10}+\frac{3b^{3}x^{\frac{13}{3}}}{13}$	36
default	$\frac{3a^{3}x^{\frac{4}{3}}}{4}+\frac{9a^{2}b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^{2}x^{\frac{10}{3}}}{10}+\frac{3b^{3}x^{\frac{13}{3}}}{13}$	36
trager	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36
risch	$\frac{3x^{\frac{4}{3}}\left(140b^{3}x^{3}+546a\,b^{2}x^{2}+780a^{2}bx+455a^{3}\right)}{1820}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 3/4*a^3*x^(4/3)+9/7*a^2*b*x^(7/3)+9/10*a*b^2*x^(10/3)+3/13*b^3*x^(13/3)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^{3}x^{\frac{13}{3}}+\frac{9}{10}\,ab^{2}x^{\frac{10}{3}}+\frac{9}{7}\,a^{2}bx^{\frac{7}{3}}+\frac{3}{4}\,a^{3}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="maxima")
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 0.75

$$\frac{3}{1820}\left(140\,b^{3}x^{4}+546\,ab^{2}x^{3}+780\,a^{2}bx^{2}+455\,a^{3}x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^3*x^4 + 546*a*b^2*x^3 + 780*a^2*b*x^2 + 455*a^3*x)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.62, size = 5012, normalized size = 98.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**3,x)
```

```
[Out] Piecewise((-243*a**(73/3)*(-1 + b*(a/b + x)/a)**(1/3)/(1820*a**20*b**(4/3)
- 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 364
00*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 1092
0*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a
**(73/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x)
+ 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 +
1820*a**14*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(-1 + b*(a/b + x)/a)*
*(1/3)*(a/b + x)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(18
20*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*
(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(
a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/
b + x)**6) - 3213*a**(67/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(
1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3
)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)
*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(
a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a**20*b**
(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2
 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4
- 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) +
 3927*a**(64/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(1820*a**20*b
**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)*
*2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**
4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6)
 - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a**20*b**(4/3) - 1092
0*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**
17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**1
5*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 2163*a**(61
/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(1820*a**20*b**(4/3) - 10
920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a
```

17*b(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a*
*15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(
61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(
7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)
*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*
(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1827*a**(58/3)*b**5*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(1820*a**20*b**(4/3) - 10920*a**19*b*
(7/3)(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/
3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3
)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(
a/b + x)**5*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b +
x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**
3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5
 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 6573*a**(55/3)*b**6*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**6/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)
3 + 27300*a16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)*
*5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*e
xp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a*
*18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14
*b**(22/3)*(a/b + x)**6) - 8787*a**(52/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*
(a/b + x)**7/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*
a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a
16*b(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**
14*b**(22/3)*(a/b + x)**6) + 6498*a**(49/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3
)*(a/b + x)**8/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 2730
0*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300
*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a
14*b(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(-1 + b*(a/b + x)/a)**(1
/3)*(a/b + x)**9/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(-1 + b*(a/b + x)/a)**
(1/3)*(a/b + x)**10/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) +
 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1
820*a**14*b**(22/3)*(a/b + x)**6), Abs(b*(a/b + x)/a) > 1), (-243*a**(73/3)
(1 - b(a/b + x)/a)**(1/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*
b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(1
3/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19
/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(73/3)*exp(I
*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*
b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b
(16/3)*(a/b + x)4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**

(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x
)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300
*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*
a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a*
*14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(1820*
a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/
b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
 + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
 x)**6) - 3213*a**(67/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I
*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*
b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b
(16/3)*(a/b + x)4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**
(22/3)*(a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a
20*b(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
 + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
+ x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
x)**6) + 3927*a**(64/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*
pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b
(10/3)*(a/b + x)2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b*
(16/3)(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(
22/3)*(a/b + x)**6) - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a*
*20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
+ x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b +
 x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x
)**6) - 2163*a**(61/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*p
i/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b*
(10/3)(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**
(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(2
2/3)*(a/b + x)**6) + 3645*a**(61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**
20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b +
 x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b +
x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)
6) - 1827*a(58/3)*b**5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi
/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**
(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(
16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22
/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(a/b + x)**5*exp(I*pi/3)/(1820*a**2
0*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b +
x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x
)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)*
*6) + 6573*a**(55/3)*b**6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/
3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(
10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(1
6/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/
3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*exp(I*pi/3)/(1820*a**20*

```
b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)
**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)*
*4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6
) - 8787*a**(52/3)*b**7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(I*pi/3)
/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10
/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/
3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)
*(a/b + x)**6) + 6498*a**(49/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**
8*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300
*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*
a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a*
*14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(1 - b*(a/b + x)/a)**(1/3
)*(a/b + x)**9*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
 + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x
)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)
**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(1 - b*(a/b
+ x)/a)**(1/3)*(a/b + x)**10*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19
*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(
13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(1
9/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6), True))
```

Giac [A]
time = 0.00, size = 61, normalized size = 1.20

$$\frac{3}{13}b^3x^{\frac{1}{3}}x^4+\frac{9}{10}ab^2x^{\frac{1}{3}}x^3+\frac{9}{7}a^2bx^{\frac{1}{3}}x^2+\frac{3}{4}a^3x^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x)
```

```
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{4/3}}{4}+\frac{3\,b^3\,x^{13/3}}{13}+\frac{9\,a^2\,b\,x^{7/3}}{7}+\frac{9\,a\,b^2\,x^{10/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(4/3))/4 + (3*b^3*x^(13/3))/13 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^
(10/3))/10
```

3.670 $\int \frac{(a+bx)^3}{\sqrt[3]{x}} dx$

Optimal. Leaf size=51

$$\frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}$$

[Out] 3/2*a^3*x^(2/3)+9/5*a^2*b*x^(5/3)+9/8*a*b^2*x^(8/3)+3/11*b^3*x^(11/3)

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3/x^(1/3),x]

[Out] (3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(1
1/3))/11

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^3}{\sqrt[3]{x}}+3a^2bx^{2/3}+3ab^2x^{5/3}+b^3x^{8/3}\right)dx\\ &= \frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.76

$$\frac{3}{440}x^{2/3}\left(220a^3+264a^2bx+165ab^2x^2+40b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(220*a^3 + 264*a^2*b*x + 165*a*b^2*x^2 + 40*b^3*x^3))/440
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 32.86, size = 3003, normalized size = 58.88

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^3/x^(1/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (2 / 3) (-81 -1 ^ (2 / 3) a ^ 3 + 220 a ^ 3 (b x / a) ^ (
2 / 3) + 264 a ^ 2 b x (b x / a) ^ (2 / 3) + 165 a b ^ 2 x ^ 2 (b x / a) ^
(2 / 3) + 40 b ^ 3 x ^ 3 (b x / a) ^ (2 / 3)) / (440 b ^ (2 / 3)), Abs[(a +
 b x) / a] > 1}}, -243 a ^ (71 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) / (44
0 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3
) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 88
00 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3)
 a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 /
 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^
6) + 243 a ^ (71 / 3) / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (
I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (
8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x
) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^
(I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14
b ^ (20 / 3) (a / b + x) ^ 6) - 1458 a ^ (68 / 3) b (a / b + x) / (440 E ^
(I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a /
 b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^
 (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 1
6 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a
 / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) + 1
296 a ^ (68 / 3) b (a / b + x) (1 - b (a / b + x) / a) ^ (2 / 3) / (440 E ^
 (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a
/ b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E
^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^
16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (
a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) -
2808 a ^ (65 / 3) b ^ 2 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 2 /
 (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5
 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2
- 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi
/ 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (
17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x
) ^ 6) + 3645 a ^ (65 / 3) b ^ 2 (a / b + x) ^ 2 / (440 E ^ (I Pi / 3) a ^
20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600
```

E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) - 4860 a ^ (62 / 3) b ^ 3 (a / b + x) ^ 3 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 1 4 b ^ (20 / 3) (a / b + x) ^ 6) + 3120 a ^ (62 / 3) b ^ 3 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 3 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) - 1710 a ^ (59 / 3) b ^ 4 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 4 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) + 3645 a ^ (59 / 3) b ^ 4 (a / b + x) ^ 4 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) - 1458 a ^ (56 / 3) b ^ 5 (a / b + x) ^ 5 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) - 72 a ^ (56 / 3) b ^ 5 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 5 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) + 243 a ^ (53 / 3) b ^ 6 (a / b + x) ^ 6 / (440 E ^ (I Pi / 3) a ^ 2 0 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) + 1104 a ^ (53 / 3) b ^ 6 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 6 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) - 1152 a ^ (50 / 3) b ^ 7 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 7 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) + 585 a ^ (47 / 3) b ^ 8 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 8 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 19 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6) - 120 a ^ (44 / 3) b ^ 9 (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 9 / (440 E ^ (I Pi / 3) a ^ 20 b ^ (2 / 3) - 2640 E ^ (I Pi / 3) a ^ 1 9 b ^ (5 / 3) (a / b + x) + 6600 E ^ (I Pi / 3) a ^ 18 b ^ (8 / 3) (a / b + x) ^ 2 - 8800 E ^ (I Pi / 3) a ^ 17 b ^ (11 / 3) (a / b + x) ^ 3 + 6600 E ^ (I Pi / 3) a ^ 16 b ^ (14 / 3) (a / b + x) ^ 4 - 2640 E ^ (I Pi / 3) a ^ 15 b ^ (17 / 3) (a / b + x) ^ 5 + 440 E ^ (I Pi / 3) a ^ 14 b ^ (20 / 3) (a / b + x) ^ 6)]

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
trager	$\left(\frac{3}{11}b^{3}x^{3}+\frac{9}{8}a\,b^{2}x^{2}+\frac{9}{5}a^{2}bx+\frac{3}{2}a^{3}\right)x^{\frac{2}{3}}$	35
gosper	$\frac{3x^{\frac{2}{3}}\left(40b^{3}x^{3}+165a\,b^{2}x^{2}+264a^{2}bx+220a^{3}\right)}{440}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{2}{3}}}{2}+\frac{9a^{2}b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^{2}x^{\frac{8}{3}}}{8}+\frac{3b^{3}x^{\frac{11}{3}}}{11}$	36
default	$\frac{3a^{3}x^{\frac{2}{3}}}{2}+\frac{9a^{2}b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^{2}x^{\frac{8}{3}}}{8}+\frac{3b^{3}x^{\frac{11}{3}}}{11}$	36
risch	$\frac{3x^{\frac{2}{3}}\left(40b^{3}x^{3}+165a\,b^{2}x^{2}+264a^{2}bx+220a^{3}\right)}{440}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/x^(1/3),x,method=_RETURNVERBOSE)

[Out] 3/2*a^3*x^(2/3)+9/5*a^2*b*x^(5/3)+9/8*a*b^2*x^(8/3)+3/11*b^3*x^(11/3)

Maxima [A]

time = 0.27, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^3x^{\frac{11}{3}}+\frac{9}{8}\,ab^2x^{\frac{8}{3}}+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+\frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.69

$$\frac{3}{440}\left(40\,b^3x^3+165\,ab^2x^2+264\,a^2bx+220\,a^3\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 165*a*b^2*x^2 + 264*a^2*b*x + 220*a^3)*x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.57, size = 6246, normalized size = 122.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(440*a**20
*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a*
*18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*e
xp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**
(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi
/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1296*a**(68/3)*b*(-1 + b*(a/b + x)/a)
**(2/3)*(a/b + x)*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(
440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*
```

a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6
*exp(I*pi/3)) + 2808*a**(65/3)*b**2*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**
2*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b +
x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*
b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(
I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/
3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)) - 3120*a**(62/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I*pi
/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*p
i/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*
(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
x)**6*exp(I*pi/3)) - 4860*a**(62/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*e
xp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3
)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3)
+ 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b
 + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1710
*a**(59/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(I*pi/3)/(440*a
20*b(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3)
 - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x
)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**
16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*e
xp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 72*a**(56/3)*b
5*(-1 + b*(a/b + x)/a)(2/3)*(a/b + x)**5*exp(I*pi/3)/(440*a**20*b**(2/3
)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(
8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/
3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(
a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1
458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**1
9*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*p
i/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)
*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) +
 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*(-1 +
b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi
/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*
a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**

5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a**(53/
3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*(-1 + b*(a/b + x)
/a)**(2/3)*(a/b + x)**7*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*
a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp
(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(1
4/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/
3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*(-1
 + b*(a/b + x)/a)**(2/3)*(a/b + x)**8*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 120*a**(
44/3)*b**9*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**9*exp(I*pi/3)/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)), Abs(b*(a/b + x)/a) > 1), (-243*a**(71/3)*(1 - b*(a/b + x)/a)**(2/3)/(4
40*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) +
 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b +
 x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a
15*b(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*
exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b
(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a18*b**(8/3)*(a/b + x)**2*exp(I*pi/3
) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 44
0*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1296*a**(68/3)*b*(1 - b*(a/b
+ x)/a)**(2/3)*(a/b + x)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5
/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b +
 x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a*
*14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(440*a
20*b(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) - 2808*a**(65/3)*b**2*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(440
*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6
600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x
)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**
15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*ex

p(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*b**(2/3)*exp(I*pi/
3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b +
x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a
16*b(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5
*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3120*a**(62/
3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*p
i/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600
*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)*
*5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 4860*a**(6
2/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3
)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 88
00*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x
)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**1
4*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*(1 - b*(a/b + x
)/a)**(2/3)*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5
/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b +
x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a*
*14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/
(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640
*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**
6*exp(I*pi/3)) - 72*a**(56/3)*b**5*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**5/
(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640
*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**
6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1104*a**
(53/3)*b**6*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(440*a**20*b**(2/3)*exp
(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*
(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) +
6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b +
x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a*
*(53/3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(
5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b
+ x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a
14*b(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1152*a**(50/3)*b**7*(1 - b*(a/b
+ x)/a)**(2/3)*(a/b + x)**7/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b*

```
*(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3)
 - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/
b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440
*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 585*a**(47/3)*b**8*(1 - b*(a/b
 + x)/a)**(2/3)*(a/b + x)**8/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b
**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3
) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 44
0*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*(1 - b*(a/
b + x)/a)**(2/3)*(a/b + x)**9/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 68, normalized size = 1.33

$$\frac{3}{11}b^{3}\left(x^{\frac{1}{3}}\right)^{2}x^{3}+\frac{9ab^{2}\left(x^{\frac{1}{3}}\right)^{2}x^{2}}{2\cdot 4}+\frac{9}{5}a^{2}b\left(x^{\frac{1}{3}}\right)^{2}x+\frac{3}{2}a^{3}\left(x^{\frac{1}{3}}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x)
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{2/3}}{2}+\frac{3\,b^3\,x^{11/3}}{11}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/3),x)
[Out] (3*a^3*x^(2/3))/2 + (3*b^3*x^(11/3))/11 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(8/3))/8
```

3.671 $\int \frac{(a+bx)^3}{x^{2/3}} \, dx$

Optimal. Leaf size=49

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

```
[Out] 3*a^3*x^(1/3)+9/4*a^2*b*x^(4/3)+9/7*a*b^2*x^(7/3)+3/10*b^3*x^(10/3)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(2/3),x]
```

```
[Out] 3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3)
)/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{2/3}}\,dx &= \int \left(\frac{a^3}{x^{2/3}}+3a^2b\sqrt[3]{x}+3ab^2x^{4/3}+b^3x^{7/3}\right)dx\\ &= 3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.80

$$\frac{3}{140}\sqrt[3]{x}\left(140a^3+105a^2bx+60ab^2x^2+14b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(140*a^3 + 105*a^2*b*x + 60*a*b^2*x^2 + 14*b^3*x^3))/140
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 33.41, size = 3003, normalized size = 61.29

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^3/x^(2/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (1 / 3) (-81 -1 ^ (1 / 3) a ^ 3 + 140 a ^ 3 (b x / a) ^ (
1 / 3) + 105 a ^ 2 b x (b x / a) ^ (1 / 3) + 60 a b ^ 2 x ^ 2 (b x / a) ^ (
1 / 3) + 14 b ^ 3 x ^ 3 (b x / a) ^ (1 / 3)) / (140 b ^ (1 / 3)), Abs[(a +
b x) / a] > 1}}, -243 a ^ (70 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) / (140
 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 /
 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2
- 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2
 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15
b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a
/ b + x) ^ 6) + 243 a ^ (70 / 3) / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3)
 - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi /
 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (
10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b
+ x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E
 ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) - 1458 a ^ (67 / 3) b
(a / b + x) / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi /
3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3
) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^
 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I
 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 1
4 b ^ (19 / 3) (a / b + x) ^ 6) + 1377 a ^ (67 / 3) b (a / b + x) (1 - b (a
 / b + x) / a) ^ (1 / 3) / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E
 ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^
18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3)
(a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4
 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2
Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) - 3213 a ^ (64 / 3) b ^ 2 (1 -
 b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 2 / (140 E ^ (I 2 Pi / 3) a ^ 2
0 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100
E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3)
 a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13
 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x
) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) + 3645 a
^ (64 / 3) b ^ 2 (a / b + x) ^ 2 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3)
```

- 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) - 4860 a ^ (61 / 3) b ^ 3 (a / b + x) ^ 3 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) + 3927 a ^ (61 / 3) b ^ 3 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 3 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) - 2583 a ^ (58 / 3) b ^ 4 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 4 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) + 3645 a ^ (58 / 3) b ^ 4 (a / b + x) ^ 4 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) - 1458 a ^ (55 / 3) b ^ 5 (a / b + x) ^ 5 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) + 693 a ^ (55 / 3) b ^ 5 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 5 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) + 243 a ^ (52 / 3) b ^ 6 (a / b + x) ^ 6 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3

```
) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^
5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) + 273 a ^ (52
 / 3) b ^ 6 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 6 / (140 E ^ (I
 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a
/ b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800
E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3
) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16
 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x
) ^ 6) - 387 a ^ (49 / 3) b ^ 7 (1 - b (a / b + x) / a) ^ (1 / 3) (a / b +
x) ^ 7 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a
^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a
/ b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 +
2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi
 / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^
 (19 / 3) (a / b + x) ^ 6) + 198 a ^ (46 / 3) b ^ 8 (1 - b (a / b + x) / a)
 ^ (1 / 3) (a / b + x) ^ 8 / (140 E ^ (I 2 Pi / 3) a ^ 20 b ^ (1 / 3) - 840
 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100 E ^ (I 2 Pi / 3) a
^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3) a ^ 17 b ^ (10 / 3
) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13 / 3) (a / b + x) ^
 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x) ^ 5 + 140 E ^ (I
2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6) - 42 a ^ (43 / 3) b ^ 9 (1 -
 b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 9 / (140 E ^ (I 2 Pi / 3) a ^ 2
0 b ^ (1 / 3) - 840 E ^ (I 2 Pi / 3) a ^ 19 b ^ (4 / 3) (a / b + x) + 2100
E ^ (I 2 Pi / 3) a ^ 18 b ^ (7 / 3) (a / b + x) ^ 2 - 2800 E ^ (I 2 Pi / 3)
 a ^ 17 b ^ (10 / 3) (a / b + x) ^ 3 + 2100 E ^ (I 2 Pi / 3) a ^ 16 b ^ (13
 / 3) (a / b + x) ^ 4 - 840 E ^ (I 2 Pi / 3) a ^ 15 b ^ (16 / 3) (a / b + x
) ^ 5 + 140 E ^ (I 2 Pi / 3) a ^ 14 b ^ (19 / 3) (a / b + x) ^ 6)]
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.73

method	result	size
trager	$\left(\frac{3}{10}b^3x^3+\frac{9}{7}a\,b^2x^2+\frac{9}{4}a^2bx+3a^3\right)x^{\frac{1}{3}}$	35
gosper	$\frac{3x^{\frac{1}{3}}\left(14b^3x^3+60a\,b^2x^2+105a^2bx+140a^3\right)}{140}$	36
derivativedivides	$3a^3x^{\frac{1}{3}}+\frac{9a^2b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{3b^3x^{\frac{10}{3}}}{10}$	36
default	$3a^3x^{\frac{1}{3}}+\frac{9a^2b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{3b^3x^{\frac{10}{3}}}{10}$	36
risch	$\frac{3x^{\frac{1}{3}}\left(14b^3x^3+60a\,b^2x^2+105a^2bx+140a^3\right)}{140}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(2/3),x,method=_RETURNVERBOSE)
[Out] 3*a^3*x^(1/3)+9/4*a^2*b*x^(4/3)+9/7*a*b^2*x^(7/3)+3/10*b^3*x^(10/3)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.71

$$\frac{3}{10}\,b^3x^{\frac{10}{3}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+3\,a^3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.71

$$\frac{3}{140}\left(14\,b^3x^3+60\,ab^2x^2+105\,a^2bx+140\,a^3\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 + 60*a*b^2*x^2 + 105*a^2*b*x + 140*a^3)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.58, size = 6667, normalized size = 136.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(2/3),x)
```

```
[Out] Piecewise((243*a**(70/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*
a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*e
xp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16
*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*ex
p(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1377*a**(67
/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(140*a**20*b**(1/
3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*
b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp
(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b*
*(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2
*I*pi/3)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)*
```

*2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a
16*b(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**
5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3213*a*
*(64/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(140*a*
*20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2
100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
 x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 84
0*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x
)**6*exp(2*I*pi/3)) + 3645*a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 3927*a**(61/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2
*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*e
xp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*
b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*ex
p(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b*
(19/3)(a/b + x)**6*exp(2*I*pi/3)) - 4860*a**(61/3)*b**3*(a/b + x)**3/(140
*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3)
 + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/
b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b
 + x)**6*exp(2*I*pi/3)) + 2583*a**(58/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a
/b + x)**4*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(
4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/
3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)
*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3
) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*a**(58/3)*b**4*(
a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x
)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**
17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4
*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14
*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 693*a**(55/3)*b**5*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**5*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)
2*exp(2*I*pi/3) - 2800*a17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*
a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)*
*5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1458*a
(55/3)*b5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
(4/3)(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 273*a**(52/3)*b**6*
(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(140*a**20*b**(1/3)*

exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) + 243*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
5*exp(2*I*pi/3) + 140*a14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 387*a
(49/3)*b7*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(2*I*pi/3)/(140*a
20*b(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
+ x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 8
40*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
x)**6*exp(2*I*pi/3)) - 198*a**(46/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**8*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3
)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)
- 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a
/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) +
140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 42*a**(43/3)*b**9*(-1 +
b*(a/b + x)/a)**(1/3)*(a/b + x)**9*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*
I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*
(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3
) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(
a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
, Abs(b*(a/b + x)/a) > 1), (-243*a**(70/3)*(1 - b*(a/b + x)/a)**(1/3)/(140*
a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
+ x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
x)**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 8
40*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**
2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a*
*16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5
*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 1377*a**
(67/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*p
i/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/
b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) +
2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b
+ x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) -
1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
(4/3)(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 3213*a**(64/3)*b**2

(1 - b(a/b + x)/a)**(1/3)*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
5*exp(2*I*pi/3) + 140*a14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*
a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b
(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a18*b**(7/3)*(a/b + x)**2*exp(2*I*
pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13
/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*p
i/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3927*a**(61/3)*b**
3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3)
 - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 210
0*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x
)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860
*a**(61/3)*b**3*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*
b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I
*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(1
3/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*
pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 2583*a**(58/3)*b*
4(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 364
5*a**(58/3)*b**4*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19
*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*
I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(
13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I
*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 693*a**(55/3)*b*
5(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 145
8*a**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19
*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*
I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(
13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I
*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 273*a**(52/3)*b*
6(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 243

```
*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*
b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I
*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(1
3/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*
pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 387*a**(49/3)*b**
7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(140*a**20*b**(1/3)*exp(2*I*pi/3)
 - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 210
0*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x
)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 198*
a**(46/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**8/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 42*a**(43/3)*b**9*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**9/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)), True))
```

Giac [A]
time = 0.00, size = 56, normalized size = 1.14

$$\frac{3}{10}b^3x^{\frac{1}{3}}x^3 + \frac{9}{7}ab^2x^{\frac{1}{3}}x^2 + \frac{9}{4}a^2bx^{\frac{1}{3}}x + 3a^3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x)
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$3\,a^3\,x^{1/3}+\frac{3\,b^3\,x^{10/3}}{10}+\frac{9\,a^2\,b\,x^{4/3}}{4}+\frac{9\,a\,b^2\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(2/3),x)
[Out] 3*a^3*x^(1/3) + (3*b^3*x^(10/3))/10 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3
))/7
```

3.672 $\int \frac{(a+bx)^3}{x^{4/3}} dx$

Optimal. Leaf size=49

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

```
[Out] -3*a^3/x^(1/3)+9/2*a^2*b*x^(2/3)+9/5*a*b^2*x^(5/3)+3/8*b^3*x^(8/3)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8/
3))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{4/3}}\,dx &= \int \left(\frac{a^3}{x^{4/3}}+\frac{3a^2b}{\sqrt[3]{x}}+3ab^2x^{2/3}+b^3x^{5/3}\right)dx\\ &= -\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$-\frac{3\left(40a^3-60a^2bx-24ab^2x^2-5b^3x^3\right)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*(40*a^3 - 60*a^2*b*x - 24*a*b^2*x^2 - 5*b^3*x^3))/(40*x^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 23.09, size = 2278, normalized size = 46.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^3/x^(4/3),x]')
```

```
[Out] Piecewise[{{3 a ^ (2 / 3) (-40 a ^ 3 (b x / a) ^ (2 / 3) - 81 -1 ^ (2 / 3)
a ^ 2 b x + 60 a ^ 2 b x (b x / a) ^ (2 / 3) + 24 a b ^ 2 x ^ 2 (b x / a) ^
 (2 / 3) + 5 b ^ 3 x ^ 3 (b x / a) ^ (2 / 3)) / (40 b ^ (2 / 3) x), Abs[(a
+ b x) / a] > 1}}, -243 E ^ (I 2 Pi / 3) a ^ (68 / 3) b ^ (1 / 3) / (40 a ^
 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^
 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b
 ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 243 E ^ (I 2 Pi /
 3) a ^ (68 / 3) b ^ (1 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) / (40 a ^ 20
 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17
 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^
5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 1296 E ^ (I 2 Pi / 3
) a ^ (65 / 3) b ^ (4 / 3) (a / b + x) (1 - b (a / b + x) / a) ^ (2 / 3) /
(40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 -
800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a
 ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 1458 E ^ (
I 2 Pi / 3) a ^ (65 / 3) b ^ (4 / 3) (a / b + x) / (40 a ^ 20 - 240 a ^ 19
b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b
+ x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x)
^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 3645 E ^ (I 2 Pi / 3) a ^ (62 / 3)
 b ^ (7 / 3) (a / b + x) ^ 2 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600
a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 1
6 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^
6 (a / b + x) ^ 6) + 2808 E ^ (I 2 Pi / 3) a ^ (62 / 3) b ^ (7 / 3) (1 - b
(a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 2 / (40 a ^ 20 - 240 a ^ 19 b (a /
 b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^
 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 +
40 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 3120 E ^ (I 2 Pi / 3) a ^ (59 / 3) b ^ (
10 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 3 / (40 a ^ 20 - 24
0 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^
3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a
/ b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 4860 E ^ (I 2 Pi / 3) a ^
 (59 / 3) b ^ (10 / 3) (a / b + x) ^ 3 / (40 a ^ 20 - 240 a ^ 19 b (a / b +
```

x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 3645 E ^ (I 2 Pi / 3) a ^ (56 / 3) b ^ (13 / 3) (a / b + x) ^ 4 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 1830 E ^ (I 2 Pi / 3) a ^ (56 / 3) b ^ (13 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 4 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 1 4 b ^ 6 (a / b + x) ^ 6) - 528 E ^ (I 2 Pi / 3) a ^ (53 / 3) b ^ (16 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 5 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 1458 E ^ (I 2 Pi / 3) a ^ (53 / 3) b ^ (16 / 3) (a / b + x) ^ 5 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 243 E ^ (I 2 Pi / 3) a ^ (50 / 3) b ^ (19 / 3) (a / b + x) ^ 6 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 96 E ^ (I 2 Pi / 3) a ^ (50 / 3) b ^ (19 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 6 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 48 E ^ (I 2 Pi / 3) a ^ (47 / 3) b ^ (22 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 7 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 60 0 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 15 E ^ (I 2 Pi / 3) a ^ (44 / 3) b ^ (25 / 3) (1 - b (a / b + x) / a) ^ (2 / 3) (a / b + x) ^ 8 / (40 a ^ 20 - 240 a ^ 19 b (a / b + x) + 600 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 800 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 600 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 240 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 40 a ^ 14 b ^ 6 (a / b + x) ^ 6)]

Maple [A]
time = 0.10, size = 36, normalized size = 0.73

method	result	size
gosper	$-\frac{3\left(-5 b^{3} x^{3}-24 a\,b^{2} x^{2}-60 a^{2} b x +40 a^{3}\right)}{40 x^{\frac{1}{3}}}$	36
derivativedivides	$-\frac{3 a^{3}}{x^{\frac{1}{3}}}+\frac{9 a^{2} b\,x^{\frac{2}{3}}}{2}+\frac{9 a\,b^{2} x^{\frac{5}{3}}}{5}+\frac{3 b^{3} x^{\frac{8}{3}}}{8}$	36
default	$-\frac{3 a^{3}}{x^{\frac{1}{3}}}+\frac{9 a^{2} b\,x^{\frac{2}{3}}}{2}+\frac{9 a\,b^{2} x^{\frac{5}{3}}}{5}+\frac{3 b^{3} x^{\frac{8}{3}}}{8}$	36

trager	$-\frac{3(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3)}{40x^{\frac{1}{3}}}$	36
risch	$-\frac{3(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3)}{40x^{\frac{1}{3}}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(4/3),x,method=_RETURNVERBOSE)
[Out] -3*a^3/x^(1/3)+9/2*a^2*b*x^(2/3)+9/5*a*b^2*x^(5/3)+3/8*b^3*x^(8/3)
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="maxima")
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Fricas [A]
time = 0.31, size = 35, normalized size = 0.71

$$\frac{3\,(5\,b^3x^3+24\,ab^2x^2+60\,a^2bx-40\,a^3)}{40\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="fricas")
[Out] 3/40*(5*b^3*x^3 + 24*a*b^2*x^2 + 60*a^2*b*x - 40*a^3)/x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.63, size = 4004, normalized size = 81.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(4/3),x)
[Out] Piecewise((243*a**(68/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)/(40*a**20 - 2
40*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b +
x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**1
4*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
```

/b + x)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b +
x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**
15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(62/3)*b**(7/3
)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(40*a**20 - 240*a**19*b*(a/b + x
) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b
4*(a/b + x)4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6
) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**1
9*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*
(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b +
 x)**3/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 80
0*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a
/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b
 + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*
a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(
13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(40*a**20 - 240*a**19*b*(a/b
 + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**
16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x
)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
3 + 600*a16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**5/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*
(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 96*a**(50/3)*b*
(19/3)(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(40*a**20 - 240*a**19*b*(a
/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a
16*b4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
 x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**7/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(-1 +
 b*(a/b + x)/a)**(2/3)*(a/b + x)**8/(40*a**20 - 240*a**19*b*(a/b + x) + 600
*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/
b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), Abs(

b*(a/b + x)/a) > 1), (243*a**(68/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/3)*exp
(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2
*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 -
800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*
(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(1 - b
*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b +
x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16
*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)*
*6) + 1458*a**(65/3)*b**(4/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19
b(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(
a/b + x)**6) + 2808*a**(62/3)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)
2*exp(2*I*pi/3)/(40*a20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a*
*15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3645*a**(62/3)*b**(7/
3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18
*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)
4 - 240*a15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3120*a**(
59/3)*b**(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(2*I*pi/3)/(40*a
20 - 240*a19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3
*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 +
40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(13/3)*(1 - b
*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/
b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a*
*16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
x)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(1 - b*(a/b + x)/a)**(2/3)*(a
/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2
*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)
*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 6
00*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(
a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 9
6*a**(50/3)*b**(19/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(2*I*pi/3)
/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**1
7*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x
)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*e
xp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**

```
2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b
**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(1
- b*(a/b + x)/a)**(2/3)*(a/b + x)**7*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*
(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600
*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b
 + x)**6) + 15*a**(44/3)*b**(25/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**8*
exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)*
*2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*
b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), True))
```

Giac [A]
time = 0.00, size = 61, normalized size = 1.24

$$\frac{3}{8}\left(x^{\frac{1}{3}}\right)^{2}x^{2}b^{3}+\frac{9}{5}\left(x^{\frac{1}{3}}\right)^{2}xb^{2}a+\frac{9}{2}\left(x^{\frac{1}{3}}\right)^{2}ba^{2}-\frac{3a^{3}}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x)
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{8/3}}{8}-\frac{3\,a^3}{x^{1/3}}+\frac{9\,a^2\,b\,x^{2/3}}{2}+\frac{9\,a\,b^2\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(4/3),x)
[Out] (3*b^3*x^(8/3))/8 - (3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3
))/5
```

3.673 $\int \frac{(a+bx)^3}{x^{5/3}} dx$

Optimal. Leaf size=49

$$-\frac{3a^3}{2x^{2/3}} + 9a^2b\sqrt[3]{x} + \frac{9}{4}ab^2x^{4/3} + \frac{3}{7}b^3x^{7/3}$$

```
[Out] -3/2*a^3/x^(2/3)+9*a^2*b*x^(1/3)+9/4*a*b^2*x^(4/3)+3/7*b^3*x^(7/3)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3}{2x^{2/3}} + 9a^2b\sqrt[3]{x} + \frac{9}{4}ab^2x^{4/3} + \frac{3}{7}b^3x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7/
3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/3}} dx &= \int \left(\frac{a^3}{x^{5/3}} + \frac{3a^2b}{x^{2/3}} + 3ab^2\sqrt[3]{x} + b^3x^{4/3}\right) dx \\ &= -\frac{3a^3}{2x^{2/3}} + 9a^2b\sqrt[3]{x} + \frac{9}{4}ab^2x^{4/3} + \frac{3}{7}b^3x^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$-\frac{3(14a^3 - 84a^2bx - 21ab^2x^2 - 4b^3x^3)}{28x^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/x^(5/3),x]

[Out] (-3*(14*a^3 - 84*a^2*b*x - 21*a*b^2*x^2 - 4*b^3*x^3))/(28*x^(2/3))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 23.09, size = 2278, normalized size = 46.49

result too large to display

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^3/x^(5/3),x]')

[Out] Piecewise[{{3 a ^ (1 / 3) (-14 a ^ 3 (b x / a) ^ (1 / 3) - 81 -1 ^ (1 / 3) a ^ 2 b x + 84 a ^ 2 b x (b x / a) ^ (1 / 3) + 21 a b ^ 2 x ^ 2 (b x / a) ^ (1 / 3) + 4 b ^ 3 x ^ 3 (b x / a) ^ (1 / 3)) / (28 b ^ (1 / 3) x), Abs[(a + b x) / a] > 1}}, -243 E ^ (I Pi / 3) a ^ (67 / 3) b ^ (2 / 3) / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 243 E ^ (I Pi / 3) a ^ (67 / 3) b ^ (2 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 1377 E ^ (I Pi / 3) a ^ (64 / 3) b ^ (5 / 3) (a / b + x) (1 - b (a / b + x) / a) ^ (1 / 3) / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 1458 E ^ (I Pi / 3) a ^ (64 / 3) b ^ (5 / 3) (a / b + x) / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 3645 E ^ (I Pi / 3) a ^ (61 / 3) b ^ (8 / 3) (a / b + x) ^ 2 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 3213 E ^ (I Pi / 3) a ^ (61 / 3) b ^ (8 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 2 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 3927 E ^ (I Pi / 3) a ^ (58 / 3) b ^ (11 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 3 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 4860 E ^ (I Pi / 3) a ^ (58 / 3) b ^ (11 / 3) (a / b + x) ^ 3 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a /

b + x) ^ 6) - 3645 E ^ (I Pi / 3) a ^ (55 / 3) b ^ (14 / 3) (a / b + x) ^ 4 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 2625 E ^ (I Pi / 3) a ^ (55 / 3) b ^ (14 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 4 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 903 E ^ (I Pi / 3) a ^ (52 / 3) b ^ (17 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 5 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 1458 E ^ (I Pi / 3) a ^ (52 / 3) b ^ (17 / 3) (a / b + x) ^ 5 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 24 3 E ^ (I Pi / 3) a ^ (49 / 3) b ^ (20 / 3) (a / b + x) ^ 6 / (28 a ^ 20 - 1 68 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 147 E ^ (I Pi / 3) a ^ (49 / 3) b ^ (20 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 6 / (2 8 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 56 0 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) - 33 E ^ (I Pi / 3) a ^ (46 / 3) b ^ (23 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 7 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6) + 1 2 E ^ (I Pi / 3) a ^ (43 / 3) b ^ (26 / 3) (1 - b (a / b + x) / a) ^ (1 / 3) (a / b + x) ^ 8 / (28 a ^ 20 - 168 a ^ 19 b (a / b + x) + 420 a ^ 18 b ^ 2 (a / b + x) ^ 2 - 560 a ^ 17 b ^ 3 (a / b + x) ^ 3 + 420 a ^ 16 b ^ 4 (a / b + x) ^ 4 - 168 a ^ 15 b ^ 5 (a / b + x) ^ 5 + 28 a ^ 14 b ^ 6 (a / b + x) ^ 6)]

Maple [A]
time = 0.11, size = 36, normalized size = 0.73

method	result	size
gosper	$-\frac{3\left(-4b^{3}x^{3}-21a\,b^{2}x^{2}-84a^{2}bx+14a^{3}\right)}{28x^{\frac{2}{3}}}$	36
derivativedivides	$-\frac{3a^{3}}{2x^{\frac{2}{3}}}+9a^{2}b\,x^{\frac{1}{3}}+\frac{9a\,b^{2}x^{\frac{4}{3}}}{4}+\frac{3b^{3}x^{\frac{7}{3}}}{7}$	36
default	$-\frac{3a^{3}}{2x^{\frac{2}{3}}}+9a^{2}b\,x^{\frac{1}{3}}+\frac{9a\,b^{2}x^{\frac{4}{3}}}{4}+\frac{3b^{3}x^{\frac{7}{3}}}{7}$	36
trager	$-\frac{3\left(-4b^{3}x^{3}-21a\,b^{2}x^{2}-84a^{2}bx+14a^{3}\right)}{28x^{\frac{2}{3}}}$	36

risch	$-\frac{3(-4b^3x^3-21a\,b^2x^2-84a^2bx+14a^3)}{28x^{\frac{2}{3}}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a^3/x^(2/3)+9*a^2*b*x^(1/3)+9/4*a*b^2*x^(4/3)+3/7*b^3*x^(7/3)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Fricas [A]
time = 0.31, size = 35, normalized size = 0.71

$$\frac{3\,(4\,b^3x^3+21\,ab^2x^2+84\,a^2bx-14\,a^3)}{28\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^3*x^3 + 21*a*b^2*x^2 + 84*a^2*b*x - 14*a^3)/x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.63, size = 3964, normalized size = 80.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/3),x)
```

```
[Out] Piecewise((243*a**(67/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)/(28*a**20 - 1
68*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b +
x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**1
4*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 168*a
**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**
3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b*
*6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
 + x)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560
*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/
```

b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)*(a/b + x)
*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**
2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b
5*(a/b + x)5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3)*b**(8/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a
/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a
16*b4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
 x)**6) - 3927*a**(58/3)*b**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
5(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(55/3)*b**(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(
a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*
a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b
 + x)**6) - 903*a**(52/3)*b**(17/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
5(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(2
8*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b
3*(a/b + x)3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**
5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(-1 + b*(a/b + x)/
a)**(1/3)*(a/b + x)**8/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(
a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 1
68*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6), Abs(b*(a/b + x)/a
) > 1), (243*a**(67/3)*b**(2/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(28*
a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**

3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3927*a**(58/3)*b**(11/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3645*a**(55/3)*b**(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 903*a**(52/3)*b**(17/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 243*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 33*a**(46/3)*b**(23/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp

```
(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 -
560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*
(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(1 - b*
(a/b + x)/a)**(1/3)*(a/b + x)**8*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b +
 x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16
*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)*
*6), True))
```

Giac [A]
time = 0.00, size = 58, normalized size = 1.18

$$\frac{3}{7}x^{\frac{1}{3}}x^2b^3+\frac{9}{4}x^{\frac{1}{3}}xb^2a+9x^{\frac{1}{3}}ba^2-\frac{\frac{1}{2}\cdot 3a^3}{\left(x^{\frac{1}{3}}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x)
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{7/3}}{7}-\frac{3\,a^3}{2\,x^{2/3}}+9\,a^2\,b\,x^{1/3}+\frac{9\,a\,b^2\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/3),x)
[Out] (3*b^3*x^(7/3))/7 - (3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3
))/4
```

3.674 $\int \frac{x^{5/3}}{a+bx}\,dx$

Optimal. Leaf size=125

$$-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}$$

```
[Out] -3/2*a*x^(2/3)/b^2+3/5*x^(5/3)/b-3/2*a^(5/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^
(8/3)+1/2*a^(5/3)*ln(b*x+a)/b^(8/3)-a^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/b^(8/3)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 58, 631, 210, 31}

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(2/3))/(2*b^2) + (3*x^(5/3))/(5*b) - (Sqrt[3]*a^(5/3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(8/3) - (3*a^(5/3)*Log[a^(1/3
) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) + (a^(5/3)*Log[a + b*x])/(2*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{3x^{5/3}}{5b} - \frac{a\int \frac{x^{2/3}}{a+bx}\,dx}{b}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b^2}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^3} - \frac{(3a^{5/3}}{}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{(3a^{5/3})\,\text{Subst}\left(\int \frac{1}{-3-}\right.}{b}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log}{2}$$

Mathematica [A]

time = 0.09, size = 140, normalized size = 1.12

$$\frac{-15ab^{2/3}x^{2/3} + 6b^{5/3}x^{5/3} - 10\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) + 5a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{10b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x),x]
```

```
[Out] (-15*a*b^(2/3)*x^(2/3) + 6*b^(5/3)*x^(5/3) - 10*Sqrt[3]*a^(5/3)*ArcTan[(1 -
 (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 10*a^(5/3)*Log[a^(1/3) + b^(1/3)*x
^(1/3)] + 5*a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)
])/(10*b^(8/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 45.16, size = 173, normalized size = 1.38

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{5}{3}}\right], a==0\&\&b==0\right\}, \left\{\frac{3x^{\frac{8}{3}}}{8a}, b==0\right\}, \left\{\frac{3x^{\frac{5}{3}}}{5b}, a==0\right\}\right\}, -\frac{a^2\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,a^2\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^2\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3ax^{\frac{2}{3}}}{2b^2}+\frac{3x^{\frac{5}{3}}}{5b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/3)/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (5 / 3)], a == 0 && b == 0}, {3 x ^ (8 / 3
) / (8 a), b == 0}, {3 x ^ (5 / 3) / (5 b), a == 0}}, -a ^ 2 Log[4 x ^ (1 /
 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (2 b ^ 3 (
-a / b) ^ (1 / 3)) + Sqrt[3] a ^ 2 ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 /
3) / (3 (-a / b) ^ (1 / 3))] / (b ^ 3 (-a / b) ^ (1 / 3)) + a ^ 2 Log[x ^ (
1 / 3) - (-a / b) ^ (1 / 3)] / (b ^ 3 (-a / b) ^ (1 / 3)) - 3 a x ^ (2 / 3)
 / (2 b ^ 2) + 3 x ^ (5 / 3) / (5 b)]
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.99

method	result
risch	$-\frac{3(-2bx+5a)x^{\frac{2}{3}}}{10b^2}-\frac{a^2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$-\frac{3\left(-\frac{b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{\frac{2}{3}}}{2}\right)}{b^2}+\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^2}{b^2}$

default	$-\frac{3\left(-\frac{b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{\frac{2}{3}}}{2}\right)}{b^{2}}+\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^{2}}{b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b^2*(-1/5*b*x^(5/3)+1/2*a*x^(2/3))+3*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b
)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a^2/b^
2
```

Maxima [A]
time = 0.35, size = 130, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\left(2\,bx^{\frac{5}{3}}-5\,ax^{\frac{2}{3}}\right)}{10\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(1/3)) + 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(1/3)) - a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/10*(2
*b*x^(5/3) - 5*a*x^(2/3))/b^2
```

Fricas [A]
time = 0.32, size = 147, normalized size = 1.18

$$\frac{10\,\sqrt{3}\,a\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3\,a}\right)-5\,a\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}-a\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)+10\,a\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(b\left(-\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)+3\,(2\,bx-5\,a)x^{\frac{2}{3}}}{10\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/10*(10*sqrt(3)*a*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a^2/b
^2)^(1/3) + sqrt(3)*a)/a) - 5*a*(-a^2/b^2)^(1/3)*log(-b*x^(1/3)*(-a^2/b^2)^
```

```
(2/3) + a*x^(2/3) - a*(-a^2/b^2)^(1/3)) + 10*a*(-a^2/b^2)^(1/3)*log(b*(-a^2
/b^2)^(2/3) + a*x^(1/3)) + 3*(2*b*x - 5*a)*x^(2/3))/b^2
```

Sympy [A]
time = 46.95, size = 180, normalized size = 1.44

$$\begin{cases} \tilde{\infty} x^{\frac{5}{3}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{3x^{\frac{8}{3}}}{8a} & \text{for}\ b = 0 \\ \frac{3x^{\frac{5}{3}}}{5b} & \text{for}\ a = 0 \\ \frac{a^2 \log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{b^3 \sqrt[3]{-\frac{a}{b}}} - \frac{a^2 \log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^3 \sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}\, a^2 \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{b^3 \sqrt[3]{-\frac{a}{b}}} - \frac{3ax^{\frac{2}{3}}}{2b^2} + \frac{3x^{\frac{5}{3}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a), Eq(b, 0))
, (3*x**(5/3)/(5*b), Eq(a, 0)), (a**2*log(x**(1/3) - (-a/b)**(1/3))/(b**3*(
-a/b)**(1/3)) - a**2*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**
(2/3))/(2*b**3*(-a/b)**(1/3)) + sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)/(3*(-a
/b)**(1/3)) + sqrt(3)/3)/(b**3*(-a/b)**(1/3)) - 3*a*x**(2/3)/(2*b**2) + 3*x
**(5/3)/(5*b), True))
```

Giac [A]
time = 0.01, size = 214, normalized size = 1.71

$$3\left(\frac{a\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2 \ln\left(\left(x^{\frac{1}{3}}\right)^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^4} - \frac{a\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2 \arctan\left(\frac{2\left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\, b^4} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}} b^3 a^2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3b^5 a} + \frac{\frac{1}{5}\left(x^{\frac{1}{3}}\right)^2 x b^4 - \frac{1}{2}\left(x^{\frac{1}{3}}\right)^2 b^3 a}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x)
```

```
[Out] -a*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - sqrt(3)*(-a*b^2)^(2/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/2*
(-a*b^2)^(2/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 3
/10*(2*b^4*x^(5/3) - 5*a*b^3*x^(2/3))/b^5
```

Mupad [B]
time = 0.24, size = 151, normalized size = 1.21

$$\frac{3\,x^{5/3}}{5\,b} + \frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3} - \frac{9\,{\left(-a\right)}^{13/3}}{b^{10/3}}\right)}{b^{8/3}} - \frac{3\,a\,x^{2/3}}{2\,b^2} + \frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3} - \frac{9\,{\left(-a\right)}^{13/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{8/3}} - \frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3} - \frac{9\,{\left(-a\right)}^{13/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x),x)
```

```
[Out] (3*x^(5/3))/(5*b) + ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3))/b
^(10/3)))/b^(8/3) - (3*a*x^(2/3))/(2*b^2) + ((-a)^(5/3)*log((9*a^4*x^(1/3))
/b^3 - (9*(-a)^(13/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 -
 1/2))/b^(8/3) - ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3)*((3^(
1/2)*1i)/2 + 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 + 1/2))/b^(8/3)
```

3.675 $\int \frac{x^{4/3}}{a+bx}\,dx$

Optimal. Leaf size=123

$$-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}+\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}$$

```
[Out] -3*a*x^(1/3)/b^2+3/4*x^(4/3)/b+3/2*a^(4/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(7
/3)-1/2*a^(4/3)*ln(b*x+a)/b^(7/3)-a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(
1/3))/a^(1/3)*3^(1/2))*3^(1/2)/b^(7/3)
```

Rubi [A]

time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 60, 631, 210, 31}

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(4/3))/(4*b) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (3*a^(4/3)*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*b^(7/3)) - (a^(4/3)*Log[a + b*x])/(2*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{4/3}}{a+bx}\,dx &= \frac{3x^{4/3}}{4b}-\frac{a\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{b}\\
&= -\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}+\frac{a^2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b^2}\\
&= -\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}+\frac{\left(3a^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{(3a
\end{aligned}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}+\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}+\frac{\left(3a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-3-}\right.}{b}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}}+\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log}{2}$$

Mathematica [A]
time = 0.07, size = 140, normalized size = 1.14

$$\frac{-12a\sqrt[3]{b}\sqrt[3]{x}+3b^{4/3}x^{4/3}-4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)-2a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{4b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x),x]
```

```
[Out] (-12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x^(
1/3)] - 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])
/(4*b^(7/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 10.51, size = 168, normalized size = 1.37

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{4}{3}}\right], a==0\&\&b==0\right\}, \left\{\frac{3x^{\frac{7}{3}}}{7a}, b==0\right\}, \left\{\frac{3x^{\frac{4}{3}}}{4b}, a==0\right\}\right\}, -\frac{a\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{b^2}+\frac{a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2b^2}+\frac{\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{b^2}-\frac{3ax^{\frac{1}{3}}}{b^2}+\frac{3x^{\frac{4}{3}}}{4b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(4/3)/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (4 / 3)], a == 0 && b == 0}, {3 x ^ (7 / 3
) / (7 a), b == 0}, {3 x ^ (4 / 3) / (4 b), a == 0}}, -a Log[x ^ (1 / 3) -
(-a / b) ^ (1 / 3)] (-a / b) ^ (1 / 3) / b ^ 2 + a Log[4 x ^ (1 / 3) (-a /
b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b) ^ (1 / 3) / (
2 b ^ 2) + Sqrt[3] a ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a /
b) ^ (1 / 3))] (-a / b) ^ (1 / 3) / b ^ 2 - 3 a x ^ (1 / 3) / b ^ 2 + 3 x ^
 (4 / 3) / (4 b)]
```

Maple [A]
time = 0.17, size = 123, normalized size = 1.00

method	result	siz
derivativedivides	$-\frac{3\left(-\frac{b\,x^{\frac{4}{3}}}{4}+a\,x^{\frac{1}{3}}\right)}{b^2}+\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^2}{b^2}$	12

default	$-\frac{3\left(-\frac{b\,x^{\frac{4}{3}}}{4}+a\,x^{\frac{1}{3}}\right)}{b^{2}}+\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^{2}}{b^{2}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b^2*(-1/4*b*x^(4/3)+a*x^(1/3))+3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/
3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a^2/b^2
```

Maxima [A]
time = 0.35, size = 128, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\left(bx^{\frac{4}{3}}-4\,ax^{\frac{1}{3}}\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(2/3)) - 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(2/3)) + a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3/4*(b*
x^(4/3) - 4*a*x^(1/3))/b^2
```

Fricas [A]
time = 0.32, size = 116, normalized size = 0.94

$$\frac{4\,\sqrt{3}\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3\,(bx-4\,a)x^{\frac{1}{3}}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a/b)^(2/3) -
sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(
2/3)) + 4*a*(a/b)^(1/3)*log(x^(1/3) + (a/b)^(1/3)) + 3*(b*x - 4*a)*x^(1/3))
/b^2
```

Sympy [A]
time = 21.26, size = 173, normalized size = 1.41

$$\begin{cases} \tilde{\infty}x^{\frac{4}{3}} & \text{for } a=0\wedge b=0 \\ \frac{3x^{\frac{7}{3}}}{7a} & \text{for } b=0 \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{for } a=0 \\ -\frac{3a\sqrt[3]{x}}{b^2}-\frac{a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^2}+\frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^2}+\frac{\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b^2}+\frac{3x^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a), Eq(b, 0))
, (3*x**(4/3)/(4*b), Eq(a, 0)), (-3*a*x**(1/3)/b**2 - a*(-a/b)**(1/3)*log(x
**(1/3) - (-a/b)**(1/3))/b**2 + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)
*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b**2) + sqrt(3)*a*(-a/b)**(1/3)*atan(2
*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/b**2 + 3*x**(4/3)/(4*b), T
rue))
```

Giac [A]
time = 0.00, size = 194, normalized size = 1.58

$$3\left(\frac{a\left(-ab^2\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^3}+\frac{a\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^3}-\frac{b^2a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3b^4a}+\frac{\frac{1}{4}x^{\frac{1}{3}}xb^3-x^{\frac{1}{3}}b^2a}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x)
```

```
[Out] -a*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + sqrt(3)*(-a*b^2)^(1/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*
(-a*b^2)^(1/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 3
/4*(b^3*x^(4/3) - 4*a*b^2*x^(1/3))/b^4
```

Mupad [B]
time = 0.07, size = 126, normalized size = 1.02

$$\frac{3\,x^{4/3}}{4\,b}-\frac{3\,a\,x^{1/3}}{b^2}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{1/3}}+9\,a^2\,x^{1/3}\right)}{b^{7/3}}+\frac{a^{4/3}\,\ln\left(9\,a^2\,x^{1/3}+\frac{9\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{7/3}}-\frac{a^{4/3}\,\ln\left(9\,a^2\,x^{1/3}-\frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x),x)
```

```
[Out] (3*x^(4/3))/(4*b) - (3*a*x^(1/3))/b^2 + (a^(4/3)*log((9*a^(7/3))/b^(1/3) +
9*a^2*x^(1/3)))/b^(7/3) + (a^(4/3)*log(9*a^2*x^(1/3) + (9*a^(7/3)*((3^(1/2)
*1i)/2 - 1/2))/b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(7/3) - (a^(4/3)*log(9*a^
2*x^(1/3) - (9*a^(7/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1
/2))/b^(7/3)
```

3.676 $\int \frac{x^{2/3}}{a+bx} \, dx$

Optimal. Leaf size=111

$$\frac{3x^{2/3}}{2b}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

```
[Out] 3/2*x^(2/3)/b+3/2*a^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(5/3)-1/2*a^(2/3)*l
n(b*x+a)/b^(5/3)+a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(
1/2))*3^(1/2)/b^(5/3)
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 58, 631, 210, 31}

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(
Sqrt[3]*a^(1/3))])/b^(5/3) + (3*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*
b^(5/3)) - (a^(2/3)*Log[a + b*x])/(2*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{3x^{2/3}}{2b} - \frac{a\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b}$$

$$= \frac{3x^{2/3}}{2b} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{(3a)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^2} + \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{}{}\right.}{2b}$$

$$= \frac{3x^{2/3}}{2b} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

Mathematica [A]
time = 0.06, size = 127, normalized size = 1.14

$$\frac{3b^{2/3}x^{2/3} + 2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*b^(2/3)*x^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1
/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - a^(2/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(5/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.45, size = 160, normalized size = 1.44

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{2}{3}}\right], a==0\&\&b==0\right\}, \left\{\frac{3x^{\frac{2}{3}}}{2b}, a==0\right\}, \left\{\frac{3x^{\frac{5}{3}}}{5a}, b==0\right\}\right\}, -\frac{\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{2b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{3x^{\frac{2}{3}}}{2b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(2/3)/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (2 / 3)], a == 0 && b == 0}, {3 x ^ (2 / 3
) / (2 b), a == 0}, {3 x ^ (5 / 3) / (5 a), b == 0}}, -Sqrt[3] a ArcTan[Sqr
t[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (b ^ 2 (-a / b
) ^ (1 / 3)) - a Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] / (b ^ 2 (-a / b) ^
(1 / 3)) + a Log[4 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a /
 b) ^ (2 / 3)] / (2 b ^ 2 (-a / b) ^ (1 / 3)) + 3 x ^ (2 / 3) / (2 b)]
```

Maple [A]
time = 0.11, size = 112, normalized size = 1.01

method	result	size
risch	$\frac{3x^{\frac{2}{3}}}{2b}+\frac{a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	107
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2b}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	112

default	$\frac{3x^{\frac{2}{3}}}{2b}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*x^(2/3)/b-3*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/
3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a/b
```

Maxima [A]
time = 0.35, size = 114, normalized size = 1.03

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(1/3)) + 3/2*x^(2/3)/b - 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/
b)^(2/3))/(b^2*(a/b)^(1/3)) + a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(1/3)
)
```

Fricas [A]
time = 0.31, size = 128, normalized size = 1.15

$$-\frac{2\sqrt{3}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)-2\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3\,x^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^(
1/3) - sqrt(3)*a)/a) + (a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a*x
^(2/3) + a*(a^2/b^2)^(1/3)) - 2*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x
^(1/3)) - 3*x^(2/3))/b
```

Sympy [A]
time = 4.33, size = 162, normalized size = 1.46

$$\begin{cases} \tilde{\infty}x^{\frac{2}{3}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{5}{3}}}{5a} & \text{for } b = 0 \\ \frac{3x^{\frac{2}{3}}}{2b} & \text{for } a = 0 \\ -\frac{a\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{b^2\sqrt[3]{-\frac{a}{b}}} + \frac{a\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^2\sqrt[3]{-\frac{a}{b}}} - \frac{\sqrt{3}\,a\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{b^2\sqrt[3]{-\frac{a}{b}}} + \frac{3x^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a), Eq(b, 0))
, (3*x**(2/3)/(2*b), Eq(a, 0)), (-a*log(x**(1/3) - (-a/b)**(1/3))/(b**2*(-a
/b)**(1/3)) + a*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3)
)/(2*b**2*(-a/b)**(1/3)) - sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/
3)) + sqrt(3)/3)/(b**2*(-a/b)**(1/3)) + 3*x**(2/3)/(2*b), True))
```

Giac [A]
time = 0.01, size = 187, normalized size = 1.68

$$3\left(-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\ln\left(\left(x^{\frac{1}{3}}\right)^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^3} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3b^2a} + \frac{\left(x^{\frac{1}{3}}\right)^2 b}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x)
```

```
[Out] (-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b + 3/2*x^(2/3)/b + sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^3 - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
b^3
```

Mupad [B]
time = 0.15, size = 130, normalized size = 1.17

$$\frac{3\,x^{2/3}}{2\,b} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{4/3}} + \frac{9\,a^2\,x^{1/3}}{b}\right)}{b^{5/3}} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}} + \frac{9\,a^2\,x^{1/3}}{b}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{5/3}} - \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}} + \frac{9\,a^2\,x^{1/3}}{b}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x),x)
```

```
[Out] (3*x^(2/3))/(2*b) + (a^(2/3)*log((9*a^(7/3))/b^(4/3) + (9*a^2*x^(1/3))/b))/
b^(5/3) + (a^(2/3)*log((9*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(4/3) + (9*a^
2*x^(1/3))/b)*((3^(1/2)*1i)/2 - 1/2))/b^(5/3) - (a^(2/3)*log((9*a^(7/3)*((3
^(1/2)*1i)/2 + 1/2)^2)/b^(4/3) + (9*a^2*x^(1/3))/b)*((3^(1/2)*1i)/2 + 1/2))
/b^(5/3)
```

3.677 $\int \frac{\sqrt[3]{x}}{a+bx} \, dx$

Optimal. Leaf size=109

$$\frac{3\sqrt[3]{x}}{b}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}$$

```
[Out] 3*x^(1/3)/b-3/2*a^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(4/3)+1/2*a^(1/3)*ln(
b*x+a)/b^(4/3)+a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/
2))*3^(1/2)/b^(4/3)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 60, 631, 210, 31}

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt
[3]*a^(1/3))])/b^(4/3) - (3*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(4
/3)) + (a^(1/3)*Log[a + b*x])/(2*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{3\sqrt[3]{x}}{b} - \frac{a\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{a}\,\log(a+bx)}{2b^{4/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}}{}$$

$$= \frac{3\sqrt[3]{x}}{b} - \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\,\log(a+bx)}{2b^{4/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\,\log(a+bx)}{2b^{4/3}}$$

Mathematica [A]
time = 0.06, size = 126, normalized size = 1.16

$$\frac{6\sqrt[3]{b}\,\sqrt[3]{x} + 2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) + \sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x),x]
```

```
[Out] (6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1
/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + a^(1/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(4/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.66, size = 156, normalized size = 1.43

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{1}{3}}\right],a{=}{=}0\&\&b{=}{=}0\right\},\left\{\frac{3x^{\frac{1}{3}}}{b},a{=}{=}0\right\},\left\{\frac{3x^{\frac{4}{3}}}{4a},b{=}{=}0\right\}\right\},-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{b}-\frac{\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2b}+\frac{\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{b}+\frac{3x^{\frac{1}{3}}}{b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1/3)/(a + b*x),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (1 / 3)], a == 0 && b == 0}, {3 x ^ (1 / 3
) / b, a == 0}, {3 x ^ (4 / 3) / (4 a), b == 0}}, -Sqrt[3] ArcTan[Sqrt[3] /
 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] (-a / b) ^ (1 / 3) / b
 - Log[4 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 /
 3)] (-a / b) ^ (1 / 3) / (2 b) + Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] (-a
 / b) ^ (1 / 3) / b + 3 x ^ (1 / 3) / b]
```

Maple [A]
time = 0.15, size = 112, normalized size = 1.03

method	result	size
derivativedivides	$\frac{3x^{\frac{1}{3}}}{b}-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	112

default	$\frac{3x^{\frac{1}{3}}}{b}-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)/b-3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*
ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a/b
```

Maxima [A]
time = 0.40, size = 115, normalized size = 1.06

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\,x^{\frac{1}{3}}}{b}+\frac{a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(2/3)) + 3*x^(1/3)/b + 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)
^(2/3))/(b^2*(a/b)^(2/3)) - a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Fricas [A]
time = 0.32, size = 114, normalized size = 1.05

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6\,x^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b)^(2/3) -
sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2
/3)) + 2*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^(1/3)) + 6*x^(1/3))/b
```

Sympy [A]
time = 2.33, size = 148, normalized size = 1.36

$$\begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{for } a = 0 \\ \frac{3x^{\frac{4}{3}}}{4a} & \text{for } b = 0 \\ \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b} - \frac{\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b} - \frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/b, Eq(a, 0)), (3
*x**(4/3)/(4*a), Eq(b, 0)), (3*x**(1/3)/b + (-a/b)**(1/3)*log(x**(1/3) - (-
a/b)**(1/3))/b - (-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) +
4*(-a/b)**(2/3))/(2*b) - sqrt(3)*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(
-a/b)**(1/3)) + sqrt(3)/3)/b, True))
```

Giac [A]
time = 0.00, size = 165, normalized size = 1.51

$$3\left(-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^{2}}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}}+\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3ba}+\frac{x^{\frac{1}{3}}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x)
```

```
[Out] (-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b - sqrt(3)*(-a*b^2)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 3*x^(1/3)/b
 - 1/2*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
2
```

Mupad [B]
time = 0.07, size = 126, normalized size = 1.16

$$\frac{3\,x^{1/3}}{b}+\frac{{\left(-a\right)}^{1/3}\,\ln\left(9\,{\left(-a\right)}^{4/3}\,b^{2/3}+9\,a\,b\,x^{1/3}\right)}{b^{4/3}}+\frac{{\left(-a\right)}^{1/3}\,\ln\left(9\,{\left(-a\right)}^{4/3}\,b^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+9\,a\,b\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{4/3}}-\frac{{\left(-a\right)}^{1/3}\,\ln\left(9\,{\left(-a\right)}^{4/3}\,b^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-9\,a\,b\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x),x)
```

```
[Out] (3*x^(1/3))/b + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3) + 9*a*b*x^(1/3)))/b^(4
/3) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 - 1/2) + 9*a*b*x
^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(4/3) - ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2
/3)*((3^(1/2)*1i)/2 + 1/2) - 9*a*b*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/b^(4/3)
```

3.678 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -3/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(2/3)+1/2*ln(b*x+a)/a^(1/3)/b^(2
/3)-arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
/b^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {58, 631, 210, 31}

$$-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3
)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[
a + b*x]/(2*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx = \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,b^{2/3}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}$$

Mathematica [A]
time = 0.05, size = 103, normalized size = 1.03

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/
3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(
2/3)])/(2*a^(1/3)*b^(2/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 4.03, size = 147, normalized size = 1.47

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{1}{3}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-3}{bx^{\frac{1}{3}}}, a==0\right\}, \left\{\frac{3x^{\frac{2}{3}}}{2a}, b==0\right\}\right\}, -\frac{\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(1/3)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (1 / 3)], a == 0 && b == 0}, {-3 / (b
x ^ (1 / 3)), a == 0}, {3 x ^ (2 / 3) / (2 a), b == 0}}, -Log[4 x ^ (1 / 3)
 (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (2 b (-a / b)
 ^ (1 / 3)) + Sqrt[3] ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a /
 b) ^ (1 / 3))] / (b (-a / b) ^ (1 / 3)) + Log[x ^ (1 / 3) - (-a / b) ^ (1
/ 3)] / (b (-a / b) ^ (1 / 3))]
```

Maple [A]

time = 0.12, size = 96, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	96
default	$-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)
^(1/3)*x^(1/3)+(a/b)^(2/3))+3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x^(1/3)-1))
```

Maxima [A]

time = 0.34, size = 103, normalized size = 1.03

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(1/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3
)) - log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Fricas [A]
time = 0.33, size = 313, normalized size = 3.13

$$\left[\frac{\sqrt{3}\,ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+\sqrt{3}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)}{2\,ab^2},\ \frac{2\sqrt{3}\,ab\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{3}\left(2\,bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{3\,b}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)}{2\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x - a*b + sqrt(3)*(a*b*
x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x + a)) + (-a*b^2)^(2/3)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/
3) - (-a*b^2)^(1/3)))/(a*b^2), 1/2*(2*sqrt(3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*a
rctan(1/3*sqrt(3)*(2*b*x^(1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 + (-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/
3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A]
time = 2.63, size = 141, normalized size = 1.41

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{3}{b \sqrt[3]{x}} & \text{for}\: a = 0 \\ \frac{3 x^{\frac{2}{3}}}{2 a} & \text{for}\: b = 0 \\ \frac{\log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{b \sqrt[3]{- \frac{a}{b}}} - \frac{\log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{2 b \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{b \sqrt[3]{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (-3/(b*x**(1/3)), Eq(a, 0)),
 (3*x**(2/3)/(2*a), Eq(b, 0)), (log(x**(1/3) - (-a/b)**(1/3))/(b*(-a/b)**(1
/3)) - log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b*(-
a/b)**(1/3)) + sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/
3)/(b*(-a/b)**(1/3)), True))
```

Giac [A]
time = 0.00, size = 171, normalized size = 1.71

$$3\left(\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6ab^{2}}-\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab^{2}}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x)
```

```
[Out] -(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/2*(-
a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B]
time = 0.11, size = 120, normalized size = 1.20

$$\frac{\ln\left(9\,b\,x^{1/3}-9\,{(-a)}^{1/3}\,b^{2/3}\right)}{{(-a)}^{1/3}\,b^{2/3}}+\frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,{(-a)}^{1/3}\,b^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{(-a)}^{1/3}\,b^{2/3}}-\frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,{(-a)}^{1/3}\,b^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{(-a)}^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)),x)
```

```
[Out] log(9*b*x^(1/3) - 9*(-a)^(1/3)*b^(2/3))/((-a)^(1/3)*b^(2/3)) + (log(9*b*x^(
1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-
a)^(1/3)*b^(2/3)) - (log(9*b*x^(1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i +
1)^2)/4)*(3^(1/2)*1i + 1))/(2*(-a)^(1/3)*b^(2/3))
```

3.679 $\int \frac{1}{x^{2/3}(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

```
[Out] 3/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(1/3)-1/2*ln(b*x+a)/a^(2/3)/b^(1/
3)-arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)/
b^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {60, 631, 210, 31}

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2/3
)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[
a + b*x]/(2*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = -\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{}\right.}{2a^{2/3}\sqrt[3]{b}}$$

$$= \frac{3\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}} + \frac{3\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.05, size = 103, normalized size = 1.03

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right) + \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{2a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)),x]
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)
*x^(2/3)])/(a^(2/3)*b^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.41, size = 148, normalized size = 1.48

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{2}{3}}}\right], a{=}{=}0\&\&b{=}{=}0\right\}, \left\{\frac{3x^{\frac{1}{3}}}{a}, b{=}{=}0\right\}, \left\{\frac{-3}{2bx^{\frac{2}{3}}}, a{=}{=}0\right\}\right\}, -\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4x^{\frac{2}{3}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\text{Log}\left[x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(2/3)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (2 / 3)], a == 0 && b == 0}, {3 x ^ (1
 / 3) / a, b == 0}, {-3 / (2 b x ^ (2 / 3)), a == 0}}, -Sqrt[3] ArcTan[Sqrt
[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (b (-a / b) ^ (
2 / 3)) - Log[4 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b)
 ^ (2 / 3)] / (2 b (-a / b) ^ (2 / 3)) + Log[x ^ (1 / 3) - (-a / b) ^ (1 /
3)] / (b (-a / b) ^ (2 / 3))]
```

Maple [A]
time = 0.10, size = 95, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	95
default	$\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^
(1/3)*x^(1/3)+(a/b)^(2/3))+1/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x^(1/3)-1))
```

Maxima [A]
time = 0.34, size = 102, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(2/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3
)) + log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.32, size = 307, normalized size = 3.07

$$\left[\frac{\sqrt{3}\,ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+\sqrt{3}\left(2\,abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3\,(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)}{2\,a^2b}, \frac{2\sqrt{3}\,ab\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{3}\left((a^2b)^{\frac{1}{3}}a-2\,(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{3\,a^2}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + sqrt(3)*(2*a*
b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b)
 - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) - (a^2*b)^(2/3)*log(a*b*x^(2/3) +
(a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3)
+ (a^2*b)^(2/3)))/(a^2*b), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(
-1/3*sqrt(3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)
/b)/a^2) - (a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*
x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A]
time = 6.55, size = 141, normalized size = 1.41

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a} & \text{for}\: b = 0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for}\: a = 0 \\ \frac{\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a, Eq(b, 0)), (-
3/(2*b*x**(2/3)), Eq(a, 0)), (log(x**(1/3) - (-a/b)**(1/3))/(b*(-a/b)**(2/3
)) - log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b*(-a/
b)**(2/3)) - sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)
/(b*(-a/b)**(2/3)), True))
```

Giac [A]

time = 0.00, size = 153, normalized size = 1.53

$$3\left(\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6ab}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x)
```

```
[Out] -(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a + sqrt(3)*(-a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/2*(-a*
b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B]
time = 0.21, size = 110, normalized size = 1.10

$$\frac{\ln\left(9\,a^{1/3}\,b^{5/3}+9\,b^2\,x^{1/3}\right)}{a^{2/3}\,b^{1/3}}+\frac{\ln\left(9\,b^2\,x^{1/3}+\frac{9\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(9\,b^2\,x^{1/3}-\frac{9\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)),x)
```

```
[Out] log(9*a^(1/3)*b^(5/3) + 9*b^2*x^(1/3))/(a^(2/3)*b^(1/3)) + (log(9*b^2*x^(1/
3) + (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^(2/3)*b
^(1/3)) - (log(9*b^2*x^(1/3) - (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(
1/2)*1i + 1))/(2*a^(2/3)*b^(1/3))
```

3.680 $\int \frac{1}{x^{4/3}(a+bx)}\,dx$

Optimal. Leaf size=109

$$-\frac{3}{a\sqrt[3]{x}}+\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}}+\frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}}$$

```
[Out] -3/a/x^(1/3)+3/2*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(4/3)-1/2*b^(1/3)*ln
(b*x+a)/a^(4/3)+b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1
/2))*3^(1/2)/a^(4/3)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {53, 58, 631, 210, 31}

$$\frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqr
t[3]*a^(1/3))])/a^(4/3) + (3*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
4/3)) - (b^(1/3)*Log[a + b*x])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{3}{a\sqrt[3]{x}} - \frac{b\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{a}$$

$$= -\frac{3}{a\sqrt[3]{x}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a} + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}}{}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{a^{4/3}}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+}{2a^{4/3}}$$

Mathematica [A]
time = 0.08, size = 127, normalized size = 1.17

$$\frac{-\frac{6\sqrt[3]{a}}{\sqrt[3]{x}} + 2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - \sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] ((-6*a^(1/3))/x^(1/3) + 2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a
^(1/3))/Sqrt[3]] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - b^(1/3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a^(4/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 10.15, size = 157, normalized size = 1.44

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{4}{3}}}\right],a==0\&\&b==0\right\},\left\{\frac{-3}{4bx^{\frac{4}{3}}},a==0\right\},\left\{\frac{-3}{ax^{\frac{1}{3}}},b==0\right\}\right\},-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{2a\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3}{ax^{\frac{1}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(4/3)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (4 / 3)], a == 0 && b == 0}, {-3 / (4
b x ^ (4 / 3)), a == 0}, {-3 / (a x ^ (1 / 3)), b == 0}}, -Sqrt[3] ArcTan[S
qrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (a (-a / b)
^ (1 / 3)) - Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] / (a (-a / b) ^ (1 / 3))
 + Log[4 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 /
 3)] / (2 a (-a / b) ^ (1 / 3)) - 3 / (a x ^ (1 / 3))]
```

Maple [A]
time = 0.12, size = 112, normalized size = 1.03

method	result	size
risch	$-\frac{3}{a\,x^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	104
derivativedivides	$-\frac{3}{a\,x^{\frac{1}{3}}}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	112

default	$-\frac{3}{a\,x^{\frac{1}{3}}}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	112

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(4/3)/(b*x+a),x,method=_RETURNVERBOSE)

[Out] -3/a/x^(1/3)-3*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*b/a

Maxima [A]
time = 0.35, size = 111, normalized size = 1.02

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3}{ax^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="maxima")

[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 3/(a*x^(1/3))

Fricas [A]
time = 0.32, size = 113, normalized size = 1.04

$$-\frac{2\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+6x^{\frac{2}{3}}}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="fricas")

[Out] -1/2*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + x*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 6*x^(2/3))/(a*x)

Sympy [A]
time = 23.79, size = 151, normalized size = 1.39

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for } a = 0 \\ -\frac{3}{a\sqrt[3]{x}} & \text{for } b = 0 \\ -\frac{\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{a\sqrt[3]{-\frac{a}{b}}} + \frac{\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\sqrt[3]{-\frac{a}{b}}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{a\sqrt[3]{-\frac{a}{b}}} - \frac{3}{a\sqrt[3]{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b*x**(4/3)), Eq(a, 0)
), (-3/(a*x**(1/3)), Eq(b, 0)), (-log(x**(1/3) - (-a/b)**(1/3))/(a*(-a/b)**
(1/3)) + log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*a*
(-a/b)**(1/3)) - sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3
)/3)/(a*(-a/b)**(1/3)) - 3/(a*x**(1/3)), True))
```

Giac [A]
time = 0.00, size = 183, normalized size = 1.68

$$3\left(-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\ln\left(\left(x^{\frac{1}{3}}\right)^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2b} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3a^2} - \frac{1}{ax^{\frac{1}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x)
```

```
[Out] b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + sqrt(3)*(-a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 3/(
a*x^(1/3)) - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a^2*b)
```

Mupad [B]
time = 0.15, size = 124, normalized size = 1.14

$$\frac{b^{1/3}\ln\left(9\,a^{4/3}\,b^{8/3} + 9\,a\,b^3\,x^{1/3}\right)}{a^{4/3}} - \frac{3}{a\,x^{1/3}} + \frac{b^{1/3}\ln\left(9\,a\,b^3\,x^{1/3} + 9\,a^{4/3}\,b^{8/3}\left(-\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)^2\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)}{a^{4/3}} - \frac{b^{1/3}\ln\left(9\,a\,b^3\,x^{1/3} + 9\,a^{4/3}\,b^{8/3}\left(\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)^2\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1i}{2}\right)}{a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)),x)
```

```
[Out] (b^(1/3)*log(9*a^(4/3)*b^(8/3) + 9*a*b^3*x^(1/3)))/a^(4/3) - 3/(a*x^(1/3))
+ (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2)^2
)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3) - (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3
)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 + 1/2))/a^(4/3)
```

3.681 $\int \frac{1}{x^{5/3}(a+bx)}\,dx$

Optimal. Leaf size=111

$$-\frac{3}{2ax^{2/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}$$

```
[Out] -3/2/a/x^(2/3)-3/2*b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(5/3)+1/2*b^(2/3)*
ln(b*x+a)/a^(5/3)+b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^
(1/2))*3^(1/2)/a^(5/3)
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {53, 60, 631, 210, 31}

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(5/3) - (3*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a
^(5/3)) + (b^(2/3)*Log[a + b*x])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/3}(a+bx)}\,dx &= -\frac{3}{2ax^{2/3}} - \frac{b\int \frac{1}{x^{2/3}(a+bx)}\,dx}{a}\\
&= -\frac{3}{2ax^{2/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}} - \frac{(3b^{2/3})\cdots}{\cdots}\\
&= -\frac{3}{2ax^{2/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{(3b^{2/3})\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\cdots\right.}{a^{5/3}}\\
&= -\frac{3}{2ax^{2/3}} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+\cdots}{2a^{5/3}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 1.14

$$\frac{-\frac{3a^{2/3}}{x^{2/3}} + 2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] ((-3*a^(2/3))/x^(2/3) + 2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a
^(1/3))/Sqrt[3]] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + b^(2/3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a^(5/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 41.43, size = 156, normalized size = 1.41

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{3}}}\right],a{=}{=}0\&\&b{=}{=}0\right\},\left\{\frac{-3}{5bx^{\frac{5}{3}}},a{=}{=}0\right\},\left\{\frac{-3}{2ax^{\frac{2}{3}}},b{=}{=}0\right\}\right\},-\frac{\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{2a\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3}{2ax^{\frac{2}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/3)*(a + b*x)),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 3)], a == 0 && b == 0}, {-3 / (5
b x ^ (5 / 3)), a == 0}, {-3 / (2 a x ^ (2 / 3)), b == 0}}, -Log[x ^ (1 / 3
) - (-a / b) ^ (1 / 3)] / (a (-a / b) ^ (2 / 3)) + Log[4 x ^ (1 / 3) (-a /
b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (2 a (-a / b) ^ (2 /
 3)) + Sqrt[3] ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (
1 / 3))] / (a (-a / b) ^ (2 / 3)) - 3 / (2 a x ^ (2 / 3))]
```

Maple [A]
time = 0.17, size = 112, normalized size = 1.01

method	result	size
derivativedivides	$-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}-\frac{3}{2a\,x^{\frac{2}{3}}}$	112

default	$-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}-\frac{3}{2a\,x^{\frac{2}{3}}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-
(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*b/a-3/2/a/x^(2/3)
```

Maxima [A]
time = 0.35, size = 112, normalized size = 1.01

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3}{2\,ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(2/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/
3)) - log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 3/2/(a*x^(2/3))
```

Fricas [A]
time = 0.31, size = 147, normalized size = 1.32

$$\frac{2\sqrt{3}\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right)-x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,x^{\frac{1}{3}}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2
)^(2/3) - sqrt(3)*b)/b) - x*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*
(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x*(-b^2/a^2)^(1/3)*log(b*x^(1/
3) - a*(-b^2/a^2)^(1/3)) - 3*x^(1/3))/(a*x)
```

Sympy [A]
time = 41.42, size = 155, normalized size = 1.40

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}} & \text{for } b = 0 \\ -\frac{\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(-\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{3}{2ax^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(5*b*x**(5/3)), Eq(a, 0)
), (-3/(2*a*x**(2/3)), Eq(b, 0)), (-log(x**(1/3) - (-a/b)**(1/3))/(a*(-a/b)
**(2/3)) + log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*
a*(-a/b)**(2/3)) + sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/(a*(-a/b)**(2/3)) - 3/(2*a*x**(2/3)), True))
```

Giac [A]
time = 0.00, size = 173, normalized size = 1.56

$$3\left(-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^{2} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^{2}} - \frac{\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{2}} + \frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3a^{2}} - \frac{\frac{1}{2}}{a\left(x^{\frac{1}{3}}\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x)
```

```
[Out] b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 - sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/2*(-a
*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/2/(a
*x^(2/3))
```

Mupad [B]
time = 0.07, size = 138, normalized size = 1.24

$$\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3} - 9\,a^2\,b^3\,x^{1/3}\right)}{(-a)^{5/3}} - \frac{3}{2\,a\,x^{2/3}} + \frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - 9\,a^2\,b^3\,x^{1/3}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}} - \frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + 9\,a^2\,b^3\,x^{1/3}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)),x)
```

```
[Out] (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3) - 9*a^2*b^3*x^(1/3)))/(-a)^(5/3) - 3/(2*a
*x^(2/3)) + (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2) - 9*a^
2*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(-a)^(5/3) - (b^(2/3)*log(9*(-a)^(7/
3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 9*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/
2))/(-a)^(5/3)
```

3.682 $\int \frac{x^{5/3}}{(a+bx)^2} \, dx$

Optimal. Leaf size=129

$$\frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}$$

```
[Out] 5/2*x^(2/3)/b^2-x^(5/3)/b/(b*x+a)+5/2*a^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b
^(8/3)-5/6*a^(2/3)*ln(b*x+a)/b^(8/3)+5/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x^(1/3))/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 58, 631, 210, 31}

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (5*x^(2/3))/(2*b^2) - x^(5/3)/(b*(a + b*x)) + (5*a^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) - (5*a^(2/3)*Log[a + b*x])/(6*b^(8/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/3}}{(a+bx)^2}\,dx &= -\frac{x^{5/3}}{b(a+bx)}+\frac{5\int \frac{x^{2/3}}{a+bx}\,dx}{3b} \\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}-\frac{(5a)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b^2} \\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}-\frac{(5a)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2b^3}+\ldots \\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}-\frac{\left(5a^{2/3}\right)\text{Subst}\left(\ldots\right.}{\ldots} \\
&= \frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}+\frac{5a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\ldots}{\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 142, normalized size = 1.10

$$
\frac{\frac{3b^{2/3}x^{2/3}(5a+3bx)}{a+bx}+10\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)-5a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{6b^{8/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] ((3*b^(2/3)*x^(2/3)*(5*a + 3*b*x))/(a + b*x) + 10*Sqrt[3]*a^(2/3)*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 10*a^(2/3)*Log[a^(1/3) + b^(1/3)
*x^(1/3)] - 5*a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/
3)])/(6*b^(8/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 85.40, size = 566, normalized size = 4.39

$$
\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{2}{3}}\right],a==0\&\&b==0\right\},\left\{\frac{3x^{\frac{2}{3}}}{2b^2},a==0\right\},\left\{\frac{3x^{\frac{8}{3}}}{8a^2},b==0\right\}\right\},\frac{-10\sqrt{3}\,a^2\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10a^2\text{Log}[2]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10a^2\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5a^2\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{15abx^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10\sqrt{3}\,abx\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10abx\text{Log}[2]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10abx\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5abx\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{9b^2x^{\frac{5}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6b^4x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/3)/(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (2 / 3)], a == 0 && b == 0}, {3 x ^ (2 / 3
) / (2 b ^ 2), a == 0}, {3 x ^ (8 / 3) / (8 a ^ 2), b == 0}}, -10 Sqrt[3] a
 ^ 2 ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] /
 (6 a b ^ 3 (-a / b) ^ (1 / 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) - 10 a ^ 2 L
og[2] / (6 a b ^ 3 (-a / b) ^ (1 / 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) - 10
a ^ 2 Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] / (6 a b ^ 3 (-a / b) ^ (1 / 3)
 + 6 b ^ 4 x (-a / b) ^ (1 / 3)) + 5 a ^ 2 Log[4 x ^ (1 / 3) (-a / b) ^ (1
/ 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (6 a b ^ 3 (-a / b) ^ (1 / 3
) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) + 15 a b x ^ (2 / 3) (-a / b) ^ (1 / 3) /
 (6 a b ^ 3 (-a / b) ^ (1 / 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) - 10 Sqrt[3]
 a b x ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))]
 / (6 a b ^ 3 (-a / b) ^ (1 / 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) - 10 a b x
 Log[2] / (6 a b ^ 3 (-a / b) ^ (1 / 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) - 1
0 a b x Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] / (6 a b ^ 3 (-a / b) ^ (1 /
3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) + 5 a b x Log[4 x ^ (1 / 3) (-a / b) ^ (
1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (6 a b ^ 3 (-a / b) ^ (1 /
 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3)) + 9 b ^ 2 x ^ (5 / 3) (-a / b) ^ (1 / 3
) / (6 a b ^ 3 (-a / b) ^ (1 / 3) + 6 b ^ 4 x (-a / b) ^ (1 / 3))]
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.96

method	result
risch	$\frac{3x^{\frac{2}{3}}}{2b^{2}}+\frac{a\,x^{\frac{2}{3}}}{b^{2}(bx+a)}+\frac{5a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5a\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2b^{2}}-\frac{3a\left(-\frac{x^{\frac{2}{3}}}{3(bx+a)}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$

default	$\frac{3x^{\frac{2}{3}}}{2b^2}-\frac{3a\left(-\frac{x^{\frac{2}{3}}}{3(bx+a)}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 3/2*x^(2/3)/b^2-3*a/b^2*(-1/3*x^(2/3)/(b*x+a)-5/9/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+5/18/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+5/9*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))

Maxima [A]
time = 0.35, size = 133, normalized size = 1.03

$$\frac{ax^{\frac{2}{3}}}{b^3x+ab^2}-\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,x^{\frac{2}{3}}}{2\,b^2}-\frac{5\,a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="maxima")

[Out] a*x^(2/3)/(b^3*x + a*b^2) - 5/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/2*x^(2/3)/b^2 - 5/6*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) + 5/3*a*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))

Fricas [A]
time = 0.33, size = 162, normalized size = 1.26

$$-\frac{10\sqrt{3}\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3a}\right)+5\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3\,(3\,bx+5\,a)x^{\frac{2}{3}}}{6\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="fricas")

[Out] -1/6*(10*sqrt(3)*(b*x + a)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(b*x + a)*(a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a*x^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(b*x + a)*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x^(1/3)) - 3*(3*b*x + 5*a)*x^(2/3))/(b^3*x + a*b^2)

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 217, normalized size = 1.68

$$3\left(\frac{\left(x^{\frac{1}{3}}\right)^{2}b^{2}}{2b^{4}}+\frac{\frac{1}{3}\left(x^{\frac{1}{3}}\right)^{2}a}{b^{2}\left(xb+a\right)}-\frac{5\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18b^{4}}+\frac{\frac{1}{3}\cdot5\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{4}}+\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot3b^{2}a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x)
```

```
[Out] 5/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + a*x^(2/3)/((b*x + a
)*b^2) + 3/2*x^(2/3)/b^2 + 5/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2
*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 5/6*(-a*b^2)^(2/3)*log(x^(2/3)
 + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
```

Mupad [B]
time = 0.26, size = 150, normalized size = 1.16

$$\frac{3\,x^{2/3}}{2\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)}{3\,b^{8/3}}+\frac{a\,x^{2/3}}{x\,b^3+a\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}-\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(2/3))/(2*b^2) + (5*a^(2/3)*log((25*a^(7/3))/b^(10/3) + (25*a^2*x^(1/3
))/b^3))/(3*b^(8/3)) + (a*x^(2/3))/(a*b^2 + b^3*x) + (5*a^(2/3)*log((25*a^(
7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1
i)/2 - 1/2))/(3*b^(8/3)) - (5*a^(2/3)*log((25*a^(7/3)*((3^(1/2)*1i)/2 + 1/2
)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(8/3))
```

3.683 $\int \frac{x^{4/3}}{(a+bx)^2} dx$

Optimal. Leaf size=125

$$\frac{4\sqrt[3]{x}}{b^2}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}$$

```
[Out] 4*x^(1/3)/b^2-x^(4/3)/b/(b*x+a)-2*a^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(7/
3)+2/3*a^(1/3)*ln(b*x+a)/b^(7/3)+4/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x^(1/3))/a^(1/3)*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 60, 631, 210, 31}

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (4*x^(1/3))/b^2 - x^(4/3)/(b*(a + b*x)) + (4*a^(1/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (2*a^(1/3)*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/b^(7/3) + (2*a^(1/3)*Log[a + b*x])/(3*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = -\frac{x^{4/3}}{b(a+bx)} + \frac{4\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{3b}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{(4a)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b^2}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{b^{8/3}} -$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{4\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{7/3}} + 2\sqrt[3]{a}$$

Mathematica [A]
time = 0.18, size = 142, normalized size = 1.14

$$\frac{\frac{3\sqrt[3]{b}\,\sqrt[3]{x}\,(4a+3bx)}{a+bx} + 4\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) + 2\sqrt[3]{a}\,\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{3b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] ((3*b^(1/3)*x^(1/3)*(4*a + 3*b*x))/(a + b*x) + 4*Sqrt[3]*a^(1/3)*ArcTan[(1
- (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 4*a^(1/3)*Log[a^(1/3) + b^(1/3)*x
^(1/3)] + 2*a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)
])/(3*b^(7/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 66.08, size = 439, normalized size = 3.51

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[x^{\frac{1}{3}}\right], a==0\&\&b==0\right\}, \left\{\frac{3x^{\frac{7}{3}}}{7a^2}, b==0\right\}, \left\{\frac{3x^{\frac{1}{3}}}{b^2}, a==0\right\}\right\}, \frac{-4\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} - \frac{2a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4x^{\frac{2}{3}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} + \frac{4a\text{Log}[2]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} + \frac{4a\text{Log}\left[x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} + \frac{12ax^{\frac{1}{3}}}{3ab^2+3b^3x} - \frac{4\sqrt{3}\,bx\text{ArcTan}\left[\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} - \frac{2bx\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4x^{\frac{2}{3}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} + \frac{4bx\text{Log}[2]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} + \frac{4bx\text{Log}\left[x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2+3b^3x} + \frac{9bx^{\frac{4}{3}}}{3ab^2+3b^3x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(4/3)/(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ (1 / 3)], a == 0 && b == 0}, {3 x ^ (7 / 3
) / (7 a ^ 2), b == 0}, {3 x ^ (1 / 3) / b ^ 2, a == 0}}, -4 Sqrt[3] a ArcT
an[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] (-a / b) ^
 (1 / 3) / (3 a b ^ 2 + 3 b ^ 3 x) - 2 a Log[4 x ^ (1 / 3) (-a / b) ^ (1 /
3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b) ^ (1 / 3) / (3 a b ^ 2
+ 3 b ^ 3 x) + 4 a Log[2] (-a / b) ^ (1 / 3) / (3 a b ^ 2 + 3 b ^ 3 x) + 4
a Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] (-a / b) ^ (1 / 3) / (3 a b ^ 2 + 3
 b ^ 3 x) + 12 a x ^ (1 / 3) / (3 a b ^ 2 + 3 b ^ 3 x) - 4 Sqrt[3] b x ArcT
an[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] (-a / b) ^
 (1 / 3) / (3 a b ^ 2 + 3 b ^ 3 x) - 2 b x Log[4 x ^ (1 / 3) (-a / b) ^ (1
/ 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b) ^ (1 / 3) / (3 a b ^
2 + 3 b ^ 3 x) + 4 b x Log[2] (-a / b) ^ (1 / 3) / (3 a b ^ 2 + 3 b ^ 3 x)
+ 4 b x Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] (-a / b) ^ (1 / 3) / (3 a b ^
 2 + 3 b ^ 3 x) + 9 b x ^ (4 / 3) / (3 a b ^ 2 + 3 b ^ 3 x)]
```

Maple [A]
time = 0.26, size = 124, normalized size = 0.99

method	result	size
derivativedivides	$\frac{3x^{\frac{1}{3}}}{b^{2}}-\frac{3a\left(-\frac{x^{\frac{1}{3}}}{3(bx+a)}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$	124
default	$\frac{3x^{\frac{1}{3}}}{b^{2}}-\frac{3a\left(-\frac{x^{\frac{1}{3}}}{3(bx+a)}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)/b^2-3*a/b^2*(-1/3*x^(1/3)/(b*x+a)+4/9/b/(a/b)^(2/3)*ln(x^(1/3)+(a
/b)^(1/3))-2/9/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+4/
9/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.35, size = 133, normalized size = 1.06

$$\frac{ax^{\frac{1}{3}}}{b^3x+ab^2}-\frac{4\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\,x^{\frac{1}{3}}}{b^2}+\frac{2\,a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4\,a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(1/3)/(b^3*x + a*b^2) - 4/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3*x^(1/3)/b^2 + 2/3*a*log(x^(2
/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4/3*a*log(x^(1
/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [A]
time = 0.33, size = 147, normalized size = 1.18

$$\frac{4\sqrt{3}\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+3\,(3\,bx+4\,a)x^{\frac{1}{3}}}{3\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*x + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b
)^(2/3) - sqrt(3)*a)/a) - 2*(b*x + a)*(-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-
a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x + a)*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^
(1/3)) + 3*(3*b*x + 4*a)*x^(1/3))/(b^3*x + a*b^2)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 196, normalized size = 1.57

$$3\left(\frac{x^{\frac{1}{3}}}{b^2}+\frac{\frac{1}{3}x^{\frac{1}{3}}a}{b^2\,(xb+a)}-\frac{2\,(-ab^2)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^3}-\frac{\frac{1}{3}\cdot 4\,(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^3}+\frac{4a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot 3b^2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x)
```

```
[Out] 4/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - 4/3*sqrt(3)*(-a*b^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + a
*x^(1/3)/((b*x + a)*b^2) + 3*x^(1/3)/b^2 - 2/3*(-a*b^2)^(1/3)*log(x^(2/3) +
 x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

Mupad [B]
time = 0.15, size = 142, normalized size = 1.14

$$\frac{3\,x^{1/3}}{b^2}+\frac{a\,x^{1/3}}{x\,b^3+a\,b^2}+\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{12\,{\left(-a\right)}^{4/3}}{b^{1/3}}+12\,a\,x^{1/3}\right)}{3\,b^{7/3}}-\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}-\frac{12\,{\left(-a\right)}^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{7/3}}+\frac{{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}+\frac{9\,{\left(-a\right)}^{4/3}\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{1/3}}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(1/3))/b^2 + (a*x^(1/3))/(a*b^2 + b^3*x) + (4*(-a)^(1/3)*log((12*(-a)^
(4/3))/b^(1/3) + 12*a*x^(1/3)))/(3*b^(7/3)) - (4*(-a)^(1/3)*log(12*a*x^(1/3
) - (12*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1/2))
/(3*b^(7/3)) + ((-a)^(1/3)*log(12*a*x^(1/3) + (9*(-a)^(4/3)*((3^(1/2)*2i)/3
 - 2/3))/b^(1/3))*((3^(1/2)*2i)/3 - 2/3))/b^(7/3)
```

3.684 $\int \frac{x^{2/3}}{(a+bx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{x^{2/3}}{b(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}$$

```
[Out] -x^(2/3)/b/(b*x+a)-ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(5/3)+1/3*ln(b*x+a
)/a^(1/3)/b^(5/3)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2
))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 58, 631, 210, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(
1/3)*b^(5/3)) + Log[a + b*x]/(3*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = -\frac{x^{2/3}}{b(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b}$$

$$= -\frac{x^{2/3}}{b(a+bx)} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx \ldots\right.}{\sqrt[3]{a}\,b^{5/3}}$$

$$= -\frac{x^{2/3}}{b(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}} \ldots\right.}{\sqrt[3]{a}\,b^{5/3}}$$

$$= -\frac{x^{2/3}}{b(a+bx)} - \frac{2\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}$$

Mathematica [A]
time = 0.16, size = 133, normalized size = 1.16

$$\frac{-\frac{3b^{2/3}x^{2/3}}{a+bx} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}} + \frac{\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{\sqrt[3]{a}}}{3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] ((-3*b^(2/3)*x^(2/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3)
)/a^(1/3))/Sqrt[3]])/a^(1/3) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(1/3) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/a^(1/3))/(3*b^(5/
3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 94.06, size = 506, normalized size = 4.40

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{1}{3}}}\right],a==0\&\&b==0\right\},\left\{\frac{-3}{b^2x^{\frac{1}{3}}},a==0\right\},\left\{\frac{3x^{\frac{5}{3}}}{5a^2},b==0\right\}\right\},-\frac{a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2a\text{Log}[2]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2a\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3bx^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{bx\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,bx\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2bx\text{Log}[2]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2bx\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{3ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3b^3x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(2/3)/(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (1 / 3)], a == 0 && b == 0}, {-3 / (b
^ 2 x ^ (1 / 3)), a == 0}, {3 x ^ (5 / 3) / (5 a ^ 2), b == 0}}, -a Log[4 x
 ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (3
a b ^ 2 (-a / b) ^ (1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) + 2 Sqrt[3] a Ar
cTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (3 a b
 ^ 2 (-a / b) ^ (1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) + 2 a Log[2] / (3 a
 b ^ 2 (-a / b) ^ (1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) + 2 a Log[x ^ (1
/ 3) - (-a / b) ^ (1 / 3)] / (3 a b ^ 2 (-a / b) ^ (1 / 3) + 3 b ^ 3 x (-a
/ b) ^ (1 / 3)) - 3 b x ^ (2 / 3) (-a / b) ^ (1 / 3) / (3 a b ^ 2 (-a / b)
^ (1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) - b x Log[4 x ^ (1 / 3) (-a / b)
^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (3 a b ^ 2 (-a / b) ^ (
1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) + 2 Sqrt[3] b x ArcTan[Sqrt[3] / 3 +
 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (3 a b ^ 2 (-a / b) ^ (1
 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) + 2 b x Log[2] / (3 a b ^ 2 (-a / b)
^ (1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)) + 2 b x Log[x ^ (1 / 3) - (-a / b
) ^ (1 / 3)] / (3 a b ^ 2 (-a / b) ^ (1 / 3) + 3 b ^ 3 x (-a / b) ^ (1 / 3)
)]
```

Maple [A]
time = 0.11, size = 118, normalized size = 1.03

method	result	size
derivativedivides	$-\frac{x^{\frac{2}{3}}}{b(bx+a)}+\frac{-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	118

default	$-\frac{x^{\frac{2}{3}}}{b(bx+a)}+\frac{-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(2/3)/b/(b*x+a)+2/b*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a
/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.36, size = 120, normalized size = 1.04

$$-\frac{x^{\frac{2}{3}}}{b^2x+ab}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(2/3)/(b^2*x + a*b) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/
3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) - 2/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(84) = 168.
time = 0.33, size = 394, normalized size = 3.43

$$\left[-\frac{3\,ab^2x^{\frac{2}{3}}-3\,\sqrt{\frac{1}{3}}\,(ab^2x+a^2b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x-ab+3\,\sqrt{\frac{1}{3}}\,\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)-(-ab^2)^{\frac{2}{3}}\,(bx+a)\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)+2\,(-ab^2)^{\frac{2}{3}}\,(bx+a)\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{3\,(ab^4x+a^2b^3)},\ -\frac{3\,ab^2x^{\frac{2}{3}}-6\,\sqrt{\frac{1}{3}}\,(ab^2x+a^2b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\,\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)-(-ab^2)^{\frac{2}{3}}\,(bx+a)\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)+2\,(-ab^2)^{\frac{2}{3}}\,(bx+a)\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{3\,(ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a*b^2*x^(2/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*
x + a)) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/
3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^
```

```
(1/3)))/(a*b^4*x + a^2*b^3), -1/3*(3*a*b^2*x^(2/3) - 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^4*x + a^2*b^3)]
```

Sympy [A]
time = 95.94, size = 527, normalized size = 4.58

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{5}{3}}}{5a^2} & \text{for}\: b = 0 \\ -\frac{3}{b^2\sqrt[3]{x}} & \text{for}\: a = 0 \\ \frac{2a\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{a\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}a\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2a\log(2)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{3bx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{bx\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}bx\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log(2)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a**2), Eq(b,
0)), (-3/(b**2*x**(1/3)), Eq(a, 0)), (2*a*log(x**(1/3) - (-a/b)**(1/3))/(3*
a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - a*log(4*x**(2/3) + 4*x**(1
/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a
/b)**(1/3)) + 2*sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(
3)/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*a*log(2)/(3*a*b
**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - 3*b*x**(2/3)*(-a/b)**(1/3)/(3
*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*b*x*log(x**(1/3) - (-a/
b)**(1/3))/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - b*x*log(4*x*
*(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2*(-a/b)**(1/3
) + 3*b**3*x*(-a/b)**(1/3)) + 2*sqrt(3)*b*x*atan(2*sqrt(3)*x**(1/3)/(3*(-a/
b)**(1/3)) + sqrt(3)/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) +
 2*b*x*log(2)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)), True))
```

Giac [A]
time = 0.01, size = 198, normalized size = 1.72

$$3\left(\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^3}-\frac{\frac{1}{3}\cdot 2\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab^3}-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot 3ba}-\frac{\frac{1}{3}\left(x^{\frac{1}{3}}\right)^2}{b\left(xb+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x)
```

```
[Out] -2/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) - x^(2/3)/((b*x +
a)*b) - 2/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

Mupad [B]
time = 0.24, size = 142, normalized size = 1.23

$$\frac{2\ln\left(\frac{4x^{1/3}}{b}-\frac{4(-a)^{1/3}}{b^{4/3}}\right)}{3(-a)^{1/3}b^{5/3}}-\frac{x^{2/3}}{b\,(a+b\,x)}+\frac{\ln\left(\frac{4x^{1/3}}{b}-\frac{(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{b^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3(-a)^{1/3}b^{5/3}}-\frac{\ln\left(\frac{4x^{1/3}}{b}-\frac{(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{b^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3(-a)^{1/3}b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^2,x)
```

```
[Out] (2*log((4*x^(1/3))/b - (4*(-a)^(1/3))/b^(4/3)))/(3*(-a)^(1/3)*b^(5/3)) - x^
(2/3)/(b*(a + b*x)) + (log((4*x^(1/3))/b - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2)/
b^(4/3))*(3^(1/2)*1i - 1))/(3*(-a)^(1/3)*b^(5/3)) - (log((4*x^(1/3))/b - ((
-a)^(1/3)*(3^(1/2)*1i + 1)^2)/b^(4/3))*(3^(1/2)*1i + 1))/(3*(-a)^(1/3)*b^(5
/3))
```

3.685 $\int \frac{\sqrt[3]{x}}{(a+bx)^2} dx$

Optimal. Leaf size=117

$$-\frac{\sqrt[3]{x}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

```
[Out] -x^(1/3)/b/(b*x+a)+1/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(4/3)-1/6*ln(b
*x+a)/a^(2/3)/b^(4/3)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^
(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 60, 631, 210, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(2/
3)*b^(4/3)) - Log[a + b*x]/(6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{\sqrt[3]{x}}{b(a+bx)}+\frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{5/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx\right.}{2a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right.}{a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

Mathematica [A]
time = 0.15, size = 134, normalized size = 1.15

$$\frac{-\frac{6\sqrt[3]{b}\,\sqrt[3]{x}}{a+bx}-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{a^{2/3}}}{6b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] ((-6*b^(1/3)*x^(1/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3)
)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(2/3) -
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/a^(2/3))/(6*b^(4/
3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 59.10, size = 425, normalized size = 3.63

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{2}{3}}}\right],a==0\&\&b==0\right\},\left\{\frac{-3}{2b^2x^{\frac{2}{3}}},a==0\right\},\left\{\frac{3x^{\frac{4}{3}}}{4a^2},b==0\right\}\right\},\frac{-2a\text{Log}[2]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}-\frac{2a\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}+\frac{2\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}+\frac{a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}-\frac{6ax^{\frac{1}{3}}}{6a^2b+6ab^2x}-\frac{2bx\text{Log}[2]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}-\frac{2bx\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}+\frac{2\sqrt{3}\,bx\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}+\frac{bx\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b+6ab^2x}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1/3)/(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (2 / 3)], a == 0 && b == 0}, {-3 / (2
b ^ 2 x ^ (2 / 3)), a == 0}, {3 x ^ (4 / 3) / (4 a ^ 2), b == 0}}, -2 a Log
[2] (-a / b) ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x) - 2 a Log[x ^ (1 / 3) -
(-a / b) ^ (1 / 3)] (-a / b) ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x) + 2 Sqrt
[3] a ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))]
(-a / b) ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x) + a Log[4 x ^ (1 / 3) (-a /
b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b) ^ (1 / 3) / (
6 a ^ 2 b + 6 a b ^ 2 x) - 6 a x ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x) - 2
b x Log[2] (-a / b) ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x) - 2 b x Log[x ^ (
1 / 3) - (-a / b) ^ (1 / 3)] (-a / b) ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x)
 + 2 Sqrt[3] b x ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^
 (1 / 3))] (-a / b) ^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x) + b x Log[4 x ^ (
1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b)
^ (1 / 3) / (6 a ^ 2 b + 6 a b ^ 2 x)]
```

Maple [A]
time = 0.10, size = 117, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{x^{\frac{1}{3}}}{b(bx+a)}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b}$	117

default	$-\frac{x^{\frac{1}{3}}}{b(bx+a)}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/3)/b/(b*x+a)+1/b*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.35, size = 120, normalized size = 1.03

$$-\frac{x^{\frac{1}{3}}}{b^2x+ab}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(1/3)/(b^2*x + a*b) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/
3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(84) = 168.
time = 0.33, size = 389, normalized size = 3.32

$$\left[-\frac{6a^2bx^{\frac{1}{3}}-3\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)+(a^2b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)-2(a^2b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)}{6(a^2b^3x+a^3b^2)},-\frac{6a^2bx^{\frac{1}{3}}-6\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a-2(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)+(a^2b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)-2(a^2b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)}{6(a^2b^3x+a^3b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a^2*b*x^(1/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/
b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2
*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
 + a)) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b
)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3
```

```
)))/(a^2*b^3*x + a^3*b^2), -1/6*(6*a^2*b*x^(1/3) - 6*sqrt(1/3)*(a*b^2*x + a
^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^
(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(b*x + a)*log(a*b
*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x
+ a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [A]
time = 60.30, size = 450, normalized size = 3.85

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a^2} & \text{for}\: b = 0 \\ -\frac{3}{2b^2x^{\frac{2}{3}}} & \text{for}\: a = 0 \\ -\frac{6a\sqrt[3]{x}}{6a^2b+6ab^2x} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^2b+6ab^2x} + \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b+6ab^2x} + \frac{2\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^2b+6ab^2x} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log(2)}{6a^2b+6ab^2x} - \frac{2bx\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^2b+6ab^2x} + \frac{bx\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b+6ab^2x} + \frac{2\sqrt{3}bx\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^2b+6ab^2x} - \frac{2bx\sqrt[3]{-\frac{a}{b}}\log(2)}{6a^2b+6ab^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**2), Eq(b,
0)), (-3/(2*b**2*x**(2/3)), Eq(a, 0)), (-6*a*x**(1/3)/(6*a**2*b + 6*a*b**2*
x) - 2*a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(6*a**2*b + 6*a*b**2*x
) + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(
2/3))/(6*a**2*b + 6*a*b**2*x) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x*
*(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**2*b + 6*a*b**2*x) - 2*a*(-a/b)*
*(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x) - 2*b*x*(-a/b)**(1/3)*log(x**(1/3) -
(-a/b)**(1/3))/(6*a**2*b + 6*a*b**2*x) + b*x*(-a/b)**(1/3)*log(4*x**(2/3) +
 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b + 6*a*b**2*x) + 2*sq
rt(3)*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)
/3)/(6*a**2*b + 6*a*b**2*x) - 2*b*x*(-a/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b*
*2*x), True))
```

Giac [A]
time = 0.01, size = 180, normalized size = 1.54

$$3\left(\frac{(-ab^2)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18ab^2}+\frac{\frac{1}{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab^2}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot 3ba}-\frac{\frac{1}{3}x^{\frac{1}{3}}}{b(xb+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x)
```

```
[Out] -1/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^2) - x^(1/3)/((b*x + a)*b) + 1/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B]
time = 0.06, size = 120, normalized size = 1.03

$$\frac{\ln\left(3\,b\,x^{1/3}+3\,a^{1/3}\,b^{2/3}\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{x^{1/3}}{b\,\left(a+b\,x\right)}+\frac{\ln\left(3\,b\,x^{1/3}+\frac{3\,a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(3\,b\,x^{1/3}-\frac{3\,a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^2,x)
```

```
[Out] log(3*b*x^(1/3) + 3*a^(1/3)*b^(2/3))/(3*a^(2/3)*b^(4/3)) - x^(1/3)/(b*(a +
b*x)) + (log(3*b*x^(1/3) + (3*a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)
*1i - 1))/(6*a^(2/3)*b^(4/3)) - (log(3*b*x^(1/3) - (3*a^(1/3)*b^(2/3)*(3^(1
/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(4/3))
```

3.686 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx$

Optimal. Leaf size=116

$$\frac{x^{2/3}}{a(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}$$

```
[Out] x^(2/3)/a/(b*x+a)-1/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(4/3)/b^(2/3)+1/6*ln(b*
x+a)/a^(4/3)/b^(2/3)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(
1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 58, 631, 210, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/
3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(4/3)*
b^(2/3)) + Log[a + b*x]/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx &= \frac{x^{2/3}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a} \\
&= \frac{x^{2/3}}{a(a+bx)} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2ab} - \frac{\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\right.}{2a^{4/3}b^{2}} \\
&= \frac{x^{2/3}}{a(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,\sqrt[3]{}}{\sqrt[3]{a}}\right.}{a^{4/3}b^{2/3}} \\
&= \frac{x^{2/3}}{a(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 133, normalized size = 1.15

$$
\frac{\frac{6\sqrt[3]{a}\,x^{2/3}}{a+bx} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{b^{2/3}}}{6a^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] ((6*a^(1/3)*x^(2/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))
/a^(1/3))/Sqrt[3]])/b^(2/3) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(2/3) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/b^(2/3))/(6*a^(4/3
))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 60.09, size = 515, normalized size = 4.44

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{4}{3}}}\right],a==0\&\&b==0\right\},\left\{\frac{-3}{4b^2x^{\frac{4}{3}}},a==0\right\},\left\{\frac{3x^{\frac{2}{3}}}{2a^2},b==0\right\}\right\},-\frac{a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2a\text{Log}[2]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2a\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{6bx^{\frac{2}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{bx\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,bx\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2bx\text{Log}[2]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2bx\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{6a^2b\left(-\frac{a}{b}\right)^{\frac{1}{3}}+6ab^2x\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(1/3)*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (4 / 3)], a == 0 && b == 0}, {-3 / (4
b ^ 2 x ^ (4 / 3)), a == 0}, {3 x ^ (2 / 3) / (2 a ^ 2), b == 0}}, -a Log[4
 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (
6 a ^ 2 b (-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) + 2 Sqrt[3]
a ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (6
 a ^ 2 b (-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) + 2 a Log[2]
/ (6 a ^ 2 b (-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) + 2 a Log
[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] / (6 a ^ 2 b (-a / b) ^ (1 / 3) + 6 a b
^ 2 x (-a / b) ^ (1 / 3)) + 6 b x ^ (2 / 3) (-a / b) ^ (1 / 3) / (6 a ^ 2 b
 (-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) - b x Log[4 x ^ (1 /
3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] / (6 a ^ 2 b
(-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) + 2 Sqrt[3] b x ArcTan
[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (6 a ^ 2 b
 (-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) + 2 b x Log[2] / (6 a
 ^ 2 b (-a / b) ^ (1 / 3) + 6 a b ^ 2 x (-a / b) ^ (1 / 3)) + 2 b x Log[x ^
 (1 / 3) - (-a / b) ^ (1 / 3)] / (6 a ^ 2 b (-a / b) ^ (1 / 3) + 6 a b ^ 2
x (-a / b) ^ (1 / 3))]
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.00

method	result	size
derivativedivides	$\frac{x^{\frac{2}{3}}}{a(bx+a)}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{a}$	116

default	$\frac{x^{\frac{2}{3}}}{a(bx+a)}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(2/3)/a/(b*x+a)+1/a*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/
b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.36, size = 127, normalized size = 1.09

$$\frac{x^{\frac{2}{3}}}{abx+a^{2}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(2/3)/(a*b*x + a^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*(a/b)^(1/3)) - 1/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(83) = 166.
time = 0.33, size = 396, normalized size = 3.41

$$\left[\frac{6\,ab^{2}x^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\,(ab^{2}x+a^{2}b)\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\log\left(\frac{2b^{2}x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^{2})^{\frac{1}{3}}a+2\,(-ab^{2})^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^{2})^{\frac{1}{3}}}{a}}-3\,(-ab^{2})^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+(-ab^{2})^{\frac{2}{3}}(bx+a)\log\left(b^{2}x^{\frac{2}{3}}+(-ab^{2})^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^{2})^{\frac{2}{3}}\right)-2\,(-ab^{2})^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-(-ab^{2})^{\frac{1}{3}}\right)}{6\,(a^{2}b^{3}x+a^{3}b^{2})},\ \frac{6\,ab^{2}x^{\frac{2}{3}}+6\sqrt{\frac{1}{3}}\,(ab^{2}x+a^{2}b)\sqrt{-\frac{(-ab^{2})^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+(-ab^{2})^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^{2})^{\frac{1}{3}}}{a}}}{b}\right)+(-ab^{2})^{\frac{2}{3}}(bx+a)\log\left(b^{2}x^{\frac{2}{3}}+(-ab^{2})^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^{2})^{\frac{2}{3}}\right)-2\,(-ab^{2})^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-(-ab^{2})^{\frac{1}{3}}\right)}{6\,(a^{2}b^{3}x+a^{3}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*b^2*x^(2/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3
) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^(
```

```
1/3)))/(a^2*b^3*x + a^3*b^2), 1/6*(6*a*b^2*x^(2/3) + 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [**A**]
time = 61.77, size = 544, normalized size = 4.69

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a^2} & \text{for}\: b = 0 \\ -\frac{3}{4b^2x^{\frac{4}{3}}} & \text{for}\: a = 0 \\ \frac{2a\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} - \frac{a\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}\, a\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2a\log(2)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{6bx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} - \frac{bx\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}\, bx\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log(2)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**2), Eq(b,
0)), (-3/(4*b**2*x**(4/3)), Eq(a, 0)), (2*a*log(x**(1/3) - (-a/b)**(1/3))/(
6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) - a*log(4*x**(2/3) + 4*x
**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2
*x*(-a/b)**(1/3)) + 2*sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) +
 sqrt(3)/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*a*log(2
)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 6*b*x**(2/3)*(-a/b)
**(1/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*b*x*log(x**
(1/3) - (-a/b)**(1/3))/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3))
- b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*
b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*sqrt(3)*b*x*atan(2*sqrt(3)*
x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*
x*(-a/b)**(1/3)) + 2*b*x*log(2)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)
**(1/3)), True))
```

Giac [**A**]
time = 0.01, size = 200, normalized size = 1.72

$$3\left(\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\ln\left(\left(x^{\frac{1}{3}}\right)^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18a^2b^2} - \frac{\frac{1}{3}\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2b^2} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot 3a^2} + \frac{\frac{1}{3}\left(x^{\frac{1}{3}}\right)^2}{a\left(xb + a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x)
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + x^(2/3)/((b*x + a)
*a) - 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/
```

```
3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

Mupad [B]
time = 0.36, size = 144, normalized size = 1.24

$$\frac{x^{2/3}}{a\,\left(a+b\,x\right)}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{{\left(-1\right)}^{2/3}\,b^{2/3}}{a^{5/3}}+\frac{b\,x^{1/3}}{a^2}\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{4/3}\,b^{2/3}}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{9\,{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{a^{5/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{4/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^2),x)
```

```
[Out] x^(2/3)/(a*(a + b*x)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/a^(5/3) + (b*x
^(1/3))/a^2))/(3*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((b*x^(1/3))/a^2 + ((-1)
^(2/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(5/3))*((3^(1/2)*1i)/2 + 1/2))/(
3*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*log((b*x^(1/3))/a^2 + (9*(-1)^(2/3)*b^(2/3
)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(5/3))*((3^(1/2)*1i)/6 - 1/6))/(a^(4/3)*b^(2/
3))
```

3.687 $\int \frac{1}{x^{2/3}(a+bx)^2} \, dx$

Optimal. Leaf size=113

$$\frac{\sqrt[3]{x}}{a(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}$$

```
[Out] x^(1/3)/a/(b*x+a)+ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(5/3)/b^(1/3)-1/3*ln(b*x+a)
/a^(5/3)/b^(1/3)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2)
)/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 60, 631, 210, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(5/3
)*b^(1/3)) - Log[a + b*x]/(3*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)^2}\,dx = \frac{\sqrt[3]{x}}{a(a+bx)} + \frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a}$$

$$= \frac{\sqrt[3]{x}}{a(a+bx)} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x \cdots}\right.}{a^{5/3}\sqrt[3]{b}}$$

$$= \frac{\sqrt[3]{x}}{a(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}} + \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b} \cdots}{\sqrt[3]{a}}\right.}{a^{5/3}\sqrt[3]{b}}$$

$$= \frac{\sqrt[3]{x}}{a(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.14, size = 134, normalized size = 1.19

$$\frac{\frac{3a^{2/3}\sqrt[3]{x}}{a+bx} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{b}}}{3a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] ((3*a^(2/3)*x^(1/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))
/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(1/3) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/b^(1/3))/(3*a^(5/3
))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 39.78, size = 416, normalized size = 3.68

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{5}{3}}}\right],a==0\&\&b==0\right\},\left\{\frac{-3}{5b^2x^{\frac{5}{3}}},a==0\right\},\left\{\frac{3x^{\frac{1}{3}}}{a^2},b==0\right\}\right\},\frac{-2a\text{Log}[2]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}-\frac{2a\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}+\frac{2\sqrt{3}\,a\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}+\frac{a\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}+\frac{3ax^{\frac{1}{3}}}{3a^3+3a^2bx}-\frac{2bx\text{Log}[2]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}-\frac{2bx\text{Log}\left[x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}+\frac{2\sqrt{3}\,bx\text{ArcTan}\left[\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}+\frac{bx\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4x^{\frac{2}{3}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3+3a^2bx}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(2/3)*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (5 / 3)], a == 0 && b == 0}, {-3 / (5
b ^ 2 x ^ (5 / 3)), a == 0}, {3 x ^ (1 / 3) / a ^ 2, b == 0}}, -2 a Log[2]
(-a / b) ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) - 2 a Log[x ^ (1 / 3) - (-a /
b) ^ (1 / 3)] (-a / b) ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) + 2 Sqrt[3] a Ar
cTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] (-a / b)
 ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) + a Log[4 x ^ (1 / 3) (-a / b) ^ (1 /
3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b) ^ (1 / 3) / (3 a ^ 3 +
3 a ^ 2 b x) + 3 a x ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) - 2 b x Log[2] (-a
 / b) ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) - 2 b x Log[x ^ (1 / 3) - (-a / b
) ^ (1 / 3)] (-a / b) ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) + 2 Sqrt[3] b x A
rcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] (-a / b
) ^ (1 / 3) / (3 a ^ 3 + 3 a ^ 2 b x) + b x Log[4 x ^ (1 / 3) (-a / b) ^ (1
 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^ (2 / 3)] (-a / b) ^ (1 / 3) / (3 a ^ 3
 + 3 a ^ 2 b x)]
```

Maple [A]
time = 0.11, size = 117, normalized size = 1.04

method	result	size
derivativedivides	$\frac{x^{\frac{1}{3}}}{a(bx+a)}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	117

default	$\frac{x^{\frac{1}{3}}}{a(bx+a)}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/3)/a/(b*x+a)+2/a*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b
)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.35, size = 127, normalized size = 1.12

$$\frac{x^{\frac{1}{3}}}{abx+a^2}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(1/3)/(a*b*x + a^2) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(82) = 164$.
time = 0.32, size = 387, normalized size = 3.42

$$\left[\frac{3\,a^2bx^{\frac{1}{3}}+3\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{3\left(a^3b^2x+a^4b\right)},\ \frac{3\,a^2bx^{\frac{1}{3}}+6\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{3\left(a^3b^2x+a^4b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^2*b*x^(1/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)
^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)
```

))/(a^3*b^2*x + a^4*b), 1/3*(3*a^2*b*x^(1/3) + 6*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^3*b^2*x + a^4*b)]

Sympy [A]
time = 109.52, size = 434, normalized size = 3.84

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3 \sqrt[3]{x}}{a^{2}} & \text{for}\: b = 0 \\ - \frac{3}{5 b^{2} x^{\frac{5}{3}}} & \text{for}\: a = 0 \\ \frac{3 a \sqrt[3]{x}}{3 a^{3} + 3 a^{2} b x} - \frac{2 a \sqrt[3]{- \frac{a}{b}} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{3 a^{3} + 3 a^{2} b x} + \frac{a \sqrt[3]{- \frac{a}{b}} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{3 a^{3} + 3 a^{2} b x} + \frac{2 \sqrt{3} a \sqrt[3]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{3 a^{3} + 3 a^{2} b x} - \frac{2 a \sqrt[3]{- \frac{a}{b}} \log{\left(2 \right)}}{3 a^{3} + 3 a^{2} b x} - \frac{2 b x \sqrt[3]{- \frac{a}{b}} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{3 a^{3} + 3 a^{2} b x} + \frac{b x \sqrt[3]{- \frac{a}{b}} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{3 a^{3} + 3 a^{2} b x} + \frac{2 \sqrt{3} b x \sqrt[3]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{3 a^{3} + 3 a^{2} b x} - \frac{2 b x \sqrt[3]{- \frac{a}{b}} \log{\left(2 \right)}}{3 a^{3} + 3 a^{2} b x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(2/3)/(b*x+a)**2,x)

[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a**2, Eq(b, 0)), (-3/(5*b**2*x**(5/3)), Eq(a, 0)), (3*a*x**(1/3)/(3*a**3 + 3*a**2*b*x) - 2*a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a**3 + 3*a**2*b*x) + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a**3 + 3*a**2*b*x) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3 + 3*a**2*b*x) - 2*a*(-a/b)**(1/3)*log(2)/(3*a**3 + 3*a**2*b*x) - 2*b*x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a**3 + 3*a**2*b*x) + b*x*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a**3 + 3*a**2*b*x) + 2*sqrt(3)*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3 + 3*a**2*b*x) - 2*b*x*(-a/b)**(1/3)*log(2)/(3*a**3 + 3*a**2*b*x), True))

Giac [A]
time = 0.00, size = 182, normalized size = 1.61

$$3 \left(\frac{\left(-ab^{2}\right)^{\frac{1}{3}} \ln \left(\left(x^{\frac{1}{3}}\right)^{2} + \left(-\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{9a^{2}b} + \frac{\frac{1}{3} \cdot 2 \left(-ab^{2}\right)^{\frac{1}{3}} \arctan \left(\frac{2 \left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2} \right)}{\sqrt{3} \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{\sqrt{3}\, a^{2}b} - \frac{2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \ln \left| x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right|}{3 \cdot 3a^{2}} + \frac{\frac{1}{3} x^{\frac{1}{3}}}{a \left(xb + a\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(2/3)/(b*x+a)^2,x)

[Out] -2/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + 2/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) + x^(1/3)/((b*x + a)*a) + 1/3*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b)

Mupad [B]
time = 0.22, size = 134, normalized size = 1.19

$$\frac{2\ln\left(\frac{6b^{5/3}}{a^{2/3}}+\frac{6b^2x^{1/3}}{a}\right)}{3a^{5/3}b^{1/3}}+\frac{x^{1/3}}{a\,(a+b\,x)}+\frac{\ln\left(\frac{6b^2x^{1/3}}{a}+\frac{3b^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3a^{5/3}b^{1/3}}-\frac{\ln\left(\frac{6b^2x^{1/3}}{a}-\frac{3b^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3a^{5/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^2),x)
```

```
[Out] (2*log((6*b^(5/3))/a^(2/3) + (6*b^2*x^(1/3))/a))/(3*a^(5/3)*b^(1/3)) + x^(1
/3)/(a*(a + b*x)) + (log((6*b^2*x^(1/3))/a + (3*b^(5/3)*(3^(1/2)*1i - 1))/a
^(2/3))*(3^(1/2)*1i - 1))/(3*a^(5/3)*b^(1/3)) - (log((6*b^2*x^(1/3))/a - (3
*b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(3*a^(5/3)*b^(1/3))
```

3.688 $\int \frac{1}{x^{4/3}(a+bx)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}\,(a+bx)}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}}$$

```
[Out] -4/a^2/x^(1/3)+1/a/x^(1/3)/(b*x+a)+2*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^
(7/3)-2/3*b^(1/3)*ln(b*x+a)/a^(7/3)+4/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 58, 631, 210, 31}

$$\frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] -4/(a^2*x^(1/3)) + 1/(a*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/a^(7/3) - (2*b^(1/3)*Log[a + b*x])/(3*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\int \frac{1}{x^{4/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{(4b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right.}{a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{\left(4\sqrt[3]{}\right.}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right.}{a^{7/3}}$$

Mathematica [A]
time = 0.19, size = 142, normalized size = 1.15

$$\frac{-\frac{3\sqrt[3]{a}\,(3a+4bx)}{\sqrt[3]{x}\,(a+bx)} + 4\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) - 2\sqrt[3]{b}\,\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{3a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] ((-3*a^(1/3)*(3*a + 4*b*x))/(x^(1/3)*(a + b*x)) + 4*Sqrt[3]*b^(1/3)*ArcTan[
(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3
)*x^(1/3)] - 2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2
/3)])/(3*a^(7/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 83.36, size = 616, normalized size = 4.97

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{x^{\frac{7}{3}}}\right], a==0\&\&b==0\right\}, \left\{\frac{-3}{7b^2x^{\frac{7}{3}}}, a==0\right\}, \left\{\frac{-3}{a^2x^{\frac{1}{3}}}, b==0\right\}\right\}, \frac{-9a\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\sqrt{3}\,ax^{\frac{1}{3}}\text{ArcTan}\left[\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4ax^{\frac{1}{3}}\text{Log}[2]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4ax^{\frac{1}{3}}\text{Log}\left[x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2ax^{\frac{1}{3}}\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4x^{\frac{2}{3}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{12bx\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\sqrt{3}\,bx^{\frac{4}{3}}\text{ArcTan}\left[\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\,x^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4bx^{\frac{4}{3}}\text{Log}[2]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4bx^{\frac{4}{3}}\text{Log}\left[x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2bx^{\frac{4}{3}}\text{Log}\left[4x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 4x^{\frac{2}{3}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right]}{3a^3x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3a^2bx^{\frac{4}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(4/3)*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / x ^ (7 / 3)], a == 0 && b == 0}, {-3 / (7
b ^ 2 x ^ (7 / 3)), a == 0}, {-3 / (a ^ 2 x ^ (1 / 3)), b == 0}}, -9 a (-a
/ b) ^ (1 / 3) / (3 a ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4
 / 3) (-a / b) ^ (1 / 3)) - 4 Sqrt[3] a x ^ (1 / 3) ArcTan[Sqrt[3] / 3 + 2
Sqrt[3] x ^ (1 / 3) / (3 (-a / b) ^ (1 / 3))] / (3 a ^ 3 x ^ (1 / 3) (-a /
b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3) (-a / b) ^ (1 / 3)) - 4 a x ^ (1 / 3)
Log[2] / (3 a ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3) (-
a / b) ^ (1 / 3)) - 4 a x ^ (1 / 3) Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] /
 (3 a ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3) (-a / b) ^
 (1 / 3)) + 2 a x ^ (1 / 3) Log[4 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2
 / 3) + 4 (-a / b) ^ (2 / 3)] / (3 a ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3
 a ^ 2 b x ^ (4 / 3) (-a / b) ^ (1 / 3)) - 12 b x (-a / b) ^ (1 / 3) / (3 a
 ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3) (-a / b) ^ (1 /
 3)) - 4 Sqrt[3] b x ^ (4 / 3) ArcTan[Sqrt[3] / 3 + 2 Sqrt[3] x ^ (1 / 3) /
 (3 (-a / b) ^ (1 / 3))] / (3 a ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^
2 b x ^ (4 / 3) (-a / b) ^ (1 / 3)) - 4 b x ^ (4 / 3) Log[2] / (3 a ^ 3 x ^
 (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3) (-a / b) ^ (1 / 3)) - 4
 b x ^ (4 / 3) Log[x ^ (1 / 3) - (-a / b) ^ (1 / 3)] / (3 a ^ 3 x ^ (1 / 3)
 (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3) (-a / b) ^ (1 / 3)) + 2 b x ^ (
4 / 3) Log[4 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 4 x ^ (2 / 3) + 4 (-a / b) ^
(2 / 3)] / (3 a ^ 3 x ^ (1 / 3) (-a / b) ^ (1 / 3) + 3 a ^ 2 b x ^ (4 / 3)
(-a / b) ^ (1 / 3))]
```

Maple [A]

time = 0.14, size = 124, normalized size = 1.00

method	result
risch	$-\frac{3}{a^{2} x^{\frac{1}{3}}}-\frac{b\,x^{\frac{2}{3}}}{a^{2} \left(b x +a \right)}+\frac{4 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 a^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3 a^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 a^{2} \left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$-\frac{3}{a^{2} x^{\frac{1}{3}}}-\frac{3 b \left(\frac{x^{\frac{2}{3}}}{3 b x +3 a}-\frac{4 \ln \left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2 \ln \left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$

default	$-\frac{3}{a^{2}x^{\frac{1}{3}}}-\frac{3b\left(\frac{x^{\frac{2}{3}}}{3bx+3a}-\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/a^2/x^(1/3)-3*b/a^2*(1/3*x^(2/3)/(b*x+a)-4/9/b/(a/b)^(1/3)*ln(x^(1/3)+(a
/b)^(1/3))+2/9/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+4/
9*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.35, size = 132, normalized size = 1.06

$$-\frac{4\,bx+3\,a}{a^{2}bx^{\frac{4}{3}}+a^{3}x^{\frac{1}{3}}}-\frac{4\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x + 3*a)/(a^2*b*x^(4/3) + a^3*x^(1/3)) - 4/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/3*log(x^(2/
3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/3*log(x^(1/3)
 + (a/b)^(1/3))/(a^2*(a/b)^(1/3))
```

Fricas [A]
time = 0.32, size = 156, normalized size = 1.26

$$-\frac{4\,\sqrt{3}\,(bx^{2}+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+2\,(bx^{2}+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-4\,(bx^{2}+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\,(4\,bx+3\,a)x^{\frac{2}{3}}}{3\,(a^{2}bx^{2}+a^{3}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*(b*x^2 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^
(1/3) - 1/3*sqrt(3)) + 2*(b*x^2 + a*x)*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/
3) + b*x^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^2 + a*x)*(b/a)^(1/3)*log(a*(b/a)^(
2/3) + b*x^(1/3)) + 3*(4*b*x + 3*a)*x^(2/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 215, normalized size = 1.73

$$3\left(-\frac{2\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^{3}b}+\frac{\frac{1}{3}\cdot 4\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{3}b}+\frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot 3a^{3}}+\frac{\frac{1}{3}\left(-4xb-3a\right)}{a^{2}\left(x^{\frac{1}{3}}xb+x^{\frac{1}{3}}a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x)
```

```
[Out] 4/3*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 4/3*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*
b) - (4*b*x + 3*a)/((b*x^(4/3) + a*x^(1/3))*a^2) - 2/3*(-a*b^2)^(2/3)*log(x
^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

Mupad [B]
time = 0.15, size = 151, normalized size = 1.22

$$\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}+16\,a^2\,b^3\,x^{1/3}\right)}{3\,a^{7/3}}-\frac{\frac{3}{a}+\frac{4\,b\,x}{a^2}}{a\,x^{1/3}+b\,x^{4/3}}-\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+16\,a^2\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{7/3}}+\frac{b^{1/3}\,\ln\left(9\,a^{7/3}\,b^{8/3}\,{\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}^2+16\,a^2\,b^3\,x^{1/3}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^2),x)
```

```
[Out] (4*b^(1/3)*log(16*a^(7/3)*b^(8/3) + 16*a^2*b^3*x^(1/3)))/(3*a^(7/3)) - (3/a
 + (4*b*x)/a^2)/(a*x^(1/3) + b*x^(4/3)) - (4*b^(1/3)*log(16*a^(7/3)*b^(8/3)
*((3^(1/2)*1i)/2 + 1/2)^2 + 16*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*
a^(7/3)) + (b^(1/3)*log(9*a^(7/3)*b^(8/3)*((3^(1/2)*2i)/3 - 2/3)^2 + 16*a^2
*b^3*x^(1/3))*((3^(1/2)*2i)/3 - 2/3))/a^(7/3)
```

3.689 $\int \frac{1}{x^{5/3}(a+bx)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}$$

```
[Out] -5/2/a^2/x^(2/3)+1/a/x^(2/3)/(b*x+a)-5/2*b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3)
)/a^(8/3)+5/6*b^(2/3)*ln(b*x+a)/a^(8/3)+5/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 60, 631, 210, 31}

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] -5/(2*a^2*x^(2/3)) + 1/(a*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(8/3)) + (5*b^(2/3)*Log[a + b*x])/(6*a^(8/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{1}{ax^{2/3}(a+bx)} + \frac{5\int \frac{1}{x^{5/3}(a+bx)}\,dx}{3a}$$
$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{(5b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a^2}$$
$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx\right.}{2a^{7/3}}$$
$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{(5b^{2/}}{}$$
$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right.}{2a^{8/3}}$$

Mathematica [A]
time = 0.20, size = 142, normalized size = 1.11

$$\frac{-\frac{3a^{2/3}(3a+5bx)}{x^{2/3}(a+bx)} + 10\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10b^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) + 5b^{2/3}\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{6a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] ((-3*a^(2/3)*(3*a + 5*b*x))/(x^(2/3)*(a + b*x)) + 10*Sqrt[3]*b^(2/3)*ArcTan
[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 10*b^(2/3)*Log[a^(1/3) + b^(1
/3)*x^(1/3)] + 5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^
(2/3)])/(6*a^(8/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/3)*(a + b*x)^2),x]')
```

[Out] Timed out

Maple [A]

time = 0.22, size = 124, normalized size = 0.97

method	result	siz
derivativedivides	$-\frac{3}{2a^{2}x^{\frac{2}{3}}}-\frac{3b\left(\frac{x^{\frac{1}{3}}}{3bx+3a}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{2}}$	12
default	$-\frac{3}{2a^{2}x^{\frac{2}{3}}}-\frac{3b\left(\frac{x^{\frac{1}{3}}}{3bx+3a}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -3/2/a^2/x^(2/3)-3*b/a^2*(1/3*x^(1/3)/(b*x+a)+5/9/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-5/18/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+5/9/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))

Maxima [A]

time = 0.34, size = 132, normalized size = 1.03

$$-\frac{5\,bx+3\,a}{2\left(a^{2}bx^{\frac{5}{3}}+a^{3}x^{\frac{2}{3}}\right)}-\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*(5*b*x + 3*a)/(a^2*b*x^(5/3) + a^3*x^(2/3)) - 5/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/6*log(x

```
^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/3*log(x^(
1/3) + (a/b)^(1/3))/(a^2*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(91) = 182$.
time = 0.32, size = 189, normalized size = 1.48

$$\frac{10\sqrt{3}\,(bx^2+ax)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-5\,(bx^2+ax)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\,(bx^2+ax)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,(5\,bx+3\,a)x^{\frac{1}{3}}}{6\,(a^2bx^2+a^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*sqrt(3)*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1
/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*log
(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 10*(b
*x^2 + a*x)*(-b^2/a^2)^(1/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(5*b*x
 + 3*a)*x^(1/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 203, normalized size = 1.59

$$3\left(-\frac{\frac{1}{3}x^{\frac{1}{3}}b}{a^2\,(xb+a)}-\frac{\frac{1}{2}}{a^2\left(x^{\frac{1}{3}}\right)^2}-\frac{5\,(-ab^2)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18a^3}-\frac{\frac{1}{3}\cdot 5\,(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^3}+\frac{5b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{3\cdot 3a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x)
```

```
[Out] 5/3*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 5/3*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 -
 b*x^(1/3)/((b*x + a)*a^2) - 5/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^3 - 3/2/(a^2*x^(2/3))
```

Mupad [B]
time = 0.17, size = 166, normalized size = 1.30

$$\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}-15\,a^4\,b^3\,x^{1/3}\right)}{3\,a^{8/3}}-\frac{\frac{3}{2\,a}+\frac{5\,b\,x}{2\,a^2}}{a\,x^{2/3}+b\,x^{5/3}}+\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}-15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}-\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}+15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^2),x)
```

```
[Out] (5*(-1)^(1/3)*b^(2/3)*log(15*(-1)^(1/3)*a^(13/3)*b^(8/3) - 15*a^4*b^3*x^(1/
3)))/(3*a^(8/3)) - (3/(2*a) + (5*b*x)/(2*a^2))/(a*x^(2/3) + b*x^(5/3)) + (5
*(-1)^(1/3)*b^(2/3)*log(15*a^4*b^3*x^(1/3) - 15*(-1)^(1/3)*a^(13/3)*b^(8/3)
*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(8/3)) - (5*(-1)^(1/3
)*b^(2/3)*log(15*a^4*b^3*x^(1/3) + 15*(-1)^(1/3)*a^(13/3)*b^(8/3)*((3^(1/2)
*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3))
```

3.690 $\int \frac{x^{5/3}}{(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}$$

```
[Out] -1/2*x^(5/3)/b/(b*x+a)^2-5/6*x^(2/3)/b^2/(b*x+a)-5/6*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(1/3)/b^(8/3)+5/18*ln(b*x+a)/a^(1/3)/b^(8/3)-5/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 58, 631, 210, 31}

$$-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{x^{5/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(5/3)/(b*(a + b*x)^2) - (5*x^(2/3))/(6*b^2*(a + b*x)) - (5*ArcTan[(a
^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3))
 - (5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(1/3)*b^(8/3)) + (5*Log[a + b*x]
)/(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/3}}{(a+bx)^3}\,dx &= -\frac{x^{5/3}}{2b(a+bx)^2}+\frac{5\int \frac{x^{2/3}}{(a+bx)^2}\,dx}{6b}\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}+\frac{5\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9b^2}\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}+\frac{5\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{6b^3}-\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}+\frac{5\text{Subst}\left(\int \frac{1}{-3-}\right.}{3}\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log}{18}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 142, normalized size = 1.01

$$\frac{-\frac{3b^{2/3}x^{2/3}(5a+8bx)}{(a+bx)^2}-\frac{10\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}}+\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{a}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] ((-3*b^(2/3)*x^(2/3)*(5*a + 8*b*x))/(a + b*x)^2 - (10*Sqrt[3]*ArcTan[(1 - (
2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - (10*Log[a^(1/3) + b^(1/3)*x
^(1/3)])/a^(1/3) + (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/
3)])/a^(1/3))/(18*b^(8/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(5/3)/(a + b*x)^3,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.12, size = 130, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^2}}{(bx+a)^2}+\frac{-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^2}$	130
default	$\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^2}}{(bx+a)^2}+\frac{-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^2}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(-4/9*x^(5/3)/b-5/18*a*x^(2/3)/b^2)/(b*x+a)^2+5/3/b^2*(-1/3/b/(a/b)^(1/3)
*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(
a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(
1/3)-1)))
```

Maxima [A]
time = 0.36, size = 143, normalized size = 1.02

$$-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 5/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1
/3)) + 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/
3)) - 5/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(99) = 198.
time = 0.33, size = 506, normalized size = 3.61

$$\left[\frac{15\,\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x-ab+3\,\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+5\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-10\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)-3\,(8\,ab^3x+5\,a^2b^2)x^{\frac{2}{3}}}{18\,(ab^6x^2+2\,a^2b^5x+a^3b^4)},\ \frac{30\,\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+5\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-10\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)-3\,(8\,ab^3x+5\,a^2b^2)x^{\frac{2}{3}}}{18\,(ab^6x^2+2\,a^2b^5x+a^3b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b
^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^
2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3
))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
 - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 221, normalized size = 1.58

$$3\left(\frac{5\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{54ab^{4}}-\frac{\frac{1}{9}\cdot 5\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab^{4}}-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot 3b^{2}a}+\frac{\frac{1}{18}\left(-8\left(x^{\frac{1}{3}}\right)^{2}xb-5\left(x^{\frac{1}{3}}\right)^{2}a\right)}{b^{2}\left(xb+a\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x)
```

```
[Out] -5/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) - 5/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^4) - 1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/((b*x + a)^2*b^2) + 5/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

Mupad [B]
time = 0.17, size = 165, normalized size = 1.18

$$\frac{5\,\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{25\,{\left(-a\right)}^{1/3}}{9\,b^{10/3}}\right)}{9\,{\left(-a\right)}^{1/3}\,b^{8/3}}-\frac{\frac{4\,x^{5/3}}{3\,b}+\frac{5\,a\,x^{2/3}}{6\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{{\left(-a\right)}^{1/3}\,{\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}^2}{36\,b^{10/3}}\right)\,\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,{\left(-a\right)}^{1/3}\,b^{8/3}}-\frac{\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{{\left(-a\right)}^{1/3}\,{\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}^2}{36\,b^{10/3}}\right)\,\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,{\left(-a\right)}^{1/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^3,x)
```

```
[Out] (5*log((25*x^(1/3))/(9*b^3) - (25*(-a)^(1/3))/(9*b^(10/3))))/(9*(-a)^(1/3)*
b^(8/3)) - ((4*x^(5/3))/(3*b) + (5*a*x^(2/3))/(6*b^2))/(a^2 + b^2*x^2 + 2*a
*b*x) + (log((25*x^(1/3))/(9*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(36*b^(
10/3)))*(3^(1/2)*5i - 5))/(18*(-a)^(1/3)*b^(8/3)) - (log((25*x^(1/3))/(9*b^
3) - ((-a)^(1/3)*(3^(1/2)*5i + 5)^2)/(36*b^(10/3)))*(3^(1/2)*5i + 5))/(18*(
-a)^(1/3)*b^(8/3))
```

3.691 $\int \frac{x^{4/3}}{(a+bx)^3} \, dx$

Optimal. Leaf size=140

$$-\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}$$

```
[Out] -1/2*x^(4/3)/b/(b*x+a)^2-2/3*x^(1/3)/b^2/(b*x+a)+1/3*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(2/3)/b^(7/3)-1/9*ln(b*x+a)/a^(2/3)/b^(7/3)-2/9*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 60, 631, 210, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(4/3)/(b*(a + b*x)^2) - (2*x^(1/3))/(3*b^2*(a + b*x)) - (2*ArcTan[(a
^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3))
 + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(7/3)) - Log[a + b*x]/(9*a^(
2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{4/3}}{(a+bx)^3}\,dx &= -\frac{x^{4/3}}{2b(a+bx)^2}+\frac{2\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx}{3b}\\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9b^2}\\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{3\sqrt[3]{a}\,b^{8/3}}+\text{S}\\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{3a}\\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a}{9a^2}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 142, normalized size = 1.01

$$\frac{-\frac{3\sqrt[3]{b}\sqrt[3]{x}(4a+7bx)}{(a+bx)^2}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{2/3}}-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{a^{2/3}}}{18b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] ((-3*b^(1/3)*x^(1/3)*(4*a + 7*b*x))/(a + b*x)^2 - (4*Sqrt[3]*ArcTan[(1 - (2
*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*Log[a^(1/3) + b^(1/3)*x^(
1/3)])/a^(2/3) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)
])/a^(2/3))/(18*b^(7/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(4/3)/(a + b*x)^3,x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.12, size = 130, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^2}}{(bx+a)^2}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^2}$	130
default	$\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^2}}{(bx+a)^2}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^2}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(-7/18*x^(4/3)/b-2/9*a*x^(1/3)/b^2)/(b*x+a)^2+2/3/b^2*(1/3/b/(a/b)^(2/3)*
ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a
/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1
/3)-1)))
```

Maxima [A]
time = 0.36, size = 143, normalized size = 1.02

$$-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 2/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2
/3)) - 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3
)) + 2/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(99) = 198.
time = 0.32, size = 503, normalized size = 3.59

$$\left[\frac{6\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3\,(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-2\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+4\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)-3\,(7\,a^2b^2x+4\,a^3b)x^{\frac{1}{3}}}{18\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)},\ \frac{12\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a-2\,(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-2\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+4\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)-3\,(7\,a^2b^2x+4\,a^3b)x^{\frac{1}{3}}}{18\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^
(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3
)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2
/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*
b^3)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 202, normalized size = 1.44

$$3\left(\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27ab^{3}}+\frac{\frac{1}{9}\cdot 2\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab^{3}}-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot 3b^{2}a}+\frac{\frac{1}{18}\left(-7x^{\frac{1}{3}}xb-4x^{\frac{1}{3}}a\right)}{b^{2}\left(xb+a\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x)
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) + 2/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^3) + 1/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a*b^3) - 1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.07, size = 139, normalized size = 0.99

$$\frac{2\,\ln\left(2\,x^{1/3}+\frac{2\,a^{1/3}}{b^{1/3}}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\frac{7\,x^{4/3}}{6\,b}+\frac{2\,a\,x^{1/3}}{3\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(2\,x^{1/3}+\frac{a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\ln\left(2\,x^{1/3}-\frac{a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^3,x)
```

```
[Out] (2*log(2*x^(1/3) + (2*a^(1/3))/b^(1/3)))/(9*a^(2/3)*b^(7/3)) - ((7*x^(4/3))
/(6*b) + (2*a*x^(1/3))/(3*b^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (log(2*x^(1/3)
+ (a^(1/3)*(3^(1/2)*1i - 1))/b^(1/3))*(3^(1/2)*1i - 1))/(9*a^(2/3)*b^(7/3))
 - (log(2*x^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1))/b^(1/3))*(3^(1/2)*1i + 1))/(
9*a^(2/3)*b^(7/3))
```

3.692 $\int \frac{x^{2/3}}{(a+bx)^3} dx$

Optimal. Leaf size=143

$$-\frac{x^{2/3}}{2b(a+bx)^2}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}$$

```
[Out] -1/2*x^(2/3)/b/(b*x+a)^2+1/3*x^(2/3)/a/b/(b*x+a)-1/6*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(4/3)/b^(5/3)+1/18*ln(b*x+a)/a^(4/3)/b^(5/3)-1/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 58, 631, 210, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(2/3)/(b*(a + b*x)^2) + x^(2/3)/(3*a*b*(a + b*x)) - ArcTan[(a^(1/3)
- 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a
^(1/3) + b^(1/3)*x^(1/3)]/(6*a^(4/3)*b^(5/3)) + Log[a + b*x]/(18*a^(4/3)*b^
(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx}{3b}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9ab}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6ab^2} - \cdots \quad \text{S}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\, \cdots\right.}{3a\cdots}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a\cdots}{18a^{4/}\cdots}$$

Mathematica [A]
time = 0.20, size = 133, normalized size = 0.93

$$\frac{-\frac{3\sqrt[3]{a}\,b^{2/3}x^{2/3}(a-2bx)}{(a+bx)^2} - 2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right) + \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{18a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] ((-3*a^(1/3)*b^(2/3)*x^(2/3)*(a - 2*b*x))/(a + b*x)^2 - 2*Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*x^(1/3)]
 + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(4/3)*b^
(5/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^(2/3)/(a + b*x)^3,x]')

[Out] Timed out

Maple [A]
time = 0.10, size = 132, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^{2}}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3ab}$	132
default	$\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^{2}}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3ab}$	132

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 3*(1/9/a*x^(5/3)-1/18*x^(2/3)/b)/(b*x+a)^2+1/3/a/b*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))

Maxima [A]
time = 0.34, size = 153, normalized size = 1.07

$$\frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\,(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="maxima")

[Out] 1/6*(2*b*x^(5/3) - a*x^(2/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(102) = 204.

time = 0.32, size = 508, normalized size = 3.55

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)+3\left(2ab^3x-a^2b^2\right)x^{\frac{2}{3}}}{18\left(a^2b^5x^2+2a^3b^4x+a^4b^3\right)}, \frac{6\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)+3\left(2ab^3x-a^2b^2\right)x^{\frac{2}{3}}}{18\left(a^2b^5x^2+2a^3b^4x+a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)
^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^
2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^
2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*
x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*
a*b^3*x - a^2*b^2)*x^(2/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.01, size = 220, normalized size = 1.54

$$3\left(\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{54a^2b^3}-\frac{\frac{1}{9}\left(\left(-ab^2\right)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2b^3}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot 3ba^2}+\frac{\frac{1}{18}\left(2\left(x^{\frac{1}{3}}\right)^2xb-\left(x^{\frac{1}{3}}\right)^2a\right)}{ba\left(xb+a\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x)
```

```
[Out] -1/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) - 1/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^3) + 1/6*(2*b*x^(5/3) - a*x^(2/3))/((b*x + a)^2*a*b) + 1/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

Mupad [B]
time = 0.26, size = 172, normalized size = 1.20

$$\frac{\frac{x^{5/3}}{3a}-\frac{x^{2/3}}{6b}}{a^2+2abx+b^2x^2}+\frac{\ln\left(\frac{1}{9a^{5/3}(-b)^{4/3}}+\frac{x^{1/3}}{9a^2b}\right)}{9a^{4/3}(-b)^{5/3}}+\frac{\ln\left(\frac{x^{1/3}}{9a^2b}+\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{36a^{5/3}(-b)^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18a^{4/3}(-b)^{5/3}}-\frac{\ln\left(\frac{x^{1/3}}{9a^2b}+\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{36a^{5/3}(-b)^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18a^{4/3}(-b)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^3,x)
```

```
[Out] (x^(5/3)/(3*a) - x^(2/3)/(6*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(1/(9*a^(5/3
)*(-b)^(4/3)) + x^(1/3)/(9*a^2*b))/(9*a^(4/3)*(-b)^(5/3)) + (log(x^(1/3)/(9
*a^2*b) + (3^(1/2)*1i - 1)^2/(36*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/(18
*a^(4/3)*(-b)^(5/3)) - (log(x^(1/3)/(9*a^2*b) + (3^(1/2)*1i + 1)^2/(36*a^(5
/3)*(-b)^(4/3)))*(3^(1/2)*1i + 1))/(18*a^(4/3)*(-b)^(5/3))
```

3.693 $\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}$$

```
[Out] -1/2*x^(1/3)/b/(b*x+a)^2+1/6*x^(1/3)/a/b/(b*x+a)+1/6*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(5/3)/b^(4/3)-1/18*ln(b*x+a)/a^(5/3)/b^(4/3)-1/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 60, 631, 210, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(1/3)/(b*(a + b*x)^2) + x^(1/3)/(6*a*b*(a + b*x)) - ArcTan[(a^(1/3)
- 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a
^(1/3) + b^(1/3)*x^(1/3)]/(6*a^(5/3)*b^(4/3)) - Log[a + b*x]/(18*a^(5/3)*b^
(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6b}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9ab}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + - \text{S}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3a}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a}{18a^{5/}}$$

Mathematica [A]
time = 0.19, size = 136, normalized size = 0.95

$$\frac{\frac{3a^{2/3}\sqrt[3]{b}\,\sqrt[3]{x}\,(-2a+bx)}{(a+bx)^2} - 2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) - \log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] ((3*a^(2/3)*b^(1/3)*x^(1/3)*(-2*a + b*x))/(a + b*x)^2 - 2*Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + b^(1/3)*x^(1/3)]
 - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(5/3)*b^
(4/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1/3)/(a + b*x)^3,x]')
```

[Out] Timed out

Maple [A]
time = 0.11, size = 132, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^{2}}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3ab}$	132
default	$\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^{2}}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3ab}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(1/18/a*x^(4/3)-1/9*x^(1/3)/b)/(b*x+a)^2+1/3/a/b*(1/3/b/(a/b)^(2/3)*ln(x^
(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1
)))
```

Maxima [A]
time = 0.34, size = 152, normalized size = 1.06

$$\frac{bx^{\frac{4}{3}}-2\,ax^{\frac{1}{3}}}{6\,(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^(4/3) - 2*a*x^(1/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
2/3)) - 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(2/3)) + 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(102) = 204$.

time = 0.34, size = 501, normalized size = 3.50

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)+3\left(a^2b^2x-2a^3b\right)x^{\frac{1}{3}}}{18\left(a^3b^4x^2+2a^4b^3x+a^5b^2\right)}, \frac{6\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)+3\left(a^2b^2x-2a^3b\right)x^{\frac{1}{3}}}{18\left(a^3b^4x^2+2a^4b^3x+a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - (b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1
/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*
log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^
4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x
+ a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*
b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^2 + 2*a*b*x + a^2)*(a
^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*
(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) +
3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 202, normalized size = 1.41

$$3\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{54a^2b^2}+\frac{\frac{1}{9}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2b^2}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot 3ba^2}+\frac{\frac{1}{18}\left(x^{\frac{1}{3}}xb-2x^{\frac{1}{3}}a\right)}{ba\left(xb+a\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x)
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
```

```
^2*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^2*b^2) + 1/6*(b*x^(4/3) - 2*a*x^(1/3))/((b*x + a)^2*a*b)
```

Mupad [B]
time = 0.24, size = 146, normalized size = 1.02

$$\frac{\frac{x^{4/3}}{6\,a}-\frac{x^{1/3}}{3\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{b^{2/3}}{a^{2/3}}+\frac{b\,x^{1/3}}{a}\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(\frac{b\,x^{1/3}}{a}+\frac{b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(\frac{b\,x^{1/3}}{a}-\frac{b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^3,x)
```

```
[Out] (x^(4/3)/(6*a) - x^(1/3)/(3*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(b^(2/3)/a^(
2/3) + (b*x^(1/3))/a)/(9*a^(5/3)*b^(4/3)) + (log((b*x^(1/3))/a + (b^(2/3)*(
3^(1/2)*1i - 1))/(2*a^(2/3)))*(3^(1/2)*1i - 1))/(18*a^(5/3)*b^(4/3)) - (log
((b*x^(1/3))/a - (b^(2/3)*(3^(1/2)*1i + 1))/(2*a^(2/3)))*(3^(1/2)*1i + 1))/
(18*a^(5/3)*b^(4/3))
```

3.694 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{x^{2/3}}{2a(a+bx)^2}+\frac{2x^{2/3}}{3a^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}$$

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/3*x^(2/3)/a^2/(b*x+a)-1/3*ln(a^(1/3)+b^(1/3)*x^(1
/3))/a^(7/3)/b^(2/3)+1/9*ln(b*x+a)/a^(7/3)/b^(2/3)-2/9*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 58, 631, 210, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] x^(2/3)/(2*a*(a + b*x)^2) + (2*x^(2/3))/(3*a^2*(a + b*x)) - (2*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) -
Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(7/3)*b^(2/3)) + Log[a + b*x]/(9*a^(7/3
)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}\,(a+bx)^3}\,dx = \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9a^2}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{3a^2b} - $$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{2\text{Subst}\left(\int \frac{1}{-3-x}\right.}{3}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log}{9a}$$

Mathematica [A]

time = 0.14, size = 142, normalized size = 1.01

$$\frac{\frac{3\sqrt[3]{a}\,x^{2/3}(7a+4bx)}{(a+bx)^2}-\frac{4\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{2/3}}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{b^{2/3}}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] ((3*a^(1/3)*x^(2/3)*(7*a + 4*b*x))/(a + b*x)^2 - (4*Sqrt[3]*ArcTan[(1 - (2*
b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*x^(1
/3)])/b^(2/3) + (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]
)/b^(2/3))/(18*a^(7/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(1/3)*(a + b*x)^3),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.11, size = 139, normalized size = 0.99

method	result	si
derivativedivides	$\frac{x^{\frac{2}{3}}}{2a(bx+a)^2}+\frac{\frac{2x^{\frac{2}{3}}}{3a(bx+a)}+\frac{2\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a}}{a}$	1

default	$\frac{\frac{x^{\frac{2}{3}}}{2a(bx+a)^2}+\frac{\frac{2x^{\frac{2}{3}}}{3a(bx+a)}+\frac{2\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a}}{a}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/a*(1/3*x^(2/3)/a/(b*x+a)+1/3/a*(-1/3/b/(a/b)^(1/3
)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+
(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^
(1/3)-1))))
```

Maxima [A]
time = 0.35, size = 151, normalized size = 1.08

$$\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 2/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
1/3)) + 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(
1/3)) - 2/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(99) = 198.
time = 0.33, size = 510, normalized size = 3.64

$$\left[\frac{6\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+2\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-4\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)+3\,(4\,ab^3x+7\,a^2b^2)x^{\frac{2}{3}}}{18\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)},\ \frac{12\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+2\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-4\,(b^2x^2+2\,abx+a^2)\,(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)+3\,(4\,ab^3x+7\,a^2b^2)x^{\frac{2}{3}}}{18\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
```

```
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^
2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)
^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))
/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
+ 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 221, normalized size = 1.58

$$3\left(\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^{3}b^{2}}-\frac{\frac{1}{9}\cdot 2\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{3}b^{2}}-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot 3a^{3}}+\frac{\frac{1}{18}\left(4\left(x^{\frac{1}{3}}\right)^{2}xb+7\left(x^{\frac{1}{3}}\right)^{2}a\right)}{a^{2}\left(xb+a\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x)
```

```
[Out] -2/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 2/9*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^2) + 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/((b*x + a)^2*a^2) + 1/9*(-a*b^2)^(2/3
)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

Mupad [B]
time = 0.19, size = 167, normalized size = 1.19

$$\frac{\frac{7\,x^{2/3}}{6\,a}+\frac{2\,b\,x^{5/3}}{3\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{2\,\ln\left(\frac{4\,b\,x^{1/3}}{9\,a^4}-\frac{4\,b^{2/3}}{9\,{\left(-a\right)}^{11/3}}\right)}{9\,{\left(-a\right)}^{7/3}\,b^{2/3}}+\frac{\ln\left(\frac{4\,b\,x^{1/3}}{9\,a^4}-\frac{b^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{9\,{\left(-a\right)}^{11/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,{\left(-a\right)}^{7/3}\,b^{2/3}}-\frac{\ln\left(\frac{4\,b\,x^{1/3}}{9\,a^4}-\frac{b^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{9\,{\left(-a\right)}^{11/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,{\left(-a\right)}^{7/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^3),x)
```

```
[Out] ((7*x^(2/3))/(6*a) + (2*b*x^(5/3))/(3*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (2*
log((4*b*x^(1/3))/(9*a^4) - (4*b^(2/3))/(9*(-a)^(11/3))))/(9*(-a)^(7/3)*b^(
2/3)) + (log((4*b*x^(1/3))/(9*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(9*(-a)^(
11/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(7/3)*b^(2/3)) - (log((4*b*x^(1/3))/(9*a^
4) - (b^(2/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(9*(-a
)^(7/3)*b^(2/3))
```

3.695 $\int \frac{1}{x^{2/3}(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}$$

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/6*x^(1/3)/a^2/(b*x+a)+5/6*ln(a^(1/3)+b^(1/3)*x^(1
/3))/a^(8/3)/b^(1/3)-5/18*ln(b*x+a)/a^(8/3)/b^(1/3)-5/9*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 60, 631, 210, 31}

$$\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] x^(1/3)/(2*a*(a + b*x)^2) + (5*x^(1/3))/(6*a^2*(a + b*x)) - (5*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) +
(5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(8/3)*b^(1/3)) - (5*Log[a + b*x])/(
18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{2/3}(a+bx)^3}\,dx &= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6a} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^2} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{6a^{7/3}b^{2/3}} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\text{Subst}\left(\int \right.}{} \\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-
\end{aligned}
$$

Mathematica [**A**]

time = 0.14, size = 142, normalized size = 1.01

$$\frac{\frac{3a^{2/3}\sqrt[3]{x}\,(8a+5bx)}{(a+bx)^2}-\frac{10\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{b}}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{b}}}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] ((3*a^(2/3)*x^(1/3)*(8*a + 5*b*x))/(a + b*x)^2 - (10*Sqrt[3]*ArcTan[(1 - (2
*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*Log[a^(1/3) + b^(1/3)*x^
(1/3)])/b^(1/3) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3
)])/b^(1/3))/(18*a^(8/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(2/3)*(a + b*x)^3),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.13, size = 139, normalized size = 0.99

method	result	s
derivativedivides	$\frac{\frac{x^{\frac{1}{3}}}{2a(bx+a)^2}+\frac{\frac{5x^{\frac{1}{3}}}{6a(bx+a)}+\frac{5\left(\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2a}}{a}$	1

default	$\frac{\frac{x^{\frac{1}{3}}}{2a(bx+a)^2}+\frac{\frac{5x^{\frac{1}{3}}}{6a(bx+a)}+\frac{5\left(\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2a}}{a}}{}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/2/a*(1/3*x^(1/3)/a/(b*x+a)+2/3/a*(1/3/b/(a/b)^(2/
3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
^(1/3)-1))))
```

Maxima [A]
time = 0.37, size = 151, normalized size = 1.08

$$\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 5/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
2/3)) - 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(2/3)) + 5/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(99) = 198.
time = 0.34, size = 499, normalized size = 3.56

$$\left[\frac{15\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3\,(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-5\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\,(5\,a^2b^2x+8\,a^3b)x^{\frac{1}{3}}}{18\,(a^4b^3x^2+2\,a^5b^2x+a^6b)},\ \frac{30\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a-2\,(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-5\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\,(b^2x^2+2\,abx+a^2)\,(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\,(5\,a^2b^2x+8\,a^3b)x^{\frac{1}{3}}}{18\,(a^4b^3x^2+2\,a^5b^2x+a^6b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
```

```
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - 5*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)
^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2
/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(
a^4*b^3*x^2 + 2*a^5*b^2*x + a^6*b), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(
2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(a^4*b^3*x^2 + 2*a^5*b^2*x + a^6
*b)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 200, normalized size = 1.43

$$3\left(\frac{5\left(-ab^{2}\right)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{54a^{3}b}+\frac{\frac{1}{9}\cdot5\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{3}b}-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot3a^{3}}+\frac{\frac{1}{18}\left(5x^{\frac{1}{3}}xb+8x^{\frac{1}{3}}a\right)}{a^{2}\left(xb+a\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x)
```

```
[Out] -5/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 5/9*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
) + 5/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a^3*b) + 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/((b*x + a)^2*a^2)
```

Mupad [B]
time = 0.24, size = 157, normalized size = 1.12

$$\frac{\frac{4\,x^{1/3}}{3\,a}+\frac{5\,b\,x^{4/3}}{6\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{5\,\ln\left(\frac{5\,b^{5/3}}{a^{5/3}}+\frac{5\,b^2\,x^{1/3}}{a^2}\right)}{9\,a^{8/3}\,b^{1/3}}+\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}+\frac{b^{5/3}\,\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}-\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}-\frac{b^{5/3}\,\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^3),x)
```

```
[Out] ((4*x^(1/3))/(3*a) + (5*b*x^(4/3))/(6*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (5*
log((5*b^(5/3))/a^(5/3) + (5*b^2*x^(1/3))/a^2))/(9*a^(8/3)*b^(1/3)) + (log(
(5*b^2*x^(1/3))/a^2 + (b^(5/3)*(3^(1/2)*5i - 5))/(2*a^(5/3)))*(3^(1/2)*5i -
 5))/(18*a^(8/3)*b^(1/3)) - (log((5*b^2*x^(1/3))/a^2 - (b^(5/3)*(3^(1/2)*5i
 + 5))/(2*a^(5/3)))*(3^(1/2)*5i + 5))/(18*a^(8/3)*b^(1/3))
```

3.696 $\int \frac{1}{x^{4/3}(a+bx)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}\,(a+bx)^2}+\frac{7}{6a^2\sqrt[3]{x}\,(a+bx)}+\frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right.}{3a^{10/3}}$$

```
[Out] -14/3/a^3/x^(1/3)+1/2/a/x^(1/3)/(b*x+a)^2+7/6/a^2/x^(1/3)/(b*x+a)+7/3*b^(1/
3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(10/3)-7/9*b^(1/3)*ln(b*x+a)/a^(10/3)+14/9
*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(10/3)*3
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 58, 631, 210, 31}

$$\frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\,\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{7}{6a^2\sqrt[3]{x}\,(a+bx)}+\frac{1}{2a\sqrt[3]{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] -14/(3*a^3*x^(1/3)) + 1/(2*a*x^(1/3)*(a + b*x)^2) + 7/(6*a^2*x^(1/3)*(a + b
*x)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(10/3)) + (7*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(10
/3)) - (7*b^(1/3)*Log[a + b*x])/(9*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7\int \frac{1}{x^{4/3}(a+bx)^2}\,dx}{6a}$$

$$= \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\int \frac{1}{x^{4/3}(a+bx)}\,dx}{9a^2}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{(14b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9a^3}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{7\sqrt[3]{b}\,\log(a+bx)}{9a^{10/3}} - \frac{7\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}} - \frac{7\sqrt{\cdots}}{\cdots}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{7\cdots}{\cdots}$$

Mathematica [A]
time = 0.24, size = 153, normalized size = 1.01

$$\frac{-\frac{3\sqrt[3]{a}\,(18a^2+49abx+28b^2x^2)}{\sqrt[3]{x}\,(a+bx)^2} + 28\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - 14\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] ((-3*a^(1/3)*(18*a^2 + 49*a*b*x + 28*b^2*x^2))/(x^(1/3)*(a + b*x)^2) + 28*S
qrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^(1/
3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(10/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(x^(4/3)*(a + b*x)^3),x]')

[Out] Timed out

Maple [A]

time = 0.14, size = 133, normalized size = 0.88

method	result
derivativedivides	$-\frac{3b\left(\frac{\frac{5bx^{\frac{5}{3}}}{9}+\frac{13ax^{\frac{2}{3}}}{18}}{(bx+a)^2}-\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3}-\frac{3}{a^3x}$
default	$-\frac{3b\left(\frac{\frac{5bx^{\frac{5}{3}}}{9}+\frac{13ax^{\frac{2}{3}}}{18}}{(bx+a)^2}-\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3}-\frac{3}{a^3x}$
risch	$-\frac{3}{a^3x^{\frac{1}{3}}}-\frac{5b^2x^{\frac{5}{3}}}{3a^3(bx+a)^2}-\frac{13bx^{\frac{2}{3}}}{6a^2(bx+a)^2}+\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{14\sqrt{3}\,\arctan\left(\right.}{9a^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(4/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] -3/a^3*b*((5/9*b*x^(5/3)+13/18*a*x^(2/3))/(b*x+a)^2-14/27/b/(a/b)^(1/3)*ln(
x^(1/3)+(a/b)^(1/3))+7/27/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b
)^(2/3))+14/27*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1
/3)-1)))-3/a^3/x^(1/3)

Maxima [A]

time = 0.36, size = 154, normalized size = 1.01

$$-\frac{28\,b^{2}x^{2}+49\,abx+18\,a^{2}}{6\left(a^{3}b^{2}x^{\frac{7}{3}}+2\,a^{4}bx^{\frac{4}{3}}+a^{5}x^{\frac{1}{3}}\right)}-\frac{14\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(28*b^2*x^2 + 49*a*b*x + 18*a^2)/(a^3*b^2*x^(7/3) + 2*a^4*b*x^(4/3) +
a^5*x^(1/3)) - 14/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a
/b)^(1/3))/(a^3*(a/b)^(1/3)) - 7/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) + 14/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(1/
3))
```

Fricas [A]
time = 0.32, size = 211, normalized size = 1.39

$$-\frac{28\,\sqrt{3}\,(b^{2}x^{3}+2\,abx^{2}+a^{2}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\,(b^{2}x^{3}+2\,abx^{2}+a^{2}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\,(b^{2}x^{3}+2\,abx^{2}+a^{2}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\,(28\,b^{2}x^{2}+49\,abx+18\,a^{2})x^{\frac{2}{3}}}{18\,(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*arctan(2/3*sqrt
(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(
b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 28*(b^
2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 3*(
28*b^2*x^2 + 49*a*b*x + 18*a^2)*x^(2/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x
)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 235, normalized size = 1.55

$$3\left(-\frac{1}{a^{3}x^{\frac{1}{3}}}+\frac{\frac{1}{18}\left(-10\left(x^{\frac{1}{3}}\right)^{2}xb^{2}-13\left(x^{\frac{1}{3}}\right)^{2}ba\right)}{a^{3}\left(xb+a\right)^{2}}-\frac{7\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\ln\left(\left(x^{\frac{1}{3}}\right)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^{4}b}+\frac{\frac{1}{9}\cdot14\left(\left(-ab^{2}\right)^{\frac{1}{3}}\right)^{2}\arctan\left(\frac{2\left(x^{\frac{1}{3}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^{4}b}+\frac{14\left(-\frac{a}{b}\right)^{\frac{1}{3}}b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot3a^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x)
```

```
[Out] 14/9*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 + 14/9*sqrt(3)*(-a
*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
4*b) - 3/(a^3*x^(1/3)) - 7/9*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^4*b) - 1/6*(10*b^2*x^(5/3) + 13*a*b*x^(2/3))/((b*x +
 a)^2*a^3)
```

Mupad [B]

time = 0.09, size = 174, normalized size = 1.14

$$\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}+588\,a^3\,b^3\,x^{1/3}\right)}{9\,a^{10/3}}-\frac{\frac{3}{a}+\frac{14\,b^2\,x^2}{3\,a^3}+\frac{49\,b\,x}{6\,a^2}}{a^2\,x^{1/3}+b^2\,x^{7/3}+2\,a\,b\,x^{4/3}}+\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+588\,a^3\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+588\,a^3\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^3),x)
```

```
[Out] (14*b^(1/3)*log(588*a^(10/3)*b^(8/3) + 588*a^3*b^3*x^(1/3)))/(9*a^(10/3)) -
 (3/a + (14*b^2*x^2)/(3*a^3) + (49*b*x)/(6*a^2))/(a^2*x^(1/3) + b^2*x^(7/3)
 + 2*a*b*x^(4/3)) + (14*b^(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 -
1/2)^2 + 588*a^3*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (14*b^
(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2 + 588*a^3*b^3*x^(1/
3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

3.697 $\int \frac{1}{x^{5/3}(a+bx)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}$$

```
[Out] -10/3/a^3/x^(2/3)+1/2/a/x^(2/3)/(b*x+a)^2+4/3/a^2/x^(2/3)/(b*x+a)-10/3*b^(2
/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(11/3)+10/9*b^(2/3)*ln(b*x+a)/a^(11/3)+20
/9*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(11/3)
*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 60, 631, 210, 31}

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}-\frac{10}{3a^3x^{2/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] -10/(3*a^3*x^(2/3)) + 1/(2*a*x^(2/3)*(a + b*x)^2) + 4/(3*a^2*x^(2/3)*(a + b
*x)) + (20*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(11/3)) - (10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(1
1/3)) + (10*b^(2/3)*Log[a + b*x])/(9*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4\int \frac{1}{x^{5/3}(a+bx)^2}\,dx}{3a}$$

$$= \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20\int \frac{1}{x^{5/3}(a+bx)}\,dx}{9a^2}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} - \frac{(20b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^3}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{10b^{2/3}\log(a+bx)}{9a^{11/3}} - \frac{\left(10\sqrt[3]{b}\right)\text{Su}}{}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} - \frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}} + \frac{10}{}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{11/3}} - \frac{10}{}$$

Mathematica [A]
time = 0.24, size = 153, normalized size = 1.01

$$\frac{-\frac{3a^{2/3}(9a^2+32abx+20b^2x^2)}{x^{2/3}(a+bx)^2} + 40\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 40b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + 20b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] ((-3*a^(2/3)*(9*a^2 + 32*a*b*x + 20*b^2*x^2))/(x^(2/3)*(a + b*x)^2) + 40*Sq
rt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 40*b^(2/3
)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(11/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^(5/3)*(a + b*x)^3),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 133, normalized size = 0.88

method	result
derivativedivides	$-\frac{3b\left(\frac{\frac{11b\,x^{\frac{4}{3}}}{18}+\frac{7a\,x^{\frac{1}{3}}}{9}}{(bx+a)^{2}}+\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{3}}-\frac{}{2a}$
default	$-\frac{3b\left(\frac{\frac{11b\,x^{\frac{4}{3}}}{18}+\frac{7a\,x^{\frac{1}{3}}}{9}}{(bx+a)^{2}}+\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{3}}-\frac{}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/a^3*b*((11/18*b*x^(4/3)+7/9*a*x^(1/3))/(b*x+a)^2+20/27/b/(a/b)^(2/3)*ln(
x^(1/3)+(a/b)^(1/3))-10/27/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/
b)^(2/3))+20/27/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(
1/3)-1)))-3/2/a^3/x^(2/3)
```

Maxima [A]
time = 0.35, size = 154, normalized size = 1.01

$$-\frac{20\,b^{2}x^{2}+32\,abx+9\,a^{2}}{6\left(a^{3}b^{2}x^{\frac{8}{3}}+2\,a^{4}bx^{\frac{5}{3}}+a^{5}x^{\frac{2}{3}}\right)}-\frac{20\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/(a^3*b^2*x^(8/3) + 2*a^4*b*x^(5/3) + a
^5*x^(2/3)) - 20/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*(a/b)^(2/3)) + 10/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(2/3)) - 20/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(2/
3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(107) = 214$.
time = 0.31, size = 244, normalized size = 1.61

$$\frac{40\sqrt{3}\,(b^2x^3 + 2\,abx^2 + a^2x)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3}\,b}{3\,b}\right) - 20\,(b^2x^3 + 2\,abx^2 + a^2x)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}} + abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}} + a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right) + 40\,(b^2x^3 + 2\,abx^2 + a^2x)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}} - a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right) - 3\,(20\,b^2x^2 + 32\,abx + 9\,a^2)x^{\frac{1}{3}}}{18\,(a^3b^2x^3 + 2\,a^4bx^2 + a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(40*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(b^2*x^3 + 2*a*b
*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/
3) + a^2*(-b^2/a^2)^(2/3)) + 40*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1
/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(20*b^2*x^2 + 32*a*b*x + 9*a^2)
*x^(1/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 214, normalized size = 1.41

$$3\left(-\frac{10\,(-ab^2)^{\frac{1}{3}}\ln\left(\left(x^{\frac{1}{3}}\right)^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^4} - \frac{\frac{1}{9}\cdot 20\,(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{2\left(x^{\frac{1}{3}} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^4} + \frac{20b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\ln\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|}{9\cdot 3a^4} + \frac{\frac{1}{18}\left(-20x^2b^2 - 32xba - 9a^2\right)}{a^3\left(x^{\frac{1}{3}}xb + x^{\frac{1}{3}}a\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x)
```

```
[Out] 20/9*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 - 20/9*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4
```

```
 - 10/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a
^4 - 1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/((b*x^(4/3) + a*x^(1/3))^2*a^3)
```

Mupad [B]
time = 0.17, size = 182, normalized size = 1.20

$$\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}-540\,a^6\,b^3\,x^{1/3}\right)}{9\,{\left(-a\right)}^{11/3}}-\frac{\frac{3}{2\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{16\,b\,x}{3\,a^2}}{a^2\,x^{2/3}+b^2\,x^{8/3}+2\,a\,b\,x^{5/3}}+\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-540\,a^6\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{\left(-a\right)}^{11/3}}-\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+540\,a^6\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{\left(-a\right)}^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^3),x)
```

```
[Out] (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3) - 540*a^6*b^3*x^(1/3)))/(9*(-a)^(11
/3)) - (3/(2*a) + (10*b^2*x^2)/(3*a^3) + (16*b*x)/(3*a^2))/(a^2*x^(2/3) + b
^2*x^(8/3) + 2*a*b*x^(5/3)) + (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(
1/2)*1i)/2 - 1/2) - 540*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*(-a)^(1
1/3)) - (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 54
0*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*(-a)^(11/3))
```

3.698 $\int \frac{\sqrt[4]{1-x}}{1+x} \, dx$

Optimal. Leaf size=58

$$4\sqrt[4]{1-x} - 2\sqrt[4]{2} \tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right) - 2\sqrt[4]{2} \tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

```
[Out] 4*(1-x)^(1/4)-2*2^(1/4)*arctan(1/2*(1-x)^(1/4)*2^(3/4))-2*2^(1/4)*arctanh(1
/2*(1-x)^(1/4)*2^(3/4))
```

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 65, 218, 212, 209}

$$4\sqrt[4]{1-x} - 2\sqrt[4]{2} \tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right) - 2\sqrt[4]{2} \tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{1-x}}{1+x}\,dx &= 4\sqrt[4]{1-x}+2\int \frac{1}{(1-x)^{3/4}(1+x)}\,dx\\ &= 4\sqrt[4]{1-x}-8\text{Subst}\left(\int \frac{1}{2-x^4}\,dx,x,\sqrt[4]{1-x}\right)\\ &= 4\sqrt[4]{1-x}-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{1-x}\right)-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\right.\\ &= 4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 1.00

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.03, size = 98, normalized size = 1.69

$$4-2^{\frac{1}{4}}\text{Log}\left[1-\frac{2^{\frac{3}{4}}(-1+x)^{\frac{1}{4}}\text{exp\_polar}\left[\frac{5I}{4}\text{Pi}\right]}{2}\right]-(-1+x)^{\frac{1}{4}}-I2^{\frac{1}{4}}\text{Log}\left[1-\frac{2^{\frac{3}{4}}(-1+x)^{\frac{1}{4}}\text{exp\_polar}\left[\frac{3I}{4}\text{Pi}\right]}{2}\right]+I2^{\frac{1}{4}}\text{Log}\left[1-\frac{2^{\frac{3}{4}}(-1+x)^{\frac{1}{4}}\text{exp\_polar}\left[\frac{7I}{4}\text{Pi}\right]}{2}\right]+2^{\frac{1}{4}}\text{Log}\left[1-\frac{2^{\frac{3}{4}}(-1+x)^{\frac{1}{4}}\text{exp\_polar}\left[\frac{I}{4}\text{Pi}\right]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/4)/(1 + x),x]')
```

```
[Out] -2 ^ (1 / 4) Log[1 - 2 ^ (3 / 4) (-1 + x) ^ (1 / 4) exp_polar[5 I / 4 Pi] /
 2] - I 2 ^ (1 / 4) Log[1 - 2 ^ (3 / 4) (-1 + x) ^ (1 / 4) exp_polar[3 I /
4 Pi] / 2] + I 2 ^ (1 / 4) Log[1 - 2 ^ (3 / 4) (-1 + x) ^ (1 / 4) exp_polar
[7 I / 4 Pi] / 2] + 4 -1 ^ (1 / 4) (-1 + x) ^ (1 / 4) + 2 ^ (1 / 4) Log[1 -
 2 ^ (3 / 4) (-1 + x) ^ (1 / 4) exp_polar[I / 4 Pi] / 2]
```

Maple [A]
time = 1.21, size = 60, normalized size = 1.03

method	result
derivativedivides	$4\left(1-x\right)^{\frac{1}{4}}-2^{\frac{1}{4}}\left(\ln\left(\frac{\left(1-x\right)^{\frac{1}{4}}+2^{\frac{1}{4}}}{\left(1-x\right)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\left(1-x\right)^{\frac{1}{4}}2^{\frac{3}{4}}}{2}\right)\right)$
default	$4\left(1-x\right)^{\frac{1}{4}}-2^{\frac{1}{4}}\left(\ln\left(\frac{\left(1-x\right)^{\frac{1}{4}}+2^{\frac{1}{4}}}{\left(1-x\right)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\left(1-x\right)^{\frac{1}{4}}2^{\frac{3}{4}}}{2}\right)\right)$
trager	$4\left(1-x\right)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}-2\right)^{2}\right)\ln\left(\frac{-x\,\mathrm{RootOf}\left(\_Z^{4}-2\right)^{2}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{R}\right.}{}\right.$
risch	$-\frac{4(-1+x)}{(1-x)^{\frac{3}{4}}}+\frac{\left(\mathrm{RootOf}\left(\_Z^{4}-2\right)\ln\left(\frac{2\sqrt{-x^{3}+3x^{2}-3x+1}\,\mathrm{RootOf}\left(\_Z^{4}-2\right)^{3}x-2\sqrt{-x^{3}+3x^{2}-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/4)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(1-x)^(1/4)-2^(1/4)*(ln(((1-x)^(1/4)+2^(1/4))/((1-x)^(1/4)-2^(1/4)))+2*ar
ctan(1/2*(1-x)^(1/4)*2^(3/4)))
```

Maxima [A]
time = 0.37, size = 61, normalized size = 1.05

$$-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}-(-x+1)^{\frac{1}{4}}}{2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="maxima")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) + 2^(1/4)*log(-(2^(1/4) - (-x
 + 1)^(1/4))/(2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

Fricas [A]
time = 0.31, size = 82, normalized size = 1.41

$$4\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-x+1}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 4*2^(1/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-x + 1)) - 1/2*2^(3/4)*(-x
 + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x + 1)^(1/4)) + 2^(1/4)*log(-2^(1/4)
 + (-x + 1)^(1/4)) + 4*(-x + 1)^(1/4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.28, size = 243, normalized size = 4.19

$$\frac{5\sqrt[4]{-1}\,\sqrt[4]{x-1}\,\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\,(-1)^{\frac{3}{4}}\cdot\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{3i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{5i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\,(-1)^{\frac{3}{4}}\cdot\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{7i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/4)/(1+x),x)
```

```
[Out] 5*(-1)**(1/4)*(x - 1)**(1/4)*gamma(5/4)/gamma(9/4) + 5*(-2)**(1/4)*exp(-I*p
i/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(I*pi/4)/2 + 1)*gamma(5/4)/(4*ga
mma(9/4)) - 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4
)*exp_polar(3*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4)) - 5*(-2)**(1/4)*exp(
-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(5*I*pi/4)/2 + 1)*gamma(5/4)
/(4*gamma(9/4)) + 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)
**(1/4)*exp_polar(7*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4))
```

Giac [A]
time = 0.01, size = 80, normalized size = 1.38

$$2^{\frac{1}{4}}\ln\left|(-x+1)^{\frac{1}{4}}-2^{\frac{1}{4}}\right|-2^{\frac{1}{4}}\ln\left((-x+1)^{\frac{1}{4}}+2^{\frac{1}{4}}\right)-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{(-x+1)^{\frac{1}{4}}}{2^{\frac{1}{4}}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x)
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x +
 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

Mupad [B]
time = 0.07, size = 46, normalized size = 0.79

$$4\,{\left(1-x\right)}^{1/4}-2\,2^{1/4}\,\mathrm{atanh}\left(\frac{2^{3/4}\,{\left(1-x\right)}^{1/4}}{2}\right)-2\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(1-x\right)}^{1/4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/4)/(x + 1),x)
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*atanh((2^(3/4)*(1 - x)^(1/4))/2) - 2*2^(1/4)*at
an((2^(3/4)*(1 - x)^(1/4))/2)
```

3.699 $\int x^m(a+bx)^{10}\,dx$

Optimal. Leaf size=187

$$\frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}+\frac{210a^4b^6x^{7+m}}{7+m}+\frac{120a^3b^7x^{8+m}}{8+m}$$

```
[Out] a^10*x^(1+m)/(1+m)+10*a^9*b*x^(2+m)/(2+m)+45*a^8*b^2*x^(3+m)/(3+m)+120*a^7*
b^3*x^(4+m)/(4+m)+210*a^6*b^4*x^(5+m)/(5+m)+252*a^5*b^5*x^(6+m)/(6+m)+210*a
^4*b^6*x^(7+m)/(7+m)+120*a^3*b^7*x^(8+m)/(8+m)+45*a^2*b^8*x^(9+m)/(9+m)+10*
a*b^9*x^(10+m)/(10+m)+b^10*x^(11+m)/(11+m)
```

Rubi [A]
time = 0.06, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^{10}x^{m+1}}{m+1}+\frac{10a^9bx^{m+2}}{m+2}+\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\frac{45a^2b^8x^{m+9}}{m+9}+\frac{10ab^9x^{m+10}}{m+10}+\frac{b^{10}x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^10,x]
```

```
[Out] (a^10*x^(1 + m))/(1 + m) + (10*a^9*b*x^(2 + m))/(2 + m) + (45*a^8*b^2*x^(3
+ m))/(3 + m) + (120*a^7*b^3*x^(4 + m))/(4 + m) + (210*a^6*b^4*x^(5 + m))/(
5 + m) + (252*a^5*b^5*x^(6 + m))/(6 + m) + (210*a^4*b^6*x^(7 + m))/(7 + m)
+ (120*a^3*b^7*x^(8 + m))/(8 + m) + (45*a^2*b^8*x^(9 + m))/(9 + m) + (10*a*
b^9*x^(10 + m))/(10 + m) + (b^10*x^(11 + m))/(11 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{10}\,dx&=\int\left(a^{10}x^m+10a^9bx^{1+m}+45a^8b^2x^{2+m}+120a^7b^3x^{3+m}+210a^6b^4x^{4+m}+252a^5b^5x^{5+m}+\right.\\&=\frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 166, normalized size = 0.89

$$x^{1+m}\left(\frac{a^{10}}{1+m}+\frac{10a^9bx}{2+m}+\frac{45a^8b^2x^2}{3+m}+\frac{120a^7b^3x^3}{4+m}+\frac{210a^6b^4x^4}{5+m}+\frac{252a^5b^5x^5}{6+m}+\frac{210a^4b^6x^6}{7+m}+\frac{120a^3b^7x^7}{8+m}+\frac{45a^2b^8x^8}{9+m}+\frac{10ab^9x^9}{10+m}+\frac{b^{10}x^{10}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^10,x]
```

```
[Out] x^(1 + m)*(a^10/(1 + m) + (10*a^9*b*x)/(2 + m) + (45*a^8*b^2*x^2)/(3 + m) +
 (120*a^7*b^3*x^3)/(4 + m) + (210*a^6*b^4*x^4)/(5 + m) + (252*a^5*b^5*x^5)/
(6 + m) + (210*a^4*b^6*x^6)/(7 + m) + (120*a^3*b^7*x^7)/(8 + m) + (45*a^2*b
^8*x^8)/(9 + m) + (10*a*b^9*x^9)/(10 + m) + (b^10*x^10)/(11 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 8.78, size = 9691, normalized size = 51.82

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^m*(a + b*x)^10,x]')
```

```
[Out] Piecewise[{{(-252 a ^ 10 - 2800 a ^ 9 b x - 14175 a ^ 8 b ^ 2 x ^ 2 - 43200
 a ^ 7 b ^ 3 x ^ 3 - 88200 a ^ 6 b ^ 4 x ^ 4 - 127008 a ^ 5 b ^ 5 x ^ 5 - 1
32300 a ^ 4 b ^ 6 x ^ 6 - 100800 a ^ 3 b ^ 7 x ^ 7 - 56700 a ^ 2 b ^ 8 x ^
8 - 25200 a b ^ 9 x ^ 9 + 2520 b ^ 10 x ^ 10 Log[x]) / (2520 x ^ 10), m ==
-11}, {-a ^ 10 / (9 x ^ 9) - 5 a ^ 9 b / (4 x ^ 8) - 45 a ^ 8 b ^ 2 / (7 x
^ 7) - 20 a ^ 7 b ^ 3 / x ^ 6 - 42 a ^ 6 b ^ 4 / x ^ 5 - 63 a ^ 5 b ^ 5 / x
 ^ 4 - 70 a ^ 4 b ^ 6 / x ^ 3 - 60 a ^ 3 b ^ 7 / x ^ 2 - 45 a ^ 2 b ^ 8 / x
 + 10 a b ^ 9 Log[x] + b ^ 10 x, m == -10}, {(-7 a ^ 10 - 80 a ^ 9 b x - 42
0 a ^ 8 b ^ 2 x ^ 2 - 1344 a ^ 7 b ^ 3 x ^ 3 - 2940 a ^ 6 b ^ 4 x ^ 4 - 470
4 a ^ 5 b ^ 5 x ^ 5 - 5880 a ^ 4 b ^ 6 x ^ 6 - 6720 a ^ 3 b ^ 7 x ^ 7 + 28
b ^ 8 x ^ 8 (90 a ^ 2 Log[x] + 20 a b x + b ^ 2 x ^ 2)) / (56 x ^ 8), m ==
-9}, {-a ^ 10 / (7 x ^ 7) - 5 a ^ 9 b / (3 x ^ 6) - 9 a ^ 8 b ^ 2 / x ^ 5 -
 30 a ^ 7 b ^ 3 / x ^ 4 - 70 a ^ 6 b ^ 4 / x ^ 3 - 126 a ^ 5 b ^ 5 / x ^ 2
- 210 a ^ 4 b ^ 6 / x + 120 a ^ 3 b ^ 7 Log[x] + 45 a ^ 2 b ^ 8 x + 5 a b ^
 9 x ^ 2 + b ^ 10 x ^ 3 / 3, m == -8}, {(-2 a ^ 10 - 24 a ^ 9 b x - 135 a ^
 8 b ^ 2 x ^ 2 - 480 a ^ 7 b ^ 3 x ^ 3 - 1260 a ^ 6 b ^ 4 x ^ 4 - 3024 a ^
5 b ^ 5 x ^ 5 + b ^ 6 x ^ 6 (2520 a ^ 4 Log[x] + 1440 a ^ 3 b x + 270 a ^ 2
 b ^ 2 x ^ 2 + 40 a b ^ 3 x ^ 3 + 3 b ^ 4 x ^ 4)) / (12 x ^ 6), m == -7}, {
(-2 a ^ 10 - 25 a ^ 9 b x - 150 a ^ 8 b ^ 2 x ^ 2 - 600 a ^ 7 b ^ 3 x ^ 3 -
 2100 a ^ 6 b ^ 4 x ^ 4 + b ^ 5 x ^ 5 (2520 a ^ 5 Log[x] + 2100 a ^ 4 b x +
 600 a ^ 3 b ^ 2 x ^ 2 + 150 a ^ 2 b ^ 3 x ^ 3 + 25 a b ^ 4 x ^ 4 + 2 b ^ 5
 x ^ 5)) / (10 x ^ 5), m == -6}, {(-3 a ^ 10 - 40 a ^ 9 b x - 270 a ^ 8 b ^
 2 x ^ 2 - 1440 a ^ 7 b ^ 3 x ^ 3 + b ^ 4 x ^ 4 (2520 a ^ 6 Log[x] + 3024 a
 ^ 5 b x + 1260 a ^ 4 b ^ 2 x ^ 2 + 480 a ^ 3 b ^ 3 x ^ 3 + 135 a ^ 2 b ^ 4
 x ^ 4 + 24 a b ^ 5 x ^ 5 + 2 b ^ 6 x ^ 6)) / (12 x ^ 4), m == -5}, {-a ^ 1
0 / (3 x ^ 3) - 5 a ^ 9 b / x ^ 2 - 45 a ^ 8 b ^ 2 / x + 120 a ^ 7 b ^ 3 Lo
g[x] + 210 a ^ 6 b ^ 4 x + 126 a ^ 5 b ^ 5 x ^ 2 + 70 a ^ 4 b ^ 6 x ^ 3 + 3
0 a ^ 3 b ^ 7 x ^ 4 + 9 a ^ 2 b ^ 8 x ^ 5 + 5 a b ^ 9 x ^ 6 / 3 + b ^ 10 x
^ 7 / 7, m == -4}, {(-28 a ^ 10 - 560 a ^ 9 b x + b ^ 2 x ^ 2 (2520 a ^ 8 L
```

og[x] + 6720 a ^ 7 b x + 5880 a ^ 6 b ^ 2 x ^ 2 + 4704 a ^ 5 b ^ 3 x ^ 3 + 2940 a ^ 4 b ^ 4 x ^ 4 + 1344 a ^ 3 b ^ 5 x ^ 5 + 420 a ^ 2 b ^ 6 x ^ 6 + 80 a b ^ 7 x ^ 7 + 7 b ^ 8 x ^ 8)) / (56 x ^ 2), m == -3}, {(-a ^ 10 + b x (2520 a ^ 9 Log[x] + 11340 a ^ 8 b x + 15120 a ^ 7 b ^ 2 x ^ 2 + 17640 a ^ 6 b ^ 3 x ^ 3 + 15876 a ^ 5 b ^ 4 x ^ 4 + 10584 a ^ 4 b ^ 5 x ^ 5 + 5040 a ^ 3 b ^ 6 x ^ 6 + 1620 a ^ 2 b ^ 7 x ^ 7 + 315 a b ^ 8 x ^ 8 + 28 b ^ 9 x ^ 9) / 252) / x, m == -2}, {a ^ 10 Log[x] + 10 a ^ 9 b x + 45 a ^ 8 b ^ 2 x ^ 2 / 2 + 40 a ^ 7 b ^ 3 x ^ 3 + 105 a ^ 6 b ^ 4 x ^ 4 / 2 + 252 a ^ 5 b ^ 5 x ^ 5 / 5 + 35 a ^ 4 b ^ 6 x ^ 6 + 120 a ^ 3 b ^ 7 x ^ 7 / 7 + 45 a ^ 2 b ^ 8 x ^ 8 / 8 + 10 a b ^ 9 x ^ 9 / 9 + b ^ 10 x ^ 10 / 10, m == -1}}, 39916800 a ^ 10 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 80627040 a ^ 10 m x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 70290936 a ^ 10 m ^ 2 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 34967140 a ^ 10 m ^ 3 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 11028590 a ^ 10 m ^ 4 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2310945 a ^ 10 m ^ 5 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 326613 a ^ 10 m ^ 6 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 30810 a ^ 10 m ^ 7 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1860 a ^ 10 m ^ 8 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 65 a ^ 10 m ^ 9 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + a ^ 10 m ^ 10 x x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 199584000 a ^ 9 b x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 502927200 a ^ 9 b m x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m

^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 503126280 a ^ 9 b m ^ 2 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 274727240 a ^ 9 b m ^ 3 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 92615030 a ^ 9 b m ^ 4 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 20390160 a ^ 9 b m ^ 5 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2992710 a ^ 9 b m ^ 6 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 290760 a ^ 9 b m ^ 7 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 17970 a ^ 9 b m ^ 8 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 640 a ^ 9 b m ^ 9 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 10 a ^ 9 b m ^ 10 x ^ 2 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 598752000 a ^ 8 b ^ 2 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1608573600 a ^ 8 b ^ 2 m x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1727578440 a ^ 8 b ^ 2 m ^ 2 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1003011660 a ^ 8 b ^ 2 m ^ 3 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 355598730 a ^ 8 b ^ 2 m ^ 4 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 81560115 a ^ 8 b ^ 2 m ^ 5 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 12376665 a ^ 8 b ^ 2 m ^ 6 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m

^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1235790 a ^ 8 b ^ 2 m ^ 7 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 78120 a ^ 8 b ^ 2 m ^ 8 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2835 a ^ 8 b ^ 2 m ^ 9 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 45 a ^ 8 b ^ 2 m ^ 10 x ^ 3 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1197504000 a ^ 7 b ^ 3 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3316939200 a ^ 7 b ^ 3 m x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3698304480 a ^ 7 b ^ 3 m ^ 2 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2233166160 a ^ 7 b ^ 3 m ^ 3 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 821580360 a ^ 7 b ^ 3 m ^ 4 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 194790960 a ^ 7 b ^ 3 m ^ 5 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 30429000 a ^ 7 b ^ 3 m ^ 6 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3115440 a ^ 7 b ^ 3 m ^ 7 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 201240 a ^ 7 b ^ 3 m ^ 8 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 7440 a ^ 7 b ^ 3 m ^ 9 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 120 a ^ 7 b ^ 3 m ^ 10 x ^ 4 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1676505600 a ^ 6 b ^ 4 x ^ 5 x ^ m / (39916800 + 12054384

0 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 4727540160 a ^ 6 b ^ 4 m x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 5393046960 a ^ 6 b ^ 4 m ^ 2 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3342229800 a ^ 6 b ^ 4 m ^ 3 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1263374700 a ^ 6 b ^ 4 m ^ 4 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 307585530 a ^ 6 b ^ 4 m ^ 5 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 49260330 a ^ 6 b ^ 4 m ^ 6 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 5159700 a ^ 6 b ^ 4 m ^ 7 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 340200 a ^ 6 b ^ 4 m ^ 8 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 12810 a ^ 6 b ^ 4 m ^ 9 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 210 a ^ 6 b ^ 4 m ^ 10 x ^ 5 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1676505600 a ^ 5 b ^ 5 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 4783423680 a ^ 5 b ^ 5 m x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 5541317712 a ^ 5 b ^ 5 m ^ 2 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3497286240 a ^ 5 b ^ 5 m ^ 3 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1348939620 a ^ 5 b ^ 5 m ^ 4 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925

m ^ 9 + 66 m ^ 10 + m ^ 11) + 335437200 a ^ 5 b ^ 5 m ^ 5 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 54871236 a ^ 5 b ^ 5 m ^ 6 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 5866560 a ^ 5 b ^ 5 m ^ 7 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 394380 a ^ 5 b ^ 5 m ^ 8 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 15120 a ^ 5 b ^ 5 m ^ 9 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 252 a ^ 5 b ^ 5 m ^ 10 x ^ 6 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1197504000 a ^ 4 b ^ 6 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3445243200 a ^ 4 b ^ 6 m x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 4035361680 a ^ 4 b ^ 6 m ^ 2 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2581262040 a ^ 4 b ^ 6 m ^ 3 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1011120180 a ^ 4 b ^ 6 m ^ 4 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 255740310 a ^ 4 b ^ 6 m ^ 5 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 42592410 a ^ 4 b ^ 6 m ^ 6 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 4638060 a ^ 4 b ^ 6 m ^ 7 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 317520 a ^ 4 b ^ 6 m ^ 8 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 12390 a ^ 4 b ^ 6 m ^ 9 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 +

45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 210 a ^ 4 b ^ 6 m ^ 10 x ^ 7 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 598752000 a ^ 3 b ^ 7 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1733313600 a ^ 3 b ^ 7 m x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2047105440 a ^ 3 b ^ 7 m ^ 2 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1322982960 a ^ 3 b ^ 7 m ^ 3 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 524563080 a ^ 3 b ^ 7 m ^ 4 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 134522640 a ^ 3 b ^ 7 m ^ 5 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 22748040 a ^ 3 b ^ 7 m ^ 6 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2517840 a ^ 3 b ^ 7 m ^ 7 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 175320 a ^ 3 b ^ 7 m ^ 8 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 6960 a ^ 3 b ^ 7 m ^ 9 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 120 a ^ 3 b ^ 7 m ^ 10 x ^ 8 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 199584000 a ^ 2 b ^ 8 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 580543200 a ^ 2 b ^ 8 m x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 690085080 a ^ 2 b ^ 8 m ^ 2 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 449614260 a ^ 2 b ^ 8 m ^ 3 x

^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 180021510 a ^ 2 b ^ 8 m ^ 4 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 46695285 a ^ 2 b ^ 8 m ^ 5 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 7999425 a ^ 2 b ^ 8 m ^ 6 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 898290 a ^ 2 b ^ 8 m ^ 7 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 63540 a ^ 2 b ^ 8 m ^ 8 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 2565 a ^ 2 b ^ 8 m ^ 9 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 45 a ^ 2 b ^ 8 m ^ 10 x ^ 9 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 39916800 a b ^ 9 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 116552160 a b ^ 9 m x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 139262760 a b ^ 9 m ^ 2 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 91331800 a b ^ 9 m ^ 3 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 36862550 a b ^ 9 m ^ 4 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 9653280 a b ^ 9 m ^ 5 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 1672230 a b ^ 9 m ^ 6 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 190200 a b ^ 9 m ^ 7 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66

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m ^ 10 + m ^ 11) + 13650 a b ^ 9 m ^ 8 x ^ 10 x ^ m / (39916800 + 120543840
 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 +
2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 1
1) + 560 a b ^ 9 m ^ 9 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m
 ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 +
357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 10 a b ^ 9
m ^ 10 x ^ 10 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076
 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 3
2670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3628800 b ^ 10 x ^ 11 x ^ m
 / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m
 ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m
 ^ 9 + 66 m ^ 10 + m ^ 11) + 10628640 b ^ 10 m x ^ 11 x ^ m / (39916800 + 1
20543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535
m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10
 + m ^ 11) + 12753576 b ^ 10 m ^ 2 x ^ 11 x ^ m / (39916800 + 120543840 m +
 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637
558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) +
 8409500 b ^ 10 m ^ 3 x ^ 11 x ^ m / (39916800 + 120543840 m + 150917976 m
^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 3
57423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 3416930 b ^
10 m ^ 4 x ^ 11 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 1052580
76 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 +
 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 902055 b ^ 10 m ^ 5 x ^ 1
1 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 459
95730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 +
 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 157773 b ^ 10 m ^ 6 x ^ 11 x ^ m / (399
16800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 +
13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 +
66 m ^ 10 + m ^ 11) + 18150 b ^ 10 m ^ 7 x ^ 11 x ^ m / (39916800 + 1205438
40 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5
+ 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^
 11) + 1320 b ^ 10 m ^ 8 x ^ 11 x ^ m / (39916800 + 120543840 m + 150917976
 m ^ 2 + 105258076 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6
+ 357423 m ^ 7 + 32670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + 55 b ^ 10
 m ^ 9 x ^ 11 x ^ m / (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076
 m ^ 3 + 45995730 m ^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 3
2670 m ^ 8 + 1925 m ^ 9 + 66 m ^ 10 + m ^ 11) + b ^ 10 m ^ 10 x ^ 11 x ^ m
/ (39916800 + 120543840 m + 150917976 m ^ 2 + 105258076 m ^ 3 + 45995730 m
^ 4 + 13339535 m ^ 5 + 2637558 m ^ 6 + 357423 m ^ 7 + 32670 m ^ 8 + 1925 m
^ 9 + 66 m ^ 10 + m ^ 11)]
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Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1533 vs. $2(187) = 374$.
time = 0.11, size = 1534, normalized size = 8.20

method	result	size
risch	Expression too large to display	1534
gosper	Expression too large to display	1535

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^10*m^10*x^10+10*a*b^9*m^10*x^9+55*b^10*m^9*x^10+45*a^2*b^8*m^10*x^8+56
0*a*b^9*m^9*x^9+1320*b^10*m^8*x^10+120*a^3*b^7*m^10*x^7+2565*a^2*b^8*m^9*x^
8+13650*a*b^9*m^8*x^9+18150*b^10*m^7*x^10+210*a^4*b^6*m^10*x^6+6960*a^3*b^7
*m^9*x^7+63540*a^2*b^8*m^8*x^8+190200*a*b^9*m^7*x^9+157773*b^10*m^6*x^10+25
2*a^5*b^5*m^10*x^5+12390*a^4*b^6*m^9*x^6+175320*a^3*b^7*m^8*x^7+898290*a^2*
b^8*m^7*x^8+1672230*a*b^9*m^6*x^9+902055*b^10*m^5*x^10+210*a^6*b^4*m^10*x^4
+15120*a^5*b^5*m^9*x^5+317520*a^4*b^6*m^8*x^6+2517840*a^3*b^7*m^7*x^7+79994
25*a^2*b^8*m^6*x^8+9653280*a*b^9*m^5*x^9+3416930*b^10*m^4*x^10+120*a^7*b^3*
m^10*x^3+12810*a^6*b^4*m^9*x^4+394380*a^5*b^5*m^8*x^5+4638060*a^4*b^6*m^7*x
^6+22748040*a^3*b^7*m^6*x^7+46695285*a^2*b^8*m^5*x^8+36862550*a*b^9*m^4*x^9
+8409500*b^10*m^3*x^10+45*a^8*b^2*m^10*x^2+7440*a^7*b^3*m^9*x^3+340200*a^6*
b^4*m^8*x^4+5866560*a^5*b^5*m^7*x^5+42592410*a^4*b^6*m^6*x^6+134522640*a^3*
b^7*m^5*x^7+180021510*a^2*b^8*m^4*x^8+91331800*a*b^9*m^3*x^9+12753576*b^10*
m^2*x^10+10*a^9*b*m^10*x+2835*a^8*b^2*m^9*x^2+201240*a^7*b^3*m^8*x^3+515970
0*a^6*b^4*m^7*x^4+54871236*a^5*b^5*m^6*x^5+255740310*a^4*b^6*m^5*x^6+524563
080*a^3*b^7*m^4*x^7+449614260*a^2*b^8*m^3*x^8+139262760*a*b^9*m^2*x^9+10628
640*b^10*m*x^10+a^10*m^10+640*a^9*b*m^9*x+78120*a^8*b^2*m^8*x^2+3115440*a^7
*b^3*m^7*x^3+49260330*a^6*b^4*m^6*x^4+335437200*a^5*b^5*m^5*x^5+1011120180*
a^4*b^6*m^4*x^6+1322982960*a^3*b^7*m^3*x^7+690085080*a^2*b^8*m^2*x^8+116552
160*a*b^9*m*x^9+3628800*b^10*x^10+65*a^10*m^9+17970*a^9*b*m^8*x+1235790*a^8
*b^2*m^7*x^2+30429000*a^7*b^3*m^6*x^3+307585530*a^6*b^4*m^5*x^4+1348939620*
a^5*b^5*m^4*x^5+2581262040*a^4*b^6*m^3*x^6+2047105440*a^3*b^7*m^2*x^7+58054
3200*a^2*b^8*m*x^8+39916800*a*b^9*x^9+1860*a^10*m^8+290760*a^9*b*m^7*x+1237
6665*a^8*b^2*m^6*x^2+194790960*a^7*b^3*m^5*x^3+1263374700*a^6*b^4*m^4*x^4+3
497286240*a^5*b^5*m^3*x^5+4035361680*a^4*b^6*m^2*x^6+1733313600*a^3*b^7*m*x
^7+199584000*a^2*b^8*x^8+30810*a^10*m^7+2992710*a^9*b*m^6*x+81560115*a^8*b^
2*m^5*x^2+821580360*a^7*b^3*m^4*x^3+3342229800*a^6*b^4*m^3*x^4+5541317712*a
^5*b^5*m^2*x^5+3445243200*a^4*b^6*m*x^6+598752000*a^3*b^7*x^7+326613*a^10*m
^6+20390160*a^9*b*m^5*x+355598730*a^8*b^2*m^4*x^2+2233166160*a^7*b^3*m^3*x^
3+5393046960*a^6*b^4*m^2*x^4+4783423680*a^5*b^5*m*x^5+1197504000*a^4*b^6*x^
6+2310945*a^10*m^5+92615030*a^9*b*m^4*x+1003011660*a^8*b^2*m^3*x^2+36983044
80*a^7*b^3*m^2*x^3+4727540160*a^6*b^4*m*x^4+1676505600*a^5*b^5*x^5+11028590
*a^10*m^4+274727240*a^9*b*m^3*x+1727578440*a^8*b^2*m^2*x^2+3316939200*a^7*b
^3*m*x^3+1676505600*a^6*b^4*x^4+34967140*a^10*m^3+503126280*a^9*b*m^2*x+160
8573600*a^8*b^2*m*x^2+1197504000*a^7*b^3*x^3+70290936*a^10*m^2+502927200*a^
9*b*m*x+598752000*a^8*b^2*x^2+80627040*a^10*m+199584000*a^9*b*x+39916800*a^
```

```
10)*x^m/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [A]
time = 0.27, size = 187, normalized size = 1.00

$$\frac{b^{10}x^{m+11}}{m+11}+\frac{10\,ab^9x^{m+10}}{m+10}+\frac{45\,a^2b^8x^{m+9}}{m+9}+\frac{120\,a^3b^7x^{m+8}}{m+8}+\frac{210\,a^4b^6x^{m+7}}{m+7}+\frac{252\,a^5b^5x^{m+6}}{m+6}+\frac{210\,a^6b^4x^{m+5}}{m+5}+\frac{120\,a^7b^3x^{m+4}}{m+4}+\frac{45\,a^8b^2x^{m+3}}{m+3}+\frac{10\,a^9bx^{m+2}}{m+2}+\frac{a^{10}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] b^10*x^(m + 11)/(m + 11) + 10*a*b^9*x^(m + 10)/(m + 10) + 45*a^2*b^8*x^(m +
 9)/(m + 9) + 120*a^3*b^7*x^(m + 8)/(m + 8) + 210*a^4*b^6*x^(m + 7)/(m + 7)
 + 252*a^5*b^5*x^(m + 6)/(m + 6) + 210*a^6*b^4*x^(m + 5)/(m + 5) + 120*a^7*
b^3*x^(m + 4)/(m + 4) + 45*a^8*b^2*x^(m + 3)/(m + 3) + 10*a^9*b*x^(m + 2)/(
m + 2) + a^10*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(187) = 374$.
time = 0.33, size = 1277, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] ((b^10*m^10 + 55*b^10*m^9 + 1320*b^10*m^8 + 18150*b^10*m^7 + 157773*b^10*m^
6 + 902055*b^10*m^5 + 3416930*b^10*m^4 + 8409500*b^10*m^3 + 12753576*b^10*m
^2 + 10628640*b^10*m + 3628800*b^10)*x^11 + 10*(a*b^9*m^10 + 56*a*b^9*m^9 +
 1365*a*b^9*m^8 + 19020*a*b^9*m^7 + 167223*a*b^9*m^6 + 965328*a*b^9*m^5 + 3
686255*a*b^9*m^4 + 9133180*a*b^9*m^3 + 13926276*a*b^9*m^2 + 11655216*a*b^9*
m + 3991680*a*b^9)*x^10 + 45*(a^2*b^8*m^10 + 57*a^2*b^8*m^9 + 1412*a^2*b^8*
m^8 + 19962*a^2*b^8*m^7 + 177765*a^2*b^8*m^6 + 1037673*a^2*b^8*m^5 + 400047
8*a^2*b^8*m^4 + 9991428*a^2*b^8*m^3 + 15335224*a^2*b^8*m^2 + 12900960*a^2*b
^8*m + 4435200*a^2*b^8)*x^9 + 120*(a^3*b^7*m^10 + 58*a^3*b^7*m^9 + 1461*a^3
*b^7*m^8 + 20982*a^3*b^7*m^7 + 189567*a^3*b^7*m^6 + 1121022*a^3*b^7*m^5 + 4
371359*a^3*b^7*m^4 + 11024858*a^3*b^7*m^3 + 17059212*a^3*b^7*m^2 + 14444280
*a^3*b^7*m + 4989600*a^3*b^7)*x^8 + 210*(a^4*b^6*m^10 + 59*a^4*b^6*m^9 + 15
12*a^4*b^6*m^8 + 22086*a^4*b^6*m^7 + 202821*a^4*b^6*m^6 + 1217811*a^4*b^6*m
^5 + 4814858*a^4*b^6*m^4 + 12291724*a^4*b^6*m^3 + 19216008*a^4*b^6*m^2 + 16
405920*a^4*b^6*m + 5702400*a^4*b^6)*x^7 + 252*(a^5*b^5*m^10 + 60*a^5*b^5*m^
9 + 1565*a^5*b^5*m^8 + 23280*a^5*b^5*m^7 + 217743*a^5*b^5*m^6 + 1331100*a^5
*b^5*m^5 + 5352935*a^5*b^5*m^4 + 13878120*a^5*b^5*m^3 + 21989356*a^5*b^5*m^
2 + 18981840*a^5*b^5*m + 6652800*a^5*b^5)*x^6 + 210*(a^6*b^4*m^10 + 61*a^6*
b^4*m^9 + 1620*a^6*b^4*m^8 + 24570*a^6*b^4*m^7 + 234573*a^6*b^4*m^6 + 14646
93*a^6*b^4*m^5 + 6016070*a^6*b^4*m^4 + 15915380*a^6*b^4*m^3 + 25681176*a^6*
b^4*m^2 + 22512096*a^6*b^4*m + 7983360*a^6*b^4)*x^5 + 120*(a^7*b^3*m^10 + 6
2*a^7*b^3*m^9 + 1677*a^7*b^3*m^8 + 25962*a^7*b^3*m^7 + 253575*a^7*b^3*m^6 +
```

```
 1623258*a^7*b^3*m^5 + 6846503*a^7*b^3*m^4 + 18609718*a^7*b^3*m^3 + 3081920
4*a^7*b^3*m^2 + 27641160*a^7*b^3*m + 9979200*a^7*b^3)*x^4 + 45*(a^8*b^2*m^1
0 + 63*a^8*b^2*m^9 + 1736*a^8*b^2*m^8 + 27462*a^8*b^2*m^7 + 275037*a^8*b^2*
m^6 + 1812447*a^8*b^2*m^5 + 7902194*a^8*b^2*m^4 + 22289148*a^8*b^2*m^3 + 38
390632*a^8*b^2*m^2 + 35746080*a^8*b^2*m + 13305600*a^8*b^2)*x^3 + 10*(a^9*b
*m^10 + 64*a^9*b*m^9 + 1797*a^9*b*m^8 + 29076*a^9*b*m^7 + 299271*a^9*b*m^6
+ 2039016*a^9*b*m^5 + 9261503*a^9*b*m^4 + 27472724*a^9*b*m^3 + 50312628*a^9
*b*m^2 + 50292720*a^9*b*m + 19958400*a^9*b)*x^2 + (a^10*m^10 + 65*a^10*m^9
+ 1860*a^10*m^8 + 30810*a^10*m^7 + 326613*a^10*m^6 + 2310945*a^10*m^5 + 110
28590*a^10*m^4 + 34967140*a^10*m^3 + 70290936*a^10*m^2 + 80627040*a^10*m +
39916800*a^10)*x)*x^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 +
 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2
+ 120543840*m + 39916800)
```

Sympy [A]
time = 1.23, size = 9996, normalized size = 53.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**10,x)
```

```
[Out] Piecewise((-a**10/(10*x**10) - 10*a**9*b/(9*x**9) - 45*a**8*b**2/(8*x**8) -
 120*a**7*b**3/(7*x**7) - 35*a**6*b**4/x**6 - 252*a**5*b**5/(5*x**5) - 105*
a**4*b**6/(2*x**4) - 40*a**3*b**7/x**3 - 45*a**2*b**8/(2*x**2) - 10*a*b**9/
x + b**10*log(x), Eq(m, -11)), (-a**10/(9*x**9) - 5*a**9*b/(4*x**8) - 45*a*
*8*b**2/(7*x**7) - 20*a**7*b**3/x**6 - 42*a**6*b**4/x**5 - 63*a**5*b**5/x**
4 - 70*a**4*b**6/x**3 - 60*a**3*b**7/x**2 - 45*a**2*b**8/x + 10*a*b**9*log(
x) + b**10*x, Eq(m, -10)), (-a**10/(8*x**8) - 10*a**9*b/(7*x**7) - 15*a**8*
b**2/(2*x**6) - 24*a**7*b**3/x**5 - 105*a**6*b**4/(2*x**4) - 84*a**5*b**5/x
**3 - 105*a**4*b**6/x**2 - 120*a**3*b**7/x + 45*a**2*b**8*log(x) + 10*a*b**
9*x + b**10*x**2/2, Eq(m, -9)), (-a**10/(7*x**7) - 5*a**9*b/(3*x**6) - 9*a*
*8*b**2/x**5 - 30*a**7*b**3/x**4 - 70*a**6*b**4/x**3 - 126*a**5*b**5/x**2 -
 210*a**4*b**6/x + 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 +
b**10*x**3/3, Eq(m, -8)), (-a**10/(6*x**6) - 2*a**9*b/x**5 - 45*a**8*b**2/(
4*x**4) - 40*a**7*b**3/x**3 - 105*a**6*b**4/x**2 - 252*a**5*b**5/x + 210*a*
*4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x**3/3 +
 b**10*x**4/4, Eq(m, -7)), (-a**10/(5*x**5) - 5*a**9*b/(2*x**4) - 15*a**8*b
**2/x**3 - 60*a**7*b**3/x**2 - 210*a**6*b**4/x + 252*a**5*b**5*log(x) + 210
*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x**3 + 5*a*b**9*x**4/2 + b*
*10*x**5/5, Eq(m, -6)), (-a**10/(4*x**4) - 10*a**9*b/(3*x**3) - 45*a**8*b**
2/(2*x**2) - 120*a**7*b**3/x + 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105
*a**4*b**6*x**2 + 40*a**3*b**7*x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 +
 b**10*x**6/6, Eq(m, -5)), (-a**10/(3*x**3) - 5*a**9*b/x**2 - 45*a**8*b**2/
x + 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b
**6*x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x
```

7/7, Eq(m, -4)), (-a10/(2*x**2) - 10*a**9*b/x + 45*a**8*b**2*log(x) + 1
20*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x**3 + 105*a**4*b**6*x**
4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a*b**9*x**7/7 + b**10*x*
*8/8, Eq(m, -3)), (-a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b
3*x2 + 70*a**6*b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a
3*b7*x**6 + 45*a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9, Eq(m,
 -2)), (a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**
3 + 105*a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a
3*b7*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10,
Eq(m, -1)), (a**10*m**10*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 65*a**10*m**9*x*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1860*a**10*m**8*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m
3 + 150917976*m2 + 120543840*m + 39916800) + 30810*a**10*m**7*x*x**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 326613*a**10*m**6*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2310945*a**10*m**5*x
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 11028590*a**10*m**4*x*x**m/(m**11 + 66*m**10 + 1925*m
9 + 32670*m8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*a
10*m3*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 70290936*a**10*m**2*x*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 80627040*a**10*m*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35742
3*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15
0917976*m**2 + 120543840*m + 39916800) + 39916800*a**10*x*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 10*a**9*b*m**10*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 640*a**9*b*m**9*x**2*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 17970*a**9*b*m**8*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32
670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105
258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 290760*a**9*b*m**7

*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 2992710*a**9*b*m**6*x**2*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 20
390160*a**9*b*m**5*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3
57423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
+ 150917976*m**2 + 120543840*m + 39916800) + 92615030*a**9*b*m**4*x**2*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 274727240*a**9*b*m**3*x**2*x**m/(m**11 + 66*m**10 + 1925*m
9 + 32670*m8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 503126280*
a**9*b*m**2*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
7 + 2637558*m6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 502927200*a**9*b*m*x**2*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 199584000*a**9*b*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45*a**8*b**2*m**10*x**3*
x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**
6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12054
3840*m + 39916800) + 2835*a**8*b**2*m**9*x**3*x**m/(m**11 + 66*m**10 + 1925
*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*
m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 78120*a*
*8*b**2*m**8*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*
m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509
17976*m**2 + 120543840*m + 39916800) + 1235790*a**8*b**2*m**7*x**3*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 12376665*a**8*b**2*m**6*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 81560115*a**
8*b**2*m**5*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
7 + 2637558*m6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 355598730*a**8*b**2*m**4*x**3*x**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 1003011660*a**8*b**2*m**3*x**3*x**m/(m**11 + 66*m**10 + 1925*
m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m
4 + 105258076*m3 + 150917976*m**2 + 120543840*m + 39916800) + 172757844
0*a**8*b**2*m**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 1608573600*a**8*b**2*m*x**3*x**m

/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 598752000*a**8*b**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**7*b**
3*m**10*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7440*a**7*b**3*m**9*x**4*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 201240*a**7*b**3*m**8*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3115440*a**7*b**3*m**7
*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 30429000*a**7*b**3*m**6*x**4*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
 + 194790960*a**7*b**3*m**5*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 821580360*a**7*b**3*m*
*4*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 2233166160*a**7*b**3*m**3*x**4*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 3698304480*a**7*b**3*m**2*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3316939200*a**7*b
3*m*x4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
2 + 120543840*m + 39916800) + 1197504000*a7*b**3*x**4*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 210*a**6*b**4*m**10*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12810*a**6*b**4*m**9*x**5
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 340200*a**6*b**4*m**8*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 51597
00*a**6*b**4*m**7*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35
7423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
 150917976*m**2 + 120543840*m + 39916800) + 49260330*a**6*b**4*m**6*x**5*x*

*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 307585530*a**6*b**4*m**5*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12633
74700*a**6*b**4*m**4*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 3342229800*a**6*b**4*m**3*x*
*5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 5393046960*a**6*b**4*m**2*x**5*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
4727540160*a**6*b**4*m*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505600*a**6*b**4*x**5
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 252*a**5*b**5*m**10*x**6*x**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 15120*a
5*b5*m**9*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 394380*a**5*b**5*m**8*x**6*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
39916800) + 5866560*a**5*b**5*m**7*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 54871236*a**5
*b**5*m**6*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 335437200*a**5*b**5*m**5*x**6*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
39916800) + 1348939620*a**5*b**5*m**4*x**6*x**m/(m**11 + 66*m**10 + 1925*m
9 + 32670*m8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3497286240
*a**5*b**5*m**3*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3574
23*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1
50917976*m**2 + 120543840*m + 39916800) + 5541317712*a**5*b**5*m**2*x**6*x*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 4783423680*a**5*b**5*m*x**6*x**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505

600*a**5*b**5*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 210*a**4*b**6*m**10*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 12390*a**4*b**6*m**9*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 317520*a**4*b**6*
m**8*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 4638060*a**4*b**6*m**7*x**7*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 42592410*a**4*b**6*m**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 255740310*a**4*b**6*
m**5*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 1011120180*a**4*b**6*m**4*x**7*x**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 2581262040*a**4*b**6*m**3*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4035361680*a**4
*b**6*m**2*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 3445243200*a**4*b**6*m*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 1197504000*a**4*b**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**3*b**7*m**
10*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
+ 120543840*m + 39916800) + 6960*a**3*b**7*m**9*x**8*x**m/(m**11 + 66*m**1
0 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4
5995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
175320*a**3*b**7*m**8*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 2517840*a**3*b**7*m**7*x**8
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 22748040*a**3*b**7*m**6*x**8*x**m/(m**11 + 66*m**10 +
1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 134

522640*a**3*b**7*m**5*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 524563080*a**3*b**7*m**4*x*
*8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 1322982960*a**3*b**7*m**3*x**8*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
2047105440*a**3*b**7*m**2*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1733313600*a**3*b**7*m
*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
120543840*m + 39916800) + 598752000*a**3*b**7*x**8*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45
*a**2*b**8*m**10*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 2565*a**2*b**8*m**9*x**9*x**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 63540*a**2*b**8*m**8*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 898290*a**2*b*
*8*m**7*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7999425*a**2*b**8*m**6*x**9*x**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 46695285*a**2*b**8*m**5*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 180021510*a**2*b*
*8*m**4*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 449614260*a**2*b**8*m**3*x**9*x**m/(m**11
+ 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 690085080*a**2*b**8*m**2*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 580543200*a**2
*b**8*m*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 199584000*a**2*b**8*x**9*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680

0) + 10*a*b**9*m**10*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 560*a*b**9*m**9*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 13650*a*b**9*m**8*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 190200*a*b**9*m**7*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1672230*a*b**9*m**6*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 9653280*a*b**9*m**5*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 36862550*a*b**9*m**4*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 91331800*a*b**9*m**3*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 139262760*a*b**9*m**2*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 116552160*a*b**9*m*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 39916800*a*b**9*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + b**10*m**10*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 55*b**10*m**9*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1320*b**10*m**8*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 18150*b**10*m**7*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 157773*b**10*m**6*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 902055*b**10*m**5*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3416930*b**10*m**4*x**11*x**m/(m**11 + 66*m**10 + 1925*m****

```
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 8409500*b**1
0*m**3*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 12753576*b**10*m**2*x**11*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 10628640*b**10*m*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3628800*b**10*x**11*x**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(187) = 374$.
time = 0.02, size = 2166, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x)
```

```
[Out] (b^10*m^10*x^11*x^m + 10*a*b^9*m^10*x^10*x^m + 55*b^10*m^9*x^11*x^m + 45*a^
2*b^8*m^10*x^9*x^m + 560*a*b^9*m^9*x^10*x^m + 1320*b^10*m^8*x^11*x^m + 120*
a^3*b^7*m^10*x^8*x^m + 2565*a^2*b^8*m^9*x^9*x^m + 13650*a*b^9*m^8*x^10*x^m
+ 18150*b^10*m^7*x^11*x^m + 210*a^4*b^6*m^10*x^7*x^m + 6960*a^3*b^7*m^9*x^8
*x^m + 63540*a^2*b^8*m^8*x^9*x^m + 190200*a*b^9*m^7*x^10*x^m + 157773*b^10*
m^6*x^11*x^m + 252*a^5*b^5*m^10*x^6*x^m + 12390*a^4*b^6*m^9*x^7*x^m + 17532
0*a^3*b^7*m^8*x^8*x^m + 898290*a^2*b^8*m^7*x^9*x^m + 1672230*a*b^9*m^6*x^10
*x^m + 902055*b^10*m^5*x^11*x^m + 210*a^6*b^4*m^10*x^5*x^m + 15120*a^5*b^5*
m^9*x^6*x^m + 317520*a^4*b^6*m^8*x^7*x^m + 2517840*a^3*b^7*m^7*x^8*x^m + 79
99425*a^2*b^8*m^6*x^9*x^m + 9653280*a*b^9*m^5*x^10*x^m + 3416930*b^10*m^4*x
^11*x^m + 120*a^7*b^3*m^10*x^4*x^m + 12810*a^6*b^4*m^9*x^5*x^m + 394380*a^5
*b^5*m^8*x^6*x^m + 4638060*a^4*b^6*m^7*x^7*x^m + 22748040*a^3*b^7*m^6*x^8*x
^m + 46695285*a^2*b^8*m^5*x^9*x^m + 36862550*a*b^9*m^4*x^10*x^m + 8409500*b
^10*m^3*x^11*x^m + 45*a^8*b^2*m^10*x^3*x^m + 7440*a^7*b^3*m^9*x^4*x^m + 340
200*a^6*b^4*m^8*x^5*x^m + 5866560*a^5*b^5*m^7*x^6*x^m + 42592410*a^4*b^6*m^
6*x^7*x^m + 134522640*a^3*b^7*m^5*x^8*x^m + 180021510*a^2*b^8*m^4*x^9*x^m +
 91331800*a*b^9*m^3*x^10*x^m + 12753576*b^10*m^2*x^11*x^m + 10*a^9*b*m^10*x
^2*x^m + 2835*a^8*b^2*m^9*x^3*x^m + 201240*a^7*b^3*m^8*x^4*x^m + 5159700*a^
6*b^4*m^7*x^5*x^m + 54871236*a^5*b^5*m^6*x^6*x^m + 255740310*a^4*b^6*m^5*x^
7*x^m + 524563080*a^3*b^7*m^4*x^8*x^m + 449614260*a^2*b^8*m^3*x^9*x^m + 139
262760*a*b^9*m^2*x^10*x^m + 10628640*b^10*m*x^11*x^m + a^10*m^10*x*x^m + 64
0*a^9*b*m^9*x^2*x^m + 78120*a^8*b^2*m^8*x^3*x^m + 3115440*a^7*b^3*m^7*x^4*x
^m + 49260330*a^6*b^4*m^6*x^5*x^m + 335437200*a^5*b^5*m^5*x^6*x^m + 1011120
```

```
180*a^4*b^6*m^4*x^7*x^m + 1322982960*a^3*b^7*m^3*x^8*x^m + 690085080*a^2*b^
8*m^2*x^9*x^m + 116552160*a*b^9*m*x^10*x^m + 3628800*b^10*x^11*x^m + 65*a^1
0*m^9*x*x^m + 17970*a^9*b*m^8*x^2*x^m + 1235790*a^8*b^2*m^7*x^3*x^m + 30429
000*a^7*b^3*m^6*x^4*x^m + 307585530*a^6*b^4*m^5*x^5*x^m + 1348939620*a^5*b^
5*m^4*x^6*x^m + 2581262040*a^4*b^6*m^3*x^7*x^m + 2047105440*a^3*b^7*m^2*x^8
*x^m + 580543200*a^2*b^8*m*x^9*x^m + 39916800*a*b^9*x^10*x^m + 1860*a^10*m^
8*x*x^m + 290760*a^9*b*m^7*x^2*x^m + 12376665*a^8*b^2*m^6*x^3*x^m + 1947909
60*a^7*b^3*m^5*x^4*x^m + 1263374700*a^6*b^4*m^4*x^5*x^m + 3497286240*a^5*b^
5*m^3*x^6*x^m + 4035361680*a^4*b^6*m^2*x^7*x^m + 1733313600*a^3*b^7*m*x^8*x
^m + 199584000*a^2*b^8*x^9*x^m + 30810*a^10*m^7*x*x^m + 2992710*a^9*b*m^6*x
^2*x^m + 81560115*a^8*b^2*m^5*x^3*x^m + 821580360*a^7*b^3*m^4*x^4*x^m + 334
2229800*a^6*b^4*m^3*x^5*x^m + 5541317712*a^5*b^5*m^2*x^6*x^m + 3445243200*a
^4*b^6*m*x^7*x^m + 598752000*a^3*b^7*x^8*x^m + 326613*a^10*m^6*x*x^m + 2039
0160*a^9*b*m^5*x^2*x^m + 355598730*a^8*b^2*m^4*x^3*x^m + 2233166160*a^7*b^3
*m^3*x^4*x^m + 5393046960*a^6*b^4*m^2*x^5*x^m + 4783423680*a^5*b^5*m*x^6*x^
m + 1197504000*a^4*b^6*x^7*x^m + 2310945*a^10*m^5*x*x^m + 92615030*a^9*b*m^
4*x^2*x^m + 1003011660*a^8*b^2*m^3*x^3*x^m + 3698304480*a^7*b^3*m^2*x^4*x^m
 + 4727540160*a^6*b^4*m*x^5*x^m + 1676505600*a^5*b^5*x^6*x^m + 11028590*a^1
0*m^4*x*x^m + 274727240*a^9*b*m^3*x^2*x^m + 1727578440*a^8*b^2*m^2*x^3*x^m
+ 3316939200*a^7*b^3*m*x^4*x^m + 1676505600*a^6*b^4*x^5*x^m + 34967140*a^10
*m^3*x*x^m + 503126280*a^9*b*m^2*x^2*x^m + 1608573600*a^8*b^2*m*x^3*x^m + 1
197504000*a^7*b^3*x^4*x^m + 70290936*a^10*m^2*x*x^m + 502927200*a^9*b*m*x^2
*x^m + 598752000*a^8*b^2*x^3*x^m + 80627040*a^10*m*x*x^m + 199584000*a^9*b*
x^2*x^m + 39916800*a^10*x*x^m)/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357
423*m^7 + 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 15091
7976*m^2 + 120543840*m + 39916800)
```

Mupad [B]

time = 1.37, size = 1274, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^10,x)
```

```
[Out] (a^10*x*x^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590*m^4 + 2310
945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39916800))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (b^10*x^m*x^11*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 +
 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (45*a^2*b^8*x^m*x^9*(12900960*m + 15335224*m^2 + 9991428*m^3 + 40004
78*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 + 57*m^9 + m^10 +
4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133
```

39535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^1
1 + 39916800) + (120*a^3*b^7*x^m*x^8*(14444280*m + 17059212*m^2 + 11024858*
m^3 + 4371359*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^
9 + m^10 + 4989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599573
0*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66
*m^10 + m^11 + 39916800) + (210*a^4*b^6*x^m*x^7*(16405920*m + 19216008*m^2
+ 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821*m^6 + 22086*m^7 + 1512*
m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 150917976*m^2 + 105258076*m^
3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 19
25*m^9 + 66*m^10 + m^11 + 39916800) + (252*a^5*b^5*x^m*x^6*(18981840*m + 21
989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1331100*m^5 + 217743*m^6 + 23280*
m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/(120543840*m + 150917976*m^2 + 1
05258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 326
70*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (210*a^6*b^4*x^m*x^5*(2251
2096*m + 25681176*m^2 + 15915380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m
^6 + 24570*m^7 + 1620*m^8 + 61*m^9 + m^10 + 7983360))/(120543840*m + 150917
976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742
3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (120*a^7*b^3*x^
m*x^4*(27641160*m + 30819204*m^2 + 18609718*m^3 + 6846503*m^4 + 1623258*m^5
 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9 + m^10 + 9979200))/(120543840
*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*
m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (45*
a^8*b^2*x^m*x^3*(35746080*m + 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1
812447*m^5 + 275037*m^6 + 27462*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (10*a*b^9*x^m*x^10*(11655216*m + 13926276*m^2 + 9133180*m^3 + 368625
5*m^4 + 965328*m^5 + 167223*m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 39
91680))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339
535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
+ 39916800) + (10*a^9*b*x^m*x^2*(50292720*m + 50312628*m^2 + 27472724*m^3 +
 9261503*m^4 + 2039016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m
^10 + 19958400))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^
4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^1
0 + m^11 + 39916800)

3.700 $\int x^m(a+bx)^7\,dx$

Optimal. Leaf size=133

$$\frac{a^7x^{1+m}}{1+m}+\frac{7a^6bx^{2+m}}{2+m}+\frac{21a^5b^2x^{3+m}}{3+m}+\frac{35a^4b^3x^{4+m}}{4+m}+\frac{35a^3b^4x^{5+m}}{5+m}+\frac{21a^2b^5x^{6+m}}{6+m}+\frac{7ab^6x^{7+m}}{7+m}+\frac{b^7x^{8+m}}{8+m}$$

```
[Out] a^7*x^(1+m)/(1+m)+7*a^6*b*x^(2+m)/(2+m)+21*a^5*b^2*x^(3+m)/(3+m)+35*a^4*b^3
*x^(4+m)/(4+m)+35*a^3*b^4*x^(5+m)/(5+m)+21*a^2*b^5*x^(6+m)/(6+m)+7*a*b^6*x^
(7+m)/(7+m)+b^7*x^(8+m)/(8+m)
```

Rubi [A]
time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^7x^{m+1}}{m+1}+\frac{7a^6bx^{m+2}}{m+2}+\frac{21a^5b^2x^{m+3}}{m+3}+\frac{35a^4b^3x^{m+4}}{m+4}+\frac{35a^3b^4x^{m+5}}{m+5}+\frac{21a^2b^5x^{m+6}}{m+6}+\frac{7ab^6x^{m+7}}{m+7}+\frac{b^7x^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^7,x]
```

```
[Out] (a^7*x^(1 + m))/(1 + m) + (7*a^6*b*x^(2 + m))/(2 + m) + (21*a^5*b^2*x^(3 +
m))/(3 + m) + (35*a^4*b^3*x^(4 + m))/(4 + m) + (35*a^3*b^4*x^(5 + m))/(5 +
m) + (21*a^2*b^5*x^(6 + m))/(6 + m) + (7*a*b^6*x^(7 + m))/(7 + m) + (b^7*x^
(8 + m))/(8 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^7\,dx &= \int \left(a^7x^m+7a^6bx^{1+m}+21a^5b^2x^{2+m}+35a^4b^3x^{3+m}+35a^3b^4x^{4+m}+21a^2b^5x^{5+m}+7ab^6\right.\\ &= \frac{a^7x^{1+m}}{1+m}+\frac{7a^6bx^{2+m}}{2+m}+\frac{21a^5b^2x^{3+m}}{3+m}+\frac{35a^4b^3x^{4+m}}{4+m}+\frac{35a^3b^4x^{5+m}}{5+m}+\frac{21a^2b^5x^{6+m}}{6+m}+\frac{7a}{7}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 118, normalized size = 0.89

$$x^{1+m}\left(\frac{a^7}{1+m}+\frac{7a^6bx}{2+m}+\frac{21a^5b^2x^2}{3+m}+\frac{35a^4b^3x^3}{4+m}+\frac{35a^3b^4x^4}{5+m}+\frac{21a^2b^5x^5}{6+m}+\frac{7ab^6x^6}{7+m}+\frac{b^7x^7}{8+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^7,x]
```

```
[Out] x^(1 + m)*(a^7/(1 + m) + (7*a^6*b*x)/(2 + m) + (21*a^5*b^2*x^2)/(3 + m) + (
35*a^4*b^3*x^3)/(4 + m) + (35*a^3*b^4*x^4)/(5 + m) + (21*a^2*b^5*x^5)/(6 +
m) + (7*a*b^6*x^6)/(7 + m) + (b^7*x^7)/(8 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.37, size = 4128, normalized size = 31.04

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^m*(a + b*x)^7,x]')
```

```
[Out] Piecewise[{{(-60 a ^ 7 - 490 a ^ 6 b x - 1764 a ^ 5 b ^ 2 x ^ 2 - 3675 a ^
4 b ^ 3 x ^ 3 - 4900 a ^ 3 b ^ 4 x ^ 4 - 4410 a ^ 2 b ^ 5 x ^ 5 - 2940 a b
^ 6 x ^ 6 + 420 b ^ 7 x ^ 7 Log[x]) / (420 x ^ 7), m == -8}, {(-10 a ^ 7 -
84 a ^ 6 b x - 315 a ^ 5 b ^ 2 x ^ 2 - 700 a ^ 4 b ^ 3 x ^ 3 - 1050 a ^ 3 b
 ^ 4 x ^ 4 - 1260 a ^ 2 b ^ 5 x ^ 5 + 60 b ^ 6 x ^ 6 (7 a Log[x] + b x)) /
(60 x ^ 6), m == -7}, {(-4 a ^ 7 - 35 a ^ 6 b x - 140 a ^ 5 b ^ 2 x ^ 2 - 3
50 a ^ 4 b ^ 3 x ^ 3 - 700 a ^ 3 b ^ 4 x ^ 4 + 10 b ^ 5 x ^ 5 (42 a ^ 2 Log
[x] + 14 a b x + b ^ 2 x ^ 2)) / (20 x ^ 5), m == -6}, {(-3 a ^ 7 - 28 a ^
6 b x - 126 a ^ 5 b ^ 2 x ^ 2 - 420 a ^ 4 b ^ 3 x ^ 3 + 2 b ^ 4 x ^ 4 (210
a ^ 3 Log[x] + 126 a ^ 2 b x + 21 a b ^ 2 x ^ 2 + 2 b ^ 3 x ^ 3)) / (12 x ^
 4), m == -5}, {(-4 a ^ 7 - 42 a ^ 6 b x - 252 a ^ 5 b ^ 2 x ^ 2 + b ^ 3 x
^ 3 (420 a ^ 4 Log[x] + 420 a ^ 3 b x + 126 a ^ 2 b ^ 2 x ^ 2 + 28 a b ^ 3
x ^ 3 + 3 b ^ 4 x ^ 4)) / (12 x ^ 3), m == -4}, {(-10 a ^ 7 - 140 a ^ 6 b x
 + b ^ 2 x ^ 2 (420 a ^ 5 Log[x] + 700 a ^ 4 b x + 350 a ^ 3 b ^ 2 x ^ 2 +
140 a ^ 2 b ^ 3 x ^ 3 + 35 a b ^ 4 x ^ 4 + 4 b ^ 5 x ^ 5)) / (20 x ^ 2), m
== -3}, {(-a ^ 7 + b x (420 a ^ 6 Log[x] + 1260 a ^ 5 b x + 1050 a ^ 4 b ^
2 x ^ 2 + 700 a ^ 3 b ^ 3 x ^ 3 + 315 a ^ 2 b ^ 4 x ^ 4 + 84 a b ^ 5 x ^ 5
+ 10 b ^ 6 x ^ 6) / 60) / x, m == -2}, {a ^ 7 Log[x] + 7 a ^ 6 b x + 21 a ^
 5 b ^ 2 x ^ 2 / 2 + 35 a ^ 4 b ^ 3 x ^ 3 / 3 + 35 a ^ 3 b ^ 4 x ^ 4 / 4 +
21 a ^ 2 b ^ 5 x ^ 5 / 5 + 7 a b ^ 6 x ^ 6 / 6 + b ^ 7 x ^ 7 / 7, m == -1}}
, 40320 a ^ 7 x x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22
449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 69264 a ^ 7 m x x
^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m
 ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 48860 a ^ 7 m ^ 2 x x ^ m / (40320 +
 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^
 6 + 36 m ^ 7 + m ^ 8) + 18424 a ^ 7 m ^ 3 x x ^ m / (40320 + 109584 m + 11
8124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7
+ m ^ 8) + 4025 a ^ 7 m ^ 4 x x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67
284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 511
a ^ 7 m ^ 5 x x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 2244
```

9 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 35 a ^ 7 m ^ 6 x x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + a ^ 7 m ^ 7 x x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 141120 a ^ 6 b x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 312984 a ^ 6 b m x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 256942 a ^ 6 b m ^ 2 x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 107023 a ^ 6 b m ^ 3 x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 25060 a ^ 6 b m ^ 4 x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 3346 a ^ 6 b m ^ 5 x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 238 a ^ 6 b m ^ 6 x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 7 a ^ 6 b m ^ 7 x ^ 2 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 282240 a ^ 5 b ^ 2 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 673008 a ^ 5 b ^ 2 m x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 602532 a ^ 5 b ^ 2 m ^ 2 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 270144 a ^ 5 b ^ 2 m ^ 3 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 67095 a ^ 5 b ^ 2 m ^ 4 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 9387 a ^ 5 b ^ 2 m ^ 5 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 693 a ^ 5 b ^ 2 m ^ 6 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 21 a ^ 5 b ^ 2 m ^ 7 x ^ 3 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 352800 a ^ 4 b ^ 3 x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 870660 a ^ 4 b ^ 3 m x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 815920 a ^ 4 b ^ 3 m ^ 2 x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 384755 a ^ 4 b ^ 3 m ^ 3 x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 100240 a ^ 4 b ^ 3 m ^ 4 x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 14630 a ^ 4 b ^ 3 m ^ 5 x ^ 4 x ^ m / (40320 + 109584

m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 1120 a ^ 4 b ^ 3 m ^ 6 x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 35 a ^ 4 b ^ 3 m ^ 7 x ^ 4 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 282240 a ^ 3 b ^ 4 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 710640 a ^ 3 b ^ 4 m x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 684740 a ^ 3 b ^ 4 m ^ 2 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 334040 a ^ 3 b ^ 4 m ^ 3 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 90335 a ^ 3 b ^ 4 m ^ 4 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 13685 a ^ 3 b ^ 4 m ^ 5 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 1085 a ^ 3 b ^ 4 m ^ 6 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 35 a ^ 3 b ^ 4 m ^ 7 x ^ 5 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 141120 a ^ 2 b ^ 5 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 360024 a ^ 2 b ^ 5 m x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 353430 a ^ 2 b ^ 5 m ^ 2 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 176589 a ^ 2 b ^ 5 m ^ 3 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 49140 a ^ 2 b ^ 5 m ^ 4 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 7686 a ^ 2 b ^ 5 m ^ 5 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 630 a ^ 2 b ^ 5 m ^ 6 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 21 a ^ 2 b ^ 5 m ^ 7 x ^ 6 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 40320 a b ^ 6 x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 103824 a b ^ 6 m x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 103292 a b ^ 6 m ^ 2 x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 52528 a b ^ 6 m ^ 3 x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 14945 a b ^ 6 m ^ 4 x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6

```
+ 36 m ^ 7 + m ^ 8) + 2401 a b ^ 6 m ^ 5 x ^ 7 x ^ m / (40320 + 109584 m +
118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^
7 + m ^ 8) + 203 a b ^ 6 m ^ 6 x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^
 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8)
 + 7 a b ^ 6 m ^ 7 x ^ 7 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m
 ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 5040 b ^
7 x ^ 8 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^
4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 13068 b ^ 7 m x ^ 8 x ^ m
/ (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5
 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 13132 b ^ 7 m ^ 2 x ^ 8 x ^ m / (40320 +
 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^
 6 + 36 m ^ 7 + m ^ 8) + 6769 b ^ 7 m ^ 3 x ^ 8 x ^ m / (40320 + 109584 m +
 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^
 7 + m ^ 8) + 1960 b ^ 7 m ^ 4 x ^ 8 x ^ m / (40320 + 109584 m + 118124 m ^
 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8)
 + 322 b ^ 7 m ^ 5 x ^ 8 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m
 ^ 3 + 22449 m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + 28 b ^ 7
m ^ 6 x ^ 8 x ^ m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449
m ^ 4 + 4536 m ^ 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8) + b ^ 7 m ^ 7 x ^ 8 x ^
m / (40320 + 109584 m + 118124 m ^ 2 + 67284 m ^ 3 + 22449 m ^ 4 + 4536 m ^
 5 + 546 m ^ 6 + 36 m ^ 7 + m ^ 8)]
```

Maple [A]
time = 0.10, size = 156, normalized size = 1.17

method	result
norman	$\frac{a^{7} x\,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{7} x^{8} {\mathrm e}^{m \ln \left(x \right)}}{8+m}+\frac{7 a\,b^{6} x^{7} {\mathrm e}^{m \ln \left(x \right)}}{7+m}+\frac{21 a^{2} b^{5} x^{6} {\mathrm e}^{m \ln \left(x \right)}}{6+m}+\frac{35 a^{3} b^{4} x^{5} {\mathrm e}^{m \ln \left(x \right)}}{5+m}+\frac{35 a^{4} b^{3} x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}+\frac{21 a^{5} b^{2} x^{3} {\mathrm e}^{m}}{3+m}$
risch	$\frac{x \left(b^{7} m^{7} x^{7}+7 a\,b^{6} m^{7} x^{6}+28 b^{7} m^{6} x^{7}+21 a^{2} b^{5} m^{7} x^{5}+203 a\,b^{6} m^{6} x^{6}+322 b^{7} m^{5} x^{7}+35 a^{3} b^{4} m^{7} x^{4}+630 a^{2} b^{5} m^{6} x^{5}+2401 a\,b^{6} m^{5} x^{6}+1960 b^{7}}{}$
gosper	$\frac{x^{1+m} \left(b^{7} m^{7} x^{7}+7 a\,b^{6} m^{7} x^{6}+28 b^{7} m^{6} x^{7}+21 a^{2} b^{5} m^{7} x^{5}+203 a\,b^{6} m^{6} x^{6}+322 b^{7} m^{5} x^{7}+35 a^{3} b^{4} m^{7} x^{4}+630 a^{2} b^{5} m^{6} x^{5}+2401 a\,b^{6} m^{5} x^{6}+19}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] a^7/(1+m)*x*exp(m*ln(x))+b^7/(8+m)*x^8*exp(m*ln(x))+7*a*b^6/(7+m)*x^7*exp(m
*ln(x))+21*a^2*b^5/(6+m)*x^6*exp(m*ln(x))+35*a^3*b^4/(5+m)*x^5*exp(m*ln(x))
+35*a^4*b^3/(4+m)*x^4*exp(m*ln(x))+21*a^5*b^2/(3+m)*x^3*exp(m*ln(x))+7*a^6*
b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 133, normalized size = 1.00

$$\frac{b^7x^{m+8}}{m+8}+\frac{7\,ab^6x^{m+7}}{m+7}+\frac{21\,a^2b^5x^{m+6}}{m+6}+\frac{35\,a^3b^4x^{m+5}}{m+5}+\frac{35\,a^4b^3x^{m+4}}{m+4}+\frac{21\,a^5b^2x^{m+3}}{m+3}+\frac{7\,a^6bx^{m+2}}{m+2}+\frac{a^7x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] b^7*x^(m + 8)/(m + 8) + 7*a*b^6*x^(m + 7)/(m + 7) + 21*a^2*b^5*x^(m + 6)/(m
 + 6) + 35*a^3*b^4*x^(m + 5)/(m + 5) + 35*a^4*b^3*x^(m + 4)/(m + 4) + 21*a^
5*b^2*x^(m + 3)/(m + 3) + 7*a^6*b*x^(m + 2)/(m + 2) + a^7*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(133) = 266$.
time = 0.32, size = 665, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] ((b^7*m^7 + 28*b^7*m^6 + 322*b^7*m^5 + 1960*b^7*m^4 + 6769*b^7*m^3 + 13132*
b^7*m^2 + 13068*b^7*m + 5040*b^7)*x^8 + 7*(a*b^6*m^7 + 29*a*b^6*m^6 + 343*a
*b^6*m^5 + 2135*a*b^6*m^4 + 7504*a*b^6*m^3 + 14756*a*b^6*m^2 + 14832*a*b^6*
m + 5760*a*b^6)*x^7 + 21*(a^2*b^5*m^7 + 30*a^2*b^5*m^6 + 366*a^2*b^5*m^5 +
2340*a^2*b^5*m^4 + 8409*a^2*b^5*m^3 + 16830*a^2*b^5*m^2 + 17144*a^2*b^5*m +
 6720*a^2*b^5)*x^6 + 35*(a^3*b^4*m^7 + 31*a^3*b^4*m^6 + 391*a^3*b^4*m^5 + 2
581*a^3*b^4*m^4 + 9544*a^3*b^4*m^3 + 19564*a^3*b^4*m^2 + 20304*a^3*b^4*m +
8064*a^3*b^4)*x^5 + 35*(a^4*b^3*m^7 + 32*a^4*b^3*m^6 + 418*a^4*b^3*m^5 + 28
64*a^4*b^3*m^4 + 10993*a^4*b^3*m^3 + 23312*a^4*b^3*m^2 + 24876*a^4*b^3*m +
10080*a^4*b^3)*x^4 + 21*(a^5*b^2*m^7 + 33*a^5*b^2*m^6 + 447*a^5*b^2*m^5 + 3
195*a^5*b^2*m^4 + 12864*a^5*b^2*m^3 + 28692*a^5*b^2*m^2 + 32048*a^5*b^2*m +
 13440*a^5*b^2)*x^3 + 7*(a^6*b*m^7 + 34*a^6*b*m^6 + 478*a^6*b*m^5 + 3580*a^
6*b*m^4 + 15289*a^6*b*m^3 + 36706*a^6*b*m^2 + 44712*a^6*b*m + 20160*a^6*b)*
x^2 + (a^7*m^7 + 35*a^7*m^6 + 511*a^7*m^5 + 4025*a^7*m^4 + 18424*a^7*m^3 +
48860*a^7*m^2 + 69264*a^7*m + 40320*a^7)*x)*x^m/(m^8 + 36*m^7 + 546*m^6 + 4
536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)
```

Sympy [A]
time = 0.71, size = 4257, normalized size = 32.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**7,x)
```

```
[Out] Piecewise((-a**7/(7*x**7) - 7*a**6*b/(6*x**6) - 21*a**5*b**2/(5*x**5) - 35*
a**4*b**3/(4*x**4) - 35*a**3*b**4/(3*x**3) - 21*a**2*b**5/(2*x**2) - 7*a*b*
*6/x + b**7*log(x), Eq(m, -8)), (-a**7/(6*x**6) - 7*a**6*b/(5*x**5) - 21*a*
*5*b**2/(4*x**4) - 35*a**4*b**3/(3*x**3) - 35*a**3*b**4/(2*x**2) - 21*a**2*
b**5/x + 7*a*b**6*log(x) + b**7*x, Eq(m, -7)), (-a**7/(5*x**5) - 7*a**6*b/(
4*x**4) - 7*a**5*b**2/x**3 - 35*a**4*b**3/(2*x**2) - 35*a**3*b**4/x + 21*a*
*2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2, Eq(m, -6)), (-a**7/(4*x**4) - 7*
```

a**6*b/(3*x**3) - 21*a**5*b**2/(2*x**2) - 35*a**4*b**3/x + 35*a**3*b**4*log
(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3, Eq(m, -5)), (-a**7/(3
*x**3) - 7*a**6*b/(2*x**2) - 21*a**5*b**2/x + 35*a**4*b**3*log(x) + 35*a**3
*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/3 + b**7*x**4/4, Eq(m, -4)),
(-a**7/(2*x**2) - 7*a**6*b/x + 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a*
*3*b**4*x**2/2 + 7*a**2*b**5*x**3 + 7*a*b**6*x**4/4 + b**7*x**5/5, Eq(m, -3
)), (-a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*
a**3*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6, Eq(
m, -2)), (a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**
3/3 + 35*a**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x*
*7/7, Eq(m, -1)), (a**7*m**7*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**7*m**6*
x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 1
18124*m**2 + 109584*m + 40320) + 511*a**7*m**5*x*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 4025*a**7*m**4*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m
4 + 67284*m3 + 118124*m**2 + 109584*m + 40320) + 18424*a**7*m**3*x*x**m
/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*
m**2 + 109584*m + 40320) + 48860*a**7*m**2*x*x**m/(m**8 + 36*m**7 + 546*m**
6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
69264*a**7*m*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 +
67284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a**7*x*x**m/(m**8 + 36
*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1095
84*m + 40320) + 7*a**6*b*m**7*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
5 + 22449*m4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 238*a**6*
b*m**6*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6728
4*m**3 + 118124*m**2 + 109584*m + 40320) + 3346*a**6*b*m**5*x**2*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + 25060*a**6*b*m**4*x**2*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
107023*a**6*b*m**3*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 256942*a**6*b*m**2*x
2*xm/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 312984*a**6*b*m*x**2*x**m/(m**8 + 36*m**
7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m
+ 40320) + 141120*a**6*b*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**5*b**2*
m**7*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 693*a**5*b**2*m**6*x**3*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + 9387*a**5*b**2*m**5*x**3*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 67095*a**5*b**2*m**4*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2
2449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 270144*a**5*b**2
*m**3*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284

*m**3 + 118124*m**2 + 109584*m + 40320) + 602532*a**5*b**2*m**2*x**3*x**m/(
m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m*
*2 + 109584*m + 40320) + 673008*a**5*b**2*m*x**3*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 282240*a**5*b**2*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**4*b**3*m**7
*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
 + 118124*m**2 + 109584*m + 40320) + 1120*a**4*b**3*m**6*x**4*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 14630*a**4*b**3*m**5*x**4*x**m/(m**8 + 36*m**7 + 546*m**6
 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
100240*a**4*b**3*m**4*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 384755*a**4*b**3*
m**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 815920*a**4*b**3*m**2*x**4*x**m/(m
8 + 36*m7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**
2 + 109584*m + 40320) + 870660*a**4*b**3*m*x**4*x**m/(m**8 + 36*m**7 + 546*
m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320
) + 352800*a**4*b**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 224
49*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**3*b**4*m**7*
x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
+ 118124*m**2 + 109584*m + 40320) + 1085*a**3*b**4*m**6*x**5*x**m/(m**8 + 3
6*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109
584*m + 40320) + 13685*a**3*b**4*m**5*x**5*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 9
0335*a**3*b**4*m**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 334040*a**3*b**4*m*
*3*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320) + 684740*a**3*b**4*m**2*x**5*x**m/(m**
8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2
+ 109584*m + 40320) + 710640*a**3*b**4*m*x**5*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 282240*a**3*b**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**2*b**5*m**7*x*
*6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 630*a**2*b**5*m**6*x**6*x**m/(m**8 + 36*m
7 + 546*m6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 7686*a**2*b**5*m**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 45
36*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 49140
*a**2*b**5*m**4*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m*
*4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 176589*a**2*b**5*m**3*x
6*xm/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 353430*a**2*b**5*m**2*x**6*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 360024*a**2*b**5*m*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 +

```
 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14
1120*a**2*b**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**
4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 7*a*b**6*m**7*x**7*x**m/
(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m
**2 + 109584*m + 40320) + 203*a*b**6*m**6*x**7*x**m/(m**8 + 36*m**7 + 546*m
**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
 + 2401*a*b**6*m**5*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14945*a*b**6*m**4*x
**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 52528*a*b**6*m**3*x**7*x**m/(m**8 + 36*m
**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 103292*a*b**6*m**2*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 453
6*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 103824
*a*b**6*m*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6
7284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a*b**6*x**7*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + b**7*m**7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 28*b**7*m
**6*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m
**3 + 118124*m**2 + 109584*m + 40320) + 322*b**7*m**5*x**8*x**m/(m**8 + 36*
m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10958
4*m + 40320) + 1960*b**7*m**4*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 6769*b**7
*m**3*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 13132*b**7*m**2*x**8*x**m/(m**8 +
 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1
09584*m + 40320) + 13068*b**7*m*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536
*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 5040*b*
*7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(133) = 266.

time = 0.01, size = 1119, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x)
```

```
[Out] (b^7*m^7*x^8*x^m + 7*a*b^6*m^7*x^7*x^m + 28*b^7*m^6*x^8*x^m + 21*a^2*b^5*m^
7*x^6*x^m + 203*a*b^6*m^6*x^7*x^m + 322*b^7*m^5*x^8*x^m + 35*a^3*b^4*m^7*x^
5*x^m + 630*a^2*b^5*m^6*x^6*x^m + 2401*a*b^6*m^5*x^7*x^m + 1960*b^7*m^4*x^8
*x^m + 35*a^4*b^3*m^7*x^4*x^m + 1085*a^3*b^4*m^6*x^5*x^m + 7686*a^2*b^5*m^5
*x^6*x^m + 14945*a*b^6*m^4*x^7*x^m + 6769*b^7*m^3*x^8*x^m + 21*a^5*b^2*m^7*
x^3*x^m + 1120*a^4*b^3*m^6*x^4*x^m + 13685*a^3*b^4*m^5*x^5*x^m + 49140*a^2*
```

```
b^5*m^4*x^6*x^m + 52528*a*b^6*m^3*x^7*x^m + 13132*b^7*m^2*x^8*x^m + 7*a^6*b
*m^7*x^2*x^m + 693*a^5*b^2*m^6*x^3*x^m + 14630*a^4*b^3*m^5*x^4*x^m + 90335*
a^3*b^4*m^4*x^5*x^m + 176589*a^2*b^5*m^3*x^6*x^m + 103292*a*b^6*m^2*x^7*x^m
 + 13068*b^7*m*x^8*x^m + a^7*m^7*x*x^m + 238*a^6*b*m^6*x^2*x^m + 9387*a^5*b
^2*m^5*x^3*x^m + 100240*a^4*b^3*m^4*x^4*x^m + 334040*a^3*b^4*m^3*x^5*x^m +
353430*a^2*b^5*m^2*x^6*x^m + 103824*a*b^6*m*x^7*x^m + 5040*b^7*x^8*x^m + 35
*a^7*m^6*x*x^m + 3346*a^6*b*m^5*x^2*x^m + 67095*a^5*b^2*m^4*x^3*x^m + 38475
5*a^4*b^3*m^3*x^4*x^m + 684740*a^3*b^4*m^2*x^5*x^m + 360024*a^2*b^5*m*x^6*x
^m + 40320*a*b^6*x^7*x^m + 511*a^7*m^5*x*x^m + 25060*a^6*b*m^4*x^2*x^m + 27
0144*a^5*b^2*m^3*x^3*x^m + 815920*a^4*b^3*m^2*x^4*x^m + 710640*a^3*b^4*m*x^
5*x^m + 141120*a^2*b^5*x^6*x^m + 4025*a^7*m^4*x*x^m + 107023*a^6*b*m^3*x^2*
x^m + 602532*a^5*b^2*m^2*x^3*x^m + 870660*a^4*b^3*m*x^4*x^m + 282240*a^3*b^
4*x^5*x^m + 18424*a^7*m^3*x*x^m + 256942*a^6*b*m^2*x^2*x^m + 673008*a^5*b^2
*m*x^3*x^m + 352800*a^4*b^3*x^4*x^m + 48860*a^7*m^2*x*x^m + 312984*a^6*b*m*
x^2*x^m + 282240*a^5*b^2*x^3*x^m + 69264*a^7*m*x*x^m + 141120*a^6*b*x^2*x^m
 + 40320*a^7*x*x^m)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*
m^3 + 118124*m^2 + 109584*m + 40320)
```

Mupad [B]
time = 0.78, size = 683, normalized size = 5.14

$$\frac{a^7\,x\,x^m\,\left(m^7+35\,m^6+511\,m^5+4025\,m^4+18424\,m^3+48860\,m^2+69264\,m+40320\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{b^7\,x^m\,x^8\,\left(m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{21\,a^2\,b^5\,x^m\,x^6\,\left(m^7+30\,m^6+366\,m^5+2340\,m^4+8409\,m^3+16830\,m^2+17144\,m+6720\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{35\,a^3\,b^4\,x^m\,x^5\,\left(m^7+31\,m^6+391\,m^5+2581\,m^4+9544\,m^3+19564\,m^2+20304\,m+8064\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{35\,a^4\,b^3\,x^m\,x^4\,\left(m^7+32\,m^6+418\,m^5+2864\,m^4+10993\,m^3+23312\,m^2+24876\,m+10080\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{21\,a^5\,b^2\,x^m\,x^3\,\left(m^7+33\,m^6+447\,m^5+3195\,m^4+12864\,m^3+28692\,m^2+32048\,m+13440\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{7\,a\,b^6\,x^m\,x^7\,\left(m^7+29\,m^6+343\,m^5+2135\,m^4+7504\,m^3+14756\,m^2+14832\,m+5760\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{7\,a^6\,b\,x^m\,x^2\,\left(m^7+34\,m^6+478\,m^5+3580\,m^4+15289\,m^3+36706\,m^2+44712\,m+20160\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^7,x)
```

```
[Out] (a^7*x*x^m*(69264*m + 48860*m^2 + 18424*m^3 + 4025*m^4 + 511*m^5 + 35*m^6 +
 m^7 + 40320))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (b^7*x^m*x^8*(13068*m + 13132*m^2 + 6769*
m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(109584*m + 118124*m^2 + 6
7284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^2
*b^5*x^m*x^6*(17144*m + 16830*m^2 + 8409*m^3 + 2340*m^4 + 366*m^5 + 30*m^6
+ m^7 + 6720))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (35*a^3*b^4*x^m*x^5*(20304*m + 19564*m^2
+ 9544*m^3 + 2581*m^4 + 391*m^5 + 31*m^6 + m^7 + 8064))/(109584*m + 118124*
m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) +
(35*a^4*b^3*x^m*x^4*(24876*m + 23312*m^2 + 10993*m^3 + 2864*m^4 + 418*m^5 +
 32*m^6 + m^7 + 10080))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 45
36*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^5*b^2*x^m*x^3*(32048*m + 2
8692*m^2 + 12864*m^3 + 3195*m^4 + 447*m^5 + 33*m^6 + m^7 + 13440))/(109584*
m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8
+ 40320) + (7*a*b^6*x^m*x^7*(14832*m + 14756*m^2 + 7504*m^3 + 2135*m^4 + 34
3*m^5 + 29*m^6 + m^7 + 5760))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^
4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (7*a^6*b*x^m*x^2*(44712*m
+ 36706*m^2 + 15289*m^3 + 3580*m^4 + 478*m^5 + 34*m^6 + m^7 + 20160))/(1095
```

```
84*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m
^8 + 40320)
```

3.701 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(2+m)/(2+m)+3*a*b^2*x^(3+m)/(3+m)+b^3*x^(4+m)/(
4+m)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx&=\int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)\,dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.89

$$x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx}{2+m}+\frac{3ab^2x^2}{3+m}+\frac{b^3x^3}{4+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.49, size = 662, normalized size = 10.85

$$\text{Piecewise}\left[\left\{\left\{-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\text{Log}[x],m==-4\right\},\left\{-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\text{Log}[x]+b^3x,m==-3\right\},\left\{\frac{-a^3+\frac{bx(6a^2\text{Log}[x]+6abx+b^2x^2)}{2}}{x},m==-2\right\},\left\{a^3\text{Log}[x]+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3},m==-1\right\}\right\},\frac{24a^3xx^m}{24+50m+35m^2+10m^3+m^4}+\frac{26a^3mxx^m}{24+50m+35m^2+10m^3+m^4}+\frac{9a^3m^2xx^m}{24+50m+35m^2+10m^3+m^4}+\frac{a^3m^3xx^m}{24+50m+35m^2+10m^3+m^4}+\frac{36a^2bx^2x^m}{24+50m+35m^2+10m^3+m^4}+\frac{57a^2bmx^2x^m}{24+50m+35m^2+10m^3+m^4}+\frac{24a^2bm^2x^2x^m}{24+50m+35m^2+10m^3+m^4}+\frac{3a^2bm^3x^2x^m}{24+50m+35m^2+10m^3+m^4}+\frac{24ab^2x^3x^m}{24+50m+35m^2+10m^3+m^4}+\frac{42ab^2mx^3x^m}{24+50m+35m^2+10m^3+m^4}+\frac{21ab^2m^2x^3x^m}{24+50m+35m^2+10m^3+m^4}+\frac{3ab^2m^3x^3x^m}{24+50m+35m^2+10m^3+m^4}+\frac{6b^3x^4x^m}{24+50m+35m^2+10m^3+m^4}+\frac{11b^3mx^4x^m}{24+50m+35m^2+10m^3+m^4}+\frac{6b^3m^2x^4x^m}{24+50m+35m^2+10m^3+m^4}+\frac{b^3m^3x^4x^m}{24+50m+35m^2+10m^3+m^4}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^m*(a + b*x)^3,x]')
```

```
[Out] Piecewise[{{-a ^ 3 / (3 x ^ 3) - 3 a ^ 2 b / (2 x ^ 2) - 3 a b ^ 2 / x + b
^ 3 Log[x], m == -4}, {-a ^ 3 / (2 x ^ 2) - 3 a ^ 2 b / x + 3 a b ^ 2 Log[x
] + b ^ 3 x, m == -3}, {(-a ^ 3 + b x (6 a ^ 2 Log[x] + 6 a b x + b ^ 2 x ^
 2) / 2) / x, m == -2}, {a ^ 3 Log[x] + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 / 2 +
 b ^ 3 x ^ 3 / 3, m == -1}}, 24 a ^ 3 x x ^ m / (24 + 50 m + 35 m ^ 2 + 10
m ^ 3 + m ^ 4) + 26 a ^ 3 m x x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m
^ 4) + 9 a ^ 3 m ^ 2 x x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) +
a ^ 3 m ^ 3 x x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 36 a ^ 2
b x ^ 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 57 a ^ 2 b m x
^ 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 24 a ^ 2 b m ^ 2 x
^ 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 3 a ^ 2 b m ^ 3 x ^
 2 x ^ m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 24 a b ^ 2 x ^ 3 x ^
 m / (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 42 a b ^ 2 m x ^ 3 x ^ m /
 (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 21 a b ^ 2 m ^ 2 x ^ 3 x ^ m /
 (24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 3 a b ^ 2 m ^ 3 x ^ 3 x ^ m /
(24 + 50 m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 6 b ^ 3 x ^ 4 x ^ m / (24 + 50
m + 35 m ^ 2 + 10 m ^ 3 + m ^ 4) + 11 b ^ 3 m x ^ 4 x ^ m / (24 + 50 m + 35
 m ^ 2 + 10 m ^ 3 + m ^ 4) + 6 b ^ 3 m ^ 2 x ^ 4 x ^ m / (24 + 50 m + 35 m
^ 2 + 10 m ^ 3 + m ^ 4) + b ^ 3 m ^ 3 x ^ 4 x ^ m / (24 + 50 m + 35 m ^ 2 +
 10 m ^ 3 + m ^ 4)]
```

Maple [A]
time = 0.10, size = 72, normalized size = 1.18

method	result
norman	$\frac{a^{3} x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{3} x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}+\frac{3 a \,b^{2} x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}+\frac{3 a^{2} b \,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}$
risch	$\frac{x \left(b^{3} m^{3} x^{3}+3 a \,b^{2} m^{3} x^{2}+6 b^{3} m^{2} x^{3}+3 a^{2} b \,m^{3} x +21 a \,b^{2} m^{2} x^{2}+11 b^{3} m \,x^{3}+a^{3} m^{3}+24 a^{2} b \,m^{2} x +42 a \,b^{2} m \,x^{2}+6 b^{3} x^{3}+9 a^{3} m^{2}+57 a^{2} b m x +\right.}{\left(4+m \right) \left(3+m \right) \left(2+m \right) \left(1+m \right)}$
gosper	$\frac{x^{1+m} \left(b^{3} m^{3} x^{3}+3 a \,b^{2} m^{3} x^{2}+6 b^{3} m^{2} x^{3}+3 a^{2} b \,m^{3} x +21 a \,b^{2} m^{2} x^{2}+11 b^{3} m \,x^{3}+a^{3} m^{3}+24 a^{2} b \,m^{2} x +42 a \,b^{2} m \,x^{2}+6 b^{3} x^{3}+9 a^{3} m^{2}+57 a^{2} b\right.}{\left(4+m \right) \left(3+m \right) \left(2+m \right) \left(1+m \right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/(1+m)*x*exp(m*ln(x))+b^3/(4+m)*x^4*exp(m*ln(x))+3*a*b^2/(3+m)*x^3*exp(m
*ln(x))+3*a^2*b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+4}}{m+4}+\frac{3\,ab^2x^{m+3}}{m+3}+\frac{3\,a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.
time = 0.33, size = 157, normalized size = 2.57

$$\frac{((b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3)x^4+3\,(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2)x^3+3\,(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b)x^2+(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3)x)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

Sympy [A]
time = 0.28, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x) & \text{for}\: m=-4 \\ -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x & \text{for}\: m=-3 \\ -\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2} & \text{for}\: m=-2 \\ a^3\log(x)+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3} & \text{for}\: m=-1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
```

```
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(61) = 122$.
time = 0.00, size = 255, normalized size = 4.18

$$\frac{a^3m^3xe^{m\ln x}+9a^3m^2xe^{m\ln x}+26a^3mxe^{m\ln x}+24a^3xe^{m\ln x}+3a^2bm^3x^2e^{m\ln x}+24a^2bm^2x^2e^{m\ln x}+57a^2bmx^2e^{m\ln x}+36a^2bx^2e^{m\ln x}+3ab^2m^3x^3e^{m\ln x}+21ab^2m^2x^3e^{m\ln x}+42ab^2mx^3e^{m\ln x}+24ab^2x^3e^{m\ln x}+b^3m^3x^4e^{m\ln x}+6b^3m^2x^4e^{m\ln x}+11b^3mx^4e^{m\ln x}+6b^3x^4e^{m\ln x}}{m^4+10m^3+35m^2+50m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x)
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B]
time = 0.44, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3x\left(m^3+9m^2+26m+24\right)}{m^4+10m^3+35m^2+50m+24}+\frac{b^3x^4\left(m^3+6m^2+11m+6\right)}{m^4+10m^3+35m^2+50m+24}+\frac{3ab^2x^3\left(m^3+7m^2+14m+8\right)}{m^4+10m^3+35m^2+50m+24}+\frac{3a^2bx^2\left(m^3+8m^2+19m+12\right)}{m^4+10m^3+35m^2+50m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

3.702 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(2+m)/(2+m)+b^2*x^(3+m)/(3+m)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx&=\int\left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)\,dx\\&=\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.88

$$x^{1+m}\left(\frac{a^2}{1+m}+\frac{2abx}{2+m}+\frac{b^2x^2}{3+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.98, size = 311, normalized size = 7.23

$$\text{Piecewise}\left[\left\{\left\{-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\text{Log}\left[x\right],m==-3\right\},\left\{\frac{-a^2+bx\left(2a\text{Log}\left[x\right]+bx\right)}{x},m==-2\right\},\left\{a^2\text{Log}\left[x\right]+2abx+\frac{b^2x^2}{2},m==-1\right\}\right\},\frac{6a^2xx^m}{6+11m+6m^2+m^3}+\frac{5a^2mxx^m}{6+11m+6m^2+m^3}+\frac{a^2m^2xx^m}{6+11m+6m^2+m^3}+\frac{6abx^2x^m}{6+11m+6m^2+m^3}+\frac{8abmx^2x^m}{6+11m+6m^2+m^3}+\frac{2abm^2x^2x^m}{6+11m+6m^2+m^3}+\frac{2b^2x^3x^m}{6+11m+6m^2+m^3}+\frac{3b^2mx^3x^m}{6+11m+6m^2+m^3}+\frac{b^2m^2x^3x^m}{6+11m+6m^2+m^3}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^m*(a + b*x)^2,x]')
```

```
[Out] Piecewise[{{-a ^ 2 / (2 x ^ 2) - 2 a b / x + b ^ 2 Log[x], m == -3}, {(-a ^
 2 + b x (2 a Log[x] + b x)) / x, m == -2}, {a ^ 2 Log[x] + 2 a b x + b ^ 2
 x ^ 2 / 2, m == -1}}, 6 a ^ 2 x x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + 5 a
 ^ 2 m x x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + a ^ 2 m ^ 2 x x ^ m / (6 +
11 m + 6 m ^ 2 + m ^ 3) + 6 a b x ^ 2 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3)
+ 8 a b m x ^ 2 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + 2 a b m ^ 2 x ^ 2 x
^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + 2 b ^ 2 x ^ 3 x ^ m / (6 + 11 m + 6 m
^ 2 + m ^ 3) + 3 b ^ 2 m x ^ 3 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3) + b ^ 2
 m ^ 2 x ^ 3 x ^ m / (6 + 11 m + 6 m ^ 2 + m ^ 3)]
```

Maple [A]
time = 0.12, size = 51, normalized size = 1.19

method	result	size
norman	$\frac{a^{2} x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{2} x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}+\frac{2 a b \,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}$	51
risch	$\frac{x \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 a b m x +2 x^{2} b^{2}+5 a^{2} m +6 a b x +6 a^{2}\right) x^{m}}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$	86
gosper	$\frac{x^{1+m} \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 a b m x +2 x^{2} b^{2}+5 a^{2} m +6 a b x +6 a^{2}\right)}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(1+m)*x*exp(m*ln(x))+b^2/(3+m)*x^3*exp(m*ln(x))+2*a*b/(2+m)*x^2*exp(m*l
n(x))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+3}}{m+3}+\frac{2\,abx^{m+2}}{m+2}+\frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

Fricas [**A**]
time = 0.32, size = 85, normalized size = 1.98

$$\frac{((b^2m^2 + 3\,b^2m + 2\,b^2)x^3 + 2\,(abm^2 + 4\,abm + 3\,ab)x^2 + (a^2m^2 + 5\,a^2m + 6\,a^2)x)x^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [**A**]
time = 0.19, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x) & \text{for } m = -3 \\ -\frac{a^2}{x} + 2ab\log(x) + b^2x & \text{for } m = -2 \\ a^2\log(x) + 2abx + \frac{b^2x^2}{2} & \text{for } m = -1 \\ \frac{a^2m^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2mxx^m}{m^3+6m^2+11m+6} + \frac{6a^2xx^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2m^2x^3x^m}{m^3+6m^2+11m+6} + \frac{3b^2mx^3x^m}{m^3+6m^2+11m+6} + \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.
time = 0.00, size = 134, normalized size = 3.12

$$\frac{a^2m^2xe^{m\ln x} + 5a^2mxe^{m\ln x} + 6a^2xe^{m\ln x} + 2abm^2x^2e^{m\ln x} + 8abmx^2e^{m\ln x} + 6abx^2e^{m\ln x} + b^2m^2x^3e^{m\ln x} + 3b^2mx^3e^{m\ln x} + 2b^2x^3e^{m\ln x}}{m^3 + 6m^2 + 11m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x)
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

Mupad [B]
time = 0.37, size = 93, normalized size = 2.16

$$x^m\,\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

3.703 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(2+m)/(2+m)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx &= \int \left(ax^m+bx^{1+m}\right)\,dx\\ &= \frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.88

$$x^{1+m}\left(\frac{a}{1+m}+\frac{bx}{2+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
```

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.68, size = 101, normalized size = 4.04

$$\text{Piecewise}\left[\left\{\left\{-\frac{a}{x}+b\text{Log}\left[x\right],m==-2\right\},\left\{a\text{Log}\left[x\right]+bx,m==-1\right\}\right\},\frac{2axx^m}{2+3m+m^2}+\frac{amxx^m}{2+3m+m^2}+\frac{bx^2x^m}{2+3m+m^2}+\frac{bmx^2x^m}{2+3m+m^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(a + b*x)^1,x]')
```

```
[Out] Piecewise[{{-a / x + b Log[x], m == -2}, {a Log[x] + b x, m == -1}}, 2 a x
x ^ m / (2 + 3 m + m ^ 2) + a m x x ^ m / (2 + 3 m + m ^ 2) + b x ^ 2 x ^ m
 / (2 + 3 m + m ^ 2) + b m x ^ 2 x ^ m / (2 + 3 m + m ^ 2)]
```

Maple [A]
time = 0.01, size = 30, normalized size = 1.20

method	result	size
norman	$\frac{a x\,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b\,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}$	30
risch	$\frac{x \left(b m x +a m +b x +2 a \right) x^{m}}{\left(2+m \right) \left(1+m \right)}$	30
gosper	$\frac{x^{1+m} \left(b m x +a m +b x +2 a \right)}{\left(2+m \right) \left(1+m \right)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] a/(1+m)*x*exp(m*ln(x))+b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.27, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.31, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [A]
time = 0.13, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x} + b\log(x) & \text{for}\: m = -2 \\ a\log(x) + bx & \text{for}\: m = -1 \\ \frac{amxx^m}{m^2+3m+2} + \frac{2axx^m}{m^2+3m+2} + \frac{bmx^2x^m}{m^2+3m+2} + \frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

Giac [A]
time = 0.00, size = 50, normalized size = 2.00

$$\frac{amxe^{m\ln x} + 2axe^{m\ln x} + bmx^2e^{m\ln x} + bx^2e^{m\ln x}}{m^2 + 3m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x)
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [B]
time = 0.30, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,\left(2\,a+a\,m+b\,x+b\,m\,x\right)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
```

```
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

3.704 $\int \frac{x^m}{a+bx}\,dx$

Optimal. Leaf size=29

$$\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*hypergeom([1, 1+m],[2+m],-b*x/a)/a/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {66}

$$\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{bx}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x),x]
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{a+bx}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x),x]
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.22, size = 24, normalized size = 0.83

$$\frac{\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[I\text{Pi}]}{a},1,1+m\right]x^{1+m}}{a}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/(a + b*x)^1,x]')
[Out] LerchPhi[b x exp_polar[I Pi] / a, 1, 1 + m] x ^ (1 + m) / a
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a),x)
[Out] int(x^m/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="maxima")
[Out] integrate(x^m/(b*x + a), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="fricas")
[Out] integral(x^m/(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 61, normalized size = 2.10

$$\frac{mxx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma\left(m+1\right)}{a\Gamma\left(m+2\right)}+\frac{xx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma\left(m+1\right)}{a\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*gamma(m
+ 2)) + x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*ga
mma(m + 2))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x),x)
```

```
[Out] int(x^m/(a + b*x), x)
```

3.705 $\int \frac{x^m}{(a+bx)^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{a}\right)}{a^2(1+m)}$$

```
[Out] x^(1+m)*hypergeom([2, 1+m],[2+m],-b*x/a)/a^2/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{66\}$

$$\frac{x^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{bx}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{(a+bx)^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{a}\right)}{a^2(1+m)}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{a}\right)}{a^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 4.01, size = 57, normalized size = 1.97

$$\frac{\left(a - am\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}, 1, 1+m\right] - bmx\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}, 1, 1+m\right]\right) x^{1+m}}{a^2 (a+bx)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/(a + b*x)^2,x]')
```

```
[Out] (a - a m LerchPhi[b x exp_polar[I Pi] / a, 1, 1 + m] - b m x LerchPhi[b x e
xp_polar[I Pi] / a, 1, 1 + m]) x ^ (1 + m) / (a ^ 2 (a + b x))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx+a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^2,x)
```

```
[Out] int(x^m/(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^2, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 262, normalized size = 9.03

$$-\frac{am^2xx^m\Phi\left(\frac{bxe^{i\pi}}{a}, 1, m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} - \frac{amxx^m\Phi\left(\frac{bxe^{i\pi}}{a}, 1, m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} + \frac{amxx^m\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} + \frac{axx^m\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} - \frac{bm^2x^2x^m\Phi\left(\frac{bxe^{i\pi}}{a}, 1, m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)} - \frac{bmx^2x^m\Phi\left(\frac{bxe^{i\pi}}{a}, 1, m+1\right)\Gamma(m+1)}{a^3\Gamma(m+2)+a^2bx\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**2,x)
```

```
[Out] -a*m**2*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3
*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - a*m*x*x**m*lerchphi(b*x*exp_polar(
I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
 + a*m*x*x**m*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) + a*
x*x**m*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - b*m**2*x*
*2*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(
m + 2) + a**2*b*x*gamma(m + 2)) - b*m*x**2*x**m*lerchphi(b*x*exp_polar(I*pi
)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^2,x)
```

```
[Out] int(x^m/(a + b*x)^2, x)
```

3.706 $\int \frac{x^m}{(a+bx)^3} \, dx$

Optimal. Leaf size=29

$$\frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

```
[Out] x^(1+m)*hypergeom([3, 1+m],[2+m],-b*x/a)/a^3/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{66\}$

$$\frac{x^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{bx}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{(a+bx)^3} \, dx = \frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 7.46, size = 155, normalized size = 5.34

$$\frac{a^2\left(2+m-m\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[\imath\text{Pi}]}{a},1,1+m\right]-m^2+m^3\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[\imath\text{Pi}]}{a},1,1+m\right]\right)x^{1+m}+ab\left(1-2m\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[\imath\text{Pi}]}{a},1,1+m\right]-m^2+2m^3\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[\imath\text{Pi}]}{a},1,1+m\right]\right)x^{2+m}+b^2m\text{LerchPhi}\left[\frac{bx\text{exp\_polar}[\imath\text{Pi}]}{a},1,1+m\right](-1+m^2)x^{3+m}}{2a^3(1+m)(a+bx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/(a + b*x)^3,x]')
```

```
[Out] (a ^ 2 (2 + m - m LerchPhi[b x exp_polar[I Pi] / a, 1, 1 + m] - m ^ 2 + m ^
 3 LerchPhi[b x exp_polar[I Pi] / a, 1, 1 + m]) x ^ (1 + m) + a b (1 - 2 m
LerchPhi[b x exp_polar[I Pi] / a, 1, 1 + m] - m ^ 2 + 2 m ^ 3 LerchPhi[b x
exp_polar[I Pi] / a, 1, 1 + m]) x ^ (2 + m) + b ^ 2 m LerchPhi[b x exp_pola
r[I Pi] / a, 1, 1 + m] (-1 + m ^ 2) x ^ (3 + m)) / (2 a ^ 3 (1 + m) (a + b
x) ^ 2)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^3,x)
```

```
[Out] int(x^m/(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^3, x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.68, size = 717, normalized size = 24.72

$$\frac{a^2m^3xx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}-\frac{a^2m^2xx^m\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}-\frac{a^2mxx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}+\frac{a^2mxx^m\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}+\frac{2a^2xx^m\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}+\frac{2abm^3x^2x^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}-\frac{abm^2x^2x^m\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}-\frac{2abmx^2x^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}+\frac{abx^2x^m\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}+\frac{b^2m^3x^3x^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}-\frac{b^2mx^3x^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^5\Gamma(m+2)+4a^4bx\Gamma(m+2)+2a^3b^2x^2\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**3,x)
```

```
[Out] a**2*m**3*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*
a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)
) - a**2*m**2*x*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m
 + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a**2*m*x*x**m*lerchphi(b*x*exp_pol
ar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(
m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + a**2*m*x*x**m*gamma(m + 1)/(2*a**
5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) +
 2*a**2*x*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2)
+ 2*a**3*b**2*x**2*gamma(m + 2)) + 2*a*b*m**3*x**2*x**m*lerchphi(b*x*exp_po
lar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma
(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a*b*m**2*x**2*x**m*gamma(m + 1)/
(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m +
 2)) - 2*a*b*m*x**2*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m
+ 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamm
a(m + 2)) + a*b*x**2*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*ga
mma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + b**2*m**3*x**3*x**m*lerchphi(
b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4
*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - b**2*m*x**3*x**m*lerch
phi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*
a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^3,x)
```

```
[Out] int(x^m/(a + b*x)^3, x)
```

3.707 $\int x^m(a+bx)^{5/2}\,dx$

Optimal. Leaf size=48

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}$$

```
[Out] 2/7*x^m*(b*x+a)^(7/2)*hypergeom([7/2, -m],[9/2],1+b*x/a)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m(a+bx)^{7/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};\frac{bx}{a}+1\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^(5/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{5/2}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m(a+bx)^{5/2}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x)^(5/2),x]

[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(-((b*x)/a))^m)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 6.87, size = 34, normalized size = 0.71

$$\frac{a^{\frac{5}{2}}x^{1+m}\text{hyper}\left[\left\{-\frac{5}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^m*(a + b*x)^(5/2),x]')

[Out] a ^ (5 / 2) x ^ (1 + m) hyper[{-5 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / (1 + m)

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m\,(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(b*x+a)^(5/2),x)

[Out] int(x^m*(b*x+a)^(5/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate((b*x + a)^(5/2)*x^m, x)

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 6.46, size = 37, normalized size = 0.77

$$\frac{a^{\frac{5}{2}} x x^{m} \Gamma(m+1)\, {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(5/2),x)
```

```
[Out] a**(5/2)*x*x**m*gamma(m + 1)*hyper((-5/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^(5/2),x)
```

```
[Out] int(x^m*(a + b*x)^(5/2), x)
```

3.708 $\int x^m(a+bx)^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}$$

```
[Out] 2/5*x^m*(b*x+a)^(5/2)*hypergeom([5/2, -m],[7/2],1+b*x/a)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {69, 67}

$$\frac{2x^m(a+bx)^{5/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};\frac{bx}{a}+1\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^(3/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{3/2}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m(a+bx)^{3/2}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^(3/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.31, size = 34, normalized size = 0.71

$$\frac{a^{\frac{3}{2}}x^{1+m}\text{hyper}\left[\left\{-\frac{3}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(a + b*x)^(3/2),x]')
```

```
[Out] a ^ (3 / 2) x ^ (1 + m) hyper[{-3 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi]
 / a] / (1 + m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(3/2),x)
```

```
[Out] int(x^m*(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*x^m, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.84, size = 37, normalized size = 0.77

$$\frac{a^{\frac{3}{2}}xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m + 1)*hyper((-3/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^(3/2),x)
```

```
[Out] int(x^m*(a + b*x)^(3/2), x)
```

3.709 $\int x^m \sqrt{a+bx}\, dx$

Optimal. Leaf size=48

$$\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} (a+bx)^{3/2} {}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}$$

```
[Out] 2/3*x^m*(b*x+a)^(3/2)*hypergeom([3/2, -m],[5/2],1+b*x/a)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m (a+bx)^{3/2} \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{bx}{a}+1\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int x^m \sqrt{a+bx}\, dx &= \left(x^m \left(-\frac{bx}{a}\right)^{-m}\right) \int \left(-\frac{bx}{a}\right)^m \sqrt{a+bx}\, dx\\ &= \frac{2x^m \left(-\frac{bx}{a}\right)^{-m} (a+bx)^{3/2} {}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(
-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.46, size = 34, normalized size = 0.71

$$\frac{\sqrt{a}\,x^{1+m}\text{hyper}\left[\left\{-\frac{1}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(a + b*x)^(1/2),x]')
```

```
[Out] Sqrt[a] x ^ (1 + m) hyper[{-1 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a
] / (1 + m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^m\sqrt{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*x^m, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.81, size = 37, normalized size = 0.77

$$\frac{\sqrt{a}\,xx^m\Gamma\left(m+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(1/2),x)
[Out] sqrt(a)*x*x**m*gamma(m + 1)*hyper((-1/2, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\sqrt{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^(1/2),x)
[Out] int(x^m*(a + b*x)^(1/2), x)
```

3.710 $\int \frac{x^m}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=46

$$\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx} \, {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

```
[Out] 2*x^m*hypergeom([1/2, -m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m \sqrt{a+bx} \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx}} \, dx &= \left(x^m \left(-\frac{bx}{a}\right)^{-m}\right) \int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}} \, dx \\ &= \frac{2x^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx} \, {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.40, size = 34, normalized size = 0.74

$$\frac{x^{1+m}\text{hyper}\left[\left\{\frac{1}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\,(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/(a + b*x)^(1/2),x]')
```

```
[Out] x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / (Sqrt
[a] (1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.72, size = 36, normalized size = 0.78

$$\frac{xx^m\Gamma\left(m+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(s
qrt(a)*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x)^(1/2), x)
```

3.711 $\int \frac{x^m}{(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=46

$$-\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}$$

```
[Out] -2*x^m*hypergeom([-1/2, -m],[1/2],1+b*x/a)/b/((-b*x/a)^m)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$-\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx)^{3/2}} \, dx &= \left(x^m \left(-\frac{bx}{a}\right)^{-m}\right) \int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}} \, dx \\ &= -\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 1.00

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.57, size = 34, normalized size = 0.74

$$\frac{x^{1+m}\text{hyper}\left[\left\{\frac{3}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{a^{\frac{3}{2}}(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/(a + b*x)^(3/2),x]')
```

```
[Out] x ^ (1 + m) hyper[{3 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / (a ^
(3 / 2) (1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{x^m}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*x^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.93, size = 36, normalized size = 0.78

$$\frac{xx^{m}\Gamma\left(m+1\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{2}}\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(3/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(a
**(3/2)*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(3/2),x)
```

```
[Out] int(x^m/(a + b*x)^(3/2), x)
```

3.712 $\int \frac{x^m}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}$$

```
[Out] -2/3*x^m*hypergeom([-3/2, -m],[-1/2],1+b*x/a)/b/((-b*x/a)^m)/(b*x+a)^(3/2)
```

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};\frac{bx}{a}+1\right)}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx)^{5/2}}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{5/2}}\,dx\\ &= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.00

$$-\frac{2x^m \left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.34, size = 34, normalized size = 0.71

$$\frac{x^{1+m}\text{hyper}\left[\left\{\frac{5}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{a^{\frac{5}{2}}(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/(a + b*x)^(5/2),x]')
```

```
[Out] x ^ (1 + m) hyper[{5 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / (a ^
(5 / 2) (1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(5/2),x)
```

```
[Out] int(x^m/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*x^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.93, size = 36, normalized size = 0.75

$$\frac{xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(5/2),x)
[Out] x*x**m*gamma(m + 1)*hyper((5/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(a
**(5/2)*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(5/2),x)
[Out] int(x^m/(a + b*x)^(5/2), x)
```

3.713 $\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}$$

```
[Out] 2*a^2*x^m*hypergeom([1/2, -2-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b^3/((-b*x/a)^
m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$\frac{2a^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m-2;\frac{3}{2};\frac{bx}{a}+1\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx &= \frac{\left(a^2x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{2+m}}{\sqrt{a+bx}}\,dx}{b^2}\\ &= \frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 1.00

$$\frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.25, size = 34, normalized size = 0.67

$$\frac{x^{3+m}\text{hyper}\left[\left\{\frac{1}{2},3+m\right\},\{4+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\ (3+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(2 + m)/Sqrt[a + b*x],x]')
```

```
[Out] x ^ (3 + m) hyper[{1 / 2, 3 + m}, {4 + m}, b x exp_polar[I Pi] / a] / (Sqrt
[a] (3 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{2+m}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(2+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m + 2)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.73, size = 37, normalized size = 0.73

$$\frac{x^3 x^m \Gamma(m+3)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+3\\ m+4\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**3*x**m*gamma(m + 3)*hyper((1/2, m + 3), (m + 4,), b*x*exp_polar(I*pi)/a)
/(sqrt(a)*gamma(m + 4))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m + 2)/(a + b*x)^(1/2), x)
```

3.714 $\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=49

$$-\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}$$

```
[Out] -2*a*x^m*hypergeom([1/2, -1-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b^2/((-b*x/a)^m
)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$-\frac{2ax^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m-1;\frac{3}{2};\frac{bx}{a}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(ax^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{1+m}}{\sqrt{a+bx}}\,dx}{b}\\ &= -\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.00

$$-\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.83, size = 34, normalized size = 0.69

$$\frac{x^{2+m}\text{hyper}\left[\left\{\frac{1}{2},2+m\right\},\{3+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\,(2+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1 + m)/Sqrt[a + b*x],x]')
```

```
[Out] x ^ (2 + m) hyper[{1 / 2, 2 + m}, {3 + m}, b x exp_polar[I Pi] / a] / (Sqrt
[a] (2 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{x^{1+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(1+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m + 1)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.24, size = 37, normalized size = 0.76

$$\frac{x^2x^m\Gamma(m+2)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+2\\m+3\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)/(b*x+a)**(1/2),x)
[Out] x**2*x**m*gamma(m + 2)*hyper((1/2, m + 2), (m + 3,), b*x*exp_polar(I*pi)/a)
/(sqrt(a)*gamma(m + 3))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+1}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)/(a + b*x)^(1/2),x)
[Out] int(x^(m + 1)/(a + b*x)^(1/2), x)
```

3.715 $\int \frac{x^m}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=46

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

```
[Out] 2*x^m*hypergeom([1/2, -m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {69, 67}

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx}}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.39, size = 34, normalized size = 0.74

$$\frac{x^{1+m}\text{hyper}\left[\left\{\frac{1}{2},1+m\right\},\left\{2+m\right\},\frac{bx\text{exp\_polar}\left[I\text{Pi}\right]}{a}\right]}{\sqrt{a}\,(1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(0 + m)/Sqrt[a + b*x],x]')
```

```
[Out] x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / (Sqrt
[a] (1 + m))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.64, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.72, size = 36, normalized size = 0.78

$$\frac{xx^m\Gamma\left(m+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(s
qrt(a)*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x)^(1/2), x)
```

3.716 $\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}$$

```
[Out] -2*x^m*hypergeom([1/2, 1-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/a/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$-\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-1+m}}{\sqrt{a+bx}}\,dx}{a}\\ &= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.00

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.38, size = 28, normalized size = 0.58

$$\frac{x^m\text{hyper}\left[\left\{\frac{1}{2},m\right\},\{1+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\,m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-1 + m)/Sqrt[a + b*x],x]')
```

```
[Out] x ^ m hyper[{1 / 2, m}, {1 + m}, b x exp_polar[I Pi] / a] / (Sqrt[a] m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(-1+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^(m - 1)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.98, size = 31, normalized size = 0.65

$$\frac{x^m\Gamma\left(m\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*gam
ma(m + 1))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-1}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m - 1)/(a + b*x)^(1/2), x)
```

3.717 $\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=49

$$\frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}$$

```
[Out] 2*b*x^m*hypergeom([1/2, 2-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/a^2/((-b*x/a)^m)
```

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$\frac{2bx^m\sqrt{a+bx}\ \left(-\frac{bx}{a}\right)^{-m}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx &= \frac{\left(b^2x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-2+m}}{\sqrt{a+bx}}\,dx}{a^2}\\ &= \frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.00

$$\frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 6.33, size = 32, normalized size = 0.65

$$\frac{x^{-1+m}\text{hyper}\left[\left\{\frac{1}{2},-1+m\right\},\{m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\ (-1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-2 + m)/Sqrt[a + b*x],x]')
```

```
[Out] x ^ (-1 + m) hyper[{1 / 2, -1 + m}, {m}, b x exp_polar[I Pi] / a] / (Sqrt[a
] (-1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-2+m}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(-2+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m - 2)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.58, size = 32, normalized size = 0.65

$$\frac{x^m\Gamma(m-1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m-1\\ m\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,x\Gamma(m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)/(b*x+a)**(1/2),x)
[Out] x**m*gamma(m - 1)*hyper((1/2, m - 1), (m,), b*x*exp_polar(I*pi)/a)/(sqrt(a)
*x*gamma(m))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)/(a + b*x)^(1/2),x)
[Out] int(x^(m - 2)/(a + b*x)^(1/2), x)
```

3.718 $\int \frac{x^{-3+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$-\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}$$

```
[Out] -2*b^2*x^m*hypergeom([1/2, 3-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/a^3/((-b*x/a)^
m)
```

Rubi [A]

time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$-\frac{2b^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-3+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(b^3x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-3+m}}{\sqrt{a+bx}}\,dx}{a^3}\\ &= -\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 1.00

$$-\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 14.49, size = 34, normalized size = 0.67

$$\frac{x^{-2+m}\text{hyper}\left[\left\{\frac{1}{2},-2+m\right\},\{-1+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\ (-2+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-3 + m)/Sqrt[a + b*x],x]')
```

```
[Out] x ^ (-2 + m) hyper[{1 / 2, -2 + m}, {-1 + m}, b x exp_polar[I Pi] / a] / (S
qrt[a] (-2 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{x^{-3+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(x^(-3+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 3)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m - 3)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 16.57, size = 37, normalized size = 0.73

$$\frac{x^m\Gamma(m-2)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m-2\\ m-1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,x^2\Gamma(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)/(b*x+a)**(1/2),x)
[Out] x**m*gamma(m - 2)*hyper((1/2, m - 2), (m - 1,), b*x*exp_polar(I*pi)/a)/(sqr
t(a)*x**2*gamma(m - 1))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-3}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)/(a + b*x)^(1/2),x)
[Out] int(x^(m - 3)/(a + b*x)^(1/2), x)
```

3.719 $\int \frac{x^m}{\sqrt{2+3x}}\,dx$

Optimal. Leaf size=31

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] 1/2*x^(1+m)*hypergeom([1/2, 1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {66}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[2 + 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx=\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[2 + 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.27, size = 32, normalized size = 1.03

$$\frac{\sqrt{2}\,x^{1+m}\mathrm{hyper}\left[\left\{\frac{1}{2},1+m\right\},\{2+m\},\frac{3x\mathrm{exp\_polar}[I\mathrm{Pi}]}{2}\right]}{2+2m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/Sqrt[2 + 3*x],x]')
```

```
[Out] Sqrt[2] x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, 3 x exp_polar[I Pi] / 2]
 / (2 (1 + m))
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.94

method	result	size
meijerg	$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)\sqrt{2}}{2+2m}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(3*x + 2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 37, normalized size = 1.19

$$\frac{\sqrt{2}\, xx^m \Gamma\left(m+1\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{3xe^{i\pi}}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(2+3*x)**(1/2),x)
```

```
[Out] sqrt(2)*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x*exp_polar(I*p
i)/2)/(2*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(3*x + 2)^(1/2),x)
```

```
[Out] int(x^m/(3*x + 2)^(1/2), x)
```

3.720 $\int \frac{x^m}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=31

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] 1/2*x^(1+m)*hypergeom([1/2, 1+m],[2+m],3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {66}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[2 - 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[2 - 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.28, size = 33, normalized size = 1.06

$$-I\left(\frac{2}{3}\right)^{1+m}\sqrt{-2+3x}\,\text{hyper}\left[\left\{\frac{1}{2},-m\right\},\left\{\frac{3}{2}\right\},\frac{(-2+3x)\,\text{exp\_polar}\,[I\text{Pi}]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/Sqrt[2 - 3*x],x]')
[Out] -I (2 / 3) ^ (1 + m) Sqrt[-2 + 3 x] hyper[{1 / 2, -m}, {3 / 2}, (-2 + 3 x)
exp_polar[I Pi] / 2]
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.94

method	result	size
meijerg	$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)\sqrt{2}}{2+2m}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],3/2*x)/(1+m)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(-3*x + 2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="fricas")
[Out] integral(-x^m*sqrt(-3*x + 2)/(3*x - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 46, normalized size = 1.48

$$-\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}i\sqrt{x-\frac{2}{3}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*I*sqrt(x - 2/3)*hyper((1/2, -m), (3/2,), 3*(x - 2/3)*exp_po
lar(I*pi)/2)/(3*3**m)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{2-3\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2 - 3*x)^(1/2),x)
```

```
[Out] int(x^m/(2 - 3*x)^(1/2), x)
```

3.721 $\int \frac{x^m}{\sqrt{-2+3x}}\,dx$

Optimal. Leaf size=36

$$\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

```
[Out] (3/2)^(-1-m)*hypergeom([1/2, -m],[3/2],1-3/2*x)*(-2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {67}

$$\left(\frac{3}{2}\right)^{-m-1}\sqrt{3x-2}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx=\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 1.00

$$\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[-2 + 3*x],x]
```

[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.26, size = 30, normalized size = 0.83

$$\frac{-I\sqrt{2}\,x^{1+m}\text{hyper}\left[\left\{\frac{1}{2},1+m\right\},\{2+m\},\frac{3x}{2}\right]}{2+2m}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[x^m/Sqrt[-2 + 3*x],x]')

[Out] -I Sqrt[2] x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, 3 x / 2] / (2 + 2 m)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.13, size = 43, normalized size = 1.19

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\text{signum}\left(x-\frac{2}{3}\right)}\,x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2\sqrt{\text{signum}\left(x-\frac{2}{3}\right)}\,(1+m)}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*2^(1/2)/signum(x-2/3)^(1/2)*(-signum(x-2/3))^(1/2)/(1+m)*x^(1+m)*hyperg
eom([1/2,1+m],[2+m],3/2*x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^m/sqrt(3*x - 2), x)

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="fricas")

```
[Out] integral(x^m/sqrt(3*x - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 36, normalized size = 1.00

$$-\frac{\sqrt{2}\, i x x^{m} \Gamma\left(m+1\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{3x}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-2+3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x/2)/(2*gamm
a(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(3*x - 2)^(1/2),x)
```

```
[Out] int(x^m/(3*x - 2)^(1/2), x)
```

3.722 $\int \frac{x^m}{\sqrt{-2-3x}}\,dx$

Optimal. Leaf size=50

$$-2^{1+m}3^{-1-m}\sqrt{-2-3x}\,(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

```
[Out] -2^(1+m)*3^(-1-m)*x^m*hypergeom([1/2, -m],[3/2],1+3/2*x)*(-2-3*x)^(1/2)/((-
x)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {69, 12, 67}

$$-2^{m+1}3^{-m-1}\sqrt{-3x-2}\,(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[-2 - 3*x],x]
```

```
[Out] -((2^(1 + m)*3^(-1 - m)*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2,
1 + (3*x)/2])/(-x)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \left(\left(\frac{2}{3}\right)^m (-x)^{-m}x^m\right)\int \frac{\left(\frac{3}{2}\right)^m (-x)^m}{\sqrt{-2-3x}}\,dx$$
$$= \left((-x)^{-m}x^m\right)\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$$
$$= -2^{1+m}3^{-1-m}\sqrt{-2-3x}\,(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.96

$$-\frac{2}{3}\left(1+\frac{1}{2}(-2-3x)\right)^{-m}\sqrt{-2-3x}\,x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[-2 - 3*x],x]
```

```
[Out] (-2*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(3*(1
+ (-2 - 3*x)/2)^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.28, size = 34, normalized size = 0.68

$$\frac{-I\sqrt{2}\,x^{1+m}\text{hyper}\left[\left\{\frac{1}{2},1+m\right\},\{2+m\},\frac{3x\text{exp\_polar}[I\text{Pi}]}{2}\right]}{2+2m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m/Sqrt[-2 - 3*x],x]')
```

```
[Out] -I Sqrt[2] x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, 3 x exp_polar[I Pi] /
 2] / (2 + 2 m)
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 30, normalized size = 0.60

method	result	size
meijerg	$-\frac{i x^{1+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)\sqrt{2}}{2(1+m)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(-3*x - 2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-x^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 41, normalized size = 0.82

$$-\frac{\sqrt{2}\,ixx^{m}\Gamma\left(m+1\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{3xe^{i\pi}}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-2-3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x*exp_polar(
I*pi)/2)/(2*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{-3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(- 3*x - 2)^(1/2),x)
[Out] int(x^m/(- 3*x - 2)^(1/2), x)
```

3.723 $\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$\frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

```
[Out] 2*(-x)^m*hypergeom([1/2, -m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$\frac{2(-x)^m\sqrt{a+bx}\,\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[a + b*x],x]
```

```
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx &= \left((-x)^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}\,dx\\ &= \frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$\frac{2(-x)^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[a + b*x],x]
```

```
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-
((b*x)/a))^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.44, size = 40, normalized size = 0.83

$$\frac{E^{I\text{Pi}m}x^{1+m}\text{hyper}\left[\left\{\frac{1}{2},1+m\right\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{a}\ (1+m)}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-x)^m/Sqrt[a + b*x],x]')
```

```
[Out] E ^ (I Pi m) x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, b x exp_polar[I Pi]
 / a] / (Sqrt[a] (1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-x)^m}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(b*x+a)^(1/2),x)
```

```
[Out] int((-x)^m/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.73, size = 42, normalized size = 0.88

$$\frac{xx^me^{i\pi m}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar
(I*pi)/a)/(sqrt(a)*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(-x)^m}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(a + b*x)^(1/2),x)
```

```
[Out] int((-x)^m/(a + b*x)^(1/2), x)
```

3.724 $\int \frac{(-x)^m}{\sqrt{2+3x}} \, dx$

Optimal. Leaf size=34

$$-\frac{(-x)^{1+m} \, _2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] -1/2*(-x)^(1+m)*hypergeom([1/2, 1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {66}

$$-\frac{(-x)^{m+1} \, _2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[2 + 3*x],x]
```

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1
 + m)))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{2+3x}} \, dx = -\frac{(-x)^{1+m} \, _2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.94

$$\frac{(-x)^m x \, _2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[2 + 3*x],x]
```

```
[Out] ((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.28, size = 32, normalized size = 0.94

$$\left(\frac{2}{3}\right)^{1+m}\sqrt{2+3x}\,\text{hyper}\left[\left\{\frac{1}{2},-m\right\},\left\{\frac{3}{2}\right\},\frac{(2+3x)\,\text{exp\_polar}\,[2I\text{Pi}]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-x)^m/Sqrt[2 + 3*x],x]')
```

```
[Out] (2 / 3) ^ (1 + m) Sqrt[2 + 3 x] hyper[{1 / 2, -m}, {3 / 2}, (2 + 3 x) exp_p
olar[2 I Pi] / 2]
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.88

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^m x\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)}{2+2m}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(3*x + 2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 44, normalized size = 1.29

$$\frac{2 \cdot 2^m \sqrt{3} \cdot 3^{-m} \sqrt{x + \frac{2}{3}}\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -m \\ \frac{3}{2} \end{matrix}\middle| \frac{3(x+\frac{2}{3})e^{2i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(2+3*x)**(1/2),x)
```

```
[Out] 2*2**m*sqrt(3)*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*exp_polar
(2*I*pi)/2)/(3*3**m)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(-x)^m}{\sqrt{3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(3*x + 2)^(1/2),x)
```

```
[Out] int((-x)^m/(3*x + 2)^(1/2), x)
```

3.725 $\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=34

$$-\frac{(-x)^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] -1/2*(-x)^(1+m)*hypergeom([1/2, 1+m],[2+m],3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {66}

$$-\frac{(-x)^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[2 - 3*x],x]
```

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1
+ m)))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx = -\frac{(-x)^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.94

$$\frac{(-x)^m x\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[2 - 3*x],x]
```

```
[Out] ((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.36, size = 39, normalized size = 1.15

$$-I\left(\frac{2}{3}\right)^{1+m}E^{I\text{Pi}m}\sqrt{-2+3x}\text{ hyper}\left[\left\{\frac{1}{2},-m\right\},\left\{\frac{3}{2}\right\},\frac{(-2+3x)\text{ exp\_polar}\left[I\text{Pi}\right]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-x)^m/Sqrt[2 - 3*x],x]')
```

```
[Out] -I (2 / 3) ^ (1 + m) E ^ (I Pi m) Sqrt[-2 + 3 x] hyper[{1 / 2, -m}, {3 / 2}
, (-2 + 3 x) exp_polar[I Pi] / 2]
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.88

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^{m} x\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2+2m}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(-3*x + 2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="fricas")
```

[Out] integral(-(-x)^m*sqrt(-3*x + 2)/(3*x - 2), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 53, normalized size = 1.56

$$-\frac{2 \cdot 2^{m} \sqrt{3} \cdot 3^{-m} i \sqrt{x - \frac{2}{3}}\, e^{i\pi m} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, -m \\ \frac{3}{2} \end{matrix}\middle| \frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)**m/(2-3*x)**(1/2),x)

[Out] -2*2**m*sqrt(3)*I*sqrt(x - 2/3)*exp(I*pi*m)*hyper((1/2, -m), (3/2,), 3*(x - 2/3)*exp_polar(I*pi)/2)/(3*3**m)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)^m/(2-3*x)^(1/2),x)

[Out] Could not integrate

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(-x)^m}{\sqrt{2-3\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x)^m/(2 - 3*x)^(1/2),x)

[Out] int((-x)^m/(2 - 3*x)^(1/2), x)

3.726 $\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx$

Optimal. Leaf size=49

$$2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

[Out] 2^(1+m)*3^(-1-m)*(-x)^m*hypergeom([1/2, -m],[3/2],1-3/2*x)*(-2+3*x)^(1/2)/(x^m)

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {69, 12, 67}

$$2^{m+1}3^{-m-1}\sqrt{3x-2}\,(-x)^m x^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(-x)^m/Sqrt[-2 + 3*x],x]

[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2])/x^m

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \left(\left(\frac{2}{3}\right)^m (-x)^m x^{-m}\right) \int \frac{\left(\frac{3}{2}\right)^m x^m}{\sqrt{-2+3x}}\,dx$$
$$= \left((-x)^m x^{-m}\right) \int \frac{x^m}{\sqrt{-2+3x}}\,dx$$
$$= 2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2,
 1 - (3*x)/2])/x^m
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.31, size = 36, normalized size = 0.73

$$\frac{-I\sqrt{2}\,E^{I\text{Pi}m}x^{1+m}\text{hyper}\left[\left\{\frac{1}{2},1+m\right\},\{2+m\},\frac{3x}{2}\right]}{2+2m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-x)^m/Sqrt[-2 + 3*x],x]')
```

```
[Out] -I Sqrt[2] E ^ (I Pi m) x ^ (1 + m) hyper[{1 / 2, 1 + m}, {2 + m}, 3 x / 2]
 / (2 + 2 m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.13, size = 44, normalized size = 0.90

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^m\sqrt{-\text{signum}\left(x-\frac{2}{3}\right)}\,x\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2\sqrt{\text{signum}\left(x-\frac{2}{3}\right)}\,(1+m)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/signum(x-2/3)^(1/2)*(-signum(x-2/3))^(1/2)/(1+m)*x*hyper
geom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(3*x - 2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(3*x - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 42, normalized size = 0.86

$$-\frac{\sqrt{2}\, ix x^{m} e^{i\pi m}\Gamma\left(m+1\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, m+1 \\ m+2 \end{matrix}\middle| {\frac{3x}{2}} \right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(-2+3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*
x/2)/(2*gamma(m + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(3*x - 2)^(1/2),x)
```

```
[Out] int((-x)^m/(3*x - 2)^(1/2), x)
```

3.727 $\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$

Optimal. Leaf size=37

$$-\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2-3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

```
[Out] -(3/2)^(-1-m)*hypergeom([1/2, -m],[3/2],1+3/2*x)*(-2-3*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {67}

$$-\left(\frac{3}{2}\right)^{-m-1}\sqrt{-3x-2}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[-2 - 3*x],x]
```

```
[Out] -((3/2)^(-1 - m)*Sqrt[-2 - 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2
])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx = -\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2-3x}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.54

$$-\frac{2}{3}\left(1+\frac{1}{2}(-2-3x)\right)^{-m}\sqrt{-2-3x}\,x^{-m}\left(-x^2\right)^m\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[-2 - 3*x],x]
```

```
[Out] (-2*Sqrt[-2 - 3*x]*(-x^2)^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(
3*(1 + (-2 - 3*x)/2)^m*x^m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.30, size = 33, normalized size = 0.89

$$-I\left(\frac{2}{3}\right)^{1+m}\sqrt{2+3x}\,\text{hyper}\left[\left\{\frac{1}{2},-m\right\},\left\{\frac{3}{2}\right\},\frac{(2+3x)\,\text{exp\_polar}\,[2I\text{Pi}]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-x)^m/Sqrt[-2 - 3*x],x]')
```

```
[Out] -I (2 / 3) ^ (1 + m) Sqrt[2 + 3 x] hyper[{1 / 2, -m}, {3 / 2}, (2 + 3 x) ex
p_polar[2 I Pi] / 2]
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 31, normalized size = 0.84

method	result	size
meijerg	$-\frac{i\sqrt{2}\,(-x)^{m} x\,\text{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)}{2(1+m)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(-3*x - 2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(-x)^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 48, normalized size = 1.30

$$-\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}i\sqrt{x+\frac{2}{3}}\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\right|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(-2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*I*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*exp_po
lar(2*I*pi)/2)/(3*3**m)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(-x)^m}{\sqrt{-3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(- 3*x - 2)^(1/2),x)
```

```
[Out] int((-x)^m/(- 3*x - 2)^(1/2), x)
```

3.728 $\int \frac{x^n}{\sqrt{1-x}} dx$

Optimal. Leaf size=26

$$-2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

```
[Out] -2*hypergeom([1/2, -n],[3/2],1-x)*(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {67}

$$-2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^n/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{x^n}{\sqrt{1-x}} dx = -2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$-2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.14, size = 23, normalized size = 0.88

$$-2I\sqrt{-1+x}\,\text{hyper}\left[\left\{\frac{1}{2},-n\right\},\left\{\frac{3}{2}\right\},(-1+x)\,\text{exp\_polar}\,[I\text{Pi}]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^n/Sqrt[1 - x],x]')
[Out] -2 I Sqrt[-1 + x] hyper[{1 / 2, -n}, {3 / 2}, (-1 + x) exp_polar[I Pi]]
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.88

method	result	size
meijerg	$\frac{x^{1+n}\,\text{hypergeom}\left(\left[\frac{1}{2},1+n\right],[2+n],x\right)}{1+n}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(1+n)*x^(1+n)*hypergeom([1/2,1+n],[2+n],x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^n/sqrt(-x + 1), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="fricas")
[Out] integral(-x^n*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 26, normalized size = 1.00

$$-2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/(1-x)**(1/2),x)
```

```
[Out] -2*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^n}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(1 - x)^(1/2),x)
```

```
[Out] int(x^n/(1 - x)^(1/2), x)
```

3.729 $\int \frac{x^n}{\sqrt{a-ax}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{a-ax}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

```
[Out] -2*hypergeom([1/2, -n],[3/2],1-x)*(-a*x+a)^(1/2)/a
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{67\}$

$$-\frac{2\sqrt{a-ax}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^n/Sqrt[a - a*x],x]
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{x^n}{\sqrt{a-ax}}\,dx = -\frac{2\sqrt{a-ax}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Mathematica [A]
time = 0.10, size = 30, normalized size = 1.00

$$-\frac{2\sqrt{a-ax}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/Sqrt[a - a*x],x]
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.19, size = 26, normalized size = 0.87

$$\frac{-2I\sqrt{-1+x}\,\text{hyper}\left[\left\{\frac{1}{2},-n\right\},\left\{\frac{3}{2}\right\},(-1+x)\,\text{exp\_polar}\,[I\text{Pi}]\right]}{\sqrt{a}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^n/Sqrt[a - a*x],x]')
```

```
[Out] -2 I Sqrt[-1 + x] hyper[{1 / 2, -n}, {3 / 2}, (-1 + x) exp_polar[I Pi]] / S
qrt[a]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^n}{\sqrt{-ax+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(-a*x+a)^(1/2),x)
```

```
[Out] int(x^n/(-a*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^n/sqrt(-a*x + a), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x + a)*x^n/(a*x - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 31, normalized size = 1.03

$$-\frac{2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/(-a*x+a)**(1/2),x)
[Out] -2*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi))/sqrt(a)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^n}{\sqrt{a-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(a - a*x)^(1/2),x)
[Out] int(x^n/(a - a*x)^(1/2), x)
```

3.730 $\int x^m(a+bx)^n\,dx$

Optimal. Leaf size=47

$$\frac{x^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}$$

```
[Out] x^(1+m)*(b*x+a)^n*hypergeom([-n, 1+m],[2+m],-b*x/a)/(1+m)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {68, 66}

$$\frac{x^{m+1}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^m\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{x^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$\frac{x^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\ {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.37, size = 36, normalized size = 0.77

$$\frac{a^n x^{1+m}\text{hyper}\left[\{-n,1+m\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(a + b*x)^n,x]')
```

```
[Out] a ^ n x ^ (1 + m) hyper[{-n, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a] / (1
 + m)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^m\left(bx+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^n,x)
```

```
[Out] int(x^m*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^m, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.73, size = 34, normalized size = 0.72

$$\frac{a^n x x^m \Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**n,x)
```

```
[Out] a**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a
)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(a+b\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^n,x)
```

```
[Out] int(x^m*(a + b*x)^n, x)
```

3.731 $\int (cx)^m (a+bx)^n \, dx$

Optimal. Leaf size=52

$$\frac{(cx)^{1+m}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{c(1+m)}$$

```
[Out] (c*x)^(1+m)*(b*x+a)^n*hypergeom([-n, 1+m],[2+m],-b*x/a)/c/(1+m)/((1+b*x/a)^
n)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{(cx)^{m+1}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x)^n,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])
/(c*(1 + m)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (cx)^m (a+bx)^n \, dx &= \left((a+bx)^n \left(1+\frac{bx}{a}\right)^{-n}\right) \int (cx)^m \left(1+\frac{bx}{a}\right)^n \, dx\\ &= \frac{(cx)^{1+m}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.92

$$\frac{x(cx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x)^n,x]
[Out] (x*(c*x)^m*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.16, size = 39, normalized size = 0.75

$$\frac{a^n c^m x^{1+m}\text{hyper}\left[\{-n,1+m\},\{2+m\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x)^m*(a + b*x)^n,x]')
[Out] a ^ n c ^ m x ^ (1 + m) hyper[{-n, 1 + m}, {2 + m}, b x exp_polar[I Pi] / a
] / (1 + m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^m (bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x+a)^n,x)
[Out] int((c*x)^m*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*(c*x)^m, x)
```

Fricas [F]
time = 0.74, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*(c*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.52, size = 37, normalized size = 0.71

$$\frac{a^n c^m x x^m \Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x+a)**n,x)
```

```
[Out] a**n*c**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m\,(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x)^n,x)
```

```
[Out] int((c*x)^m*(a + b*x)^n, x)
```

3.732 $\int x^3(a+bx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}$$

```
[Out] -a^3*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*(b*x+a)^(2+n)/b^4/(2+n)-3*a*(b*x+a)^(3+n
)/b^4/(3+n)+(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^n,x]
```

```
[Out] -((a^3*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (3*a^2*(a + b*x)^(2 + n))/(b^4*(
2 + n)) - (3*a*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (a + b*x)^(4 + n)/(b^4*(4
 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^n\,dx &= \int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.81

$$\frac{(a+bx)^{1+n}\left(-\frac{a^3}{1+n}+\frac{3a^2(a+bx)}{2+n}-\frac{3a(a+bx)^2}{3+n}+\frac{(a+bx)^3}{4+n}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-(a^3/(1 + n)) + (3*a^2*(a + b*x))/(2 + n) - (3*a*(a +
b*x)^2)/(3 + n) + (a + b*x)^3/(4 + n)))/b^4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.73, size = 929, normalized size = 11.19

$$\text{Piecewise}\left[\left\{\left\{\frac{x^4a^n}{4}, b==0\right\}, \left\{\frac{a^3\left(11+6\text{Log}\left[\frac{a+bx}{b}\right]\right)+9a^2bx\left(3+2\text{Log}\left[\frac{a+bx}{b}\right]\right)+18ab^2x^2\left(1+\text{Log}\left[\frac{a+bx}{b}\right]\right)+6b^3x^3\text{Log}\left[\frac{a+bx}{b}\right]}{6b^4\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}, n==-4\right\}, \ldots\right.\right.$$ [illegible]

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(a + b*x)^n,x]')
```

```
[Out] Piecewise[{{x ^ 4 a ^ n / 4, b == 0}, {(a ^ 3 (11 + 6 Log[(a + b x) / b]) +
 9 a ^ 2 b x (3 + 2 Log[(a + b x) / b]) + 18 a b ^ 2 x ^ 2 (1 + Log[(a + b
x) / b]) + 6 b ^ 3 x ^ 3 Log[(a + b x) / b]) / (6 b ^ 4 (a ^ 3 + 3 a ^ 2 b
x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)), n == -4}, {(-3 a ^ 3 (3 + 2 Log[(a + b
 x) / b]) / 2 - 6 a ^ 2 b x (1 + Log[(a + b x) / b]) - 3 a b ^ 2 x ^ 2 Log[
(a + b x) / b] + b ^ 3 x ^ 3) / (b ^ 4 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)), n
== -3}, {(6 a ^ 3 (1 + Log[(a + b x) / b]) + 6 a ^ 2 b x Log[(a + b x) / b]
 - 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) / (2 b ^ 4 (a + b x)), n == -2}, {(-a ^ 3
 Log[a / b + x] + a ^ 2 b x - a b ^ 2 x ^ 2 / 2 + b ^ 3 x ^ 3 / 3) / b ^ 4,
 n == -1}}, -6 a ^ 4 (a + b x) ^ n / (24 b ^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^
2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4) + 6 a ^ 3 b n x (a + b x) ^ n / (24 b ^ 4
 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4) - 3 a ^ 2 b
^ 2 n x ^ 2 (a + b x) ^ n / (24 b ^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b
^ 4 n ^ 3 + b ^ 4 n ^ 4) - 3 a ^ 2 b ^ 2 n ^ 2 x ^ 2 (a + b x) ^ n / (24 b
^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4) + 2 a b ^
 3 n x ^ 3 (a + b x) ^ n / (24 b ^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^
 4 n ^ 3 + b ^ 4 n ^ 4) + 3 a b ^ 3 n ^ 2 x ^ 3 (a + b x) ^ n / (24 b ^ 4 +
 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4) + a b ^ 3 n ^
3 x ^ 3 (a + b x) ^ n / (24 b ^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^ 4
n ^ 3 + b ^ 4 n ^ 4) + 6 b ^ 4 x ^ 4 (a + b x) ^ n / (24 b ^ 4 + 50 b ^ 4 n
 + 35 b ^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4) + 11 b ^ 4 n x ^ 4 (a + b
 x) ^ n / (24 b ^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4
n ^ 4) + 6 b ^ 4 n ^ 2 x ^ 4 (a + b x) ^ n / (24 b ^ 4 + 50 b ^ 4 n + 35 b
^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4) + b ^ 4 n ^ 3 x ^ 4 (a + b x) ^ n
 / (24 b ^ 4 + 50 b ^ 4 n + 35 b ^ 4 n ^ 2 + 10 b ^ 4 n ^ 3 + b ^ 4 n ^ 4)]
```

Maple [A]
time = 0.11, size = 126, normalized size = 1.52

method	result

gosper	$-\frac{(bx+a)^{1+n}(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3)}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4)(bx+a)^n}{(3+n)(4+n)(2+n)(1+n)b^4}$
norman	$\frac{x^4e^{n\ln(bx+a)}}{4+n}+\frac{an\,x^3e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{6a^4e^{n\ln(bx+a)}}{b^4(n^4+10n^3+35n^2+50n+24)}-\frac{3a^2n\,x^2e^{n\ln(bx+a)}}{b^2(n^3+9n^2+26n+24)}+\frac{6n\,a^3x\,e^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9*a
*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/b^4/(n^4+10*n
^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.27, size = 101, normalized size = 1.22

$$\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.33, size = 143, normalized size = 1.72

$$\frac{(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*(b*
x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [A]
time = 0.63, size = 1318, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a
**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2
*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log
(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 1
8*a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2
+ 6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2
*b*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a
**3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) -
 3*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(
n, -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3
*b), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*
a**2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(
b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n
*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)*
*n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4
*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(83) = 166.
time = 0.00, size = 250, normalized size = 3.01

$$\frac{-6a^4e^{n\ln(a+bx)} + 6a^3bnxe^{n\ln(a+bx)} - 3a^2b^2n^2x^2e^{n\ln(a+bx)} - 3a^2b^2nx^2e^{n\ln(a+bx)} + ab^3n^3x^3e^{n\ln(a+bx)} + 3ab^3n^2x^3e^{n\ln(a+bx)} + 2ab^3nx^3e^{n\ln(a+bx)} + b^4n^3x^4e^{n\ln(a+bx)} + 6b^4n^2x^4e^{n\ln(a+bx)} + 11b^4nx^4e^{n\ln(a+bx)} + 6b^4x^4e^{n\ln(a+bx)}}{b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x)
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
```

```
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [B]
time = 0.53, size = 176, normalized size = 2.12

$$(a+bx)^n\left(\frac{x^4\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4}{b^4\,(n^4+10\,n^3+35\,n^2+50\,n+24)}+\frac{6\,a^3\,n\,x}{b^3\,(n^4+10\,n^3+35\,n^2+50\,n+24)}+\frac{a\,n\,x^3\,(n^2+3\,n+2)}{b\,(n^4+10\,n^3+35\,n^2+50\,n+24)}-\frac{3\,a^2\,n\,x^2\,(n+1)}{b^2\,(n^4+10\,n^3+35\,n^2+50\,n+24)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 +
 24) - (6*a^4)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x)/(b^3
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^3*(3*n + n^2 + 2))/(b*(50*n
+ 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2
+ 10*n^3 + n^4 + 24)))
```

3.733 $\int x^2(a+bx)^n\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}$$

```
[Out] a^2*(b*x+a)^(1+n)/b^3/(1+n)-2*a*(b*x+a)^(2+n)/b^3/(2+n)+(b*x+a)^(3+n)/b^3/(
3+n)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n,x]
```

```
[Out] (a^2*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*x)^(2 + n))/(b^3*(2 + n
)) + (a + b*x)^(3 + n)/(b^3*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\,dx&=\int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.32, size = 496, normalized size = 8.27

$$\text{Piecewise}\left[\left\{\left\{\frac{x^3a^n}{3},b==0\right\},\left\{\frac{\frac{a^2\left(3+2\text{Log}\left[\frac{a+bx}{b}\right]\right)}{2}+2abx\left(1+\text{Log}\left[\frac{a+bx}{b}\right]\right)+b^2x^2\text{Log}\left[\frac{a+bx}{b}\right]}{b^3\left(a^2+2abx+b^2x^2\right)},n==-3\right\},\left\{\frac{-2a^2\left(1+\text{Log}\left[\frac{a+bx}{b}\right]\right)-2abx\text{Log}\left[\frac{a+bx}{b}\right]+b^2x^2}{b^3\left(a+bx\right)},n==-2\right\},\left\{\frac{a^2\text{Log}\left[\frac{a}{b}+x\right]-abx+\frac{b^2x^2}{2}}{b^3},n==-1\right\}\right\},\frac{2a^3\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}-\frac{2a^2bnx\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}+\frac{ab^2nx^2\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}+\frac{ab^2n^2x^2\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}+\frac{2b^3x^3\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}+\frac{3b^3nx^3\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}+\frac{b^3n^2x^3\left(a+bx\right)^n}{6b^3+11b^3n+6b^3n^2+b^3n^3}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(a + b*x)^n,x]')
```

```
[Out] Piecewise[{{x ^ 3 a ^ n / 3, b == 0}, {(a ^ 2 (3 + 2 Log[(a + b x) / b]) /
2 + 2 a b x (1 + Log[(a + b x) / b]) + b ^ 2 x ^ 2 Log[(a + b x) / b]) / (b
 ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)), n == -3}, {(-2 a ^ 2 (1 + Log[(a + b
 x) / b]) - 2 a b x Log[(a + b x) / b] + b ^ 2 x ^ 2) / (b ^ 3 (a + b x)),
n == -2}, {(a ^ 2 Log[a / b + x] - a b x + b ^ 2 x ^ 2 / 2) / b ^ 3, n == -
1}}, 2 a ^ 3 (a + b x) ^ n / (6 b ^ 3 + 11 b ^ 3 n + 6 b ^ 3 n ^ 2 + b ^ 3
n ^ 3) - 2 a ^ 2 b n x (a + b x) ^ n / (6 b ^ 3 + 11 b ^ 3 n + 6 b ^ 3 n ^
2 + b ^ 3 n ^ 3) + a b ^ 2 n x ^ 2 (a + b x) ^ n / (6 b ^ 3 + 11 b ^ 3 n +
6 b ^ 3 n ^ 2 + b ^ 3 n ^ 3) + a b ^ 2 n ^ 2 x ^ 2 (a + b x) ^ n / (6 b ^ 3
 + 11 b ^ 3 n + 6 b ^ 3 n ^ 2 + b ^ 3 n ^ 3) + 2 b ^ 3 x ^ 3 (a + b x) ^ n
/ (6 b ^ 3 + 11 b ^ 3 n + 6 b ^ 3 n ^ 2 + b ^ 3 n ^ 3) + 3 b ^ 3 n x ^ 3 (a
 + b x) ^ n / (6 b ^ 3 + 11 b ^ 3 n + 6 b ^ 3 n ^ 2 + b ^ 3 n ^ 3) + b ^ 3
n ^ 2 x ^ 3 (a + b x) ^ n / (6 b ^ 3 + 11 b ^ 3 n + 6 b ^ 3 n ^ 2 + b ^ 3 n
 ^ 3)]
```

Maple [A]
time = 0.11, size = 73, normalized size = 1.22

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)}{b^3\left(n^3+6n^2+11n+6\right)}$	73
risch	$\frac{\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{(2+n)(3+n)(1+n)b^3}$	88
norman	$\frac{x^3\mathrm{e}^{n\ln(bx+a)}}{3+n}+\frac{an\,x^2\mathrm{e}^{n\ln(bx+a)}}{b(n^2+5n+6)}+\frac{2a^3\mathrm{e}^{n\ln(bx+a)}}{b^3(n^3+6n^2+11n+6)}-\frac{2n\,a^2x\,\mathrm{e}^{n\ln(bx+a)}}{b^2(n^3+6n^2+11n+6)}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 68, normalized size = 1.13

$$\frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.32, size = 96, normalized size = 1.60

$$-\frac{(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2)(bx+a)^n}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [A]
time = 0.41, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^nx^3}{3} & \text{for}\: b=0 \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for}\: n=-3 \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2}{ab^3+b^4x}-\frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}+\frac{b^2x^2}{ab^3+b^4x} & \text{for}\: n=-2 \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b} & \text{for}\: n=-1 \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{b^3n^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{3b^3nx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{2b^3x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b**3 +
b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x)
 + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x/b
**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*
```

```
n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3
*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b
**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(60) = 120$.
time = 0.00, size = 154, normalized size = 2.57

$$\frac{2a^3e^{n\ln(a+bx)} - 2a^2bnxe^{n\ln(a+bx)} + ab^2n^2x^2e^{n\ln(a+bx)} + ab^2nx^2e^{n\ln(a+bx)} + b^3n^2x^3e^{n\ln(a+bx)} + 3b^3nx^3e^{n\ln(a+bx)} + 2b^3x^3e^{n\ln(a+bx)}}{b^3n^3 + 6b^3n^2 + 11b^3n + 6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x)
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Mupad [B]
time = 0.56, size = 192, normalized size = 3.20

$$\begin{cases} \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3} & \text{if } n=-1 \\ \frac{x}{b^2}-\frac{a^2}{b^3\,(a+b\,x)}-\frac{2\,a\,\ln(a+b\,x)}{b^3} & \text{if } n=-2 \\ \frac{\ln(a+b\,x)+\frac{2\,a}{a+b\,x}-\frac{a^2}{2\,(a+b\,x)^2}}{b^3} & \text{if } n=-3 \\ \frac{2\,(a+b\,x)^{n+1}\,(8\,a^2-8\,a\,b\,n\,x-8\,a\,b\,x+4\,b^2\,n^2\,x^2+12\,b^2\,n\,x^2+8\,b^2\,x^2)}{b^3\,(8\,n^3+48\,n^2+88\,n+48)} & \text{if } n\neq -1\wedge n\neq -2\wedge n\neq -3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), n == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, n == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, n ~= -1 & n ~= -2 & n ~=
 -3, (2*(a + b*x)^(n + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*n*x^2 - 8*a*b*x + 4*b
^2*n^2*x^2 - 8*a*b*n*x))/(b^3*(88*n + 48*n^2 + 8*n^3 + 48)))
```

3.734 $\int x(a+bx)^n \, dx$

Optimal. Leaf size=39

$$-\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}$$

```
[Out] -a*(b*x+a)^(1+n)/b^2/(1+n)+(b*x+a)^(2+n)/b^2/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = $\{45\}$

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n,x]
```

```
[Out] -((a*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (a + b*x)^(2 + n)/(b^2*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n \, dx &= \int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right) dx\\ &= -\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.40, size = 213, normalized size = 5.46

$$\text{Piecewise}\left[\left\{\left\{\frac{x^2a^n}{2},b{=}{=}0\right\},\left\{\frac{a+a\text{Log}\left[\frac{a}{b}+x\right]+bx\text{Log}\left[\frac{a}{b}+x\right]}{b^2\left(a+bx\right)},n{=}{=}-2\right\},\left\{\frac{-a\text{Log}\left[\frac{a}{b}+x\right]+bx}{b^2},n{=}{=}-1\right\}\right\},-\frac{a^2\left(a+bx\right)^n}{2b^2+3b^2n+b^2n^2}+\frac{abnx\left(a+bx\right)^n}{2b^2+3b^2n+b^2n^2}+\frac{b^2x^2\left(a+bx\right)^n}{2b^2+3b^2n+b^2n^2}+\frac{b^2nx^2\left(a+bx\right)^n}{2b^2+3b^2n+b^2n^2}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^1*(a + b*x)^n,x]')
```

```
[Out] Piecewise[{{x ^ 2 a ^ n / 2, b == 0}, {(a + a Log[a / b + x] + b x Log[a /
b + x]) / (b ^ 2 (a + b x)), n == -2}, {(-a Log[a / b + x] + b x) / b ^ 2,
n == -1}}, -a ^ 2 (a + b x) ^ n / (2 b ^ 2 + 3 b ^ 2 n + b ^ 2 n ^ 2) + a b
 n x (a + b x) ^ n / (2 b ^ 2 + 3 b ^ 2 n + b ^ 2 n ^ 2) + b ^ 2 x ^ 2 (a +
 b x) ^ n / (2 b ^ 2 + 3 b ^ 2 n + b ^ 2 n ^ 2) + b ^ 2 n x ^ 2 (a + b x) ^
 n / (2 b ^ 2 + 3 b ^ 2 n + b ^ 2 n ^ 2)]
```

Maple [A]
time = 0.12, size = 36, normalized size = 0.92

method	result	size
gosper	$-\frac{\left(bx+a\right)^{1+n}\left(-xnb-bx+a\right)}{b^{2}\left(n^{2}+3n+2\right)}$	36
risch	$-\frac{\left(-b^{2}n\,x^{2}-abnx-x^{2}b^{2}+a^{2}\right)\left(bx+a\right)^{n}}{b^{2}\left(2+n\right)\left(1+n\right)}$	50
norman	$\frac{x^{2}{\mathrm e}^{n\ln\left(bx+a\right)}}{2+n}+\frac{nax\,{\mathrm e}^{n\ln\left(bx+a\right)}}{b\left(n^{2}+3n+2\right)}-\frac{a^{2}{\mathrm e}^{n\ln\left(bx+a\right)}}{b^{2}\left(n^{2}+3n+2\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(-b*n*x-b*x+a)/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.26, size = 42, normalized size = 1.08

$$\frac{{\left(b^{2}{\left(n+1\right)}x^{2}+abnx-a^{2}\right)}{\left(bx+a\right)}^{n}}{{\left(n^{2}+3\,n+2\right)}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.31, size = 53, normalized size = 1.36

$$\frac{{\left(abnx+{\left(b^{2}n+b^{2}\right)}x^{2}-a^{2}\right)}{\left(bx+a\right)}^{n}}{b^{2}n^{2}+3\,b^{2}n+2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*(b*x + a)^n/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Sympy [A]
time = 0.25, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^n x^2}{2} & \text{for}\: b = 0 \\ \frac{a \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\: n = -2 \\ -\frac{a \log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for}\: n = -1 \\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n +
 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*n*x*
*2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2), True))
```

Giac [A]
time = 0.00, size = 83, normalized size = 2.13

$$\frac{-a^2 e^{n \ln(a+bx)} + abnxe^{n \ln(a+bx)} + b^2nx^2e^{n \ln(a+bx)} + b^2x^2e^{n \ln(a+bx)}}{b^2n^2 + 3b^2n + 2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x)
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Mupad [B]
time = 0.38, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if}\ \ n=-1 \\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if}\ \ n=-2 \\ \frac{2\left(\frac{(a+b\,x)^{n+2}}{2\,n+4}-\frac{a\,(a+b\,x)^{n+1}}{2\,n+2}\right)}{b^2} & \text{if}\ \ n\neq -1\land n\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, -(a*log(a + b*x) - b*x)/b^2, n == -2, (log(a + b*x) + a/
(a + b*x))/b^2, n ~= -1 & n ~= -2, (2*((a + b*x)^(n + 2)/(2*n + 4) - (a*(a
+ b*x)^(n + 1))/(2*n + 2)))/b^2)
```

3.735 $\int (a+bx)^n \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{1+n}}{b(1+n)}$$

```
[Out] (b*x+a)^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n,x]
[Out] (a + b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^n \, dx = \frac{(a+bx)^{1+n}}{b(1+n)}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{1+n}}{b+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n,x]
[Out] (a + b*x)^(1 + n)/(b + b*n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^0*(a + b*x)^n,x]')
```

```
[Out] cought exception:
```

Maple [A]
time = 0.11, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+n}}{b(1+n)}$	19
default	$\frac{(bx+a)^{1+n}}{b(1+n)}$	19
risch	$\frac{(bx+a)(bx+a)^{n}}{b(1+n)}$	22
norman	$\frac{x\,e^{n\ln(bx+a)}}{1+n}+\frac{a\,e^{n\ln(bx+a)}}{b(1+n)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 0.31, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^{n}}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^n/(b*n + b)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n,x)
[Out] Piecewise(((a + b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*x), True))/b
```

Giac [A]
time = 0.00, size = 16, normalized size = 0.89

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x)
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 0.20, size = 18, normalized size = 1.00

$$\frac{(a+b\,x)^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n,x)
[Out] (a + b*x)^(n + 1)/(b*(n + 1))
```

3.736 $\int \frac{(a+bx)^n}{x} dx$

Optimal. Leaf size=35

$$-\frac{(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] -(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {67}

$$-\frac{(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x} dx = -\frac{(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$-\frac{(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.87, size = 40, normalized size = 1.14

$$\frac{\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right](-a-bx)\left(\frac{a+bx}{b}\right)^n b^n}{a}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/x^1,x]')
[Out] LerchPhi[(a + b x) / a, 1, 1 + n] (-a - b x) ((a + b x) / b) ^ n b ^ n / a
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x,x)
[Out] int((b*x+a)^n/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="maxima")
[Out] integrate((b*x + a)^n/x, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="fricas")
[Out] integral((b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.
time = 0.64, size = 83, normalized size = 2.37

$$-\frac{bb^n n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n \Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}-\frac{bb^n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n \Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x,x)
[Out] -b*b**n*n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n
+ 1)/(a*gamma(n + 2)) - b*b**n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/
a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,x)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x,x)
[Out] int((a + b*x)^n/x, x)
```

3.737 $\int \frac{(a+bx)^n}{x^2} dx$

Optimal. Leaf size=35

$$\frac{b(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

```
[Out] b*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {67}

$$\frac{b(a+bx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^2,x]
```

```
[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*
(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x^2} dx = \frac{b(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$\frac{b(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^2,x]
```

```
[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*
(1 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 5.61, size = 75, normalized size = 2.14

$$\frac{\left(-a^2-abx-abnx\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right]-b^2nx^2\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right]\right)\left(\frac{a+bx}{b}\right)^nb^n}{a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/x^2,x]')
```

```
[Out] (-a ^ 2 - a b x - a b n x LerchPhi[(a + b x) / a, 1, 1 + n] - b ^ 2 n x ^ 2
 LerchPhi[(a + b x) / a, 1, 1 + n]) ((a + b x) / b) ^ n b ^ n / (a ^ 2 x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^2,x)
```

```
[Out] int((b*x+a)^n/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^2, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^2, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(27) = 54$.
time = 0.96, size = 354, normalized size = 10.11

$$\frac{ab^2b^nn^2\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}+\frac{ab^2b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}-\frac{ab^2b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}-\frac{ab^2b^n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}-\frac{b^3b^nn^2\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}-\frac{b^3b^nn\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma(n+2)+a^2b\left(\frac{a}{b}+x\right)\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**2,x)
```

```
[Out] a*b**2*b**n*n**2*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*g
amma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma(n + 2)) + a*b**2*b
**n*n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)
/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma(n + 2)) - a*b**2*b**n*n*(a/b
+ x)*(a/b + x)**n*gamma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma
(n + 2)) - a*b**2*b**n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(-a**3*gamma(n +
 2) + a**2*b*(a/b + x)*gamma(n + 2)) - b**3*b**n*n**2*(a/b + x)**2*(a/b + x
)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(-a**3*gamma(n + 2) + a
**2*b*(a/b + x)*gamma(n + 2)) - b**3*b**n*n*(a/b + x)**2*(a/b + x)**n*lerch
phi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b
 + x)*gamma(n + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{(a+b\,x)}^n}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^2,x)
```

```
[Out] int((a + b*x)^n/x^2, x)
```

3.738 $\int \frac{(a+bx)^n}{x^3} \, dx$

Optimal. Leaf size=38

$$-\frac{b^2(a+bx)^{1+n} {}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

```
[Out] -b^2*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {67}

$$-\frac{b^2(a+bx)^{n+1} {}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^3,x]
```

```
[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x^3} \, dx = -\frac{b^2(a+bx)^{1+n} {}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$-\frac{b^2(a+bx)^{1+n} {}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^3,x]
```

```
[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 11.58, size = 150, normalized size = 3.95

$$\frac{\left(-a^3-a^2bnx+ab^2x^2-ab^2nx^2+ab^2nx^2\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right]-ab^2n^2x^2\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right]+b^3nx^3\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right]-b^3n^2x^3\text{LerchPhi}\left[\frac{a+bx}{a},1,1+n\right]\right)\left(\frac{a+bx}{b}\right)^n b^n}{2a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/x^3,x]')
```

```
[Out] (-a ^ 3 - a ^ 2 b n x + a b ^ 2 x ^ 2 - a b ^ 2 n x ^ 2 + a b ^ 2 n x ^ 2 L
erchPhi[(a + b x) / a, 1, 1 + n] - a b ^ 2 n ^ 2 x ^ 2 LerchPhi[(a + b x) /
 a, 1, 1 + n] + b ^ 3 n x ^ 3 LerchPhi[(a + b x) / a, 1, 1 + n] - b ^ 3 n ^
 2 x ^ 3 LerchPhi[(a + b x) / a, 1, 1 + n]) ((a + b x) / b) ^ n b ^ n / (2
a ^ 3 x ^ 2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^3,x)
```

```
[Out] int((b*x+a)^n/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^3, x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^3, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(31) = 62$.

time = 1.99, size = 918, normalized size = 24.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**3,x)
```

```
[Out] -a**2*b**3*b**n*n**3*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n +
1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*
a**3*b**2*(a/b + x)**2*gamma(n + 2)) + a**2*b**3*b**n*n**2*(a/b + x)*(a/b +
 x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2)
+ 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) + a**2*b**3*b**n*n*(a/b + x)*(a/b
+ x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2)
 - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2))
 - a**2*b**3*b**n*n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2
) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)
) - 2*a**2*b**3*b**n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n +
2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2
)) + 2*a*b**4*b**n*n**3*(a/b + x)**2*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1
, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2
) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) - a*b**4*b**n*n**2*(a/b + x)**2*
(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n
 + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) - 2*a*b**4*b**n*n*(a/b + x)*
*2*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamm
a(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma
(n + 2)) + a*b**4*b**n*(a/b + x)**2*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma
(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(
n + 2)) - b**5*b**n*n**3*(a/b + x)**3*(a/b + x)**n*lerchphi(b*(a/b + x)/a,
1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n +
2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) + b**5*b**n*n*(a/b + x)**3*(a/b
 + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2
) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)
)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,x)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^3,x)
```

```
[Out] int((a + b*x)^n/x^3, x)
```

3.739 $\int x^{-4+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=110

$$-\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}+\frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}$$

```
[Out] -x^(-3+n)*(b*x+a)^(1-n)/a/(3-n)+2*b*x^(-2+n)*(b*x+a)^(1-n)/a^2/(2-n)/(3-n)-
2*b^2*x^(-1+n)*(b*x+a)^(1-n)/a^3/(3-n)/(n^2-3*n+2)
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-3 + n)*(a + b*x)^(1 - n))/(a*(3 - n))) + (2*b*x^(-2 + n)*(a + b*x)^(
1 - n))/(a^2*(2 - n)*(3 - n)) - (2*b^2*x^(-1 + n)*(a + b*x)^(1 - n))/(a^3*(
1 - n)*(2 - n)*(3 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int x^{-4+n}(a+bx)^{-n}\,dx = -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} - \frac{(2b)\int x^{-3+n}(a+bx)^{-n}\,dx}{a(3-n)}$$
$$= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} + \frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)} + \frac{(2b^2)\int x^{-2+n}(a+bx)^{-n}\,dx}{a^2(2-n)(3-n)}$$
$$= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} + \frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)} - \frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}$$

Mathematica [**A**]
time = 0.04, size = 64, normalized size = 0.58

$$\frac{x^{-3+n}(a+bx)^{1-n}\left(a^2\left(2-3n+n^2\right)+2ab(-1+n)x+2b^2x^2\right)}{a^3(-3+n)(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-3 + n)*(a + b*x)^(1 - n)*(a^2*(2 - 3*n + n^2) + 2*a*b*(-1 + n)*x + 2*b
^2*x^2))/(a^3*(-3 + n)*(-2 + n)*(-1 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-4 + n)/(a + b*x)^n,x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Maple [**A**]
time = 0.13, size = 77, normalized size = 0.70

method	result	size
gosper	$\frac{(bx+a)x^{-3+n}\left(a^2n^2+2abnx+2x^2b^2-3a^2n-2abx+2a^2\right)(bx+a)^{-n}}{(-3+n)(-2+n)(-1+n)a^3}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4+n)/((b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)*x^(-3+n)*(a^2*n^2+2*a*b*n*x+2*b^2*x^2-3*a^2*n-2*a*b*x+2*a^2)/((b*x+
a)^n)/(-3+n)/(-2+n)/(-1+n)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

Fricas [A]
time = 0.32, size = 104, normalized size = 0.95

$$\frac{(2\,ab^2nx^3 + 2\,b^3x^4 + (a^2bn^2 - a^2bn)x^2 + (a^3n^2 - 3\,a^3n + 2\,a^3)x)x^{n-4}}{(a^3n^3 - 6\,a^3n^2 + 11\,a^3n - 6\,a^3)(bx + a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 + 2*b^3*x^4 + (a^2*b*n^2 - a^2*b*n)*x^2 + (a^3*n^2 - 3*a^3*n
 + 2*a^3)*x)*x^(n - 4)/((a^3*n^3 - 6*a^3*n^2 + 11*a^3*n - 6*a^3)*(b*x + a)^
n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4+n)/((b*x+a)**n),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.52, size = 136, normalized size = 1.24

$$\frac{\frac{x\,x^{n-4}\,\left(n^2-3\,n+2\right)}{n^3-6\,n^2+11\,n-6}+\frac{2\,b^3\,x^{n-4}\,x^4}{a^3\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{2\,b^2\,n\,x^{n-4}\,x^3}{a^2\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{b\,n\,x^{n-4}\,x^2\,\left(n-1\right)}{a\,\left(n^3-6\,n^2+11\,n-6\right)}}{{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 4)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 4)*(n^2 - 3*n + 2))/(11*n - 6*n^2 + n^3 - 6) + (2*b^3*x^(n - 4)*
x^4)/(a^3*(11*n - 6*n^2 + n^3 - 6)) + (2*b^2*n*x^(n - 4)*x^3)/(a^2*(11*n -
6*n^2 + n^3 - 6)) + (b*n*x^(n - 4)*x^2*(n - 1))/(a*(11*n - 6*n^2 + n^3 - 6)
))/(a + b*x)^n
```

3.740 $\int x^{-3+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=64

$$-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}$$

```
[Out] -x^(-2+n)*(b*x+a)^(1-n)/a/(2-n)+b*x^(-1+n)*(b*x+a)^(1-n)/a^2/(1-n)/(2-n)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-2 + n)*(a + b*x)^(1 - n))/(a*(2 - n))) + (b*x^(-1 + n)*(a + b*x)^(1
- n))/(a^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3+n}(a+bx)^{-n}\,dx &= -\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}-\frac{b\int x^{-2+n}(a+bx)^{-n}\,dx}{a(2-n)}\\ &= -\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.61

$$\frac{x^{-2+n}(a+bx)^{1-n}(a(-1+n)+bx)}{a^2(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + n)/(a + b*x)^n,x]
[Out] (x^(-2 + n)*(a + b*x)^(1 - n)*(a*(-1 + n) + b*x))/(a^2*(-2 + n)*(-1 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 190.94, size = 344, normalized size = 5.38

$$\text{Piecewise}\left[\left\{\left\{-\frac{x^{-2+n}(bx)^{-n}}{2},a==0\right\},\left\{\frac{-a+bx\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}[x]\right)}{a^2x},n==1\right\},\left\{\frac{a\left(1+\text{Log}[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)+bx\left(\text{Log}[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^2(a+bx)},n==2\right\}\right\},-\frac{a^2x^n}{2a^2x^2(a+bx)^n-3a^2nx^2(a+bx)^n+a^2n^2x^2(a+bx)^n}+\frac{a^2nx^n}{2a^2x^2(a+bx)^n-3a^2nx^2(a+bx)^n+a^2n^2x^2(a+bx)^n}+\frac{abnxx^n}{2a^2x^2(a+bx)^n-3a^2nx^2(a+bx)^n+a^2n^2x^2(a+bx)^n}+\frac{b^2x^2x^n}{2a^2x^2(a+bx)^n-3a^2nx^2(a+bx)^n+a^2n^2x^2(a+bx)^n}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-3 + n)/(a + b*x)^n,x]')
[Out] Piecewise[{{-x ^ (-2 + n) (b x) ^ (-n) / 2, a == 0}, {(-a + b x (Log[(a + b
 x) / b] - Log[x])) / (a ^ 2 x), n == 1}, {(a (1 + Log[x] - Log[(a + b x) /
 b]) + b x (Log[x] - Log[(a + b x) / b])) / (a ^ 2 (a + b x)), n == 2}}, -a
 ^ 2 x ^ n / (2 a ^ 2 x ^ 2 (a + b x) ^ n - 3 a ^ 2 n x ^ 2 (a + b x) ^ n +
 a ^ 2 n ^ 2 x ^ 2 (a + b x) ^ n) + a ^ 2 n x ^ n / (2 a ^ 2 x ^ 2 (a + b x
) ^ n - 3 a ^ 2 n x ^ 2 (a + b x) ^ n + a ^ 2 n ^ 2 x ^ 2 (a + b x) ^ n) +
a b n x x ^ n / (2 a ^ 2 x ^ 2 (a + b x) ^ n - 3 a ^ 2 n x ^ 2 (a + b x) ^
n + a ^ 2 n ^ 2 x ^ 2 (a + b x) ^ n) + b ^ 2 x ^ 2 x ^ n / (2 a ^ 2 x ^ 2 (
a + b x) ^ n - 3 a ^ 2 n x ^ 2 (a + b x) ^ n + a ^ 2 n ^ 2 x ^ 2 (a + b x)
^ n)]
```

Maple [A]
time = 0.12, size = 44, normalized size = 0.69

method	result	size
gosper	$\frac{x^{-2+n}(an+bx-a)(bx+a)(bx+a)^{-n}}{(-2+n)(-1+n)a^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+n)/((b*x+a)^n),x,method=_RETURNVERBOSE)
[Out] x^(-2+n)*(a*n+b*x-a)*(b*x+a)/((b*x+a)^n)/(-2+n)/(-1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

Fricas [A]
time = 0.32, size = 64, normalized size = 1.00

$$\frac{(abnx^2 + b^2x^3 + (a^2n - a^2)x)x^{n-3}}{(a^2n^2 - 3\,a^2n + 2\,a^2)(bx + a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (a*b*n*x^2 + b^2*x^3 + (a^2*n - a^2)*x)*x^(n - 3)/((a^2*n^2 - 3*a^2*n + 2*a
^2)*(b*x + a)^n)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.45, size = 80, normalized size = 1.25

$$\frac{\frac{x\,x^{n-3}\,(n-1)}{n^2-3\,n+2} + \frac{b^2\,x^{n-3}\,x^3}{a^2\,(n^2-3\,n+2)} + \frac{b\,n\,x^{n-3}\,x^2}{a\,(n^2-3\,n+2)}}{(a+b\,x)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 3)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 3)*(n - 1))/(n^2 - 3*n + 2) + (b^2*x^(n - 3)*x^3)/(a^2*(n^2 - 3*
n + 2)) + (b*n*x^(n - 3)*x^2)/(a*(n^2 - 3*n + 2)))/(a + b*x)^n
```

3.741 $\int x^{-2+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=28

$$-\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

```
[Out] -x^(-1+n)*(b*x+a)^(1-n)/a/(1-n)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + n)/(a + b*x)^n,x]
[Out] -((x^(-1 + n)*(a + b*x)^(1 - n))/(a*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-2+n}(a+bx)^{-n}\,dx = -\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.89

$$\frac{x^{-1+n}(a+bx)^{1-n}}{a(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + n)/(a + b*x)^n,x]
[Out] (x^(-1 + n)*(a + b*x)^(1 - n))/(a*(-1 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 169.75, size = 120, normalized size = 4.29

$$\text{Piecewise}\left[\left\{\left\{-\frac{1}{bx},a==0\&\&n==1\right\},\left\{-x^{-1+n}(bx)^{-n},a==0\right\},\left\{\frac{\text{Log}[x]-\text{Log}\left[\frac{a}{b}+x\right]}{a},n==1\right\}\right\},\frac{ax^n}{-ax(a+bx)^n+anx(a+bx)^n}+\frac{bxx^n}{-ax(a+bx)^n+anx(a+bx)^n}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-2 + n)/(a + b*x)^n,x]')
```

```
[Out] Piecewise[{{-1 / (b x), a == 0 && n == 1}, {-x ^ (-1 + n) (b x) ^ (-n), a =
= 0}, {(Log[x] - Log[a / b + x]) / a, n == 1}}, a x ^ n / (-a x (a + b x) ^
 n + a n x (a + b x) ^ n) + b x x ^ n / (-a x (a + b x) ^ n + a n x (a + b
x) ^ n)]
```

Maple [A]
time = 0.12, size = 29, normalized size = 1.04

method	result	size
gosper	$\frac{x^{-1+n}(bx+a)(bx+a)^{-n}}{a(-1+n)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+n)/((b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x^(-1+n)*(b*x+a)/a/(-1+n)/((b*x+a)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

Fricas [A]
time = 0.32, size = 33, normalized size = 1.18

$$\frac{(bx^2+ax)x^{n-2}}{(an-a)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*x^(n - 2)/((a*n - a)*(b*x + a)^n)
```

Sympy [**A**]
time = 98.36, size = 85, normalized size = 3.04

$$\begin{cases} -\frac{1}{bx} & \text{for } a = 0 \wedge n = 1 \\ -\frac{x^n (bx)^{-n}}{x} & \text{for } a = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for } n = 1 \\ \frac{ax^n}{anx(a+bx)^n - ax(a+bx)^n} + \frac{bxx^n}{anx(a+bx)^n - ax(a+bx)^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+n)/((b*x+a)**n),x)
```

```
[Out] Piecewise((-1/(b*x), Eq(a, 0) & Eq(n, 1)), (-x**n/(x*(b*x)**n), Eq(a, 0)),
(log(x)/a - log(a/b + x)/a, Eq(n, 1)), (a*x**n/(a*n*x*(a + b*x)**n - a*x*(a
 + b*x)**n) + b*x*x**n/(a*n*x*(a + b*x)**n - a*x*(a + b*x)**n), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.35, size = 29, normalized size = 1.04

$$\frac{x^n\,\left(a+b\,x\right)}{a\,x\,\left(n-1\right)\,{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 2)/(a + b*x)^n,x)
```

```
[Out] (x^n*(a + b*x))/(a*x*(n - 1)*(a + b*x)^n)
```

3.742 $\int x^{-1+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=39

$$\frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}$$

```
[Out] x^n*(1+b*x/a)^n*hypergeom([n, n],[1+n],-b*x/a)/n/((b*x+a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {68, 66}

$$\frac{x^n(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n;n+1;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*
x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^{-1+n}\left(1+\frac{bx}{a}\right)^{-n}\,dx\\ &= \frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*
x)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 154.03, size = 30, normalized size = 0.77

$$\frac{a^{-n}x^n\text{hyper}\left[\{n,n\},\{1+n\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{n}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-1 + n)/(a + b*x)^n,x]')
```

```
[Out] a ^ (-n) x ^ n hyper[{n, n}, {1 + n}, b x exp_polar[I Pi] / a] / n
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1+n}\left(bx+a\right)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/((b*x+a)^n),x)
```

```
[Out] int(x^(-1+n)/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 1)/(b*x + a)^n, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^(n - 1)/(b*x + a)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 148.89, size = 27, normalized size = 0.69

$$\frac{a^{-n}x^{n}\Gamma\left(n\right){}_{2}F_{1}\left(\begin{matrix} n,n \\ n+1 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/((b*x+a)**n),x)
```

```
[Out] x**n*gamma(n)*hyper((n, n), (n + 1,), b*x*exp_polar(I*pi)/a)/(a**n*gamma(n
+ 1))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{n-1}}{{\left(a+b\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x)^n,x)
```

```
[Out] int(x^(n - 1)/(a + b*x)^n, x)
```

3.743 $\int x^n(a+bx)^{-n}\,dx$

Optimal. Leaf size=45

$$\frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}$$

```
[Out] x^(1+n)*(1+b*x/a)^n*hypergeom([n, 1+n],[2+n],-b*x/a)/(1+n)/((b*x+a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {68, 66}

$$\frac{x^{n+1}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{bx}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n/(a + b*x)^n,x]
```

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^n(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^n\left(1+\frac{bx}{a}\right)^{-n}dx\\ &= \frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\,{}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/(a + b*x)^n,x]
```

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 15.25, size = 36, normalized size = 0.80

$$\frac{a^{-n}x^{1+n}\text{hyper}\left[\{n,1+n\},\{2+n\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{1+n}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(0 + n)/(a + b*x)^n,x]')
```

```
[Out] a ^ (-n) x ^ (1 + n) hyper[{n, 1 + n}, {2 + n}, b x exp_polar[I Pi] / a] /
(1 + n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{n}\left(bx+a\right)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/((b*x+a)^n),x)
```

```
[Out] int(x^n/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^n/(b*x + a)^n, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="fricas")
[Out] integral(x^n/(b*x + a)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.84, size = 32, normalized size = 0.71

$$\frac{a^{-n}xx^n\Gamma(n+1)\,{}_2F_1\left(\begin{matrix}n,n+1\\n+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/((b*x+a)**n),x)
[Out] x*x**n*gamma(n + 1)*hyper((n, n + 1), (n + 2,), b*x*exp_polar(I*pi)/a)/(a**
n*gamma(n + 2))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^n}{(a+b\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(a + b*x)^n,x)
[Out] int(x^n/(a + b*x)^n, x)
```

3.744 $\int x^{1+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=45

$$\frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\,{}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}$$

```
[Out] x^(2+n)*(1+b*x/a)^n*hypergeom([n, 2+n],[3+n],-b*x/a)/(2+n)/((b*x+a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {68, 66}

$$\frac{x^{n+2}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^{n}\,{}_2F_1\left(n,n+2;n+3;-\frac{bx}{a}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/
((2 + n)*(a + b*x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^{1+n}(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\right)\int x^{1+n}\left(1+\frac{bx}{a}\right)^{-n}\,dx\\ &= \frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\,{}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/
((2 + n)*(a + b*x)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 109.86, size = 36, normalized size = 0.80

$$\frac{a^{-n}x^{2+n}\text{hyper}\left[\{n,2+n\},\{3+n\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{2+n}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1 + n)/(a + b*x)^n,x]')
```

```
[Out] a ^ (-n) x ^ (2 + n) hyper[{n, 2 + n}, {3 + n}, b x exp_polar[I Pi] / a] /
(2 + n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{1+n}\left(bx+a\right)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+n)/((b*x+a)^n),x)
```

```
[Out] int(x^(1+n)/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n + 1)/(b*x + a)^n, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^(n + 1)/(b*x + a)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 156.36, size = 34, normalized size = 0.76

$$\frac{a^{-n}x^2x^n\Gamma\left(n+2\right){}_2F_1\left(\begin{matrix} n,n+2 \\ n+3 \end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(n+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+n)/((b*x+a)**n),x)
```

```
[Out] x**2*x**n*gamma(n + 2)*hyper((n, n + 2), (n + 3,), b*x*exp_polar(I*pi)/a)/(
a**n*gamma(n + 3))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{n+1}}{{\left(a+b\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n + 1)/(a + b*x)^n,x)
```

```
[Out] int(x^(n + 1)/(a + b*x)^n, x)
```

3.745 $\int x^{3/2}(a+bx)^n\,dx$

Optimal. Leaf size=45

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

```
[Out] 2/5*x^(5/2)*(b*x+a)^n*hypergeom([5/2, -n],[7/2],-b*x/a)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^n,x]
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{3/2}\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 1.00

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^n,x]
```

```
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 39.61, size = 26, normalized size = 0.58

$$\frac{2x^{\frac{5}{2}}a^n\text{hyper}\left[\left\{\frac{5}{2},-n\right\},\left\{\frac{7}{2}\right\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{5}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(3/2)*(a + b*x)^n,x]')
```

```
[Out] 2 x ^ (5 / 2) a ^ n hyper[{5 / 2, -n}, {7 / 2}, b x exp_polar[I Pi] / a] /
5
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\left(bx+a\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^n,x)
```

```
[Out] int(x^(3/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(3/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 77.31, size = 27, normalized size = 0.60

$$\frac{2a^n x^{\frac{5}{2}} {}_2F_1\left(\begin{matrix} \frac{5}{2}, -n \\ \frac{7}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**n,x)
```

```
[Out] 2*a**n*x**(5/2)*hyper((5/2, -n), (7/2,), b*x*exp_polar(I*pi)/a)/5
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3/2}\,{\left(a+b\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^n,x)
```

```
[Out] int(x^(3/2)*(a + b*x)^n, x)
```

3.746 $\int \sqrt{x}\,(a+bx)^n\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

```
[Out] 2/3*x^(3/2)*(b*x+a)^n*hypergeom([3/2, -n],[5/2],-b*x/a)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {68, 66}

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^n,x]
```

```
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \sqrt{x}\,\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.00

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^n,x]
```

```
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 5.18, size = 26, normalized size = 0.58

$$\frac{2x^{\frac{3}{2}}a^n\text{hyper}\left[\left\{\frac{3}{2},-n\right\},\left\{\frac{5}{2}\right\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{3}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(1/2)*(a + b*x)^n,x]')
```

```
[Out] 2 x ^ (3 / 2) a ^ n hyper[{3 / 2, -n}, {5 / 2}, b x exp_polar[I Pi] / a] /
3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\left(bx+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^n,x)
```

```
[Out] int(x^(1/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*sqrt(x), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="fricas")
[Out] integral((b*x + a)^n*sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.77, size = 27, normalized size = 0.60

$$\frac{2a^n x^{\frac{3}{2}} {}_2F_1\left(\begin{matrix} \frac{3}{2}, -n \\ \frac{5}{2} \end{matrix} \middle| \frac{bxe^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**n,x)
[Out] 2*a**n*x**(3/2)*hyper((3/2, -n), (5/2,), b*x*exp_polar(I*pi)/a)/3
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x}\,\left(a+b\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^n,x)
[Out] int(x^(1/2)*(a + b*x)^n, x)
```

3.747 $\int \frac{(a+bx)^n}{\sqrt{x}}\,dx$

Optimal. Leaf size=43

$$2\sqrt{x}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

```
[Out] 2*(b*x+a)^n*hypergeom([1/2, -n],[3/2],-b*x/a)*x^(1/2)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$2\sqrt{x}\,(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{\sqrt{x}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{\sqrt{x}}\,dx\\ &= 2\sqrt{x}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 1.00

$$2\sqrt{x}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/Sqrt[x],x]
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.87, size = 26, normalized size = 0.60

$$2\sqrt{x}\,a^n\text{hyper}\left[\left\{\frac{1}{2},-n\right\},\left\{\frac{3}{2}\right\},\frac{bx\text{exp\_polar}\,[I\text{Pi}]}{a}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/x^(1/2),x]')
[Out] 2 Sqrt[x] a ^ n hyper[{1 / 2, -n}, {3 / 2}, b x exp_polar[I Pi] / a]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(1/2),x)
[Out] int((b*x+a)^n/x^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/sqrt(x), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="fricas")
[Out] integral((b*x + a)^n/sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.35, size = 26, normalized size = 0.60

$$2a^n\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(1/2),x)
[Out] 2*a**n*sqrt(x)*hyper((1/2, -n), (3/2,), b*x*exp_polar(I*pi)/a)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(1/2),x)
[Out] int((a + b*x)^n/x^(1/2), x)
```

3.748 $\int \frac{(a+bx)^n}{x^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

```
[Out] -2*(b*x+a)^n*hypergeom([-1/2, -n],[1/2],-b*x/a)/((1+b*x/a)^n)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^(3/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^n}{x^{3/2}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{x^{3/2}}\,dx\\
&= -\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 43, normalized size = 1.00

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^(3/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 15.30, size = 26, normalized size = 0.60

$$\frac{-2a^n\text{hyper}\left[\left\{-\frac{1}{2},-n\right\},\left\{\frac{1}{2}\right\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/x^(3/2),x]')
```

```
[Out] -2 a ^ n hyper[{-1 / 2, -n}, {1 / 2}, b x exp_polar[I Pi] / a] / Sqrt[x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(3/2),x)
```

```
[Out] int((b*x+a)^n/x^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^(3/2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.71, size = 29, normalized size = 0.67

$$-\frac{2a^n {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -n \\ \frac{1}{2} \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(3/2),x)
```

```
[Out] -2*a**n*hyper((-1/2, -n), (1/2,), b*x*exp_polar(I*pi)/a)/sqrt(x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(3/2),x)
```

```
[Out] int((a + b*x)^n/x^(3/2), x)
```

3.749 $\int \frac{(a+bx)^n}{x^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

```
[Out] -2/3*(b*x+a)^n*hypergeom([-3/2, -n],[-1/2],-b*x/a)/x^(3/2)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {68, 66}

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^{5/2}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{x^{5/2}}\,dx\\ &= -\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 1.00

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 172.89, size = 26, normalized size = 0.58

$$\frac{-2a^n\text{hyper}\left[\left\{-\frac{3}{2},-n\right\},\left\{-\frac{1}{2}\right\},\frac{bx\text{exp\_polar}[I\text{Pi}]}{a}\right]}{3x^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/x^(5/2),x]')
```

```
[Out] -2 a ^ n hyper[{-3 / 2, -n}, {-1 / 2}, b x exp_polar[I Pi] / a] / (3 x ^ (3
 / 2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(5/2),x)
```

```
[Out] int((b*x+a)^n/x^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^(5/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^(5/2), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(5/2),x)
```

```
[Out] int((a + b*x)^n/x^(5/2), x)
```

3.750 $\int (bx)^m(2+dx)^n\,dx$

Optimal. Leaf size=35

$$\frac{2^n(bx)^{1+m}\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{b(1+m)}$$

```
[Out] 2^n*(b*x)^(1+m)*hypergeom([-n, 1+m],[2+m],-1/2*d*x)/b/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{66\}$

$$\frac{2^n(bx)^{m+1}\,{}_2F_1\left(m+1,-n;m+2;-\frac{dx}{2}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(2 + d*x)^n,x]
```

```
[Out] (2^n*(b*x)^(1 + m)*Hypergeometric2F1[1 + m, -n, 2 + m, -1/2*(d*x)])/(b*(1 +
 m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int (bx)^m(2+dx)^n\,dx=\frac{2^n(bx)^{1+m}\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{b(1+m)}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.89

$$\frac{2^n x(bx)^m\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(2 + d*x)^n,x]
```

```
[Out] (2^n*x*(b*x)^m*Hypergeometric2F1[1 + m, -n, 2 + m, -1/2*(d*x)])/(1 + m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.87, size = 37, normalized size = 1.06

$$\frac{2^n b^m x^{1+m} \text{hyper}\left[\{-n,1+m\},\{2+m\},\frac{dx\text{exp\_polar}[I\text{Pi}]}{2}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(b*x)^m*(2 + d*x)^n,x]')
```

```
[Out] 2 ^ n b ^ m x ^ (1 + m) hyper[{-n, 1 + m}, {2 + m}, d x exp_polar[I Pi] / 2
] / (1 + m)
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.91

method	result	size
meijerg	$\frac{2^n (bx)^m x\, \text{hypergeom}\left([-n,1+m],[2+m],-\frac{dx}{2}\right)}{1+m}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(d*x+2)^n,x,method=_RETURNVERBOSE)
```

```
[Out] 2^n*(b*x)^m/(1+m)*x*hypergeom([-n,1+m],[2+m],-1/2*d*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*(d*x + 2)^n, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="fricas")
```

[Out] integral((b*x)^m*(d*x + 2)^n, x)

Sympy [C] Result contains complex when optimal does not.
time = 1.24, size = 37, normalized size = 1.06

$$\frac{2^n b^m x x^m \Gamma(m+1)\,{}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{dxe^{i\pi}}{2}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)**m*(d*x+2)**n,x)

[Out] 2**n*b**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), d*x*exp_polar(I*pi)/2)/gamma(m + 2)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^m*(d*x+2)^n,x)

[Out] Could not integrate

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (b\,x)^m\,(d\,x+2)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x)^m*(d*x + 2)^n,x)

[Out] int((b*x)^m*(d*x + 2)^n, x)

3.751 $\int (bx)^m(c-bcx)^n\,dx$

Optimal. Leaf size=40

$$-\frac{(c-bcx)^{1+n}\,{}_2F_1(-m,1+n;2+n;1-bx)}{bc(1+n)}$$

```
[Out] -(-b*c*x+c)^(1+n)*hypergeom([-m, 1+n],[2+n],-b*x+1)/b/c/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{67\}$

$$-\frac{(c-bcx)^{n+1}\,{}_2F_1(-m,n+1;n+2;1-bx)}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(c - b*c*x)^n,x]
```

```
[Out] -(((c - b*c*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, 1 - b*x])/(b*c*(
1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int (bx)^m(c-bcx)^n\,dx = -\frac{(c-bcx)^{1+n}\,{}_2F_1(-m,1+n;2+n;1-bx)}{bc(1+n)}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 1.10

$$\frac{x(bx)^m(1-bx)^{-n}(c-bcx)^n\,{}_2F_1(1+m,-n;2+m;bx)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(c - b*c*x)^n,x]
```

```
[Out] (x*(b*x)^m*(c - b*c*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, b*x])/((1 + m)
*(1 - b*x)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.86, size = 36, normalized size = 0.90

$$\frac{b^m c^n x^{1+m} \text{hyper}\left[\{-n,1+m\},\{2+m\},bx\text{exp\_polar}[2I\text{Pi}]\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(b*x)^m*(c - b*c*x)^n,x]')
```

```
[Out] b ^ m c ^ n x ^ (1 + m) hyper[{-n, 1 + m}, {2 + m}, b x exp_polar[2 I Pi]]
/ (1 + m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx)^m (-bcx+c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(-b*c*x+c)^n,x)
```

```
[Out] int((b*x)^m*(-b*c*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + c)^n*(b*x)^m, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + c)^n*(b*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.27, size = 37, normalized size = 0.92

$$\frac{b^m c^n x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix} \middle| b x e^{2i\pi}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(-b*c*x+c)**n,x)
```

```
[Out] b**m*c**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(2*
I*pi))/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,x)^m\,(c-b\,c\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(c - b*c*x)^n,x)
```

```
[Out] int((b*x)^m*(c - b*c*x)^n, x)
```

3.752 $\int (bx)^m (c+dx)^n \, dx$

Optimal. Leaf size=52

$$\frac{(bx)^{1+m}(c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{b(1+m)}$$

```
[Out] (b*x)^(1+m)*(d*x+c)^n*hypergeom([-n, 1+m],[2+m],-d*x/c)/b/(1+m)/((1+d*x/c)^
n)
```

Rubi [A]

time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{(bx)^{m+1}(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{c}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])
/(b*(1 + m)*(1 + (d*x)/c)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (bx)^m(c+dx)^n\,dx &= \left((c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}\right)\int (bx)^m\left(1+\frac{dx}{c}\right)^n\,dx\\ &= \frac{(bx)^{1+m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{b(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.92

$$\frac{x(bx)^m(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(c + d*x)^n,x]
```

```
[Out] (x*(b*x)^m*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])/((1
 + m)*(1 + (d*x)/c)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 3.15, size = 39, normalized size = 0.75

$$\frac{b^m c^n x^{1+m}\text{hyper}\left[\{-n,1+m\},\{2+m\},\frac{dx\text{exp\_polar}[I\text{Pi}]}{c}\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(b*x)^m*(c + d*x)^n,x]')
```

```
[Out] b ^ m c ^ n x ^ (1 + m) hyper[{-n, 1 + m}, {2 + m}, d x exp_polar[I Pi] / c
] / (1 + m)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (bx)^m (dx+c)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(d*x+c)^n,x)
```

```
[Out] int((b*x)^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*(d*x + c)^n, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="fricas")
[Out] integral((b*x)^m*(d*x + c)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.55, size = 37, normalized size = 0.71

$$\frac{b^m c^n x x^m \Gamma(m+1) \, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{dxe^{i\pi}}{c}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(d*x+c)**n,x)
[Out] b**m*c**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), d*x*exp_polar(I*
pi)/c)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(c + d*x)^n,x)
[Out] int((b*x)^m*(c + d*x)^n, x)
```

3.753 $\int x^{-1+n}(a+bx)^{-1-n}\,dx$

Optimal. Leaf size=19

$$\frac{x^n(a+bx)^{-n}}{an}$$

```
[Out] x^n/a/n/((b*x+a)^n)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx = \frac{x^n(a+bx)^{-n}}{an}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 202.58, size = 236, normalized size = 12.42

$$\text{Piecewise}\left[\left\{\left\{-\frac{x^{-1+n}(bx)^{-n}}{b}, a==0\right\}, \left\{\frac{x^n \text{ComplexInfinity}^{1+n}}{n}, a==-bx\right\}, \left\{\frac{x^n\left(0^{\frac{1}{n}}\right)^{-1-n}}{n}, a==-bx+0^{\frac{1}{n}}\right\}, \left\{\frac{\text{Log}[x]-\text{Log}\left[\frac{a}{b}+x\right]}{a}, n==0\right\}\right\}, \frac{a^2x^n}{a^3n(a+bx)^n+2a^2bnx(a+bx)^n+ab^2nx^2(a+bx)^n}+\frac{bxx^n}{a^2n(a+bx)^n+abnx(a+bx)^n}+\frac{abxx^n}{a^3n(a+bx)^n+2a^2bnx(a+bx)^n+ab^2nx^2(a+bx)^n}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^(-1 + n)*(a + b*x)^(-1 - n),x]')
```

```
[Out] Piecewise[{{-x ^ (-1 + n) (b x) ^ (-n) / b, a == 0}, {x ^ n ComplexInfinity
 ^ (1 + n) / n, a == -b x}, {x ^ n (0 ^ (1 / n)) ^ (-1 - n) / n, a == -b x
+ 0 ^ (1 / n)}, {(Log[x] - Log[a / b + x]) / a, n == 0}}, a ^ 2 x ^ n / (a
^ 3 n (a + b x) ^ n + 2 a ^ 2 b n x (a + b x) ^ n + a b ^ 2 n x ^ 2 (a + b
x) ^ n) + b x x ^ n / (a ^ 2 n (a + b x) ^ n + a b n x (a + b x) ^ n) + a b
 x x ^ n / (a ^ 3 n (a + b x) ^ n + 2 a ^ 2 b n x (a + b x) ^ n + a b ^ 2 n
 x ^ 2 (a + b x) ^ n)]
```

Maple [A]
time = 0.11, size = 20, normalized size = 1.05

method	result	size
gosper	$\frac{x^n(bx+a)^{-n}}{an}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b*x+a)^(-1-n),x,method=_RETURNVERBOSE)
```

```
[Out] x^n*(b*x+a)^(-n)/a/n
```

Maxima [A]
time = 0.33, size = 22, normalized size = 1.16

$$\frac{e^{(-n\log(bx+a)+n\log(x))}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="maxima")
```

```
[Out] e^(-n*log(b*x + a) + n*log(x))/(a*n)
```

Fricas [A]
time = 0.32, size = 32, normalized size = 1.68

$$\frac{(bx^2+ax)(bx+a)^{-n-1}x^{n-1}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(b*x + a)^(-n - 1)*x^(n - 1)/(a*n)
```

Sympy [A]
time = 128.51, size = 197, normalized size = 10.37

$$\begin{cases} -\frac{x^n(bx)^{-n}}{bx} & \text{for } a=0 \\ \frac{0^{-n-1}x^n}{n} & \text{for } a=-bx \\ \frac{x^n\left(0^{\frac{1}{n}}\right)^{-n-1}}{n} & \text{for } a=0^{\frac{1}{n}}-bx \\ \frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for } n=0 \\ \frac{a^2x^n}{a^3n(a+bx)^n+2a^2bnx(a+bx)^n+ab^2nx^2(a+bx)^n}+\frac{abxx^n}{a^3n(a+bx)^n+2a^2bnx(a+bx)^n+ab^2nx^2(a+bx)^n}+\frac{bxx^n}{a^2n(a+bx)^n+abnx(a+bx)^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b*x+a)**(-1-n),x)
[Out] Piecewise((-x**n/(b*x*(b*x)**n), Eq(a, 0)), (0**(-n - 1)*x**n/n, Eq(a, -b*x
)), (x**n*(0**(1/n))**(-n - 1)/n, Eq(a, 0**(1/n) - b*x)), (log(x)/a - log(a
/b + x)/a, Eq(n, 0)), (a**2*x**n/(a**3*n*(a + b*x)**n + 2*a**2*b*n*x*(a + b
*x)**n + a*b**2*n*x**2*(a + b*x)**n) + a*b*x*x**n/(a**3*n*(a + b*x)**n + 2*
a**2*b*n*x*(a + b*x)**n + a*b**2*n*x**2*(a + b*x)**n) + b*x*x**n/(a**2*n*(a
 + b*x)**n + a*b*n*x*(a + b*x)**n), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x)
[Out] Could not integrate
```

Mupad [B]
time = 0.50, size = 19, normalized size = 1.00

$$\frac{x^n}{a\,n\,{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x)^(n + 1),x)
[Out] x^n/(a*n*(a + b*x)^n)
```

3.754 $\int x^{-3-n}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

```
[Out] -x^(-2-n)*(b*x+a)^(1+n)/a/(2+n)+b*x^(-1-n)*(b*x+a)^(1+n)/a^2/(1+n)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3-n}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.69

$$-\frac{x^{-2-n}(a+an-bx)(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - n)*(a + b*x)^n,x]
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 41.60, size = 302, normalized size = 5.21

$$\text{Piecewise}\left[\left\{\left\{-\frac{x^{-2-n}(bx)^n}{2},a==0\right\},\left\{\frac{a\left(1+\text{Log}[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)+bx\left(\text{Log}[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^2(a+bx)},n==-2\right\},\left\{\frac{-a+bx\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}[x]\right)}{a^2x},n==-1\right\}\right\},-\frac{a^2(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}-\frac{a^2n(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}-\frac{abnx(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}+\frac{b^2x^2(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-3 - n)*(a + b*x)^n,x]')
[Out] Piecewise[{{-x ^ (-2 - n) (b x) ^ n / 2, a == 0}, {(a (1 + Log[x] - Log[(a
+ b x) / b]) + b x (Log[x] - Log[(a + b x) / b])) / (a ^ 2 (a + b x)), n ==
 -2}, {(-a + b x (Log[(a + b x) / b] - Log[x])) / (a ^ 2 x), n == -1}}, -a
^ 2 (a + b x) ^ n / (2 a ^ 2 x ^ 2 x ^ n + 3 a ^ 2 n x ^ 2 x ^ n + a ^ 2 n
^ 2 x ^ 2 x ^ n) - a ^ 2 n (a + b x) ^ n / (2 a ^ 2 x ^ 2 x ^ n + 3 a ^ 2 n
 x ^ 2 x ^ n + a ^ 2 n ^ 2 x ^ 2 x ^ n) - a b n x (a + b x) ^ n / (2 a ^ 2
x ^ 2 x ^ n + 3 a ^ 2 n x ^ 2 x ^ n + a ^ 2 n ^ 2 x ^ 2 x ^ n) + b ^ 2 x ^
2 (a + b x) ^ n / (2 a ^ 2 x ^ 2 x ^ n + 3 a ^ 2 n x ^ 2 x ^ n + a ^ 2 n ^
2 x ^ 2 x ^ n)]
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^{-2-n}(an-bx+a)}{(2+n)(1+n)a^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x^(-2-n)*(a*n-b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Fricas [A]
time = 0.32, size = 64, normalized size = 1.10

$$-\frac{(abnx^2 - b^2x^3 + (a^2n + a^2)x)(bx + a)^n x^{-n-3}}{a^2n^2 + 3\,a^2n + 2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

Sympy [A]
time = 17.45, size = 328, normalized size = 5.66

$$\begin{cases} -\frac{x^{-n}(bx)^n}{2x^2} & \text{for}\ a = 0 \\ \frac{a\log(x)}{a^3+a^2bx} - \frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} + \frac{a}{a^3+a^2bx} + \frac{bx\log(x)}{a^3+a^2bx} - \frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for}\ n = -2 \\ -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for}\ n = -1 \\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} + \frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-(b*x)**n/(2*x**2*x**n), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x)
- a*log(a/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3
 + a**2*b*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) -
b*log(x)/a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**
2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)*
*n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*
(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n)
+ b**2*x**2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2
*x**2*x**n), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.50, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\,\left(\frac{x\,\left(n+1\right)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

3.755 $\int x^{2n-3(1+n)}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

```
[Out] -x^(-2-n)*(b*x+a)^(1+n)/a/(2+n)+b*x^(-1-n)*(b*x+a)^(1+n)/a^2/(n^2+3*n+2)
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {47, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{2n-3(1+n)}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 0.69

$$-\frac{x^{-2-n}(a+an-bx)(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 18.00, size = 302, normalized size = 5.21

$$\text{Piecewise}\left[\left\{\left\{-\frac{x^{-2-n}(bx)^n}{2},a==0\right\},\left\{\frac{a\left(1+\text{Log}[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)+bx\left(\text{Log}[x]-\text{Log}\left[\frac{a+bx}{b}\right]\right)}{a^2(a+bx)},n==-2\right\},\left\{\frac{-a+bx\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}[x]\right)}{a^2x},n==-1\right\}\right\},-\frac{a^2(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}-\frac{a^2n(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}-\frac{abnx(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}+\frac{b^2x^2(a+bx)^n}{2a^2x^2x^n+3a^2nx^2x^n+a^2n^2x^2x^n}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]')
```

```
[Out] Piecewise[{{-x ^ (-2 - n) (b x) ^ n / 2, a == 0}, {(a (1 + Log[x] - Log[(a
+ b x) / b]) + b x (Log[x] - Log[(a + b x) / b])) / (a ^ 2 (a + b x)), n ==
 -2}, {(-a + b x (Log[(a + b x) / b] - Log[x])) / (a ^ 2 x), n == -1}}, -a
^ 2 (a + b x) ^ n / (2 a ^ 2 x ^ 2 x ^ n + 3 a ^ 2 n x ^ 2 x ^ n + a ^ 2 n
^ 2 x ^ 2 x ^ n) - a ^ 2 n (a + b x) ^ n / (2 a ^ 2 x ^ 2 x ^ n + 3 a ^ 2 n
 x ^ 2 x ^ n + a ^ 2 n ^ 2 x ^ 2 x ^ n) - a b n x (a + b x) ^ n / (2 a ^ 2
x ^ 2 x ^ n + 3 a ^ 2 n x ^ 2 x ^ n + a ^ 2 n ^ 2 x ^ 2 x ^ n) + b ^ 2 x ^
2 (a + b x) ^ n / (2 a ^ 2 x ^ 2 x ^ n + 3 a ^ 2 n x ^ 2 x ^ n + a ^ 2 n ^
2 x ^ 2 x ^ n)]
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^{-2-n}(an-bx+a)}{(2+n)(1+n)a^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x^(-2-n)*(a*n-b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Fricas [A]
time = 0.32, size = 64, normalized size = 1.10

$$-\frac{(abnx^2 - b^2x^3 + (a^2n + a^2)x)(bx + a)^n x^{-n-3}}{a^2n^2 + 3\,a^2n + 2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

Sympy [A]
time = 17.52, size = 328, normalized size = 5.66

$$\begin{cases} -\frac{x^{-n}(bx)^n}{2x^2} & \text{for}\: a = 0 \\ \frac{a\log(x)}{a^3+a^2bx} - \frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} + \frac{a}{a^3+a^2bx} + \frac{bx\log(x)}{a^3+a^2bx} - \frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for}\: n = -2 \\ -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for}\: n = -1 \\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} + \frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-(b*x)**n/(2*x**2*x**n), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x)
- a*log(a/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3
 + a**2*b*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) -
b*log(x)/a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**
2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)*
*n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*
(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n)
+ b**2*x**2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2
*x**2*x**n), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.00, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\,\left(\frac{x\,\left(n+1\right)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

3.756 $\int x^3 \sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

```
[Out] 1/5*a*x^4*(c*x^2)^(1/2)+1/6*b*x^5*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^4*Sqrt[c*x^2])/5 + (b*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x^4(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(ax^4+bx^5\right)\,dx}{x}\\ &= \frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}\,(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

Mathics [A]
time = 1.73, size = 19, normalized size = 0.54

$$x^4\left(\frac{a}{5}+\frac{bx}{6}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]')
[Out] x ^ 4 (a / 5 + b x / 6) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^{4}(5bx+6a)\sqrt{c\,x^{2}}}{30}$	21
default	$\frac{x^{4}(5bx+6a)\sqrt{c\,x^{2}}}{30}$	21
risch	$\frac{a\,x^{4}\sqrt{c\,x^{2}}}{5}+\frac{b\,x^{5}\sqrt{c\,x^{2}}}{6}$	28
trager	$\frac{\left(5b\,x^{5}+6a\,x^{4}+5b\,x^{4}+6a\,x^{3}+5b\,x^{3}+6a\,x^{2}+5x^{2}b+6ax+5bx+6a+5b\right)(-1+x)\sqrt{c\,x^{2}}}{30x}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*x^4*(5*b*x+6*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.94

$$\frac{(cx^2)^{\frac{3}{2}}\,bx^3}{6\,c}+\frac{(cx^2)^{\frac{3}{2}}\,ax^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(3/2)*b*x^3/c + 1/5*(c*x^2)^(3/2)*a*x^2/c
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^{5}+6\,ax^{4}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*x^5 + 6*a*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.15, size = 29, normalized size = 0.83

$$\frac{ax^{4}\sqrt{cx^{2}}}{5}+\frac{bx^{5}\sqrt{cx^{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*x**4*sqrt(c*x**2)/5 + b*x**5*sqrt(c*x**2)/6
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.71

$$\sqrt{c}\,\left(\frac{1}{5}ax^{5}\mathrm{sign}\left(x\right)+\frac{1}{6}bx^{6}\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^{3}\,\sqrt{c\,x^{2}}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x), x)
```

3.757 $\int x^2\sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

```
[Out] 1/4*a*x^3*(c*x^2)^(1/2)+1/5*b*x^4*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^3*Sqrt[c*x^2])/4 + (b*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}\,(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^3(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^3+bx^4\right)\,dx}{x}\\&=\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}\,(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

Mathics [A]
time = 1.72, size = 19, normalized size = 0.54

$$x^3\left(\frac{a}{4}+\frac{bx}{5}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]')
[Out] x ^ 3 (a / 4 + b x / 5) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^{3}(4bx+5a)\sqrt{c\,x^{2}}}{20}$	21
default	$\frac{x^{3}(4bx+5a)\sqrt{c\,x^{2}}}{20}$	21
risch	$\frac{a\,x^{3}\sqrt{c\,x^{2}}}{4}+\frac{b\,x^{4}\sqrt{c\,x^{2}}}{5}$	28
trager	$\frac{(4b\,x^{4}+5a\,x^{3}+4b\,x^{3}+5a\,x^{2}+4x^{2}b+5ax+4bx+5a+4b)(-1+x)\sqrt{c\,x^{2}}}{20x}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^3*(4*b*x+5*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 31, normalized size = 0.89

$$\frac{(cx^2)^{\frac{3}{2}}\,bx^2}{5\,c}+\frac{(cx^2)^{\frac{3}{2}}\,ax}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(3/2)*b*x^2/c + 1/4*(c*x^2)^(3/2)*a*x/c
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^4 + 5\,ax^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/20*(4*b*x^4 + 5*a*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.12, size = 29, normalized size = 0.83

$$\frac{ax^3\sqrt{cx^2}}{4} + \frac{bx^4\sqrt{cx^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*(c*x**2)**(1/2),x)
[Out] a*x**3*sqrt(c*x**2)/4 + b*x**4*sqrt(c*x**2)/5
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.71

$$\sqrt{c}\left(\frac{1}{4}ax^4\mathrm{sign}\left(x\right) + \frac{1}{5}bx^5\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x)
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x),x)
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x), x)
```

3.758 $\int x\sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

```
[Out] 1/3*a*x^2*(c*x^2)^(1/2)+1/4*b*x^3*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {15, 45}

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^2*Sqrt[c*x^2])/3 + (b*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x^2(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(ax^2+bx^3\right)dx}{x}\\ &= \frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Mathics [A]
time = 1.69, size = 19, normalized size = 0.54

$$x^2\left(\frac{a}{3}+\frac{bx}{4}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*Sqrt[c*x^2]*(a + b*x),x]')
[Out] x ^ 2 (a / 3 + b x / 4) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^2(3bx+4a)\sqrt{c\,x^2}}{12}$	21
default	$\frac{x^2(3bx+4a)\sqrt{c\,x^2}}{12}$	21
risch	$\frac{a\,x^2\sqrt{c\,x^2}}{3}+\frac{b\,x^3\sqrt{c\,x^2}}{4}$	28
trager	$\frac{(3b\,x^3+4a\,x^2+3x^2b+4ax+3bx+4a+3b)(-1+x)\sqrt{c\,x^2}}{12x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x^2*(3*b*x+4*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.80

$$\frac{(cx^2)^{\frac{3}{2}}\,bx}{4\,c}+\frac{(cx^2)^{\frac{3}{2}}\,a}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*b*x/c + 1/3*(c*x^2)^(3/2)*a/c
```

Fricas [**A**]
time = 0.30, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^{3}+4\,ax^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.10, size = 29, normalized size = 0.83

$$\frac{ax^{2}\sqrt{cx^{2}}}{3}+\frac{bx^{3}\sqrt{cx^{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*x**2*sqrt(c*x**2)/3 + b*x**3*sqrt(c*x**2)/4
```

Giac [**A**]
time = 0.00, size = 25, normalized size = 0.71

$$\sqrt{c}\,\left(\frac{1}{3}ax^{3}\mathrm{sign}\left(x\right)+\frac{1}{4}bx^{4}\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\sqrt{c\,x^{2}}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(1/2)*(a + b*x), x)
```

3.759 $\int \sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

```
[Out] 1/2*a*x*(c*x^2)^(1/2)+1/3*b*x^2*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x*Sqrt[c*x^2])/2 + (b*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(ax+bx^2\right)\,dx}{x}\\
&= \frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Mathics [A]
time = 1.66, size = 18, normalized size = 0.55

$$\frac{x\,(3a+2bx)\,\sqrt{cx^2}}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[c*x^2]*(a + b*x),x]')
[Out] x (3 a + 2 b x) Sqrt[c x ^ 2] / 6
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.58

method	result	size
gosper	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
default	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
risch	$\frac{ax\sqrt{c\,x^2}}{2}+\frac{b\,x^2\sqrt{c\,x^2}}{3}$	26
trager	$\frac{(2x^2b+3ax+2bx+3a+2b)(-1+x)\sqrt{c\,x^2}}{6x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*x*(2*b*x+3*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{cx^2}\,ax+\frac{(cx^2)^{\frac{3}{2}}\,b}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(c*x^2)*a*x + 1/3*(c*x^2)^(3/2)*b/c
```

Fricas [A]
time = 0.30, size = 20, normalized size = 0.61

$$\frac{1}{6}\left(2\,bx^2+3\,ax\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.09, size = 27, normalized size = 0.82

$$\frac{ax\sqrt{cx^2}}{2}+\frac{bx^2\sqrt{cx^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2),x)
[Out] a*x*sqrt(c*x**2)/2 + b*x**2*sqrt(c*x**2)/3
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.76

$$\sqrt{c}\,\left(\frac{1}{2}ax^2\mathrm{sign}\left(x\right)+\frac{1}{3}bx^3\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x)
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.54, size = 20, normalized size = 0.61

$$\frac{\sqrt{c}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,\left|x\right|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x),x)
[Out] (c^(1/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

3.760 $\int \frac{\sqrt{cx^2}(a+bx)}{x}\,dx$

Optimal. Leaf size=27

$$a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}$$

```
[Out] a*(c*x^2)^(1/2)+1/2*b*x*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = $\{15\}$

$$a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] a*Sqrt[c*x^2] + (b*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)}{x}\,dx &= \frac{\sqrt{cx^2}\,\int (a+bx)\,dx}{x}\\ &= a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.89

$$\frac{cx^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] (c*x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Mathics [A]
time = 1.69, size = 14, normalized size = 0.52

$$\left(a+\frac{bx}{2}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]')
```

```
[Out] (a + b x / 2) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.63

method	result	size
gosper	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
default	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
risch	$a\sqrt{c\,x^2}+\frac{bx\sqrt{c\,x^2}}{2}$	22
trager	$\frac{(bx+2a+b)(-1+x)\sqrt{c\,x^2}}{2x}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+2*a)*(c*x^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 16, normalized size = 0.59

$$\frac{1}{2}\sqrt{cx^2}\,(bx+2\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)
```

Sympy [A]
time = 0.09, size = 22, normalized size = 0.81

$$a\sqrt{cx^2} + \frac{bx\sqrt{cx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x,x)
```

```
[Out] a*sqrt(c*x**2) + b*x*sqrt(c*x**2)/2
```

Giac [A]
time = 0.00, size = 19, normalized size = 0.70

$$\sqrt{c}\,\left(\frac{1}{2}bx^2 + ax\right)\mathrm{sign}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sqrt(c)*sgn(x)
```

Mupad [B]
time = 0.19, size = 14, normalized size = 0.52

$$\frac{\sqrt{c}\,\,|x|\,\,(2\,a + b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x,x)
```

```
[Out] (c^(1/2)*abs(x)*(2*a + b*x))/2
```

3.761 $\int \frac{\sqrt{cx^2}(a+bx)}{x^2} dx$

Optimal. Leaf size=28

$$b\sqrt{cx^2}+\frac{a\sqrt{cx^2}\log(x)}{x}$$

```
[Out] b*(c*x^2)^(1/2)+a*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]

time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] b*Sqrt[c*x^2] + (a*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x^2} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x} dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(b+\frac{a}{x}\right) dx}{x}\\ &= b\sqrt{cx^2}+\frac{a\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.71

$$\frac{cx(bx + a\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.71

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(bx+a\ln(x))}{x}$	20
risch	$b\sqrt{c\,x^{2}}+\frac{a\ln(x)\sqrt{c\,x^{2}}}{x}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)/x*(b*x+a*ln(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.31, size = 19, normalized size = 0.68

$$\frac{\sqrt{cx^2}\,(bx + a\log(x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a + bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 19, normalized size = 0.68

$$\sqrt{c}\,(a\mathrm{sign}\,(x)\ln|x| + bx\mathrm{sign}\,(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\,(a + b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^2, x)
```

3.762 $\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx$

Optimal. Leaf size=32

$$-\frac{a\sqrt{cx^2}}{x^2}+\frac{b\sqrt{cx^2}\log(x)}{x}$$

```
[Out] -a*(c*x^2)^(1/2)/x^2+b*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{b\sqrt{cx^2}\log(x)}{x}-\frac{a\sqrt{cx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

```
[Out] -((a*Sqrt[c*x^2])/x^2) + (b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^2} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(\frac{a}{x^2}+\frac{b}{x}\right) dx}{x} \\
&= -\frac{a\sqrt{cx^2}}{x^2}+\frac{b\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.62

$$\frac{c(-a+bx\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

```
[Out] (c*(-a + b*x*Log[x]))/Sqrt[c*x^2]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.66

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(bx\ln(x)-a)}{x^2}$	21
risch	$-\frac{a\sqrt{c\,x^2}}{x^2}+\frac{b\ln(x)\sqrt{c\,x^2}}{x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)*(b*x*ln(x)-a)/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.31, size = 20, normalized size = 0.62

$$\frac{\sqrt{cx^2}\,(bx\log(x)-a)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}\,(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**3, x)
```

Giac [A]
time = 0.00, size = 21, normalized size = 0.66

$$\sqrt{c}\left(-\frac{a\mathrm{sign}\,(x)}{x}+b\mathrm{sign}\,(x)\ln|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x)
[Out] (b*log(abs(x))*sgn(x) - a*sgn(x)/x)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{c\,x^2}\,(a+b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^3,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^3, x)
```

3.763 $\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

```
[Out] -1/2*(b*x+a)^2*(c*x^2)^(1/2)/a/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -1/2*(Sqrt[c*x^2]*(a + b*x)^2)/(a*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx &= \frac{\sqrt{cx^2} \int \frac{a+bx}{x^3} dx}{x} \\ &= -\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.85

$$-\frac{\sqrt{cx^2}(a+2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
[Out] -1/2*(Sqrt[c*x^2]*(a + 2*b*x))/x^3
```

Mathics [A]
time = 1.74, size = 19, normalized size = 0.73

$$\frac{\left(-\frac{a}{2}-bx\right)\sqrt{cx^2}}{x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]')
[Out] (-a / 2 - b x) Sqrt[c x ^ 2] / x ^ 3
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.73

method	result	size
gosper	$-\frac{(2bx+a)\sqrt{c\,x^2}}{2x^3}$	19
default	$-\frac{(2bx+a)\sqrt{c\,x^2}}{2x^3}$	19
risch	$\frac{\left(-bx-\frac{a}{2}\right)\sqrt{c\,x^2}}{x^3}$	20
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^2}}{2x^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2*(2*b*x+a)*(c*x^2)^(1/2)/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.31, size = 18, normalized size = 0.69

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/x^3
```

Sympy [A]
time = 0.20, size = 29, normalized size = 1.12

$$-\frac{a\sqrt{cx^2}}{2x^3}-\frac{b\sqrt{cx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] -a*sqrt(c*x**2)/(2*x**3) - b*sqrt(c*x**2)/x**2
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.96

$$\frac{\sqrt{c}\,\left(-a\mathrm{sign}\left(x\right)-2bx\mathrm{sign}\left(x\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(2*b*x*sgn(x) + a*sgn(x))*sqrt(c)/x^2
```

Mupad [B]
time = 0.14, size = 28, normalized size = 1.08

$$-\frac{a\,\sqrt{c}\,x^2+2\,b\,\sqrt{c}\,x^3}{2\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^4,x)
```

```
[Out] -(a*c^(1/2)*x^2 + 2*b*c^(1/2)*x^3)/(2*x*(x^2)^(3/2))
```

3.764 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

```
[Out] 1/7*a*c*x^6*(c*x^2)^(1/2)+1/8*b*c*x^7*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^6*Sqrt[c*x^2])/7 + (b*c*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^6+bx^7\right)\,dx}{x}\\
&= \frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

Mathics [A]
time = 1.86, size = 19, normalized size = 0.51

$$x^4\left(\frac{a}{7}+\frac{bx}{8}\right)\left(cx^2\right)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]')
[Out] x ^ 4 (a / 7 + b x / 8) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.57

method	result	size
gosper	$\frac{x^4(7bx+8a)(c\,x^2)^{\frac{3}{2}}}{56}$	21
default	$\frac{x^4(7bx+8a)(c\,x^2)^{\frac{3}{2}}}{56}$	21
risch	$\frac{ac\,x^6\sqrt{c\,x^2}}{7}+\frac{bc\,x^7\sqrt{c\,x^2}}{8}$	30
trager	$\frac{c(7b\,x^7+8a\,x^6+7b\,x^6+8a\,x^5+7b\,x^5+8a\,x^4+7b\,x^4+8a\,x^3+7b\,x^3+8a\,x^2+7x^2b+8ax+7bx+8a+7b)(-1+x)\sqrt{c\,x^2}}{56x}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/56*x^4*(7*b*x+8*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.89

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^3}{8\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b*x^3/c + 1/7*(c*x^2)^(5/2)*a*x^2/c
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.65

$$\frac{1}{56}\left(7\,bcx^7+8\,acx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/56*(7*b*c*x^7 + 8*a*c*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.26, size = 29, normalized size = 0.78

$$\frac{ax^4\left(cx^2\right)^{\frac{3}{2}}}{7}+\frac{bx^5\left(cx^2\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*x**4*(c*x**2)**(3/2)/7 + b*x**5*(c*x**2)**(3/2)/8
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.70

$$\sqrt{c}\,c\left(\frac{1}{7}ax^7\mathrm{sign}\left(x\right)+\frac{1}{8}bx^8\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\,{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x), x)
```

3.765 $\int x^2 (cx^2)^{3/2} (a+bx) \, dx$

Optimal. Leaf size=37

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

```
[Out] 1/6*a*c*x^5*(c*x^2)^(1/2)+1/7*b*c*x^6*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^5*Sqrt[c*x^2])/6 + (b*c*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(cx^2\right)^{3/2} (a+bx) \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x}\\
&= \frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

Mathics [A]
time = 1.83, size = 19, normalized size = 0.51

$$x^3\left(\frac{a}{6}+\frac{bx}{7}\right)\left(cx^2\right)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]')
[Out] x ^ 3 (a / 6 + b x / 7) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.57

method	result	size
gosper	$\frac{x^{3}\left(6bx+7a\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
default	$\frac{x^{3}\left(6bx+7a\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
risch	$\frac{ac\,x^{5}\sqrt{c\,x^{2}}}{6}+\frac{bc\,x^{6}\sqrt{c\,x^{2}}}{7}$	30
trager	$\frac{c\left(6b\,x^{6}+7a\,x^{5}+6b\,x^{5}+7a\,x^{4}+6b\,x^{4}+7a\,x^{3}+6b\,x^{3}+7a\,x^{2}+6x^{2}b+7ax+6bx+7a+6b\right)\left(-1+x\right)\sqrt{c\,x^{2}}}{42x}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/42*x^3*(6*b*x+7*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.84

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/7*(c*x^2)^(5/2)*b*x^2/c + 1/6*(c*x^2)^(5/2)*a*x/c
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.65

$$\frac{1}{42}\left(6\,bcx^6+7\,acx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/42*(6*b*c*x^6 + 7*a*c*x^5)*sqrt(c*x^2)
```

Sympy [A]
time = 0.22, size = 29, normalized size = 0.78

$$\frac{ax^3\left(cx^2\right)^{\frac{3}{2}}}{6}+\frac{bx^4\left(cx^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a),x)
[Out] a*x**3*(c*x**2)**(3/2)/6 + b*x**4*(c*x**2)**(3/2)/7
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.70

$$\sqrt{c}\,c\left(\frac{1}{6}ax^6\mathrm{sign}\left(x\right)+\frac{1}{7}bx^7\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x)
[Out] 1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x),x)
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x), x)
```

3.766 $\int x\left(cx^2\right)^{3/2}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

```
[Out] 1/5*a*c*x^4*(c*x^2)^(1/2)+1/6*b*c*x^5*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {15, 45}

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^4*Sqrt[c*x^2])/5 + (b*c*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^4+bx^5\right)\,dx}{x}\\&=\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Mathics [A]
time = 1.79, size = 19, normalized size = 0.51

$$x^2\left(\frac{a}{5}+\frac{bx}{6}\right)\left(cx^2\right)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]')
[Out] x ^ 2 (a / 5 + b x / 6) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.57

method	result	size
gosper	$\frac{x^{2}(5bx+6a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{30}$	21
default	$\frac{x^{2}(5bx+6a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{30}$	21
risch	$\frac{a c\,x^{4}\sqrt{c\,x^{2}}}{5}+\frac{b c\,x^{5}\sqrt{c\,x^{2}}}{6}$	30
trager	$\frac{c\left(5b\,x^{5}+6a\,x^{4}+5b\,x^{4}+6a\,x^{3}+5b\,x^{3}+6a\,x^{2}+5x^{2}b+6ax+5bx+6a+5b\right)(-1+x)\sqrt{c\,x^{2}}}{30x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/30*x^2*(5*b*x+6*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx}{6\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*b*x/c + 1/5*(c*x^2)^(5/2)*a/c
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.65

$$\frac{1}{30}\left(5\,bcx^5+6\,acx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*c*x^5 + 6*a*c*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.20, size = 29, normalized size = 0.78

$$\frac{ax^2\left(cx^2\right)^{\frac{3}{2}}}{5}+\frac{bx^3\left(cx^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*x**2*(c*x**2)**(3/2)/5 + b*x**3*(c*x**2)**(3/2)/6
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.70

$$\sqrt{c}\,c\left(\frac{1}{5}ax^5\mathrm{sign}\left(x\right)+\frac{1}{6}bx^6\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(3/2)*(a + b*x), x)
```

3.767 $\int (cx^2)^{3/2} (a+bx) \, dx$

Optimal. Leaf size=37

$$\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}$$

```
[Out] 1/4*a*c*x^3*(c*x^2)^(1/2)+1/5*b*c*x^4*(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^3*Sqrt[c*x^2])/4 + (b*c*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (cx^2)^{3/2} (a+bx) \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx) \, dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right) \, dx}{x}\\
&= \frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.59

$$\frac{1}{20}x\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Mathics [A]
time = 1.76, size = 18, normalized size = 0.49

$$\frac{x(5a+4bx)\left(cx^2\right)^{\frac{3}{2}}}{20}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c*x^2)^(3/2)*(a + b*x),x]')
```

```
[Out] x (5 a + 4 b x) (c x ^ 2) ^ (3 / 2) / 20
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.51

method	result	size
gosper	$\frac{x(4bx+5a)\left(c\,x^2\right)^{\frac{3}{2}}}{20}$	19
default	$\frac{x(4bx+5a)\left(c\,x^2\right)^{\frac{3}{2}}}{20}$	19
risch	$\frac{ac\,x^3\sqrt{c\,x^2}}{4}+\frac{bc\,x^4\sqrt{c\,x^2}}{5}$	30
trager	$\frac{c\left(4b\,x^4+5a\,x^3+4b\,x^3+5a\,x^2+4x^2b+5ax+4bx+5a+4b\right)(-1+x)\sqrt{c\,x^2}}{20x}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*x*(4*b*x+5*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.68

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}ax+\frac{\left(cx^2\right)^{\frac{5}{2}}b}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*a*x + 1/5*(c*x^2)^(5/2)*b/c
```

Fricas [A]
time = 0.30, size = 24, normalized size = 0.65

$$\frac{1}{20}\left(4\,bcx^4+5\,acx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/20*(4*b*c*x^4 + 5*a*c*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.16, size = 27, normalized size = 0.73

$$\frac{ax\left(cx^2\right)^{\frac{3}{2}}}{4}+\frac{bx^2\left(cx^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a),x)
[Out] a*x*(c*x**2)**(3/2)/4 + b*x**2*(c*x**2)**(3/2)/5
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.70

$$\sqrt{c}\,c\left(\frac{1}{4}ax^4\mathrm{sign}\left(x\right)+\frac{1}{5}bx^5\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x)
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x),x)
[Out] int((c*x^2)^(3/2)*(a + b*x), x)
```

3.768 $\int \frac{(cx^2)^{3/2}(a+bx)}{x} dx$

Optimal. Leaf size=37

$$\frac{1}{3}acx^2\sqrt{cx^2} + \frac{1}{4}bcx^3\sqrt{cx^2}$$

```
[Out] 1/3*a*c*x^2*(c*x^2)^(1/2)+1/4*b*c*x^3*(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{3}acx^2\sqrt{cx^2} + \frac{1}{4}bcx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (a*c*x^2*Sqrt[c*x^2])/3 + (b*c*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int (ax^2+bx^3)\,dx}{x}\\ &= \frac{1}{3}acx^2\sqrt{cx^2} + \frac{1}{4}bcx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.68

$$\frac{1}{12}cx^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]
[Out] (c*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Mathics [A]
time = 1.75, size = 16, normalized size = 0.43

$$\left(\frac{a}{3}+\frac{bx}{4}\right)(cx^2)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]')
[Out] (a / 3 + b x / 4) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.49

method	result	size
gosper	$\frac{(3bx+4a)(c\,x^2)^{\frac{3}{2}}}{12}$	18
default	$\frac{(3bx+4a)(c\,x^2)^{\frac{3}{2}}}{12}$	18
risch	$\frac{ac\,x^2\sqrt{c\,x^2}}{3}+\frac{bc\,x^3\sqrt{c\,x^2}}{4}$	30
trager	$\frac{c(3b\,x^3+4a\,x^2+3x^2b+4ax+3bx+4a+3b)(-1+x)\sqrt{c\,x^2}}{12x}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/12*(3*b*x+4*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.59

$$\frac{1}{4}\,(cx^2)^{\frac{3}{2}}\,bx+\frac{1}{3}\,(cx^2)^{\frac{3}{2}}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*b*x + 1/3*(c*x^2)^(3/2)*a
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.65

$$\frac{1}{12}\left(3\,bcx^3+4\,acx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c*x^3 + 4*a*c*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.17, size = 24, normalized size = 0.65

$$\frac{a\left(cx^2\right)^{\frac{3}{2}}}{3}+\frac{bx\left(cx^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x,x)
```

```
[Out] a*(c*x**2)**(3/2)/3 + b*x*(c*x**2)**(3/2)/4
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.70

$$\sqrt{c}\,c\left(\frac{1}{3}ax^3\mathrm{sign}\left(x\right)+\frac{1}{4}bx^4\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x)
```

```
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x, x)
```

3.769 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} \, dx$

Optimal. Leaf size=35

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

```
[Out] 1/2*a*c*x*(c*x^2)^(1/2)+1/3*b*c*x^2*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c*x*Sqrt[c*x^2])/2 + (b*c*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx) \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax+bx^2\right) \, dx}{x} \\
&= \frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.66

$$\frac{1}{6}cx\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Mathics [A]
time = 1.76, size = 19, normalized size = 0.54

$$\frac{\left(\frac{a}{2}+\frac{bx}{3}\right)\left(cx^2\right)^{\frac{3}{2}}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]')
```

```
[Out] (a / 2 + b x / 3) (c x ^ 2) ^ (3 / 2) / x
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{3}{2}}}{6x}$	21
default	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{3}{2}}}{6x}$	21
risch	$\frac{acx\sqrt{c\,x^2}}{2}+\frac{bc\,x^2\sqrt{c\,x^2}}{3}$	28
trager	$\frac{c\left(2x^2b+3ax+2bx+3a+2b\right)(-1+x)\sqrt{c\,x^2}}{6x}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/x*(2*b*x+3*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bcx^2+3\,acx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c*x^2 + 3*a*c*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.17, size = 24, normalized size = 0.69

$$\frac{a\left(cx^2\right)^{\frac{3}{2}}}{2x}+\frac{b\left(cx^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**2,x)
```

```
[Out] a*(c*x**2)**(3/2)/(2*x) + b*(c*x**2)**(3/2)/3
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.74

$$\sqrt{c}\,c\left(\frac{1}{2}ax^2\mathrm{sign}\left(x\right)+\frac{1}{3}bx^3\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x)
```

```
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(3/2)
```

Mupad [B]
time = 0.27, size = 20, normalized size = 0.57

$$\frac{c^{3/2}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,\left|x\right|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^2,x)
```

```
[Out] (c^(3/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

3.770 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx$

Optimal. Leaf size=29

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

```
[Out] a*c*(c*x^2)^(1/2)+1/2*b*c*x*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] a*c*Sqrt[c*x^2] + (b*c*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)\,dx}{x} \\ &= ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.72

$$\frac{1}{2}c\sqrt{cx^2}\,(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(2*a + b*x))/2
```

Mathics [A]
time = 1.78, size = 17, normalized size = 0.59

$$\frac{\left(a+\frac{bx}{2}\right)\left(cx^2\right)^{\frac{3}{2}}}{x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]')
```

```
[Out] (a + b x / 2) (c x ^ 2) ^ (3 / 2) / x ^ 2
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.69

method	result	size
gosper	$\frac{(bx+2a)\left(c\,x^2\right)^{\frac{3}{2}}}{2x^2}$	20
default	$\frac{(bx+2a)\left(c\,x^2\right)^{\frac{3}{2}}}{2x^2}$	20
risch	$ac\sqrt{c\,x^2}+\frac{bcx\sqrt{c\,x^2}}{2}$	24
trager	$\frac{c(bx+2a+b)(-1+x)\sqrt{c\,x^2}}{2x}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^2*(b*x+2*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 18, normalized size = 0.62

$$\frac{1}{2}\left(bcx+2\,ac\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x + 2*a*c)*sqrt(c*x^2)
```

Sympy [A]
time = 0.24, size = 26, normalized size = 0.90

$$\frac{a\left(cx^{2}\right)^{\frac{3}{2}}}{x^{2}}+\frac{b\left(cx^{2}\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**3,x)
```

```
[Out] a*(c*x**2)**(3/2)/x**2 + b*(c*x**2)**(3/2)/(2*x)
```

Giac [A]
time = 0.00, size = 20, normalized size = 0.69

$$\sqrt{c}\,c\left(\frac{1}{2}bx^{2}+ax\right)\operatorname{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)*sgn(x)
```

Mupad [B]
time = 0.22, size = 14, normalized size = 0.48

$$\frac{c^{3/2}\,\left|x\right|\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^3,x)
```

```
[Out] (c^(3/2)*abs(x)*(2*a + b*x))/2
```

3.771 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^4} \, dx$

Optimal. Leaf size=30

$$bc\sqrt{cx^2} + \frac{ac\sqrt{cx^2}\log(x)}{x}$$

```
[Out] b*c*(c*x^2)^(1/2)+a*c*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{ac\sqrt{cx^2}\log(x)}{x} + bc\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] b*c*Sqrt[c*x^2] + (a*c*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^4} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{a+bx}{x} \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(b+\frac{a}{x}\right) \, dx}{x} \\
&= bc\sqrt{cx^2} + \frac{ac\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.70

$$\frac{(cx^2)^{3/2}(bx + a\log(x))}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
[Out] ((c*x^2)^(3/2)*(b*x + a*Log[x]))/x^3
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.67

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(bx+a\ln(x))}{x^3}$	20
risch	$bc\sqrt{c\,x^2}+\frac{ac\ln(x)\sqrt{c\,x^2}}{x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(3/2)/x^3*(b*x+a*ln(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.70

$$\frac{(bcx + ac\log(x))\sqrt{cx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c*log(x))*sqrt(c*x^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)/x**4, x)
```

Giac [A]
time = 0.00, size = 20, normalized size = 0.67

$$\sqrt{c}\,c\,(a\mathrm{sign}\,(x)\ln|x| + bx\mathrm{sign}\,(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x)
```

```
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,x^2)^{3/2}\,(a+b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^4,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x^4, x)
```

3.772 $\int x^3 \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

```
[Out] 1/9*a*c^2*x^8*(c*x^2)^(1/2)+1/10*b*c^2*x^9*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^8*Sqrt[c*x^2])/9 + (b*c^2*x^9*Sqrt[c*x^2])/10
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(cx^2\right)^{5/2} (a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^8(a+bx)\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^8+bx^9\right)\,dx}{x}\\
&= \frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

Mathics [A]
time = 2.00, size = 19, normalized size = 0.46

$$x^4\left(\frac{a}{9}+\frac{bx}{10}\right)\left(cx^2\right)^{\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]')
[Out] x ^ 4 (a / 9 + b x / 10) (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result
gosper	$\frac{x^{4}(9bx+10a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{90}$
default	$\frac{x^{4}(9bx+10a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{90}$
risch	$\frac{a\,c^{2}x^{8}\sqrt{c\,x^{2}}}{9}+\frac{b\,c^{2}x^{9}\sqrt{c\,x^{2}}}{10}$
trager	$\frac{c^{2}\left(9b\,x^{9}+10a\,x^{8}+9b\,x^{8}+10a\,x^{7}+9b\,x^{7}+10a\,x^{6}+9b\,x^{6}+10a\,x^{5}+9b\,x^{5}+10a\,x^{4}+9b\,x^{4}+10a\,x^{3}+9b\,x^{3}+10a\,x^{2}+9x^{2}b+10ax+9bx+10a+\cdots\right.}{90x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/90*x^4*(9*b*x+10*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.80

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^3}{10\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}ax^2}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/10*(c*x^2)^(7/2)*b*x^3/c + 1/9*(c*x^2)^(7/2)*a*x^2/c
```

Fricas [A]
time = 0.31, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^9+10\,ac^2x^8\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/90*(9*b*c^2*x^9 + 10*a*c^2*x^8)*sqrt(c*x^2)
```

Sympy [A]
time = 0.47, size = 29, normalized size = 0.71

$$\frac{ax^4\left(cx^2\right)^{\frac{5}{2}}}{9}+\frac{bx^5\left(cx^2\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*x**4*(c*x**2)**(5/2)/9 + b*x**5*(c*x**2)**(5/2)/10
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\left(\frac{1}{9}ac^2x^9\mathrm{sign}\left(x\right)+\frac{1}{10}bc^2x^{10}\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/90*(9*b*c^2*x^10*sgn(x) + 10*a*c^2*x^9*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,{\left(c\,x^2\right)}^{5/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(5/2)*(a + b*x), x)
```

3.773 $\int x^2 (cx^2)^{5/2} (a+bx) \, dx$

Optimal. Leaf size=41

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

```
[Out] 1/8*a*c^2*x^7*(c*x^2)^(1/2)+1/9*b*c^2*x^8*(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^7*Sqrt[c*x^2])/8 + (b*c^2*x^8*Sqrt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 (cx^2)^{5/2} (a+bx) \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^7(a+bx) \, dx}{x} \\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^7+bx^8\right) \, dx}{x} \\ &= \frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

Mathics [A]
time = 1.96, size = 19, normalized size = 0.46

$$x^3\left(\frac{a}{8}+\frac{bx}{9}\right)\left(cx^2\right)^{\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]')
[Out] x ^ 3 (a / 8 + b x / 9) (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result
gosper	$\frac{x^3(8bx+9a)(c\,x^2)^{\frac{5}{2}}}{72}$
default	$\frac{x^3(8bx+9a)(c\,x^2)^{\frac{5}{2}}}{72}$
risch	$\frac{a\,c^2x^7\sqrt{c\,x^2}}{8}+\frac{b\,c^2x^8\sqrt{c\,x^2}}{9}$
trager	$\frac{c^2\left(8b\,x^8+9a\,x^7+8b\,x^7+9a\,x^6+8b\,x^6+9a\,x^5+8b\,x^5+9a\,x^4+8b\,x^4+9a\,x^3+8b\,x^3+9a\,x^2+8x^2b+9ax+8bx+9a+8b\right)(-1+x)\sqrt{c\,x^2}}{72x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/72*x^3*(8*b*x+9*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.25, size = 31, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^2}{9\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}ax}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/9*(c*x^2)^(7/2)*b*x^2/c + 1/8*(c*x^2)^(7/2)*a*x/c
```

Fricas [A]
time = 0.30, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^8+9\,ac^2x^7\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/72*(8*b*c^2*x^8 + 9*a*c^2*x^7)*sqrt(c*x^2)
```

Sympy [A]
time = 0.41, size = 29, normalized size = 0.71

$$\frac{ax^3\left(cx^2\right)^{\frac{5}{2}}}{8}+\frac{bx^4\left(cx^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(5/2)*(b*x+a),x)
[Out] a*x**3*(c*x**2)**(5/2)/8 + b*x**4*(c*x**2)**(5/2)/9
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\left(\frac{1}{8}ac^2x^8\mathrm{sign}\left(x\right)+\frac{1}{9}bc^2x^9\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/72*(8*b*c^2*x^9*sgn(x) + 9*a*c^2*x^8*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,{\left(c\,x^2\right)}^{5/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(a + b*x),x)
[Out] int(x^2*(c*x^2)^(5/2)*(a + b*x), x)
```

3.774 $\int x\left(cx^2\right)^{5/2}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

```
[Out] 1/7*a*c^2*x^6*(c*x^2)^(1/2)+1/8*b*c^2*x^7*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {15, 45}

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^6*Sqrt[c*x^2])/7 + (b*c^2*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)\,dx}{x}\\&=\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

Mathics [A]
time = 1.92, size = 19, normalized size = 0.46

$$x^2\left(\frac{a}{7}+\frac{bx}{8}\right)\left(cx^2\right)^{\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]')
[Out] x ^ 2 (a / 7 + b x / 8) (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$\frac{x^2(7bx+8a)\left(c\,x^2\right)^{\frac{5}{2}}}{56}$	21
default	$\frac{x^2(7bx+8a)\left(c\,x^2\right)^{\frac{5}{2}}}{56}$	21
risch	$\frac{a\,c^2x^6\sqrt{c\,x^2}}{7}+\frac{b\,c^2x^7\sqrt{c\,x^2}}{8}$	34
trager	$\frac{c^2\left(7b\,x^7+8a\,x^6+7b\,x^6+8a\,x^5+7b\,x^5+8a\,x^4+7b\,x^4+8a\,x^3+7b\,x^3+8a\,x^2+7x^2b+8ax+7bx+8a+7b\right)(-1+x)\sqrt{c\,x^2}}{56x}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/56*x^2*(7*b*x+8*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.25, size = 28, normalized size = 0.68

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx}{8\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}a}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(7/2)*b*x/c + 1/7*(c*x^2)^(7/2)*a/c
```

Fricas [A]
time = 0.30, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^{2}x^{7}+8\,ac^{2}x^{6}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/56*(7*b*c^2*x^7 + 8*a*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.35, size = 29, normalized size = 0.71

$$\frac{ax^{2}\left(cx^{2}\right)^{\frac{5}{2}}}{7}+\frac{bx^{3}\left(cx^{2}\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*x**2*(c*x**2)**(5/2)/7 + b*x**3*(c*x**2)**(5/2)/8
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\left(\frac{1}{7}ac^{2}x^{7}\mathrm{sign}\left(x\right)+\frac{1}{8}bc^{2}x^{8}\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/56*(7*b*c^2*x^8*sgn(x) + 8*a*c^2*x^7*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,{\left(c\,x^{2}\right)}^{5/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x), x)
```

3.775 $\int (cx^2)^{5/2} (a+bx)\, dx$

Optimal. Leaf size=41

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

```
[Out] 1/6*a*c^2*x^5*(c*x^2)^(1/2)+1/7*b*c^2*x^6*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^5*Sqrt[c*x^2])/6 + (b*c^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (cx^2)^{5/2} (a+bx)\, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)\, dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\, dx}{x}\\
&= \frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

Mathics [A]
time = 1.86, size = 18, normalized size = 0.44

$$\frac{x\,(7a+6bx)\,(cx^2)^{\frac{5}{2}}}{42}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c*x^2)^(5/2)*(a + b*x),x]')
[Out] x (7 a + 6 b x) (c x ^ 2) ^ (5 / 2) / 42
```

Maple [A]
time = 0.03, size = 19, normalized size = 0.46

method	result	size
gosper	$\frac{x(6bx+7a)(c\,x^2)^{\frac{5}{2}}}{42}$	19
default	$\frac{x(6bx+7a)(c\,x^2)^{\frac{5}{2}}}{42}$	19
risch	$\frac{a\,c^2x^5\sqrt{c\,x^2}}{6}+\frac{b\,c^2x^6\sqrt{c\,x^2}}{7}$	34
trager	$\frac{c^2\left(6b\,x^6+7a\,x^5+6b\,x^5+7a\,x^4+6b\,x^4+7a\,x^3+6b\,x^3+7a\,x^2+6x^2b+7ax+6bx+7a+6b\right)(-1+x)\sqrt{c\,x^2}}{42x}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/42*x*(6*b*x+7*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.61

$$\frac{1}{6}\,\left(cx^2\right)^{\frac{5}{2}}ax+\frac{\left(cx^2\right)^{\frac{7}{2}}b}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*a*x + 1/7*(c*x^2)^(7/2)*b/c
```

Fricas [A]
time = 0.31, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^2x^6+7\,ac^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/42*(6*b*c^2*x^6 + 7*a*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A]
time = 0.30, size = 27, normalized size = 0.66

$$\frac{ax\left(cx^2\right)^{\frac{5}{2}}}{6}+\frac{bx^2\left(cx^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a),x)
[Out] a*x*(c*x**2)**(5/2)/6 + b*x**2*(c*x**2)**(5/2)/7
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\left(\frac{1}{6}ac^2x^6\mathrm{sign}\left(x\right)+\frac{1}{7}bc^2x^7\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/42*(6*b*c^2*x^7*sgn(x) + 7*a*c^2*x^6*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2\right)}^{5/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x),x)
[Out] int((c*x^2)^(5/2)*(a + b*x), x)
```

3.776 $\int \frac{(cx^2)^{5/2}(a+bx)}{x} dx$

Optimal. Leaf size=41

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

```
[Out] 1/5*a*c^2*x^4*(c*x^2)^(1/2)+1/6*b*c^2*x^5*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (a*c^2*x^4*Sqrt[c*x^2])/5 + (b*c^2*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)}{x} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^4+bx^5\right)\,dx}{x}\\
&= \frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.61

$$\frac{1}{30}cx^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]
[Out] (c*x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Mathics [A]
time = 1.86, size = 16, normalized size = 0.39

$$\left(\frac{a}{5}+\frac{bx}{6}\right)\left(cx^2\right)^{\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]')
[Out] (a / 5 + b x / 6) (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.44

method	result	size
gosper	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
default	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
risch	$\frac{a\,c^2x^4\sqrt{c\,x^2}}{5}+\frac{b\,c^2x^5\sqrt{c\,x^2}}{6}$	34
trager	$\frac{c^2\left(5b\,x^5+6a\,x^4+5b\,x^4+6a\,x^3+5b\,x^3+6a\,x^2+5x^2b+6ax+5bx+6a+5b\right)(-1+x)\sqrt{c\,x^2}}{30x}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/30*(5*b*x+6*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.54

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}bx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*b*x + 1/5*(c*x^2)^(5/2)*a
```

Fricas [A]
time = 0.62, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^5+6\,ac^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*c^2*x^5 + 6*a*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.30, size = 24, normalized size = 0.59

$$\frac{a\left(cx^2\right)^{\frac{5}{2}}}{5}+\frac{bx\left(cx^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x,x)
```

```
[Out] a*(c*x**2)**(5/2)/5 + b*x*(c*x**2)**(5/2)/6
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\left(\frac{1}{5}ac^2x^5\mathrm{sign}\left(x\right)+\frac{1}{6}bc^2x^6\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x)
```

```
[Out] 1/30*(5*b*c^2*x^6*sgn(x) + 6*a*c^2*x^5*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x))/x, x)
```

3.777 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^2} dx$

Optimal. Leaf size=41

$$\frac{1}{4}ac^2x^3\sqrt{cx^2} + \frac{1}{5}bc^2x^4\sqrt{cx^2}$$

```
[Out] 1/4*a*c^2*x^3*(c*x^2)^(1/2)+1/5*b*c^2*x^4*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{4}ac^2x^3\sqrt{cx^2} + \frac{1}{5}bc^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c^2*x^3*Sqrt[c*x^2])/4 + (b*c^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^{5/2}(a+bx)}{x^2} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)\,dx}{x} \\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right)\,dx}{x} \\ &= \frac{1}{4}ac^2x^3\sqrt{cx^2} + \frac{1}{5}bc^2x^4\sqrt{cx^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.56

$$\frac{1}{20}cx\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Mathics [A]
time = 1.86, size = 19, normalized size = 0.46

$$\frac{\left(\frac{a}{4}+\frac{bx}{5}\right)\left(cx^2\right)^{\frac{5}{2}}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]')
```

```
[Out] (a / 4 + b x / 5) (c x ^ 2) ^ (5 / 2) / x
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.51

method	result	size
gosper	$\frac{(4bx+5a)\left(c\,x^2\right)^{\frac{5}{2}}}{20x}$	21
default	$\frac{(4bx+5a)\left(c\,x^2\right)^{\frac{5}{2}}}{20x}$	21
risch	$\frac{a\,c^2x^3\sqrt{c\,x^2}}{4}+\frac{b\,c^2x^4\sqrt{c\,x^2}}{5}$	34
trager	$\frac{c^2\left(4b\,x^4+5a\,x^3+4b\,x^3+5a\,x^2+4x^2b+5ax+4bx+5a+4b\right)(-1+x)\sqrt{c\,x^2}}{20x}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20/x*(4*b*x+5*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.59

$$\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}b+\frac{\left(cx^2\right)^{\frac{5}{2}}a}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2)^(5/2)*b + 1/4*(c*x^2)^(5/2)*a/x
```

Fricas [A]
time = 0.29, size = 28, normalized size = 0.68

$$\frac{1}{20}\left(4\,bc^2x^4+5\,ac^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^2*x^4 + 5*a*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.32, size = 24, normalized size = 0.59

$$\frac{a\left(cx^2\right)^{\frac{5}{2}}}{4x}+\frac{b\left(cx^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**2,x)
```

```
[Out] a*(c*x**2)**(5/2)/(4*x) + b*(c*x**2)**(5/2)/5
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\,\left(\frac{1}{4}ac^2x^4\mathrm{sign}\left(x\right)+\frac{1}{5}bc^2x^5\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x)
```

```
[Out] 1/20*(4*b*c^2*x^5*sgn(x) + 5*a*c^2*x^4*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.28, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\,\left(4\,b\,\sqrt{x^{10}}+5\,a\,x^3\,\sqrt{x^2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^2,x)
```

```
[Out] (c^(5/2)*(4*b*(x^10)^(1/2) + 5*a*x^3*(x^2)^(1/2)))/20
```

3.778 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^3} dx$

Optimal. Leaf size=41

$$\frac{1}{3}ac^2x^2\sqrt{cx^2} + \frac{1}{4}bc^2x^3\sqrt{cx^2}$$

```
[Out] 1/3*a*c^2*x^2*(c*x^2)^(1/2)+1/4*b*c^2*x^3*(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{3}ac^2x^2\sqrt{cx^2} + \frac{1}{4}bc^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (a*c^2*x^2*Sqrt[c*x^2])/3 + (b*c^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)}{x^3} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int (ax^2+bx^3)\,dx}{x} \\
&= \frac{1}{3}ac^2x^2\sqrt{cx^2} + \frac{1}{4}bc^2x^3\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.66

$$\frac{1}{12}c^2x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
[Out] (c^2*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Mathics [A]
time = 1.90, size = 19, normalized size = 0.46

$$\frac{\left(\frac{a}{3}+\frac{bx}{4}\right)(cx^2)^{\frac{5}{2}}}{x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]')
[Out] (a / 3 + b x / 4) (c x ^ 2) ^ (5 / 2) / x ^ 2
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.51

method	result	size
gosper	$\frac{(3bx+4a)(c\,x^2)^{\frac{5}{2}}}{12x^2}$	21
default	$\frac{(3bx+4a)(c\,x^2)^{\frac{5}{2}}}{12x^2}$	21
risch	$\frac{a\,c^2x^2\sqrt{c\,x^2}}{3}+\frac{b\,c^2x^3\sqrt{c\,x^2}}{4}$	34
trager	$\frac{c^2(3b\,x^3+4a\,x^2+3x^2b+4ax+3bx+4a+3b)(-1+x)\sqrt{c\,x^2}}{12x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/12/x^2*(3*b*x+4*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^3+4\,ac^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^2*x^3 + 4*a*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.39, size = 27, normalized size = 0.66

$$\frac{a\left(cx^2\right)^{\frac{5}{2}}}{3x^2}+\frac{b\left(cx^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**3,x)
```

```
[Out] a*(c*x**2)**(5/2)/(3*x**2) + b*(c*x**2)**(5/2)/(4*x)
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.76

$$\sqrt{c}\,\left(\frac{1}{3}ac^2x^3\mathrm{sign}\left(x\right)+\frac{1}{4}bc^2x^4\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/12*(3*b*c^2*x^4*sgn(x) + 4*a*c^2*x^3*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.27, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\,\left(4\,a\,\sqrt{x^6}+3\,b\,x^3\,\sqrt{x^2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^3,x)
```

```
[Out] (c^(5/2)*(4*a*(x^6)^(1/2) + 3*b*x^3*(x^2)^(1/2)))/12
```

3.779 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx$

Optimal. Leaf size=39

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

```
[Out] 1/2*a*c^2*x*(c*x^2)^(1/2)+1/3*b*c^2*x^2*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (a*c^2*x*Sqrt[c*x^2])/2 + (b*c^2*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.64

$$\frac{1}{6}c^2x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
[Out] (c^2*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Mathics [A]
time = 1.90, size = 19, normalized size = 0.49

$$\frac{\left(\frac{a}{2}+\frac{bx}{3}\right)\left(cx^2\right)^{\frac{5}{2}}}{x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]')
[Out] (a / 2 + b x / 3) (c x ^ 2) ^ (5 / 2) / x ^ 3
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.54

method	result	size
gosper	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
default	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
risch	$\frac{a\,c^2x\sqrt{c\,x^2}}{2}+\frac{b\,c^2x^2\sqrt{c\,x^2}}{3}$	32
trager	$\frac{c^2\left(2x^2b+3ax+2bx+3a+2b\right)(-1+x)\sqrt{c\,x^2}}{6x}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/6/x^3*(2*b*x+3*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 26, normalized size = 0.67

$$\frac{1}{6}\left(2\,bc^2x^2+3\,ac^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^2*x^2 + 3*a*c^2*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.38, size = 29, normalized size = 0.74

$$\frac{a\left(cx^2\right)^{\frac{5}{2}}}{2x^3}+\frac{b\left(cx^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**4,x)
```

```
[Out] a*(c*x**2)**(5/2)/(2*x**3) + b*(c*x**2)**(5/2)/(3*x**2)
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.79

$$\sqrt{c}\left(\frac{1}{2}ac^2x^2\operatorname{sign}(x)+\frac{1}{3}bc^2x^3\operatorname{sign}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x)
```

```
[Out] 1/6*(2*b*c^2*x^3*sgn(x) + 3*a*c^2*x^2*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.26, size = 20, normalized size = 0.51

$$\frac{c^{5/2}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,\left|x\right|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^4,x)
```

```
[Out] (c^(5/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

3.780 $\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

```
[Out] 1/3*a*x^4/(c*x^2)^(1/2)+1/4*b*x^5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^4)/(3*Sqrt[c*x^2]) + (b*x^5)/(4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax^2+bx^3\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^4(4a+3bx)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(4*a + 3*b*x))/(12*Sqrt[c*x^2])
```

Mathics [A]
time = 1.77, size = 19, normalized size = 0.54

$$\frac{x^4\left(\frac{a}{3}+\frac{bx}{4}\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]')
```

```
[Out] x ^ 4 (a / 3 + b x / 4) / Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^{4}(3bx+4a)}{12\sqrt{c\,x^{2}}}$	21
default	$\frac{x^{4}(3bx+4a)}{12\sqrt{c\,x^{2}}}$	21
risch	$\frac{a\,x^{4}}{3\sqrt{c\,x^{2}}}+\frac{b\,x^{5}}{4\sqrt{c\,x^{2}}}$	28
trager	$\frac{(3b\,x^{3}+4a\,x^{2}+3x^{2}b+4ax+3bx+4a+3b)(-1+x)\sqrt{c\,x^{2}}}{12cx}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^4*(3*b*x+4*a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.94

$$\frac{\sqrt{cx^2}\,bx^3}{4\,c}+\frac{\sqrt{cx^2}\,ax^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b*x^3/c + 1/3*sqrt(c*x^2)*a*x^2/c
```

Fricas [A]
time = 0.31, size = 25, normalized size = 0.71

$$\frac{(3\,bx^3 + 4\,ax^2)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.25, size = 29, normalized size = 0.83

$$\frac{ax^4}{3\sqrt{cx^2}} + \frac{bx^5}{4\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**4/(3*sqrt(c*x**2)) + b*x**5/(4*sqrt(c*x**2))
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.71

$$\frac{\frac{1}{4}bx^4 + \frac{1}{3}ax^3}{\sqrt{c}\,\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/12*(3*b*x^4 + 4*a*x^3)/(sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\ (a+b\,x)}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(1/2), x)
```

3.781 $\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

```
[Out] 1/2*a*x^3/(c*x^2)^(1/2)+1/3*b*x^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{15, 45\}$

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^3)/(2*Sqrt[c*x^2]) + (b*x^4)/(3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax+bx^2\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^3(3a+2bx)}{6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x^3*(3*a + 2*b*x))/(6*Sqrt[c*x^2])
```

Mathics [A]
time = 1.74, size = 19, normalized size = 0.54

$$\frac{x^3\left(\frac{a}{2}+\frac{bx}{3}\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]')
[Out] x ^ 3 (a / 2 + b x / 3) / Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^3(2bx+3a)}{6\sqrt{c\,x^2}}$	21
default	$\frac{x^3(2bx+3a)}{6\sqrt{c\,x^2}}$	21
risch	$\frac{a\,x^3}{2\sqrt{c\,x^2}}+\frac{b\,x^4}{3\sqrt{c\,x^2}}$	28
trager	$\frac{(2x^2b+3ax+2bx+3a+2b)(-1+x)\sqrt{c\,x^2}}{6cx}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*x^3*(2*b*x+3*a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.74

$$\frac{\sqrt{cx^2}\,bx^2}{3\,c}+\frac{ax^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*b*x^2/c + 1/2*a*x^2/sqrt(c)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.66

$$\frac{(2\,bx^2 + 3\,ax)\sqrt{cx^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.23, size = 29, normalized size = 0.83

$$\frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**3/(2*sqrt(c*x**2)) + b*x**4/(3*sqrt(c*x**2))
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.71

$$\frac{\frac{1}{3}bx^3 + \frac{1}{2}ax^2}{\sqrt{c}\,\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/6*(2*b*x^3 + 3*a*x^2)/(sqrt(c)*sgn(x))
```

Mupad [B]
time = 0.25, size = 23, normalized size = 0.66

$$\frac{2\,b\,\sqrt{x^6} + 3\,a\,x\,\sqrt{x^2}}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (2*b*(x^6)^(1/2) + 3*a*x*(x^2)^(1/2))/(6*c^(1/2))
```

3.782 $\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=32

$$\frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}$$

```
[Out] a*x^2/(c*x^2)^(1/2)+1/2*b*x^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = $\{15\}$

$$\frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^2)/Sqrt[c*x^2] + (b*x^3)/(2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^2}{\sqrt{cx^2}}+\frac{bx^3}{2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.72

$$\frac{x^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Mathics [A]
time = 1.75, size = 17, normalized size = 0.53

$$\frac{x^2\left(a+\frac{bx}{2}\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^1*(a + b*x))/Sqrt[c*x^2],x]')
[Out] x ^ 2 (a + b x / 2) / Sqrt[c x ^ 2]
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.62

method	result	size
gosper	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
default	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
trager	$\frac{(bx+2a+b)(-1+x)\sqrt{c\,x^{2}}}{2cx}$	27
risch	$\frac{a\,x^{2}}{\sqrt{c\,x^{2}}}+\frac{b\,x^{3}}{2\sqrt{c\,x^{2}}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*(b*x+2*a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.69

$$\frac{bx^2}{2\sqrt{c}}+\frac{\sqrt{cx^2}\,a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*x^2/sqrt(c) + sqrt(c*x^2)*a/c
```

Fricas [A]
time = 0.30, size = 19, normalized size = 0.59

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c
```

Sympy [A]
time = 0.21, size = 27, normalized size = 0.84

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**2/sqrt(c*x**2) + b*x**3/(2*sqrt(c*x**2))
```

Giac [A]
time = 0.00, size = 21, normalized size = 0.66

$$\frac{\frac{1}{2}bx^2 + ax}{\sqrt{c}\,\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/(sqrt(c)*sgn(x))
```

Mupad [B]
time = 0.22, size = 19, normalized size = 0.59

$$\frac{2\,a\,|x| + b\,x\,\sqrt{x^2}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (2*a*abs(x) + b*x*(x^2)^(1/2))/(2*c^(1/2))
```

3.783 $\int \frac{a+bx}{\sqrt{cx^2}} dx$

Optimal. Leaf size=29

$$\frac{bx^2}{\sqrt{cx^2}} + \frac{ax\log(x)}{\sqrt{cx^2}}$$

```
[Out] b*x^2/(c*x^2)^(1/2)+a*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{ax\log(x)}{\sqrt{cx^2}} + \frac{bx^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (b*x^2)/Sqrt[c*x^2] + (a*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{\sqrt{cx^2}} dx &= \frac{x \int \frac{a+bx}{x} dx}{\sqrt{cx^2}} \\
&= \frac{x \int \left(b + \frac{a}{x}\right) dx}{\sqrt{cx^2}} \\
&= \frac{bx^2}{\sqrt{cx^2}} + \frac{ax\log(x)}{\sqrt{cx^2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 19, normalized size = 0.66

$$\frac{x(bx + a\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/Sqrt[c*x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.03, size = 18, normalized size = 0.62

method	result	size
default	$\frac{x(bx+a\ln(x))}{\sqrt{c\,x^2}}$	18
risch	$\frac{b\,x^2}{\sqrt{c\,x^2}}+\frac{ax\ln(x)}{\sqrt{c\,x^2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^2)^(1/2)*x*(b*x+a*ln(x))
```

Maxima [**A**]
time = 0.27, size = 20, normalized size = 0.69

$$\frac{a\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x)/sqrt(c) + sqrt(c*x^2)*b/c
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.76

$$\frac{\sqrt{cx^2}\,(bx + a\log(x))}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)/sqrt(c*x**2), x)
```

Giac [A]
time = 0.00, size = 22, normalized size = 0.76

$$\frac{\frac{a\ln|x|}{\mathrm{sign}(x)} + \frac{bx}{\mathrm{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x)
[Out] (b*x/sgn(x) + a*log(abs(x))/sgn(x))/sqrt(c)
```

Mupad [B]
time = 0.51, size = 17, normalized size = 0.59

$$\frac{b\,|x| + a\,\ln(c\,x)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(1/2),x)
[Out] (b*abs(x) + a*log(c*x)*sign(x))/c^(1/2)
```

3.784 $\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}$$

```
[Out] -a/(c*x^2)^(1/2)+b*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a/Sqrt[c*x^2]) + (b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{\sqrt{cx^2}}\\ &= -\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.85

$$\frac{cx^2(-a+bx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*Sqrt[c*x^2]),x]
[Out] (c*x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)/(x^1*Sqrt[c*x^2]),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.67

method	result	size
default	$\frac{bx\ln(x)-a}{\sqrt{c\,x^{2}}}$	18
risch	$-\frac{a}{\sqrt{c\,x^{2}}}+\frac{bx\ln(x)}{\sqrt{c\,x^{2}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (b*x*ln(x)-a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.63

$$\frac{b\log(x)}{\sqrt{c}}-\frac{a}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b*log(x)/sqrt(c) - a/(sqrt(c)*x)
```

Fricas [A]
time = 0.31, size = 23, normalized size = 0.85

$$\frac{\sqrt{cx^2}\,(bx\log{(x)} - a)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)/(x*sqrt(c*x**2)), x)
```

Giac [A]
time = 0.00, size = 24, normalized size = 0.89

$$\frac{-\frac{a}{x\mathrm{sign}(x)} + \frac{b\ln|x|}{\mathrm{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x)
[Out] (b*log(abs(x))/sgn(x) - a/(x*sgn(x)))/sqrt(c)
```

Mupad [B]
time = 1.22, size = 22, normalized size = 0.81

$$-\frac{\frac{a}{\sqrt{x^2}} - b\,\ln{(c\,x)}\,\mathrm{sign}\,(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(1/2)),x)
[Out] -(a/(x^2)^(1/2) - b*log(c*x)*sign(x))/c^(1/2)
```

3.785 $\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

```
[Out] -1/2*(b*x+a)^2/a/x/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -1/2*(a + b*x)^2/(a*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ax\sqrt{cx^2}} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.88

$$\frac{cx(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (c*x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.76, size = 22, normalized size = 0.85

$$\frac{\left(-\frac{a}{2}-bx\right)\sqrt{cx^2}}{cx^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]')
```

```
[Out] (-a / 2 - b x) Sqrt[c x ^ 2] / (c x ^ 3)
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.73

method	result	size
gosper	$-\frac{2bx+a}{2x\sqrt{c\,x^2}}$	19
default	$-\frac{2bx+a}{2x\sqrt{c\,x^2}}$	19
risch	$\frac{-bx-\frac{a}{2}}{x\sqrt{c\,x^2}}$	20
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^2}}{2c\,x^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b*x+a)/x/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.73

$$-\frac{b}{\sqrt{c}\,x}-\frac{a}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c)*x) - 1/2*a/(sqrt(c)*x^2)
```

Fricas [A]
time = 0.30, size = 21, normalized size = 0.81

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c*x^3)
```

Sympy [A]
time = 0.24, size = 24, normalized size = 0.92

$$-\frac{a}{2x\sqrt{cx^2}}-\frac{b}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(1/2),x)
```

```
[Out] -a/(2*x*sqrt(c*x**2)) - b/sqrt(c*x**2)
```

Giac [A]
time = 0.00, size = 24, normalized size = 0.92

$$\frac{-2bx-a}{\sqrt{c}\cdot 2\left(x^2\operatorname{sign}\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/2*(2*b*x + a)/(sqrt(c)*x^2*sgn(x))
```

Mupad [B]
time = 0.16, size = 25, normalized size = 0.96

$$-\frac{2\,b\,x^3+a\,x^2}{2\,\sqrt{c}\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(1/2)*x*(x^2)^(3/2))
```

3.786 $\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

```
[Out] -1/3*a/x^2/(c*x^2)^(1/2)-1/2*b/x/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -1/3*a/(x^2*Sqrt[c*x^2]) - b/(2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)\,dx}{\sqrt{cx^2}}\\
&= -\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.63

$$\frac{c(-2a-3bx)}{6\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
[Out] (c*(-2*a - 3*b*x))/(6*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.79, size = 22, normalized size = 0.63

$$\frac{\left(-\frac{a}{3}-\frac{bx}{2}\right)\sqrt{cx^2}}{cx^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]')
[Out] (-a / 3 - b x / 2) Sqrt[c x ^ 2] / (c x ^ 4)
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^2\sqrt{c\,x^2}}$	20
gosper	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
default	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
trager	$\frac{(-1+x)\left(2a\,x^2+3x^2b+2ax+3bx+2a\right)\sqrt{c\,x^2}}{6c\,x^4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(3*b*x+2*a)/x^2/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.54

$$-\frac{b}{2\,\sqrt{c}\,x^2}-\frac{a}{3\,\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(sqrt(c)*x^2) - 1/3*a/(sqrt(c)*x^3)
```

Fricas [**A**]
time = 0.30, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c*x^4)
```

Sympy [**A**]
time = 0.26, size = 29, normalized size = 0.83

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a/(3*x**2*sqrt(c*x**2)) - b/(2*x*sqrt(c*x**2))
```

Giac [**A**]
time = 0.00, size = 26, normalized size = 0.74

$$\frac{-3bx-2a}{\sqrt{c}\cdot 6\,(x^3\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x)
```

```
[Out] -1/6*(3*b*x + 2*a)/(sqrt(c)*x^3*sgn(x))
```

Mupad [**B**]
time = 0.15, size = 26, normalized size = 0.74

$$-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(1/2)),x)
```

```
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(1/2)*x^4)
```

3.787 $\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

```
[Out] -1/4*a/x^3/(c*x^2)^(1/2)-1/3*b/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -1/4*a/(x^3*Sqrt[c*x^2]) - b/(3*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.69

$$\frac{-3a-4bx}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a - 4*b*x)/(12*x^3*Sqrt[c*x^2])
```

Mathics [A]
time = 1.82, size = 22, normalized size = 0.63

$$\frac{\left(-\frac{a}{4}-\frac{bx}{3}\right)\sqrt{cx^2}}{cx^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]')
```

```
[Out] (-a / 4 - b x / 3) Sqrt[c x ^ 2] / (c x ^ 5)
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^3\sqrt{c\,x^2}}$	20
gosper	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
default	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
trager	$\frac{(-1+x)\left(3a\,x^3+4b\,x^3+3a\,x^2+4x^2b+3ax+4bx+3a\right)\sqrt{c\,x^2}}{12c\,x^5}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(4*b*x+3*a)/x^3/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.54

$$-\frac{b}{3\sqrt{c}\,x^3}-\frac{a}{4\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(sqrt(c)*x^3) - 1/4*a/(sqrt(c)*x^4)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c*x^5)
```

Sympy [A]
time = 0.28, size = 31, normalized size = 0.89

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a/(4*x**3*sqrt(c*x**2)) - b/(3*x**2*sqrt(c*x**2))
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.74

$$\frac{-4bx-3a}{\sqrt{c}\cdot 12\,(x^4\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x)
```

```
[Out] -1/12*(4*b*x + 3*a)/(sqrt(c)*x^4*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.74

$$-\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(1/2)),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(1/2)*x^5)
```

3.788 $\int \frac{x^3(a+bx)}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{ax^2}{c\sqrt{cx^2}} + \frac{bx^3}{2c\sqrt{cx^2}}$$

```
[Out] a*x^2/c/(c*x^2)^(1/2)+1/2*b*x^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {15}

$$\frac{ax^2}{c\sqrt{cx^2}} + \frac{bx^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (a*x^2)/(c*Sqrt[c*x^2]) + (b*x^3)/(2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{(cx^2)^{3/2}} dx &= \frac{x\int (a+bx)\,dx}{c\sqrt{cx^2}} \\ &= \frac{ax^2}{c\sqrt{cx^2}} + \frac{bx^3}{2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.78, size = 17, normalized size = 0.45

$$\frac{x^4\left(a+\frac{bx}{2}\right)}{(cx^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]')
[Out] x ^ 4 (a + b x / 2) / (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.53

method	result	size
gosper	$\frac{x^4(bx+2a)}{2(c\,x^2)^{\frac{3}{2}}}$	20
default	$\frac{x^4(bx+2a)}{2(c\,x^2)^{\frac{3}{2}}}$	20
trager	$\frac{(bx+2a+b)(-1+x)\sqrt{c\,x^2}}{2c^2x}$	27
risch	$\frac{a\,x^2}{c\sqrt{c\,x^2}}+\frac{b\,x^3}{2c\sqrt{c\,x^2}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^4*(b*x+2*a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.32, size = 32, normalized size = 0.84

$$\frac{bx^3}{2\sqrt{cx^2}\,c}+\frac{ax^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*b*x^3/(sqrt(c*x^2)*c) + a*x^2/(sqrt(c*x^2)*c)
```

Fricas [A]
time = 0.30, size = 19, normalized size = 0.50

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c^2
```

Sympy [A]
time = 0.26, size = 27, normalized size = 0.71

$$\frac{ax^4}{(cx^2)^{\frac{3}{2}}} + \frac{bx^5}{2\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] a*x**4/(c*x**2)**(3/2) + b*x**5/(2*(c*x**2)**(3/2))
```

Giac [A]
time = 0.00, size = 23, normalized size = 0.61

$$\frac{\frac{1}{2}bx^2 + ax}{\sqrt{c}\, c\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/(c^(3/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(3/2), x)
```

3.789 $\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{bx^2}{c\sqrt{cx^2}} + \frac{ax\log(x)}{c\sqrt{cx^2}}$$

```
[Out] b*x^2/c/(c*x^2)^(1/2)+a*x*ln(x)/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{ax\log(x)}{c\sqrt{cx^2}} + \frac{bx^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^2)/(c*Sqrt[c*x^2]) + (a*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(b+\frac{a}{x}\right)\,dx}{c\sqrt{cx^2}}\\
&= \frac{bx^2}{c\sqrt{cx^2}} + \frac{ax\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.60

$$\frac{x^3(bx + a\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^3*(b*x + a*Log[x]))/(c*x^2)^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.57

method	result	size
default	$\frac{x^3(bx+a\ln(x))}{(c\,x^2)^{\frac{3}{2}}}$	20
risch	$\frac{b\,x^2}{c\sqrt{c\,x^2}} + \frac{ax\ln(x)}{c\sqrt{c\,x^2}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/(c*x^2)^(3/2)*x^3*(b*x+a*ln(x))
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.66

$$\frac{bx^2}{\sqrt{cx^2}\,c} + \frac{a\log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b*x^2/(sqrt(c*x^2)*c) + a*log(x)/c^(3/2)
```

Fricas [A]
time = 0.32, size = 22, normalized size = 0.63

$$\frac{\sqrt{cx^2}\,(bx + a\log(x))}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,(a + bx)}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(3/2),x)
[Out] Integral(x**2*(a + b*x)/(c*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{\frac{a\ln|x|}{\mathrm{sign}(x)} + \frac{bx}{\mathrm{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x)
[Out] (b*x/sgn(x) + a*log(abs(x))/sgn(x))/c^(3/2)
```

Mupad [B]
time = 0.32, size = 30, normalized size = 0.86

$$\frac{b\,|x|}{c^{3/2}} + \frac{a\,\ln(x + |x|)}{c^{3/2}} - \frac{a\,x}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(3/2),x)
[Out] (b*abs(x))/c^(3/2) + (a*log(x + abs(x)))/c^(3/2) - (a*x)/(c^(3/2)*(x^2)^(1/
2))
```

3.790 $\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}$$

```
[Out] -a/c/(c*x^2)^(1/2)+b*x*ln(x)/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 45}

$$\frac{bx\log(x)}{c\sqrt{cx^2}}-\frac{a}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -(a/(c*Sqrt[c*x^2])) + (b*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{c\sqrt{cx^2}}\\
&= -\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.67

$$\frac{x^2(-a+bx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^1*(a + b*x))/(c*x^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.64

method	result	size
default	$\frac{x^2(bx\ln(x)-a)}{(c\,x^2)^{\frac{3}{2}}}$	21
risch	$-\frac{a}{c\sqrt{c\,x^2}}+\frac{bx\ln(x)}{c\sqrt{c\,x^2}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(b*x*ln(x)-a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.25, size = 21, normalized size = 0.64

$$\frac{b\log(x)}{c^{\frac{3}{2}}}-\frac{a}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*log(x)/c^(3/2) - a/(sqrt(c*x^2)*c)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\,(bx\log(x)-a)}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+bx)}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)/(c*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.79

$$\frac{-\frac{a}{x\mathrm{sign}(x)}+\frac{b\ln|x|}{\mathrm{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] (b*log(abs(x))/sgn(x) - a/(x*sgn(x)))/c^(3/2)
```

Mupad [B]
time = 0.25, size = 28, normalized size = 0.85

$$-\frac{a+b\,x-b\,\ln\left(x+|x|\right)\,\sqrt{x^2}}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] -(a + b*x - b*log(x + abs(x))*(x^2)^(1/2))/(c^(3/2)*(x^2)^(1/2))
```

3.791 $\int \frac{a+bx}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

[Out] -1/2*(b*x+a)^2/a/c/x/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c*x^2)^(3/2),x]

[Out] -1/2*(a + b*x)^2/(a*c*x*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(cx^2)^{3/2}} dx &= \frac{x\int \frac{a+bx}{x^3} dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2acx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.76

$$\frac{x(-a-2bx)}{2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(3/2),x]
[Out] (x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.77, size = 18, normalized size = 0.62

$$\frac{x(-a-2bx)}{2(cx^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(c*x^2)^(3/2),x]')
[Out] x (-a - 2 b x) / (2 (c x ^ 2) ^ (3 / 2))
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.59

method	result	size
gosper	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
default	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
risch	$\frac{-bx-\frac{a}{2}}{cx\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^2}}{2c^2x^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*x*(2*b*x+a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 0.79

$$-\frac{b}{\sqrt{cx^2}\,c}-\frac{a}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -b/(sqrt(c*x^2)*c) - 1/2*a/(c^(3/2)*x^2)
```

Fricas [A]
time = 0.29, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^2*x^3)
```

Sympy [A]
time = 0.24, size = 27, normalized size = 0.93

$$-\frac{ax}{2\,(cx^2)^{\frac{3}{2}}}-\frac{bx^2}{(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(3/2),x)
[Out] -a*x/(2*(c*x**2)**(3/2)) - b*x**2/(c*x**2)**(3/2)
```

Giac [A]
time = 0.00, size = 26, normalized size = 0.90

$$\frac{-2bx-a}{\sqrt{c}\,c\cdot 2\,(x^2\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x)
[Out] -1/2*(2*b*x + a)/(c^(3/2)*x^2*sgn(x))
```

Mupad [B]
time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3+a\,x^2}{2\,c^{3/2}\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(3/2),x)
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(3/2)*x*(x^2)^(3/2))
```

3.792 $\int \frac{a+bx}{x(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

```
[Out] -1/3*a/c/x^2/(c*x^2)^(1/2)-1/2*b/c/x/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -1/3*a/(c*x^2*Sqrt[c*x^2]) - b/(2*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.61

$$\frac{cx^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.77, size = 16, normalized size = 0.39

$$\frac{-\frac{a}{3}-\frac{bx}{2}}{\left(cx^2\right)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^1*(c*x^2)^(3/2)),x]')
```

```
[Out] (-a / 3 - b x / 2) / (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.44

method	result	size
gosper	$-\frac{3bx+2a}{6\left(c\,x^{2}\right)^{\frac{3}{2}}}$	18
default	$-\frac{3bx+2a}{6\left(c\,x^{2}\right)^{\frac{3}{2}}}$	18
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{c\,x^{2}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)\left(2a\,x^{2}+3x^{2}b+2ax+3bx+2a\right)\sqrt{c\,x^{2}}}{6c^{2}x^{4}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(3*b*x+2*a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{2\,c^{\frac{3}{2}}x^2}-\frac{a}{3\,c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(c^(3/2)*x^2) - 1/3*a/(c^(3/2)*x^3)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^2*x^4)
```

Sympy [A]
time = 0.26, size = 26, normalized size = 0.63

$$-\frac{a}{3\,(cx^2)^{\frac{3}{2}}}-\frac{bx}{2\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(3/2),x)
```

```
[Out] -a/(3*(c*x**2)**(3/2)) - b*x/(2*(c*x**2)**(3/2))
```

Giac [A]
time = 0.00, size = 28, normalized size = 0.68

$$\frac{-3bx-2a}{\sqrt{c}\,c\cdot 6\,(x^3\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x)
```

```
[Out] -1/6*(3*b*x + 2*a)/(c^(3/2)*x^3*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{3/2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(3/2)),x)
```

```
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(3/2)*x^4)
```

3.793 $\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

```
[Out] -1/4*a/c/x^3/(c*x^2)^(1/2)-1/3*b/c/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/4*a/(c*x^3*Sqrt[c*x^2]) - b/(3*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)\,dx}{c\sqrt{cx^2}}\\
&= -\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^2*x^5)
```

Mathics [A]
time = 1.78, size = 22, normalized size = 0.54

$$\frac{\left(-\frac{a}{4}-\frac{bx}{3}\right)(cx^2)^{\frac{3}{2}}}{c^3x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]')
[Out] (-a / 4 - b x / 3) (c x ^ 2) ^ (3 / 2) / (c ^ 3 x ^ 7)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{4bx+3a}{12x\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
default	$-\frac{4bx+3a}{12x\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{c\,x^{3}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)\left(3a\,x^{3}+4b\,x^{3}+3a\,x^{2}+4x^{2}b+3ax+4bx+3a\right)\sqrt{c\,x^{2}}}{12c^{2}x^{5}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(4*b*x+3*a)/x/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{3\,c^{\frac{3}{2}}x^{3}}-\frac{a}{4\,c^{\frac{3}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(c^(3/2)*x^3) - 1/4*a/(c^(3/2)*x^4)
```

Fricas [A]
time = 0.28, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^2*x^5)
```

Sympy [A]
time = 0.28, size = 26, normalized size = 0.63

$$-\frac{a}{4x\,(cx^2)^{\frac{3}{2}}}-\frac{b}{3\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(3/2),x)
```

```
[Out] -a/(4*x*(c*x**2)**(3/2)) - b/(3*(c*x**2)**(3/2))
```

Giac [A]
time = 0.00, size = 28, normalized size = 0.68

$$\frac{-4bx-3a}{\sqrt{c}\,c\cdot 12\,(x^4\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x)
```

```
[Out] -1/12*(4*b*x + 3*a)/(c^(3/2)*x^4*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(3/2)),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(3/2)*x^5)
```

3.794 $\int \frac{a+bx}{x^3 (cx^2)^{3/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

```
[Out] -1/5*a/c/x^4/(c*x^2)^(1/2)-1/4*b/c/x^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -1/5*a/(c*x^4*Sqrt[c*x^2]) - b/(4*c*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3 (cx^2)^{3/2}} \, dx &= \frac{x \int \frac{a+bx}{x^6} \, dx}{c\sqrt{cx^2}} \\
&= \frac{x \int \left(\frac{a}{x^6}+\frac{b}{x^5}\right) \, dx}{c\sqrt{cx^2}} \\
&= -\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-4a-5bx)}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
[Out] (c*(-4*a - 5*b*x))/(20*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.83, size = 22, normalized size = 0.54

$$\frac{\left(-\frac{a}{5}-\frac{bx}{4}\right)\left(cx^2\right)^{\frac{3}{2}}}{c^3x^8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]')
[Out] (-a / 5 - b x / 4) (c x ^ 2) ^ (3 / 2) / (c ^ 3 x ^ 8)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{5bx+4a}{20x^2\left(c\,x^2\right)^{\frac{3}{2}}}$	21
default	$-\frac{5bx+4a}{20x^2\left(c\,x^2\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{4}-\frac{a}{5}}{c\,x^4\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)\left(4a\,x^4+5b\,x^4+4a\,x^3+5b\,x^3+4a\,x^2+5x^2b+4ax+5bx+4a\right)\sqrt{c\,x^2}}{20c^2x^6}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/20*(5*b*x+4*a)/x^2/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{3}{2}}x^4}-\frac{a}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(3/2)*x^4) - 1/5*a/(c^(3/2)*x^5)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^2*x^6)
```

Sympy [A]
time = 0.31, size = 29, normalized size = 0.71

$$-\frac{a}{5x^2\,(cx^2)^{\frac{3}{2}}}-\frac{b}{4x\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a/(5*x**2*(c*x**2)**(3/2)) - b/(4*x*(c*x**2)**(3/2))
```

Giac [A]
time = 0.00, size = 28, normalized size = 0.68

$$\frac{-5bx-4a}{\sqrt{c}\,c\cdot 20\,(x^5\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x)
```

```
[Out] -1/20*(5*b*x + 4*a)/(c^(3/2)*x^5*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2}+5\,b\,x\,\sqrt{x^2}}{20\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(3/2)),x)
```

```
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(3/2)*x^6)
```

3.795 $\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

```
[Out] -1/6*a/c/x^5/(c*x^2)^(1/2)-1/5*b/c/x^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -1/6*a/(c*x^5*Sqrt[c*x^2]) - b/(5*c*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.85, size = 22, normalized size = 0.54

$$\frac{\left(-\frac{a}{6}-\frac{bx}{5}\right)\left(cx^2\right)^{\frac{3}{2}}}{c^3x^9}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]')
[Out] (-a / 6 - b x / 5) (c x ^ 2) ^ (3 / 2) / (c ^ 3 x ^ 9)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{6bx+5a}{30x^3\left(c\,x^2\right)^{\frac{3}{2}}}$	21
default	$-\frac{6bx+5a}{30x^3\left(c\,x^2\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{5}-\frac{a}{6}}{c\,x^5\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)\left(5a\,x^5+6b\,x^5+5a\,x^4+6b\,x^4+5a\,x^3+6b\,x^3+5a\,x^2+6x^2b+5ax+6bx+5a\right)\sqrt{c\,x^2}}{30c^2x^7}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/30*(6*b*x+5*a)/x^3/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{3}{2}}x^5}-\frac{a}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(3/2)*x^5) - 1/6*a/(c^(3/2)*x^6)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^2*x^7)
```

Sympy [A]
time = 0.35, size = 31, normalized size = 0.76

$$-\frac{a}{6x^3\,(cx^2)^{\frac{3}{2}}}-\frac{b}{5x^2\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a/(6*x**3*(c*x**2)**(3/2)) - b/(5*x**2*(c*x**2)**(3/2))
```

Giac [A]
time = 0.00, size = 28, normalized size = 0.68

$$\frac{-6bx-5a}{\sqrt{c}\,c\cdot 30\,(x^6\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x)
```

```
[Out] -1/30*(6*b*x + 5*a)/(c^(3/2)*x^6*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(3/2)),x)
```

```
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(3/2)*x^7)
```

3.796 $\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] -a/c^2/(c*x^2)^(1/2)+b*x*ln(x)/c^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a/(c^2*Sqrt[c*x^2])) + (b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.67

$$\frac{-a+bx\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (-a + b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.64

method	result	size
default	$\frac{x^4(bx\ln(x)-a)}{(c\,x^2)^{\frac{5}{2}}}$	21
risch	$-\frac{a}{c^2\sqrt{c\,x^2}}+\frac{bx\ln(x)}{c^2\sqrt{c\,x^2}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] x^4*(b*x*ln(x)-a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.30, size = 24, normalized size = 0.73

$$-\frac{ax^2}{(cx^2)^{\frac{3}{2}}\,c}+\frac{b\log(x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -a*x^2/((c*x^2)^(3/2)*c) + b*log(x)/c^(5/2)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\,(bx\log(x)-a)}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\,(a+bx)}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(5/2),x)
[Out] Integral(x**3*(a + b*x)/(c*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 31, normalized size = 0.94

$$\frac{-\frac{a}{c^2x\mathrm{sign}(x)}+\frac{b\ln|x|}{c^2\mathrm{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x)
[Out] (b*log(abs(x))/(c^2*sgn(x)) - a/(c^2*x*sgn(x)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x^3\,(a+b\,x)}{{(c\,x^2)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(5/2),x)
[Out] int((x^3*(a + b*x))/(c*x^2)^(5/2), x)
```

3.797 $\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

```
[Out] -1/2*(b*x+a)^2/a/c^2/x/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -1/2*(a + b*x)^2/(a*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.83

$$\frac{x^3(-a-2bx)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-a - 2*b*x))/(2*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.85, size = 19, normalized size = 0.66

$$\frac{x^3\left(-\frac{a}{2}-bx\right)}{\left(cx^2\right)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]')
```

```
[Out] x ^ 3 (-a / 2 - b x) / (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.66

method	result	size
gosper	$-\frac{x^{3}(2bx+a)}{2\left(c\,x^{2}\right)^{\frac{5}{2}}}$	19
default	$-\frac{x^{3}(2bx+a)}{2\left(c\,x^{2}\right)^{\frac{5}{2}}}$	19
risch	$\frac{-bx-\frac{a}{2}}{c^{2}x\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^{2}}}{2c^{3}x^{3}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^3*(2*b*x+a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.25, size = 26, normalized size = 0.90

$$-\frac{bx^2}{\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b*x^2/((c*x^2)^(3/2)*c) - 1/2*a/(c^(5/2)*x^2)
```

Fricas [A]
time = 0.29, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^3*x^3)
```

Sympy [A]
time = 0.32, size = 29, normalized size = 1.00

$$-\frac{ax^3}{2\,(cx^2)^{\frac{5}{2}}}-\frac{bx^4}{(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**3/(2*(c*x**2)**(5/2)) - b*x**4/(c*x**2)**(5/2)
```

Giac [A]
time = 0.00, size = 27, normalized size = 0.93

$$\frac{-2bx-a}{\sqrt{c}\cdot 2\,(c^2x^2\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/2*(2*b*x + a)/(c^(5/2)*x^2*sgn(x))
```

Mupad [B]
time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3+a\,x^2}{2\,c^{5/2}\,x\,{(x^2)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(5/2)*x*(x^2)^(3/2))
```

3.798 $\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

```
[Out] -1/3*a/c^2/x^2/(c*x^2)^(1/2)-1/2*b/c^2/x/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 45}

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -1/3*a/(c^2*x^2*Sqrt[c*x^2]) - b/(2*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{x^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.83, size = 19, normalized size = 0.46

$$\frac{x^2\left(-\frac{a}{3}-\frac{bx}{2}\right)}{\left(cx^2\right)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]')
[Out] x ^ 2 (-a / 3 - b x / 2) / (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{x^{2}(3bx+2a)}{6(c\,x^{2})^{\frac{5}{2}}}$	21
default	$-\frac{x^{2}(3bx+2a)}{6(c\,x^{2})^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{c^{2}x^{2}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)(2a\,x^{2}+3x^{2}b+2ax+3bx+2a)\sqrt{c\,x^{2}}}{6c^{3}x^{4}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/6*x^2*(3*b*x+2*a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 0.56

$$-\frac{a}{3\,\left(cx^2\right)^{\frac{3}{2}}\,c}-\frac{b}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a/((c*x^2)^(3/2)*c) - 1/2*b/(c^(5/2)*x^2)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^3*x^4)
```

Sympy [A]
time = 0.32, size = 31, normalized size = 0.76

$$-\frac{ax^2}{3\,(cx^2)^{\frac{5}{2}}}-\frac{bx^3}{2\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**2/(3*(c*x**2)**(5/2)) - b*x**3/(2*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{-3bx-2a}{\sqrt{c}\cdot 6\,(c^2x^3\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/6*(3*b*x + 2*a)/(c^(5/2)*x^3*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{5/2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(5/2)*x^4)
```

3.799 $\int \frac{a+bx}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

```
[Out] -1/4*a/c^2/x^3/(c*x^2)^(1/2)-1/3*b/c^2/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(5/2),x]
```

```
[Out] -1/4*a/(c^2*x^3*Sqrt[c*x^2]) - b/(3*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(5/2),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^3*x^5)
```

Mathics [A]
time = 1.83, size = 18, normalized size = 0.44

$$\frac{x\,(-3a-4bx)}{12(cx^2)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(c*x^2)^(5/2),x]')
[Out] x (-3 a - 4 b x) / (12 (c x ^ 2) ^ (5 / 2))
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.46

method	result	size
gosper	$-\frac{x(4bx+3a)}{12(c\,x^{2})^{\frac{5}{2}}}$	19
default	$-\frac{x(4bx+3a)}{12(c\,x^{2})^{\frac{5}{2}}}$	19
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{c^{2}x^{3}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)(3a\,x^{3}+4b\,x^{3}+3a\,x^{2}+4x^{2}b+3ax+4bx+3a)\sqrt{c\,x^{2}}}{12c^{3}x^{5}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12*x*(4*b*x+3*a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.56

$$-\frac{b}{3\,(cx^2)^{\frac{3}{2}}\,c}-\frac{a}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/((c*x^2)^(3/2)*c) - 1/4*a/(c^(5/2)*x^4)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^3*x^5)
```

Sympy [A]
time = 0.31, size = 29, normalized size = 0.71

$$-\frac{ax}{4\,(cx^2)^{\frac{5}{2}}}-\frac{bx^2}{3\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x/(4*(c*x**2)**(5/2)) - b*x**2/(3*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{-4bx-3a}{\sqrt{c}\cdot 12\,(c^2x^4\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/12*(4*b*x + 3*a)/(c^(5/2)*x^4*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(5/2),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(5/2)*x^5)
```

3.800 $\int \frac{a+bx}{x(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

[Out] -1/5*a/c^2/x^4/(c*x^2)^(1/2)-1/4*b/c^2/x^3/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(x*(c*x^2)^(5/2)),x]

[Out] -1/5*a/(c^2*x^4*Sqrt[c*x^2]) - b/(4*c^2*x^3*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4a+5bx)}{20c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]
[Out] -1/20*(Sqrt[c*x^2]*(4*a + 5*b*x))/(c^3*x^6)
```

Mathics [A]
time = 1.83, size = 16, normalized size = 0.39

$$\frac{-\frac{a}{5}-\frac{bx}{4}}{(cx^2)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]')
[Out] (-a / 5 - b x / 4) / (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.44

method	result	size
gosper	$-\frac{5bx+4a}{20(c\,x^2)^{\frac{5}{2}}}$	18
default	$-\frac{5bx+4a}{20(c\,x^2)^{\frac{5}{2}}}$	18
risch	$\frac{-\frac{bx}{4}-\frac{a}{5}}{c^2x^4\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(4a\,x^4+5b\,x^4+4a\,x^3+5b\,x^3+4a\,x^2+5x^2b+4ax+5bx+4a)\sqrt{c\,x^2}}{20c^3x^6}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/20*(5*b*x+4*a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{5}{2}}x^4}-\frac{a}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(5/2)*x^4) - 1/5*a/(c^(5/2)*x^5)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^3*x^6)
```

Sympy [A]
time = 0.35, size = 26, normalized size = 0.63

$$-\frac{a}{5\,(cx^2)^{\frac{5}{2}}}-\frac{bx}{4\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(5/2),x)
```

```
[Out] -a/(5*(c*x**2)**(5/2)) - b*x/(4*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{-5bx-4a}{\sqrt{c}\cdot 20\,(c^2x^5\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x)
```

```
[Out] -1/20*(5*b*x + 4*a)/(c^(5/2)*x^5*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2}+5\,b\,x\,\sqrt{x^2}}{20\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(5/2)),x)
```

```
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(5/2)*x^6)
```

3.801 $\int \frac{a+bx}{x^2(cx^2)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

```
[Out] -1/6*a/c^2/x^5/(c*x^2)^(1/2)-1/5*b/c^2/x^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/6*a/(c^2*x^5*Sqrt[c*x^2]) - b/(5*c^2*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(5a+6bx)}{30c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
[Out] -1/30*(Sqrt[c*x^2]*(5*a + 6*b*x))/(c^3*x^7)
```

Mathics [A]
time = 1.90, size = 22, normalized size = 0.54

$$\frac{\left(-\frac{a}{6}-\frac{bx}{5}\right)(cx^2)^{\frac{5}{2}}}{c^5x^{11}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]')
[Out] (-a / 6 - b x / 5) (c x ^ 2) ^ (5 / 2) / (c ^ 5 x ^ 11)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{6bx+5a}{30x(c\,x^2)^{\frac{5}{2}}}$	21
default	$-\frac{6bx+5a}{30x(c\,x^2)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{5}-\frac{a}{6}}{c^2x^5\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(5a\,x^5+6b\,x^5+5a\,x^4+6b\,x^4+5a\,x^3+6b\,x^3+5a\,x^2+6x^2b+5ax+6bx+5a)\sqrt{c\,x^2}}{30c^3x^7}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/30*(6*b*x+5*a)/x/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{5}{2}}x^5}-\frac{a}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(5/2)*x^5) - 1/6*a/(c^(5/2)*x^6)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^3*x^7)
```

Sympy [A]
time = 0.39, size = 26, normalized size = 0.63

$$-\frac{a}{6x\,(cx^2)^{\frac{5}{2}}}-\frac{b}{5\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a/(6*x*(c*x**2)**(5/2)) - b/(5*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{-6bx-5a}{\sqrt{c}\cdot 30\,(c^2x^6\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/30*(6*b*x + 5*a)/(c^(5/2)*x^6*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(5/2)),x)
```

```
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(5/2)*x^7)
```

3.802 $\int \frac{a+bx}{x^3(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

```
[Out] -1/7*a/c^2/x^6/(c*x^2)^(1/2)-1/6*b/c^2/x^5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -1/7*a/(c^2*x^6*Sqrt[c*x^2]) - b/(6*c^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^8}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^8}+\frac{b}{x^7}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-6a-7bx)}{42\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
[Out] (c*(-6*a - 7*b*x))/(42*(c*x^2)^(7/2))
```

Mathics [A]
time = 1.94, size = 22, normalized size = 0.54

$$\frac{\left(-\frac{a}{7}-\frac{bx}{6}\right)\left(cx^2\right)^{\frac{5}{2}}}{c^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]')
[Out] (-a / 7 - b x / 6) (c x ^ 2) ^ (5 / 2) / (c ^ 5 x ^ 12)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{7bx+6a}{42x^2\left(c\,x^2\right)^{\frac{5}{2}}}$	21
default	$-\frac{7bx+6a}{42x^2\left(c\,x^2\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{6}-\frac{a}{7}}{c^2x^6\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)\left(6a\,x^6+7b\,x^6+6a\,x^5+7b\,x^5+6a\,x^4+7b\,x^4+6a\,x^3+7b\,x^3+6a\,x^2+7x^2b+6ax+7bx+6a\right)\sqrt{c\,x^2}}{42c^3x^8}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/42*(7*b*x+6*a)/x^2/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{6\,c^{\frac{5}{2}}x^6}-\frac{a}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b/(c^(5/2)*x^6) - 1/7*a/(c^(5/2)*x^7)
```

Fricas [**A**]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(7\,bx+6\,a)}{42\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/42*sqrt(c*x^2)*(7*b*x + 6*a)/(c^3*x^8)
```

Sympy [**A**]
time = 0.43, size = 29, normalized size = 0.71

$$-\frac{a}{7x^2\,(cx^2)^{\frac{5}{2}}}-\frac{b}{6x\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a/(7*x**2*(c*x**2)**(5/2)) - b/(6*x*(c*x**2)**(5/2))
```

Giac [**A**]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{-7bx-6a}{\sqrt{c}\cdot 42\,(c^2x^7\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x)
```

```
[Out] -1/42*(7*b*x + 6*a)/(c^(5/2)*x^7*sgn(x))
```

Mupad [**B**]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{6\,a\,\sqrt{x^2}+7\,b\,x\,\sqrt{x^2}}{42\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(5/2)),x)
```

```
[Out] -(6*a*(x^2)^(1/2) + 7*b*x*(x^2)^(1/2))/(42*c^(5/2)*x^8)
```

3.803 $\int \frac{a+bx}{x^4(cx^2)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

```
[Out] -1/8*a/c^2/x^7/(c*x^2)^(1/2)-1/7*b/c^2/x^6/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -1/8*a/(c^2*x^7*Sqrt[c*x^2]) - b/(7*c^2*x^6*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^9}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^9}+\frac{b}{x^8}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.97, size = 22, normalized size = 0.54

$$\frac{\left(-\frac{a}{8}-\frac{bx}{7}\right)(cx^2)^{\frac{5}{2}}}{c^5x^{13}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]')
[Out] (-a / 8 - b x / 7) (c x ^ 2) ^ (5 / 2) / (c ^ 5 x ^ 13)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{8bx+7a}{56x^3(cx^2)^{\frac{5}{2}}}$	21
default	$-\frac{8bx+7a}{56x^3(cx^2)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{7}-\frac{a}{8}}{c^2x^7\sqrt{cx^2}}$	23
trager	$\frac{(-1+x)(7a x^7+8b x^7+7a x^6+8b x^6+7a x^5+8b x^5+7a x^4+8b x^4+7a x^3+8b x^3+7a x^2+8x^2b+7ax+8bx+7a)\sqrt{cx^2}}{56c^3x^9}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/56*(8*b*x+7*a)/x^3/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{7c^{\frac{5}{2}}x^7}-\frac{a}{8c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/7*b/(c^(5/2)*x^7) - 1/8*a/(c^(5/2)*x^8)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(8\,bx+7\,a)}{56\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/56*sqrt(c*x^2)*(8*b*x + 7*a)/(c^3*x^9)
```

Sympy [A]
time = 0.46, size = 31, normalized size = 0.76

$$-\frac{a}{8x^3\,(cx^2)^{\frac{5}{2}}}-\frac{b}{7x^2\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a/(8*x**3*(c*x**2)**(5/2)) - b/(7*x**2*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{-8bx-7a}{\sqrt{c}\cdot 56\,(c^2x^8\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/56*(8*b*x + 7*a)/(c^(5/2)*x^8*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{7\,a\,\sqrt{x^2}+8\,b\,x\,\sqrt{x^2}}{56\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(7*a*(x^2)^(1/2) + 8*b*x*(x^2)^(1/2))/(56*c^(5/2)*x^9)
```

3.804 $\int x^3 \sqrt{cx^2} (a + bx)^2 \, dx$

Optimal. Leaf size=57

$$\frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}$$

```
[Out] 1/5*a^2*x^4*(c*x^2)^(1/2)+1/3*a*b*x^5*(c*x^2)^(1/2)+1/7*b^2*x^6*(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4*Sqrt[c*x^2])/5 + (a*b*x^5*Sqrt[c*x^2])/3 + (b^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^4(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int\left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\
&= \frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Mathics [A]
time = 1.87, size = 30, normalized size = 0.53

$$x^4\left(\frac{a^2}{5}+\frac{abx}{3}+\frac{b^2x^2}{7}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]')
[Out] x ^ 4 (a ^ 2 / 5 + a b x / 3 + b ^ 2 x ^ 2 / 7) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.56

method	result
gosper	$\frac{x^4\left(15x^2b^2+35abx+21a^2\right)\sqrt{c\,x^2}}{105}$
default	$\frac{x^4\left(15x^2b^2+35abx+21a^2\right)\sqrt{c\,x^2}}{105}$
risch	$\frac{a^2x^4\sqrt{c\,x^2}}{5}+\frac{ab\,x^5\sqrt{c\,x^2}}{3}+\frac{b^2x^6\sqrt{c\,x^2}}{7}$
trager	$\frac{(15b^2x^6+35ab\,x^5+15b^2x^5+21a^2x^4+35ab\,x^4+15b^2x^4+21a^2x^3+35ab\,x^3+15b^2x^3+21a^2x^2+35ab\,x^2+15x^2b^2+21a^2x+35abx+15b^2x+2}{105x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/105*x^4*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.95

$$\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x^4}{7\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}abx^3}{3\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}a^2x^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(3/2)*b^2*x^4/c + 1/3*(c*x^2)^(3/2)*a*b*x^3/c + 1/5*(c*x^2)^(3/
2)*a^2*x^2/c
```

Fricas [A]
time = 0.30, size = 33, normalized size = 0.58

$$\frac{1}{105}\left(15\,b^2x^6+35\,abx^5+21\,a^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*x^6 + 35*a*b*x^5 + 21*a^2*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.17, size = 49, normalized size = 0.86

$$\frac{a^2x^4\sqrt{cx^2}}{5}+\frac{abx^5\sqrt{cx^2}}{3}+\frac{b^2x^6\sqrt{cx^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4*sqrt(c*x**2)/5 + a*b*x**5*sqrt(c*x**2)/3 + b**2*x**6*sqrt(c*x**2)
/7
```

Giac [A]
time = 0.00, size = 39, normalized size = 0.68

$$\sqrt{c}\left(\frac{1}{5}a^2x^5\mathrm{sign}\left(x\right)+\frac{1}{7}b^2x^7\mathrm{sign}\left(x\right)+\frac{1}{3}abx^6\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.805 $\int x^2 \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=57

$$\frac{1}{4}a^2x^3\sqrt{cx^2} + \frac{2}{5}abx^4\sqrt{cx^2} + \frac{1}{6}b^2x^5\sqrt{cx^2}$$

```
[Out] 1/4*a^2*x^3*(c*x^2)^(1/2)+2/5*a*b*x^4*(c*x^2)^(1/2)+1/6*b^2*x^5*(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{4}a^2x^3\sqrt{cx^2} + \frac{2}{5}abx^4\sqrt{cx^2} + \frac{1}{6}b^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*x^4*Sqrt[c*x^2])/5 + (b^2*x^5*Sqrt[c*x^2])
/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^3(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int\left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\
&= \frac{1}{4}a^2x^3\sqrt{cx^2} + \frac{2}{5}abx^4\sqrt{cx^2} + \frac{1}{6}b^2x^5\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\,\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Mathics [A]
time = 1.85, size = 31, normalized size = 0.54

$$\frac{x^3\left(15a^2+24abx+10b^2x^2\right)\sqrt{cx^2}}{60}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]')
[Out] x ^ 3 (15 a ^ 2 + 24 a b x + 10 b ^ 2 x ^ 2) Sqrt[c x ^ 2] / 60
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.56

method	result
gosper	$\frac{x^3\left(10x^2b^2+24abx+15a^2\right)\sqrt{c\,x^2}}{60}$
default	$\frac{x^3\left(10x^2b^2+24abx+15a^2\right)\sqrt{c\,x^2}}{60}$
risch	$\frac{a^2x^3\sqrt{c\,x^2}}{4}+\frac{2ab\,x^4\sqrt{c\,x^2}}{5}+\frac{b^2x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{\left(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24ab\,x^3+10b^2x^3+15a^2x^2+24ab\,x^2+10x^2b^2+15a^2x+24abx+10b^2x+15a^2+24ab+10b^2\right)\left(-1+x\right)}{60x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/60*x^3*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.91

$$\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x^3}{6\,c}+\frac{2\,\left(cx^2\right)^{\frac{3}{2}}abx^2}{5\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}a^2x}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(3/2)*b^2*x^3/c + 2/5*(c*x^2)^(3/2)*a*b*x^2/c + 1/4*(c*x^2)^(3/
2)*a^2*x/c
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^5+24\,abx^4+15\,a^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*x^5 + 24*a*b*x^4 + 15*a^2*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.15, size = 51, normalized size = 0.89

$$\frac{a^2x^3\sqrt{cx^2}}{4}+\frac{2abx^4\sqrt{cx^2}}{5}+\frac{b^2x^5\sqrt{cx^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3*sqrt(c*x**2)/4 + 2*a*b*x**4*sqrt(c*x**2)/5 + b**2*x**5*sqrt(c*x**
2)/6
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.70

$$\sqrt{c}\left(\frac{1}{4}a^2x^4\mathrm{sign}\left(x\right)+\frac{1}{6}b^2x^6\mathrm{sign}\left(x\right)+\frac{2}{5}abx^5\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.806 $\int x\sqrt{cx^2}\,(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

```
[Out] 1/3*a^2*x^2*(c*x^2)^(1/2)+1/2*a*b*x^3*(c*x^2)^(1/2)+1/5*b^2*x^4*(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2*Sqrt[c*x^2])/3 + (a*b*x^3*Sqrt[c*x^2])/2 + (b^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}\,(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(a^2x^2+2abx^3+b^2x^4\right)\,dx}{x}\\&=\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\,\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Mathics [A]
time = 1.81, size = 30, normalized size = 0.53

$$x^2\left(\frac{a^2}{3}+\frac{abx}{2}+\frac{b^2x^2}{5}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]')
[Out] x ^ 2 (a ^ 2 / 3 + a b x / 2 + b ^ 2 x ^ 2 / 5) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.56

method	result	size
gosper	$\frac{x^2(6x^2b^2+15abx+10a^2)\sqrt{c\,x^2}}{30}$	32
default	$\frac{x^2(6x^2b^2+15abx+10a^2)\sqrt{c\,x^2}}{30}$	32
risch	$\frac{a^2x^2\sqrt{c\,x^2}}{3}+\frac{ab\,x^3\sqrt{c\,x^2}}{2}+\frac{b^2x^4\sqrt{c\,x^2}}{5}$	46
trager	$\frac{(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2)(-1+x)\sqrt{c\,x^2}}{30x}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.86

$$\frac{(cx^2)^{\frac{3}{2}}\,b^2x^2}{5\,c}+\frac{(cx^2)^{\frac{3}{2}}\,abx}{2\,c}+\frac{(cx^2)^{\frac{3}{2}}\,a^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2)^(3/2)*b^2*x^2/c + 1/2*(c*x^2)^(3/2)*a*b*x/c + 1/3*(c*x^2)^(3/2)
*a^2/c
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^4+15\,abx^3+10\,a^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.13, size = 49, normalized size = 0.86

$$\frac{a^2x^2\sqrt{cx^2}}{3}+\frac{abx^3\sqrt{cx^2}}{2}+\frac{b^2x^4\sqrt{cx^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**2*sqrt(c*x**2)/3 + a*b*x**3*sqrt(c*x**2)/2 + b**2*x**4*sqrt(c*x**2)
/5
```

Giac [A]
time = 0.00, size = 39, normalized size = 0.68

$$\sqrt{c}\,\left(\frac{1}{3}a^2x^3\mathrm{sign}\left(x\right)+\frac{1}{5}b^2x^5\mathrm{sign}\left(x\right)+\frac{1}{2}abx^4\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.807 $\int \sqrt{cx^2}(a+bx)^2 dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

```
[Out] 1/2*a^2*x*(c*x^2)^(1/2)+2/3*a*b*x^2*(c*x^2)^(1/2)+1/4*b^2*x^3*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x*Sqrt[c*x^2])/2 + (2*a*b*x^2*Sqrt[c*x^2])/3 + (b^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(a^2x+2abx^2+b^2x^3\right)\,dx}{x}\\ &= \frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 0.60

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Mathics [A]
time = 1.80, size = 29, normalized size = 0.53

$$\frac{x\left(6a^2+8abx+3b^2x^2\right)\sqrt{cx^2}}{12}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]')
[Out] x (6 a ^ 2 + 8 a b x + 3 b ^ 2 x ^ 2) Sqrt[c x ^ 2] / 12
```

Maple [A]
time = 0.11, size = 30, normalized size = 0.55

method	result	size
gosper	$\frac{x\left(3x^2b^2+8abx+6a^2\right)\sqrt{c\,x^2}}{12}$	30
default	$\frac{x\left(3x^2b^2+8abx+6a^2\right)\sqrt{c\,x^2}}{12}$	30
risch	$\frac{a^2x\sqrt{c\,x^2}}{2}+\frac{2ab\,x^2\sqrt{c\,x^2}}{3}+\frac{b^2x^3\sqrt{c\,x^2}}{4}$	44
trager	$\frac{\left(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2\right)(-1+x)\sqrt{c\,x^2}}{12x}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2}\,a^2x+\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x}{4\,c}+\frac{2\,\left(cx^2\right)^{\frac{3}{2}}ab}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2)*a^2*x + 1/4*(c*x^2)^(3/2)*b^2*x/c + 2/3*(c*x^2)^(3/2)*a*b/c
```

Fricas [**A**]
time = 0.29, size = 31, normalized size = 0.56

$$\frac{1}{12}\left(3\,b^2x^3+8\,abx^2+6\,a^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.11, size = 49, normalized size = 0.89

$$\frac{a^2x\sqrt{cx^2}}{2}+\frac{2abx^2\sqrt{cx^2}}{3}+\frac{b^2x^3\sqrt{cx^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x*sqrt(c*x**2)/2 + 2*a*b*x**2*sqrt(c*x**2)/3 + b**2*x**3*sqrt(c*x**2)/
4
```

Giac [**A**]
time = 0.00, size = 40, normalized size = 0.73

$$\sqrt{c}\,\left(\frac{1}{2}a^2x^2\mathrm{sign}\left(x\right)+\frac{1}{4}b^2x^4\mathrm{sign}\left(x\right)+\frac{2}{3}abx^3\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.808 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x} dx$

Optimal. Leaf size=26

$$\frac{\sqrt{cx^2}(a+bx)^3}{3bx}$$

```
[Out] 1/3*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^2}{x} dx &= \frac{\sqrt{cx^2}\int (a+bx)^2 dx}{x}\\ &= \frac{\sqrt{cx^2}(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.96

$$\frac{cx(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (c*x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Mathics [A]
time = 1.79, size = 24, normalized size = 0.92

$$\left(a^2+abx+\frac{b^2x^2}{3}\right)\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]')
```

```
[Out] (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3) Sqrt[c x ^ 2]
```

Maple [A]
time = 0.10, size = 23, normalized size = 0.88

method	result	size
default	$\frac{(bx+a)^3\sqrt{c\,x^2}}{3bx}$	23
risch	$\frac{(bx+a)^3\sqrt{c\,x^2}}{3bx}$	23
gosper	$\frac{(x^2b^2+3abx+3a^2)\sqrt{c\,x^2}}{3}$	28
trager	$\frac{(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c\,x^2}}{3x}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 27, normalized size = 1.04

$$\frac{1}{3}\left(b^{2}x^{2}+3\,abx+3\,a^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 0.11, size = 41, normalized size = 1.58

$$a^{2}\sqrt{cx^{2}}+abx\sqrt{cx^{2}}+\frac{b^{2}x^{2}\sqrt{cx^{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x,x)
```

```
[Out] a**2*sqrt(c*x**2) + a*b*x*sqrt(c*x**2) + b**2*x**2*sqrt(c*x**2)/3
```

Giac [A]
time = 0.00, size = 32, normalized size = 1.23

$$\sqrt{c}\left(-\frac{a^{3}\mathrm{sign}\left(x\right)}{3b}+\frac{\left(bx+a\right)^{3}\mathrm{sign}\left(x\right)}{3b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^{2}}\,\left(a+b\,x\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x, x)
```

3.809 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=49

$$2ab\sqrt{cx^2} + \frac{1}{2}b^2x\sqrt{cx^2} + \frac{a^2\sqrt{cx^2}\log(x)}{x}$$

```
[Out] 2*a*b*(c*x^2)^(1/2)+1/2*b^2*x*(c*x^2)^(1/2)+a^2*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2\sqrt{cx^2}\log(x)}{x} + 2ab\sqrt{cx^2} + \frac{1}{2}b^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c*x^2] + (b^2*x*Sqrt[c*x^2])/2 + (a^2*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(2ab + \frac{a^2}{x} + b^2x\right) dx}{x} \\
&= 2ab\sqrt{cx^2} + \frac{1}{2}b^2x\sqrt{cx^2} + \frac{a^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.67

$$\frac{cx\left(bx(4a+bx)+2a^2\log(x)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.67

method	result	size
default	$\frac{\sqrt{c\,x^2}\,\left(x^2b^2+2a^2\ln(x)+4abx\right)}{2x}$	33
risch	$\frac{\sqrt{c\,x^2}\,b\left(\frac{1}{2}x^2b+2ax\right)}{x}+\frac{a^2\ln(x)\sqrt{c\,x^2}}{x}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 32, normalized size = 0.65

$$\frac{(b^2x^2 + 4\,abx + 2\,a^2\log(x))\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**2, x)
```

Giac [A]
time = 0.00, size = 34, normalized size = 0.69

$$\sqrt{c}\,\left(a^2\mathrm{sign}\,(x)\ln|x| + \frac{1}{2}b^2x^2\mathrm{sign}\,(x) + 2abx\mathrm{sign}\,(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2, x)
```

3.810 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=49

$$b^2\sqrt{cx^2}-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}$$

```
[Out] b^2*(c*x^2)^(1/2)-a^2*(c*x^2)^(1/2)/x^2+2*a*b*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}+b^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

```
[Out] b^2*Sqrt[c*x^2] - (a^2*Sqrt[c*x^2])/x^2 + (2*a*b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^2} dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right) dx}{x}\\
&= b^2\sqrt{cx^2}-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.63

$$\frac{c(-a^2+b^2x^2+2abx\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

```
[Out] (c*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/Sqrt[c*x^2]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.65

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(2ab\ln(x)x+x^{2}b^{2}-a^{2})}{x^{2}}$	32
risch	$b^{2}\sqrt{c\,x^{2}}-\frac{a^{2}\sqrt{c\,x^{2}}}{x^{2}}+\frac{2ab\ln(x)\sqrt{c\,x^{2}}}{x}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)*(2*a*b*ln(x)*x+x^2*b^2-a^2)/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 31, normalized size = 0.63

$$\frac{(b^2x^2 + 2\,abx\log{(x)} - a^2)\sqrt{cx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}\,(a+bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**3, x)
```

Giac [A]
time = 0.00, size = 32, normalized size = 0.65

$$\sqrt{c}\left(-\frac{a^2\mathrm{sign}\,(x)}{x} + b^2x\mathrm{sign}\,(x) + 2ab\mathrm{sign}\,(x)\ln|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] (b^2*x*sgn(x) + 2*a*b*log(abs(x))*sgn(x) - a^2*sgn(x)/x)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c\,x^2}\,{(a+b\,x)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3, x)
```

3.811 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=54

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

```
[Out] -1/2*a^2*(c*x^2)^(1/2)/x^3-2*a*b*(c*x^2)^(1/2)/x^2+b^2*ln(x)*(c*x^2)^(1/2)/
x
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] -1/2*(a^2*Sqrt[c*x^2])/x^3 - (2*a*b*Sqrt[c*x^2])/x^2 + (b^2*Sqrt[c*x^2]*Log
[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^3} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right) dx}{x} \\
&= -\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.67

$$\frac{\sqrt{cx^2}\,(-a(a+4bx)+2b^2x^2\log(x))}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.63

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(2b^2\ln(x)x^2-4abx-a^2)}{2x^3}$	34
risch	$\frac{\sqrt{c\,x^2}\,(-\frac{1}{2}a^2-2abx)}{x^3}+\frac{b^2\ln(x)\sqrt{c\,x^2}}{x}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.61

$$\frac{(2\,b^2x^2\log{(x)}-4\,abx-a^2)\sqrt{cx^2}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}\,\left(a+bx\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**4, x)
```

Giac [A]
time = 0.00, size = 39, normalized size = 0.72

$$\sqrt{c}\,\left(\frac{-a^2\mathrm{sign}\,(x)-4abx\mathrm{sign}\,(x)}{2x^2}+b^2\mathrm{sign}\,(x)\ln{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] 1/2*(2*b^2*log(abs(x))*sgn(x) - (4*a*b*x*sgn(x) + a^2*sgn(x))/x^2)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4, x)
```

3.812 $\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

```
[Out] 1/7*a^2*c*x^6*(c*x^2)^(1/2)+1/4*a*b*c*x^7*(c*x^2)^(1/2)+1/9*b^2*c*x^8*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^6*Sqrt[c*x^2])/7 + (a*b*c*x^7*Sqrt[c*x^2])/4 + (b^2*c*x^8*Sqrt[c*x
^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 (cx^2)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)^2 \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^6+2abx^7+b^2x^8\right) dx}{x}\\ &= \frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

Mathics [A]
time = 2.01, size = 30, normalized size = 0.50

$$x^4\left(\frac{a^2}{7}+\frac{abx}{4}+\frac{b^2x^2}{9}\right)\left(cx^2\right)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]')
```

```
[Out] x ^ 4 (a ^ 2 / 7 + a b x / 4 + b ^ 2 x ^ 2 / 9) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.53

method	result
gosper	$\frac{x^4\left(28x^2b^2+63abx+36a^2\right)\left(cx^2\right)^{\frac{3}{2}}}{252}$
default	$\frac{x^4\left(28x^2b^2+63abx+36a^2\right)\left(cx^2\right)^{\frac{3}{2}}}{252}$
risch	$\frac{a^2cx^6\sqrt{cx^2}}{7}+\frac{abcx^7\sqrt{cx^2}}{4}+\frac{b^2cx^8\sqrt{cx^2}}{9}$
trager	$\frac{c(28b^2x^8+63ab\,x^7+28b^2x^7+36a^2x^6+63ab\,x^6+28b^2x^6+36a^2x^5+63ab\,x^5+28b^2x^5+36a^2x^4+63ab\,x^4+28b^2x^4+36a^2x^3+63ab\,x^3+28b^2}{252x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/252*x^4*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.90

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^4}{9\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}abx^3}{4\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2x^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(5/2)*b^2*x^4/c + 1/4*(c*x^2)^(5/2)*a*b*x^3/c + 1/7*(c*x^2)^(5/
2)*a^2*x^2/c
```

Fricas [A]
time = 0.28, size = 36, normalized size = 0.60

$$\frac{1}{252}\left(28\,b^2cx^8+63\,abcx^7+36\,a^2cx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c*x^8 + 63*a*b*c*x^7 + 36*a^2*c*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.31, size = 49, normalized size = 0.82

$$\frac{a^2x^4\left(cx^2\right)^{\frac{3}{2}}}{7}+\frac{abx^5\left(cx^2\right)^{\frac{3}{2}}}{4}+\frac{b^2x^6\left(cx^2\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**4*(c*x**2)**(3/2)/7 + a*b*x**5*(c*x**2)**(3/2)/4 + b**2*x**6*(c*x**
2)**(3/2)/9
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.67

$$\sqrt{c}\,c\left(\frac{1}{7}a^2x^7\mathrm{sign}\left(x\right)+\frac{1}{9}b^2x^9\mathrm{sign}\left(x\right)+\frac{1}{4}abx^8\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.813 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

```
[Out] 1/6*a^2*c*x^5*(c*x^2)^(1/2)+2/7*a*b*c*x^6*(c*x^2)^(1/2)+1/8*b^2*c*x^7*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^5*Sqrt[c*x^2])/6 + (2*a*b*c*x^6*Sqrt[c*x^2])/7 + (b^2*c*x^7*Sqrt[c
*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right) dx}{x}\\
&= \frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

Mathics [A]
time = 1.98, size = 31, normalized size = 0.52

$$\frac{x^3\left(28a^2+48abx+21b^2x^2\right)\left(cx^2\right)^{\frac{3}{2}}}{168}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]')
[Out] x ^ 3 (28 a ^ 2 + 48 a b x + 21 b ^ 2 x ^ 2) (c x ^ 2) ^ (3 / 2) / 168
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.53

method	result
gosper	$\frac{x^3\left(21x^2b^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{168}$
default	$\frac{x^3\left(21x^2b^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{168}$
risch	$\frac{a^2c\,x^5\sqrt{c\,x^2}}{6}+\frac{2abc\,x^6\sqrt{c\,x^2}}{7}+\frac{b^2c\,x^7\sqrt{c\,x^2}}{8}$
trager	$\frac{c(21b^2x^7+48ab\,x^6+21b^2x^6+28a^2x^5+48ab\,x^5+21b^2x^5+28a^2x^4+48ab\,x^4+21b^2x^4+28a^2x^3+48ab\,x^3+21b^2x^3+28a^2x^2+48ab\,x^2+21}{168x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/168*x^3*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 52, normalized size = 0.87

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^3}{8\,c}+\frac{2\,\left(cx^2\right)^{\frac{5}{2}}abx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2x}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b^2*x^3/c + 2/7*(c*x^2)^(5/2)*a*b*x^2/c + 1/6*(c*x^2)^(5/
2)*a^2*x/c
```

Fricas [A]
time = 0.29, size = 36, normalized size = 0.60

$$\frac{1}{168}\left(21\,b^2cx^7+48\,abcx^6+28\,a^2cx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c*x^7 + 48*a*b*c*x^6 + 28*a^2*c*x^5)*sqrt(c*x^2)
```

Sympy [A]
time = 0.26, size = 51, normalized size = 0.85

$$\frac{a^2x^3\left(cx^2\right)^{\frac{3}{2}}}{6}+\frac{2abx^4\left(cx^2\right)^{\frac{3}{2}}}{7}+\frac{b^2x^5\left(cx^2\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**3*(c*x**2)**(3/2)/6 + 2*a*b*x**4*(c*x**2)**(3/2)/7 + b**2*x**5*(c*x
**2)**(3/2)/8
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.68

$$\sqrt{c}\,c\left(\frac{1}{6}a^2x^6\mathrm{sign}\left(x\right)+\frac{1}{8}b^2x^8\mathrm{sign}\left(x\right)+\frac{2}{7}abx^7\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.814 $\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

```
[Out] 1/5*a^2*c*x^4*(c*x^2)^(1/2)+1/3*a*b*c*x^5*(c*x^2)^(1/2)+1/7*b^2*c*x^6*(c*x^
2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^4*Sqrt[c*x^2])/5 + (a*b*c*x^5*Sqrt[c*x^2])/3 + (b^2*c*x^6*Sqrt[c*x
^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^4+2abx^5+b^2x^6\right)\,dx}{x}\\
&= \frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Mathics [A]
time = 1.93, size = 30, normalized size = 0.50

$$x^2\left(\frac{a^2}{5}+\frac{abx}{3}+\frac{b^2x^2}{7}\right)\left(cx^2\right)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]')
[Out] x ^ 2 (a ^ 2 / 5 + a b x / 3 + b ^ 2 x ^ 2 / 7) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.53

method	result
gosper	$\frac{x^2\left(15x^2b^2+35abx+21a^2\right)\left(cx^2\right)^{\frac{3}{2}}}{105}$
default	$\frac{x^2\left(15x^2b^2+35abx+21a^2\right)\left(cx^2\right)^{\frac{3}{2}}}{105}$
risch	$\frac{a^2cx^4\sqrt{cx^2}}{5}+\frac{abcx^5\sqrt{cx^2}}{3}+\frac{b^2cx^6\sqrt{cx^2}}{7}$
trager	$\frac{c(15b^2x^6+35ab\,x^5+15b^2x^5+21a^2x^4+35ab\,x^4+15b^2x^4+21a^2x^3+35ab\,x^3+15b^2x^3+21a^2x^2+35ab\,x^2+15x^2b^2+21a^2x+35abx+15b^2x+}{105x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/105*x^2*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 49, normalized size = 0.82

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}abx}{3\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(5/2)*b^2*x^2/c + 1/3*(c*x^2)^(5/2)*a*b*x/c + 1/5*(c*x^2)^(5/2)
*a^2/c
```

Fricas [A]
time = 0.29, size = 36, normalized size = 0.60

$$\frac{1}{105}\left(15\,b^2cx^6+35\,abcx^5+21\,a^2cx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*c*x^6 + 35*a*b*c*x^5 + 21*a^2*c*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.23, size = 49, normalized size = 0.82

$$\frac{a^2x^2\left(cx^2\right)^{\frac{3}{2}}}{5}+\frac{abx^3\left(cx^2\right)^{\frac{3}{2}}}{3}+\frac{b^2x^4\left(cx^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**2*(c*x**2)**(3/2)/5 + a*b*x**3*(c*x**2)**(3/2)/3 + b**2*x**4*(c*x**
2)**(3/2)/7
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.67

$$\sqrt{c}\,c\left(\frac{1}{5}a^2x^5\mathrm{sign}\left(x\right)+\frac{1}{7}b^2x^7\mathrm{sign}\left(x\right)+\frac{1}{3}abx^6\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.815 $\int (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}$$

```
[Out] 1/4*a^2*c*x^3*(c*x^2)^(1/2)+2/5*a*b*c*x^4*(c*x^2)^(1/2)+1/6*b^2*c*x^5*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^3*Sqrt[c*x^2])/4 + (2*a*b*c*x^4*Sqrt[c*x^2])/5 + (b^2*c*x^5*Sqrt[c
*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (cx^2)^{3/2}(a+bx)^2\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\ &= \frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Mathics [A]
time = 1.87, size = 29, normalized size = 0.48

$$\frac{x\left(15a^2+24abx+10b^2x^2\right)\left(cx^2\right)^{\frac{3}{2}}}{60}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]')
[Out] x (15 a ^ 2 + 24 a b x + 10 b ^ 2 x ^ 2) (c x ^ 2) ^ (3 / 2) / 60
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.50

method	result
gosper	$\frac{x\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{60}$
default	$\frac{x\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{60}$
risch	$\frac{a^2c\,x^3\sqrt{c\,x^2}}{4}+\frac{2abc\,x^4\sqrt{c\,x^2}}{5}+\frac{b^2c\,x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{c\left(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24ab\,x^3+10b^2x^3+15a^2x^2+24ab\,x^2+10x^2b^2+15a^2x+24abx+10b^2x+15a^2+24ab+10b^2\right)(-1+x}{60x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/60*x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 44, normalized size = 0.73

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}a^2x+\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x}{6\,c}+\frac{2\left(cx^2\right)^{\frac{5}{2}}ab}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*a^2*x + 1/6*(c*x^2)^(5/2)*b^2*x/c + 2/5*(c*x^2)^(5/2)*a*b
/c
```

Fricas [A]
time = 0.29, size = 36, normalized size = 0.60

$$\frac{1}{60}\left(10\,b^2cx^5+24\,abcx^4+15\,a^2cx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c*x^5 + 24*a*b*c*x^4 + 15*a^2*c*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.20, size = 49, normalized size = 0.82

$$\frac{a^2x\left(cx^2\right)^{\frac{3}{2}}}{4}+\frac{2abx^2\left(cx^2\right)^{\frac{3}{2}}}{5}+\frac{b^2x^3\left(cx^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x*(c*x**2)**(3/2)/4 + 2*a*b*x**2*(c*x**2)**(3/2)/5 + b**2*x**3*(c*x**2
)**(3/2)/6
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.68

$$\sqrt{c}\,c\left(\frac{1}{4}a^2x^4\mathrm{sign}\left(x\right)+\frac{1}{6}b^2x^6\mathrm{sign}\left(x\right)+\frac{2}{5}abx^5\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.816 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x} dx$

Optimal. Leaf size=60

$$\frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}$$

```
[Out] 1/3*a^2*c*x^2*(c*x^2)^(1/2)+1/2*a*b*c*x^3*(c*x^2)^(1/2)+1/5*b^2*c*x^4*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c*x^2*Sqrt[c*x^2])/3 + (a*b*c*x^3*Sqrt[c*x^2])/2 + (b^2*c*x^4*Sqrt[c*x
^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^2}{x} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^2 dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int (a^2x^2+2abx^3+b^2x^4) dx}{x} \\
&= \frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.60

$$\frac{1}{30}cx^2\sqrt{cx^2}\,\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (c*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Mathics [A]
time = 1.88, size = 27, normalized size = 0.45

$$\left(\frac{a^2}{3}+\frac{abx}{2}+\frac{b^2x^2}{5}\right)\left(cx^2\right)^{\frac{3}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]')
```

```
[Out] (a ^ 2 / 3 + a b x / 2 + b ^ 2 x ^ 2 / 5) (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.48

method	result	size
gosper	$\frac{\left(6x^2b^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{30}$	29
default	$\frac{\left(6x^2b^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{30}$	29
risch	$\frac{a^2c\,x^2\sqrt{c\,x^2}}{3}+\frac{abc\,x^3\sqrt{c\,x^2}}{2}+\frac{b^2c\,x^4\sqrt{c\,x^2}}{5}$	49
trager	$\frac{c\left(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2\right)\left(-1+x\right)\sqrt{c\,x^2}}{30x}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.67

$$\frac{1}{2}\left(cx^2\right)^{\frac{3}{2}}abx+\frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a^2+\frac{\left(cx^2\right)^{\frac{5}{2}}b^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*(c*x^2)^(3/2)*a*b*x + 1/3*(c*x^2)^(3/2)*a^2 + 1/5*(c*x^2)^(5/2)*b^2/c
```

Fricas [A]
time = 0.29, size = 36, normalized size = 0.60

$$\frac{1}{30}\left(6\,b^2cx^4+15\,abcx^3+10\,a^2cx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/30*(6*b^2*c*x^4 + 15*a*b*c*x^3 + 10*a^2*c*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.20, size = 44, normalized size = 0.73

$$\frac{a^2\left(cx^2\right)^{\frac{3}{2}}}{3}+\frac{abx\left(cx^2\right)^{\frac{3}{2}}}{2}+\frac{b^2x^2\left(cx^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x,x)
[Out] a**2*(c*x**2)**(3/2)/3 + a*b*x*(c*x**2)**(3/2)/2 + b**2*x**2*(c*x**2)**(3/2
)/5
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.67

$$\sqrt{c}\,c\left(\frac{1}{3}a^2x^3\mathrm{sign}\left(x\right)+\frac{1}{5}b^2x^5\mathrm{sign}\left(x\right)+\frac{1}{2}abx^4\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x)
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x, x)
```

3.817 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=58

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

```
[Out] 1/2*a^2*c*x*(c*x^2)^(1/2)+2/3*a*b*c*x^2*(c*x^2)^(1/2)+1/4*b^2*c*x^3*(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c*x*Sqrt[c*x^2])/2 + (2*a*b*c*x^2*Sqrt[c*x^2])/3 + (b^2*c*x^3*Sqrt[c*x
^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)\,dx}{x}\\ &= \frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.59

$$\frac{1}{12}cx\sqrt{cx^2}\,(6a^2+8abx+3b^2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
[Out] (c*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Mathics [A]
time = 1.88, size = 29, normalized size = 0.50

$$\frac{(6a^2+bx(8a+3bx))(cx^2)^{\frac{3}{2}}}{12x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]')
[Out] (6 a ^ 2 + b x (8 a + 3 b x)) (c x ^ 2) ^ (3 / 2) / (12 x)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.55

method	result	size
gosper	$\frac{(3x^2b^2+8abx+6a^2)(cx^2)^{\frac{3}{2}}}{12x}$	32
default	$\frac{(3x^2b^2+8abx+6a^2)(cx^2)^{\frac{3}{2}}}{12x}$	32
risch	$\frac{a^2cx\sqrt{cx^2}}{2}+\frac{2abcx^2\sqrt{cx^2}}{3}+\frac{b^2cx^3\sqrt{cx^2}}{4}$	47
trager	$\frac{c(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2)(-1+x)\sqrt{cx^2}}{12x}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/12/x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.59

$$\frac{1}{12}\left(3\,b^2cx^3+8\,abcx^2+6\,a^2cx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c*x^3 + 8*a*b*c*x^2 + 6*a^2*c*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.21, size = 44, normalized size = 0.76

$$\frac{a^2\left(cx^2\right)^{\frac{3}{2}}}{2x}+\frac{2ab\left(cx^2\right)^{\frac{3}{2}}}{3}+\frac{b^2x\left(cx^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*(c*x**2)**(3/2)/(2*x) + 2*a*b*(c*x**2)**(3/2)/3 + b**2*x*(c*x**2)**(3/
2)/4
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.71

$$\sqrt{c}\,c\left(\frac{1}{2}a^2x^2\mathrm{sign}\left(x\right)+\frac{1}{4}b^2x^4\mathrm{sign}\left(x\right)+\frac{2}{3}abx^3\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x)
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2, x)
```

3.818 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=27

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

```
[Out] 1/3*c*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\,(a+bx)^3}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^3)/(3*b*x^3)
```

Mathics [A]
time = 1.91, size = 27, normalized size = 1.00

$$\frac{\left(a^2+abx+\frac{b^2x^2}{3}\right)(cx^2)^{\frac{3}{2}}}{x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]')
```

```
[Out] (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3) (c x ^ 2) ^ (3 / 2) / x ^ 2
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.85

method	result	size
default	$\frac{(cx^2)^{\frac{3}{2}}(bx+a)^3}{3x^3b}$	23
risch	$\frac{c(bx+a)^3\sqrt{cx^2}}{3bx}$	24
gosper	$\frac{(x^2b^2+3abx+3a^2)(cx^2)^{\frac{3}{2}}}{3x^2}$	31
trager	$\frac{c(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{cx^2}}{3x}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2)^(3/2)/x^3*(b*x+a)^3/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 30, normalized size = 1.11

$$\frac{1}{3}\left(b^{2} c x^{2}+3\,abcx+3\,a^{2} c\right)\sqrt{c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c*x^2 + 3*a*b*c*x + 3*a^2*c)*sqrt(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.
time = 0.27, size = 41, normalized size = 1.52

$$\frac{a^{2}\left(c x^{2}\right)^{\frac{3}{2}}}{x^{2}}+\frac{ab\left(c x^{2}\right)^{\frac{3}{2}}}{x}+\frac{b^{2}\left(c x^{2}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*(c*x**2)**(3/2)/x**2 + a*b*(c*x**2)**(3/2)/x + b**2*(c*x**2)**(3/2)/3
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.22

$$\sqrt{c}\,c\left(-\frac{a^{3}\mathrm{sign}\left(x\right)}{3b}+\frac{\left(bx+a\right)^{3}\mathrm{sign}\left(x\right)}{3b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,x^{2}\right)^{3/2}\,\left(a+b\,x\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3, x)
```

3.819 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx$

Optimal. Leaf size=52

$$2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}+\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}$$

```
[Out] 2*a*b*c*(c*x^2)^(1/2)+1/2*b^2*c*x*(c*x^2)^(1/2)+a^2*c*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}+2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] 2*a*b*c*Sqrt[c*x^2] + (b^2*c*x*Sqrt[c*x^2])/2 + (a^2*c*Sqrt[c*x^2]*Log[x])/
x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}\\ &= 2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}+\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.65

$$\frac{(cx^2)^{3/2}\,(bx(4a+bx)+2a^2\log(x))}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
[Out] ((c*x^2)^(3/2)*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.63

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\,(x^2b^2+2a^2\ln(x)+4abx)}{2x^3}$	33
risch	$\frac{c\sqrt{c\,x^2}\;b(\frac{1}{2}x^2b+2ax)}{x}+\frac{a^2c\ln(x)\sqrt{c\,x^2}}{x}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(3/2)*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.67

$$\frac{\left(b^2cx^2 + 4\,abcx + 2\,a^2c\log\left(x\right)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] 1/2*(b^2*c*x^2 + 4*a*b*c*x + 2*a^2*c*log(x))*sqrt(c*x^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}\left(a + bx\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**4,x)
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**2/x**4, x)
```

Giac [A]
time = 0.00, size = 35, normalized size = 0.67

$$\sqrt{c}\,c\left(a^2\mathrm{sign}\left(x\right)\ln|x| + \frac{1}{2}b^2x^2\mathrm{sign}\left(x\right) + 2abx\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x)
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a + b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4, x)
```

3.820 $\int x (cx^2)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=66

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

```
[Out] 1/7*a^2*c^2*x^6*(c*x^2)^(1/2)+1/4*a*b*c^2*x^7*(c*x^2)^(1/2)+1/9*b^2*c^2*x^8
*(c*x^2)^(1/2)
```

Rubi [**A**]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^6*Sqrt[c*x^2])/7 + (a*b*c^2*x^7*Sqrt[c*x^2])/4 + (b^2*c^2*x^8*Sq
rt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x (cx^2)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^6+2abx^7+b^2x^8\right) dx}{x}\\
&= \frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

Mathics [A]
time = 2.06, size = 30, normalized size = 0.45

$$x^2\left(\frac{a^2}{7}+\frac{abx}{4}+\frac{b^2x^2}{9}\right)\left(cx^2\right)^{\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]')
[Out] x ^ 2 (a ^ 2 / 7 + a b x / 4 + b ^ 2 x ^ 2 / 9) (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result
gosper	$\frac{x^2\left(28x^2b^2+63abx+36a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{252}$
default	$\frac{x^2\left(28x^2b^2+63abx+36a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{252}$
risch	$\frac{a^2c^2x^6\sqrt{c\,x^2}}{7}+\frac{ab\,c^2x^7\sqrt{c\,x^2}}{4}+\frac{b^2c^2x^8\sqrt{c\,x^2}}{9}$
trager	$\frac{c^2\left(28b^2x^8+63ab\,x^7+28b^2x^7+36a^2x^6+63ab\,x^6+28b^2x^6+36a^2x^5+63ab\,x^5+28b^2x^5+36a^2x^4+63ab\,x^4+28b^2x^4+36a^2x^3+63ab\,x^3+28b\right.}{252x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/252*x^2*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.74

$$\frac{\left(cx^2\right)^{\frac{7}{2}}b^2x^2}{9\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}abx}{4\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}a^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(7/2)*b^2*x^2/c + 1/4*(c*x^2)^(7/2)*a*b*x/c + 1/7*(c*x^2)^(7/2)
*a^2/c
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.64

$$\frac{1}{252}\left(28\,b^2c^2x^8+63\,abc^2x^7+36\,a^2c^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c^2*x^8 + 63*a*b*c^2*x^7 + 36*a^2*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.41, size = 49, normalized size = 0.74

$$\frac{a^2x^2\left(cx^2\right)^{\frac{5}{2}}}{7}+\frac{abx^3\left(cx^2\right)^{\frac{5}{2}}}{4}+\frac{b^2x^4\left(cx^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**2*(c*x**2)**(5/2)/7 + a*b*x**3*(c*x**2)**(5/2)/4 + b**2*x**4*(c*x**
2)**(5/2)/9
```

Giac [A]
time = 0.00, size = 48, normalized size = 0.73

$$\sqrt{c}\left(\frac{1}{7}a^2c^2x^7\mathrm{sign}\left(x\right)+\frac{1}{9}b^2c^2x^9\mathrm{sign}\left(x\right)+\frac{1}{4}abc^2x^8\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*(28*b^2*c^2*x^9*sgn(x) + 63*a*b*c^2*x^8*sgn(x) + 36*a^2*c^2*x^7*sgn(x
))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x)^2, x)
```

3.821 $\int (cx^2)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=66

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

```
[Out] 1/6*a^2*c^2*x^5*(c*x^2)^(1/2)+2/7*a*b*c^2*x^6*(c*x^2)^(1/2)+1/8*b^2*c^2*x^7
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^5*Sqrt[c*x^2])/6 + (2*a*b*c^2*x^6*Sqrt[c*x^2])/7 + (b^2*c^2*x^7*
Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (cx^2)^{5/2}(a+bx)^2\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right)dx}{x}\\ &= \frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.50

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

Mathics [A]
time = 2.00, size = 29, normalized size = 0.44

$$\frac{x\left(28a^2+48abx+21b^2x^2\right)\left(cx^2\right)^{\frac{5}{2}}}{168}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]')
[Out] x (28 a ^ 2 + 48 a b x + 21 b ^ 2 x ^ 2) (c x ^ 2) ^ (5 / 2) / 168
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.45

method	result
gosper	$\frac{x\left(21x^2b^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{168}$
default	$\frac{x\left(21x^2b^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{168}$
risch	$\frac{a^2c^2x^5\sqrt{c\,x^2}}{6}+\frac{2ab\,c^2x^6\sqrt{c\,x^2}}{7}+\frac{b^2c^2x^7\sqrt{c\,x^2}}{8}$
trager	$\frac{c^2\left(21b^2x^7+48ab\,x^6+21b^2x^6+28a^2x^5+48ab\,x^5+21b^2x^5+28a^2x^4+48ab\,x^4+21b^2x^4+28a^2x^3+48ab\,x^3+21b^2x^3+28a^2x^2+48ab\,x^2+2\right.}{168x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/168*x*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 44, normalized size = 0.67

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}a^2x+\frac{\left(cx^2\right)^{\frac{7}{2}}b^2x}{8\,c}+\frac{2\left(cx^2\right)^{\frac{7}{2}}ab}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*a^2*x + 1/8*(c*x^2)^(7/2)*b^2*x/c + 2/7*(c*x^2)^(7/2)*a*b
/c
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.64

$$\frac{1}{168}\left(21\,b^2c^2x^7+48\,abc^2x^6+28\,a^2c^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c^2*x^7 + 48*a*b*c^2*x^6 + 28*a^2*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A]
time = 0.35, size = 49, normalized size = 0.74

$$\frac{a^2x\left(cx^2\right)^{\frac{5}{2}}}{6}+\frac{2abx^2\left(cx^2\right)^{\frac{5}{2}}}{7}+\frac{b^2x^3\left(cx^2\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x*(c*x**2)**(5/2)/6 + 2*a*b*x**2*(c*x**2)**(5/2)/7 + b**2*x**3*(c*x**2
)**(5/2)/8
```

Giac [A]
time = 0.00, size = 49, normalized size = 0.74

$$\sqrt{c}\,\left(\frac{1}{6}a^2c^2x^6\mathrm{sign}\left(x\right)+\frac{1}{8}b^2c^2x^8\mathrm{sign}\left(x\right)+\frac{2}{7}abc^2x^7\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*(21*b^2*c^2*x^8*sgn(x) + 48*a*b*c^2*x^7*sgn(x) + 28*a^2*c^2*x^6*sgn(x
))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(5/2)*(a + b*x)^2, x)
```

3.822 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=66

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

```
[Out] 1/5*a^2*c^2*x^4*(c*x^2)^(1/2)+1/3*a*b*c^2*x^5*(c*x^2)^(1/2)+1/7*b^2*c^2*x^6
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c^2*x^4*Sqrt[c*x^2])/5 + (a*b*c^2*x^5*Sqrt[c*x^2])/3 + (b^2*c^2*x^6*Sq
rt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\ &= \frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.55

$$\frac{1}{105}cx^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
[Out] (c*x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Mathics [A]
time = 2.00, size = 27, normalized size = 0.41

$$\left(\frac{a^2}{5}+\frac{abx}{3}+\frac{b^2x^2}{7}\right)\left(cx^2\right)^{\frac{5}{2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]')
[Out] (a ^ 2 / 5 + a b x / 3 + b ^ 2 x ^ 2 / 7) (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.11, size = 29, normalized size = 0.44

method	result
gosper	$\frac{\left(15x^2b^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
default	$\frac{\left(15x^2b^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
risch	$\frac{a^2c^2x^4\sqrt{c\,x^2}}{5}+\frac{ab\,c^2x^5\sqrt{c\,x^2}}{3}+\frac{b^2c^2x^6\sqrt{c\,x^2}}{7}$
trager	$\frac{c^2\left(15b^2x^6+35ab\,x^5+15b^2x^5+21a^2x^4+35ab\,x^4+15b^2x^4+21a^2x^3+35ab\,x^3+15b^2x^3+21a^2x^2+35ab\,x^2+15x^2b^2+21a^2x+35abx+15b^2x-\right.}{105x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/105*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.61

$$\frac{1}{3}\left(cx^2\right)^{\frac{5}{2}}abx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a^2+\frac{\left(cx^2\right)^{\frac{7}{2}}b^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2)^(5/2)*a*b*x + 1/5*(c*x^2)^(5/2)*a^2 + 1/7*(c*x^2)^(7/2)*b^2/c
```

Fricas [**A**]
time = 0.29, size = 42, normalized size = 0.64

$$\frac{1}{105}\left(15\,b^2c^2x^6+35\,abc^2x^5+21\,a^2c^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*c^2*x^6 + 35*a*b*c^2*x^5 + 21*a^2*c^2*x^4)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.36, size = 44, normalized size = 0.67

$$\frac{a^2\left(cx^2\right)^{\frac{5}{2}}}{5}+\frac{abx\left(cx^2\right)^{\frac{5}{2}}}{3}+\frac{b^2x^2\left(cx^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/5 + a*b*x*(c*x**2)**(5/2)/3 + b**2*x**2*(c*x**2)**(5/2
)/7
```

Giac [**A**]
time = 0.00, size = 48, normalized size = 0.73

$$\sqrt{c}\,\left(\frac{1}{5}a^2c^2x^5\mathrm{sign}\left(x\right)+\frac{1}{7}b^2c^2x^7\mathrm{sign}\left(x\right)+\frac{1}{3}abc^2x^6\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x)
```

```
[Out] 1/105*(15*b^2*c^2*x^7*sgn(x) + 35*a*b*c^2*x^6*sgn(x) + 21*a^2*c^2*x^5*sgn(x
))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x, x)
```

3.823 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

```
[Out] 1/4*a^2*c^2*x^3*(c*x^2)^(1/2)+2/5*a*b*c^2*x^4*(c*x^2)^(1/2)+1/6*b^2*c^2*x^5
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*c^2*x^4*Sqrt[c*x^2])/5 + (b^2*c^2*x^5*
Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^2\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx}{x}\\&=\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.52

$$\frac{1}{60}cx\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
[Out] (c*x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Mathics [A]
time = 2.02, size = 29, normalized size = 0.44

$$\frac{\left(\frac{a^2}{4}+\frac{bx(12a+5bx)}{30}\right)\left(cx^2\right)^{\frac{5}{2}}}{x}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]')
[Out] (a ^ 2 / 4 + b x (12 a + 5 b x) / 30) (c x ^ 2) ^ (5 / 2) / x
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.48

method	result
gosper	$\frac{\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{60x}$
default	$\frac{\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{60x}$
risch	$\frac{a^2c^2x^3\sqrt{c\,x^2}}{4}+\frac{2ab\,c^2x^4\sqrt{c\,x^2}}{5}+\frac{b^2c^2x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{c^2\left(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24ab\,x^3+10b^2x^3+15a^2x^2+24ab\,x^2+10x^2b^2+15a^2x+24abx+10b^2x+15a^2+24ab+10b^2\right)(-1+}{60x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/60/x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.61

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}b^2x+\frac{2}{5}\left(cx^2\right)^{\frac{5}{2}}ab+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*b^2*x + 2/5*(c*x^2)^(5/2)*a*b + 1/4*(c*x^2)^(5/2)*a^2/x
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.64

$$\frac{1}{60}\left(10\,b^2c^2x^5+24\,abc^2x^4+15\,a^2c^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c^2*x^5 + 24*a*b*c^2*x^4 + 15*a^2*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.36, size = 44, normalized size = 0.67

$$\frac{a^2\left(cx^2\right)^{\frac{5}{2}}}{4x}+\frac{2ab\left(cx^2\right)^{\frac{5}{2}}}{5}+\frac{b^2x\left(cx^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/(4*x) + 2*a*b*(c*x**2)**(5/2)/5 + b**2*x*(c*x**2)**(5/
2)/6
```

Giac [A]
time = 0.00, size = 49, normalized size = 0.74

$$\sqrt{c}\,\left(\frac{1}{4}a^2c^2x^4\mathrm{sign}\left(x\right)+\frac{1}{6}b^2c^2x^6\mathrm{sign}\left(x\right)+\frac{2}{5}abc^2x^5\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x)
```

```
[Out] 1/60*(10*b^2*c^2*x^6*sgn(x) + 24*a*b*c^2*x^5*sgn(x) + 15*a^2*c^2*x^4*sgn(x)
)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2, x)
```

3.824 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=66

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

```
[Out] 1/3*a^2*c^2*x^2*(c*x^2)^(1/2)+1/2*a*b*c^2*x^3*(c*x^2)^(1/2)+1/5*b^2*c^2*x^4
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (a^2*c^2*x^2*Sqrt[c*x^2])/3 + (a*b*c^2*x^3*Sqrt[c*x^2])/2 + (b^2*c^2*x^4*Sq
rt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\ &= \frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 0.58

$$\frac{1}{30}c^2x^2\sqrt{cx^2}\,\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
[Out] (c^2*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Mathics [A]
time = 2.00, size = 30, normalized size = 0.45

$$\frac{\left(\frac{a^2}{3}+\frac{abx}{2}+\frac{b^2x^2}{5}\right)(cx^2)^{\frac{5}{2}}}{x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]')
[Out] (a ^ 2 / 3 + a b x / 2 + b ^ 2 x ^ 2 / 5) (c x ^ 2) ^ (5 / 2) / x ^ 2
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result	size
gosper	$\frac{(6x^2b^2+15abx+10a^2)(cx^2)^{\frac{5}{2}}}{30x^2}$	32
default	$\frac{(6x^2b^2+15abx+10a^2)(cx^2)^{\frac{5}{2}}}{30x^2}$	32
risch	$\frac{a^2c^2x^2\sqrt{cx^2}}{3}+\frac{ab\,c^2x^3\sqrt{cx^2}}{2}+\frac{b^2c^2x^4\sqrt{cx^2}}{5}$	55
trager	$\frac{c^2(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2)(-1+x)\sqrt{cx^2}}{30x}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
[Out] 1/30/x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.64

$$\frac{1}{30}\left(6\,b^2c^2x^4+15\,abc^2x^3+10\,a^2c^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*c^2*x^4 + 15*a*b*c^2*x^3 + 10*a^2*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.44, size = 44, normalized size = 0.67

$$\frac{a^2\left(cx^2\right)^{\frac{5}{2}}}{3x^2}+\frac{ab\left(cx^2\right)^{\frac{5}{2}}}{2x}+\frac{b^2\left(cx^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/(3*x**2) + a*b*(c*x**2)**(5/2)/(2*x) + b**2*(c*x**2)**
(5/2)/5
```

Giac [A]
time = 0.00, size = 48, normalized size = 0.73

$$\sqrt{c}\left(\frac{1}{3}a^2c^2x^3\mathrm{sign}\left(x\right)+\frac{1}{5}b^2c^2x^5\mathrm{sign}\left(x\right)+\frac{1}{2}abc^2x^4\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/30*(6*b^2*c^2*x^5*sgn(x) + 15*a*b*c^2*x^4*sgn(x) + 10*a^2*c^2*x^3*sgn(x))
*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3, x)
```

3.825 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=64

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

```
[Out] 1/2*a^2*c^2*x*(c*x^2)^(1/2)+2/3*a*b*c^2*x^2*(c*x^2)^(1/2)+1/4*b^2*c^2*x^3*(
c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c*x^2])/2 + (2*a*b*c^2*x^2*Sqrt[c*x^2])/3 + (b^2*c^2*x^3*Sq
rt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)\,dx}{x}\\
&= \frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.56

$$\frac{1}{12}c^2x\sqrt{cx^2}\,\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Mathics [A]
time = 2.03, size = 31, normalized size = 0.48

$$\frac{(6a^2+8abx+3b^2x^2)\,(cx^2)^{\frac{5}{2}}}{12x^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]')
```

```
[Out] (6 a ^ 2 + 8 a b x + 3 b ^ 2 x ^ 2) (c x ^ 2) ^ (5 / 2) / (12 x ^ 3)
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.50

method	result	size
gosper	$\frac{(3x^2b^2+8abx+6a^2)\,(c\,x^2)^{\frac{5}{2}}}{12x^3}$	32
default	$\frac{(3x^2b^2+8abx+6a^2)\,(c\,x^2)^{\frac{5}{2}}}{12x^3}$	32
risch	$\frac{a^2c^2x\sqrt{c\,x^2}}{2}+\frac{2ab\,c^2x^2\sqrt{c\,x^2}}{3}+\frac{b^2c^2x^3\sqrt{c\,x^2}}{4}$	53
trager	$\frac{c^2\,(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2)\,(-1+x)\sqrt{c\,x^2}}{12x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/x^3*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 40, normalized size = 0.62

$$\frac{1}{12}\left(3\,b^2c^2x^3+8\,abc^2x^2+6\,a^2c^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^2*x^3 + 8*a*b*c^2*x^2 + 6*a^2*c^2*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.43, size = 49, normalized size = 0.77

$$\frac{a^2\left(cx^2\right)^{\frac{5}{2}}}{2x^3}+\frac{2ab\left(cx^2\right)^{\frac{5}{2}}}{3x^2}+\frac{b^2\left(cx^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**4,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/(2*x**3) + 2*a*b*(c*x**2)**(5/2)/(3*x**2) + b**2*(c*x*
*2)**(5/2)/(4*x)
```

Giac [A]
time = 0.00, size = 49, normalized size = 0.77

$$\sqrt{c}\,\left(\frac{1}{2}a^2c^2x^2\mathrm{sign}\left(x\right)+\frac{1}{4}b^2c^2x^4\mathrm{sign}\left(x\right)+\frac{2}{3}abc^2x^3\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x)
```

```
[Out] 1/12*(3*b^2*c^2*x^4*sgn(x) + 8*a*b*c^2*x^3*sgn(x) + 6*a^2*c^2*x^2*sgn(x))*s
qrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4, x)
```

3.826 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5} dx$

Optimal. Leaf size=29

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

```
[Out] 1/3*c^2*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Rubi [A]

time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 26, normalized size = 0.90

$$\frac{(cx^2)^{5/2}\,(a+bx)^3}{3bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(a + b*x)^3)/(3*b*x^5)
```

Mathics [**A**]
time = 2.03, size = 27, normalized size = 0.93

$$\frac{\left(a^2 + abx + \frac{b^2x^2}{3}\right)(cx^2)^{\frac{5}{2}}}{x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]')
```

```
[Out] (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3) (c x ^ 2) ^ (5 / 2) / x ^ 4
```

Maple [**A**]
time = 0.12, size = 23, normalized size = 0.79

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}(bx+a)^3}{3x^5b}$	23
risch	$\frac{c^2(bx+a)^3\sqrt{c\,x^2}}{3bx}$	26
gosper	$\frac{(x^2b^2+3abx+3a^2)(c\,x^2)^{\frac{5}{2}}}{3x^4}$	31
trager	$\frac{c^2(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c\,x^2}}{3x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2)^(5/2)/x^5*(b*x+a)^3/b
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 36, normalized size = 1.24

$$\frac{1}{3}\left(b^2c^2x^2 + 3\,abc^2x + 3\,a^2c^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^2 + 3*a*b*c^2*x + 3*a^2*c^2)*sqrt(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.44, size = 46, normalized size = 1.59

$$\frac{a^2\,(cx^2)^{\frac{5}{2}}}{x^4} + \frac{ab\,(cx^2)^{\frac{5}{2}}}{x^3} + \frac{b^2\,(cx^2)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**5,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/x**4 + a*b*(c*x**2)**(5/2)/x**3 + b**2*(c*x**2)**(5/2)
/(3*x**2)
```

Giac [A]
time = 0.00, size = 42, normalized size = 1.45

$$\sqrt{c}\,\left(a^2c^2x\mathrm{sign}\,(x) + \frac{1}{3}b^2c^2x^3\mathrm{sign}\,(x) + abc^2x^2\mathrm{sign}\,(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x)
```

```
[Out] 1/3*(b^2*c^2*x^3*sgn(x) + 3*a*b*c^2*x^2*sgn(x) + 3*a^2*c^2*x*sgn(x))*sqrt(c
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5, x)
```

3.827 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6} dx$

Optimal. Leaf size=58

$$2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2} + \frac{a^2c^2\sqrt{cx^2}\log(x)}{x}$$

```
[Out] 2*a*b*c^2*(c*x^2)^(1/2)+1/2*b^2*c^2*x*(c*x^2)^(1/2)+a^2*c^2*ln(x)*(c*x^2)^(
1/2)/x
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c^2\sqrt{cx^2}\log(x)}{x} + 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] 2*a*b*c^2*Sqrt[c*x^2] + (b^2*c^2*x*Sqrt[c*x^2])/2 + (a^2*c^2*Sqrt[c*x^2]*Lo
g[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x} dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right) dx}{x} \\
&= 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2} + \frac{a^2c^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.60

$$\frac{c^3x\left(bx(4a+bx)+2a^2\log(x)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] (c^3*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.57

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(x^{2}b^{2}+2a^{2}\ln\left(x\right)+4abx\right)}{2x^{5}}$	33
risch	$\frac{c^{2}\sqrt{c\,x^{2}}\,b\left(\frac{1}{2}x^{2}b+2ax\right)}{x}+\frac{a^{2}c^{2}\ln\left(x\right)\sqrt{c\,x^{2}}}{x}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(5/2)*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/x^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.29, size = 41, normalized size = 0.71

$$\frac{\left(b^2c^2x^2 + 4\,abc^2x + 2\,a^2c^2\log\left(x\right)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="fricas")
[Out] 1/2*(b^2*c^2*x^2 + 4*a*b*c^2*x + 2*a^2*c^2*log(x))*sqrt(c*x^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}\left(a+bx\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**6,x)
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**2/x**6, x)
```

Giac [A]
time = 0.00, size = 43, normalized size = 0.74

$$\sqrt{c}\,\left(a^2c^2\mathrm{sign}\left(x\right)\ln\left|x\right| + \frac{1}{2}b^2c^2x^2\mathrm{sign}\left(x\right) + 2abc^2x\mathrm{sign}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x)
[Out] 1/2*(b^2*c^2*x^2*sgn(x) + 4*a*b*c^2*x*sgn(x) + 2*a^2*c^2*log(abs(x))*sgn(x)
)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6, x)
```

3.828 $\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

```
[Out] 1/3*a^2*x^4/(c*x^2)^(1/2)+1/2*a*b*x^5/(c*x^2)^(1/2)+1/5*b^2*x^6/(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^4)/(3*Sqrt[c*x^2]) + (a*b*x^5)/(2*Sqrt[c*x^2]) + (b^2*x^6)/(5*Sqrt[c
*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)^2\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx}{\sqrt{cx^2}}\\
&= \frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{x^4 (10a^2 + 15abx + 6b^2x^2)}{30\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^4*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*Sqrt[c*x^2])
```

Mathics [A]
time = 1.92, size = 30, normalized size = 0.53

$$\frac{x^4 \left(\frac{a^2}{3} + \frac{abx}{2} + \frac{b^2x^2}{5}\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]')
[Out] x ^ 4 (a ^ 2 / 3 + a b x / 2 + b ^ 2 x ^ 2 / 5) / Sqrt[c x ^ 2]
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.56

method	result	size
gosper	$\frac{x^4(6x^2b^2+15abx+10a^2)}{30\sqrt{c\,x^2}}$	32
default	$\frac{x^4(6x^2b^2+15abx+10a^2)}{30\sqrt{c\,x^2}}$	32
risch	$\frac{a^2x^4}{3\sqrt{c\,x^2}}+\frac{ab\,x^5}{2\sqrt{c\,x^2}}+\frac{b^2x^6}{5\sqrt{c\,x^2}}$	46
trager	$\frac{(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2)(-1+x)\sqrt{c\,x^2}}{30cx}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*x^4*(6*b^2*x^2+15*a*b*x+10*a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.95

$$\frac{\sqrt{cx^2}\,b^2x^4}{5\,c} + \frac{\sqrt{cx^2}\,abx^3}{2\,c} + \frac{\sqrt{cx^2}\,a^2x^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2)*b^2*x^4/c + 1/2*sqrt(c*x^2)*a*b*x^3/c + 1/3*sqrt(c*x^2)*a^2
*x^2/c
```

Fricas [A]
time = 0.29, size = 36, normalized size = 0.63

$$\frac{(6\,b^2x^4+15\,abx^3+10\,a^2x^2)\sqrt{cx^2}}{30\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.27, size = 49, normalized size = 0.86

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4/(3*sqrt(c*x**2)) + a*b*x**5/(2*sqrt(c*x**2)) + b**2*x**6/(5*sqrt(
c*x**2))
```

Giac [A]
time = 0.00, size = 37, normalized size = 0.65

$$\frac{\frac{1}{5}b^2x^5+\frac{1}{2}abx^4+\frac{1}{3}a^2x^3}{\sqrt{c}\,\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/30*(6*b^2*x^5 + 15*a*b*x^4 + 10*a^2*x^3)/(sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^3\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

3.829 $\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}$$

```
[Out] 1/2*a^2*x^3/(c*x^2)^(1/2)+2/3*a*b*x^4/(c*x^2)^(1/2)+1/4*b^2*x^5/(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^3)/(2*Sqrt[c*x^2]) + (2*a*b*x^4)/(3*Sqrt[c*x^2]) + (b^2*x^5)/(4*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(a^2x+2abx^2+b^2x^3\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{x^3\left(6a^2+8abx+3b^2x^2\right)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^3*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*Sqrt[c*x^2])
```

Mathics [A]
time = 1.88, size = 31, normalized size = 0.54

$$\frac{x^3\left(6a^2+8abx+3b^2x^2\right)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]')
[Out] x ^ 3 (6 a ^ 2 + 8 a b x + 3 b ^ 2 x ^ 2) / (12 Sqrt[c x ^ 2])
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.56

method	result	size
gosper	$\frac{x^3(3x^2b^2+8abx+6a^2)}{12\sqrt{c\,x^2}}$	32
default	$\frac{x^3(3x^2b^2+8abx+6a^2)}{12\sqrt{c\,x^2}}$	32
risch	$\frac{a^2x^3}{2\sqrt{c\,x^2}}+\frac{2ab\,x^4}{3\sqrt{c\,x^2}}+\frac{b^2x^5}{4\sqrt{c\,x^2}}$	46
trager	$\frac{(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2)(-1+x)\sqrt{c\,x^2}}{12cx}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x^3*(3*b^2*x^2+8*a*b*x+6*a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 47, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,b^2x^3}{4\,c}+\frac{2\,\sqrt{cx^2}\,abx^2}{3\,c}+\frac{a^2x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b^2*x^3/c + 2/3*sqrt(c*x^2)*a*b*x^2/c + 1/2*a^2*x^2/sqrt(c)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.60

$$\frac{(3\,b^2x^3 + 8\,abx^2 + 6\,a^2x)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.25, size = 51, normalized size = 0.89

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3/(2*sqrt(c*x**2)) + 2*a*b*x**4/(3*sqrt(c*x**2)) + b**2*x**5/(4*sqr
t(c*x**2))
```

Giac [A]
time = 0.00, size = 38, normalized size = 0.67

$$\frac{\frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}a^2x^2}{\sqrt{c}\,\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)/(sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

3.830 $\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

[Out] 1/3*x*(b*x+a)^3/b/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x)^2)/Sqrt[c*x^2],x]

[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Mathics [A]
time = 1.84, size = 27, normalized size = 1.12

$$\frac{x^2\left(a^2+abx+\frac{b^2x^2}{3}\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^1*(a + b*x)^2)/Sqrt[c*x^2],x]')
[Out] x ^ 2 (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3) / Sqrt[c x ^ 2]
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.88

method	result	size
default	$\frac{x(bx+a)^3}{3b\sqrt{c\,x^2}}$	21
risch	$\frac{x(bx+a)^3}{3b\sqrt{c\,x^2}}$	21
gosper	$\frac{x^2(x^2b^2+3abx+3a^2)}{3\sqrt{c\,x^2}}$	31
trager	$\frac{(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c\,x^2}}{3cx}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(b*x+a)^3/b/(c*x^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 0.26, size = 42, normalized size = 1.75

$$\frac{\sqrt{cx^2}\,b^2x^2}{3\,c}+\frac{abx^2}{\sqrt{c}}+\frac{\sqrt{cx^2}\,a^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(c*x^2)*b^2*x^2/c + a*b*x^2/sqrt(c) + sqrt(c*x^2)*a^2/c
```

Fricas [A]
time = 0.28, size = 30, normalized size = 1.25

$$\frac{\left(b^{2} x^{2} + 3 \, a b x + 3 \, a^{2}\right) \sqrt{c x^{2}}}{3 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.23, size = 46, normalized size = 1.92

$$\frac{a^{2} x^{2}}{\sqrt{c x^{2}}} + \frac{a b x^{3}}{\sqrt{c x^{2}}} + \frac{b^{2} x^{4}}{3 \sqrt{c x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**2/sqrt(c*x**2) + a*b*x**3/sqrt(c*x**2) + b**2*x**4/(3*sqrt(c*x**2))
```

Giac [A]
time = 0.00, size = 31, normalized size = 1.29

$$\frac{\frac{1}{3} \, b^{2} x^{3} + a b x^{2} + a^{2} x}{\sqrt{c} \mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/(sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

3.831 $\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}$$

```
[Out] 2*a*b*x^2/(c*x^2)^(1/2)+1/2*b^2*x^3/(c*x^2)^(1/2)+a^2*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 45}

$$\frac{a^2x\log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (2*a*b*x^2)/Sqrt[c*x^2] + (b^2*x^3)/(2*Sqrt[c*x^2]) + (a^2*x*Log[x])/Sqrt[c
*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 0.62

$$\frac{x(bx(4a+bx)+2a^2\log(x))}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c*x^2],x]
[Out] (x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/Sqrt[c*x^2],x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.12, size = 31, normalized size = 0.60

method	result	size
default	$\frac{x(x^2b^2+2a^2\ln(x)+4abx)}{2\sqrt{cx^2}}$	31
risch	$\frac{xb(\frac{1}{2}x^2b+2ax)}{\sqrt{cx^2}}+\frac{a^2x\ln(x)}{\sqrt{cx^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.67

$$\frac{b^2x^2}{2\sqrt{c}}+\frac{a^2\log(x)}{\sqrt{c}}+\frac{2\sqrt{cx^2}ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*b^2*x^2/sqrt(c) + a^2*log(x)/sqrt(c) + 2*sqrt(c*x^2)*a*b/c
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.67

$$\frac{\left(b^{2} x^{2} + 4 \, a b x + 2 \, a^{2} \log\left(x\right)\right) \sqrt{c x^{2}}}{2 \, c x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b x\right)^{2}}{\sqrt{c x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/sqrt(c*x**2), x)
```

Giac [A]
time = 0.00, size = 43, normalized size = 0.83

$$\frac{\frac{\frac{1}{2} \, b^{2} x^{2} \mathrm{sign}\left(x\right) + 2 \, a b x \mathrm{sign}\left(x\right)}{\mathrm{sign}\left(x\right)^{2}} + \frac{a^{2} \ln\left|x\right|}{\mathrm{sign}\left(x\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*(2*a^2*log(abs(x))/sgn(x) + (b^2*x^2*sgn(x) + 4*a*b*x*sgn(x))/sgn(x)^2)
/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(1/2),x)
```

```
[Out] int((a + b*x)^2/(c*x^2)^(1/2), x)
```

3.832 $\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}$$

```
[Out] -a^2/(c*x^2)^(1/2)+b^2*x^2/(c*x^2)^(1/2)+2*a*b*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a^2/Sqrt[c*x^2]) + (b^2*x^2)/Sqrt[c*x^2] + (2*a*b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx&=\frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{\sqrt{cx^2}}\\&=\frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx}{\sqrt{cx^2}}\\&=-\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.72

$$\frac{cx^2\left(-a^2+b^2x^2+2abx\log(x)\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] (c*x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.62

method	result	size
default	$\frac{2ab\ln(x)x+x^2b^2-a^2}{\sqrt{c\,x^2}}$	29
risch	$-\frac{a^2}{\sqrt{c\,x^2}}+\frac{b^2x^2}{\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{\sqrt{c\,x^2}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*a*b*ln(x)*x+x^2*b^2-a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.74

$$\frac{2\,ab\log\left(x\right)}{\sqrt{c}}+\frac{\sqrt{cx^2}\,b^2}{c}-\frac{a^2}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*a*b*log(x)/sqrt(c) + sqrt(c*x^2)*b^2/c - a^2/(sqrt(c)*x)
```

Fricas [A]
time = 0.31, size = 34, normalized size = 0.72

$$\frac{(b^2x^2 + 2\,abx\log{(x)} - a^2)\sqrt{cx^2}}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^2}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)**2/(x*sqrt(c*x**2)), x)
```

Giac [A]
time = 0.00, size = 36, normalized size = 0.77

$$\frac{-\frac{a^2}{x\mathrm{sign}(x)} + \frac{b^2x}{\mathrm{sign}(x)} + \frac{2ab\ln|x|}{\mathrm{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x)
[Out] (b^2*x/sgn(x) + 2*a*b*log(abs(x))/sgn(x) - a^2/(x*sgn(x)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x)^2}{x\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(1/2)),x)
[Out] int((a + b*x)^2/(x*(c*x^2)^(1/2)), x)
```

3.833 $\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

```
[Out] -2*a*b/(c*x^2)^(1/2)-1/2*a^2/x/(c*x^2)^(1/2)+b^2*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (-2*a*b)/Sqrt[c*x^2] - a^2/(2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.71

$$\frac{cx\left(-a(a+4bx)+2b^2x^2\log(x)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
[Out] (c*x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.69

method	result	size
default	$\frac{2b^2\ln(x)x^2-4abx-a^2}{2x\sqrt{c\,x^2}}$	34
risch	$\frac{-\frac{1}{2}a^2-2abx}{\sqrt{c\,x^2}\,x}+\frac{b^2x\ln(x)}{\sqrt{c\,x^2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 31, normalized size = 0.63

$$\frac{b^2\log(x)}{\sqrt{c}}-\frac{2\,ab}{\sqrt{c}\,x}-\frac{a^2}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b^2*log(x)/sqrt(c) - 2*a*b/(sqrt(c)*x) - 1/2*a^2/(sqrt(c)*x^2)
```

Fricas [A]
time = 0.30, size = 36, normalized size = 0.73

$$\frac{\left(2\, b^{2} x^{2} \log\left(x\right) - 4\, a b x - a^{2}\right) \sqrt{c x^{2}}}{2\, c x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx\right)^{2}}{x^{2} \sqrt{cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x**2*sqrt(c*x**2)), x)
```

Giac [A]
time = 0.00, size = 39, normalized size = 0.80

$$\frac{\frac{-4abx-a^{2}}{2(x^{2}\mathrm{sign}(x))} + \frac{b^{2}\ln|x|}{\mathrm{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*(2*b^2*log(abs(x))/sgn(x) - (4*a*b*x + a^2)/(x^2*sgn(x)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,x\right)}^2}{x^2\,\sqrt{c\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)), x)
```

3.834 $\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

```
[Out] -1/3*(b*x+a)^3/a/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -1/3*(a + b*x)^3/(a*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.27

$$\frac{c(-a^2 - 3abx - 3b^2x^2)}{3(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] (c*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.88, size = 29, normalized size = 1.12

$$\frac{c(-a^2 - 3abx - 3b^2x^2)}{3(cx^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]')
```

```
[Out] c (-a ^ 2 - 3 a b x - 3 b ^ 2 x ^ 2) / (3 (c x ^ 2) ^ (3 / 2))
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.15

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
default	$-\frac{3x^2b^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{x^2\sqrt{c\,x^2}}$	31
trager	$\frac{(-1+x)(a^2x^2+3ab\,x^2+3x^2b^2+a^2x+3abx+a^2)\sqrt{c\,x^2}}{3c\,x^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^2/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.25, size = 33, normalized size = 1.27

$$-\frac{b^2}{\sqrt{c}\,x} - \frac{ab}{\sqrt{c}\,x^2} - \frac{a^2}{3\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c)*x) - a*b/(sqrt(c)*x^2) - 1/3*a^2/(sqrt(c)*x^3)
```

Fricas [A]
time = 0.30, size = 32, normalized size = 1.23

$$-\frac{(3\,b^2x^2+3\,abx+a^2)\sqrt{cx^2}}{3\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c*x^4)
```

Sympy [A]
time = 0.26, size = 42, normalized size = 1.62

$$-\frac{a^2}{3x^2\sqrt{cx^2}}-\frac{ab}{x\sqrt{cx^2}}-\frac{b^2}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(3*x**2*sqrt(c*x**2)) - a*b/(x*sqrt(c*x**2)) - b**2/sqrt(c*x**2)
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.38

$$\frac{-3b^2x^2-3abx-a^2}{\sqrt{c}\cdot 3\left(x^3\mathrm{sign}\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(sqrt(c)*x^3*sgn(x))
```

Mupad [B]
time = 0.18, size = 33, normalized size = 1.27

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,\sqrt{c}\,\left(x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(1/2)),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(1/2)*(x^2)^(5/2))
```

3.835 $\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

```
[Out] -1/4*a^2/x^3/(c*x^2)^(1/2)-2/3*a*b/x^2/(c*x^2)^(1/2)-1/2*b^2/x/(c*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -1/4*a^2/(x^3*Sqrt[c*x^2]) - (2*a*b)/(3*x^2*Sqrt[c*x^2]) - b^2/(2*x*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)\,dx}{\sqrt{cx^2}}$$
$$= -\frac{a^2}{4x^3\sqrt{cx^2}} - \frac{2ab}{3x^2\sqrt{cx^2}} - \frac{b^2}{2x\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.61

$$\frac{-3a^2-8abx-6b^2x^2}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x^3*Sqrt[c*x^2])
```

Mathics [A]
time = 1.96, size = 32, normalized size = 0.56

$$\frac{c\,(-3a^2-8abx-6b^2x^2)}{12x(cx^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]')
[Out] c (-3 a ^ 2 - 8 a b x - 6 b ^ 2 x ^ 2) / (12 x (c x ^ 2) ^ (3 / 2))
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.56

method	result	size
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^3\sqrt{c\,x^2}}$	31
gosper	$-\frac{6x^2b^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32
default	$-\frac{6x^2b^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32

trager	$\frac{(-1+x)(3a^2x^3+8ab\,x^3+6b^2x^3+3a^2x^2+8ab\,x^2+6x^2b^2+3a^2x+8abx+3a^2)\sqrt{c\,x^2}}{12c\,x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x^3/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.58

$$-\frac{b^2}{2\sqrt{c}\,x^2}-\frac{2\,ab}{3\sqrt{c}\,x^3}-\frac{a^2}{4\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(sqrt(c)*x^2) - 2/3*a*b/(sqrt(c)*x^3) - 1/4*a^2/(sqrt(c)*x^4)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.60

$$-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c*x^5)
```

Sympy [A]
time = 0.29, size = 51, normalized size = 0.89

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(4*x**3*sqrt(c*x**2)) - 2*a*b/(3*x**2*sqrt(c*x**2)) - b**2/(2*x*sqrt(
c*x**2))
```

Giac [A]
time = 0.00, size = 38, normalized size = 0.67

$$\frac{-6b^2x^2-8abx-3a^2}{\sqrt{c}\cdot 12\,(x^4\text{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x)
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/(sqrt(c)*x^4*sgn(x))
```

Mupad [B]
time = 0.19, size = 42, normalized size = 0.74

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(1/2)),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(1
/2)*x^5)
```

3.836 $\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

[Out] 1/3*x*(b*x+a)^3/b/c/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]

[Out] (x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{x^3(a+bx)^3}{3b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^3)/(3*b*(c*x^2)^(3/2))
```

Mathics [A]
time = 1.90, size = 27, normalized size = 1.00

$$\frac{x^4\left(a^2+abx+\frac{b^2x^2}{3}\right)}{(cx^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]')
```

```
[Out] x ^ 4 (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3) / (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.85

method	result	size
default	$\frac{(bx+a)^3x^3}{3(c\,x^2)^{\frac{3}{2}}b}$	23
risch	$\frac{x(bx+a)^3}{3bc\sqrt{c\,x^2}}$	24
gosper	$\frac{x^4(x^2b^2+3abx+3a^2)}{3(c\,x^2)^{\frac{3}{2}}}$	31
trager	$\frac{(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c\,x^2}}{3c^2x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/(c*x^2)^(3/2)*x^3/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(23) = 46.
time = 0.26, size = 52, normalized size = 1.93

$$\frac{b^2x^4}{3\sqrt{cx^2}\,c}+\frac{abx^3}{\sqrt{cx^2}\,c}+\frac{a^2x^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^4/(sqrt(c*x^2)*c) + a*b*x^3/(sqrt(c*x^2)*c) + a^2*x^2/(sqrt(c*x^2
)*c)
```

Fricas [A]
time = 0.29, size = 30, normalized size = 1.11

$$\frac{(b^2x^2 + 3\,abx + 3\,a^2)\sqrt{cx^2}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.
time = 0.28, size = 46, normalized size = 1.70

$$\frac{a^2x^4}{(cx^2)^{\frac{3}{2}}} + \frac{abx^5}{(cx^2)^{\frac{3}{2}}} + \frac{b^2x^6}{3\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(3/2),x)
[Out] a**2*x**4/(c*x**2)**(3/2) + a*b*x**5/(c*x**2)**(3/2) + b**2*x**6/(3*(c*x**2
)**(3/2))
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.22

$$\frac{\frac{1}{3}b^2x^3 + abx^2 + a^2x}{\sqrt{c}\,c\mathrm{sign}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x)
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/(c^(3/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3\,{\left(a+b\,x\right)}^2}{{\left(c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2),x)
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

3.837 $\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}$$

```
[Out] 2*a*b*x^2/c/(c*x^2)^(1/2)+1/2*b^2*x^3/c/(c*x^2)^(1/2)+a^2*x*ln(x)/c/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (2*a*b*x^2)/(c*Sqrt[c*x^2]) + (b^2*x^3)/(2*c*Sqrt[c*x^2]) + (a^2*x*Log[x])/
(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx}{c\sqrt{cx^2}}\\
&= \frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.56

$$\frac{x^3 (bx(4a + bx) + 2a^2 \log(x))}{2 (cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x^3*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.54

method	result	size
default	$\frac{x^3 (x^2 b^2+2a^2 \ln(x)+4abx)}{2(c x^2)^{\frac{3}{2}}}$	33
risch	$\frac{xb(\frac{1}{2}x^2 b+2ax)}{c\sqrt{c x^2}}+\frac{a^2 x \ln(x)}{c\sqrt{c x^2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^3*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.74

$$\frac{b^2 x^3}{2 \sqrt{cx^2} c} + \frac{2 abx^2}{\sqrt{cx^2} c} + \frac{a^2 \log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*b^2*x^3/(sqrt(c*x^2)*c) + 2*a*b*x^2/(sqrt(c*x^2)*c) + a^2*log(x)/c^(3/2
)
```

Fricas [A]
time = 0.29, size = 35, normalized size = 0.57

$$\frac{(b^2x^2 + 4\,abx + 2\,a^2\log{(x)})\sqrt{cx^2}}{2\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 45, normalized size = 0.74

$$\frac{\frac{\frac{1}{2}b^2x^2\mathrm{sign}(x)+2abx\mathrm{sign}(x)}{\mathrm{sign}(x)^2}+\frac{a^2\ln|x|}{\mathrm{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*(2*a^2*log(abs(x))/sgn(x) + (b^2*x^2*sgn(x) + 4*a*b*x*sgn(x))/sgn(x)^2)
/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^2\,(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

3.838 $\int \frac{x(a+bx)^2}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}$$

```
[Out] -a^2/c/(c*x^2)^(1/2)+b^2*x^2/c/(c*x^2)^(1/2)+2*a*b*x*ln(x)/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -(a^2/(c*Sqrt[c*x^2])) + (b^2*x^2)/(c*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(c*Sq
rt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a^2}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 0.59

$$\frac{x^2(-a^2+b^2x^2+2abx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^1*(a + b*x)^2)/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.57

method	result	size
default	$\frac{x^2(2ab\ln(x)x+x^2b^2-a^2)}{(c\,x^2)^{\frac{3}{2}}}$	32
risch	$-\frac{a^2}{c\sqrt{c\,x^2}}+\frac{b^2x^2}{c\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{c\sqrt{c\,x^2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] x^2*(2*a*b*ln(x)*x+x^2*b^2-a^2)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 42, normalized size = 0.75

$$\frac{b^2x^2}{\sqrt{cx^2}\,c}+\frac{2\,ab\log(x)}{c^{\frac{3}{2}}}-\frac{a^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b^2*x^2/(sqrt(c*x^2)*c) + 2*a*b*log(x)/c^(3/2) - a^2/(sqrt(c*x^2)*c)
```

Fricas [A]
time = 0.30, size = 34, normalized size = 0.61

$$\frac{(b^2x^2 + 2\,abx\log\left(x\right) - a^2)\sqrt{cx^2}}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(a + bx\right)^2}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 38, normalized size = 0.68

$$\frac{-\frac{a^2}{x\mathrm{sign}(x)} + \frac{b^2x}{\mathrm{sign}(x)} + \frac{2ab\ln|x|}{\mathrm{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] (b^2*x/sgn(x) + 2*a*b*log(abs(x))/sgn(x) - a^2/(x*sgn(x)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x\left(a + b\,x\right)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

3.839 $\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2ab}{c\sqrt{cx^2}}-\frac{a^2}{2cx\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

```
[Out] -2*a*b/c/(c*x^2)^(1/2)-1/2*a^2/c/x/(c*x^2)^(1/2)+b^2*x*ln(x)/c/(c*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 45}

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (-2*a*b)/(c*Sqrt[c*x^2]) - a^2/(2*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{2ab}{c\sqrt{cx^2}} - \frac{a^2}{2cx\sqrt{cx^2}} + \frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.59

$$\frac{x\left(-a(a+4bx)+2b^2x^2\log(x)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]
[Out] (x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.55

method	result	size
default	$\frac{x\left(2b^2\ln(x)x^2-4abx-a^2\right)}{2\left(c\,x^2\right)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{2}a^2-2abx}{cx\sqrt{c\,x^2}}+\frac{b^2x\ln(x)}{c\sqrt{c\,x^2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.60

$$\frac{b^2 \log(x)}{c^{\frac{3}{2}}} - \frac{2ab}{\sqrt{cx^2}\,c} - \frac{a^2}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b^2*log(x)/c^(3/2) - 2*a*b/(sqrt(c*x^2)*c) - 1/2*a^2/(c^(3/2)*x^2)
```

Fricas [A]
time = 0.30, size = 36, normalized size = 0.62

$$\frac{(2\,b^2x^2\log(x) - 4\,abx - a^2)\sqrt{cx^2}}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(3/2),x)
[Out] Integral((a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.71

$$\frac{\frac{-4abx-a^2}{2(x^2\operatorname{sign}(x))} + \frac{b^2\ln|x|}{\operatorname{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x)
[Out] 1/2*(2*b^2*log(abs(x))/sgn(x) - (4*a*b*x + a^2)/(x^2*sgn(x)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(3/2),x)
[Out] int((a + b*x)^2/(c*x^2)^(3/2), x)
```

3.840 $\int \frac{(a+bx)^2}{x(cx^2)^{3/2}} dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

```
[Out] -1/3*(b*x+a)^3/a/c/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -1/3*(a + b*x)^3/(a*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x(cx^2)^{3/2}} dx &= \frac{x\int \frac{(a+bx)^2}{x^4} dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.24

$$\frac{cx^2(-a^2-3abx-3b^2x^2)}{3(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.87, size = 27, normalized size = 0.93

$$\frac{-\frac{a^2}{3}-abx-b^2x^2}{(cx^2)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^1*(c*x^2)^(3/2)),x]')
```

```
[Out] (-a ^ 2 / 3 - a b x - b ^ 2 x ^ 2) / (c x ^ 2) ^ (3 / 2)
```

Maple [A]
time = 0.10, size = 27, normalized size = 0.93

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3(cx^2)^{\frac{3}{2}}}$	27
default	$-\frac{3x^2b^2+3abx+a^2}{3(cx^2)^{\frac{3}{2}}}$	27
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{cx^2\sqrt{cx^2}}$	34
trager	$\frac{(-1+x)(a^2x^2+3ab\,x^2+3x^2b^2+a^2x+3abx+a^2)\sqrt{cx^2}}{3c^2x^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 1.28

$$-\frac{b^2}{\sqrt{cx^2}\,c}-\frac{ab}{c^{\frac{3}{2}}x^2}-\frac{a^2}{3\,c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c*x^2)*c) - a*b/(c^(3/2)*x^2) - 1/3*a^2/(c^(3/2)*x^3)
```

Fricas [A]
time = 0.29, size = 32, normalized size = 1.10

$$-\frac{(3\,b^2x^2+3\,abx+a^2)\sqrt{cx^2}}{3\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^2*x^4)
```

Sympy [A]
time = 0.26, size = 42, normalized size = 1.45

$$-\frac{a^2}{3\,(cx^2)^{\frac{3}{2}}}-\frac{abx}{(cx^2)^{\frac{3}{2}}}-\frac{b^2x^2}{(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(3*(c*x**2)**(3/2)) - a*b*x/(c*x**2)**(3/2) - b**2*x**2/(c*x**2)**(3/
2)
```

Giac [A]
time = 0.00, size = 38, normalized size = 1.31

$$\frac{-3b^2x^2-3abx-a^2}{\sqrt{c}\,c\cdot 3\,(x^3\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x)
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(c^(3/2)*x^3*sgn(x))
```

Mupad [B]
time = 0.19, size = 33, normalized size = 1.14

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{3/2}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(3/2)),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(3/2)*(x^2)^(5/2))
```

3.841 $\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

```
[Out] -1/4*a^2/c/x^3/(c*x^2)^(1/2)-2/3*a*b/c/x^2/(c*x^2)^(1/2)-1/2*b^2/c/x/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/4*a^2/(c*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c*x^2*Sqrt[c*x^2]) - b^2/(2*c*x*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{3/2}} \, dx = \frac{x \int \frac{(a+bx)^2}{x^5} \, dx}{c\sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^5} + \frac{2ab}{x^4} + \frac{b^2}{x^3}\right) dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{4cx^3\sqrt{cx^2}} - \frac{2ab}{3cx^2\sqrt{cx^2}} - \frac{b^2}{2cx\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(3a^2 + 8abx + 6b^2x^2)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^2*x^5)
```

Mathics [**A**]
time = 1.90, size = 32, normalized size = 0.48

$$\frac{\left(-\frac{a^2}{4} + \frac{bx(-4a-3bx)}{6}\right)(cx^2)^{\frac{3}{2}}}{c^3x^7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]')
[Out] (-a ^ 2 / 4 + b x (-4 a - 3 b x) / 6) (c x ^ 2) ^ (3 / 2) / (c ^ 3 x ^ 7)
```

Maple [**A**]
time = 0.12, size = 32, normalized size = 0.48

method	result	size
gosper	$-\frac{6x^2b^2+8abx+3a^2}{12x(cx^2)^{\frac{3}{2}}}$	32
default	$-\frac{6x^2b^2+8abx+3a^2}{12x(cx^2)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{cx^3\sqrt{cx^2}}$	34

trager	$\frac{(-1+x)(3a^2x^3+8ab\,x^3+6b^2x^3+3a^2x^2+8ab\,x^2+6x^2b^2+3a^2x+8abx+3a^2)\sqrt{c\,x^2}}{12c^2x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.50

$$-\frac{b^2}{2\,c^{\frac{3}{2}}x^2}-\frac{2\,ab}{3\,c^{\frac{3}{2}}x^3}-\frac{a^2}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b^2/(c^(3/2)*x^2) - 2/3*a*b/(c^(3/2)*x^3) - 1/4*a^2/(c^(3/2)*x^4)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^2*x^5)
```

Sympy [A]
time = 0.28, size = 46, normalized size = 0.70

$$-\frac{a^2}{4x\,(cx^2)^{\frac{3}{2}}}-\frac{2ab}{3\,(cx^2)^{\frac{3}{2}}}-\frac{b^2x}{2\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(3/2),x)
[Out] -a**2/(4*x*(c*x**2)**(3/2)) - 2*a*b/(3*(c*x**2)**(3/2)) - b**2*x/(2*(c*x**2
)**(3/2))
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.61

$$\frac{-6b^2x^2-8abx-3a^2}{\sqrt{c}\,c\cdot 12\,(x^4\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x)
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/(c^(3/2)*x^4*sgn(x))
```

Mupad [B]
time = 0.19, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(3/2)),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(3
/2)*x^5)
```

3.842 $\int \frac{(a+bx)^2}{x^3(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

```
[Out] -1/5*a^2/c/x^4/(c*x^2)^(1/2)-1/2*a*b/c/x^3/(c*x^2)^(1/2)-1/3*b^2/c/x^2/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -1/5*a^2/(c*x^4*Sqrt[c*x^2]) - (a*b)/(2*c*x^3*Sqrt[c*x^2]) - b^2/(3*c*x^2*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{3/2}} dx = \frac{x \int \frac{(a+bx)^2}{x^6} dx}{c\sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^6} + \frac{2ab}{x^5} + \frac{b^2}{x^4}\right) dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{5cx^4\sqrt{cx^2}} - \frac{ab}{2cx^3\sqrt{cx^2}} - \frac{b^2}{3cx^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.50

$$\frac{c(-6a^2 - 15abx - 10b^2x^2)}{30 (cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
[Out] (c*(-6*a^2 - 15*a*b*x - 10*b^2*x^2))/(30*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.98, size = 29, normalized size = 0.44

$$\frac{c(-6a^2 - 15abx - 10b^2x^2)}{30(cx^2)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]')
[Out] c (-6 a ^ 2 - 15 a b x - 10 b ^ 2 x ^ 2) / (30 (c x ^ 2) ^ (5 / 2))
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result	size
gosper	$-\frac{10x^2b^2+15abx+6a^2}{30x^2(cx^2)^{\frac{3}{2}}}$	32
default	$-\frac{10x^2b^2+15abx+6a^2}{30x^2(cx^2)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{3}x^2b^2-\frac{1}{2}abx-\frac{1}{5}a^2}{cx^4\sqrt{cx^2}}$	34

trager	$\frac{(-1+x)\left(6a^2x^4+15ab\,x^4+10b^2x^4+6a^2x^3+15ab\,x^3+10b^2x^3+6a^2x^2+15ab\,x^2+10x^2b^2+6a^2x+15abx+6a^2\right)\sqrt{c\,x^2}}{30c^2x^6}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/x^2/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.50

$$-\frac{b^2}{3\,c^{\frac{3}{2}}x^3}-\frac{ab}{2\,c^{\frac{3}{2}}x^4}-\frac{a^2}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2/(c^(3/2)*x^3) - 1/2*a*b/(c^(3/2)*x^4) - 1/5*a^2/(c^(3/2)*x^5)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{\left(10\,b^2x^2+15\,abx+6\,a^2\right)\sqrt{cx^2}}{30\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^2*x^6)
```

Sympy [A]
time = 0.32, size = 46, normalized size = 0.70

$$-\frac{a^2}{5x^2\left(cx^2\right)^{\frac{3}{2}}}-\frac{ab}{2x\left(cx^2\right)^{\frac{3}{2}}}-\frac{b^2}{3\left(cx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(5*x**2*(c*x**2)**(3/2)) - a*b/(2*x*(c*x**2)**(3/2)) - b**2/(3*(c*x**
2)**(3/2))
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.61

$$\frac{-10b^2x^2-15abx-6a^2}{\sqrt{c}\,c\cdot 30\left(x^5\mathrm{sign}\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x)
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/(c^(3/2)*x^5*sgn(x))
```

Mupad [B]
time = 0.20, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(3/2)),x)
```

```
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(3/2)*x^6)
```

3.843 $\int \frac{(a+bx)^2}{x^4(cx^2)^{3/2}} dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

```
[Out] -1/6*a^2/c/x^5/(c*x^2)^(1/2)-2/5*a*b/c/x^4/(c*x^2)^(1/2)-1/4*b^2/c/x^3/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -1/6*a^2/(c*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c*x^4*Sqrt[c*x^2]) - b^2/(4*c*x^3
*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{6cx^5\sqrt{cx^2}} - \frac{2ab}{5cx^4\sqrt{cx^2}} - \frac{b^2}{4cx^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.53

$$\frac{-10a^2-24abx-15b^2x^2}{60x^3 (cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

Mathics [A]
time = 2.02, size = 32, normalized size = 0.48

$$\frac{c^2x\left(-10a^2-24abx-15b^2x^2\right)}{60(cx^2)^{\frac{7}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]')
[Out] c ^ 2 x (-10 a ^ 2 - 24 a b x - 15 b ^ 2 x ^ 2) / (60 (c x ^ 2) ^ (7 / 2))
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{15x^2b^2+24abx+10a^2}{60x^3(cx^2)^{\frac{3}{2}}}$
default	$-\frac{15x^2b^2+24abx+10a^2}{60x^3(cx^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{1}{4}x^2b^2-\frac{2}{5}abx-\frac{1}{6}a^2}{cx^5\sqrt{cx^2}}$

trager	$\frac{(-1+x)(10a^2x^5+24ab\,x^5+15b^2x^5+10a^2x^4+24ab\,x^4+15b^2x^4+10a^2x^3+24ab\,x^3+15b^2x^3+10a^2x^2+24ab\,x^2+15x^2b^2+10a^2x+24abx+}{60c^2x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x^3/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{3}{2}}x^4}-\frac{2\,ab}{5\,c^{\frac{3}{2}}x^5}-\frac{a^2}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(c^(3/2)*x^4) - 2/5*a*b/(c^(3/2)*x^5) - 1/6*a^2/(c^(3/2)*x^6)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(15\,b^2x^2+24\,abx+10\,a^2)\sqrt{cx^2}}{60\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^2*x^7)
```

Sympy [A]
time = 0.35, size = 51, normalized size = 0.77

$$-\frac{a^2}{6x^3\,(cx^2)^{\frac{3}{2}}}-\frac{2ab}{5x^2\,(cx^2)^{\frac{3}{2}}}-\frac{b^2}{4x\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(6*x**3*(c*x**2)**(3/2)) - 2*a*b/(5*x**2*(c*x**2)**(3/2)) - b**2/(4*x
*(c*x**2)**(3/2))
```

Giac [A]
time = 0.00, size = 40, normalized size = 0.61

$$\frac{-15b^2x^2-24abx-10a^2}{\sqrt{c}\,c\cdot 60\,(x^6\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x)
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/(c^(3/2)*x^6*sgn(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}+15\,b^2\,x^2\,\sqrt{x^2}+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(3/2)),x)
```

```
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(3/2)*x^7)
```

3.844 $\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] -a^2/c^2/(c*x^2)^(1/2)+b^2*x^2/c^2/(c*x^2)^(1/2)+2*a*b*x*ln(x)/c^2/(c*x^2)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a^2/(c^2*Sqrt[c*x^2])) + (b^2*x^2)/(c^2*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(
c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.59

$$\frac{-a^2+b^2x^2+2abx\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (-a^2 + b^2*x^2 + 2*a*b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.57

method	result	size
default	$\frac{x^4(2ab\ln(x)x+x^2b^2-a^2)}{(c\,x^2)^{\frac{5}{2}}}$	32
risch	$-\frac{a^2}{c^2\sqrt{c\,x^2}}+\frac{b^2x^2}{c^2\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{c^2\sqrt{c\,x^2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^4*(2*a*b*ln(x)*x+x^2*b^2-a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.80

$$\frac{b^2x^4}{(cx^2)^{\frac{3}{2}}\,c} - \frac{a^2x^2}{(cx^2)^{\frac{3}{2}}\,c} + \frac{2\,ab\log\left(x\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] b^2*x^4/((c*x^2)^(3/2)*c) - a^2*x^2/((c*x^2)^(3/2)*c) + 2*a*b*log(x)/c^(5/2
)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.61

$$\frac{\left(b^2x^2 + 2\,abx\log\left(x\right) - a^2\right)\sqrt{cx^2}}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a + bx\right)^2}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(5/2),x)
[Out] Integral(x**3*(a + b*x)**2/(c*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 47, normalized size = 0.84

$$\frac{-\frac{a^2}{c^2x\mathrm{sign}\left(x\right)} + \frac{b^2x}{c^2\mathrm{sign}\left(x\right)} + \frac{2ab\ln|x|}{c^2\mathrm{sign}\left(x\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x)
[Out] (b^2*x/(c^2*sgn(x)) + 2*a*b*log(abs(x))/(c^2*sgn(x)) - a^2/(c^2*x*sgn(x)))/
sqrt(c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,(a+b\,x)^2}{(c\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2),x)
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

3.845 $\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=58

$$-\frac{2ab}{c^2\sqrt{cx^2}}-\frac{a^2}{2c^2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] -2*a*b/c^2/(c*x^2)^(1/2)-1/2*a^2/c^2/x/(c*x^2)^(1/2)+b^2*x*ln(x)/c^2/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-2*a*b)/(c^2*Sqrt[c*x^2]) - a^2/(2*c^2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c^
2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{2ab}{c^2\sqrt{cx^2}} - \frac{a^2}{2c^2x\sqrt{cx^2}} + \frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.62

$$\frac{x^3\left(-a(a+4bx)+2b^2x^2\log(x)\right)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (x^3*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.11, size = 34, normalized size = 0.59

method	result	size
default	$\frac{x^3(2b^2\ln(x)x^2-4abx-a^2)}{2(cx^2)^{\frac{5}{2}}}$	34
risch	$\frac{-\frac{1}{2}a^2-2abx}{c^2x\sqrt{cx^2}}+\frac{b^2x\ln(x)}{c^2\sqrt{cx^2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^3*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.66

$$-\frac{2\,abx^2}{\left(cx^2\right)^{\frac{3}{2}}\,c}+\frac{b^2\log\left(x\right)}{c^{\frac{5}{2}}}-\frac{a^2}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*x^2/((c*x^2)^(3/2)*c) + b^2*log(x)/c^(5/2) - 1/2*a^2/(c^(5/2)*x^2)
```

Fricas [A]
time = 0.30, size = 36, normalized size = 0.62

$$\frac{\left(2\,b^2x^2\log\left(x\right)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+bx\right)^2}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(5/2), x)
```

Giac [A]
time = 0.00, size = 46, normalized size = 0.79

$$\frac{\frac{-4abx-a^2}{2\left(c^2x^2\mathrm{sign}\left(x\right)\right)}+\frac{b^2\ln|x|}{c^2\mathrm{sign}\left(x\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] 1/2*(2*b^2*log(abs(x))/(c^2*sgn(x)) - (4*a*b*x + a^2)/(c^2*x^2*sgn(x)))/sqr
t(c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 (a + b x)^2}{(c x^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2),x)
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

3.846 $\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} \, dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

[Out] -1/3*(b*x+a)^3/a/c^2/x^2/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]

[Out] -1/3*(a + b*x)^3/(a*c^2*x^2*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} \, dx &= \frac{x\int \frac{(a+bx)^2}{x^4} \, dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.21

$$\frac{x^2 (-a^2 - 3abx - 3b^2x^2)}{3 (cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

Mathics [A]
time = 1.93, size = 30, normalized size = 1.03

$$\frac{x^2 \left(-\frac{a^2}{3} - abx - b^2x^2\right)}{(cx^2)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]')
[Out] x ^ 2 (-a ^ 2 / 3 - a b x - b ^ 2 x ^ 2) / (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.03

method	result	size
gosper	$-\frac{x^2(3x^2b^2+3abx+a^2)}{3(c\,x^2)^{\frac{5}{2}}}$	30
default	$-\frac{x^2(3x^2b^2+3abx+a^2)}{3(c\,x^2)^{\frac{5}{2}}}$	30
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{c^2x^2\sqrt{c\,x^2}}$	34
trager	$\frac{(-1+x)(a^2x^2+3ab\,x^2+3x^2b^2+a^2x+3abx+a^2)\sqrt{c\,x^2}}{3c^3x^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3*x^2*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 44, normalized size = 1.52

$$-\frac{b^2x^2}{(cx^2)^{\frac{3}{2}}\,c} - \frac{a^2}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{ab}{c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*x^2/((c*x^2)^(3/2)*c) - 1/3*a^2/((c*x^2)^(3/2)*c) - a*b/(c^(5/2)*x^2)
```

Fricas [A]
time = 0.29, size = 32, normalized size = 1.10

$$-\frac{(3\,b^2x^2+3\,abx+a^2)\sqrt{cx^2}}{3\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^3*x^4)
```

Sympy [A]
time = 0.31, size = 48, normalized size = 1.66

$$-\frac{a^2x^2}{3\,(cx^2)^{\frac{5}{2}}}-\frac{abx^3}{(cx^2)^{\frac{5}{2}}}-\frac{b^2x^4}{(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x**2/(3*(c*x**2)**(5/2)) - a*b*x**3/(c*x**2)**(5/2) - b**2*x**4/(c*x*
*2)**(5/2)
```

Giac [A]
time = 0.00, size = 39, normalized size = 1.34

$$\frac{-3b^2x^2-3abx-a^2}{\sqrt{c}\cdot 3\left(c^2x^3\mathrm{sign}\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(c^(5/2)*x^3*sgn(x))
```

Mupad [B]
time = 0.18, size = 33, normalized size = 1.14

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{5/2}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(5/2)*(x^2)^(5/2))
```

3.847 $\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

```
[Out] -1/4*a^2/c^2/x^3/(c*x^2)^(1/2)-2/3*a*b/c^2/x^2/(c*x^2)^(1/2)-1/2*b^2/c^2/x/
(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(5/2),x]
```

```
[Out] -1/4*a^2/(c^2*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c^2*x^2*Sqrt[c*x^2]) - b^2/(2*c
^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)\,dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(3a^2+8abx+6b^2x^2)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^3*x^5)
```

Mathics [A]
time = 1.90, size = 29, normalized size = 0.44

$$\frac{x\,(-3a^2-8abx-6b^2x^2)}{12(cx^2)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]')
[Out] x (-3 a ^ 2 - 8 a b x - 6 b ^ 2 x ^ 2) / (12 (c x ^ 2) ^ (5 / 2))
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.45

method	result	size
gosper	$-\frac{x\left(6x^2b^2+8abx+3a^2\right)}{12\left(c\,x^2\right)^{\frac{5}{2}}}$	30
default	$-\frac{x\left(6x^2b^2+8abx+3a^2\right)}{12\left(c\,x^2\right)^{\frac{5}{2}}}$	30
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{c^2x^3\sqrt{c\,x^2}}$	34

trager	$\frac{(-1+x)(3a^2x^3+8ab\,x^3+6b^2x^3+3a^2x^2+8ab\,x^2+6x^2b^2+3a^2x+8abx+3a^2)\sqrt{c\,x^2}}{12c^3x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*x*(6*b^2*x^2+8*a*b*x+3*a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.56

$$-\frac{2\,ab}{3\,(cx^2)^{\frac{3}{2}}\,c}-\frac{b^2}{2\,c^{\frac{5}{2}}x^2}-\frac{a^2}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*a*b/((c*x^2)^(3/2)*c) - 1/2*b^2/(c^(5/2)*x^2) - 1/4*a^2/(c^(5/2)*x^4)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^3*x^5)
```

Sympy [A]
time = 0.32, size = 51, normalized size = 0.77

$$-\frac{a^2x}{4\,(cx^2)^{\frac{5}{2}}}-\frac{2abx^2}{3\,(cx^2)^{\frac{5}{2}}}-\frac{b^2x^3}{2\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x/(4*(c*x**2)**(5/2)) - 2*a*b*x**2/(3*(c*x**2)**(5/2)) - b**2*x**3/(2
*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.62

$$\frac{-6b^2x^2-8abx-3a^2}{\sqrt{c}\cdot 12\,(c^2x^4\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/(c^(5/2)*x^4*sgn(x))
```

Mupad [B]
time = 0.17, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(5/2),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(5
/2)*x^5)
```

3.848 $\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

```
[Out] -1/5*a^2/c^2/x^4/(c*x^2)^(1/2)-1/2*a*b/c^2/x^3/(c*x^2)^(1/2)-1/3*b^2/c^2/x^
2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/5*a^2/(c^2*x^4*Sqrt[c*x^2]) - (a*b)/(2*c^2*x^3*Sqrt[c*x^2]) - b^2/(3*c^2
*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)\,dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{5c^2x^4\sqrt{cx^2}} - \frac{ab}{2c^2x^3\sqrt{cx^2}} - \frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(6a^2+15abx+10b^2x^2)}{30c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/30*(Sqrt[c*x^2]*(6*a^2 + 15*a*b*x + 10*b^2*x^2))/(c^3*x^6)
```

Mathics [A]
time = 1.93, size = 27, normalized size = 0.41

$$\frac{-\frac{a^2}{5}-\frac{abx}{2}-\frac{b^2x^2}{3}}{\left(cx^2\right)^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]')
```

```
[Out] (-a ^ 2 / 5 - a b x / 2 - b ^ 2 x ^ 2 / 3) / (c x ^ 2) ^ (5 / 2)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.44

method	result	size
gosper	$-\frac{10x^2b^2+15abx+6a^2}{30(c\,x^2)^{\frac{5}{2}}}$	29
default	$-\frac{10x^2b^2+15abx+6a^2}{30(c\,x^2)^{\frac{5}{2}}}$	29
risch	$\frac{-\frac{1}{3}x^2b^2-\frac{1}{2}abx-\frac{1}{5}a^2}{c^2x^4\sqrt{c\,x^2}}$	34

trager	$\frac{(-1+x)(6a^2x^4+15ab\,x^4+10b^2x^4+6a^2x^3+15ab\,x^3+10b^2x^3+6a^2x^2+15ab\,x^2+10x^2b^2+6a^2x+15abx+6a^2)\sqrt{c\,x^2}}{30c^3x^6}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.56

$$-\frac{b^2}{3\,(cx^2)^{\frac{3}{2}}\,c}-\frac{ab}{2\,c^{\frac{5}{2}}x^4}-\frac{a^2}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2/((c*x^2)^(3/2)*c) - 1/2*a*b/(c^(5/2)*x^4) - 1/5*a^2/(c^(5/2)*x^5)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(10\,b^2x^2+15\,abx+6\,a^2)\sqrt{cx^2}}{30\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^3*x^6)
```

Sympy [A]
time = 0.34, size = 46, normalized size = 0.70

$$-\frac{a^2}{5\,(cx^2)^{\frac{5}{2}}}-\frac{abx}{2\,(cx^2)^{\frac{5}{2}}}-\frac{b^2x^2}{3\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(5*(c*x**2)**(5/2)) - a*b*x/(2*(c*x**2)**(5/2)) - b**2*x**2/(3*(c*x**
2)**(5/2))
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.62

$$\frac{-10b^2x^2-15abx-6a^2}{\sqrt{c}\cdot 30\,(c^2x^5\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x)
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/(c^(5/2)*x^5*sgn(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(5/2)),x)
```

```
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(5/2)*x^6)
```

3.849 $\int \frac{(a+bx)^2}{x^2(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

```
[Out] -1/6*a^2/c^2/x^5/(c*x^2)^(1/2)-2/5*a*b/c^2/x^4/(c*x^2)^(1/2)-1/4*b^2/c^2/x^
3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/6*a^2/(c^2*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c^2*x^4*Sqrt[c*x^2]) - b^2/(4*c
^2*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^2 (cx^2)^{5/2}} \, dx = \frac{x \int \frac{(a+bx)^2}{x^7} \, dx}{c^2 \sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^7} + \frac{2ab}{x^6} + \frac{b^2}{x^5}\right) dx}{c^2 \sqrt{cx^2}}$$
$$= -\frac{a^2}{6c^2 x^5 \sqrt{cx^2}} - \frac{2ab}{5c^2 x^4 \sqrt{cx^2}} - \frac{b^2}{4c^2 x^3 \sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2} \, (10a^2 + 24abx + 15b^2x^2)}{60c^3 x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/60*(Sqrt[c*x^2]*(10*a^2 + 24*a*b*x + 15*b^2*x^2))/(c^3*x^7)
```

Mathics [A]
time = 2.00, size = 32, normalized size = 0.48

$$\frac{\left(-\frac{a^2}{6} + \frac{bx(-8a-5bx)}{20}\right) (cx^2)^{\frac{5}{2}}}{c^5 x^{11}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]')
```

```
[Out] (-a ^ 2 / 6 + b x (-8 a - 5 b x) / 20) (c x ^ 2) ^ (5 / 2) / (c ^ 5 x ^ 11)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{15x^2b^2+24abx+10a^2}{60x(cx^2)^{\frac{5}{2}}}$
default	$-\frac{15x^2b^2+24abx+10a^2}{60x(cx^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{4}x^2b^2-\frac{2}{5}abx-\frac{1}{6}a^2}{c^2x^5\sqrt{cx^2}}$

trager	$\frac{(-1+x)(10a^2x^5+24ab\,x^5+15b^2x^5+10a^2x^4+24ab\,x^4+15b^2x^4+10a^2x^3+24ab\,x^3+15b^2x^3+10a^2x^2+24ab\,x^2+15x^2b^2+10a^2x+24abx}{60c^3x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{5}{2}}x^4}-\frac{2\,ab}{5\,c^{\frac{5}{2}}x^5}-\frac{a^2}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(5/2)*x^4) - 2/5*a*b/(c^(5/2)*x^5) - 1/6*a^2/(c^(5/2)*x^6)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(15\,b^2x^2+24\,abx+10\,a^2)\sqrt{cx^2}}{60\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^3*x^7)
```

Sympy [A]
time = 0.38, size = 46, normalized size = 0.70

$$-\frac{a^2}{6x\,(cx^2)^{\frac{5}{2}}}-\frac{2ab}{5\,(cx^2)^{\frac{5}{2}}}-\frac{b^2x}{4\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(5/2),x)
[Out] -a**2/(6*x*(c*x**2)**(5/2)) - 2*a*b/(5*(c*x**2)**(5/2)) - b**2*x/(4*(c*x**2
)**(5/2))
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.62

$$\frac{-15b^2x^2-24abx-10a^2}{\sqrt{c}\cdot 60\,(c^2x^6\text{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/(c^(5/2)*x^6*sgn(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}+15\,b^2\,x^2\,\sqrt{x^2}+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(5/2)),x)
```

```
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(5/2)*x^7)
```

3.850 $\int \frac{(a+bx)^2}{x^3 (cx^2)^{5/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

```
[Out] -1/7*a^2/c^2/x^6/(c*x^2)^(1/2)-1/3*a*b/c^2/x^5/(c*x^2)^(1/2)-1/5*b^2/c^2/x^
4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -1/7*a^2/(c^2*x^6*Sqrt[c*x^2]) - (a*b)/(3*c^2*x^5*Sqrt[c*x^2]) - b^2/(5*c^2
*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^3 (cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^8}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{7c^2x^6\sqrt{cx^2}} - \frac{ab}{3c^2x^5\sqrt{cx^2}} - \frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.50

$$\frac{c\left(-15a^2-35abx-21b^2x^2\right)}{105\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
[Out] (c*(-15*a^2 - 35*a*b*x - 21*b^2*x^2))/(105*(c*x^2)^(7/2))
```

Mathics [A]
time = 2.03, size = 29, normalized size = 0.44

$$\frac{c\left(-15a^2-35abx-21b^2x^2\right)}{105(cx^2)^{\frac{7}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]')
[Out] c (-15 a ^ 2 - 35 a b x - 21 b ^ 2 x ^ 2) / (105 (c x ^ 2) ^ (7 / 2))
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{21x^2b^2+35abx+15a^2}{105x^2(cx^2)^{\frac{5}{2}}}$
default	$-\frac{21x^2b^2+35abx+15a^2}{105x^2(cx^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{5}x^2b^2-\frac{1}{3}abx-\frac{1}{7}a^2}{c^2x^6\sqrt{cx^2}}$

trager	$\frac{(-1+x)(15a^2x^6+35ab\,x^6+21b^2x^6+15a^2x^5+35ab\,x^5+21b^2x^5+15a^2x^4+35ab\,x^4+21b^2x^4+15a^2x^3+35ab\,x^3+21b^2x^3+15a^2x^2+35ab}{105c^3x^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(21*b^2*x^2+35*a*b*x+15*a^2)/x^2/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.50

$$-\frac{b^2}{5\,c^{\frac{5}{2}}x^5}-\frac{ab}{3\,c^{\frac{5}{2}}x^6}-\frac{a^2}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(c^(5/2)*x^5) - 1/3*a*b/(c^(5/2)*x^6) - 1/7*a^2/(c^(5/2)*x^7)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(21\,b^2x^2+35\,abx+15\,a^2)\sqrt{cx^2}}{105\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)*sqrt(c*x^2)/(c^3*x^8)
```

Sympy [A]
time = 0.43, size = 46, normalized size = 0.70

$$-\frac{a^2}{7x^2\,(cx^2)^{\frac{5}{2}}}-\frac{ab}{3x\,(cx^2)^{\frac{5}{2}}}-\frac{b^2}{5\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(7*x**2*(c*x**2)**(5/2)) - a*b/(3*x*(c*x**2)**(5/2)) - b**2/(5*(c*x**
2)**(5/2))
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.62

$$\frac{-21b^2x^2-35abx-15a^2}{\sqrt{c}\cdot 105\,(c^2x^7\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x)
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/(c^(5/2)*x^7*sgn(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{15\,a^2\,\sqrt{x^2}+21\,b^2\,x^2\,\sqrt{x^2}+35\,a\,b\,x\,\sqrt{x^2}}{105\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(5/2)),x)
```

```
[Out] -(15*a^2*(x^2)^(1/2) + 21*b^2*x^2*(x^2)^(1/2) + 35*a*b*x*(x^2)^(1/2))/(105*
c^(5/2)*x^8)
```

3.851 $\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

```
[Out] -1/8*a^2/c^2/x^7/(c*x^2)^(1/2)-2/7*a*b/c^2/x^6/(c*x^2)^(1/2)-1/6*b^2/c^2/x^
5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -1/8*a^2/(c^2*x^7*Sqrt[c*x^2]) - (2*a*b)/(7*c^2*x^6*Sqrt[c*x^2]) - b^2/(6*c
^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^9}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^9}+\frac{2ab}{x^8}+\frac{b^2}{x^7}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.53

$$\frac{-21a^2-48abx-28b^2x^2}{168x^3 (cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

Mathics [A]
time = 2.07, size = 32, normalized size = 0.48

$$\frac{c^2x\left(-21a^2-48abx-28b^2x^2\right)}{168(cx^2)^{\frac{9}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]')
[Out] c ^ 2 x (-21 a ^ 2 - 48 a b x - 28 b ^ 2 x ^ 2) / (168 (c x ^ 2) ^ (9 / 2))
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{28x^2b^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
default	$-\frac{28x^2b^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{6}x^2b^2-\frac{2}{7}abx-\frac{1}{8}a^2}{c^2x^7\sqrt{c\,x^2}}$

trager	$\frac{(-1+x)(21a^2x^7+48ab\,x^7+28b^2x^7+21a^2x^6+48ab\,x^6+28b^2x^6+21a^2x^5+48ab\,x^5+28b^2x^5+21a^2x^4+48ab\,x^4+28b^2x^4+21a^2x^3+48ab}{168c^3x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/168*(28*b^2*x^2+48*a*b*x+21*a^2)/x^3/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.50

$$-\frac{b^2}{6\,c^{\frac{5}{2}}x^6}-\frac{2\,ab}{7\,c^{\frac{5}{2}}x^7}-\frac{a^2}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/6*b^2/(c^(5/2)*x^6) - 2/7*a*b/(c^(5/2)*x^7) - 1/8*a^2/(c^(5/2)*x^8)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.52

$$-\frac{(28\,b^2x^2+48\,abx+21\,a^2)\sqrt{cx^2}}{168\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)*sqrt(c*x^2)/(c^3*x^9)
```

Sympy [A]
time = 0.47, size = 51, normalized size = 0.77

$$-\frac{a^2}{8x^3\,(cx^2)^{\frac{5}{2}}}-\frac{2ab}{7x^2\,(cx^2)^{\frac{5}{2}}}-\frac{b^2}{6x\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(5/2),x)
[Out] -a**2/(8*x**3*(c*x**2)**(5/2)) - 2*a*b/(7*x**2*(c*x**2)**(5/2)) - b**2/(6*x
*(c*x**2)**(5/2))
```

Giac [A]
time = 0.00, size = 41, normalized size = 0.62

$$\frac{-28b^2x^2-48abx-21a^2}{\sqrt{c}\cdot 168\,(c^2x^8\mathrm{sign}\,(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)/(c^(5/2)*x^8*sgn(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{21\,a^2\,\sqrt{x^2}+28\,b^2\,x^2\,\sqrt{x^2}+48\,a\,b\,x\,\sqrt{x^2}}{168\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(21*a^2*(x^2)^(1/2) + 28*b^2*x^2*(x^2)^(1/2) + 48*a*b*x*(x^2)^(1/2))/(168*
c^(5/2)*x^9)
```

3.852 $\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=102

$$-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] -a^3*(c*x^2)^(1/2)/b^4+1/2*a^2*x*(c*x^2)^(1/2)/b^3-1/3*a*x^2*(c*x^2)^(1/2)/
b^2+1/4*x^3*(c*x^2)^(1/2)/b+a^4*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a^3*Sqrt[c*x^2])/b^4) + (a^2*x*Sqrt[c*x^2])/(2*b^3) - (a*x^2*Sqrt[c*x^2]
)/(3*b^2) + (x^3*Sqrt[c*x^2])/(4*b) + (a^4*Sqrt[c*x^2]*Log[a + b*x])/(b^5*x
)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{cx^2}\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.62

$$\frac{cx\left(bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)+12a^4\log(a+bx)\right)}{12b^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^4*Log[a +
b*x]))/(12*b^5*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.62

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx)}{12b^5x}$	63
risch	$\frac{\sqrt{c\,x^2}\,(\frac{1}{4}b^3x^4-\frac{1}{3}a\,b^2x^3+\frac{1}{2}a^2b\,x^2-a^3x)}{x\,b^4}+\frac{a^4\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*x^2)^(1/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x
```

Maxima [A]
time = 0.28, size = 128, normalized size = 1.25

$$\frac{(-1)^{\frac{2cx}{b}}a^4\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}a^4\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{\sqrt{cx^2}\,a^2x}{2b^3}+\frac{(cx^2)^{\frac{3}{2}}x}{4bc}-\frac{\sqrt{cx^2}\,a^3}{b^4}-\frac{(cx^2)^{\frac{3}{2}}a}{3b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*sqrt(c)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/2*sqrt(c*x^2)*a^2*x/b^3 + 1/4*(c*x^2
)^(3/2)*x/(b*c) - sqrt(c*x^2)*a^3/b^4 - 1/3*(c*x^2)^(3/2)*a/(b^2*c)
```

Fricas [A]
time = 0.29, size = 62, normalized size = 0.61

$$\frac{(3b^4x^4-4ab^3x^3+6a^2b^2x^2-12a^3bx+12a^4\log(bx+a))\sqrt{cx^2}}{12b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 88, normalized size = 0.86

$$\sqrt{c}\left(\frac{-a^3x\mathrm{sign}(x)+\frac{1}{4}b^3x^4\mathrm{sign}(x)-\frac{1}{3}ab^2x^3\mathrm{sign}(x)+\frac{1}{2}a^2bx^2\mathrm{sign}(x)}{b^4}+\frac{a^4\mathrm{sign}(x)\ln|bx+a|}{b^5}-\frac{a^4\ln|a|\cdot\mathrm{sign}(x)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x)
```

[Out] 1/12*sqrt(c)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a^3*x*sgn(x))/b^4)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(c*x^2)^(1/2))/(a + b*x),x)

[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x), x)

3.853 $\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=80

$$\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] a^2*(c*x^2)^(1/2)/b^3-1/2*a*x*(c*x^2)^(1/2)/b^2+1/3*x^2*(c*x^2)^(1/2)/b-a^3
*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (a^2*Sqrt[c*x^2])/b^3 - (a*x*Sqrt[c*x^2])/(2*b^2) + (x^2*Sqrt[c*x^2])/(3*b)
 - (a^3*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.65

$$\frac{cx\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]
[Out] (c*x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[
c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 52, normalized size = 0.65

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,\left(-2b^{3}x^{3}+3a\,b^{2}x^{2}+6a^{3}\ln\left(bx+a\right)-6a^{2}bx\right)}{6x\,b^{4}}$	52
risch	$\frac{\sqrt{c\,x^{2}}\,\left(\frac{1}{3}b^{2}x^{3}-\frac{1}{2}ab\,x^{2}+a^{2}x\right)}{x\,b^{3}}-\frac{a^{3}\ln\left(bx+a\right)\sqrt{c\,x^{2}}}{b^{4}x}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/6*(c*x^2)^(1/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x/b^4
```

Maxima [A]
time = 0.27, size = 110, normalized size = 1.38

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a^{3}\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^{4}}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^{3}\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^{4}}-\frac{\sqrt{cx^{2}}\,ax}{2\,b^{2}}+\frac{\sqrt{cx^{2}}\,a^{2}}{b^{3}}+\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*sqrt(c)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*x/b^2 + sqrt(c*x^2)
*a^2/b^3 + 1/3*(c*x^2)^(3/2)/(b*c)
```

Fricas [A]
time = 0.30, size = 51, normalized size = 0.64

$$\frac{(2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log{(bx + a)})\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{cx^2}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 73, normalized size = 0.91

$$\sqrt{c}\left(\frac{a^2x\mathrm{sign}\,(x) + \frac{1}{3}b^2x^3\mathrm{sign}\,(x) - \frac{1}{2}abx^2\mathrm{sign}\,(x)}{b^3} - \frac{a^3\mathrm{sign}\,(x)\ln|bx + a|}{b^4} + \frac{a^3\ln|a|\cdot\mathrm{sign}\,(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -1/6*sqrt(c)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x), x)
```

3.854 $\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=58

$$-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] -a*(c*x^2)^(1/2)/b^2+1/2*x*(c*x^2)^(1/2)/b+a^2*ln(b*x+a)*(c*x^2)^(1/2)/b^3/
x
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a*Sqrt[c*x^2])/b^2) + (x*Sqrt[c*x^2])/(2*b) + (a^2*Sqrt[c*x^2]*Log[a + b
*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^2}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\ &= -\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.69

$$\frac{cx\left(bx(-2a+bx)+2a^2\log(a+bx)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.69

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(x^2b^2+2a^2\ln(bx+a)-2abx)}{2b^3x}$	40
risch	$\frac{\sqrt{c\,x^2}\,(\frac{1}{2}x^2b-ax)}{x\,b^2}+\frac{a^2\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x
```

Maxima [A]
time = 0.26, size = 91, normalized size = 1.57

$$\frac{(-1)^{\frac{2cx}{b}}a^2\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2acx}{b}}a^2\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,x}{2\,b}-\frac{\sqrt{cx^2}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*x/b - sqrt(c*x^2)*a/b^
2
```

Fricas [A]
time = 0.29, size = 39, normalized size = 0.67

$$\frac{\left(b^2x^2 - 2\,abx + 2\,a^2\log\left(bx + a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{cx^2}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 59, normalized size = 1.02

$$\sqrt{c}\left(\frac{-ax\mathrm{sign}\left(x\right) + \frac{1}{2}bx^2\mathrm{sign}\left(x\right)}{b^2} + \frac{a^2\mathrm{sign}\left(x\right)\ln\left|bx + a\right|}{b^3} - \frac{a^2\ln\left|a\right|\cdot\mathrm{sign}\left(x\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/2*sqrt(c)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x\,\sqrt{c\,x^2}}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x), x)
```

3.855 $\int \frac{\sqrt{cx^2}}{a+bx} dx$

Optimal. Leaf size=38

$$\frac{\sqrt{cx^2}}{b} - \frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] (c*x^2)^(1/2)/b-a*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{\sqrt{cx^2}}{b} - \frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]/b - (a*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\ &= \frac{\sqrt{cx^2}}{b} - \frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.74

$$\frac{cx(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x),x]
[Out] (c*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(a + b*x),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.76

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,(a\ln(bx+a)-bx)}{b^{2}x}$	29
risch	$\frac{\sqrt{c\,x^{2}}}{b}-\frac{a\ln(bx+a)\sqrt{c\,x^{2}}}{b^{2}x}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(1/2)*(a*ln(b*x+a)-b*x)/b^2/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.
time = 0.28, size = 74, normalized size = 1.95

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^2}-\frac{(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}+\frac{\sqrt{cx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*sqrt(c)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)/b
```

Fricas [A]
time = 0.29, size = 27, normalized size = 0.71

$$\frac{\sqrt{cx^2}\,(bx - a\log(bx + a))}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a),x)
[Out] Integral(sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 40, normalized size = 1.05

$$\sqrt{c}\left(\frac{x\mathrm{sign}\,(x)}{b} - \frac{a\mathrm{sign}\,(x)\ln|bx + a|}{b^2} + \frac{a\ln|a|\cdot\mathrm{sign}\,(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x)
[Out] sqrt(c)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x),x)
[Out] int((c*x^2)^(1/2)/(a + b*x), x)
```

3.856 $\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

```
[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 31\}$

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.95

$$\frac{cx\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (c*x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**A**]
time = 0.12, size = 21, normalized size = 0.95

method	result	size
default	$\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{bx}$	21
risch	$\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{bx}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.29, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^2}\,\log\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*log(b*x + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a),x)
[Out] Integral(sqrt(c*x**2)/(x*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 28, normalized size = 1.27

$$\sqrt{c}\left(-\frac{\ln|a|\cdot\operatorname{sign}(x)}{b}+\frac{\operatorname{sign}(x)\ln|bx+a|}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x)
[Out] sqrt(c)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c\,x^2}}{x\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)),x)
[Out] int((c*x^2)^(1/2)/(x*(a + b*x)), x)
```

3.857 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)} dx$

Optimal. Leaf size=42

$$\frac{\sqrt{cx^2}\log(x)}{ax} - \frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

```
[Out] ln(x)*(c*x^2)^(1/2)/a/x-ln(b*x+a)*(c*x^2)^(1/2)/a/x
```

Rubi [A]
time = 0.00, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 36, 29, 31}

$$\frac{\sqrt{cx^2}\log(x)}{ax} - \frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[x])/(a*x) - (Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \frac{1}{x}\,dx}{ax} - \frac{\left(b\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{\sqrt{cx^2}\log(x)}{ax} - \frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.62

$$\frac{cx(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (c*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 26, normalized size = 0.62

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(\ln(x)-\ln(bx+a))}{ax}$	26
risch	$\frac{\sqrt{c\,x^2}\,\ln(-x)}{xa} - \frac{\ln(bx+a)\sqrt{c\,x^2}}{ax}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] (c*x^2)^(1/2)*(ln(x)-ln(b*x+a))/a/x

Maxima [A]
time = 0.29, size = 24, normalized size = 0.57

$$-\frac{\sqrt{c}\,\log\left(bx+a\right)}{a}+\frac{\sqrt{c}\,\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="maxima")

[Out] -sqrt(c)*log(b*x + a)/a + sqrt(c)*log(x)/a

Fricas [A]
time = 0.29, size = 64, normalized size = 1.52

$$\left[\frac{\sqrt{cx^2}\,\log\left(\frac{x}{bx+a}\right)}{ax},\ \frac{2\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="fricas")

[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x + a)*sqrt(-c)/(a*c*x))/a]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a),x)

[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x)

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)), x)
```

3.858 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] -(c*x^2)^(1/2)/a/x^2-b*ln(x)*(c*x^2)^(1/2)/a^2/x+b*ln(b*x+a)*(c*x^2)^(1/2)/
a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{b\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -(Sqrt[c*x^2]/(a*x^2)) - (b*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*Sqrt[c*x^2]*Lo
g[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\
&= -\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\,\log(a+bx)}{a^2x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.52

$$-\frac{c(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
[Out] -((c*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.54

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,\left(bx\ln\left(x\right)-b\ln\left(bx+a\right)x+a\right)}{a^{2}x^{2}}$	33
risch	$-\frac{\sqrt{c\,x^{2}}}{a\,x^{2}}+\frac{\sqrt{c\,x^{2}}\,b\ln\left(-bx-a\right)}{x\,a^{2}}-\frac{b\ln\left(x\right)\sqrt{c\,x^{2}}}{a^{2}x}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(1/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x^2
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.61

$$\frac{b\sqrt{c}\,\log\left(bx+a\right)}{a^2}-\frac{b\sqrt{c}\,\log\left(x\right)}{a^2}-\frac{\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="maxima")
[Out] b*sqrt(c)*log(b*x + a)/a^2 - b*sqrt(c)*log(x)/a^2 - sqrt(c)/(a*x)
```

Fricas [A]
time = 0.30, size = 31, normalized size = 0.51

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c\,x^2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)), x)
```

3.859 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}$$

```
[Out] -1/2*(c*x^2)^(1/2)/a/x^3+b*(c*x^2)^(1/2)/a^2/x^2+b^2*ln(x)*(c*x^2)^(1/2)/a^
3/x-b^2*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] -1/2*Sqrt[c*x^2]/(a*x^3) + (b*Sqrt[c*x^2])/(a^2*x^2) + (b^2*Sqrt[c*x^2]*Log
[x])/(a^3*x) - (b^2*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\
&= -\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.63

$$\frac{\sqrt{cx^2}\,(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx))}{2a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
[Out] (Sqrt[c*x^2]*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x])
)/(2*a^3*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.61

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2)}{2a^3x^3}$	51
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{bx}{a^2}-\frac{1}{2a}\right)}{x^3}+\frac{\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^3}-\frac{b^2\ln(bx+a)\sqrt{c\,x^2}}{a^3x}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^3
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.62

$$-\frac{b^2\sqrt{c}\,\log(bx+a)}{a^3}+\frac{b^2\sqrt{c}\,\log(x)}{a^3}+\frac{2\,b\sqrt{c}\,x-a\sqrt{c}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="maxima")
[Out] -b^2*sqrt(c)*log(b*x + a)/a^3 + b^2*sqrt(c)*log(x)/a^3 + 1/2*(2*b*sqrt(c)*x
 - a*sqrt(c))/(a^2*x^2)
```

Fricas [A]
time = 0.30, size = 44, normalized size = 0.52

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^4\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c\,x^2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)), x)
```

3.860 $\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=107

$$-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] -a^3*c*(c*x^2)^(1/2)/b^4+1/2*a^2*c*x*(c*x^2)^(1/2)/b^3-1/3*a*c*x^2*(c*x^2)^
(1/2)/b^2+1/4*c*x^3*(c*x^2)^(1/2)/b+a^4*c*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2])/b^4) + (a^2*c*x*Sqrt[c*x^2])/(2*b^3) - (a*c*x^2*Sqrt[
c*x^2])/(3*b^2) + (c*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c*Sqrt[c*x^2]*Log[a + b*
x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x (cx^2)^{3/2}}{a+bx} dx = \frac{\left(c\sqrt{cx^2}\right) \int \frac{x^4}{a+bx} dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right) \int \left(-\frac{a^3}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4(a+bx)}\right) dx}{x}$$
$$= -\frac{a^3c\sqrt{cx^2}}{b^4} + \frac{a^2cx\sqrt{cx^2}}{2b^3} - \frac{acx^2\sqrt{cx^2}}{3b^2} + \frac{cx^3\sqrt{cx^2}}{4b} + \frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.60

$$\frac{(cx^2)^{3/2} (bx (-12a^3 + 6a^2bx - 4ab^2x^2 + 3b^3x^3) + 12a^4 \log(a+bx))}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^
4*Log[a + b*x]))/(12*b^5*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 63, normalized size = 0.59

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}} (3b^4x^4 - 4a\,b^3x^3 + 6a^2b^2x^2 + 12a^4 \ln(bx+a) - 12a^3bx)}{12b^5x^3}$	63
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{4}b^3x^4 - \frac{1}{3}a\,b^2x^3 + \frac{1}{2}a^2b\,x^2 - a^3x\right)}{x\,b^4} + \frac{a^4c\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*x^2)^(3/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x^3
```

Maxima [A]
time = 0.28, size = 124, normalized size = 1.16

$$\frac{(-1)^{\frac{2cx}{b}}a^4c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}a^4c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{(cx^2)^{\frac{3}{2}}x}{4b}+\frac{\sqrt{cx^2}\,a^2cx}{2b^3}-\frac{(cx^2)^{\frac{3}{2}}a}{3b^2}-\frac{\sqrt{cx^2}\,a^3c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(3/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*x/b + 1/2*sqrt(c*x^2
)*a^2*c*x/b^3 - 1/3*(c*x^2)^(3/2)*a/b^2 - sqrt(c*x^2)*a^3*c/b^4
```

Fricas [A]
time = 0.29, size = 67, normalized size = 0.63

$$\frac{(3b^4cx^4-4ab^3cx^3+6a^2b^2cx^2-12a^3bcx+12a^4c\log(bx+a))\sqrt{cx^2}}{12b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c*x^4 - 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 - 12*a^3*b*c*x + 12*a^4
*c*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(cx^2)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 89, normalized size = 0.83

$$\sqrt{c}\,c\left(\frac{-a^3x\mathrm{sign}(x)+\frac{1}{4}b^3x^4\mathrm{sign}(x)-\frac{1}{3}ab^2x^3\mathrm{sign}(x)+\frac{1}{2}a^2bx^2\mathrm{sign}(x)}{b^4}+\frac{a^4\mathrm{sign}(x)\ln|bx+a|}{b^5}-\frac{a^4\ln|a|\cdot\mathrm{sign}(x)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/12*c^(3/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(c\,x^2)^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x), x)
```

3.861 $\int \frac{(cx^2)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=84

$$\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] a^2*c*(c*x^2)^(1/2)/b^3-1/2*a*c*x*(c*x^2)^(1/2)/b^2+1/3*c*x^2*(c*x^2)^(1/2)
/b-a^3*c*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]

time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{15, 45\}$

$$-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (a^2*c*Sqrt[c*x^2])/b^3 - (a*c*x*Sqrt[c*x^2])/(2*b^2) + (c*x^2*Sqrt[c*x^2])
/(3*b) - (a^3*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{a+bx}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.63

$$\frac{(cx^2)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6
*b^4*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(a + b*x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.11, size = 52, normalized size = 0.62

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6x^3b^4}$	52
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{x\,b^3}-\frac{a^3c\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^3/b
^4
```

Maxima [A]
time = 0.27, size = 109, normalized size = 1.30

$$-\frac{(-1)^{\frac{2cx}{b}}a^3c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2acx}{b}}a^3c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,acx}{2\,b^2}+\frac{(cx^2)^{\frac{3}{2}}}{3\,b}+\frac{\sqrt{cx^2}\,a^2c}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(3/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c*x/b^2 + 1/3*(c*x^
2)^(3/2)/b + sqrt(c*x^2)*a^2*c/b^3
```

Fricas [A]
time = 0.30, size = 55, normalized size = 0.65

$$\frac{\left(2\,b^{3}cx^{3}-3\,ab^{2}cx^{2}+6\,a^{2}bcx-6\,a^{3}c\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{6\,b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c*x^3 - 3*a*b^2*c*x^2 + 6*a^2*b*c*x - 6*a^3*c*log(b*x + a))*sqrt
(c*x^2)/(b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 74, normalized size = 0.88

$$\sqrt{c}\,c\left(\frac{a^{2}x\mathrm{sign}\left(x\right)+\frac{1}{3}b^{2}x^{3}\mathrm{sign}\left(x\right)-\frac{1}{2}abx^{2}\mathrm{sign}\left(x\right)}{b^{3}}-\frac{a^{3}\mathrm{sign}\left(x\right)\ln\left|bx+a\right|}{b^{4}}+\frac{a^{3}\ln\left|a\right|\cdot\mathrm{sign}\left(x\right)}{b^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*c^(3/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^{2}\right)}^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(3/2)/(a + b*x), x)
```

3.862 $\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx$

Optimal. Leaf size=61

$$-\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}+\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] -a*c*(c*x^2)^(1/2)/b^2+1/2*c*x*(c*x^2)^(1/2)/b+a^2*c*ln(b*x+a)*(c*x^2)^(1/2
)/b^3/x
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] -((a*c*Sqrt[c*x^2])/b^2) + (c*x*Sqrt[c*x^2])/(2*b) + (a^2*c*Sqrt[c*x^2]*Log
[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}$$
$$= -\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}+\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.69

$$\frac{c^2x\left(bx(-2a+bx)+2a^2\log(a+bx)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]
[Out] (c^2*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.11, size = 40, normalized size = 0.66

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\,(x^2b^2+2a^2\ln(bx+a)-2abx)}{2b^3x^3}$	40
risch	$\frac{c\sqrt{c\,x^2}\,(\frac{1}{2}x^2b-ax)}{x\,b^2}+\frac{a^2c\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(3/2)*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^3
```

Maxima [A]
time = 0.28, size = 93, normalized size = 1.52

$$\frac{(-1)^{\frac{2cx}{b}}a^2c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2acx}{b}}a^2c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,cx}{2b}-\frac{\sqrt{cx^2}\,ac}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c*x/b - sqrt(c*x^2)*a*
c/b^2
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.69

$$\frac{(b^2cx^2-2\,abcx+2\,a^2c\log(bx+a))\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*x^2 - 2*a*b*c*x + 2*a^2*c*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{3}{2}}}{x\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 60, normalized size = 0.98

$$\sqrt{c}\,c\left(\frac{-ax\mathrm{sign}\,(x)+\frac{1}{2}bx^2\mathrm{sign}\,(x)}{b^2}+\frac{a^2\mathrm{sign}\,(x)\ln|bx+a|}{b^3}-\frac{a^2\ln|a|\cdot\mathrm{sign}\,(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x)
```

```
[Out] 1/2*c^(3/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^2)^{3/2}}{x\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)), x)
```

3.863 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=40

$$\frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

```
[Out] c*(c*x^2)^(1/2)/b-a*c*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2])/b - (a*c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{a+bx}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\
&= \frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\,\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 30, normalized size = 0.75

$$\frac{c^2x(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c^2*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**A**]
time = 0.13, size = 29, normalized size = 0.72

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(a\ln\left(bx+a\right)-bx\right)}{b^{2}x^{3}}$	29
risch	$\frac{c\sqrt{c\,x^{2}}}{b}-\frac{ac\ln\left(bx+a\right)\sqrt{c\,x^{2}}}{b^{2}x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(3/2)*(a*ln(b*x+a)-b*x)/b^2/x^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(36) = 72.
time = 0.32, size = 75, normalized size = 1.88

$$-\frac{(-1)^{\frac{2cx}{b}}ac^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}ac^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(3/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c/b
```

Fricas [A]
time = 0.30, size = 29, normalized size = 0.72

$$\frac{(bcx - ac\log(bx + a))\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x - a*c*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 41, normalized size = 1.02

$$\sqrt{c}\,c\left(\frac{x\mathrm{sign}(x)}{b} - \frac{a\mathrm{sign}(x)\ln|bx + a|}{b^2} + \frac{a\ln|a|\cdot\mathrm{sign}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x)
```

```
[Out] c^(3/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^2)^{3/2}}{x^2\,(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)), x)
```

3.864 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)} dx$

Optimal. Leaf size=23

$$\frac{c\sqrt{cx^2}\,\log(a+bx)}{bx}$$

```
[Out] c*ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{c\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,\log(a+bx)}{bx} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.13, size = 21, normalized size = 0.91

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \ln \left(b x +a \right)}{x^{3} b}$	21
risch	$\frac{c \ln \left(b x +a \right) \sqrt{c\,x^{2}}}{b x}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)/x^3*ln(b*x+a)/b
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.57

$$\frac{c^{\frac{3}{2}} \log \left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] c^(3/2)*log(b*x + a)/b
```

Fricas [A]
time = 0.29, size = 21, normalized size = 0.91

$$\frac{\sqrt{cx^2}\, c \log \left(bx + a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c*log(b*x + a)/(b*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x)
```

Giac [**A**]
time = 0.00, size = 29, normalized size = 1.26

$$\sqrt{c}\,c\left(-\frac{\ln|a|\cdot\mathrm{sign}\,(x)}{b}+\frac{\mathrm{sign}\,(x)\ln|bx+a|}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x)
```

```
[Out] c^(3/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,x^2)^{3/2}}{x^3\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^3*(a + b*x)), x)
```

3.865 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{c\sqrt{cx^2}\,\log(x)}{ax}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

[Out] c*ln(x)*(c*x^2)^(1/2)/a/x-c*ln(b*x+a)*(c*x^2)^(1/2)/a/x

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c\sqrt{cx^2}\,\log(x)}{ax}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]

[Out] (c*Sqrt[c*x^2]*Log[x])/(a*x) - (c*Sqrt[c*x^2]*Log[a + b*x])/(a*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax} - \frac{\left(bc\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{c\sqrt{cx^2}\log(x)}{ax} - \frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

Mathematica [**A**]
time = 0.01, size = 27, normalized size = 0.61

$$\frac{(cx^2)^{3/2}(\log(x)-\log(a+bx))}{ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
[Out] ((c*x^2)^(3/2)*(Log[x] - Log[a + b*x]))/(a*x^3)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.11, size = 26, normalized size = 0.59

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(\ln(x)-\ln(bx+a))}{a\,x^3}$	26
risch	$\frac{c\sqrt{c\,x^2}\,\ln(-x)}{xa}-\frac{c\ln(bx+a)\sqrt{c\,x^2}}{ax}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)*(ln(x)-ln(b*x+a))/a/x^3
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.55

$$-\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{a}+\frac{c^{\frac{3}{2}}\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(3/2)*log(b*x + a)/a + c^(3/2)*log(x)/a
```

Fricas [A]
time = 0.30, size = 66, normalized size = 1.50

$$\left[\frac{\sqrt{cx^2}\,c\log\left(\frac{x}{bx+a}\right)}{ax},\ \frac{2\sqrt{-c}\,c\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^4\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)), x)
```

3.866 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx$

Optimal. Leaf size=64

$$-\frac{c\sqrt{cx^2}}{ax^2}-\frac{bc\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] -c*(c*x^2)^(1/2)/a/x^2-b*c*ln(x)*(c*x^2)^(1/2)/a^2/x+b*c*ln(b*x+a)*(c*x^2)^
(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 46}

$$-\frac{bc\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a*x^2)) - (b*c*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c*Sqrt[c
*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{ax^2} - \frac{bc\sqrt{cx^2}\,\log(x)}{a^2x} + \frac{bc\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

Mathematica [**A**]
time = 0.01, size = 34, normalized size = 0.53

$$-\frac{c^2(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
[Out] -((c^2*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.11, size = 33, normalized size = 0.52

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}(bx\ln(x)-b\ln(bx+a)x+a)}{a^2x^4}$	33
risch	$-\frac{c\sqrt{c\,x^2}}{a\,x^2}+\frac{c\sqrt{c\,x^2}\;b\ln(-bx-a)}{x\,a^2}-\frac{bc\ln(x)\sqrt{c\,x^2}}{a^2x}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(3/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x^4
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.58

$$\frac{bc^{\frac{3}{2}}\log(bx+a)}{a^2} - \frac{bc^{\frac{3}{2}}\log(x)}{a^2} - \frac{c^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="maxima")
[Out] b*c^(3/2)*log(b*x + a)/a^2 - b*c^(3/2)*log(x)/a^2 - c^(3/2)/(a*x)
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.52

$$\frac{\left(bcx\log\left(\frac{bx+a}{x}\right) - ac\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="fricas")
[Out] (b*c*x*log((b*x + a)/x) - a*c)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a),x)
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x)
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^5\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)), x)
```

3.867 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx$

Optimal. Leaf size=88

$$-\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}$$

```
[Out] -1/2*c*(c*x^2)^(1/2)/a/x^3+b*c*(c*x^2)^(1/2)/a^2/x^2+b^2*c*ln(x)*(c*x^2)^(1
/2)/a^3/x-b^2*c*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] -1/2*(c*Sqrt[c*x^2])/(a*x^3) + (b*c*Sqrt[c*x^2])/(a^2*x^2) + (b^2*c*Sqrt[c*
x^2]*Log[x])/(a^3*x) - (b^2*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.60

$$\frac{(cx^2)^{3/2}\left(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx)\right)}{2a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
[Out] ((c*x^2)^(3/2)*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x
]))/(2*a^3*x^5)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.58

method	result	size
default	$\frac{(cx^2)^{\frac{3}{2}}(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2)}{2a^3x^5}$	51
risch	$\frac{c\sqrt{cx^2}\left(\frac{bx}{a^2}-\frac{1}{2a}\right)}{x^3}+\frac{c\sqrt{cx^2}\,b^2\ln(-x)}{x\,a^3}-\frac{b^2c\ln(bx+a)\sqrt{cx^2}}{a^3x}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] 1/2*(c*x^2)^(3/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^5

Maxima [A]
time = 0.29, size = 52, normalized size = 0.59

$$-\frac{b^2c^{\frac{3}{2}}\log(bx+a)}{a^3}+\frac{b^2c^{\frac{3}{2}}\log(x)}{a^3}+\frac{2\,bc^{\frac{3}{2}}x-ac^{\frac{3}{2}}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="maxima")

[Out] -b^2*c^(3/2)*log(b*x + a)/a^3 + b^2*c^(3/2)*log(x)/a^3 + 1/2*(2*b*c^(3/2)*x - a*c^(3/2))/(a^2*x^2)

Fricas [A]
time = 0.30, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2cx^2\log\left(\frac{x}{bx+a}\right)+2\,abcx-a^2c\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="fricas")

[Out] 1/2*(2*b^2*c*x^2*log(x/(b*x + a)) + 2*a*b*c*x - a^2*c)*sqrt(c*x^2)/(a^3*x^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{3}{2}}}{x^6\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a),x)

[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x)

[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^6\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)), x)
```

3.868 $\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=112

$$-\frac{c\sqrt{cx^2}}{3ax^4}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

```
[Out] -1/3*c*(c*x^2)^(1/2)/a/x^4+1/2*b*c*(c*x^2)^(1/2)/a^2/x^3-b^2*c*(c*x^2)^(1/2
)/a^3/x^2-b^3*c*ln(x)*(c*x^2)^(1/2)/a^4/x+b^3*c*ln(b*x+a)*(c*x^2)^(1/2)/a^4
/x
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -1/3*(c*Sqrt[c*x^2])/(a*x^4) + (b*c*Sqrt[c*x^2])/(2*a^2*x^3) - (b^2*c*Sqrt[
c*x^2])/(a^3*x^2) - (b^3*c*Sqrt[c*x^2]*Log[x])/(a^4*x) + (b^3*c*Sqrt[c*x^2]
*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^4(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{3ax^4} + \frac{bc\sqrt{cx^2}}{2a^2x^3} - \frac{b^2c\sqrt{cx^2}}{a^3x^2} - \frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x} + \frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.58

$$-\frac{(cx^2)^{3/2}\left(a\left(2a^2-3abx+6b^2x^2\right)+6b^3x^3\log(x)-6b^3x^3\log(a+bx)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
[Out] -1/6*((c*x^2)^(3/2)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] - 6
*b^3*x^3*Log[a + b*x]))/(a^4*x^6)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.13, size = 62, normalized size = 0.55

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3\right)}{6a^4x^6}$	62
risch	$\frac{c\sqrt{c\,x^2}\left(-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}\right)}{x^4}-\frac{b^3c\ln(x)\sqrt{c\,x^2}}{a^4x}+\frac{c\sqrt{c\,x^2}\,b^3\ln(-bx-a)}{x\,a^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^7/(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] -1/6*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/a^4/x^6

Maxima [A]
time = 0.26, size = 66, normalized size = 0.59

$$\frac{b^3c^{\frac{3}{2}}\log\left(bx+a\right)}{a^4}-\frac{b^3c^{\frac{3}{2}}\log\left(x\right)}{a^4}-\frac{6\,b^2c^{\frac{3}{2}}x^2-3\,abc^{\frac{3}{2}}x+2\,a^2c^{\frac{3}{2}}}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="maxima")

[Out] b^3*c^(3/2)*log(b*x + a)/a^4 - b^3*c^(3/2)*log(x)/a^4 - 1/6*(6*b^2*c^(3/2)*x^2 - 3*a*b*c^(3/2)*x + 2*a^2*c^(3/2))/(a^3*x^3)

Fricas [A]
time = 0.30, size = 59, normalized size = 0.53

$$\frac{\left(6\,b^3cx^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2cx^2+3\,a^2bcx-2\,a^3c\right)\sqrt{cx^2}}{6\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="fricas")

[Out] 1/6*(6*b^3*c*x^3*log((b*x + a)/x) - 6*a*b^2*c*x^2 + 3*a^2*b*c*x - 2*a^3*c)*sqrt(c*x^2)/(a^4*x^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^7\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(3/2)/x**7/(b*x+a),x)

[Out] Integral((c*x**2)**(3/2)/(x**7*(a + b*x)), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x)

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^2)^{3/2}}{x^7\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^7*(a + b*x)), x)
```

3.869 $\int \frac{(cx^2)^{5/2}}{a+bx} \, dx$

Optimal. Leaf size=142

$$\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}$$

```
[Out] a^4*c^2*(c*x^2)^(1/2)/b^5-1/2*a^3*c^2*x*(c*x^2)^(1/2)/b^4+1/3*a^2*c^2*x^2*(
c*x^2)^(1/2)/b^3-1/4*a*c^2*x^3*(c*x^2)^(1/2)/b^2+1/5*c^2*x^4*(c*x^2)^(1/2)/
b-a^5*c^2*ln(b*x+a)*(c*x^2)^(1/2)/b^6/x
```

Rubi [A]
time = 0.03, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}+\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2])/b^5 - (a^3*c^2*x*Sqrt[c*x^2])/(2*b^4) + (a^2*c^2*x^2*
Sqrt[c*x^2])/(3*b^3) - (a*c^2*x^3*Sqrt[c*x^2])/(4*b^2) + (c^2*x^4*Sqrt[c*x^
2])/(5*b) - (a^5*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^6*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^5}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a^5}{b^5(a+bx)}\right)dx}{x}$$
$$= \frac{a^4c^2\sqrt{cx^2}}{b^5} - \frac{a^3c^2x\sqrt{cx^2}}{2b^4} + \frac{a^2c^2x^2\sqrt{cx^2}}{3b^3} - \frac{ac^2x^3\sqrt{cx^2}}{4b^2} + \frac{c^2x^4\sqrt{cx^2}}{5b} - \frac{a^5c^2\sqrt{cx^2}\log(a}{b^6x}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 0.54

$$\frac{c^3x\left(bx\left(60a^4 - 30a^3bx + 20a^2b^2x^2 - 15ab^3x^3 + 12b^4x^4\right) - 60a^5\log(a+bx)\right)}{60b^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (c^3*x*(b*x*(60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^4*x
^4) - 60*a^5*Log[a + b*x]))/(60*b^6*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(a + b*x),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.52

method	result	size
default	$-\frac{(c\,x^2)^{\frac{5}{2}}\left(-12b^5x^5+15a\,b^4x^4-20a^2b^3x^3+30a^3b^2x^2+60a^5\ln(bx+a)-60a^4bx\right)}{60x^5b^6}$	74
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{5}b^4x^5-\frac{1}{4}a\,b^3x^4+\frac{1}{3}a^2b^2x^3-\frac{1}{2}a^3b\,x^2+a^4x\right)}{x\,b^5}-\frac{a^5c^2\ln(bx+a)\sqrt{c\,x^2}}{b^6x}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(c*x^2)^(5/2)*(-12*b^5*x^5+15*a*b^4*x^4-20*a^2*b^3*x^3+30*a^3*b^2*x^2
+60*a^5*ln(b*x+a)-60*a^4*b*x)/x^5/b^6
```

Maxima [A]
time = 0.28, size = 146, normalized size = 1.03

$$-\frac{(-1)^{\frac{2cx}{b}}a^5c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^6}-\frac{(-1)^{\frac{2acx}{b}}a^5c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^6}-\frac{(cx^2)^{\frac{3}{2}}acx}{4b^2}-\frac{\sqrt{cx^2}\,a^3c^2x}{2b^4}+\frac{(cx^2)^{\frac{5}{2}}}{5b}+\frac{(cx^2)^{\frac{3}{2}}a^2c}{3b^3}+\frac{\sqrt{cx^2}\,a^4c^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^5*c^(5/2)*log(2*c*x/b)/b^6 - (-1)^(2*a*c*x/b)*a^5*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^6 - 1/4*(c*x^2)^(3/2)*a*c*x/b^2 - 1/2*sqr
t(c*x^2)*a^3*c^2*x/b^4 + 1/5*(c*x^2)^(5/2)/b + 1/3*(c*x^2)^(3/2)*a^2*c/b^3
+ sqrt(c*x^2)*a^4*c^2/b^5
```

Fricas [A]
time = 0.29, size = 91, normalized size = 0.64

$$\frac{\left(12\,b^5c^2x^5-15\,ab^4c^2x^4+20\,a^2b^3c^2x^3-30\,a^3b^2c^2x^2+60\,a^4bc^2x-60\,a^5c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{60\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*c^2*x^5 - 15*a*b^4*c^2*x^4 + 20*a^2*b^3*c^2*x^3 - 30*a^3*b^2*c
^2*x^2 + 60*a^4*b*c^2*x - 60*a^5*c^2*log(b*x + a))*sqrt(c*x^2)/(b^6*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(a + b*x), x)
```

Giac [A]
time = 0.00, size = 123, normalized size = 0.87

$$\sqrt{c}\left(\frac{a^4c^2x\mathrm{sign}\left(x\right)+\frac{1}{5}b^4c^2x^5\mathrm{sign}\left(x\right)-\frac{1}{4}ab^3c^2x^4\mathrm{sign}\left(x\right)+\frac{1}{3}a^2b^2c^2x^3\mathrm{sign}\left(x\right)-\frac{1}{2}a^3bc^2x^2\mathrm{sign}\left(x\right)}{b^5}-\frac{a^5c^2\mathrm{sign}\left(x\right)\ln|bx+a|}{b^6}+\frac{a^5c^2\ln|a|\cdot\mathrm{sign}\left(x\right)}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x)
```

```
[Out] -1/60*(60*a^5*c^2*log(abs(b*x + a))*sgn(x)/b^6 - 60*a^5*c^2*log(abs(a))*sgn
(x)/b^6 - (12*b^4*c^2*x^5*sgn(x) - 15*a*b^3*c^2*x^4*sgn(x) + 20*a^2*b^2*c^2
*x^3*sgn(x) - 30*a^3*b*c^2*x^2*sgn(x) + 60*a^4*c^2*x*sgn(x))/b^5)*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(5/2)/(a + b*x), x)
```

3.870 $\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] -a^3*c^2*(c*x^2)^(1/2)/b^4+1/2*a^2*c^2*x*(c*x^2)^(1/2)/b^3-1/3*a*c^2*x^2*(c
*x^2)^(1/2)/b^2+1/4*c^2*x^3*(c*x^2)^(1/2)/b+a^4*c^2*ln(b*x+a)*(c*x^2)^(1/2)
/b^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2])/b^4) + (a^2*c^2*x*Sqrt[c*x^2])/(2*b^3) - (a*c^2*x^2
*Sqrt[c*x^2])/(3*b^2) + (c^2*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c^2*Sqrt[c*x^2]*
Log[a + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 0.56

$$\frac{c\,(cx^2)^{3/2}\,(bx\,(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3)+12a^4\log(a+bx))}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]
[Out] (c*(c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*
a^4*Log[a + b*x]))/(12*b^5*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.54

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}\,(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx)}{12b^5x^5}$	63
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{4}b^3x^4-\frac{1}{3}a\,b^2x^3+\frac{1}{2}a^2b\,x^2-a^3x\right)}{x\,b^4}+\frac{a^4c^2\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*x^2)^(5/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x^5
```

Maxima [A]
time = 0.29, size = 130, normalized size = 1.11

$$\frac{(-1)^{\frac{2cx}{b}}a^4c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}a^4c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{(cx^2)^{\frac{3}{2}}cx}{4b}+\frac{\sqrt{cx^2}a^2c^2x}{2b^3}-\frac{(cx^2)^{\frac{3}{2}}ac}{3b^2}-\frac{\sqrt{cx^2}a^3c^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(5/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*c*x/b + 1/2*sqrt(c*x
^2)*a^2*c^2*x/b^3 - 1/3*(c*x^2)^(3/2)*a*c/b^2 - sqrt(c*x^2)*a^3*c^2/b^4
```

Fricas [A]
time = 0.29, size = 77, normalized size = 0.66

$$\frac{(3b^4c^2x^4-4ab^3c^2x^3+6a^2b^2c^2x^2-12a^3bc^2x+12a^4c^2\log(bx+a))\sqrt{cx^2}}{12b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c^2*x^4 - 4*a*b^3*c^2*x^3 + 6*a^2*b^2*c^2*x^2 - 12*a^3*b*c^2*x
+ 12*a^4*c^2*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}}{x(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 106, normalized size = 0.91

$$\sqrt{c}\left(\frac{-a^3c^2x\mathrm{sign}(x)+\frac{1}{4}b^3c^2x^4\mathrm{sign}(x)-\frac{1}{3}ab^2c^2x^3\mathrm{sign}(x)+\frac{1}{2}a^2bc^2x^2\mathrm{sign}(x)}{b^4}+\frac{a^4c^2\mathrm{sign}(x)\ln|bx+a|}{b^5}-\frac{a^4c^2\ln|a|\cdot\mathrm{sign}(x)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x)
```

```
[Out] 1/12*(12*a^4*c^2*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*c^2*log(abs(a))*sgn(
x)/b^5 + (3*b^3*c^2*x^4*sgn(x) - 4*a*b^2*c^2*x^3*sgn(x) + 6*a^2*b*c^2*x^2*s
gn(x) - 12*a^3*c^2*x*sgn(x))/b^4)*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{5/2}}{x\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x*(a + b*x)), x)
```

3.871 $\int \frac{(cx^2)^{5/2}}{x^2(a+bx)} dx$

Optimal. Leaf size=92

$$\frac{a^2c^2\sqrt{cx^2}}{b^3} - \frac{ac^2x\sqrt{cx^2}}{2b^2} + \frac{c^2x^2\sqrt{cx^2}}{3b} - \frac{a^3c^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] a^2*c^2*(c*x^2)^(1/2)/b^3-1/2*a*c^2*x*(c*x^2)^(1/2)/b^2+1/3*c^2*x^2*(c*x^2)
^(1/2)/b-a^3*c^2*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3c^2\sqrt{cx^2}\,\log(a+bx)}{b^4x} + \frac{a^2c^2\sqrt{cx^2}}{b^3} - \frac{ac^2x\sqrt{cx^2}}{2b^2} + \frac{c^2x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2])/b^3 - (a*c^2*x*Sqrt[c*x^2])/(2*b^2) + (c^2*x^2*Sqrt[c
*x^2])/(3*b) - (a^3*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}$$
$$= \frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}-\frac{a^3c^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 0.59

$$\frac{c\,(cx^2)^{3/2}\,(bx\,(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx))}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
[Out] (c*(c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/
(6*b^4*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.57

method	result	size
default	$-\frac{(c\,x^2)^{\frac{5}{2}}\,(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx)}{6b^4x^5}$	52
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{x\,b^3}-\frac{a^3c^2\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^2)^(5/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/b^4/x
^5
```

Maxima [A]
time = 0.30, size = 114, normalized size = 1.24

$$-\frac{(-1)^{\frac{2cx}{b}}a^3c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2acx}{b}}a^3c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ac^2x}{2\,b^2}+\frac{(cx^2)^{\frac{3}{2}}\,c}{3\,b}+\frac{\sqrt{cx^2}\,a^2c^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(5/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c^2*x/b^2 + 1/3*(c*
x^2)^(3/2)*c/b + sqrt(c*x^2)*a^2*c^2/b^3
```

Fricas [A]
time = 0.29, size = 63, normalized size = 0.68

$$\frac{(2\,b^3c^2x^3-3\,ab^2c^2x^2+6\,a^2bc^2x-6\,a^3c^2\log(bx+a))\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c^2*x^3 - 3*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - 6*a^3*c^2*log(b*x +
a))*sqrt(c*x^2)/(b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}}{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**2*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 88, normalized size = 0.96

$$\sqrt{c}\left(\frac{a^2c^2x\mathrm{sign}(x)+\frac{1}{3}b^2c^2x^3\mathrm{sign}(x)-\frac{1}{2}abc^2x^2\mathrm{sign}(x)}{b^3}-\frac{a^3c^2\mathrm{sign}(x)\ln|bx+a|}{b^4}+\frac{a^3c^2\ln|a|\cdot\mathrm{sign}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x)
```

```
[Out] -1/6*(6*a^3*c^2*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*c^2*log(abs(a))*sgn(x)
/b^4 - (2*b^2*c^2*x^3*sgn(x) - 3*a*b*c^2*x^2*sgn(x) + 6*a^2*c^2*x*sgn(x))/b
^3)*sqrt(c)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^2)^{5/2}}{x^2\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^2*(a + b*x)), x)
```

3.872 $\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=67

$$-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}$$

```
[Out] -a*c^2*(c*x^2)^(1/2)/b^2+1/2*c^2*x*(c*x^2)^(1/2)/b+a^2*c^2*ln(b*x+a)*(c*x^2
)^(1/2)/b^3/x
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2])/b^2) + (c^2*x*Sqrt[c*x^2])/(2*b) + (a^2*c^2*Sqrt[c*x^
2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}$$
$$= -\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.63

$$\frac{c^3x\left(bx(-2a+bx)+2a^2\log(a+bx)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
[Out] (c^3*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.17, size = 40, normalized size = 0.60

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}(x^2b^2+2a^2\ln(bx+a)-2abx)}{2b^3x^5}$	40
risch	$\frac{c^2\sqrt{c\,x^2}\,(\frac{1}{2}x^2b-ax)}{x\,b^2}+\frac{a^2c^2\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(5/2)*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^5
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.45

$$\frac{(-1)^{\frac{2cx}{b}}a^2c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2acx}{b}}a^2c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,c^2x}{2b}-\frac{\sqrt{cx^2}\,ac^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*c^(5/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c^2*x/b - sqrt(c*x^2)*
a*c^2/b^2
```

Fricas [A]
time = 0.29, size = 48, normalized size = 0.72

$$\frac{(b^2c^2x^2-2abc^2x+2a^2c^2\log(bx+a))\sqrt{cx^2}}{2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 - 2*a*b*c^2*x + 2*a^2*c^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}}{x^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**3*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 71, normalized size = 1.06

$$\sqrt{c}\left(\frac{-ac^2x\mathrm{sign}(x)+\frac{1}{2}bc^2x^2\mathrm{sign}(x)}{b^2}+\frac{a^2c^2\mathrm{sign}(x)\ln|bx+a|}{b^3}-\frac{a^2c^2\ln|a|\cdot\mathrm{sign}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x)
```

```
[Out] 1/2*(2*a^2*c^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*c^2*log(abs(a))*sgn(x)/
b^3 + (b*c^2*x^2*sgn(x) - 2*a*c^2*x*sgn(x))/b^2)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^3*(a + b*x)),x)
[Out] int((c*x^2)^(5/2)/(x^3*(a + b*x)), x)
```

3.873 $\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{c^2\sqrt{cx^2}}{b}-\frac{ac^2\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

```
[Out] c^2*(c*x^2)^(1/2)/b-a*c^2*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{c^2\sqrt{cx^2}}{b}-\frac{ac^2\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2])/b - (a*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\
&= \frac{c^2\sqrt{cx^2}}{b}-\frac{ac^2\sqrt{cx^2}\,\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 0.68

$$\frac{c^3x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
[Out] (c^3*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.66

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(a\ln \left(b x +a \right)-b x \right)}{b^{2} x^{5}}$	29
risch	$\frac{c^{2} \sqrt{c\,x^{2}}}{b}-\frac{a\,c^{2} \ln \left(b x +a \right) \sqrt{c\,x^{2}}}{b^{2} x}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(5/2)*(a*ln(b*x+a)-b*x)/b^2/x^5
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.75

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^2}-\frac{(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}+\frac{\sqrt{cx^2}\,c^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="maxima")
[Out] -(-1)^(2*c*x/b)*a*c^(5/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(5/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c^2/b
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.75

$$\frac{\left(bc^2x - ac^2 \log\left(bx + a\right)\right)\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x - a*c^2*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**4*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 49, normalized size = 1.11

$$\sqrt{c}\left(\frac{c^2x\mathrm{sign}\left(x\right)}{b} - \frac{ac^2\mathrm{sign}\left(x\right)\ln\left|bx + a\right|}{b^2} + \frac{ac^2\ln\left|a\right|\cdot\mathrm{sign}\left(x\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x)
```

```
[Out] (c^2*x*sgn(x)/b - a*c^2*log(abs(b*x + a))*sgn(x)/b^2 + a*c^2*log(abs(a))*sg
n(x)/b^2)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}}{x^4\,\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^4*(a + b*x)), x)
```

3.874 $\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} \, dx$

Optimal. Leaf size=25

$$\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}$$

[Out] c^2*ln(b*x+a)*(c*x^2)^(1/2)/b/x

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]

[Out] (c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^{5/2}}{x^5(a+bx)} \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{a+bx} \, dx}{x} \\ &= \frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.13, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \ln \left(b x +a \right)}{x^{5} b}$	21
risch	$\frac{c^{2} \ln \left(b x +a \right) \sqrt{c\,x^{2}}}{b x}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(5/2)/x^5*ln(b*x+a)/b
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.52

$$\frac{c^{\frac{5}{2}} \log \left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="maxima")
[Out] c^(5/2)*log(b*x + a)/b
```

Fricas [A]
time = 0.30, size = 23, normalized size = 0.92

$$\frac{\sqrt{cx^2}\,c^2 \log \left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c^2*log(b*x + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}}{x^5\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 34, normalized size = 1.36

$$\sqrt{c}\,\left(-\frac{c^2\ln|a|\cdot\operatorname{sign}(x)}{b}+\frac{c^2\operatorname{sign}(x)\ln|bx+a|}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x)
```

```
[Out] (c^2*log(abs(b*x + a))*sgn(x)/b - c^2*log(abs(a))*sgn(x)/b)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,x^2)^{5/2}}{x^5\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^5*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^5*(a + b*x)), x)
```

3.875 $\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx$

Optimal. Leaf size=48

$$\frac{c^2\sqrt{cx^2}\,\log(x)}{ax}-\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

[Out] c^2*ln(x)*(c*x^2)^(1/2)/a/x-c^2*ln(b*x+a)*(c*x^2)^(1/2)/a/x

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c^2\sqrt{cx^2}\,\log(x)}{ax}-\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]

[Out] (c^2*Sqrt[c*x^2]*Log[x])/(a*x) - (c^2*Sqrt[c*x^2]*Log[a + b*x])/(a*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax} - \frac{\left(bc^2\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}$$

Mathematica [**A**]
time = 0.01, size = 28, normalized size = 0.58

$$\frac{c^3x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^3*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.14, size = 26, normalized size = 0.54

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}(\ln(x)-\ln(bx+a))}{a\,x^5}$	26
risch	$\frac{c^2\sqrt{c\,x^2}\,\ln(-x)}{xa} - \frac{c^2\ln(bx+a)\sqrt{c\,x^2}}{ax}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^6/(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] (c*x^2)^(5/2)*(ln(x)-ln(b*x+a))/a/x^5

Maxima [A]
time = 0.26, size = 24, normalized size = 0.50

$$-\frac{c^{\frac{5}{2}}\log\left(bx+a\right)}{a}+\frac{c^{\frac{5}{2}}\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="maxima")

[Out] -c^(5/2)*log(b*x + a)/a + c^(5/2)*log(x)/a

Fricas [A]
time = 0.30, size = 70, normalized size = 1.46

$$\left[\frac{\sqrt{cx^2}\,c^2\log\left(\frac{x}{bx+a}\right)}{ax},\ \frac{2\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="fricas")

[Out] [sqrt(c*x^2)*c^2*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c^2*arctan(sqrt(c*x^2)*(2*b*x + a)*sqrt(-c)/(a*c*x))/a]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^6\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(5/2)/x**6/(b*x+a),x)

[Out] Integral((c*x**2)**(5/2)/(x**6*(a + b*x)), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x)

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^6\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^6*(a + b*x)), x)
```

3.876 $\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{c^2\sqrt{cx^2}}{ax^2}-\frac{bc^2\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] -c^2*(c*x^2)^(1/2)/a/x^2-b*c^2*ln(x)*(c*x^2)^(1/2)/a^2/x+b*c^2*ln(b*x+a)*(c
*x^2)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{bc^2\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^2*Sqrt[c*x^2])/(a*x^2)) - (b*c^2*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c^2*
Sqrt[c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx}{x}$$
$$= -\frac{c^2\sqrt{cx^2}}{ax^2} - \frac{bc^2\sqrt{cx^2}\log(x)}{a^2x} + \frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.49

$$-\frac{c^3(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
[Out] -((c^3*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.47

method	result	size
default	$-\frac{(c\,x^2)^{\frac{5}{2}}(bx\ln(x)-b\ln(bx+a)x+a)}{a^2x^6}$	33
risch	$-\frac{c^2\sqrt{c\,x^2}}{a\,x^2}+\frac{c^2\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^2}-\frac{b\,c^2\ln(x)\sqrt{c\,x^2}}{a^2x}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^7/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(5/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x^6
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.53

$$\frac{bc^{\frac{5}{2}}\log\left(bx+a\right)}{a^2}-\frac{bc^{\frac{5}{2}}\log\left(x\right)}{a^2}-\frac{c^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="maxima")
[Out] b*c^(5/2)*log(b*x + a)/a^2 - b*c^(5/2)*log(x)/a^2 - c^(5/2)/(a*x)
```

Fricas [A]
time = 0.30, size = 37, normalized size = 0.53

$$\frac{\left(bc^2x\log\left(\frac{bx+a}{x}\right)-ac^2\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="fricas")
[Out] (b*c^2*x*log((b*x + a)/x) - a*c^2)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^7\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**7/(b*x+a),x)
[Out] Integral((c*x**2)**(5/2)/(x**7*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x)
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^2\right)}^{5/2}}{x^7\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^7*(a + b*x)), x)
```

3.877 $\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=83

$$\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}$$

```
[Out] a^2*x^2/b^3/(c*x^2)^(1/2)-1/2*a*x^3/b^2/(c*x^2)^(1/2)+1/3*x^4/b/(c*x^2)^(1/
2)-a^3*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}+\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*Sqrt[c*x^2]) + x^4/(3*b*Sqrt[c
*x^2]) - (a^3*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x\int \frac{x^3}{a+bx}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.61

$$\frac{x\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[c*
x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 50, normalized size = 0.60

method	result	size
default	$-\frac{x\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6\sqrt{c\,x^2}\,b^4}$	50
risch	$\frac{x\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{\sqrt{c\,x^2}\,b^3}-\frac{a^3x\ln(bx+a)}{b^4\sqrt{c\,x^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(1/2)/b^4
```

Maxima [A]
time = 0.27, size = 142, normalized size = 1.71

$$\frac{\sqrt{cx^2}\,x^2}{3\,bc} - \frac{7\,ax^2}{6\,b^2\sqrt{c}} - \frac{(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}} + \frac{2\,\sqrt{cx^2}\,ax}{3\,b^2c} - \frac{14\,a^2x}{3\,b^3\sqrt{c}} - \frac{a^3\log\left(bx\right)}{b^4\sqrt{c}} + \frac{17\,\sqrt{cx^2}\,a^2}{3\,b^3c} - \frac{7\,a^3}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*x^2/(b*c) - 7/6*a*x^2/(b^2*sqrt(c)) - (-1)^(2*a*c*x/b)*a^3*
log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^2*c)
- 14/3*a^2*x/(b^3*sqrt(c)) - a^3*log(b*x)/(b^4*sqrt(c)) + 17/3*sqrt(c*x^2)*
a^2/(b^3*c) - 7/2*a^3/(b^4*sqrt(c))
```

Fricas [A]
time = 0.29, size = 54, normalized size = 0.65

$$\frac{\left(2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log\left(bx + a\right)\right)\sqrt{cx^2}}{6\,b^4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^2}\,\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 86, normalized size = 1.04

$$\frac{\frac{\frac{1}{3}b^2x^3\mathrm{sign}(x)^2 - \frac{1}{2}abx^2\mathrm{sign}(x)^2 + a^2x\mathrm{sign}(x)^2}{b^3\mathrm{sign}(x)^3} - \frac{a^3\ln|bx+a|}{b^4\mathrm{sign}(x)} + \frac{a^3\ln|a|\cdot\mathrm{sign}(x)}{b^4}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/6*(6*a^3*log(abs(a))*sgn(x)/b^4 - 6*a^3*log(abs(b*x + a))/(b^4*sgn(x)) +
(2*b^2*x^3*sgn(x)^2 - 3*a*b*x^2*sgn(x)^2 + 6*a^2*x*sgn(x)^2)/(b^3*sgn(x)^3)
)/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.878 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=61

$$-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}$$

```
[Out] -a*x^2/b^2/(c*x^2)^(1/2)+1/2*x^3/b/(c*x^2)^(1/2)+a^2*x*ln(b*x+a)/b^3/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[c*x^2])) + x^3/(2*b*Sqrt[c*x^2]) + (a^2*x*Log[a + b*x])
/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{x^2}{a+bx}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.64

$$\frac{x\,(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.62

method	result	size
default	$\frac{x\left(x^2b^2+2a^2\ln(bx+a)-2abx\right)}{2\sqrt{c\,x^2}\;b^3}$	38
risch	$\frac{x\left(\frac{1}{2}x^2b-ax\right)}{\sqrt{c\,x^2}\;b^2}+\frac{a^2x\ln(bx+a)}{b^3\sqrt{c\,x^2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(1/2)/b^3
```

Maxima [A]
time = 0.30, size = 100, normalized size = 1.64

$$\frac{x^2}{2\,b\sqrt{c}}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}}+\frac{2\,ax}{b^2\sqrt{c}}+\frac{a^2\log\left(bx\right)}{b^3\sqrt{c}}-\frac{3\,\sqrt{cx^2}\,a}{b^2c}+\frac{3\,a^2}{2\,b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/(b*sqrt(c)) + (-1)^(2*a*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^3*sqrt(c)) + 2*a*x/(b^2*sqrt(c)) + a^2*log(b*x)/(b^3*sqrt(c)) - 3*sqrt(c*
x^2)*a/(b^2*c) + 3/2*a^2/(b^3*sqrt(c))
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.69

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{cx^2}\,\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.08

$$\frac{\frac{-ax\mathrm{sign}(x)+\frac{1}{2}bx^2\mathrm{sign}(x)}{b^2\mathrm{sign}(x)^2}+\frac{a^2\ln|bx+a|}{b^3\mathrm{sign}(x)}-\frac{a^2\ln|a|\cdot\mathrm{sign}(x)}{b^3}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/2*(2*a^2*log(abs(a))*sgn(x)/b^3 - 2*a^2*log(abs(b*x + a))/(b^3*sgn(x)) -
 (b*x^2*sgn(x) - 2*a*x*sgn(x))/(b^2*sgn(x)^2))/sqrt(c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{c\,x^2}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.879 $\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

```
[Out] x^2/b/(c*x^2)^(1/2)-a*x*ln(b*x+a)/b^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] x^2/(b*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{\sqrt{cx^2}}\\
&= \frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.69

$$\frac{x(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 27, normalized size = 0.69

method	result	size
default	$-\frac{x(a\ln(bx+a)-bx)}{\sqrt{c\,x^2}\,b^2}$	27
risch	$\frac{x^2}{b\sqrt{c\,x^2}}-\frac{ax\ln(bx+a)}{b^2\sqrt{c\,x^2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(a*ln(b*x+a)-b*x)/(c*x^2)^(1/2)/b^2
```

Maxima [A]
time = 0.31, size = 64, normalized size = 1.64

$$-\frac{(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}}-\frac{a\log\left(bx\right)}{b^2\sqrt{c}}+\frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^2*sqrt(c)) - a*log(b*
x)/(b^2*sqrt(c)) + sqrt(c*x^2)/(b*c)
```

Fricas [A]
time = 0.29, size = 30, normalized size = 0.77

$$\frac{\sqrt{cx^2}\,(bx-a\log{(bx+a)})}{b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 43, normalized size = 1.10

$$\frac{\frac{x}{b\mathrm{sign}\,(x)}-\frac{a\ln|bx+a|}{b^2\mathrm{sign}\,(x)}+\frac{a\ln|a|\cdot\mathrm{sign}\,(x)}{b^2}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] (a*log(abs(a))*sgn(x)/b^2 + x/(b*sgn(x)) - a*log(abs(b*x + a))/(b^2*sgn(x))
)/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x^2}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.880 $\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=20

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[c*x^2]*(a + b*x)),x]

[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.13, size = 19, normalized size = 0.95

method	result	size
default	$\frac{x \ln \left(b x +a \right)}{b \sqrt{c \,x^{2}}}$	19
risch	$\frac{x \ln \left(b x +a \right)}{b \sqrt{c \,x^{2}}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 0.28, size = 46, normalized size = 2.30

$$\frac{\left(-1\right)^{\frac{2 \, a c x}{b}} \log \left(-\frac{2 \, a c x}{b {\left| b x + a \right|}}\right)}{b \sqrt{c}} + \frac{\log \left(b x\right)}{b \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*sqrt(c)) + log(b*x)/(b*s
qrt(c))
```

Fricas [**A**]
time = 0.30, size = 23, normalized size = 1.15

$$\frac{\sqrt{c x^{2}} \log \left(b x + a\right)}{b c x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 30, normalized size = 1.50

$$\frac{-\frac{\ln|a|\cdot\mathrm{sign}(x)}{b}+\frac{\ln|bx+a|}{b\mathrm{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -(log(abs(a))*sgn(x)/b - log(abs(b*x + a))/(b*sgn(x)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.881 $\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

```
[Out] x*ln(x)/a/(c*x^2)^(1/2)-x*ln(b*x+a)/a/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \frac{1}{x}\,dx}{a\sqrt{cx^2}} - \frac{(bx)\int \frac{1}{a+bx}\,dx}{a\sqrt{cx^2}}$$
$$= \frac{x\log(x)}{a\sqrt{cx^2}} - \frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.66

$$\frac{x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.11, size = 24, normalized size = 0.63

method	result	size
default	$\frac{x(\ln(x)-\ln(bx+a))}{\sqrt{c\,x^2}\,a}$	24
risch	$\frac{x\ln(-x)}{\sqrt{c\,x^2}\,a} - \frac{x\ln(bx+a)}{a\sqrt{c\,x^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(ln(x)-ln(b*x+a))/(c*x^2)^(1/2)/a
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.92

$$-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*sqrt(c))

Fricas [A]
time = 0.30, size = 70, normalized size = 1.84

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{acx}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")

[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x + a)*sqrt(-c)/(a*c*x))/(a*c)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(c*x**2)**(1/2),x)

[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x)

[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Mupad [F]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.882 $\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}$$

```
[Out] -1/a/(c*x^2)^(1/2)-b*x*ln(x)/a^2/(c*x^2)^(1/2)+b*x*ln(b*x+a)/a^2/(c*x^2)^(1
/2)
```

Rubi [A]

time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 46}

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(1/(a*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*Sqrt[c*x^2]) + (b*x*Log[a + b*x])/
(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.67

$$\frac{cx^2(-a-bx\log(x)+bx\log(a+bx))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.56

method	result	size
default	$-\frac{bx\ln(x)-b\ln(bx+a)x+a}{\sqrt{c\,x^2}\,a^2}$	30
risch	$-\frac{1}{a\sqrt{c\,x^2}}+\frac{xb\ln(-bx-a)}{\sqrt{c\,x^2}\,a^2}-\frac{bx\ln(x)}{a^2\sqrt{c\,x^2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x*ln(x)-b*ln(b*x+a)*x+a)/(c*x^2)^(1/2)/a^2
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.69

$$\frac{b\log\left(bx+a\right)}{a^2\sqrt{c}}-\frac{b\log\left(x\right)}{a^2\sqrt{c}}-\frac{1}{a\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(a^2*sqrt(c)) - b*log(x)/(a^2*sqrt(c)) - 1/(a*sqrt(c)*x)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)), x)
```

3.883 $\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=77

$$\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}$$

```
[Out] b/a^2/(c*x^2)^(1/2)-1/2/a/x/(c*x^2)^(1/2)+b^2*x*ln(x)/a^3/(c*x^2)^(1/2)-b^2
*x*ln(b*x+a)/a^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}+\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] b/(a^2*Sqrt[c*x^2]) - 1/(2*a*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*Sqrt[c*x^
2]) - (b^2*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.68

$$\frac{cx\left(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
[Out] (c*x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3
*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.66

method	result	size
default	$\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2}{2x\sqrt{c\,x^2}\,a^3}$	51
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{\sqrt{c\,x^2}\,x}+\frac{x\,b^2\ln(-x)}{\sqrt{c\,x^2}\,a^3}-\frac{b^2x\ln(bx+a)}{a^3\sqrt{c\,x^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/(c*x^2)^(1/2)/a^3
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.71

$$-\frac{b^2\log\left(bx+a\right)}{a^3\sqrt{c}}+\frac{b^2\log\left(x\right)}{a^3\sqrt{c}}+\frac{2\,b\sqrt{c}\,x-a\sqrt{c}}{2\,a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/(a^3*sqrt(c)) + b^2*log(x)/(a^3*sqrt(c)) + 1/2*(2*b*sqrt(
c)*x - a*sqrt(c))/(a^2*c*x^2)
```

Fricas [A]
time = 0.30, size = 47, normalized size = 0.61

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{cx^2}\,\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)), x)
```

3.884 $\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2}{a^3\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}$$

```
[Out] -b^2/a^3/(c*x^2)^(1/2)-1/3/a/x^2/(c*x^2)^(1/2)+1/2*b/a^2/x/(c*x^2)^(1/2)-b^
3*x*ln(x)/a^4/(c*x^2)^(1/2)+b^3*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}-\frac{b^2}{a^3\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*Sqrt[c*x^2])) - 1/(3*a*x^2*Sqrt[c*x^2]) + b/(2*a^2*x*Sqrt[c*x^2]
) - (b^3*x*Log[x])/(a^4*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*Sqrt[c*x^2
])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{b^2}{a^3\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 63, normalized size = 0.63

$$\frac{c\,(a\,(-2a^2+3abx-6b^2x^2)-6b^3x^3\log(x)+6b^3x^3\log(a+bx))}{6a^4\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log[a +
 b*x]))/(6*a^4*(c*x^2)^(3/2))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.14, size = 62, normalized size = 0.62

method	result	size
default	$-\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3}{6x^2\sqrt{c\,x^2}\;a^4}$	62
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{\sqrt{c\,x^2}\;x^2}-\frac{b^3x\ln(x)}{a^4\sqrt{c\,x^2}}+\frac{x\,b^3\ln(-bx-a)}{\sqrt{c\,x^2}\;a^4}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/6/x^2*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/(c*x^2)^(1/2)/a^4

Maxima [A]
time = 0.27, size = 69, normalized size = 0.69

$$\frac{b^3 \log (bx + a)}{a^4 \sqrt{c}} - \frac{b^3 \log (x)}{a^4 \sqrt{c}} - \frac{6\, b^2 \sqrt{c}\, x^2 - 3\, ab \sqrt{c}\, x + 2\, a^2 \sqrt{c}}{6\, a^3 c x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] b^3*log(b*x + a)/(a^4*sqrt(c)) - b^3*log(x)/(a^4*sqrt(c)) - 1/6*(6*b^2*sqrt(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c*x^3)

Fricas [A]
time = 0.30, size = 58, normalized size = 0.58

$$\frac{\left(6\, b^3 x^3 \log \left(\frac{bx+a}{x}\right) - 6\, ab^2 x^2 + 3\, a^2 bx - 2\, a^3\right) \sqrt{cx^2}}{6\, a^4 c x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x^2)/(a^4*c*x^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{cx^2}\, (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x+a)/(c*x**2)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(c*x**2)*(a + b*x)), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x)

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \sqrt{c x^2} (a + b x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)), x)
```

3.885 $\int \frac{x^6}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=95

$$\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}$$

```
[Out] a^2*x^2/b^3/c/(c*x^2)^(1/2)-1/2*a*x^3/b^2/c/(c*x^2)^(1/2)+1/3*x^4/b/c/(c*x^
2)^(1/2)-a^3*x*ln(b*x+a)/b^4/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}+\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*c*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*c*Sqrt[c*x^2]) + x^4/(3*b*c*
Sqrt[c*x^2]) - (a^3*x*Log[a + b*x])/(b^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{x^3}{a+bx}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.56

$$\frac{x^3\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*(c*x^
2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.55

method	result	size
default	$-\frac{x^3\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6\left(c\,x^2\right)^{\frac{3}{2}}b^4}$	52
risch	$\frac{x\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{c\sqrt{c\,x^2}\;b^3}-\frac{a^3x\ln(bx+a)}{b^4c\sqrt{c\,x^2}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x^3*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(3/2)/b
^4
```

Maxima [A]
time = 0.31, size = 162, normalized size = 1.71

$$\frac{x^4}{3\sqrt{cx^2}\,bc}-\frac{ax^3}{2\sqrt{cx^2}\,b^2c}+\frac{a^2x^2}{\sqrt{cx^2}\,b^3c}-\frac{(-1)^{\frac{2acx}{b}}\,a^3\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4c^{\frac{3}{2}}}+\frac{29\,a^3x}{6\sqrt{cx^2}\,b^4c}-\frac{a^3\log\left(bx\right)}{b^4c^{\frac{3}{2}}}-\frac{2\,a^4}{\sqrt{cx^2}\,b^5c}+\frac{2\,a^4}{b^5c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/(sqrt(c*x^2)*b*c) - 1/2*a*x^3/(sqrt(c*x^2)*b^2*c) + a^2*x^2/(sqrt(c
*x^2)*b^3*c) - (-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*c^(
3/2)) + 29/6*a^3*x/(sqrt(c*x^2)*b^4*c) - a^3*log(b*x)/(b^4*c^(3/2)) - 2*a^4
/(sqrt(c*x^2)*b^5*c) + 2*a^4/(b^5*c^(3/2)*x)
```

Fricas [A]
time = 0.29, size = 54, normalized size = 0.57

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 88, normalized size = 0.93

$$\frac{\frac{\frac{1}{3}b^2x^3\mathrm{sign}(x)^2-\frac{1}{2}abx^2\mathrm{sign}(x)^2+a^2x\mathrm{sign}(x)^2}{b^3\mathrm{sign}(x)^3}-\frac{a^3\ln|bx+a|}{b^4\mathrm{sign}(x)}+\frac{a^3\ln|a|\cdot\mathrm{sign}(x)}{b^4}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/6*(6*a^3*log(abs(a))*sgn(x)/b^4 - 6*a^3*log(abs(b*x + a))/(b^4*sgn(x)) +
(2*b^2*x^3*sgn(x)^2 - 3*a*b*x^2*sgn(x)^2 + 6*a^2*x*sgn(x)^2)/(b^3*sgn(x)^3)
)/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^6/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.886 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}$$

```
[Out] -a*x^2/b^2/c/(c*x^2)^(1/2)+1/2*x^3/b/c/(c*x^2)^(1/2)+a^2*x*ln(b*x+a)/b^3/c/
(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*c*Sqrt[c*x^2])) + x^3/(2*b*c*Sqrt[c*x^2]) + (a^2*x*Log[a + b
*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{x^2}{a+bx}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.59

$$\frac{x^3\left(bx(-2a+bx)+2a^2\log(a+bx)\right)}{2b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.57

method	result	size
default	$\frac{x^3\left(x^2b^2+2a^2\ln(bx+a)-2abx\right)}{2\left(cx^2\right)^{\frac{3}{2}}b^3}$	40
risch	$\frac{x\left(\frac{1}{2}x^2b-ax\right)}{c\sqrt{cx^2}\,b^2}+\frac{a^2x\ln(bx+a)}{b^3c\sqrt{cx^2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x^3*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(3/2)/b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.
time = 0.30, size = 140, normalized size = 2.00

$$\frac{x^3}{2\sqrt{cx^2}bc} - \frac{ax^2}{\sqrt{cx^2}b^2c} + \frac{(-1)^{\frac{2acx}{b}}a^2\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}} - \frac{7a^2x}{2\sqrt{cx^2}b^3c} + \frac{a^2\log(bx)}{b^3c^{\frac{3}{2}}} + \frac{2a^3}{\sqrt{cx^2}b^4c} - \frac{2a^3}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(c*x^2)*b*c) - a*x^2/(sqrt(c*x^2)*b^2*c) + (-1)^(2*a*c*x/b)*a^
2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) - 7/2*a^2*x/(sqrt(c*x^2)*b^3
*c) + a^2*log(b*x)/(b^3*c^(3/2)) + 2*a^3/(sqrt(c*x^2)*b^4*c) - 2*a^3/(b^4*c
^(3/2)*x)
```

Fricas [A]
time = 0.29, size = 42, normalized size = 0.60

$$\frac{(b^2x^2 - 2abx + 2a^2\log(bx+a))\sqrt{cx^2}}{2b^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 68, normalized size = 0.97

$$\frac{\frac{-ax\mathrm{sign}(x)+\frac{1}{2}bx^2\mathrm{sign}(x)}{b^2\mathrm{sign}(x)^2} + \frac{a^2\ln|bx+a|}{b^3\mathrm{sign}(x)} - \frac{a^2\ln|a|\cdot\mathrm{sign}(x)}{b^3}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/2*(2*a^2*log(abs(a))*sgn(x)/b^3 - 2*a^2*log(abs(b*x + a))/(b^3*sgn(x)) -
 (b*x^2*sgn(x) - 2*a*x*sgn(x))/(b^2*sgn(x)^2))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.887 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=45

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

```
[Out] x^2/b/c/(c*x^2)^(1/2)-a*x*ln(b*x+a)/b^2/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] x^2/(b*c*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.64

$$\frac{x^3(bx - a\log(a + bx))}{b^2 (cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x - a*Log[a + b*x]))/(b^2*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.64

method	result	size
default	$-\frac{x^{3}\left(a \ln \left(b x+a\right)-b x\right)}{\left(c \,x^{2}\right)^{\frac{3}{2}} b^{2}}$	29
risch	$\frac{x^{2}}{b c \sqrt{c \,x^{2}}}-\frac{a x \ln \left(b x+a\right)}{b^{2} c \sqrt{c \,x^{2}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -x^3*(a*ln(b*x+a)-b*x)/(c*x^2)^(3/2)/b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(41) = 82.
time = 0.28, size = 116, normalized size = 2.58

$$\frac{x^2}{\sqrt{cx^2}\,bc} - \frac{(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}} + \frac{2\,ax}{\sqrt{cx^2}\,b^2c} - \frac{a\log(bx)}{b^2c^{\frac{3}{2}}} - \frac{2\,a^2}{\sqrt{cx^2}\,b^3c} + \frac{2\,a^2}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(c*x^2)*b*c) - (-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^2*c^(3/2)) + 2*a*x/(sqrt(c*x^2)*b^2*c) - a*log(b*x)/(b^2*c^(3/2)) - 2*a^2
/(sqrt(c*x^2)*b^3*c) + 2*a^2/(b^3*c^(3/2)*x)
```

Fricas [A]
time = 0.29, size = 30, normalized size = 0.67

$$\frac{\sqrt{cx^2}\,(bx - a\log{(bx + a)})}{b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^2)^{\frac{3}{2}}\,(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 45, normalized size = 1.00

$$\frac{\frac{x}{b\operatorname{sign}(x)} - \frac{a\ln|bx+a|}{b^2\operatorname{sign}(x)} + \frac{a\ln|a|\cdot\operatorname{sign}(x)}{b^2}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] (a*log(abs(a))*sgn(x)/b^2 + x/(b*sgn(x)) - a*log(abs(b*x + a))/(b^2*sgn(x))
)/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.888 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

[Out] x*ln(b*x+a)/b/c/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)),x]

[Out] (x*Log[a + b*x])/(b*c*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{x^3\log(a+bx)}{b\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.13, size = 21, normalized size = 0.91

method	result	size
default	$\frac{x^{3} \ln \left(b x +a \right)}{\left(c \,x^{2}\right)^{\frac{3}{2}} b}$	21
risch	$\frac{x \ln \left(b x +a \right)}{b c \sqrt{c \,x^{2}}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*ln(b*x+a)/b
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(21) = 42$.
time = 0.29, size = 74, normalized size = 3.22

$$\frac{(-1)^{\frac{2\,acx}{b}} \log\left(-\frac{2\,acx}{b|bx+a|}\right)}{bc^{\frac{3}{2}}} + \frac{\log\left(bx\right)}{bc^{\frac{3}{2}}} + \frac{2\,a}{\sqrt{cx^2}\,b^2c} - \frac{2\,a}{b^2c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*c^(3/2)) + log(b*x)/(b*c
^(3/2)) + 2*a/(sqrt(c*x^2)*b^2*c) - 2*a/(b^2*c^(3/2)*x)
```

Fricas [**A**]
time = 0.29, size = 23, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,\log\left(bx+a\right)}{bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 32, normalized size = 1.39

$$\frac{-\frac{\ln|a|\cdot\mathrm{sign}(x)}{b}+\frac{\ln|bx+a|}{b\mathrm{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -(log(abs(a))*sgn(x)/b - log(abs(b*x + a))/(b*sgn(x)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.889 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

```
[Out] x*ln(x)/a/c/(c*x^2)^(1/2)-x*ln(b*x+a)/a/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*c*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{1}{x(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \frac{1}{x}\,dx}{ac\sqrt{cx^2}} - \frac{(bx)\int \frac{1}{a+bx}\,dx}{ac\sqrt{cx^2}}$$
$$= \frac{x\log(x)}{ac\sqrt{cx^2}} - \frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.61

$$\frac{x^3(\log(x)-\log(a+bx))}{a\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(Log[x] - Log[a + b*x]))/(a*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 26, normalized size = 0.59

method	result	size
default	$\frac{x^3(\ln(x)-\ln(bx+a))}{(c\,x^2)^{\frac{3}{2}}a}$	26
risch	$\frac{x\ln(-x)}{c\sqrt{c\,x^2}\,a} - \frac{x\ln(bx+a)}{ac\sqrt{c\,x^2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(ln(x)-ln(b*x+a))/(c*x^2)^(3/2)/a
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.80

$$-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*c^(3/2))
```

Fricas [A]
time = 0.30, size = 70, normalized size = 1.59

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ac^2x}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}\,(2bx+a)\sqrt{-c}}{acx}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c^2*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/(a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.890 $\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{ac\sqrt{cx^2}}-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}$$

```
[Out] -1/a/c/(c*x^2)^(1/2)-b*x*ln(x)/a^2/c/(c*x^2)^(1/2)+b*x*ln(b*x+a)/a^2/c/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 46}

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(1/(a*c*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*c*Sqrt[c*x^2]) + (b*x*Log[a + b*
x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{1}{ac\sqrt{cx^2}}-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.56

$$\frac{x^2(-a - bx\log(x) + bx\log(a + bx))}{a^2 (cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.11, size = 33, normalized size = 0.52

method	result	size
default	$-\frac{x^2(bx\ln(x)-b\ln(bx+a)x+a)}{(cx^2)^{\frac{3}{2}}a^2}$	33
risch	$-\frac{1}{ac\sqrt{cx^2}}+\frac{xb\ln(-bx-a)}{c\sqrt{cx^2}\,a^2}-\frac{bx\ln(x)}{a^2c\sqrt{cx^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -x^2*(b*x*ln(x)-b*ln(b*x+a)*x+a)/(c*x^2)^(3/2)/a^2
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{(-1)^{\frac{2acx}{b}}b\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}} - \frac{1}{\sqrt{cx^2}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] (-1)^(2*a*c*x/b)*b*log(-2*a*c*x/(b*abs(b*x + a)))/(a^2*c^(3/2)) - 1/(sqrt(c
*x^2)*a*c)
```

Fricas [A]
time = 0.30, size = 34, normalized size = 0.54

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a),x)
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x)
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)),x)
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.891 $\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=89

$$\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}$$

```
[Out] b/a^2/c/(c*x^2)^(1/2)-1/2/a/c/x/(c*x^2)^(1/2)+b^2*x*ln(x)/a^3/c/(c*x^2)^(1/
2)-b^2*x*ln(b*x+a)/a^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 46}

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] b/(a^2*c*Sqrt[c*x^2]) - 1/(2*a*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*c*Sqr
t[c*x^2]) - (b^2*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 51, normalized size = 0.57

$$\frac{x\left(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3*(
c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.11, size = 49, normalized size = 0.55

method	result	size
default	$\frac{x\left(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2\right)}{2\left(c\,x^2\right)^{\frac{3}{2}}a^3}$	49
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{cx\sqrt{c\,x^2}}+\frac{x\,b^2\ln(-x)}{c\sqrt{c\,x^2}\,a^3}-\frac{b^2x\ln(bx+a)}{a^3c\sqrt{c\,x^2}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/(c*x^2)^(3/2)/a^3
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.73

$$-\frac{(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}+\frac{b}{\sqrt{cx^2}\,a^2c}-\frac{1}{2\,ac^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*b^2*log(-2*a*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) + b/(sqr
t(c*x^2)*a^2*c) - 1/2/(a*c^(3/2)*x^2)
```

Fricas [**A**]
time = 0.30, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c^2*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a),x)
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x)
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)),x)
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.892 $\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=115

$$-\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

```
[Out] -b^2/a^3/c/(c*x^2)^(1/2)-1/3/a/c/x^2/(c*x^2)^(1/2)+1/2*b/a^2/c/x/(c*x^2)^(1
/2)-b^3*x*ln(x)/a^4/c/(c*x^2)^(1/2)+b^3*x*ln(b*x+a)/a^4/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}-\frac{b^2}{a^3c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*c*Sqrt[c*x^2])) - 1/(3*a*c*x^2*Sqrt[c*x^2]) + b/(2*a^2*c*x*Sqrt[
c*x^2]) - (b^3*x*Log[x])/(a^4*c*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*c*
Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\,(cx^2)^{3/2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{b^2}{a^3c\sqrt{cx^2}} - \frac{1}{3acx^2\sqrt{cx^2}} + \frac{b}{2a^2cx\sqrt{cx^2}} - \frac{b^3x\log(x)}{a^4c\sqrt{cx^2}} + \frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 0.57

$$\frac{cx^2\left(a\left(-2a^2+3abx-6b^2x^2\right)-6b^3x^3\log(x)+6b^3x^3\log(a+bx)\right)}{6a^4\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (c*x^2*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log
[a + b*x]))/(6*a^4*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.14, size = 59, normalized size = 0.51

method	result	size
default	$-\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3}{6(c\,x^2)^{\frac{3}{2}}a^4}$	59
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{c\,x^2\sqrt{c\,x^2}}-\frac{b^3x\ln(x)}{a^4c\sqrt{c\,x^2}}+\frac{x\,b^3\ln(-bx-a)}{c\sqrt{c\,x^2}\;a^4}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/(c*x
^2)^(3/2)/a^4
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.60

$$\frac{b^3 \log (bx+a)}{a^4 c^{\frac{3}{2}}} - \frac{b^3 \log (x)}{a^4 c^{\frac{3}{2}}} - \frac{6\,b^2 \sqrt{c}\,x^2 - 3\,ab\sqrt{c}\,x + 2\,a^2\sqrt{c}}{6\,a^3 c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*c^(3/2)) - b^3*log(x)/(a^4*c^(3/2)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c^2*x^3)
```

Fricas [A]
time = 0.30, size = 58, normalized size = 0.50

$$\frac{\left(6\,b^3 x^3 \log \left(\frac{bx+a}{x}\right) - 6\,ab^2 x^2 + 3\,a^2 bx - 2\,a^3\right)\sqrt{cx^2}}{6\,a^4 c^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/(x*(c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(c\,x^2)^{3/2}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(3/2)*(a + b*x)), x)
```

3.893 $\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=106

$$\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] 3*a^2*(c*x^2)^(1/2)/b^4-a*x*(c*x^2)^(1/2)/b^3+1/3*x^2*(c*x^2)^(1/2)/b^2-a^4
*(c*x^2)^(1/2)/b^5/x/(b*x+a)-4*a^3*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (3*a^2*Sqrt[c*x^2])/b^4 - (a*x*Sqrt[c*x^2])/b^3 + (x^2*Sqrt[c*x^2])/(3*b^2)
 - (a^4*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*Sqrt[c*x^2]*Log[a + b*x])/(
b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{x}$$
$$= \frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [**A**]
time = 0.02, size = 81, normalized size = 0.76

$$\frac{cx\left(-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4-12a^3(a+bx)\log(a+bx)\right)}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(
a + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.14, size = 88, normalized size = 0.83

method	result	size
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-ab\,x^2+3a^2x\right)}{x\,b^4}-\frac{a^4\sqrt{c\,x^2}}{b^5x(bx+a)}-\frac{4a^3\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	87
default	$-\frac{\sqrt{c\,x^2}\,\left(-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)-9a^3bx+3a^4\right)}{3x\,b^5(bx+a)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^2)^(1/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x/b^5/(b*x+a)
```

Maxima [A]
time = 0.28, size = 135, normalized size = 1.27

$$\frac{\sqrt{cx^2}\,a^3}{b^5x+ab^4}-\frac{4\,(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^5}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}-\frac{\sqrt{cx^2}\,ax}{b^3}+\frac{3\,\sqrt{cx^2}\,a^2}{b^4}+\frac{(cx^2)^{\frac{3}{2}}}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*x + a*b^4) - 4*(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
sqrt(c*x^2)*a*x/b^3 + 3*sqrt(c*x^2)*a^2/b^4 + 1/3*(c*x^2)^(3/2)/(b^2*c)
```

Fricas [A]
time = 0.30, size = 83, normalized size = 0.78

$$\frac{\left(b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\,(b^6x^2+ab^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\sqrt{cx^2}}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 101, normalized size = 0.95

$$\sqrt{c}\,\left(\frac{\frac{1}{3}b^4x^3\mathrm{sign}\left(x\right)-ab^3x^2\mathrm{sign}\left(x\right)+3a^2b^2x\mathrm{sign}\left(x\right)}{b^6}-\frac{a^4\mathrm{sign}\left(x\right)}{b^5\left(bx+a\right)}-\frac{4a^3\mathrm{sign}\left(x\right)\ln|bx+a|}{b^5}+\frac{\left(4a^3\ln|a|+a^3\right)\mathrm{sign}\left(x\right)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*sqrt(c)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

3.894 $\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

```
[Out] -2*a*(c*x^2)^(1/2)/b^3+1/2*x*(c*x^2)^(1/2)/b^2+a^3*(c*x^2)^(1/2)/b^4/x/(b*x
+a)+3*a^2*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (-2*a*Sqrt[c*x^2])/b^3 + (x*Sqrt[c*x^2])/(2*b^2) + (a^3*Sqrt[c*x^2])/(b^4*x
*(a + b*x)) + (3*a^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}$$
$$= -\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

Mathematica [A]
time = 0.01, size = 70, normalized size = 0.82

$$\frac{cx\left(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx)\right)}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
[Out] (c*x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b
*x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.12, size = 76, normalized size = 0.89

method	result	size
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{1}{2}x^2b-2ax\right)}{x\,b^3}+\frac{a^3\sqrt{c\,x^2}}{b^4x(bx+a)}+\frac{3a^2\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	75
default	$\frac{\sqrt{c\,x^2}\,\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2x\,b^4(bx+a)}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x/b^4/(b*x+a)
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.39

$$-\frac{\sqrt{cx^2}\,a^2}{b^4x+ab^3}+\frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^4}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}+\frac{\sqrt{cx^2}\,x}{2\,b^2}-\frac{2\,\sqrt{cx^2}\,a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*x + a*b^3) + 3*(-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b
)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 +
 1/2*sqrt(c*x^2)*x/b^2 - 2*sqrt(c*x^2)*a/b^3
```

Fricas [A]
time = 0.30, size = 72, normalized size = 0.85

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,(b^5x^2+ab^4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 87, normalized size = 1.02

$$\sqrt{c}\left(\frac{\frac{1}{2}b^2x^2\mathrm{sign}\left(x\right)-2abx\mathrm{sign}\left(x\right)}{b^4}+\frac{a^3\mathrm{sign}\left(x\right)}{b^4\left(bx+a\right)}+\frac{3a^2\mathrm{sign}\left(x\right)\ln|bx+a|}{b^4}+\frac{\left(-3a^2\ln|a|-a^2\right)\mathrm{sign}\left(x\right)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*sqrt(c)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

3.895 $\int \frac{x\sqrt{cx^2}}{(a+bx)^2} dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx^2}}{b^2}-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}$$

```
[Out] (c*x^2)^(1/2)/b^2-a^2*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*ln(b*x+a)*(c*x^2)^(1/
2)/b^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]/b^2 - (a^2*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*Sqrt[c*x^2]*Lo
g[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}$$
$$= \frac{\sqrt{cx^2}}{b^2}-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

Mathematica [**A**]
time = 0.01, size = 53, normalized size = 0.82

$$\frac{cx\left(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx)\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2
]*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 62, normalized size = 0.95

method	result	size
risch	$\frac{\sqrt{c\,x^2}}{b^2}-\frac{a^2\sqrt{c\,x^2}}{b^3x(bx+a)}-\frac{2a\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	60
default	$-\frac{\sqrt{c\,x^2}\,\left(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2\right)}{x\,b^3\,(bx+a)}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x/b^3/
(b*x+a)
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.48

$$\frac{\sqrt{cx^2}\,a}{b^3x+ab^2}-\frac{2\,(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^3
 - 2*(-1)^(2*a*c*x/b)*a*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt(c
*x^2)/b^2
```

Fricas [A]
time = 0.32, size = 57, normalized size = 0.88

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*x^2
 + a*b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{cx^2}}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 64, normalized size = 0.98

$$\sqrt{c}\left(\frac{x\mathrm{sign}\left(x\right)}{b^2}-\frac{a^2\mathrm{sign}\left(x\right)}{b^3\left(bx+a\right)}-\frac{2a\mathrm{sign}\left(x\right)\ln|bx+a|}{b^3}+\frac{\left(2a\ln|a|+a\right)\mathrm{sign}\left(x\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] sqrt(c)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\sqrt{c\,x^2}}{\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

3.896 $\int \frac{\sqrt{cx^2}}{(a+bx)^2} dx$

Optimal. Leaf size=47

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] a*(c*x^2)^(1/2)/b^2/x/(b*x+a)+ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (a*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{(a+bx)^2} dx &= \frac{\sqrt{cx^2} \int \frac{x}{(a+bx)^2} dx}{x} \\
&= \frac{\sqrt{cx^2} \int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx}{x} \\
&= \frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.77

$$\frac{cx(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.87

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(b\ln(bx+a)x+a\ln(bx+a)+a)}{x\,b^{2}(bx+a)}$	41
risch	$\frac{a\sqrt{c\,x^{2}}}{b^{2}x(bx+a)}+\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{b^{2}x}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x/b^2/(b*x+a)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 1.68

$$\frac{(-1)^{\frac{2\,cx}{b}}\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^2}+\frac{(-1)^{\frac{2\,acx}{b}}\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}-\frac{\sqrt{cx^2}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*sqrt(c)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*sqrt(c)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)/(b^2*x + a*b)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 0.81

$$\frac{\sqrt{cx^2}\,((bx+a)\log(bx+a)+a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a)**2,x)
[Out] Integral(sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 48, normalized size = 1.02

$$\sqrt{c}\left(\frac{a\operatorname{sign}(x)}{b^2(bx+a)}+\frac{\operatorname{sign}(x)\ln|bx+a|}{b^2}+\frac{(-\ln|a|-1)\operatorname{sign}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x)
[Out] -sqrt(c)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{{(a+b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x)^2,x)
[Out] int((c*x^2)^(1/2)/(a + b*x)^2, x)
```

3.897 $\int \frac{\sqrt{cx^2}}{x(a+bx)^2} dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

```
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx^2}}{x(a+bx)^2} dx &= \frac{\sqrt{cx^2} \int \frac{1}{(a+bx)^2} dx}{x} \\ &= -\frac{\sqrt{cx^2}}{bx(a+bx)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.96

$$-\frac{cx}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -((c*x)/(b*Sqrt[c*x^2]*(a + b*x)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

time = 1.85, size = 39, normalized size = 1.62

$$\text{Piecewise}\left[\left\{\left\{-\frac{\sqrt{cx^2}}{bx\,(a+bx)}, b!{=}0\right\}\right\}, \frac{\sqrt{cx^2}}{a^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{-Sqrt[c x ^ 2] / (b x (a + b x)), b != 0}}, Sqrt[c x ^ 2] / a ^
 2]
```

Maple [A]

time = 0.12, size = 23, normalized size = 0.96

method	result	size
gosper	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
default	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
risch	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
trager	$\frac{(-1+x)\sqrt{c\,x^{2}}}{(bx+a)(a+b)x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

Maxima [A]

time = 0.27, size = 16, normalized size = 0.67

$$-\frac{\sqrt{c}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c)/(b^2*x + a*b)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 0.96

$$-\frac{\sqrt{cx^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [A]
time = 0.32, size = 32, normalized size = 1.33

$$\begin{cases} -\frac{\sqrt{cx^2}}{abx+b^2x^2} & \text{for } b \neq 0 \\ \frac{\sqrt{cx^2}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] Piecewise((-sqrt(c*x**2)/(a*b*x + b**2*x**2), Ne(b, 0)), (sqrt(c*x**2)/a**2
, True))
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.08

$$\sqrt{c}\left(\frac{\text{sign}(x)}{ab}-\frac{\text{sign}(x)}{b(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x)
```

```
[Out] -sqrt(c)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

Mupad [B]
time = 0.16, size = 22, normalized size = 0.92

$$-\frac{\sqrt{c\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*x*(a + b*x))
```

3.898 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2} dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx^2}}{ax(a+bx)}+\frac{\sqrt{cx^2}\log(x)}{a^2x}-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}$$

```
[Out] (c*x^2)^(1/2)/a/x/(b*x+a)+ln(x)*(c*x^2)^(1/2)/a^2/x-ln(b*x+a)*(c*x^2)^(1/2)
/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]/(a*x*(a + b*x)) + (Sqrt[c*x^2]*Log[x])/(a^2*x) - (Sqrt[c*x^2]*L
og[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{x}$$
$$= \frac{\sqrt{cx^2}}{ax(a+bx)} + \frac{\sqrt{cx^2}\log(x)}{a^2x} - \frac{\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [**A**]
time = 0.01, size = 45, normalized size = 0.69

$$\frac{cx(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a +
 b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 52, normalized size = 0.80

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{x\,a^2(bx+a)}$	52
risch	$\frac{\sqrt{c\,x^2}}{ax(bx+a)} + \frac{\sqrt{c\,x^2}\,\ln(-x)}{x\,a^2} - \frac{\ln(bx+a)\sqrt{c\,x^2}}{a^2x}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/x/a^2/(b*x+a)
```

Maxima [A]
time = 0.27, size = 38, normalized size = 0.58

$$\frac{\sqrt{c}}{abx + a^2} - \frac{\sqrt{c}\,\log(bx + a)}{a^2} + \frac{\sqrt{c}\,\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c)/(a*b*x + a^2) - sqrt(c)*log(b*x + a)/a^2 + sqrt(c)*log(x)/a^2
```

Fricas [A]
time = 0.30, size = 42, normalized size = 0.65

$$\frac{\sqrt{cx^2}\left((bx + a)\log\left(\frac{x}{bx+a}\right) + a\right)}{a^2bx^2 + a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*x^2 + a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^2\,(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^2\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2),x)
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2), x)
```

3.899 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{cx^2}}{a^2x^2}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

```
[Out] -(c*x^2)^(1/2)/a^2/x^2-b*(c*x^2)^(1/2)/a^2/x/(b*x+a)-2*b*ln(x)*(c*x^2)^(1/2
)/a^3/x+2*b*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(a^2*x^2)) - (b*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*Sqrt[c*
x^2]*Log[x])/(a^3*x) + (2*b*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{\sqrt{cx^2}}{a^2x^2} - \frac{b\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2b\sqrt{cx^2}\log(x)}{a^3x} + \frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.66

$$-\frac{c(a(a+2bx)+2bx(a+bx)\log(x)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
[Out] -((c*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*x]
))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.11, size = 74, normalized size = 0.85

method	result	size
default	$-\frac{\sqrt{c\,x^2}\,(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2)}{x^2a^3(bx+a)}$	74
risch	$\frac{\sqrt{c\,x^2}\,\left(-\frac{2bx}{a^2}-\frac{1}{a}\right)}{x^2(bx+a)}-\frac{2b\ln(x)\sqrt{c\,x^2}}{a^3x}+\frac{2\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+
a)*a*b*x+2*a*b*x+a^2)/x^2/a^3/(b*x+a)
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.67

$$-\frac{2\,b\sqrt{c}\,x+a\sqrt{c}}{a^2bx^2+a^3x}+\frac{2\,b\sqrt{c}\,\log\left(bx+a\right)}{a^3}-\frac{2\,b\sqrt{c}\,\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*sqrt(c)*x + a*sqrt(c))/(a^2*b*x^2 + a^3*x) + 2*b*sqrt(c)*log(b*x + a)
/a^3 - 2*b*sqrt(c)*log(x)/a^3
```

Fricas [A]
time = 0.30, size = 60, normalized size = 0.69

$$-\frac{\left(2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3+a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
x^3 + a^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^3\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x)
```

[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^3\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2),x)
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2), x)
```

3.900 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{cx^2}}{2a^2x^3}+\frac{2b\sqrt{cx^2}}{a^3x^2}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

```
[Out] -1/2*(c*x^2)^(1/2)/a^2/x^3+2*b*(c*x^2)^(1/2)/a^3/x^2+b^2*(c*x^2)^(1/2)/a^3/
x/(b*x+a)+3*b^2*ln(x)*(c*x^2)^(1/2)/a^4/x-3*b^2*ln(b*x+a)*(c*x^2)^(1/2)/a^4
/x
```

Rubi [A]
time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/2*Sqrt[c*x^2]/(a^2*x^3) + (2*b*Sqrt[c*x^2])/(a^3*x^2) + (b^2*Sqrt[c*x^2]
)/(a^3*x*(a + b*x)) + (3*b^2*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*Sqrt[c*x^
2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2} \, dx = \frac{\sqrt{cx^2} \int \frac{1}{x^3(a+bx)^2} \, dx}{x}$$
$$= \frac{\sqrt{cx^2} \int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right) dx}{x}$$
$$= -\frac{\sqrt{cx^2}}{2a^2x^3} + \frac{2b\sqrt{cx^2}}{a^3x^2} + \frac{b^2\sqrt{cx^2}}{a^3x(a+bx)} + \frac{3b^2\sqrt{cx^2}\log(x)}{a^4x} - \frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 0.73

$$\frac{\sqrt{cx^2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] (Sqrt[c*x^2]*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] -
 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^3*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.13, size = 95, normalized size = 0.85

method	result	size
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}\right)}{x^3(bx+a)} - \frac{3b^2\ln(bx+a)\sqrt{c\,x^2}}{a^4x} + \frac{3\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^4}$	90
default	$\frac{\sqrt{c\,x^2}\,\left(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3\right)}{2x^3a^4(bx+a)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/2*(c*x^2)^(1/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*ln(b*x+a)*a*b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^3/a^4/(b*x+a)

Maxima [A]
time = 0.25, size = 79, normalized size = 0.71

$$\frac{6\,b^2\sqrt{c}\,x^2+3\,ab\sqrt{c}\,x-a^2\sqrt{c}}{2\,(a^3bx^3+a^4x^2)}-\frac{3\,b^2\sqrt{c}\,\log\left(bx+a\right)}{a^4}+\frac{3\,b^2\sqrt{c}\,\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*(6*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x - a^2*sqrt(c))/(a^3*b*x^3 + a^4*x^2) - 3*b^2*sqrt(c)*log(b*x + a)/a^4 + 3*b^2*sqrt(c)*log(x)/a^4

Fricas [A]
time = 0.30, size = 77, normalized size = 0.69

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,(b^3x^3+ab^2x^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bx^4+a^5x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^4\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a)**2,x)

[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)**2), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x)

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^4\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2), x)
```

3.901 $\int \frac{x(cx^2)^{3/2}}{(a+bx)^2} \, dx$

Optimal. Leaf size=111

$$\frac{3a^2c\sqrt{cx^2}}{b^4} - \frac{acx\sqrt{cx^2}}{b^3} + \frac{cx^2\sqrt{cx^2}}{3b^2} - \frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3c\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] 3*a^2*c*(c*x^2)^(1/2)/b^4-a*c*x*(c*x^2)^(1/2)/b^3+1/3*c*x^2*(c*x^2)^(1/2)/b
^2-a^4*c*(c*x^2)^(1/2)/b^5/x/(b*x+a)-4*a^3*c*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3c\sqrt{cx^2}\,\log(a+bx)}{b^5x} + \frac{3a^2c\sqrt{cx^2}}{b^4} - \frac{acx\sqrt{cx^2}}{b^3} + \frac{cx^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (3*a^2*c*Sqrt[c*x^2])/b^4 - (a*c*x*Sqrt[c*x^2])/b^3 + (c*x^2*Sqrt[c*x^2])/(
3*b^2) - (a^4*c*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*c*Sqrt[c*x^2]*Log[a
 + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right)dx}{x}$$
$$= \frac{3a^2c\sqrt{cx^2}}{b^4} - \frac{acx\sqrt{cx^2}}{b^3} + \frac{cx^2\sqrt{cx^2}}{3b^2} - \frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 0.74

$$\frac{(cx^2)^{3/2}\left(-3a^4 + 9a^3bx + 6a^2b^2x^2 - 2ab^3x^3 + b^4x^4 - 12a^3(a+bx)\log(a+bx)\right)}{3b^5x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
[Out] ((c*x^2)^(3/2)*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4
- 12*a^3*(a + b*x)*Log[a + b*x]))/(3*b^5*x^3*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.16, size = 88, normalized size = 0.79

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)-9a^3bx+3a^4\right)}{3x^3b^5(bx+a)}$	88
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-ab\,x^2+3a^2x\right)}{x\,b^4} - \frac{a^4c\sqrt{c\,x^2}}{b^5x(bx+a)} - \frac{4a^3c\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^2)^(3/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x^3/b^5/(b*x+a)
```

Maxima [A]
time = 0.27, size = 132, normalized size = 1.19

$$\frac{(cx^2)^{\frac{3}{2}}a}{b^3x+ab^2}-\frac{4\,(-1)^{\frac{2\,cx}{b}}\,a^3c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}-\frac{2\,\sqrt{cx^2}\,acx}{b^3}+\frac{(cx^2)^{\frac{3}{2}}}{3\,b^2}+\frac{4\,\sqrt{cx^2}\,a^2c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (c*x^2)^(3/2)*a/(b^3*x + a*b^2) - 4*(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
2*sqrt(c*x^2)*a*c*x/b^3 + 1/3*(c*x^2)^(3/2)/b^2 + 4*sqrt(c*x^2)*a^2*c/b^4
```

Fricas [A]
time = 0.30, size = 91, normalized size = 0.82

$$\frac{\left(b^4cx^4-2\,ab^3cx^3+6\,a^2b^2cx^2+9\,a^3bcx-3\,a^4c-12\,(a^3bcx+a^4c)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\,(b^6x^2+ab^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*c*x^4 - 2*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 9*a^3*b*c*x - 3*a^4*c -
12*(a^3*b*c*x + a^4*c)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\,(cx^2)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 102, normalized size = 0.92

$$\sqrt{c}\,c\left(\frac{\frac{1}{3}b^4x^3\mathrm{sign}\,(x)-ab^3x^2\mathrm{sign}\,(x)+3a^2b^2x\mathrm{sign}\,(x)}{b^6}-\frac{a^4\mathrm{sign}\,(x)}{b^5\,(bx+a)}-\frac{4a^3\mathrm{sign}\,(x)\ln|bx+a|}{b^5}+\frac{(4a^3\ln|a|+a^3)\,\mathrm{sign}\,(x)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*c^(3/2)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(c\,x^2\right)}^{3/2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x)^2, x)
```

3.902 $\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=89

$$-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}$$

```
[Out] -2*a*c*(c*x^2)^(1/2)/b^3+1/2*c*x*(c*x^2)^(1/2)/b^2+a^3*c*(c*x^2)^(1/2)/b^4/
x/(b*x+a)+3*a^2*c*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c*Sqrt[c*x^2])/b^3 + (c*x*Sqrt[c*x^2])/(2*b^2) + (a^3*c*Sqrt[c*x^2])/
(b^4*x*(a + b*x)) + (3*a^2*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}$$
$$= -\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 0.80

$$\frac{(cx^2)^{3/2}\left(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx)\right)}{2b^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)
*Log[a + b*x]))/(2*b^4*x^3*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 76, normalized size = 0.85

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2x^3b^4(bx+a)}$	76
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{2}x^2b-2ax\right)}{x\,b^3}+\frac{a^3c\sqrt{c\,x^2}}{b^4x(bx+a)}+\frac{3a^2c\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/2*(c*x^2)^(3/2)*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*a^3)/x^3/b^4/(b*x+a)

Maxima [A]
time = 0.27, size = 115, normalized size = 1.29

$$\frac{3 (-1)^{\frac{2 cx}{b}} a^2 c^{\frac{3}{2}} \log\left(\frac{2 cx}{b}\right)}{b^4} + \frac{3 (-1)^{\frac{2 acx}{b}} a^2 c^{\frac{3}{2}} \log\left(-\frac{2 acx}{b|bx+a|}\right)}{b^4} - \frac{(cx^2)^{\frac{3}{2}}}{b^2 x + ab} + \frac{3 \sqrt{cx^2}\, cx}{2 b^2} - \frac{3 \sqrt{cx^2}\, ac}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")

[Out] 3*(-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - (c*x^2)^(3/2)/(b^2*x + a*b) + 3/2*sqrt(c*x^2)*c*x/b^2 - 3*sqrt(c*x^2)*a*c/b^3

Fricas [A]
time = 0.31, size = 79, normalized size = 0.89

$$\frac{\left(b^3 cx^3 - 3 ab^2 cx^2 - 4 a^2 bcx + 2 a^3 c + 6 (a^2 bcx + a^3 c) \log(bx + a)\right) \sqrt{cx^2}}{2 (b^5 x^2 + ab^4 x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(b^3*c*x^3 - 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 2*a^3*c + 6*(a^2*b*c*x + a^3*c)*log(b*x + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(3/2)/(b*x+a)**2,x)

[Out] Integral((c*x**2)**(3/2)/(a + b*x)**2, x)

Giac [A]
time = 0.00, size = 88, normalized size = 0.99

$$\sqrt{c}\, c \left(\frac{\frac{1}{2} b^2 x^2 \mathrm{sign}(x) - 2abx\mathrm{sign}(x)}{b^4} + \frac{a^3 \mathrm{sign}(x)}{b^4 (bx + a)} + \frac{3a^2 \mathrm{sign}(x) \ln|bx + a|}{b^4} + \frac{(-3a^2 \ln|a| - a^2)\, \mathrm{sign}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*c^(3/2)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^2)^{3/2}}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)/(a + b*x)^2, x)
```

3.903 $\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx$

Optimal. Leaf size=68

$$\frac{c\sqrt{cx^2}}{b^2}-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] c*(c*x^2)^(1/2)/b^2-a^2*c*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*c*ln(b*x+a)*(c*x^
2)^(1/2)/b^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/b^2 - (a^2*c*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*c*Sqrt[c
*x^2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}$$
$$= \frac{c\sqrt{cx^2}}{b^2}-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

Mathematica [**A**]
time = 0.01, size = 55, normalized size = 0.81

$$\frac{c^2x\left(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx)\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
[Out] (c^2*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x
^2]*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 62, normalized size = 0.91

method	result	size
default	$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2\right)}{x^3b^3(bx+a)}$	62
risch	$\frac{c\sqrt{c\,x^2}}{b^2}-\frac{a^2c\sqrt{c\,x^2}}{b^3x(bx+a)}-\frac{2ac\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(3/2)*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x^3/b^
3/(b*x+a)
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.44

$$\frac{\sqrt{cx^2}\,ac}{b^3x+ab^2}-\frac{2\left(-1\right)^{\frac{2\,cx}{b}}ac^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}-\frac{2\left(-1\right)^{\frac{2\,acx}{b}}ac^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,c}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a*c/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b
^3 - 2*(-1)^(2*a*c*x/b)*a*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt
(c*x^2)*c/b^2
```

Fricas [A]
time = 0.30, size = 63, normalized size = 0.93

$$\frac{\left(b^2cx^2+abcx-a^2c-2\left(abcx+a^2c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x^2 + a*b*c*x - a^2*c - 2*(a*b*c*x + a^2*c)*log(b*x + a))*sqrt(c*x^2
)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 65, normalized size = 0.96

$$\sqrt{c}\,c\left(\frac{x\mathrm{sign}\left(x\right)}{b^2}-\frac{a^2\mathrm{sign}\left(x\right)}{b^3\left(bx+a\right)}-\frac{2a\mathrm{sign}\left(x\right)\ln|bx+a|}{b^3}+\frac{\left(2a\ln|a|+a\right)\mathrm{sign}\left(x\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x)
```

```
[Out] c^(3/2)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^2)^{3/2}}{x\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)^2), x)
```

3.904 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx$

Optimal. Leaf size=49

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)} + \frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] a*c*(c*x^2)^(1/2)/b^2/x/(b*x+a)+c*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)} + \frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (a*c*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{(a+bx)^2} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx}{x} \\
&= \frac{ac\sqrt{cx^2}}{b^2x(a+bx)} + \frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 0.78

$$\frac{c^2x(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (c^2*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 41, normalized size = 0.84

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(b\ln(bx+a)x+a\ln(bx+a)+a)}{x^3b^2(bx+a)}$	41
risch	$\frac{ac\sqrt{c\,x^2}}{b^2x(bx+a)}+\frac{c\ln(bx+a)\sqrt{c\,x^2}}{b^2x}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x^3/b^2/(b*x+a)
```

Maxima [A]
time = 0.27, size = 80, normalized size = 1.63

$$\frac{(-1)^{\frac{2cx}{b}}c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2}+\frac{(-1)^{\frac{2acx}{b}}c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2}-\frac{\sqrt{cx^2}\,c}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*c^(3/2)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*c^(3/2)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)*c/(b^2*x + a*b)
```

Fricas [A]
time = 0.30, size = 43, normalized size = 0.88

$$\frac{\sqrt{cx^2}\,(ac + (bcx + ac)\log(bx + a))}{b^3x^2 + ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(b*x + a))/(b^3*x^2 + a*b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^2\,(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 49, normalized size = 1.00

$$\sqrt{c}\,c\left(\frac{a\mathrm{sign}\,(x)}{b^2\,(bx + a)} + \frac{\mathrm{sign}\,(x)\ln|bx + a|}{b^2} + \frac{(-\ln|a| - 1)\,\mathrm{sign}\,(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x)
```

```
[Out] -c^(3/2)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^2)^{3/2}}{x^2\,(a + b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2), x)
```

3.905 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

```
[Out] -c*(c*x^2)^(1/2)/b/x/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{(a+bx)^2}\,dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.23, size = 42, normalized size = 1.68

$$\text{Piecewise}\left[\left\{\left\{-\frac{(cx^2)^{\frac{3}{2}}}{bx^3(a+bx)}, b!=0\right\}\right\}, \frac{(cx^2)^{\frac{3}{2}}}{a^2x^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{-(c x ^ 2) ^ (3 / 2) / (b x ^ 3 (a + b x)), b != 0}}, (c x ^ 2)
 ^ (3 / 2) / (a ^ 2 x ^ 2)]
```

Maple [A]
time = 0.12, size = 23, normalized size = 0.92

method	result	size
gosper	$-\frac{(c\,x^2)^{\frac{3}{2}}}{(bx+a)b\,x^3}$	23
default	$-\frac{(c\,x^2)^{\frac{3}{2}}}{(bx+a)b\,x^3}$	23
risch	$-\frac{c\sqrt{c\,x^2}}{bx(bx+a)}$	24
trager	$\frac{c(-1+x)\sqrt{c\,x^2}}{(bx+a)(a+b)x}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*x+a)/b*(c*x^2)^(3/2)/x^3
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.64

$$-\frac{c^{\frac{3}{2}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -c^(3/2)/(b^2*x + a*b)
```

Fricas [A]
time = 0.29, size = 24, normalized size = 0.96

$$-\frac{\sqrt{cx^2}\, c}{b^2x^2 + abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
[Out] -sqrt(c*x^2)*c/(b^2*x^2 + a*b*x)
```

Sympy [A]
time = 0.64, size = 37, normalized size = 1.48

$$\begin{cases} -\frac{(cx^2)^{\frac{3}{2}}}{abx^3+b^2x^4} & \text{for}\ b \neq 0 \\ \frac{(cx^2)^{\frac{3}{2}}}{a^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a)**2,x)
[Out] Piecewise((-(c*x**2)**(3/2)/(a*b*x**3 + b**2*x**4), Ne(b, 0)), ((c*x**2)**(
3/2)/(a**2*x**2), True))
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.08

$$\sqrt{c}\, c \left(\frac{\operatorname{sign}(x)}{ab} - \frac{\operatorname{sign}(x)}{b\,(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x)
[Out] -c^(3/2)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

Mupad [B]
time = 0.15, size = 24, normalized size = 0.96

$$-\frac{c^{3/2}\,\sqrt{x^2}}{b^2\,x^2 + a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)^2),x)
[Out] -(c^(3/2)*(x^2)^(1/2))/(b^2*x^2 + a*b*x)
```

3.906 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2} dx$

Optimal. Leaf size=68

$$\frac{c\sqrt{cx^2}}{ax(a+bx)} + \frac{c\sqrt{cx^2}\log(x)}{a^2x} - \frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}$$

```
[Out] c*(c*x^2)^(1/2)/a/x/(b*x+a)+c*ln(x)*(c*x^2)^(1/2)/a^2/x-c*ln(b*x+a)*(c*x^2)
^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x} + \frac{c\sqrt{cx^2}\log(x)}{a^2x} + \frac{c\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/(a*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[x])/(a^2*x) - (c*Sqrt[
c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{x}$$
$$= \frac{c\sqrt{cx^2}}{ax(a+bx)} + \frac{c\sqrt{cx^2}\log(x)}{a^2x} - \frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.68

$$\frac{(cx^2)^{3/2}\,(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*x^3*(a
 + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.76

method	result	size
default	$\frac{(cx^2)^{\frac{3}{2}}(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{x^3a^2(bx+a)}$	52
risch	$\frac{c\sqrt{cx^2}}{ax(bx+a)}+\frac{c\sqrt{cx^2}\,\ln(-x)}{x\,a^2}-\frac{c\ln(bx+a)\sqrt{cx^2}}{a^2x}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/x^3/a^2/(b*x+
a)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 0.56

$$\frac{c^{\frac{3}{2}}}{abx+a^2}-\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{a^2}+\frac{c^{\frac{3}{2}}\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] c^(3/2)/(a*b*x + a^2) - c^(3/2)*log(b*x + a)/a^2 + c^(3/2)*log(x)/a^2
```

Fricas [A]
time = 0.29, size = 47, normalized size = 0.69

$$\frac{\sqrt{cx^2}\,\left(ac+(bcx+ac)\log\left(\frac{x}{bx+a}\right)\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(x/(b*x + a)))/(a^2*b*x^2 + a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^4\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2), x)
```

3.907 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2} \, dx$

Optimal. Leaf size=91

$$-\frac{c\sqrt{cx^2}}{a^2x^2} - \frac{bc\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2bc\sqrt{cx^2}\log(x)}{a^3x} + \frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}$$

```
[Out] -c*(c*x^2)^(1/2)/a^2/x^2-b*c*(c*x^2)^(1/2)/a^2/x/(b*x+a)-2*b*c*ln(x)*(c*x^2
)^(1/2)/a^3/x+2*b*c*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x} + \frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x} - \frac{bc\sqrt{cx^2}}{a^2x(a+bx)} - \frac{c\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a^2*x^2)) - (b*c*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*c
*Sqrt[c*x^2]*Log[x])/(a^3*x) + (2*b*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{a^2x^2} - \frac{bc\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x} + \frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

Mathematica [A]
time = 0.02, size = 59, normalized size = 0.65

$$-\frac{c^2(a(a+2bx)+2bx(a+bx)\log(x)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
[Out] -((c^2*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*
x]))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.81

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2)}{x^4a^3(bx+a)}$	74
risch	$\frac{c\sqrt{c\,x^2}\,\left(-\frac{2bx}{a^2}-\frac{1}{a}\right)}{x^2(bx+a)} - \frac{2bc\ln(x)\sqrt{c\,x^2}}{a^3x} + \frac{2c\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^3}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(3/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+
a)*a*b*x+2*a*b*x+a^2)/x^4/a^3/(b*x+a)
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.64

$$\frac{2\,bc^{\frac{3}{2}}\log\left(bx+a\right)}{a^{3}}-\frac{2\,bc^{\frac{3}{2}}\log\left(x\right)}{a^{3}}-\frac{2\,bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}}{a^{2}bx^{2}+a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*b*c^(3/2)*log(b*x + a)/a^3 - 2*b*c^(3/2)*log(x)/a^3 - (2*b*c^(3/2)*x + a*
c^(3/2))/(a^2*b*x^2 + a^3*x)
```

Fricas [A]
time = 0.30, size = 65, normalized size = 0.71

$$-\frac{\left(2\,abcx+a^{2}c-2\left(b^{2}cx^{2}+abcx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^{2}}}{a^{3}bx^{3}+a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*c*x + a^2*c - 2*(b^2*c*x^2 + a*b*c*x)*log((b*x + a)/x))*sqrt(c*x^2)
/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{5}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^5\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2),x)
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2), x)
```

3.908 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

```
[Out] -1/2*c*(c*x^2)^(1/2)/a^2/x^3+2*b*c*(c*x^2)^(1/2)/a^3/x^2+b^2*c*(c*x^2)^(1/2
)/a^3/x/(b*x+a)+3*b^2*c*ln(x)*(c*x^2)^(1/2)/a^4/x-3*b^2*c*ln(b*x+a)*(c*x^2)
^(1/2)/a^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] -1/2*(c*Sqrt[c*x^2])/(a^2*x^3) + (2*b*c*Sqrt[c*x^2])/(a^3*x^2) + (b^2*c*Sqr
t[c*x^2])/(a^3*x*(a + b*x)) + (3*b^2*c*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2
*c*Sqrt[c*x^2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 0.70

$$\frac{(cx^2)^{3/2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
[Out] ((c*x^2)^(3/2)*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x]
 - 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^5*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.14, size = 95, normalized size = 0.81

method	result	size
risch	$\frac{c\sqrt{c\,x^2}\left(\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}\right)}{x^3(bx+a)}-\frac{3b^2c\ln(bx+a)\sqrt{c\,x^2}}{a^4x}+\frac{3c\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^4}$	93
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3\right)}{2x^5a^4(bx+a)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*
ln(b*x+a)*a*b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^5/a^4/(b*x+a)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.68

$$-\frac{3\,b^2c^{\frac{3}{2}}\log\left(bx+a\right)}{a^4}+\frac{3\,b^2c^{\frac{3}{2}}\log\left(x\right)}{a^4}+\frac{6\,b^2c^{\frac{3}{2}}x^2+3\,abc^{\frac{3}{2}}x-a^2c^{\frac{3}{2}}}{2\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*b^2*c^(3/2)*log(b*x + a)/a^4 + 3*b^2*c^(3/2)*log(x)/a^4 + 1/2*(6*b^2*c^(
3/2)*x^2 + 3*a*b*c^(3/2)*x - a^2*c^(3/2))/(a^3*b*x^3 + a^4*x^2)
```

Fricas [A]
time = 0.30, size = 82, normalized size = 0.70

$$\frac{\left(6\,ab^2cx^2+3\,a^2bcx-a^3c+6\left(b^3cx^3+ab^2cx^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*c*x^2 + 3*a^2*b*c*x - a^3*c + 6*(b^3*c*x^3 + a*b^2*c*x^2)*log(
x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^2)^{3/2}}{x^6\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2), x)
```

3.909 $\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=107

$$\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}$$

```
[Out] 3*a^2*x^2/b^4/(c*x^2)^(1/2)-a*x^3/b^3/(c*x^2)^(1/2)+1/3*x^4/b^2/(c*x^2)^(1/
2)-a^4*x/b^5/(b*x+a)/(c*x^2)^(1/2)-4*a^3*x*ln(b*x+a)/b^5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}+\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (3*a^2*x^2)/(b^4*Sqrt[c*x^2]) - (a*x^3)/(b^3*Sqrt[c*x^2]) + x^4/(3*b^2*Sqrt
[c*x^2]) - (a^4*x)/(b^5*Sqrt[c*x^2]*(a + b*x)) - (4*a^3*x*Log[a + b*x])/(b^
5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^4}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{3a^2x^2}{b^4\sqrt{cx^2}} - \frac{ax^3}{b^3\sqrt{cx^2}} + \frac{x^4}{3b^2\sqrt{cx^2}} - \frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)} - \frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 80, normalized size = 0.75

$$\frac{x\left(-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4-12a^3(a+bx)\log(a+bx)\right)}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(a
+ b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 86, normalized size = 0.80

method	result	size
risch	$\frac{x\left(\frac{1}{3}b^2x^3-ab\,x^2+3a^2x\right)}{\sqrt{c\,x^2}\,b^4} - \frac{a^4x}{b^5(bx+a)\sqrt{c\,x^2}} - \frac{4a^3x\ln(bx+a)}{b^5\sqrt{c\,x^2}}$	81
default	$-\frac{x\left(-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)-9a^3bx+3a^4\right)}{3\sqrt{c\,x^2}\,b^5(bx+a)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^2+12*a^4*ln(b
*x+a)-9*a^3*b*x+3*a^4)/(c*x^2)^(1/2)/b^5/(b*x+a)
```

Maxima [A]
time = 0.30, size = 168, normalized size = 1.57

$$\frac{\sqrt{cx^2}\,a^3}{b^5cx+ab^4c}+\frac{\sqrt{cx^2}\,x^2}{3\,b^2c}-\frac{5\,ax^2}{3\,b^3\sqrt{c}}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5\sqrt{c}}+\frac{2\,\sqrt{cx^2}\,ax}{3\,b^3c}-\frac{20\,a^2x}{3\,b^4\sqrt{c}}-\frac{4\,a^3\log\left(bx\right)}{b^5\sqrt{c}}+\frac{29\,\sqrt{cx^2}\,a^2}{3\,b^4c}-\frac{5\,a^3}{b^5\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*c*x + a*b^4*c) + 1/3*sqrt(c*x^2)*x^2/(b^2*c) - 5/3*a*x
^2/(b^3*sqrt(c)) - 4*(-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b
^5*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^3*c) - 20/3*a^2*x/(b^4*sqrt(c)) - 4*a^
3*log(b*x)/(b^5*sqrt(c)) + 29/3*sqrt(c*x^2)*a^2/(b^4*c) - 5*a^3/(b^5*sqrt(c
))
```

Fricas [A]
time = 0.29, size = 85, normalized size = 0.79

$$\frac{\left(b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,\left(a^3bx+a^4\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\,\left(b^6cx^2+ab^5cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*c*x^2 + a*b^5*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 114, normalized size = 1.07

$$\frac{-\frac{a^4}{b^5(bx+a)\mathrm{sign}(x)}+\frac{\frac{1}{3}b^4x^3\mathrm{sign}(x)^2-ab^3x^2\mathrm{sign}(x)^2+3a^2b^2x\mathrm{sign}(x)^2}{b^6\mathrm{sign}(x)^3}-\frac{4a^3\ln|bx+a|}{b^5\mathrm{sign}(x)}+\frac{(4a^3\ln|a|+a^3)\mathrm{sign}(x)}{b^5}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*(12*a^3*log(abs(b*x + a))/(b^5*sgn(x)) + 3*a^4/((b*x + a)*b^5*sgn(x))
- 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x)^2 - 3*a*b^3*x^2*
sgn(x)^2 + 9*a^2*b^2*x*sgn(x)^2)/(b^6*sgn(x)^3))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.910 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=86

$$-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}$$

```
[Out] -2*a*x^2/b^3/(c*x^2)^(1/2)+1/2*x^3/b^2/(c*x^2)^(1/2)+a^3*x/b^4/(b*x+a)/(c*x
^2)^(1/2)+3*a^2*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (-2*a*x^2)/(b^3*Sqrt[c*x^2]) + x^3/(2*b^2*Sqrt[c*x^2]) + (a^3*x)/(b^4*Sqrt[
c*x^2]*(a + b*x)) + (3*a^2*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^3}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 69, normalized size = 0.80

$$\frac{x\left(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx)\right)}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b*x
]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 74, normalized size = 0.86

method	result	size
risch	$\frac{x\left(\frac{1}{2}x^2b-2ax\right)}{\sqrt{c\,x^2}\;b^3}+\frac{a^3x}{b^4(bx+a)\sqrt{c\,x^2}}+\frac{3a^2x\ln(bx+a)}{b^4\sqrt{c\,x^2}}$	69
default	$\frac{x\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2\sqrt{c\,x^2}\;b^4(bx+a)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*
a^3)/(c*x^2)^(1/2)/b^4/(b*x+a)
```

Maxima [A]
time = 0.27, size = 129, normalized size = 1.50

$$-\frac{\sqrt{cx^2}\,a^2}{b^4cx+ab^3c}+\frac{x^2}{2\,b^2\sqrt{c}}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}}+\frac{2\,ax}{b^3\sqrt{c}}+\frac{3\,a^2\log\left(bx\right)}{b^4\sqrt{c}}-\frac{4\,\sqrt{cx^2}\,a}{b^3c}+\frac{3\,a^2}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*c*x + a*b^3*c) + 1/2*x^2/(b^2*sqrt(c)) + 3*(-1)^(2*a*
c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2*a*x/(b^3*sqrt(c
)) + 3*a^2*log(b*x)/(b^4*sqrt(c)) - 4*sqrt(c*x^2)*a/(b^3*c) + 3/2*a^2/(b^4*
sqrt(c))
```

Fricas [A]
time = 0.30, size = 74, normalized size = 0.86

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,\left(b^5cx^2+ab^4cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*c*x^2 + a*b^4*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 94, normalized size = 1.09

$$\frac{\frac{a^3}{b^4(bx+a)\mathrm{sign}(x)} + \frac{\frac{1}{2}b^2x^2\mathrm{sign}(x)-2abx\mathrm{sign}(x)}{b^4\mathrm{sign}(x)^2} + \frac{3a^2\ln|bx+a|}{b^4\mathrm{sign}(x)} + \frac{(-3a^2\ln|a|-a^2)\mathrm{sign}(x)}{b^4}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*(6*a^2*log(abs(b*x + a))/(b^4*sgn(x)) - 2*(3*a^2*log(abs(a)) + a^2)*sgn
(x)/b^4 + 2*a^3/((b*x + a)*b^4*sgn(x)) + (b^2*x^2*sgn(x) - 4*a*b*x*sgn(x))/
(b^4*sgn(x)^2))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.911 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=64

$$\frac{x^2}{b^2\sqrt{cx^2}}-\frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}$$

```
[Out] x^2/b^2/(c*x^2)^(1/2)-a^2*x/b^3/(b*x+a)/(c*x^2)^(1/2)-2*a*x*ln(b*x+a)/b^3/(
c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*Sqrt[c*x^2]) - (a^2*x)/(b^3*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Log[a
+ b*x])/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{x^2}{b^2\sqrt{cx^2}} - \frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)} - \frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.81

$$\frac{x\left(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx)\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2]*
(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 60, normalized size = 0.94

method	result	size
risch	$\frac{x^2}{b^2\sqrt{c\,x^2}} - \frac{a^2x}{b^3(bx+a)\sqrt{c\,x^2}} - \frac{2ax\ln(bx+a)}{b^3\sqrt{c\,x^2}}$	59
default	$-\frac{x\left(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2\right)}{\sqrt{c\,x^2}\,b^3(bx+a)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(1/2)/b^3/
(b*x+a)
```

Maxima [A]
time = 0.28, size = 88, normalized size = 1.38

$$\frac{\sqrt{cx^2}\,a}{b^3cx+ab^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}}-\frac{2\,a\log\left(bx\right)}{b^3\sqrt{c}}+\frac{\sqrt{cx^2}}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*c*x + a*b^2*c) - 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*ab
s(b*x + a)))/(b^3*sqrt(c)) - 2*a*log(b*x)/(b^3*sqrt(c)) + sqrt(c*x^2)/(b^2*
c)
```

Fricas [A]
time = 0.30, size = 59, normalized size = 0.92

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4cx^2+ab^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c*x
^2 + a*b^3*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 67, normalized size = 1.05

$$\frac{\frac{x}{b^2\mathrm{sign}(x)}-\frac{a^2}{b^3(bx+a)\mathrm{sign}(x)}-\frac{2a\ln|bx+a|}{b^3\mathrm{sign}(x)}+\frac{(2a\ln|a|+a)\mathrm{sign}(x)}{b^3}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] ((2*a*log(abs(a)) + a)*sgn(x)/b^3 + x/(b^2*sgn(x)) - 2*a*log(abs(b*x + a))/
(b^3*sgn(x)) - a^2/((b*x + a)*b^3*sgn(x)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.912 $\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=43

$$\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

```
[Out] a*x/b^2/(b*x+a)/(c*x^2)^(1/2)+x*ln(b*x+a)/b^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{x}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx}{\sqrt{cx^2}}\\
&= \frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.81

$$\frac{x(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.13, size = 39, normalized size = 0.91

method	result	size
default	$\frac{x(b\ln(bx+a)x+a\ln(bx+a)+a)}{\sqrt{c\,x^2}\,b^2(bx+a)}$	39
risch	$\frac{ax}{b^2(bx+a)\sqrt{c\,x^2}}+\frac{x\ln(bx+a)}{b^2\sqrt{c\,x^2}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(1/2)/b^2/(b*x+a)
```

Maxima [A]
time = 0.26, size = 68, normalized size = 1.58

$$-\frac{\sqrt{cx^2}}{b^2cx+abc}+\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}}+\frac{\log(bx)}{b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(c*x^2)/(b^2*c*x + a*b*c) + (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x +
 a)))/(b^2*sqrt(c)) + log(b*x)/(b^2*sqrt(c))
```

Fricas [A]
time = 0.29, size = 40, normalized size = 0.93

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(bx+a\right)+a\right)}{b^3cx^2+ab^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c*x^2 + a*b^2*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 50, normalized size = 1.16

$$\frac{\frac{a}{b^2(bx+a)\mathrm{sign}(x)}+\frac{\ln|bx+a|}{b^2\mathrm{sign}(x)}+\frac{(-\ln|a|-1)\mathrm{sign}(x)}{b^2}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))/(b^2*sgn(x)) - a/((b*x +
 a)*b^2*sgn(x)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.913 $\int \frac{x}{\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=22

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

```
[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 32}

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= -\frac{x}{b\sqrt{cx^2}\,(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.08, size = 83, normalized size = 3.77

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{1}{\sqrt{cx^2}}\right], a==0\&\&b==0\right\}, \left\{\text{DirectedInfinity}\left[\frac{x^2}{\sqrt{cx^2}}\right], a==-bx\right\}, \left\{\frac{x^2}{a^2\sqrt{cx^2}}, b==0\right\}\right\}, -\frac{x}{ab\sqrt{cx^2}+b^2x\sqrt{cx^2}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[1 / Sqrt[c x ^ 2]], a == 0 && b == 0}, {Direct
edInfinity[x ^ 2 / Sqrt[c x ^ 2]], a == -b x}, {x ^ 2 / (a ^ 2 Sqrt[c x ^ 2
]), b == 0}}, -x / (a b Sqrt[c x ^ 2] + b ^ 2 x Sqrt[c x ^ 2])]
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.95

method	result	size
gosper	$-\frac{x}{b(bx+a)\sqrt{c\,x^{2}}}$	21
default	$-\frac{x}{b(bx+a)\sqrt{c\,x^{2}}}$	21
risch	$-\frac{x}{b(bx+a)\sqrt{c\,x^{2}}}$	21
trager	$\frac{(-1+x)\sqrt{c\,x^{2}}}{c(bx+a)(a+b)x}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.95

$$\frac{\sqrt{cx^2}}{abcx+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)/(a*b*c*x + a^2*c)
```

Fricas [A]
time = 0.29, size = 25, normalized size = 1.14

$$-\frac{\sqrt{cx^2}}{b^2cx^2 + abcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^2)/(b^2*c*x^2 + a*b*c*x)
```

Sympy [A]
time = 0.44, size = 68, normalized size = 3.09

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{cx^2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^2}{\sqrt{cx^2}} & \text{for } a = -bx \\ \frac{x^2}{a^2\sqrt{cx^2}} & \text{for } b = 0 \\ -\frac{x}{ab\sqrt{cx^2} + b^2x\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Piecewise((zoo/sqrt(c*x**2), Eq(a, 0) & Eq(b, 0)), (zoo*x**2/sqrt(c*x**2),
Eq(a, -b*x)), (x**2/(a**2*sqrt(c*x**2)), Eq(b, 0)), (-x/(a*b*sqrt(c*x**2) +
 b**2*x*sqrt(c*x**2)), True))
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.18

$$\frac{\frac{\operatorname{sign}(x)}{ab} - \frac{1}{b(bx+a)\operatorname{sign}(x)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x)
[Out] (sgn(x)/(a*b) - 1/((b*x + a)*b*sgn(x)))/sqrt(c)
```

Mupad [B]
time = 0.16, size = 25, normalized size = 1.14

$$-\frac{\sqrt{c\,x^2}}{b\,c\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] -(c*x^2)^(1/2)/(b*c*x*(a + b*x))
```

3.914 $\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=59

$$\frac{x}{a\sqrt{cx^2}(a+bx)} + \frac{x\log(x)}{a^2\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2\sqrt{cx^2}}$$

```
[Out] x/a/(b*x+a)/(c*x^2)^(1/2)+x*ln(x)/a^2/(c*x^2)^(1/2)-x*ln(b*x+a)/a^2/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 46}

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}} + \frac{x\log(x)}{a^2\sqrt{cx^2}} + \frac{x}{a\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x/(a*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*Sqrt[c*x^2]) - (x*Log[a + b*x
])/(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx = \frac{x\int \frac{1}{x(a+bx)^2} dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right) dx}{\sqrt{cx^2}}$$
$$= \frac{x}{a\sqrt{cx^2}(a+bx)} + \frac{x\log(x)}{a^2\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 0.75

$$\frac{x(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a + b
*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 50, normalized size = 0.85

method	result	size
default	$\frac{x(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{\sqrt{cx^2}\,a^2(bx+a)}$	50
risch	$\frac{x}{a(bx+a)\sqrt{cx^2}}+\frac{x\ln(-x)}{\sqrt{cx^2}\,a^2}-\frac{x\ln(bx+a)}{a^2\sqrt{cx^2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2/(b*x+a)
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.03

$$-\frac{\sqrt{cx^2}\,b}{a^2bcx+a^3c}-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*b/(a^2*b*c*x + a^3*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b
*x + a)))/(a^2*sqrt(c))
```

Fricas [A]
time = 0.31, size = 44, normalized size = 0.75

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bcx^2+a^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c*x^2 + a^3*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.915 $\int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{1}{a^2\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}$$

```
[Out] -1/a^2/(c*x^2)^(1/2)-b*x/a^2/(b*x+a)/(c*x^2)^(1/2)-2*b*x*ln(x)/a^3/(c*x^2)^
(1/2)+2*b*x*ln(b*x+a)/a^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*Sqrt[c*x^2])) - (b*x)/(a^2*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log[x])
/(a^3*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{1}{a^2\sqrt{cx^2}} - \frac{bx}{a^2\sqrt{cx^2}\,(a+bx)} - \frac{2bx\log(x)}{a^3\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 0.77

$$\frac{cx^2(-a(a+2bx)-2bx(a+bx)\log(x)+2bx(a+bx)\log(a+bx))}{a^3\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (c*x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a +
 b*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.11, size = 71, normalized size = 0.91

method	result	size
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{\sqrt{c\,x^2}\,(bx+a)} - \frac{2bx\ln(x)}{a^3\sqrt{c\,x^2}} + \frac{2xb\ln(-bx-a)}{\sqrt{c\,x^2}\,a^3}$	69
default	$-\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2}{\sqrt{c\,x^2}\,a^3(bx+a)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+a)*a*b*x+2*a*b
*x+a^2)/(c*x^2)^(1/2)/a^3/(b*x+a)
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.73

$$-\frac{2\,bx+a}{a^2b\sqrt{c}\,x^2+a^3\sqrt{c}\,x}+\frac{2\,b\log\left(bx+a\right)}{a^3\sqrt{c}}-\frac{2\,b\log\left(x\right)}{a^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*sqrt(c)*x^2 + a^3*sqrt(c)*x) + 2*b*log(b*x + a)/(a^3*sq
rt(c)) - 2*b*log(x)/(a^3*sqrt(c))
```

Fricas [A]
time = 0.30, size = 62, normalized size = 0.79

$$-\frac{\left(2\,abx+a^2-2\,\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bcx^3+a^4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c*x^3 + a^4*c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.916 $\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=103

$$\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}$$

```
[Out] 2*b/a^3/(c*x^2)^(1/2)-1/2/a^2/x/(c*x^2)^(1/2)+b^2*x/a^3/(b*x+a)/(c*x^2)^(1/
2)+3*b^2*x*ln(x)/a^4/(c*x^2)^(1/2)-3*b^2*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 46}

$$\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*Sqrt[c*x^2]) - 1/(2*a^2*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*Sqrt[c*x^2
]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*Sqrt[c*x^2]) - (3*b^2*x*Log[a + b*x])/
(a^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{2b}{a^3\sqrt{cx^2}} - \frac{1}{2a^2x\sqrt{cx^2}} + \frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)} + \frac{3b^2x\log(x)}{a^4\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 81, normalized size = 0.79

$$\frac{cx\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x
^2*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.14, size = 95, normalized size = 0.92

method	result	size
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{\sqrt{c\,x^2}\;x(bx+a)} - \frac{3b^2x\ln(bx+a)}{a^4\sqrt{c\,x^2}} + \frac{3x\,b^2\ln(-x)}{\sqrt{c\,x^2}\;a^4}$	86
default	$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3}{2x\sqrt{c\,x^2}\;a^4(bx+a)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*ln(b*x+a)*a*
b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(1/2)/a^4/(b*x+a)
```

Maxima [A]
time = 0.27, size = 76, normalized size = 0.74

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\,\left(a^3b\sqrt{c}\,x^3+a^4\sqrt{c}\,x^2\right)}-\frac{3\,b^2\log\left(bx+a\right)}{a^4\sqrt{c}}+\frac{3\,b^2\log\left(x\right)}{a^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*sqrt(c)*x^3 + a^4*sqrt(c)*x^2) - 3*b
^2*log(b*x + a)/(a^4*sqrt(c)) + 3*b^2*log(x)/(a^4*sqrt(c))
```

Fricas [A]
time = 0.30, size = 79, normalized size = 0.77

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,\left(a^4bcx^4+a^5cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c*x^4 + a^5*c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.917 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=73

$$\frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}$$

```
[Out] x^2/b^2/c/(c*x^2)^(1/2)-a^2*x/b^3/c/(b*x+a)/(c*x^2)^(1/2)-2*a*x*ln(b*x+a)/b
^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*c*Sqrt[c*x^2]) - (a^2*x)/(b^3*c*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Lo
g[a + b*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{x^2}{b^2c\sqrt{cx^2}} - \frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)} - \frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 54, normalized size = 0.74

$$\frac{x^3\left(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx)\right)}{b^3(cx^2)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*(c*x^2)^(3
/2)*(a + b*x))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.13, size = 62, normalized size = 0.85

method	result	size
default	$-\frac{x^3\left(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2\right)}{(cx^2)^{\frac{3}{2}}b^3(bx+a)}$	62
risch	$\frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c(bx+a)\sqrt{cx^2}}-\frac{2ax\ln(bx+a)}{b^3c\sqrt{cx^2}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^3*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(3/2)/b^
3/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(67) = 134$.
time = 0.29, size = 149, normalized size = 2.04

$$\frac{a^3}{\sqrt{cx^2}\,b^5cx+\sqrt{cx^2}\,ab^4c}+\frac{x^2}{\sqrt{cx^2}\,b^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{cx^2}\,b^3c}-\frac{2\,a\log\left(bx\right)}{b^3c^{\frac{3}{2}}}-\frac{5\,a^2}{\sqrt{cx^2}\,b^4c}+\frac{4\,a^2}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(sqrt(c*x^2)*b^5*c*x + sqrt(c*x^2)*a*b^4*c) + x^2/(sqrt(c*x^2)*b^2*c) -
 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) + 2*a*x/
(sqrt(c*x^2)*b^3*c) - 2*a*log(b*x)/(b^3*c^(3/2)) - 5*a^2/(sqrt(c*x^2)*b^4*c
) + 4*a^2/(b^4*c^(3/2)*x)
```

Fricas [A]
time = 0.29, size = 63, normalized size = 0.86

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4c^2x^2+ab^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c^2
*x^2 + a*b^3*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 69, normalized size = 0.95

$$\frac{\frac{x}{b^2\mathrm{sign}(x)}-\frac{a^2}{b^3(bx+a)\mathrm{sign}(x)}-\frac{2a\ln|bx+a|}{b^3\mathrm{sign}(x)}+\frac{(2a\ln|a|+a)\mathrm{sign}(x)}{b^3}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] ((2*a*log(abs(a)) + a)*sgn(x)/b^3 + x/(b^2*sgn(x)) - 2*a*log(abs(b*x + a))/
(b^3*sgn(x)) - a^2/((b*x + a)*b^3*sgn(x)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(c\,x^2)^{3/2}\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.918 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=49

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

```
[Out] a*x/b^2/c/(b*x+a)/(c*x^2)^(1/2)+x*ln(b*x+a)/b^2/c/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{x}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.76

$$\frac{x^3(a+(a+bx)\log(a+bx))}{b^2(cx^2)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(a + (a + b*x)*Log[a + b*x]))/(b^2*(c*x^2)^(3/2)*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.84

method	result	size
default	$\frac{x^3(b\ln(bx+a)x+a\ln(bx+a)+a)}{(cx^2)^{\frac{3}{2}}b^2(bx+a)}$	41
risch	$\frac{ax}{b^2c(bx+a)\sqrt{cx^2}}+\frac{x\ln(bx+a)}{b^2c\sqrt{cx^2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(3/2)/b^2/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(45) = 90.
time = 0.29, size = 108, normalized size = 2.20

$$-\frac{a^2}{\sqrt{cx^2}\,b^4cx+\sqrt{cx^2}\,ab^3c}+\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}}+\frac{\log(bx)}{b^2c^{\frac{3}{2}}}+\frac{3\,a}{\sqrt{cx^2}\,b^3c}-\frac{2\,a}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(sqrt(c*x^2)*b^4*c*x + sqrt(c*x^2)*a*b^3*c) + (-1)^(2*a*c*x/b)*log(-2*
a*c*x/(b*abs(b*x + a)))/(b^2*c^(3/2)) + log(b*x)/(b^2*c^(3/2)) + 3*a/(sqrt(
c*x^2)*b^3*c) - 2*a/(b^3*c^(3/2)*x)
```

Fricas [A]
time = 0.29, size = 44, normalized size = 0.90

$$\frac{\sqrt{cx^2}\,((bx+a)\log(bx+a)+a)}{b^3c^2x^2+ab^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c^2*x^2 + a*b^2*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 52, normalized size = 1.06

$$\frac{\frac{a}{b^2(bx+a)\operatorname{sign}(x)}+\frac{\ln|bx+a|}{b^2\operatorname{sign}(x)}+\frac{(-\ln|a|-1)\operatorname{sign}(x)}{b^2}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))/(b^2*sgn(x)) - a/((b*x +
 a)*b^2*sgn(x)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.919 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=25

$$-\frac{x}{bc\sqrt{cx^2}\,(a+bx)}$$

```
[Out] -x/b/c/(b*x+a)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$-\frac{x}{bc\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x/(b*c*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{x}{bc\sqrt{cx^2}\,(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.96

$$-\frac{x^3}{b\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.22, size = 89, normalized size = 3.56

$$\text{Piecewise}\left[\left\{\left\{\text{DirectedInfinity}\left[\frac{x^2}{(cx^2)^{\frac{3}{2}}}\right], a==0\&\&b==0\right\}, \left\{\text{DirectedInfinity}\left[\frac{x^4}{(cx^2)^{\frac{3}{2}}}\right], a==-bx\right\}, \left\{\frac{x^4}{a^2(cx^2)^{\frac{3}{2}}}, b==0\right\}\right\}, -\frac{x^3}{ab(cx^2)^{\frac{3}{2}}+b^2x(cx^2)^{\frac{3}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]')
```

```
[Out] Piecewise[{{DirectedInfinity[x ^ 2 / (c x ^ 2) ^ (3 / 2)], a == 0 && b == 0
}, {DirectedInfinity[x ^ 4 / (c x ^ 2) ^ (3 / 2)], a == -b x}, {x ^ 4 / (a
^ 2 (c x ^ 2) ^ (3 / 2)), b == 0}}, -x ^ 3 / (a b (c x ^ 2) ^ (3 / 2) + b ^
 2 x (c x ^ 2) ^ (3 / 2))]
```

Maple [A]
time = 0.14, size = 23, normalized size = 0.92

method	result	size
gosper	$-\frac{x^3}{(bx+a)b(cx^2)^{\frac{3}{2}}}$	23
default	$-\frac{x^3}{(bx+a)b(cx^2)^{\frac{3}{2}}}$	23
risch	$-\frac{x}{bc(bx+a)\sqrt{cx^2}}$	24
trager	$\frac{(-1+x)\sqrt{cx^2}}{c^2(bx+a)(a+b)x}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*x+a)/b*x^3/(c*x^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(23) = 46.
time = 0.28, size = 47, normalized size = 1.88

$$\frac{a}{\sqrt{cx^2}\,b^3cx+\sqrt{cx^2}\,ab^2c}-\frac{1}{\sqrt{cx^2}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a/(sqrt(c*x^2)*b^3*c*x + sqrt(c*x^2)*a*b^2*c) - 1/(sqrt(c*x^2)*b^2*c)
```

Fricas [A]
time = 0.29, size = 29, normalized size = 1.16

$$-\frac{\sqrt{cx^2}}{b^2c^2x^2 + abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c^2*x^2 + a*b*c^2*x)
```

Sympy [A]
time = 0.57, size = 73, normalized size = 2.92

$$\begin{cases} \frac{\tilde{\infty}x^2}{(cx^2)^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^4}{(cx^2)^{\frac{3}{2}}} & \text{for } a = -bx \\ \frac{x^4}{a^2(cx^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{x^3}{ab(cx^2)^{\frac{3}{2}} + b^2x(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2/(c*x**2)**(3/2), Eq(a, 0) & Eq(b, 0)), (zoo*x**4/(c*x**
2)**(3/2), Eq(a, -b*x)), (x**4/(a**2*(c*x**2)**(3/2)), Eq(b, 0)), (-x**3/(a
*b*(c*x**2)**(3/2) + b**2*x*(c*x**2)**(3/2)), True))
```

Giac [A]
time = 0.00, size = 28, normalized size = 1.12

$$\frac{\frac{\operatorname{sign}(x)}{ab} - \frac{1}{b(bx+a)\operatorname{sign}(x)}}{\sqrt{c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] (sgn(x)/(a*b) - 1/((b*x + a)*b*sgn(x)))/c^(3/2)
```

Mupad [B]
time = 0.17, size = 25, normalized size = 1.00

$$-\frac{\sqrt{c\,x^2}}{b\,c^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] -(c*x^2)^(1/2)/(b*c^2*x*(a + b*x))
```

3.920 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=68

$$\frac{x}{ac\sqrt{cx^2}\,(a+bx)}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}$$

```
[Out] x/a/c/(b*x+a)/(c*x^2)^(1/2)+x*ln(x)/a^2/c/(c*x^2)^(1/2)-x*ln(b*x+a)/a^2/c/(
c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x/(a*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*c*Sqrt[c*x^2]) - (x*Log[a +
 b*x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{x}{ac\sqrt{cx^2}\,(a+bx)} + \frac{x\log(x)}{a^2c\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.68

$$\frac{x^3(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*(c*x^2)^(3/2)*(a
 + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.76

method	result	size
default	$\frac{x^3(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{(c\,x^2)^{\frac{3}{2}}\,a^2(bx+a)}$	52
risch	$\frac{x}{ac(bx+a)\sqrt{c\,x^2}}+\frac{x\ln(-x)}{c\sqrt{c\,x^2}\,a^2}-\frac{x\ln(bx+a)}{a^2c\sqrt{c\,x^2}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2/(b*x+
a)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.21

$$-\frac{1}{\sqrt{cx^2}\,b^2cx+\sqrt{cx^2}\,abc}-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}}+\frac{1}{\sqrt{cx^2}\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2)*b^2*c*x + sqrt(c*x^2)*a*b*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*
x/(b*abs(b*x + a)))/(a^2*c^(3/2)) + 1/(sqrt(c*x^2)*a*b*c)
```

Fricas [A]
time = 0.29, size = 48, normalized size = 0.71

$$\frac{\sqrt{cx^2}\,\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bc^2x^2+a^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c^2*x^2 + a^3*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(c\,x^2)^{3/2}\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.921 $\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{1}{a^2c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}$$

```
[Out] -1/a^2/c/(c*x^2)^(1/2)-b*x/a^2/c/(b*x+a)/(c*x^2)^(1/2)-2*b*x*ln(x)/a^3/c/(c
*x^2)^(1/2)+2*b*x*ln(b*x+a)/a^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 46}

$$-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*c*Sqrt[c*x^2])) - (b*x)/(a^2*c*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log
[x])/(a^3*c*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{1}{a^2c\sqrt{cx^2}} - \frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)} - \frac{2bx\log(x)}{a^3c\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 0.66

$$\frac{x^2(-a(a+2bx) - 2bx(a+bx)\log(x) + 2bx(a+bx)\log(a+bx))}{a^3\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a + b
*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.82

method	result	size
default	$-\frac{x^2(2b^2\ln(x)x^2 - 2b^2\ln(bx+a)x^2 + 2ab\ln(x)x - 2\ln(bx+a)abx + 2abx + a^2)}{(cx^2)^{\frac{3}{2}}a^3(bx+a)}$	74
risch	$\frac{-\frac{2bx}{a^2} - \frac{1}{a}}{c\sqrt{cx^2}\,(bx+a)} - \frac{2bx\ln(x)}{a^3c\sqrt{cx^2}} + \frac{2xb\ln(-bx-a)}{c\sqrt{cx^2}\,a^3}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^2*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+a)*a*b*x+2
*a*b*x+a^2)/(c*x^2)^(3/2)/a^3/(b*x+a)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.88

$$\frac{1}{\sqrt{cx^2}\,abcx+\sqrt{cx^2}\,a^2c}+\frac{2\,(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}-\frac{2}{\sqrt{cx^2}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/(sqrt(c*x^2)*a*b*c*x + sqrt(c*x^2)*a^2*c) + 2*(-1)^(2*a*c*x/b)*b*log(-2*a
*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) - 2/(sqrt(c*x^2)*a^2*c)
```

Fricas [A]
time = 0.30, size = 66, normalized size = 0.73

$$-\frac{\left(2\,abx+a^2-2\,(b^2x^2+abx)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bc^2x^3+a^4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c^2*x^3 + a^4*c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.922 $\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=118

$$\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}+\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

```
[Out] 2*b/a^3/c/(c*x^2)^(1/2)-1/2/a^2/c/x/(c*x^2)^(1/2)+b^2*x/a^3/c/(b*x+a)/(c*x^
2)^(1/2)+3*b^2*x*ln(x)/a^4/c/(c*x^2)^(1/2)-3*b^2*x*ln(b*x+a)/a^4/c/(c*x^2)^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 46}

$$\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*c*Sqrt[c*x^2]) - 1/(2*a^2*c*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*c*Sqrt
[c*x^2]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*c*Sqrt[c*x^2]) - (3*b^2*x*Log[a
+ b*x])/(a^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{2b}{a^3c\sqrt{cx^2}} - \frac{1}{2a^2cx\sqrt{cx^2}} + \frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)} + \frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 80, normalized size = 0.68

$$\frac{x\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x^2
*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 93, normalized size = 0.79

method	result	size
default	$\frac{x\left(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3\right)}{2\left(c\,x^2\right)^{\frac{3}{2}}a^4(bx+a)}$	93
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{cx\sqrt{c\,x^2}\,(bx+a)}-\frac{3b^2x\ln(bx+a)}{a^4c\sqrt{c\,x^2}}+\frac{3x\,b^2\ln(-x)}{c\sqrt{c\,x^2}\,a^4}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*ln(b*x+a)*a*
b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(3/2)/a^4/(b*x+a)
```

Maxima [A]
time = 0.27, size = 98, normalized size = 0.83

$$-\frac{b}{\sqrt{cx^2}\,a^2bcx+\sqrt{cx^2}\,a^3c}-\frac{3\,(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^4c^{\frac{3}{2}}}+\frac{3\,b}{\sqrt{cx^2}\,a^3c}-\frac{1}{2\,a^2c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*a^2*b*c*x + sqrt(c*x^2)*a^3*c) - 3*(-1)^(2*a*c*x/b)*b^2*log
(-2*a*c*x/(b*abs(b*x + a)))/(a^4*c^(3/2)) + 3*b/(sqrt(c*x^2)*a^3*c) - 1/2/(
a^2*c^(3/2)*x^2)
```

Fricas [A]
time = 0.30, size = 83, normalized size = 0.70

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,(b^3x^3+ab^2x^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bc^2x^4+a^5c^2x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c^2*x^4 + a^5*c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.923 $\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}$$

```
[Out] -a^3*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^4/(1+n)/x+3*a^2*(b*x+a)^(2+n)*(c*x^2)^(1
/2)/b^4/(2+n)/x-3*a*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^4/(3+n)/x+(b*x+a)^(4+n)*(
c*x^2)^(1/2)/b^4/(4+n)/x
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a^3*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(
b^4*(3 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx = \frac{\sqrt{cx^2}\,\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\,\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}}{b^4(}$$

Mathematica [A]
time = 0.04, size = 97, normalized size = 0.74

$$\frac{cx(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
[Out] (c*x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 136, normalized size = 1.04

method	result
gosper	$-\frac{\sqrt{c\,x^2}\,(bx+a)^{1+n}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{x\,b^4\left(n^4+10n^3+35n^2+50n+24\right)}$
risch	$-\frac{\sqrt{c\,x^2}\,\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)^{}}{x(3+n)(4+n)(2+n)(1+n)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(1+n)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.89

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4\sqrt{c}\,x^4+(n^3+3\,n^2+2\,n)ab^3\sqrt{c}\,x^3-3\,(n^2+n)a^2b^2\sqrt{c}\,x^2+6\,a^3b\sqrt{c}\,nx-6\,a^4\sqrt{c}\,\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*sqrt(c)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*sqrt(
c)*x^3 - 3*(n^2 + n)*a^2*b^2*sqrt(c)*x^2 + 6*a^3*b*sqrt(c)*n*x - 6*a^4*sqrt
(c))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.30, size = 153, normalized size = 1.17

$$\frac{\left(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^3 \sqrt{cx^2}}{4} & \text{for}\: b = 0 \\ \int \frac{x^2\sqrt{cx^2}}{(a+bx)^4}\,dx & \text{for}\: n = -4 \\ \int \frac{x^2\sqrt{cx^2}}{(a+bx)^3}\,dx & \text{for}\: n = -3 \\ \int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx & \text{for}\: n = -2 \\ \int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx & \text{for}\: n = -1 \\ -\frac{6a^4\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{6a^3bnx\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} - \frac{3a^2b^2n^2x^2\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} - \frac{3a^2b^2nx^2\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{ab^3n^3x^3\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{3ab^3n^2x^3\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{2ab^3nx^3\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{b^4n^3x^4\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{6b^4n^2x^4\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{11b^4nx^4\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{6b^4x^4\sqrt{cx^2}(a+bx)^n}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**3*sqrt(c*x**2)/4, Eq(b, 0)), (Integral(x**2*sqrt(c*x**2)
/(a + b*x)**4, x), Eq(n, -4)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**3, x)
```

```
, Eq(n, -3)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x), Eq(n, -2)), (In
tegral(x**2*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (-6*a**4*sqrt(c*x**2)*(
a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x +
24*b**4*x) + 6*a**3*b*n*x*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*
n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) - 3*a**2*b**2*n**2*x**2*
sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x +
50*b**4*n*x + 24*b**4*x) - 3*a**2*b**2*n*x**2*sqrt(c*x**2)*(a + b*x)**n/(b*
*4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + a*
b**3*n**3*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35
*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 3*a*b**3*n**2*x**3*sqrt(c*x**2)*(
a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x +
24*b**4*x) + 2*a*b**3*n*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b*
*4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + b**4*n**3*x**4*sqrt
(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b
**4*n*x + 24*b**4*x) + 6*b**4*n**2*x**4*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**
4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 11*b**4*
n*x**4*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n*
*2*x + 50*b**4*n*x + 24*b**4*x) + 6*b**4*x**4*sqrt(c*x**2)*(a + b*x)**n/(b*
*4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(123) = 246$.
time = 0.00, size = 327, normalized size = 2.50

$$\sqrt{c}\left(\frac{6a^4e^{n\ln a}\mathrm{sign}\left(x\right)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{-6a^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+6b^4x^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+11b^4nx^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+6b^4n^2x^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+b^4n^3x^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+2ab^3nx^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+3ab^3n^2x^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+ab^3n^3x^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-3a^2b^2nx^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-3a^2b^2n^2x^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+6a^3bnx\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*sqrt(c)
```

Mupad [B]
time = 0.35, size = 214, normalized size = 1.63

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) - (6*a^4*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^3 +
n^4 + 24)) + (6*a^3*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (a*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3
+ n^4 + 24)) - (3*a^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24))))/x
```

3.924 $\int x\sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=96

$$\frac{a^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2a\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

```
[Out] a^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^3/(1+n)/x-2*a*(b*x+a)^(2+n)*(c*x^2)^(1/2)
/b^3/(2+n)/x+(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^3/(3+n)/x
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{a^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (a^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*Sqrt[c*x^2]*(a +
 b*x)^(2 + n))/(b^3*(2 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3*(3 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\sqrt{cx^2}(a+bx)^n\,dx = \frac{\sqrt{cx^2}\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int\left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2a\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x} + \frac{\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.71

$$\frac{cx(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^n,x]
[Out] (c*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1*Sqrt[c*x^2]*(a + b*x)^n,x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.11, size = 83, normalized size = 0.86

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)\sqrt{c\,x^2}}{x\,b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{\sqrt{c\,x^2}\,\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{x(2+n)(3+n)(1+n)b^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(1/2)/x/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.26, size = 80, normalized size = 0.83

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.30, size = 106, normalized size = 1.10

$$-\frac{\left(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^2 \sqrt{cx^2}}{3} & \text{for}\: b = 0 \\ \int \frac{x \sqrt{cx^2}}{(a+bx)^3}\, dx & \text{for}\: n = -3 \\ \int \frac{x \sqrt{cx^2}}{(a+bx)^2}\, dx & \text{for}\: n = -2 \\ \int \frac{x \sqrt{cx^2}}{a+bx}\, dx & \text{for}\: n = -1 \\ \frac{2a^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} - \frac{2a^2bnx\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{ab^2n^2x^2\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{ab^2nx^2\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{b^3n^2x^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{3b^3nx^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{2b^3x^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**2*sqrt(c*x**2)/3, Eq(b, 0)), (Integral(x*sqrt(c*x**2)/(a
 + b*x)**3, x), Eq(n, -3)), (Integral(x*sqrt(c*x**2)/(a + b*x)**2, x), Eq(n
, -2)), (Integral(x*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (2*a**3*sqrt(c*
x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) -
 2*a**2*b*n*x*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b
**3*n*x + 6*b**3*x) + a*b**2*n**2*x**2*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3
```

```
*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + a*b**2*n*x**2*sqrt(c*x**2)*(
a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + b**3*n
**2*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n
*x + 6*b**3*x) + 3*b**3*n*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b
**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + 2*b**3*x**3*sqrt(c*x**2)*(a + b*x)**
n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(90) = 180.
time = 0.00, size = 216, normalized size = 2.25

$$\sqrt{c}\left(-\frac{2a^3e^{n\ln a}\mathrm{sign}\left(x\right)}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{2a^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+2b^3x^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+3b^3nx^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+b^3n^2x^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+ab^2nx^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+ab^2n^2x^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-2a^2bnx\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}}{b^3n^3+6b^3n^2+11b^3n+6b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] -(2*a^3*a^n*sgn(x)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3) - ((b*x + a)^n*
b^3*n^2*x^3*sgn(x) + (b*x + a)^n*a*b^2*n^2*x^2*sgn(x) + 3*(b*x + a)^n*b^3*n
*x^3*sgn(x) + (b*x + a)^n*a*b^2*n*x^2*sgn(x) + 2*(b*x + a)^n*b^3*x^3*sgn(x)
 - 2*(b*x + a)^n*a^2*b*n*x*sgn(x) + 2*(b*x + a)^n*a^3*sgn(x))/(b^3*n^3 + 6*
b^3*n^2 + 11*b^3*n + 6*b^3))*sqrt(c)
```

Mupad [B]
time = 0.25, size = 142, normalized size = 1.48

$$\frac{(a+b\,x)^n\,\left(\frac{2\,a^3\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (x^3*(
c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*n*x*(c*x^2)
^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b
*(11*n + 6*n^2 + n^3 + 6))))/x
```

3.925 $\int \sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=63

$$-\frac{a\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

```
[Out] -a*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^2/(1+n)/x+(b*x+a)^(2+n)*(c*x^2)^(1/2)/b^2/
(2+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (Sqrt[c*x^2]*(a + b*
x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\,\int x(a+bx)^n\,dx}{x}\\
&= \frac{\sqrt{cx^2}\,\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\
&= -\frac{a\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.70

$$\frac{cx(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2]
)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[c*x^2]*(a + b*x)^n,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 46, normalized size = 0.73

method	result	size
gosper	$-\frac{\sqrt{c\,x^{2}}\,(bx+a)^{1+n}(-bnx-bx+a)}{x\,b^{2}(n^{2}+3n+2)}$	46
risch	$-\frac{\sqrt{c\,x^{2}}\,(-b^{2}n\,x^{2}-abnx-x^{2}b^{2}+a^{2})(bx+a)^{n}}{x\,b^{2}(2+n)(1+n)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(1+n)*(-b*n*x-b*x+a)/x/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.81

$$\frac{\left(b^2\sqrt{c}\,(n+1)x^2+ab\sqrt{c}\,nx-a^2\sqrt{c}\,\right)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*sqrt(c)*(n + 1)*x^2 + a*b*sqrt(c)*n*x - a^2*sqrt(c))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.30, size = 63, normalized size = 1.00

$$\frac{(abnx + (b^2n + b^2)x^2 - a^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2n^2 + 3\,b^2n + 2\,b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n^2 + 3*b
^2*n + 2*b^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x\sqrt{cx^2}}{2} & \text{for}\ b = 0 \\ \int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{\sqrt{cx^2}}{a+bx}\,dx & \text{for}\ n = -1 \\ -\frac{a^2\sqrt{cx^2}\,(a+bx)^n}{b^2n^2x+3b^2nx+2b^2x} + \frac{abnx\sqrt{cx^2}\,(a+bx)^n}{b^2n^2x+3b^2nx+2b^2x} + \frac{b^2nx^2\sqrt{cx^2}\,(a+bx)^n}{b^2n^2x+3b^2nx+2b^2x} + \frac{b^2x^2\sqrt{cx^2}\,(a+bx)^n}{b^2n^2x+3b^2nx+2b^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x*sqrt(c*x**2)/2, Eq(b, 0)), (Integral(sqrt(c*x**2)/(a + b*
x)**2, x), Eq(n, -2)), (Integral(sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (-
a**2*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x) + a*b*
n*x*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x) + b**2*
n*x**2*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x) + b*
*2*x**2*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(59) = 118.
time = 0.00, size = 129, normalized size = 2.05

$$\sqrt{c}\left(\frac{a^2e^{n\ln a}\text{sign}\,(x)}{b^2n^2+3b^2n+2b^2} + \frac{-a^2\text{sign}\,(x)\,e^{n\ln(bx+a)} + b^2x^2\text{sign}\,(x)\,e^{n\ln(bx+a)} + b^2nx^2\text{sign}\,(x)\,e^{n\ln(bx+a)} + abnx\text{sign}\,(x)\,e^{n\ln(bx+a)}}{b^2n^2+3b^2n+2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*sqrt(c)
```

Mupad [B]
time = 0.22, size = 85, normalized size = 1.35

$$\frac{(a+b\,x)^n\,\left(\frac{x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,\sqrt{c\,x^2}}{b^2\,(n^2+3\,n+2)}+\frac{a\,n\,x\,\sqrt{c\,x^2}}{b\,(n^2+3\,n+2)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*(c*x^2)^(1
/2))/(b^2*(3*n + n^2 + 2)) + (a*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2))))/x
```

3.926 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}$$

```
[Out] (b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x}\,dx &= \frac{\sqrt{cx^2}\,\int (a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.97

$$\frac{cx(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^1,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.97

method	result	size
gosper	$\frac{(bx+a)^{1+n}\sqrt{c\,x^2}}{b(1+n)x}$	29
risch	$\frac{(bx+a)(bx+a)^{n}\sqrt{c\,x^2}}{b(1+n)x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.93

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\,\right)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A]
time = 0.30, size = 30, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\sqrt{cx^2}}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n\sqrt{cx^2} & \text{for } b = 0 \\ \int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx & \text{for } n = -1 \\ \frac{a\sqrt{cx^2}\,(a+bx)^n}{bnx+bx} + \frac{bx\sqrt{cx^2}\,(a+bx)^n}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(c*x**2)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sqrt(c*x**2), Eq(b,
 0)), (Integral(sqrt(c*x**2)/(x*(a + b*x)), x), Eq(n, -1)), (a*sqrt(c*x**2)
*(a + b*x)**n/(b*n*x + b*x) + b*x*sqrt(c*x**2)*(a + b*x)**n/(b*n*x + b*x),
True))
```

Giac [A]
time = 0.00, size = 40, normalized size = 1.33

$$\sqrt{c}\left(-\frac{a^{n+1}\mathrm{sign}\left(x\right)}{bn+b}+\frac{\left(bx+a\right)^{n+1}\mathrm{sign}\left(x\right)}{b\left(n+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x)
```

```
[Out] -sqrt(c)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

Mupad [B]
time = 0.23, size = 31, normalized size = 1.03

$$\frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^n\,\left(a+b\,x\right)}{b\,x\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((c*x^2)^(1/2)*(a + b*x)^n*(a + b*x))/(b*x*(n + 1))
```

3.927 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x^2} dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{cx^2}(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}$$

```
[Out] -(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a/(1+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{\sqrt{cx^2}(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]
```

```
[Out] -((Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*
x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^n}{x^2} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^n}{x} dx}{x}\\ &= -\frac{\sqrt{cx^2}(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.98

$$-\frac{cx(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]
[Out] -((c*x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*Sqrt[c*x^2]))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\sqrt{c\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\ (a+bx)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**2, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\ (a+b\,x)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^2, x)
```

3.928 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x^3} dx$

Optimal. Leaf size=47

$$\frac{b\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

```
[Out] b*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^2/(1+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{b\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]
```

```
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^3}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{b\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$\frac{b\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\sqrt{c\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**3, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^3,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^3, x)
```

3.929 $\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{b^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}$$

```
[Out] -b^2*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^3/(1+n
)/x
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]
```

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^n}{x^3}\,dx}{x}\\ &= -\frac{b^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{b^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]
```

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\sqrt{c\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**4, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^4,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^4, x)
```

3.930 $\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx$

Optimal. Leaf size=169

$$\frac{a^4c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac\sqrt{cx^2}\,(a+bx)^{4+n}}{b^5(4+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{5+n}}{b^5(5+n)x}$$

```
[Out] a^4*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^5/(1+n)/x-4*a^3*c*(b*x+a)^(2+n)*(c*x^2)
^(1/2)/b^5/(2+n)/x+6*a^2*c*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^5/(3+n)/x-4*a*c*(b
*x+a)^(4+n)*(c*x^2)^(1/2)/b^5/(4+n)/x+c*(b*x+a)^(5+n)*(c*x^2)^(1/2)/b^5/(5+
n)/x
```

Rubi [A]
time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{a^4c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (a^4*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c*Sqrt[c*x^2
]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c*Sqrt[c*x^2]*(a + b*x)^(3 +
n))/(b^5*(3 + n)*x) - (4*a*c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4 + n)*x)
 + (c*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int\left(\frac{a^4(a+bx)^n}{b^4} - \frac{4a^3(a+bx)^{1+n}}{b^4} + \frac{6a^2(a+bx)^{2+n}}{b^4} - \frac{4a(a+bx)^{3+n}}{b^4} + \frac{(a+bx)^{4+n}}{b^4}\right)dx}{x}$$
$$= \frac{a^4c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x} - \frac{4a^3c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x} + \frac{6a^2c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x} - \frac{4ac\cdots}{\cdots}$$

Mathematica [A]
time = 0.07, size = 132, normalized size = 0.78

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{1+n}\left(24a^4-24a^3b(1+n)x+12a^2b^2\left(2+3n+n^2\right)x^2-4ab^3\left(6+11n+6n^2+n^3\right)x^3+b^4\left(24+50n+35n^2+10n^3+n^4\right)x^4\right)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^2*
(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50*n
 + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n
)*x^3)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1*(c*x^2)^(3/2)*(a + b*x)^n,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.13, size = 199, normalized size = 1.18

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44x^3an\,b^3+24b^4x^4+36a^2n\,x^2b^2-24a\,b^3\cdots\right.}{x^3b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}$
risch	$\frac{c\sqrt{c\,x^2}\,\left(b^5n^4x^5+a\,b^4n^4x^4+10b^5n^3x^5+6a\,b^4n^3x^4+35b^5n^2x^5-4a^2b^3n^3x^3+11a\,b^4n^2x^4+50b^5n\,x^5-12a^2b^3n^2x^3+6x^4an\,b^4+24b^5x^5\cdots\right.}{x(4+n)(5+n)(3+n)(2+n)(1+n)b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(3/2)/x^3/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A]
time = 0.26, size = 157, normalized size = 0.93

$$\frac{\left((n^4+10\,n^3+35\,n^2+50\,n+24)b^5c^{\frac{3}{2}}x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4c^{\frac{3}{2}}x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3c^{\frac{3}{2}}x^3+12\,(n^2+n)a^3b^2c^{\frac{3}{2}}x^2-24\,a^4bc^{\frac{3}{2}}nx+24\,a^5c^{\frac{3}{2}}\right)(bx+a)^n}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(3/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(3/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(3/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(3/2)*x^2 - 24*a^4*b*c^(3/2)*n*x + 24*a^5*c^(3/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [A]
time = 0.31, size = 233, normalized size = 1.38

$$-\frac{(24\,a^4bcnx-24\,a^5c-(b^5cn^4+10\,b^5cn^3+35\,b^5cn^2+50\,b^5cn+24\,b^5c)x^5-(ab^4cn^4+6\,ab^4cn^3+11\,ab^4cn^2+6\,ab^4cn)x^4+4\,(a^2b^3cn^3+3\,a^2b^3cn^2+2\,a^2b^3cn)x^3-12\,(a^3b^2cn^2+a^3b^2cn)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c*n*x - 24*a^5*c - (b^5*c*n^4 + 10*b^5*c*n^3 + 35*b^5*c*n^2 + 50
*b^5*c*n + 24*b^5*c)*x^5 - (a*b^4*c*n^4 + 6*a*b^4*c*n^3 + 11*a*b^4*c*n^2 +
6*a*b^4*c*n)*x^4 + 4*(a^2*b^3*c*n^3 + 3*a^2*b^3*c*n^2 + 2*a^2*b^3*c*n)*x^3
- 12*(a^3*b^2*c*n^2 + a^3*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^5*n^5 +
 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(cx^2\right)^{\frac{3}{2}}(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)*(a + b*x)**n, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(159) = 318.
time = 0.00, size = 464, normalized size = 2.75

$$\sqrt{c}\,c\left(-\frac{24a^5e^{n\ln a}\mathrm{sign}\left(x\right)}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{24a^5\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+24b^5x^5\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+50b^5nx^5\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+35b^5n^2x^5\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+10b^5n^3x^5\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+b^5n^4x^5\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+6ab^4nx^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+11ab^4n^2x^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+6ab^4n^3x^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+ab^4n^4x^4\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-8a^2b^3nx^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-12a^2b^3n^2x^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-4a^2b^3n^3x^3\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+12a^3b^2nx^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+12a^3b^2n^2x^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}-24a^4bnx\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] -(24*a^5*a^n*sgn(x)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*
b^5*n + 120*b^5) - ((b*x + a)^n*b^5*n^4*x^5*sgn(x) + (b*x + a)^n*a*b^4*n^4*
x^4*sgn(x) + 10*(b*x + a)^n*b^5*n^3*x^5*sgn(x) + 6*(b*x + a)^n*a*b^4*n^3*x^
4*sgn(x) + 35*(b*x + a)^n*b^5*n^2*x^5*sgn(x) - 4*(b*x + a)^n*a^2*b^3*n^3*x^
3*sgn(x) + 11*(b*x + a)^n*a*b^4*n^2*x^4*sgn(x) + 50*(b*x + a)^n*b^5*n*x^5*s
gn(x) - 12*(b*x + a)^n*a^2*b^3*n^2*x^3*sgn(x) + 6*(b*x + a)^n*a*b^4*n*x^4*s
gn(x) + 24*(b*x + a)^n*b^5*x^5*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n^2*x^2*sgn(
x) - 8*(b*x + a)^n*a^2*b^3*n*x^3*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n*x^2*sgn(
x) - 24*(b*x + a)^n*a^4*b*n*x*sgn(x) + 24*(b*x + a)^n*a^5*sgn(x))/(b^5*n^5
+ 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5))*c^(3/2)
```

Mupad [B]
time = 0.41, size = 307, normalized size = 1.82

$$\frac{(a+b\,x)^n\,\left(\frac{24\,a^5\,c\,\sqrt{c\,x^2}}{b^5\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}+\frac{c\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}-\frac{24\,a^4\,c\,n\,x\,\sqrt{c\,x^2}}{b^4\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}+\frac{a\,c\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}+\frac{12\,a^3\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}-\frac{4\,a^2\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((24*a^5*c*(c*x^2)^(1/2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120)) + (c*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24
))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) - (24*a^4*c*n*x*(c*x^2)^
(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c*n*x^4*(
c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^
4 + n^5 + 120)) + (12*a^3*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c*n*x^3*(c*x^2)^(1/2)*(3*n + n
^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))))/x
```

3.931 $\int (cx^2)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=135

$$-\frac{a^3 c \sqrt{cx^2} (a+bx)^{1+n}}{b^4 (1+n) x}+\frac{3 a^2 c \sqrt{cx^2} (a+bx)^{2+n}}{b^4 (2+n) x}-\frac{3 a c \sqrt{cx^2} (a+bx)^{3+n}}{b^4 (3+n) x}+\frac{c \sqrt{cx^2} (a+bx)^{4+n}}{b^4 (4+n) x}$$

```
[Out] -a^3*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^4/(1+n)/x+3*a^2*c*(b*x+a)^(2+n)*(c*x^2
)^(1/2)/b^4/(2+n)/x-3*a*c*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^4/(3+n)/x+c*(b*x+a)
^(4+n)*(c*x^2)^(1/2)/b^4/(4+n)/x
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^3 c \sqrt{cx^2} (a+bx)^{n+1}}{b^4 (n+1) x}+\frac{3 a^2 c \sqrt{cx^2} (a+bx)^{n+2}}{b^4 (n+2) x}-\frac{3 a c \sqrt{cx^2} (a+bx)^{n+3}}{b^4 (n+3) x}+\frac{c \sqrt{cx^2} (a+bx)^{n+4}}{b^4 (n+4) x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c*Sqrt[c*
x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c*Sqrt[c*x^2]*(a + b*x)^(3 +
 n))/(b^4*(3 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(cx^2\right)^{3/2} (a+bx)^n\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3} + \frac{3a^2(a+bx)^{1+n}}{b^3} - \frac{3a(a+bx)^{2+n}}{b^3} + \frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x} + \frac{3a^2c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x} - \frac{3ac\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x} + \frac{c\sqrt{c}}{b}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.73

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 +
 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(
3 + n)*(4 + n)*x^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(3/2)*(a + b*x)^n,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 136, normalized size = 1.01

method	result
gosper	$-\frac{(bx+a)^{1+n}\left(c\,x^2\right)^{\frac{3}{2}}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{x^3b^4\left(n^4+10n^3+35n^2+50n+24\right)}$
risch	$-\frac{c\sqrt{c\,x^2}\,\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{x(3+n)(4+n)(2+n)(1+n)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(3/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^3/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.27, size = 116, normalized size = 0.86

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4c^{\frac{3}{2}}x^4+(n^3+3\,n^2+2\,n)ab^3c^{\frac{3}{2}}x^3-3\,(n^2+n)a^2b^2c^{\frac{3}{2}}x^2+6\,a^3bc^{\frac{3}{2}}nx-6\,a^4c^{\frac{3}{2}}\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(3/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(3/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(3/2)*x^2 + 6*a^3*b*c^(3/2)*n*x - 6*a^4*c^(3
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.30, size = 164, normalized size = 1.21

$$\frac{(6\,a^3bcnx-6\,a^4c+(b^4cn^3+6\,b^4cn^2+11\,b^4cn+6\,b^4c)x^4+(ab^3cn^3+3\,ab^3cn^2+2\,ab^3cn)x^3-3\,(a^2b^2cn^2+a^2b^2cn)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c*n*x - 6*a^4*c + (b^4*c*n^3 + 6*b^4*c*n^2 + 11*b^4*c*n + 6*b^4*c)
*x^4 + (a*b^3*c*n^3 + 3*a*b^3*c*n^2 + 2*a*b^3*c*n)*x^3 - 3*(a^2*b^2*c*n^2 +
 a^2*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*
n^2 + 50*b^4*n + 24*b^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(127) = 254.
time = 0.00, size = 328, normalized size = 2.43

$$\sqrt{c}\,c\left(\frac{6a^4e^{n\ln a}\mathrm{sign}\,(x)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{-6a^4\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+6b^4x^4\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+11b^4nx^4\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+6b^4n^2x^4\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+b^4n^3x^4\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+2ab^3nx^3\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+3ab^3n^2x^3\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+ab^3n^3x^3\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}-3a^2b^2nx^2\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}-3a^2b^2n^2x^2\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}+6a^3bnx\mathrm{sign}\,(x)\,e^{n\ln(bx+a)}}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*c^(3/2)
```

Mupad [B]
time = 0.32, size = 219, normalized size = 1.62

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2
 + 10*n^3 + n^4 + 24) - (6*a^4*c*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^
3 + n^4 + 24)) + (6*a^3*c*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (3*a^2*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (a*c*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24))))/x
```

3.932 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=99

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

```
[Out] a^2*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^3/(1+n)/x-2*a*c*(b*x+a)^(2+n)*(c*x^2)^(
1/2)/b^3/(2+n)/x+c*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^3/(3+n)/x
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c*Sqrt[c*x^2]*
(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3
*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.71

$$\frac{c^2x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
[Out] (c^2*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^1,x]')
[Out] Timed out
```

Maple [A]
time = 0.14, size = 83, normalized size = 0.84

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{x^3b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{c\sqrt{c\,x^2}\,\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{x(2+n)(3+n)(1+n)b^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(3/2)/x^3/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 80, normalized size = 0.81

$$\frac{\left((n^2+3\,n+2)b^3c^{\frac{3}{2}}x^3+(n^2+n)ab^2c^{\frac{3}{2}}x^2-2\,a^2bc^{\frac{3}{2}}nx+2\,a^3c^{\frac{3}{2}}\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(3/2)*x^3 + (n^2 + n)*a*b^2*c^(3/2)*x^2 - 2*a^2*b*c^
(3/2)*n*x + 2*a^3*c^(3/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.30, size = 113, normalized size = 1.14

$$-\frac{(2\,a^2bcnx-2\,a^3c-(b^3cn^2+3\,b^3cn+2\,b^3c)x^3-(ab^2cn^2+ab^2cn)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c*n*x - 2*a^3*c - (b^3*c*n^2 + 3*b^3*c*n + 2*b^3*c)*x^3 - (a*b^2*
c*n^2 + a*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*
b^3*n + 6*b^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{3}{2}}\,(a+bx)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.26, size = 146, normalized size = 1.47

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,c\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,c\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6
) + (2*a^3*c*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*c*n*x*(
c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c*n*x^2*(c*x^2)^(1/2)*(n
+ 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

3.933 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2} dx$

Optimal. Leaf size=65

$$-\frac{ac\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

```
[Out] -a*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^2/(1+n)/x+c*(b*x+a)^(2+n)*(c*x^2)^(1/2)/
b^2/(2+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c*Sqrt[c*x^2]*(a
+ b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}$$
$$= -\frac{ac\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.71

$$\frac{c^2x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
[Out] (c^2*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.11, size = 46, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(cx^2)^{\frac{3}{2}}(-bnx-bx+a)}{x^3b^2(n^2+3n+2)}$	46
risch	$-\frac{c\sqrt{cx^2}\,(-b^2nx^2-abnx-x^2b^2+a^2)(bx+a)^n}{xb^2(2+n)(1+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(3/2)*(-b*n*x-b*x+a)/x^3/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.78

$$\frac{\left(b^2c^{\frac{3}{2}}(n+1)x^2+abc^{\frac{3}{2}}nx-a^2c^{\frac{3}{2}}\right)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^(3/2)*(n + 1)*x^2 + a*b*c^(3/2)*n*x - a^2*c^(3/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.30, size = 68, normalized size = 1.05

$$\frac{(abcnx-a^2c+(b^2cn+b^2c)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2n^2+3\,b^2n+2\,b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (a*b*c*n*x - a^2*c + (b^2*c*n + b^2*c)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n
^2 + 3*b^2*n + 2*b^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n(cx^2)^{\frac{3}{2}}}{2x} & \text{for}\ b=0 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a+bx)}\,dx & \text{for}\ n=-1 \\ -\frac{a^2(cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3}+\frac{abnx(cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3}+\frac{b^2nx^2(cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3}+\frac{b^2x^2(cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Piecewise((a**n*(c*x**2)**(3/2)/(2*x), Eq(b, 0)), (Integral((c*x**2)**(3/2)
/(x**2*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(3/2)/(x**2*(a +
b*x)), x), Eq(n, -1)), (-a**2*(c*x**2)**(3/2)*(a + b*x)**n/(b**2*n**2*x**3
+ 3*b**2*n*x**3 + 2*b**2*x**3) + a*b*n*x*(c*x**2)**(3/2)*(a + b*x)**n/(b**2
```

```
*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + b**2*n*x**2*(c*x**2)**(3/2)*(a
+ b*x)**n/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + b**2*x**2*(c*x**
2)**(3/2)*(a + b*x)**n/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3), True
))
```

Giac [A]
time = 0.00, size = 130, normalized size = 2.00

$$\sqrt{c}\,c\left(\frac{a^2e^{n\ln a}\mathrm{sign}\left(x\right)}{b^2n^2+3b^2n+2b^2}+\frac{-a^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+b^2x^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+b^2nx^2\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}+abnx\mathrm{sign}\left(x\right)e^{n\ln(bx+a)}}{b^2n^2+3b^2n+2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x)
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*c^(3/2)
```

Mupad [B]
time = 0.23, size = 88, normalized size = 1.35

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c\,\sqrt{c\,x^2}}{b^2\,(n^2+3\,n+2)}+\frac{a\,c\,n\,x\,\sqrt{c\,x^2}}{b\,(n^2+3\,n+2)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c*(c*x^2
)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2)
))/x
```

3.934 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3} dx$

Optimal. Leaf size=31

$$\frac{c\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}$$

```
[Out] c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}\,(a+bx)^n}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.97

$$\frac{(cx^2)^{3/2}\,(a+bx)^{1+n}}{b(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n))/(b*(1 + n)*x^3)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 29, normalized size = 0.94

method	result	size
gosper	$\frac{(bx+a)^{1+n}(c\,x^{2})^{\frac{3}{2}}}{b(1+n)x^{3}}$	29
risch	$\frac{c\sqrt{c\,x^{2}}\,(bx+a)(bx+a)^{n}}{xb(1+n)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*(c*x^2)^(3/2)/x^3
```

Maxima [**A**]
time = 0.29, size = 28, normalized size = 0.90

$$\frac{\left(bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}\right)(bx+a)^{n}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] (b*c^(3/2)*x + a*c^(3/2))*(b*x + a)^n/(b*(n + 1))
```

Fricas [**A**]
time = 0.30, size = 33, normalized size = 1.06

$$\frac{(bcx+ac)\sqrt{cx^{2}}\,(bx+a)^{n}}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{(cx^2)^{\frac{3}{2}}}{ax^2} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n (cx^2)^{\frac{3}{2}}}{x^2} & \text{for } b = 0 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^3(a+bx)}\, dx & \text{for } n = -1 \\ \frac{a(cx^2)^{\frac{3}{2}}(a+bx)^n}{bnx^3+bx^3} + \frac{bx(cx^2)^{\frac{3}{2}}(a+bx)^n}{bnx^3+bx^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Piecewise(((c*x**2)**(3/2)/(a*x**2), Eq(b, 0) & Eq(n, -1)), (a**n*(c*x**2)*
*(3/2)/x**2, Eq(b, 0)), (Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x), Eq(
n, -1)), (a*(c*x**2)**(3/2)*(a + b*x)**n/(b*n*x**3 + b*x**3) + b*x*(c*x**2)
**(3/2)*(a + b*x)**n/(b*n*x**3 + b*x**3), True))
```

Giac [A]
time = 0.00, size = 41, normalized size = 1.32

$$\sqrt{c}\, c \left(-\frac{a^{n+1}\mathrm{sign}\,(x)}{bn+b} + \frac{(bx+a)^{n+1}\,\mathrm{sign}\,(x)}{b\,(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x)
```

```
[Out] -c^(3/2)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

Mupad [B]
time = 0.23, size = 45, normalized size = 1.45

$$\frac{\left(\frac{c\,x\,\sqrt{c\,x^2}}{n+1} + \frac{a\,c\,\sqrt{c\,x^2}}{b\,(n+1)}\right)\,(a+b\,x)^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] (((c*x*(c*x^2)^(1/2))/(n + 1) + (a*c*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b*x)^
n)/x
```

3.935 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}$$

```
[Out] -c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a/(1+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x}\,dx}{x}\\ &= -\frac{c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$-\frac{(cx^2)^{3/2}(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -(((c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x^3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2)^{\frac{3}{2}}(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^4, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}} (a+bx)^n}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**4,x)
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**4, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^n}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^4, x)
```

3.936 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^5} dx$

Optimal. Leaf size=48

$$\frac{bc\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

```
[Out] b*c*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^2/(1+n)
/x
```

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bc\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (b*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^5}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{bc\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$\frac{b\left(cx^2\right)^{3/2}(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (b*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x^3)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(bx+a\right)^{n}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^5, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}} (a+bx)^n}{x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**5, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^n}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^5, x)
```

3.937 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^6}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}$$

```
[Out] -b^2*c*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^3/(1
+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2c\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((b^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^6}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^3}\,dx}{x}\\ &= -\frac{b^2c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.98

$$-\frac{b^2 (cx^2)^{3/2} (a+bx)^{1+n} {}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((b^2*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x^3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2)^{\frac{3}{2}} (bx+a)^n}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^6, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}} (a+bx)^n}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**6,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**6, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^n}{x^6} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^6, x)
```

3.938 $\int (cx^2)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=217

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^6(1+n)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^6(2+n)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^6(3+n)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{4+n}}{b^6(4+n)x}-5a$$

```
[Out] -a^5*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^6/(1+n)/x+5*a^4*c^2*(b*x+a)^(2+n)*(c
*x^2)^(1/2)/b^6/(2+n)/x-10*a^3*c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^6/(3+n)/x+
10*a^2*c^2*(b*x+a)^(4+n)*(c*x^2)^(1/2)/b^6/(4+n)/x-5*a*c^2*(b*x+a)^(5+n)*(c
*x^2)^(1/2)/b^6/(5+n)/x+c^2*(b*x+a)^(6+n)*(c*x^2)^(1/2)/b^6/(6+n)/x
```

Rubi [A]
time = 0.05, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^6(n+5)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+6}}{b^6(n+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] -((a^5*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^6*(1 + n)*x)) + (5*a^4*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^6*(2 + n)*x) - (10*a^3*c^2*Sqrt[c*x^2]*(a +
b*x)^(3 + n))/(b^6*(3 + n)*x) + (10*a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/
(b^6*(4 + n)*x) - (5*a*c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^6*(5 + n)*x) +
 (c^2*Sqrt[c*x^2]*(a + b*x)^(6 + n))/(b^6*(6 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(cx^2\right)^{5/2}(a+bx)^n\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^5(a+bx)^n}{b^5}+\frac{5a^4(a+bx)^{1+n}}{b^5}-\frac{10a^3(a+bx)^{2+n}}{b^5}+\frac{10a^2(a+bx)^{3+n}}{b^5}-\frac{5a(a+bx)^{4+}}{b^5}\right.}{x}$$
$$= -\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^6(1+n)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^6(2+n)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^6(3+n)x}+$$

Mathematica [A]
time = 0.09, size = 172, normalized size = 0.79

$$\frac{c^3x(a+bx)^{1+n}\left(-120a^5+120a^4b(1+n)x-60a^3b^2\left(2+3n+n^2\right)x^2+20a^2b^3\left(6+11n+6n^2+n^3\right)x^3-5ab^4\left(24+50n+35n^2+10n^3+n^4\right)x^4+b^5\left(120+274n+225n^2+85n^3+15n^4+n^5\right)x^5\right)}{b^6(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-120*a^5 + 120*a^4*b*(1 + n)*x - 60*a^3*b^2*(2 +
3*n + n^2)*x^2 + 20*a^2*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 - 5*a*b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4 + b^5*(120 + 274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5)*x^5))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*Sqrt[
c*x^2])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^(5/2)*(a + b*x)^n,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 280, normalized size = 1.29

method	result
gosper	$-\frac{(bx+a)^{1+n}\left(c\,x^2\right)^{\frac{5}{2}}\left(-b^5n^5x^5-15b^5n^4x^5+5a\,b^4n^4x^4-85b^5n^3x^5+50a\,b^4n^3x^4-225b^5n^2x^5-20a^2b^3n^3x^3+175a\,b^4n^2x^4-274b^5n\,x^5-\right.}{x^5b^6\left(n^6+21n^5+175\right.}$
risch	$-\frac{c^2\sqrt{c\,x^2}\,\left(-b^6n^5x^6-a\,b^5n^5x^5-15b^6n^4x^6-10a\,b^5n^4x^5-85b^6n^3x^6+5a^2b^4n^4x^4-35a\,b^5n^3x^5-225b^6n^2x^6+30a^2b^4n^3x^4-50a\,b^5n^2\right.}{x(5+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b^5*n^5*x^5-15*b^5*n^4*x^5+5*a*b^4*n^4*x^4-8
5*b^5*n^3*x^5+50*a*b^4*n^3*x^4-225*b^5*n^2*x^5-20*a^2*b^3*n^3*x^3+175*a*b^4
*n^2*x^4-274*b^5*n*x^5-120*a^2*b^3*n^2*x^3+250*a*b^4*n*x^4-120*b^5*x^5+60*a
^3*b^2*n^2*x^2-220*a^2*b^3*n*x^3+120*a*b^4*x^4+180*a^3*b^2*n*x^2-120*a^2*b^
3*x^3-120*a^4*b*n*x+120*a^3*b^2*x^2-120*a^4*b*x+120*a^5)/x^5/b^6/(n^6+21*n^
5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

Maxima [A]
time = 0.29, size = 203, normalized size = 0.94

$$\frac{\left((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6c^{\frac{5}{2}}x^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5c^{\frac{5}{2}}x^5-5\,(n^4+6\,n^3+11\,n^2+6\,n)a^2b^4c^{\frac{5}{2}}x^4+20\,(n^3+3\,n^2+2\,n)a^3b^3c^{\frac{5}{2}}x^3-60\,(n^2+n)a^4b^2c^{\frac{5}{2}}x^2+120\,a^5bc^{\frac{5}{2}}nx-120\,a^6c^{\frac{5}{2}}\right)(bx+a)^n}{(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*c^(5/2)*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*c^(5/2)*x^5 - 5*(n^4 + 6*n^3 + 11*n^2
 + 6*n)*a^2*b^4*c^(5/2)*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*c^(5/2)*x^3 -
60*(n^2 + n)*a^4*b^2*c^(5/2)*x^2 + 120*a^5*b*c^(5/2)*n*x - 120*a^6*c^(5/2))
*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6)
```

Fricas [A]
time = 0.30, size = 352, normalized size = 1.62

$$\frac{(120\,a^5bc^2nx-120\,a^6c^2+(b^6c^2n^5+15\,b^6c^2n^4+85\,b^6c^2n^3+225\,b^6c^2n^2+274\,b^6c^2n+120\,b^6c^2)x^6+(ab^5c^2n^5+10\,ab^5c^2n^4+35\,ab^5c^2n^3+50\,ab^5c^2n^2+24\,ab^5c^2n)x^5-5\,(a^2b^4c^2n^4+6\,a^2b^4c^2n^3+11\,a^2b^4c^2n^2+6\,a^2b^4c^2n)x^4+20\,(a^3b^3c^2n^3+3\,a^3b^3c^2n^2+2\,a^3b^3c^2n)x^3-60\,(a^4b^2c^2n^2+a^4b^2c^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^6n^6+21\,b^6n^5+175\,b^6n^4+735\,b^6n^3+1624\,b^6n^2+1764\,b^6n+720\,b^6)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (120*a^5*b*c^2*n*x - 120*a^6*c^2 + (b^6*c^2*n^5 + 15*b^6*c^2*n^4 + 85*b^6*c
^2*n^3 + 225*b^6*c^2*n^2 + 274*b^6*c^2*n + 120*b^6*c^2)*x^6 + (a*b^5*c^2*n^
5 + 10*a*b^5*c^2*n^4 + 35*a*b^5*c^2*n^3 + 50*a*b^5*c^2*n^2 + 24*a*b^5*c^2*n
)*x^5 - 5*(a^2*b^4*c^2*n^4 + 6*a^2*b^4*c^2*n^3 + 11*a^2*b^4*c^2*n^2 + 6*a^2
*b^4*c^2*n)*x^4 + 20*(a^3*b^3*c^2*n^3 + 3*a^3*b^3*c^2*n^2 + 2*a^3*b^3*c^2*n
)*x^3 - 60*(a^4*b^2*c^2*n^2 + a^4*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/(
(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6
*n + 720*b^6)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{5}{2}} (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(205) = 410.
time = 0.01, size = 693, normalized size = 3.19

$$\sqrt{c}\left(\frac{120a^6c^2e^{n\ln a}\operatorname{sign}(x)}{b^6n^6+21b^6n^5+175b^6n^4+735b^6n^3+1624b^6n^2+1764b^6n+720b^6}+\frac{-120a^6c^2\operatorname{sign}(x)e^{n\ln(bx+a)}+120b^6c^2x^6\operatorname{sign}(x)e^{n\ln(bx+a)}+274b^6c^2nx^6\operatorname{sign}(x)e^{n\ln(bx+a)}+225b^6c^2n^2x^6\operatorname{sign}(x)e^{n\ln(bx+a)}+85b^6c^2n^3x^6\operatorname{sign}(x)\cdots}{\cdots}\right)$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] (120*a^6*a^n*c^2*sgn(x)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 +
 1624*b^6*n^2 + 1764*b^6*n + 720*b^6) + ((b*x + a)^n*b^6*c^2*n^5*x^6*sgn(x)
 + (b*x + a)^n*a*b^5*c^2*n^5*x^5*sgn(x) + 15*(b*x + a)^n*b^6*c^2*n^4*x^6*sg
n(x) + 10*(b*x + a)^n*a*b^5*c^2*n^4*x^5*sgn(x) + 85*(b*x + a)^n*b^6*c^2*n^3
*x^6*sgn(x) - 5*(b*x + a)^n*a^2*b^4*c^2*n^4*x^4*sgn(x) + 35*(b*x + a)^n*a*b
^5*c^2*n^3*x^5*sgn(x) + 225*(b*x + a)^n*b^6*c^2*n^2*x^6*sgn(x) - 30*(b*x +
a)^n*a^2*b^4*c^2*n^3*x^4*sgn(x) + 50*(b*x + a)^n*a*b^5*c^2*n^2*x^5*sgn(x) +
 274*(b*x + a)^n*b^6*c^2*n*x^6*sgn(x) + 20*(b*x + a)^n*a^3*b^3*c^2*n^3*x^3*
sgn(x) - 55*(b*x + a)^n*a^2*b^4*c^2*n^2*x^4*sgn(x) + 24*(b*x + a)^n*a*b^5*c
^2*n*x^5*sgn(x) + 120*(b*x + a)^n*b^6*c^2*x^6*sgn(x) + 60*(b*x + a)^n*a^3*b
^3*c^2*n^2*x^3*sgn(x) - 30*(b*x + a)^n*a^2*b^4*c^2*n*x^4*sgn(x) - 60*(b*x +
 a)^n*a^4*b^2*c^2*n^2*x^2*sgn(x) + 40*(b*x + a)^n*a^3*b^3*c^2*n*x^3*sgn(x)
- 60*(b*x + a)^n*a^4*b^2*c^2*n*x^2*sgn(x) + 120*(b*x + a)^n*a^5*b*c^2*n*x*s
gn(x) - 120*(b*x + a)^n*a^6*c^2*sgn(x))/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6))*sqrt(c)
```

Mupad [B]
time = 0.50, size = 424, normalized size = 1.95

$$\frac{(a+bx)^n\left(\frac{c^2x^6\sqrt{cx^2}\,(n^5+15n^4+85n^3+225n^2+274n+120)}{n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720}-\frac{120a^6c^2\sqrt{cx^2}}{b^6\,(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{120a^5c^2nx\sqrt{cx^2}}{b^5\,(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}-\frac{5a^2c^2nx^4\sqrt{cx^2}\,(n^3+6n^2+11n+6)}{b^2\,(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}-\frac{60a^4c^2nx^2\sqrt{cx^2}\,(n+1)}{b^4\,(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{ac^2nx^5\sqrt{cx^2}\,(n^4+10n^3+35n^2+50n+24)}{b\,(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}+\frac{20a^3c^2nx^3\sqrt{cx^2}\,(n^2+3n+2)}{b^3\,(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^6*(c*x^2)^(1/2)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n
^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (
120*a^6*c^2*(c*x^2)^(1/2))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21
*n^5 + n^6 + 720)) + (120*a^5*c^2*n*x*(c*x^2)^(1/2))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (5*a^2*c^2*n*x^4*(c*x^2)^(1/
2)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
21*n^5 + n^6 + 720)) - (60*a^4*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^4*(1764*
```

n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*c^2*n*x^5*(c*x
^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (20*a^3*c^2*n*x^3*(c*x^2)^(1/2)*(3*n
 + n^2 + 2))/(b^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 7
20))))/x

3.939 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x} \, dx$

Optimal. Leaf size=179

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{4+n}}{b^5(4+n)x}+\frac{c^2\sqrt{cx^2}}{b^5(5}$$

```
[Out] a^4*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^5/(1+n)/x-4*a^3*c^2*(b*x+a)^(2+n)*(c*
x^2)^(1/2)/b^5/(2+n)/x+6*a^2*c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^5/(3+n)/x-4*
a*c^2*(b*x+a)^(4+n)*(c*x^2)^(1/2)/b^5/(4+n)/x+c^2*(b*x+a)^(5+n)*(c*x^2)^(1/
2)/b^5/(5+n)/x
```

Rubi [A]

time = 0.04, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c^2*Sqrt[c
*x^2]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c^2*Sqrt[c*x^2]*(a + b*x)
^(3 + n))/(b^5*(3 + n)*x) - (4*a*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4
 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4(a+bx)^n}{b^4} - \frac{4a^3(a+bx)^{1+n}}{b^4} + \frac{6a^2(a+bx)^{2+n}}{b^4} - \frac{4a(a+bx)^{3+n}}{b^4} + \frac{(a+bx)^{4+n}}{b^4}\right) d}{x}$$
$$= \frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x} - \frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x} + \frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x} - 4$$

Mathematica [A]
time = 0.01, size = 133, normalized size = 0.74

$$\frac{c\,(cx^2)^{3/2}\,(a+bx)^{1+n}\,(24a^4 - 24a^3b(1+n)x + 12a^2b^2\,(2+3n+n^2)\,x^2 - 4ab^3\,(6+11n+6n^2+n^3)\,x^3 + b^4\,(24+50n+35n^2+10n^3+n^4)\,x^4)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^
2*(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*x^3)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^1,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 199, normalized size = 1.11

method	result
gosper	$\frac{(bx+a)^{1+n}\,(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44x^3an\,b^3+24b^4x^4+36a^2n\,x^2b^2-24a\,b}{x^5b^5(n^5+15n^4+85n^3+225n^2+274n+120)}$
risch	$\frac{c^2\sqrt{c\,x^2}\,\,(b^5n^4x^5+a\,b^4n^4x^4+10b^5n^3x^5+6a\,b^4n^3x^4+35b^5n^2x^5-4a^2b^3n^3x^3+11a\,b^4n^2x^4+50b^5n\,x^5-12a^2b^3n^2x^3+6x^4an\,b^4+24b^5}{x(4+n)(5+n)(3+n)(2+n)(1+n)b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(5/2)/x^5/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A]
time = 0.27, size = 157, normalized size = 0.88

$$\frac{\left((n^4+10\,n^3+35\,n^2+50\,n+24)b^5c^{\frac{5}{2}}x^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4c^{\frac{5}{2}}x^4-4\,(n^3+3\,n^2+2\,n)a^2b^3c^{\frac{5}{2}}x^3+12\,(n^2+n)a^3b^2c^{\frac{5}{2}}x^2-24\,a^4bc^{\frac{5}{2}}nx+24\,a^5c^{\frac{5}{2}}\right)(bx+a)^n}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(5/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(5/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(5/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(5/2)*x^2 - 24*a^4*b*c^(5/2)*n*x + 24*a^5*c^(5/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [A]
time = 0.31, size = 265, normalized size = 1.48

$$-\frac{(24\,a^4bc^2nx-24\,a^5c^2-(b^5c^2n^4+10\,b^5c^2n^3+35\,b^5c^2n^2+50\,b^5c^2n+24\,b^5c^2)x^5-(ab^4c^2n^4+6\,ab^4c^2n^3+11\,ab^4c^2n^2+6\,ab^4c^2n)x^4+4\,(a^2b^3c^2n^3+3\,a^2b^3c^2n^2+2\,a^2b^3c^2n)x^3-12\,(a^3b^2c^2n^2+a^3b^2c^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c^2*n*x - 24*a^5*c^2 - (b^5*c^2*n^4 + 10*b^5*c^2*n^3 + 35*b^5*c^
2*n^2 + 50*b^5*c^2*n + 24*b^5*c^2)*x^5 - (a*b^4*c^2*n^4 + 6*a*b^4*c^2*n^3 +
 11*a*b^4*c^2*n^2 + 6*a*b^4*c^2*n)*x^4 + 4*(a^2*b^3*c^2*n^3 + 3*a^2*b^3*c^2
*n^2 + 2*a^2*b^3*c^2*n)*x^3 - 12*(a^3*b^2*c^2*n^2 + a^3*b^2*c^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 27
4*b^5*n + 120*b^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}\left(a+bx\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.38, size = 319, normalized size = 1.78

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{24\,a^5\,c^2\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{24\,a^4\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(
274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (24*a^5*c^2*(c*x^2)^(1/2))
/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (24*a^4*c^2*n*x*(c
*x^2)^(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c^2
*n*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3
 + 15*n^4 + n^5 + 120)) + (12*a^3*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c^2*n*x^3*(c*x^2)^(1
/2)*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
)/x
```

3.940 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2} \, dx$

Optimal. Leaf size=143

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}$$

```
[Out] -a^3*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^4/(1+n)/x+3*a^2*c^2*(b*x+a)^(2+n)*(c
*x^2)^(1/2)/b^4/(2+n)/x-3*a*c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^4/(3+n)/x+c^2
*(b*x+a)^(4+n)*(c*x^2)^(1/2)/b^4/(4+n)/x
```

Rubi [A]
time = 0.03, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c^2*Sqrt[c*x^2]*(a + b*x
)^(3 + n))/(b^4*(3 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+$$

Mathematica [A]
time = 0.01, size = 99, normalized size = 0.69

$$\frac{c\,(cx^2)^{3/2}(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2
 + 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)
*(3 + n)*(4 + n)*x^3)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]')
[Out] Timed out
```

Maple [A]
time = 0.14, size = 136, normalized size = 0.95

method	result
gosper	$-\frac{(bx+a)^{1+n}(c\,x^2)^{\frac{5}{2}}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{x^5b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{c^2\sqrt{c\,x^2}\,\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+}{x(3+n)(4+n)(2+n)(1+n)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^5/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.27, size = 116, normalized size = 0.81

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4c^{\frac{5}{2}}x^4+(n^3+3\,n^2+2\,n)ab^3c^{\frac{5}{2}}x^3-3\,(n^2+n)a^2b^2c^{\frac{5}{2}}x^2+6\,a^3bc^{\frac{5}{2}}nx-6\,a^4c^{\frac{5}{2}}\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(5/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(5/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(5/2)*x^2 + 6*a^3*b*c^(5/2)*n*x - 6*a^4*c^(5
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.30, size = 186, normalized size = 1.30

$$\frac{(6\,a^3bc^2nx-6\,a^4c^2+(b^4c^2n^3+6\,b^4c^2n^2+11\,b^4c^2n+6\,b^4c^2)x^4+(ab^3c^2n^3+3\,ab^3c^2n^2+2\,ab^3c^2n)x^3-3\,(a^2b^2c^2n^2+a^2b^2c^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c^2*n*x - 6*a^4*c^2 + (b^4*c^2*n^3 + 6*b^4*c^2*n^2 + 11*b^4*c^2*n
+ 6*b^4*c^2)*x^4 + (a*b^3*c^2*n^3 + 3*a*b^3*c^2*n^2 + 2*a*b^3*c^2*n)*x^3 -
3*(a^2*b^2*c^2*n^2 + a^2*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4
+ 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**2, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.32, size = 229, normalized size = 1.60

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c^2\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n
^2 + 10*n^3 + n^4 + 24) - (6*a^4*c^2*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (6*a^3*c^2*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24)) + (a*c^2*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n +
35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/x
```

3.941 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=105

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

```
[Out] a^2*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^3/(1+n)/x-2*a*c^2*(b*x+a)^(2+n)*(c*x^
2)^(1/2)/b^3/(2+n)/x+c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^3/(3+n)/x
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c^2*Sqrt[c*x
^2]*(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(3 + n)
)/(b^3*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2ac^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x} + \frac{c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

Mathematica [**A**]
time = 0.03, size = 70, normalized size = 0.67

$$\frac{c^3x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.12, size = 83, normalized size = 0.79

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{x^5b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{x(2+n)(3+n)(1+n)b^3}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(5/2)/x^5/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.26, size = 80, normalized size = 0.76

$$\frac{\left((n^2+3\,n+2)b^3c^{\frac{5}{2}}x^3+(n^2+n)ab^2c^{\frac{5}{2}}x^2-2\,a^2bc^{\frac{5}{2}}nx+2\,a^3c^{\frac{5}{2}}\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(5/2)*x^3 + (n^2 + n)*a*b^2*c^(5/2)*x^2 - 2*a^2*b*c^
(5/2)*n*x + 2*a^3*c^(5/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.30, size = 127, normalized size = 1.21

$$-\frac{(2\,a^2bc^2nx-2\,a^3c^2-(b^3c^2n^2+3\,b^3c^2n+2\,b^3c^2)x^3-(ab^2c^2n^2+ab^2c^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c^2*n*x - 2*a^3*c^2 - (b^3*c^2*n^2 + 3*b^3*c^2*n + 2*b^3*c^2)*x^3
 - (a*b^2*c^2*n^2 + a*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6
*b^3*n^2 + 11*b^3*n + 6*b^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**3, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.27, size = 154, normalized size = 1.47

$$\frac{(a+b\,x)^n\,\left(\frac{2\,a^3\,c^2\,\sqrt{c\,x^2}}{b^3\,(n^3+6\,n^2+11\,n+6)}+\frac{c^2\,x^3\,\sqrt{c\,x^2}\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,(n^3+6\,n^2+11\,n+6)}+\frac{a\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,(n^3+6\,n^2+11\,n+6)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*c^2*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (c
^2*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*c^2
*n*x*(c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c^2*n*x^2*(c*x^2)^(
1/2)*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

3.942 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=69

$$-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

```
[Out] -a*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^2/(1+n)/x+c^2*(b*x+a)^(2+n)*(c*x^2)^(1
/2)/b^2/(2+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^4} dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^n dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b} + \frac{(a+bx)^{1+n}}{b}\right) dx}{x}$$
$$= -\frac{ac^2\sqrt{cx^2}(a+bx)^{1+n}}{b^2(1+n)x} + \frac{c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^2(2+n)x}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.67

$$\frac{c^3x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.16, size = 46, normalized size = 0.67

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(c\,x^2)^{\frac{5}{2}}(-bnx-bx+a)}{x^5b^2(n^2+3n+2)}$	46
risch	$-\frac{c^2\sqrt{c\,x^2}\,(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{x\,b^2(2+n)(1+n)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b*n*x-b*x+a)/x^5/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.74

$$\frac{\left(b^2c^{\frac{5}{2}}(n+1)x^2+abc^{\frac{5}{2}}nx-a^2c^{\frac{5}{2}}\right)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*c^(5/2)*(n + 1)*x^2 + a*b*c^(5/2)*n*x - a^2*c^(5/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.30, size = 76, normalized size = 1.10

$$\frac{(abc^2nx-a^2c^2+(b^2c^2n+b^2c^2)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2n^2+3\,b^2n+2\,b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
```

```
[Out] (a*b*c^2*n*x - a^2*c^2 + (b^2*c^2*n + b^2*c^2)*x^2)*sqrt(c*x^2)*(b*x + a)^n
/((b^2*n^2 + 3*b^2*n + 2*b^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n\left(cx^2\right)^{\frac{5}{2}}}{2x^3} & \text{for}\: b=0 \\ \int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4(a+bx)^2}\,dx & \text{for}\: n=-2 \\ \int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4(a+bx)}\,dx & \text{for}\: n=-1 \\ -\frac{a^2\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5}+\frac{abnx\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5}+\frac{b^2nx^2\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5}+\frac{b^2x^2\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Piecewise((a**n*(c*x**2)**(5/2)/(2*x**3), Eq(b, 0)), (Integral((c*x**2)**(5
/2)/(x**4*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(5/2)/(x**4*(a
 + b*x)), x), Eq(n, -1)), (-a**2*(c*x**2)**(5/2)*(a + b*x)**n/(b**2*n**2*x*
*5 + 3*b**2*n*x**5 + 2*b**2*x**5) + a*b*n*x*(c*x**2)**(5/2)*(a + b*x)**n/(b
```

```
**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + b**2*n*x**2*(c*x**2)**(5/2)*
(a + b*x)**n/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + b**2*x**2*(c*
x**2)**(5/2)*(a + b*x)**n/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5), T
rue))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x)
[Out] Could not integrate
```

Mupad [B]
time = 0.24, size = 94, normalized size = 1.36

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c^2\,\sqrt{c\,x^2}}{b^2\,(n^2+3\,n+2)}+\frac{a\,c^2\,n\,x\,\sqrt{c\,x^2}}{b\,(n^2+3\,n+2)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x)
[Out] ((a + b*x)^n*((c^2*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c^2*(c
*x^2)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c^2*n*x*(c*x^2)^(1/2))/(b*(3*n + n^
2 + 2))))/x
```

3.943 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx$

Optimal. Leaf size=33

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}$$

[Out] c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x

Rubi [A]

time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 31, normalized size = 0.94

$$\frac{c^3x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.12, size = 29, normalized size = 0.88

method	result	size
gosper	$\frac{(bx+a)^{1+n} (c\,x^{2})^{\frac{5}{2}}}{b(1+n)x^{5}}$	29
risch	$\frac{c^{2} \sqrt{c\,x^{2}}\, (bx+a)(bx+a)^{n}}{xb(1+n)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*(c*x^2)^(5/2)/x^5
```

Maxima [**A**]
time = 0.27, size = 28, normalized size = 0.85

$$\frac{\left(bc^{\frac{5}{2}}x + ac^{\frac{5}{2}}\right)(bx+a)^{n}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
```

```
[Out] (b*c^(5/2)*x + a*c^(5/2))*(b*x + a)^n/(b*(n + 1))
```

Fricas [**A**]
time = 0.30, size = 37, normalized size = 1.12

$$\frac{(bc^{2}x + ac^{2})\sqrt{cx^{2}}\,(bx+a)^{n}}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
```

```
[Out] (b*c^2*x + a*c^2)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{ax^{4}} & \text{for}\: b = 0 \wedge n = -1 \\ \frac{a^{n} \left(cx^{2}\right)^{\frac{5}{2}}}{x^{4}} & \text{for}\: b = 0 \\ \int \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{5} \left(a + bx\right)}\, dx & \text{for}\: n = -1 \\ \frac{a \left(cx^{2}\right)^{\frac{5}{2}} \left(a + bx\right)^{n}}{bnx^{5} + bx^{5}} + \frac{bx \left(cx^{2}\right)^{\frac{5}{2}} \left(a + bx\right)^{n}}{bnx^{5} + bx^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Piecewise((((c*x**2)**(5/2)/(a*x**4), Eq(b, 0) & Eq(n, -1)), (a**n*(c*x**2)*
*(5/2)/x**4, Eq(b, 0)), (Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x), Eq(
n, -1)), (a*(c*x**2)**(5/2)*(a + b*x)**n/(b*n*x**5 + b*x**5) + b*x*(c*x**2)
**(5/2)*(a + b*x)**n/(b*n*x**5 + b*x**5), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.23, size = 49, normalized size = 1.48

$$\frac{\left(\frac{c^2\,x\,\sqrt{c\,x^2}}{n+1}+\frac{a\,c^2\,\sqrt{c\,x^2}}{b\,\left(n+1\right)}\right)\,{\left(a+b\,x\right)}^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x)
```

```
[Out] (((c^2*x*(c*x^2)^(1/2))/(n + 1) + (a*c^2*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b
*x)^n)/x
```

3.944 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}$$

```
[Out] -c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a/(1+n)/
x
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^6}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x}\,dx}{x}\\ &= -\frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.94

$$-\frac{(cx^2)^{5/2} (a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -(((c*x^2)^(5/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x^5))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2)^{\frac{5}{2}} (bx+a)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
```

```
[Out] int((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^6, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**6, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^6, x)
```

3.945 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^7} dx$

Optimal. Leaf size=50

$$\frac{bc^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

```
[Out] b*c^2*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^2/(1+
n)/x
```

Rubi [A]

time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bc^2\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]
```

```
[Out] (b*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^7}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{bc^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.94

$$\frac{b\,(cx^2)^{5/2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]
[Out] (b*(c*x^2)^(5/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x^5)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]')
[Out] Timed out
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(bx+a\right)^{n}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
[Out] int((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="maxima")
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^7, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c^2/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^n}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**7,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**7, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^n}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^7, x)
```

3.946 $\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}$$

```
[Out] -a^3*x*(b*x+a)^(1+n)/b^4/(1+n)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+n)/b^4/(2+n
)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+n)/b^4/(3+n)/(c*x^2)^(1/2)+x*(b*x+a)^(4+n)
/b^4/(4+n)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*x)
^(2 + n))/(b^4*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*(3 + n
)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 96, normalized size = 0.78

$$\frac{x(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.14, size = 134, normalized size = 1.09

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x\left(-b^3n^3x^3-6b^3n^2x^3+3ab^2n^2x^2-11b^3nx^3+9ab^2nx^2-6b^3x^3-6a^2bnx+6ab^2x^2-6a^2bx+6a^3\right)}{\sqrt{cx^2}\,b^4(n^4+10n^3+35n^2+50n+24)}$	134
risch	$-\frac{x\left(-b^4n^3x^4-ab^3n^3x^3-6b^4n^2x^4-3ab^3n^2x^3-11b^4nx^4+3a^2b^2n^2x^2-2x^3anb^3-6b^4x^4+3a^2nx^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{\sqrt{cx^2}\,(3+n)(4+n)(2+n)(1+n)b^4}$	154

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9
*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(1/
2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.26, size = 104, normalized size = 0.85

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*sqrt(c))
```

Fricas [A]
time = 0.31, size = 158, normalized size = 1.28

$$\frac{\left(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^4cn^4+10\,b^4cn^3+35\,b^4cn^2+50\,b^4cn+24\,b^4c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c*n^4 + 10*b^4*c*n^3 + 35*b^4*c*n^2 + 50*b^4*c*n
 + 24*b^4*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**5/(4*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**4/(sqrt(c*x*
*2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**
3), x), Eq(n, -3)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x), Eq(n, -
2)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1)), (-6*a**4*x*(a
 + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n*
*2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 6*a**3*b
```

```
*n*x**2*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) +
35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2))
- 3*a**2*b**2*n**2*x**3*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3
*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**
4*sqrt(c*x**2)) - 3*a**2*b**2*n*x**3*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) +
 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x
**2) + 24*b**4*sqrt(c*x**2)) + a*b**3*n**3*x**4*(a + b*x)**n/(b**4*n**4*sqr
t(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4
*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 3*a*b**3*n**2*x**4*(a + b*x)**n/(
b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x*
*2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 2*a*b**3*n*x**4*(a +
 b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2
*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + b**4*n**3*
x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*
b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 6
*b**4*n**2*x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*
x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c
*x**2)) + 11*b**4*n*x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**
3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b*
*4*sqrt(c*x**2)) + 6*b**4*x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b*
*4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) +
 24*b**4*sqrt(c*x**2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[3,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[3,1,0]} /{1,[0
,0,1] Error: Bad Argument Value
```

Mupad [**B**]
time = 0.37, size = 186, normalized size = 1.51

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,x}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
+ 24) - (6*a^4*x)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x^2)
/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 + 2))/(b*(
50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

3.947 $\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}$$

```
[Out] a^2*x*(b*x+a)^(1+n)/b^3/(1+n)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+n)/b^3/(2+n)/(
c*x^2)^(1/2)+x*(b*x+a)^(3+n)/b^3/(3+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2 +
 n))/(b^3*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*(3 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 0.74

$$\frac{x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 81, normalized size = 0.90

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)x}{\sqrt{c\,x^2}\;b^3\left(n^3+6n^2+11n+6\right)}$	81
risch	$\frac{x\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{\sqrt{c\,x^2}\;(2+n)(3+n)(1+n)b^3}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
/(c*x^2)^(1/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.27, size = 83, normalized size = 0.92

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c)
```

Fricas [A]
time = 0.30, size = 110, normalized size = 1.22

$$-\frac{\left(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^3cn^3+6\,b^3cn^2+11\,b^3cn+6\,b^3c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c*n^3 + 6*b^3*c*n^2 + 11*b^3*c*n +
6*b^3*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^4}{3\sqrt{cx^2}} & \text{for}\ b=0 \\ \int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^3}\,dx & \text{for}\ n=-3 \\ \int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for}\ n=-1 \\ \frac{2a^3x(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} - \frac{2a^2bnx^2(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} + \frac{ab^2n^2x^3(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} + \frac{ab^2nx^3(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} + \frac{b^3n^2x^4(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} + \frac{3b^3nx^4(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} + \frac{2b^3x^4(a+bx)^n}{b^3n^3\sqrt{cx^2}+6b^3n^2\sqrt{cx^2}+11b^3n\sqrt{cx^2}+6b^3\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**4/(3*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**3/(sqrt(c*x*
*2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)**
2), x), Eq(n, -2)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1))
, (2*a**3*x*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2)
 + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) - 2*a**2*b*n*x**2*(a + b*x
)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*
x**2) + 6*b**3*sqrt(c*x**2)) + a*b**2*n**2*x**3*(a + b*x)**n/(b**3*n**3*sqr
t(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt
```

```
(c*x**2)) + a*b**2*n*x**3*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**
2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) + b**3*n**2*
x**4*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b
**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) + 3*b**3*n*x**4*(a + b*x)**n/(b**
3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6
*b**3*sqrt(c*x**2)) + 2*b**3*x**4*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*
b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)), Tru
e))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[2,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[2,1,0]} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [**B**]
time = 0.29, size = 121, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,x}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (2*a^3*x)/(b
^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*(11*n + 6*n^2 + n^3 + 6))
 + (a*n*x^3*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

3.948 $\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=59

$$-\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}$$

```
[Out] -a*x*(b*x+a)^(1+n)/b^2/(1+n)/(c*x^2)^(1/2)+x*(b*x+a)^(2+n)/b^2/(2+n)/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n)
)/(b^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.01, size = 43, normalized size = 0.73

$$\frac{x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.14, size = 44, normalized size = 0.75

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x(-bnx-bx+a)}{\sqrt{c\,x^2}\;b^2(n^2+3n+2)}$	44
risch	$-\frac{x(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{\sqrt{c\,x^2}\;b^2(2+n)(1+n)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x*(-b*n*x-b*x+a)/(c*x^2)^(1/2)/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.76

$$\frac{(b^2(n+1)x^2 + abnx - a^2)(bx+a)^n}{(n^2+3n+2)b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*sqrt(c))
```

Fricas [A]
time = 0.30, size = 66, normalized size = 1.12

$$\frac{(abnx + (b^2n + b^2)x^2 - a^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2cn^2 + 3\,b^2cn + 2\,b^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c*n^2 + 3
*b^2*c*n + 2*b^2*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^3}{2\sqrt{cx^2}} & \text{for } b=0 \\ \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx & \text{for } n=-2 \\ \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } n=-1 \\ -\frac{a^2x(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{abnx^2(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{b^2nx^3(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{b^2x^3(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(1/2),x)
[Out] Piecewise((a**n*x**3/(2*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**2/(sqrt(c*x*
*2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**2/(sqrt(c*x**2)*(a + b*x)),
 x), Eq(n, -1)), (-a**2*x*(a + b*x)**n/(b**2*n**2*sqrt(c*x**2) + 3*b**2*n*s
qrt(c*x**2) + 2*b**2*sqrt(c*x**2)) + a*b*n*x**2*(a + b*x)**n/(b**2*n**2*sqr
t(c*x**2) + 3*b**2*n*sqrt(c*x**2) + 2*b**2*sqrt(c*x**2)) + b**2*n*x**3*(a +
 b*x)**n/(b**2*n**2*sqrt(c*x**2) + 3*b**2*n*sqrt(c*x**2) + 2*b**2*sqrt(c*x*
*2)) + b**2*x**3*(a + b*x)**n/(b**2*n**2*sqrt(c*x**2) + 3*b**2*n*sqrt(c*x**
2) + 2*b**2*sqrt(c*x**2)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[1,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[1,1,0} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [B]
time = 0.28, size = 71, normalized size = 1.20

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,x}{b^2\,(n^2+3\,n+2)}+\frac{a\,n\,x^2}{b\,(n^2+3\,n+2)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(3*n + n^2 + 2) - (a^2*x)/(b^2*(3*n + n^2 + 2))
 + (a*n*x^2)/(b*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

3.949 $\int \frac{x(a+bx)^n}{\sqrt{cx^2}} dx$

Optimal. Leaf size=28

$$\frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

```
[Out] x*(b*x+a)^(1+n)/b/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx)^n}{\sqrt{cx^2}} dx &= \frac{x \int (a+bx)^n dx}{\sqrt{cx^2}} \\ &= \frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$\frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x*(a + b*x)^n)/Sqrt[c*x^2],x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.13, size = 27, normalized size = 0.96

method	result	size
gosper	$\frac{x(bx+a)^{1+n}}{b(1+n)\sqrt{c\,x^{2}}}$	27
risch	$\frac{(bx+a)x(bx+a)^{n}}{b(1+n)\sqrt{c\,x^{2}}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*x+a)^(1+n)/b/(1+n)/(c*x^2)^(1/2)
```

Maxima [**A**]
time = 0.27, size = 31, normalized size = 1.11

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\,\right)(bx+a)^{n}}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c*(n + 1))
```

Fricas [**A**]
time = 0.30, size = 33, normalized size = 1.18

$$\frac{\sqrt{cx^{2}}\,(bx+a)(bx+a)^{n}}{(bcn+bc)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c*n + b*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{a\sqrt{cx^2}} & \text{for } b=0 \wedge n=-1 \\ \frac{a^n x^2}{\sqrt{cx^2}} & \text{for } b=0 \\ \int \frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } n=-1 \\ \frac{ax(a+bx)^n}{bn\sqrt{cx^2}+b\sqrt{cx^2}} + \frac{bx^2(a+bx)^n}{bn\sqrt{cx^2}+b\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((x**2/(a*sqrt(c*x**2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**2/sqrt(c*
x**2), Eq(b, 0)), (Integral(x/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1)), (a*
x*(a + b*x)**n/(b*n*sqrt(c*x**2) + b*sqrt(c*x**2)) + b*x**2*(a + b*x)**n/(b
*n*sqrt(c*x**2) + b*sqrt(c*x**2)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[0,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[0,1,0]} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [B]
time = 0.22, size = 36, normalized size = 1.29

$$\frac{\left(\frac{x^2}{n+1}+\frac{a\,x}{b\,(n+1)}\right)\,{(a+b\,x)}^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((x^2/(n + 1) + (a*x)/(b*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

3.950 $\int \frac{(a+bx)^n}{\sqrt{cx^2}} \, dx$

Optimal. Leaf size=45

$$-\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}$$

[Out] -x*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)/(c*x^2)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 67}

$$-\frac{x(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^n/Sqrt[c*x^2],x]

[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n)*Sqrt[c*x^2]))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{\sqrt{cx^2}} \, dx &= \frac{x \int \frac{(a+bx)^n}{x} \, dx}{\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.00

$$-\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/Sqrt[c*x^2],x]
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n)*Sqrt[c*x^2]))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/Sqrt[c*x^2],x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/(c*x^2)^(1/2),x)
[Out] int((b*x+a)^n/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/sqrt(c*x^2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/sqrt(c*x**2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,x\right)}^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c*x^2)^(1/2),x)
```

```
[Out] int((a + b*x)^n/(c*x^2)^(1/2), x)
```

3.951 $\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=45

$$\frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\sqrt{cx^2}}$$

```
[Out] b*x*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x*Sqrt[c*x^2]),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.07

$$\frac{bcx^3(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x*Sqrt[c*x^2]),x]
```

```
[Out] (b*c*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^2*(1 + n)*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(x*Sqrt[c*x^2]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

```
[Out] int((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/(x*sqrt(c*x**2)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(x*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^n/(x*(c*x^2)^(1/2)), x)
```

3.952 $\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\sqrt{cx^2}}$$

```
[Out] -b^2*x*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/(1+n)/(c*x^2)^(1
/2)
```

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.06

$$-\frac{b^2cx^3(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]
[Out] -((b^2*c*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)
/a])/(a^3*(1 + n)*(c*x^2)^(3/2)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^2\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
[Out] int((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x^2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/(x**2*sqrt(c*x**2)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x^2\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^n/(x^2*(c*x^2)^(1/2)), x)
```

3.953 $\int \frac{x^6(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}$$

```
[Out] -a^3*x*(b*x+a)^(1+n)/b^4/c/(1+n)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+n)/b^4/c/
(2+n)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+n)/b^4/c/(3+n)/(c*x^2)^(1/2)+x*(b*x+a)
^(4+n)/b^4/c/(4+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*
x)^(2 + n))/(b^4*c*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*c*
(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.73

$$\frac{x^3(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.14, size = 136, normalized size = 1.01

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^3\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{\left(c\,x^2\right)^{\frac{3}{2}}b^4\left(n^4+10n^3+35n^2+50n+24\right)}$	136
risch	$-\frac{x\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{c\sqrt{c\,x^2}\,(3+n)(4+n)(2+n)(1+n)b^4}$	157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x^3*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
3/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.26, size = 104, normalized size = 0.77

$$\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(3/2))
```

Fricas [A]
time = 0.30, size = 168, normalized size = 1.24

$$\frac{(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^4c^2n^4+10\,b^4c^2n^3+35\,b^4c^2n^2+50\,b^4c^2n+24\,b^4c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^2*n^4 + 10*b^4*c^2*n^3 + 35*b^4*c^2*n^2 + 50*b
^4*c^2*n + 24*b^4*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**7/(4*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**6/((c*x**
2)**(3/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**6/((c*x**2)**(3/2)*(a
 + b*x)**3), x), Eq(n, -3)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)**2),
```

```
x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x), Eq(n, -1))
, (-6*a**4*x**3*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x
**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24
*b**4*(c*x**2)**(3/2)) + 6*a**3*b*n*x**4*(a + b*x)**n/(b**4*n**4*(c*x**2)**
(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b*
*4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) - 3*a**2*b**2*n**2*x**5*(a
+ b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b*
*4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/
2)) - 3*a**2*b**2*n*x**5*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*
n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(
3/2) + 24*b**4*(c*x**2)**(3/2)) + a*b**3*n**3*x**6*(a + b*x)**n/(b**4*n**4*
(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/
2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + 3*a*b**3*n**2*x
**6*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2)
+ 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**
2)**(3/2)) + 2*a*b**3*n*x**6*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b
**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2
)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + b**4*n**3*x**7*(a + b*x)**n/(b**4*n**
4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(
3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + 6*b**4*n**2*x
**7*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2)
+ 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**
2)**(3/2)) + 11*b**4*n*x**7*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b*
*4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)
**(3/2) + 24*b**4*(c*x**2)**(3/2)) + 6*b**4*x**7*(a + b*x)**n/(b**4*n**4*(c
*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2)
 + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[3,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[3,1,0]} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [**B**]
time = 0.40, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,\left(n+1\right)}{b^2\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24)) - (6*a^4*x)/(b^4*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*
n*x^2)/(b^3*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 +
2))/(b*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*
c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

3.954 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}$$

```
[Out] a^2*x*(b*x+a)^(1+n)/b^3/c/(1+n)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+n)/b^3/c/(2+
n)/(c*x^2)^(1/2)+x*(b*x+a)^(3+n)/b^3/c/(3+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2
 + n))/(b^3*c*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c*(3 + n)*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 0.70

$$\frac{x^3(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(3+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] (x^3*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.13, size = 83, normalized size = 0.84

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)x^3}{(c\,x^2)^{\frac{3}{2}}b^3(n^3+6n^2+11n+6)}$	83
risch	$\frac{x\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{c\sqrt{c\,x^2}\,(2+n)(3+n)(1+n)b^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^3/(c*x^2)^(3/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.84

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^2)
```

Fricas [A]
time = 0.30, size = 118, normalized size = 1.19

$$-\frac{\left(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^3c^2n^3+6\,b^3c^2n^2+11\,b^3c^2n+6\,b^3c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^2*n^3 + 6*b^3*c^2*n^2 + 11*b^3*c^
2*n + 6*b^3*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^6}{3(cx^2)^{\frac{3}{2}}} & \text{for}\ b=0 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^3}\,dx & \text{for}\ n=-3 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for}\ n=-1 \\ \frac{2a^3x^3(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} - \frac{2a^2bnx^4(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{ab^2n^2x^5(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{ab^2nx^5(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{b^3n^2x^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{3b^3nx^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{2b^3x^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**6/(3*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**5/((c*x**
2)**(3/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**5/((c*x**2)**(3/2)*(a
 + b*x)**2), x), Eq(n, -2)), (Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
, Eq(n, -1)), (2*a**3*x**3*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3
*n**2*(c*x**2)**(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2))
 - 2*a**2*b*n*x**4*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c
*x**2)**(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + a*b**
2*n**2*x**5*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)*
```

```
*(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + a*b**2*n*x**
5*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 1
1*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + b**3*n**2*x**6*(a + b*
x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 11*b**3*n*
(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + 3*b**3*n*x**6*(a + b*x)**n/(b**
3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 11*b**3*n*(c*x**2)**
(3/2) + 6*b**3*(c*x**2)**(3/2)) + 2*b**3*x**6*(a + b*x)**n/(b**3*n**3*(c*x*
*2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b*
*3*(c*x**2)**(3/2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[2,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[2,1,0]} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [**B**]
time = 0.31, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{c\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,\left(n+1\right)}{b\,c\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3*x
)/(b^3*c*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c*(11*n + 6*n^2 + n
^3 + 6)) + (a*n*x^3*(n + 1))/(b*c*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

3.955 $\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}} \, dx$

Optimal. Leaf size=65

$$-\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}$$

```
[Out] -a*x*(b*x+a)^(1+n)/b^2/c/(1+n)/(c*x^2)^(1/2)+x*(b*x+a)^(2+n)/b^2/c/(2+n)/(c
*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 +
n))/(b^2*c*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.69

$$\frac{x^3(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] (x^3*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*(c*x^2)^(3/
2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.13, size = 46, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^3(-bnx-bx+a)}{(cx^2)^{\frac{3}{2}}b^2(n^2+3n+2)}$	46
risch	$-\frac{x(-b^2nx^2-abnx-x^2b^2+a^2)(bx+a)^n}{c\sqrt{cx^2}\,b^2(2+n)(1+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x^3*(-b*n*x-b*x+a)/(c*x^2)^(3/2)/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.26, size = 45, normalized size = 0.69

$$\frac{(b^2(n+1)x^2 + abnx - a^2)(bx+a)^n}{(n^2+3\,n+2)b^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(3/2))
```

Fricas [A]
time = 0.30, size = 72, normalized size = 1.11

$$\frac{(abnx + (b^2n+b^2)x^2 - a^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2c^2n^2 + 3\,b^2c^2n + 2\,b^2c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^2*n^2 +
 3*b^2*c^2*n + 2*b^2*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^5}{2(cx^2)^{\frac{3}{2}}} & \text{for } b=0 \\ \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx & \text{for } n=-2 \\ \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } n=-1 \\ -\frac{a^2x^3(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{abnx^4(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{b^2nx^5(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{b^2x^5(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**5/(2*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**4/((c*x**
2)**(3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**4/((c*x**2)**(3/2)*(a
 + b*x)), x), Eq(n, -1)), (-a**2*x**3*(a + b*x)**n/(b**2*n**2*(c*x**2)**(3/
2) + 3*b**2*n*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)) + a*b*n*x**4*(a + b
*x)**n/(b**2*n**2*(c*x**2)**(3/2) + 3*b**2*n*(c*x**2)**(3/2) + 2*b**2*(c*x*
*2)**(3/2)) + b**2*n*x**5*(a + b*x)**n/(b**2*n**2*(c*x**2)**(3/2) + 3*b**2*
n*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)) + b**2*x**5*(a + b*x)**n/(b**2*
n**2*(c*x**2)**(3/2) + 3*b**2*n*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)),
True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[1,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[1,1,0]} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [B]
time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{c\,(n^2+3\,n+2)}-\frac{a^2\,x}{b^2\,c\,(n^2+3\,n+2)}+\frac{a\,n\,x^2}{b\,c\,(n^2+3\,n+2)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c*(3*n + n^2
 + 2)) + (a*n*x^2)/(b*c*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

3.956 $\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}$$

[Out] x*(b*x+a)^(1+n)/b/c/(1+n)/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]

[Out] (x*(a + b*x)^(1 + n))/(b*c*(1 + n)*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.97

$$\frac{x^3(a+bx)^{1+n}}{b(1+n)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n))/(b*(1 + n)*(c*x^2)^(3/2))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.14, size = 29, normalized size = 0.94

method	result	size
gosper	$\frac{(bx+a)^{1+n} x^{3}}{b(1+n)(c\,x^{2})^{\frac{3}{2}}}$	29
risch	$\frac{x(bx+a)(bx+a)^{n}}{c\sqrt{c\,x^{2}}\,b(1+n)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*x^3/(c*x^2)^(3/2)
```

Maxima [**A**]
time = 0.26, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\,\right)(bx+a)^{n}}{bc^{2}(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^2*(n + 1))
```

Fricas [**A**]
time = 0.30, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^{2}}\,(bx+a)(bx+a)^{n}}{(bc^{2}n+bc^{2})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^2*n + b*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4}{a(cx^2)^{\frac{3}{2}}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^4}{(cx^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ \int \frac{x^3}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ \frac{ax^3(a+bx)^n}{bn(cx^2)^{\frac{3}{2}}+b(cx^2)^{\frac{3}{2}}} + \frac{bx^4(a+bx)^n}{bn(cx^2)^{\frac{3}{2}}+b(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(3/2),x)
[Out] Piecewise((x**4/(a*(c*x**2)**(3/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**4/(c*x
**2)**(3/2), Eq(b, 0)), (Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x), Eq(
n, -1)), (a*x**3*(a + b*x)**n/(b*n*(c*x**2)**(3/2) + b*(c*x**2)**(3/2)) + b
*x**4*(a + b*x)**n/(b*n*(c*x**2)**(3/2) + b*(c*x**2)**(3/2)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[0,1,0] / 1,[0,0,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x)
[Out] Unable to divide, perhaps due to rounding error{1,[0,1,0]} / {1,[0
,0,1]} Error: Bad Argument Value
```

Mupad [B]
time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c\,(n+1)}+\frac{a\,x}{b\,c\,(n+1)}\right)\,{(a+b\,x)}^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(3/2),x)
[Out] ((x^2/(c*(n + 1)) + (a*x)/(b*c*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

3.957 $\int \frac{x^2(a+bx)^n}{(cx^2)^{3/2}} \, dx$

Optimal. Leaf size=48

$$-\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac(1+n)\sqrt{cx^2}}$$

```
[Out] -x*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/c/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{x(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^n}{(cx^2)^{3/2}} \, dx &= \frac{x\int \frac{(a+bx)^n}{x} \, dx}{c\sqrt{cx^2}} \\
&= -\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac(1+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$-\frac{x^3(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*(c*x^2)^(3/2)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(bx+a\right)^n}{\left(c\,x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*x)^n)/(c*x^2)^(3/2), x)
```

3.958 $\int \frac{x(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=48

$$\frac{bx(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c(1+n)\sqrt{cx^2}}$$

[Out] b*x*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/c/(1+n)/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 67}

$$\frac{bx(a+bx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x)^n)/(c*x^2)^(3/2),x]

[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*c*(1 + n)*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 + d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{(cx^2)^{3/2}} dx &= \frac{x \int \frac{(a+bx)^n}{x^2} dx}{c\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 47, normalized size = 0.98

$$\frac{bx^3(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] (b*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^2*(1 + n)*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^1*(a + b*x)^n)/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x\left(bx+a\right)^{n}}{\left(c\,x^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(3/2),x)
[Out] int(x*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(a+b\,x)^n}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*x)^n)/(c*x^2)^(3/2), x)
```

3.959 $\int \frac{(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c(1+n)\sqrt{cx^2}}$$

```
[Out] -b^2*x*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/c/(1+n)/(c*x^2)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 67}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c*x^2)^(3/2),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 0.98

$$-\frac{b^2x^3(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c*x^2)^(3/2),x]
```

```
[Out] -((b^2*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^3*(1 + n)*(c*x^2)^(3/2)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c*x^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{(c\,x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((a + b*x)^n/(c*x^2)^(3/2), x)
```

3.960 $\int \frac{(a+bx)^n}{x(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c(1+n)\sqrt{cx^2}}$$

```
[Out] b^3*x*(b*x+a)^(1+n)*hypergeom([4, 1+n],[2+n],1+b*x/a)/a^4/c/(1+n)/(c*x^2)^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x\,(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= \frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 1.00

$$\frac{b^3cx^5(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (b^3*c*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^4*(1 + n)*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(x^1*(c*x^2)^(3/2)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x\left(c\,x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

```
[Out] int((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/((c*x^2)^(3/2)*x), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^5), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{x\,(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**n/(x*(c*x**2)**(3/2)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x\,(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(x*(c*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x)^n/(x*(c*x^2)^(3/2)), x)
```

3.961 $\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}} \, dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}$$

```
[Out] -a^3*x*(b*x+a)^(1+n)/b^4/c^2/(1+n)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+n)/b^4/
c^2/(2+n)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+n)/b^4/c^2/(3+n)/(c*x^2)^(1/2)+x*(
b*x+a)^(4+n)/b^4/c^2/(4+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c^2*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a +
b*x)^(2 + n))/(b^4*c^2*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^
4*c^2*(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c^2*(4 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 99, normalized size = 0.73

$$\frac{x(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4c^2(1+n)(2+n)(3+n)(4+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*c^2*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 136, normalized size = 1.01

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^5\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{(c\,x^2)^{\frac{5}{2}}b^4(n^4+10n^3+35n^2+50n+24)}$	136
risch	$-\frac{x\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{c^2\sqrt{c\,x^2}\,(3+n)(4+n)(2+n)(1+n)b^4}$	157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x^5*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
5/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.27, size = 104, normalized size = 0.77

$$\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(5/2))
```

Fricas [A]
time = 0.30, size = 168, normalized size = 1.24

$$\frac{(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^4c^3n^4+10\,b^4c^3n^3+35\,b^4c^3n^2+50\,b^4c^3n+24\,b^4c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^3*n^4 + 10*b^4*c^3*n^3 + 35*b^4*c^3*n^2 + 50*b
^4*c^3*n + 24*b^4*c^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**9/(4*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**8/((c*x**
2)**(5/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**8/((c*x**2)**(5/2)*(a
 + b*x)**3), x), Eq(n, -3)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)**2),
```

```
x), Eq(n, -2)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)), x), Eq(n, -1))
, (-6*a**4*x**5*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x
**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24
*b**4*(c*x**2)**(5/2)) + 6*a**3*b*n*x**6*(a + b*x)**n/(b**4*n**4*(c*x**2)**
(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b*
*4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) - 3*a**2*b**2*n**2*x**7*(a
+ b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b*
*4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/
2)) - 3*a**2*b**2*n*x**7*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*
n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(
5/2) + 24*b**4*(c*x**2)**(5/2)) + a*b**3*n**3*x**8*(a + b*x)**n/(b**4*n**4*
(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/
2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + 3*a*b**3*n**2*x
**8*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2)
+ 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**
2)**(5/2)) + 2*a*b**3*n*x**8*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b
**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2
)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + b**4*n**3*x**9*(a + b*x)**n/(b**4*n**
4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(
5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + 6*b**4*n**2*x
**9*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2)
+ 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**
2)**(5/2)) + 11*b**4*n*x**9*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b*
*4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)
**(5/2) + 24*b**4*(c*x**2)**(5/2)) + 6*b**4*x**9*(a + b*x)**n/(b**4*n**4*(c
*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2)
 + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[3,1,0,0] / 1,[0,0,2,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[3,1,0,0]} / {1,
[0,0,2,1]} Error: Bad Argument Value
```

Mupad [**B**]
time = 0.41, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,\left(n+1\right)}{b^2\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c^2*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (6*a^4*x)/(b^4*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*
a^3*n*x^2)/(b^3*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n +
n^2 + 2))/(b*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1
))/(b^2*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

3.962 $\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}$$

```
[Out] a^2*x*(b*x+a)^(1+n)/b^3/c^2/(1+n)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+n)/b^3/c^2
/(2+n)/(c*x^2)^(1/2)+x*(b*x+a)^(3+n)/b^3/c^2/(3+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c^2*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^
(2 + n))/(b^3*c^2*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c^2*(3
+ n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.71

$$\frac{x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3c^2(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*c^2*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 83, normalized size = 0.84

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)x^5}{\left(c\,x^2\right)^{\frac{5}{2}}b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{x\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{c^2\sqrt{c\,x^2}\,(2+n)(3+n)(1+n)b^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^5/(c*x^2)^(5/2)/b^3/(n^3+6*n^2+11*n+6)

Maxima [A]
time = 0.26, size = 83, normalized size = 0.84

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")

[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^3)

Fricas [A]
time = 0.30, size = 118, normalized size = 1.19

$$-\frac{(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^3c^3n^3+6\,b^3c^3n^2+11\,b^3c^3n+6\,b^3c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")

[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^3*n^3 + 6*b^3*c^3*n^2 + 11*b^3*c^
3*n + 6*b^3*c^3)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^8}{3(cx^2)^{\frac{5}{2}}} & \text{for } b=0 \\ \int \frac{x^7}{(cx^2)^{\frac{5}{2}}(a+bx)^3}\,dx & \text{for } n=-3 \\ \int \frac{x^7}{(cx^2)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for } n=-2 \\ \int \frac{x^7}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx & \text{for } n=-1 \\ \frac{2a^3x^5(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} - \frac{2a^2bnx^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{ab^2n^2x^7(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{ab^2nx^7(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{b^3n^2x^8(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{3b^3nx^8(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{2b^3x^8(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b*x+a)**n/(c*x**2)**(5/2),x)

[Out] Piecewise((a**n*x**8/(3*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**7/((c*x**
2)**(5/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**7/((c*x**2)**(5/2)*(a
 + b*x)**2), x), Eq(n, -2)), (Integral(x**7/((c*x**2)**(5/2)*(a + b*x)), x)
, Eq(n, -1)), (2*a**3*x**5*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3
*n**2*(c*x**2)**(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2))
 - 2*a**2*b*n*x**6*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c
*x**2)**(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + a*b**
2*n**2*x**7*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)*

```
*(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + a*b**2*n*x**
7*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 1
1*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + b**3*n**2*x**8*(a + b*
x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 11*b**3*n*
(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + 3*b**3*n*x**8*(a + b*x)**n/(b**
3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 11*b**3*n*(c*x**2)**
(5/2) + 6*b**3*(c*x**2)**(5/2)) + 2*b**3*x**8*(a + b*x)**n/(b**3*n**3*(c*x*
*2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b*
*3*(c*x**2)**(5/2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[2,1,0,0] / 1,[0,0,2,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[2,1,0,0]} / {1,
[0,0,2,1]} Error: Bad Argument Value
```

Mupad [**B**]
time = 0.35, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{c^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,\left(n+1\right)}{b\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c^2*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3
*x)/(b^3*c^2*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c^2*(11*n + 6*n
^2 + n^3 + 6)) + (a*n*x^3*(n + 1))/(b*c^2*(11*n + 6*n^2 + n^3 + 6))))/(c*x^
2)^(1/2)
```

3.963 $\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}} dx$

Optimal. Leaf size=65

$$-\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}$$

```
[Out] -a*x*(b*x+a)^(1+n)/b^2/c^2/(1+n)/(c*x^2)^(1/2)+x*(b*x+a)^(2+n)/b^2/c^2/(2+n
)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2
+ n))/(b^2*c^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.71

$$\frac{x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2c^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*c^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.14, size = 46, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^5(-bnx-bx+a)}{(cx^2)^{\frac{5}{2}}b^2(n^2+3n+2)}$	46
risch	$-\frac{x(-b^2nx^2-abnx-x^2b^2+a^2)(bx+a)^n}{c^2\sqrt{cx^2}\,b^2(2+n)(1+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] -(b*x+a)^(1+n)*x^5*(-b*n*x-b*x+a)/(c*x^2)^(5/2)/b^2/(n^2+3*n+2)

Maxima [A]
time = 0.27, size = 45, normalized size = 0.69

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n}{(n^2+3\,n+2)b^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")

[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(5/2))

Fricas [A]
time = 0.30, size = 72, normalized size = 1.11

$$\frac{(abnx+(b^2n+b^2)x^2-a^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2c^3n^2+3\,b^2c^3n+2\,b^2c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")

[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^3*n^2 + 3*b^2*c^3*n + 2*b^2*c^3)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^7}{2(cx^2)^{\frac{5}{2}}} & \text{for}\ b=0 \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx & \text{for}\ n=-1 \\ -\frac{a^2x^5(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}}+\frac{abnx^6(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}}+\frac{b^2nx^7(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}}+\frac{b^2x^7(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(5/2),x)

[Out] Piecewise((a**n*x**7/(2*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**6/((c*x**2)**(5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(5/2)*(a + b*x)), x), Eq(n, -1)), (-a**2*x**5*(a + b*x)**n/(b**2*n**2*(c*x**2)**(5/2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + a*b*n*x**6*(a + b*x)**n/(b**2*n**2*(c*x**2)**(5/2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + b**2*n*x**7*(a + b*x)**n/(b**2*n**2*(c*x**2)**(5/2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + b**2*x**7*(a + b*x)**n/(b**2*n**2*(c*x**2)**(5/2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)), True))

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[1,1,0,0] / 1,[0,0,2,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[1,1,0,0]} / {1,
[0,0,2,1]} Error: Bad Argument Value
```

Mupad [B]
time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{c^2\,(n^2+3\,n+2)}-\frac{a^2\,x}{b^2\,c^2\,(n^2+3\,n+2)}+\frac{a\,n\,x^2}{b\,c^2\,(n^2+3\,n+2)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c^2*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c^2*(3*n +
 n^2 + 2)) + (a*n*x^2)/(b*c^2*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

3.964 $\int \frac{x^5(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}$$

[Out] x*(b*x+a)^(1+n)/b/c^2/(1+n)/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]

[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**A**]
time = 0.13, size = 29, normalized size = 0.94

method	result	size
gosper	$\frac{(bx+a)^{1+n} x^5}{b(1+n)(c x^2)^{\frac{5}{2}}}$	29
risch	$\frac{x(bx+a)(bx+a)^n}{c^2\sqrt{c x^2}\, b(1+n)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*x^5/(c*x^2)^(5/2)
```

Maxima [**A**]
time = 0.27, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x + a\sqrt{c}\,\right)(bx+a)^n}{bc^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^3*(n + 1))
```

Fricas [**A**]
time = 0.30, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bc^3n+bc^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^3*n + b*c^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^{6}}{a(cx^{2})^{\frac{5}{2}}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^{n}x^{6}}{(cx^{2})^{\frac{5}{2}}} & \text{for } b = 0 \\ \int \frac{x^{5}}{(cx^{2})^{\frac{5}{2}}(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ax^{5}(a+bx)^{n}}{bn(cx^{2})^{\frac{5}{2}}+b(cx^{2})^{\frac{5}{2}}} + \frac{bx^{6}(a+bx)^{n}}{bn(cx^{2})^{\frac{5}{2}}+b(cx^{2})^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((x**6/(a*(c*x**2)**(5/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**6/(c*x
**2)**(5/2), Eq(b, 0)), (Integral(x**5/((c*x**2)**(5/2)*(a + b*x)), x), Eq(
n, -1)), (a*x**5*(a + b*x)**n/(b*n*(c*x**2)**(5/2) + b*(c*x**2)**(5/2)) + b
*x**6*(a + b*x)**n/(b*n*(c*x**2)**(5/2) + b*(c*x**2)**(5/2)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Unable to divide, perhaps due to rounding error1,[0,1,0,0] / 1,[0,0,2,1] Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Unable to divide, perhaps due to rounding error{1,[0,1,0,0]} / {1,
[0,0,2,1]} Error: Bad Argument Value
```

Mupad [B]
time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c^2\,(n+1)}+\frac{a\,x}{b\,c^2\,(n+1)}\right)\,{(a+b\,x)}^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((x^2/(c^2*(n + 1)) + (a*x)/(b*c^2*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

3.965 $\int \frac{x^4(a+bx)^n}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=48

$$-\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac^2(1+n)\sqrt{cx^2}}$$

```
[Out] -x*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/c^2/(1+n)/(c*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{x(a+bx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c^2*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(a+bx)^n}{(cx^2)^{5/2}} dx &= \frac{x\int \frac{(a+bx)^n}{x} dx}{c^2\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$-\frac{x^5(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*(c*x^2)^(5/2)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(bx+a\right)^n}{\left(c\,x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + bx)^n}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.966 $\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c^2(1+n)\sqrt{cx^2}}$$

```
[Out] b*x*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/c^2/(1+n)/(c*x^2)^(
1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$\frac{bx^5(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^2*(1 + n)*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(b x +a \right)^{n}}{\left(c \,x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + bx)^n}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\, (a + b\, x)^n}{(c\, x^2)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.967 $\int \frac{x^2(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c^2(1+n)\sqrt{cx^2}}$$

```
[Out] -b^2*x*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/c^2/(1+n)/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c^2*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.98

$$-\frac{b^2x^5(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((b^2*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^3*(1 + n)*(c*x^2)^(5/2)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(b x +a \right)^{n}}{\left(c \,x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx)^n}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\, (a + b\, x)^n}{(c\, x^2)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.968 $\int \frac{x(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c^2(1+n)\sqrt{cx^2}}$$

```
[Out] b^3*x*(b*x+a)^(1+n)*hypergeom([4, 1+n],[2+n],1+b*x/a)/a^4/c^2/(1+n)/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 67}

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 0.98

$$\frac{b^3x^5(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b^3*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^4*(1 + n)*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(x^1*(a + b*x)^n)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x\left(bx+a\right)^{n}}{\left(c\,x^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+bx)^n}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(a+b\,x)^n}{(c\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.969 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\, dx$

Optimal. Leaf size=65

$$\frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}$$

```
[Out] a*c^2*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x+b*c^2*(d*x)^(7+m)*(c*x^2)^(1/2)
/d^7/(7+m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{ac^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (b*c^2*(d*x)^(7 + m)*Sq
rt[c*x^2])/(d^7*(7 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)\,dx}{d^5x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a(dx)^{5+m}+\frac{b(dx)^{6+m}}{d}\right)dx}{d^5x}$$
$$= \frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.58

$$\frac{x(dx)^m \left(cx^2\right)^{5/2}(a(7+m)+b(6+m)x)}{(6+m)(7+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a*(7 + m) + b*(6 + m)*x))/((6 + m)*(7 + m))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.62

method	result	size
gosper	$\frac{x(bmx+am+6bx+7a)(dx)^m\left(cx^2\right)^{\frac{5}{2}}}{(7+m)(6+m)}$	40
risch	$\frac{c^2x^5\sqrt{cx^2}\,(bmx+am+6bx+7a)(dx)^m}{(7+m)(6+m)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x+a*m+6*b*x+7*a)*(d*x)^m*(c*x^2)^(5/2)/(7+m)/(6+m)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.60

$$\frac{bc^{\frac{5}{2}}d^m x^7 x^m}{m+7} + \frac{ac^{\frac{5}{2}}d^m x^6 x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a*c^(5/2)*d^m*x^6*x^m/(m + 6)
```

Fricas [A]
time = 0.31, size = 58, normalized size = 0.89

$$\frac{\left((bc^2m + 6\,bc^2)x^6 + (ac^2m + 7\,ac^2)x^5\right)\sqrt{cx^2}\,(dx)^m}{m^2 + 13\,m + 42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c^2*m + 6*b*c^2)*x^6 + (a*c^2*m + 7*a*c^2)*x^5)*sqrt(c*x^2)*(d*x)^m/(m^
2 + 13*m + 42)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\left(cx^2\right)^{\frac{5}{2}}}{x^7}\,dx + \int \frac{b\left(cx^2\right)^{\frac{5}{2}}}{x^6}\,dx}{d^7} & \text{for}\: m = -7 \\ \frac{\int \frac{a\left(cx^2\right)^{\frac{5}{2}}}{x^6}\,dx + \int \frac{b\left(cx^2\right)^{\frac{5}{2}}}{x^5}\,dx}{d^6} & \text{for}\: m = -6 \\ \frac{amx\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42} + \frac{7ax\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42} + \frac{bmx^2\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42} + \frac{6bx^2\left(cx^2\right)^{\frac{5}{2}}(dx)^m}{m^2+13m+42} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(5/2)/x**7, x) + Integral(b*(c*x**2)**(5/2
)/x**6, x))/d**7, Eq(m, -7)), ((Integral(a*(c*x**2)**(5/2)/x**6, x) + Integ
```

```
ral(b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a*m*x*(c*x**2)**(5/2)*(d
*x)**m/(m**2 + 13*m + 42) + 7*a*x*(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m + 4
2) + b*m*x**2*(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m + 42) + 6*b*x**2*(c*x**
2)**(5/2)*(d*x)**m/(m**2 + 13*m + 42), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [B]
time = 0.27, size = 44, normalized size = 0.68

$$\frac{c^2\,x^5\,{\left(d\,x\right)}^m\,\sqrt{c\,x^2}\,\left(7\,a+a\,m+6\,b\,x+b\,m\,x\right)}{m^2+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] (c^2*x^5*(d*x)^m*(c*x^2)^(1/2)*(7*a + a*m + 6*b*x + b*m*x))/(13*m + m^2 + 4
2)
```

3.970 $\int (dx)^m (cx^2)^{3/2} (a+bx)\, dx$

Optimal. Leaf size=61

$$\frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}$$

```
[Out] a*c*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x+b*c*(d*x)^(5+m)*(c*x^2)^(1/2)/d^5
/(5+m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{ac\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (b*c*(d*x)^(5 + m)*Sqrt[c
*x^2])/(d^5*(5 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)\,dx}{d^3x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(a(dx)^{3+m}+\frac{b(dx)^{4+m}}{d}\right)dx}{d^3x}$$
$$= \frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.62

$$\frac{x(dx)^m \left(cx^2\right)^{3/2} (a(5+m)+b(4+m)x)}{(4+m)(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a*(5 + m) + b*(4 + m)*x))/((4 + m)*(5 + m))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.66

method	result	size
gosper	$\frac{x(bmx+am+4bx+5a)(dx)^m\left(cx^2\right)^{\frac{3}{2}}}{(5+m)(4+m)}$	40
risch	$\frac{cx^3\sqrt{cx^2}\,(bmx+am+4bx+5a)(dx)^m}{(5+m)(4+m)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x+a*m+4*b*x+5*a)*(d*x)^m*(c*x^2)^(3/2)/(5+m)/(4+m)
```

Maxima [A]
time = 0.30, size = 39, normalized size = 0.64

$$\frac{bc^{\frac{3}{2}}d^m x^5 x^m}{m+5} + \frac{ac^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a*c^(3/2)*d^m*x^4*x^m/(m + 4)
```

Fricas [A]
time = 0.30, size = 50, normalized size = 0.82

$$\frac{((bcm+4\,bc)x^4+(acm+5\,ac)x^3)\sqrt{cx^2}\,(dx)^m}{m^2+9\,m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c*m + 4*b*c)*x^4 + (a*c*m + 5*a*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^2 + 9*m
+ 20)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^5}\,dx+\int \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx}{d^5} & \text{for}\ m=-5 \\ \frac{\int \frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx+\int \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^3}\,dx}{d^4} & \text{for}\ m=-4 \\ \frac{amx\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20}+\frac{5ax\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20}+\frac{bmx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20}+\frac{4bx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(3/2)/x**5, x) + Integral(b*(c*x**2)**(3/2
)/x**4, x))/d**5, Eq(m, -5)), ((Integral(a*(c*x**2)**(3/2)/x**4, x) + Integ
```

```
ral(b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a*m*x*(c*x**2)**(3/2)*(d
*x)**m/(m**2 + 9*m + 20) + 5*a*x*(c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20)
 + b*m*x**2*(c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20) + 4*b*x**2*(c*x**2)*
*(3/2)*(d*x)**m/(m**2 + 9*m + 20), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**B**]
time = 0.24, size = 42, normalized size = 0.69

$$\frac{c\,x^3\,{\left(d\,x\right)}^m\,\sqrt{c\,x^2}\,\left(5\,a+a\,m+4\,b\,x+b\,m\,x\right)}{m^2+9\,m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] (c*x^3*(d*x)^m*(c*x^2)^(1/2)*(5*a + a*m + 4*b*x + b*m*x))/(9*m + m^2 + 20)
```

3.971 $\int (dx)^m \sqrt{cx^2} (a + bx) \, dx$

Optimal. Leaf size=59

$$\frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x} + \frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}$$

```
[Out] a*(d*x)^(2+m)*(c*x^2)^(1/2)/d^2/(2+m)/x+b*(d*x)^(3+m)*(c*x^2)^(1/2)/d^3/(3+
m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{a\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x} + \frac{b\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (b*(d*x)^(3 + m)*Sqrt[c*x^2
])/(d^3*(3 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)\,dx}{dx}\\ &= \frac{\sqrt{cx^2}\,\int \left(a(dx)^{1+m}+\frac{b(dx)^{2+m}}{d}\right)\,dx}{dx}\\ &= \frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.64

$$\frac{x(dx)^m\sqrt{cx^2}\,(a(3+m)+b(2+m)x)}{(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a*(3 + m) + b*(2 + m)*x))/((2 + m)*(3 + m))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(1/2)*(a + b*x),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.68

method	result	size
gosper	$\frac{x(bmx+am+2bx+3a)(dx)^m\sqrt{c\,x^2}}{(3+m)(2+m)}$	40
risch	$\frac{x(bmx+am+2bx+3a)(dx)^m\sqrt{c\,x^2}}{(3+m)(2+m)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x+a*m+2*b*x+3*a)*(d*x)^m*(c*x^2)^(1/2)/(3+m)/(2+m)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.66

$$\frac{b\sqrt{c}\,d^m x^3 x^m}{m+3}+\frac{a\sqrt{c}\,d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a*sqrt(c)*d^m*x^2*x^m/(m + 2)
```

Fricas [A]
time = 0.30, size = 44, normalized size = 0.75

$$\frac{((bm+2\,b)x^2+(am+3\,a)x)\sqrt{cx^2}\,(dx)^m}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + 2*b)*x^2 + (a*m + 3*a)*x)*sqrt(c*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\sqrt{cx^2}}{x^3}\,dx+\int \frac{b\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for}\ m=-3 \\ \frac{\int \frac{a\sqrt{cx^2}}{x^2}\,dx+\int \frac{b\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for}\ m=-2 \\ \frac{amx\sqrt{cx^2}\,(dx)^m}{m^2+5m+6}+\frac{3ax\sqrt{cx^2}\,(dx)^m}{m^2+5m+6}+\frac{bmx^2\sqrt{cx^2}\,(dx)^m}{m^2+5m+6}+\frac{2bx^2\sqrt{cx^2}\,(dx)^m}{m^2+5m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*sqrt(c*x**2)/x**3, x) + Integral(b*sqrt(c*x**2)/x**2
, x))/d**3, Eq(m, -3)), ((Integral(a*sqrt(c*x**2)/x**2, x) + Integral(b*sqr
t(c*x**2)/x, x))/d**2, Eq(m, -2)), (a*m*x*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m
 + 6) + 3*a*x*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m + 6) + b*m*x**2*sqrt(c*x**2
```

```
)*(d*x)**m/(m**2 + 5*m + 6) + 2*b*x**2*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m +
6), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**B**]
time = 0.21, size = 39, normalized size = 0.66

$$\frac{x\,(d\,x)^m\,\sqrt{c\,x^2}\,\left(3\,a+a\,m+2\,b\,x+b\,m\,x\right)}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] (x*(d*x)^m*(c*x^2)^(1/2)*(3*a + a*m + 2*b*x + b*m*x))/(5*m + m^2 + 6)
```

3.972 $\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}$$

```
[Out] a*x*(d*x)^m/m/(c*x^2)^(1/2)+b*x*(d*x)^(1+m)/d/(1+m)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int (dx)^{-1+m}(a+bx)\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int \left(a(dx)^{-1+m}+\frac{b(dx)^m}{d}\right)\,dx}{\sqrt{cx^2}}$$
$$= \frac{ax(dx)^m}{m\sqrt{cx^2}} + \frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.02, size = 33, normalized size = 0.69

$$\frac{x(dx)^m(a+am+bmx)}{m(1+m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a + a*m + b*m*x))/(m*(1 + m)*Sqrt[c*x^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)/(c*x^2)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.01, size = 32, normalized size = 0.67

method	result	size
gosper	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32
risch	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x+a*m+a)*(d*x)^m/(1+m)/m/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 32, normalized size = 0.67

$$\frac{bd^m x x^m}{\sqrt{c}\,(m+1)} + \frac{ad^m x^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a*d^m*x^m/(sqrt(c)*m)
```

Fricas [A]
time = 0.30, size = 36, normalized size = 0.75

$$\frac{(bmx + am + a)\sqrt{cx^2}\,(dx)^m}{(cm^2 + cm)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*m*x + a*m + a)*sqrt(c*x^2)*(d*x)^m/((c*m^2 + c*m)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b}{\sqrt{cx^2}}\,dx + \int \frac{a}{x\sqrt{cx^2}}\,dx}{d} & \text{for } m = -1 \\ \int \frac{a+bx}{\sqrt{cx^2}}\,dx & \text{for } m = 0 \\ \frac{amx(dx)^m}{m^2\sqrt{cx^2} + m\sqrt{cx^2}} + \frac{ax(dx)^m}{m^2\sqrt{cx^2} + m\sqrt{cx^2}} + \frac{bmx^2(dx)^m}{m^2\sqrt{cx^2} + m\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b/sqrt(c*x**2), x) + Integral(a/(x*sqrt(c*x**2)), x))/
d, Eq(m, -1)), (Integral((a + b*x)/sqrt(c*x**2), x), Eq(m, 0)), (a*m*x*(d*x
)**m/(m**2*sqrt(c*x**2) + m*sqrt(c*x**2)) + a*x*(d*x)**m/(m**2*sqrt(c*x**2)
```

```
 + m*sqrt(c*x**2)) + b*m*x**2*(d*x)**m/(m**2*sqrt(c*x**2) + m*sqrt(c*x**2))
, True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [B]
time = 0.21, size = 30, normalized size = 0.62

$$\frac{\left(\frac{a\,x}{m}+\frac{b\,x^2}{m+1}\right)\,{\left(d\,x\right)}^m}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (((a*x)/m + (b*x^2)/(m + 1))*(d*x)^m)/(c*x^2)^(1/2)
```

3.973 $\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

```
[Out] -a*d^2*x*(d*x)^(-2+m)/c/(2-m)/(c*x^2)^(1/2)-b*d*x*(d*x)^(-1+m)/c/(1-m)/(c*x
^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{15, 16, 45\}$

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -((a*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (b*d*x*(d*x)^(-1 + m)
)/(c*(1 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int \left(a(dx)^{-3+m} + \frac{b(dx)^{-2+m}}{d}\right)\,dx}{c\sqrt{cx^2}}$$
$$= -\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}} - \frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.58

$$\frac{x(dx)^m(a(-1+m)+b(-2+m)x)}{(-2+m)(-1+m)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a*(-1 + m) + b*(-2 + m)*x))/((-2 + m)*(-1 + m)*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)/(c*x^2)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.62

method	result	size
gosper	$\frac{x(bmx+am-2bx-a)(dx)^m}{(-1+m)(-2+m)(cx^2)^{\frac{3}{2}}}$	40
risch	$\frac{(bmx+am-2bx-a)(dx)^m}{cx\sqrt{cx^2}\,(-1+m)(-2+m)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x+a*m-2*b*x-a)*(d*x)^m/(-1+m)/(-2+m)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.30, size = 39, normalized size = 0.60

$$\frac{bd^m x^m}{c^{\frac{3}{2}}(m-1)x} + \frac{ad^m x^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a*d^m*x^m/(c^(3/2)*(m - 2)*x^2)
```

Fricas [A]
time = 0.30, size = 53, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,(am + (bm - 2\,b)x - a)\,(dx)^m}{(c^2m^2 - 3\,c^2m + 2\,c^2)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 2*b)*x - a)*(d*x)^m/((c^2*m^2 - 3*c^2*m + 2*c^2)*
x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d\left(\int \frac{ax}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{bx^2}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for}\: m = 1 \\ d^2\left(\int \frac{ax^2}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{bx^3}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for}\: m = 2 \\ \frac{amx(dx)^m}{m^2(cx^2)^{\frac{3}{2}} - 3m(cx^2)^{\frac{3}{2}} + 2(cx^2)^{\frac{3}{2}}} - \frac{ax(dx)^m}{m^2(cx^2)^{\frac{3}{2}} - 3m(cx^2)^{\frac{3}{2}} + 2(cx^2)^{\frac{3}{2}}} + \frac{bmx^2(dx)^m}{m^2(cx^2)^{\frac{3}{2}} - 3m(cx^2)^{\frac{3}{2}} + 2(cx^2)^{\frac{3}{2}}} - \frac{2bx^2(dx)^m}{m^2(cx^2)^{\frac{3}{2}} - 3m(cx^2)^{\frac{3}{2}} + 2(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((d*(Integral(a*x/(c*x**2)**(3/2), x) + Integral(b*x**2/(c*x**2)**
(3/2), x)), Eq(m, 1)), (d**2*(Integral(a*x**2/(c*x**2)**(3/2), x) + Integra
l(b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a*m*x*(d*x)**m/(m**2*(c*x**2)**(
3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) - a*x*(d*x)**m/(m**2*(c*x**
```

```
2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) + b*m*x**2*(d*x)**m/(m
**2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) - 2*b*x**2*(
d*x)**m/(m**2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)), T
rue))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.26, size = 48, normalized size = 0.74

$$\frac{b\,{\left(d\,x\right)}^m}{c\,\sqrt{c\,x^2}\,\left(m-1\right)}+\frac{a\,{\left(d\,x\right)}^m}{c\,x\,\sqrt{c\,x^2}\,\left(m-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] (b*(d*x)^m)/(c*(c*x^2)^(1/2)*(m - 1)) + (a*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m -
 2))
```

3.974 $\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

```
[Out] -a*d^4*x*(d*x)^(-4+m)/c^2/(4-m)/(c*x^2)^(1/2)-b*d^3*x*(d*x)^(-3+m)/c^2/(3-m
)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -((a*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (b*d^3*x*(d*x)^(-3
+ m))/(c^2*(3 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int \left(a(dx)^{-5+m}+\frac{b(dx)^{-4+m}}{d}\right)\,dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}} - \frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.57

$$\frac{x(dx)^m(a(-3+m)+b(-4+m)x)}{(-4+m)(-3+m)\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a*(-3 + m) + b*(-4 + m)*x))/((-4 + m)*(-3 + m)*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.60

method	result	size
gosper	$\frac{x(bmx+am-4bx-3a)(dx)^m}{(-3+m)(-4+m)(c\,x^2)^{\frac{5}{2}}}$	40
risch	$\frac{(bmx+am-4bx-3a)(dx)^m}{c^2x^3\sqrt{c\,x^2}\,(-3+m)(-4+m)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x+a*m-4*b*x-3*a)*(d*x)^m/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 39, normalized size = 0.58

$$\frac{bd^m x^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{ad^m x^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Fricas [A]
time = 0.30, size = 53, normalized size = 0.79

$$\frac{\sqrt{cx^2}\,(am+(bm-4\,b)x-3\,a)\,(dx)^m}{(c^3m^2-7\,c^3m+12\,c^3)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 4*b)*x - 3*a)*(d*x)^m/((c^3*m^2 - 7*c^3*m + 12*c^
3)*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d^3\left(\int \frac{ax^3}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{bx^4}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for } m=3 \\ d^4\left(\int \frac{ax^4}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{bx^5}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for } m=4 \\ \frac{amx(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}} - \frac{3ax(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}} + \frac{bmx^2(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}} - \frac{4bx^2(dx)^m}{m^2(cx^2)^{\frac{5}{2}}-7m(cx^2)^{\frac{5}{2}}+12(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**3*(Integral(a*x**3/(c*x**2)**(5/2), x) + Integral(b*x**4/(c*x
**2)**(5/2), x)), Eq(m, 3)), (d**4*(Integral(a*x**4/(c*x**2)**(5/2), x) + I
ntegral(b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a*m*x*(d*x)**m/(m**2*(c*x*
*2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) - 3*a*x*(d*x)**m/(m*
*2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) + b*m*x**2*(
d*x)**m/(m**2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) -
 4*b*x**2*(d*x)**m/(m**2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2
)**(5/2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.28, size = 47, normalized size = 0.70

$$-\frac{(d\,x)^m\,\left(3\,a-a\,m+4\,b\,x-b\,m\,x\right)}{c^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2-7\,m+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -((d*x)^m*(3*a - a*m + 4*b*x - b*m*x))/(c^2*x^3*(c*x^2)^(1/2)*(m^2 - 7*m +
12))
```

3.975 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}+\frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

```
[Out] a^2*c^2*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x+2*a*b*c^2*(d*x)^(7+m)*(c*x^2)
^(1/2)/d^7/(7+m)/x+b^2*c^2*(d*x)^(8+m)*(c*x^2)^(1/2)/d^8/(8+m)/x
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2c^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}\,(dx)^{m+8}}{d^8(m+8)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (2*a*b*c^2*(d*x)^(7 +
 m)*Sqrt[c*x^2])/(d^7*(7 + m)*x) + (b^2*c^2*(d*x)^(8 + m)*Sqrt[c*x^2])/(d^8
*(8 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)^2\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)^2\,dx}{d^5x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2(dx)^{5+m}+\frac{2ab(dx)^{6+m}}{d}+\frac{b^2(dx)^{7+m}}{d^2}\right)dx}{d^5x}$$
$$= \frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}+\frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.47

$$x(dx)^m \left(cx^2\right)^{5/2}\left(\frac{a^2}{6+m}+\frac{2abx}{7+m}+\frac{b^2x^2}{8+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
[Out] x*(d*x)^m*(c*x^2)^(5/2)*(a^2/(6 + m) + (2*a*b*x)/(7 + m) + (b^2*x^2)/(8 + m
))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.10, size = 95, normalized size = 0.92

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+13m\,x^2b^2+a^2m^2+28abmx+42x^2b^2+15a^2m+96abx+56a^2\right)(dx)^m\left(c\,x^2\right)^{\frac{5}{2}}}{(8+m)(7+m)(6+m)}$	95
risch	$\frac{c^2x^5\sqrt{c\,x^2}\,\left(b^2m^2x^2+2ab\,m^2x+13m\,x^2b^2+a^2m^2+28abmx+42x^2b^2+15a^2m+96abx+56a^2\right)(dx)^m}{(8+m)(7+m)(6+m)}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+13*b^2*m*x^2+a^2*m^2+28*a*b*m*x+42*b^2*x^2+15*a^
2*m+96*a*b*x+56*a^2)*(d*x)^m*(c*x^2)^(5/2)/(8+m)/(7+m)/(6+m)
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.62

$$\frac{b^2c^{\frac{5}{2}}d^mx^8x^m}{m+8}+\frac{2\,abc^{\frac{5}{2}}d^mx^7x^m}{m+7}+\frac{a^2c^{\frac{5}{2}}d^mx^6x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(5/2)*d^m*x^8*x^m/(m + 8) + 2*a*b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a^2*c
^(5/2)*d^m*x^6*x^m/(m + 6)
```

Fricas [A]
time = 0.30, size = 123, normalized size = 1.19

$$\frac{((b^2c^2m^2+13\,b^2c^2m+42\,b^2c^2)x^7+2\,(abc^2m^2+14\,abc^2m+48\,abc^2)x^6+(a^2c^2m^2+15\,a^2c^2m+56\,a^2c^2)x^5)\sqrt{cx^2}\,\,(dx)^m}{m^3+21\,m^2+146\,m+336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*m^2 + 13*b^2*c^2*m + 42*b^2*c^2)*x^7 + 2*(a*b*c^2*m^2 + 14*a*b*c^
2*m + 48*a*b*c^2)*x^6 + (a^2*c^2*m^2 + 15*a^2*c^2*m + 56*a^2*c^2)*x^5)*sqrt
(c*x^2)*(d*x)^m/(m^3 + 21*m^2 + 146*m + 336)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases}\frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^8}\,dx+\int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^6}\,dx+\int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^7}\,dx}{d^8} & \text{for}\: m=-8\\ \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^7}\,dx+\int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^5}\,dx+\int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^6}\,dx}{d^7} & \text{for}\: m=-7\\ \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^6}\,dx+\int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^4}\,dx+\int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^5}\,dx}{d^6} & \text{for}\: m=-6\\ \frac{a^2m^2x(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{15a^2mx(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{56a^2x(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{2abm^2x^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{28abmx^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{96abx^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{b^2m^2x^3(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{13b^2mx^3(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{42b^2x^3(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(5/2)/x**8, x) + Integral(b**2*(c*x**2)
**(5/2)/x**6, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**7, x))/d**8, Eq(m, -8)
```

```
), ((Integral(a**2*(c*x**2)**(5/2)/x**7, x) + Integral(b**2*(c*x**2)**(5/2)
/x**5, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**6, x))/d**7, Eq(m, -7)), ((In
tegral(a**2*(c*x**2)**(5/2)/x**6, x) + Integral(b**2*(c*x**2)**(5/2)/x**4,
x) + Integral(2*a*b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a**2*m**2*
x*(c*x**2)**(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 15*a**2*m*x*(c*
x**2)**(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 56*a**2*x*(c*x**2)**
(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 2*a*b*m**2*x**2*(c*x**2)**(
5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 28*a*b*m*x**2*(c*x**2)**(5/2
)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 96*a*b*x**2*(c*x**2)**(5/2)*(d*
x)**m/(m**3 + 21*m**2 + 146*m + 336) + b**2*m**2*x**3*(c*x**2)**(5/2)*(d*x)
**m/(m**3 + 21*m**2 + 146*m + 336) + 13*b**2*m*x**3*(c*x**2)**(5/2)*(d*x)**
m/(m**3 + 21*m**2 + 146*m + 336) + 42*b**2*x**3*(c*x**2)**(5/2)*(d*x)**m/(m
**3 + 21*m**2 + 146*m + 336), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**B**]
time = 0.31, size = 127, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,c^2\,x^5\,\sqrt{c\,x^2}\,\left(m^2+15\,m+56\right)}{m^3+21\,m^2+146\,m+336}+\frac{b^2\,c^2\,x^7\,\sqrt{c\,x^2}\,\left(m^2+13\,m+42\right)}{m^3+21\,m^2+146\,m+336}+\frac{2\,a\,b\,c^2\,x^6\,\sqrt{c\,x^2}\,\left(m^2+14\,m+48\right)}{m^3+21\,m^2+146\,m+336}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c^2*x^5*(c*x^2)^(1/2)*(15*m + m^2 + 56))/(146*m + 21*m^2 + m^
3 + 336) + (b^2*c^2*x^7*(c*x^2)^(1/2)*(13*m + m^2 + 42))/(146*m + 21*m^2 +
m^3 + 336) + (2*a*b*c^2*x^6*(c*x^2)^(1/2)*(14*m + m^2 + 48))/(146*m + 21*m^
2 + m^3 + 336))
```

3.976 $\int (dx)^m (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=97

$$\frac{a^2c(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}+\frac{b^2c(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}$$

```
[Out] a^2*c*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x+2*a*b*c*(d*x)^(5+m)*(c*x^2)^(1/
2)/d^5/(5+m)/x+b^2*c*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2c\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (2*a*b*c*(d*x)^(5 + m)*
Sqrt[c*x^2])/(d^5*(5 + m)*x) + (b^2*c*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 +
m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx = \frac{\left(c\sqrt{cx^2}\right) \int x^3 (dx)^m (a+bx)^2 \, dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right) \int (dx)^{3+m} (a+bx)^2 \, dx}{d^3 x}$$
$$= \frac{\left(c\sqrt{cx^2}\right) \int \left(a^2 (dx)^{3+m} + \frac{2ab(dx)^{4+m}}{d} + \frac{b^2 (dx)^{5+m}}{d^2}\right) dx}{d^3 x}$$
$$= \frac{a^2 c (dx)^{4+m} \sqrt{cx^2}}{d^4 (4+m) x} + \frac{2abc(dx)^{5+m} \sqrt{cx^2}}{d^5 (5+m) x} + \frac{b^2 c (dx)^{6+m} \sqrt{cx^2}}{d^6 (6+m) x}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.49

$$x(dx)^m \left(cx^2\right)^{3/2} \left(\frac{a^2}{4+m} + \frac{2abx}{5+m} + \frac{b^2 x^2}{6+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] x*(d*x)^m*(c*x^2)^(3/2)*(a^2/(4 + m) + (2*a*b*x)/(5 + m) + (b^2*x^2)/(6 + m
))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.11, size = 95, normalized size = 0.98

method	result	size
gosper	$\frac{x\left(b^2 m^2 x^2+2ab\, m^2 x+9m\, x^2 b^2+a^2 m^2+20abmx+20x^2 b^2+11a^2 m+48abx+30a^2\right)(dx)^m \left(c\, x^2\right)^{\frac{3}{2}}}{(6+m)(5+m)(4+m)}$	95

risch	$\frac{c\,x^{3}\sqrt{c\,x^{2}}\,\left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x+9 m\,x^{2} b^{2}+a^{2} m^{2}+20 a b m x+20 x^{2} b^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(d x\right)^{m}}{\left(6+m\right) \left(5+m\right) \left(4+m\right)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+9*b^2*m*x^2+a^2*m^2+20*a*b*m*x+20*b^2*x^2+11*a^2
*m+48*a*b*x+30*a^2)*(d*x)^m*(c*x^2)^(3/2)/(6+m)/(5+m)/(4+m)
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.66

$$\frac{b^{2} c^{\frac{3}{2}} d^{m} x^{6} x^{m}}{m+6}+\frac{2\,abc^{\frac{3}{2}} d^{m} x^{5} x^{m}}{m+5}+\frac{a^{2} c^{\frac{3}{2}} d^{m} x^{4} x^{m}}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(3/2)*d^m*x^6*x^m/(m + 6) + 2*a*b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a^2*c
^(3/2)*d^m*x^4*x^m/(m + 4)
```

Fricas [A]
time = 0.31, size = 105, normalized size = 1.08

$$\frac{\left(\left(b^{2} c m^{2}+9\,b^{2} c m+20\,b^{2} c\right) x^{5}+2\,\left(a b c m^{2}+10\,a b c m+24\,a b c\right) x^{4}+\left(a^{2} c m^{2}+11\,a^{2} c m+30\,a^{2} c\right) x^{3}\right) \sqrt{c x^{2}}\,\left(d x\right)^{m}}{m^{3}+15\,m^{2}+74\,m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c*m^2 + 9*b^2*c*m + 20*b^2*c)*x^5 + 2*(a*b*c*m^2 + 10*a*b*c*m + 24*a*
b*c)*x^4 + (a^2*c*m^2 + 11*a^2*c*m + 30*a^2*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^
3 + 15*m^2 + 74*m + 120)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^{2} \left(c x^{2}\right)^{\frac{3}{2}}}{x^{6}}\, dx + \int \frac{b^{2} \left(c x^{2}\right)^{\frac{3}{2}}}{x^{4}}\, dx + \int \frac{2 a b \left(c x^{2}\right)^{\frac{3}{2}}}{x^{5}}\, dx}{d^{6}} & \text{for}\: m = -6 \\ \frac{\int \frac{a^{2} \left(c x^{2}\right)^{\frac{3}{2}}}{x^{5}}\, dx + \int \frac{b^{2} \left(c x^{2}\right)^{\frac{3}{2}}}{x^{3}}\, dx + \int \frac{2 a b \left(c x^{2}\right)^{\frac{3}{2}}}{x^{4}}\, dx}{d^{5}} & \text{for}\: m = -5 \\ \frac{\int \frac{a^{2} \left(c x^{2}\right)^{\frac{3}{2}}}{x^{4}}\, dx + \int \frac{b^{2} \left(c x^{2}\right)^{\frac{3}{2}}}{x^{2}}\, dx + \int \frac{2 a b \left(c x^{2}\right)^{\frac{3}{2}}}{x^{3}}\, dx}{d^{4}} & \text{for}\: m = -4 \\ \frac{a^{2} m^{2} x \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{11 a^{2} m x \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{30 a^{2} x \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{2 a b m^{2} x^{2} \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{20 a b m x^{2} \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{48 a b x^{2} \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{b^{2} m^{2} x^{3} \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{9 b^{2} m x^{3} \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} + \frac{20 b^{2} x^{3} \left(c x^{2}\right)^{\frac{3}{2}} \left(d x\right)^{m}}{m^{3} + 15 m^{2} + 74 m + 120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(3/2)/x**6, x) + Integral(b**2*(c*x**2)
**(3/2)/x**4, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**5, x))/d**6, Eq(m, -6)
), ((Integral(a**2*(c*x**2)**(3/2)/x**5, x) + Integral(b**2*(c*x**2)**(3/2)
/x**3, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**4, x))/d**5, Eq(m, -5)), ((In
tegral(a**2*(c*x**2)**(3/2)/x**4, x) + Integral(b**2*(c*x**2)**(3/2)/x**2,
x) + Integral(2*a*b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a**2*m**2*
x*(c*x**2)**(3/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 11*a**2*m*x*(c*x
**2)**(3/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 30*a**2*x*(c*x**2)**(3
/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 2*a*b*m**2*x**2*(c*x**2)**(3/2
)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 20*a*b*m*x**2*(c*x**2)**(3/2)*(d
*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 48*a*b*x**2*(c*x**2)**(3/2)*(d*x)**m
/(m**3 + 15*m**2 + 74*m + 120) + b**2*m**2*x**3*(c*x**2)**(3/2)*(d*x)**m/(m
**3 + 15*m**2 + 74*m + 120) + 9*b**2*m*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3
+ 15*m**2 + 74*m + 120) + 20*b**2*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3 + 15*
m**2 + 74*m + 120), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**B**]
time = 0.28, size = 121, normalized size = 1.25

$$(d\,x)^m\,\left(\frac{a^2\,c\,x^3\,\sqrt{c\,x^2}\,\left(m^2+11\,m+30\right)}{m^3+15\,m^2+74\,m+120}+\frac{b^2\,c\,x^5\,\sqrt{c\,x^2}\,\left(m^2+9\,m+20\right)}{m^3+15\,m^2+74\,m+120}+\frac{2\,a\,b\,c\,x^4\,\sqrt{c\,x^2}\,\left(m^2+10\,m+24\right)}{m^3+15\,m^2+74\,m+120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c*x^3*(c*x^2)^(1/2)*(11*m + m^2 + 30))/(74*m + 15*m^2 + m^3 +
 120) + (b^2*c*x^5*(c*x^2)^(1/2)*(9*m + m^2 + 20))/(74*m + 15*m^2 + m^3 + 1
20) + (2*a*b*c*x^4*(c*x^2)^(1/2)*(10*m + m^2 + 24))/(74*m + 15*m^2 + m^3 +
120))
```

3.977 $\int (dx)^m \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=94

$$\frac{a^2 (dx)^{2+m} \sqrt{cx^2}}{d^2 (2+m) x} + \frac{2ab (dx)^{3+m} \sqrt{cx^2}}{d^3 (3+m) x} + \frac{b^2 (dx)^{4+m} \sqrt{cx^2}}{d^4 (4+m) x}$$

```
[Out] a^2*(d*x)^(2+m)*(c*x^2)^(1/2)/d^2/(2+m)/x+2*a*b*(d*x)^(3+m)*(c*x^2)^(1/2)/d
^3/(3+m)/x+b^2*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2 \sqrt{cx^2} (dx)^{m+2}}{d^2 (m+2) x} + \frac{2ab \sqrt{cx^2} (dx)^{m+3}}{d^3 (m+3) x} + \frac{b^2 \sqrt{cx^2} (dx)^{m+4}}{d^4 (m+4) x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (2*a*b*(d*x)^(3 + m)*Sqrt
[c*x^2])/(d^3*(3 + m)*x) + (b^2*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{cx^2}\,(a+bx)^2\,dx = \frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)^2\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)^2\,dx}{dx}$$
$$= \frac{\sqrt{cx^2}\,\int \left(a^2(dx)^{1+m} + \frac{2ab(dx)^{2+m}}{d} + \frac{b^2(dx)^{3+m}}{d^2}\right)dx}{dx}$$
$$= \frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x} + \frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x} + \frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.77

$$\frac{x(dx)^m\sqrt{cx^2}\,\left(a^2\left(12+7m+m^2\right)+2ab\left(8+6m+m^2\right)x+b^2\left(6+5m+m^2\right)x^2\right)}{(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a^2*(12 + 7*m + m^2) + 2*a*b*(8 + 6*m + m^2)*x + b^
2*(6 + 5*m + m^2)*x^2))/((2 + m)*(3 + m)*(4 + m))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(1/2)*(a + b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 95, normalized size = 1.01

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+5m\,x^2b^2+a^2m^2+12abmx+6x^2b^2+7a^2m+16abx+12a^2\right)(dx)^m\sqrt{c\,x^2}}{(4+m)(3+m)(2+m)}$	95
risch	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+5m\,x^2b^2+a^2m^2+12abmx+6x^2b^2+7a^2m+16abx+12a^2\right)(dx)^m\sqrt{c\,x^2}}{(4+m)(3+m)(2+m)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+5*b^2*m*x^2+a^2*m^2+12*a*b*m*x+6*b^2*x^2+7*a^2*m
+16*a*b*x+12*a^2)*(d*x)^m*(c*x^2)^(1/2)/(4+m)/(3+m)/(2+m)
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.68

$$\frac{b^2\sqrt{c}\,d^m x^4 x^m}{m+4}+\frac{2\,ab\sqrt{c}\,d^m x^3 x^m}{m+3}+\frac{a^2\sqrt{c}\,d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(c)*d^m*x^4*x^m/(m + 4) + 2*a*b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a^2*s
qrt(c)*d^m*x^2*x^m/(m + 2)
```

Fricas [A]
time = 0.30, size = 94, normalized size = 1.00

$$\frac{((b^2m^2+5\,b^2m+6\,b^2)x^3+2\,(abm^2+6\,abm+8\,ab)x^2+(a^2m^2+7\,a^2m+12\,a^2)x)\sqrt{cx^2}\;(dx)^m}{m^3+9\,m^2+26\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 6*b^2)*x^3 + 2*(a*b*m^2 + 6*a*b*m + 8*a*b)*x^2 + (a^2
*m^2 + 7*a^2*m + 12*a^2)*x)*sqrt(c*x^2)*(d*x)^m/(m^3 + 9*m^2 + 26*m + 24)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2\sqrt{cx^2}}{x^4}\,dx+\int \frac{b^2\sqrt{cx^2}}{x^2}\,dx+\int \frac{2ab\sqrt{cx^2}}{x^3}\,dx}{d^4} & \text{for}\ m=-4 \\ \frac{\int \frac{a^2\sqrt{cx^2}}{x^3}\,dx+\int \frac{b^2\sqrt{cx^2}}{x}\,dx+\int \frac{2ab\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for}\ m=-3 \\ \frac{\int b^2\sqrt{cx^2}\,dx+\int \frac{a^2\sqrt{cx^2}}{x^2}\,dx+\int \frac{2ab\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for}\ m=-2 \\ \frac{a^2m^2x\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{7a^2mx\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{12a^2x\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{2abm^2x^2\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{12abmx^2\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{16abx^2\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{b^2m^2x^3\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{5b^2mx^3\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24}+\frac{6b^2x^3\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*sqrt(c*x**2)/x**4, x) + Integral(b**2*sqrt(c*x**2
)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)/x**3, x))/d**4, Eq(m, -4)), ((Inte
gral(a**2*sqrt(c*x**2)/x**3, x) + Integral(b**2*sqrt(c*x**2)/x, x) + Integr
```

```
al(2*a*b*sqrt(c*x**2)/x**2, x))/d**3, Eq(m, -3)), ((Integral(b**2*sqrt(c*x*
*2), x) + Integral(a**2*sqrt(c*x**2)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)
/x, x))/d**2, Eq(m, -2)), (a**2*m**2*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2
 + 26*m + 24) + 7*a**2*m*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24
) + 12*a**2*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 2*a*b*m**
2*x**2*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 12*a*b*m*x**2*sq
rt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 16*a*b*x**2*sqrt(c*x**2)*
(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + b**2*m**2*x**3*sqrt(c*x**2)*(d*x)**m
/(m**3 + 9*m**2 + 26*m + 24) + 5*b**2*m*x**3*sqrt(c*x**2)*(d*x)**m/(m**3 +
9*m**2 + 26*m + 24) + 6*b**2*x**3*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26
*m + 24), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [B]
time = 0.26, size = 116, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,x\,\sqrt{c\,x^2}\,\left(m^2+7\,m+12\right)}{m^3+9\,m^2+26\,m+24}+\frac{b^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2+5\,m+6\right)}{m^3+9\,m^2+26\,m+24}+\frac{2\,a\,b\,x^2\,\sqrt{c\,x^2}\,\left(m^2+6\,m+8\right)}{m^3+9\,m^2+26\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*x*(c*x^2)^(1/2)*(7*m + m^2 + 12))/(26*m + 9*m^2 + m^3 + 24) +
 (b^2*x^3*(c*x^2)^(1/2)*(5*m + m^2 + 6))/(26*m + 9*m^2 + m^3 + 24) + (2*a*b
*x^2*(c*x^2)^(1/2)*(6*m + m^2 + 8))/(26*m + 9*m^2 + m^3 + 24))
```

3.978 $\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}+\frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}$$

```
[Out] a^2*x*(d*x)^m/m/(c*x^2)^(1/2)+2*a*b*x*(d*x)^(1+m)/d/(1+m)/(c*x^2)^(1/2)+b^2
*x*(d*x)^(2+m)/d^2/(2+m)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (2*a*b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c
*x^2]) + (b^2*x*(d*x)^(2 + m))/(d^2*(2 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int (dx)^{-1+m}(a+bx)^2\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int \left(a^2(dx)^{-1+m} + \frac{2ab(dx)^m}{d} + \frac{b^2(dx)^{1+m}}{d^2}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x(dx)^m}{m\sqrt{cx^2}} + \frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}} + \frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.77

$$\frac{x(dx)^m\left(a^2\left(2+3m+m^2\right)+2abm(2+m)x+b^2m(1+m)x^2\right)}{m(1+m)(2+m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x*(d*x)^m*(a^2*(2 + 3*m + m^2) + 2*a*b*m*(2 + m)*x + b^2*m*(1 + m)*x^2))/(
m*(1 + m)*(2 + m)*Sqrt[c*x^2])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)^2/(c*x^2)^(1/2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.13, size = 79, normalized size = 0.98

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+m\,x^2b^2+a^2m^2+4abmx+3a^2m+2a^2\right)(dx)^m}{(2+m)(1+m)m\sqrt{c\,x^2}}$	79
risch	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+m\,x^2b^2+a^2m^2+4abmx+3a^2m+2a^2\right)(dx)^m}{(2+m)(1+m)m\sqrt{c\,x^2}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+b^2*m*x^2+a^2*m^2+4*a*b*m*x+3*a^2*m+2*a^2)*(d*x)
^m/(2+m)/(1+m)/m/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.70

$$\frac{b^2 d^m x^2 x^m}{\sqrt{c}\,(m+2)} + \frac{2\,abd^m x x^m}{\sqrt{c}\,(m+1)} + \frac{a^2 d^m x^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^2*x^m/(sqrt(c)*(m + 2)) + 2*a*b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a^2
*d^m*x^m/(sqrt(c)*m)
```

Fricas [A]
time = 0.30, size = 85, normalized size = 1.05

$$\frac{\left(a^2m^2 + 3\,a^2m + (b^2m^2 + b^2m)x^2 + 2\,a^2 + 2\,(abm^2 + 2\,abm)x\right)\sqrt{cx^2}\,\left(dx\right)^m}{(cm^3 + 3\,cm^2 + 2\,cm)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 + 3*a^2*m + (b^2*m^2 + b^2*m)*x^2 + 2*a^2 + 2*(a*b*m^2 + 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c*m^3 + 3*c*m^2 + 2*c*m)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b^2}{\sqrt{cx^2}}\,dx + \int \frac{a^2}{x^2\sqrt{cx^2}}\,dx + \int \frac{2ab}{x\sqrt{cx^2}}\,dx}{d^2} & \text{for}\: m = -2 \\ \frac{\int \frac{2ab}{\sqrt{cx^2}}\,dx + \int \frac{a^2}{x\sqrt{cx^2}}\,dx + \int \frac{b^2x}{\sqrt{cx^2}}\,dx}{d} & \text{for}\: m = -1 \\ \int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx & \text{for}\: m = 0 \\ \frac{a^2m^2x(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{3a^2mx(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{2a^2x(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{2abm^2x^2(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{4abmx^2(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{b^2m^2x^3(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{b^2mx^3(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b**2/sqrt(c*x**2), x) + Integral(a**2/(x**2*sqrt(c*x**
2)), x) + Integral(2*a*b/(x*sqrt(c*x**2)), x))/d**2, Eq(m, -2)), ((Integral
(2*a*b/sqrt(c*x**2), x) + Integral(a**2/(x*sqrt(c*x**2)), x) + Integral(b**
```

```
2*x/sqrt(c*x**2), x))/d, Eq(m, -1)), (Integral((a + b*x)**2/sqrt(c*x**2), x
), Eq(m, 0)), (a**2*m**2*x*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2
) + 2*m*sqrt(c*x**2)) + 3*a**2*m*x*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqr
t(c*x**2) + 2*m*sqrt(c*x**2)) + 2*a**2*x*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m*
*2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 2*a*b*m**2*x**2*(d*x)**m/(m**3*sqrt(c
*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 4*a*b*m*x**2*(d*x)**m/(m
**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + b**2*m**2*x**3
*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + b*
*2*m*x**3*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x*
*2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

Mupad [**B**]
time = 0.26, size = 62, normalized size = 0.77

$$\frac{(d\,x)^m\,\left(\frac{a^2\,x}{m}+\frac{b^2\,x^3\,(m+1)}{m^2+3\,m+2}+\frac{2\,a\,b\,x^2\,(m+2)}{m^2+3\,m+2}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] ((d*x)^m*((a^2*x)/m + (b^2*x^3*(m + 1))/(3*m + m^2 + 2) + (2*a*b*x^2*(m + 2
))/(3*m + m^2 + 2)))/(c*x^2)^(1/2)
```

3.979 $\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

```
[Out] -a^2*d^2*x*(d*x)^(-2+m)/c/(2-m)/(c*x^2)^(1/2)-2*a*b*d*x*(d*x)^(-1+m)/c/(1-m
)/(c*x^2)^(1/2)+b^2*x*(d*x)^m/c/m/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^2*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (2*a*b*d*x*(d*x)^(-
1 + m))/(c*(1 - m)*Sqrt[c*x^2]) + (b^2*x*(d*x)^m)/(c*m*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)^2\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int \left(a^2(dx)^{-3+m} + \frac{2ab(dx)^{-2+m}}{d} + \frac{b^2(dx)^{-1+m}}{d^2}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}} - \frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}} + \frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.67

$$\frac{x(dx)^m\left(a^2(-1+m)m + 2ab(-2+m)mx + b^2\left(2-3m+m^2\right)x^2\right)}{(-2+m)(-1+m)m\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x*(d*x)^m*(a^2*(-1 + m)*m + 2*a*b*(-2 + m)*m*x + b^2*(2 - 3*m + m^2)*x^2))
/((-2 + m)*(-1 + m)*m*(c*x^2)^(3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)^2/(c*x^2)^(3/2),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.12, size = 83, normalized size = 0.89

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x-3m\,x^2b^2+a^2m^2-4abmx+2x^2b^2-a^2m\right)(dx)^m}{m(-1+m)(-2+m)(c\,x^2)^{\frac{3}{2}}}$	83
risch	$\frac{\left(b^2m^2x^2+2ab\,m^2x-3m\,x^2b^2+a^2m^2-4abmx+2x^2b^2-a^2m\right)(dx)^m}{cx\sqrt{c\,x^2}\,m(-1+m)(-2+m)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-3*b^2*m*x^2+a^2*m^2-4*a*b*m*x+2*b^2*x^2-a^2*m)*(
d*x)^m/m/(-1+m)/(-2+m)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.63

$$\frac{b^2 d^m x^m}{c^{\frac{3}{2}} m} + \frac{2\,abd^m x^m}{c^{\frac{3}{2}}(m-1)x} + \frac{a^2 d^m x^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(3/2)*m) + 2*a*b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a^2*d^m*x^m/(
c^(3/2)*(m - 2)*x^2)
```

Fricas [A]
time = 0.30, size = 92, normalized size = 0.99

$$\frac{(a^2m^2 - a^2m + (b^2m^2 - 3\,b^2m + 2\,b^2)x^2 + 2\,(abm^2 - 2\,abm)x)\sqrt{cx^2}\,(dx)^m}{(c^2m^3 - 3\,c^2m^2 + 2\,c^2m)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - a^2*m + (b^2*m^2 - 3*b^2*m + 2*b^2)*x^2 + 2*(a*b*m^2 - 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c^2*m^3 - 3*c^2*m^2 + 2*c^2*m)*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \frac{(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx & \text{for}\: m = 0 \\ d\left(\int \frac{a^2x}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{b^2x^3}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{2abx^2}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for}\: m = 1 \\ d^2\left(\int \frac{a^2x^2}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{b^2x^4}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{2abx^3}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for}\: m = 2 \\ \frac{a^2m^2x(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} - \frac{a^2mx(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} + \frac{2abm^2x^2(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} - \frac{4abmx^2(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} + \frac{b^2m^2x^3(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} - \frac{3b^2mx^3(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} + \frac{2b^2x^3(dx)^m}{m^3(cx^2)^{\frac{3}{2}} - 3m^2(cx^2)^{\frac{3}{2}} + 2m(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((Integral((a + b*x)**2/(c*x**2)**(3/2), x), Eq(m, 0)), (d*(Integr
al(a**2*x/(c*x**2)**(3/2), x) + Integral(b**2*x**3/(c*x**2)**(3/2), x) + In
tegral(2*a*b*x**2/(c*x**2)**(3/2), x)), Eq(m, 1)), (d**2*(Integral(a**2*x**
```

```
2/(c*x**2)**(3/2), x) + Integral(b**2*x**4/(c*x**2)**(3/2), x) + Integral(2
*a*b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a**2*m**2*x*(d*x)**m/(m**3*(c*x
**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) - a**2*m*x*(d*x
)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2))
+ 2*a*b*m**2*x**2*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) +
 2*m*(c*x**2)**(3/2)) - 4*a*b*m*x**2*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**
2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) + b**2*m**2*x**3*(d*x)**m/(m**3*(c
*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) - 3*b**2*m*x*
*3*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**
(3/2)) + 2*b**2*x**3*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2
) + 2*m*(c*x**2)**(3/2)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.32, size = 66, normalized size = 0.71

$$\frac{a^2\,{\left(d\,x\right)}^m}{c\,x\,\sqrt{c\,x^2}\,\left(m-2\right)}+\frac{b\,{\left(d\,x\right)}^m\,\left(2\,a\,m-b\,x+b\,m\,x\right)}{c\,m\,\sqrt{c\,x^2}\,\left(m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m - 2)) + (b*(d*x)^m*(2*a*m - b*x + b*m*x
))/(c*m*(c*x^2)^(1/2)*(m - 1))
```

3.980 $\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

```
[Out] -a^2*d^4*x*(d*x)^(-4+m)/c^2/(4-m)/(c*x^2)^(1/2)-2*a*b*d^3*x*(d*x)^(-3+m)/c^
2/(3-m)/(c*x^2)^(1/2)-b^2*d^2*x*(d*x)^(-2+m)/c^2/(2-m)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^2*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (2*a*b*d^3*x*(d*x
)^(-3 + m))/(c^2*(3 - m)*Sqrt[c*x^2]) - (b^2*d^2*x*(d*x)^(-2 + m))/(c^2*(2
- m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)^2\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int \left(a^2(dx)^{-5+m} + \frac{2ab(dx)^{-4+m}}{d} + \frac{b^2(dx)^{-3+m}}{d^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}} - \frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}} - \frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.69

$$\frac{x(dx)^m\left(a^2\left(6-5m+m^2\right)+2ab\left(8-6m+m^2\right)x+b^2\left(12-7m+m^2\right)x^2\right)}{(-4+m)(-3+m)(-2+m)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(6 - 5*m + m^2) + 2*a*b*(8 - 6*m + m^2)*x + b^2*(12 - 7*m +
 m^2)*x^2))/((-4 + m)*(-3 + m)*(-2 + m)*(c*x^2)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)^2/(c*x^2)^(5/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.11, size = 95, normalized size = 0.90

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x-7m\,x^2b^2+a^2m^2-12abmx+12x^2b^2-5a^2m+16abx+6a^2\right)(dx)^m}{(-2+m)(-3+m)(-4+m)(c\,x^2)^{\frac{5}{2}}}$	95
risch	$\frac{\left(b^2m^2x^2+2ab\,m^2x-7m\,x^2b^2+a^2m^2-12abmx+12x^2b^2-5a^2m+16abx+6a^2\right)(dx)^m}{c^2x^3\sqrt{c\,x^2}\,(-2+m)(-3+m)(-4+m)}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-7*b^2*m*x^2+a^2*m^2-12*a*b*m*x+12*b^2*x^2-5*a^2*
m+16*a*b*x+6*a^2)*(d*x)^m/(-2+m)/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.61

$$\frac{b^2 d^m x^m}{c^{\frac{5}{2}}(m-2)x^2} + \frac{2\,abd^m x^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{a^2 d^m x^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(5/2)*(m - 2)*x^2) + 2*a*b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a
^2*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Fricas [A]
time = 0.31, size = 106, normalized size = 1.01

$$\frac{(a^2m^2 - 5\,a^2m + (b^2m^2 - 7\,b^2m + 12\,b^2)x^2 + 6\,a^2 + 2\,(abm^2 - 6\,abm + 8\,ab)x)\sqrt{cx^2}\,(dx)^m}{(c^3m^3 - 9\,c^3m^2 + 26\,c^3m - 24\,c^3)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - 5*a^2*m + (b^2*m^2 - 7*b^2*m + 12*b^2)*x^2 + 6*a^2 + 2*(a*b*m^2
- 6*a*b*m + 8*a*b)*x)*sqrt(c*x^2)*(d*x)^m/((c^3*m^3 - 9*c^3*m^2 + 26*c^3*m
- 24*c^3)*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d^2\left(\int \frac{a^2x^2}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{b^2x^4}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{2abx^3}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for}\: m = 2 \\ d^3\left(\int \frac{a^2x^3}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{b^2x^5}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{2abx^4}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for}\: m = 3 \\ d^4\left(\int \frac{a^2x^4}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{b^2x^6}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{2abx^5}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for}\: m = 4 \\ \frac{a^2m^2x(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} - \frac{5a^2mx(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} + \frac{6a^2x(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} + \frac{2abm^2x^2(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} - \frac{12abmx^2(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} + \frac{16abx^2(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} + \frac{b^2m^2x^3(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} - \frac{7b^2mx^3(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} + \frac{12b^2x^3(dx)^m}{m^3(cx^2)^{\frac{5}{2}} - 9m^2(cx^2)^{\frac{5}{2}} + 26m(cx^2)^{\frac{5}{2}} - 24(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**2*(Integral(a**2*x**2/(c*x**2)**(5/2), x) + Integral(b**2*x**
4/(c*x**2)**(5/2), x) + Integral(2*a*b*x**3/(c*x**2)**(5/2), x)), Eq(m, 2))
, (d**3*(Integral(a**2*x**3/(c*x**2)**(5/2), x) + Integral(b**2*x**5/(c*x**
```

```
2)**(5/2), x) + Integral(2*a*b*x**4/(c*x**2)**(5/2), x)), Eq(m, 3)), (d**4*
(Integral(a**2*x**4/(c*x**2)**(5/2), x) + Integral(b**2*x**6/(c*x**2)**(5/2
), x) + Integral(2*a*b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a**2*m**2*x*(
d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/
2) - 24*(c*x**2)**(5/2)) - 5*a**2*m*x*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m*
*2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) + 6*a**2*x*
(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5
/2) - 24*(c*x**2)**(5/2)) + 2*a*b*m**2*x**2*(d*x)**m/(m**3*(c*x**2)**(5/2)
- 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) - 12*
a*b*m*x**2*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(
c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) + 16*a*b*x**2*(d*x)**m/(m**3*(c*x**2)*
*(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)
) + b**2*m**2*x**3*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2)
+ 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) - 7*b**2*m*x**3*(d*x)**m/(m**3
*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x*
*2)**(5/2)) + 12*b**2*x**3*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)
**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)), True))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.34, size = 82, normalized size = 0.78

$$\frac{a^2\,{\left(d\,x\right)}^m}{c^2\,x^3\,\sqrt{c\,x^2}\,\left(m-4\right)}+\frac{b^2\,{\left(d\,x\right)}^m}{c^2\,x\,\sqrt{c\,x^2}\,\left(m-2\right)}+\frac{2\,a\,b\,{\left(d\,x\right)}^m}{c^2\,x^2\,\sqrt{c\,x^2}\,\left(m-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c^2*x^3*(c*x^2)^(1/2)*(m - 4)) + (b^2*(d*x)^m)/(c^2*x*(c*x^2
)^(1/2)*(m - 2)) + (2*a*b*(d*x)^m)/(c^2*x^2*(c*x^2)^(1/2)*(m - 3))
```

3.981 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=67

$$\frac{c^2(dx)^{6+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{d^6(6+m)x}$$

```
[Out] c^2*(d*x)^(6+m)*(b*x+a)^n*hypergeom([-n, 6+m],[7+m],-b*x/a)*(c*x^2)^(1/2)/d
^6/(6+m)/x/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{c^2\sqrt{cx^2}\,(dx)^{m+6}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+6,-n;m+7;-\frac{bx}{a}\right)}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^2*(d*x)^(6 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 +
 m, -((b*x)/a)])/(d^6*(6 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^n\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)^n\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)^n\,dx}{d^5x}\\ &= \frac{\left(c^2\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{5+m}\left(1+\frac{bx}{a}\right)^n\,dx}{d^5x}\\ &= \frac{c^2(dx)^{6+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{d^6(6+m)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.85

$$\frac{x(dx)^m\left(cx^2\right)^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{6+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 + m, -(
(b*x)/a)])/((6 + m)*(1 + (b*x)/a)^n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m\left(c\,x^2\right)^{\frac{5}{2}}(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m*c^2*x^4, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(c\,x^2\right)^{5/2}\,(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x)
[Out] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n, x)
```

3.982 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=65

$$\frac{c(dx)^{4+m}\sqrt{cx^2}\,(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{d^4(4+m)x}$$

```
[Out] c*(d*x)^(4+m)*(b*x+a)^n*hypergeom([-n, 4+m],[5+m],-b*x/a)*(c*x^2)^(1/2)/d^4
/(4+m)/x/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{c\sqrt{cx^2}\,(dx)^{m+4}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+4,-n;m+5;-\frac{bx}{a}\right)}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (c*(d*x)^(4 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m
, -((b*x)/a)])/(d^4*(4 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^n\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)^n\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)^n\,dx}{d^3x}\\ &= \frac{\left(c\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{3+m}\left(1+\frac{bx}{a}\right)^n\,dx}{d^3x}\\ &= \frac{c(dx)^{4+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{d^4(4+m)x}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 57, normalized size = 0.88

$$\frac{x(dx)^m\left(cx^2\right)^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{4+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m, -(
(b*x)/a)])/((4 + m)*(1 + (b*x)/a)^n)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]')
[Out] Timed out
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^2\right)^{\frac{3}{2}} (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m*c*x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{3}{2}} \left(dx\right)^m \left(a+bx\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(d*x)**m*(a + b*x)**n, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\,x)^m\,\left(c\,x^2\right)^{3/2}\,(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x)
[Out] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n, x)
```

3.983 $\int (dx)^m \sqrt{cx^2} (a+bx)^n \, dx$

Optimal. Leaf size=64

$$\frac{(dx)^{2+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{d^2(2+m)x}$$

```
[Out] (d*x)^(2+m)*(b*x+a)^n*hypergeom([-n, 2+m],[3+m],-b*x/a)*(c*x^2)^(1/2)/d^2/(
2+m)/x/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{\sqrt{cx^2}\,(dx)^{m+2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+2,-n;m+3;-\frac{bx}{a}\right)}{d^2(m+2)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] ((d*x)^(2 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m,
-((b*x)/a)])/(d^2*(2 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)^n\,dx}{x}\\
&= \frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)^n\,dx}{dx}\\
&= \frac{\left(\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{1+m}\left(1+\frac{bx}{a}\right)^n\,dx}{dx}\\
&= \frac{(dx)^{2+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{d^2(2+m)x}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 57, normalized size = 0.89

$$\frac{x(dx)^m\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{2+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n,x]
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m, -((b
*x)/a)])/((2 + m)*(1 + (b*x)/a)^n)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^(1/2)*(a + b*x)^n,x]')
[Out] Timed out
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{c\,x^2}\;(bx+a)^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2} (dx)^m (a + bx)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**n,x)
```

```
[Out] Integral(sqrt(c*x**2)*(d*x)**m*(a + b*x)**n, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\,x)^m\,\sqrt{c\,x^2}\,\,{\left(a+b\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^n, x)
```

3.984 $\int \frac{(dx)^m(a+bx)^n}{\sqrt{cx^2}} dx$

Optimal. Leaf size=53

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

```
[Out] x*(d*x)^m*(b*x+a)^n*hypergeom([m, -n],[1+m],-b*x/a)/m/((1+b*x/a)^n)/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m,-n;m+1;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x}\,dx}{\sqrt{cx^2}}\\
&= \frac{(dx)\int (dx)^{-1+m}(a+bx)^n\,dx}{\sqrt{cx^2}}\\
&= \frac{\left(dx(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-1+m}\left(1+\frac{bx}{a}\right)^n\,dx}{\sqrt{cx^2}}\\
&= \frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 53, normalized size = 1.00

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)^n/(c*x^2)^(1/2),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\,(bx+a)^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/sqrt(c*x^2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (a + bx)^n}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/sqrt(c*x**2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m\,(a+b\,x)^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(1/2),x)
[Out] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(1/2), x)
```

3.985 $\int \frac{(dx)^m(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{d^2x(dx)^{-2+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

```
[Out] -d^2*x*(d*x)^(-2+m)*(b*x+a)^n*hypergeom([-n, -2+m],[-1+m],-b*x/a)/c/(2-m)/(
(1+b*x/a)^n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$-\frac{d^2x(dx)^{m-2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-2,-n;m-1;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((d^2*x*(d*x)^(-2 + m)*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -
((b*x)/a)])/(c*(2 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x^3}\,dx}{c\sqrt{cx^2}}\\
&= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)^n\,dx}{c\sqrt{cx^2}}\\
&= \frac{\left(d^3x(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-3+m}\left(1+\frac{bx}{a}\right)^n\,dx}{c\sqrt{cx^2}}\\
&= -\frac{d^2x(dx)^{-2+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 57, normalized size = 0.84

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{(-2+m)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -((b*x)/a)])/(
(-2 + m)*(c*x^2)^(3/2)*(1 + (b*x)/a)^n)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)^n/(c*x^2)^(3/2),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\,(bx+a)^n}{(c\,x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(3/2), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c^2*x^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (a+bx)^n}{(cx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m\,(a+b\,x)^n}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x)
[Out] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2), x)
```

3.986 $\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{d^4x(dx)^{-4+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

```
[Out] -d^4*x*(d*x)^(-4+m)*(b*x+a)^n*hypergeom([-n, -4+m],[-3+m],-b*x/a)/c^2/(4-m)
/((1+b*x/a)^n)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$-\frac{d^4x(dx)^{m-4}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-4,-n;m-3;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((d^4*x*(d*x)^(-4 + m)*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -
((b*x)/a)])/(c^2*(4 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x^5}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{\left(d^5x(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-5+m}\left(1+\frac{bx}{a}\right)^n\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{d^4x(dx)^{-4+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.84

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{(-4+m)\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -((b*x)/a)])/(
(-4 + m)*(c*x^2)^(5/2)*(1 + (b*x)/a)^n)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(a + b*x)^n/(c*x^2)^(5/2),x]')
[Out] Timed out
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\,(bx+a)^n}{(c\,x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c^3*x^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (a+bx)^n}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m\,(a+b\,x)^n}{(c\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.987 $\int x^3 (cx^2)^p (a+bx)^{-5-2p} \, dx$

Optimal. Leaf size=33

$$\frac{x^4 (cx^2)^p (a+bx)^{-2(2+p)}}{2a(2+p)}$$

```
[Out] 1/2*x^4*(c*x^2)^p/a/(2+p)/((b*x+a)^(4+2*p))
```

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$\frac{x^4 (cx^2)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p)/(2*a*(2 + p)*(a + b*x)^(2*(2 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int x^3 (cx^2)^p (a+bx)^{-5-2p} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{3+2p} (a+bx)^{-5-2p} \, dx \\ &= \frac{x^4 (cx^2)^p (a+bx)^{-2(2+p)}}{2a(2+p)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 32, normalized size = 0.97

$$\frac{x^4 (cx^2)^p (a+bx)^{-4-2p}}{a(4+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
[Out] (x^4*(c*x^2)^p*(a + b*x)^(-4 - 2*p))/(a*(4 + 2*p))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^3*(c*x^2)^p/(a + b*x)^(2*p + 5),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Maple [A]
time = 0.14, size = 32, normalized size = 0.97

method	result	size
gosper	$\frac{x^{4}(bx+a)^{-4-2p}(c\,x^{2})^{p}}{2a(2+p)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x,method=_RETURNVERBOSE)
[Out] 1/2*x^4*(b*x+a)^(-4-2*p)/a/(2+p)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 5)*x^3, x)
```

Fricas [A]
time = 0.31, size = 40, normalized size = 1.21

$$\frac{(bx^5 + ax^4)(cx^2)^p(bx + a)^{-2p-5}}{2(ap + 2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^5 + a*x^4)*(c*x^2)^p*(b*x + a)^(-2*p - 5)/(a*p + 2*a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**p*(b*x+a)**(-5-2*p),x)
[Out] Exception raised: SystemError
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(33) = 66.
time = 0.01, size = 81, normalized size = 2.45

$$\frac{ax^4e^{-2p\ln(a+bx)-5\ln(a+bx)}e^{p\ln(cx^2)}+bx^5e^{-2p\ln(a+bx)-5\ln(a+bx)}e^{p\ln(cx^2)}}{2ap+4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x)
[Out] 1/2*((c*x^2)^p*b*x^5*e^(-2*p*log(b*x + a) - 5*log(b*x + a)) + (c*x^2)^p*a*x
^4*e^(-2*p*log(b*x + a) - 5*log(b*x + a)))/(a*p + 2*a)
```

Mupad [B]
time = 0.27, size = 33, normalized size = 1.00

$$\frac{x^4\,{\left(c\,x^2\right)}^p}{2\,a\,\left(p+2\right)\,{\left(a+b\,x\right)}^{2\,p+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^p)/(a + b*x)^(2*p + 5),x)
[Out] (x^4*(c*x^2)^p)/(2*a*(p + 2)*(a + b*x)^(2*p + 4))
```

3.988 $\int x^2 (cx^2)^p (a+bx)^{-4-2p} dx$

Optimal. Leaf size=32

$$\frac{x^3 (cx^2)^p (a+bx)^{-3-2p}}{a(3+2p)}$$

[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$\frac{x^3 (cx^2)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]

[Out] (x^3*(c*x^2)^p*(a + b*x)^(-3 - 2*p))/(a*(3 + 2*p))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^2 (cx^2)^p (a+bx)^{-4-2p} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{2+2p}(a+bx)^{-4-2p} dx \\ &= \frac{x^3 (cx^2)^p (a+bx)^{-3-2p}}{a(3+2p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 1.06

$$\frac{x^3 (cx^2)^p (a+bx)^{1-2(2+p)}}{a(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(1 - 2*(2 + p)))/(a*(3 + 2*p))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^2*(c*x^2)^p/(a + b*x)^(2*p + 4),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.14, size = 33, normalized size = 1.03

method	result	size
gosper	$\frac{x^{3} \left(c\,x^{2}\right)^{p} \left(b x +a \right)^{-3-2 p}}{a \left(3+2 p \right)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

Fricas [**A**]
time = 0.30, size = 40, normalized size = 1.25

$$\frac{\left(bx^4 + ax^3\right)\left(cx^2\right)^p \left(bx + a\right)^{-2\,p-4}}{2\,ap + 3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^4 + a*x^3)*(c*x^2)^p*(b*x + a)^(-2*p - 4)/(2*a*p + 3*a)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**p*(b*x+a)**(-4-2*p),x)
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x)
[Out] Could not integrate
```

Mupad [B]
time = 0.24, size = 34, normalized size = 1.06

$$\frac{x^3\,{\left(c\,x^2\right)}^p}{a\,\left(2\,p+3\right)\,{\left(a+b\,x\right)}^{2\,p+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^p)/(a + b*x)^(2*p + 4),x)
[Out] (x^3*(c*x^2)^p)/(a*(2*p + 3)*(a + b*x)^(2*p + 3))
```

3.989 $\int x \left(cx^2\right)^p (a+bx)^{-3-2p}\,dx$

Optimal. Leaf size=33

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(1+p)}}{2a(1+p)}$$

[Out] 1/2*x^2*(c*x^2)^p/a/(1+p)/((b*x+a)^(2+2*p))

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]

[Out] (x^2*(c*x^2)^p)/(2*a*(1 + p)*(a + b*x)^(2*(1 + p)))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int x \left(cx^2\right)^p (a+bx)^{-3-2p}\,dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{1+2p}(a+bx)^{-3-2p}\,dx \\ &= \frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(1+p)}}{2a(1+p)} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2-2p}}{a(2+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p*(a + b*x)^(-2 - 2*p))/(a*(2 + 2*p))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 141.62, size = 1406, normalized size = 42.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^1*(c*x^2)^p/(a + b*x)^(2*p + 3),x]')
```

```
[Out] Piecewise[{{-(b x) ^ (-2 p) (c x ^ 2) ^ p / (b ^ 3 x), a == 0}, {x ^ 2 (c x
 ^ 2) ^ p ComplexInfinity ^ (1 + 2 p) / (1 + p), a == -b x}, {x ^ 2 (c x ^
2) ^ p (0 ^ (1 / p)) ^ (-3 - 2 p) / (2 (1 + p)), a == -b x + 0 ^ (1 / p)},
{(Log[x] - Log[a / b + x]) / (a c), p == -1}}, a ^ 4 x ^ 2 (c x ^ 2) ^ p /
(2 a ^ 7 (a + b x) ^ (2 p) + 2 a ^ 7 p (a + b x) ^ (2 p) + 12 a ^ 6 b x (a
+ b x) ^ (2 p) + 12 a ^ 6 b p x (a + b x) ^ (2 p) + 30 a ^ 5 b ^ 2 x ^ 2 (a
 + b x) ^ (2 p) + 30 a ^ 5 b ^ 2 p x ^ 2 (a + b x) ^ (2 p) + 40 a ^ 4 b ^ 3
 x ^ 3 (a + b x) ^ (2 p) + 40 a ^ 4 b ^ 3 p x ^ 3 (a + b x) ^ (2 p) + 30 a
^ 3 b ^ 4 x ^ 4 (a + b x) ^ (2 p) + 30 a ^ 3 b ^ 4 p x ^ 4 (a + b x) ^ (2 p
) + 12 a ^ 2 b ^ 5 x ^ 5 (a + b x) ^ (2 p) + 12 a ^ 2 b ^ 5 p x ^ 5 (a + b
x) ^ (2 p) + 2 a b ^ 6 x ^ 6 (a + b x) ^ (2 p) + 2 a b ^ 6 p x ^ 6 (a + b x
) ^ (2 p)) + b x ^ 3 (c x ^ 2) ^ p / (2 a ^ 4 (a + b x) ^ (2 p) + 2 a ^ 4 p
 (a + b x) ^ (2 p) + 6 a ^ 3 b x (a + b x) ^ (2 p) + 6 a ^ 3 b p x (a + b x
) ^ (2 p) + 6 a ^ 2 b ^ 2 x ^ 2 (a + b x) ^ (2 p) + 6 a ^ 2 b ^ 2 p x ^ 2 (
a + b x) ^ (2 p) + 2 a b ^ 3 x ^ 3 (a + b x) ^ (2 p) + 2 a b ^ 3 p x ^ 3 (a
 + b x) ^ (2 p)) + 3 a ^ 3 b x ^ 3 (c x ^ 2) ^ p / (2 a ^ 7 (a + b x) ^ (2
p) + 2 a ^ 7 p (a + b x) ^ (2 p) + 12 a ^ 6 b x (a + b x) ^ (2 p) + 12 a ^
6 b p x (a + b x) ^ (2 p) + 30 a ^ 5 b ^ 2 x ^ 2 (a + b x) ^ (2 p) + 30 a ^
 5 b ^ 2 p x ^ 2 (a + b x) ^ (2 p) + 40 a ^ 4 b ^ 3 x ^ 3 (a + b x) ^ (2 p)
 + 40 a ^ 4 b ^ 3 p x ^ 3 (a + b x) ^ (2 p) + 30 a ^ 3 b ^ 4 x ^ 4 (a + b x
) ^ (2 p) + 30 a ^ 3 b ^ 4 p x ^ 4 (a + b x) ^ (2 p) + 12 a ^ 2 b ^ 5 x ^ 5
 (a + b x) ^ (2 p) + 12 a ^ 2 b ^ 5 p x ^ 5 (a + b x) ^ (2 p) + 2 a b ^ 6 x
 ^ 6 (a + b x) ^ (2 p) + 2 a b ^ 6 p x ^ 6 (a + b x) ^ (2 p)) + 3 a ^ 2 b ^
 2 x ^ 4 (c x ^ 2) ^ p / (2 a ^ 7 (a + b x) ^ (2 p) + 2 a ^ 7 p (a + b x) ^
 (2 p) + 12 a ^ 6 b x (a + b x) ^ (2 p) + 12 a ^ 6 b p x (a + b x) ^ (2 p)
+ 30 a ^ 5 b ^ 2 x ^ 2 (a + b x) ^ (2 p) + 30 a ^ 5 b ^ 2 p x ^ 2 (a + b x)
 ^ (2 p) + 40 a ^ 4 b ^ 3 x ^ 3 (a + b x) ^ (2 p) + 40 a ^ 4 b ^ 3 p x ^ 3
(a + b x) ^ (2 p) + 30 a ^ 3 b ^ 4 x ^ 4 (a + b x) ^ (2 p) + 30 a ^ 3 b ^ 4
 p x ^ 4 (a + b x) ^ (2 p) + 12 a ^ 2 b ^ 5 x ^ 5 (a + b x) ^ (2 p) + 12 a
^ 2 b ^ 5 p x ^ 5 (a + b x) ^ (2 p) + 2 a b ^ 6 x ^ 6 (a + b x) ^ (2 p) + 2
 a b ^ 6 p x ^ 6 (a + b x) ^ (2 p)) + a b ^ 3 x ^ 5 (c x ^ 2) ^ p / (2 a ^
7 (a + b x) ^ (2 p) + 2 a ^ 7 p (a + b x) ^ (2 p) + 12 a ^ 6 b x (a + b x)
```

```
^ (2 p) + 12 a ^ 6 b p x (a + b x) ^ (2 p) + 30 a ^ 5 b ^ 2 x ^ 2 (a + b x)
 ^ (2 p) + 30 a ^ 5 b ^ 2 p x ^ 2 (a + b x) ^ (2 p) + 40 a ^ 4 b ^ 3 x ^ 3
(a + b x) ^ (2 p) + 40 a ^ 4 b ^ 3 p x ^ 3 (a + b x) ^ (2 p) + 30 a ^ 3 b ^
 4 x ^ 4 (a + b x) ^ (2 p) + 30 a ^ 3 b ^ 4 p x ^ 4 (a + b x) ^ (2 p) + 12
a ^ 2 b ^ 5 x ^ 5 (a + b x) ^ (2 p) + 12 a ^ 2 b ^ 5 p x ^ 5 (a + b x) ^ (2
 p) + 2 a b ^ 6 x ^ 6 (a + b x) ^ (2 p) + 2 a b ^ 6 p x ^ 6 (a + b x) ^ (2
p))]
```

Maple [A]
time = 0.14, size = 32, normalized size = 0.97

method	result	size
gosper	$\frac{x^{2} \left(b x +a \right)^{-2-2 p} \left(c \,x^{2}\right)^{p}}{2 a \left(1+p \right)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*(b*x+a)^(-2-2*p)/a/(1+p)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 3)*x, x)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 1.15

$$\frac{(bx^3 + ax^2)\,(cx^2)^p\,(bx + a)^{-2\,p-3}}{2\,(ap + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="fricas")
[Out] 1/2*(b*x^3 + a*x^2)*(c*x^2)^p*(b*x + a)^(-2*p - 3)/(a*p + a)
```

Sympy [A]
time = 147.57, size = 1409, normalized size = 42.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**p*(b*x+a)**(-3-2*p),x)
```

```
[Out] Piecewise((-(c*x**2)**p/(b**3*x*(b*x)**(2*p)), Eq(a, 0)), (0**(-2*p - 3)*x**
*2*(c*x**2)**p/(2*p + 2), Eq(a, -b*x)), (x**2*(c*x**2)**p*(0**(1/p))**(-2*p
 - 3)/(2*p + 2), Eq(a, 0**(1/p) - b*x)), ((log(x)/a - log(a/b + x)/a)/c, Eq
(p, -1)), (a**4*x**2*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b
*x)**(2*p) + 12*a**6*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p)
+ 30*a**5*b**2*p*x**2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p)
 + 40*a**4*b**3*p*x**3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p
) + 30*a**3*b**4*p*x**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*
p) + 12*a**2*b**5*p*x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2
*p) + 2*a*b**6*p*x**6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) +
3*a**3*b*x**3*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b*x)**(2
*p) + 12*a**6*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p) + 30*a*
*5*b**2*p*x**2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p) + 40*a
**4*b**3*p*x**3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p) + 30*
a**3*b**4*p*x**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*p) + 12
*a**2*b**5*p*x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2*p) + 2
*a*b**6*p*x**6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) + 3*a**2*
b**2*x**4*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b*x)**(2*p)
+ 12*a**6*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p) + 30*a**5*b
**2*p*x**2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p) + 40*a**4*
b**3*p*x**3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p) + 30*a**3
*b**4*p*x**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*p) + 12*a**
2*b**5*p*x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2*p) + 2*a*b
**6*p*x**6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) + a*b**3*x**5
*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b*x)**(2*p) + 12*a**6
*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p) + 30*a**5*b**2*p*x**
2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p) + 40*a**4*b**3*p*x*
*3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p) + 30*a**3*b**4*p*x
**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*p) + 12*a**2*b**5*p*
x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2*p) + 2*a*b**6*p*x**
6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) + b*x**3*(c*x**2)**p/(
2*a**4*p*(a + b*x)**(2*p) + 2*a**4*(a + b*x)**(2*p) + 6*a**3*b*p*x*(a + b*x
)**(2*p) + 6*a**3*b*x*(a + b*x)**(2*p) + 6*a**2*b**2*p*x**2*(a + b*x)**(2*p
) + 6*a**2*b**2*x**2*(a + b*x)**(2*p) + 2*a*b**3*p*x**3*(a + b*x)**(2*p) +
2*a*b**3*x**3*(a + b*x)**(2*p)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(33) = 66$.
time = 0.01, size = 81, normalized size = 2.45

$$\frac{ax^2e^{-2p\ln(a+bx)-3\ln(a+bx)}e^{p\ln(cx^2)}+bx^3e^{-2p\ln(a+bx)-3\ln(a+bx)}e^{p\ln(cx^2)}}{2ap+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x)
```

```
[Out] 1/2*((c*x^2)^p*b*x^3*e^(-2*p*log(b*x + a) - 3*log(b*x + a)) + (c*x^2)^p*a*x
^2*e^(-2*p*log(b*x + a) - 3*log(b*x + a)))/(a*p + a)
```

Mupad [B]
time = 0.22, size = 33, normalized size = 1.00

$$\frac{x^2\,{\left(c\,x^2\right)}^p}{2\,a\,\left(p+1\right)\,{\left(a+b\,x\right)}^{2\,p+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^p)/(a + b*x)^(2*p + 3),x)
```

```
[Out] (x^2*(c*x^2)^p)/(2*a*(p + 1)*(a + b*x)^(2*p + 2))
```

3.990 $\int (cx^2)^p (a+bx)^{-2-2p} dx$

Optimal. Leaf size=30

$$\frac{x(cx^2)^p(a+bx)^{-1-2p}}{a(1+2p)}$$

```
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 37}

$$\frac{x(cx^2)^p(a+bx)^{-2p-1}}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (cx^2)^p (a+bx)^{-2-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(a+bx)^{-2-2p}\,dx\\ &= \frac{x(cx^2)^p(a+bx)^{-1-2p}}{a(1+2p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.93

$$\frac{x(cx^2)^p(a+bx)^{-1-2p}}{a+2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a + 2*a*p)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^0*(c*x^2)^p/(a + b*x)^(2*p + 2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.13, size = 31, normalized size = 1.03

method	result	size
gosper	$\frac{x\left(c\,x^{2}\right)^{p}\left(bx+a\right)^{-1-2p}}{a\left(1+2p\right)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

Fricas [**A**]
time = 0.30, size = 36, normalized size = 1.20

$$\frac{\left(bx^{2}+ax\right)\left(cx^{2}\right)^{p}\left(bx+a\right)^{-2\,p-2}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="fricas")
```

[Out] (b*x^2 + a*x)*(c*x^2)^p*(b*x + a)^(-2*p - 2)/(2*a*p + a)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{(bx)^{-2p}(cx^2)^p}{b^2x} & \text{for } a = 0 \\ \frac{0^{-2p-2}x(cx^2)^p}{2p+1} & \text{for } a = -bx \\ \frac{x(cx^2)^p\left(0^{\frac{1}{p}}\right)^{-2p-2}}{2p+1} & \text{for } a = 0^{\frac{1}{p}} - bx \\ \int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } p = -\frac{1}{2} \\ \frac{a^3x(cx^2)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}} + \frac{2a^2bx^2(cx^2)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}} + \frac{ab^2x^3(cx^2)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}} + \frac{bx^2(cx^2)^p}{2a^3p(a+bx)^{2p}+a^3(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2a^2bx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}+ab^2x^2(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**p*(b*x+a)**(-2-2*p),x)

[Out] Piecewise((-(c*x**2)**p/(b**2*x*(b*x)**(2*p)), Eq(a, 0)), (0**(-2*p - 2)*x*(c*x**2)**p/(2*p + 1), Eq(a, -b*x)), (x*(c*x**2)**p*(0**(1/p))**(-2*p - 2)/(2*p + 1), Eq(a, 0**(1/p) - b*x)), (Integral(1/(sqrt(c*x**2)*(a + b*x)), x), Eq(p, -1/2)), (a**3*x*(c*x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + 2*a**2*b*x**2*(c*x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + a*b**2*x**3*(c*x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + b*x**2*(c*x**2)**p/(2*a**3*p*(a + b*x)**(2*p) + a**3*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a**2*b*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p) + a*b**2*x**2*(a + b*x)**(2*p)), True))

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x)

[Out] Could not integrate

Mupad [B]
time = 0.20, size = 32, normalized size = 1.07

$$\frac{x\,{\left(c\,x^2\right)}^p}{a\,\left(2\,p+1\right)\,{\left(a+b\,x\right)}^{2\,p+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(a + b*x)^(2*p + 2),x)
[Out] (x*(c*x^2)^p)/(a*(2*p + 1)*(a + b*x)^(2*p + 1))
```

3.991 $\int \frac{(cx^2)^p(a+bx)^{-1-2p}}{x} dx$

Optimal. Leaf size=26

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

```
[Out] 1/2*(c*x^2)^p/a/p/((b*x+a)^(2*p))
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{-1+2p}(a+bx)^{-1-2p} dx\\ &= \frac{(cx^2)^p (a+bx)^{-2p}}{2ap}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 19.40, size = 284, normalized size = 10.92

$$\text{Piecewise}\left[\left\{\left\{-\frac{(bx)^{-2p}(cx^2)^p}{bx}, a==0\right\}, \left\{\frac{(cx^2)^p\text{ComplexInfinity}^{1+2p}}{p}, a==-bx\right\}, \left\{\frac{(cx^2)^p\left(0^{\frac{1}{p}}\right)^{-1-2p}}{2p}, a==-bx+0^{\frac{1}{p}}\right\}, \left\{\frac{\text{Log}[x]-\text{Log}\left[\frac{a}{b}+x\right]}{a}, p==0\right\}\right\}, \frac{a^2(cx^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}}+\frac{bx(cx^2)^p}{2a^2p(a+bx)^{2p}+2abpx(a+bx)^{2p}}+\frac{abx(cx^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^p/(a + b*x)^(2*p + 1)/x^1,x]')
```

```
[Out] Piecewise[{{-(b x) ^ (-2 p) (c x ^ 2) ^ p / (b x), a == 0}, {(c x ^ 2) ^ p
ComplexInfinity ^ (1 + 2 p) / p, a == -b x}, {(c x ^ 2) ^ p (0 ^ (1 / p)) ^
 (-1 - 2 p) / (2 p), a == -b x + 0 ^ (1 / p)}, {(Log[x] - Log[a / b + x]) /
 a, p == 0}}, a ^ 2 (c x ^ 2) ^ p / (2 a ^ 3 p (a + b x) ^ (2 p) + 4 a ^ 2
b p x (a + b x) ^ (2 p) + 2 a b ^ 2 p x ^ 2 (a + b x) ^ (2 p)) + b x (c x ^
 2) ^ p / (2 a ^ 2 p (a + b x) ^ (2 p) + 2 a b p x (a + b x) ^ (2 p)) + a b
 x (c x ^ 2) ^ p / (2 a ^ 3 p (a + b x) ^ (2 p) + 4 a ^ 2 b p x (a + b x) ^
 (2 p) + 2 a b ^ 2 p x ^ 2 (a + b x) ^ (2 p))]
```

Maple [A]
time = 0.12, size = 25, normalized size = 0.96

method	result	size
gosper	$\frac{(bx+a)^{-2p}(c x^2)^p}{2ap}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(-2*p)/a/p*(c*x^2)^p
```

Maxima [A]
time = 0.26, size = 27, normalized size = 1.04

$$\frac{c^p e^{(-2\,p\log(bx+a)+2\,p\log(x))}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c^p*e^(-2*p*log(b*x + a) + 2*p*log(x))/(a*p)
```

Fricas [A]
time = 0.31, size = 31, normalized size = 1.19

$$\frac{(bx + a)(cx^2)^p (bx + a)^{-2p-1}}{2ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p - 1)/(a*p)
```

Sympy [A]
time = 18.54, size = 250, normalized size = 9.62

$$\begin{cases} -\frac{(bx)^{-2p}(cx^2)^p}{bx} & \text{for } a = 0 \\ \frac{0^{-2p-1}(cx^2)^p}{2p} & \text{for } a = -bx \\ \frac{(cx^2)^p\left(0^{\frac{1}{p}}\right)^{-2p-1}}{2p} & \text{for } a = 0^{\frac{1}{p}} - bx \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for } p = 0 \\ \frac{a^2(cx^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}} + \frac{abx(cx^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}} + \frac{bx(cx^2)^p}{2a^2p(a+bx)^{2p}+2abpx(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-1-2*p)/x,x)
```

```
[Out] Piecewise((-(c*x**2)**p/(b*x*(b*x)**(2*p)), Eq(a, 0)), (0**(-2*p - 1)*(c*x*
*2)**p/(2*p), Eq(a, -b*x)), ((c*x**2)**p*(0**(1/p))**(-2*p - 1)/(2*p), Eq(a
, 0**(1/p) - b*x)), (log(x)/a - log(a/b + x)/a, Eq(p, 0)), (a**2*(c*x**2)**
p/(2*a**3*p*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x
**2*(a + b*x)**(2*p)) + a*b*x*(c*x**2)**p/(2*a**3*p*(a + b*x)**(2*p) + 4*a*
*2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p)) + b*x*(c*x**2
)**p/(2*a**2*p*(a + b*x)**(2*p) + 2*a*b*p*x*(a + b*x)**(2*p)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.26, size = 26, normalized size = 1.00

$$\frac{(c\,x^2)^p}{2\,a\,p\,(a+b\,x)^{2\,p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x*(a + b*x)^(2*p + 1)),x)
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

3.992 $\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

```
[Out] -(c*x^2)^p*(b*x+a)^(1-2*p)/a/(1-2*p)/x
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(1 - 2*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{-2+2p}(a+bx)^{-2p} dx \\ &= -\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{1-2p}}{a(-1+2p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(-1 + 2*p)*x)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^p/(a + b*x)^(2*p + 0)/x^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.15, size = 38, normalized size = 1.15

method	result
gosper	$\frac{(bx+a)\left(c\,x^{2}\right)^{p}(bx+a)^{-2p}}{xa(2p-1)}$
risch	$\frac{(bx+a)(bx+a)^{-2p}\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix^{2}\right)^{3}+2i\pi\mathrm{csgn}\left(ix^{2}\right)^{2}\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}(ix)^{2}+i\pi\,\mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}\left(ic\,x^{2}\right)^{2}-i\pi\,\mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}\left(ic\,x^{2}\right)\mathrm{csg}}{2}}}{(2p-1)ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/x^2/((b*x+a)^(2*p)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x*(b*x+a)/a/(2*p-1)*(c*x^2)^p/((b*x+a)^(2*p))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

Fricas [A]
time = 0.31, size = 37, normalized size = 1.12

$$\frac{(bx+a)\left(cx^{2}\right)^{p}}{(2\,ap-a)(bx+a)^{2\,p}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="fricas")
```

```
[Out] (b*x + a)*(c*x^2)^p/((2*a*p - a)*(b*x + a)^(2*p)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{\sqrt{cx^2}}{bx^2} & \text{for } a = 0 \wedge p = \frac{1}{2} \\ -\frac{(bx)^{-2p}(cx^2)^p}{x} & \text{for } a = 0 \\ \int \frac{\sqrt{cx^2}}{x^2(a+bx)}\, dx & \text{for } p = \frac{1}{2} \\ \frac{a(cx^2)^p}{2apx(a+bx)^{2p} - ax(a+bx)^{2p}} + \frac{bx(cx^2)^p}{2apx(a+bx)^{2p} - ax(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p/x**2/((b*x+a)**(2*p)),x)
```

```
[Out] Piecewise((-sqrt(c*x**2)/(b*x**2), Eq(a, 0) & Eq(p, 1/2)), (-(c*x**2)**p/(x
*(b*x)**(2*p)), Eq(a, 0)), (Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x), Eq(
p, 1/2)), (a*(c*x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(a + b*x)**(2*p))
+ b*x*(c*x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(a + b*x)**(2*p)), True))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.24, size = 32, normalized size = 0.97

$$\frac{(c\,x^2)^p\,(a+b\,x)^{1-2\,p}}{a\,x\,(2\,p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x^2*(a + b*x)^(2*p)),x)
```

```
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*x*(2*p - 1))
```

3.993 $\int \frac{(cx^2)^p(a+bx)^{1-2p}}{x^3} dx$

Optimal. Leaf size=35

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

```
[Out] -1/2*(c*x^2)^p*(b*x+a)^(2-2*p)/a/(1-p)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] -1/2*((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(1 - p)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{-3+2p}(a+bx)^{1-2p} dx \\ &= -\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.91

$$\frac{(cx^2)^p (a+bx)^{2-2p}}{a(-2+2p)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(-2 + 2*p)*x^2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^p/(a + b*x)^(2*p - 1)/x^3,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.15, size = 32, normalized size = 0.91

method	result
gosper	$\frac{(bx+a)^{2-2p}(c\,x^2)^p}{2x^2a(p-1)}$
risch	$\frac{(bx+a)^{1-2p}(bx+a)\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix^2\right)^3+2i\pi\mathrm{csgn}\left(ix^2\right)^2\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}(ix)^2+i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}\left(ic\,x^2\right)^2-i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}\left(ic\,x^2\right)\mathrm{csgn}(}{2}}}{2x^2a(p-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^2*(b*x+a)^(2-2*p)/a/(p-1)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

Fricas [A]
time = 0.30, size = 37, normalized size = 1.06

$$\frac{(bx+a)\left(cx^2\right)^p(bx+a)^{-2\,p+1}}{2\,(ap-a)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 1)/((a*p - a)*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(1-2*p)/x**3,x)
```

```
[Out] Integral((c*x**2)**p*(a + b*x)**(1 - 2*p)/x**3, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.25, size = 50, normalized size = 1.43

$$\frac{\left(\frac{(c\,x^2)^p}{2\,(p-1)}+\frac{b\,x\,(c\,x^2)^p}{2\,a\,(p-1)}\right)\,(a+b\,x)^{1-2\,p}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x)
```

```
[Out] (((c*x^2)^p/(2*(p - 1)) + (b*x*(c*x^2)^p)/(2*a*(p - 1)))*(a + b*x)^(1 - 2*p
))/x^2
```

3.994 $\int \frac{(cx^2)^p(a+bx)^{2-2p}}{x^4}\,dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(3-2p)x^3}$$

```
[Out] -(c*x^2)^p*(b*x+a)^(3-2*p)/a/(3-2*p)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$-\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(3-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(3 - 2*p)*x^3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p\,(a+bx)^{2-2p}}{x^4}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{-4+2p}(a+bx)^{2-2p}\,dx\\ &= -\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(3-2p)x^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(-3+2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
[Out] ((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(-3 + 2*p)*x^3)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c*x^2)^p/(a + b*x)^(2*p - 2)/x^4,x]')
[Out] Timed out
```

Maple [A]
time = 0.19, size = 33, normalized size = 1.00

method	result
gosper	$\frac{(bx+a)^{3-2p}(c\,x^{2})^{p}}{x^{3}a(2p-3)}$
risch	$\frac{(bx+a)^{2-2p}(bx+a)\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix^{2}\right)^{3}+2i\pi\mathrm{csgn}\left(ix^{2}\right)^{2}\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}(ix)^{2}+i\pi\,\mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}\left(ic\,x^{2}\right)^{2}-i\pi\,\mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}\left(ic\,x^{2}\right)\mathrm{csg}}{2}}}{x^{3}a(2p-3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/x^3*(b*x+a)^(3-2*p)/a/(2*p-3)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

Fricas [A]
time = 0.30, size = 37, normalized size = 1.12

$$\frac{(bx+a)\,(cx^2)^p\,(bx+a)^{-2\,p+2}}{(2\,ap-3\,a)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 2)/((2*a*p - 3*a)*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p\,(a+bx)^{2-2p}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(2-2*p)/x**4,x)
```

```
[Out] Integral((c*x**2)**p*(a + b*x)**(2 - 2*p)/x**4, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.25, size = 51, normalized size = 1.55

$$\frac{\left(\frac{(c\,x^2)^p}{2\,p-3}+\frac{b\,x\,(c\,x^2)^p}{a\,(2\,p-3)}\right)\,(a+b\,x)^{2-2\,p}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x)
```

```
[Out] (((c*x^2)^p/(2*p - 3) + (b*x*(c*x^2)^p)/(a*(2*p - 3)))*(a + b*x)^(2 - 2*p))
/x^3
```

3.995 $\int x^m (cx^2)^p (a+bx)^{-2-m-2p} \, dx$

Optimal. Leaf size=38

$$\frac{x^{1+m} (cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

```
[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {15, 37}

$$\frac{x^{m+1} (cx^2)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int x^m (cx^2)^p (a+bx)^{-2-m-2p} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{m+2p}(a+bx)^{-2-m-2p} \, dx \\ &= \frac{x^{1+m} (cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)} \end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 1.00

$$\frac{x^{1+m} (cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(c*x^2)^p/(a + b*x)^(2*p + m + 2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8012 deep
```

Maple [**A**]
time = 0.16, size = 39, normalized size = 1.03

method	result	size
gosper	$\frac{x^{1+m} \left(c\,x^{2}\right)^{p} \left(b x +a \right)^{-1-m-2 p}}{a \left(1+m +2 p \right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

Fricas [**A**]
time = 0.31, size = 49, normalized size = 1.29

$$\frac{(bx^2 + ax)(bx + a)^{-m-2\,p-2} x^m e^{(p\log(c)+2\,p\log(x))}}{am + 2\,ap + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*x^m*e^(p*log(c) + 2*p*log(x))/(a*m +
 2*a*p + a)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
[Out] Could not integrate
```

Mupad [**B**]
time = 0.34, size = 50, normalized size = 1.32

$$\frac{x\,x^m\,{\left(c\,x^2\right)}^p}{a\,{\left(a+b\,x\right)}^m\,{\left(a+b\,x\right)}^{2\,p}\,\left(a+b\,x\right)\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
[Out] (x*x^m*(c*x^2)^p)/(a*(a + b*x)^m*(a + b*x)^(2*p)*(a + b*x)*(m + 2*p + 1))
```

3.996 $\int (dx)^m (cx^2)^p (a+bx)^{-2-m-2p} \, dx$

Optimal. Leaf size=39

$$\frac{x(dx)^m (cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

```
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 20, 37}

$$\frac{x (cx^2)^p (dx)^m (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^{-2-m-2p}\,dx \\ &= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx \\ &= \frac{x(dx)^m\left(cx^2\right)^p(a+bx)^{-1-m-2p}}{a(1+m+2p)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{x(dx)^m\left(cx^2\right)^p(a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^p/(a + b*x)^(2*p + m + 2),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 8012 deep
```

Maple [A]
time = 0.21, size = 40, normalized size = 1.03

method	result	size
gosper	$\frac{x(dx)^m\left(c\,x^2\right)^p(bx+a)^{-1-m-2p}}{a(1+m+2p)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x,method=_RETURNVERBOSE)
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

Fricas [A]
time = 0.31, size = 57, normalized size = 1.46

$$\frac{(bx^2 + ax)(bx + a)^{-m-2\,p-2}\,(dx)^m\,e^{\left(2\,p\log(dx)+p\log\left(\frac{c}{d^2}\right)\right)}}{am + 2\,ap + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*(d*x)^m*e^(2*p*log(d*x) + p*log(c/d^
2))/(a*m + 2*a*p + a)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.26, size = 39, normalized size = 1.00

$$\frac{x\,{\left(d\,x\right)}^m\,{\left(c\,x^2\right)}^p}{a\,{\left(a+b\,x\right)}^{m+2\,p+1}\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
```

```
[Out] (x*(d*x)^m*(c*x^2)^p)/(a*(a + b*x)^(m + 2*p + 1)*(m + 2*p + 1))
```

3.997 $\int x^m \left(cx^2\right)^p (a+bx)^n \, dx$

Optimal. Leaf size=63

$$\frac{x^{1+m} \left(cx^2\right)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

```
[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^n*hypergeom([-n, 1+m+2*p],[2+m+2*p],-b*x/a)/(1+m+
2*p)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {15, 68, 66}

$$\frac{x^{m+1} \left(cx^2\right)^p (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\int x^m (cx^2)^p (a+bx)^n\,dx = (x^{-2p}(cx^2)^p)\int x^{m+2p}(a+bx)^n\,dx$$
$$= \left(x^{-2p}(cx^2)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{m+2p}\left(1+\frac{bx}{a}\right)^n\,dx$$
$$= \frac{x^{1+m}(cx^2)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$\frac{x^{1+m}(cx^2)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^n,x]
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[x^m*(c*x^2)^p*(a + b*x)^n,x]')
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m (c\,x^2)^p (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^n,x)
[Out] int(x^m*(c*x^2)^p*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(cx^2\right)^p (a + bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**n,x)
```

```
[Out] Integral(x**m*(c*x**2)**p*(a + b*x)**n, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(c\,x^2\right)^p (a + b\,x)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(a + b*x)^n,x)
```

```
[Out] int(x^m*(c*x^2)^p*(a + b*x)^n, x)
```

3.998 $\int (dx)^m (cx^2)^p (a+bx)^n \, dx$

Optimal. Leaf size=68

$$\frac{(dx)^{1+m} (cx^2)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{d(1+m+2p)}$$

```
[Out] (d*x)^(1+m)*(c*x^2)^p*(b*x+a)^n*hypergeom([-n, 1+m+2*p],[2+m+2*p],-b*x/a)/d
/(1+m+2*p)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 68, 66}

$$\frac{x (cx^2)^p (dx)^m (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^p (a+bx)^n\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^n\,dx\\ &= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^n\,dx\\ &= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{m+2p}\left(1+\frac{bx}{a}\right)^n dx\\ &= \frac{x(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.94

$$\frac{x(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]')
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^2\right)^p (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (dx)^m (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**p*(d*x)**m*(a + b*x)**n, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(c\,x^2\right)^p\,(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(a + b*x)^n,x)
[Out] int((d*x)^m*(c*x^2)^p*(a + b*x)^n, x)
```

3.999 $\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$\frac{b^2(a+bx)^3}{3d^3}$$

[Out] 1/3*b^2*(b*x+a)^3/d^3

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/((a*d)/b + d*x)^3,x]

[Out] (b^2*(a + b*x)^3)/(3*d^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int (a+bx)^2\,dx}{d^3}\\ &= \frac{b^2(a+bx)^3}{3d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Mathics [A]
time = 1.66, size = 27, normalized size = 1.59

$$\frac{b^3x\,(3a^2+3abx+b^2x^2)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*d/b + d*x)^3,x]')
```

```
[Out] b ^ 3 x (3 a ^ 2 + 3 a b x + b ^ 2 x ^ 2) / (3 d ^ 3)
```

Maple [A]
time = 0.13, size = 16, normalized size = 0.94

method	result	size
default	$\frac{b^{2}\left(bx+a\right)^{3}}{3d^{3}}$	16
gosper	$\frac{b^{3}x\left(x^{2}b^{2}+3abx+3a^{2}\right)}{3d^{3}}$	28
risch	$\frac{b^{5}x^{3}}{3d^{3}}+\frac{b^{4}a\,x^{2}}{d^{3}}+\frac{b^{3}a^{2}x}{d^{3}}+\frac{b^{2}a^{3}}{3d^{3}}$	46
norman	$\frac{\frac{b^{7}x^{5}}{3d}+\frac{5a\,b^{6}x^{4}}{3d}+\frac{10a^{2}b^{5}x^{3}}{3d}-\frac{3a^{5}b^{2}}{d}-\frac{5a^{4}b^{3}x}{d}}{d^{2}\left(bx+a\right)^{2}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*(b*x+a)^3/d^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.
time = 0.26, size = 31, normalized size = 1.82

$$\frac{b^5x^3+3\,ab^4x^2+3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.29, size = 31, normalized size = 1.82

$$\frac{b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.06, size = 34, normalized size = 2.00

$$\frac{a^2b^3x}{d^3} + \frac{ab^4x^2}{d^3} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*d/b+d*x)**3,x)
```

```
[Out] a**2*b**3*x/d**3 + a*b**4*x**2/d**3 + b**5*x**3/(3*d**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.00, size = 32, normalized size = 1.88

$$\frac{\frac{1}{3}x^3b^5 + x^2b^4a + xb^3a^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x)
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Mupad [B]
time = 0.05, size = 27, normalized size = 1.59

$$\frac{b^3\,x\,\left(3\,a^2 + 3\,a\,b\,x + b^2\,x^2\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*d^3)
```

3.1000 $\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=23

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

[Out] a*b^3*x/d^3+1/2*b^4*x^2/d^3

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {21}

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^4/((a*d)/b + d*x)^3,x]

[Out] (a*b^3*x)/d^3 + (b^4*x^2)/(2*d^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx)\,dx}{d^3} \\ &= \frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^3\left(ax+\frac{bx^2}{2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] (b^3*(a*x + (b*x^2)/2))/d^3
```

Mathics [A]
time = 1.61, size = 16, normalized size = 0.70

$$\frac{b^3x\,(2a+bx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(a*d/b + d*x)^3,x]')
[Out] b ^ 3 x (2 a + b x) / (2 d ^ 3)
```

Maple [A]
time = 0.14, size = 18, normalized size = 0.78

method	result	size
gosper	$\frac{b^{3} x (b x +2 a)}{2 d^{3}}$	17
default	$\frac{b^{3} \left(\frac{1}{2} x^{2} b +a x \right)}{d^{3}}$	18
risch	$\frac{a\,b^{3} x}{d^{3}}+\frac{b^{4} x^{2}}{2 d^{3}}$	22
norman	$\frac{\frac{b^{6} x^{4}}{2 d}+\frac{2 a\,b^{5} x^{3}}{d}-\frac{5 a^{4} b^{2}}{2 d}-\frac{4 a^{3} b^{3} x}{d}}{d^{2} \left(b x +a \right)^{2}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
[Out] b^3/d^3*(1/2*x^2*b+a*x)
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.87

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Fricas [A]
time = 0.29, size = 20, normalized size = 0.87

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.87

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*d/b+d*x)**3,x)
```

```
[Out] a*b**3*x/d**3 + b**4*x**2/(2*d**3)
```

Giac [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{\frac{1}{2}x^2b^4 + xb^3a}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x)
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^3\,x\,\left(2\,a+b\,x\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x*(2*a + b*x))/(2*d^3)
```

3.1001 $\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

```
[Out] b^3*x/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int 1 \, dx}{d^3} \\ &= \frac{b^3x}{d^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Mathics [A]
time = 1.54, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/(a*d/b + d*x)^3,x]')
```

```
[Out] b ^ 3 x / d ^ 3
```

Maple [A]
time = 0.15, size = 9, normalized size = 1.12

method	result	size
default	$\frac{b^3x}{d^3}$	9
risch	$\frac{b^3x}{d^3}$	9
norman	$\frac{\frac{b^5x^3}{d}-\frac{2a^3b^2}{d}-\frac{3a^2b^3x}{d}}{d^2(bx+a)^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3
```

Maxima [A]
time = 0.25, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

Fricas [A]
time = 0.29, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] b**3*x/d**3
```

Giac [A]
time = 0.00, size = 9, normalized size = 1.12

$$\frac{xb^3}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] b^3*x/d^3
```

Mupad [B]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x)/d^3
```

3.1002 $\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=13

$$\frac{b^2 \log(a+bx)}{d^3}$$

```
[Out] b^2*ln(b*x+a)/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 31}

$$\frac{b^2 \log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int \frac{1}{a+bx} dx}{d^3} \\ &= \frac{b^2 \log(a+bx)}{d^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2 \log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Mathics [A]
time = 1.63, size = 17, normalized size = 1.31

$$\frac{b^2 \text{Log}\left[d^3\left(a+bx\right)\right]}{d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*d/b + d*x)^3,x]')
```

```
[Out] b ^ 2 Log[d ^ 3 (a + b x)] / d ^ 3
```

Maple [A]
time = 0.15, size = 14, normalized size = 1.08

method	result	size
default	$\frac{b^{2} \ln \left(b x +a \right)}{d^{3}}$	14
norman	$\frac{b^{2} \ln \left(b x +a \right)}{d^{3}}$	14
risch	$\frac{b^{2} \ln \left(b x +a \right)}{d^{3}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(b*x+a)/d^3
```

Maxima [A]
time = 0.27, size = 13, normalized size = 1.00

$$\frac{b^2 \log\left(bx+a\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/d^3
```

Fricas [A]
time = 0.30, size = 13, normalized size = 1.00

$$\frac{b^2 \log\left(bx+a\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(b*x + a)/d^3
```

Sympy [A]
time = 0.05, size = 19, normalized size = 1.46

$$\frac{b^2 \log \left(a d^3 + b d^3 x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] b**2*log(a*d**3 + b*d**3*x)/d**3
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.31

$$\frac{b^3 \ln |xb + a|}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] b^2*log(abs(b*x + a))/d^3
```

Mupad [B]
time = 0.05, size = 13, normalized size = 1.00

$$\frac{b^2 \ln (a + b\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^2*log(a + b*x))/d^3
```

3.1003 $\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{b^2}{d^3(a+bx)}$$

[Out] -b^2/d^3/(b*x+a)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/((a*d)/b + d*x)^3,x]

[Out] -(b^2/(d^3*(a + b*x)))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^2}\,dx}{d^3}\\ &= -\frac{b^2}{d^3(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((a*d)/b + d*x)^3,x]
[Out] -(b^2/(d^3*(a + b*x)))
```

Mathics [A]
time = 1.67, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1/(a*d/b + d*x)^3,x]')
[Out] -b ^ 2 / (d ^ 3 (a + b x))
```

Maple [A]
time = 0.13, size = 16, normalized size = 1.07

method	result	size
gosper	$-\frac{b^2}{d^3(bx+a)}$	16
default	$-\frac{b^2}{d^3(bx+a)}$	16
risch	$-\frac{b^2}{d^3(bx+a)}$	16
norman	$\frac{-\frac{a\,b^2}{d}-\frac{b^3x}{d}}{d^2(bx+a)^2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
[Out] -b^2/d^3/(b*x+a)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x+ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] -b^2/(b*d^3*x + a*d^3)
```

Fricas [A]
time = 0.29, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x+ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -b^2/(b*d^3*x + a*d^3)
```

Sympy [A]
time = 0.08, size = 19, normalized size = 1.27

$$-\frac{b^3}{abd^3 + b^2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(a*b*d**3 + b**2*d**3*x)
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3(xb + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x)
```

```
[Out] -b^2/((b*x + a)*d^3)
```

Mupad [B]
time = 0.04, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3(a + b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d*x + (a*d)/b)^3,x)
```

```
[Out] -b^2/(d^3*(a + b*x))
```

3.1004 $\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=17

$$-\frac{b^2}{3d^3(a+bx)^3}$$

[Out] -1/3*b^2/d^3/(b*x+a)^3

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)*((a*d)/b + d*x)^3),x]

[Out] -1/3*b^2/(d^3*(a + b*x)^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int \frac{1}{(a+bx)^4} dx}{d^3}\\ &= -\frac{b^2}{3d^3(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/3*b^2/(d^3*(a + b*x)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.
time = 1.94, size = 37, normalized size = 2.18

$$-\frac{b^2}{3d^3 (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)*(a*d/b + d*x)^3),x]')
```

```
[Out] -b ^ 2 / (3 d ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{b^2}{3d^3(bx+a)^3}$	16
default	$-\frac{b^2}{3d^3(bx+a)^3}$	16
norman	$-\frac{b^2}{3d^3(bx+a)^3}$	16
risch	$-\frac{b^2}{3d^3(bx+a)^3}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2/d^3/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.
time = 0.26, size = 47, normalized size = 2.76

$$-\frac{b^2}{3 (b^3d^3x^3 + 3\, ab^2d^3x^2 + 3\, a^2bd^3x + a^3d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.29, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\,(b^3d^3x^3 + 3\,ab^2d^3x^2 + 3\,a^2bd^3x + a^3d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.14, size = 53, normalized size = 3.12

$$-\frac{b^3}{3a^3bd^3 + 9a^2b^2d^3x + 9ab^3d^3x^2 + 3b^4d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(3*a**3*b*d**3 + 9*a**2*b**2*d**3*x + 9*a*b**3*d**3*x**2 + 3*b**4*d**
3*x**3)
```

Giac [A]
time = 0.00, size = 18, normalized size = 1.06

$$-\frac{b^2}{3d^3\left(xb + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x)
```

```
[Out] -1/3*b^2/((b*x + a)^3*d^3)
```

Mupad [B]
time = 0.15, size = 49, normalized size = 2.88

$$-\frac{b^2}{3\,\left(a^3\,d^3 + 3\,a^2\,b\,d^3\,x + 3\,a\,b^2\,d^3\,x^2 + b^3\,d^3\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)),x)
```

```
[Out] -b^2/(3*(a^3*d^3 + b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x))
```

3.1005 $\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{4d^3(a+bx)^4}$$

[Out] -1/4*b^2/d^3/(b*x+a)^4

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/4*b^2/(d^3*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^5}\,dx}{d^3}\\ &= -\frac{b^2}{4d^3(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/4*b^2/(d^3*(a + b*x)^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(17) = 34.
time = 2.07, size = 48, normalized size = 2.82

$$-\frac{b^2}{4d^3\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^2*(a*d/b + d*x)^3),x]')
```

```
[Out] -b ^ 2 / (4 d ^ 3 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x
^ 3 + b ^ 4 x ^ 4))
```

Maple [A]
time = 0.14, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{b^2}{4d^3(bx+a)^4}$	16
default	$-\frac{b^2}{4d^3(bx+a)^4}$	16
norman	$-\frac{b^2}{4d^3(bx+a)^4}$	16
risch	$-\frac{b^2}{4d^3(bx+a)^4}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^2/d^3/(b*x+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(15) = 30.
time = 0.25, size = 61, normalized size = 3.59

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

time = 0.29, size = 61, normalized size = 3.59

$$-\frac{b^2}{4\,(b^4d^3x^4 + 4\,ab^3d^3x^3 + 6\,a^2b^2d^3x^2 + 4\,a^3bd^3x + a^4d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(15) = 30$.

time = 0.17, size = 68, normalized size = 4.00

$$-\frac{b^3}{4a^4bd^3 + 16a^3b^2d^3x + 24a^2b^3d^3x^2 + 16ab^4d^3x^3 + 4b^5d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(4*a**4*b*d**3 + 16*a**3*b**2*d**3*x + 24*a**2*b**3*d**3*x**2 + 16*a*
b**4*d**3*x**3 + 4*b**5*d**3*x**4)
```

Giac [A]

time = 0.00, size = 18, normalized size = 1.06

$$-\frac{b^2}{4d^3\,(xb + a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] -1/4*b^2/((b*x + a)^4*d^3)
```

Mupad [B]

time = 0.06, size = 63, normalized size = 3.71

$$-\frac{b^2}{4\,\left(a^4\,d^3 + 4\,a^3\,b\,d^3\,x + 6\,a^2\,b^2\,d^3\,x^2 + 4\,a\,b^3\,d^3\,x^3 + b^4\,d^3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^2),x)
```

```
[Out] -b^2/(4*(a^4*d^3 + b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^
3*b*d^3*x))
```

3.1006 $\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{5d^3(a+bx)^5}$$

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{21, 32\}$

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/5*b^2/(d^3*(a + b*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^6}\,dx}{d^3}\\ &= -\frac{b^2}{5d^3(a+bx)^5}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/5*b^2/(d^3*(a + b*x)^5)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.
time = 2.22, size = 59, normalized size = 3.47

$$-\frac{b^2}{5d^3\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^3*(a*d/b + d*x)^3),x]')
```

```
[Out] -b ^ 2 / (5 d ^ 3 (a ^ 5 + 5 a ^ 4 b x + 10 a ^ 3 b ^ 2 x ^ 2 + 10 a ^ 2 b
^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 + b ^ 5 x ^ 5))
```

Maple [A]
time = 0.14, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{b^2}{5d^3(bx+a)^5}$	16
default	$-\frac{b^2}{5d^3(bx+a)^5}$	16
norman	$-\frac{b^2}{5d^3(bx+a)^5}$	16
risch	$-\frac{b^2}{5d^3(bx+a)^5}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.26, size = 75, normalized size = 4.41

$$-\frac{b^2}{5\left(b^5d^3x^5+5\,ab^4d^3x^4+10\,a^2b^3d^3x^3+10\,a^3b^2d^3x^2+5\,a^4bd^3x+a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.29, size = 75, normalized size = 4.41

$$-\frac{b^2}{5\left(b^5d^3x^5 + 5\,ab^4d^3x^4 + 10\,a^2b^3d^3x^3 + 10\,a^3b^2d^3x^2 + 5\,a^4bd^3x + a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(15) = 30$.
time = 0.21, size = 83, normalized size = 4.88

$$-\frac{b^3}{5a^5bd^3 + 25a^4b^2d^3x + 50a^3b^3d^3x^2 + 50a^2b^4d^3x^3 + 25ab^5d^3x^4 + 5b^6d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(5*a**5*b*d**3 + 25*a**4*b**2*d**3*x + 50*a**3*b**3*d**3*x**2 + 50*a*
*2*b**4*d**3*x**3 + 25*a*b**5*d**3*x**4 + 5*b**6*d**3*x**5)
```

Giac [A]
time = 0.00, size = 18, normalized size = 1.06

$$-\frac{b^2}{5d^3\left(xb + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] -1/5*b^2/((b*x + a)^5*d^3)
```

Mupad [B]
time = 0.05, size = 77, normalized size = 4.53

$$-\frac{b^2}{5\,\left(a^5\,d^3 + 5\,a^4\,b\,d^3\,x + 10\,a^3\,b^2\,d^3\,x^2 + 10\,a^2\,b^3\,d^3\,x^3 + 5\,a\,b^4\,d^3\,x^4 + b^5\,d^3\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^3),x)
```

```
[Out] -b^2/(5*(a^5*d^3 + b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^3*b^2*d^3*x^2 + 10*
a^2*b^3*d^3*x^3 + 5*a^4*b*d^3*x))
```

3.1007 $\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{b^5(c+dx)^3}{3d^6}$$

[Out] 1/3*b^5*(d*x+c)^3/d^6

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

[In] Int[((b*c)/d + b*x)^5/(c + d*x)^3,x]

[Out] (b^5*(c + d*x)^3)/(3*d^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx &= \frac{b^5\int (c+dx)^2\,dx}{d^5}\\ &= \frac{b^5(c+dx)^3}{3d^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Mathics [A]
time = 1.69, size = 27, normalized size = 1.59

$$\frac{b^5x\left(3c^2+3cdx+d^2x^2\right)}{3d^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(b*c/d + b*x)^5/(c + d*x)^3,x]')
```

```
[Out] b ^ 5 x (3 c ^ 2 + 3 c d x + d ^ 2 x ^ 2) / (3 d ^ 5)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{b^{5}\left(d x+c\right)^{3}}{3 d^{6}}$	16
gosper	$\frac{b^{5} x \left(d^{2} x^{2}+3 c d x+3 c^{2}\right)}{3 d^{5}}$	28
risch	$\frac{b^{5} x^{3}}{3 d^{3}}+\frac{b^{5} c \,x^{2}}{d^{4}}+\frac{b^{5} c^{2} x}{d^{5}}+\frac{b^{5} c^{3}}{3 d^{6}}$	46
norman	$\frac{\frac{b^{5} d^{3} x^{5}}{3}+\frac{5 c^{3} b^{5} x^{2}}{2}-\frac{c^{5} b^{5}}{2 d^{2}}+\frac{5 b^{5} c \,d^{2} x^{4}}{3}+\frac{10 b^{5} c^{2} d \,x^{3}}{3}}{d^{4} \left(d x+c\right)^{2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^5/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^5*(d*x+c)^3/d^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.27, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3+3\,b^5cdx^2+3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.29, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3 + 3\,b^5cdx^2 + 3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.06, size = 34, normalized size = 2.00

$$\frac{b^5c^2x}{d^5} + \frac{b^5cx^2}{d^4} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**5/(d*x+c)**3,x)
```

```
[Out] b**5*c**2*x/d**5 + b**5*c*x**2/d**4 + b**5*x**3/(3*d**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.00, size = 36, normalized size = 2.12

$$\frac{\frac{1}{3}x^3b^5d^2 + x^2b^5cd + xb^5c^2}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x)
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Mupad [B]
time = 0.16, size = 27, normalized size = 1.59

$$\frac{b^5\,x\,\left(3\,c^2+3\,c\,d\,x+d^2\,x^2\right)}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^5/(c + d*x)^3,x)
```

```
[Out] (b^5*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/(3*d^5)
```

3.1008 $\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx$

Optimal. Leaf size=23

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

```
[Out] b^4*c*x/d^4+1/2*b^4*x^2/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx &= \frac{b^4\int (c+dx)\,dx}{d^4}\\ &= \frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^4\left(cx+\frac{dx^2}{2}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*(c*x + (d*x^2)/2))/d^4
```

Mathics [A]
time = 1.60, size = 16, normalized size = 0.70

$$\frac{b^4x\,(2c+dx)}{2d^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(b*c/d + b*x)^4/(c + d*x)^3,x]')
```

```
[Out] b ^ 4 x (2 c + d x) / (2 d ^ 4)
```

Maple [A]
time = 0.13, size = 18, normalized size = 0.78

method	result	size
gosper	$\frac{b^4x(dx+2c)}{2d^4}$	17
default	$\frac{b^4\left(cx+\frac{1}{2}d\,x^2\right)}{d^4}$	18
risch	$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$	22
norman	$\frac{\frac{b^4d^2x^4}{2}-\frac{5c^4b^4}{2d^2}+2b^4cd\,x^3-\frac{4c^3b^4x}{d}}{d^3(dx+c)^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^4/d^4*(c*x+1/2*d*x^2)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.91

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Fricas [A]
time = 0.29, size = 21, normalized size = 0.91

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.87

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**4/(d*x+c)**3,x)
```

```
[Out] b**4*c*x/d**4 + b**4*x**2/(2*d**3)
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{\frac{1}{2}x^2b^4d+xb^4c}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x)
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^4\,x\,\left(2\,c+d\,x\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^4/(c + d*x)^3,x)
```

```
[Out] (b^4*x*(2*c + d*x))/(2*d^4)
```

3.1009 $\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3} \, dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

```
[Out] b^3*x/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3} \, dx &= \frac{b^3 \int 1 \, dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Mathics [A]
time = 1.51, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(b*c/d + b*x)^3/(c + d*x)^3,x]')
```

```
[Out] b ^ 3 x / d ^ 3
```

Maple [A]
time = 0.14, size = 9, normalized size = 1.12

method	result	size
default	$\frac{b^3x}{d^3}$	9
risch	$\frac{b^3x}{d^3}$	9
norman	$\frac{b^3dx^3-\frac{2c^3b^3}{d^2}-\frac{3c^2b^3x}{d}}{d^2(dx+c)^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3
```

Maxima [A]
time = 0.25, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

Fricas [A]
time = 0.29, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

<u>Sympy</u> [A]
time = 0.05, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3
```

<u>Giac</u> [A]
time = 0.00, size = 9, normalized size = 1.12

$$\frac{xb^3}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x)
```

```
[Out] b^3*x/d^3
```

<u>Mupad</u> [B]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^3/(c + d*x)^3,x)
```

```
[Out] (b^3*x)/d^3
```

3.1010 $\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} \, dx$

Optimal. Leaf size=13

$$\frac{b^2 \log(c+dx)}{d^3}$$

```
[Out] b^2*ln(d*x+c)/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 31}

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} \, dx &= \frac{b^2 \int \frac{1}{c+dx} \, dx}{d^2} \\ &= \frac{b^2 \log(c+dx)}{d^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Mathics [A]
time = 1.61, size = 17, normalized size = 1.31

$$\frac{b^2\mathrm{Log}\left[d^2\,(c+dx)\right]}{d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(b*c/d + b*x)^2/(c + d*x)^3,x]')
```

```
[Out] b ^ 2 Log[d ^ 2 (c + d x)] / d ^ 3
```

Maple [A]
time = 0.16, size = 14, normalized size = 1.08

method	result	size
default	$\frac{b^2\ln(dx+c)}{d^3}$	14
norman	$\frac{b^2\ln(dx+c)}{d^3}$	14
risch	$\frac{b^2\ln(dx+c)}{d^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(d*x+c)/d^3
```

Maxima [A]
time = 0.26, size = 13, normalized size = 1.00

$$\frac{b^2\log\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(d*x + c)/d^3
```

Fricas [A]
time = 0.29, size = 13, normalized size = 1.00

$$\frac{b^2\log\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(d*x + c)/d^3
```

Sympy [A]
time = 0.05, size = 17, normalized size = 1.31

$$\frac{b^2 \log \left(cd^2 + d^3 x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c*d**2 + d**3*x)/d**3
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.15

$$\frac{b^2 \ln |xd + c|}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x)
```

```
[Out] b^2*log(abs(d*x + c))/d^3
```

Mupad [B]
time = 0.14, size = 13, normalized size = 1.00

$$\frac{b^2 \ln \left(c + d\,x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3
```

3.1011 $\int \frac{\frac{bc}{d}+bx}{(c+dx)^3} \, dx$

Optimal. Leaf size=13

$$-\frac{b}{d^2(c+dx)}$$

```
[Out] -b/d^2/(d*x+c)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{bc}{d}+bx}{(c+dx)^3} \, dx &= \frac{b\int \frac{1}{(c+dx)^2} \, dx}{d} \\
&= -\frac{b}{d^2(c+dx)}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)/(c + d*x)^3,x]
[Out] -(b/(d^2*(c + d*x)))
```

Mathics [A]
time = 1.61, size = 13, normalized size = 1.00

$$-\frac{b}{d^2\left(c+dx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(b*c/d + b*x)^1/(c + d*x)^3,x]')
[Out] -b / (d ^ 2 (c + d x))
```

Maple [A]
time = 0.13, size = 14, normalized size = 1.08

method	result	size
gosper	$-\frac{b}{d^2(dx+c)}$	14
default	$-\frac{b}{d^2(dx+c)}$	14
risch	$-\frac{b}{d^2(dx+c)}$	14
norman	$\frac{-\frac{cb}{d^2}-\frac{bx}{d}}{(dx+c)^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -b/d^2/(d*x+c)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
[Out] -b/(d^3*x + c*d^2)
```

Fricas [A]
time = 0.29, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -b/(d^3*x + c*d^2)
```

Sympy [A]
time = 0.08, size = 12, normalized size = 0.92

$$-\frac{b}{cd^2 + d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -b/(c*d**2 + d**3*x)
```

Giac [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{b}{d^2\left(xd + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x)
```

```
[Out] -b/((d*x + c)*d^2)
```

Mupad [B]
time = 0.04, size = 13, normalized size = 1.00

$$-\frac{b}{d^2\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)/(c + d*x)^3,x)
```

```
[Out] -b/(d^2*(c + d*x))
```

3.1012 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(c+dx)^3}$$

[Out] -1/3/b/(d*x+c)^3

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

[In] Int[1/(((b*c)/d + b*x)*(c + d*x)^3),x]

[Out] -1/3*1/(b*(c + d*x)^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx &= \frac{d\int \frac{1}{(c+dx)^4}\,dx}{b}\\
&= -\frac{1}{3b(c+dx)^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/3*1/(b*(c + d*x)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.
time = 1.85, size = 34, normalized size = 2.43

$$-\frac{1}{3b\left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((b*c/d + b*x)*(c + d*x)^3),x]')
```

```
[Out] -1 / (3 b (c ^ 3 + 3 c ^ 2 d x + 3 c d ^ 2 x ^ 2 + d ^ 3 x ^ 3))
```

Maple [A]
time = 0.15, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{3b(dx+c)^3}$	13
default	$-\frac{1}{3b(dx+c)^3}$	13
norman	$-\frac{1}{3b(dx+c)^3}$	13
risch	$-\frac{1}{3b(dx+c)^3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/(d*x+c)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.
time = 0.26, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.
time = 0.29, size = 36, normalized size = 2.57

$$-\frac{1}{3\,(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.
time = 0.14, size = 44, normalized size = 3.14

$$-\frac{d}{3bc^3d+9bc^2d^2x+9bcd^3x^2+3bd^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -d/(3*b*c**3*d + 9*b*c**2*d**2*x + 9*b*c*d**3*x**2 + 3*b*d**4*x**3)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{3b\,(xd+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x)
```

```
[Out] -1/3/((d*x + c)^3*b)
```

Mupad [B]
time = 0.05, size = 38, normalized size = 2.71

$$-\frac{1}{3\,b\,c^3+9\,b\,c^2\,d\,x+9\,b\,c\,d^2\,x^2+3\,b\,d^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)*(c + d*x)^3),x)
```

```
[Out] -1/(3*b*c^3 + 3*b*d^3*x^3 + 9*b*c^2*d*x + 9*b*c*d^2*x^2)
```

3.1013 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2(c+dx)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{d}{4b^2(c+dx)^4}$$

[Out] -1/4*d/b^2/(d*x+c)^4

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

[In] Int[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]

[Out] -1/4*d/(b^2*(c + d*x)^4)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2(c+dx)^3}\,dx &= \frac{d^2\int \frac{1}{(c+dx)^5}\,dx}{b^2}\\ &= -\frac{d}{4b^2(c+dx)^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/4*d/(b^2*(c + d*x)^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 2.06, size = 46, normalized size = 3.07

$$-\frac{d}{4b^2\,(c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((b*c/d + b*x)^2*(c + d*x)^3),x]')
```

```
[Out] -d / (4 b ^ 2 (c ^ 4 + 4 c ^ 3 d x + 6 c ^ 2 d ^ 2 x ^ 2 + 4 c d ^ 3 x ^ 3
+ d ^ 4 x ^ 4))
```

Maple [A]
time = 0.13, size = 14, normalized size = 0.93

method	result	size
gosper	$-\frac{d}{4b^2(dx+c)^4}$	14
default	$-\frac{d}{4b^2(dx+c)^4}$	14
norman	$-\frac{d}{4b^2(dx+c)^4}$	14
risch	$-\frac{d}{4b^2(dx+c)^4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/b^2/(d*x+c)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.
time = 0.25, size = 59, normalized size = 3.93

$$-\frac{d}{4\,(b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.
time = 0.29, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^2d^4x^4 + 4\,b^2cd^3x^3 + 6\,b^2c^2d^2x^2 + 4\,b^2c^3dx + b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 0.18, size = 68, normalized size = 4.53

$$-\frac{d^2}{4b^2c^4d + 16b^2c^3d^2x + 24b^2c^2d^3x^2 + 16b^2cd^4x^3 + 4b^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] -d**2/(4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2 + 16*b*
*2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

Giac [A]
time = 0.00, size = 16, normalized size = 1.07

$$-\frac{d}{4b^2\left(xd + c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x)
```

```
[Out] -1/4*d/((d*x + c)^4*b^2)
```

Mupad [B]
time = 0.05, size = 61, normalized size = 4.07

$$-\frac{d}{4\left(b^2\,c^4 + 4\,b^2\,c^3\,d\,x + 6\,b^2\,c^2\,d^2\,x^2 + 4\,b^2\,c\,d^3\,x^3 + b^2\,d^4\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^2*(c + d*x)^3),x)
```

```
[Out] -d/(4*(b^2*c^4 + b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*
c^3*d*x))
```

3.1014 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3(c+dx)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{d^2}{5b^3(c+dx)^5}$$

[Out] -1/5*d^2/b^3/(d*x+c)^5

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

[In] Int[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]

[Out] -1/5*d^2/(b^3*(c + d*x)^5)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3(c+dx)^3}\,dx &= \frac{d^3\int \frac{1}{(c+dx)^6}\,dx}{b^3}\\ &= -\frac{d^2}{5b^3(c+dx)^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/5*d^2/(b^3*(c + d*x)^5)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.
time = 2.20, size = 59, normalized size = 3.47

$$-\frac{d^2}{5b^3\left(c^5 + 5c^4dx + 10c^3d^2x^2 + 10c^2d^3x^3 + 5cd^4x^4 + d^5x^5\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((b*c/d + b*x)^3*(c + d*x)^3),x]')
```

```
[Out] -d ^ 2 / (5 b ^ 3 (c ^ 5 + 5 c ^ 4 d x + 10 c ^ 3 d ^ 2 x ^ 2 + 10 c ^ 2 d
^ 3 x ^ 3 + 5 c d ^ 4 x ^ 4 + d ^ 5 x ^ 5))
```

Maple [A]
time = 0.16, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{d^2}{5b^3(dx+c)^5}$	16
default	$-\frac{d^2}{5b^3(dx+c)^5}$	16
norman	$-\frac{d^2}{5b^3(dx+c)^5}$	16
risch	$-\frac{d^2}{5b^3(dx+c)^5}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*d^2/b^3/(d*x+c)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.25, size = 75, normalized size = 4.41

$$-\frac{d^2}{5\left(b^3d^5x^5 + 5\,b^3cd^4x^4 + 10\,b^3c^2d^3x^3 + 10\,b^3c^3d^2x^2 + 5\,b^3c^4dx + b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.29, size = 75, normalized size = 4.41

$$-\frac{d^2}{5\left(b^3d^5x^5 + 5\,b^3cd^4x^4 + 10\,b^3c^2d^3x^3 + 10\,b^3c^3d^2x^2 + 5\,b^3c^4dx + b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(15) = 30$.
time = 0.21, size = 83, normalized size = 4.88

$$-\frac{d^3}{5b^3c^5d + 25b^3c^4d^2x + 50b^3c^3d^3x^2 + 50b^3c^2d^4x^3 + 25b^3cd^5x^4 + 5b^3d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] -d**3/(5*b**3*c**5*d + 25*b**3*c**4*d**2*x + 50*b**3*c**3*d**3*x**2 + 50*b*
*3*c**2*d**4*x**3 + 25*b**3*c*d**5*x**4 + 5*b**3*d**6*x**5)
```

Giac [A]
time = 0.00, size = 18, normalized size = 1.06

$$-\frac{d^2}{5b^3\left(xd + c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x)
```

```
[Out] -1/5*d^2/((d*x + c)^5*b^3)
```

Mupad [B]
time = 0.17, size = 77, normalized size = 4.53

$$-\frac{d^2}{5\,\left(b^3\,c^5 + 5\,b^3\,c^4\,d\,x + 10\,b^3\,c^3\,d^2\,x^2 + 10\,b^3\,c^2\,d^3\,x^3 + 5\,b^3\,c\,d^4\,x^4 + b^3\,d^5\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^3*(c + d*x)^3),x)
```

```
[Out] -d^2/(5*(b^3*c^5 + b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^3*d^2*x^2 + 10*
b^3*c^2*d^3*x^3 + 5*b^3*c^4*d*x))
```

3.1015 $\int (a+bx)^5(ac+bcx)^n\,dx$

Optimal. Leaf size=24

$$\frac{(ac+bcx)^{6+n}}{bc^6(6+n)}$$

```
[Out] (b*c*x+a*c)^(6+n)/b/c^6/(6+n)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] (a*c + b*c*x)^(6 + n)/(b*c^6*(6 + n))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^n\,dx &= \frac{\int (ac+bcx)^{5+n}\,dx}{c^5}\\ &= \frac{(ac+bcx)^{6+n}}{bc^6(6+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.04

$$\frac{(a+bx)^6(c(a+bx))^n}{b(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] ((a + b*x)^6*(c*(a + b*x))^n)/(b*(6 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.27, size = 245, normalized size = 10.21

$$\text{Piecewise}\left[\left\{\left\{\frac{x}{ac^6}, b{=}{=}0\&\&n{=}{=}-6\right\}, \left\{a^5x\,(ac)^n, b{=}{=}0\right\}, \left\{\frac{\text{Log}\left[\frac{a}{b}+x\right]}{bc^6}, n{=}{=}-6\right\}\right\}, \frac{a^6\,(ac+bcx)^n}{6b+bn} + \frac{6a^5bx\,(ac+bcx)^n}{6b+bn} + \frac{15a^4b^2x^2\,(ac+bcx)^n}{6b+bn} + \frac{20a^3b^3x^3\,(ac+bcx)^n}{6b+bn} + \frac{15a^2b^4x^4\,(ac+bcx)^n}{6b+bn} + \frac{6ab^5x^5\,(ac+bcx)^n}{6b+bn} + \frac{b^6x^6\,(ac+bcx)^n}{6b+bn}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5*(a*c + b*c*x)^n,x]')
```

```
[Out] Piecewise[{{x / (a c ^ 6), b == 0 && n == -6}, {a ^ 5 x (a c) ^ n, b == 0},
 {Log[a / b + x] / (b c ^ 6), n == -6}}, a ^ 6 (a c + b c x) ^ n / (6 b + b
 n) + 6 a ^ 5 b x (a c + b c x) ^ n / (6 b + b n) + 15 a ^ 4 b ^ 2 x ^ 2 (a
 c + b c x) ^ n / (6 b + b n) + 20 a ^ 3 b ^ 3 x ^ 3 (a c + b c x) ^ n / (6
 b + b n) + 15 a ^ 2 b ^ 4 x ^ 4 (a c + b c x) ^ n / (6 b + b n) + 6 a b ^
5 x ^ 5 (a c + b c x) ^ n / (6 b + b n) + b ^ 6 x ^ 6 (a c + b c x) ^ n / (
6 b + b n)]
```

Maple [A]
time = 0.17, size = 27, normalized size = 1.12

method	result
gosper	$\frac{(bx+a)^6(bcx+ac)^n}{b(6+n)}$
risch	$\frac{(x^6b^6+6a\,x^5b^5+15a^2x^4b^4+20a^3b^3x^3+15a^4x^2b^2+6a^5xb+a^6)(c(bx+a))^n}{b(6+n)}$
norman	$\frac{a^6\mathrm{e}^{n\ln(bcx+ac)}}{b(6+n)}+\frac{b^5x^6\mathrm{e}^{n\ln(bcx+ac)}}{6+n}+\frac{6a^5x\,\mathrm{e}^{n\ln(bcx+ac)}}{6+n}+\frac{6a\,b^4x^5\mathrm{e}^{n\ln(bcx+ac)}}{6+n}+\frac{15a^2b^3x^4\mathrm{e}^{n\ln(bcx+ac)}}{6+n}+\frac{20a^3b^2x^3\mathrm{e}^{n\ln(b}}{6+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^6/b/(6+n)*(b*c*x+a*c)^n
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(24) = 48.
time = 0.29, size = 649, normalized size = 27.04

$$\frac{5\,(b^2c^n(n+1)x^2+abc^nnx-a^2c^n)(bx+a)^na^4}{(n^2+3n+2)b}+\frac{10\,((n^2+3n+2)b^3c^nx^3+(n^2+n)ab^2c^nx^2-2a^2bc^nnx+2a^3c^n)(bx+a)^na^3}{(n^3+6n^2+11n+6)b}+\frac{(bcx+ac)^{n+1}a^5}{bc(n+1)}+\frac{10\,((n^3+6n^2+11n+6)b^4c^nx^4+(n^3+3n^2+2n)ab^3c^nx^3-3\,(n^2+n)a^2b^2c^nx^2+6a^3bc^nnx-6a^4c^n)(bx+a)^na^2}{(n^4+10n^3+35n^2+50n+24)b}+\frac{5\,((n^4+10n^3+35n^2+50n+24)b^5c^nx^5+(n^4+6n^3+11n^2+6n)ab^4c^nx^4-4\,(n^3+3n^2+2n)a^2b^3c^nx^3+12\,(n^2+n)a^3b^2c^nx^2-24a^4bc^nnx+24a^5c^n)(bx+a)^na}{(n^5+15n^4+85n^3+225n^2+274n+120)b}+\frac{((n^5+15n^4+85n^3+225n^2+274n+120)b^6c^nx^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5c^nx^5-5\,(n^4+6n^3+11n^2+6n)a^2b^4c^nx^4+20\,(n^3+3n^2+2n)a^3b^3c^nx^3-60\,(n^2+n)a^4b^2c^nx^2+120a^5bc^nnx-120a^6c^n)(bx+a)^n}{(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 5*(b^2*c^n*(n + 1)*x^2 + a*b*c^n*n*x - a^2*c^n)*(b*x + a)^n*a^4/((n^2 + 3*n
 + 2)*b) + 10*((n^2 + 3*n + 2)*b^3*c^n*x^3 + (n^2 + n)*a*b^2*c^n*x^2 - 2*a^
2*b*c^n*n*x + 2*a^3*c^n)*(b*x + a)^n*a^3/((n^3 + 6*n^2 + 11*n + 6)*b) + (b*
c*x + a*c)^(n + 1)*a^5/(b*c*(n + 1)) + 10*((n^3 + 6*n^2 + 11*n + 6)*b^4*c^n
*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^n*x^3 - 3*(n^2 + n)*a^2*b^2*c^n*x^2 + 6*
a^3*b*c^n*n*x - 6*a^4*c^n)*(b*x + a)^n*a^2/((n^4 + 10*n^3 + 35*n^2 + 50*n +
 24)*b) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^n*x^5 + (n^4 + 6*n^3
 + 11*n^2 + 6*n)*a*b^4*c^n*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^n*x^3 + 12
*(n^2 + n)*a^3*b^2*c^n*x^2 - 24*a^4*b*c^n*n*x + 24*a^5*c^n)*(b*x + a)^n*a/(
(n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b) + ((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^6*c^n*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 +
24*n)*a*b^5*c^n*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*c^n*x^4 + 20*(
n^3 + 3*n^2 + 2*n)*a^3*b^3*c^n*x^3 - 60*(n^2 + n)*a^4*b^2*c^n*x^2 + 120*a^5
*b*c^n*n*x - 120*a^6*c^n)*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.
time = 0.30, size = 80, normalized size = 3.33

$$\frac{(b^6x^6 + 6\,ab^5x^5 + 15\,a^2b^4x^4 + 20\,a^3b^3x^3 + 15\,a^4b^2x^2 + 6\,a^5bx + a^6)(bcx + ac)^n}{bn + 6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] (b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 +
 6*a^5*b*x + a^6)*(b*c*x + a*c)^n/(b*n + 6*b)
```

Sympy [A]
time = 0.65, size = 212, normalized size = 8.83

$$\begin{cases} \frac{x}{ac^6} & \text{for}\: b = 0 \wedge n = -6 \\ a^5x\,(ac)^n & \text{for}\: b = 0 \\ \frac{\log\left(\frac{a}{b} + x\right)}{bc^6} & \text{for}\: n = -6 \\ \frac{a^6(ac+bcx)^n}{bn+6b} + \frac{6a^5bx(ac+bcx)^n}{bn+6b} + \frac{15a^4b^2x^2(ac+bcx)^n}{bn+6b} + \frac{20a^3b^3x^3(ac+bcx)^n}{bn+6b} + \frac{15a^2b^4x^4(ac+bcx)^n}{bn+6b} + \frac{6ab^5x^5(ac+bcx)^n}{bn+6b} + \frac{b^6x^6(ac+bcx)^n}{bn+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((x/(a*c**6), Eq(b, 0) & Eq(n, -6)), (a**5*x*(a*c)**n, Eq(b, 0)),
(log(a/b + x)/(b*c**6), Eq(n, -6)), (a**6*(a*c + b*c*x)**n/(b*n + 6*b) + 6*
a**5*b*x*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**4*b**2*x**2*(a*c + b*c*x)**n/
(b*n + 6*b) + 20*a**3*b**3*x**3*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**2*b**4
*x**4*(a*c + b*c*x)**n/(b*n + 6*b) + 6*a*b**5*x**5*(a*c + b*c*x)**n/(b*n +
6*b) + b**6*x**6*(a*c + b*c*x)**n/(b*n + 6*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(24) = 48.
time = 0.01, size = 154, normalized size = 6.42

$$\frac{a^{6}e^{n\ln(ac+bcx)}+6a^{5}bxe^{n\ln(ac+bcx)}+15a^{4}b^{2}x^{2}e^{n\ln(ac+bcx)}+20a^{3}b^{3}x^{3}e^{n\ln(ac+bcx)}+15a^{2}b^{4}x^{4}e^{n\ln(ac+bcx)}+6ab^{5}x^{5}e^{n\ln(ac+bcx)}+b^{6}x^{6}e^{n\ln(ac+bcx)}}{bn+6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x)
```

```
[Out] ((b*c*x + a*c)^n*b^6*x^6 + 6*(b*c*x + a*c)^n*a*b^5*x^5 + 15*(b*c*x + a*c)^n
*a^2*b^4*x^4 + 20*(b*c*x + a*c)^n*a^3*b^3*x^3 + 15*(b*c*x + a*c)^n*a^4*b^2*
x^2 + 6*(b*c*x + a*c)^n*a^5*b*x + (b*c*x + a*c)^n*a^6)/(b*n + 6*b)
```

Mupad [B]
time = 0.33, size = 107, normalized size = 4.46

$$(a\,c+b\,c\,x)^n\,\left(\frac{a^6}{b\,\left(n+6\right)}+\frac{b^5\,x^6}{n+6}+\frac{6\,a^5\,x}{n+6}+\frac{15\,a^4\,b\,x^2}{n+6}+\frac{6\,a\,b^4\,x^5}{n+6}+\frac{20\,a^3\,b^2\,x^3}{n+6}+\frac{15\,a^2\,b^3\,x^4}{n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^n*(a + b*x)^5,x)
```

```
[Out] (a*c + b*c*x)^n*(a^6/(b*(n + 6)) + (b^5*x^6)/(n + 6) + (6*a^5*x)/(n + 6) +
(15*a^4*b*x^2)/(n + 6) + (6*a*b^4*x^5)/(n + 6) + (20*a^3*b^2*x^3)/(n + 6) +
 (15*a^2*b^3*x^4)/(n + 6))
```

3.1016 $\int (a+bx)^5(ac+bcx)^3\,dx$

Optimal. Leaf size=17

$$\frac{c^3(a+bx)^9}{9b}$$

```
[Out] 1/9*c^3*(b*x+a)^9/b
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^3\,dx &= c^3\int (a+bx)^8\,dx\\ &= \frac{c^3(a+bx)^9}{9b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^3,x]

[Out] (c^3*(a + b*x)^9)/(9*b)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 92 vs. $2(17) = 34$.
time = 2.03, size = 90, normalized size = 5.29

$$\frac{c^3x\left(9a^8+36a^7bx+84a^6b^2x^2+126a^5b^3x^3+126a^4b^4x^4+84a^3b^5x^5+36a^2b^6x^6+9ab^7x^7+b^8x^8\right)}{9}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^5*(a*c + b*c*x)^3,x]')

[Out] c ^ 3 x (9 a ^ 8 + 36 a ^ 7 b x + 84 a ^ 6 b ^ 2 x ^ 2 + 126 a ^ 5 b ^ 3 x ^ 3 + 126 a ^ 4 b ^ 4 x ^ 4 + 84 a ^ 3 b ^ 5 x ^ 5 + 36 a ^ 2 b ^ 6 x ^ 6 + 9 a b ^ 7 x ^ 7 + b ^ 8 x ^ 8) / 9

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.12, size = 114, normalized size = 6.71

method	result
gosper	$\frac{x\left(b^8x^8+9a\,b^7x^7+36a^2x^6b^6+84a^3x^5b^5+126a^4x^4b^4+126a^5x^3b^3+84a^6x^2b^2+36a^7xb+9a^8\right)c^3}{9}$
default	$\frac{1}{9}b^8c^3x^9+a\,b^7c^3x^8+4a^2b^6c^3x^7+\frac{28}{3}a^3b^5c^3x^6+14a^4b^4c^3x^5+14a^5b^3c^3x^4+\frac{28}{3}a^6c^3b^2x^3+4a^7c^3b\,x$
norman	$\frac{1}{9}b^8c^3x^9+a\,b^7c^3x^8+4a^2b^6c^3x^7+\frac{28}{3}a^3b^5c^3x^6+14a^4b^4c^3x^5+14a^5b^3c^3x^4+\frac{28}{3}a^6c^3b^2x^3+4a^7c^3b\,x$
risch	$\frac{b^8c^3x^9}{9}+a\,b^7c^3x^8+4a^2b^6c^3x^7+\frac{28a^3b^5c^3x^6}{3}+14a^4b^4c^3x^5+14a^5b^3c^3x^4+\frac{28a^6c^3b^2x^3}{3}+4a^7c^3b\,x^2+a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^5*(b*c*x+a*c)^3,x,method=_RETURNVERBOSE)

[Out] 1/9*b^8*c^3*x^9+a*b^7*c^3*x^8+4*a^2*b^6*c^3*x^7+28/3*a^3*b^5*c^3*x^6+14*a^4*b^4*c^3*x^5+14*a^5*b^3*c^3*x^4+28/3*a^6*c^3*b^2*x^3+4*a^7*c^3*b*x^2+a^8*c^3*x

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.26, size = 113, normalized size = 6.65

$$\frac{1}{9}\,b^8c^3x^9+ab^7c^3x^8+4\,a^2b^6c^3x^7+\frac{28}{3}\,a^3b^5c^3x^6+14\,a^4b^4c^3x^5+14\,a^5b^3c^3x^4+\frac{28}{3}\,a^6b^2c^3x^3+4\,a^7bc^3x^2+a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="maxima")

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.29, size = 113, normalized size = 6.65

$$\frac{1}{9}\,b^{8}c^{3}x^{9}+ab^{7}c^{3}x^{8}+4\,a^{2}b^{6}c^{3}x^{7}+\frac{28}{3}\,a^{3}b^{5}c^{3}x^{6}+14\,a^{4}b^{4}c^{3}x^{5}+14\,a^{5}b^{3}c^{3}x^{4}+\frac{28}{3}\,a^{6}b^{2}c^{3}x^{3}+4\,a^{7}bc^{3}x^{2}+a^{8}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(12) = 24$.
time = 0.05, size = 124, normalized size = 7.29

$$a^{8}c^{3}x+4a^{7}bc^{3}x^{2}+\frac{28a^{6}b^{2}c^{3}x^{3}}{3}+14a^{5}b^{3}c^{3}x^{4}+14a^{4}b^{4}c^{3}x^{5}+\frac{28a^{3}b^{5}c^{3}x^{6}}{3}+4a^{2}b^{6}c^{3}x^{7}+ab^{7}c^{3}x^{8}+\frac{b^{8}c^{3}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**3,x)
```

```
[Out] a**8*c**3*x + 4*a**7*b*c**3*x**2 + 28*a**6*b**2*c**3*x**3/3 + 14*a**5*b**3*
c**3*x**4 + 14*a**4*b**4*c**3*x**5 + 28*a**3*b**5*c**3*x**6/3 + 4*a**2*b**6
*c**3*x**7 + a*b**7*c**3*x**8 + b**8*c**3*x**9/9
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.00, size = 119, normalized size = 7.00

$$\frac{1}{9}x^{9}b^{8}c^{3}+x^{8}b^{7}ac^{3}+4x^{7}b^{6}a^{2}c^{3}+\frac{28}{3}x^{6}b^{5}a^{3}c^{3}+14x^{5}b^{4}a^{4}c^{3}+14x^{4}b^{3}a^{5}c^{3}+\frac{28}{3}x^{3}b^{2}a^{6}c^{3}+4x^{2}ba^{7}c^{3}+xa^{8}c^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x)
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Mupad [B]
time = 0.05, size = 113, normalized size = 6.65

$$a^{8}\,c^{3}\,x+4\,a^{7}\,b\,c^{3}\,x^{2}+\frac{28\,a^{6}\,b^{2}\,c^{3}\,x^{3}}{3}+14\,a^{5}\,b^{3}\,c^{3}\,x^{4}+14\,a^{4}\,b^{4}\,c^{3}\,x^{5}+\frac{28\,a^{3}\,b^{5}\,c^{3}\,x^{6}}{3}+4\,a^{2}\,b^{6}\,c^{3}\,x^{7}+a\,b^{7}\,c^{3}\,x^{8}+\frac{b^{8}\,c^{3}\,x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^3*(a + b*x)^5,x)
```

```
[Out] a^8*c^3*x + (b^8*c^3*x^9)/9 + 4*a^7*b*c^3*x^2 + a*b^7*c^3*x^8 + (28*a^6*b^2
*c^3*x^3)/3 + 14*a^5*b^3*c^3*x^4 + 14*a^4*b^4*c^3*x^5 + (28*a^3*b^5*c^3*x^6
)/3 + 4*a^2*b^6*c^3*x^7
```

3.1017 $\int (a+bx)^5(ac+bcx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(a+bx)^8}{8b}$$

```
[Out] 1/8*c^2*(b*x+a)^8/b
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^2\,dx &= c^2\int (a+bx)^7\,dx\\ &= \frac{c^2(a+bx)^8}{8b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(17) = 34$.
time = 1.94, size = 79, normalized size = 4.65

$$\frac{c^2x\left(8a^7+28a^6bx+56a^5b^2x^2+70a^4b^3x^3+56a^3b^4x^4+28a^2b^5x^5+8ab^6x^6+b^7x^7\right)}{8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(a*c + b*c*x)^2,x]')
```

```
[Out] c ^ 2 x (8 a ^ 7 + 28 a ^ 6 b x + 56 a ^ 5 b ^ 2 x ^ 2 + 70 a ^ 4 b ^ 3 x ^
 3 + 56 a ^ 3 b ^ 4 x ^ 4 + 28 a ^ 2 b ^ 5 x ^ 5 + 8 a b ^ 6 x ^ 6 + b ^ 7
x ^ 7) / 8
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.14, size = 100, normalized size = 5.88

method	result	size
gosper	$\frac{x\left(b^7x^7+8a\,b^6x^6+28a^2b^5x^5+56a^3b^4x^4+70a^4b^3x^3+56a^5b^2x^2+28a^6bx+8a^7\right)c^2}{8}$	80
default	$\frac{1}{8}b^7c^2x^8+a\,b^6c^2x^7+\frac{7}{2}a^2b^5c^2x^6+7a^3b^4c^2x^5+\frac{35}{4}a^4b^3c^2x^4+7a^5b^2c^2x^3+\frac{7}{2}a^6b\,c^2x^2+a^7c^2x$	100
norman	$\frac{1}{8}b^7c^2x^8+a\,b^6c^2x^7+\frac{7}{2}a^2b^5c^2x^6+7a^3b^4c^2x^5+\frac{35}{4}a^4b^3c^2x^4+7a^5b^2c^2x^3+\frac{7}{2}a^6b\,c^2x^2+a^7c^2x$	100
risch	$\frac{b^7c^2x^8}{8}+a\,b^6c^2x^7+\frac{7a^2b^5c^2x^6}{2}+7a^3b^4c^2x^5+\frac{35a^4b^3c^2x^4}{4}+7a^5b^2c^2x^3+\frac{7a^6b\,c^2x^2}{2}+a^7c^2x+\frac{c^2a^8}{8b}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^7*c^2*x^8+a*b^6*c^2*x^7+7/2*a^2*b^5*c^2*x^6+7*a^3*b^4*c^2*x^5+35/4*a^
4*b^3*c^2*x^4+7*a^5*b^2*c^2*x^3+7/2*a^6*b*c^2*x^2+a^7*c^2*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.27, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^7c^2x^8+ab^6c^2x^7+\frac{7}{2}\,a^2b^5c^2x^6+7\,a^3b^4c^2x^5+\frac{35}{4}\,a^4b^3c^2x^4+7\,a^5b^2c^2x^3+\frac{7}{2}\,a^6bc^2x^2+a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.29, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}\,a^2b^5c^2x^6 + 7\,a^3b^4c^2x^5 + \frac{35}{4}\,a^4b^3c^2x^4 + 7\,a^5b^2c^2x^3 + \frac{7}{2}\,a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(12) = 24$.
time = 0.04, size = 110, normalized size = 6.47

$$a^7c^2x + \frac{7a^6bc^2x^2}{2} + 7a^5b^2c^2x^3 + \frac{35a^4b^3c^2x^4}{4} + 7a^3b^4c^2x^5 + \frac{7a^2b^5c^2x^6}{2} + ab^6c^2x^7 + \frac{b^7c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**2,x)
```

```
[Out] a**7*c**2*x + 7*a**6*b*c**2*x**2/2 + 7*a**5*b**2*c**2*x**3 + 35*a**4*b**3*c
**2*x**4/4 + 7*a**3*b**4*c**2*x**5 + 7*a**2*b**5*c**2*x**6/2 + a*b**6*c**2*
x**7 + b**7*c**2*x**8/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.00, size = 107, normalized size = 6.29

$$\frac{1}{8}x^8b^7c^2 + x^7b^6ac^2 + \frac{7}{2}x^6b^5a^2c^2 + 7x^5b^4a^3c^2 + \frac{35}{4}x^4b^3a^4c^2 + 7x^3b^2a^5c^2 + \frac{7}{2}x^2ba^6c^2 + xa^7c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x)
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Mupad [B]
time = 0.04, size = 99, normalized size = 5.82

$$a^7\,c^2\,x + \frac{7\,a^6\,b\,c^2\,x^2}{2} + 7\,a^5\,b^2\,c^2\,x^3 + \frac{35\,a^4\,b^3\,c^2\,x^4}{4} + 7\,a^3\,b^4\,c^2\,x^5 + \frac{7\,a^2\,b^5\,c^2\,x^6}{2} + a\,b^6\,c^2\,x^7 + \frac{b^7\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^2*(a + b*x)^5,x)
```

```
[Out] a^7*c^2*x + (b^7*c^2*x^8)/8 + (7*a^6*b*c^2*x^2)/2 + a*b^6*c^2*x^7 + 7*a^5*b
^2*c^2*x^3 + (35*a^4*b^3*c^2*x^4)/4 + 7*a^3*b^4*c^2*x^5 + (7*a^2*b^5*c^2*x^
6)/2
```

3.1018 $\int (a+bx)^5(ac+bcx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^7}{7b}$$

```
[Out] 1/7*c*(b*x+a)^7/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {21, 32}

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)\,dx &= c\int (a+bx)^6\,dx\\ &= \frac{c(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 68 vs. $2(15) = 30$.
time = 1.80, size = 66, normalized size = 4.40

$$\frac{cx\left(7a^6 + 21a^5bx + 35a^4b^2x^2 + 35a^3b^3x^3 + 21a^2b^4x^4 + 7ab^5x^5 + b^6x^6\right)}{7}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(a*c + b*c*x)^1,x]')
```

```
[Out] c x (7 a ^ 6 + 21 a ^ 5 b x + 35 a ^ 4 b ^ 2 x ^ 2 + 35 a ^ 3 b ^ 3 x ^ 3 +
 21 a ^ 2 b ^ 4 x ^ 4 + 7 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6) / 7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.13, size = 72, normalized size = 4.80

method	result	size
gosper	$\frac{cx\left(x^6b^6+7a\,x^5b^5+21a^2x^4b^4+35a^3b^3x^3+35a^4x^2b^2+21a^5xb+7a^6\right)}{7}$	67
default	$\frac{1}{7}b^6c\,x^7+a\,b^5c\,x^6+3a^2b^4c\,x^5+5a^3b^3c\,x^4+5a^4b^2c\,x^3+3a^5bc\,x^2+a^6cx$	72
norman	$\frac{1}{7}b^6c\,x^7+a\,b^5c\,x^6+3a^2b^4c\,x^5+5a^3b^3c\,x^4+5a^4b^2c\,x^3+3a^5bc\,x^2+a^6cx$	72
risch	$\frac{b^6c\,x^7}{7}+a\,b^5c\,x^6+3a^2b^4c\,x^5+5a^3b^3c\,x^4+5a^4b^2c\,x^3+3a^5bc\,x^2+a^6cx+\frac{c\,a^7}{7b}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^6*c*x^7+a*b^5*c*x^6+3*a^2*b^4*c*x^5+5*a^3*b^3*c*x^4+5*a^4*b^2*c*x^3+3
*a^5*b*c*x^2+a^6*c*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.25, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^6cx^7 + ab^5cx^6 + 3\,a^2b^4cx^5 + 5\,a^3b^3cx^4 + 5\,a^4b^2cx^3 + 3\,a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.29, size = 71, normalized size = 4.73

$$\frac{1}{7}b^6cx^7 + ab^5cx^6 + 3a^2b^4cx^5 + 5a^3b^3cx^4 + 5a^4b^2cx^3 + 3a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(10) = 20$.
time = 0.04, size = 78, normalized size = 5.20

$$a^6cx + 3a^5bcx^2 + 5a^4b^2cx^3 + 5a^3b^3cx^4 + 3a^2b^4cx^5 + ab^5cx^6 + \frac{b^6cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c),x)
```

```
[Out] a**6*c*x + 3*a**5*b*c*x**2 + 5*a**4*b**2*c*x**3 + 5*a**3*b**3*c*x**4 + 3*a*
*2*b**4*c*x**5 + a*b**5*c*x**6 + b**6*c*x**7/7
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.00, size = 73, normalized size = 4.87

$$\frac{1}{7}x^7b^6c + x^6b^5ac + 3x^5b^4a^2c + 5x^4b^3a^3c + 5x^3b^2a^4c + 3x^2ba^5c + xa^6c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x)
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Mupad [B]
time = 0.03, size = 71, normalized size = 4.73

$$c\,a^6\,x + 3\,c\,a^5\,b\,x^2 + 5\,c\,a^4\,b^2\,x^3 + 5\,c\,a^3\,b^3\,x^4 + 3\,c\,a^2\,b^4\,x^5 + c\,a\,b^5\,x^6 + \frac{c\,b^6\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)*(a + b*x)^5,x)
```

```
[Out] (b^6*c*x^7)/7 + a^6*c*x + 5*a^4*b^2*c*x^3 + 5*a^3*b^3*c*x^4 + 3*a^2*b^4*c*x
^5 + 3*a^5*b*c*x^2 + a*b^5*c*x^6
```

3.1019 $\int \frac{(a+bx)^5}{ac+bcx} dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^5}{5bc}$$

[Out] 1/5*(b*x+a)^5/b/c

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x),x]

[Out] (a + b*x)^5/(5*b*c)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{ac+bcx}\,dx &= \frac{\int (a+bx)^4\,dx}{c}\\ &= \frac{(a+bx)^5}{5bc}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.
time = 1.75, size = 46, normalized size = 2.71

$$\frac{x\left(5a^4+10a^3bx+10a^2b^2x^2+5ab^3x^3+b^4x^4\right)}{5c}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^1,x]')
```

```
[Out] x (5 a ^ 4 + 10 a ^ 3 b x + 10 a ^ 2 b ^ 2 x ^ 2 + 5 a b ^ 3 x ^ 3 + b ^ 4
x ^ 4) / (5 c)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{(bx+a)^5}{5bc}$	16
gosper	$\frac{x(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4)}{5c}$	47
norman	$\frac{a^4x}{c}+\frac{a\,b^3x^4}{c}+\frac{b^4x^5}{5c}+\frac{2a^3b\,x^2}{c}+\frac{2b^2a^2x^3}{c}$	58
risch	$\frac{b^4x^5}{5c}+\frac{a\,b^3x^4}{c}+\frac{2b^2a^2x^3}{c}+\frac{2a^3b\,x^2}{c}+\frac{a^4x}{c}+\frac{a^5}{5bc}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(b*x+a)^5/b/c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.28, size = 48, normalized size = 2.82

$$\frac{b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.29, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(10) = 20$.
time = 0.05, size = 51, normalized size = 3.00

$$\frac{a^4x}{c} + \frac{2a^3bx^2}{c} + \frac{2a^2b^2x^3}{c} + \frac{ab^3x^4}{c} + \frac{b^4x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c),x)
```

```
[Out] a**4*x/c + 2*a**3*b*x**2/c + 2*a**2*b**2*x**3/c + a*b**3*x**4/c + b**4*x**5
/(5*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.00, size = 47, normalized size = 2.76

$$\frac{\frac{1}{5}x^5b^4 + x^4b^3a + 2x^3b^2a^2 + 2x^2ba^3 + xa^4}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x)
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Mupad [B]
time = 0.03, size = 57, normalized size = 3.35

$$\frac{a^4\,x}{c} + \frac{b^4\,x^5}{5\,c} + \frac{2\,a^3\,b\,x^2}{c} + \frac{a\,b^3\,x^4}{c} + \frac{2\,a^2\,b^2\,x^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x),x)
```

```
[Out] (a^4*x)/c + (b^4*x^5)/(5*c) + (2*a^3*b*x^2)/c + (a*b^3*x^4)/c + (2*a^2*b^2*
x^3)/c
```

3.1020 $\int \frac{(a+bx)^5}{(ac+bcx)^2} \, dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^4}{4bc^2}$$

[Out] 1/4*(b*x+a)^4/b/c^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^2,x]

[Out] (a + b*x)^4/(4*b*c^2)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^2} \, dx &= \frac{\int (a+bx)^3 \, dx}{c^2}\\ &= \frac{(a+bx)^4}{4bc^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 1.70, size = 35, normalized size = 2.06

$$\frac{x\left(4a^3 + 6a^2bx + 4ab^2x^2 + b^3x^3\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^2,x]')
```

```
[Out] x (4 a ^ 3 + 6 a ^ 2 b x + 4 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) / (4 c ^ 2)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{(bx+a)^4}{4b\,c^2}$	16
gosper	$\frac{x\left(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3\right)}{4c^2}$	36
risch	$\frac{b^3x^4}{4c^2}+\frac{b^2a\,x^3}{c^2}+\frac{3b\,a^2x^2}{2c^2}+\frac{a^3x}{c^2}+\frac{a^4}{4b\,c^2}$	55
norman	$\frac{\frac{a^4x}{c}+\frac{b^4x^5}{4c}+\frac{5a\,b^3x^4}{4c}+\frac{5a^3b\,x^2}{2c}+\frac{5b^2a^2x^3}{2c}}{c(bx+a)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4/b/c^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.26, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.29, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.
time = 0.05, size = 46, normalized size = 2.71

$$\frac{a^3x}{c^2} + \frac{3a^2bx^2}{2c^2} + \frac{ab^2x^3}{c^2} + \frac{b^3x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**2,x)
```

```
[Out] a**3*x/c**2 + 3*a**2*b*x**2/(2*c**2) + a*b**2*x**3/c**2 + b**3*x**4/(4*c**2
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.00, size = 40, normalized size = 2.35

$$\frac{\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x)
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Mupad [B]
time = 0.05, size = 43, normalized size = 2.53

$$\frac{a^3\,x}{c^2} + \frac{b^3\,x^4}{4\,c^2} + \frac{3\,a^2\,b\,x^2}{2\,c^2} + \frac{a\,b^2\,x^3}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^2,x)
```

```
[Out] (a^3*x)/c^2 + (b^3*x^4)/(4*c^2) + (3*a^2*b*x^2)/(2*c^2) + (a*b^2*x^3)/c^2
```

3.1021 $\int \frac{(a+bx)^5}{(ac+bcx)^3} dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^3}{3bc^3}$$

[Out] 1/3*(b*x+a)^3/b/c^3

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^3,x]

[Out] (a + b*x)^3/(3*b*c^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^3} dx &= \frac{\int (a+bx)^2 dx}{c^3} \\ &= \frac{(a+bx)^3}{3bc^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] (a + b*x)^3/(3*b*c^3)
```

Mathics [A]
time = 1.64, size = 24, normalized size = 1.41

$$\frac{x\left(3a^2+3abx+b^2x^2\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^3,x]')
```

```
[Out] x (3 a ^ 2 + 3 a b x + b ^ 2 x ^ 2) / (3 c ^ 3)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{(bx+a)^3}{3b\,c^3}$	16
gosper	$\frac{x\left(x^2b^2+3abx+3a^2\right)}{3c^3}$	25
risch	$\frac{b^2x^3}{3c^3}+\frac{ba\,x^2}{c^3}+\frac{a^2x}{c^3}+\frac{a^3}{3b\,c^3}$	41
norman	$\frac{\frac{b^4x^5}{3c}+\frac{5a\,b^3x^4}{3c}-\frac{3a^5}{bc}+\frac{10b^2a^2x^3}{3c}-\frac{5a^4x}{c}}{c^2(bx+a)^2}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/b/c^3
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.53

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Fricas [A]
time = 0.29, size = 26, normalized size = 1.53

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.05, size = 29, normalized size = 1.71

$$\frac{a^2x}{c^3} + \frac{abx^2}{c^3} + \frac{b^2x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**3,x)
```

```
[Out] a**2*x/c**3 + a*b*x**2/c**3 + b**2*x**3/(3*c**3)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.59

$$\frac{\frac{1}{3}x^3b^2 + x^2ba + xa^2}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x)
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Mupad [B]
time = 0.04, size = 24, normalized size = 1.41

$$\frac{x\,\left(3\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^3,x)
```

```
[Out] (x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*c^3)
```

3.1022 $\int \frac{(a+bx)^5}{(ac+bcx)^4} dx$

Optimal. Leaf size=18

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

[Out] a*x/c^4+1/2*b*x^2/c^4

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {21}

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] (a*x)/c^4 + (b*x^2)/(2*c^4)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^4} dx &= \frac{\int (a+bx) dx}{c^4} \\ &= \frac{ax}{c^4} + \frac{bx^2}{2c^4} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.89

$$\frac{ax + \frac{bx^2}{2}}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^4,x]

```
[Out] (a*x + (b*x^2)/2)/c^4
```

Mathics [A]
time = 1.57, size = 13, normalized size = 0.72

$$\frac{x\,(2a+bx)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^4,x]')
[Out] x (2 a + b x) / (2 c ^ 4)
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{x(bx+2a)}{2c^4}$	14
default	$\frac{\frac{1}{2}x^2b+ax}{c^4}$	15
risch	$\frac{ax}{c^4}+\frac{b\,x^2}{2c^4}$	17
norman	$\frac{\frac{b^4x^5}{2c}+\frac{5a\,b^3x^4}{2c}-\frac{9a^5}{2bc}-\frac{10a^3b\,x^2}{c}-\frac{25a^4x}{2c}}{c^3(bx+a)^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
[Out] 1/c^4*(1/2*x^2*b+a*x)
```

Maxima [A]
time = 0.26, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Fricas [A]
time = 0.29, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.83

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**4,x)
```

```
[Out] a*x/c**4 + b*x**2/(2*c**4)
```

Giac [A]
time = 0.00, size = 17, normalized size = 0.94

$$\frac{\frac{1}{2}x^2b + xa}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x)
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(2\,a+b\,x\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^4,x)
```

```
[Out] (x*(2*a + b*x))/(2*c^4)
```

3.1023 $\int \frac{(a+bx)^5}{(ac+bcx)^5} dx$

Optimal. Leaf size=5

$$\frac{x}{c^5}$$

[Out] x/c^5

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 8}

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^5,x]

[Out] x/c^5

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^5} dx &= \frac{\int 1 \, dx}{c^5}\\ &= \frac{x}{c^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^5,x]
```

```
[Out] x/c^5
```

Mathics [A]
time = 1.51, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^5,x]')
```

```
[Out] x / c ^ 5
```

Maple [A]
time = 0.15, size = 6, normalized size = 1.20

method	result	size
default	$\frac{x}{c^5}$	6
risch	$\frac{x}{c^5}$	6
norman	$\frac{\frac{b^4 x^5}{c}+\frac{a^4 x}{c}+\frac{4 a\,b^3 x^4}{c}+\frac{4 a^3 b\,x^2}{c}+\frac{6 b^2 a^2 x^3}{c}}{c^4 (b x+a)^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^5
```

Maxima [A]
time = 0.26, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] x/c^5
```

Fricas [A]
time = 0.29, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] x/c^5
```

Sympy [A]
time = 0.05, size = 3, normalized size = 0.60

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**5,x)
```

```
[Out] x/c**5
```

Giac [A]
time = 0.00, size = 6, normalized size = 1.20

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x)
```

```
[Out] x/c^5
```

Mupad [B]
time = 0.01, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^5,x)
```

```
[Out] x/c^5
```

3.1024 $\int \frac{(a+bx)^5}{(ac+bcx)^6} dx$

Optimal. Leaf size=13

$$\frac{\log(a+bx)}{bc^6}$$

[Out] ln(b*x+a)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 31}

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^6,x]

[Out] Log[a + b*x]/(b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^6} dx &= \frac{\int \frac{1}{a+bx} dx}{c^6} \\ &= \frac{\log(a+bx)}{bc^6} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Mathics [A]
time = 1.61, size = 17, normalized size = 1.31

$$\frac{\text{Log}\left[c^6\left(a+bx\right)\right]}{bc^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^6,x]')
```

```
[Out] Log[c ^ 6 (a + b x)] / (b c ^ 6)
```

Maple [A]
time = 0.14, size = 14, normalized size = 1.08

method	result	size
default	$\frac{\ln(bx+a)}{b\,c^6}$	14
norman	$\frac{\ln(bx+a)}{b\,c^6}$	14
risch	$\frac{\ln(bx+a)}{b\,c^6}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b/c^6
```

Maxima [A]
time = 0.27, size = 13, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c^6)
```

Fricas [A]
time = 0.29, size = 13, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] log(b*x + a)/(b*c^6)
```

Sympy [A]
time = 0.06, size = 17, normalized size = 1.31

$$\frac{\log\left(ac^6 + bc^6x\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**6,x)
```

```
[Out] log(a*c**6 + b*c**6*x)/(b*c**6)
```

Giac [A]
time = 0.00, size = 14, normalized size = 1.08

$$\frac{\ln|xb + a|}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x)
```

```
[Out] log(abs(b*x + a))/(b*c^6)
```

Mupad [B]
time = 0.04, size = 13, normalized size = 1.00

$$\frac{\ln\left(a + b\,x\right)}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^6,x)
```

```
[Out] log(a + b*x)/(b*c^6)
```

3.1025 $\int \frac{(a+bx)^5}{(ac+bcx)^7} dx$

Optimal. Leaf size=15

$$-\frac{1}{bc^7(a+bx)}$$

```
[Out] -1/b/c^7/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^7} dx &= \frac{\int \frac{1}{(a+bx)^2} dx}{c^7} \\ &= -\frac{1}{bc^7(a+bx)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^7,x]
[Out] -(1/(b*c^7*(a + b*x)))
```

Mathics [A]
time = 1.65, size = 15, normalized size = 1.00

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^7,x]')
[Out] -1 / (b c ^ 7 (a + b x))
```

Maple [A]
time = 0.15, size = 16, normalized size = 1.07

method	result	size
gosper	$-\frac{1}{b\,c^7(bx+a)}$	16
default	$-\frac{1}{b\,c^7(bx+a)}$	16
risch	$-\frac{1}{b\,c^7(bx+a)}$	16
norman	$\frac{-\frac{a^5}{bc}-\frac{b^4x^5}{c}-\frac{5a^4x}{c}-\frac{5a\,b^3x^4}{c}-\frac{10b^2a^2x^3}{c}-\frac{10a^3b\,x^2}{c}}{c^6(bx+a)^6}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^7,x,method=_RETURNVERBOSE)
[Out] -1/b/c^7/(b*x+a)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.27

$$-\frac{1}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="maxima")
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

Fricas [A]
time = 0.29, size = 19, normalized size = 1.27

$$-\frac{1}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

Sympy [A]
time = 0.10, size = 17, normalized size = 1.13

$$-\frac{1}{abc^7 + b^2c^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**7,x)
```

```
[Out] -1/(a*b*c**7 + b**2*c**7*x)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.80

$$-\frac{1}{bc^7\,(xb+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x)
```

```
[Out] -1/((b*x + a)*b*c^7)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 1.27

$$-\frac{1}{x\,b^2\,c^7 + a\,b\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^7,x)
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

3.1026 $\int \frac{(a+bx)^5}{(ac+bcx)^8}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2bc^8(a+bx)^2}$$

```
[Out] -1/2/b/c^8/(b*x+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/2*1/(b*c^8*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^8}\,dx &= \frac{\int \frac{1}{(a+bx)^3}\,dx}{c^8} \\ &= -\frac{1}{2bc^8(a+bx)^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/2*1/(b*c^8*(a + b*x)^2)
```

Mathics [A]
time = 1.80, size = 26, normalized size = 1.53

$$-\frac{1}{2bc^8\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^8,x]')
```

```
[Out] -1 / (2 b c ^ 8 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.12, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{2b\,c^{8}\left(bx+a\right)^{2}}$	16
default	$-\frac{1}{2b\,c^{8}\left(bx+a\right)^{2}}$	16
risch	$-\frac{1}{2b\,c^{8}\left(bx+a\right)^{2}}$	16
norman	$\frac{-\frac{5a^{3}b\,x^{2}}{c}-\frac{a^{5}}{2bc}-\frac{b^{4}x^{5}}{2c}-\frac{5a\,b^{3}x^{4}}{2c}-\frac{5b^{2}a^{2}x^{3}}{c}-\frac{5a^{4}x}{2c}}{c^{7}\left(bx+a\right)^{7}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/c^8/(b*x+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.26, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^3c^8x^2+2\,ab^2c^8x+a^2bc^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.29, size = 33, normalized size = 1.94

$$-\frac{1}{2\,(b^3c^8x^2 + 2\,ab^2c^8x + a^2bc^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.
time = 0.13, size = 36, normalized size = 2.12

$$-\frac{1}{2a^2bc^8 + 4ab^2c^8x + 2b^3c^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**8,x)
```

```
[Out] -1/(2*a**2*b*c**8 + 4*a*b**2*c**8*x + 2*b**3*c**8*x**2)
```

Giac [A]
time = 0.00, size = 15, normalized size = 0.88

$$-\frac{1}{2bc^8\,(xb + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x)
```

```
[Out] -1/2/((b*x + a)^2*b*c^8)
```

Mupad [B]
time = 0.15, size = 35, normalized size = 2.06

$$-\frac{1}{2\,a^2\,b\,c^8 + 4\,a\,b^2\,c^8\,x + 2\,b^3\,c^8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^8,x)
```

```
[Out] -1/(2*a^2*b*c^8 + 2*b^3*c^8*x^2 + 4*a*b^2*c^8*x)
```

3.1027 $\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{2+3x}\,\log(2+3x)}{3\sqrt{-2-3x}}$$

[Out] 1/3*ln(2+3*x)*(2+3*x)^(1/2)/(-2-3*x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {23, 31}

$$\frac{\sqrt{3x+2}\,\log(3x+2)}{3\sqrt{-3x-2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]

[Out] (Sqrt[2 + 3*x]*Log[2 + 3*x])/(3*Sqrt[-2 - 3*x])

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx &= \frac{\sqrt{2+3x}\int \frac{1}{2+3x}\,dx}{\sqrt{-2-3x}}\\ &= \frac{\sqrt{2+3x}\,\log(2+3x)}{3\sqrt{-2-3x}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.39, size = 94, normalized size = 3.36

$$\text{Piecewise}\left[\left\{\left\{0,\text{Abs}\left[\frac{2}{3}+x\right]<1\&\&\frac{3}{\text{Abs}[2+3x]}<1\right\},\left\{\left(-\frac{I}{3}\right)\text{Log}\left[\frac{2}{3}+x\right],\text{Abs}\left[\frac{2}{3}+x\right]<1\right\},\left\{\frac{I\text{Log}\left[\frac{3}{2+3x}\right]}{3},\frac{3}{\text{Abs}[2+3x]}<1\right\}\right\},-\frac{I\text{meijerg}\left[\{\{1,1\},\{\}\},\{\{\},\{0,0\}\},\frac{2}{3}+x\right]}{3}+\frac{I\text{meijerg}\left[\{\{\},\{1,1\}\},\{\{0,0\},\{\}\},\frac{2}{3}+x\right]}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]')
```

```
[Out] Piecewise[{{0, Abs[2 / 3 + x] < 1 && 3 / Abs[2 + 3 x] < 1}, {(-I / 3) Log[2
 / 3 + x], Abs[2 / 3 + x] < 1}, {I / 3 Log[3 / (2 + 3 x)], 3 / Abs[2 + 3 x]
 < 1}}, -I meijerg[{{1, 1}, {}}, {{}, {0, 0}}, 2 / 3 + x] / 3 + I meijerg[{
{}, {1, 1}}, {{0, 0}, {}}, 2 / 3 + x] / 3]
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.82

method	result	size
meijerg	$-\frac{i \ln\left(1+\frac{3x}{2}\right)}{3}$	10
default	$\frac{\ln(2+3x)\sqrt{2+3x}}{3\sqrt{-2-3x}}$	23
risch	$-\frac{i\sqrt{\frac{-2-3x}{2+3x}}\,\sqrt{2+3x}\,\ln(2+3x)}{3\sqrt{-2-3x}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(2+3*x)*(2+3*x)^(1/2)/(-2-3*x)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 6, normalized size = 0.21

$$\frac{1}{3}i\,\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x + 2/3)
```

Fricas [A]
time = 0.30, size = 1, normalized size = 0.04

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [A]
time = 0.59, size = 66, normalized size = 2.36

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|x+\frac{2}{3}\right|}<1\wedge\left|x+\frac{2}{3}\right|<1 \\ -\frac{i\log\left(x+\frac{2}{3}\right)}{3} & \text{for}\ \left|x+\frac{2}{3}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{3}}\right)}{3} & \text{for}\ \frac{1}{\left|x+\frac{2}{3}\right|}<1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|x+\frac{2}{3}\right)}{3}-\frac{iG_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|x+\frac{2}{3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)**(1/2)/(2+3*x)**(1/2),x)
[Out] Piecewise((0, (Abs(x + 2/3) < 1) & (1/Abs(x + 2/3) < 1)), (-I*log(x + 2/3)/
3, Abs(x + 2/3) < 1), (I*log(1/(x + 2/3))/3, 1/Abs(x + 2/3) < 1), (I*meijer
g(((), (1, 1)), ((0, 0), ()), x + 2/3)/3 - I*meijerg(((1, 1), ()), ((), (0,
 0)), x + 2/3)/3, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.00, size = 14, normalized size = 0.50

$$-\frac{1}{3}\mathrm{sign}\left(x\right)\mathrm{i}\ln\left|-3x-2\right|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x)
[Out] -1/3*I*log(abs(-3*x - 2))*sgn(x)
```

Mupad [B]
time = 0.22, size = 35, normalized size = 1.25

$$-\frac{4\,\mathrm{atan}\left(\frac{-\sqrt{-3\,x-2}+\sqrt{2}\,1\mathrm{i}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3*x - 2)^(1/2)*(3*x + 2)^(1/2)),x)
[Out] -(4*atan((2^(1/2)*1i - (- 3*x - 2)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

3.1028 $\int (a+bx)(ac-bcx)^3 \, dx$

Optimal. Leaf size=38

$$-\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}$$

```
[Out] -1/2*a*c^3*(-b*x+a)^4/b+1/5*c^3*(-b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] -1/2*(a*c^3*(a - b*x)^4)/b + (c^3*(a - b*x)^5)/(5*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int (a+bx)(ac-bcx)^3 \, dx &= \int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right) dx \\ &= -\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.05

$$c^3\left(a^4x-a^3bx^2+\frac{1}{2}ab^3x^4-\frac{b^4x^5}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

Mathics [A]
time = 1.69, size = 36, normalized size = 0.95

$$\frac{c^3x\left(10a^4-10a^3bx+5ab^3x^3-2b^4x^4\right)}{10}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)*(a*c - b*c*x)^3,x]')
[Out] c ^ 3 x (10 a ^ 4 - 10 a ^ 3 b x + 5 a b ^ 3 x ^ 3 - 2 b ^ 4 x ^ 4) / 10
```

Maple [A]
time = 0.14, size = 45, normalized size = 1.18

method	result	size
gosper	$\frac{x\left(-2b^4x^4+5a\,b^3x^3-10a^3bx+10a^4\right)c^3}{10}$	37
default	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45
norman	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45
risch	$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/5*b^4*c^3*x^5+1/2*a*b^3*c^3*x^4-a^3*b*c^3*x^2+a^4*c^3*x
```

Maxima [A]
time = 0.31, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Fricas [A]
time = 0.29, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Sympy [A]
time = 0.04, size = 44, normalized size = 1.16

$$a^4c^3x - a^3bc^3x^2 + \frac{ab^3c^3x^4}{2} - \frac{b^4c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**3,x)
```

```
[Out] a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

Giac [A]
time = 0.00, size = 49, normalized size = 1.29

$$-\frac{1}{5}x^5b^4c^3 + \frac{1}{2}x^4b^3ac^3 - x^2ba^3c^3 + xa^4c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x)
```

```
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Mupad [B]
time = 0.16, size = 44, normalized size = 1.16

$$a^4\,c^3\,x - a^3\,b\,c^3\,x^2 + \frac{a\,b^3\,c^3\,x^4}{2} - \frac{b^4\,c^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x),x)
```

```
[Out] a^4*c^3*x - (b^4*c^3*x^5)/5 - a^3*b*c^3*x^2 + (a*b^3*c^3*x^4)/2
```

3.1029 $\int (a+bx)(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$-\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}$$

```
[Out] -2/3*a*c^2*(-b*x+a)^3/b+1/4*c^2*(-b*x+a)^4/b
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {45}

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^2,x]
```

```
[Out] (-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^2\,dx &= \int \left(2a(ac-bcx)^2-\frac{(ac-bcx)^3}{c}\right)dx\\ &= -\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.11

$$c^2\left(a^3x-\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^2,x]
```

```
[Out] c^2*(a^3*x - (a^2*b*x^2)/2 - (a*b^2*x^3)/3 + (b^3*x^4)/4)
```

Mathics [A]
time = 1.68, size = 36, normalized size = 0.95

$$\frac{c^2x\left(12a^3-6a^2bx-4ab^2x^2+3b^3x^3\right)}{12}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)*(a*c - b*c*x)^2,x]')
[Out] c ^ 2 x (12 a ^ 3 - 6 a ^ 2 b x - 4 a b ^ 2 x ^ 2 + 3 b ^ 3 x ^ 3) / 12
```

Maple [A]
time = 0.11, size = 45, normalized size = 1.18

method	result	size
gosper	$\frac{x\left(3b^3x^3-4a\,b^2x^2-6a^2bx+12a^3\right)c^2}{12}$	37
default	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2b\,c^2x^2+a^3c^2x$	45
norman	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2b\,c^2x^2+a^3c^2x$	45
risch	$\frac{1}{4}b^3c^2x^4-\frac{1}{3}a\,b^2c^2x^3-\frac{1}{2}a^2b\,c^2x^2+a^3c^2x$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*b^3*c^2*x^4-1/3*a*b^2*c^2*x^3-1/2*a^2*b*c^2*x^2+a^3*c^2*x
```

Maxima [A]
time = 0.26, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^3c^2x^4-\frac{1}{3}\,ab^2c^2x^3-\frac{1}{2}\,a^2bc^2x^2+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="maxima")
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Fricas [A]
time = 0.28, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^3c^2x^4-\frac{1}{3}\,ab^2c^2x^3-\frac{1}{2}\,a^2bc^2x^2+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Sympy [A]
time = 0.04, size = 46, normalized size = 1.21

$$a^3c^2x - \frac{a^2bc^2x^2}{2} - \frac{ab^2c^2x^3}{3} + \frac{b^3c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**2,x)
```

```
[Out] a**3*c**2*x - a**2*b*c**2*x**2/2 - a*b**2*c**2*x**3/3 + b**3*c**2*x**4/4
```

Giac [A]
time = 0.00, size = 52, normalized size = 1.37

$$\frac{1}{4}x^4b^3c^2 - \frac{1}{3}x^3b^2ac^2 - \frac{1}{2}x^2ba^2c^2 + xa^3c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Mupad [B]
time = 0.05, size = 44, normalized size = 1.16

$$a^3\,c^2\,x - \frac{a^2\,b\,c^2\,x^2}{2} - \frac{a\,b^2\,c^2\,x^3}{3} + \frac{b^3\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x),x)
```

```
[Out] a^3*c^2*x + (b^3*c^2*x^4)/4 - (a^2*b*c^2*x^2)/2 - (a*b^2*c^2*x^3)/3
```

3.1030 $\int (a+bx)(ac-bcx)\,dx$

Optimal. Leaf size=18

$$a^2cx-\frac{1}{3}b^2cx^3$$

```
[Out] a^2*c*x-1/3*b^2*c*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {41}

$$a^2cx-\frac{1}{3}b^2cx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x),x]
```

```
[Out] a^2*c*x - (b^2*c*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)\,dx &= \int \left(a^2c-b^2cx^2\right)\,dx\\ &= a^2cx-\frac{1}{3}b^2cx^3\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$c\left(a^2x-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x),x]
```

```
[Out] c*(a^2*x - (b^2*x^3)/3)
```

Mathics [A]
time = 1.56, size = 18, normalized size = 1.00

$$\frac{cx\left(3a^2-b^2x^2\right)}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)*(a*c - b*c*x)^1,x]')
```

```
[Out] c x (3 a ^ 2 - b ^ 2 x ^ 2) / 3
```

Maple [A]
time = 0.04, size = 17, normalized size = 0.94

method	result	size
default	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
norman	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
risch	$a^2cx-\frac{1}{3}b^2c\,x^3$	17
gosper	$\frac{cx\left(-x^2b^2+3a^2\right)}{3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*c*x-1/3*b^2*c*x^3
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.89

$$-\frac{1}{3}\,b^2cx^3+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Fricas [A]
time = 0.29, size = 16, normalized size = 0.89

$$-\frac{1}{3}\,b^2cx^3+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.83

$$a^2cx - \frac{b^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x)
[Out] a**2*c*x - b**2*c*x**3/3
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.06

$$-\frac{1}{3}x^3b^2c + xa^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x)
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Mupad [B]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{c\,x\,\left(3\,a^2-b^2\,x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x),x)
[Out] (c*x*(3*a^2 - b^2*x^2))/3
```

3.1031 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx = ax+\frac{bx^2}{2}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Mathics [A]
time = 1.52, size = 10, normalized size = 0.83

$$\frac{x\,(2a+bx)}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)*(a*c - b*c*x)^0,x]')
```

```
[Out] x (2 a + b x) / 2
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{1}{2}x^2b+ax$	11
default	$\frac{1}{2}x^2b+ax$	11
norman	$\frac{1}{2}x^2b+ax$	11
risch	$\frac{1}{2}x^2b+ax$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*b+a*x
```

Maxima [A]
time = 0.25, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.26, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A]
time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

3.1032 $\int \frac{a+bx}{ac-bcx} dx$

Optimal. Leaf size=23

$$-\frac{x}{c}-\frac{2a\log(a-bx)}{bc}$$

[Out] -x/c-2*a*ln(-b*x+a)/b/c

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(a*c - b*c*x),x]

[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{ac-bcx}\,dx &= \int \left(-\frac{1}{c}+\frac{2a}{c(a-bx)}\right)dx\\ &= -\frac{x}{c}-\frac{2a\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{x}{c}-\frac{2a\log(a-bx)}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(a*c - b*c*x),x]

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Mathics [A]
time = 1.65, size = 23, normalized size = 1.00

$$\frac{-2a\text{Log}\left[-a+bx\right]-bx}{bc}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(a*c - b*c*x)^1,x]')
```

```
[Out] (-2 a Log[-a + b x] - b x) / (b c)
```

Maple [A]
time = 0.11, size = 22, normalized size = 0.96

method	result	size
default	$\frac{-x-\frac{2a\ln(-bx+a)}{b}}{c}$	22
norman	$-\frac{x}{c}-\frac{2a\ln(-bx+a)}{bc}$	24
risch	$-\frac{x}{c}-\frac{2a\ln(-bx+a)}{bc}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-x-2*a/b*ln(-b*x+a))
```

Maxima [A]
time = 0.26, size = 24, normalized size = 1.04

$$-\frac{x}{c}-\frac{2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -x/c - 2*a*log(b*x - a)/(b*c)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 1.00

$$-\frac{bx+2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

Sympy [A]
time = 0.07, size = 17, normalized size = 0.74

$$-\frac{2a\log\left(-a+bx\right)}{bc}-\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x)
[Out] -2*a*log(-a + b*x)/(b*c) - x/c
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.09

$$-\frac{xb}{bc}-\frac{2a\ln|xb-a|}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x)
[Out] -x/c - 2*a*log(abs(b*x - a))/(b*c)
```

Mupad [B]
time = 0.05, size = 23, normalized size = 1.00

$$-\frac{b\,x+2\,a\,\ln\left(b\,x-a\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x),x)
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

3.1033 $\int \frac{a+bx}{(ac-bcx)^2} \, dx$

Optimal. Leaf size=32

$$\frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2}$$

```
[Out] 2*a/b/c^2/(-b*x+a)+ln(-b*x+a)/b/c^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$\frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] (2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{(ac-bcx)^2} \, dx &= \int \left(\frac{2a}{c^2(a-bx)^2} - \frac{1}{c^2(a-bx)}\right) dx \\ &= \frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.88

$$\frac{\frac{2a}{a-bx} + \log(c(a-bx))}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] ((2*a)/(a - b*x) + Log[c*(a - b*x)])/(b*c^2)
```

Mathics [A]
time = 1.82, size = 34, normalized size = 1.06

$$\frac{2a + \text{Log}\left[-a + bx\right](a - bx)}{bc^2(a - bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(a*c - b*c*x)^2,x]')
```

```
[Out] (2 a + Log[-a + b x] (a - b x)) / (b c ^ 2 (a - b x))
```

Maple [A]
time = 0.12, size = 31, normalized size = 0.97

method	result	size
default	$\frac{\frac{2a}{b(-bx+a)}+\frac{\ln(-bx+a)}{b}}{c^2}$	31
norman	$\frac{2a}{b\,c^2(-bx+a)}+\frac{\ln(-bx+a)}{b\,c^2}$	33
risch	$\frac{2a}{b\,c^2(-bx+a)}+\frac{\ln(-bx+a)}{b\,c^2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(2*a/b/(-b*x+a)+1/b*ln(-b*x+a))
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.16

$$-\frac{2\,a}{b^2c^2x - abc^2} + \frac{\log\left(bx - a\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -2*a/(b^2*c^2*x - a*b*c^2) + log(b*x - a)/(b*c^2)
```

Fricas [A]
time = 0.29, size = 39, normalized size = 1.22

$$\frac{(bx - a)\log\left(bx - a\right) - 2\,a}{b^2c^2x - abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] ((b*x - a)*log(b*x - a) - 2*a)/(b^2*c^2*x - a*b*c^2)
```

Sympy [**A**]
time = 0.10, size = 29, normalized size = 0.91

$$-\frac{2a}{-abc^2+b^2c^2x}+\frac{\log(-a+bx)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**2,x)
[Out] -2*a/(-a*b*c**2 + b**2*c**2*x) + log(-a + b*x)/(b*c**2)
```

Giac [**A**]
time = 0.00, size = 33, normalized size = 1.03

$$-\frac{2a}{c^2b(xb-a)}+\frac{\ln|xb-a|}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x)
[Out] log(abs(b*x - a))/(b*c^2) - 2*a/((b*x - a)*b*c^2)
```

Mupad [**B**]
time = 0.05, size = 37, normalized size = 1.16

$$\frac{\ln(b\,x-a)}{b\,c^2}+\frac{2\,a}{b\,(a\,c^2-b\,c^2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^2,x)
[Out] log(b*x - a)/(b*c^2) + (2*a)/(b*(a*c^2 - b*c^2*x))
```

3.1034 $\int \frac{a+bx}{(ac-bcx)^3} \, dx$

Optimal. Leaf size=13

$$\frac{x}{c^3(a-bx)^2}$$

```
[Out] x/c^3/(-b*x+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{34\}$

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^3,x]
```

```
[Out] x/(c^3*(a - b*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{a+bx}{(ac-bcx)^3} \, dx = \frac{x}{c^3(a-bx)^2}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^3,x]
```

```
[Out] x/(c^3*(a - b*x)^2)
```

Mathics [A]
time = 1.76, size = 23, normalized size = 1.77

$$\frac{x}{c^3 (a^2 - 2abx + b^2x^2)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(a*c - b*c*x)^3,x]')
```

```
[Out] x / (c ^ 3 (a ^ 2 - 2 a b x + b ^ 2 x ^ 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.
time = 0.14, size = 32, normalized size = 2.46

method	result	size
gosper	$\frac{x}{c^3(-bx+a)^2}$	14
norman	$\frac{x}{c^3(-bx+a)^2}$	14
risch	$\frac{x}{c^3(-bx+a)^2}$	14
default	$\frac{\frac{a}{b(-bx+a)^2}-\frac{1}{b(-bx+a)}}{c^3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/b/(-b*x+a)^2-1/b/(-b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.25, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.29, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.
time = 0.12, size = 27, normalized size = 2.08

$$\frac{x}{a^2c^3 - 2abc^3x + b^2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] x/(a**2*c**3 - 2*a*b*c**3*x + b**2*c**3*x**2)
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.15

$$\frac{x}{c^3\left(xb - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] x/((b*x - a)^2*c^3)
```

Mupad [B]
time = 0.15, size = 13, normalized size = 1.00

$$\frac{x}{c^3\left(a - b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^3,x)
```

```
[Out] x/(c^3*(a - b*x)^2)
```

3.1035 $\int \frac{a+bx}{(ac-bcx)^4}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}$$

```
[Out] 2/3*a/b/c^4/(-b*x+a)^3-1/2/b/c^4/(-b*x+a)^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] (2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^4}\,dx &= \int \left(\frac{2a}{c^4(a-bx)^4}-\frac{1}{c^4(a-bx)^3}\right)dx\\ &= \frac{2a}{3bc^4(a-bx)^3}-\frac{1}{2bc^4(a-bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.66

$$-\frac{a+3bx}{6bc^4(-a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b*c^4*(-a + b*x)^3)
```

Mathics [A]
time = 1.99, size = 44, normalized size = 1.16

$$\frac{a + 3bx}{6bc^4\left(a^3 - 3a^2bx + 3ab^2x^2 - b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(a*c - b*c*x)^4,x]')
```

```
[Out] (a + 3 b x) / (6 b c ^ 4 (a ^ 3 - 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 - b ^ 3 x ^
 3))
```

Maple [A]
time = 0.11, size = 33, normalized size = 0.87

method	result	size
gosper	$\frac{3bx+a}{6(-bx+a)^3c^4b}$	23
risch	$\frac{\frac{x}{2}+\frac{a}{6b}}{c^4(-bx+a)^3}$	23
norman	$\frac{\frac{a}{6bc}+\frac{x}{2c}}{c^3(-bx+a)^3}$	29
default	$\frac{-\frac{1}{2b(-bx+a)^2}+\frac{2a}{3b(-bx+a)^3}}{c^4}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2/b/(-b*x+a)^2+2/3*a/b/(-b*x+a)^3)
```

Maxima [A]
time = 0.26, size = 54, normalized size = 1.42

$$-\frac{3\,bx + a}{6\left(b^4c^4x^3 - 3\,ab^3c^4x^2 + 3\,a^2b^2c^4x - a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

Fricas [A]
time = 0.29, size = 54, normalized size = 1.42

$$-\frac{3\,bx + a}{6\left(b^4c^4x^3 - 3\,ab^3c^4x^2 + 3\,a^2b^2c^4x - a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

Sympy [A]
time = 0.15, size = 56, normalized size = 1.47

$$\frac{-a-3bx}{-6a^3bc^4+18a^2b^2c^4x-18ab^3c^4x^2+6b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a - 3*b*x)/(-6*a**3*b*c**4 + 18*a**2*b**2*c**4*x - 18*a*b**3*c**4*x**2 +
6*b**4*c**4*x**3)
```

Giac [A]
time = 0.00, size = 25, normalized size = 0.66

$$-\frac{-3xb-a}{6bc^4\left(-xb+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x)
```

```
[Out] -1/6*(3*b*x + a)/((b*x - a)^3*b*c^4)
```

Mupad [B]
time = 0.05, size = 54, normalized size = 1.42

$$\frac{\frac{x}{2}+\frac{a}{6\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^4,x)
```

```
[Out] (x/2 + a/(6*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*x)
```

3.1036 $\int \frac{a+bx}{(ac-bcx)^5} \, dx$

Optimal. Leaf size=38

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

```
[Out] 1/2*a/b/c^5/(-b*x+a)^4-1/3/b/c^5/(-b*x+a)^3
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^5,x]
```

```
[Out] a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^5} \, dx &= \int \left(\frac{2a}{c^5(a-bx)^5}-\frac{1}{c^5(a-bx)^4}\right) dx\\ &= \frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.63

$$\frac{a+2bx}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^5,x]
```

```
[Out] (a + 2*b*x)/(6*b*c^5*(a - b*x)^4)
```

Mathics [A]
time = 2.10, size = 54, normalized size = 1.42

$$\frac{a+2bx}{6bc^5\left(a^4-4a^3bx+6a^2b^2x^2-4ab^3x^3+b^4x^4\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(a*c - b*c*x)^5,x]')
```

```
[Out] (a + 2 b x) / (6 b c ^ 5 (a ^ 4 - 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 - 4 a b
 ^ 3 x ^ 3 + b ^ 4 x ^ 4))
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.87

method	result	size
gosper	$\frac{2bx+a}{6(-bx+a)^4c^5b}$	23
risch	$\frac{\frac{x}{3}+\frac{a}{6b}}{c^5(-bx+a)^4}$	23
norman	$\frac{\frac{a}{6bc}+\frac{x}{3c}}{c^4(-bx+a)^4}$	29
default	$\frac{-\frac{1}{3b(-bx+a)^3}+\frac{a}{2b(-bx+a)^4}}{c^5}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-1/3/b/(-b*x+a)^3+1/2*a/b/(-b*x+a)^4)
```

Maxima [A]
time = 0.26, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\,\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

Fricas [A]
time = 0.29, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\,\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(29) = 58$.
time = 0.19, size = 73, normalized size = 1.92

$$-\frac{-a-2bx}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a - 2*b*x)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a**2*b**3*c**5*x**2
 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Giac [A]
time = 0.00, size = 22, normalized size = 0.58

$$\frac{2xb+a}{6bc^5\left(xb-a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/6*(2*b*x + a)/((b*x - a)^4*b*c^5)
```

Mupad [B]
time = 0.17, size = 67, normalized size = 1.76

$$\frac{\frac{x}{3}+\frac{a}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^5,x)
```

```
[Out] (x/3 + a/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3 + 6*a^2*b^2*c^5*x^
2 - 4*a^3*b*c^5*x)
```

3.1037 $\int \frac{a+bx}{(ac-bcx)^6}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

```
[Out] 2/5*a/b/c^6/(-b*x+a)^5-1/4/b/c^6/(-b*x+a)^4
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = $\{45\}$

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] (2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^6}\,dx &= \int \left(\frac{2a}{c^6(a-bx)^6}-\frac{1}{c^6(a-bx)^5}\right)dx\\ &= \frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{3a+5bx}{20bc^6(-a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] -1/20*(3*a + 5*b*x)/(b*c^6*(-a + b*x)^5)
```

Mathics [A]
time = 2.25, size = 68, normalized size = 1.79

$$\frac{3a+5bx}{20bc^6\left(a^5-5a^4bx+10a^3b^2x^2-10a^2b^3x^3+5ab^4x^4-b^5x^5\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)/(a*c - b*c*x)^6,x]')
[Out] (3 a + 5 b x) / (20 b c ^ 6 (a ^ 5 - 5 a ^ 4 b x + 10 a ^ 3 b ^ 2 x ^ 2 - 1
0 a ^ 2 b ^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 - b ^ 5 x ^ 5))
```

Maple [A]
time = 0.14, size = 33, normalized size = 0.87

method	result	size
risch	$\frac{\frac{x}{4}+\frac{3a}{20b}}{c^6(-bx+a)^5}$	23
gosper	$\frac{5bx+3a}{20(-bx+a)^5c^6b}$	25
norman	$\frac{\frac{3a}{20bc}+\frac{x}{4c}}{c^5(-bx+a)^5}$	29
default	$\frac{-\frac{1}{4b(-bx+a)^4}+\frac{2a}{5b(-bx+a)^5}}{c^6}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
[Out] 1/c^6*(-1/4/b/(-b*x+a)^4+2/5*a/b/(-b*x+a)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.
time = 0.27, size = 84, normalized size = 2.21

$$-\frac{5\,bx+3\,a}{20\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="maxima")
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.
time = 0.29, size = 84, normalized size = 2.21

$$-\frac{5\,bx+3\,a}{20\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(31) = 62$.
time = 0.23, size = 88, normalized size = 2.32

$$\frac{-3a - 5bx}{-20a^5bc^6 + 100a^4b^2c^6x - 200a^3b^3c^6x^2 + 200a^2b^4c^6x^3 - 100ab^5c^6x^4 + 20b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**6,x)
```

```
[Out] (-3*a - 5*b*x)/(-20*a**5*b*c**6 + 100*a**4*b**2*c**6*x - 200*a**3*b**3*c**6
*x**2 + 200*a**2*b**4*c**6*x**3 - 100*a*b**5*c**6*x**4 + 20*b**6*c**6*x**5)
```

Giac [A]
time = 0.00, size = 27, normalized size = 0.71

$$-\frac{-5xb - 3a}{20bc^6\left(-xb + a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x)
```

```
[Out] -1/20*(5*b*x + 3*a)/((b*x - a)^5*b*c^6)
```

Mupad [B]
time = 0.08, size = 82, normalized size = 2.16

$$\frac{\frac{x}{4} + \frac{3\,a}{20\,b}}{a^5\,c^6 - 5\,a^4\,b\,c^6\,x + 10\,a^3\,b^2\,c^6\,x^2 - 10\,a^2\,b^3\,c^6\,x^3 + 5\,a\,b^4\,c^6\,x^4 - b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^6,x)
```

```
[Out] (x/4 + (3*a)/(20*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6*x^4 + 10*a^3*b^2*
c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

3.1038 $\int (a+bx)^2(ac-bcx)^3\,dx$

Optimal. Leaf size=57

$$-\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}$$

```
[Out] -a^2*c^3*(-b*x+a)^4/b+4/5*a*c^3*(-b*x+a)^5/b-1/6*c^3*(-b*x+a)^6/b
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] -((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^6
)/(6*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^3\,dx &= \int \left(4a^2(ac-bcx)^3-\frac{4a(ac-bcx)^4}{c}+\frac{(ac-bcx)^5}{c^2}\right)dx\\ &= -\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.19

$$c^3\left(a^5x-\frac{1}{2}a^4bx^2-\frac{2}{3}a^3b^2x^3+\frac{1}{2}a^2b^3x^4+\frac{1}{5}ab^4x^5-\frac{b^5x^6}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^5*x - (a^4*b*x^2)/2 - (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 + (a*b^4*x
^5)/5 - (b^5*x^6)/6)
```

Mathics [A]
time = 1.80, size = 58, normalized size = 1.02

$$\frac{c^3x\left(30a^5-15a^4bx-20a^3b^2x^2+15a^2b^3x^3+6ab^4x^4-5b^5x^5\right)}{30}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]')
```

```
[Out] c ^ 3 x (30 a ^ 5 - 15 a ^ 4 b x - 20 a ^ 3 b ^ 2 x ^ 2 + 15 a ^ 2 b ^ 3 x
^ 3 + 6 a b ^ 4 x ^ 4 - 5 b ^ 5 x ^ 5) / 30
```

Maple [A]
time = 0.17, size = 73, normalized size = 1.28

method	result	size
gosper	$\frac{x\left(-5b^5x^5+6a\,b^4x^4+15a^2b^3x^3-20a^3b^2x^2-15a^4bx+30a^5\right)c^3}{30}$	59
default	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73
norman	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73
risch	$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3c^3b^2x^3-\frac{1}{2}a^4c^3b\,x^2+a^5c^3x$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b^5*c^3*x^6+1/5*a*b^4*c^3*x^5+1/2*a^2*b^3*c^3*x^4-2/3*a^3*c^3*b^2*x^3-
1/2*a^4*c^3*b*x^2+a^5*c^3*x
```

Maxima [A]
time = 0.25, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4-\frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Fricas [A]
time = 0.29, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4-\frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Sympy [A]
time = 0.04, size = 78, normalized size = 1.37

$$a^5c^3x - \frac{a^4bc^3x^2}{2} - \frac{2a^3b^2c^3x^3}{3} + \frac{a^2b^3c^3x^4}{2} + \frac{ab^4c^3x^5}{5} - \frac{b^5c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**3,x)
```

```
[Out] a**5*c**3*x - a**4*b*c**3*x**2/2 - 2*a**3*b**2*c**3*x**3/3 + a**2*b**3*c**3
*x**4/2 + a*b**4*c**3*x**5/5 - b**5*c**3*x**6/6
```

Giac [A]
time = 0.00, size = 85, normalized size = 1.49

$$-\frac{1}{6}x^6b^5c^3 + \frac{1}{5}x^5b^4ac^3 + \frac{1}{2}x^4b^3a^2c^3 - \frac{2}{3}x^3b^2a^3c^3 - \frac{1}{2}x^2ba^4c^3 + xa^5c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x)
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Mupad [B]
time = 0.03, size = 72, normalized size = 1.26

$$a^5\,c^3\,x - \frac{a^4\,b\,c^3\,x^2}{2} - \frac{2\,a^3\,b^2\,c^3\,x^3}{3} + \frac{a^2\,b^3\,c^3\,x^4}{2} + \frac{a\,b^4\,c^3\,x^5}{5} - \frac{b^5\,c^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x)^2,x)
```

```
[Out] a^5*c^3*x - (b^5*c^3*x^6)/6 - (a^4*b*c^3*x^2)/2 + (a*b^4*c^3*x^5)/5 - (2*a^
3*b^2*c^3*x^3)/3 + (a^2*b^3*c^3*x^4)/2
```

3.1039 $\int (a+bx)^2(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

```
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 200}

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^2\,dx &= \int \left(a^2c-b^2cx^2\right)^2\,dx\\ &= \int \left(a^4c^2-2a^2b^2c^2x^2+b^4c^2x^4\right)\,dx\\ &= a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.00

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Mathics [A]
time = 1.64, size = 31, normalized size = 0.82

$$\frac{c^2x\left(15a^4-10a^2b^2x^2+3b^4x^4\right)}{15}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]')
```

```
[Out] c ^ 2 x (15 a ^ 4 - 10 a ^ 2 b ^ 2 x ^ 2 + 3 b ^ 4 x ^ 4) / 15
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.92

method	result	size
gosper	$\frac{x\left(3b^4x^4-10a^2b^2x^2+15a^4\right)c^2}{15}$	32
default	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
norman	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
risch	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5-\frac{2}{3}\,a^2b^2c^2x^3+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5-\frac{2}{3}\,a^2b^2c^2x^3+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Sympy [A]
time = 0.04, size = 36, normalized size = 0.95

$$a^4c^2x - \frac{2a^2b^2c^2x^3}{3} + \frac{b^4c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**2,x)
```

```
[Out] a**4*c**2*x - 2*a**2*b**2*c**2*x**3/3 + b**4*c**2*x**5/5
```

Giac [A]
time = 0.00, size = 39, normalized size = 1.03

$$\frac{1}{5}x^5b^4c^2 - \frac{2}{3}x^3b^2a^2c^2 + xa^4c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x)
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.82

$$\frac{c^2\,x\,\left(15\,a^4-10\,a^2\,b^2\,x^2+3\,b^4\,x^4\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x)^2,x)
```

```
[Out] (c^2*x*(15*a^4 + 3*b^4*x^4 - 10*a^2*b^2*x^2))/15
```

3.1040 $\int (a+bx)^2(ac-bcx)\,dx$

Optimal. Leaf size=32

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

```
[Out] 2/3*a*c*(b*x+a)^3/b-1/4*c*(b*x+a)^4/b
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {45}

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] (2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)\,dx &= \int \left(2ac(a+bx)^2-c(a+bx)^3\right)\,dx\\ &= \frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.25

$$c\left(a^3x+\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] c*(a^3*x + (a^2*b*x^2)/2 - (a*b^2*x^3)/3 - (b^3*x^4)/4)
```

Mathics [A]
time = 1.65, size = 34, normalized size = 1.06

$$\frac{cx\,(12a^3+6a^2bx-4ab^2x^2-3b^3x^3)}{12}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(a*c - b*c*x)^1,x]')
[Out] c x (12 a ^ 3 + 6 a ^ 2 b x - 4 a b ^ 2 x ^ 2 - 3 b ^ 3 x ^ 3) / 12
```

Maple [A]
time = 0.12, size = 37, normalized size = 1.16

method	result	size
gosper	$\frac{cx(-3b^3x^3-4a\,b^2x^2+6a^2bx+12a^3)}{12}$	35
default	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37
norman	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37
risch	$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] -1/4*b^3*c*x^4-1/3*a*b^2*c*x^3+1/2*a^2*b*c*x^2+a^3*c*x
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="maxima")
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Fricas [A]
time = 0.29, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Sympy [A]
time = 0.03, size = 39, normalized size = 1.22

$$a^{3}cx + \frac{a^{2}bcx^{2}}{2} - \frac{ab^{2}cx^{3}}{3} - \frac{b^{3}cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c),x)
```

```
[Out] a**3*c*x + a**2*b*c*x**2/2 - a*b**2*c*x**3/3 - b**3*c*x**4/4
```

Giac [A]
time = 0.00, size = 44, normalized size = 1.38

$$-\frac{1}{4}x^{4}b^{3}c - \frac{1}{3}x^{3}b^{2}ac + \frac{1}{2}x^{2}ba^{2}c + xa^{3}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x)
```

```
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Mupad [B]
time = 0.05, size = 36, normalized size = 1.12

$$c\,a^{3}\,x + \frac{c\,a^{2}\,b\,x^{2}}{2} - \frac{c\,a\,b^{2}\,x^{3}}{3} - \frac{c\,b^{3}\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x)^2,x)
```

```
[Out] a^3*c*x - (b^3*c*x^4)/4 + (a^2*b*c*x^2)/2 - (a*b^2*c*x^3)/3
```

3.1041 $\int (a+bx)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

[Out] 1/3*(b*x+a)^3/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2\,dx = \frac{(a+bx)^3}{3b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Mathics [A]
time = 1.58, size = 18, normalized size = 1.29

$$x\left(a^2+abx+\frac{b^2x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(a*c - b*c*x)^0,x]')
```

```
[Out] x (a ^ 2 + a b x + b ^ 2 x ^ 2 / 3)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
gosper	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
norman	$\frac{1}{3}b^2x^3+ab\,x^2+a^2x$	21
risch	$\frac{b^2x^3}{3}+ab\,x^2+a^2x+\frac{a^3}{3b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/b
```

Maxima [A]
time = 0.29, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A]
time = 0.28, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.
time = 0.03, size = 19, normalized size = 1.36

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x)
```

```
[Out] 1/3*(b*x + a)^3/b
```

Mupad [B]
time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
```

```
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

3.1042 $\int \frac{(a+bx)^2}{ac-bcx} \, dx$

Optimal. Leaf size=43

$$-\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}$$

```
[Out] -2*a*x/c-1/2*(b*x+a)^2/b/c-4*a^2*ln(-b*x+a)/b/c
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x),x]
```

```
[Out] (-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{ac-bcx}\,dx &= \int \left(-\frac{2a}{c}-\frac{a+bx}{c}+\frac{4a^2}{ac-bcx}\right)dx\\ &= -\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.86

$$-\frac{3ax}{c}-\frac{bx^2}{2c}-\frac{4a^2\log(a-bx)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x),x]
```

```
[Out] (-3*a*x)/c - (b*x^2)/(2*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Mathics [A]
time = 1.73, size = 33, normalized size = 0.77

$$\frac{-8a^2\text{Log}\left[-a+bx\right]+bx\left(-6a-bx\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^1,x]')
```

```
[Out] (-8 a ^ 2 Log[-a + b x] + b x (-6 a - b x)) / (2 b c)
```

Maple [A]
time = 0.18, size = 31, normalized size = 0.72

method	result	size
default	$\frac{-\frac{x^2 b}{2}-3ax-\frac{4a^2\ln(-bx+a)}{b}}{c}$	31
norman	$-\frac{3ax}{c}-\frac{b\,x^2}{2c}-\frac{4a^2\ln(-bx+a)}{bc}$	36
risch	$-\frac{3ax}{c}-\frac{b\,x^2}{2c}-\frac{4a^2\ln(-bx+a)}{bc}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*x^2*b-3*a*x-4*a^2/b*ln(-b*x+a))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.81

$$-\frac{4\,a^2\log\left(bx-a\right)}{bc}-\frac{bx^2+6\,ax}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -4*a^2*log(b*x - a)/(b*c) - 1/2*(b*x^2 + 6*a*x)/c
```

Fricas [A]
time = 0.29, size = 34, normalized size = 0.79

$$-\frac{b^2x^2+6\,abx+8\,a^2\log\left(bx-a\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/(b*c)
```

Sympy [A]
time = 0.08, size = 31, normalized size = 0.72

$$-\frac{4a^2\log(-a+bx)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c),x)
[Out] -4*a**2*log(-a + b*x)/(b*c) - 3*a*x/c - b*x**2/(2*c)
```

Giac [A]
time = 0.00, size = 50, normalized size = 1.16

$$\frac{-\frac{1}{2}x^2b^3c-3xb^2ac}{b^2c^2}-\frac{4a^2\ln|xb-a|}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x)
[Out] -4*a^2*log(abs(b*x - a))/(b*c) - 1/2*(b^3*c*x^2 + 6*a*b^2*c*x)/(b^2*c^2)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.79

$$-\frac{8\,a^2\,\ln(b\,x-a)+b^2\,x^2+6\,a\,b\,x}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x),x)
[Out] -(8*a^2*log(b*x - a) + b^2*x^2 + 6*a*b*x)/(2*b*c)
```

3.1043 $\int \frac{(a+bx)^2}{(ac-bcx)^2} dx$

Optimal. Leaf size=41

$$\frac{x}{c^2} + \frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2}$$

```
[Out] x/c^2+4*a^2/b/c^2/(-b*x+a)+4*a*ln(-b*x+a)/b/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$\frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2} + \frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{(ac-bcx)^2} dx &= \int \left(\frac{1}{c^2} + \frac{4a^2}{c^2(a-bx)^2} - \frac{4a}{c^2(a-bx)}\right) dx \\
&= \frac{x}{c^2} + \frac{4a^2}{bc^2(a-bx)} + \frac{4a\log(a-bx)}{bc^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.85

$$\frac{x + \frac{4a^2}{b(a-bx)} + \frac{4a\log(a-bx)}{b}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] (x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b)/c^2
```

Mathics [A]
time = 1.89, size = 47, normalized size = 1.15

$$\frac{4a\text{Log}\left[-a+bx\right](a-bx)+4a^2+bx(a-bx)}{bc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]')
```

```
[Out] (4 a Log[-a + b x] (a - b x) + 4 a ^ 2 + b x (a - b x)) / (b c ^ 2 (a - b x
))
```

Maple [A]
time = 0.16, size = 36, normalized size = 0.88

method	result	size
default	$\frac{x+\frac{4a^2}{b(-bx+a)}+\frac{4a\ln(-bx+a)}{b}}{c^2}$	36
risch	$\frac{x}{c^2}+\frac{4a^2}{b\,c^2(-bx+a)}+\frac{4a\ln(-bx+a)}{b\,c^2}$	42
norman	$\frac{\frac{5a^2}{bc}-\frac{b\,x^2}{c}}{c(-bx+a)}+\frac{4a\ln(-bx+a)}{b\,c^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(x+4*a^2/b/(-b*x+a)+4*a/b*ln(-b*x+a))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 1.12

$$-\frac{4\,a^2}{b^2c^2x-abc^2}+\frac{x}{c^2}+\frac{4\,a\log\left(bx-a\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2/(b^2*c^2*x - a*b*c^2) + x/c^2 + 4*a*log(b*x - a)/(b*c^2)
```

Fricas [A]
time = 0.29, size = 57, normalized size = 1.39

$$\frac{b^2x^2-abx-4\,a^2+4\,(abx-a^2)\log\left(bx-a\right)}{b^2c^2x-abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*c^2*x - a*b*c
^2)
```

Sympy [A]
time = 0.10, size = 39, normalized size = 0.95

$$-\frac{4a^2}{-abc^2+b^2c^2x}+\frac{4a\log\left(-a+bx\right)}{bc^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -4*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*log(-a + b*x)/(b*c**2) + x/c**2
```

Giac [A]
time = 0.00, size = 50, normalized size = 1.22

$$\frac{xb^2}{b^2c^2}-\frac{4a^2}{bc^2\left(xb-a\right)}+\frac{4a\ln\left|xb-a\right|}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] x/c^2 + 4*a*log(abs(b*x - a))/(b*c^2) - 4*a^2/((b*x - a)*b*c^2)
```

Mupad [B]
time = 0.15, size = 46, normalized size = 1.12

$$\frac{x}{c^2}+\frac{4\,a^2}{b\,\left(a\,c^2-b\,c^2\,x\right)}+\frac{4\,a\,\ln\left(b\,x-a\right)}{b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^2,x)
```

```
[Out] x/c^2 + (4*a^2)/(b*(a*c^2 - b*c^2*x)) + (4*a*log(b*x - a))/(b*c^2)
```

3.1044 $\int \frac{(a+bx)^2}{(ac-bcx)^3} dx$

Optimal. Leaf size=52

$$\frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}$$

[Out] 2*a^2/b/c^3/(-b*x+a)^2-4*a/b/c^3/(-b*x+a)-ln(-b*x+a)/b/c^3

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/(a*c - b*c*x)^3,x]

[Out] (2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^3
)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx &= \int \left(\frac{4a^2}{c^3(a-bx)^3} - \frac{4a}{c^3(a-bx)^2} + \frac{1}{c^3(a-bx)}\right)dx\\ &= \frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.63

$$-\frac{\frac{2a(a-2bx)}{(a-bx)^2} + \log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] -(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/(b*c^3))
```

Mathics [A]
time = 2.06, size = 61, normalized size = 1.17

$$\frac{-2a\,(a-2bx)-\mathrm{Log}\,[-a+bx]\,(a^2-2abx+b^2x^2)}{bc^3\,(a^2-2abx+b^2x^2)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]')
```

```
[Out] (-2 a (a - 2 b x) - Log[-a + b x] (a ^ 2 - 2 a b x + b ^ 2 x ^ 2)) / (b c ^
 3 (a ^ 2 - 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.92

method	result	size
risch	$\frac{4ax-\frac{2a^2}{b}}{c^3(-bx+a)^2}-\frac{\ln(-bx+a)}{b\,c^3}$	42
default	$\frac{-\frac{4a}{b(-bx+a)}-\frac{\ln(-bx+a)}{b}+\frac{2a^2}{b(-bx+a)^2}}{c^3}$	48
norman	$\frac{-\frac{2a^2}{bc}+\frac{4ax}{c}}{c^2(-bx+a)^2}-\frac{\ln(-bx+a)}{b\,c^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-4*a/b/(-b*x+a)-1/b*ln(-b*x+a)+2*a^2/b/(-b*x+a)^2)
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.17

$$\frac{2\,(2\,abx-a^2)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}-\frac{\log{(bx-a)}}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*x - a^2)/(b^3*c^3*x^2 - 2*a*b^2*c^3*x + a^2*b*c^3) - log(b*x - a)/
(b*c^3)
```

Fricas [A]
time = 0.30, size = 69, normalized size = 1.33

$$\frac{4\,abx - 2\,a^2 - (b^2x^2 - 2\,abx + a^2)\log(bx - a)}{b^3c^3x^2 - 2\,ab^2c^3x + a^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] (4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*c^3*x^2 - 2
*a*b^2*c^3*x + a^2*b*c^3)
```

Sympy [A]
time = 0.16, size = 54, normalized size = 1.04

$$-\frac{2a^2 - 4abx}{a^2bc^3 - 2ab^2c^3x + b^3c^3x^2} - \frac{\log(-a + bx)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(2*a**2 - 4*a*b*x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) - log(
-a + b*x)/(b*c**3)
```

Giac [A]
time = 0.00, size = 48, normalized size = 0.92

$$\frac{\frac{1}{2}\left(8bax - 4a^2\right)}{c^3b\left(-xb + a\right)^2} - \frac{\ln|xb - a|}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] -log(abs(b*x - a))/(b*c^3) + 2*(2*a*b*x - a^2)/((b*x - a)^2*b*c^3)
```

Mupad [B]
time = 0.17, size = 59, normalized size = 1.13

$$\frac{4\,a\,x - \frac{2\,a^2}{b}}{a^2\,c^3 - 2\,a\,b\,c^3\,x + b^2\,c^3\,x^2} - \frac{\ln(b\,x - a)}{b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^3,x)
```

```
[Out] (4*a*x - (2*a^2)/b)/(a^2*c^3 + b^2*c^3*x^2 - 2*a*b*c^3*x) - log(b*x - a)/(b
*c^3)
```

3.1045 $\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

```
[Out] 1/6*(b*x+a)^3/a/b/c^4/(-b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] (a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx = \frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.11

$$-\frac{a^2+3b^2x^2}{3bc^4(-a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*b^2*x^2)/(b*c^4*(-a + b*x)^3)
```

Mathics [A]
time = 2.04, size = 50, normalized size = 1.79

$$\frac{\frac{a^2}{3}+b^2x^2}{bc^4\left(a^3-3a^2bx+3ab^2x^2-b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]')
```

```
[Out] (a ^ 2 / 3 + b ^ 2 x ^ 2) / (b c ^ 4 (a ^ 3 - 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2
 - b ^ 3 x ^ 3))
```

Maple [A]
time = 0.14, size = 48, normalized size = 1.71

method	result	size
risch	$\frac{x^2b+\frac{a^2}{3b}}{c^4(-bx+a)^3}$	27
gosper	$\frac{3x^2b^2+a^2}{3(-bx+a)^3c^4b}$	29
norman	$\frac{\frac{a^2}{3bc}+\frac{b\,x^2}{c}}{c^3(-bx+a)^3}$	33
default	$\frac{-\frac{2a}{b(-bx+a)^2}+\frac{4a^2}{3b(-bx+a)^3}+\frac{1}{b(-bx+a)}}{c^4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-2*a/b/(-b*x+a)^2+4/3*a^2/b/(-b*x+a)^3+1/b/(-b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.26, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.29, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.
time = 0.18, size = 61, normalized size = 2.18

$$\frac{-a^2 - 3b^2x^2}{-3a^3bc^4 + 9a^2b^2c^4x - 9ab^3c^4x^2 + 3b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a**2 - 3*b**2*x**2)/(-3*a**3*b*c**4 + 9*a**2*b**2*c**4*x - 9*a*b**3*c**4*
x**2 + 3*b**4*c**4*x**3)
```

Giac [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{3x^2b^2 + a^2}{3bc^4(-xb + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x)
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/((b*x - a)^3*b*c^4)
```

Mupad [B]
time = 0.05, size = 58, normalized size = 2.07

$$\frac{b\,x^2 + \frac{a^2}{3\,b}}{a^3\,c^4 - 3\,a^2\,b\,c^4\,x + 3\,a\,b^2\,c^4\,x^2 - b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^4,x)
```

```
[Out] (b*x^2 + a^2/(3*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*
x)
```

3.1046 $\int \frac{(a+bx)^2}{(ac-bcx)^5} dx$

Optimal. Leaf size=56

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

```
[Out] a^2/b/c^5/(-b*x+a)^4-4/3*a/b/c^5/(-b*x+a)^3+1/2/b/c^5/(-b*x+a)^2
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {45}

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*x
)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^5}\,dx &= \int \left(\frac{4a^2}{c^5(a-bx)^5}-\frac{4a}{c^5(a-bx)^4}+\frac{1}{c^5(a-bx)^3}\right)dx\\ &= \frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.62

$$\frac{a^2+2abx+3b^2x^2}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] (a^2 + 2*a*b*x + 3*b^2*x^2)/(6*b*c^5*(a - b*x)^4)
```

Mathics [A]
time = 2.19, size = 65, normalized size = 1.16

$$\frac{a^2 + 2abx + 3b^2x^2}{6bc^5\left(a^4 - 4a^3bx + 6a^2b^2x^2 - 4ab^3x^3 + b^4x^4\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]')
```

```
[Out] (a ^ 2 + 2 a b x + 3 b ^ 2 x ^ 2) / (6 b c ^ 5 (a ^ 4 - 4 a ^ 3 b x + 6 a ^
 2 b ^ 2 x ^ 2 - 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4))
```

Maple [A]
time = 0.15, size = 48, normalized size = 0.86

method	result	size
risch	$\frac{\frac{x^2 b}{2}+\frac{a x}{3}+\frac{a^2}{6b}}{c^5(-bx+a)^4}$	32
gosper	$\frac{3x^2b^2+2abx+a^2}{6(-bx+a)^4c^5b}$	34
norman	$\frac{\frac{a^2}{6bc}+\frac{b\,x^2}{2c}+\frac{ax}{3c}}{c^4(-bx+a)^4}$	41
default	$\frac{\frac{1}{2b(-bx+a)^2}-\frac{4a}{3b(-bx+a)^3}+\frac{a^2}{b(-bx+a)^4}}{c^5}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/2/b/(-b*x+a)^2-4/3*a/b/(-b*x+a)^3+a^2/b/(-b*x+a)^4)
```

Maxima [A]
time = 0.26, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2 + 2\,abx + a^2}{6\left(b^5c^5x^4 - 4\,ab^4c^5x^3 + 6\,a^2b^3c^5x^2 - 4\,a^3b^2c^5x + a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Fricas [A]
time = 0.29, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Sympy [A]
time = 0.22, size = 85, normalized size = 1.52

$$-\frac{-a^2-2abx-3b^2x^2}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a**2 - 2*a*b*x - 3*b**2*x**2)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*
a**2*b**3*c**5*x**2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Giac [A]
time = 0.00, size = 33, normalized size = 0.59

$$\frac{3x^2b^2+2xba+a^2}{6bc^5\,(xb-a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/((b*x - a)^4*b*c^5)
```

Mupad [B]
time = 0.05, size = 76, normalized size = 1.36

$$\frac{\frac{a\,x}{3}+\frac{b\,x^2}{2}+\frac{a^2}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^5,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/2 + a^2/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3
+ 6*a^2*b^2*c^5*x^2 - 4*a^3*b*c^5*x)
```

3.1047 $\int \frac{(a+bx)^2}{(ac-bcx)^6} dx$

Optimal. Leaf size=57

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

```
[Out] 4/5*a^2/b/c^6/(-b*x+a)^5-a/b/c^6/(-b*x+a)^4+1/3/b/c^6/(-b*x+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] (4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*x
)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx &= \int \left(\frac{4a^2}{c^6(a-bx)^6}-\frac{4a}{c^6(a-bx)^5}+\frac{1}{c^6(a-bx)^4}\right)dx\\ &= \frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.67

$$-\frac{2a^2+5abx+5b^2x^2}{15bc^6(-a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] -1/15*(2*a^2 + 5*a*b*x + 5*b^2*x^2)/(b*c^6*(-a + b*x)^5)
```

Mathics [A]
time = 2.32, size = 79, normalized size = 1.39

$$\frac{2a^2 + 5abx + 5b^2x^2}{15bc^6\left(a^5 - 5a^4bx + 10a^3b^2x^2 - 10a^2b^3x^3 + 5ab^4x^4 - b^5x^5\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]')
```

```
[Out] (2 a ^ 2 + 5 a b x + 5 b ^ 2 x ^ 2) / (15 b c ^ 6 (a ^ 5 - 5 a ^ 4 b x + 10
 a ^ 3 b ^ 2 x ^ 2 - 10 a ^ 2 b ^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 - b ^ 5 x ^ 5))
```

Maple [A]
time = 0.15, size = 49, normalized size = 0.86

method	result	size
risch	$\frac{\frac{x^2 b}{3}+\frac{a x}{3}+\frac{2 a^2}{15 b}}{c^6 \left(-b x+a\right)^5}$	32
gosper	$\frac{5 x^2 b^2+5 a b x+2 a^2}{15 \left(-b x+a\right)^5 c^6 b}$	36
norman	$\frac{\frac{2 a^2}{15 b c}+\frac{b\,x^2}{3 c}+\frac{a x}{3 c}}{c^5 \left(-b x+a\right)^5}$	41
default	$\frac{\frac{1}{3 b \left(-b x+a\right)^3}+\frac{4 a^2}{5 b \left(-b x+a\right)^5}-\frac{a}{b \left(-b x+a\right)^4}}{c^6}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/3/b/(-b*x+a)^3+4/5*a^2/b/(-b*x+a)^5-a/b/(-b*x+a)^4)
```

Maxima [A]
time = 0.27, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2 + 5\,abx + 2\,a^2}{15\,\left(b^6c^6x^5 - 5\,ab^5c^6x^4 + 10\,a^2b^4c^6x^3 - 10\,a^3b^3c^6x^2 + 5\,a^4b^2c^6x - a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Fricas [A]
time = 0.29, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.
time = 0.25, size = 100, normalized size = 1.75

$$\frac{-2a^2-5abx-5b^2x^2}{-15a^5bc^6+75a^4b^2c^6x-150a^3b^3c^6x^2+150a^2b^4c^6x^3-75ab^5c^6x^4+15b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**6,x)
```

```
[Out] (-2*a**2 - 5*a*b*x - 5*b**2*x**2)/(-15*a**5*b*c**6 + 75*a**4*b**2*c**6*x -
150*a**3*b**3*c**6*x**2 + 150*a**2*b**4*c**6*x**3 - 75*a*b**5*c**6*x**4 + 1
5*b**6*c**6*x**5)
```

Giac [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{5x^2b^2+5xba+2a^2}{15bc^6\,(-xb+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x)
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x - a)^5*b*c^6)
```

Mupad [B]
time = 0.19, size = 91, normalized size = 1.60

$$\frac{\frac{a\,x}{3}+\frac{b\,x^2}{3}+\frac{2\,a^2}{15\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^6,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/3 + (2*a^2)/(15*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6
*x^4 + 10*a^3*b^2*c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

3.1048 $\int \frac{(a+bx)^2}{(ac-bcx)^7} dx$

Optimal. Leaf size=59

$$\frac{2a^2}{3bc^7(a-bx)^6} - \frac{4a}{5bc^7(a-bx)^5} + \frac{1}{4bc^7(a-bx)^4}$$

```
[Out] 2/3*a^2/b/c^7/(-b*x+a)^6-4/5*a/b/c^7/(-b*x+a)^5+1/4/b/c^7/(-b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$\frac{2a^2}{3bc^7(a-bx)^6} - \frac{4a}{5bc^7(a-bx)^5} + \frac{1}{4bc^7(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(a
 - b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{(ac-bcx)^7} dx &= \int \left(\frac{4a^2}{c^7(a-bx)^7} - \frac{4a}{c^7(a-bx)^6} + \frac{1}{c^7(a-bx)^5}\right) dx \\
&= \frac{2a^2}{3bc^7(a-bx)^6} - \frac{4a}{5bc^7(a-bx)^5} + \frac{1}{4bc^7(a-bx)^4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.63

$$\frac{7a^2 + 18abx + 15b^2x^2}{60bc^7(a-bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (7*a^2 + 18*a*b*x + 15*b^2*x^2)/(60*b*c^7*(a - b*x)^6)
```

Mathics [A]
time = 2.56, size = 89, normalized size = 1.51

$$\frac{7a^2 + 18abx + 15b^2x^2}{60bc^7\left(a^6 - 6a^5bx + 15a^4b^2x^2 - 20a^3b^3x^3 + 15a^2b^4x^4 - 6ab^5x^5 + b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]')
```

```
[Out] (7 a ^ 2 + 18 a b x + 15 b ^ 2 x ^ 2) / (60 b c ^ 7 (a ^ 6 - 6 a ^ 5 b x +
15 a ^ 4 b ^ 2 x ^ 2 - 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 - 6 a b
^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.16, size = 49, normalized size = 0.83

method	result	size
risch	$\frac{\frac{x^2 b}{4}+\frac{3ax}{10}+\frac{7a^2}{60b}}{c^7(-bx+a)^6}$	32
gosper	$\frac{15x^2b^2+18abx+7a^2}{60(-bx+a)^6 c^7 b}$	36
norman	$\frac{\frac{7a^2}{60bc}+\frac{b\,x^2}{4c}+\frac{3ax}{10c}}{c^6(-bx+a)^6}$	41
default	$\frac{\frac{1}{4b(-bx+a)^4}+\frac{2a^2}{3b(-bx+a)^6}-\frac{4a}{5b(-bx+a)^5}}{c^7}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^7*(1/4/b/(-b*x+a)^4+2/3*a^2/b/(-b*x+a)^6-4/5*a/b/(-b*x+a)^5)
```

Maxima [A]
time = 0.30, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2 + 18\,abx + 7\,a^2}{60\left(b^7c^7x^6 - 6\,ab^6c^7x^5 + 15\,a^2b^5c^7x^4 - 20\,a^3b^4c^7x^3 + 15\,a^4b^3c^7x^2 - 6\,a^5b^2c^7x + a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

Fricas [**A**]
time = 0.29, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.
time = 0.29, size = 117, normalized size = 1.98

$$-\frac{-7a^2-18abx-15b^2x^2}{60a^6bc^7-360a^5b^2c^7x+900a^4b^3c^7x^2-1200a^3b^4c^7x^3+900a^2b^5c^7x^4-360ab^6c^7x^5+60b^7c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**7,x)
```

```
[Out] -(-7*a**2 - 18*a*b*x - 15*b**2*x**2)/(60*a**6*b*c**7 - 360*a**5*b**2*c**7*x
 + 900*a**4*b**3*c**7*x**2 - 1200*a**3*b**4*c**7*x**3 + 900*a**2*b**5*c**7*
x**4 - 360*a*b**6*c**7*x**5 + 60*b**7*c**7*x**6)
```

Giac [**A**]
time = 0.00, size = 35, normalized size = 0.59

$$\frac{15x^2b^2+18xba+7a^2}{60bc^7\left(xb-a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x)
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/((b*x - a)^6*b*c^7)
```

Mupad [**B**]
time = 0.11, size = 104, normalized size = 1.76

$$\frac{\frac{3\,a\,x}{10}+\frac{b\,x^2}{4}+\frac{7\,a^2}{60\,b}}{a^6\,c^7-6\,a^5\,b\,c^7\,x+15\,a^4\,b^2\,c^7\,x^2-20\,a^3\,b^3\,c^7\,x^3+15\,a^2\,b^4\,c^7\,x^4-6\,a\,b^5\,c^7\,x^5+b^6\,c^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^7,x)
```

```
[Out] ((3*a*x)/10 + (b*x^2)/4 + (7*a^2)/(60*b))/(a^6*c^7 + b^6*c^7*x^6 - 6*a*b^5*
c^7*x^5 + 15*a^4*b^2*c^7*x^2 - 20*a^3*b^3*c^7*x^3 + 15*a^2*b^4*c^7*x^4 - 6*
a^5*b*c^7*x)
```

3.1049 $\int \frac{(ac-bcx)^3}{a+bx}\,dx$

Optimal. Leaf size=61

$$-4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}$$

```
[Out] -4*a^2*c^3*x+a*c^3*(-b*x+a)^2/b+1/3*c^3*(-b*x+a)^3/b+8*a^3*c^3*ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] -4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^3
*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{a+bx}\,dx &= \int \left(-4a^2c^3+\frac{8a^3c^3}{a+bx}-2ac^2(ac-bcx)-c(ac-bcx)^2\right)dx\\ &= -4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.69

$$c^3\left(-7a^2x+2abx^2-\frac{b^2x^3}{3}+\frac{8a^3\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] c^3*(-7*a^2*x + 2*a*b*x^2 - (b^2*x^3)/3 + (8*a^3*Log[a + b*x])/b)
```

Mathics [A]
time = 1.79, size = 42, normalized size = 0.69

$$\frac{c^3\left(24a^3\mathrm{Log}\left[a+bx\right]+bx\left(-21a^2+6abx-b^2x^2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)*(a*c - b*c*x)^3,x]')
```

```
[Out] c ^ 3 (24 a ^ 3 Log[a + b x] + b x (-21 a ^ 2 + 6 a b x - b ^ 2 x ^ 2)) / (
3 b)
```

Maple [A]
time = 0.15, size = 41, normalized size = 0.67

method	result	size
default	$c^3\left(-\frac{b^2x^3}{3}+2ab\,x^2-7a^2x+\frac{8a^3\ln(bx+a)}{b}\right)$	41
norman	$-7a^2c^3x-\frac{b^2c^3x^3}{3}+2a\,c^3b\,x^2+\frac{8a^3c^3\ln(bx+a)}{b}$	49
risch	$-7a^2c^3x-\frac{b^2c^3x^3}{3}+2a\,c^3b\,x^2+\frac{8a^3c^3\ln(bx+a)}{b}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*b^2*x^3+2*a*b*x^2-7*a^2*x+8/b*a^3*ln(b*x+a))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.79

$$-\frac{1}{3}\,b^2c^3x^3+2\,abc^3x^2-7\,a^2c^3x+\frac{8\,a^3c^3\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*c^3*x^3 + 2*a*b*c^3*x^2 - 7*a^2*c^3*x + 8*a^3*c^3*log(b*x + a)/b
```

Fricas [A]
time = 0.29, size = 52, normalized size = 0.85

$$-\frac{b^3c^3x^3-6\,ab^2c^3x^2+21\,a^2bc^3x-24\,a^3c^3\log\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 21*a^2*b*c^3*x - 24*a^3*c^3*log(b*x +
 a))/b
```

Sympy [A]
time = 0.10, size = 49, normalized size = 0.80

$$\frac{8a^3c^3\log{(a+bx)}}{b} - 7a^2c^3x + 2abc^3x^2 - \frac{b^2c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a),x)
```

```
[Out] 8*a**3*c**3*log(a + b*x)/b - 7*a**2*c**3*x + 2*a*b*c**3*x**2 - b**2*c**3*x*
*3/3
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.03

$$\frac{-\frac{1}{3}x^3b^5c^3 + 2x^2b^4c^3a - 7xb^3c^3a^2}{b^3} + \frac{8c^3a^3\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x)
```

```
[Out] 8*a^3*c^3*log(abs(b*x + a))/b - 1/3*(b^5*c^3*x^3 - 6*a*b^4*c^3*x^2 + 21*a^2
*b^3*c^3*x)/b^3
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.79

$$\frac{8\,a^3\,c^3\,\ln{(a+b\,x)}}{b} - \frac{b^2\,c^3\,x^3}{3} - 7\,a^2\,c^3\,x + 2\,a\,b\,c^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x),x)
```

```
[Out] (8*a^3*c^3*log(a + b*x))/b - (b^2*c^3*x^3)/3 - 7*a^2*c^3*x + 2*a*b*c^3*x^2
```

3.1050 $\int \frac{(ac-bcx)^2}{a+bx} \, dx$

Optimal. Leaf size=43

$$-2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b}$$

```
[Out] -2*a*c^2*x+1/2*c^2*(-b*x+a)^2/b+4*a^2*c^2*ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] -2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(ac-bcx)^2}{a+bx} \, dx &= \int \left(-2ac^2+\frac{4a^2c^2}{a+bx}-c(ac-bcx)\right) dx \\ &= -2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 0.72

$$c^2\left(-3ax+\frac{bx^2}{2}+\frac{4a^2\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] c^2*(-3*a*x + (b*x^2)/2 + (4*a^2*Log[a + b*x])/b)
```

Mathics [A]
time = 1.70, size = 30, normalized size = 0.70

$$\frac{c^2\left(8a^2\text{Log}\left[a+bx\right]+bx\left(-6a+bx\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)*(a*c - b*c*x)^2,x]')
```

```
[Out] c ^ 2 (8 a ^ 2 Log[a + b x] + b x (-6 a + b x)) / (2 b)
```

Maple [A]
time = 0.15, size = 30, normalized size = 0.70

method	result	size
default	$c^2\left(\frac{x^2 b}{2}-3ax+\frac{4a^2\ln\left(bx+a\right)}{b}\right)$	30
norman	$-3a\,c^2x+\frac{b\,c^2x^2}{2}+\frac{4a^2c^2\ln\left(bx+a\right)}{b}$	35
risch	$-3a\,c^2x+\frac{b\,c^2x^2}{2}+\frac{4a^2c^2\ln\left(bx+a\right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*x^2*b-3*a*x+4*a^2/b*ln(b*x+a))
```

Maxima [A]
time = 0.26, size = 34, normalized size = 0.79

$$\frac{1}{2}\,bc^2x^2-3\,ac^2x+\frac{4\,a^2c^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*x^2 - 3*a*c^2*x + 4*a^2*c^2*log(b*x + a)/b
```

Fricas [A]
time = 0.30, size = 38, normalized size = 0.88

$$\frac{b^2c^2x^2-6\,abc^2x+8\,a^2c^2\log\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 - 6*a*b*c^2*x + 8*a^2*c^2*log(b*x + a))/b
```

Sympy [A]
time = 0.09, size = 34, normalized size = 0.79

$$\frac{4a^2c^2\log\left(a+bx\right)}{b}-3ac^2x+\frac{bc^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a),x)
```

```
[Out] 4*a**2*c**2*log(a + b*x)/b - 3*a*c**2*x + b*c**2*x**2/2
```

Giac [A]
time = 0.00, size = 48, normalized size = 1.12

$$\frac{\frac{1}{2}x^2b^3c^2-3xb^2c^2a}{b^2}+\frac{4c^2a^2\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x)
```

```
[Out] 4*a^2*c^2*log(abs(b*x + a))/b + 1/2*(b^3*c^2*x^2 - 6*a*b^2*c^2*x)/b^2
```

Mupad [B]
time = 0.15, size = 32, normalized size = 0.74

$$\frac{c^2\,\left(8\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-6\,a\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x),x)
```

```
[Out] (c^2*(8*a^2*log(a + b*x) + b^2*x^2 - 6*a*b*x))/(2*b)
```

3.1051 $\int \frac{ac-bcx}{a+bx}\,dx$

Optimal. Leaf size=18

$$-cx+\frac{2ac\log(a+bx)}{b}$$

[Out] -c*x+2*a*c*ln(b*x+a)/b

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\frac{2ac\log(a+bx)}{b}-cx$$

Antiderivative was successfully verified.

[In] Int[(a*c - b*c*x)/(a + b*x),x]

[Out] -(c*x) + (2*a*c*Log[a + b*x])/b

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{a+bx}\,dx &= \int \left(-c+\frac{2ac}{a+bx}\right)dx\\ &= -cx+\frac{2ac\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$c\left(-x+\frac{2a\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a*c - b*c*x)/(a + b*x),x]

```
[Out] c*(-x + (2*a*Log[a + b*x])/b)
```

Mathics [A]
time = 1.61, size = 19, normalized size = 1.06

$$\frac{c\left(2a\mathrm{Log}\left[a+bx\right]-bx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)*(a*c - b*c*x)^1,x]')
[Out] c (2 a Log[a + b x] - b x) / b
```

Maple [A]
time = 0.14, size = 19, normalized size = 1.06

method	result	size
default	$c\left(-x+\frac{2a\ln(bx+a)}{b}\right)$	19
norman	$-cx+\frac{2ac\ln(bx+a)}{b}$	19
risch	$-cx+\frac{2ac\ln(bx+a)}{b}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] c*(-x+2*a/b*ln(b*x+a))
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.00

$$-cx+\frac{2\,ac\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="maxima")
[Out] -c*x + 2*a*c*log(b*x + a)/b
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.11

$$-\frac{bcx-2\,ac\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*c*x - 2*a*c*log(b*x + a))/b
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.83

$$\frac{2ac\log{(a+bx)}}{b}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x)
[Out] 2*a*c*log(a + b*x)/b - c*x
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.17

$$-\frac{xbc}{b}+\frac{2ca\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x)
[Out] -c*x + 2*a*c*log(abs(b*x + a))/b
```

Mupad [B]
time = 0.04, size = 18, normalized size = 1.00

$$\frac{2\,a\,c\,\ln{(a+b\,x)}}{b}-c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x),x)
[Out] (2*a*c*log(a + b*x))/b - c*x
```

3.1052 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Mathics [A]
time = 1.54, size = 10, normalized size = 1.00

$$\frac{\text{Log}\,[a+bx]}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)*(a*c - b*c*x)^0,x]')
```

```
[Out] Log[a + b x] / b
```

Maple [A]
time = 0.13, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A]
time = 0.27, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A]
time = 0.28, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.70

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

3.1053 $\int \frac{1}{(a+bx)(ac-bcx)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

[Out] arctanh(b*x/a)/a/b/c

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {35, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)*(a*c - b*c*x)),x]

[Out] ArcTanh[(b*x)/a]/(a*b*c)

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)}\,dx &= \int \frac{1}{a^2c-b^2cx^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Mathics [A]
time = 1.80, size = 31, normalized size = 1.82

$$\frac{\text{Log}\left[\frac{a}{b}+x\right]-\text{Log}\left[-\frac{a}{b}+x\right]}{2abc}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)/(a*c - b*c*x)^1,x]')
```

```
[Out] (Log[a / b + x] - Log[-a / b + x]) / (2 a b c)
```

Maple [A]
time = 0.16, size = 35, normalized size = 2.06

method	result	size
default	$\frac{-\frac{\ln(-bx+a)}{2ab}+\frac{\ln(bx+a)}{2ab}}{c}$	35
norman	$-\frac{\ln(-bx+a)}{2abc}+\frac{\ln(bx+a)}{2abc}$	37
risch	$-\frac{\ln(-bx+a)}{2abc}+\frac{\ln(bx+a)}{2abc}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2/a/b*ln(-b*x+a)+1/2*ln(b*x+a)/a/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.
time = 0.30, size = 37, normalized size = 2.18

$$\frac{\log(bx+a)}{2\,abc}-\frac{\log(bx-a)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)/(a*b*c) - 1/2*log(b*x - a)/(a*b*c)
```

Fricas [A]
time = 0.30, size = 28, normalized size = 1.65

$$\frac{\log(bx+a)-\log(bx-a)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b*c)
```

Sympy [A]
time = 0.09, size = 22, normalized size = 1.29

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.
time = 0.00, size = 37, normalized size = 2.18

$$-\frac{b\ln|xb-a|}{2b^2ac}+\frac{b\ln|xb+a|}{2b^2ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] 1/2*log(abs(b*x + a))/(a*b*c) - 1/2*log(abs(b*x - a))/(a*b*c)
```

Mupad [B]
time = 0.17, size = 17, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)),x)
```

```
[Out] atanh((b*x)/a)/(a*b*c)
```

3.1054 $\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx$

Optimal. Leaf size=42

$$\frac{1}{2abc^2(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}$$

```
[Out] 1/2/a/b/c^2/(-b*x+a)+1/2*arctanh(b*x/a)/a^2/b/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {46, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] 1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx &= \int \left(\frac{1}{2ac^2(a-bx)^2}+\frac{1}{2ac^2\left(a^2-b^2x^2\right)}\right)dx\\
&= \frac{1}{2abc^2(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac^2}\\
&= \frac{1}{2abc^2(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.26

$$\frac{2a+(-a+bx)\log(a-bx)+(a-bx)\log(a+bx)}{4a^2bc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*c^2*(a -
b*x))
```

Mathics [A]
time = 2.12, size = 55, normalized size = 1.31

$$\frac{2a+(a-bx)\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}\left[\frac{-a+bx}{b}\right]\right)}{4a^2bc^2\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)/(a*c - b*c*x)^2,x]')
```

```
[Out] (2 a + (a - b x) (Log[(a + b x) / b] - Log[(-a + b x) / b])) / (4 a ^ 2 b c
 ^ 2 (a - b x))
```

Maple [A]
time = 0.15, size = 51, normalized size = 1.21

method	result	size
default	$\frac{-\frac{\ln(-bx+a)}{4a^2b}+\frac{1}{2ab(-bx+a)}+\frac{\ln(bx+a)}{4a^2b}}{c^2}$	51
norman	$\frac{1}{2ab\,c^2(-bx+a)}-\frac{\ln(-bx+a)}{4a^2b\,c^2}+\frac{\ln(bx+a)}{4a^2b\,c^2}$	56
risch	$\frac{1}{2ab\,c^2(-bx+a)}-\frac{\ln(-bx+a)}{4a^2b\,c^2}+\frac{\ln(bx+a)}{4a^2b\,c^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4/a^2/b*ln(-b*x+a)+1/2/a/b/(-b*x+a)+1/4/a^2/b*ln(b*x+a))
```

Maxima [A]
time = 0.25, size = 60, normalized size = 1.43

$$-\frac{1}{2\left(ab^2c^2x-a^2bc^2\right)}+\frac{\log\left(bx+a\right)}{4\,a^2bc^2}-\frac{\log\left(bx-a\right)}{4\,a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c^2*x - a^2*b*c^2) + 1/4*log(b*x + a)/(a^2*b*c^2) - 1/4*log(b*x
 - a)/(a^2*b*c^2)
```

Fricas [A]
time = 0.30, size = 60, normalized size = 1.43

$$\frac{(bx-a)\log(bx+a)-(bx-a)\log(bx-a)-2a}{4\left(a^2b^2c^2x-a^3bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*c^2*x
- a^3*b*c^2)
```

Sympy [A]
time = 0.15, size = 48, normalized size = 1.14

$$-\frac{1}{-2a^2bc^2+2ab^2c^2x}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -1/(-2*a**2*b*c**2 + 2*a*b**2*c**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)
/(a**2*b*c**2)
```

Giac [A]
time = 0.00, size = 63, normalized size = 1.50

$$\frac{\ln|xb+a|}{4ba^2c^2}-\frac{\ln|xb-a|}{4ba^2c^2}-\frac{\frac{1}{4}\cdot 2a}{c^2a^2b\,(xb-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4*log(abs(b*x + a))/(a^2*b*c^2) - 1/4*log(abs(b*x - a))/(a^2*b*c^2) - 1/2
/((b*x - a)*a*b*c^2)
```

Mupad [B]
time = 0.07, size = 42, normalized size = 1.00

$$\frac{1}{2\,a\,b\,\left(a\,c^2-b\,c^2\,x\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)),x)
```

```
[Out] 1/(2*a*b*(a*c^2 - b*c^2*x)) + atanh((b*x)/a)/(2*a^2*b*c^2)
```

3.1055 $\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}$$

```
[Out] 1/4/a/b/c^3/(-b*x+a)^2+1/4/a^2/b/c^3/(-b*x+a)+1/4*arctanh(b*x/a)/a^3/b/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {46, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4a^2bc^3(a-bx)}+\frac{1}{4abc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(4
*a^3*b*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx &= \int \left(\frac{1}{2ac^3(a-bx)^3}+\frac{1}{4a^2c^3(a-bx)^2}+\frac{1}{4a^2c^3\left(a^2-b^2x^2\right)}\right)dx\\
&= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2c^3}\\
&= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.03

$$\frac{2a(2a-bx)-(a-bx)^2\log(a-bx)+(a-bx)^2\log(a+bx)}{8a^3bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] (2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8*
a^3*b*c^3*(a - b*x)^2)
```

Mathics [A]
time = 2.40, size = 83, normalized size = 1.32

$$\frac{2a\left(2a-bx\right)+\left(a^2-2abx+b^2x^2\right)\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}\left[\frac{-a+bx}{b}\right]\right)}{8a^3bc^3\left(a^2-2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)/(a*c - b*c*x)^3,x]')
```

```
[Out] (2 a (2 a - b x) + (a ^ 2 - 2 a b x + b ^ 2 x ^ 2) (Log[(a + b x) / b] - Lo
g[(-a + b x) / b])) / (8 a ^ 3 b c ^ 3 (a ^ 2 - 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.18, size = 67, normalized size = 1.06

method	result	size
risch	$\frac{-\frac{x}{4a^2}+\frac{1}{2ab}}{c^3(-bx+a)^2}-\frac{\ln(-bx+a)}{8a^3b\,c^3}+\frac{\ln(bx+a)}{8a^3b\,c^3}$	64
default	$\frac{-\frac{\ln(-bx+a)}{8a^3b}+\frac{1}{4a^2b(-bx+a)}+\frac{1}{4ab(-bx+a)^2}+\frac{\ln(bx+a)}{8a^3b}}{c^3}$	67
norman	$\frac{\frac{3x}{4a^2c}-\frac{b\,x^2}{2a^3c}}{c^2(-bx+a)^2}-\frac{\ln(-bx+a)}{8a^3b\,c^3}+\frac{\ln(bx+a)}{8a^3b\,c^3}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/8/a^3/b*ln(-b*x+a)+1/4/a^2/b/(-b*x+a)+1/4/a/b/(-b*x+a)^2+1/8/a^3/
b*ln(b*x+a))
```

Maxima [A]
time = 0.26, size = 82, normalized size = 1.30

$$-\frac{bx-2\,a}{4\left(a^2b^3c^3x^2-2\,a^3b^2c^3x+a^4bc^3\right)}+\frac{\log\left(bx+a\right)}{8\,a^3bc^3}-\frac{\log\left(bx-a\right)}{8\,a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x - 2*a)/(a^2*b^3*c^3*x^2 - 2*a^3*b^2*c^3*x + a^4*b*c^3) + 1/8*log(
b*x + a)/(a^3*b*c^3) - 1/8*log(b*x - a)/(a^3*b*c^3)
```

Fricas [A]
time = 0.29, size = 98, normalized size = 1.56

$$-\frac{2\,abx-4\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log\left(bx+a\right)+\left(b^2x^2-2\,abx+a^2\right)\log\left(bx-a\right)}{8\,\left(a^3b^3c^3x^2-2\,a^4b^2c^3x+a^5bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 -
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*c^3*x^2 - 2*a^4*b^2*c^3*x + a^5*b*c^
3)
```

Sympy [A]
time = 0.19, size = 71, normalized size = 1.13

$$-\frac{-2a+bx}{4a^4bc^3-8a^3b^2c^3x+4a^2b^3c^3x^2}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8}-\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a + b*x)/(4*a**4*b*c**3 - 8*a**3*b**2*c**3*x + 4*a**2*b**3*c**3*x**2)
- (log(-a/b + x)/8 - log(a/b + x)/8)/(a**3*b*c**3)
```

Giac [A]
time = 0.00, size = 74, normalized size = 1.17

$$\frac{\ln|xb+a|}{8ba^3c^3}-\frac{\ln|xb-a|}{8ba^3c^3}+\frac{\frac{1}{16}\left(-4bax+8a^2\right)}{c^3a^3b\left(-xb+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] 1/8*log(abs(b*x + a))/(a^3*b*c^3) - 1/8*log(abs(b*x - a))/(a^3*b*c^3) - 1/4
*(a*b*x - 2*a^2)/((b*x - a)^2*a^3*b*c^3)
```

Mupad [B]
time = 0.08, size = 64, normalized size = 1.02

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b\,c^3}-\frac{\frac{x}{4\,a^2}-\frac{1}{2\,a\,b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)),x)
```

```
[Out] atanh((b*x)/a)/(4*a^3*b*c^3) - (x/(4*a^2) - 1/(2*a*b))/(a^2*c^3 + b^2*c^3*x
^2 - 2*a*b*c^3*x)
```

3.1056 $\int \frac{(ac-bcx)^3}{(a+bx)^2} dx$

Optimal. Leaf size=54

$$5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}$$

```
[Out] 5*a*c^3*x-1/2*b*c^3*x^2-8*a^3*c^3/b/(b*x+a)-12*a^2*c^3*ln(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a +
 b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx &= \int \left(5ac^3-bc^3x+\frac{8a^3c^3}{(a+bx)^2}-\frac{12a^2c^3}{a+bx}\right)dx\\ &= 5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.85

$$c^3\left(5ax-\frac{bx^2}{2}-\frac{8a^3}{b(a+bx)}-\frac{12a^2\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] c^3*(5*a*x - (b*x^2)/2 - (8*a^3)/(b*(a + b*x)) - (12*a^2*Log[a + b*x])/b)
```

Mathics [A]
time = 1.89, size = 53, normalized size = 0.98

$$\frac{c^3\left(-24a^2\mathrm{Log}\left[a+bx\right]\left(a+bx\right)-16a^3+bx\left(a+bx\right)\left(10a-bx\right)\right)}{2b\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2*(a*c - b*c*x)^3,x]')
```

```
[Out] c ^ 3 (-24 a ^ 2 Log[a + b x] (a + b x) - 16 a ^ 3 + b x (a + b x) (10 a -
b x)) / (2 b (a + b x))
```

Maple [A]
time = 0.15, size = 45, normalized size = 0.83

method	result	size
default	$c^{3}\left(-\frac{x^{2} b}{2}+5 a x-\frac{8 a^{3}}{b \left(b x+a\right)}-\frac{12 a^{2} \ln \left(b x+a\right)}{b}\right)$	45
risch	$5 a\,c^{3} x-\frac{b\,c^{3} x^{2}}{2}-\frac{8 a^{3} c^{3}}{b \left(b x+a\right)}-\frac{12 a^{2} c^{3} \ln \left(b x+a\right)}{b}$	53
norman	$\frac{13 a^{2} c^{3} x-\frac{1}{2} b^{2} c^{3} x^{3}+\frac{9}{2} a\,c^{3} b\,x^{2}}{b x+a}-\frac{12 a^{2} c^{3} \ln \left(b x+a\right)}{b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/2*x^2*b+5*a*x-8/b*a^3/(b*x+a)-12*a^2/b*ln(b*x+a))
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.98

$$-\frac{1}{2}\,bc^3x^2-\frac{8\,a^3c^3}{b^2x+ab}+5\,ac^3x-\frac{12\,a^2c^3\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^3*x^2 - 8*a^3*c^3/(b^2*x + a*b) + 5*a*c^3*x - 12*a^2*c^3*log(b*x +
 a)/b
```

Fricas [A]
time = 0.29, size = 79, normalized size = 1.46

$$-\frac{b^3c^3x^3-9\,ab^2c^3x^2-10\,a^2bc^3x+16\,a^3c^3+24\left(a^2bc^3x+a^3c^3\right)\log\left(bx+a\right)}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3*x^3 - 9*a*b^2*c^3*x^2 - 10*a^2*b*c^3*x + 16*a^3*c^3 + 24*(a^2
*b*c^3*x + a^3*c^3)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A]
time = 0.12, size = 51, normalized size = 0.94

$$-\frac{8a^3c^3}{ab+b^2x}-\frac{12a^2c^3\log{(a+bx)}}{b}+5ac^3x-\frac{bc^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a)**2,x)
```

```
[Out] -8*a**3*c**3/(a*b + b**2*x) - 12*a**2*c**3*log(a + b*x)/b + 5*a*c**3*x - b*
c**3*x**2/2
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.22

$$\frac{-\frac{1}{2}x^2b^5c^3+5xb^4c^3a}{b^4}-\frac{8c^3a^3}{b(xb+a)}-\frac{12c^3a^2\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x)
```

```
[Out] -12*a^2*c^3*log(abs(b*x + a))/b - 8*a^3*c^3/((b*x + a)*b) - 1/2*(b^5*c^3*x^
2 - 10*a*b^4*c^3*x)/b^4
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.96

$$5\,a\,c^3\,x-\frac{b\,c^3\,x^2}{2}-\frac{12\,a^2\,c^3\,\ln\left(a+b\,x\right)}{b}-\frac{8\,a^3\,c^3}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x)^2,x)
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (12*a^2*c^3*log(a + b*x))/b - (8*a^3*c^3)/(b*(a
 + b*x))
```

3.1057 $\int \frac{(ac-bcx)^2}{(a+bx)^2} dx$

Optimal. Leaf size=39

$$c^2x - \frac{4a^2c^2}{b(a+bx)} - \frac{4ac^2\log(a+bx)}{b}$$

```
[Out] c^2*x-4*a^2*c^2/b/(b*x+a)-4*a*c^2*ln(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$-\frac{4a^2c^2}{b(a+bx)} - \frac{4ac^2\log(a+bx)}{b} + c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx &= \int \left(c^2 + \frac{4a^2c^2}{(a+bx)^2} - \frac{4ac^2}{a+bx}\right) dx\\ &= c^2x - \frac{4a^2c^2}{b(a+bx)} - \frac{4ac^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.85

$$c^2\left(x - \frac{4a^2}{b(a+bx)} - \frac{4a\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*(x - (4*a^2)/(b*(a + b*x)) - (4*a*Log[a + b*x])/b)
```

Mathics [A]
time = 1.80, size = 42, normalized size = 1.08

$$\frac{c^2\left(-4a\text{Log}\left[a+bx\right]\left(a+bx\right)-4a^2+bx\left(a+bx\right)\right)}{b\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2*(a*c - b*c*x)^2,x]')
```

```
[Out] c ^ 2 (-4 a Log[a + b x] (a + b x) - 4 a ^ 2 + b x (a + b x)) / (b (a + b x
))
```

Maple [A]
time = 0.15, size = 34, normalized size = 0.87

method	result	size
default	$c^{2}\left(x-\frac{4a^{2}}{b(bx+a)}-\frac{4a\ln(bx+a)}{b}\right)$	34
risch	$c^{2}x-\frac{4a^{2}c^{2}}{b(bx+a)}-\frac{4a\,c^{2}\ln(bx+a)}{b}$	40
norman	$\frac{b\,c^{2}x^{2}+5a\,c^{2}x}{bx+a}-\frac{4a\,c^{2}\ln(bx+a)}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(x-4*a^2/b/(b*x+a)-4*a/b*ln(b*x+a))
```

Maxima [A]
time = 0.26, size = 40, normalized size = 1.03

$$-\frac{4\,a^2c^2}{b^2x+ab}+c^2x-\frac{4\,ac^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2*c^2/(b^2*x + a*b) + c^2*x - 4*a*c^2*log(b*x + a)/b
```

Fricas [A]
time = 0.29, size = 61, normalized size = 1.56

$$\frac{b^2c^2x^2+abc^2x-4\,a^2c^2-4\left(abc^2x+a^2c^2\right)\log\left(bx+a\right)}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*x^2 + a*b*c^2*x - 4*a^2*c^2 - 4*(a*b*c^2*x + a^2*c^2)*log(b*x + a)
)/(b^2*x + a*b)
```

Sympy [A]
time = 0.11, size = 36, normalized size = 0.92

$$-\frac{4a^2c^2}{ab+b^2x}-\frac{4ac^2\log{(a+bx)}}{b}+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a)**2,x)
```

```
[Out] -4*a**2*c**2/(a*b + b**2*x) - 4*a*c**2*log(a + b*x)/b + c**2*x
```

Giac [A]
time = 0.00, size = 46, normalized size = 1.18

$$\frac{xb^2c^2}{b^2}-\frac{4c^2a^2}{b\,(xb+a)}-\frac{4c^2a\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x)
```

```
[Out] c^2*x - 4*a*c^2*log(abs(b*x + a))/b - 4*a^2*c^2/((b*x + a)*b)
```

Mupad [B]
time = 0.17, size = 39, normalized size = 1.00

$$c^2\,x-\frac{4\,a\,c^2\,\ln\left(a+b\,x\right)}{b}-\frac{4\,a^2\,c^2}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x)^2,x)
```

```
[Out] c^2*x - (4*a*c^2*log(a + b*x))/b - (4*a^2*c^2)/(b*(a + b*x))
```

3.1058 $\int \frac{ac-bcx}{(a+bx)^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

```
[Out] -2*a*c/b/(b*x+a)-c*ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = $\{45\}$

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{(a+bx)^2}\,dx &= \int \left(\frac{2ac}{(a+bx)^2}-\frac{c}{a+bx}\right)dx\\ &= -\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.85

$$-\frac{c\left(\frac{2a}{a+bx}+\log(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] -((c*((2*a)/(a + b*x) + Log[a + b*x]))/b)
```

Mathics [A]
time = 1.74, size = 29, normalized size = 1.07

$$\frac{c\left(-2a - \text{Log}\left[a + bx\right]\left(a + bx\right)\right)}{b\left(a + bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2*(a*c - b*c*x)^1,x]')
[Out] c (-2 a - Log[a + b x] (a + b x)) / (b (a + b x))
```

Maple [A]
time = 0.14, size = 28, normalized size = 1.04

method	result	size
norman	$\frac{2cx}{bx+a} - \frac{c\ln(bx+a)}{b}$	25
default	$c\left(-\frac{2a}{b(bx+a)} - \frac{\ln(bx+a)}{b}\right)$	28
risch	$-\frac{2ac}{b(bx+a)} - \frac{c\ln(bx+a)}{b}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(-2*a/b/(b*x+a)-ln(b*x+a)/b)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 1.04

$$-\frac{2\,ac}{b^2x + ab} - \frac{c\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="maxima")
[Out] -2*a*c/(b^2*x + a*b) - c*log(b*x + a)/b
```

Fricas [A]
time = 0.29, size = 33, normalized size = 1.22

$$-\frac{2\,ac + (bcx + ac)\log\left(bx + a\right)}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c + (b*c*x + a*c)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [**A**]
time = 0.09, size = 24, normalized size = 0.89

$$-\frac{2ac}{ab+b^2x}-\frac{c\log{(a+bx)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)**2,x)
```

```
[Out] -2*a*c/(a*b + b**2*x) - c*log(a + b*x)/b
```

Giac [**A**]
time = 0.00, size = 26, normalized size = 0.96

$$-\frac{2ca}{b\,(xb+a)}-\frac{c\ln|xb+a|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x)
```

```
[Out] -c*log(abs(b*x + a))/b - 2*a*c/((b*x + a)*b)
```

Mupad [**B**]
time = 0.04, size = 27, normalized size = 1.00

$$-\frac{c\,\ln{(a+b\,x)}}{b}-\frac{2\,a\,c}{b\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x)^2,x)
```

```
[Out] - (c*log(a + b*x))/b - (2*a*c)/(b*(a + b*x))
```

3.1059 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
[Out] -(1/(b*(a + b*x)))
```

Mathics [A]
time = 1.60, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2*(a*c - b*c*x)^0,x]')
```

```
[Out] -1 / (b (a + b x))
```

Maple [A]
time = 0.13, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A]
time = 0.29, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A]
time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{b\,(xb+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x)
```

```
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

3.1060 $\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}$$

```
[Out] -1/2/a/b/c/(b*x+a)+1/2*arctanh(b*x/a)/a^2/b/c
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {46, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] -1/2*1/(a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx &= \int \left(\frac{1}{2ac(a+bx)^2}+\frac{1}{2ac\left(a^2-b^2x^2\right)}\right)dx\\
&= -\frac{1}{2abc(a+bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac}\\
&= -\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.22

$$\frac{-2a-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)}{4a^2bc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] (-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*c*(a + b*
x))
```

Mathics [A]
time = 2.09, size = 53, normalized size = 1.29

$$\frac{-2a+(a+bx)\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}\left[\frac{-a+bx}{b}\right]\right)}{4a^2bc\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2/(a*c - b*c*x)^1,x]')
```

```
[Out] (-2 a + (a + b x) (Log[(a + b x) / b] - Log[(-a + b x) / b])) / (4 a ^ 2 b
c (a + b x))
```

Maple [A]
time = 0.15, size = 50, normalized size = 1.22

method	result	size
default	$\frac{-\frac{\ln(-bx+a)}{4a^2b}+\frac{\ln(bx+a)}{4a^2b}-\frac{1}{2ab(bx+a)}}{c}$	50
norman	$-\frac{1}{2abc(bx+a)}-\frac{\ln(-bx+a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$	55
risch	$-\frac{1}{2abc(bx+a)}-\frac{\ln(-bx+a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/4/a^2/b*ln(-b*x+a)+1/4/a^2/b*ln(b*x+a)-1/2/a/b/(b*x+a))
```

Maxima [A]
time = 0.26, size = 55, normalized size = 1.34

$$-\frac{1}{2\,(ab^2cx+a^2bc)}+\frac{\log(bx+a)}{4\,a^2bc}-\frac{\log(bx-a)}{4\,a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c*x + a^2*b*c) + 1/4*log(b*x + a)/(a^2*b*c) - 1/4*log(b*x - a)/
(a^2*b*c)
```

Fricas [A]
time = 0.30, size = 51, normalized size = 1.24

$$\frac{(bx+a)\log(bx+a)-(bx+a)\log(bx-a)-2\,a}{4\,(a^2b^2cx+a^3bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*c*x +
a^3*b*c)
```

Sympy [A]
time = 0.14, size = 44, normalized size = 1.07

$$-\frac{1}{2a^2bc+2ab^2cx}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4}-\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -1/(2*a**2*b*c + 2*a*b**2*c*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b
*c)
```

Giac [A]
time = 0.00, size = 56, normalized size = 1.37

$$-\frac{\ln|xb-a|}{4ba^2c}+\frac{\ln|xb+a|}{4ba^2c}-\frac{\frac{1}{4}\cdot 2a}{a^2cb\,(xb+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x)
```

```
[Out] 1/4*log(abs(b*x + a))/(a^2*b*c) - 1/4*log(abs(b*x - a))/(a^2*b*c) - 1/2/((b
*x + a)*a*b*c)
```

Mupad [B]
time = 0.18, size = 37, normalized size = 0.90

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c}-\frac{1}{2\,a\,b\,\left(a\,c+b\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)^2),x)
```

```
[Out] atanh((b*x)/a)/(2*a^2*b*c) - 1/(2*a*b*(a*c + b*c*x))
```

3.1061 $\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

```
[Out] 1/2*x/a^2/c^2/(-b^2*x^2+a^2)+1/2*arctanh(b*x/a)/a^3/b/c^2
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {41, 205, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}+\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = \int \frac{1}{\left(a^2c-b^2cx^2\right)^2}\,dx$$
$$= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\int \frac{1}{a^2c-b^2cx^2}\,dx}{2a^2c}$$
$$= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.61

$$\frac{2abx+\left(-a^2+b^2x^2\right)\log(a-bx)+\left(a^2-b^2x^2\right)\log(a+bx)}{4a^3bc^2(a-bx)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a*b*x + (-a^2 + b^2*x^2)*Log[a - b*x] + (a^2 - b^2*x^2)*Log[a + b*x])/(4
*a^3*b*c^2*(a - b*x)*(a + b*x))
```

Mathics [A]
time = 2.17, size = 69, normalized size = 1.50

$$\frac{2abx+\left(a^2-b^2x^2\right)\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}\left[\frac{-a+bx}{b}\right]\right)}{4a^3bc^2\left(a^2-b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2/(a*c - b*c*x)^2,x]')
```

```
[Out] (2 a b x + (a ^ 2 - b ^ 2 x ^ 2) (Log[(a + b x) / b] - Log[(-a + b x) / b])
) / (4 a ^ 3 b c ^ 2 (a ^ 2 - b ^ 2 x ^ 2))
```

Maple [A]
time = 0.16, size = 66, normalized size = 1.43

method	result	size
norman	$\frac{x}{2a^2c^2(bx+a)(-bx+a)}-\frac{\ln(-bx+a)}{4a^3bc^2}+\frac{\ln(bx+a)}{4a^3bc^2}$	61
risch	$\frac{x}{2a^2c^2(bx+a)(-bx+a)}-\frac{\ln(-bx+a)}{4a^3bc^2}+\frac{\ln(bx+a)}{4a^3bc^2}$	61
default	$\frac{-\frac{\ln(-bx+a)}{4a^3b}+\frac{1}{4a^2b(-bx+a)}+\frac{\ln(bx+a)}{4a^3b}-\frac{1}{4a^2b(bx+a)}}{c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4/a^3/b*ln(-b*x+a)+1/4/a^2/b/(-b*x+a)+1/4/a^3/b*ln(b*x+a)-1/4/a^2
/b/(b*x+a))
```

Maxima [A]
time = 0.27, size = 64, normalized size = 1.39

$$-\frac{x}{2\,(a^2b^2c^2x^2-a^4c^2)}+\frac{\log\left(bx+a\right)}{4\,a^3bc^2}-\frac{\log\left(bx-a\right)}{4\,a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(a^2*b^2*c^2*x^2 - a^4*c^2) + 1/4*log(b*x + a)/(a^3*b*c^2) - 1/4*log
(b*x - a)/(a^3*b*c^2)
```

Fricas [A]
time = 0.30, size = 76, normalized size = 1.65

$$-\frac{2\,abx-(b^2x^2-a^2)\log\left(bx+a\right)+(b^2x^2-a^2)\log\left(bx-a\right)}{4\,(a^3b^3c^2x^2-a^5bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a)
)/(a^3*b^3*c^2*x^2 - a^5*b*c^2)
```

Sympy [A]
time = 0.13, size = 49, normalized size = 1.07

$$-\frac{x}{-2a^4c^2+2a^2b^2c^2x^2}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -x/(-2*a**4*c**2 + 2*a**2*b**2*c**2*x**2) + (-log(-a/b + x)/4 + log(a/b + x
)/4)/(a**3*b*c**2)
```

Giac [A]
time = 0.00, size = 68, normalized size = 1.48

$$-\frac{b\ln|xb-a|}{4b^2a^3c^2}+\frac{b\ln|xb+a|}{4b^2a^3c^2}+\frac{x}{2a^2c^2\,(-x^2b^2+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] -1/2*x/((b^2*x^2 - a^2)*a^2*c^2) + 1/4*log(abs(b*x + a))/(a^3*b*c^2) - 1/4*
log(abs(b*x - a))/(a^3*b*c^2)
```

Mupad [B]
time = 0.18, size = 46, normalized size = 1.00

$$\frac{x}{2\,a^2\,\left(a^2\,c^2-b^2\,c^2\,x^2\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)^2),x)
```

```
[Out] x/(2*a^2*(a^2*c^2 - b^2*c^2*x^2)) + atanh((b*x)/a)/(2*a^3*b*c^2)
```

3.1062 $\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$

Optimal. Leaf size=83

$$\frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

```
[Out] 1/8/a^2/b/c^3/(-b*x+a)^2+1/4/a^3/b/c^3/(-b*x+a)-1/8/a^3/b/c^3/(b*x+a)+3/8*a
rctanh(b*x/a)/a^4/b/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {46, 214}

$$\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(a
 + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx &= \int \left(\frac{1}{4a^2c^3(a-bx)^3}+\frac{1}{4a^3c^3(a-bx)^2}+\frac{1}{8a^3c^3(a+bx)^2}+\frac{3}{8a^3c^3\left(a^2-b^2x^2\right)}\right)\\
&= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3c^3}\\
&= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.82

$$\frac{\frac{2a(2a^2+3abx-3b^2x^2)}{(a-bx)^2(a+bx)}-3\log(a-bx)+3\log(a+bx)}{16a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] ((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b*
x] + 3*Log[a + b*x])/(16*a^4*b*c^3)
```

Mathics [A]
time = 2.78, size = 117, normalized size = 1.41

$$\frac{2a\left(2a^2+3abx-3b^2x^2\right)+3\left(a^3-a^2bx-ab^2x^2+b^3x^3\right)\left(\text{Log}\left[\frac{a+bx}{b}\right]-\text{Log}\left[\frac{-a+bx}{b}\right]\right)}{16a^4bc^3\left(a^3-a^2bx-ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2/(a*c - b*c*x)^3,x]')
```

```
[Out] (2 a (2 a ^ 2 + 3 a b x - 3 b ^ 2 x ^ 2) + 3 (a ^ 3 - a ^ 2 b x - a b ^ 2 x
 ^ 2 + b ^ 3 x ^ 3) (Log[(a + b x) / b] - Log[(-a + b x) / b])) / (16 a ^ 4
 b c ^ 3 (a ^ 3 - a ^ 2 b x - a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [A]
time = 0.18, size = 82, normalized size = 0.99

method	result	size
risch	$\frac{-\frac{3b\,x^2}{8a^3}+\frac{3x}{8a^2}+\frac{1}{4ab}}{(bx+a)c^3(-bx+a)^2}-\frac{3\ln(-bx+a)}{16a^4c^3b}+\frac{3\ln(bx+a)}{16a^4c^3b}$	80
default	$\frac{-\frac{3\ln(-bx+a)}{16a^4b}+\frac{1}{4a^3b(-bx+a)}+\frac{1}{8a^2b(-bx+a)^2}+\frac{3\ln(bx+a)}{16a^4b}-\frac{1}{8a^3b(bx+a)}}{c^3}$	82
norman	$\frac{\frac{1}{4abc}+\frac{3x}{8a^2c}-\frac{3b\,x^2}{8a^3c}}{(bx+a)c^2(-bx+a)^2}-\frac{3\ln(-bx+a)}{16a^4c^3b}+\frac{3\ln(bx+a)}{16a^4c^3b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-3/16/a^4/b*ln(-b*x+a)+1/4/a^3/b/(-b*x+a)+1/8/a^2/b/(-b*x+a)^2+3/16/
a^4/b*ln(b*x+a)-1/8/a^3/b/(b*x+a))
```

Maxima [A]
time = 0.26, size = 108, normalized size = 1.30

$$-\frac{3\,b^2x^2-3\,abx-2\,a^2}{8\,(a^3b^4c^3x^3-a^4b^3c^3x^2-a^5b^2c^3x+a^6bc^3)}+\frac{3\,\log\left(bx+a\right)}{16\,a^4bc^3}-\frac{3\,\log\left(bx-a\right)}{16\,a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*c^3*x^3 - a^4*b^3*c^3*x^2 - a^5
*b^2*c^3*x + a^6*b*c^3) + 3/16*log(b*x + a)/(a^4*b*c^3) - 3/16*log(b*x - a)
/(a^4*b*c^3)
```

Fricas [A]
time = 0.30, size = 146, normalized size = 1.76

$$-\frac{6\,ab^2x^2-6\,a^2bx-4\,a^3-3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log(bx+a)+3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log(bx-a)}{16\,(a^4b^4c^3x^3-a^5b^3c^3x^2-a^6b^2c^3x+a^7bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a))/
(a^4*b^4*c^3*x^3 - a^5*b^3*c^3*x^2 - a^6*b^2*c^3*x + a^7*b*c^3)
```

Sympy [A]
time = 0.25, size = 104, normalized size = 1.25

$$-\frac{-2a^2-3abx+3b^2x^2}{8a^6bc^3-8a^5b^2c^3x-8a^4b^3c^3x^2+8a^3b^4c^3x^3}-\frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b*c**3 - 8*a**5*b**2*c**3*x - 8*
a**4*b**3*c**3*x**2 + 8*a**3*b**4*c**3*x**3) - (3*log(-a/b + x)/16 - 3*log(
a/b + x)/16)/(a**4*b*c**3)
```

Giac [A]
time = 0.00, size = 93, normalized size = 1.12

$$\frac{3\ln|xb+a|}{16ba^4c^3}-\frac{3\ln|xb-a|}{16ba^4c^3}+\frac{\frac{1}{32}\left(-12b^2ax^2+12ba^2x+8a^3\right)}{a^4c^3b\left(-xb+a\right)^2(xb+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] 3/16*log(abs(b*x + a))/(a^4*b*c^3) - 3/16*log(abs(b*x - a))/(a^4*b*c^3) - 1
/8*(3*a*b^2*x^2 - 3*a^2*b*x - 2*a^3)/((b*x + a)*(b*x - a)^2*a^4*b*c^3)
```

Mupad [B]
time = 0.10, size = 86, normalized size = 1.04

$$\frac{\frac{3x}{8a^2}+\frac{1}{4ab}-\frac{3bx^2}{8a^3}}{a^3c^3-a^2bc^3x-ab^2c^3x^2+b^3c^3x^3}+\frac{3\operatorname{atanh}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)^2),x)
```

```
[Out] ((3*x)/(8*a^2) + 1/(4*a*b) - (3*b*x^2)/(8*a^3))/(a^3*c^3 + b^3*c^3*x^3 - a*
b^2*c^3*x^2 - a^2*b*c^3*x) + (3*atanh((b*x)/a))/(8*a^4*b*c^3)
```

3.1063 $\int (1-x)^{9/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=108

$$\frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}$$

```
[Out] 7/8*(1-x)^(3/2)*(1+x)^(3/2)+21/40*(1-x)^(5/2)*(1+x)^(3/2)+3/10*(1-x)^(7/2)*
(1+x)^(3/2)+1/6*(1-x)^(9/2)*(1+x)^(3/2)+21/16*arcsin(x)+21/16*x*(1-x)^(1/2)
*(1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{21}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (21*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/8 + (21
*(1 - x)^(5/2)*(1 + x)^(3/2))/40 + (3*(1 - x)^(7/2)*(1 + x)^(3/2))/10 + ((1
 - x)^(9/2)*(1 + x)^(3/2))/6 + (21*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{9/2}\sqrt{1+x}\,dx &= \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}+\frac{3}{2}\int (1-x)^{7/2}\sqrt{1+x}\,dx \\
&= \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}+\frac{21}{10}\int (1-x)^{5/2}\sqrt{1+x}\,dx \\
&= \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}+\frac{21}{8}\int \\
&= \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.68

$$
-\frac{\sqrt{1+x}\,(-448+523x+181x^2-606x^3+542x^4-232x^5+40x^6)}{240\sqrt{1-x}}+\frac{21}{8}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] -1/240*(Sqrt[1 + x]*(-448 + 523*x + 181*x^2 - 606*x^3 + 542*x^4 - 232*x^5 +
 40*x^6))/Sqrt[1 - x] + (21*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/8
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 70.67, size = 196, normalized size = 1.81

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(-5894(1+x)^{\frac{7}{2}}-5225(1+x)^{\frac{3}{2}}-630\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-472(1+x)^{\frac{11}{2}}+40(1+x)^{\frac{13}{2}}+630\sqrt{1+x}+2302(1+x)^{\frac{9}{2}}+8171(1+x)^{\frac{5}{2}}\right)}{240\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-8171(1+x)^{\frac{5}{2}}}{240\sqrt{1-x}}-\frac{1151(1+x)^{\frac{9}{2}}}{120\sqrt{1-x}}-\frac{21\sqrt{1+x}}{8\sqrt{1-x}}-\frac{(1+x)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{59(1+x)^{\frac{11}{2}}}{30\sqrt{1-x}}+\frac{21\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{8}+\frac{1045(1+x)^{\frac{3}{2}}}{48\sqrt{1-x}}+\frac{2947(1+x)^{\frac{7}{2}}}{120\sqrt{1-x}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(9/2)*(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 240 (-5894 (1 + x) ^ (7 / 2) - 5225 (1 + x) ^ (3 / 2) - 630
 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 472 (1 + x) ^ (11 / 2) + 4
0 (1 + x) ^ (13 / 2) + 630 Sqrt[1 + x] + 2302 (1 + x) ^ (9 / 2) + 8171 (1 +
 x) ^ (5 / 2)) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -8171 (1 + x) ^ (5 / 2) /
(240 Sqrt[1 - x]) - 1151 (1 + x) ^ (9 / 2) / (120 Sqrt[1 - x]) - 21 Sqrt[1
+ x] / (8 Sqrt[1 - x]) - (1 + x) ^ (13 / 2) / (6 Sqrt[1 - x]) + 59 (1 + x)
^ (11 / 2) / (30 Sqrt[1 - x]) + 21 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 8 + 10
45 (1 + x) ^ (3 / 2) / (48 Sqrt[1 - x]) + 2947 (1 + x) ^ (7 / 2) / (120 Sqr
t[1 - x])]
```

Maple [A]
time = 0.16, size = 113, normalized size = 1.05

method	result
risch	$-\frac{\left(40x^5-192x^4+350x^3-256x^2-75x+448\right)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{240\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{21\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}}}{6}+\frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}{10}+\frac{21(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{40}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{8}+\frac{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{16}-\frac{21\sqrt{1-x}\,\sqrt{1}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(9/2)*(1+x)^(3/2)+3/10*(1-x)^(7/2)*(1+x)^(3/2)+21/40*(1-x)^(5/2)*
(1+x)^(3/2)+7/8*(1-x)^(3/2)*(1+x)^(3/2)+21/16*(1-x)^(1/2)*(1+x)^(3/2)-21/16
*(1-x)^(1/2)*(1+x)^(1/2)+21/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*
arcsin(x)
```

Maxima [A]
time = 0.35, size = 68, normalized size = 0.63

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{3}{2}}x^3+\frac{4}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{13}{8}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{28}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{21}{16}\sqrt{-x^2+1}\,x+\frac{21}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(3/2)*x^3 + 4/5*(-x^2 + 1)^(3/2)*x^2 - 13/8*(-x^2 + 1)^(3/2
)*x + 28/15*(-x^2 + 1)^(3/2) + 21/16*sqrt(-x^2 + 1)*x + 21/16*arcsin(x)
```

Fricas [A]
time = 0.31, size = 62, normalized size = 0.57

$$\frac{1}{240}\left(40\,x^5-192\,x^4+350\,x^3-256\,x^2-75\,x+448\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{21}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/240*(40*x^5 - 192*x^4 + 350*x^3 - 256*x^2 - 75*x + 448)*sqrt(x + 1)*sqrt(
-x + 1) - 21/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 96.65, size = 287, normalized size = 2.66

$$\begin{cases} -\frac{21i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}-\frac{59i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}}+\frac{1151i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}}-\frac{2947i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}}+\frac{8171i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}}-\frac{1045i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{21i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{21\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{59(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}}-\frac{1151(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}}+\frac{2947(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}}-\frac{8171(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}}+\frac{1045(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{21\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-21*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt
(x - 1)) - 59*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) + 1151*I*(x + 1)**(9/2)/(1
20*sqrt(x - 1)) - 2947*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) + 8171*I*(x + 1)*
*(5/2)/(240*sqrt(x - 1)) - 1045*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 21*I*sq
rt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1) > 2), (21*asin(sqrt(2)*sqrt(x + 1)/2)
/8 - (x + 1)**(13/2)/(6*sqrt(1 - x)) + 59*(x + 1)**(11/2)/(30*sqrt(1 - x))
- 1151*(x + 1)**(9/2)/(120*sqrt(1 - x)) + 2947*(x + 1)**(7/2)/(120*sqrt(1 -
 x)) - 8171*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 1045*(x + 1)**(3/2)/(48*sqrt
(1 - x)) - 21*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(76) = 152.
time = 0.04, size = 582, normalized size = 5.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x)
```

```
[Out] 1/240*((2*((4*(5*x + 26)*(x - 1) + 321)*(x - 1) + 451)*(x - 1) + 745)*(x -
1) + 405)*sqrt(x + 1)*sqrt(-x + 1) - 1/24*((2*(3*(4*x + 17)*(x - 1) + 133)*
(x - 1) + 295)*(x - 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 5/12*((2*(3*x + 10
)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) - 5/3*((2*x + 5)*(x
- 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 5/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) -
 sqrt(x + 1)*sqrt(-x + 1) - 21/8*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(1/2),x)
[Out] int((1 - x)^(9/2)*(x + 1)^(1/2), x)
```

3.1064 $\int (1-x)^{7/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=88

$$\frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{8}\sin^{-1}(x)$$

```
[Out] 7/12*(1-x)^(3/2)*(1+x)^(3/2)+7/20*(1-x)^(5/2)*(1+x)^(3/2)+1/5*(1-x)^(7/2)*(
1+x)^(3/2)+7/8*arcsin(x)+7/8*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + (7*(
1 - x)^(5/2)*(1 + x)^(3/2))/20 + ((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*ArcSi
n[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{7/2}\sqrt{1+x}\,dx &= \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{5}\int (1-x)^{5/2}\sqrt{1+x}\,dx \\
&= \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx \\
&= \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{4}\int \surd \\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x \\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x \\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 0.77

$$\frac{\sqrt{1+x}\,(136-121x-127x^2+202x^3-114x^4+24x^5)}{120\sqrt{1-x}}+\frac{7}{4}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(136 - 121*x - 127*x^2 + 202*x^3 - 114*x^4 + 24*x^5))/(120*Sqr
t[1 - x]) + (7*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 24.89, size = 175, normalized size = 1.99

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-1315(1+x)^{\frac{3}{2}}-898(1+x)^{\frac{7}{2}}-210\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-24(1+x)^{\frac{11}{2}}+210\sqrt{1+x}+234(1+x)^{\frac{9}{2}}+1657(1+x)^{\frac{5}{2}}\right)}{120\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-1657(1+x)^{\frac{5}{2}}}{120\sqrt{1-x}}-\frac{39(1+x)^{\frac{9}{2}}}{20\sqrt{1-x}}-\frac{7\sqrt{1+x}}{4\sqrt{1-x}}+\frac{(1+x)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{7\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4}+\frac{449(1+x)^{\frac{7}{2}}}{60\sqrt{1-x}}+\frac{263(1+x)^{\frac{3}{2}}}{24\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(7/2)*(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 120 (-1315 (1 + x) ^ (3 / 2) - 898 (1 + x) ^ (7 / 2) - 210
ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 24 (1 + x) ^ (11 / 2) + 210
```

```
 Sqrt[1 + x] + 234 (1 + x) ^ (9 / 2) + 1657 (1 + x) ^ (5 / 2)) / Sqrt[-1 +
x], Abs[1 + x] > 2}}, -1657 (1 + x) ^ (5 / 2) / (120 Sqrt[1 - x]) - 39 (1 +
 x) ^ (9 / 2) / (20 Sqrt[1 - x]) - 7 Sqrt[1 + x] / (4 Sqrt[1 - x]) + (1 + x
) ^ (11 / 2) / (5 Sqrt[1 - x]) + 7 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 4 + 44
9 (1 + x) ^ (7 / 2) / (60 Sqrt[1 - x]) + 263 (1 + x) ^ (3 / 2) / (24 Sqrt[1
 - x])]
```

Maple [A]
time = 0.15, size = 99, normalized size = 1.12

method	result
risch	$\frac{\left(24 x^{4}-90 x^{3}+112 x^{2}-15 x-136\right) \sqrt{1+x}\, \left(-1+x\right) \sqrt{\left(1+x\right) \left(1-x\right)}}{120 \sqrt{-\left(1+x\right) \left(-1+x\right)}\, \sqrt{1-x}}+\frac{7 \sqrt{\left(1+x\right) \left(1-x\right)}\, \arcsin \left(x\right)}{8 \sqrt{1+x}\, \sqrt{1-x}}$
default	$\frac{\left(1-x\right)^{\frac{7}{2}} \left(1+x\right)^{\frac{3}{2}}}{5}+\frac{7 \left(1-x\right)^{\frac{5}{2}} \left(1+x\right)^{\frac{3}{2}}}{20}+\frac{7 \left(1-x\right)^{\frac{3}{2}} \left(1+x\right)^{\frac{3}{2}}}{12}+\frac{7 \sqrt{1-x}\, \left(1+x\right)^{\frac{3}{2}}}{8}-\frac{7 \sqrt{1-x}\, \sqrt{1+x}}{8}+\frac{7 \sqrt{\left(1+x\right)\dots}}{8 \sqrt{1\dots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1-x)^(7/2)*(1+x)^(3/2)+7/20*(1-x)^(5/2)*(1+x)^(3/2)+7/12*(1-x)^(3/2)*(
1+x)^(3/2)+7/8*(1-x)^(1/2)*(1+x)^(3/2)-7/8*(1-x)^(1/2)*(1+x)^(1/2)+7/8*((1+
x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.37, size = 54, normalized size = 0.61

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{-x^2+1}\,x+\frac{7}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2)
+ 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.65

$$-\frac{1}{120}\left(24\,x^4-90\,x^3+112\,x^2-15\,x-136\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{7}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x + 1)*sqrt(-x + 1) -
7/4*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 30.39, size = 252, normalized size = 2.86

$$\begin{cases} -\frac{7 i \operatorname{acosh}{\left(\frac{\sqrt{2} \sqrt{x + 1}}{2} \right)}}{4} - \frac{i \left(x + 1\right)^{\frac{11}{2}}}{5 \sqrt{x - 1}} + \frac{39 i \left(x + 1\right)^{\frac{9}{2}}}{20 \sqrt{x - 1}} - \frac{449 i \left(x + 1\right)^{\frac{7}{2}}}{60 \sqrt{x - 1}} + \frac{1657 i \left(x + 1\right)^{\frac{5}{2}}}{120 \sqrt{x - 1}} - \frac{263 i \left(x + 1\right)^{\frac{3}{2}}}{24 \sqrt{x - 1}} + \frac{7 i \sqrt{x + 1}}{4 \sqrt{x - 1}} & \text{for}\: \left|{x + 1}\right| > 2 \\ \frac{7 \operatorname{asin}{\left(\frac{\sqrt{2} \sqrt{x + 1}}{2} \right)}}{4} + \frac{\left(x + 1\right)^{\frac{11}{2}}}{5 \sqrt{1 - x}} - \frac{39 \left(x + 1\right)^{\frac{9}{2}}}{20 \sqrt{1 - x}} + \frac{449 \left(x + 1\right)^{\frac{7}{2}}}{60 \sqrt{1 - x}} - \frac{1657 \left(x + 1\right)^{\frac{5}{2}}}{120 \sqrt{1 - x}} + \frac{263 \left(x + 1\right)^{\frac{3}{2}}}{24 \sqrt{1 - x}} - \frac{7 \sqrt{x + 1}}{4 \sqrt{1 - x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 39*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 449*I*(x + 1)**(7/2)/(60*s
qrt(x - 1)) + 1657*I*(x + 1)**(5/2)/(120*sqrt(x - 1)) - 263*I*(x + 1)**(3/2
)/(24*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (7*a
sin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 39*(x + 1)
**(9/2)/(20*sqrt(1 - x)) + 449*(x + 1)**(7/2)/(60*sqrt(1 - x)) - 1657*(x +
1)**(5/2)/(120*sqrt(1 - x)) + 263*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 7*sqrt(
x + 1)/(4*sqrt(1 - x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(62) = 124.
time = 0.02, size = 429, normalized size = 4.88

$$-2\left(2\left(\left(\left(\left(\frac{1}{20}\sqrt{-x+1}\sqrt{-x+1}-\frac{21}{80}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{133}{240}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{59}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)+8\left(2\left(\left(\left(\frac{13}{48}-\frac{1}{16}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-12\left(2\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)+8\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x)
```

```
[Out] -1/120*((2*(3*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295)*(x - 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) + 1/6*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x
 + 1)*sqrt(-x + 1) - ((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 2*(
x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - 7/4*arcsin(1/2
*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(1/2), x)
```

3.1065 $\int (1-x)^{5/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=68

$$\frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x)$$

```
[Out] 5/12*(1-x)^(3/2)*(1+x)^(3/2)+1/4*(1-x)^(5/2)*(1+x)^(3/2)+5/8*arcsin(x)+5/8*
x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + ((1
- x)^(5/2)*(1 + x)^(3/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{5/2}\sqrt{1+x}\,dx &= \frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\
&= \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{}{\sqrt{1}}\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{}{\sqrt{1}}\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.93

$$-\frac{\sqrt{1+x}\,\left(-16+7x+25x^2-22x^3+6x^4\right)}{24\sqrt{1-x}}+\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] -1/24*(Sqrt[1 + x]*(-16 + 7*x + 25*x^2 - 22*x^3 + 6*x^4))/Sqrt[1 - x] + (5*
ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 9.83, size = 154, normalized size = 2.26

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-133(1+x)^{\frac{3}{2}}-46(1+x)^{\frac{7}{2}}-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}+6(1+x)^{\frac{9}{2}}+30\sqrt{1+x}+127(1+x)^{\frac{5}{2}}\right)}{24\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-127(1+x)^{\frac{5}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{1+x}}{4\sqrt{1-x}}-\frac{(1+x)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4}+\frac{23(1+x)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{133(1+x)^{\frac{3}{2}}}{24\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(5/2)*(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 24 (-133 (1 + x) ^ (3 / 2) - 46 (1 + x) ^ (7 / 2) - 30 ArcC
osh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] + 6 (1 + x) ^ (9 / 2) + 30 Sqrt[1
 + x] + 127 (1 + x) ^ (5 / 2)) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -127 (1 +
x) ^ (5 / 2) / (24 Sqrt[1 - x]) - 5 Sqrt[1 + x] / (4 Sqrt[1 - x]) - (1 + x)
 ^ (9 / 2) / (4 Sqrt[1 - x]) + 5 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 4 + 23 (
1 + x) ^ (7 / 2) / (12 Sqrt[1 - x]) + 133 (1 + x) ^ (3 / 2) / (24 Sqrt[1 -
x])]
```

Maple [A]
time = 0.16, size = 85, normalized size = 1.25

method	result
risch	$-\frac{(6x^3-16x^2+9x+16)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{4}+\frac{5(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{12}+\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1-x)^(5/2)*(1+x)^(3/2)+5/12*(1-x)^(3/2)*(1+x)^(3/2)+5/8*(1-x)^(1/2)*(1
+x)^(3/2)-5/8*(1-x)^(1/2)*(1+x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(
1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.36, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}\,x+\frac{5}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6x^3-16x^2+9x+16\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 9.33, size = 216, normalized size = 3.18

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}+\frac{127i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{133i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}-\frac{127(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{133(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 23*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) + 127*I*(x + 1)**(5/2)/(24*sq
rt(x - 1)) - 133*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqr
t(x - 1)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/
2)/(4*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(12*sqrt(1 - x)) - 127*(x + 1)**(5/2
)/(24*sqrt(1 - x)) + 133*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4
*sqrt(1 - x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(48) = 96$.
time = 0.02, size = 298, normalized size = 4.38

$$2\left(2\left(\left(\left(\left(\frac{13}{48}-\frac{1}{16}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-6\left(2\left(\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)+6\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x)
```

```
[Out] 1/24*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) -
1/2*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*(x + 2)*sqrt(x +
 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - 5/4*arcsin(1/2*sqrt(2)*sqrt(-
x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(1/2), x)
```

3.1066 $\int (1-x)^{3/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*arcsin(x)+1/2*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{3/2}\sqrt{1+x}\,dx &= \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 52, normalized size = 1.08

$$
\frac{\sqrt{1+x}\,(2+x-5x^2+2x^3)}{6\sqrt{1-x}}+\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(2 + x - 5*x^2 + 2*x^3))/(6*Sqrt[1 - x]) + ArcTan[Sqrt[1 + x]/
Sqrt[1 - x]]
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.09, size = 129, normalized size = 2.69

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(\sqrt{1+x}-\frac{17(1+x)^{\frac{3}{2}}}{6}-\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\sqrt{-1+x}-\frac{(1+x)^{\frac{7}{2}}}{3}+\frac{11(1+x)^{\frac{5}{2}}}{6}\right)}{\sqrt{-1+x}},\text{Abs}\,[1+x]>2\right\}\right\},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]-\frac{11\,(1+x)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{1+x}}{\sqrt{1-x}}+\frac{(1+x)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{17\,(1+x)^{\frac{3}{2}}}{6\sqrt{1-x}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(3/2)*(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I (Sqrt[1 + x] - 17 (1 + x) ^ (3 / 2) / 6 - ArcCosh[Sqrt[2] Sqr
t[1 + x] / 2] Sqrt[-1 + x] - (1 + x) ^ (7 / 2) / 3 + 11 (1 + x) ^ (5 / 2) /
 6) / Sqrt[-1 + x], Abs[1 + x] > 2}}, ArcSin[Sqrt[2] Sqrt[1 + x] / 2] - 11
(1 + x) ^ (5 / 2) / (6 Sqrt[1 - x]) - Sqrt[1 + x] / Sqrt[1 - x] + (1 + x) ^
 (7 / 2) / (3 Sqrt[1 - x]) + 17 (1 + x) ^ (3 / 2) / (6 Sqrt[1 - x])]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.
time = 0.16, size = 71, normalized size = 1.48

method	result	size
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{3}+\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$\frac{(2x^2-3x-2)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+
x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 28, normalized size = 0.58

$$\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 47, normalized size = 0.98

$$-\frac{1}{6}\left(2\,x^2-3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 - 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.40, size = 167, normalized size = 3.48

$$\begin{cases} -i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}+\frac{11i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{17i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}-\frac{11(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{17(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) + 11*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 17*I*(x + 1)**(3/2)/(6*sqrt(x -
1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (asin(sqrt(2)*sqrt(x + 1)
/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) - 11*(x + 1)**(5/2)/(6*sqrt(1 - x)) +
17*(x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(34) = 68$.
time = 0.01, size = 189, normalized size = 3.94

$$-2\left(2\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)+4\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x)
```

```
[Out] -1/6*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + (x + 2)*sqrt(x + 1)
*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(1/2), x)
```

3.1067 $\int \sqrt{1-x}\,\sqrt{1+x}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {38, 41, 222}

$$\frac{1}{2}\sqrt{1-x}\,\sqrt{x+1}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ArcSin[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\,\sqrt{1+x}\,dx &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 1.32

$$\frac{1}{2}x\sqrt{1-x^2} - \tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[1 + x],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.42, size = 106, normalized size = 3.79

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-x\sqrt{1+x}+x^2\sqrt{1+x}-2\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}\right)}{2\sqrt{-1+x}}, \text{Abs}[1+x]>2\right\}\right\}, \text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]-\frac{\sqrt{1+x}}{\sqrt{1-x}}-\frac{(1+x)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{3(1+x)^{\frac{3}{2}}}{2\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/2)*(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 2 (-x Sqrt[1 + x] + x ^ 2 Sqrt[1 + x] - 2 ArcCosh[Sqrt[2] S
qrt[1 + x] / 2] Sqrt[-1 + x]) / Sqrt[-1 + x], Abs[1 + x] > 2}}, ArcSin[Sqrt
[2] Sqrt[1 + x] / 2] - Sqrt[1 + x] / Sqrt[1 - x] - (1 + x) ^ (5 / 2) / (2 S
qrt[1 - x]) + 3 (1 + x) ^ (3 / 2) / (2 Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20) = 40$.
time = 0.16, size = 57, normalized size = 2.04

method	result	size
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$-\frac{x\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1+x)*(1-x))^(
1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.36, size = 17, normalized size = 0.61

$$\frac{1}{2}\sqrt{-x^2+1}\,x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 1.36

$$\frac{1}{2}\sqrt{x+1}\,x\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.61, size = 131, normalized size = 4.68

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{3i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{3(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x - 1
)) - 3*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x
+ 1) > 2), (asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) +
3*(x + 1)**(3/2)/(2*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.
time = 0.01, size = 103, normalized size = 3.68

$$2\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\,\sqrt{-x+1}\right)\sqrt{-x+1}\,\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\,\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x)
```

```
[Out] 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - arcsin(1/
2*sqrt(2)*sqrt(-x + 1))
```

Mupad [B]
time = 0.20, size = 37, normalized size = 1.32

$$\frac{x\sqrt{1-x}\ \sqrt{x+1}}{2}-\frac{\ln\left(x-\sqrt{1-x}\ \sqrt{x+1}\ 1\mathrm{i}\right)\ 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] (x*(1 - x)^(1/2)*(x + 1)^(1/2))/2 - (log(x - (1 - x)^(1/2)*(x + 1)^(1/2)*1i
)*1i)/2
```

3.1068 $\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=21

$$-\sqrt{1-x}\,\sqrt{1+x}+\sin^{-1}(x)$$

```
[Out] arcsin(x)-(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\sin^{-1}(x)-\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x],x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\sqrt{1-x}\,\sqrt{1+x} + \sin^{-1}(x)$$

Mathematica [A]
time = 0.06, size = 34, normalized size = 1.62

$$-\sqrt{1-x^2} + 2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x],x]
```

```
[Out] -Sqrt[1 - x^2] + 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.73, size = 90, normalized size = 4.29

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-2\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x} - (1+x)^{\frac{3}{2}} + 2\sqrt{1+x}\right)}{\sqrt{-1+x}}, \text{Abs}[1+x] > 2\right\}\right\}, \frac{-2\sqrt{1+x}}{\sqrt{1-x}} + 2\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right] + \frac{(1+x)^{\frac{3}{2}}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(1/2),x]')
```

```
[Out] Piecewise[{{I (-2 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - (1 + x) ^
 (3 / 2) + 2 Sqrt[1 + x]) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -2 Sqrt[1 + x]
/ Sqrt[1 - x] + 2 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + (1 + x) ^ (3 / 2) / Sqr
t[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.17, size = 42, normalized size = 2.00

method	result	size
default	$-\sqrt{1-x}\,\sqrt{1+x} + \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	42

risch	$\frac{\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin
(x)
```

Maxima [A]
time = 0.36, size = 14, normalized size = 0.67

$$-\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.29, size = 37, normalized size = 1.76

$$-\sqrt{x+1}\,\sqrt{-x+1}-2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.94, size = 99, normalized size = 4.71

$$\begin{cases}-2i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}+\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2\\ 2\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
 + 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (2*asin(sqrt(2)*sqrt(x + 1
)/2) + (x + 1)**(3/2)/sqrt(1 - x) - 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.00, size = 35, normalized size = 1.67

$$-\sqrt{x+1}\,\sqrt{-x+1}+2\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x)
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 0.14, size = 14, normalized size = 0.67

$$\operatorname{asin}\left(x\right)-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(1/2),x)
```

```
[Out] asin(x) - (1 - x^2)^(1/2)
```

3.1069 $\int \frac{\sqrt{1+x}}{(1-x)^{3/2}} \, dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x)$$

[Out] -arcsin(x)+2*(1+x)^(1/2)/(1-x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x]/(1 - x)^(3/2),x]

[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x)$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.70

$$\frac{2\sqrt{1+x}}{\sqrt{1-x}} - 2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(3/2),x]
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.48, size = 63, normalized size = 2.74

$$\text{Piecewise}\left[\left\{\left\{\frac{-2I\sqrt{1+x}}{\sqrt{-1+x}} + 2I\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right], \text{Abs}[1+x] > 2\right\}\right\}, -2\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right] + \frac{2\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(3/2),x]')
[Out] Piecewise[{{-2 I Sqrt[1 + x] / Sqrt[-1 + x] + 2 I ArcCosh[Sqrt[2] Sqrt[1 +
x] / 2], Abs[1 + x] > 2}}, -2 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 2 Sqrt[1 +
x] / Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.
time = 0.19, size = 64, normalized size = 2.78

method	result	size
risch	$\frac{2\sqrt{1+x}\,\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} - \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+x)^(1/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)-((1+x)*
(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x-1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.
time = 0.29, size = 48, normalized size = 2.09

$$\frac{2\left((x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x - 1)
```

Sympy [A]
time = 0.76, size = 70, normalized size = 3.04

$$\begin{cases} 2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ -2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1),
Abs(x + 1) > 2), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x
), True))
```

Giac [**A**]
time = 0.00, size = 41, normalized size = 1.78

$$\frac{2\sqrt{x+1}\,\sqrt{-x+1}}{-x+1} - 2\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x)
[Out] -2*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(3/2),x)
[Out] int((x + 1)^(1/2)/(1 - x)^(3/2), x)
```

3.1070 $\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx$

Optimal. Leaf size=20

$$\frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

```
[Out] 1/3*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

```
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$\frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

```
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.52, size = 52, normalized size = 2.60

$$\text{Piecewise}\left[\left\{\left\{\frac{I(1+x)^{\frac{3}{2}}}{3(-1+x)^{\frac{3}{2}}}, \text{Abs}[1+x]>2\right\}\right\}, -\frac{(1+x)^{\frac{3}{2}}}{-6\sqrt{1-x}+3(1+x)\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 3 (1 + x) ^ (3 / 2) / (-1 + x) ^ (3 / 2), Abs[1 + x] > 2}},
 -(1 + x) ^ (3 / 2) / (-6 Sqrt[1 - x] + 3 (1 + x) Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.
time = 0.16, size = 30, normalized size = 1.50

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}}{3(1-x)^{\frac{3}{2}}}$	15
default	$\frac{2\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{3\sqrt{1-x}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^2+2x+1)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+x)^(1/2)/(1-x)^(3/2)-1/3*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.
time = 0.26, size = 38, normalized size = 1.90

$$\frac{2\sqrt{-x^2+1}}{3(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 0.30, size = 33, normalized size = 1.65

$$\frac{x^2 + (x+1)^{\frac{3}{2}}\sqrt{-x+1} - 2x + 1}{3(x^2 - 2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + (x + 1)^(3/2)*sqrt(-x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [A]
time = 0.92, size = 60, normalized size = 3.00

$$\begin{cases} \frac{i(x+1)^{\frac{3}{2}}}{3\sqrt{x-1}\,(x+1)-6\sqrt{x-1}} & \text{for } |x+1| > 2 \\ -\frac{(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}\,(x+1)-6\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(3/2)/(3*sqrt(x - 1)*(x + 1) - 6*sqrt(x - 1)), Abs(x
+ 1) > 2), (-(x + 1)**(3/2)/(3*sqrt(1 - x)*(x + 1) - 6*sqrt(1 - x)), True))
```

Giac [A]
time = 0.01, size = 39, normalized size = 1.95

$$\frac{\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{3(-x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x)
```

```
[Out] 1/3*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^2
```

Mupad [B]
time = 0.27, size = 34, normalized size = 1.70

$$\frac{\left(\frac{x\sqrt{x+1}}{3} + \frac{\sqrt{x+1}}{3}\right)\sqrt{1-x}}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(5/2),x)
```

```
[Out] (((x*(x + 1)^(1/2))/3 + (x + 1)^(1/2)/3)*(1 - x)^(1/2))/(x^2 - 2*x + 1)
```

3.1071 $\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx$

Optimal. Leaf size=41

$$\frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}$$

```
[Out] 1/5*(1+x)^(3/2)/(1-x)^(5/2)+1/15*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(7/2),x]
[Out] (1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx\\ &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.56

$$-\frac{(-4+x)(1+x)^{3/2}}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

```
[Out] -1/15*((-4 + x)*(1 + x)^(3/2))/(1 - x)^(5/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.66, size = 125, normalized size = 3.05

$$\text{Piecewise}\left[\left\{\left\{\frac{I\,(-4+x)\,(1+x)^{\frac{3}{2}}}{15\sqrt{-1+x}\,(1-2x+x^2)},\text{Abs}\,[1+x]>2\right\}\right\},-\frac{(1+x)^{\frac{5}{2}}}{-60\,(1+x)\,\sqrt{1-x}+15\,(1+x)^2\,\sqrt{1-x}+60\sqrt{1-x}}+\frac{5\,(1+x)^{\frac{3}{2}}}{-60\,(1+x)\,\sqrt{1-x}+15\,(1+x)^2\,\sqrt{1-x}+60\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(7/2),x]')
```

```
[Out] Piecewise[{{I / 15 (-4 + x) (1 + x) ^ (3 / 2) / (Sqrt[-1 + x] (1 - 2 x + x
^ 2)), Abs[1 + x] > 2}}, -(1 + x) ^ (5 / 2) / (-60 (1 + x) Sqrt[1 - x] + 15
 (1 + x) ^ 2 Sqrt[1 - x] + 60 Sqrt[1 - x]) + 5 (1 + x) ^ (3 / 2) / (-60 (1
+ x) Sqrt[1 - x] + 15 (1 + x) ^ 2 Sqrt[1 - x] + 60 Sqrt[1 - x])]
```

Maple [A]
time = 0.15, size = 44, normalized size = 1.07

method	result	size
gosper	$-\frac{(1+x)^{\frac{3}{2}}(x-4)}{15(1-x)^{\frac{5}{2}}}$	18
default	$\frac{2\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{15(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{15\sqrt{1-x}}$	44
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^3-2x^2-7x-4)}{15\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(1+x)^(1/2)/(1-x)^(5/2)-1/15*(1+x)^(1/2)/(1-x)^(3/2)-1/15*(1+x)^(1/2)/(
1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.

time = 0.25, size = 64, normalized size = 1.56

$$-\frac{2\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)}-\frac{\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)}+\frac{\sqrt{-x^2+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 53, normalized size = 1.29

$$\frac{4\,x^3-12\,x^2+(x^2-3\,x-4)\sqrt{x+1}\,\sqrt{-x+1}+12\,x-4}{15\,(x^3-3\,x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x - 4)
/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.77, size = 172, normalized size = 4.20

$$\begin{cases} \frac{i(x+1)^{\frac{5}{2}}}{15\sqrt{x-1}\,(x+1)^2-60\sqrt{x-1}\,(x+1)+60\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{15\sqrt{x-1}\,(x+1)^2-60\sqrt{x-1}\,(x+1)+60\sqrt{x-1}} & \text{for } |x+1|>2 \\ -\frac{(x+1)^{\frac{5}{2}}}{15\sqrt{1-x}\,(x+1)^2-60\sqrt{1-x}\,(x+1)+60\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{15\sqrt{1-x}\,(x+1)^2-60\sqrt{1-x}\,(x+1)+60\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(5/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60*sqrt(x - 1)*(x
+ 1) + 60*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60
*sqrt(x - 1)*(x + 1) + 60*sqrt(x - 1)), Abs(x + 1) > 2), (-(x + 1)**(5/2)/(
15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(1 - x)) + 5*(x
 + 1)**(3/2)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(
1 - x)), True))
```

Giac [A]
time = 0.01, size = 59, normalized size = 1.44

$$\frac{2\left(\frac{1}{6}-\frac{1}{30}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x)
```

```
[Out] 1/15*(x + 1)^(3/2)*(x - 4)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B]
time = 0.24, size = 50, normalized size = 1.22

$$-\frac{\sqrt{1-x}\,\left(\frac{x\,\sqrt{x+1}}{5}+\frac{4\,\sqrt{x+1}}{15}-\frac{x^2\,\sqrt{x+1}}{15}\right)}{x^3-3\,x^2+3\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((x*(x + 1)^(1/2))/5 + (4*(x + 1)^(1/2))/15 - (x^2*(x + 1)^
(1/2))/15))/(3*x - 3*x^2 + x^3 - 1)
```

3.1072 $\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=61

$$\frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{35(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}$$

```
[Out] 1/7*(1+x)^(3/2)/(1-x)^(7/2)+2/35*(1+x)^(3/2)/(1-x)^(5/2)+2/105*(1+x)^(3/2)/
(1-x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*(1 + x)^(3/2))/(35*(1 - x)^(5/2)) + (2
*(1 + x)^(3/2))/(105*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2}{7}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx \\ &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2}{35}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx \\ &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.49

$$\frac{(1+x)^{3/2}\left(23-10x+2x^2\right)}{105(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] ((1 + x)^(3/2)*(23 - 10*x + 2*x^2))/(105*(1 - x)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 15.47, size = 344, normalized size = 5.64

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(23+13x-8x^2+2x^3\right)\sqrt{1+x}}{105\sqrt{-1+x}\left(-1+3x-3x^2+x^3\right)},\text{Abs}\left[1+x\right]>2\right\}\right\},\frac{-63\left(1+x\right)^{\frac{5}{2}}}{-3360\left(1+x\right)\sqrt{1-x}-840\left(1+x\right)^3\sqrt{1-x}+105\left(1+x\right)^4\sqrt{1-x}+1680\sqrt{1-x}+2520\left(1+x\right)^2\sqrt{1-x}}-\frac{2\left(1+x\right)^{\frac{9}{2}}}{-3360\left(1+x\right)\sqrt{1-x}-840\left(1+x\right)^3\sqrt{1-x}+105\left(1+x\right)^4\sqrt{1-x}+1680\sqrt{1-x}+2520\left(1+x\right)^2\sqrt{1-x}}+\frac{18\left(1+x\right)^{\frac{7}{2}}}{-3360\left(1+x\right)\sqrt{1-x}-840\left(1+x\right)^3\sqrt{1-x}+105\left(1+x\right)^4\sqrt{1-x}+1680\sqrt{1-x}+2520\left(1+x\right)^2\sqrt{1-x}}+\frac{70\left(1+x\right)^{\frac{3}{2}}}{-3360\left(1+x\right)\sqrt{1-x}-840\left(1+x\right)^3\sqrt{1-x}+105\left(1+x\right)^4\sqrt{1-x}+1680\sqrt{1-x}+2520\left(1+x\right)^2\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(9/2),x]')
```

```
[Out] Piecewise[{{I / 105 (23 + 13 x - 8 x ^ 2 + 2 x ^ 3) Sqrt[1 + x] / (Sqrt[-1
+ x] (-1 + 3 x - 3 x ^ 2 + x ^ 3)), Abs[1 + x] > 2}}, -63 (1 + x) ^ (5 / 2)
 / (-3360 (1 + x) Sqrt[1 - x] - 840 (1 + x) ^ 3 Sqrt[1 - x] + 105 (1 + x) ^
 4 Sqrt[1 - x] + 1680 Sqrt[1 - x] + 2520 (1 + x) ^ 2 Sqrt[1 - x]) - 2 (1 +
x) ^ (9 / 2) / (-3360 (1 + x) Sqrt[1 - x] - 840 (1 + x) ^ 3 Sqrt[1 - x] + 1
05 (1 + x) ^ 4 Sqrt[1 - x] + 1680 Sqrt[1 - x] + 2520 (1 + x) ^ 2 Sqrt[1 - x
]) + 18 (1 + x) ^ (7 / 2) / (-3360 (1 + x) Sqrt[1 - x] - 840 (1 + x) ^ 3 Sq
rt[1 - x] + 105 (1 + x) ^ 4 Sqrt[1 - x] + 1680 Sqrt[1 - x] + 2520 (1 + x) ^
 2 Sqrt[1 - x]) + 70 (1 + x) ^ (3 / 2) / (-3360 (1 + x) Sqrt[1 - x] - 840 (
1 + x) ^ 3 Sqrt[1 - x] + 105 (1 + x) ^ 4 Sqrt[1 - x] + 1680 Sqrt[1 - x] + 2
520 (1 + x) ^ 2 Sqrt[1 - x])]
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.95

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}(2x^{2}-10x+23)}{105(1-x)^{\frac{7}{2}}}$	25
default	$\frac{2\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{35(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{105(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{105\sqrt{1-x}}$	58
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(2x^{4}-6x^{3}+5x^{2}+36x+23)}{105\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{3}\sqrt{-(1+x)(-1+x)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(1+x)^(1/2)/(1-x)^(7/2)-1/35*(1+x)^(1/2)/(1-x)^(5/2)-2/105*(1+x)^(1/2)/
(1-x)^(3/2)-2/105*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(43) = 86$.
time = 0.28, size = 95, normalized size = 1.56

$$\frac{2\sqrt{-x^{2}+1}}{7(x^{4}-4x^{3}+6x^{2}-4x+1)}+\frac{\sqrt{-x^{2}+1}}{35(x^{3}-3x^{2}+3x-1)}-\frac{2\sqrt{-x^{2}+1}}{105(x^{2}-2x+1)}+\frac{2\sqrt{-x^{2}+1}}{105(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt(-
x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 70, normalized size = 1.15

$$\frac{23x^{4}-92x^{3}+138x^{2}+(2x^{3}-8x^{2}+13x+23)\sqrt{x+1}\,\sqrt{-x+1}-92x+23}{105(x^{4}-4x^{3}+6x^{2}-4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x + 1)*
sqrt(-x + 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.41, size = 566, normalized size = 9.28

$$\begin{cases} \frac{2i(x+1)^{\frac{9}{2}}}{105\sqrt{x-1}(x+1)^{4}-840\sqrt{x-1}(x+1)^{3}+2520\sqrt{x-1}(x+1)^{2}-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}}-\frac{18i(x+1)^{\frac{7}{2}}}{105\sqrt{x-1}(x+1)^{4}-840\sqrt{x-1}(x+1)^{3}+2520\sqrt{x-1}(x+1)^{2}-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}}+\frac{63i(x+1)^{\frac{5}{2}}}{105\sqrt{x-1}(x+1)^{4}-840\sqrt{x-1}(x+1)^{3}+2520\sqrt{x-1}(x+1)^{2}-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}}-\frac{70i(x+1)^{\frac{3}{2}}}{105\sqrt{x-1}(x+1)^{4}-840\sqrt{x-1}(x+1)^{3}+2520\sqrt{x-1}(x+1)^{2}-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ -\frac{2(x+1)^{\frac{9}{2}}}{105\sqrt{1-x}(x+1)^{4}-840\sqrt{1-x}(x+1)^{3}+2520\sqrt{1-x}(x+1)^{2}-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}}+\frac{18(x+1)^{\frac{7}{2}}}{105\sqrt{1-x}(x+1)^{4}-840\sqrt{1-x}(x+1)^{3}+2520\sqrt{1-x}(x+1)^{2}-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}}-\frac{63(x+1)^{\frac{5}{2}}}{105\sqrt{1-x}(x+1)^{4}-840\sqrt{1-x}(x+1)^{3}+2520\sqrt{1-x}(x+1)^{2}-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}}+\frac{70(x+1)^{\frac{3}{2}}}{105\sqrt{1-x}(x+1)^{4}-840\sqrt{1-x}(x+1)^{3}+2520\sqrt{1-x}(x+1)^{2}-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(9/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(x - 1)
*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1) + 1680
*sqrt(x - 1)) - 18*I*(x + 1)**(7/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(
x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1)
+ 1680*sqrt(x - 1)) + 63*I*(x + 1)**(5/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840
*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x
 + 1) + 1680*sqrt(x - 1)) - 70*I*(x + 1)**(3/2)/(105*sqrt(x - 1)*(x + 1)**4
 - 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x -
 1)*(x + 1) + 1680*sqrt(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(9/2)/(105*s
qrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(x +
1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 18*(x + 1)**(7/2)/(1
05*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(
x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) - 63*(x + 1)**(5/2
)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 -
x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 70*(x + 1)**
(3/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(
1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)), True))
```

Giac [A]
time = 0.01, size = 77, normalized size = 1.26

$$\frac{2\left(\left(\left(\frac{1}{105}\sqrt{x+1}\,\sqrt{x+1}-\frac{1}{15}\right)\sqrt{x+1}\,\sqrt{x+1}+\frac{1}{6}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x)
```

```
[Out] 1/105*(2*(x + 1)*(x - 6) + 35)*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.27, size = 64, normalized size = 1.05

$$\frac{\sqrt{1-x}\,\left(\frac{13\,x\,\sqrt{x+1}}{105}+\frac{23\,\sqrt{x+1}}{105}-\frac{8\,x^2\,\sqrt{x+1}}{105}+\frac{2\,x^3\,\sqrt{x+1}}{105}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((13*x*(x + 1)^(1/2))/105 + (23*(x + 1)^(1/2))/105 - (8*x^2*
(x + 1)^(1/2))/105 + (2*x^3*(x + 1)^(1/2))/105))/(6*x^2 - 4*x - 4*x^3 + x^4
 + 1)
```

3.1073 $\int \frac{\sqrt{1+x}}{(1-x)^{11/2}} dx$

Optimal. Leaf size=81

$$\frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}$$

```
[Out] 1/9*(1+x)^(3/2)/(1-x)^(9/2)+1/21*(1+x)^(3/2)/(1-x)^(7/2)+2/105*(1+x)^(3/2)/
(1-x)^(5/2)+2/315*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(3/2)/(9*(1 - x)^(9/2)) + (1 + x)^(3/2)/(21*(1 - x)^(7/2)) + (2*(1
+ x)^(3/2))/(105*(1 - x)^(5/2)) + (2*(1 + x)^(3/2))/(315*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{1}{3}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2}{21}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2}{105}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 0.43

$$\frac{(1+x)^{3/2}\left(58-33x+12x^2-2x^3\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(3/2)*(58 - 33*x + 12*x^2 - 2*x^3))/(315*(1 - x)^(9/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 43.37, size = 867, normalized size = 10.70

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(11/2),x]')
```

```
[Out] Piecewise[{{I / 315 (-58 - 25 x + 21 x ^ 2 - 10 x ^ 3 + 2 x ^ 4) Sqrt[1 + x
] / (Sqrt[-1 + x] (1 - 4 x + 6 x ^ 2 - 4 x ^ 3 + x ^ 4)), Abs[1 + x] > 2}},
 -1530 (1 + x) ^ (7 / 2) / (-211680 (1 + x) ^ 2 Sqrt[1 - x] - 88200 (1 + x)
 ^ 4 Sqrt[1 - x] - 40320 Sqrt[1 - x] - 4410 (1 + x) ^ 6 Sqrt[1 - x] + 315 (
1 + x) ^ 7 Sqrt[1 - x] + 26460 (1 + x) ^ 5 Sqrt[1 - x] + 141120 (1 + x) Sqr
t[1 - x] + 176400 (1 + x) ^ 3 Sqrt[1 - x]) - 840 (1 + x) ^ (3 / 2) / (-2116
80 (1 + x) ^ 2 Sqrt[1 - x] - 88200 (1 + x) ^ 4 Sqrt[1 - x] - 40320 Sqrt[1 -
 x] - 4410 (1 + x) ^ 6 Sqrt[1 - x] + 315 (1 + x) ^ 7 Sqrt[1 - x] + 26460 (1
 + x) ^ 5 Sqrt[1 - x] + 141120 (1 + x) Sqrt[1 - x] + 176400 (1 + x) ^ 3 Sqr
t[1 - x]) - 195 (1 + x) ^ (11 / 2) / (-211680 (1 + x) ^ 2 Sqrt[1 - x] - 882
00 (1 + x) ^ 4 Sqrt[1 - x] - 40320 Sqrt[1 - x] - 4410 (1 + x) ^ 6 Sqrt[1 -
x] + 315 (1 + x) ^ 7 Sqrt[1 - x] + 26460 (1 + x) ^ 5 Sqrt[1 - x] + 141120 (
```

```
1 + x) Sqrt[1 - x] + 176400 (1 + x) ^ 3 Sqrt[1 - x]) - 2 (1 + x) ^ (15 / 2)
 / (-211680 (1 + x) ^ 2 Sqrt[1 - x] - 88200 (1 + x) ^ 4 Sqrt[1 - x] - 40320
 Sqrt[1 - x] - 4410 (1 + x) ^ 6 Sqrt[1 - x] + 315 (1 + x) ^ 7 Sqrt[1 - x] +
 26460 (1 + x) ^ 5 Sqrt[1 - x] + 141120 (1 + x) Sqrt[1 - x] + 176400 (1 + x
) ^ 3 Sqrt[1 - x]) + 30 (1 + x) ^ (13 / 2) / (-211680 (1 + x) ^ 2 Sqrt[1 -
x] - 88200 (1 + x) ^ 4 Sqrt[1 - x] - 40320 Sqrt[1 - x] - 4410 (1 + x) ^ 6 S
qrt[1 - x] + 315 (1 + x) ^ 7 Sqrt[1 - x] + 26460 (1 + x) ^ 5 Sqrt[1 - x] +
141120 (1 + x) Sqrt[1 - x] + 176400 (1 + x) ^ 3 Sqrt[1 - x]) + 715 (1 + x)
^ (9 / 2) / (-211680 (1 + x) ^ 2 Sqrt[1 - x] - 88200 (1 + x) ^ 4 Sqrt[1 - x
] - 40320 Sqrt[1 - x] - 4410 (1 + x) ^ 6 Sqrt[1 - x] + 315 (1 + x) ^ 7 Sqrt
[1 - x] + 26460 (1 + x) ^ 5 Sqrt[1 - x] + 141120 (1 + x) Sqrt[1 - x] + 1764
00 (1 + x) ^ 3 Sqrt[1 - x]) + 1764 (1 + x) ^ (5 / 2) / (-211680 (1 + x) ^ 2
 Sqrt[1 - x] - 88200 (1 + x) ^ 4 Sqrt[1 - x] - 40320 Sqrt[1 - x] - 4410 (1
+ x) ^ 6 Sqrt[1 - x] + 315 (1 + x) ^ 7 Sqrt[1 - x] + 26460 (1 + x) ^ 5 Sqrt
[1 - x] + 141120 (1 + x) Sqrt[1 - x] + 176400 (1 + x) ^ 3 Sqrt[1 - x])]
```

Maple [A]
time = 0.17, size = 72, normalized size = 0.89

method	result	size
gosper	$-\frac{(1+x)^{\frac{3}{2}} (2x^3-12x^2+33x-58)}{315(1-x)^{\frac{9}{2}}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(2x^5-8x^4+11x^3-4x^2-83x-58)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{2\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}-\frac{\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{315\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(1+x)^(1/2)/(1-x)^(9/2)-1/63*(1+x)^(1/2)/(1-x)^(7/2)-1/105*(1+x)^(1/2)/
(1-x)^(5/2)-2/315*(1+x)^(1/2)/(1-x)^(3/2)-2/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(57) = 114.
time = 0.26, size = 131, normalized size = 1.62

$$-\frac{2\sqrt{-x^2+1}}{9(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{\sqrt{-x^2+1}}{63(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{105(x^3-3x^2+3x-1)}-\frac{2\sqrt{-x^2+1}}{315(x^2-2x+1)}+\frac{2\sqrt{-x^2+1}}{315(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 1/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
```

```
2 + 3*x - 1) - 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/315*sqrt(-x^2 + 1)/
(x - 1)
```

Fricas [A]
time = 0.29, size = 85, normalized size = 1.05

$$\frac{58\,x^5-290\,x^4+580\,x^3-580\,x^2+\left(2\,x^4-10\,x^3+21\,x^2-25\,x-58\right)\sqrt{x+1}\,\sqrt{-x+1}+290\,x-58}{315\,\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(58*x^5 - 290*x^4 + 580*x^3 - 580*x^2 + (2*x^4 - 10*x^3 + 21*x^2 - 25
*x - 58)*sqrt(x + 1)*sqrt(-x + 1) + 290*x - 58)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 61.44, size = 1561, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(15/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x -
1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4
 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*s
qrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 30*I*(x + 1)**(13/2)/(315*sqrt(x
- 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**
5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*s
qrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1
95*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)
**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*
sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)
*(x + 1) - 40320*sqrt(x - 1)) - 715*I*(x + 1)**(9/2)/(315*sqrt(x - 1)*(x +
1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*
sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)
*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1530*I*(x +
 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 2646
0*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1
)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) -
 40320*sqrt(x - 1)) - 1764*I*(x + 1)**(5/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4
410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x -
1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)*
*2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 840*I*(x + 1)**(3/2)
/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x -
 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)*
```

```
*3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqr
t(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(15/2)/(315*sqrt(1 - x)*(x + 1)**7
 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(
1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x +
 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 30*(x + 1)**(13/
2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1
 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1
)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*s
qrt(1 - x)) - 195*(x + 1)**(11/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1
 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)
**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 14112
0*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 715*(x + 1)**(9/2)/(315*sqrt(1
 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)*
*5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*
sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) -
1530*(x + 1)**(7/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)*
*6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*s
qrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*
(x + 1) - 40320*sqrt(1 - x)) + 1764*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)
**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sq
rt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(
x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) - 840*(x + 1)**
(3/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqr
t(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x
+ 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 4032
0*sqrt(1 - x)), True))
```

Giac [A]
time = 0.02, size = 96, normalized size = 1.19

$$\frac{2\left(\left(\left(\left(\frac{1}{35}-\frac{1}{315}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}-\frac{1}{10}\right)\sqrt{x+1}\,\sqrt{x+1}+\frac{1}{6}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x)
[Out] 1/315*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1)^(3/2)*sqrt(-x + 1)/(
x - 1)^5
```

Mupad [B]
time = 0.28, size = 80, normalized size = 0.99

$$-\frac{\sqrt{1-x}\,\left(\frac{5\,x\,\sqrt{x+1}}{63}+\frac{58\,\sqrt{x+1}}{315}-\frac{x^2\,\sqrt{x+1}}{15}+\frac{2\,x^3\,\sqrt{x+1}}{63}-\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((5*x*(x + 1)^(1/2))/63 + (58*(x + 1)^(1/2))/315 - (x^2*(x
+ 1)^(1/2))/15 + (2*x^3*(x + 1)^(1/2))/63 - (2*x^4*(x + 1)^(1/2))/315))/(5*
x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

3.1074 $\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}}+\frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}$$

```
[Out] 1/11*(1+x)^(3/2)/(1-x)^(11/2)+4/99*(1+x)^(3/2)/(1-x)^(9/2)+4/231*(1+x)^(3/2
)/(1-x)^(7/2)+8/1155*(1+x)^(3/2)/(1-x)^(5/2)+8/3465*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*(1 + x)^(3/2))/(99*(1 - x)^(9/2)) +
(4*(1 + x)^(3/2))/(231*(1 - x)^(7/2)) + (8*(1 + x)^(3/2))/(1155*(1 - x)^(5/
2)) + (8*(1 + x)^(3/2))/(3465*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx = \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4}{11}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4}{33}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8}{231}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx$$
$$= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx}{1155}$$
$$= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.40

$$\frac{(1+x)^{3/2}\left(547-364x+180x^2-56x^3+8x^4\right)}{3465(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(3/2)*(547 - 364*x + 180*x^2 - 56*x^3 + 8*x^4))/(3465*(1 - x)^(11/
2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 112.43, size = 1953, normalized size = 19.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(1/2)/(1 - x)^(13/2),x]')
```

```
[Out] Piecewise[{{I / 3465 (547 + 183 x - 184 x ^ 2 + 124 x ^ 3 - 48 x ^ 4 + 8 x
^ 5) Sqrt[1 + x] / (Sqrt[-1 + x] (-1 + 5 x - 10 x ^ 2 + 10 x ^ 3 - 5 x ^ 4
+ x ^ 5)), Abs[1 + x] > 2}}, -479952 (1 + x) ^ (7 / 2) / (-146361600 (1 + x
) ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560 (1 + x) ^ 6
 Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1 - x] - 7623
0 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 762300 (1 + x)
 ^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (1 + x) Sqrt
[1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x) ^ 3 Sqrt[1
- x]) - 329588 (1 + x) ^ (11 / 2) / (-146361600 (1 + x) ^ 4 Sqrt[1 - x] - 9
```

7574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560 (1 + x) ^ 6 Sqrt[1 - x] - 709632
0 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1 - x] - 76230 (1 + x) ^ 10 Sqrt[1
 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 762300 (1 + x) ^ 9 Sqrt[1 - x] + 18
295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (1 + x) Sqrt[1 - x] + 102453120 (
1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x) ^ 3 Sqrt[1 - x]) - 73920 (1 + x)
 ^ (3 / 2) / (-146361600 (1 + x) ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2 Sqr
t[1 - x] - 51226560 (1 + x) ^ 6 Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 4573800
 (1 + x) ^ 8 Sqrt[1 - x] - 76230 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x) ^
11 Sqrt[1 - x] + 762300 (1 + x) ^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7 Sqrt
[1 - x] + 39029760 (1 + x) Sqrt[1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 - x]
+ 146361600 (1 + x) ^ 3 Sqrt[1 - x]) - 52003 (1 + x) ^ (15 / 2) / (-1463616
00 (1 + x) ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560 (1
 + x) ^ 6 Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1 -
x] - 76230 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 76230
0 (1 + x) ^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (1
+ x) Sqrt[1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x) ^
3 Sqrt[1 - x]) - 1932 (1 + x) ^ (19 / 2) / (-146361600 (1 + x) ^ 4 Sqrt[1 -
 x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560 (1 + x) ^ 6 Sqrt[1 - x] -
 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1 - x] - 76230 (1 + x) ^ 10
 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 762300 (1 + x) ^ 9 Sqrt[1 -
x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (1 + x) Sqrt[1 - x] + 1024
53120 (1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x) ^ 3 Sqrt[1 - x]) - 8 (1 +
 x) ^ (23 / 2) / (-146361600 (1 + x) ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2
 Sqrt[1 - x] - 51226560 (1 + x) ^ 6 Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 457
3800 (1 + x) ^ 8 Sqrt[1 - x] - 76230 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x
) ^ 11 Sqrt[1 - x] + 762300 (1 + x) ^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7
Sqrt[1 - x] + 39029760 (1 + x) Sqrt[1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 -
 x] + 146361600 (1 + x) ^ 3 Sqrt[1 - x]) + 184 (1 + x) ^ (21 / 2) / (-14636
1600 (1 + x) ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560
(1 + x) ^ 6 Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1
- x] - 76230 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 762
300 (1 + x) ^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (
1 + x) Sqrt[1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x)
^ 3 Sqrt[1 - x]) + 12236 (1 + x) ^ (17 / 2) / (-146361600 (1 + x) ^ 4 Sqrt[
1 - x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560 (1 + x) ^ 6 Sqrt[1 - x
] - 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1 - x] - 76230 (1 + x) ^
 10 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 762300 (1 + x) ^ 9 Sqrt[1
 - x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (1 + x) Sqrt[1 - x] + 1
02453120 (1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x) ^ 3 Sqrt[1 - x]) + 155
316 (1 + x) ^ (13 / 2) / (-146361600 (1 + x) ^ 4 Sqrt[1 - x] - 97574400 (1
+ x) ^ 2 Sqrt[1 - x] - 51226560 (1 + x) ^ 6 Sqrt[1 - x] - 7096320 Sqrt[1 -
x] - 4573800 (1 + x) ^ 8 Sqrt[1 - x] - 76230 (1 + x) ^ 10 Sqrt[1 - x] + 346
5 (1 + x) ^ 11 Sqrt[1 - x] + 762300 (1 + x) ^ 9 Sqrt[1 - x] + 18295200 (1 +
 x) ^ 7 Sqrt[1 - x] + 39029760 (1 + x) Sqrt[1 - x] + 102453120 (1 + x) ^ 5
Sqrt[1 - x] + 146361600 (1 + x) ^ 3 Sqrt[1 - x]) + 280896 (1 + x) ^ (5 / 2)

```
 / (-146361600 (1 + x) ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] -
 51226560 (1 + x) ^ 6 Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^
 8 Sqrt[1 - x] - 76230 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1
- x] + 762300 (1 + x) ^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] +
39029760 (1 + x) Sqrt[1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 - x] + 14636160
0 (1 + x) ^ 3 Sqrt[1 - x]) + 488224 (1 + x) ^ (9 / 2) / (-146361600 (1 + x)
 ^ 4 Sqrt[1 - x] - 97574400 (1 + x) ^ 2 Sqrt[1 - x] - 51226560 (1 + x) ^ 6
Sqrt[1 - x] - 7096320 Sqrt[1 - x] - 4573800 (1 + x) ^ 8 Sqrt[1 - x] - 76230
 (1 + x) ^ 10 Sqrt[1 - x] + 3465 (1 + x) ^ 11 Sqrt[1 - x] + 762300 (1 + x)
^ 9 Sqrt[1 - x] + 18295200 (1 + x) ^ 7 Sqrt[1 - x] + 39029760 (1 + x) Sqrt[
1 - x] + 102453120 (1 + x) ^ 5 Sqrt[1 - x] + 146361600 (1 + x) ^ 3 Sqrt[1 -
 x])]
```

Maple [**A**]
time = 0.16, size = 86, normalized size = 0.85

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}\left(8x^{4}-56x^{3}+180x^{2}-364x+547\right)}{3465(1-x)^{\frac{11}{2}}}$	35
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(8x^{6}-40x^{5}+76x^{4}-60x^{3}-x^{2}+730x+547\right)}{3465\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{5}\sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{2\sqrt{1+x}}{11(1-x)^{\frac{11}{2}}}-\frac{\sqrt{1+x}}{99(1-x)^{\frac{9}{2}}}-\frac{4\sqrt{1+x}}{693(1-x)^{\frac{7}{2}}}-\frac{4\sqrt{1+x}}{1155(1-x)^{\frac{5}{2}}}-\frac{8\sqrt{1+x}}{3465(1-x)^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{3465\sqrt{1-x}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*(1+x)^(1/2)/(1-x)^(11/2)-1/99*(1+x)^(1/2)/(1-x)^(9/2)-4/693*(1+x)^(1/2
)/(1-x)^(7/2)-4/1155*(1+x)^(1/2)/(1-x)^(5/2)-8/3465*(1+x)^(1/2)/(1-x)^(3/2)
-8/3465*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.
time = 0.26, size = 172, normalized size = 1.70

$$\frac{2\sqrt{-x^2+1}}{11\left(x^6-6x^5+15x^4-20x^3+15x^2-6x+1\right)}+\frac{\sqrt{-x^2+1}}{99\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}-\frac{4\sqrt{-x^2+1}}{693\left(x^4-4x^3+6x^2-4x+1\right)}+\frac{4\sqrt{-x^2+1}}{1155\left(x^3-3x^2+3x-1\right)}-\frac{8\sqrt{-x^2+1}}{3465\left(x^2-2x+1\right)}+\frac{8\sqrt{-x^2+1}}{3465\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 2/11*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 1/
99*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 4/693*sqrt(-x
^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/1155*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 8/3465*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/3465*sqrt(-x^2 + 1
)/(x - 1)
```

Fricas [A]
time = 0.30, size = 100, normalized size = 0.99

$$\frac{547\,x^{6}-3282\,x^{5}+8205\,x^{4}-10940\,x^{3}+8205\,x^{2}+\left(8\,x^{5}-48\,x^{4}+124\,x^{3}-184\,x^{2}+183\,x+547\right)\sqrt{x+1}\,\sqrt{-x+1}-3282\,x+547}{3465\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(547*x^6 - 3282*x^5 + 8205*x^4 - 10940*x^3 + 8205*x^2 + (8*x^5 - 48*
x^4 + 124*x^3 - 184*x^2 + 183*x + 547)*sqrt(x + 1)*sqrt(-x + 1) - 3282*x +
547)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 178.23, size = 3648, normalized size = 36.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((8*I*(x + 1)**(23/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 184*I*(x + 1)**(21/2)/(3465*s
qrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)
*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)
**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 -
146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574
400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x
- 1)) + 1932*I*(x + 1)**(19/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 12236*I*(x + 1)**(17/2)/(3465
*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x -
1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x +
1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5
- 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 975
74400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(
x - 1)) + 52003*I*(x + 1)**(15/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqr
t(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*
(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)
**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 +
```

146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029
760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 155316*I*(x + 1)**(13/2)/(
3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(
x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(
x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)
5 - 146361600*sqrt(x - 1)*(x + 1)4 + 146361600*sqrt(x - 1)*(x + 1)**3 -
97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*s
qrt(x - 1)) + 329588*I*(x + 1)**(11/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7623
0*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x
- 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x
+ 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)
4 + 146361600*sqrt(x - 1)*(x + 1)3 - 97574400*sqrt(x - 1)*(x + 1)**2 +
39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 488224*I*(x + 1)**(9/
2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*s
qrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x -
1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x
+ 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)*
*3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70963
20*sqrt(x - 1)) + 479952*I*(x + 1)**(7/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7
6230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt
(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)
*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x +
1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2
+ 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 280896*I*(x + 1)**
(5/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 76230
0*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x
- 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*
(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x +
1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70
96320*sqrt(x - 1)) + 73920*I*(x + 1)**(3/2)/(3465*sqrt(x - 1)*(x + 1)**11 -
76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sq
rt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x -
1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x
+ 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)*
*2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)), Abs(x + 1) > 2),
(-8*(x + 1)**(23/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x +
1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 1
8295200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 10245312
0*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqr
t(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x
)*(x + 1) - 7096320*sqrt(1 - x)) + 184*(x + 1)**(21/2)/(3465*sqrt(1 - x)*(x
+ 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -

x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 1932*
(x + 1)**(19/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**
10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 18295
200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sq
rt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1
- x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x
+ 1) - 7096320*sqrt(1 - x)) + 12236*(x + 1)**(17/2)/(3465*sqrt(1 - x)*(x +
1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4
573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560*
sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(
1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)
*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 52003*(
x + 1)**(15/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**1
0 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182952
00*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqr
t(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 -
x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x
+ 1) - 7096320*sqrt(1 - x)) + 155316*(x + 1)**(13/2)/(3465*sqrt(1 - x)*(x +
1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4
573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560*
sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(
1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)
*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 329588*
(x + 1)**(11/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**
10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 18295
200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sq
rt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1
- x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x
+ 1) - 7096320*sqrt(1 - x)) + 488224*(x + 1)**(9/2)/(3465*sqrt(1 - x)*(x +
1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4
573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560*
sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(
1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)
*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 479952*
(x + 1)**(7/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**1
0 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182952
00*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqr
t(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 -
x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x
+ 1) - 7096320*sqrt(1 - x)) + 280896*(x + 1)**(5/2)/(3465*sqrt(1 - x)*(x +
1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 45
73800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560*s
qrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1
- x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*
(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 73920*(x

```
 + 1)**(3/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10
+ 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200
*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(
1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x
)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x +
1) - 7096320*sqrt(1 - x)), True))
```

Giac [A]
time = 0.02, size = 119, normalized size = 1.18

$$\frac{2\left(\left(\left(\left(\left(\frac{4}{3465}\sqrt{x+1}\sqrt{x+1}-\frac{4}{315}\right)\sqrt{x+1}\sqrt{x+1}+\frac{2}{35}\right)\sqrt{x+1}\sqrt{x+1}-\frac{2}{15}\right)\sqrt{x+1}\sqrt{x+1}+\frac{1}{6}\right)\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x)
```

```
[Out] 1/3465*(4*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1) + 1155)*(x + 1)
^(3/2)*sqrt(-x + 1)/(x - 1)^6
```

Mupad [B]
time = 0.29, size = 94, normalized size = 0.93

$$\frac{\sqrt{1-x}\,\left(\frac{61\,x\,\sqrt{x+1}}{1155}+\frac{547\,\sqrt{x+1}}{3465}-\frac{184\,x^2\,\sqrt{x+1}}{3465}+\frac{124\,x^3\,\sqrt{x+1}}{3465}-\frac{16\,x^4\,\sqrt{x+1}}{1155}+\frac{8\,x^5\,\sqrt{x+1}}{3465}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((61*x*(x + 1)^(1/2))/1155 + (547*(x + 1)^(1/2))/3465 - (184
*x^2*(x + 1)^(1/2))/3465 + (124*x^3*(x + 1)^(1/2))/3465 - (16*x^4*(x + 1)^(
1/2))/1155 + (8*x^5*(x + 1)^(1/2))/3465))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 -
 6*x^5 + x^6 + 1)
```

3.1075 $\int (1-x)^{9/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=109

$$\frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(1-x)^{7/2}(1+x)^{5/2}+\frac{1}{7}(1-x)^{9/2}(1+x)^{5/}$$

```
[Out] 3/8*(1-x)^(3/2)*x*(1+x)^(3/2)+3/10*(1-x)^(5/2)*(1+x)^(5/2)+3/14*(1-x)^(7/2)
*(1+x)^(5/2)+1/7*(1-x)^(9/2)*(1+x)^(5/2)+9/16*arcsin(x)+9/16*x*(1-x)^(1/2)*
(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{9}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (9*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (3*(1 - x)^(3/2)*x*(1 + x)^(3/2))/8 + (3
*(1 - x)^(5/2)*(1 + x)^(5/2))/10 + (3*(1 - x)^(7/2)*(1 + x)^(5/2))/14 + ((1
 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{9/2}(1+x)^{3/2}\,dx &= \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2}+\frac{9}{7}\int (1-x)^{7/2}(1+x)^{3/2}\,dx\\
&= \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2}+\frac{1}{7}(1-x)^{9/2}(1+x)^{5/2}+\frac{3}{2}\int (1-x)^{5/2}(1+x)^{3/2}\,dx\\
&= \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(1-x)^{7/2}(1+x)^{5/2}+\frac{1}{7}(1-x)^{9/2}(1+x)^{5/2}+\frac{3}{2}\int\\
&= \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(1-x)^{7/2}(1+x)^{5/2}+\frac{1}{7}\\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(\\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(\\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(\end{aligned}$$

Mathematica [A]
time = 0.13, size = 78, normalized size = 0.72

$$\frac{\sqrt{1-x}\,(368+613x-411x^2-306x^3+558x^4-72x^5-200x^6+80x^7)}{560\sqrt{1+x}}-\frac{9}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(368 + 613*x - 411*x^2 - 306*x^3 + 558*x^4 - 72*x^5 - 200*x^6
+ 80*x^7))/(560*Sqrt[1 + x]) - (9*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 127, normalized size = 1.17

method	result
risch	$-\frac{(80x^6-280x^5+208x^4+350x^3-656x^2+245x+368)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{560\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{9\sqrt{(1+x)(1-x)}\,\operatorname{arcs}}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{5}{2}}}{7}+\frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{14}+\frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{10}+\frac{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{8}+\frac{3\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{8}-\frac{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1-x)^(9/2)*(1+x)^(5/2)+3/14*(1-x)^(7/2)*(1+x)^(5/2)+3/10*(1-x)^(5/2)*(
1+x)^(5/2)+3/8*(1-x)^(3/2)*(1+x)^(5/2)+3/8*(1-x)^(1/2)*(1+x)^(5/2)-3/16*(1-
x)^(1/2)*(1+x)^(3/2)-9/16*(1-x)^(1/2)*(1+x)^(1/2)+9/16*((1+x)*(1-x))^(1/2)/
(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 66, normalized size = 0.61

$$\frac{1}{7}\left(-x^2+1\right)^{\frac{5}{2}}x^2-\frac{1}{2}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{23}{35}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{3}{8}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{9}{16}\sqrt{-x^2+1}\,x+\frac{9}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(5/2)*x^2 - 1/2*(-x^2 + 1)^(5/2)*x + 23/35*(-x^2 + 1)^(5/2)
+ 3/8*(-x^2 + 1)^(3/2)*x + 9/16*sqrt(-x^2 + 1)*x + 9/16*arcsin(x)
```

Fricas [A]
time = 0.30, size = 67, normalized size = 0.61

$$\frac{1}{560}\left(80\,x^6-280\,x^5+208\,x^4+350\,x^3-656\,x^2+245\,x+368\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{9}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(80*x^6 - 280*x^5 + 208*x^4 + 350*x^3 - 656*x^2 + 245*x + 368)*sqrt(x
 + 1)*sqrt(-x + 1) - 9/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(77) = 154.
time = 0.04, size = 647, normalized size = 5.94

$$2\left(2\left(\left(\left(\left(\left(\left(\frac{1}{28}\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{168}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{661}{840}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{1517}{1120}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{683}{480}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{181}{192}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{27}{64}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{5}{16}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-8\left(2\left(\left(\left(\left(\left(\frac{31}{120}-\frac{1}{24}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{107}{160}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{451}{480}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{149}{192}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{27}{64}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{5}{16}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)+10\left(2\left(\left(\left(\left(\frac{1}{20}\sqrt{-x+1}\sqrt{-x+1}-\frac{21}{80}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{133}{240}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{59}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-10\left(2\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)+8\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x)
```

```
[Out] 1/1680*((2*((4*(5*(6*x + 37)*(x - 1) + 661)*(x - 1) + 4551)*(x - 1) + 4781)
*(x - 1) + 6335)*(x - 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/60*((2*((4*(5
*x + 26)*(x - 1) + 321)*(x - 1) + 451)*(x - 1) + 745)*(x - 1) + 405)*sqrt(x
 + 1)*sqrt(-x + 1) + 1/24*((2*(3*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295)*(
x - 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 5/6*((2*x + 5)*(x - 1) + 9)*sqrt(x
 + 1)*sqrt(-x + 1) + 2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(
-x + 1) - 9/8*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(3/2), x)
```

3.1076 $\int (1-x)^{7/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{16}\sin^{-1}(x)$$

```
[Out] 7/24*(1-x)^(3/2)*x*(1+x)^(3/2)+7/30*(1-x)^(5/2)*(1+x)^(5/2)+1/6*(1-x)^(7/2)
*(1+x)^(5/2)+7/16*arcsin(x)+7/16*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
7*(1 - x)^(5/2)*(1 + x)^(5/2))/30 + ((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*Ar
cSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{7/2}(1+x)^{3/2}\,dx &= \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{6}\int (1-x)^{5/2}(1+x)^{3/2}\,dx\\
&= \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{8}\\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(\\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(\\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.82

$$\frac{\sqrt{1-x}\,\left(96+231x-57x^2-182x^3+106x^4+56x^5-40x^6\right)}{240\sqrt{1+x}}-\frac{7}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(96 + 231*x - 57*x^2 - 182*x^3 + 106*x^4 + 56*x^5 - 40*x^6))/(
240*Sqrt[1 + x]) - (7*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 98.88, size = 196, normalized size = 2.20

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-1543(1+x)^{\frac{5}{2}}-1366(1+x)^{\frac{9}{2}}-210\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-40(1+x)^{\frac{13}{2}}-35(1+x)^{\frac{3}{2}}+210\sqrt{1+x}+376(1+x)^{\frac{11}{2}}+2302(1+x)^{\frac{7}{2}}\right)}{240\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-1151(1+x)^{\frac{7}{2}}}{120\sqrt{1-x}}-\frac{47(1+x)^{\frac{11}{2}}}{30\sqrt{1-x}}-\frac{7\sqrt{1+x}}{8\sqrt{1-x}}+\frac{7(1+x)^{\frac{3}{2}}}{48\sqrt{1-x}}+\frac{(1+x)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{7\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{8}+\frac{683(1+x)^{\frac{9}{2}}}{120\sqrt{1-x}}+\frac{1543(1+x)^{\frac{5}{2}}}{240\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 240 (-1543 (1 + x) ^ (5 / 2) - 1366 (1 + x) ^ (9 / 2) - 210
 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 40 (1 + x) ^ (13 / 2) - 35
```

```
(1 + x) ^ (3 / 2) + 210 Sqrt[1 + x] + 376 (1 + x) ^ (11 / 2) + 2302 (1 + x
) ^ (7 / 2)) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -1151 (1 + x) ^ (7 / 2) / (1
20 Sqrt[1 - x]) - 47 (1 + x) ^ (11 / 2) / (30 Sqrt[1 - x]) - 7 Sqrt[1 + x]
/ (8 Sqrt[1 - x]) + 7 (1 + x) ^ (3 / 2) / (48 Sqrt[1 - x]) + (1 + x) ^ (13
/ 2) / (6 Sqrt[1 - x]) + 7 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 8 + 683 (1 + x
) ^ (9 / 2) / (120 Sqrt[1 - x]) + 1543 (1 + x) ^ (5 / 2) / (240 Sqrt[1 - x]
)]
```

Maple [A]
time = 0.16, size = 113, normalized size = 1.27

method	result
risch	$\frac{(40x^5-96x^4-10x^3+192x^2-135x-96)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{240\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{7\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{6}+\frac{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{30}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{24}+\frac{7\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{7\sqrt{1-x}\,\sqrt{1}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(7/2)*(1+x)^(5/2)+7/30*(1-x)^(5/2)*(1+x)^(5/2)+7/24*(1-x)^(3/2)*(
1+x)^(5/2)+7/24*(1-x)^(1/2)*(1+x)^(5/2)-7/48*(1-x)^(1/2)*(1+x)^(3/2)-7/16*(
1-x)^(1/2)*(1+x)^(1/2)+7/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arc
sin(x)
```

Maxima [A]
time = 0.35, size = 52, normalized size = 0.58

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{2}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{7}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{7}{16}\sqrt{-x^2+1}\,x+\frac{7}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)*x + 2/5*(-x^2 + 1)^(5/2) + 7/24*(-x^2 + 1)^(3/2)*x +
7/16*sqrt(-x^2 + 1)*x + 7/16*arcsin(x)
```

Fricas [A]
time = 0.29, size = 62, normalized size = 0.70

$$-\frac{1}{240}\left(40\,x^5-96\,x^4-10\,x^3+192\,x^2-135\,x-96\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{7}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="fricas")
```

[Out] -1/240*(40*x^5 - 96*x^4 - 10*x^3 + 192*x^2 - 135*x - 96)*sqrt(x + 1)*sqrt(-x + 1) - 7/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

Sympy [A]
time = 114.36, size = 287, normalized size = 3.22

$$\begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} + \frac{47i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}} - \frac{683i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}} + \frac{1151i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}} - \frac{1543i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}} - \frac{7i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{7i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} - \frac{47(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}} + \frac{683(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}} - \frac{1151(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}} + \frac{1543(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}} + \frac{7(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{7\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(7/2)*(1+x)**(3/2),x)

[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(13/2)/(6*sqrt(x - 1)) + 47*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) - 683*I*(x + 1)**(9/2)/(120*sqrt(x - 1)) + 1151*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) - 1543*I*(x + 1)**(5/2)/(240*sqrt(x - 1)) - 7*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1) > 2), (7*asin(sqrt(2)*sqrt(x + 1)/2)/8 + (x + 1)**(13/2)/(6*sqrt(1 - x)) - 47*(x + 1)**(11/2)/(30*sqrt(1 - x)) + 683*(x + 1)**(9/2)/(120*sqrt(1 - x)) - 1151*(x + 1)**(7/2)/(120*sqrt(1 - x)) + 1543*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 7*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 7*sqrt(x + 1)/(8*sqrt(1 - x)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(63) = 126.
time = 0.04, size = 583, normalized size = 6.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x)

[Out] -1/240*((2*((4*(5*x + 26)*(x - 1) + 321)*(x - 1) + 451)*(x - 1) + 745)*(x - 1) + 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295)*(x - 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - 7/8*arcsin(1/2*sqrt(2)*sqrt(-x + 1))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(7/2)*(x + 1)^(3/2),x)

[Out] int((1 - x)^(7/2)*(x + 1)^(3/2), x)

3.1077 $\int (1-x)^{5/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}\;+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] 1/4*(1-x)^(3/2)*x*(1+x)^(3/2)+1/5*(1-x)^(5/2)*(1+x)^(5/2)+3/8*arcsin(x)+3/8
*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}\;+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{5/2}(1+x)^{3/2}\,dx &= \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\ &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-}\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 68, normalized size = 0.99

$$\frac{\sqrt{1-x}\,\left(8+33x+9x^2-26x^3-2x^4+8x^5\right)}{40\sqrt{1+x}}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(8 + 33*x + 9*x^2 - 26*x^3 - 2*x^4 + 8*x^5))/(40*Sqrt[1 + x])
- (3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 35.72, size = 175, normalized size = 2.54

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-129(1+x)^{\frac{5}{2}}-58(1+x)^{\frac{9}{2}}-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-5(1+x)^{\frac{3}{2}}+8(1+x)^{\frac{11}{2}}+30\sqrt{1+x}+146(1+x)^{\frac{7}{2}}\right)}{40\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-73(1+x)^{\frac{7}{2}}}{20\sqrt{1-x}}-\frac{3\sqrt{1+x}}{4\sqrt{1-x}}-\frac{(1+x)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{(1+x)^{\frac{3}{2}}}{8\sqrt{1-x}}+\frac{3\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4}+\frac{29(1+x)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{129(1+x)^{\frac{5}{2}}}{40\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 40 (-129 (1 + x) ^ (5 / 2) - 58 (1 + x) ^ (9 / 2) - 30 ArcC
osh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 5 (1 + x) ^ (3 / 2) + 8 (1 + x)
 ^ (11 / 2) + 30 Sqrt[1 + x] + 146 (1 + x) ^ (7 / 2)) / Sqrt[-1 + x], Abs[1
 + x] > 2}}, -73 (1 + x) ^ (7 / 2) / (20 Sqrt[1 - x]) - 3 Sqrt[1 + x] / (4
Sqrt[1 - x]) - (1 + x) ^ (11 / 2) / (5 Sqrt[1 - x]) + (1 + x) ^ (3 / 2) / (
8 Sqrt[1 - x]) + 3 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 4 + 29 (1 + x) ^ (9 /
2) / (20 Sqrt[1 - x]) + 129 (1 + x) ^ (5 / 2) / (40 Sqrt[1 - x])]
```

Maple [A]
time = 0.14, size = 99, normalized size = 1.43

method	result
risch	$-\frac{(8x^4-10x^3-16x^2+25x+8)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{40\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{5}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)}}{8\sqrt{1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1-x)^(5/2)*(1+x)^(5/2)+1/4*(1-x)^(3/2)*(1+x)^(5/2)+1/4*(1-x)^(1/2)*(1+
x)^(5/2)-1/8*(1-x)^(1/2)*(1+x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+x)
*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 40, normalized size = 0.58

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*
arcsin(x)
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.83

$$\frac{1}{40}\left(8\,x^4-10\,x^3-16\,x^2+25\,x+8\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 10*x^3 - 16*x^2 + 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*ar
ctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 35.95, size = 248, normalized size = 3.59

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}-\frac{29i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}+\frac{73i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{129i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{29(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}-\frac{73(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{129(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) - 29*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) + 73*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - 129*I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sq
rt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (3*asin(sqrt
(2)*sqrt(x + 1)/2)/4 - (x + 1)**(11/2)/(5*sqrt(1 - x)) + 29*(x + 1)**(9/2)/
(20*sqrt(1 - x)) - 73*(x + 1)**(7/2)/(20*sqrt(1 - x)) + 129*(x + 1)**(5/2)/
(40*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1
 - x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.
time = 0.02, size = 343, normalized size = 4.97

$$2\left(2\left(\left(\left(\left(\frac{1}{20}\sqrt{-x+1}\sqrt{-x+1}-\frac{21}{80}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{133}{240}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{59}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-4\left(2\left(\left(\left(\frac{13}{48}-\frac{1}{16}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)+4\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x)
```

```
[Out] 1/120*((2*(3*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295)*(x - 1) + 195)*sqrt(x
 + 1)*sqrt(-x + 1) - 1/12*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x
 + 1)*sqrt(-x + 1) + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x
 + 1) - 3/4*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(3/2), x)
```

3.1078 $\int (1-x)^{3/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] 1/4*(1-x)^(3/2)*x*(1+x)^(3/2)+3/8*arcsin(x)+3/8*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {38, 41, 222}

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + (3*Ar
cSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{3/2}(1+x)^{3/2}\,dx &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx \\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.94

$$-\frac{1}{8}x\sqrt{1-x^2}\,(-5+2x^2) - \frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] -1/8*(x*Sqrt[1 - x^2]*(-5 + 2*x^2)) - (3*ArcTan[Sqrt[1 - x^2]/(1 + x)])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 13.50, size = 154, normalized size = 3.14

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-13(1+x)^{\frac{5}{2}} - 6\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x} - 2(1+x)^{\frac{9}{2}} - (1+x)^{\frac{3}{2}} + 6\sqrt{1+x} + 10(1+x)^{\frac{7}{2}}\right)}{8\sqrt{-1+x}}, \text{Abs}[1+x] > 2\right\}\right\}, \frac{-5(1+x)^{\frac{7}{2}}}{4\sqrt{1-x}} - \frac{3\sqrt{1+x}}{4\sqrt{1-x}} + \frac{(1+x)^{\frac{3}{2}}}{8\sqrt{1-x}} + \frac{(1+x)^{\frac{9}{2}}}{4\sqrt{1-x}} + \frac{3\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4} + \frac{13(1+x)^{\frac{5}{2}}}{8\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 8 (-13 (1 + x) ^ (5 / 2) - 6 ArcCosh[Sqrt[2] Sqrt[1 + x] /
2] Sqrt[-1 + x] - 2 (1 + x) ^ (9 / 2) - (1 + x) ^ (3 / 2) + 6 Sqrt[1 + x] +
 10 (1 + x) ^ (7 / 2)) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -5 (1 + x) ^ (7 /
2) / (4 Sqrt[1 - x]) - 3 Sqrt[1 + x] / (4 Sqrt[1 - x]) + (1 + x) ^ (3 / 2)
/ (8 Sqrt[1 - x]) + (1 + x) ^ (9 / 2) / (4 Sqrt[1 - x]) + 3 ArcSin[Sqrt[2]
Sqrt[1 + x] / 2] / 4 + 13 (1 + x) ^ (5 / 2) / (8 Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.
time = 0.14, size = 85, normalized size = 1.73

method	result

risch	$\frac{x(2x^2-5)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{8\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1-x)^(3/2)*(1+x)^(5/2)+1/4*(1-x)^(1/2)*(1+x)^(5/2)-1/8*(1-x)^(1/2)*(1+
x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1
-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.36, size = 29, normalized size = 0.59

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*arcsin(x)
```

Fricas [A]
time = 0.30, size = 46, normalized size = 0.94

$$-\frac{1}{8}\left(2x^3-5x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 - 5*x)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.65, size = 212, normalized size = 4.33

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}+\frac{5i(x+1)^{\frac{7}{2}}}{4\sqrt{x-1}}-\frac{13i(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}-\frac{5(x+1)^{\frac{7}{2}}}{4\sqrt{1-x}}+\frac{13(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) + 5*I*(x + 1)**(7/2)/(4*sqrt(x - 1)) - 13*I*(x + 1)**(5/2)/(8*sqrt(x
 - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1))
, Abs(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*sqr
t(1 - x)) - 5*(x + 1)**(7/2)/(4*sqrt(1 - x)) + 13*(x + 1)**(5/2)/(8*sqrt(1
- x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(35) = 70$.
time = 0.02, size = 298, normalized size = 6.08

$$-2\left(2\left(\left(\left(\left(\frac{13}{48}-\frac{1}{16}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)\right)+2\left(2\left(\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)\right)+2\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x)
```

```
[Out] -1/24*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) +
 1/6*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*(x + 2)*sqrt(x
+ 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - 3/4*arcsin(1/2*sqrt(2)*sqrt(
-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(3/2), x)
```

3.1079 $\int \sqrt{1-x}\,(1+x)^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*arcsin(x)+1/2*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 - ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-x}\,(1+x)^{3/2}\,dx &= -\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 1.12

$$\frac{\sqrt{1-x}\,(-2+x+5x^2+2x^3)}{6\sqrt{1+x}}-\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(-2 + x + 5*x^2 + 2*x^3))/(6*Sqrt[1 + x]) - ArcTan[Sqrt[1 - x]
/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.72, size = 129, normalized size = 2.69

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(\sqrt{1+x}-\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\sqrt{-1+x}-\frac{5(1+x)^{\frac{5}{2}}}{6}-\frac{(1+x)^{\frac{3}{2}}}{6}+\frac{(1+x)^{\frac{7}{2}}}{3}\right)}{\sqrt{-1+x}},\text{Abs}\,[1+x]>2\right\}\right\},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]-\frac{\sqrt{1+x}}{\sqrt{1-x}}-\frac{(1+x)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{(1+x)^{\frac{3}{2}}}{6\sqrt{1-x}}+\frac{5\,(1+x)^{\frac{5}{2}}}{6\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/2)*(1 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I (Sqrt[1 + x] - ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x]
- 5 (1 + x) ^ (5 / 2) / 6 - (1 + x) ^ (3 / 2) / 6 + (1 + x) ^ (7 / 2) / 3)
/ Sqrt[-1 + x], Abs[1 + x] > 2}}, ArcSin[Sqrt[2] Sqrt[1 + x] / 2] - Sqrt[1
+ x] / Sqrt[1 - x] - (1 + x) ^ (7 / 2) / (3 Sqrt[1 - x]) + (1 + x) ^ (3 / 2
) / (6 Sqrt[1 - x]) + 5 (1 + x) ^ (5 / 2) / (6 Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.
time = 0.14, size = 71, normalized size = 1.48

method	result	size
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{3}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{6}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$-\frac{(2x^2+3x-2)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(1-x)^(1/2)*(1+x)^(5/2)-1/6*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+
x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 28, normalized size = 0.58

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
[Out] -1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.31, size = 47, normalized size = 0.98

$$\frac{1}{6}\left(2\,x^2+3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.73, size = 163, normalized size = 3.40

$$\begin{cases} -i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{5i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ \mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{5(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) - 5*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - I*(x + 1)**(3/2)/(6*sqrt(x - 1))
+ I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (asin(sqrt(2)*sqrt(x + 1)/2)
- (x + 1)**(7/2)/(3*sqrt(1 - x)) + 5*(x + 1)**(5/2)/(6*sqrt(1 - x)) + (x +
1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.01, size = 124, normalized size = 2.58

$$2\left(2\left(\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x)
```

```
[Out] 1/6*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x
+ 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{1-x}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(3/2), x)
```

3.1080 $\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=47

$$-\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}\,-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)-1/2*(1-x)^(1/2)*(1+x)^(3/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx &= -\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.04

$$-\frac{\sqrt{1-x}\,(4+5x+x^2)}{2\sqrt{1+x}}-3\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] -1/2*(Sqrt[1 - x]*(4 + 5*x + x^2))/Sqrt[1 + x] - 3*ArcTan[Sqrt[1 - x]/Sqrt[
1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.36, size = 112, normalized size = 2.38

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-6\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\sqrt{-1+x}-(1+x)^{\frac{3}{2}}-(1+x)^{\frac{5}{2}}+6\sqrt{1+x}\right)}{2\sqrt{-1+x}},\text{Abs}\,[1+x]>2\right\}\right\},\frac{-3\sqrt{1+x}}{\sqrt{1-x}}+\frac{(1+x)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{(1+x)^{\frac{5}{2}}}{2\sqrt{1-x}}+3\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 2 (-6 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - (1 +
x) ^ (3 / 2) - (1 + x) ^ (5 / 2) + 6 Sqrt[1 + x]) / Sqrt[-1 + x], Abs[1 + x
] > 2}}, -3 Sqrt[1 + x] / Sqrt[1 - x] + (1 + x) ^ (3 / 2) / (2 Sqrt[1 - x])
 + (1 + x) ^ (5 / 2) / (2 Sqrt[1 - x]) + 3 ArcSin[Sqrt[2] Sqrt[1 + x] / 2]]
```

Maple [A]
time = 0.16, size = 57, normalized size = 1.21

method	result	size

default	$-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$\frac{(4+x)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(1-x)^(1/2)*(1+x)^(3/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^
(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.36, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x-2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 40, normalized size = 0.85

$$-\frac{1}{2}(x+4)\sqrt{x+1}\,\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

Sympy [A]
time = 2.36, size = 134, normalized size = 2.85

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{3\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 3*I*sqrt(x + 1)/sqrt(x - 1), Abs(
x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 - x)
) + (x + 1)**(3/2)/(2*sqrt(1 - x)) - 3*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.01, size = 58, normalized size = 1.23

$$2\left(-\frac{3}{4}-\frac{1}{4}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{-x+1}\,+3\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x)
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(1/2), x)
```

3.1081 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}} \, dx$

Optimal. Leaf size=41

$$3\sqrt{1-x}\,\sqrt{1+x}+\frac{2(1+x)^{3/2}}{\sqrt{1-x}}-3\sin^{-1}(x)$$

```
[Out] -3*arcsin(x)+2*(1+x)^(3/2)/(1-x)^(1/2)+3*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}+3\sqrt{1-x}\,\sqrt{x+1}-3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] 3*Sqrt[1 - x]*Sqrt[1 + x] + (2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\ &= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 41, normalized size = 1.00

$$\frac{(-5+x)\sqrt{1-x^2}}{-1+x} - 6\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]
[Out] ((-5 + x)*Sqrt[1 - x^2])/(-1 + x) - 6*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.67, size = 89, normalized size = 2.17

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left((1+x)^{\frac{3}{2}} - 6\sqrt{1+x} + 6\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}\right)}{\sqrt{-1+x}}, \text{Abs}[1+x] > 2\right\}\right\}, -6\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right] - \frac{(1+x)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{6\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]')
[Out] Piecewise[{{I ((1 + x) ^ (3 / 2) - 6 Sqrt[1 + x] + 6 ArcCosh[Sqrt[2] Sqrt[1
 + x] / 2] Sqrt[-1 + x]) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -6 ArcSin[Sqrt[2
] Sqrt[1 + x] / 2] - (1 + x) ^ (3 / 2) / Sqrt[1 - x] + 6 Sqrt[1 + x] / Sqrt
[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.
time = 0.16, size = 72, normalized size = 1.76

method	result	size
risch	$-\frac{(x^2-4x-5)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(x^2-4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1
/2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 42, normalized size = 1.02

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{x^2-2x+1}-\frac{6\sqrt{-x^2+1}}{x-1}-3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(3/2)/(x^2 - 2*x + 1) - 6*sqrt(-x^2 + 1)/(x - 1) - 3*arcsin(x)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 1.27

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1}+6\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+5\,x-5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x + 1)*(x - 5)*sqrt(-x + 1) + 6*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x +
 1) - 1)/x) + 5*x - 5)/(x - 1)
```

Sympy [A]
time = 1.67, size = 99, normalized size = 2.41

$$\begin{cases} 6i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{6i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{6\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
- 6*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (-6*asin(sqrt(2)*sqrt(x + 1
)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 6*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [**A**]
time = 0.01, size = 61, normalized size = 1.49

$$\frac{2\left(-\frac{1}{2}\sqrt{x+1}\,\sqrt{x+1}+3\right)\sqrt{x+1}\,\sqrt{-x+1}}{-x+1}-6\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x)
```

```
[Out] sqrt(x + 1)*(x - 5)*sqrt(-x + 1)/(x - 1) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1)
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(3/2), x)
```

3.1082 $\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{2\sqrt{1+x}}{\sqrt{1-x}}+\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 2/3*(1+x)^(3/2)/(1-x)^(3/2)+arcsin(x)-2*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} - \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx \\
&= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.12

$$
\frac{4\sqrt{1+x}\,(-1+2x)}{3(1-x)^{3/2}} - 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (4*Sqrt[1 + x]*(-1 + 2*x))/(3*(1 - x)^(3/2)) - 2*ArcTan[Sqrt[1 - x]/Sqrt[1
+ x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.39, size = 266, normalized size = 6.49

$$
\text{Piecewise}\left[\left\{\left\{\frac{4I(-1+x)(-1+2x)\sqrt{1+x}+3\left(-2\text{Pi}+\text{Pi}(1+x)-2I\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](1+x)+4I\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\right)(-1+x)^{\frac{3}{2}}}{3(-1+x)^{\frac{5}{2}}},\text{Abs}[1+x]>2\right\}\right\},\frac{-12\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](1+x)^{\frac{13}{2}}\sqrt{1-x}}{-6(1+x)^{\frac{13}{2}}\sqrt{1-x}+3(1+x)^{\frac{15}{2}}\sqrt{1-x}}-\frac{8(1+x)^{8}}{-6(1+x)^{\frac{13}{2}}\sqrt{1-x}+3(1+x)^{\frac{15}{2}}\sqrt{1-x}}+\frac{6\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](1+x)^{\frac{15}{2}}\sqrt{1-x}}{-6(1+x)^{\frac{13}{2}}\sqrt{1-x}+3(1+x)^{\frac{15}{2}}\sqrt{1-x}}+\frac{12(1+x)^{7}}{-6(1+x)^{\frac{13}{2}}\sqrt{1-x}+3(1+x)^{\frac{15}{2}}\sqrt{1-x}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]')
```

```
[Out] Piecewise[{{(4 I (-1 + x) (-1 + 2 x) Sqrt[1 + x] + 3 (-2 Pi + Pi (1 + x) -
2 I ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] (1 + x) + 4 I ArcCosh[Sqrt[2] Sqrt[1 +
 x] / 2]) (-1 + x) ^ (3 / 2)) / (3 (-1 + x) ^ (5 / 2)), Abs[1 + x] > 2}}, -
12 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] (1 + x) ^ (13 / 2) Sqrt[1 - x] / (-6 (1
+ x) ^ (13 / 2) Sqrt[1 - x] + 3 (1 + x) ^ (15 / 2) Sqrt[1 - x]) - 8 (1 + x)
 ^ 8 / (-6 (1 + x) ^ (13 / 2) Sqrt[1 - x] + 3 (1 + x) ^ (15 / 2) Sqrt[1 - x
]) + 6 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] (1 + x) ^ (15 / 2) Sqrt[1 - x] / (-6
 (1 + x) ^ (13 / 2) Sqrt[1 - x] + 3 (1 + x) ^ (15 / 2) Sqrt[1 - x]) + 12 (1
 + x) ^ 7 / (-6 (1 + x) ^ (13 / 2) Sqrt[1 - x] + 3 (1 + x) ^ (15 / 2) Sqrt[
1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.
time = 0.16, size = 76, normalized size = 1.85

method	result	size
risch	$-\frac{4(2x^2+x-1)\sqrt{(1+x)(1-x)}}{3(-1+x)\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(2*x^2+x-1)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/
2)/(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 0.36, size = 66, normalized size = 1.61

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)}+\frac{7\,\sqrt{-x^2+1}}{3\,(x-1)}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2 + 3*x - 1) + 2/3*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x - 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 0.30, size = 71, normalized size = 1.73

$$-\frac{2\left(2\,x^2-2\,(2\,x-1)\sqrt{x+1}\,\sqrt{-x+1}+3\,(x^2-2\,x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-4\,x+2\right)}{3\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*x^2 - 2*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*(x^2 - 2*x + 1)*arct
an((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.18, size = 498, normalized size = 12.15

$$\begin{cases} -\frac{6i\sqrt{x-1}\,(x+1)^{\frac{15}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}-6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}+\frac{3\pi\sqrt{x-1}\,(x+1)^{\frac{15}{2}}}{3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}-6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}+\frac{12i\sqrt{x-1}\,(x+1)^{\frac{13}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}-6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}-\frac{6\pi\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}{3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}-6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}+\frac{8i(x+1)^{8}}{3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}-6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}-\frac{12i(x+1)^{7}}{3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}-6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} & \text{for}\ |x+1|>2 \\ \frac{6\sqrt{1-x}\,(x+1)^{\frac{15}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}}-\frac{12\sqrt{1-x}\,(x+1)^{\frac{13}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}}-\frac{8(x+1)^{8}}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}}+\frac{12(x+1)^{7}}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((-6*I*sqrt(x - 1)*(x + 1)**(15/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(3
*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) + 3*pi*sqrt(x
 - 1)*(x + 1)**(15/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1
)**(13/2)) + 12*I*sqrt(x - 1)*(x + 1)**(13/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/
(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) - 6*pi*sqrt
(x - 1)*(x + 1)**(13/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x +
 1)**(13/2)) + 8*I*(x + 1)**8/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1
)*(x + 1)**(13/2)) - 12*I*(x + 1)**7/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqr
t(x - 1)*(x + 1)**(13/2)), Abs(x + 1) > 2), (6*sqrt(1 - x)*(x + 1)**(15/2)*
asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*
(x + 1)**(13/2)) - 12*sqrt(1 - x)*(x + 1)**(13/2)*asin(sqrt(2)*sqrt(x + 1)/
2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) - 8*(x +
 1)**8/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) + 12
*(x + 1)**7/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2))
, True))
```

Giac [A]
time = 0.01, size = 62, normalized size = 1.51

$$\frac{2\left(\frac{4}{3}\sqrt{x+1}\sqrt{x+1}-2\right)\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^2}+2\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x)
```

```
[Out] 4/3*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 2*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(5/2), x)
```

3.1083 $\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx$

Optimal. Leaf size=20

$$\frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

```
[Out] 1/5*(1+x)^(5/2)/(1-x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

Mathematica [A]
time = 0.04, size = 20, normalized size = 1.00

$$\frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.62, size = 75, normalized size = 3.75

$$\text{Piecewise}\left[\left\{\left\{\frac{\left(-\frac{I}{5}\right)(1+x)^{\frac{5}{2}}}{\sqrt{-1+x}\,(1-2x+x^2)}, \text{Abs}[1+x]>2\right\}\right\}, \frac{(1+x)^{\frac{5}{2}}}{-20(1+x)\sqrt{1-x}+5(1+x)^2\sqrt{1-x}+20\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]')

[Out] Piecewise[{{(-I / 5) (1 + x) ^ (5 / 2) / (Sqrt[-1 + x] (1 - 2 x + x ^ 2)), Abs[1 + x] > 2}}, (1 + x) ^ (5 / 2) / (-20 (1 + x) Sqrt[1 - x] + 5 (1 + x) ^ 2 Sqrt[1 - x] + 20 Sqrt[1 - x])]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(14) = 28$.
time = 0.13, size = 57, normalized size = 2.85

method	result	size
gosper	$\frac{(1+x)^{\frac{5}{2}}}{5(1-x)^{\frac{5}{2}}}$	15
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^3+3x^2+3x+1)}{5\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54
default	$\frac{(1+x)^{\frac{3}{2}}}{(1-x)^{\frac{5}{2}}}-\frac{6\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}+\frac{\sqrt{1+x}}{5(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{5\sqrt{1-x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(3/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)

[Out] (1+x)^(3/2)/(1-x)^(5/2)-6/5*(1+x)^(1/2)/(1-x)^(5/2)+1/5*(1+x)^(1/2)/(1-x)^(3/2)+1/5*(1+x)^(1/2)/(1-x)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(14) = 28$.
time = 0.27, size = 94, normalized size = 4.70

$$\frac{(-x^2+1)^{\frac{3}{2}}}{x^4-4x^3+6x^2-4x+1}+\frac{6\sqrt{-x^2+1}}{5(x^3-3x^2+3x-1)}+\frac{\sqrt{-x^2+1}}{5(x^2-2x+1)}-\frac{\sqrt{-x^2+1}}{5(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="maxima")

```
[Out] (-x^2 + 1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 6/5*sqrt(-x^2 + 1)/(x^3
- 3*x^2 + 3*x - 1) + 1/5*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/5*sqrt(-x^2 + 1
)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(14) = 28$.
time = 0.30, size = 52, normalized size = 2.60

$$\frac{x^3 - 3\,x^2 - (x^2 + 2\,x + 1)\sqrt{x+1}\,\sqrt{-x+1} + 3\,x - 1}{5\,(x^3 - 3\,x^2 + 3\,x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="fricas")
[Out] 1/5*(x^3 - 3*x^2 - (x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*x - 1)/(x^3
 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.73, size = 87, normalized size = 4.35

$$\begin{cases} -\frac{i(x+1)^{\frac{5}{2}}}{5\sqrt{x-1}\,(x+1)^2 - 20\sqrt{x-1}\,(x+1) + 20\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \frac{(x+1)^{\frac{5}{2}}}{5\sqrt{1-x}\,(x+1)^2 - 20\sqrt{1-x}\,(x+1) + 20\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(7/2),x)
[Out] Piecewise((-I*(x + 1)**(5/2)/(5*sqrt(x - 1)*(x + 1)**2 - 20*sqrt(x - 1)*(x
+ 1) + 20*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**(5/2)/(5*sqrt(1 - x)*(x
+ 1)**2 - 20*sqrt(1 - x)*(x + 1) + 20*sqrt(1 - x)), True))
```

Giac [A]
time = 0.01, size = 53, normalized size = 2.65

$$\frac{\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{5\,(-x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x)
[Out] -1/5*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B]
time = 0.25, size = 50, normalized size = 2.50

$$-\frac{\sqrt{1-x}\,\left(\frac{2\,x\,\sqrt{x+1}}{5} + \frac{\sqrt{x+1}}{5} + \frac{x^2\,\sqrt{x+1}}{5}\right)}{x^3 - 3\,x^2 + 3\,x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((2*x*(x + 1)^(1/2))/5 + (x + 1)^(1/2)/5 + (x^2*(x + 1)^(1/
2))/5))/(3*x - 3*x^2 + x^3 - 1)
```

3.1084 $\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=41

$$\frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}}$$

```
[Out] 1/7*(1+x)^(5/2)/(1-x)^(7/2)+1/35*(1+x)^(5/2)/(1-x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(9/2),x]
[Out] (1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{1}{7}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.56

$$-\frac{(-6+x)(1+x)^{5/2}}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] -1/35*((-6 + x)*(1 + x)^(5/2))/(1 - x)^(7/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 14.75, size = 159, normalized size = 3.88

$$\text{Piecewise}\left[\left\{\left\{\frac{I(6-x)(1+x)^{\frac{5}{2}}}{35\sqrt{-1+x}\,(-1+3x-3x^2+x^3)},\text{Abs}[1+x]>2\right\}\right\},\frac{-7(1+x)^{\frac{5}{2}}}{-280\sqrt{1-x}-210(1+x)^2\sqrt{1-x}+35(1+x)^3\sqrt{1-x}+420(1+x)\sqrt{1-x}}+\frac{(1+x)^{\frac{7}{2}}}{-280\sqrt{1-x}-210(1+x)^2\sqrt{1-x}+35(1+x)^3\sqrt{1-x}+420(1+x)\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]')
```

```
[Out] Piecewise[{{I / 35 (6 - x) (1 + x) ^ (5 / 2) / (Sqrt[-1 + x] (-1 + 3 x - 3
x ^ 2 + x ^ 3)), Abs[1 + x] > 2}}, -7 (1 + x) ^ (5 / 2) / (-280 Sqrt[1 - x]
 - 210 (1 + x) ^ 2 Sqrt[1 - x] + 35 (1 + x) ^ 3 Sqrt[1 - x] + 420 (1 + x) S
qrt[1 - x]) + (1 + x) ^ (7 / 2) / (-280 Sqrt[1 - x] - 210 (1 + x) ^ 2 Sqrt[
1 - x] + 35 (1 + x) ^ 3 Sqrt[1 - x] + 420 (1 + x) Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(29) = 58$.
time = 0.14, size = 72, normalized size = 1.76

method	result	size
gosper	$-\frac{(1+x)^{\frac{5}{2}}(-6+x)}{35(1-x)^{\frac{7}{2}}}$	18
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^4-3x^3-15x^2-17x-6)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59
default	$\frac{(1+x)^{\frac{3}{2}}}{2(1-x)^{\frac{7}{2}}}-\frac{3\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70(1-x)^{\frac{5}{2}}}+\frac{\sqrt{1+x}}{35(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{35\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(3/2)/(1-x)^(7/2)-3/7*(1+x)^(1/2)/(1-x)^(7/2)+3/70*(1+x)^(1/2)/(1
-x)^(5/2)+1/35*(1+x)^(1/2)/(1-x)^(3/2)+1/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(29) = 58$.
time = 0.28, size = 131, normalized size = 3.20

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{2\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}-\frac{3\,\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{3\,\sqrt{-x^2+1}}{70\,(x^3-3\,x^2+3\,x-1)}+\frac{\sqrt{-x^2+1}}{35\,(x^2-2\,x+1)}-\frac{\sqrt{-x^2+1}}{35\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 3/7*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/70*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 1/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/35*sqrt(-x^2 + 1)/(x
 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 0.30, size = 69, normalized size = 1.68

$$\frac{6\,x^4-24\,x^3+36\,x^2-(x^3-4\,x^2-11\,x-6)\sqrt{x+1}\,\sqrt{-x+1}-24\,x+6}{35\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(6*x^4 - 24*x^3 + 36*x^2 - (x^3 - 4*x^2 - 11*x - 6)*sqrt(x + 1)*sqrt(-
x + 1) - 24*x + 6)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.37, size = 226, normalized size = 5.51

$$\begin{cases} -\frac{i(x+1)^{\frac{7}{2}}}{35\sqrt{x-1}\,(x+1)^3-210\sqrt{x-1}\,(x+1)^2+420\sqrt{x-1}\,(x+1)-280\sqrt{x-1}}+\frac{7i(x+1)^{\frac{5}{2}}}{35\sqrt{x-1}\,(x+1)^3-210\sqrt{x-1}\,(x+1)^2+420\sqrt{x-1}\,(x+1)-280\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{(x+1)^{\frac{7}{2}}}{35\sqrt{1-x}\,(x+1)^3-210\sqrt{1-x}\,(x+1)^2+420\sqrt{1-x}\,(x+1)-280\sqrt{1-x}}-\frac{7(x+1)^{\frac{5}{2}}}{35\sqrt{1-x}\,(x+1)^3-210\sqrt{1-x}\,(x+1)^2+420\sqrt{1-x}\,(x+1)-280\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(7/2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(
x + 1)**2 + 420*sqrt(x - 1)*(x + 1) - 280*sqrt(x - 1)) + 7*I*(x + 1)**(5/2)
/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(x + 1)**2 + 420*sqrt(x - 1)*
(x + 1) - 280*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**(7/2)/(35*sqrt(1 - x
)*(x + 1)**3 - 210*sqrt(1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*s
qrt(1 - x)) - 7*(x + 1)**(5/2)/(35*sqrt(1 - x)*(x + 1)**3 - 210*sqrt(1 - x)
*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*sqrt(1 - x)), True))
```

Giac [A]
time = 0.01, size = 73, normalized size = 1.78

$$\frac{2\left(\frac{1}{10}-\frac{1}{70}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x)
```

```
[Out] -1/35*(x + 1)^(5/2)*(x - 6)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.27, size = 64, normalized size = 1.56

$$\frac{\sqrt{1-x}\,\left(\frac{11\,x\,\sqrt{x+1}}{35}+\frac{6\,\sqrt{x+1}}{35}+\frac{4\,x^2\,\sqrt{x+1}}{35}-\frac{x^3\,\sqrt{x+1}}{35}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11*x*(x + 1)^(1/2))/35 + (6*(x + 1)^(1/2))/35 + (4*x^2*(x
+ 1)^(1/2))/35 - (x^3*(x + 1)^(1/2))/35))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

3.1085 $\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\, dx$

Optimal. Leaf size=61

$$\frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}$$

```
[Out] 1/9*(1+x)^(5/2)/(1-x)^(9/2)+2/63*(1+x)^(5/2)/(1-x)^(7/2)+2/315*(1+x)^(5/2)/
(1-x)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*(1 + x)^(5/2))/(63*(1 - x)^(7/2)) + (2
*(1 + x)^(5/2))/(315*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx \\
&= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2}{63}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx \\
&= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.49

$$\frac{(1+x)^{5/2}\left(47-14x+2x^2\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(5/2)*(47 - 14*x + 2*x^2))/(315*(1 - x)^(9/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 40.83, size = 410, normalized size = 6.72

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-47-80x-21x^2+10x^3-2x^4\right)\sqrt{1+x}}{315\sqrt{-1+x}\left(1-4x+6x^2-4x^3+x^4\right)},\text{Abs}[1+x]>2\right\}\right\},\frac{-126(1+x)^{\frac{5}{2}}}{-25200(1+x)^2\sqrt{1-x}-10080\sqrt{1-x}-3150(1+x)^4\sqrt{1-x}+315(1+x)^5\sqrt{1-x}+12600(1+x)^3\sqrt{1-x}+25200(1+x)\sqrt{1-x}}-\frac{22(1+x)^{\frac{9}{2}}}{-25200(1+x)^2\sqrt{1-x}-10080\sqrt{1-x}-3150(1+x)^4\sqrt{1-x}+315(1+x)^5\sqrt{1-x}+12600(1+x)^3\sqrt{1-x}+25200(1+x)\sqrt{1-x}}+\frac{2(1+x)^{\frac{11}{2}}}{-25200(1+x)^2\sqrt{1-x}-10080\sqrt{1-x}-3150(1+x)^4\sqrt{1-x}+315(1+x)^5\sqrt{1-x}+12600(1+x)^3\sqrt{1-x}+25200(1+x)\sqrt{1-x}}+\frac{99(1+x)^{\frac{7}{2}}}{-25200(1+x)^2\sqrt{1-x}-10080\sqrt{1-x}-3150(1+x)^4\sqrt{1-x}+315(1+x)^5\sqrt{1-x}+12600(1+x)^3\sqrt{1-x}+25200(1+x)\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]')
```

```
[Out] Piecewise[{{I / 315 (-47 - 80 x - 21 x ^ 2 + 10 x ^ 3 - 2 x ^ 4) Sqrt[1 + x
] / (Sqrt[-1 + x] (1 - 4 x + 6 x ^ 2 - 4 x ^ 3 + x ^ 4)), Abs[1 + x] > 2}},
 -126 (1 + x) ^ (5 / 2) / (-25200 (1 + x) ^ 2 Sqrt[1 - x] - 10080 Sqrt[1 -
x] - 3150 (1 + x) ^ 4 Sqrt[1 - x] + 315 (1 + x) ^ 5 Sqrt[1 - x] + 12600 (1
+ x) ^ 3 Sqrt[1 - x] + 25200 (1 + x) Sqrt[1 - x]) - 22 (1 + x) ^ (9 / 2) /
(-25200 (1 + x) ^ 2 Sqrt[1 - x] - 10080 Sqrt[1 - x] - 3150 (1 + x) ^ 4 Sqrt
[1 - x] + 315 (1 + x) ^ 5 Sqrt[1 - x] + 12600 (1 + x) ^ 3 Sqrt[1 - x] + 252
00 (1 + x) Sqrt[1 - x]) + 2 (1 + x) ^ (11 / 2) / (-25200 (1 + x) ^ 2 Sqrt[1
 - x] - 10080 Sqrt[1 - x] - 3150 (1 + x) ^ 4 Sqrt[1 - x] + 315 (1 + x) ^ 5
Sqrt[1 - x] + 12600 (1 + x) ^ 3 Sqrt[1 - x] + 25200 (1 + x) Sqrt[1 - x]) +
99 (1 + x) ^ (7 / 2) / (-25200 (1 + x) ^ 2 Sqrt[1 - x] - 10080 Sqrt[1 - x]
- 3150 (1 + x) ^ 4 Sqrt[1 - x] + 315 (1 + x) ^ 5 Sqrt[1 - x] + 12600 (1 + x
) ^ 3 Sqrt[1 - x] + 25200 (1 + x) Sqrt[1 - x])]
```

Maple [A]
time = 0.18, size = 86, normalized size = 1.41

method	result	size
gosper	$\frac{(1+x)^{\frac{5}{2}}(2x^2-14x+47)}{315(1-x)^{\frac{9}{2}}}$	25
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^5-8x^4+11x^3+101x^2+127x+47)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{(1+x)^{\frac{3}{2}}}{3(1-x)^{\frac{9}{2}}}-\frac{2\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}+\frac{\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}+\frac{\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{315\sqrt{1-x}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+x)^(3/2)/(1-x)^(9/2)-2/9*(1+x)^(1/2)/(1-x)^(9/2)+1/63*(1+x)^(1/2)/(1
-x)^(7/2)+1/105*(1+x)^(1/2)/(1-x)^(5/2)+2/315*(1+x)^(1/2)/(1-x)^(3/2)+2/315
*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(43) = 86$.
time = 0.28, size = 172, normalized size = 2.82

$$\frac{(-x^2+1)^{\frac{3}{2}}}{3(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}+\frac{2\sqrt{-x^2+1}}{9(x^5-5x^4+10x^3-10x^2+5x-1)}+\frac{\sqrt{-x^2+1}}{63(x^4-4x^3+6x^2-4x+1)}-\frac{\sqrt{-x^2+1}}{105(x^3-3x^2+3x-1)}+\frac{2\sqrt{-x^2+1}}{315(x^2-2x+1)}-\frac{2\sqrt{-x^2+1}}{315(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 2
/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/63*sqrt(-x^
2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^2
+ 3*x - 1) + 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/315*sqrt(-x^2 + 1)/(x
 - 1)
```

Fricas [A]
time = 0.30, size = 86, normalized size = 1.41

$$\frac{47x^5-235x^4+470x^3-470x^2-(2x^4-10x^3+21x^2+80x+47)\sqrt{x+1}\,\sqrt{-x+1}+235x-47}{315(x^5-5x^4+10x^3-10x^2+5x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(47*x^5 - 235*x^4 + 470*x^3 - 470*x^2 - (2*x^4 - 10*x^3 + 21*x^2 + 80
*x + 47)*sqrt(x + 1)*sqrt(-x + 1) + 235*x - 47)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 61.16, size = 675, normalized size = 11.07

$$\begin{cases} -\frac{2i(x+1)^{\frac{11}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}} + \frac{22i(x+1)^{\frac{9}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}} - \frac{99i(x+1)^{\frac{7}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}} + \frac{126i(x+1)^{\frac{5}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{2(x+1)^{\frac{11}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}} - \frac{22(x+1)^{\frac{9}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}} + \frac{99(x+1)^{\frac{7}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}} - \frac{126(x+1)^{\frac{5}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x -
 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**
2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 22*I*(x + 1)**(9/2)/(3
15*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)
*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10
080*sqrt(x - 1)) - 99*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*s
qrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x
 + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 126*I*(x + 1)**
(5/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqr
t(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x +
 1) - 10080*sqrt(x - 1)), Abs(x + 1) > 2), (2*(x + 1)**(11/2)/(315*sqrt(1 -
 x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3
 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1
- x)) - 22*(x + 1)**(9/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x
 + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25
200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)) + 99*(x + 1)**(7/2)/(315*sqrt(
1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)
**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt
(1 - x)) - 126*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x
)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2
+ 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)), True))
```

Giac [A]
time = 0.02, size = 91, normalized size = 1.49

$$\frac{2\left(\left(\frac{1}{315}\sqrt{x+1}\sqrt{x+1}-\frac{1}{35}\right)\sqrt{x+1}\sqrt{x+1}+\frac{1}{10}\right)\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x)
```

```
[Out] -1/315*(2*(x + 1)*(x - 8) + 63)*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.32, size = 80, normalized size = 1.31

$$-\frac{\sqrt{1-x}\,\left(\frac{16\,x\,\sqrt{x+1}}{63}+\frac{47\,\sqrt{x+1}}{315}+\frac{x^2\,\sqrt{x+1}}{15}-\frac{2\,x^3\,\sqrt{x+1}}{63}+\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((16*x*(x + 1)^(1/2))/63 + (47*(x + 1)^(1/2))/315 + (x^2*(x
 + 1)^(1/2))/15 - (2*x^3*(x + 1)^(1/2))/63 + (2*x^4*(x + 1)^(1/2))/315))/(5
*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

3.1086 $\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx$

Optimal. Leaf size=81

$$\frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}$$

```
[Out] 1/11*(1+x)^(5/2)/(1-x)^(11/2)+1/33*(1+x)^(5/2)/(1-x)^(9/2)+2/231*(1+x)^(5/2
)/(1-x)^(7/2)+2/1155*(1+x)^(5/2)/(1-x)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (1 + x)^(5/2)/(33*(1 - x)^(9/2)) + (2*(
1 + x)^(5/2))/(231*(1 - x)^(7/2)) + (2*(1 + x)^(5/2))/(1155*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2}{33}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2}{231}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 0.43

$$\frac{(1+x)^{5/2}\,(152-61x+16x^2-2x^3)}{1155(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(5/2)*(152 - 61*x + 16*x^2 - 2*x^3))/(1155*(1 - x)^(11/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 107.60, size = 975, normalized size = 12.04

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]')
```

```
[Out] Piecewise[{{I / 1155 (152 + 243 x + 46 x ^ 2 - 31 x ^ 3 + 12 x ^ 4 - 2 x ^
5) Sqrt[1 + x] / (Sqrt[-1 + x] (-1 + 5 x - 10 x ^ 2 + 10 x ^ 3 - 5 x ^ 4 +
x ^ 5)), Abs[1 + x] > 2}}, -3564 (1 + x) ^ (7 / 2) / (-2069760 (1 + x) ^ 3
Sqrt[1 - x] - 1182720 (1 + x) Sqrt[1 - x] - 517440 (1 + x) ^ 5 Sqrt[1 - x]
- 18480 (1 + x) ^ 7 Sqrt[1 - x] + 1155 (1 + x) ^ 8 Sqrt[1 - x] + 129360 (1
+ x) ^ 6 Sqrt[1 - x] + 295680 Sqrt[1 - x] + 1293600 (1 + x) ^ 4 Sqrt[1 - x]
 + 2069760 (1 + x) ^ 2 Sqrt[1 - x]) - 1105 (1 + x) ^ (11 / 2) / (-2069760 (
1 + x) ^ 3 Sqrt[1 - x] - 1182720 (1 + x) Sqrt[1 - x] - 517440 (1 + x) ^ 5 S
qrt[1 - x] - 18480 (1 + x) ^ 7 Sqrt[1 - x] + 1155 (1 + x) ^ 8 Sqrt[1 - x] +
 129360 (1 + x) ^ 6 Sqrt[1 - x] + 295680 Sqrt[1 - x] + 1293600 (1 + x) ^ 4
Sqrt[1 - x] + 2069760 (1 + x) ^ 2 Sqrt[1 - x]) - 34 (1 + x) ^ (15 / 2) / (-
2069760 (1 + x) ^ 3 Sqrt[1 - x] - 1182720 (1 + x) Sqrt[1 - x] - 517440 (1 +
```

```
 x) ^ 5 Sqrt[1 - x] - 18480 (1 + x) ^ 7 Sqrt[1 - x] + 1155 (1 + x) ^ 8 Sqrt
[1 - x] + 129360 (1 + x) ^ 6 Sqrt[1 - x] + 295680 Sqrt[1 - x] + 1293600 (1
+ x) ^ 4 Sqrt[1 - x] + 2069760 (1 + x) ^ 2 Sqrt[1 - x]) + 2 (1 + x) ^ (17 /
 2) / (-2069760 (1 + x) ^ 3 Sqrt[1 - x] - 1182720 (1 + x) Sqrt[1 - x] - 517
440 (1 + x) ^ 5 Sqrt[1 - x] - 18480 (1 + x) ^ 7 Sqrt[1 - x] + 1155 (1 + x)
^ 8 Sqrt[1 - x] + 129360 (1 + x) ^ 6 Sqrt[1 - x] + 295680 Sqrt[1 - x] + 129
3600 (1 + x) ^ 4 Sqrt[1 - x] + 2069760 (1 + x) ^ 2 Sqrt[1 - x]) + 255 (1 +
x) ^ (13 / 2) / (-2069760 (1 + x) ^ 3 Sqrt[1 - x] - 1182720 (1 + x) Sqrt[1
- x] - 517440 (1 + x) ^ 5 Sqrt[1 - x] - 18480 (1 + x) ^ 7 Sqrt[1 - x] + 115
5 (1 + x) ^ 8 Sqrt[1 - x] + 129360 (1 + x) ^ 6 Sqrt[1 - x] + 295680 Sqrt[1
- x] + 1293600 (1 + x) ^ 4 Sqrt[1 - x] + 2069760 (1 + x) ^ 2 Sqrt[1 - x]) +
 1848 (1 + x) ^ (5 / 2) / (-2069760 (1 + x) ^ 3 Sqrt[1 - x] - 1182720 (1 +
x) Sqrt[1 - x] - 517440 (1 + x) ^ 5 Sqrt[1 - x] - 18480 (1 + x) ^ 7 Sqrt[1
- x] + 1155 (1 + x) ^ 8 Sqrt[1 - x] + 129360 (1 + x) ^ 6 Sqrt[1 - x] + 2956
80 Sqrt[1 - x] + 1293600 (1 + x) ^ 4 Sqrt[1 - x] + 2069760 (1 + x) ^ 2 Sqrt
[1 - x]) + 2750 (1 + x) ^ (9 / 2) / (-2069760 (1 + x) ^ 3 Sqrt[1 - x] - 118
2720 (1 + x) Sqrt[1 - x] - 517440 (1 + x) ^ 5 Sqrt[1 - x] - 18480 (1 + x) ^
 7 Sqrt[1 - x] + 1155 (1 + x) ^ 8 Sqrt[1 - x] + 129360 (1 + x) ^ 6 Sqrt[1 -
 x] + 295680 Sqrt[1 - x] + 1293600 (1 + x) ^ 4 Sqrt[1 - x] + 2069760 (1 + x
) ^ 2 Sqrt[1 - x])]
```

Maple [A]
time = 0.16, size = 100, normalized size = 1.23

method	result	size
gosper	$-\frac{(1+x)^{\frac{5}{2}} (2x^3-16x^2+61x-152)}{1155(1-x)^{\frac{11}{2}}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^6-10x^5+19x^4-15x^3-289x^2-395x-152)}{1155\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^5\sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{(1+x)^{\frac{3}{2}}}{4(1-x)^{\frac{11}{2}}}-\frac{3\sqrt{1+x}}{22(1-x)^{\frac{11}{2}}}+\frac{\sqrt{1+x}}{132(1-x)^{\frac{9}{2}}}+\frac{\sqrt{1+x}}{231(1-x)^{\frac{7}{2}}}+\frac{\sqrt{1+x}}{385(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{1155(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{1155\sqrt{1-x}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1+x)^(3/2)/(1-x)^(11/2)-3/22*(1+x)^(1/2)/(1-x)^(11/2)+1/132*(1+x)^(1/2
)/(1-x)^(9/2)+1/231*(1+x)^(1/2)/(1-x)^(7/2)+1/385*(1+x)^(1/2)/(1-x)^(5/2)+2
/1155*(1+x)^(1/2)/(1-x)^(3/2)+2/1155*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(57) = 114$.
time = 0.27, size = 218, normalized size = 2.69

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{4\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}-\frac{3\,\sqrt{-x^2+1}}{22\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}-\frac{\sqrt{-x^2+1}}{132\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{\sqrt{-x^2+1}}{231\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{\sqrt{-x^2+1}}{385\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{1155\,(x^2-2\,x+1)}-\frac{2\,\sqrt{-x^2+1}}{1155\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 3/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
+ 1) - 1/132*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/2
31*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/385*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/1155*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/1155*sqrt
(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 101, normalized size = 1.25

$$\frac{152\,x^6-912\,x^5+2280\,x^4-3040\,x^3+2280\,x^2-\left(2\,x^5-12\,x^4+31\,x^3-46\,x^2-243\,x-152\right)\sqrt{x+1}\,\sqrt{-x+1}\,-912\,x+152}{1155\,\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(152*x^6 - 912*x^5 + 2280*x^4 - 3040*x^3 + 2280*x^2 - (2*x^5 - 12*x^
4 + 31*x^3 - 46*x^2 - 243*x - 152)*sqrt(x + 1)*sqrt(-x + 1) - 912*x + 152)/
(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 177.97, size = 1751, normalized size = 21.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(17/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x
 - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x +
1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2
069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x
 - 1)) + 34*I*(x + 1)**(15/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)
**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206
9760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
 1)) - 255*I*(x + 1)**(13/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) + 1105*I*(x + 1)**(11/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
```

```
1)) - 2750*I*(x + 1)**(9/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1
)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**
5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 20697
60*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1
)) + 3564*I*(x + 1)**(7/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)
*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206976
0*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1)
) - 1848*I*(x + 1)**(5/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)*
(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
+ 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069760
*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1))
, Abs(x + 1) > 2), (2*(x + 1)**(17/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*
sqrt(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)
*(x + 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)*
*3 + 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*
sqrt(1 - x)) - 34*(x + 1)**(15/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt
(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x
+ 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 +
 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt
(1 - x)) + 255*(x + 1)**(13/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1
- x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1
)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 20
69760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1
- x)) - 1105*(x + 1)**(11/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 -
x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)*
*5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069
760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 -
x)) + 2750*(x + 1)**(9/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*
(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5
+ 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760
*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x))
 - 3564*(x + 1)**(7/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(x
+ 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 + 1
293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*sq
rt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x)) +
1848*(x + 1)**(5/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(x + 1
)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 + 1293
600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*sqrt(
1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x)), True
))
```

Giac [A]

time = 0.02, size = 111, normalized size = 1.37

$$\frac{2\left(\left(\left(\left(\frac{1}{105}-\frac{1}{1155}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}-\frac{3}{70}\right)\sqrt{x+1}\,\sqrt{x+1}+\frac{1}{10}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x)
```

```
[Out] -1/1155*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1)^(5/2)*sqrt(-x + 1
)/(x - 1)^6
```

Mupad [B]
time = 0.31, size = 94, normalized size = 1.16

$$\frac{\sqrt{1-x}\,\left(\frac{81\,x\,\sqrt{x+1}}{385}+\frac{152\,\sqrt{x+1}}{1155}+\frac{46\,x^2\,\sqrt{x+1}}{1155}-\frac{31\,x^3\,\sqrt{x+1}}{1155}+\frac{4\,x^4\,\sqrt{x+1}}{385}-\frac{2\,x^5\,\sqrt{x+1}}{1155}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((81*x*(x + 1)^(1/2))/385 + (152*(x + 1)^(1/2))/1155 + (46*x
^2*(x + 1)^(1/2))/1155 - (31*x^3*(x + 1)^(1/2))/1155 + (4*x^4*(x + 1)^(1/2)
)/385 - (2*x^5*(x + 1)^(1/2))/1155))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^
5 + x^6 + 1)
```

3.1087 $\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4(1+x)^{5/2}}{143(1-x)^{11/2}}+\frac{4(1+x)^{5/2}}{429(1-x)^{9/2}}+\frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}}+\frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}$$

```
[Out] 1/13*(1+x)^(5/2)/(1-x)^(13/2)+4/143*(1+x)^(5/2)/(1-x)^(11/2)+4/429*(1+x)^(5
/2)/(1-x)^(9/2)+8/3003*(1+x)^(5/2)/(1-x)^(7/2)+8/15015*(1+x)^(5/2)/(1-x)^(5
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*(1 + x)^(5/2))/(143*(1 - x)^(11/2))
+ (4*(1 + x)^(5/2))/(429*(1 - x)^(9/2)) + (8*(1 + x)^(5/2))/(3003*(1 - x)^(
7/2)) + (8*(1 + x)^(5/2))/(15015*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4}{13}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx \\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{12}{143}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx \\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8}{429}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx \\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx}{3003} \\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.40

$$\frac{(1+x)^{5/2}\,(1763-852x+308x^2-72x^3+8x^4)}{15015(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(5/2)*(1763 - 852*x + 308*x^2 - 72*x^3 + 8*x^4))/(15015*(1 - x)^(1
3/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Maple [A]
time = 0.14, size = 114, normalized size = 1.13

method	result
gosper	$\frac{(1+x)^{\frac{5}{2}}\,(8x^4-72x^3+308x^2-852x+1763)}{15015(1-x)^{\frac{13}{2}}}$

risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^7-48x^6+116x^5-136x^4+59x^3+3041x^2+4437x+1763)}{15015\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^6\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{3}{2}}}{5(1-x)^{\frac{13}{2}}}-\frac{6\sqrt{1+x}}{65(1-x)^{\frac{13}{2}}}+\frac{3\sqrt{1+x}}{715(1-x)^{\frac{11}{2}}}+\frac{\sqrt{1+x}}{429(1-x)^{\frac{9}{2}}}+\frac{4\sqrt{1+x}}{3003(1-x)^{\frac{7}{2}}}+\frac{4\sqrt{1+x}}{5005(1-x)^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{15015(1-x)^{\frac{3}{2}}}+\frac{8\sqrt{1+}}{15015\sqrt{1-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1+x)^(3/2)/(1-x)^(13/2)-6/65*(1+x)^(1/2)/(1-x)^(13/2)+3/715*(1+x)^(1/2
)/(1-x)^(11/2)+1/429*(1+x)^(1/2)/(1-x)^(9/2)+4/3003*(1+x)^(1/2)/(1-x)^(7/2)
+4/5005*(1+x)^(1/2)/(1-x)^(5/2)+8/15015*(1+x)^(1/2)/(1-x)^(3/2)+8/15015*(1+
x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(71) = 142$.
time = 0.28, size = 269, normalized size = 2.66

$$\frac{(-x^2+1)^{\frac{3}{2}}}{5\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}+\frac{6\,\sqrt{-x^2+1}}{65\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}+\frac{3\,\sqrt{-x^2+1}}{715\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}-\frac{\sqrt{-x^2+1}}{429\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{4\,\sqrt{-x^2+1}}{3003\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{4\,\sqrt{-x^2+1}}{5005\,(x^3-3\,x^2+3\,x-1)}+\frac{8\,\sqrt{-x^2+1}}{15015\,(x^2-2\,x+1)}-\frac{8\,\sqrt{-x^2+1}}{15015\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 6/65*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^
3 - 21*x^2 + 7*x - 1) + 3/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3
 + 15*x^2 - 6*x + 1) - 1/429*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
+ 5*x - 1) + 4/3003*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/5005
*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 8/15015*sqrt(-x^2 + 1)/(x^2 - 2*x
 + 1) - 8/15015*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.31, size = 116, normalized size = 1.15

$$\frac{1763\,x^7-12341\,x^6+37023\,x^5-61705\,x^4+61705\,x^3-37023\,x^2-(8\,x^6-56\,x^5+172\,x^4-308\,x^3+367\,x^2+2674\,x+1763)\sqrt{x+1}\,\sqrt{-x+1}+12341\,x-1763}{15015\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1763*x^7 - 12341*x^6 + 37023*x^5 - 61705*x^4 + 61705*x^3 - 37023*x
^2 - (8*x^6 - 56*x^5 + 172*x^4 - 308*x^3 + 367*x^2 + 2674*x + 1763)*sqrt(x
+ 1)*sqrt(-x + 1) + 12341*x - 1763)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3
 - 21*x^2 + 7*x - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(15/2),x)
```

```
[Out] Exception raised: SystemError
```

Giac [A]
time = 0.03, size = 133, normalized size = 1.32

$$\frac{2\left(\left(\left(\left(\frac{4}{15015}\sqrt{x+1}\sqrt{x+1}-\frac{4}{1155}\right)\sqrt{x+1}\sqrt{x+1}+\frac{2}{105}\right)\sqrt{x+1}\sqrt{x+1}-\frac{2}{35}\right)\sqrt{x+1}\sqrt{x+1}+\frac{1}{10}\right)\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x)
```

```
[Out] -1/15015*(4*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1) + 3003)*(x +
 1)^(5/2)*sqrt(-x + 1)/(x - 1)^7
```

Mupad [B]
time = 0.33, size = 110, normalized size = 1.09

$$-\frac{\sqrt{1-x}\,\left(\frac{382\,x\,\sqrt{x+1}}{2145}+\frac{1763\,\sqrt{x+1}}{15015}+\frac{367\,x^2\,\sqrt{x+1}}{15015}-\frac{4\,x^3\,\sqrt{x+1}}{195}+\frac{172\,x^4\,\sqrt{x+1}}{15015}-\frac{8\,x^5\,\sqrt{x+1}}{2145}+\frac{8\,x^6\,\sqrt{x+1}}{15015}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((382*x*(x + 1)^(1/2))/2145 + (1763*(x + 1)^(1/2))/15015 +
(367*x^2*(x + 1)^(1/2))/15015 - (4*x^3*(x + 1)^(1/2))/195 + (172*x^4*(x + 1
)^(1/2))/15015 - (8*x^5*(x + 1)^(1/2))/2145 + (8*x^6*(x + 1)^(1/2))/15015))
/(7*x - 21*x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

3.1088 $\int (1-x)^{11/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2}+\frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+\frac{11}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{11}{72}(1-x)^{9/2}($$

```
[Out] 55/192*(1-x)^(3/2)*x*(1+x)^(3/2)+11/48*(1-x)^(5/2)*x*(1+x)^(5/2)+11/56*(1-x
)^(7/2)*(1+x)^(7/2)+11/72*(1-x)^(9/2)*(1+x)^(7/2)+1/9*(1-x)^(11/2)*(1+x)^(7
/2)+55/128*arcsin(x)+55/128*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}+\frac{55}{128}x\sqrt{x+1}\sqrt{1-x}+\frac{55}{128}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (55*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (55*(1 - x)^(3/2)*x*(1 + x)^(3/2))/192
 + (11*(1 - x)^(5/2)*x*(1 + x)^(5/2))/48 + (11*(1 - x)^(7/2)*(1 + x)^(7/2))
/56 + (11*(1 - x)^(9/2)*(1 + x)^(7/2))/72 + ((1 - x)^(11/2)*(1 + x)^(7/2))/
9 + (55*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{11/2}(1+x)^{5/2}\,dx = \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{9}\int (1-x)^{9/2}(1+x)^{5/2}\,dx$$
$$= \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx$$
$$= \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}$$
$$= \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}$$
$$= \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} +$$
$$= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} +$$
$$= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} +$$
$$= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} +$$

Mathematica [A]
time = 0.17, size = 88, normalized size = 0.68

$$\frac{\sqrt{1-x}\,(3712+8311x-5641x^2-7174x^3+11514x^4+1224x^5-8248x^6+2000x^7+2128x^8-896x^9)}{8064\sqrt{1+x}} - \frac{55}{64}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(3712 + 8311*x - 5641*x^2 - 7174*x^3 + 11514*x^4 + 1224*x^5 -
8248*x^6 + 2000*x^7 + 2128*x^8 - 896*x^9))/(8064*Sqrt[1 + x]) - (55*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]])/64
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]')
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 4060 deep

Maple [A]
time = 0.14, size = 155, normalized size = 1.19

method	result
risch	$\frac{(896x^8-3024x^7+1024x^6+7224x^5-8448x^4-3066x^3+10240x^2-4599x-3712)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{8064\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{55\sqrt{}}{}$
default	$\frac{(1-x)^{\frac{11}{2}}(1+x)^{\frac{7}{2}}}{9}+\frac{11(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{72}+\frac{11(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56}+\frac{11(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{48}+\frac{11(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{48}+\frac{11\sqrt{1-x}\,(1+}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(11/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(1-x)^(11/2)*(1+x)^(7/2)+11/72*(1-x)^(9/2)*(1+x)^(7/2)+11/56*(1-x)^(7/2
)*(1+x)^(7/2)+11/48*(1-x)^(5/2)*(1+x)^(7/2)+11/48*(1-x)^(3/2)*(1+x)^(7/2)+1
1/64*(1-x)^(1/2)*(1+x)^(7/2)-11/192*(1-x)^(1/2)*(1+x)^(5/2)-55/384*(1-x)^(1
/2)*(1+x)^(3/2)-55/128*(1-x)^(1/2)*(1+x)^(1/2)+55/128*((1+x)*(1-x))^(1/2)/(
1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 78, normalized size = 0.60

$$\frac{1}{9}\left(-x^2+1\right)^{\frac{7}{2}}x^2-\frac{3}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{29}{63}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{11}{48}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{55}{192}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{55}{128}\sqrt{-x^2+1}\,x+\frac{55}{128}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(-x^2 + 1)^(7/2)*x^2 - 3/8*(-x^2 + 1)^(7/2)*x + 29/63*(-x^2 + 1)^(7/2)
+ 11/48*(-x^2 + 1)^(5/2)*x + 55/192*(-x^2 + 1)^(3/2)*x + 55/128*sqrt(-x^2 +
 1)*x + 55/128*arcsin(x)
```

Fricas [A]
time = 0.29, size = 77, normalized size = 0.59

$$-\frac{1}{8064}\left(896\,x^8-3024\,x^7+1024\,x^6+7224\,x^5-8448\,x^4-3066\,x^3+10240\,x^2-4599\,x-3712\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{55}{64}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*x^8 - 3024*x^7 + 1024*x^6 + 7224*x^5 - 8448*x^4 - 3066*x^3 + 1
0240*x^2 - 4599*x - 3712)*sqrt(x + 1)*sqrt(-x + 1) - 55/64*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(11/2)*(1+x)**(5/2),x)
```

```
[Out] Exception raised: SystemError
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(92) = 184.
time = 0.06, size = 1174, normalized size = 9.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x)
```

```
[Out] -1/40320*((2*((4*(5*(2*(7*(8*x + 65)*(x - 1) + 2073)*(x - 1) + 9833)*(x - 1
) + 75293)*(x - 1) + 310203)*(x - 1) + 216993)*(x - 1) + 205275)*(x - 1) +
69615)*sqrt(x + 1)*sqrt(-x + 1) + 1/3360*((2*((4*(5*(6*(7*x + 50)*(x - 1) +
 1219)*(x - 1) + 12463)*(x - 1) + 64233)*(x - 1) + 53963)*(x - 1) + 59465)*
(x - 1) + 23205)*sqrt(x + 1)*sqrt(-x + 1) - 1/420*((2*((4*(5*(6*x + 37)*(x
- 1) + 661)*(x - 1) + 4551)*(x - 1) + 4781)*(x - 1) + 6335)*(x - 1) + 2835)
*sqrt(x + 1)*sqrt(-x + 1) - 1/60*((2*((4*(5*x + 26)*(x - 1) + 321)*(x - 1)
+ 451)*(x - 1) + 745)*(x - 1) + 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3
*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295)*(x - 1) + 195)*sqrt(x + 1)*sqrt(-
x + 1) - 1/6*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x
 + 1) - 2/3*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 2*(x + 2)*sq
rt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - 55/64*arcsin(1/2*sqrt(2
)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{11/2} (x+1)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(11/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(11/2)*(x + 1)^(5/2), x)
```

3.1089 $\int (1-x)^{9/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=110

$$\frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1-$$

```
[Out] 15/64*(1-x)^(3/2)*x*(1+x)^(3/2)+3/16*(1-x)^(5/2)*x*(1+x)^(5/2)+9/56*(1-x)^(
7/2)*(1+x)^(7/2)+1/8*(1-x)^(9/2)*(1+x)^(7/2)+45/128*arcsin(x)+45/128*x*(1-x
)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\sqrt{1-x}+\frac{45}{128}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (45*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (15*(1 - x)^(3/2)*x*(1 + x)^(3/2))/64
+ (3*(1 - x)^(5/2)*x*(1 + x)^(5/2))/16 + (9*(1 - x)^(7/2)*(1 + x)^(7/2))/56
 + ((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (45*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{9/2}(1+x)^{5/2}\,dx &= \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{9}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx\\ &= \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{9}{8}\int (1-x)^{5/2}(1+x)^{5/2}\,dx\\ &= \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{15}{16}\\ &= \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\\ &= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\\ &= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\\ &= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\end{aligned}$$

Mathematica [A]
time = 0.14, size = 82, normalized size = 0.75

$$\frac{1}{896}\left(\frac{\sqrt{1-x}\,(256+837x-187x^2-978x^3+558x^4+600x^5-424x^6-144x^7+112x^8)}{\sqrt{1+x}}-630\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]
[Out] ((Sqrt[1 - x]*(256 + 837*x - 187*x^2 - 978*x^3 + 558*x^4 + 600*x^5 - 424*x^
6 - 144*x^7 + 112*x^8))/Sqrt[1 + x] - 630*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/
896
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]')
[Out] Timed out
```

Maple [A]
time = 0.14, size = 141, normalized size = 1.28

method	result
risch	$-\frac{(112x^7-256x^6-168x^5+768x^4-210x^3-768x^2+581x+256)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{896\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{45\sqrt{(1+x)(1}}{128\sqrt{1+x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{8}+\frac{9(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56}+\frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{16}+\frac{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{16}+\frac{9\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{64}-\frac{3\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(1-x)^(9/2)*(1+x)^(7/2)+9/56*(1-x)^(7/2)*(1+x)^(7/2)+3/16*(1-x)^(5/2)*(
1+x)^(7/2)+3/16*(1-x)^(3/2)*(1+x)^(7/2)+9/64*(1-x)^(1/2)*(1+x)^(7/2)-3/64*(
1-x)^(1/2)*(1+x)^(5/2)-15/128*(1-x)^(1/2)*(1+x)^(3/2)-45/128*(1-x)^(1/2)*(1
+x)^(1/2)+45/128*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 64, normalized size = 0.58

$$-\frac{1}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{2}{7}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{3}{16}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{15}{64}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{45}{128}\sqrt{-x^2+1}\,x+\frac{45}{128}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^2 + 1)^(7/2)*x + 2/7*(-x^2 + 1)^(7/2) + 3/16*(-x^2 + 1)^(5/2)*x +
15/64*(-x^2 + 1)^(3/2)*x + 45/128*sqrt(-x^2 + 1)*x + 45/128*arcsin(x)
```

Fricas [A]
time = 0.30, size = 72, normalized size = 0.65

$$\frac{1}{896}\left(112\,x^7-256\,x^6-168\,x^5+768\,x^4-210\,x^3-768\,x^2+581\,x+256\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{45}{64}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/896*(112*x^7 - 256*x^6 - 168*x^5 + 768*x^4 - 210*x^3 - 768*x^2 + 581*x +
256)*sqrt(x + 1)*sqrt(-x + 1) - 45/64*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(78) = 156.
time = 0.05, size = 954, normalized size = 8.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x)
```

```
[Out] 1/13440*((2*((4*(5*(6*(7*x + 50)*(x - 1) + 1219)*(x - 1) + 12463)*(x - 1) +
 64233)*(x - 1) + 53963)*(x - 1) + 59465)*(x - 1) + 23205)*sqrt(x + 1)*sqrt
(-x + 1) - 1/560*((2*((4*(5*(6*x + 37)*(x - 1) + 661)*(x - 1) + 4551)*(x -
1) + 4781)*(x - 1) + 6335)*(x - 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) + 1/240
*((2*((4*(5*x + 26)*(x - 1) + 321)*(x - 1) + 451)*(x - 1) + 745)*(x - 1) +
405)*sqrt(x + 1)*sqrt(-x + 1) + 1/24*((2*(3*(4*x + 17)*(x - 1) + 133)*(x -
1) + 295)*(x - 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 5/24*((2*(3*x + 10)*(x
- 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/6*((2*x + 5)*(x - 1)
+ 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt
(x + 1)*sqrt(-x + 1) - 45/64*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2} (x+1)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(5/2), x)
```

3.1090 $\int (1-x)^{7/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=90

$$\frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\frac{5}{16}\sin^{-1}(x)$$

```
[Out] 5/24*(1-x)^(3/2)*x*(1+x)^(3/2)+1/6*(1-x)^(5/2)*x*(1+x)^(5/2)+1/7*(1-x)^(7/2
)*(1+x)^(7/2)+5/16*arcsin(x)+5/16*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + ((1 - x)^(7/2)*(1 + x)^(7/2))/7 + (5*Arc
Sin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{7/2}(1+x)^{5/2}\,dx &= \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\int (1-x)^{5/2}(1+x)^{5/2}\,dx\\
&= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\frac{5}{8}\int\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 78, normalized size = 0.87

$$\frac{\sqrt{1-x}\,\left(48+279x+87x^2-326x^3-38x^4+200x^5+8x^6-48x^7\right)}{336\sqrt{1+x}}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]
[Out] (Sqrt[1 - x]*(48 + 279*x + 87*x^2 - 326*x^3 - 38*x^4 + 200*x^5 + 8*x^6 - 48
*x^7))/(336*Sqrt[1 + x]) - (5*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 127, normalized size = 1.41

method	result
risch	$\frac{(48x^6-56x^5-144x^4+182x^3+144x^2-231x-48)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{336\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{7}+\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{8}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1-x)^(7/2)*(1+x)^(7/2)+1/6*(1-x)^(5/2)*(1+x)^(7/2)+1/6*(1-x)^(3/2)*(1+
x)^(7/2)+1/8*(1-x)^(1/2)*(1+x)^(7/2)-1/24*(1-x)^(1/2)*(1+x)^(5/2)-5/48*(1-x
)^(1/2)*(1+x)^(3/2)-5/16*(1-x)^(1/2)*(1+x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(
1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 52, normalized size = 0.58

$$\frac{1}{7}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}\,x+\frac{5}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(7/2) + 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5
/16*sqrt(-x^2 + 1)*x + 5/16*arcsin(x)
```

Fricas [A]
time = 0.30, size = 67, normalized size = 0.74

$$-\frac{1}{336}\left(48\,x^6-56\,x^5-144\,x^4+182\,x^3+144\,x^2-231\,x-48\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/336*(48*x^6 - 56*x^5 - 144*x^4 + 182*x^3 + 144*x^2 - 231*x - 48)*sqrt(x
+ 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(64) = 128.
time = 0.04, size = 756, normalized size = 8.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x)
```

```
[Out] -1/1680*((2*((4*(5*(6*x + 37)*(x - 1) + 661)*(x - 1) + 4551)*(x - 1) + 4781
)*(x - 1) + 6335)*(x - 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*((4*
(5*x + 26)*(x - 1) + 321)*(x - 1) + 451)*(x - 1) + 745)*(x - 1) + 405)*sqrt
(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295
)*(x - 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/6*((2*(3*x + 10)*(x - 1) + 43
)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/6*((2*x + 5)*(x - 1) + 9)*sqrt
(x + 1)*sqrt(-x + 1) + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(
-x + 1) - 5/8*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(5/2), x)
```

3.1091 $\int (1-x)^{5/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\sin^{-1}(x)$$

```
[Out] 5/24*(1-x)^(3/2)*x*(1+x)^(3/2)+1/6*(1-x)^(5/2)*x*(1+x)^(5/2)+5/16*arcsin(x)
+5/16*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 222}

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + (5*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{5/2}(1+x)^{5/2}\,dx &= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx \\
&= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{8}\int \sqrt{1-x}\,\sqrt{1+x}\,dx \\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16} \\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16} \\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.73

$$
\frac{1}{48}x\sqrt{1-x^2}\left(33-26x^2+8x^4\right)-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (x*Sqrt[1 - x^2]*(33 - 26*x^2 + 8*x^4))/48 - (5*ArcTan[Sqrt[1 - x^2]/(1 + x
)])/8
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 194.11, size = 196, normalized size = 2.80

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(-110(1+x)^{\frac{7}{2}}-56(1+x)^{\frac{11}{2}}-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-5(1+x)^{\frac{3}{2}}-(1+x)^{\frac{5}{2}}+8(1+x)^{\frac{13}{2}}+30\sqrt{1+x}+134(1+x)^{\frac{9}{2}}\right)}{48\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-67(1+x)^{\frac{9}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{1+x}}{8\sqrt{1-x}}-\frac{(1+x)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{(1+x)^{\frac{5}{2}}}{48\sqrt{1-x}}+\frac{5(1+x)^{\frac{3}{2}}}{48\sqrt{1-x}}+\frac{5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{8}+\frac{7(1+x)^{\frac{11}{2}}}{6\sqrt{1-x}}+\frac{55(1+x)^{\frac{7}{2}}}{24\sqrt{1-x}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 48 (-110 (1 + x) ^ (7 / 2) - 56 (1 + x) ^ (11 / 2) - 30 Arc
Cosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 5 (1 + x) ^ (3 / 2) - (1 + x)
^ (5 / 2) + 8 (1 + x) ^ (13 / 2) + 30 Sqrt[1 + x] + 134 (1 + x) ^ (9 / 2))
/ Sqrt[-1 + x], Abs[1 + x] > 2}}, -67 (1 + x) ^ (9 / 2) / (24 Sqrt[1 - x])
- 5 Sqrt[1 + x] / (8 Sqrt[1 - x]) - (1 + x) ^ (13 / 2) / (6 Sqrt[1 - x]) +
(1 + x) ^ (5 / 2) / (48 Sqrt[1 - x]) + 5 (1 + x) ^ (3 / 2) / (48 Sqrt[1 - x
]) + 5 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 8 + 7 (1 + x) ^ (11 / 2) / (6 Sqrt
[1 - x]) + 55 (1 + x) ^ (7 / 2) / (24 Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.
time = 0.15, size = 113, normalized size = 1.61

method	result
risch	$-\frac{x(8x^4-26x^2+33)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{48\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{8}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{5\sqrt{1-x}\,\sqrt{}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(5/2)*(1+x)^(7/2)+1/6*(1-x)^(3/2)*(1+x)^(7/2)+1/8*(1-x)^(1/2)*(1+
x)^(7/2)-1/24*(1-x)^(1/2)*(1+x)^(5/2)-5/48*(1-x)^(1/2)*(1+x)^(3/2)-5/16*(1-
x)^(1/2)*(1+x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Maxima [A]
time = 0.41, size = 41, normalized size = 0.59

$$\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}\,x+\frac{5}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5/16*sqrt(-x^2 + 1)*x +
5/16*arcsin(x)
```

Fricas [A]
time = 0.31, size = 51, normalized size = 0.73

$$\frac{1}{48}\left(8\,x^5-26\,x^3+33\,x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 26*x^3 + 33*x)*sqrt(x + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(50) = 100.

time = 0.03, size = 582, normalized size = 8.31

$$2\left(2\left(\left(\left(\left(\left(\frac{31}{120}-\frac{1}{24}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{107}{160}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{451}{480}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{149}{192}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{27}{64}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{5}{16}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-2\left(2\left(\left(\left(\left(\frac{1}{20}\sqrt{-x+1}\sqrt{-x+1}-\frac{21}{80}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{133}{240}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{59}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)-4\left(2\left(\left(\left(\frac{13}{48}-\frac{1}{16}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)+4\left(2\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)+2\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x)
```

```
[Out] 1/240*((2*((4*(5*x + 26)*(x - 1) + 321)*(x - 1) + 451)*(x - 1) + 745)*(x -
1) + 405)*sqrt(x + 1)*sqrt(-x + 1) - 1/120*((2*(3*(4*x + 17)*(x - 1) + 133)
*(x - 1) + 295)*(x - 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x + 1
0)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/3*((2*x + 5)*(x
 - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1)
- sqrt(x + 1)*sqrt(-x + 1) - 5/8*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(5/2), x)
```

3.1092 $\int (1-x)^{3/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] 1/4*(1-x)^(3/2)*x*(1+x)^(3/2)-1/5*(1-x)^(5/2)*(1+x)^(5/2)+3/8*arcsin(x)+3/8
*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {51, 38, 41, 222}

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 - ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{3/2}(1+x)^{5/2}\,dx &= -\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{}{\sqrt{}}\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{}{\sqrt{}}\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 68, normalized size = 0.99

$$\frac{\sqrt{1-x}\,(-8+17x+41x^2+6x^3-18x^4-8x^5)}{40\sqrt{1+x}}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-8 + 17*x + 41*x^2 + 6*x^3 - 18*x^4 - 8*x^5))/(40*Sqrt[1 + x]
) - (3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 64.07, size = 175, normalized size = 2.54

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-46(1+x)^{\frac{7}{2}}-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-8(1+x)^{\frac{11}{2}}-5(1+x)^{\frac{3}{2}}-(1+x)^{\frac{5}{2}}+30\sqrt{1+x}+38(1+x)^{\frac{9}{2}}\right)}{40\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-19(1+x)^{\frac{9}{2}}}{20\sqrt{1-x}}-\frac{3\sqrt{1+x}}{4\sqrt{1-x}}+\frac{(1+x)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(1+x)^{\frac{3}{2}}}{8\sqrt{1-x}}+\frac{(1+x)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{3\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4}+\frac{23(1+x)^{\frac{7}{2}}}{20\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 40 (-46 (1 + x) ^ (7 / 2) - 30 ArcCosh[Sqrt[2] Sqrt[1 + x]
/ 2] Sqrt[-1 + x] - 8 (1 + x) ^ (11 / 2) - 5 (1 + x) ^ (3 / 2) - (1 + x) ^
(5 / 2) + 30 Sqrt[1 + x] + 38 (1 + x) ^ (9 / 2)) / Sqrt[-1 + x], Abs[1 + x]
 > 2}}, -19 (1 + x) ^ (9 / 2) / (20 Sqrt[1 - x]) - 3 Sqrt[1 + x] / (4 Sqrt[
1 - x]) + (1 + x) ^ (5 / 2) / (40 Sqrt[1 - x]) + (1 + x) ^ (3 / 2) / (8 Sqr
t[1 - x]) + (1 + x) ^ (11 / 2) / (5 Sqrt[1 - x]) + 3 ArcSin[Sqrt[2] Sqrt[1
+ x] / 2] / 4 + 23 (1 + x) ^ (7 / 2) / (20 Sqrt[1 - x])]
```

Maple [A]
time = 0.16, size = 99, normalized size = 1.43

method	result
risch	$\frac{(8x^4+10x^3-16x^2-25x+8)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{40\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{5}+\frac{3\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{20}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{20}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1-x)^(3/2)*(1+x)^(7/2)+3/20*(1-x)^(1/2)*(1+x)^(7/2)-1/20*(1-x)^(1/2)*(
1+x)^(5/2)-1/8*(1-x)^(1/2)*(1+x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+
x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 40, normalized size = 0.58

$$-\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8
*arcsin(x)
```

Fricas [A]
time = 0.29, size = 57, normalized size = 0.83

$$-\frac{1}{40}\left(8\,x^4+10\,x^3-16\,x^2-25\,x+8\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/40*(8*x^4 + 10*x^3 - 16*x^2 - 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*a
rctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 61.20, size = 245, normalized size = 3.55

$$\begin{cases}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{19i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1|>2\\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{19(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 19*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 23*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x
 - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (3*asin(sqrt(2)*
sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 19*(x + 1)**(9/2)/(20*
sqrt(1 - x)) + 23*(x + 1)**(7/2)/(20*sqrt(1 - x)) + (x + 1)**(5/2)/(40*sqrt
(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)),
True))
```

Giac [A]
time = 0.01, size = 255, normalized size = 3.70

$$-2\left(2\left(\left(\left(\left(\left(\frac{1}{20}\sqrt{-x+1}\sqrt{-x+1}-\frac{21}{80}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{133}{240}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{59}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)\right)+4\left(2\left(\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x)
```

```
[Out] -1/120*((2*(3*(4*x + 17)*(x - 1) + 133)*(x - 1) + 295)*(x - 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) + 1/3*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1)
- sqrt(x + 1)*sqrt(-x + 1) - 3/4*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{3/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(5/2), x)
```

3.1093 $\int \sqrt{1-x}\,(1+x)^{5/2}\,dx$

Optimal. Leaf size=68

$$\frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\sin^{-1}(x)$$

```
[Out] -5/12*(1-x)^(3/2)*(1+x)^(3/2)-1/4*(1-x)^(3/2)*(1+x)^(5/2)+5/8*arcsin(x)+5/8
*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 - (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 - ((1
- x)^(3/2)*(1 + x)^(5/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\,(1+x)^{5/2}\,dx &= -\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\,(1+x)^{3/2}\,dx\\ &= -\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{}{\sqrt{1}}\ldots\\ &= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{}{\sqrt{1}}\ldots\\ &= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\sin^{-1}(\ldots\end{aligned}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.93

$$\frac{\sqrt{1-x}\,\left(-16-7x+25x^2+22x^3+6x^4\right)}{24\sqrt{1+x}}-\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-16 - 7*x + 25*x^2 + 22*x^3 + 6*x^4))/(24*Sqrt[1 + x]) - (5*A
rcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 22.85, size = 154, normalized size = 2.26

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-14(1+x)^{\frac{7}{2}}-5(1+x)^{\frac{3}{2}}-(1+x)^{\frac{5}{2}}+6(1+x)^{\frac{9}{2}}+30\sqrt{1+x}\right)}{24\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-5\sqrt{1+x}}{4\sqrt{1-x}}-\frac{(1+x)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{(1+x)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{5(1+x)^{\frac{3}{2}}}{24\sqrt{1-x}}+\frac{7(1+x)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/2)*(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 24 (-30 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 14
(1 + x) ^ (7 / 2) - 5 (1 + x) ^ (3 / 2) - (1 + x) ^ (5 / 2) + 6 (1 + x) ^ (
9 / 2) + 30 Sqrt[1 + x]) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -5 Sqrt[1 + x] /
 (4 Sqrt[1 - x]) - (1 + x) ^ (9 / 2) / (4 Sqrt[1 - x]) + (1 + x) ^ (5 / 2)
/ (24 Sqrt[1 - x]) + 5 (1 + x) ^ (3 / 2) / (24 Sqrt[1 - x]) + 7 (1 + x) ^ (
7 / 2) / (12 Sqrt[1 - x]) + 5 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] / 4]
```

Maple [A]
time = 0.14, size = 85, normalized size = 1.25

method	result
risch	$-\frac{(6x^3+16x^2+9x-16)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{12}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{24}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{5\sqrt{(1+x)(1-x)}\,a}{8\sqrt{1+x}\,\sqrt{1-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1-x)^(1/2)*(1+x)^(7/2)-1/12*(1-x)^(1/2)*(1+x)^(5/2)-5/24*(1-x)^(1/2)*(
1+x)^(3/2)-5/8*(1-x)^(1/2)*(1+x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.34, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}\,x+\frac{5}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6\,x^3+16\,x^2+9\,x-16\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 21.50, size = 212, normalized size = 3.12

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{7i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{7(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 7*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - I*(x + 1)**(5/2)/(24*sqrt(x
- 1)) - 5*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqrt(x - 1
)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/2)/(4*s
qrt(1 - x)) + 7*(x + 1)**(7/2)/(12*sqrt(1 - x)) + (x + 1)**(5/2)/(24*sqrt(1
 - x)) + 5*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4*sqrt(1 - x)),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(48) = 96$.
time = 0.02, size = 298, normalized size = 4.38

$$2\left(2\left(\left(\left(\left(\frac{13}{48}-\frac{1}{16}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{43}{96}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{13}{32}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{3}{8}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)\right)+2\left(2\left(\left(\left(\frac{1}{12}\sqrt{-x+1}\sqrt{-x+1}-\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{3}{8}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(2\left(\frac{3}{8}-\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}\right)\sqrt{-x+1}\sqrt{x+1}+\frac{\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)}{2}\right)-2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{x+1}+\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x)
```

```
[Out] 1/24*((2*(3*x + 10)*(x - 1) + 43)*(x - 1) + 39)*sqrt(x + 1)*sqrt(-x + 1) +
1/6*((2*x + 5)*(x - 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*(x + 2)*sqrt(x +
 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x + 1) - 5/4*arcsin(1/2*sqrt(2)*sqrt(-
x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-x}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(5/2), x)
```

3.1094 $\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=67

$$-\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}\,-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] 5/2*arcsin(x)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-1/3*(1-x)^(1/2)*(1+x)^(5/2)-5/2*(
1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
- x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx &= -\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}} \\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.84

$$-\frac{\sqrt{1-x}\,(22+31x+11x^2+2x^3)}{6\sqrt{1+x}}-5\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] -1/6*(Sqrt[1 - x]*(22 + 31*x + 11*x^2 + 2*x^3))/Sqrt[1 + x] - 5*ArcTan[Sqrt
[1 - x]/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 10.52, size = 133, normalized size = 1.99

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-5(1+x)^{\frac{3}{2}}-2(1+x)^{\frac{7}{2}}-(1+x)^{\frac{5}{2}}+30\sqrt{1+x}\right)}{6\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-5\sqrt{1+x}}{\sqrt{1-x}}+\frac{(1+x)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{(1+x)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{5(1+x)^{\frac{3}{2}}}{6\sqrt{1-x}}+5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 6 (-30 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - 5 (1
 + x) ^ (3 / 2) - 2 (1 + x) ^ (7 / 2) - (1 + x) ^ (5 / 2) + 30 Sqrt[1 + x])
 / Sqrt[-1 + x], Abs[1 + x] > 2}}, -5 Sqrt[1 + x] / Sqrt[1 - x] + (1 + x) ^
 (5 / 2) / (6 Sqrt[1 - x]) + (1 + x) ^ (7 / 2) / (3 Sqrt[1 - x]) + 5 (1 + x
) ^ (3 / 2) / (6 Sqrt[1 - x]) + 5 ArcSin[Sqrt[2] Sqrt[1 + x] / 2]]
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.06

method	result	size
default	$-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{3}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{6}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$\frac{(2x^2+9x+22)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(1-x)^(1/2)*(1+x)^(5/2)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-5/2*(1-x)^(1/2)*(1
+x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.34, size = 42, normalized size = 0.63

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\frac{3}{2}\sqrt{-x^2+1}\,x-\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

Fricas [A]
time = 0.30, size = 47, normalized size = 0.70

$$-\frac{1}{6}\left(2x^2+9x+22\right)\sqrt{x+1}\,\sqrt{-x+1}-5\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sq
rt(-x + 1) - 1)/x)
```

Sympy [A]
time = 9.51, size = 170, normalized size = 2.54

$$\begin{cases} -5i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 5\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(6*sqrt(x - 1)
) + 5*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x +
1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) + (x + 1)**(5/2)/(6*sqrt(1 - x)) + 5
*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 5*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.00, size = 78, normalized size = 1.16

$$2\left(\left(\left(-\frac{5}{12}-\frac{1}{6}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}-\frac{5}{4}\right)\sqrt{x+1}\,\sqrt{-x+1}+5\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x)
```

```
[Out] -1/6*((2*x + 7)*(x + 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(
2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{5/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(1/2), x)
```

3.1095 $\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{2(1+x)^{5/2}}{\sqrt{1-x}}-\frac{15}{2}\sin^{-1}(x)$$

```
[Out] -15/2*arcsin(x)+2*(1+x)^(5/2)/(1-x)^(1/2)+5/2*(1-x)^(1/2)*(1+x)^(3/2)+15/2*
(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 52, 41, 222}

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}\,(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\,\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] (15*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*Sqrt[1 - x]*(1 + x)^(3/2))/2 + (2*(1 +
x)^(5/2))/Sqrt[1 - x] - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - 5\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.75

$$-\frac{\sqrt{1+x}\,\left(-24+7x+x^2\right)}{2\sqrt{1-x}} + 15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 + x]*(-24 + 7*x + x^2))/Sqrt[1 - x] + 15*ArcTan[Sqrt[1 - x]/Sq
rt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 7.25, size = 110, normalized size = 1.69

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left((1+x)^{\frac{5}{2}} - 30\sqrt{1+x} + 5(1+x)^{\frac{3}{2}} + 30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}\right)}{2\sqrt{-1+x}}, \text{Abs}[1+x] > 2\right\}\right\}, -15\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right] - \frac{5(1+x)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{(1+x)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{15\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 2 ((1 + x) ^ (5 / 2) - 30 Sqrt[1 + x] + 5 (1 + x) ^ (3 / 2)
 + 30 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x]) / Sqrt[-1 + x], Abs[1
+ x] > 2}}, -15 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] - 5 (1 + x) ^ (3 / 2) / (2
Sqrt[1 - x]) - (1 + x) ^ (5 / 2) / (2 Sqrt[1 - x]) + 15 Sqrt[1 + x] / Sqrt[
1 - x]]
```

Maple [A]
time = 0.18, size = 77, normalized size = 1.18

method	result	size
risch	$-\frac{\left(x^{3}+8 x^{2}-17 x -24\right) \sqrt{\left(1+x \right) \left(1-x \right)}}{2 \sqrt{-\left(1+x \right) \left(-1+x \right)}\, \sqrt{1-x}\, \sqrt{1+x}}-\frac{15 \sqrt{\left(1+x \right) \left(1-x \right)}\, \arcsin \left(x \right)}{2 \sqrt{1+x}\, \sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^3+8*x^2-17*x-24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.38, size = 56, normalized size = 0.86

$$-\frac{x^{3}}{2 \sqrt{-x^{2}+1}}-\frac{4 x^{2}}{\sqrt{-x^{2}+1}}+\frac{17 x}{2 \sqrt{-x^{2}+1}}+\frac{12}{\sqrt{-x^{2}+1}}-\frac{15}{2} \arcsin \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) - 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) + 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 58, normalized size = 0.89

$$\frac{\left(x^{2}+7 x-24\right) \sqrt{x+1} \sqrt{-x+1}+30 \left(x-1\right) \arctan \left(\frac{\sqrt{x+1} \sqrt{-x+1}-1}{x}\right)+24 x-24}{2 \left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x - 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) + 24*x - 24)/(x - 1)
```

Sympy [A]
time = 6.18, size = 138, normalized size = 2.12

$$\begin{cases} 15i\,\text{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{5i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}-\frac{15i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ -15\,\text{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{5(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{15\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 5*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 15*I*sqrt(x + 1)/sqrt(x - 1), A
bs(x + 1) > 2), (-15*asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1
 - x)) - 5*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 15*sqrt(x + 1)/sqrt(1 - x), Tru
e))
```

Giac [A]
time = 0.01, size = 84, normalized size = 1.29

$$\frac{2\left(\left(-\frac{5}{4}-\frac{1}{4}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}+\frac{15}{2}\right)\sqrt{x+1}\,\sqrt{-x+1}}{-x+1}-15\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x)
```

```
[Out] 1/2*((x + 6)*(x + 1) - 30)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 15*arcsin(1/2
*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(3/2), x)
```

3.1096 $\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} \, dx$

Optimal. Leaf size=63

$$-5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\sin^{-1}(x)$$

```
[Out] 2/3*(1+x)^(5/2)/(1-x)^(3/2)+5*arcsin(x)-10/3*(1+x)^(3/2)/(1-x)^(1/2)-5*(1-x
)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}} - \frac{10(x+1)^{3/2}}{3\sqrt{1-x}} - 5\sqrt{1-x}\,\sqrt{x+1} + 5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] -5*Sqrt[1 - x]*Sqrt[1 + x] - (10*(1 + x)^(3/2))/(3*Sqrt[1 - x]) + (2*(1 + x
)^(5/2))/(3*(1 - x)^(3/2)) + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} - \frac{5}{3}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx \\
&= -\frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 51, normalized size = 0.81

$$-\frac{\sqrt{1-x^2}\,(23-34x+3x^2)}{3(-1+x)^2} + 10\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[1 - x^2]*(23 - 34*x + 3*x^2))/(-1 + x)^2 + 10*ArcTan[Sqrt[1 + x]
/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 9.32, size = 311, normalized size = 4.94

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-20+40x-3(1+x)^2\right)(-1+x)\sqrt{1+x}+15\left(-2\text{Pi}+\text{Pi}(1+x)-2I\,\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](1+x)+4I\,\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\right)(-1+x)^{\frac{3}{2}}}{3(-1+x)^{\frac{5}{2}}},\text{Abs}[1+x]>2\right\}\right\},\frac{-60\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](1+x)^{\frac{25}{2}}\sqrt{1-x}}{-6(1+x)^{\frac{25}{2}}\sqrt{1-x}+3(1+x)^{\frac{27}{2}}\sqrt{1-x}}-\frac{40(1+x)^{14}}{-6(1+x)^{\frac{25}{2}}\sqrt{1-x}+3(1+x)^{\frac{27}{2}}\sqrt{1-x}}+\frac{3(1+x)^{15}}{-6(1+x)^{\frac{25}{2}}\sqrt{1-x}+3(1+x)^{\frac{27}{2}}\sqrt{1-x}}+\frac{30\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](1+x)^{\frac{27}{2}}\sqrt{1-x}}{-6(1+x)^{\frac{25}{2}}\sqrt{1-x}+3(1+x)^{\frac{27}{2}}\sqrt{1-x}}+\frac{60(1+x)^{13}}{-6(1+x)^{\frac{25}{2}}\sqrt{1-x}+3(1+x)^{\frac{27}{2}}\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]')
```

```
[Out] Piecewise[{{(I (-20 + 40 x - 3 (1 + x) ^ 2) (-1 + x) Sqrt[1 + x] + 15 (-2 P
i + Pi (1 + x) - 2 I ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] (1 + x) + 4 I ArcCosh
[Sqrt[2] Sqrt[1 + x] / 2]) (-1 + x) ^ (3 / 2)) / (3 (-1 + x) ^ (5 / 2)), Ab
s[1 + x] > 2}}, -60 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] (1 + x) ^ (25 / 2) Sqrt
[1 - x] / (-6 (1 + x) ^ (25 / 2) Sqrt[1 - x] + 3 (1 + x) ^ (27 / 2) Sqrt[1
- x]) - 40 (1 + x) ^ 14 / (-6 (1 + x) ^ (25 / 2) Sqrt[1 - x] + 3 (1 + x) ^
(27 / 2) Sqrt[1 - x]) + 3 (1 + x) ^ 15 / (-6 (1 + x) ^ (25 / 2) Sqrt[1 - x]
 + 3 (1 + x) ^ (27 / 2) Sqrt[1 - x]) + 30 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] (
1 + x) ^ (27 / 2) Sqrt[1 - x] / (-6 (1 + x) ^ (25 / 2) Sqrt[1 - x] + 3 (1 +
 x) ^ (27 / 2) Sqrt[1 - x]) + 60 (1 + x) ^ 13 / (-6 (1 + x) ^ (25 / 2) Sqrt
[1 - x] + 3 (1 + x) ^ (27 / 2) Sqrt[1 - x])]
```

Maple [A]
time = 0.16, size = 84, normalized size = 1.33

method	result	size
risch	$\frac{(3x^3-31x^2-11x+23)\sqrt{(1+x)(1-x)}}{3(-1+x)\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*x^3-31*x^2-11*x+23)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)
/(1-x)^(1/2)/(1+x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcs
in(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(47) = 94.
time = 0.35, size = 99, normalized size = 1.57

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1}-\frac{5\,(-x^2+1)^{\frac{3}{2}}}{3\,(x^3-3\,x^2+3\,x-1)}+\frac{10\,\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)}+\frac{35\,\sqrt{-x^2+1}}{3\,(x-1)}+5\,\arcsin{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(5/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x
^3 - 3*x^2 + 3*x - 1) + 10/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 35/3*sqrt(-x^
2 + 1)/(x - 1) + 5*arcsin(x)
```

Fricas [A]
time = 0.30, size = 75, normalized size = 1.19

$$-\frac{23\,x^2+(3\,x^2-34\,x+23)\sqrt{x+1}\,\sqrt{-x+1}+30\,(x^2-2\,x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-46\,x+23}{3\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(23*x^2 + (3*x^2 - 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x^2 - 2*x
 + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 46*x + 23)/(x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.10, size = 575, normalized size = 9.13

$$\begin{cases} -\frac{30i\sqrt{x-1}(x+1)^{\frac{27}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{15\pi\sqrt{x-1}(x+1)^{\frac{27}{2}}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{60i\sqrt{x-1}(x+1)^{\frac{25}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{30\pi\sqrt{x-1}(x+1)^{\frac{25}{2}}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{3i(x+1)^{15}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{40i(x+1)^{14}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{60i(x+1)^{13}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}} & \text{for}\ |x+1|>2 \\ \frac{30\sqrt{1-x}(x+1)^{\frac{27}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}-\frac{60\sqrt{1-x}(x+1)^{\frac{25}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}+\frac{3(x+1)^{15}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}-\frac{40(x+1)^{14}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}+\frac{60(x+1)^{13}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((-30*I*sqrt(x - 1)*(x + 1)**(27/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(
3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) + 15*pi*sqrt
(x - 1)*(x + 1)**(27/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x +
 1)**(25/2)) + 60*I*sqrt(x - 1)*(x + 1)**(25/2)*acosh(sqrt(2)*sqrt(x + 1)/2
)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) - 30*pi*s
qrt(x - 1)*(x + 1)**(25/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(
x + 1)**(25/2)) - 3*I*(x + 1)**15/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x
 - 1)*(x + 1)**(25/2)) + 40*I*(x + 1)**14/(3*sqrt(x - 1)*(x + 1)**(27/2) -
6*sqrt(x - 1)*(x + 1)**(25/2)) - 60*I*(x + 1)**13/(3*sqrt(x - 1)*(x + 1)**(
27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)), Abs(x + 1) > 2), (30*sqrt(1 - x)*(x
 + 1)**(27/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) -
6*sqrt(1 - x)*(x + 1)**(25/2)) - 60*sqrt(1 - x)*(x + 1)**(25/2)*asin(sqrt(2
)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x + 1)**(2
5/2)) + 3*(x + 1)**15/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x + 1
)**(25/2)) - 40*(x + 1)**14/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*
(x + 1)**(25/2)) + 60*(x + 1)**13/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1
 - x)*(x + 1)**(25/2)), True))
```

Giac [A]
time = 0.01, size = 82, normalized size = 1.30

$$\frac{2\left(\left(\frac{20}{3}-\frac{1}{2}\sqrt{x+1}\sqrt{x+1}\right)\sqrt{x+1}\sqrt{x+1}-10\right)\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^2}+10\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x)
```

```
[Out] -1/3*((3*x - 37)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 10*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(5/2), x)
```

3.1097 $\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}} dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \sin^{-1}(x)$$

```
[Out] -2/3*(1+x)^(3/2)/(1-x)^(3/2)+2/5*(1+x)^(5/2)/(1-x)^(5/2)-arcsin(x)+2*(1+x)^
(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}} - \frac{2(x+1)^{3/2}}{3(1-x)^{3/2}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1 +
 x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx &= \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx \\
&= -\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} + \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.81

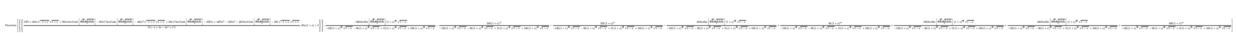
$$
\frac{2\sqrt{1+x}\,(13-24x+23x^2)}{15(1-x)^{5/2}} + 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x]*(13 - 24*x + 23*x^2))/(15*(1 - x)^(5/2)) + 2*ArcTan[Sqrt[1 -
 x]/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 18.35, size = 746, normalized size = 11.84



Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]')
```

```
[Out] Piecewise[{{(15 Pi + 48 I x Sqrt[-1 + x] Sqrt[1 + x] + 90 I x ArcCosh[Sqrt[
2] Sqrt[1 + x] / 2] - 90 I x ^ 2 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] - 46 I x
^ 2 Sqrt[-1 + x] Sqrt[1 + x] + 30 I x ^ 3 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2]
- 45 Pi x + 45 Pi x ^ 2 - 15 Pi x ^ 3 - 30 I ArcCosh[Sqrt[2] Sqrt[1 + x] /
2] - 26 I Sqrt[-1 + x] Sqrt[1 + x]) / (15 (-1 + 3 x - 3 x ^ 2 + x ^ 3)), Ab
s[1 + x] > 2}}, -360 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] (1 + x) ^ (31 / 2) Sqr
t[1 - x] / (-120 (1 + x) ^ (29 / 2) Sqrt[1 - x] - 90 (1 + x) ^ (33 / 2) Sqr
t[1 - x] + 15 (1 + x) ^ (35 / 2) Sqrt[1 - x] + 180 (1 + x) ^ (31 / 2) Sqrt[
```

```
1 - x]) - 240 (1 + x) ^ 15 / (-120 (1 + x) ^ (29 / 2) Sqrt[1 - x] - 90 (1 +
 x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (35 / 2) Sqrt[1 - x] + 180 (1 + x
) ^ (31 / 2) Sqrt[1 - x]) - 232 (1 + x) ^ 17 / (-120 (1 + x) ^ (29 / 2) Sqr
t[1 - x] - 90 (1 + x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (35 / 2) Sqrt[1
 - x] + 180 (1 + x) ^ (31 / 2) Sqrt[1 - x]) - 30 ArcSin[Sqrt[2] Sqrt[1 + x]
 / 2] (1 + x) ^ (35 / 2) Sqrt[1 - x] / (-120 (1 + x) ^ (29 / 2) Sqrt[1 - x]
 - 90 (1 + x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (35 / 2) Sqrt[1 - x] +
180 (1 + x) ^ (31 / 2) Sqrt[1 - x]) + 46 (1 + x) ^ 18 / (-120 (1 + x) ^ (29
 / 2) Sqrt[1 - x] - 90 (1 + x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (35 /
2) Sqrt[1 - x] + 180 (1 + x) ^ (31 / 2) Sqrt[1 - x]) + 180 ArcSin[Sqrt[2] S
qrt[1 + x] / 2] (1 + x) ^ (33 / 2) Sqrt[1 - x] / (-120 (1 + x) ^ (29 / 2) S
qrt[1 - x] - 90 (1 + x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (35 / 2) Sqrt
[1 - x] + 180 (1 + x) ^ (31 / 2) Sqrt[1 - x]) + 240 ArcSin[Sqrt[2] Sqrt[1 +
 x] / 2] (1 + x) ^ (29 / 2) Sqrt[1 - x] / (-120 (1 + x) ^ (29 / 2) Sqrt[1 -
 x] - 90 (1 + x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (35 / 2) Sqrt[1 - x]
 + 180 (1 + x) ^ (31 / 2) Sqrt[1 - x]) + 400 (1 + x) ^ 16 / (-120 (1 + x) ^
 (29 / 2) Sqrt[1 - x] - 90 (1 + x) ^ (33 / 2) Sqrt[1 - x] + 15 (1 + x) ^ (3
5 / 2) Sqrt[1 - x] + 180 (1 + x) ^ (31 / 2) Sqrt[1 - x])]
```

Maple [A]
time = 0.16, size = 84, normalized size = 1.33

method	result	size
risch	$\frac{2(23x^3-x^2-11x+13)\sqrt{(1+x)(1-x)}}{15(-1+x)^2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(23*x^3-x^2-11*x+13)/(-1+x)^2/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2
)/(1-x)^(1/2)/(1+x)^(1/2)-((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(47) = 94$.
time = 0.36, size = 160, normalized size = 2.54

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{5(x^5-5x^4+10x^3-10x^2+5x-1)}+\frac{(-x^2+1)^{\frac{3}{2}}}{x^4-4x^3+6x^2-4x+1}+\frac{(-x^2+1)^{\frac{3}{2}}}{3(x^3-3x^2+3x-1)}+\frac{6\sqrt{-x^2+1}}{5(x^3-3x^2+3x-1)}-\frac{7\sqrt{-x^2+1}}{15(x^2-2x+1)}-\frac{38\sqrt{-x^2+1}}{15(x-1)}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + (-x^2 + 1
)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2
```

```
+ 3*x - 1) + 6/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 7/15*sqrt(-x^2 +
1)/(x^2 - 2*x + 1) - 38/15*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [**A**]
time = 0.29, size = 91, normalized size = 1.44

$$\frac{2\left(13\,x^3-39\,x^2-(23\,x^2-24\,x+13)\sqrt{x+1}\,\sqrt{-x+1}+15\,(x^3-3\,x^2+3\,x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+39\,x-13\right)}{15\,(x^3-3\,x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(13*x^3 - 39*x^2 - (23*x^2 - 24*x + 13)*sqrt(x + 1)*sqrt(-x + 1) + 15*
(x^3 - 3*x^2 + 3*x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 39*x - 1
3)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 8.54, size = 1606, normalized size = 25.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(35/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(1
5*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 15*pi*sqrt(x - 1
)*(x + 1)**(35/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)*
*(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)
) - 180*I*sqrt(x - 1)*(x + 1)**(33/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt
(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*
(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 90*pi*sqrt(x - 1)*(x +
 1)**(33/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2
) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 36
0*I*sqrt(x - 1)*(x + 1)**(31/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1
)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1
)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 180*pi*sqrt(x - 1)*(x + 1)**
(31/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 1
80*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 240*I*s
qrt(x - 1)*(x + 1)**(29/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1)*(x
+ 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(3
1/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 120*pi*sqrt(x - 1)*(x + 1)**(29/2
)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sq
rt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 46*I*(x + 1)
**18/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180
*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 232*I*(x
+ 1)**17/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) +
```

```
 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 400*I
*(x + 1)**16/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/
2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 2
40*I*(x + 1)**15/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**
(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2))
, Abs(x + 1) > 2), (-30*sqrt(1 - x)*(x + 1)**(35/2)*asin(sqrt(2)*sqrt(x + 1
)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180
*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 180*sqrt(
1 - x)*(x + 1)**(33/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)*
*(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2)
- 120*sqrt(1 - x)*(x + 1)**(29/2)) - 360*sqrt(1 - x)*(x + 1)**(31/2)*asin(s
qrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x +
 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(2
9/2)) + 240*sqrt(1 - x)*(x + 1)**(29/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqr
t(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)
*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 46*(x + 1)**18/(15*sq
rt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x
)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 232*(x + 1)**17/(15*
sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
 x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 400*(x + 1)**16/(1
5*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1
 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 240*(x + 1)**15/
(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt
(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)), True))
```

Giac [A]
time = 0.01, size = 83, normalized size = 1.32

$$\frac{2\left(\left(\frac{23}{15}\sqrt{x+1}\,\sqrt{x+1}-\frac{14}{3}\right)\sqrt{x+1}\,\sqrt{x+1}+4\right)\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^3}-2\arcsin\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x)
[Out] -2/15*((23*x - 47)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3 - 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(7/2),x)
[Out] int((x + 1)^(5/2)/(1 - x)^(7/2), x)
```

3.1098 $\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

```
[Out] 1/7*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

Mathematica [A]
time = 0.04, size = 20, normalized size = 1.00

$$\frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 14.13, size = 95, normalized size = 4.75

$$\text{Piecewise}\left[\left\{\left\{\frac{I(1+x)^{\frac{7}{2}}}{7\sqrt{-1+x}\,(-1+3x-3x^2+x^3)},\text{Abs}[1+x]>2\right\}\right\},-\frac{(1+x)^{\frac{7}{2}}}{-56\sqrt{1-x}-42(1+x)^2\sqrt{1-x}+7(1+x)^3\sqrt{1-x}+84(1+x)\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]')
```

```
[Out] Piecewise[{{I / 7 (1 + x) ^ (7 / 2) / (Sqrt[-1 + x] (-1 + 3 x - 3 x ^ 2 + x
 ^ 3)), Abs[1 + x] > 2}}, -(1 + x) ^ (7 / 2) / (-56 Sqrt[1 - x] - 42 (1 + x
) ^ 2 Sqrt[1 - x] + 7 (1 + x) ^ 3 Sqrt[1 - x] + 84 (1 + x) Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(14) = 28.
time = 0.14, size = 85, normalized size = 4.25

method	result	size
gosper	$\frac{(1+x)^{\frac{7}{2}}}{7(1-x)^{\frac{7}{2}}}$	15
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^4+4x^3+6x^2+4x+1)}{7\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59
default	$\frac{(1+x)^{\frac{5}{2}}}{(1-x)^{\frac{7}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{2(1-x)^{\frac{7}{2}}}+\frac{15\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}-\frac{3\sqrt{1+x}}{14(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{7(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{7\sqrt{1-x}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)^(5/2)/(1-x)^(7/2)-5/2*(1+x)^(3/2)/(1-x)^(7/2)+15/7*(1+x)^(1/2)/(1-x)^
(7/2)-3/14*(1+x)^(1/2)/(1-x)^(5/2)-1/7*(1+x)^(1/2)/(1-x)^(3/2)-1/7*(1+x)^(1
/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(14) = 28.
time = 0.26, size = 171, normalized size = 8.55

$$\frac{(-x^2+1)^{\frac{5}{2}}}{x^6-6x^5+15x^4-20x^3+15x^2-6x+1}+\frac{5(-x^2+1)^{\frac{3}{2}}}{2(x^5-5x^4+10x^3-10x^2+5x-1)}+\frac{15\sqrt{-x^2+1}}{7(x^4-4x^3+6x^2-4x+1)}+\frac{3\sqrt{-x^2+1}}{14(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{7(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{7(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/2*(
-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 15/7*sqrt(-x^2
```

```
+ 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 3/14*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3
*x - 1) - 1/7*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/7*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(14) = 28$.
time = 0.30, size = 66, normalized size = 3.30

$$\frac{x^4 - 4x^3 + 6x^2 + (x^3 + 3x^2 + 3x + 1)\sqrt{x+1}\sqrt{-x+1} - 4x + 1}{7(x^4 - 4x^3 + 6x^2 - 4x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="fricas")
[Out] 1/7*(x^4 - 4*x^3 + 6*x^2 + (x^3 + 3*x^2 + 3*x + 1)*sqrt(x + 1)*sqrt(-x + 1)
 - 4*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.71, size = 114, normalized size = 5.70

$$\begin{cases} \frac{i(x+1)^{\frac{7}{2}}}{7\sqrt{x-1}\,(x+1)^3 - 42\sqrt{x-1}\,(x+1)^2 + 84\sqrt{x-1}\,(x+1) - 56\sqrt{x-1}} & \text{for } |x+1| > 2 \\ -\frac{(x+1)^{\frac{7}{2}}}{7\sqrt{1-x}\,(x+1)^3 - 42\sqrt{1-x}\,(x+1)^2 + 84\sqrt{1-x}\,(x+1) - 56\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(9/2),x)
[Out] Piecewise((I*(x + 1)**(7/2)/(7*sqrt(x - 1)*(x + 1)**3 - 42*sqrt(x - 1)*(x +
 1)**2 + 84*sqrt(x - 1)*(x + 1) - 56*sqrt(x - 1)), Abs(x + 1) > 2), (-(x +
1)**(7/2)/(7*sqrt(1 - x)*(x + 1)**3 - 42*sqrt(1 - x)*(x + 1)**2 + 84*sqrt(1
 - x)*(x + 1) - 56*sqrt(1 - x)), True))
```

Giac [A]
time = 0.01, size = 67, normalized size = 3.35

$$\frac{\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{7(-x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x)
[Out] 1/7*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.28, size = 64, normalized size = 3.20

$$\frac{\sqrt{1-x}\,\left(\frac{3x\sqrt{x+1}}{7} + \frac{\sqrt{x+1}}{7} + \frac{3x^2\sqrt{x+1}}{7} + \frac{x^3\sqrt{x+1}}{7}\right)}{x^4 - 4x^3 + 6x^2 - 4x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((3*x*(x + 1)^(1/2))/7 + (x + 1)^(1/2)/7 + (3*x^2*(x + 1)^(1
/2))/7 + (x^3*(x + 1)^(1/2))/7))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

3.1099 $\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=41

$$\frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}$$

```
[Out] 1/9*(1+x)^(7/2)/(1-x)^(9/2)+1/63*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]

time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(11/2),x]
[Out] (1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx&=\frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{1}{9}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx\\&=\frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.56

$$-\frac{(-8+x)(1+x)^{7/2}}{63(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]
```

```
[Out] -1/63*((-8 + x)*(1 + x)^(7/2))/(1 - x)^(9/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 37.91, size = 191, normalized size = 4.66

$$\text{Piecewise}\left[\left\{\left\{\frac{I(-8+x)(1+x)^{\frac{7}{2}}}{63\sqrt{-1+x}\,(1-4x+6x^2-4x^3+x^4)},\text{Abs}[1+x]>2\right\}\right\},-\frac{(1+x)^{\frac{9}{2}}}{-2016(1+x)\sqrt{1-x}-504(1+x)^3\sqrt{1-x}+63(1+x)^4\sqrt{1-x}+1008\sqrt{1-x}+1512(1+x)^2\sqrt{1-x}}+\frac{9(1+x)^{\frac{7}{2}}}{-2016(1+x)\sqrt{1-x}-504(1+x)^3\sqrt{1-x}+63(1+x)^4\sqrt{1-x}+1008\sqrt{1-x}+1512(1+x)^2\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]')
```

```
[Out] Piecewise[{{I / 63 (-8 + x) (1 + x) ^ (7 / 2) / (Sqrt[-1 + x] (1 - 4 x + 6
x ^ 2 - 4 x ^ 3 + x ^ 4)), Abs[1 + x] > 2}}, -(1 + x) ^ (9 / 2) / (-2016 (1
 + x) Sqrt[1 - x] - 504 (1 + x) ^ 3 Sqrt[1 - x] + 63 (1 + x) ^ 4 Sqrt[1 - x
] + 1008 Sqrt[1 - x] + 1512 (1 + x) ^ 2 Sqrt[1 - x]) + 9 (1 + x) ^ (7 / 2)
/ (-2016 (1 + x) Sqrt[1 - x] - 504 (1 + x) ^ 3 Sqrt[1 - x] + 63 (1 + x) ^ 4
 Sqrt[1 - x] + 1008 Sqrt[1 - x] + 1512 (1 + x) ^ 2 Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.
time = 0.14, size = 100, normalized size = 2.44

method	result	size
gosper	$-\frac{(1+x)^{\frac{7}{2}}(x-8)}{63(1-x)^{\frac{9}{2}}}$	18
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^5-4x^4-26x^3-44x^2-31x-8)}{63\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	64
default	$\frac{(1+x)^{\frac{5}{2}}}{2(1-x)^{\frac{9}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{6(1-x)^{\frac{9}{2}}}+\frac{5\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{126(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{42(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{63(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{63\sqrt{1-x}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(5/2)/(1-x)^(9/2)-5/6*(1+x)^(3/2)/(1-x)^(9/2)+5/9*(1+x)^(1/2)/(1-
x)^(9/2)-5/126*(1+x)^(1/2)/(1-x)^(7/2)-1/42*(1+x)^(1/2)/(1-x)^(5/2)-1/63*(1
+x)^(1/2)/(1-x)^(3/2)-1/63*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(29) = 58$.
time = 0.28, size = 218, normalized size = 5.32

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{2(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}-\frac{5(-x^2+1)^{\frac{3}{2}}}{6(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}-\frac{5\sqrt{-x^2+1}}{9(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{5\sqrt{-x^2+1}}{126(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{42(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{63(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{63(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(5/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 5/6*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
 + 1) - 5/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/12
6*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/42*sqrt(-x^2 + 1)/(x^3
 - 3*x^2 + 3*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/63*sqrt(-x^2
+ 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.
time = 0.29, size = 83, normalized size = 2.02

$$\frac{8x^5-40x^4+80x^3-80x^2+(x^4-5x^3-21x^2-23x-8)\sqrt{x+1}\sqrt{-x+1}+40x-8}{63(x^5-5x^4+10x^3-10x^2+5x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/63*(8*x^5 - 40*x^4 + 80*x^3 - 80*x^2 + (x^4 - 5*x^3 - 21*x^2 - 23*x - 8)*
sqrt(x + 1)*sqrt(-x + 1) + 40*x - 8)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x -
 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 67.28, size = 280, normalized size = 6.83

$$\begin{cases} \frac{i(x+1)^{\frac{9}{2}}}{63\sqrt{x-1}(x+1)^4-504\sqrt{x-1}(x+1)^3+1512\sqrt{x-1}(x+1)^2-2016\sqrt{x-1}(x+1)+1008\sqrt{x-1}}-\frac{9i(x+1)^{\frac{7}{2}}}{63\sqrt{x-1}(x+1)^4-504\sqrt{x-1}(x+1)^3+1512\sqrt{x-1}(x+1)^2-2016\sqrt{x-1}(x+1)+1008\sqrt{x-1}} & \text{for } |x+1|>2 \\ -\frac{(x+1)^{\frac{9}{2}}}{63\sqrt{1-x}(x+1)^4-504\sqrt{1-x}(x+1)^3+1512\sqrt{1-x}(x+1)^2-2016\sqrt{1-x}(x+1)+1008\sqrt{1-x}}+\frac{9(x+1)^{\frac{7}{2}}}{63\sqrt{1-x}(x+1)^4-504\sqrt{1-x}(x+1)^3+1512\sqrt{1-x}(x+1)^2-2016\sqrt{1-x}(x+1)+1008\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(9/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1)*(x
 + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 1008*sq
rt(x - 1)) - 9*I*(x + 1)**(7/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1
)*(x + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 100
8*sqrt(x - 1)), Abs(x + 1) > 2), (-(x + 1)**(9/2)/(63*sqrt(1 - x)*(x + 1)**
4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqrt(1
```

```
- x)*(x + 1) + 1008*sqrt(1 - x)) + 9*(x + 1)**(7/2)/(63*sqrt(1 - x)*(x + 1)
**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqrt(
1 - x)*(x + 1) + 1008*sqrt(1 - x)), True))
```

Giac [A]
time = 0.02, size = 87, normalized size = 2.12

$$\frac{2\left(\frac{1}{14}-\frac{1}{126}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x)
```

```
[Out] 1/63*(x + 1)^(7/2)*(x - 8)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.30, size = 80, normalized size = 1.95

$$-\frac{\sqrt{1-x}\,\left(\frac{23\,x\,\sqrt{x+1}}{63}+\frac{8\,\sqrt{x+1}}{63}+\frac{x^2\,\sqrt{x+1}}{3}+\frac{5\,x^3\,\sqrt{x+1}}{63}-\frac{x^4\,\sqrt{x+1}}{63}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((23*x*(x + 1)^(1/2))/63 + (8*(x + 1)^(1/2))/63 + (x^2*(x +
 1)^(1/2))/3 + (5*x^3*(x + 1)^(1/2))/63 - (x^4*(x + 1)^(1/2))/63))/(5*x - 1
0*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

3.1100 $\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} \, dx$

Optimal. Leaf size=61

$$\frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}$$

```
[Out] 1/11*(1+x)^(7/2)/(1-x)^(11/2)+2/99*(1+x)^(7/2)/(1-x)^(9/2)+2/693*(1+x)^(7/2
)/(1-x)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}} + \frac{2(x+1)^{7/2}}{99(1-x)^{9/2}} + \frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*(1 + x)^(7/2))/(99*(1 - x)^(9/2)) +
(2*(1 + x)^(7/2))/(693*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2}{11}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2}{99}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}
\end{aligned}
$$

<u>Mathematica</u> [**A**]
time = 0.06, size = 30, normalized size = 0.49

$$
\frac{(1+x)^{7/2}\left(79-18x+2x^2\right)}{693(1-x)^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(7/2)*(79 - 18*x + 2*x^2))/(693*(1 - x)^(11/2))
```

<u>Mathics</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 103.28, size = 475, normalized size = 7.79

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\left(-55+143x-26(1+x)^2+2(1+x)^3\right)(1+x)^{\frac{7}{2}}}{693\sqrt{-1+x}\left(1-6x+15x^2-20x^3+15x^4-6x^5+x^6\right)},\text{Abs}[1+x]>2\right\}\right\},\frac{-143(1+x)^{\frac{9}{2}}}{-133056(1+x)\sqrt{1-x}-110880(1+x)^3\sqrt{1-x}-8316(1+x)^5\sqrt{1-x}+693(1+x)^6\sqrt{1-x}+41580(1+x)^4\sqrt{1-x}+44352\sqrt{1-x}+166320(1+x)^2\sqrt{1-x}}-\frac{2(1+x)^{\frac{13}{2}}}{-133056(1+x)\sqrt{1-x}-110880(1+x)^3\sqrt{1-x}-8316(1+x)^5\sqrt{1-x}+693(1+x)^6\sqrt{1-x}+41580(1+x)^4\sqrt{1-x}+44352\sqrt{1-x}+166320(1+x)^2\sqrt{1-x}}+\frac{26(1+x)^{\frac{11}{2}}}{-133056(1+x)\sqrt{1-x}-110880(1+x)^3\sqrt{1-x}-8316(1+x)^5\sqrt{1-x}+693(1+x)^6\sqrt{1-x}+41580(1+x)^4\sqrt{1-x}+44352\sqrt{1-x}+166320(1+x)^2\sqrt{1-x}}+\frac{198(1+x)^{\frac{7}{2}}}{-133056(1+x)\sqrt{1-x}-110880(1+x)^3\sqrt{1-x}-8316(1+x)^5\sqrt{1-x}+693(1+x)^6\sqrt{1-x}+41580(1+x)^4\sqrt{1-x}+44352\sqrt{1-x}+166320(1+x)^2\sqrt{1-x}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]')
```

```
[Out] Piecewise[{{I / 693 (-55 + 143 x - 26 (1 + x) ^ 2 + 2 (1 + x) ^ 3) (1 + x)
^ (7 / 2) / (Sqrt[-1 + x] (1 - 6 x + 15 x ^ 2 - 20 x ^ 3 + 15 x ^ 4 - 6 x ^
 5 + x ^ 6)), Abs[1 + x] > 2}}, -143 (1 + x) ^ (9 / 2) / (-133056 (1 + x) S
qrt[1 - x] - 110880 (1 + x) ^ 3 Sqrt[1 - x] - 8316 (1 + x) ^ 5 Sqrt[1 - x]
+ 693 (1 + x) ^ 6 Sqrt[1 - x] + 41580 (1 + x) ^ 4 Sqrt[1 - x] + 44352 Sqrt[
1 - x] + 166320 (1 + x) ^ 2 Sqrt[1 - x]) - 2 (1 + x) ^ (13 / 2) / (-133056
(1 + x) Sqrt[1 - x] - 110880 (1 + x) ^ 3 Sqrt[1 - x] - 8316 (1 + x) ^ 5 Sqr
t[1 - x] + 693 (1 + x) ^ 6 Sqrt[1 - x] + 41580 (1 + x) ^ 4 Sqrt[1 - x] + 44
352 Sqrt[1 - x] + 166320 (1 + x) ^ 2 Sqrt[1 - x]) + 26 (1 + x) ^ (11 / 2) /
 (-133056 (1 + x) Sqrt[1 - x] - 110880 (1 + x) ^ 3 Sqrt[1 - x] - 8316 (1 +
x) ^ 5 Sqrt[1 - x] + 693 (1 + x) ^ 6 Sqrt[1 - x] + 41580 (1 + x) ^ 4 Sqrt[1
 - x] + 44352 Sqrt[1 - x] + 166320 (1 + x) ^ 2 Sqrt[1 - x]) + 198 (1 + x) ^
 (7 / 2) / (-133056 (1 + x) Sqrt[1 - x] - 110880 (1 + x) ^ 3 Sqrt[1 - x] -
8316 (1 + x) ^ 5 Sqrt[1 - x] + 693 (1 + x) ^ 6 Sqrt[1 - x] + 41580 (1 + x)
^ 4 Sqrt[1 - x] + 44352 Sqrt[1 - x] + 166320 (1 + x) ^ 2 Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(43) = 86$.
time = 0.14, size = 114, normalized size = 1.87

method	result
gosper	$\frac{(1+x)^{\frac{7}{2}}(2x^2-18x+79)}{693(1-x)^{\frac{11}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(2x^6-10x^5+19x^4+216x^3+404x^2+298x+79)}{693\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^5\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{3(1-x)^{\frac{11}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{12(1-x)^{\frac{11}{2}}}+\frac{5\sqrt{1+x}}{22(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{396(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{693(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{231(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{693(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{693\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+x)^(5/2)/(1-x)^(11/2)-5/12*(1+x)^(3/2)/(1-x)^(11/2)+5/22*(1+x)^(1/2)
/(1-x)^(11/2)-5/396*(1+x)^(1/2)/(1-x)^(9/2)-5/693*(1+x)^(1/2)/(1-x)^(7/2)-1
/231*(1+x)^(1/2)/(1-x)^(5/2)-2/693*(1+x)^(1/2)/(1-x)^(3/2)-2/693*(1+x)^(1/2
)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(43) = 86$.
time = 0.26, size = 269, normalized size = 4.41

$$\frac{(-x^2+1)^{\frac{5}{2}}}{3(x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1)}+\frac{5(-x^2+1)^{\frac{3}{2}}}{12(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}+\frac{5\sqrt{-x^2+1}}{22(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}+\frac{5\sqrt{-x^2+1}}{396(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{5\sqrt{-x^2+1}}{693(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{231(x^3-3x^2+3x-1)}-\frac{2\sqrt{-x^2+1}}{693(x^2-2x+1)}+\frac{2\sqrt{-x^2+1}}{693(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(5/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 5/12*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1) + 5/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^
3 + 15*x^2 - 6*x + 1) + 5/396*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
 + 5*x - 1) - 5/693*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/231*
sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/693*sqrt(-x^2 + 1)/(x^2 - 2*x +
1) + 2/693*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(43) = 86$.
time = 0.30, size = 100, normalized size = 1.64

$$\frac{79\,x^6-474\,x^5+1185\,x^4-1580\,x^3+1185\,x^2+(2\,x^5-12\,x^4+31\,x^3+185\,x^2+219\,x+79)\sqrt{x+1}\,\sqrt{-x+1}-474\,x+79}{693\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/693*(79*x^6 - 474*x^5 + 1185*x^4 - 1580*x^3 + 1185*x^2 + (2*x^5 - 12*x^4
+ 31*x^3 + 185*x^2 + 219*x + 79)*sqrt(x + 1)*sqrt(-x + 1) - 474*x + 79)/(x^
6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.02, size = 105, normalized size = 1.72

$$\frac{2\left(\left(\frac{1}{693}\sqrt{x+1}\,\sqrt{x+1}-\frac{1}{63}\right)\sqrt{x+1}\,\sqrt{x+1}+\frac{1}{14}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x)
```

```
[Out] 1/693*(2*(x + 1)*(x - 10) + 99)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^6
```

Mupad [B]
time = 0.31, size = 94, normalized size = 1.54

$$\frac{\sqrt{1-x}\,\left(\frac{73\,x\,\sqrt{x+1}}{231}+\frac{79\,\sqrt{x+1}}{693}+\frac{185\,x^2\,\sqrt{x+1}}{693}+\frac{31\,x^3\,\sqrt{x+1}}{693}-\frac{4\,x^4\,\sqrt{x+1}}{231}+\frac{2\,x^5\,\sqrt{x+1}}{693}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((73*x*(x + 1)^(1/2))/231 + (79*(x + 1)^(1/2))/693 + (185*x^
2*(x + 1)^(1/2))/693 + (31*x^3*(x + 1)^(1/2))/693 - (4*x^4*(x + 1)^(1/2))/2
31 + (2*x^5*(x + 1)^(1/2))/693))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^5 +
x^6 + 1)
```

3.1101 $\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} dx$

Optimal. Leaf size=81

$$\frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}$$

```
[Out] 1/13*(1+x)^(7/2)/(1-x)^(13/2)+3/143*(1+x)^(7/2)/(1-x)^(11/2)+2/429*(1+x)^(7
/2)/(1-x)^(9/2)+2/3003*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*(1 + x)^(7/2))/(143*(1 - x)^(11/2))
+ (2*(1 + x)^(7/2))/(429*(1 - x)^(9/2)) + (2*(1 + x)^(7/2))/(3003*(1 - x)^(
7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3}{13}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{6}{143}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{429(1-x)^{9/2}} + \frac{2}{429}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{429(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}$$

Mathematica [**A**]
time = 0.06, size = 35, normalized size = 0.43

$$\frac{(1+x)^{7/2}\,(310-97x+20x^2-2x^3)}{3003(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]
[Out] ((1 + x)^(7/2)*(310 - 97*x + 20*x^2 - 2*x^3))/(3003*(1 - x)^(13/2))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.
time = 0.16, size = 128, normalized size = 1.58

method	result
gosper	$-\frac{(1+x)^{\frac{7}{2}}\left(2x^3-20x^2+97x-310\right)}{3003(1-x)^{\frac{13}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(2x^7-12x^6+29x^5-34x^4-736x^3-1492x^2-1143x-310\right)}{3003\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^6\sqrt{-(1+x)(-1+x)}}$

default	$\frac{(1+x)^{\frac{5}{2}}}{4(1-x)^{\frac{13}{2}}} - \frac{(1+x)^{\frac{3}{2}}}{4(1-x)^{\frac{13}{2}}} + \frac{3\sqrt{1+x}}{26(1-x)^{\frac{13}{2}}} - \frac{3\sqrt{1+x}}{572(1-x)^{\frac{11}{2}}} - \frac{5\sqrt{1+x}}{1716(1-x)^{\frac{9}{2}}} - \frac{5\sqrt{1+x}}{3003(1-x)^{\frac{7}{2}}} - \frac{\sqrt{1+x}}{1001(1-x)^{\frac{5}{2}}} - \frac{2\sqrt{1+x}}{3003(1-x)^{\frac{3}{2}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1+x)^(5/2)/(1-x)^(13/2)-1/4*(1+x)^(3/2)/(1-x)^(13/2)+3/26*(1+x)^(1/2)/
(1-x)^(13/2)-3/572*(1+x)^(1/2)/(1-x)^(11/2)-5/1716*(1+x)^(1/2)/(1-x)^(9/2)-
5/3003*(1+x)^(1/2)/(1-x)^(7/2)-1/1001*(1+x)^(1/2)/(1-x)^(5/2)-2/3003*(1+x)^
(1/2)/(1-x)^(3/2)-2/3003*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(57) = 114$.
time = 0.27, size = 325, normalized size = 4.01

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{4(x^9-9x^8+36x^7-84x^6+126x^5-126x^4+84x^3-36x^2+9x-1)} - \frac{(-x^2+1)^{\frac{3}{2}}}{4(x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1)} - \frac{3\sqrt{-x^2+1}}{26(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)} - \frac{3\sqrt{-x^2+1}}{572(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)} + \frac{5\sqrt{-x^2+1}}{1716(x^5-5x^4+10x^3-10x^2+5x-1)} - \frac{5\sqrt{-x^2+1}}{3003(x^4-4x^3+6x^2-4x+1)} + \frac{\sqrt{-x^2+1}}{1001(x^3-3x^2+3x-1)} - \frac{2\sqrt{-x^2+1}}{3003(x^2-2x+1)} + \frac{2\sqrt{-x^2+1}}{3003(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(5/2)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 +
84*x^3 - 36*x^2 + 9*x - 1) - 1/4*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 5
6*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) - 3/26*sqrt(-x^2 + 1)/(x^7 - 7*
x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1) - 3/572*sqrt(-x^2 + 1)/(
x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/1716*sqrt(-x^2 + 1)/(
x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/3003*sqrt(-x^2 + 1)/(x^4 - 4*x
^3 + 6*x^2 - 4*x + 1) + 1/1001*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/3
003*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/3003*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.
time = 0.30, size = 115, normalized size = 1.42

$$\frac{310x^7 - 2170x^6 + 6510x^5 - 10850x^4 + 10850x^3 - 6510x^2 + (2x^6 - 14x^5 + 43x^4 - 77x^3 - 659x^2 - 833x - 310)\sqrt{x+1}\sqrt{-x+1} + 2170x - 310}{3003(x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(310*x^7 - 2170*x^6 + 6510*x^5 - 10850*x^4 + 10850*x^3 - 6510*x^2 +
(2*x^6 - 14*x^5 + 43*x^4 - 77*x^3 - 659*x^2 - 833*x - 310)*sqrt(x + 1)*sqrt
(-x + 1) + 2170*x - 310)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 +
 7*x - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(15/2),x)
```

```
[Out] Exception raised: SystemError
```

Giac [A]
time = 0.03, size = 124, normalized size = 1.53

$$\frac{2\left(\left(\left(\frac{1}{231}-\frac{1}{3003}\sqrt{x+1}\,\sqrt{x+1}\right)\sqrt{x+1}\,\sqrt{x+1}-\frac{1}{42}\right)\sqrt{x+1}\,\sqrt{x+1}+\frac{1}{14}\right)\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{x+1}\,\sqrt{-x+1}}{(-x+1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x)
```

```
[Out] 1/3003*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1)^(7/2)*sqrt(-x + 1
)/(x - 1)^7
```

Mupad [B]
time = 0.31, size = 110, normalized size = 1.36

$$-\frac{\sqrt{1-x}\,\left(\frac{119\,x\,\sqrt{x+1}}{429}+\frac{310\,\sqrt{x+1}}{3003}+\frac{659\,x^2\,\sqrt{x+1}}{3003}+\frac{x^3\,\sqrt{x+1}}{39}-\frac{43\,x^4\,\sqrt{x+1}}{3003}+\frac{2\,x^5\,\sqrt{x+1}}{429}-\frac{2\,x^6\,\sqrt{x+1}}{3003}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((119*x*(x + 1)^(1/2))/429 + (310*(x + 1)^(1/2))/3003 + (65
9*x^2*(x + 1)^(1/2))/3003 + (x^3*(x + 1)^(1/2))/39 - (43*x^4*(x + 1)^(1/2))
/3003 + (2*x^5*(x + 1)^(1/2))/429 - (2*x^6*(x + 1)^(1/2))/3003))/(7*x - 21*
x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

3.1102 $\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}} dx$

Optimal. Leaf size=101

$$\frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}$$

```
[Out] 1/15*(1+x)^(7/2)/(1-x)^(15/2)+4/195*(1+x)^(7/2)/(1-x)^(13/2)+4/715*(1+x)^(7
/2)/(1-x)^(11/2)+8/6435*(1+x)^(7/2)/(1-x)^(9/2)+8/45045*(1+x)^(7/2)/(1-x)^(
7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] (1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*(1 + x)^(7/2))/(195*(1 - x)^(13/2))
+ (4*(1 + x)^(7/2))/(715*(1 - x)^(11/2)) + (8*(1 + x)^(7/2))/(6435*(1 - x)^
(9/2)) + (8*(1 + x)^(7/2))/(45045*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx = \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4}{15}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4}{65}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8}{715}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}} + \frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx}{6435}$$
$$= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}} + \frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.40

$$\frac{(1+x)^{7/2}\,(4243-1628x+468x^2-88x^3+8x^4)}{45045(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] ((1 + x)^(7/2)*(4243 - 1628*x + 468*x^2 - 88*x^3 + 8*x^4))/(45045*(1 - x)^(
15/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Maple [A]
time = 0.16, size = 142, normalized size = 1.41

method	result
gosper	$\frac{(1+x)^{\frac{7}{2}}\,(8x^4-88x^3+468x^2-1628x+4243)}{45045(1-x)^{\frac{15}{2}}}$

risch	$-\frac{\sqrt{(1+x)(1-x)}\,(8x^8-56x^7+164x^6-252x^5+195x^4+8988x^3+19414x^2+15344x+4243)}{45045\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^7\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{5(1-x)^{\frac{15}{2}}}-\frac{(1+x)^{\frac{3}{2}}}{6(1-x)^{\frac{15}{2}}}+\frac{\sqrt{1+x}}{15(1-x)^{\frac{15}{2}}}-\frac{\sqrt{1+x}}{390(1-x)^{\frac{13}{2}}}-\frac{\sqrt{1+x}}{715(1-x)^{\frac{11}{2}}}-\frac{\sqrt{1+x}}{1287(1-x)^{\frac{9}{2}}}-\frac{4\sqrt{1+x}}{9009(1-x)^{\frac{7}{2}}}-\frac{4\sqrt{1+x}}{15015(1-x)^{\frac{5}{2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(17/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1+x)^(5/2)/(1-x)^(15/2)-1/6*(1+x)^(3/2)/(1-x)^(15/2)+1/15/(1-x)^(15/2)
*(1+x)^(1/2)-1/390*(1+x)^(1/2)/(1-x)^(13/2)-1/715*(1+x)^(1/2)/(1-x)^(11/2)-
1/1287*(1+x)^(1/2)/(1-x)^(9/2)-4/9009*(1+x)^(1/2)/(1-x)^(7/2)-4/15015*(1+x)
^(1/2)/(1-x)^(5/2)-8/45045*(1+x)^(1/2)/(1-x)^(3/2)-8/45045*(1+x)^(1/2)/(1-x
)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(71) = 142$.
time = 0.26, size = 386, normalized size = 3.82

$$\frac{(-x^2+1)^{\frac{5}{2}}}{5\,(x^{10}-10\,x^9+45\,x^8-120\,x^7+210\,x^6-252\,x^5+210\,x^4-120\,x^3+45\,x^2-10\,x+1)}+\frac{(-x^2+1)^{\frac{3}{2}}}{6\,(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1)}+\frac{\sqrt{-x^2+1}}{15\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}+\frac{\sqrt{-x^2+1}}{390\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}-\frac{\sqrt{-x^2+1}}{715\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}+\frac{\sqrt{-x^2+1}}{1287\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}-\frac{4\,\sqrt{-x^2+1}}{9009\,(x^4-4\,x^3+6\,x^2-4\,x+1)}+\frac{4\,\sqrt{-x^2+1}}{15015\,(x^3-3\,x^2+3\,x-1)}-\frac{8\,\sqrt{-x^2+1}}{45045\,(x^2-2\,x+1)}+\frac{8\,\sqrt{-x^2+1}}{45045\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5
+ 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1) + 1/6*(-x^2 + 1)^(3/2)/(x^9 - 9*x^
8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) + 1/15
*sqrt(-x^2 + 1)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 -
 8*x + 1) + 1/390*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 -
21*x^2 + 7*x - 1) - 1/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 1
5*x^2 - 6*x + 1) + 1/1287*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5
*x - 1) - 4/9009*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/15015*s
qrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 8/45045*sqrt(-x^2 + 1)/(x^2 - 2*x +
 1) + 8/45045*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 130, normalized size = 1.29

$$\frac{4243\,x^8-33944\,x^7+118804\,x^6-237608\,x^5+297010\,x^4-237608\,x^3+118804\,x^2+(8\,x^7-64\,x^6+228\,x^5-480\,x^4+675\,x^3+8313\,x^2+11101\,x+4243)\sqrt{x+1}\,\sqrt{-x+1}-33944\,x+4243}{45045\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="fricas")
```

```
[Out] 1/45045*(4243*x^8 - 33944*x^7 + 118804*x^6 - 237608*x^5 + 297010*x^4 - 2376
08*x^3 + 118804*x^2 + (8*x^7 - 64*x^6 + 228*x^5 - 480*x^4 + 675*x^3 + 8313*
```

```
x^2 + 11101*x + 4243)*sqrt(x + 1)*sqrt(-x + 1) - 33944*x + 4243)/(x^8 - 8*x
^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(17/2),x)
```

```
[Out] Exception raised: SystemError
```

Giac [A]
time = 0.03, size = 147, normalized size = 1.46

$$\frac{2\left(\left(\left(\left(\frac{4}{45045}\sqrt{x+1}\sqrt{x+1}-\frac{4}{3003}\right)\sqrt{x+1}\sqrt{x+1}+\frac{2}{231}\right)\sqrt{x+1}\sqrt{x+1}-\frac{2}{63}\right)\sqrt{x+1}\sqrt{x+1}+\frac{1}{14}\right)\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x)
```

```
[Out] 1/45045*(4*((2*(x + 1)*(x - 14) + 195)*(x + 1) - 715)*(x + 1) + 6435)*(x +
1)^(7/2)*sqrt(-x + 1)/(x - 1)^8
```

Mupad [B]
time = 0.35, size = 124, normalized size = 1.23

$$\frac{\sqrt{1-x}\,\left(\frac{11101\,x\,\sqrt{x+1}}{45045}+\frac{4243\,\sqrt{x+1}}{45045}+\frac{2771\,x^2\,\sqrt{x+1}}{15015}+\frac{15\,x^3\,\sqrt{x+1}}{1001}-\frac{32\,x^4\,\sqrt{x+1}}{3003}+\frac{76\,x^5\,\sqrt{x+1}}{15015}-\frac{64\,x^6\,\sqrt{x+1}}{45045}+\frac{8\,x^7\,\sqrt{x+1}}{45045}\right)}{x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(17/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11101*x*(x + 1)^(1/2))/45045 + (4243*(x + 1)^(1/2))/45045
+ (2771*x^2*(x + 1)^(1/2))/15015 + (15*x^3*(x + 1)^(1/2))/1001 - (32*x^4*(x
 + 1)^(1/2))/3003 + (76*x^5*(x + 1)^(1/2))/15015 - (64*x^6*(x + 1)^(1/2))/4
5045 + (8*x^7*(x + 1)^(1/2))/45045))/(28*x^2 - 8*x - 56*x^3 + 70*x^4 - 56*x
^5 + 28*x^6 - 8*x^7 + x^8 + 1)
```

3.1103 $\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}} dx$

Optimal. Leaf size=121

$$\frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{153153(1-x)^{7/2}}$$

```
[Out] 1/17*(1+x)^(7/2)/(1-x)^(17/2)+1/51*(1+x)^(7/2)/(1-x)^(15/2)+4/663*(1+x)^(7/
2)/(1-x)^(13/2)+4/2431*(1+x)^(7/2)/(1-x)^(11/2)+8/21879*(1+x)^(7/2)/(1-x)^(
9/2)+8/153153*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] (1 + x)^(7/2)/(17*(1 - x)^(17/2)) + (1 + x)^(7/2)/(51*(1 - x)^(15/2)) + (4*
(1 + x)^(7/2))/(663*(1 - x)^(13/2)) + (4*(1 + x)^(7/2))/(2431*(1 - x)^(11/2
)) + (8*(1 + x)^(7/2))/(21879*(1 - x)^(9/2)) + (8*(1 + x)^(7/2))/(153153*(1
 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}}\,dx &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{5}{17}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4}{51}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4}{221}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx}{2431}\\ &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}}+\\ &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}}+\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 45, normalized size = 0.37

$$\frac{(1+x)^{7/2}\left(13252-5871x+2096x^2-556x^3+96x^4-8x^5\right)}{153153(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(7/2)*(13252 - 5871*x + 2096*x^2 - 556*x^3 + 96*x^4 - 8*x^5))/(153
153*(1 - x)^(17/2))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^(5/2)/(1 - x)^(19/2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8437 deep
```

Maple [**A**]
time = 0.18, size = 156, normalized size = 1.29

method	result

gosper	$-\frac{(1+x)^{\frac{7}{2}}(8x^5-96x^4+556x^3-2096x^2+5871x-13252)}{153153(1-x)^{\frac{17}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(8x^9-64x^8+220x^7-416x^6+447x^5-216x^4-25610x^3-58124x^2-47137x-13252)}{153153\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^8\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{6(1-x)^{\frac{17}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{42(1-x)^{\frac{17}{2}}}+\frac{5\sqrt{1+x}}{119(1-x)^{\frac{17}{2}}}-\frac{\sqrt{1+x}}{714(1-x)^{\frac{15}{2}}}-\frac{\sqrt{1+x}}{1326(1-x)^{\frac{13}{2}}}-\frac{\sqrt{1+x}}{2431(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{21879(1-x)^{\frac{9}{2}}}-\frac{20\sqrt{1+}}{153153(1-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(19/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)^(5/2)/(1-x)^(17/2)-5/42*(1+x)^(3/2)/(1-x)^(17/2)+5/119/(1-x)^(17/
2)*(1+x)^(1/2)-1/714/(1-x)^(15/2)*(1+x)^(1/2)-1/1326*(1+x)^(1/2)/(1-x)^(13/
2)-1/2431*(1+x)^(1/2)/(1-x)^(11/2)-5/21879*(1+x)^(1/2)/(1-x)^(9/2)-20/15315
3*(1+x)^(1/2)/(1-x)^(7/2)-4/51051*(1+x)^(1/2)/(1-x)^(5/2)-8/153153*(1+x)^(1
/2)/(1-x)^(3/2)-8/153153*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(85) = 170$.
time = 0.27, size = 452, normalized size = 3.74

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{6(x^{11}-11x^{10}+55x^9-165x^8+330x^7-462x^6+462x^5-330x^4+165x^3-55x^2+11x-1)}-\frac{5(-x^2+1)^{\frac{3}{2}}}{42(x^{10}-10x^9+45x^8-120x^7+210x^6-252x^5+210x^4-120x^3+45x^2-10x+1)}-\frac{5\sqrt{-x^2+1}}{119(x^9-9x^8+36x^7-84x^6+126x^5-126x^4+84x^3-36x^2+9x-1)}-\frac{\sqrt{-x^2+1}}{714(x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1)}+\frac{\sqrt{-x^2+1}}{1326(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}-\frac{\sqrt{-x^2+1}}{2431(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}+\frac{5\sqrt{-x^2+1}}{21879(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{20\sqrt{-x^2+1}}{153153(x^4-4x^3+6x^2-4x+1)}+\frac{4\sqrt{-x^2+1}}{51051(x^3-3x^2+3x-1)}-\frac{8\sqrt{-x^2+1}}{153153(x^2-2x+1)}+\frac{8\sqrt{-x^2+1}}{153153(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^
6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1) - 5/42*(-x^2 + 1)^(3/2
)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5 + 210*x^4 - 120*x^3
 + 45*x^2 - 10*x + 1) - 5/119*sqrt(-x^2 + 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6
 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) - 1/714*sqrt(-x^2 + 1)/(x
^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) + 1/1326
*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1)
 - 1/2431*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
 + 5/21879*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 20/15
3153*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/51051*sqrt(-x^2 + 1
)/(x^3 - 3*x^2 + 3*x - 1) - 8/153153*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/153
153*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 145, normalized size = 1.20

$$\frac{13252x^9-119268x^8+477072x^7-1113168x^6+1669752x^5-1669752x^4+1113168x^3-477072x^2+(8x^8-72x^7+292x^6-708x^5+1155x^4-1371x^3-24239x^2-33885x-13252)\sqrt{x+1}\sqrt{-x+1}+119268x-13252}{153153(x^9-9x^8+36x^7-84x^6+126x^5-126x^4+84x^3-36x^2+9x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="fricas")
```

```
[Out] 1/153153*(13252*x^9 - 119268*x^8 + 477072*x^7 - 1113168*x^6 + 1669752*x^5 -
 1669752*x^4 + 1113168*x^3 - 477072*x^2 + (8*x^8 - 72*x^7 + 292*x^6 - 708*x
^5 + 1155*x^4 - 1371*x^3 - 24239*x^2 - 33885*x - 13252)*sqrt(x + 1)*sqrt(-x
 + 1) + 119268*x - 13252)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^
4 + 84*x^3 - 36*x^2 + 9*x - 1)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(19/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.04, size = 167, normalized size = 1.38

$$\frac{2\left(\left(\left(\left(\left(\left(-\frac{4}{153153}\sqrt{x+1}\sqrt{x+1}+\frac{4}{9009}\right)\sqrt{x+1}\sqrt{x+1}-\frac{10}{3003}\right)\sqrt{x+1}\sqrt{x+1}+\frac{10}{693}\right)\sqrt{x+1}\sqrt{x+1}-\frac{5}{126}\right)\sqrt{x+1}\sqrt{x+1}+\frac{1}{14}\right)\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{x+1}\sqrt{-x+1}}{(-x+1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x)
```

```
[Out] 1/153153*((4*((2*(x + 1)*(x - 16) + 255)*(x + 1) - 1105)*(x + 1) + 12155)*(
x + 1) - 21879)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^9
```

Mupad [**B**]
time = 0.37, size = 140, normalized size = 1.16

$$-\frac{\sqrt{1-x}\,\left(\frac{3765\,x\,\sqrt{x+1}}{17017}+\frac{13252\,\sqrt{x+1}}{153153}+\frac{24239\,x^2\,\sqrt{x+1}}{153153}+\frac{457\,x^3\,\sqrt{x+1}}{51051}-\frac{5\,x^4\,\sqrt{x+1}}{663}+\frac{236\,x^5\,\sqrt{x+1}}{51051}-\frac{292\,x^6\,\sqrt{x+1}}{153153}+\frac{8\,x^7\,\sqrt{x+1}}{17017}-\frac{8\,x^8\,\sqrt{x+1}}{153153}\right)}{x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(19/2),x)
```

```
[Out] -((1 - x)^(1/2)*((3765*x*(x + 1)^(1/2))/17017 + (13252*(x + 1)^(1/2))/15315
3 + (24239*x^2*(x + 1)^(1/2))/153153 + (457*x^3*(x + 1)^(1/2))/51051 - (5*x
^4*(x + 1)^(1/2))/663 + (236*x^5*(x + 1)^(1/2))/51051 - (292*x^6*(x + 1)^(1
/2))/153153 + (8*x^7*(x + 1)^(1/2))/17017 - (8*x^8*(x + 1)^(1/2))/153153))/
(9*x - 36*x^2 + 84*x^3 - 126*x^4 + 126*x^5 - 84*x^6 + 36*x^7 - 9*x^8 + x^9
- 1)
```

3.1104 $\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=64

$$-\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

```
[Out] 3/2*arcsin(a*x)/a-1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*
x+1)^(1/2)/a
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {52, 41, 222}

$$-\frac{\sqrt{1-ax}\,(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\,\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] (-3*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a) - (Sqrt[1 - a*x]*(1 + a*x)^(3/2))/(2
*a) + (3*ArcSin[a*x])/(2*a)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3\sin^{-1}(ax)}{2a}$$

Mathematica [**A**]
time = 0.09, size = 64, normalized size = 1.00

$$-\frac{\frac{\sqrt{1-ax}\,(4+5ax+a^2x^2)}{\sqrt{1+ax}} + 6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
[Out] -1/2*((Sqrt[1 - a*x]*(4 + 5*a*x + a^2*x^2))/Sqrt[1 + a*x] + 6*ArcTan[Sqrt[1
 - a*x]/Sqrt[1 + a*x]])/a
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]')
[Out] cought exception:
```

Maple [**A**]
time = 0.15, size = 98, normalized size = 1.53

method	result
default	$-\frac{(ax+1)^{\frac{3}{2}}\sqrt{-ax+1}}{2a} - \frac{3\sqrt{-ax+1}\,\sqrt{ax+1}}{2a} + \frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}$

risch	$\frac{(ax+4)\sqrt{ax+1}\,(ax-1)\sqrt{(ax+1)(-ax+1)}}{2a\sqrt{-(ax+1)(ax-1)}\,\sqrt{-ax+1}}+\frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a+3/2*
((a*x+1)*(-a*x+1))^(1/2)/(a*x+1)^(1/2)/(-a*x+1)^(1/2)/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.37, size = 42, normalized size = 0.66

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Fricas [A]
time = 0.30, size = 55, normalized size = 0.86

$$-\frac{(ax+4)\sqrt{ax+1}\,\sqrt{-ax+1}+6\arctan\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) + 6*arctan((sqrt(a*x + 1)*sqrt
(-a*x + 1) - 1)/(a*x)))/a
```

Sympy [A]
time = 16.28, size = 88, normalized size = 1.38

$$\begin{cases}\frac{2\left(\begin{cases}-\frac{ax\sqrt{-ax+1}\,\sqrt{ax+1}}{4}-\sqrt{-ax+1}\,\sqrt{ax+1}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{ax+1}}{2}\right)}{2} & \text{for}\ \sqrt{ax+1}>-\sqrt{2}\wedge\sqrt{ax+1}<\sqrt{2}\end{cases}\right)}{a} & \text{for}\ a\neq 0\\ x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((-a*x*sqrt(-a*x + 1)*sqrt(a*x + 1)/4 - sqrt(-a*x + 1
)*sqrt(a*x + 1) + 3*asin(sqrt(2)*sqrt(a*x + 1)/2)/2, (sqrt(a*x + 1) < sqrt(
2)) & (sqrt(a*x + 1) > -sqrt(2))))/a, Ne(a, 0)), (x, True))
```

Giac [A]
time = 0.00, size = 71, normalized size = 1.11

$$\frac{2\left(-\frac{3}{4}-\frac{1}{4}\sqrt{ax+1}\,\sqrt{ax+1}\right)\sqrt{ax+1}\,\sqrt{-ax+1}+3\arcsin\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 6*arcsin(1/2*sqrt(2)*sqrt(a*
x + 1)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,x+1)^{3/2}}{\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2),x)
```

```
[Out] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2), x)
```

3.1105 $\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx$

Optimal. Leaf size=62

$$-\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}+\frac{3\sin^{-1}(ax)}{2a}$$

```
[Out] -1/2*(-a^2*x^2+1)^(3/2)/a/(-a*x+1)+3/2*arcsin(a*x)/a-3/2*(-a^2*x^2+1)^(1/2)
/a
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {809, 679, 222}

$$-\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*a) - (1 - a^2*x^2)^(3/2)/(2*a*(1 - a*x)) + (3*Arc
Sin[a*x])/(2*a)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx &= -\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}+\frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx\\ &= -\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}+\frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}+\frac{3\sin^{-1}(ax)}{2a}\end{aligned}$$

Mathematica [**A**]
time = 0.16, size = 69, normalized size = 1.11

$$\frac{(-4-ax)\sqrt{1-a^2x^2}}{2a}-\frac{3\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{2\sqrt{-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
[Out] ((-4 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) - (3*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^
2*x^2]])/(2*Sqrt[-a^2])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]')
[Out] cought exception:
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(52) = 104.
time = 0.16, size = 120, normalized size = 1.94

method	result
risch	$\frac{(ax+4)(a^2x^2-1)}{2a\sqrt{-a^2x^2+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$

default	$-\frac{x\sqrt{-a^2x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-\frac{2\left(\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}-\frac{a\arctan\left(\frac{}{\sqrt{-a^2\left(x-\right.}}\right.}{}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(-a^2*x^2+1)^(1/2)-1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-2/a*((-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)))
```

Maxima [A]
time = 0.34, size = 42, normalized size = 0.68

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin\left(ax\right)}{2\,a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Fricas [A]
time = 0.30, size = 48, normalized size = 0.77

$$-\frac{\sqrt{-a^2x^2+1}\,(ax+4)+6\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a
```

Sympy [A]
time = 3.11, size = 76, normalized size = 1.23

$$-\begin{cases}-\frac{-\sqrt{-a^2x^2+1}+\operatorname{asin}(ax)}{a} & \text{for}\: ax>-1\wedge ax<1\end{cases}-\begin{cases}-\frac{-\frac{ax\sqrt{-a^2x^2+1}}{2}-\sqrt{-a^2x^2+1}+\frac{\operatorname{asin}(ax)}{2}}{a} & \text{for}\: ax>-1\wedge ax<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

```
[Out] -Piecewise((-(-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, (a*x > -1) & (a*x < 1))
) - Piecewise((-(-a*x*sqrt(-a**2*x**2 + 1)/2 - sqrt(-a**2*x**2 + 1) + asin(
a*x)/2)/a, (a*x > -1) & (a*x < 1)))
```

Giac [A]
time = 0.01, size = 47, normalized size = 0.76

$$2\left(-\frac{2ax}{8a}-\frac{8}{8a}\right)\sqrt{-a^2x^2+1}+\frac{3\operatorname{sign}\left(a\right)\arcsin\left(ax\right)}{2\left|a\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x)
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

Mupad [B]
time = 0.15, size = 55, normalized size = 0.89

$$\frac{\frac{3\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{2}+\sqrt{1-a^2x^2}\left(\frac{2a}{\sqrt{-a^2}}-\frac{x\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - a^2*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] ((3*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a)/(-a^2)^(1/2) - (x
*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

3.1106 $\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=87

$$\frac{35}{8}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{24}(1-x)^{3/2}\sqrt{1+x}+\frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{8}\sin^{-1}(x)$$

```
[Out] 35/8*arcsin(x)+35/24*(1-x)^(3/2)*(1+x)^(1/2)+7/12*(1-x)^(5/2)*(1+x)^(1/2)+1
/4*(1-x)^(7/2)*(1+x)^(1/2)+35/8*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\frac{1}{4}\sqrt{x+1}\,(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (35*Sqrt[1 - x]*Sqrt[1 + x])/8 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/24 + (7*(1
 - x)^(5/2)*Sqrt[1 + x])/12 + ((1 - x)^(7/2)*Sqrt[1 + x])/4 + (35*ArcSin[x])
/8
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx &= \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{7}{4}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{12}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} \\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} \\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x}
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 63, normalized size = 0.72

$$\frac{\sqrt{1+x}\,(160-241x+113x^2-38x^3+6x^4)}{24\sqrt{1-x}} + \frac{35}{4}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(160 - 241*x + 113*x^2 - 38*x^3 + 6*x^4))/(24*Sqrt[1 - x]) + (
35*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 14.28, size = 157, normalized size = 1.80

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-210\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]-163\sqrt{-1+x}\,(1+x)^{\frac{3}{2}}-6\sqrt{-1+x}\,(1+x)^{\frac{7}{2}}+50\sqrt{-1+x}\,(1+x)^{\frac{5}{2}}+279\sqrt{-1+x}\,\sqrt{1+x}\right)}{24},\text{Abs}[1+x]>2\right\}\right\},\frac{-605\,(1+x)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{31\,(1+x)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{(1+x)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{35\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]}{4}+\frac{263\,(1+x)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{93\sqrt{1+x}}{4\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(7/2)/(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 24 (-210 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] - 163 Sqrt[-1 + x
] (1 + x) ^ (3 / 2) - 6 Sqrt[-1 + x] (1 + x) ^ (7 / 2) + 50 Sqrt[-1 + x] (1
 + x) ^ (5 / 2) + 279 Sqrt[-1 + x] Sqrt[1 + x]), Abs[1 + x] > 2}}, -605 (1
+ x) ^ (3 / 2) / (24 Sqrt[1 - x]) - 31 (1 + x) ^ (7 / 2) / (12 Sqrt[1 - x])
 + (1 + x) ^ (9 / 2) / (4 Sqrt[1 - x]) + 35 ArcSin[Sqrt[2] Sqrt[1 + x] / 2]
 / 4 + 263 (1 + x) ^ (5 / 2) / (24 Sqrt[1 - x]) + 93 Sqrt[1 + x] / (4 Sqrt[
1 - x])]
```

Maple [A]
time = 0.14, size = 85, normalized size = 0.98

method	result
risch	$\frac{(6x^3-32x^2+81x-160)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}\sqrt{1+x}}{4}+\frac{7(1-x)^{\frac{5}{2}}\sqrt{1+x}}{12}+\frac{35(1-x)^{\frac{3}{2}}\sqrt{1+x}}{24}+\frac{35\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{35\sqrt{(1+x)(1-x)}}{8\sqrt{1+x}\,\sqrt{1-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(1-x)^(7/2)*(1+x)^(1/2)+7/12*(1-x)^(5/2)*(1+x)^(1/2)+35/24*(1-x)^(3/2)*
(1+x)^(1/2)+35/8*(1-x)^(1/2)*(1+x)^(1/2)+35/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/
2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.34, size = 56, normalized size = 0.64

$$-\frac{1}{4}\sqrt{-x^2+1}\,x^3+\frac{4}{3}\sqrt{-x^2+1}\,x^2-\frac{27}{8}\sqrt{-x^2+1}\,x+\frac{20}{3}\sqrt{-x^2+1}+\frac{35}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(-x^2 + 1)*x^3 + 4/3*sqrt(-x^2 + 1)*x^2 - 27/8*sqrt(-x^2 + 1)*x +
20/3*sqrt(-x^2 + 1) + 35/8*arcsin(x)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 0.60

$$-\frac{1}{24}\left(6\,x^3-32\,x^2+81\,x-160\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{35}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] -1/24*(6*x^3 - 32*x^2 + 81*x - 160)*sqrt(x + 1)*sqrt(-x + 1) - 35/4*arctan(
(sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 16.46, size = 197, normalized size = 2.26

$$\begin{cases} -\frac{i\sqrt{x-1}\,(x+1)^{\frac{7}{2}}}{4}+\frac{25i\sqrt{x-1}\,(x+1)^{\frac{5}{2}}}{12}-\frac{163i\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{24}+\frac{93i\sqrt{x-1}\,\sqrt{x+1}}{8}-\frac{35i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4} & \text{for } |x+1|>2 \\ \frac{35\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}-\frac{31(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{263(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}-\frac{605(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}+\frac{93\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x - 1)*(x + 1)**(7/2)/4 + 25*I*sqrt(x - 1)*(x + 1)**(5/2
)/12 - 163*I*sqrt(x - 1)*(x + 1)**(3/2)/24 + 93*I*sqrt(x - 1)*sqrt(x + 1)/8
 - 35*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4, Abs(x + 1) > 2), (35*asin(sqrt(2)*s
qrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*sqrt(1 - x)) - 31*(x + 1)**(7/2)/(12*sq
rt(1 - x)) + 263*(x + 1)**(5/2)/(24*sqrt(1 - x)) - 605*(x + 1)**(3/2)/(24*s
qrt(1 - x)) + 93*sqrt(x + 1)/(4*sqrt(1 - x)), True))
```

Giac [A]
time = 0.01, size = 107, normalized size = 1.23

$$2\left(\left(\left(\left(\frac{1}{8}\sqrt{-x+1}\sqrt{-x+1}+\frac{7}{24}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{35}{48}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{35}{16}\right)\sqrt{-x+1}\sqrt{x+1}-\frac{35}{4}\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x)
```

```
[Out] -1/24*((2*(3*x - 10)*(x - 1) + 35)*(x - 1) - 105)*sqrt(x + 1)*sqrt(-x + 1)
- 35/4*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(1/2), x)
```

3.1107 $\int \frac{(1-x)^{5/2}}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=67

$$\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] 5/2*arcsin(x)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+1/3*(1-x)^(5/2)*(1+x)^(1/2)+5/2*(
1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\frac{1}{3}\sqrt{x+1}\,(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*(1 - x)^(3/2)*Sqrt[1 + x])/6 + ((1 - x)^
(5/2)*Sqrt[1 + x])/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx &= \frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\ &= \frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\ &= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}} \\ &= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.84

$$\frac{\sqrt{1+x}\,(22-31x+11x^2-2x^3)}{6\sqrt{1-x}}+5\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(22 - 31*x + 11*x^2 - 2*x^3))/(6*Sqrt[1 - x]) + 5*ArcTan[Sqrt[
1 + x]/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.90, size = 133, normalized size = 1.99

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-66\sqrt{1+x}-30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{-1+x}-17(1+x)^{\frac{5}{2}}+2(1+x)^{\frac{7}{2}}+59(1+x)^{\frac{3}{2}}\right)}{6\sqrt{-1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-59(1+x)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{(1+x)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{17(1+x)^{\frac{5}{2}}}{6\sqrt{1-x}}+5\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]+\frac{11\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(5/2)/(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 6 (-66 Sqrt[1 + x] - 30 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sq
rt[-1 + x] - 17 (1 + x) ^ (5 / 2) + 2 (1 + x) ^ (7 / 2) + 59 (1 + x) ^ (3 /
 2)) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -59 (1 + x) ^ (3 / 2) / (6 Sqrt[1 -
x]) - (1 + x) ^ (7 / 2) / (3 Sqrt[1 - x]) + 17 (1 + x) ^ (5 / 2) / (6 Sqrt[
1 - x]) + 5 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 11 Sqrt[1 + x] / Sqrt[1 - x]]
```

Maple [A]
time = 0.16, size = 71, normalized size = 1.06

method	result	size
default	$\frac{(1-x)^{\frac{5}{2}}\sqrt{1+x}}{3}+\frac{5(1-x)^{\frac{3}{2}}\sqrt{1+x}}{6}+\frac{5\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$-\frac{(2x^2-9x+22)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1-x)^(5/2)*(1+x)^(1/2)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+5/2*(1-x)^(1/2)*(1+
x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.36, size = 42, normalized size = 0.63

$$\frac{1}{3}\sqrt{-x^2+1}\,x^2-\frac{3}{2}\sqrt{-x^2+1}\,x+\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x + 11/3*sqrt(-x^2 + 1) + 5/2*a
rcsin(x)
```

Fricas [A]
time = 0.31, size = 47, normalized size = 0.70

$$\frac{1}{6}\left(2x^2-9x+22\right)\sqrt{x+1}\,\sqrt{-x+1}-5\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sqr
t(-x + 1) - 1)/x)
```

Sympy [A]
time = 4.54, size = 173, normalized size = 2.58

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{17i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{59i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}-\frac{11i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{17(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{59(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}+\frac{11\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - 17*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 59*I*(x + 1)**(3/2)/(6*sqrt(x
- 1)) - 11*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt
(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 17*(x + 1)**(5/2)/(6*sqrt(1 -
 x)) - 59*(x + 1)**(3/2)/(6*sqrt(1 - x)) + 11*sqrt(x + 1)/sqrt(1 - x), True
))
```

Giac [A]
time = 0.01, size = 83, normalized size = 1.24

$$2\left(\left(\frac{1}{6}\sqrt{-x+1}\sqrt{-x+1}+\frac{5}{12}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{5}{4}\right)\sqrt{-x+1}\sqrt{x+1}-5\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x)
```

```
[Out] 1/6*((2*x - 7)*(x - 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) - 5*arcsin(1/2*sqrt(2
)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(1/2), x)
```

3.1108 $\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\, dx$

Optimal. Leaf size=47

$$\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}\,+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}\,+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)+1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\frac{1}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\,\sqrt{1-x}\,+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 - x]*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*ArcSin[x
])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx &= \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.04

$$\frac{\sqrt{1+x}\,(4-5x+x^2)}{2\sqrt{1-x}} + 3\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)^(3/2)/Sqrt[1 + x],x]

[Out] (Sqrt[1 + x]*(4 - 5*x + x^2))/(2*Sqrt[1 - x]) + 3*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.60, size = 112, normalized size = 2.38

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-10\sqrt{1+x} - 6\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\sqrt{-1+x} - (1+x)^{\frac{5}{2}} + 7(1+x)^{\frac{3}{2}}\right)}{2\sqrt{-1+x}}, \text{Abs}[1+x] > 2\right\}\right\}, \frac{-7(1+x)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{(1+x)^{\frac{5}{2}}}{2\sqrt{1-x}} + 3\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right] + \frac{5\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 - x)^(3/2)/(1 + x)^(1/2),x]')

[Out] Piecewise[{{I / 2 (-10 Sqrt[1 + x] - 6 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[-1 + x] - (1 + x) ^ (5 / 2) + 7 (1 + x) ^ (3 / 2)) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -7 (1 + x) ^ (3 / 2) / (2 Sqrt[1 - x]) + (1 + x) ^ (5 / 2) / (2 Sqrt[1 - x]) + 3 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 5 Sqrt[1 + x] / Sqrt[1 - x]]

Maple [A]
time = 0.16, size = 57, normalized size = 1.21

method	result	size

default	$\frac{(1-x)^{\frac{3}{2}}\sqrt{1+x}}{2}+\frac{3\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{3\sqrt{(1+x)\,(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$\frac{(x-4)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)\,(1-x)}}{2\sqrt{-(1+x)\,(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)\,(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^(
1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x+2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x + 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 40, normalized size = 0.85

$$-\frac{1}{2}\sqrt{x+1}\,(x-4)\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

Sympy [A]
time = 1.53, size = 138, normalized size = 2.94

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{7i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}-\frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{7(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 7*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 5*I*sqrt(x + 1)/sqrt(x - 1), Ab
s(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 -
x)) - 7*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 5*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.01, size = 61, normalized size = 1.30

$$2\left(\frac{1}{4}\sqrt{-x+1}\,\sqrt{-x+1} + \frac{3}{4}\right)\sqrt{-x+1}\,\sqrt{x+1} - 3\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x)
```

```
[Out] -1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(1/2), x)
```

3.1109 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{1-x}\,\sqrt{1+x}+\sin^{-1}(x)$$

```
[Out] arcsin(x)+(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 41, 222}

$$\sqrt{1-x}\,\sqrt{x+1}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x]*Sqrt[1 + x] + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = \sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \sqrt{1-x}\,\sqrt{1+x} + \sin^{-1}(x)$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 1.60

$$\sqrt{1-x^2} + 2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x^2] + 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.96, size = 89, normalized size = 4.45

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left((1+x)^{\frac{3}{2}} - 2\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\sqrt{-1+x} - 2\sqrt{1+x}\right)}{\sqrt{-1+x}}, \text{Abs}\,[1+x] > 2\right\}\right\}, -\frac{(1+x)^{\frac{3}{2}}}{\sqrt{1-x}} + 2\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right] + \frac{2\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/2)/(1 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I ((1 + x) ^ (3 / 2) - 2 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[
-1 + x] - 2 Sqrt[1 + x]) / Sqrt[-1 + x], Abs[1 + x] > 2}}, -(1 + x) ^ (3 /
2) / Sqrt[1 - x] + 2 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 2 Sqrt[1 + x] / Sqrt
[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(16) = 32$.
time = 0.16, size = 41, normalized size = 2.05

method	result	size
default	$\sqrt{1-x}\,\sqrt{1+x} + \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	41

risch	$-\frac{\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(
x)
```

Maxima [A]
time = 0.35, size = 12, normalized size = 0.60

$$\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(16) = 32.
time = 0.30, size = 36, normalized size = 1.80

$$\sqrt{x+1}\,\sqrt{-x+1}-2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.79, size = 99, normalized size = 4.95

$$\begin{cases} -2i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 2\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
 - 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (2*asin(sqrt(2)*sqrt(x + 1
)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.80

$$\sqrt{-x+1}\,\sqrt{x+1}\; - 2\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [B]
time = 0.12, size = 12, normalized size = 0.60

$$\operatorname{asin}(x) + \sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] asin(x) + (1 - x^2)^(1/2)
```

3.1110 $\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] arcsin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {41, 222}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.02, size = 14, normalized size = 7.00

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcTan[x/Sqrt[1 - x^2]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.31, size = 35, normalized size = 17.50

$$\text{Piecewise}\left[\left\{\left\{-2I\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right],\text{Abs}\,[1+x]>2\right\}\right\},2\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(1/2)*(1 + x)^(1/2)),x]')
```

```
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[2] Sqrt[1 + x] / 2], Abs[1 + x] > 2}}, 2 ArcS
in[Sqrt[2] Sqrt[1 + x] / 2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(2) = 4.
time = 0.14, size = 27, normalized size = 13.50

method	result	size
default	$\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.37, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(2) = 4.
time = 0.30, size = 22, normalized size = 11.00

$$-2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 39, normalized size = 19.50

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } |x+1| > 2 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1) > 2), (2*asin(sqrt
(2)*sqrt(x + 1)/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(2) = 4.
time = 0.00, size = 19, normalized size = 9.50

$$-2 \arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] -2*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [B]
time = 0.08, size = 22, normalized size = 11.00

$$-4 \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] -4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1))
```

3.1111 $\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{1+x}}{\sqrt{1-x}}$$

```
[Out] (1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.66, size = 48, normalized size = 2.82

$$\text{Piecewise}\left[\left\{\left\{-1+\frac{2}{1+x},\left\{\frac{1-x}{1+x}!{=}0\right\}\&\&\frac{1}{\text{Abs}\,[1+x]}>\frac{1}{2}\right\}\right\},-\frac{I}{\sqrt{1-\frac{2}{1+x}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(3/2)*(1 + x)^(1/2)),x]')
```

```
[Out] Piecewise[{{-1 + 2 / (1 + x), {(1 - x) / (1 + x) != 0} && 1 / Abs[1 + x] >
1 / 2}}, -I / Sqrt[1 - 2 / (1 + x)]]
```

Maple [A]
time = 0.13, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{\sqrt{1+x}}{\sqrt{1-x}}$	14
default	$\frac{\sqrt{1+x}}{\sqrt{1-x}}$	14
risch	$\frac{\sqrt{(1+x)(1-x)}\,\sqrt{1+x}}{\sqrt{1-x}\,\sqrt{-(1+x)(-1+x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.35, size = 16, normalized size = 0.94

$$-\frac{\sqrt{-x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 1.35

$$\frac{x - \sqrt{x+1}\,\sqrt{-x+1} - 1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] (x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x - 1)
```

Sympy [A]
time = 0.49, size = 31, normalized size = 1.82

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(1/2),x)
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)), 1/Abs(x + 1) > 1/2), (-I/sqrt(1 - 2/(x +
 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(13) = 26$.
time = 0.00, size = 64, normalized size = 3.76

$$2\left(\frac{\sqrt{-x+1}}{2\left(-2\sqrt{x+1}+2\sqrt{2}\right)} - \frac{-2\sqrt{x+1}+2\sqrt{2}}{8\sqrt{-x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x)
[Out] -1/2*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) + 1/2*sqrt(-x + 1)/(sqrt(2) - sqr
t(x + 1))
```

Mupad [B]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(1/2)),x)
[Out] (x + 1)^(1/2)/(1 - x)^(1/2)
```

3.1112 $\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}$$

```
[Out] 1/3*(1+x)^(1/2)/(1-x)^(3/2)+1/3*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.56

$$-\frac{(-2+x)\sqrt{1+x}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] -1/3*((-2 + x)*Sqrt[1 + x])/(1 - x)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.87, size = 106, normalized size = 2.59

$$\text{Piecewise}\left[\left\{\left\{\frac{-2+x}{3(-1+x)\sqrt{\frac{1-x}{1+x}}},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},-\frac{I(1+x)}{-6\sqrt{1-\frac{2}{1+x}}+3(1+x)\sqrt{1-\frac{2}{1+x}}}+\frac{I3}{-6\sqrt{1-\frac{2}{1+x}}+3(1+x)\sqrt{1-\frac{2}{1+x}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(5/2)*(1 + x)^(1/2)),x]')
```

```
[Out] Piecewise[{{(-2 + x) / (3 (-1 + x) Sqrt[(1 - x) / (1 + x)]), 1 / Abs[1 + x]
 > 1 / 2}}, -I (1 + x) / (-6 Sqrt[1 - 2 / (1 + x)] + 3 (1 + x) Sqrt[1 - 2 /
 (1 + x)]) + I 3 / (-6 Sqrt[1 - 2 / (1 + x)] + 3 (1 + x) Sqrt[1 - 2 / (1 +
x)])]
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.73

method	result	size
gosper	$-\frac{\sqrt{1+x}\,(-2+x)}{3(1-x)^{\frac{3}{2}}}$	18
default	$\frac{\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^2-x-2)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+x)^(1/2)/(1-x)^(3/2)+1/3*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.35, size = 38, normalized size = 0.93

$$\frac{\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)}-\frac{\sqrt{-x^2+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 39, normalized size = 0.95

$$\frac{2\,x^2-\sqrt{x+1}\,(x-2)\sqrt{-x+1}\,-4\,x+2}{3\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x^2 - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [A]
time = 1.33, size = 128, normalized size = 3.12

$$\begin{cases} \frac{x+1}{3\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6\sqrt{-1+\frac{2}{x+1}}}-\frac{3}{3\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{i(x+1)}{3\sqrt{1-\frac{2}{x+1}}\,(x+1)-6\sqrt{1-\frac{2}{x+1}}}+\frac{3i}{3\sqrt{1-\frac{2}{x+1}}\,(x+1)-6\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise(((x + 1)/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))
) - 3/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))), 1/Abs(x +
1) > 1/2), (-I*(x + 1)/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1
))) + 3*I/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(29) = 58$.
time = 0.00, size = 149, normalized size = 3.63

$$-2\left(\frac{-\frac{1024}{3}\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3+\frac{1536\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{32768}+\frac{9\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2+1}{96\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x)
```

```
[Out] -1/48*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) - 3/16*(sqrt(2) - sqrt(x + 1
))/sqrt(-x + 1) - 1/48*(-x + 1)^(3/2)*(9*(sqrt(2) - sqrt(x + 1))^2/(x - 1)
- 1)/(sqrt(2) - sqrt(x + 1))^3
```

Mupad [B]
time = 0.31, size = 43, normalized size = 1.05

$$\frac{x\sqrt{1-x}+2\sqrt{1-x}-x^2\sqrt{1-x}}{3\,(x-1)^2\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x + 1
)^(1/2))
```

3.1113 $\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}$$

```
[Out] 1/5*(1+x)^(1/2)/(1-x)^(5/2)+2/15*(1+x)^(1/2)/(1-x)^(3/2)+2/15*(1+x)^(1/2)/(
1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*Sqrt[1 + x])/(15*(1 - x)^(3/2)) + (2*Sqr
t[1 + x])/(15*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2}{15}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 30, normalized size = 0.49

$$
\frac{\sqrt{1+x}\,(7-6x+2x^2)}{15(1-x)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(7 - 6*x + 2*x^2))/(15*(1 - x)^(5/2))
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 8.26, size = 211, normalized size = 3.46

$$
\text{Piecewise}\left[\left\{\left\{\frac{7-6x+2x^2}{15\left(1-2x+x^2\right)\sqrt{\frac{1-x}{1+x}}},\frac{1}{\text{Abs}\left[1+x\right]}>\frac{1}{2}\right\}\right\},\frac{-15I}{-60\left(1+x\right)\sqrt{1-\frac{2}{1+x}}+15\left(1+x\right)^2\sqrt{1-\frac{2}{1+x}}+60\sqrt{1-\frac{2}{1+x}}}-\frac{2I\left(1+x\right)^2}{-60\left(1+x\right)\sqrt{1-\frac{2}{1+x}}+15\left(1+x\right)^2\sqrt{1-\frac{2}{1+x}}+60\sqrt{1-\frac{2}{1+x}}}+\frac{I10\left(1+x\right)}{-60\left(1+x\right)\sqrt{1-\frac{2}{1+x}}+15\left(1+x\right)^2\sqrt{1-\frac{2}{1+x}}+60\sqrt{1-\frac{2}{1+x}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(7/2)*(1 + x)^(1/2)),x]')
```

```
[Out] Piecewise[{{(7 - 6 x + 2 x ^ 2) / (15 (1 - 2 x + x ^ 2) Sqrt[(1 - x) / (1 +
 x)]), 1 / Abs[1 + x] > 1 / 2}}, -15 I / (-60 (1 + x) Sqrt[1 - 2 / (1 + x)]
 + 15 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] + 60 Sqrt[1 - 2 / (1 + x)]) - 2 I (
1 + x) ^ 2 / (-60 (1 + x) Sqrt[1 - 2 / (1 + x)] + 15 (1 + x) ^ 2 Sqrt[1 - 2
 / (1 + x)] + 60 Sqrt[1 - 2 / (1 + x)]) + I 10 (1 + x) / (-60 (1 + x) Sqrt[
1 - 2 / (1 + x)] + 15 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] + 60 Sqrt[1 - 2 / (
1 + x)])]
```

Maple [**A**]
time = 0.14, size = 44, normalized size = 0.72

method	result	size

gosper	$\frac{\sqrt{1+x}\,(2x^2-6x+7)}{15(1-x)^{\frac{5}{2}}}$	25
default	$\frac{\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{15(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}$	44
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^3-4x^2+x+7)}{15\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1+x)^(1/2)/(1-x)^(5/2)+2/15*(1+x)^(1/2)/(1-x)^(3/2)+2/15*(1+x)^(1/2)/(
1-x)^(1/2)
```

Maxima [A]
time = 0.35, size = 64, normalized size = 1.05

$$-\frac{\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)}-\frac{2\,\sqrt{-x^2+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 2/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) - 2/15*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.30, size = 56, normalized size = 0.92

$$\frac{7\,x^3-21\,x^2-(2\,x^2-6\,x+7)\sqrt{x+1}\,\sqrt{-x+1}+21\,x-7}{15\,(x^3-3\,x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3 - 21*x^2 - (2*x^2 - 6*x + 7)*sqrt(x + 1)*sqrt(-x + 1) + 21*x -
7)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.54, size = 303, normalized size = 4.97

$$\begin{cases} \frac{2(x+1)^2}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)+60\sqrt{-1+\frac{2}{x+1}}}-\frac{10(x+1)}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)+60\sqrt{-1+\frac{2}{x+1}}}+\frac{15}{15\sqrt{-1+\frac{2}{x+1}}(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}(x+1)+60\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i(x+1)^2}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-\frac{2}{x+1}}}+\frac{10i(x+1)}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-\frac{2}{x+1}}}-\frac{15i}{15\sqrt{1-\frac{2}{x+1}}(x+1)^2-60\sqrt{1-\frac{2}{x+1}}(x+1)+60\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*(x + 1)**2/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 +
2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x + 1))) - 10*(x + 1)/(15*sqrt(-1 + 2/
(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x +
 1))) + 15/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x
 + 1) + 60*sqrt(-1 + 2/(x + 1))), 1/Abs(x + 1) > 1/2), (-2*I*(x + 1)**2/(15
*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(
1 - 2/(x + 1))) + 10*I*(x + 1)/(15*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt
(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1))) - 15*I/(15*sqrt(1 - 2/(x
+ 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1)))
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(43) = 86.
time = 0.01, size = 222, normalized size = 3.64

$$2\left(\frac{\frac{1}{5}\cdot 4294967296\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}+\frac{1}{3}\cdot 21474836480\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}-\frac{21474836480\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{1099511627776}+\frac{-150\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}-25\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}-3}{3840\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x)
```

```
[Out] -1/640*(sqrt(2) - sqrt(x + 1))^5/(-x + 1)^(5/2) - 5/384*(sqrt(2) - sqrt(x +
 1))^3/(-x + 1)^(3/2) - 5/64*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) + 1/1920*
(150*(sqrt(2) - sqrt(x + 1))^4/(x - 1)^2 - 25*(sqrt(2) - sqrt(x + 1))^2/(x
- 1) + 3)*(-x + 1)^(5/2)/(sqrt(2) - sqrt(x + 1))^5
```

Mupad [B]
time = 0.32, size = 55, normalized size = 0.90

$$-\frac{x\,\sqrt{1-x}+7\,\sqrt{1-x}-4\,x^2\,\sqrt{1-x}+2\,x^3\,\sqrt{1-x}}{15\,{\left(x-1\right)}^3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(1/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 7*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(15*(x - 1)^3*(x + 1)^(1/2))
```

3.1114 $\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}$$

```
[Out] 1/7*(1+x)^(1/2)/(1-x)^(7/2)+3/35*(1+x)^(1/2)/(1-x)^(5/2)+2/35*(1+x)^(1/2)/(
1-x)^(3/2)+2/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*Sqrt[1 + x])/(35*(1 - x)^(5/2)) + (2*Sqr
t[1 + x])/(35*(1 - x)^(3/2)) + (2*Sqrt[1 + x])/(35*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{6}{35}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2}{35}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 0.43

$$
\frac{\sqrt{1+x}\,(12-13x+8x^2-2x^3)}{35(1-x)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(12 - 13*x + 8*x^2 - 2*x^3))/(35*(1 - x)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 20.02, size = 350, normalized size = 4.32

$$
\text{Piecewise}\left[\left\{\left\{\frac{-12+13x-8x^2+2x^3}{35\left(-1+3x-3x^2+x^3\right)\sqrt{\frac{1-x}{1+x}}},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},\frac{-35I(1+x)}{-280\sqrt{1-\frac{2}{1+x}}-210(1+x)^2\sqrt{1-\frac{2}{1+x}}+35(1+x)^3\sqrt{1-\frac{2}{1+x}}+420(1+x)\sqrt{1-\frac{2}{1+x}}}-\frac{2I(1+x)^3}{-280\sqrt{1-\frac{2}{1+x}}-210(1+x)^2\sqrt{1-\frac{2}{1+x}}+35(1+x)^3\sqrt{1-\frac{2}{1+x}}+420(1+x)\sqrt{1-\frac{2}{1+x}}}+\frac{I14(1+x)^2}{-280\sqrt{1-\frac{2}{1+x}}-210(1+x)^2\sqrt{1-\frac{2}{1+x}}+35(1+x)^3\sqrt{1-\frac{2}{1+x}}+420(1+x)\sqrt{1-\frac{2}{1+x}}}+\frac{I35}{-280\sqrt{1-\frac{2}{1+x}}-210(1+x)^2\sqrt{1-\frac{2}{1+x}}+35(1+x)^3\sqrt{1-\frac{2}{1+x}}+420(1+x)\sqrt{1-\frac{2}{1+x}}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(9/2)*(1 + x)^(1/2)),x]')
```

```
[Out] Piecewise[{{(-12 + 13 x - 8 x ^ 2 + 2 x ^ 3) / (35 (-1 + 3 x - 3 x ^ 2 + x
^ 3) Sqrt[(1 - x) / (1 + x)]), 1 / Abs[1 + x] > 1 / 2}}, -35 I (1 + x) / (-
280 Sqrt[1 - 2 / (1 + x)] - 210 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] + 35 (1 +
 x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 420 (1 + x) Sqrt[1 - 2 / (1 + x)]) - 2 I (1
 + x) ^ 3 / (-280 Sqrt[1 - 2 / (1 + x)] - 210 (1 + x) ^ 2 Sqrt[1 - 2 / (1 +
 x)] + 35 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 420 (1 + x) Sqrt[1 - 2 / (1 +
 x)]) + I 14 (1 + x) ^ 2 / (-280 Sqrt[1 - 2 / (1 + x)] - 210 (1 + x) ^ 2 Sq
rt[1 - 2 / (1 + x)] + 35 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 420 (1 + x) Sq
rt[1 - 2 / (1 + x)]) + I 35 / (-280 Sqrt[1 - 2 / (1 + x)] - 210 (1 + x) ^ 2
 Sqrt[1 - 2 / (1 + x)] + 35 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 420 (1 + x)
 Sqrt[1 - 2 / (1 + x)])]
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.72

method	result	size
gosper	$-\frac{\sqrt{1+x}\,(2x^3-8x^2+13x-12)}{35(1-x)^{\frac{7}{2}}}$	30
default	$\frac{\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{35(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{35(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}$	58
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^4-6x^3+5x^2+x-12)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1+x)^(1/2)/(1-x)^(7/2)+3/35*(1+x)^(1/2)/(1-x)^(5/2)+2/35*(1+x)^(1/2)/(
1-x)^(3/2)+2/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.35, size = 95, normalized size = 1.17

$$\frac{\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{3\,\sqrt{-x^2+1}}{35\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{35\,(x^2-2\,x+1)}-\frac{2\,\sqrt{-x^2+1}}{35\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/35*sqrt(-x^
2 + 1)/(x - 1)
```

Fricas [A]
time = 0.29, size = 71, normalized size = 0.88

$$\frac{12\,x^4-48\,x^3+72\,x^2-(2\,x^3-8\,x^2+13\,x-12)\sqrt{x+1}\,\sqrt{-x+1}-48\,x+12}{35\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(12*x^4 - 48*x^3 + 72*x^2 - (2*x^3 - 8*x^2 + 13*x - 12)*sqrt(x + 1)*sq
rt(-x + 1) - 48*x + 12)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.20, size = 542, normalized size = 6.69

$$\begin{cases} \frac{2(x+1)^3}{35\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}\,(x+1)-280\sqrt{-1+\frac{2}{x+1}}}-\frac{14(x+1)^2}{35\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}\,(x+1)-280\sqrt{-1+\frac{2}{x+1}}}+\frac{35(x+1)}{35\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}\,(x+1)-280\sqrt{-1+\frac{2}{x+1}}}-\frac{35}{35\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}\,(x+1)-280\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i(x+1)^3}{35\sqrt{1-\frac{2}{x+1}}\,(x+1)^3-210\sqrt{1-\frac{2}{x+1}}\,(x+1)^2+420\sqrt{1-\frac{2}{x+1}}\,(x+1)-280\sqrt{1-\frac{2}{x+1}}}+\frac{14i(x+1)^2}{35\sqrt{1-\frac{2}{x+1}}\,(x+1)^3-210\sqrt{1-\frac{2}{x+1}}\,(x+1)^2+420\sqrt{1-\frac{2}{x+1}}\,(x+1)-280\sqrt{1-\frac{2}{x+1}}}-\frac{35i(x+1)}{35\sqrt{1-\frac{2}{x+1}}\,(x+1)^3-210\sqrt{1-\frac{2}{x+1}}\,(x+1)^2+420\sqrt{1-\frac{2}{x+1}}\,(x+1)-280\sqrt{1-\frac{2}{x+1}}}+\frac{35i}{35\sqrt{1-\frac{2}{x+1}}\,(x+1)^3-210\sqrt{1-\frac{2}{x+1}}\,(x+1)^2+420\sqrt{1-\frac{2}{x+1}}\,(x+1)-280\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*(x + 1)**3/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1 +
 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 + 2
/(x + 1))) - 14*(x + 1)**2/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-
1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))) + 35*(x + 1)/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-
1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))) - 35/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1 + 2/(x
 + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 + 2/(x +
 1))), 1/Abs(x + 1) > 1/2), (-2*I*(x + 1)**3/(35*sqrt(1 - 2/(x + 1))*(x + 1
)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1)
 - 280*sqrt(1 - 2/(x + 1))) + 14*I*(x + 1)**2/(35*sqrt(1 - 2/(x + 1))*(x +
1)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1
) - 280*sqrt(1 - 2/(x + 1))) - 35*I*(x + 1)/(35*sqrt(1 - 2/(x + 1))*(x + 1)
**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1)
- 280*sqrt(1 - 2/(x + 1))) + 35*I/(35*sqrt(1 - 2/(x + 1))*(x + 1)**3 - 210*
sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1) - 280*sqrt
(1 - 2/(x + 1))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(57) = 114.

time = 0.01, size = 293, normalized size = 3.62

$$-2\left(\frac{-\frac{1}{7}\cdot 73786976294838206464\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{7}-\frac{1}{5}\cdot 51650883406386744524 8\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}-516508834063867445248\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}+\frac{129127208515966861312 0\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{151115727451828646838272}+\frac{1225\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{6}+245\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}+49\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}+5}{71680\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{7}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x)
```

```
[Out] -1/7168*(sqrt(2) - sqrt(x + 1))^7/(-x + 1)^(7/2) - 7/5120*(sqrt(2) - sqrt(x
 + 1))^5/(-x + 1)^(5/2) - 7/1024*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) -
 35/1024*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/35840*(1225*(sqrt(2) - sq
rt(x + 1))^6/(x - 1)^3 - 245*(sqrt(2) - sqrt(x + 1))^4/(x - 1)^2 + 49*(sqrt
(2) - sqrt(x + 1))^2/(x - 1) - 5)*(-x + 1)^(7/2)/(sqrt(2) - sqrt(x + 1))^7
```

Mupad [B]

time = 0.34, size = 67, normalized size = 0.83

$$-\frac{x\sqrt{1-x}-12\sqrt{1-x}+5\,x^2\sqrt{1-x}-6\,x^3\sqrt{1-x}+2\,x^4\sqrt{1-x}}{35\,{\left(x-1\right)}^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(1/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) - 12*(1 - x)^(1/2) + 5*x^2*(1 - x)^(1/2) - 6*x^3*(1 - x)^
(1/2) + 2*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

3.1115 $\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}$$

```
[Out] 1/9*(1+x)^(1/2)/(1-x)^(9/2)+4/63*(1+x)^(1/2)/(1-x)^(7/2)+4/105*(1+x)^(1/2)/
(1-x)^(5/2)+8/315*(1+x)^(1/2)/(1-x)^(3/2)+8/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*Sqrt[1 + x])/(63*(1 - x)^(7/2)) + (4*Sqr
t[1 + x])/(105*(1 - x)^(5/2)) + (8*Sqrt[1 + x])/(315*(1 - x)^(3/2)) + (8*Sq
rt[1 + x])/(315*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4}{9}\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4}{21}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8}{105}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8}{315}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 40, normalized size = 0.40

$$\frac{\sqrt{1+x}\,\left(83-100x+84x^2-40x^3+8x^4\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(83 - 100*x + 84*x^2 - 40*x^3 + 8*x^4))/(315*(1 - x)^(9/2))
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 49.64, size = 525, normalized size = 5.20

$$\text{Piecewise}\left[\left\{\left\{\frac{83-100x+84x^2-40x^3+8x^4}{315\left(1-4x+6x^2-4x^3+x^4\right)\sqrt{\frac{1-x}{1+x}}},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},\frac{-315I}{-10080(1+x)\sqrt{1-\frac{2}{1+x}}-2520(1+x)^3\sqrt{1-\frac{2}{1+x}}+315(1+x)^4\sqrt{1-\frac{2}{1+x}}+5040\sqrt{1-\frac{2}{1+x}}+7560(1+x)^2\sqrt{1-\frac{2}{1+x}}}-\frac{252I(1+x)^2}{-10080(1+x)\sqrt{1-\frac{2}{1+x}}-2520(1+x)^3\sqrt{1-\frac{2}{1+x}}+315(1+x)^4\sqrt{1-\frac{2}{1+x}}+5040\sqrt{1-\frac{2}{1+x}}+7560(1+x)^2\sqrt{1-\frac{2}{1+x}}}-\frac{8I(1+x)^4}{-10080(1+x)\sqrt{1-\frac{2}{1+x}}-2520(1+x)^3\sqrt{1-\frac{2}{1+x}}+315(1+x)^4\sqrt{1-\frac{2}{1+x}}+5040\sqrt{1-\frac{2}{1+x}}+7560(1+x)^2\sqrt{1-\frac{2}{1+x}}}+\frac{I72(1+x)^3}{-10080(1+x)\sqrt{1-\frac{2}{1+x}}-2520(1+x)^3\sqrt{1-\frac{2}{1+x}}+315(1+x)^4\sqrt{1-\frac{2}{1+x}}+5040\sqrt{1-\frac{2}{1+x}}+7560(1+x)^2\sqrt{1-\frac{2}{1+x}}}+\frac{I420(1+x)}{-10080(1+x)\sqrt{1-\frac{2}{1+x}}-2520(1+x)^3\sqrt{1-\frac{2}{1+x}}+315(1+x)^4\sqrt{1-\frac{2}{1+x}}+5040\sqrt{1-\frac{2}{1+x}}+7560(1+x)^2\sqrt{1-\frac{2}{1+x}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(11/2)*(1 + x)^(1/2)),x]')
```

```
[Out] Piecewise[{{(83 - 100 x + 84 x ^ 2 - 40 x ^ 3 + 8 x ^ 4) / (315 (1 - 4 x +
6 x ^ 2 - 4 x ^ 3 + x ^ 4) Sqrt[(1 - x) / (1 + x)]), 1 / Abs[1 + x] > 1 / 2
}}, -315 I / (-10080 (1 + x) Sqrt[1 - 2 / (1 + x)] - 2520 (1 + x) ^ 3 Sqrt[
1 - 2 / (1 + x)] + 315 (1 + x) ^ 4 Sqrt[1 - 2 / (1 + x)] + 5040 Sqrt[1 - 2
/ (1 + x)] + 7560 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)]) - 252 I (1 + x) ^ 2 /
(-10080 (1 + x) Sqrt[1 - 2 / (1 + x)] - 2520 (1 + x) ^ 3 Sqrt[1 - 2 / (1 +
x)] + 315 (1 + x) ^ 4 Sqrt[1 - 2 / (1 + x)] + 5040 Sqrt[1 - 2 / (1 + x)] +
7560 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)]) - 8 I (1 + x) ^ 4 / (-10080 (1 + x)
 Sqrt[1 - 2 / (1 + x)] - 2520 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 315 (1 +
```

```
x) ^ 4 Sqrt[1 - 2 / (1 + x)] + 5040 Sqrt[1 - 2 / (1 + x)] + 7560 (1 + x) ^
2 Sqrt[1 - 2 / (1 + x)]) + I 72 (1 + x) ^ 3 / (-10080 (1 + x) Sqrt[1 - 2 /
(1 + x)] - 2520 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 315 (1 + x) ^ 4 Sqrt[1
- 2 / (1 + x)] + 5040 Sqrt[1 - 2 / (1 + x)] + 7560 (1 + x) ^ 2 Sqrt[1 - 2 /
 (1 + x)]) + I 420 (1 + x) / (-10080 (1 + x) Sqrt[1 - 2 / (1 + x)] - 2520 (
1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] + 315 (1 + x) ^ 4 Sqrt[1 - 2 / (1 + x)] +
5040 Sqrt[1 - 2 / (1 + x)] + 7560 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)])]
```

Maple [A]
time = 0.16, size = 72, normalized size = 0.71

method	result	size
gosper	$\frac{\sqrt{1+x}\,(8x^4-40x^3+84x^2-100x+83)}{315(1-x)^{\frac{9}{2}}}$	35
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-32x^4+44x^3-16x^2-17x+83)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}+\frac{4\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}+\frac{4\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(1+x)^(1/2)/(1-x)^(9/2)+4/63*(1+x)^(1/2)/(1-x)^(7/2)+4/105*(1+x)^(1/2)/
(1-x)^(5/2)+8/315*(1+x)^(1/2)/(1-x)^(3/2)+8/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.35, size = 131, normalized size = 1.30

$$-\frac{\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{4\,\sqrt{-x^2+1}}{63\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{4\,\sqrt{-x^2+1}}{105\,(x^3-3\,x^2+3\,x-1)}+\frac{8\,\sqrt{-x^2+1}}{315\,(x^2-2\,x+1)}-\frac{8\,\sqrt{-x^2+1}}{315\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 4/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 8/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 8/315*sqrt(-x^2 + 1)/
(x - 1)
```

Fricas [A]
time = 0.30, size = 86, normalized size = 0.85

$$\frac{83\,x^5-415\,x^4+830\,x^3-830\,x^2-(8\,x^4-40\,x^3+84\,x^2-100\,x+83)\sqrt{x+1}\,\sqrt{-x+1}+415\,x-83}{315\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(83*x^5 - 415*x^4 + 830*x^3 - 830*x^2 - (8*x^4 - 40*x^3 + 84*x^2 - 10
0*x + 83)*sqrt(x + 1)*sqrt(-x + 1) + 415*x - 83)/(x^5 - 5*x^4 + 10*x^3 - 10
*x^2 + 5*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 57.54, size = 850, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((8*(x + 1)**4/(315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1
 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqr
t(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1))) - 72*(x + 1)**3/(315
*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7
560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) +
5040*sqrt(-1 + 2/(x + 1))) + 252*(x + 1)**2/(315*sqrt(-1 + 2/(x + 1))*(x +
1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x
 + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1)))
- 420*(x + 1)/(315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1 + 2/(x +
1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(
x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1))) + 315/(315*sqrt(-1 + 2/(x + 1)
)*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x +
 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x
+ 1))), 1/Abs(x + 1) > 1/2), (-8*I*(x + 1)**4/(315*sqrt(1 - 2/(x + 1))*(x +
 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x
+ 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) + 7
2*I*(x + 1)**3/(315*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1
))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x +
 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) - 252*I*(x + 1)**2/(315*sqrt(1 - 2
/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 -
2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2
/(x + 1))) + 420*I*(x + 1)/(315*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*sqrt(
1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*sqr
t(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) - 315*I/(315*sqrt(1 -
2/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 -
 2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 -
2/(x + 1))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(71) = 142.
time = 0.02, size = 366, normalized size = 3.62

$$2\left(\frac{\frac{1}{9}\cdot 5192296858534827628530496329220096\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{9}+\frac{1}{7}\cdot 46730671726813448656774466962980864\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{7}+\frac{1}{5}\cdot 186922686907253794627097867851923456\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}+145384312038975173598853897218162688\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}-\frac{327114702087694140597421268740866048\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{85070591730234615865843651857942052864}+\frac{-39690\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{8}-8820\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{6}-2268\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}-405\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}-35}{5160960\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x)
```

```
[Out] -1/73728*(sqrt(2) - sqrt(x + 1))^9/(-x + 1)^(9/2) - 9/57344*(sqrt(2) - sqrt
(x + 1))^7/(-x + 1)^(7/2) - 9/10240*(sqrt(2) - sqrt(x + 1))^5/(-x + 1)^(5/2
) - 7/2048*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) - 63/4096*(sqrt(2) - sq
rt(x + 1))/sqrt(-x + 1) + 1/2580480*(39690*(sqrt(2) - sqrt(x + 1))^8/(x - 1
)^4 - 8820*(sqrt(2) - sqrt(x + 1))^6/(x - 1)^3 + 2268*(sqrt(2) - sqrt(x + 1
))^4/(x - 1)^2 - 405*(sqrt(2) - sqrt(x + 1))^2/(x - 1) + 35)*(-x + 1)^(9/2)
/(sqrt(2) - sqrt(x + 1))^9
```

Mupad [B]
time = 0.36, size = 80, normalized size = 0.79

$$\frac{17\,x\,\sqrt{1-x}-83\,\sqrt{1-x}+16\,x^2\,\sqrt{1-x}-44\,x^3\,\sqrt{1-x}+32\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{315\,{\left(x-1\right)}^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(1/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 83*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(315*(x - 1)^5*(x +
 1)^(1/2))
```

3.1116 $\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2(1-x)^{7/2}}{\sqrt{1+x}}-\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}-\frac{7}{3}(1-x)^{5/2}\sqrt{1+x}-\frac{35}{2}\sin^{-1}(x)$$

```
[Out] -35/2*arcsin(x)-2*(1-x)^(7/2)/(1+x)^(1/2)-35/6*(1-x)^(3/2)*(1+x)^(1/2)-7/3*
(1-x)^(5/2)*(1+x)^(1/2)-35/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}\,(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/Sqrt[1 + x] - (35*Sqrt[1 - x]*Sqrt[1 + x])/2 - (35*(1 -
x)^(3/2)*Sqrt[1 + x])/6 - (7*(1 - x)^(5/2)*Sqrt[1 + x])/3 - (35*ArcSin[x])/
2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - 7\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2} \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2} \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.66

$$-\frac{\sqrt{1-x}\,(166+55x-13x^2+2x^3)}{6\sqrt{1+x}} + 35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] -1/6*(Sqrt[1 - x]*(166 + 55*x - 13*x^2 + 2*x^3))/Sqrt[1 + x] + 35*ArcTan[Sq
rt[1 - x]/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 14.87, size = 154, normalized size = 1.81

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-55x\sqrt{-1+x}+13x^2\sqrt{-1+x}-2x^3\sqrt{-1+x}-166\sqrt{-1+x}+210\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{1+x}\right)}{6\sqrt{1+x}},\text{Abs}[1+x]>2\right\}\right\},-35\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]-\frac{32}{\sqrt{1+x}\sqrt{1-x}}-\frac{13\sqrt{1+x}}{\sqrt{1-x}}-\frac{23(1+x)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{(1+x)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{125(1+x)^{\frac{3}{2}}}{6\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 6 (-55 x Sqrt[-1 + x] + 13 x ^ 2 Sqrt[-1 + x] - 2 x ^ 3 Sqr
t[-1 + x] - 166 Sqrt[-1 + x] + 210 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[1
+ x]) / Sqrt[1 + x], Abs[1 + x] > 2}}, -35 ArcSin[Sqrt[2] Sqrt[1 + x] / 2]
- 32 / (Sqrt[1 + x] Sqrt[1 - x]) - 13 Sqrt[1 + x] / Sqrt[1 - x] - 23 (1 + x
) ^ (5 / 2) / (6 Sqrt[1 - x]) + (1 + x) ^ (7 / 2) / (3 Sqrt[1 - x]) + 125 (
1 + x) ^ (3 / 2) / (6 Sqrt[1 - x])]
```

Maple [A]
time = 0.14, size = 84, normalized size = 0.99

method	result	size
risch	$\frac{(2x^4-15x^3+68x^2+111x-166)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*x^4-15*x^3+68*x^2+111*x-166)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/
2)/(1-x)^(1/2)/(1+x)^(1/2)-35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Maxima [A]
time = 0.35, size = 70, normalized size = 0.82

$$\frac{x^4}{3\sqrt{-x^2+1}}-\frac{5x^3}{2\sqrt{-x^2+1}}+\frac{34x^2}{3\sqrt{-x^2+1}}+\frac{37x}{2\sqrt{-x^2+1}}-\frac{83}{3\sqrt{-x^2+1}}-\frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/sqrt(-x^2 + 1) - 5/2*x^3/sqrt(-x^2 + 1) + 34/3*x^2/sqrt(-x^2 + 1) +
 37/2*x/sqrt(-x^2 + 1) - 83/3/sqrt(-x^2 + 1) - 35/2*arcsin(x)
```

Fricas [A]
time = 0.31, size = 65, normalized size = 0.76

$$-\frac{(2x^3-13x^2+55x+166)\sqrt{x+1}\,\sqrt{-x+1}-210(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+166x+166}{6(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*((2*x^3 - 13*x^2 + 55*x + 166)*sqrt(x + 1)*sqrt(-x + 1) - 210*(x + 1)*
arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 166*x + 166)/(x + 1)
```

Sympy [A]
time = 16.72, size = 206, normalized size = 2.42

$$\begin{cases} 35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} + \frac{23i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{125i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{13i\sqrt{x+1}}{\sqrt{x-1}} + \frac{32i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1| > 2 \\ -35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} - \frac{23(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{125(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{13\sqrt{x+1}}{\sqrt{1-x}} - \frac{32}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) + 23*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 125*I*(x + 1)**(3/2)/(6*sqrt(x
 - 1)) + 13*I*sqrt(x + 1)/sqrt(x - 1) + 32*I/(sqrt(x - 1)*sqrt(x + 1)), Abs
(x + 1) > 2), (-35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 -
 x)) - 23*(x + 1)**(5/2)/(6*sqrt(1 - x)) + 125*(x + 1)**(3/2)/(6*sqrt(1 - x
)) - 13*sqrt(x + 1)/sqrt(1 - x) - 32/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [A]
time = 0.01, size = 108, normalized size = 1.27

$$\frac{2\left(\left(\left(\left(\frac{1}{6}\sqrt{-x+1}\sqrt{-x+1}+\frac{7}{12}\right)\sqrt{-x+1}\sqrt{-x+1}+\frac{35}{12}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{35}{2}\right)\sqrt{-x+1}\sqrt{x+1}}{x+1}+35\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x)
```

```
[Out] -1/6*(((2*x - 9)*(x - 1) + 35)*(x - 1) + 210)*sqrt(-x + 1)/sqrt(x + 1) + 35
*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(3/2), x)
```

3.1117 $\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{2}(1-x)^{3/2}\sqrt{1+x}-\frac{15}{2}\sin^{-1}(x)$$

```
[Out] -15/2*arcsin(x)-2*(1-x)^(5/2)/(1+x)^(1/2)-5/2*(1-x)^(3/2)*(1+x)^(1/2)-15/2*
(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}\,(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/Sqrt[1 + x] - (15*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*(1 - x
)^(3/2)*Sqrt[1 + x])/2 - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - 5\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.75

$$\frac{\sqrt{1-x}\,(-24-7x+x^2)}{2\sqrt{1+x}} + 15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x]*(-24 - 7*x + x^2))/(2*Sqrt[1 + x]) + 15*ArcTan[Sqrt[1 - x]/Sqr
t[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.24, size = 129, normalized size = 1.98

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-7x\sqrt{-1+x}+x^2\sqrt{-1+x}-24\sqrt{-1+x}+30\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{1+x}\right)}{2\sqrt{1+x}},\text{Abs}[1+x]>2\right\}\right\},\frac{-16}{\sqrt{1+x}\sqrt{1-x}}-15\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]-\frac{\sqrt{1+x}}{\sqrt{1-x}}-\frac{(1+x)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{11(1+x)^{\frac{3}{2}}}{2\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 2 (-7 x Sqrt[-1 + x] + x ^ 2 Sqrt[-1 + x] - 24 Sqrt[-1 + x]
 + 30 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] Sqrt[1 + x]) / Sqrt[1 + x], Abs[1 +
x] > 2}}, -16 / (Sqrt[1 + x] Sqrt[1 - x]) - 15 ArcSin[Sqrt[2] Sqrt[1 + x] /
 2] - Sqrt[1 + x] / Sqrt[1 - x] - (1 + x) ^ (5 / 2) / (2 Sqrt[1 - x]) + 11
(1 + x) ^ (3 / 2) / (2 Sqrt[1 - x])]
```

Maple [A]
time = 0.16, size = 77, normalized size = 1.18

method	result	size
risch	$-\frac{\left(x^{3}-8 x^{2}-17 x+24\right) \sqrt{\left(1+x\right) \left(1-x\right)}}{2 \sqrt{-\left(1+x\right) \left(-1+x\right)}\, \sqrt{1-x}\, \sqrt{1+x}}-\frac{15 \sqrt{\left(1+x\right) \left(1-x\right)}\, \arcsin\left(x\right)}{2 \sqrt{1+x}\, \sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^3-8*x^2-17*x+24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.35, size = 56, normalized size = 0.86

$$-\frac{x^{3}}{2 \sqrt{-x^{2}+1}}+\frac{4 x^{2}}{\sqrt{-x^{2}+1}}+\frac{17 x}{2 \sqrt{-x^{2}+1}}-\frac{12}{\sqrt{-x^{2}+1}}-\frac{15}{2} \arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) + 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) - 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Fricas [A]
time = 0.29, size = 58, normalized size = 0.89

$$\frac{\left(x^{2}-7 x-24\right) \sqrt{x+1}\, \sqrt{-x+1}+30 \left(x+1\right) \arctan\left(\frac{\sqrt{x+1}\, \sqrt{-x+1}-1}{x}\right)-24 x-24}{2 \left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x + 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) - 24*x - 24)/(x + 1)
```

Sympy [A]
time = 4.89, size = 167, normalized size = 2.57

$$\begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{11i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} + \frac{16i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1| > 2 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{11(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} - \frac{16}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - 11*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1) + 16
*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) > 2), (-15*asin(sqrt(2)*sqrt(x + 1
)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) + 11*(x + 1)**(3/2)/(2*sqrt(1 - x)) -
 sqrt(x + 1)/sqrt(1 - x) - 16/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [A]
time = 0.01, size = 86, normalized size = 1.32

$$\frac{2\left(\left(\left(\frac{1}{4}\sqrt{-x+1}\sqrt{-x+1}+\frac{5}{4}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{15}{2}\right)\sqrt{-x+1}\sqrt{x+1}\right)}{x+1}+15\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x)
```

```
[Out] 1/2*((x - 1)*(x - 6) - 30)*sqrt(-x + 1)/sqrt(x + 1) + 15*arcsin(1/2*sqrt(2)
*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(3/2), x)
```

3.1118 $\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\sin^{-1}(x)$$

[Out] -3*arcsin(x)-2*(1-x)^(3/2)/(1+x)^(1/2)-3*(1-x)^(1/2)*(1+x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}-3\sqrt{x+1}\,\sqrt{1-x}-3\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x)^(3/2)/(1 + x)^(3/2),x]

[Out] (-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*Sqrt[1 - x]*Sqrt[1 + x] - 3*ArcSin[x]

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 43, normalized size = 1.05

$$\frac{(-5-x)\sqrt{1-x}}{\sqrt{1+x}}-6\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]
[Out] ((-5 - x)*Sqrt[1 - x])/Sqrt[1 + x] - 6*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.42, size = 105, normalized size = 2.56

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-x\sqrt{-1+x}-5\sqrt{-1+x}+6\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\sqrt{1+x}\right)}{\sqrt{1+x}},\text{Abs}\left[1+x\right]>2\right\}\right\},\frac{-8}{\sqrt{1+x}\sqrt{1-x}}-6\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]+\frac{2\sqrt{1+x}}{\sqrt{1-x}}+\frac{(1+x)^{\frac{3}{2}}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]')
[Out] Piecewise[{{I (-x Sqrt[-1 + x] - 5 Sqrt[-1 + x] + 6 ArcCosh[Sqrt[2] Sqrt[1
+ x] / 2] Sqrt[1 + x]) / Sqrt[1 + x], Abs[1 + x] > 2}}, -8 / (Sqrt[1 + x] S
qrt[1 - x]) - 6 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 2 Sqrt[1 + x] / Sqrt[1 -
x] + (1 + x) ^ (3 / 2) / Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(33) = 66$.
time = 0.16, size = 71, normalized size = 1.73

method	result	size
risch	$\frac{(x^2+4x-5)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} - \frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x^2+4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/
2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.36, size = 41, normalized size = 1.00

$$\frac{(-x^2+1)^{\frac{3}{2}}}{x^2+2x+1} - \frac{6\sqrt{-x^2+1}}{x+1} - 3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^2 + 2*x + 1) - 6*sqrt(-x^2 + 1)/(x + 1) - 3*arcsin(x)
```

Fricas [A]
time = 0.30, size = 53, normalized size = 1.29

$$-\frac{(x+5)\sqrt{x+1}\,\sqrt{-x+1} - 6(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) + 5x+5}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x + 5)*sqrt(x + 1)*sqrt(-x + 1) - 6*(x + 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + 5*x + 5)/(x + 1)
```

Sympy [A]
time = 1.38, size = 131, normalized size = 3.20

$$\begin{cases} 6i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{8i}{\sqrt{x-1}\,\sqrt{x+1}} & \text{for } |x+1| > 2 \\ -6\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{8}{\sqrt{1-x}\,\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
- 2*I*sqrt(x + 1)/sqrt(x - 1) + 8*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) >
 2), (-6*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(
x + 1)/sqrt(1 - x) - 8/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [**A**]
time = 0.01, size = 61, normalized size = 1.49

$$\frac{2\left(\frac{1}{2}\sqrt{-x+1}\sqrt{-x+1}-3\right)\sqrt{-x+1}\sqrt{x+1}}{x+1}+6\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] -(x + 5)*sqrt(-x + 1)/sqrt(x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(3/2), x)
```

3.1119 $\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \sin^{-1}(x)$$

[Out] -arcsin(x)-2*(1-x)^(1/2)/(1+x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}} - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x]/(1 + x)^(3/2),x]

[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \sin^{-1}(x)$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.70

$$-\frac{2\sqrt{1-x}}{\sqrt{1+x}} + 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(3/2),x]
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] + 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.77, size = 77, normalized size = 3.35

$$\text{Piecewise}\left[\left\{\left\{\frac{-2I\sqrt{-1+x}}{\sqrt{1+x}} + 2I\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right], \text{Abs}\left[1+x\right] > 2\right\}\right\}, \frac{-4}{\sqrt{1+x}\sqrt{1-x}} - 2\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right] + \frac{2\sqrt{1+x}}{\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/2)/(1 + x)^(3/2),x]')
[Out] Piecewise[{{-2 I Sqrt[-1 + x] / Sqrt[1 + x] + 2 I ArcCosh[Sqrt[2] Sqrt[1 +
x] / 2], Abs[1 + x] > 2}}, -4 / (Sqrt[1 + x] Sqrt[1 - x]) - 2 ArcSin[Sqrt[2
] Sqrt[1 + x] / 2] + 2 Sqrt[1 + x] / Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(19) = 38.
time = 0.14, size = 67, normalized size = 2.91

method	result	size
risch	$\frac{2(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\sqrt{1-x}\sqrt{1+x}} - \frac{\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1+x}\sqrt{1-x}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-
((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.34, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x+1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.
time = 0.30, size = 50, normalized size = 2.17

$$\frac{2\left((x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 102, normalized size = 4.43

$$\begin{cases} 2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{2i\sqrt{x+1}}{\sqrt{x-1}}+\frac{4i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1|>2 \\ -2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\frac{4}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1) +
 4*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) > 2), (-2*asin(sqrt(2)*sqrt(x +
1)/2) + 2*sqrt(x + 1)/sqrt(1 - x) - 4/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [**A**]
time = 0.00, size = 41, normalized size = 1.78

$$-\frac{2\sqrt{-x+1}\,\sqrt{x+1}}{x+1}+2\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x)
[Out] -2*sqrt(-x + 1)/sqrt(x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-x}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(3/2),x)
[Out] int((1 - x)^(1/2)/(x + 1)^(3/2), x)
```

3.1120 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

```
[Out] -(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.25, size = 39, normalized size = 2.17

$$\text{Piecewise}\left[\left\{\left\{-\sqrt{\frac{1-x}{1+x}}, \frac{1}{\text{Abs}\,[1+x]} > \frac{1}{2}\right\}\right\}, -I\sqrt{1-\frac{2}{1+x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(1/2)*(1 + x)^(3/2)),x]')
[Out] Piecewise[{{-Sqrt[(1 - x) / (1 + x)], 1 / Abs[1 + x] > 1 / 2}}, -I Sqrt[1 -
 2 / (1 + x)]]
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	15
default	$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	15
risch	$\frac{(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.34, size = 16, normalized size = 0.89

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A]
time = 0.29, size = 23, normalized size = 1.28

$$-\frac{x+\sqrt{x+1}\,\sqrt{-x+1}+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(x + 1)*sqrt(-x + 1) + 1)/(x + 1)
```

Sympy [A]
time = 0.51, size = 31, normalized size = 1.72

$$\begin{cases} -\sqrt{-1 + \frac{2}{x+1}} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -i\sqrt{1 - \frac{2}{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1)), 1/Abs(x + 1) > 1/2), (-I*sqrt(1 - 2/(x +
1)), True))
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.17

$$-\frac{\sqrt{-x+1}\,\sqrt{x+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x)
```

```
[Out] -sqrt(-x + 1)/sqrt(x + 1)
```

Mupad [B]
time = 0.36, size = 14, normalized size = 0.78

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(1 - x)^(1/2)/(x + 1)^(1/2)
```

3.1121 $\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}} dx$

Optimal. Leaf size=18

$$\frac{x}{\sqrt{1-x}\sqrt{1+x}}$$

```
[Out] x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{39\}$

$$\frac{x}{\sqrt{1-x}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}} dx = \frac{x}{\sqrt{1-x}\sqrt{1+x}}$$

Mathematica [A]
time = 0.03, size = 13, normalized size = 0.72

$$\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/Sqrt[1 - x^2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.01, size = 64, normalized size = 3.56

$$\text{Piecewise}\left[\left\{\left\{-\frac{x\sqrt{\frac{1-x}{1+x}}}{-1+x}, \frac{1}{\text{Abs}[1+x]} > \frac{1}{2}\right\}\right\}, -\frac{I}{\sqrt{1-\frac{2}{1+x}}} + \frac{I}{(1+x)\sqrt{1-\frac{2}{1+x}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{-x Sqrt[(1 - x) / (1 + x)] / (-1 + x), 1 / Abs[1 + x] > 1 / 2}}
, -I / Sqrt[1 - 2 / (1 + x)] + I / ((1 + x) Sqrt[1 - 2 / (1 + x)])]
```

Maple [A]
time = 0.15, size = 29, normalized size = 1.61

method	result	size
gosper	$\frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$	15
default	$\frac{1}{\sqrt{1-x}\,\sqrt{1+x}}-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	29
risch	$\frac{\sqrt{(1+x)(1-x)}\,x}{\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-(1+x)(-1+x)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1-x)^(1/2)/(1+x)^(1/2)-(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.61

$$\frac{x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 1)
```

Fricas [A]
time = 0.30, size = 22, normalized size = 1.22

$$-\frac{\sqrt{x+1}\,x\sqrt{-x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.97, size = 63, normalized size = 3.50

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{x-1}+\frac{\sqrt{-1+\frac{2}{x+1}}}{x-1} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}}+\frac{i}{\sqrt{1-\frac{2}{x+1}}\,(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))*(x + 1)/(x - 1) + sqrt(-1 + 2/(x + 1))/(x
- 1), 1/Abs(x + 1) > 1/2), (-I/sqrt(1 - 2/(x + 1)) + I/(sqrt(1 - 2/(x + 1))
*(x + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.
time = 0.00, size = 87, normalized size = 4.83

$$2\left(\frac{\sqrt{-x+1}}{4\left(-2\sqrt{x+1}+2\sqrt{2}\right)}-\frac{-2\sqrt{x+1}+2\sqrt{2}}{16\sqrt{-x+1}}-\frac{\sqrt{-x+1}\,\sqrt{x+1}}{4\,(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x)
```

```
[Out] -1/4*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/2*sqrt(-x + 1)/sqrt(x + 1) +
1/4*sqrt(-x + 1)/(sqrt(2) - sqrt(x + 1))
```

Mupad [B]
time = 0.31, size = 14, normalized size = 0.78

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] x/((1 - x)^(1/2)*(x + 1)^(1/2))
```

3.1122 $\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/3/(1-x)^(3/2)/(1+x)^(1/2)+2/3*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 39}

$$\frac{2x}{3\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.71

$$\frac{-1-2x+2x^2}{3(-1+x)\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-1 - 2*x + 2*x^2)/(3*(-1 + x)*Sqrt[1 - x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.37, size = 133, normalized size = 3.17

$$\text{Piecewise}\left[\left\{\left\{\frac{(1+2x-2x^2)\sqrt{\frac{1-x}{1+x}}}{3\,(1-2x+x^2)},\frac{1}{\text{Abs}\,[1+x]}>\frac{1}{2}\right\}\right\},\frac{-3I\sqrt{1-\frac{2}{1+x}}}{-12x+3\,(1+x)^2}-\frac{2I\,(1+x)^2\sqrt{1-\frac{2}{1+x}}}{-12x+3\,(1+x)^2}+\frac{I6\,(1+x)\sqrt{1-\frac{2}{1+x}}}{-12x+3\,(1+x)^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{(1 + 2 x - 2 x ^ 2) Sqrt[(1 - x) / (1 + x)] / (3 (1 - 2 x + x ^
 2)), 1 / Abs[1 + x] > 1 / 2}}, -3 I Sqrt[1 - 2 / (1 + x)] / (-12 x + 3 (1
+ x) ^ 2) - 2 I (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] / (-12 x + 3 (1 + x) ^ 2)
 + I 6 (1 + x) Sqrt[1 - 2 / (1 + x)] / (-12 x + 3 (1 + x) ^ 2)]
```

Maple [A]
time = 0.14, size = 44, normalized size = 1.05

method	result	size
gosper	$-\frac{2x^2-2x-1}{3\sqrt{1+x}\,(1-x)^{\frac{3}{2}}}$	25
default	$\frac{1}{3(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{2}{3\sqrt{1-x}\,\sqrt{1+x}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	44
risch	$\frac{\sqrt{(1+x)\,(1-x)}\,(2x^2-2x-1)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)\,(-1+x)}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(1-x)^(3/2)/(1+x)^(1/2)+2/3/(1-x)^(1/2)/(1+x)^(1/2)-2/3*(1-x)^(1/2)/(1+
x)^(1/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.95

$$\frac{2x}{3\sqrt{-x^2+1}} - \frac{1}{3\left(\sqrt{-x^2+1}\,x - \sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.29, size = 54, normalized size = 1.29

$$\frac{x^3 - x^2 - (2x^2 - 2x - 1)\sqrt{x+1}\,\sqrt{-x+1} - x + 1}{3\,(x^3 - x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
[Out] 1/3*(x^3 - x^2 - (2*x^2 - 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x + 1)/(x^3 -
 x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.47, size = 160, normalized size = 3.81

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-12x+3(x+1)^2} + \frac{6\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^2} - \frac{3\sqrt{-1+\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-12x+3(x+1)^2} + \frac{6i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^2} - \frac{3i\sqrt{1-\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(3/2),x)
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*sq
rt(-1 + 2/(x + 1))*(x + 1)/(-12*x + 3*(x + 1)**2) - 3*sqrt(-1 + 2/(x + 1))/
(-12*x + 3*(x + 1)**2), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x +
 1)**2/(-12*x + 3*(x + 1)**2) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-12*x + 3*
(x + 1)**2) - 3*I*sqrt(1 - 2/(x + 1))/(-12*x + 3*(x + 1)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(30) = 60.
time = 0.01, size = 171, normalized size = 4.07

$$-2\left(\frac{-\frac{4096}{3}\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3 + \frac{14336\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{262144} + \frac{21\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2 + 1}{192\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3} + \frac{\sqrt{-x+1}\,\sqrt{x+1}}{8\,(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x)
```

```
[Out] -1/96*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) - 7/32*(sqrt(2) - sqrt(x + 1
))/sqrt(-x + 1) - 1/4*sqrt(-x + 1)/sqrt(x + 1) - 1/96*(-x + 1)^(3/2)*(21*(s
qrt(2) - sqrt(x + 1))^2/(x - 1) - 1)/(sqrt(2) - sqrt(x + 1))^3
```

Mupad [B]
time = 0.32, size = 42, normalized size = 1.00

$$\frac{2\,x\,\sqrt{1-x}\,+\sqrt{1-x}\,-2\,x^2\,\sqrt{1-x}}{3\,{\left(x-1\right)}^2\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(3/2)),x)
```

```
[Out] (2*x*(1 - x)^(1/2) + (1 - x)^(1/2) - 2*x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x +
 1)^(1/2))
```

3.1123 $\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{5\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(1/2)+1/5/(1-x)^(3/2)/(1+x)^(1/2)+2/5*x/(1-x)^(1/2)/(
1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 39}

$$\frac{2x}{5\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + 1/(5*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(
5*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{3}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{1}{5(1-x)^{3/2}\sqrt{1+x}} + \frac{2}{5}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{1}{5(1-x)^{3/2}\sqrt{1+x}} + \frac{2x}{5\sqrt{1-x}\,\sqrt{1+x}}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 0.53

$$\frac{2+x-4x^2+2x^3}{5(-1+x)^2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] (2 + x - 4*x^2 + 2*x^3)/(5*(-1 + x)^2*Sqrt[1 - x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 13.12, size = 206, normalized size = 3.32

$$\text{Piecewise}\left[\left\{\left\{\frac{(-2-x+4x^2-2x^3)\sqrt{\frac{1-x}{1+x}}}{5(-1+3x-3x^2+x^3)}, \frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\}, \frac{-15I(1+x)\sqrt{1-\frac{2}{1+x}}}{20+60x-30(1+x)^2+5(1+x)^3} - \frac{2I(1+x)^3\sqrt{1-\frac{2}{1+x}}}{20+60x-30(1+x)^2+5(1+x)^3} + \frac{I5\sqrt{1-\frac{2}{1+x}}}{20+60x-30(1+x)^2+5(1+x)^3} + \frac{I10(1+x)^2\sqrt{1-\frac{2}{1+x}}}{20+60x-30(1+x)^2+5(1+x)^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{(-2 - x + 4 x ^ 2 - 2 x ^ 3) Sqrt[(1 - x) / (1 + x)] / (5 (-1 +
 3 x - 3 x ^ 2 + x ^ 3)), 1 / Abs[1 + x] > 1 / 2}}, -15 I (1 + x) Sqrt[1 -
2 / (1 + x)] / (20 + 60 x - 30 (1 + x) ^ 2 + 5 (1 + x) ^ 3) - 2 I (1 + x) ^
 3 Sqrt[1 - 2 / (1 + x)] / (20 + 60 x - 30 (1 + x) ^ 2 + 5 (1 + x) ^ 3) + I
 5 Sqrt[1 - 2 / (1 + x)] / (20 + 60 x - 30 (1 + x) ^ 2 + 5 (1 + x) ^ 3) + I
 10 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] / (20 + 60 x - 30 (1 + x) ^ 2 + 5 (1
+ x) ^ 3)]
```

Maple [A]
time = 0.16, size = 58, normalized size = 0.94

method	result	size

gosper	$\frac{2x^3-4x^2+x+2}{5\sqrt{1+x}\,(1-x)^{\frac{5}{2}}}$	28
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^3-4x^2+x+2)}{5\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54
default	$\frac{1}{5(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{1}{5(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{2}{5\sqrt{1-x}\,\sqrt{1+x}}-\frac{2\sqrt{1-x}}{5\sqrt{1+x}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(1/2)+1/5/(1-x)^(3/2)/(1+x)^(1/2)+2/5/(1-x)^(1/2)/(1+
x)^(1/2)-2/5*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.26, size = 79, normalized size = 1.27

$$\frac{2x}{5\sqrt{-x^2+1}}+\frac{1}{5\left(\sqrt{-x^2+1}\,x^2-2\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{1}{5\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/sqrt(-x^2 + 1) + 1/5/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(
-x^2 + 1)) - 1/5/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.30, size = 59, normalized size = 0.95

$$\frac{2x^4-4x^3-(2x^3-4x^2+x+2)\sqrt{x+1}\,\sqrt{-x+1}+4x-2}{5(x^4-2x^3+2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4 - 4*x^3 - (2*x^3 - 4*x^2 + x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x
 - 2)/(x^4 - 2*x^3 + 2*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.06, size = 284, normalized size = 4.58

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{10\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20}-\frac{15\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{5\sqrt{-1+\frac{2}{x+1}}}{60x+5(x+1)^3-30(x+1)^2+20} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{10i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20}-\frac{15i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{5i\sqrt{1-\frac{2}{x+1}}}{60x+5(x+1)^3-30(x+1)^2+20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(60*x + 5*(x + 1)**3 - 30*(x
+ 1)**2 + 20) + 10*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(60*x + 5*(x + 1)**3 - 3
0*(x + 1)**2 + 20) - 15*sqrt(-1 + 2/(x + 1))*(x + 1)/(60*x + 5*(x + 1)**3 -
 30*(x + 1)**2 + 20) + 5*sqrt(-1 + 2/(x + 1))/(60*x + 5*(x + 1)**3 - 30*(x
+ 1)**2 + 20), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(6
0*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 10*I*sqrt(1 - 2/(x + 1))*(x + 1)
**2/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) - 15*I*sqrt(1 - 2/(x + 1))*(
x + 1)/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 5*I*sqrt(1 - 2/(x + 1))
/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(44) = 88$.

time = 0.01, size = 243, normalized size = 3.92

$$2\left(\frac{\frac{1}{5}\cdot 68719476736\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}+206158430208\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}-\frac{1305670057984\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{35184372088832}+\frac{-190\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}-15\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}-1}{2560\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}}-\frac{\sqrt{-x+1}\sqrt{x+1}}{16(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x)
```

```
[Out] -1/1280*(sqrt(2) - sqrt(x + 1))^5/(-x + 1)^(5/2) - 3/256*(sqrt(2) - sqrt(x
+ 1))^3/(-x + 1)^(3/2) - 19/128*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/8*
sqrt(-x + 1)/sqrt(x + 1) + 1/1280*(190*(sqrt(2) - sqrt(x + 1))^4/(x - 1)^2
- 15*(sqrt(2) - sqrt(x + 1))^2/(x - 1) + 1)*(-x + 1)^(5/2)/(sqrt(2) - sqrt(
x + 1))^5
```

Mupad [B]
time = 0.34, size = 55, normalized size = 0.89

$$-\frac{x\sqrt{1-x}+2\sqrt{1-x}-4x^2\sqrt{1-x}+2x^3\sqrt{1-x}}{5(x-1)^3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(5*(x - 1)^3*(x + 1)^(1/2))
```

3.1124 $\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8x}{35\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(1/2)+4/35/(1-x)^(5/2)/(1+x)^(1/2)+4/35/(1-x)^(3/2)/(
1+x)^(1/2)+8/35*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 39}

$$\frac{8x}{35\sqrt{1-x}\,\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(35*(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(35*
(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(35*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx &= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{12}{35}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8}{35}\int \\ &= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{}{35\sqrt{1}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.49

$$\frac{-13+4x+20x^2-24x^3+8x^4}{35(-1+x)^3\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-13 + 4*x + 20*x^2 - 24*x^3 + 8*x^4)/(35*(-1 + x)^3*Sqrt[1 - x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 34.25, size = 290, normalized size = 3.54

$$\text{Piecewise}\left[\left\{\left\{\frac{(13-4x-20x^2+24x^3-8x^4)\sqrt{\frac{1-x}{1+x}}}{35(1-4x+6x^2-4x^3+x^4)},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},\frac{-140I(1+x)^2\sqrt{1-\frac{2}{1+x}}}{-560-1120x-280(1+x)^3+35(1+x)^4+840(1+x)^2}-\frac{35I\sqrt{1-\frac{2}{1+x}}}{-560-1120x-280(1+x)^3+35(1+x)^4+840(1+x)^2}-\frac{8I(1+x)^4\sqrt{1-\frac{2}{1+x}}}{-560-1120x-280(1+x)^3+35(1+x)^4+840(1+x)^2}+\frac{I56(1+x)^3\sqrt{1-\frac{2}{1+x}}}{-560-1120x-280(1+x)^3+35(1+x)^4+840(1+x)^2}+\frac{I140(1+x)\sqrt{1-\frac{2}{1+x}}}{-560-1120x-280(1+x)^3+35(1+x)^4+840(1+x)^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{(13 - 4 x - 20 x ^ 2 + 24 x ^ 3 - 8 x ^ 4) Sqrt[(1 - x) / (1 +
x)] / (35 (1 - 4 x + 6 x ^ 2 - 4 x ^ 3 + x ^ 4)), 1 / Abs[1 + x] > 1 / 2}},
 -140 I (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] / (-560 - 1120 x - 280 (1 + x) ^
3 + 35 (1 + x) ^ 4 + 840 (1 + x) ^ 2) - 35 I Sqrt[1 - 2 / (1 + x)] / (-560
- 1120 x - 280 (1 + x) ^ 3 + 35 (1 + x) ^ 4 + 840 (1 + x) ^ 2) - 8 I (1 + x
) ^ 4 Sqrt[1 - 2 / (1 + x)] / (-560 - 1120 x - 280 (1 + x) ^ 3 + 35 (1 + x)
 ^ 4 + 840 (1 + x) ^ 2) + I 56 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] / (-560 -
1120 x - 280 (1 + x) ^ 3 + 35 (1 + x) ^ 4 + 840 (1 + x) ^ 2) + I 140 (1 + x
) Sqrt[1 - 2 / (1 + x)] / (-560 - 1120 x - 280 (1 + x) ^ 3 + 35 (1 + x) ^ 4
 + 840 (1 + x) ^ 2)]
```

Maple [A]
time = 0.14, size = 72, normalized size = 0.88

method	result	size
gosper	$-\frac{8x^4-24x^3+20x^2+4x-13}{35\sqrt{1+x}\,(1-x)^{\frac{7}{2}}}$	35
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^4-24x^3+20x^2+4x-13)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	61
default	$\frac{1}{7(1-x)^{\frac{7}{2}}\sqrt{1+x}}+\frac{4}{35(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{4}{35(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{8}{35\sqrt{1-x}\,\sqrt{1+x}}-\frac{8\sqrt{1-x}}{35\sqrt{1+x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(1/2)+4/35/(1-x)^(5/2)/(1+x)^(1/2)+4/35/(1-x)^(3/2)/(
1+x)^(1/2)+8/35/(1-x)^(1/2)/(1+x)^(1/2)-8/35*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(58) = 116$.
time = 0.27, size = 134, normalized size = 1.63

$$\frac{8x}{35\sqrt{-x^2+1}}-\frac{1}{7\left(\sqrt{-x^2+1}\,x^3-3\sqrt{-x^2+1}\,x^2+3\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}+\frac{4}{35\left(\sqrt{-x^2+1}\,x^2-2\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{4}{35\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/35*x/sqrt(-x^2 + 1) - 1/7/(sqrt(-x^2 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*
sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4/35/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2
 + 1)*x + sqrt(-x^2 + 1)) - 4/35/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.29, size = 86, normalized size = 1.05

$$\frac{13x^5-39x^4+26x^3+26x^2-(8x^4-24x^3+20x^2+4x-13)\sqrt{x+1}\,\sqrt{-x+1}-39x+13}{35(x^5-3x^4+2x^3+2x^2-3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(13*x^5 - 39*x^4 + 26*x^3 + 26*x^2 - (8*x^4 - 24*x^3 + 20*x^2 + 4*x -
13)*sqrt(x + 1)*sqrt(-x + 1) - 39*x + 13)/(x^5 - 3*x^4 + 2*x^3 + 2*x^2 - 3*
x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 42.08, size = 425, normalized size = 5.18

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{56\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{140\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{140\sqrt{-1+\frac{2}{x+1}}(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{35\sqrt{-1+\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{56i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{140i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{140i\sqrt{1-\frac{2}{x+1}}(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{35i\sqrt{1-\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 28
0*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(
-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*sqrt
(-1 + 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840
*(x + 1)**2 - 560) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)/(-1120*x + 35*(x + 1)
**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*sqrt(-1 + 2/(x + 1))/(-11
20*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), 1/Abs(x + 1)
 > 1/2), (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 28
0*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/
(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*I*s
qrt(1 - 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 8
40*(x + 1)**2 - 560) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-1120*x + 35*(x +
 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*I*sqrt(1 - 2/(x + 1))/
(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(58) = 116$.

time $= 0.02$, size $= 315$, normalized size $= 3.84$

$$-2\left(\frac{-\frac{1}{7}\cdot 4722366482869645213696\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{7}-\frac{1}{5}\cdot 5194603131156609735065 6\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}-897249631745232590602 24\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}+\frac{441541266148311827480576\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{1934281311383406679529881 6}+\frac{6545\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{6}+665\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}+77\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}+5}{143360\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{7}}+\frac{\sqrt{-x+1}\sqrt{x+1}}{32\,(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x)
```

```
[Out] -1/14336*(sqrt(2) - sqrt(x + 1))^7/(-x + 1)^(7/2) - 11/10240*(sqrt(2) - sqr
t(x + 1))^5/(-x + 1)^(5/2) - 19/2048*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/
2) - 187/2048*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/16*sqrt(-x + 1)/sqrt
(x + 1) - 1/71680*(6545*(sqrt(2) - sqrt(x + 1))^6/(x - 1)^3 - 665*(sqrt(2)
- sqrt(x + 1))^4/(x - 1)^2 + 77*(sqrt(2) - sqrt(x + 1))^2/(x - 1) - 5)*(-x
+ 1)^(7/2)/(sqrt(2) - sqrt(x + 1))^7
```

Mupad [B]

time $= 0.36$, size $= 68$, normalized size $= 0.83$

$$-\frac{4\,x\,\sqrt{1-x}-13\,\sqrt{1-x}+20\,x^2\,\sqrt{1-x}-24\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{35\,{\left(x-1\right)}^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(4*x*(1 - x)^(1/2) - 13*(1 - x)^(1/2) + 20*x^2*(1 - x)^(1/2) - 24*x^3*(1 -
 x)^(1/2) + 8*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

3.1125 $\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{3/2}\sqrt{1+x}}+\frac{8x}{63\sqrt{1-x}\sqrt{1+x}}$$

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(1/2)+5/63/(1-x)^(7/2)/(1+x)^(1/2)+4/63/(1-x)^(5/2)/(
1+x)^(1/2)+4/63/(1-x)^(3/2)/(1+x)^(1/2)+8/63*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 39}

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*Sqrt[1 + x]) + 5/(63*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(63*
(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(63*(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(63*S
qrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx = \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{9}\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{20}{63}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{21}\int$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{}{63(1-}$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{}{63(1-}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.44

$$\frac{20-17x-16x^2+44x^3-32x^4+8x^5}{63(-1+x)^4\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] (20 - 17*x - 16*x^2 + 44*x^3 - 32*x^4 + 8*x^5)/(63*(-1 + x)^4*Sqrt[1 - x^2]
)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 88.37, size = 388, normalized size = 3.80

$$\text{Piecewise}\left[\left\{\left\{\frac{(-20+17x+16x^2-44x^3+32x^4-8x^5)\sqrt{\frac{1-x}{1+x}}}{63(-1+5x-10x^2+10x^3-5x^4+x^5)}, \frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\}, \frac{-315I(1+x)\sqrt{1-\frac{2}{1+x}}}{3024+5040x-5040(1+x)^2-630(1+x)^4+63(1+x)^5+2520(1+x)^3} - \frac{252I(1+x)^3\sqrt{1-\frac{2}{1+x}}}{3024+5040x-5040(1+x)^2-630(1+x)^4+63(1+x)^5+2520(1+x)^3} - \frac{8I(1+x)^5\sqrt{1-\frac{2}{1+x}}}{3024+5040x-5040(1+x)^2-630(1+x)^4+63(1+x)^5+2520(1+x)^3} + \frac{I63\sqrt{1-\frac{2}{1+x}}}{3024+5040x-5040(1+x)^2-630(1+x)^4+63(1+x)^5+2520(1+x)^3} + \frac{I72(1+x)^4\sqrt{1-\frac{2}{1+x}}}{3024+5040x-5040(1+x)^2-630(1+x)^4+63(1+x)^5+2520(1+x)^3} + \frac{I420(1+x)^2\sqrt{1-\frac{2}{1+x}}}{3024+5040x-5040(1+x)^2-630(1+x)^4+63(1+x)^5+2520(1+x)^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{(-20 + 17 x + 16 x ^ 2 - 44 x ^ 3 + 32 x ^ 4 - 8 x ^ 5) Sqrt[(1
 - x) / (1 + x)] / (63 (-1 + 5 x - 10 x ^ 2 + 10 x ^ 3 - 5 x ^ 4 + x ^ 5)),
 1 / Abs[1 + x] > 1 / 2}}, -315 I (1 + x) Sqrt[1 - 2 / (1 + x)] / (3024 + 5
040 x - 5040 (1 + x) ^ 2 - 630 (1 + x) ^ 4 + 63 (1 + x) ^ 5 + 2520 (1 + x)
^ 3) - 252 I (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] / (3024 + 5040 x - 5040 (1 +
 x) ^ 2 - 630 (1 + x) ^ 4 + 63 (1 + x) ^ 5 + 2520 (1 + x) ^ 3) - 8 I (1 + x
) ^ 5 Sqrt[1 - 2 / (1 + x)] / (3024 + 5040 x - 5040 (1 + x) ^ 2 - 630 (1 +
x) ^ 4 + 63 (1 + x) ^ 5 + 2520 (1 + x) ^ 3) + I 63 Sqrt[1 - 2 / (1 + x)] /
(3024 + 5040 x - 5040 (1 + x) ^ 2 - 630 (1 + x) ^ 4 + 63 (1 + x) ^ 5 + 2520
```

```
(1 + x) ^ 3) + I 72 (1 + x) ^ 4 Sqrt[1 - 2 / (1 + x)] / (3024 + 5040 x - 5
040 (1 + x) ^ 2 - 630 (1 + x) ^ 4 + 63 (1 + x) ^ 5 + 2520 (1 + x) ^ 3) + I
420 (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] / (3024 + 5040 x - 5040 (1 + x) ^ 2 -
 630 (1 + x) ^ 4 + 63 (1 + x) ^ 5 + 2520 (1 + x) ^ 3)]
```

Maple [A]
time = 0.14, size = 86, normalized size = 0.84

method	result
gosper	$\frac{8x^5-32x^4+44x^3-16x^2-17x+20}{63\sqrt{1+x}\,(1-x)^{\frac{9}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-32x^4+44x^3-16x^2-17x+20)}{63\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$
default	$\frac{1}{9(1-x)^{\frac{9}{2}}\sqrt{1+x}}+\frac{5}{63(1-x)^{\frac{7}{2}}\sqrt{1+x}}+\frac{4}{63(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{4}{63(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{8}{63\sqrt{1-x}\,\sqrt{1+x}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(1/2)+5/63/(1-x)^(7/2)/(1+x)^(1/2)+4/63/(1-x)^(5/2)/(
1+x)^(1/2)+4/63/(1-x)^(3/2)/(1+x)^(1/2)+8/63/(1-x)^(1/2)/(1+x)^(1/2)-8/63*(
1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(72) = 144$.
time = 0.27, size = 201, normalized size = 1.97

$$\frac{8x}{63\sqrt{-x^2+1}}+\frac{1}{9\left(\sqrt{-x^2+1}\,x^4-4\sqrt{-x^2+1}\,x^3+6\sqrt{-x^2+1}\,x^2-4\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{5}{63\left(\sqrt{-x^2+1}\,x^3-3\sqrt{-x^2+1}\,x^2+3\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}+\frac{4}{63\left(\sqrt{-x^2+1}\,x^2-2\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{4}{63\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/63*x/sqrt(-x^2 + 1) + 1/9/(sqrt(-x^2 + 1)*x^4 - 4*sqrt(-x^2 + 1)*x^3 + 6*
sqrt(-x^2 + 1)*x^2 - 4*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 5/63/(sqrt(-x^2
 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4
/63/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 4/63/(sqrt
(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.29, size = 91, normalized size = 0.89

$$\frac{20\,x^6-80\,x^5+100\,x^4-100\,x^2-(8\,x^5-32\,x^4+44\,x^3-16\,x^2-17\,x+20)\sqrt{x+1}\,\sqrt{-x+1}+80\,x-20}{63\,(x^6-4\,x^5+5\,x^4-5\,x^2+4\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*(20*x^6 - 80*x^5 + 100*x^4 - 100*x^2 - (8*x^5 - 32*x^4 + 44*x^3 - 16*x
^2 - 17*x + 20)*sqrt(x + 1)*sqrt(-x + 1) + 80*x - 20)/(x^6 - 4*x^5 + 5*x^4
- 5*x^2 + 4*x - 1)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 117.56, size = 593, normalized size = 5.81

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^5}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} + \frac{72\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} - \frac{252\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} + \frac{420\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} - \frac{315\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} + \frac{63\sqrt{-1+\frac{2}{x+1}}}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^5}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} + \frac{72i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} - \frac{252i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} + \frac{420i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} - \frac{315i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} + \frac{63i\sqrt{1-\frac{2}{x+1}}}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(5040*x + 63*(x + 1)**5 - 630
*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) + 72*sqrt(-1 + 2/(x
 + 1))*(x + 1)**4/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**
3 - 5040*(x + 1)**2 + 3024) - 252*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(5040*x +
 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024)
 + 420*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(5040*x + 63*(x + 1)**5 - 630*(x + 1
)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) - 315*sqrt(-1 + 2/(x + 1))
*(x + 1)/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*
(x + 1)**2 + 3024) + 63*sqrt(-1 + 2/(x + 1))/(5040*x + 63*(x + 1)**5 - 630*
(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024), 1/Abs(x + 1) > 1/2)
, (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**5/(5040*x + 63*(x + 1)**5 - 630*(x + 1
)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) + 72*I*sqrt(1 - 2/(x + 1))
*(x + 1)**4/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 50
40*(x + 1)**2 + 3024) - 252*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(5040*x + 63*(
x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) + 42
0*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4
 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) - 315*I*sqrt(1 - 2/(x + 1))*(x
 + 1)/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x
+ 1)**2 + 3024) + 63*I*sqrt(1 - 2/(x + 1))/(5040*x + 63*(x + 1)**5 - 630*(x
 + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(72) = 144.
time = 0.02, size = 387, normalized size = 3.79

$$2\left(\frac{\frac{1}{9}\cdot 132922799578491587290380706028034457 6\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^9+\frac{1}{7}\cdot 172799639452039063477494917836444794 88\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^7+21267647932558653966460912964485513216\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^5+138239711561631250781995934269155835904\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3-\frac{580872634158008236458963685342510579712\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{4355614296588012332331194975126633106 6368}+\frac{-55062\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^8-6552\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^6-1008\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^4-117\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2-7}{2064384\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^9}-\frac{\sqrt{-x+1}\sqrt{x+1}}{64\,(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x)
```

```
[Out] -1/147456*(sqrt(2) - sqrt(x + 1))^9/(-x + 1)^(9/2) - 13/114688*(sqrt(2) - s
qrt(x + 1))^7/(-x + 1)^(7/2) - 1/1024*(sqrt(2) - sqrt(x + 1))^5/(-x + 1)^(5
```

```
/2) - 13/2048*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) - 437/8192*(sqrt(2)
- sqrt(x + 1))/sqrt(-x + 1) - 1/32*sqrt(-x + 1)/sqrt(x + 1) + 1/1032192*(55
062*(sqrt(2) - sqrt(x + 1))^8/(x - 1)^4 - 6552*(sqrt(2) - sqrt(x + 1))^6/(x
 - 1)^3 + 1008*(sqrt(2) - sqrt(x + 1))^4/(x - 1)^2 - 117*(sqrt(2) - sqrt(x
+ 1))^2/(x - 1) + 7)*(-x + 1)^(9/2)/(sqrt(2) - sqrt(x + 1))^9
```

Mupad [B]
time = 0.36, size = 80, normalized size = 0.78

$$\frac{17\,x\,\sqrt{1-x}\,-20\,\sqrt{1-x}\,+16\,x^2\,\sqrt{1-x}\,-44\,x^3\,\sqrt{1-x}\,+32\,x^4\,\sqrt{1-x}\,-8\,x^5\,\sqrt{1-x}}{63\,{\left(x-1\right)}^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(3/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 20*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(63*(x - 1)^5*(x +
1)^(1/2))
```

3.1126 $\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{105}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/2}\sqrt{1+x}+\frac{105}{2}\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(9/2)/(1+x)^(3/2)+105/2*arcsin(x)+6*(1-x)^(7/2)/(1+x)^(1/2)+35/2
*(1-x)^(3/2)*(1+x)^(1/2)+7*(1-x)^(5/2)*(1+x)^(1/2)+105/2*(1-x)^(1/2)*(1+x)^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) + (6*(1 - x)^(7/2))/Sqrt[1 + x] + (105
*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/2 + 7*(1 - x)^
(5/2)*Sqrt[1 + x] + (105*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}-3\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx\\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+21\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+7(1-x)^{5/2}\sqrt{1+x}+35\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/2}\sqrt{1+x}+\frac{105}{2}\int \frac{\sqrt{1-}}{\sqrt{1-}}\\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{105}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/}\\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{105}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/}\\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{105}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 61, normalized size = 0.59

$$\frac{\sqrt{1-x}\,\left(494+679x+102x^2-17x^3+2x^4\right)}{6(1+x)^{3/2}}-105\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(494 + 679*x + 102*x^2 - 17*x^3 + 2*x^4))/(6*(1 + x)^(3/2)) -
105*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 38.27, size = 193, normalized size = 1.87

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-630\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](-1+x)^{\frac{3}{2}}(1+x)^2+128(-1+x)\sqrt{1+x}+896(1-x)(1+x)^{\frac{3}{2}}+(-1+x)\left(301+215x-29(1+x)^2+2(1+x)^3\right)(1+x)^{\frac{5}{2}}\right)}{6(-1+x)^{\frac{3}{2}}(1+x)^2},\text{Abs}[1+x]>2\right\}\right\},\frac{-215(1+x)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{64}{3(1+x)^{\frac{3}{2}}\sqrt{1-x}}-\frac{43\sqrt{1+x}}{3\sqrt{1-x}}-\frac{(1+x)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{29(1+x)^{\frac{5}{2}}}{6\sqrt{1-x}}+105\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]+\frac{448}{3\sqrt{1+x}\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 6 (-630 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] (-1 + x) ^ (3 / 2)
 (1 + x) ^ 2 + 128 (-1 + x) Sqrt[1 + x] + 896 (1 - x) (1 + x) ^ (3 / 2) + (
-1 + x) (301 + 215 x - 29 (1 + x) ^ 2 + 2 (1 + x) ^ 3) (1 + x) ^ (5 / 2)) /
 ((-1 + x) ^ (3 / 2) (1 + x) ^ 2), Abs[1 + x] > 2}}, -215 (1 + x) ^ (3 / 2)
 / (6 Sqrt[1 - x]) - 64 / (3 (1 + x) ^ (3 / 2) Sqrt[1 - x]) - 43 Sqrt[1 + x
] / (3 Sqrt[1 - x]) - (1 + x) ^ (7 / 2) / (3 Sqrt[1 - x]) + 29 (1 + x) ^ (5
 / 2) / (6 Sqrt[1 - x]) + 105 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 448 / (3 Sq
rt[1 + x] Sqrt[1 - x])]
```

Maple [A]
time = 0.17, size = 89, normalized size = 0.86

method	result	size
risch	$-\frac{(2x^5-19x^4+119x^3+577x^2-185x-494)\sqrt{(1+x)(1-x)}}{6(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\sqrt{1-x}}+\frac{105\sqrt{(1+x)(1-x)}\arcsin(x)}{2\sqrt{1+x}\sqrt{1-x}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*x^5-19*x^4+119*x^3+577*x^2-185*x-494)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(
1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+105/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.34, size = 125, normalized size = 1.21

$$\frac{x^6}{3(-x^2+1)^{\frac{3}{2}}}-\frac{7x^5}{2(-x^2+1)^{\frac{3}{2}}}+\frac{23x^4}{(-x^2+1)^{\frac{3}{2}}}+\frac{35}{2}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{143x}{6\sqrt{-x^2+1}}-\frac{127x^2}{(-x^2+1)^{\frac{3}{2}}}+\frac{22x}{3(-x^2+1)^{\frac{3}{2}}}+\frac{247}{3(-x^2+1)^{\frac{3}{2}}}+\frac{105}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^6/(-x^2 + 1)^(3/2) - 7/2*x^5/(-x^2 + 1)^(3/2) + 23*x^4/(-x^2 + 1)^(3/
2) + 35/2*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 143/6*x/sqrt(-x
^2 + 1) - 127*x^2/(-x^2 + 1)^(3/2) + 22/3*x/(-x^2 + 1)^(3/2) + 247/3/(-x^2
+ 1)^(3/2) + 105/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 85, normalized size = 0.83

$$\frac{494\,x^2 + (2\,x^4 - 17\,x^3 + 102\,x^2 + 679\,x + 494)\sqrt{x+1}\,\sqrt{-x+1} - 630\,(x^2 + 2\,x + 1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) + 988\,x + 494}{6\,(x^2 + 2\,x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(494*x^2 + (2*x^4 - 17*x^3 + 102*x^2 + 679*x + 494)*sqrt(x + 1)*sqrt(-x
 + 1) - 630*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 988*
x + 494)/(x^2 + 2*x + 1)
```

Sympy [A]
time = 53.13, size = 248, normalized size = 2.41

$$\begin{cases} -105i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{29i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} + \frac{215i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{43i\sqrt{x+1}}{3\sqrt{x-1}} - \frac{448i}{3\sqrt{x-1}\,\sqrt{x+1}} + \frac{64i}{3\sqrt{x-1}\,(x+1)^{\frac{3}{2}}} & \text{for}\ |x+1| > 2 \\ 105\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{29(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} - \frac{215(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{43\sqrt{x+1}}{3\sqrt{1-x}} + \frac{448}{3\sqrt{1-x}\,\sqrt{x+1}} - \frac{64}{3\sqrt{1-x}\,(x+1)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-105*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x
 - 1)) - 29*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 215*I*(x + 1)**(3/2)/(6*sqrt
(x - 1)) + 43*I*sqrt(x + 1)/(3*sqrt(x - 1)) - 448*I/(3*sqrt(x - 1)*sqrt(x +
 1)) + 64*I/(3*sqrt(x - 1)*(x + 1)**(3/2)), Abs(x + 1) > 2), (105*asin(sqrt
(2)*sqrt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 29*(x + 1)**(5/2)/(6*
sqrt(1 - x)) - 215*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 43*sqrt(x + 1)/(3*sqrt(
1 - x)) + 448/(3*sqrt(1 - x)*sqrt(x + 1)) - 64/(3*sqrt(1 - x)*(x + 1)**(3/2
)), True))
```

Giac [A]
time = 0.01, size = 127, normalized size = 1.23

$$\frac{2\left(\left(\left(\left(\frac{1}{6}\sqrt{-x+1}\,\sqrt{-x+1} + \frac{3}{4}\right)\sqrt{-x+1}\,\sqrt{-x+1} + \frac{21}{4}\right)\sqrt{-x+1}\,\sqrt{-x+1} - 70\right)\sqrt{-x+1}\,\sqrt{-x+1} + 105\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x+1)^2} - 105\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x)
```

```
[Out] 1/6*((((2*x - 11)*(x - 1) + 63)*(x - 1) + 840)*(x - 1) + 1260)*sqrt(-x + 1)
/(x + 1)^(3/2) - 105*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{9/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)/(x + 1)^(5/2),x)
[Out] int((1 - x)^(9/2)/(x + 1)^(5/2), x)
```

3.1127 $\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(7/2)/(1+x)^(3/2)+35/2*arcsin(x)+14/3*(1-x)^(5/2)/(1+x)^(1/2)+35
/6*(1-x)^(3/2)*(1+x)^(1/2)+35/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 52, 41, 222}

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) + (14*(1 - x)^(5/2))/(3*Sqrt[1 + x]) +
 (35*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/6 + (35*Ar
cSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}-\frac{7}{3}\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\int \frac{}{\sqrt{1}} \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\int \frac{}{\sqrt{1}} \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\sin^{-1}(
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.64

$$\frac{\sqrt{1-x}\,\left(164+229x+30x^2-3x^3\right)}{6(1+x)^{3/2}}-35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(164 + 229*x + 30*x^2 - 3*x^3))/(6*(1 + x)^(3/2)) - 35*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 14.60, size = 172, normalized size = 1.98

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-210\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right](-1+x)^{\frac{3}{2}}(1+x)^2+64(-1+x)\sqrt{1+x}+352(1-x)(1+x)^{\frac{3}{2}}+(-1+x)\left(127+45x-3(1+x)^2\right)(1+x)^{\frac{5}{2}}\right)}{6(-1+x)^{\frac{3}{2}}(1+x)^2},\text{Abs}[1+x]>2\right\}\right\},\frac{-41\sqrt{1+x}}{3\sqrt{1-x}}-\frac{32}{3(1+x)^{\frac{3}{2}}\sqrt{1-x}}-\frac{15(1+x)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{(1+x)^{\frac{5}{2}}}{2\sqrt{1-x}}+35\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]+\frac{176}{3\sqrt{1+x}\sqrt{1-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{I / 6 (-210 ArcCosh[Sqrt[2] Sqrt[1 + x] / 2] (-1 + x) ^ (3 / 2)
 (1 + x) ^ 2 + 64 (-1 + x) Sqrt[1 + x] + 352 (1 - x) (1 + x) ^ (3 / 2) + (-
1 + x) (127 + 45 x - 3 (1 + x) ^ 2) (1 + x) ^ (5 / 2)) / ((-1 + x) ^ (3 / 2
) (1 + x) ^ 2), Abs[1 + x] > 2}}, -41 Sqrt[1 + x] / (3 Sqrt[1 - x]) - 32 /
(3 (1 + x) ^ (3 / 2) Sqrt[1 - x]) - 15 (1 + x) ^ (3 / 2) / (2 Sqrt[1 - x])
+ (1 + x) ^ (5 / 2) / (2 Sqrt[1 - x]) + 35 ArcSin[Sqrt[2] Sqrt[1 + x] / 2]
+ 176 / (3 Sqrt[1 + x] Sqrt[1 - x])]
```

Maple [A]
time = 0.16, size = 84, normalized size = 0.97

method	result	size
risch	$\frac{(3x^4-33x^3-199x^2+65x+164)\sqrt{(1+x)(1-x)}}{6(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(3*x^4-33*x^3-199*x^2+65*x+164)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x
)*(1-x))^(1/2)/(1-x)^(1/2)+35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Maxima [A]
time = 0.34, size = 111, normalized size = 1.28

$$-\frac{x^5}{2\,(-x^2+1)^{\frac{3}{2}}}+\frac{6\,x^4}{(-x^2+1)^{\frac{3}{2}}}+\frac{35}{6}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{61\,x}{6\,\sqrt{-x^2+1}}-\frac{44\,x^2}{(-x^2+1)^{\frac{3}{2}}}+\frac{16\,x}{3\,(-x^2+1)^{\frac{3}{2}}}+\frac{82}{3\,(-x^2+1)^{\frac{3}{2}}}+\frac{35}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/(-x^2 + 1)^(3/2) + 6*x^4/(-x^2 + 1)^(3/2) + 35/6*x*(3*x^2/(-x^2 +
1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 61/6*x/sqrt(-x^2 + 1) - 44*x^2/(-x^2 + 1)^
(3/2) + 16/3*x/(-x^2 + 1)^(3/2) + 82/3/(-x^2 + 1)^(3/2) + 35/2*arcsin(x)
```

Fricas [A]
time = 0.30, size = 81, normalized size = 0.93

$$\frac{164\,x^2-(3\,x^3-30\,x^2-229\,x-164)\sqrt{x+1}\,\sqrt{-x+1}-210\,(x^2+2\,x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+328\,x+164}{6\,(x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(164*x^2 - (3*x^3 - 30*x^2 - 229*x - 164)*sqrt(x + 1)*sqrt(-x + 1) - 21
0*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 328*x + 164)/(
x^2 + 2*x + 1)
```

Sympy [A]
time = 16.63, size = 212, normalized size = 2.44

$$\begin{cases} -35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{15i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{41i\sqrt{x+1}}{3\sqrt{x-1}} - \frac{176i}{3\sqrt{x-1}\sqrt{x+1}} + \frac{32i}{3\sqrt{x-1}(x+1)^{\frac{3}{2}}} & \text{for } |x+1| > 2 \\ 35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{15(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{41\sqrt{x+1}}{3\sqrt{1-x}} + \frac{176}{3\sqrt{1-x}\sqrt{x+1}} - \frac{32}{3\sqrt{1-x}(x+1)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x
- 1)) + 15*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 41*I*sqrt(x + 1)/(3*sqrt(x -
1)) - 176*I/(3*sqrt(x - 1)*sqrt(x + 1)) + 32*I/(3*sqrt(x - 1)*(x + 1)**(3/2
)), Abs(x + 1) > 2), (35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sq
rt(1 - x)) - 15*(x + 1)**(3/2)/(2*sqrt(1 - x)) - 41*sqrt(x + 1)/(3*sqrt(1 -
 x)) + 176/(3*sqrt(1 - x)*sqrt(x + 1)) - 32/(3*sqrt(1 - x)*(x + 1)**(3/2)),
 True))
```

Giac [A]
time = 0.01, size = 108, normalized size = 1.24

$$\frac{2\left(\left(\left(\left(\frac{1}{4}\sqrt{-x+1}\sqrt{-x+1}+\frac{7}{4}\right)\sqrt{-x+1}\sqrt{-x+1}-\frac{70}{3}\right)\sqrt{-x+1}\sqrt{-x+1}+35\right)\sqrt{-x+1}\sqrt{x+1}}{(x+1)^2} - 35\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x)
```

```
[Out] -1/6*((3*(x - 1)*(x - 8) - 280)*(x - 1) - 420)*sqrt(-x + 1)/(x + 1)^(3/2) -
 35*arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(5/2), x)
```

3.1128 $\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(5/2)/(1+x)^(3/2)+5*arcsin(x)+10/3*(1-x)^(3/2)/(1+x)^(1/2)+5*(1-
x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\,\sqrt{1-x}+5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) + (10*(1 - x)^(3/2))/(3*Sqrt[1 + x]) +
 5*Sqrt[1 - x]*Sqrt[1 + x] + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}-\frac{5}{3}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 51, normalized size = 0.81

$$\frac{\sqrt{1-x}\,(23+34x+3x^2)}{3(1+x)^{3/2}}+10\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]
[Out] (Sqrt[1 - x]*(23 + 34*x + 3*x^2))/(3*(1 + x)^(3/2)) + 10*ArcTan[Sqrt[1 + x]
/Sqrt[1 - x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.47, size = 171, normalized size = 2.71

$$\text{Piecewise}\left[\left\{\left\{\frac{-8\sqrt{\frac{1-x}{1+x}}+(1+x)\left(15I\text{Log}[1+x]+15I\text{Log}\left[\frac{1}{1+x}\right]+3(1+x)\sqrt{\frac{1-x}{1+x}}+28\sqrt{\frac{1-x}{1+x}}+30\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{1+x}}{2}\right]\right)}{3(1+x)},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},-10I\text{Log}\left[1+\sqrt{1-\frac{2}{1+x}}\right]-\frac{8I\sqrt{1-\frac{2}{1+x}}}{3(1+x)}+I5\text{Log}\left[\frac{1}{1+x}\right]+\frac{I28\sqrt{1-\frac{2}{1+x}}}{3}+I(1+x)\sqrt{1-\frac{2}{1+x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{(-8 Sqrt[(1 - x) / (1 + x)] + (1 + x) (15 I Log[1 + x] + 15 I L
og[1 / (1 + x)] + 3 (1 + x) Sqrt[(1 - x) / (1 + x)] + 28 Sqrt[(1 - x) / (1
+ x)] + 30 ArcSin[Sqrt[2] Sqrt[1 + x] / 2])) / (3 (1 + x)), 1 / Abs[1 + x]
> 1 / 2}}, -10 I Log[1 + Sqrt[1 - 2 / (1 + x)]] - 8 I Sqrt[1 - 2 / (1 + x)]
 / (3 (1 + x)) + I 5 Log[1 / (1 + x)] + I 28 Sqrt[1 - 2 / (1 + x)] / 3 + I
(1 + x) Sqrt[1 - 2 / (1 + x)]]
```

Maple [A]
time = 0.17, size = 79, normalized size = 1.25

method	result	size
risch	$-\frac{\left(3 x^{3}+31 x^{2}-11 x -23\right) \sqrt{\left(1+x \right) \left(1-x \right)}}{3 \left(1+x \right)^{\frac{3}{2}} \sqrt{-\left(1+x \right) \left(-1+x \right)}\, \sqrt{1-x}}+\frac{5 \sqrt{\left(1+x \right) \left(1-x \right)}\, \arcsin \left(x \right)}{\sqrt{1+x}\, \sqrt{1-x}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*x^3+31*x^2-11*x-23)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))
^(1/2)/(1-x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.
time = 0.35, size = 98, normalized size = 1.56

$$\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^4+4\,x^3+6\,x^2+4\,x+1}-\frac{5\,\left(-x^2+1\right)^{\frac{3}{2}}}{3\,\left(x^3+3\,x^2+3\,x+1\right)}-\frac{10\,\sqrt{-x^2+1}}{3\,\left(x^2+2\,x+1\right)}+\frac{35\,\sqrt{-x^2+1}}{3\,(x+1)}+5\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x^
3 + 3*x^2 + 3*x + 1) - 10/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 35/3*sqrt(-x^2
 + 1)/(x + 1) + 5*arcsin(x)
```

Fricas [A]
time = 0.29, size = 75, normalized size = 1.19

$$\frac{23\,x^2+\left(3\,x^2+34\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}-30\,\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+46\,x+23}{3\,\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(23*x^2 + (3*x^2 + 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) - 30*(x^2 + 2*x
+ 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 46*x + 23)/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.83, size = 162, normalized size = 2.57

$$\begin{cases} \sqrt{-1+\frac{2}{x+1}}\,(x+1)+\frac{28\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{8\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)+5i\log(x+1)+10\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ i\sqrt{1-\frac{2}{x+1}}\,(x+1)+\frac{28i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{8i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)-10i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))*(x + 1) + 28*sqrt(-1 + 2/(x + 1))/3 - 8*sqr
t(-1 + 2/(x + 1))/(3*(x + 1)) + 5*I*log(1/(x + 1)) + 5*I*log(x + 1) + 10*as
in(sqrt(2)*sqrt(x + 1)/2), 1/Abs(x + 1) > 1/2), (I*sqrt(1 - 2/(x + 1))*(x +
 1) + 28*I*sqrt(1 - 2/(x + 1))/3 - 8*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) + 5*
I*log(1/(x + 1)) - 10*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Giac [A]
time = 0.01, size = 86, normalized size = 1.37

$$\frac{2\left(\left(\left(\frac{1}{2}\sqrt{-x+1}\,\sqrt{-x+1}-\frac{20}{3}\right)\sqrt{-x+1}\,\sqrt{-x+1}+10\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x+1)^2}-10\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x)
```

```
[Out] 1/3*((3*x + 37)*(x - 1) + 60)*sqrt(-x + 1)/(x + 1)^(3/2) - 10*arcsin(1/2*sq
rt(2)*sqrt(-x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(5/2), x)
```

3.1129 $\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{1+x}}+\sin^{-1}(x)$$

[Out] -2/3*(1-x)^(3/2)/(1+x)^(3/2)+arcsin(x)+2*(1-x)^(1/2)/(1+x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x)^(3/2)/(1 + x)^(5/2),x]

[Out] (-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSin[x]

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} - \int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx \\ &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.12

$$\frac{4\sqrt{1-x}\,(1+2x)}{3(1+x)^{3/2}} - 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (4*Sqrt[1 - x]*(1 + 2*x))/(3*(1 + x)^(3/2)) - 2*ArcTan[Sqrt[1 - x]/Sqrt[1 +
 x]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.15, size = 137, normalized size = 3.34

$$\text{Piecewise}\left[\left\{\left\{\frac{-4\sqrt{\frac{1-x}{1+x}} + (1+x)\left(3I\text{Log}\,[1+x] + 3I\text{Log}\left[\frac{1}{1+x}\right] + 6\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{1+x}}{2}\right] + 8\sqrt{\frac{1-x}{1+x}}\right)}{3\,(1+x)}, \frac{1}{\text{Abs}\,[1+x]} > \frac{1}{2}\right\}\right\}, -2I\text{Log}\left[1+\sqrt{1-\frac{2}{1+x}}\right] - \frac{4I\sqrt{1-\frac{2}{1+x}}}{3\,(1+x)} + \frac{I8\sqrt{1-\frac{2}{1+x}}}{3} + I\text{Log}\left[\frac{1}{1+x}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{(-4 Sqrt[(1 - x) / (1 + x)] + (1 + x) (3 I Log[1 + x] + 3 I Log
[1 / (1 + x)] + 6 ArcSin[Sqrt[2] Sqrt[1 + x] / 2] + 8 Sqrt[(1 - x) / (1 + x
)])) / (3 (1 + x)), 1 / Abs[1 + x] > 1 / 2}}, -2 I Log[1 + Sqrt[1 - 2 / (1
+ x)]] - 4 I Sqrt[1 - 2 / (1 + x)] / (3 (1 + x)) + I 8 Sqrt[1 - 2 / (1 + x)
] / 3 + I Log[1 / (1 + x)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.
time = 0.16, size = 73, normalized size = 1.78

method	result	size
risch	$-\frac{4(2x^2-x-1)\sqrt{(1+x)(1-x)}}{3(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\arcsin(x)}{\sqrt{1+x}\sqrt{1-x}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(2*x^2-x-1)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x
)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 0.36, size = 66, normalized size = 1.61

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{3(x^3+3x^2+3x+1)}-\frac{2\sqrt{-x^2+1}}{3(x^2+2x+1)}+\frac{7\sqrt{-x^2+1}}{3(x+1)}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 + 3*x^2 + 3*x + 1) - 2/3*sqrt(-x^2 + 1)/(x^2 + 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 0.30, size = 71, normalized size = 1.73

$$\frac{2\left(2x^2+2(2x+1)\sqrt{x+1}\sqrt{-x+1}-3(x^2+2x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+4x+2\right)}{3(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + 2*(2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) - 3*(x^2 + 2*x + 1)*arcta
n((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.06, size = 128, normalized size = 3.12

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{4\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+i\log\left(\frac{1}{x+1}\right)+i\log(x+1)+2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ \frac{8i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{4i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+i\log\left(\frac{1}{x+1}\right)-2i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))/3 - 4*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
I*log(1/(x + 1)) + I*log(x + 1) + 2*asin(sqrt(2)*sqrt(x + 1)/2), 1/Abs(x +
1) > 1/2), (8*I*sqrt(1 - 2/(x + 1))/3 - 4*I*sqrt(1 - 2/(x + 1))/(3*(x + 1))
 + I*log(1/(x + 1)) - 2*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Giac [A]
time = 0.01, size = 65, normalized size = 1.59

$$\frac{2\left(-\frac{4}{3}\sqrt{-x+1}\sqrt{-x+1}+2\right)\sqrt{-x+1}\sqrt{x+1}}{(x+1)^{2}}-2\arcsin\left(\frac{\sqrt{-x+1}}{\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x)
```

```
[Out] 4/3*(2*x + 1)*sqrt(-x + 1)/(x + 1)^(3/2) - 2*arcsin(1/2*sqrt(2)*sqrt(-x + 1
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(5/2), x)
```

3.1130 $\int \frac{\sqrt{1-x}}{(1+x)^{5/2}} \, dx$

Optimal. Leaf size=20

$$-\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

```
[Out] -1/3*(1-x)^(3/2)/(1+x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

```
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}} \, dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

Mathematica [A]
time = 0.04, size = 20, normalized size = 1.00

$$-\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

```
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.93, size = 66, normalized size = 3.30

$$\text{Piecewise}\left[\left\{\left\{\frac{(-1+x)\sqrt{\frac{1-x}{1+x}}}{3\,(1+x)}, \frac{1}{\text{Abs}\,[1+x]} > \frac{1}{2}\right\}\right\}, \frac{-2I\sqrt{1-\frac{2}{1+x}}}{3\,(1+x)} + \frac{I\sqrt{1-\frac{2}{1+x}}}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/2)/(1 + x)^(5/2),x]')
```

```
[Out] Piecewise[{{(-1 + x) Sqrt[(1 - x) / (1 + x)] / (3 (1 + x)), 1 / Abs[1 + x]
> 1 / 2}}, -2 I Sqrt[1 - 2 / (1 + x)] / (3 (1 + x)) + I Sqrt[1 - 2 / (1 + x
)] / 3]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 0.16, size = 30, normalized size = 1.50

method	result	size
gosper	$-\frac{(1-x)^{\frac{3}{2}}}{3(1+x)^{\frac{3}{2}}}$	15
default	$-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{1+x}}$	30
risch	$-\frac{\sqrt{(1+x)\,(1-x)}\,(x^2-2x+1)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)\,(-1+x)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1-x)^(1/2)/(1+x)^(3/2)+1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.
time = 0.26, size = 38, normalized size = 1.90

$$-\frac{2\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)}+\frac{\sqrt{-x^2+1}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 0.29, size = 37, normalized size = 1.85

$$-\frac{x^2 - \sqrt{x+1}\,(x-1)\sqrt{-x+1} + 2\,x + 1}{3\,(x^2 + 2\,x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(x^2 - sqrt(x + 1)*(x - 1)*sqrt(-x + 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

Sympy [A]
time = 0.95, size = 66, normalized size = 3.30

$$\begin{cases} \frac{\sqrt{-1 + \frac{2}{x+1}}}{3} - \frac{2\sqrt{-1 + \frac{2}{x+1}}}{3(x+1)} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ \frac{i\sqrt{1 - \frac{2}{x+1}}}{3} - \frac{2i\sqrt{1 - \frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(5/2),x)
[Out] Piecewise((sqrt(-1 + 2/(x + 1))/3 - 2*sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 1/A
bs(x + 1) > 1/2), (I*sqrt(1 - 2/(x + 1))/3 - 2*I*sqrt(1 - 2/(x + 1))/(3*(x
+ 1)), True))
```

Giac [A]
time = 0.01, size = 41, normalized size = 2.05

$$-\frac{\sqrt{-x+1}\,\sqrt{-x+1}\,\sqrt{-x+1}\,\sqrt{x+1}}{3\,(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x)
[Out] 1/3*(x - 1)*sqrt(-x + 1)/(x + 1)^(3/2)
```

Mupad [B]
time = 0.26, size = 32, normalized size = 1.60

$$\frac{x\,\sqrt{1-x} - \sqrt{1-x}}{(3\,x+3)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(5/2),x)
[Out] (x*(1 - x)^(1/2) - (1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

3.1131 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}$$

```
[Out] -1/3*(1-x)^(1/2)/(1+x)^(3/2)-1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {47, 37}

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -1/3*Sqrt[1 - x]/(1 + x)^(3/2) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx&=-\frac{\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx\\&=-\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.56

$$-\frac{\sqrt{1-x}\,(2+x)}{3(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -1/3*(Sqrt[1 - x]*(2 + x))/(1 + x)^(3/2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.42, size = 68, normalized size = 1.66

$$\text{Piecewise}\left[\left\{\left\{\frac{(-2-x)\sqrt{\frac{1-x}{1+x}}}{3\,(1+x)},\frac{1}{\text{Abs}\,[1+x]}>\frac{1}{2}\right\}\right\},-\frac{I\sqrt{1-\frac{2}{1+x}}}{3\,(1+x)}-\frac{I\sqrt{1-\frac{2}{1+x}}}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(1/2)*(1 + x)^(5/2)),x]')
```

```
[Out] Piecewise[{{(-2 - x) Sqrt[(1 - x) / (1 + x)] / (3 (1 + x)), 1 / Abs[1 + x]
> 1 / 2}}, -I Sqrt[1 - 2 / (1 + x)] / (3 (1 + x)) - I Sqrt[1 - 2 / (1 + x)]
 / 3]
```

Maple [A]
time = 0.16, size = 30, normalized size = 0.73

method	result	size
gosper	$-\frac{(2+x)\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}$	18
default	$-\frac{\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^{2}+x-2)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(1-x)^(1/2)/(1+x)^(3/2)-1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.34, size = 38, normalized size = 0.93

$$-\frac{\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)}-\frac{\sqrt{-x^2+1}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 0.93

$$-\frac{2\,x^2+(x+2)\sqrt{x+1}\,\sqrt{-x+1}\,+4\,x+2}{3\,(x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^2 + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [A]
time = 1.37, size = 66, normalized size = 1.61

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))/3 - sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 1/Ab
s(x + 1) > 1/2), (-I*sqrt(1 - 2/(x + 1))/3 - I*sqrt(1 - 2/(x + 1))/(3*(x +
1)), True))
```

Giac [A]
time = 0.00, size = 47, normalized size = 1.15

$$-\frac{4\left(\frac{1}{4}-\frac{1}{12}\sqrt{-x+1}\,\sqrt{-x+1}\,\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x)
```

```
[Out] -1/3*(x + 2)*sqrt(-x + 1)/(x + 1)^(3/2)
```

Mupad [B]
time = 0.31, size = 33, normalized size = 0.80

$$-\frac{x\,\sqrt{1-x}+2\,\sqrt{1-x}}{\left(3\,x+3\right)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

3.1132 $\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$$

```
[Out] 1/(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(Sqrt[1 - x]*(1 + x)^(3/2)) - (2*Sqrt[1 - x])/(3*(1 + x)^(3/2)) - (2*Sqrt
[1 - x])/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} + 2\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx$$
$$= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3(1+x)^{3/2}} + \frac{2}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx$$
$$= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3\sqrt{1+x}}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.52

$$\frac{-1+2x+2x^2}{3\sqrt{1-x}\,(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
[Out] (-1 + 2*x + 2*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.46, size = 133, normalized size = 2.29

$$\text{Piecewise}\left[\left\{\left\{\frac{(1-2x-2x^2)\sqrt{\frac{1-x}{1+x}}}{3(-1+x^2)}, \frac{1}{\text{Abs}[1+x]} > \frac{1}{2}\right\}\right\}, \frac{-2I(1+x)^2\sqrt{1-\frac{2}{1+x}}}{-6-6x+3(1+x)^2} + \frac{I2(1+x)\sqrt{1-\frac{2}{1+x}}}{-6-6x+3(1+x)^2} + \frac{I\sqrt{1-\frac{2}{1+x}}}{-6-6x+3(1+x)^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]')
[Out] Piecewise[{{(1 - 2 x - 2 x ^ 2) Sqrt[(1 - x) / (1 + x)] / (3 (-1 + x ^ 2)),
 1 / Abs[1 + x] > 1 / 2}}, -2 I (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] / (-6 - 6
 x + 3 (1 + x) ^ 2) + I 2 (1 + x) Sqrt[1 - 2 / (1 + x)] / (-6 - 6 x + 3 (1
+ x) ^ 2) + I Sqrt[1 - 2 / (1 + x)] / (-6 - 6 x + 3 (1 + x) ^ 2)]
```

Maple [A]
time = 0.16, size = 43, normalized size = 0.74

method	result	size
gosper	$\frac{2x^2+2x-1}{3(1+x)^{\frac{3}{2}}\sqrt{1-x}}$	25
default	$\frac{1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	43

risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^2+2x-1)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)
^(1/2)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 0.66

$$\frac{2x}{3\sqrt{-x^2+1}}-\frac{1}{3\left(\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.30, size = 49, normalized size = 0.84

$$-\frac{x^3+x^2+(2x^2+2x-1)\sqrt{x+1}\,\sqrt{-x+1}-x-1}{3(x^3+x^2-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + x^2 + (2*x^2 + 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x - 1)/(x^3
+ x^2 - x - 1)
```

Sympy [A]
time = 3.49, size = 167, normalized size = 2.88

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-6x+3(x+1)^2-6}+\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-6x+3(x+1)^2-6}+\frac{\sqrt{-1+\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-6x+3(x+1)^2-6}+\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-6x+3(x+1)^2-6}+\frac{i\sqrt{1-\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2
*sqrt(-1 + 2/(x + 1))*(x + 1)/(-6*x + 3*(x + 1)**2 - 6) + sqrt(-1 + 2/(x +
1))/(-6*x + 3*(x + 1)**2 - 6), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1
))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2*I*sqrt(1 - 2/(x + 1))*(x + 1)/(
-6*x + 3*(x + 1)**2 - 6) + I*sqrt(1 - 2/(x + 1))/(-6*x + 3*(x + 1)**2 - 6),
 True))
```

Giac [A]
time = 0.00, size = 111, normalized size = 1.91

$$2\left(\frac{\sqrt{-x+1}}{8\left(-2\sqrt{x+1}+2\sqrt{2}\right)}-\frac{-2\sqrt{x+1}+2\sqrt{2}}{32\sqrt{-x+1}}+\frac{2\left(\frac{5}{48}\sqrt{-x+1}\sqrt{-x+1}-\frac{1}{4}\right)\sqrt{-x+1}\sqrt{x+1}}{(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x)
```

```
[Out] -1/8*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/12*(5*x + 7)*sqrt(-x + 1)/(x
+ 1)^(3/2) + 1/8*sqrt(-x + 1)/(sqrt(2) - sqrt(x + 1))
```

Mupad [B]
time = 0.34, size = 48, normalized size = 0.83

$$-\frac{2\,x\,\sqrt{1-x}-\sqrt{1-x}+2\,x^2\,\sqrt{1-x}}{\left(3\,x^2-3\right)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(2*x*(1 - x)^(1/2) - (1 - x)^(1/2) + 2*x^2*(1 - x)^(1/2))/((3*x^2 - 3)*(x
+ 1)^(1/2))
```

3.1133 $\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}} dx$

Optimal. Leaf size=43

$$\frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/3*x/(1-x)^(3/2)/(1+x)^(3/2)+2/3*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {40, 39}

$$\frac{2x}{3\sqrt{1-x}\,\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
[Out] x/(3*(1 - x)^(3/2)*(1 + x)^(3/2)) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 0.56

$$\frac{3x-2x^3}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] (3*x - 2*x^3)/(3*(1 - x^2)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 8.56, size = 199, normalized size = 4.63

$$\text{Piecewise}\left[\left\{\left\{\frac{x\left(3-2x^2\right)\sqrt{\frac{1-x}{1+x}}}{3\left(1-x-x^2+x^3\right)},\frac{1}{\text{Abs}\left[1+x\right]}>\frac{1}{2}\right\}\right\},\frac{-3I\left(1+x\right)\sqrt{1-\frac{2}{1+x}}}{12+12x-12\left(1+x\right)^2+3\left(1+x\right)^3}-\frac{2I\left(1+x\right)^3\sqrt{1-\frac{2}{1+x}}}{12+12x-12\left(1+x\right)^2+3\left(1+x\right)^3}-\frac{I\sqrt{1-\frac{2}{1+x}}}{12+12x-12\left(1+x\right)^2+3\left(1+x\right)^3}+\frac{I6\left(1+x\right)^2\sqrt{1-\frac{2}{1+x}}}{12+12x-12\left(1+x\right)^2+3\left(1+x\right)^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]')
```

```
[Out] Piecewise[{{x (3 - 2 x ^ 2) Sqrt[(1 - x) / (1 + x)] / (3 (1 - x - x ^ 2 + x
 ^ 3)), 1 / Abs[1 + x] > 1 / 2}}, -3 I (1 + x) Sqrt[1 - 2 / (1 + x)] / (12
+ 12 x - 12 (1 + x) ^ 2 + 3 (1 + x) ^ 3) - 2 I (1 + x) ^ 3 Sqrt[1 - 2 / (1
+ x)] / (12 + 12 x - 12 (1 + x) ^ 2 + 3 (1 + x) ^ 3) - I Sqrt[1 - 2 / (1 +
x)] / (12 + 12 x - 12 (1 + x) ^ 2 + 3 (1 + x) ^ 3) + I 6 (1 + x) ^ 2 Sqrt[1
 - 2 / (1 + x)] / (12 + 12 x - 12 (1 + x) ^ 2 + 3 (1 + x) ^ 3)]
```

Maple [A]
time = 0.15, size = 57, normalized size = 1.33

method	result	size
gosper	$-\frac{x\left(2x^2-3\right)}{3(1+x)^{\frac{3}{2}}(1-x)^{\frac{3}{2}}}$	23
default	$\frac{1}{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(1-x)^(3/2)/(1+x)^(3/2)+1/(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)
^(3/2)-2/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.58

$$\frac{2\,x}{3\,\sqrt{-x^2+1}}+\frac{x}{3\,\left(-x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

[Out] 2/3*x/sqrt(-x^2 + 1) + 1/3*x/(-x^2 + 1)^(3/2)

Fricas [A]
time = 0.29, size = 35, normalized size = 0.81

$$-\frac{\left(2\,x^{3}-3\,x\right)\sqrt{x+1}\,\sqrt{-x+1}}{3\,\left(x^{4}-2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")

[Out] -1/3*(2*x^3 - 3*x)*sqrt(x + 1)*sqrt(-x + 1)/(x^4 - 2*x^2 + 1)

Sympy [C] Result contains complex when optimal does not.
time = 6.71, size = 280, normalized size = 6.51

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12}+\frac{6\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{3\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{\sqrt{-1+\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12}+\frac{6i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{3i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{i\sqrt{1-\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)**(5/2)/(1+x)**(5/2),x)

[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*sqrt(-1 + 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - sqrt(-1 + 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*I*sqrt(1 - 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - I*sqrt(1 - 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(31) = 62.
time = 0.01, size = 197, normalized size = 4.58

$$-2\left(\frac{-\frac{16384}{3}\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3+\frac{90112\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{2097152}+\frac{33\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2+1}{384\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3}+\frac{2\left(\frac{3}{16}-\frac{1}{12}\sqrt{-x+1}\,\sqrt{-x+1}\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x)

[Out] -1/192*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) - 11/64*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/12*(4*x + 5)*sqrt(-x + 1)/(x + 1)^(3/2) - 1/192*(-x +

```
 1)^(3/2)*(33*(sqrt(2) - sqrt(x + 1))^2/(x - 1) - 1)/(sqrt(2) - sqrt(x + 1)
)^3
```

Mupad [B]
time = 0.37, size = 41, normalized size = 0.95

$$\frac{3\,x\,\sqrt{1-x}-2\,x^3\,\sqrt{1-x}}{\left(3\,x+3\right)\,{\left(x-1\right)}^2\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(5/2)),x)
[Out] (3*x*(1 - x)^(1/2) - 2*x^3*(1 - x)^(1/2))/((3*x + 3)*(x - 1)^2*(x + 1)^(1/2
))
```

3.1134 $\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{15\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(3/2)+4/15*x/(1-x)^(3/2)/(1+x)^(3/2)+8/15*x/(1-x)^(1/
2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {47, 40, 39}

$$\frac{8x}{15\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*x)/(15*(1 - x)^(3/2)*(1 + x)^(3/2))
+ (8*x)/(15*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\
&= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{15}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{15\sqrt{1-x}\,\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.63

$$\frac{3+12x-12x^2-8x^3+8x^4}{15(1-x)^{5/2}(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] (3 + 12*x - 12*x^2 - 8*x^3 + 8*x^4)/(15*(1 - x)^(5/2)*(1 + x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 22.17, size = 285, normalized size = 4.52

$$\text{Piecewise}\left[\left\{\left\{\frac{(-3-12x+12x^2+8x^3-8x^4)\sqrt{\frac{1-x}{1+x}}}{15(-1+2x-2x^3+x^4)},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},\frac{-60I(1+x)^2\sqrt{1-\frac{2}{1+x}}}{-120-120x-90(1+x)^3+15(1+x)^4+180(1+x)^2}-\frac{8I(1+x)^4\sqrt{1-\frac{2}{1+x}}}{-120-120x-90(1+x)^3+15(1+x)^4+180(1+x)^2}+\frac{I5\sqrt{1-\frac{2}{1+x}}}{-120-120x-90(1+x)^3+15(1+x)^4+180(1+x)^2}+\frac{I20(1+x)\sqrt{1-\frac{2}{1+x}}}{-120-120x-90(1+x)^3+15(1+x)^4+180(1+x)^2}+\frac{I40(1+x)^3\sqrt{1-\frac{2}{1+x}}}{-120-120x-90(1+x)^3+15(1+x)^4+180(1+x)^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]')
```

```
[Out] Piecewise[{{(-3 - 12 x + 12 x ^ 2 + 8 x ^ 3 - 8 x ^ 4) Sqrt[(1 - x) / (1 +
x)] / (15 (-1 + 2 x - 2 x ^ 3 + x ^ 4)), 1 / Abs[1 + x] > 1 / 2}}, -60 I (1
 + x) ^ 2 Sqrt[1 - 2 / (1 + x)] / (-120 - 120 x - 90 (1 + x) ^ 3 + 15 (1 +
x) ^ 4 + 180 (1 + x) ^ 2) - 8 I (1 + x) ^ 4 Sqrt[1 - 2 / (1 + x)] / (-120 -
 120 x - 90 (1 + x) ^ 3 + 15 (1 + x) ^ 4 + 180 (1 + x) ^ 2) + I 5 Sqrt[1 -
2 / (1 + x)] / (-120 - 120 x - 90 (1 + x) ^ 3 + 15 (1 + x) ^ 4 + 180 (1 + x
) ^ 2) + I 20 (1 + x) Sqrt[1 - 2 / (1 + x)] / (-120 - 120 x - 90 (1 + x) ^
3 + 15 (1 + x) ^ 4 + 180 (1 + x) ^ 2) + I 40 (1 + x) ^ 3 Sqrt[1 - 2 / (1 +
x)] / (-120 - 120 x - 90 (1 + x) ^ 3 + 15 (1 + x) ^ 4 + 180 (1 + x) ^ 2)]
```

Maple [A]
time = 0.16, size = 72, normalized size = 1.14

method	result	size
gosper	$\frac{8x^4-8x^3-12x^2+12x+3}{15(1+x)^{\frac{3}{2}}(1-x)^{\frac{5}{2}}}$	35
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^4-8x^3-12x^2+12x+3)}{15\sqrt{1-x}\,(1+x)^{\frac{3}{2}}(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	61
default	$\frac{1}{5(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{15(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{15(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{15\sqrt{1+x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(3/2)+4/15/(1-x)^(3/2)/(1+x)^(3/2)+4/5/(1-x)^(1/2)/(1
+x)^(3/2)-8/15*(1-x)^(1/2)/(1+x)^(3/2)-8/15*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.26, size = 52, normalized size = 0.83

$$\frac{8\,x}{15\,\sqrt{-x^2+1}}+\frac{4\,x}{15\,(-x^2+1)^{\frac{3}{2}}}-\frac{1}{5\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-x^2 + 1) + 4/15*x/(-x^2 + 1)^(3/2) - 1/5/((-x^2 + 1)^(3/2)*x -
 (-x^2 + 1)^(3/2))
```

Fricas [A]
time = 0.29, size = 84, normalized size = 1.33

$$\frac{3\,x^5-3\,x^4-6\,x^3+6\,x^2-(8\,x^4-8\,x^3-12\,x^2+12\,x+3)\sqrt{x+1}\,\sqrt{-x+1}+3\,x-3}{15\,(x^5-x^4-2\,x^3+2\,x^2+x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5 - 3*x^4 - 6*x^3 + 6*x^2 - (8*x^4 - 8*x^3 - 12*x^2 + 12*x + 3)*s
qrt(x + 1)*sqrt(-x + 1) + 3*x - 3)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 23.86, size = 425, normalized size = 6.75

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{40\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}-\frac{60\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{20\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{5\sqrt{-1+\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{40i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}-\frac{60i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{20i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{5i\sqrt{1-\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 40*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-1
20*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*sqrt(-1 +
 2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1
)**2 - 120) + 20*sqrt(-1 + 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*sqrt(-1 + 2/(x + 1))/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), 1/Abs(x + 1) > 1/2), (-8*
I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 +
180*(x + 1)**2 - 120) + 40*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*I*sqrt(1 - 2/(x + 1))
*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120)
 + 20*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3
 + 180*(x + 1)**2 - 120) + 5*I*sqrt(1 - 2/(x + 1))/(-120*x + 15*(x + 1)**4
- 90*(x + 1)**3 + 180*(x + 1)**2 - 120), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(45) = 90$.
time = 0.01, size = 269, normalized size = 4.27

$$2\left(\frac{\frac{1}{5}\cdot 1099511627776\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}+\frac{1}{3}\cdot 14293651161088\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}-\frac{45079976738816\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{1125899906842624}+\frac{-1230\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}-65\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}-3}{15360\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}}+\frac{2\left(\frac{11}{192}\sqrt{-x+1}\sqrt{-x+1}-\frac{1}{8}\right)\sqrt{-x+1}\sqrt{x+1}}{(x+1)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x)
```

```
[Out] -1/2560*(sqrt(2) - sqrt(x + 1))^5/(-x + 1)^(5/2) - 13/1536*(sqrt(2) - sqrt(
x + 1))^3/(-x + 1)^(3/2) - 41/256*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/
48*(11*x + 13)*sqrt(-x + 1)/(x + 1)^(3/2) + 1/7680*(1230*(sqrt(2) - sqrt(x
+ 1))^4/(x - 1)^2 - 65*(sqrt(2) - sqrt(x + 1))^2/(x - 1) + 3)*(-x + 1)^(5/2
)/(sqrt(2) - sqrt(x + 1))^5
```

Mupad [**B**]
time = 0.38, size = 75, normalized size = 1.19

$$-\frac{12\,x\,\sqrt{1-x}+3\,\sqrt{1-x}-12\,x^2\,\sqrt{1-x}-8\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{(15\,x+15)\,{(x-1)}^3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(12*x*(1 - x)^(1/2) + 3*(1 - x)^(1/2) - 12*x^2*(1 - x)^(1/2) - 8*x^3*(1 -
x)^(1/2) + 8*x^4*(1 - x)^(1/2))/((15*x + 15)*(x - 1)^3*(x + 1)^(1/2))
```

3.1135 $\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}} \, dx$

Optimal. Leaf size=83

$$\frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{21\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(3/2)+1/7/(1-x)^(5/2)/(1+x)^(3/2)+4/21*x/(1-x)^(3/2)/
(1+x)^(3/2)+8/21*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {47, 40, 39}

$$\frac{8x}{21\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + 1/(7*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*
x)/(21*(1 - x)^(3/2)*(1 + x)^(3/2)) + (8*x)/(21*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{5}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{7}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{21}\int\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{}{21\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.54

$$\frac{6+9x-24x^2+4x^3+16x^4-8x^5}{21(1-x)^{7/2}(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] (6 + 9*x - 24*x^2 + 4*x^3 + 16*x^4 - 8*x^5)/(21*(1 - x)^(7/2)*(1 + x)^(3/2)
)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 56.75, size = 388, normalized size = 4.67

$$\text{Piecewise}\left[\left\{\left\{\frac{(6+9x-24x^2+4x^3+16x^4-8x^5)\sqrt{\frac{1-x}{1+x}}}{21(1-3x+2x^2+2x^3-3x^4+x^5)},\frac{1}{\text{Abs}[1+x]}>\frac{1}{2}\right\}\right\},\frac{-140I(1+x)^3\sqrt{1-\frac{2}{1+x}}}{336+336x-672(1+x)^2-168(1+x)^4+21(1+x)^5+504(1+x)^3}-\frac{35I(1+x)\sqrt{1-\frac{2}{1+x}}}{336+336x-672(1+x)^2-168(1+x)^4+21(1+x)^5+504(1+x)^3}-\frac{8I(1+x)^5\sqrt{1-\frac{2}{1+x}}}{336+336x-672(1+x)^2-168(1+x)^4+21(1+x)^5+504(1+x)^3}-\frac{7I\sqrt{1-\frac{2}{1+x}}}{336+336x-672(1+x)^2-168(1+x)^4+21(1+x)^5+504(1+x)^3}+\frac{I56(1+x)^4\sqrt{1-\frac{2}{1+x}}}{336+336x-672(1+x)^2-168(1+x)^4+21(1+x)^5+504(1+x)^3}+\frac{I140(1+x)^2\sqrt{1-\frac{2}{1+x}}}{336+336x-672(1+x)^2-168(1+x)^4+21(1+x)^5+504(1+x)^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]')
```

```
[Out] Piecewise[{{(6 + 9 x - 24 x ^ 2 + 4 x ^ 3 + 16 x ^ 4 - 8 x ^ 5) Sqrt[(1 - x
) / (1 + x)] / (21 (1 - 3 x + 2 x ^ 2 + 2 x ^ 3 - 3 x ^ 4 + x ^ 5)), 1 / Ab
s[1 + x] > 1 / 2}}, -140 I (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)] / (336 + 336 x
 - 672 (1 + x) ^ 2 - 168 (1 + x) ^ 4 + 21 (1 + x) ^ 5 + 504 (1 + x) ^ 3) -
35 I (1 + x) Sqrt[1 - 2 / (1 + x)] / (336 + 336 x - 672 (1 + x) ^ 2 - 168 (
1 + x) ^ 4 + 21 (1 + x) ^ 5 + 504 (1 + x) ^ 3) - 8 I (1 + x) ^ 5 Sqrt[1 - 2
 / (1 + x)] / (336 + 336 x - 672 (1 + x) ^ 2 - 168 (1 + x) ^ 4 + 21 (1 + x)
 ^ 5 + 504 (1 + x) ^ 3) - 7 I Sqrt[1 - 2 / (1 + x)] / (336 + 336 x - 672 (1
 + x) ^ 2 - 168 (1 + x) ^ 4 + 21 (1 + x) ^ 5 + 504 (1 + x) ^ 3) + I 56 (1 +
 x) ^ 4 Sqrt[1 - 2 / (1 + x)] / (336 + 336 x - 672 (1 + x) ^ 2 - 168 (1 + x
```

```
) ^ 4 + 21 (1 + x) ^ 5 + 504 (1 + x) ^ 3) + I 140 (1 + x) ^ 2 Sqrt[1 - 2 /
(1 + x)] / (336 + 336 x - 672 (1 + x) ^ 2 - 168 (1 + x) ^ 4 + 21 (1 + x) ^
5 + 504 (1 + x) ^ 3)]
```

Maple [A]
time = 0.16, size = 86, normalized size = 1.04

method	result	size
gosper	$-\frac{8x^5-16x^4-4x^3+24x^2-9x-6}{21(1+x)^{\frac{3}{2}}(1-x)^{\frac{7}{2}}}$	40
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-16x^4-4x^3+24x^2-9x-6)}{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{1}{7(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}+\frac{1}{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{21(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{21(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{21\sqrt{1+x}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(3/2)+1/7/(1-x)^(5/2)/(1+x)^(3/2)+4/21/(1-x)^(3/2)/(1
+x)^(3/2)+4/7/(1-x)^(1/2)/(1+x)^(3/2)-8/21*(1-x)^(1/2)/(1+x)^(3/2)-8/21*(1-
x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.10

$$\frac{8\,x}{21\,\sqrt{-x^2+1}}+\frac{4\,x}{21\,(-x^2+1)^{\frac{3}{2}}}+\frac{1}{7\left((-x^2+1)^{\frac{3}{2}}x^2-2\,(-x^2+1)^{\frac{3}{2}}x+(-x^2+1)^{\frac{3}{2}}\right)}-\frac{1}{7\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/21*x/sqrt(-x^2 + 1) + 4/21*x/(-x^2 + 1)^(3/2) + 1/7/((-x^2 + 1)^(3/2)*x^2
 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 1/7/((-x^2 + 1)^(3/2)*x - (-x
^2 + 1)^(3/2))
```

Fricas [A]
time = 0.30, size = 101, normalized size = 1.22

$$\frac{6\,x^6-12\,x^5-6\,x^4+24\,x^3-6\,x^2-(8\,x^5-16\,x^4-4\,x^3+24\,x^2-9\,x-6)\sqrt{x+1}\,\sqrt{-x+1}-12\,x+6}{21\,(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(6*x^6 - 12*x^5 - 6*x^4 + 24*x^3 - 6*x^2 - (8*x^5 - 16*x^4 - 4*x^3 + 2
4*x^2 - 9*x - 6)*sqrt(x + 1)*sqrt(-x + 1) - 12*x + 6)/(x^6 - 2*x^5 - x^4 +
4*x^3 - x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 71.34, size = 593, normalized size = 7.14

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^5}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} + \frac{56\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} - \frac{140\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} + \frac{140\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} - \frac{35\sqrt{-1+\frac{2}{x+1}}(x+1)}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} - \frac{7\sqrt{-1+\frac{2}{x+1}}}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^5}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} + \frac{56i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} - \frac{140i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} + \frac{140i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} - \frac{35i\sqrt{1-\frac{2}{x+1}}(x+1)}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} - \frac{7i\sqrt{1-\frac{2}{x+1}}}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(336*x + 21*(x + 1)**5 - 168*
(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 56*sqrt(-1 + 2/(x + 1
))*(x + 1)**4/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 67
2*(x + 1)**2 + 336) - 140*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(336*x + 21*(x +
1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 140*sqrt(
-1 + 2/(x + 1))*(x + 1)**2/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x
 + 1)**3 - 672*(x + 1)**2 + 336) - 35*sqrt(-1 + 2/(x + 1))*(x + 1)/(336*x +
 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) -
7*sqrt(-1 + 2/(x + 1))/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1
)**3 - 672*(x + 1)**2 + 336), 1/Abs(x + 1) > 1/2), (-8*I*sqrt(1 - 2/(x + 1)
)*(x + 1)**5/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672
*(x + 1)**2 + 336) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(336*x + 21*(x + 1
)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) - 140*I*sqrt
(1 - 2/(x + 1))*(x + 1)**3/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x
 + 1)**3 - 672*(x + 1)**2 + 336) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(33
6*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 33
6) - 35*I*sqrt(1 - 2/(x + 1))*(x + 1)/(336*x + 21*(x + 1)**5 - 168*(x + 1)*
*4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) - 7*I*sqrt(1 - 2/(x + 1))/(336*
x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(59) = 118.
time = 0.02, size = 341, normalized size = 4.11

$$-2\left(\frac{-\frac{1}{7}\cdot 302231454903657293676544\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^7 - 906694364710971881029632\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^5 - \frac{1}{3}\cdot 329432285844986450107432 96\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^3 + \frac{778245996376917531217100 80\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{2475880078570760549798248448} + \frac{10815\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^6 + 763\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^4 + 63\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^2 + 3}{172032\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^7} + \frac{2\left(-\frac{7}{192}\sqrt{-x+1}\sqrt{-x+1}+\frac{5}{64}\right)\sqrt{-x+1}\sqrt{x+1}}{(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x)
```

```
[Out] -1/28672*(sqrt(2) - sqrt(x + 1))^7/(-x + 1)^(7/2) - 3/4096*(sqrt(2) - sqrt(
x + 1))^5/(-x + 1)^(5/2) - 109/12288*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/
2) - 515/4096*(sqrt(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/48*(7*x + 8)*sqrt(-x
 + 1)/(x + 1)^(3/2) - 1/86016*(10815*(sqrt(2) - sqrt(x + 1))^6/(x - 1)^3 -
763*(sqrt(2) - sqrt(x + 1))^4/(x - 1)^2 + 63*(sqrt(2) - sqrt(x + 1))^2/(x -
 1) - 3)*(-x + 1)^(7/2)/(sqrt(2) - sqrt(x + 1))^7
```

Mupad [B]
time = 0.41, size = 86, normalized size = 1.04

$$\frac{9\,x\,\sqrt{1-x}+6\,\sqrt{1-x}-24\,x^2\,\sqrt{1-x}+4\,x^3\,\sqrt{1-x}+16\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{(21\,x+21)\,(x-1)^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(5/2)),x)
```

```
[Out] (9*x*(1 - x)^(1/2) + 6*(1 - x)^(1/2) - 24*x^2*(1 - x)^(1/2) + 4*x^3*(1 - x)
^(1/2) + 16*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/((21*x + 21)*(x - 1)^4
*(x + 1)^(1/2))
```

3.1136 $\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=103

$$\frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63(1-x)^{3/2}(1+x)^{3/2}}+\frac{16x}{63\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(3/2)+2/21/(1-x)^(7/2)/(1+x)^(3/2)+2/21/(1-x)^(5/2)/(
1+x)^(3/2)+8/63*x/(1-x)^(3/2)/(1+x)^(3/2)+16/63*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {47, 40, 39}

$$\frac{16x}{63\sqrt{1-x}\,\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + 2/(21*(1 - x)^(7/2)*(1 + x)^(3/2)) + 2/
(21*(1 - x)^(5/2)*(1 + x)^(3/2)) + (8*x)/(63*(1 - x)^(3/2)*(1 + x)^(3/2)) +
 (16*x)/(63*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx = \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{3}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{10}{21}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}} \ldots$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{5/2}(1+x)^{3/2}} + \ldots$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{5/2}(1+x)^{3/2}} + \ldots$$
$$= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{7/2}(1+x)^{3/2}} + \frac{2}{21(1-x)^{5/2}(1+x)^{3/2}} + \ldots$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.49

$$\frac{19+6x-66x^2+56x^3+24x^4-48x^5+16x^6}{63(1-x)^{9/2}(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] (19 + 6*x - 66*x^2 + 56*x^3 + 24*x^4 - 48*x^5 + 16*x^6)/(63*(1 - x)^(9/2)*(
1 + x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 167.48, size = 495, normalized size = 4.81

[illegible]

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]')
```

```
[Out] Piecewise[{{(-19 - 6 x + 66 x ^ 2 - 56 x ^ 3 - 24 x ^ 4 + 48 x ^ 5 - 16 x ^
 6) Sqrt[(1 - x) / (1 + x)] / (63 (-1 + 4 x - 5 x ^ 2 + 5 x ^ 4 - 4 x ^ 5 +
 x ^ 6)), 1 / Abs[1 + x] > 1 / 2}}, -630 I (1 + x) ^ 2 Sqrt[1 - 2 / (1 + x)
] / (-2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x) ^ 6 +
 2520 (1 + x) ^ 4 + 5040 (1 + x) ^ 2) - 504 I (1 + x) ^ 4 Sqrt[1 - 2 / (1 +
 x)] / (-2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x) ^
```

```
6 + 2520 (1 + x) ^ 4 + 5040 (1 + x) ^ 2) - 16 I (1 + x) ^ 6 Sqrt[1 - 2 / (1
 + x)] / (-2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x)
^ 6 + 2520 (1 + x) ^ 4 + 5040 (1 + x) ^ 2) + I 21 Sqrt[1 - 2 / (1 + x)] / (
-2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x) ^ 6 + 2520
 (1 + x) ^ 4 + 5040 (1 + x) ^ 2) + I 126 (1 + x) Sqrt[1 - 2 / (1 + x)] / (-
2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x) ^ 6 + 2520
(1 + x) ^ 4 + 5040 (1 + x) ^ 2) + I 144 (1 + x) ^ 5 Sqrt[1 - 2 / (1 + x)] /
 (-2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x) ^ 6 + 25
20 (1 + x) ^ 4 + 5040 (1 + x) ^ 2) + I 840 (1 + x) ^ 3 Sqrt[1 - 2 / (1 + x)
] / (-2016 - 2016 x - 5040 (1 + x) ^ 3 - 630 (1 + x) ^ 5 + 63 (1 + x) ^ 6 +
 2520 (1 + x) ^ 4 + 5040 (1 + x) ^ 2)]
```

Maple [A]

time = 0.16, size = 100, normalized size = 0.97

method	result
gosper	$\frac{16x^{6}-48x^{5}+24x^{4}+56x^{3}-66x^{2}+6x+19}{63(1+x)^{\frac{3}{2}}(1-x)^{\frac{9}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,(16x^{6}-48x^{5}+24x^{4}+56x^{3}-66x^{2}+6x+19)}{63\sqrt{1-x}\,(1+x)^{\frac{3}{2}}(-1+x)^{4}\sqrt{-(1+x)(-1+x)}}$
default	$\frac{1}{9(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}}}+\frac{2}{21(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}+\frac{2}{21(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{8}{63(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{8}{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{16\sqrt{1-x}}{63(1+x)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(3/2)+2/21/(1-x)^(7/2)/(1+x)^(3/2)+2/21/(1-x)^(5/2)/(
1+x)^(3/2)+8/63/(1-x)^(3/2)/(1+x)^(3/2)+8/21/(1-x)^(1/2)/(1+x)^(3/2)-16/63*
(1-x)^(1/2)/(1+x)^(3/2)-16/63*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]

time = 0.26, size = 146, normalized size = 1.42

$$\frac{16\,x}{63\,\sqrt{-x^{2}+1}}+\frac{8\,x}{63\,(-x^{2}+1)^{\frac{3}{2}}}-\frac{1}{9\left((-x^{2}+1)^{\frac{3}{2}}x^{3}-3\,(-x^{2}+1)^{\frac{3}{2}}x^{2}+3\,(-x^{2}+1)^{\frac{3}{2}}x-(-x^{2}+1)^{\frac{3}{2}}\right)}+\frac{2}{21\left((-x^{2}+1)^{\frac{3}{2}}x^{2}-2\,(-x^{2}+1)^{\frac{3}{2}}x+(-x^{2}+1)^{\frac{3}{2}}\right)}-\frac{2}{21\left((-x^{2}+1)^{\frac{3}{2}}x-(-x^{2}+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/63*x/sqrt(-x^2 + 1) + 8/63*x/(-x^2 + 1)^(3/2) - 1/9/((-x^2 + 1)^(3/2)*x^
3 - 3*(-x^2 + 1)^(3/2)*x^2 + 3*(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2)) + 2/2
1/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 2/21/(
(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))
```

Fricas [A]

time = 0.30, size = 114, normalized size = 1.11

$$\frac{19\,x^{7}-57\,x^{6}+19\,x^{5}+95\,x^{4}-95\,x^{3}-19\,x^{2}-(16\,x^{6}-48\,x^{5}+24\,x^{4}+56\,x^{3}-66\,x^{2}+6\,x+19)\sqrt{x+1}\,\sqrt{-x+1}+57\,x-19}{63\,(x^{7}-3\,x^{6}+x^{5}+5\,x^{4}-5\,x^{3}-x^{2}+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(19*x^7 - 57*x^6 + 19*x^5 + 95*x^4 - 95*x^3 - 19*x^2 - (16*x^6 - 48*x^
5 + 24*x^4 + 56*x^3 - 66*x^2 + 6*x + 19)*sqrt(x + 1)*sqrt(-x + 1) + 57*x -
19)/(x^7 - 3*x^6 + x^5 + 5*x^4 - 5*x^3 - x^2 + 3*x - 1)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(73) = 146.
time = 0.02, size = 414, normalized size = 4.02

$$2\left(\frac{\frac{1}{9}\cdot 340282366920938463463374607431768211456\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{9}+\frac{1}{7}\cdot 578480023765595387887736832634005959 4752\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{7}+9527906273786276976974489008089509920768\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{5}+\frac{1}{3}\cdot 2545312104568619706706042063589626221690 88\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{3}-\frac{499194232273016725900770549102403966205952\left(-2\sqrt{x+1}+2\sqrt{2}\right)}{\sqrt{-x+1}}}{22300745198530623141535718272648361505980416}+\frac{-184842\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{8}-15708\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{6}-1764\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{4}-153\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{2}-7}{4128768\left(-\frac{-2\sqrt{x+1}+2\sqrt{2}}{2\sqrt{-x+1}}\right)^{9}}+\frac{2\left(\frac{17}{768}\sqrt{-x+1}\,\sqrt{-x+1}-\frac{3}{64}\right)\sqrt{-x+1}\,\sqrt{x+1}}{(x+1)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x)
```

```
[Out] -1/294912*(sqrt(2) - sqrt(x + 1))^9/(-x + 1)^(9/2) - 17/229376*(sqrt(2) - s
qrt(x + 1))^7/(-x + 1)^(7/2) - 7/8192*(sqrt(2) - sqrt(x + 1))^5/(-x + 1)^(5
/2) - 187/24576*(sqrt(2) - sqrt(x + 1))^3/(-x + 1)^(3/2) - 1467/16384*(sqrt
(2) - sqrt(x + 1))/sqrt(-x + 1) - 1/192*(17*x + 19)*sqrt(-x + 1)/(x + 1)^(3
/2) + 1/2064384*(184842*(sqrt(2) - sqrt(x + 1))^8/(x - 1)^4 - 15708*(sqrt(2
) - sqrt(x + 1))^6/(x - 1)^3 + 1764*(sqrt(2) - sqrt(x + 1))^4/(x - 1)^2 - 1
53*(sqrt(2) - sqrt(x + 1))^2/(x - 1) + 7)*(-x + 1)^(9/2)/(sqrt(2) - sqrt(x
+ 1))^9
```

Mupad [**B**]
time = 0.42, size = 99, normalized size = 0.96

$$-\frac{6\,x\,\sqrt{1-x}+19\,\sqrt{1-x}-66\,x^2\,\sqrt{1-x}+56\,x^3\,\sqrt{1-x}+24\,x^4\,\sqrt{1-x}-48\,x^5\,\sqrt{1-x}+16\,x^6\,\sqrt{1-x}}{\left(63\,x+63\right)\,{\left(x-1\right)}^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(6*x*(1 - x)^(1/2) + 19*(1 - x)^(1/2) - 66*x^2*(1 - x)^(1/2) + 56*x^3*(1 -
 x)^(1/2) + 24*x^4*(1 - x)^(1/2) - 48*x^5*(1 - x)^(1/2) + 16*x^6*(1 - x)^(1
/2))/((63*x + 63)*(x - 1)^5*(x + 1)^(1/2))
```

3.1137 $\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx$

Optimal. Leaf size=126

$$\frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2}+\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\dots}}{\sqrt{a}\,\sqrt{c-\dots}}\right.$$

```
[Out] 5/24*a*c*x*(a*x+a)^(3/2)*(-c*x+c)^(3/2)+1/6*x*(a*x+a)^(5/2)*(-c*x+c)^(5/2)+
5/8*a^(5/2)*c^(5/2)*arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))+5/
16*a^2*c^2*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {38, 65, 223, 209}

$$\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{5}{16}a^2c^2x\sqrt{ax+a}\,\sqrt{c-cx}+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (5*a^2*c^2*x*Sqrt[a + a*x]*Sqrt[c - c*x])/16 + (5*a*c*x*(a + a*x)^(3/2)*(c
- c*x)^(3/2))/24 + (x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a^(5/2)*c^(5/
2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{6}(5ac)\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$$

$$= \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{8}\left(5a^2c^2\right)\int \sqrt{a-$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}($$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}($$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}($$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}($$

Mathematica [A]
time = 0.23, size = 86, normalized size = 0.68

$$\frac{c^2(a(1+x))^{5/2}\left(x\sqrt{1+x}\,\sqrt{c-cx}\,(33-26x^2+8x^4)-30\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{c}\,\sqrt{1+x}}\right)\right)}{48(1+x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
[Out] (c^2*(a*(1 + x))^(5/2)*(x*Sqrt[1 + x]*Sqrt[c - c*x]*(33 - 26*x^2 + 8*x^4) -
 30*Sqrt[c]*ArcTan[Sqrt[c - c*x]/(Sqrt[c]*Sqrt[1 + x])]))/(48*(1 + x)^(5/2)
)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(94) = 188$.

time = 0.19, size = 198, normalized size = 1.57

method	result
risch	$-\frac{x(8x^4-26x^2+33)(1+x)(-1+x)a^3c^3}{48\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}+\frac{5\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)a^3c^3\sqrt{-a(1+x)c(-1+x)}}{16\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$

$$5c\left(\frac{(-cx+c)^{\frac{3}{2}}\sqrt{a}}{2a}\right.$$

$$a\left(\frac{(-cx+c)^{\frac{5}{2}}\sqrt{ax+a}}{3a}+\right.$$

$$3a\left(-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}}}{4c}+\right.$$

$$5a\left(-\frac{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}}}{5c}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/c*(a*x+a)^(5/2)*(-c*x+c)^(7/2)+5/6*a*(-1/5/c*(a*x+a)^(3/2)*(-c*x+c)^(7
/2)+3/5*a*(-1/4/c*(a*x+a)^(1/2)*(-c*x+c)^(7/2)+1/4*a*(1/3/a*(-c*x+c)^(5/2)*
(a*x+a)^(1/2)+5/3*c*(1/2/a*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+3/2*c*(1/a*(-c*x+c)
^(1/2)*(a*x+a)^(1/2)+c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2
)/(a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))))))
```

Maxima [A]
time = 0.36, size = 72, normalized size = 0.57

$$\frac{5\,a^3c^3\arcsin\left(x\right)}{16\,\sqrt{ac}}+\frac{5}{16}\,\sqrt{-acx^2+ac}\,a^2c^2x+\frac{5}{24}\,\left(-acx^2+ac\right)^{\frac{3}{2}}acx+\frac{1}{6}\,\left(-acx^2+ac\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*c^3*arcsin(x)/sqrt(a*c) + 5/16*sqrt(-a*c*x^2 + a*c)*a^2*c^2*x + 5/
24*(-a*c*x^2 + a*c)^(3/2)*a*c*x + 1/6*(-a*c*x^2 + a*c)^(5/2)*x
```

Fricas [A]
time = 0.30, size = 201, normalized size = 1.60

$$\left[\frac{5}{32}\,\sqrt{-ac}\,a^2c^2\log\left(2\,acx^2+2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)+\frac{1}{48}\,\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c}\,,-\frac{5}{16}\,\sqrt{ac}\,a^2c^2\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)+\frac{1}{48}\,\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [5/32*sqrt(-a*c)*a^2*c^2*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c
*x + c)*x - a*c) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2*x^3 + 33*a^2*c^2*x)*sqr
t(a*x + a)*sqrt(-c*x + c), -5/16*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt(a*
x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a*c)) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2
*x^3 + 33*a^2*c^2*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(x+1\right)\right)^{\frac{5}{2}}\left(-c\left(x-1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(5/2)*(-c*x+c)**(5/2),x)
```

```
[Out] Integral((a*(x + 1))**(5/2)*(-c*(x - 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(94) = 188.
time = 0.11, size = 1165, normalized size = 9.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x)
```

```
[Out] 1/240*(150*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*((a*x + a)*(4*(a
*x + a)*(5*(a*x + a)/a^5 - 31/a^4) + 321/a^3) - 451/a^2)*(a*x + a) + 745/a)
*(a*x + a) - 405)*sqrt(a*x + a))*c^2*abs(a) - 1/120*(90*a^2*c*log(abs(-sqrt
(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-
(a*x + a)*a*c + 2*a^2*c)*((2*(a*x + a)*(3*(a*x + a)*(4*(a*x + a)/a^4 - 21/a
^3) + 133/a^2) - 295/a)*(a*x + a) + 195)*sqrt(a*x + a))*c^2*abs(a) - 1/12*(
18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)
))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x + a)*(3*(
a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c^2*abs(a) + 1/3*(6*a^2
*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqr
t(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2*(a*x +
 a)/a^2 - 7/a) + 9))*c^2*abs(a) - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a
) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a
^2*c)*sqrt(a*x + a))*c^2*abs(a) + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x
 + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c +
 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c^2*abs(a)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,x)^{5/2}\,(c-c\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(5/2)*(c - c*x)^(5/2),x)
```

```
[Out] int((a + a*x)^(5/2)*(c - c*x)^(5/2), x)
```

3.1138 $\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)$$

```
[Out] 1/4*x*(a*x+a)^(3/2)*(-c*x+c)^(3/2)+3/4*a^(3/2)*c^(3/2)*arctan(c^(1/2)*(a*x+
a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))+3/8*a*c*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {38, 65, 223, 209}

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\,\sqrt{c-cx}+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (3*a*c*x*Sqrt[a + a*x]*Sqrt[c - c*x])/8 + (x*(a + a*x)^(3/2)*(c - c*x)^(3/2
))/4 + (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}(3ac)\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx$$

$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{8}\left(3a^2c^2\right)\int \frac{}{\sqrt{a+ax}}$$

$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int -\right.$$

$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{}{1}\right.$$

$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.$$

Mathematica [**A**]
time = 0.16, size = 79, normalized size = 0.82

$$-\frac{c(a(1+x))^{3/2}\left(x\sqrt{1+x}\,\sqrt{c-cx}\,\left(-5+2x^2\right)+6\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{c}\,\sqrt{1+x}}\right)\right)}{8(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] -1/8*(c*(a*(1 + x))^(3/2)*(x*Sqrt[1 + x]*Sqrt[c - c*x]*(-5 + 2*x^2) + 6*Sqr
t[c]*ArcTan[Sqrt[c - c*x]/(Sqrt[c]*Sqrt[1 + x])]))/(1 + x)^(3/2)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]')
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(70) = 140.
time = 0.18, size = 150, normalized size = 1.56

method	result
risch	$\frac{x(2x^2-5)(1+x)(-1+x)a^2c^2}{8\sqrt{a(1+x)}\sqrt{-c(-1+x)}}+\frac{3\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)a^2c^2\sqrt{-a(1+x)c(-1+x)}}{8\sqrt{ac}\sqrt{a(1+x)}\sqrt{-c(-1+x)}}$
default	$-\frac{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}}}{4c}+\frac{3a\left(-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{5}{2}}}{3c}+\frac{a\left(\frac{(-cx+c)^{\frac{3}{2}}\sqrt{ax+a}}{2a}+\frac{3c\left(\frac{\sqrt{-cx+c}\sqrt{ax+a}}{a}+\frac{c\sqrt{(-cx}}{}\right)}{}\right)}{3}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4/c*(a*x+a)^(3/2)*(-c*x+c)^(5/2)+3/4*a*(-1/3/c*(a*x+a)^(1/2)*(-c*x+c)^(5/2)+1/3*a*(1/2/a*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+3/2*c*(1/a*(-c*x+c)^(1/2)*(a*x+a)^(1/2)+c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2)))))

Maxima [A]
time = 0.33, size = 50, normalized size = 0.52

$$\frac{3\,a^2c^2\arcsin(x)}{8\sqrt{ac}}+\frac{3}{8}\sqrt{-acx^2+ac}\,acx+\frac{1}{4}\left(-acx^2+ac\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="maxima")

[Out] 3/8*a^2*c^2*arcsin(x)/sqrt(a*c) + 3/8*sqrt(-a*c*x^2 + a*c)*a*c*x + 1/4*(-a*c*x^2 + a*c)^(3/2)*x

Fricas [A]
time = 0.31, size = 155, normalized size = 1.61

$$\left[\frac{3}{16}\sqrt{-ac}\,ac\log\left(2\,acx^2+2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}\,x-ac\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\sqrt{-cx+c}\,,-\frac{3}{8}\sqrt{ac}\,ac\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}\,x}{acx^2-ac}\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/16*sqrt(-a*c)*a*c*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - a*c) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c), -3/8
*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a
*c)) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(x+1\right)\right)^{\frac{3}{2}}\left(-c\left(x-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(3/2)*(-c*x+c)**(3/2),x)
```

```
[Out] Integral((a*(x + 1))**(3/2)*(-c*(x - 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(70) = 140.
time = 0.06, size = 623, normalized size = 6.49

$$-\frac{2ac|a|\left(2\left(\left(\left(\frac{\text{[illegible]}\cdot 5760a^3\sqrt{ax+a}\sqrt{ax+a}}{a^6}-\frac{\text{[illegible]}\cdot 24960a^4}{a^6}\right)\sqrt{ax+a}\sqrt{ax+a}+\frac{\text{[illegible]}\cdot 41280a^5}{a^6}\right)\sqrt{ax+a}\sqrt{ax+a}-\frac{\text{[illegible]}\cdot 37440a^6}{a^6}\right)\sqrt{ax+a}\sqrt{2a^2c-ac(ax+a)}+\frac{6a^2c\ln\left|\sqrt{2a^2c-ac(ax+a)}-\sqrt{-ac}\sqrt{ax+a}\right|}{16\sqrt{-ac}}\right)}{a^2}-\frac{2ac|a|\left(2\left(\left(\frac{\text{[illegible]}\cdot 24a\sqrt{ax+a}\sqrt{ax+a}}{a^3}-\frac{\text{[illegible]}\cdot 84a^2}{a^3}\right)\sqrt{ax+a}\sqrt{ax+a}+\frac{\text{[illegible]}\cdot 108a^3}{a^3}\right)\sqrt{ax+a}\sqrt{2a^2c-ac(ax+a)}-\frac{2a^2c\ln\left|\sqrt{2a^2c-ac(ax+a)}-\sqrt{-ac}\sqrt{ax+a}\right|}{4\sqrt{-ac}}\right)}{a^2}+\frac{2ac|a|\left(2\left(\frac{1}{8}\sqrt{ax+a}\sqrt{ax+a}-\frac{12}{32}a\right)\sqrt{ax+a}\sqrt{2a^2c-ac(ax+a)}+\frac{2a^2c\ln\left|\sqrt{2a^2c-ac(ax+a)}-\sqrt{-ac}\sqrt{ax+a}\right|}{4\sqrt{-ac}}\right)}{a^2a}+\frac{2ac|a|\left(\frac{1}{2}\sqrt{ax+a}\sqrt{2a^2c-ac(ax+a)}-\frac{2a^2c\ln\left|\sqrt{2a^2c-ac(ax+a)}-\sqrt{-ac}\sqrt{ax+a}\right|}{2\sqrt{-ac}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x)
```

```
[Out] -1/24*(18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2
*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x +
a)*(3*(a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c*abs(a)/a + 1/6
*(6*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2
*(a*x + a)/a^2 - 7/a) + 9))*c*abs(a)/a - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(
a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*
c + 2*a^2*c)*sqrt(a*x + a))*c*abs(a)/a + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*s
qrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a
)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c*abs(a)/a^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,x\right)}^{3/2}\,{\left(c-c\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(3/2)*(c - c*x)^(3/2),x)
[Out] int((a + a*x)^(3/2)*(c - c*x)^(3/2), x)
```

3.1139 $\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)$$

```
[Out] arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))*a^(1/2)*c^(1/2)+1/2*x*
(a*x+a)^(1/2)*(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 65, 223, 209}

$$\frac{1}{2}x\sqrt{ax+a}\,\sqrt{c-cx}+\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{2}(ac)\int \frac{1}{\sqrt{a+ax}\,\sqrt{c-cx}}\,dx\\ &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+c\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx,x,\sqrt{a+ax}\right)\\ &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+c\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx,x,\frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)\\ &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)\end{aligned}$$

Mathematica [A]
time = 0.16, size = 69, normalized size = 1.03

$$\frac{1}{2}\sqrt{c}\,\sqrt{a(1+x)}\left(\frac{x\sqrt{c-cx}}{\sqrt{c}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{1+x}}{\sqrt{c-cx}}\right)}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*x]*Sqrt[c - c*x],x]
[Out] (Sqrt[c]*Sqrt[a*(1 + x)]*((x*Sqrt[c - c*x])/Sqrt[c] + (2*ArcTan[(Sqrt[c]*Sq
rt[1 + x])/Sqrt[c - c*x]])/Sqrt[1 + x]))/2
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + a*x)^(1/2)*(c - c*x)^(1/2),x]')
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(49) = 98.
time = 0.14, size = 102, normalized size = 1.52

method	result
risch	$-\frac{x(1+x)(-1+x)ac}{2\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}+\frac{\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)ac\sqrt{-a(1+x)\,c(-1+x)}}{2\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$
default	$-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}}{2c}+\frac{a\left(\frac{\sqrt{-cx+c}\,\sqrt{ax+a}}{a}+\frac{c\sqrt{(-cx+c)(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{-cx+c}\,\sqrt{ax+a}\,\sqrt{ac}}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c*(a*x+a)^(1/2)*(-c*x+c)^(3/2)+1/2*a*(1/a*(-c*x+c)^(1/2)*(a*x+a)^(1/2)
+c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*arctan
((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2)))
```

Maxima [A]
time = 0.36, size = 28, normalized size = 0.42

$$\frac{ac\arcsin(x)}{2\sqrt{ac}}+\frac{1}{2}\sqrt{-acx^2+ac}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*arcsin(x)/sqrt(a*c) + 1/2*sqrt(-a*c*x^2 + a*c)*x
```

Fricas [A]
time = 0.31, size = 127, normalized size = 1.90

$$\left[\frac{1}{2}\sqrt{ax+a}\,\sqrt{-cx+c}\,x+\frac{1}{4}\sqrt{-ac}\,\log\left(2\,acx^2+2\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right),\frac{1}{2}\sqrt{ax+a}\,\sqrt{-cx+c}\,x-\frac{1}{2}\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*x + a)*sqrt(-c*x + c)*x + 1/4*sqrt(-a*c)*log(2*a*c*x^2 + 2*sqrt
(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x - a*c), 1/2*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - 1/2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*
x^2 - a*c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(x+1)}\,\sqrt{-c(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(1/2)*(-c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(x + 1))*sqrt(-c*(x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(49) = 98.

time = 0.02, size = 231, normalized size = 3.45

$$\frac{2\,|a|\left(2\left(\frac{1}{8}\sqrt{ax+a}\,\sqrt{ax+a}-\frac{12}{32}a\right)\sqrt{ax+a}\,\sqrt{2a^2c-ac\,(ax+a)}+\frac{2a^3c\ln\left|\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right|}{4\sqrt{-ac}}\right)}{a^2a}+\frac{2\,|a|\left(\frac{1}{2}\sqrt{ax+a}\,\sqrt{2a^2c-ac\,(ax+a)}-\frac{2a^2c\ln\left|\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right|}{2\sqrt{-ac}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x)
```

```
[Out] -(2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*abs(a)/a^2 +
 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a
^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*
a))*abs(a)/a^3
```

Mupad [B]

time = 0.30, size = 59, normalized size = 0.88

$$\frac{x\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}{2}-\frac{\sqrt{a}\,\sqrt{-c}\,\ln\left(\sqrt{-c}\,\sqrt{a\,(x+1)}\,\sqrt{-c\,(x-1)}-\sqrt{a}\,c\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(1/2)*(c - c*x)^(1/2),x)
```

```
[Out] (x*(a + a*x)^(1/2)*(c - c*x)^(1/2))/2 - (a^(1/2)*(-c)^(1/2)*log((-c)^(1/2)*
(a*(x + 1))^(1/2)*(-c*(x - 1))^(1/2) - a^(1/2)*c*x))/2
```

3.1140 $\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] 2*arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {65, 223, 209}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[c
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right)}{a}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)}{a}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.09

$$\frac{2\sqrt{1+x}\,\tan^{-1}\left(\frac{\sqrt{c}\sqrt{1+x}}{\sqrt{c-cx}}\right)}{\sqrt{c}\,\sqrt{a(1+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*Sqrt[1 + x]*ArcTan[(Sqrt[c]*Sqrt[1 + x])/Sqrt[c - c*x]])/(Sqrt[c]*Sqrt[a
*(1 + x)])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.53, size = 63, normalized size = 1.47

$$\frac{-I\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\left\{\right\}\right\},\frac{1}{x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\left\{\right\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{\text{exp\_polar}\left[-2I\text{Pi}\right]}{x^2}\right]}{4\text{Pi}^{\frac{3}{2}}\sqrt{a}\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + a*x)^(1/2)*(c - c*x)^(1/2)),x]')
```

```
[Out] (-I meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1 / 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 /
4, 1, 0}, {}}, 1 / x ^ 2] + meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 / 2, 1},
{}}, {{-1 / 4, 1 / 4}, {-1 / 2, 0, 0, 0}}, exp_polar[-2 I Pi] / x ^ 2]) / (
4 Pi ^ (3 / 2) Sqrt[a] Sqrt[c])
```

Maple [A]
time = 0.14, size = 57, normalized size = 1.33

method	result	size
default	$\frac{\sqrt{(-cx+c)(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{ax+a}\,\sqrt{-cx+c}\,\sqrt{ac}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-c*x+c)*(a*x+a))^(1/2)/(a*x+a)^(1/2)/(-c*x+c)^(1/2)/(a*c)^(1/2)*arctan((a
*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Maxima [A]
time = 0.36, size = 8, normalized size = 0.19

$$\frac{\arcsin(x)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(a*c)
```

Fricas [A]
time = 0.30, size = 101, normalized size = 2.35

$$\left[-\frac{\sqrt{-ac}\,\log\left(2\,acx^2-2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)}{2\,ac},-\frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(2*a*c*x^2 - 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*
x - a*c)/(a*c), -sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/
(a*c*x^2 - a*c))/(a*c)]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.20, size = 85, normalized size = 1.98

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\,\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(1/2)/(-c*x+c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
x**(-2))/(4*pi**(3/2)*sqrt(a)*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**2)/(4*pi**(
3/2)*sqrt(a)*sqrt(c))
```

Giac [A]
time = 0.01, size = 62, normalized size = 1.44

$$-\frac{2a^2 \ln \left| \sqrt{2a^2c - ac\,(ax + a)} - \sqrt{-ac}\,\sqrt{ax + a} \right|}{|a|\,a\sqrt{-ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x)
```

```
[Out] -2*a*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/(
sqrt(-a*c)*abs(a))
```

Mupad [B]
time = 0.18, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{a\left(\sqrt{c-c\,x}-\sqrt{c}\right)}{\sqrt{a\,c}\,\left(\sqrt{a+a\,x}-\sqrt{a}\right)}\right)}{\sqrt{a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(1/2)*(c - c*x)^(1/2)),x)
```

```
[Out] -(4*atan((a*((c - c*x)^(1/2) - c^(1/2)))/((a*c)^(1/2)*((a + a*x)^(1/2) - a^
(1/2)))))/(a*c)^(1/2)
```

3.1141 $\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{ac\sqrt{a+ax}\,\sqrt{c-cx}}$$

```
[Out] x/a/c/(a*x+a)^(1/2)/(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{39\}$

$$\frac{x}{ac\sqrt{ax+a}\,\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{ac\sqrt{a+ax}\,\sqrt{c-cx}}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 1.00

$$\frac{x(1+x)}{c(a(1+x))^{3/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] (x*(1 + x))/(c*(a*(1 + x))^(3/2)*Sqrt[c - c*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.45, size = 63, normalized size = 2.33

$$\frac{-I\text{meijerg}\left[\left\{\left\{\frac{3}{4},\frac{5}{4},1\right\},\left\{\frac{1}{2},\frac{3}{2},2\right\}\right\},\left\{\left\{\frac{3}{4},1,\frac{5}{4},\frac{3}{2},2\right\},\{0\}\right\},\frac{1}{x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\},\{\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{-\frac{1}{2},0,1,0\right\}\right\},\frac{\text{exp\_polar}[-2I\text{Pi}]}{x^2}\right]}{2\text{Pi}^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]')
```

```
[Out] (-I meijerg[{{3 / 4, 5 / 4, 1}, {1 / 2, 3 / 2, 2}}, {{3 / 4, 1, 5 / 4, 3 /
2, 2}, {0}}, 1 / x ^ 2] + meijerg[{{-1 / 2, 0, 1 / 4, 1 / 2, 3 / 4, 1}, {}}
, {{1 / 4, 3 / 4}, {-1 / 2, 0, 1, 0}}, exp_polar[-2 I Pi] / x ^ 2]) / (2 Pi
 ^ (3 / 2) a ^ (3 / 2) c ^ (3 / 2))
```

Maple [A]
time = 0.14, size = 47, normalized size = 1.74

method	result	size
risch	$\frac{x}{ac\sqrt{a(1+x)}\sqrt{-c(-1+x)}}$	24
gosper	$-\frac{(1+x)(-1+x)x}{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}$	25
default	$-\frac{1}{ac\sqrt{ax+a}\sqrt{-cx+c}}+\frac{\sqrt{ax+a}}{c a^2\sqrt{-cx+c}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/c/(a*x+a)^(1/2)/(-c*x+c)^(1/2)+1/c/a^2/(-c*x+c)^(1/2)*(a*x+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.78

$$\frac{x}{\sqrt{-acx^2+ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-a*c*x^2 + a*c)*a*c)
```

Fricas [A]
time = 0.29, size = 39, normalized size = 1.44

$$-\frac{\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{a^2c^2x^2-a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*x + a)*sqrt(-c*x + c)*x/(a^2*c^2*x^2 - a^2*c^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.25, size = 82, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(3/2)/(-c*x+c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), x
**(-2))/(2*pi**(3/2)*a**(3/2)*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4,
 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/x**2)/(2*pi**(3
/2)*a**(3/2)*c**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(23) = 46.
time = 0.01, size = 131, normalized size = 4.85

$$-2\left(-\frac{\frac{1}{8}\cdot 2\sqrt{ax+a}\,\sqrt{2a^2c-ac\,(ax+a)}}{c\,|a|\,(2a^2c-ac\,(ax+a))}-\frac{2a\sqrt{-ac}}{2c\,|a|\left(\left(\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^2-2ca^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x)
```

```
[Out] -2*sqrt(-a*c)*a/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)*c*abs(a)) - 1/2*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a
)/(((a*x + a)*a*c - 2*a^2*c)*c*abs(a))
```

Mupad [B]
time = 0.39, size = 23, normalized size = 0.85

$$\frac{x}{a\,c\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x)
```

```
[Out] x/(a*c*(a + a*x)^(1/2)*(c - c*x)^(1/2))
```

3.1142 $\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\,\sqrt{c-cx}}$$

```
[Out] 1/3*x/a/c/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+2/3*x/a^2/c^2/(a*x+a)^(1/2)/(-c*x+c)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {40, 39}

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*
Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{3ac}\\ &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\,\sqrt{c-cx}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 42, normalized size = 0.69

$$\frac{x(1+x)\left(-3+2x^2\right)}{3c^2(-1+x)(a(1+x))^{5/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] (x*(1 + x)*(-3 + 2*x^2))/(3*c^2*(-1 + x)*(a*(1 + x))^(5/2)*Sqrt[c - c*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 9.62, size = 63, normalized size = 1.03

$$\frac{I\text{meijerg}\left[\left\{\left\{\frac{5}{4},\frac{7}{4},1\right\},\left\{\frac{1}{2},\frac{5}{2},3\right\}\right\},\left\{\left\{\frac{5}{4},\frac{7}{4},2,\frac{5}{2},3\right\},\{0\}\right\},\frac{1}{x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1\right\},\{\}\right\},\left\{\left\{\frac{3}{4},\frac{5}{4}\right\},\left\{-\frac{1}{2},0,2,0\right\}\right\},\frac{\text{exp\_polar}[-2I\text{Pi}]}{x^2}\right]}{3\text{Pi}^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]')
```

```
[Out] (I meijerg[{{5 / 4, 7 / 4, 1}, {1 / 2, 5 / 2, 3}}, {{5 / 4, 7 / 4, 2, 5 / 2
, 3}, {0}}, 1 / x ^ 2] + meijerg[{{-1 / 2, 0, 1 / 2, 3 / 4, 5 / 4, 1}, {}},
 {{3 / 4, 5 / 4}, {-1 / 2, 0, 2, 0}}, exp_polar[-2 I Pi] / x ^ 2]) / (3 Pi
^ (3 / 2) a ^ (5 / 2) c ^ (5 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(49) = 98.
time = 0.14, size = 105, normalized size = 1.72

method	result	size
gosper	$\frac{(1+x)(-1+x)x\left(2x^2-3\right)}{3(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}$	32
default	$-\frac{1}{3ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}+\frac{-\frac{1}{ac\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{ax+a}}{3ac(-cx+c)^{\frac{3}{2}}}+\frac{2\sqrt{ax+a}}{3a\,c^2\sqrt{-cx+c}}}{a}}{a}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/c/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+1/a*(-1/a/c/(a*x+a)^(1/2)/(-c*x+c)^(3
/2)+2/a*(1/3/a/c/(-c*x+c)^(3/2)*(a*x+a)^(1/2)+1/3/a/c^2/(-c*x+c)^(1/2)*(a*x
+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.74

$$\frac{x}{3\left(-acx^{2}+ac\right)^{\frac{3}{2}}ac}+\frac{2\,x}{3\,\sqrt{-acx^{2}+ac}\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-a*c*x^2 + a*c)^(3/2)*a*c) + 2/3*x/(sqrt(-a*c*x^2 + a*c)*a^2*c^2)
```

Fricas [A]
time = 0.29, size = 57, normalized size = 0.93

$$-\frac{\left(2\,x^{3}-3\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{3\left(a^{3}c^{3}x^{4}-2\,a^{3}c^{3}x^{2}+a^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^3*c^3*x^4 - 2*a^3*c^3*x^
2 + a^3*c^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.48, size = 82, normalized size = 1.34

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3\\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0\end{matrix}\middle|\,\frac{1}{x^{2}}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0\end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^{2}}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(5/2)/(-c*x+c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), x*
*(-2))/(3*pi**(3/2)*a**(5/2)*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4, 5/4,
1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), exp_polar(-2*I*pi)/x**2)/(3*pi**(3/
2)*a**(5/2)*c**(5/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(49) = 98.
time = 0.04, size = 303, normalized size = 4.97

$$2\left(\frac{2\left(-\frac{\frac{1}{2304}\cdot 192ac|a|\sqrt{ax+a}\,\sqrt{ax+a}}{ac^{2}a^{2}}+\frac{\frac{1}{2304}\cdot 432a^{2}c|a|}{ac^{2}a^{2}}\right)\sqrt{ax+a}\,\sqrt{2a^{2}c-ac\left(ax+a\right)}}{\left(2a^{2}c-ac\left(ax+a\right)\right)^{2}}+\frac{2\left(3\sqrt{-ac}\,\left(\sqrt{2a^{2}c-ac\left(ax+a\right)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^{4}-18a^{2}c\sqrt{-ac}\,\left(\sqrt{2a^{2}c-ac\left(ax+a\right)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^{2}+16a^{4}c^{2}\sqrt{-ac}\right)}{12c^{2}\left|a\right|\left(\left(\sqrt{2a^{2}c-ac\left(ax+a\right)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^{2}-2a^{2}c\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x)
```

```
[Out] -1/12*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(4*(a*x + a)*abs(a)/(a^2
*c) - 9*abs(a)/(a*c))/((a*x + a)*a*c - 2*a^2*c)^2 - 1/3*(16*sqrt(-a*c)*a^4*
c^2 - 18*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2
*c))^2*a^2*c + 3*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^4)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)^3*c^2*abs(a))
```

Mupad [B]
time = 0.41, size = 62, normalized size = 1.02

$$-\frac{3\,x\,\sqrt{c-c\,x}-2\,x^3\,\sqrt{c-c\,x}}{\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^2\,\left(3\,a^2\,\left(c-c\,x\right)-6\,a^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x)
```

```
[Out] -(3*x*(c - c*x)^(1/2) - 2*x^3*(c - c*x)^(1/2))/((a + a*x)^(1/2)*(c - c*x)^2
*(3*a^2*(c - c*x) - 6*a^2*c))
```

3.1143 $\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}\,\sqrt{c-cx}}$$

```
[Out] 1/5*x/a/c/(a*x+a)^(5/2)/(-c*x+c)^(5/2)+4/15*x/a^2/c^2/(a*x+a)^(3/2)/(-c*x+c
)^(3/2)+8/15*x/a^3/c^3/(a*x+a)^(1/2)/(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {40, 39}

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*x)/(15*a^2*c^2*(a + a*x)^(3/
2)*(c - c*x)^(3/2)) + (8*x)/(15*a^3*c^3*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{5ac}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{15a^2c^2}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 49, normalized size = 0.54

$$\frac{x(15-20x^2+8x^4)}{15a^3c^3\sqrt{a(1+x)}\sqrt{c-cx}\,(-1+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] (x*(15 - 20*x^2 + 8*x^4))/(15*a^3*c^3*Sqrt[a*(1 + x)]*Sqrt[c - c*x]*(-1 + x
^2)^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 38.96, size = 63, normalized size = 0.69

$$\frac{2\left(-I\text{meijerg}\left[\{\{\frac{7}{4},\frac{9}{4},1\},\{\frac{1}{2},\frac{7}{2},4\}\},\{\{\frac{7}{4},\frac{9}{4},3,\frac{7}{2},4\},\{0\}\},\frac{1}{x^2}\right]+\text{meijerg}\left[\{\{-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1\},\{\}\},\{\{\frac{5}{4},\frac{7}{4}\},\{-\frac{1}{2},0,3,0\}\},\frac{\text{exp\_polar}[-2I\text{Pi}]}{x^2}\right]\right)}{15\text{Pi}^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]')
```

```
[Out] 2 (-I meijerg[{{7 / 4, 9 / 4, 1}, {1 / 2, 7 / 2, 4}}, {{7 / 4, 9 / 4, 3, 7
/ 2, 4}, {0}}, 1 / x ^ 2] + meijerg[{{-1 / 2, 0, 1 / 2, 5 / 4, 7 / 4, 1}, {
}}, {{5 / 4, 7 / 4}, {-1 / 2, 0, 3, 0}}, exp_polar[-2 I Pi] / x ^ 2]) / (15
 Pi ^ (3 / 2) a ^ (7 / 2) c ^ (7 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(73) = 146.
time = 0.16, size = 163, normalized size = 1.79

method	result
gosper	$-\frac{(1+x)(-1+x)x(8x^4-20x^2+15)}{15(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}$
default	$-\frac{1}{5ac(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}+\frac{-\frac{1}{3ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}}}+\frac{-\frac{4}{3ac\sqrt{ax+a}\,(-cx+c)^{\frac{5}{2}}}+\frac{4\left(\frac{3\sqrt{ax+a}}{5ac(-cx+c)^{\frac{5}{2}}}+\frac{3\left(\frac{2\sqrt{ax+a}}{15ac(-cx+c)^{\frac{3}{2}}}+\frac{2\sqrt{\cdots}}{15a\,c^2\cdots}\right.}{c}\right.}{3a}}{a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/c/(a*x+a)^(5/2)/(-c*x+c)^(5/2)+1/a*(-1/3/a/c/(a*x+a)^(3/2)/(-c*x+c)^
(5/2)+4/3/a*(-1/a/c/(a*x+a)^(1/2)/(-c*x+c)^(5/2)+3/a*(1/5/a/c/(-c*x+c)^(5/2
)*(a*x+a)^(1/2)+2/5/c*(1/3/a/c/(-c*x+c)^(3/2)*(a*x+a)^(1/2)+1/3/a/c^2/(-c*x
+c)^(1/2)*(a*x+a)^(1/2)))))
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.74

$$\frac{x}{5\left(-acx^{2}+ac\right)^{\frac{5}{2}}ac}+\frac{4\,x}{15\left(-acx^{2}+ac\right)^{\frac{3}{2}}a^{2}c^{2}}+\frac{8\,x}{15\,\sqrt{-acx^{2}+ac}\,a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-a*c*x^2 + a*c)^(5/2)*a*c) + 4/15*x/((-a*c*x^2 + a*c)^(3/2)*a^2*c^2
) + 8/15*x/(sqrt(-a*c*x^2 + a*c)*a^3*c^3)
```

Fricas [A]
time = 0.30, size = 74, normalized size = 0.81

$$-\frac{\left(8\,x^{5}-20\,x^{3}+15\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{15\left(a^{4}c^{4}x^{6}-3\,a^{4}c^{4}x^{4}+3\,a^{4}c^{4}x^{2}-a^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*x^5 - 20*x^3 + 15*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^4*c^4*x^6 - 3
*a^4*c^4*x^4 + 3*a^4*c^4*x^2 - a^4*c^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 46.54, size = 85, normalized size = 0.93

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\frac{1}{x^{2}}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^{2}}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(7/2)/(-c*x+c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 x**(-2))/(15*pi**(3/2)*a**(7/2)*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/2, 5/4,
 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), exp_polar(-2*I*pi)/x**2)/(15*
pi**(3/2)*a**(7/2)*c**(7/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(73) = 146.
time = 0.07, size = 459, normalized size = 5.04

$$-2\left(\frac{2\left(\left(\left(-\frac{\frac{1}{14745600}\cdot 983040a^2c^2a^2\sqrt{ax+a}\,\sqrt{ax+a}}{a^2c^3|a|a^2}+\frac{\frac{1}{14745600}\cdot 4224000a^3c^2a^2}{a^2c^3|a|a^2}\right)\sqrt{ax+a}\,\sqrt{ax+a}-\frac{\frac{1}{14745600}\cdot 4608000a^4c^2a^2}{a^2c^3|a|a^2}\right)\sqrt{ax+a}\,\sqrt{2a^2c-ac\,(ax+a)}}{(2a^2c-ac\,(ax+a))^3}+\frac{2\left(-45\left(\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^8+450a^2c\left(\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^6-1660a^4c^2\left(\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^4+2200a^6c^3\left(\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^2-1024a^8c^4\right)}{240c^2\sqrt{-ac}\,|a|\left(-\left(\sqrt{2a^2c-ac\,(ax+a)}-\sqrt{-ac}\,\sqrt{ax+a}\right)^2+2a^2c\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x)
```

```
[Out] -1/240*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(64*(a*x + a
)/(c*abs(a)) - 275*a/(c*abs(a))) + 300*a^2/(c*abs(a)))/((a*x + a)*a*c - 2*a
^2*c)^3 + 1/60*(1024*a^8*c^4 - 2200*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
+ a)*a*c + 2*a^2*c))^2*a^6*c^3 + 1660*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*
x + a)*a*c + 2*a^2*c))^4*a^4*c^2 - 450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a
*x + a)*a*c + 2*a^2*c))^6*a^2*c + 45*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^8)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^2)^5*sqrt(-a*c)*c^2*abs(a))
```

Mupad [B]
time = 0.44, size = 50, normalized size = 0.55

$$\frac{x\,\left(8\,x^4-20\,x^2+15\right)}{15\,a^3\,\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^{5/2}\,\left(c+3\,c\,x-x\,\left(c-c\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x)
```

```
[Out] (x*(8*x^4 - 20*x^2 + 15))/(15*a^3*(a + a*x)^(1/2)*(c - c*x)^(5/2)*(c + 3*c*
x - x*(c - c*x)))
```

3.1144 $\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}} dx$

Optimal. Leaf size=121

$$\frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{16x}{35a^4c^4\sqrt{a+ax}\sqrt{c}$$

```
[Out] 1/7*x/a/c/(a*x+a)^(7/2)/(-c*x+c)^(7/2)+6/35*x/a^2/c^2/(a*x+a)^(5/2)/(-c*x+c
)^(5/2)+8/35*x/a^3/c^3/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+16/35*x/a^4/c^4/(a*x+a)
^(1/2)/(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {40, 39}

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*x)/(35*a^2*c^2*(a + a*x)^(5/
2)*(c - c*x)^(5/2)) + (8*x)/(35*a^3*c^3*(a + a*x)^(3/2)*(c - c*x)^(3/2)) +
(16*x)/(35*a^4*c^4*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx}{7ac}$$

$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{24\int \frac{1}{(a+ax)^{5/2}(c-}}{35a^2c^2}$$

$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)}$$

$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)}$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.45

$$\frac{x(-35+70x^2-56x^4+16x^6)}{35a^4c^4\sqrt{a(1+x)}\sqrt{c-cx}\,(-1+x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] (x*(-35 + 70*x^2 - 56*x^4 + 16*x^6))/(35*a^4*c^4*Sqrt[a*(1 + x)]*Sqrt[c - c
*x]*(-1 + x^2)^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 188.87, size = 63, normalized size = 0.52

$$\frac{4\left(I\text{meijerg}\left[\{\{\frac{9}{4},\frac{11}{4},1\},\{\frac{1}{2},\frac{9}{2},5\}\},\{\{\frac{9}{4},\frac{11}{4},4,\frac{9}{2},5\},\{0\}\},\frac{1}{x^2}\right]+\text{meijerg}\left[\{\{-\frac{1}{2},0,\frac{1}{2},\frac{7}{4},\frac{9}{4},1\},\{\}\},\{\{\frac{7}{4},\frac{9}{4}\},\{-\frac{1}{2},0,4,0\}\},\frac{\text{exp\_polar}[-2I\text{Pi}]}{x^2}\right]\right)}{105\text{Pi}^{\frac{3}{2}}a^{\frac{9}{2}}c^{\frac{9}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]')
```

```
[Out] 4 (I meijerg[{{9 / 4, 11 / 4, 1}, {1 / 2, 9 / 2, 5}}, {{9 / 4, 11 / 4, 4, 9
 / 2, 5}, {0}}, 1 / x ^ 2] + meijerg[{{-1 / 2, 0, 1 / 2, 7 / 4, 9 / 4, 1},
{}}, {{7 / 4, 9 / 4}, {-1 / 2, 0, 4, 0}}, exp_polar[-2 I Pi] / x ^ 2]) / (1
05 Pi ^ (3 / 2) a ^ (9 / 2) c ^ (9 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(97) = 194.
time = 0.14, size = 221, normalized size = 1.83

method	result
gosper	$\frac{(1+x)(-1+x)x(16x^6-56x^4+70x^2-35)}{35(ax+a)^{\frac{9}{2}}(-cx+c)^{\frac{9}{2}}}$
default	$-\frac{1}{7ac(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{-\frac{1}{5ac(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{-\frac{2}{5ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{6\left(-\frac{5}{3ac\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}}}+\frac{5\left(\frac{4\sqrt{ax+a}}{7ac(-cx+c)^{\frac{7}{2}}}+\cdots\right)}{\cdots}\right)}{\cdots}}{a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/a/c/(a*x+a)^(7/2)/(-c*x+c)^(7/2)+1/a*(-1/5/a/c/(a*x+a)^(5/2)/(-c*x+c)^
(7/2)+6/5/a*(-1/3/a/c/(a*x+a)^(3/2)/(-c*x+c)^(7/2)+5/3/a*(-1/a/c/(a*x+a)^(1
/2)/(-c*x+c)^(7/2)+4/a*(1/7/a/c/(-c*x+c)^(7/2)*(a*x+a)^(1/2)+3/7/c*(1/5/a/c
/(-c*x+c)^(5/2)*(a*x+a)^(1/2)+2/5/c*(1/3/a/c/(-c*x+c)^(3/2)*(a*x+a)^(1/2)+1
/3/a/c^2/(-c*x+c)^(1/2)*(a*x+a)^(1/2))))))
```

Maxima [A]
time = 0.28, size = 89, normalized size = 0.74

$$\frac{x}{7\,(-acx^2+ac)^{\frac{7}{2}}ac}+\frac{6\,x}{35\,(-acx^2+ac)^{\frac{5}{2}}a^2c^2}+\frac{8\,x}{35\,(-acx^2+ac)^{\frac{3}{2}}a^3c^3}+\frac{16\,x}{35\,\sqrt{-acx^2+ac}\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-a*c*x^2 + a*c)^(7/2)*a*c) + 6/35*x/((-a*c*x^2 + a*c)^(5/2)*a^2*c^2
) + 8/35*x/((-a*c*x^2 + a*c)^(3/2)*a^3*c^3) + 16/35*x/(sqrt(-a*c*x^2 + a*c)
*a^4*c^4)
```

Fricas [A]
time = 0.30, size = 89, normalized size = 0.74

$$-\frac{\left(16\,x^{7}-56\,x^{5}+70\,x^{3}-35\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{35\,\left(a^{5}c^{5}x^{8}-4\,a^{5}c^{5}x^{6}+6\,a^{5}c^{5}x^{4}-4\,a^{5}c^{5}x^{2}+a^{5}c^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*x^7 - 56*x^5 + 70*x^3 - 35*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^5*c
^5*x^8 - 4*a^5*c^5*x^6 + 6*a^5*c^5*x^4 - 4*a^5*c^5*x^2 + a^5*c^5)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(9/2)/(-c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(97) = 194.
time = 0.18, size = 607, normalized size = 5.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x)
```

```
[Out] -1/1120*sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*((a*x + a)*(256*(a*x + a)
*abs(a)/(a^2*c) - 1617*abs(a)/(a*c)) + 3430*abs(a)/c) - 2450*a*abs(a)/c)*sq
rt(a*x + a)/((a*x + a)*a*c - 2*a^2*c)^4 + 1/280*(16384*a^12*c^6 - 51744*(sq
rt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2*a^10*c^5 + 66416
*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^4*a^8*c^4 - 43
120*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^6*a^6*c^3 +
 14280*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^8*a^4*c^
2 - 2450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^10*a^2
*c + 175*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^12)/((
2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2)^7*
sqrt(-a*c)*a*c^3*abs(a))
```

Mupad [B]
time = 0.48, size = 66, normalized size = 0.55

$$-\frac{x\,\left(16\,x^6-56\,x^4+70\,x^2-35\right)}{35\,a^4\,\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^{7/2}\,\left(c-x^2\,\left(c-c\,x\right)+7\,c\,x-4\,x\,\left(c-c\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x)
```

```
[Out] -(x*(70*x^2 - 56*x^4 + 16*x^6 - 35))/(35*a^4*(a + a*x)^(1/2)*(c - c*x)^(7/2
)*(c - x^2*(c - c*x) + 7*c*x - 4*x*(c - c*x)))
```

3.1145 $\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}+\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}$$

```
[Out] 5/24*a^2*c*x*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)+1/6*x*(b*x+a)^(5/2)*(-b*c*x+a
*c)^(5/2)+5/8*a^6*c^(5/2)*arctan(c^(1/2)*(b*x+a)^(1/2)/(c*(-b*x+a))^(1/2))/
b+5/16*a^4*c^2*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {38, 65, 223, 209}

$$\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (5*a^4*c^2*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/16 + (5*a^2*c*x*(a + b*x)^(3/
2)*(a*c - b*c*x)^(3/2))/24 + (x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5
*a^6*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(8*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{1}{6}\left(5a^2c\right)\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$$

$$= \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{1}{8}\left(5a^4c^2\right)\int$$

$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx$$

$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx$$

$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx$$

$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx$$

Mathematica [A]
time = 0.17, size = 103, normalized size = 0.76

$$\frac{(c(a-bx))^{5/2}\left(bx\sqrt{a-bx}\,\sqrt{a+bx}\,\left(33a^4-26a^2b^2x^2+8b^4x^4\right)+30a^6\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{48b(a-bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] ((c*(a - b*x))^(5/2)*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(33*a^4 - 26*a^2*b^2*
x^2 + 8*b^4*x^4) + 30*a^6*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(48*b*(a -
b*x)^(5/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(107) = 214$.
time = 0.16, size = 242, normalized size = 1.79

method	result
risch	$\frac{x(8b^4x^4-26a^2b^2x^2+33a^4)\sqrt{bx+a}\,(-bx+a)c^3}{48\sqrt{-c(bx-a)}}+\frac{5a^6\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)\,c\,(bx-a)}\,c^3}{16\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c\,(bx-a)}}$

$$5a\left(-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}}{5bc}+\frac{3a\left(-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{7}{2}}}{4bc}+\frac{a\left(\frac{(-bcx+ac)^{\frac{5}{2}}\sqrt{bx+a}}{3b}+\frac{5ac\left(\frac{(-bcx+a}{}\right.}{}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b/c*(b*x+a)^(5/2)*(-b*c*x+a*c)^(7/2)+5/6*a*(-1/5/b/c*(b*x+a)^(3/2)*(-b
*c*x+a*c)^(7/2)+3/5*a*(-1/4/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(7/2)+1/4*a*(1/3
/b*(-b*c*x+a*c)^(5/2)*(b*x+a)^(1/2)+5/3*a*c*(1/2/b*(-b*c*x+a*c)^(3/2)*(b*x+
a)^(1/2)+3/2*a*c*(1/b*(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+a*c*((b*x+a)*(-b*c*x
+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^
(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2)))))))
```

Maxima [A]
time = 0.36, size = 89, normalized size = 0.66

$$\frac{5\,a^6c^{\frac{5}{2}}\arcsin\left(\frac{bx}{a}\right)}{16\,b}+\frac{5}{16}\,\sqrt{-b^2cx^2+a^2c}\,a^4c^2x+\frac{5}{24}\,\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^2cx+\frac{1}{6}\,\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*c^(5/2)*arcsin(b*x/a)/b + 5/16*sqrt(-b^2*c*x^2 + a^2*c)*a^4*c^2*x
+ 5/24*(-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c*x + 1/6*(-b^2*c*x^2 + a^2*c)^(5/2)*
x
```

Fricas [A]
time = 0.31, size = 232, normalized size = 1.72

$$\left[\frac{15\,a^6\sqrt{-c}\,c^2\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{96\,b},-\frac{15\,a^6c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^6*sqrt(-c)*c^2*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(8*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4
*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b, -1/48*(15*a^6*c^(5/2)*arctan
(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (8*b^5
*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\left(-a+bx\right)\right)^{\frac{5}{2}}\left(a+bx\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(-b*c*x+a*c)**(5/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(5/2)*(a + b*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(109) = 218.
time = 0.11, size = 951, normalized size = 7.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x)
```

```
[Out] -1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^5*c^2 - 120*(
2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*a^4*c^2 - 80*(
6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*
x + a))*a^3*c^2 + 20*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
 + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^
2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^2*c^2 + 2*(90*a^5
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
- (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))
*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a*c^2 - (150*a^6*c*lo
g(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (40
5*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a) + 321*a^2)*(b*x
 + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*c^2)
/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{5/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2), x)
```

3.1146 $\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}$$

```
[Out] 1/4*x*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)+3/4*a^4*c^(3/2)*arctan(c^(1/2)*(b*x+
a)^(1/2)/(c*(-b*x+a))^(1/2))/b+3/8*a^2*c*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.174$, Rules used = {38, 65, 223, 209}

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (3*a^2*c*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 + (x*(a + b*x)^(3/2)*(a*c - b
*c*x)^(3/2))/4 + (3*a^4*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a -
b*x)]])/(4*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{4}\left(3a^2c\right)\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{8}\left(3a^4c^2\right)\int \frac{}{\sqrt{a}}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{\left(3a^4c^2\right)\text{Subst}\left(\right.}{}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{\left(3a^4c^2\right)\text{Subst}\left(\right.}{}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{3a^4c^{3/2}\tan^{-1}\left(\right.}{4b}$$

Mathematica [A]
time = 0.12, size = 92, normalized size = 0.90

$$\frac{(c(a-bx))^{3/2}\left(bx\sqrt{a-bx}\,\sqrt{a+bx}\,\left(5a^2-2b^2x^2\right)+6a^4\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{8b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
[Out] ((c*(a - b*x))^(3/2)*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(5*a^2 - 2*b^2*x^2) +
 6*a^4*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(8*b*(a - b*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]')

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(80) = 160$.

time = 0.17, size = 184, normalized size = 1.80

method	result
risch	$\frac{x(-2x^2b^2+5a^2)\sqrt{bx+a}\,(-bx+a)c^2}{8\sqrt{-c(bx-a)}}+\frac{3a^4\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)\,c\,(bx-a)}\,c^2}{8\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}}{4bc}+\frac{3a\left(-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}}{3bc}+\frac{a\left(\frac{(-bcx+ac)^{\frac{3}{2}}\sqrt{bx+a}}{2b}+\frac{3ac\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}}{b}+\cdots\right)}{\cdots}\right)}{\cdots}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4/b/c*(b*x+a)^(3/2)*(-b*c*x+a*c)^(5/2)+3/4*a*(-1/3/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(5/2)+1/3*a*(1/2/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+3/2*a*c*(1/b*(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+a*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2)))))

Maxima [A]

time = 0.36, size = 63, normalized size = 0.62

$$\frac{3\,a^4c^{\frac{3}{2}}\arcsin\left(\frac{bx}{a}\right)}{8\,b}+\frac{3}{8}\sqrt{-b^2cx^2+a^2c}\,a^2cx+\frac{1}{4}\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="maxima")

```
[Out] 3/8*a^4*c^(3/2)*arcsin(b*x/a)/b + 3/8*sqrt(-b^2*c*x^2 + a^2*c)*a^2*c*x + 1/
4*(-b^2*c*x^2 + a^2*c)^(3/2)*x
```

Fricas [A]
time = 0.31, size = 193, normalized size = 1.89

$$\left[\frac{3\,a^{4}\sqrt{-c}\,c\log\left(2\,b^{2}cx^{2}+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^{2}c\right)-2\,(2\,b^{3}cx^{3}-5\,a^{2}bcx)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{16\,b},\ -\frac{3\,a^{4}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^{2}cx^{2}-a^{2}c}\right)+(2\,b^{3}cx^{3}-5\,a^{2}bcx)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^4*sqrt(-c)*c*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) - 2*(2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/b, -1/8*(3*a^4*c^(3/2)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c
*x + a*c)*sqrt(b*x + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-a+bx\right)\right)^{\frac{3}{2}}\left(a+bx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**(3/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(3/2)*(a + b*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(82) = 164.
time = 0.06, size = 516, normalized size = 5.06

$$\frac{2a^{2}c\left(2\left(\frac{1}{8}\sqrt{a+bx}\sqrt{a+bx}-\frac{12}{32}a\right)\sqrt{a+bx}\sqrt{2ac-c(a+bx)}+\frac{2a^{2}c\ln\left|\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right|}{4\sqrt{-c}}\right)-2c\left(2\left(\left(\left(\frac{1}{16}\sqrt{a+bx}\sqrt{a+bx}-\frac{24960}{92160}a\right)\sqrt{a+bx}\sqrt{a+bx}+\frac{41280}{92160}a^{2}\right)\sqrt{a+bx}\sqrt{a+bx}-\frac{37440}{92160}a^{3}\right)\sqrt{a+bx}\sqrt{2ac-c(a+bx)}+\frac{6a^{4}c\ln\left|\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right|}{16\sqrt{-c}}\right)+2a^{3}c\left(\frac{1}{2}\sqrt{a+bx}\sqrt{2ac-c(a+bx)}-\frac{2ac\ln\left|\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right|}{2\sqrt{-c}}\right)-2ac\left(2\left(\left(\frac{1}{12}\sqrt{a+bx}\sqrt{a+bx}-\frac{84}{288}a\right)\sqrt{a+bx}\sqrt{a+bx}+\frac{108}{288}a^{2}\right)\sqrt{a+bx}\sqrt{2ac-c(a+bx)}-\frac{2a^{3}c\ln\left|\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right|}{4\sqrt{-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x)
```

```
[Out] -1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^3*c - 12*(2*a^2
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
+ sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*a^2*c - 4*(6*a^3*c*
log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (
(2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*
a*c + (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c
)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*s
qrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*c)/b
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{3/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2),x)
[Out] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2), x)
```

3.1147 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx$

Optimal. Leaf size=68

$$\frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$

```
[Out] a^2*arctan(c^(1/2)*(b*x+a)^(1/2)/(c*(-b*x+a))^(1/2))*c^(1/2)/b+1/2*x*(b*x+a
)^(1/2)*(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {38, 65, 223, 209}

$$\frac{a^2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + b*x])/Sqrt[c*(a - b*x)]])/b
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx &= \frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{2}\left(a^2c\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= \frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\\ &= \frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 1.15

$$\frac{\sqrt{c(a-bx)}\,\left(bx\sqrt{a-bx}\,\sqrt{a+bx}+2a^2\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{2b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
[Out] (Sqrt[c*(a - b*x)]*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x] + 2*a^2*ArcTan[Sqrt[a +
 b*x]/Sqrt[a - b*x]]))/(2*b*Sqrt[a - b*x])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(a*c - b*c*x)^(1/2),x]')
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.
time = 0.18, size = 126, normalized size = 1.85

method	result
risch	$\frac{x(-bx+a)\sqrt{bx+a}\,c}{2\sqrt{-c(bx-a)}}+\frac{a^2\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}\,c}{2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}{2bc}+\frac{a\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}}{b}+\frac{ac\sqrt{(bx+a)(-bcx+ac)}\,\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a}}\right.}{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,\sqrt{b^2c}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(3/2)+1/2*a*(1/b*(-b*c*x+a*c)^(1/2)*(b*
x+a)^(1/2)+a*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2
)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2)))
```

Maxima [A]
time = 0.36, size = 39, normalized size = 0.57

$$\frac{a^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^2cx^2+a^2c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*sqrt(c)*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*x
```

Fricas [A]
time = 0.32, size = 159, normalized size = 2.34

$$\left[\frac{a^2\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,bx}{4\,b},-\frac{a^2\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\sqrt{-bcx+ac}\,\sqrt{bx+a}\,bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a^2*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(-c)*x - a^2*c) + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b, -1/2*(a^2*s
qrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2
*c)) - sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\,(-a+bx)}\,\sqrt{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(56) = 112.
time = 0.02, size = 199, normalized size = 2.93

$$\frac{2\left(2\left(\frac{1}{8}\sqrt{a+bx}\,\sqrt{a+bx}-\frac{12}{32}a\right)\sqrt{a+bx}\,\sqrt{2ac-c\,(a+bx)}+\frac{2a^2c\ln\left|\sqrt{2ac-c\,(a+bx)}-\sqrt{-c}\,\sqrt{a+bx}\right|}{4\sqrt{-c}}\right)+2a\left(\frac{1}{2}\sqrt{a+bx}\,\sqrt{2ac-c\,(a+bx)}-\frac{2ac\ln\left|\sqrt{2ac-c\,(a+bx)}-\sqrt{-c}\,\sqrt{a+bx}\right|}{2\sqrt{-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/2*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a) - 2*(2*a*c
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a)/b
```

Mupad [B]
time = 0.20, size = 72, normalized size = 1.06

$$\frac{x\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2}-\frac{a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,(a-b\,x)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{(-b\,c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] (x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (a^2*b^(1/2)*c^2*log((-b*c)^(1/
2)*(c*(a - b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

3.1148 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

```
[Out] 2*arctan(c^(1/2)*(b*x+a)^(1/2)/(c*(-b*x+a))^(1/2))/b/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {65, 223, 209}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(b*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 1.26

$$\frac{2\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (2*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(b*Sqrt[c*(a - b*x)])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.77, size = 76, normalized size = 2.00

$$\frac{-I\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{a^2}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{a^2\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(a*c - b*c*x)^(1/2)),x]')
```

```
[Out] (-I meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1 / 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 /
4, 1, 0}, {}}, a ^ 2 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4,
 1 / 2, 1}, {}}, {{-1 / 4, 1 / 4}, {-1 / 2, 0, 0, 0}}, a ^ 2 exp_polar[-2 I
 Pi] / (b ^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b Sqrt[c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.
time = 0.19, size = 71, normalized size = 1.87

method	result	size
default	$\frac{\sqrt{(bx+a)(-bcx+ac)}\,\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{\sqrt{bx+a}\,\sqrt{-bcx+ac}\,\sqrt{b^2c}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(-b*c*x+a*c))^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(b^2*c)^(1/2)
*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

Maxima [A]
time = 0.34, size = 14, normalized size = 0.37

$$\frac{\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/a)/(b*sqrt(c))
```

Fricas [A]
time = 0.31, size = 108, normalized size = 2.84

$$\left[-\frac{\sqrt{-c}\,\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)}{2\,bc}, -\frac{\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
-c)*x - a^2*c)/(b*c), -arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/
(b^2*c*x^2 - a^2*c))/(b*sqrt(c))]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.41, size = 90, normalized size = 2.37

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{a^2}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{a^2e^{-2i\pi}}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*meijerg((((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b**2*
x**2))/(4*pi**(3/2)*b*sqrt(c))
```

Giac [A]
time = 0.01, size = 49, normalized size = 1.29

$$-\frac{2\ln\left|\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right|}{\sqrt{-c}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -2*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/(b*sqrt(-
c))
```

Mupad [B]
time = 0.18, size = 53, normalized size = 1.39

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(b^2*c)^(1/2)
```

3.1149 $\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

```
[Out] x/a^2/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {39}

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 0.97

$$\frac{x}{a^2c\sqrt{c(a-bx)}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.89, size = 79, normalized size = 2.63

$$-\frac{I\text{meijerg}\left[\left\{\left\{\frac{3}{4},\frac{5}{4},1\right\},\left\{\frac{1}{2},\frac{3}{2},2\right\}\right\},\left\{\left\{\frac{3}{4},1,\frac{5}{4},\frac{3}{2},2\right\},\left\{0\right\}\right\},\frac{a^2}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\},\left\{\right\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{-\frac{1}{2},0,1,0\right\}\right\},\frac{a^2\text{exp\_polar}\left[-2I\text{Pi}\right]}{b^2x^2}\right]}{2\text{Pi}^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]')
```

```
[Out] (-I meijerg[{{3 / 4, 5 / 4, 1}, {1 / 2, 3 / 2, 2}}, {{3 / 4, 1, 5 / 4, 3 / 2, 2}, {0}}, a ^ 2 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, 0, 1 / 4, 1 / 2, 3 / 4, 1}, {}}, {{1 / 4, 3 / 4}, {-1 / 2, 0, 1, 0}}, a ^ 2 exp_polar[-2 I Pi] / (b ^ 2 x ^ 2)]) / (2 Pi ^ (3 / 2) a ^ 2 b c ^ (3 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.
time = 0.15, size = 59, normalized size = 1.97

method	result	size
gosper	$\frac{(-bx+a)x}{\sqrt{bx+a}\,a^2(-bcx+ac)^{\frac{3}{2}}}$	30
default	$-\frac{1}{abc\sqrt{bx+a}\,\sqrt{-bcx+ac}}+\frac{\sqrt{bx+a}}{bc\,a^2\sqrt{-bcx+ac}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/b/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/b/c/a^2/(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.83

$$\frac{x}{\sqrt{-b^2cx^2+a^2c}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-b^2*c*x^2 + a^2*c)*a^2*c)
```

Fricas [A]
time = 0.29, size = 45, normalized size = 1.50

$$-\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,x}{a^2b^2c^2x^2-a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="fricas")

[Out] -sqrt(-b*c*x + a*c)*sqrt(b*x + a)*x/(a^2*b^2*c^2*x^2 - a^4*c^2)

Sympy [C] Result contains complex when optimal does not.
time = 2.51, size = 94, normalized size = 3.13

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(3/2)/(-b*c*x+a*c)**(3/2),x)

[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), a **2/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(26) = 52.
time = 0.01, size = 112, normalized size = 3.73

$$\frac{2\left(\frac{\sqrt{a+bx}\sqrt{2ac-c(a+bx)}}{4ca^2(2ac-c(a+bx))}+\frac{2\sqrt{-c}}{2ac\left(\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^2-2ac\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x)

[Out] 1/2*(4*sqrt(-c)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2 - 2*a*c)*a*c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)/(((b*x + a)*c - 2*a*c)*a^2*c))/b

Mupad [B]
time = 0.50, size = 26, normalized size = 0.87

$$\frac{x}{a^2\,c\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2)),x)

[Out] x/(a^2*c*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))

3.1150 $\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

```
[Out] 1/3*x/a^2/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+2/3*x/a^4/c^2/(b*x+a)^(1/2)/(-
b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{3a^2c}\\ &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.69

$$\frac{3a^2x - 2b^2x^3}{3a^4c(c(a-bx))^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] (3*a^2*x - 2*b^2*x^3)/(3*a^4*c*(c*(a - b*x))^(3/2)*(a + b*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 10.81, size = 79, normalized size = 1.18

$$\frac{I\text{meijerg}\left[\left\{\left\{\frac{5}{4},\frac{7}{4},1\right\},\left\{\frac{1}{2},\frac{5}{2},3\right\}\right\},\left\{\left\{\frac{5}{4},\frac{7}{4},2,\frac{5}{2},3\right\},\{0\}\right\},\frac{a^2}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1\right\},\{\}\right\},\left\{\left\{\frac{3}{4},\frac{5}{4}\right\},\left\{-\frac{1}{2},0,2,0\right\}\right\},\frac{a^2\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]}{3\text{Pi}^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]')
```

```
[Out] (I meijerg[{{5 / 4, 7 / 4, 1}, {1 / 2, 5 / 2, 3}}, {{5 / 4, 7 / 4, 2, 5 / 2
, 3}, {0}}, a ^ 2 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, 0, 1 / 2, 3 / 4, 5 /
 4, 1}, {}}, {{3 / 4, 5 / 4}, {-1 / 2, 0, 2, 0}}, a ^ 2 exp_polar[-2 I Pi]
/ (b ^ 2 x ^ 2)]) / (3 Pi ^ (3 / 2) a ^ 4 b c ^ (5 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(55) = 110.
time = 0.14, size = 129, normalized size = 1.93

method	result	size
gosper	$\frac{(-bx+a)x(-2x^2b^2+3a^2)}{3(bx+a)^{\frac{3}{2}}a^4(-bcx+ac)^{\frac{5}{2}}}$	45
default	$-\frac{1}{3abc(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{3}{2}}}+\frac{-\frac{1}{abc\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{bx+a}}{3abc(-bcx+ac)^{\frac{3}{2}}}+\frac{2\sqrt{bx+a}}{3b\,a^2c^2\sqrt{-bcx+ac}}}{a}}{a}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/b/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+1/a*(-1/a/b/c/(b*x+a)^(1/2)/(-b
*c*x+a*c)^(3/2)+2/a*(1/3/a/b/c/(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+1/3/b/a^2/c
^2/(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.79

$$\frac{x}{3\,(-b^2cx^2+a^2c)^{\frac{3}{2}}a^2c}+\frac{2\,x}{3\,\sqrt{-b^2cx^2+a^2c}\,a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c) + 2/3*x/(sqrt(-b^2*c*x^2 + a^2*c)*
a^4*c^2)
```

Fricas [A]
time = 0.30, size = 72, normalized size = 1.07

$$-\frac{(2\,b^2x^3-3\,a^2x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{3\,(a^4b^4c^3x^4-2\,a^6b^2c^3x^2+a^8c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^3 - 3*a^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^4*b^4*c^3*x^
4 - 2*a^6*b^2*c^3*x^2 + a^8*c^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.48, size = 94, normalized size = 1.40

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3\\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0\end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(-b*c*x+a*c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), a*
*2/(b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4
, 5/4, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), a**2*exp_polar(-2*I*pi)/(b**
2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(55) = 110.
time = 0.03, size = 256, normalized size = 3.82

$$\frac{2\left(\frac{2\left(-\frac{192ca^3\sqrt{a+bx}\sqrt{a+bx}}{2304c^2a^7}+\frac{432ca^4}{2304c^2a^7}\right)\sqrt{a+bx}\,\sqrt{2ac-c\,(a+bx)}}{(2ac-c(a+bx))^2}-\frac{2\left(-3\left(\sqrt{2ac-c\,(a+bx)}-\sqrt{-c}\,\sqrt{a+bx}\right)^4+18c\left(\sqrt{2ac-c\,(a+bx)}-\sqrt{-c}\,\sqrt{a+bx}\right)^2a-16c^2a^2\right)}{12c\sqrt{-c}\,a^3\left(-\left(\sqrt{2ac-c\,(a+bx)}-\sqrt{-c}\,\sqrt{a+bx}\right)^2+2ca\right)^3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x)
```

```
[Out] -1/12*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(4*(b*x + a)/(a^4*c) - 9/(a
^3*c))/((b*x + a)*c - 2*a*c)^2 + 4*(3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x
+ a)*c + 2*a*c))^4 - 18*a*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^2*c + 16*a^2*c^2)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*
c))^2 - 2*a*c)^3*a^3*sqrt(-c)*c))/b
```

Mupad [B]
time = 0.58, size = 80, normalized size = 1.19

$$-\frac{3\,a^2\,x\,\sqrt{a\,c-b\,c\,x}-2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^2\,\left(3\,a^4\,\left(a\,c-b\,c\,x\right)-6\,a^5\,c\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(3*a^2*x*(a*c - b*c*x)^(1/2) - 2*b^2*x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*
x)^2*(3*a^4*(a*c - b*c*x) - 6*a^5*c)*(a + b*x)^(1/2))
```

3.1151 $\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] 1/5*x/a^2/c/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2)+4/15*x/a^4/c^2/(b*x+a)^(3/2)/(
-b*c*x+a*c)^(3/2)+8/15*x/a^6/c^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {40, 39}

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*x)/(15*a^4*c^2*(a + b*
x)^(3/2)*(a*c - b*c*x)^(3/2)) + (8*x)/(15*a^6*c^3*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx}{5a^2c}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8\int \frac{}{(a+bx}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{}{15a^6c^3\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 57, normalized size = 0.57

$$\frac{15a^4x - 20a^2b^2x^3 + 8b^4x^5}{15a^6c(c(a - bx))^{5/2}(a + bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] (15*a^4*x - 20*a^2*b^2*x^3 + 8*b^4*x^5)/(15*a^6*c*(c*(a - b*x))^(5/2)*(a + 
b*x)^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 41.02, size = 79, normalized size = 0.79

$$\frac{2\left(-I\text{meijerg}\left[\left\{\left\{\frac{7}{4},\frac{9}{4},1\right\},\left\{\frac{1}{2},\frac{7}{2},4\right\}\right\},\left\{\left\{\frac{7}{4},\frac{9}{4},3,\frac{7}{2},4\right\},\{0\}\right\},\frac{a^2}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1\right\},\{\}\right\},\left\{\left\{\frac{5}{4},\frac{7}{4}\right\},\left\{-\frac{1}{2},0,3,0\right\}\right\},\frac{a^2\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]\right)}{15\text{Pi}^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]')
```

```
[Out] 2 (-I meijerg[{{7 / 4, 9 / 4, 1}, {1 / 2, 7 / 2, 4}}, {{7 / 4, 9 / 4, 3, 7
/ 2, 4}, {0}}, a ^ 2 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, 0, 1 / 2, 5 / 4,
7 / 4, 1}, {}}, {{5 / 4, 7 / 4}, {-1 / 2, 0, 3, 0}}, a ^ 2 exp_polar[-2 I P
i] / (b ^ 2 x ^ 2)]) / (15 Pi ^ (3 / 2) a ^ 6 b c ^ (7 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(82) = 164$.
time = 0.17, size = 202, normalized size = 2.02

method	result
gosper	$\frac{(-bx+a)x(8b^4x^4-20a^2b^2x^2+15a^4)}{15(bx+a)^{\frac{5}{2}}a^6(-bcx+ac)^{\frac{7}{2}}}$
default	$-\frac{1}{5abc(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{5}{2}}}+\frac{-\frac{1}{3abc(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}}+\frac{-\frac{4}{3abc\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}}+\frac{4\left(\frac{3\sqrt{bx+a}}{5abc(-bcx+ac)^{\frac{5}{2}}}+\frac{3\left(\frac{2\sqrt{bx+a}}{15abc(-bcx+ac)\cdots}\right.}{\cdots}\right.}{3a\cdots}}{a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/b/c/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2)+1/a*(-1/3/a/b/c/(b*x+a)^(3/2)/(
-b*c*x+a*c)^(5/2)+4/3/a*(-1/a/b/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(5/2)+3/a*(1/5
```

```
/a/b/c/(-b*c*x+a*c)^(5/2)*(b*x+a)^(1/2)+2/5/a/c*(1/3/a/b/c/(-b*c*x+a*c)^(3/
2)*(b*x+a)^(1/2)+1/3/b/a^2/c^2/(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)))))
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.79

$$\frac{x}{5\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^2c}+\frac{4x}{15\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^4c^2}+\frac{8x}{15\sqrt{-b^2cx^2+a^2c}\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-b^2*c*x^2 + a^2*c)^(5/2)*a^2*c) + 4/15*x/((-b^2*c*x^2 + a^2*c)^(3/
2)*a^4*c^2) + 8/15*x/(sqrt(-b^2*c*x^2 + a^2*c)*a^6*c^3)
```

Fricas [A]
time = 0.31, size = 98, normalized size = 0.98

$$-\frac{\left(8\,b^4x^5-20\,a^2b^2x^3+15\,a^4x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{15\left(a^6b^6c^4x^6-3\,a^8b^4c^4x^4+3\,a^{10}b^2c^4x^2-a^{12}c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^4*x^5 - 20*a^2*b^2*x^3 + 15*a^4*x)*sqrt(-b*c*x + a*c)*sqrt(b*x +
 a)/(a^6*b^6*c^4*x^6 - 3*a^8*b^4*c^4*x^4 + 3*a^10*b^2*c^4*x^2 - a^12*c^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 48.59, size = 97, normalized size = 0.97

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(-b*c*x+a*c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 a**2/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/
2, 5/4, 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), a**2*exp_polar(-2*I*pi
)/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(82) = 164.
time = 0.07, size = 389, normalized size = 3.89

$$\frac{2\left(\frac{2\left(\left(\frac{983040c^2a^9\sqrt{a+bx}\sqrt{a+bx}}{14745600c^3a^{15}}-\frac{4224000c^2a^{10}}{14745600c^3a^{15}}\right)\sqrt{a+bx}\sqrt{a+bx}+\frac{4608000c^2a^{11}}{14745600c^3a^{15}}\right)\sqrt{a+bx}\sqrt{2ac-c(a+bx)}}{(2ac-c(a+bx))^3}+\frac{2\left(45\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^8-450c\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^6a+1660c^2\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^4a^2-2200c^3\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^2a^3+1024c^4a^4\right)}{240c^2\sqrt{-c}\,a^5\left(-\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^2+2ca\right)^5}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x)
```

```
[Out] -1/240*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*(64*(b*x + a)/(
a^6*c) - 275/(a^5*c)) + 300/(a^4*c))/((b*x + a)*c - 2*a*c)^3 + 4*(45*(sqrt(
b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^8 - 450*a*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*c + 1660*a^2*(sqrt(b*x + a)*sqrt(-c)
 - sqrt(-(b*x + a)*c + 2*a*c))^4*c^2 - 2200*a^3*(sqrt(b*x + a)*sqrt(-c) - s
qrt(-(b*x + a)*c + 2*a*c))^2*c^3 + 1024*a^4*c^4)/(((sqrt(b*x + a)*sqrt(-c)
- sqrt(-(b*x + a)*c + 2*a*c))^2 - 2*a*c)^5*a^5*sqrt(-c)*c^2))/b
```

Mupad [B]
time = 0.65, size = 111, normalized size = 1.11

$$\frac{15\,a^4\,x\,\sqrt{a\,c-b\,c\,x}+8\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}-20\,a^2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^3\,\left(60\,a^8\,c-\left(a\,c-b\,c\,x\right)\,\left(45\,a^7+15\,b\,x\,a^6\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(7/2)*(a + b*x)^(7/2)),x)
```

```
[Out] (15*a^4*x*(a*c - b*c*x)^(1/2) + 8*b^4*x^5*(a*c - b*c*x)^(1/2) - 20*a^2*b^2*
x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*x)^3*(60*a^8*c - (a*c - b*c*x)*(45*a^7
 + 15*a^6*b*x))*(a + b*x)^(1/2))
```

3.1152 $\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx$

Optimal. Leaf size=133

$$\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{}{35a^8c^4\sqrt{a}\cdots}$$

```
[Out] 1/7*x/a^2/c/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2)+6/35*x/a^4/c^2/(b*x+a)^(5/2)/(
-b*c*x+a*c)^(5/2)+8/35*x/a^6/c^3/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+16/35*x/a
^8/c^4/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {40, 39}

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*x)/(35*a^4*c^2*(a + b*
x)^(5/2)*(a*c - b*c*x)^(5/2)) + (8*x)/(35*a^6*c^3*(a + b*x)^(3/2)*(a*c - b*
c*x)^(3/2)) + (16*x)/(35*a^8*c^4*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx}{7a^2c}$$

$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{24\int \frac{1}{(a+bx}$$

$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{}{35a^6c^3(a}$$

$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{}{35a^6c^3(a}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.57

$$\frac{\sqrt{c(a-bx)}\,(35a^6x-70a^4b^2x^3+56a^2b^4x^5-16b^6x^7)}{35a^8c^5(a-bx)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(35*a^6*x - 70*a^4*b^2*x^3 + 56*a^2*b^4*x^5 - 16*b^6*x^7
))/(35*a^8*c^5*(a - b*x)^4*(a + b*x)^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 190.34, size = 79, normalized size = 0.59

$$\frac{4\left(I\mathrm{meijerg}\left[\left\{\left\{\frac{9}{4},\frac{11}{4},1\right\},\left\{\frac{1}{2},\frac{9}{2},5\right\}\right\},\left\{\left\{\frac{9}{4},\frac{11}{4},4,\frac{9}{2},5\right\},\{0\}\right\},\frac{a^2}{b^2x^2}\right]+\mathrm{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{2},\frac{7}{4},\frac{9}{4},1\right\},\{\}\right\},\left\{\left\{\frac{7}{4},\frac{9}{4}\right\},\left\{-\frac{1}{2},0,4,0\right\}\right\},\frac{a^2\mathrm{exp\_polar}[-2I\mathrm{Pi}]}{b^2x^2}\right]\right)}{105\mathrm{Pi}^{\frac{3}{2}}a^8bc^{\frac{9}{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]')
```

```
[Out] 4 (I meijerg[{{9 / 4, 11 / 4, 1}, {1 / 2, 9 / 2, 5}}, {{9 / 4, 11 / 4, 4, 9
 / 2, 5}, {0}}, a ^ 2 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, 0, 1 / 2, 7 / 4,
 9 / 4, 1}, {}}, {{7 / 4, 9 / 4}, {-1 / 2, 0, 4, 0}}, a ^ 2 exp_polar[-2 I
Pi] / (b ^ 2 x ^ 2)]) / (105 Pi ^ (3 / 2) a ^ 8 b c ^ (9 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(109) = 218.
time = 0.15, size = 275, normalized size = 2.07

method	result

gosper	$\frac{(-bx+a)x(-16x^6b^6+56a^2x^4b^4-70a^4x^2b^2+35a^6)}{35(bx+a)^{\frac{7}{2}}a^8(-bcx+ac)^{\frac{9}{2}}}$
default	$-\frac{1}{7abc(bx+a)^{\frac{7}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{-\frac{1}{5abc(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{-\frac{2}{5abc(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{6\left(-\frac{5}{3abc\sqrt{bx+a}\,(-bcx+ac)^{\frac{7}{2}}}+\frac{5\left(\frac{4\sqrt{\cdots}}{7ab\cdots}\cdots\right)}{\cdots}\right)}{\cdots}}{\cdots}}{a}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/a/b/c/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2)+1/a*(-1/5/a/b/c/(b*x+a)^(5/2)/(
-b*c*x+a*c)^(7/2)+6/5/a*(-1/3/a/b/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(7/2)+5/3/a*
(-1/a/b/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(7/2)+4/a*(1/7/a/b/c/(-b*c*x+a*c)^(7/2
)*(b*x+a)^(1/2)+3/7/a/c*(1/5/a/b/c/(-b*c*x+a*c)^(5/2)*(b*x+a)^(1/2)+2/5/a/c
*(1/3/a/b/c/(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+1/3/b/a^2/c^2/(-b*c*x+a*c)^(1/
2)*(b*x+a)^(1/2)))))))
```

Maxima [A]
time = 0.27, size = 105, normalized size = 0.79

$$\frac{x}{7\left(-b^2cx^2+a^2c\right)^{\frac{7}{2}}a^2c}+\frac{6\,x}{35\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^4c^2}+\frac{8\,x}{35\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^6c^3}+\frac{16\,x}{35\sqrt{-b^2cx^2+a^2c}\,a^8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-b^2*c*x^2 + a^2*c)^(7/2)*a^2*c) + 6/35*x/((-b^2*c*x^2 + a^2*c)^(5/
2)*a^4*c^2) + 8/35*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^6*c^3) + 16/35*x/(sqrt(-
b^2*c*x^2 + a^2*c)*a^8*c^4)
```

Fricas [A]
time = 0.33, size = 122, normalized size = 0.92

$$-\frac{\left(16\,b^{6}x^{7}-56\,a^{2}b^{4}x^{5}+70\,a^{4}b^{2}x^{3}-35\,a^{6}x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{35\,\left(a^{8}b^{8}c^{5}x^{8}-4\,a^{10}b^{6}c^{5}x^{6}+6\,a^{12}b^{4}c^{5}x^{4}-4\,a^{14}b^{2}c^{5}x^{2}+a^{16}c^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*b^6*x^7 - 56*a^2*b^4*x^5 + 70*a^4*b^2*x^3 - 35*a^6*x)*sqrt(-b*c*x
 + a*c)*sqrt(b*x + a)/(a^8*b^8*c^5*x^8 - 4*a^10*b^6*c^5*x^6 + 6*a^12*b^4*c^
5*x^4 - 4*a^14*b^2*c^5*x^2 + a^16*c^5)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(-b*c*x+a*c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(109) = 218.
time = 0.15, size = 521, normalized size = 3.92

$$\frac{2\left(\frac{2\left(\left(\left(\left(-\frac{7046430720c^{3}a^{18}\sqrt{a+bx}\sqrt{a+bx}}{123312537600c^{4}a^{26}}+\frac{44508119040c^{3}a^{19}}{123312537600c^{4}a^{26}}\right)\sqrt{a+bx}\sqrt{a+bx}-\frac{94411161600c^{3}a^{20}}{123312537600c^{4}a^{26}}\right)\sqrt{a+bx}\sqrt{a+bx}+\frac{67436544000c^{3}a^{21}}{123312537600c^{4}a^{26}}\right)\sqrt{a+bx}\sqrt{2ac-c(a+bx)}\right)}{(2ac-c(a+bx))^{4}}-\frac{2\left(-175\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{12}+2450c\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{10}a-14280c^{2}\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{8}a^{2}+43120c^{3}\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{6}a^{3}-66416c^{4}\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{4}a^{4}+51744c^{5}\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{2}a^{5}-16384c^{6}a^{6}\right)}{1120c^{3}\sqrt{-c}\,a^{7}\left(-\left(\sqrt{2ac-c(a+bx)}-\sqrt{-c}\sqrt{a+bx}\right)^{2}+2ca\right)^{7}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x)
```

```
[Out] -1/1120*(((b*x + a)*((b*x + a)*(256*(b*x + a)/(a^8*c) - 1617/(a^7*c)) + 343
0/(a^6*c)) - 2450/(a^5*c))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)/((b*x +
 a)*c - 2*a*c)^4 + 4*(175*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^12 - 2450*a*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^10*c
 + 14280*a^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^8*c^2 -
43120*a^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*c^3 + 664
16*a^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*c^4 - 51744*
a^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c^5 + 16384*a^6
*c^6)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2 - 2*a*c)^7*
a^7*sqrt(-c)*c^3))/b
```

Mupad [B]
time = 0.71, size = 170, normalized size = 1.28

$$-\frac{35\,a^6\,x\,\sqrt{a\,c-b\,c\,x}-16\,b^6\,x^7\,\sqrt{a\,c-b\,c\,x}-70\,a^4\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}+56\,a^2\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}}{\left(\left(70\,a^9\,{\left(a\,c-b\,c\,x\right)}^5+35\,a^8\,{\left(a\,c-b\,c\,x\right)}^5\,\left(a+b\,x\right)\right)\,\left(a+b\,x\right)+{\left(a\,c-b\,c\,x\right)}^4\,\left(140\,a^{10}\,\left(a\,c-b\,c\,x\right)-280\,a^{11}\,c\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*c - b*c*x)^(9/2)*(a + b*x)^(9/2)),x)

[Out] -(35*a^6*x*(a*c - b*c*x)^(1/2) - 16*b^6*x^7*(a*c - b*c*x)^(1/2) - 70*a^4*b^2*x^3*(a*c - b*c*x)^(1/2) + 56*a^2*b^4*x^5*(a*c - b*c*x)^(1/2))/(((70*a^9*(a*c - b*c*x)^5 + 35*a^8*(a*c - b*c*x)^5*(a + b*x))*(a + b*x) + (a*c - b*c*x)^4*(140*a^10*(a*c - b*c*x) - 280*a^11*c))*(a + b*x)^(1/2))

3.1153 $\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2}+\frac{45}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] 15/2*(1-2*x)^(3/2)*x*(1+2*x)^(3/2)*6^(1/2)+45/8*arcsin(2*x)*6^(1/2)+6*(1-2*
x)^(5/2)*x*(1+2*x)^(5/2)*6^(1/2)+45/4*x*6^(1/2)*(1-2*x)^(1/2)*(1+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {38, 41, 222}

$$6\sqrt{6}(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (45*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 15*Sqrt[3/2]*(1 - 2*x)^(3/
2)*x*(1 + 2*x)^(3/2) + 6*Sqrt[6]*(1 - 2*x)^(5/2)*x*(1 + 2*x)^(5/2) + (45*Sq
rt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = 6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2} + 5\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$$
$$= 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{45}{2}\int \sqrt{3-$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2$$

Mathematica [A]
time = 1.11, size = 193, normalized size = 1.93

$$-\frac{3\sqrt{3-6x}\,x\,(33-104x^2+128x^4)\left(8119+45112x+91052x^2+80768x^3+30160x^4+3712x^5+64x^6-\sqrt{2+4x}\,(5741+26158x+41096x^2+26224x^3+6160x^4+352x^5)\right)}{2\left(11482+52316x+82192x^2+52448x^3+12320x^4+704x^5-\sqrt{2+4x}\,(8119+28874x+33304x^2+14160x^3+1840x^4+32x^5)\right)}+45\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{-\sqrt{2}+\sqrt{1+2x}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[3 - 6*x]*x*(33 - 104*x^2 + 128*x^4)*(8119 + 45112*x + 91052*x^2 +
80768*x^3 + 30160*x^4 + 3712*x^5 + 64*x^6 - Sqrt[2 + 4*x]*(5741 + 26158*x +
 41096*x^2 + 26224*x^3 + 6160*x^4 + 352*x^5)))/(2*(11482 + 52316*x + 82192*
x^2 + 52448*x^3 + 12320*x^4 + 704*x^5 - Sqrt[2 + 4*x]*(8119 + 28874*x + 333
04*x^2 + 14160*x^3 + 1840*x^4 + 32*x^5))) + 45*Sqrt[3/2]*ArcTan[(-Sqrt[2] +
 Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.18, size = 134, normalized size = 1.34

method	result

risch	$-\frac{3x(128x^4-104x^2+33)(2x-1)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{4\sqrt{-(2x-1)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{45\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{8\sqrt{2+4x}\,\sqrt{3-6x}}$
default	$\frac{(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{7}{2}}}{24}+\frac{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{7}{2}}}{8}+\frac{9\sqrt{3-6x}\,(2+4x)^{\frac{7}{2}}}{32}-\frac{3(2+4x)^{\frac{5}{2}}\sqrt{3-6x}}{16}-\frac{15(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{16}-$ 45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(5/2)*(2+4*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(3-6*x)^(5/2)*(2+4*x)^(7/2)+1/8*(3-6*x)^(3/2)*(2+4*x)^(7/2)+9/32*(3-6*
x)^(1/2)*(2+4*x)^(7/2)-3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-15/16*(2+4*x)^(3/2)
*(3-6*x)^(1/2)-45/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+45/8*((2+4*x)*(3-6*x))^(1/2
)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2)
```

Maxima [A]
time = 0.35, size = 46, normalized size = 0.46

$$\frac{1}{6}\left(-24\,x^2+6\right)^{\frac{5}{2}}x+\frac{5}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x+\frac{45}{4}\sqrt{-24\,x^2+6}\,x+\frac{45}{8}\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-24*x^2 + 6)^(5/2)*x + 5/4*(-24*x^2 + 6)^(3/2)*x + 45/4*sqrt(-24*x^2 +
 6)*x + 45/8*sqrt(6)*arcsin(2*x)
```

Fricas [A]
time = 0.30, size = 65, normalized size = 0.65

$$\frac{3}{4}\left(128\,x^5-104\,x^3+33\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{45}{8}\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 3/4*(128*x^5 - 104*x^3 + 33*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 45/8*sqrt(3)*
sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(5/2)*(4*x+2)**(5/2),x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(70) = 140.
time = 0.04, size = 690, normalized size = 6.90

$$\sqrt{3}\sqrt{2}\left(1152\left(2\left(\left(\left(\left(\left(\frac{31}{3840}-\frac{1}{768}\sqrt{-2x+1}\sqrt{-2x+1}\right)\sqrt{-2x+1}\sqrt{-2x+1}-\frac{107}{5120}\right)\sqrt{-2x+1}\sqrt{-2x+1}+\frac{451}{15360}\right)\sqrt{-2x+1}\sqrt{-2x+1}-\frac{149}{6144}\right)\sqrt{-2x+1}\sqrt{-2x+1}+\frac{27}{2048}\right)\sqrt{-2x+1}\sqrt{2x+1}+\frac{5}{512}\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)-576\left(2\left(\left(\left(\left(-\frac{21}{1280}+\frac{1}{320}\sqrt{-2x+1}\sqrt{-2x+1}\right)\sqrt{-2x+1}\sqrt{-2x+1}+\frac{133}{3840}\right)\sqrt{-2x+1}\sqrt{-2x+1}-\frac{59}{1536}\right)\sqrt{-2x+1}\sqrt{-2x+1}+\frac{13}{512}\right)\sqrt{-2x+1}\sqrt{2x+1}+\frac{3}{128}\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)-576\left(2\left(\left(\left(\frac{13}{384}-\frac{1}{128}\sqrt{-2x+1}\sqrt{-2x+1}\right)\sqrt{-2x+1}\sqrt{-2x+1}-\frac{43}{768}\right)\sqrt{-2x+1}\sqrt{-2x+1}+\frac{13}{256}\right)\sqrt{-2x+1}\sqrt{2x+1}+\frac{3}{64}\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)+288\left(2\left(\left(\frac{1}{48}\sqrt{-2x+1}\sqrt{-2x+1}-\frac{7}{96}\right)\sqrt{-2x+1}\sqrt{-2x+1}+\frac{3}{32}\right)\sqrt{-2x+1}\sqrt{2x+1}+\frac{\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{8}\right)-36\left(2\left(\frac{1}{8}\sqrt{-2x+1}\sqrt{-2x+1}-\frac{3}{8}\right)\sqrt{-2x+1}\sqrt{2x+1}-\frac{\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{2}\right)-36\left(\frac{1}{2}\sqrt{-2x+1}\sqrt{2x+1}+\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x)
```

```
[Out] 3/40*sqrt(3)*sqrt(2)*(((2*((8*(5*x + 13)*(2*x - 1) + 321)*(2*x - 1) + 451)*
(2*x - 1) + 745)*(2*x - 1) + 405)*sqrt(2*x + 1)*sqrt(-2*x + 1) - 2*((2*(3*(
8*x + 17)*(2*x - 1) + 133)*(2*x - 1) + 295)*(2*x - 1) + 195)*sqrt(2*x + 1)*
sqrt(-2*x + 1) - 20*((4*(3*x + 5)*(2*x - 1) + 43)*(2*x - 1) + 39)*sqrt(2*x
+ 1)*sqrt(-2*x + 1) + 80*((4*x + 5)*(2*x - 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
+ 1) + 240*sqrt(2*x + 1)*(x + 1)*sqrt(-2*x + 1) - 240*sqrt(2*x + 1)*sqrt(-2
*x + 1) - 150*arcsin(1/2*sqrt(2)*sqrt(-2*x + 1)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (4\,x+2)^{5/2}\,(3-6\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2),x)
```

```
[Out] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2), x)
```

3.1154 $\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$

Optimal. Leaf size=74

$$\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] 3/2*(1-2*x)^(3/2)*x*(1+2*x)^(3/2)*6^(1/2)+9/8*arcsin(2*x)*6^(1/2)+9/4*x*6^(
1/2)*(1-2*x)^(1/2)*(1+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {38, 41, 222}

$$3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (9*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 3*Sqrt[3/2]*(1 - 2*x)^(3/2)
*x*(1 + 2*x)^(3/2) + (9*Sqrt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{9}{2}\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{27}{2}\int \frac{}{\sqrt{3-6}}$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{27}{2}\int \frac{1}{\sqrt{6-2}}$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{9}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(74) = 148.
time = 0.81, size = 179, normalized size = 2.42

$$\frac{-12\sqrt{6}\,\sqrt{1-2x}\,x\sqrt{1+2x}\,(-5+8x^2)\,(-169-490x-364x^2-56x^3)-12\sqrt{3}\,\sqrt{1-2x}\,x\,(-5+8x^2)\,(239+932x+1088x^2+368x^3+16x^4)}{-2704-7840x-5824x^2-896x^3+\sqrt{2}\,\sqrt{1+2x}\,(1912+3632x+1440x^2+64x^3)}+9\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{-\sqrt{2}+\sqrt{1+2x}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (-12*Sqrt[6]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x]*(-5 + 8*x^2)*(-169 - 490*x - 364
*x^2 - 56*x^3) - 12*Sqrt[3]*Sqrt[1 - 2*x]*x*(-5 + 8*x^2)*(239 + 932*x + 108
8*x^2 + 368*x^3 + 16*x^4))/(-2704 - 7840*x - 5824*x^2 - 896*x^3 + Sqrt[2]*S
qrt[1 + 2*x]*(1912 + 3632*x + 1440*x^2 + 64*x^3)) + 9*Sqrt[3/2]*ArcTan[(-Sq
rt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.
time = 0.14, size = 102, normalized size = 1.38

method	result

default	$\frac{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}}{16}+\frac{3(2+4x)^{\frac{5}{2}}\sqrt{3-6x}}{16}-\frac{3(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{16}-\frac{9\sqrt{3-6x}\,\sqrt{2+4x}}{8}+\frac{9\sqrt{(2+4x)\,(3-}}{8\sqrt{2+4x}}$
risch	$\frac{3x(8x^2-5)(2x-1)(1+2x)\sqrt{(2+4x)\,(3-6x)}\,\sqrt{6}}{4\sqrt{-(2x-1)\,(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{9\sqrt{(2+4x)\,(3-6x)}\,\arcsin(2x)\sqrt{6}}{8\sqrt{2+4x}\,\sqrt{3-6x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(3/2)*(2+4*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(3-6*x)^(3/2)*(2+4*x)^(5/2)+3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-3/16*(2+4
*x)^(3/2)*(3-6*x)^(1/2)-9/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+9/8*((2+4*x)*(3-6*x
))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2)
```

Maxima [A]
time = 0.39, size = 34, normalized size = 0.46

$$\frac{1}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x+\frac{9}{4}\,\sqrt{-24\,x^2+6}\,x+\frac{9}{8}\,\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-24*x^2 + 6)^(3/2)*x + 9/4*sqrt(-24*x^2 + 6)*x + 9/8*sqrt(6)*arcsin(2*
x)
```

Fricas [A]
time = 0.29, size = 60, normalized size = 0.81

$$-\frac{3}{4}\left(8\,x^3-5\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{9}{8}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -3/4*(8*x^3 - 5*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 9/8*sqrt(3)*sqrt(2)*arcta
n(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(3/2)*(4*x+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(50) = 100$.
time = 0.02, size = 358, normalized size = 4.84

$$\sqrt{3}\,\sqrt{2}\left(-48\left(2\left(\left(\left(\left(\frac{13}{384}-\frac{1}{128}\sqrt{-2x+1}\,\sqrt{-2x+1}\right)\sqrt{-2x+1}\,\sqrt{-2x+1}-\frac{43}{768}\right)\sqrt{-2x+1}\,\sqrt{-2x+1}+\frac{13}{256}\right)\sqrt{-2x+1}\,\sqrt{2x+1}+\frac{3}{64}\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)\right)+24\left(2\left(\left(\left(\frac{1}{48}\sqrt{-2x+1}\,\sqrt{-2x+1}-\frac{7}{96}\right)\sqrt{-2x+1}\,\sqrt{-2x+1}+\frac{3}{32}\right)\sqrt{-2x+1}\,\sqrt{2x+1}+\frac{\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{8}\right)\right)-6\left(2\left(\frac{1}{8}\sqrt{-2x+1}\,\sqrt{-2x+1}-\frac{3}{8}\right)\sqrt{-2x+1}\,\sqrt{2x+1}-\frac{\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{2}\right)-6\left(\frac{1}{2}\sqrt{-2x+1}\,\sqrt{2x+1}+\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x)
```

```
[Out] -1/8*sqrt(3)*sqrt(2)*(((4*(3*x + 5)*(2*x - 1) + 43)*(2*x - 1) + 39)*sqrt(2*
x + 1)*sqrt(-2*x + 1) - 4*((4*x + 5)*(2*x - 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
 + 1) - 24*sqrt(2*x + 1)*(x + 1)*sqrt(-2*x + 1) + 24*sqrt(2*x + 1)*sqrt(-2*
x + 1) + 18*arcsin(1/2*sqrt(2)*sqrt(-2*x + 1)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (4\,x+2)^{3/2}\,(3-6\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2),x)
```

```
[Out] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2), x)
```

3.1155 $\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

[Out] 1/4*arcsin(2*x)*6^(1/2)+1/2*x*6^(1/2)*(1-2*x)^(1/2)*(1+2*x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {38, 41, 222}

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\,\sqrt{2x+1}\,x+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]

[Out] Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + (Sqrt[3/2]*ArcSin[2*x])/2

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx &= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}+3\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx \\
&= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}+3\int \frac{1}{\sqrt{6-24x^2}}\,dx \\
&= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}+\frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.
time = 0.66, size = 108, normalized size = 2.51

$$
\frac{\sqrt{3-6x}\,x\left(7+4x^2-5\sqrt{2+4x}+x\left(16-6\sqrt{2+4x}\right)\right)}{-10+7\sqrt{2+4x}+2x\left(-6+\sqrt{2+4x}\right)}+\sqrt{6}\,\tan^{-1}\left(\frac{-\sqrt{2}+\sqrt{1+2x}}{\sqrt{1-2x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (Sqrt[3 - 6*x]*x*(7 + 4*x^2 - 5*Sqrt[2 + 4*x] + x*(16 - 6*Sqrt[2 + 4*x])))/
(-10 + 7*Sqrt[2 + 4*x] + 2*x*(-6 + Sqrt[2 + 4*x])) + Sqrt[6]*ArcTan[(-Sqrt[
2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.33, size = 129, normalized size = 3.00

$$
\text{Piecewise}\left[\left\{\left\{\frac{I\sqrt{6}\left(-x\sqrt{1+2x}+2x^2\sqrt{1+2x}-\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{1+2x}}{2}\right]\sqrt{-1+2x}\right)}{2\sqrt{-1+2x}},\text{Abs}\left[\frac{1}{2}+x\right]>1\right\}\right\},-\frac{\sqrt{6}\left(\frac{1}{2}+x\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}}-\frac{\sqrt{6}\sqrt{\frac{1}{2}+x}}{2\sqrt{\frac{1}{2}-x}}+\frac{\sqrt{6}\,\text{ArcSin}\left[\sqrt{\frac{1}{2}+x}\right]}{2}+\frac{3\sqrt{6}\left(\frac{1}{2}+x\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}}\right]
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - 6*x)^(1/2)*(2 + 4*x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 2 Sqrt[6] (-x Sqrt[1 + 2 x] + 2 x ^ 2 Sqrt[1 + 2 x] - ArcCo
sh[Sqrt[2] Sqrt[1 + 2 x] / 2] Sqrt[-1 + 2 x]) / Sqrt[-1 + 2 x], Abs[1 / 2 +
 x] > 1}}, -Sqrt[6] (1 / 2 + x) ^ (5 / 2) / Sqrt[1 / 2 - x] - Sqrt[6] Sqrt[
1 / 2 + x] / (2 Sqrt[1 / 2 - x]) + Sqrt[6] ArcSin[Sqrt[1 / 2 + x]] / 2 + 3
Sqrt[6] (1 / 2 + x) ^ (3 / 2) / (2 Sqrt[1 / 2 - x])]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 0.18, size = 70, normalized size = 1.63

method	result	size
default	$\frac{(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{8}-\frac{\sqrt{3-6x}\,\sqrt{2+4x}}{4}+\frac{\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{4\sqrt{2+4x}\,\sqrt{3-6x}}$	70
risch	$-\frac{x(2x-1)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{2\sqrt{-(2x-1)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{4\sqrt{2+4x}\,\sqrt{3-6x}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(1/2)*(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2+4*x)^(3/2)*(3-6*x)^(1/2)-1/4*(3-6*x)^(1/2)*(2+4*x)^(1/2)+1/4*((2+4*x
)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2)
```

Maxima [A]
time = 0.35, size = 22, normalized size = 0.51

$$\frac{1}{2}\sqrt{-24\,x^2+6}\,x+\frac{1}{4}\sqrt{6}\arcsin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-24*x^2 + 6)*x + 1/4*sqrt(6)*arcsin(2*x)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 1.21

$$\frac{1}{2}\sqrt{4\,x+2}\,x\sqrt{-6\,x+3}-\frac{1}{4}\sqrt{3}\,\sqrt{2}\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x + 2)*x*sqrt(-6*x + 3) - 1/4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3
)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.15, size = 187, normalized size = 4.35

$$\begin{cases} -\frac{\sqrt{6}\,i\,\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{2}+\frac{\sqrt{6}\,i\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{x-\frac{1}{2}}}-\frac{3\sqrt{6}\,i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{x-\frac{1}{2}}}+\frac{\sqrt{6}\,i\sqrt{x+\frac{1}{2}}}{2\sqrt{x-\frac{1}{2}}} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ \frac{\sqrt{6}\,\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{2}-\frac{\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}}+\frac{3\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}}-\frac{\sqrt{6}\sqrt{x+\frac{1}{2}}}{2\sqrt{\frac{1}{2}-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(1/2)*(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/2 + sqrt(6)*I*(x + 1/2)**(5/2)/s
qrt(x - 1/2) - 3*sqrt(6)*I*(x + 1/2)**(3/2)/(2*sqrt(x - 1/2)) + sqrt(6)*I*s
qrt(x + 1/2)/(2*sqrt(x - 1/2)), Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x + 1
/2))/2 - sqrt(6)*(x + 1/2)**(5/2)/sqrt(1/2 - x) + 3*sqrt(6)*(x + 1/2)**(3/2
)/(2*sqrt(1/2 - x)) - sqrt(6)*sqrt(x + 1/2)/(2*sqrt(1/2 - x)), True))
```

Giac [A]
time = 0.01, size = 125, normalized size = 2.91

$$\sqrt{3}\,\sqrt{2}\left(-2\left(\frac{1}{8}\sqrt{-2x+1}\,\sqrt{-2x+1}-\frac{3}{8}\right)\sqrt{-2x+1}\,\sqrt{2x+1}+\frac{\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{2}-\frac{1}{2}\sqrt{-2x+1}\,\sqrt{2x+1}-\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x)
```

```
[Out] 1/2*sqrt(3)*sqrt(2)*(sqrt(2*x + 1)*(x + 1)*sqrt(-2*x + 1) - sqrt(2*x + 1)*s
qrt(-2*x + 1) - arcsin(1/2*sqrt(2)*sqrt(-2*x + 1)))
```

Mupad [B]
time = 0.26, size = 44, normalized size = 1.02

$$\frac{x\,\sqrt{4\,x+2}\,\sqrt{3-6\,x}}{2}-\frac{\sqrt{6}\,\ln\left(x-\frac{\sqrt{1-2\,x}\,\sqrt{2\,x+1}\,1{}\mathrm{i}}{2}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(1/2)*(3 - 6*x)^(1/2),x)
```

```
[Out] (x*(4*x + 2)^(1/2)*(3 - 6*x)^(1/2))/2 - (6^(1/2)*log(x - ((1 - 2*x)^(1/2)*(
2*x + 1)^(1/2)*1i)/2)*1i)/4
```

3.1156 $\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx$

Optimal. Leaf size=13

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

```
[Out] 1/12*arcsin(2*x)*6^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {41, 222}

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] ArcSin[2*x]/(2*Sqrt[6])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx &= \int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \frac{\sin^{-1}(2x)}{2\sqrt{6}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

time = 0.04, size = 27, normalized size = 2.08

$$-\frac{\tan^{-1}\left(\frac{\sqrt{1-4x^2}}{1+2x}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] -(ArcTan[Sqrt[1 - 4*x^2]/(1 + 2*x)]/Sqrt[6])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.80, size = 38, normalized size = 2.92

$$\text{Piecewise}\left[\left\{\left\{\left(-\frac{I}{6}\right)\sqrt{6}\,\text{ArcCosh}\left[\frac{\sqrt{2}\,\sqrt{1+2x}}{2}\right],\text{Abs}\left[\frac{1}{2}+x\right]>1\right\}\right\},\frac{\sqrt{6}\,\text{ArcSin}\left[\sqrt{\frac{1}{2}+x}\right]}{6}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - 6*x)^(1/2)*(2 + 4*x)^(1/2)),x]')
```

```
[Out] Piecewise[{{(-I / 6) Sqrt[6] ArcCosh[Sqrt[2] Sqrt[1 + 2 x] / 2], Abs[1 / 2
+ x] > 1}}, Sqrt[6] ArcSin[Sqrt[1 / 2 + x]] / 6]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(9) = 18$.
time = 0.16, size = 37, normalized size = 2.85

method	result	size
default	$\frac{\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{12\sqrt{2+4x}\,\sqrt{3-6x}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((2+4*x)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2
)
```

Maxima [A]
time = 0.35, size = 9, normalized size = 0.69

$$\frac{1}{12}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*arcsin(2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(9) = 18.
time = 0.30, size = 28, normalized size = 2.15

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="fricas")
[Out] -1/12*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [A]
time = 1.06, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{\sqrt{6}i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{for } \left|x+\frac{1}{2}\right|>1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(1/2)/(4*x+2)**(1/2),x)
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/6, Abs(x + 1/2) > 1), (sqrt(6)*a
sin(sqrt(x + 1/2))/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.00, size = 32, normalized size = 2.46

$$-\frac{\arcsin\left(\frac{\sqrt{-2x+1}}{\sqrt{2}}\right)}{\sqrt{3}\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x)
[Out] -1/6*sqrt(3)*sqrt(2)*arcsin(1/2*sqrt(2)*sqrt(-2*x + 1))
```

Mupad [B]
time = 0.05, size = 40, normalized size = 3.08

$$-\frac{\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{24}\left(\sqrt{3}-\sqrt{3-6x}\right)}{6\left(\sqrt{2}-\sqrt{4x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(1/2)*(3 - 6*x)^(1/2)),x)
[Out] -(6^(1/2)*atan((24^(1/2)*(3^(1/2) - (3 - 6*x)^(1/2)))/(6*(2^(1/2) - (4*x +
2)^(1/2)))))/3
```

3.1157 $\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}} dx$

Optimal. Leaf size=28

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}$$

[Out] 1/36*x*6^(1/2)/(1-2*x)^(1/2)/(1+2*x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{39\}$

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]

[Out] x/(6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}} dx = \frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.
time = 0.69, size = 59, normalized size = 2.11

$$\frac{x\left(3+2x-2\sqrt{2+4x}\right)}{6\sqrt{3-6x}\left(-4+3\sqrt{2+4x}+2x\left(-4+\sqrt{2+4x}\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]

```
[Out] (x*(3 + 2*x - 2*Sqrt[2 + 4*x]))/(6*Sqrt[3 - 6*x]*(-4 + 3*Sqrt[2 + 4*x] + 2*
x*(-4 + Sqrt[2 + 4*x])))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 37.42, size = 90, normalized size = 3.21

$$\text{Piecewise}\left[\left\{\left\{\frac{\left(-\frac{I}{36}\right)\sqrt{6}\,x}{\sqrt{-1+2x}\,\sqrt{1+2x}},\text{Abs}\left[\frac{1}{2}+x\right]>1\right\}\right\},\frac{-2\sqrt{6}\,\left(\frac{1}{2}+x\right)\sqrt{\frac{1}{2}-x}}{-144\sqrt{\frac{1}{2}+x}+144\left(\frac{1}{2}+x\right)^{\frac{3}{2}}}+\frac{\sqrt{6}\,\sqrt{\frac{1}{2}-x}}{-144\sqrt{\frac{1}{2}+x}+144\left(\frac{1}{2}+x\right)^{\frac{3}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]')
[Out] Piecewise[{{(-I / 36) Sqrt[6] x / (Sqrt[-1 + 2 x] Sqrt[1 + 2 x]), Abs[1 / 2
 + x] > 1}}, -2 Sqrt[6] (1 / 2 + x) Sqrt[1 / 2 - x] / (-144 Sqrt[1 / 2 + x]
 + 144 (1 / 2 + x) ^ (3 / 2)) + Sqrt[6] Sqrt[1 / 2 - x] / (-144 Sqrt[1 / 2
+ x] + 144 (1 / 2 + x) ^ (3 / 2))]
```

Maple [A]
time = 0.15, size = 34, normalized size = 1.21

method	result	size
gosper	$-\frac{(2x-1)(1+2x)x}{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}$	28
default	$\frac{1}{12\sqrt{3-6x}\,\sqrt{2+4x}}-\frac{\sqrt{3-6x}}{36\sqrt{2+4x}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/12/(3-6*x)^(1/2)/(2+4*x)^(1/2)-1/36/(2+4*x)^(1/2)*(3-6*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.43

$$\frac{x}{6\sqrt{-24\,x^2+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="maxima")
[Out] 1/6*x/sqrt(-24*x^2 + 6)
```

Fricas [A]
time = 0.29, size = 26, normalized size = 0.93

$$-\frac{\sqrt{4\,x+2}\,x\sqrt{-6\,x+3}}{36\,(4\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 44.74, size = 156, normalized size = 5.57

$$\begin{cases} -\frac{2\sqrt{6}\,i\sqrt{x-\frac{1}{2}}\,(x+\frac{1}{2})}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\,i\sqrt{x-\frac{1}{2}}}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{for } \left|x+\frac{1}{2}\right|>1 \\ -\frac{2\sqrt{6}\,\sqrt{\frac{1}{2}-x}\,(x+\frac{1}{2})}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\,\sqrt{\frac{1}{2}-x}}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(3/2)/(4*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(6)*I*sqrt(x - 1/2)*(x + 1/2)/(144*(x + 1/2)**(3/2) - 144
*sqrt(x + 1/2)) + sqrt(6)*I*sqrt(x - 1/2)/(144*(x + 1/2)**(3/2) - 144*sqrt(
x + 1/2)), Abs(x + 1/2) > 1), (-2*sqrt(6)*sqrt(1/2 - x)*(x + 1/2)/(144*(x +
 1/2)**(3/2) - 144*sqrt(x + 1/2)) + sqrt(6)*sqrt(1/2 - x)/(144*(x + 1/2)**(
3/2) - 144*sqrt(x + 1/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(20) = 40.
time = 0.00, size = 112, normalized size = 4.00

$$\frac{\frac{\sqrt{-2x+1}}{24\left(-2\sqrt{2x+1}+2\sqrt{2}\right)}-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{96\sqrt{-2x+1}}-\frac{\sqrt{-2x+1}\,\sqrt{2x+1}}{24(2x+1)}}{\sqrt{3}\,\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x)
```

```
[Out] -1/288*sqrt(3)*sqrt(2)*((sqrt(2) - sqrt(2*x + 1))/sqrt(-2*x + 1) + 2*sqrt(-
2*x + 1)/sqrt(2*x + 1) - sqrt(-2*x + 1)/(sqrt(2) - sqrt(2*x + 1)))
```

Mupad [B]
time = 0.46, size = 24, normalized size = 0.86

$$-\frac{x\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(36\,x-18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(3/2)*(3 - 6*x)^(3/2)),x)
```

```
[Out] -(x*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(36*x - 18))
```

3.1158 $\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}$$

```
[Out] 1/648*x/(1-2*x)^(3/2)/(1+2*x)^(3/2)*6^(1/2)+1/324*x*6^(1/2)/(1-2*x)^(1/2)/(
1+2*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] x/(108*Sqrt[6]*(1 - 2*x)^(3/2)*(1 + 2*x)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 2*
x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx &= \frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{1}{9}\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx\\ &= \frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}\end{aligned}$$

Mathematica [A]
time = 0.80, size = 107, normalized size = 1.88

$$\frac{x\left(-3+8x^2\right)\left(99+8x^3-70\sqrt{2+4x}+x\left(246-104\sqrt{2+4x}\right)+x^2\left(132-24\sqrt{2+4x}\right)\right)}{54\sqrt{3-6x}\,(-1+2x)\left(-4+3\sqrt{2+4x}+2x\left(-4+\sqrt{2+4x}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] (x*(-3 + 8*x^2)*(99 + 8*x^3 - 70*Sqrt[2 + 4*x] + x*(246 - 104*Sqrt[2 + 4*x]
) + x^2*(132 - 24*Sqrt[2 + 4*x])))/(54*Sqrt[3 - 6*x]*(-1 + 2*x)*(-4 + 3*Sqr
t[2 + 4*x] + 2*x*(-4 + Sqrt[2 + 4*x]))^3)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Maple [A]
time = 0.15, size = 66, normalized size = 1.16

method	result	size
gosper	$\frac{(2x-1)(1+2x)x(8x^2-3)}{3(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}$	35
default	$\frac{1}{36(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}+\frac{1}{36\sqrt{3-6x}\,(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{162(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{324\sqrt{2+4x}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36/(3-6*x)^(3/2)/(2+4*x)^(3/2)+1/36/(3-6*x)^(1/2)/(2+4*x)^(3/2)-1/162/(2+
4*x)^(3/2)*(3-6*x)^(1/2)-1/324/(2+4*x)^(1/2)*(3-6*x)^(1/2)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.44

$$\frac{x}{54\sqrt{-24x^2+6}}+\frac{x}{18\left(-24x^2+6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/54*x/sqrt(-24*x^2 + 6) + 1/18*x/(-24*x^2 + 6)^(3/2)
```

Fricas [A]
time = 0.29, size = 39, normalized size = 0.68

$$-\frac{(8\,x^3-3\,x)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{648\,(16\,x^4-8\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/648*(8*x^3 - 3*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(16*x^4 - 8*x^2 + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(5/2)/(4*x+2)**(5/2),x)
```

```
[Out] Exception raised: SystemError
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(41) = 82.
time = 0.01, size = 232, normalized size = 4.07

$$-\frac{\frac{-7077888\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^3+\frac{116785152\left(-2\sqrt{2x+1}+2\sqrt{2}\right)}{\sqrt{-2x+1}}}{97844723712}-\frac{33\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^2+1}{13824\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^3}-\frac{2\left(\frac{1}{192}-\frac{1}{432}\sqrt{-2x+1}\,\sqrt{-2x+1}\right)\sqrt{-2x+1}\,\sqrt{2x+1}}{(2x+1)^2}}{\sqrt{3}\,\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x)
```

```
[Out] -1/82944*sqrt(3)*sqrt(2)*((sqrt(2) - sqrt(2*x + 1))^3/(-2*x + 1)^(3/2) + 33
*(sqrt(2) - sqrt(2*x + 1))/sqrt(-2*x + 1) + 16*(8*x + 5)*sqrt(-2*x + 1)/(2*
x + 1)^(3/2) + (-2*x + 1)^(3/2)*(33*(sqrt(2) - sqrt(2*x + 1))^2/(2*x - 1) -
 1)/(sqrt(2) - sqrt(2*x + 1))^3)
```

Mupad [B]
time = 0.31, size = 49, normalized size = 0.86

$$-\frac{3\,x\,\sqrt{3-6\,x}-8\,x^3\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(-2592\,x^3+1296\,x^2+648\,x-324\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(5/2)*(3 - 6*x)^(5/2)),x)
```

```
[Out] -(3*x*(3 - 6*x)^(1/2) - 8*x^3*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(648*x + 12
96*x^2 - 2592*x^3 - 324))
```

3.1159 $\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}} dx$

Optimal. Leaf size=85

$$\frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{405\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}$$

```
[Out] 1/6480*x/(1-2*x)^(5/2)/(1+2*x)^(5/2)*6^(1/2)+1/4860*x/(1-2*x)^(3/2)/(1+2*x)
^(3/2)*6^(1/2)+1/2430*x*6^(1/2)/(1-2*x)^(1/2)/(1+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {40, 39}

$$\frac{x}{405\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}(1-2x)^{5/2}(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] x/(1080*Sqrt[6]*(1 - 2*x)^(5/2)*(1 + 2*x)^(5/2)) + x/(810*Sqrt[6]*(1 - 2*x)
^(3/2)*(1 + 2*x)^(3/2)) + x/(405*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}} dx &= \frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{2}{15}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}} dx\\ &= \frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{2}{135}\int\\ &= \frac{x}{1080\sqrt{6}(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{}{405\sqrt{6}}\end{aligned}$$

Mathematica [A]
time = 1.05, size = 149, normalized size = 1.75

$$\frac{x\left(15-80x^2+128x^4\right)\left(3363+32x^5-2378\sqrt{2+4x}+x\left(13930-7472\sqrt{2+4x}\right)-80x^4\left(-19+2\sqrt{2+4x}\right)-80x^3\left(-121+28\sqrt{2+4x}\right)-8x^2\left(-2375+894\sqrt{2+4x}\right)\right)}{810\sqrt{3-6x}\,(1-2x)^2\left(-4+3\sqrt{2+4x}+2x\left(-4+\sqrt{2+4x}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] (x*(15 - 80*x^2 + 128*x^4)*(3363 + 32*x^5 - 2378*Sqrt[2 + 4*x] + x*(13930 -
 7472*Sqrt[2 + 4*x]) - 80*x^4*(-19 + 2*Sqrt[2 + 4*x]) - 80*x^3*(-121 + 28*S
qrt[2 + 4*x]) - 8*x^2*(-2375 + 894*Sqrt[2 + 4*x])))/(810*Sqrt[3 - 6*x]*(1 -
 2*x)^2*(-4 + 3*Sqrt[2 + 4*x] + 2*x*(-4 + Sqrt[2 + 4*x]))^5)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 98, normalized size = 1.15

method	result
gosper	$-\frac{(2x-1)(1+2x)x(128x^4-80x^2+15)}{15(3-6x)^{\frac{7}{2}}(2+4x)^{\frac{7}{2}}}$
default	$\frac{1}{60(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}+\frac{1}{108(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}}+\frac{1}{81\sqrt{3-6x}\,(2+4x)^{\frac{5}{2}}}-\frac{\sqrt{3-6x}}{405(2+4x)^{\frac{5}{2}}}-\frac{\sqrt{3-6x}}{1215(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{2430\sqrt{2+4x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/(3-6*x)^(5/2)/(2+4*x)^(5/2)+1/108/(3-6*x)^(3/2)/(2+4*x)^(5/2)+1/81/(3-
6*x)^(1/2)/(2+4*x)^(5/2)-1/405/(2+4*x)^(5/2)*(3-6*x)^(1/2)-1/1215/(2+4*x)^(
3/2)*(3-6*x)^(1/2)-1/2430/(2+4*x)^(1/2)*(3-6*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.44

$$\frac{x}{405\sqrt{-24\,x^2+6}}+\frac{x}{135\left(-24\,x^2+6\right)^{\frac{3}{2}}}+\frac{x}{30\left(-24\,x^2+6\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/405*x/sqrt(-24*x^2 + 6) + 1/135*x/(-24*x^2 + 6)^(3/2) + 1/30*x/(-24*x^2 +
 6)^(5/2)
```

Fricas [A]
time = 0.29, size = 49, normalized size = 0.58

$$-\frac{(128\,x^5-80\,x^3+15\,x)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{19440\,(64\,x^6-48\,x^4+12\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/19440*(128*x^5 - 80*x^3 + 15*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(64*x^6 - 4
8*x^4 + 12*x^2 - 1)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(7/2)/(4*x+2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(61) = 122.
time = 0.02, size = 339, normalized size = 3.99

$$\frac{\frac{\frac{1}{5}\cdot 382943598331104602357 76\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^5+217001372387625941336064\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^3-\frac{271889954815084267674 0096\left(-2\sqrt{2x+1}+2\sqrt{2}\right)}{\sqrt{-2x+1}}}{16940199370653408073579757568}+\frac{-2130\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^4-85\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^2-3}{6635520\left(-\frac{-2\sqrt{2x+1}+2\sqrt{2}}{2\sqrt{-2x+1}}\right)^5}+\frac{2\left(\left(\frac{55}{41472}-\frac{1}{3240}\sqrt{-2x+1}\sqrt{-2x+1}\right)\sqrt{-2x+1}\sqrt{-2x+1}-\frac{5}{3456}\right)\sqrt{-2x+1}\sqrt{2x+1}}{(2x+1)^3}}{\sqrt{3}\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x)
```

```
[Out] -1/39813120*sqrt(3)*sqrt(2)*(3*(sqrt(2) - sqrt(2*x + 1))^5/(-2*x + 1)^(5/2)
 + 85*(sqrt(2) - sqrt(2*x + 1))^3/(-2*x + 1)^(3/2) + 2130*(sqrt(2) - sqrt(2
*x + 1))/sqrt(-2*x + 1) + 64*((128*x + 211)*(2*x - 1) + 300)*sqrt(-2*x + 1)
/(2*x + 1)^(5/2) - (2130*(sqrt(2) - sqrt(2*x + 1))^4/(2*x - 1)^2 - 85*(sqrt
(2) - sqrt(2*x + 1))^2/(2*x - 1) + 3)*(-2*x + 1)^(5/2)/(sqrt(2) - sqrt(2*x
+ 1))^5)
```

Mupad [B]
time = 0.45, size = 66, normalized size = 0.78

$$-\frac{15\,x\,\sqrt{3-6\,x}-80\,x^3\,\sqrt{3-6\,x}+128\,x^5\,\sqrt{3-6\,x}}{\left((6\,x-3)\,\left(240\,x+360\right)+1440\right)\,\sqrt{4\,x+2}\,{\left(6\,x-3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(7/2)*(3 - 6*x)^(7/2)),x)
```

```
[Out] -(15*x*(3 - 6*x)^(1/2) - 80*x^3*(3 - 6*x)^(1/2) + 128*x^5*(3 - 6*x)^(1/2))/
(((6*x - 3)*(240*x + 360) + 1440)*(4*x + 2)^(1/2)*(6*x - 3)^3)
```

3.1160 $\int (3-x)^{3/2}(-2+x)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{3}{64}\sqrt{3-x}\,\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}-\frac{3}{128}\sin^{-1}(5-2$$

[Out] -1/4*(3-x)^(5/2)*(-2+x)^(3/2)+3/128*arcsin(-5+2*x)+1/32*(3-x)^(3/2)*(-2+x)^(1/2)-1/8*(3-x)^(5/2)*(-2+x)^(1/2)+3/64*(3-x)^(1/2)*(-2+x)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {52, 55, 633, 222}

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}\,(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}\,(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\,\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

[In] Int[(3 - x)^(3/2)*(-2 + x)^(3/2),x]

[Out] (3*Sqrt[3 - x]*Sqrt[-2 + x])/64 + ((3 - x)^(3/2)*Sqrt[-2 + x])/32 - ((3 - x)^(5/2)*Sqrt[-2 + x])/8 - ((3 - x)^(5/2)*(-2 + x)^(3/2))/4 - (3*ArcSin[5 - 2*x])/128

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int (3-x)^{3/2}(-2+x)^{3/2}\,dx = -\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{8}\int (3-x)^{3/2}\sqrt{-2+x}\,dx$$
$$= -\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{1}{16}\int \frac{(3-x)^{3/2}}{\sqrt{-2+x}}\,dx$$
$$= \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{6\cdots}$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(\cdots$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(\cdots$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(\cdots$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(\cdots$$

Mathematica [A]
time = 0.08, size = 82, normalized size = 0.90

$$-\frac{\sqrt{-6+5x-x^2}\left(\sqrt{-2+x}\,(675-1095x+650x^2-168x^3+16x^4)+3\sqrt{-3+x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-3+x}{-2+x}}}\right)\right)}{64(-3+x)\sqrt{-2+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
[Out] -1/64*(Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(675 - 1095*x + 650*x^2 - 168*x^3
 + 16*x^4) + 3*Sqrt[-3 + x]*ArcTanh[1/Sqrt[(-3 + x)/(-2 + x)]]))/((-3 + x)*
Sqrt[-2 + x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 13.98, size = 144, normalized size = 1.58

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-26(-2+x)^{\frac{5}{2}}-16(-2+x)^{\frac{9}{2}}-3\text{ArcCosh}\left[\sqrt{-2+x}\right]\sqrt{-3+x}-(-2+x)^{\frac{3}{2}}+3\sqrt{-2+x}+40(-2+x)^{\frac{7}{2}}\right)}{64\sqrt{-3+x}},\text{Abs}[-2+x]>1\right\}\right\},\frac{-5(-2+x)^{\frac{7}{2}}}{8\sqrt{3-x}}-\frac{3\sqrt{-2+x}}{64\sqrt{3-x}}+\frac{(-2+x)^{\frac{3}{2}}}{64\sqrt{3-x}}+\frac{3\text{ArcSin}\left[\sqrt{-2+x}\right]}{64}+\frac{(-2+x)^{\frac{9}{2}}}{4\sqrt{3-x}}+\frac{13(-2+x)^{\frac{5}{2}}}{32\sqrt{3-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - x)^(3/2)*(-2 + x)^(3/2),x]')
```

```
[Out] Piecewise[{{I / 64 (-26 (-2 + x) ^ (5 / 2) - 16 (-2 + x) ^ (9 / 2) - 3 ArcC
osh[Sqrt[-2 + x]] Sqrt[-3 + x] - (-2 + x) ^ (3 / 2) + 3 Sqrt[-2 + x] + 40 (
-2 + x) ^ (7 / 2)) / Sqrt[-3 + x], Abs[-2 + x] > 1}}, -5 (-2 + x) ^ (7 / 2)
 / (8 Sqrt[3 - x]) - 3 Sqrt[-2 + x] / (64 Sqrt[3 - x]) + (-2 + x) ^ (3 / 2)
 / (64 Sqrt[3 - x]) + 3 ArcSin[Sqrt[-2 + x]] / 64 + (-2 + x) ^ (9 / 2) / (4
 Sqrt[3 - x]) + 13 (-2 + x) ^ (5 / 2) / (32 Sqrt[3 - x])]
```

Maple [A]
time = 0.16, size = 89, normalized size = 0.98

method	result
risch	$\frac{(16x^3-120x^2+290x-225)(-3+x)\sqrt{-2+x}\,\sqrt{(-2+x)(3-x)}}{64\sqrt{-(-3+x)(-2+x)}\,\sqrt{3-x}}+\frac{3\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{128\sqrt{-2+x}\,\sqrt{3-x}}$
default	$\frac{(3-x)^{\frac{3}{2}}(-2+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{3-x}\,(-2+x)^{\frac{5}{2}}}{8}-\frac{\sqrt{3-x}\,(-2+x)^{\frac{3}{2}}}{32}-\frac{3\sqrt{3-x}\,\sqrt{-2+x}}{64}+\frac{3\sqrt{(-2+x)(3-}}{128\sqrt{-2+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(3/2)*(-2+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(3-x)^(3/2)*(-2+x)^(5/2)+1/8*(3-x)^(1/2)*(-2+x)^(5/2)-1/32*(3-x)^(1/2)*
(-2+x)^(3/2)-3/64*(3-x)^(1/2)*(-2+x)^(1/2)+3/128*((-2+x)*(3-x))^(1/2)/(-2+x
)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A]
time = 0.38, size = 67, normalized size = 0.74

$$\frac{1}{4}\left(-x^2+5x-6\right)^{\frac{3}{2}}x-\frac{5}{8}\left(-x^2+5x-6\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^2+5x-6}\,x-\frac{15}{64}\sqrt{-x^2+5x-6}+\frac{3}{128}\arcsin\left(2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 5*x - 6)^(3/2)*x - 5/8*(-x^2 + 5*x - 6)^(3/2) + 3/32*sqrt(-x^2
+ 5*x - 6)*x - 15/64*sqrt(-x^2 + 5*x - 6) + 3/128*arcsin(2*x - 5)
```

Fricas [A]
time = 0.30, size = 62, normalized size = 0.68

$$-\frac{1}{64}\left(16x^3-120x^2+290x-225\right)\sqrt{x-2}\,\sqrt{-x+3}-\frac{3}{128}\arctan\left(\frac{(2x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2\left(x^2-5x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 120*x^2 + 290*x - 225)*sqrt(x - 2)*sqrt(-x + 3) - 3/128*arc
tan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [A]
time = 12.09, size = 199, normalized size = 2.19

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\sqrt{x-2}\right)}{64} - \frac{i(x-2)^{\frac{9}{2}}}{4\sqrt{x-3}} + \frac{5i(x-2)^{\frac{7}{2}}}{8\sqrt{x-3}} - \frac{13i(x-2)^{\frac{5}{2}}}{32\sqrt{x-3}} - \frac{i(x-2)^{\frac{3}{2}}}{64\sqrt{x-3}} + \frac{3i\sqrt{x-2}}{64\sqrt{x-3}} & \text{for}\ |x-2| > 1 \\ \frac{3\operatorname{asin}\left(\sqrt{x-2}\right)}{64} + \frac{(x-2)^{\frac{9}{2}}}{4\sqrt{3-x}} - \frac{5(x-2)^{\frac{7}{2}}}{8\sqrt{3-x}} + \frac{13(x-2)^{\frac{5}{2}}}{32\sqrt{3-x}} + \frac{(x-2)^{\frac{3}{2}}}{64\sqrt{3-x}} - \frac{3\sqrt{x-2}}{64\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(3/2)*(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(x - 2))/64 - I*(x - 2)**(9/2)/(4*sqrt(x - 3)) +
5*I*(x - 2)**(7/2)/(8*sqrt(x - 3)) - 13*I*(x - 2)**(5/2)/(32*sqrt(x - 3)) -
 I*(x - 2)**(3/2)/(64*sqrt(x - 3)) + 3*I*sqrt(x - 2)/(64*sqrt(x - 3)), Abs(
x - 2) > 1), (3*asin(sqrt(x - 2))/64 + (x - 2)**(9/2)/(4*sqrt(3 - x)) - 5*(
x - 2)**(7/2)/(8*sqrt(3 - x)) + 13*(x - 2)**(5/2)/(32*sqrt(3 - x)) + (x - 2
)**(3/2)/(64*sqrt(3 - x)) - 3*sqrt(x - 2)/(64*sqrt(3 - x)), True))
```

Giac [A]
time = 0.02, size = 275, normalized size = 3.02

$$-2\left(2\left(\left(\left(\left(\frac{73}{96}-\frac{1}{16}\sqrt{-x+3}\sqrt{-x+3}\right)\sqrt{-x+3}\sqrt{-x+3}-\frac{1363}{384}\right)\sqrt{-x+3}\sqrt{-x+3}+\frac{2093}{256}\right)\sqrt{-x+3}\sqrt{x-2}+\frac{1363}{128}\arcsin\left(\sqrt{-x+3}\right)\right)+16\left(2\left(\left(\left(\frac{1}{12}\sqrt{-x+3}\sqrt{-x+3}-\frac{37}{48}\right)\sqrt{-x+3}\sqrt{-x+3}+\frac{83}{32}\right)\sqrt{-x+3}\sqrt{x-2}+\frac{61}{16}\arcsin\left(\sqrt{-x+3}\right)\right)-42\left(2\left(\frac{13}{16}-\frac{1}{8}\sqrt{-x+3}\sqrt{-x+3}\right)\sqrt{-x+3}\sqrt{x-2}+\frac{11}{8}\arcsin\left(\sqrt{-x+3}\right)\right)+36\left(\frac{1}{2}\sqrt{-x+3}\sqrt{x-2}+\frac{\arcsin\left(\sqrt{-x+3}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x)
```

```
[Out] -1/192*(2*(4*(6*x + 55)*(x - 3) + 1363)*(x - 3) + 6279)*sqrt(x - 2)*sqrt(-x
 + 3) + 1/3*(2*(4*x + 25)*(x - 3) + 249)*sqrt(x - 2)*sqrt(-x + 3) - 21/4*(2
*x + 7)*sqrt(x - 2)*sqrt(-x + 3) + 18*sqrt(x - 2)*sqrt(-x + 3) - 3/64*arcsi
n(sqrt(-x + 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (x-2)^{3/2}\,(3-x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(3/2)*(3 - x)^(3/2),x)
```

```
[Out] int((x - 2)^(3/2)*(3 - x)^(3/2), x)
```

3.1161 $\int \sqrt{3-x}\,\sqrt{-2+x}\,dx$

Optimal. Leaf size=51

$$\frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}\,-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}\,-\frac{1}{8}\sin^{-1}(5-2x)$$

```
[Out] 1/8*arcsin(-5+2*x)-1/2*(3-x)^(3/2)*(-2+x)^(1/2)+1/4*(3-x)^(1/2)*(-2+x)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {52, 55, 633, 222}

$$-\frac{1}{2}\sqrt{x-2}\,(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\,\sqrt{3-x}\,-\frac{1}{8}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[3 - x]*Sqrt[-2 + x])/4 - ((3 - x)^(3/2)*Sqrt[-2 + x])/2 - ArcSin[5 -
2*x]/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x}\,\sqrt{-2+x}\,dx &= -\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{4}\int \frac{\sqrt{3-x}}{\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-\right.\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\sin^{-1}(5-2x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 1.41

$$\frac{\sqrt{-6+5x-x^2}\left(\sqrt{-2+x}\,(15-11x+2x^2)-\sqrt{-3+x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-3+x}{-2+x}}}\right)\right)}{4(-3+x)\sqrt{-2+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(15 - 11*x + 2*x^2) - Sqrt[-3 + x]*ArcT
anh[1/Sqrt[(-3 + x)/(-2 + x)]]))/(4*(-3 + x)*Sqrt[-2 + x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.02, size = 100, normalized size = 1.96

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(\sqrt{-2+x}-3\,(-2+x)^{\frac{3}{2}}-\text{ArcCosh}\left[\sqrt{-2+x}\right]\sqrt{-3+x}+2\,(-2+x)^{\frac{5}{2}}\right)}{4\sqrt{-3+x}},\text{Abs}\,[-2+x]>1\right\}\right\},-\frac{(-2+x)^{\frac{5}{2}}}{2\sqrt{3-x}}-\frac{\sqrt{-2+x}}{4\sqrt{3-x}}+\frac{\text{ArcSin}\left[\sqrt{-2+x}\right]}{4}+\frac{3\,(-2+x)^{\frac{3}{2}}}{4\sqrt{3-x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - x)^(1/2)*(-2 + x)^(1/2),x]')
```

```
[Out] Piecewise[{{I / 4 (Sqrt[-2 + x] - 3 (-2 + x) ^ (3 / 2) - ArcCosh[Sqrt[-2 +
x]] Sqrt[-3 + x] + 2 (-2 + x) ^ (5 / 2)) / Sqrt[-3 + x], Abs[-2 + x] > 1}},
```

```
-(-2 + x) ^ (5 / 2) / (2 Sqrt[3 - x]) - Sqrt[-2 + x] / (4 Sqrt[3 - x]) + A
rcSin[Sqrt[-2 + x]] / 4 + 3 (-2 + x) ^ (3 / 2) / (4 Sqrt[3 - x])]
```

Maple [A]
time = 0.17, size = 61, normalized size = 1.20

method	result	size
default	$\frac{\sqrt{3-x}\,(-2+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{3-x}\,\sqrt{-2+x}}{4}+\frac{\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{8\sqrt{-2+x}\,\sqrt{3-x}}$	61
risch	$-\frac{(2x-5)(-3+x)\sqrt{-2+x}\,\sqrt{(-2+x)(3-x)}}{4\sqrt{-(-3+x)(-2+x)}\,\sqrt{3-x}}+\frac{\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{8\sqrt{-2+x}\,\sqrt{3-x}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(1/2)*(-2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(3-x)^(1/2)*(-2+x)^(3/2)-1/4*(3-x)^(1/2)*(-2+x)^(1/2)+1/8*((-2+x)*(3-x)
)^(1/2)/(-2+x)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A]
time = 0.35, size = 38, normalized size = 0.75

$$\frac{1}{2}\sqrt{-x^2+5x-6}\,x-\frac{5}{4}\sqrt{-x^2+5x-6}+\frac{1}{8}\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 5*x - 6)*x - 5/4*sqrt(-x^2 + 5*x - 6) + 1/8*arcsin(2*x - 5)
```

Fricas [A]
time = 0.30, size = 52, normalized size = 1.02

$$\frac{1}{4}(2x-5)\sqrt{x-2}\,\sqrt{-x+3}-\frac{1}{8}\arctan\left(\frac{(2x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2(x^2-5x+6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="fricas")
[Out] 1/4*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3) - 1/8*arctan(1/2*(2*x - 5)*sqrt(x -
2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [A]
time = 1.88, size = 124, normalized size = 2.43

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{x-2}\right)}{4}+\frac{i(x-2)^{\frac{5}{2}}}{2\sqrt{x-3}}-\frac{3i(x-2)^{\frac{3}{2}}}{4\sqrt{x-3}}+\frac{i\sqrt{x-2}}{4\sqrt{x-3}} & \text{for } |x-2|>1 \\ \frac{\operatorname{asin}\left(\sqrt{x-2}\right)}{4}-\frac{(x-2)^{\frac{5}{2}}}{2\sqrt{3-x}}+\frac{3(x-2)^{\frac{3}{2}}}{4\sqrt{3-x}}-\frac{\sqrt{x-2}}{4\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(1/2)*(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(x - 2))/4 + I*(x - 2)**(5/2)/(2*sqrt(x - 3)) - 3*I
*(x - 2)**(3/2)/(4*sqrt(x - 3)) + I*sqrt(x - 2)/(4*sqrt(x - 3)), Abs(x - 2)
 > 1), (asin(sqrt(x - 2))/4 - (x - 2)**(5/2)/(2*sqrt(3 - x)) + 3*(x - 2)**(
3/2)/(4*sqrt(3 - x)) - sqrt(x - 2)/(4*sqrt(3 - x)), True))
```

Giac [A]
time = 0.00, size = 93, normalized size = 1.82

$$2\left(2\left(\frac{13}{16}-\frac{1}{8}\sqrt{-x+3}\,\sqrt{-x+3}\right)\sqrt{-x+3}\,\sqrt{x-2}+\frac{11}{8}\arcsin\left(\sqrt{-x+3}\right)\right)-6\left(\frac{1}{2}\sqrt{-x+3}\,\sqrt{x-2}+\frac{\arcsin\left(\sqrt{-x+3}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x)
```

```
[Out] 1/4*(2*x + 7)*sqrt(x - 2)*sqrt(-x + 3) - 3*sqrt(x - 2)*sqrt(-x + 3) - 1/4*a
rcsin(sqrt(-x + 3))
```

Mupad [B]
time = 0.21, size = 41, normalized size = 0.80

$$\left(\frac{x}{2}-\frac{5}{4}\right)\sqrt{x-2}\,\sqrt{3-x}-\frac{\ln\left(x-\frac{5}{2}-\sqrt{x-2}\,\sqrt{3-x}\,1\mathrm{i}\right)1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(1/2)*(3 - x)^(1/2),x)
```

```
[Out] (x/2 - 5/4)*(x - 2)^(1/2)*(3 - x)^(1/2) - (log(x - (x - 2)^(1/2)*(3 - x)^(1
/2)*1i - 5/2)*1i)/8
```

3.1162 $\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(5-2x)$$

```
[Out] arcsin(-5+2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 633, 222}

$$-\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
```

```
[Out] -ArcSin[5 - 2*x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx &= \int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-2x\right)\\ &= -\sin^{-1}(5-2x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(8) = 16.
time = 0.03, size = 44, normalized size = 5.50

$$\frac{2\sqrt{-3+x}\,\sqrt{-2+x}\,\tanh^{-1}\left(\frac{\sqrt{-2+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(-2+x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
[Out] (2*Sqrt[-3 + x]*Sqrt[-2 + x]*ArcTanh[Sqrt[-2 + x]/Sqrt[-3 + x]])/Sqrt[-((-3
 + x)*(-2 + x))]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.56, size = 25, normalized size = 3.12

$$\text{Piecewise}\left[\left\{\left\{-2I\text{ArcCosh}\left[\sqrt{-2+x}\right],\text{Abs}\left[-2+x\right]>1\right\}\right\},2\text{ArcSin}\left[\sqrt{-2+x}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - x)^(1/2)*(-2 + x)^(1/2)),x]')
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[-2 + x]], Abs[-2 + x] > 1}}, 2 ArcSin[Sqrt[-2
 + x]]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(6) = 12.
time = 0.17, size = 31, normalized size = 3.88

method	result	size
default	$\frac{\sqrt{(-2+x)\,(3-x)}\,\arcsin(2x-5)}{\sqrt{-2+x}\,\sqrt{3-x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(1/2)/(-2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-2+x)*(3-x))^(1/2)/(-2+x)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A]
time = 0.34, size = 6, normalized size = 0.75

$$\arcsin\left(2\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(6) = 12$.
time = 0.30, size = 32, normalized size = 4.00

$$-\arctan\left(\frac{(2x-5)\sqrt{x-2}\sqrt{-x+3}}{2(x^2-5x+6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [A]
time = 0.79, size = 26, normalized size = 3.25

$$\begin{cases} -2i\operatorname{acosh}\left(\sqrt{x-2}\right) & \text{for } |x-2|>1 \\ 2\operatorname{asin}\left(\sqrt{x-2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(-2+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(x - 2)), Abs(x - 2) > 1), (2*asin(sqrt(x - 2)),
True))
```

Giac [A]
time = 0.00, size = 12, normalized size = 1.50

$$-2\arcsin\left(\sqrt{-x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x)
[Out] -2*arcsin(sqrt(-x + 3))
```

Mupad [B]
time = 0.18, size = 31, normalized size = 3.88

$$-4\operatorname{atan}\left(\frac{\sqrt{x-2}-\sqrt{2}\,1\mathrm{i}}{\sqrt{3}-\sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(1/2)*(3 - x)^(1/2)),x)
[Out] -4*atan(((x - 2)^(1/2) - 2^(1/2)*1i)/(3^(1/2) - (3 - x)^(1/2)))
```

3.1163 $\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}$$

```
[Out] 2/(3-x)^(1/2)/(-2+x)^(1/2)-4*(3-x)^(1/2)/(-2+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2}{\sqrt{3-x}\,\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] 2/(Sqrt[3 - x]*Sqrt[-2 + x]) - (4*Sqrt[3 - x])/Sqrt[-2 + x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx &= \frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}+2\int \frac{1}{\sqrt{3-x}\,(-2+x)^{3/2}}\,dx\\ &= \frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 0.57

$$\frac{2(-5+2x)}{\sqrt{-6+5x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] (2*(-5 + 2*x))/Sqrt[-6 + 5*x - x^2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.25, size = 78, normalized size = 2.11

$$\text{Piecewise}\left[\left\{\left\{\frac{I\,(10-4x)}{\sqrt{-3+x}\,\sqrt{-2+x}},\text{Abs}\,[-2+x]>1\right\}\right\},\frac{-4\,(-2+x)\,\sqrt{3-x}}{(-2+x)^{\frac{3}{2}}-\sqrt{-2+x}}+\frac{2\sqrt{3-x}}{(-2+x)^{\frac{3}{2}}-\sqrt{-2+x}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{I (10 - 4 x) / (Sqrt[-3 + x] Sqrt[-2 + x]), Abs[-2 + x] > 1}},
-4 (-2 + x) Sqrt[3 - x] / ((-2 + x) ^ (3 / 2) - Sqrt[-2 + x]) + 2 Sqrt[3 -
x] / ((-2 + x) ^ (3 / 2) - Sqrt[-2 + x])]
```

Maple [A]
time = 0.16, size = 30, normalized size = 0.81

method	result	size
gosper	$\frac{4x-10}{\sqrt{3-x}\,\sqrt{-2+x}}$	20
default	$\frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}$	30
risch	$\frac{2\sqrt{(-2+x)\,(3-x)}\,(2x-5)}{\sqrt{3-x}\,\sqrt{-2+x}\,\sqrt{-(-3+x)\,(-2+x)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(3/2)/(-2+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(3-x)^(1/2)/(-2+x)^(1/2)-4*(3-x)^(1/2)/(-2+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.81

$$\frac{4\,x}{\sqrt{-x^2+5\,x-6}}-\frac{10}{\sqrt{-x^2+5\,x-6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 5*x - 6) - 10/sqrt(-x^2 + 5*x - 6)
```

Fricas [A]
time = 0.30, size = 29, normalized size = 0.78

$$-\frac{2\,(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}}{x^2-5\,x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6)
```

Sympy [A]
time = 1.28, size = 100, normalized size = 2.70

$$\begin{cases} -\frac{4i\sqrt{x-3}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}}+\frac{2i\sqrt{x-3}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{for } |x-2|>1 \\ -\frac{4\sqrt{3-x}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}}+\frac{2\sqrt{3-x}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-4*I*sqrt(x - 3)*(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*I*sq
rt(x - 3)/((x - 2)**(3/2) - sqrt(x - 2)), Abs(x - 2) > 1), (-4*sqrt(3 - x)*
(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*sqrt(3 - x)/((x - 2)**(3/2) - sq
rt(x - 2)), True))
```

Giac [A]
time = 0.00, size = 71, normalized size = 1.92

$$2\left(\frac{\sqrt{-x+3}}{-2\sqrt{x-2}+2}-\frac{-2\sqrt{x-2}+2}{4\sqrt{-x+3}}-\frac{\sqrt{-x+3}\,\sqrt{x-2}}{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x)
```

```
[Out] (sqrt(x - 2) - 1)/sqrt(-x + 3) - 2*sqrt(-x + 3)/sqrt(x - 2) - sqrt(-x + 3)/
(sqrt(x - 2) - 1)
```

Mupad [B]
time = 0.25, size = 32, normalized size = 0.86

$$-\frac{4\,x\,\sqrt{3-x}-10\,\sqrt{3-x}}{\sqrt{x-2}\,\left(x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(3/2)*(3 - x)^(3/2)),x)
[Out] -(4*x*(3 - x)^(1/2) - 10*(3 - x)^(1/2))/((x - 2)^(1/2)*(x - 3))
```

3.1164 $\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}}-\frac{16\sqrt{3-x}}{3(-2+x)^{3/2}}-\frac{32\sqrt{3-x}}{3\sqrt{-2+x}}$$

```
[Out] 2/3/(3-x)^(3/2)/(-2+x)^(3/2)+4/(-2+x)^(3/2)/(3-x)^(1/2)-16/3*(3-x)^(1/2)/(-
2+x)^(3/2)-32/3*(3-x)^(1/2)/(-2+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] 2/(3*(3 - x)^(3/2)*(-2 + x)^(3/2)) + 4/(Sqrt[3 - x]*(-2 + x)^(3/2)) - (16*S
qrt[3 - x])/(3*(-2 + x)^(3/2)) - (32*Sqrt[3 - x])/(3*Sqrt[-2 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx = \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + 2\int \frac{1}{(3-x)^{3/2}(-2+x)^{5/2}}\,dx$$

$$= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} + 8\int \frac{1}{\sqrt{3-x}\,(-2+x)^{5/2}}\,d$$

$$= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} - \frac{16\sqrt{3-x}}{3(-2+x)^{3/2}} + \frac{16}{3}\int \frac{}{\sqrt{3}}$$

$$= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} - \frac{16\sqrt{3-x}}{3(-2+x)^{3/2}} - \frac{32\sqrt{3-}}{3\sqrt{-2+}}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.54

$$\frac{2\left(-235+294x-120x^2+16x^3\right)}{3(-3+x)(-2+x)\sqrt{-6+5x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (2*(-235 + 294*x - 120*x^2 + 16*x^3))/(3*(-3 + x)*(-2 + x)*Sqrt[-6 + 5*x - 
x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 8.75, size = 206, normalized size = 2.61

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(235-294x+120x^2-16x^3\right)\sqrt{\frac{3-x}{-2+x}}}{3\left(-18+21x-8x^2+x^3\right)}, \frac{1}{\text{Abs}[-2+x]}>1\right\}\right\}, \frac{-32I(-2+x)^3\sqrt{1-\frac{1}{-2+x}}}{-6+3x-6(-2+x)^2+3(-2+x)^3} - \frac{12I(-2+x)\sqrt{1-\frac{1}{-2+x}}}{-6+3x-6(-2+x)^2+3(-2+x)^3} - \frac{2I\sqrt{1-\frac{1}{-2+x}}}{-6+3x-6(-2+x)^2+3(-2+x)^3} + \frac{I48(-2+x)^2\sqrt{1-\frac{1}{-2+x}}}{-6+3x-6(-2+x)^2+3(-2+x)^3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]')
```

```
[Out] Piecewise[{{2 (235 - 294 x + 120 x ^ 2 - 16 x ^ 3) Sqrt[(3 - x) / (-2 + x)]
 / (3 (-18 + 21 x - 8 x ^ 2 + x ^ 3)), 1 / Abs[-2 + x] > 1}}, -32 I (-2 + x
) ^ 3 Sqrt[1 - 1 / (-2 + x)] / (-6 + 3 x - 6 (-2 + x) ^ 2 + 3 (-2 + x) ^ 3)
 - 12 I (-2 + x) Sqrt[1 - 1 / (-2 + x)] / (-6 + 3 x - 6 (-2 + x) ^ 2 + 3 (-
2 + x) ^ 3) - 2 I Sqrt[1 - 1 / (-2 + x)] / (-6 + 3 x - 6 (-2 + x) ^ 2 + 3 (
-2 + x) ^ 3) + I 48 (-2 + x) ^ 2 Sqrt[1 - 1 / (-2 + x)] / (-6 + 3 x - 6 (-2
 + x) ^ 2 + 3 (-2 + x) ^ 3)]
```

Maple [A]
time = 0.17, size = 58, normalized size = 0.73

method	result	size
gosper	$-\frac{2(16x^3-120x^2+294x-235)}{3(-2+x)^{\frac{3}{2}}(3-x)^{\frac{3}{2}}}$	30
default	$\frac{2}{3(3-x)^{\frac{3}{2}}(-2+x)^{\frac{3}{2}}}+\frac{4}{(-2+x)^{\frac{3}{2}}\sqrt{3-x}}-\frac{16\sqrt{3-x}}{3(-2+x)^{\frac{3}{2}}}-\frac{32\sqrt{3-x}}{3\sqrt{-2+x}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(5/2)/(-2+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(3-x)^(3/2)/(-2+x)^(3/2)+4/(-2+x)^(3/2)/(3-x)^(1/2)-16/3*(3-x)^(1/2)/(-
2+x)^(3/2)-32/3*(3-x)^(1/2)/(-2+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.75

$$\frac{32\,x}{3\,\sqrt{-x^2+5\,x-6}}-\frac{80}{3\,\sqrt{-x^2+5\,x-6}}+\frac{4\,x}{3\,(-x^2+5\,x-6)^{\frac{3}{2}}}-\frac{10}{3\,(-x^2+5\,x-6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x/sqrt(-x^2 + 5*x - 6) - 80/3/sqrt(-x^2 + 5*x - 6) + 4/3*x/(-x^2 + 5*x
 - 6)^(3/2) - 10/3/(-x^2 + 5*x - 6)^(3/2)
```

Fricas [A]
time = 0.31, size = 49, normalized size = 0.62

$$-\frac{2\,(16\,x^3-120\,x^2+294\,x-235)\sqrt{x-2}\,\sqrt{-x+3}}{3\,(x^4-10\,x^3+37\,x^2-60\,x+36)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*x^3 - 120*x^2 + 294*x - 235)*sqrt(x - 2)*sqrt(-x + 3)/(x^4 - 10*x^
3 + 37*x^2 - 60*x + 36)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.05, size = 282, normalized size = 3.57

$$\begin{cases} -\frac{32\sqrt{-1+\frac{1}{x-2}}\,(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6}+\frac{48\sqrt{-1+\frac{1}{x-2}}\,(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{12\sqrt{-1+\frac{1}{x-2}}\,(x-2)}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{2\sqrt{-1+\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{for}\ \frac{1}{|x-2|}>1 \\ -\frac{32i\sqrt{1-\frac{1}{x-2}}\,(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6}+\frac{48i\sqrt{1-\frac{1}{x-2}}\,(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{12i\sqrt{1-\frac{1}{x-2}}\,(x-2)}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{2i\sqrt{1-\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(5/2)/(-2+x)**(5/2),x)
```

```
[Out] Piecewise((-32*sqrt(-1 + 1/(x - 2))*(x - 2)**3/(3*x + 3*(x - 2)**3 - 6*(x -
 2)**2 - 6) + 48*sqrt(-1 + 1/(x - 2))*(x - 2)**2/(3*x + 3*(x - 2)**3 - 6*(x
 - 2)**2 - 6) - 12*sqrt(-1 + 1/(x - 2))*(x - 2)/(3*x + 3*(x - 2)**3 - 6*(x
- 2)**2 - 6) - 2*sqrt(-1 + 1/(x - 2))/(3*x + 3*(x - 2)**3 - 6*(x - 2)**2 -
6), 1/Abs(x - 2) > 1), (-32*I*sqrt(1 - 1/(x - 2))*(x - 2)**3/(3*x + 3*(x -
2)**3 - 6*(x - 2)**2 - 6) + 48*I*sqrt(1 - 1/(x - 2))*(x - 2)**2/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 12*I*sqrt(1 - 1/(x - 2))*(x - 2)/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 2*I*sqrt(1 - 1/(x - 2))/(3*x + 3*(x - 2)**3
- 6*(x - 2)**2 - 6), True))
```

Giac [A]
time = 0.01, size = 173, normalized size = 2.19

$$-2\left(\frac{-\frac{64}{3}\left(-\frac{-2\sqrt{x-2}+2}{2\sqrt{-x+3}}\right)^{3}+\frac{352\left(-2\sqrt{x-2}+2\right)}{\sqrt{-x+3}}}{512}+\frac{33\left(-\frac{-2\sqrt{x-2}+2}{2\sqrt{-x+3}}\right)^{2}+1}{24\left(-\frac{-2\sqrt{x-2}+2}{2\sqrt{-x+3}}\right)^{3}}+\frac{2\left(-\frac{4}{3}\sqrt{-x+3}\sqrt{-x+3}+\frac{3}{2}\right)\sqrt{-x+3}\sqrt{x-2}}{(x-2)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x)
```

```
[Out] 1/12*(sqrt(x - 2) - 1)^3/(-x + 3)^(3/2) + 11/4*(sqrt(x - 2) - 1)/sqrt(-x +
3) - 2/3*(8*x - 15)*sqrt(-x + 3)/(x - 2)^(3/2) + 1/12*(-x + 3)^(3/2)*(33*(s
qrt(x - 2) - 1)^2/(x - 3) - 1)/(sqrt(x - 2) - 1)^3
```

Mupad [B]
time = 0.37, size = 69, normalized size = 0.87

$$-\frac{32\,(x-2)^3\,\sqrt{3-x}-48\,(x-2)^2\,\sqrt{3-x}+2\,\sqrt{3-x}+12\,(x-2)\,\sqrt{3-x}}{(3\,x-6)\,\sqrt{x-2}\,(x-3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(5/2)*(3 - x)^(5/2)),x)
```

```
[Out] -(32*(x - 2)^3*(3 - x)^(1/2) - 48*(x - 2)^2*(3 - x)^(1/2) + 2*(3 - x)^(1/2)
 + 12*(x - 2)*(3 - x)^(1/2))/((3*x - 6)*(x - 2)^(1/2)*(x - 3)^2)
```

3.1165 $\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x}{9\sqrt{3-x}\,\sqrt{3+x}}$$

```
[Out] 1/9*x/(3-x)^(1/2)/(3+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {39}

$$\frac{x}{9\sqrt{3-x}\,\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
[Out] x/(9*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{3-x}\,\sqrt{3+x}}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{9-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
[Out] x/(9*Sqrt[9 - x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.00, size = 66, normalized size = 3.14

$$\text{Piecewise}\left[\left\{\left\{-\frac{x\sqrt{\frac{3-x}{3+x}}}{-27+9x},\frac{1}{\text{Abs}\,[3+x]}>\frac{1}{6}\right\}\right\},-\frac{I}{9\sqrt{1-\frac{6}{3+x}}}+\frac{I}{3\sqrt{1-\frac{6}{3+x}}\,(3+x)}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]')
[Out] Piecewise[{{-x Sqrt[(3 - x) / (3 + x)] / (-27 + 9 x), 1 / Abs[3 + x] > 1 /
6}}, -I / (9 Sqrt[1 - 6 / (3 + x)]) + I / (3 Sqrt[1 - 6 / (3 + x)] (3 + x))
]
```

Maple [A]
time = 0.16, size = 30, normalized size = 1.43

method	result	size
gosper	$\frac{x}{9\sqrt{3-x}\,\sqrt{3+x}}$	16
default	$\frac{1}{3\sqrt{3-x}\,\sqrt{3+x}}-\frac{\sqrt{3-x}}{9\sqrt{3+x}}$	30
risch	$\frac{\sqrt{(3+x)\,(3-x)}\,x}{9\sqrt{3-x}\,\sqrt{3+x}\,\sqrt{-(-3+x)\,(3+x)}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(3/2)/(3+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3/(3-x)^(1/2)/(3+x)^(1/2)-1/9/(3+x)^(1/2)*(3-x)^(1/2)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.57

$$\frac{x}{9\sqrt{-x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
[Out] 1/9*x/sqrt(-x^2 + 9)
```

Fricas [A]
time = 0.29, size = 22, normalized size = 1.05

$$-\frac{\sqrt{x+3}\,x\sqrt{-x+3}}{9\,(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(x + 3)*x*sqrt(-x + 3)/(x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.99, size = 71, normalized size = 3.38

$$\begin{cases} -\frac{\sqrt{-1+\frac{6}{x+3}}\,(x+3)}{9x-27}+\frac{3\sqrt{-1+\frac{6}{x+3}}}{9x-27} & \text{for}\ \frac{1}{|x+3|}>\frac{1}{6} \\ -\frac{i}{9\sqrt{1-\frac{6}{x+3}}}+\frac{i}{3\sqrt{1-\frac{6}{x+3}}\,(x+3)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 6/(x + 3))*(x + 3)/(9*x - 27) + 3*sqrt(-1 + 6/(x + 3)
)/(9*x - 27), 1/Abs(x + 3) > 1/6), (-I/(9*sqrt(1 - 6/(x + 3))) + I/(3*sqrt(
1 - 6/(x + 3))*(x + 3)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(15) = 30.
time = 0.00, size = 87, normalized size = 4.14

$$2\left(\frac{\sqrt{-x+3}}{36\left(2\sqrt{6}-2\sqrt{x+3}\right)}-\frac{2\sqrt{6}-2\sqrt{x+3}}{144\sqrt{-x+3}}-\frac{\sqrt{-x+3}\sqrt{x+3}}{36\,(x+3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x)
```

```
[Out] -1/36*(sqrt(6) - sqrt(x + 3))/sqrt(-x + 3) - 1/18*sqrt(-x + 3)/sqrt(x + 3)
+ 1/36*sqrt(-x + 3)/(sqrt(6) - sqrt(x + 3))
```

Mupad [B]
time = 0.36, size = 22, normalized size = 1.05

$$-\frac{x\,\sqrt{3-x}}{(9\,x-27)\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - x)^(1/2))/((9*x - 27)*(x + 3)^(1/2))
```

3.1166 $\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{9\sqrt{3-bx}\,\sqrt{3+bx}}$$

```
[Out] 1/9*x/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{39\}$

$$\frac{x}{9\sqrt{3-bx}\,\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{9\sqrt{3-bx}\,\sqrt{3+bx}}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{9-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[9 - b^2*x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.27, size = 69, normalized size = 2.88

$$\frac{-I\text{meijerg}\left[\left\{\left\{\frac{3}{4},\frac{5}{4},1\right\},\left\{\frac{1}{2},\frac{3}{2},2\right\}\right\},\left\{\left\{\frac{3}{4},1,\frac{5}{4},\frac{3}{2},2\right\},\{0\}\right\},\frac{9}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\},\{\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{-\frac{1}{2},0,1,0\right\}\right\},\frac{9\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]}{18\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]')
```

```
[Out] (-I meijerg[{{3 / 4, 5 / 4, 1}, {1 / 2, 3 / 2, 2}}, {{3 / 4, 1, 5 / 4, 3 /
2, 2}, {0}}, 9 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, 0, 1 / 4, 1 / 2, 3 / 4,
 1}, {}}, {{1 / 4, 3 / 4}, {-1 / 2, 0, 1, 0}}, 9 exp_polar[-2 I Pi] / (b ^
2 x ^ 2)]) / (18 Pi ^ (3 / 2) b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.
time = 0.15, size = 42, normalized size = 1.75

method	result	size
gosper	$\frac{x}{9\sqrt{-bx+3}\,\sqrt{bx+3}}$	19
default	$\frac{1}{3b\sqrt{-bx+3}\,\sqrt{bx+3}}-\frac{\sqrt{-bx+3}}{9b\sqrt{bx+3}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/(-b*x+3)^(1/2)/(b*x+3)^(1/2)-1/9/b/(b*x+3)^(1/2)*(-b*x+3)^(1/2)
```

Maxima [A]
time = 0.26, size = 15, normalized size = 0.62

$$\frac{x}{9\sqrt{-b^2x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-b^2*x^2 + 9)
```

Fricas [A]
time = 0.29, size = 29, normalized size = 1.21

$$-\frac{\sqrt{bx+3}\,\sqrt{-bx+3}\,x}{9\,(b^2x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(b*x + 3)*sqrt(-b*x + 3)*x/(b^2*x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.40, size = 73, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), 9
/(b**2*x**2))/(18*pi**(3/2)*b) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()),
 ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*x**2))/(18*pi**(
3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(18) = 36.
time = 0.00, size = 103, normalized size = 4.29

$$\frac{2\left(\frac{\sqrt{-bx+3}}{36\left(2\sqrt{6}-2\sqrt{bx+3}\right)}-\frac{2\sqrt{6}-2\sqrt{bx+3}}{144\sqrt{-bx+3}}-\frac{\sqrt{-bx+3}\sqrt{bx+3}}{36(bx+3)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] -1/36*((sqrt(6) - sqrt(b*x + 3))/sqrt(-b*x + 3) + 2*sqrt(-b*x + 3)/sqrt(b*x
 + 3) - sqrt(-b*x + 3)/(sqrt(6) - sqrt(b*x + 3)))/b
```

Mupad [B]
time = 0.46, size = 26, normalized size = 1.08

$$-\frac{x\sqrt{3-b\,x}}{\sqrt{b\,x+3}\,\left(9\,b\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - b*x)^(3/2)*(b*x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - b*x)^(1/2))/((b*x + 3)^(1/2)*(9*b*x - 27))
```

3.1167 $\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{3+x}}$$

[Out] 1/36*x*2^(1/2)/(3-x)^(1/2)/(3+x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{39\}$

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

[In] Int[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]

[Out] x/(18*Sqrt[2]*Sqrt[3 - x]*Sqrt[3 + x])

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{3+x}}$$

Mathematica [A]
time = 0.09, size = 21, normalized size = 0.81

$$\frac{x}{18\sqrt{6-2x}\sqrt{3+x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]

[Out] x/(18*Sqrt[6 - 2*x]*Sqrt[3 + x])

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 10.34, size = 75, normalized size = 2.88

$$\text{Piecewise}\left[\left\{\left\{-\frac{\sqrt{2}\,x\sqrt{\frac{3-x}{3+x}}}{-108+36x},\frac{1}{\text{Abs}\,[3+x]}>\frac{1}{6}\right\}\right\},-\frac{I\sqrt{2}}{36\sqrt{1-\frac{6}{3+x}}}+\frac{I\sqrt{2}}{12\sqrt{1-\frac{6}{3+x}}\,(3+x)}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]')
```

```
[Out] Piecewise[{{-Sqrt[2] x Sqrt[(3 - x) / (3 + x)] / (-108 + 36 x), 1 / Abs[3 +
 x] > 1 / 6}}, -I Sqrt[2] / (36 Sqrt[1 - 6 / (3 + x)]) + I Sqrt[2] / (12 Sq
rt[1 - 6 / (3 + x)] (3 + x))]
```

Maple [A]
time = 0.14, size = 30, normalized size = 1.15

method	result	size
gosper	$-\frac{(-3+x)x}{9\sqrt{3+x}\,(6-2x)^{\frac{3}{2}}}$	19
default	$\frac{1}{6\sqrt{6-2x}\,\sqrt{3+x}}-\frac{\sqrt{6-2x}}{36\sqrt{3+x}}$	30
risch	$\frac{\sqrt{(3+x)\,(6-2x)}\,\sqrt{2}\,x}{36\sqrt{3+x}\,\sqrt{6-2x}\,\sqrt{-(-3+x)\,(3+x)}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6-2*x)^(3/2)/(3+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(6-2*x)^(1/2)/(3+x)^(1/2)-1/36/(3+x)^(1/2)*(6-2*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.46

$$\frac{x}{18\sqrt{-2\,x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*x^2 + 18)
```

Fricas [A]
time = 0.29, size = 22, normalized size = 0.85

$$-\frac{\sqrt{x+3}\,x\sqrt{-2\,x+6}}{36\,(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(x + 3)*x*sqrt(-2*x + 6)/(x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.94, size = 92, normalized size = 3.54

$$\begin{cases} -\frac{\sqrt{2}\sqrt{-1+\frac{6}{x+3}}\,(x+3)}{36x-108}+\frac{3\sqrt{2}\sqrt{-1+\frac{6}{x+3}}}{36x-108} & \text{for } \frac{1}{|x+3|}>\frac{1}{6} \\ -\frac{\sqrt{2}\,i}{36\sqrt{1-\frac{6}{x+3}}}+\frac{\sqrt{2}\,i}{12\sqrt{1-\frac{6}{x+3}}\,(x+3)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(2)*sqrt(-1 + 6/(x + 3))*(x + 3)/(36*x - 108) + 3*sqrt(2)*s
qrt(-1 + 6/(x + 3))/(36*x - 108), 1/Abs(x + 3) > 1/6), (-sqrt(2)*I/(36*sqrt
(1 - 6/(x + 3))) + sqrt(2)*I/(12*sqrt(1 - 6/(x + 3))*(x + 3)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(18) = 36$.
time = 0.00, size = 92, normalized size = 3.54

$$\frac{\frac{\sqrt{-x+3}}{36\left(2\sqrt{6}-2\sqrt{x+3}\right)}-\frac{2\sqrt{6}-2\sqrt{x+3}}{144\sqrt{-x+3}}-\frac{\sqrt{-x+3}\,\sqrt{x+3}}{36(x+3)}}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x)
```

```
[Out] -1/144*sqrt(2)*((sqrt(6) - sqrt(x + 3))/sqrt(-x + 3) + 2*sqrt(-x + 3)/sqrt(
x + 3) - sqrt(-x + 3)/(sqrt(6) - sqrt(x + 3)))
```

Mupad [B]
time = 0.37, size = 22, normalized size = 0.85

$$-\frac{x\,\sqrt{6-2\,x}}{\left(36\,x-108\right)\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((6 - 2*x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*x)^(1/2))/((36*x - 108)*(x + 3)^(1/2))
```

3.1168 $\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx$

Optimal. Leaf size=29

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{3+bx}}$$

```
[Out] 1/36*x*2^(1/2)/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{39\}$

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx = \frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{3+bx}}$$

Mathematica [A]
time = 0.16, size = 19, normalized size = 0.66

$$\frac{x}{18\sqrt{18-2b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[18 - 2*b^2*x^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 12.46, size = 72, normalized size = 2.48

$$\frac{\sqrt{2}\left(-I\text{meijerg}\left[\left\{\left\{\frac{3}{4},\frac{5}{4},1\right\},\left\{\frac{1}{2},\frac{3}{2},2\right\}\right\},\left\{\left\{\frac{3}{4},1,\frac{5}{4},\frac{3}{2},2\right\},\{0\}\right\},\frac{9}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\},\{\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{-\frac{1}{2},0,1,0\right\}\right\},\frac{9\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]\right)}{72\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]')
```

```
[Out] Sqrt[2] (-I meijerg[{{3 / 4, 5 / 4, 1}, {1 / 2, 3 / 2, 2}}, {{3 / 4, 1, 5 /
 4, 3 / 2, 2}, {0}}, 9 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, 0, 1 / 4, 1 / 2
, 3 / 4, 1}, {}}, {{1 / 4, 3 / 4}, {-1 / 2, 0, 1, 0}}, 9 exp_polar[-2 I Pi]
 / (b ^ 2 x ^ 2)]) / (72 Pi ^ (3 / 2) b)
```

Maple [A]
time = 0.18, size = 42, normalized size = 1.45

method	result	size
gosper	$-\frac{(bx-3)x}{9\sqrt{bx+3}\,(-2bx+6)^{\frac{3}{2}}}$	24
default	$\frac{1}{6b\sqrt{-2bx+6}\,\sqrt{bx+3}}-\frac{\sqrt{-2bx+6}}{36b\sqrt{bx+3}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b/(-2*b*x+6)^(1/2)/(b*x+3)^(1/2)-1/36/b/(b*x+3)^(1/2)*(-2*b*x+6)^(1/2)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.52

$$\frac{x}{18\sqrt{-2\,b^2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*b^2*x^2 + 18)
```

Fricas [A]
time = 0.29, size = 29, normalized size = 1.00

$$-\frac{\sqrt{bx+3}\,\sqrt{-2\,bx+6}\,x}{36\,(b^2x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
[Out] -1/36*sqrt(b*x + 3)*sqrt(-2*b*x + 6)*x/(b^2*x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.05, size = 83, normalized size = 2.86

$$-\frac{\sqrt{2}\, iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix} \middle| \frac{9}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b} + \frac{\sqrt{2}\, G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix} \middle| \frac{9e^{-2i\pi}}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)**(3/2)/(b*x+3)**(3/2),x)
[Out] -sqrt(2)*I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2),
(0,)), 9/(b**2*x**2))/(72*pi**(3/2)*b) + sqrt(2)*meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*
x**2))/(72*pi**(3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(21) = 42$.
time = 0.00, size = 108, normalized size = 3.72

$$\frac{\frac{\sqrt{-bx+3}}{36\left(2\sqrt{6}-2\sqrt{bx+3}\right)} - \frac{2\sqrt{6}-2\sqrt{bx+3}}{144\sqrt{-bx+3}} - \frac{\sqrt{-bx+3}\,\sqrt{bx+3}}{36(bx+3)}}{\sqrt{2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x)
[Out] -1/144*sqrt(2)*((sqrt(6) - sqrt(b*x + 3))/sqrt(-b*x + 3) + 2*sqrt(-b*x + 3)
/sqrt(b*x + 3) - sqrt(-b*x + 3)/(sqrt(6) - sqrt(b*x + 3)))/b
```

Mupad [B]
time = 0.32, size = 26, normalized size = 0.90

$$-\frac{x\,\sqrt{6-2\,b\,x}}{\sqrt{b\,x+3}\,\left(36\,b\,x-108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 3)^(3/2)*(6 - 2*b*x)^(3/2)),x)
[Out] -(x*(6 - 2*b*x)^(1/2))/((b*x + 3)^(1/2)*(36*b*x - 108))
```

3.1169 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{-ad+bdx}}\,dx$

Optimal. Leaf size=39

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2))/b/d^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]

[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{-2ad+dx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

Mathematica [**A**]
time = 0.06, size = 39, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-ad+bdx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-(a*d) + b*d*x]/(Sqrt[d]*Sqrt[a + b*x])])/(b*Sqrt[d])
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.37, size = 76, normalized size = 1.95

$$\frac{-I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{a^2\text{exp\_polar}[2I\text{Pi}]}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{a^2}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[a + b*x]*Sqrt[-a*d + b*d*x]),x]')
```

```
[Out] (-I meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 / 2, 1}, {}}, {{-1 / 4, 1 / 4}, {
-1 / 2, 0, 0, 0}}, a ^ 2 exp_polar[2 I Pi] / (b ^ 2 x ^ 2)] + meijerg[{{1 /
 4, 3 / 4}, {1 / 2, 1 / 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 / 4, 1, 0}, {}}, a
^ 2 / (b ^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b Sqrt[d])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.
time = 0.17, size = 76, normalized size = 1.95

method	result	size
default	$\frac{\sqrt{(bx+a)(bdx-ad)}\,\ln\left(\frac{b^2dx}{\sqrt{b^2d}}+\sqrt{b^2d\,x^2-a^2d}\right)}{\sqrt{bx+a}\,\sqrt{bdx-ad}\,\sqrt{b^2d}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(b*d*x-a*d))^(1/2)/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2)*ln(b^2*d*x/(b^2
*d)^(1/2)+(b^2*d*x^2-a^2*d)^(1/2))/(b^2*d)^(1/2)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 1.00

$$\frac{\log\left(2\,b^2dx+2\,\sqrt{b^2dx^2-a^2d}\,b\sqrt{d}\right)}{b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*d*x + 2*sqrt(b^2*d*x^2 - a^2*d)*b*sqrt(d))/(b*sqrt(d))
```

Fricas [A]
time = 0.31, size = 108, normalized size = 2.77

$$\left[\frac{\log\left(2\,b^2dx^2+2\,\sqrt{bdx-ad}\,\sqrt{bx+a}\,b\sqrt{d}\,x-a^2d\right)}{2\,b\sqrt{d}},-\frac{\sqrt{-d}\,\arctan\left(\frac{\sqrt{bdx-ad}\,\sqrt{bx+a}\,b\sqrt{-d}\,x}{b^2dx^2-a^2d}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b^2*d*x^2 + 2*sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(d)*x - a^2*
d)/(b*sqrt(d)), -sqrt(-d)*arctan(sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(-d)
*x/(b^2*d*x^2 - a^2*d))/(b*d)]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.18, size = 88, normalized size = 2.26

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{a^2e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*d*x-a*d)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a**
2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(d)) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(2*I*pi)/(b**2*x*
*2))/(4*pi**(3/2)*b*sqrt(d))
```

Giac [**A**]
time = 0.01, size = 47, normalized size = 1.21

$$-\frac{2\ln\left|\sqrt{-2ad+d\,(a+bx)}-\sqrt{d}\,\sqrt{a+bx}\right|}{\sqrt{d}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x)
```

```
[Out] -2*log(abs(-sqrt(b*x + a)*sqrt(d) + sqrt((b*x + a)*d - 2*a*d)))/(b*sqrt(d))
```

Mupad [**B**]
time = 0.22, size = 56, normalized size = 1.44

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,d\,x-a\,d}-\sqrt{-a\,d}\right)}{\sqrt{-b^2\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*d*x - a*d)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*d*x - a*d)^(1/2) - (-a*d)^(1/2)))/((-b^2*d)^(1/2)*((a + b*x
)^(1/2) - a^(1/2)))))/(-b^2*d)^(1/2)
```

3.1170 $\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}\,e}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}\,e}-\frac{\log\left(\frac{\sqrt{6-3ex}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{2+ex}}{\sqrt{2+ex}}\right.}{\sqrt{2}\sqrt[4]{3}\,e}$$

```
[Out] -1/6*ln(3^(1/2)-(-e*x+2)^(1/4)*6^(1/2)/(e*x+2)^(1/4)+3^(1/2)*(-e*x+2)^(1/2)
/(e*x+2)^(1/2))*3^(3/4)/e*2^(1/2)+1/6*ln(3^(1/2)+(-e*x+2)^(1/4)*6^(1/2)/(e*
x+2)^(1/4)+3^(1/2)*(-e*x+2)^(1/2)/(e*x+2)^(1/2))*3^(3/4)/e*2^(1/2)-1/3*arct
an(-1+(-e*x+2)^(1/4)*2^(1/2)/(e*x+2)^(1/4))*2^(1/2)*3^(3/4)/e-1/3*arctan(1+
(-e*x+2)^(1/4)*2^(1/2)/(e*x+2)^(1/4))*2^(1/2)*3^(3/4)/e
```

Rubi [A]
time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}\,e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}\,e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}\,e}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt[4]{3}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx = -\frac{4\text{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$

$$= -\frac{4\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$

$$= \frac{2\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right.}{3e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}x\right.}{e}$$

$$= -\frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\log\left(\sqrt{2-ex}+\sqrt{2}\right.}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\log\left(\right.}{}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 42, normalized size = 0.17

$$-\frac{\sqrt{2}(6-3ex)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{12}(6-3ex)\right)}{9e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] -1/9*(Sqrt[2]*(6 - 3*e*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (6 - 3*e*x
)/12])/e
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3ex+6\right)^{\frac{1}{4}} \left(ex+2\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

```
[Out] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x*e + 2)^(3/4)*(-3*x*e + 6)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(184) = 368.
time = 0.31, size = 452, normalized size = 1.88

$$2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe^4-2e^3)\sqrt{\frac{3\sqrt{\frac{1}{3}}(xe^3-2e^2)e^{(-2)}+\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}}{xe-2}}\,e^{(-3)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-xe+2}{xe-2}\right)e^{(-1)}+2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe^4-2e^3)\sqrt{\frac{3\sqrt{\frac{1}{3}}(xe^3-2e^2)e^{(-2)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}}{xe-2}}\,e^{(-3)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}+xe-2}{xe-2}\right)e^{(-1)}-\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}e^{(-1)}\log\left(\frac{3\left(3\sqrt{\frac{1}{3}}(xe^3-2e^2)e^{(-2)}+\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}\right)}{xe-2}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}e^{(-1)}\log\left(\frac{3\left(3\sqrt{\frac{1}{3}}(xe^3-2e^2)e^{(-2)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}\right)}{xe-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(1/3)^(1/4)*arctan((sqrt(3)*sqrt(2)*(1/3)^(3/4)*(x*e^4 - 2*e^3)*s
qrt((3*sqrt(1/3)*(x*e^3 - 2*e^2)*e^(-2) + sqrt(2)*(1/3)^(1/4)*(x*e + 2)^(1/
4)*(-3*x*e + 6)^(3/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e - 2))*e^(-3) -
 sqrt(2)*(1/3)^(3/4)*(x*e + 2)^(1/4)*(-3*x*e + 6)^(3/4) - x*e + 2)/(x*e - 2
))*e^(-1) + 2*sqrt(2)*(1/3)^(1/4)*arctan((sqrt(3)*sqrt(2)*(1/3)^(3/4)*(x*e^
4 - 2*e^3)*sqrt((3*sqrt(1/3)*(x*e^3 - 2*e^2)*e^(-2) - sqrt(2)*(1/3)^(1/4)*(
x*e + 2)^(1/4)*(-3*x*e + 6)^(3/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e -
2))*e^(-3) - sqrt(2)*(1/3)^(3/4)*(x*e + 2)^(1/4)*(-3*x*e + 6)^(3/4) + x*e -
 2)/(x*e - 2))*e^(-1) - 1/2*sqrt(2)*(1/3)^(1/4)*e^(-1)*log(3*(3*sqrt(1/3)*(
x*e^3 - 2*e^2)*e^(-2) + sqrt(2)*(1/3)^(1/4)*(x*e + 2)^(1/4)*(-3*x*e + 6)^(3
/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e - 2)) + 1/2*sqrt(2)*(1/3)^(1/4)*
e^(-1)*log(3*(3*sqrt(1/3)*(x*e^3 - 2*e^2)*e^(-2) - sqrt(2)*(1/3)^(1/4)*(x*e
 + 2)^(1/4)*(-3*x*e + 6)^(3/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e - 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{3^{\frac{3}{4}} \int \frac{1}{\sqrt[4]{-ex+2}\,(ex+2)^{\frac{3}{4}}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)**(1/4)/(e*x+2)**(3/4),x)
```

```
[Out] 3**(3/4)*Integral(1/((-e*x + 2)**(1/4)*(e*x + 2)**(3/4)), x)/3
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(e\,x+2)}^{3/4}\,{(6-3\,e\,x)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)),x)
```

```
[Out] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)), x)
```

3.1171 $\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=144

$$\frac{14a^2x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-\frac{14a^2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x\right.}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 14/5*a^2*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-14/15*I*(a-I*a*x)^(3/4)*(a+I*a*x
)^(3/4)-2/5*I*(a-I*a*x)^(7/4)*(a+I*a*x)^(3/4)/a-14/5*a^2*(x^2+1)^(1/4)*(cos
(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^
(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 42, 235, 233, 202}

$$-\frac{14a^2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{14a^2x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (14*a^2*x)/(5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - ((14*I)/15)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(3/4) - (((2*I)/5)*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
/a - (14*a^2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4
)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{1}{5}(7a)\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx\\
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{1}{5}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a
\end{aligned}$$

$$= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{\left(7a^2\sqrt[4]{a^2+a^2x^2}\right)\int \frac{}{\sqrt[4]{a^2}}}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+}}$$

$$= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{\left(7a^2\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x}}}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

$$= \frac{14a^2x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-$$

$$= \frac{14a^2x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-\frac{1}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.49

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{11/4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[1/
4, 11/4, 15/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.22, size = 104, normalized size = 0.72

method	result	size
risch	$-\frac{2(10i+3x)(x-i)(x+i)a^2}{15(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)a^2\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(10*I+3*x)*(x-I)*(x+I)*a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+7/5/
(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*a^2*(-a^2*(-1+I*x)*(1+I*x))^(
1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(1/4), x)
```

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 10*I*x - 21) - 15*x*
integral(14/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(x^4 + x^2), x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{7}{4}}}{\sqrt[4]{ia(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(1/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(1/4), x)
```

3.1172 $\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=106

$$\frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{2a\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2*a*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(3/4)
/a-2*a*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*Ellipt
icE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 42, 235, 233, 202}

$$-\frac{2a\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (2*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (((2*I)/3)*(a - I*a*x)^(3/4
)*(a + I*a*x)^(3/4))/a - (2*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((
a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+a\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx \\ &= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\ &= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(a\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\ &= \frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{\left(a\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\ &= \frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{2a\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.66

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{7/4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[1/4,
 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]')

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Maple [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 94, normalized size = 0.89

method	result	size
risch	$-\frac{2i(x-i)(x+i)a}{3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)a\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)

[Out] -2/3*I*(x-I)*(x+I)*a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+1/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(1/4), x)

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")

[Out] 1/3*(3*a*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^2), x) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(I*x - 3))/(a*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}{\sqrt[4]{ia\left(x-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{3/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(1/4),x)
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(1/4), x)
```

3.1173 $\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=71

$$\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] 2*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^
(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1
/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {42, 235, 233, 202}

$$\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (2*(1 + x^2)^(1/4)*EllipticE[
ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\,dx &= \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{\sqrt[4]{1+x^2}\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= \frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.99

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
[Out] (((2*I)/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 3.94, size = 74, normalized size = 1.04

$$\frac{\frac{I}{4}\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{3}{8},0,\frac{1}{8},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{3}{8},\frac{1}{8}\right\},\left\{-\frac{1}{2},-\frac{1}{4},0,0\right\}\right\},\frac{\text{exp\_polar}[-I\text{Pi}]}{x^2}\right]}{\text{Pi}\sqrt{a}\,\text{Gamma}\left[\frac{1}{4}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]')
```

```
[Out] (-1 ^ (3 / 4) meijerg[{{1 / 8, 5 / 8, 1}, {1 / 4, 1 / 2, 3 / 4}}, {{-1 / 4,
 1 / 8, 1 / 4, 5 / 8, 3 / 4}, {0}}, exp_polar[-3 I Pi] / x ^ 2] + I meijerg
[{{-1 / 2, -3 / 8, 0, 1 / 8, 1 / 2, 1}, {}}, {{-3 / 8, 1 / 8}, {-1 / 2, -1
/ 4, 0, 0}}, exp_polar[-I Pi] / x ^ 2]) / (4 Pi Sqrt[a] Gamma[1 / 4])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{1}{4}} \left(iax+a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (a^2*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2),
 x) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^2*x)
```

Sympy [A]
time = 1.95, size = 102, normalized size = 1.44

$$-\frac{i G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{8}, \frac{5}{8}, 1 & \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \\ -\frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{5}{8}, \frac{3}{4} & 0 \end{matrix} \middle| {\frac{e^{-3i\pi}}{x^{2}}} \right) e^{\frac{i\pi}{4}}}{4\pi\sqrt{a}\,\Gamma\left(\frac{1}{4}\right)} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{3}{8}, 0, \frac{1}{8}, \frac{1}{2}, 1 & \\ -\frac{3}{8}, \frac{1}{8} & -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| {\frac{e^{-i\pi}}{x^{2}}} \right)}{4\pi\sqrt{a}\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] -I*meijerg(((1/8, 5/8, 1), (1/4, 1/2, 3/4)), ((-1/4, 1/8, 1/4, 5/8, 3/4), (
0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*sqrt(a)*gamma(1/4)) + I*me
ijerg(((-1/2, -3/8, 0, 1/8, 1/2, 1), ()), ((-3/8, 1/8), (-1/2, -1/4, 0, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*sqrt(a)*gamma(1/4))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(1/4)), x)
```

3.1174 $\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=78

$$-\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^
2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*
x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {50, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (-2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ellipti
cE[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 50

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+a\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 0.87

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
[Out] ((-2*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, 1/2 - (I/
2)*x])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 94, normalized size = 1.21

method	result	size
risch	$\frac{2x-2i}{a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{\left(a^{2}\right)^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x-I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] ((a^3*x^2 + I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x) - 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 + I
*a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\,\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(1/4)), x)
```

3.1175 $\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=82

$$-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -4/5*I/a/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+2/5*(x^2+1)^(1/4)*(cos(1/2*arctan(
x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^2/(
a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/5)/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ell
ipticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx &= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.85

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
[Out] (((-2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]')
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 105, normalized size = 1.28

method	result	size
risch	$\frac{\frac{2}{5}x^2+\frac{2}{5}ix+\frac{4}{5}}{(x+i)a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(x^2+2+I*x)/(x+I)/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-
a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x + 2*I) + 5*(a^4*x^2 + 2*I*a^
4*x - a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^
4), x))/(a^4*x^2 + 2*I*a^4*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia(x-i)}\,(-ia(x+i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{9/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(1/4)), x)
```

3.1176 $\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=115

$$-\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -4/15*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)-2/9*I*(a+I*a*x)^(3/4)/a^2/(a-I*
a*x)^(9/4)+2/15*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x
))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4
)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}-\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/15)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/9)*(a + I*a
*x)^(3/4))/(a^2*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]
/2, 2])/(15*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx}{3a}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{15a}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{15a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\
&= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\right)}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.61

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

[Out] (((-2*I)/9)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, 1/2 - (I/2)*x])/(a*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 113, normalized size = 0.98

method	result	size
risch	$\frac{\frac{2}{15}x^3+\frac{4}{15}ix^2-\frac{4}{45}x+\frac{22}{45}i}{(x+i)^2a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{15(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)

[Out] 2/45*(6*I*x^2+3*x^3-2*x+11*I)/(x+I)^2/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(13/4)), x)

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")

```
[Out] 1/45*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 9*I*x - 11) + 45*(a^5
*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I*a^5)*integral(-1/15*(I*a*x + a)^(3/4)*(-I*
a*x + a)^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I*a^
5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\left(-ia\left(x+i\right)\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(13/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{13/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(1/4)), x)
```

3.1177 $\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=148

$$-\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{39a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -4/39*I/a^3/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)-2/13*I*(a+I*a*x)^(3/4)/a^2/(a-I
*a*x)^(13/4)-10/117*I*(a+I*a*x)^(3/4)/a^3/(a-I*a*x)^(9/4)+2/39*(x^2+1)^(1/4
)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(
x)),2^(1/2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{39a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}-\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/39)/(a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/13)*(a + I*
a*x)^(3/4))/(a^2*(a - I*a*x)^(13/4)) - (((10*I)/117)*(a + I*a*x)^(3/4))/(a^
3*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(39*a^
4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} + \frac{5\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx}{13a}$$

$$= -\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{5\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx}{39a^2}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \int$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \sqrt[4]{}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{}{3}$$

$$= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}} - \frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}} + \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.47

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{13}{4},\frac{1}{4};-\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13a(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/13)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, 1/
2 - (I/2)*x])/(a*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5458 deep
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 114, normalized size = 0.77

method	result	size
risch	$\frac{\frac{2}{39}x^4+\frac{2}{13}ix^3-\frac{16}{117}x^2-\frac{40}{117}}{(x+i)^3 a^4 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{39(a^2)^{\frac{1}{4}} a^4 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/117*(9*I*x^3+3*x^4-20-8*x^2)/(x+I)^3/a^4/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^
(1/4)-1/39/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(-1+I*x)
*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(17/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/117*(2*(3*x^3 + 12*I*x^2 - 20*x - 20*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3
/4) + 117*(a^6*x^4 + 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)*integral(-1
/39*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(a^6*x^4 + 4*
I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{17/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(1/4)), x)
```

3.1178 $\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=256

$$-\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{i\log\left(1+\frac{\sqrt{a-}}{\sqrt{a-}}\right.}{}$$

```
[Out] -I*(a-I*a*x)^(1/4)*(a+I*a*x)^(3/4)/a-1/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)
/(a+I*a*x)^(1/4))*2^(1/2)+1/2*I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^
(1/4))*2^(1/2)-1/4*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)
^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)+1/4*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x
)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (I*ArcTan[1 + (Sqrt[2]*(a - I*a
*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqr
t[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((
I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+2i\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx,x,\sqrt[4]{a-iax}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+2i\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+i\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\right.$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{1}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{1}{2}i\text{Subst}$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}+\frac{i\log\left(1+\frac{\sqrt{a}}{\sqrt{a}}\right.}{}$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-ia}}{\sqrt[4]{a+iax}}\right.}{\sqrt{2}}$$

Mathematica [A]
time = 0.35, size = 111, normalized size = 0.43

$$\frac{\sqrt[4]{-i+x}\,\sqrt[4]{a-iax}\,\left((-i+x)^{3/4}\sqrt[4]{i+x}\,+i\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+i\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{\sqrt[4]{i+x}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-I + x)^(1/4)*(a - I*a*x)^(1/4)*((-I + x)^(3/4)*(I + x)^(1/4) + I*ArcTan[
(I + x)^(1/4)/(-I + x)^(1/4)] + I*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)]))/(
(I + x)^(1/4)*(a + I*a*x)^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.43, size = 479, normalized size = 1.87

method	result
risch	$\frac{i(x-i)(x+i)(-a(ix-1))^{\frac{1}{4}}}{(ix-1)(a(ix+1))^{\frac{1}{4}}} - \frac{\left(\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(-\frac{\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)x^2+i\,\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2ix+\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] I*(x-I)*(x+I)*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)-(1/2*RootOf(_Z
^2-I)*ln(-((1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2+I*RootOf(_Z^2-I)*
(1-x^4-2*I*x^3-2*I*x)^(3/4)+x^3+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+2*I*RootOf(
_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+2*I*x^2-(1-x^4-2*I*x^3-2*I*x)^(1/2)-R
ootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-x)/(-1+I*x)^2)+1/2*I*RootOf(_Z^2-
I)*ln(-(I*(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(
1-x^4-2*I*x^3-2*I*x)^(1/4)*x+x^3+RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)
-I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x-I*RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/
4)+2*I*x^2+(1-x^4-2*I*x^3-2*I*x)^(1/2)-x)/(-1+I*x)^2))*(-a*(-1+I*x))^(1/4)/
(-1+I*x)*(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

Fricas [A]

time = 0.31, size = 194, normalized size = 0.76

$$\frac{\sqrt{i}\, a \log\left(\frac{\sqrt{i}\,(ax-i\,a)+(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right) - \sqrt{i}\, a \log\left(-\frac{\sqrt{i}\,(ax-i\,a)-(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right) + \sqrt{-i}\, a \log\left(\frac{\sqrt{-i}\,(ax-i\,a)+(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right) - \sqrt{-i}\, a \log\left(-\frac{\sqrt{-i}\,(ax-i\,a)-(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right) - 2i\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(I)*a*log((sqrt(I)*(a*x - I*a) + (I*a*x + a)^(3/4)*(-I*a*x + a)^(1
/4))/(x - I)) - sqrt(I)*a*log(-(sqrt(I)*(a*x - I*a) - (I*a*x + a)^(3/4)*(-I
*a*x + a)^(1/4))/(x - I)) + sqrt(-I)*a*log((sqrt(-I)*(a*x - I*a) + (I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-I)*a*log(-(sqrt(-I)*(a*x - I
*a) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 2*I*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/a
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(1/4), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4),x)
```

```
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4), x)
```

3.1179 $\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=233

$$-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}\,a}$$

```
[Out] -1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*
x)^(1/2))/a*2^(1/2)+1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I
*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(
a+I*a*x)^(1/4))*2^(1/2)/a+I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4
))*2^(1/2)/a
```

Rubi [A]

time = 0.10, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a -
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$
$$= \frac{(4i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= -\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a}$$
$$= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} -$$

Mathematica [A]
time = 0.15, size = 77, normalized size = 0.33

$$\frac{2\sqrt[4]{-1}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)-i\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
[Out] (2*(-1)^(1/4)*(ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)] -
I*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]))/a
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]')
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{3}{4}} \left(iax+a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A]
time = 0.32, size = 221, normalized size = 0.95

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}+2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}-2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}+2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}-2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 1/2*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3
/4)*(-I*a*x + a)^(1/4))/(x - I)) - 1/2*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I*a
^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 1/2*
sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/(x - I)) - 1/2*sqrt(-4*I/a^2)*log(-1/2*((a^2*x - I*a^
2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\,\left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)), x)
```

3.1180 $\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

```
[Out] -2/3*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 1.00

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]')

[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep

Maple [**A**]
time = 0.16, size = 31, normalized size = 0.94

method	result	size
risch	$\frac{\frac{2x}{3}-\frac{2i}{3}}{a(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)

[Out] 2/3/a/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(x-I)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)

Fricas [**A**]
time = 0.29, size = 31, normalized size = 0.94

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,(a^3x+i\,a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")

[Out] 2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x + I*a^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,\left(-ia\,(x+i)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x)
[Out] Could not integrate
```

Mupad [B]
time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(x-\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{3\,a^2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(1/4)),x)
[Out] -(2*(x - 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^2*(x*1i - 1)*(a*(x*1i + 1))^(1/4))
```

3.1181 $\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=67

$$-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}$$

```
[Out] -2/7*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(7/4)-4/21*I*(a+I*a*x)^(3/4)/a^3/(a-I*
a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I
*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}} + \frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{7a}$$
$$= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}} - \frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.67

$$\frac{2(5-2ix)(a+iax)^{3/4}}{21a^3(i+x)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
[Out] (2*(5 - (2*I)*x)*(a + I*a*x)^(3/4))/(21*a^3*(I + x)*(a - I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.66

method	result	size
risch	$\frac{\frac{4}{21}x^2+\frac{2}{7}ix+\frac{10}{21}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
[Out] 2/21/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(2*x^2+5+3*I*x)/(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [A]
time = 0.31, size = 44, normalized size = 0.66

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+5i)}{21\,(a^4x^2+2i\,a^4x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 2/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + 5*I)/(a^4*x^2 + 2*I*a^4*x
 - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x)
[Out] Could not integrate
```

Mupad [B]
time = 0.67, size = 46, normalized size = 0.69

$$-\frac{(-a\,\left(-1+x\,1\mathrm{i}\right))^{1/4}\,\left(2\,x^2+x\,3\mathrm{i}+5\right)\,2\mathrm{i}}{21\,a^3\,{\left(-1+x\,1\mathrm{i}\right)}^2\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(1/4)),x)
[Out] -((-a*(x*1i - 1))^(1/4)*(x*3i + 2*x^2 + 5)*2i)/(21*a^3*(x*1i - 1)^2*(a*(x*1
i + 1))^(1/4))
```

3.1182 $\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}$$

```
[Out] -2/11*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(11/4)-8/77*I*(a+I*a*x)^(3/4)/a^3/(a-I*a*x)^(7/4)-16/231*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(3/4)
```

Rubi [A]

time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) - (((8*I)/77)*(a + I*a*x)^(3/4))/(a^3*(a - I*a*x)^(7/4)) - (((16*I)/231)*(a + I*a*x)^(3/4))/(a^4*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} + \frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{11a}$$
$$= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} - \frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}} + \frac{8\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{77a^2}$$
$$= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} - \frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}} - \frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.52

$$\frac{2(a+iax)^{3/4}\,(41i+28x-8ix^2)}{231a^4(i+x)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
[Out] (2*(a + I*a*x)^(3/4)*(41*I + 28*x - (8*I)*x^2))/(231*a^4*(I + x)^2*(a - I*a
*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Maple [A]
time = 0.16, size = 50, normalized size = 0.50

method	result	size
risch	$\frac{\frac{16}{231}x^3+\frac{40}{231}ix^2-\frac{26}{231}x+\frac{82}{231}i}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
[Out] 2/231/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(20*I*x^2+8*x^3-13*x+41*I)/
(x+I)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

Fricas [**A**]
time = 0.30, size = 57, normalized size = 0.57

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^{2}+28i\,x-41)}{231\,(a^{5}x^{3}+3i\,a^{5}x^{2}-3\,a^{5}x-i\,a^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 2/231*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 28*I*x - 41)/(a^5*x^3 +
 3*I*a^5*x^2 - 3*a^5*x - I*a^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(1/4),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x)
[Out] Could not integrate
```

Mupad [**B**]
time = 0.75, size = 51, normalized size = 0.51

$$\frac{(x-\mathrm{i})^{4}\,(-a\,\,(-1+x\,1\mathrm{i}))^{1/4}\,\,(8\,x^{2}+x\,28\mathrm{i}-41)\,\,2\mathrm{i}}{231\,a^{4}\,(x^{2}+1)^{3}\,(a\,\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] ((x - 1i)^4*(-a*(x*1i - 1))^(1/4)*(x*28i + 8*x^2 - 41)*2i)/(231*a^4*(x^2 +
1)^3*(a*(x*1i + 1))^(1/4))
```

3.1183 $\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=133

$$-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}$$

```
[Out] -2/15*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(15/4)-4/55*I*(a+I*a*x)^(3/4)/a^3/(a-
I*a*x)^(11/4)-16/385*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(7/4)-32/1155*I*(a+I*a
*x)^(3/4)/a^5/(a-I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {47, 37}

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) - (((4*I)/55)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(11/4)) - (((16*I)/385)*(a + I*a*x)^(3/4))/
(a^4*(a - I*a*x)^(7/4)) - (((32*I)/1155)*(a + I*a*x)^(3/4))/(a^5*(a - I*a*x
)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} + \frac{2\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx}{5a}$$

$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} + \frac{8\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{55a^2}$$

$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} + \frac{16\int \frac{}{(a-iax)}}{3}$$

$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} - \frac{32i(a+i}{1155a^5(a-}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.43

$$\frac{2(a+iax)^{3/4}\left(-159+138ix+72x^2-16ix^3\right)}{1155a^5(i+x)^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(-159 + (138*I)*x + 72*x^2 - (16*I)*x^3))/(1155*a^5*(I
 + x)^3*(a - I*a*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7772 deep
```

Maple [A]
time = 0.17, size = 55, normalized size = 0.41

method	result	size
risch	$\frac{\frac{32}{1155}x^4+\frac{16}{165}ix^3-\frac{4}{35}x^2-\frac{2}{55}ix-\frac{106}{385}}{a^4(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/a^4/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(56*I*x^3+16*x^4-21*I*x-15
9-66*x^2)/(x+I)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

Fricas [A]
time = 0.30, size = 68, normalized size = 0.51

$$\frac{2\left(16\,x^{3}+72i\,x^{2}-138\,x-159i\right)\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{1155\left(a^{6}x^{4}+4i\,a^{6}x^{3}-6\,a^{6}x^{2}-4i\,a^{6}x+a^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*x^3 + 72*I*x^2 - 138*x - 159*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(
1/4)/(a^6*x^4 + 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(19/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.79, size = 57, normalized size = 0.43

$$-\frac{(x-\mathrm{i})^5\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,\left(-16\,x^3-x^2\,72\mathrm{i}+138\,x+159\mathrm{i}\right)\,2\mathrm{i}}{1155\,a^5\,(x^2+1)^4\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(19/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -((x - 1i)^5*(-a*(x*1i - 1))^(1/4)*(138*x - x^2*72i - 16*x^3 + 159i)*2i)/(1
155*a^5*(x^2 + 1)^4*(a*(x*1i + 1))^(1/4))
```

3.1184 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=256

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\log\left(1+\right.}{}$$

```
[Out] -I*(a-I*a*x)^(3/4)*(a+I*a*x)^(1/4)/a-3/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)
/(a+I*a*x)^(1/4))*2^(1/2)+3/2*I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^
(1/4))*2^(1/2)+3/4*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)
^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)-3/4*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x
)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = $\{52, 65, 338, 303, 1176, 631, 210, 1179, 642\}$

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((3*I)*ArcTan[1 + (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((3*I)/2)*Log[1 + Sqrt[a
- I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
Sqrt[2] - (((3*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a
- I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{1}{2}(3a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-3i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+3i\text{Subst}\left(\int \frac{1}{1}\right.$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{3}{2}i\text{Sub}$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}-\frac{3i\log\left(1+\right.}{}$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{a+}}\right.}{\sqrt{2}}$$

Mathematica [A]
time = 0.41, size = 111, normalized size = 0.43

$$\frac{(-i+x)^{3/4}(a-iax)^{3/4}\left(\sqrt[4]{-i+x}\,(i+x)^{3/4}-3i\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+3i\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{(i+x)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I + x)^(3/4)*(a - I*a*x)^(3/4)*((-I + x)^(1/4)*(I + x)^(3/4) - (3*I)*Arc
Tan[(I + x)^(1/4)/(-I + x)^(1/4)] + (3*I)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1
/4)]))/((I + x)^(3/4)*(a + I*a*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.60, size = 465, normalized size = 1.82

method	result
risch	$-\frac{i(x-i)(x+i)a}{(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}+\frac{\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_Z^{2}-i\right)\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}x^{2}-i\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\left(-x^{4}+2ix\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(x-I)*(x+I)/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*a+(-3/2*RootOf(_Z^2-I)
*ln(-(-RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x^2-I*RootOf(_Z^2-I)*(1-x
^4+2*I*x^3+2*I*x)^(3/4)+x^3+2*I*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*
x+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x-2*I*x^2+RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I
*x)^(1/4)+(1-x^4+2*I*x^3+2*I*x)^(1/2)-x)/(1+I*x)^2)-3/2*I*RootOf(_Z^2-I)*ln
(-(-I*(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(1-x^
4+2*I*x^3+2*I*x)^(1/4)*x+x^3-RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)-I*(
1-x^4+2*I*x^3+2*I*x)^(1/2)*x+I*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)-2
*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)-x)/(1+I*x)^2))/(a*(1+I*x))^(3/4)*(-(-1+I
*x)*(1+I*x)^3)^(1/4)/(-a*(-1+I*x))^(1/4)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

Fricas [A]
time = 0.31, size = 198, normalized size = 0.77

$$\frac{\sqrt{9i}\,a\log\left(\frac{\sqrt{9i}\,(ax+ia)+3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-\sqrt{9i}\,a\log\left(-\frac{\sqrt{9i}\,(ax+ia)-3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)+\sqrt{-9i}\,a\log\left(\frac{\sqrt{-9i}\,(ax+ia)+3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-\sqrt{-9i}\,a\log\left(-\frac{\sqrt{-9i}\,(ax+ia)-3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-2i\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(9*I)*a*log(1/3*(sqrt(9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a
*x + a)^(3/4))/(x + I)) - sqrt(9*I)*a*log(-1/3*(sqrt(9*I)*(a*x + I*a) - 3*(
I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) + sqrt(-9*I)*a*log(1/3*(sqrt(
-9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - sqrt
(-9*I)*a*log(-1/3*(sqrt(-9*I)*(a*x + I*a) - 3*(I*a*x + a)^(1/4)*(-I*a*x + a
)^(3/4))/(x + I)) - 2*I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(a-a\,x\,1\mathrm{i})}^{3/4}}{{(a+a\,x\,1\mathrm{i})}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4), x)
```

3.1185 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=233

$$-\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-\cdots}}{\sqrt[4]{a+i\cdots}}\right.}{\sqrt{2}\,a}$$

```
[Out] 1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x
)^(1/2))/a*2^(1/2)-1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*
a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a
+I*a*x)^(1/4))*2^(1/2)/a+I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)
)*2^(1/2)/a
```

Rubi [A]
time = 0.10, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a +
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx = \frac{(4i)\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$
$$= \frac{(4i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= -\frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a}$$
$$= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} +$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 0.33

$$-\frac{2\sqrt[4]{-1}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)-i\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (-2*(-1)^(1/4)*(ArcTanh[((-1)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)] -
 I*ArcTanh[((-1)^(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)]))/a
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{1}{4}} \left(iax+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [A]
time = 0.31, size = 221, normalized size = 0.95

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(1
/4)*(-I*a*x + a)^(3/4))/(x + I)) - 1/2*sqrt(4*I/a^2)*log(-1/2*((a^2*x + I*a
^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) + 1/2*
sqrt(-4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4
)*(-I*a*x + a)^(3/4))/(x + I)) - 1/2*sqrt(-4*I/a^2)*log(-1/2*((a^2*x + I*a^
2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}} \sqrt[4]{-ia\left(x+i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{1/4}\,(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)), x)
```

3.1186 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}} dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

```
[Out] -2*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Maple [A]
time = 0.16, size = 31, normalized size = 1.00

method	result	size
risch	$\frac{2x-2i}{a(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
[Out] 2/a/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(x-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [A]
time = 0.30, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x+i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x + I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)), x)
```

3.1187 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}$$

```
[Out] -2/5*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(5/4)-4/5*I*(a+I*a*x)^(1/4)/a^3/(a-I*a
*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}} + \frac{2\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{5a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}} - \frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.67

$$\frac{2(3-2ix)\sqrt[4]{a+iax}}{5a^3(i+x)\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
[Out] (2*(3 - (2*I)*x)*(a + I*a*x)^(1/4))/(5*a^3*(I + x)*(a - I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.66

method	result	size
risch	$\frac{\frac{4}{5}x^2+\frac{2}{5}ix+\frac{6}{5}}{a^2(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
[Out] 2/5/a^2/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(2*x^2+3+I*x)/(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [**A**]
time = 0.30, size = 44, normalized size = 0.66

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x+3i)}{5\,(a^4x^2+2i\,a^4x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(2*x + 3*I)/(a^4*x^2 + 2*I*a^4*x -
a^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{(a-a\,x\,1\mathrm{i})}^{9/4}\,{(a+a\,x\,1\mathrm{i})}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)), x)
```

3.1188 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}$$

```
[Out] -2/9*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(9/4)-8/45*I*(a+I*a*x)^(1/4)/a^3/(a-I*
a*x)^(5/4)-16/45*I*(a+I*a*x)^(1/4)/a^4/(a-I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) - (((8*I)/45)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(5/4)) - (((16*I)/45)*(a + I*a*x)^(1/4))/(a^4
*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} + \frac{4\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx}{9a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} - \frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{45a^2}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} - \frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}} - \frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.52

$$\frac{2\sqrt[4]{a+iax}\,(17i+20x-8ix^2)}{45a^4(i+x)^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(a + I*a*x)^(1/4)*(17*I + 20*x - (8*I)*x^2))/(45*a^4*(I + x)^2*(a - I*a*
x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 50, normalized size = 0.50

method	result	size
risch	$\frac{\frac{16}{45}x^3+\frac{8}{15}ix^2+\frac{2}{15}x+\frac{34}{45}i}{a^3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/a^3/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(12*I*x^2+8*x^3+3*x+17*I)/(x
+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

Fricas [A]
time = 0.30, size = 57, normalized size = 0.57

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(8\,x^2+20i\,x-17)}{45\,(a^5x^3+3i\,a^5x^2-3\,a^5x-i\,a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 2/45*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 20*I*x - 17)/(a^5*x^3 +
3*I*a^5*x^2 - 3*a^5*x - I*a^5)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(3/4),x)
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)), x)
```

3.1189 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=112

$$-\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{10a^2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -10/3*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4)-2/3*I*(a-I*a*x)^(5/4)*(a+I*a*x)^(1/
4)/a+10/3*a^2*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))
*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {52, 42, 239, 237}

$$\frac{10a^2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-10*I)/3)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4) - (((2*I)/3)*(a - I*a*x)^(5
/4)*(a + I*a*x)^(1/4))/a + (10*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2
])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx = -\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a} + \frac{1}{3}(5a)\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} - \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a} + \frac{1}{3}\left(5a^2\right)\int \frac{1}{(a-iax)^{3/4}(a+$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} - \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a} + \frac{\left(5a^2\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{}{(a^2+a}}{3(a-iax)^{3/4}(a+iax)}$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} - \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a} + \frac{\left(5a^2\left(1+x^2\right)^{3/4}\right)\int \frac{1}{(1+x^2)^{3/4}}}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} - \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a} + \frac{10a^2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(\right.}{3(a-iax)^{3/4}(a+iax)^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.62

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{9/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
[Out] (((2*I)/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[3/4,
 9/4, 13/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{5}{4}}}{(iax+a)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(3/4), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x + 6*I) + integral(5/3*(I*a*x +
 a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia(x-i))^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(3/4), x)
```

3.1190 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{2a\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4)/a+2*a*(x^2+1)^(3/4)*(cos(1/2*arctan(x)
)^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*
x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {52, 42, 239, 237}

$$\frac{2a\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-2*I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*Ellip
ticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+a\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx\\
&= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{\left(a\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{\left(a\left(1+x^2\right)^{3/4}\right)\int \frac{1}{(1+x^2)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{2a\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.92

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[3/4,
 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-iax + a)^{\frac{1}{4}}}{(iax + a)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(3/4), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] (a*integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^2 + a), x) - 2*I*(I*a
*x + a)^(1/4)*(-I*a*x + a)^(1/4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x + i)}}{(ia\,(x - i))^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(3/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{1/4}}{(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(3/4), x)
```

3.1191 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}} \, dx$

Optimal. Leaf size=43

$$\frac{2 (1+x^2)^{3/4} F\left(\frac{1}{2} \tan^{-1}(x)\middle| 2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 2*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(s
in(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]

time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {42, 239, 237}

$$\frac{2 (x^2+1)^{3/4} F\left(\frac{1}{2} \tan^{-1}(x)\middle| 2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x
)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx &= \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= \frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 1.58

$$
\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}\sqrt[4]{a-iax}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a(a+iax)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((2*I)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 3/4
, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 4.48, size = 73, normalized size = 1.70

$$
\frac{-\text{meijerg}\left[\left\{\left\{\frac{3}{8},\frac{7}{8},1\right\},\left\{\frac{1}{2},\frac{3}{4},\frac{5}{4}\right\}\right\},\left\{\left\{\frac{1}{4},\frac{3}{8},\frac{3}{4},\frac{7}{8},\frac{5}{4}\right\},\{0\}\right\},\frac{\text{exp\_polar}[-3I\text{Pi}]}{x^2}\right]+I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{8},0,\frac{3}{8},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{8},\frac{3}{8}\right\},\left\{-\frac{1}{2},0,\frac{1}{4},0\right\}\right\},\frac{\text{exp\_polar}[-I\text{Pi}]}{x^2}\right]}{4\text{Pi}a^{\frac{3}{2}}\text{Gamma}\left[\frac{3}{4}\right]}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]')
```

```
[Out] (I meijerg[{{-1 / 2, -1 / 8, 0, 3 / 8, 1 / 2, 1}, {}}, {{-1 / 8, 3 / 8}, {-
1 / 2, 0, 1 / 4, 0}}, exp_polar[-I Pi] / x ^ 2] + -1 ^ (1 / 4) meijerg[{{3
/ 8, 7 / 8, 1}, {1 / 2, 3 / 4, 5 / 4}}, {{1 / 4, 3 / 8, 3 / 4, 7 / 8, 5 / 4
}, {0}}, exp_polar[-3 I Pi] / x ^ 2]) / (4 Pi a ^ (3 / 2) Gamma[3 / 4])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(-iax+a\right)^{\frac{3}{4}}\left(iax+a\right)^{\frac{3}{4}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a^2*x^2 + a^2), x)
```

Sympy [**A**]
time = 2.56, size = 100, normalized size = 2.33

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{8},\frac{7}{8},1 & \frac{1}{2},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{3}{8},\frac{3}{4},\frac{7}{8},\frac{5}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{\frac{3i\pi}{4}}}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{8},0,\frac{3}{8},\frac{1}{2},1 & \\ -\frac{1}{8},\frac{3}{8} & -\frac{1}{2},0,\frac{1}{4},0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
[Out] -I*meijerg(((3/8, 7/8, 1), (1/2, 3/4, 5/4)), ((1/4, 3/8, 3/4, 7/8, 5/4), (0
,)), exp_polar(-3*I*pi)/x**2)*exp(3*I*pi/4)/(4*pi*a**(3/2)*gamma(3/4)) + I*
meijerg(((-1/2, -1/8, 0, 3/8, 1/2, 1), ()), ((-1/8, 3/8), (-1/2, 0, 1/4, 0)
), exp_polar(-I*pi)/x**2)/(4*pi*a**(3/2)*gamma(3/4))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(3/4)), x)
```

3.1192 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2/3*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*(cos(1/2*arcta
n(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a/(
a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]

time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 239, 237}

$$\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)
*EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{3a}\\
&= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.85

$$-\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
[Out] (((-2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, 1/2 -
 (I/2)*x])/(a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]')
```

[Out] Timed out

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{7}{4}} \left(iax+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
[Out] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 1/3*(3*(a^3*x + I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x + I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}} \left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(3/4),x)
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{7/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(3/4)), x)
```

3.1193 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=115

$$-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2/7*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(7/4)-2/7*I*(a+I*a*x)^(1/4)/a^3/(a-I*a
*x)^(3/4)+2/7*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))
*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 239, 237}

$$-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(7/4)) - (((2*I)/7)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x
]/2, 2])/(7*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}+\frac{3\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx}{7a}\\
&= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{7a^2}\\
&= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{(a^2+a^2x^2)^{3/4}\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{(1+x^2)^{3/4}\int \frac{1}{(1+x^2)^{3/4}}\,dx}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.61

$$-\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]')
[Out] Timed out
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{11}{4}} \left(iax+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
[Out] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 1/7*(7*(a^4*x^2 + 2*I*a^4*x - a^4)*integral(1/7*(I*a*x + a)^(1/4)*(-I*a*x +
 a)^(1/4)/(a^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x +
 2*I))/(a^4*x^2 + 2*I*a^4*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}} \left(-ia\left(x+i\right)\right)^{\frac{11}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(3/4),x)
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{11/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(3/4)), x)
```

3.1194 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=291

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}+\frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

```
[Out] 4/3*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(3/4)+7/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(1/
4)/a+7/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-7/2*I*
arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-7/4*I*ln(1-(a-I*a
*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)+
7/4*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x
)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {49, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}\,(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) + (((7*I)/3)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + ((7*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4)
)/(a + I*a*x)^(1/4)])/Sqrt[2] - ((7*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((7*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a
+ I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((7*I
)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 303

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 338

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m + 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegersQ[m, p + (m + 1)/n]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} - \frac{7}{3}\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{1}{2}(7a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-ia$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + 7i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) -$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-}}{\sqrt[4]{a+}}$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}$$
$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + \frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{7i\tan^{-}}{}$$

Mathematica [A]

time = 0.64, size = 121, normalized size = 0.42

$$-\frac{(a-iax)^{3/4}\left((i+x)^{3/4}(-11i+3x)-21i(-i+x)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+21i(-i+x)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{3(i+x)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] -1/3*((a - I*a*x)^(3/4)*((I + x)^(3/4)*(-11*I + 3*x) - (21*I)*(-I + x)^(3/4
)*ArcTan[(I + x)^(1/4)/(-I + x)^(1/4)] + (21*I)*(-I + x)^(3/4)*ArcTanh[(I +
 x)^(1/4)/(-I + x)^(1/4)]))/((I + x)^(3/4)*(a + I*a*x)^(3/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]')
```

```
[Out] Timed out
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.61, size = 465, normalized size = 1.60

method	result
risch	$\frac{i(3x^2-8ix+11)a}{3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}+\frac{\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2-x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4+\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*(-8*I*x+3*x^2+11)/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*a+(-7/2*RootO
f(_Z^2+I)*ln((-(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-x^3+I*RootOf(
_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+2*I*Roo
tOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x+2*I*x^2+(1-x^4+2*I*x^3+2*I*x)^(1/
2)+RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)+x)/(1+I*x)^2)-7/2*I*RootOf(_Z
^2+I)*ln((-I*(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I
)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x-x^3-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+RootOf(
_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^
```

(1/4)+2*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)+x)/(1+I*x)^2))/(a*(1+I*x))^(3/4)*
(-(-1+I*x)*(1+I*x)^3)^(1/4)/(-a*(-1+I*x))^(1/4)*a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(7/4), x)
```

Fricas [A]
time = 0.31, size = 235, normalized size = 0.81

$$-\frac{3\sqrt{49i}\,(ax-ia)\log\left(\frac{\sqrt{49i}\,(ax+ia)+7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)-3\sqrt{49i}\,(ax-ia)\log\left(-\frac{\sqrt{49i}\,(ax+ia)-7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)+3\sqrt{-49i}\,(ax-ia)\log\left(\frac{\sqrt{-49i}\,(ax+ia)+7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)-3\sqrt{-49i}\,(ax-ia)\log\left(-\frac{\sqrt{-49i}\,(ax+ia)-7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)+2\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}(-3ix-11)}{6\,(ax-ia)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(49*I)*(a*x - I*a)*log(1/7*(sqrt(49*I)*(a*x + I*a) + 7*(I*a*x +
 a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*sqrt(49*I)*(a*x - I*a)*log(-1/7*
(sqrt(49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
+ 3*sqrt(-49*I)*(a*x - I*a)*log(1/7*(sqrt(-49*I)*(a*x + I*a) + 7*(I*a*x + a
)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*sqrt(-49*I)*(a*x - I*a)*log(-1/7*(
sqrt(-49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
+ 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(-3*I*x - 11))/(a*x - I*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(7/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4), x)
```

3.1195 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=266

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}+\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log\left(1+\frac{\sqrt{a-ia}}{\sqrt{a+ia}}\right.}{\sqrt{}}$$

```
[Out] 4/3*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(3/4)-1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/
(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)+1/2*I*ln(1+(a-I*
a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/
2)+I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a-I*arctan(1
+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a
```

Rubi [A]

time = 0.10, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {49, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (I*Sqrt[2]*ArcTan[1 -
 (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 +
(Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a
*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt
[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^
(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a}}{\sqrt[4]{a}}\right.}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,\right.}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\right.}{\sqrt{2}\,a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{a}$$

Mathematica [A]
time = 0.19, size = 112, normalized size = 0.42

$$\frac{2\left(\frac{2i(a-iax)^{3/4}}{(a+iax)^{3/4}}+3\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)-3(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (2*(((2*I)*(a - I*a*x)^(3/4))/(a + I*a*x)^(3/4) + 3*(-1)^(1/4)*ArcTanh[((-1
)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)] - 3*(-1)^(3/4)*ArcTanh[((-1)^
(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)]))/(3*a)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.30, size = 455, normalized size = 1.71

method	result
risch	$\frac{\frac{4x}{3}+\frac{4i}{3}}{(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}-\frac{\left(\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2-x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4+2ix\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*(x+I)/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)-(RootOf(_Z^2+I)*ln((-(1-x^4
+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-x^3+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^
3+2*I*x)^(3/4)+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+2*I*RootOf(_Z^2+I)*(1-x^4+2*
I*x^3+2*I*x)^(1/4)*x+2*I*x^2+(1-x^4+2*I*x^3+2*I*x)^(1/2)+RootOf(_Z^2+I)*(1-
x^4+2*I*x^3+2*I*x)^(1/4)+x)/(1+I*x)^2)+I*RootOf(_Z^2+I)*ln((-I*(1-x^4+2*I*x
^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(
1/4)*x-x^3-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*
I*x)^(3/4)+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)+2*I*x^2-(1-x^4+2*I*
x^3+2*I*x)^(1/2)+x)/(1+I*x)^2))/(a*(1+I*x))^(3/4)*(-(-1+I*x)*(1+I*x)^3)^(1/
4)/(-a*(-1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

Fricas [A]
time = 0.31, size = 298, normalized size = 1.12

$$-\frac{3\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-3\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)+3\left(a^2x-i\,a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+i\,a^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-3\left(a^2x-i\,a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+i\,a^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-8\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{6\left(a^2x-i\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2*x - I*a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2
) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*(a^2*x - I*a^2)*sq
rt(4*I/a^2)*log(-1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(
-I*a*x + a)^(3/4))/(x + I)) + 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(1/2*((a^
2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x +
I)) - 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x + I*a^2)*sqrt(-4*I/
a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 8*(I*a*x + a)^(1/
4)*(-I*a*x + a)^(3/4))/(a^2*x - I*a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(7/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4), x)
```

3.1196 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

```
[Out] 2/3*I*(a-I*a*x)^(3/4)/a^2/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{37\}$

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx=\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Maple [**A**]
time = 0.15, size = 31, normalized size = 0.94

method	result	size
risch	$\frac{\frac{2x}{3}+\frac{2i}{3}}{a\left(a\left(ix+1\right)\right)^{\frac{3}{4}}\left(-a\left(ix-1\right)\right)^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
[Out] 2/3/a/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(x+I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [**A**]
time = 0.30, size = 31, normalized size = 0.94

$$\frac{2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}{3\left(a^{3}x-i\,a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x - I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)),x)
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)), x)
```

3.1197 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=65

$$-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}$$

```
[Out] -2*I/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4)+4/3*I*(a-I*a*x)^(3/4)/a^3/(a+I*a*x
)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {47, 37}

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(
3/4))/(a^3*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{2\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx}{a}$$
$$= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.69

$$\frac{2(1+2ix)(a-iax)^{3/4}}{3a^3(i+x)(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
[Out] (2*(1 + (2*I)*x)*(a - I*a*x)^(3/4))/(3*a^3*(I + x)*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.15, size = 33, normalized size = 0.51

method	result	size
risch	$\frac{-\frac{2i}{3}+\frac{4x}{3}}{a^2(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
[Out] 2/3/a^2/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(-I+2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [A]
time = 0.29, size = 36, normalized size = 0.55

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x-i)}{3\,(a^4x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(2*x - I)/(a^4*x^2 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)),x)
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)), x)
```

3.1198 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}-\frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}$$

```
[Out] -2/5*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4)-8/5*I/a^3/(a-I*a*x)^(1/4)/(a+I*a
*x)^(3/4)+16/15*I*(a-I*a*x)^(3/4)/a^4/(a+I*a*x)^(3/4)
```

Rubi [A]

time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {47, 37}

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) - ((8*I)/5)/(a^3*(a -
I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((16*I)/15)*(a - I*a*x)^(3/4))/(a^4*(a +
 I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} + \frac{4\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx}{5a}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{8\int \frac{}{\sqrt[4]{a-iax}}}{5a}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{16i(a-iax)}{15a^4(a+iax}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.52

$$\frac{2i(a-iax)^{3/4}\left(7+4ix+8x^2\right)}{15a^4(i+x)^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/15)*(a - I*a*x)^(3/4)*(7 + (4*I)*x + 8*x^2))/(a^4*(I + x)^2*(a + I*
a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.44

method	result	size
risch	$\frac{\frac{16}{15}x^2+\frac{8}{15}ix+\frac{14}{15}}{a^3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^3/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(8*x^2+4*I*x+7)/(x+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.30, size = 56, normalized size = 0.56

$$\frac{2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}\left(8\,x^2+4i\,x+7\right)}{15\left(a^5x^3+i\,a^5x^2+a^5x+i\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/15*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 4*I*x + 7)/(a^5*x^3 + I*
a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{9/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)),x)
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)), x)
```

3.1199 $\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}} \, dx$

Optimal. Leaf size=139

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-\frac{10a^2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 4/3*I*(a-I*a*x)^(9/4)/a/(a+I*a*x)^(3/4)+10*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4
)+2*I*(a-I*a*x)^(5/4)*(a+I*a*x)^(1/4)/a-10*a^2*(x^2+1)^(3/4)*(cos(1/2*arcta
n(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-
I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 52, 42, 239, 237}

$$-\frac{10a^2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a+iax}\,(a-iax)^{5/4}}{a}+10i\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(9/4))/(a*(a + I*a*x)^(3/4)) + (10*I)*(a - I*a*x)^(1
/4)*(a + I*a*x)^(1/4) + ((2*I)*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))/a - (10
*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a
*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] && !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} - 3\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx \\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - (5a)\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx \\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \left(5a^2\right)\int \frac{}{(a-}\ldots \\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{\left(5a^2\left(a^2+a^2x\ldots\right.\right.}{(a-iax\ldots} \\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{\left(5a^2\left(1+x^2\right)^{3}\ldots\right.}{(a-iax)^{3/}\ldots} \\
&= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{10a^2\left(1+x^2\right)^{3}\ldots}{(a-iax)\ldots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.50

$$
\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{13/4}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13a^2(a+iax)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/13)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(13/4)*Hypergeometric2F1[7/4, 1
3/4, 17/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{9}{4}}}{(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(9/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

[Out] 1/3*(3*(x - I)*integral(-5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x^2 + 11*I*x + 20))/(x - I)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{9}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)

[Out] Integral((-I*a*(x + I))**(9/4)/(I*a*(x - I))**(7/4), x)

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)

[Out] Could not integrate

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{9/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a*x*1i)^(9/4)/(a + a*x*1i)^(7/4),x)

[Out] int((a - a*x*1i)^(9/4)/(a + a*x*1i)^(7/4), x)

3.1200 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=113

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{10a\,(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 4/3*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(3/4)+10/3*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1
/4)/a-10/3*a*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*
EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]

time = 0.02, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 52, 42, 239, 237}

$$-\frac{10a\,(x^2+1)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{3(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(3/4)) + (((10*I)/3)*(a - I*a*
x)^(1/4)*(a + I*a*x)^(1/4))/a - (10*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{1}{3}(5a)\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{\left(5a\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{\left(5a\left(1+x^2\right)^{3/4}\right)\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{10a\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.62

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{9/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[7/4, 9/4
, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-iax + a)^{\frac{5}{4}}}{(iax + a)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 1/3*(3*(a*x - I*a)*integral(-5/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^
2 + a), x) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(-3*I*x - 7))/(a*x - I*
a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(7/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(7/4), x)
```

3.1201 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=79

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 4/3*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(3/4)-2/3*(x^2+1)^(3/4)*(cos(1/2*arctan(x
))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a
*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {49, 42, 239, 237}

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(3/4)) - (2*(1 + x^2)^(3/4)*El
lipticF[ArcTan[x]/2, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx &= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{1}{3}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx\\ &= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.89

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
[Out] ((I/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 7/4
, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x - I*a^2)*integral(-1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(
a^2*x^2 + a^2), x) + 4*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^2*x - I*a^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(7/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{1/4}}{(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(7/4), x)
```

3.1202 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 2/3*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*(cos(1/2*arctan
(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a/(a
-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 239, 237}

$$\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*
EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{3a}\\ &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{(a^2+a^2x^2)^{3/4}\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{(1+x^2)^{3/4}\int \frac{1}{(1+x^2)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 0.83

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\sqrt[4]{a-iax}\;{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
[Out] (I*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 7/4, 5/
4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{3}{4}} \left(iax+a\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 1/3*(3*(a^3*x - I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x - I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}} \left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{3/4}\,(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(7/4)), x)
```

3.1203 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}} dx$

Optimal. Leaf size=81

$$\frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 2/3*x/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*(cos(1/2*arctan
(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a^2/
(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {42, 205, 239, 237}

$$\frac{2\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (2*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*Elli
pticF[ArcTan[x]/2, 2])/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx &= \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{7/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.86

$$-\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
[Out] ((-1/3*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 7/4, 1/4, 1/2 - (
I/2)*x])/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 23.10, size = 74, normalized size = 0.91

$$\frac{\frac{I}{3}\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{3}{8},\frac{1}{2},\frac{7}{8},1\right\},\{\}\right\},\left\{\left\{\frac{3}{8},\frac{7}{8}\right\},\left\{-\frac{1}{2},0,\frac{5}{4},0\right\}\right\},\frac{\text{exp\_polar}[-I\text{Pi}]}{x^2}\right]}{\text{Pi}a^{\frac{7}{2}}\text{Gamma}\left[\frac{3}{4}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]')
```

```
[Out] (-1 ^ (1 / 4) meijerg[{{7 / 8, 11 / 8, 1}, {1 / 2, 7 / 4, 9 / 4}}, {{7 / 8,
 5 / 4, 11 / 8, 7 / 4, 9 / 4}, {0}}, exp_polar[-3 I Pi] / x ^ 2] + I meijer
g[{{-1 / 2, 0, 3 / 8, 1 / 2, 7 / 8, 1}, {}}, {{3 / 8, 7 / 8}, {-1 / 2, 0, 5
 / 4, 0}}, exp_polar[-I Pi] / x ^ 2]) / (3 Pi a ^ (7 / 2) Gamma[3 / 4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{7}{4}} \left(iax+a\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
[Out] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 1/3*(3*(a^4*x^2 + a^4)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*x)/(a^4*x^2 + a^
4)
```

Sympy [A]
time = 26.80, size = 95, normalized size = 1.17

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{8},\frac{11}{8},1 & \frac{1}{2},\frac{7}{4},\frac{9}{4}\\ \frac{7}{8},\frac{5}{4},\frac{11}{8},\frac{7}{4},\frac{9}{4} & 0\end{matrix}\middle|\,\frac{e^{-3i\pi}}{x^{2}}\right)e^{-\frac{i\pi}{4}}}{4\pi a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{3}{8},\frac{1}{2},\frac{7}{8},1 & \\ \frac{3}{8},\frac{7}{8} & -\frac{1}{2},0,\frac{5}{4},0\end{matrix}\middle|\,\frac{e^{-i\pi}}{x^{2}}\right)}{4\pi a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] -I*meijerg((((7/8, 11/8, 1), (1/2, 7/4, 9/4)), ((7/8, 5/4, 11/8, 7/4, 9/4),
(0,)), exp_polar(-3*I*pi)/x**2)*exp(-I*pi/4)/(4*pi*a**(7/2)*gamma(7/4)) + I
*meijerg(((-1/2, 0, 3/8, 1/2, 7/8, 1), ()), ((3/8, 7/8), (-1/2, 0, 5/4, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*a**(7/2)*gamma(7/4))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{7/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(7/4)), x)
```

3.1204 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}} \, dx$

Optimal. Leaf size=114

$$-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10\,(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4)+10/21*x/a^3/(a-I*a*x)^(3/4)/(a+I
*a*x)^(3/4)+10/21*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan
(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a^3/(a-I*a*x)^(3/4)/(a+I*a*x)^(3
/4)
```

Rubi [A]
time = 0.02, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 239, 237}

$$\frac{10\,(x^2+1)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(21*a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(21*a^3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{5\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx}{7a} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{\left(5\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{(a^2+a^2x^2)^{7/4}}\,dx}{7a(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{\left(5\left(a^2+\right.\right.}{21a^3(a} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{\left(5\left(1+x\right.\right.}{21a^3(a-} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10\left(1+x^2\right.}{21a^3(a-}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.61

$$-\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{7}{4};-\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

[Out] ((-1/7*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 7/4, -3/4, 1/2 - (I/2)*x])/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]')

[Out] Timed out

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{11}{4}} \left(iax+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)

[Out] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")

[Out] 1/21*(21*(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)*integral(5/21*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a^5*x^2 + a^5), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(5*x^2 + 5*I*x + 3))/(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1\mathrm{i}\right)}^{11/4}\,{\left(a+a\,x\,1\mathrm{i}\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(7/4)), x)
```

3.1205 $\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=147

$$-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}-\frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10\left(1+x^2\right)^{3}}{33a^4(a-}$$

```
[Out] -2/11*I/a^2/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4)-2/11*I/a^3/(a-I*a*x)^(7/4)/(a+
I*a*x)^(3/4)+10/33*x/a^4/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)+10/33*(x^2+1)^(3/4
)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(
x)),2^(1/2))/a^4/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 239, 237}

$$\frac{10\left(x^2+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/11)/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)) - ((2*I)/11)/(a^3*(a
 - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(33*a^4*(a - I*a*x)^(3/4)*(a +
I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/(33*a^4*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} + \frac{7\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx}{11a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{5\int \frac{}{(a-i}}{}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{\left(5\,(a^2\right.}{11a^2}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{}{33a^4(a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{}{33a^4(a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{}{33a^4(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.48

$$-\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{7}{4};-\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-1/11*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-11/4, 7/4, -7/4, 1/2
- (I/2)*x])/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{15}{4}}(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/33*(33*(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*x - a^6)*integral(5/33*(I*a*x + a
)^(1/4)*(-I*a*x + a)^(1/4)/(a^6*x^2 + a^6), x) + 2*(5*x^3 + 10*I*x^2 - 2*x
+ 6*I)*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a
^6*x - a^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{15/4}\,(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(7/4)), x)
```

3.1206 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=137

$$-\frac{14ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{14a\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -14*a*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+4*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(1/
4)+14/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(3/4)/a+14*a*(x^2+1)^(1/4)*(cos(1/2*arc
tan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/(
a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {49, 52, 42, 235, 233, 202}

$$\frac{14a\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a+iax)^{3/4}(a-iax)^{3/4}}{3a}-\frac{14ax}{\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (-14*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(7/4))
/(a*(a + I*a*x)^(1/4)) + (((14*I)/3)*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))/a
 + (14*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a +
 I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx &= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} - 7\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - (7a)\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - \frac{\left(7a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - \frac{\left(7a\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= -\frac{14ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} + \frac{\left(7a\sqrt[4]{1+x^2}\right.}{\sqrt[4]{a-iax}} \\
&= -\frac{14ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} + \frac{14a\sqrt[4]{1+x^2}}{\sqrt[4]{a-ia}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.51

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{11/4}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[5/4, 1
1/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 96, normalized size = 0.70

method	result	size
risch	$\frac{2i(x^2-12ix+13)a}{3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{7x\,\mathrm{hypergeom}([\frac{1}{4},\frac{1}{2}],[\frac{3}{2}],-x^2)a(-a^2(ix-1)(ix+1))^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(x^2+13-12*I*x)*a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-7/(a^2)^(1/4)
*x*hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+
I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(5/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
[Out] -1/3*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(-I*x^2 + 8*x - 21*I) - 3*(a*x
^2 - I*a*x)*integral(-14*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^
2), x))/(a*x^2 - I*a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{7/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(5/4),x)
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(5/4), x)
```

3.1207 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=102

$$-\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -6*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+4*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(1/4)+
6*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(s
in(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]

time = 0.01, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 42, 235, 233, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (-6*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(3/4))/(a
*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I
*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-3\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx\\
&= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}-\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= -\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= -\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.69

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{7/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[5/4, 7/4
, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Maple [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 88, normalized size = 0.86

method	result	size
risch	$\frac{4x+4i}{(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
[Out] 4*(x+I)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+
I*x))^(1/4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(5/4), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
[Out] -(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x - 3*I) - (a^2*x^2 - I*a^2*x)*in
tegral(-6*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2), x))/(a^
2*x^2 - I*a^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(5/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(5/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(5/4), x)
```

3.1208 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2
)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*x
)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {50, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Elliptic
E[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 50

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{5/4}}\,dx &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+a\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.90

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
[Out] ((I/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 5/4
, 7/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 94, normalized size = 1.21

method	result	size
risch	$\frac{2x+2i}{a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x+I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] ((a^3*x^2 - I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x) + 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 - I
*a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{5}{4}}\sqrt[4]{-ia(x+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(5/4)), x)
```

3.1209 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(s
in(1/2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(a^2*(a - I*a*x)^(1/4)*(a + I
*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$
$$= \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$
$$= \frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 1.48

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, 1/2 - (I/2)
*x])/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 7.73, size = 72, normalized size = 1.57

$$\frac{-\text{meijerg}\left[\left\{\left\{\frac{5}{8},\frac{9}{8},1\right\},\left\{\frac{1}{2},\frac{5}{4},\frac{7}{4}\right\}\right\},\left\{\left\{\frac{5}{8},\frac{3}{4},\frac{9}{8},\frac{5}{4},\frac{7}{4}\right\},\{0\}\right\},\frac{\text{exp\_polar}[-3I\text{Pi}]}{x^2}\right]+I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{8},\frac{1}{2},\frac{5}{8},1\right\},\{\}\right\},\left\{\left\{\frac{1}{8},\frac{5}{8}\right\},\left\{-\frac{1}{2},0,\frac{3}{4},0\right\}\right\},\frac{\text{exp\_polar}[-I\text{Pi}]}{x^2}\right]}{\text{Pi}a^{\frac{5}{2}}\text{Gamma}\left[\frac{1}{4}\right]}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]')
```

```
[Out] (I meijerg[{{-1 / 2, 0, 1 / 8, 1 / 2, 5 / 8, 1}, {}}, {{1 / 8, 5 / 8}, {-1
/ 2, 0, 3 / 4, 0}}, exp_polar[-I Pi] / x ^ 2] + -1 ^ (3 / 4) meijerg[{{5 /
8, 9 / 8, 1}, {1 / 2, 5 / 4, 7 / 4}}, {{5 / 8, 3 / 4, 9 / 8, 5 / 4, 7 / 4},
 {0}}, exp_polar[-3 I Pi] / x ^ 2]) / (Pi a ^ (5 / 2) Gamma[1 / 4])
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 91, normalized size = 1.98

method	result	size

risch	$\frac{2x}{a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}} - \frac{x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)(-a^2(ix-1)(ix+1))^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] (2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*x + (a^4*x^2 + a^4)*integral(-(I*a*
x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^4), x))/(a^4*x^2 + a^4)
```

Sympy [A]
time = 6.48, size = 97, normalized size = 2.11

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{8},\frac{9}{8},1 & \frac{1}{2},\frac{5}{4},\frac{7}{4}\\ \frac{5}{8},\frac{3}{4},\frac{9}{8},\frac{5}{4},\frac{7}{4} & 0\end{matrix}\middle|\,\frac{e^{-3i\pi}}{x^2}\right)e^{-\frac{3i\pi}{4}}}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{8},\frac{1}{2},\frac{5}{8},1 & \\ \frac{1}{8},\frac{5}{8} & -\frac{1}{2},0,\frac{3}{4},0\end{matrix}\middle|\,\frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] -I*meijerg(((5/8, 9/8, 1), (1/2, 5/4, 7/4)), ((5/8, 3/4, 9/8, 5/4, 7/4), (0
,)), exp_polar(-3*I*pi)/x**2)*exp(-3*I*pi/4)/(4*pi*a**(5/2)*gamma(5/4)) + I
```

```
*meijerg(((-1/2, 0, 1/8, 1/2, 5/8, 1), ()), ((1/8, 5/8), (-1/2, 0, 3/4, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*a**(5/2)*gamma(5/4))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(5/4)), x)
```

3.1210 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2/5*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+6/5*(x^2+1)^(1/4)*(cos(1/2*arcta
n(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3
/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 203, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*E
llipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{3\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{5a}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.85

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-1/5*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 107, normalized size = 1.30

method	result	size
risch	$\frac{\frac{6}{5}x^2+\frac{6}{5}ix+\frac{2}{5}}{(x+i)a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
[Out] 2/5*(3*I*x+3*x^2+1)/(x+I)/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/5/(a^
2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/4
)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 3*I*x + 1) + 5*(a^5*x^
3 + I*a^5*x^2 + a^5*x + I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)
^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{9/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(5/4)), x)
```

3.1211 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=115

$$-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2/9*I/a^2/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4)-2/9*I/a^3/(a-I*a*x)^(5/4)/(a+I*a
*x)^(1/4)+2/3*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))
*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)) - ((2*I)/9)/(a^3*(a -
I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
 2])/(3*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}+\frac{5\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx}{9a}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\int \frac{1}{(a-iax)^{5/4}(a+}}{3a^2}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{a^2+a^2x^2}\,\int}{3a^2\sqrt[4]{a-ia}}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\,\int}{3a^4\sqrt[4]{a-iax}}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E(}{3a^4\sqrt[4]{a-ia}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.61

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
[Out] ((-1/9*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 113, normalized size = 0.98

method	result	size
risch	$\frac{\frac{2}{3}x^3+\frac{4}{3}ix^2-\frac{4}{9}x+\frac{4}{9}i}{(x+i)^2a^4(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{3(a^2)^{\frac{1}{4}}a^4(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(6*I*x^2+3*x^3-2*x+2*I)/(x+I)^2/a^4/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/
4)-1/3/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(-1+I*x)*(1+
I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(3*x^3 + 6*I*x^2 - 2*x + 2*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) +
 9*(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*x - a^6)*integral(-1/3*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*
x - a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{13/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(5/4)), x)
```

3.1212 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}} \, dx$

Optimal. Leaf size=287

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

```
[Out] 4*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(1/4)+5*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(3/4)/a
+5/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-5/2*I*arct
an(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)+5/4*I*ln(1-(a-I*a*x)^
(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)-5/4*
I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1
/2))*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {49, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) + ((5*I)*(a - I*a*x)^(1/4)*
(a + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I
*a*x)^(1/4)])/Sqrt[2] - ((5*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
I*a*x)^(1/4)])/Sqrt[2] + (((5*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((5*I)/2)*Log
[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a
*x)^(1/4)])/Sqrt[2]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} - 5\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{1}{2}(5a)\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right.$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 5i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) -$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{5}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-}}{\sqrt[4]{a+}}\right.$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{2\sqrt{2}}$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{5i\tan}{}$$

Mathematica [A]

time = 0.54, size = 117, normalized size = 0.41

$$-\frac{\sqrt[4]{a-iax}\left(\sqrt[4]{i+x}\,(-9i+x)+5i\sqrt[4]{-i+x}\,\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+5i\sqrt[4]{-i+x}\,\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{\sqrt[4]{i+x}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] -(((a - I*a*x)^(1/4)*((I + x)^(1/4)*(-9*I + x) + (5*I)*(-I + x)^(1/4)*ArcTa
n[(I + x)^(1/4)/(-I + x)^(1/4)] + (5*I)*(-I + x)^(1/4)*ArcTanh[(I + x)^(1/4
)/(-I + x)^(1/4)]))/((I + x)^(1/4)*(a + I*a*x)^(1/4)))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.57, size = 480, normalized size = 1.67

method	result
risch	$-\frac{i(x^2-8ix+9)(-a(ix-1))^{\frac{1}{4}}}{(ix-1)(a(ix+1))^{\frac{1}{4}}}-\frac{\left(5\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{-\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2+x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4-\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(x^2+9-8*I*x)*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)-(5/2*RootOf
(_Z^2+I)*ln(-(-(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+x^3+I*RootOf(
_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x-2*I*Roo
tOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+2*I*x^2+(1-x^4-2*I*x^3-2*I*x)^(1/
2)+RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-x)/(-1+I*x)^2)+5/2*I*RootOf(_
Z^2+I)*ln(-(-I*(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+2*RootOf(_Z^2
+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+x^3+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+RootO
f(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)+I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x
```

```
)^(1/4)+2*I*x^2-(1-x^4-2*I*x^3-2*I*x)^(1/2)-x)/(-1+I*x)^2))*(-a*(-1+I*x))^(
1/4)/(-1+I*x)*(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(5/4), x)
```

Fricas [A]
time = 0.30, size = 233, normalized size = 0.81

$$-\frac{\sqrt{25i}\,(ax-i\,a)\log\left(\frac{\sqrt{25i}\,(ax-i\,a)+5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)-\sqrt{25i}\,(ax-i\,a)\log\left(-\frac{\sqrt{25i}\,(ax-i\,a)-5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)+\sqrt{-25i}\,(ax-i\,a)\log\left(\frac{\sqrt{-25i}\,(ax-i\,a)+5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)-\sqrt{-25i}\,(ax-i\,a)\log\left(-\frac{\sqrt{-25i}\,(ax-i\,a)-5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(-i\,x-9)}{2\,(ax-i\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(25*I)*(a*x - I*a)*log(1/5*(sqrt(25*I)*(a*x - I*a) + 5*(I*a*x + a
)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(25*I)*(a*x - I*a)*log(-1/5*(sqr
t(25*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + sq
rt(-25*I)*(a*x - I*a)*log(1/5*(sqrt(-25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-25*I)*(a*x - I*a)*log(-1/5*(sqrt(-25
*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 2*(I*a
*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(-I*x - 9))/(a*x - I*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4), x)
```

3.1213 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=264

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}\right.}{\sqrt{}}$$

```
[Out] 4*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(1/4)+1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a
+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-1/2*I*ln(1+(a-I*a*
x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)
+I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a-I*arctan(1+(
a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {49, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) + (I*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (I*Log[1 + Sqrt[a - I*a*x]/
Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*
a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a}}{\sqrt[4]{a}}\right.}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,\right.}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} + \right.}{\sqrt{2}\,a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{a}$$

Mathematica [A]
time = 0.19, size = 109, normalized size = 0.41

$$\frac{2\left(\frac{2i\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} - \sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) + (-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (2*(((2*I)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4) - (-1)^(1/4)*ArcTanh[((-1)^
(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)] + (-1)^(3/4)*ArcTanh[((-1)^(3/4
)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]))/a
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.15, size = 478, normalized size = 1.81

method	result
risch	$-\frac{4(x+i)(-a(ix-1))^{\frac{1}{4}}}{a(ix-1)(a(ix+1))^{\frac{1}{4}}}+\frac{\left(\operatorname{RootOf}\left(\_Z^2-i\right)\ln\left(-\frac{\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2-i\right)x^2+i\operatorname{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2ix+\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(x+I)/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)+(RootOf(_Z^2-I)*l
n(-((1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2+I*RootOf(_Z^2-I)*(1-x^4-
2*I*x^3-2*I*x)^(3/4)+x^3+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+2*I*RootOf(_Z^2-I)
*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+2*I*x^2-(1-x^4-2*I*x^3-2*I*x)^(1/2)-RootOf(_
Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-x)/(-1+I*x)^2)+I*RootOf(_Z^2-I)*ln(-(I*(
1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(1-x^4-2*I*x
^3-2*I*x)^(1/4)*x+x^3+RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-I*(1-x^4-2
*I*x^3-2*I*x)^(1/2)*x-I*RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)+2*I*x^2+
(1-x^4-2*I*x^3-2*I*x)^(1/2)-x)/(-1+I*x)^2))/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)*
(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

Fricas [A]
time = 0.32, size = 296, normalized size = 1.12

$$-\frac{(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)+(a^2x-i\,a^2)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x-i\,a^2)\sqrt{-\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-(a^2x-i\,a^2)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x-i\,a^2)\sqrt{-\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-8\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(a^2x-i\,a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")

[Out] -1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - (a^2*x - I*a^2)*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 8*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(a^2*x - I*a^2)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)

[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(5/4), x)

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x)

[Out] Could not integrate

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4),x)

[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4), x)

3.1214 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}} \, dx$

Optimal. Leaf size=31

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

[Out] 2*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(1/4)

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]

[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}} \, dx = \frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 1.00

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]

[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Maple [**A**]
time = 0.18, size = 31, normalized size = 1.00

method	result	size
risch	$\frac{2x+2i}{a(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(x+I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [**A**]
time = 0.30, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a^3x-i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x - I*a^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.16, size = 27, normalized size = 0.87

$$\frac{(-a\ (-1+x\,1\mathrm{i}))^{1/4}\,2\mathrm{i}}{a^2\,(a\ (1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] ((-a*(x*1i - 1))^(1/4)*2i)/(a^2*(a*(x*1i + 1))^(1/4))
```

3.1215 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}$$

```
[Out] -2/3*I/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4)+4/3*I*(a-I*a*x)^(1/4)/a^3/(a+I*a
*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*
x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{3a}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.67

$$\frac{2(1-2ix)(a+iax)^{3/4}}{3a^3(-i+x)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
[Out] (2*(1 - (2*I)*x)*(a + I*a*x)^(3/4))/(3*a^3*(-I + x)*(a - I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.15, size = 33, normalized size = 0.49

method	result	size
risch	$\frac{\frac{2i}{3}+\frac{4x}{3}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
[Out] 2/3/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(I+2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [A]
time = 0.30, size = 36, normalized size = 0.54

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+i)}{3\,(a^4x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + I)/(a^4*x^2 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.60, size = 40, normalized size = 0.60

$$-\frac{2\,\left(2\,x+1\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{3\,a^3\,\left(-1+x\,1\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -(2*(2*x + 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^3*(x*1i - 1)*(a*(x*1i + 1))^(1/4
))
```

3.1216 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}-\frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}$$

```
[Out] -2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4)-8/21*I/a^3/(a-I*a*x)^(3/4)/(a+I*
a*x)^(1/4)+16/21*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}\,(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}\,(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) - ((8*I)/21)/(a^3*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((16*I)/21)*(a - I*a*x)^(1/4))/(a^4*(a
+ I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} + \frac{4\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx}{7a}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{8\int \frac{1}{(a-iax)^{3/}}}{2}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{16i\sqrt[4]{a-i}}{21a^4\sqrt[4]{a+}}$$

Mathematica [**A**]
time = 0.10, size = 52, normalized size = 0.52

$$\frac{2(a+iax)^{3/4}\left(i+12x-8ix^2\right)}{21a^4(a-iax)^{3/4}\left(1+x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(I + 12*x - (8*I)*x^2))/(21*a^4*(a - I*a*x)^(3/4)*(1 +
 x^2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.15, size = 44, normalized size = 0.44

method	result	size
risch	$\frac{\frac{16}{21}x^2+\frac{8}{7}ix-\frac{2}{21}}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(8*x^2+12*I*x-1)/(x+I)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.30, size = 56, normalized size = 0.56

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^2+12i\,x-1)}{21\,(a^5x^3+i\,a^5x^2+a^5x+i\,a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 12*I*x - 1)/(a^5*x^3 + I
*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.76, size = 46, normalized size = 0.46

$$-\frac{(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,(8\,x^2+x\,12\mathrm{i}-1)\,2\mathrm{i}}{21\,a^4\,(-1+x\,1\mathrm{i})^2\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -((-a*(x*1i - 1))^(1/4)*(x*12i + 8*x^2 - 1)*2i)/(21*a^4*(x*1i - 1)^2*(a*(x*
1i + 1))^(1/4))
```

3.1217 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}} dx$

Optimal. Leaf size=141

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}+\frac{42x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}-\frac{42\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 4/5*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(5/4)+42/5*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1
/4)-28/5*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(1/4)-42/5*(x^2+1)^(1/4)*(cos(1/2*ar
ctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/
(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 42, 235, 233, 202}

$$-\frac{42\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{42x}{5\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(5/4)) + (42*x)/(5*(a - I*a*x)
^(1/4)*(a + I*a*x)^(1/4)) - (((28*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(
1/4)) - (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4)
*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{7}{5}\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx\\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{21}{5}\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx\\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{\left(21\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{\left(21\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}+\frac{42x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}-\frac{\left(21\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}+\frac{42x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}-\frac{42\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(\ldots\right.}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+ia\ldots}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.50

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{11/4}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]

[Out] ((I/11)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[9/4, 11/4, 15/4, 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]')

[Out] Timed out

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 101, normalized size = 0.72

method	result	size
risch	$-\frac{8(4x^2+ix+3)}{5(x-i)(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{21x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	101

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)

[Out] -8/5*(4*x^2+3+I*x)/(x-I)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+21/5/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(9/4), x)

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*x^2 - 30*I*x - 21) + 5*(a^2*
x^3 - 2*I*a^2*x^2 - a^2*x)*integral(42/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/
4)/(a^2*x^4 + a^2*x^2), x))/(a^2*x^3 - 2*I*a^2*x^2 - a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(9/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(9/4),x)
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(9/4), x)
```

3.1218 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=115

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 4/5*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(5/4)-6/5*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(
1/4)-6/5*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*Elli
pticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 50, 42, 203, 202}

$$-\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(5/4)) - ((6*I)/5)/(a*(a - I*a
*x)^(1/4)*(a + I*a*x)^(1/4)) - (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2]
)/(5*a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
```

```
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{3}{5}\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{1}{5}(3a)\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{\left(3a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.61

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
[Out] ((I/7)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 9/4, 11/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.21, size = 107, normalized size = 0.93

method	result	size
risch	$-\frac{2(3x^2+2ix+1)}{5(x-i)a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(3*x^2+1+2*I*x)/(x-I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+3/5/(a^2
)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(
-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(9/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*I*x + 3) - 5*(a^3*x^3 - 2*I
*a^3*x^2 - a^3*x)*integral(6/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*x^
4 + a^3*x^2), x))/(a^3*x^3 - 2*I*a^3*x^2 - a^3*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(9/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(9/4), x)
```

3.1219 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=82

$$\frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 4/5*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4)+2/5*(x^2+1)^(1/4)*(cos(1/2*arctan(x
))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^2/(a
-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((4*I)/5)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (2*(1 + x^2)^(1/4)*Elli
pticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{9/4}}\,dx &= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.85

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
[Out] ((I/3)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 9/4, 7/4, 1
/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]')
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 105, normalized size = 1.28

method	result	size
risch	$\frac{\frac{2}{5}x^2-\frac{2}{5}ix+\frac{4}{5}}{(x-i)a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(x^2+2-I*x)/(x-I)/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-
a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x - 2*I) + 5*(a^4*x^2 - 2*I*a^
4*x - a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^
4), x))/(a^4*x^2 - 2*I*a^4*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\sqrt[4]{-ia\left(x+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(9/4)), x)
```

3.1220 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2/5*I/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4)+6/5*(x^2+1)^(1/4)*(cos(1/2*arctan
(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3/
(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 48, 42, 203, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((2*I)/5)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (6*(1 + x^2)^(1/4)*El
lipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{3\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{9/4}}\,dx}{a}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{3\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{5a}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 68, normalized size = 0.83

$$-\frac{i\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{\sqrt[4]{2}\,a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 9/4, 3/4, 1/2 - (I/2)*x])/(2^
(1/4)*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]')
```

```
[Out] Timed out
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 107, normalized size = 1.30

method	result	size
risch	$\frac{\frac{6}{5}x^2-\frac{6}{5}ix+\frac{2}{5}}{(x-i)a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(-3*I*x+3*x^2+1)/(x-I)/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/5/(a
^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/
4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 - 3*I*x + 1) + 5*(a^5*x^
3 - I*a^5*x^2 + a^5*x - I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)
^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 - I*a^5*x^2 + a^5*x - I*a^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1\mathrm{i}\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(9/4)),x)
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(9/4)), x)
```

3.1221 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=88

$$\frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2/5*x/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)/(x^2+1)+6/5*(x^2+1)^(1/4)*(cos(1/
2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/
2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {42, 205, 203, 202}

$$\frac{2x}{5a^4\,(x^2+1)\,\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*x)/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (6*(1 + x^2)^
(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)
)
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx &= \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{9/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)} + \frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)} + \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)} + \frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.80

$$
-\frac{i\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5\sqrt[4]{2}\,a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-1/5*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, 1/2 - (I/2)*x]
)/(2^(1/4)*a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 94.50, size = 74, normalized size = 0.84

$$
\frac{-4-\text{meijerg}\left[\left\{\left\{\frac{9}{8},\frac{13}{8},1\right\},\left\{\frac{1}{2},\frac{9}{4},\frac{11}{4}\right\}\right\},\left\{\left\{\frac{9}{8},\frac{13}{8},\frac{7}{4},\frac{9}{4},\frac{11}{4}\right\},\{0\}\right\},\frac{\text{exp\_polar}[-3I\text{Pi}]}{x^2}\right]+4I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},0,\frac{1}{2},\frac{5}{8},\frac{9}{8},1\right\},\{\}\right\},\left\{\left\{\frac{5}{8},\frac{9}{8}\right\},\left\{-\frac{1}{2},0,\frac{7}{4},0\right\}\right\},\frac{\text{exp\_polar}[-I\text{Pi}]}{x^2}\right]}{5\text{Pi}a^{\frac{9}{2}}\text{Gamma}\left[\frac{1}{4}\right]}
$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]')
[Out] (-4 -1 ^ (3 / 4) meijerg[{{9 / 8, 13 / 8, 1}, {1 / 2, 9 / 4, 11 / 4}}, {{9
/ 8, 13 / 8, 7 / 4, 9 / 4, 11 / 4}, {0}}, exp_polar[-3 I Pi] / x ^ 2] + 4 I
 meijerg[{{-1 / 2, 0, 1 / 2, 5 / 8, 9 / 8, 1}, {}}, {{5 / 8, 9 / 8}, {-1 /
2, 0, 7 / 4, 0}}, exp_polar[-I Pi] / x ^ 2]) / (5 Pi a ^ (9 / 2) Gamma[1 /
4])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{9}{4}} \left(iax+a\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
[Out] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
[Out] 1/5*(2*(3*x^3 + 4*x)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + 5*(a^6*x^4 + 2*
a^6*x^2 + a^6)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2
+ a^6), x))/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [A]
time = 125.79, size = 95, normalized size = 1.08

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{8}, \frac{13}{8}, 1 & \frac{1}{2}, \frac{9}{4}, \frac{11}{4} \\ \frac{9}{8}, \frac{13}{8}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4} & 0 \end{matrix} \middle| {\frac{e^{-3i\pi}}{x^{2}}} \right) e^{\frac{i\pi}{4}}}{4\pi a^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{5}{8}, \frac{9}{8}, 1 & \\ \frac{5}{8}, \frac{9}{8} & -\frac{1}{2}, 0, \frac{7}{4}, 0 \end{matrix} \middle| {\frac{e^{-i\pi}}{x^{2}}} \right)}{4\pi a^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] -I*meijerg(((9/8, 13/8, 1), (1/2, 9/4, 11/4)), ((9/8, 13/8, 7/4, 9/4, 11/4)
, (0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*a**(9/2)*gamma(9/4)) +
I*meijerg(((-1/2, 0, 1/2, 5/8, 9/8, 1), ()), ((5/8, 9/8), (-1/2, 0, 7/4, 0)
), exp_polar(-I*pi)/x**2)/(4*pi*a**(9/2)*gamma(9/4))
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(9/4)), x)
```

3.1222 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=121

$$-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{45a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}\,(1+x^2)}+\frac{14\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] -2/9*I/a^2/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4)+14/45*x/a^5/(a-I*a*x)^(1/4)/(a+I
*a*x)^(1/4)/(x^2+1)+14/15*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/
2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^5/(a-I*a*x)^(1/4)/(a+I
*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 203, 202}

$$\frac{14x}{45a^5(x^2+1)\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(45*a^5*(a -
I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (14*(1 + x^2)^(1/4)*EllipticE[A
rcTan[x]/2, 2])/(15*a^5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{7\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx}{9a}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{\left(7\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{9/4}}\,dx}{9a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)} + \frac{(7$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)} + \frac{(7}{15}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)} + \frac{14}{1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.58

$$-\frac{i\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{9}{4},\frac{9}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}\,a^3(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-1/9*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 9/4, -5/4, 1/2 - (I/2)*x]
)/(2^(1/4)*a^3*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 5458 deep
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.21, size = 124, normalized size = 1.02

method	result	size
risch	$\frac{\frac{14}{15}x^4+\frac{14}{15}ix^3+\frac{56}{45}x^2+\frac{56}{45}ix+\frac{2}{9}}{(x-i)(x+i)^2a^5(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{15(a^2)^{\frac{1}{4}}a^5(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
[Out] 2/45*(21*I*x^3+21*x^4+28*I*x+28*x^2+5)/(x-I)/(x+I)^2/a^5/(-a*(-1+I*x))^(1/4
)/(a*(1+I*x))^(1/4)-7/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^5*
(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(21*x^4 + 21*I*x^3 + 28*x^2 + 28*I*x + 5)*(I*a*x + a)^(3/4)*(-I*a*x
 + a)^(3/4) + 45*(a^7*x^5 + I*a^7*x^4 + 2*a^7*x^3 + 2*I*a^7*x^2 + a^7*x + I
*a^7)*integral(-7/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^7*x^2 + a^7),
x))/(a^7*x^5 + I*a^7*x^4 + 2*a^7*x^3 + 2*I*a^7*x^2 + a^7*x + I*a^7)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(9/4),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{13/4}\,(a + a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(9/4)),x)
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(9/4)), x)
```

3.1223 $\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=154

$$-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{65a^6\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{42\sqrt[4]{1+}}{65a^6\sqrt[4]{}}$$

```
[Out] -2/13*I/a^2/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4)-2/13*I/a^3/(a-I*a*x)^(9/4)/(a+
I*a*x)^(5/4)+14/65*x/a^6/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)/(x^2+1)+42/65*(x^2
+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2
*arctan(x)),2^(1/2))/a^6/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 203, 202}

$$\frac{14x}{65a^6\left(x^2+1\right)\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{42\sqrt[4]{x^2+1}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{65a^6\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/13)/(a^2*(a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)) - ((2*I)/13)/(a^3*(a
 - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(65*a^6*(a - I*a*x)^(1/4)*(a +
I*a*x)^(1/4)*(1 + x^2)) + (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(6
5*a^6*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}+\frac{9\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx}{13a}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{7\int \frac{}{(a-\cdots}}{\cdots}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{\left(7\sqrt[4]{a\cdots}\right.}{13\cdots}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{\cdots}{65a^6\sqrt[4]{\cdots}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{\cdots}{65a^6\sqrt[4]{\cdots}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{\cdots}{65a^6\sqrt[4]{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.45

$$-\frac{i\sqrt[4]{1+ix}\ {}_2F_1\left(-\frac{13}{4},\frac{9}{4};-\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13\sqrt[4]{2}\,a^3(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-1/13*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 9/4, -9/4, 1/2 - (I/2)*
x])/(2^(1/4)*a^3*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 10662 deep
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.21, size = 130, normalized size = 0.84

method	result	size
risch	$\frac{\frac{42}{65}x^5+\frac{84}{65}ix^4+\frac{14}{65}x^3+\frac{112}{65}ix^2-\frac{46}{65}x+\frac{4}{13}i}{(x-i)(x+i)^3a^6(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{21x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{65(a^2)^{\frac{1}{4}}a^6(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/65*(42*I*x^4+21*x^5+56*I*x^2-23*x+7*x^3+10*I)/(x-I)/(x+I)^3/a^6/(-a*(-1+I
*x))^(1/4)/(a*(1+I*x))^(1/4)-21/65/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],
-x^2)/a^6*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/
4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/65*(2*(21*x^5 + 42*I*x^4 + 7*x^3 + 56*I*x^2 - 23*x + 10*I)*(I*a*x + a)^(3
/4)*(-I*a*x + a)^(3/4) + 65*(a^8*x^6 + 2*I*a^8*x^5 + a^8*x^4 + 4*I*a^8*x^3
- a^8*x^2 + 2*I*a^8*x - a^8)*integral(-21/65*(I*a*x + a)^(3/4)*(-I*a*x + a)
^(3/4)/(a^8*x^2 + a^8), x))/(a^8*x^6 + 2*I*a^8*x^5 + a^8*x^4 + 4*I*a^8*x^3
- a^8*x^2 + 2*I*a^8*x - a^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{17/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(9/4)), x)
```

3.1224 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=297

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log}{}$$

```
[Out] 4/5*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(5/4)-4*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(1/
4)-1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*
a*x)^(1/2))/a*2^(1/2)+1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a
-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)
/(a+I*a*x)^(1/4))*2^(1/2)/a+I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1
/4))*2^(1/2)/a
```

Rubi [A]

time = 0.10, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {49, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(1
/4))/(a*(a + I*a*x)^(1/4)) - (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/
4))/(a + I*a*x)^(1/4)])/a + (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[
a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]
)/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\text{Subst}\left(\int\right.}{}$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\text{Subst}\left(\right.}{}$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1-\right.}{}$$
$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1-\right.}{}$$

Mathematica [A]
time = 0.29, size = 127, normalized size = 0.43

$$\frac{2\left(\frac{4(2i-3x)\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a(-i+x)^2} + 5\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 5(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (2*((4*(2*I - 3*x)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/(a*(-I + x)^2) + 5*
(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)] - 5*(-
1)^(3/4)*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]))/(5*a)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]')
```

```
[Out] Timed out
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.34, size = 490, normalized size = 1.65

method	result
risch	$\frac{8(3x^2+ix+2)(-a(ix-1))^{\frac{1}{4}}}{5(x-i)a(ix-1)(a(ix+1))^{\frac{1}{4}}} - \frac{\left(\operatorname{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+i\right)x^2+i\operatorname{RootOf}\left(\_Z^2+i\right)\left(-x^4-2ix^3\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 8/5*(3*x^2+2+I*x)/(x-I)/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)-(R
ootOf(_Z^2+I)*ln((-(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+I*RootOf(
_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2
*I*x)^(1/4)*x+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(1-x^4
-2*I*x^3-2*I*x)^(1/4)-(1-x^4-2*I*x^3-2*I*x)^(1/2)+x)/(-1+I*x)^2)+I*RootOf(_
Z^2+I)*ln((-I*(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+2*RootOf(_Z^2+
I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x-x^3+RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(
3/4)-I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)
^(1/4)-2*I*x^2+(1-x^4-2*I*x^3-2*I*x)^(1/2)+x)/(-1+I*x)^2))/a*(-a*(-1+I*x))^
(1/4)/(-1+I*x)*(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(9/4), x)
```

Fricas [A]
time = 0.31, size = 343, normalized size = 1.15

$$\frac{5\left(a^{2}x^{2}-2i\,a^{2}x-a^{2}\right)\sqrt{\frac{4i}{a^{2}}}\log\left(\frac{\left(a^{2}x-i\,a^{2}\right)\sqrt{\frac{4i}{a^{2}}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)-5\left(a^{2}x^{2}-2i\,a^{2}x-a^{2}\right)\sqrt{\frac{4i}{a^{2}}}\log\left(-\frac{\left(a^{2}x-i\,a^{2}\right)\sqrt{\frac{4i}{a^{2}}}-2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)+5\left(a^{2}x^{2}-2i\,a^{2}x-a^{2}\right)\sqrt{-\frac{4i}{a^{2}}}\log\left(\frac{\left(a^{2}x-i\,a^{2}\right)\sqrt{-\frac{4i}{a^{2}}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)-5\left(a^{2}x^{2}-2i\,a^{2}x-a^{2}\right)\sqrt{-\frac{4i}{a^{2}}}\log\left(-\frac{\left(a^{2}x-i\,a^{2}\right)\sqrt{-\frac{4i}{a^{2}}}-2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\left(x-i\right)}\right)-16\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}\left(3\,x-2i\right)}{10\left(a^{2}x^{2}-2i\,a^{2}x-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^2*x^2 - 2*I*a^2*x - a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*
sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 5*(a^2*x
^2 - 2*I*a^2*x - a^2)*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)
 - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 5*(a^2*x^2 - 2*I*a^2*
x - a^2)*sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x
+ a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 5*(a^2*x^2 - 2*I*a^2*x - a^2)*sqr
t(-4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(x - I)) - 16*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(3*x
 - 2*I))/(a^2*x^2 - 2*I*a^2*x - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(9/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4), x)
```

3.1225 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

```
[Out] 2/5*I*(a-I*a*x)^(5/4)/a^2/(a+I*a*x)^(5/4)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]')

[Out] Timed out

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.
time = 0.15, size = 50, normalized size = 1.52

method	result	size
risch	$\frac{2(-a(ix-1))^{\frac{1}{4}}(x^2+2ix-1)}{5a^2(ix-1)(a(ix+1))^{\frac{1}{4}}(x-i)}$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)

[Out] 2/5/a^2*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)*(2*I*x+x^2-1)/(x-I)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)

Fricas [**A**]
time = 0.30, size = 42, normalized size = 1.27

$$-\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(x+i)}{5\,(a^3x^2-2i\,a^3x-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")

[Out] -2/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(x + I)/(a^3*x^2 - 2*I*a^3*x - a^
3)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(9/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x)
[Out] Could not integrate
```

Mupad [**B**]
time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^2\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(9/4),x)
[Out] -(2*(x*1i - 1)*(-a*(x*1i - 1))^(1/4))/(5*a^2*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

3.1226 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=67

$$\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}+\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}$$

```
[Out] 2/5*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(5/4)+4/5*I*(a-I*a*x)^(1/4)/a^3/(a+I*a*
x)^(1/4)
```

Rubi [A]

time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {47, 37}

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{5a}$$
$$= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.67

$$\frac{2(3+2ix)\sqrt[4]{a-iax}}{5a^3(-i+x)\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
[Out] (2*(3 + (2*I)*x)*(a - I*a*x)^(1/4))/(5*a^3*(-I + x)*(a + I*a*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.17, size = 44, normalized size = 0.66

method	result	size
risch	$\frac{\frac{4}{5}x^2-\frac{2}{5}ix+\frac{6}{5}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
[Out] 2/5/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(2*x^2+3-I*x)/(x-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A]
time = 0.29, size = 44, normalized size = 0.66

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x-3i)}{5\,(a^4x^2-2i\,a^4x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x - 3*I)/(a^4*x^2 - 2*I*a^4*x -
 a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{9}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.63, size = 38, normalized size = 0.57

$$\frac{2\,\left(3+x\,2\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^3\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(x*2i + 3)*(-a*(x*1i - 1))^(1/4))/(5*a^3*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

3.1227 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}$$

```
[Out] -2/3*I/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4)+8/15*I*(a-I*a*x)^(1/4)/a^3/(a+I*
a*x)^(5/4)+16/15*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((8*I)/15)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(5/4)) + (((16*I)/15)*(a - I*a*x)^(1/4))/(a^4*(
a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{4\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx}{3a}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,d}{15a^2}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 0.52

$$-\frac{2i(a+iax)^{3/4}\left(7-4ix+8x^2\right)}{15a^4(-i+x)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4)*(7 - (4*I)*x + 8*x^2))/(a^4*(-I + x)^2*(a - I*a*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.44

method	result	size
risch	$\frac{\frac{16}{15}x^2-\frac{8}{15}ix+\frac{14}{15}}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(8*x^2-4*I*x+7)/(x-I)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.30, size = 56, normalized size = 0.56

$$\frac{2\,{\left(i\,ax+a\right)}^{\frac{3}{4}}{\left(-i\,ax+a\right)}^{\frac{1}{4}}{\left(8\,x^{2}-4i\,x+7\right)}}{15\,{\left(a^{5}x^{3}-i\,a^{5}x^{2}+a^{5}x-i\,a^{5}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 - 4*I*x + 7)/(a^5*x^3 - I*
a^5*x^2 + a^5*x - I*a^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.53, size = 45, normalized size = 0.45

$$\frac{2\,{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)}^{1/4}\,\left(x^2\,8{}\mathrm{i}+4\,x+7{}\mathrm{i}\right)}{15\,a^4\,\left(x^2+1\right)\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(4*x + x^2*8i + 7i))/(15*a^4*(x^2 + 1)*(a*(x*1i +
1))^(1/4))
```

3.1228 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=133

$$-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}+\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}$$

```
[Out] -2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4)-4/7*I/a^3/(a-I*a*x)^(3/4)/(a+I*a
*x)^(5/4)+16/35*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(5/4)+32/35*I*(a-I*a*x)^(1/
4)/a^5/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) - ((4*I)/7)/(a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((16*I)/35)*(a - I*a*x)^(1/4))/(a^4*(a +
 I*a*x)^(5/4)) + (((32*I)/35)*(a - I*a*x)^(1/4))/(a^5*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} + \frac{6\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx}{7a} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8\int \frac{}{(a-iax)^3}}{7} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-}}{35a^4(a+i} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-}}{35a^4(a+i}
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 64, normalized size = 0.48

$$\frac{2(a+iax)^{3/4}\left(9-22ix+8x^2-16ix^3\right)}{35a^5(-i+x)^2(i+x)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(9 - (22*I)*x + 8*x^2 - (16*I)*x^3))/(35*a^5*(-I + x)^
2*(I + x)*(a - I*a*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Maple [**A**]
time = 0.16, size = 56, normalized size = 0.42

method	result	size
risch	$\frac{\frac{32}{35}x^3+\frac{16}{35}ix^2+\frac{44}{35}x+\frac{18}{35}i}{a^4(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)(x+i)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/a^4/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(16*x^3+8*I*x^2+22*x+9*I)/(x
-I)/(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [A]
time = 0.31, size = 54, normalized size = 0.41

$$\frac{2\,(16\,x^3+8i\,x^2+22\,x+9i)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{35\,(a^6x^4+2\,a^6x^2+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
[Out] 2/35*(16*x^3 + 8*I*x^2 + 22*x + 9*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(
a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(9/4),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x)
[Out] Could not integrate
```

Mupad [B]
time = 0.69, size = 56, normalized size = 0.42

$$\frac{2\,{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)}^{1/4}\,\left(x^4\,16{}\mathrm{i}+8\,x^3+x^2\,30{}\mathrm{i}+13\,x+9{}\mathrm{i}\right)}{35\,a^5\,{\left(x^2+1\right)}^2\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(13*x + x^2*30i + 8*x^3 + x^4*16i + 9i))/(35*a^5*(
x^2 + 1)^2*(a*(x*1i + 1))^(1/4))
```

3.1229 $\int (a+bx)^2(ac-bcx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}$$

```
[Out] -4*a^2*(-b*c*x+a*c)^(1+n)/b/c/(1+n)+4*a*(-b*c*x+a*c)^(2+n)/b/c^2/(2+n)-(-b*
c*x+a*c)^(3+n)/b/c^3/(3+n)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = $\{45\}$

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] (-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))/
(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^n\,dx &= \int \left(4a^2(ac-bcx)^n-\frac{4a(ac-bcx)^{1+n}}{c}+\frac{(ac-bcx)^{2+n}}{c^2}\right)dx\\ &= -\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 77, normalized size = 0.93

$$\frac{(c(a-bx))^n(-a+bx)\left(a^2\left(14+7n+n^2\right)+2ab\left(4+5n+n^2\right)x+b^2\left(2+3n+n^2\right)x^2\right)}{b(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a^2*(14 + 7*n + n^2) + 2*a*b*(4 + 5*n + n^2)*x
 + b^2*(2 + 3*n + n^2)*x^2))/(b*(1 + n)*(2 + n)*(3 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.95, size = 693, normalized size = 8.35

$$\text{Piecewise}\left[\left\{\left\{a^2x\,(ac)^n, b==0\right\}, \left\{\frac{-a^2\left(2+\text{Log}\left[\frac{-a+bx}{b}\right]\right)+bx\left(2a\text{Log}\left[\frac{-a+bx}{b}\right]+4a-bx\text{Log}\left[\frac{-a+bx}{b}\right]\right)}{bc^3\left(a^2-2abx+b^2x^2\right)}, n==-3\right\}, \left\{\frac{a^2\left(5+4\text{Log}\left[\frac{-a+bx}{b}\right]\right)-bx\left(4a\text{Log}\left[\frac{-a+bx}{b}\right]+bx\right)}{bc^2\,(a-bx)}, n==-2\right\}, \left\{\frac{-8a^2\text{Log}\left[\frac{-a+bx}{b}\right]+bx\,(-6a-bx)}{2bc}, n==-1\right\}\right\}, \frac{-14a^3\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}-\frac{7a^3n\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}-\frac{a^3n^2\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{6a^2bx\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}-\frac{3a^2bnx\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}-\frac{a^2bn^2x\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{6ab^2x^2\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{7ab^2nx^2\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{ab^2n^2x^2\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{2b^3x^3\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{3b^3nx^3\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}+\frac{b^3n^2x^3\,(ac-bcx)^n}{6b+11bn+6bn^2+bn^3}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]')
```

```
[Out] Piecewise[{{a ^ 2 x (a c) ^ n, b == 0}, {(-a ^ 2 (2 + Log[(-a + b x) / b])
+ b x (2 a Log[(-a + b x) / b] + 4 a - b x Log[(-a + b x) / b])) / (b c ^ 3
 (a ^ 2 - 2 a b x + b ^ 2 x ^ 2)), n == -3}, {(a ^ 2 (5 + 4 Log[(-a + b x)
/ b]) - b x (4 a Log[(-a + b x) / b] + b x)) / (b c ^ 2 (a - b x)), n == -2
}, {(-8 a ^ 2 Log[(-a + b x) / b] + b x (-6 a - b x)) / (2 b c), n == -1}},
 -14 a ^ 3 (a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) - 7 a ^
 3 n (a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) - a ^ 3 n ^ 2
 (a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + 6 a ^ 2 b x (a
c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) - 3 a ^ 2 b n x (a c
- b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) - a ^ 2 b n ^ 2 x (a c
- b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + 6 a b ^ 2 x ^ 2 (a c
- b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + 7 a b ^ 2 n x ^ 2 (a
c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + a b ^ 2 n ^ 2 x ^ 2
 (a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + 2 b ^ 3 x ^ 3 (
a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + 3 b ^ 3 n x ^ 3 (
a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3) + b ^ 3 n ^ 2 x ^ 3
 (a c - b c x) ^ n / (6 b + 11 b n + 6 b n ^ 2 + b n ^ 3)]
```

Maple [A]
time = 0.17, size = 103, normalized size = 1.24

method	result	size
gosper	$-\frac{(-bx+a)\left(b^2n^2x^2+2ab\,n^2x+3b^2n\,x^2+a^2n^2+10abnx+2x^2b^2+7a^2n+8abx+14a^2\right)(-bcx+ac)^n}{b\left(n^3+6n^2+11n+6\right)}$	103
risch	$-\frac{\left(-b^3n^2x^3-a\,b^2n^2x^2-3n\,b^3x^3+a^2b\,n^2x-7a\,b^2n\,x^2-2b^3x^3+a^3n^2+3a^2bnx-6a\,b^2x^2+7a^3n-6a^2bx+14a^3\right)(c(-bx+a))^n}{(2+n)(3+n)b(1+n)}$	133
norman	$\frac{b^2x^3\mathrm{e}^{n\ln(-bcx+ac)}}{3+n}+\frac{ab(6+n)x^2\mathrm{e}^{n\ln(-bcx+ac)}}{n^2+5n+6}-\frac{a^2\left(n^2+3n-6\right)x\,\mathrm{e}^{n\ln(-bcx+ac)}}{n^3+6n^2+11n+6}-\frac{a^3\left(n^2+7n+14\right)\mathrm{e}^{n\ln(-bcx+ac)}}{b\left(n^3+6n^2+11n+6\right)}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+a)*(b^2*n^2*x^2+2*a*b*n^2*x+3*b^2*n*x^2+a^2*n^2+10*a*b*n*x+2*b^2*x^2
+7*a^2*n+8*a*b*x+14*a^2)*(-b*c*x+a*c)^n/b/(n^3+6*n^2+11*n+6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(83) = 166$.
time = 0.27, size = 167, normalized size = 2.01

$$\frac{2\left(b^{2}c^{n}(n+1)x^{2}-abc^{n}nx-a^{2}c^{n}\right)(-bx+a)^{n}a}{(n^{2}+3\,n+2)b}+\frac{\left((n^{2}+3\,n+2)b^{3}c^{n}x^{3}-(n^{2}+n)ab^{2}c^{n}x^{2}-2\,a^{2}bc^{n}nx-2\,a^{3}c^{n}\right)(-bx+a)^{n}}{(n^{3}+6\,n^{2}+11\,n+6)b}-\frac{(-bcx+ac)^{n+1}a^{2}}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 2*(b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n*a/((n^2 + 3*n
+ 2)*b) + ((n^2 + 3*n + 2)*b^3*c^n*x^3 - (n^2 + n)*a*b^2*c^n*x^2 - 2*a^2*b*
c^n*n*x - 2*a^3*c^n)*(-b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b) - (-b*c*x +
a*c)^(n + 1)*a^2/(b*c*(n + 1))
```

Fricas [A]
time = 0.30, size = 128, normalized size = 1.54

$$-\frac{\left(a^{3}n^{2}+7\,a^{3}n-(b^{3}n^{2}+3\,b^{3}n+2\,b^{3})x^{3}+14\,a^{3}-(ab^{2}n^{2}+7\,ab^{2}n+6\,ab^{2})x^{2}+(a^{2}bn^{2}+3\,a^{2}bn-6\,a^{2}b)x\right)(-bcx+ac)^{n}}{bn^{3}+6\,bn^{2}+11\,bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^2 + 7*a^3*n - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + 14*a^3 - (a*b^2*n^2
 + 7*a*b^2*n + 6*a*b^2)*x^2 + (a^2*b*n^2 + 3*a^2*b*n - 6*a^2*b)*x)*(-b*c*x
+ a*c)^n/(b*n^3 + 6*b*n^2 + 11*b*n + 6*b)
```

Sympy [A]
time = 0.44, size = 819, normalized size = 9.87

$$\begin{cases} a^{2}x\left(ac\right)^{n} & \text{for}\: b=0 \\ -\frac{a^{2}\log\left(-\frac{a}{b}+x\right)}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}}-\frac{2a^{2}}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}}+\frac{2abx\log\left(-\frac{a}{b}+x\right)}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}}+\frac{4abx}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}}-\frac{b^{2}x^{2}\log\left(-\frac{a}{b}+x\right)}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}} & \text{for}\: n=-3 \\ -\frac{4a^{2}\log\left(-\frac{a}{b}+x\right)}{-abc^{2}+b^{2}c^{2}x}-\frac{5a^{2}}{-abc^{2}+b^{2}c^{2}x}+\frac{4abx\log\left(-\frac{a}{b}+x\right)}{-abc^{2}+b^{2}c^{2}x}+\frac{b^{2}x^{2}}{-abc^{2}+b^{2}c^{2}x} & \text{for}\: n=-2 \\ -\frac{4a^{2}\log\left(-\frac{a}{b}+x\right)}{bc}-\frac{3ax}{c}-\frac{bx^{2}}{2c} & \text{for}\: n=-1 \\ -\frac{a^{3}n^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}-\frac{7a^{3}n(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}-\frac{14a^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}-\frac{a^{2}bn^{2}x(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}-\frac{3a^{2}bnx(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{6a^{2}bx(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{ab^{2}n^{2}x^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{7ab^{2}nx^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{6ab^{2}x^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{b^{3}n^{2}x^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{3b^{3}nx^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b}+\frac{2b^{3}x^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a**2*x*(a*c)**n, Eq(b, 0)), (-a**2*log(-a/b + x)/(a**2*b*c**3 -
2*a*b**2*c**3*x + b**3*c**3*x**2) - 2*a**2/(a**2*b*c**3 - 2*a*b**2*c**3*x +
 b**3*c**3*x**2) + 2*a*b*x*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b
**3*c**3*x**2) + 4*a*b*x/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) -
 b**2*x**2*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2),
Eq(n, -3)), (-4*a**2*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) - 5*a**2/(-a*b
*c**2 + b**2*c**2*x) + 4*a*b*x*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) + b*
```

```
*2*x**2/(-a*b*c**2 + b**2*c**2*x), Eq(n, -2)), (-4*a**2*log(-a/b + x)/(b*c)
 - 3*a*x/c - b*x**2/(2*c), Eq(n, -1)), (-a**3*n**2*(a*c - b*c*x)**n/(b*n**3
 + 6*b*n**2 + 11*b*n + 6*b) - 7*a**3*n*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2
+ 11*b*n + 6*b) - 14*a**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*
b) - a**2*b*n**2*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 3*
a**2*b*n*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a**2*b*x
*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + a*b**2*n**2*x**2*(a*
c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 7*a*b**2*n*x**2*(a*c - b
*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a*b**2*x**2*(a*c - b*c*x)**
n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + b**3*n**2*x**3*(a*c - b*c*x)**n/(b*n
**3 + 6*b*n**2 + 11*b*n + 6*b) + 3*b**3*n*x**3*(a*c - b*c*x)**n/(b*n**3 + 6
*b*n**2 + 11*b*n + 6*b) + 2*b**3*x**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 +
 11*b*n + 6*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(83) = 166$.
time = 0.00, size = 282, normalized size = 3.40

$$\frac{-a^3n^2e^{n\ln(ac-bcx)} - 7a^3ne^{n\ln(ac-bcx)} - 14a^3e^{n\ln(ac-bcx)} - a^2bn^2xe^{n\ln(ac-bcx)} - 3a^2bnxe^{n\ln(ac-bcx)} + 6a^2bxe^{n\ln(ac-bcx)} + ab^2n^2x^2e^{n\ln(ac-bcx)} + 7ab^2nx^2e^{n\ln(ac-bcx)} + 6ab^2x^2e^{n\ln(ac-bcx)} + b^3n^2x^3e^{n\ln(ac-bcx)} + 3b^3nx^3e^{n\ln(ac-bcx)} + 2b^3x^3e^{n\ln(ac-bcx)}}{bn^3 + 6bn^2 + 11bn + 6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x)
```

```
[Out] ((-b*c*x + a*c)^n*b^3*n^2*x^3 + (-b*c*x + a*c)^n*a*b^2*n^2*x^2 + 3*(-b*c*x
+ a*c)^n*b^3*n*x^3 - (-b*c*x + a*c)^n*a^2*b*n^2*x + 7*(-b*c*x + a*c)^n*a*b^
2*n*x^2 + 2*(-b*c*x + a*c)^n*b^3*x^3 - (-b*c*x + a*c)^n*a^3*n^2 - 3*(-b*c*x
 + a*c)^n*a^2*b*n*x + 6*(-b*c*x + a*c)^n*a*b^2*x^2 - 7*(-b*c*x + a*c)^n*a^3
*n + 6*(-b*c*x + a*c)^n*a^2*b*x - 14*(-b*c*x + a*c)^n*a^3)/(b*n^3 + 6*b*n^2
 + 11*b*n + 6*b)
```

Mupad [B]
time = 0.49, size = 133, normalized size = 1.60

$$-(a\,c-b\,c\,x)^n\,\left(\frac{a^2\,x\,\left(n^2+3\,n-6\right)}{n^3+6\,n^2+11\,n+6}+\frac{a^3\,\left(n^2+7\,n+14\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{b^2\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}-\frac{a\,b\,x^2\,\left(n^2+7\,n+6\right)}{n^3+6\,n^2+11\,n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x)^2,x)
```

```
[Out] -(a*c - b*c*x)^n*((a^2*x*(3*n + n^2 - 6))/(11*n + 6*n^2 + n^3 + 6) + (a^3*(
7*n + n^2 + 14))/(b*(11*n + 6*n^2 + n^3 + 6)) - (b^2*x^3*(3*n + n^2 + 2))/(
11*n + 6*n^2 + n^3 + 6) - (a*b*x^2*(7*n + n^2 + 6))/(11*n + 6*n^2 + n^3 + 6
))
```

3.1230 $\int (a+bx)(ac-bcx)^n\,dx$

Optimal. Leaf size=53

$$-\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}$$

```
[Out] -2*a*(-b*c*x+a*c)^(1+n)/b/c/(1+n)+(-b*c*x+a*c)^(2+n)/b/c^2/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = $\{45\}$

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] (-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*(
2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^n\,dx &= \int \left(2a(ac-bcx)^n-\frac{(ac-bcx)^{1+n}}{c}\right)dx\\ &= -\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.81

$$\frac{(c(a-bx))^n(-a+bx)(a(3+n)+b(1+n)x)}{b(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a*(3 + n) + b*(1 + n)*x))/(b*(1 + n)*(2 + n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.80, size = 256, normalized size = 4.83

$$\text{Piecewise}\left[\left\{\left\{ax\,(ac)^n, b==0\right\}, \left\{\frac{a\left(2+\text{Log}\left[\frac{-a+bx}{b}\right]\right)-bx\text{Log}\left[\frac{-a+bx}{b}\right]}{bc^2\,(a-bx)}, n==-2\right\}, \left\{\frac{-2a\text{Log}\left[-\frac{a}{b}+x\right]-bx}{bc}, n==-1\right\}\right\}, \frac{-3a^2\,(ac-bcx)^n}{2b+3bn+bn^2}-\frac{a^2n\,(ac-bcx)^n}{2b+3bn+bn^2}+\frac{2abx\,(ac-bcx)^n}{2b+3bn+bn^2}+\frac{b^2x^2\,(ac-bcx)^n}{2b+3bn+bn^2}+\frac{b^2nx^2\,(ac-bcx)^n}{2b+3bn+bn^2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^1*(a*c - b*c*x)^n,x]')
```

```
[Out] Piecewise[{{a x (a c) ^ n, b == 0}, {(a (2 + Log[(-a + b x) / b]) - b x Log
[(-a + b x) / b]) / (b c ^ 2 (a - b x)), n == -2}, {(-2 a Log[-a / b + x] -
 b x) / (b c), n == -1}}, -3 a ^ 2 (a c - b c x) ^ n / (2 b + 3 b n + b n ^
 2) - a ^ 2 n (a c - b c x) ^ n / (2 b + 3 b n + b n ^ 2) + 2 a b x (a c -
b c x) ^ n / (2 b + 3 b n + b n ^ 2) + b ^ 2 x ^ 2 (a c - b c x) ^ n / (2 b
 + 3 b n + b n ^ 2) + b ^ 2 n x ^ 2 (a c - b c x) ^ n / (2 b + 3 b n + b n
^ 2)]
```

Maple [A]
time = 0.12, size = 47, normalized size = 0.89

method	result	size
gosper	$-\frac{(-bcx+ac)^n(bnx+an+bx+3a)(-bx+a)}{b(n^2+3n+2)}$	47
risch	$-\frac{(-b^2n\,x^2-x^2b^2+a^2n-2abx+3a^2)(c(-bx+a))^n}{(2+n)(1+n)b}$	59
norman	$\frac{b\,x^2\mathrm{e}^{n\ln(-bcx+ac)}}{2+n}+\frac{2ax\,\mathrm{e}^{n\ln(-bcx+ac)}}{n^2+3n+2}-\frac{a^2(3+n)\mathrm{e}^{n\ln(-bcx+ac)}}{b(n^2+3n+2)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*c*x+a*c)^n*(b*n*x+a*n+b*x+3*a)*(-b*x+a)/b/(n^2+3*n+2)
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.53

$$\frac{(b^2c^n(n+1)x^2-abc^nnx-a^2c^n)(-bx+a)^n}{(n^2+3\,n+2)b}-\frac{(-bcx+ac)^{n+1}a}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] (b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n/((n^2 + 3*n + 2)
*b) - (-b*c*x + a*c)^(n + 1)*a/(b*c*(n + 1))
```

Fricas [A]
time = 0.31, size = 58, normalized size = 1.09

$$-\frac{(a^2n - 2\,abx - (b^2n + b^2)x^2 + 3\,a^2)(-bcx + ac)^n}{bn^2 + 3\,bn + 2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n - 2*a*b*x - (b^2*n + b^2)*x^2 + 3*a^2)*(-b*c*x + a*c)^n/(b*n^2 + 3*
b*n + 2*b)
```

Sympy [A]
time = 0.28, size = 245, normalized size = 4.62

$$\begin{cases} ax\,(ac)^n & \text{for}\ b = 0 \\ -\frac{a\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{2a}{-abc^2+b^2c^2x} + \frac{bx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} & \text{for}\ n = -2 \\ -\frac{2a\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{x}{c} & \text{for}\ n = -1 \\ -\frac{a^2n(ac-bcx)^n}{bn^2+3bn+2b} - \frac{3a^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{2abx(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2nx^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2x^2(ac-bcx)^n}{bn^2+3bn+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a*x*(a*c)**n, Eq(b, 0)), (-a*log(-a/b + x)/(-a*b*c**2 + b**2*c**
2*x) - 2*a/(-a*b*c**2 + b**2*c**2*x) + b*x*log(-a/b + x)/(-a*b*c**2 + b**2*
c**2*x), Eq(n, -2)), (-2*a*log(-a/b + x)/(b*c) - x/c, Eq(n, -1)), (-a**2*n*
(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) - 3*a**2*(a*c - b*c*x)**n/(b*n**2 +
 3*b*n + 2*b) + 2*a*b*x*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*n*x*
*2*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*x**2*(a*c - b*c*x)**n/(b*
n**2 + 3*b*n + 2*b), True))
```

Giac [A]
time = 0.00, size = 113, normalized size = 2.13

$$\frac{-a^2ne^{n\ln(ac-bcx)} - 3a^2e^{n\ln(ac-bcx)} + 2abxe^{n\ln(ac-bcx)} + b^2nx^2e^{n\ln(ac-bcx)} + b^2x^2e^{n\ln(ac-bcx)}}{bn^2 + 3bn + 2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x)
```

```
[Out] ((-b*c*x + a*c)^n*b^2*n*x^2 + (-b*c*x + a*c)^n*b^2*x^2 - (-b*c*x + a*c)^n*a
^2*n + 2*(-b*c*x + a*c)^n*a*b*x - 3*(-b*c*x + a*c)^n*a^2)/(b*n^2 + 3*b*n +
2*b)
```

Mupad [B]
time = 0.32, size = 66, normalized size = 1.25

$$(a\,c-b\,c\,x)^n\,\left(\frac{2\,a\,x}{n^2+3\,n+2}-\frac{a^2\,\left(n+3\right)}{b\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,\left(n+1\right)}{n^2+3\,n+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x),x)
```

```
[Out] (a*c - b*c*x)^n*((2*a*x)/(3*n + n^2 + 2) - (a^2*(n + 3))/(b*(3*n + n^2 + 2)
) + (b*x^2*(n + 1))/(3*n + n^2 + 2))
```

3.1231 $\int \frac{(ac-bcx)^n}{a+bx} dx$

Optimal. Leaf size=52

$$-\frac{(ac-bcx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2abc(1+n)}$$

```
[Out] -1/2*(-b*c*x+a*c)^(1+n)*hypergeom([1, 1+n],[2+n],1/2*(-b*x+a)/a)/a/b/c/(1+n
)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{70\}$

$$-\frac{(ac-bcx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{a-bx}{2a}\right)}{2abc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^n/(a + b*x),x]
```

```
[Out] -1/2*((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x)/(2
*a)])/(a*b*c*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(ac-bcx)^n}{a+bx} dx = -\frac{(ac-bcx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2abc(1+n)}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 1.00

$$-\frac{(a-bx)(c(a-bx))^n {}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2ab(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^n/(a + b*x),x]
```

```
[Out] -1/2*((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x
)/(2*a)])/(a*b*(1 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(a + b*x)^1*(a*c - b*c*x)^n,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-bcx + ac)^n}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^n/(b*x+a),x)
```

```
[Out] int((-b*c*x+a*c)^n/(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^n/(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(-a+bx))^n}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**n/(b*x+a),x)
[Out] Integral((-c*(-a + b*x))**n/(a + b*x), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,c-b\,c\,x)^n}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n/(a + b*x),x)
[Out] int((a*c - b*c*x)^n/(a + b*x), x)
```

3.1232 $\int \frac{(ac-bcx)^n}{(a+bx)^2} dx$

Optimal. Leaf size=52

$$-\frac{(ac-bcx)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2bc(1+n)}$$

```
[Out] -1/4*(-b*c*x+a*c)^(1+n)*hypergeom([2, 1+n],[2+n],1/2*(-b*x+a)/a)/a^2/b/c/(1
+n)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {70}

$$-\frac{(ac-bcx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{a-bx}{2a}\right)}{4a^2bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

```
[Out] -1/4*((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x)/(2
*a)])/(a^2*b*c*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(ac-bcx)^n}{(a+bx)^2} dx = -\frac{(ac-bcx)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2bc(1+n)}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 1.00

$$-\frac{(a-bx)(c(a-bx))^n {}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

```
[Out] -1/4*((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x
)/(2*a)])/(a^2*b*(1 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(a + b*x)^2*(a*c - b*c*x)^n,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-bcx + ac)^n}{(bx + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

```
[Out] int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-a+bx\right)\right)^{n}}{\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**n/(b*x+a)**2,x)
```

```
[Out] Integral((-c*(-a + b*x))**n/(a + b*x)**2, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a\,c-b\,c\,x\right)}^n}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n/(a + b*x)^2,x)
```

```
[Out] int((a*c - b*c*x)^n/(a + b*x)^2, x)
```

3.1233 $\int (a+ax)^m(c-cx)^m \, dx$

Optimal. Leaf size=41

$$x(a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

```
[Out] x*(a*x+a)^m*(-c*x+c)^m*hypergeom([1/2, -m],[3/2],x^2)/((-x^2+1)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {42, 252, 251}

$$x\left(1-x^2\right)^{-m}(ax+a)^m(c-cx)^m{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^m*(c - c*x)^m,x]
```

```
[Out] (x*(a + a*x)^m*(c - c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, x^2])/(1 - x^2)^
m
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] && !IntegerQ[2*m]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+ax)^m(c-cx)^m\,dx = \left((a+ax)^m(c-cx)^m\left(ac-acx^2\right)^{-m}\right)\int \left(ac-acx^2\right)^m\,dx$$
$$= \left((a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\right)\int \left(1-x^2\right)^m\,dx$$
$$= x(a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.29

$$\frac{2^m(-1+x)(1+x)^{-m}(a(1+x))^m(c-cx)^m\,{}_2F_1\left(-m,1+m;2+m;\frac{1}{2}-\frac{x}{2}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^m*(c - c*x)^m,x]
```

```
[Out] (2^m*(-1 + x)*(a*(1 + x))^m*(c - c*x)^m*Hypergeometric2F1[-m, 1 + m, 2 + m,
 1/2 - x/2])/((1 + m)*(1 + x)^m)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + a*x)^m*(c - c*x)^m,x]')
```

```
[Out] Exception raised: AttributeError >> 'SympyExpression' object has no attribu
te 'expr'
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (ax+a)^m\,(-cx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^m*(-c*x+c)^m,x)
```

```
[Out] int((a*x+a)^m*(-c*x+c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + a)^m*(-c*x + c)^m, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + a)^m*(-c*x + c)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.07, size = 124, normalized size = 3.02

$$\frac{a^{m}c^{m}G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2}, \frac{1}{2}-\frac{m}{2}, 1 & \frac{1}{2}, -m, \frac{1}{2}-m \\ -m-\frac{1}{2}, -m, -\frac{m}{2}, \frac{1}{2}-m, \frac{1}{2}-\frac{m}{2} & 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^{2}}\right)e^{-i\pi m}}{4\pi\Gamma\left(-m\right)} - \frac{a^{m}c^{m}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2}, 1 & \\ -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2} & -\frac{1}{2}, 0, -m-\frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{x^{2}}\right)}{4\pi\Gamma\left(-m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**m*(-c*x+c)**m,x)
```

```
[Out] a**m*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m - 1/2, -
m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), exp_polar(-2*I*pi)/x**2)*exp(-I*pi*m)/
(4*pi*gamma(-m)) - a**m*c**m*meijerg(((-1/2, 0, 1/2, -m/2 - 1/2, -m/2, 1),
()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0)), x**(-2))/(4*pi*gamma(-m)
)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,x\right)}^m\,{\left(c-c\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^m*(c - c*x)^m,x)
```

```
[Out] int((a + a*x)^m*(c - c*x)^m, x)
```

3.1234 $\int (a+bx)^m(ac-bcx)^m\,dx$

Optimal. Leaf size=57

$$x(a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

```
[Out] x*(b*x+a)^m*(-b*c*x+a*c)^m*hypergeom([1/2, -m],[3/2],b^2*x^2/a^2)/((1-b^2*x
^2/a^2)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {42, 252, 251}

$$x(a+bx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}(ac-bcx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

```
[Out] (x*(a + b*x)^m*(a*c - b*c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, (b^2*x^2)/a^
2])/(1 - (b^2*x^2)/a^2)^m
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+bx)^m(ac-bcx)^m\,dx = \left((a+bx)^m(ac-bcx)^m\left(a^2c-b^2cx^2\right)^{-m}\right)\int \left(a^2c-b^2cx^2\right)^m\,dx$$
$$= \left((a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}\right)\int \left(1-\frac{b^2x^2}{a^2}\right)^m\,dx$$
$$= x(a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.26

$$-\frac{2^m(a-bx)(c(a-bx))^m(a+bx)^m\left(\frac{a+bx}{a}\right)^{-m}\,{}_2F_1\left(-m,1+m;2+m;\frac{a-bx}{2a}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

```
[Out] -((2^m*(a - b*x)*(c*(a - b*x))^m*(a + b*x)^m*Hypergeometric2F1[-m, 1 + m, 2
 + m, (a - b*x)/(2*a)])/(b*(1 + m)*((a + b*x)/a)^m))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m*(a*c - b*c*x)^m,x]')
```

```
[Out] Exception raised: AttributeError >> 'SympyExpression' object has no attribu
te 'expr'
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(-bcx+ac)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

```
[Out] int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="maxima")
[Out] integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="fricas")
[Out] integral((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.74, size = 146, normalized size = 2.56

$$\frac{aa^{2m}c^{m}G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2}, \frac{1}{2} - \frac{m}{2}, 1 & \frac{1}{2}, -m, \frac{1}{2} - m \\ -m - \frac{1}{2}, -m, -\frac{m}{2}, \frac{1}{2} - m, \frac{1}{2} - \frac{m}{2} & 0 \end{matrix} \middle| \frac{a^{2}e^{-2i\pi}}{b^{2}x^{2}}\right)e^{-i\pi m}}{4\pi b\Gamma\left(-m\right)} - \frac{aa^{2m}c^{m}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, -\frac{m}{2} - \frac{1}{2}, -\frac{m}{2}, 1 & \\ -\frac{m}{2} - \frac{1}{2}, -\frac{m}{2} & -\frac{1}{2}, 0, -m - \frac{1}{2}, 0 \end{matrix} \middle| \frac{a^{2}}{b^{2}x^{2}}\right)}{4\pi b\Gamma\left(-m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b*c*x+a*c)**m,x)
[Out] a*a**(2*m)*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m -
1/2, -m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), a**2*exp_polar(-2*I*pi)/(b**2*x*
*2))*exp(-I*pi*m)/(4*pi*b*gamma(-m)) - a*a**(2*m)*c**m*meijerg(((-1/2, 0, 1
/2, -m/2 - 1/2, -m/2, 1), ()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0))
, a**2/(b**2*x**2))/(4*pi*b*gamma(-m))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a\,c-b\,c\,x)^m\,(a+b\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^m*(a + b*x)^m,x)
[Out] int((a*c - b*c*x)^m*(a + b*x)^m, x)
```

3.1235 $\int (3-6x)^m(2+4x)^m\,dx$

Optimal. Leaf size=20

$$6^m x\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)$$

```
[Out] 6^m*x*hypergeom([1/2, -m],[3/2],4*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {41, 251}

$$6^m x\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (3-6x)^m(2+4x)^m\,dx &= \int \left(6-24x^2\right)^m\,dx\\ &= 6^m x\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$6^m x\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - 6*x)^m*(2 + 4*x)^m,x]')
```

```
[Out] Exception raised: AttributeError >> 'SympyExpression' object has no attribu
te 'expr'
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (3-6x)^{m} (2+4x)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^m*(2+4*x)^m,x)
```

```
[Out] int((3-6*x)^m*(2+4*x)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="fricas")
```

```
[Out] integral((4*x + 2)^m*(-6*x + 3)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.28, size = 42, normalized size = 2.10

$$\frac{24^{m}\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}\right)^{m}\Gamma\left(m+1\right){}_{2}F_{1}\left(\begin{matrix}-m,m+1\\m+2\end{matrix}\middle|\left(x+\frac{1}{2}\right)e^{2i\pi}\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**m*(4*x+2)**m,x)
```

```
[Out] 24**m*(x + 1/2)*(x + 1/2)**m*gamma(m + 1)*hyper((-m, m + 1), (m + 2,), (x +
 1/2)*exp_polar(2*I*pi))/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int (4\,x+2)^m\,(3-6\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^m*(3 - 6*x)^m,x)
```

```
[Out] int((4*x + 2)^m*(3 - 6*x)^m, x)
```

3.1236 $\int (a+bx)^4(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

```
[Out] 1/5*(-a*d+b*c)*(b*x+a)^5/b^2+1/6*d*(b*x+a)^6/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)\,dx&=\int \left(\frac{(bc-ad)(a+bx)^4}{b}+\frac{d(a+bx)^5}{b}\right)\,dx\\&=\frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(38) = 76.
time = 0.01, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^4(2c+dx)+20a^3bx(3c+2dx)+15a^2b^2x^2(4c+3dx)+6ab^3x^3(5c+4dx)+b^4x^4(6c+5dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x),x]
```

```
[Out] (x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c + 3
*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(38) = 76.
time = 2.13, size = 85, normalized size = 2.24

$$\frac{x\left(30a^4c+15a^3x\left(ad+4bc\right)+20a^2bx^2\left(2ad+3bc\right)+15ab^2x^3\left(3ad+2bc\right)+6b^3x^4\left(4ad+bc\right)+5b^4dx^5\right)}{30}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x),x]')
```

```
[Out] x (30 a ^ 4 c + 15 a ^ 3 x (a d + 4 b c) + 20 a ^ 2 b x ^ 2 (2 a d + 3 b c)
 + 15 a b ^ 2 x ^ 3 (3 a d + 2 b c) + 6 b ^ 3 x ^ 4 (4 a d + b c) + 5 b ^ 4
 d x ^ 5) / 30
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(34) = 68.
time = 0.12, size = 97, normalized size = 2.55

method	result
norman	$\frac{b^4 d\,x^6}{6}+\left(\frac{4}{5}a\,b^3 d+\frac{1}{5}b^4 c\right)x^5+\left(\frac{3}{2}b^2 a^2 d+a\,b^3 c\right)x^4+\left(\frac{4}{3}a^3 bd+2b^2 a^2 c\right)x^3+\left(\frac{1}{2}a^4 d+2a^3 bc\right)x^2+a^4$
default	$\frac{b^4 d\,x^6}{6}+\frac{(4a\,b^3 d+b^4 c)x^5}{5}+\frac{(6b^2 a^2 d+4a\,b^3 c)x^4}{4}+\frac{(4a^3 bd+6b^2 a^2 c)x^3}{3}+\frac{(a^4 d+4a^3 bc)x^2}{2}+a^4 cx$
gosper	$\frac{1}{6}b^4 d\,x^6+\frac{4}{5}x^5 a\,b^3 d+\frac{1}{5}x^5 b^4 c+\frac{3}{2}x^4 b^2 a^2 d+x^4 a\,b^3 c+\frac{4}{3}x^3 a^3 bd+2x^3 b^2 a^2 c+\frac{1}{2}x^2 a^4 d+2x^2 a^3 bc+a$
risch	$\frac{1}{6}b^4 d\,x^6+\frac{4}{5}x^5 a\,b^3 d+\frac{1}{5}x^5 b^4 c+\frac{3}{2}x^4 b^2 a^2 d+x^4 a\,b^3 c+\frac{4}{3}x^3 a^3 bd+2x^3 b^2 a^2 c+\frac{1}{2}x^2 a^4 d+2x^2 a^3 bc+a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^4*d*x^6+1/5*(4*a*b^3*d+b^4*c)*x^5+1/4*(6*a^2*b^2*d+4*a*b^3*c)*x^4+1/3
*(4*a^3*b*d+6*a^2*b^2*c)*x^3+1/2*(a^4*d+4*a^3*b*c)*x^2+a^4*c*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(34) = 68.
time = 0.28, size = 96, normalized size = 2.53

$$\frac{1}{6}\,b^4dx^6+a^4cx+\frac{1}{5}\,\left(b^4c+4\,ab^3d\right)x^5+\frac{1}{2}\,\left(2\,ab^3c+3\,a^2b^2d\right)x^4+\frac{2}{3}\,\left(3\,a^2b^2c+2\,a^3bd\right)x^3+\frac{1}{2}\,\left(4\,a^3bc+a^4d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*
a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d
)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.
time = 0.29, size = 96, normalized size = 2.53

$$\frac{1}{6}b^4dx^6 + a^4cx + \frac{1}{5}(b^4c + 4ab^3d)x^5 + \frac{1}{2}(2ab^3c + 3a^2b^2d)x^4 + \frac{2}{3}(3a^2b^2c + 2a^3bd)x^3 + \frac{1}{2}(4a^3bc + a^4d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*
a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d
)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.
time = 0.04, size = 100, normalized size = 2.63

$$a^4cx + \frac{b^4dx^6}{6} + x^5 \cdot \left(\frac{4ab^3d}{5} + \frac{b^4c}{5}\right) + x^4 \cdot \left(\frac{3a^2b^2d}{2} + ab^3c\right) + x^3 \cdot \left(\frac{4a^3bd}{3} + 2a^2b^2c\right) + x^2\left(\frac{a^4d}{2} + 2a^3bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c),x)
```

```
[Out] a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b*
*2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 +
 2*a**3*b*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.
time = 0.00, size = 109, normalized size = 2.87

$$\frac{1}{6}x^6b^4d + \frac{1}{5}x^5b^4c + \frac{4}{5}x^5b^3ad + x^4b^3ac + \frac{3}{2}x^4b^2a^2d + 2x^3b^2a^2c + \frac{4}{3}x^3ba^3d + 2x^2ba^3c + \frac{1}{2}x^2a^4d + xa^4c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x)
```

```
[Out] 1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^2
*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^2
+ a^4*c*x
```

Mupad [B]
time = 0.19, size = 88, normalized size = 2.32

$$x^5\left(\frac{c\,b^4}{5} + \frac{4\,a\,d\,b^3}{5}\right) + x^2\left(\frac{d\,a^4}{2} + 2\,b\,c\,a^3\right) + \frac{b^4\,d\,x^6}{6} + a^4\,c\,x + \frac{2\,a^2\,b\,x^3\,(2\,a\,d + 3\,b\,c)}{3} + \frac{a\,b^2\,x^4\,(3\,a\,d + 2\,b\,c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x),x)
```

```
[Out] x^5*((b^4*c)/5 + (4*a*b^3*d)/5) + x^2*((a^4*d)/2 + 2*a^3*b*c) + (b^4*d*x^6)
/6 + a^4*c*x + (2*a^2*b*x^3*(2*a*d + 3*b*c))/3 + (a*b^2*x^4*(3*a*d + 2*b*c)
)/2
```

3.1237 $\int (a+bx)^3(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

```
[Out] 1/4*(-a*d+b*c)*(b*x+a)^4/b^2+1/5*d*(b*x+a)^5/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^3}{b}+\frac{d(a+bx)^4}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.76

$$a^3cx+\frac{1}{2}a^2(3bc+ad)x^2+ab(bc+ad)x^3+\frac{1}{4}b^2(bc+3ad)x^4+\frac{1}{5}b^3dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*a
*d)*x^4)/4 + (b^3*d*x^5)/5
```

Mathics [A]
time = 2.01, size = 60, normalized size = 1.58

$$x\left(a^3c+\frac{a^2x\,(ad+3bc)}{2}+abx^2\,(ad+bc)+\frac{b^2x^3\,(3ad+bc)}{4}+\frac{b^3dx^4}{5}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x),x]')
```

```
[Out] x (a ^ 3 c + a ^ 2 x (a d + 3 b c) / 2 + a b x ^ 2 (a d + b c) + b ^ 2 x ^
3 (3 a d + b c) / 4 + b ^ 3 d x ^ 4 / 5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(34) = 68.
time = 0.12, size = 73, normalized size = 1.92

method	result	size
norman	$\frac{b^3 d\,x^5}{5}+\left(\frac{3}{4}a\,b^2 d+\frac{1}{4}b^3 c\right)x^4+\left(a^2 bd+a\,b^2 c\right)x^3+\left(\frac{1}{2}a^3 d+\frac{3}{2}a^2 bc\right)x^2+a^3 cx$	70
gosper	$\frac{1}{5}b^3 d\,x^5+\frac{3}{4}x^4 a\,b^2 d+\frac{1}{4}x^4 b^3 c+a^2 bd\,x^3+a\,b^2 c\,x^3+\frac{1}{2}x^2 a^3 d+\frac{3}{2}a^2 bc\,x^2+a^3 cx$	73
default	$\frac{b^3 d\,x^5}{5}+\frac{(3a\,b^2 d+b^3 c)x^4}{4}+\frac{(3a^2 bd+3a\,b^2 c)x^3}{3}+\frac{(a^3 d+3a^2 bc)x^2}{2}+a^3 cx$	73
risch	$\frac{1}{5}b^3 d\,x^5+\frac{3}{4}x^4 a\,b^2 d+\frac{1}{4}x^4 b^3 c+a^2 bd\,x^3+a\,b^2 c\,x^3+\frac{1}{2}x^2 a^3 d+\frac{3}{2}a^2 bc\,x^2+a^3 cx$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^3*d*x^5+1/4*(3*a*b^2*d+b^3*c)*x^4+1/3*(3*a^2*b*d+3*a*b^2*c)*x^3+1/2*(
a^3*d+3*a^2*b*c)*x^2+a^3*c*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.
time = 0.26, size = 69, normalized size = 1.82

$$\frac{1}{5}\,b^3dx^5+a^3cx+\frac{1}{4}\,(b^3c+3\,ab^2d)x^4+(ab^2c+a^2bd)x^3+\frac{1}{2}\,(3\,a^2bc+a^3d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.29, size = 69, normalized size = 1.82

$$\frac{1}{5}\,b^3dx^5 + a^3cx + \frac{1}{4}\,\left(b^3c + 3\,ab^2d\right)x^4 + \left(ab^2c + a^2bd\right)x^3 + \frac{1}{2}\,\left(3\,a^2bc + a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 0.04, size = 73, normalized size = 1.92

$$a^3cx + \frac{b^3dx^5}{5} + x^4\cdot\left(\frac{3ab^2d}{4} + \frac{b^3c}{4}\right) + x^3\left(a^2bd + ab^2c\right) + x^2\left(\frac{a^3d}{2} + \frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
+ a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.
time = 0.00, size = 82, normalized size = 2.16

$$\frac{1}{5}x^5b^3d + \frac{1}{4}x^4b^3c + \frac{3}{4}x^4b^2ad + x^3b^2ac + x^3ba^2d + \frac{3}{2}x^2ba^2c + \frac{1}{2}x^2a^3d + xa^3c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x)
```

```
[Out] 1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^3
 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [**B**]
time = 0.16, size = 65, normalized size = 1.71

$$x^4\,\left(\frac{c\,b^3}{4} + \frac{3\,a\,d\,b^2}{4}\right) + x^2\,\left(\frac{d\,a^3}{2} + \frac{3\,b\,c\,a^2}{2}\right) + \frac{b^3\,d\,x^5}{5} + a^3\,c\,x + a\,b\,x^3\,\left(a\,d + b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x),x)
```

```
[Out] x^4*((b^3*c)/4 + (3*a*b^2*d)/4) + x^2*((a^3*d)/2 + (3*a^2*b*c)/2) + (b^3*d*
x^5)/5 + a^3*c*x + a*b*x^3*(a*d + b*c)
```

3.1238 $\int (a+bx)^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

[Out] 1/3*(-a*d+b*c)*(b*x+a)^3/b^2+1/4*d*(b*x+a)^4/b^2

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2*(c + d*x),x]

[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)\,dx&=\int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx\\&=\frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2*(c + d*x),x]

[Out] (x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12

Mathics [A]
time = 1.81, size = 43, normalized size = 1.13

$$x\left(a^2c+\frac{ax\left(ad+2bc\right)}{2}+\frac{bx^2\left(2ad+bc\right)}{3}+\frac{b^2dx^3}{4}\right)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^2*(c + d*x),x]')

[Out] x (a ^ 2 c + a x (a d + 2 b c) / 2 + b x ^ 2 (2 a d + b c) / 3 + b ^ 2 d x ^ 3 / 4)

Maple [A]
time = 0.12, size = 49, normalized size = 1.29

method	result	size
norman	$\frac{b^2dx^4}{4}+\left(\frac{2}{3}abd+\frac{1}{3}b^2c\right)x^3+\left(\frac{1}{2}a^2d+abc\right)x^2+a^2cx$	48
default	$\frac{b^2dx^4}{4}+\frac{(2abd+b^2c)x^3}{3}+\frac{(a^2d+2abc)x^2}{2}+a^2cx$	49
gosper	$\frac{1}{4}b^2d\,x^4+\frac{2}{3}x^3abd+\frac{1}{3}b^2c\,x^3+\frac{1}{2}x^2a^2d+x^2abc+a^2cx$	50
risch	$\frac{1}{4}b^2d\,x^4+\frac{2}{3}x^3abd+\frac{1}{3}b^2c\,x^3+\frac{1}{2}x^2a^2d+x^2abc+a^2cx$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/4*b^2*d*x^4+1/3*(2*a*b*d+b^2*c)*x^3+1/2*(a^2*d+2*a*b*c)*x^2+a^2*c*x

Maxima [A]
time = 0.26, size = 48, normalized size = 1.26

$$\frac{1}{4}\,b^2dx^4+a^2cx+\frac{1}{3}\left(b^2c+2\,abd\right)x^3+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="maxima")

[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)*x^2

Fricas [A]
time = 0.28, size = 48, normalized size = 1.26

$$\frac{1}{4}\,b^2dx^4+a^2cx+\frac{1}{3}\left(b^2c+2\,abd\right)x^3+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

Sympy [A]
time = 0.03, size = 49, normalized size = 1.29

$$a^2cx + \frac{b^2dx^4}{4} + x^3 \cdot \left(\frac{2abd}{3} + \frac{b^2c}{3}\right) + x^2\left(\frac{a^2d}{2} + abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 + a
*b*c)
```

Giac [A]
time = 0.00, size = 57, normalized size = 1.50

$$\frac{1}{4}x^4b^2d + \frac{1}{3}x^3b^2c + \frac{2}{3}x^3bad + x^2bac + \frac{1}{2}x^2a^2d + xa^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x)
```

```
[Out] 1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 a^2*c*x
```

Mupad [B]
time = 0.05, size = 47, normalized size = 1.24

$$x^2\left(\frac{d\,a^2}{2} + b\,c\,a\right) + x^3\left(\frac{c\,b^2}{3} + \frac{2\,a\,d\,b}{3}\right) + \frac{b^2\,d\,x^4}{4} + a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x),x)
```

```
[Out] x^2*((a^2*d)/2 + a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) + (b^2*d*x^4)/4 + a
^2*c*x
```

3.1239 $\int (a+bx)(c+dx)\,dx$

Optimal. Leaf size=28

$$acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3$$

```
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)\,dx &= \int \left(ac+(bc+ad)x+bdx^2\right)\,dx\\ &= acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Mathics [A]
time = 1.70, size = 23, normalized size = 0.82

$$x\left(ac+\frac{x\left(ad+bc\right)}{2}+\frac{bdx^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x),x]')
```

```
[Out] x (a c + x (a d + b c) / 2 + b d x ^ 2 / 3)
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^2}{2}+\frac{bd\,x^3}{3}$	25
norman	$\frac{bd\,x^3}{3}+\left(\frac{ad}{2}+\frac{bc}{2}\right)x^2+acx$	26
gosper	$\frac{1}{3}bd\,x^3+\frac{1}{2}x^2ad+\frac{1}{2}bc\,x^2+acx$	27
risch	$\frac{1}{3}bd\,x^3+\frac{1}{2}x^2ad+\frac{1}{2}bc\,x^2+acx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.86

$$\frac{1}{3}\,bdx^3+acx+\frac{1}{2}\,(bc+ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Fricas [A]
time = 0.26, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3db+\frac{1}{2}x^2cb+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*d*b + 1/2*x^2*c*b + 1/2*x^2*d*a + x*c*a
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.93

$$acx + \frac{bdx^3}{3} + x^2 \left(\frac{ad}{2} + \frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x)
```

```
[Out] a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

Giac [A]
time = 0.00, size = 32, normalized size = 1.14

$$\frac{1}{3}x^3bd + \frac{1}{2}x^2bc + \frac{1}{2}x^2ad + xac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x)
```

```
[Out] 1/3*b*d*x^3 + 1/2*b*c*x^2 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^3}{3} + \left(\frac{a\,d}{2} + \frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x),x)
```

```
[Out] x^2*((a*d)/2 + (b*c)/2) + a*c*x + (b*d*x^3)/3
```

3.1240 $\int (c+dx)\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[c + d*x,x]
[Out] c*x + (d*x^2)/2
```

Rubi steps

$$\int (c+dx)\,dx = cx+\frac{dx^2}{2}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[c + d*x,x]
[Out] c*x + (d*x^2)/2
```

Mathics [A]
time = 1.54, size = 10, normalized size = 0.83

$$\frac{x\,(2c+dx)}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x),x]')
```

```
[Out] x (2 c + d x) / 2
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
gosper	$cx+\frac{1}{2}d\,x^{2}$	11
default	$cx+\frac{1}{2}d\,x^{2}$	11
norman	$cx+\frac{1}{2}d\,x^{2}$	11
risch	$cx+\frac{1}{2}d\,x^{2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c,x,method=_RETURNVERBOSE)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A]
time = 0.31, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^{2}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A]
time = 0.26, size = 10, normalized size = 0.83

$$\frac{1}{2}x^{2}d+xc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*d + x*c
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$cx+\frac{dx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A]
time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x)
```

```
[Out] 1/2*d*x^2 + c*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c + d*x,x)
```

```
[Out] c*x + (d*x^2)/2
```

3.1241 $\int \frac{c+dx}{a+bx} dx$

Optimal. Leaf size=25

$$\frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}$$

[Out] d*x/b+(-a*d+b*c)*ln(b*x+a)/b^2

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x),x]

[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+bx}\,dx &= \int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx\\ &= \frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x),x]

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Mathics [A]
time = 1.71, size = 25, normalized size = 1.00

$$\frac{bdx - \text{Log}\left[a + bx\right]\left(ad - bc\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)/(a + b*x)^1,x]')
[Out] (b d x - Log[a + b x] (a d - b c)) / b ^ 2
```

Maple [A]
time = 0.12, size = 26, normalized size = 1.04

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\ln(bx+a)}{b^2}$	26
norman	$\frac{dx}{b}-\frac{(ad-bc)\ln(bx+a)}{b^2}$	27
risch	$\frac{dx}{b}-\frac{\ln(bx+a)ad}{b^2}+\frac{c\ln(bx+a)}{b}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] d*x/b+(-a*d+b*c)*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.27, size = 25, normalized size = 1.00

$$\frac{dx}{b} + \frac{(bc - ad)\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="maxima")
[Out] d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

Fricas [A]
time = 0.29, size = 24, normalized size = 0.96

$$\frac{bdx + (bc - ad)\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

Sympy [A]
time = 0.08, size = 20, normalized size = 0.80

$$\frac{dx}{b} - \frac{(ad - bc)\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x)
[Out] d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.04

$$\frac{xd}{b} + \frac{(-da + cb)\ln|xb + a|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x)
[Out] d*x/b + (b*c - a*d)*log(abs(b*x + a))/b^2
```

Mupad [B]
time = 0.05, size = 26, normalized size = 1.04

$$\frac{d\,x}{b} - \frac{\ln(a + b\,x)\,(a\,d - b\,c)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x),x)
[Out] (d*x)/b - (log(a + b*x)*(a*d - b*c))/b^2
```

3.1242 $\int \frac{c+dx}{(a+bx)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{bc-ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}$$

[Out] (a*d-b*c)/b^2/(b*x+a)+d*ln(b*x+a)/b^2

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*x)^2,x]

[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^2}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^2}+\frac{d}{b(a+bx)}\right)dx\\ &= -\frac{bc-ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.97

$$\frac{-bc+ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + b*x)^2,x]

```
[Out] (-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

Mathics [A]
time = 1.86, size = 32, normalized size = 1.00

$$\frac{ad - bc + d\text{Log}\left[a + bx\right](a + bx)}{b^2 (a + bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)/(a + b*x)^2,x]')
[Out] (a d - b c + d Log[a + b x] (a + b x)) / (b ^ 2 (a + b x))
```

Maple [A]
time = 0.13, size = 33, normalized size = 1.03

method	result	size
norman	$\frac{ad-bc}{b^2(bx+a)} + \frac{d\ln(bx+a)}{b^2}$	32
default	$-\frac{-ad+bc}{b^2(bx+a)} + \frac{d\ln(bx+a)}{b^2}$	33
risch	$\frac{ad}{b^2(bx+a)} - \frac{c}{b(bx+a)} + \frac{d\ln(bx+a)}{b^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -(-a*d+b*c)/b^2/(b*x+a)+d*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.28, size = 35, normalized size = 1.09

$$-\frac{bc - ad}{b^3x + ab^2} + \frac{d\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="maxima")
[Out] -(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

Fricas [A]
time = 0.29, size = 39, normalized size = 1.22

$$-\frac{bc - ad - (bdx + ad)\log(bx + a)}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.11, size = 27, normalized size = 0.84

$$\frac{ad-bc}{ab^2+b^3x}+\frac{d\log\left(a+bx\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**2,x)
```

```
[Out] (a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.03

$$\frac{da-cb}{bb\left(xb+a\right)}+\frac{d\ln\left|xb+a\right|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x)
```

```
[Out] d*log(abs(b*x + a))/b^2 - (b*c - a*d)/((b*x + a)*b^2)
```

Mupad [B]
time = 0.17, size = 31, normalized size = 0.97

$$\frac{a\,d-b\,c}{b^2\,\left(a+b\,x\right)}+\frac{d\,\ln\left(a+b\,x\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^2,x)
```

```
[Out] (a*d - b*c)/(b^2*(a + b*x)) + (d*log(a + b*x))/b^2
```

3.1243 $\int \frac{c+dx}{(a+bx)^3} \, dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

```
[Out] -1/2*(d*x+c)^2/(-a*d+b*c)/(b*x+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{37\}$

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -1/2*(c + d*x)^2/((b*c - a*d)*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx)^3} \, dx = -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -1/2*(a*d + b*(c + 2*d*x))/(b^2*(a + b*x)^2)
```

Mathics [A]
time = 1.86, size = 37, normalized size = 1.32

$$\frac{-ad-bc-2bdx}{2b^2\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)/(a + b*x)^3,x]')
[Out] (-a d - b c - 2 b d x) / (2 b ^ 2 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.12, size = 35, normalized size = 1.25

method	result	size
gosper	$-\frac{2bdx+ad+bc}{2(bx+a)^2b^2}$	25
risch	$\frac{-\frac{dx}{b}-\frac{ad+bc}{2b^2}}{(bx+a)^2}$	29
norman	$\frac{-\frac{dx}{b}+\frac{-ad-bc}{2b^2}}{(bx+a)^2}$	31
default	$-\frac{d}{b^2(bx+a)}-\frac{-ad+bc}{2b^2(bx+a)^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -d/b^2/(b*x+a)-1/2*(-a*d+b*c)/b^2/(b*x+a)^2
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [A]
time = 0.30, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A]
time = 0.15, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Giac [A]
time = 0.00, size = 29, normalized size = 1.04

$$\frac{-2xdb-da-cb}{2b^2\left(xb+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x)
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.16, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,b^2}+\frac{d\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*b^2) + (d*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.1244 $\int \frac{c+dx}{(a+bx)^4} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

```
[Out] 1/3*(a*d-b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -1/3*(b*c - a*d)/(b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^4} dx &= \int \left(\frac{bc-ad}{b(a+bx)^4}+\frac{d}{b(a+bx)^3}\right) dx\\ &= -\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{2bc+ad+3bdx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -1/6*(2*b*c + a*d + 3*b*d*x)/(b^2*(a + b*x)^3)
```

Mathics [A]
time = 2.03, size = 48, normalized size = 1.26

$$\frac{-ad-2bc-3bdx}{6b^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)/(a + b*x)^4,x]')
```

```
[Out] (-a d - 2 b c - 3 b d x) / (6 b ^ 2 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2
+ b ^ 3 x ^ 3))
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{3bdx+ad+2bc}{6b^2(bx+a)^3}$	26
risch	$\frac{-\frac{dx}{2b}-\frac{ad+2bc}{6b^2}}{(bx+a)^3}$	30
norman	$\frac{-\frac{dx}{2b}+\frac{-abd-2b^2c}{6b^3}}{(bx+a)^3}$	34
default	$-\frac{d}{2b^2(bx+a)^2}-\frac{-ad+bc}{3b^2(bx+a)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/b^2/(b*x+a)^2-1/3*(-a*d+b*c)/b^2/(b*x+a)^3
```

Maxima [A]
time = 0.29, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Fricas [A]
time = 0.29, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

Sympy [A]
time = 0.19, size = 53, normalized size = 1.39

$$\frac{-ad - 2bc - 3bdx}{6a^3b^2 + 18a^2b^3x + 18ab^4x^2 + 6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**4,x)
```

```
[Out] (-a*d - 2*b*c - 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

Giac [A]
time = 0.00, size = 30, normalized size = 0.79

$$\frac{-3xdb - da - 2cb}{6b^2\left(xb + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x)
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/((b*x + a)^3*b^2)
```

Mupad [B]
time = 0.17, size = 52, normalized size = 1.37

$$-\frac{\frac{a\,d+2\,b\,c}{6\,b^2}+\frac{d\,x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^4,x)
```

```
[Out] -((a*d + 2*b*c)/(6*b^2) + (d*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x)
```

3.1245 $\int \frac{c+dx}{(a+bx)^5}\,dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

```
[Out] 1/4*(a*d-b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -1/4*(b*c - a*d)/(b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^5}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^5}+\frac{d}{b(a+bx)^4}\right)dx\\ &= -\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{3bc+ad+4bdx}{12b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -1/12*(3*b*c + a*d + 4*b*d*x)/(b^2*(a + b*x)^4)
```

Mathics [A]
time = 2.12, size = 59, normalized size = 1.55

$$\frac{-ad - 3bc - 4bdx}{12b^2 (a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)/(a + b*x)^5,x]')
```

```
[Out] (-a d - 3 b c - 4 b d x) / (12 b ^ 2 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x
 ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4))
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{4bdx+ad+3bc}{12b^2(bx+a)^4}$	26
risch	$\frac{-\frac{dx}{3b}-\frac{ad+3bc}{12b^2}}{(bx+a)^4}$	30
default	$-\frac{-ad+bc}{4b^2(bx+a)^4}-\frac{d}{3b^2(bx+a)^3}$	35
norman	$\frac{-\frac{dx}{3b}+\frac{-a\,b^2d-3b^3c}{12b^4}}{(bx+a)^4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-a*d+b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.61

$$-\frac{4\,bdx + 3\,bc + ad}{12\,(b^6x^4 + 4\,ab^5x^3 + 6\,a^2b^4x^2 + 4\,a^3b^3x + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Fricas [A]
time = 0.28, size = 61, normalized size = 1.61

$$-\frac{4\,bdx + 3\,bc + ad}{12\,(b^6x^4 + 4\,ab^5x^3 + 6\,a^2b^4x^2 + 4\,a^3b^3x + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.
time = 0.25, size = 65, normalized size = 1.71

$$\frac{-ad - 3bc - 4bdx}{12a^4b^2 + 48a^3b^3x + 72a^2b^4x^2 + 48ab^5x^3 + 12b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**5,x)
```

```
[Out] (-a*d - 3*b*c - 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2
 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

Giac [A]
time = 0.00, size = 30, normalized size = 0.79

$$\frac{-4xdb - da - 3cb}{12b^2 (xb + a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x)
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/((b*x + a)^4*b^2)
```

Mupad [B]
time = 0.04, size = 63, normalized size = 1.66

$$-\frac{\frac{a\,d+3\,b\,c}{12\,b^2}+\frac{d\,x}{3\,b}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^5,x)
```

```
[Out] -((a*d + 3*b*c)/(12*b^2) + (d*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)
```

3.1246 $\int (a+bx)^4(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

```
[Out] 1/5*(-a*d+b*c)^2*(b*x+a)^5/b^3+1/3*d*(-a*d+b*c)*(b*x+a)^6/b^3+1/7*d^2*(b*x+
a)^7/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^4}{b^2}+\frac{2d(bc-ad)(a+bx)^5}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(65) = 130.
time = 0.02, size = 148, normalized size = 2.28

$$a^4c^2x+a^3c(2bc+ad)x^2+\frac{1}{3}a^2\left(6b^2c^2+8abcd+a^2d^2\right)x^3+ab\left(b^2c^2+3abcd+a^2d^2\right)x^4+\frac{1}{5}b^2\left(b^2c^2+8abcd+6a^2d^2\right)x^5+\frac{1}{3}b^3d(bc+2ad)x^6+\frac{1}{7}b^4d^2x^7$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^4*(c + d*x)^2,x]

[Out] a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*b* c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 147 vs. $2(65) = 130$.
time = 2.60, size = 139, normalized size = 2.14

$$x\left(a^4c^2 + a^3cx\,(ad + 2bc) + \frac{a^2x^2\,(a^2d^2 + 8abcd + 6b^2c^2)}{3} + abx^3\,(a^2d^2 + 3abcd + b^2c^2) + \frac{b^2x^4\,(6a^2d^2 + 8abcd + b^2c^2)}{5} + \frac{b^3dx^5\,(2ad + bc)}{3} + \frac{b^4d^2x^6}{7}\right)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^2,x]')

[Out] x (a ^ 4 c ^ 2 + a ^ 3 c x (a d + 2 b c) + a ^ 2 x ^ 2 (a ^ 2 d ^ 2 + 8 a b c d + 6 b ^ 2 c ^ 2) / 3 + a b x ^ 3 (a ^ 2 d ^ 2 + 3 a b c d + b ^ 2 c ^ 2) + b ^ 2 x ^ 4 (6 a ^ 2 d ^ 2 + 8 a b c d + b ^ 2 c ^ 2) / 5 + b ^ 3 d x ^ 5 (2 a d + b c) / 3 + b ^ 4 d ^ 2 x ^ 6 / 7)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(59) = 118$.
time = 0.13, size = 163, normalized size = 2.51

method	result
norman	$\frac{b^4d^2x^7}{7} + \left(\frac{2}{3}a\,b^3d^2 + \frac{1}{3}b^4cd\right)x^6 + \left(\frac{6}{5}b^2a^2d^2 + \frac{8}{5}a\,b^3cd + \frac{1}{5}b^4c^2\right)x^5 + \left(a^3b\,d^2 + 3b^2a^2cd + a\,b^3c^2\right)x^4 +$
default	$\frac{b^4d^2x^7}{7} + \frac{(4a\,b^3d^2+2b^4cd)x^6}{6} + \frac{(6b^2a^2d^2+8a\,b^3cd+b^4c^2)x^5}{5} + \frac{(4a^3b\,d^2+12b^2a^2cd+4a\,b^3c^2)x^4}{4} + \frac{(a^4d^2+8a^3bcd+6a^2b^2c^2)x^3}{3}$
gosper	$\frac{1}{7}b^4d^2x^7 + \frac{2}{3}x^6a\,b^3d^2 + \frac{1}{3}x^6b^4cd + \frac{6}{5}x^5b^2a^2d^2 + \frac{8}{5}x^5a\,b^3cd + \frac{1}{5}b^4c^2x^5 + a^3b\,d^2x^4 + 3a^2b^2cd\,x^4 + a\,b^3$
risch	$\frac{1}{7}b^4d^2x^7 + \frac{2}{3}x^6a\,b^3d^2 + \frac{1}{3}x^6b^4cd + \frac{6}{5}x^5b^2a^2d^2 + \frac{8}{5}x^5a\,b^3cd + \frac{1}{5}b^4c^2x^5 + a^3b\,d^2x^4 + 3a^2b^2cd\,x^4 + a\,b^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^4*(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/7*b^4*d^2*x^7+1/6*(4*a*b^3*d^2+2*b^4*c*d)*x^6+1/5*(6*a^2*b^2*d^2+8*a*b^3*c*d+b^4*c^2)*x^5+1/4*(4*a^3*b*d^2+12*a^2*b^2*c*d+4*a*b^3*c^2)*x^4+1/3*(a^4*d^2+8*a^3*b*c*d+6*a^2*b^2*c^2)*x^3+1/2*(2*a^4*c*d+4*a^3*b*c^2)*x^2+a^4*c^2*x

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.
time = 0.28, size = 156, normalized size = 2.40

$$\frac{1}{7}\,b^4d^2x^7 + a^4c^2x + \frac{1}{3}\,(b^4cd + 2\,ab^3d^2)x^6 + \frac{1}{5}\,(b^4c^2 + 8\,ab^3cd + 6\,a^2b^2d^2)x^5 + (ab^3c^2 + 3\,a^2b^2cd + a^3bd^2)x^4 + \frac{1}{3}\,(6\,a^2b^2c^2 + 8\,a^3bcd + a^4d^2)x^3 + (2\,a^3bc^2 + a^4cd)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.
time = 0.28, size = 156, normalized size = 2.40

$$\frac{1}{7}b^4d^2x^7+a^4c^2x+\frac{1}{3}\left(b^4cd+2\,ab^3d^2\right)x^6+\frac{1}{5}\left(b^4c^2+8\,ab^3cd+6\,a^2b^2d^2\right)x^5+\left(ab^3c^2+3\,a^2b^2cd+a^3bd^2\right)x^4+\frac{1}{3}\left(6\,a^2b^2c^2+8\,a^3bcd+a^4d^2\right)x^3+\left(2\,a^3bc^2+a^4cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(54) = 108$.
time = 0.05, size = 168, normalized size = 2.58

$$a^4c^2x+\frac{b^4d^2x^7}{7}+x^6\cdot\left(\frac{2ab^3d^2}{3}+\frac{b^4cd}{3}\right)+x^5\cdot\left(\frac{6a^2b^2d^2}{5}+\frac{8ab^3cd}{5}+\frac{b^4c^2}{5}\right)+x^4\left(a^3bd^2+3a^2b^2cd+ab^3c^2\right)+x^3\left(\frac{a^4d^2}{3}+\frac{8a^3bcd}{3}+2a^2b^2c^2\right)+x^2\left(a^4cd+2a^3bc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**2,x)
```

```
[Out] a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**5
*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2 +
3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*a**
2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(59) = 118$.
time = 0.00, size = 186, normalized size = 2.86

$$\frac{1}{7}x^7b^4d^2+\frac{1}{3}x^6b^4dc+\frac{2}{3}x^6b^3ad^2+\frac{1}{5}x^5b^4c^2+\frac{8}{5}x^5b^3adc+\frac{6}{5}x^5b^2a^2d^2+x^4b^3ac^2+3x^4b^2a^2dc+x^4ba^3d^2+2x^3b^2a^2c^2+\frac{8}{3}x^3ba^3dc+\frac{1}{3}x^3a^4d^2+2x^2ba^3c^2+x^2a^4dc+xa^4c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x)
```

```
[Out] 1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 + 8
/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^4
+ a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^3 +
 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

Mupad [B]
time = 0.07, size = 144, normalized size = 2.22

$$x^3\left(\frac{a^4\,d^2}{3}+\frac{8\,a^3\,b\,c\,d}{3}+2\,a^2\,b^2\,c^2\right)+x^5\left(\frac{6\,a^2\,b^2\,d^2}{5}+\frac{8\,a\,b^3\,c\,d}{5}+\frac{b^4\,c^2}{5}\right)+a^4\,c^2\,x+\frac{b^4\,d^2\,x^7}{7}+a^3\,c\,x^2\,\left(a\,d+2\,b\,c\right)+\frac{b^3\,d\,x^6\,\left(2\,a\,d+b\,c\right)}{3}+a\,b\,x^4\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^2,x)
```

```
[Out] x^3*((a^4*d^2)/3 + 2*a^2*b^2*c^2 + (8*a^3*b*c*d)/3) + x^5*((b^4*c^2)/5 + (6
*a^2*b^2*d^2)/5 + (8*a*b^3*c*d)/5) + a^4*c^2*x + (b^4*d^2*x^7)/7 + a^3*c*x^
2*(a*d + 2*b*c) + (b^3*d*x^6*(2*a*d + b*c))/3 + a*b*x^4*(a^2*d^2 + b^2*c^2
+ 3*a*b*c*d)
```

3.1247 $\int (a+bx)^3(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

```
[Out] 1/4*(-a*d+b*c)^2*(b*x+a)^4/b^3+2/5*d*(-a*d+b*c)*(b*x+a)^5/b^3+1/6*d^2*(b*x+
a)^6/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^3}{b^2}+\frac{2d(bc-ad)(a+bx)^4}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 122, normalized size = 1.88

$$a^3c^2x+\frac{1}{2}a^2c(3bc+2ad)x^2+\frac{1}{3}a\left(3b^2c^2+6abcd+a^2d^2\right)x^3+\frac{1}{4}b\left(b^2c^2+6abcd+3a^2d^2\right)x^4+\frac{1}{5}b^2d(2bc+3ad)x^5+\frac{1}{6}b^3d^2x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*c
+ 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6
```

Mathics [**A**]
time = 2.32, size = 111, normalized size = 1.71

$$x\left(a^3c^2+\frac{a^2cx\,(2ad+3bc)}{2}+\frac{ax^2\,(a^2d^2+6abcd+3b^2c^2)}{3}+\frac{bx^3\,(3a^2d^2+6abcd+b^2c^2)}{4}+\frac{b^2dx^4\,(3ad+2bc)}{5}+\frac{b^3d^2x^5}{6}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^2,x]')
```

```
[Out] x (a ^ 3 c ^ 2 + a ^ 2 c x (2 a d + 3 b c) / 2 + a x ^ 2 (a ^ 2 d ^ 2 + 6 a
 b c d + 3 b ^ 2 c ^ 2) / 3 + b x ^ 3 (3 a ^ 2 d ^ 2 + 6 a b c d + b ^ 2 c
^ 2) / 4 + b ^ 2 d x ^ 4 (3 a d + 2 b c) / 5 + b ^ 3 d ^ 2 x ^ 5 / 6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(59) = 118.
time = 0.13, size = 125, normalized size = 1.92

method	result
norman	$\frac{b^3d^2x^6}{6}+\left(\frac{3}{5}a\,b^2d^2+\frac{2}{5}b^3cd\right)x^5+\left(\frac{3}{4}a^2b\,d^2+\frac{3}{2}a\,b^2cd+\frac{1}{4}b^3c^2\right)x^4+\left(\frac{1}{3}a^3d^2+2a^2bcd+a\,b^2c^2\right)x^3+($
default	$\frac{b^3d^2x^6}{6}+\frac{(3a\,b^2d^2+2b^3cd)x^5}{5}+\frac{(3a^2b\,d^2+6a\,b^2cd+b^3c^2)x^4}{4}+\frac{(a^3d^2+6a^2bcd+3a\,b^2c^2)x^3}{3}+\frac{(2a^3cd+3a^2b\,c^2)x^2}{2}+a^3c^2x$
gosper	$\frac{1}{6}b^3d^2x^6+\frac{3}{5}x^5a\,b^2d^2+\frac{2}{5}x^5b^3cd+\frac{3}{4}x^4a^2b\,d^2+\frac{3}{2}x^4a\,b^2cd+\frac{1}{4}x^4b^3c^2+\frac{1}{3}x^3a^3d^2+2x^3a^2bcd+x^3a\,b^2$
risch	$\frac{1}{6}b^3d^2x^6+\frac{3}{5}x^5a\,b^2d^2+\frac{2}{5}x^5b^3cd+\frac{3}{4}x^4a^2b\,d^2+\frac{3}{2}x^4a\,b^2cd+\frac{1}{4}x^4b^3c^2+\frac{1}{3}x^3a^3d^2+2x^3a^2bcd+x^3a\,b^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^3*d^2*x^6+1/5*(3*a*b^2*d^2+2*b^3*c*d)*x^5+1/4*(3*a^2*b*d^2+6*a*b^2*c*
d+b^3*c^2)*x^4+1/3*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^3+1/2*(2*a^3*c*d+3*a
^2*b*c^2)*x^2+a^3*c^2*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(59) = 118.
time = 0.26, size = 124, normalized size = 1.91

$$\frac{1}{6}\,b^3d^2x^6+a^3c^2x+\frac{1}{5}\,(2\,b^3cd+3\,ab^2d^2)x^5+\frac{1}{4}\,(b^3c^2+6\,ab^2cd+3\,a^2bd^2)x^4+\frac{1}{3}\,(3\,ab^2c^2+6\,a^2bcd+a^3d^2)x^3+\frac{1}{2}\,(3\,a^2bc^2+2\,a^3cd)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.29, size = 124, normalized size = 1.91

$$\frac{1}{6}b^3d^2x^6 + a^3c^2x + \frac{1}{5}\left(2b^3cd + 3ab^2d^2\right)x^5 + \frac{1}{4}\left(b^3c^2 + 6ab^2cd + 3a^2bd^2\right)x^4 + \frac{1}{3}\left(3ab^2c^2 + 6a^2bcd + a^3d^2\right)x^3 + \frac{1}{2}\left(3a^2bc^2 + 2a^3cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.
time = 0.04, size = 133, normalized size = 2.05

$$a^3c^2x + \frac{b^3d^2x^6}{6} + x^5 \cdot \left(\frac{3ab^2d^2}{5} + \frac{2b^3cd}{5}\right) + x^4 \cdot \left(\frac{3a^2bd^2}{4} + \frac{3ab^2cd}{2} + \frac{b^3c^2}{4}\right) + x^3\left(\frac{a^3d^2}{3} + 2a^2bcd + ab^2c^2\right) + x^2\left(a^3cd + \frac{3a^2bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x*
*4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 + 2
*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.
time = 0.00, size = 146, normalized size = 2.25

$$\frac{1}{6}x^6b^3d^2 + \frac{2}{5}x^5b^3dc + \frac{3}{5}x^5b^2ad^2 + \frac{1}{4}x^4b^3c^2 + \frac{3}{2}x^4b^2adc + \frac{3}{4}x^4ba^2d^2 + x^3b^2ac^2 + 2x^3ba^2dc + \frac{1}{3}x^3a^3d^2 + \frac{3}{2}x^2ba^2c^2 + x^2a^3dc + xa^3c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x)
```

```
[Out] 1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 + 3
/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 + 1/
3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

Mupad [B]
time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(\frac{a^3d^2}{3} + 2a^2bcd + ab^2c^2\right) + x^4\left(\frac{3a^2bd^2}{4} + \frac{3ab^2cd}{2} + \frac{b^3c^2}{4}\right) + a^3c^2x + \frac{b^3d^2x^6}{6} + \frac{a^2cx^2\left(2ad + 3bc\right)}{2} + \frac{b^2dx^5\left(3ad + 2bc\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] x^3*((a^3*d^2)/3 + a*b^2*c^2 + 2*a^2*b*c*d) + x^4*((b^3*c^2)/4 + (3*a^2*b*d
^2)/4 + (3*a*b^2*c*d)/2) + a^3*c^2*x + (b^3*d^2*x^6)/6 + (a^2*c*x^2*(2*a*d
+ 3*b*c))/2 + (b^2*d*x^5*(3*a*d + 2*b*c))/5
```

3.1248 $\int (a+bx)^2(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

```
[Out] 1/3*(-a*d+b*c)^2*(b*x+a)^3/b^3+1/2*d*(-a*d+b*c)*(b*x+a)^4/b^3+1/5*d^2*(b*x+
a)^5/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x)^4)/(2*b^3) +
 (d^2*(a + b*x)^5)/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.22

$$a^2c^2x+ac(bc+ad)x^2+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3 +
 (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

Mathics [A]
time = 2.08, size = 72, normalized size = 1.11

$$x\left(a^2c^2 + acx\,(ad+bc) + \frac{x^2\,(a^2d^2+4abcd+b^2c^2)}{3} + \frac{bdx^3\,(ad+bc)}{2} + \frac{b^2d^2x^4}{5}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^2,x]')
```

```
[Out] x (a ^ 2 c ^ 2 + a c x (a d + b c) + x ^ 2 (a ^ 2 d ^ 2 + 4 a b c d + b ^ 2
 c ^ 2) / 3 + b d x ^ 3 (a d + b c) / 2 + b ^ 2 d ^ 2 x ^ 4 / 5)
```

Maple [A]
time = 0.14, size = 87, normalized size = 1.34

method	result	size
norman	$\frac{b^2d^2x^5}{5} + \left(\frac{1}{2}ab\,d^2 + \frac{1}{2}b^2cd\right)x^4 + \left(\frac{1}{3}a^2d^2 + \frac{4}{3}abcd + \frac{1}{3}b^2c^2\right)x^3 + \left(a^2cd + ab\,c^2\right)x^2 + a^2c^2x$	84
default	$\frac{b^2d^2x^5}{5} + \frac{(2ab\,d^2+2b^2cd)x^4}{4} + \frac{(a^2d^2+4abcd+b^2c^2)x^3}{3} + \frac{(2a^2cd+2ab\,c^2)x^2}{2} + a^2c^2x$	87
gosper	$\frac{1}{5}b^2d^2x^5 + \frac{1}{2}x^4ab\,d^2 + \frac{1}{2}x^4b^2cd + \frac{1}{3}x^3a^2d^2 + \frac{4}{3}x^3abcd + \frac{1}{3}x^3b^2c^2 + a^2cd\,x^2 + ab\,c^2x^2 + a^2c^2x$	90
risch	$\frac{1}{5}b^2d^2x^5 + \frac{1}{2}x^4ab\,d^2 + \frac{1}{2}x^4b^2cd + \frac{1}{3}x^3a^2d^2 + \frac{4}{3}x^3abcd + \frac{1}{3}x^3b^2c^2 + a^2cd\,x^2 + ab\,c^2x^2 + a^2c^2x$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*d^2*x^5+1/4*(2*a*b*d^2+2*b^2*c*d)*x^4+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^3+1/2*(2*a^2*c*d+2*a*b*c^2)*x^2+a^2*c^2*x
```

Maxima [A]
time = 0.29, size = 81, normalized size = 1.25

$$\frac{1}{5}\,b^2d^2x^5 + a^2c^2x + \frac{1}{2}\,\left(b^2cd + abd^2\right)x^4 + \frac{1}{3}\,\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^3 + \left(abc^2 + a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Fricas [A]
time = 0.29, size = 81, normalized size = 1.25

$$\frac{1}{5}\,b^2d^2x^5 + a^2c^2x + \frac{1}{2}\,\left(b^2cd + abd^2\right)x^4 + \frac{1}{3}\,\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^3 + \left(abc^2 + a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A]
time = 0.04, size = 87, normalized size = 1.34

$$a^2c^2x + \frac{b^2d^2x^5}{5} + x^4\left(\frac{abd^2}{2} + \frac{b^2cd}{2}\right) + x^3\left(\frac{a^2d^2}{3} + \frac{4abcd}{3} + \frac{b^2c^2}{3}\right) + x^2\left(a^2cd + abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a**
2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

Giac [A]
time = 0.00, size = 101, normalized size = 1.55

$$\frac{1}{5}x^5b^2d^2 + \frac{1}{2}x^4b^2dc + \frac{1}{2}x^4bad^2 + \frac{1}{3}x^3b^2c^2 + \frac{4}{3}x^3badc + \frac{1}{3}x^3a^2d^2 + x^2bac^2 + x^2a^2dc + xa^2c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x)
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

Mupad [B]
time = 0.17, size = 74, normalized size = 1.14

$$x^3\,\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^5}{5}+a\,c\,x^2\,\left(a\,d+b\,c\right)+\frac{b\,d\,x^4\,\left(a\,d+b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) + a^2*c^2*x + (b^2*d^2*x^5)
/5 + a*c*x^2*(a*d + b*c) + (b*d*x^4*(a*d + b*c))/2
```

3.1249 $\int (a+bx)(c+dx)^2\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}$$

```
[Out] -1/3*(-a*d+b*c)*(d*x+c)^3/d^2+1/4*b*(d*x+c)^4/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] -1/3*((b*c - a*d)*(c + d*x)^3)/d^2 + (b*(c + d*x)^4)/(4*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^2\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^2}{d}+\frac{b(c+dx)^3}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.24

$$\frac{1}{12}x\left(12ac^2+6c(bc+2ad)x+4d(2bc+ad)x^2+3bd^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^2,x]
```

[Out] (x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))/
12

Mathics [A]
time = 1.78, size = 43, normalized size = 1.13

$$x\left(ac^2+\frac{cx\,(2ad+bc)}{2}+\frac{dx^2\,(ad+2bc)}{3}+\frac{bd^2x^3}{4}\right)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^2,x]')

[Out] x (a c ^ 2 + c x (2 a d + b c) / 2 + d x ^ 2 (a d + 2 b c) / 3 + b d ^ 2 x
^ 3 / 4)

Maple [A]
time = 0.12, size = 49, normalized size = 1.29

method	result	size
norman	$\frac{b\,d^2x^4}{4}+\left(\frac{1}{3}a\,d^2+\frac{2}{3}bdc\right)x^3+\left(acd+\frac{1}{2}b\,c^2\right)x^2+a\,c^2x$	48
default	$\frac{b\,d^2x^4}{4}+\frac{(a\,d^2+2bdc)x^3}{3}+\frac{(2acd+b\,c^2)x^2}{2}+a\,c^2x$	49
gosper	$\frac{1}{4}b\,d^2x^4+\frac{1}{3}x^3a\,d^2+\frac{2}{3}x^3bdc+x^2acd+\frac{1}{2}b\,c^2x^2+a\,c^2x$	50
risch	$\frac{1}{4}b\,d^2x^4+\frac{1}{3}x^3a\,d^2+\frac{2}{3}x^3bdc+x^2acd+\frac{1}{2}b\,c^2x^2+a\,c^2x$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/4*b*d^2*x^4+1/3*(a*d^2+2*b*c*d)*x^3+1/2*(2*a*c*d+b*c^2)*x^2+a*c^2*x

Maxima [A]
time = 0.28, size = 48, normalized size = 1.26

$$\frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\,\left(2\,bcd+ad^2\right)x^3+\frac{1}{2}\,\left(bc^2+2\,acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="maxima")

[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2

Fricas [A]
time = 0.28, size = 48, normalized size = 1.26

$$\frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\,\left(2\,bcd+ad^2\right)x^3+\frac{1}{2}\,\left(bc^2+2\,acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

Sympy [A]
time = 0.03, size = 49, normalized size = 1.29

$$ac^2x + \frac{bd^2x^4}{4} + x^3\left(\frac{ad^2}{3} + \frac{2bcd}{3}\right) + x^2\left(acd + \frac{bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c*
*2/2)
```

Giac [A]
time = 0.00, size = 57, normalized size = 1.50

$$\frac{1}{4}x^4bd^2 + \frac{2}{3}x^3bdc + \frac{1}{3}x^3ad^2 + \frac{1}{2}x^2bc^2 + x^2adc + xac^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x)
```

```
[Out] 1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2 +
 a*c^2*x
```

Mupad [B]
time = 0.04, size = 47, normalized size = 1.24

$$x^2\left(\frac{b\,c^2}{2} + a\,d\,c\right) + x^3\left(\frac{a\,d^2}{3} + \frac{2\,b\,c\,d}{3}\right) + \frac{b\,d^2\,x^4}{4} + a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^2,x)
```

```
[Out] x^2*((b*c^2)/2 + a*c*d) + x^3*((a*d^2)/3 + (2*b*c*d)/3) + (b*d^2*x^4)/4 + a
*c^2*x
```

3.1250 $\int (c+dx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^3}{3d}$$

```
[Out] 1/3*(d*x+c)^3/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2 \, dx = \frac{(c+dx)^3}{3d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Mathics [A]
time = 1.60, size = 18, normalized size = 1.29

$$x\left(c^2 + cdx + \frac{d^2x^2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^2,x]')
[Out] x (c ^ 2 + c d x + d ^ 2 x ^ 2 / 3)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(dx+c)^3}{3d}$	13
gosper	$\frac{1}{3}d^2x^3 + cd\,x^2 + c^2x$	21
norman	$\frac{1}{3}d^2x^3 + cd\,x^2 + c^2x$	21
risch	$\frac{d^2x^3}{3} + cd\,x^2 + c^2x + \frac{c^3}{3d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(d*x+c)^3/d
```

Maxima [A]
time = 0.31, size = 20, normalized size = 1.43

$$\frac{1}{3}\,d^2x^3 + cdx^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="maxima")
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Fricas [A]
time = 0.28, size = 20, normalized size = 1.43

$$\frac{1}{3}\,d^2x^3 + cdx^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="fricas")
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.03, size = 19, normalized size = 1.36

$$c^2x + cdx^2 + \frac{d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2,x)
```

```
[Out] c**2*x + c*d*x**2 + d**2*x**3/3
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(dx+c)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x)
```

```
[Out] 1/3*(d*x + c)^3/d
```

Mupad [B]
time = 0.03, size = 20, normalized size = 1.43

$$c^2\,x+c\,d\,x^2+\frac{d^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2,x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + c*d*x^2
```

3.1251 $\int \frac{(c+dx)^2}{a+bx}\,dx$

Optimal. Leaf size=49

$$\frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}$$

```
[Out] d*(-a*d+b*c)*x/b^2+1/2*(d*x+c)^2/b+(-a*d+b*c)^2*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx\\ &= \frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.88

$$\frac{bdx(4bc-2ad+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x),x]
[Out] (b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

Mathics [A]
time = 1.95, size = 46, normalized size = 0.94

$$\frac{-bdx\,(ad-2bc)+\frac{b^2d^2x^2}{2}+\mathrm{Log}\,[a+bx]\,(ad-bc)^2}{b^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^1,x]')
[Out] (-b d x (a d - 2 b c) + b ^ 2 d ^ 2 x ^ 2 / 2 + Log[a + b x] (a d - b c) ^
2) / b ^ 3
```

Maple [A]
time = 0.17, size = 56, normalized size = 1.14

method	result	size
default	$-\frac{d\left(-\frac{1}{2}bd\,x^2+adx-2bcx\right)}{b^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(bx+a\right)}{b^3}$	56
norman	$\frac{d^2x^2}{2b}-\frac{d(ad-2bc)x}{b^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(bx+a\right)}{b^3}$	59
risch	$\frac{d^2x^2}{2b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}+\frac{\ln(bx+a)a^2d^2}{b^3}-\frac{2\ln(bx+a)acd}{b^2}+\frac{\ln(bx+a)c^2}{b}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -d/b^2*(-1/2*b*d*x^2+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3*ln(b*x+
a)
```

Maxima [A]
time = 0.26, size = 61, normalized size = 1.24

$$\frac{bd^2x^2+2\,(2\,bcd-ad^2)x}{2\,b^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log\,(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="maxima")
[Out] 1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(b*x + a)/b^3
```

Fricas [A]
time = 0.29, size = 63, normalized size = 1.29

$$\frac{b^2d^2x^2+2\,(2\,b^2cd-abd^2)x+2\,(b^2c^2-2\,abcd+a^2d^2)\log\,(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(b*x + a))/b^3
```

Sympy [A]
time = 0.13, size = 44, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\frac{d^2x^2}{2b}+\frac{(ad-bc)^2\log(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + d**2*x**2/(2*b) + (a*d - b*c)**2*log(a + b*x)/
b**3
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.33

$$\frac{\frac{1}{2}x^2d^2b-xd^2a+2xdcb}{b^2}+\frac{(d^2a^2-2dcba+c^2b^2)\ln|xb+a|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x)
```

```
[Out] 1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(b*x + a))/b^3
```

Mupad [B]
time = 0.19, size = 62, normalized size = 1.27

$$\frac{\ln(a+b\,x)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{b^3}-x\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{d^2\,x^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x),x)
```

```
[Out] (log(a + b*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/b^3 - x*((a*d^2)/b^2 - (2*c*
d)/b) + (d^2*x^2)/(2*b)
```

3.1252 $\int \frac{(c+dx)^2}{(a+bx)^2}\,dx$

Optimal. Leaf size=51

$$\frac{d^2x}{b^2}-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}$$

```
[Out] d^2*x/b^2-(-a*d+b*c)^2/b^3/(b*x+a)+2*d*(-a*d+b*c)*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right)dx\\ &= \frac{d^2x}{b^2}-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.92

$$\frac{bd^2x-\frac{(bc-ad)^2}{a+bx}+2d(bc-ad)\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

Mathics [A]
time = 2.15, size = 67, normalized size = 1.31

$$\frac{-a^2d^2 - b^2c^2 - 2d\mathrm{Log}\left[a + bx\right](a + bx)(ad - bc) + 2abcd + bd^2x(a + bx)}{b^3(a + bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^2,x]')
```

```
[Out] (-a ^ 2 d ^ 2 - b ^ 2 c ^ 2 - 2 d Log[a + b x] (a + b x) (a d - b c) + 2 a
b c d + b d ^ 2 x (a + b x)) / (b ^ 3 (a + b x))
```

Maple [A]
time = 0.14, size = 63, normalized size = 1.24

method	result	size
default	$\frac{d^2x}{b^2} - \frac{a^2d^2-2abcd+b^2c^2}{b^3(bx+a)} - \frac{2d(ad-bc)\ln(bx+a)}{b^3}$	63
norman	$\frac{\frac{d^2x^2}{b} - \frac{2a^2d^2-2abcd+b^2c^2}{b^3}}{bx+a} - \frac{2d(ad-bc)\ln(bx+a)}{b^3}$	68
risch	$\frac{d^2x}{b^2} - \frac{a^2d^2}{b^3(bx+a)} + \frac{2acd}{b^2(bx+a)} - \frac{c^2}{b(bx+a)} - \frac{2d^2\ln(bx+a)a}{b^3} + \frac{2d\ln(bx+a)c}{b^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2-(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)-2/b^3*d*(a*d-b*c)*ln(b*x+
a)
```

Maxima [A]
time = 0.26, size = 67, normalized size = 1.31

$$\frac{d^2x}{b^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{b^4x + ab^3} + \frac{2\,(bcd - ad^2)\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a*
d^2)*log(b*x + a)/b^3
```

Fricas [A]
time = 0.29, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2 + abd^2x - b^2c^2 + 2\,abcd - a^2d^2 + 2\,(abcd - a^2d^2 + (b^2cd - abd^2)x)\log{(bx + a)}}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^2
*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A]
time = 0.20, size = 60, normalized size = 1.18

$$\frac{-a^2d^2 + 2abcd - b^2c^2}{ab^3 + b^4x} + \frac{d^2x}{b^2} - \frac{2d\,(ad - bc)\log{(a + bx)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**2,x)
```

```
[Out] (-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d*
(a*d - b*c)*log(a + b*x)/b**3
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.35

$$\frac{xd^2}{b^2} + \frac{-d^2a^2 + 2dbac - b^2c^2}{b^3\,(xb + a)} + \frac{(-2d^2a + 2dcb)\ln{|xb + a|}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x)
```

```
[Out] d^2*x/b^2 + 2*(b*c*d - a*d^2)*log(abs(b*x + a))/b^3 - (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/((b*x + a)*b^3)
```

Mupad [B]
time = 0.20, size = 71, normalized size = 1.39

$$\frac{d^2\,x}{b^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{b\,\left(x\,b^3 + a\,b^2\right)} - \frac{\ln{(a + b\,x)}\,\left(2\,a\,d^2 - 2\,b\,c\,d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^2,x)
```

```
[Out] (d^2*x)/b^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(b*(a*b^2 + b^3*x)) - (log(a
+ b*x)*(2*a*d^2 - 2*b*c*d))/b^3
```

3.1253 $\int \frac{(c+dx)^2}{(a+bx)^3} dx$

Optimal. Leaf size=59

$$-\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}$$

```
[Out] -1/2*(-a*d+b*c)^2/b^3/(b*x+a)^2-2*d*(-a*d+b*c)/b^3/(b*x+a)+d^2*ln(b*x+a)/b^
3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*c - a*d)^2/(b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) +
(d^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^3}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{d^2}{b^2(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.83

$$\frac{-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2}+2d^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] (-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x])
/(2*b^3)
```

Mathics [A]
time = 2.40, size = 84, normalized size = 1.42

$$\frac{-abcd + d^2\text{Log}\left[a+bx\right]\left(a^2+2abx+b^2x^2\right)+\frac{3a^2d^2}{2}-\frac{b^2c^2}{2}+2bdx\left(ad-bc\right)}{b^3\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^3,x]')
```

```
[Out] (-a b c d + d ^ 2 Log[a + b x] (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) + 3 a ^ 2 d
^ 2 / 2 - b ^ 2 c ^ 2 / 2 + 2 b d x (a d - b c)) / (b ^ 3 (a ^ 2 + 2 a b x
+ b ^ 2 x ^ 2))
```

Maple [A]
time = 0.14, size = 69, normalized size = 1.17

method	result	size
risch	$\frac{\frac{2d(ad-bc)x}{b^2}+\frac{3a^2d^2-2abcd-b^2c^2}{2b^3}}{(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	67
default	$\frac{2d(ad-bc)}{b^3(bx+a)}-\frac{a^2d^2-2abcd+b^2c^2}{2b^3(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	69
norman	$\frac{\frac{3a^2d^2-2abcd-b^2c^2}{2b^3}+\frac{2\left(a\,d^2-bdc\right)x}{b^2}}{(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*d*(a*d-b*c)/(b*x+a)-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^2+d^2
*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.34

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\,(b^2cd-abd^2)x}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}+\frac{d^2\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

Fricas [A]
time = 0.29, size = 99, normalized size = 1.68

$$-\frac{b^2c^2 + 2\,abcd - 3\,a^2d^2 + 4\,(b^2cd - abd^2)x - 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log{(bx + a)}}{2\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
)
```

Sympy [A]
time = 0.27, size = 80, normalized size = 1.36

$$\frac{3a^2d^2 - 2abcd - b^2c^2 + x\,(4abd^2 - 4b^2cd)}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{d^2\log{(a + bx)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

Giac [A]
time = 0.00, size = 75, normalized size = 1.27

$$\frac{\frac{1}{2}\left((4d^2a - 4dbc)\,x + \frac{3d^2a^2 - 2dbac - b^2c^2}{b}\right)}{b^2\,(xb + a)^2} + \frac{d^2\ln{|xb + a|}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x)
```

```
[Out] d^2*log(abs(b*x + a))/b^3 - 1/2*(4*(b*c*d - a*d^2)*x + (b^2*c^2 + 2*a*b*c*d
 - 3*a^2*d^2)/b)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.20, size = 77, normalized size = 1.31

$$\frac{d^2\,\ln{(a + b\,x)}}{b^3} - \frac{\frac{-3\,a^2\,d^2 + 2\,a\,b\,c\,d + b^2\,c^2}{2\,b^3} - \frac{2\,d\,x\,(a\,d - b\,c)}{b^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^3,x)
```

```
[Out] (d^2*log(a + b*x))/b^3 - ((b^2*c^2 - 3*a^2*d^2 + 2*a*b*c*d)/(2*b^3) - (2*d*
x*(a*d - b*c))/b^2)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.1254 $\int \frac{(c+dx)^2}{(a+bx)^4} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

```
[Out] -1/3*(d*x+c)^3/(-a*d+b*c)/(b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(c + d*x)^3/((b*c - a*d)*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+bx)^4} dx = -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.89

$$-\frac{a^2d^2+abd(c+3dx)+b^2(c^2+3cdx+3d^2x^2)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(b^3*(
a + b*x)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 82 vs. $2(28) = 56$.
time = 2.39, size = 80, normalized size = 2.86

$$\frac{-a^2d^2 - abcd - b^2c^2 - 3bdx\,(ad + bc) - 3b^2d^2x^2}{3b^3\,(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^4,x]')
```

```
[Out] (-a ^ 2 d ^ 2 - a b c d - b ^ 2 c ^ 2 - 3 b d x (a d + b c) - 3 b ^ 2 d ^ 2
 x ^ 2) / (3 b ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(26) = 52$.
time = 0.14, size = 70, normalized size = 2.50

method	result	size
gosper	$-\frac{3d^2x^2b^2+3ab\,d^2x+3b^2cdx+a^2d^2+abcd+b^2c^2}{3(bx+a)^3b^3}$	60
risch	$\frac{-\frac{d^2x^2}{b}-\frac{d(ad+bc)x}{b^2}-\frac{a^2d^2+abcd+b^2c^2}{3b^3}}{(bx+a)^3}$	60
norman	$\frac{-\frac{d^2x^2}{b}+\frac{\left(-a\,d^2-bdc\right)x}{b^2}+\frac{-a^2d^2-abcd-b^2c^2}{3b^3}}{(bx+a)^3}$	66
default	$-\frac{d^2}{b^3(bx+a)}+\frac{d(ad-bc)}{b^3(bx+a)^2}-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3/(b*x+a)+1/b^3*d*(a*d-b*c)/(b*x+a)^2-1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/b^3/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.
time = 0.29, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2 + b^2c^2 + abcd + a^2d^2 + 3\,(b^2cd + abd^2)x}{3\,(b^6x^3 + 3\,ab^5x^2 + 3\,a^2b^4x + a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.
time = 0.30, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2 + b^2c^2 + abcd + a^2d^2 + 3\,(b^2cd + abd^2)x}{3\,(b^6x^3 + 3\,ab^5x^2 + 3\,a^2b^4x + a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(22) = 44$.
time = 0.35, size = 88, normalized size = 3.14

$$\frac{-a^2d^2 - abcd - b^2c^2 - 3b^2d^2x^2 + x\left(-3abd^2 - 3b^2cd\right)}{3a^3b^3 + 9a^2b^4x + 9ab^5x^2 + 3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**4,x)
```

```
[Out] (-a**2*d**2 - a*b*c*d - b**2*c**2 - 3*b**2*d**2*x**2 + x*(-3*a*b*d**2 - 3*b
**2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.
time = 0.00, size = 67, normalized size = 2.39

$$\frac{-3x^2d^2b^2 - 3xd^2ba - 3xdcb^2 - d^2a^2 - dcba - c^2b^2}{3b^3\left(xb + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x)
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*d
^2)/((b*x + a)^3*b^3)
```

Mupad [B]
time = 0.04, size = 80, normalized size = 2.86

$$-\frac{\frac{a^2\,d^2+a\,b\,c\,d+b^2\,c^2}{3\,b^3}+\frac{d^2\,x^2}{b}+\frac{d\,x\,\left(a\,d+b\,c\right)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^4,x)
```

```
[Out] -((a^2*d^2 + b^2*c^2 + a*b*c*d)/(3*b^3) + (d^2*x^2)/b + (d*x*(a*d + b*c))/b
^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.1255 $\int \frac{(c+dx)^2}{(a+bx)^5} dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}$$

```
[Out] -1/4*(-a*d+b*c)^2/b^3/(b*x+a)^4-2/3*d*(-a*d+b*c)/b^3/(b*x+a)^3-1/2*d^2/b^3/
(b*x+a)^2
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -1/4*(b*c - a*d)^2/(b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3
) - d^2/(2*b^3*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^5}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{d^2}{b^2(a+bx)^3}\right)dx\\ &= -\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.86

$$-\frac{a^2d^2+2abd(c+2dx)+b^2\left(3c^2+8cdx+6d^2x^2\right)}{12b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -1/12*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/(
b^3*(a + b*x)^4)
```

Mathics [**A**]
time = 2.65, size = 92, normalized size = 1.42

$$\frac{-a^2d^2 - 2abcd - 3b^2c^2 - 4bdx\,(ad + 2bc) - 6b^2d^2x^2}{12b^3\,(a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^5,x]')
```

```
[Out] (-a ^ 2 d ^ 2 - 2 a b c d - 3 b ^ 2 c ^ 2 - 4 b d x (a d + 2 b c) - 6 b ^ 2
 d ^ 2 x ^ 2) / (12 b ^ 3 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a
b ^ 3 x ^ 3 + b ^ 4 x ^ 4))
```

Maple [**A**]
time = 0.14, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{6d^2x^2b^2+4ab\,d^2x+8b^2cdx+a^2d^2+2abcd+3b^2c^2}{12b^3(bx+a)^4}$	62
risch	$\frac{-\frac{d^2x^2}{2b}-\frac{d(ad+2bc)x}{3b^2}-\frac{a^2d^2+2abcd+3b^2c^2}{12b^3}}{(bx+a)^4}$	63
default	$-\frac{a^2d^2-2abcd+b^2c^2}{4b^3(bx+a)^4}-\frac{d^2}{2b^3(bx+a)^2}+\frac{2d(ad-bc)}{3b^3(bx+a)^3}$	71
norman	$\frac{-\frac{d^2x^2}{2b}+\frac{\left(-ab\,d^2-2b^2cd\right)x}{3b^3}+\frac{-a^2b\,d^2-2a\,b^2cd-3b^3c^2}{12b^4}}{(bx+a)^4}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^4-1/2*d^2/b^3/(b*x+a)^2+2/3/b^
3*d*(a*d-b*c)/(b*x+a)^3
```

Maxima [**A**]
time = 0.27, size = 98, normalized size = 1.51

$$-\frac{6\,b^2d^2x^2 + 3\,b^2c^2 + 2\,abcd + a^2d^2 + 4\,(2\,b^2cd + abd^2)x}{12\,(b^7x^4 + 4\,ab^6x^3 + 6\,a^2b^5x^2 + 4\,a^3b^4x + a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Fricas [A]
time = 0.29, size = 98, normalized size = 1.51

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\,(2\,b^2cd+abd^2)x}{12\,(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [A]
time = 0.44, size = 104, normalized size = 1.60

$$\frac{-a^2d^2-2abcd-3b^2c^2-6b^2d^2x^2+x\left(-4abd^2-8b^2cd\right)}{12a^4b^3+48a^3b^4x+72a^2b^5x^2+48ab^6x^3+12b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**5,x)
```

```
[Out] (-a**2*d**2 - 2*a*b*c*d - 3*b**2*c**2 - 6*b**2*d**2*x**2 + x*(-4*a*b*d**2 -
 8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b**
6*x**3 + 12*b**7*x**4)
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.06

$$\frac{-6x^2d^2b^2-4xd^2ba-8xdcb^2-d^2a^2-2dcba-3c^2b^2}{12b^3\left(xb+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x)
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*a*b*d^2*x + 3*b^2*c^2 + 2*a*b*c*d +
a^2*d^2)/((b*x + a)^4*b^3)
```

Mupad [B]
time = 0.19, size = 39, normalized size = 0.60

$$\frac{{\left(c+d\,x\right)}^3\,\left(4\,a\,d-3\,b\,c+b\,d\,x\right)}{12\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^5,x)
```

```
[Out] ((c + d*x)^3*(4*a*d - 3*b*c + b*d*x))/(12*(a*d - b*c)^2*(a + b*x)^4)
```

3.1256 $\int \frac{(c+dx)^2}{(a+bx)^6} dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}$$

```
[Out] -1/5*(-a*d+b*c)^2/b^3/(b*x+a)^5-1/2*d*(-a*d+b*c)/b^3/(b*x+a)^4-1/3*d^2/b^3/
(b*x+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -1/5*(b*c - a*d)^2/(b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4)
- d^2/(3*b^3*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^6}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^4}\right)dx\\ &= -\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(3c+5dx)+b^2\left(6c^2+15cdx+10d^2x^2\right)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -1/30*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))
/(b^3*(a + b*x)^5)
```

Mathics [A]
time = 2.89, size = 103, normalized size = 1.58

$$\frac{-a^2d^2 - 3abcd - 6b^2c^2 - 5bdx\,(ad + 3bc) - 10b^2d^2x^2}{30b^3\,(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^6,x]')
```

```
[Out] (-a ^ 2 d ^ 2 - 3 a b c d - 6 b ^ 2 c ^ 2 - 5 b d x (a d + 3 b c) - 10 b ^
2 d ^ 2 x ^ 2) / (30 b ^ 3 (a ^ 5 + 5 a ^ 4 b x + 10 a ^ 3 b ^ 2 x ^ 2 + 10
 a ^ 2 b ^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 + b ^ 5 x ^ 5))
```

Maple [A]
time = 0.15, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{10d^2x^2b^2+5ab\,d^2x+15b^2cdx+a^2d^2+3abcd+6b^2c^2}{30b^3(bx+a)^5}$	62
risch	$\frac{-\frac{d^2x^2}{3b}-\frac{d(ad+3bc)x}{6b^2}-\frac{a^2d^2+3abcd+6b^2c^2}{30b^3}}{(bx+a)^5}$	63
default	$\frac{d(ad-bc)}{2b^3(bx+a)^4}-\frac{a^2d^2-2abcd+b^2c^2}{5b^3(bx+a)^5}-\frac{d^2}{3b^3(bx+a)^3}$	71
norman	$\frac{-\frac{d^2x^2}{3b}+\frac{\left(-a\,b^2d^2-3b^3cd\right)x}{6b^4}+\frac{-b^2a^2d^2-3a\,b^3cd-6b^4c^2}{30b^5}}{(bx+a)^5}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^3*d*(a*d-b*c)/(b*x+a)^4-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5
-1/3*d^2/b^3/(b*x+a)^3
```

Maxima [A]
time = 0.27, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2 + 6\,b^2c^2 + 3\,abcd + a^2d^2 + 5\,(3\,b^2cd + abd^2)x}{30\,(b^8x^5 + 5\,ab^7x^4 + 10\,a^2b^6x^3 + 10\,a^3b^5x^2 + 5\,a^4b^4x + a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Fricas [A]
time = 0.28, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\,(3\,b^2cd+abd^2)x}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.
time = 0.55, size = 116, normalized size = 1.78

$$\frac{-a^2d^2-3abcd-6b^2c^2-10b^2d^2x^2+x\,(-5abd^2-15b^2cd)}{30a^5b^3+150a^4b^4x+300a^3b^5x^2+300a^2b^6x^3+150ab^7x^4+30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**6,x)
```

```
[Out] (-a**2*d**2 - 3*a*b*c*d - 6*b**2*c**2 - 10*b**2*d**2*x**2 + x*(-5*a*b*d**2
- 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*
a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.06

$$\frac{-10x^2d^2b^2-5xd^2ba-15xdcb^2-d^2a^2-3dcba-6c^2b^2}{30b^3\,(xb+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x)
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
+ a^2*d^2)/((b*x + a)^5*b^3)
```

Mupad [B]
time = 0.20, size = 107, normalized size = 1.65

$$-\frac{\frac{a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2}{30\,b^3}+\frac{d^2\,x^2}{3\,b}+\frac{d\,x\,(a\,d+3\,b\,c)}{6\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^6,x)
```

```
[Out] -((a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d)/(30*b^3) + (d^2*x^2)/(3*b) + (d*x*(a*d
+ 3*b*c))/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b
^3*x^3 + 5*a^4*b*x)
```

3.1257 $\int \frac{(c+dx)^2}{(a+bx)^7} dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}$$

```
[Out] -1/6*(-a*d+b*c)^2/b^3/(b*x+a)^6-2/5*d*(-a*d+b*c)/b^3/(b*x+a)^5-1/4*d^2/b^3/
(b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -1/6*(b*c - a*d)^2/(b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5
) - d^2/(4*b^3*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 0.89

$$-\frac{a^2d^2+2abd(2c+3dx)+b^2\left(10c^2+24cdx+15d^2x^2\right)}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^
2))/(b^3*(a + b*x)^6)
```

Mathics [A]
time = 3.09, size = 114, normalized size = 1.75

$$\frac{-a^2d^2 - 4abcd - 10b^2c^2 - 6bdx\,(ad + 4bc) - 15b^2d^2x^2}{60b^3\,(a^6 + 6a^5bx + 15a^4b^2x^2 + 20a^3b^3x^3 + 15a^2b^4x^4 + 6ab^5x^5 + b^6x^6)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^2/(a + b*x)^7,x]')
```

```
[Out] (-a ^ 2 d ^ 2 - 4 a b c d - 10 b ^ 2 c ^ 2 - 6 b d x (a d + 4 b c) - 15 b ^
 2 d ^ 2 x ^ 2) / (60 b ^ 3 (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 2
0 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6)
)
```

Maple [A]
time = 0.16, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{15d^2x^2b^2+6ab\,d^2x+24b^2cdx+a^2d^2+4abcd+10b^2c^2}{60b^3(bx+a)^6}$	62
risch	$\frac{-\frac{d^2x^2}{4b}-\frac{d(ad+4bc)x}{10b^2}-\frac{a^2d^2+4abcd+10b^2c^2}{60b^3}}{(bx+a)^6}$	63
default	$-\frac{d^2}{4b^3(bx+a)^4}+\frac{2d(ad-bc)}{5b^3(bx+a)^5}-\frac{a^2d^2-2abcd+b^2c^2}{6b^3(bx+a)^6}$	71
norman	$\frac{-\frac{d^2x^2}{4b}+\frac{\left(-a\,b^3d^2-4b^4cd\right)x}{10b^5}+\frac{-a^2b^3d^2-4a\,b^4cd-10c^2b^5}{60b^6}}{(bx+a)^6}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2/b^3/(b*x+a)^4+2/5/b^3*d*(a*d-b*c)/(b*x+a)^5-1/6*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/b^3/(b*x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.
time = 0.28, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2 + 10\,b^2c^2 + 4\,abcd + a^2d^2 + 6\,(4\,b^2cd + abd^2)x}{60\,(b^9x^6 + 6\,ab^8x^5 + 15\,a^2b^7x^4 + 20\,a^3b^6x^3 + 15\,a^4b^5x^2 + 6\,a^5b^4x + a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.
time = 0.28, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\,(4\,b^2cd+abd^2)x}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(58) = 116$.
time = 0.67, size = 128, normalized size = 1.97

$$\frac{-a^2d^2-4abcd-10b^2c^2-15b^2d^2x^2+x\,(-6abd^2-24b^2cd)}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**7,x)
```

```
[Out] (-a**2*d**2 - 4*a*b*c*d - 10*b**2*c**2 - 15*b**2*d**2*x**2 + x*(-6*a*b*d**2
 - 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 120
0*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.06

$$\frac{-15x^2d^2b^2-6xd^2ba-24xdcb^2-d^2a^2-4dcba-10c^2b^2}{60b^3\,(xb+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x)
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)/((b*x + a)^6*b^3)
```

Mupad [B]
time = 0.09, size = 118, normalized size = 1.82

$$-\frac{\frac{a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2}{60\,b^3}+\frac{d^2\,x^2}{4\,b}+\frac{d\,x\,\left(a\,d+4\,b\,c\right)}{10\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^7,x)
```

```
[Out] -((a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d)/(60*b^3) + (d^2*x^2)/(4*b) + (d*x*(a*d
 + 4*b*c))/(10*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3
*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

3.1258 $\int (a+bx)^5(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

```
[Out] 1/6*(-a*d+b*c)^3*(b*x+a)^6/b^4+3/7*d*(-a*d+b*c)^2*(b*x+a)^7/b^4+3/8*d^2*(-a
*d+b*c)*(b*x+a)^8/b^4+1/9*d^3*(b*x+a)^9/b^4
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^5}{b^3}+\frac{3d(bc-ad)^2(a+bx)^6}{b^3}+\frac{3d^2(bc-ad)(a+bx)^7}{b^3}+\frac{d^3}{}\right.\\ &= \frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+}{9b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(92) = 184.
time = 0.05, size = 235, normalized size = 2.55

$$\frac{1}{504}x\left(126a^5\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+126a^4bx\left(10c^3+20c^2dx+15cd^2x^2+4d^3x^3\right)+84a^3b^2x^2\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(35c^3+84c^2dx+70cd^2x^2+20d^3x^3\right)+9ab^4x^4\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)+b^5x^5\left(84c^3+216c^2dx+189cd^2x^2+56d^3x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] (x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45*c
^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*x +
 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2
*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3
*x^3)))/504
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 266 vs. $2(92) = 184$.
time = 3.59, size = 252, normalized size = 2.74

$$x\left(a^5c^3+\frac{a^4c^2x(3ad+5bc)}{2}+\frac{a^3cx^2(3a^2d^2+15abcd+10b^2c^2)}{3}+\frac{a^2x^3(a^3d^3+15a^2bcd^2+30ab^2c^2d+10b^3c^3)}{4}+abx^4(a^3d^3+6a^2bcd^2+6ab^2c^2d+b^3c^3)+\frac{b^2x^5(10a^3d^3+30a^2bcd^2+15ab^2c^2d+b^3c^3)}{6}+\frac{b^3dx^6(10a^2d^2+15abcd+3b^2c^2)}{7}+\frac{b^4d^2x^7(5ad+3bc)}{8}+\frac{b^5d^3x^8}{9}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(c + d*x)^3,x]')
```

```
[Out] x (a ^ 5 c ^ 3 + a ^ 4 c ^ 2 x (3 a d + 5 b c) / 2 + a ^ 3 c x ^ 2 (3 a ^ 2
 d ^ 2 + 15 a b c d + 10 b ^ 2 c ^ 2) / 3 + a ^ 2 x ^ 3 (a ^ 3 d ^ 3 + 15 a
 ^ 2 b c d ^ 2 + 30 a b ^ 2 c ^ 2 d + 10 b ^ 3 c ^ 3) / 4 + a b x ^ 4 (a ^
3 d ^ 3 + 6 a ^ 2 b c d ^ 2 + 6 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) + b ^ 2 x ^
5 (10 a ^ 3 d ^ 3 + 30 a ^ 2 b c d ^ 2 + 15 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3)
/ 6 + b ^ 3 d x ^ 6 (10 a ^ 2 d ^ 2 + 15 a b c d + 3 b ^ 2 c ^ 2) / 7 + b ^
 4 d ^ 2 x ^ 7 (5 a d + 3 b c) / 8 + b ^ 5 d ^ 3 x ^ 8 / 9)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(84) = 168$.
time = 0.13, size = 281, normalized size = 3.05

method	result
norman	$\frac{b^5d^3x^9}{9}+\left(\frac{5}{8}a\,b^4d^3+\frac{3}{8}b^5c\,d^2\right)x^8+\left(\frac{10}{7}a^2b^3d^3+\frac{15}{7}a\,b^4c\,d^2+\frac{3}{7}b^5c^2d\right)x^7+\left(\frac{5}{3}a^3b^2d^3+5a^2b^3c\,d^2+\frac{5}{2}a\right.$
default	$\frac{b^5d^3x^9}{9}+\frac{(5a\,b^4d^3+3b^5c\,d^2)x^8}{8}+\frac{(10a^2b^3d^3+15a\,b^4c\,d^2+3b^5c^2d)x^7}{7}+\frac{(10a^3b^2d^3+30a^2b^3c\,d^2+15a\,b^4c^2d+b^5c^3)x^6}{6}+\frac{(5a^4b\,d^3}{}$
gosper	$\frac{1}{9}b^5d^3x^9+\frac{5}{8}x^8a\,b^4d^3+\frac{3}{8}x^8b^5c\,d^2+\frac{10}{7}x^7a^2b^3d^3+\frac{15}{7}x^7a\,b^4c\,d^2+\frac{3}{7}x^7b^5c^2d+\frac{5}{3}x^6a^3b^2d^3+5x^6a^2b^3c$
risch	$\frac{1}{9}b^5d^3x^9+\frac{5}{8}x^8a\,b^4d^3+\frac{3}{8}x^8b^5c\,d^2+\frac{10}{7}x^7a^2b^3d^3+\frac{15}{7}x^7a\,b^4c\,d^2+\frac{3}{7}x^7b^5c^2d+\frac{5}{3}x^6a^3b^2d^3+5x^6a^2b^3c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*d^3*x^9+1/8*(5*a*b^4*d^3+3*b^5*c*d^2)*x^8+1/7*(10*a^2*b^3*d^3+15*a*
b^4*c*d^2+3*b^5*c^2*d)*x^7+1/6*(10*a^3*b^2*d^3+30*a^2*b^3*c*d^2+15*a*b^4*c^
2*d+b^5*c^3)*x^6+1/5*(5*a^4*b*d^3+30*a^3*b^2*c*d^2+30*a^2*b^3*c^2*d+5*a*b^4
*c^3)*x^5+1/4*(a^5*d^3+15*a^4*b*c*d^2+30*a^3*b^2*c^2*d+10*a^2*b^3*c^3)*x^4+
```

```
1/3*(3*a^5*c*d^2+15*a^4*b*c^2*d+10*a^3*b^2*c^3)*x^3+1/2*(3*a^5*c^2*d+5*a^4*
b*c^3)*x^2+a^5*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(84) = 168$.
time = 0.28, size = 277, normalized size = 3.01

$$\frac{1}{9}b^5d^3x^9 + a^5c^3x + \frac{1}{8}(3b^5cd^2 + 5ab^4d^3)x^8 + \frac{1}{7}(3b^5c^2d + 15ab^4cd^2 + 10a^2b^3d^3)x^7 + \frac{1}{6}(b^5c^3 + 15ab^4c^2d + 30a^2b^3cd^2 + 10a^3b^2d^3)x^6 + (ab^4c^3 + 6a^2b^3c^2d + 6a^3b^2cd^2 + a^4bd^3)x^5 + \frac{1}{4}(10a^2b^3c^3 + 30a^3b^2c^2d + 15a^4bcd^2 + a^5d^3)x^4 + \frac{1}{3}(10a^3b^2c^3 + 15a^4bc^2d + 3a^5cd^2)x^3 + \frac{1}{2}(5a^4bc^3 + 3a^5c^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(84) = 168$.
time = 0.29, size = 277, normalized size = 3.01

$$\frac{1}{9}b^5d^3x^9 + a^5c^3x + \frac{1}{8}(3b^5cd^2 + 5ab^4d^3)x^8 + \frac{1}{7}(3b^5c^2d + 15ab^4cd^2 + 10a^2b^3d^3)x^7 + \frac{1}{6}(b^5c^3 + 15ab^4c^2d + 30a^2b^3cd^2 + 10a^3b^2d^3)x^6 + (ab^4c^3 + 6a^2b^3c^2d + 6a^3b^2cd^2 + a^4bd^3)x^5 + \frac{1}{4}(10a^2b^3c^3 + 30a^3b^2c^2d + 15a^4bcd^2 + a^5d^3)x^4 + \frac{1}{3}(10a^3b^2c^3 + 15a^4bc^2d + 3a^5cd^2)x^3 + \frac{1}{2}(5a^4bc^3 + 3a^5c^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.
time = 0.06, size = 308, normalized size = 3.35

$$a^5c^3x + \frac{b^5d^3x^9}{9} + x^8 \cdot \left(\frac{5ab^4d^3}{8} + \frac{3b^5cd^2}{8}\right) + x^7 \cdot \left(\frac{10a^2b^3d^3}{7} + \frac{15ab^4cd^2}{7} + \frac{3b^5c^2d}{7}\right) + x^6 \cdot \left(\frac{5a^3b^2d^3}{3} + 5a^2b^3cd^2 + \frac{5ab^4c^2d}{2} + \frac{b^5c^3}{6}\right) + x^5\left(a^4bd^3 + 6a^3b^2cd^2 + 6a^2b^3c^2d + ab^4c^3\right) + x^4\left(\frac{a^5d^3}{4} + \frac{15a^4bcd^2}{4} + \frac{15a^3b^2c^2d}{2} + \frac{5a^2b^3c^3}{2}\right) + x^3\left(a^5cd^2 + 5a^4bc^2d + \frac{10a^3b^2c^3}{3}\right) + x^2 \cdot \left(\frac{3a^5c^2d}{2} + \frac{5a^4bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**3,x)
```

```
[Out] a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8) +
 x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6*(
```

```
5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/6)
+ x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*c**3
) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5*a**2
*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c**3/3)
+ x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(84) = 168.
time = 0.00, size = 335, normalized size = 3.64

$$\frac{1}{9}x^9b^5d^3 + \frac{3}{8}x^8b^5d^2c + \frac{5}{8}x^8b^4ad^3 + \frac{3}{7}x^7b^5dc^2 + \frac{15}{7}x^7b^4ad^2c + \frac{10}{7}x^7b^3a^2d^3 + \frac{1}{6}x^6b^5c^3 + \frac{5}{2}x^6b^4adc^2 + 5x^6b^3a^2d^2c + \frac{5}{3}x^6b^2a^3d^3 + x^5b^4ac^3 + 6x^5b^3a^2dc^2 + 6x^5b^2a^3d^2c + x^5ba^4d^3 + \frac{5}{2}x^4b^3a^2c^3 + \frac{15}{2}x^4b^2a^3dc^2 + \frac{15}{4}x^4ba^4d^2c + \frac{1}{4}x^4a^5d^3 + \frac{10}{3}x^3b^2a^3c^3 + 5x^3ba^4dc^2 + x^3a^5d^2c + \frac{5}{2}x^2ba^4c^3 + \frac{3}{2}x^2a^5dc^2 + xa^5c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x)
```

```
[Out] 1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^7
 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*b^
4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5 + 6
*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*c^3*
x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4 + 10/
3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3*x^2 +
 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

Mupad [**B**]
time = 0.24, size = 261, normalized size = 2.84

$$x^5\,\left(a^4\,b\,d^3+6\,a^3\,b^2\,c\,d^2+6\,a^2\,b^3\,c^2\,d+a\,b^4\,c^3\right)+x^4\,\left(\frac{a^5\,d^3}{4}+\frac{15\,a^4\,b\,c\,d^2}{4}+\frac{15\,a^3\,b^2\,c^2\,d}{2}+\frac{5\,a^2\,b^3\,c^3}{2}\right)+x^6\,\left(\frac{5\,a^3\,b^2\,d^3}{3}+5\,a^2\,b^3\,c\,d^2+\frac{5\,a\,b^4\,c^2\,d}{2}+\frac{b^5\,c^3}{6}\right)+a^5\,c^3\,x+\frac{b^5\,d^3\,x^9}{9}+\frac{a^4\,c^2\,x^2\,\left(3\,a\,d+5\,b\,c\right)}{2}+\frac{b^4\,d^2\,x^8\,\left(5\,a\,d+3\,b\,c\right)}{8}+\frac{a^3\,c\,x^3\,\left(3\,a^2\,d^2+15\,a\,b\,c\,d+10\,b^2\,c^2\right)}{3}+\frac{b^3\,d\,x^7\,\left(10\,a^2\,d^2+15\,a\,b\,c\,d+3\,b^2\,c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^3,x)
```

```
[Out] x^5*(a*b^4*c^3 + a^4*b*d^3 + 6*a^2*b^3*c^2*d + 6*a^3*b^2*c*d^2) + x^4*((a^5
*d^3)/4 + (5*a^2*b^3*c^3)/2 + (15*a^3*b^2*c^2*d)/2 + (15*a^4*b*c*d^2)/4) +
x^6*((b^5*c^3)/6 + (5*a^3*b^2*d^3)/3 + 5*a^2*b^3*c*d^2 + (5*a*b^4*c^2*d)/2)
 + a^5*c^3*x + (b^5*d^3*x^9)/9 + (a^4*c^2*x^2*(3*a*d + 5*b*c))/2 + (b^4*d^2
*x^8*(5*a*d + 3*b*c))/8 + (a^3*c*x^3*(3*a^2*d^2 + 10*b^2*c^2 + 15*a*b*c*d))
/3 + (b^3*d*x^7*(10*a^2*d^2 + 3*b^2*c^2 + 15*a*b*c*d))/7
```

3.1259 $\int (a+bx)^4(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

```
[Out] 1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-a
*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^4}{b^3}+\frac{3d(bc-ad)^2(a+bx)^5}{b^3}+\frac{3d^2(bc-ad)(a+bx)^6}{b^3}+\frac{d^3\cdots}{\cdots}\right.\\ &= \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+\cdots}{8b^4}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.
time = 0.02, size = 217, normalized size = 2.36

$$a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2+a^2c\left(2b^2c^2+4abcd+a^2d^2\right)x^3+\frac{1}{4}a\left(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3\right)x^4+\frac{1}{5}b\left(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3\right)x^5+\frac{1}{2}b^2d\left(b^2c^2+4abcd+2a^2d^2\right)x^6+\frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)
*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5)/5
 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c + 4*a*
d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 216 vs. $2(92) = 184$.
time = 3.18, size = 204, normalized size = 2.22

$$x\left(a^4c^3 + \frac{a^3c^2x\,(3ad+4bc)}{2} + a^2cx^2\left(a^2d^2+4abcd+2b^2c^2\right) + \frac{ax^3\left(a^3d^3+12a^2bcd^2+18ab^2c^2d+4b^3c^3\right)}{4} + \frac{bx^4\left(4a^3d^3+18a^2bcd^2+12ab^2c^2d+b^3c^3\right)}{5} + \frac{b^2dx^5\left(2a^2d^2+4abcd+b^2c^2\right)}{2} + \frac{b^3d^2x^6\,(4ad+3bc)}{7} + \frac{b^4d^3x^7}{8}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^3,x]')
```

```
[Out] x (a ^ 4 c ^ 3 + a ^ 3 c ^ 2 x (3 a d + 4 b c) / 2 + a ^ 2 c x ^ 2 (a ^ 2 d
 ^ 2 + 4 a b c d + 2 b ^ 2 c ^ 2) + a x ^ 3 (a ^ 3 d ^ 3 + 12 a ^ 2 b c d ^
 2 + 18 a b ^ 2 c ^ 2 d + 4 b ^ 3 c ^ 3) / 4 + b x ^ 4 (4 a ^ 3 d ^ 3 + 18
a ^ 2 b c d ^ 2 + 12 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) / 5 + b ^ 2 d x ^ 5 (2
a ^ 2 d ^ 2 + 4 a b c d + b ^ 2 c ^ 2) / 2 + b ^ 3 d ^ 2 x ^ 6 (4 a d + 3 b
 c) / 7 + b ^ 4 d ^ 3 x ^ 7 / 8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(84) = 168$.
time = 0.13, size = 229, normalized size = 2.49

method	result
norman	$\frac{b^4d^3x^8}{8} + \left(\frac{4}{7}a\,b^3d^3 + \frac{3}{7}b^4c\,d^2\right)x^7 + \left(b^2a^2d^3 + 2a\,b^3c\,d^2 + \frac{1}{2}b^4c^2d\right)x^6 + \left(\frac{4}{5}a^3b\,d^3 + \frac{18}{5}b^2a^2c\,d^2 + \frac{12}{5}a\,b^3\right.$
default	$\frac{b^4d^3x^8}{8} + \frac{(4a\,b^3d^3+3b^4c\,d^2)x^7}{7} + \frac{(6b^2a^2d^3+12a\,b^3c\,d^2+3b^4c^2d)x^6}{6} + \frac{(4a^3b\,d^3+18b^2a^2c\,d^2+12a\,b^3c^2d+b^4c^3)x^5}{5} + \frac{(a^4d^3+12a^3}{}$
gosper	$\frac{1}{8}b^4d^3x^8 + \frac{4}{7}x^7a\,b^3d^3 + \frac{3}{7}x^7b^4c\,d^2 + x^6b^2a^2d^3 + 2x^6a\,b^3c\,d^2 + \frac{1}{2}x^6b^4c^2d + \frac{4}{5}x^5a^3b\,d^3 + \frac{18}{5}x^5b^2a^2c\,d^2$
risch	$\frac{1}{8}b^4d^3x^8 + \frac{4}{7}x^7a\,b^3d^3 + \frac{3}{7}x^7b^4c\,d^2 + x^6b^2a^2d^3 + 2x^6a\,b^3c\,d^2 + \frac{1}{2}x^6b^4c^2d + \frac{4}{5}x^5a^3b\,d^3 + \frac{18}{5}x^5b^2a^2c\,d^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^4*d^3*x^8+1/7*(4*a*b^3*d^3+3*b^4*c*d^2)*x^7+1/6*(6*a^2*b^2*d^3+12*a*b
^3*c*d^2+3*b^4*c^2*d)*x^6+1/5*(4*a^3*b*d^3+18*a^2*b^2*c*d^2+12*a*b^3*c^2*d+
b^4*c^3)*x^5+1/4*(a^4*d^3+12*a^3*b*c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3)*x^4+
1/3*(3*a^4*c*d^2+12*a^3*b*c^2*d+6*a^2*b^2*c^3)*x^3+1/2*(3*a^4*c^2*d+4*a^3*b
*c^3)*x^2+a^4*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.
time = 0.26, size = 225, normalized size = 2.45

$$\frac{1}{8}b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3b^4cd^2 + 4ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d + 4ab^3cd^2 + 2a^2b^2d^3\right)x^6 + \frac{1}{5}\left(b^4c^3 + 12ab^3c^2d + 18a^2b^2cd^2 + 4a^3bd^3\right)x^5 + \frac{1}{4}\left(4ab^3c^3 + 18a^2b^2c^2d + 12a^3bcd^2 + a^4d^3\right)x^4 + \left(2a^2b^2c^3 + 4a^3bc^2d + a^4cd^2\right)x^3 + \frac{1}{2}\left(4a^3bc^3 + 3a^4c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.
time = 0.29, size = 225, normalized size = 2.45

$$\frac{1}{8}b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3b^4cd^2 + 4ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d + 4ab^3cd^2 + 2a^2b^2d^3\right)x^6 + \frac{1}{5}\left(b^4c^3 + 12ab^3c^2d + 18a^2b^2cd^2 + 4a^3bd^3\right)x^5 + \frac{1}{4}\left(4ab^3c^3 + 18a^2b^2c^2d + 12a^3bcd^2 + a^4d^3\right)x^4 + \left(2a^2b^2c^3 + 4a^3bc^2d + a^4cd^2\right)x^3 + \frac{1}{2}\left(4a^3bc^3 + 3a^4c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.
time = 0.05, size = 243, normalized size = 2.64

$$a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\cdot\left(\frac{4ab^3d^3}{7} + \frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3 + 2ab^3cd^2 + \frac{b^4c^2d}{2}\right) + x^5\cdot\left(\frac{4a^3bd^3}{5} + \frac{18a^2b^2cd^2}{5} + \frac{12ab^3c^2d}{5} + \frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4} + 3a^3bcd^2 + \frac{9a^2b^2c^2d}{2} + ab^3c^3\right) + x^3\left(a^4cd^2 + 4a^3bc^2d + 2a^2b^2c^3\right) + x^2\cdot\left(\frac{3a^4c^2d}{2} + 2a^3bc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**3,x)
```

```
[Out] a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) +
 x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d
**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a
**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(
a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 +
 2*a**3*b*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(84) = 168.
time = 0.00, size = 267, normalized size = 2.90

$$\frac{1}{8}x^8b^4d^3+\frac{3}{7}x^7b^4d^2c+\frac{4}{7}x^7b^3ad^3+\frac{1}{2}x^6b^4dc^2+2x^6b^3ad^2c+x^6b^2a^2d^3+\frac{1}{5}x^5b^4c^3+\frac{12}{5}x^5b^3adc^2+\frac{18}{5}x^5b^2a^2d^2c+\frac{4}{5}x^5ba^3d^3+x^4b^3ac^3+\frac{9}{2}x^4b^2a^2dc^2+3x^4ba^3d^2c+\frac{1}{4}x^4a^4d^3+2x^3b^2a^2c^3+4x^3ba^3dc^2+x^3a^4d^2c+2x^2ba^3c^3+\frac{3}{2}x^2a^4dc^2+xa^4c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x)
```

```
[Out] 1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^6
 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2*d
*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*a^2
*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^3 +
4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^2 + a
^4*c^3*x
```

Mupad [B]
time = 0.21, size = 208, normalized size = 2.26

$$x^4\left(\frac{a^4d^3}{4}+3a^3bcd^2+\frac{9a^2b^2c^2d}{2}+ab^3c^3\right)+x^5\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right)+a^4c^3x+\frac{b^4d^3x^8}{8}+\frac{a^3c^2x^2\,(3ad+4bc)}{2}+\frac{b^3d^2x^7\,(4ad+3bc)}{7}+a^2cx^3\,(a^2d^2+4abcd+2b^2c^2)+\frac{b^2dx^6\,(2a^2d^2+4abcd+b^2c^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^3,x)
```

```
[Out] x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*(
(b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5)
+ a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d^2*
x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) + (b^2
*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

3.1260 $\int (a+bx)^3(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

```
[Out] 1/4*(-a*d+b*c)^3*(b*x+a)^4/b^4+3/5*d*(-a*d+b*c)^2*(b*x+a)^5/b^4+1/2*d^2*(-a
*d+b*c)*(b*x+a)^6/b^4+1/7*d^3*(b*x+a)^7/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^4)/(4*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^5)/(5*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^6)/(2*b^4) + (d^3*(a + b*x)^7)/(7*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^3}{b^3}+\frac{3d(bc-ad)^2(a+bx)^4}{b^3}+\frac{3d^2(bc-ad)(a+bx)^5}{b^3}+\frac{d^3\end{aligned}$$

$$= \frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+b}{7b^4}$$

Mathematica [A]
time = 0.01, size = 161, normalized size = 1.75

$$a^3c^3x+\frac{3}{2}a^2c^2(bc+ad)x^2+ac\left(b^2c^2+3abcd+a^2d^2\right)x^3+\frac{1}{4}\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^4+\frac{3}{5}bd\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{2}b^2d^2(bc+ad)x^6+\frac{1}{7}b^3d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2*
d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 + (3
*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/2 +
 (b^3*d^3*x^7)/7
```

Mathics [A]
time = 2.67, size = 153, normalized size = 1.66

$$\frac{x\left(140a^3c^3+210a^2c^2x\left(ad+bc\right)+140acx^2\left(a^2d^2+3abcd+b^2c^2\right)+35x^3\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)+84bdx^4\left(a^2d^2+3abcd+b^2c^2\right)+70b^2d^2x^5\left(ad+bc\right)+20b^3d^3x^6\right)}{140}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^3,x]')
```

```
[Out] x (140 a ^ 3 c ^ 3 + 210 a ^ 2 c ^ 2 x (a d + b c) + 140 a c x ^ 2 (a ^ 2 d
 ^ 2 + 3 a b c d + b ^ 2 c ^ 2) + 35 x ^ 3 (a ^ 3 d ^ 3 + 9 a ^ 2 b c d ^ 2
 + 9 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) + 84 b d x ^ 4 (a ^ 2 d ^ 2 + 3 a b c d
 + b ^ 2 c ^ 2) + 70 b ^ 2 d ^ 2 x ^ 5 (a d + b c) + 20 b ^ 3 d ^ 3 x ^ 6)
/ 140
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(84) = 168$.
time = 0.14, size = 177, normalized size = 1.92

method	result
norman	$\frac{b^3d^3x^7}{7}+\left(\frac{1}{2}a\,b^2d^3+\frac{1}{2}b^3c\,d^2\right)x^6+\left(\frac{3}{5}a^2b\,d^3+\frac{9}{5}a\,b^2c\,d^2+\frac{3}{5}b^3c^2d\right)x^5+\left(\frac{1}{4}a^3d^3+\frac{9}{4}a^2bc\,d^2+\frac{9}{4}a\,b^2c^2$
default	$\frac{b^3d^3x^7}{7}+\frac{\left(3a\,b^2d^3+3b^3c\,d^2\right)x^6}{6}+\frac{\left(3a^2b\,d^3+9a\,b^2c\,d^2+3b^3c^2d\right)x^5}{5}+\frac{\left(a^3d^3+9a^2bc\,d^2+9a\,b^2c^2d+b^3c^3\right)x^4}{4}+\frac{\left(3a^3c\,d^2+9a^2b\,c^2d+\right.}{3}$
gosper	$\frac{1}{7}b^3d^3x^7+\frac{1}{2}x^6a\,b^2d^3+\frac{1}{2}x^6b^3c\,d^2+\frac{3}{5}x^5a^2b\,d^3+\frac{9}{5}x^5a\,b^2c\,d^2+\frac{3}{5}x^5b^3c^2d+\frac{1}{4}x^4a^3d^3+\frac{9}{4}x^4a^2bc\,d^2+$
risch	$\frac{1}{7}b^3d^3x^7+\frac{1}{2}x^6a\,b^2d^3+\frac{1}{2}x^6b^3c\,d^2+\frac{3}{5}x^5a^2b\,d^3+\frac{9}{5}x^5a\,b^2c\,d^2+\frac{3}{5}x^5b^3c^2d+\frac{1}{4}x^4a^3d^3+\frac{9}{4}x^4a^2bc\,d^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^3*d^3*x^7+1/6*(3*a*b^2*d^3+3*b^3*c*d^2)*x^6+1/5*(3*a^2*b*d^3+9*a*b^2*
c*d^2+3*b^3*c^2*d)*x^5+1/4*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*x^
4+1/3*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^3+1/2*(3*a^3*c^2*d+3*a^2*b*
c^3)*x^2+a^3*c^3*x
```

Maxima [A]
time = 0.26, size = 167, normalized size = 1.82

$$\frac{1}{7}\,b^3d^3x^7+a^3c^3x+\frac{1}{2}\,{\left(b^3cd^2+ab^2d^3\right)}x^6+\frac{3}{5}\,{\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)}x^5+\frac{1}{4}\,{\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)}x^4+{\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)}x^3+\frac{3}{2}\,{\left(a^2bc^3+a^3c^2d\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Fricas [**A**]
time = 0.28, size = 167, normalized size = 1.82

$$\frac{1}{7}\,b^3d^3x^7+a^3c^3x+\frac{1}{2}\,(b^3cd^2+ab^2d^3)x^6+\frac{3}{5}\,(b^3c^2d+3\,ab^2cd^2+a^2bd^3)x^5+\frac{1}{4}\,(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)x^4+(ab^2c^3+3\,a^2bc^2d+a^3cd^2)x^3+\frac{3}{2}\,(a^2bc^3+a^3c^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(80) = 160$.
time = 0.05, size = 190, normalized size = 2.07

$$a^3c^3x+\frac{b^3d^3x^7}{7}+x^6\left(\frac{ab^2d^3}{2}+\frac{b^3cd^2}{2}\right)+x^5\cdot\left(\frac{3a^2bd^3}{5}+\frac{9ab^2cd^2}{5}+\frac{3b^3c^2d}{5}\right)+x^4\left(\frac{a^3d^3}{4}+\frac{9a^2bcd^2}{4}+\frac{9ab^2c^2d}{4}+\frac{b^3c^3}{4}\right)+x^3\left(a^3cd^2+3a^2bc^2d+ab^2c^3\right)+x^2\cdot\left(\frac{3a^3c^2d}{2}+\frac{3a^2bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x**
5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d**3
/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c*d*
*2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*c**3
/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(84) = 168$.
time = 0.00, size = 212, normalized size = 2.30

$$\frac{1}{7}x^7b^3d^3+\frac{1}{2}x^6b^3d^2c+\frac{1}{2}x^6b^2ad^3+\frac{3}{5}x^5b^3dc^2+\frac{9}{5}x^5b^2ad^2c+\frac{3}{5}x^5ba^2d^3+\frac{1}{4}x^4b^3c^3+\frac{9}{4}x^4b^2adc^2+\frac{9}{4}x^4ba^2d^2c+\frac{1}{4}x^4a^3d^3+x^3b^2ac^3+3x^3ba^2dc^2+x^3a^3d^2c+\frac{3}{2}x^2ba^2c^3+\frac{3}{2}x^2a^3dc^2+xa^3c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x)
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^5
 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*c^
2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*b*c
```

```
^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*
x
```

Mupad [B]
time = 0.06, size = 152, normalized size = 1.65

$$x^4\left(\frac{a^3d^3}{4}+\frac{9a^2bcd^2}{4}+\frac{9ab^2c^2d}{4}+\frac{b^3c^3}{4}\right)+a^3c^3x+\frac{b^3d^3x^7}{7}+acx^3\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3bdx^5\left(a^2d^2+3abcd+b^2c^2\right)}{5}+\frac{3a^2c^2x^2\left(ad+bc\right)}{2}+\frac{b^2d^2x^6\left(ad+bc\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] x^4*((a^3*d^3)/4 + (b^3*c^3)/4 + (9*a*b^2*c^2*d)/4 + (9*a^2*b*c*d^2)/4) + a
^3*c^3*x + (b^3*d^3*x^7)/7 + a*c*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3*b
*d*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5 + (3*a^2*c^2*x^2*(a*d + b*c))/2 +
 (b^2*d^2*x^6*(a*d + b*c))/2
```

3.1261 $\int (a+bx)^2(c+dx)^3\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

```
[Out] 1/4*(-a*d+b*c)^2*(d*x+c)^4/d^3-2/5*b*(-a*d+b*c)*(d*x+c)^5/d^3+1/6*b^2*(d*x+
c)^6/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 122, normalized size = 1.88

$$a^2c^3x+\frac{1}{2}ac^2(2bc+3ad)x^2+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4+\frac{1}{5}bd^2(3bc+2ad)x^5+\frac{1}{6}b^2d^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c
+ 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

Mathics [**A**]
time = 2.35, size = 111, normalized size = 1.71

$$x\left(a^2c^3+\frac{ac^2x\,(3ad+2bc)}{2}+\frac{bd^2x^4\,(2ad+3bc)}{5}+\frac{cx^2\,(3a^2d^2+6abcd+b^2c^2)}{3}+\frac{dx^3\,(a^2d^2+6abcd+3b^2c^2)}{4}+\frac{b^2d^3x^5}{6}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^3,x]')
```

```
[Out] x (a ^ 2 c ^ 3 + a c ^ 2 x (3 a d + 2 b c) / 2 + b d ^ 2 x ^ 4 (2 a d + 3 b
 c) / 5 + c x ^ 2 (3 a ^ 2 d ^ 2 + 6 a b c d + b ^ 2 c ^ 2) / 3 + d x ^ 3 (
a ^ 2 d ^ 2 + 6 a b c d + 3 b ^ 2 c ^ 2) / 4 + b ^ 2 d ^ 3 x ^ 5 / 6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(59) = 118.
time = 0.14, size = 125, normalized size = 1.92

method	result
norman	$\frac{b^2d^3x^6}{6}+\left(\frac{2}{5}ab\,d^3+\frac{3}{5}b^2c\,d^2\right)x^5+\left(\frac{1}{4}a^2d^3+\frac{3}{2}abc\,d^2+\frac{3}{4}b^2c^2d\right)x^4+\left(a^2c\,d^2+2ab\,c^2d+\frac{1}{3}b^2c^3\right)x^3+$
default	$\frac{b^2d^3x^6}{6}+\frac{(2ab\,d^3+3b^2c\,d^2)x^5}{5}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^4}{4}+\frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^3}{3}+\frac{(3a^2c^2d+2a\,c^3b)x^2}{2}+a^2c^3x$
gosper	$\frac{1}{6}b^2d^3x^6+\frac{2}{5}x^5ab\,d^3+\frac{3}{5}x^5b^2c\,d^2+\frac{1}{4}x^4a^2d^3+\frac{3}{2}x^4abc\,d^2+\frac{3}{4}x^4b^2c^2d+x^3a^2c\,d^2+2x^3ab\,c^2d+\frac{1}{3}b^2c^3$
risch	$\frac{1}{6}b^2d^3x^6+\frac{2}{5}x^5ab\,d^3+\frac{3}{5}x^5b^2c\,d^2+\frac{1}{4}x^4a^2d^3+\frac{3}{2}x^4abc\,d^2+\frac{3}{4}x^4b^2c^2d+x^3a^2c\,d^2+2x^3ab\,c^2d+\frac{1}{3}b^2c^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*d^3*x^6+1/5*(2*a*b*d^3+3*b^2*c*d^2)*x^5+1/4*(a^2*d^3+6*a*b*c*d^2+3*
b^2*c^2*d)*x^4+1/3*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^3+1/2*(3*a^2*c^2*d+2
*a*b*c^3)*x^2+a^2*c^3*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(59) = 118.
time = 0.27, size = 124, normalized size = 1.91

$$\frac{1}{6}\,b^2d^3x^6+a^2c^3x+\frac{1}{5}\,(3\,b^2cd^2+2\,abd^3)x^5+\frac{1}{4}\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^4+\frac{1}{3}\,(b^2c^3+6\,abc^2d+3\,a^2cd^2)x^3+\frac{1}{2}\,(2\,abc^3+3\,a^2c^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.29, size = 124, normalized size = 1.91

$$\frac{1}{6}\,b^2d^3x^6 + a^2c^3x + \frac{1}{5}\,(3\,b^2cd^2 + 2\,abd^3)x^5 + \frac{1}{4}\,(3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^4 + \frac{1}{3}\,(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^3 + \frac{1}{2}\,(2\,abc^3 + 3\,a^2c^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.
time = 0.04, size = 133, normalized size = 2.05

$$a^2c^3x + \frac{b^2d^3x^6}{6} + x^5\cdot\left(\frac{2abd^3}{5} + \frac{3b^2cd^2}{5}\right) + x^4\left(\frac{a^2d^3}{4} + \frac{3abcd^2}{2} + \frac{3b^2c^2d}{4}\right) + x^3\left(a^2cd^2 + 2abc^2d + \frac{b^2c^3}{3}\right) + x^2\cdot\left(\frac{3a^2c^2d}{2} + abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x*
*4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 + 2
*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.
time = 0.00, size = 146, normalized size = 2.25

$$\frac{1}{6}x^6b^2d^3 + \frac{3}{5}x^5b^2d^2c + \frac{2}{5}x^5bad^3 + \frac{3}{4}x^4b^2dc^2 + \frac{3}{2}x^4bad^2c + \frac{1}{4}x^4a^2d^3 + \frac{1}{3}x^3b^2c^3 + 2x^3badc^2 + x^3a^2d^2c + x^2bac^3 + \frac{3}{2}x^2a^2dc^2 + xa^2c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x)
```

```
[Out] 1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

Mupad [B]
time = 0.05, size = 115, normalized size = 1.77

$$x^3\,\left(a^2\,c\,d^2+2\,a\,b\,c^2\,d+\frac{b^2\,c^3}{3}\right)+x^4\,\left(\frac{a^2\,d^3}{4}+\frac{3\,a\,b\,c\,d^2}{2}+\frac{3\,b^2\,c^2\,d}{4}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^6}{6}+\frac{a\,c^2\,x^2\,\left(3\,a\,d+2\,b\,c\right)}{2}+\frac{b\,d^2\,x^5\,\left(2\,a\,d+3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b^2*c^2
*d)/4 + (3*a*b*c*d^2)/2) + a^2*c^3*x + (b^2*d^3*x^6)/6 + (a*c^2*x^2*(3*a*d
+ 2*b*c))/2 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

3.1262 $\int (a+bx)(c+dx)^3\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}$$

```
[Out] -1/4*(-a*d+b*c)*(d*x+c)^4/d^2+1/5*b*(d*x+c)^5/d^2
```

Rubi [A]

time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*(c + d*x)^4)/d^2 + (b*(c + d*x)^5)/(5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^3}{d}+\frac{b(c+dx)^4}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 67, normalized size = 1.76

$$ac^3x+\frac{1}{2}c^2(bc+3ad)x^2+cd(bc+ad)x^3+\frac{1}{4}d^2(3bc+ad)x^4+\frac{1}{5}bd^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c + a
*d)*x^4)/4 + (b*d^3*x^5)/5
```

Mathics [A]
time = 1.98, size = 60, normalized size = 1.58

$$x\left(ac^3+\frac{c^2x\left(3ad+bc\right)}{2}+\frac{d^2x^3\left(ad+3bc\right)}{4}+\frac{bd^3x^4}{5}+cdx^2\left(ad+bc\right)\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^3,x]')
```

```
[Out] x (a c ^ 3 + c ^ 2 x (3 a d + b c) / 2 + d ^ 2 x ^ 3 (a d + 3 b c) / 4 + b
d ^ 3 x ^ 4 / 5 + c d x ^ 2 (a d + b c))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(34) = 68.
time = 0.12, size = 73, normalized size = 1.92

method	result	size
norman	$\frac{b\,d^3x^5}{5}+\left(\frac{1}{4}a\,d^3+\frac{3}{4}bc\,d^2\right)x^4+\left(ac\,d^2+b\,c^2d\right)x^3+\left(\frac{3}{2}a\,c^2d+\frac{1}{2}b\,c^3\right)x^2+a\,c^3x$	70
gosper	$\frac{1}{5}b\,d^3x^5+\frac{1}{4}x^4a\,d^3+\frac{3}{4}x^4bc\,d^2+ac\,d^2x^3+b\,c^2d\,x^3+\frac{3}{2}x^2a\,c^2d+\frac{1}{2}b\,c^3x^2+a\,c^3x$	73
default	$\frac{b\,d^3x^5}{5}+\frac{\left(a\,d^3+3bc\,d^2\right)x^4}{4}+\frac{\left(3ac\,d^2+3b\,c^2d\right)x^3}{3}+\frac{\left(3a\,c^2d+b\,c^3\right)x^2}{2}+a\,c^3x$	73
risch	$\frac{1}{5}b\,d^3x^5+\frac{1}{4}x^4a\,d^3+\frac{3}{4}x^4bc\,d^2+ac\,d^2x^3+b\,c^2d\,x^3+\frac{3}{2}x^2a\,c^2d+\frac{1}{2}b\,c^3x^2+a\,c^3x$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b*d^3*x^5+1/4*(a*d^3+3*b*c*d^2)*x^4+1/3*(3*a*c*d^2+3*b*c^2*d)*x^3+1/2*(
3*a*c^2*d+b*c^3)*x^2+a*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.
time = 0.26, size = 69, normalized size = 1.82

$$\frac{1}{5}\,bd^3x^5+ac^3x+\frac{1}{4}\,\left(3\,bcd^2+ad^3\right)x^4+\left(bc^2d+acd^2\right)x^3+\frac{1}{2}\,\left(bc^3+3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.30, size = 69, normalized size = 1.82

$$\frac{1}{5}\,bd^3x^5 + ac^3x + \frac{1}{4}\,\left(3\,bcd^2 + ad^3\right)x^4 + \left(bc^2d + acd^2\right)x^3 + \frac{1}{2}\,\left(bc^3 + 3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 0.04, size = 73, normalized size = 1.92

$$ac^3x + \frac{bd^3x^5}{5} + x^4\left(\frac{ad^3}{4} + \frac{3bcd^2}{4}\right) + x^3\left(acd^2 + bc^2d\right) + x^2\cdot\left(\frac{3ac^2d}{2} + \frac{bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
+ b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.
time = 0.00, size = 82, normalized size = 2.16

$$\frac{1}{5}x^5bd^3 + \frac{3}{4}x^4bd^2c + \frac{1}{4}x^4ad^3 + x^3bdc^2 + x^3ad^2c + \frac{1}{2}x^2bc^3 + \frac{3}{2}x^2adc^2 + xac^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x)
```

```
[Out] 1/5*b*d^3*x^5 + 3/4*b*c*d^2*x^4 + 1/4*a*d^3*x^4 + b*c^2*d*x^3 + a*c*d^2*x^3
 + 1/2*b*c^3*x^2 + 3/2*a*c^2*d*x^2 + a*c^3*x
```

Mupad [B]
time = 0.03, size = 65, normalized size = 1.71

$$x^2\,\left(\frac{b\,c^3}{2}+\frac{3\,a\,d\,c^2}{2}\right)+x^4\,\left(\frac{a\,d^3}{4}+\frac{3\,b\,c\,d^2}{4}\right)+\frac{b\,d^3\,x^5}{5}+a\,c^3\,x+c\,d\,x^3\,\left(a\,d+b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^3,x)
```

```
[Out] x^2*((b*c^3)/2 + (3*a*c^2*d)/2) + x^4*((a*d^3)/4 + (3*b*c*d^2)/4) + (b*d^3*
x^5)/5 + a*c^3*x + c*d*x^3*(a*d + b*c)
```

3.1263 $\int (c+dx)^3\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^4}{4d}$$

[Out] 1/4*(d*x+c)^4/d

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3,x]

[Out] (c + d*x)^4/(4*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3\,dx = \frac{(c+dx)^4}{4d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3,x]

[Out] (c + d*x)^4/(4*d)

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.
time = 1.64, size = 32, normalized size = 2.29

$$\frac{x\,(4c^3+6c^2dx+4cd^2x^2+d^3x^3)}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^3,x]')
```

```
[Out] x (4 c ^ 3 + 6 c ^ 2 d x + 4 c d ^ 2 x ^ 2 + d ^ 3 x ^ 3) / 4
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(dx+c)^4}{4d}$	13
gosper	$\frac{1}{4}d^3x^4+c\,d^2x^3+\frac{3}{2}c^2d\,x^2+c^3x$	32
norman	$\frac{1}{4}d^3x^4+c\,d^2x^3+\frac{3}{2}c^2d\,x^2+c^3x$	32
risch	$\frac{d^3x^4}{4}+c\,d^2x^3+\frac{3c^2d\,x^2}{2}+c^3x+\frac{c^4}{4d}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(d*x+c)^4/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.27, size = 31, normalized size = 2.21

$$\frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\,c^2dx^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.29, size = 31, normalized size = 2.21

$$\frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\,c^2dx^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.
time = 0.03, size = 32, normalized size = 2.29

$$c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3,x)
```

```
[Out] c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(dx + c)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x)
```

```
[Out] 1/4*(d*x + c)^4/d
```

Mupad [B]
time = 0.04, size = 31, normalized size = 2.21

$$c^3\,x + \frac{3\,c^2\,d\,x^2}{2} + c\,d^2\,x^3 + \frac{d^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3,x)
```

```
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3
```

3.1264 $\int \frac{(c+dx)^3}{a+bx}\,dx$

Optimal. Leaf size=73

$$\frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}$$

```
[Out] d*(-a*d+b*c)^2*x/b^3+1/2*(-a*d+b*c)*(d*x+c)^2/b^2+1/3*(d*x+c)^3/b+(-a*d+b*c
)^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx\\ &= \frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.01

$$\frac{bdx\left(6a^2d^2-3abd(6c+dx)+b^2\left(18c^2+9cdx+2d^2x^2\right)\right)+6(bc-ad)^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^2
)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

Mathics [A]
time = 2.34, size = 80, normalized size = 1.10

$$\frac{bdx\left(a^2d^2-3abcd+3b^2c^2\right)+\frac{b^2d^2x^2(-ad+3bc)}{2}+\frac{b^3d^3x^3}{3}-\text{Log}\left[a+bx\right]\left(ad-bc\right)^3}{b^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^1,x]')
```

```
[Out] (b d x (a ^ 2 d ^ 2 - 3 a b c d + 3 b ^ 2 c ^ 2) + b ^ 2 d ^ 2 x ^ 2 (-a d
+ 3 b c) / 2 + b ^ 3 d ^ 3 x ^ 3 / 3 - Log[a + b x] (a d - b c) ^ 3) / b ^
4
```

Maple [A]
time = 0.14, size = 109, normalized size = 1.49

method	result
norman	$\frac{d(a^2d^2-3abcd+3b^2c^2)x}{b^3}+\frac{d^3x^3}{3b}-\frac{d^2(ad-3bc)x^2}{2b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(bx+a)}{b^4}$
default	$\frac{d(\frac{1}{3}d^2x^3b^2-\frac{1}{2}ab\,d^2x^2+\frac{3}{2}b^2cd\,x^2+a^2d^2x-3abcdx+3b^2c^2x)}{b^3}+\frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\ln(bx+a)}{b^4}$
risch	$\frac{d^3x^3}{3b}-\frac{d^3a\,x^2}{2b^2}+\frac{3d^2c\,x^2}{2b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}-\frac{\ln(bx+a)a^3d^3}{b^4}+\frac{3\ln(bx+a)a^2c\,d^2}{b^3}-\frac{3\ln(bx+a)a\,c^2d}{b^2}+\frac{\ln(bx+}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/3*d^2*x^3*b^2-1/2*a*b*d^2*x^2+3/2*b^2*c*d*x^2+a^2*d^2*x-3*a*b*c*d*
x+3*b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4*ln(b*x+a)
```

Maxima [A]
time = 0.26, size = 114, normalized size = 1.56

$$\frac{2\,b^2d^3x^3+3\,(3\,b^2cd^2-abd^3)x^2+6\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{6\,b^3}+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*b
*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(b*x + a)/b^4
```

Fricas [A]
time = 0.29, size = 116, normalized size = 1.59

$$\frac{2\,b^3d^3x^3+3\,(3\,b^3cd^2-ab^2d^3)x^2+6\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x+6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(b*x + a))/b^4
```

Sympy [A]
time = 0.18, size = 83, normalized size = 1.14

$$x^2\left(-\frac{ad^3}{2b^2}+\frac{3cd^2}{2b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\frac{d^3x^3}{3b}-\frac{(ad-bc)^3\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a),x)
```

```
[Out] x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + d**3*x**3/(3*b) - (a*d - b*c)**3*log(a + b*x)/b**4
```

Giac [A]
time = 0.00, size = 124, normalized size = 1.70

$$\frac{\frac{1}{3}x^3d^3b^2-\frac{1}{2}x^2d^3ba+\frac{3}{2}x^2d^2cb^2+xd^3a^2-3xd^2cba+3xdc^2b^2}{b^3}+\frac{(-d^3a^3+3d^2cba^2-3dc^2b^2a+c^3b^3)\ln|xb+a|}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x)
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18*
a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(b*x + a))/b^4
```

Mupad [B]
time = 0.20, size = 118, normalized size = 1.62

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^2\,\left(\frac{a\,d^3}{2\,b^2}-\frac{3\,c\,d^2}{2\,b}\right)+\frac{d^3\,x^3}{3\,b}-\frac{\ln\left(a+b\,x\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^2*((a*d^3)/(2*b^2)
- (3*c*d^2)/(2*b)) + (d^3*x^3)/(3*b) - (log(a + b*x)*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/b^4
```

3.1265 $\int \frac{(c+dx)^3}{(a+bx)^2}\,dx$

Optimal. Leaf size=75

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/2*d^3*x^2/b^2-(-a*d+b*c)^3/b^4/(b*x+a)+3*d*(-a*d
+b*c)^2*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^2}\,dx&=\int\left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx\\&=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.96

$$\frac{2bd^2(3bc-2ad)x+b^2d^3x^2-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6*
d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

Mathics [A]
time = 2.68, size = 108, normalized size = 1.44

$$\frac{3d\text{Log}\left[a+bx\right]\left(a+bx\right)\left(ad-bc\right)^2+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3-bd^2x\left(a+bx\right)\left(2ad-3bc\right)+\frac{b^2d^3x^2(a+bx)}{2}}{b^4\left(a+bx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^2,x]')
```

```
[Out] (3 d Log[a + b x] (a + b x) (a d - b c) ^ 2 + a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^
 2 + 3 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3 - b d ^ 2 x (a + b x) (2 a d - 3 b c)
+ b ^ 2 d ^ 3 x ^ 2 (a + b x) / 2) / (b ^ 4 (a + b x))
```

Maple [A]
time = 0.17, size = 109, normalized size = 1.45

method	result
default	$-\frac{d^2\left(-\frac{1}{2}bd\,x^2+2adx-3bcx\right)}{b^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{b^4(bx+a)}+\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)\ln(bx+a)}{b^4}$
norman	$\frac{\frac{3a^3d^3-6a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{b^4}+\frac{d^3x^3}{2b}-\frac{3d^2(ad-2bc)x^2}{2b^2}}{bx+a}+\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)\ln(bx+a)}{b^4}$
risch	$\frac{d^3x^2}{2b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}+\frac{a^3d^3}{b^4(bx+a)}-\frac{3a^2c\,d^2}{b^3(bx+a)}+\frac{3a\,c^2d}{b^2(bx+a)}-\frac{c^3}{b(bx+a)}+\frac{3d^3\ln(bx+a)a^2}{b^4}-\frac{6d^2\ln(bx+a)ac}{b^3}+$ 3d ln(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3*(-1/2*b*d*x^2+2*a*d*x-3*b*c*x)-1/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b
^2*c^2*d+b^3*c^3)/(b*x+a)+3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(b*x+a)
```

Maxima [A]
time = 0.27, size = 118, normalized size = 1.57

$$-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{b^5x+ab^4}+\frac{bd^3x^2+2\left(3\,bcd^2-2\,ad^3\right)x}{2\,b^3}+\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2*
(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(b*x + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(73) = 146$.
time = 0.30, size = 173, normalized size = 2.31

$$\frac{b^3d^3x^3 - 2\,b^3c^3 + 6\,ab^2c^2d - 6\,a^2bcd^2 + 2\,a^3d^3 + 3\,(2\,b^3cd^2 - ab^2d^3)x^2 + 2\,(3\,ab^2cd^2 - 2\,a^2bd^3)x + 6\,(ab^2c^2d - 2\,a^2bcd^2 + a^3d^3 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x)\log(bx + a)}{2\,(b^5x + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(a*
b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A]
time = 0.30, size = 102, normalized size = 1.36

$$x\left(-\frac{2ad^3}{b^3} + \frac{3cd^2}{b^2}\right) + \frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{ab^4 + b^5x} + \frac{d^3x^2}{2b^2} + \frac{3d\,(ad - bc)^2\log(a + bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3)/(a*b**4 + b**5*x) + d**3*x**2/(2*b**2) + 3*d*(a*d - b
*c)**2*log(a + b*x)/b**4
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.68

$$\frac{\frac{1}{2}x^2d^3b^2 - 2xd^3ba + 3xd^2cb^2}{b^4} + \frac{d^3a^3 - 3d^2ba^2c + 3db^2ac^2 - b^3c^3}{b^4\,(xb + a)} + \frac{(3d^3a^2 - 6d^2cba + 3dc^2b^2)\ln|xb + a|}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x)
```

```
[Out] 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a))/b^4 + 1/2*(b^2*d^3*
x^2 + 6*b^2*c*d^2*x - 4*a*b*d^3*x)/b^4 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)/((b*x + a)*b^4)
```

Mupad [B]
time = 0.21, size = 123, normalized size = 1.64

$$\frac{\ln(a + b\,x)\,\left(3\,a^2\,d^3 - 6\,a\,b\,c\,d^2 + 3\,b^2\,c^2\,d\right)}{b^4} - x\,\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right) + \frac{a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3}{b\,\left(x\,b^4 + a\,b^3\right)} + \frac{d^3\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^2,x)
```

```
[Out] (log(a + b*x)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/b^4 - x*((2*a*d^3)/b
^3 - (3*c*d^2)/b^2) + (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
b*(a*b^3 + b^4*x)) + (d^3*x^2)/(2*b^2)
```

3.1266 $\int \frac{(c+dx)^3}{(a+bx)^3}\,dx$

Optimal. Leaf size=78

$$\frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}$$

```
[Out] d^3*x/b^3-1/2*(-a*d+b*c)^3/b^4/(b*x+a)^2-3*d*(-a*d+b*c)^2/b^4/(b*x+a)+3*d^2
*(-a*d+b*c)*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^3}\,dx &= \int \left(\frac{d^3}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)^3}+\frac{3d(bc-ad)^2}{b^3(a+bx)^2}+\frac{3d^2(bc-ad)}{b^3(a+bx)}\right)dx\\ &= \frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.46

$$\frac{-5a^3d^3+a^2bd^2(9c-4dx)+ab^2d\left(-3c^2+12cdx+4d^2x^2\right)-b^3\left(c^3+6c^2dx-2d^3x^3\right)-6d^2(-bc+ad)(a+bx)^2\log(a+bx)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2*
x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

Mathics [A]
time = 3.07, size = 144, normalized size = 1.85

$$\frac{-5a^3d^3+9a^2bcd^2-3ab^2c^2d-b^3c^3-6bdx\left(a^2d^2-2abcd+b^2c^2\right)-6d^2\text{Log}\left[a+bx\right]\left(a^2+2abx+b^2x^2\right)(ad-bc)+2bd^3x\left(a^2+2abx+b^2x^2\right)}{2b^4\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^3,x]')
```

```
[Out] (-5 a ^ 3 d ^ 3 + 9 a ^ 2 b c d ^ 2 - 3 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3 - 6 b
 d x (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) - 6 d ^ 2 Log[a + b x] (a ^ 2
+ 2 a b x + b ^ 2 x ^ 2) (a d - b c) + 2 b d ^ 3 x (a ^ 2 + 2 a b x + b ^ 2
 x ^ 2)) / (2 b ^ 4 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [A]
time = 0.14, size = 114, normalized size = 1.46

method	result	size
default	$\frac{d^3x}{b^3}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{2b^4(bx+a)^2}-\frac{3d^2(ad-bc)\ln(bx+a)}{b^4}$	114
norman	$\frac{\frac{d^3x^3}{b}-\frac{9a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3}{2b^4}-\frac{\left(6a^2d^3-6abc\,d^2+3b^2c^2d\right)x}{b^3}}{(bx+a)^2}-\frac{3d^2(ad-bc)\ln(bx+a)}{b^4}$	116
risch	$\frac{d^3x}{b^3}+\frac{(-3a^2d^3+6abc\,d^2-3b^2c^2d)x-\frac{5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3}{2b}}{b^3(bx+a)^2}-\frac{3d^3\ln(bx+a)a}{b^4}+\frac{3d^2\ln(bx+a)c}{b^3}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*x/b^3-3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)-1/2/b^4*(-a^3*d^3+3*a
^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(b*x+a)^2-3/b^4*d^2*(a*d-b*c)*ln(b*x+a)
```

Maxima [A]
time = 0.28, size = 125, normalized size = 1.60

$$\frac{d^3x}{b^3}-\frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}+\frac{3\,(bcd^2-ad^3)\log{(bx+a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(76) = 152.
time = 0.31, size = 188, normalized size = 2.41

$$\frac{2\,b^{3}d^{3}x^{3}+4\,ab^{2}d^{3}x^{2}-b^{3}c^{3}-3\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}-5\,a^{3}d^{3}-2\,(3\,b^{3}c^{2}d-6\,ab^{2}cd^{2}+2\,a^{2}bd^{3})x+6\,(a^{2}bcd^{2}-a^{3}d^{3}+(b^{3}cd^{2}-ab^{2}d^{3})x^{2}+2\,(ab^{2}cd^{2}-a^{2}bd^{3})x)\log\left(bx+a\right)}{2\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^2*
b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b*d^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A]
time = 0.50, size = 128, normalized size = 1.64

$$\frac{-5a^{3}d^{3}+9a^{2}bcd^{2}-3ab^{2}c^{2}d-b^{3}c^{3}+x\left(-6a^{2}bd^{3}+12ab^{2}cd^{2}-6b^{3}c^{2}d\right)}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}}+\frac{d^{3}x}{b^{3}}-\frac{3d^{2}\left(ad-bc\right)\log\left(a+bx\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**3,x)
```

```
[Out] (-5*a**3*d**3 + 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - b**3*c**3 + x*(-6*a**2*
b*d**3 + 12*a*b**2*c*d**2 - 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

Giac [A]
time = 0.00, size = 125, normalized size = 1.60

$$\frac{xd^{3}}{b^{3}}+\frac{\frac{1}{2}\left(-5d^{3}a^{3}+9d^{2}ba^{2}c-3db^{2}ac^{2}-b^{3}c^{3}+\left(-6d^{3}ba^{2}+12d^{2}b^{2}ac-6db^{3}c^{2}\right)x\right)}{b^{4}\left(xb+a\right)^{2}}+\frac{\left(-3d^{3}a+3d^{2}cb\right)\ln\left|xb+a\right|}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x)
```

```
[Out] d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a*
b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((b*x + a)^2*b^4)
```

Mupad [B]
time = 0.82, size = 130, normalized size = 1.67

$$\frac{d^{3}\,x}{b^{3}}-\frac{\ln\left(a+b\,x\right)\,\left(3\,a\,d^{3}-3\,b\,c\,d^{2}\right)}{b^{4}}-\frac{\frac{5\,a^{3}\,d^{3}-9\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d+b^{3}\,c^{3}}{2\,b}+x\,\left(3\,a^{2}\,d^{3}-6\,a\,b\,c\,d^{2}+3\,b^{2}\,c^{2}\,d\right)}{a^{2}\,b^{3}+2\,a\,b^{4}\,x+b^{5}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^3,x)
```

```
[Out] (d^3*x)/b^3 - (log(a + b*x)*(3*a*d^3 - 3*b*c*d^2))/b^4 - ((5*a^3*d^3 + b^3*
c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2)/(2*b) + x*(3*a^2*d^3 + 3*b^2*c^2*d - 6
*a*b*c*d^2))/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x)
```

3.1267 $\int \frac{(c+dx)^3}{(a+bx)^4} dx$

Optimal. Leaf size=86

$$-\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}$$

```
[Out] -1/3*(-a*d+b*c)^3/b^4/(b*x+a)^3-3/2*d*(-a*d+b*c)^2/b^4/(b*x+a)^2-3*d^2*(-a*
d+b*c)/b^4/(b*x+a)+d^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] -1/3*(b*c - a*d)^3/(b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)
^2) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^4}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{d^3}{b^3(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.93

$$\frac{-\frac{(bc-ad)\left(11a^2d^2+abd(5c+27dx)+b^2\left(2c^2+9cdx+18d^2x^2\right)\right)}{(a+bx)^3}+6d^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3/(a + b*x)^4,x]

[Out] (-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x + 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)

Mathics [A]
time = 3.45, size = 155, normalized size = 1.80

$$\frac{-6a^2bcd^2-3ab^2c^2d-9bdx\left(-3a^2d^2+2abcd+b^2c^2\right)+18b^2d^2x^2(ad-bc)+6d^3\text{Log}\left[a+bx\right]\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)+11a^3d^3-2b^3c^3}{6b^4\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^4,x]')

[Out] (-6 a ^ 2 b c d ^ 2 - 3 a b ^ 2 c ^ 2 d - 9 b d x (-3 a ^ 2 d ^ 2 + 2 a b c d + b ^ 2 c ^ 2) + 18 b ^ 2 d ^ 2 x ^ 2 (a d - b c) + 6 d ^ 3 Log[a + b x] (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) + 11 a ^ 3 d ^ 3 - 2 b ^ 3 c ^ 3) / (6 b ^ 4 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))

Maple [A]
time = 0.14, size = 120, normalized size = 1.40

method	result	size
risch	$\frac{\frac{3d^2(ad-bc)x^2}{b^2}+\frac{3d\left(3a^2d^2-2abcd-b^2c^2\right)x}{2b^3}+\frac{11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3}{6b^4}}{(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}$	115
norman	$\frac{\frac{11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3}{6b^4}+\frac{3\left(a\,d^3-bc\,d^2\right)x^2}{b^2}+\frac{3\left(3a^2d^3-2abc\,d^2-b^2c^2d\right)x}{2b^3}}{(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}$	119
default	$\frac{3d^2(ad-bc)}{b^4(bx+a)}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{2b^4(bx+a)^2}+\frac{d^3\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{3b^4(bx+a)^3}$	120

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3/(b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] 3/b^4*d^2*(a*d-b*c)/(b*x+a)-3/2/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2+d^3*ln(b*x+a)/b^4-1/3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^3

Maxima [A]
time = 0.28, size = 142, normalized size = 1.65

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\,(b^3cd^2-ab^2d^3)x^2+9\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}+\frac{d^3\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.
time = 0.30, size = 176, normalized size = 2.05

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\,(b^3cd^2-ab^2d^3)x^2+9\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x-6\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\log(bx+a)}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A]
time = 0.66, size = 148, normalized size = 1.72

$$\frac{11a^3d^3-6a^2bcd^2-3ab^2c^2d-2b^3c^3+x^2\cdot(18ab^2d^3-18b^3cd^2)+x\,(27a^2bd^3-18ab^2cd^2-9b^3c^2d)}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}+\frac{d^3\log(a+bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18*
a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*b*
*3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 d**3*log(a + b*x)/b**4
```

Giac [A]
time = 0.00, size = 128, normalized size = 1.49

$$\frac{\frac{1}{6}\left((18d^3ba-18d^2b^2c)\,x^2+(27d^3a^2-18d^2bac-9db^2c^2)\,x+\frac{11d^3a^3-6d^2ba^2c-3db^2ac^2-2b^3c^3}{b}\right)}{b^3\,(xb+a)^3}+\frac{d^3\ln|xb+a|}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x)
```

```
[Out] d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2*
d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2
 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

Mupad [B]
time = 0.25, size = 138, normalized size = 1.60

$$\frac{d^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{\frac{-11\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{6\,b^4}+\frac{3\,x\,\left(-3\,a^2\,d^3+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{2\,b^3}-\frac{3\,d^2\,x^2\,\left(a\,d-b\,c\right)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^4,x)
```

```
[Out] (d^3*log(a + b*x))/b^4 - ((2*b^3*c^3 - 11*a^3*d^3 + 3*a*b^2*c^2*d + 6*a^2*b
*c*d^2)/(6*b^4) + (3*x*(b^2*c^2*d - 3*a^2*d^3 + 2*a*b*c*d^2))/(2*b^3) - (3*
d^2*x^2*(a*d - b*c))/b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.1268 $\int \frac{(c+dx)^3}{(a+bx)^5} \, dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

```
[Out] -1/4*(d*x+c)^4/(-a*d+b*c)/(b*x+a)^4
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -1/4*(c + d*x)^4/((b*c - a*d)*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^5} \, dx = -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(28) = 56.
time = 0.02, size = 91, normalized size = 3.25

$$-\frac{a^3d^3 + a^2bd^2(c+4dx) + ab^2d\left(c^2+4cdx+6d^2x^2\right) + b^3\left(c^3+4c^2dx+6cd^2x^2+4d^3x^3\right)}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -1/4*(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2)
 + b^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(b^4*(a + b*x)^4)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(28) = 56.
time = 3.49, size = 138, normalized size = 4.93

$$\frac{-a^3d^3 - a^2bcd^2 - ab^2c^2d - b^3c^3 - 4bdx\left(a^2d^2 + abcd + b^2c^2\right) + 6b^2d^2x^2\left(-ad - bc\right) - 4b^3d^3x^3}{4b^4\left(a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^5,x]')
```

```
[Out] (-a ^ 3 d ^ 3 - a ^ 2 b c d ^ 2 - a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3 - 4 b d x (
a ^ 2 d ^ 2 + a b c d + b ^ 2 c ^ 2) + 6 b ^ 2 d ^ 2 x ^ 2 (-a d - b c) - 4
 b ^ 3 d ^ 3 x ^ 3) / (4 b ^ 4 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 +
 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(26) = 52.
time = 0.14, size = 122, normalized size = 4.36

method	result	size
risch	$\frac{-\frac{d^3x^3}{b}-\frac{3d^2(ad+bc)x^2}{2b^2}-\frac{d\left(a^2d^2+abcd+b^2c^2\right)x}{b^3}-\frac{a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3}{4b^4}}{(bx+a)^4}$	104
gosper	$-\frac{4d^3x^3b^3+6a\,b^2d^3x^2+6b^3c\,d^2x^2+4a^2b\,d^3x+4a\,b^2c\,d^2x+4b^3c^2dx+a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3}{4(bx+a)^4b^4}$	112
norman	$\frac{-\frac{d^3x^3}{b}+\frac{3\left(-a\,d^3-bc\,d^2\right)x^2}{2b^2}+\frac{\left(-a^2d^3-abc\,d^2-b^2c^2d\right)x}{b^3}+\frac{-a^3d^3-a^2bc\,d^2-a\,b^2c^2d-b^3c^3}{4b^4}}{(bx+a)^4}$	116
default	$-\frac{d^3}{b^4(bx+a)}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}+\frac{3d^2(ad-bc)}{2b^4(bx+a)^2}-\frac{d(a^2d^2-2abcd+b^2c^2)}{b^4(bx+a)^3}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3/b^4/(b*x+a)-1/4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*
x+a)^4+3/2/b^4*d^2*(a*d-b*c)/(b*x+a)^2-1/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(26) = 52.
time = 0.27, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3 + b^3c^3 + ab^2c^2d + a^2bcd^2 + a^3d^3 + 6\,(b^3cd^2 + ab^2d^3)x^2 + 4\,(b^3c^2d + ab^2cd^2 + a^2bd^3)x}{4\,(b^8x^4 + 4\,ab^7x^3 + 6\,a^2b^6x^2 + 4\,a^3b^5x + a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(26) = 52$.
time = 0.29, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+ab^2cd^2+a^2bd^3)x}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(22) = 44$.
time = 0.91, size = 155, normalized size = 5.54

$$\frac{-a^3d^3-a^2bcd^2-ab^2c^2d-b^3c^3-4b^3d^3x^3+x^2\left(-6ab^2d^3-6b^3cd^2\right)+x\left(-4a^2bd^3-4ab^2cd^2-4b^3c^2d\right)}{4a^4b^4+16a^3b^5x+24a^2b^6x^2+16ab^7x^3+4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**5,x)
```

```
[Out] (-a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d - b**3*c**3 - 4*b**3*d**3*x**3
+ x**2*(-6*a*b**2*d**3 - 6*b**3*c*d**2) + x*(-4*a**2*b*d**3 - 4*a*b**2*c*d*
*2 - 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 + 16
*a*b**7*x**3 + 4*b**8*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(26) = 52$.
time = 0.00, size = 123, normalized size = 4.39

$$\frac{-4x^3d^3b^3-6x^2d^3b^2a-6x^2d^2cb^3-4xd^3ba^2-4xd^2cb^2a-4xdc^2b^3-d^3a^3-d^2cba^2-dc^2b^2a-c^3b^3}{4b^4\left(xb+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x)
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 4
*a*b^2*c*d^2*x + 4*a^2*b*d^3*x + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*
d^3)/((b*x + a)^4*b^4)
```

Mupad [B]
time = 0.07, size = 135, normalized size = 4.82

$$-\frac{\frac{a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3}{4\,b^4}+\frac{d^3\,x^3}{b}+\frac{d\,x\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+b\,c\right)}{2\,b^2}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^5,x)
```

```
[Out] -((a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)/(4*b^4) + (d^3*x^3)/b + (
d*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/b^3 + (3*d^2*x^2*(a*d + b*c))/(2*b^2))/(
a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)
```

3.1269 $\int \frac{(c+dx)^3}{(a+bx)^6} \, dx$

Optimal. Leaf size=58

$$-\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5}+\frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

```
[Out] -1/5*(d*x+c)^4/(-a*d+b*c)/(b*x+a)^5+1/20*d*(d*x+c)^4/(-a*d+b*c)^2/(b*x+a)^4
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2}-\frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -1/5*(c + d*x)^4/((b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d
)^2*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^6} dx = -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} - \frac{d\int \frac{(c+dx)^3}{(a+bx)^5} dx}{5(bc-ad)}$$
$$= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} + \frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.67

$$-\frac{a^3d^3 + a^2bd^2(2c+5dx) + ab^2d\left(3c^2+10cdx+10d^2x^2\right) + b^3\left(4c^3+15c^2dx+20cd^2x^2+10d^3x^3\right)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -1/20*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*d
^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(b^4*(a + b
*x)^5)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 153 vs. $2(58) = 116$.
time = 3.85, size = 151, normalized size = 2.60

$$\frac{-a^3d^3 - 2a^2bcd^2 - 3ab^2c^2d - 4b^3c^3 - 5bdx\left(a^2d^2+2abcd+3b^2c^2\right) + 10b^2d^2x^2\left(-ad-2bc\right) - 10b^3d^3x^3}{20b^4\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^6,x]')
```

```
[Out] (-a ^ 3 d ^ 3 - 2 a ^ 2 b c d ^ 2 - 3 a b ^ 2 c ^ 2 d - 4 b ^ 3 c ^ 3 - 5 b
 d x (a ^ 2 d ^ 2 + 2 a b c d + 3 b ^ 2 c ^ 2) + 10 b ^ 2 d ^ 2 x ^ 2 (-a d
 - 2 b c) - 10 b ^ 3 d ^ 3 x ^ 3) / (20 b ^ 4 (a ^ 5 + 5 a ^ 4 b x + 10 a ^
 3 b ^ 2 x ^ 2 + 10 a ^ 2 b ^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 + b ^ 5 x ^ 5))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(54) = 108$.
time = 0.14, size = 121, normalized size = 2.09

method	result	size
risch	$\frac{-\frac{d^3x^3}{2b} - \frac{d^2(ad+2bc)x^2}{2b^2} - \frac{d\left(a^2d^2+2abcd+3b^2c^2\right)x}{4b^3} - \frac{a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3}{20b^4}}{(bx+a)^5}$	110
gosper	$-\frac{10d^3x^3b^3+10a\,b^2d^3x^2+20b^3c\,d^2x^2+5a^2b\,d^3x+10a\,b^2c\,d^2x+15b^3c^2dx+a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3}{20b^4(bx+a)^5}$	115

default	$-\frac{3d(a^2d^2-2abcd+b^2c^2)}{4b^4(bx+a)^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}-\frac{d^3}{2b^4(bx+a)^2}+\frac{d^2(ad-bc)}{b^4(bx+a)^3}$	121
norman	$\frac{-\frac{d^3x^3}{2b}+\frac{(-ab\,d^3-2b^2c\,d^2)x^2}{2b^3}+\frac{(-a^2b\,d^3-2a\,b^2c\,d^2-3b^3c^2d)x}{4b^4}+\frac{-a^3b\,d^3-2b^2a^2c\,d^2-3a\,b^3c^2d-4b^4c^3}{20b^5}}{(bx+a)^5}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4-1/5*(-a^3*d^3+3*a^2*b*c*d^
2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^5-1/2*d^3/b^4/(b*x+a)^2+1/b^4*d^2*(a*d
-b*c)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.
time = 0.26, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\,(2\,b^3cd^2+ab^2d^3)x^2+5\,(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3)x}{20\,(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.
time = 0.29, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\,(2\,b^3cd^2+ab^2d^3)x^2+5\,(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3)x}{20\,(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(46) = 92$.
time = 1.15, size = 172, normalized size = 2.97

$$\frac{-a^3d^3-2a^2bcd^2-3ab^2c^2d-4b^3c^3-10b^3d^3x^3+x^2\left(-10ab^2d^3-20b^3cd^2\right)+x\left(-5a^2bd^3-10ab^2cd^2-15b^3c^2d\right)}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**6,x)
```

```
[Out] (-a**3*d**3 - 2*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 4*b**3*c**3 - 10*b**3*d**
3*x**3 + x**2*(-10*a*b**2*d**3 - 20*b**3*c*d**2) + x*(-5*a**2*b*d**3 - 10*a
*b**2*c*d**2 - 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*
b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(54) = 108.
time = 0.00, size = 126, normalized size = 2.17

$$\frac{-10x^3d^3b^3 - 10x^2d^3b^2a - 20x^2d^2cb^3 - 5xd^3ba^2 - 10xd^2cb^2a - 15xdc^2b^3 - d^3a^3 - 2d^2cba^2 - 3dc^2b^2a - 4c^3b^3}{20b^4\left(xb+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x)
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d*
x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*
c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

Mupad [B]
time = 0.08, size = 39, normalized size = 0.67

$$\frac{{\left(c+d\,x\right)}^4\,\left(5\,a\,d-4\,b\,c+b\,d\,x\right)}{20\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^6,x)
```

```
[Out] ((c + d*x)^4*(5*a*d - 4*b*c + b*d*x))/(20*(a*d - b*c)^2*(a + b*x)^5)
```

3.1270 $\int \frac{(c+dx)^3}{(a+bx)^7} dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}$$

```
[Out] -1/6*(-a*d+b*c)^3/b^4/(b*x+a)^6-3/5*d*(-a*d+b*c)^2/b^4/(b*x+a)^5-3/4*d^2*(-
a*d+b*c)/b^4/(b*x+a)^4-1/3*d^3/b^4/(b*x+a)^3
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -1/6*(b*c - a*d)^3/(b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)
^5) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+bx)^7} dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{d^3}{b^3(a+bx)^4}\right) dx\\
&= -\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+3a^2bd^2(c+2dx)+3ab^2d\left(2c^2+6cdx+5d^2x^2\right)+b^3\left(10c^3+36c^2dx+45cd^2x^2+20d^3x^3\right)}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*d
^2*x^2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(b^4*(a +
b*x)^6)
```

Mathics [A]
time = 4.17, size = 162, normalized size = 1.76

$$\frac{-a^3d^3-3a^2bcd^2-6ab^2c^2d-10b^3c^3-6bdx\left(a^2d^2+3abcd+6b^2c^2\right)+15b^2d^2x^2\left(-ad-3bc\right)-20b^3d^3x^3}{60b^4\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^7,x]')
```

```
[Out] (-a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 - 6 a b ^ 2 c ^ 2 d - 10 b ^ 3 c ^ 3 - 6
b d x (a ^ 2 d ^ 2 + 3 a b c d + 6 b ^ 2 c ^ 2) + 15 b ^ 2 d ^ 2 x ^ 2 (-a
d - 3 b c) - 20 b ^ 3 d ^ 3 x ^ 3) / (60 b ^ 4 (a ^ 6 + 6 a ^ 5 b x + 15 a
^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x
 ^ 5 + b ^ 6 x ^ 6))
```

Maple [A]
time = 0.14, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{d^3x^3}{3b}-\frac{d^2(ad+3bc)x^2}{4b^2}-\frac{d\left(a^2d^2+3abcd+6b^2c^2\right)x}{10b^3}-\frac{a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3}{60b^4}}{(bx+a)^6}$	110
gosper	$-\frac{20d^3x^3b^3+15a\,b^2d^3x^2+45b^3c\,d^2x^2+6a^2b\,d^3x+18a\,b^2c\,d^2x+36b^3c^2dx+a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3}{60b^4(bx+a)^6}$	115
default	$\frac{3d^2(ad-bc)}{4b^4(bx+a)^4}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{5b^4(bx+a)^5}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{6b^4(bx+a)^6}-\frac{d^3}{3b^4(bx+a)^3}$	122
norman	$\frac{-\frac{d^3x^3}{3b}+\frac{\left(-a\,b^2d^3-3b^3c\,d^2\right)x^2}{4b^4}+\frac{\left(-b^2a^2d^3-3a\,b^3c\,d^2-6b^4c^2d\right)x}{10b^5}+\frac{-a^3b^2d^3-3a^2b^3c\,d^2-6a\,b^4c^2d-10b^5c^3}{60b^6}}{(bx+a)^6}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/b^4*d^2*(a*d-b*c)/(b*x+a)^4-3/5/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+
a)^5-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6-1/3*d
^3/b^4/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.
time = 0.34, size = 171, normalized size = 1.86

$$-\frac{20\,b^3d^3x^3+10\,b^3c^3+6\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+15\,(3\,b^3cd^2+ab^2d^3)x^2+6\,(6\,b^3c^2d+3\,ab^2cd^2+a^2bd^3)x}{60\,(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.
time = 0.29, size = 171, normalized size = 1.86

$$-\frac{20\,b^3d^3x^3+10\,b^3c^3+6\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+15\,(3\,b^3cd^2+ab^2d^3)x^2+6\,(6\,b^3c^2d+3\,ab^2cd^2+a^2bd^3)x}{60\,(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(83) = 166$.
time = 1.50, size = 184, normalized size = 2.00

$$\frac{-a^3d^3-3a^2bcd^2-6ab^2c^2d-10b^3c^3-20b^3d^3x^3+x^2\left(-15ab^2d^3-45b^3cd^2\right)+x\left(-6a^2bd^3-18ab^2cd^2-36b^3c^2d\right)}{60a^6b^4+360a^5b^5x+900a^4b^6x^2+1200a^3b^7x^3+900a^2b^8x^4+360ab^9x^5+60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**7,x)
```

```
[Out] (-a**3*d**3 - 3*a**2*b*c*d**2 - 6*a*b**2*c**2*d - 10*b**3*c**3 - 20*b**3*d*
*3*x**3 + x**2*(-15*a*b**2*d**3 - 45*b**3*c*d**2) + x*(-6*a**2*b*d**3 - 18*
a*b**2*c*d**2 - 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4
*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 6
0*b**10*x**6)
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.37

$$\frac{-20x^3d^3b^3-15x^2d^3b^2a-45x^2d^2cb^3-6xd^3ba^2-18xd^2cb^2a-36xdc^2b^3-d^3a^3-3d^2cba^2-6dc^2b^2a-10c^3b^3}{60b^4\left(xb+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x)
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d*
x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b
*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

Mupad [B]
time = 0.22, size = 165, normalized size = 1.79

$$-\frac{\frac{a^3\,d^3+3\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d+10\,b^3\,c^3}{60\,b^4}+\frac{d^3\,x^3}{3\,b}+\frac{d\,x\,\left(a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2\right)}{10\,b^3}+\frac{d^2\,x^2\,\left(a\,d+3\,b\,c\right)}{4\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^7,x)
```

```
[Out] -((a^3*d^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(60*b^4) + (d^3*x^
3)/(3*b) + (d*x*(a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d))/(10*b^3) + (d^2*x^2*(a*d
 + 3*b*c))/(4*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3*
b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

3.1271 $\int \frac{(c+dx)^3}{(a+bx)^8}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}$$

```
[Out] -1/7*(-a*d+b*c)^3/b^4/(b*x+a)^7-1/2*d*(-a*d+b*c)^2/b^4/(b*x+a)^6-3/5*d^2*(-
a*d+b*c)/b^4/(b*x+a)^5-1/4*d^3/b^4/(b*x+a)^4
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -1/7*(b*c - a*d)^3/(b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6
) - (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(4c+7dx)+ab^2d\left(10c^2+28cdx+21d^2x^2\right)+b^3\left(20c^3+70c^2dx+84cd^2x^2+35d^3x^3\right)}{140b^4(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -1/140*(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 21
*d^2*x^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(b^4*(a
+ b*x)^7)
```

Mathics [A]
time = 4.54, size = 173, normalized size = 1.88

$$\frac{-a^3d^3 - 4a^2bcd^2 - 10ab^2c^2d - 20b^3c^3 - 7bdx\left(a^2d^2 + 4abcd + 10b^2c^2\right) + 21b^2d^2x^2\left(-ad - 4bc\right) - 35b^3d^3x^3}{140b^4\left(a^7 + 7a^6bx + 21a^5b^2x^2 + 35a^4b^3x^3 + 35a^3b^4x^4 + 21a^2b^5x^5 + 7ab^6x^6 + b^7x^7\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^8,x]')
```

```
[Out] (-a ^ 3 d ^ 3 - 4 a ^ 2 b c d ^ 2 - 10 a b ^ 2 c ^ 2 d - 20 b ^ 3 c ^ 3 - 7
 b d x (a ^ 2 d ^ 2 + 4 a b c d + 10 b ^ 2 c ^ 2) + 21 b ^ 2 d ^ 2 x ^ 2 (-
a d - 4 b c) - 35 b ^ 3 d ^ 3 x ^ 3) / (140 b ^ 4 (a ^ 7 + 7 a ^ 6 b x + 21
 a ^ 5 b ^ 2 x ^ 2 + 35 a ^ 4 b ^ 3 x ^ 3 + 35 a ^ 3 b ^ 4 x ^ 4 + 21 a ^ 2
 b ^ 5 x ^ 5 + 7 a b ^ 6 x ^ 6 + b ^ 7 x ^ 7))
```

Maple [A]
time = 0.16, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{d^3x^3}{4b}-\frac{3d^2(ad+4bc)x^2}{20b^2}-\frac{d\left(a^2d^2+4abcd+10b^2c^2\right)x}{20b^3}-\frac{a^3d^3+4a^2bc\,d^2+10a\,b^2c^2d+20b^3c^3}{140b^4}}{(bx+a)^7}$	110
gosper	$-\frac{35d^3x^3b^3+21a\,b^2d^3x^2+84b^3c\,d^2x^2+7a^2b\,d^3x+28a\,b^2c\,d^2x+70b^3c^2dx+a^3d^3+4a^2bc\,d^2+10a\,b^2c^2d+20b^3c^3}{140b^4(bx+a)^7}$	115
default	$-\frac{d^3}{4b^4(bx+a)^4}+\frac{3d^2(ad-bc)}{5b^4(bx+a)^5}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{7b^4(bx+a)^7}-\frac{d(a^2d^2-2abcd+b^2c^2)}{2b^4(bx+a)^6}$	122
norman	$\frac{-\frac{d^3x^3}{4b}+\frac{3\left(-a\,b^3d^3-4b^4c\,d^2\right)x^2}{20b^5}+\frac{\left(-a^2b^3d^3-4a\,b^4c\,d^2-10b^5c^2d\right)x}{20b^6}+\frac{-a^3b^3d^3-4a^2b^4c\,d^2-10a\,b^5c^2d-20c^3b^6}{140b^7}}{(bx+a)^7}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3/b^4/(b*x+a)^4+3/5/b^4*d^2*(a*d-b*c)/(b*x+a)^5-1/7*(-a^3*d^3+3*a^2*
b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^7-1/2/b^4*d*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(b*x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(84) = 168.
time = 0.28, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3 + 20\,b^3c^3 + 10\,ab^2c^2d + 4\,a^2bcd^2 + a^3d^3 + 21\,(4\,b^3cd^2 + ab^2d^3)x^2 + 7\,(10\,b^3c^2d + 4\,ab^2cd^2 + a^2bd^3)x}{140\,(b^{11}x^7 + 7\,ab^{10}x^6 + 21\,a^2b^9x^5 + 35\,a^3b^8x^4 + 35\,a^4b^7x^3 + 21\,a^5b^6x^2 + 7\,a^6b^5x + a^7b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.
time = 0.29, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\,(4\,b^3cd^2+ab^2d^3)x^2+7\,(10\,b^3c^2d+4\,ab^2cd^2+a^2bd^3)x}{140\,(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(82) = 164$.
time = 1.96, size = 196, normalized size = 2.13

$$\frac{-a^3d^3-4a^2bcd^2-10ab^2c^2d-20b^3c^3-35b^3d^3x^3+x^2\left(-21ab^2d^3-84b^3cd^2\right)+x\left(-7a^2bd^3-28ab^2cd^2-70b^3c^2d\right)}{140a^7b^4+980a^6b^5x+2940a^5b^6x^2+4900a^4b^7x^3+4900a^3b^8x^4+2940a^2b^9x^5+980ab^{10}x^6+140b^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**8,x)
```

```
[Out] (-a**3*d**3 - 4*a**2*b*c*d**2 - 10*a*b**2*c**2*d - 20*b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-21*a*b**2*d**3 - 84*b**3*c*d**2) + x*(-7*a**2*b*d**3 - 28
*a*b**2*c*d**2 - 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 2940*a
**5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b**9*
x**5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.37

$$\frac{-35x^3d^3b^3-21x^2d^3b^2a-84x^2d^2cb^3-7xd^3ba^2-28xd^2cb^2a-70xdc^2b^3-d^3a^3-4d^2cba^2-10dc^2b^2a-20c^3b^3}{140b^4\left(xb+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x)
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*d
*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

Mupad [B]
time = 0.11, size = 176, normalized size = 1.91

$$-\frac{\frac{a^3\,d^3+4\,a^2\,b\,c\,d^2+10\,a\,b^2\,c^2\,d+20\,b^3\,c^3}{140\,b^4}+\frac{d^3\,x^3}{4\,b}+\frac{d\,x\,\left(a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2\right)}{20\,b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+4\,b\,c\right)}{20\,b^2}}{a^7+7\,a^6\,b\,x+21\,a^5\,b^2\,x^2+35\,a^4\,b^3\,x^3+35\,a^3\,b^4\,x^4+21\,a^2\,b^5\,x^5+7\,a\,b^6\,x^6+b^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^8,x)
```

```
[Out] -((a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2)/(140*b^4) + (d^3*
x^3)/(4*b) + (d*x*(a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d))/(20*b^3) + (3*d^2*x^2
*(a*d + 4*b*c))/(20*b^2))/(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5*b^2*x^2 + 3
5*a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x)
```

3.1272 $\int \frac{(c+dx)^3}{(a+bx)^9}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}$$

```
[Out] -1/8*(-a*d+b*c)^3/b^4/(b*x+a)^8-3/7*d*(-a*d+b*c)^2/b^4/(b*x+a)^7-1/2*d^2*(-
a*d+b*c)/b^4/(b*x+a)^6-1/5*d^3/b^4/(b*x+a)^5
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -1/8*(b*c - a*d)^3/(b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)
^7) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^9}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^6}\right)dx\\ &= -\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(5c+8dx)+ab^2d\left(15c^2+40cdx+28d^2x^2\right)+b^3\left(35c^3+120c^2dx+140cd^2x^2+56d^3x^3\right)}{280b^4(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -1/280*(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 28
*d^2*x^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(b^4*(
a + b*x)^8)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(92) = 184.
time = 5.01, size = 184, normalized size = 2.00

$$\frac{-a^3d^3-5a^2bcd^2-15ab^2c^2d-35b^3c^3-8bdx\left(a^2d^2+5abcd+15b^2c^2\right)+28b^2d^2x^2\left(-ad-5bc\right)-56b^3d^3x^3}{280b^4\left(a^8+8a^7bx+28a^6b^2x^2+56a^5b^3x^3+70a^4b^4x^4+56a^3b^5x^5+28a^2b^6x^6+8ab^7x^7+b^8x^8\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^3/(a + b*x)^9,x]')
```

```
[Out] (-a ^ 3 d ^ 3 - 5 a ^ 2 b c d ^ 2 - 15 a b ^ 2 c ^ 2 d - 35 b ^ 3 c ^ 3 - 8
 b d x (a ^ 2 d ^ 2 + 5 a b c d + 15 b ^ 2 c ^ 2) + 28 b ^ 2 d ^ 2 x ^ 2 (-
a d - 5 b c) - 56 b ^ 3 d ^ 3 x ^ 3) / (280 b ^ 4 (a ^ 8 + 8 a ^ 7 b x + 28
 a ^ 6 b ^ 2 x ^ 2 + 56 a ^ 5 b ^ 3 x ^ 3 + 70 a ^ 4 b ^ 4 x ^ 4 + 56 a ^ 3
 b ^ 5 x ^ 5 + 28 a ^ 2 b ^ 6 x ^ 6 + 8 a b ^ 7 x ^ 7 + b ^ 8 x ^ 8))
```

Maple [A]
time = 0.14, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{d^3x^3}{5b}-\frac{d^2(ad+5bc)x^2}{10b^2}-\frac{d\left(a^2d^2+5abcd+15b^2c^2\right)x}{35b^3}-\frac{a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3}{280b^4}}{(bx+a)^8}$	110
gosper	$-\frac{56d^3x^3b^3+28a\,b^2d^3x^2+140b^3c\,d^2x^2+8a^2b\,d^3x+40a\,b^2c\,d^2x+120b^3c^2dx+a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3}{280b^4(bx+a)^8}$	115
default	$-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{8b^4(bx+a)^8}-\frac{d^3}{5b^4(bx+a)^5}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{7b^4(bx+a)^7}+\frac{d^2(ad-bc)}{2b^4(bx+a)^6}$	122
norman	$\frac{-\frac{d^3x^3}{5b}+\frac{\left(-a\,b^4d^3-5b^5c\,d^2\right)x^2}{10b^6}+\frac{\left(-a^2b^4d^3-5a\,b^5c\,d^2-15c^2d\,b^6\right)x}{35b^7}+\frac{-a^3b^4d^3-5a^2b^5c\,d^2-15a\,b^6c^2d-35c^3b^7}{280b^8}}{(bx+a)^8}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^8-1/5*d^3/b
^4/(b*x+a)^5-3/7/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^7+1/2/b^4*d^2*(a
*d-b*c)/(b*x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(84) = 168.
time = 0.27, size = 193, normalized size = 2.10

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\,(5\,b^3cd^2+ab^2d^3)x^2+8\,(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3)x}{280\,(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.
time = 0.29, size = 193, normalized size = 2.10

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\,(5\,b^3cd^2+ab^2d^3)x^2+8\,(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3)x}{280\,(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(82) = 164$.
time = 3.12, size = 207, normalized size = 2.25

$$\frac{-a^3d^3-5a^2bcd^2-15ab^2c^2d-35b^3c^3-56b^3d^3x^3+x^2\left(-28ab^2d^3-140b^3cd^2\right)+x\left(-8a^2bd^3-40ab^2cd^2-120b^3c^2d\right)}{280a^8b^4+2240a^7b^5x+7840a^6b^6x^2+15680a^5b^7x^3+19600a^4b^8x^4+15680a^3b^9x^5+7840a^2b^{10}x^6+2240ab^{11}x^7+280b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**9,x)
```

```
[Out] (-a**3*d**3 - 5*a**2*b*c*d**2 - 15*a*b**2*c**2*d - 35*b**3*c**3 - 56*b**3*d
**3*x**3 + x**2*(-28*a*b**2*d**3 - 140*b**3*c*d**2) + x*(-8*a**2*b*d**3 - 4
0*a*b**2*c*d**2 - 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x + 784
0*a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*a**3
*b**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8)
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.37

$$\frac{-56x^3d^3b^3-28x^2d^3b^2a-140x^2d^2cb^3-8xd^3ba^2-40xd^2cb^2a-120xdc^2b^3-d^3a^3-5d^2cba^2-15dc^2b^2a-35c^3b^3}{280b^4\left(xb+a\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x)
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^2
*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

Mupad [B]
time = 0.23, size = 187, normalized size = 2.03

$$-\frac{\frac{a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3}{280\,b^4}+\frac{d^3\,x^3}{5\,b}+\frac{d\,x\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{35\,b^3}+\frac{d^2\,x^2\,\left(a\,d+5\,b\,c\right)}{10\,b^2}}{a^8+8\,a^7\,b\,x+28\,a^6\,b^2\,x^2+56\,a^5\,b^3\,x^3+70\,a^4\,b^4\,x^4+56\,a^3\,b^5\,x^5+28\,a^2\,b^6\,x^6+8\,a\,b^7\,x^7+b^8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^9,x)
```

```
[Out] -((a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2)/(280*b^4) + (d^3*
x^3)/(5*b) + (d*x*(a^2*d^2 + 15*b^2*c^2 + 5*a*b*c*d))/(35*b^3) + (d^2*x^2*(
a*d + 5*b*c))/(10*b^2))/(a^8 + b^8*x^8 + 8*a*b^7*x^7 + 28*a^6*b^2*x^2 + 56*
a^5*b^3*x^3 + 70*a^4*b^4*x^4 + 56*a^3*b^5*x^5 + 28*a^2*b^6*x^6 + 8*a^7*b*x)
```

3.1273 $\int (a+bx)^9(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35d^3(bc-ad)^4(a+bx)^{13}}{13b^8}+\frac{5d^4(bc$$

```
[Out] 1/10*(-a*d+b*c)^7*(b*x+a)^10/b^8+7/11*d*(-a*d+b*c)^6*(b*x+a)^11/b^8+7/4*d^2
*(-a*d+b*c)^5*(b*x+a)^12/b^8+35/13*d^3*(-a*d+b*c)^4*(b*x+a)^13/b^8+5/2*d^4*
(-a*d+b*c)^3*(b*x+a)^14/b^8+7/5*d^5*(-a*d+b*c)^2*(b*x+a)^15/b^8+7/16*d^6*(-
a*d+b*c)*(b*x+a)^16/b^8+1/17*d^7*(b*x+a)^17/b^8
```

Rubi [A]
time = 0.46, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx)^{12}(bc-ad)^5}{4b^8}+\frac{7d(a+bx)^{11}(bc-ad)^6}{11b^8}+\frac{(a+bx)^{10}(bc-ad)^7}{10b^8}+\frac{d^7(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^10)/(10*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^11)/(1
1*b^8) + (7*d^2*(b*c - a*d)^5*(a + b*x)^12)/(4*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^13)/(13*b^8) + (5*d^4*(b*c - a*d)^3*(a + b*x)^14)/(2*b^8) + (7*d
^5*(b*c - a*d)^2*(a + b*x)^15)/(5*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^16)/(
16*b^8) + (d^7*(a + b*x)^17)/(17*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^9(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^9}{b^7}+\frac{7d(bc-ad)^6(a+bx)^{10}}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{b^7}\right.\\ &= \frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35c}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 993 vs. 2(200) = 400.

time = 0.09, size = 993, normalized size = 4.96

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] a^9*c^7*x + (a^8*c^6*(9*b*c + 7*a*d)*x^2)/2 + a^7*c^5*(12*b^2*c^2 + 21*a*b*
c*d + 7*a^2*d^2)*x^3 + (7*a^6*c^4*(12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^5*c^3*(18*b^4*c^4 + 84*a*b^3*c^3*d + 108*a^
2*b^2*c^2*d^2 + 45*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^4*c^2*(6*b^5*c^5
+ 42*a*b^4*c^4*d + 84*a^2*b^3*c^3*d^2 + 60*a^3*b^2*c^2*d^3 + 15*a^4*b*c*d^4
 + a^5*d^5)*x^6)/2 + a^3*c*(12*b^6*c^6 + 126*a*b^5*c^5*d + 378*a^2*b^4*c^4*
d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 27*a^5*b*c*d^5 + a^6*d^6)
*x^7 + (a^2*(36*b^7*c^7 + 588*a*b^6*c^6*d + 2646*a^2*b^5*c^5*d^2 + 4410*a^3
*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 756*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6
+ a^7*d^7)*x^8)/8 + a*b*(b^7*c^7 + 28*a*b^6*c^6*d + 196*a^2*b^5*c^5*d^2 + 4
90*a^3*b^4*c^4*d^3 + 490*a^4*b^3*c^3*d^4 + 196*a^5*b^2*c^2*d^5 + 28*a^6*b*c
*d^6 + a^7*d^7)*x^9 + (b^2*(b^7*c^7 + 63*a*b^6*c^6*d + 756*a^2*b^5*c^5*d^2
+ 2940*a^3*b^4*c^4*d^3 + 4410*a^4*b^3*c^3*d^4 + 2646*a^5*b^2*c^2*d^5 + 588*
a^6*b*c*d^6 + 36*a^7*d^7)*x^10)/10 + (7*b^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 1
80*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*
c*d^5 + 12*a^6*d^6)*x^11)/11 + (7*b^4*d^2*(b^5*c^5 + 15*a*b^4*c^4*d + 60*a^
2*b^3*c^3*d^2 + 84*a^3*b^2*c^2*d^3 + 42*a^4*b*c*d^4 + 6*a^5*d^5)*x^12)/4 +
(7*b^5*d^3*(5*b^4*c^4 + 45*a*b^3*c^3*d + 108*a^2*b^2*c^2*d^2 + 84*a^3*b*c*d
^3 + 18*a^4*d^4)*x^13)/13 + (b^6*d^4*(5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b
*c*d^2 + 12*a^3*d^3)*x^14)/2 + (b^7*d^5*(7*b^2*c^2 + 21*a*b*c*d + 12*a^2*d^
2)*x^15)/5 + (b^8*d^6*(7*b*c + 9*a*d)*x^16)/16 + (b^9*d^7*x^17)/17
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 966 vs. $2(200) = 400$.
time = 9.56, size = 964, normalized size = 4.82

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^9*(c + d*x)^7,x]')
```

```
[Out] x (194480 a ^ 9 c ^ 7 + 97240 a ^ 8 c ^ 6 x (7 a d + 9 b c) + a ^ 7 c ^ 5 x
 ^ 2 (1361360 a ^ 2 d ^ 2 + 4084080 a b c d + 2333760 b ^ 2 c ^ 2) + a ^ 6
c ^ 4 x ^ 3 (1701700 a ^ 3 d ^ 3 + 9189180 a ^ 2 b c d ^ 2 + 12252240 a b ^
 2 c ^ 2 d + 4084080 b ^ 3 c ^ 3) + a ^ 5 c ^ 3 x ^ 4 (1361360 a ^ 4 d ^ 4
+ 12252240 a ^ 3 b c d ^ 3 + 29405376 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 22870848 a
b ^ 3 c ^ 3 d + 4900896 b ^ 4 c ^ 4) + a ^ 4 c ^ 2 x ^ 5 (680680 a ^ 5 d ^
5 + 10210200 a ^ 4 b c d ^ 4 + 40840800 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 57177120
a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 28588560 a b ^ 4 c ^ 4 d + 4084080 b ^ 5 c ^ 5) +
```

```
194480 a ^ 3 c x ^ 6 (a ^ 6 d ^ 6 + 27 a ^ 5 b c d ^ 5 + 180 a ^ 4 b ^ 2 c
 ^ 2 d ^ 4 + 420 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 378 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 12
6 a b ^ 5 c ^ 5 d + 12 b ^ 6 c ^ 6) + a ^ 2 x ^ 7 (24310 a ^ 7 d ^ 7 + 1531
530 a ^ 6 b c d ^ 6 + 18378360 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 71471400 a ^ 4 b ^
 3 c ^ 3 d ^ 4 + 107207100 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 64324260 a ^ 2 b ^ 5 c
 ^ 5 d ^ 2 + 14294280 a b ^ 6 c ^ 6 d + 875160 b ^ 7 c ^ 7) + 194480 a b x
^ 8 (a ^ 7 d ^ 7 + 28 a ^ 6 b c d ^ 6 + 196 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 490 a
 ^ 4 b ^ 3 c ^ 3 d ^ 4 + 490 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 196 a ^ 2 b ^ 5 c ^
5 d ^ 2 + 28 a b ^ 6 c ^ 6 d + b ^ 7 c ^ 7) + b ^ 2 x ^ 9 (700128 a ^ 7 d ^
 7 + 11435424 a ^ 6 b c d ^ 6 + 51459408 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 85765680
 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 57177120 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 14702688 a ^
2 b ^ 5 c ^ 5 d ^ 2 + 1225224 a b ^ 6 c ^ 6 d + 19448 b ^ 7 c ^ 7) + 123760
 b ^ 3 d x ^ 10 (12 a ^ 6 d ^ 6 + 126 a ^ 5 b c d ^ 5 + 378 a ^ 4 b ^ 2 c ^
 2 d ^ 4 + 420 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 180 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 27 a
 b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + b ^ 4 d ^ 2 x ^ 11 (2042040 a ^ 5 d ^ 5 + 1
4294280 a ^ 4 b c d ^ 4 + 28588560 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 20420400 a ^ 2
 b ^ 3 c ^ 3 d ^ 2 + 5105100 a b ^ 4 c ^ 4 d + 340340 b ^ 5 c ^ 5) + b ^ 5
d ^ 3 x ^ 12 (1884960 a ^ 4 d ^ 4 + 8796480 a ^ 3 b c d ^ 3 + 11309760 a ^
2 b ^ 2 c ^ 2 d ^ 2 + 4712400 a b ^ 3 c ^ 3 d + 523600 b ^ 4 c ^ 4) + b ^ 6
 d ^ 4 x ^ 13 (1166880 a ^ 3 d ^ 3 + 3500640 a ^ 2 b c d ^ 2 + 2625480 a b
^ 2 c ^ 2 d + 486200 b ^ 3 c ^ 3) + b ^ 7 d ^ 5 x ^ 14 (466752 a ^ 2 d ^ 2
+ 816816 a b c d + 272272 b ^ 2 c ^ 2) + b ^ 8 d ^ 6 x ^ 15 (109395 a d + 8
5085 b c) + 11440 b ^ 9 d ^ 7 x ^ 16) / 194480
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(184) = 368$.

time = 0.14, size = 1033, normalized size = 5.16

method	result
norman	$a^9c^7x + \left(\frac{7}{2}a^9c^6d + \frac{9}{2}a^8b\,c^7\right)x^2 + \left(7a^9c^5d^2 + 21a^8b\,c^6d + 12a^7b^2c^7\right)x^3 + \left(\frac{35}{4}a^9c^4d^3 + \frac{189}{4}a^8b\,c^5d^2\right.$
default	$\frac{b^9d^7x^{17}}{17} + \frac{(9a\,b^8d^7+7b^9c\,d^6)x^{16}}{16} + \frac{(36a^2b^7d^7+63a\,b^8c\,d^6+21b^9c^2d^5)x^{15}}{15} + \frac{(84a^3b^6d^7+252a^2b^7c\,d^6+189a\,b^8c^2d^5+35b^9c^3d^4)}{14}$
gosper	$\frac{7}{2}x^2a^9c^6d + a\,b^8c^7x^9 + \frac{35}{13}x^{13}b^9c^4d^3 + 6x^{14}a^3b^6d^7 + \frac{5}{2}x^{14}b^9c^3d^4 + \frac{12}{5}x^{15}a^2b^7d^7 + \frac{7}{5}x^{15}b^9c^2d^5 + \frac{9}{16}x$
risch	$\frac{7}{2}x^2a^9c^6d + a\,b^8c^7x^9 + \frac{35}{13}x^{13}b^9c^4d^3 + 6x^{14}a^3b^6d^7 + \frac{5}{2}x^{14}b^9c^3d^4 + \frac{12}{5}x^{15}a^2b^7d^7 + \frac{7}{5}x^{15}b^9c^2d^5 + \frac{9}{16}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*b^9*d^7*x^17+1/16*(9*a*b^8*d^7+7*b^9*c*d^6)*x^16+1/15*(36*a^2*b^7*d^7+
63*a*b^8*c*d^6+21*b^9*c^2*d^5)*x^15+1/14*(84*a^3*b^6*d^7+252*a^2*b^7*c*d^6+
189*a*b^8*c^2*d^5+35*b^9*c^3*d^4)*x^14+1/13*(126*a^4*b^5*d^7+588*a^3*b^6*c*
d^6+756*a^2*b^7*c^2*d^5+315*a*b^8*c^3*d^4+35*b^9*c^4*d^3)*x^13+1/12*(126*a^
5*b^4*d^7+882*a^4*b^5*c*d^6+1764*a^3*b^6*c^2*d^5+1260*a^2*b^7*c^3*d^4+315*a
*b^8*c^4*d^3+21*b^9*c^5*d^2)*x^12+1/11*(84*a^6*b^3*d^7+882*a^5*b^4*c*d^6+26
```

```
46*a^4*b^5*c^2*d^5+2940*a^3*b^6*c^3*d^4+1260*a^2*b^7*c^4*d^3+189*a*b^8*c^5*
d^2+7*b^9*c^6*d)*x^11+1/10*(36*a^7*b^2*d^7+588*a^6*b^3*c*d^6+2646*a^5*b^4*c
^2*d^5+4410*a^4*b^5*c^3*d^4+2940*a^3*b^6*c^4*d^3+756*a^2*b^7*c^5*d^2+63*a*b
^8*c^6*d+b^9*c^7)*x^10+1/9*(9*a^8*b*d^7+252*a^7*b^2*c*d^6+1764*a^6*b^3*c^2*
d^5+4410*a^5*b^4*c^3*d^4+4410*a^4*b^5*c^4*d^3+1764*a^3*b^6*c^5*d^2+252*a^2*
b^7*c^6*d+9*a*b^8*c^7)*x^9+1/8*(a^9*d^7+63*a^8*b*c*d^6+756*a^7*b^2*c^2*d^5+
2940*a^6*b^3*c^3*d^4+4410*a^5*b^4*c^4*d^3+2646*a^4*b^5*c^5*d^2+588*a^3*b^6*
c^6*d+36*a^2*b^7*c^7)*x^8+1/7*(7*a^9*c*d^6+189*a^8*b*c^2*d^5+1260*a^7*b^2*c
^3*d^4+2940*a^6*b^3*c^4*d^3+2646*a^5*b^4*c^5*d^2+882*a^4*b^5*c^6*d+84*a^3*b
^6*c^7)*x^7+1/6*(21*a^9*c^2*d^5+315*a^8*b*c^3*d^4+1260*a^7*b^2*c^4*d^3+1764
*a^6*b^3*c^5*d^2+882*a^5*b^4*c^6*d+126*a^4*b^5*c^7)*x^6+1/5*(35*a^9*c^3*d^4
+315*a^8*b*c^4*d^3+756*a^7*b^2*c^5*d^2+588*a^6*b^3*c^6*d+126*a^5*b^4*c^7)*x
^5+1/4*(35*a^9*c^4*d^3+189*a^8*b*c^5*d^2+252*a^7*b^2*c^6*d+84*a^6*b^3*c^7)*
x^4+1/3*(21*a^9*c^5*d^2+63*a^8*b*c^6*d+36*a^7*b^2*c^7)*x^3+1/2*(7*a^9*c^6*d
+9*a^8*b*c^7)*x^2+a^9*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(184) = 368$.
time = 0.28, size = 1023, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
```

```
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(184) = 368$.
time = 0.30, size = 1023, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(184) = 368$.
time = 0.11, size = 1163, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**7,x)
```

```
[Out] a**9*c**7*x + b**9*d**7*x**17/17 + x**16*(9*a*b**8*d**7/16 + 7*b**9*c*d**6/
16) + x**15*(12*a**2*b**7*d**7/5 + 21*a*b**8*c*d**6/5 + 7*b**9*c**2*d**5/5)
```

```
 + x**14*(6*a**3*b**6*d**7 + 18*a**2*b**7*c*d**6 + 27*a*b**8*c**2*d**5/2 +
5*b**9*c**3*d**4/2) + x**13*(126*a**4*b**5*d**7/13 + 588*a**3*b**6*c*d**6/1
3 + 756*a**2*b**7*c**2*d**5/13 + 315*a*b**8*c**3*d**4/13 + 35*b**9*c**4*d**
3/13) + x**12*(21*a**5*b**4*d**7/2 + 147*a**4*b**5*c*d**6/2 + 147*a**3*b**6
*c**2*d**5 + 105*a**2*b**7*c**3*d**4 + 105*a*b**8*c**4*d**3/4 + 7*b**9*c**5
*d**2/4) + x**11*(84*a**6*b**3*d**7/11 + 882*a**5*b**4*c*d**6/11 + 2646*a**
4*b**5*c**2*d**5/11 + 2940*a**3*b**6*c**3*d**4/11 + 1260*a**2*b**7*c**4*d**
3/11 + 189*a*b**8*c**5*d**2/11 + 7*b**9*c**6*d/11) + x**10*(18*a**7*b**2*d*
*7/5 + 294*a**6*b**3*c*d**6/5 + 1323*a**5*b**4*c**2*d**5/5 + 441*a**4*b**5*
c**3*d**4 + 294*a**3*b**6*c**4*d**3 + 378*a**2*b**7*c**5*d**2/5 + 63*a*b**8
*c**6*d/10 + b**9*c**7/10) + x**9*(a**8*b*d**7 + 28*a**7*b**2*c*d**6 + 196*
a**6*b**3*c**2*d**5 + 490*a**5*b**4*c**3*d**4 + 490*a**4*b**5*c**4*d**3 + 1
96*a**3*b**6*c**5*d**2 + 28*a**2*b**7*c**6*d + a*b**8*c**7) + x**8*(a**9*d*
*7/8 + 63*a**8*b*c*d**6/8 + 189*a**7*b**2*c**2*d**5/2 + 735*a**6*b**3*c**3*
d**4/2 + 2205*a**5*b**4*c**4*d**3/4 + 1323*a**4*b**5*c**5*d**2/4 + 147*a**3
*b**6*c**6*d/2 + 9*a**2*b**7*c**7/2) + x**7*(a**9*c*d**6 + 27*a**8*b*c**2*d
**5 + 180*a**7*b**2*c**3*d**4 + 420*a**6*b**3*c**4*d**3 + 378*a**5*b**4*c**
5*d**2 + 126*a**4*b**5*c**6*d + 12*a**3*b**6*c**7) + x**6*(7*a**9*c**2*d**5
/2 + 105*a**8*b*c**3*d**4/2 + 210*a**7*b**2*c**4*d**3 + 294*a**6*b**3*c**5*
d**2 + 147*a**5*b**4*c**6*d + 21*a**4*b**5*c**7) + x**5*(7*a**9*c**3*d**4 +
 63*a**8*b*c**4*d**3 + 756*a**7*b**2*c**5*d**2/5 + 588*a**6*b**3*c**6*d/5 +
 126*a**5*b**4*c**7/5) + x**4*(35*a**9*c**4*d**3/4 + 189*a**8*b*c**5*d**2/4
 + 63*a**7*b**2*c**6*d + 21*a**6*b**3*c**7) + x**3*(7*a**9*c**5*d**2 + 21*a
**8*b*c**6*d + 12*a**7*b**2*c**7) + x**2*(7*a**9*c**6*d/2 + 9*a**8*b*c**7/2
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(184) = 368$.
time = 0.00, size = 1269, normalized size = 6.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x)
```

```
[Out] 1/17*b^9*d^7*x^17 + 7/16*b^9*c*d^6*x^16 + 9/16*a*b^8*d^7*x^16 + 7/5*b^9*c^2
*d^5*x^15 + 21/5*a*b^8*c*d^6*x^15 + 12/5*a^2*b^7*d^7*x^15 + 5/2*b^9*c^3*d^4
*x^14 + 27/2*a*b^8*c^2*d^5*x^14 + 18*a^2*b^7*c*d^6*x^14 + 6*a^3*b^6*d^7*x^1
4 + 35/13*b^9*c^4*d^3*x^13 + 315/13*a*b^8*c^3*d^4*x^13 + 756/13*a^2*b^7*c^2
*d^5*x^13 + 588/13*a^3*b^6*c*d^6*x^13 + 126/13*a^4*b^5*d^7*x^13 + 7/4*b^9*c
^5*d^2*x^12 + 105/4*a*b^8*c^4*d^3*x^12 + 105*a^2*b^7*c^3*d^4*x^12 + 147*a^3
*b^6*c^2*d^5*x^12 + 147/2*a^4*b^5*c*d^6*x^12 + 21/2*a^5*b^4*d^7*x^12 + 7/11
*b^9*c^6*d*x^11 + 189/11*a*b^8*c^5*d^2*x^11 + 1260/11*a^2*b^7*c^4*d^3*x^11
+ 2940/11*a^3*b^6*c^3*d^4*x^11 + 2646/11*a^4*b^5*c^2*d^5*x^11 + 882/11*a^5*
b^4*c*d^6*x^11 + 84/11*a^6*b^3*d^7*x^11 + 1/10*b^9*c^7*x^10 + 63/10*a*b^8*c
```

```
^6*d*x^10 + 378/5*a^2*b^7*c^5*d^2*x^10 + 294*a^3*b^6*c^4*d^3*x^10 + 441*a^4
*b^5*c^3*d^4*x^10 + 1323/5*a^5*b^4*c^2*d^5*x^10 + 294/5*a^6*b^3*c*d^6*x^10
+ 18/5*a^7*b^2*d^7*x^10 + a*b^8*c^7*x^9 + 28*a^2*b^7*c^6*d*x^9 + 196*a^3*b^
6*c^5*d^2*x^9 + 490*a^4*b^5*c^4*d^3*x^9 + 490*a^5*b^4*c^3*d^4*x^9 + 196*a^6
*b^3*c^2*d^5*x^9 + 28*a^7*b^2*c*d^6*x^9 + a^8*b*d^7*x^9 + 9/2*a^2*b^7*c^7*x
^8 + 147/2*a^3*b^6*c^6*d*x^8 + 1323/4*a^4*b^5*c^5*d^2*x^8 + 2205/4*a^5*b^4*
c^4*d^3*x^8 + 735/2*a^6*b^3*c^3*d^4*x^8 + 189/2*a^7*b^2*c^2*d^5*x^8 + 63/8*
a^8*b*c*d^6*x^8 + 1/8*a^9*d^7*x^8 + 12*a^3*b^6*c^7*x^7 + 126*a^4*b^5*c^6*d*
x^7 + 378*a^5*b^4*c^5*d^2*x^7 + 420*a^6*b^3*c^4*d^3*x^7 + 180*a^7*b^2*c^3*d
^4*x^7 + 27*a^8*b*c^2*d^5*x^7 + a^9*c*d^6*x^7 + 21*a^4*b^5*c^7*x^6 + 147*a^
5*b^4*c^6*d*x^6 + 294*a^6*b^3*c^5*d^2*x^6 + 210*a^7*b^2*c^4*d^3*x^6 + 105/2
*a^8*b*c^3*d^4*x^6 + 7/2*a^9*c^2*d^5*x^6 + 126/5*a^5*b^4*c^7*x^5 + 588/5*a^
6*b^3*c^6*d*x^5 + 756/5*a^7*b^2*c^5*d^2*x^5 + 63*a^8*b*c^4*d^3*x^5 + 7*a^9*
c^3*d^4*x^5 + 21*a^6*b^3*c^7*x^4 + 63*a^7*b^2*c^6*d*x^4 + 189/4*a^8*b*c^5*d
^2*x^4 + 35/4*a^9*c^4*d^3*x^4 + 12*a^7*b^2*c^7*x^3 + 21*a^8*b*c^6*d*x^3 + 7
*a^9*c^5*d^2*x^3 + 9/2*a^8*b*c^7*x^2 + 7/2*a^9*c^6*d*x^2 + a^9*c^7*x
```

Mupad [B]
time = 0.55, size = 997, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^7,x)
```

```
[Out] x^5*((126*a^5*b^4*c^7)/5 + 7*a^9*c^3*d^4 + (588*a^6*b^3*c^6*d)/5 + 63*a^8*b
*c^4*d^3 + (756*a^7*b^2*c^5*d^2)/5) + x^13*((126*a^4*b^5*d^7)/13 + (35*b^9*
c^4*d^3)/13 + (315*a*b^8*c^3*d^4)/13 + (588*a^3*b^6*c*d^6)/13 + (756*a^2*b^
7*c^2*d^5)/13) + x^8*((a^9*d^7)/8 + (9*a^2*b^7*c^7)/2 + (147*a^3*b^6*c^6*d)
/2 + (1323*a^4*b^5*c^5*d^2)/4 + (2205*a^5*b^4*c^4*d^3)/4 + (735*a^6*b^3*c^3
*d^4)/2 + (189*a^7*b^2*c^2*d^5)/2 + (63*a^8*b*c*d^6)/8) + x^10*((b^9*c^7)/1
0 + (18*a^7*b^2*d^7)/5 + (294*a^6*b^3*c*d^6)/5 + (378*a^2*b^7*c^5*d^2)/5 +
294*a^3*b^6*c^4*d^3 + 441*a^4*b^5*c^3*d^4 + (1323*a^5*b^4*c^2*d^5)/5 + (63*
a*b^8*c^6*d)/10) + x^6*(21*a^4*b^5*c^7 + (7*a^9*c^2*d^5)/2 + 147*a^5*b^4*c^
6*d + (105*a^8*b*c^3*d^4)/2 + 294*a^6*b^3*c^5*d^2 + 210*a^7*b^2*c^4*d^3) +
x^12*((21*a^5*b^4*d^7)/2 + (7*b^9*c^5*d^2)/4 + (105*a*b^8*c^4*d^3)/4 + (147
*a^4*b^5*c*d^6)/2 + 105*a^2*b^7*c^3*d^4 + 147*a^3*b^6*c^2*d^5) + x^7*(a^9*c
*d^6 + 12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 27*a^8*b*c^2*d^5 + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4) + x^11*((7*b^9*c^6*d)/
11 + (84*a^6*b^3*d^7)/11 + (189*a*b^8*c^5*d^2)/11 + (882*a^5*b^4*c*d^6)/11
+ (1260*a^2*b^7*c^4*d^3)/11 + (2940*a^3*b^6*c^3*d^4)/11 + (2646*a^4*b^5*c^2
*d^5)/11) + x^9*(a*b^8*c^7 + a^8*b*d^7 + 28*a^2*b^7*c^6*d + 28*a^7*b^2*c*d^
6 + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d^3 + 490*a^5*b^4*c^3*d^4 + 196*a
^6*b^3*c^2*d^5) + a^9*c^7*x + (b^9*d^7*x^17)/17 + (7*a^6*c^4*x^4*(5*a^3*d^3
 + 12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c*d^2))/4 + (b^6*d^4*x^14*(12*a^3
```

```
*d^3 + 5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b*c*d^2))/2 + (a^8*c^6*x^2*(7*a*
d + 9*b*c))/2 + (b^8*d^6*x^16*(9*a*d + 7*b*c))/16 + a^7*c^5*x^3*(7*a^2*d^2
+ 12*b^2*c^2 + 21*a*b*c*d) + (b^7*d^5*x^15*(12*a^2*d^2 + 7*b^2*c^2 + 21*a*b
*c*d))/5
```

3.1274 $\int (a+bx)^8(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35d^3(bc-ad)^4(a+bx)^{12}}{12b^8}+\frac{35d^4(b}{}$$

```
[Out] 1/9*(-a*d+b*c)^7*(b*x+a)^9/b^8+7/10*d*(-a*d+b*c)^6*(b*x+a)^10/b^8+21/11*d^2
*(-a*d+b*c)^5*(b*x+a)^11/b^8+35/12*d^3*(-a*d+b*c)^4*(b*x+a)^12/b^8+35/13*d^
4*(-a*d+b*c)^3*(b*x+a)^13/b^8+3/2*d^5*(-a*d+b*c)^2*(b*x+a)^14/b^8+7/15*d^6*
(-a*d+b*c)*(b*x+a)^15/b^8+1/16*d^7*(b*x+a)^16/b^8
```

Rubi [A]
time = 0.39, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+bx)^{11}(bc-ad)^5}{11b^8}+\frac{7d(a+bx)^{10}(bc-ad)^6}{10b^8}+\frac{(a+bx)^9(bc-ad)^7}{9b^8}+\frac{d^7(a+bx)^{16}}{16b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^9)/(9*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^10)/(10*
b^8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^11)/(11*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^12)/(12*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^13)/(13*b^8) + (3
*d^5*(b*c - a*d)^2*(a + b*x)^14)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^15)
/(15*b^8) + (d^7*(a + b*x)^16)/(16*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^8(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^8}{b^7}+\frac{7d(bc-ad)^6(a+bx)^9}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{b^7}+\right.\\ &= \frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 897 vs. 2(200) = 400.

time = 0.06, size = 897, normalized size = 4.48

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^8*(c + d*x)^7,x]

[Out] a^8*c^7*x + (a^7*c^6*(8*b*c + 7*a*d)*x^2)/2 + (7*a^6*c^5*(4*b^2*c^2 + 8*a*b *c*d + 3*a^2*d^2)*x^3)/3 + (7*a^5*c^4*(8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2* b*c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^4*c^3*(10*b^4*c^4 + 56*a*b^3*c^3*d + 84* a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^3*c^2*(8*b^5*c^ 5 + 70*a*b^4*c^4*d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c *d^4 + 3*a^5*d^5)*x^6)/6 + a^2*c*(4*b^6*c^6 + 56*a*b^5*c^5*d + 210*a^2*b^4* c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 140*a^4*b^2*c^2*d^4 + 24*a^5*b*c*d^5 + a^6* d^6)*x^7 + (a*(8*b^7*c^7 + 196*a*b^6*c^6*d + 1176*a^2*b^5*c^5*d^2 + 2450*a^ 3*b^4*c^4*d^3 + 1960*a^4*b^3*c^3*d^4 + 588*a^5*b^2*c^2*d^5 + 56*a^6*b*c*d^6 + a^7*d^7)*x^8)/8 + (b*(b^7*c^7 + 56*a*b^6*c^6*d + 588*a^2*b^5*c^5*d^2 + 1 960*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 + 1176*a^5*b^2*c^2*d^5 + 196*a^6 *b*c*d^6 + 8*a^7*d^7)*x^9)/9 + (7*b^2*d*(b^6*c^6 + 24*a*b^5*c^5*d + 140*a^2 *b^4*c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 210*a^4*b^2*c^2*d^4 + 56*a^5*b*c*d^5 + 4*a^6*d^6)*x^10)/10 + (7*b^3*d^2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3 *c^3*d^2 + 168*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^11)/11 + (7* b^4*d^3*(5*b^4*c^4 + 40*a*b^3*c^3*d + 84*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 10*a^4*d^4)*x^12)/12 + (7*b^5*d^4*(5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a^2*b*c *d^2 + 8*a^3*d^3)*x^13)/13 + (b^6*d^5*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x ^14)/2 + (b^7*d^6*(7*b*c + 8*a*d)*x^15)/15 + (b^8*d^7*x^16)/16

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 864 vs. $2(200) = 400$.
time = 8.57, size = 862, normalized size = 4.31

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^8*(c + d*x)^7,x]')

[Out] x (102960 a ^ 8 c ^ 7 + 51480 a ^ 7 c ^ 6 x (7 a d + 8 b c) + a ^ 6 c ^ 5 x ^ 2 (720720 a ^ 2 d ^ 2 + 1921920 a b c d + 960960 b ^ 2 c ^ 2) + a ^ 5 c ^ 4 x ^ 3 (900900 a ^ 3 d ^ 3 + 4324320 a ^ 2 b c d ^ 2 + 5045040 a b ^ 2 c ^ 2 d + 1441440 b ^ 3 c ^ 3) + a ^ 4 c ^ 3 x ^ 4 (720720 a ^ 4 d ^ 4 + 576 5760 a ^ 3 b c d ^ 3 + 12108096 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 8072064 a b ^ 3 c ^ 3 d + 1441440 b ^ 4 c ^ 4) + a ^ 3 c ^ 2 x ^ 5 (360360 a ^ 5 d ^ 5 + 480 4800 a ^ 4 b c d ^ 4 + 16816800 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 20180160 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 8408400 a b ^ 4 c ^ 4 d + 960960 b ^ 5 c ^ 5) + 102960 a ^ 2 c x ^ 6 (a ^ 6 d ^ 6 + 24 a ^ 5 b c d ^ 5 + 140 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 280 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 210 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 56 a b ^ 5 c

```
^ 5 d + 4 b ^ 6 c ^ 6) + 12870 a x ^ 7 (a ^ 7 d ^ 7 + 56 a ^ 6 b c d ^ 6 +
588 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 1960 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 2450 a ^ 3 b
^ 4 c ^ 4 d ^ 3 + 1176 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 196 a b ^ 6 c ^ 6 d + 8 b
^ 7 c ^ 7) + 11440 b x ^ 8 (8 a ^ 7 d ^ 7 + 196 a ^ 6 b c d ^ 6 + 1176 a ^
5 b ^ 2 c ^ 2 d ^ 5 + 2450 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 1960 a ^ 3 b ^ 4 c ^ 4
d ^ 3 + 588 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 56 a b ^ 6 c ^ 6 d + b ^ 7 c ^ 7) +
72072 b ^ 2 d x ^ 9 (4 a ^ 6 d ^ 6 + 56 a ^ 5 b c d ^ 5 + 210 a ^ 4 b ^ 2 c
^ 2 d ^ 4 + 280 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 140 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 24
a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + b ^ 3 d ^ 2 x ^ 10 (524160 a ^ 5 d ^ 5 +
4586400 a ^ 4 b c d ^ 4 + 11007360 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 9172800 a ^ 2
b ^ 3 c ^ 3 d ^ 2 + 2620800 a b ^ 4 c ^ 4 d + 196560 b ^ 5 c ^ 5) + b ^ 4 d
^ 3 x ^ 11 (600600 a ^ 4 d ^ 4 + 3363360 a ^ 3 b c d ^ 3 + 5045040 a ^ 2 b
^ 2 c ^ 2 d ^ 2 + 2402400 a b ^ 3 c ^ 3 d + 300300 b ^ 4 c ^ 4) + b ^ 5 d
^ 4 x ^ 12 (443520 a ^ 3 d ^ 3 + 1552320 a ^ 2 b c d ^ 2 + 1330560 a b ^ 2
c ^ 2 d + 277200 b ^ 3 c ^ 3) + b ^ 6 d ^ 5 x ^ 13 (205920 a ^ 2 d ^ 2 + 41
1840 a b c d + 154440 b ^ 2 c ^ 2) + b ^ 7 d ^ 6 x ^ 14 (54912 a d + 48048
b c) + 6435 b ^ 8 d ^ 7 x ^ 15) / 102960
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(184) = 368$.

time = 0.15, size = 925, normalized size = 4.62

method	result
norman	$a^8c^7x+\left(\frac{7}{2}a^8c^6d+4a^7b\,c^7\right)x^2+\left(7a^8c^5d^2+\frac{56}{3}a^7b\,c^6d+\frac{28}{3}a^6b^2c^7\right)x^3+\left(\frac{35}{4}a^8c^4d^3+42a^7b\,c^5d^2+\right.$
default	$\frac{b^8d^7x^{16}}{16}+\frac{(8a\,b^7d^7+7b^8c\,d^6)x^{15}}{15}+\frac{(28a^2b^6d^7+56a\,b^7c\,d^6+21b^8c^2d^5)x^{14}}{14}+\frac{(56a^3b^5d^7+196a^2b^6c\,d^6+168a\,b^7c^2d^5+35b^8c^3d^4)x}{13}$
gosper	$\frac{196}{3}x^9a^2b^6c^5d^2+\frac{56}{9}x^9a\,b^7c^6d+\frac{196}{5}x^{10}a^5b^3c\,d^6+147x^{10}a^4b^4c^2d^5+196x^{10}a^3b^5c^3d^4+98x^{10}a^2b^6c^4d$
risch	$\frac{196}{3}x^9a^2b^6c^5d^2+\frac{56}{9}x^9a\,b^7c^6d+\frac{196}{5}x^{10}a^5b^3c\,d^6+147x^{10}a^4b^4c^2d^5+196x^{10}a^3b^5c^3d^4+98x^{10}a^2b^6c^4d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^8*d^7*x^16+1/15*(8*a*b^7*d^7+7*b^8*c*d^6)*x^15+1/14*(28*a^2*b^6*d^7+
56*a*b^7*c*d^6+21*b^8*c^2*d^5)*x^14+1/13*(56*a^3*b^5*d^7+196*a^2*b^6*c*d^6+
168*a*b^7*c^2*d^5+35*b^8*c^3*d^4)*x^13+1/12*(70*a^4*b^4*d^7+392*a^3*b^5*c*d
^6+588*a^2*b^6*c^2*d^5+280*a*b^7*c^3*d^4+35*b^8*c^4*d^3)*x^12+1/11*(56*a^5*
b^3*d^7+490*a^4*b^4*c*d^6+1176*a^3*b^5*c^2*d^5+980*a^2*b^6*c^3*d^4+280*a*b^
7*c^4*d^3+21*b^8*c^5*d^2)*x^11+1/10*(28*a^6*b^2*d^7+392*a^5*b^3*c*d^6+1470*
a^4*b^4*c^2*d^5+1960*a^3*b^5*c^3*d^4+980*a^2*b^6*c^4*d^3+168*a*b^7*c^5*d^2+
7*b^8*c^6*d)*x^10+1/9*(8*a^7*b*d^7+196*a^6*b^2*c*d^6+1176*a^5*b^3*c^2*d^5+2
450*a^4*b^4*c^3*d^4+1960*a^3*b^5*c^4*d^3+588*a^2*b^6*c^5*d^2+56*a*b^7*c^6*d
+b^8*c^7)*x^9+1/8*(a^8*d^7+56*a^7*b*c*d^6+588*a^6*b^2*c^2*d^5+1960*a^5*b^3*
c^3*d^4+2450*a^4*b^4*c^4*d^3+1176*a^3*b^5*c^5*d^2+196*a^2*b^6*c^6*d+8*a*b^7
*c^7)*x^8+1/7*(7*a^8*c*d^6+168*a^7*b*c^2*d^5+980*a^6*b^2*c^3*d^4+1960*a^5*b
```

```
^3*c^4*d^3+1470*a^4*b^4*c^5*d^2+392*a^3*b^5*c^6*d+28*a^2*b^6*c^7)*x^7+1/6*(
21*a^8*c^2*d^5+280*a^7*b*c^3*d^4+980*a^6*b^2*c^4*d^3+1176*a^5*b^3*c^5*d^2+4
90*a^4*b^4*c^6*d+56*a^3*b^5*c^7)*x^6+1/5*(35*a^8*c^3*d^4+280*a^7*b*c^4*d^3+
588*a^6*b^2*c^5*d^2+392*a^5*b^3*c^6*d+70*a^4*b^4*c^7)*x^5+1/4*(35*a^8*c^4*d
^3+168*a^7*b*c^5*d^2+196*a^6*b^2*c^6*d+56*a^5*b^3*c^7)*x^4+1/3*(21*a^8*c^5*
d^2+56*a^7*b*c^6*d+28*a^6*b^2*c^7)*x^3+1/2*(7*a^8*c^6*d+8*a^7*b*c^7)*x^2+a^
8*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(184) = 368$.
time = 0.27, size = 921, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(184) = 368$.
time = 0.30, size = 921, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(184) = 368$.
time = 0.10, size = 1046, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**7,x)
```

```
[Out] a**8*c**7*x + b**8*d**7*x**16/16 + x**15*(8*a*b**7*d**7/15 + 7*b**8*c*d**6/
15) + x**14*(2*a**2*b**6*d**7 + 4*a*b**7*c*d**6 + 3*b**8*c**2*d**5/2) + x**
13*(56*a**3*b**5*d**7/13 + 196*a**2*b**6*c*d**6/13 + 168*a*b**7*c**2*d**5/1
3 + 35*b**8*c**3*d**4/13) + x**12*(35*a**4*b**4*d**7/6 + 98*a**3*b**5*c*d**
6/3 + 49*a**2*b**6*c**2*d**5 + 70*a*b**7*c**3*d**4/3 + 35*b**8*c**4*d**3/12
) + x**11*(56*a**5*b**3*d**7/11 + 490*a**4*b**4*c*d**6/11 + 1176*a**3*b**5*
c**2*d**5/11 + 980*a**2*b**6*c**3*d**4/11 + 280*a*b**7*c**4*d**3/11 + 21*b*
*8*c**5*d**2/11) + x**10*(14*a**6*b**2*d**7/5 + 196*a**5*b**3*c*d**6/5 + 14
7*a**4*b**4*c**2*d**5 + 196*a**3*b**5*c**3*d**4 + 98*a**2*b**6*c**4*d**3 +
84*a*b**7*c**5*d**2/5 + 7*b**8*c**6*d/10) + x**9*(8*a**7*b*d**7/9 + 196*a**
6*b**2*c*d**6/9 + 392*a**5*b**3*c**2*d**5/3 + 2450*a**4*b**4*c**3*d**4/9 +
1960*a**3*b**5*c**4*d**3/9 + 196*a**2*b**6*c**5*d**2/3 + 56*a*b**7*c**6*d/9
 + b**8*c**7/9) + x**8*(a**8*d**7/8 + 7*a**7*b*c*d**6 + 147*a**6*b**2*c**2*
d**5/2 + 245*a**5*b**3*c**3*d**4 + 1225*a**4*b**4*c**4*d**3/4 + 147*a**3*b*
*5*c**5*d**2 + 49*a**2*b**6*c**6*d/2 + a*b**7*c**7) + x**7*(a**8*c*d**6 + 2
```

```
4*a**7*b*c**2*d**5 + 140*a**6*b**2*c**3*d**4 + 280*a**5*b**3*c**4*d**3 + 21
0*a**4*b**4*c**5*d**2 + 56*a**3*b**5*c**6*d + 4*a**2*b**6*c**7) + x**6*(7*a
**8*c**2*d**5/2 + 140*a**7*b*c**3*d**4/3 + 490*a**6*b**2*c**4*d**3/3 + 196*
a**5*b**3*c**5*d**2 + 245*a**4*b**4*c**6*d/3 + 28*a**3*b**5*c**7/3) + x**5*
(7*a**8*c**3*d**4 + 56*a**7*b*c**4*d**3 + 588*a**6*b**2*c**5*d**2/5 + 392*a
**5*b**3*c**6*d/5 + 14*a**4*b**4*c**7) + x**4*(35*a**8*c**4*d**3/4 + 42*a**
7*b*c**5*d**2 + 49*a**6*b**2*c**6*d + 14*a**5*b**3*c**7) + x**3*(7*a**8*c**
5*d**2 + 56*a**7*b*c**6*d/3 + 28*a**6*b**2*c**7/3) + x**2*(7*a**8*c**6*d/2
+ 4*a**7*b*c**7)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(184) = 368$.
time = 0.00, size = 1140, normalized size = 5.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x)
```

```
[Out] 1/16*b^8*d^7*x^16 + 7/15*b^8*c*d^6*x^15 + 8/15*a*b^7*d^7*x^15 + 3/2*b^8*c^2
*d^5*x^14 + 4*a*b^7*c*d^6*x^14 + 2*a^2*b^6*d^7*x^14 + 35/13*b^8*c^3*d^4*x^1
3 + 168/13*a*b^7*c^2*d^5*x^13 + 196/13*a^2*b^6*c*d^6*x^13 + 56/13*a^3*b^5*d
^7*x^13 + 35/12*b^8*c^4*d^3*x^12 + 70/3*a*b^7*c^3*d^4*x^12 + 49*a^2*b^6*c^2
*d^5*x^12 + 98/3*a^3*b^5*c*d^6*x^12 + 35/6*a^4*b^4*d^7*x^12 + 21/11*b^8*c^5
*d^2*x^11 + 280/11*a*b^7*c^4*d^3*x^11 + 980/11*a^2*b^6*c^3*d^4*x^11 + 1176/
11*a^3*b^5*c^2*d^5*x^11 + 490/11*a^4*b^4*c*d^6*x^11 + 56/11*a^5*b^3*d^7*x^1
1 + 7/10*b^8*c^6*d*x^10 + 84/5*a*b^7*c^5*d^2*x^10 + 98*a^2*b^6*c^4*d^3*x^10
 + 196*a^3*b^5*c^3*d^4*x^10 + 147*a^4*b^4*c^2*d^5*x^10 + 196/5*a^5*b^3*c*d^
6*x^10 + 14/5*a^6*b^2*d^7*x^10 + 1/9*b^8*c^7*x^9 + 56/9*a*b^7*c^6*d*x^9 + 1
96/3*a^2*b^6*c^5*d^2*x^9 + 1960/9*a^3*b^5*c^4*d^3*x^9 + 2450/9*a^4*b^4*c^3*
d^4*x^9 + 392/3*a^5*b^3*c^2*d^5*x^9 + 196/9*a^6*b^2*c*d^6*x^9 + 8/9*a^7*b*d
^7*x^9 + a*b^7*c^7*x^8 + 49/2*a^2*b^6*c^6*d*x^8 + 147*a^3*b^5*c^5*d^2*x^8 +
 1225/4*a^4*b^4*c^4*d^3*x^8 + 245*a^5*b^3*c^3*d^4*x^8 + 147/2*a^6*b^2*c^2*d
^5*x^8 + 7*a^7*b*c*d^6*x^8 + 1/8*a^8*d^7*x^8 + 4*a^2*b^6*c^7*x^7 + 56*a^3*b
^5*c^6*d*x^7 + 210*a^4*b^4*c^5*d^2*x^7 + 280*a^5*b^3*c^4*d^3*x^7 + 140*a^6*
b^2*c^3*d^4*x^7 + 24*a^7*b*c^2*d^5*x^7 + a^8*c*d^6*x^7 + 28/3*a^3*b^5*c^7*x
^6 + 245/3*a^4*b^4*c^6*d*x^6 + 196*a^5*b^3*c^5*d^2*x^6 + 490/3*a^6*b^2*c^4*
d^3*x^6 + 140/3*a^7*b*c^3*d^4*x^6 + 7/2*a^8*c^2*d^5*x^6 + 14*a^4*b^4*c^7*x^
5 + 392/5*a^5*b^3*c^6*d*x^5 + 588/5*a^6*b^2*c^5*d^2*x^5 + 56*a^7*b*c^4*d^3*
x^5 + 7*a^8*c^3*d^4*x^5 + 14*a^5*b^3*c^7*x^4 + 49*a^6*b^2*c^6*d*x^4 + 42*a^
7*b*c^5*d^2*x^4 + 35/4*a^8*c^4*d^3*x^4 + 28/3*a^6*b^2*c^7*x^3 + 56/3*a^7*b*
c^6*d*x^3 + 7*a^8*c^5*d^2*x^3 + 4*a^7*b*c^7*x^2 + 7/2*a^8*c^6*d*x^2 + a^8*c
^7*x
```

Mupad [**B**]

time = 0.36, size = 892, normalized size = 4.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^7,x)
```

```
[Out] x^8*((a^8*d^7)/8 + a*b^7*c^7 + (49*a^2*b^6*c^6*d)/2 + 147*a^3*b^5*c^5*d^2 +
 (1225*a^4*b^4*c^4*d^3)/4 + 245*a^5*b^3*c^3*d^4 + (147*a^6*b^2*c^2*d^5)/2 +
 7*a^7*b*c*d^6) + x^9*((b^8*c^7)/9 + (8*a^7*b*d^7)/9 + (196*a^6*b^2*c*d^6)/
9 + (196*a^2*b^6*c^5*d^2)/3 + (1960*a^3*b^5*c^4*d^3)/9 + (2450*a^4*b^4*c^3*
d^4)/9 + (392*a^5*b^3*c^2*d^5)/3 + (56*a*b^7*c^6*d)/9) + x^5*(14*a^4*b^4*c^
7 + 7*a^8*c^3*d^4 + (392*a^5*b^3*c^6*d)/5 + 56*a^7*b*c^4*d^3 + (588*a^6*b^2
*c^5*d^2)/5) + x^12*((35*a^4*b^4*d^7)/6 + (35*b^8*c^4*d^3)/12 + (70*a*b^7*c
^3*d^4)/3 + (98*a^3*b^5*c*d^6)/3 + 49*a^2*b^6*c^2*d^5) + x^6*((28*a^3*b^5*c
^7)/3 + (7*a^8*c^2*d^5)/2 + (245*a^4*b^4*c^6*d)/3 + (140*a^7*b*c^3*d^4)/3 +
 196*a^5*b^3*c^5*d^2 + (490*a^6*b^2*c^4*d^3)/3) + x^11*((56*a^5*b^3*d^7)/11
 + (21*b^8*c^5*d^2)/11 + (280*a*b^7*c^4*d^3)/11 + (490*a^4*b^4*c*d^6)/11 +
(980*a^2*b^6*c^3*d^4)/11 + (1176*a^3*b^5*c^2*d^5)/11) + x^7*(a^8*c*d^6 + 4*
a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 24*a^7*b*c^2*d^5 + 210*a^4*b^4*c^5*d^2 + 2
80*a^5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4) + x^10*((7*b^8*c^6*d)/10 + (14*a^
6*b^2*d^7)/5 + (84*a*b^7*c^5*d^2)/5 + (196*a^5*b^3*c*d^6)/5 + 98*a^2*b^6*c^
4*d^3 + 196*a^3*b^5*c^3*d^4 + 147*a^4*b^4*c^2*d^5) + a^8*c^7*x + (b^8*d^7*x
^16)/16 + (7*a^5*c^4*x^4*(5*a^3*d^3 + 8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*b
*c*d^2))/4 + (7*b^5*d^4*x^13*(8*a^3*d^3 + 5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a
^2*b*c*d^2))/13 + (a^7*c^6*x^2*(7*a*d + 8*b*c))/2 + (b^7*d^6*x^15*(8*a*d +
7*b*c))/15 + (7*a^6*c^5*x^3*(3*a^2*d^2 + 4*b^2*c^2 + 8*a*b*c*d))/3 + (b^6*d
^5*x^14*(4*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/2
```

3.1275 $\int (a+bx)^7 (c+dx)^7 \, dx$

Optimal. Leaf size=200

$$\frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d^3(bc-ad)^4(a+bx)^{11}}{11b^8}+\frac{35d^4(bc}{}$$

```
[Out] 1/8*(-a*d+b*c)^7*(b*x+a)^8/b^8+7/9*d*(-a*d+b*c)^6*(b*x+a)^9/b^8+21/10*d^2*(
-a*d+b*c)^5*(b*x+a)^10/b^8+35/11*d^3*(-a*d+b*c)^4*(b*x+a)^11/b^8+35/12*d^4*
(-a*d+b*c)^3*(b*x+a)^12/b^8+21/13*d^5*(-a*d+b*c)^2*(b*x+a)^13/b^8+1/2*d^6*(
-a*d+b*c)*(b*x+a)^14/b^8+1/15*d^7*(b*x+a)^15/b^8
```

Rubi [A]
time = 0.31, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+bx)^{10}(bc-ad)^5}{10b^8}+\frac{7d(a+bx)^9(bc-ad)^6}{9b^8}+\frac{(a+bx)^8(bc-ad)^7}{8b^8}+\frac{d^7(a+bx)^{15}}{15b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^8)/(8*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^9)/(9*b^
8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^10)/(10*b^8) + (35*d^3*(b*c - a*d)^4*(
a + b*x)^11)/(11*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^12)/(12*b^8) + (21*
d^5*(b*c - a*d)^2*(a + b*x)^13)/(13*b^8) + (d^6*(b*c - a*d)*(a + b*x)^14)/(
2*b^8) + (d^7*(a + b*x)^15)/(15*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^7(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^7}{b^7}+\frac{7d(bc-ad)^6(a+bx)^8}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^9}{b^7}+3\right.\\ &= \frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d^3(}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 785 vs. 2(200) = 400.

time = 0.04, size = 785, normalized size = 3.92

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] a^7*c^7*x + (7*a^6*c^6*(b*c + a*d)*x^2)/2 + (7*a^5*c^5*(3*b^2*c^2 + 7*a*b*c
*d + 3*a^2*d^2)*x^3)/3 + (7*a^4*c^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*
c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^3*c^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2
*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^2*c^2*(3*b^5*c^5 +
 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^
4 + 3*a^5*d^5)*x^6)/6 + a*c*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2
 + 175*a^3*b^3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^
7 + ((b^7*c^7 + 49*a*b^6*c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3
 + 1225*a^4*b^3*c^3*d^4 + 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x
^8)/8 + (7*b*d*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 + 175*a^3*b^
3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^9)/9 + (7*b^2
*d^2*(3*b^5*c^5 + 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^
3 + 35*a^4*b*c*d^4 + 3*a^5*d^5)*x^10)/10 + (7*b^3*d^3*(5*b^4*c^4 + 35*a*b^3
*c^3*d + 63*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^11)/11 + (7*b^4
*d^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2 + 5*a^3*d^3)*x^12)/12 + (
7*b^5*d^5*(3*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x^13)/13 + (b^6*d^6*(b*c + a*
d)*x^14)/2 + (b^7*d^7*x^15)/15
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 757 vs. 2(200) = 400.
time = 7.60, size = 755, normalized size = 3.78

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7*(c + d*x)^7,x]')
```

```
[Out] x (51480 a ^ 7 c ^ 7 + 180180 a ^ 6 c ^ 6 x (a d + b c) + a ^ 5 c ^ 5 x ^ 2
 (360360 a ^ 2 d ^ 2 + 840840 a b c d + 360360 b ^ 2 c ^ 2) + a ^ 4 c ^ 4 x
 ^ 3 (450450 a ^ 3 d ^ 3 + 1891890 a ^ 2 b c d ^ 2 + 1891890 a b ^ 2 c ^ 2
d + 450450 b ^ 3 c ^ 3) + a ^ 3 c ^ 3 x ^ 4 (360360 a ^ 4 d ^ 4 + 2522520 a
 ^ 3 b c d ^ 3 + 4540536 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 2522520 a b ^ 3 c ^ 3 d
+ 360360 b ^ 4 c ^ 4) + a ^ 2 c ^ 2 x ^ 5 (180180 a ^ 5 d ^ 5 + 2102100 a ^
 4 b c d ^ 4 + 6306300 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 6306300 a ^ 2 b ^ 3 c ^ 3
d ^ 2 + 2102100 a b ^ 4 c ^ 4 d + 180180 b ^ 5 c ^ 5) + 51480 a c x ^ 6 (a
^ 6 d ^ 6 + 21 a ^ 5 b c d ^ 5 + 105 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 175 a ^ 3 b
^ 3 c ^ 3 d ^ 3 + 105 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 21 a b ^ 5 c ^ 5 d + b ^ 6
c ^ 6) + x ^ 7 (6435 a ^ 7 d ^ 7 + 315315 a ^ 6 b c d ^ 6 + 2837835 a ^ 5 b
 ^ 2 c ^ 2 d ^ 5 + 7882875 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 7882875 a ^ 3 b ^ 4 c
```

```
^ 4 d ^ 3 + 2837835 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 315315 a b ^ 6 c ^ 6 d + 6435
 b ^ 7 c ^ 7) + 40040 b d x ^ 8 (a ^ 6 d ^ 6 + 21 a ^ 5 b c d ^ 5 + 105 a ^
 4 b ^ 2 c ^ 2 d ^ 4 + 175 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 105 a ^ 2 b ^ 4 c ^ 4
d ^ 2 + 21 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + b ^ 2 d ^ 2 x ^ 9 (108108 a ^ 5
 d ^ 5 + 1261260 a ^ 4 b c d ^ 4 + 3783780 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 378378
0 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 1261260 a b ^ 4 c ^ 4 d + 108108 b ^ 5 c ^ 5) +
 b ^ 3 d ^ 3 x ^ 10 (163800 a ^ 4 d ^ 4 + 1146600 a ^ 3 b c d ^ 3 + 2063880
 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 1146600 a b ^ 3 c ^ 3 d + 163800 b ^ 4 c ^ 4) +
b ^ 4 d ^ 4 x ^ 11 (150150 a ^ 3 d ^ 3 + 630630 a ^ 2 b c d ^ 2 + 630630 a
b ^ 2 c ^ 2 d + 150150 b ^ 3 c ^ 3) + b ^ 5 d ^ 5 x ^ 12 (83160 a ^ 2 d ^ 2
 + 194040 a b c d + 83160 b ^ 2 c ^ 2) + b ^ 6 d ^ 6 x ^ 13 (25740 a d + 25
740 b c) + 3432 b ^ 7 d ^ 7 x ^ 14) / 51480
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(184) = 368$.
time = 0.15, size = 817, normalized size = 4.08

method	result
norman	$a^7c^7x + \left(\frac{7}{2}a^7c^6d + \frac{7}{2}a^6b\,c^7\right)x^2 + \left(7a^7c^5d^2 + \frac{49}{3}a^6b\,c^6d + 7a^5b^2c^7\right)x^3 + \left(\frac{35}{4}a^7c^4d^3 + \frac{147}{4}a^6b\,c^5d^2 + \frac{1}{}\right.$
default	$\frac{b^7d^7x^{15}}{15} + \frac{(7a\,b^6d^7+7b^7c\,d^6)x^{14}}{14} + \frac{(21a^2b^5d^7+49a\,b^6c\,d^6+21b^7c^2d^5)x^{13}}{13} + \frac{(35a^3b^4d^7+147a^2b^5c\,d^6+147a\,b^6c^2d^5+35b^7c^3d^4)x^{1}}{12}$
gosper	$7x^5a^3b^4c^7 + \frac{7}{2}x^6a^7c^2d^5 + \frac{7}{2}x^6a^2b^5c^7 + \frac{7}{9}x^9a^6b\,d^7 + \frac{7}{9}x^9b^7c^6d + \frac{21}{10}x^{10}a^5b^2d^7 + \frac{21}{10}x^{10}b^7c^5d^2 + \frac{35}{11}x^{11}a$
risch	$7x^5a^3b^4c^7 + \frac{7}{2}x^6a^7c^2d^5 + \frac{7}{2}x^6a^2b^5c^7 + \frac{7}{9}x^9a^6b\,d^7 + \frac{7}{9}x^9b^7c^6d + \frac{21}{10}x^{10}a^5b^2d^7 + \frac{21}{10}x^{10}b^7c^5d^2 + \frac{35}{11}x^{11}a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^7*d^7*x^15+1/14*(7*a*b^6*d^7+7*b^7*c*d^6)*x^14+1/13*(21*a^2*b^5*d^7+
49*a*b^6*c*d^6+21*b^7*c^2*d^5)*x^13+1/12*(35*a^3*b^4*d^7+147*a^2*b^5*c*d^6+
147*a*b^6*c^2*d^5+35*b^7*c^3*d^4)*x^12+1/11*(35*a^4*b^3*d^7+245*a^3*b^4*c*d
^6+441*a^2*b^5*c^2*d^5+245*a*b^6*c^3*d^4+35*b^7*c^4*d^3)*x^11+1/10*(21*a^5*
b^2*d^7+245*a^4*b^3*c*d^6+735*a^3*b^4*c^2*d^5+735*a^2*b^5*c^3*d^4+245*a*b^6
*c^4*d^3+21*b^7*c^5*d^2)*x^10+1/9*(7*a^6*b*d^7+147*a^5*b^2*c*d^6+735*a^4*b^
3*c^2*d^5+1225*a^3*b^4*c^3*d^4+735*a^2*b^5*c^4*d^3+147*a*b^6*c^5*d^2+7*b^7*
c^6*d)*x^9+1/8*(a^7*d^7+49*a^6*b*c*d^6+441*a^5*b^2*c^2*d^5+1225*a^4*b^3*c^3
*d^4+1225*a^3*b^4*c^4*d^3+441*a^2*b^5*c^5*d^2+49*a*b^6*c^6*d+b^7*c^7)*x^8+1
/7*(7*a^7*c*d^6+147*a^6*b*c^2*d^5+735*a^5*b^2*c^3*d^4+1225*a^4*b^3*c^4*d^3+
735*a^3*b^4*c^5*d^2+147*a^2*b^5*c^6*d+7*a*b^6*c^7)*x^7+1/6*(21*a^7*c^2*d^5+
245*a^6*b*c^3*d^4+735*a^5*b^2*c^4*d^3+735*a^4*b^3*c^5*d^2+245*a^3*b^4*c^6*d
+21*a^2*b^5*c^7)*x^6+1/5*(35*a^7*c^3*d^4+245*a^6*b*c^4*d^3+441*a^5*b^2*c^5*
d^2+245*a^4*b^3*c^6*d+35*a^3*b^4*c^7)*x^5+1/4*(35*a^7*c^4*d^3+147*a^6*b*c^5
*d^2+147*a^5*b^2*c^6*d+35*a^4*b^3*c^7)*x^4+1/3*(21*a^7*c^5*d^2+49*a^6*b*c^6
*d+21*a^5*b^2*c^7)*x^3+1/2*(7*a^7*c^6*d+7*a^6*b*c^7)*x^2+a^7*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(184) = 368$.
time = 0.27, size = 807, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(184) = 368$.
time = 0.29, size = 807, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
```

```
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(182) = 364.
time = 0.09, size = 935, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**7,x)
```

```
[Out] a**7*c**7*x + b**7*d**7*x**15/15 + x**14*(a*b**6*d**7/2 + b**7*c*d**6/2) +
x**13*(21*a**2*b**5*d**7/13 + 49*a*b**6*c*d**6/13 + 21*b**7*c**2*d**5/13) +
 x**12*(35*a**3*b**4*d**7/12 + 49*a**2*b**5*c*d**6/4 + 49*a*b**6*c**2*d**5/
4 + 35*b**7*c**3*d**4/12) + x**11*(35*a**4*b**3*d**7/11 + 245*a**3*b**4*c*d
**6/11 + 441*a**2*b**5*c**2*d**5/11 + 245*a*b**6*c**3*d**4/11 + 35*b**7*c**
4*d**3/11) + x**10*(21*a**5*b**2*d**7/10 + 49*a**4*b**3*c*d**6/2 + 147*a**3
*b**4*c**2*d**5/2 + 147*a**2*b**5*c**3*d**4/2 + 49*a*b**6*c**4*d**3/2 + 21*
b**7*c**5*d**2/10) + x**9*(7*a**6*b*d**7/9 + 49*a**5*b**2*c*d**6/3 + 245*a*
*4*b**3*c**2*d**5/3 + 1225*a**3*b**4*c**3*d**4/9 + 245*a**2*b**5*c**4*d**3/
3 + 49*a*b**6*c**5*d**2/3 + 7*b**7*c**6*d/9) + x**8*(a**7*d**7/8 + 49*a**6*
b*c*d**6/8 + 441*a**5*b**2*c**2*d**5/8 + 1225*a**4*b**3*c**3*d**4/8 + 1225*
a**3*b**4*c**4*d**3/8 + 441*a**2*b**5*c**5*d**2/8 + 49*a*b**6*c**6*d/8 + b*
*7*c**7/8) + x**7*(a**7*c*d**6 + 21*a**6*b*c**2*d**5 + 105*a**5*b**2*c**3*d
**4 + 175*a**4*b**3*c**4*d**3 + 105*a**3*b**4*c**5*d**2 + 21*a**2*b**5*c**6
*d + a*b**6*c**7) + x**6*(7*a**7*c**2*d**5/2 + 245*a**6*b*c**3*d**4/6 + 245
*a**5*b**2*c**4*d**3/2 + 245*a**4*b**3*c**5*d**2/2 + 245*a**3*b**4*c**6*d/6
 + 7*a**2*b**5*c**7/2) + x**5*(7*a**7*c**3*d**4 + 49*a**6*b*c**4*d**3 + 441
*a**5*b**2*c**5*d**2/5 + 49*a**4*b**3*c**6*d + 7*a**3*b**4*c**7) + x**4*(35
*a**7*c**4*d**3/4 + 147*a**6*b*c**5*d**2/4 + 147*a**5*b**2*c**6*d/4 + 35*a*
*4*b**3*c**7/4) + x**3*(7*a**7*c**5*d**2 + 49*a**6*b*c**6*d/3 + 7*a**5*b**2
*c**7) + x**2*(7*a**7*c**6*d/2 + 7*a**6*b*c**7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(184) = 368.
time = 0.00, size = 1024, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x)
```

```
[Out] 1/15*b^7*d^7*x^15 + 1/2*b^7*c*d^6*x^14 + 1/2*a*b^6*d^7*x^14 + 21/13*b^7*c^2
*d^5*x^13 + 49/13*a*b^6*c*d^6*x^13 + 21/13*a^2*b^5*d^7*x^13 + 35/12*b^7*c^3
*d^4*x^12 + 49/4*a*b^6*c^2*d^5*x^12 + 49/4*a^2*b^5*c*d^6*x^12 + 35/12*a^3*b
^4*d^7*x^12 + 35/11*b^7*c^4*d^3*x^11 + 245/11*a*b^6*c^3*d^4*x^11 + 441/11*a
^2*b^5*c^2*d^5*x^11 + 245/11*a^3*b^4*c*d^6*x^11 + 35/11*a^4*b^3*d^7*x^11 +
21/10*b^7*c^5*d^2*x^10 + 49/2*a*b^6*c^4*d^3*x^10 + 147/2*a^2*b^5*c^3*d^4*x^
10 + 147/2*a^3*b^4*c^2*d^5*x^10 + 49/2*a^4*b^3*c*d^6*x^10 + 21/10*a^5*b^2*d
^7*x^10 + 7/9*b^7*c^6*d*x^9 + 49/3*a*b^6*c^5*d^2*x^9 + 245/3*a^2*b^5*c^4*d^
3*x^9 + 1225/9*a^3*b^4*c^3*d^4*x^9 + 245/3*a^4*b^3*c^2*d^5*x^9 + 49/3*a^5*b
^2*c*d^6*x^9 + 7/9*a^6*b*d^7*x^9 + 1/8*b^7*c^7*x^8 + 49/8*a*b^6*c^6*d*x^8 +
 441/8*a^2*b^5*c^5*d^2*x^8 + 1225/8*a^3*b^4*c^4*d^3*x^8 + 1225/8*a^4*b^3*c^
3*d^4*x^8 + 441/8*a^5*b^2*c^2*d^5*x^8 + 49/8*a^6*b*c*d^6*x^8 + 1/8*a^7*d^7*
x^8 + a*b^6*c^7*x^7 + 21*a^2*b^5*c^6*d*x^7 + 105*a^3*b^4*c^5*d^2*x^7 + 175*
a^4*b^3*c^4*d^3*x^7 + 105*a^5*b^2*c^3*d^4*x^7 + 21*a^6*b*c^2*d^5*x^7 + a^7*
c*d^6*x^7 + 7/2*a^2*b^5*c^7*x^6 + 245/6*a^3*b^4*c^6*d*x^6 + 245/2*a^4*b^3*c
^5*d^2*x^6 + 245/2*a^5*b^2*c^4*d^3*x^6 + 245/6*a^6*b*c^3*d^4*x^6 + 7/2*a^7*
c^2*d^5*x^6 + 7*a^3*b^4*c^7*x^5 + 49*a^4*b^3*c^6*d*x^5 + 441/5*a^5*b^2*c^5*
d^2*x^5 + 49*a^6*b*c^4*d^3*x^5 + 7*a^7*c^3*d^4*x^5 + 35/4*a^4*b^3*c^7*x^4 +
 147/4*a^5*b^2*c^6*d*x^4 + 147/4*a^6*b*c^5*d^2*x^4 + 35/4*a^7*c^4*d^3*x^4 +
 7*a^5*b^2*c^7*x^3 + 49/3*a^6*b*c^6*d*x^3 + 7*a^7*c^5*d^2*x^3 + 7/2*a^6*b*c
^7*x^2 + 7/2*a^7*c^6*d*x^2 + a^7*c^7*x
```

Mupad [B]
time = 0.40, size = 781, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^7,x)
```

```
[Out] x^8*((a^7*d^7)/8 + (b^7*c^7)/8 + (441*a^2*b^5*c^5*d^2)/8 + (1225*a^3*b^4*c^
4*d^3)/8 + (1225*a^4*b^3*c^3*d^4)/8 + (441*a^5*b^2*c^2*d^5)/8 + (49*a*b^6*c
^6*d)/8 + (49*a^6*b*c*d^6)/8) + x^5*(7*a^3*b^4*c^7 + 7*a^7*c^3*d^4 + 49*a^4
*b^3*c^6*d + 49*a^6*b*c^4*d^3 + (441*a^5*b^2*c^5*d^2)/5) + x^11*((35*a^4*b^
3*d^7)/11 + (35*b^7*c^4*d^3)/11 + (245*a*b^6*c^3*d^4)/11 + (245*a^3*b^4*c*d
^6)/11 + (441*a^2*b^5*c^2*d^5)/11) + x^7*(a*b^6*c^7 + a^7*c*d^6 + 21*a^2*b^
5*c^6*d + 21*a^6*b*c^2*d^5 + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 10
5*a^5*b^2*c^3*d^4) + x^9*((7*a^6*b*d^7)/9 + (7*b^7*c^6*d)/9 + (49*a*b^6*c^5
*d^2)/3 + (49*a^5*b^2*c*d^6)/3 + (245*a^2*b^5*c^4*d^3)/3 + (1225*a^3*b^4*c^
3*d^4)/9 + (245*a^4*b^3*c^2*d^5)/3) + x^6*((7*a^2*b^5*c^7)/2 + (7*a^7*c^2*d
^5)/2 + (245*a^3*b^4*c^6*d)/6 + (245*a^6*b*c^3*d^4)/6 + (245*a^4*b^3*c^5*d^
2)/2 + (245*a^5*b^2*c^4*d^3)/2) + x^10*((21*a^5*b^2*d^7)/10 + (21*b^7*c^5*d
^2)/10 + (49*a*b^6*c^4*d^3)/2 + (49*a^4*b^3*c*d^6)/2 + (147*a^2*b^5*c^3*d^4
)/2 + (147*a^3*b^4*c^2*d^5)/2) + a^7*c^7*x + (b^7*d^7*x^15)/15 + (7*a^4*c^4
```

```
*x^4*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/4 + (7*b^4*
d^4*x^12*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/12 + (7
*a^6*c^6*x^2*(a*d + b*c))/2 + (b^6*d^6*x^14*(a*d + b*c))/2 + (7*a^5*c^5*x^3
*(3*a^2*d^2 + 3*b^2*c^2 + 7*a*b*c*d))/3 + (7*b^5*d^5*x^13*(3*a^2*d^2 + 3*b^
2*c^2 + 7*a*b*c*d))/13
```

3.1276 $\int (a+bx)^6(c+dx)^7\,dx$

Optimal. Leaf size=173

$$\frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(bc-ad)^3(c+dx)^{11}}{11d^7}+\frac{5b^4(bc-$$

```
[Out] 1/8*(-a*d+b*c)^6*(d*x+c)^8/d^7-2/3*b*(-a*d+b*c)^5*(d*x+c)^9/d^7+3/2*b^2*(-a
*d+b*c)^4*(d*x+c)^10/d^7-20/11*b^3*(-a*d+b*c)^3*(d*x+c)^11/d^7+5/4*b^4*(-a*
d+b*c)^2*(d*x+c)^12/d^7-6/13*b^5*(-a*d+b*c)*(d*x+c)^13/d^7+1/14*b^6*(d*x+c)
^14/d^7
```

Rubi [A]

time = 0.30, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)^9(bc-ad)^5}{3d^7}+\frac{(c+dx)^8(bc-ad)^6}{8d^7}+\frac{b^6(c+dx)^{14}}{14d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^8)/(8*d^7) - (2*b*(b*c - a*d)^5*(c + d*x)^9)/(3*d^
7) + (3*b^2*(b*c - a*d)^4*(c + d*x)^10)/(2*d^7) - (20*b^3*(b*c - a*d)^3*(c
+ d*x)^11)/(11*d^7) + (5*b^4*(b*c - a*d)^2*(c + d*x)^12)/(4*d^7) - (6*b^5*(
b*c - a*d)*(c + d*x)^13)/(13*d^7) + (b^6*(c + d*x)^14)/(14*d^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^6(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^6(c+dx)^7}{d^6}-\frac{6b(bc-ad)^5(c+dx)^8}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^9}{d^6}-\right.$$

$$=\frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 684 vs. 2(173) = 346.

time = 0.05, size = 684, normalized size = 3.95

$$a^6c^7x+\frac{1}{2}a^5c^6(6bc+7ad)x^2+a^4c^5\left(5b^2c^2+14abcd+7a^2d^2\right)x^3+\frac{1}{4}a^3c^4\left(20b^3c^3+105ab^2c^2d+126a^2bcd^2+35a^3d^3\right)x^4+a^2c^3\left(3b^4c^4+28ab^3c^3d+63a^2b^2c^2d^2+42a^3bcd^3+7a^4d^4\right)x^5+\frac{1}{2}ac^2\left(2b^5c^5+35ab^4c^4d+140a^2b^3c^3d^2+175a^3b^2c^2d^3+70a^4bcd^4+7a^5d^5\right)x^6+\frac{1}{7}c\left(b^6c^6+42ab^5c^5d+315a^2b^4c^4d^2+700a^3b^3c^3d^3+525a^4b^2c^2d^4+126a^5bcd^5+7a^6d^6\right)x^7+\frac{1}{8}d\left(7b^6c^6+126ab^5c^5d+525a^2b^4c^4d^2+700a^3b^3c^3d^3+315a^4b^2c^2d^4+42a^5bcd^5+a^6d^6\right)x^8+\frac{1}{3}bd^2\left(7b^5c^5+70ab^4c^4d+175a^2b^3c^3d^2+140a^3b^2c^2d^3+35a^4bcd^4+2a^5d^5\right)x^9+\frac{1}{2}b^2d^3\left(7b^4c^4+42ab^3c^3d+63a^2b^2c^2d^2+28a^3bcd^3+3a^4d^4\right)x^{10}+\frac{1}{11}b^3d^4\left(35b^3c^3+126ab^2c^2d+105a^2bcd^2+20a^3d^3\right)x^{11}+\frac{1}{4}b^4d^5\left(7b^2c^2+14abcd+5a^2d^2\right)x^{12}+\frac{1}{13}b^5d^6(7bc+6ad)x^{13}+\frac{1}{14}b^6d^7x^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] a^6*c^7*x + (a^5*c^6*(6*b*c + 7*a*d)*x^2)/2 + a^4*c^5*(5*b^2*c^2 + 14*a*b*c
*d + 7*a^2*d^2)*x^3 + (a^3*c^4*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + a^2*c^3*(3*b^4*c^4 + 28*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 42*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (a*c^2*(2*b^5*c^5 + 35*a*b^4*c
^4*d + 140*a^2*b^3*c^3*d^2 + 175*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d
^5)*x^6)/2 + (c*(b^6*c^6 + 42*a*b^5*c^5*d + 315*a^2*b^4*c^4*d^2 + 700*a^3*b
^3*c^3*d^3 + 525*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 7*a^6*d^6)*x^7)/7 + (d
*(7*b^6*c^6 + 126*a*b^5*c^5*d + 525*a^2*b^4*c^4*d^2 + 700*a^3*b^3*c^3*d^3 +
 315*a^4*b^2*c^2*d^4 + 42*a^5*b*c*d^5 + a^6*d^6)*x^8)/8 + (b*d^2*(7*b^5*c^5
 + 70*a*b^4*c^4*d + 175*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 35*a^4*b*c*
d^4 + 2*a^5*d^5)*x^9)/3 + (b^2*d^3*(7*b^4*c^4 + 42*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 28*a^3*b*c*d^3 + 3*a^4*d^4)*x^10)/2 + (b^3*d^4*(35*b^3*c^3 + 126
*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 20*a^3*d^3)*x^11)/11 + (b^4*d^5*(7*b^2*c^2
 + 14*a*b*c*d + 5*a^2*d^2)*x^12)/4 + (b^5*d^6*(7*b*c + 6*a*d)*x^13)/13 + (b
^6*d^7*x^14)/14
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 683 vs. 2(173) = 346.
time = 6.86, size = 661, normalized size = 3.82

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^6*(c + d*x)^7,x]')
```

```
[Out] x (a ^ 6 c ^ 7 + a ^ 5 c ^ 6 x (7 a d + 6 b c) / 2 + a ^ 4 c ^ 5 x ^ 2 (7 a
 ^ 2 d ^ 2 + 14 a b c d + 5 b ^ 2 c ^ 2) + a ^ 3 c ^ 4 x ^ 3 (35 a ^ 3 d ^
3 + 126 a ^ 2 b c d ^ 2 + 105 a b ^ 2 c ^ 2 d + 20 b ^ 3 c ^ 3) / 4 + a ^ 2
 c ^ 3 x ^ 4 (7 a ^ 4 d ^ 4 + 42 a ^ 3 b c d ^ 3 + 63 a ^ 2 b ^ 2 c ^ 2 d ^
 2 + 28 a b ^ 3 c ^ 3 d + 3 b ^ 4 c ^ 4) + a c ^ 2 x ^ 5 (7 a ^ 5 d ^ 5 + 7
0 a ^ 4 b c d ^ 4 + 175 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 140 a ^ 2 b ^ 3 c ^ 3 d ^
 2 + 35 a b ^ 4 c ^ 4 d + 2 b ^ 5 c ^ 5) / 2 + b ^ 3 d ^ 4 x ^ 10 (20 a ^ 3
 d ^ 3 + 105 a ^ 2 b c d ^ 2 + 126 a b ^ 2 c ^ 2 d + 35 b ^ 3 c ^ 3) / 11 +
 c x ^ 6 (7 a ^ 6 d ^ 6 + 126 a ^ 5 b c d ^ 5 + 525 a ^ 4 b ^ 2 c ^ 2 d ^ 4
 + 700 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 315 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 42 a b ^ 5 c
 ^ 5 d + b ^ 6 c ^ 6) / 7 + d x ^ 7 (a ^ 6 d ^ 6 + 42 a ^ 5 b c d ^ 5 + 315
 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 700 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 525 a ^ 2 b ^ 4 c
^ 4 d ^ 2 + 126 a b ^ 5 c ^ 5 d + 7 b ^ 6 c ^ 6) / 8 + b d ^ 2 x ^ 8 (2 a ^
 5 d ^ 5 + 35 a ^ 4 b c d ^ 4 + 140 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 175 a ^ 2 b ^
 3 c ^ 3 d ^ 2 + 70 a b ^ 4 c ^ 4 d + 7 b ^ 5 c ^ 5) / 3 + b ^ 2 d ^ 3 x ^
9 (3 a ^ 4 d ^ 4 + 28 a ^ 3 b c d ^ 3 + 63 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 42 a b
 ^ 3 c ^ 3 d + 7 b ^ 4 c ^ 4) / 2 + b ^ 4 d ^ 5 x ^ 11 (5 a ^ 2 d ^ 2 + 14
a b c d + 7 b ^ 2 c ^ 2) / 4 + b ^ 5 d ^ 6 x ^ 12 (6 a d + 7 b c) / 13 + b
^ 6 d ^ 7 x ^ 13 / 14)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(159) = 318$.
time = 0.14, size = 709, normalized size = 4.10

method	result
norman	$a^6c^7x + \left(\frac{7}{2}a^6c^6d + 3a^5b\,c^7\right)x^2 + \left(7a^6c^5d^2 + 14a^5b\,c^6d + 5a^4b^2c^7\right)x^3 + \left(\frac{35}{4}a^6c^4d^3 + \frac{63}{2}a^5b\,c^5d^2 + \right.$
default	$\frac{b^6d^7x^{14}}{14} + \frac{(6a\,b^5d^7+7b^6c\,d^6)x^{13}}{13} + \frac{(15a^2b^4d^7+42a\,b^5c\,d^6+21b^6c^2d^5)x^{12}}{12} + \frac{(20a^3b^3d^7+105a^2b^4c\,d^6+126a\,b^5c^2d^5+35b^6c^3d^4)}{11}$
gosper	$18x^7a^5b\,c^2d^5 + \frac{5}{4}x^{12}a^2b^4d^7 + 7a^6c^5d^2x^3 + 5a^4b^2c^7x^3 + 7a^6c^3d^4x^5 + 3a^2b^4c^7x^5 + \frac{7}{4}x^{12}b^6c^2d^5 + \frac{6}{13}$
risch	$18x^7a^5b\,c^2d^5 + \frac{5}{4}x^{12}a^2b^4d^7 + 7a^6c^5d^2x^3 + 5a^4b^2c^7x^3 + 7a^6c^3d^4x^5 + 3a^2b^4c^7x^5 + \frac{7}{4}x^{12}b^6c^2d^5 + \frac{6}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^6*d^7*x^14+1/13*(6*a*b^5*d^7+7*b^6*c*d^6)*x^13+1/12*(15*a^2*b^4*d^7+
42*a*b^5*c*d^6+21*b^6*c^2*d^5)*x^12+1/11*(20*a^3*b^3*d^7+105*a^2*b^4*c*d^6+
126*a*b^5*c^2*d^5+35*b^6*c^3*d^4)*x^11+1/10*(15*a^4*b^2*d^7+140*a^3*b^3*c*d
^6+315*a^2*b^4*c^2*d^5+210*a*b^5*c^3*d^4+35*b^6*c^4*d^3)*x^10+1/9*(6*a^5*b*
d^7+105*a^4*b^2*c*d^6+420*a^3*b^3*c^2*d^5+525*a^2*b^4*c^3*d^4+210*a*b^5*c^4
*d^3+21*b^6*c^5*d^2)*x^9+1/8*(a^6*d^7+42*a^5*b*c*d^6+315*a^4*b^2*c^2*d^5+70
0*a^3*b^3*c^3*d^4+525*a^2*b^4*c^4*d^3+126*a*b^5*c^5*d^2+7*b^6*c^6*d)*x^8+1/
7*(7*a^6*c*d^6+126*a^5*b*c^2*d^5+525*a^4*b^2*c^3*d^4+700*a^3*b^3*c^4*d^3+31
5*a^2*b^4*c^5*d^2+42*a*b^5*c^6*d+b^6*c^7)*x^7+1/6*(21*a^6*c^2*d^5+210*a^5*b
*c^3*d^4+525*a^4*b^2*c^4*d^3+420*a^3*b^3*c^5*d^2+105*a^2*b^4*c^6*d+6*a*b^5*
c^7)*x^6+1/5*(35*a^6*c^3*d^4+210*a^5*b*c^4*d^3+315*a^4*b^2*c^5*d^2+140*a^3*
b^3*c^6*d+15*a^2*b^4*c^7)*x^5+1/4*(35*a^6*c^4*d^3+126*a^5*b*c^5*d^2+105*a^4
*b^2*c^6*d+20*a^3*b^3*c^7)*x^4+1/3*(21*a^6*c^5*d^2+42*a^5*b*c^6*d+15*a^4*b^
2*c^7)*x^3+1/2*(7*a^6*c^6*d+6*a^5*b*c^7)*x^2+a^6*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(159) = 318$.
time = 0.28, size = 706, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
```

```
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(159) = 318$.
time = 0.29, size = 706, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(158) = 316$.
time = 0.09, size = 796, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**7,x)
```

```
[Out] a**6*c**7*x + b**6*d**7*x**14/14 + x**13*(6*a*b**5*d**7/13 + 7*b**6*c*d**6/
13) + x**12*(5*a**2*b**4*d**7/4 + 7*a*b**5*c*d**6/2 + 7*b**6*c**2*d**5/4) +
```

```
 x**11*(20*a**3*b**3*d**7/11 + 105*a**2*b**4*c*d**6/11 + 126*a*b**5*c**2*d*
*5/11 + 35*b**6*c**3*d**4/11) + x**10*(3*a**4*b**2*d**7/2 + 14*a**3*b**3*c*
d**6 + 63*a**2*b**4*c**2*d**5/2 + 21*a*b**5*c**3*d**4 + 7*b**6*c**4*d**3/2)
 + x**9*(2*a**5*b*d**7/3 + 35*a**4*b**2*c*d**6/3 + 140*a**3*b**3*c**2*d**5/
3 + 175*a**2*b**4*c**3*d**4/3 + 70*a*b**5*c**4*d**3/3 + 7*b**6*c**5*d**2/3)
 + x**8*(a**6*d**7/8 + 21*a**5*b*c*d**6/4 + 315*a**4*b**2*c**2*d**5/8 + 175
*a**3*b**3*c**3*d**4/2 + 525*a**2*b**4*c**4*d**3/8 + 63*a*b**5*c**5*d**2/4
+ 7*b**6*c**6*d/8) + x**7*(a**6*c*d**6 + 18*a**5*b*c**2*d**5 + 75*a**4*b**2
*c**3*d**4 + 100*a**3*b**3*c**4*d**3 + 45*a**2*b**4*c**5*d**2 + 6*a*b**5*c*
*6*d + b**6*c**7/7) + x**6*(7*a**6*c**2*d**5/2 + 35*a**5*b*c**3*d**4 + 175*
a**4*b**2*c**4*d**3/2 + 70*a**3*b**3*c**5*d**2 + 35*a**2*b**4*c**6*d/2 + a*
b**5*c**7) + x**5*(7*a**6*c**3*d**4 + 42*a**5*b*c**4*d**3 + 63*a**4*b**2*c*
*5*d**2 + 28*a**3*b**3*c**6*d + 3*a**2*b**4*c**7) + x**4*(35*a**6*c**4*d**3
/4 + 63*a**5*b*c**5*d**2/2 + 105*a**4*b**2*c**6*d/4 + 5*a**3*b**3*c**7) + x
**3*(7*a**6*c**5*d**2 + 14*a**5*b*c**6*d + 5*a**4*b**2*c**7) + x**2*(7*a**6
*c**6*d/2 + 3*a**5*b*c**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(159) = 318.
time = 0.00, size = 866, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x)
```

```
[Out] 1/14*b^6*d^7*x^14 + 7/13*b^6*c*d^6*x^13 + 6/13*a*b^5*d^7*x^13 + 7/4*b^6*c^2
*d^5*x^12 + 7/2*a*b^5*c*d^6*x^12 + 5/4*a^2*b^4*d^7*x^12 + 35/11*b^6*c^3*d^4
*x^11 + 126/11*a*b^5*c^2*d^5*x^11 + 105/11*a^2*b^4*c*d^6*x^11 + 20/11*a^3*b
^3*d^7*x^11 + 7/2*b^6*c^4*d^3*x^10 + 21*a*b^5*c^3*d^4*x^10 + 63/2*a^2*b^4*c
^2*d^5*x^10 + 14*a^3*b^3*c*d^6*x^10 + 3/2*a^4*b^2*d^7*x^10 + 7/3*b^6*c^5*d^
2*x^9 + 70/3*a*b^5*c^4*d^3*x^9 + 175/3*a^2*b^4*c^3*d^4*x^9 + 140/3*a^3*b^3*
c^2*d^5*x^9 + 35/3*a^4*b^2*c*d^6*x^9 + 2/3*a^5*b*d^7*x^9 + 7/8*b^6*c^6*d*x^
8 + 63/4*a*b^5*c^5*d^2*x^8 + 525/8*a^2*b^4*c^4*d^3*x^8 + 175/2*a^3*b^3*c^3*
d^4*x^8 + 315/8*a^4*b^2*c^2*d^5*x^8 + 21/4*a^5*b*c*d^6*x^8 + 1/8*a^6*d^7*x^
8 + 1/7*b^6*c^7*x^7 + 6*a*b^5*c^6*d*x^7 + 45*a^2*b^4*c^5*d^2*x^7 + 100*a^3*
b^3*c^4*d^3*x^7 + 75*a^4*b^2*c^3*d^4*x^7 + 18*a^5*b*c^2*d^5*x^7 + a^6*c*d^6
*x^7 + a*b^5*c^7*x^6 + 35/2*a^2*b^4*c^6*d*x^6 + 70*a^3*b^3*c^5*d^2*x^6 + 17
5/2*a^4*b^2*c^4*d^3*x^6 + 35*a^5*b*c^3*d^4*x^6 + 7/2*a^6*c^2*d^5*x^6 + 3*a^
2*b^4*c^7*x^5 + 28*a^3*b^3*c^6*d*x^5 + 63*a^4*b^2*c^5*d^2*x^5 + 42*a^5*b*c^
4*d^3*x^5 + 7*a^6*c^3*d^4*x^5 + 5*a^3*b^3*c^7*x^4 + 105/4*a^4*b^2*c^6*d*x^4
 + 63/2*a^5*b*c^5*d^2*x^4 + 35/4*a^6*c^4*d^3*x^4 + 5*a^4*b^2*c^7*x^3 + 14*a
^5*b*c^6*d*x^3 + 7*a^6*c^5*d^2*x^3 + 3*a^5*b*c^7*x^2 + 7/2*a^6*c^6*d*x^2 +
a^6*c^7*x
```

Mupad [B]

time = 0.26, size = 683, normalized size = 3.95

$$x^{5} \left(7 a^{6} c^{3} d^{4}+42 a^{5} b c^{4} d^{3}+63 a^{4} b^{2} c^{5} d^{2}+28 a^{3} b^{3} c^{6} d+3 a^{2} b^{4} c^{7}\right)+x^{10} \left(\frac{3 a^{4} b^{2} d^{7}}{2}+14 a^{3} b^{3} c d^{6}+\frac{63 a^{2} b^{4} c^{2} d^{5}}{2}+21 a b^{5} c^{3} d^{4}+\frac{7 b^{6} c^{4} d^{3}}{2}\right)+x^{6} \left(\frac{7 a^{6} c^{2} d^{5}}{2}+35 a^{5} b c^{3} d^{4}+\frac{175 a^{4} b^{2} c^{4} d^{3}}{2}+70 a^{3} b^{3} c^{5} d^{2}+\frac{35 a^{2} b^{4} c^{6} d}{2}+a b^{5} c^{7}\right)+x^{9} \left(\frac{2 a^{5} b d^{7}}{3}+\frac{35 a^{4} b^{2} c d^{6}}{3}+\frac{140 a^{3} b^{3} c^{2} d^{5}}{3}+\frac{175 a^{2} b^{4} c^{3} d^{4}}{3}+\frac{70 a b^{5} c^{4} d^{3}}{3}+\frac{7 b^{6} c^{5} d^{2}}{3}\right)+x^{7} \left(a^{6} c d^{6}+18 a^{5} b c^{2} d^{5}+75 a^{4} b^{2} c^{3} d^{4}+100 a^{3} b^{3} c^{4} d^{3}+45 a^{2} b^{4} c^{5} d^{2}+6 a b^{5} c^{6} d+\frac{b^{6} c^{7}}{7}\right)+x^{8} \left(\frac{a^{6} d^{7}}{8}+\frac{21 a^{5} b c d^{6}}{4}+\frac{315 a^{4} b^{2} c^{2} d^{5}}{8}+\frac{175 a^{3} b^{3} c^{3} d^{4}}{2}+\frac{525 a^{2} b^{4} c^{4} d^{3}}{8}+\frac{63 a b^{5} c^{5} d^{2}}{4}+\frac{7 b^{6} c^{6} d}{8}\right)+x^{4} \left(\frac{35 a^{6} c^{4} d^{3}}{4}+\frac{63 a^{5} b c^{5} d^{2}}{2}+\frac{105 a^{4} b^{2} c^{6} d}{4}+5 a^{3} b^{3} c^{7}\right)+x^{11} \left(\frac{20 a^{3} b^{3} d^{7}}{11}+\frac{105 a^{2} b^{4} c d^{6}}{11}+\frac{126 a b^{5} c^{2} d^{5}}{11}+\frac{35 b^{6} c^{3} d^{4}}{11}\right)+a^{6} c^{7} x+\frac{b^{6} d^{7} x^{14}}{14}+\frac{a^{5} c^{6} x^{2} \left(7 a d+6 b c\right)}{2}+\frac{b^{5} d^{6} x^{13} \left(6 a d+7 b c\right)}{13}+a^{4} c^{5} x^{3} \left(7 a^{2} d^{2}+14 a b c d+5 b^{2} c^{2}\right)+\frac{b^{4} d^{5} x^{12} \left(5 a^{2} d^{2}+14 a b c d+7 b^{2} c^{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^7,x)
```

```
[Out] x^5*(3*a^2*b^4*c^7 + 7*a^6*c^3*d^4 + 28*a^3*b^3*c^6*d + 42*a^5*b*c^4*d^3 +
63*a^4*b^2*c^5*d^2) + x^10*((3*a^4*b^2*d^7)/2 + (7*b^6*c^4*d^3)/2 + 21*a*b^
5*c^3*d^4 + 14*a^3*b^3*c*d^6 + (63*a^2*b^4*c^2*d^5)/2) + x^6*(a*b^5*c^7 + (
7*a^6*c^2*d^5)/2 + (35*a^2*b^4*c^6*d)/2 + 35*a^5*b*c^3*d^4 + 70*a^3*b^3*c^5
*d^2 + (175*a^4*b^2*c^4*d^3)/2) + x^9*((2*a^5*b*d^7)/3 + (7*b^6*c^5*d^2)/3
+ (70*a*b^5*c^4*d^3)/3 + (35*a^4*b^2*c*d^6)/3 + (175*a^2*b^4*c^3*d^4)/3 + (
140*a^3*b^3*c^2*d^5)/3) + x^7*((b^6*c^7)/7 + a^6*c*d^6 + 18*a^5*b*c^2*d^5 +
 45*a^2*b^4*c^5*d^2 + 100*a^3*b^3*c^4*d^3 + 75*a^4*b^2*c^3*d^4 + 6*a*b^5*c^
6*d) + x^8*((a^6*d^7)/8 + (7*b^6*c^6*d)/8 + (63*a*b^5*c^5*d^2)/4 + (525*a^2
*b^4*c^4*d^3)/8 + (175*a^3*b^3*c^3*d^4)/2 + (315*a^4*b^2*c^2*d^5)/8 + (21*a
^5*b*c*d^6)/4) + x^4*(5*a^3*b^3*c^7 + (35*a^6*c^4*d^3)/4 + (105*a^4*b^2*c^6
*d)/4 + (63*a^5*b*c^5*d^2)/2) + x^11*((20*a^3*b^3*d^7)/11 + (35*b^6*c^3*d^4
)/11 + (126*a*b^5*c^2*d^5)/11 + (105*a^2*b^4*c*d^6)/11) + a^6*c^7*x + (b^6*
d^7*x^14)/14 + (a^5*c^6*x^2*(7*a*d + 6*b*c))/2 + (b^5*d^6*x^13*(6*a*d + 7*b
*c))/13 + a^4*c^5*x^3*(7*a^2*d^2 + 14*a*b*c*d) + (b^4*d^5*x^12*
(5*a^2*d^2 + 7*b^2*c^2 + 14*a*b*c*d))/4
```

3.1277 $\int (a+bx)^5(c+dx)^7\,dx$

Optimal. Leaf size=144

$$-\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3(bc-ad)^2(c+dx)^{11}}{11d^6}-\frac{5b^4(bc-$$

```
[Out] -1/8*(-a*d+b*c)^5*(d*x+c)^8/d^6+5/9*b*(-a*d+b*c)^4*(d*x+c)^9/d^6-b^2*(-a*d+
b*c)^3*(d*x+c)^10/d^6+10/11*b^3*(-a*d+b*c)^2*(d*x+c)^11/d^6-5/12*b^4*(-a*d+
b*c)*(d*x+c)^12/d^6+1/13*b^5*(d*x+c)^13/d^6
```

Rubi [A]
time = 0.25, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-ad)^5}{8d^6}+\frac{b^5(c+dx)^{13}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] -1/8*((b*c - a*d)^5*(c + d*x)^8)/d^6 + (5*b*(b*c - a*d)^4*(c + d*x)^9)/(9*d
^6) - (b^2*(b*c - a*d)^3*(c + d*x)^10)/d^6 + (10*b^3*(b*c - a*d)^2*(c + d*x
)^11)/(11*d^6) - (5*b^4*(b*c - a*d)*(c + d*x)^12)/(12*d^6) + (b^5*(c + d*x)
^13)/(13*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^5(c+dx)^7}{d^5}+\frac{5b(bc-ad)^4(c+dx)^8}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^9}{d^5}+\right.$$

$$= -\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 574 vs. 2(144) = 288.
time = 0.04, size = 574, normalized size = 3.99

$$a^5c^7x+\frac{1}{2}a^4c^6(5bc+7ad)x^2+\frac{1}{3}a^3c^5\left(10b^2c^2+35abcd+21a^2d^2\right)x^3+\frac{5}{4}a^2c^4\left(2b^3c^3+14ab^2c^2d+21a^2bcd^2+7a^3d^3\right)x^4+ac^3\left(b^4c^4+14ab^3c^3d+42a^2b^2c^2d^2+35a^3bcd^3+7a^4d^4\right)x^5+\frac{1}{6}c^2\left(b^5c^5+35ab^4c^4d+210a^2b^3c^3d^2+350a^3b^2c^2d^3+175a^4bcd^4+21a^5d^5\right)x^6+cd\left(b^5c^5+15ab^4c^4d+50a^2b^3c^3d^2+50a^3b^2c^2d^3+15a^4bcd^4+a^5d^5\right)x^7+\frac{1}{8}d^2\left(21b^5c^5+175ab^4c^4d+350a^2b^3c^3d^2+210a^3b^2c^2d^3+35a^4bcd^4+a^5d^5\right)x^8+\frac{5}{9}bd^3\left(7b^4c^4+35ab^3c^3d+42a^2b^2c^2d^2+14a^3bcd^3+a^4d^4\right)x^9+\frac{1}{2}b^2d^4\left(7b^3c^3+21ab^2c^2d+14a^2bcd^2+2a^3d^3\right)x^{10}+\frac{1}{11}b^3d^5\left(21b^2c^2+35abcd+10a^2d^2\right)x^{11}+\frac{1}{12}b^4d^6(7bc+5ad)x^{12}+\frac{1}{13}b^5d^7x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] a^5*c^7*x + (a^4*c^6*(5*b*c + 7*a*d)*x^2)/2 + (a^3*c^5*(10*b^2*c^2 + 35*a*b
*c*d + 21*a^2*d^2)*x^3)/3 + (5*a^2*c^4*(2*b^3*c^3 + 14*a*b^2*c^2*d + 21*a^2
*b*c*d^2 + 7*a^3*d^3)*x^4)/4 + a*c^3*(b^4*c^4 + 14*a*b^3*c^3*d + 42*a^2*b^2
*c^2*d^2 + 35*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (c^2*(b^5*c^5 + 35*a*b^4*c^4*d
 + 210*a^2*b^3*c^3*d^2 + 350*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5
)*x^6)/6 + c*d*(b^5*c^5 + 15*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 + 50*a^3*b^2*
c^2*d^3 + 15*a^4*b*c*d^4 + a^5*d^5)*x^7 + (d^2*(21*b^5*c^5 + 175*a*b^4*c^4*
d + 350*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 + a^5*d^5)*x
^8)/8 + (5*b*d^3*(7*b^4*c^4 + 35*a*b^3*c^3*d + 42*a^2*b^2*c^2*d^2 + 14*a^3*
b*c*d^3 + a^4*d^4)*x^9)/9 + (b^2*d^4*(7*b^3*c^3 + 21*a*b^2*c^2*d + 14*a^2*b
*c*d^2 + 2*a^3*d^3)*x^10)/2 + (b^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^
2)*x^11)/11 + (b^4*d^6*(7*b*c + 5*a*d)*x^12)/12 + (b^5*d^7*x^13)/13
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 573 vs. 2(144) = 288.
time = 5.98, size = 553, normalized size = 3.84

$$x\left(a^5c^7+\frac{a^4c^6x\,(7ad+5bc)}{2}+\frac{a^3c^5x^2\,(21a^2d^2+35abcd+10b^2c^2)}{3}+\frac{5a^2c^4x^3\,(7a^3d^3+21a^2bcd^2+14ab^2c^2d+2b^3c^3)}{4}+ac^3x^4\,(7a^4d^4+35a^3bcd^3+42a^2b^2c^2d^2+14ab^3c^3d+b^4c^4)+\frac{5bd^3x^8\,(a^4d^4+14a^3bcd^3+42a^2b^2c^2d^2+35ab^3c^3d+7b^4c^4)}{9}+\frac{b^2d^4x^9\,(2a^3d^3+14a^2bcd^2+21ab^2c^2d+7b^3c^3)}{2}+\frac{b^4d^6x^{11}\,(5ad+7bc)}{12}+\frac{c^2x^5\,(21a^5d^5+175a^4bcd^4+350a^3b^2c^2d^3+210a^2b^3c^3d^2+35ab^4c^4d+b^5c^5)}{6}+\frac{d^2x^7\,(a^5d^5+35a^4bcd^4+210a^3b^2c^2d^3+350a^2b^3c^3d^2+175ab^4c^4d+21b^5c^5)}{8}+\frac{b^3d^5x^{10}\,(10a^2d^2+35abcd+21b^2c^2)}{11}+\frac{b^5d^7x^{12}}{13}+cdx^6\,(a^5d^5+15a^4bcd^4+50a^3b^2c^2d^3+50a^2b^3c^3d^2+15ab^4c^4d+b^5c^5)\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(c + d*x)^7,x]')
```

```
[Out] x (a ^ 5 c ^ 7 + a ^ 4 c ^ 6 x (7 a d + 5 b c) / 2 + a ^ 3 c ^ 5 x ^ 2 (21
a ^ 2 d ^ 2 + 35 a b c d + 10 b ^ 2 c ^ 2) / 3 + 5 a ^ 2 c ^ 4 x ^ 3 (7 a ^
 3 d ^ 3 + 21 a ^ 2 b c d ^ 2 + 14 a b ^ 2 c ^ 2 d + 2 b ^ 3 c ^ 3) / 4 + a
 c ^ 3 x ^ 4 (7 a ^ 4 d ^ 4 + 35 a ^ 3 b c d ^ 3 + 42 a ^ 2 b ^ 2 c ^ 2 d ^
 2 + 14 a b ^ 3 c ^ 3 d + b ^ 4 c ^ 4) + 5 b d ^ 3 x ^ 8 (a ^ 4 d ^ 4 + 14
a ^ 3 b c d ^ 3 + 42 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 35 a b ^ 3 c ^ 3 d + 7 b ^ 4
 c ^ 4) / 9 + b ^ 2 d ^ 4 x ^ 9 (2 a ^ 3 d ^ 3 + 14 a ^ 2 b c d ^ 2 + 21 a
b ^ 2 c ^ 2 d + 7 b ^ 3 c ^ 3) / 2 + b ^ 4 d ^ 6 x ^ 11 (5 a d + 7 b c) / 1
2 + c ^ 2 x ^ 5 (21 a ^ 5 d ^ 5 + 175 a ^ 4 b c d ^ 4 + 350 a ^ 3 b ^ 2 c ^
 2 d ^ 3 + 210 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 35 a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5)
/ 6 + d ^ 2 x ^ 7 (a ^ 5 d ^ 5 + 35 a ^ 4 b c d ^ 4 + 210 a ^ 3 b ^ 2 c ^ 2
 d ^ 3 + 350 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 175 a b ^ 4 c ^ 4 d + 21 b ^ 5 c ^ 5
) / 8 + b ^ 3 d ^ 5 x ^ 10 (10 a ^ 2 d ^ 2 + 35 a b c d + 21 b ^ 2 c ^ 2) /
 11 + b ^ 5 d ^ 7 x ^ 12 / 13 + c d x ^ 6 (a ^ 5 d ^ 5 + 15 a ^ 4 b c d ^ 4
 + 50 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 50 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 15 a b ^ 4 c ^
 4 d + b ^ 5 c ^ 5))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(134) = 268.
time = 0.14, size = 601, normalized size = 4.17

method	result
norman	$\frac{b^5d^7x^{13}}{13}+\left(\frac{5}{12}a\,b^4d^7+\frac{7}{12}b^5c\,d^6\right)x^{12}+\left(\frac{10}{11}a^2b^3d^7+\frac{35}{11}a\,b^4c\,d^6+\frac{21}{11}b^5c^2d^5\right)x^{11}+\left(a^3b^2d^7+7a^2b^3c\,d^6\right.$
default	$\frac{b^5d^7x^{13}}{13}+\frac{(5a\,b^4d^7+7b^5c\,d^6)x^{12}}{12}+\frac{(10a^2b^3d^7+35a\,b^4c\,d^6+21b^5c^2d^5)x^{11}}{11}+\frac{(10a^3b^2d^7+70a^2b^3c\,d^6+105a\,b^4c^2d^5+35b^5c^3d^4)x}{10}$
gosper	$\frac{10}{11}x^{11}a^2b^3d^7+\frac{21}{11}x^{11}b^5c^2d^5+x^{10}a^3b^2d^7+\frac{7}{2}x^{10}b^5c^3d^4+\frac{5}{9}x^9a^4b\,d^7+b^5c^6d\,x^7+7a^5c^3d^4x^5+a\,b^4c^7$
risch	$\frac{10}{11}x^{11}a^2b^3d^7+\frac{21}{11}x^{11}b^5c^2d^5+x^{10}a^3b^2d^7+\frac{7}{2}x^{10}b^5c^3d^4+\frac{5}{9}x^9a^4b\,d^7+b^5c^6d\,x^7+7a^5c^3d^4x^5+a\,b^4c^7$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^5*d^7*x^13+1/12*(5*a*b^4*d^7+7*b^5*c*d^6)*x^12+1/11*(10*a^2*b^3*d^7+
35*a*b^4*c*d^6+21*b^5*c^2*d^5)*x^11+1/10*(10*a^3*b^2*d^7+70*a^2*b^3*c*d^6+1
05*a*b^4*c^2*d^5+35*b^5*c^3*d^4)*x^10+1/9*(5*a^4*b*d^7+70*a^3*b^2*c*d^6+210
*a^2*b^3*c^2*d^5+175*a*b^4*c^3*d^4+35*b^5*c^4*d^3)*x^9+1/8*(a^5*d^7+35*a^4*
b*c*d^6+210*a^3*b^2*c^2*d^5+350*a^2*b^3*c^3*d^4+175*a*b^4*c^4*d^3+21*b^5*c^
5*d^2)*x^8+1/7*(7*a^5*c*d^6+105*a^4*b*c^2*d^5+350*a^3*b^2*c^3*d^4+350*a^2*b
^3*c^4*d^3+105*a*b^4*c^5*d^2+7*b^5*c^6*d)*x^7+1/6*(21*a^5*c^2*d^5+175*a^4*b
*c^3*d^4+350*a^3*b^2*c^4*d^3+210*a^2*b^3*c^5*d^2+35*a*b^4*c^6*d+b^5*c^7)*x^
6+1/5*(35*a^5*c^3*d^4+175*a^4*b*c^4*d^3+210*a^3*b^2*c^5*d^2+70*a^2*b^3*c^6*
d+5*a*b^4*c^7)*x^5+1/4*(35*a^5*c^4*d^3+105*a^4*b*c^5*d^2+70*a^3*b^2*c^6*d+1
0*a^2*b^3*c^7)*x^4+1/3*(21*a^5*c^5*d^2+35*a^4*b*c^6*d+10*a^3*b^2*c^7)*x^3+1
/2*(7*a^5*c^6*d+5*a^4*b*c^7)*x^2+a^5*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(134) = 268$.
time = 0.28, size = 594, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
```

```
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(134) = 268$.
time = 0.29, size = 594, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(129) = 258$.
time = 0.08, size = 673, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**7,x)
```

```
[Out] a**5*c**7*x + b**5*d**7*x**13/13 + x**12*(5*a*b**4*d**7/12 + 7*b**5*c*d**6/
12) + x**11*(10*a**2*b**3*d**7/11 + 35*a*b**4*c*d**6/11 + 21*b**5*c**2*d**5
/11) + x**10*(a**3*b**2*d**7 + 7*a**2*b**3*c*d**6 + 21*a*b**4*c**2*d**5/2 +
 7*b**5*c**3*d**4/2) + x**9*(5*a**4*b*d**7/9 + 70*a**3*b**2*c*d**6/9 + 70*a
**2*b**3*c**2*d**5/3 + 175*a*b**4*c**3*d**4/9 + 35*b**5*c**4*d**3/9) + x**8
*(a**5*d**7/8 + 35*a**4*b*c*d**6/8 + 105*a**3*b**2*c**2*d**5/4 + 175*a**2*b
**3*c**3*d**4/4 + 175*a*b**4*c**4*d**3/8 + 21*b**5*c**5*d**2/8) + x**7*(a**
5*c*d**6 + 15*a**4*b*c**2*d**5 + 50*a**3*b**2*c**3*d**4 + 50*a**2*b**3*c**4
*d**3 + 15*a*b**4*c**5*d**2 + b**5*c**6*d) + x**6*(7*a**5*c**2*d**5/2 + 175
*a**4*b*c**3*d**4/6 + 175*a**3*b**2*c**4*d**3/3 + 35*a**2*b**3*c**5*d**2 +
35*a*b**4*c**6*d/6 + b**5*c**7/6) + x**5*(7*a**5*c**3*d**4 + 35*a**4*b*c**4
```

```
*d**3 + 42*a**3*b**2*c**5*d**2 + 14*a**2*b**3*c**6*d + a*b**4*c**7) + x**4*
(35*a**5*c**4*d**3/4 + 105*a**4*b*c**5*d**2/4 + 35*a**3*b**2*c**6*d/2 + 5*a
**2*b**3*c**7/2) + x**3*(7*a**5*c**5*d**2 + 35*a**4*b*c**6*d/3 + 10*a**3*b*
*2*c**7/3) + x**2*(7*a**5*c**6*d/2 + 5*a**4*b*c**7/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(134) = 268.
time = 0.00, size = 734, normalized size = 5.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x)
```

```
[Out] 1/13*b^5*d^7*x^13 + 7/12*b^5*c*d^6*x^12 + 5/12*a*b^4*d^7*x^12 + 21/11*b^5*c
^2*d^5*x^11 + 35/11*a*b^4*c*d^6*x^11 + 10/11*a^2*b^3*d^7*x^11 + 7/2*b^5*c^3
*d^4*x^10 + 21/2*a*b^4*c^2*d^5*x^10 + 7*a^2*b^3*c*d^6*x^10 + a^3*b^2*d^7*x^
10 + 35/9*b^5*c^4*d^3*x^9 + 175/9*a*b^4*c^3*d^4*x^9 + 70/3*a^2*b^3*c^2*d^5*
x^9 + 70/9*a^3*b^2*c*d^6*x^9 + 5/9*a^4*b*d^7*x^9 + 21/8*b^5*c^5*d^2*x^8 + 1
75/8*a*b^4*c^4*d^3*x^8 + 175/4*a^2*b^3*c^3*d^4*x^8 + 105/4*a^3*b^2*c^2*d^5*
x^8 + 35/8*a^4*b*c*d^6*x^8 + 1/8*a^5*d^7*x^8 + b^5*c^6*d*x^7 + 15*a*b^4*c^5
*d^2*x^7 + 50*a^2*b^3*c^4*d^3*x^7 + 50*a^3*b^2*c^3*d^4*x^7 + 15*a^4*b*c^2*d
^5*x^7 + a^5*c*d^6*x^7 + 1/6*b^5*c^7*x^6 + 35/6*a*b^4*c^6*d*x^6 + 35*a^2*b^
3*c^5*d^2*x^6 + 175/3*a^3*b^2*c^4*d^3*x^6 + 175/6*a^4*b*c^3*d^4*x^6 + 7/2*a
^5*c^2*d^5*x^6 + a*b^4*c^7*x^5 + 14*a^2*b^3*c^6*d*x^5 + 42*a^3*b^2*c^5*d^2*
x^5 + 35*a^4*b*c^4*d^3*x^5 + 7*a^5*c^3*d^4*x^5 + 5/2*a^2*b^3*c^7*x^4 + 35/2
*a^3*b^2*c^6*d*x^4 + 105/4*a^4*b*c^5*d^2*x^4 + 35/4*a^5*c^4*d^3*x^4 + 10/3*
a^3*b^2*c^7*x^3 + 35/3*a^4*b*c^6*d*x^3 + 7*a^5*c^5*d^2*x^3 + 5/2*a^4*b*c^7*
x^2 + 7/2*a^5*c^6*d*x^2 + a^5*c^7*x
```

Mupad [**B**]
time = 0.21, size = 570, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^7,x)
```

```
[Out] x^7*(a^5*c*d^6 + b^5*c^6*d + 15*a*b^4*c^5*d^2 + 15*a^4*b*c^2*d^5 + 50*a^2*b
^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4) + x^6*((b^5*c^7)/6 + (7*a^5*c^2*d^5)/2 + (
175*a^4*b*c^3*d^4)/6 + 35*a^2*b^3*c^5*d^2 + (175*a^3*b^2*c^4*d^3)/3 + (35*a
*b^4*c^6*d)/6) + x^8*((a^5*d^7)/8 + (21*b^5*c^5*d^2)/8 + (175*a*b^4*c^4*d^3
)/8 + (175*a^2*b^3*c^3*d^4)/4 + (105*a^3*b^2*c^2*d^5)/4 + (35*a^4*b*c*d^6)/
8) + x^5*(a*b^4*c^7 + 7*a^5*c^3*d^4 + 14*a^2*b^3*c^6*d + 35*a^4*b*c^4*d^3 +
 42*a^3*b^2*c^5*d^2) + x^9*((5*a^4*b*d^7)/9 + (35*b^5*c^4*d^3)/9 + (175*a*b
^4*c^3*d^4)/9 + (70*a^3*b^2*c*d^6)/9 + (70*a^2*b^3*c^2*d^5)/3) + a^5*c^7*x
```

```
+ (b^5*d^7*x^13)/13 + (5*a^2*c^4*x^4*(7*a^3*d^3 + 2*b^3*c^3 + 14*a*b^2*c^2*
d + 21*a^2*b*c*d^2))/4 + (b^2*d^4*x^10*(2*a^3*d^3 + 7*b^3*c^3 + 21*a*b^2*c^
2*d + 14*a^2*b*c*d^2))/2 + (a^4*c^6*x^2*(7*a*d + 5*b*c))/2 + (b^4*d^6*x^12*
(5*a*d + 7*b*c))/12 + (a^3*c^5*x^3*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*b*c*d))/
3 + (b^3*d^5*x^11*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/11
```

3.1278 $\int (a+bx)^4(c+dx)^7\,dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(bc-ad)(c+dx)^{11}}{11d^5}+\frac{b^4(c+dx)^{1}}{12d^5}$$

```
[Out] 1/8*(-a*d+b*c)^4*(d*x+c)^8/d^5-4/9*b*(-a*d+b*c)^3*(d*x+c)^9/d^5+3/5*b^2*(-a
*d+b*c)^2*(d*x+c)^10/d^5-4/11*b^3*(-a*d+b*c)*(d*x+c)^11/d^5+1/12*b^4*(d*x+c
)^12/d^5
```

Rubi [A]
time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^8)/(8*d^5) - (4*b*(b*c - a*d)^3*(c + d*x)^9)/(9*d^
5) + (3*b^2*(b*c - a*d)^2*(c + d*x)^10)/(5*d^5) - (4*b^3*(b*c - a*d)*(c + d
*x)^11)/(11*d^5) + (b^4*(c + d*x)^12)/(12*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^7}{d^4}-\frac{4b(bc-ad)^3(c+dx)^8}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^9}{d^4}-\right.\\ &= \frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(b}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 473 vs. 2(119) = 238.
time = 0.03, size = 473, normalized size = 3.97

$$a^4c^7x+\frac{1}{2}a^3c^6(4bc+7ad)x^2+\frac{1}{3}a^2c^5\left(6b^2c^2+28abcd+21a^2d^2\right)x^3+\frac{1}{4}ac^4\left(4b^3c^3+42ab^2c^2d+84a^2bcd^2+35a^3d^3\right)x^4+\frac{1}{5}c^3\left(b^4c^4+28ab^3c^3d+126a^2b^2c^2d^2+140a^3bcd^3+35a^4d^4\right)x^5+\frac{7}{6}c^2d\left(b^4c^4+12ab^3c^3d+30a^2b^2c^2d^2+20a^3bcd^3+3a^4d^4\right)x^6+cd^2\left(3b^4c^4+20ab^3c^3d+30a^2b^2c^2d^2+12a^3bcd^3+a^4d^4\right)x^7+\frac{1}{8}d^3\left(35b^4c^4+140ab^3c^3d+126a^2b^2c^2d^2+28a^3bcd^3+a^4d^4\right)x^8+\frac{1}{9}bd^4\left(35b^3c^3+84ab^2c^2d+42a^2bcd^2+4a^3d^3\right)x^9+\frac{1}{10}b^2d^5\left(21b^2c^2+28abcd+6a^2d^2\right)x^{10}+\frac{1}{11}b^3d^6(7bc+4ad)x^{11}+\frac{1}{12}b^4d^7x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] a^4*c^7*x + (a^3*c^6*(4*b*c + 7*a*d)*x^2)/2 + (a^2*c^5*(6*b^2*c^2 + 28*a*b*
c*d + 21*a^2*d^2)*x^3)/3 + (a*c^4*(4*b^3*c^3 + 42*a*b^2*c^2*d + 84*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + (c^3*(b^4*c^4 + 28*a*b^3*c^3*d + 126*a^2*b^2*c^2
*d^2 + 140*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + (7*c^2*d*(b^4*c^4 + 12*a*b^3*
c^3*d + 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 + 3*a^4*d^4)*x^6)/6 + c*d^2*(3*
b^4*c^4 + 20*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)*x
^7 + (d^3*(35*b^4*c^4 + 140*a*b^3*c^3*d + 126*a^2*b^2*c^2*d^2 + 28*a^3*b*c*
d^3 + a^4*d^4)*x^8)/8 + (b*d^4*(35*b^3*c^3 + 84*a*b^2*c^2*d + 42*a^2*b*c*d^
2 + 4*a^3*d^3)*x^9)/9 + (b^2*d^5*(21*b^2*c^2 + 28*a*b*c*d + 6*a^2*d^2)*x^10
)/10 + (b^3*d^6*(7*b*c + 4*a*d)*x^11)/11 + (b^4*d^7*x^12)/12
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 472 vs. $2(119) = 238$.
time = 5.17, size = 452, normalized size = 3.80

$$x\left(a^4c^7+\frac{a^3c^6x(7ad+4bc)}{2}+\frac{a^2c^5x^2(21a^2d^2+28abcd+6b^2c^2)}{3}+\frac{ac^4x^3(35a^3d^3+84a^2bcd^2+42ab^2c^2d+4b^3c^3)}{4}+\frac{bd^4x^8(4a^3d^3+42a^2bcd^2+84ab^2c^2d+35b^3c^3)}{9}+\frac{b^3d^6x^{10}(4ad+7bc)}{11}+\frac{c^3x^4(35a^4d^4+140a^3bcd^3+126a^2b^2c^2d^2+28ab^3c^3d+b^4c^4)}{5}+\frac{d^3x^7(a^4d^4+28a^3bcd^3+126a^2b^2c^2d^2+140ab^3c^3d+35b^4c^4)}{8}+\frac{b^2d^5x^9(6a^2d^2+28abcd+21b^2c^2)}{10}+\frac{b^4d^7x^{11}}{12}+\frac{7c^2dx^5(3a^4d^4+20a^3bcd^3+30a^2b^2c^2d^2+12ab^3c^3d+b^4c^4)}{6}+cd^2x^6(a^4d^4+12a^3bcd^3+30a^2b^2c^2d^2+20ab^3c^3d+3b^4c^4)\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^7,x]')
```

```
[Out] x (a ^ 4 c ^ 7 + a ^ 3 c ^ 6 x (7 a d + 4 b c) / 2 + a ^ 2 c ^ 5 x ^ 2 (21
a ^ 2 d ^ 2 + 28 a b c d + 6 b ^ 2 c ^ 2) / 3 + a c ^ 4 x ^ 3 (35 a ^ 3 d ^
 3 + 84 a ^ 2 b c d ^ 2 + 42 a b ^ 2 c ^ 2 d + 4 b ^ 3 c ^ 3) / 4 + b d ^ 4
 x ^ 8 (4 a ^ 3 d ^ 3 + 42 a ^ 2 b c d ^ 2 + 84 a b ^ 2 c ^ 2 d + 35 b ^ 3
c ^ 3) / 9 + b ^ 3 d ^ 6 x ^ 10 (4 a d + 7 b c) / 11 + c ^ 3 x ^ 4 (35 a ^
4 d ^ 4 + 140 a ^ 3 b c d ^ 3 + 126 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 28 a b ^ 3 c
^ 3 d + b ^ 4 c ^ 4) / 5 + d ^ 3 x ^ 7 (a ^ 4 d ^ 4 + 28 a ^ 3 b c d ^ 3 +
126 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 140 a b ^ 3 c ^ 3 d + 35 b ^ 4 c ^ 4) / 8 + b
 ^ 2 d ^ 5 x ^ 9 (6 a ^ 2 d ^ 2 + 28 a b c d + 21 b ^ 2 c ^ 2) / 10 + b ^ 4
 d ^ 7 x ^ 11 / 12 + 7 c ^ 2 d x ^ 5 (3 a ^ 4 d ^ 4 + 20 a ^ 3 b c d ^ 3 +
30 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 12 a b ^ 3 c ^ 3 d + b ^ 4 c ^ 4) / 6 + c d ^
2 x ^ 6 (a ^ 4 d ^ 4 + 12 a ^ 3 b c d ^ 3 + 30 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 20
 a b ^ 3 c ^ 3 d + 3 b ^ 4 c ^ 4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(109) = 218$.
time = 0.14, size = 493, normalized size = 4.14

method	result
norman	$\frac{b^4d^7x^{12}}{12}+\left(\frac{4}{11}a\,b^3d^7+\frac{7}{11}b^4c\,d^6\right)x^{11}+\left(\frac{3}{5}b^2a^2d^7+\frac{14}{5}a\,b^3c\,d^6+\frac{21}{10}b^4c^2d^5\right)x^{10}+\left(\frac{4}{9}a^3b\,d^7+\frac{14}{3}b^2a^2c\,d^6\right.$
default	$\frac{b^4d^7x^{12}}{12}+\frac{(4a\,b^3d^7+7b^4c\,d^6)x^{11}}{11}+\frac{(6b^2a^2d^7+28a\,b^3c\,d^6+21b^4c^2d^5)x^{10}}{10}+\frac{(4a^3b\,d^7+42b^2a^2c\,d^6+84a\,b^3c^2d^5+35b^4c^3d^4)x^9}{9}+$
gosper	$7x^3a^4c^5d^2+2x^3b^2a^2c^7+\frac{7}{2}x^2a^4c^6d+2x^2a^3b\,c^7+a^4c\,d^6x^7+3b^4c^5d^2x^7+\frac{35}{4}x^4a^4c^4d^3+x^4a\,b^3c^7+$

risch	$7x^3a^4c^5d^2+2x^3b^2a^2c^7+\frac{7}{2}x^2a^4c^6d+2x^2a^3b\,c^7+a^4c\,d^6x^7+3b^4c^5d^2x^7+\frac{35}{4}x^4a^4c^4d^3+x^4a\,b^3c^7+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^4*d^7*x^12+1/11*(4*a*b^3*d^7+7*b^4*c*d^6)*x^11+1/10*(6*a^2*b^2*d^7+2
8*a*b^3*c*d^6+21*b^4*c^2*d^5)*x^10+1/9*(4*a^3*b*d^7+42*a^2*b^2*c*d^6+84*a*b
^3*c^2*d^5+35*b^4*c^3*d^4)*x^9+1/8*(a^4*d^7+28*a^3*b*c*d^6+126*a^2*b^2*c^2*
d^5+140*a*b^3*c^3*d^4+35*b^4*c^4*d^3)*x^8+1/7*(7*a^4*c*d^6+84*a^3*b*c^2*d^5
+210*a^2*b^2*c^3*d^4+140*a*b^3*c^4*d^3+21*b^4*c^5*d^2)*x^7+1/6*(21*a^4*c^2*
d^5+140*a^3*b*c^3*d^4+210*a^2*b^2*c^4*d^3+84*a*b^3*c^5*d^2+7*b^4*c^6*d)*x^6
+1/5*(35*a^4*c^3*d^4+140*a^3*b*c^4*d^3+126*a^2*b^2*c^5*d^2+28*a*b^3*c^6*d+b
^4*c^7)*x^5+1/4*(35*a^4*c^4*d^3+84*a^3*b*c^5*d^2+42*a^2*b^2*c^6*d+4*a*b^3*c
^7)*x^4+1/3*(21*a^4*c^5*d^2+28*a^3*b*c^6*d+6*a^2*b^2*c^7)*x^3+1/2*(7*a^4*c^
6*d+4*a^3*b*c^7)*x^2+a^4*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(109) = 218$.
time = 0.27, size = 489, normalized size = 4.11

$$\frac{1}{12}\,b^4d^7x^{12}+a^4c^7x+\frac{1}{11}\,(7\,b^4cd^6+4\,ab^3d^7)x^{11}+\frac{1}{10}\,(21\,b^4c^2d^5+28\,ab^3cd^6+6\,a^2b^2d^7)x^{10}+\frac{1}{9}\,(35\,b^4c^3d^4+84\,ab^3c^2d^5+42\,a^2b^2cd^6+4\,a^3bd^7)x^9+\frac{1}{8}\,(35\,b^4c^4d^3+140\,ab^3c^3d^4+126\,a^2b^2c^2d^5+28\,a^3bcd^6+a^4d^7)x^8+(3\,b^4c^5d^2+20\,ab^3c^4d^3+30\,a^2b^2c^3d^4+12\,a^3bc^2d^5+a^4cd^6)x^7+\frac{7}{6}\,(b^4c^6d+12\,ab^3c^5d^2+30\,a^2b^2c^4d^3+20\,a^3bc^3d^4+3\,a^4c^2d^5)x^6+\frac{1}{5}\,(b^4c^7+28\,ab^3c^6d+126\,a^2b^2c^5d^2+140\,a^3bc^4d^3+35\,a^4c^3d^4)x^5+\frac{1}{4}\,(4\,ab^3c^7+42\,a^2b^2c^6d+84\,a^3bc^5d^2+35\,a^4c^4d^3)x^4+\frac{1}{3}\,(6\,a^2b^2c^7+28\,a^3bc^6d+21\,a^4c^5d^2)x^3+\frac{1}{2}\,(4\,a^3bc^7+7\,a^4c^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(109) = 218$.
time = 0.29, size = 489, normalized size = 4.11

$$\frac{1}{12}\,b^4d^7x^{12}+a^4c^7x+\frac{1}{11}\,(7\,b^4cd^6+4\,ab^3d^7)x^{11}+\frac{1}{10}\,(21\,b^4c^2d^5+28\,ab^3cd^6+6\,a^2b^2d^7)x^{10}+\frac{1}{9}\,(35\,b^4c^3d^4+84\,ab^3c^2d^5+42\,a^2b^2cd^6+4\,a^3bd^7)x^9+\frac{1}{8}\,(35\,b^4c^4d^3+140\,ab^3c^3d^4+126\,a^2b^2c^2d^5+28\,a^3bcd^6+a^4d^7)x^8+(3\,b^4c^5d^2+20\,ab^3c^4d^3+30\,a^2b^2c^3d^4+12\,a^3bc^2d^5+a^4cd^6)x^7+\frac{7}{6}\,(b^4c^6d+12\,ab^3c^5d^2+30\,a^2b^2c^4d^3+20\,a^3bc^3d^4+3\,a^4c^2d^5)x^6+\frac{1}{5}\,(b^4c^7+28\,ab^3c^6d+126\,a^2b^2c^5d^2+140\,a^3bc^4d^3+35\,a^4c^3d^4)x^5+\frac{1}{4}\,(4\,ab^3c^7+42\,a^2b^2c^6d+84\,a^3bc^5d^2+35\,a^4c^4d^3)x^4+\frac{1}{3}\,(6\,a^2b^2c^7+28\,a^3bc^6d+21\,a^4c^5d^2)x^3+\frac{1}{2}\,(4\,a^3bc^7+7\,a^4c^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(107) = 214$.
time = 0.07, size = 549, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**7,x)
```

```
[Out] a**4*c**7*x + b**4*d**7*x**12/12 + x**11*(4*a*b**3*d**7/11 + 7*b**4*c*d**6/
11) + x**10*(3*a**2*b**2*d**7/5 + 14*a*b**3*c*d**6/5 + 21*b**4*c**2*d**5/10
) + x**9*(4*a**3*b*d**7/9 + 14*a**2*b**2*c*d**6/3 + 28*a*b**3*c**2*d**5/3 +
 35*b**4*c**3*d**4/9) + x**8*(a**4*d**7/8 + 7*a**3*b*c*d**6/2 + 63*a**2*b**
2*c**2*d**5/4 + 35*a*b**3*c**3*d**4/2 + 35*b**4*c**4*d**3/8) + x**7*(a**4*c
*d**6 + 12*a**3*b*c**2*d**5 + 30*a**2*b**2*c**3*d**4 + 20*a*b**3*c**4*d**3
+ 3*b**4*c**5*d**2) + x**6*(7*a**4*c**2*d**5/2 + 70*a**3*b*c**3*d**4/3 + 35
*a**2*b**2*c**4*d**3 + 14*a*b**3*c**5*d**2 + 7*b**4*c**6*d/6) + x**5*(7*a**
4*c**3*d**4 + 28*a**3*b*c**4*d**3 + 126*a**2*b**2*c**5*d**2/5 + 28*a*b**3*c
**6*d/5 + b**4*c**7/5) + x**4*(35*a**4*c**4*d**3/4 + 21*a**3*b*c**5*d**2 +
21*a**2*b**2*c**6*d/2 + a*b**3*c**7) + x**3*(7*a**4*c**5*d**2 + 28*a**3*b*c
**6*d/3 + 2*a**2*b**2*c**7) + x**2*(7*a**4*c**6*d/2 + 2*a**3*b*c**7)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(109) = 218$.
time = 0.00, size = 596, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x)
```

```
[Out] 1/12*b^4*d^7*x^12 + 7/11*b^4*c*d^6*x^11 + 4/11*a*b^3*d^7*x^11 + 21/10*b^4*c
^2*d^5*x^10 + 14/5*a*b^3*c*d^6*x^10 + 3/5*a^2*b^2*d^7*x^10 + 35/9*b^4*c^3*d
^4*x^9 + 28/3*a*b^3*c^2*d^5*x^9 + 14/3*a^2*b^2*c*d^6*x^9 + 4/9*a^3*b*d^7*x^
```

```
9 + 35/8*b^4*c^4*d^3*x^8 + 35/2*a*b^3*c^3*d^4*x^8 + 63/4*a^2*b^2*c^2*d^5*x^
8 + 7/2*a^3*b*c*d^6*x^8 + 1/8*a^4*d^7*x^8 + 3*b^4*c^5*d^2*x^7 + 20*a*b^3*c^
4*d^3*x^7 + 30*a^2*b^2*c^3*d^4*x^7 + 12*a^3*b*c^2*d^5*x^7 + a^4*c*d^6*x^7 +
 7/6*b^4*c^6*d*x^6 + 14*a*b^3*c^5*d^2*x^6 + 35*a^2*b^2*c^4*d^3*x^6 + 70/3*a
^3*b*c^3*d^4*x^6 + 7/2*a^4*c^2*d^5*x^6 + 1/5*b^4*c^7*x^5 + 28/5*a*b^3*c^6*d
*x^5 + 126/5*a^2*b^2*c^5*d^2*x^5 + 28*a^3*b*c^4*d^3*x^5 + 7*a^4*c^3*d^4*x^5
 + a*b^3*c^7*x^4 + 21/2*a^2*b^2*c^6*d*x^4 + 21*a^3*b*c^5*d^2*x^4 + 35/4*a^4
*c^4*d^3*x^4 + 2*a^2*b^2*c^7*x^3 + 28/3*a^3*b*c^6*d*x^3 + 7*a^4*c^5*d^2*x^3
 + 2*a^3*b*c^7*x^2 + 7/2*a^4*c^6*d*x^2 + a^4*c^7*x
```

Mupad [B]
time = 0.31, size = 470, normalized size = 3.95

$$x^5\left(7a^4c^3d^4+28a^3bc^4d^3+\frac{126a^2b^2c^5d^2}{5}+\frac{28ab^3c^6d}{5}+\frac{b^4c^7}{5}\right)+x^8\left(\frac{a^4d^7}{8}+\frac{7a^3bcd^6}{2}+\frac{63a^2b^2c^2d^5}{4}+\frac{35ab^3c^3d^4}{2}+\frac{35b^4c^4d^3}{8}\right)+x^4\left(\frac{35a^4c^4d^3}{4}+21a^3bc^5d^2+\frac{21a^2b^2c^6d}{2}+ab^3c^7\right)+x^9\left(\frac{4a^3bd^7}{9}+\frac{14a^2b^2cd^6}{3}+\frac{28ab^3c^2d^5}{3}+\frac{35b^4c^3d^4}{9}\right)+x^7\left(a^4cd^6+12a^3bc^2d^5+30a^2b^2c^3d^4+20ab^3c^4d^3+3b^4c^5d^2\right)+x^6\left(\frac{7a^4c^2d^5}{2}+\frac{70a^3bc^3d^4}{3}+35a^2b^2c^4d^3+14ab^3c^5d^2+\frac{7b^4c^6d}{6}\right)+a^4c^7x+\frac{b^4d^7x^{12}}{12}+\frac{a^3c^6x^2\left(7ad+4bc\right)}{2}+\frac{b^3d^6x^{11}\left(4ad+7bc\right)}{11}+\frac{a^2c^5x^3\left(21a^2d^2+28abcd+6b^2c^2\right)}{3}+\frac{b^2d^5x^{10}\left(6a^2d^2+28abcd+21b^2c^2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^7,x)
```

```
[Out] x^5*((b^4*c^7)/5 + 7*a^4*c^3*d^4 + 28*a^3*b*c^4*d^3 + (126*a^2*b^2*c^5*d^2)
/5 + (28*a*b^3*c^6*d)/5) + x^8*((a^4*d^7)/8 + (35*b^4*c^4*d^3)/8 + (35*a*b^
3*c^3*d^4)/2 + (63*a^2*b^2*c^2*d^5)/4 + (7*a^3*b*c*d^6)/2) + x^4*(a*b^3*c^7
 + (35*a^4*c^4*d^3)/4 + (21*a^2*b^2*c^6*d)/2 + 21*a^3*b*c^5*d^2) + x^9*((4*
a^3*b*d^7)/9 + (35*b^4*c^3*d^4)/9 + (28*a*b^3*c^2*d^5)/3 + (14*a^2*b^2*c*d^
6)/3) + x^7*(a^4*c*d^6 + 3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 12*a^3*b*c^2*d^
5 + 30*a^2*b^2*c^3*d^4) + x^6*((7*b^4*c^6*d)/6 + (7*a^4*c^2*d^5)/2 + 14*a*b
^3*c^5*d^2 + (70*a^3*b*c^3*d^4)/3 + 35*a^2*b^2*c^4*d^3) + a^4*c^7*x + (b^4*
d^7*x^12)/12 + (a^3*c^6*x^2*(7*a*d + 4*b*c))/2 + (b^3*d^6*x^11*(4*a*d + 7*b
*c))/11 + (a^2*c^5*x^3*(21*a^2*d^2 + 6*b^2*c^2 + 28*a*b*c*d))/3 + (b^2*d^5*
x^10*(6*a^2*d^2 + 21*b^2*c^2 + 28*a*b*c*d))/10
```

3.1279 $\int (a+bx)^3(c+dx)^7\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

```
[Out] -1/8*(-a*d+b*c)^3*(d*x+c)^8/d^4+1/3*b*(-a*d+b*c)^2*(d*x+c)^9/d^4-3/10*b^2*(
-a*d+b*c)*(d*x+c)^10/d^4+1/11*b^3*(d*x+c)^11/d^4
```

Rubi [A]
time = 0.15, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] -1/8*((b*c - a*d)^3*(c + d*x)^8)/d^4 + (b*(b*c - a*d)^2*(c + d*x)^9)/(3*d^4
) - (3*b^2*(b*c - a*d)*(c + d*x)^10)/(10*d^4) + (b^3*(c + d*x)^11)/(11*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^3(c+dx)^7}{d^3}+\frac{3b(bc-ad)^2(c+dx)^8}{d^3}-\frac{3b^2(bc-ad)(c+dx)^9}{d^3}+\frac{b^3}{}\right.$$

$$= -\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\frac{b^3(c+}{11d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 360 vs. 2(92) = 184.
time = 0.03, size = 360, normalized size = 3.91

$$a^3c^7x+\frac{1}{2}a^2c^6(3bc+7ad)x^2+ac^5\left(b^2c^2+7abcd+7a^2d^2\right)x^3+\frac{1}{4}c^4\left(b^3c^3+21ab^2c^2d+63a^2bcd^2+35a^3d^3\right)x^4+\frac{7}{5}c^3d\left(b^3c^3+9ab^2c^2d+15a^2bcd^2+5a^3d^3\right)x^5+\frac{7}{2}c^2d^2\left(b^3c^3+5ab^2c^2d+5a^2bcd^2+a^3d^3\right)x^6+cd^3\left(5b^3c^3+15ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^7+\frac{1}{8}d^4\left(35b^3c^3+63ab^2c^2d+21a^2bcd^2+a^3d^3\right)x^8+\frac{1}{3}bd^5\left(7b^2c^2+7abcd+a^2d^2\right)x^9+\frac{1}{10}b^2d^6(7bc+3ad)x^{10}+\frac{1}{11}b^3d^7x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] a^3*c^7*x + (a^2*c^6*(3*b*c + 7*a*d)*x^2)/2 + a*c^5*(b^2*c^2 + 7*a*b*c*d +
7*a^2*d^2)*x^3 + (c^4*(b^3*c^3 + 21*a*b^2*c^2*d + 63*a^2*b*c*d^2 + 35*a^3*d
^3)*x^4)/4 + (7*c^3*d*(b^3*c^3 + 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
)*x^5)/5 + (7*c^2*d^2*(b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*d^3)*x
^6)/2 + c*d^3*(5*b^3*c^3 + 15*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7 +
(d^4*(35*b^3*c^3 + 63*a*b^2*c^2*d + 21*a^2*b*c*d^2 + a^3*d^3)*x^8)/8 + (b*d
^5*(7*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*x^9)/3 + (b^2*d^6*(7*b*c + 3*a*d)*x^10
)/10 + (b^3*d^7*x^11)/11
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 359 vs. $2(92) = 184$.
time = 4.35, size = 343, normalized size = 3.73

$$x\left(a^3c^7+\frac{a^2c^6x\left(7ad+3bc\right)}{2}+ac^5x^2\left(7a^2d^2+7abcd+b^2c^2\right)+\frac{bd^5x^8\left(a^2d^2+7abcd+7b^2c^2\right)}{3}+\frac{b^2d^6x^9\left(3ad+7bc\right)}{10}+\frac{c^4x^3\left(35a^3d^3+63a^2bcd^2+21ab^2c^2d+b^3c^3\right)}{4}+\frac{d^4x^7\left(a^3d^3+21a^2bcd^2+63ab^2c^2d+35b^3c^3\right)}{8}+\frac{b^3d^7x^{10}}{11}+\frac{7c^3dx^4\left(5a^3d^3+15a^2bcd^2+9ab^2c^2d+b^3c^3\right)}{5}+\frac{7c^2d^2x^5\left(a^3d^3+5a^2bcd^2+5ab^2c^2d+b^3c^3\right)}{2}+cd^3x^6\left(a^3d^3+9a^2bcd^2+15ab^2c^2d+5b^3c^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^7,x]')
```

```
[Out] x (a ^ 3 c ^ 7 + a ^ 2 c ^ 6 x (7 a d + 3 b c) / 2 + a c ^ 5 x ^ 2 (7 a ^ 2
 d ^ 2 + 7 a b c d + b ^ 2 c ^ 2) + b d ^ 5 x ^ 8 (a ^ 2 d ^ 2 + 7 a b c d
+ 7 b ^ 2 c ^ 2) / 3 + b ^ 2 d ^ 6 x ^ 9 (3 a d + 7 b c) / 10 + c ^ 4 x ^ 3
 (35 a ^ 3 d ^ 3 + 63 a ^ 2 b c d ^ 2 + 21 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) /
 4 + d ^ 4 x ^ 7 (a ^ 3 d ^ 3 + 21 a ^ 2 b c d ^ 2 + 63 a b ^ 2 c ^ 2 d + 3
5 b ^ 3 c ^ 3) / 8 + b ^ 3 d ^ 7 x ^ 10 / 11 + 7 c ^ 3 d x ^ 4 (5 a ^ 3 d ^
 3 + 15 a ^ 2 b c d ^ 2 + 9 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) / 5 + 7 c ^ 2 d
^ 2 x ^ 5 (a ^ 3 d ^ 3 + 5 a ^ 2 b c d ^ 2 + 5 a b ^ 2 c ^ 2 d + b ^ 3 c ^
3) / 2 + c d ^ 3 x ^ 6 (a ^ 3 d ^ 3 + 9 a ^ 2 b c d ^ 2 + 15 a b ^ 2 c ^ 2
d + 5 b ^ 3 c ^ 3))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(84) = 168$.
time = 0.14, size = 385, normalized size = 4.18

method	result
norman	$\frac{b^3d^7x^{11}}{11}+\left(\frac{3}{10}a\,b^2d^7+\frac{7}{10}b^3c\,d^6\right)x^{10}+\left(\frac{1}{3}a^2b\,d^7+\frac{7}{3}a\,b^2c\,d^6+\frac{7}{3}b^3c^2d^5\right)x^9+\left(\frac{1}{8}a^3d^7+\frac{21}{8}a^2bc\,d^6+\right.$
default	$\frac{b^3d^7x^{11}}{11}+\frac{\left(3a\,b^2d^7+7b^3c\,d^6\right)x^{10}}{10}+\frac{\left(3a^2b\,d^7+21a\,b^2c\,d^6+21b^3c^2d^5\right)x^9}{9}+\frac{\left(a^3d^7+21a^2bc\,d^6+63a\,b^2c^2d^5+35b^3c^3d^4\right)x^8}{8}+$
gosper	$5b^3c^4d^3x^7+7a^3c^5d^2x^3+a\,b^2c^7x^3+\frac{3}{10}x^{10}a\,b^2d^7+\frac{7}{10}x^{10}b^3c\,d^6+\frac{1}{3}x^9a^2b\,d^7+\frac{7}{3}x^9b^3c^2d^5+\frac{35}{8}x^8b^3$
risch	$5b^3c^4d^3x^7+7a^3c^5d^2x^3+a\,b^2c^7x^3+\frac{3}{10}x^{10}a\,b^2d^7+\frac{7}{10}x^{10}b^3c\,d^6+\frac{1}{3}x^9a^2b\,d^7+\frac{7}{3}x^9b^3c^2d^5+\frac{35}{8}x^8b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^3*d^7*x^11+1/10*(3*a*b^2*d^7+7*b^3*c*d^6)*x^10+1/9*(3*a^2*b*d^7+21*a
*b^2*c*d^6+21*b^3*c^2*d^5)*x^9+1/8*(a^3*d^7+21*a^2*b*c*d^6+63*a*b^2*c^2*d^5
+35*b^3*c^3*d^4)*x^8+1/7*(7*a^3*c*d^6+63*a^2*b*c^2*d^5+105*a*b^2*c^3*d^4+35
*b^3*c^4*d^3)*x^7+1/6*(21*a^3*c^2*d^5+105*a^2*b*c^3*d^4+105*a*b^2*c^4*d^3+2
1*b^3*c^5*d^2)*x^6+1/5*(35*a^3*c^3*d^4+105*a^2*b*c^4*d^3+63*a*b^2*c^5*d^2+7
*b^3*c^6*d)*x^5+1/4*(35*a^3*c^4*d^3+63*a^2*b*c^5*d^2+21*a*b^2*c^6*d+b^3*c^7
)*x^4+1/3*(21*a^3*c^5*d^2+21*a^2*b*c^6*d+3*a*b^2*c^7)*x^3+1/2*(7*a^3*c^6*d+
3*a^2*b*c^7)*x^2+a^3*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(84) = 168$.
time = 0.27, size = 376, normalized size = 4.09

$$\frac{1}{11}\,b^{3}d^{7}x^{11}+a^{3}c^{7}x+\frac{1}{10}\,(7\,b^{3}cd^{6}+3\,ab^{2}d^{7})x^{10}+\frac{1}{3}\,(7\,b^{3}c^{2}d^{5}+7\,ab^{2}cd^{6}+a^{2}bd^{7})x^{9}+\frac{1}{8}\,(35\,b^{3}c^{3}d^{4}+63\,ab^{2}c^{2}d^{5}+21\,a^{2}bcd^{6}+a^{3}d^{7})x^{8}+(5\,b^{3}c^{4}d^{3}+15\,ab^{2}c^{3}d^{4}+9\,a^{2}bc^{2}d^{5}+a^{3}cd^{6})x^{7}+\frac{7}{2}\,(b^{3}c^{5}d^{2}+5\,ab^{2}c^{4}d^{3}+5\,a^{2}bc^{3}d^{4}+a^{3}c^{2}d^{5})x^{6}+\frac{7}{5}\,(b^{3}c^{6}d+9\,ab^{2}c^{5}d^{2}+15\,a^{2}bc^{4}d^{3}+5\,a^{3}c^{3}d^{4})x^{5}+\frac{1}{4}\,(b^{3}c^{7}+21\,ab^{2}c^{6}d+63\,a^{2}bc^{5}d^{2}+35\,a^{3}c^{4}d^{3})x^{4}+(ab^{2}c^{7}+7\,a^{2}bc^{6}d+7\,a^{3}c^{5}d^{2})x^{3}+\frac{1}{2}\,(3\,a^{2}bc^{7}+7\,a^{3}c^{6}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(84) = 168$.
time = 0.29, size = 376, normalized size = 4.09

$$\frac{1}{11}\,b^{3}d^{7}x^{11}+a^{3}c^{7}x+\frac{1}{10}\,(7\,b^{3}cd^{6}+3\,ab^{2}d^{7})x^{10}+\frac{1}{3}\,(7\,b^{3}c^{2}d^{5}+7\,ab^{2}cd^{6}+a^{2}bd^{7})x^{9}+\frac{1}{8}\,(35\,b^{3}c^{3}d^{4}+63\,ab^{2}c^{2}d^{5}+21\,a^{2}bcd^{6}+a^{3}d^{7})x^{8}+(5\,b^{3}c^{4}d^{3}+15\,ab^{2}c^{3}d^{4}+9\,a^{2}bc^{2}d^{5}+a^{3}cd^{6})x^{7}+\frac{7}{2}\,(b^{3}c^{5}d^{2}+5\,ab^{2}c^{4}d^{3}+5\,a^{2}bc^{3}d^{4}+a^{3}c^{2}d^{5})x^{6}+\frac{7}{5}\,(b^{3}c^{6}d+9\,ab^{2}c^{5}d^{2}+15\,a^{2}bc^{4}d^{3}+5\,a^{3}c^{3}d^{4})x^{5}+\frac{1}{4}\,(b^{3}c^{7}+21\,ab^{2}c^{6}d+63\,a^{2}bc^{5}d^{2}+35\,a^{3}c^{4}d^{3})x^{4}+(ab^{2}c^{7}+7\,a^{2}bc^{6}d+7\,a^{3}c^{5}d^{2})x^{3}+\frac{1}{2}\,(3\,a^{2}bc^{7}+7\,a^{3}c^{6}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(80) = 160.
time = 0.06, size = 427, normalized size = 4.64

$$a^3c^7x + \frac{b^3d^7x^{11}}{11} + x^{10}\cdot\left(\frac{3ab^2d^7}{10} + \frac{7b^3cd^6}{10}\right) + x^9\left(\frac{a^2bd^7}{3} + \frac{7ab^2cd^6}{3} + \frac{7b^3c^2d^5}{3}\right) + x^8\left(\frac{a^3d^7}{8} + \frac{21a^2bcd^6}{8} + \frac{63ab^2c^2d^5}{8} + \frac{35b^3c^3d^4}{8}\right) + x^7\left(a^3cd^6 + 9a^2bc^2d^5 + 15ab^2c^3d^4 + 5b^3c^4d^3\right) + x^6\cdot\left(\frac{7a^3c^2d^5}{2} + \frac{35a^2bc^3d^4}{2} + \frac{35ab^2c^4d^3}{2} + \frac{7b^3c^5d^2}{2}\right) + x^5\cdot\left(7a^3c^3d^4 + 21a^2bc^4d^3 + \frac{63ab^2c^5d^2}{5} + \frac{7b^3c^6d}{5}\right) + x^4\cdot\left(\frac{35a^3c^4d^3}{4} + \frac{63a^2bc^5d^2}{4} + \frac{21ab^2c^6d}{4} + \frac{b^3c^7}{4}\right) + x^3\cdot\left(7a^3c^5d^2 + 7a^2bc^6d + ab^2c^7\right) + x^2\cdot\left(\frac{7a^3c^6d}{2} + \frac{3a^2bc^7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**7,x)
```

```
[Out] a**3*c**7*x + b**3*d**7*x**11/11 + x**10*(3*a*b**2*d**7/10 + 7*b**3*c*d**6/
10) + x**9*(a**2*b*d**7/3 + 7*a*b**2*c*d**6/3 + 7*b**3*c**2*d**5/3) + x**8*
(a**3*d**7/8 + 21*a**2*b*c*d**6/8 + 63*a*b**2*c**2*d**5/8 + 35*b**3*c**3*d*
*4/8) + x**7*(a**3*c*d**6 + 9*a**2*b*c**2*d**5 + 15*a*b**2*c**3*d**4 + 5*b*
*3*c**4*d**3) + x**6*(7*a**3*c**2*d**5/2 + 35*a**2*b*c**3*d**4/2 + 35*a*b**
2*c**4*d**3/2 + 7*b**3*c**5*d**2/2) + x**5*(7*a**3*c**3*d**4 + 21*a**2*b*c*
*4*d**3 + 63*a*b**2*c**5*d**2/5 + 7*b**3*c**6*d/5) + x**4*(35*a**3*c**4*d**
3/4 + 63*a**2*b*c**5*d**2/4 + 21*a*b**2*c**6*d/4 + b**3*c**7/4) + x**3*(7*a
**3*c**5*d**2 + 7*a**2*b*c**6*d + a*b**2*c**7) + x**2*(7*a**3*c**6*d/2 + 3*
a**2*b*c**7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(84) = 168.
time = 0.00, size = 464, normalized size = 5.04

$$\frac{1}{11}x^{11}b^3d^7 + \frac{7}{10}x^{10}b^3d^6c + \frac{3}{10}x^{10}b^2ad^7 + \frac{7}{3}x^9b^3d^5c^2 + \frac{7}{3}x^9b^2ad^6c + \frac{1}{3}x^9ba^2d^7 + \frac{35}{8}x^8b^3d^4c^3 + \frac{63}{8}x^8b^2ad^5c^2 + \frac{21}{8}x^8ba^2d^6c + \frac{1}{8}x^8a^3d^7 + 5x^7b^3d^3c^4 + 15x^7b^2ad^4c^3 + 9x^7ba^2d^5c^2 + x^7a^3d^6c + \frac{7}{2}x^6b^3d^2c^5 + \frac{35}{2}x^6b^2ad^3c^4 + \frac{35}{2}x^6ba^2d^4c^3 + \frac{7}{2}x^6a^3d^5c^2 + \frac{7}{5}x^5b^3dc^6 + \frac{63}{5}x^5b^2ad^2c^5 + 21x^5ba^2d^3c^4 + 7x^5a^3d^4c^3 + \frac{1}{4}x^4b^3c^7 + \frac{21}{4}x^4b^2adc^6 + \frac{63}{4}x^4ba^2d^2c^5 + \frac{35}{4}x^4a^3d^3c^4 + x^3b^2ac^7 + 7x^3ba^2dc^6 + 7x^3a^3d^2c^5 + \frac{3}{2}x^2ba^2c^7 + \frac{7}{2}x^2a^3dc^6 + xa^3c^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x)
```

```
[Out] 1/11*b^3*d^7*x^11 + 7/10*b^3*c*d^6*x^10 + 3/10*a*b^2*d^7*x^10 + 7/3*b^3*c^2
*d^5*x^9 + 7/3*a*b^2*c*d^6*x^9 + 1/3*a^2*b*d^7*x^9 + 35/8*b^3*c^3*d^4*x^8 +
 63/8*a*b^2*c^2*d^5*x^8 + 21/8*a^2*b*c*d^6*x^8 + 1/8*a^3*d^7*x^8 + 5*b^3*c^
4*d^3*x^7 + 15*a*b^2*c^3*d^4*x^7 + 9*a^2*b*c^2*d^5*x^7 + a^3*c*d^6*x^7 + 7/
2*b^3*c^5*d^2*x^6 + 35/2*a*b^2*c^4*d^3*x^6 + 35/2*a^2*b*c^3*d^4*x^6 + 7/2*a
^3*c^2*d^5*x^6 + 7/5*b^3*c^6*d*x^5 + 63/5*a*b^2*c^5*d^2*x^5 + 21*a^2*b*c^4*
d^3*x^5 + 7*a^3*c^3*d^4*x^5 + 1/4*b^3*c^7*x^4 + 21/4*a*b^2*c^6*d*x^4 + 63/4
*a^2*b*c^5*d^2*x^4 + 35/4*a^3*c^4*d^3*x^4 + a*b^2*c^7*x^3 + 7*a^2*b*c^6*d*x
^3 + 7*a^3*c^5*d^2*x^3 + 3/2*a^2*b*c^7*x^2 + 7/2*a^3*c^6*d*x^2 + a^3*c^7*x
```

Mupad [B]
time = 0.27, size = 356, normalized size = 3.87

$$x^7\left(a^3\,c\,d^6+9\,a^2\,b\,c^2\,d^5+15\,a\,b^2\,c^3\,d^4+5\,b^3\,c^4\,d^3\right)+x^5\left(7\,a^3\,c^3\,d^4+21\,a^2\,b\,c^4\,d^3+\frac{63\,a\,b^2\,c^5\,d^2}{5}+\frac{7\,b^3\,c^6\,d}{5}\right)+x^4\left(\frac{35\,a^3\,c^4\,d^3}{4}+\frac{63\,a^2\,b\,c^5\,d^2}{4}+\frac{21\,a\,b^2\,c^6\,d}{4}+\frac{b^3\,c^7}{4}\right)+x^8\left(\frac{a^3\,d^7}{8}+\frac{21\,a^2\,b\,c\,d^6}{8}+\frac{63\,a\,b^2\,c^2\,d^5}{8}+\frac{35\,b^3\,c^3\,d^4}{8}\right)+a^3\,c^7\,x+\frac{b^3\,d^7\,x^{11}}{11}+\frac{7\,c^2\,d^2\,x^6\,\left(a^3\,d^3+5\,a^2\,b\,c\,d^2+5\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{2}+\frac{a^2\,c^6\,x^2\,\left(7\,a\,d+3\,b\,c\right)}{2}+\frac{b^2\,d^6\,x^{10}\,\left(3\,a\,d+7\,b\,c\right)}{10}+a\,c^5\,x^3\,\left(7\,a^2\,d^2+7\,a\,b\,c\,d+b^2\,c^2\right)+\frac{b\,d^5\,x^9\,\left(a^2\,d^2+7\,a\,b\,c\,d+7\,b^2\,c^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^7,x)

[Out] x^7*(a^3*c*d^6 + 5*b^3*c^4*d^3 + 15*a*b^2*c^3*d^4 + 9*a^2*b*c^2*d^5) + x^5*
((7*b^3*c^6*d)/5 + 7*a^3*c^3*d^4 + (63*a*b^2*c^5*d^2)/5 + 21*a^2*b*c^4*d^3)
 + x^4*((b^3*c^7)/4 + (35*a^3*c^4*d^3)/4 + (63*a^2*b*c^5*d^2)/4 + (21*a*b^2
*c^6*d)/4) + x^8*((a^3*d^7)/8 + (35*b^3*c^3*d^4)/8 + (63*a*b^2*c^2*d^5)/8 +
 (21*a^2*b*c*d^6)/8) + a^3*c^7*x + (b^3*d^7*x^11)/11 + (7*c^2*d^2*x^6*(a^3*
d^3 + b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2))/2 + (a^2*c^6*x^2*(7*a*d + 3
*b*c))/2 + (b^2*d^6*x^10*(3*a*d + 7*b*c))/10 + a*c^5*x^3*(7*a^2*d^2 + b^2*c
^2 + 7*a*b*c*d) + (b*d^5*x^9*(a^2*d^2 + 7*b^2*c^2 + 7*a*b*c*d))/3
```

3.1280 $\int (a+bx)^2(c+dx)^7\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

```
[Out] 1/8*(-a*d+b*c)^2*(d*x+c)^8/d^3-2/9*b*(-a*d+b*c)*(d*x+c)^9/d^3+1/10*b^2*(d*x
+c)^10/d^3
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = $\{45\}$

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^8)/(8*d^3) - (2*b*(b*c - a*d)*(c + d*x)^9)/(9*d^3)
 + (b^2*(c + d*x)^10)/(10*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^7}{d^2}-\frac{2b(bc-ad)(c+dx)^8}{d^2}+\frac{b^2(c+dx)^9}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(65) = 130.
time = 0.02, size = 261, normalized size = 4.02

$$a^2c^7x+\frac{1}{2}ac^6(2bc+7ad)x^2+\frac{1}{3}c^5\left(b^2c^2+14abcd+21a^2d^2\right)x^3+\frac{7}{4}c^4d\left(b^2c^2+6abcd+5a^2d^2\right)x^4+\frac{7}{5}c^3d^2\left(3b^2c^2+10abcd+5a^2d^2\right)x^5+\frac{7}{6}c^2d^3\left(5b^2c^2+10abcd+3a^2d^2\right)x^6+cd^4\left(5b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{8}d^5\left(21b^2c^2+14abcd+a^2d^2\right)x^8+\frac{1}{9}bd^6(7bc+2ad)x^9+\frac{1}{10}b^2d^7x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] a^2*c^7*x + (a*c^6*(2*b*c + 7*a*d)*x^2)/2 + (c^5*(b^2*c^2 + 14*a*b*c*d + 21
*a^2*d^2)*x^3)/3 + (7*c^4*d*(b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*x^4)/4 + (7*c
^3*d^2*(3*b^2*c^2 + 10*a*b*c*d + 5*a^2*d^2)*x^5)/5 + (7*c^2*d^3*(5*b^2*c^2
+ 10*a*b*c*d + 3*a^2*d^2)*x^6)/6 + c*d^4*(5*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^7 + (d^5*(21*b^2*c^2 + 14*a*b*c*d + a^2*d^2)*x^8)/8 + (b*d^6*(7*b*c + 2*a
*d)*x^9)/9 + (b^2*d^7*x^10)/10
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 247 vs. $2(65) = 130$.
time = 3.55, size = 245, normalized size = 3.77

$$\frac{x\left(360a^2c^7+180ac^6x\left(7ad+2bc\right)+c^5x^2\left(2520a^2d^2+1680abcd+120b^2c^2\right)+630c^4dx^3\left(5a^2d^2+6abcd+b^2c^2\right)+c^3d^2x^4\left(2520a^2d^2+5040abcd+1512b^2c^2\right)+360cd^4x^6\left(a^2d^2+6abcd+5b^2c^2\right)+d^5x^7\left(45a^2d^2+630abcd+945b^2c^2\right)+40bd^6x^8\left(2ad+7bc\right)+36b^2d^7x^9+c^2d^3x^5\left(1260a^2d^2+4200abcd+2100b^2c^2\right)\right)}{360}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^7,x]')
```

```
[Out] x (360 a ^ 2 c ^ 7 + 180 a c ^ 6 x (7 a d + 2 b c) + c ^ 5 x ^ 2 (2520 a ^
2 d ^ 2 + 1680 a b c d + 120 b ^ 2 c ^ 2) + 630 c ^ 4 d x ^ 3 (5 a ^ 2 d ^
2 + 6 a b c d + b ^ 2 c ^ 2) + c ^ 3 d ^ 2 x ^ 4 (2520 a ^ 2 d ^ 2 + 5040 a
 b c d + 1512 b ^ 2 c ^ 2) + 360 c d ^ 4 x ^ 6 (a ^ 2 d ^ 2 + 6 a b c d + 5
 b ^ 2 c ^ 2) + d ^ 5 x ^ 7 (45 a ^ 2 d ^ 2 + 630 a b c d + 945 b ^ 2 c ^ 2
) + 40 b d ^ 6 x ^ 8 (2 a d + 7 b c) + 36 b ^ 2 d ^ 7 x ^ 9 + c ^ 2 d ^ 3 x
 ^ 5 (1260 a ^ 2 d ^ 2 + 4200 a b c d + 2100 b ^ 2 c ^ 2)) / 360
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(59) = 118$.
time = 0.13, size = 277, normalized size = 4.26

method	result
norman	$\frac{b^2d^7x^{10}}{10}+\left(\frac{2}{9}ab\,d^7+\frac{7}{9}b^2c\,d^6\right)x^9+\left(\frac{1}{8}a^2d^7+\frac{7}{4}abc\,d^6+\frac{21}{8}b^2c^2d^5\right)x^8+\left(a^2c\,d^6+6ab\,c^2d^5+5b^2c^3d^4\right)$
default	$\frac{b^2d^7x^{10}}{10}+\frac{\left(2ab\,d^7+7b^2c\,d^6\right)x^9}{9}+\frac{\left(a^2d^7+14abc\,d^6+21b^2c^2d^5\right)x^8}{8}+\frac{\left(7a^2c\,d^6+42ab\,c^2d^5+35b^2c^3d^4\right)x^7}{7}+\frac{(21a^2c^2d^5+70ab\,c^3d^4-}{6}$
gosper	$\frac{1}{10}b^2d^7x^{10}+\frac{2}{9}x^9ab\,d^7+\frac{7}{9}x^9b^2c\,d^6+\frac{1}{8}x^8a^2d^7+\frac{7}{4}x^8abc\,d^6+\frac{21}{8}x^8b^2c^2d^5+a^2c\,d^6x^7+6ab\,c^2d^5x^7+$
risch	$\frac{1}{10}b^2d^7x^{10}+\frac{2}{9}x^9ab\,d^7+\frac{7}{9}x^9b^2c\,d^6+\frac{1}{8}x^8a^2d^7+\frac{7}{4}x^8abc\,d^6+\frac{21}{8}x^8b^2c^2d^5+a^2c\,d^6x^7+6ab\,c^2d^5x^7+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d^7*x^10+1/9*(2*a*b*d^7+7*b^2*c*d^6)*x^9+1/8*(a^2*d^7+14*a*b*c*d^6
+21*b^2*c^2*d^5)*x^8+1/7*(7*a^2*c*d^6+42*a*b*c^2*d^5+35*b^2*c^3*d^4)*x^7+1/
6*(21*a^2*c^2*d^5+70*a*b*c^3*d^4+35*b^2*c^4*d^3)*x^6+1/5*(35*a^2*c^3*d^4+70
*a*b*c^4*d^3+21*b^2*c^5*d^2)*x^5+1/4*(35*a^2*c^4*d^3+42*a*b*c^5*d^2+7*b^2*c
```

```
^6*d)*x^4+1/3*(21*a^2*c^5*d^2+14*a*b*c^6*d+b^2*c^7)*x^3+1/2*(7*a^2*c^6*d+2*
a*b*c^7)*x^2+a^2*c^7*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(59) = 118$.
time = 0.27, size = 273, normalized size = 4.20

$$\frac{1}{10}\,b^2d^7x^{10}+a^2c^7x+\frac{1}{9}\,(7\,b^2cd^6+2\,abd^7)x^9+\frac{1}{8}\,(21\,b^2c^2d^5+14\,abcd^6+a^2d^7)x^8+(5\,b^2c^3d^4+6\,abc^2d^5+a^2cd^6)x^7+\frac{7}{6}\,(5\,b^2c^4d^3+10\,abc^3d^4+3\,a^2c^2d^5)x^6+\frac{7}{5}\,(3\,b^2c^5d^2+10\,abc^4d^3+5\,a^2c^3d^4)x^5+\frac{7}{4}\,(b^2c^6d+6\,abc^5d^2+5\,a^2c^4d^3)x^4+\frac{1}{3}\,(b^2c^7+14\,abc^6d+21\,a^2c^5d^2)x^3+\frac{1}{2}\,(2\,abc^7+7\,a^2c^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(59) = 118$.
time = 0.28, size = 273, normalized size = 4.20

$$\frac{1}{10}\,b^2d^7x^{10}+a^2c^7x+\frac{1}{9}\,(7\,b^2cd^6+2\,abd^7)x^9+\frac{1}{8}\,(21\,b^2c^2d^5+14\,abcd^6+a^2d^7)x^8+(5\,b^2c^3d^4+6\,abc^2d^5+a^2cd^6)x^7+\frac{7}{6}\,(5\,b^2c^4d^3+10\,abc^3d^4+3\,a^2c^2d^5)x^6+\frac{7}{5}\,(3\,b^2c^5d^2+10\,abc^4d^3+5\,a^2c^3d^4)x^5+\frac{7}{4}\,(b^2c^6d+6\,abc^5d^2+5\,a^2c^4d^3)x^4+\frac{1}{3}\,(b^2c^7+14\,abc^6d+21\,a^2c^5d^2)x^3+\frac{1}{2}\,(2\,abc^7+7\,a^2c^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(56) = 112$.
time = 0.06, size = 303, normalized size = 4.66

$$a^2c^7x+\frac{b^2d^7x^{10}}{10}+x^9\cdot\left(\frac{2abd^7}{9}+\frac{7b^2cd^6}{9}\right)+x^8\left(\frac{a^2d^7}{8}+\frac{7abcd^6}{4}+\frac{21b^2c^2d^5}{8}\right)+x^7\left(a^2cd^6+6abc^2d^5+5b^2c^3d^4\right)+x^6\cdot\left(\frac{7a^2c^2d^5}{2}+\frac{35abc^3d^4}{3}+\frac{35b^2c^4d^3}{6}\right)+x^5\cdot\left(7a^2c^3d^4+14abc^4d^3+\frac{21b^2c^5d^2}{5}\right)+x^4\cdot\left(\frac{35a^2c^4d^3}{4}+\frac{21abc^5d^2}{2}+\frac{7b^2c^6d}{4}\right)+x^3\cdot\left(7a^2c^5d^2+\frac{14abc^6d}{3}+\frac{b^2c^7}{3}\right)+x^2\cdot\left(\frac{7a^2c^6d}{2}+abc^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**7,x)
```

```
[Out] a**2*c**7*x + b**2*d**7*x**10/10 + x**9*(2*a*b*d**7/9 + 7*b**2*c*d**6/9) +
x**8*(a**2*d**7/8 + 7*a*b*c*d**6/4 + 21*b**2*c**2*d**5/8) + x**7*(a**2*c*d*
```

```
*6 + 6*a*b*c**2*d**5 + 5*b**2*c**3*d**4) + x**6*(7*a**2*c**2*d**5/2 + 35*a*
b*c**3*d**4/3 + 35*b**2*c**4*d**3/6) + x**5*(7*a**2*c**3*d**4 + 14*a*b*c**4
*d**3 + 21*b**2*c**5*d**2/5) + x**4*(35*a**2*c**4*d**3/4 + 21*a*b*c**5*d**2
/2 + 7*b**2*c**6*d/4) + x**3*(7*a**2*c**5*d**2 + 14*a*b*c**6*d/3 + b**2*c**
7/3) + x**2*(7*a**2*c**6*d/2 + a*b*c**7)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(59) = 118$.
time = 0.00, size = 326, normalized size = 5.02

$$\frac{1}{10}x^{10}b^2d^7 + \frac{7}{9}x^9b^2d^6c + \frac{2}{9}x^9bad^7 + \frac{21}{8}x^8b^2d^5c^2 + \frac{7}{4}x^8bad^6c + \frac{1}{8}x^8a^2d^7 + 5x^7b^2d^4c^3 + 6x^7bad^5c^2 + x^7a^2d^6c + \frac{35}{6}x^6b^2d^3c^4 + \frac{35}{3}x^6bad^4c^3 + \frac{7}{2}x^6a^2d^5c^2 + \frac{21}{5}x^5b^2d^2c^5 + 14x^5bad^3c^4 + 7x^5a^2d^4c^3 + \frac{7}{4}x^4b^2dc^6 + \frac{21}{2}x^4bad^2c^5 + \frac{35}{4}x^4a^2d^3c^4 + \frac{1}{3}x^3b^2c^7 + \frac{14}{3}x^3badc^6 + 7x^3a^2d^2c^5 + x^2bac^7 + \frac{7}{2}x^2a^2dc^6 + xa^2c^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x)
```

```
[Out] 1/10*b^2*d^7*x^10 + 7/9*b^2*c*d^6*x^9 + 2/9*a*b*d^7*x^9 + 21/8*b^2*c^2*d^5*
x^8 + 7/4*a*b*c*d^6*x^8 + 1/8*a^2*d^7*x^8 + 5*b^2*c^3*d^4*x^7 + 6*a*b*c^2*d
^5*x^7 + a^2*c*d^6*x^7 + 35/6*b^2*c^4*d^3*x^6 + 35/3*a*b*c^3*d^4*x^6 + 7/2*
a^2*c^2*d^5*x^6 + 21/5*b^2*c^5*d^2*x^5 + 14*a*b*c^4*d^3*x^5 + 7*a^2*c^3*d^4
*x^5 + 7/4*b^2*c^6*d*x^4 + 21/2*a*b*c^5*d^2*x^4 + 35/4*a^2*c^4*d^3*x^4 + 1/
3*b^2*c^7*x^3 + 14/3*a*b*c^6*d*x^3 + 7*a^2*c^5*d^2*x^3 + a*b*c^7*x^2 + 7/2*
a^2*c^6*d*x^2 + a^2*c^7*x
```

Mupad [**B**]
time = 0.11, size = 249, normalized size = 3.83

$$x^3\left(7a^2c^5d^2 + \frac{14abc^6d}{3} + \frac{b^2c^7}{3}\right) + x^8\left(\frac{a^2d^7}{8} + \frac{7abcd^6}{4} + \frac{21b^2c^2d^5}{8}\right) + a^2c^7x + \frac{b^2d^7x^{10}}{10} + \frac{ac^6x^2\,(7ad + 2bc)}{2} + \frac{bd^6x^9\,(2ad + 7bc)}{9} + \frac{7c^4dx^4\,(5a^2d^2 + 6abcd + b^2c^2)}{4} + cd^4x^7\,(a^2d^2 + 6abcd + 5b^2c^2) + \frac{7c^3d^2x^5\,(5a^2d^2 + 10abcd + 3b^2c^2)}{5} + \frac{7c^2d^3x^6\,(3a^2d^2 + 10abcd + 5b^2c^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^7,x)
```

```
[Out] x^3*((b^2*c^7)/3 + 7*a^2*c^5*d^2 + (14*a*b*c^6*d)/3) + x^8*((a^2*d^7)/8 + (
21*b^2*c^2*d^5)/8 + (7*a*b*c*d^6)/4) + a^2*c^7*x + (b^2*d^7*x^10)/10 + (a*c
^6*x^2*(7*a*d + 2*b*c))/2 + (b*d^6*x^9*(2*a*d + 7*b*c))/9 + (7*c^4*d*x^4*(5
*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/4 + c*d^4*x^7*(a^2*d^2 + 5*b^2*c^2 + 6*a*b
*c*d) + (7*c^3*d^2*x^5*(5*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/5 + (7*c^2*d^3
*x^6*(3*a^2*d^2 + 5*b^2*c^2 + 10*a*b*c*d))/6
```

3.1281 $\int (a+bx)(c+dx)^7\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}$$

```
[Out] -1/8*(-a*d+b*c)*(d*x+c)^8/d^2+1/9*b*(d*x+c)^9/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] -1/8*((b*c - a*d)*(c + d*x)^8)/d^2 + (b*(c + d*x)^9)/(9*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^7}{d}+\frac{b(c+dx)^8}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(38) = 76.
time = 0.01, size = 151, normalized size = 3.97

$$ac^7x+\frac{1}{2}c^6(bc+7ad)x^2+\frac{7}{3}c^5d(bc+3ad)x^3+\frac{7}{4}c^4d^2(3bc+5ad)x^4+7c^3d^3(bc+ad)x^5+\frac{7}{6}c^2d^4(5bc+3ad)x^6+cd^5(3bc+ad)x^7+\frac{1}{8}d^6(7bc+ad)x^8+\frac{1}{9}bd^7x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] a*c^7*x + (c^6*(b*c + 7*a*d)*x^2)/2 + (7*c^5*d*(b*c + 3*a*d)*x^3)/3 + (7*c^
4*d^2*(3*b*c + 5*a*d)*x^4)/4 + 7*c^3*d^3*(b*c + a*d)*x^5 + (7*c^2*d^4*(5*b*
c + 3*a*d)*x^6)/6 + c*d^5*(3*b*c + a*d)*x^7 + (d^6*(7*b*c + a*d)*x^8)/8 + (
b*d^7*x^9)/9
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 142 vs. 2(38) = 76.
time = 2.68, size = 140, normalized size = 3.68

$$\frac{x\left(72ac^7+d^6x^7\left(9ad+63bc\right)+8bd^7x^8+36c^6x\left(7ad+bc\right)+168c^5dx^2\left(3ad+bc\right)+c^4d^2x^3\left(630ad+378bc\right)+c^3d^3x^4\left(504ad+504bc\right)+c^2d^4x^5\left(252ad+420bc\right)+72cd^5x^6\left(ad+3bc\right)\right)}{72}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^7,x]')
```

```
[Out] x (72 a c ^ 7 + d ^ 6 x ^ 7 (9 a d + 63 b c) + 8 b d ^ 7 x ^ 8 + 36 c ^ 6 x
 (7 a d + b c) + 168 c ^ 5 d x ^ 2 (3 a d + b c) + c ^ 4 d ^ 2 x ^ 3 (630 a
 d + 378 b c) + c ^ 3 d ^ 3 x ^ 4 (504 a d + 504 b c) + c ^ 2 d ^ 4 x ^ 5 (
252 a d + 420 b c) + 72 c d ^ 5 x ^ 6 (a d + 3 b c)) / 72
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(34) = 68.
time = 0.12, size = 169, normalized size = 4.45

method	result
norman	$\frac{b\,d^7x^9}{9}+\left(\frac{1}{8}a\,d^7+\frac{7}{8}bc\,d^6\right)x^8+\left(ac\,d^6+3b\,c^2d^5\right)x^7+\left(\frac{7}{2}a\,c^2d^5+\frac{35}{6}b\,c^3d^4\right)x^6+\left(7a\,c^3d^4+7b\,c^4d^3\right)x$
default	$\frac{b\,d^7x^9}{9}+\frac{\left(a\,d^7+7bc\,d^6\right)x^8}{8}+\frac{\left(7ac\,d^6+21b\,c^2d^5\right)x^7}{7}+\frac{\left(21a\,c^2d^5+35b\,c^3d^4\right)x^6}{6}+\frac{\left(35a\,c^3d^4+35b\,c^4d^3\right)x^5}{5}+\frac{\left(35a\,c^4d^3+21b\,c^5d^2\right.}{4}$
gosper	$\frac{1}{9}b\,d^7x^9+\frac{1}{8}x^8a\,d^7+\frac{7}{8}x^8bc\,d^6+ac\,d^6x^7+3b\,c^2d^5x^7+\frac{7}{2}x^6a\,c^2d^5+\frac{35}{6}x^6b\,c^3d^4+7a\,c^3d^4x^5+7b\,c^4d^3$
risch	$\frac{1}{9}b\,d^7x^9+\frac{1}{8}x^8a\,d^7+\frac{7}{8}x^8bc\,d^6+ac\,d^6x^7+3b\,c^2d^5x^7+\frac{7}{2}x^6a\,c^2d^5+\frac{35}{6}x^6b\,c^3d^4+7a\,c^3d^4x^5+7b\,c^4d^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d^7*x^9+1/8*(a*d^7+7*b*c*d^6)*x^8+1/7*(7*a*c*d^6+21*b*c^2*d^5)*x^7+1/
6*(21*a*c^2*d^5+35*b*c^3*d^4)*x^6+1/5*(35*a*c^3*d^4+35*b*c^4*d^3)*x^5+1/4*(
35*a*c^4*d^3+21*b*c^5*d^2)*x^4+1/3*(21*a*c^5*d^2+7*b*c^6*d)*x^3+1/2*(7*a*c^
6*d+b*c^7)*x^2+a*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(34) = 68.
time = 0.26, size = 163, normalized size = 4.29

$$\frac{1}{9}bd^7x^9+ac^7x+\frac{1}{8}\left(7\,bcd^6+ad^7\right)x^8+\left(3\,bc^2d^5+acd^6\right)x^7+\frac{7}{6}\left(5\,bc^3d^4+3\,ac^2d^5\right)x^6+7\left(bc^4d^3+ac^3d^4\right)x^5+\frac{7}{4}\left(3\,bc^5d^2+5\,ac^4d^3\right)x^4+\frac{7}{3}\left(bc^6d+3\,ac^5d^2\right)x^3+\frac{1}{2}\left(bc^7+7\,ac^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(34) = 68$.
time = 0.30, size = 163, normalized size = 4.29

$$\frac{1}{9}\,bd^7x^9 + ac^7x + \frac{1}{8}\,(7\,bcd^6 + ad^7)x^8 + (3\,bc^2d^5 + acd^6)x^7 + \frac{7}{6}\,(5\,bc^3d^4 + 3\,ac^2d^5)x^6 + 7\,(bc^4d^3 + ac^3d^4)x^5 + \frac{7}{4}\,(3\,bc^5d^2 + 5\,ac^4d^3)x^4 + \frac{7}{3}\,(bc^6d + 3\,ac^5d^2)x^3 + \frac{1}{2}\,(bc^7 + 7\,ac^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(32) = 64$.
time = 0.05, size = 178, normalized size = 4.68

$$ac^7x + \frac{bd^7x^9}{9} + x^8\left(\frac{ad^7}{8} + \frac{7bcd^6}{8}\right) + x^7\left(acd^6 + 3bc^2d^5\right) + x^6\cdot\left(\frac{7ac^2d^5}{2} + \frac{35bc^3d^4}{6}\right) + x^5\cdot\left(7ac^3d^4 + 7bc^4d^3\right) + x^4\cdot\left(\frac{35ac^4d^3}{4} + \frac{21bc^5d^2}{4}\right) + x^3\cdot\left(7ac^5d^2 + \frac{7bc^6d}{3}\right) + x^2\cdot\left(\frac{7ac^6d}{2} + \frac{bc^7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**7,x)
```

```
[Out] a*c**7*x + b*d**7*x**9/9 + x**8*(a*d**7/8 + 7*b*c*d**6/8) + x**7*(a*c*d**6
+ 3*b*c**2*d**5) + x**6*(7*a*c**2*d**5/2 + 35*b*c**3*d**4/6) + x**5*(7*a*c*
*3*d**4 + 7*b*c**4*d**3) + x**4*(35*a*c**4*d**3/4 + 21*b*c**5*d**2/4) + x**
3*(7*a*c**5*d**2 + 7*b*c**6*d/3) + x**2*(7*a*c**6*d/2 + b*c**7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(34) = 68$.
time = 0.00, size = 189, normalized size = 4.97

$$\frac{1}{9}x^9bd^7 + \frac{7}{8}x^8bd^6c + \frac{1}{8}x^8ad^7 + 3x^7bd^5c^2 + x^7ad^6c + \frac{35}{6}x^6bd^4c^3 + \frac{7}{2}x^6ad^5c^2 + 7x^5bd^3c^4 + 7x^5ad^4c^3 + \frac{21}{4}x^4bd^2c^5 + \frac{35}{4}x^4ad^3c^4 + \frac{7}{3}x^3bdc^6 + 7x^3ad^2c^5 + \frac{1}{2}x^2bc^7 + \frac{7}{2}x^2adc^6 + xac^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x)
```

```
[Out] 1/9*b*d^7*x^9 + 7/8*b*c*d^6*x^8 + 1/8*a*d^7*x^8 + 3*b*c^2*d^5*x^7 + a*c*d^6
*x^7 + 35/6*b*c^3*d^4*x^6 + 7/2*a*c^2*d^5*x^6 + 7*b*c^4*d^3*x^5 + 7*a*c^3*d
^4*x^5 + 21/4*b*c^5*d^2*x^4 + 35/4*a*c^4*d^3*x^4 + 7/3*b*c^6*d*x^3 + 7*a*c^
5*d^2*x^3 + 1/2*b*c^7*x^2 + 7/2*a*c^6*d*x^2 + a*c^7*x
```

Mupad [B]
time = 0.08, size = 143, normalized size = 3.76

$$x^2\left(\frac{b\,c^7}{2}+\frac{7\,a\,d\,c^6}{2}\right)+x^8\left(\frac{a\,d^7}{8}+\frac{7\,b\,c\,d^6}{8}\right)+\frac{b\,d^7\,x^9}{9}+a\,c^7\,x+\frac{7\,c^5\,d\,x^3\,\left(3\,a\,d+b\,c\right)}{3}+c\,d^5\,x^7\,\left(a\,d+3\,b\,c\right)+7\,c^3\,d^3\,x^5\,\left(a\,d+b\,c\right)+\frac{7\,c^4\,d^2\,x^4\,\left(5\,a\,d+3\,b\,c\right)}{4}+\frac{7\,c^2\,d^4\,x^6\,\left(3\,a\,d+5\,b\,c\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^7,x)
```

```
[Out] x^2*((b*c^7)/2 + (7*a*c^6*d)/2) + x^8*((a*d^7)/8 + (7*b*c*d^6)/8) + (b*d^7*
x^9)/9 + a*c^7*x + (7*c^5*d*x^3*(3*a*d + b*c))/3 + c*d^5*x^7*(a*d + 3*b*c)
+ 7*c^3*d^3*x^5*(a*d + b*c) + (7*c^4*d^2*x^4*(5*a*d + 3*b*c))/4 + (7*c^2*d^
4*x^6*(3*a*d + 5*b*c))/6
```

3.1282 $\int (c+dx)^7 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^8}{8d}$$

```
[Out] 1/8*(d*x+c)^8/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^7 \, dx = \frac{(c+dx)^8}{8d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(14) = 28.
time = 1.87, size = 76, normalized size = 5.43

$$\frac{x\,(8c^7+28c^6dx+56c^5d^2x^2+70c^4d^3x^3+56c^3d^4x^4+28c^2d^5x^5+8cd^6x^6+d^7x^7)}{8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^7,x]')
```

```
[Out] x (8 c ^ 7 + 28 c ^ 6 d x + 56 c ^ 5 d ^ 2 x ^ 2 + 70 c ^ 4 d ^ 3 x ^ 3 + 5
6 c ^ 3 d ^ 4 x ^ 4 + 28 c ^ 2 d ^ 5 x ^ 5 + 8 c d ^ 6 x ^ 6 + d ^ 7 x ^ 7)
 / 8
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(dx+c)^8}{8d}$	13
gosper	$\frac{1}{8}d^7x^8 + c\,d^6x^7 + \frac{7}{2}c^2d^5x^6 + 7c^3d^4x^5 + \frac{35}{4}c^4d^3x^4 + 7c^5d^2x^3 + \frac{7}{2}c^6d\,x^2 + c^7x$	76
norman	$\frac{1}{8}d^7x^8 + c\,d^6x^7 + \frac{7}{2}c^2d^5x^6 + 7c^3d^4x^5 + \frac{35}{4}c^4d^3x^4 + 7c^5d^2x^3 + \frac{7}{2}c^6d\,x^2 + c^7x$	76
risch	$\frac{d^7x^8}{8} + c\,d^6x^7 + \frac{7c^2d^5x^6}{2} + 7c^3d^4x^5 + \frac{35c^4d^3x^4}{4} + 7c^5d^2x^3 + \frac{7c^6d\,x^2}{2} + c^7x + \frac{c^8}{8d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x+c)^8/d
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{(dx+c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^8/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(12) = 24$.
time = 0.29, size = 75, normalized size = 5.36

$$\frac{1}{8}\,d^7x^8 + cd^6x^7 + \frac{7}{2}\,c^2d^5x^6 + 7\,c^3d^4x^5 + \frac{35}{4}\,c^4d^3x^4 + 7\,c^5d^2x^3 + \frac{7}{2}\,c^6dx^2 + c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^7*x^8 + c*d^6*x^7 + 7/2*c^2*d^5*x^6 + 7*c^3*d^4*x^5 + 35/4*c^4*d^3*x^
4 + 7*c^5*d^2*x^3 + 7/2*c^6*d*x^2 + c^7*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.
time = 0.04, size = 83, normalized size = 5.93

$$c^7x + \frac{7c^6dx^2}{2} + 7c^5d^2x^3 + \frac{35c^4d^3x^4}{4} + 7c^3d^4x^5 + \frac{7c^2d^5x^6}{2} + cd^6x^7 + \frac{d^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7,x)
```

```
[Out] c**7*x + 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*
d**4*x**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(dx + c)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x)
```

```
[Out] 1/8*(d*x + c)^8/d
```

Mupad [B]
time = 0.06, size = 75, normalized size = 5.36

$$c^7\,x + \frac{7\,c^6\,d\,x^2}{2} + 7\,c^5\,d^2\,x^3 + \frac{35\,c^4\,d^3\,x^4}{4} + 7\,c^3\,d^4\,x^5 + \frac{7\,c^2\,d^5\,x^6}{2} + c\,d^6\,x^7 + \frac{d^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7,x)
```

```
[Out] c^7*x + (d^7*x^8)/8 + (7*c^6*d*x^2)/2 + c*d^6*x^7 + 7*c^5*d^2*x^3 + (35*c^4
*d^3*x^4)/4 + 7*c^3*d^4*x^5 + (7*c^2*d^5*x^6)/2
```

3.1283 $\int \frac{(c+dx)^7}{a+bx}\,dx$

Optimal. Leaf size=169

$$\frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc-ad)^2(c+dx)^5}{5b^3}+\frac{(bc-
$$

```
[Out] d*(-a*d+b*c)^6*x/b^7+1/2*(-a*d+b*c)^5*(d*x+c)^2/b^6+1/3*(-a*d+b*c)^4*(d*x+c
)^3/b^5+1/4*(-a*d+b*c)^3*(d*x+c)^4/b^4+1/5*(-a*d+b*c)^2*(d*x+c)^5/b^3+1/6*(
-a*d+b*c)*(d*x+c)^6/b^2+1/7*(d*x+c)^7/b+(-a*d+b*c)^7*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{(bc-ad)^7\log(a+bx)}{b^8}+\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(bc-ad)^2}{5b^3}+\frac{(c+dx)^6(bc-ad)}{6b^2}+\frac{(c+dx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^6*x)/b^7 + ((b*c - a*d)^5*(c + d*x)^2)/(2*b^6) + ((b*c - a*d
)^4*(c + d*x)^3)/(3*b^5) + ((b*c - a*d)^3*(c + d*x)^4)/(4*b^4) + ((b*c - a*
d)^2*(c + d*x)^5)/(5*b^3) + ((b*c - a*d)*(c + d*x)^6)/(6*b^2) + (c + d*x)^7
/(7*b) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^6}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)}+\frac{d(bc-ad)^5(c+dx)}{b^6}+\frac{d(bc-ad)^4(c+dx)^2}{b^5}+\frac{d(bc-ad\right.\\ &= \frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc-\end{aligned}$$

Mathematica [A]
time = 0.09, size = 304, normalized size = 1.80

$$\frac{dx\left(420a^6d^6-210a^5bd^5(14c+dx)+70a^4b^2d^4\left(126c^2+21cdx+2d^2x^2\right)-35a^3b^3d^3\left(420c^3+126c^2dx+28cd^2x^2+3d^3x^3\right)+21a^2b^4d^2\left(700c^4+350c^3dx+140c^2d^2x^2+35cd^3x^3+4d^4x^4\right)-7ab^5d\left(1260c^5+1050c^4dx+700c^3d^2x^2+315c^2d^3x^3+84cd^4x^4+10d^5x^5\right)+b^6\left(2940c^6+4410c^5dx+4900c^4d^2x^2+3675c^3d^3x^3+1764c^2d^4x^4+490cd^5x^5+60d^6x^6\right)\right)}{420b^7}+\frac{(bc-ad)^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*x*(420*a^6*d^6 - 210*a^5*b*d^5*(14*c + d*x) + 70*a^4*b^2*d^4*(126*c^2 +
21*c*d*x + 2*d^2*x^2) - 35*a^3*b^3*d^3*(420*c^3 + 126*c^2*d*x + 28*c*d^2*x^
2 + 3*d^3*x^3) + 21*a^2*b^4*d^2*(700*c^4 + 350*c^3*d*x + 140*c^2*d^2*x^2 +
35*c*d^3*x^3 + 4*d^4*x^4) - 7*a*b^5*d*(1260*c^5 + 1050*c^4*d*x + 700*c^3*d^
2*x^2 + 315*c^2*d^3*x^3 + 84*c*d^4*x^4 + 10*d^5*x^5) + b^6*(2940*c^6 + 4410
*c^5*d*x + 4900*c^4*d^2*x^2 + 3675*c^3*d^3*x^3 + 1764*c^2*d^4*x^4 + 490*c*d
^5*x^5 + 60*d^6*x^6)))/(420*b^7) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 358 vs. $2(169) = 338$.
time = 5.10, size = 356, normalized size = 2.11

$$\frac{420bdx\left(a^6d^6-7a^5bcd^5+21a^4b^2c^2d^4-35a^3b^3c^3d^3+35a^2b^4c^4d^2-21ab^5c^5d+7b^6c^6\right)+210b^2d^2x^2\left(-a^5d^5+7a^4bcd^4-21a^3b^2c^2d^3+35a^2b^3c^3d^2-35ab^4c^4d+21b^5c^5\right)+140b^3d^3x^3\left(a^4d^4-7a^3bcd^3+21a^2b^2c^2d^2-35ab^3c^3d+35b^4c^4\right)+105b^4d^4x^4\left(-a^3d^3+7a^2bcd^2-21ab^2c^2d+35b^3c^3\right)+84b^5d^5x^5\left(a^2d^2-7abcd+21b^2c^2\right)+70b^6d^6x^6\left(-ad+7bc\right)+60b^7d^7x^7-420\mathrm{Log}\left[a+bx\right]\left(ad-bc\right)^7}{420b^8}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^1,x]')
```

```
[Out] (420 b d x (a ^ 6 d ^ 6 - 7 a ^ 5 b c d ^ 5 + 21 a ^ 4 b ^ 2 c ^ 2 d ^ 4 -
35 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 35 a ^ 2 b ^ 4 c ^ 4 d ^ 2 - 21 a b ^ 5 c ^ 5
d + 7 b ^ 6 c ^ 6) + 210 b ^ 2 d ^ 2 x ^ 2 (-a ^ 5 d ^ 5 + 7 a ^ 4 b c d ^
4 - 21 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 35 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 35 a b ^ 4 c
^ 4 d + 21 b ^ 5 c ^ 5) + 140 b ^ 3 d ^ 3 x ^ 3 (a ^ 4 d ^ 4 - 7 a ^ 3 b c
d ^ 3 + 21 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 35 a b ^ 3 c ^ 3 d + 35 b ^ 4 c ^ 4) +
 105 b ^ 4 d ^ 4 x ^ 4 (-a ^ 3 d ^ 3 + 7 a ^ 2 b c d ^ 2 - 21 a b ^ 2 c ^ 2
 d + 35 b ^ 3 c ^ 3) + 84 b ^ 5 d ^ 5 x ^ 5 (a ^ 2 d ^ 2 - 7 a b c d + 21 b
 ^ 2 c ^ 2) + 70 b ^ 6 d ^ 6 x ^ 6 (-a d + 7 b c) + 60 b ^ 7 d ^ 7 x ^ 7 -
420 Log[a + b x] (a d - b c) ^ 7) / (420 b ^ 8)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(157) = 314$.
time = 0.14, size = 491, normalized size = 2.91

method	result
norman	$\frac{d\left(a^6d^6-7a^5bc\,d^5+21a^4b^2c^2d^4-35a^3b^3c^3d^3+35a^2b^4c^4d^2-21a\,b^5c^5d+7b^6c^6\right)x}{b^7}+\frac{d^7x^7}{7b}-\frac{d^2\left(a^5d^5-7a^4bc\,d^4+21a^3b^2c^2d^3-35a^2b\right.}{2b^6}$
default	$\frac{d\left(-\frac{1}{6}a\,b^5d^6x^6+\frac{7}{6}b^6c\,d^5x^6+\frac{1}{5}a^2b^4d^6x^5+\frac{21}{5}b^6c^2d^4x^5-\frac{1}{4}a^3b^3d^6x^4+\frac{35}{4}b^6c^3d^3x^4+\frac{1}{3}a^4b^2d^6x^3+\frac{35}{3}b^6c^4d^2x^3-\frac{1}{2}a^5b\,d^6x^2+\frac{1}{7}d^6x^7b^6+a\right.}{}$
risch	$\frac{\ln(bx+a)c^7}{b}-\frac{d^7a\,x^6}{6b^2}+\frac{7d^6c\,x^6}{6b}+\frac{d^7a^2x^5}{5b^3}+\frac{21d^5c^2x^5}{5b}-\frac{d^7a^3x^4}{4b^4}+\frac{35d^4c^3x^4}{4b}+\frac{d^7a^4x^3}{3b^5}+\frac{35d^3c^4x^3}{3b}-\frac{d^7a^5x^2}{2b^6}+\frac{d^7}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^7*(-1/6*a*b^5*d^6*x^6+7/6*b^6*c*d^5*x^6+1/5*a^2*b^4*d^6*x^5+21/5*b^6*c^
2*d^4*x^5-1/4*a^3*b^3*d^6*x^4+35/4*b^6*c^3*d^3*x^4+1/3*a^4*b^2*d^6*x^3+35/3
```

```
*b^6*c^4*d^2*x^3-1/2*a^5*b*d^6*x^2+1/7*d^6*x^7*b^6+a^6*d^6*x+7*b^6*c^6*x-21
/4*a*b^5*c^2*d^4*x^4-7/3*a^3*b^3*c*d^5*x^3+7*a^2*b^4*c^2*d^4*x^3-35/3*a*b^5
*c^3*d^3*x^3+7/2*a^4*b^2*c*d^5*x^2-21/2*a^3*b^3*c^2*d^4*x^2+35/2*a^2*b^4*c^
3*d^3*x^2-35/2*a*b^5*c^4*d^2*x^2-7*a^5*b*c*d^5*x+21*a^4*b^2*c^2*d^4*x-35*a^
3*b^3*c^3*d^3*x+35*a^2*b^4*c^4*d^2*x-21*a*b^5*c^5*d*x+21/2*b^6*c^5*d*x^2-7/
5*a*b^5*c*d^5*x^5+7/4*a^2*b^4*c*d^5*x^4)+(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2
*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c
^6*d+b^7*c^7)/b^8*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(157) = 314$.
time = 0.28, size = 460, normalized size = 2.72

$$\frac{60 b^6 d^7 x^7 + 70 (7 b^6 c d^6 - a b^5 d^7) x^6 + 84 (21 b^6 c^2 d^5 - 7 a b^5 c d^6 + a^2 b^4 d^7) x^5 + 105 (35 b^6 c^3 d^4 - 21 a b^5 c^2 d^5 + 7 a^2 b^4 c d^6 - a^3 b^3 d^7) x^4 + 140 (35 b^6 c^4 d^3 - 35 a b^5 c^3 d^4 + 21 a^2 b^4 c^2 d^5 - 7 a^3 b^3 c d^6 + a^4 b^2 d^7) x^3 + 210 (21 b^6 c^5 d^2 - 35 a b^5 c^4 d^3 + 35 a^2 b^4 c^3 d^4 - 21 a^3 b^3 c^2 d^5 + 7 a^4 b^2 c d^6 - a^5 b d^7) x^2 + 420 (7 b^6 c^6 d - 21 a b^5 c^5 d^2 + 35 a^2 b^4 c^4 d^3 - 35 a^3 b^3 c^3 d^4 + 21 a^4 b^2 c^2 d^5 - 7 a^5 b c d^6 + a^6 d^7) x}{420 b^7} + \frac{(b^7 c^7 - 7 a b^6 c^6 d + 21 a^2 b^5 c^5 d^2 - 35 a^3 b^4 c^4 d^3 + 35 a^4 b^3 c^3 d^4 - 21 a^5 b^2 c^2 d^5 + 7 a^6 b c d^6 - a^7 d^7) \log(bx + a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 70*(7*b^6*c*d^6 - a*b^5*d^7)*x^6 + 84*(21*b^6*c^2*d
^5 - 7*a*b^5*c*d^6 + a^2*b^4*d^7)*x^5 + 105*(35*b^6*c^3*d^4 - 21*a*b^5*c^2*
d^5 + 7*a^2*b^4*c*d^6 - a^3*b^3*d^7)*x^4 + 140*(35*b^6*c^4*d^3 - 35*a*b^5*c
^3*d^4 + 21*a^2*b^4*c^2*d^5 - 7*a^3*b^3*c*d^6 + a^4*b^2*d^7)*x^3 + 210*(21*
b^6*c^5*d^2 - 35*a*b^5*c^4*d^3 + 35*a^2*b^4*c^3*d^4 - 21*a^3*b^3*c^2*d^5 +
7*a^4*b^2*c*d^6 - a^5*b*d^7)*x^2 + 420*(7*b^6*c^6*d - 21*a*b^5*c^5*d^2 + 35
*a^2*b^4*c^4*d^3 - 35*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6
+ a^6*d^7)*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*
b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7
*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(157) = 314$.
time = 0.32, size = 462, normalized size = 2.73

$$\frac{60 b^7 d^7 x^7 + 70 (7 b^7 c d^6 - a b^6 d^7) x^6 + 84 (21 b^7 c^2 d^5 - 7 a b^6 c d^6 + a^2 b^5 d^7) x^5 + 105 (35 b^7 c^3 d^4 - 21 a b^6 c^2 d^5 + 7 a^2 b^5 c d^6 - a^3 b^4 d^7) x^4 + 140 (35 b^7 c^4 d^3 - 35 a b^6 c^3 d^4 + 21 a^2 b^5 c^2 d^5 - 7 a^3 b^4 c d^6 + a^4 b^3 d^7) x^3 + 210 (21 b^7 c^5 d^2 - 35 a b^6 c^4 d^3 + 35 a^2 b^5 c^3 d^4 - 21 a^3 b^4 c^2 d^5 + 7 a^4 b^3 c d^6 - a^5 b^2 d^7) x^2 + 420 (7 b^7 c^6 d - 21 a b^6 c^5 d^2 + 35 a^2 b^5 c^4 d^3 - 35 a^3 b^4 c^3 d^4 + 21 a^4 b^3 c^2 d^5 - 7 a^5 b^2 c d^6 + a^6 b d^7) x + 420 (b^7 c^7 - 7 a b^6 c^6 d + 21 a^2 b^5 c^5 d^2 - 35 a^3 b^4 c^4 d^3 + 35 a^4 b^3 c^3 d^4 - 21 a^5 b^2 c^2 d^5 + 7 a^6 b c d^6 - a^7 d^7) \log(bx + a)}{420 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/420*(60*b^7*d^7*x^7 + 70*(7*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(21*b^7*c^2*d
^5 - 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(35*b^7*c^3*d^4 - 21*a*b^6*c^2*
d^5 + 7*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(35*b^7*c^4*d^3 - 35*a*b^6*c
^3*d^4 + 21*a^2*b^5*c^2*d^5 - 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(21*
b^7*c^5*d^2 - 35*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 - 21*a^3*b^4*c^2*d^5 +
7*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 420*(7*b^7*c^6*d - 21*a*b^6*c^5*d^2 +
35*a^2*b^5*c^4*d^3 - 35*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 - 7*a^5*b^2*c*
d^6 + a^6*b*d^7)*x + 420*(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35
```

```
*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6
- a^7*d^7)*log(b*x + a))/b^8
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(144) = 288$.
time = 0.49, size = 408, normalized size = 2.41

$$x^6\left(-\frac{ad^7}{6b^2}+\frac{7cd^6}{6b}\right)+x^5\left(\frac{a^2d^7}{5b^3}-\frac{7acd^6}{5b^2}+\frac{21c^2d^5}{5b}\right)+x^4\left(-\frac{a^3d^7}{4b^4}+\frac{7a^2cd^6}{4b^3}-\frac{21ac^2d^5}{4b^2}+\frac{35c^3d^4}{4b}\right)+x^3\left(\frac{a^4d^7}{3b^5}-\frac{7a^3cd^6}{3b^4}+\frac{7a^2c^2d^5}{b^3}-\frac{35ac^3d^4}{3b^2}+\frac{35c^4d^3}{3b}\right)+x^2\left(-\frac{a^5d^7}{2b^6}+\frac{7a^4cd^6}{2b^5}-\frac{21a^3c^2d^5}{2b^4}+\frac{35a^2c^3d^4}{2b^3}-\frac{35ac^4d^3}{2b^2}+\frac{21c^5d^2}{2b}\right)+x\left(\frac{a^6d^7}{b^7}-\frac{7a^5cd^6}{b^6}+\frac{21a^4c^2d^5}{b^5}-\frac{35a^3c^3d^4}{b^4}+\frac{35a^2c^4d^3}{b^3}-\frac{21ac^5d^2}{b^2}+\frac{7c^6d}{b}\right)+\frac{d^7x^7}{7b}-\frac{(ad-bc)^7\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a),x)
```

```
[Out] x**6*(-a*d**7/(6*b**2) + 7*c*d**6/(6*b)) + x**5*(a**2*d**7/(5*b**3) - 7*a*c
*d**6/(5*b**2) + 21*c**2*d**5/(5*b)) + x**4*(-a**3*d**7/(4*b**4) + 7*a**2*c
*d**6/(4*b**3) - 21*a*c**2*d**5/(4*b**2) + 35*c**3*d**4/(4*b)) + x**3*(a**4
*d**7/(3*b**5) - 7*a**3*c*d**6/(3*b**4) + 7*a**2*c**2*d**5/b**3 - 35*a*c**3
*d**4/(3*b**2) + 35*c**4*d**3/(3*b)) + x**2*(-a**5*d**7/(2*b**6) + 7*a**4*c
*d**6/(2*b**5) - 21*a**3*c**2*d**5/(2*b**4) + 35*a**2*c**3*d**4/(2*b**3) -
35*a*c**4*d**3/(2*b**2) + 21*c**5*d**2/(2*b)) + x*(a**6*d**7/b**7 - 7*a**5*
c*d**6/b**6 + 21*a**4*c**2*d**5/b**5 - 35*a**3*c**3*d**4/b**4 + 35*a**2*c**
4*d**3/b**3 - 21*a*c**5*d**2/b**2 + 7*c**6*d/b) + d**7*x**7/(7*b) - (a*d -
b*c)**7*log(a + b*x)/b**8
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(157) = 314$.
time = 0.00, size = 552, normalized size = 3.27

$$\frac{\frac{1}{7}x^7d^7b^6-\frac{1}{6}x^6d^7b^5a+\frac{7}{6}x^6d^6cb^6+\frac{1}{5}x^5d^7b^4a^2-\frac{7}{5}x^5d^6cb^5a+\frac{21}{5}x^5d^5c^2b^6-\frac{1}{4}x^4d^7b^3a^3+\frac{7}{4}x^4d^6cb^4a^2-\frac{21}{4}x^4d^5c^2b^5a+\frac{35}{4}x^4d^4c^3b^6+\frac{1}{3}x^3d^7b^2a^4-\frac{7}{3}x^3d^6cb^3a^3+7x^3d^5c^2b^4a^2-\frac{35}{3}x^3d^4c^3b^5a+\frac{35}{3}x^3d^3c^4b^6-\frac{1}{2}x^2d^7ba^5+\frac{7}{2}x^2d^6cb^2a^4-\frac{21}{2}x^2d^5c^2b^3a^3+\frac{35}{2}x^2d^4c^3b^4a^2-\frac{35}{2}x^2d^3c^4b^5a+\frac{21}{2}x^2d^2c^5b^6+xd^7a^6-7xd^6cba^5+21xd^5c^2b^2a^4-35xd^4c^3b^3a^3+35xd^3c^4b^4a^2-21xd^2c^5b^5a+7xdc^6b^6}{b^7}+\frac{(-d^7a^7+7d^6cba^6-21d^5c^2b^2a^5+35d^4c^3b^3a^4-35d^3c^4b^4a^3+21d^2c^5b^5a^2-7dc^6b^6a+c^7b^7)\ln|xb+a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x)
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 490*b^6*c*d^6*x^6 - 70*a*b^5*d^7*x^6 + 1764*b^6*c^2
*d^5*x^5 - 588*a*b^5*c*d^6*x^5 + 84*a^2*b^4*d^7*x^5 + 3675*b^6*c^3*d^4*x^4
- 2205*a*b^5*c^2*d^5*x^4 + 735*a^2*b^4*c*d^6*x^4 - 105*a^3*b^3*d^7*x^4 + 49
00*b^6*c^4*d^3*x^3 - 4900*a*b^5*c^3*d^4*x^3 + 2940*a^2*b^4*c^2*d^5*x^3 - 98
0*a^3*b^3*c*d^6*x^3 + 140*a^4*b^2*d^7*x^3 + 4410*b^6*c^5*d^2*x^2 - 7350*a*b
^5*c^4*d^3*x^2 + 7350*a^2*b^4*c^3*d^4*x^2 - 4410*a^3*b^3*c^2*d^5*x^2 + 1470
*a^4*b^2*c*d^6*x^2 - 210*a^5*b*d^7*x^2 + 2940*b^6*c^6*d*x - 8820*a*b^5*c^5*
d^2*x + 14700*a^2*b^4*c^4*d^3*x - 14700*a^3*b^3*c^3*d^4*x + 8820*a^4*b^2*c^
2*d^5*x - 2940*a^5*b*c*d^6*x + 420*a^6*d^7*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*
d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b
^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*log(abs(b*x + a))/b^8
```

Mupad [B]
time = 0.22, size = 509, normalized size = 3.01

$$x\left(\frac{7c^6d}{b}-\frac{a\left(\frac{a\left(\frac{a\left(\frac{35c^3d^4}{b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{b}\right)}{b}-\frac{35c^4d^3}{b}\right)}{b}+\frac{21c^5d^2}{b}\right)}{b}\right)-x^6\left(\frac{ad^7}{6b^2}-\frac{7cd^6}{6b}\right)+x^4\left(\frac{35c^3d^4}{4b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{4b}\right)+x^2\left(\frac{a\left(\frac{a\left(\frac{35c^3d^4}{b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{b}\right)}{b}-\frac{35c^4d^3}{b}\right)}{2b}+\frac{21c^5d^2}{2b}\right)+x^5\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{5b}+\frac{21c^2d^5}{5b}\right)-x^3\left(\frac{a\left(\frac{35c^3d^4}{b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{b}\right)}{3b}-\frac{35c^4d^3}{3b}\right)-\frac{\ln(a+bx)\,(a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7)}{b^8}+\frac{d^7x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x),x)
```

```
[Out] x*((7*c^6*d)/b - (a*((a*((a*((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^
6)/b))/b + (21*c^2*d^5)/b))/b))/b - (35*c^4*d^3)/b))/b + (21*c^5*d^2)/b))/b
) - x^6*((a*d^7)/(6*b^2) - (7*c*d^6)/(6*b)) + x^4*((35*c^3*d^4)/(4*b) - (a*
((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/(4*b)) + x^2*((a*((a*
((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/
b))/b - (35*c^4*d^3)/b))/(2*b) + (21*c^5*d^2)/(2*b)) + x^5*((a*((a*d^7)/b^2
 - (7*c*d^6)/b))/(5*b) + (21*c^2*d^5)/(5*b)) - x^3*((a*((35*c^3*d^4)/b - (a
*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/b))/(3*b) - (35*c^4*
d^3)/(3*b)) - (log(a + b*x)*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6))/b^8 + (d^7*x^7)/(7*b)
```

3.1284 $\int \frac{(c+dx)^7}{(a+bx)^2}\,dx$

Optimal. Leaf size=187

$$\frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\frac{21d^5(bc-ad)^2(a+bx)^4}{4b^8}$$

```
[Out] 21*d^2*(-a*d+b*c)^5*x/b^7-(-a*d+b*c)^7/b^8/(b*x+a)+35/2*d^3*(-a*d+b*c)^4*(b
*x+a)^2/b^8+35/3*d^4*(-a*d+b*c)^3*(b*x+a)^3/b^8+21/4*d^5*(-a*d+b*c)^2*(b*x+
a)^4/b^8+7/5*d^6*(-a*d+b*c)*(b*x+a)^5/b^8+1/6*d^7*(b*x+a)^6/b^8+7*d*(-a*d+b
*c)^6*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{7d(bc-ad)^6\log(a+bx)}{b^8}+\frac{d^7(a+bx)^6}{6b^8}+\frac{21d^2x(bc-ad)^5}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (21*d^2*(b*c - a*d)^5*x)/b^7 - (b*c - a*d)^7/(b^8*(a + b*x)) + (35*d^3*(b*c
 - a*d)^4*(a + b*x)^2)/(2*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^3)/(3*b^8)
 + (21*d^5*(b*c - a*d)^2*(a + b*x)^4)/(4*b^8) + (7*d^6*(b*c - a*d)*(a + b*x
)^5)/(5*b^8) + (d^7*(a + b*x)^6)/(6*b^8) + (7*d*(b*c - a*d)^6*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^2}\,dx &= \int \left(\frac{21d^2(bc-ad)^5}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^2}+\frac{7d(bc-ad)^6}{b^7(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)}{b^7}+\frac{35d^4(bc-\right.\\ &= \frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 388 vs. 2(187) = 374.
time = 0.08, size = 388, normalized size = 2.07

$$\frac{60a^7d^7-60a^6bd^6(7c+6dx)+210a^5b^2d^5(6c^2+10cdx-d^2x^2)+70a^4b^3d^4(-30c^3-72c^2dx+18cd^2x^2+d^3x^3)-35a^3b^4d^3(-60c^4-180c^3dx+90c^2d^2x^2+12cd^3x^3+d^4x^4)+21a^2b^5d^2(-60c^5-200c^4dx+200c^3d^2x^2+50c^2d^3x^3+10cd^4x^4+d^5x^5)-7ab^6d(-60c^6-180c^5dx+450c^4d^2x^2+200c^3d^3x^3+75c^2d^4x^4+18cd^5x^5+2d^6x^6)+b^7(-60c^7+1260c^5d^2x^2+1050c^4d^3x^3+700c^3d^4x^4+315c^2d^5x^5+84cd^6x^6+10d^7x^7)+420d(bc-ad)^6(a+bx)\log(a+bx)}{60b^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (60*a^7*d^7 - 60*a^6*b*d^6*(7*c + 6*d*x) + 210*a^5*b^2*d^5*(6*c^2 + 10*c*d*
x - d^2*x^2) + 70*a^4*b^3*d^4*(-30*c^3 - 72*c^2*d*x + 18*c*d^2*x^2 + d^3*x^
3) - 35*a^3*b^4*d^3*(-60*c^4 - 180*c^3*d*x + 90*c^2*d^2*x^2 + 12*c*d^3*x^3
+ d^4*x^4) + 21*a^2*b^5*d^2*(-60*c^5 - 200*c^4*d*x + 200*c^3*d^2*x^2 + 50*c
^2*d^3*x^3 + 10*c*d^4*x^4 + d^5*x^5) - 7*a*b^6*d*(-60*c^6 - 180*c^5*d*x + 4
50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 75*c^2*d^4*x^4 + 18*c*d^5*x^5 + 2*d^6*x^
6) + b^7*(-60*c^7 + 1260*c^5*d^2*x^2 + 1050*c^4*d^3*x^3 + 700*c^3*d^4*x^4 +
 315*c^2*d^5*x^5 + 84*c*d^6*x^6 + 10*d^7*x^7) + 420*d*(b*c - a*d)^6*(a + b*
x)*Log[a + b*x])/(60*b^8*(a + b*x))
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 414 vs. 2(187) = 374.
time = 5.56, size = 404, normalized size = 2.16

$$\frac{7d\text{Log}\left[a+bx\right](a+bx)(ad-bc)^6+a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7-bd^2x(a+bx)(6a^5d^5-35a^4bcd^4+84a^3b^2c^2d^3-105a^2b^3c^3d^2+70ab^4c^4d-21b^5c^5)+\frac{b^2d^3x^2(a+bx)(5a^4d^4-28a^3bcd^3+63a^2b^2c^2d^2-70ab^3c^3d+35b^4c^4)}{2}-\frac{b^3d^4x^3(a+bx)(4a^3d^3-21a^2bcd^2+42ab^2c^2d-35b^3c^3)}{3}+\frac{b^4d^5x^4(a+bx)(3a^2d^2-14abcd+21b^2c^2)}{4}-\frac{b^5d^6x^5(a+bx)(2ad-7bc)}{5}+\frac{b^6d^7x^6(a+bx)}{6}}{b^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^2,x]')
```

```
[Out] (7 d Log[a + b x] (a + b x) (a d - b c) ^ 6 + a ^ 7 d ^ 7 - 7 a ^ 6 b c d ^
 6 + 21 a ^ 5 b ^ 2 c ^ 2 d ^ 5 - 35 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 35 a ^ 3 b ^
 4 c ^ 4 d ^ 3 - 21 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 7 a b ^ 6 c ^ 6 d - b ^ 7 c ^
 7 - b d ^ 2 x (a + b x) (6 a ^ 5 d ^ 5 - 35 a ^ 4 b c d ^ 4 + 84 a ^ 3 b ^
 2 c ^ 2 d ^ 3 - 105 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 70 a b ^ 4 c ^ 4 d - 21 b ^
5 c ^ 5) + b ^ 2 d ^ 3 x ^ 2 (a + b x) (5 a ^ 4 d ^ 4 - 28 a ^ 3 b c d ^ 3
+ 63 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 70 a b ^ 3 c ^ 3 d + 35 b ^ 4 c ^ 4) / 2 - b
 ^ 3 d ^ 4 x ^ 3 (a + b x) (4 a ^ 3 d ^ 3 - 21 a ^ 2 b c d ^ 2 + 42 a b ^ 2
 c ^ 2 d - 35 b ^ 3 c ^ 3) / 3 + b ^ 4 d ^ 5 x ^ 4 (a + b x) (3 a ^ 2 d ^ 2
 - 14 a b c d + 21 b ^ 2 c ^ 2) / 4 - b ^ 5 d ^ 6 x ^ 5 (a + b x) (2 a d -
7 b c) / 5 + b ^ 6 d ^ 7 x ^ 6 (a + b x) / 6) / (b ^ 8 (a + b x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(177) = 354.
time = 0.17, size = 479, normalized size = 2.56

method	result

norman	$\frac{d^7x^7}{6b}-\frac{7d^2\left(a^5d^5-6a^4bc\,d^4+15a^3b^2c^2d^3-20a^2b^3c^3d^2+15a\,b^4c^4d-6b^5c^5\right)x^2}{2b^6}+\frac{7d^3\left(a^4d^4-6a^3bc\,d^3+15a^2b^2c^2d^2-20a\,b^3c^3d+15b^4c^4\right)x^3}{6b^5}-\frac{7d^4(}{}$
default	$-\frac{d^2(-\frac{1}{6}d^5x^6b^5+\frac{2}{5}a\,b^4d^5x^5-\frac{7}{5}b^5c\,d^4x^5-\frac{3}{4}a^2b^3d^5x^4+\frac{7}{2}a\,b^4c\,d^4x^4-\frac{21}{4}b^5c^2d^3x^4+\frac{4}{3}a^3b^2d^5x^3-7a^2b^3c\,d^4x^3+14a\,b^4c^2d^3x^3-\frac{35}{3}b^5c^3}{}$
risch	$\frac{d^7x^6}{6b^2}-\frac{c^7}{b(bx+a)}-\frac{42d^6\ln(bx+a)a^5c}{b^7}+\frac{105d^5\ln(bx+a)a^4c^2}{b^6}-\frac{140d^4\ln(bx+a)a^3c^3}{b^5}+\frac{105d^3\ln(bx+a)a^2c^4}{b^4}-\frac{42d^2\ln(bx+}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^7*(-1/6*d^5*x^6*b^5+2/5*a*b^4*d^5*x^5-7/5*b^5*c*d^4*x^5-3/4*a^2*b^3*
d^5*x^4+7/2*a*b^4*c*d^4*x^4-21/4*b^5*c^2*d^3*x^4+4/3*a^3*b^2*d^5*x^3-7*a^2*
b^3*c*d^4*x^3+14*a*b^4*c^2*d^3*x^3-35/3*b^5*c^3*d^2*x^3-5/2*a^4*b*d^5*x^2+1
4*a^3*b^2*c*d^4*x^2-63/2*a^2*b^3*c^2*d^3*x^2+35*a*b^4*c^3*d^2*x^2-35/2*b^5*
c^4*d*x^2+6*a^5*d^5*x-35*a^4*b*c*d^4*x+84*a^3*b^2*c^2*d^3*x-105*a^2*b^3*c^3
*d^2*x+70*a*b^4*c^4*d*x-21*b^5*c^5*x)-1/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*
b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^
6*c^6*d+b^7*c^7)/(b*x+a)+7/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-
20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(177) = 354$.
time = 0.28, size = 467, normalized size = 2.50

$$-\frac{b^7c^7-7\,ab^6c^6d+21\,a^2b^5c^5d^2-35\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4-21\,a^5b^2c^2d^5+7\,a^6bcd^6-a^7d^7}{b^9x+ab^8}+\frac{10\,b^5d^7x^6+12\,(7\,b^5cd^6-2\,ab^4d^7)x^5+15\,(21\,b^5c^2d^5-14\,ab^4cd^6+3\,a^2b^3d^7)x^4+20\,(35\,b^5c^3d^4-42\,ab^4c^2d^5+21\,a^2b^3cd^6-4\,a^3b^2d^7)x^3+30\,(35\,b^5c^4d^3-70\,ab^4c^3d^4+63\,a^2b^3c^2d^5-28\,a^3b^2cd^6+5\,a^4bd^7)x^2+60\,(21\,b^5c^5d^2-70\,ab^4c^4d^3+105\,a^2b^3c^3d^4-84\,a^3b^2c^2d^5+35\,a^4bcd^6-6\,a^5d^7)x}{60\,b^7}+\frac{7\,(b^6c^6d-6\,ab^5c^5d^2+15\,a^2b^4c^4d^3-20\,a^3b^3c^3d^4+15\,a^4b^2c^2d^5-6\,a^5bcd^6+a^6d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(b^9*x + a*b^
8) + 1/60*(10*b^5*d^7*x^6 + 12*(7*b^5*c*d^6 - 2*a*b^4*d^7)*x^5 + 15*(21*b^5
*c^2*d^5 - 14*a*b^4*c*d^6 + 3*a^2*b^3*d^7)*x^4 + 20*(35*b^5*c^3*d^4 - 42*a*
b^4*c^2*d^5 + 21*a^2*b^3*c*d^6 - 4*a^3*b^2*d^7)*x^3 + 30*(35*b^5*c^4*d^3 -
70*a*b^4*c^3*d^4 + 63*a^2*b^3*c^2*d^5 - 28*a^3*b^2*c*d^6 + 5*a^4*b*d^7)*x^2
 + 60*(21*b^5*c^5*d^2 - 70*a*b^4*c^4*d^3 + 105*a^2*b^3*c^3*d^4 - 84*a^3*b^2
*c^2*d^5 + 35*a^4*b*c*d^6 - 6*a^5*d^7)*x)/b^7 + 7*(b^6*c^6*d - 6*a*b^5*c^5*
d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*
b*c*d^6 + a^6*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(177) = 354$.
time = 0.29, size = 632, normalized size = 3.38

$$\frac{10\,b^7d^7x^7-60\,b^7c^7+420\,ab^6c^6d-1260\,a^2b^5c^5d^2+2100\,a^3b^4c^4d^3-2100\,a^4b^3c^3d^4+1260\,a^5b^2c^2d^5-420\,a^6bcd^6+60\,a^7d^7+14\,(6\,b^7cd^6-ab^6d^7)x^6+21\,(15\,b^7c^2d^5-6\,ab^6cd^6+a^2b^5d^7)x^5+35\,(20\,b^7c^3d^4-15\,ab^6c^2d^5+6\,a^2b^5cd^6-a^3b^4d^7)x^4+70\,(15\,b^7c^4d^3-20\,ab^6c^3d^4+15\,a^2b^5c^2d^5-6\,a^3b^4cd^6+a^4b^3d^7)x^3+210\,(6\,b^7c^5d^2-15\,ab^6c^4d^3+20\,a^2b^5c^3d^4-15\,a^3b^4c^2d^5+6\,a^4b^3cd^6-a^5b^2d^7)x^2+60\,(21\,ab^6c^5d^2-70\,a^2b^5c^4d^3+105\,a^3b^4c^3d^4-84\,a^4b^3c^2d^5+35\,a^5b^2cd^6-6\,a^6bd^7)x+420\,(ab^6c^6d-6\,a^2b^5c^5d^2+15\,a^3b^4c^4d^3-20\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5-6\,a^6bcd^6+a^7d^7+(b^7c^6d-6\,ab^6c^5d^2+15\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5-6\,a^5b^2cd^6+a^6bd^7)x)\log(bx+a)}{60\,(b^9x+ab^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*d^7*x^7 - 60*b^7*c^7 + 420*a*b^6*c^6*d - 1260*a^2*b^5*c^5*d^2
+ 2100*a^3*b^4*c^4*d^3 - 2100*a^4*b^3*c^3*d^4 + 1260*a^5*b^2*c^2*d^5 - 420*
a^6*b*c*d^6 + 60*a^7*d^7 + 14*(6*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(15*b^7*c^
2*d^5 - 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(20*b^7*c^3*d^4 - 15*a*b^6*c^
2*d^5 + 6*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 70*(15*b^7*c^4*d^3 - 20*a*b^6*
c^3*d^4 + 15*a^2*b^5*c^2*d^5 - 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(6*
b^7*c^5*d^2 - 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 15*a^3*b^4*c^2*d^5 +
6*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 60*(21*a*b^6*c^5*d^2 - 70*a^2*b^5*c^4*
d^3 + 105*a^3*b^4*c^3*d^4 - 84*a^4*b^3*c^2*d^5 + 35*a^5*b^2*c*d^6 - 6*a^6*b
*d^7)*x + 420*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d -
 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2
*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^9*x + a*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(172) = 344$.
time = 0.85, size = 428, normalized size = 2.29

$$x^{5}\left(-\frac{2ad^{7}}{5b^{3}}+\frac{7cd^{6}}{5b^{2}}\right)+x^{4}\cdot\left(\frac{3a^{2}d^{7}}{4b^{4}}-\frac{7acd^{6}}{2b^{3}}+\frac{21c^{2}d^{5}}{4b^{2}}\right)+x^{3}\left(-\frac{4a^{3}d^{7}}{3b^{5}}+\frac{7a^{2}cd^{6}}{b^{4}}-\frac{14ac^{2}d^{5}}{b^{3}}+\frac{35c^{3}d^{4}}{3b^{2}}\right)+x^{2}\cdot\left(\frac{5a^{4}d^{7}}{2b^{6}}-\frac{14a^{3}cd^{6}}{b^{5}}+\frac{63a^{2}c^{2}d^{5}}{2b^{4}}-\frac{35ac^{3}d^{4}}{b^{3}}+\frac{35c^{4}d^{3}}{2b^{2}}\right)+x\left(-\frac{6a^{5}d^{7}}{b^{7}}+\frac{35a^{4}cd^{6}}{b^{6}}-\frac{84a^{3}c^{2}d^{5}}{b^{5}}+\frac{105a^{2}c^{3}d^{4}}{b^{4}}-\frac{70ac^{4}d^{3}}{b^{3}}+\frac{21c^{5}d^{2}}{b^{2}}\right)+\frac{a^{7}d^{7}-7a^{6}bcd^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35a^{3}b^{4}c^{4}d^{3}-21a^{2}b^{5}c^{5}d^{2}+7ab^{6}c^{6}d-b^{7}c^{7}}{ab^{8}+b^{9}x}+\frac{d^{7}x^{6}}{6b^{2}}+\frac{7d\left(ad-bc\right)^{6}\log\left(a+bx\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**2,x)
```

```
[Out] x**5*(-2*a*d**7/(5*b**3) + 7*c*d**6/(5*b**2)) + x**4*(3*a**2*d**7/(4*b**4)
- 7*a*c*d**6/(2*b**3) + 21*c**2*d**5/(4*b**2)) + x**3*(-4*a**3*d**7/(3*b**5
) + 7*a**2*c*d**6/b**4 - 14*a*c**2*d**5/b**3 + 35*c**3*d**4/(3*b**2)) + x**
2*(5*a**4*d**7/(2*b**6) - 14*a**3*c*d**6/b**5 + 63*a**2*c**2*d**5/(2*b**4)
- 35*a*c**3*d**4/b**3 + 35*c**4*d**3/(2*b**2)) + x*(-6*a**5*d**7/b**7 + 35*
a**4*c*d**6/b**6 - 84*a**3*c**2*d**5/b**5 + 105*a**2*c**3*d**4/b**4 - 70*a*
c**4*d**3/b**3 + 21*c**5*d**2/b**2) + (a**7*d**7 - 7*a**6*b*c*d**6 + 21*a**
5*b**2*c**2*d**5 - 35*a**4*b**3*c**3*d**4 + 35*a**3*b**4*c**4*d**3 - 21*a**
2*b**5*c**5*d**2 + 7*a*b**6*c**6*d - b**7*c**7)/(a*b**8 + b**9*x) + d**7*x*
*6/(6*b**2) + 7*d*(a*d - b*c)**6*log(a + b*x)/b**8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(177) = 354$.
time = 0.00, size = 530, normalized size = 2.83

$$\frac{\frac{1}{6}x^{6}d^{7}b^{10}-\frac{2}{5}x^{5}d^{7}b^{9}a+\frac{7}{5}x^{5}d^{6}cb^{10}+\frac{3}{4}x^{4}d^{7}b^{8}a^{2}-\frac{7}{2}x^{4}d^{6}cb^{9}a+\frac{21}{4}x^{4}d^{5}c^{2}b^{10}-\frac{4}{3}x^{3}d^{7}b^{7}a^{3}+7x^{3}d^{6}cb^{8}a^{2}-14x^{3}d^{5}c^{2}b^{9}a+\frac{35}{3}x^{3}d^{4}c^{3}b^{10}+\frac{5}{2}x^{2}d^{7}b^{6}a^{4}-14x^{2}d^{6}cb^{7}a^{3}+\frac{63}{2}x^{2}d^{5}c^{2}b^{8}a^{2}-35x^{2}d^{4}c^{3}b^{9}a+\frac{35}{2}x^{2}d^{3}c^{4}b^{10}-6xd^{7}b^{5}a^{5}+35xd^{6}cb^{6}a^{4}-84xd^{5}c^{2}b^{7}a^{3}+105xd^{4}c^{3}b^{8}a^{2}-70xd^{3}c^{4}b^{9}a+21xd^{2}c^{5}b^{10}}{b^{12}}+\frac{d^{7}a^{7}-7d^{6}ba^{6}c+21d^{5}b^{2}a^{5}c^{2}-35d^{4}b^{3}a^{4}c^{3}+35d^{3}b^{4}a^{3}c^{4}-21d^{2}b^{5}a^{2}c^{5}+7db^{6}ac^{6}-b^{7}c^{7}}{b^{8}\left(xb+a\right)}+\frac{\left(7d^{7}a^{6}-42d^{6}cba^{5}+105d^{5}c^{2}b^{2}a^{4}-140d^{4}c^{3}b^{3}a^{3}+105d^{3}c^{4}b^{4}a^{2}-42d^{2}c^{5}b^{5}a+7dc^{6}b^{6}\right)\ln\left|xb+a\right|}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x)
```

```
[Out] 7*(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 +
15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*log(abs(b*x + a))/b^8 - (b^7*
c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*
c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/((b*x + a)*b^8) + 1
/60*(10*b^10*d^7*x^6 + 84*b^10*c*d^6*x^5 - 24*a*b^9*d^7*x^5 + 315*b^10*c^2*
d^5*x^4 - 210*a*b^9*c*d^6*x^4 + 45*a^2*b^8*d^7*x^4 + 700*b^10*c^3*d^4*x^3 -
 840*a*b^9*c^2*d^5*x^3 + 420*a^2*b^8*c*d^6*x^3 - 80*a^3*b^7*d^7*x^3 + 1050*
b^10*c^4*d^3*x^2 - 2100*a*b^9*c^3*d^4*x^2 + 1890*a^2*b^8*c^2*d^5*x^2 - 840*
a^3*b^7*c*d^6*x^2 + 150*a^4*b^6*d^7*x^2 + 1260*b^10*c^5*d^2*x - 4200*a*b^9*
c^4*d^3*x + 6300*a^2*b^8*c^3*d^4*x - 5040*a^3*b^7*c^2*d^5*x + 2100*a^4*b^6*
c*d^6*x - 360*a^5*b^5*d^7*x)/b^12
```

Mupad [B]

time = 0.24, size = 841, normalized size = 4.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^2,x)
```

```
[Out] x^4*((a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(2*b) - (a^2*d^7)/(4*b^4) + (21*c^
2*d^5)/(4*b^2)) - x^2*((a*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3 - (
7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^7)/b
^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)/(2*b^2) + (a^2*((2*a*((2*a*d^7)
/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/(2*b^2)) - x^
5*((2*a*d^7)/(5*b^3) - (7*c*d^6)/(5*b^2)) + x*((2*a*((2*a*((35*c^3*d^4)/b^2
 - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*
d^5)/b^2))/b + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)
/b^2 + (a^2*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*
c^2*d^5)/b^2))/b^2))/b - (a^2*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3
 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^
7)/b^3 - (7*c*d^6)/b^2))/b^2))/b^2 + (21*c^5*d^2)/b^2) + x^3*((35*c^3*d^4)/
(3*b^2) - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (
21*c^2*d^5)/b^2))/(3*b) + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(3*b^2)) +
(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c
^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)/(b*(a*b^7 + b^
8*x)) + (d^7*x^6)/(6*b^2) + (log(a + b*x)*(7*a^6*d^7 + 7*b^6*c^6*d - 42*a*b
^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4 + 105*a^4*b^2*c^2*d^
5 - 42*a^5*b*c*d^6))/b^8
```

3.1285 $\int \frac{(c+dx)^7}{(a+bx)^3}\,dx$

Optimal. Leaf size=185

$$\frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-ad)^2(a+bx)^3}{b^8}+\frac{7d^6(bc-$$

```
[Out] 35*d^3*(-a*d+b*c)^4*x/b^7-1/2*(-a*d+b*c)^7/b^8/(b*x+a)^2-7*d*(-a*d+b*c)^6/b
^8/(b*x+a)+35/2*d^4*(-a*d+b*c)^3*(b*x+a)^2/b^8+7*d^5*(-a*d+b*c)^2*(b*x+a)^3
/b^8+7/4*d^6*(-a*d+b*c)*(b*x+a)^4/b^8+1/5*d^7*(b*x+a)^5/b^8+21*d^2*(-a*d+b*
c)^5*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{21d^2(bc-ad)^5\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{b^8(a+bx)}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}+\frac{d^7(a+bx)^5}{5b^8}+\frac{35d^3x(bc-ad)^4}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (35*d^3*(b*c - a*d)^4*x)/b^7 - (b*c - a*d)^7/(2*b^8*(a + b*x)^2) - (7*d*(b*
c - a*d)^6)/(b^8*(a + b*x)) + (35*d^4*(b*c - a*d)^3*(a + b*x)^2)/(2*b^8) +
(7*d^5*(b*c - a*d)^2*(a + b*x)^3)/b^8 + (7*d^6*(b*c - a*d)*(a + b*x)^4)/(4*
b^8) + (d^7*(a + b*x)^5)/(5*b^8) + (21*d^2*(b*c - a*d)^5*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^3}\,dx=\int \left(\frac{35d^3(bc-ad)^4}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^3}+\frac{7d(bc-ad)^6}{b^7(a+bx)^2}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)}+\frac{35d^4(bc-ad)^3(a+\right.$$
$$=\frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-a}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(185) = 370.

time = 0.08, size = 389, normalized size = 2.10

$$\frac{-130a^7d^7+10a^6bd^6(77c+16dx)+10a^5b^2d^5\left(-189c^2-56cdx+50d^2x^2\right)+70a^4b^3d^4\left(35c^3+6c^2dx-34cd^2x^2+2d^3x^3\right)-35a^3b^4d^3\left(50c^4-20c^3dx-126c^2d^2x^2+20cd^3x^3+d^4x^4\right)+7a^2b^5d^2\left(90c^5-200c^4dx-550c^3d^2x^2+200c^2d^3x^3+25cd^4x^4+2d^5x^5\right)-7ab^6d\left(10c^6-120c^5dx-200c^4d^2x^2+200c^3d^3x^3+50c^2d^4x^4+10cd^5x^5+d^6x^6\right)+b^7\left(-10c^7-140c^6dx+700c^4d^3x^3+350c^3d^4x^4+140c^2d^5x^5+35cd^6x^6+4d^7x^7\right)-420d^2(-bc+ad)^5(a+bx)^2\log(a+bx)}{20b^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (-130*a^7*d^7 + 10*a^6*b*d^6*(77*c + 16*d*x) + 10*a^5*b^2*d^5*(-189*c^2 - 5
6*c*d*x + 50*d^2*x^2) + 70*a^4*b^3*d^4*(35*c^3 + 6*c^2*d*x - 34*c*d^2*x^2 +
 2*d^3*x^3) - 35*a^3*b^4*d^3*(50*c^4 - 20*c^3*d*x - 126*c^2*d^2*x^2 + 20*c*
d^3*x^3 + d^4*x^4) + 7*a^2*b^5*d^2*(90*c^5 - 200*c^4*d*x - 550*c^3*d^2*x^2
+ 200*c^2*d^3*x^3 + 25*c*d^4*x^4 + 2*d^5*x^5) - 7*a*b^6*d*(10*c^6 - 120*c^5
*d*x - 200*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 50*c^2*d^4*x^4 + 10*c*d^5*x^5 +
d^6*x^6) + b^7*(-10*c^7 - 140*c^6*d*x + 700*c^4*d^3*x^3 + 350*c^3*d^4*x^4 +
 140*c^2*d^5*x^5 + 35*c*d^6*x^6 + 4*d^7*x^7) - 420*d^2*(-(b*c) + a*d)^5*(a
+ b*x)^2*Log[a + b*x])/(20*b^8*(a + b*x)^2)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 488 vs. 2(185) = 370.
time = 6.86, size = 486, normalized size = 2.63

$$\frac{-420d^2\mathrm{Log}\left[a+bx\right]\left(a^2+2abx+b^2x^2\right)(ad-bc)^5-130a^7d^7+770a^6bcd^6-1890a^5b^2c^2d^5+2450a^4b^3c^3d^4-1750a^3b^4c^4d^3+630a^2b^5c^5d^2-70ab^6c^6d-10b^7c^7-140bdx\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)+20bd^3x\left(a^2+2abx+b^2x^2\right)\left(15a^4d^4-70a^3bcd^3+126a^2b^2c^2d^2-105ab^3c^3d+35b^4c^4\right)-10b^2d^4x^2\left(a^2+2abx+b^2x^2\right)\left(10a^3d^3-42a^2bcd^2+63ab^2c^2d-35b^3c^3\right)+20b^3d^5x^3\left(a^2+2abx+b^2x^2\right)\left(2a^2d^2-7abcd+7b^2c^2\right)-5b^4d^6x^4\left(a^2+2abx+b^2x^2\right)(3ad-7bc)+4b^5d^7x^5\left(a^2+2abx+b^2x^2\right)}{20b^8\left(a^2+2abx+b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^3,x]')
```

```
[Out] (-420 d ^ 2 Log[a + b x] (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (a d - b c) ^ 5 -
130 a ^ 7 d ^ 7 + 770 a ^ 6 b c d ^ 6 - 1890 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 2450
 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 1750 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 630 a ^ 2 b ^ 5 c
 ^ 5 d ^ 2 - 70 a b ^ 6 c ^ 6 d - 10 b ^ 7 c ^ 7 - 140 b d x (a ^ 6 d ^ 6 -
 6 a ^ 5 b c d ^ 5 + 15 a ^ 4 b ^ 2 c ^ 2 d ^ 4 - 20 a ^ 3 b ^ 3 c ^ 3 d ^
3 + 15 a ^ 2 b ^ 4 c ^ 4 d ^ 2 - 6 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + 20 b d
^ 3 x (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (15 a ^ 4 d ^ 4 - 70 a ^ 3 b c d ^ 3
+ 126 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 105 a b ^ 3 c ^ 3 d + 35 b ^ 4 c ^ 4) - 10
b ^ 2 d ^ 4 x ^ 2 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (10 a ^ 3 d ^ 3 - 42 a ^
2 b c d ^ 2 + 63 a b ^ 2 c ^ 2 d - 35 b ^ 3 c ^ 3) + 20 b ^ 3 d ^ 5 x ^ 3 (
a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (2 a ^ 2 d ^ 2 - 7 a b c d + 7 b ^ 2 c ^ 2)
- 5 b ^ 4 d ^ 6 x ^ 4 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (3 a d - 7 b c) + 4 b
 ^ 5 d ^ 7 x ^ 5 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)) / (20 b ^ 8 (a ^ 2 + 2 a
b x + b ^ 2 x ^ 2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(177) = 354.
time = 0.15, size = 467, normalized size = 2.52

method	result

norman	$-\frac{63a^7d^7-315a^6bc\,d^6+630a^5b^2c^2d^5-630a^4b^3c^3d^4+315a^3b^4c^4d^3-63a^2b^5c^5d^2+7a\,b^6c^6d+b^7c^7}{2b^8}+\frac{d^7x^7}{5b}-\frac{(42a^6d^7-210a^5bc\,d^6+420a^4b^2c^2d^5-420}{}$
default	$\frac{d^3\left(\frac{1}{5}d^4x^5b^4-\frac{3}{4}a\,b^3d^4x^4+\frac{7}{4}b^4c\,d^3x^4+2a^2b^2d^4x^3-7a\,b^3c\,d^3x^3+7b^4c^2d^2x^3-5a^3b\,d^4x^2+21a^2b^2c\,d^3x^2-\frac{63}{2}a\,b^3c^2d^2x^2+\frac{35}{2}b^4c^3d\,x^2+15a\right.}{b^7}$
risch	$\frac{d^7x^5}{5b^3}-\frac{3d^7a\,x^4}{4b^4}+\frac{7d^6c\,x^4}{4b^3}+\frac{2d^7a^2x^3}{b^5}-\frac{7d^6ac\,x^3}{b^4}+\frac{7d^5c^2x^3}{b^3}-\frac{5d^7a^3x^2}{b^6}+\frac{21d^6a^2c\,x^2}{b^5}-\frac{63d^5a\,c^2x^2}{2b^4}+\frac{35d^4c^3x^2}{2b^3}+15$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/b^7*(1/5*d^4*x^5*b^4-3/4*a*b^3*d^4*x^4+7/4*b^4*c*d^3*x^4+2*a^2*b^2*d^4*
x^3-7*a*b^3*c*d^3*x^3+7*b^4*c^2*d^2*x^3-5*a^3*b*d^4*x^2+21*a^2*b^2*c*d^3*x^
2-63/2*a*b^3*c^2*d^2*x^2+35/2*b^4*c^3*d*x^2+15*a^4*d^4*x-70*a^3*b*c*d^3*x+1
26*a^2*b^2*c^2*d^2*x-105*a*b^3*c^3*d*x+35*b^4*c^4*x)-7/b^8*d*(a^6*d^6-6*a^5
*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c
^5*d+b^6*c^6)/(b*x+a)-1/2/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35
*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^
7)/(b*x+a)^2-21/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^
3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(177) = 354$.
time = 0.27, size = 473, normalized size = 2.56

$$-\frac{b^7c^7+7ab^6c^6d-63a^2b^5c^5d^2+175a^3b^4c^4d^3-245a^4b^3c^3d^4+189a^5b^2c^2d^5-77a^6bcd^6+13a^7d^7+14\left(b^7c^6d-6ab^6c^5d^2+15a^2b^5c^4d^3-20a^3b^4c^3d^4+15a^4b^3c^2d^5-6a^5b^2cd^6+a^6bd^7\right)x}{2\left(b^{10}x^2+2ab^9x+a^2b^8\right)}+\frac{4b^4d^7x^5+5\left(7b^4cd^6-3ab^3d^7\right)x^4+20\left(7b^4c^2d^5-7ab^3cd^6+2a^2b^2d^7\right)x^3+10\left(35b^4c^3d^4-63ab^3c^2d^5+42a^2b^2cd^6-10a^3bd^7\right)x^2+20\left(35b^4c^4d^3-105ab^3c^3d^4+126a^2b^2c^2d^5-70a^3bcd^6+15a^4d^7\right)x}{20b^7}+\frac{21\left(b^5c^5d^2-5ab^4c^4d^3+10a^2b^3c^3d^4-10a^3b^2c^2d^5+5a^4bcd^6-a^5d^7\right)\log\left(bx+a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*c^7 + 7*a*b^6*c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 -
245*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 1
4*(b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 +
15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^10*x^2 + 2*a*b^9*x
+ a^2*b^8) + 1/20*(4*b^4*d^7*x^5 + 5*(7*b^4*c*d^6 - 3*a*b^3*d^7)*x^4 + 20*(
7*b^4*c^2*d^5 - 7*a*b^3*c*d^6 + 2*a^2*b^2*d^7)*x^3 + 10*(35*b^4*c^3*d^4 - 6
3*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 - 10*a^3*b*d^7)*x^2 + 20*(35*b^4*c^4*d^3
 - 105*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 - 70*a^3*b*c*d^6 + 15*a^4*d^7)*x
)/b^7 + 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2
*c^2*d^5 + 5*a^4*b*c*d^6 - a^5*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(177) = 354$.
time = 0.29, size = 703, normalized size = 3.80

$$\frac{4b^7d^7x^7-10b^7c^7-70ab^6c^6d+630a^2b^5c^5d^2-1750a^3b^4c^4d^3+2450a^4b^3c^3d^4-1890a^5b^2c^2d^5+770a^6bcd^6-130a^7d^7+7\left(5b^7cd^6-ab^6d^7\right)x^6+14\left(10b^7c^2d^5-5ab^6cd^6+a^2b^5d^7\right)x^5+35\left(10b^7c^3d^4-10ab^6c^2d^5+5a^2b^5cd^6-a^3b^4d^7\right)x^4+140\left(5b^7c^4d^3-10ab^6c^3d^4+10a^2b^5c^2d^5-5a^3b^4cd^6+a^4b^3d^7\right)x^3+10\left(140ab^6c^4d^3-385a^2b^5c^3d^4+441a^3b^4c^2d^5-238a^4b^3cd^6+50a^5b^2d^7\right)x^2-20\left(7b^7c^6d-42ab^6c^5d^2+70a^2b^5c^4d^3-35a^3b^4c^3d^4-21a^4b^3c^2d^5+28a^5b^2cd^6-8a^6bd^7\right)x+420\left(a^2b^5c^5d^2-5a^3b^4c^4d^3+10a^4b^3c^3d^4-10a^5b^2c^2d^5+5a^6bcd^6-a^7d^7+\left(b^7c^5d^2-5ab^6c^4d^3+10a^2b^5c^3d^4-10a^3b^4c^2d^5+5a^4b^3cd^6-a^5b^2d^7\right)x^2+2\left(ab^6c^5d^2-5a^2b^5c^4d^3+10a^3b^4c^3d^4-10a^4b^3c^2d^5+5a^5b^2cd^6-a^6bd^7\right)x\right)\log\left(bx+a\right)}{20\left(b^{10}x^2+2ab^9x+a^2b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*d^7*x^7 - 10*b^7*c^7 - 70*a*b^6*c^6*d + 630*a^2*b^5*c^5*d^2 - 1
750*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 - 1890*a^5*b^2*c^2*d^5 + 770*a^6
*b*c*d^6 - 130*a^7*d^7 + 7*(5*b^7*c*d^6 - a*b^6*d^7)*x^6 + 14*(10*b^7*c^2*d
^5 - 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(10*b^7*c^3*d^4 - 10*a*b^6*c^2*d
^5 + 5*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(5*b^7*c^4*d^3 - 10*a*b^6*c^3
*d^4 + 10*a^2*b^5*c^2*d^5 - 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 10*(140*a*
b^6*c^4*d^3 - 385*a^2*b^5*c^3*d^4 + 441*a^3*b^4*c^2*d^5 - 238*a^4*b^3*c*d^6
 + 50*a^5*b^2*d^7)*x^2 - 20*(7*b^7*c^6*d - 42*a*b^6*c^5*d^2 + 70*a^2*b^5*c^
4*d^3 - 35*a^3*b^4*c^3*d^4 - 21*a^4*b^3*c^2*d^5 + 28*a^5*b^2*c*d^6 - 8*a^6*
b*d^7)*x + 420*(a^2*b^5*c^5*d^2 - 5*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 -
10*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 - a^7*d^7 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d
^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^
7)*x^2 + 2*(a*b^6*c^5*d^2 - 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 - 10*a^4
*b^3*c^2*d^5 + 5*a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^10*x^2 + 2*
a*b^9*x + a^2*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(170) = 340$.
time = 1.73, size = 447, normalized size = 2.42

$$x^4\left(-\frac{3ad^7}{4b^4}+\frac{7cd^6}{4b^3}\right)+x^3\cdot\left(\frac{2a^2d^7}{b^5}-\frac{7acd^6}{b^4}+\frac{7c^2d^5}{b^3}\right)+x^2\left(-\frac{5a^3d^7}{b^6}+\frac{21a^2cd^6}{b^5}-\frac{63ac^2d^5}{2b^4}+\frac{35c^3d^4}{2b^3}\right)+x\left(\frac{15a^4d^7}{b^7}-\frac{70a^3cd^6}{b^6}+\frac{126a^2c^2d^5}{b^5}-\frac{105ac^3d^4}{b^4}+\frac{35c^4d^3}{b^3}\right)+\frac{-13a^7d^7+77a^6bcd^6-189a^5b^2c^2d^5+245a^4b^3c^3d^4-175a^3b^4c^4d^3+63a^2b^5c^5d^2-7ab^6c^6d-b^7c^7+x\left(-14a^6bd^7+84a^5b^2cd^6-210a^4b^3c^2d^5+280a^3b^4c^3d^4-210a^2b^5c^4d^3+84ab^6c^5d^2-14b^7c^6d\right)}{2a^2b^8+4ab^9x+2b^{10}x^2}+\frac{d^7x^5}{5b^3}-\frac{21d^2\left(ad-bc\right)^5\log\left(a+bx\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**3,x)
```

```
[Out] x**4*(-3*a*d**7/(4*b**4) + 7*c*d**6/(4*b**3)) + x**3*(2*a**2*d**7/b**5 - 7*
a*c*d**6/b**4 + 7*c**2*d**5/b**3) + x**2*(-5*a**3*d**7/b**6 + 21*a**2*c*d**
6/b**5 - 63*a*c**2*d**5/(2*b**4) + 35*c**3*d**4/(2*b**3)) + x*(15*a**4*d**7
/b**7 - 70*a**3*c*d**6/b**6 + 126*a**2*c**2*d**5/b**5 - 105*a*c**3*d**4/b**
4 + 35*c**4*d**3/b**3) + (-13*a**7*d**7 + 77*a**6*b*c*d**6 - 189*a**5*b**2*
c**2*d**5 + 245*a**4*b**3*c**3*d**4 - 175*a**3*b**4*c**4*d**3 + 63*a**2*b**
5*c**5*d**2 - 7*a*b**6*c**6*d - b**7*c**7 + x*(-14*a**6*b*d**7 + 84*a**5*b*
*2*c*d**6 - 210*a**4*b**3*c**2*d**5 + 280*a**3*b**4*c**3*d**4 - 210*a**2*b*
*5*c**4*d**3 + 84*a*b**6*c**5*d**2 - 14*b**7*c**6*d))/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + d**7*x**5/(5*b**3) - 21*d**2*(a*d - b*c)**5*log(a +
b*x)/b**8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(177) = 354$.
time = 0.00, size = 511, normalized size = 2.76

$$\frac{\frac{1}{5}x^5d^7b^{12}-\frac{3}{4}x^4d^7b^{11}a+\frac{7}{4}x^4d^6cb^{12}+2x^3d^7b^{10}a^2-7x^3d^6cb^{11}a+7x^3d^5c^2b^{12}-5x^2d^7b^9a^3+21x^2d^6cb^{10}a^2-\frac{63}{2}x^2d^5c^2b^{11}a+\frac{35}{2}x^2d^4c^3b^{12}+15xd^7b^8a^4-70xd^6cb^9a^3+126xd^5c^2b^{10}a^2-105xd^4c^3b^{11}a+35xd^3c^4b^{12}}{b^{15}}+\frac{\frac{1}{2}\left(-13d^7a^7+77d^6ba^6c-189d^5b^2a^5c^2+245d^4b^3a^4c^3-175d^3b^4a^3c^4+63d^2b^5a^2c^5-7db^6ac^6-b^7c^7+\left(-14d^7ba^6+84d^6b^2a^5c-210d^5b^3a^4c^2+280d^4b^4a^3c^3-210d^3b^5a^2c^4+84d^2b^6ac^5-14db^7c^6\right)x\right)}{b^8\left(xb+a\right)^2}+\frac{\left(-21d^7a^5+105d^6cba^4-210d^5c^2b^2a^3+210d^4c^3b^3a^2-105d^3c^4b^4a+21d^2c^5b^5\right)\ln\left|xb+a\right|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x)
```

```
[Out] 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2*c^2*d^5
 + 5*a^4*b*c*d^6 - a^5*d^7)*log(abs(b*x + a))/b^8 - 1/2*(b^7*c^7 + 7*a*b^6*
c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 14*(b^7*c^6*d - 6*a*b^6*c
^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a
^5*b^2*c*d^6 + a^6*b*d^7)*x)/((b*x + a)^2*b^8) + 1/20*(4*b^12*d^7*x^5 + 35*
b^12*c*d^6*x^4 - 15*a*b^11*d^7*x^4 + 140*b^12*c^2*d^5*x^3 - 140*a*b^11*c*d^
6*x^3 + 40*a^2*b^10*d^7*x^3 + 350*b^12*c^3*d^4*x^2 - 630*a*b^11*c^2*d^5*x^2
 + 420*a^2*b^10*c*d^6*x^2 - 100*a^3*b^9*d^7*x^2 + 700*b^12*c^4*d^3*x - 2100
*a*b^11*c^3*d^4*x + 2520*a^2*b^10*c^2*d^5*x - 1400*a^3*b^9*c*d^6*x + 300*a^
4*b^8*d^7*x)/b^15
```

Mupad [B]
time = 0.27, size = 690, normalized size = 3.73

$$x\left(\frac{3a\left(\frac{3a\left(\frac{3a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{3a^2d^7}{b^5}+\frac{21c^2d^5}{b^3}\right)}{b}+\frac{a^3d^7}{b^6}-\frac{35c^3d^4}{b^3}-\frac{3a^2\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b^2}\right)}{b}+\frac{35c^4d^3}{b^3}+\frac{a^3\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b^3}-\frac{3a^2\left(\frac{3a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{3a^2d^7}{b^5}+\frac{21c^2d^5}{b^3}\right)}{b^2}\right)-x^4\left(\frac{3ad^7}{4b^4}-\frac{7cd^6}{4b^3}\right)-x^2\left(\frac{3a\left(\frac{3a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{3a^2d^7}{b^5}+\frac{21c^2d^5}{b^3}\right)}{2b}+\frac{a^3d^7}{2b^6}-\frac{35c^3d^4}{2b^3}-\frac{3a^2\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{2b^2}\right)+x^3\left(\frac{a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{a^2d^7}{b^5}+\frac{7c^2d^5}{b^3}\right)-\frac{\frac{13a^7d^7-77a^6bcd^6+189a^5b^2c^2d^5-245a^4b^3c^3d^4+175a^3b^4c^4d^3-63a^2b^5c^5d^2+7ab^6c^6d+b^7c^7}{2b}+x\left(7a^6d^7-42a^5bcd^6+105a^4b^2c^2d^5-140a^3b^3c^3d^4+105a^2b^4c^4d^3-42ab^5c^5d^2+7b^6c^6d\right)}{a^2b^7+2ab^8x+b^9x^2}+\frac{d^7x^5}{5b^3}-\frac{\ln\left(a+bx\right)\left(21a^5d^7-105a^4bcd^6+210a^3b^2c^2d^5-210a^2b^3c^3d^4+105ab^4c^4d^3-21b^5c^5d^2\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^3,x)
```

```
[Out] x*((3*a*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)/b^5 +
(21*c^2*d^5)/b^3))/b + (a^3*d^7)/b^6 - (35*c^3*d^4)/b^3 - (3*a^2*((3*a*d^7)
/b^4 - (7*c*d^6)/b^3))/b^2))/b + (35*c^4*d^3)/b^3 + (a^3*((3*a*d^7)/b^4 - (
7*c*d^6)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a
^2*d^7)/b^5 + (21*c^2*d^5)/b^3))/b^2) - x^4*((3*a*d^7)/(4*b^4) - (7*c*d^6)/
(4*b^3)) - x^2*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)
/b^5 + (21*c^2*d^5)/b^3))/(2*b) + (a^3*d^7)/(2*b^6) - (35*c^3*d^4)/(2*b^3)
- (3*a^2*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/(2*b^2)) + x^3*((a*((3*a*d^7)/b^4
 - (7*c*d^6)/b^3))/b - (a^2*d^7)/b^5 + (7*c^2*d^5)/b^3) - ((13*a^7*d^7 + b^
7*c^7 - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 77*a^6*b*c*d^6)/(2*b) + x*(7*a^6*d^7 +
7*b^6*c^6*d - 42*a*b^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4
+ 105*a^4*b^2*c^2*d^5 - 42*a^5*b*c*d^6))/(a^2*b^7 + b^9*x^2 + 2*a*b^8*x) +
(d^7*x^5)/(5*b^3) - (log(a + b*x)*(21*a^5*d^7 - 21*b^5*c^5*d^2 + 105*a*b^4*
c^4*d^3 - 210*a^2*b^3*c^3*d^4 + 210*a^3*b^2*c^2*d^5 - 105*a^4*b*c*d^6))/b^8
```

3.1286 $\int \frac{(c+dx)^7}{(a+bx)^4} dx$

Optimal. Leaf size=187

$$\frac{35d^4(bc-ad)^3x}{b^7} - \frac{(bc-ad)^7}{3b^8(a+bx)^3} - \frac{7d(bc-ad)^6}{2b^8(a+bx)^2} - \frac{21d^2(bc-ad)^5}{b^8(a+bx)} + \frac{21d^5(bc-ad)^2(a+bx)^2}{2b^8} + \frac{7d^6(bc-ad)(a+}{3b^8}$$

```
[Out] 35*d^4*(-a*d+b*c)^3*x/b^7-1/3*(-a*d+b*c)^7/b^8/(b*x+a)^3-7/2*d*(-a*d+b*c)^6
/b^8/(b*x+a)^2-21*d^2*(-a*d+b*c)^5/b^8/(b*x+a)+21/2*d^5*(-a*d+b*c)^2*(b*x+a
)^2/b^8+7/3*d^6*(-a*d+b*c)*(b*x+a)^3/b^8+1/4*d^7*(b*x+a)^4/b^8+35*d^3*(-a*d
+b*c)^4*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8} + \frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8} + \frac{35d^3(bc-ad)^4\log(a+bx)}{b^8} - \frac{21d^2(bc-ad)^5}{b^8(a+bx)} - \frac{7d(bc-ad)^6}{2b^8(a+bx)^2} - \frac{(bc-ad)^7}{3b^8(a+bx)^3} + \frac{d^7(a+bx)^4}{4b^8} + \frac{35d^4x(bc-ad)^3}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (35*d^4*(b*c - a*d)^3*x)/b^7 - (b*c - a*d)^7/(3*b^8*(a + b*x)^3) - (7*d*(b*
c - a*d)^6)/(2*b^8*(a + b*x)^2) - (21*d^2*(b*c - a*d)^5)/(b^8*(a + b*x)) +
(21*d^5*(b*c - a*d)^2*(a + b*x)^2)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^3
)/(3*b^8) + (d^7*(a + b*x)^4)/(4*b^8) + (35*d^3*(b*c - a*d)^4*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^4} dx = \int \left(\frac{35d^4(bc-ad)^3}{b^7} + \frac{(bc-ad)^7}{b^7(a+bx)^4} + \frac{7d(bc-ad)^6}{b^7(a+bx)^3} + \frac{21d^2(bc-ad)^5}{b^7(a+bx)^2} + \frac{35d^3(bc-ad)^4}{b^7(a+bx)} + \right.$$
$$= \frac{35d^4(bc-ad)^3x}{b^7} - \frac{(bc-ad)^7}{3b^8(a+bx)^3} - \frac{7d(bc-ad)^6}{2b^8(a+bx)^2} - \frac{21d^2(bc-ad)^5}{b^8(a+bx)} + \frac{21d^5(bc-ad)^2(a+}{2b^8}$$

Mathematica [A]
time = 0.07, size = 199, normalized size = 1.06

$$\frac{12bd^4\left(35b^3c^3-84ab^2c^2d+70a^2bcd^2-20a^3d^3\right)x+6b^2d^5\left(21b^2c^2-28abcd+10a^2d^2\right)x^2+4b^3d^6(7bc-4ad)x^3+3b^4d^7x^4-\frac{4(bc-ad)^7}{(a+bx)^3}-\frac{42d(bc-ad)^6}{(a+bx)^2}+\frac{252d^2(-bc+ad)^5}{a+bx}+420d^3(bc-ad)^4\log(a+bx)}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (12*b*d^4*(35*b^3*c^3 - 84*a*b^2*c^2*d + 70*a^2*b*c*d^2 - 20*a^3*d^3)*x + 6
*b^2*d^5*(21*b^2*c^2 - 28*a*b*c*d + 10*a^2*d^2)*x^2 + 4*b^3*d^6*(7*b*c - 4*
a*d)*x^3 + 3*b^4*d^7*x^4 - (4*(b*c - a*d)^7)/(a + b*x)^3 - (42*d*(b*c - a*d
)^6)/(a + b*x)^2 + (252*d^2*(-(b*c) + a*d)^5)/(a + b*x) + 420*d^3*(b*c - a*
d)^4*Log[a + b*x])/(12*b^8)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 552 vs. $2(187) = 374$.
time = 9.09, size = 550, normalized size = 2.94

$$\frac{214a^7d^7-1036a^6bcd^6+1974a^5b^2c^2d^5-1820a^4b^3c^3d^4+770a^3b^4c^4d^3-84a^2b^5c^5d^2-14ab^6c^6d-4b^7c^7-42bdx\left(-11a^6d^6+54a^5bcd^5-105a^4b^2c^2d^4+100a^3b^3c^3d^3-45a^2b^4c^4d^2+6ab^5c^5d+b^6c^6\right)+252b^2d^2x^2\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)+420d^3\mathrm{Log}\left[a+bx\right]\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)(ad-bc)^4-12bd^4x\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(20a^3d^3-70a^2bcd^2+84ab^2c^2d-35b^3c^3\right)+6b^2d^5x^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(10a^2d^2-28abcd+21b^2c^2\right)-4b^3d^6x^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)(4ad-7bc)+3b^4d^7x^4\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}{12b^8\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^4,x]')
```

```
[Out] (214 a ^ 7 d ^ 7 - 1036 a ^ 6 b c d ^ 6 + 1974 a ^ 5 b ^ 2 c ^ 2 d ^ 5 - 18
20 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 770 a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 84 a ^ 2 b ^ 5 c
 ^ 5 d ^ 2 - 14 a b ^ 6 c ^ 6 d - 4 b ^ 7 c ^ 7 - 42 b d x (-11 a ^ 6 d ^ 6
 + 54 a ^ 5 b c d ^ 5 - 105 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 100 a ^ 3 b ^ 3 c ^ 3
 d ^ 3 - 45 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 6 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + 25
2 b ^ 2 d ^ 2 x ^ 2 (a ^ 5 d ^ 5 - 5 a ^ 4 b c d ^ 4 + 10 a ^ 3 b ^ 2 c ^ 2
 d ^ 3 - 10 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 5 a b ^ 4 c ^ 4 d - b ^ 5 c ^ 5) + 42
0 d ^ 3 Log[a + b x] (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)
(a d - b c) ^ 4 - 12 b d ^ 4 x (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^
 3 x ^ 3) (20 a ^ 3 d ^ 3 - 70 a ^ 2 b c d ^ 2 + 84 a b ^ 2 c ^ 2 d - 35 b
^ 3 c ^ 3) + 6 b ^ 2 d ^ 5 x ^ 2 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b
 ^ 3 x ^ 3) (10 a ^ 2 d ^ 2 - 28 a b c d + 21 b ^ 2 c ^ 2) - 4 b ^ 3 d ^ 6
x ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (4 a d - 7 b c)
 + 3 b ^ 4 d ^ 7 x ^ 4 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3
)) / (12 b ^ 8 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(177) = 354$.
time = 0.14, size = 459, normalized size = 2.45

method	result
norman	$\frac{385a^7d^7-1540a^6bc\,d^6+2310a^5b^2c^2d^5-1540a^4b^3c^3d^4+385a^3b^4c^4d^3-42a^2b^5c^5d^2-7a\,b^6c^6d-2b^7c^7}{6b^8}+\frac{d^7x^7}{4b}+\frac{3\left(35a^5d^7-140a^4bc\,d^6+210a^3b^2c^2d^5\right.}{}$

default	$-\frac{d^4\left(-\frac{1}{4}d^3x^4b^3+\frac{4}{3}a\,b^2d^3x^3-\frac{7}{3}b^3c\,d^2x^3-5a^2b\,d^3x^2+14a\,b^2c\,d^2x^2-\frac{21}{2}b^3c^2d\,x^2+20a^3d^3x-70a^2bc\,d^2x+84a\,b^2c^2dx-35b^3c^3x\right)}{b^7}+ 2$
risch	$\frac{d^7x^4}{4b^4}-\frac{4d^7a\,x^3}{3b^5}+\frac{7d^6c\,x^3}{3b^4}+\frac{5d^7a^2x^2}{b^6}-\frac{14d^6ac\,x^2}{b^5}+\frac{21d^5c^2x^2}{2b^4}-\frac{20d^7a^3x}{b^7}+\frac{70d^6a^2cx}{b^6}-\frac{84d^5a\,c^2x}{b^5}+\frac{35d^4c^3x}{b^4}+ ($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -d^4/b^7*(-1/4*d^3*x^4*b^3+4/3*a*b^2*d^3*x^3-7/3*b^3*c*d^2*x^3-5*a^2*b*d^3*
x^2+14*a*b^2*c*d^2*x^2-21/2*b^3*c^2*d*x^2+20*a^3*d^3*x-70*a^2*b*c*d^2*x+84*
a*b^2*c^2*d*x-35*b^3*c^3*x)+21/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^
2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)-7/2/b^8*d*(a^6*d^6-
6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*
b^5*c^5*d+b^6*c^6)/(b*x+a)^2+35/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^
2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(b*x+a)-1/3/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a
^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a
*b^6*c^6*d+b^7*c^7)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(177) = 354$.
time = 0.28, size = 484, normalized size = 2.59

$$-\frac{2\,b^7c^7+7\,ab^6c^6d+42\,a^2b^5c^5d^2-385\,a^3b^4c^4d^3+910\,a^4b^3c^3d^4-987\,a^5b^2c^2d^5+518\,a^6bcd^6-107\,a^7d^7+126\,(b^7c^5d^2-5\,ab^6c^4d^3+10\,a^2b^5c^3d^4-10\,a^3b^4c^2d^5+5\,a^4b^3cd^6-a^5b^2d^7)x^2+21\,(b^7c^6d+6\,ab^6c^5d^2-45\,a^2b^5c^4d^3+100\,a^3b^4c^3d^4-105\,a^4b^3c^2d^5+54\,a^5b^2cd^6-11\,a^6bd^7)x}{6\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}+\frac{3\,b^3d^7x^4+4\,(7\,b^3cd^6-4\,ab^2d^7)x^3+6\,(21\,b^3c^2d^5-28\,ab^2cd^6+10\,a^2bd^7)x^2+12\,(35\,b^3c^3d^4-84\,ab^2c^2d^5+70\,a^2bcd^6-20\,a^3d^7)x}{12\,b^7}+\frac{35\,(b^4c^4d^3-4\,ab^3c^3d^4+6\,a^2b^2c^2d^5-4\,a^3bcd^6+a^4d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3
+ 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 + 518*a^6*b*c*d^6 - 107*a^7*d^7
 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2
*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21*(b^7*c^6*d + 6*a*b^6*c^5*d^2
 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 - 105*a^4*b^3*c^2*d^5 + 54*a^5*
b^2*c*d^6 - 11*a^6*b*d^7)*x)/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b
^8) + 1/12*(3*b^3*d^7*x^4 + 4*(7*b^3*c*d^6 - 4*a*b^2*d^7)*x^3 + 6*(21*b^3*c
^2*d^5 - 28*a*b^2*c*d^6 + 10*a^2*b*d^7)*x^2 + 12*(35*b^3*c^3*d^4 - 84*a*b^2
*c^2*d^5 + 70*a^2*b*c*d^6 - 20*a^3*d^7)*x)/b^7 + 35*(b^4*c^4*d^3 - 4*a*b^3*
c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(177) = 354$.
time = 0.31, size = 739, normalized size = 3.95

$$\frac{3\,b^7d^7x^7-4\,b^7c^7-14\,ab^6c^6d-84\,a^2b^5c^5d^2+770\,a^3b^4c^4d^3-1820\,a^4b^3c^3d^4+1974\,a^5b^2c^2d^5-1036\,a^6bcd^6+214\,a^7d^7+7\,(4\,b^7cd^6-ab^6d^7)x^6+21\,(6\,b^7c^2d^5-4\,ab^6cd^6+a^2b^5d^7)x^5+105\,(4\,b^7c^3d^4-6\,ab^6c^2d^5+4\,a^2b^5cd^6-a^3b^4d^7)x^4+2\,(630\,ab^6c^3d^4-1323\,a^2b^5c^2d^5+1022\,a^3b^4cd^6-278\,a^4b^3d^7)x^3-6\,(42\,b^7c^5d^2-210\,ab^6c^4d^3+210\,a^2b^5c^3d^4+63\,a^3b^4c^2d^5-182\,a^4b^3cd^6+68\,a^5b^2d^7)x^2-6\,(7\,b^7c^6d+42\,ab^6c^5d^2-315\,a^2b^5c^4d^3+630\,a^3b^4c^3d^4-567\,a^4b^3c^2d^5+238\,a^5b^2cd^6-37\,a^6bd^7)x+420\,(a^3b^4c^4d^3-4\,a^4b^3c^3d^4+6\,a^5b^2c^2d^5-4\,a^6bcd^6+a^7d^7+(b^7c^4d^3-4\,ab^6c^3d^4+6\,a^2b^5c^2d^5-4\,a^3b^4cd^6+a^4b^3d^7)x^3+3\,(ab^6c^4d^3-4\,a^2b^5c^3d^4+6\,a^3b^4c^2d^5-4\,a^4b^3cd^6+a^5b^2d^7)x^2+3\,(a^2b^5c^4d^3-4\,a^3b^4c^3d^4+6\,a^4b^3c^2d^5-4\,a^5b^2cd^6+a^6bd^7)x)\log(bx+a)}{12\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*d^7*x^7 - 4*b^7*c^7 - 14*a*b^6*c^6*d - 84*a^2*b^5*c^5*d^2 + 770
*a^3*b^4*c^4*d^3 - 1820*a^4*b^3*c^3*d^4 + 1974*a^5*b^2*c^2*d^5 - 1036*a^6*b
*c*d^6 + 214*a^7*d^7 + 7*(4*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(6*b^7*c^2*d^5
- 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(4*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 +
 4*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 2*(630*a*b^6*c^3*d^4 - 1323*a^2*b^5*c
^2*d^5 + 1022*a^3*b^4*c*d^6 - 278*a^4*b^3*d^7)*x^3 - 6*(42*b^7*c^5*d^2 - 21
0*a*b^6*c^4*d^3 + 210*a^2*b^5*c^3*d^4 + 63*a^3*b^4*c^2*d^5 - 182*a^4*b^3*c*
d^6 + 68*a^5*b^2*d^7)*x^2 - 6*(7*b^7*c^6*d + 42*a*b^6*c^5*d^2 - 315*a^2*b^5
*c^4*d^3 + 630*a^3*b^4*c^3*d^4 - 567*a^4*b^3*c^2*d^5 + 238*a^5*b^2*c*d^6 -
37*a^6*b*d^7)*x + 420*(a^3*b^4*c^4*d^3 - 4*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*
d^5 - 4*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*
c^2*d^5 - 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 3*(a*b^6*c^4*d^3 - 4*a^2*b^5
*c^3*d^4 + 6*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 3*(a^2*
b^5*c^4*d^3 - 4*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 - 4*a^5*b^2*c*d^6 + a^6
*b*d^7)*x)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b^8)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(172) = 344$.
time = 22.14, size = 474, normalized size = 2.53

$$x^3\left(-\frac{4ad^7}{3b^5}+\frac{7cd^6}{3b^4}\right)+x^2\cdot\left(\frac{5a^2d^7}{b^6}-\frac{14acd^6}{b^5}+\frac{21c^2d^5}{2b^4}\right)+x\left(-\frac{20a^3d^7}{b^7}+\frac{70a^2cd^6}{b^6}-\frac{84ac^2d^5}{b^5}+\frac{35c^3d^4}{b^4}\right)+\frac{107a^7d^7-518a^6bcd^6+987a^5b^2c^2d^5-910a^4b^3c^3d^4+385a^3b^4c^4d^3-42a^2b^5c^5d^2-7ab^6c^6d-2b^7c^7+x^2\cdot(126a^5b^2d^7-630a^4b^3cd^6+1260a^3b^4c^2d^5-1260a^2b^5c^3d^4+630ab^6c^4d^3-126b^7c^5d^2)+x(231a^6bd^7-1134a^5b^2cd^6+2205a^4b^3c^2d^5-2100a^3b^4c^3d^4+945a^2b^5c^4d^3-126ab^6c^5d^2-21b^7c^6d)}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3}+\frac{d^7x^4}{4b^4}+\frac{35d^3(ad-bc)^4\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**4,x)
```

```
[Out] x**3*(-4*a*d**7/(3*b**5) + 7*c*d**6/(3*b**4)) + x**2*(5*a**2*d**7/b**6 - 14
*a*c*d**6/b**5 + 21*c**2*d**5/(2*b**4)) + x*(-20*a**3*d**7/b**7 + 70*a**2*c
*d**6/b**6 - 84*a*c**2*d**5/b**5 + 35*c**3*d**4/b**4) + (107*a**7*d**7 - 51
8*a**6*b*c*d**6 + 987*a**5*b**2*c**2*d**5 - 910*a**4*b**3*c**3*d**4 + 385*a
**3*b**4*c**4*d**3 - 42*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 2*b**7*c**7
 + x**2*(126*a**5*b**2*d**7 - 630*a**4*b**3*c*d**6 + 1260*a**3*b**4*c**2*d*
*5 - 1260*a**2*b**5*c**3*d**4 + 630*a*b**6*c**4*d**3 - 126*b**7*c**5*d**2)
+ x*(231*a**6*b*d**7 - 1134*a**5*b**2*c*d**6 + 2205*a**4*b**3*c**2*d**5 - 2
100*a**3*b**4*c**3*d**4 + 945*a**2*b**5*c**4*d**3 - 126*a*b**6*c**5*d**2 -
21*b**7*c**6*d))/(6*a**3*b**8 + 18*a**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*
x**3) + d**7*x**4/(4*b**4) + 35*d**3*(a*d - b*c)**4*log(a + b*x)/b**8
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(177) = 354$.
time = 0.00, size = 502, normalized size = 2.68

$$\frac{\frac{1}{4}x^4d^7b^{12}-\frac{4}{3}x^3d^7b^{11}a+\frac{7}{3}x^3d^6cb^{12}+5x^2d^7b^{10}a^2-14x^2d^6cb^{11}a+\frac{21}{2}x^2d^5c^2b^{12}-20xd^7b^9a^3+70xd^6cb^{10}a^2-84xd^5c^2b^{11}a+35xd^4c^3b^{12}}{b^{16}}+\frac{\frac{1}{6}\left((126d^7b^2a^5-630d^6b^3a^4c+1260d^5b^4a^3c^2-1260d^4b^5a^2c^3+630d^3b^6ac^4-126d^2b^7c^5)x^2+(231d^7ba^6-1134d^6b^2a^5c+2205d^5b^3a^4c^2-2100d^4b^4a^3c^3+945d^3b^5a^2c^4-126d^2b^6ac^5-21db^7c^6)x+107d^7a^7-518d^6ba^6c+987d^5b^2a^5c^2-910d^4b^3a^4c^3+385d^3b^4a^3c^4-42d^2b^5a^2c^5-7db^6ac^6-2b^7c^7\right)}{b^8(xb+a)^3}+\frac{(35d^7a^4-140d^6cba^3+210d^5c^2b^2a^2-140d^4c^3b^3a+35d^3c^4b^4)\ln|xb+a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x)
```

```
[Out] 35*(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4
*d^7)*log(abs(b*x + a))/b^8 - 1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c
^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 +
518*a^6*b*c*d^6 - 107*a^7*d^7 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2
*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21
*(b^7*c^6*d + 6*a*b^6*c^5*d^2 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 -
105*a^4*b^3*c^2*d^5 + 54*a^5*b^2*c*d^6 - 11*a^6*b*d^7)*x)/((b*x + a)^3*b^8)
 + 1/12*(3*b^12*d^7*x^4 + 28*b^12*c*d^6*x^3 - 16*a*b^11*d^7*x^3 + 126*b^12*
c^2*d^5*x^2 - 168*a*b^11*c*d^6*x^2 + 60*a^2*b^10*d^7*x^2 + 420*b^12*c^3*d^4
*x - 1008*a*b^11*c^2*d^5*x + 840*a^2*b^10*c*d^6*x - 240*a^3*b^9*d^7*x)/b^16
```

Mupad [B]
time = 0.29, size = 559, normalized size = 2.99

$$x^2\left(\frac{2a\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b}-\frac{3a^2d^7}{b^6}+\frac{21c^2d^5}{2b^4}\right)-x^3\left(\frac{4ad^7}{3b^5}-\frac{7cd^6}{3b^4}\right)-\frac{\frac{-107a^7d^7+518a^6bcd^6-987a^5b^2c^2d^5+910a^4b^3c^3d^4-385a^3b^4c^4d^3+42a^2b^5c^5d^2+7ab^6c^6d+2b^7c^7}{6b}+x\left(-\frac{77a^6d^7}{2}+189a^5bcd^6-\frac{735a^4b^2c^2d^5}{2}+350a^3b^3c^3d^4-\frac{315a^2b^4c^4d^3}{2}+21ab^5c^5d^2+\frac{7b^6c^6d}{2}\right)-x^2\left(21a^5bd^7-105a^4b^2cd^6+210a^3b^3c^2d^5-210a^2b^4c^3d^4+105ab^5c^4d^3-21b^6c^5d^2\right)}{a^3b^7+3a^2b^8x+3ab^9x^2+b^{10}x^3}-x\left(\frac{4a\left(\frac{4a\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b}-\frac{6a^2d^7}{b^6}+\frac{21c^2d^5}{b^4}\right)}{b}+\frac{4a^3d^7}{b^7}-\frac{35c^3d^4}{b^4}-\frac{6a^2\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b^2}\right)+\frac{\ln(a+bx)\left(35a^4d^7-140a^3bcd^6+210a^2b^2c^2d^5-140ab^3c^3d^4+35b^4c^4d^3\right)}{b^8}+\frac{d^7x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^4,x)
```

```
[Out] x^2*((2*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (3*a^2*d^7)/b^6 + (21*c^2*d^
5)/(2*b^4)) - x^3*((4*a*d^7)/(3*b^5) - (7*c*d^6)/(3*b^4)) - ((2*b^7*c^7 - 1
07*a^7*d^7 + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4
 - 987*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d + 518*a^6*b*c*d^6)/(6*b) + x*((7*b^6
*c^6*d)/2 - (77*a^6*d^7)/2 + 21*a*b^5*c^5*d^2 - (315*a^2*b^4*c^4*d^3)/2 + 3
50*a^3*b^3*c^3*d^4 - (735*a^4*b^2*c^2*d^5)/2 + 189*a^5*b*c*d^6) - x^2*(21*a
^5*b*d^7 - 21*b^6*c^5*d^2 + 105*a*b^5*c^4*d^3 - 105*a^4*b^2*c*d^6 - 210*a^2
*b^4*c^3*d^4 + 210*a^3*b^3*c^2*d^5))/(a^3*b^7 + b^10*x^3 + 3*a^2*b^8*x + 3*
a*b^9*x^2) - x*((4*a*((4*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (6*a^2*d^7)
/b^6 + (21*c^2*d^5)/b^4))/b + (4*a^3*d^7)/b^7 - (35*c^3*d^4)/b^4 - (6*a^2*(
(4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b^2) + (log(a + b*x)*(35*a^4*d^7 + 35*b^4*c
^4*d^3 - 140*a*b^3*c^3*d^4 + 210*a^2*b^2*c^2*d^5 - 140*a^3*b*c*d^6))/b^8 +
(d^7*x^4)/(4*b^4)
```

3.1287 $\int \frac{(c+dx)^7}{(a+bx)^5} dx$

Optimal. Leaf size=187

$$\frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+\frac{7d^6(bc-ad)(a+bx)^2}{2b^8}+\frac{d^7}{}$$

```
[Out] 21*d^5*(-a*d+b*c)^2*x/b^7-1/4*(-a*d+b*c)^7/b^8/(b*x+a)^4-7/3*d*(-a*d+b*c)^6
/b^8/(b*x+a)^3-21/2*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^2-35*d^3*(-a*d+b*c)^4/b^8/
(b*x+a)+7/2*d^6*(-a*d+b*c)*(b*x+a)^2/b^8+1/3*d^7*(b*x+a)^3/b^8+35*d^4*(-a*d
+b*c)^3*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}+\frac{d^7(a+bx)^3}{3b^8}+\frac{21d^5x(bc-ad)^2}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (21*d^5*(b*c - a*d)^2*x)/b^7 - (b*c - a*d)^7/(4*b^8*(a + b*x)^4) - (7*d*(b*
c - a*d)^6)/(3*b^8*(a + b*x)^3) - (21*d^2*(b*c - a*d)^5)/(2*b^8*(a + b*x)^2
) - (35*d^3*(b*c - a*d)^4)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*(a + b*x)^2
)/(2*b^8) + (d^7*(a + b*x)^3)/(3*b^8) + (35*d^4*(b*c - a*d)^3*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^5}\,dx &= \int \left(\frac{21d^5(bc-ad)^2}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^5}+\frac{7d(bc-ad)^6}{b^7(a+bx)^4}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^3}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^2}+\right.\\ &= \frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+\frac{7d}{}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 173, normalized size = 0.93

$$\frac{12bd^5\left(21b^2c^2-35abcd+15a^2d^2\right)x+6b^2d^6(7bc-5ad)x^2+4b^3d^7x^3-\frac{3(bc-ad)^7}{(a+bx)^4}-\frac{28d(bc-ad)^6}{(a+bx)^3}+\frac{126d^2(-bc+ad)^5}{(a+bx)^2}-\frac{420d^3(bc-ad)^4}{a+bx}+420d^4(bc-ad)^3\log(a+bx)}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(21*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2)*x + 6*b^2*d^6*(7*b*c - 5*a
*d)*x^2 + 4*b^3*d^7*x^3 - (3*(b*c - a*d)^7)/(a + b*x)^4 - (28*d*(b*c - a*d)
^6)/(a + b*x)^3 + (126*d^2*(-(b*c) + a*d)^5)/(a + b*x)^2 - (420*d^3*(b*c -
a*d)^4)/(a + b*x) + 420*d^4*(b*c - a*d)^3*Log[a + b*x])/(12*b^8)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 595 vs. 2(187) = 374.
time = 19.49, size = 593, normalized size = 3.17

$$\frac{-420d^4\text{Log}[a+bx]\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)(ad-bc)^3-319a^7d^7+1197a^6bcd^6-1617a^5b^2c^2d^5+875a^4b^3c^3d^4-105a^3b^4c^4d^3-21a^2b^5c^5d^2-7ab^6c^6d-3b^7c^7-28bdx\left(37a^6d^6-141a^5bcd^5+195a^4b^2c^2d^4-110a^3b^3c^3d^3+15a^2b^4c^4d^2+3ab^5c^5d+b^6c^6\right)+126b^2d^2x^2\left(-9a^5d^5+35a^4bcd^4-50a^3b^2c^2d^3+30a^2b^3c^3d^2-5ab^4c^4d-b^5c^5\right)+420b^3d^3x^3\left(-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d-b^4c^4\right)+12bd^5x\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\left(15a^2d^2-35abcd+21b^2c^2\right)-6b^2d^6x^2\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)(5ad-7bc)+4b^3d^7x^3\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)}{12b^8\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^5,x]')
```

```
[Out] (-420 d ^ 4 Log[a + b x] (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b
 ^ 3 x ^ 3 + b ^ 4 x ^ 4) (a d - b c) ^ 3 - 319 a ^ 7 d ^ 7 + 1197 a ^ 6 b
c d ^ 6 - 1617 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 875 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 105
a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 21 a ^ 2 b ^ 5 c ^ 5 d ^ 2 - 7 a b ^ 6 c ^ 6 d -
3 b ^ 7 c ^ 7 - 28 b d x (37 a ^ 6 d ^ 6 - 141 a ^ 5 b c d ^ 5 + 195 a ^ 4
b ^ 2 c ^ 2 d ^ 4 - 110 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 15 a ^ 2 b ^ 4 c ^ 4 d ^
2 + 3 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + 126 b ^ 2 d ^ 2 x ^ 2 (-9 a ^ 5 d ^
5 + 35 a ^ 4 b c d ^ 4 - 50 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 30 a ^ 2 b ^ 3 c ^ 3
d ^ 2 - 5 a b ^ 4 c ^ 4 d - b ^ 5 c ^ 5) + 420 b ^ 3 d ^ 3 x ^ 3 (-a ^ 4 d
^ 4 + 4 a ^ 3 b c d ^ 3 - 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 4 a b ^ 3 c ^ 3 d - b
 ^ 4 c ^ 4) + 12 b d ^ 5 x (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a
 b ^ 3 x ^ 3 + b ^ 4 x ^ 4) (15 a ^ 2 d ^ 2 - 35 a b c d + 21 b ^ 2 c ^ 2)
- 6 b ^ 2 d ^ 6 x ^ 2 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^
3 x ^ 3 + b ^ 4 x ^ 4) (5 a d - 7 b c) + 4 b ^ 3 d ^ 7 x ^ 3 (a ^ 4 + 4 a ^
 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4)) / (12 b ^ 8
(a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(177) = 354.
time = 0.14, size = 453, normalized size = 2.42

method	result

norman	$-\frac{875a^7d^7-2625a^6bc\,d^6+2625a^5b^2c^2d^5-875a^4b^3c^3d^4+105a^3b^4c^4d^3+21a^2b^5c^5d^2+7a\,b^6c^6d+3b^7c^7}{12b^8}+\frac{d^7x^7}{3b}-\frac{\left(140a^4d^7-420a^3bc\,d^6+420b^2a^2c^2d^5\right.}{b^5}$
default	$\frac{d^5\left(\frac{1}{3}d^2x^3b^2-\frac{5}{2}ab\,d^2x^2+\frac{7}{2}b^2cd\,x^2+15a^2d^2x-35abcdx+21b^2c^2x\right)}{b^7}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{b^8(bx+a)}-\frac{-a^7d^7+}{}$
risch	$\frac{d^7x^3}{3b^5}-\frac{5d^7a\,x^2}{2b^6}+\frac{7d^6c\,x^2}{2b^5}+\frac{15d^7a^2x}{b^7}-\frac{35d^6acx}{b^6}+\frac{21d^5c^2x}{b^5}+\frac{\left(-35a^4b^2d^7+140a^3b^3c\,d^6-210a^2b^4c^2d^5+140a\,b^5c^3d^4-35b^6\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^5/b^7*(1/3*d^2*x^3*b^2-5/2*a*b*d^2*x^2+7/2*b^2*c*d*x^2+15*a^2*d^2*x-35*a*
b*c*d*x+21*b^2*c^2*x)-35/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)/(b*x+a)-1/4/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^
2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*
d+b^7*c^7)/(b*x+a)^4+21/2/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3
-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^2-35/b^8*d^4*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*x+a)-7/3/b^8*d*(a^6*d^6-6*a^5*b*c*d
^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b
^6*c^6)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(177) = 354$.
time = 0.30, size = 494, normalized size = 2.64

$$-\frac{3b^7c^7+7ab^6c^6d+21a^2b^5c^5d^2+105a^3b^4c^4d^3-875a^4b^3c^3d^4+1617a^5b^2c^2d^5-1197a^6bcd^6+319a^7d^7+420\left(b^7c^4d^3-4ab^6c^3d^4+6a^2b^5c^2d^5-4a^3b^4cd^6+a^4b^3d^7\right)x^3+126\left(b^7c^5d^2+5ab^6c^4d^3-30a^2b^5c^3d^4+50a^3b^4c^2d^5-35a^4b^3cd^6+9a^5b^2d^7\right)x^2+28\left(b^7c^6d+3ab^6c^5d^2+15a^2b^5c^4d^3-110a^3b^4c^3d^4+195a^4b^3c^2d^5-141a^5b^2cd^6+37a^6bd^7\right)x}{12\left(b^{12}x^4+4ab^{11}x^3+6a^2b^{10}x^2+4a^3b^9x+a^4b^8\right)}+\frac{2b^2d^7x^3+3\left(7b^2cd^6-5abd^7\right)x^2+6\left(21b^2c^2d^5-35abcd^6+15a^2d^7\right)x}{6b^7}+\frac{35\left(b^3c^3d^4-3ab^2c^2d^5+3a^2bcd^6-a^3d^7\right)\log\left(bx+a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^7*c^7 + 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3
 - 875*a^4*b^3*c^3*d^4 + 1617*a^5*b^2*c^2*d^5 - 1197*a^6*b*c*d^6 + 319*a^7*
d^7 + 420*(b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a^3*b^4*c*
d^6 + a^4*b^3*d^7)*x^3 + 126*(b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 - 30*a^2*b^5*c^
3*d^4 + 50*a^3*b^4*c^2*d^5 - 35*a^4*b^3*c*d^6 + 9*a^5*b^2*d^7)*x^2 + 28*(b^
7*c^6*d + 3*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 110*a^3*b^4*c^3*d^4 + 195*
a^4*b^3*c^2*d^5 - 141*a^5*b^2*c*d^6 + 37*a^6*b*d^7)*x)/(b^12*x^4 + 4*a*b^11
*x^3 + 6*a^2*b^10*x^2 + 4*a^3*b^9*x + a^4*b^8) + 1/6*(2*b^2*d^7*x^3 + 3*(7*
b^2*c*d^6 - 5*a*b*d^7)*x^2 + 6*(21*b^2*c^2*d^5 - 35*a*b*c*d^6 + 15*a^2*d^7)
*x)/b^7 + 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(
b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(177) = 354$.
time = 0.30, size = 754, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*d^7*x^7 - 3*b^7*c^7 - 7*a*b^6*c^6*d - 21*a^2*b^5*c^5*d^2 - 105*
a^3*b^4*c^4*d^3 + 875*a^4*b^3*c^3*d^4 - 1617*a^5*b^2*c^2*d^5 + 1197*a^6*b*c
*d^6 - 319*a^7*d^7 + 14*(3*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(3*b^7*c^2*d^5 -
 3*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 4*(252*a*b^6*c^2*d^5 - 357*a^2*b^5*c*d^
6 + 139*a^3*b^4*d^7)*x^4 - 4*(105*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4 + 252*a^2
*b^5*c^2*d^5 + 168*a^3*b^4*c*d^6 - 136*a^4*b^3*d^7)*x^3 - 6*(21*b^7*c^5*d^2
 + 105*a*b^6*c^4*d^3 - 630*a^2*b^5*c^3*d^4 + 882*a^3*b^4*c^2*d^5 - 462*a^4*
b^3*c*d^6 + 74*a^5*b^2*d^7)*x^2 - 4*(7*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 105*a
^2*b^5*c^4*d^3 - 770*a^3*b^4*c^3*d^4 + 1302*a^4*b^3*c^2*d^5 - 882*a^5*b^2*c
*d^6 + 214*a^6*b*d^7)*x + 420*(a^4*b^3*c^3*d^4 - 3*a^5*b^2*c^2*d^5 + 3*a^6*
b*c*d^6 - a^7*d^7 + (b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*
b^4*d^7)*x^4 + 4*(a*b^6*c^3*d^4 - 3*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 - a^4
*b^3*d^7)*x^3 + 6*(a^2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*d^6 -
a^5*b^2*d^7)*x^2 + 4*(a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6
 - a^6*b*d^7)*x)*log(b*x + a))/(b^12*x^4 + 4*a*b^11*x^3 + 6*a^2*b^10*x^2 +
4*a^3*b^9*x + a^4*b^8)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(177) = 354.
time = 0.00, size = 499, normalized size = 2.67

$$\frac{\frac{1}{3}x^3d^7b^{10}-\frac{5}{2}x^2d^7b^9a+\frac{7}{2}x^2d^6cb^{10}+15xd^7b^8a^2-35xd^6cb^9a+21xd^5c^2b^{10}}{b^{15}}+\frac{\frac{1}{12}\left(\left(-420d^7b^3a^4+1680d^6b^4a^3c-2520d^5b^5a^2c^2+1680d^4b^6ac^3-420d^3b^7c^4\right)x^3+\left(-1134d^7b^2a^5+4410d^6b^3a^4c-6300d^5b^4a^3c^2+3780d^4b^5a^2c^3-630d^3b^6ac^4-126d^2b^7c^5\right)x^2+\left(-1036d^7ba^6+3948d^6b^2a^5c-5460d^5b^3a^4c^2+3080d^4b^4a^3c^3-420d^3b^5a^2c^4-84d^2b^6ac^5-28db^7c^6\right)x-319d^7a^7+1197d^6ba^6c-1617d^5b^2a^5c^2+875d^4b^3a^4c^3-105d^3b^4a^3c^4-21d^2b^5a^2c^5-7db^6ac^6-3b^7c^7\right)}{b^8\left(xb+a\right)^4}+\frac{\left(-35d^7a^3+105d^6cba^2-105d^5c^2b^2a+35d^4c^3b^3\right)\ln\left|xb+a\right|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x)
```

```
[Out] 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(abs(b*x +
a))/b^8 - 1/12*(3*b^7*c^7 + 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^
4*c^4*d^3 - 875*a^4*b^3*c^3*d^4 + 1617*a^5*b^2*c^2*d^5 - 1197*a^6*b*c*d^6 +
 319*a^7*d^7 + 420*(b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a
^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 126*(b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 - 30*a
^2*b^5*c^3*d^4 + 50*a^3*b^4*c^2*d^5 - 35*a^4*b^3*c*d^6 + 9*a^5*b^2*d^7)*x^2
```

```
 + 28*(b^7*c^6*d + 3*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 110*a^3*b^4*c^3*d
^4 + 195*a^4*b^3*c^2*d^5 - 141*a^5*b^2*c*d^6 + 37*a^6*b*d^7)*x)/((b*x + a)^
4*b^8) + 1/6*(2*b^10*d^7*x^3 + 21*b^10*c*d^6*x^2 - 15*a*b^9*d^7*x^2 + 126*b
^10*c^2*d^5*x - 210*a*b^9*c*d^6*x + 90*a^2*b^8*d^7*x)/b^15
```

Mupad [B]
time = 0.77, size = 512, normalized size = 2.74

$$x\left(\frac{5\,a\,\left(\frac{5\,a\,d^7}{b^6}-\frac{7\,c\,d^6}{b^5}\right)}{b}-\frac{10\,a^2\,d^7}{b^7}+\frac{21\,c^2\,d^5}{b^5}\right)-x^2\,\left(\frac{5\,a\,d^7}{2\,b^6}-\frac{7\,c\,d^6}{2\,b^5}\right)-\frac{\frac{319\,a^7\,d^7-1197\,a^6\,b\,c\,d^6+1617\,a^5\,b^2\,c^2\,d^5-875\,a^4\,b^3\,c^3\,d^4+105\,a^3\,b^4\,c^4\,d^3+21\,a^2\,b^5\,c^5\,d^2+7\,a\,b^6\,c^6\,d+3\,b^7\,c^7}{12\,b}+x\,\left(\frac{259\,a^6\,d^7}{3}-329\,a^5\,b\,c\,d^6+455\,a^4\,b^2\,c^2\,d^5-\frac{770\,a^3\,b^3\,c^3\,d^4}{3}+35\,a^2\,b^4\,c^4\,d^3+7\,a\,b^5\,c^5\,d^2+\frac{7\,b^6\,c^6\,d}{3}\right)+x^3\,\left(35\,a^4\,b^2\,d^7-140\,a^3\,b^3\,c\,d^6+210\,a^2\,b^4\,c^2\,d^5-140\,a\,b^5\,c^3\,d^4+35\,b^6\,c^4\,d^3\right)+x^2\,\left(\frac{189\,a^5\,b\,d^7}{2}-\frac{735\,a^4\,b^2\,c\,d^6}{2}+525\,a^3\,b^3\,c^2\,d^5-315\,a^2\,b^4\,c^3\,d^4+\frac{105\,a\,b^5\,c^4\,d^3}{2}+\frac{21\,b^6\,c^5\,d^2}{2}\right)}{a^4\,b^7+4\,a^3\,b^8\,x+6\,a^2\,b^9\,x^2+4\,a\,b^{10}\,x^3+b^{11}\,x^4}-\frac{\ln\left(a+b\,x\right)\,\left(35\,a^3\,d^7-105\,a^2\,b\,c\,d^6+105\,a\,b^2\,c^2\,d^5-35\,b^3\,c^3\,d^4\right)}{b^8}+\frac{d^7\,x^3}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^5,x)
```

```
[Out] x*((5*a*((5*a*d^7)/b^6 - (7*c*d^6)/b^5))/b - (10*a^2*d^7)/b^7 + (21*c^2*d^5
)/b^5) - x^2*((5*a*d^7)/(2*b^6) - (7*c*d^6)/(2*b^5)) - ((319*a^7*d^7 + 3*b^
7*c^7 + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 - 875*a^4*b^3*c^3*d^4 + 16
17*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 1197*a^6*b*c*d^6)/(12*b) + x*((259*a^6
*d^7)/3 + (7*b^6*c^6*d)/3 + 7*a*b^5*c^5*d^2 + 35*a^2*b^4*c^4*d^3 - (770*a^3
*b^3*c^3*d^4)/3 + 455*a^4*b^2*c^2*d^5 - 329*a^5*b*c*d^6) + x^3*(35*a^4*b^2*
d^7 + 35*b^6*c^4*d^3 - 140*a*b^5*c^3*d^4 - 140*a^3*b^3*c*d^6 + 210*a^2*b^4*
c^2*d^5) + x^2*((189*a^5*b*d^7)/2 + (21*b^6*c^5*d^2)/2 + (105*a*b^5*c^4*d^3
)/2 - (735*a^4*b^2*c*d^6)/2 - 315*a^2*b^4*c^3*d^4 + 525*a^3*b^3*c^2*d^5))/(
a^4*b^7 + b^11*x^4 + 4*a^3*b^8*x + 4*a*b^10*x^3 + 6*a^2*b^9*x^2) - (log(a +
 b*x)*(35*a^3*d^7 - 35*b^3*c^3*d^4 + 105*a*b^2*c^2*d^5 - 105*a^2*b*c*d^6))/
b^8 + (d^7*x^3)/(3*b^5)
```

3.1288 $\int \frac{(c+dx)^7}{(a+bx)^6}\,dx$

Optimal. Leaf size=181

$$\frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}+\frac{21c}{}$$

```
[Out] d^6*(-6*a*d+7*b*c)*x/b^7+1/2*d^7*x^2/b^6-1/5*(-a*d+b*c)^7/b^8/(b*x+a)^5-7/4
*d*(-a*d+b*c)^6/b^8/(b*x+a)^4-7*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^3-35/2*d^3*(-a
*d+b*c)^4/b^8/(b*x+a)^2-35*d^4*(-a*d+b*c)^3/b^8/(b*x+a)+21*d^5*(-a*d+b*c)^2
*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}+\frac{d^6x(7bc-6ad)}{b^7}+\frac{d^7x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (d^6*(7*b*c - 6*a*d)*x)/b^7 + (d^7*x^2)/(2*b^6) - (b*c - a*d)^7/(5*b^8*(a +
 b*x)^5) - (7*d*(b*c - a*d)^6)/(4*b^8*(a + b*x)^4) - (7*d^2*(b*c - a*d)^5)/
(b^8*(a + b*x)^3) - (35*d^3*(b*c - a*d)^4)/(2*b^8*(a + b*x)^2) - (35*d^4*(b
*c - a*d)^3)/(b^8*(a + b*x)) + (21*d^5*(b*c - a*d)^2*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^6}\,dx &= \int \left(\frac{d^6(7bc-6ad)}{b^7}+\frac{d^7x}{b^6}+\frac{(bc-ad)^7}{b^7(a+bx)^6}+\frac{7d(bc-ad)^6}{b^7(a+bx)^5}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^4}+\frac{35d^3(bc-}{b^7(a+bx}\right.\\ &= \frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad}{2b^8(a+bx)^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(181) = 362.

time = 0.09, size = 389, normalized size = 2.15

$$\frac{459a^7d^7 + 3a^6bd^6(-406c + 625dx) + a^5b^2d^5\left(959c^2 - 5250cdx + 2700d^2x^2\right) + 5a^4b^3d^4\left(-28c^3 + 875c^2dx - 1680cd^2x^2 + 260d^3x^3\right) - 5a^3b^4d^3\left(7c^4 + 140c^3dx - 1540c^2d^2x^2 + 1120cd^3x^3 + 80d^4x^4\right) - a^2b^5d^2\left(14c^5 + 175c^4dx + 1400c^3d^2x^2 - 6300c^2d^3x^3 + 700cd^4x^4 + 500d^5x^5\right) - 7ab^6d\left(c^6 + 10c^5dx + 50c^4d^2x^2 + 200c^3d^3x^3 - 300c^2d^4x^4 - 100cd^5x^5 + 10d^6x^6\right) - b^7\left(4c^7 + 35c^6dx + 140c^5d^2x^2 + 350c^4d^3x^3 + 700c^3d^4x^4 - 140cd^6x^6 - 10d^7x^7\right) + 420d^5(bc - ad)^2(a + bx)^5\log(a + bx)}{20b^8(a + bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (459*a^7*d^7 + 3*a^6*b*d^6*(-406*c + 625*d*x) + a^5*b^2*d^5*(959*c^2 - 5250
*c*d*x + 2700*d^2*x^2) + 5*a^4*b^3*d^4*(-28*c^3 + 875*c^2*d*x - 1680*c*d^2*
x^2 + 260*d^3*x^3) - 5*a^3*b^4*d^3*(7*c^4 + 140*c^3*d*x - 1540*c^2*d^2*x^2
+ 1120*c*d^3*x^3 + 80*d^4*x^4) - a^2*b^5*d^2*(14*c^5 + 175*c^4*d*x + 1400*c
^3*d^2*x^2 - 6300*c^2*d^3*x^3 + 700*c*d^4*x^4 + 500*d^5*x^5) - 7*a*b^6*d*(c
^6 + 10*c^5*d*x + 50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 - 300*c^2*d^4*x^4 - 100*
c*d^5*x^5 + 10*d^6*x^6) - b^7*(4*c^7 + 35*c^6*d*x + 140*c^5*d^2*x^2 + 350*c
^4*d^3*x^3 + 700*c^3*d^4*x^4 - 140*c*d^6*x^6 - 10*d^7*x^7) + 420*d^5*(b*c -
 a*d)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 614 vs. 2(181) = 362.
time = 66.68, size = 612, normalized size = 3.38

$$\frac{459a^7d^7 - 1218a^6bcd^6 + 959a^5b^2c^2d^5 - 140a^4b^3c^3d^4 - 35a^3b^4c^4d^3 - 14a^2b^5c^5d^2 - 7ab^6c^6d - 4b^7c^7 - 35bdx\left(-57a^6d^6 + 154a^5bcd^5 - 125a^4b^2c^2d^4 + 20a^3b^3c^3d^3 + 5a^2b^4c^4d^2 + 2ab^5c^5d + b^6c^6\right) + 70b^2d^2x^2\left(47a^5d^5 - 130a^4bcd^4 + 110a^3b^2c^2d^3 - 20a^2b^3c^3d^2 - 5ab^4c^4d - 2b^5c^5\right) + 350b^3d^3x^3\left(7a^4d^4 - 20a^3bcd^3 + 18a^2b^2c^2d^2 - 4ab^3c^3d - b^4c^4\right) + 700b^4d^4x^4\left(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3\right) + 420d^5\mathrm{Log}[a + bx]\left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)(ad - bc)^2 - 20bd^6x\left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)(6ad - 7bc) + 10b^2d^7x^2\left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)}{20b^8\left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^6,x]')
```

```
[Out] (459 a ^ 7 d ^ 7 - 1218 a ^ 6 b c d ^ 6 + 959 a ^ 5 b ^ 2 c ^ 2 d ^ 5 - 140
 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 35 a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 14 a ^ 2 b ^ 5 c ^
5 d ^ 2 - 7 a b ^ 6 c ^ 6 d - 4 b ^ 7 c ^ 7 - 35 b d x (-57 a ^ 6 d ^ 6 + 1
54 a ^ 5 b c d ^ 5 - 125 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 20 a ^ 3 b ^ 3 c ^ 3 d ^
 3 + 5 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 2 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + 70 b ^
2 d ^ 2 x ^ 2 (47 a ^ 5 d ^ 5 - 130 a ^ 4 b c d ^ 4 + 110 a ^ 3 b ^ 2 c ^ 2
 d ^ 3 - 20 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 5 a b ^ 4 c ^ 4 d - 2 b ^ 5 c ^ 5) +
350 b ^ 3 d ^ 3 x ^ 3 (7 a ^ 4 d ^ 4 - 20 a ^ 3 b c d ^ 3 + 18 a ^ 2 b ^ 2
c ^ 2 d ^ 2 - 4 a b ^ 3 c ^ 3 d - b ^ 4 c ^ 4) + 700 b ^ 4 d ^ 4 x ^ 4 (a ^
 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3) + 420 d ^ 5
 Log[a + b x] (a ^ 5 + 5 a ^ 4 b x + 10 a ^ 3 b ^ 2 x ^ 2 + 10 a ^ 2 b ^ 3
x ^ 3 + 5 a b ^ 4 x ^ 4 + b ^ 5 x ^ 5) (a d - b c) ^ 2 - 20 b d ^ 6 x (a ^
5 + 5 a ^ 4 b x + 10 a ^ 3 b ^ 2 x ^ 2 + 10 a ^ 2 b ^ 3 x ^ 3 + 5 a b ^ 4 x
 ^ 4 + b ^ 5 x ^ 5) (6 a d - 7 b c) + 10 b ^ 2 d ^ 7 x ^ 2 (a ^ 5 + 5 a ^ 4
 b x + 10 a ^ 3 b ^ 2 x ^ 2 + 10 a ^ 2 b ^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 + b ^
5 x ^ 5)) / (20 b ^ 8 (a ^ 5 + 5 a ^ 4 b x + 10 a ^ 3 b ^ 2 x ^ 2 + 10 a ^
2 b ^ 3 x ^ 3 + 5 a b ^ 4 x ^ 4 + b ^ 5 x ^ 5))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(173) = 346.
time = 0.18, size = 451, normalized size = 2.49

method	result
default	$-\frac{d^{6}\left(-\frac{1}{2}bd\,x^{2}+6adx-7bcx\right)}{b^{7}}+\frac{35d^{4}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}{b^{8}(bx+a)}-\frac{7d\left(a^{6}d^{6}-6a^{5}bc\,d^{5}+15a^{4}b^{2}c^{2}d^{4}-20a^{3}b^{3}c^{3}d^{3}+15a^{2}b^{4}c^{4}\cdots\right.}{4b^{8}(bx+a)^{4}}$
norman	$\frac{959a^{7}d^{7}-1918a^{6}bc\,d^{6}+959a^{5}b^{2}c^{2}d^{5}-140a^{4}b^{3}c^{3}d^{4}-35a^{3}b^{4}c^{4}d^{3}-14a^{2}b^{5}c^{5}d^{2}-7a\,b^{6}c^{6}d-4b^{7}c^{7}}{20b^{8}}+\frac{d^{7}x^{7}}{2b}+\frac{5\left(21a^{3}d^{7}-42a^{2}bc\,d^{6}+21a\,b^{2}c^{2}d^{5}-7b^{3}\cdots\right.}{b^{4}}$
risch	$\frac{d^{7}x^{2}}{2b^{6}}-\frac{6d^{7}ax}{b^{7}}+\frac{7d^{6}cx}{b^{6}}+\frac{\left(35a^{3}b^{3}d^{7}-105a^{2}b^{4}c\,d^{6}+105a\,b^{5}c^{2}d^{5}-35b^{6}c^{3}d^{4}\right)x^{4}+\frac{35b^{2}d^{3}\left(7a^{4}d^{4}-20a^{3}bc\,d^{3}+18a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d-\cdots\right.}{2}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -d^6/b^7*(-1/2*b*d*x^2+6*a*d*x-7*b*c*x)+35/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)/(b*x+a)-7/4/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c
^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)
^4-1/5/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35
*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/(b*x+a)^5-35/2/b
^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x
+a)^2+21/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(b*x+a)+7/b^8*d^2*(a^5*d^5-5
*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(
b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(173) = 346$.
time = 0.31, size = 504, normalized size = 2.78

$$-\frac{4\,b^{7}c^{7}+7\,ab^{6}c^{6}d+14\,a^{2}b^{5}c^{5}d^{2}+35\,a^{3}b^{4}c^{4}d^{3}+140\,a^{4}b^{3}c^{3}d^{4}-959\,a^{5}b^{2}c^{2}d^{5}+1218\,a^{6}bcd^{6}-459\,a^{7}d^{7}+700\,(b^{7}c^{3}d^{4}-3\,ab^{6}c^{2}d^{5}+3\,a^{2}b^{5}cd^{6}-a^{3}b^{4}d^{7})x^{4}+350\,(b^{7}c^{4}d^{3}+4\,ab^{6}c^{3}d^{4}-18\,a^{2}b^{5}c^{2}d^{5}+20\,a^{3}b^{4}cd^{6}-7\,a^{4}b^{3}d^{7})x^{3}+70\,(2\,b^{7}c^{5}d^{2}+5\,ab^{6}c^{4}d^{3}+20\,a^{2}b^{5}c^{3}d^{4}-110\,a^{3}b^{4}c^{2}d^{5}+130\,a^{4}b^{3}cd^{6}-47\,a^{5}b^{2}d^{7})x^{2}+35\,(b^{7}c^{6}d+2\,ab^{6}c^{5}d^{2}+5\,a^{2}b^{5}c^{4}d^{3}+20\,a^{3}b^{4}c^{3}d^{4}-125\,a^{4}b^{3}c^{2}d^{5}+154\,a^{5}b^{2}cd^{6}-57\,a^{6}bd^{7})x}{20\,(b^{13}x^{5}+5\,ab^{12}x^{4}+10\,a^{2}b^{11}x^{3}+10\,a^{3}b^{10}x^{2}+5\,a^{4}b^{9}x+a^{5}b^{8})}+\frac{bd^{7}x^{2}+2\,(7\,bcd^{6}-6\,ad^{7})x}{2\,b^{7}}+\frac{21\,(b^{2}c^{2}d^{5}-2\,abcd^{6}+a^{2}d^{7})\log(bx+a)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(4*b^7*c^7 + 7*a*b^6*c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3
+ 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^
7 + 700*(b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4
 + 350*(b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d
^6 - 7*a^4*b^3*d^7)*x^3 + 70*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*
c^3*d^4 - 110*a^3*b^4*c^2*d^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 3
5*(b^7*c^6*d + 2*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 1
25*a^4*b^3*c^2*d^5 + 154*a^5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/(b^13*x^5 + 5*a*b
^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*
(b*d^7*x^2 + 2*(7*b*c*d^6 - 6*a*d^7)*x)/b^7 + 21*(b^2*c^2*d^5 - 2*a*b*c*d^6
 + a^2*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(173) = 346$.

time = 0.29, size = 732, normalized size = 4.04

$$\frac{10b^7d^7x^7 - 4b^7c^7 - 7ab^6c^6d - 14a^2b^5c^5d^2 - 35a^3b^4c^4d^3 - 140a^4b^3c^3d^4 + 959a^5b^2c^2d^5 - 1218a^6bcd^6 + 459a^7d^7 + 70(2b^7cd^6 - ab^6d^7)x^6 + 100(7ab^6cd^6 - 5a^2b^5d^7)x^5 - 100(7b^7c^3d^4 - 21ab^6c^2d^5 + 7a^2b^5cd^6 + 4a^3b^4d^7)x^4 - 50(7b^7c^4d^3 + 28ab^6c^3d^4 - 126a^2b^5c^2d^5 + 112a^3b^4cd^6 - 26a^4b^3d^7)x^3 - 10(14b^7c^5d^2 + 35ab^6c^4d^3 + 140a^2b^5c^3d^4 - 770a^3b^4c^2d^5 + 840a^4b^3cd^6 - 270a^5b^2d^7)x^2 - 5(7b^7c^6d + 14ab^6c^5d^2 + 35a^2b^5c^4d^3 + 140a^3b^4c^3d^4 - 875a^4b^3c^2d^5 + 1050a^5b^2cd^6 - 375a^6bd^7)x + 420(a^5b^2c^2d^5 - 2a^6bcd^6 + a^7d^7 + (b^7c^2d^5 - 2ab^6cd^6 + a^2b^5d^7)x^5 + 5(ab^6c^2d^5 - 2a^2b^5cd^6 + a^3b^4d^7)x^4 + 10(a^2b^5c^2d^5 - 2a^3b^4cd^6 + a^4b^3d^7)x^3 + 10(a^3b^4c^2d^5 - 2a^4b^3cd^6 + a^5b^2d^7)x^2 + 5(a^4b^3c^2d^5 - 2a^5b^2cd^6 + a^6bd^7)x)\log(bx + a)}{20(b^{13}x^5 + 5ab^{12}x^4 + 10a^2b^{11}x^3 + 10a^3b^{10}x^2 + 5a^4b^9x + a^5b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*d^7*x^7 - 4*b^7*c^7 - 7*a*b^6*c^6*d - 14*a^2*b^5*c^5*d^2 - 35*
a^3*b^4*c^4*d^3 - 140*a^4*b^3*c^3*d^4 + 959*a^5*b^2*c^2*d^5 - 1218*a^6*b*c*
d^6 + 459*a^7*d^7 + 70*(2*b^7*c*d^6 - a*b^6*d^7)*x^6 + 100*(7*a*b^6*c*d^6 -
 5*a^2*b^5*d^7)*x^5 - 100*(7*b^7*c^3*d^4 - 21*a*b^6*c^2*d^5 + 7*a^2*b^5*c*d
^6 + 4*a^3*b^4*d^7)*x^4 - 50*(7*b^7*c^4*d^3 + 28*a*b^6*c^3*d^4 - 126*a^2*b^
5*c^2*d^5 + 112*a^3*b^4*c*d^6 - 26*a^4*b^3*d^7)*x^3 - 10*(14*b^7*c^5*d^2 +
35*a*b^6*c^4*d^3 + 140*a^2*b^5*c^3*d^4 - 770*a^3*b^4*c^2*d^5 + 840*a^4*b^3*
c*d^6 - 270*a^5*b^2*d^7)*x^2 - 5*(7*b^7*c^6*d + 14*a*b^6*c^5*d^2 + 35*a^2*b
^5*c^4*d^3 + 140*a^3*b^4*c^3*d^4 - 875*a^4*b^3*c^2*d^5 + 1050*a^5*b^2*c*d^6
 - 375*a^6*b*d^7)*x + 420*(a^5*b^2*c^2*d^5 - 2*a^6*b*c*d^6 + a^7*d^7 + (b^7
*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 5*(a*b^6*c^2*d^5 - 2*a^2*b^5*
c*d^6 + a^3*b^4*d^7)*x^4 + 10*(a^2*b^5*c^2*d^5 - 2*a^3*b^4*c*d^6 + a^4*b^3*
d^7)*x^3 + 10*(a^3*b^4*c^2*d^5 - 2*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 5*(a^
4*b^3*c^2*d^5 - 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^13*x^5 + 5
*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(173) = 346.
time = 0.00, size = 493, normalized size = 2.72

$$\frac{\frac{1}{2}x^2d^7b^6 - 6xd^7b^5a + 7xd^6cb^6}{b^{12}} + \frac{\frac{1}{20}\left((700d^7b^4a^3 - 2100d^6b^5a^2c + 2100d^5b^6ac^2 - 700d^4b^7c^3)x^4 + (2450d^7b^3a^4 - 7000d^6b^4a^3c + 6300d^5b^5a^2c^2 - 1400d^4b^6ac^3 - 350d^3b^7c^4)x^3 + (3290d^7b^2a^5 - 9100d^6b^3a^4c + 7700d^5b^4a^3c^2 - 1400d^4b^5a^2c^3 - 350d^3b^6ac^4 - 140d^2b^7c^5)x^2 + (1995d^7ba^6 - 5390d^6b^2a^5c + 4375d^5b^3a^4c^2 - 700d^4b^4a^3c^3 - 175d^3b^5a^2c^4 - 70d^2b^6ac^5 - 35db^7c^6)x + 459d^7a^7 - 1218d^6ba^6c + 959d^5b^2a^5c^2 - 140d^4b^3a^4c^3 - 35d^3b^4a^3c^4 - 14d^2b^5a^2c^5 - 7db^6ac^6 - 4b^7c^7\right)}{b^8(xb + a)^5} + \frac{(21d^7a^2 - 42d^6cba + 21d^5c^2b^2)\ln|xb + a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x)
```

```
[Out] 21*(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*d
^7*x^2 + 14*b^6*c*d^6*x - 12*a*b^5*d^7*x)/b^12 - 1/20*(4*b^7*c^7 + 7*a*b^6*
c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959
```

```
*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^7 + 700*(b^7*c^3*d^4 - 3*a*
b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 350*(b^7*c^4*d^3 + 4*a*b
^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d^6 - 7*a^4*b^3*d^7)*x^3 + 7
0*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 110*a^3*b^4*c^2*d
^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 35*(b^7*c^6*d + 2*a*b^6*c^5*
d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 125*a^4*b^3*c^2*d^5 + 154*a^
5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/((b*x + a)^5*b^8)
```

Mupad [B]

time = 0.34, size = 508, normalized size = 2.81

$$\frac{\ln(a+bx)\,(21\,a^2\,d^7-42\,a\,b\,c\,d^6+21\,b^2\,c^2\,d^5)}{b^8}-x\left(\frac{6\,a\,d^7}{b^7}-\frac{7\,c\,d^6}{b^6}\right)-\frac{\frac{-459\,a^7\,d^7+1218\,a^6\,b\,c\,d^6-959\,a^5\,b^2\,c^2\,d^5+140\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+14\,a^2\,b^5\,c^5\,d^2+7\,a\,b^6\,c^6\,d+4\,b^7\,c^7}{20\,b}+x\left(-\frac{399\,a^6\,d^7}{4}+\frac{539\,a^5\,b\,c\,d^6}{2}-\frac{875\,a^4\,b^2\,c^2\,d^5}{4}+35\,a^3\,b^3\,c^3\,d^4+\frac{35\,a^2\,b^4\,c^4\,d^3}{4}+\frac{7\,a\,b^5\,c^5\,d^2}{2}+\frac{7\,b^6\,c^6\,d}{4}\right)+x^3\left(-\frac{245\,a^4\,b^2\,d^7}{2}+350\,a^3\,b^3\,c\,d^6-315\,a^2\,b^4\,c^2\,d^5+70\,a\,b^5\,c^3\,d^4+\frac{35\,b^6\,c^4\,d^3}{2}\right)+x^2\left(-\frac{329\,a^5\,b\,d^7}{2}+455\,a^4\,b^2\,c\,d^6-385\,a^3\,b^3\,c^2\,d^5+70\,a^2\,b^4\,c^3\,d^4+\frac{35\,a\,b^5\,c^4\,d^3}{2}+7\,b^6\,c^5\,d^2\right)-x^4\,(35\,a^3\,b^3\,d^7-105\,a^2\,b^4\,c\,d^6+105\,a\,b^5\,c^2\,d^5-35\,b^6\,c^3\,d^4)}{a^5\,b^7+5\,a^4\,b^8\,x+10\,a^3\,b^9\,x^2+10\,a^2\,b^{10}\,x^3+5\,a\,b^{11}\,x^4+b^{12}\,x^5}+\frac{d^7\,x^2}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^6,x)
```

```
[Out] (log(a + b*x)*(21*a^2*d^7 + 21*b^2*c^2*d^5 - 42*a*b*c*d^6))/b^8 - x*((6*a*d
^7)/b^7 - (7*c*d^6)/b^6) - ((4*b^7*c^7 - 459*a^7*d^7 + 14*a^2*b^5*c^5*d^2 +
 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 7*a*b^6*c
^6*d + 1218*a^6*b*c*d^6)/(20*b) + x*((7*b^6*c^6*d)/4 - (399*a^6*d^7)/4 + (7
*a*b^5*c^5*d^2)/2 + (35*a^2*b^4*c^4*d^3)/4 + 35*a^3*b^3*c^3*d^4 - (875*a^4*
b^2*c^2*d^5)/4 + (539*a^5*b*c*d^6)/2) + x^3*((35*b^6*c^4*d^3)/2 - (245*a^4*
b^2*d^7)/2 + 70*a*b^5*c^3*d^4 + 350*a^3*b^3*c*d^6 - 315*a^2*b^4*c^2*d^5) +
x^2*(7*b^6*c^5*d^2 - (329*a^5*b*d^7)/2 + (35*a*b^5*c^4*d^3)/2 + 455*a^4*b^2
*c*d^6 + 70*a^2*b^4*c^3*d^4 - 385*a^3*b^3*c^2*d^5) - x^4*(35*a^3*b^3*d^7 -
35*b^6*c^3*d^4 + 105*a*b^5*c^2*d^5 - 105*a^2*b^4*c*d^6))/(a^5*b^7 + b^12*x^
5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*a^2*b^10*x^3) + (d^7*x
^2)/(2*b^6)
```

3.1289 $\int \frac{(c+dx)^7}{(a+bx)^7} dx$

Optimal. Leaf size=186

$$\frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}+\frac{7d^6(\cdots}{}$$

```
[Out] d^7*x/b^7-1/6*(-a*d+b*c)^7/b^8/(b*x+a)^6-7/5*d*(-a*d+b*c)^6/b^8/(b*x+a)^5-2
1/4*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^4-35/3*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^3-35/2
*d^4*(-a*d+b*c)^3/b^8/(b*x+a)^2-21*d^5*(-a*d+b*c)^2/b^8/(b*x+a)+7*d^6*(-a*d
+b*c)*ln(b*x+a)/b^8
```

Rubi [A]

time = 0.11, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}+\frac{d^7x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] (d^7*x)/b^7 - (b*c - a*d)^7/(6*b^8*(a + b*x)^6) - (7*d*(b*c - a*d)^6)/(5*b^
8*(a + b*x)^5) - (21*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^4) - (35*d^3*(b*c
- a*d)^4)/(3*b^8*(a + b*x)^3) - (35*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^2)
- (21*d^5*(b*c - a*d)^2)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^7}\,dx=\int \left(\frac{d^7}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^7}+\frac{7d(bc-ad)^6}{b^7(a+bx)^6}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^5}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^4}+\frac{35d^4(bc-ad}{b^7(a+bx)^3}\cdots\right.$$
$$=\frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}\cdots$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 390 vs. $2(186) = 372$.
time = 0.13, size = 390, normalized size = 2.10

$$-\frac{669a^7d^7+3a^6bd^6(-343c+1198dx)+3a^5b^2d^5\left(70c^2-1918cdx+2575d^2x^2\right)+5a^4b^3d^4\left(14c^3+252c^2dx-2625cd^2x^2+1640d^3x^3\right)+5a^3b^4d^3\left(7c^4+84c^3dx+630c^2d^2x^2-3080cd^3x^3+810d^4x^4\right)+3a^2b^5d^2\left(7c^5+70c^4dx+350c^3d^2x^2+1400c^2d^3x^3-3150cd^4x^4+120d^5x^5\right)+ab^6d\left(14c^6+126c^5dx+525c^4d^2x^2+1400c^3d^3x^3+3150c^2d^4x^4-2520cd^5x^5-360d^6x^6\right)+b^7\left(10c^7+84c^6dx+315c^5d^2x^2+700c^4d^3x^3+1050c^3d^4x^4+1260c^2d^5x^5-60d^7x^7\right)+420d^6(-bc+ad)(a+bx)^6\log(a+bx)}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7*d^7 + 3*a^6*b*d^6*(-343*c + 1198*d*x) + 3*a^5*b^2*d^5*(70*c^
2 - 1918*c*d*x + 2575*d^2*x^2) + 5*a^4*b^3*d^4*(14*c^3 + 252*c^2*d*x - 2625
*c*d^2*x^2 + 1640*d^3*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 84*c^3*d*x + 630*c^2*d^
2*x^2 - 3080*c*d^3*x^3 + 810*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 70*c^4*d*x +
 350*c^3*d^2*x^2 + 1400*c^2*d^3*x^3 - 3150*c*d^4*x^4 + 120*d^5*x^5) + a*b^6
*d*(14*c^6 + 126*c^5*d*x + 525*c^4*d^2*x^2 + 1400*c^3*d^3*x^3 + 3150*c^2*d^
4*x^4 - 2520*c*d^5*x^5 - 360*d^6*x^6) + b^7*(10*c^7 + 84*c^6*d*x + 315*c^5*
d^2*x^2 + 700*c^4*d^3*x^3 + 1050*c^3*d^4*x^4 + 1260*c^2*d^5*x^5 - 60*d^7*x^
7) + 420*d^6*(-(b*c) + a*d)*(a + b*x)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 612 vs. $2(186) = 372$.
time = 236.59, size = 610, normalized size = 3.28

$$\frac{-669a^7d^7+1029a^6bcd^6-210a^5b^2c^2d^5-70a^4b^3c^3d^4-35a^3b^4c^4d^3-21a^2b^5c^5d^2-14ab^6c^6d-10b^7c^7-42bdx\left(87a^6d^6-137a^5bcd^5+30a^4b^2c^2d^4+10a^3b^3c^3d^3+5a^2b^4c^4d^2+3ab^5c^5d+2b^6c^6\right)+105b^2d^2x^2\left(-77a^5d^5+125a^4bcd^4-30a^3b^2c^2d^3-10a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5\right)+700b^3d^3x^3\left(-13a^4d^4+22a^3bcd^3-6a^2b^2c^2d^2-2ab^3c^3d-b^4c^4\right)+1050b^4d^4x^4\left(-5a^3d^3+9a^2bcd^2-3ab^2c^2d-b^3c^3\right)+1260b^5d^5x^5\left(-a^2d^2+2abcd-b^2c^2\right)-420d^6\mathrm{Log}\left[a+bx\right]\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)(ad-bc)+60bd^7x\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}{60b^8\left(a^6+6a^5bx+15a^4b^2x^2+20a^3b^3x^3+15a^2b^4x^4+6ab^5x^5+b^6x^6\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^7,x]')
```

```
[Out] (-669 a ^ 7 d ^ 7 + 1029 a ^ 6 b c d ^ 6 - 210 a ^ 5 b ^ 2 c ^ 2 d ^ 5 - 70
 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 35 a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 21 a ^ 2 b ^ 5 c ^
5 d ^ 2 - 14 a b ^ 6 c ^ 6 d - 10 b ^ 7 c ^ 7 - 42 b d x (87 a ^ 6 d ^ 6 -
137 a ^ 5 b c d ^ 5 + 30 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 10 a ^ 3 b ^ 3 c ^ 3 d ^
 3 + 5 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 3 a b ^ 5 c ^ 5 d + 2 b ^ 6 c ^ 6) + 105 b
 ^ 2 d ^ 2 x ^ 2 (-77 a ^ 5 d ^ 5 + 125 a ^ 4 b c d ^ 4 - 30 a ^ 3 b ^ 2 c
^ 2 d ^ 3 - 10 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 5 a b ^ 4 c ^ 4 d - 3 b ^ 5 c ^ 5)
 + 700 b ^ 3 d ^ 3 x ^ 3 (-13 a ^ 4 d ^ 4 + 22 a ^ 3 b c d ^ 3 - 6 a ^ 2 b
^ 2 c ^ 2 d ^ 2 - 2 a b ^ 3 c ^ 3 d - b ^ 4 c ^ 4) + 1050 b ^ 4 d ^ 4 x ^ 4
 (-5 a ^ 3 d ^ 3 + 9 a ^ 2 b c d ^ 2 - 3 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3) + 1
260 b ^ 5 d ^ 5 x ^ 5 (-a ^ 2 d ^ 2 + 2 a b c d - b ^ 2 c ^ 2) - 420 d ^ 6
Log[a + b x] (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x
 ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6) (a d - b c) +
60 b d ^ 7 x (a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x
 ^ 3 + 15 a ^ 2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6)) / (60 b ^ 8 (
a ^ 6 + 6 a ^ 5 b x + 15 a ^ 4 b ^ 2 x ^ 2 + 20 a ^ 3 b ^ 3 x ^ 3 + 15 a ^
2 b ^ 4 x ^ 4 + 6 a b ^ 5 x ^ 5 + b ^ 6 x ^ 6))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(176) = 352$.

time = 0.15, size = 456, normalized size = 2.45

method	result
risch	$\frac{d^7 x}{b^7} + \frac{(-21a^2b^4d^7+42a\,b^5c\,d^6-21b^6c^2d^5)x^5-\frac{35b^3d^4\left(5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)x^4}{2}-\frac{35b^2d^3\left(13a^4d^4-22a^3bc\,d^3+6a^2b^2c^2d^2+2a\,b^3 \right.}{3}}{}$
default	$\frac{d^7 x}{b^7} - \frac{21d^5\left(a^2d^2-2abcd+b^2c^2\right)}{b^8(bx+a)} + \frac{21d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{4b^8(bx+a)^4} - \frac{7d\left(a^6d^6-6a^5bc\,d^5+15a^4b\right.}{}$
norman	$\frac{\frac{d^7x^7}{b} - \frac{1029a^7d^7-1029a^6bc\,d^6+210a^5b^2c^2d^5+70a^4b^3c^3d^4+35a^3b^4c^4d^3+21a^2b^5c^5d^2+14a\,b^6c^6d+10b^7c^7}{60b^8} - \frac{3\left(14a^2d^7-14abc\,d^6+7b^2c^2d^5\right)x^5}{b^3} - \frac{5(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] d^7*x/b^7-21/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+21/4/b^8*d^2*(a^5*
d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c
^5)/(b*x+a)^4-7/5/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^
3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^5+35/2/b^8*d^4*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^2-1/6/b^8*(-a^7*d^7+7
*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^
2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/(b*x+a)^6-7/b^8*d^6*(a*d-b*c)*ln(b*x+a
)-35/3/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c
^4)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(176) = 352$.

time = 0.32, size = 516, normalized size = 2.77

$$\frac{d^7x}{b^7} - \frac{10\,b^7c^7 + 14\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 35\,a^3b^4c^4d^3 + 70\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 - 1029\,a^6bcd^6 + 669\,a^7d^7 + 1260\,(b^7c^2d^5 - 2\,ab^6cd^6 + a^2b^5d^7)x^5 + 1050\,(b^7c^3d^4 + 3\,ab^6c^2d^5 - 9\,a^2b^5cd^6 + 5\,a^3b^4d^7)x^4 + 700\,(b^7c^4d^3 + 2\,ab^6c^3d^4 + 6\,a^2b^5c^2d^5 - 22\,a^3b^4cd^6 + 13\,a^4b^3d^7)x^3 + 105\,(3\,b^7c^5d^2 + 5\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 + 30\,a^3b^4c^2d^5 - 125\,a^4b^3cd^6 + 77\,a^5b^2d^7)x^2 + 42\,(2\,b^7c^6d + 3\,ab^6c^5d^2 + 5\,a^2b^5c^4d^3 + 10\,a^3b^4c^3d^4 + 30\,a^4b^3c^2d^5 - 137\,a^5b^2cd^6 + 87\,a^6bd^7)x}{60\,(b^{14}x^6 + 6\,ab^{13}x^5 + 15\,a^2b^{12}x^4 + 20\,a^3b^{11}x^3 + 15\,a^4b^{10}x^2 + 6\,a^5b^9x + a^6b^8)} + \frac{7\,(bcd^6 - ad^7)\log(bx + a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] d^7*x/b^7 - 1/60*(10*b^7*c^7 + 14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3
*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6
+ 669*a^7*d^7 + 1260*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050
*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 70
0*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 1
3*a^4*b^3*d^7)*x^3 + 105*(3*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*
d^4 + 30*a^3*b^4*c^2*d^5 - 125*a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*
b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a
^4*b^3*c^2*d^5 - 137*a^5*b^2*c*d^6 + 87*a^6*b*d^7)*x)/(b^14*x^6 + 6*a*b^13*
x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a
^6*b^8) + 7*(b*c*d^6 - a*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(176) = 352$.
time = 0.29, size = 692, normalized size = 3.72

$$\frac{60 b^7 d^7 x^7 + 360 a b^6 d^7 x^6 - 10 b^7 c^7 - 14 a b^6 c^6 d - 21 a^2 b^5 c^5 d^2 - 35 a^3 b^4 c^4 d^3 - 70 a^4 b^3 c^3 d^4 - 210 a^5 b^2 c^2 d^5 + 1029 a^6 b c d^6 - 669 a^7 d^7 - 180 (7 b^7 c^2 d^5 - 14 a b^6 c d^6 + 2 a^2 b^5 d^7) x^5 - 150 (7 b^7 c^3 d^4 + 21 a b^6 c^2 d^5 - 63 a^2 b^5 c d^6 + 27 a^3 b^4 d^7) x^4 - 100 (7 b^7 c^4 d^3 + 14 a b^6 c^3 d^4 + 42 a^2 b^5 c^2 d^5 - 154 a^3 b^4 c d^6 + 82 a^4 b^3 d^7) x^3 - 15 (21 b^7 c^5 d^2 + 35 a b^6 c^4 d^3 + 70 a^2 b^5 c^3 d^4 + 210 a^3 b^4 c^2 d^5 - 875 a^4 b^3 c d^6 + 515 a^5 b^2 d^7) x^2 - 6 (14 b^7 c^6 d + 21 a b^6 c^5 d^2 + 35 a^2 b^5 c^4 d^3 + 70 a^3 b^4 c^3 d^4 + 210 a^4 b^3 c^2 d^5 - 959 a^5 b^2 c d^6 + 599 a^6 b d^7) x + 420 (a^6 b c d^6 - a^7 d^7 + (b^7 c d^6 - a b^6 d^7) x^6 + 6 (a b^6 c d^6 - a^2 b^5 d^7) x^5 + 15 (a^2 b^5 c d^6 - a^3 b^4 d^7) x^4 + 20 (a^3 b^4 c d^6 - a^4 b^3 d^7) x^3 + 15 (a^4 b^3 c d^6 - a^5 b^2 d^7) x^2 + 6 (a^5 b^2 c d^6 - a^6 b d^7) x) \log (b x + a)}{60 (b^{14} x^6 + 6 a b^{13} x^5 + 15 a^2 b^{12} x^4 + 20 a^3 b^{11} x^3 + 15 a^4 b^{10} x^2 + 6 a^5 b^9 x + a^6 b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*d^7*x^7 + 360*a*b^6*d^7*x^6 - 10*b^7*c^7 - 14*a*b^6*c^6*d - 21
*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 - 70*a^4*b^3*c^3*d^4 - 210*a^5*b^2*c^
2*d^5 + 1029*a^6*b*c*d^6 - 669*a^7*d^7 - 180*(7*b^7*c^2*d^5 - 14*a*b^6*c*d^
6 + 2*a^2*b^5*d^7)*x^5 - 150*(7*b^7*c^3*d^4 + 21*a*b^6*c^2*d^5 - 63*a^2*b^5
*c*d^6 + 27*a^3*b^4*d^7)*x^4 - 100*(7*b^7*c^4*d^3 + 14*a*b^6*c^3*d^4 + 42*a
^2*b^5*c^2*d^5 - 154*a^3*b^4*c*d^6 + 82*a^4*b^3*d^7)*x^3 - 15*(21*b^7*c^5*d
^2 + 35*a*b^6*c^4*d^3 + 70*a^2*b^5*c^3*d^4 + 210*a^3*b^4*c^2*d^5 - 875*a^4*
b^3*c*d^6 + 515*a^5*b^2*d^7)*x^2 - 6*(14*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 35*
a^2*b^5*c^4*d^3 + 70*a^3*b^4*c^3*d^4 + 210*a^4*b^3*c^2*d^5 - 959*a^5*b^2*c*
d^6 + 599*a^6*b*d^7)*x + 420*(a^6*b*c*d^6 - a^7*d^7 + (b^7*c*d^6 - a*b^6*d^
7)*x^6 + 6*(a*b^6*c*d^6 - a^2*b^5*d^7)*x^5 + 15*(a^2*b^5*c*d^6 - a^3*b^4*d^
7)*x^4 + 20*(a^3*b^4*c*d^6 - a^4*b^3*d^7)*x^3 + 15*(a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^2 + 6*(a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^14*x^6 + 6*a*
b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*
x + a^6*b^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(176) = 352$.
time = 0.00, size = 492, normalized size = 2.65

$$\frac{x d^7}{b^7} + \frac{\frac{1}{60} \left((-1260 d^7 b^5 a^2 + 2520 d^6 b^6 a c - 1260 d^5 b^7 c^2) x^5 + (-5250 d^7 b^4 a^3 + 9450 d^6 b^5 a^2 c - 3150 d^5 b^6 a c^2 - 1050 d^4 b^7 c^3) x^4 + (-9100 d^7 b^3 a^4 + 15400 d^6 b^4 a^3 c - 4200 d^5 b^5 a^2 c^2 - 1400 d^4 b^6 a c^3 - 700 d^3 b^7 c^4) x^3 + (-8085 d^7 b^2 a^5 + 13125 d^6 b^3 a^4 c - 3150 d^5 b^4 a^3 c^2 - 1050 d^4 b^5 a^2 c^3 - 525 d^3 b^6 a c^4 - 315 d^2 b^7 c^5) x^2 + (-3654 d^7 b a^6 + 5754 d^6 b^2 a^5 c - 1260 d^5 b^3 a^4 c^2 - 420 d^4 b^4 a^3 c^3 - 210 d^3 b^5 a^2 c^4 - 126 d^2 b^6 a c^5 - 84 d b^7 c^6) x - 669 d^7 a^7 + 1029 d^6 b a^6 c - 210 d^5 b^2 a^5 c^2 - 70 d^4 b^3 a^4 c^3 - 35 d^3 b^4 a^3 c^4 - 21 d^2 b^5 a^2 c^5 - 14 d b^6 a c^6 - 10 b^7 c^7 \right)}{b^8 (x b + a)^6} + \frac{(-7 d^7 a + 7 d^6 c b) \ln |x b + a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x)
```

```
[Out] d^7*x/b^7 + 7*(b*c*d^6 - a*d^7)*log(abs(b*x + a))/b^8 - 1/60*(10*b^7*c^7 +
14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d
```

```
^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6 + 669*a^7*d^7 + 1260*(b^7*c^2*d
^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5
 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 700*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^
4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 13*a^4*b^3*d^7)*x^3 + 105*(3*b^7
*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 - 125*
a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5
*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 - 137*a^5*b^2*c*
d^6 + 87*a^6*b*d^7)*x)/((b*x + a)^6*b^8)
```

Mupad [B]
time = 0.37, size = 517, normalized size = 2.78

$$\frac{d^7\,x}{b^7}-\frac{\ln\left(a+b\,x\right)\,\left(7\,a\,d^7-7\,b\,c\,d^6\right)}{b^8}-\frac{\frac{669\,a^7\,d^7-1029\,a^6\,b\,c\,d^6+210\,a^5\,b^2\,c^2\,d^5+70\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+21\,a^2\,b^5\,c^5\,d^2+14\,a\,b^6\,c^6\,d+10\,b^7\,c^7}{60\,b}+x\,\left(\frac{609\,a^6\,d^7}{10}-\frac{959\,a^5\,b\,c\,d^6}{10}+21\,a^4\,b^2\,c^2\,d^5+7\,a^3\,b^3\,c^3\,d^4+\frac{7\,a^2\,b^4\,c^4\,d^3}{2}+\frac{21\,a\,b^5\,c^5\,d^2}{10}+\frac{7\,b^6\,c^6\,d}{5}\right)+x^3\,\left(\frac{455\,a^4\,b^2\,d^7}{3}-\frac{770\,a^3\,b^3\,c\,d^6}{3}+70\,a^2\,b^4\,c^2\,d^5+\frac{70\,a\,b^5\,c^3\,d^4}{3}+\frac{35\,b^6\,c^4\,d^3}{3}\right)+x^2\,\left(\frac{539\,a^5\,b\,d^7}{4}-\frac{875\,a^4\,b^2\,c\,d^6}{4}+\frac{105\,a^3\,b^3\,c^2\,d^5}{2}+\frac{35\,a^2\,b^4\,c^3\,d^4}{2}+\frac{35\,a\,b^5\,c^4\,d^3}{4}+\frac{21\,b^6\,c^5\,d^2}{4}\right)+x^5\,\left(21\,a^2\,b^4\,d^7-42\,a\,b^5\,c\,d^6+21\,b^6\,c^2\,d^5\right)+x^4\,\left(\frac{175\,a^3\,b^3\,d^7}{2}-\frac{315\,a^2\,b^4\,c\,d^6}{2}+\frac{105\,a\,b^5\,c^2\,d^5}{2}+\frac{35\,b^6\,c^3\,d^4}{2}\right)}{a^6\,b^7+6\,a^5\,b^8\,x+15\,a^4\,b^9\,x^2+20\,a^3\,b^{10}\,x^3+15\,a^2\,b^{11}\,x^4+6\,a\,b^{12}\,x^5+b^{13}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^7,x)
```

```
[Out] (d^7*x)/b^7 - (log(a + b*x)*(7*a*d^7 - 7*b*c*d^6))/b^8 - ((669*a^7*d^7 + 10
*b^7*c^7 + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 2
10*a^5*b^2*c^2*d^5 + 14*a*b^6*c^6*d - 1029*a^6*b*c*d^6)/(60*b) + x*((609*a^
6*d^7)/10 + (7*b^6*c^6*d)/5 + (21*a*b^5*c^5*d^2)/10 + (7*a^2*b^4*c^4*d^3)/2
 + 7*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - (959*a^5*b*c*d^6)/10) + x^3*((4
55*a^4*b^2*d^7)/3 + (35*b^6*c^4*d^3)/3 + (70*a*b^5*c^3*d^4)/3 - (770*a^3*b^
3*c*d^6)/3 + 70*a^2*b^4*c^2*d^5) + x^2*((539*a^5*b*d^7)/4 + (21*b^6*c^5*d^2
)/4 + (35*a*b^5*c^4*d^3)/4 - (875*a^4*b^2*c*d^6)/4 + (35*a^2*b^4*c^3*d^4)/2
 + (105*a^3*b^3*c^2*d^5)/2) + x^5*(21*a^2*b^4*d^7 + 21*b^6*c^2*d^5 - 42*a*b
^5*c*d^6) + x^4*((175*a^3*b^3*d^7)/2 + (35*b^6*c^3*d^4)/2 + (105*a*b^5*c^2*
d^5)/2 - (315*a^2*b^4*c*d^6)/2))/(a^6*b^7 + b^13*x^6 + 6*a^5*b^8*x + 6*a*b^
12*x^5 + 15*a^4*b^9*x^2 + 20*a^3*b^10*x^3 + 15*a^2*b^11*x^4)
```

3.1290 $\int \frac{(c+dx)^7}{(a+bx)^8}\,dx$

Optimal. Leaf size=194

$$-\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{7d^6(bc-ad)}{b^8(a+bx)}$$

```
[Out] -1/7*(-a*d+b*c)^7/b^8/(b*x+a)^7-7/6*d*(-a*d+b*c)^6/b^8/(b*x+a)^6-21/5*d^2*(
-a*d+b*c)^5/b^8/(b*x+a)^5-35/4*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^4-35/3*d^4*(-a*
d+b*c)^3/b^8/(b*x+a)^3-21/2*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^2-7*d^6*(-a*d+b*c)
/b^8/(b*x+a)+d^7*ln(b*x+a)/b^8
```

Rubi [A]

time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+\frac{d^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -1/7*(b*c - a*d)^7/(b^8*(a + b*x)^7) - (7*d*(b*c - a*d)^6)/(6*b^8*(a + b*x)
^6) - (21*d^2*(b*c - a*d)^5)/(5*b^8*(a + b*x)^5) - (35*d^3*(b*c - a*d)^4)/(
4*b^8*(a + b*x)^4) - (35*d^4*(b*c - a*d)^3)/(3*b^8*(a + b*x)^3) - (21*d^5*(
b*c - a*d)^2)/(2*b^8*(a + b*x)^2) - (7*d^6*(b*c - a*d))/(b^8*(a + b*x)) + (
d^7*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^8}+\frac{7d(bc-ad)^6}{b^7(a+bx)^7}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^6}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^5}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^4}+\right.\\ &= -\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\end{aligned}$$

Mathematica [A]
time = 0.10, size = 308, normalized size = 1.59

$$-\frac{(bc-ad)\left(1089a^6d^6+3a^5bd^5(223c+2401dx)+3a^4b^2d^4(153c^2+1421cdx+6713d^2x^2)+a^3b^3d^3(319c^3+2793c^2dx+11319cd^2x^2+30625d^3x^3)+a^2b^4d^2(214c^4+1813c^3dx+6909c^2d^2x^2+15925cd^3x^3+26950d^4x^4)+ab^5d(130c^5+1078c^4dx+3969c^3d^2x^2+8575c^2d^3x^3+12250cd^4x^4+13230d^5x^5)+b^6(60c^6+490c^5dx+1764c^4d^2x^2+3675c^3d^3x^3+4900c^2d^4x^4+4410cd^5x^5+2940d^6x^6)\right)}{420b^8(a+bx)^7}+\frac{d^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -1/420*((b*c - a*d)*(1089*a^6*d^6 + 3*a^5*b*d^5*(223*c + 2401*d*x) + 3*a^4*
b^2*d^4*(153*c^2 + 1421*c*d*x + 6713*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 2793
*c^2*d*x + 11319*c*d^2*x^2 + 30625*d^3*x^3) + a^2*b^4*d^2*(214*c^4 + 1813*c
^3*d*x + 6909*c^2*d^2*x^2 + 15925*c*d^3*x^3 + 26950*d^4*x^4) + a*b^5*d*(130
*c^5 + 1078*c^4*d*x + 3969*c^3*d^2*x^2 + 8575*c^2*d^3*x^3 + 12250*c*d^4*x^4
 + 13230*d^5*x^5) + b^6*(60*c^6 + 490*c^5*d*x + 1764*c^4*d^2*x^2 + 3675*c^3
*d^3*x^3 + 4900*c^2*d^4*x^4 + 4410*c*d^5*x^5 + 2940*d^6*x^6)))/(b^8*(a + b*
x)^7) + (d^7*Log[a + b*x])/b^8
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^8,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(182) = 364.
time = 0.15, size = 462, normalized size = 2.38

method	result
risch	$\frac{7d^6(ad-bc)x^6}{b^2}+\frac{21d^5\left(3a^2d^2-2abcd-b^2c^2\right)x^5}{2b^3}+\frac{35d^4\left(11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3\right)x^4}{6b^4}+\frac{35d^3\left(25a^4d^4-12a^3bc\,d^3-6a^2b^2c^2d^2-4a\,b^3c^3d-3\right.}{12b^5}$
norman	$\frac{1089a^7d^7-420a^6bc\,d^6-210a^5b^2c^2d^5-140a^4b^3c^3d^4-105a^3b^4c^4d^3-84a^2b^5c^5d^2-70a\,b^6c^6d-60b^7c^7}{420b^8}+\frac{7\left(a\,d^7-bc\,d^6\right)x^6}{b^2}+\frac{21\left(3a^2d^7-2abc\,d^6-b^2c^2\right.}{2b^3}$
default	$\frac{7d^6(ad-bc)}{b^8(bx+a)}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{4b^8(bx+a)^4}+\frac{21d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c\right.}{5b^8(bx+a)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 7/b^8*d^6*(a*d-b*c)/(b*x+a)-35/4/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^4+21/5/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4
+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^5-1/7
```

```
*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c
^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^7-21/2/b^8*d^5
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2-7/6/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*
a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6
)/(b*x+a)^6+d^7*ln(b*x+a)/b^8+35/3/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(182) = 364$.
time = 0.28, size = 534, normalized size = 2.75

$$-\frac{60\,b^{7}c^{7} + 70\,ab^{6}c^{6}d + 84\,a^{2}b^{5}c^{5}d^{2} + 105\,a^{3}b^{4}c^{4}d^{3} + 140\,a^{4}b^{3}c^{3}d^{4} + 210\,a^{5}b^{2}c^{2}d^{5} + 420\,a^{6}bcd^{6} - 1089\,a^{7}d^{7} + 2940\,(b^{7}cd^{6} - ab^{6}d^{7})x^{6} + 4410\,(b^{7}c^{2}d^{5} + 2\,ab^{6}cd^{6} - 3\,a^{2}b^{5}d^{7})x^{5} + 2450\,(2\,b^{7}c^{3}d^{4} + 3\,ab^{6}c^{2}d^{5} + 6\,a^{2}b^{5}cd^{6} - 11\,a^{3}b^{4}d^{7})x^{4} + 1225\,(3\,b^{7}c^{4}d^{3} + 4\,ab^{6}c^{3}d^{4} + 6\,a^{2}b^{5}c^{2}d^{5} + 12\,a^{3}b^{4}cd^{6} - 25\,a^{4}b^{3}d^{7})x^{3} + 147\,(12\,b^{7}c^{5}d^{2} + 15\,ab^{6}c^{4}d^{3} + 20\,a^{2}b^{5}c^{3}d^{4} + 30\,a^{3}b^{4}c^{2}d^{5} + 60\,a^{4}b^{3}cd^{6} - 137\,a^{5}b^{2}d^{7})x^{2} + 49\,(10\,b^{7}c^{6}d + 12\,ab^{6}c^{5}d^{2} + 15\,a^{2}b^{5}c^{4}d^{3} + 20\,a^{3}b^{4}c^{3}d^{4} + 30\,a^{4}b^{3}c^{2}d^{5} + 60\,a^{5}b^{2}cd^{6} - 147\,a^{6}bd^{7})x}{420\,(b^{15}x^{7} + 7\,ab^{14}x^{6} + 21\,a^{2}b^{13}x^{5} + 35\,a^{3}b^{12}x^{4} + 35\,a^{4}b^{11}x^{3} + 21\,a^{5}b^{10}x^{2} + 7\,a^{6}b^{9}x + a^{7}b^{8})} + \frac{d^{7}\log\left(bx + a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x)/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 + 35*a^3*b^12
*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8) + d^7*log
(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(182) = 364$.
time = 0.30, size = 624, normalized size = 3.22

$$-\frac{60\,b^{7}c^{7} + 70\,ab^{6}c^{6}d + 84\,a^{2}b^{5}c^{5}d^{2} + 105\,a^{3}b^{4}c^{4}d^{3} + 140\,a^{4}b^{3}c^{3}d^{4} + 210\,a^{5}b^{2}c^{2}d^{5} + 420\,a^{6}bcd^{6} - 1089\,a^{7}d^{7} + 2940\,(b^{7}cd^{6} - ab^{6}d^{7})x^{6} + 4410\,(b^{7}c^{2}d^{5} + 2\,ab^{6}cd^{6} - 3\,a^{2}b^{5}d^{7})x^{5} + 2450\,(2\,b^{7}c^{3}d^{4} + 3\,ab^{6}c^{2}d^{5} + 6\,a^{2}b^{5}cd^{6} - 11\,a^{3}b^{4}d^{7})x^{4} + 1225\,(3\,b^{7}c^{4}d^{3} + 4\,ab^{6}c^{3}d^{4} + 6\,a^{2}b^{5}c^{2}d^{5} + 12\,a^{3}b^{4}cd^{6} - 25\,a^{4}b^{3}d^{7})x^{3} + 147\,(12\,b^{7}c^{5}d^{2} + 15\,ab^{6}c^{4}d^{3} + 20\,a^{2}b^{5}c^{3}d^{4} + 30\,a^{3}b^{4}c^{2}d^{5} + 60\,a^{4}b^{3}cd^{6} - 137\,a^{5}b^{2}d^{7})x^{2} + 49\,(10\,b^{7}c^{6}d + 12\,ab^{6}c^{5}d^{2} + 15\,a^{2}b^{5}c^{4}d^{3} + 20\,a^{3}b^{4}c^{3}d^{4} + 30\,a^{4}b^{3}c^{2}d^{5} + 60\,a^{5}b^{2}cd^{6} - 147\,a^{6}bd^{7})x - 420\,(b^{7}d^{7}x^{7} + 7\,ab^{6}d^{7}x^{6} + 21\,a^{2}b^{5}d^{7}x^{5} + 35\,a^{3}b^{4}d^{7}x^{4} + 35\,a^{4}b^{3}d^{7}x^{3} + 21\,a^{5}b^{2}d^{7}x^{2} + 7\,a^{6}bd^{7}x + a^{7}d^{7})\log\left(bx + a\right)}{420\,(b^{15}x^{7} + 7\,ab^{14}x^{6} + 21\,a^{2}b^{13}x^{5} + 35\,a^{3}b^{12}x^{4} + 35\,a^{4}b^{11}x^{3} + 21\,a^{5}b^{10}x^{2} + 7\,a^{6}b^{9}x + a^{7}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
```

```
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x - 420*(b^7*d^7*x^7 + 7*a*b^6*d^7*x^6 + 21*a^2*b^5*d^7*x^5
 + 35*a^3*b^4*d^7*x^4 + 35*a^4*b^3*d^7*x^3 + 21*a^5*b^2*d^7*x^2 + 7*a^6*b*d
^7*x + a^7*d^7)*log(b*x + a))/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 +
35*a^3*b^12*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8
)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(182) = 364.
time = 0.00, size = 494, normalized size = 2.55

$$\frac{\frac{1}{420}\left(\left(2940d^7b^5a-2940d^6b^6c\right)x^6+\left(13230d^7b^4a^2-8820d^6b^5ac-4410d^5b^6c^2\right)x^5+\left(26950d^7b^3a^3-14700d^6b^4a^2c-7350d^5b^5ac^2-4900d^4b^6c^3\right)x^4+\left(30625d^7b^2a^4-14700d^6b^3a^3c-7350d^5b^4a^2c^2-4900d^4b^5ac^3-3675d^3b^6c^4\right)x^3+\left(20139d^7ba^5-8820d^6b^2a^4c-4410d^5b^3a^3c^2-2940d^4b^4a^2c^3-2205d^3b^5ac^4-1764d^2b^6c^5\right)x^2+\left(7203d^7a^6-2940d^6ba^5c-1470d^5b^2a^4c^2-980d^4b^3a^3c^3-735d^3b^4a^2c^4-588d^2b^5ac^5-490db^6c^6\right)x+\frac{1089d^7a^7-420d^6ba^6c-210d^5b^2a^5c^2-140d^4b^3a^4c^3-105d^3b^4a^3c^4-84d^2b^5a^2c^5-70db^6ac^6-60b^7c^7}{b}\right)}{b^7\left(xb+a\right)^7}+\frac{d^7\ln|xb+a|}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x)
```

```
[Out] d^7*log(abs(b*x + a))/b^8 - 1/420*(2940*(b^6*c*d^6 - a*b^5*d^7)*x^6 + 4410*
(b^6*c^2*d^5 + 2*a*b^5*c*d^6 - 3*a^2*b^4*d^7)*x^5 + 2450*(2*b^6*c^3*d^4 + 3
*a*b^5*c^2*d^5 + 6*a^2*b^4*c*d^6 - 11*a^3*b^3*d^7)*x^4 + 1225*(3*b^6*c^4*d^
3 + 4*a*b^5*c^3*d^4 + 6*a^2*b^4*c^2*d^5 + 12*a^3*b^3*c*d^6 - 25*a^4*b^2*d^7
)*x^3 + 147*(12*b^6*c^5*d^2 + 15*a*b^5*c^4*d^3 + 20*a^2*b^4*c^3*d^4 + 30*a^
3*b^3*c^2*d^5 + 60*a^4*b^2*c*d^6 - 137*a^5*b*d^7)*x^2 + 49*(10*b^6*c^6*d +
12*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 + 30*a^4*b^2*c^2
*d^5 + 60*a^5*b*c*d^6 - 147*a^6*d^7)*x + (60*b^7*c^7 + 70*a*b^6*c^6*d + 84*
a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c
^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^7*d^7)/b)/((b*x + a)^7*b^7)
```

Mupad [B]
time = 0.35, size = 461, normalized size = 2.38

$$\frac{d^7\,\ln\left(a+b\,x\right)}{b^8}-\frac{x\,\left(-\frac{343\,a^6\,b\,d^7}{20}+7\,a^5\,b^2\,c\,d^6+\frac{7\,a^4\,b^3\,c^2\,d^5}{2}+\frac{7\,a^3\,b^4\,c^3\,d^4}{3}+\frac{7\,a^2\,b^5\,c^4\,d^3}{4}+\frac{7\,a\,b^6\,c^5\,d^2}{5}+\frac{7\,b^7\,c^6\,d}{6}\right)-x^6\,\left(7\,a\,b^6\,d^7-7\,b^7\,c\,d^6\right)+x^3\,\left(-\frac{875\,a^4\,b^3\,d^7}{12}+35\,a^3\,b^4\,c\,d^6+\frac{35\,a^2\,b^5\,c^2\,d^5}{2}+\frac{35\,a\,b^6\,c^3\,d^4}{3}+\frac{35\,b^7\,c^4\,d^3}{4}\right)+x^5\,\left(-\frac{63\,a^2\,b^5\,d^7}{2}+21\,a\,b^6\,c\,d^6+\frac{21\,b^7\,c^2\,d^5}{2}\right)+x^2\,\left(-\frac{959\,a^5\,b^2\,d^7}{20}+21\,a^4\,b^3\,c\,d^6+\frac{21\,a^3\,b^4\,c^2\,d^5}{2}+7\,a^2\,b^5\,c^3\,d^4+\frac{21\,a\,b^6\,c^4\,d^3}{4}+\frac{21\,b^7\,c^5\,d^2}{5}\right)-\frac{363\,a^7\,d^7}{140}+\frac{b^7\,c^7}{7}+x^4\,\left(-\frac{385\,a^3\,b^4\,d^7}{6}+35\,a^2\,b^5\,c\,d^6+\frac{35\,a\,b^6\,c^2\,d^5}{2}+\frac{35\,b^7\,c^3\,d^4}{3}\right)+\frac{a^2\,b^5\,c^5\,d^2}{5}+\frac{a^3\,b^4\,c^4\,d^3}{4}+\frac{a^4\,b^3\,c^3\,d^4}{3}+\frac{a^5\,b^2\,c^2\,d^5}{2}+\frac{a\,b^6\,c^6\,d}{6}+a^6\,b\,c\,d^6}{b^8\,{\left(a+b\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^8,x)
```

```
[Out] (d^7*log(a + b*x))/b^8 - (x*((7*b^7*c^6*d)/6 - (343*a^6*b*d^7)/20 + (7*a*b^
6*c^5*d^2)/5 + 7*a^5*b^2*c*d^6 + (7*a^2*b^5*c^4*d^3)/4 + (7*a^3*b^4*c^3*d^4
```

```
)/3 + (7*a^4*b^3*c^2*d^5)/2) - x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*b
^7*c^4*d^3)/4 - (875*a^4*b^3*d^7)/12 + (35*a*b^6*c^3*d^4)/3 + 35*a^3*b^4*c*
d^6 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*b^7*c^2*d^5)/2 - (63*a^2*b^5*d^7)/
2 + 21*a*b^6*c*d^6) + x^2*((21*b^7*c^5*d^2)/5 - (959*a^5*b^2*d^7)/20 + (21*
a*b^6*c^4*d^3)/4 + 21*a^4*b^3*c*d^6 + 7*a^2*b^5*c^3*d^4 + (21*a^3*b^4*c^2*d
^5)/2) - (363*a^7*d^7)/140 + (b^7*c^7)/7 + x^4*((35*b^7*c^3*d^4)/3 - (385*a
^3*b^4*d^7)/6 + (35*a*b^6*c^2*d^5)/2 + 35*a^2*b^5*c*d^6) + (a^2*b^5*c^5*d^2
)/5 + (a^3*b^4*c^4*d^3)/4 + (a^4*b^3*c^3*d^4)/3 + (a^5*b^2*c^2*d^5)/2 + (a*
b^6*c^6*d)/6 + a^6*b*c*d^6)/(b^8*(a + b*x)^7)
```

3.1291 $\int \frac{(c+dx)^7}{(a+bx)^9}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

```
[Out] -1/8*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^8
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -1/8*(c + d*x)^8/((b*c - a*d)*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^9}\,dx = -\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(28) = 56.
time = 0.08, size = 353, normalized size = 12.61

$$-\frac{a^7d^7+a^6bd^6(c+8dx)+a^5b^2d^5\left(c^2+8cdx+28d^2x^2\right)+a^4b^3d^4\left(c^3+8c^2dx+28cd^2x^2+56d^3x^3\right)+a^3b^4d^3\left(c^4+8c^3dx+28c^2d^2x^2+56cd^3x^3+70d^4x^4\right)+a^2b^5d^2\left(c^5+8c^4dx+28c^3d^2x^2+56c^2d^3x^3+70cd^4x^4+56d^5x^5\right)+ab^6d\left(c^6+8c^5dx+28c^4d^2x^2+56c^3d^3x^3+70c^2d^4x^4+56cd^5x^5+28d^6x^6\right)+b^7\left(c^7+8c^6dx+28c^5d^2x^2+56c^4d^3x^3+70c^3d^4x^4+56c^2d^5x^5+28cd^6x^6+8d^7x^7\right)}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -1/8*(a^7*d^7 + a^6*b*d^6*(c + 8*d*x) + a^5*b^2*d^5*(c^2 + 8*c*d*x + 28*d^2
*x^2) + a^4*b^3*d^4*(c^3 + 8*c^2*d*x + 28*c*d^2*x^2 + 56*d^3*x^3) + a^3*b^4
*d^3*(c^4 + 8*c^3*d*x + 28*c^2*d^2*x^2 + 56*c*d^3*x^3 + 70*d^4*x^4) + a^2*b
^5*d^2*(c^5 + 8*c^4*d*x + 28*c^3*d^2*x^2 + 56*c^2*d^3*x^3 + 70*c*d^4*x^4 +
56*d^5*x^5) + a*b^6*d*(c^6 + 8*c^5*d*x + 28*c^4*d^2*x^2 + 56*c^3*d^3*x^3 +
70*c^2*d^4*x^4 + 56*c*d^5*x^5 + 28*d^6*x^6) + b^7*(c^7 + 8*c^6*d*x + 28*c^5
*d^2*x^2 + 56*c^4*d^3*x^3 + 70*c^3*d^4*x^4 + 56*c^2*d^5*x^5 + 28*c*d^6*x^6
+ 8*d^7*x^7))/(b^8*(a + b*x)^8)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^9,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(26) = 52$.
time = 0.14, size = 464, normalized size = 16.57

method	result
risch	$-\frac{d^7x^7}{b}-\frac{7d^6(ad+bc)x^6}{2b^2}-\frac{7d^5\left(a^2d^2+abcd+b^2c^2\right)x^5}{b^3}-\frac{35d^4\left(a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^4}{4b^4}-\frac{7d^3\left(a^4d^4+a^3bc\,d^3+a^2b^2c^2d^2+a\,b^3c^3d+b^4c^4\right)x}{b^5}$
norman	$-\frac{d^7x^7}{b}+\frac{7\left(-a\,d^7-bc\,d^6\right)x^6}{2b^2}+\frac{7\left(-a^2d^7-abc\,d^6-b^2c^2d^5\right)x^5}{b^3}+\frac{35\left(-a^3d^7-a^2bc\,d^6-a\,b^2c^2d^5-b^3c^3d^4\right)x^4}{4b^4}+\frac{7\left(-a^4d^7-a^3bc\,d^6-b^2a^2c^2d^5-a\,b^3c\right.}{b^5}$
default	$-\frac{d^7}{b^8(bx+a)}+\frac{35d^4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{4b^8(bx+a)^4}-\frac{-a^7d^7+7a^6bc\,d^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-}{8b^8(bx+a)^8}$
gosper	$-\frac{8d^7x^7b^7+28a\,b^6d^7x^6+28b^7c\,d^6x^6+56a^2b^5d^7x^5+56a\,b^6c\,d^6x^5+56b^7c^2d^5x^5+70a^3b^4d^7x^4+70a^2b^5c\,d^6x^4+70a\,b^6c^2d^5x^4+70b^7c^3d^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

```
[Out] -d^7/b^8/(b*x+a)+35/4/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/(b*x+a)^4-1/8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^
4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^
8-7/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
/(b*x+a)^5-1/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3
*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^7+7/2/b^8*d^6*(a*d-b
*c)/(b*x+a)^2+7/2/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*
b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^6-7/b^8*d^5*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(26) = 52$.
time = 0.29, size = 509, normalized size = 18.18

$$-\frac{8 b^{7} d^{7} x^{7} + b^{7} c^{7} + a b^{6} c^{6} d + a^{2} b^{5} c^{5} d^{2} + a^{3} b^{4} c^{4} d^{3} + a^{4} b^{3} c^{3} d^{4} + a^{5} b^{2} c^{2} d^{5} + a^{6} b c d^{6} + a^{7} d^{7} + 28 \left(b^{7} c d^{6} + a b^{6} d^{7}\right) x^{6} + 56 \left(b^{7} c^{2} d^{5} + a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right) x^{5} + 70 \left(b^{7} c^{3} d^{4} + a b^{6} c^{2} d^{5} + a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right) x^{4} + 56 \left(b^{7} c^{4} d^{3} + a b^{6} c^{3} d^{4} + a^{2} b^{5} c^{2} d^{5} + a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right) x^{3} + 28 \left(b^{7} c^{5} d^{2} + a b^{6} c^{4} d^{3} + a^{2} b^{5} c^{3} d^{4} + a^{3} b^{4} c^{2} d^{5} + a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right) x^{2} + 8 \left(b^{7} c^{6} d + a b^{6} c^{5} d^{2} + a^{2} b^{5} c^{4} d^{3} + a^{3} b^{4} c^{3} d^{4} + a^{4} b^{3} c^{2} d^{5} + a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right) x}{8 \left(b^{16} x^{8} + 8 a b^{15} x^{7} + 28 a^{2} b^{14} x^{6} + 56 a^{3} b^{13} x^{5} + 70 a^{4} b^{12} x^{4} + 56 a^{5} b^{11} x^{3} + 28 a^{6} b^{10} x^{2} + 8 a^{7} b^{9} x + a^{8} b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(26) = 52$.
time = 0.30, size = 509, normalized size = 18.18

$$-\frac{8 b^{7} d^{7} x^{7} + b^{7} c^{7} + a b^{6} c^{6} d + a^{2} b^{5} c^{5} d^{2} + a^{3} b^{4} c^{4} d^{3} + a^{4} b^{3} c^{3} d^{4} + a^{5} b^{2} c^{2} d^{5} + a^{6} b c d^{6} + a^{7} d^{7} + 28 \left(b^{7} c d^{6} + a b^{6} d^{7}\right) x^{6} + 56 \left(b^{7} c^{2} d^{5} + a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right) x^{5} + 70 \left(b^{7} c^{3} d^{4} + a b^{6} c^{2} d^{5} + a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right) x^{4} + 56 \left(b^{7} c^{4} d^{3} + a b^{6} c^{3} d^{4} + a^{2} b^{5} c^{2} d^{5} + a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right) x^{3} + 28 \left(b^{7} c^{5} d^{2} + a b^{6} c^{4} d^{3} + a^{2} b^{5} c^{3} d^{4} + a^{3} b^{4} c^{2} d^{5} + a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right) x^{2} + 8 \left(b^{7} c^{6} d + a b^{6} c^{5} d^{2} + a^{2} b^{5} c^{4} d^{3} + a^{3} b^{4} c^{3} d^{4} + a^{4} b^{3} c^{2} d^{5} + a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right) x}{8 \left(b^{16} x^{8} + 8 a b^{15} x^{7} + 28 a^{2} b^{14} x^{6} + 56 a^{3} b^{13} x^{5} + 70 a^{4} b^{12} x^{4} + 56 a^{5} b^{11} x^{3} + 28 a^{6} b^{10} x^{2} + 8 a^{7} b^{9} x + a^{8} b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**9,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(26) = 52.
time = 0.00, size = 527, normalized size = 18.82

$$\frac{-8x^7d^7b^7-28x^6d^7b^6a-28x^6d^6cb^7-56x^5d^7b^5a^2-56x^5d^6cb^6a-56x^5d^5c^2b^7-70x^4d^7b^4a^3-70x^4d^6cb^5a^2-70x^4d^5c^2b^6a-70x^4d^4c^3b^7-56x^3d^7b^3a^4-56x^3d^6cb^4a^3-56x^3d^5c^2b^5a^2-56x^3d^4c^3b^6a-56x^3d^3c^4b^7-28x^2d^7b^2a^5-28x^2d^6cb^3a^4-28x^2d^5c^2b^4a^3-28x^2d^4c^3b^5a^2-28x^2d^3c^4b^6a-28x^2d^2c^5b^7-8xd^7ba^6-8xd^6cb^2a^5-8xd^5c^2b^3a^4-8xd^4c^3b^4a^3-8xd^3c^4b^5a^2-8xd^2c^5b^6a-8xdc^6b^7-d^7a^7-d^6cba^6-d^5c^2b^2a^5-d^4c^3b^3a^4-d^3c^4b^4a^3-d^2c^5b^5a^2-dc^6b^6a-c^7b^7}{8b^8(xb+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x)
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + 28*b^7*c*d^6*x^6 + 28*a*b^6*d^7*x^6 + 56*b^7*c^2*d^5*
x^5 + 56*a*b^6*c*d^6*x^5 + 56*a^2*b^5*d^7*x^5 + 70*b^7*c^3*d^4*x^4 + 70*a*b
^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4 + 70*a^3*b^4*d^7*x^4 + 56*b^7*c^4*d^3
*x^3 + 56*a*b^6*c^3*d^4*x^3 + 56*a^2*b^5*c^2*d^5*x^3 + 56*a^3*b^4*c*d^6*x^3
 + 56*a^4*b^3*d^7*x^3 + 28*b^7*c^5*d^2*x^2 + 28*a*b^6*c^4*d^3*x^2 + 28*a^2*
b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 28*a^4*b^3*c*d^6*x^2 + 28*a^5*b^
2*d^7*x^2 + 8*b^7*c^6*d*x + 8*a*b^6*c^5*d^2*x + 8*a^2*b^5*c^4*d^3*x + 8*a^3
*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 8*a^5*b^2*c*d^6*x + 8*a^6*b*d^7*x +
b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4
 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^8*b^8)
```

Mupad [B]
time = 0.17, size = 571, normalized size = 20.39

$$-\frac{a^7d^7+a^6bcd^6+8a^6bd^7x+a^5b^2c^2d^5+8a^5b^2cd^6x+28a^5b^2d^7x^2+a^4b^3c^3d^4+8a^4b^3c^2d^5x+28a^4b^3cd^6x^2+56a^4b^3d^7x^3+a^3b^4c^4d^3+8a^3b^4c^3d^4x+28a^3b^4c^2d^5x^2+56a^3b^4cd^6x^3+70a^3b^4d^7x^4+a^2b^5c^5d^2+8a^2b^5c^4d^3x+28a^2b^5c^3d^4x^2+56a^2b^5c^2d^5x^3+70a^2b^5cd^6x^4+56a^2b^5d^7x^5+ab^6c^6d+8ab^6c^5d^2x+28ab^6c^4d^3x^2+56ab^6c^3d^4x^3+70ab^6c^2d^5x^4+56ab^6cd^6x^5+28ab^6d^7x^6+b^7c^7+8b^7c^6dx+28b^7c^5d^2x^2+56b^7c^4d^3x^3+70b^7c^3d^4x^4+56b^7c^2d^5x^5+28b^7cd^6x^6+8b^7d^7x^7}{8a^8b^8+64a^7b^9x+224a^6b^{10}x^2+448a^5b^{11}x^3+560a^4b^{12}x^4+448a^3b^{13}x^5+224a^2b^{14}x^6+64ab^{15}x^7+8b^{16}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^9,x)
```

```
[Out] -(a^7*d^7 + b^7*c^7 + 8*b^7*d^7*x^7 + 28*a*b^6*d^7*x^6 + 28*b^7*c*d^6*x^6 +
 a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + 28
*a^5*b^2*d^7*x^2 + 56*a^4*b^3*d^7*x^3 + 70*a^3*b^4*d^7*x^4 + 56*a^2*b^5*d^7
*x^5 + 28*b^7*c^5*d^2*x^2 + 56*b^7*c^4*d^3*x^3 + 70*b^7*c^3*d^4*x^4 + 56*b^
7*c^2*d^5*x^5 + a*b^6*c^6*d + a^6*b*c*d^6 + 8*a^6*b*d^7*x + 8*b^7*c^6*d*x +
 28*a^2*b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 56*a^2*b^5*c^2*d^5*x^3 +
 8*a*b^6*c^5*d^2*x + 8*a^5*b^2*c*d^6*x + 56*a*b^6*c*d^6*x^5 + 8*a^2*b^5*c^4
*d^3*x + 8*a^3*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 28*a*b^6*c^4*d^3*x^2 +
 28*a^4*b^3*c*d^6*x^2 + 56*a*b^6*c^3*d^4*x^3 + 56*a^3*b^4*c*d^6*x^3 + 70*a*
b^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4)/(8*a^8*b^8 + 8*b^16*x^8 + 64*a^7*b^
9*x + 64*a*b^15*x^7 + 224*a^6*b^10*x^2 + 448*a^5*b^11*x^3 + 560*a^4*b^12*x^
4 + 448*a^3*b^13*x^5 + 224*a^2*b^14*x^6)
```

3.1292 $\int \frac{(c+dx)^7}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=58

$$-\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9}+\frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}$$

[Out] -1/9*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^9+1/72*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a)^8

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2}-\frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^7/(a + b*x)^10,x]

[Out] -1/9*(c + d*x)^8/((b*c - a*d)*(a + b*x)^9) + (d*(c + d*x)^8)/(72*(b*c - a*d)^2*(a + b*x)^8)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx = -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{9(bc-ad)}$$
$$= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9} + \frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 367 vs. $2(58) = 116$.
time = 0.07, size = 367, normalized size = 6.33

$$-\frac{a^7d^7 + a^6bd^6(2c+9dx) + 3a^5b^2d^5\left(c^2+6cdx+12d^2x^2\right) + a^4b^3d^4\left(4c^3+27c^2dx+72cd^2x^2+84d^3x^3\right) + a^3b^4d^3\left(5c^4+36c^3dx+108c^2d^2x^2+168cd^3x^3+126d^4x^4\right) + 3a^2b^5d^2\left(2c^5+15c^4dx+48c^3d^2x^2+84c^2d^3x^3+84cd^4x^4+42d^5x^5\right) + ab^6d\left(7c^6+54c^5dx+180c^4d^2x^2+336c^3d^3x^3+378c^2d^4x^4+252cd^5x^5+84d^6x^6\right) + b^7\left(8c^7+63c^6dx+216c^5d^2x^2+420c^4d^3x^3+504c^3d^4x^4+378c^2d^5x^5+168cd^6x^6+36d^7x^7\right)}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7*d^7 + a^6*b*d^6*(2*c + 9*d*x) + 3*a^5*b^2*d^5*(c^2 + 6*c*d*x + 1
2*d^2*x^2) + a^4*b^3*d^4*(4*c^3 + 27*c^2*d*x + 72*c*d^2*x^2 + 84*d^3*x^3) +
 a^3*b^4*d^3*(5*c^4 + 36*c^3*d*x + 108*c^2*d^2*x^2 + 168*c*d^3*x^3 + 126*d^
4*x^4) + 3*a^2*b^5*d^2*(2*c^5 + 15*c^4*d*x + 48*c^3*d^2*x^2 + 84*c^2*d^3*x^
3 + 84*c*d^4*x^4 + 42*d^5*x^5) + a*b^6*d*(7*c^6 + 54*c^5*d*x + 180*c^4*d^2*
x^2 + 336*c^3*d^3*x^3 + 378*c^2*d^4*x^4 + 252*c*d^5*x^5 + 84*d^6*x^6) + b^7
*(8*c^7 + 63*c^6*d*x + 216*c^5*d^2*x^2 + 420*c^4*d^3*x^3 + 504*c^3*d^4*x^4
+ 378*c^2*d^5*x^5 + 168*c*d^6*x^6 + 36*d^7*x^7))/(b^8*(a + b*x)^9)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^10,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(54) = 108$.
time = 0.18, size = 464, normalized size = 8.00

method	result
risch	$-\frac{d^7x^7}{2b} - \frac{7d^6(ad+2bc)x^6}{6b^2} - \frac{7d^5\left(a^2d^2+2abcd+3b^2c^2\right)x^5}{4b^3} - \frac{7d^4\left(a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3\right)x^4}{4b^4} - \frac{7d^3\left(a^4d^4+2a^3bc\,d^3+3a^2b^2c^2d^2+4a\,b^3c^3 \right.}{6b^5}$

default	$-\frac{21d^5(a^2d^2-2abcd+b^2c^2)}{4b^8(bx+a)^4}-\frac{-a^7d^7+7a^6bc\,d^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7a\,b^6c^6d+b^7c^7}{9b^8(bx+a)^9}-\frac{7d(a^6d^6-$
norman	$\frac{-\frac{d^7x^7}{2b}+\frac{7(-ab\,d^7-2b^2c\,d^6)x^6}{6b^3}+\frac{7(-a^2b\,d^7-2a\,b^2c\,d^6-3b^3c^2d^5)x^5}{4b^4}+\frac{7(-a^3b\,d^7-2b^2a^2c\,d^6-3a\,b^3c^2d^5-4b^4c^3d^4)x^4}{4b^5}+\frac{7(-a^4b\,d^7-2a^3b^2c\,d^6-3}{}$
gosper	$-\frac{36d^7x^7b^7+84a\,b^6d^7x^6+168b^7c\,d^6x^6+126a^2b^5d^7x^5+252a\,b^6c\,d^6x^5+378b^7c^2d^5x^5+126a^3b^4d^7x^4+252a^2b^5c\,d^6x^4+378a\,b^6c^2d^5x^4+5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -21/4/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4-1/9*(-a^7*d^7+7*a^6*b*c
*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^
5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^9-7/8/b^8*d*(a^6*d^6-6*a^5*b*c*d^5
+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6
*c^6)/(b*x+a)^8+7/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*
x+a)^5+3/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d
^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^7-1/2*d^7/b^8/(b*x+a)^2-35/6/b^8*d^3*(a^4
*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^6+7/3/b
^8*d^6*(a*d-b*c)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(54) = 108$.
time = 0.30, size = 548, normalized size = 9.45

$$-\frac{36 b^7 d^7 x^7 + 8 b^7 c^7 + 7 a b^6 c^6 d + 6 a^2 b^5 c^5 d^2 + 5 a^3 b^4 c^4 d^3 + 4 a^4 b^3 c^3 d^4 + 3 a^5 b^2 c^2 d^5 + 2 a^6 b c d^6 + a^7 d^7 + 84 (2 b^7 c d^6 + a b^6 d^7) x^6 + 126 (3 b^7 c^2 d^5 + 2 a b^6 c d^6 + a^2 b^5 d^7) x^5 + 126 (4 b^7 c^3 d^4 + 3 a b^6 c^2 d^5 + 2 a^2 b^5 c d^6 + a^3 b^4 d^7) x^4 + 84 (5 b^7 c^4 d^3 + 4 a b^6 c^3 d^4 + 3 a^2 b^5 c^2 d^5 + 2 a^3 b^4 c d^6 + a^4 b^3 d^7) x^3 + 36 (6 b^7 c^5 d^2 + 5 a b^6 c^4 d^3 + 4 a^2 b^5 c^3 d^4 + 3 a^3 b^4 c^2 d^5 + 2 a^4 b^3 c d^6 + a^5 b^2 d^7) x^2 + 9 (7 b^7 c^6 d + 6 a b^6 c^5 d^2 + 5 a^2 b^5 c^4 d^3 + 4 a^3 b^4 c^3 d^4 + 3 a^4 b^3 c^2 d^5 + 2 a^5 b^2 c d^6 + a^6 b d^7) x}{72 (b^{17} x^9 + 9 a b^{16} x^8 + 36 a^2 b^{15} x^7 + 84 a^3 b^{14} x^6 + 126 a^4 b^{13} x^5 + 126 a^5 b^{12} x^4 + 84 a^6 b^{11} x^3 + 36 a^7 b^{10} x^2 + 9 a^8 b^9 x + a^9 b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(54) = 108$.
time = 0.29, size = 548, normalized size = 9.45

$$-\frac{36 b^7 d^7 x^7 + 8 b^7 c^7 + 7 a b^6 c^6 d + 6 a^2 b^5 c^5 d^2 + 5 a^3 b^4 c^4 d^3 + 4 a^4 b^3 c^3 d^4 + 3 a^5 b^2 c^2 d^5 + 2 a^6 b c d^6 + a^7 d^7 + 84 (2 b^7 c d^6 + a b^6 d^7) x^6 + 126 (3 b^7 c^2 d^5 + 2 a b^6 c d^6 + a^2 b^5 d^7) x^5 + 126 (4 b^7 c^3 d^4 + 3 a b^6 c^2 d^5 + 2 a^2 b^5 c d^6 + a^3 b^4 d^7) x^4 + 84 (5 b^7 c^4 d^3 + 4 a b^6 c^3 d^4 + 3 a^2 b^5 c^2 d^5 + 2 a^3 b^4 c d^6 + a^4 b^3 d^7) x^3 + 36 (6 b^7 c^5 d^2 + 5 a b^6 c^4 d^3 + 4 a^2 b^5 c^3 d^4 + 3 a^3 b^4 c^2 d^5 + 2 a^4 b^3 c d^6 + a^5 b^2 d^7) x^2 + 9 (7 b^7 c^6 d + 6 a b^6 c^5 d^2 + 5 a^2 b^5 c^4 d^3 + 4 a^3 b^4 c^3 d^4 + 3 a^4 b^3 c^2 d^5 + 2 a^5 b^2 c d^6 + a^6 b d^7) x}{72 (b^{17} x^9 + 9 a b^{16} x^8 + 36 a^2 b^{15} x^7 + 84 a^3 b^{14} x^6 + 126 a^4 b^{13} x^5 + 126 a^5 b^{12} x^4 + 84 a^6 b^{11} x^3 + 36 a^7 b^{10} x^2 + 9 a^8 b^9 x + a^9 b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**10,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(54) = 108.
time = 0.00, size = 534, normalized size = 9.21

$$\frac{-36x^7d^7b^7 - 84x^6d^7b^6a - 168x^6d^6cb^7 - 126x^5d^7b^5a^2 - 252x^5d^6cb^6a - 378x^5d^5c^2b^7 - 126x^4d^7b^4a^3 - 252x^4d^6cb^5a^2 - 378x^4d^5c^2b^6a - 504x^4d^4c^3b^7 - 84x^3d^7b^3a^4 - 168x^3d^6cb^4a^3 - 252x^3d^5c^2b^5a^2 - 336x^3d^4c^3b^6a - 420x^3d^3c^4b^7 - 36x^2d^7b^2a^5 - 72x^2d^6cb^3a^4 - 108x^2d^5c^2b^4a^3 - 144x^2d^4c^3b^5a^2 - 180x^2d^3c^4b^6a - 216x^2d^2c^5b^7 - 9xd^7ba^6 - 18xd^6cb^2a^5 - 27xd^5c^2b^3a^4 - 36xd^4c^3b^4a^3 - 45xd^3c^4b^5a^2 - 54xd^2c^5b^6a - 63xdc^6b^7 - d^7a^7 - 2d^6cba^6 - 3d^5c^2b^2a^5 - 4d^4c^3b^3a^4 - 5d^3c^4b^4a^3 - 6d^2c^5b^5a^2 - 7dc^6b^6a - 8c^7b^7}{72b^8\left(xb+a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x)
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 168*b^7*c*d^6*x^6 + 84*a*b^6*d^7*x^6 + 378*b^7*c^2*
d^5*x^5 + 252*a*b^6*c*d^6*x^5 + 126*a^2*b^5*d^7*x^5 + 504*b^7*c^3*d^4*x^4 +
 378*a*b^6*c^2*d^5*x^4 + 252*a^2*b^5*c*d^6*x^4 + 126*a^3*b^4*d^7*x^4 + 420*
b^7*c^4*d^3*x^3 + 336*a*b^6*c^3*d^4*x^3 + 252*a^2*b^5*c^2*d^5*x^3 + 168*a^3
*b^4*c*d^6*x^3 + 84*a^4*b^3*d^7*x^3 + 216*b^7*c^5*d^2*x^2 + 180*a*b^6*c^4*d
^3*x^2 + 144*a^2*b^5*c^3*d^4*x^2 + 108*a^3*b^4*c^2*d^5*x^2 + 72*a^4*b^3*c*d
^6*x^2 + 36*a^5*b^2*d^7*x^2 + 63*b^7*c^6*d*x + 54*a*b^6*c^5*d^2*x + 45*a^2*
b^5*c^4*d^3*x + 36*a^3*b^4*c^3*d^4*x + 27*a^4*b^3*c^2*d^5*x + 18*a^5*b^2*c*
d^6*x + 9*a^6*b*d^7*x + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7)/((b*x + a)^9*b^8)
```

Mupad [B]
time = 0.15, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^8\,\left(9\,a\,d-8\,b\,c+b\,d\,x\right)}{72\,\left(a\,d-b\,c\right)^2\,\left(a+b\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^10,x)
[Out] ((c + d*x)^8*(9*a*d - 8*b*c + b*d*x))/(72*(a*d - b*c)^2*(a + b*x)^9)
```

3.1293 $\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx$

Optimal. Leaf size=89

$$-\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}+\frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9}-\frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}$$

```
[Out] -1/10*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^10+1/45*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a)
^9-1/360*d^2*(d*x+c)^8/(-a*d+b*c)^3/(b*x+a)^8
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -1/10*(c + d*x)^8/((b*c - a*d)*(a + b*x)^10) + (d*(c + d*x)^8)/(45*(b*c - a
*d)^2*(a + b*x)^9) - (d^2*(c + d*x)^8)/(360*(b*c - a*d)^3*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx = -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{5(bc-ad)}$$
$$= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{45(bc-ad)^2}$$
$$= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} - \frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 371 vs. 2(89) = 178.
time = 0.08, size = 371, normalized size = 4.17

$$-\frac{a^7d^7 + a^6bd^6(3c+10dx) + 3a^5b^2d^5\left(2c^2+10cdx+15d^2x^2\right) + 5a^4b^3d^4\left(2c^3+12c^2dx+27cd^2x^2+24d^3x^3\right) + 5a^3b^4d^3\left(3c^4+20c^3dx+54c^2d^2x^2+72cd^3x^3+42d^4x^4\right) + 3a^2b^5d^2\left(7c^5+50c^4dx+150c^3d^2x^2+240c^2d^3x^3+210cd^4x^4+84d^5x^5\right) + ab^6d\left(28c^6+210c^5dx+675c^4d^2x^2+1200c^3d^3x^3+1260c^2d^4x^4+756cd^5x^5+210d^6x^6\right) + b^7\left(36c^7+280c^6dx+945c^5d^2x^2+1800c^4d^3x^3+2100c^3d^4x^4+1512c^2d^5x^5+630cd^6x^6+120d^7x^7\right)}{360b^8(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -1/360*(a^7*d^7 + a^6*b*d^6*(3*c + 10*d*x) + 3*a^5*b^2*d^5*(2*c^2 + 10*c*d*
x + 15*d^2*x^2) + 5*a^4*b^3*d^4*(2*c^3 + 12*c^2*d*x + 27*c*d^2*x^2 + 24*d^3
*x^3) + 5*a^3*b^4*d^3*(3*c^4 + 20*c^3*d*x + 54*c^2*d^2*x^2 + 72*c*d^3*x^3 +
 42*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 50*c^4*d*x + 150*c^3*d^2*x^2 + 240*c^
2*d^3*x^3 + 210*c*d^4*x^4 + 84*d^5*x^5) + a*b^6*d*(28*c^6 + 210*c^5*d*x + 6
75*c^4*d^2*x^2 + 1200*c^3*d^3*x^3 + 1260*c^2*d^4*x^4 + 756*c*d^5*x^5 + 210*
d^6*x^6) + b^7*(36*c^7 + 280*c^6*d*x + 945*c^5*d^2*x^2 + 1800*c^4*d^3*x^3 +
 2100*c^3*d^4*x^4 + 1512*c^2*d^5*x^5 + 630*c*d^6*x^6 + 120*d^7*x^7))/(b^8*(
a + b*x)^10)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^11,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(83) = 166.
time = 0.14, size = 464, normalized size = 5.21

method	result
risch	$-\frac{d^7x^7}{3b}-\frac{7d^6(ad+3bc)x^6}{12b^2}-\frac{7d^5\left(a^2d^2+3abcd+6b^2c^2\right)x^5}{10b^3}-\frac{7d^4\left(a^3d^3+3a^2bcd^2+6ab^2c^2d+10b^3c^3\right)x^4}{12b^4}-\frac{d^3\left(a^4d^4+3a^3bcd^3+6a^2b^2c^2d^2+10ab^3c^3 \cdots\right.}{3b^5}$
default	$\frac{7d^6(ad-bc)}{4b^8(bx+a)^4}-\frac{7d\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)}{9b^8(bx+a)^9}+\frac{21d^2\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3- \cdots\right.}{8b^8(bx+ \cdots}$
norman	$-\frac{d^7x^7}{3b}+\frac{7\left(-ab^2d^7-3b^3cd^6\right)x^6}{12b^4}+\frac{7\left(-b^2a^2d^7-3ab^3cd^6-6b^4c^2d^5\right)x^5}{10b^5}+\frac{7\left(-a^3b^2d^7-3a^2b^3cd^6-6ab^4c^2d^5-10b^5c^3d^4\right)x^4}{12b^6}+\frac{\left(-a^4b^2d^7-3a^3b^3 \cdots\right.}{}$
gosper	$-\frac{120d^7x^7b^7+210ab^6d^7x^6+630b^7cd^6x^6+252a^2b^5d^7x^5+756ab^6cd^6x^5+1512b^7c^2d^5x^5+210a^3b^4d^7x^4+630a^2b^5cd^6x^4+1260ab^6c^2 \cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^11,x,method=_RETURNVERBOSE)
```

```
[Out] 7/4/b^8*d^6*(a*d-b*c)/(b*x+a)^4-7/9/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2
*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+
a)^9+21/8/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*
d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^8-21/5/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^
2)/(b*x+a)^5-5/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3
*d+b^4*c^4)/(b*x+a)^7-1/10*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^
4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/
b^8/(b*x+a)^10+35/6/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(
b*x+a)^6-1/3*d^7/b^8/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(83) = 166$.
time = 0.28, size = 559, normalized size = 6.28

$$-\frac{120\,b^7d^7x^7+36\,b^7c^7+28\,ab^6c^6d+21\,a^2b^5c^5d^2+15\,a^3b^4c^4d^3+10\,a^4b^3c^3d^4+6\,a^5b^2c^2d^5+3\,a^6bcd^6+a^7d^7+210\,(3\,b^7cd^6+ab^6d^7)x^6+252\,(6\,b^7c^2d^5+3\,ab^6cd^6+a^2b^5d^7)x^5+210\,(10\,b^7c^3d^4+6\,ab^6c^2d^5+3\,a^2b^5cd^6+a^3b^4d^7)x^4+120\,(15\,b^7c^4d^3+10\,ab^6c^3d^4+6\,a^2b^5c^2d^5+3\,a^3b^4cd^6+a^4b^3d^7)x^3+45\,(21\,b^7c^5d^2+15\,ab^6c^4d^3+10\,a^2b^5c^3d^4+6\,a^3b^4c^2d^5+3\,a^4b^3cd^6+a^5b^2d^7)x^2+10\,(28\,b^7c^6d+21\,ab^6c^5d^2+15\,a^2b^5c^4d^3+10\,a^3b^4c^3d^4+6\,a^4b^3c^2d^5+3\,a^5b^2cd^6+a^6bd^7)x}{360\,(b^{18}x^{10}+10\,ab^{17}x^9+45\,a^2b^{16}x^8+120\,a^3b^{15}x^7+210\,a^4b^{14}x^6+252\,a^5b^{13}x^5+210\,a^6b^{12}x^4+120\,a^7b^{11}x^3+45\,a^8b^{10}x^2+10\,a^9b^9x+a^{10}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(83) = 166$.
time = 0.29, size = 559, normalized size = 6.28

$$-\frac{120\,b^{7}d^{7}x^{7}+36\,b^{7}c^{7}+28\,ab^{6}c^{6}d+21\,a^{2}b^{5}c^{5}d^{2}+15\,a^{3}b^{4}c^{4}d^{3}+10\,a^{4}b^{3}c^{3}d^{4}+6\,a^{5}b^{2}c^{2}d^{5}+3\,a^{6}bcd^{6}+a^{7}d^{7}+210\,(3\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+252\,(6\,b^{7}c^{2}d^{5}+3\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+210\,(10\,b^{7}c^{3}d^{4}+6\,ab^{6}c^{2}d^{5}+3\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+120\,(15\,b^{7}c^{4}d^{3}+10\,ab^{6}c^{3}d^{4}+6\,a^{2}b^{5}c^{2}d^{5}+3\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+45\,(21\,b^{7}c^{5}d^{2}+15\,ab^{6}c^{4}d^{3}+10\,a^{2}b^{5}c^{3}d^{4}+6\,a^{3}b^{4}c^{2}d^{5}+3\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+10\,(28\,b^{7}c^{6}d+21\,ab^{6}c^{5}d^{2}+15\,a^{2}b^{5}c^{4}d^{3}+10\,a^{3}b^{4}c^{3}d^{4}+6\,a^{4}b^{3}c^{2}d^{5}+3\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{360\,(b^{18}x^{10}+10\,ab^{17}x^{9}+45\,a^{2}b^{16}x^{8}+120\,a^{3}b^{15}x^{7}+210\,a^{4}b^{14}x^{6}+252\,a^{5}b^{13}x^{5}+210\,a^{6}b^{12}x^{4}+120\,a^{7}b^{11}x^{3}+45\,a^{8}b^{10}x^{2}+10\,a^{9}b^{9}x+a^{10}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**11,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(83) = 166$.
time = 0.00, size = 534, normalized size = 6.00

$$\frac{-120x^{7}d^{7}b^{7}-210x^{6}d^{7}b^{6}a-630x^{6}d^{6}cb^{7}-252x^{5}d^{7}b^{5}a^{2}-756x^{5}d^{6}cb^{6}a-1512x^{5}d^{5}c^{2}b^{7}-210x^{4}d^{7}b^{4}a^{3}-630x^{4}d^{6}cb^{5}a^{2}-1260x^{4}d^{5}c^{2}b^{6}a-2100x^{4}d^{4}c^{3}b^{7}-120x^{3}d^{7}b^{3}a^{4}-360x^{3}d^{6}cb^{4}a^{3}-720x^{3}d^{5}c^{2}b^{5}a^{2}-1200x^{3}d^{4}c^{3}b^{6}a-1800x^{3}d^{3}c^{4}b^{7}-45x^{2}d^{7}b^{2}a^{5}-135x^{2}d^{6}cb^{3}a^{4}-270x^{2}d^{5}c^{2}b^{4}a^{3}-450x^{2}d^{4}c^{3}b^{5}a^{2}-675x^{2}d^{3}c^{4}b^{6}a-945x^{2}d^{2}c^{5}b^{7}-10xd^{7}ba^{6}-30xd^{6}cb^{2}a^{5}-60xd^{5}c^{2}b^{3}a^{4}-100xd^{4}c^{3}b^{4}a^{3}-150xd^{3}c^{4}b^{5}a^{2}-210xd^{2}c^{5}b^{6}a-280xdc^{6}b^{7}-d^{7}a^{7}-3d^{6}cba^{6}-6d^{5}c^{2}b^{2}a^{5}-10d^{4}c^{3}b^{3}a^{4}-15d^{3}c^{4}b^{4}a^{3}-21d^{2}c^{5}b^{5}a^{2}-28dc^{6}b^{6}a-36c^{7}b^{7}}{360b^{8}\left(xb+a\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x)
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 630*b^7*c*d^6*x^6 + 210*a*b^6*d^7*x^6 + 1512*b^7*
c^2*d^5*x^5 + 756*a*b^6*c*d^6*x^5 + 252*a^2*b^5*d^7*x^5 + 2100*b^7*c^3*d^4*
x^4 + 1260*a*b^6*c^2*d^5*x^4 + 630*a^2*b^5*c*d^6*x^4 + 210*a^3*b^4*d^7*x^4
+ 1800*b^7*c^4*d^3*x^3 + 1200*a*b^6*c^3*d^4*x^3 + 720*a^2*b^5*c^2*d^5*x^3 +
```

```
 360*a^3*b^4*c*d^6*x^3 + 120*a^4*b^3*d^7*x^3 + 945*b^7*c^5*d^2*x^2 + 675*a*
b^6*c^4*d^3*x^2 + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*c^2*d^5*x^2 + 135*a
^4*b^3*c*d^6*x^2 + 45*a^5*b^2*d^7*x^2 + 280*b^7*c^6*d*x + 210*a*b^6*c^5*d^2
*x + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x + 60*a^4*b^3*c^2*d^5*x +
 30*a^5*b^2*c*d^6*x + 10*a^6*b*d^7*x + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2
*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5
+ 3*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^10*b^8)
```

Mupad [B]
time = 0.45, size = 600, normalized size = 6.74

$$-\frac{a^7 d^7 + 3 a^6 b c d^6 + 10 a^6 b d^7 x + 6 a^5 b^2 c^2 d^5 + 30 a^5 b^2 c d^6 x + 45 a^5 b^2 d^7 x^2 + 10 a^4 b^3 c^3 d^4 + 60 a^4 b^3 c^2 d^5 x + 135 a^4 b^3 c d^6 x^2 + 120 a^4 b^3 d^7 x^3 + 15 a^3 b^4 c^4 d^3 + 100 a^3 b^4 c^3 d^4 x + 270 a^3 b^4 c^2 d^5 x^2 + 360 a^3 b^4 c d^6 x^3 + 210 a^3 b^4 d^7 x^4 + 21 a^2 b^5 c^5 d^2 + 150 a^2 b^5 c^4 d^3 x + 450 a^2 b^5 c^3 d^4 x^2 + 720 a^2 b^5 c^2 d^5 x^3 + 630 a^2 b^5 c d^6 x^4 + 252 a^2 b^5 d^7 x^5 + 28 a b^6 c^6 d + 210 a b^6 c^5 d^2 x + 675 a b^6 c^4 d^3 x^2 + 1200 a b^6 c^3 d^4 x^3 + 1260 a b^6 c^2 d^5 x^4 + 756 a b^6 c d^6 x^5 + 210 a b^6 d^7 x^6 + 36 b^7 c^7 + 280 b^7 c^6 d x + 945 b^7 c^5 d^2 x^2 + 1800 b^7 c^4 d^3 x^3 + 2100 b^7 c^3 d^4 x^4 + 1512 b^7 c^2 d^5 x^5 + 630 b^7 c d^6 x^6 + 120 b^7 d^7 x^7}{360 a^{10} b^8 + 3600 a^9 b^9 x + 16200 a^8 b^{10} x^2 + 43200 a^7 b^{11} x^3 + 75600 a^6 b^{12} x^4 + 90720 a^5 b^{13} x^5 + 75600 a^4 b^{14} x^6 + 43200 a^3 b^{15} x^7 + 16200 a^2 b^{16} x^8 + 3600 a b^{17} x^9 + 360 b^{18} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^11,x)
```

```
[Out] -(a^7*d^7 + 36*b^7*c^7 + 120*b^7*d^7*x^7 + 210*a*b^6*d^7*x^6 + 630*b^7*c*d^
6*x^6 + 21*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^
5*b^2*c^2*d^5 + 45*a^5*b^2*d^7*x^2 + 120*a^4*b^3*d^7*x^3 + 210*a^3*b^4*d^7*
x^4 + 252*a^2*b^5*d^7*x^5 + 945*b^7*c^5*d^2*x^2 + 1800*b^7*c^4*d^3*x^3 + 21
00*b^7*c^3*d^4*x^4 + 1512*b^7*c^2*d^5*x^5 + 28*a*b^6*c^6*d + 3*a^6*b*c*d^6
+ 10*a^6*b*d^7*x + 280*b^7*c^6*d*x + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*
c^2*d^5*x^2 + 720*a^2*b^5*c^2*d^5*x^3 + 210*a*b^6*c^5*d^2*x + 30*a^5*b^2*c*
d^6*x + 756*a*b^6*c*d^6*x^5 + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x
 + 60*a^4*b^3*c^2*d^5*x + 675*a*b^6*c^4*d^3*x^2 + 135*a^4*b^3*c*d^6*x^2 + 1
200*a*b^6*c^3*d^4*x^3 + 360*a^3*b^4*c*d^6*x^3 + 1260*a*b^6*c^2*d^5*x^4 + 63
0*a^2*b^5*c*d^6*x^4)/(360*a^10*b^8 + 360*b^18*x^10 + 3600*a^9*b^9*x + 3600*
a*b^17*x^9 + 16200*a^8*b^10*x^2 + 43200*a^7*b^11*x^3 + 75600*a^6*b^12*x^4 +
 90720*a^5*b^13*x^5 + 75600*a^4*b^14*x^6 + 43200*a^3*b^15*x^7 + 16200*a^2*b
^16*x^8)
```

3.1294 $\int \frac{(c+dx)^7}{(a+bx)^{12}} dx$

Optimal. Leaf size=120

$$-\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}-\frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9}+\frac{d^3(c+dx)^8}{1320(bc-ad)^4(a+bx)^8}$$

```
[Out] -1/11*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^11+3/110*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a
)^10-1/165*d^2*(d*x+c)^8/(-a*d+b*c)^3/(b*x+a)^9+1/1320*d^3*(d*x+c)^8/(-a*d+
b*c)^4/(b*x+a)^8
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -1/11*(c + d*x)^8/((b*c - a*d)*(a + b*x)^11) + (3*d*(c + d*x)^8)/(110*(b*c
- a*d)^2*(a + b*x)^10) - (d^2*(c + d*x)^8)/(165*(b*c - a*d)^3*(a + b*x)^9)
+ (d^3*(c + d*x)^8)/(1320*(b*c - a*d)^4*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx &= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} - \frac{(3d)\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)} \\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} + \frac{(3d^2)\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{55(bc-ad)^2} \\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} - \frac{d^3\int \frac{(c-}{(a-}}{165(bc} \\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} + \frac{c}{1320(b}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(120) = 240.
time = 0.08, size = 369, normalized size = 3.08

$$
-\frac{a^7d^7+a^6bd^6(4c+11dx)+a^5b^2d^5(10c^2+44cdx+55d^2x^2)+5a^4b^3d^4(4c^3+22c^2dx+44cd^2x^2+33d^3x^3)+5a^3b^4d^3(7c^4+44c^3dx+110c^2d^2x^2+132cd^3x^3+66d^4x^4)+a^2b^5d^2(56c^5+385c^4dx+1100c^3d^2x^2+1650c^2d^3x^3+1320cd^4x^4+462d^5x^5)+ab^6d(84c^6+616c^5dx+1925c^4d^2x^2+3300c^3d^3x^3+3300c^2d^4x^4+1848cd^5x^5+462d^6x^6)+b^7(120c^7+924c^6dx+3080c^5d^2x^2+5775c^4d^3x^3+6600c^3d^4x^4+4620c^2d^5x^5+1848cd^6x^6+330d^7x^7)}{1320b^8(a+bx)^{11}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -1/1320*(a^7*d^7 + a^6*b*d^6*(4*c + 11*d*x) + a^5*b^2*d^5*(10*c^2 + 44*c*d*
x + 55*d^2*x^2) + 5*a^4*b^3*d^4*(4*c^3 + 22*c^2*d*x + 44*c*d^2*x^2 + 33*d^3
*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 44*c^3*d*x + 110*c^2*d^2*x^2 + 132*c*d^3*x^3
 + 66*d^4*x^4) + a^2*b^5*d^2*(56*c^5 + 385*c^4*d*x + 1100*c^3*d^2*x^2 + 165
0*c^2*d^3*x^3 + 1320*c*d^4*x^4 + 462*d^5*x^5) + a*b^6*d*(84*c^6 + 616*c^5*d
*x + 1925*c^4*d^2*x^2 + 3300*c^3*d^3*x^3 + 3300*c^2*d^4*x^4 + 1848*c*d^5*x^
5 + 462*d^6*x^6) + b^7*(120*c^7 + 924*c^6*d*x + 3080*c^5*d^2*x^2 + 5775*c^4
*d^3*x^3 + 6600*c^3*d^4*x^4 + 4620*c^2*d^5*x^5 + 1848*c*d^6*x^6 + 330*d^7*x
^7))/(b^8*(a + b*x)^11)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^12,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(112) = 224.
time = 0.14, size = 464, normalized size = 3.87

method	result
risch	$\frac{-\frac{d^7x^7}{4b}-\frac{7d^6(ad+4bc)x^6}{20b^2}-\frac{7d^5\left(a^2d^2+4abcd+10b^2c^2\right)x^5}{20b^3}-\frac{d^4\left(a^3d^3+4a^2bc\,d^2+10a\,b^2c^2d+20b^3c^3\right)x^4}{4b^4}-\frac{d^3\left(a^4d^4+4a^3bc\,d^3+10a^2b^2c^2d^2+20a\,b^3c^3 \right.}{8b^5}}{}$
default	$-\frac{d^7}{4b^8(bx+a)^4}+\frac{7d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{3b^8(bx+a)^9}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4 \right.}{8b^8(bx+a)^8}$
norman	$\frac{-\frac{d^7x^7}{4b}+\frac{7\left(-a\,b^3d^7-4b^4c\,d^6\right)x^6}{20b^5}+\frac{7\left(-a^2b^3d^7-4a\,b^4c\,d^6-10b^5c^2d^5\right)x^5}{20b^6}+\frac{\left(-a^3b^3d^7-4a^2b^4c\,d^6-10a\,b^5c^2d^5-20b^6c^3d^4\right)x^4}{4b^7}+\frac{\left(-a^4b^3d^7-4a^3b^4c \right.}{}}{}$
gosper	$-\frac{330d^7x^7b^7+462a\,b^6d^7x^6+1848b^7c\,d^6x^6+462a^2b^5d^7x^5+1848a\,b^6c\,d^6x^5+4620b^7c^2d^5x^5+330a^3b^4d^7x^4+1320a^2b^5c\,d^6x^4+3300a\,b^6c^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^7/(b*x+a)^12,x,method=_RETURNVERBOSE)

[Out] -1/4*d^7/b^8/(b*x+a)^4+7/3/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^9-35/8/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^8+7/5/b^8*d^6*(a*d-b*c)/(b*x+a)^5+5/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^7-7/10/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^10-7/2/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^6-1/11*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^11

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(112) = 224$.
time = 0.30, size = 570, normalized size = 4.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="maxima")

[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 + 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^12*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(112) = 224$.

time = 0.30, size = 570, normalized size = 4.75

$$-\frac{330\,b^{7}d^{7}x^{7}+120\,b^{7}c^{7}+84\,ab^{6}c^{6}d+56\,a^{2}b^{5}c^{5}d^{2}+35\,a^{3}b^{4}c^{4}d^{3}+20\,a^{4}b^{3}c^{3}d^{4}+10\,a^{5}b^{2}c^{2}d^{5}+4\,a^{6}bcd^{6}+a^{7}d^{7}+462\,(4\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+462\,(10\,b^{7}c^{2}d^{5}+4\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+330\,(20\,b^{7}c^{3}d^{4}+10\,ab^{6}c^{2}d^{5}+4\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+165\,(35\,b^{7}c^{4}d^{3}+20\,ab^{6}c^{3}d^{4}+10\,a^{2}b^{5}c^{2}d^{5}+4\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+55\,(56\,b^{7}c^{5}d^{2}+35\,ab^{6}c^{4}d^{3}+20\,a^{2}b^{5}c^{3}d^{4}+10\,a^{3}b^{4}c^{2}d^{5}+4\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+11\,(84\,b^{7}c^{6}d+56\,ab^{6}c^{5}d^{2}+35\,a^{2}b^{5}c^{4}d^{3}+20\,a^{3}b^{4}c^{3}d^{4}+10\,a^{4}b^{3}c^{2}d^{5}+4\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{1320\,(b^{19}x^{11}+11\,ab^{18}x^{10}+55\,a^{2}b^{17}x^{9}+165\,a^{3}b^{16}x^{8}+330\,a^{4}b^{15}x^{7}+462\,a^{5}b^{14}x^{6}+462\,a^{6}b^{13}x^{5}+330\,a^{7}b^{12}x^{4}+165\,a^{8}b^{11}x^{3}+55\,a^{9}b^{10}x^{2}+11\,a^{10}b^{9}x+a^{11}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

Sympy [**F(-1)**]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**12,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(112) = 224$.

time = 0.00, size = 534, normalized size = 4.45

$$\frac{-330x^{7}d^{7}b^{7}-462x^{6}d^{7}b^{6}a-1848x^{6}d^{6}cb^{7}-462x^{5}d^{7}b^{5}a^{2}-1848x^{5}d^{6}cb^{6}a-4620x^{5}d^{5}c^{2}b^{7}-330x^{4}d^{7}b^{4}a^{3}-1320x^{4}d^{6}cb^{5}a^{2}-3300x^{4}d^{5}c^{2}b^{6}a-6600x^{4}d^{4}c^{3}b^{7}-165x^{3}d^{7}b^{3}a^{4}-660x^{3}d^{6}cb^{4}a^{3}-1650x^{3}d^{5}c^{2}b^{5}a^{2}-3300x^{3}d^{4}c^{3}b^{6}a-5775x^{3}d^{3}c^{4}b^{7}-55x^{2}d^{7}b^{2}a^{5}-220x^{2}d^{6}cb^{3}a^{4}-550x^{2}d^{5}c^{2}b^{4}a^{3}-1100x^{2}d^{4}c^{3}b^{5}a^{2}-1925x^{2}d^{3}c^{4}b^{6}a-3080x^{2}d^{2}c^{5}b^{7}-11xd^{7}ba^{6}-44xd^{6}cb^{2}a^{5}-110xd^{5}c^{2}b^{3}a^{4}-220xd^{4}c^{3}b^{4}a^{3}-385xd^{3}c^{4}b^{5}a^{2}-616xd^{2}c^{5}b^{6}a-924xdc^{6}b^{7}-d^{7}a^{7}-4d^{6}cba^{6}-10d^{5}c^{2}b^{2}a^{5}-20d^{4}c^{3}b^{3}a^{4}-35d^{3}c^{4}b^{4}a^{3}-56d^{2}c^{5}b^{5}a^{2}-84dc^{6}b^{6}a-120c^{7}b^{7}}{1320b^{8}\,(xb+a)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x)
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 1848*b^7*c*d^6*x^6 + 462*a*b^6*d^7*x^6 + 4620*b^
7*c^2*d^5*x^5 + 1848*a*b^6*c*d^6*x^5 + 462*a^2*b^5*d^7*x^5 + 6600*b^7*c^3*d
^4*x^4 + 3300*a*b^6*c^2*d^5*x^4 + 1320*a^2*b^5*c*d^6*x^4 + 330*a^3*b^4*d^7*
x^4 + 5775*b^7*c^4*d^3*x^3 + 3300*a*b^6*c^3*d^4*x^3 + 1650*a^2*b^5*c^2*d^5*
```

x^3 + 660*a^3*b^4*c*d^6*x^3 + 165*a^4*b^3*d^7*x^3 + 3080*b^7*c^5*d^2*x^2 + 1925*a*b^6*c^4*d^3*x^2 + 1100*a^2*b^5*c^3*d^4*x^2 + 550*a^3*b^4*c^2*d^5*x^2 + 220*a^4*b^3*c*d^6*x^2 + 55*a^5*b^2*d^7*x^2 + 924*b^7*c^6*d*x + 616*a*b^6*c^5*d^2*x + 385*a^2*b^5*c^4*d^3*x + 220*a^3*b^4*c^3*d^4*x + 110*a^4*b^3*c^2*d^5*x + 44*a^5*b^2*c*d^6*x + 11*a^6*b*d^7*x + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^11*b^8)

Mupad [B]
time = 0.52, size = 548, normalized size = 4.57

$$-\frac{\frac{a^7\,d^7+4\,a^6\,b\,c\,d^6+10\,a^5\,b^2\,c^2\,d^5+20\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+56\,a^2\,b^5\,c^5\,d^2+84\,a\,b^6\,c^6\,d+120\,b^7\,c^7}{1320\,b^8}+\frac{d^7\,x^7}{4\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+4\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3+20\,a^2\,b^3\,c^3\,d^2+35\,a\,b^4\,c^4\,d+56\,b^5\,c^5\right)}{24\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+4\,a^2\,b\,c\,d^2+10\,a\,b^2\,c^2\,d+20\,b^3\,c^3\right)}{4\,b^4}+\frac{7\,d^6\,x^6\,\left(a\,d+4\,b\,c\right)}{20\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+4\,a^3\,b\,c\,d^3+10\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d+35\,b^4\,c^4\right)}{8\,b^5}+\frac{d\,x\,\left(a^6\,d^6+4\,a^5\,b\,c\,d^5+10\,a^4\,b^2\,c^2\,d^4+20\,a^3\,b^3\,c^3\,d^3+35\,a^2\,b^4\,c^4\,d^2+56\,a\,b^5\,c^5\,d+84\,b^6\,c^6\right)}{120\,b^7}+\frac{7\,d^5\,x^5\,\left(a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2\right)}{20\,b^3}}{a^{11}+11\,a^{10}\,b\,x+55\,a^9\,b^2\,x^2+165\,a^8\,b^3\,x^3+330\,a^7\,b^4\,x^4+462\,a^6\,b^5\,x^5+462\,a^5\,b^6\,x^6+330\,a^4\,b^7\,x^7+165\,a^3\,b^8\,x^8+55\,a^2\,b^9\,x^9+11\,a\,b^{10}\,x^{10}+b^{11}\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^7/(a + b*x)^12,x)

[Out] -((a^7*d^7 + 120*b^7*c^7 + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 84*a*b^6*c^6*d + 4*a^6*b*c*d^6)/(1320*b^8) + (d^7*x^7)/(4*b) + (d^2*x^2*(a^5*d^5 + 56*b^5*c^5 + 20*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a*b^4*c^4*d + 4*a^4*b*c*d^4))/(24*b^6) + (d^4*x^4*(a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2))/(4*b^4) + (7*d^6*x^6*(a*d + 4*b*c))/(20*b^2) + (d^3*x^3*(a^4*d^4 + 35*b^4*c^4 + 10*a^2*b^2*c^2*d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3))/(8*b^5) + (d*x*(a^6*d^6 + 84*b^6*c^6 + 35*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 10*a^4*b^2*c^2*d^4 + 56*a*b^5*c^5*d + 4*a^5*b*c*d^5))/(120*b^7) + (7*d^5*x^5*(a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d))/(20*b^3))/(a^11 + b^11*x^11 + 11*a*b^10*x^10 + 55*a^9*b^2*x^2 + 165*a^8*b^3*x^3 + 330*a^7*b^4*x^4 + 462*a^6*b^5*x^5 + 462*a^5*b^6*x^6 + 330*a^4*b^7*x^7 + 165*a^3*b^8*x^8 + 55*a^2*b^9*x^9 + 11*a^10*b*x)

3.1295 $\int \frac{(c+dx)^7}{(a+bx)^{13}}\,dx$

Optimal. Leaf size=151

$$-\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}}-\frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}}+\frac{d^3(c+dx)^8}{495(bc-ad)^4(a+bx)^9}-\frac{d^4(c+dx)^8}{3960(bc-ad)^5(a+bx)^8}$$

```
[Out] -1/12*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^12+1/33*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a)
^11-1/110*d^2*(d*x+c)^8/(-a*d+b*c)^3/(b*x+a)^10+1/495*d^3*(d*x+c)^8/(-a*d+b
*c)^4/(b*x+a)^9-1/3960*d^4*(d*x+c)^8/(-a*d+b*c)^5/(b*x+a)^8
```

Rubi [A]

time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{47, 37\}$

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^8}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -1/12*(c + d*x)^8/((b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^8)/(33*(b*c - a
*d)^2*(a + b*x)^11) - (d^2*(c + d*x)^8)/(110*(b*c - a*d)^3*(a + b*x)^10) +
(d^3*(c + d*x)^8)/(495*(b*c - a*d)^4*(a + b*x)^9) - (d^4*(c + d*x)^8)/(3960
*(b*c - a*d)^5*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{13}}\,dx = -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx}{3(bc-ad)}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)^2}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} - \frac{d^3\int \frac{(c+d}{(a+b}}{55(bc-}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(}{495(bc-}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(}{495(bc-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 371 vs. 2(151) = 302.

time = 0.08, size = 371, normalized size = 2.46

$$-\frac{a^7d^7+a^6bd^6(5c+12dx)+3a^5b^2d^5(5c^2+20cdx+22d^2x^2)+5a^4b^3d^4(7c^3+36c^2dx+66cd^2x^2+44d^3x^3)+5a^3b^4d^3(14c^4+84c^3dx+198c^2d^2x^2+220cd^3x^3+99d^4x^4)+3a^2b^5d^2(42c^5+280c^4dx+770c^3d^2x^2+1100c^2d^3x^3+825cd^4x^4+264d^5x^5)+ab^6d(210c^6+1512c^5dx+4620c^4d^2x^2+7700c^3d^3x^3+7425c^2d^4x^4+3960cd^5x^5+924d^6x^6)+b^7(330c^7+2520c^6dx+8316c^5d^2x^2+15400c^4d^3x^3+17325c^3d^4x^4+11880c^2d^5x^5+4620cd^6x^6+792d^7x^7)}{3960b^8(a+bx)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -1/3960*(a^7*d^7 + a^6*b*d^6*(5*c + 12*d*x) + 3*a^5*b^2*d^5*(5*c^2 + 20*c*d
*x + 22*d^2*x^2) + 5*a^4*b^3*d^4*(7*c^3 + 36*c^2*d*x + 66*c*d^2*x^2 + 44*d^
3*x^3) + 5*a^3*b^4*d^3*(14*c^4 + 84*c^3*d*x + 198*c^2*d^2*x^2 + 220*c*d^3*x
^3 + 99*d^4*x^4) + 3*a^2*b^5*d^2*(42*c^5 + 280*c^4*d*x + 770*c^3*d^2*x^2 +
1100*c^2*d^3*x^3 + 825*c*d^4*x^4 + 264*d^5*x^5) + a*b^6*d*(210*c^6 + 1512*c
^5*d*x + 4620*c^4*d^2*x^2 + 7700*c^3*d^3*x^3 + 7425*c^2*d^4*x^4 + 3960*c*d^
5*x^5 + 924*d^6*x^6) + b^7*(330*c^7 + 2520*c^6*d*x + 8316*c^5*d^2*x^2 + 154
00*c^4*d^3*x^3 + 17325*c^3*d^4*x^4 + 11880*c^2*d^5*x^5 + 4620*c*d^6*x^6 + 7
92*d^7*x^7))/(b^8*(a + b*x)^12)
```

Mathics [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^13,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(141) = 282$.

time = 0.14, size = 464, normalized size = 3.07

method	result
risch	$-\frac{d^7x^7}{5b}-\frac{7d^6(ad+5bc)x^6}{30b^2}-\frac{d^5\left(a^2d^2+5abcd+15b^2c^2\right)x^5}{5b^3}-\frac{d^4\left(a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3\right)x^4}{8b^4}-\frac{d^3\left(a^4d^4+5a^3bc\,d^3+15a^2b^2c^2d^2+35a\,b^3c \ldots\right.}{18b^5}$ …
default	$-\frac{-a^7d^7+7a^6bc\,d^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7a\,b^6c^6d+b^7c^7}{12b^8(bx+a)^{12}}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b \ldots\right.}{9b^8(bx+a)^9}$ …
norman	$\frac{-\frac{d^7x^7}{5b}+\frac{7\left(-a\,b^4d^7-5b^5c\,d^6\right)x^6}{30b^6}+\frac{\left(-a^2b^4d^7-5a\,b^5c\,d^6-15b^6c^2d^5\right)x^5}{5b^7}+\frac{\left(-a^3b^4d^7-5a^2b^5c\,d^6-15a\,b^6c^2d^5-35b^7c^3d^4\right)x^4}{8b^8}+\frac{\left(-a^4b^4d^7-5a^3b^5 \ldots\right.}{\ldots}}{\ldots}$ …
gosper	$-\frac{792d^7x^7b^7+924a\,b^6d^7x^6+4620b^7c\,d^6x^6+792a^2b^5d^7x^5+3960a\,b^6c\,d^6x^5+11880b^7c^2d^5x^5+495a^3b^4d^7x^4+2475a^2b^5c\,d^6x^4+7425a\,b \ldots}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*
b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^12-35/9/b
^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x
+a)^9+35/8/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^8-
1/5*d^7/b^8/(b*x+a)^5-3/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^7+21/10
/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d-b^5*c^5)/(b*x+a)^10+7/6/b^8*d^6*(a*d-b*c)/(b*x+a)^6-7/11/b^8*d*(a^
6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^
2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(141) = 282$.

time = 0.30, size = 581, normalized size = 3.85

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924\,(5\,b^7cd^6+ab^6d^7)x^6+792\,(15\,b^7c^2d^5+5\,ab^6cd^6+a^2b^5d^7)x^5+495\,(35\,b^7c^3d^4+15\,ab^6c^2d^5+5\,a^2b^5cd^6+a^3b^4d^7)x^4+220\,(70\,b^7c^4d^3+35\,ab^6c^3d^4+15\,a^2b^5c^2d^5+5\,a^3b^4cd^6+a^4b^3d^7)x^3+66\,(126\,b^7c^5d^2+70\,ab^6c^4d^3+35\,a^2b^5c^3d^4+15\,a^3b^4c^2d^5+5\,a^4b^3cd^6+a^5b^2d^7)x^2+12\,(210\,b^7c^6d+126\,ab^6c^5d^2+70\,a^2b^5c^4d^3+35\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5+5\,a^5b^2cd^6+a^6bd^7)x}{3960\,(b^{20}x^{12}+12\,ab^{19}x^{11}+66\,a^2b^{18}x^{10}+220\,a^3b^{17}x^9+495\,a^4b^{16}x^8+792\,a^5b^{15}x^7+924\,a^6b^{14}x^6+792\,a^7b^{13}x^5+495\,a^8b^{12}x^4+220\,a^9b^{11}x^3+66\,a^{10}b^{10}x^2+12\,a^{11}b^9x+a^{12}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
```

```
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(141) = 282$.
time = 0.30, size = 581, normalized size = 3.85

$$-\frac{792\,b^{7}d^{7}x^{7}+330\,b^{7}c^{7}+210\,ab^{6}c^{6}d+126\,a^{2}b^{5}c^{5}d^{2}+70\,a^{3}b^{4}c^{4}d^{3}+35\,a^{4}b^{3}c^{3}d^{4}+15\,a^{5}b^{2}c^{2}d^{5}+5\,a^{6}bcd^{6}+a^{7}d^{7}+924\,(5\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+792\,(15\,b^{7}c^{2}d^{5}+5\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+495\,(35\,b^{7}c^{3}d^{4}+15\,ab^{6}c^{2}d^{5}+5\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+220\,(70\,b^{7}c^{4}d^{3}+35\,ab^{6}c^{3}d^{4}+15\,a^{2}b^{5}c^{2}d^{5}+5\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+66\,(126\,b^{7}c^{5}d^{2}+70\,ab^{6}c^{4}d^{3}+35\,a^{2}b^{5}c^{3}d^{4}+15\,a^{3}b^{4}c^{2}d^{5}+5\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+12\,(210\,b^{7}c^{6}d+126\,ab^{6}c^{5}d^{2}+70\,a^{2}b^{5}c^{4}d^{3}+35\,a^{3}b^{4}c^{3}d^{4}+15\,a^{4}b^{3}c^{2}d^{5}+5\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{3960\,(b^{20}x^{12}+12\,ab^{19}x^{11}+66\,a^{2}b^{18}x^{10}+220\,a^{3}b^{17}x^{9}+495\,a^{4}b^{16}x^{8}+792\,a^{5}b^{15}x^{7}+924\,a^{6}b^{14}x^{6}+792\,a^{7}b^{13}x^{5}+495\,a^{8}b^{12}x^{4}+220\,a^{9}b^{11}x^{3}+66\,a^{10}b^{10}x^{2}+12\,a^{11}b^{9}x+a^{12}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**13,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(141) = 282$.
time = 0.00, size = 534, normalized size = 3.54

$$\frac{-792x^{7}d^{7}b^{7}-924x^{6}d^{7}b^{6}a-4620x^{6}d^{6}cb^{7}-792x^{5}d^{7}b^{5}a^{2}-3960x^{5}d^{6}cb^{6}a-11880x^{5}d^{5}c^{2}b^{7}-495x^{4}d^{7}b^{4}a^{3}-2475x^{4}d^{6}cb^{5}a^{2}-7425x^{4}d^{5}c^{2}b^{6}a-17325x^{4}d^{4}c^{3}b^{7}-220x^{3}d^{7}b^{3}a^{4}-1100x^{3}d^{6}cb^{4}a^{3}-3300x^{3}d^{5}c^{2}b^{5}a^{2}-7700x^{3}d^{4}c^{3}b^{6}a-15400x^{3}d^{3}c^{4}b^{7}-66x^{2}d^{7}b^{2}a^{5}-330x^{2}d^{6}cb^{3}a^{4}-990x^{2}d^{5}c^{2}b^{4}a^{3}-2310x^{2}d^{4}c^{3}b^{5}a^{2}-4620x^{2}d^{3}c^{4}b^{6}a-8316x^{2}d^{2}c^{5}b^{7}-12xd^{7}ba^{6}-60xd^{6}cb^{2}a^{5}-180xd^{5}c^{2}b^{3}a^{4}-420xd^{4}c^{3}b^{4}a^{3}-840xd^{3}c^{4}b^{5}a^{2}-1512xd^{2}c^{5}b^{6}a-2520xdc^{6}b^{7}-d^{7}a^{7}-5d^{6}cba^{6}-15d^{5}c^{2}b^{2}a^{5}-35d^{4}c^{3}b^{3}a^{4}-70d^{3}c^{4}b^{4}a^{3}-126d^{2}c^{5}b^{5}a^{2}-210dc^{6}b^{6}a-330c^{7}b^{7}}{3960b^{8}\left(xb+a\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x)
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 4620*b^7*c*d^6*x^6 + 924*a*b^6*d^7*x^6 + 11880*b
^7*c^2*d^5*x^5 + 3960*a*b^6*c*d^6*x^5 + 792*a^2*b^5*d^7*x^5 + 17325*b^7*c^3
*d^4*x^4 + 7425*a*b^6*c^2*d^5*x^4 + 2475*a^2*b^5*c*d^6*x^4 + 495*a^3*b^4*d^
7*x^4 + 15400*b^7*c^4*d^3*x^3 + 7700*a*b^6*c^3*d^4*x^3 + 3300*a^2*b^5*c^2*d
^5*x^3 + 1100*a^3*b^4*c*d^6*x^3 + 220*a^4*b^3*d^7*x^3 + 8316*b^7*c^5*d^2*x^
2 + 4620*a*b^6*c^4*d^3*x^2 + 2310*a^2*b^5*c^3*d^4*x^2 + 990*a^3*b^4*c^2*d^5
*x^2 + 330*a^4*b^3*c*d^6*x^2 + 66*a^5*b^2*d^7*x^2 + 2520*b^7*c^6*d*x + 1512
*a*b^6*c^5*d^2*x + 840*a^2*b^5*c^4*d^3*x + 420*a^3*b^4*c^3*d^4*x + 180*a^4*
b^3*c^2*d^5*x + 60*a^5*b^2*c*d^6*x + 12*a^6*b*d^7*x + 330*b^7*c^7 + 210*a*b
^6*c^6*d + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 +
15*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^12*b^8)
```

Mupad [B]
time = 0.23, size = 559, normalized size = 3.70

$$-\frac{\frac{a^7\,d^7+5\,a^6\,b\,c\,d^6+15\,a^5\,b^2\,c^2\,d^5+35\,a^4\,b^3\,c^3\,d^4+70\,a^3\,b^4\,c^4\,d^3+126\,a^2\,b^5\,c^5\,d^2+210\,a\,b^6\,c^6\,d+330\,b^7\,c^7}{3960\,b^8}+\frac{d^7\,x^7}{5\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+5\,a^4\,b\,c\,d^4+15\,a^3\,b^2\,c^2\,d^3+35\,a^2\,b^3\,c^3\,d^2+70\,a\,b^4\,c^4\,d+126\,b^5\,c^5\right)}{60\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3\right)}{8\,b^4}+\frac{7\,d^6\,x^6\,\left(a\,d+5\,b\,c\right)}{30\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+5\,a^3\,b\,c\,d^3+15\,a^2\,b^2\,c^2\,d^2+35\,a\,b^3\,c^3\,d+70\,b^4\,c^4\right)}{18\,b^5}+\frac{d\,x\,\left(a^6\,d^6+5\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4+35\,a^3\,b^3\,c^3\,d^3+70\,a^2\,b^4\,c^4\,d^2+126\,a\,b^5\,c^5\,d+210\,b^6\,c^6\right)}{330\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{5\,b^3}}{a^{12}+12\,a^{11}\,b\,x+66\,a^{10}\,b^2\,x^2+220\,a^9\,b^3\,x^3+495\,a^8\,b^4\,x^4+792\,a^7\,b^5\,x^5+924\,a^6\,b^6\,x^6+792\,a^5\,b^7\,x^7+495\,a^4\,b^8\,x^8+220\,a^3\,b^9\,x^9+66\,a^2\,b^{10}\,x^{10}+12\,a\,b^{11}\,x^{11}+b^{12}\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^13,x)
```

```
[Out] -((a^7*d^7 + 330*b^7*c^7 + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 210*a*b^6*c^6*d + 5*a^6*b*c*d^6)/(3960
*b^8) + (d^7*x^7)/(5*b) + (d^2*x^2*(a^5*d^5 + 126*b^5*c^5 + 35*a^2*b^3*c^3*
d^2 + 15*a^3*b^2*c^2*d^3 + 70*a*b^4*c^4*d + 5*a^4*b*c*d^4))/(60*b^6) + (d^4
*x^4*(a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2))/(8*b^4) + (7*
d^6*x^6*(a*d + 5*b*c))/(30*b^2) + (d^3*x^3*(a^4*d^4 + 70*b^4*c^4 + 15*a^2*b
^2*c^2*d^2 + 35*a*b^3*c^3*d + 5*a^3*b*c*d^3))/(18*b^5) + (d*x*(a^6*d^6 + 21
0*b^6*c^6 + 70*a^2*b^4*c^4*d^2 + 35*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 +
126*a*b^5*c^5*d + 5*a^5*b*c*d^5))/(330*b^7) + (d^5*x^5*(a^2*d^2 + 15*b^2*c^
2 + 5*a*b*c*d))/(5*b^3))/(a^12 + b^12*x^12 + 12*a*b^11*x^11 + 66*a^10*b^2*x
^2 + 220*a^9*b^3*x^3 + 495*a^8*b^4*x^4 + 792*a^7*b^5*x^5 + 924*a^6*b^6*x^6
+ 792*a^5*b^7*x^7 + 495*a^4*b^8*x^8 + 220*a^3*b^9*x^9 + 66*a^2*b^10*x^10 +
12*a^11*b*x)
```

3.1296 $\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx$

Optimal. Leaf size=198

$$-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{d^6(bc-ad)}{b^8(a+bx)^7}$$

```
[Out] -1/13*(-a*d+b*c)^7/b^8/(b*x+a)^13-7/12*d*(-a*d+b*c)^6/b^8/(b*x+a)^12-21/11*
d^2*(-a*d+b*c)^5/b^8/(b*x+a)^11-7/2*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^10-35/9*d^
4*(-a*d+b*c)^3/b^8/(b*x+a)^9-21/8*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^8-d^6*(-a*d+
b*c)/b^8/(b*x+a)^7-1/6*d^7/b^8/(b*x+a)^6
```

Rubi [A]
time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{d^7}{6b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -1/13*(b*c - a*d)^7/(b^8*(a + b*x)^13) - (7*d*(b*c - a*d)^6)/(12*b^8*(a + b
*x)^12) - (21*d^2*(b*c - a*d)^5)/(11*b^8*(a + b*x)^11) - (7*d^3*(b*c - a*d)
^4)/(2*b^8*(a + b*x)^10) - (35*d^4*(b*c - a*d)^3)/(9*b^8*(a + b*x)^9) - (21
*d^5*(b*c - a*d)^2)/(8*b^8*(a + b*x)^8) - (d^6*(b*c - a*d))/(b^8*(a + b*x)^
7) - d^7/(6*b^8*(a + b*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{14}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{13}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{12}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{11}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{10}}+\right.\\ &= -\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 369, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(6c+13dx)+3a^5b^2d^5(7c^2+26cdx+26d^2x^2)+a^4b^3d^4(56c^3+273c^2dx+468cd^2x^2+286d^3x^3)+a^3b^4d^3(126c^4+728c^3dx+1638c^2d^2x^2+1716cd^3x^3+715d^4x^4)+3a^2b^5d^2(84c^5+546c^4dx+1456c^3d^2x^2+2002c^2d^3x^3+1430cd^4x^4+429d^5x^5)+ab^6d(462c^6+3276c^5dx+9828c^4d^2x^2+16016c^3d^3x^3+15015c^2d^4x^4+7722cd^5x^5+1716d^6x^6)+b^7(792c^7+6006c^6dx+19656c^5d^2x^2+36036c^4d^3x^3+40040c^3d^4x^4+27027c^2d^5x^5+10296cd^6x^6+1716d^7x^7)}{10296b^8(a+bx)^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -1/10296*(a^7*d^7 + a^6*b*d^6*(6*c + 13*d*x) + 3*a^5*b^2*d^5*(7*c^2 + 26*c*
d*x + 26*d^2*x^2) + a^4*b^3*d^4*(56*c^3 + 273*c^2*d*x + 468*c*d^2*x^2 + 286
*d^3*x^3) + a^3*b^4*d^3*(126*c^4 + 728*c^3*d*x + 1638*c^2*d^2*x^2 + 1716*c*
d^3*x^3 + 715*d^4*x^4) + 3*a^2*b^5*d^2*(84*c^5 + 546*c^4*d*x + 1456*c^3*d^2
*x^2 + 2002*c^2*d^3*x^3 + 1430*c*d^4*x^4 + 429*d^5*x^5) + a*b^6*d*(462*c^6
+ 3276*c^5*d*x + 9828*c^4*d^2*x^2 + 16016*c^3*d^3*x^3 + 15015*c^2*d^4*x^4 +
 7722*c*d^5*x^5 + 1716*d^6*x^6) + b^7*(792*c^7 + 6006*c^6*d*x + 19656*c^5*d
^2*x^2 + 36036*c^4*d^3*x^3 + 40040*c^3*d^4*x^4 + 27027*c^2*d^5*x^5 + 10296*
c*d^6*x^6 + 1716*d^7*x^7))/(b^8*(a + b*x)^13)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^14,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(184) = 368$.
time = 0.17, size = 463, normalized size = 2.34

method	result
risch	$-\frac{d^7x^7}{6b}-\frac{d^6(ad+6bc)x^6}{6b^2}-\frac{d^5\left(a^2d^2+6abcd+21b^2c^2\right)x^5}{8b^3}-\frac{5d^4\left(a^3d^3+6a^2bcd^2+21ab^2c^2d+56b^3c^3\right)x^4}{72b^4}-\frac{d^3\left(a^4d^4+6a^3bcd^3+21a^2b^2c^2d^2+56ab^3c\cdots\right.}{36b^5}\cdots$
default	$-\frac{-a^7d^7+7a^6bcd^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7ab^6c^6d+b^7c^7}{13b^8(bx+a)^{13}}-\frac{7d\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3\cdots\right.}{12b^8(bx\cdots}$
norman	$-\frac{d^7x^7}{6b}+\frac{\left(-ab^5d^7-6b^6cd^6\right)x^6}{6b^7}+\frac{\left(-a^2b^5d^7-6ab^6cd^6-21b^7c^2d^5\right)x^5}{8b^8}+\frac{5\left(-a^3b^5d^7-6a^2b^6cd^6-21ab^7c^2d^5-56b^8c^3d^4\right)x^4}{72b^9}+\frac{\left(-a^4b^5d^7-6a^3b^6\cdots\right.}{\cdots}$
gosper	$-\frac{1716d^7x^7b^7+1716ab^6d^7x^6+10296b^7cd^6x^6+1287a^2b^5d^7x^5+7722ab^6cd^6x^5+27027b^7c^2d^5x^5+715a^3b^4d^7x^4+4290a^2b^5cd^6x^4+15\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*
b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^13-7/12/b
^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^
4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^12+35/9/b^8*d^4*(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^9-21/8/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/(b*x+a)^8+1/b^8*d^6*(a*d-b*c)/(b*x+a)^7-7/2/b^8*d^3*(a^4*d^4-4*a^3*b*c*
d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^10-1/6*d^7/b^8/(b*x+a)
^6+21/11/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d
^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(184) = 368$.
time = 0.31, size = 592, normalized size = 2.99

$$-\frac{1716\,b^7d^7x^7+792\,b^7c^7+462\,ab^6c^6d+252\,a^2b^5c^5d^2+126\,a^3b^4c^4d^3+56\,a^4b^3c^3d^4+21\,a^5b^2c^2d^5+6\,a^6bcd^6+a^7d^7+1716\,(6\,b^7cd^6+ab^6d^7)x^6+1287\,(21\,b^7c^2d^5+6\,ab^6cd^6+a^2b^5d^7)x^5+715\,(56\,b^7c^3d^4+21\,ab^6c^2d^5+6\,a^2b^5cd^6+a^3b^4d^7)x^4+286\,(126\,b^7c^4d^3+56\,ab^6c^3d^4+21\,a^2b^5c^2d^5+6\,a^3b^4cd^6+a^4b^3d^7)x^3+78\,(252\,b^7c^5d^2+126\,ab^6c^4d^3+56\,a^2b^5c^3d^4+21\,a^3b^4c^2d^5+6\,a^4b^3cd^6+a^5b^2d^7)x^2+13\,(462\,b^7c^6d+252\,ab^6c^5d^2+126\,a^2b^5c^4d^3+56\,a^3b^4c^3d^4+21\,a^4b^3c^2d^5+6\,a^5b^2cd^6+a^6bd^7)x}{10296\,(b^{21}x^{13}+13\,ab^{20}x^{12}+78\,a^2b^{19}x^{11}+286\,a^3b^{18}x^{10}+715\,a^4b^{17}x^9+1287\,a^5b^{16}x^8+1716\,a^6b^{15}x^7+1716\,a^7b^{14}x^6+1287\,a^8b^{13}x^5+715\,a^9b^{12}x^4+286\,a^{10}b^{11}x^3+78\,a^{11}b^{10}x^2+13\,a^{12}b^9x+a^{13}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(184) = 368$.
time = 0.30, size = 592, normalized size = 2.99

$$-\frac{1716\,b^7d^7x^7+792\,b^7c^7+462\,ab^6c^6d+252\,a^2b^5c^5d^2+126\,a^3b^4c^4d^3+56\,a^4b^3c^3d^4+21\,a^5b^2c^2d^5+6\,a^6bcd^6+a^7d^7+1716\,(6\,b^7cd^6+ab^6d^7)x^6+1287\,(21\,b^7c^2d^5+6\,ab^6cd^6+a^2b^5d^7)x^5+715\,(56\,b^7c^3d^4+21\,ab^6c^2d^5+6\,a^2b^5cd^6+a^3b^4d^7)x^4+286\,(126\,b^7c^4d^3+56\,ab^6c^3d^4+21\,a^2b^5c^2d^5+6\,a^3b^4cd^6+a^4b^3d^7)x^3+78\,(252\,b^7c^5d^2+126\,ab^6c^4d^3+56\,a^2b^5c^3d^4+21\,a^3b^4c^2d^5+6\,a^4b^3cd^6+a^5b^2d^7)x^2+13\,(462\,b^7c^6d+252\,ab^6c^5d^2+126\,a^2b^5c^4d^3+56\,a^3b^4c^3d^4+21\,a^4b^3c^2d^5+6\,a^5b^2cd^6+a^6bd^7)x}{10296\,(b^{21}x^{13}+13\,ab^{20}x^{12}+78\,a^2b^{19}x^{11}+286\,a^3b^{18}x^{10}+715\,a^4b^{17}x^9+1287\,a^5b^{16}x^8+1716\,a^6b^{15}x^7+1716\,a^7b^{14}x^6+1287\,a^8b^{13}x^5+715\,a^9b^{12}x^4+286\,a^{10}b^{11}x^3+78\,a^{11}b^{10}x^2+13\,a^{12}b^9x+a^{13}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
```

```
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**14,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(184) = 368.
time = 0.00, size = 534, normalized size = 2.70

$$\frac{-1716x^7d^7b^7 - 1716x^6d^7b^6a - 10296x^6d^6cb^7 - 1287x^5d^7b^5a^2 - 7722x^5d^6cb^6a - 27027x^5d^5c^2b^7 - 715x^4d^7b^4a^3 - 4290x^4d^6cb^5a^2 - 15015x^4d^5c^2b^6a - 40040x^4d^4c^3b^7 - 286x^3d^7b^3a^4 - 1716x^3d^6cb^4a^3 - 6006x^3d^5c^2b^5a^2 - 16016x^3d^4c^3b^6a - 36036x^3d^3c^4b^7 - 78x^2d^7b^2a^5 - 468x^2d^6cb^3a^4 - 1638x^2d^5c^2b^4a^3 - 4368x^2d^4c^3b^5a^2 - 9828x^2d^3c^4b^6a - 19656x^2d^2c^5b^7 - 13xd^7ba^6 - 78xd^6cb^2a^5 - 273xd^5c^2b^3a^4 - 728xd^4c^3b^4a^3 - 1638xd^3c^4b^5a^2 - 3276xd^2c^5b^6a - 6006xdc^6b^7 - d^7a^7 - 6d^6cba^6 - 21d^5c^2b^2a^5 - 56d^4c^3b^3a^4 - 126d^3c^4b^4a^3 - 252d^2c^5b^5a^2 - 462dc^6b^6a - 792c^7b^7}{10296b^8\left(xb+a\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x)
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 10296*b^7*c*d^6*x^6 + 1716*a*b^6*d^7*x^6 + 270
27*b^7*c^2*d^5*x^5 + 7722*a*b^6*c*d^6*x^5 + 1287*a^2*b^5*d^7*x^5 + 40040*b^
7*c^3*d^4*x^4 + 15015*a*b^6*c^2*d^5*x^4 + 4290*a^2*b^5*c*d^6*x^4 + 715*a^3*
b^4*d^7*x^4 + 36036*b^7*c^4*d^3*x^3 + 16016*a*b^6*c^3*d^4*x^3 + 6006*a^2*b^
5*c^2*d^5*x^3 + 1716*a^3*b^4*c*d^6*x^3 + 286*a^4*b^3*d^7*x^3 + 19656*b^7*c^
5*d^2*x^2 + 9828*a*b^6*c^4*d^3*x^2 + 4368*a^2*b^5*c^3*d^4*x^2 + 1638*a^3*b^
4*c^2*d^5*x^2 + 468*a^4*b^3*c*d^6*x^2 + 78*a^5*b^2*d^7*x^2 + 6006*b^7*c^6*d
*x + 3276*a*b^6*c^5*d^2*x + 1638*a^2*b^5*c^4*d^3*x + 728*a^3*b^4*c^3*d^4*x
+ 273*a^4*b^3*c^2*d^5*x + 78*a^5*b^2*c*d^6*x + 13*a^6*b*d^7*x + 792*b^7*c^7
 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3
*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^13*b^8)
```

Mupad [**B**]
time = 0.40, size = 570, normalized size = 2.88

$$-\frac{\frac{a^7\,d^7+6\,a^6\,b\,c\,d^6+21\,a^5\,b^2\,c^2\,d^5+56\,a^4\,b^3\,c^3\,d^4+126\,a^3\,b^4\,c^4\,d^3+252\,a^2\,b^5\,c^5\,d^2+462\,a\,b^6\,c^6\,d+792\,b^7\,c^7}{10296\,b^8}+\frac{d^7\,x^7}{6\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+6\,a^4\,b\,c\,d^4+21\,a^3\,b^2\,c^2\,d^3+56\,a^2\,b^3\,c^3\,d^2+126\,a\,b^4\,c^4\,d+252\,b^5\,c^5\right)}{132\,b^6}+\frac{5\,d^4\,x^4\,\left(a^3\,d^3+6\,a^2\,b\,c\,d^2+21\,a\,b^2\,c^2\,d+56\,b^3\,c^3\right)}{72\,b^4}+\frac{d^6\,x^6\,\left(a\,d+6\,b\,c\right)}{6\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+6\,a^3\,b\,c\,d^3+21\,a^2\,b^2\,c^2\,d^2+56\,a\,b^3\,c^3\,d+126\,b^4\,c^4\right)}{36\,b^5}+\frac{d\,x\,\left(a^6\,d^6+6\,a^5\,b\,c\,d^5+21\,a^4\,b^2\,c^2\,d^4+56\,a^3\,b^3\,c^3\,d^3+126\,a^2\,b^4\,c^4\,d^2+252\,a\,b^5\,c^5\,d+462\,b^6\,c^6\right)}{792\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2\right)}{8\,b^3}}{a^{13}+13\,a^{12}\,b\,x+78\,a^{11}\,b^2\,x^2+286\,a^{10}\,b^3\,x^3+715\,a^9\,b^4\,x^4+1287\,a^8\,b^5\,x^5+1716\,a^7\,b^6\,x^6+1716\,a^6\,b^7\,x^7+1287\,a^5\,b^8\,x^8+715\,a^4\,b^9\,x^9+286\,a^3\,b^{10}\,x^{10}+78\,a^2\,b^{11}\,x^{11}+13\,a\,b^{12}\,x^{12}+b^{13}\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^14,x)
```

```
[Out] -((a^7*d^7 + 792*b^7*c^7 + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a
^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 462*a*b^6*c^6*d + 6*a^6*b*c*d^6)/(102
96*b^8) + (d^7*x^7)/(6*b) + (d^2*x^2*(a^5*d^5 + 252*b^5*c^5 + 56*a^2*b^3*c^
3*d^2 + 21*a^3*b^2*c^2*d^3 + 126*a*b^4*c^4*d + 6*a^4*b*c*d^4))/(132*b^6) +
(5*d^4*x^4*(a^3*d^3 + 56*b^3*c^3 + 21*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(72*b^4
) + (d^6*x^6*(a*d + 6*b*c))/(6*b^2) + (d^3*x^3*(a^4*d^4 + 126*b^4*c^4 + 21*
a^2*b^2*c^2*d^2 + 56*a*b^3*c^3*d + 6*a^3*b*c*d^3))/(36*b^5) + (d*x*(a^6*d^6
 + 462*b^6*c^6 + 126*a^2*b^4*c^4*d^2 + 56*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*
d^4 + 252*a*b^5*c^5*d + 6*a^5*b*c*d^5))/(792*b^7) + (d^5*x^5*(a^2*d^2 + 21*
b^2*c^2 + 6*a*b*c*d))/(8*b^3))/(a^13 + b^13*x^13 + 13*a*b^12*x^12 + 78*a^11
*b^2*x^2 + 286*a^10*b^3*x^3 + 715*a^9*b^4*x^4 + 1287*a^8*b^5*x^5 + 1716*a^7
*b^6*x^6 + 1716*a^6*b^7*x^7 + 1287*a^5*b^8*x^8 + 715*a^4*b^9*x^9 + 286*a^3*
b^10*x^10 + 78*a^2*b^11*x^11 + 13*a^12*b*x)
```

3.1297 $\int \frac{(c+dx)^7}{(a+bx)^{15}} dx$

Optimal. Leaf size=200

$$-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^6(bc}{8b^8(a-}$$

[Out] -1/14*(-a*d+b*c)^7/b^8/(b*x+a)^14-7/13*d*(-a*d+b*c)^6/b^8/(b*x+a)^13-7/4*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^12-35/11*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^11-7/2*d^4*(-a*d+b*c)^3/b^8/(b*x+a)^10-7/3*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^9-7/8*d^6*(-a*d+b*c)/b^8/(b*x+a)^8-1/7*d^7/b^8/(b*x+a)^7

Rubi [A]
time = 0.10, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{d^7}{7b^8(a+bx)^7}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^7/(a + b*x)^15,x]

[Out] -1/14*(b*c - a*d)^7/(b^8*(a + b*x)^14) - (7*d*(b*c - a*d)^6)/(13*b^8*(a + b*x)^13) - (7*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^12) - (35*d^3*(b*c - a*d)^4)/(11*b^8*(a + b*x)^11) - (7*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^10) - (7*d^5*(b*c - a*d)^2)/(3*b^8*(a + b*x)^9) - (7*d^6*(b*c - a*d))/(8*b^8*(a + b*x)^8) - d^7/(7*b^8*(a + b*x)^7)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{15}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{14}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{13}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{12}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{11}}\right.\\ &= -\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+7a^6bd^6(c+2dx)+7a^5b^2d^5\left(4c^2+14cdx+13d^2x^2\right)+7a^4b^3d^4\left(12c^3+56c^2dx+91cd^2x^2+52d^3x^3\right)+7a^3b^4d^3\left(30c^4+168c^3dx+364c^2d^2x^2+364cd^3x^3+143d^4x^4\right)+7a^2b^5d^2\left(66c^5+420c^4dx+1092c^3d^2x^2+1456c^2d^3x^3+1001cd^4x^4+286d^5x^5\right)+7ab^6d\left(132c^6+924c^5dx+2730c^4d^2x^2+4368c^3d^3x^3+4004c^2d^4x^4+2002cd^5x^5+429d^6x^6\right)+b^7\left(1716c^7+12936c^6dx+42042c^5d^2x^2+76440c^4d^3x^3+84084c^3d^4x^4+56056c^2d^5x^5+21021cd^6x^6+3432d^7x^7\right)}{24024b^8(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -1/24024*(a^7*d^7 + 7*a^6*b*d^6*(c + 2*d*x) + 7*a^5*b^2*d^5*(4*c^2 + 14*c*d
*x + 13*d^2*x^2) + 7*a^4*b^3*d^4*(12*c^3 + 56*c^2*d*x + 91*c*d^2*x^2 + 52*d
^3*x^3) + 7*a^3*b^4*d^3*(30*c^4 + 168*c^3*d*x + 364*c^2*d^2*x^2 + 364*c*d^3
*x^3 + 143*d^4*x^4) + 7*a^2*b^5*d^2*(66*c^5 + 420*c^4*d*x + 1092*c^3*d^2*x^
2 + 1456*c^2*d^3*x^3 + 1001*c*d^4*x^4 + 286*d^5*x^5) + 7*a*b^6*d*(132*c^6 +
 924*c^5*d*x + 2730*c^4*d^2*x^2 + 4368*c^3*d^3*x^3 + 4004*c^2*d^4*x^4 + 200
2*c*d^5*x^5 + 429*d^6*x^6) + b^7*(1716*c^7 + 12936*c^6*d*x + 42042*c^5*d^2*
x^2 + 76440*c^4*d^3*x^3 + 84084*c^3*d^4*x^4 + 56056*c^2*d^5*x^5 + 21021*c*d
^6*x^6 + 3432*d^7*x^7))/(b^8*(a + b*x)^14)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^15,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(184) = 368$.
time = 0.16, size = 464, normalized size = 2.32

method	result
risch	$\frac{-\frac{d^7x^7}{7b}-\frac{d^6(ad+7bc)x^6}{8b^2}-\frac{d^5\left(a^2d^2+7abcd+28b^2c^2\right)x^5}{12b^3}-\frac{d^4\left(a^3d^3+7a^2bc\,d^2+28a\,b^2c^2d+84b^3c^3\right)x^4}{24b^4}-\frac{d^3\left(a^4d^4+7a^3bc\,d^3+28a^2b^2c^2d^2+84a\,b^3c^3d \right.}{66b^5}$ …
default	$-\frac{7d\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)}{13b^8(bx+a)^{13}}+\frac{7d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a \right.}{4b^8(bx+a)^{12}}$ …
norman	$\frac{\frac{-a^7b^6d^7-7a^6b^7c\,d^6-28a^5b^8c^2d^5-84a^4b^9c^3d^4-210a^3b^{10}c^4d^3-462a^2b^{11}c^5d^2-924a\,b^{12}c^6d-1716b^{13}c^7}{24024b^{14}}+\left(-a^6b^6d^7-7a^5b^7c\,d^6-28a^4b^8c^2d^5-84 \right.}{}$ …
gosper	$-\frac{3432d^7x^7b^7+3003a\,b^6d^7x^6+21021b^7c\,d^6x^6+2002a^2b^5d^7x^5+14014a\,b^6c\,d^6x^5+56056b^7c^2d^5x^5+1001a^3b^4d^7x^4+7007a^2b^5c\,d^6x^4+28}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^15,x,method=_RETURNVERBOSE)
```

```
[Out] -7/13/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15
*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^13+7/4/b^8*d^2*(a^5*d^5-5*a
^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*
x+a)^12-7/3/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^9+7/8/b^8*d^6*(a*d-
b*c)/(b*x+a)^8-1/14*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c
^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*
x+a)^14-1/7*d^7/b^8/(b*x+a)^7+7/2/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
2*d-b^3*c^3)/(b*x+a)^10-35/11/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*
d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(184) = 368$.
time = 0.29, size = 603, normalized size = 3.02

$$-\frac{3432\,b^7d^7x^7+1716\,b^7c^7+924\,ab^6c^6d+462\,a^2b^5c^5d^2+210\,a^3b^4c^4d^3+84\,a^4b^3c^3d^4+28\,a^5b^2c^2d^5+7\,a^6bcd^6+a^7d^7+3003\,(7\,b^7cd^6+ab^6d^7)x^6+2002\,(28\,b^7c^2d^5+7\,ab^6cd^6+a^2b^5d^7)x^5+1001\,(84\,b^7c^3d^4+28\,ab^6c^2d^5+7\,a^2b^5cd^6+a^3b^4d^7)x^4+364\,(210\,b^7c^4d^3+84\,ab^6c^3d^4+28\,a^2b^5c^2d^5+7\,a^3b^4cd^6+a^4b^3d^7)x^3+91\,(462\,b^7c^5d^2+210\,ab^6c^4d^3+84\,a^2b^5c^3d^4+28\,a^3b^4c^2d^5+7\,a^4b^3cd^6+a^5b^2d^7)x^2+14\,(924\,b^7c^6d+462\,ab^6c^5d^2+210\,a^2b^5c^4d^3+84\,a^3b^4c^3d^4+28\,a^4b^3c^2d^5+7\,a^5b^2cd^6+a^6bd^7)x}{24024\,(b^{22}x^{14}+14\,ab^{21}x^{13}+91\,a^2b^{20}x^{12}+364\,a^3b^{19}x^{11}+1001\,a^4b^{18}x^{10}+2002\,a^5b^{17}x^9+3003\,a^6b^{16}x^8+3432\,a^7b^{15}x^7+3003\,a^8b^{14}x^6+2002\,a^9b^{13}x^5+1001\,a^{10}b^{12}x^4+364\,a^{11}b^{11}x^3+91\,a^{12}b^{10}x^2+14\,a^{13}b^9x+a^{14}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(184) = 368$.
time = 0.30, size = 603, normalized size = 3.02

$$-\frac{3432\,b^7d^7x^7+1716\,b^7c^7+924\,ab^6c^6d+462\,a^2b^5c^5d^2+210\,a^3b^4c^4d^3+84\,a^4b^3c^3d^4+28\,a^5b^2c^2d^5+7\,a^6bcd^6+a^7d^7+3003\,(7\,b^7cd^6+ab^6d^7)x^6+2002\,(28\,b^7c^2d^5+7\,ab^6cd^6+a^2b^5d^7)x^5+1001\,(84\,b^7c^3d^4+28\,ab^6c^2d^5+7\,a^2b^5cd^6+a^3b^4d^7)x^4+364\,(210\,b^7c^4d^3+84\,ab^6c^3d^4+28\,a^2b^5c^2d^5+7\,a^3b^4cd^6+a^4b^3d^7)x^3+91\,(462\,b^7c^5d^2+210\,ab^6c^4d^3+84\,a^2b^5c^3d^4+28\,a^3b^4c^2d^5+7\,a^4b^3cd^6+a^5b^2d^7)x^2+14\,(924\,b^7c^6d+462\,ab^6c^5d^2+210\,a^2b^5c^4d^3+84\,a^3b^4c^3d^4+28\,a^4b^3c^2d^5+7\,a^5b^2cd^6+a^6bd^7)x}{24024\,(b^{22}x^{14}+14\,ab^{21}x^{13}+91\,a^2b^{20}x^{12}+364\,a^3b^{19}x^{11}+1001\,a^4b^{18}x^{10}+2002\,a^5b^{17}x^9+3003\,a^6b^{16}x^8+3432\,a^7b^{15}x^7+3003\,a^8b^{14}x^6+2002\,a^9b^{13}x^5+1001\,a^{10}b^{12}x^4+364\,a^{11}b^{11}x^3+91\,a^{12}b^{10}x^2+14\,a^{13}b^9x+a^{14}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
```

```
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**15,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(184) = 368.
time = 0.00, size = 534, normalized size = 2.67

$$\frac{-3432x^7d^7b^7-3003x^6d^7b^6a-21021x^6d^6cb^7-2002x^5d^7b^5a^2-14014x^5d^6cb^6a-56056x^5d^5c^2b^7-1001x^4d^7b^4a^3-7007x^4d^6cb^5a^2-28028x^4d^5c^2b^6a-84084x^4d^4c^3b^7-364x^3d^7b^3a^4-2548x^3d^6cb^4a^3-10192x^3d^5c^2b^5a^2-30576x^3d^4c^3b^6a-76440x^3d^3c^4b^7-91x^2d^7b^2a^5-637x^2d^6cb^3a^4-2548x^2d^5c^2b^4a^3-7644x^2d^4c^3b^5a^2-19110x^2d^3c^4b^6a-42042x^2d^2c^5b^7-14xd^7ba^6-98xd^6cb^2a^5-392xd^5c^2b^3a^4-1176xd^4c^3b^4a^3-2940xd^3c^4b^5a^2-6468xd^2c^5b^6a-12936xdc^6b^7-d^7a^7-7d^6cba^6-28d^5c^2b^2a^5-84d^4c^3b^3a^4-210d^3c^4b^4a^3-462d^2c^5b^5a^2-924dc^6b^6a-1716c^7b^7}{24024b^8\left(xb+a\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x)
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 21021*b^7*c*d^6*x^6 + 3003*a*b^6*d^7*x^6 + 560
56*b^7*c^2*d^5*x^5 + 14014*a*b^6*c*d^6*x^5 + 2002*a^2*b^5*d^7*x^5 + 84084*b
^7*c^3*d^4*x^4 + 28028*a*b^6*c^2*d^5*x^4 + 7007*a^2*b^5*c*d^6*x^4 + 1001*a^
3*b^4*d^7*x^4 + 76440*b^7*c^4*d^3*x^3 + 30576*a*b^6*c^3*d^4*x^3 + 10192*a^2
*b^5*c^2*d^5*x^3 + 2548*a^3*b^4*c*d^6*x^3 + 364*a^4*b^3*d^7*x^3 + 42042*b^7
*c^5*d^2*x^2 + 19110*a*b^6*c^4*d^3*x^2 + 7644*a^2*b^5*c^3*d^4*x^2 + 2548*a^
3*b^4*c^2*d^5*x^2 + 637*a^4*b^3*c*d^6*x^2 + 91*a^5*b^2*d^7*x^2 + 12936*b^7*
c^6*d*x + 6468*a*b^6*c^5*d^2*x + 2940*a^2*b^5*c^4*d^3*x + 1176*a^3*b^4*c^3*
d^4*x + 392*a^4*b^3*c^2*d^5*x + 98*a^5*b^2*c*d^6*x + 14*a^6*b*d^7*x + 1716*
b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^
14*b^8)
```

Mupad [B]
time = 1.24, size = 581, normalized size = 2.90

$$-\frac{\frac{a^7\,d^7+7\,a^6\,b\,c\,d^6+28\,a^5\,b^2\,c^2\,d^5+84\,a^4\,b^3\,c^3\,d^4+210\,a^3\,b^4\,c^4\,d^3+462\,a^2\,b^5\,c^5\,d^2+924\,a\,b^6\,c^6\,d+1716\,b^7\,c^7}{24024\,b^8}+\frac{d^7\,x^7}{7\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+7\,a^4\,b\,c\,d^4+28\,a^3\,b^2\,c^2\,d^3+84\,a^2\,b^3\,c^3\,d^2+210\,a\,b^4\,c^4\,d+462\,b^5\,c^5\right)}{264\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+7\,a^2\,b\,c\,d^2+28\,a\,b^2\,c^2\,d+84\,b^3\,c^3\right)}{24\,b^4}+\frac{d^6\,x^6\,\left(a\,d+7\,b\,c\right)}{8\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+7\,a^3\,b\,c\,d^3+28\,a^2\,b^2\,c^2\,d^2+84\,a\,b^3\,c^3\,d+210\,b^4\,c^4\right)}{66\,b^5}+\frac{d\,x\,\left(a^6\,d^6+7\,a^5\,b\,c\,d^5+28\,a^4\,b^2\,c^2\,d^4+84\,a^3\,b^3\,c^3\,d^3+210\,a^2\,b^4\,c^4\,d^2+462\,a\,b^5\,c^5\,d+924\,b^6\,c^6\right)}{1716\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+7\,a\,b\,c\,d+28\,b^2\,c^2\right)}{12\,b^3}}{a^{14}+14\,a^{13}\,b\,x+91\,a^{12}\,b^2\,x^2+364\,a^{11}\,b^3\,x^3+1001\,a^{10}\,b^4\,x^4+2002\,a^9\,b^5\,x^5+3003\,a^8\,b^6\,x^6+3432\,a^7\,b^7\,x^7+3003\,a^6\,b^8\,x^8+2002\,a^5\,b^9\,x^9+1001\,a^4\,b^{10}\,x^{10}+364\,a^3\,b^{11}\,x^{11}+91\,a^2\,b^{12}\,x^{12}+14\,a\,b^{13}\,x^{13}+b^{14}\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^7/(a + b*x)^15,x)

[Out] -((a^7*d^7 + 1716*b^7*c^7 + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 924*a*b^6*c^6*d + 7*a^6*b*c*d^6)/(24024*b^8) + (d^7*x^7)/(7*b) + (d^2*x^2*(a^5*d^5 + 462*b^5*c^5 + 84*a^2*b^3*c^3*d^2 + 28*a^3*b^2*c^2*d^3 + 210*a*b^4*c^4*d + 7*a^4*b*c*d^4))/(264*b^6) + (d^4*x^4*(a^3*d^3 + 84*b^3*c^3 + 28*a*b^2*c^2*d + 7*a^2*b*c*d^2))/(24*b^4) + (d^6*x^6*(a*d + 7*b*c))/(8*b^2) + (d^3*x^3*(a^4*d^4 + 210*b^4*c^4 + 28*a^2*b^2*c^2*d^2 + 84*a*b^3*c^3*d + 7*a^3*b*c*d^3))/(66*b^5) + (d*x*(a^6*d^6 + 924*b^6*c^6 + 210*a^2*b^4*c^4*d^2 + 84*a^3*b^3*c^3*d^3 + 28*a^4*b^2*c^2*d^4 + 462*a*b^5*c^5*d + 7*a^5*b*c*d^5))/(1716*b^7) + (d^5*x^5*(a^2*d^2 + 28*b^2*c^2 + 7*a*b*c*d))/(12*b^3))/(a^14 + b^14*x^14 + 14*a*b^13*x^13 + 91*a^12*b^2*x^2 + 364*a^11*b^3*x^3 + 1001*a^10*b^4*x^4 + 2002*a^9*b^5*x^5 + 3003*a^8*b^6*x^6 + 3432*a^7*b^7*x^7 + 3003*a^6*b^8*x^8 + 2002*a^5*b^9*x^9 + 1001*a^4*b^10*x^10 + 364*a^3*b^11*x^11 + 91*a^2*b^12*x^12 + 14*a^13*b*x)

3.1298 $\int \frac{(c+dx)^7}{(a+bx)^{16}} dx$

Optimal. Leaf size=200

$$-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}$$

```
[Out] -1/15*(-a*d+b*c)^7/b^8/(b*x+a)^15-1/2*d*(-a*d+b*c)^6/b^8/(b*x+a)^14-21/13*d
^2*(-a*d+b*c)^5/b^8/(b*x+a)^13-35/12*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^12-35/11*
d^4*(-a*d+b*c)^3/b^8/(b*x+a)^11-21/10*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^10-7/9*d
^6*(-a*d+b*c)/b^8/(b*x+a)^9-1/8*d^7/b^8/(b*x+a)^8
```

Rubi [A]
time = 0.10, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d^7}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -1/15*(b*c - a*d)^7/(b^8*(a + b*x)^15) - (d*(b*c - a*d)^6)/(2*b^8*(a + b*x)
^14) - (21*d^2*(b*c - a*d)^5)/(13*b^8*(a + b*x)^13) - (35*d^3*(b*c - a*d)^4
)/(12*b^8*(a + b*x)^12) - (35*d^4*(b*c - a*d)^3)/(11*b^8*(a + b*x)^11) - (2
1*d^5*(b*c - a*d)^2)/(10*b^8*(a + b*x)^10) - (7*d^6*(b*c - a*d))/(9*b^8*(a
+ b*x)^9) - d^7/(8*b^8*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx=\int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{16}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{15}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{14}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{13}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{12}}+\right.$$
$$=-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-$$

Mathematica [**A**]
time = 0.08, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(8c+15dx)+3a^5b^2d^5\left(12c^2+40cdx+35d^2x^2\right)+5a^4b^3d^4\left(24c^3+108c^2dx+168cd^2x^2+91d^3x^3\right)+5a^3b^4d^3\left(66c^4+360c^3dx+756c^2d^2x^2+728cd^3x^3+273d^4x^4\right)+3a^2b^5d^2\left(264c^5+1650c^4dx+4200c^3d^2x^2+5460c^2d^3x^3+3640cd^4x^4+1001d^5x^5\right)+ab^6d\left(1716c^6+11880c^5dx+34650c^4d^2x^2+54600c^3d^3x^3+49140c^2d^4x^4+24024cd^5x^5+5005d^6x^6\right)+b^7\left(3432c^7+25740c^6dx+83160c^5d^2x^2+150150c^4d^3x^3+163800c^3d^4x^4+108108c^2d^5x^5+40040cd^6x^6+6435d^7x^7\right)}{51480b^8(a+bx)^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -1/51480*(a^7*d^7 + a^6*b*d^6*(8*c + 15*d*x) + 3*a^5*b^2*d^5*(12*c^2 + 40*c
*d*x + 35*d^2*x^2) + 5*a^4*b^3*d^4*(24*c^3 + 108*c^2*d*x + 168*c*d^2*x^2 +
91*d^3*x^3) + 5*a^3*b^4*d^3*(66*c^4 + 360*c^3*d*x + 756*c^2*d^2*x^2 + 728*c
*d^3*x^3 + 273*d^4*x^4) + 3*a^2*b^5*d^2*(264*c^5 + 1650*c^4*d*x + 4200*c^3*
d^2*x^2 + 5460*c^2*d^3*x^3 + 3640*c*d^4*x^4 + 1001*d^5*x^5) + a*b^6*d*(1716
*c^6 + 11880*c^5*d*x + 34650*c^4*d^2*x^2 + 54600*c^3*d^3*x^3 + 49140*c^2*d^
4*x^4 + 24024*c*d^5*x^5 + 5005*d^6*x^6) + b^7*(3432*c^7 + 25740*c^6*d*x + 8
3160*c^5*d^2*x^2 + 150150*c^4*d^3*x^3 + 163800*c^3*d^4*x^4 + 108108*c^2*d^5
*x^5 + 40040*c*d^6*x^6 + 6435*d^7*x^7))/(b^8*(a + b*x)^15)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^7/(a + b*x)^16,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(184) = 368$.
time = 0.14, size = 464, normalized size = 2.32

method	result
risch	$-\frac{a^7d^7+8a^6bc\,d^6+36a^5b^2c^2d^5+120a^4b^3c^3d^4+330a^3b^4c^4d^3+792a^2b^5c^5d^2+1716a\,b^6c^6d+3432b^7c^7}{51480b^8}-\frac{d\left(a^6d^6+8a^5bc\,d^5+36a^4b^2c^2d^4+120a^3b^3c^3\right.}{343}$
default	$\frac{21d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{13b^8(bx+a)^{13}}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{12b^8(bx+a)^{12}}+\frac{7d^6(ad}{9b^8(bx}$
norman	$\frac{-a^7b^7d^7-8a^6b^8c\,d^6-36a^5b^9c^2d^5-120a^4b^{10}c^3d^4-330a^3b^{11}c^4d^3-792a^2b^{12}c^5d^2-1716a\,b^{13}c^6d-3432b^{14}c^7}{51480b^{15}}+\frac{\left(-a^6b^7d^7-8a^5b^8c\,d^6-36a^4b^9c^2d\right.}{}$
gosper	$-\frac{6435d^7x^7b^7+5005a\,b^6d^7x^6+40040b^7c\,d^6x^6+3003a^2b^5d^7x^5+24024a\,b^6c\,d^6x^5+108108b^7c^2d^5x^5+1365a^3b^4d^7x^4+10920a^2b^5c\,d^6x^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^16,x,method=_RETURNVERBOSE)
```

[Out] 21/13/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^13-35/12/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^12+7/9/b^8*d^6*(a*d-b*c)/(b*x+a)^9-1/8*d^7/b^8/(b*x+a)^8-1/2/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^14-21/10/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^10-1/15*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^15+35/11/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^11

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(184) = 368$.
time = 0.29, size = 614, normalized size = 3.07

$$-\frac{6435\,b^{7}d^{7}x^{7}+3432\,b^{7}c^{7}+1716\,ab^{6}c^{6}d+792\,a^{2}b^{5}c^{5}d^{2}+330\,a^{3}b^{4}c^{4}d^{3}+120\,a^{4}b^{3}c^{3}d^{4}+36\,a^{5}b^{2}c^{2}d^{5}+8\,a^{6}bcd^{6}+a^{7}d^{7}+5005\,(8\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+3003\,(36\,b^{7}c^{2}d^{5}+8\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+1365\,(120\,b^{7}c^{3}d^{4}+36\,ab^{6}c^{2}d^{5}+8\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+455\,(330\,b^{7}c^{4}d^{3}+120\,ab^{6}c^{3}d^{4}+36\,a^{2}b^{5}c^{2}d^{5}+8\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+105\,(792\,b^{7}c^{5}d^{2}+330\,ab^{6}c^{4}d^{3}+120\,a^{2}b^{5}c^{3}d^{4}+36\,a^{3}b^{4}c^{2}d^{5}+8\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+15\,(1716\,b^{7}c^{6}d+792\,ab^{6}c^{5}d^{2}+330\,a^{2}b^{5}c^{4}d^{3}+120\,a^{3}b^{4}c^{3}d^{4}+36\,a^{4}b^{3}c^{2}d^{5}+8\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{51480\,(b^{23}x^{15}+15\,ab^{22}x^{14}+105\,a^{2}b^{21}x^{13}+455\,a^{3}b^{20}x^{12}+1365\,a^{4}b^{19}x^{11}+3003\,a^{5}b^{18}x^{10}+5005\,a^{6}b^{17}x^{9}+6435\,a^{7}b^{16}x^{8}+6435\,a^{8}b^{15}x^{7}+5005\,a^{9}b^{14}x^{6}+3003\,a^{10}b^{13}x^{5}+1365\,a^{11}b^{12}x^{4}+455\,a^{12}b^{11}x^{3}+105\,a^{13}b^{10}x^{2}+15\,a^{14}b^{9}x+a^{15}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="maxima")

[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 120*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 + 5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(184) = 368$.
time = 0.30, size = 614, normalized size = 3.07

$$-\frac{6435\,b^{7}d^{7}x^{7}+3432\,b^{7}c^{7}+1716\,ab^{6}c^{6}d+792\,a^{2}b^{5}c^{5}d^{2}+330\,a^{3}b^{4}c^{4}d^{3}+120\,a^{4}b^{3}c^{3}d^{4}+36\,a^{5}b^{2}c^{2}d^{5}+8\,a^{6}bcd^{6}+a^{7}d^{7}+5005\,(8\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+3003\,(36\,b^{7}c^{2}d^{5}+8\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+1365\,(120\,b^{7}c^{3}d^{4}+36\,ab^{6}c^{2}d^{5}+8\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+455\,(330\,b^{7}c^{4}d^{3}+120\,ab^{6}c^{3}d^{4}+36\,a^{2}b^{5}c^{2}d^{5}+8\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+105\,(792\,b^{7}c^{5}d^{2}+330\,ab^{6}c^{4}d^{3}+120\,a^{2}b^{5}c^{3}d^{4}+36\,a^{3}b^{4}c^{2}d^{5}+8\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+15\,(1716\,b^{7}c^{6}d+792\,ab^{6}c^{5}d^{2}+330\,a^{2}b^{5}c^{4}d^{3}+120\,a^{3}b^{4}c^{3}d^{4}+36\,a^{4}b^{3}c^{2}d^{5}+8\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{51480\,(b^{23}x^{15}+15\,ab^{22}x^{14}+105\,a^{2}b^{21}x^{13}+455\,a^{3}b^{20}x^{12}+1365\,a^{4}b^{19}x^{11}+3003\,a^{5}b^{18}x^{10}+5005\,a^{6}b^{17}x^{9}+6435\,a^{7}b^{16}x^{8}+6435\,a^{8}b^{15}x^{7}+5005\,a^{9}b^{14}x^{6}+3003\,a^{10}b^{13}x^{5}+1365\,a^{11}b^{12}x^{4}+455\,a^{12}b^{11}x^{3}+105\,a^{13}b^{10}x^{2}+15\,a^{14}b^{9}x+a^{15}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="fricas")

[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7

*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
 + 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**7/(b*x+a)**16,x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(184) = 368.
time = 0.00, size = 534, normalized size = 2.67

$$\frac{-6435x^7d^7b^7 - 5005x^6d^7b^6a - 40040x^6d^6cb^7 - 3003x^5d^7b^5a^2 - 24024x^5d^6cb^6a - 108108x^5d^5c^2b^7 - 1365x^4d^7b^4a^3 - 10920x^4d^6cb^5a^2 - 49140x^4d^5c^2b^6a - 163800x^4d^4c^3b^7 - 455x^3d^7b^3a^4 - 3640x^3d^6cb^4a^3 - 16380x^3d^5c^2b^5a^2 - 54600x^3d^4c^3b^6a - 150150x^3d^3c^4b^7 - 105x^2d^7b^2a^5 - 840x^2d^6cb^3a^4 - 3780x^2d^5c^2b^4a^3 - 12600x^2d^4c^3b^5a^2 - 34650x^2d^3c^4b^6a - 83160x^2d^2c^5b^7 - 15xd^7ba^6 - 120xd^6cb^2a^5 - 540xd^5c^2b^3a^4 - 1800xd^4c^3b^4a^3 - 4950xd^3c^4b^5a^2 - 11880xd^2c^5b^6a - 25740xdc^6b^7 - d^7a^7 - 8d^6cba^6 - 36d^5c^2b^2a^5 - 120d^4c^3b^3a^4 - 330d^3c^4b^4a^3 - 792d^2c^5b^5a^2 - 1716dc^6b^6a - 3432c^7b^7}{51480b^8\left(xb+a\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^16,x)

[Out] -1/51480*(6435*b^7*d^7*x^7 + 40040*b^7*c*d^6*x^6 + 5005*a*b^6*d^7*x^6 + 108
108*b^7*c^2*d^5*x^5 + 24024*a*b^6*c*d^6*x^5 + 3003*a^2*b^5*d^7*x^5 + 163800
*b^7*c^3*d^4*x^4 + 49140*a*b^6*c^2*d^5*x^4 + 10920*a^2*b^5*c*d^6*x^4 + 1365
*a^3*b^4*d^7*x^4 + 150150*b^7*c^4*d^3*x^3 + 54600*a*b^6*c^3*d^4*x^3 + 16380
*a^2*b^5*c^2*d^5*x^3 + 3640*a^3*b^4*c*d^6*x^3 + 455*a^4*b^3*d^7*x^3 + 83160
*b^7*c^5*d^2*x^2 + 34650*a*b^6*c^4*d^3*x^2 + 12600*a^2*b^5*c^3*d^4*x^2 + 37
80*a^3*b^4*c^2*d^5*x^2 + 840*a^4*b^3*c*d^6*x^2 + 105*a^5*b^2*d^7*x^2 + 2574
0*b^7*c^6*d*x + 11880*a*b^6*c^5*d^2*x + 4950*a^2*b^5*c^4*d^3*x + 1800*a^3*b
^4*c^3*d^4*x + 540*a^4*b^3*c^2*d^5*x + 120*a^5*b^2*c*d^6*x + 15*a^6*b*d^7*x
 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*
d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7)/(
(b*x + a)^15*b^8)

Mupad [**B**]
time = 2.20, size = 592, normalized size = 2.96

$$-\frac{\frac{a^7\,d^7+8\,a^6\,b\,c\,d^6+36\,a^5\,b^2\,c^2\,d^5+120\,a^4\,b^3\,c^3\,d^4+330\,a^3\,b^4\,c^4\,d^3+792\,a^2\,b^5\,c^5\,d^2+1716\,a\,b^6\,c^6\,d+3432\,b^7\,c^7}{51480\,b^8}+\frac{d^7\,x^7}{8\,b}+\frac{7\,d^2\,x^2\,\left(a^5\,d^5+8\,a^4\,b\,c\,d^4+36\,a^3\,b^2\,c^2\,d^3+120\,a^2\,b^3\,c^3\,d^2+330\,a\,b^4\,c^4\,d+792\,b^5\,c^5\right)}{3432\,b^6}+\frac{7\,d^4\,x^4\,\left(a^3\,d^3+8\,a^2\,b\,c\,d^2+36\,a\,b^2\,c^2\,d+120\,b^3\,c^3\right)}{264\,b^4}+\frac{7\,d^6\,x^6\,\left(a\,d+8\,b\,c\right)}{72\,b^2}+\frac{7\,d^3\,x^3\,\left(a^4\,d^4+8\,a^3\,b\,c\,d^3+36\,a^2\,b^2\,c^2\,d^2+120\,a\,b^3\,c^3\,d+330\,b^4\,c^4\right)}{792\,b^5}+\frac{d\,x\,\left(a^6\,d^6+8\,a^5\,b\,c\,d^5+36\,a^4\,b^2\,c^2\,d^4+120\,a^3\,b^3\,c^3\,d^3+330\,a^2\,b^4\,c^4\,d^2+792\,a\,b^5\,c^5\,d+1716\,b^6\,c^6\right)}{3432\,b^7}+\frac{7\,d^5\,x^5\,\left(a^2\,d^2+8\,a\,b\,c\,d+36\,b^2\,c^2\right)}{120\,b^3}}{a^{15}+15\,a^{14}\,b\,x+105\,a^{13}\,b^2\,x^2+455\,a^{12}\,b^3\,x^3+1365\,a^{11}\,b^4\,x^4+3003\,a^{10}\,b^5\,x^5+5005\,a^9\,b^6\,x^6+6435\,a^8\,b^7\,x^7+6435\,a^7\,b^8\,x^8+5005\,a^6\,b^9\,x^9+3003\,a^5\,b^{10}\,x^{10}+1365\,a^4\,b^{11}\,x^{11}+455\,a^3\,b^{12}\,x^{12}+105\,a^2\,b^{13}\,x^{13}+15\,a\,b^{14}\,x^{14}+b^{15}\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^16,x)
```

```
[Out] -((a^7*d^7 + 3432*b^7*c^7 + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120
*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 1716*a*b^6*c^6*d + 8*a^6*b*c*d^6)/(
51480*b^8) + (d^7*x^7)/(8*b) + (7*d^2*x^2*(a^5*d^5 + 792*b^5*c^5 + 120*a^2*
b^3*c^3*d^2 + 36*a^3*b^2*c^2*d^3 + 330*a*b^4*c^4*d + 8*a^4*b*c*d^4))/(3432*
b^6) + (7*d^4*x^4*(a^3*d^3 + 120*b^3*c^3 + 36*a*b^2*c^2*d + 8*a^2*b*c*d^2))
/(264*b^4) + (7*d^6*x^6*(a*d + 8*b*c))/(72*b^2) + (7*d^3*x^3*(a^4*d^4 + 330
*b^4*c^4 + 36*a^2*b^2*c^2*d^2 + 120*a*b^3*c^3*d + 8*a^3*b*c*d^3))/(792*b^5)
 + (d*x*(a^6*d^6 + 1716*b^6*c^6 + 330*a^2*b^4*c^4*d^2 + 120*a^3*b^3*c^3*d^3
 + 36*a^4*b^2*c^2*d^4 + 792*a*b^5*c^5*d + 8*a^5*b*c*d^5))/(3432*b^7) + (7*d
^5*x^5*(a^2*d^2 + 36*b^2*c^2 + 8*a*b*c*d))/(120*b^3))/(a^15 + b^15*x^15 + 1
5*a*b^14*x^14 + 105*a^13*b^2*x^2 + 455*a^12*b^3*x^3 + 1365*a^11*b^4*x^4 + 3
003*a^10*b^5*x^5 + 5005*a^9*b^6*x^6 + 6435*a^8*b^7*x^7 + 6435*a^7*b^8*x^8 +
 5005*a^6*b^9*x^9 + 3003*a^5*b^10*x^10 + 1365*a^4*b^11*x^11 + 455*a^3*b^12*
x^12 + 105*a^2*b^13*x^13 + 15*a^14*b*x)
```

3.1299 $\int (a+bx)^{12}(c+dx)^{10}\,dx$

Optimal. Leaf size=275

$$\frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\frac{15d^3(bc-ad)^7(a+bx)^{16}}{2b^{11}}+\frac{210d^4}{}$$

```
[Out] 1/13*(-a*d+b*c)^10*(b*x+a)^13/b^11+5/7*d*(-a*d+b*c)^9*(b*x+a)^14/b^11+3*d^2
*(-a*d+b*c)^8*(b*x+a)^15/b^11+15/2*d^3*(-a*d+b*c)^7*(b*x+a)^16/b^11+210/17*
d^4*(-a*d+b*c)^6*(b*x+a)^17/b^11+14*d^5*(-a*d+b*c)^5*(b*x+a)^18/b^11+210/19
*d^6*(-a*d+b*c)^4*(b*x+a)^19/b^11+6*d^7*(-a*d+b*c)^3*(b*x+a)^20/b^11+15/7*d
^8*(-a*d+b*c)^2*(b*x+a)^21/b^11+5/11*d^9*(-a*d+b*c)*(b*x+a)^22/b^11+1/23*d^
10*(b*x+a)^23/b^11
```

Rubi [A]

time = 1.01, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+bx)^{18}(bc-ad)^5}{b^{11}}+\frac{210d^4(a+bx)^{17}(bc-ad)^6}{17b^{11}}+\frac{15d^3(a+bx)^{16}(bc-ad)^7}{2b^{11}}+\frac{3d^2(a+bx)^{15}(bc-ad)^8}{b^{11}}+\frac{5d(a+bx)^{14}(bc-ad)^9}{7b^{11}}+\frac{(a+bx)^{13}(bc-ad)^{10}}{13b^{11}}+\frac{d^{10}(a+bx)^{23}}{23b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^13)/(13*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^14)/
(7*b^11) + (3*d^2*(b*c - a*d)^8*(a + b*x)^15)/b^11 + (15*d^3*(b*c - a*d)^7*
(a + b*x)^16)/(2*b^11) + (210*d^4*(b*c - a*d)^6*(a + b*x)^17)/(17*b^11) + (
14*d^5*(b*c - a*d)^5*(a + b*x)^18)/b^11 + (210*d^6*(b*c - a*d)^4*(a + b*x)^
19)/(19*b^11) + (6*d^7*(b*c - a*d)^3*(a + b*x)^20)/b^11 + (15*d^8*(b*c - a*
d)^2*(a + b*x)^21)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^22)/(11*b^11) +
(d^10*(a + b*x)^23)/(23*b^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{12}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{12}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx}{b^{10}}\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1817 vs. $2(275) = 550$.

time = 0.18, size = 1817, normalized size = 6.61

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] a^12*c^10*x + a^11*c^9*(6*b*c + 5*a*d)*x^2 + a^10*c^8*(22*b^2*c^2 + 40*a*b*
c*d + 15*a^2*d^2)*x^3 + 5*a^9*c^7*(11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 6*a^3*d^3)*x^4 + a^8*c^6*(99*b^4*c^4 + 440*a*b^3*c^3*d + 594*a^2*b^2
*c^2*d^2 + 288*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + 3*a^7*c^5*(44*b^5*c^5 + 275*
a*b^4*c^4*d + 550*a^2*b^3*c^3*d^2 + 440*a^3*b^2*c^2*d^3 + 140*a^4*b*c*d^4 +
 14*a^5*d^5)*x^6 + (3*a^6*c^4*(308*b^6*c^6 + 2640*a*b^5*c^5*d + 7425*a^2*b^
4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 4620*a^4*b^2*c^2*d^4 + 1008*a^5*b*c*d^5
+ 70*a^6*d^6)*x^7)/7 + 3*a^5*c^3*(33*b^7*c^7 + 385*a*b^6*c^6*d + 1485*a^2*b
^5*c^5*d^2 + 2475*a^3*b^4*c^4*d^3 + 1925*a^4*b^3*c^3*d^4 + 693*a^5*b^2*c^2*
d^5 + 105*a^6*b*c*d^6 + 5*a^7*d^7)*x^8 + 5*a^4*c^2*(11*b^8*c^8 + 176*a*b^7*
c^7*d + 924*a^2*b^6*c^6*d^2 + 2112*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 +
 1232*a^5*b^3*c^3*d^5 + 308*a^6*b^2*c^2*d^6 + 32*a^7*b*c*d^7 + a^8*d^8)*x^9
 + a^3*c*(22*b^9*c^9 + 495*a*b^8*c^8*d + 3564*a^2*b^7*c^7*d^2 + 11088*a^3*b
^6*c^6*d^3 + 16632*a^4*b^5*c^5*d^4 + 12474*a^5*b^4*c^4*d^5 + 4620*a^6*b^3*c
^3*d^6 + 792*a^7*b^2*c^2*d^7 + 54*a^8*b*c*d^8 + a^9*d^9)*x^10 + (a^2*(66*b^
10*c^10 + 2200*a*b^9*c^9*d + 22275*a^2*b^8*c^8*d^2 + 95040*a^3*b^7*c^7*d^3
+ 194040*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5 + 103950*a^6*b^4*c^4*d^6
+ 26400*a^7*b^3*c^3*d^7 + 2970*a^8*b^2*c^2*d^8 + 120*a^9*b*c*d^9 + a^10*d^1
0)*x^11)/11 + a*b*(b^10*c^10 + 55*a*b^9*c^9*d + 825*a^2*b^8*c^8*d^2 + 4950*
a^3*b^7*c^7*d^3 + 13860*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 + 13860*a^6
*b^4*c^4*d^6 + 4950*a^7*b^3*c^3*d^7 + 825*a^8*b^2*c^2*d^8 + 55*a^9*b*c*d^9
+ a^10*d^10)*x^12 + (b^2*(b^10*c^10 + 120*a*b^9*c^9*d + 2970*a^2*b^8*c^8*d^
2 + 26400*a^3*b^7*c^7*d^3 + 103950*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5
 + 194040*a^6*b^4*c^4*d^6 + 95040*a^7*b^3*c^3*d^7 + 22275*a^8*b^2*c^2*d^8 +
 2200*a^9*b*c*d^9 + 66*a^10*d^10)*x^13)/13 + (5*b^3*d*(b^9*c^9 + 54*a*b^8*c
^8*d + 792*a^2*b^7*c^7*d^2 + 4620*a^3*b^6*c^6*d^3 + 12474*a^4*b^5*c^5*d^4 +
 16632*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 3564*a^7*b^2*c^2*d^7 + 495
*a^8*b*c*d^8 + 22*a^9*d^9)*x^14)/7 + 3*b^4*d^2*(b^8*c^8 + 32*a*b^7*c^7*d +
308*a^2*b^6*c^6*d^2 + 1232*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 + 2112*a^
5*b^3*c^3*d^5 + 924*a^6*b^2*c^2*d^6 + 176*a^7*b*c*d^7 + 11*a^8*d^8)*x^15 +
(3*b^5*d^3*(5*b^7*c^7 + 105*a*b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1925*a^3*b^
4*c^4*d^3 + 2475*a^4*b^3*c^3*d^4 + 1485*a^5*b^2*c^2*d^5 + 385*a^6*b*c*d^6 +
 33*a^7*d^7)*x^16)/2 + (3*b^6*d^4*(70*b^6*c^6 + 1008*a*b^5*c^5*d + 4620*a^2
*b^4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 7425*a^4*b^2*c^2*d^4 + 2640*a^5*b*c*d
^5 + 308*a^6*d^6)*x^17)/17 + b^7*d^5*(14*b^5*c^5 + 140*a*b^4*c^4*d + 440*a^
2*b^3*c^3*d^2 + 550*a^3*b^2*c^2*d^3 + 275*a^4*b*c*d^4 + 44*a^5*d^5)*x^18 +
(5*b^8*d^6*(42*b^4*c^4 + 288*a*b^3*c^3*d + 594*a^2*b^2*c^2*d^2 + 440*a^3*b*
```

```
c*d^3 + 99*a^4*d^4)*x^19)/19 + b^9*d^7*(6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2
*b*c*d^2 + 11*a^3*d^3)*x^20 + (b^10*d^8*(15*b^2*c^2 + 40*a*b*c*d + 22*a^2*d
^2)*x^21)/7 + (b^11*d^9*(5*b*c + 6*a*d)*x^22)/11 + (b^12*d^10*x^23)/23
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 1816 vs. 2(275) = 550.
time = 15.99, size = 1796, normalized size = 6.53

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^12*(c + d*x)^10,x]')
```

```
[Out] x (a ^ 12 c ^ 10 + a ^ 11 c ^ 9 x (5 a d + 6 b c) + a ^ 10 c ^ 8 x ^ 2 (15
a ^ 2 d ^ 2 + 40 a b c d + 22 b ^ 2 c ^ 2) + 5 a ^ 9 c ^ 7 x ^ 3 (6 a ^ 3 d
 ^ 3 + 27 a ^ 2 b c d ^ 2 + 33 a b ^ 2 c ^ 2 d + 11 b ^ 3 c ^ 3) + a ^ 8 c
^ 6 x ^ 4 (42 a ^ 4 d ^ 4 + 288 a ^ 3 b c d ^ 3 + 594 a ^ 2 b ^ 2 c ^ 2 d ^
 2 + 440 a b ^ 3 c ^ 3 d + 99 b ^ 4 c ^ 4) + 3 a ^ 7 c ^ 5 x ^ 5 (14 a ^ 5
d ^ 5 + 140 a ^ 4 b c d ^ 4 + 440 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 550 a ^ 2 b ^ 3
 c ^ 3 d ^ 2 + 275 a b ^ 4 c ^ 4 d + 44 b ^ 5 c ^ 5) + 3 a ^ 6 c ^ 4 x ^ 6
(70 a ^ 6 d ^ 6 + 1008 a ^ 5 b c d ^ 5 + 4620 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 880
0 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 7425 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 2640 a b ^ 5 c ^
 5 d + 308 b ^ 6 c ^ 6) / 7 + 3 a ^ 5 c ^ 3 x ^ 7 (5 a ^ 7 d ^ 7 + 105 a ^
6 b c d ^ 6 + 693 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 1925 a ^ 4 b ^ 3 c ^ 3 d ^ 4 +
2475 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 1485 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 385 a b ^ 6 c
 ^ 6 d + 33 b ^ 7 c ^ 7) + 5 a ^ 4 c ^ 2 x ^ 8 (a ^ 8 d ^ 8 + 32 a ^ 7 b c
d ^ 7 + 308 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 1232 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 2310 a
 ^ 4 b ^ 4 c ^ 4 d ^ 4 + 2112 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 924 a ^ 2 b ^ 6 c ^
 6 d ^ 2 + 176 a b ^ 7 c ^ 7 d + 11 b ^ 8 c ^ 8) + a ^ 3 c x ^ 9 (a ^ 9 d ^
 9 + 54 a ^ 8 b c d ^ 8 + 792 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 4620 a ^ 6 b ^ 3 c
^ 3 d ^ 6 + 12474 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 16632 a ^ 4 b ^ 5 c ^ 5 d ^ 4 +
 11088 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 3564 a ^ 2 b ^ 7 c ^ 7 d ^ 2 + 495 a b ^ 8
 c ^ 8 d + 22 b ^ 9 c ^ 9) + a ^ 2 x ^ 10 (a ^ 10 d ^ 10 + 120 a ^ 9 b c d
^ 9 + 2970 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 26400 a ^ 7 b ^ 3 c ^ 3 d ^ 7 + 103950
 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 199584 a ^ 5 b ^ 5 c ^ 5 d ^ 5 + 194040 a ^ 4 b
^ 6 c ^ 6 d ^ 4 + 95040 a ^ 3 b ^ 7 c ^ 7 d ^ 3 + 22275 a ^ 2 b ^ 8 c ^ 8 d
 ^ 2 + 2200 a b ^ 9 c ^ 9 d + 66 b ^ 10 c ^ 10) / 11 + a b x ^ 11 (a ^ 10 d
 ^ 10 + 55 a ^ 9 b c d ^ 9 + 825 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 4950 a ^ 7 b ^ 3
 c ^ 3 d ^ 7 + 13860 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 19404 a ^ 5 b ^ 5 c ^ 5 d ^
5 + 13860 a ^ 4 b ^ 6 c ^ 6 d ^ 4 + 4950 a ^ 3 b ^ 7 c ^ 7 d ^ 3 + 825 a ^
2 b ^ 8 c ^ 8 d ^ 2 + 55 a b ^ 9 c ^ 9 d + b ^ 10 c ^ 10) + b ^ 2 x ^ 12 (6
6 a ^ 10 d ^ 10 + 2200 a ^ 9 b c d ^ 9 + 22275 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 95
040 a ^ 7 b ^ 3 c ^ 3 d ^ 7 + 194040 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 199584 a ^ 5
 b ^ 5 c ^ 5 d ^ 5 + 103950 a ^ 4 b ^ 6 c ^ 6 d ^ 4 + 26400 a ^ 3 b ^ 7 c ^
 7 d ^ 3 + 2970 a ^ 2 b ^ 8 c ^ 8 d ^ 2 + 120 a b ^ 9 c ^ 9 d + b ^ 10 c ^
10) / 13 + 5 b ^ 3 d x ^ 13 (22 a ^ 9 d ^ 9 + 495 a ^ 8 b c d ^ 8 + 3564 a
^ 7 b ^ 2 c ^ 2 d ^ 7 + 11088 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 16632 a ^ 5 b ^ 4 c
```

```
^ 4 d ^ 5 + 12474 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 4620 a ^ 3 b ^ 6 c ^ 6 d ^ 3 +
792 a ^ 2 b ^ 7 c ^ 7 d ^ 2 + 54 a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) / 7 + 3 b
^ 4 d ^ 2 x ^ 14 (11 a ^ 8 d ^ 8 + 176 a ^ 7 b c d ^ 7 + 924 a ^ 6 b ^ 2 c
^ 2 d ^ 6 + 2112 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 2310 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 1
232 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 308 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 32 a b ^ 7 c ^
7 d + b ^ 8 c ^ 8) + 3 b ^ 5 d ^ 3 x ^ 15 (33 a ^ 7 d ^ 7 + 385 a ^ 6 b c d
^ 6 + 1485 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 2475 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 1925 a
^ 3 b ^ 4 c ^ 4 d ^ 3 + 693 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 105 a b ^ 6 c ^ 6 d
+ 5 b ^ 7 c ^ 7) / 2 + 3 b ^ 6 d ^ 4 x ^ 16 (308 a ^ 6 d ^ 6 + 2640 a ^ 5 b
c d ^ 5 + 7425 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 8800 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 46
20 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 1008 a b ^ 5 c ^ 5 d + 70 b ^ 6 c ^ 6) / 17 +
b ^ 7 d ^ 5 x ^ 17 (44 a ^ 5 d ^ 5 + 275 a ^ 4 b c d ^ 4 + 550 a ^ 3 b ^ 2
c ^ 2 d ^ 3 + 440 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 140 a b ^ 4 c ^ 4 d + 14 b ^ 5
c ^ 5) + 5 b ^ 8 d ^ 6 x ^ 18 (99 a ^ 4 d ^ 4 + 440 a ^ 3 b c d ^ 3 + 594 a
^ 2 b ^ 2 c ^ 2 d ^ 2 + 288 a b ^ 3 c ^ 3 d + 42 b ^ 4 c ^ 4) / 19 + b ^ 9
d ^ 7 x ^ 19 (11 a ^ 3 d ^ 3 + 33 a ^ 2 b c d ^ 2 + 27 a b ^ 2 c ^ 2 d + 6
b ^ 3 c ^ 3) + b ^ 10 d ^ 8 x ^ 20 (22 a ^ 2 d ^ 2 + 40 a b c d + 15 b ^ 2
c ^ 2) / 7 + b ^ 11 d ^ 9 x ^ 21 (6 a d + 5 b c) / 11 + b ^ 12 d ^ 10 x ^
22 / 23)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. $2(259) = 518$.

time = 0.14, size = 1891, normalized size = 6.88

method	result	size
norman	Expression too large to display	1869
default	Expression too large to display	1891
gosper	Expression too large to display	2187
risch	Expression too large to display	2187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/23*b^12*d^10*x^23+1/22*(12*a*b^11*d^10+10*b^12*c*d^9)*x^22+1/21*(66*a^2*b
^10*d^10+120*a*b^11*c*d^9+45*b^12*c^2*d^8)*x^21+1/20*(220*a^3*b^9*d^10+660*
a^2*b^10*c*d^9+540*a*b^11*c^2*d^8+120*b^12*c^3*d^7)*x^20+1/19*(495*a^4*b^8*
d^10+2200*a^3*b^9*c*d^9+2970*a^2*b^10*c^2*d^8+1440*a*b^11*c^3*d^7+210*b^12*
c^4*d^6)*x^19+1/18*(792*a^5*b^7*d^10+4950*a^4*b^8*c*d^9+9900*a^3*b^9*c^2*d^
8+7920*a^2*b^10*c^3*d^7+2520*a*b^11*c^4*d^6+252*b^12*c^5*d^5)*x^18+1/17*(92
4*a^6*b^6*d^10+7920*a^5*b^7*c*d^9+22275*a^4*b^8*c^2*d^8+26400*a^3*b^9*c^3*d
^7+13860*a^2*b^10*c^4*d^6+3024*a*b^11*c^5*d^5+210*b^12*c^6*d^4)*x^17+1/16*(
792*a^7*b^5*d^10+9240*a^6*b^6*c*d^9+35640*a^5*b^7*c^2*d^8+59400*a^4*b^8*c^3
*d^7+46200*a^3*b^9*c^4*d^6+16632*a^2*b^10*c^5*d^5+2520*a*b^11*c^6*d^4+120*b
^12*c^7*d^3)*x^16+1/15*(495*a^8*b^4*d^10+7920*a^7*b^5*c*d^9+41580*a^6*b^6*c
^2*d^8+95040*a^5*b^7*c^3*d^7+103950*a^4*b^8*c^4*d^6+55440*a^3*b^9*c^5*d^5+1
```

```
3860*a^2*b^10*c^6*d^4+1440*a*b^11*c^7*d^3+45*b^12*c^8*d^2)*x^15+1/14*(220*a
^9*b^3*d^10+4950*a^8*b^4*c*d^9+35640*a^7*b^5*c^2*d^8+110880*a^6*b^6*c^3*d^7
+166320*a^5*b^7*c^4*d^6+124740*a^4*b^8*c^5*d^5+46200*a^3*b^9*c^6*d^4+7920*a
^2*b^10*c^7*d^3+540*a*b^11*c^8*d^2+10*b^12*c^9*d)*x^14+1/13*(66*a^10*b^2*d^
10+2200*a^9*b^3*c*d^9+22275*a^8*b^4*c^2*d^8+95040*a^7*b^5*c^3*d^7+194040*a^
6*b^6*c^4*d^6+199584*a^5*b^7*c^5*d^5+103950*a^4*b^8*c^6*d^4+26400*a^3*b^9*c
^7*d^3+2970*a^2*b^10*c^8*d^2+120*a*b^11*c^9*d+b^12*c^10)*x^13+1/12*(12*a^11
*b*d^10+660*a^10*b^2*c*d^9+9900*a^9*b^3*c^2*d^8+59400*a^8*b^4*c^3*d^7+16632
0*a^7*b^5*c^4*d^6+232848*a^6*b^6*c^5*d^5+166320*a^5*b^7*c^6*d^4+59400*a^4*b
^8*c^7*d^3+9900*a^3*b^9*c^8*d^2+660*a^2*b^10*c^9*d+12*a*b^11*c^10)*x^12+1/1
1*(a^12*d^10+120*a^11*b*c*d^9+2970*a^10*b^2*c^2*d^8+26400*a^9*b^3*c^3*d^7+1
03950*a^8*b^4*c^4*d^6+199584*a^7*b^5*c^5*d^5+194040*a^6*b^6*c^6*d^4+95040*a
^5*b^7*c^7*d^3+22275*a^4*b^8*c^8*d^2+2200*a^3*b^9*c^9*d+66*a^2*b^10*c^10)*x
^11+1/10*(10*a^12*c*d^9+540*a^11*b*c^2*d^8+7920*a^10*b^2*c^3*d^7+46200*a^9*
b^3*c^4*d^6+124740*a^8*b^4*c^5*d^5+166320*a^7*b^5*c^6*d^4+110880*a^6*b^6*c^
7*d^3+35640*a^5*b^7*c^8*d^2+4950*a^4*b^8*c^9*d+220*a^3*b^9*c^10)*x^10+1/9*(
45*a^12*c^2*d^8+1440*a^11*b*c^3*d^7+13860*a^10*b^2*c^4*d^6+55440*a^9*b^3*c^
5*d^5+103950*a^8*b^4*c^6*d^4+95040*a^7*b^5*c^7*d^3+41580*a^6*b^6*c^8*d^2+79
20*a^5*b^7*c^9*d+495*a^4*b^8*c^10)*x^9+1/8*(120*a^12*c^3*d^7+2520*a^11*b*c^
4*d^6+16632*a^10*b^2*c^5*d^5+46200*a^9*b^3*c^6*d^4+59400*a^8*b^4*c^7*d^3+35
640*a^7*b^5*c^8*d^2+9240*a^6*b^6*c^9*d+792*a^5*b^7*c^10)*x^8+1/7*(210*a^12*
c^4*d^6+3024*a^11*b*c^5*d^5+13860*a^10*b^2*c^6*d^4+26400*a^9*b^3*c^7*d^3+22
275*a^8*b^4*c^8*d^2+7920*a^7*b^5*c^9*d+924*a^6*b^6*c^10)*x^7+1/6*(252*a^12*
c^5*d^5+2520*a^11*b*c^6*d^4+7920*a^10*b^2*c^7*d^3+9900*a^9*b^3*c^8*d^2+4950
*a^8*b^4*c^9*d+792*a^7*b^5*c^10)*x^6+1/5*(210*a^12*c^6*d^4+1440*a^11*b*c^7*
d^3+2970*a^10*b^2*c^8*d^2+2200*a^9*b^3*c^9*d+495*a^8*b^4*c^10)*x^5+1/4*(120
*a^12*c^7*d^3+540*a^11*b*c^8*d^2+660*a^10*b^2*c^9*d+220*a^9*b^3*c^10)*x^4+1
/3*(45*a^12*c^8*d^2+120*a^11*b*c^9*d+66*a^10*b^2*c^10)*x^3+1/2*(10*a^12*c^9
*d+12*a^11*b*c^10)*x^2+a^12*c^10*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(259) = 518$.
time = 0.30, size = 1877, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
```

10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(259) = 518$.

time = 0.31, size = 1877, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
```

```
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. $2(255) = 510$.

time = 0.17, size = 2088, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12*(d*x+c)**10,x)
```

```
[Out] a**12*c**10*x + b**12*d**10*x**23/23 + x**22*(6*a*b**11*d**10/11 + 5*b**12*
c*d**9/11) + x**21*(22*a**2*b**10*d**10/7 + 40*a*b**11*c*d**9/7 + 15*b**12*
c**2*d**8/7) + x**20*(11*a**3*b**9*d**10 + 33*a**2*b**10*c*d**9 + 27*a*b**1
1*c**2*d**8 + 6*b**12*c**3*d**7) + x**19*(495*a**4*b**8*d**10/19 + 2200*a**
3*b**9*c*d**9/19 + 2970*a**2*b**10*c**2*d**8/19 + 1440*a*b**11*c**3*d**7/19
 + 210*b**12*c**4*d**6/19) + x**18*(44*a**5*b**7*d**10 + 275*a**4*b**8*c*d*
*9 + 550*a**3*b**9*c**2*d**8 + 440*a**2*b**10*c**3*d**7 + 140*a*b**11*c**4*
d**6 + 14*b**12*c**5*d**5) + x**17*(924*a**6*b**6*d**10/17 + 7920*a**5*b**7
*c*d**9/17 + 22275*a**4*b**8*c**2*d**8/17 + 26400*a**3*b**9*c**3*d**7/17 +
13860*a**2*b**10*c**4*d**6/17 + 3024*a*b**11*c**5*d**5/17 + 210*b**12*c**6*
d**4/17) + x**16*(99*a**7*b**5*d**10/2 + 1155*a**6*b**6*c*d**9/2 + 4455*a**
5*b**7*c**2*d**8/2 + 7425*a**4*b**8*c**3*d**7/2 + 5775*a**3*b**9*c**4*d**6/
2 + 2079*a**2*b**10*c**5*d**5/2 + 315*a*b**11*c**6*d**4/2 + 15*b**12*c**7*d
**3/2) + x**15*(33*a**8*b**4*d**10 + 528*a**7*b**5*c*d**9 + 2772*a**6*b**6*
c**2*d**8 + 6336*a**5*b**7*c**3*d**7 + 6930*a**4*b**8*c**4*d**6 + 3696*a**3
*b**9*c**5*d**5 + 924*a**2*b**10*c**6*d**4 + 96*a*b**11*c**7*d**3 + 3*b**12
*c**8*d**2) + x**14*(110*a**9*b**3*d**10/7 + 2475*a**8*b**4*c*d**9/7 + 1782
0*a**7*b**5*c**2*d**8/7 + 7920*a**6*b**6*c**3*d**7 + 11880*a**5*b**7*c**4*d
**6 + 8910*a**4*b**8*c**5*d**5 + 3300*a**3*b**9*c**6*d**4 + 3960*a**2*b**10
*c**7*d**3/7 + 270*a*b**11*c**8*d**2/7 + 5*b**12*c**9*d/7) + x**13*(66*a**1
0*b**2*d**10/13 + 2200*a**9*b**3*c*d**9/13 + 22275*a**8*b**4*c**2*d**8/13 +
 95040*a**7*b**5*c**3*d**7/13 + 194040*a**6*b**6*c**4*d**6/13 + 199584*a**5
*b**7*c**5*d**5/13 + 103950*a**4*b**8*c**6*d**4/13 + 26400*a**3*b**9*c**7*d
**3/13 + 2970*a**2*b**10*c**8*d**2/13 + 120*a*b**11*c**9*d/13 + b**12*c**10
/13) + x**12*(a**11*b*d**10 + 55*a**10*b**2*c*d**9 + 825*a**9*b**3*c**2*d**
8 + 4950*a**8*b**4*c**3*d**7 + 13860*a**7*b**5*c**4*d**6 + 19404*a**6*b**6*
c**5*d**5 + 13860*a**5*b**7*c**6*d**4 + 4950*a**4*b**8*c**7*d**3 + 825*a**3
*b**9*c**8*d**2 + 55*a**2*b**10*c**9*d + a*b**11*c**10) + x**11*(a**12*d**1
0/11 + 120*a**11*b*c*d**9/11 + 270*a**10*b**2*c**2*d**8 + 2400*a**9*b**3*c*
*3*d**7 + 9450*a**8*b**4*c**4*d**6 + 18144*a**7*b**5*c**5*d**5 + 17640*a**6
*b**6*c**6*d**4 + 8640*a**5*b**7*c**7*d**3 + 2025*a**4*b**8*c**8*d**2 + 200
*a**3*b**9*c**9*d + 6*a**2*b**10*c**10) + x**10*(a**12*c*d**9 + 54*a**11*b*
c**2*d**8 + 792*a**10*b**2*c**3*d**7 + 4620*a**9*b**3*c**4*d**6 + 12474*a**
8*b**4*c**5*d**5 + 16632*a**7*b**5*c**6*d**4 + 11088*a**6*b**6*c**7*d**3 +
3564*a**5*b**7*c**8*d**2 + 495*a**4*b**8*c**9*d + 22*a**3*b**9*c**10) + x**
9*(5*a**12*c**2*d**8 + 160*a**11*b*c**3*d**7 + 1540*a**10*b**2*c**4*d**6 +
6160*a**9*b**3*c**5*d**5 + 11550*a**8*b**4*c**6*d**4 + 10560*a**7*b**5*c**7
*d**3 + 4620*a**6*b**6*c**8*d**2 + 880*a**5*b**7*c**9*d + 55*a**4*b**8*c**1
0) + x**8*(15*a**12*c**3*d**7 + 315*a**11*b*c**4*d**6 + 2079*a**10*b**2*c**
5*d**5 + 5775*a**9*b**3*c**6*d**4 + 7425*a**8*b**4*c**7*d**3 + 4455*a**7*b*
*5*c**8*d**2 + 1155*a**6*b**6*c**9*d + 99*a**5*b**7*c**10) + x**7*(30*a**12
```

```
*c**4*d**6 + 432*a**11*b*c**5*d**5 + 1980*a**10*b**2*c**6*d**4 + 26400*a**9
*b**3*c**7*d**3/7 + 22275*a**8*b**4*c**8*d**2/7 + 7920*a**7*b**5*c**9*d/7 +
 132*a**6*b**6*c**10) + x**6*(42*a**12*c**5*d**5 + 420*a**11*b*c**6*d**4 +
1320*a**10*b**2*c**7*d**3 + 1650*a**9*b**3*c**8*d**2 + 825*a**8*b**4*c**9*d
 + 132*a**7*b**5*c**10) + x**5*(42*a**12*c**6*d**4 + 288*a**11*b*c**7*d**3
+ 594*a**10*b**2*c**8*d**2 + 440*a**9*b**3*c**9*d + 99*a**8*b**4*c**10) + x
**4*(30*a**12*c**7*d**3 + 135*a**11*b*c**8*d**2 + 165*a**10*b**2*c**9*d + 5
5*a**9*b**3*c**10) + x**3*(15*a**12*c**8*d**2 + 40*a**11*b*c**9*d + 22*a**1
0*b**2*c**10) + x**2*(5*a**12*c**9*d + 6*a**11*b*c**10)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. $2(259) = 518$.
time = 0.00, size = 2282, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x)
```

```
[Out] 1/23*b^12*d^10*x^23 + 5/11*b^12*c*d^9*x^22 + 6/11*a*b^11*d^10*x^22 + 15/7*b
^12*c^2*d^8*x^21 + 40/7*a*b^11*c*d^9*x^21 + 22/7*a^2*b^10*d^10*x^21 + 6*b^1
2*c^3*d^7*x^20 + 27*a*b^11*c^2*d^8*x^20 + 33*a^2*b^10*c*d^9*x^20 + 11*a^3*b
^9*d^10*x^20 + 210/19*b^12*c^4*d^6*x^19 + 1440/19*a*b^11*c^3*d^7*x^19 + 297
0/19*a^2*b^10*c^2*d^8*x^19 + 2200/19*a^3*b^9*c*d^9*x^19 + 495/19*a^4*b^8*d^
10*x^19 + 14*b^12*c^5*d^5*x^18 + 140*a*b^11*c^4*d^6*x^18 + 440*a^2*b^10*c^3
*d^7*x^18 + 550*a^3*b^9*c^2*d^8*x^18 + 275*a^4*b^8*c*d^9*x^18 + 44*a^5*b^7*
d^10*x^18 + 210/17*b^12*c^6*d^4*x^17 + 3024/17*a*b^11*c^5*d^5*x^17 + 13860/
17*a^2*b^10*c^4*d^6*x^17 + 26400/17*a^3*b^9*c^3*d^7*x^17 + 22275/17*a^4*b^8
*c^2*d^8*x^17 + 7920/17*a^5*b^7*c*d^9*x^17 + 924/17*a^6*b^6*d^10*x^17 + 15/
2*b^12*c^7*d^3*x^16 + 315/2*a*b^11*c^6*d^4*x^16 + 2079/2*a^2*b^10*c^5*d^5*x
^16 + 5775/2*a^3*b^9*c^4*d^6*x^16 + 7425/2*a^4*b^8*c^3*d^7*x^16 + 4455/2*a^
5*b^7*c^2*d^8*x^16 + 1155/2*a^6*b^6*c*d^9*x^16 + 99/2*a^7*b^5*d^10*x^16 + 3
*b^12*c^8*d^2*x^15 + 96*a*b^11*c^7*d^3*x^15 + 924*a^2*b^10*c^6*d^4*x^15 + 3
696*a^3*b^9*c^5*d^5*x^15 + 6930*a^4*b^8*c^4*d^6*x^15 + 6336*a^5*b^7*c^3*d^7
*x^15 + 2772*a^6*b^6*c^2*d^8*x^15 + 528*a^7*b^5*c*d^9*x^15 + 33*a^8*b^4*d^1
0*x^15 + 5/7*b^12*c^9*d*x^14 + 270/7*a*b^11*c^8*d^2*x^14 + 3960/7*a^2*b^10*
c^7*d^3*x^14 + 3300*a^3*b^9*c^6*d^4*x^14 + 8910*a^4*b^8*c^5*d^5*x^14 + 1188
0*a^5*b^7*c^4*d^6*x^14 + 7920*a^6*b^6*c^3*d^7*x^14 + 17820/7*a^7*b^5*c^2*d^
8*x^14 + 2475/7*a^8*b^4*c*d^9*x^14 + 110/7*a^9*b^3*d^10*x^14 + 1/13*b^12*c^
10*x^13 + 120/13*a*b^11*c^9*d*x^13 + 2970/13*a^2*b^10*c^8*d^2*x^13 + 26400/
13*a^3*b^9*c^7*d^3*x^13 + 103950/13*a^4*b^8*c^6*d^4*x^13 + 199584/13*a^5*b^
7*c^5*d^5*x^13 + 194040/13*a^6*b^6*c^4*d^6*x^13 + 95040/13*a^7*b^5*c^3*d^7*
x^13 + 22275/13*a^8*b^4*c^2*d^8*x^13 + 2200/13*a^9*b^3*c*d^9*x^13 + 66/13*a
^10*b^2*d^10*x^13 + a*b^11*c^10*x^12 + 55*a^2*b^10*c^9*d*x^12 + 825*a^3*b^9
*c^8*d^2*x^12 + 4950*a^4*b^8*c^7*d^3*x^12 + 13860*a^5*b^7*c^6*d^4*x^12 + 19
404*a^6*b^6*c^5*d^5*x^12 + 13860*a^7*b^5*c^4*d^6*x^12 + 4950*a^8*b^4*c^3*d^
```

```
7*x^12 + 825*a^9*b^3*c^2*d^8*x^12 + 55*a^10*b^2*c*d^9*x^12 + a^11*b*d^10*x^
12 + 6*a^2*b^10*c^10*x^11 + 200*a^3*b^9*c^9*d*x^11 + 2025*a^4*b^8*c^8*d^2*x
^11 + 8640*a^5*b^7*c^7*d^3*x^11 + 17640*a^6*b^6*c^6*d^4*x^11 + 18144*a^7*b^
5*c^5*d^5*x^11 + 9450*a^8*b^4*c^4*d^6*x^11 + 2400*a^9*b^3*c^3*d^7*x^11 + 27
0*a^10*b^2*c^2*d^8*x^11 + 120/11*a^11*b*c*d^9*x^11 + 1/11*a^12*d^10*x^11 +
22*a^3*b^9*c^10*x^10 + 495*a^4*b^8*c^9*d*x^10 + 3564*a^5*b^7*c^8*d^2*x^10 +
 11088*a^6*b^6*c^7*d^3*x^10 + 16632*a^7*b^5*c^6*d^4*x^10 + 12474*a^8*b^4*c^
5*d^5*x^10 + 4620*a^9*b^3*c^4*d^6*x^10 + 792*a^10*b^2*c^3*d^7*x^10 + 54*a^1
1*b*c^2*d^8*x^10 + a^12*c*d^9*x^10 + 55*a^4*b^8*c^10*x^9 + 880*a^5*b^7*c^9*
d*x^9 + 4620*a^6*b^6*c^8*d^2*x^9 + 10560*a^7*b^5*c^7*d^3*x^9 + 11550*a^8*b^
4*c^6*d^4*x^9 + 6160*a^9*b^3*c^5*d^5*x^9 + 1540*a^10*b^2*c^4*d^6*x^9 + 160*
a^11*b*c^3*d^7*x^9 + 5*a^12*c^2*d^8*x^9 + 99*a^5*b^7*c^10*x^8 + 1155*a^6*b^
6*c^9*d*x^8 + 4455*a^7*b^5*c^8*d^2*x^8 + 7425*a^8*b^4*c^7*d^3*x^8 + 5775*a^
9*b^3*c^6*d^4*x^8 + 2079*a^10*b^2*c^5*d^5*x^8 + 315*a^11*b*c^4*d^6*x^8 + 15
*a^12*c^3*d^7*x^8 + 132*a^6*b^6*c^10*x^7 + 7920/7*a^7*b^5*c^9*d*x^7 + 22275
/7*a^8*b^4*c^8*d^2*x^7 + 26400/7*a^9*b^3*c^7*d^3*x^7 + 1980*a^10*b^2*c^6*d^
4*x^7 + 432*a^11*b*c^5*d^5*x^7 + 30*a^12*c^4*d^6*x^7 + 132*a^7*b^5*c^10*x^6
 + 825*a^8*b^4*c^9*d*x^6 + 1650*a^9*b^3*c^8*d^2*x^6 + 1320*a^10*b^2*c^7*d^3
*x^6 + 420*a^11*b*c^6*d^4*x^6 + 42*a^12*c^5*d^5*x^6 + 99*a^8*b^4*c^10*x^5 +
 440*a^9*b^3*c^9*d*x^5 + 594*a^10*b^2*c^8*d^2*x^5 + 288*a^11*b*c^7*d^3*x^5
+ 42*a^12*c^6*d^4*x^5 + 55*a^9*b^3*c^10*x^4 + 165*a^10*b^2*c^9*d*x^4 + 135*
a^11*b*c^8*d^2*x^4 + 30*a^12*c^7*d^3*x^4 + 22*a^10*b^2*c^10*x^3 + 40*a^11*b
*c^9*d*x^3 + 15*a^12*c^8*d^2*x^3 + 6*a^11*b*c^10*x^2 + 5*a^12*c^9*d*x^2 + a
^12*c^10*x
```

Mupad [B]
time = 0.98, size = 1847, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12*(c + d*x)^10,x)
```

```
[Out] x^12*(a*b^11*c^10 + a^11*b*d^10 + 55*a^2*b^10*c^9*d + 55*a^10*b^2*c*d^9 + 8
25*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7*c^6*d^4 + 19404*a
^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3*d^7 + 825*a^9*b^3
*c^2*d^8) + x^7*(132*a^6*b^6*c^10 + 30*a^12*c^4*d^6 + (7920*a^7*b^5*c^9*d)/
7 + 432*a^11*b*c^5*d^5 + (22275*a^8*b^4*c^8*d^2)/7 + (26400*a^9*b^3*c^7*d^3
)/7 + 1980*a^10*b^2*c^6*d^4) + x^17*((924*a^6*b^6*d^10)/17 + (210*b^12*c^6*
d^4)/17 + (3024*a*b^11*c^5*d^5)/17 + (7920*a^5*b^7*c*d^9)/17 + (13860*a^2*b
^10*c^4*d^6)/17 + (26400*a^3*b^9*c^3*d^7)/17 + (22275*a^4*b^8*c^2*d^8)/17)
+ x^5*(99*a^8*b^4*c^10 + 42*a^12*c^6*d^4 + 440*a^9*b^3*c^9*d + 288*a^11*b*c
^7*d^3 + 594*a^10*b^2*c^8*d^2) + x^19*((495*a^4*b^8*d^10)/19 + (210*b^12*c^
4*d^6)/19 + (1440*a*b^11*c^3*d^7)/19 + (2200*a^3*b^9*c*d^9)/19 + (2970*a^2*
b^10*c^2*d^8)/19) + x^8*(99*a^5*b^7*c^10 + 15*a^12*c^3*d^7 + 1155*a^6*b^6*c
^9*d + 315*a^11*b*c^4*d^6 + 4455*a^7*b^5*c^8*d^2 + 7425*a^8*b^4*c^7*d^3 + 5
```

775*a^9*b^3*c^6*d^4 + 2079*a^10*b^2*c^5*d^5) + x^16*((99*a^7*b^5*d^10)/2 + (15*b^12*c^7*d^3)/2 + (315*a*b^11*c^6*d^4)/2 + (1155*a^6*b^6*c*d^9)/2 + (2079*a^2*b^10*c^5*d^5)/2 + (5775*a^3*b^9*c^4*d^6)/2 + (7425*a^4*b^8*c^3*d^7)/2 + (4455*a^5*b^7*c^2*d^8)/2) + x^11*((a^12*d^10)/11 + 6*a^2*b^10*c^10 + 200*a^3*b^9*c^9*d + 2025*a^4*b^8*c^8*d^2 + 8640*a^5*b^7*c^7*d^3 + 17640*a^6*b^6*c^6*d^4 + 18144*a^7*b^5*c^5*d^5 + 9450*a^8*b^4*c^4*d^6 + 2400*a^9*b^3*c^3*d^7 + 270*a^10*b^2*c^2*d^8 + (120*a^11*b*c*d^9)/11) + x^13*((b^12*c^10)/13 + (66*a^10*b^2*d^10)/13 + (2200*a^9*b^3*c*d^9)/13 + (2970*a^2*b^10*c^8*d^2)/13 + (26400*a^3*b^9*c^7*d^3)/13 + (103950*a^4*b^8*c^6*d^4)/13 + (199584*a^5*b^7*c^5*d^5)/13 + (194040*a^6*b^6*c^4*d^6)/13 + (95040*a^7*b^5*c^3*d^7)/13 + (22275*a^8*b^4*c^2*d^8)/13 + (120*a*b^11*c^9*d)/13) + x^6*(132*a^7*b^5*c^10 + 42*a^12*c^5*d^5 + 825*a^8*b^4*c^9*d + 420*a^11*b*c^6*d^4 + 1650*a^9*b^3*c^8*d^2 + 1320*a^10*b^2*c^7*d^3) + x^18*(44*a^5*b^7*d^10 + 14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^6 + 275*a^4*b^8*c*d^9 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8) + x^9*(55*a^4*b^8*c^10 + 5*a^12*c^2*d^8 + 880*a^5*b^7*c^9*d + 160*a^11*b*c^3*d^7 + 4620*a^6*b^6*c^8*d^2 + 10560*a^7*b^5*c^7*d^3 + 11550*a^8*b^4*c^6*d^4 + 6160*a^9*b^3*c^5*d^5 + 1540*a^10*b^2*c^4*d^6) + x^15*(33*a^8*b^4*d^10 + 3*b^12*c^8*d^2 + 96*a*b^11*c^7*d^3 + 528*a^7*b^5*c*d^9 + 924*a^2*b^10*c^6*d^4 + 3696*a^3*b^9*c^5*d^5 + 6930*a^4*b^8*c^4*d^6 + 6336*a^5*b^7*c^3*d^7 + 2772*a^6*b^6*c^2*d^8) + x^10*(a^12*c*d^9 + 22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 54*a^11*b*c^2*d^8 + 3564*a^5*b^7*c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7) + x^14*((5*b^12*c^9*d)/7 + (110*a^9*b^3*d^10)/7 + (270*a*b^11*c^8*d^2)/7 + (2475*a^8*b^4*c*d^9)/7 + (3960*a^2*b^10*c^7*d^3)/7 + 3300*a^3*b^9*c^6*d^4 + 8910*a^4*b^8*c^5*d^5 + 11880*a^5*b^7*c^4*d^6 + 7920*a^6*b^6*c^3*d^7 + (17820*a^7*b^5*c^2*d^8)/7) + a^12*c^10*x + (b^12*d^10*x^23)/23 + 5*a^9*c^7*x^4*(6*a^3*d^3 + 11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c*d^2) + b^9*d^7*x^20*(11*a^3*d^3 + 6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2*b*c*d^2) + a^11*c^9*x^2*(5*a*d + 6*b*c) + (b^11*d^9*x^22*(6*a*d + 5*b*c))/11 + a^10*c^8*x^3*(15*a^2*d^2 + 22*b^2*c^2 + 40*a*b*c*d) + (b^10*d^8*x^21*(22*a^2*d^2 + 15*b^2*c^2 + 40*a*b*c*d))/7

3.1300 $\int (a+bx)^{11}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\frac{8d^3(bc-ad)^7(a+bx)^{15}}{b^{11}}+\frac{105d^4}{}$$

```
[Out] 1/12*(-a*d+b*c)^10*(b*x+a)^12/b^11+10/13*d*(-a*d+b*c)^9*(b*x+a)^13/b^11+45/
14*d^2*(-a*d+b*c)^8*(b*x+a)^14/b^11+8*d^3*(-a*d+b*c)^7*(b*x+a)^15/b^11+105/
8*d^4*(-a*d+b*c)^6*(b*x+a)^16/b^11+252/17*d^5*(-a*d+b*c)^5*(b*x+a)^17/b^11+
35/3*d^6*(-a*d+b*c)^4*(b*x+a)^18/b^11+120/19*d^7*(-a*d+b*c)^3*(b*x+a)^19/b^
11+9/4*d^8*(-a*d+b*c)^2*(b*x+a)^20/b^11+10/21*d^9*(-a*d+b*c)*(b*x+a)^21/b^1
1+1/22*d^10*(b*x+a)^22/b^11
```

Rubi [A]
time = 0.86, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(a+bx)^{17}(bc-ad)^5}{17b^{11}}+\frac{105d^4(a+bx)^{16}(bc-ad)^6}{8b^{11}}+\frac{8d^3(a+bx)^{15}(bc-ad)^7}{b^{11}}+\frac{45d^2(a+bx)^{14}(bc-ad)^8}{14b^{11}}+\frac{10d(a+bx)^{13}(bc-ad)^9}{13b^{11}}+\frac{(a+bx)^{12}(bc-ad)^{10}}{12b^{11}}+\frac{d^{10}(a+bx)^{22}}{22b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^12)/(12*b^11) + (10*d*(b*c - a*d)^9*(a + b*x)^13)
/(13*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^14)/(14*b^11) + (8*d^3*(b*c -
a*d)^7*(a + b*x)^15)/b^11 + (105*d^4*(b*c - a*d)^6*(a + b*x)^16)/(8*b^11) +
 (252*d^5*(b*c - a*d)^5*(a + b*x)^17)/(17*b^11) + (35*d^6*(b*c - a*d)^4*(a
+ b*x)^18)/(3*b^11) + (120*d^7*(b*c - a*d)^3*(a + b*x)^19)/(19*b^11) + (9*d
^8*(b*c - a*d)^2*(a + b*x)^20)/(4*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^21)
/(21*b^11) + (d^10*(a + b*x)^22)/(22*b^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{11}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{11}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{12}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)}{b^{10}}\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1702 vs. $2(279) = 558$.
time = 0.12, size = 1702, normalized size = 6.10

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] a^11*c^10*x + (a^10*c^9*(11*b*c + 10*a*d)*x^2)/2 + (5*a^9*c^8*(11*b^2*c^2 +
 22*a*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^8*c^7*(33*b^3*c^3 + 110*a*b^2*c^2*d
+ 99*a^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 3*a^7*c^6*(22*b^4*c^4 + 110*a*b^3*c
^3*d + 165*a^2*b^2*c^2*d^2 + 88*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + (a^6*c^5*(1
54*b^5*c^5 + 1100*a*b^4*c^4*d + 2475*a^2*b^3*c^3*d^2 + 2200*a^3*b^2*c^2*d^3
 + 770*a^4*b*c*d^4 + 84*a^5*d^5)*x^6)/2 + (6*a^5*c^4*(77*b^6*c^6 + 770*a*b^
5*c^5*d + 2475*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 1925*a^4*b^2*c^2*d^
4 + 462*a^5*b*c*d^5 + 35*a^6*d^6)*x^7)/7 + (15*a^4*c^3*(11*b^7*c^7 + 154*a*
b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1320*a^3*b^4*c^4*d^3 + 1155*a^4*b^3*c^3*d
^4 + 462*a^5*b^2*c^2*d^5 + 77*a^6*b*c*d^6 + 4*a^7*d^7)*x^8)/4 + (5*a^3*c^2*
(11*b^8*c^8 + 220*a*b^7*c^7*d + 1386*a^2*b^6*c^6*d^2 + 3696*a^3*b^5*c^5*d^3
 + 4620*a^4*b^4*c^4*d^4 + 2772*a^5*b^3*c^3*d^5 + 770*a^6*b^2*c^2*d^6 + 88*a
^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 + (a^2*c*(11*b^9*c^9 + 330*a*b^8*c^8*d + 297
0*a^2*b^7*c^7*d^2 + 11088*a^3*b^6*c^6*d^3 + 19404*a^4*b^5*c^5*d^4 + 16632*a
^5*b^4*c^4*d^5 + 6930*a^6*b^3*c^3*d^6 + 1320*a^7*b^2*c^2*d^7 + 99*a^8*b*c*d
^8 + 2*a^9*d^9)*x^10)/2 + (a*(11*b^10*c^10 + 550*a*b^9*c^9*d + 7425*a^2*b^8
*c^8*d^2 + 39600*a^3*b^7*c^7*d^3 + 97020*a^4*b^6*c^6*d^4 + 116424*a^5*b^5*c
^5*d^5 + 69300*a^6*b^4*c^4*d^6 + 19800*a^7*b^3*c^3*d^7 + 2475*a^8*b^2*c^2*d
^8 + 110*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (b*(b^10*c^10 + 110*a*b^9*c^9*
d + 2475*a^2*b^8*c^8*d^2 + 19800*a^3*b^7*c^7*d^3 + 69300*a^4*b^6*c^6*d^4 +
116424*a^5*b^5*c^5*d^5 + 97020*a^6*b^4*c^4*d^6 + 39600*a^7*b^3*c^3*d^7 + 74
25*a^8*b^2*c^2*d^8 + 550*a^9*b*c*d^9 + 11*a^10*d^10)*x^12)/12 + (5*b^2*d*(2
*b^9*c^9 + 99*a*b^8*c^8*d + 1320*a^2*b^7*c^7*d^2 + 6930*a^3*b^6*c^6*d^3 + 1
6632*a^4*b^5*c^5*d^4 + 19404*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 2970
*a^7*b^2*c^2*d^7 + 330*a^8*b*c*d^8 + 11*a^9*d^9)*x^13)/13 + (15*b^3*d^2*(3*
b^8*c^8 + 88*a*b^7*c^7*d + 770*a^2*b^6*c^6*d^2 + 2772*a^3*b^5*c^5*d^3 + 462
0*a^4*b^4*c^4*d^4 + 3696*a^5*b^3*c^3*d^5 + 1386*a^6*b^2*c^2*d^6 + 220*a^7*b
*c*d^7 + 11*a^8*d^8)*x^14)/14 + 2*b^4*d^3*(4*b^7*c^7 + 77*a*b^6*c^6*d + 462
*a^2*b^5*c^5*d^2 + 1155*a^3*b^4*c^4*d^3 + 1320*a^4*b^3*c^3*d^4 + 693*a^5*b^
2*c^2*d^5 + 154*a^6*b*c*d^6 + 11*a^7*d^7)*x^15 + (3*b^5*d^4*(35*b^6*c^6 + 4
62*a*b^5*c^5*d + 1925*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 2475*a^4*b^2
*c^2*d^4 + 770*a^5*b*c*d^5 + 77*a^6*d^6)*x^16)/8 + (3*b^6*d^5*(84*b^5*c^5 +
 770*a*b^4*c^4*d + 2200*a^2*b^3*c^3*d^2 + 2475*a^3*b^2*c^2*d^3 + 1100*a^4*b
*c*d^4 + 154*a^5*d^5)*x^17)/17 + (5*b^7*d^6*(14*b^4*c^4 + 88*a*b^3*c^3*d +
165*a^2*b^2*c^2*d^2 + 110*a^3*b*c*d^3 + 22*a^4*d^4)*x^18)/6 + (5*b^8*d^7*(2
4*b^3*c^3 + 99*a*b^2*c^2*d + 110*a^2*b*c*d^2 + 33*a^3*d^3)*x^19)/19 + (b^9*
```

```
d^8*(9*b^2*c^2 + 22*a*b*c*d + 11*a^2*d^2)*x^20)/4 + (b^10*d^9*(10*b*c + 11*
a*d)*x^21)/21 + (b^11*d^10*x^22)/22
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 1652 vs. 2(279) = 558.
time = 15.62, size = 1650, normalized size = 5.91

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^11*(c + d*x)^10,x]')
```

```
[Out] x (7759752 a ^ 11 c ^ 10 + 3879876 a ^ 10 c ^ 9 x (10 a d + 11 b c) + a ^ 9
 c ^ 8 x ^ 2 (116396280 a ^ 2 d ^ 2 + 284524240 a b c d + 142262120 b ^ 2 c
 ^ 2) + a ^ 8 c ^ 7 x ^ 3 (232792560 a ^ 3 d ^ 3 + 960269310 a ^ 2 b c d ^
2 + 1066965900 a b ^ 2 c ^ 2 d + 320089770 b ^ 3 c ^ 3) + a ^ 7 c ^ 6 x ^ 4
 (325909584 a ^ 4 d ^ 4 + 2048574528 a ^ 3 b c d ^ 3 + 3841077240 a ^ 2 b ^
 2 c ^ 2 d ^ 2 + 2560718160 a b ^ 3 c ^ 3 d + 512143632 b ^ 4 c ^ 4) + a ^
6 c ^ 5 x ^ 5 (325909584 a ^ 5 d ^ 5 + 2987504520 a ^ 4 b c d ^ 4 + 8535727
200 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 9602693100 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 42678636
00 a b ^ 4 c ^ 4 d + 597500904 b ^ 5 c ^ 5) + a ^ 5 c ^ 4 x ^ 6 (232792560
a ^ 6 d ^ 6 + 3072861792 a ^ 5 b c d ^ 5 + 12803590800 a ^ 4 b ^ 2 c ^ 2 d
^ 4 + 21949012800 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 16461759600 a ^ 2 b ^ 4 c ^ 4 d
 ^ 2 + 5121436320 a b ^ 5 c ^ 5 d + 512143632 b ^ 6 c ^ 6) + a ^ 4 c ^ 3 x
^ 7 (116396280 a ^ 7 d ^ 7 + 2240628390 a ^ 6 b c d ^ 6 + 13443770340 a ^ 5
 b ^ 2 c ^ 2 d ^ 5 + 33609425850 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 38410772400 a ^
3 b ^ 4 c ^ 4 d ^ 3 + 20165655510 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 4481256780 a b
^ 6 c ^ 6 d + 320089770 b ^ 7 c ^ 7) + a ^ 3 c ^ 2 x ^ 8 (38798760 a ^ 8 d
^ 8 + 1138096960 a ^ 7 b c d ^ 7 + 9958348400 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 358
50054240 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 59750090400 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 47
800072320 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 17925027120 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 2
845242400 a b ^ 7 c ^ 7 d + 142262120 b ^ 8 c ^ 8) + 3879876 a ^ 2 c x ^ 9
(2 a ^ 9 d ^ 9 + 99 a ^ 8 b c d ^ 8 + 1320 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 6930 a
 ^ 6 b ^ 3 c ^ 3 d ^ 6 + 16632 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 19404 a ^ 4 b ^ 5
c ^ 5 d ^ 4 + 11088 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 2970 a ^ 2 b ^ 7 c ^ 7 d ^ 2
+ 330 a b ^ 8 c ^ 8 d + 11 b ^ 9 c ^ 9) + 705432 a x ^ 10 (a ^ 10 d ^ 10 +
110 a ^ 9 b c d ^ 9 + 2475 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 19800 a ^ 7 b ^ 3 c ^
3 d ^ 7 + 69300 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 116424 a ^ 5 b ^ 5 c ^ 5 d ^ 5 +
97020 a ^ 4 b ^ 6 c ^ 6 d ^ 4 + 39600 a ^ 3 b ^ 7 c ^ 7 d ^ 3 + 7425 a ^ 2
b ^ 8 c ^ 8 d ^ 2 + 550 a b ^ 9 c ^ 9 d + 11 b ^ 10 c ^ 10) + 646646 b x ^
11 (11 a ^ 10 d ^ 10 + 550 a ^ 9 b c d ^ 9 + 7425 a ^ 8 b ^ 2 c ^ 2 d ^ 8 +
 39600 a ^ 7 b ^ 3 c ^ 3 d ^ 7 + 97020 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 116424 a ^
 5 b ^ 5 c ^ 5 d ^ 5 + 69300 a ^ 4 b ^ 6 c ^ 6 d ^ 4 + 19800 a ^ 3 b ^ 7 c
^ 7 d ^ 3 + 2475 a ^ 2 b ^ 8 c ^ 8 d ^ 2 + 110 a b ^ 9 c ^ 9 d + b ^ 10 c ^
 10) + 2984520 b ^ 2 d x ^ 12 (11 a ^ 9 d ^ 9 + 330 a ^ 8 b c d ^ 8 + 2970
a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 11088 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 19404 a ^ 5 b ^ 4
 c ^ 4 d ^ 5 + 16632 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 6930 a ^ 3 b ^ 6 c ^ 6 d ^ 3
```

```
 + 1320 a ^ 2 b ^ 7 c ^ 7 d ^ 2 + 99 a b ^ 8 c ^ 8 d + 2 b ^ 9 c ^ 9) + b ^
 3 d ^ 2 x ^ 13 (91454220 a ^ 8 d ^ 8 + 1829084400 a ^ 7 b c d ^ 7 + 115232
31720 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 30728617920 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 38410
772400 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 23046463440 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 6401
795400 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 731633760 a b ^ 7 c ^ 7 d + 24942060 b ^ 8
 c ^ 8) + b ^ 4 d ^ 3 x ^ 14 (170714544 a ^ 7 d ^ 7 + 2390003616 a ^ 6 b c
d ^ 6 + 10755016272 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 20485745280 a ^ 4 b ^ 3 c ^ 3
 d ^ 4 + 17925027120 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 7170010848 a ^ 2 b ^ 5 c ^ 5
 d ^ 2 + 1195001808 a b ^ 6 c ^ 6 d + 62078016 b ^ 7 c ^ 7) + b ^ 5 d ^ 4 x
 ^ 15 (224062839 a ^ 6 d ^ 6 + 2240628390 a ^ 5 b c d ^ 5 + 7202019825 a ^
4 b ^ 2 c ^ 2 d ^ 4 + 9602693100 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 5601570975 a ^ 2
 b ^ 4 c ^ 4 d ^ 2 + 1344377034 a b ^ 5 c ^ 5 d + 101846745 b ^ 6 c ^ 6) +
b ^ 6 d ^ 5 x ^ 16 (210882672 a ^ 5 d ^ 5 + 1506304800 a ^ 4 b c d ^ 4 + 33
89185800 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 3012609600 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 105
4413360 a b ^ 4 c ^ 4 d + 115026912 b ^ 5 c ^ 5) + b ^ 7 d ^ 6 x ^ 17 (1422
62120 a ^ 4 d ^ 4 + 711310600 a ^ 3 b c d ^ 3 + 1066965900 a ^ 2 b ^ 2 c ^
2 d ^ 2 + 569048480 a b ^ 3 c ^ 3 d + 90530440 b ^ 4 c ^ 4) + b ^ 8 d ^ 7 x
 ^ 18 (67387320 a ^ 3 d ^ 3 + 224624400 a ^ 2 b c d ^ 2 + 202161960 a b ^ 2
 c ^ 2 d + 49008960 b ^ 3 c ^ 3) + b ^ 9 d ^ 8 x ^ 19 (21339318 a ^ 2 d ^ 2
 + 42678636 a b c d + 17459442 b ^ 2 c ^ 2) + b ^ 10 d ^ 9 x ^ 20 (4064632
a d + 3695120 b c) + 352716 b ^ 11 d ^ 10 x ^ 21) / 7759752
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(259) = 518$.
time = 0.14, size = 1741, normalized size = 6.24

method	result	size
norman	Expression too large to display	1721
default	Expression too large to display	1741
gosper	Expression too large to display	2011
risch	Expression too large to display	2011

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*b^11*d^10*x^22+1/21*(11*a*b^10*d^10+10*b^11*c*d^9)*x^21+1/20*(55*a^2*b
^9*d^10+110*a*b^10*c*d^9+45*b^11*c^2*d^8)*x^20+1/19*(165*a^3*b^8*d^10+550*a
^2*b^9*c*d^9+495*a*b^10*c^2*d^8+120*b^11*c^3*d^7)*x^19+1/18*(330*a^4*b^7*d^
10+1650*a^3*b^8*c*d^9+2475*a^2*b^9*c^2*d^8+1320*a*b^10*c^3*d^7+210*b^11*c^4
*d^6)*x^18+1/17*(462*a^5*b^6*d^10+3300*a^4*b^7*c*d^9+7425*a^3*b^8*c^2*d^8+6
600*a^2*b^9*c^3*d^7+2310*a*b^10*c^4*d^6+252*b^11*c^5*d^5)*x^17+1/16*(462*a^
6*b^5*d^10+4620*a^5*b^6*c*d^9+14850*a^4*b^7*c^2*d^8+19800*a^3*b^8*c^3*d^7+1
1550*a^2*b^9*c^4*d^6+2772*a*b^10*c^5*d^5+210*b^11*c^6*d^4)*x^16+1/15*(330*a
^7*b^4*d^10+4620*a^6*b^5*c*d^9+20790*a^5*b^6*c^2*d^8+39600*a^4*b^7*c^3*d^7+
34650*a^3*b^8*c^4*d^6+13860*a^2*b^9*c^5*d^5+2310*a*b^10*c^6*d^4+120*b^11*c^
```

```
7*d^3)*x^15+1/14*(165*a^8*b^3*d^10+3300*a^7*b^4*c*d^9+20790*a^6*b^5*c^2*d^8
+55440*a^5*b^6*c^3*d^7+69300*a^4*b^7*c^4*d^6+41580*a^3*b^8*c^5*d^5+11550*a^
2*b^9*c^6*d^4+1320*a*b^10*c^7*d^3+45*b^11*c^8*d^2)*x^14+1/13*(55*a^9*b^2*d^
10+1650*a^8*b^3*c*d^9+14850*a^7*b^4*c^2*d^8+55440*a^6*b^5*c^3*d^7+97020*a^5
*b^6*c^4*d^6+83160*a^4*b^7*c^5*d^5+34650*a^3*b^8*c^6*d^4+6600*a^2*b^9*c^7*d
^3+495*a*b^10*c^8*d^2+10*b^11*c^9*d)*x^13+1/12*(11*a^10*b*d^10+550*a^9*b^2*
c*d^9+7425*a^8*b^3*c^2*d^8+39600*a^7*b^4*c^3*d^7+97020*a^6*b^5*c^4*d^6+1164
24*a^5*b^6*c^5*d^5+69300*a^4*b^7*c^6*d^4+19800*a^3*b^8*c^7*d^3+2475*a^2*b^9
*c^8*d^2+110*a*b^10*c^9*d+b^11*c^10)*x^12+1/11*(a^11*d^10+110*a^10*b*c*d^9+
2475*a^9*b^2*c^2*d^8+19800*a^8*b^3*c^3*d^7+69300*a^7*b^4*c^4*d^6+116424*a^6
*b^5*c^5*d^5+97020*a^5*b^6*c^6*d^4+39600*a^4*b^7*c^7*d^3+7425*a^3*b^8*c^8*d
^2+550*a^2*b^9*c^9*d+11*a*b^10*c^10)*x^11+1/10*(10*a^11*c*d^9+495*a^10*b*c^
2*d^8+6600*a^9*b^2*c^3*d^7+34650*a^8*b^3*c^4*d^6+83160*a^7*b^4*c^5*d^5+9702
0*a^6*b^5*c^6*d^4+55440*a^5*b^6*c^7*d^3+14850*a^4*b^7*c^8*d^2+1650*a^3*b^8*
c^9*d+55*a^2*b^9*c^10)*x^10+1/9*(45*a^11*c^2*d^8+1320*a^10*b*c^3*d^7+11550*
a^9*b^2*c^4*d^6+41580*a^8*b^3*c^5*d^5+69300*a^7*b^4*c^6*d^4+55440*a^6*b^5*c
^7*d^3+20790*a^5*b^6*c^8*d^2+3300*a^4*b^7*c^9*d+165*a^3*b^8*c^10)*x^9+1/8*(
120*a^11*c^3*d^7+2310*a^10*b*c^4*d^6+13860*a^9*b^2*c^5*d^5+34650*a^8*b^3*c^
6*d^4+39600*a^7*b^4*c^7*d^3+20790*a^6*b^5*c^8*d^2+4620*a^5*b^6*c^9*d+330*a^
4*b^7*c^10)*x^8+1/7*(210*a^11*c^4*d^6+2772*a^10*b*c^5*d^5+11550*a^9*b^2*c^6
*d^4+19800*a^8*b^3*c^7*d^3+14850*a^7*b^4*c^8*d^2+4620*a^6*b^5*c^9*d+462*a^5
*b^6*c^10)*x^7+1/6*(252*a^11*c^5*d^5+2310*a^10*b*c^6*d^4+6600*a^9*b^2*c^7*d
^3+7425*a^8*b^3*c^8*d^2+3300*a^7*b^4*c^9*d+462*a^6*b^5*c^10)*x^6+1/5*(210*a
^11*c^6*d^4+1320*a^10*b*c^7*d^3+2475*a^9*b^2*c^8*d^2+1650*a^8*b^3*c^9*d+330
*a^7*b^4*c^10)*x^5+1/4*(120*a^11*c^7*d^3+495*a^10*b*c^8*d^2+550*a^9*b^2*c^9
*d+165*a^8*b^3*c^10)*x^4+1/3*(45*a^11*c^8*d^2+110*a^10*b*c^9*d+55*a^9*b^2*c
^10)*x^3+1/2*(10*a^11*c^9*d+11*a^10*b*c^10)*x^2+a^11*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(259) = 518$.
time = 0.29, size = 1740, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
```

```
462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(259) = 518$.

time = 0.31, size = 1740, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
```

```
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
 462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1965 vs. $2(258) = 516$.

time = 0.16, size = 1965, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11*(d*x+c)**10,x)
```

```
[Out] a**11*c**10*x + b**11*d**10*x**22/22 + x**21*(11*a*b**10*d**10/21 + 10*b**1
1*c*d**9/21) + x**20*(11*a**2*b**9*d**10/4 + 11*a*b**10*c*d**9/2 + 9*b**11*
c**2*d**8/4) + x**19*(165*a**3*b**8*d**10/19 + 550*a**2*b**9*c*d**9/19 + 49
```

```
5*a*b**10*c**2*d**8/19 + 120*b**11*c**3*d**7/19) + x**18*(55*a**4*b**7*d**1
0/3 + 275*a**3*b**8*c*d**9/3 + 275*a**2*b**9*c**2*d**8/2 + 220*a*b**10*c**3
*d**7/3 + 35*b**11*c**4*d**6/3) + x**17*(462*a**5*b**6*d**10/17 + 3300*a**4
*b**7*c*d**9/17 + 7425*a**3*b**8*c**2*d**8/17 + 6600*a**2*b**9*c**3*d**7/17
 + 2310*a*b**10*c**4*d**6/17 + 252*b**11*c**5*d**5/17) + x**16*(231*a**6*b*
*5*d**10/8 + 1155*a**5*b**6*c*d**9/4 + 7425*a**4*b**7*c**2*d**8/8 + 2475*a*
*3*b**8*c**3*d**7/2 + 5775*a**2*b**9*c**4*d**6/8 + 693*a*b**10*c**5*d**5/4
+ 105*b**11*c**6*d**4/8) + x**15*(22*a**7*b**4*d**10 + 308*a**6*b**5*c*d**9
 + 1386*a**5*b**6*c**2*d**8 + 2640*a**4*b**7*c**3*d**7 + 2310*a**3*b**8*c**
4*d**6 + 924*a**2*b**9*c**5*d**5 + 154*a*b**10*c**6*d**4 + 8*b**11*c**7*d**
3) + x**14*(165*a**8*b**3*d**10/14 + 1650*a**7*b**4*c*d**9/7 + 1485*a**6*b*
*5*c**2*d**8 + 3960*a**5*b**6*c**3*d**7 + 4950*a**4*b**7*c**4*d**6 + 2970*a
**3*b**8*c**5*d**5 + 825*a**2*b**9*c**6*d**4 + 660*a*b**10*c**7*d**3/7 + 45
*b**11*c**8*d**2/14) + x**13*(55*a**9*b**2*d**10/13 + 1650*a**8*b**3*c*d**9
/13 + 14850*a**7*b**4*c**2*d**8/13 + 55440*a**6*b**5*c**3*d**7/13 + 97020*a
**5*b**6*c**4*d**6/13 + 83160*a**4*b**7*c**5*d**5/13 + 34650*a**3*b**8*c**6
*d**4/13 + 6600*a**2*b**9*c**7*d**3/13 + 495*a*b**10*c**8*d**2/13 + 10*b**1
1*c**9*d/13) + x**12*(11*a**10*b*d**10/12 + 275*a**9*b**2*c*d**9/6 + 2475*a
**8*b**3*c**2*d**8/4 + 3300*a**7*b**4*c**3*d**7 + 8085*a**6*b**5*c**4*d**6
+ 9702*a**5*b**6*c**5*d**5 + 5775*a**4*b**7*c**6*d**4 + 1650*a**3*b**8*c**7
*d**3 + 825*a**2*b**9*c**8*d**2/4 + 55*a*b**10*c**9*d/6 + b**11*c**10/12) +
 x**11*(a**11*d**10/11 + 10*a**10*b*c*d**9 + 225*a**9*b**2*c**2*d**8 + 1800
*a**8*b**3*c**3*d**7 + 6300*a**7*b**4*c**4*d**6 + 10584*a**6*b**5*c**5*d**5
 + 8820*a**5*b**6*c**6*d**4 + 3600*a**4*b**7*c**7*d**3 + 675*a**3*b**8*c**8
*d**2 + 50*a**2*b**9*c**9*d + a*b**10*c**10) + x**10*(a**11*c*d**9 + 99*a**
10*b*c**2*d**8/2 + 660*a**9*b**2*c**3*d**7 + 3465*a**8*b**3*c**4*d**6 + 831
6*a**7*b**4*c**5*d**5 + 9702*a**6*b**5*c**6*d**4 + 5544*a**5*b**6*c**7*d**3
 + 1485*a**4*b**7*c**8*d**2 + 165*a**3*b**8*c**9*d + 11*a**2*b**9*c**10/2)
+ x**9*(5*a**11*c**2*d**8 + 440*a**10*b*c**3*d**7/3 + 3850*a**9*b**2*c**4*d
**6/3 + 4620*a**8*b**3*c**5*d**5 + 7700*a**7*b**4*c**6*d**4 + 6160*a**6*b**
5*c**7*d**3 + 2310*a**5*b**6*c**8*d**2 + 1100*a**4*b**7*c**9*d/3 + 55*a**3*
b**8*c**10/3) + x**8*(15*a**11*c**3*d**7 + 1155*a**10*b*c**4*d**6/4 + 3465*
a**9*b**2*c**5*d**5/2 + 17325*a**8*b**3*c**6*d**4/4 + 4950*a**7*b**4*c**7*d
**3 + 10395*a**6*b**5*c**8*d**2/4 + 1155*a**5*b**6*c**9*d/2 + 165*a**4*b**7
*c**10/4) + x**7*(30*a**11*c**4*d**6 + 396*a**10*b*c**5*d**5 + 1650*a**9*b*
*2*c**6*d**4 + 19800*a**8*b**3*c**7*d**3/7 + 14850*a**7*b**4*c**8*d**2/7 +
660*a**6*b**5*c**9*d + 66*a**5*b**6*c**10) + x**6*(42*a**11*c**5*d**5 + 385
*a**10*b*c**6*d**4 + 1100*a**9*b**2*c**7*d**3 + 2475*a**8*b**3*c**8*d**2/2
+ 550*a**7*b**4*c**9*d + 77*a**6*b**5*c**10) + x**5*(42*a**11*c**6*d**4 + 2
64*a**10*b*c**7*d**3 + 495*a**9*b**2*c**8*d**2 + 330*a**8*b**3*c**9*d + 66*
a**7*b**4*c**10) + x**4*(30*a**11*c**7*d**3 + 495*a**10*b*c**8*d**2/4 + 275
*a**9*b**2*c**9*d/2 + 165*a**8*b**3*c**10/4) + x**3*(15*a**11*c**8*d**2 + 1
10*a**10*b*c**9*d/3 + 55*a**9*b**2*c**10/3) + x**2*(5*a**11*c**9*d + 11*a**
10*b*c**10/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. $2(259) = 518$.
time = 0.00, size = 2150, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x)
```

```
[Out] 1/22*b^11*d^10*x^22 + 10/21*b^11*c*d^9*x^21 + 11/21*a*b^10*d^10*x^21 + 9/4*
b^11*c^2*d^8*x^20 + 11/2*a*b^10*c*d^9*x^20 + 11/4*a^2*b^9*d^10*x^20 + 120/1
9*b^11*c^3*d^7*x^19 + 495/19*a*b^10*c^2*d^8*x^19 + 550/19*a^2*b^9*c*d^9*x^1
9 + 165/19*a^3*b^8*d^10*x^19 + 35/3*b^11*c^4*d^6*x^18 + 220/3*a*b^10*c^3*d^
7*x^18 + 275/2*a^2*b^9*c^2*d^8*x^18 + 275/3*a^3*b^8*c*d^9*x^18 + 55/3*a^4*b
^7*d^10*x^18 + 252/17*b^11*c^5*d^5*x^17 + 2310/17*a*b^10*c^4*d^6*x^17 + 660
0/17*a^2*b^9*c^3*d^7*x^17 + 7425/17*a^3*b^8*c^2*d^8*x^17 + 3300/17*a^4*b^7*
c*d^9*x^17 + 462/17*a^5*b^6*d^10*x^17 + 105/8*b^11*c^6*d^4*x^16 + 693/4*a*b
^10*c^5*d^5*x^16 + 5775/8*a^2*b^9*c^4*d^6*x^16 + 2475/2*a^3*b^8*c^3*d^7*x^1
6 + 7425/8*a^4*b^7*c^2*d^8*x^16 + 1155/4*a^5*b^6*c*d^9*x^16 + 231/8*a^6*b^5
*d^10*x^16 + 8*b^11*c^7*d^3*x^15 + 154*a*b^10*c^6*d^4*x^15 + 924*a^2*b^9*c^
5*d^5*x^15 + 2310*a^3*b^8*c^4*d^6*x^15 + 2640*a^4*b^7*c^3*d^7*x^15 + 1386*a
^5*b^6*c^2*d^8*x^15 + 308*a^6*b^5*c*d^9*x^15 + 22*a^7*b^4*d^10*x^15 + 45/14
*b^11*c^8*d^2*x^14 + 660/7*a*b^10*c^7*d^3*x^14 + 825*a^2*b^9*c^6*d^4*x^14 +
 2970*a^3*b^8*c^5*d^5*x^14 + 4950*a^4*b^7*c^4*d^6*x^14 + 3960*a^5*b^6*c^3*d
^7*x^14 + 1485*a^6*b^5*c^2*d^8*x^14 + 1650/7*a^7*b^4*c*d^9*x^14 + 165/14*a^
8*b^3*d^10*x^14 + 10/13*b^11*c^9*d*x^13 + 495/13*a*b^10*c^8*d^2*x^13 + 6600
/13*a^2*b^9*c^7*d^3*x^13 + 34650/13*a^3*b^8*c^6*d^4*x^13 + 83160/13*a^4*b^7
*c^5*d^5*x^13 + 97020/13*a^5*b^6*c^4*d^6*x^13 + 55440/13*a^6*b^5*c^3*d^7*x^
13 + 14850/13*a^7*b^4*c^2*d^8*x^13 + 1650/13*a^8*b^3*c*d^9*x^13 + 55/13*a^9
*b^2*d^10*x^13 + 1/12*b^11*c^10*x^12 + 55/6*a*b^10*c^9*d*x^12 + 825/4*a^2*b
^9*c^8*d^2*x^12 + 1650*a^3*b^8*c^7*d^3*x^12 + 5775*a^4*b^7*c^6*d^4*x^12 + 9
702*a^5*b^6*c^5*d^5*x^12 + 8085*a^6*b^5*c^4*d^6*x^12 + 3300*a^7*b^4*c^3*d^7
*x^12 + 2475/4*a^8*b^3*c^2*d^8*x^12 + 275/6*a^9*b^2*c*d^9*x^12 + 11/12*a^10
*b*d^10*x^12 + a*b^10*c^10*x^11 + 50*a^2*b^9*c^9*d*x^11 + 675*a^3*b^8*c^8*d
^2*x^11 + 3600*a^4*b^7*c^7*d^3*x^11 + 8820*a^5*b^6*c^6*d^4*x^11 + 10584*a^6
*b^5*c^5*d^5*x^11 + 6300*a^7*b^4*c^4*d^6*x^11 + 1800*a^8*b^3*c^3*d^7*x^11 +
 225*a^9*b^2*c^2*d^8*x^11 + 10*a^10*b*c*d^9*x^11 + 1/11*a^11*d^10*x^11 + 11
/2*a^2*b^9*c^10*x^10 + 165*a^3*b^8*c^9*d*x^10 + 1485*a^4*b^7*c^8*d^2*x^10 +
 5544*a^5*b^6*c^7*d^3*x^10 + 9702*a^6*b^5*c^6*d^4*x^10 + 8316*a^7*b^4*c^5*d
^5*x^10 + 3465*a^8*b^3*c^4*d^6*x^10 + 660*a^9*b^2*c^3*d^7*x^10 + 99/2*a^10*
b*c^2*d^8*x^10 + a^11*c*d^9*x^10 + 55/3*a^3*b^8*c^10*x^9 + 1100/3*a^4*b^7*c
^9*d*x^9 + 2310*a^5*b^6*c^8*d^2*x^9 + 6160*a^6*b^5*c^7*d^3*x^9 + 7700*a^7*b
^4*c^6*d^4*x^9 + 4620*a^8*b^3*c^5*d^5*x^9 + 3850/3*a^9*b^2*c^4*d^6*x^9 + 44
0/3*a^10*b*c^3*d^7*x^9 + 5*a^11*c^2*d^8*x^9 + 165/4*a^4*b^7*c^10*x^8 + 1155
/2*a^5*b^6*c^9*d*x^8 + 10395/4*a^6*b^5*c^8*d^2*x^8 + 4950*a^7*b^4*c^7*d^3*x
^8 + 17325/4*a^8*b^3*c^6*d^4*x^8 + 3465/2*a^9*b^2*c^5*d^5*x^8 + 1155/4*a^10
```

```
*b*c^4*d^6*x^8 + 15*a^11*c^3*d^7*x^8 + 66*a^5*b^6*c^10*x^7 + 660*a^6*b^5*c^
9*d*x^7 + 14850/7*a^7*b^4*c^8*d^2*x^7 + 19800/7*a^8*b^3*c^7*d^3*x^7 + 1650*
a^9*b^2*c^6*d^4*x^7 + 396*a^10*b*c^5*d^5*x^7 + 30*a^11*c^4*d^6*x^7 + 77*a^6
*b^5*c^10*x^6 + 550*a^7*b^4*c^9*d*x^6 + 2475/2*a^8*b^3*c^8*d^2*x^6 + 1100*a
^9*b^2*c^7*d^3*x^6 + 385*a^10*b*c^6*d^4*x^6 + 42*a^11*c^5*d^5*x^6 + 66*a^7*
b^4*c^10*x^5 + 330*a^8*b^3*c^9*d*x^5 + 495*a^9*b^2*c^8*d^2*x^5 + 264*a^10*b
*c^7*d^3*x^5 + 42*a^11*c^6*d^4*x^5 + 165/4*a^8*b^3*c^10*x^4 + 275/2*a^9*b^2
*c^9*d*x^4 + 495/4*a^10*b*c^8*d^2*x^4 + 30*a^11*c^7*d^3*x^4 + 55/3*a^9*b^2*
c^10*x^3 + 110/3*a^10*b*c^9*d*x^3 + 15*a^11*c^8*d^2*x^3 + 11/2*a^10*b*c^10*
x^2 + 5*a^11*c^9*d*x^2 + a^11*c^10*x
```

Mupad [B]

time = 1.03, size = 1702, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11*(c + d*x)^10,x)
```

```
[Out] x^7*(66*a^5*b^6*c^10 + 30*a^11*c^4*d^6 + 660*a^6*b^5*c^9*d + 396*a^10*b*c^5
*d^5 + (14850*a^7*b^4*c^8*d^2)/7 + (19800*a^8*b^3*c^7*d^3)/7 + 1650*a^9*b^2
*c^6*d^4) + x^16*((231*a^6*b^5*d^10)/8 + (105*b^11*c^6*d^4)/8 + (693*a*b^10
*c^5*d^5)/4 + (1155*a^5*b^6*c*d^9)/4 + (5775*a^2*b^9*c^4*d^6)/8 + (2475*a^3
*b^8*c^3*d^7)/2 + (7425*a^4*b^7*c^2*d^8)/8) + x^11*((a^11*d^10)/11 + a*b^10
*c^10 + 50*a^2*b^9*c^9*d + 675*a^3*b^8*c^8*d^2 + 3600*a^4*b^7*c^7*d^3 + 882
0*a^5*b^6*c^6*d^4 + 10584*a^6*b^5*c^5*d^5 + 6300*a^7*b^4*c^4*d^6 + 1800*a^8
*b^3*c^3*d^7 + 225*a^9*b^2*c^2*d^8 + 10*a^10*b*c*d^9) + x^12*((b^11*c^10)/1
2 + (11*a^10*b*d^10)/12 + (275*a^9*b^2*c*d^9)/6 + (825*a^2*b^9*c^8*d^2)/4 +
 1650*a^3*b^8*c^7*d^3 + 5775*a^4*b^7*c^6*d^4 + 9702*a^5*b^6*c^5*d^5 + 8085*
a^6*b^5*c^4*d^6 + 3300*a^7*b^4*c^3*d^7 + (2475*a^8*b^3*c^2*d^8)/4 + (55*a*b
^10*c^9*d)/6) + x^5*(66*a^7*b^4*c^10 + 42*a^11*c^6*d^4 + 330*a^8*b^3*c^9*d
+ 264*a^10*b*c^7*d^3 + 495*a^9*b^2*c^8*d^2) + x^18*((55*a^4*b^7*d^10)/3 + (
35*b^11*c^4*d^6)/3 + (220*a*b^10*c^3*d^7)/3 + (275*a^3*b^8*c*d^9)/3 + (275*
a^2*b^9*c^2*d^8)/2) + x^8*((165*a^4*b^7*c^10)/4 + 15*a^11*c^3*d^7 + (1155*a
^5*b^6*c^9*d)/2 + (1155*a^10*b*c^4*d^6)/4 + (10395*a^6*b^5*c^8*d^2)/4 + 495
0*a^7*b^4*c^7*d^3 + (17325*a^8*b^3*c^6*d^4)/4 + (3465*a^9*b^2*c^5*d^5)/2) +
 x^15*(22*a^7*b^4*d^10 + 8*b^11*c^7*d^3 + 154*a*b^10*c^6*d^4 + 308*a^6*b^5*
c*d^9 + 924*a^2*b^9*c^5*d^5 + 2310*a^3*b^8*c^4*d^6 + 2640*a^4*b^7*c^3*d^7 +
 1386*a^5*b^6*c^2*d^8) + x^6*(77*a^6*b^5*c^10 + 42*a^11*c^5*d^5 + 550*a^7*b
^4*c^9*d + 385*a^10*b*c^6*d^4 + (2475*a^8*b^3*c^8*d^2)/2 + 1100*a^9*b^2*c^7
*d^3) + x^17*((462*a^5*b^6*d^10)/17 + (252*b^11*c^5*d^5)/17 + (2310*a*b^10*
c^4*d^6)/17 + (3300*a^4*b^7*c*d^9)/17 + (6600*a^2*b^9*c^3*d^7)/17 + (7425*a
^3*b^8*c^2*d^8)/17) + x^9*((55*a^3*b^8*c^10)/3 + 5*a^11*c^2*d^8 + (1100*a^4
*b^7*c^9*d)/3 + (440*a^10*b*c^3*d^7)/3 + 2310*a^5*b^6*c^8*d^2 + 6160*a^6*b^
5*c^7*d^3 + 7700*a^7*b^4*c^6*d^4 + 4620*a^8*b^3*c^5*d^5 + (3850*a^9*b^2*c^4
*d^6)/3) + x^14*((165*a^8*b^3*d^10)/14 + (45*b^11*c^8*d^2)/14 + (660*a*b^10
```

*c^7*d^3)/7 + (1650*a^7*b^4*c*d^9)/7 + 825*a^2*b^9*c^6*d^4 + 2970*a^3*b^8*c^5*d^5 + 4950*a^4*b^7*c^4*d^6 + 3960*a^5*b^6*c^3*d^7 + 1485*a^6*b^5*c^2*d^8) + x^10*(a^11*c*d^9 + (11*a^2*b^9*c^10)/2 + 165*a^3*b^8*c^9*d + (99*a^10*b*c^2*d^8)/2 + 1485*a^4*b^7*c^8*d^2 + 5544*a^5*b^6*c^7*d^3 + 9702*a^6*b^5*c^6*d^4 + 8316*a^7*b^4*c^5*d^5 + 3465*a^8*b^3*c^4*d^6 + 660*a^9*b^2*c^3*d^7) + x^13*((10*b^11*c^9*d)/13 + (55*a^9*b^2*d^10)/13 + (495*a*b^10*c^8*d^2)/13 + (1650*a^8*b^3*c*d^9)/13 + (6600*a^2*b^9*c^7*d^3)/13 + (34650*a^3*b^8*c^6*d^4)/13 + (83160*a^4*b^7*c^5*d^5)/13 + (97020*a^5*b^6*c^4*d^6)/13 + (55440*a^6*b^5*c^3*d^7)/13 + (14850*a^7*b^4*c^2*d^8)/13) + a^11*c^10*x + (b^11*d^10*x^22)/22 + (5*a^8*c^7*x^4*(24*a^3*d^3 + 33*b^3*c^3 + 110*a*b^2*c^2*d + 99*a^2*b*c*d^2))/4 + (5*b^8*d^7*x^19*(33*a^3*d^3 + 24*b^3*c^3 + 99*a*b^2*c^2*d + 110*a^2*b*c*d^2))/19 + (a^10*c^9*x^2*(10*a*d + 11*b*c))/2 + (b^10*d^9*x^21*(11*a*d + 10*b*c))/21 + (5*a^9*c^8*x^3*(9*a^2*d^2 + 11*b^2*c^2 + 22*a*b*c*d))/3 + (b^9*d^8*x^20*(11*a^2*d^2 + 9*b^2*c^2 + 22*a*b*c*d))/4

3.1301 $\int (a+bx)^{10}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\frac{60d^3(bc-ad)^7(a+bx)^{14}}{7b^{11}}+\frac{14d^4}{}$$

```
[Out] 1/11*(-a*d+b*c)^10*(b*x+a)^11/b^11+5/6*d*(-a*d+b*c)^9*(b*x+a)^12/b^11+45/13
*d^2*(-a*d+b*c)^8*(b*x+a)^13/b^11+60/7*d^3*(-a*d+b*c)^7*(b*x+a)^14/b^11+14*
d^4*(-a*d+b*c)^6*(b*x+a)^15/b^11+63/4*d^5*(-a*d+b*c)^5*(b*x+a)^16/b^11+210/
17*d^6*(-a*d+b*c)^4*(b*x+a)^17/b^11+20/3*d^7*(-a*d+b*c)^3*(b*x+a)^18/b^11+4
5/19*d^8*(-a*d+b*c)^2*(b*x+a)^19/b^11+1/2*d^9*(-a*d+b*c)*(b*x+a)^20/b^11+1/
21*d^10*(b*x+a)^21/b^11
```

Rubi [A]

time = 0.76, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a+bx)^{16}(bc-ad)^5}{4b^{11}}+\frac{14d^4(a+bx)^{15}(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^{14}(bc-ad)^7}{7b^{11}}+\frac{45d^2(a+bx)^{13}(bc-ad)^8}{13b^{11}}+\frac{5d(a+bx)^{12}(bc-ad)^9}{6b^{11}}+\frac{(a+bx)^{11}(bc-ad)^{10}}{11b^{11}}+\frac{d^{10}(a+bx)^{21}}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^11)/(11*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^12)/
(6*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^13)/(13*b^11) + (60*d^3*(b*c - a
*d)^7*(a + b*x)^14)/(7*b^11) + (14*d^4*(b*c - a*d)^6*(a + b*x)^15)/b^11 + (
63*d^5*(b*c - a*d)^5*(a + b*x)^16)/(4*b^11) + (210*d^6*(b*c - a*d)^4*(a + b
*x)^17)/(17*b^11) + (20*d^7*(b*c - a*d)^3*(a + b*x)^18)/(3*b^11) + (45*d^8*
(b*c - a*d)^2*(a + b*x)^19)/(19*b^11) + (d^9*(b*c - a*d)*(a + b*x)^20)/(2*b
^11) + (d^10*(a + b*x)^21)/(21*b^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{10}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{10}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{11}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx}{b^{10}}\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1539 vs. $2(279) = 558$.
time = 0.10, size = 1539, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] a^10*c^10*x + 5*a^9*c^9*(b*c + a*d)*x^2 + (5*a^8*c^8*(9*b^2*c^2 + 20*a*b*c*
d + 9*a^2*d^2)*x^3)/3 + (15*a^7*c^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*
c*d^2 + 4*a^3*d^3)*x^4)/2 + 3*a^6*c^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^
2*b^2*c^2*d^2 + 80*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + 2*a^5*c^5*(21*b^5*c^5 +
175*a*b^4*c^4*d + 450*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d
^4 + 21*a^5*d^5)*x^6 + (30*a^4*c^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 315*a^2*b^
4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c*d^5 + 7*
a^6*d^6)*x^7)/7 + (15*a^3*c^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5
*d^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35
*a^6*b*c*d^6 + 2*a^7*d^7)*x^8)/2 + (5*a^2*c^2*(3*b^8*c^8 + 80*a*b^7*c^7*d +
 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2016*a
^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 +
 a*c*(b^9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3
 + 5292*a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540
*a^7*b^2*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^10 + ((b^10*c^10 + 100*a*b^9
*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100*a^4*b^6*c^6*d
^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^7*b^3*c^3*d^7
+ 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (5*b*d*(b^
9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3 + 5292*
a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540*a^7*b^2
*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^12)/6 + (15*b^2*d^2*(3*b^8*c^8 + 80*
a*b^7*c^7*d + 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4
*d^4 + 2016*a^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*
d^8)*x^13)/13 + (30*b^3*d^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5*d
^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35*a
^6*b*c*d^6 + 2*a^7*d^7)*x^14)/7 + 2*b^4*d^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 3
15*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c
*d^5 + 7*a^6*d^6)*x^15 + (3*b^5*d^5*(21*b^5*c^5 + 175*a*b^4*c^4*d + 450*a^2
*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5)*x^16)/4
+ (15*b^6*d^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 80*a^3*b
*c*d^3 + 14*a^4*d^4)*x^17)/17 + (5*b^7*d^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + 4*a^3*d^3)*x^18)/3 + (5*b^8*d^8*(9*b^2*c^2 + 20*a*b*c*d + 9*
a^2*d^2)*x^19)/19 + (b^9*d^9*(b*c + a*d)*x^20)/2 + (b^10*d^10*x^21)/21
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1501 vs. $2(279) = 558$.
time = 13.77, size = 1499, normalized size = 5.37

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^10*(c + d*x)^10,x]')
```

```
[Out] x (3879876 a ^ 10 c ^ 10 + 19399380 a ^ 9 c ^ 9 x (a d + b c) + a ^ 8 c ^ 8
 x ^ 2 (58198140 a ^ 2 d ^ 2 + 129329200 a b c d + 58198140 b ^ 2 c ^ 2) +
a ^ 7 c ^ 7 x ^ 3 (116396280 a ^ 3 d ^ 3 + 436486050 a ^ 2 b c d ^ 2 + 4364
86050 a b ^ 2 c ^ 2 d + 116396280 b ^ 3 c ^ 3) + a ^ 6 c ^ 6 x ^ 4 (1629547
92 a ^ 4 d ^ 4 + 931170240 a ^ 3 b c d ^ 3 + 1571349780 a ^ 2 b ^ 2 c ^ 2 d
 ^ 2 + 931170240 a b ^ 3 c ^ 3 d + 162954792 b ^ 4 c ^ 4) + a ^ 5 c ^ 5 x ^
 5 (162954792 a ^ 5 d ^ 5 + 1357956600 a ^ 4 b c d ^ 4 + 3491888400 a ^ 3 b
 ^ 2 c ^ 2 d ^ 3 + 3491888400 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 1357956600 a b ^ 4
c ^ 4 d + 162954792 b ^ 5 c ^ 5) + a ^ 4 c ^ 4 x ^ 6 (116396280 a ^ 6 d ^ 6
 + 1396755360 a ^ 5 b c d ^ 5 + 5237832600 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 798145
9200 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 5237832600 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 1396755
360 a b ^ 5 c ^ 5 d + 116396280 b ^ 6 c ^ 6) + a ^ 3 c ^ 3 x ^ 7 (58198140
a ^ 7 d ^ 7 + 1018467450 a ^ 6 b c d ^ 6 + 5499724230 a ^ 5 b ^ 2 c ^ 2 d ^
 5 + 12221609400 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 12221609400 a ^ 3 b ^ 4 c ^ 4 d
^ 3 + 5499724230 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 1018467450 a b ^ 6 c ^ 6 d + 581
98140 b ^ 7 c ^ 7) + a ^ 2 c ^ 2 x ^ 8 (19399380 a ^ 8 d ^ 8 + 517316800 a
^ 7 b c d ^ 7 + 4073869800 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 13036383360 a ^ 5 b ^
3 c ^ 3 d ^ 5 + 19011392400 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 13036383360 a ^ 3 b ^
 5 c ^ 5 d ^ 3 + 4073869800 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 517316800 a b ^ 7 c ^
 7 d + 19399380 b ^ 8 c ^ 8) + 3879876 a c x ^ 9 (a ^ 9 d ^ 9 + 45 a ^ 8 b
c d ^ 8 + 540 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 2520 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 5292
 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 5292 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 2520 a ^ 3 b ^ 6
c ^ 6 d ^ 3 + 540 a ^ 2 b ^ 7 c ^ 7 d ^ 2 + 45 a b ^ 8 c ^ 8 d + b ^ 9 c ^
9) + x ^ 10 (352716 a ^ 10 d ^ 10 + 35271600 a ^ 9 b c d ^ 9 + 714249900 a
^ 8 b ^ 2 c ^ 2 d ^ 8 + 5079110400 a ^ 7 b ^ 3 c ^ 3 d ^ 7 + 15554775600 a
^ 6 b ^ 4 c ^ 4 d ^ 6 + 22398876864 a ^ 5 b ^ 5 c ^ 5 d ^ 5 + 15554775600 a
 ^ 4 b ^ 6 c ^ 6 d ^ 4 + 5079110400 a ^ 3 b ^ 7 c ^ 7 d ^ 3 + 714249900 a ^
 2 b ^ 8 c ^ 8 d ^ 2 + 35271600 a b ^ 9 c ^ 9 d + 352716 b ^ 10 c ^ 10) + 3
233230 b d x ^ 11 (a ^ 9 d ^ 9 + 45 a ^ 8 b c d ^ 8 + 540 a ^ 7 b ^ 2 c ^ 2
 d ^ 7 + 2520 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 5292 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 5292
 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 2520 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 540 a ^ 2 b ^ 7 c
 ^ 7 d ^ 2 + 45 a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + b ^ 2 d ^ 2 x ^ 12 (134303
40 a ^ 8 d ^ 8 + 358142400 a ^ 7 b c d ^ 7 + 2820371400 a ^ 6 b ^ 2 c ^ 2 d
 ^ 6 + 9025188480 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 13161733200 a ^ 4 b ^ 4 c ^ 4 d
 ^ 4 + 9025188480 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 2820371400 a ^ 2 b ^ 6 c ^ 6 d
^ 2 + 358142400 a b ^ 7 c ^ 7 d + 13430340 b ^ 8 c ^ 8) + b ^ 3 d ^ 3 x ^ 1
3 (33256080 a ^ 7 d ^ 7 + 581981400 a ^ 6 b c d ^ 6 + 3142699560 a ^ 5 b ^
2 c ^ 2 d ^ 5 + 6983776800 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 6983776800 a ^ 3 b ^ 4
 c ^ 4 d ^ 3 + 3142699560 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 581981400 a b ^ 6 c ^ 6
 d + 33256080 b ^ 7 c ^ 7) + b ^ 4 d ^ 4 x ^ 14 (54318264 a ^ 6 d ^ 6 + 651
```

```
819168 a ^ 5 b c d ^ 5 + 2444321880 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 3724680960 a
^ 3 b ^ 3 c ^ 3 d ^ 3 + 2444321880 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 651819168 a b
^ 5 c ^ 5 d + 54318264 b ^ 6 c ^ 6) + b ^ 5 d ^ 5 x ^ 15 (61108047 a ^ 5 d
^ 5 + 509233725 a ^ 4 b c d ^ 4 + 1309458150 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 1309
458150 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 509233725 a b ^ 4 c ^ 4 d + 61108047 b ^ 5
 c ^ 5) + b ^ 6 d ^ 6 x ^ 16 (47927880 a ^ 4 d ^ 4 + 273873600 a ^ 3 b c d
^ 3 + 462161700 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 273873600 a b ^ 3 c ^ 3 d + 47927
880 b ^ 4 c ^ 4) + b ^ 7 d ^ 7 x ^ 17 (25865840 a ^ 3 d ^ 3 + 96996900 a ^
2 b c d ^ 2 + 96996900 a b ^ 2 c ^ 2 d + 25865840 b ^ 3 c ^ 3) + b ^ 8 d ^
8 x ^ 18 (9189180 a ^ 2 d ^ 2 + 20420400 a b c d + 9189180 b ^ 2 c ^ 2) + b
 ^ 9 d ^ 9 x ^ 19 (1939938 a d + 1939938 b c) + 184756 b ^ 10 d ^ 10 x ^ 20
) / 3879876
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(259) = 518$.

time = 0.13, size = 1591, normalized size = 5.70

method	result	size
norman	Expression too large to display	1572
default	Expression too large to display	1591
gosper	Expression too large to display	1834
risch	Expression too large to display	1834

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b^10*d^10*x^21+1/20*(10*a*b^9*d^10+10*b^10*c*d^9)*x^20+1/19*(45*a^2*b^
8*d^10+100*a*b^9*c*d^9+45*b^10*c^2*d^8)*x^19+1/18*(120*a^3*b^7*d^10+450*a^2
*b^8*c*d^9+450*a*b^9*c^2*d^8+120*b^10*c^3*d^7)*x^18+1/17*(210*a^4*b^6*d^10+
1200*a^3*b^7*c*d^9+2025*a^2*b^8*c^2*d^8+1200*a*b^9*c^3*d^7+210*b^10*c^4*d^6
)*x^17+1/16*(252*a^5*b^5*d^10+2100*a^4*b^6*c*d^9+5400*a^3*b^7*c^2*d^8+5400*
a^2*b^8*c^3*d^7+2100*a*b^9*c^4*d^6+252*b^10*c^5*d^5)*x^16+1/15*(210*a^6*b^4
*d^10+2520*a^5*b^5*c*d^9+9450*a^4*b^6*c^2*d^8+14400*a^3*b^7*c^3*d^7+9450*a^
2*b^8*c^4*d^6+2520*a*b^9*c^5*d^5+210*b^10*c^6*d^4)*x^15+1/14*(120*a^7*b^3*d
^10+2100*a^6*b^4*c*d^9+11340*a^5*b^5*c^2*d^8+25200*a^4*b^6*c^3*d^7+25200*a^
3*b^7*c^4*d^6+11340*a^2*b^8*c^5*d^5+2100*a*b^9*c^6*d^4+120*b^10*c^7*d^3)*x^
14+1/13*(45*a^8*b^2*d^10+1200*a^7*b^3*c*d^9+9450*a^6*b^4*c^2*d^8+30240*a^5*
b^5*c^3*d^7+44100*a^4*b^6*c^4*d^6+30240*a^3*b^7*c^5*d^5+9450*a^2*b^8*c^6*d^
4+1200*a*b^9*c^7*d^3+45*b^10*c^8*d^2)*x^13+1/12*(10*a^9*b*d^10+450*a^8*b^2*
c*d^9+5400*a^7*b^3*c^2*d^8+25200*a^6*b^4*c^3*d^7+52920*a^5*b^5*c^4*d^6+5292
0*a^4*b^6*c^5*d^5+25200*a^3*b^7*c^6*d^4+5400*a^2*b^8*c^7*d^3+450*a*b^9*c^8*
d^2+10*b^10*c^9*d)*x^12+1/11*(a^10*d^10+100*a^9*b*c*d^9+2025*a^8*b^2*c^2*d^
8+14400*a^7*b^3*c^3*d^7+44100*a^6*b^4*c^4*d^6+63504*a^5*b^5*c^5*d^5+44100*a
^4*b^6*c^6*d^4+14400*a^3*b^7*c^7*d^3+2025*a^2*b^8*c^8*d^2+100*a*b^9*c^9*d+b
^10*c^10)*x^11+1/10*(10*a^10*c*d^9+450*a^9*b*c^2*d^8+5400*a^8*b^2*c^3*d^7+2
```

```
5200*a^7*b^3*c^4*d^6+52920*a^6*b^4*c^5*d^5+52920*a^5*b^5*c^6*d^4+25200*a^4*
b^6*c^7*d^3+5400*a^3*b^7*c^8*d^2+450*a^2*b^8*c^9*d+10*a*b^9*c^10)*x^10+1/9*
(45*a^10*c^2*d^8+1200*a^9*b*c^3*d^7+9450*a^8*b^2*c^4*d^6+30240*a^7*b^3*c^5*
d^5+44100*a^6*b^4*c^6*d^4+30240*a^5*b^5*c^7*d^3+9450*a^4*b^6*c^8*d^2+1200*a
^3*b^7*c^9*d+45*a^2*b^8*c^10)*x^9+1/8*(120*a^10*c^3*d^7+2100*a^9*b*c^4*d^6+
11340*a^8*b^2*c^5*d^5+25200*a^7*b^3*c^6*d^4+25200*a^6*b^4*c^7*d^3+11340*a^5
*b^5*c^8*d^2+2100*a^4*b^6*c^9*d+120*a^3*b^7*c^10)*x^8+1/7*(210*a^10*c^4*d^6
+2520*a^9*b*c^5*d^5+9450*a^8*b^2*c^6*d^4+14400*a^7*b^3*c^7*d^3+9450*a^6*b^4
*c^8*d^2+2520*a^5*b^5*c^9*d+210*a^4*b^6*c^10)*x^7+1/6*(252*a^10*c^5*d^5+210
0*a^9*b*c^6*d^4+5400*a^8*b^2*c^7*d^3+5400*a^7*b^3*c^8*d^2+2100*a^6*b^4*c^9*
d+252*a^5*b^5*c^10)*x^6+1/5*(210*a^10*c^6*d^4+1200*a^9*b*c^7*d^3+2025*a^8*b
^2*c^8*d^2+1200*a^7*b^3*c^9*d+210*a^6*b^4*c^10)*x^5+1/4*(120*a^10*c^7*d^3+4
50*a^9*b*c^8*d^2+450*a^8*b^2*c^9*d+120*a^7*b^3*c^10)*x^4+1/3*(45*a^10*c^8*d
^2+100*a^9*b*c^9*d+45*a^8*b^2*c^10)*x^3+1/2*(10*a^10*c^9*d+10*a^9*b*c^10)*x
^2+a^10*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(259) = 518$.
time = 0.28, size = 1581, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
```

```
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(259) = 518$.

time = 0.31, size = 1581, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
```

```
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1775 vs. $2(257) = 514$.

time = 0.16, size = 1775, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10*(d*x+c)**10,x)
```

```
[Out] a**10*c**10*x + b**10*d**10*x**21/21 + x**20*(a*b**9*d**10/2 + b**10*c*d**9
/2) + x**19*(45*a**2*b**8*d**10/19 + 100*a*b**9*c*d**9/19 + 45*b**10*c**2*d
**8/19) + x**18*(20*a**3*b**7*d**10/3 + 25*a**2*b**8*c*d**9 + 25*a*b**9*c**
2*d**8 + 20*b**10*c**3*d**7/3) + x**17*(210*a**4*b**6*d**10/17 + 1200*a**3*
b**7*c*d**9/17 + 2025*a**2*b**8*c**2*d**8/17 + 1200*a*b**9*c**3*d**7/17 + 2
10*b**10*c**4*d**6/17) + x**16*(63*a**5*b**5*d**10/4 + 525*a**4*b**6*c*d**9
/4 + 675*a**3*b**7*c**2*d**8/2 + 675*a**2*b**8*c**3*d**7/2 + 525*a*b**9*c**
4*d**6/4 + 63*b**10*c**5*d**5/4) + x**15*(14*a**6*b**4*d**10 + 168*a**5*b**
5*c*d**9 + 630*a**4*b**6*c**2*d**8 + 960*a**3*b**7*c**3*d**7 + 630*a**2*b**
8*c**4*d**6 + 168*a*b**9*c**5*d**5 + 14*b**10*c**6*d**4) + x**14*(60*a**7*b
**3*d**10/7 + 150*a**6*b**4*c*d**9 + 810*a**5*b**5*c**2*d**8 + 1800*a**4*b*
*6*c**3*d**7 + 1800*a**3*b**7*c**4*d**6 + 810*a**2*b**8*c**5*d**5 + 150*a*b
**9*c**6*d**4 + 60*b**10*c**7*d**3/7) + x**13*(45*a**8*b**2*d**10/13 + 1200
*a**7*b**3*c*d**9/13 + 9450*a**6*b**4*c**2*d**8/13 + 30240*a**5*b**5*c**3*d
**7/13 + 44100*a**4*b**6*c**4*d**6/13 + 30240*a**3*b**7*c**5*d**5/13 + 9450
*a**2*b**8*c**6*d**4/13 + 1200*a*b**9*c**7*d**3/13 + 45*b**10*c**8*d**2/13)
 + x**12*(5*a**9*b*d**10/6 + 75*a**8*b**2*c*d**9/2 + 450*a**7*b**3*c**2*d**
8 + 2100*a**6*b**4*c**3*d**7 + 4410*a**5*b**5*c**4*d**6 + 4410*a**4*b**6*c*
*5*d**5 + 2100*a**3*b**7*c**6*d**4 + 450*a**2*b**8*c**7*d**3 + 75*a*b**9*c*
*8*d**2/2 + 5*b**10*c**9*d/6) + x**11*(a**10*d**10/11 + 100*a**9*b*c*d**9/1
1 + 2025*a**8*b**2*c**2*d**8/11 + 14400*a**7*b**3*c**3*d**7/11 + 44100*a**6
*b**4*c**4*d**6/11 + 63504*a**5*b**5*c**5*d**5/11 + 44100*a**4*b**6*c**6*d*
*4/11 + 14400*a**3*b**7*c**7*d**3/11 + 2025*a**2*b**8*c**8*d**2/11 + 100*a*
```

```
b**9*c**9*d/11 + b**10*c**10/11) + x**10*(a**10*c*d**9 + 45*a**9*b*c**2*d**
8 + 540*a**8*b**2*c**3*d**7 + 2520*a**7*b**3*c**4*d**6 + 5292*a**6*b**4*c**
5*d**5 + 5292*a**5*b**5*c**6*d**4 + 2520*a**4*b**6*c**7*d**3 + 540*a**3*b**
7*c**8*d**2 + 45*a**2*b**8*c**9*d + a*b**9*c**10) + x**9*(5*a**10*c**2*d**8
 + 400*a**9*b*c**3*d**7/3 + 1050*a**8*b**2*c**4*d**6 + 3360*a**7*b**3*c**5*
d**5 + 4900*a**6*b**4*c**6*d**4 + 3360*a**5*b**5*c**7*d**3 + 1050*a**4*b**6
*c**8*d**2 + 400*a**3*b**7*c**9*d/3 + 5*a**2*b**8*c**10) + x**8*(15*a**10*c
**3*d**7 + 525*a**9*b*c**4*d**6/2 + 2835*a**8*b**2*c**5*d**5/2 + 3150*a**7*
b**3*c**6*d**4 + 3150*a**6*b**4*c**7*d**3 + 2835*a**5*b**5*c**8*d**2/2 + 52
5*a**4*b**6*c**9*d/2 + 15*a**3*b**7*c**10) + x**7*(30*a**10*c**4*d**6 + 360
*a**9*b*c**5*d**5 + 1350*a**8*b**2*c**6*d**4 + 14400*a**7*b**3*c**7*d**3/7
+ 1350*a**6*b**4*c**8*d**2 + 360*a**5*b**5*c**9*d + 30*a**4*b**6*c**10) + x
**6*(42*a**10*c**5*d**5 + 350*a**9*b*c**6*d**4 + 900*a**8*b**2*c**7*d**3 +
900*a**7*b**3*c**8*d**2 + 350*a**6*b**4*c**9*d + 42*a**5*b**5*c**10) + x**5
*(42*a**10*c**6*d**4 + 240*a**9*b*c**7*d**3 + 405*a**8*b**2*c**8*d**2 + 240
*a**7*b**3*c**9*d + 42*a**6*b**4*c**10) + x**4*(30*a**10*c**7*d**3 + 225*a*
*9*b*c**8*d**2/2 + 225*a**8*b**2*c**9*d/2 + 30*a**7*b**3*c**10) + x**3*(15*
a**10*c**8*d**2 + 100*a**9*b*c**9*d/3 + 15*a**8*b**2*c**10) + x**2*(5*a**10
*c**9*d + 5*a**9*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(259) = 518$.
time = 0.00, size = 1943, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x)
```

```
[Out] 1/21*b^10*d^10*x^21 + 1/2*b^10*c*d^9*x^20 + 1/2*a*b^9*d^10*x^20 + 45/19*b^1
0*c^2*d^8*x^19 + 100/19*a*b^9*c*d^9*x^19 + 45/19*a^2*b^8*d^10*x^19 + 20/3*b
^10*c^3*d^7*x^18 + 25*a*b^9*c^2*d^8*x^18 + 25*a^2*b^8*c*d^9*x^18 + 20/3*a^3
*b^7*d^10*x^18 + 210/17*b^10*c^4*d^6*x^17 + 1200/17*a*b^9*c^3*d^7*x^17 + 20
25/17*a^2*b^8*c^2*d^8*x^17 + 1200/17*a^3*b^7*c*d^9*x^17 + 210/17*a^4*b^6*d^
10*x^17 + 63/4*b^10*c^5*d^5*x^16 + 525/4*a*b^9*c^4*d^6*x^16 + 675/2*a^2*b^8
*c^3*d^7*x^16 + 675/2*a^3*b^7*c^2*d^8*x^16 + 525/4*a^4*b^6*c*d^9*x^16 + 63/
4*a^5*b^5*d^10*x^16 + 14*b^10*c^6*d^4*x^15 + 168*a*b^9*c^5*d^5*x^15 + 630*a
^2*b^8*c^4*d^6*x^15 + 960*a^3*b^7*c^3*d^7*x^15 + 630*a^4*b^6*c^2*d^8*x^15 +
 168*a^5*b^5*c*d^9*x^15 + 14*a^6*b^4*d^10*x^15 + 60/7*b^10*c^7*d^3*x^14 + 1
50*a*b^9*c^6*d^4*x^14 + 810*a^2*b^8*c^5*d^5*x^14 + 1800*a^3*b^7*c^4*d^6*x^1
4 + 1800*a^4*b^6*c^3*d^7*x^14 + 810*a^5*b^5*c^2*d^8*x^14 + 150*a^6*b^4*c*d^
9*x^14 + 60/7*a^7*b^3*d^10*x^14 + 45/13*b^10*c^8*d^2*x^13 + 1200/13*a*b^9*c
^7*d^3*x^13 + 9450/13*a^2*b^8*c^6*d^4*x^13 + 30240/13*a^3*b^7*c^5*d^5*x^13
+ 44100/13*a^4*b^6*c^4*d^6*x^13 + 30240/13*a^5*b^5*c^3*d^7*x^13 + 9450/13*a
^6*b^4*c^2*d^8*x^13 + 1200/13*a^7*b^3*c*d^9*x^13 + 45/13*a^8*b^2*d^10*x^13
+ 5/6*b^10*c^9*d*x^12 + 75/2*a*b^9*c^8*d^2*x^12 + 450*a^2*b^8*c^7*d^3*x^12
```

```
+ 2100*a^3*b^7*c^6*d^4*x^12 + 4410*a^4*b^6*c^5*d^5*x^12 + 4410*a^5*b^5*c^4*
d^6*x^12 + 2100*a^6*b^4*c^3*d^7*x^12 + 450*a^7*b^3*c^2*d^8*x^12 + 75/2*a^8*
b^2*c*d^9*x^12 + 5/6*a^9*b*d^10*x^12 + 1/11*b^10*c^10*x^11 + 100/11*a*b^9*c
^9*d*x^11 + 2025/11*a^2*b^8*c^8*d^2*x^11 + 14400/11*a^3*b^7*c^7*d^3*x^11 +
44100/11*a^4*b^6*c^6*d^4*x^11 + 63504/11*a^5*b^5*c^5*d^5*x^11 + 44100/11*a^
6*b^4*c^4*d^6*x^11 + 14400/11*a^7*b^3*c^3*d^7*x^11 + 2025/11*a^8*b^2*c^2*d^
8*x^11 + 100/11*a^9*b*c*d^9*x^11 + 1/11*a^10*d^10*x^11 + a*b^9*c^10*x^10 +
45*a^2*b^8*c^9*d*x^10 + 540*a^3*b^7*c^8*d^2*x^10 + 2520*a^4*b^6*c^7*d^3*x^1
0 + 5292*a^5*b^5*c^6*d^4*x^10 + 5292*a^6*b^4*c^5*d^5*x^10 + 2520*a^7*b^3*c^
4*d^6*x^10 + 540*a^8*b^2*c^3*d^7*x^10 + 45*a^9*b*c^2*d^8*x^10 + a^10*c*d^9*
x^10 + 5*a^2*b^8*c^10*x^9 + 400/3*a^3*b^7*c^9*d*x^9 + 1050*a^4*b^6*c^8*d^2*
x^9 + 3360*a^5*b^5*c^7*d^3*x^9 + 4900*a^6*b^4*c^6*d^4*x^9 + 3360*a^7*b^3*c^
5*d^5*x^9 + 1050*a^8*b^2*c^4*d^6*x^9 + 400/3*a^9*b*c^3*d^7*x^9 + 5*a^10*c^2
*d^8*x^9 + 15*a^3*b^7*c^10*x^8 + 525/2*a^4*b^6*c^9*d*x^8 + 2835/2*a^5*b^5*c
^8*d^2*x^8 + 3150*a^6*b^4*c^7*d^3*x^8 + 3150*a^7*b^3*c^6*d^4*x^8 + 2835/2*a
^8*b^2*c^5*d^5*x^8 + 525/2*a^9*b*c^4*d^6*x^8 + 15*a^10*c^3*d^7*x^8 + 30*a^4
*b^6*c^10*x^7 + 360*a^5*b^5*c^9*d*x^7 + 1350*a^6*b^4*c^8*d^2*x^7 + 14400/7*
a^7*b^3*c^7*d^3*x^7 + 1350*a^8*b^2*c^6*d^4*x^7 + 360*a^9*b*c^5*d^5*x^7 + 30
*a^10*c^4*d^6*x^7 + 42*a^5*b^5*c^10*x^6 + 350*a^6*b^4*c^9*d*x^6 + 900*a^7*b
^3*c^8*d^2*x^6 + 900*a^8*b^2*c^7*d^3*x^6 + 350*a^9*b*c^6*d^4*x^6 + 42*a^10*
c^5*d^5*x^6 + 42*a^6*b^4*c^10*x^5 + 240*a^7*b^3*c^9*d*x^5 + 405*a^8*b^2*c^8
*d^2*x^5 + 240*a^9*b*c^7*d^3*x^5 + 42*a^10*c^6*d^4*x^5 + 30*a^7*b^3*c^10*x^
4 + 225/2*a^8*b^2*c^9*d*x^4 + 225/2*a^9*b*c^8*d^2*x^4 + 30*a^10*c^7*d^3*x^4
 + 15*a^8*b^2*c^10*x^3 + 100/3*a^9*b*c^9*d*x^3 + 15*a^10*c^8*d^2*x^3 + 5*a^
9*b*c^10*x^2 + 5*a^10*c^9*d*x^2 + a^10*c^10*x
```

Mupad [B]
time = 0.69, size = 1549, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10*(c + d*x)^10,x)
```

```
[Out] x^7*(30*a^4*b^6*c^10 + 30*a^10*c^4*d^6 + 360*a^5*b^5*c^9*d + 360*a^9*b*c^5*
d^5 + 1350*a^6*b^4*c^8*d^2 + (14400*a^7*b^3*c^7*d^3)/7 + 1350*a^8*b^2*c^6*d
^4) + x^15*(14*a^6*b^4*d^10 + 14*b^10*c^6*d^4 + 168*a*b^9*c^5*d^5 + 168*a^5
*b^5*c*d^9 + 630*a^2*b^8*c^4*d^6 + 960*a^3*b^7*c^3*d^7 + 630*a^4*b^6*c^2*d^
8) + x^5*(42*a^6*b^4*c^10 + 42*a^10*c^6*d^4 + 240*a^7*b^3*c^9*d + 240*a^9*b
*c^7*d^3 + 405*a^8*b^2*c^8*d^2) + x^17*((210*a^4*b^6*d^10)/17 + (210*b^10*c
^4*d^6)/17 + (1200*a*b^9*c^3*d^7)/17 + (1200*a^3*b^7*c*d^9)/17 + (2025*a^2*
b^8*c^2*d^8)/17) + x^11*((a^10*d^10)/11 + (b^10*c^10)/11 + (2025*a^2*b^8*c^
8*d^2)/11 + (14400*a^3*b^7*c^7*d^3)/11 + (44100*a^4*b^6*c^6*d^4)/11 + (6350
4*a^5*b^5*c^5*d^5)/11 + (44100*a^6*b^4*c^4*d^6)/11 + (14400*a^7*b^3*c^3*d^7
)/11 + (2025*a^8*b^2*c^2*d^8)/11 + (100*a*b^9*c^9*d)/11 + (100*a^9*b*c*d^9)
/11) + x^8*(15*a^3*b^7*c^10 + 15*a^10*c^3*d^7 + (525*a^4*b^6*c^9*d)/2 + (52
```

5*a^9*b*c^4*d^6)/2 + (2835*a^5*b^5*c^8*d^2)/2 + 3150*a^6*b^4*c^7*d^3 + 3150
*a^7*b^3*c^6*d^4 + (2835*a^8*b^2*c^5*d^5)/2) + x^14*((60*a^7*b^3*d^10)/7 +
(60*b^10*c^7*d^3)/7 + 150*a*b^9*c^6*d^4 + 150*a^6*b^4*c*d^9 + 810*a^2*b^8*c
^5*d^5 + 1800*a^3*b^7*c^4*d^6 + 1800*a^4*b^6*c^3*d^7 + 810*a^5*b^5*c^2*d^8)
 + x^10*(a*b^9*c^10 + a^10*c*d^9 + 45*a^2*b^8*c^9*d + 45*a^9*b*c^2*d^8 + 54
0*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*
b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540*a^8*b^2*c^3*d^7) + x^12*((5*a^9*b*
d^10)/6 + (5*b^10*c^9*d)/6 + (75*a*b^9*c^8*d^2)/2 + (75*a^8*b^2*c*d^9)/2 +
450*a^2*b^8*c^7*d^3 + 2100*a^3*b^7*c^6*d^4 + 4410*a^4*b^6*c^5*d^5 + 4410*a^
5*b^5*c^4*d^6 + 2100*a^6*b^4*c^3*d^7 + 450*a^7*b^3*c^2*d^8) + x^6*(42*a^5*b
^5*c^10 + 42*a^10*c^5*d^5 + 350*a^6*b^4*c^9*d + 350*a^9*b*c^6*d^4 + 900*a^7
*b^3*c^8*d^2 + 900*a^8*b^2*c^7*d^3) + x^16*((63*a^5*b^5*d^10)/4 + (63*b^10*
c^5*d^5)/4 + (525*a*b^9*c^4*d^6)/4 + (525*a^4*b^6*c*d^9)/4 + (675*a^2*b^8*c
^3*d^7)/2 + (675*a^3*b^7*c^2*d^8)/2) + x^9*(5*a^2*b^8*c^10 + 5*a^10*c^2*d^8
 + (400*a^3*b^7*c^9*d)/3 + (400*a^9*b*c^3*d^7)/3 + 1050*a^4*b^6*c^8*d^2 + 3
360*a^5*b^5*c^7*d^3 + 4900*a^6*b^4*c^6*d^4 + 3360*a^7*b^3*c^5*d^5 + 1050*a^
8*b^2*c^4*d^6) + x^13*((45*a^8*b^2*d^10)/13 + (45*b^10*c^8*d^2)/13 + (1200*
a*b^9*c^7*d^3)/13 + (1200*a^7*b^3*c*d^9)/13 + (9450*a^2*b^8*c^6*d^4)/13 + (
30240*a^3*b^7*c^5*d^5)/13 + (44100*a^4*b^6*c^4*d^6)/13 + (30240*a^5*b^5*c^3
*d^7)/13 + (9450*a^6*b^4*c^2*d^8)/13) + a^10*c^10*x + (b^10*d^10*x^21)/21 +
 (15*a^7*c^7*x^4*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d^2))
/2 + (5*b^7*d^7*x^18*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d
^2))/3 + 5*a^9*c^9*x^2*(a*d + b*c) + (b^9*d^9*x^20*(a*d + b*c))/2 + (5*a^8*
c^8*x^3*(9*a^2*d^2 + 9*b^2*c^2 + 20*a*b*c*d))/3 + (5*b^8*d^8*x^19*(9*a^2*d^
2 + 9*b^2*c^2 + 20*a*b*c*d))/19

3.1302 $\int (a+bx)^9(c+dx)^{10}\,dx$

Optimal. Leaf size=250

$$-\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\frac{6b^3(bc-ad)^6(c+dx)^{14}}{d^{10}}-\frac{42b^4(\cdots}{}$$

```
[Out] -1/11*(-a*d+b*c)^9*(d*x+c)^11/d^10+3/4*b*(-a*d+b*c)^8*(d*x+c)^12/d^10-36/13
*b^2*(-a*d+b*c)^7*(d*x+c)^13/d^10+6*b^3*(-a*d+b*c)^6*(d*x+c)^14/d^10-42/5*b
^4*(-a*d+b*c)^5*(d*x+c)^15/d^10+63/8*b^5*(-a*d+b*c)^4*(d*x+c)^16/d^10-84/17
*b^6*(-a*d+b*c)^3*(d*x+c)^17/d^10+2*b^7*(-a*d+b*c)^2*(d*x+c)^18/d^10-9/19*b
^8*(-a*d+b*c)*(d*x+c)^19/d^10+1/20*b^9*(d*x+c)^20/d^10
```

Rubi [A]
time = 0.70, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+dx)^{15}(bc-ad)^5}{5d^{10}}+\frac{6b^3(c+dx)^{14}(bc-ad)^6}{d^{10}}-\frac{36b^2(c+dx)^{13}(bc-ad)^7}{13d^{10}}+\frac{3b(c+dx)^{12}(bc-ad)^8}{4d^{10}}-\frac{(c+dx)^{11}(bc-ad)^9}{11d^{10}}+\frac{b^9(c+dx)^{20}}{20d^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^9*(c + d*x)^11)/d^10 + (3*b*(b*c - a*d)^8*(c + d*x)^12)/
(4*d^10) - (36*b^2*(b*c - a*d)^7*(c + d*x)^13)/(13*d^10) + (6*b^3*(b*c - a*
d)^6*(c + d*x)^14)/d^10 - (42*b^4*(b*c - a*d)^5*(c + d*x)^15)/(5*d^10) + (6
3*b^5*(b*c - a*d)^4*(c + d*x)^16)/(8*d^10) - (84*b^6*(b*c - a*d)^3*(c + d*x
)^17)/(17*d^10) + (2*b^7*(b*c - a*d)^2*(c + d*x)^18)/d^10 - (9*b^8*(b*c - a
*d)*(c + d*x)^19)/(19*d^10) + (b^9*(c + d*x)^20)/(20*d^10)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^9(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^9(c+dx)^{10}}{d^9}+\frac{9b(bc-ad)^8(c+dx)^{11}}{d^9}-\frac{36b^2(bc-ad)^7(c+dx)^{\cdots}}{d^9}\right.$$

$$= -\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1397 vs. 2(250) = 500.
time = 0.10, size = 1397, normalized size = 5.59

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] a^9*c^10*x + (a^8*c^9*(9*b*c + 10*a*d)*x^2)/2 + 3*a^7*c^8*(4*b^2*c^2 + 10*a
*b*c*d + 5*a^2*d^2)*x^3 + (3*a^6*c^7*(28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^
2*b*c*d^2 + 40*a^3*d^3)*x^4)/4 + (6*a^5*c^6*(21*b^4*c^4 + 140*a*b^3*c^3*d +
 270*a^2*b^2*c^2*d^2 + 180*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + 3*a^4*c^5*(7*
b^5*c^5 + 70*a*b^4*c^4*d + 210*a^2*b^3*c^3*d^2 + 240*a^3*b^2*c^2*d^3 + 105*
a^4*b*c*d^4 + 14*a^5*d^5)*x^6 + 6*a^3*c^4*(2*b^6*c^6 + 30*a*b^5*c^5*d + 135
*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 54*a^5*b*c*d
^5 + 5*a^6*d^6)*x^7 + (3*a^2*c^3*(6*b^7*c^7 + 140*a*b^6*c^6*d + 945*a^2*b^5
*c^5*d^2 + 2520*a^3*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 1512*a^5*b^2*c^2*d
^5 + 315*a^6*b*c*d^6 + 20*a^7*d^7)*x^8)/4 + a*c^2*(b^8*c^8 + 40*a*b^7*c^7*d
 + 420*a^2*b^6*c^6*d^2 + 1680*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2352
*a^5*b^3*c^3*d^5 + 840*a^6*b^2*c^2*d^6 + 120*a^7*b*c*d^7 + 5*a^8*d^8)*x^9 +
 (c*(b^9*c^9 + 90*a*b^8*c^8*d + 1620*a^2*b^7*c^7*d^2 + 10080*a^3*b^6*c^6*d^
3 + 26460*a^4*b^5*c^5*d^4 + 31752*a^5*b^4*c^4*d^5 + 17640*a^6*b^3*c^3*d^6 +
 4320*a^7*b^2*c^2*d^7 + 405*a^8*b*c*d^8 + 10*a^9*d^9)*x^10)/10 + (d*(10*b^9
*c^9 + 405*a*b^8*c^8*d + 4320*a^2*b^7*c^7*d^2 + 17640*a^3*b^6*c^6*d^3 + 317
52*a^4*b^5*c^5*d^4 + 26460*a^5*b^4*c^4*d^5 + 10080*a^6*b^3*c^3*d^6 + 1620*a
^7*b^2*c^2*d^7 + 90*a^8*b*c*d^8 + a^9*d^9)*x^11)/11 + (3*b*d^2*(5*b^8*c^8 +
 120*a*b^7*c^7*d + 840*a^2*b^6*c^6*d^2 + 2352*a^3*b^5*c^5*d^3 + 2940*a^4*b^
4*c^4*d^4 + 1680*a^5*b^3*c^3*d^5 + 420*a^6*b^2*c^2*d^6 + 40*a^7*b*c*d^7 + a
^8*d^8)*x^12)/4 + (6*b^2*d^3*(20*b^7*c^7 + 315*a*b^6*c^6*d + 1512*a^2*b^5*c
^5*d^2 + 2940*a^3*b^4*c^4*d^3 + 2520*a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5
+ 140*a^6*b*c*d^6 + 6*a^7*d^7)*x^13)/13 + 3*b^3*d^4*(5*b^6*c^6 + 54*a*b^5*c
^5*d + 180*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 135*a^4*b^2*c^2*d^4 + 30
*a^5*b*c*d^5 + 2*a^6*d^6)*x^14 + (6*b^4*d^5*(14*b^5*c^5 + 105*a*b^4*c^4*d +
 240*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d^5)*x^
15)/5 + (3*b^5*d^6*(35*b^4*c^4 + 180*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 14
0*a^3*b*c*d^3 + 21*a^4*d^4)*x^16)/8 + (3*b^6*d^7*(40*b^3*c^3 + 135*a*b^2*c^
2*d + 120*a^2*b*c*d^2 + 28*a^3*d^3)*x^17)/17 + (b^7*d^8*(5*b^2*c^2 + 10*a*b
*c*d + 4*a^2*d^2)*x^18)/2 + (b^8*d^9*(10*b*c + 9*a*d)*x^19)/19 + (b^9*d^10*
x^20)/20
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1360 vs. 2(250) = 500.
time = 12.47, size = 1358, normalized size = 5.43

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^9*(c + d*x)^10,x]')

[Out] x (1847560 a ^ 9 c ^ 10 + 923780 a ^ 8 c ^ 9 x (10 a d + 9 b c) + a ^ 7 c ^
8 x ^ 2 (27713400 a ^ 2 d ^ 2 + 55426800 a b c d + 22170720 b ^ 2 c ^ 2) +
a ^ 6 c ^ 7 x ^ 3 (55426800 a ^ 3 d ^ 3 + 187065450 a ^ 2 b c d ^ 2 + 1662
80400 a b ^ 2 c ^ 2 d + 38798760 b ^ 3 c ^ 3) + a ^ 5 c ^ 6 x ^ 4 (77597520
a ^ 4 d ^ 4 + 399072960 a ^ 3 b c d ^ 3 + 598609440 a ^ 2 b ^ 2 c ^ 2 d ^
2 + 310390080 a b ^ 3 c ^ 3 d + 46558512 b ^ 4 c ^ 4) + a ^ 4 c ^ 5 x ^ 5 (
77597520 a ^ 5 d ^ 5 + 581981400 a ^ 4 b c d ^ 4 + 1330243200 a ^ 3 b ^ 2 c
^ 2 d ^ 3 + 1163962800 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 387987600 a b ^ 4 c ^ 4 d
+ 38798760 b ^ 5 c ^ 5) + a ^ 3 c ^ 4 x ^ 6 (55426800 a ^ 6 d ^ 6 + 598609
440 a ^ 5 b c d ^ 5 + 1995364800 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 2660486400 a ^ 3
b ^ 3 c ^ 3 d ^ 3 + 1496523600 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 332560800 a b ^ 5
c ^ 5 d + 22170720 b ^ 6 c ^ 6) + a ^ 2 c ^ 3 x ^ 7 (27713400 a ^ 7 d ^ 7
+ 436486050 a ^ 6 b c d ^ 6 + 2095133040 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 40738698
00 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 3491888400 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 130945815
0 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 193993800 a b ^ 6 c ^ 6 d + 8314020 b ^ 7 c ^ 7
) + 1847560 a c ^ 2 x ^ 8 (5 a ^ 8 d ^ 8 + 120 a ^ 7 b c d ^ 7 + 840 a ^ 6
b ^ 2 c ^ 2 d ^ 6 + 2352 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 2940 a ^ 4 b ^ 4 c ^ 4 d
^ 4 + 1680 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 420 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 40 a b
^ 7 c ^ 7 d + b ^ 8 c ^ 8) + 1385670 b d ^ 2 x ^ 11 (a ^ 8 d ^ 8 + 40 a ^ 7
b c d ^ 7 + 420 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 1680 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 2
940 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 2352 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 840 a ^ 2 b ^
6 c ^ 6 d ^ 2 + 120 a b ^ 7 c ^ 7 d + 5 b ^ 8 c ^ 8) + b ^ 6 d ^ 7 x ^ 16 (
9129120 a ^ 3 d ^ 3 + 39124800 a ^ 2 b c d ^ 2 + 44015400 a b ^ 2 c ^ 2 d +
13041600 b ^ 3 c ^ 3) + b ^ 8 d ^ 9 x ^ 18 (875160 a d + 972400 b c) + 184
756 c x ^ 9 (10 a ^ 9 d ^ 9 + 405 a ^ 8 b c d ^ 8 + 4320 a ^ 7 b ^ 2 c ^ 2
d ^ 7 + 17640 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 31752 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 264
60 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 10080 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 1620 a ^ 2 b ^
7 c ^ 7 d ^ 2 + 90 a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + 167960 d x ^ 10 (a ^ 9
d ^ 9 + 90 a ^ 8 b c d ^ 8 + 1620 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 10080 a ^ 6 b
^ 3 c ^ 3 d ^ 6 + 26460 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 31752 a ^ 4 b ^ 5 c ^ 5 d
^ 4 + 17640 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 4320 a ^ 2 b ^ 7 c ^ 7 d ^ 2 + 405 a
b ^ 8 c ^ 8 d + 10 b ^ 9 c ^ 9) + b ^ 2 d ^ 3 x ^ 12 (5116320 a ^ 7 d ^ 7
+ 119380800 a ^ 6 b c d ^ 6 + 805820400 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 214885440
0 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 2506996800 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 1289312640
a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 268606800 a b ^ 6 c ^ 6 d + 17054400 b ^ 7 c ^ 7
) + b ^ 3 d ^ 4 x ^ 13 (11085360 a ^ 6 d ^ 6 + 166280400 a ^ 5 b c d ^ 5 +
748261800 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 1330243200 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 99
7682400 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 299304720 a b ^ 5 c ^ 5 d + 27713400 b ^
6 c ^ 6) + b ^ 4 d ^ 5 x ^ 14 (15519504 a ^ 5 d ^ 5 + 155195040 a ^ 4 b c d
^ 4 + 465585120 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 532097280 a ^ 2 b ^ 3 c ^ 3 d ^
2 + 232792560 a b ^ 4 c ^ 4 d + 31039008 b ^ 5 c ^ 5) + b ^ 5 d ^ 6 x ^ 15

```
(14549535 a ^ 4 d ^ 4 + 96996900 a ^ 3 b c d ^ 3 + 187065450 a ^ 2 b ^ 2 c
^ 2 d ^ 2 + 124710300 a b ^ 3 c ^ 3 d + 24249225 b ^ 4 c ^ 4) + b ^ 7 d ^ 8
 x ^ 17 (3695120 a ^ 2 d ^ 2 + 9237800 a b c d + 4618900 b ^ 2 c ^ 2) + 923
78 b ^ 9 d ^ 10 x ^ 19) / 1847560
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(234) = 468$.
time = 0.14, size = 1441, normalized size = 5.76

method	result
norman	Expression too large to display
default	$\frac{b^{9} d^{10} x^{20}}{20}+\frac{\left(9 a \,b^{8} d^{10}+10 b^{9} c \,d^{9}\right) x^{19}}{19}+\frac{\left(36 a^{2} b^{7} d^{10}+90 a \,b^{8} c \,d^{9}+45 b^{9} c^{2} d^{8}\right) x^{18}}{18}+\frac{\left(84 a^{3} b^{6} d^{10}+360 a^{2} b^{7} c \,d^{9}+405 a \,b^{8} c^{2} d^{8}+120 b^{9} c^{3}\right.}{17}$
gosper	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*b^9*d^10*x^20+1/19*(9*a*b^8*d^10+10*b^9*c*d^9)*x^19+1/18*(36*a^2*b^7*d
^10+90*a*b^8*c*d^9+45*b^9*c^2*d^8)*x^18+1/17*(84*a^3*b^6*d^10+360*a^2*b^7*c
*d^9+405*a*b^8*c^2*d^8+120*b^9*c^3*d^7)*x^17+1/16*(126*a^4*b^5*d^10+840*a^3
*b^6*c*d^9+1620*a^2*b^7*c^2*d^8+1080*a*b^8*c^3*d^7+210*b^9*c^4*d^6)*x^16+1/
15*(126*a^5*b^4*d^10+1260*a^4*b^5*c*d^9+3780*a^3*b^6*c^2*d^8+4320*a^2*b^7*c
^3*d^7+1890*a*b^8*c^4*d^6+252*b^9*c^5*d^5)*x^15+1/14*(84*a^6*b^3*d^10+1260*
a^5*b^4*c*d^9+5670*a^4*b^5*c^2*d^8+10080*a^3*b^6*c^3*d^7+7560*a^2*b^7*c^4*d
^6+2268*a*b^8*c^5*d^5+210*b^9*c^6*d^4)*x^14+1/13*(36*a^7*b^2*d^10+840*a^6*b
^3*c*d^9+5670*a^5*b^4*c^2*d^8+15120*a^4*b^5*c^3*d^7+17640*a^3*b^6*c^4*d^6+9
072*a^2*b^7*c^5*d^5+1890*a*b^8*c^6*d^4+120*b^9*c^7*d^3)*x^13+1/12*(9*a^8*b*
d^10+360*a^7*b^2*c*d^9+3780*a^6*b^3*c^2*d^8+15120*a^5*b^4*c^3*d^7+26460*a^4
*b^5*c^4*d^6+21168*a^3*b^6*c^5*d^5+7560*a^2*b^7*c^6*d^4+1080*a*b^8*c^7*d^3+
45*b^9*c^8*d^2)*x^12+1/11*(a^9*d^10+90*a^8*b*c*d^9+1620*a^7*b^2*c^2*d^8+100
80*a^6*b^3*c^3*d^7+26460*a^5*b^4*c^4*d^6+31752*a^4*b^5*c^5*d^5+17640*a^3*b^
6*c^6*d^4+4320*a^2*b^7*c^7*d^3+405*a*b^8*c^8*d^2+10*b^9*c^9*d)*x^11+1/10*(1
0*a^9*c*d^9+405*a^8*b*c^2*d^8+4320*a^7*b^2*c^3*d^7+17640*a^6*b^3*c^4*d^6+31
752*a^5*b^4*c^5*d^5+26460*a^4*b^5*c^6*d^4+10080*a^3*b^6*c^7*d^3+1620*a^2*b^
7*c^8*d^2+90*a*b^8*c^9*d+b^9*c^10)*x^10+1/9*(45*a^9*c^2*d^8+1080*a^8*b*c^3*
d^7+7560*a^7*b^2*c^4*d^6+21168*a^6*b^3*c^5*d^5+26460*a^5*b^4*c^6*d^4+15120*
a^4*b^5*c^7*d^3+3780*a^3*b^6*c^8*d^2+360*a^2*b^7*c^9*d+9*a*b^8*c^10)*x^9+1/
8*(120*a^9*c^3*d^7+1890*a^8*b*c^4*d^6+9072*a^7*b^2*c^5*d^5+17640*a^6*b^3*c^
6*d^4+15120*a^5*b^4*c^7*d^3+5670*a^4*b^5*c^8*d^2+840*a^3*b^6*c^9*d+36*a^2*b
^7*c^10)*x^8+1/7*(210*a^9*c^4*d^6+2268*a^8*b*c^5*d^5+7560*a^7*b^2*c^6*d^4+1
0080*a^6*b^3*c^7*d^3+5670*a^5*b^4*c^8*d^2+1260*a^4*b^5*c^9*d+84*a^3*b^6*c^1
0)*x^7+1/6*(252*a^9*c^5*d^5+1890*a^8*b*c^6*d^4+4320*a^7*b^2*c^7*d^3+3780*a^
6*b^3*c^8*d^2+1260*a^5*b^4*c^9*d+126*a^4*b^5*c^10)*x^6+1/5*(210*a^9*c^6*d^4
```

```
+1080*a^8*b*c^7*d^3+1620*a^7*b^2*c^8*d^2+840*a^6*b^3*c^9*d+126*a^5*b^4*c^10
)*x^5+1/4*(120*a^9*c^7*d^3+405*a^8*b*c^8*d^2+360*a^7*b^2*c^9*d+84*a^6*b^3*c
^10)*x^4+1/3*(45*a^9*c^8*d^2+90*a^8*b*c^9*d+36*a^7*b^2*c^10)*x^3+1/2*(10*a^
9*c^9*d+9*a^8*b*c^10)*x^2+a^9*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(234) = 468$.
time = 0.28, size = 1437, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 +
 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*
c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3
/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(234) = 468$.
time = 0.30, size = 1437, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 +
 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*
c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3
/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(231) = 462$.
time = 0.14, size = 1598, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**10,x)
```

```
[Out] a**9*c**10*x + b**9*d**10*x**20/20 + x**19*(9*a*b**8*d**10/19 + 10*b**9*c*d
**9/19) + x**18*(2*a**2*b**7*d**10 + 5*a*b**8*c*d**9 + 5*b**9*c**2*d**8/2)
+ x**17*(84*a**3*b**6*d**10/17 + 360*a**2*b**7*c*d**9/17 + 405*a*b**8*c**2*
d**8/17 + 120*b**9*c**3*d**7/17) + x**16*(63*a**4*b**5*d**10/8 + 105*a**3*b
**6*c*d**9/2 + 405*a**2*b**7*c**2*d**8/4 + 135*a*b**8*c**3*d**7/2 + 105*b**
9*c**4*d**6/8) + x**15*(42*a**5*b**4*d**10/5 + 84*a**4*b**5*c*d**9 + 252*a*
*3*b**6*c**2*d**8 + 288*a**2*b**7*c**3*d**7 + 126*a*b**8*c**4*d**6 + 84*b**
9*c**5*d**5/5) + x**14*(6*a**6*b**3*d**10 + 90*a**5*b**4*c*d**9 + 405*a**4*
b**5*c**2*d**8 + 720*a**3*b**6*c**3*d**7 + 540*a**2*b**7*c**4*d**6 + 162*a*
b**8*c**5*d**5 + 15*b**9*c**6*d**4) + x**13*(36*a**7*b**2*d**10/13 + 840*a*
*6*b**3*c*d**9/13 + 5670*a**5*b**4*c**2*d**8/13 + 15120*a**4*b**5*c**3*d**7
/13 + 17640*a**3*b**6*c**4*d**6/13 + 9072*a**2*b**7*c**5*d**5/13 + 1890*a*b
**8*c**6*d**4/13 + 120*b**9*c**7*d**3/13) + x**12*(3*a**8*b*d**10/4 + 30*a*
*7*b**2*c*d**9 + 315*a**6*b**3*c**2*d**8 + 1260*a**5*b**4*c**3*d**7 + 2205*
a**4*b**5*c**4*d**6 + 1764*a**3*b**6*c**5*d**5 + 630*a**2*b**7*c**6*d**4 +
90*a*b**8*c**7*d**3 + 15*b**9*c**8*d**2/4) + x**11*(a**9*d**10/11 + 90*a**8
*b*c*d**9/11 + 1620*a**7*b**2*c**2*d**8/11 + 10080*a**6*b**3*c**3*d**7/11 +
 26460*a**5*b**4*c**4*d**6/11 + 31752*a**4*b**5*c**5*d**5/11 + 17640*a**3*b
**6*c**6*d**4/11 + 4320*a**2*b**7*c**7*d**3/11 + 405*a*b**8*c**8*d**2/11 +
10*b**9*c**9*d/11) + x**10*(a**9*c*d**9 + 81*a**8*b*c**2*d**8/2 + 432*a**7*
b**2*c**3*d**7 + 1764*a**6*b**3*c**4*d**6 + 15876*a**5*b**4*c**5*d**5/5 + 2
646*a**4*b**5*c**6*d**4 + 1008*a**3*b**6*c**7*d**3 + 162*a**2*b**7*c**8*d**
2 + 9*a*b**8*c**9*d + b**9*c**10/10) + x**9*(5*a**9*c**2*d**8 + 120*a**8*b*
c**3*d**7 + 840*a**7*b**2*c**4*d**6 + 2352*a**6*b**3*c**5*d**5 + 2940*a**5*
b**4*c**6*d**4 + 1680*a**4*b**5*c**7*d**3 + 420*a**3*b**6*c**8*d**2 + 40*a*
*2*b**7*c**9*d + a*b**8*c**10) + x**8*(15*a**9*c**3*d**7 + 945*a**8*b*c**4*
d**6/4 + 1134*a**7*b**2*c**5*d**5 + 2205*a**6*b**3*c**6*d**4 + 1890*a**5*b*
*4*c**7*d**3 + 2835*a**4*b**5*c**8*d**2/4 + 105*a**3*b**6*c**9*d + 9*a**2*b
**7*c**10/2) + x**7*(30*a**9*c**4*d**6 + 324*a**8*b*c**5*d**5 + 1080*a**7*b
**2*c**6*d**4 + 1440*a**6*b**3*c**7*d**3 + 810*a**5*b**4*c**8*d**2 + 180*a*
*4*b**5*c**9*d + 12*a**3*b**6*c**10) + x**6*(42*a**9*c**5*d**5 + 315*a**8*b
*c**6*d**4 + 720*a**7*b**2*c**7*d**3 + 630*a**6*b**3*c**8*d**2 + 210*a**5*b
**4*c**9*d + 21*a**4*b**5*c**10) + x**5*(42*a**9*c**6*d**4 + 216*a**8*b*c**
7*d**3 + 324*a**7*b**2*c**8*d**2 + 168*a**6*b**3*c**9*d + 126*a**5*b**4*c**
10/5) + x**4*(30*a**9*c**7*d**3 + 405*a**8*b*c**8*d**2/4 + 90*a**7*b**2*c**
9*d + 21*a**6*b**3*c**10) + x**3*(15*a**9*c**8*d**2 + 30*a**8*b*c**9*d + 12
*a**7*b**2*c**10) + x**2*(5*a**9*c**9*d + 9*a**8*b*c**10/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(234) = 468$.
time = 0.00, size = 1744, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^9*(d*x+c)^10,x)

[Out] 1/20*b^9*d^10*x^20 + 10/19*b^9*c*d^9*x^19 + 9/19*a*b^8*d^10*x^19 + 5/2*b^9*c^2*d^8*x^18 + 5*a*b^8*c*d^9*x^18 + 2*a^2*b^7*d^10*x^18 + 120/17*b^9*c^3*d^7*x^17 + 405/17*a*b^8*c^2*d^8*x^17 + 360/17*a^2*b^7*c*d^9*x^17 + 84/17*a^3*b^6*d^10*x^17 + 105/8*b^9*c^4*d^6*x^16 + 135/2*a*b^8*c^3*d^7*x^16 + 405/4*a^2*b^7*c^2*d^8*x^16 + 105/2*a^3*b^6*c*d^9*x^16 + 63/8*a^4*b^5*d^10*x^16 + 84/5*b^9*c^5*d^5*x^15 + 126*a*b^8*c^4*d^6*x^15 + 288*a^2*b^7*c^3*d^7*x^15 + 252*a^3*b^6*c^2*d^8*x^15 + 84*a^4*b^5*c*d^9*x^15 + 42/5*a^5*b^4*d^10*x^15 + 15*b^9*c^6*d^4*x^14 + 162*a*b^8*c^5*d^5*x^14 + 540*a^2*b^7*c^4*d^6*x^14 + 720*a^3*b^6*c^3*d^7*x^14 + 405*a^4*b^5*c^2*d^8*x^14 + 90*a^5*b^4*c*d^9*x^14 + 6*a^6*b^3*d^10*x^14 + 120/13*b^9*c^7*d^3*x^13 + 1890/13*a*b^8*c^6*d^4*x^13 + 9072/13*a^2*b^7*c^5*d^5*x^13 + 17640/13*a^3*b^6*c^4*d^6*x^13 + 15120/13*a^4*b^5*c^3*d^7*x^13 + 5670/13*a^5*b^4*c^2*d^8*x^13 + 840/13*a^6*b^3*c*d^9*x^13 + 36/13*a^7*b^2*d^10*x^13 + 15/4*b^9*c^8*d^2*x^12 + 90*a*b^8*c^7*d^3*x^12 + 630*a^2*b^7*c^6*d^4*x^12 + 1764*a^3*b^6*c^5*d^5*x^12 + 2205*a^4*b^5*c^4*d^6*x^12 + 1260*a^5*b^4*c^3*d^7*x^12 + 315*a^6*b^3*c^2*d^8*x^12 + 30*a^7*b^2*c*d^9*x^12 + 3/4*a^8*b*d^10*x^12 + 10/11*b^9*c^9*d*x^11 + 405/11*a*b^8*c^8*d^2*x^11 + 4320/11*a^2*b^7*c^7*d^3*x^11 + 17640/11*a^3*b^6*c^6*d^4*x^11 + 31752/11*a^4*b^5*c^5*d^5*x^11 + 26460/11*a^5*b^4*c^4*d^6*x^11 + 10080/11*a^6*b^3*c^3*d^7*x^11 + 1620/11*a^7*b^2*c^2*d^8*x^11 + 90/11*a^8*b*c*d^9*x^11 + 1/11*a^9*d^10*x^11 + 1/10*b^9*c^10*x^10 + 9*a*b^8*c^9*d*x^10 + 162*a^2*b^7*c^8*d^2*x^10 + 1008*a^3*b^6*c^7*d^3*x^10 + 2646*a^4*b^5*c^6*d^4*x^10 + 15876/5*a^5*b^4*c^5*d^5*x^10 + 1764*a^6*b^3*c^4*d^6*x^10 + 432*a^7*b^2*c^3*d^7*x^10 + 81/2*a^8*b*c^2*d^8*x^10 + a^9*c*d^9*x^10 + a*b^8*c^10*x^9 + 40*a^2*b^7*c^9*d*x^9 + 420*a^3*b^6*c^8*d^2*x^9 + 1680*a^4*b^5*c^7*d^3*x^9 + 2940*a^5*b^4*c^6*d^4*x^9 + 2352*a^6*b^3*c^5*d^5*x^9 + 840*a^7*b^2*c^4*d^6*x^9 + 120*a^8*b*c^3*d^7*x^9 + 5*a^9*c^2*d^8*x^9 + 9/2*a^2*b^7*c^10*x^8 + 105*a^3*b^6*c^9*d*x^8 + 2835/4*a^4*b^5*c^8*d^2*x^8 + 1890*a^5*b^4*c^7*d^3*x^8 + 2205*a^6*b^3*c^6*d^4*x^8 + 1134*a^7*b^2*c^5*d^5*x^8 + 945/4*a^8*b*c^4*d^6*x^8 + 15*a^9*c^3*d^7*x^8 + 12*a^3*b^6*c^10*x^7 + 180*a^4*b^5*c^9*d*x^7 + 810*a^5*b^4*c^8*d^2*x^7 + 1440*a^6*b^3*c^7*d^3*x^7 + 1080*a^7*b^2*c^6*d^4*x^7 + 324*a^8*b*c^5*d^5*x^7 + 30*a^9*c^4*d^6*x^7 + 21*a^4*b^5*c^10*x^6 + 210*a^5*b^4*c^9*d*x^6 + 630*a^6*b^3*c^8*d^2*x^6 + 720*a^7*b^2*c^7*d^3*x^6 + 315*a^8*b*c^6*d^4*x^6 + 42*a^9*c^5*d^5*x^6 + 126/5*a^5*b^4*c^10*x^5 + 168*a^6*b^3*c^9*d*x^5 + 324*a^7*b^2*c^8*d^2*x^5 + 216*a^8*b*c^7*d^3*x^5 + 42*a^9*c^6*d^4*x^5 + 21*a^6*b^3*c^10*x^4 + 90*a^7*b^2*c^9*d*x^4 + 405/4*a^8*b*c^8*d^2*x^4 + 30*a^9*c^7*d^3*x^4 + 12*a^7*b^2*c^10*x^3 + 30*a^8*b*c^9*d*x^3 + 15*a^9*c^8*d^2*x^3 + 9/2*a^8*b*c^10*x^2 + 5*a^9*c^9*d*x^2 + a^9*c^10*x

Mupad [B]

time = 0.79, size = 1404, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^10,x)

[Out] x^7*(12*a^3*b^6*c^10 + 30*a^9*c^4*d^6 + 180*a^4*b^5*c^9*d + 324*a^8*b*c^5*d
^5 + 810*a^5*b^4*c^8*d^2 + 1440*a^6*b^3*c^7*d^3 + 1080*a^7*b^2*c^6*d^4) + x
^14*(6*a^6*b^3*d^10 + 15*b^9*c^6*d^4 + 162*a*b^8*c^5*d^5 + 90*a^5*b^4*c*d^9
 + 540*a^2*b^7*c^4*d^6 + 720*a^3*b^6*c^3*d^7 + 405*a^4*b^5*c^2*d^8) + x^5*(
(126*a^5*b^4*c^10)/5 + 42*a^9*c^6*d^4 + 168*a^6*b^3*c^9*d + 216*a^8*b*c^7*d
^3 + 324*a^7*b^2*c^8*d^2) + x^16*((63*a^4*b^5*d^10)/8 + (105*b^9*c^4*d^6)/8
 + (135*a*b^8*c^3*d^7)/2 + (105*a^3*b^6*c*d^9)/2 + (405*a^2*b^7*c^2*d^8)/4)
 + x^8*((9*a^2*b^7*c^10)/2 + 15*a^9*c^3*d^7 + 105*a^3*b^6*c^9*d + (945*a^8*
b*c^4*d^6)/4 + (2835*a^4*b^5*c^8*d^2)/4 + 1890*a^5*b^4*c^7*d^3 + 2205*a^6*b
^3*c^6*d^4 + 1134*a^7*b^2*c^5*d^5) + x^13*((36*a^7*b^2*d^10)/13 + (120*b^9*
c^7*d^3)/13 + (1890*a*b^8*c^6*d^4)/13 + (840*a^6*b^3*c*d^9)/13 + (9072*a^2*
b^7*c^5*d^5)/13 + (17640*a^3*b^6*c^4*d^6)/13 + (15120*a^4*b^5*c^3*d^7)/13 +
 (5670*a^5*b^4*c^2*d^8)/13) + x^9*(a*b^8*c^10 + 5*a^9*c^2*d^8 + 40*a^2*b^7*
c^9*d + 120*a^8*b*c^3*d^7 + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 29
40*a^5*b^4*c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6) + x^12*((3
*a^8*b*d^10)/4 + (15*b^9*c^8*d^2)/4 + 90*a*b^8*c^7*d^3 + 30*a^7*b^2*c*d^9 +
 630*a^2*b^7*c^6*d^4 + 1764*a^3*b^6*c^5*d^5 + 2205*a^4*b^5*c^4*d^6 + 1260*a
^5*b^4*c^3*d^7 + 315*a^6*b^3*c^2*d^8) + x^6*(21*a^4*b^5*c^10 + 42*a^9*c^5*d
^5 + 210*a^5*b^4*c^9*d + 315*a^8*b*c^6*d^4 + 630*a^6*b^3*c^8*d^2 + 720*a^7*
b^2*c^7*d^3) + x^15*((42*a^5*b^4*d^10)/5 + (84*b^9*c^5*d^5)/5 + 126*a*b^8*c
^4*d^6 + 84*a^4*b^5*c*d^9 + 288*a^2*b^7*c^3*d^7 + 252*a^3*b^6*c^2*d^8) + x^
10*((b^9*c^10)/10 + a^9*c*d^9 + (81*a^8*b*c^2*d^8)/2 + 162*a^2*b^7*c^8*d^2
+ 1008*a^3*b^6*c^7*d^3 + 2646*a^4*b^5*c^6*d^4 + (15876*a^5*b^4*c^5*d^5)/5 +
 1764*a^6*b^3*c^4*d^6 + 432*a^7*b^2*c^3*d^7 + 9*a*b^8*c^9*d) + x^11*((a^9*d
^10)/11 + (10*b^9*c^9*d)/11 + (405*a*b^8*c^8*d^2)/11 + (4320*a^2*b^7*c^7*d^
3)/11 + (17640*a^3*b^6*c^6*d^4)/11 + (31752*a^4*b^5*c^5*d^5)/11 + (26460*a^
5*b^4*c^4*d^6)/11 + (10080*a^6*b^3*c^3*d^7)/11 + (1620*a^7*b^2*c^2*d^8)/11
+ (90*a^8*b*c*d^9)/11) + a^9*c^10*x + (b^9*d^10*x^20)/20 + (3*a^6*c^7*x^4*(
40*a^3*d^3 + 28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^2*b*c*d^2))/4 + (3*b^6*d^
7*x^17*(28*a^3*d^3 + 40*b^3*c^3 + 135*a*b^2*c^2*d + 120*a^2*b*c*d^2))/17 +
(a^8*c^9*x^2*(10*a*d + 9*b*c))/2 + (b^8*d^9*x^19*(9*a*d + 10*b*c))/19 + 3*a
^7*c^8*x^3*(5*a^2*d^2 + 4*b^2*c^2 + 10*a*b*c*d) + (b^7*d^8*x^18*(4*a^2*d^2
+ 5*b^2*c^2 + 10*a*b*c*d))/2
```

3.1303 $\int (a+bx)^8(c+dx)^{10}\,dx$

Optimal. Leaf size=225

$$\frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3(bc-ad)^5(c+dx)^{14}}{d^9}+\frac{14b^4(bc-$$

```
[Out] 1/11*(-a*d+b*c)^8*(d*x+c)^11/d^9-2/3*b*(-a*d+b*c)^7*(d*x+c)^12/d^9+28/13*b^
2*(-a*d+b*c)^6*(d*x+c)^13/d^9-4*b^3*(-a*d+b*c)^5*(d*x+c)^14/d^9+14/3*b^4*(-
a*d+b*c)^4*(d*x+c)^15/d^9-7/2*b^5*(-a*d+b*c)^3*(d*x+c)^16/d^9+28/17*b^6*(-a
*d+b*c)^2*(d*x+c)^17/d^9-4/9*b^7*(-a*d+b*c)*(d*x+c)^18/d^9+1/19*b^8*(d*x+c)
^19/d^9
```

Rubi [A]
time = 0.61, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+dx)^{14}(bc-ad)^5}{d^9}+\frac{28b^2(c+dx)^{13}(bc-ad)^6}{13d^9}-\frac{2b(c+dx)^{12}(bc-ad)^7}{3d^9}+\frac{(c+dx)^{11}(bc-ad)^8}{11d^9}+\frac{b^8(c+dx)^{19}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^8*(c + d*x)^11)/(11*d^9) - (2*b*(b*c - a*d)^7*(c + d*x)^12)/(3
*d^9) + (28*b^2*(b*c - a*d)^6*(c + d*x)^13)/(13*d^9) - (4*b^3*(b*c - a*d)^5
*(c + d*x)^14)/d^9 + (14*b^4*(b*c - a*d)^4*(c + d*x)^15)/(3*d^9) - (7*b^5*(
b*c - a*d)^3*(c + d*x)^16)/(2*d^9) + (28*b^6*(b*c - a*d)^2*(c + d*x)^17)/(1
7*d^9) - (4*b^7*(b*c - a*d)*(c + d*x)^18)/(9*d^9) + (b^8*(c + d*x)^19)/(19*
d^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^8(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^8(c+dx)^{10}}{d^8}-\frac{8b(bc-ad)^7(c+dx)^{11}}{d^8}+\frac{28b^2(bc-ad)^6(c+dx)^{12}}{d^8}\right.$$

$$=\frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1241 vs. $2(225) = 450$.
time = 0.09, size = 1241, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] a^8*c^10*x + a^7*c^9*(4*b*c + 5*a*d)*x^2 + (a^6*c^8*(28*b^2*c^2 + 80*a*b*c*
d + 45*a^2*d^2)*x^3)/3 + 2*a^5*c^7*(7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c
*d^2 + 15*a^3*d^3)*x^4 + 2*a^4*c^6*(7*b^4*c^4 + 56*a*b^3*c^3*d + 126*a^2*b^
2*c^2*d^2 + 96*a^3*b*c*d^3 + 21*a^4*d^4)*x^5 + (14*a^3*c^5*(2*b^5*c^5 + 25*
a*b^4*c^4*d + 90*a^2*b^3*c^3*d^2 + 120*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 + 9
*a^5*d^5)*x^6)/3 + 2*a^2*c^4*(2*b^6*c^6 + 40*a*b^5*c^5*d + 225*a^2*b^4*c^4*
d^2 + 480*a^3*b^3*c^3*d^3 + 420*a^4*b^2*c^2*d^4 + 144*a^5*b*c*d^5 + 15*a^6*
d^6)*x^7 + a*c^3*(b^7*c^7 + 35*a*b^6*c^6*d + 315*a^2*b^5*c^5*d^2 + 1050*a^3
*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*d^5 + 210*a^6*b*c*d^6
 + 15*a^7*d^7)*x^8 + (c^2*(b^8*c^8 + 80*a*b^7*c^7*d + 1260*a^2*b^6*c^6*d^2
+ 6720*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 14112*a^5*b^3*c^3*d^5 + 58
80*a^6*b^2*c^2*d^6 + 960*a^7*b*c*d^7 + 45*a^8*d^8)*x^9)/9 + c*d*(b^8*c^8 +
36*a*b^7*c^7*d + 336*a^2*b^6*c^6*d^2 + 1176*a^3*b^5*c^5*d^3 + 1764*a^4*b^4*
c^4*d^4 + 1176*a^5*b^3*c^3*d^5 + 336*a^6*b^2*c^2*d^6 + 36*a^7*b*c*d^7 + a^8
*d^8)*x^10 + (d^2*(45*b^8*c^8 + 960*a*b^7*c^7*d + 5880*a^2*b^6*c^6*d^2 + 14
112*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 6720*a^5*b^3*c^3*d^5 + 1260*a
^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + a^8*d^8)*x^11)/11 + (2*b*d^3*(15*b^7*c^7
+ 210*a*b^6*c^6*d + 882*a^2*b^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1050*a^4*b
^3*c^3*d^4 + 315*a^5*b^2*c^2*d^5 + 35*a^6*b*c*d^6 + a^7*d^7)*x^12)/3 + (14*
b^2*d^4*(15*b^6*c^6 + 144*a*b^5*c^5*d + 420*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c
^3*d^3 + 225*a^4*b^2*c^2*d^4 + 40*a^5*b*c*d^5 + 2*a^6*d^6)*x^13)/13 + 2*b^3
*d^5*(9*b^5*c^5 + 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3
 + 25*a^4*b*c*d^4 + 2*a^5*d^5)*x^14 + (2*b^4*d^6*(21*b^4*c^4 + 96*a*b^3*c^3
*d + 126*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 7*a^4*d^4)*x^15)/3 + (b^5*d^7*(
15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2 + 7*a^3*d^3)*x^16)/2 + (b^6*d^
8*(45*b^2*c^2 + 80*a*b*c*d + 28*a^2*d^2)*x^17)/17 + (b^7*d^9*(5*b*c + 4*a*d
)*x^18)/9 + (b^8*d^10*x^19)/19
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1214 vs. $2(225) = 450$.
time = 11.47, size = 1212, normalized size = 5.39

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^8*(c + d*x)^10,x]')
```

```
[Out] x (831402 a ^ 8 c ^ 10 + 831402 a ^ 7 c ^ 9 x (5 a d + 4 b c) + a ^ 6 c ^ 8
 x ^ 2 (12471030 a ^ 2 d ^ 2 + 22170720 a b c d + 7759752 b ^ 2 c ^ 2) + a
^ 5 c ^ 7 x ^ 3 (24942060 a ^ 3 d ^ 3 + 74826180 a ^ 2 b c d ^ 2 + 58198140
 a b ^ 2 c ^ 2 d + 11639628 b ^ 3 c ^ 3) + a ^ 4 c ^ 6 x ^ 4 (34918884 a ^
4 d ^ 4 + 159629184 a ^ 3 b c d ^ 3 + 209513304 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 9
3117024 a b ^ 3 c ^ 3 d + 11639628 b ^ 4 c ^ 4) + a ^ 3 c ^ 5 x ^ 5 (349188
84 a ^ 5 d ^ 5 + 232792560 a ^ 4 b c d ^ 4 + 465585120 a ^ 3 b ^ 2 c ^ 2 d
^ 3 + 349188840 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 96996900 a b ^ 4 c ^ 4 d + 775975
2 b ^ 5 c ^ 5) + a ^ 2 c ^ 4 x ^ 6 (24942060 a ^ 6 d ^ 6 + 239443776 a ^ 5
b c d ^ 5 + 698377680 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 798145920 a ^ 3 b ^ 3 c ^ 3
 d ^ 3 + 374130900 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 66512160 a b ^ 5 c ^ 5 d + 332
5608 b ^ 6 c ^ 6) + 831402 a c ^ 3 x ^ 7 (15 a ^ 7 d ^ 7 + 210 a ^ 6 b c d
^ 6 + 882 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 1470 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 1050 a ^
 3 b ^ 4 c ^ 4 d ^ 3 + 315 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 35 a b ^ 6 c ^ 6 d + b
 ^ 7 c ^ 7) + 554268 b d ^ 3 x ^ 11 (a ^ 7 d ^ 7 + 35 a ^ 6 b c d ^ 6 + 315
 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 1050 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 1470 a ^ 3 b ^ 4
c ^ 4 d ^ 3 + 882 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 210 a b ^ 6 c ^ 6 d + 15 b ^ 7
c ^ 7) + b ^ 5 d ^ 7 x ^ 15 (2909907 a ^ 3 d ^ 3 + 14549535 a ^ 2 b c d ^ 2
 + 18706545 a b ^ 2 c ^ 2 d + 6235515 b ^ 3 c ^ 3) + b ^ 7 d ^ 9 x ^ 17 (36
9512 a d + 461890 b c) + c ^ 2 x ^ 8 (4157010 a ^ 8 d ^ 8 + 88682880 a ^ 7
b c d ^ 7 + 543182640 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 1303638336 a ^ 5 b ^ 3 c ^
3 d ^ 5 + 1357956600 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 620780160 a ^ 3 b ^ 5 c ^ 5
d ^ 3 + 116396280 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 7390240 a b ^ 7 c ^ 7 d + 92378
 b ^ 8 c ^ 8) + d ^ 2 x ^ 10 (75582 a ^ 8 d ^ 8 + 6046560 a ^ 7 b c d ^ 7 +
 95233320 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 507911040 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 111
1055400 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 1066613184 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 4444
22160 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 72558720 a b ^ 7 c ^ 7 d + 3401190 b ^ 8 c
^ 8) + b ^ 2 d ^ 4 x ^ 12 (1790712 a ^ 6 d ^ 6 + 35814240 a ^ 5 b c d ^ 5 +
 201455100 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 429770880 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 37
6049520 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 128931264 a b ^ 5 c ^ 5 d + 13430340 b ^
6 c ^ 6) + b ^ 3 d ^ 5 x ^ 13 (3325608 a ^ 5 d ^ 5 + 41570100 a ^ 4 b c d ^
 4 + 149652360 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 199536480 a ^ 2 b ^ 3 c ^ 3 d ^ 2
+ 99768240 a b ^ 4 c ^ 4 d + 14965236 b ^ 5 c ^ 5) + b ^ 4 d ^ 6 x ^ 14 (38
79876 a ^ 4 d ^ 4 + 31039008 a ^ 3 b c d ^ 3 + 69837768 a ^ 2 b ^ 2 c ^ 2 d
 ^ 2 + 53209728 a b ^ 3 c ^ 3 d + 11639628 b ^ 4 c ^ 4) + b ^ 6 d ^ 8 x ^ 1
6 (1369368 a ^ 2 d ^ 2 + 3912480 a b c d + 2200770 b ^ 2 c ^ 2) + 43758 b ^
 8 d ^ 10 x ^ 18 + 831402 c d x ^ 9 (a ^ 8 d ^ 8 + 36 a ^ 7 b c d ^ 7 + 336
 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 1176 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 1764 a ^ 4 b ^ 4
c ^ 4 d ^ 4 + 1176 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 336 a ^ 2 b ^ 6 c ^ 6 d ^ 2 +
36 a b ^ 7 c ^ 7 d + b ^ 8 c ^ 8)) / 831402
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(209) = 418$.

time = 0.15, size = 1291, normalized size = 5.74

method	result
norman	$a^8c^{10}x+\left(5a^8c^9d+4a^7b\,c^{10}\right)x^2+\left(15a^8c^8d^2+\frac{80}{3}a^7b\,c^9d+\frac{28}{3}a^6b^2c^{10}\right)x^3+\left(30a^8c^7d^3+90a^7b\,c^8d\right.$
default	$\frac{b^8d^{10}x^{19}}{19}+\frac{(8a\,b^7d^{10}+10b^8c\,d^9)x^{18}}{18}+\frac{(28b^6a^2d^{10}+80a\,b^7c\,d^9+45b^8c^2d^8)x^{17}}{17}+\frac{(56a^3b^5d^{10}+280b^6a^2c\,d^9+360a\,b^7c^2d^8+120b^8}{16}$
gosper	$\frac{2}{3}x^{12}a^7b\,d^{10}+10x^{12}b^8c^7d^3+\frac{28}{13}x^{13}a^6b^2d^{10}+\frac{210}{13}x^{13}b^8c^6d^4+\frac{14}{3}x^{15}a^4b^4d^{10}+14x^{15}b^8c^4d^6+\frac{7}{2}x^{16}a^3$
risch	$\frac{2}{3}x^{12}a^7b\,d^{10}+10x^{12}b^8c^7d^3+\frac{28}{13}x^{13}a^6b^2d^{10}+\frac{210}{13}x^{13}b^8c^6d^4+\frac{14}{3}x^{15}a^4b^4d^{10}+14x^{15}b^8c^4d^6+\frac{7}{2}x^{16}a^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*b^8*d^10*x^19+1/18*(8*a*b^7*d^10+10*b^8*c*d^9)*x^18+1/17*(28*a^2*b^6*d
^10+80*a*b^7*c*d^9+45*b^8*c^2*d^8)*x^17+1/16*(56*a^3*b^5*d^10+280*a^2*b^6*c
*d^9+360*a*b^7*c^2*d^8+120*b^8*c^3*d^7)*x^16+1/15*(70*a^4*b^4*d^10+560*a^3*
b^5*c*d^9+1260*a^2*b^6*c^2*d^8+960*a*b^7*c^3*d^7+210*b^8*c^4*d^6)*x^15+1/14
*(56*a^5*b^3*d^10+700*a^4*b^4*c*d^9+2520*a^3*b^5*c^2*d^8+3360*a^2*b^6*c^3*d
^7+1680*a*b^7*c^4*d^6+252*b^8*c^5*d^5)*x^14+1/13*(28*a^6*b^2*d^10+560*a^5*b
^3*c*d^9+3150*a^4*b^4*c^2*d^8+6720*a^3*b^5*c^3*d^7+5880*a^2*b^6*c^4*d^6+201
6*a*b^7*c^5*d^5+210*b^8*c^6*d^4)*x^13+1/12*(8*a^7*b*d^10+280*a^6*b^2*c*d^9+
2520*a^5*b^3*c^2*d^8+8400*a^4*b^4*c^3*d^7+11760*a^3*b^5*c^4*d^6+7056*a^2*b^
6*c^5*d^5+1680*a*b^7*c^6*d^4+120*b^8*c^7*d^3)*x^12+1/11*(a^8*d^10+80*a^7*b*
c*d^9+1260*a^6*b^2*c^2*d^8+6720*a^5*b^3*c^3*d^7+14700*a^4*b^4*c^4*d^6+14112
*a^3*b^5*c^5*d^5+5880*a^2*b^6*c^6*d^4+960*a*b^7*c^7*d^3+45*b^8*c^8*d^2)*x^1
1+1/10*(10*a^8*c*d^9+360*a^7*b*c^2*d^8+3360*a^6*b^2*c^3*d^7+11760*a^5*b^3*c
^4*d^6+17640*a^4*b^4*c^5*d^5+11760*a^3*b^5*c^6*d^4+3360*a^2*b^6*c^7*d^3+360
*a*b^7*c^8*d^2+10*b^8*c^9*d)*x^10+1/9*(45*a^8*c^2*d^8+960*a^7*b*c^3*d^7+588
0*a^6*b^2*c^4*d^6+14112*a^5*b^3*c^5*d^5+14700*a^4*b^4*c^6*d^4+6720*a^3*b^5*
c^7*d^3+1260*a^2*b^6*c^8*d^2+80*a*b^7*c^9*d+b^8*c^10)*x^9+1/8*(120*a^8*c^3*
d^7+1680*a^7*b*c^4*d^6+7056*a^6*b^2*c^5*d^5+11760*a^5*b^3*c^6*d^4+8400*a^4*
b^4*c^7*d^3+2520*a^3*b^5*c^8*d^2+280*a^2*b^6*c^9*d+8*a*b^7*c^10)*x^8+1/7*(2
10*a^8*c^4*d^6+2016*a^7*b*c^5*d^5+5880*a^6*b^2*c^6*d^4+6720*a^5*b^3*c^7*d^3
+3150*a^4*b^4*c^8*d^2+560*a^3*b^5*c^9*d+28*a^2*b^6*c^10)*x^7+1/6*(252*a^8*c
^5*d^5+1680*a^7*b*c^6*d^4+3360*a^6*b^2*c^7*d^3+2520*a^5*b^3*c^8*d^2+700*a^4
*b^4*c^9*d+56*a^3*b^5*c^10)*x^6+1/5*(210*a^8*c^6*d^4+960*a^7*b*c^7*d^3+1260
*a^6*b^2*c^8*d^2+560*a^5*b^3*c^9*d+70*a^4*b^4*c^10)*x^5+1/4*(120*a^8*c^7*d^
3+360*a^7*b*c^8*d^2+280*a^6*b^2*c^9*d+56*a^5*b^3*c^10)*x^4+1/3*(45*a^8*c^8*
d^2+80*a^7*b*c^9*d+28*a^6*b^2*c^10)*x^3+1/2*(10*a^8*c^9*d+8*a^7*b*c^10)*x^2
+a^8*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(209) = 418$.

time = 0.29, size = 1283, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(209) = 418$.

time = 0.29, size = 1283, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
```

```
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. $2(207) = 414$.
time = 0.13, size = 1428, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**10,x)
```

```
[Out] a**8*c**10*x + b**8*d**10*x**19/19 + x**18*(4*a*b**7*d**10/9 + 5*b**8*c*d**
9/9) + x**17*(28*a**2*b**6*d**10/17 + 80*a*b**7*c*d**9/17 + 45*b**8*c**2*d*
*8/17) + x**16*(7*a**3*b**5*d**10/2 + 35*a**2*b**6*c*d**9/2 + 45*a*b**7*c**
2*d**8/2 + 15*b**8*c**3*d**7/2) + x**15*(14*a**4*b**4*d**10/3 + 112*a**3*b*
*5*c*d**9/3 + 84*a**2*b**6*c**2*d**8 + 64*a*b**7*c**3*d**7 + 14*b**8*c**4*d
**6) + x**14*(4*a**5*b**3*d**10 + 50*a**4*b**4*c*d**9 + 180*a**3*b**5*c**2*
d**8 + 240*a**2*b**6*c**3*d**7 + 120*a*b**7*c**4*d**6 + 18*b**8*c**5*d**5)
+ x**13*(28*a**6*b**2*d**10/13 + 560*a**5*b**3*c*d**9/13 + 3150*a**4*b**4*c
**2*d**8/13 + 6720*a**3*b**5*c**3*d**7/13 + 5880*a**2*b**6*c**4*d**6/13 + 2
016*a*b**7*c**5*d**5/13 + 210*b**8*c**6*d**4/13) + x**12*(2*a**7*b*d**10/3
+ 70*a**6*b**2*c*d**9/3 + 210*a**5*b**3*c**2*d**8 + 700*a**4*b**4*c**3*d**7
 + 980*a**3*b**5*c**4*d**6 + 588*a**2*b**6*c**5*d**5 + 140*a*b**7*c**6*d**4
 + 10*b**8*c**7*d**3) + x**11*(a**8*d**10/11 + 80*a**7*b*c*d**9/11 + 1260*a
**6*b**2*c**2*d**8/11 + 6720*a**5*b**3*c**3*d**7/11 + 14700*a**4*b**4*c**4*
d**6/11 + 14112*a**3*b**5*c**5*d**5/11 + 5880*a**2*b**6*c**6*d**4/11 + 960*
```

```
a*b**7*c**7*d**3/11 + 45*b**8*c**8*d**2/11) + x**10*(a**8*c*d**9 + 36*a**7*
b*c**2*d**8 + 336*a**6*b**2*c**3*d**7 + 1176*a**5*b**3*c**4*d**6 + 1764*a**
4*b**4*c**5*d**5 + 1176*a**3*b**5*c**6*d**4 + 336*a**2*b**6*c**7*d**3 + 36*
a*b**7*c**8*d**2 + b**8*c**9*d) + x**9*(5*a**8*c**2*d**8 + 320*a**7*b*c**3*
d**7/3 + 1960*a**6*b**2*c**4*d**6/3 + 1568*a**5*b**3*c**5*d**5 + 4900*a**4*
b**4*c**6*d**4/3 + 2240*a**3*b**5*c**7*d**3/3 + 140*a**2*b**6*c**8*d**2 + 8
0*a*b**7*c**9*d/9 + b**8*c**10/9) + x**8*(15*a**8*c**3*d**7 + 210*a**7*b*c*
*4*d**6 + 882*a**6*b**2*c**5*d**5 + 1470*a**5*b**3*c**6*d**4 + 1050*a**4*b*
*4*c**7*d**3 + 315*a**3*b**5*c**8*d**2 + 35*a**2*b**6*c**9*d + a*b**7*c**10
) + x**7*(30*a**8*c**4*d**6 + 288*a**7*b*c**5*d**5 + 840*a**6*b**2*c**6*d**
4 + 960*a**5*b**3*c**7*d**3 + 450*a**4*b**4*c**8*d**2 + 80*a**3*b**5*c**9*d
 + 4*a**2*b**6*c**10) + x**6*(42*a**8*c**5*d**5 + 280*a**7*b*c**6*d**4 + 56
0*a**6*b**2*c**7*d**3 + 420*a**5*b**3*c**8*d**2 + 350*a**4*b**4*c**9*d/3 +
28*a**3*b**5*c**10/3) + x**5*(42*a**8*c**6*d**4 + 192*a**7*b*c**7*d**3 + 25
2*a**6*b**2*c**8*d**2 + 112*a**5*b**3*c**9*d + 14*a**4*b**4*c**10) + x**4*(
30*a**8*c**7*d**3 + 90*a**7*b*c**8*d**2 + 70*a**6*b**2*c**9*d + 14*a**5*b**
3*c**10) + x**3*(15*a**8*c**8*d**2 + 80*a**7*b*c**9*d/3 + 28*a**6*b**2*c**1
0/3) + x**2*(5*a**8*c**9*d + 4*a**7*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1478 vs. $2(209) = 418$.

time = 0.00, size = 1558, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x)
```

```
[Out] 1/19*b^8*d^10*x^19 + 5/9*b^8*c*d^9*x^18 + 4/9*a*b^7*d^10*x^18 + 45/17*b^8*c
^2*d^8*x^17 + 80/17*a*b^7*c*d^9*x^17 + 28/17*a^2*b^6*d^10*x^17 + 15/2*b^8*c
^3*d^7*x^16 + 45/2*a*b^7*c^2*d^8*x^16 + 35/2*a^2*b^6*c*d^9*x^16 + 7/2*a^3*b
^5*d^10*x^16 + 14*b^8*c^4*d^6*x^15 + 64*a*b^7*c^3*d^7*x^15 + 84*a^2*b^6*c^2
*d^8*x^15 + 112/3*a^3*b^5*c*d^9*x^15 + 14/3*a^4*b^4*d^10*x^15 + 18*b^8*c^5*
d^5*x^14 + 120*a*b^7*c^4*d^6*x^14 + 240*a^2*b^6*c^3*d^7*x^14 + 180*a^3*b^5*
c^2*d^8*x^14 + 50*a^4*b^4*c*d^9*x^14 + 4*a^5*b^3*d^10*x^14 + 210/13*b^8*c^6
*d^4*x^13 + 2016/13*a*b^7*c^5*d^5*x^13 + 5880/13*a^2*b^6*c^4*d^6*x^13 + 672
0/13*a^3*b^5*c^3*d^7*x^13 + 3150/13*a^4*b^4*c^2*d^8*x^13 + 560/13*a^5*b^3*c
*d^9*x^13 + 28/13*a^6*b^2*d^10*x^13 + 10*b^8*c^7*d^3*x^12 + 140*a*b^7*c^6*d
^4*x^12 + 588*a^2*b^6*c^5*d^5*x^12 + 980*a^3*b^5*c^4*d^6*x^12 + 700*a^4*b^4
*c^3*d^7*x^12 + 210*a^5*b^3*c^2*d^8*x^12 + 70/3*a^6*b^2*c*d^9*x^12 + 2/3*a^
7*b*d^10*x^12 + 45/11*b^8*c^8*d^2*x^11 + 960/11*a*b^7*c^7*d^3*x^11 + 5880/1
1*a^2*b^6*c^6*d^4*x^11 + 14112/11*a^3*b^5*c^5*d^5*x^11 + 14700/11*a^4*b^4*c
^4*d^6*x^11 + 6720/11*a^5*b^3*c^3*d^7*x^11 + 1260/11*a^6*b^2*c^2*d^8*x^11 +
 80/11*a^7*b*c*d^9*x^11 + 1/11*a^8*d^10*x^11 + b^8*c^9*d*x^10 + 36*a*b^7*c^
8*d^2*x^10 + 336*a^2*b^6*c^7*d^3*x^10 + 1176*a^3*b^5*c^6*d^4*x^10 + 1764*a^
4*b^4*c^5*d^5*x^10 + 1176*a^5*b^3*c^4*d^6*x^10 + 336*a^6*b^2*c^3*d^7*x^10 +
```

```
 36*a^7*b*c^2*d^8*x^10 + a^8*c*d^9*x^10 + 1/9*b^8*c^10*x^9 + 80/9*a*b^7*c^9
*d*x^9 + 140*a^2*b^6*c^8*d^2*x^9 + 2240/3*a^3*b^5*c^7*d^3*x^9 + 4900/3*a^4*
b^4*c^6*d^4*x^9 + 1568*a^5*b^3*c^5*d^5*x^9 + 1960/3*a^6*b^2*c^4*d^6*x^9 + 3
20/3*a^7*b*c^3*d^7*x^9 + 5*a^8*c^2*d^8*x^9 + a*b^7*c^10*x^8 + 35*a^2*b^6*c^
9*d*x^8 + 315*a^3*b^5*c^8*d^2*x^8 + 1050*a^4*b^4*c^7*d^3*x^8 + 1470*a^5*b^3
*c^6*d^4*x^8 + 882*a^6*b^2*c^5*d^5*x^8 + 210*a^7*b*c^4*d^6*x^8 + 15*a^8*c^3
*d^7*x^8 + 4*a^2*b^6*c^10*x^7 + 80*a^3*b^5*c^9*d*x^7 + 450*a^4*b^4*c^8*d^2*
x^7 + 960*a^5*b^3*c^7*d^3*x^7 + 840*a^6*b^2*c^6*d^4*x^7 + 288*a^7*b*c^5*d^5
*x^7 + 30*a^8*c^4*d^6*x^7 + 28/3*a^3*b^5*c^10*x^6 + 350/3*a^4*b^4*c^9*d*x^6
 + 420*a^5*b^3*c^8*d^2*x^6 + 560*a^6*b^2*c^7*d^3*x^6 + 280*a^7*b*c^6*d^4*x^
6 + 42*a^8*c^5*d^5*x^6 + 14*a^4*b^4*c^10*x^5 + 112*a^5*b^3*c^9*d*x^5 + 252*
a^6*b^2*c^8*d^2*x^5 + 192*a^7*b*c^7*d^3*x^5 + 42*a^8*c^6*d^4*x^5 + 14*a^5*b
^3*c^10*x^4 + 70*a^6*b^2*c^9*d*x^4 + 90*a^7*b*c^8*d^2*x^4 + 30*a^8*c^7*d^3*
x^4 + 28/3*a^6*b^2*c^10*x^3 + 80/3*a^7*b*c^9*d*x^3 + 15*a^8*c^8*d^2*x^3 + 4
*a^7*b*c^10*x^2 + 5*a^8*c^9*d*x^2 + a^8*c^10*x
```

Mupad [B]
time = 0.71, size = 1253, normalized size = 5.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^10,x)
```

```
[Out] x^7*(4*a^2*b^6*c^10 + 30*a^8*c^4*d^6 + 80*a^3*b^5*c^9*d + 288*a^7*b*c^5*d^5
 + 450*a^4*b^4*c^8*d^2 + 960*a^5*b^3*c^7*d^3 + 840*a^6*b^2*c^6*d^4) + x^13*
((28*a^6*b^2*d^10)/13 + (210*b^8*c^6*d^4)/13 + (2016*a*b^7*c^5*d^5)/13 + (5
60*a^5*b^3*c*d^9)/13 + (5880*a^2*b^6*c^4*d^6)/13 + (6720*a^3*b^5*c^3*d^7)/1
3 + (3150*a^4*b^4*c^2*d^8)/13) + x^8*(a*b^7*c^10 + 15*a^8*c^3*d^7 + 35*a^2*
b^6*c^9*d + 210*a^7*b*c^4*d^6 + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3
+ 1470*a^5*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5) + x^12*((2*a^7*b*d^10)/3 + 10
*b^8*c^7*d^3 + 140*a*b^7*c^6*d^4 + (70*a^6*b^2*c*d^9)/3 + 588*a^2*b^6*c^5*d
^5 + 980*a^3*b^5*c^4*d^6 + 700*a^4*b^4*c^3*d^7 + 210*a^5*b^3*c^2*d^8) + x^1
0*(a^8*c*d^9 + b^8*c^9*d + 36*a*b^7*c^8*d^2 + 36*a^7*b*c^2*d^8 + 336*a^2*b^
6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*
d^6 + 336*a^6*b^2*c^3*d^7) + x^5*(14*a^4*b^4*c^10 + 42*a^8*c^6*d^4 + 112*a^
5*b^3*c^9*d + 192*a^7*b*c^7*d^3 + 252*a^6*b^2*c^8*d^2) + x^15*((14*a^4*b^4*
d^10)/3 + 14*b^8*c^4*d^6 + 64*a*b^7*c^3*d^7 + (112*a^3*b^5*c*d^9)/3 + 84*a^
2*b^6*c^2*d^8) + x^6*((28*a^3*b^5*c^10)/3 + 42*a^8*c^5*d^5 + (350*a^4*b^4*c
^9*d)/3 + 280*a^7*b*c^6*d^4 + 420*a^5*b^3*c^8*d^2 + 560*a^6*b^2*c^7*d^3) +
x^14*(4*a^5*b^3*d^10 + 18*b^8*c^5*d^5 + 120*a*b^7*c^4*d^6 + 50*a^4*b^4*c*d^
9 + 240*a^2*b^6*c^3*d^7 + 180*a^3*b^5*c^2*d^8) + x^9*((b^8*c^10)/9 + 5*a^8*
c^2*d^8 + (320*a^7*b*c^3*d^7)/3 + 140*a^2*b^6*c^8*d^2 + (2240*a^3*b^5*c^7*d
^3)/3 + (4900*a^4*b^4*c^6*d^4)/3 + 1568*a^5*b^3*c^5*d^5 + (1960*a^6*b^2*c^4
*d^6)/3 + (80*a*b^7*c^9*d)/9) + x^11*((a^8*d^10)/11 + (45*b^8*c^8*d^2)/11 +
```

(960*a*b^7*c^7*d^3)/11 + (5880*a^2*b^6*c^6*d^4)/11 + (14112*a^3*b^5*c^5*d^5)/11 + (14700*a^4*b^4*c^4*d^6)/11 + (6720*a^5*b^3*c^3*d^7)/11 + (1260*a^6*b^2*c^2*d^8)/11 + (80*a^7*b*c*d^9)/11) + a^8*c^10*x + (b^8*d^10*x^19)/19 + 2*a^5*c^7*x^4*(15*a^3*d^3 + 7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c*d^2) + (b^5*d^7*x^16*(7*a^3*d^3 + 15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2))/2 + a^7*c^9*x^2*(5*a*d + 4*b*c) + (b^7*d^9*x^18*(4*a*d + 5*b*c))/9 + (a^6*c^8*x^3*(45*a^2*d^2 + 28*b^2*c^2 + 80*a*b*c*d))/3 + (b^6*d^8*x^17*(28*a^2*d^2 + 45*b^2*c^2 + 80*a*b*c*d))/17

3.1304 $\int (a+bx)^7(c+dx)^{10}\,dx$

Optimal. Leaf size=200

$$-\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+\frac{5b^3(bc-ad)^4(c+dx)^{14}}{2d^8}-\frac{7b^4(b}{}$$

```
[Out] -1/11*(-a*d+b*c)^7*(d*x+c)^11/d^8+7/12*b*(-a*d+b*c)^6*(d*x+c)^12/d^8-21/13*
b^2*(-a*d+b*c)^5*(d*x+c)^13/d^8+5/2*b^3*(-a*d+b*c)^4*(d*x+c)^14/d^8-7/3*b^4
*(-a*d+b*c)^3*(d*x+c)^15/d^8+21/16*b^5*(-a*d+b*c)^2*(d*x+c)^16/d^8-7/17*b^6
*(-a*d+b*c)*(d*x+c)^17/d^8+1/18*b^7*(d*x+c)^18/d^8
```

Rubi [A]
time = 0.53, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+dx)^{13}(bc-ad)^5}{13d^8}+\frac{7b(c+dx)^{12}(bc-ad)^6}{12d^8}-\frac{(c+dx)^{11}(bc-ad)^7}{11d^8}+\frac{b^7(c+dx)^{18}}{18d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^7*(c + d*x)^11)/d^8 + (7*b*(b*c - a*d)^6*(c + d*x)^12)/(
12*d^8) - (21*b^2*(b*c - a*d)^5*(c + d*x)^13)/(13*d^8) + (5*b^3*(b*c - a*d)
^4*(c + d*x)^14)/(2*d^8) - (7*b^4*(b*c - a*d)^3*(c + d*x)^15)/(3*d^8) + (21
*b^5*(b*c - a*d)^2*(c + d*x)^16)/(16*d^8) - (7*b^6*(b*c - a*d)*(c + d*x)^17
)/(17*d^8) + (b^7*(c + d*x)^18)/(18*d^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^7(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^7(c+dx)^{10}}{d^7}+\frac{7b(bc-ad)^6(c+dx)^{11}}{d^7}-\frac{21b^2(bc-ad)^5(c+dx)}{d^7}\right.$$
$$= -\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1105 vs. 2(200) = 400.

time = 0.08, size = 1105, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] a^7*c^10*x + (a^6*c^9*(7*b*c + 10*a*d)*x^2)/2 + (a^5*c^8*(21*b^2*c^2 + 70*a
*b*c*d + 45*a^2*d^2)*x^3)/3 + (5*a^4*c^7*(7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a
^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 7*a^3*c^6*(b^4*c^4 + 10*a*b^3*c^3*d + 27*
a^2*b^2*c^2*d^2 + 24*a^3*b*c*d^3 + 6*a^4*d^4)*x^5 + (7*a^2*c^5*(3*b^5*c^5 +
 50*a*b^4*c^4*d + 225*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d
^4 + 36*a^5*d^5)*x^6)/6 + a*c^4*(b^6*c^6 + 30*a*b^5*c^5*d + 225*a^2*b^4*c^4
*d^2 + 600*a^3*b^3*c^3*d^3 + 630*a^4*b^2*c^2*d^4 + 252*a^5*b*c*d^5 + 30*a^6
*d^6)*x^7 + (c^3*(b^7*c^7 + 70*a*b^6*c^6*d + 945*a^2*b^5*c^5*d^2 + 4200*a^3
*b^4*c^4*d^3 + 7350*a^4*b^3*c^3*d^4 + 5292*a^5*b^2*c^2*d^5 + 1470*a^6*b*c*d
^6 + 120*a^7*d^7)*x^8)/8 + (5*c^2*d*(2*b^7*c^7 + 63*a*b^6*c^6*d + 504*a^2*b
^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1764*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*
d^5 + 168*a^6*b*c*d^6 + 9*a^7*d^7)*x^9)/9 + (c*d^2*(9*b^7*c^7 + 168*a*b^6*c
^6*d + 882*a^2*b^5*c^5*d^2 + 1764*a^3*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 +
504*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6 + 2*a^7*d^7)*x^10)/2 + (d^3*(120*b^7*c
^7 + 1470*a*b^6*c^6*d + 5292*a^2*b^5*c^5*d^2 + 7350*a^3*b^4*c^4*d^3 + 4200*
a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5 + 70*a^6*b*c*d^6 + a^7*d^7)*x^11)/11
+ (7*b*d^4*(30*b^6*c^6 + 252*a*b^5*c^5*d + 630*a^2*b^4*c^4*d^2 + 600*a^3*b^
3*c^3*d^3 + 225*a^4*b^2*c^2*d^4 + 30*a^5*b*c*d^5 + a^6*d^6)*x^12)/12 + (7*b
^2*d^5*(36*b^5*c^5 + 210*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 + 225*a^3*b^2*c^
2*d^3 + 50*a^4*b*c*d^4 + 3*a^5*d^5)*x^13)/13 + (5*b^3*d^6*(6*b^4*c^4 + 24*a
*b^3*c^3*d + 27*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + a^4*d^4)*x^14)/2 + (b^4*
d^7*(24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2 + 7*a^3*d^3)*x^15)/3 + (b
^5*d^8*(45*b^2*c^2 + 70*a*b*c*d + 21*a^2*d^2)*x^16)/16 + (b^6*d^9*(10*b*c +
 7*a*d)*x^17)/17 + (b^7*d^10*x^18)/18
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1072 vs. $2(200) = 400$.
time = 10.35, size = 1070, normalized size = 5.35

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^7*(c + d*x)^10,x]')
```

```
[Out] x (350064 a ^ 7 c ^ 10 + 175032 a ^ 6 c ^ 9 x (10 a d + 7 b c) + a ^ 5 c ^
8 x ^ 2 (5250960 a ^ 2 d ^ 2 + 8168160 a b c d + 2450448 b ^ 2 c ^ 2) + a ^
 4 c ^ 7 x ^ 3 (10501920 a ^ 3 d ^ 3 + 27567540 a ^ 2 b c d ^ 2 + 18378360
a b ^ 2 c ^ 2 d + 3063060 b ^ 3 c ^ 3) + a ^ 3 c ^ 6 x ^ 4 (14702688 a ^ 4
d ^ 4 + 58810752 a ^ 3 b c d ^ 3 + 66162096 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 24504
```

```
480 a b ^ 3 c ^ 3 d + 2450448 b ^ 4 c ^ 4) + a ^ 2 c ^ 5 x ^ 5 (14702688 a
^ 5 d ^ 5 + 85765680 a ^ 4 b c d ^ 4 + 147026880 a ^ 3 b ^ 2 c ^ 2 d ^ 3 +
91891800 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 20420400 a b ^ 4 c ^ 4 d + 1225224 b ^ 5
 c ^ 5) + 350064 a c ^ 4 x ^ 6 (30 a ^ 6 d ^ 6 + 252 a ^ 5 b c d ^ 5 + 630
a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 600 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 225 a ^ 2 b ^ 4 c ^
 4 d ^ 2 + 30 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + 204204 b d ^ 4 x ^ 11 (a ^ 6
 d ^ 6 + 30 a ^ 5 b c d ^ 5 + 225 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 600 a ^ 3 b ^ 3
 c ^ 3 d ^ 3 + 630 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 252 a b ^ 5 c ^ 5 d + 30 b ^ 6
 c ^ 6) + b ^ 4 d ^ 7 x ^ 14 (816816 a ^ 3 d ^ 3 + 4900896 a ^ 2 b c d ^ 2
+ 7351344 a b ^ 2 c ^ 2 d + 2800512 b ^ 3 c ^ 3) + b ^ 6 d ^ 9 x ^ 16 (1441
44 a d + 205920 b c) + c ^ 3 x ^ 7 (5250960 a ^ 7 d ^ 7 + 64324260 a ^ 6 b
c d ^ 6 + 231567336 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 321621300 a ^ 4 b ^ 3 c ^ 3 d
 ^ 4 + 183783600 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 41351310 a ^ 2 b ^ 5 c ^ 5 d ^ 2
 + 3063060 a b ^ 6 c ^ 6 d + 43758 b ^ 7 c ^ 7) + d ^ 3 x ^ 10 (31824 a ^ 7
 d ^ 7 + 2227680 a ^ 6 b c d ^ 6 + 30073680 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 13366
0800 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 233906400 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 16841260
8 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 46781280 a b ^ 6 c ^ 6 d + 3818880 b ^ 7 c ^ 7)
 + b ^ 2 d ^ 5 x ^ 12 (565488 a ^ 5 d ^ 5 + 9424800 a ^ 4 b c d ^ 4 + 42411
600 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 67858560 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 39584160 a
 b ^ 4 c ^ 4 d + 6785856 b ^ 5 c ^ 5) + b ^ 3 d ^ 6 x ^ 13 (875160 a ^ 4 d
^ 4 + 8751600 a ^ 3 b c d ^ 3 + 23629320 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 21003840
 a b ^ 3 c ^ 3 d + 5250960 b ^ 4 c ^ 4) + b ^ 5 d ^ 8 x ^ 15 (459459 a ^ 2
d ^ 2 + 1531530 a b c d + 984555 b ^ 2 c ^ 2) + 19448 b ^ 7 d ^ 10 x ^ 17 +
 194480 c ^ 2 d x ^ 8 (9 a ^ 7 d ^ 7 + 168 a ^ 6 b c d ^ 6 + 882 a ^ 5 b ^
2 c ^ 2 d ^ 5 + 1764 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 1470 a ^ 3 b ^ 4 c ^ 4 d ^ 3
 + 504 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 63 a b ^ 6 c ^ 6 d + 2 b ^ 7 c ^ 7) + 1750
32 c d ^ 2 x ^ 9 (2 a ^ 7 d ^ 7 + 63 a ^ 6 b c d ^ 6 + 504 a ^ 5 b ^ 2 c ^
2 d ^ 5 + 1470 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 1764 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 882
 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 168 a b ^ 6 c ^ 6 d + 9 b ^ 7 c ^ 7)) / 350064
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(184) = 368$.
time = 0.13, size = 1141, normalized size = 5.70

method	result
norman	$a^7c^{10}x + \left(5a^7c^9d + \frac{7}{2}a^6b\,c^{10}\right)x^2 + \left(15a^7c^8d^2 + \frac{70}{3}a^6b\,c^9d + 7a^5b^2c^{10}\right)x^3 + \left(30a^7c^7d^3 + \frac{315}{4}a^6b\,c^8d$
default	$\frac{b^7d^{10}x^{18}}{18} + \frac{(7a\,b^6d^{10}+10b^7c\,d^9)x^{17}}{17} + \frac{(21a^2b^5d^{10}+70a\,b^6c\,d^9+45b^7c^2d^8)x^{16}}{16} + \frac{(35a^3b^4d^{10}+210a^2b^5c\,d^9+315a\,b^6c^2d^8+120b^7}{15}$
gosper	$5x^9a^7c^2d^8 + 5x^2a^7c^9d + \frac{7}{2}x^2a^6b\,c^{10} + 15x^3a^7c^8d^2 + 7x^3a^5b^2c^{10} + 30x^4a^7c^7d^3 + \frac{35}{4}x^4a^4b^3c^{10} + 42x$
risch	$5x^9a^7c^2d^8 + 5x^2a^7c^9d + \frac{7}{2}x^2a^6b\,c^{10} + 15x^3a^7c^8d^2 + 7x^3a^5b^2c^{10} + 30x^4a^7c^7d^3 + \frac{35}{4}x^4a^4b^3c^{10} + 42x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*b^7*d^10*x^18+1/17*(7*a*b^6*d^10+10*b^7*c*d^9)*x^17+1/16*(21*a^2*b^5*d
^10+70*a*b^6*c*d^9+45*b^7*c^2*d^8)*x^16+1/15*(35*a^3*b^4*d^10+210*a^2*b^5*c
*d^9+315*a*b^6*c^2*d^8+120*b^7*c^3*d^7)*x^15+1/14*(35*a^4*b^3*d^10+350*a^3*
b^4*c*d^9+945*a^2*b^5*c^2*d^8+840*a*b^6*c^3*d^7+210*b^7*c^4*d^6)*x^14+1/13*
(21*a^5*b^2*d^10+350*a^4*b^3*c*d^9+1575*a^3*b^4*c^2*d^8+2520*a^2*b^5*c^3*d^
7+1470*a*b^6*c^4*d^6+252*b^7*c^5*d^5)*x^13+1/12*(7*a^6*b*d^10+210*a^5*b^2*c
*d^9+1575*a^4*b^3*c^2*d^8+4200*a^3*b^4*c^3*d^7+4410*a^2*b^5*c^4*d^6+1764*a*
b^6*c^5*d^5+210*b^7*c^6*d^4)*x^12+1/11*(a^7*d^10+70*a^6*b*c*d^9+945*a^5*b^2
*c^2*d^8+4200*a^4*b^3*c^3*d^7+7350*a^3*b^4*c^4*d^6+5292*a^2*b^5*c^5*d^5+147
0*a*b^6*c^6*d^4+120*b^7*c^7*d^3)*x^11+1/10*(10*a^7*c*d^9+315*a^6*b*c^2*d^8+
2520*a^5*b^2*c^3*d^7+7350*a^4*b^3*c^4*d^6+8820*a^3*b^4*c^5*d^5+4410*a^2*b^5
*c^6*d^4+840*a*b^6*c^7*d^3+45*b^7*c^8*d^2)*x^10+1/9*(45*a^7*c^2*d^8+840*a^6
*b*c^3*d^7+4410*a^5*b^2*c^4*d^6+8820*a^4*b^3*c^5*d^5+7350*a^3*b^4*c^6*d^4+2
520*a^2*b^5*c^7*d^3+315*a*b^6*c^8*d^2+10*b^7*c^9*d)*x^9+1/8*(120*a^7*c^3*d^
7+1470*a^6*b*c^4*d^6+5292*a^5*b^2*c^5*d^5+7350*a^4*b^3*c^6*d^4+4200*a^3*b^4
*c^7*d^3+945*a^2*b^5*c^8*d^2+70*a*b^6*c^9*d+b^7*c^10)*x^8+1/7*(210*a^7*c^4*
d^6+1764*a^6*b*c^5*d^5+4410*a^5*b^2*c^6*d^4+4200*a^4*b^3*c^7*d^3+1575*a^3*b
^4*c^8*d^2+210*a^2*b^5*c^9*d+7*a*b^6*c^10)*x^7+1/6*(252*a^7*c^5*d^5+1470*a^
6*b*c^6*d^4+2520*a^5*b^2*c^7*d^3+1575*a^4*b^3*c^8*d^2+350*a^3*b^4*c^9*d+21*
a^2*b^5*c^10)*x^6+1/5*(210*a^7*c^6*d^4+840*a^6*b*c^7*d^3+945*a^5*b^2*c^8*d^
2+350*a^4*b^3*c^9*d+35*a^3*b^4*c^10)*x^5+1/4*(120*a^7*c^7*d^3+315*a^6*b*c^8
*d^2+210*a^5*b^2*c^9*d+35*a^4*b^3*c^10)*x^4+1/3*(45*a^7*c^8*d^2+70*a^6*b*c^
9*d+21*a^5*b^2*c^10)*x^3+1/2*(10*a^7*c^9*d+7*a^6*b*c^10)*x^2+a^7*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(184) = 368$.
time = 0.29, size = 1135, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
```

```
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(184) = 368$.

time = 0.30, size = 1135, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(184) = 368$.
time = 0.12, size = 1280, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**10,x)
```

```
[Out] a**7*c**10*x + b**7*d**10*x**18/18 + x**17*(7*a*b**6*d**10/17 + 10*b**7*c*d
**9/17) + x**16*(21*a**2*b**5*d**10/16 + 35*a*b**6*c*d**9/8 + 45*b**7*c**2*
d**8/16) + x**15*(7*a**3*b**4*d**10/3 + 14*a**2*b**5*c*d**9 + 21*a*b**6*c**
2*d**8 + 8*b**7*c**3*d**7) + x**14*(5*a**4*b**3*d**10/2 + 25*a**3*b**4*c*d*
*9 + 135*a**2*b**5*c**2*d**8/2 + 60*a*b**6*c**3*d**7 + 15*b**7*c**4*d**6) +
 x**13*(21*a**5*b**2*d**10/13 + 350*a**4*b**3*c*d**9/13 + 1575*a**3*b**4*c*
*2*d**8/13 + 2520*a**2*b**5*c**3*d**7/13 + 1470*a*b**6*c**4*d**6/13 + 252*b
**7*c**5*d**5/13) + x**12*(7*a**6*b*d**10/12 + 35*a**5*b**2*c*d**9/2 + 525*
a**4*b**3*c**2*d**8/4 + 350*a**3*b**4*c**3*d**7 + 735*a**2*b**5*c**4*d**6/2
 + 147*a*b**6*c**5*d**5 + 35*b**7*c**6*d**4/2) + x**11*(a**7*d**10/11 + 70*
a**6*b*c*d**9/11 + 945*a**5*b**2*c**2*d**8/11 + 4200*a**4*b**3*c**3*d**7/11
 + 7350*a**3*b**4*c**4*d**6/11 + 5292*a**2*b**5*c**5*d**5/11 + 1470*a*b**6*
c**6*d**4/11 + 120*b**7*c**7*d**3/11) + x**10*(a**7*c*d**9 + 63*a**6*b*c**2
*d**8/2 + 252*a**5*b**2*c**3*d**7 + 735*a**4*b**3*c**4*d**6 + 882*a**3*b**4
*c**5*d**5 + 441*a**2*b**5*c**6*d**4 + 84*a*b**6*c**7*d**3 + 9*b**7*c**8*d*
*2/2) + x**9*(5*a**7*c**2*d**8 + 280*a**6*b*c**3*d**7/3 + 490*a**5*b**2*c**
4*d**6 + 980*a**4*b**3*c**5*d**5 + 2450*a**3*b**4*c**6*d**4/3 + 280*a**2*b*
*5*c**7*d**3 + 35*a*b**6*c**8*d**2 + 10*b**7*c**9*d/9) + x**8*(15*a**7*c**3
*d**7 + 735*a**6*b*c**4*d**6/4 + 1323*a**5*b**2*c**5*d**5/2 + 3675*a**4*b**
3*c**6*d**4/4 + 525*a**3*b**4*c**7*d**3 + 945*a**2*b**5*c**8*d**2/8 + 35*a*
b**6*c**9*d/4 + b**7*c**10/8) + x**7*(30*a**7*c**4*d**6 + 252*a**6*b*c**5*d
**5 + 630*a**5*b**2*c**6*d**4 + 600*a**4*b**3*c**7*d**3 + 225*a**3*b**4*c**
8*d**2 + 30*a**2*b**5*c**9*d + a*b**6*c**10) + x**6*(42*a**7*c**5*d**5 + 24
5*a**6*b*c**6*d**4 + 420*a**5*b**2*c**7*d**3 + 525*a**4*b**3*c**8*d**2/2 +
175*a**3*b**4*c**9*d/3 + 7*a**2*b**5*c**10/2) + x**5*(42*a**7*c**6*d**4 + 1
68*a**6*b*c**7*d**3 + 189*a**5*b**2*c**8*d**2 + 70*a**4*b**3*c**9*d + 7*a**
3*b**4*c**10) + x**4*(30*a**7*c**7*d**3 + 315*a**6*b*c**8*d**2/4 + 105*a**5
*b**2*c**9*d/2 + 35*a**4*b**3*c**10/4) + x**3*(15*a**7*c**8*d**2 + 70*a**6*
b*c**9*d/3 + 7*a**5*b**2*c**10) + x**2*(5*a**7*c**9*d + 7*a**6*b*c**10/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1302 vs. $2(184) = 368$.
time = 0.00, size = 1396, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x)
```

```
[Out] 1/18*b^7*d^10*x^18 + 10/17*b^7*c*d^9*x^17 + 7/17*a*b^6*d^10*x^17 + 45/16*b^
7*c^2*d^8*x^16 + 35/8*a*b^6*c*d^9*x^16 + 21/16*a^2*b^5*d^10*x^16 + 8*b^7*c^
3*d^7*x^15 + 21*a*b^6*c^2*d^8*x^15 + 14*a^2*b^5*c*d^9*x^15 + 7/3*a^3*b^4*d^
10*x^15 + 15*b^7*c^4*d^6*x^14 + 60*a*b^6*c^3*d^7*x^14 + 135/2*a^2*b^5*c^2*d
^8*x^14 + 25*a^3*b^4*c*d^9*x^14 + 5/2*a^4*b^3*d^10*x^14 + 252/13*b^7*c^5*d^
5*x^13 + 1470/13*a*b^6*c^4*d^6*x^13 + 2520/13*a^2*b^5*c^3*d^7*x^13 + 1575/1
3*a^3*b^4*c^2*d^8*x^13 + 350/13*a^4*b^3*c*d^9*x^13 + 21/13*a^5*b^2*d^10*x^1
3 + 35/2*b^7*c^6*d^4*x^12 + 147*a*b^6*c^5*d^5*x^12 + 735/2*a^2*b^5*c^4*d^6*
x^12 + 350*a^3*b^4*c^3*d^7*x^12 + 525/4*a^4*b^3*c^2*d^8*x^12 + 35/2*a^5*b^2
*c*d^9*x^12 + 7/12*a^6*b*d^10*x^12 + 120/11*b^7*c^7*d^3*x^11 + 1470/11*a*b^
6*c^6*d^4*x^11 + 5292/11*a^2*b^5*c^5*d^5*x^11 + 7350/11*a^3*b^4*c^4*d^6*x^1
1 + 4200/11*a^4*b^3*c^3*d^7*x^11 + 945/11*a^5*b^2*c^2*d^8*x^11 + 70/11*a^6*
b*c*d^9*x^11 + 1/11*a^7*d^10*x^11 + 9/2*b^7*c^8*d^2*x^10 + 84*a*b^6*c^7*d^3
*x^10 + 441*a^2*b^5*c^6*d^4*x^10 + 882*a^3*b^4*c^5*d^5*x^10 + 735*a^4*b^3*c
^4*d^6*x^10 + 252*a^5*b^2*c^3*d^7*x^10 + 63/2*a^6*b*c^2*d^8*x^10 + a^7*c*d^
9*x^10 + 10/9*b^7*c^9*d*x^9 + 35*a*b^6*c^8*d^2*x^9 + 280*a^2*b^5*c^7*d^3*x^
9 + 2450/3*a^3*b^4*c^6*d^4*x^9 + 980*a^4*b^3*c^5*d^5*x^9 + 490*a^5*b^2*c^4*
d^6*x^9 + 280/3*a^6*b*c^3*d^7*x^9 + 5*a^7*c^2*d^8*x^9 + 1/8*b^7*c^10*x^8 +
35/4*a*b^6*c^9*d*x^8 + 945/8*a^2*b^5*c^8*d^2*x^8 + 525*a^3*b^4*c^7*d^3*x^8
+ 3675/4*a^4*b^3*c^6*d^4*x^8 + 1323/2*a^5*b^2*c^5*d^5*x^8 + 735/4*a^6*b*c^4
*d^6*x^8 + 15*a^7*c^3*d^7*x^8 + a*b^6*c^10*x^7 + 30*a^2*b^5*c^9*d*x^7 + 225
*a^3*b^4*c^8*d^2*x^7 + 600*a^4*b^3*c^7*d^3*x^7 + 630*a^5*b^2*c^6*d^4*x^7 +
252*a^6*b*c^5*d^5*x^7 + 30*a^7*c^4*d^6*x^7 + 7/2*a^2*b^5*c^10*x^6 + 175/3*a
^3*b^4*c^9*d*x^6 + 525/2*a^4*b^3*c^8*d^2*x^6 + 420*a^5*b^2*c^7*d^3*x^6 + 24
5*a^6*b*c^6*d^4*x^6 + 42*a^7*c^5*d^5*x^6 + 7*a^3*b^4*c^10*x^5 + 70*a^4*b^3*
c^9*d*x^5 + 189*a^5*b^2*c^8*d^2*x^5 + 168*a^6*b*c^7*d^3*x^5 + 42*a^7*c^6*d^
4*x^5 + 35/4*a^4*b^3*c^10*x^4 + 105/2*a^5*b^2*c^9*d*x^4 + 315/4*a^6*b*c^8*d
^2*x^4 + 30*a^7*c^7*d^3*x^4 + 7*a^5*b^2*c^10*x^3 + 70/3*a^6*b*c^9*d*x^3 + 1
5*a^7*c^8*d^2*x^3 + 7/2*a^6*b*c^10*x^2 + 5*a^7*c^9*d*x^2 + a^7*c^10*x
```

Mupad [B]
time = 0.61, size = 1106, normalized size = 5.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^10,x)
```

```
[Out] x^10*(a^7*c*d^9 + (9*b^7*c^8*d^2)/2 + 84*a*b^6*c^7*d^3 + (63*a^6*b*c^2*d^8)
/2 + 441*a^2*b^5*c^6*d^4 + 882*a^3*b^4*c^5*d^5 + 735*a^4*b^3*c^4*d^6 + 252*
a^5*b^2*c^3*d^7) + x^9*((10*b^7*c^9*d)/9 + 5*a^7*c^2*d^8 + 35*a*b^6*c^8*d^2
 + (280*a^6*b*c^3*d^7)/3 + 280*a^2*b^5*c^7*d^3 + (2450*a^3*b^4*c^6*d^4)/3 +
 980*a^4*b^3*c^5*d^5 + 490*a^5*b^2*c^4*d^6) + x^5*(7*a^3*b^4*c^10 + 42*a^7*
c^6*d^4 + 70*a^4*b^3*c^9*d + 168*a^6*b*c^7*d^3 + 189*a^5*b^2*c^8*d^2) + x^1
4*((5*a^4*b^3*d^10)/2 + 15*b^7*c^4*d^6 + 60*a*b^6*c^3*d^7 + 25*a^3*b^4*c*d^
```

9 + (135*a^2*b^5*c^2*d^8)/2) + x^8*((b^7*c^10)/8 + 15*a^7*c^3*d^7 + (735*a^6*b*c^4*d^6)/4 + (945*a^2*b^5*c^8*d^2)/8 + 525*a^3*b^4*c^7*d^3 + (3675*a^4*b^3*c^6*d^4)/4 + (1323*a^5*b^2*c^5*d^5)/2 + (35*a*b^6*c^9*d)/4) + x^11*((a^7*d^10)/11 + (120*b^7*c^7*d^3)/11 + (1470*a*b^6*c^6*d^4)/11 + (5292*a^2*b^5*c^5*d^5)/11 + (7350*a^3*b^4*c^4*d^6)/11 + (4200*a^4*b^3*c^3*d^7)/11 + (945*a^5*b^2*c^2*d^8)/11 + (70*a^6*b*c*d^9)/11) + x^6*((7*a^2*b^5*c^10)/2 + 42*a^7*c^5*d^5 + (175*a^3*b^4*c^9*d)/3 + 245*a^6*b*c^6*d^4 + (525*a^4*b^3*c^8*d^2)/2 + 420*a^5*b^2*c^7*d^3) + x^13*((21*a^5*b^2*d^10)/13 + (252*b^7*c^5*d^5)/13 + (1470*a*b^6*c^4*d^6)/13 + (350*a^4*b^3*c*d^9)/13 + (2520*a^2*b^5*c^3*d^7)/13 + (1575*a^3*b^4*c^2*d^8)/13) + x^7*(a*b^6*c^10 + 30*a^7*c^4*d^6 + 30*a^2*b^5*c^9*d + 252*a^6*b*c^5*d^5 + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^3 + 630*a^5*b^2*c^6*d^4) + x^12*((7*a^6*b*d^10)/12 + (35*b^7*c^6*d^4)/2 + 147*a*b^6*c^5*d^5 + (35*a^5*b^2*c*d^9)/2 + (735*a^2*b^5*c^4*d^6)/2 + 350*a^3*b^4*c^3*d^7 + (525*a^4*b^3*c^2*d^8)/4) + a^7*c^10*x + (b^7*d^10*x^18)/18 + (5*a^4*c^7*x^4*(24*a^3*d^3 + 7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a^2*b*c*d^2))/4 + (b^4*d^7*x^15*(7*a^3*d^3 + 24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2))/3 + (a^6*c^9*x^2*(10*a*d + 7*b*c))/2 + (b^6*d^9*x^17*(7*a*d + 10*b*c))/17 + (a^5*c^8*x^3*(45*a^2*d^2 + 21*b^2*c^2 + 70*a*b*c*d))/3 + (b^5*d^8*x^16*(21*a^2*d^2 + 45*b^2*c^2 + 70*a*b*c*d))/16

3.1305 $\int (a+bx)^6(c+dx)^{10}\,dx$

Optimal. Leaf size=170

$$\frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-\frac{10b^3(bc-ad)^3(c+dx)^{14}}{7d^7}+\frac{b^4(bc-$$

```
[Out] 1/11*(-a*d+b*c)^6*(d*x+c)^11/d^7-1/2*b*(-a*d+b*c)^5*(d*x+c)^12/d^7+15/13*b^
2*(-a*d+b*c)^4*(d*x+c)^13/d^7-10/7*b^3*(-a*d+b*c)^3*(d*x+c)^14/d^7+b^4*(-a*
d+b*c)^2*(d*x+c)^15/d^7-3/8*b^5*(-a*d+b*c)*(d*x+c)^16/d^7+1/17*b^6*(d*x+c)^
17/d^7
```

Rubi [A]

time = 0.46, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^{12}(bc-ad)^5}{2d^7}+\frac{(c+dx)^{11}(bc-ad)^6}{11d^7}+\frac{b^6(c+dx)^{17}}{17d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^11)/(11*d^7) - (b*(b*c - a*d)^5*(c + d*x)^12)/(2*d
^7) + (15*b^2*(b*c - a*d)^4*(c + d*x)^13)/(13*d^7) - (10*b^3*(b*c - a*d)^3*
(c + d*x)^14)/(7*d^7) + (b^4*(b*c - a*d)^2*(c + d*x)^15)/d^7 - (3*b^5*(b*c
- a*d)*(c + d*x)^16)/(8*d^7) + (b^6*(c + d*x)^17)/(17*d^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^6(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^6(c+dx)^{10}}{d^6}-\frac{6b(bc-ad)^5(c+dx)^{11}}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)}{d^6}\right.\\ &= \frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-10\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 939 vs. 2(170) = 340.

time = 0.07, size = 939, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] a^6*c^10*x + a^5*c^9*(3*b*c + 5*a*d)*x^2 + 5*a^4*c^8*(b^2*c^2 + 4*a*b*c*d +
 3*a^2*d^2)*x^3 + (5*a^3*c^7*(2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 +
 12*a^3*d^3)*x^4)/2 + a^2*c^6*(3*b^4*c^4 + 40*a*b^3*c^3*d + 135*a^2*b^2*c^2
*d^2 + 144*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + a*c^5*(b^5*c^5 + 25*a*b^4*c^4*d
+ 150*a^2*b^3*c^3*d^2 + 300*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d^4 + 42*a^5*d^5)
*x^6 + (c^4*(b^6*c^6 + 60*a*b^5*c^5*d + 675*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*
c^3*d^3 + 3150*a^4*b^2*c^2*d^4 + 1512*a^5*b*c*d^5 + 210*a^6*d^6)*x^7)/7 + (
5*c^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 180*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d
^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 12*a^6*d^6)*x^8)/4 + 5*c^2*d^2
*(b^6*c^6 + 16*a*b^5*c^5*d + 70*a^2*b^4*c^4*d^2 + 112*a^3*b^3*c^3*d^3 + 70*
a^4*b^2*c^2*d^4 + 16*a^5*b*c*d^5 + a^6*d^6)*x^9 + c*d^3*(12*b^6*c^6 + 126*a
*b^5*c^5*d + 378*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^
4 + 27*a^5*b*c*d^5 + a^6*d^6)*x^10 + (d^4*(210*b^6*c^6 + 1512*a*b^5*c^5*d +
 3150*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*c^3*d^3 + 675*a^4*b^2*c^2*d^4 + 60*a^5
*b*c*d^5 + a^6*d^6)*x^11)/11 + (b*d^5*(42*b^5*c^5 + 210*a*b^4*c^4*d + 300*a
^2*b^3*c^3*d^2 + 150*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*x^12)/2 +
(5*b^2*d^6*(42*b^4*c^4 + 144*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 40*a^3*b*c
*d^3 + 3*a^4*d^4)*x^13)/13 + (5*b^3*d^7*(12*b^3*c^3 + 27*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 2*a^3*d^3)*x^14)/7 + b^4*d^8*(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*x^15 + (b^5*d^9*(5*b*c + 3*a*d)*x^16)/8 + (b^6*d^10*x^17)/17
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 938 vs. 2(170) = 340.
time = 9.11, size = 920, normalized size = 5.41

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^6*(c + d*x)^10,x]')
```

```
[Out] x (a ^ 6 c ^ 10 + a ^ 5 c ^ 9 x (5 a d + 3 b c) + 5 a ^ 4 c ^ 8 x ^ 2 (3 a
^ 2 d ^ 2 + 4 a b c d + b ^ 2 c ^ 2) + 5 a ^ 3 c ^ 7 x ^ 3 (12 a ^ 3 d ^ 3
+ 27 a ^ 2 b c d ^ 2 + 15 a b ^ 2 c ^ 2 d + 2 b ^ 3 c ^ 3) / 2 + a ^ 2 c ^
6 x ^ 4 (42 a ^ 4 d ^ 4 + 144 a ^ 3 b c d ^ 3 + 135 a ^ 2 b ^ 2 c ^ 2 d ^ 2
 + 40 a b ^ 3 c ^ 3 d + 3 b ^ 4 c ^ 4) + a c ^ 5 x ^ 5 (42 a ^ 5 d ^ 5 + 21
0 a ^ 4 b c d ^ 4 + 300 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 150 a ^ 2 b ^ 3 c ^ 3 d ^
 2 + 25 a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5) + b d ^ 5 x ^ 11 (a ^ 5 d ^ 5 + 25 a
 ^ 4 b c d ^ 4 + 150 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 300 a ^ 2 b ^ 3 c ^ 3 d ^ 2
+ 210 a b ^ 4 c ^ 4 d + 42 b ^ 5 c ^ 5) / 2 + 5 b ^ 2 d ^ 6 x ^ 12 (3 a ^ 4
 d ^ 4 + 40 a ^ 3 b c d ^ 3 + 135 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 144 a b ^ 3 c ^
 3 d + 42 b ^ 4 c ^ 4) / 13 + 5 b ^ 3 d ^ 7 x ^ 13 (2 a ^ 3 d ^ 3 + 15 a ^
2 b c d ^ 2 + 27 a b ^ 2 c ^ 2 d + 12 b ^ 3 c ^ 3) / 7 + b ^ 4 d ^ 8 x ^ 14
 (a ^ 2 d ^ 2 + 4 a b c d + 3 b ^ 2 c ^ 2) + c ^ 4 x ^ 6 (210 a ^ 6 d ^ 6 +
```

```
1512 a ^ 5 b c d ^ 5 + 3150 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 2400 a ^ 3 b ^ 3 c ^
3 d ^ 3 + 675 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 60 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6)
/ 7 + 5 c ^ 2 d ^ 2 x ^ 8 (a ^ 6 d ^ 6 + 16 a ^ 5 b c d ^ 5 + 70 a ^ 4 b ^
2 c ^ 2 d ^ 4 + 112 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 70 a ^ 2 b ^ 4 c ^ 4 d ^ 2 +
16 a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6) + d ^ 4 x ^ 10 (a ^ 6 d ^ 6 + 60 a ^ 5 b
c d ^ 5 + 675 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 2400 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 3150
a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 1512 a b ^ 5 c ^ 5 d + 210 b ^ 6 c ^ 6) / 11 + b
^ 5 d ^ 9 x ^ 15 (3 a d + 5 b c) / 8 + b ^ 6 d ^ 10 x ^ 16 / 17 + 5 c ^ 3
d x ^ 7 (12 a ^ 6 d ^ 6 + 126 a ^ 5 b c d ^ 5 + 378 a ^ 4 b ^ 2 c ^ 2 d ^ 4
+ 420 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 180 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 27 a b ^ 5 c
^ 5 d + b ^ 6 c ^ 6) / 4 + c d ^ 3 x ^ 9 (a ^ 6 d ^ 6 + 27 a ^ 5 b c d ^ 5
+ 180 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 420 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 378 a ^ 2 b
^ 4 c ^ 4 d ^ 2 + 126 a b ^ 5 c ^ 5 d + 12 b ^ 6 c ^ 6))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(158) = 316$.
time = 0.13, size = 991, normalized size = 5.83

method	result
norman	$a^6c^{10}x + (5a^6c^9d + 3a^5b\,c^{10})\,x^2 + (15a^6c^8d^2 + 20a^5b\,c^9d + 5a^4b^2c^{10})\,x^3 + \big(30a^6c^7d^3 + \frac{135}{2}a^5b\,c^8d$
default	$\frac{b^6d^{10}x^{17}}{17} + \frac{(6a\,b^5d^{10}+10b^6c\,d^9)x^{16}}{16} + \frac{(15a^2b^4d^{10}+60a\,b^5c\,d^9+45b^6c^2d^8)x^{15}}{15} + \frac{(20a^3b^3d^{10}+150a^2b^4c\,d^9+270a\,b^5c^2d^8+120b^6}{14}$
gosper	$\frac{1}{17}b^6d^{10}x^{17} + \frac{1}{7}x^7b^6c^{10} + \frac{1}{11}x^{11}a^6d^{10} + 3a^5b\,c^{10}x^2 + 15a^6c^8d^2x^3 + a\,b^5c^{10}x^6 + 5a^6c^2d^8x^9 + 5b^6c^8d$
risch	$\frac{1}{17}b^6d^{10}x^{17} + \frac{1}{7}x^7b^6c^{10} + \frac{1}{11}x^{11}a^6d^{10} + 3a^5b\,c^{10}x^2 + 15a^6c^8d^2x^3 + a\,b^5c^{10}x^6 + 5a^6c^2d^8x^9 + 5b^6c^8d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*b^6*d^10*x^17+1/16*(6*a*b^5*d^10+10*b^6*c*d^9)*x^16+1/15*(15*a^2*b^4*d
^10+60*a*b^5*c*d^9+45*b^6*c^2*d^8)*x^15+1/14*(20*a^3*b^3*d^10+150*a^2*b^4*c
*d^9+270*a*b^5*c^2*d^8+120*b^6*c^3*d^7)*x^14+1/13*(15*a^4*b^2*d^10+200*a^3*
b^3*c*d^9+675*a^2*b^4*c^2*d^8+720*a*b^5*c^3*d^7+210*b^6*c^4*d^6)*x^13+1/12*
(6*a^5*b*d^10+150*a^4*b^2*c*d^9+900*a^3*b^3*c^2*d^8+1800*a^2*b^4*c^3*d^7+12
60*a*b^5*c^4*d^6+252*b^6*c^5*d^5)*x^12+1/11*(a^6*d^10+60*a^5*b*c*d^9+675*a^
4*b^2*c^2*d^8+2400*a^3*b^3*c^3*d^7+3150*a^2*b^4*c^4*d^6+1512*a*b^5*c^5*d^5+
210*b^6*c^6*d^4)*x^11+1/10*(10*a^6*c*d^9+270*a^5*b*c^2*d^8+1800*a^4*b^2*c^3
*d^7+4200*a^3*b^3*c^4*d^6+3780*a^2*b^4*c^5*d^5+1260*a*b^5*c^6*d^4+120*b^6*c
^7*d^3)*x^10+1/9*(45*a^6*c^2*d^8+720*a^5*b*c^3*d^7+3150*a^4*b^2*c^4*d^6+504
0*a^3*b^3*c^5*d^5+3150*a^2*b^4*c^6*d^4+720*a*b^5*c^7*d^3+45*b^6*c^8*d^2)*x^
9+1/8*(120*a^6*c^3*d^7+1260*a^5*b*c^4*d^6+3780*a^4*b^2*c^5*d^5+4200*a^3*b^3
*c^6*d^4+1800*a^2*b^4*c^7*d^3+270*a*b^5*c^8*d^2+10*b^6*c^9*d)*x^8+1/7*(210*
a^6*c^4*d^6+1512*a^5*b*c^5*d^5+3150*a^4*b^2*c^6*d^4+2400*a^3*b^3*c^7*d^3+67
5*a^2*b^4*c^8*d^2+60*a*b^5*c^9*d+b^6*c^10)*x^7+1/6*(252*a^6*c^5*d^5+1260*a^
5*b*c^6*d^4+1800*a^4*b^2*c^7*d^3+900*a^3*b^3*c^8*d^2+150*a^2*b^4*c^9*d+6*a*
```

```
b^5*c^10)*x^6+1/5*(210*a^6*c^6*d^4+720*a^5*b*c^7*d^3+675*a^4*b^2*c^8*d^2+20
0*a^3*b^3*c^9*d+15*a^2*b^4*c^10)*x^5+1/4*(120*a^6*c^7*d^3+270*a^5*b*c^8*d^2
+150*a^4*b^2*c^9*d+20*a^3*b^3*c^10)*x^4+1/3*(45*a^6*c^8*d^2+60*a^5*b*c^9*d+
15*a^4*b^2*c^10)*x^3+1/2*(10*a^6*c^9*d+6*a^5*b*c^10)*x^2+a^6*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(158) = 316$.
time = 0.28, size = 977, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(158) = 316$.
time = 0.30, size = 977, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(153) = 306$.
time = 0.11, size = 1088, normalized size = 6.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**10,x)
```

```
[Out] a**6*c**10*x + b**6*d**10*x**17/17 + x**16*(3*a*b**5*d**10/8 + 5*b**6*c*d**
9/8) + x**15*(a**2*b**4*d**10 + 4*a*b**5*c*d**9 + 3*b**6*c**2*d**8) + x**14
*(10*a**3*b**3*d**10/7 + 75*a**2*b**4*c*d**9/7 + 135*a*b**5*c**2*d**8/7 + 6
0*b**6*c**3*d**7/7) + x**13*(15*a**4*b**2*d**10/13 + 200*a**3*b**3*c*d**9/1
3 + 675*a**2*b**4*c**2*d**8/13 + 720*a*b**5*c**3*d**7/13 + 210*b**6*c**4*d*
*6/13) + x**12*(a**5*b*d**10/2 + 25*a**4*b**2*c*d**9/2 + 75*a**3*b**3*c**2*
d**8 + 150*a**2*b**4*c**3*d**7 + 105*a*b**5*c**4*d**6 + 21*b**6*c**5*d**5)
+ x**11*(a**6*d**10/11 + 60*a**5*b*c*d**9/11 + 675*a**4*b**2*c**2*d**8/11 +
 2400*a**3*b**3*c**3*d**7/11 + 3150*a**2*b**4*c**4*d**6/11 + 1512*a*b**5*c*
*5*d**5/11 + 210*b**6*c**6*d**4/11) + x**10*(a**6*c*d**9 + 27*a**5*b*c**2*d
**8 + 180*a**4*b**2*c**3*d**7 + 420*a**3*b**3*c**4*d**6 + 378*a**2*b**4*c**
5*d**5 + 126*a*b**5*c**6*d**4 + 12*b**6*c**7*d**3) + x**9*(5*a**6*c**2*d**8
 + 80*a**5*b*c**3*d**7 + 350*a**4*b**2*c**4*d**6 + 560*a**3*b**3*c**5*d**5
+ 350*a**2*b**4*c**6*d**4 + 80*a*b**5*c**7*d**3 + 5*b**6*c**8*d**2) + x**8*
```

```
(15*a**6*c**3*d**7 + 315*a**5*b*c**4*d**6/2 + 945*a**4*b**2*c**5*d**5/2 + 5
25*a**3*b**3*c**6*d**4 + 225*a**2*b**4*c**7*d**3 + 135*a*b**5*c**8*d**2/4 +
 5*b**6*c**9*d/4) + x**7*(30*a**6*c**4*d**6 + 216*a**5*b*c**5*d**5 + 450*a*
*4*b**2*c**6*d**4 + 2400*a**3*b**3*c**7*d**3/7 + 675*a**2*b**4*c**8*d**2/7
+ 60*a*b**5*c**9*d/7 + b**6*c**10/7) + x**6*(42*a**6*c**5*d**5 + 210*a**5*b
*c**6*d**4 + 300*a**4*b**2*c**7*d**3 + 150*a**3*b**3*c**8*d**2 + 25*a**2*b*
*4*c**9*d + a*b**5*c**10) + x**5*(42*a**6*c**6*d**4 + 144*a**5*b*c**7*d**3
+ 135*a**4*b**2*c**8*d**2 + 40*a**3*b**3*c**9*d + 3*a**2*b**4*c**10) + x**4
*(30*a**6*c**7*d**3 + 135*a**5*b*c**8*d**2/2 + 75*a**4*b**2*c**9*d/2 + 5*a*
*3*b**3*c**10) + x**3*(15*a**6*c**8*d**2 + 20*a**5*b*c**9*d + 5*a**4*b**2*c
**10) + x**2*(5*a**6*c**9*d + 3*a**5*b*c**10)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(158) = 316$.
time = 0.00, size = 1186, normalized size = 6.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x)
```

```
[Out] 1/17*b^6*d^10*x^17 + 5/8*b^6*c*d^9*x^16 + 3/8*a*b^5*d^10*x^16 + 3*b^6*c^2*d
^8*x^15 + 4*a*b^5*c*d^9*x^15 + a^2*b^4*d^10*x^15 + 60/7*b^6*c^3*d^7*x^14 +
135/7*a*b^5*c^2*d^8*x^14 + 75/7*a^2*b^4*c*d^9*x^14 + 10/7*a^3*b^3*d^10*x^14
 + 210/13*b^6*c^4*d^6*x^13 + 720/13*a*b^5*c^3*d^7*x^13 + 675/13*a^2*b^4*c^2
*d^8*x^13 + 200/13*a^3*b^3*c*d^9*x^13 + 15/13*a^4*b^2*d^10*x^13 + 21*b^6*c^
5*d^5*x^12 + 105*a*b^5*c^4*d^6*x^12 + 150*a^2*b^4*c^3*d^7*x^12 + 75*a^3*b^3
*c^2*d^8*x^12 + 25/2*a^4*b^2*c*d^9*x^12 + 1/2*a^5*b*d^10*x^12 + 210/11*b^6*
c^6*d^4*x^11 + 1512/11*a*b^5*c^5*d^5*x^11 + 3150/11*a^2*b^4*c^4*d^6*x^11 +
2400/11*a^3*b^3*c^3*d^7*x^11 + 675/11*a^4*b^2*c^2*d^8*x^11 + 60/11*a^5*b*c*
d^9*x^11 + 1/11*a^6*d^10*x^11 + 12*b^6*c^7*d^3*x^10 + 126*a*b^5*c^6*d^4*x^1
0 + 378*a^2*b^4*c^5*d^5*x^10 + 420*a^3*b^3*c^4*d^6*x^10 + 180*a^4*b^2*c^3*d
^7*x^10 + 27*a^5*b*c^2*d^8*x^10 + a^6*c*d^9*x^10 + 5*b^6*c^8*d^2*x^9 + 80*a
*b^5*c^7*d^3*x^9 + 350*a^2*b^4*c^6*d^4*x^9 + 560*a^3*b^3*c^5*d^5*x^9 + 350*
a^4*b^2*c^4*d^6*x^9 + 80*a^5*b*c^3*d^7*x^9 + 5*a^6*c^2*d^8*x^9 + 5/4*b^6*c^
9*d*x^8 + 135/4*a*b^5*c^8*d^2*x^8 + 225*a^2*b^4*c^7*d^3*x^8 + 525*a^3*b^3*c
^6*d^4*x^8 + 945/2*a^4*b^2*c^5*d^5*x^8 + 315/2*a^5*b*c^4*d^6*x^8 + 15*a^6*c
^3*d^7*x^8 + 1/7*b^6*c^10*x^7 + 60/7*a*b^5*c^9*d*x^7 + 675/7*a^2*b^4*c^8*d^
2*x^7 + 2400/7*a^3*b^3*c^7*d^3*x^7 + 450*a^4*b^2*c^6*d^4*x^7 + 216*a^5*b*c^
5*d^5*x^7 + 30*a^6*c^4*d^6*x^7 + a*b^5*c^10*x^6 + 25*a^2*b^4*c^9*d*x^6 + 15
0*a^3*b^3*c^8*d^2*x^6 + 300*a^4*b^2*c^7*d^3*x^6 + 210*a^5*b*c^6*d^4*x^6 + 4
2*a^6*c^5*d^5*x^6 + 3*a^2*b^4*c^10*x^5 + 40*a^3*b^3*c^9*d*x^5 + 135*a^4*b^2
*c^8*d^2*x^5 + 144*a^5*b*c^7*d^3*x^5 + 42*a^6*c^6*d^4*x^5 + 5*a^3*b^3*c^10*
x^4 + 75/2*a^4*b^2*c^9*d*x^4 + 135/2*a^5*b*c^8*d^2*x^4 + 30*a^6*c^7*d^3*x^4
 + 5*a^4*b^2*c^10*x^3 + 20*a^5*b*c^9*d*x^3 + 15*a^6*c^8*d^2*x^3 + 3*a^5*b*c
^10*x^2 + 5*a^6*c^9*d*x^2 + a^6*c^10*x
```

Mupad [B]
time = 0.53, size = 953, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^10,x)
```

```
[Out] x^7*((b^6*c^10)/7 + 30*a^6*c^4*d^6 + 216*a^5*b*c^5*d^5 + (675*a^2*b^4*c^8*d
^2)/7 + (2400*a^3*b^3*c^7*d^3)/7 + 450*a^4*b^2*c^6*d^4 + (60*a*b^5*c^9*d)/7
) + x^11*((a^6*d^10)/11 + (210*b^6*c^6*d^4)/11 + (1512*a*b^5*c^5*d^5)/11 +
(3150*a^2*b^4*c^4*d^6)/11 + (2400*a^3*b^3*c^3*d^7)/11 + (675*a^4*b^2*c^2*d^
8)/11 + (60*a^5*b*c*d^9)/11) + x^9*(5*a^6*c^2*d^8 + 5*b^6*c^8*d^2 + 80*a*b^
5*c^7*d^3 + 80*a^5*b*c^3*d^7 + 350*a^2*b^4*c^6*d^4 + 560*a^3*b^3*c^5*d^5 +
350*a^4*b^2*c^4*d^6) + x^5*(3*a^2*b^4*c^10 + 42*a^6*c^6*d^4 + 40*a^3*b^3*c^
9*d + 144*a^5*b*c^7*d^3 + 135*a^4*b^2*c^8*d^2) + x^13*((15*a^4*b^2*d^10)/13
 + (210*b^6*c^4*d^6)/13 + (720*a*b^5*c^3*d^7)/13 + (200*a^3*b^3*c*d^9)/13 +
 (675*a^2*b^4*c^2*d^8)/13) + x^6*(a*b^5*c^10 + 42*a^6*c^5*d^5 + 25*a^2*b^4*
c^9*d + 210*a^5*b*c^6*d^4 + 150*a^3*b^3*c^8*d^2 + 300*a^4*b^2*c^7*d^3) + x^
12*((a^5*b*d^10)/2 + 21*b^6*c^5*d^5 + 105*a*b^5*c^4*d^6 + (25*a^4*b^2*c*d^9
)/2 + 150*a^2*b^4*c^3*d^7 + 75*a^3*b^3*c^2*d^8) + x^10*(a^6*c*d^9 + 12*b^6*
c^7*d^3 + 126*a*b^5*c^6*d^4 + 27*a^5*b*c^2*d^8 + 378*a^2*b^4*c^5*d^5 + 420*
a^3*b^3*c^4*d^6 + 180*a^4*b^2*c^3*d^7) + x^8*((5*b^6*c^9*d)/4 + 15*a^6*c^3*
d^7 + (135*a*b^5*c^8*d^2)/4 + (315*a^5*b*c^4*d^6)/2 + 225*a^2*b^4*c^7*d^3 +
 525*a^3*b^3*c^6*d^4 + (945*a^4*b^2*c^5*d^5)/2) + a^6*c^10*x + (b^6*d^10*x^
17)/17 + (5*a^3*c^7*x^4*(12*a^3*d^3 + 2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b
*c*d^2))/2 + (5*b^3*d^7*x^14*(2*a^3*d^3 + 12*b^3*c^3 + 27*a*b^2*c^2*d + 15*
a^2*b*c*d^2))/7 + a^5*c^9*x^2*(5*a*d + 3*b*c) + (b^5*d^9*x^16*(3*a*d + 5*b*
c))/8 + 5*a^4*c^8*x^3*(3*a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + b^4*d^8*x^15*(a^2
*d^2 + 3*b^2*c^2 + 4*a*b*c*d)
```

3.1306 $\int (a+bx)^5 (c+dx)^{10}\, dx$

Optimal. Leaf size=146

$$-\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\frac{5b^3(bc-ad)^2(c+dx)^{14}}{7d^6}-\frac{b^4(bc-}{}$$

```
[Out] -1/11*(-a*d+b*c)^5*(d*x+c)^11/d^6+5/12*b*(-a*d+b*c)^4*(d*x+c)^12/d^6-10/13*
b^2*(-a*d+b*c)^3*(d*x+c)^13/d^6+5/7*b^3*(-a*d+b*c)^2*(d*x+c)^14/d^6-1/3*b^4
*(-a*d+b*c)*(d*x+c)^15/d^6+1/16*b^5*(d*x+c)^16/d^6
```

Rubi [A]
time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc-ad)^5}{11d^6}+\frac{b^5(c+dx)^{16}}{16d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^5*(c + d*x)^11)/d^6 + (5*b*(b*c - a*d)^4*(c + d*x)^12)/(
12*d^6) - (10*b^2*(b*c - a*d)^3*(c + d*x)^13)/(13*d^6) + (5*b^3*(b*c - a*d)
^2*(c + d*x)^14)/(7*d^6) - (b^4*(b*c - a*d)*(c + d*x)^15)/(3*d^6) + (b^5*(c
 + d*x)^16)/(16*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^5(c+dx)^{10}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{11}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{12}}{d^5}\right.\\ &= -\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\ldots\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 811 vs. 2(146) = 292.
time = 0.05, size = 811, normalized size = 5.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] a^5*c^10*x + (5*a^4*c^9*(b*c + 2*a*d)*x^2)/2 + (5*a^3*c^8*(2*b^2*c^2 + 10*a
*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^2*c^7*(2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^
2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + a*c^6*(b^4*c^4 + 20*a*b^3*c^3*d + 90*a^2*b
^2*c^2*d^2 + 120*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + (c^5*(b^5*c^5 + 50*a*b^4*c
^4*d + 450*a^2*b^3*c^3*d^2 + 1200*a^3*b^2*c^2*d^3 + 1050*a^4*b*c*d^4 + 252*
a^5*d^5)*x^6)/6 + (5*c^4*d*(2*b^5*c^5 + 45*a*b^4*c^4*d + 240*a^2*b^3*c^3*d^
2 + 420*a^3*b^2*c^2*d^3 + 252*a^4*b*c*d^4 + 42*a^5*d^5)*x^7)/7 + (15*c^3*d^
2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3*c^3*d^2 + 168*a^3*b^2*c^2*d^3 +
 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^8)/8 + (5*c^2*d^3*(8*b^5*c^5 + 70*a*b^4*c^4*
d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c*d^4 + 3*a^5*d^5)
*x^9)/3 + (c*d^4*(42*b^5*c^5 + 252*a*b^4*c^4*d + 420*a^2*b^3*c^3*d^2 + 240*
a^3*b^2*c^2*d^3 + 45*a^4*b*c*d^4 + 2*a^5*d^5)*x^10)/2 + (d^5*(252*b^5*c^5 +
 1050*a*b^4*c^4*d + 1200*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 50*a^4*b*c
*d^4 + a^5*d^5)*x^11)/11 + (5*b*d^6*(42*b^4*c^4 + 120*a*b^3*c^3*d + 90*a^2*
b^2*c^2*d^2 + 20*a^3*b*c*d^3 + a^4*d^4)*x^12)/12 + (5*b^2*d^7*(24*b^3*c^3 +
 45*a*b^2*c^2*d + 20*a^2*b*c*d^2 + 2*a^3*d^3)*x^13)/13 + (5*b^3*d^8*(9*b^2*
c^2 + 10*a*b*c*d + 2*a^2*d^2)*x^14)/14 + (b^4*d^9*(2*b*c + a*d)*x^15)/3 + (
b^5*d^10*x^16)/16
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 781 vs. $2(146) = 292$.
time = 7.88, size = 779, normalized size = 5.34

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(c + d*x)^10,x]')
```

```
[Out] x (48048 a ^ 5 c ^ 10 + 120120 a ^ 4 c ^ 9 x (2 a d + b c) + a ^ 3 c ^ 8 x
^ 2 (720720 a ^ 2 d ^ 2 + 800800 a b c d + 160160 b ^ 2 c ^ 2) + a ^ 2 c ^
7 x ^ 3 (1441440 a ^ 3 d ^ 3 + 2702700 a ^ 2 b c d ^ 2 + 1201200 a b ^ 2 c
^ 2 d + 120120 b ^ 3 c ^ 3) + 48048 a c ^ 6 x ^ 4 (42 a ^ 4 d ^ 4 + 120 a ^
 3 b c d ^ 3 + 90 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 20 a b ^ 3 c ^ 3 d + b ^ 4 c ^
4) + 20020 b d ^ 6 x ^ 11 (a ^ 4 d ^ 4 + 20 a ^ 3 b c d ^ 3 + 90 a ^ 2 b ^
2 c ^ 2 d ^ 2 + 120 a b ^ 3 c ^ 3 d + 42 b ^ 4 c ^ 4) + b ^ 2 d ^ 7 x ^ 12
(36960 a ^ 3 d ^ 3 + 369600 a ^ 2 b c d ^ 2 + 831600 a b ^ 2 c ^ 2 d + 4435
20 b ^ 3 c ^ 3) + b ^ 4 d ^ 9 x ^ 14 (16016 a d + 32032 b c) + c ^ 5 x ^ 5
(2018016 a ^ 5 d ^ 5 + 8408400 a ^ 4 b c d ^ 4 + 9609600 a ^ 3 b ^ 2 c ^ 2
d ^ 3 + 3603600 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 400400 a b ^ 4 c ^ 4 d + 8008 b ^
 5 c ^ 5) + d ^ 5 x ^ 10 (4368 a ^ 5 d ^ 5 + 218400 a ^ 4 b c d ^ 4 + 19656
00 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 5241600 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 4586400 a b
^ 4 c ^ 4 d + 1100736 b ^ 5 c ^ 5) + b ^ 3 d ^ 8 x ^ 13 (34320 a ^ 2 d ^ 2
+ 171600 a b c d + 154440 b ^ 2 c ^ 2) + 3003 b ^ 5 d ^ 10 x ^ 15 + 34320 c
```

```
^ 4 d x ^ 6 (42 a ^ 5 d ^ 5 + 252 a ^ 4 b c d ^ 4 + 420 a ^ 3 b ^ 2 c ^ 2
d ^ 3 + 240 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 45 a b ^ 4 c ^ 4 d + 2 b ^ 5 c ^ 5) +
c ^ 3 d ^ 2 x ^ 7 (720720 a ^ 5 d ^ 5 + 6306300 a ^ 4 b c d ^ 4 + 15135120
a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 12612600 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 3603600 a b ^
4 c ^ 4 d + 270270 b ^ 5 c ^ 5) + c ^ 2 d ^ 3 x ^ 8 (240240 a ^ 5 d ^ 5 +
3203200 a ^ 4 b c d ^ 4 + 11211200 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 13453440 a ^ 2
b ^ 3 c ^ 3 d ^ 2 + 5605600 a b ^ 4 c ^ 4 d + 640640 b ^ 5 c ^ 5) + 24024
c d ^ 4 x ^ 9 (2 a ^ 5 d ^ 5 + 45 a ^ 4 b c d ^ 4 + 240 a ^ 3 b ^ 2 c ^ 2 d
^ 3 + 420 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 252 a b ^ 4 c ^ 4 d + 42 b ^ 5 c ^ 5))
/ 48048
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(134) = 268$.
time = 0.14, size = 841, normalized size = 5.76

method	result
norman	$a^5c^{10}x + \left(5a^5c^9d + \frac{5}{2}a^4b\,c^{10}\right)x^2 + \left(15a^5c^8d^2 + \frac{50}{3}a^4b\,c^9d + \frac{10}{3}a^3b^2c^{10}\right)x^3 + \left(30a^5c^7d^3 + \frac{225}{4}a^4b\,c^8d\right.$
default	$\frac{b^5d^{10}x^{16}}{16} + \frac{(5a\,b^4d^{10}+10b^5c\,d^9)x^{15}}{15} + \frac{(10a^2b^3d^{10}+50a\,b^4c\,d^9+45b^5c^2d^8)x^{14}}{14} + \frac{(10a^3b^2d^{10}+100a^2b^3c\,d^9+225a\,b^4c^2d^8+120b^5c^3}{13}$
gosper	$\frac{225}{4}x^4a^4b\,c^8d^2 + 25x^4a^3b^2c^9d + 175x^6a^4b\,c^6d^4 + 200x^6a^3b^2c^7d^3 + 75x^6a^2b^3c^8d^2 + \frac{25}{3}x^6a\,b^4c^9d + 180$
risch	$\frac{225}{4}x^4a^4b\,c^8d^2 + 25x^4a^3b^2c^9d + 175x^6a^4b\,c^6d^4 + 200x^6a^3b^2c^7d^3 + 75x^6a^2b^3c^8d^2 + \frac{25}{3}x^6a\,b^4c^9d + 180$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^5*d^10*x^16+1/15*(5*a*b^4*d^10+10*b^5*c*d^9)*x^15+1/14*(10*a^2*b^3*d
^10+50*a*b^4*c*d^9+45*b^5*c^2*d^8)*x^14+1/13*(10*a^3*b^2*d^10+100*a^2*b^3*c
*d^9+225*a*b^4*c^2*d^8+120*b^5*c^3*d^7)*x^13+1/12*(5*a^4*b*d^10+100*a^3*b^2
*c*d^9+450*a^2*b^3*c^2*d^8+600*a*b^4*c^3*d^7+210*b^5*c^4*d^6)*x^12+1/11*(a^
5*d^10+50*a^4*b*c*d^9+450*a^3*b^2*c^2*d^8+1200*a^2*b^3*c^3*d^7+1050*a*b^4*c
^4*d^6+252*b^5*c^5*d^5)*x^11+1/10*(10*a^5*c*d^9+225*a^4*b*c^2*d^8+1200*a^3*
b^2*c^3*d^7+2100*a^2*b^3*c^4*d^6+1260*a*b^4*c^5*d^5+210*b^5*c^6*d^4)*x^10+1
/9*(45*a^5*c^2*d^8+600*a^4*b*c^3*d^7+2100*a^3*b^2*c^4*d^6+2520*a^2*b^3*c^5*
d^5+1050*a*b^4*c^6*d^4+120*b^5*c^7*d^3)*x^9+1/8*(120*a^5*c^3*d^7+1050*a^4*b
*c^4*d^6+2520*a^3*b^2*c^5*d^5+2100*a^2*b^3*c^6*d^4+600*a*b^4*c^7*d^3+45*b^5
*c^8*d^2)*x^8+1/7*(210*a^5*c^4*d^6+1260*a^4*b*c^5*d^5+2100*a^3*b^2*c^6*d^4+
1200*a^2*b^3*c^7*d^3+225*a*b^4*c^8*d^2+10*b^5*c^9*d)*x^7+1/6*(252*a^5*c^5*d
^5+1050*a^4*b*c^6*d^4+1200*a^3*b^2*c^7*d^3+450*a^2*b^3*c^8*d^2+50*a*b^4*c^9
*d+b^5*c^10)*x^6+1/5*(210*a^5*c^6*d^4+600*a^4*b*c^7*d^3+450*a^3*b^2*c^8*d^2
+100*a^2*b^3*c^9*d+5*a*b^4*c^10)*x^5+1/4*(120*a^5*c^7*d^3+225*a^4*b*c^8*d^2
+100*a^3*b^2*c^9*d+10*a^2*b^3*c^10)*x^4+1/3*(45*a^5*c^8*d^2+50*a^4*b*c^9*d+
10*a^3*b^2*c^10)*x^3+1/2*(10*a^5*c^9*d+5*a^4*b*c^10)*x^2+a^5*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(134) = 268$.

time = 0.27, size = 835, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(134) = 268.
time = 0.30, size = 835, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
```

```
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(131) = 262$.
time = 0.10, size = 940, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**10,x)
```

```
[Out] a**5*c**10*x + b**5*d**10*x**16/16 + x**15*(a*b**4*d**10/3 + 2*b**5*c*d**9/
3) + x**14*(5*a**2*b**3*d**10/7 + 25*a*b**4*c*d**9/7 + 45*b**5*c**2*d**8/14
) + x**13*(10*a**3*b**2*d**10/13 + 100*a**2*b**3*c*d**9/13 + 225*a*b**4*c**
2*d**8/13 + 120*b**5*c**3*d**7/13) + x**12*(5*a**4*b*d**10/12 + 25*a**3*b**
2*c*d**9/3 + 75*a**2*b**3*c**2*d**8/2 + 50*a*b**4*c**3*d**7 + 35*b**5*c**4*
d**6/2) + x**11*(a**5*d**10/11 + 50*a**4*b*c*d**9/11 + 450*a**3*b**2*c**2*d
**8/11 + 1200*a**2*b**3*c**3*d**7/11 + 1050*a*b**4*c**4*d**6/11 + 252*b**5*
c**5*d**5/11) + x**10*(a**5*c*d**9 + 45*a**4*b*c**2*d**8/2 + 120*a**3*b**2*
c**3*d**7 + 210*a**2*b**3*c**4*d**6 + 126*a*b**4*c**5*d**5 + 21*b**5*c**6*d
**4) + x**9*(5*a**5*c**2*d**8 + 200*a**4*b*c**3*d**7/3 + 700*a**3*b**2*c**4
*d**6/3 + 280*a**2*b**3*c**5*d**5 + 350*a*b**4*c**6*d**4/3 + 40*b**5*c**7*d
**3/3) + x**8*(15*a**5*c**3*d**7 + 525*a**4*b*c**4*d**6/4 + 315*a**3*b**2*c
**5*d**5 + 525*a**2*b**3*c**6*d**4/2 + 75*a*b**4*c**7*d**3 + 45*b**5*c**8*d
**2/8) + x**7*(30*a**5*c**4*d**6 + 180*a**4*b*c**5*d**5 + 300*a**3*b**2*c**
6*d**4 + 1200*a**2*b**3*c**7*d**3/7 + 225*a*b**4*c**8*d**2/7 + 10*b**5*c**9
*d/7) + x**6*(42*a**5*c**5*d**5 + 175*a**4*b*c**6*d**4 + 200*a**3*b**2*c**7
*d**3 + 75*a**2*b**3*c**8*d**2 + 25*a*b**4*c**9*d/3 + b**5*c**10/6) + x**5*
(42*a**5*c**6*d**4 + 120*a**4*b*c**7*d**3 + 90*a**3*b**2*c**8*d**2 + 20*a**
2*b**3*c**9*d + a*b**4*c**10) + x**4*(30*a**5*c**7*d**3 + 225*a**4*b*c**8*d
**2/4 + 25*a**3*b**2*c**9*d + 5*a**2*b**3*c**10/2) + x**3*(15*a**5*c**8*d**
2 + 50*a**4*b*c**9*d/3 + 10*a**3*b**2*c**10/3) + x**2*(5*a**5*c**9*d + 5*a*
*4*b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(134) = 268$.
time = 0.00, size = 1024, normalized size = 7.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x)
```

```
[Out] 1/16*b^5*d^10*x^16 + 2/3*b^5*c*d^9*x^15 + 1/3*a*b^4*d^10*x^15 + 45/14*b^5*c
^2*d^8*x^14 + 25/7*a*b^4*c*d^9*x^14 + 5/7*a^2*b^3*d^10*x^14 + 120/13*b^5*c^
3*d^7*x^13 + 225/13*a*b^4*c^2*d^8*x^13 + 100/13*a^2*b^3*c*d^9*x^13 + 10/13*
a^3*b^2*d^10*x^13 + 35/2*b^5*c^4*d^6*x^12 + 50*a*b^4*c^3*d^7*x^12 + 75/2*a^
2*b^3*c^2*d^8*x^12 + 25/3*a^3*b^2*c*d^9*x^12 + 5/12*a^4*b*d^10*x^12 + 252/1
1*b^5*c^5*d^5*x^11 + 1050/11*a*b^4*c^4*d^6*x^11 + 1200/11*a^2*b^3*c^3*d^7*x
^11 + 450/11*a^3*b^2*c^2*d^8*x^11 + 50/11*a^4*b*c*d^9*x^11 + 1/11*a^5*d^10*
x^11 + 21*b^5*c^6*d^4*x^10 + 126*a*b^4*c^5*d^5*x^10 + 210*a^2*b^3*c^4*d^6*x
^10 + 120*a^3*b^2*c^3*d^7*x^10 + 45/2*a^4*b*c^2*d^8*x^10 + a^5*c*d^9*x^10 +
 40/3*b^5*c^7*d^3*x^9 + 350/3*a*b^4*c^6*d^4*x^9 + 280*a^2*b^3*c^5*d^5*x^9 +
 700/3*a^3*b^2*c^4*d^6*x^9 + 200/3*a^4*b*c^3*d^7*x^9 + 5*a^5*c^2*d^8*x^9 +
45/8*b^5*c^8*d^2*x^8 + 75*a*b^4*c^7*d^3*x^8 + 525/2*a^2*b^3*c^6*d^4*x^8 + 3
15*a^3*b^2*c^5*d^5*x^8 + 525/4*a^4*b*c^4*d^6*x^8 + 15*a^5*c^3*d^7*x^8 + 10/
7*b^5*c^9*d*x^7 + 225/7*a*b^4*c^8*d^2*x^7 + 1200/7*a^2*b^3*c^7*d^3*x^7 + 30
0*a^3*b^2*c^6*d^4*x^7 + 180*a^4*b*c^5*d^5*x^7 + 30*a^5*c^4*d^6*x^7 + 1/6*b^
5*c^10*x^6 + 25/3*a*b^4*c^9*d*x^6 + 75*a^2*b^3*c^8*d^2*x^6 + 200*a^3*b^2*c^
7*d^3*x^6 + 175*a^4*b*c^6*d^4*x^6 + 42*a^5*c^5*d^5*x^6 + a*b^4*c^10*x^5 + 2
0*a^2*b^3*c^9*d*x^5 + 90*a^3*b^2*c^8*d^2*x^5 + 120*a^4*b*c^7*d^3*x^5 + 42*a
^5*c^6*d^4*x^5 + 5/2*a^2*b^3*c^10*x^4 + 25*a^3*b^2*c^9*d*x^4 + 225/4*a^4*b*
c^8*d^2*x^4 + 30*a^5*c^7*d^3*x^4 + 10/3*a^3*b^2*c^10*x^3 + 50/3*a^4*b*c^9*d
*x^3 + 15*a^5*c^8*d^2*x^3 + 5/2*a^4*b*c^10*x^2 + 5*a^5*c^9*d*x^2 + a^5*c^10
*x
```

Mupad [B]
time = 0.34, size = 806, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^10,x)
```

```
[Out] x^10*(a^5*c*d^9 + 21*b^5*c^6*d^4 + 126*a*b^4*c^5*d^5 + (45*a^4*b*c^2*d^8)/2
 + 210*a^2*b^3*c^4*d^6 + 120*a^3*b^2*c^3*d^7) + x^7*((10*b^5*c^9*d)/7 + 30*
a^5*c^4*d^6 + (225*a*b^4*c^8*d^2)/7 + 180*a^4*b*c^5*d^5 + (1200*a^2*b^3*c^7
*d^3)/7 + 300*a^3*b^2*c^6*d^4) + x^6*((b^5*c^10)/6 + 42*a^5*c^5*d^5 + 175*a
^4*b*c^6*d^4 + 75*a^2*b^3*c^8*d^2 + 200*a^3*b^2*c^7*d^3 + (25*a*b^4*c^9*d)/
3) + x^11*((a^5*d^10)/11 + (252*b^5*c^5*d^5)/11 + (1050*a*b^4*c^4*d^6)/11 +
 (1200*a^2*b^3*c^3*d^7)/11 + (450*a^3*b^2*c^2*d^8)/11 + (50*a^4*b*c*d^9)/11
) + x^8*(15*a^5*c^3*d^7 + (45*b^5*c^8*d^2)/8 + 75*a*b^4*c^7*d^3 + (525*a^4*
b*c^4*d^6)/4 + (525*a^2*b^3*c^6*d^4)/2 + 315*a^3*b^2*c^5*d^5) + x^9*(5*a^5*
c^2*d^8 + (40*b^5*c^7*d^3)/3 + (350*a*b^4*c^6*d^4)/3 + (200*a^4*b*c^3*d^7)/
3 + 280*a^2*b^3*c^5*d^5 + (700*a^3*b^2*c^4*d^6)/3) + x^5*(a*b^4*c^10 + 42*a
```

```
^5*c^6*d^4 + 20*a^2*b^3*c^9*d + 120*a^4*b*c^7*d^3 + 90*a^3*b^2*c^8*d^2) + x
^12*((5*a^4*b*d^10)/12 + (35*b^5*c^4*d^6)/2 + 50*a*b^4*c^3*d^7 + (25*a^3*b^
2*c*d^9)/3 + (75*a^2*b^3*c^2*d^8)/2) + a^5*c^10*x + (b^5*d^10*x^16)/16 + (5
*a^2*c^7*x^4*(24*a^3*d^3 + 2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^2*b*c*d^2))/4
+ (5*b^2*d^7*x^13*(2*a^3*d^3 + 24*b^3*c^3 + 45*a*b^2*c^2*d + 20*a^2*b*c*d^2
))/13 + (5*a^4*c^9*x^2*(2*a*d + b*c))/2 + (b^4*d^9*x^15*(a*d + 2*b*c))/3 +
(5*a^3*c^8*x^3*(9*a^2*d^2 + 2*b^2*c^2 + 10*a*b*c*d))/3 + (5*b^3*d^8*x^14*(2
*a^2*d^2 + 9*b^2*c^2 + 10*a*b*c*d))/14
```

3.1307 $\int (a+bx)^4(c+dx)^{10}\,dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-\frac{2b^3(bc-ad)(c+dx)^{14}}{7d^5}+\frac{b^4(c+dx)}{15d^5}$$

```
[Out] 1/11*(-a*d+b*c)^4*(d*x+c)^11/d^5-1/3*b*(-a*d+b*c)^3*(d*x+c)^12/d^5+6/13*b^2
*(-a*d+b*c)^2*(d*x+c)^13/d^5-2/7*b^3*(-a*d+b*c)*(d*x+c)^14/d^5+1/15*b^4*(d*
x+c)^15/d^5
```

Rubi [A]
time = 0.32, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^11)/(11*d^5) - (b*(b*c - a*d)^3*(c + d*x)^12)/(3*d
^5) + (6*b^2*(b*c - a*d)^2*(c + d*x)^13)/(13*d^5) - (2*b^3*(b*c - a*d)*(c +
 d*x)^14)/(7*d^5) + (b^4*(c + d*x)^15)/(15*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^4(c+dx)^{10}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{11}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{12}}{d^4}\right.$$

$$=\frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-2b^3($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 660 vs. 2(119) = 238.
time = 0.04, size = 660, normalized size = 5.55

$$a^4c^{10}x+a^3c^9(2bc+5ad)x^2+\frac{1}{3}a^2c^8\left(6b^2c^2+40abcd+45a^2d^2\right)x^3+ac^7\left(b^3c^3+15ab^2c^2d+45a^2bcd^2+30a^3d^3\right)x^4+\frac{1}{5}c^6\left(b^4c^4+40ab^3c^3d+270a^2b^2c^2d^2+480a^3bcd^3+210a^4d^4\right)x^5+\frac{1}{3}c^5d\left(5b^4c^4+90ab^3c^3d+360a^2b^2c^2d^2+420a^3bcd^3+126a^4d^4\right)x^6+\frac{3}{7}c^4d^2\left(15b^4c^4+160ab^3c^3d+420a^2b^2c^2d^2+336a^3bcd^3+70a^4d^4\right)x^7+3c^3d^3\left(5b^4c^4+35ab^3c^3d+63a^2b^2c^2d^2+35a^3bcd^3+5a^4d^4\right)x^8+\frac{1}{3}c^2d^4\left(70b^4c^4+336ab^3c^3d+420a^2b^2c^2d^2+160a^3bcd^3+15a^4d^4\right)x^9+\frac{1}{5}cd^5\left(126b^4c^4+420ab^3c^3d+360a^2b^2c^2d^2+90a^3bcd^3+5a^4d^4\right)x^{10}+\frac{1}{11}d^6\left(210b^4c^4+480ab^3c^3d+270a^2b^2c^2d^2+40a^3bcd^3+a^4d^4\right)x^{11}+\frac{1}{3}bd^7\left(30b^3c^3+45ab^2c^2d+15a^2bcd^2+a^3d^3\right)x^{12}+\frac{1}{13}b^2d^8\left(45b^2c^2+40abcd+6a^2d^2\right)x^{13}+\frac{1}{7}b^3d^9(5bc+2ad)x^{14}+\frac{1}{15}b^4d^{10}x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] a^4*c^10*x + a^3*c^9*(2*b*c + 5*a*d)*x^2 + (a^2*c^8*(6*b^2*c^2 + 40*a*b*c*d
 + 45*a^2*d^2)*x^3)/3 + a*c^7*(b^3*c^3 + 15*a*b^2*c^2*d + 45*a^2*b*c*d^2 +
30*a^3*d^3)*x^4 + (c^6*(b^4*c^4 + 40*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 48
0*a^3*b*c*d^3 + 210*a^4*d^4)*x^5)/5 + (c^5*d*(5*b^4*c^4 + 90*a*b^3*c^3*d +
360*a^2*b^2*c^2*d^2 + 420*a^3*b*c*d^3 + 126*a^4*d^4)*x^6)/3 + (3*c^4*d^2*(1
5*b^4*c^4 + 160*a*b^3*c^3*d + 420*a^2*b^2*c^2*d^2 + 336*a^3*b*c*d^3 + 70*a^
4*d^4)*x^7)/7 + 3*c^3*d^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2*b^2*c^2*d^2
+ 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^8 + (c^2*d^4*(70*b^4*c^4 + 336*a*b^3*c^3*d
+ 420*a^2*b^2*c^2*d^2 + 160*a^3*b*c*d^3 + 15*a^4*d^4)*x^9)/3 + (c*d^5*(126*
b^4*c^4 + 420*a*b^3*c^3*d + 360*a^2*b^2*c^2*d^2 + 90*a^3*b*c*d^3 + 5*a^4*d^
4)*x^10)/5 + (d^6*(210*b^4*c^4 + 480*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 + a^4*d^4)*x^11)/11 + (b*d^7*(30*b^3*c^3 + 45*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + a^3*d^3)*x^12)/3 + (b^2*d^8*(45*b^2*c^2 + 40*a*b*c*d + 6*a^2
*d^2)*x^13)/13 + (b^3*d^9*(5*b*c + 2*a*d)*x^14)/7 + (b^4*d^10*x^15)/15
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 659 vs. 2(119) = 238.
time = 6.70, size = 637, normalized size = 5.35

$$x\left(a^4c^{10} + a^3c^9x(5ad+2bc) + \frac{a^2c^8x^2(45a^2d^2+40abcd+6b^2c^2)}{3} + ac^7x^3(30a^3d^3+45a^2bcd^2+15ab^2c^2d+b^3c^3) + \frac{bd^7x^{11}(a^3d^3+15a^2bcd^2+45ab^2c^2d+30b^3c^3)}{3} + \frac{c^6x^4(210a^4d^4+480a^3bcd^3+270a^2b^2c^2d^2+40ab^3c^3d+b^4c^4)}{5} + \frac{d^6x^{10}(a^4d^4+40a^3bcd^3+270a^2b^2c^2d^2+480ab^3c^3d+210b^4c^4)}{11} + \frac{b^2d^8x^{12}(6a^2d^2+40abcd+45b^2c^2)}{13} + \frac{b^3d^9x^{13}(2ad+5bc)}{7} + \frac{b^4d^{10}x^{14}}{15} + \frac{c^5dx^5(126a^4d^4+420a^3bcd^3+360a^2b^2c^2d^2+90ab^3c^3d+5b^4c^4)}{3} + \frac{3c^4d^2x^6(70a^4d^4+336a^3bcd^3+420a^2b^2c^2d^2+160ab^3c^3d+15b^4c^4)}{7} + 3c^3d^3x^7(5a^4d^4+35a^3bcd^3+63a^2b^2c^2d^2+35ab^3c^3d+5b^4c^4) + \frac{c^2d^4x^8(15a^4d^4+160a^3bcd^3+420a^2b^2c^2d^2+336ab^3c^3d+70b^4c^4)}{3} + \frac{cd^5x^9(5a^4d^4+90a^3bcd^3+360a^2b^2c^2d^2+420ab^3c^3d+126b^4c^4)}{5}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^10,x]')
```

```
[Out] x (a ^ 4 c ^ 10 + a ^ 3 c ^ 9 x (5 a d + 2 b c) + a ^ 2 c ^ 8 x ^ 2 (45 a ^
 2 d ^ 2 + 40 a b c d + 6 b ^ 2 c ^ 2) / 3 + a c ^ 7 x ^ 3 (30 a ^ 3 d ^ 3
+ 45 a ^ 2 b c d ^ 2 + 15 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) + b d ^ 7 x ^ 11 (
a ^ 3 d ^ 3 + 15 a ^ 2 b c d ^ 2 + 45 a b ^ 2 c ^ 2 d + 30 b ^ 3 c ^ 3) / 3
 + c ^ 6 x ^ 4 (210 a ^ 4 d ^ 4 + 480 a ^ 3 b c d ^ 3 + 270 a ^ 2 b ^ 2 c ^
 2 d ^ 2 + 40 a b ^ 3 c ^ 3 d + b ^ 4 c ^ 4) / 5 + d ^ 6 x ^ 10 (a ^ 4 d ^
4 + 40 a ^ 3 b c d ^ 3 + 270 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 480 a b ^ 3 c ^ 3 d
+ 210 b ^ 4 c ^ 4) / 11 + b ^ 2 d ^ 8 x ^ 12 (6 a ^ 2 d ^ 2 + 40 a b c d +
45 b ^ 2 c ^ 2) / 13 + b ^ 3 d ^ 9 x ^ 13 (2 a d + 5 b c) / 7 + b ^ 4 d ^ 1
0 x ^ 14 / 15 + c ^ 5 d x ^ 5 (126 a ^ 4 d ^ 4 + 420 a ^ 3 b c d ^ 3 + 360
a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 90 a b ^ 3 c ^ 3 d + 5 b ^ 4 c ^ 4) / 3 + 3 c ^ 4
 d ^ 2 x ^ 6 (70 a ^ 4 d ^ 4 + 336 a ^ 3 b c d ^ 3 + 420 a ^ 2 b ^ 2 c ^ 2
d ^ 2 + 160 a b ^ 3 c ^ 3 d + 15 b ^ 4 c ^ 4) / 7 + 3 c ^ 3 d ^ 3 x ^ 7 (5
a ^ 4 d ^ 4 + 35 a ^ 3 b c d ^ 3 + 63 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 35 a b ^ 3
c ^ 3 d + 5 b ^ 4 c ^ 4) + c ^ 2 d ^ 4 x ^ 8 (15 a ^ 4 d ^ 4 + 160 a ^ 3 b
c d ^ 3 + 420 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 336 a b ^ 3 c ^ 3 d + 70 b ^ 4 c ^
4) / 3 + c d ^ 5 x ^ 9 (5 a ^ 4 d ^ 4 + 90 a ^ 3 b c d ^ 3 + 360 a ^ 2 b ^
2 c ^ 2 d ^ 2 + 420 a b ^ 3 c ^ 3 d + 126 b ^ 4 c ^ 4) / 5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(109) = 218$.
time = 0.13, size = 691, normalized size = 5.81

method	result
norman	$a^4c^{10}x + (5a^4c^9d + 2a^3b\,c^{10})\,x^2 + \left(15a^4c^8d^2 + \frac{40}{3}a^3b\,c^9d + 2b^2a^2c^{10}\right)x^3 + (30a^4c^7d^3 + 45a^3b\,c^8d^2$
default	$\frac{b^4d^{10}x^{15}}{15} + \frac{(4a\,b^3d^{10}+10b^4c\,d^9)x^{14}}{14} + \frac{(6b^2a^2d^{10}+40a\,b^3c\,d^9+45b^4c^2d^8)x^{13}}{13} + \frac{(4a^3b\,d^{10}+60b^2a^2c\,d^9+180a\,b^3c^2d^8+120b^4c^3d^7}{12}$
gosper	$5a^4c^9d\,x^2 + \frac{126}{5}x^{10}b^4c^5d^5 + \frac{210}{11}x^{11}b^4c^4d^6 + \frac{1}{3}x^{12}a^3b\,d^{10} + 10x^{12}b^4c^3d^7 + \frac{6}{13}x^{13}b^2a^2d^{10} + \frac{45}{13}x^{13}b^4c^2$
risch	$5a^4c^9d\,x^2 + \frac{126}{5}x^{10}b^4c^5d^5 + \frac{210}{11}x^{11}b^4c^4d^6 + \frac{1}{3}x^{12}a^3b\,d^{10} + 10x^{12}b^4c^3d^7 + \frac{6}{13}x^{13}b^2a^2d^{10} + \frac{45}{13}x^{13}b^4c^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^4*d^10*x^15+1/14*(4*a*b^3*d^10+10*b^4*c*d^9)*x^14+1/13*(6*a^2*b^2*d^
10+40*a*b^3*c*d^9+45*b^4*c^2*d^8)*x^13+1/12*(4*a^3*b*d^10+60*a^2*b^2*c*d^9+
180*a*b^3*c^2*d^8+120*b^4*c^3*d^7)*x^12+1/11*(a^4*d^10+40*a^3*b*c*d^9+270*a
^2*b^2*c^2*d^8+480*a*b^3*c^3*d^7+210*b^4*c^4*d^6)*x^11+1/10*(10*a^4*c*d^9+1
80*a^3*b*c^2*d^8+720*a^2*b^2*c^3*d^7+840*a*b^3*c^4*d^6+252*b^4*c^5*d^5)*x^1
0+1/9*(45*a^4*c^2*d^8+480*a^3*b*c^3*d^7+1260*a^2*b^2*c^4*d^6+1008*a*b^3*c^5
*d^5+210*b^4*c^6*d^4)*x^9+1/8*(120*a^4*c^3*d^7+840*a^3*b*c^4*d^6+1512*a^2*b
^2*c^5*d^5+840*a*b^3*c^6*d^4+120*b^4*c^7*d^3)*x^8+1/7*(210*a^4*c^4*d^6+1008
*a^3*b*c^5*d^5+1260*a^2*b^2*c^6*d^4+480*a*b^3*c^7*d^3+45*b^4*c^8*d^2)*x^7+1
/6*(252*a^4*c^5*d^5+840*a^3*b*c^6*d^4+720*a^2*b^2*c^7*d^3+180*a*b^3*c^8*d^2
+10*b^4*c^9*d)*x^6+1/5*(210*a^4*c^6*d^4+480*a^3*b*c^7*d^3+270*a^2*b^2*c^8*d
^2+40*a*b^3*c^9*d+b^4*c^10)*x^5+1/4*(120*a^4*c^7*d^3+180*a^3*b*c^8*d^2+60*a
^2*b^2*c^9*d+4*a*b^3*c^10)*x^4+1/3*(45*a^4*c^8*d^2+40*a^3*b*c^9*d+6*a^2*b^2
*c^10)*x^3+1/2*(10*a^4*c^9*d+4*a^3*b*c^10)*x^2+a^4*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(109) = 218$.
time = 0.31, size = 686, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
```

```
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(109) = 218$.
time = 0.29, size = 686, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(105) = 210$.
time = 0.09, size = 748, normalized size = 6.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**10,x)
```

```
[Out] a**4*c**10*x + b**4*d**10*x**15/15 + x**14*(2*a*b**3*d**10/7 + 5*b**4*c*d**
9/7) + x**13*(6*a**2*b**2*d**10/13 + 40*a*b**3*c*d**9/13 + 45*b**4*c**2*d**
```

```
8/13) + x**12*(a**3*b*d**10/3 + 5*a**2*b**2*c*d**9 + 15*a*b**3*c**2*d**8 +
10*b**4*c**3*d**7) + x**11*(a**4*d**10/11 + 40*a**3*b*c*d**9/11 + 270*a**2*
b**2*c**2*d**8/11 + 480*a*b**3*c**3*d**7/11 + 210*b**4*c**4*d**6/11) + x**1
0*(a**4*c*d**9 + 18*a**3*b*c**2*d**8 + 72*a**2*b**2*c**3*d**7 + 84*a*b**3*c
**4*d**6 + 126*b**4*c**5*d**5/5) + x**9*(5*a**4*c**2*d**8 + 160*a**3*b*c**3
*d**7/3 + 140*a**2*b**2*c**4*d**6 + 112*a*b**3*c**5*d**5 + 70*b**4*c**6*d**
4/3) + x**8*(15*a**4*c**3*d**7 + 105*a**3*b*c**4*d**6 + 189*a**2*b**2*c**5*
d**5 + 105*a*b**3*c**6*d**4 + 15*b**4*c**7*d**3) + x**7*(30*a**4*c**4*d**6
+ 144*a**3*b*c**5*d**5 + 180*a**2*b**2*c**6*d**4 + 480*a*b**3*c**7*d**3/7 +
 45*b**4*c**8*d**2/7) + x**6*(42*a**4*c**5*d**5 + 140*a**3*b*c**6*d**4 + 12
0*a**2*b**2*c**7*d**3 + 30*a*b**3*c**8*d**2 + 5*b**4*c**9*d/3) + x**5*(42*a
**4*c**6*d**4 + 96*a**3*b*c**7*d**3 + 54*a**2*b**2*c**8*d**2 + 8*a*b**3*c**
9*d + b**4*c**10/5) + x**4*(30*a**4*c**7*d**3 + 45*a**3*b*c**8*d**2 + 15*a*
*2*b**2*c**9*d + a*b**3*c**10) + x**3*(15*a**4*c**8*d**2 + 40*a**3*b*c**9*d
/3 + 2*a**2*b**2*c**10) + x**2*(5*a**4*c**9*d + 2*a**3*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(109) = 218.
time = 0.00, size = 811, normalized size = 6.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x)
```

```
[Out] 1/15*b^4*d^10*x^15 + 5/7*b^4*c*d^9*x^14 + 2/7*a*b^3*d^10*x^14 + 45/13*b^4*c
^2*d^8*x^13 + 40/13*a*b^3*c*d^9*x^13 + 6/13*a^2*b^2*d^10*x^13 + 10*b^4*c^3*
d^7*x^12 + 15*a*b^3*c^2*d^8*x^12 + 5*a^2*b^2*c*d^9*x^12 + 1/3*a^3*b*d^10*x^
12 + 210/11*b^4*c^4*d^6*x^11 + 480/11*a*b^3*c^3*d^7*x^11 + 270/11*a^2*b^2*c
^2*d^8*x^11 + 40/11*a^3*b*c*d^9*x^11 + 1/11*a^4*d^10*x^11 + 126/5*b^4*c^5*d
^5*x^10 + 84*a*b^3*c^4*d^6*x^10 + 72*a^2*b^2*c^3*d^7*x^10 + 18*a^3*b*c^2*d^
8*x^10 + a^4*c*d^9*x^10 + 70/3*b^4*c^6*d^4*x^9 + 112*a*b^3*c^5*d^5*x^9 + 14
0*a^2*b^2*c^4*d^6*x^9 + 160/3*a^3*b*c^3*d^7*x^9 + 5*a^4*c^2*d^8*x^9 + 15*b^
4*c^7*d^3*x^8 + 105*a*b^3*c^6*d^4*x^8 + 189*a^2*b^2*c^5*d^5*x^8 + 105*a^3*b
*c^4*d^6*x^8 + 15*a^4*c^3*d^7*x^8 + 45/7*b^4*c^8*d^2*x^7 + 480/7*a*b^3*c^7*
d^3*x^7 + 180*a^2*b^2*c^6*d^4*x^7 + 144*a^3*b*c^5*d^5*x^7 + 30*a^4*c^4*d^6*
x^7 + 5/3*b^4*c^9*d*x^6 + 30*a*b^3*c^8*d^2*x^6 + 120*a^2*b^2*c^7*d^3*x^6 +
140*a^3*b*c^6*d^4*x^6 + 42*a^4*c^5*d^5*x^6 + 1/5*b^4*c^10*x^5 + 8*a*b^3*c^9
*d*x^5 + 54*a^2*b^2*c^8*d^2*x^5 + 96*a^3*b*c^7*d^3*x^5 + 42*a^4*c^6*d^4*x^5
 + a*b^3*c^10*x^4 + 15*a^2*b^2*c^9*d*x^4 + 45*a^3*b*c^8*d^2*x^4 + 30*a^4*c^
7*d^3*x^4 + 2*a^2*b^2*c^10*x^3 + 40/3*a^3*b*c^9*d*x^3 + 15*a^4*c^8*d^2*x^3
+ 2*a^3*b*c^10*x^2 + 5*a^4*c^9*d*x^2 + a^4*c^10*x
```

Mupad [B]
time = 0.43, size = 664, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^4*(c + d*x)^10,x)

[Out] x^5*((b^4*c^10)/5 + 42*a^4*c^6*d^4 + 96*a^3*b*c^7*d^3 + 54*a^2*b^2*c^8*d^2 + 8*a*b^3*c^9*d) + x^11*((a^4*d^10)/11 + (210*b^4*c^4*d^6)/11 + (480*a*b^3*c^3*d^7)/11 + (270*a^2*b^2*c^2*d^8)/11 + (40*a^3*b*c*d^9)/11) + x^8*(15*a^4*c^3*d^7 + 15*b^4*c^7*d^3 + 105*a*b^3*c^6*d^4 + 105*a^3*b*c^4*d^6 + 189*a^2*b^2*c^5*d^5) + x^9*(5*a^4*c^2*d^8 + (70*b^4*c^6*d^4)/3 + 112*a*b^3*c^5*d^5 + (160*a^3*b*c^3*d^7)/3 + 140*a^2*b^2*c^4*d^6) + x^7*(30*a^4*c^4*d^6 + (45*b^4*c^8*d^2)/7 + (480*a*b^3*c^7*d^3)/7 + 144*a^3*b*c^5*d^5 + 180*a^2*b^2*c^6*d^4) + x^4*(a*b^3*c^10 + 30*a^4*c^7*d^3 + 15*a^2*b^2*c^9*d + 45*a^3*b*c^8*d^2) + x^12*((a^3*b*d^10)/3 + 10*b^4*c^3*d^7 + 15*a*b^3*c^2*d^8 + 5*a^2*b^2*c*d^9) + x^10*(a^4*c*d^9 + (126*b^4*c^5*d^5)/5 + 84*a*b^3*c^4*d^6 + 18*a^3*b*c^2*d^8 + 72*a^2*b^2*c^3*d^7) + x^6*((5*b^4*c^9*d)/3 + 42*a^4*c^5*d^5 + 30*a*b^3*c^8*d^2 + 140*a^3*b*c^6*d^4 + 120*a^2*b^2*c^7*d^3) + a^4*c^10*x + (b^4*d^10*x^15)/15 + a^3*c^9*x^2*(5*a*d + 2*b*c) + (b^3*d^9*x^14*(2*a*d + 5*b*c))/7 + (a^2*c^8*x^3*(45*a^2*d^2 + 6*b^2*c^2 + 40*a*b*c*d))/3 + (b^2*d^8*x^13*(6*a^2*d^2 + 45*b^2*c^2 + 40*a*b*c*d))/13

3.1308 $\int (a+bx)^3(c+dx)^{10}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

```
[Out] -1/11*(-a*d+b*c)^3*(d*x+c)^11/d^4+1/4*b*(-a*d+b*c)^2*(d*x+c)^12/d^4-3/13*b^
2*(-a*d+b*c)*(d*x+c)^13/d^4+1/14*b^3*(d*x+c)^14/d^4
```

Rubi [A]
time = 0.25, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^3*(c + d*x)^11)/d^4 + (b*(b*c - a*d)^2*(c + d*x)^12)/(4*
d^4) - (3*b^2*(b*c - a*d)*(c + d*x)^13)/(13*d^4) + (b^3*(c + d*x)^14)/(14*d
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^3(c+dx)^{10}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{11}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{12}}{d^3}\right.$$

$$=-\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 511 vs. 2(92) = 184.
time = 0.04, size = 511, normalized size = 5.55

$$a^3c^{10}x+\frac{1}{2}a^2c^9(3bc+10ad)x^2+ac^8\left(b^2c^2+10abcd+15a^2d^2\right)x^3+\frac{1}{4}c^7\left(b^3c^3+30ab^2c^2d+135a^2bcd^2+120a^3d^3\right)x^4+c^6d\left(2b^3c^3+27ab^2c^2d+72a^2bcd^2+42a^3d^3\right)x^5+\frac{3}{2}c^5d^2\left(5b^3c^3+40ab^2c^2d+70a^2bcd^2+28a^3d^3\right)x^6+\frac{6}{7}c^4d^3\left(20b^3c^3+105ab^2c^2d+126a^2bcd^2+35a^3d^3\right)x^7+\frac{3}{4}c^3d^4\left(35b^3c^3+126ab^2c^2d+105a^2bcd^2+20a^3d^3\right)x^8+c^2d^5\left(28b^3c^3+70ab^2c^2d+40a^2bcd^2+5a^3d^3\right)x^9+\frac{1}{2}cd^6\left(42b^3c^3+72ab^2c^2d+27a^2bcd^2+2a^3d^3\right)x^{10}+\frac{1}{11}d^7\left(120b^3c^3+135ab^2c^2d+30a^2bcd^2+a^3d^3\right)x^{11}+\frac{1}{4}bd^8\left(15b^2c^2+10abcd+a^2d^2\right)x^{12}+\frac{1}{13}b^2d^9(10bc+3ad)x^{13}+\frac{1}{14}b^3d^{10}x^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] a^3*c^10*x + (a^2*c^9*(3*b*c + 10*a*d)*x^2)/2 + a*c^8*(b^2*c^2 + 10*a*b*c*d
 + 15*a^2*d^2)*x^3 + (c^7*(b^3*c^3 + 30*a*b^2*c^2*d + 135*a^2*b*c*d^2 + 120
*a^3*d^3)*x^4)/4 + c^6*d*(2*b^3*c^3 + 27*a*b^2*c^2*d + 72*a^2*b*c*d^2 + 42*
a^3*d^3)*x^5 + (3*c^5*d^2*(5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2 + 28
*a^3*d^3)*x^6)/2 + (6*c^4*d^3*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d
^2 + 35*a^3*d^3)*x^7)/7 + (3*c^3*d^4*(35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^
2*b*c*d^2 + 20*a^3*d^3)*x^8)/4 + c^2*d^5*(28*b^3*c^3 + 70*a*b^2*c^2*d + 40*
a^2*b*c*d^2 + 5*a^3*d^3)*x^9 + (c*d^6*(42*b^3*c^3 + 72*a*b^2*c^2*d + 27*a^2
*b*c*d^2 + 2*a^3*d^3)*x^10)/2 + (d^7*(120*b^3*c^3 + 135*a*b^2*c^2*d + 30*a^
2*b*c*d^2 + a^3*d^3)*x^11)/11 + (b*d^8*(15*b^2*c^2 + 10*a*b*c*d + a^2*d^2)*
x^12)/4 + (b^2*d^9*(10*b*c + 3*a*d)*x^13)/13 + (b^3*d^10*x^14)/14
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 510 vs. $2(92) = 184$.
time = 5.60, size = 490, normalized size = 5.33

$$x\left(a^3c^{10}+\frac{a^2c^9x\,(10ad+3bc)}{2}+ac^8x^2\left(15a^2d^2+10abcd+b^2c^2\right)+\frac{bd^8x^{11}\left(a^2d^2+10abcd+15b^2c^2\right)}{4}+\frac{b^2d^9x^{12}\,(3ad+10bc)}{13}+\frac{c^7x^3\left(120a^3d^3+135a^2bcd^2+30ab^2c^2d+b^3c^3\right)}{4}+\frac{6c^4d^3x^6\left(35a^3d^3+126a^2bcd^2+105ab^2c^2d+20b^3c^3\right)}{7}+\frac{3c^3d^4x^7\left(20a^3d^3+105a^2bcd^2+126ab^2c^2d+35b^3c^3\right)}{4}+c^2d^5x^8\left(5a^3d^3+40a^2bcd^2+70ab^2c^2d+28b^3c^3\right)+\frac{d^7x^{10}\left(a^3d^3+30a^2bcd^2+135ab^2c^2d+120b^3c^3\right)}{11}+\frac{b^3d^{10}x^{13}}{14}+c^6dx^4\left(42a^3d^3+72a^2bcd^2+27ab^2c^2d+2b^3c^3\right)+\frac{3c^5d^2x^5\left(28a^3d^3+70a^2bcd^2+40ab^2c^2d+5b^3c^3\right)}{2}+\frac{cd^6x^9\left(2a^3d^3+27a^2bcd^2+72ab^2c^2d+42b^3c^3\right)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^10,x]')
```

```
[Out] x (a ^ 3 c ^ 10 + a ^ 2 c ^ 9 x (10 a d + 3 b c) / 2 + a c ^ 8 x ^ 2 (15 a
^ 2 d ^ 2 + 10 a b c d + b ^ 2 c ^ 2) + b d ^ 8 x ^ 11 (a ^ 2 d ^ 2 + 10 a
b c d + 15 b ^ 2 c ^ 2) / 4 + b ^ 2 d ^ 9 x ^ 12 (3 a d + 10 b c) / 13 + c
^ 7 x ^ 3 (120 a ^ 3 d ^ 3 + 135 a ^ 2 b c d ^ 2 + 30 a b ^ 2 c ^ 2 d + b ^
 3 c ^ 3) / 4 + 6 c ^ 4 d ^ 3 x ^ 6 (35 a ^ 3 d ^ 3 + 126 a ^ 2 b c d ^ 2 +
 105 a b ^ 2 c ^ 2 d + 20 b ^ 3 c ^ 3) / 7 + 3 c ^ 3 d ^ 4 x ^ 7 (20 a ^ 3
d ^ 3 + 105 a ^ 2 b c d ^ 2 + 126 a b ^ 2 c ^ 2 d + 35 b ^ 3 c ^ 3) / 4 + c
 ^ 2 d ^ 5 x ^ 8 (5 a ^ 3 d ^ 3 + 40 a ^ 2 b c d ^ 2 + 70 a b ^ 2 c ^ 2 d +
 28 b ^ 3 c ^ 3) + d ^ 7 x ^ 10 (a ^ 3 d ^ 3 + 30 a ^ 2 b c d ^ 2 + 135 a b
 ^ 2 c ^ 2 d + 120 b ^ 3 c ^ 3) / 11 + b ^ 3 d ^ 10 x ^ 13 / 14 + c ^ 6 d x
 ^ 4 (42 a ^ 3 d ^ 3 + 72 a ^ 2 b c d ^ 2 + 27 a b ^ 2 c ^ 2 d + 2 b ^ 3 c
^ 3) + 3 c ^ 5 d ^ 2 x ^ 5 (28 a ^ 3 d ^ 3 + 70 a ^ 2 b c d ^ 2 + 40 a b ^
2 c ^ 2 d + 5 b ^ 3 c ^ 3) / 2 + c d ^ 6 x ^ 9 (2 a ^ 3 d ^ 3 + 27 a ^ 2 b
c d ^ 2 + 72 a b ^ 2 c ^ 2 d + 42 b ^ 3 c ^ 3) / 2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(84) = 168$.
time = 0.23, size = 541, normalized size = 5.88

method	result
norman	$a^3c^{10}x+\left(5a^3c^9d+\frac{3}{2}a^2b\,c^{10}\right)x^2+\left(15a^3c^8d^2+10a^2b\,c^9d+a\,b^2c^{10}\right)x^3+\left(30a^3c^7d^3+\frac{135}{4}a^2b\,c^8d^2+\right.$

default	$\frac{b^3 d^{10} x^{14}}{14}+\frac{(3 a\, b^2 d^{10}+10 b^3 c\, d^9) x^{13}}{13}+\frac{(3 a^2 b\, d^{10}+30 a\, b^2 c\, d^9+45 b^3 c^2 d^8) x^{12}}{12}+\frac{(a^3 d^{10}+30 a^2 b c\, d^9+135 a\, b^2 c^2 d^8+120 b^3 c^3 d^7) x^{11}}{11}$
gosper	$5x^2 a^3 c^9 d+\frac{3}{2} x^2 a^2 b\, c^{10}+30 x^4 a^3 c^7 d^3+42 x^6 a^3 c^5 d^5+\frac{15}{2} x^6 b^3 c^8 d^2+30 x^7 a^3 c^4 d^6+\frac{120}{7} x^7 b^3 c^7 d^3+\frac{3}{13}$
risch	$5x^2 a^3 c^9 d+\frac{3}{2} x^2 a^2 b\, c^{10}+30 x^4 a^3 c^7 d^3+42 x^6 a^3 c^5 d^5+\frac{15}{2} x^6 b^3 c^8 d^2+30 x^7 a^3 c^4 d^6+\frac{120}{7} x^7 b^3 c^7 d^3+\frac{3}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^3*d^10*x^14+1/13*(3*a*b^2*d^10+10*b^3*c*d^9)*x^13+1/12*(3*a^2*b*d^10
+30*a*b^2*c*d^9+45*b^3*c^2*d^8)*x^12+1/11*(a^3*d^10+30*a^2*b*c*d^9+135*a*b^
2*c^2*d^8+120*b^3*c^3*d^7)*x^11+1/10*(10*a^3*c*d^9+135*a^2*b*c^2*d^8+360*a*
b^2*c^3*d^7+210*b^3*c^4*d^6)*x^10+1/9*(45*a^3*c^2*d^8+360*a^2*b*c^3*d^7+630
*a*b^2*c^4*d^6+252*b^3*c^5*d^5)*x^9+1/8*(120*a^3*c^3*d^7+630*a^2*b*c^4*d^6+
756*a*b^2*c^5*d^5+210*b^3*c^6*d^4)*x^8+1/7*(210*a^3*c^4*d^6+756*a^2*b*c^5*d
^5+630*a*b^2*c^6*d^4+120*b^3*c^7*d^3)*x^7+1/6*(252*a^3*c^5*d^5+630*a^2*b*c^
6*d^4+360*a*b^2*c^7*d^3+45*b^3*c^8*d^2)*x^6+1/5*(210*a^3*c^6*d^4+360*a^2*b*
c^7*d^3+135*a*b^2*c^8*d^2+10*b^3*c^9*d)*x^5+1/4*(120*a^3*c^7*d^3+135*a^2*b*
c^8*d^2+30*a*b^2*c^9*d+b^3*c^10)*x^4+1/3*(45*a^3*c^8*d^2+30*a^2*b*c^9*d+3*a
*b^2*c^10)*x^3+1/2*(10*a^3*c^9*d+3*a^2*b*c^10)*x^2+a^3*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(84) = 168$.
time = 0.27, size = 535, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(84) = 168$.
time = 0.29, size = 535, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(80) = 160$.

time = 0.08, size = 586, normalized size = 6.37

$$a^{3}c^{10}x + \frac{b^{3}d^{10}x^{14}}{14} + x^{13}\cdot\left(\frac{3ab^{2}d^{10}}{13} + \frac{10b^{3}cd^{9}}{13}\right) + x^{12}\left(\frac{a^{2}bd^{10}}{4} + \frac{5ab^{2}cd^{9}}{2} + \frac{15b^{3}c^{2}d^{8}}{4}\right) + x^{11}\left(\frac{a^{3}d^{10}}{11} + \frac{30a^{2}bcd^{9}}{11} + \frac{135ab^{2}c^{2}d^{8}}{11} + \frac{120b^{3}c^{3}d^{7}}{11}\right) + x^{10}\left(a^{3}cd^{9} + \frac{27a^{2}bc^{2}d^{8}}{2} + 36ab^{2}c^{3}d^{7} + 21b^{3}c^{4}d^{6}\right) + x^{9}\cdot\left(5a^{3}c^{2}d^{8} + 40a^{2}bc^{3}d^{7} + 70ab^{2}c^{4}d^{6} + 28b^{3}c^{5}d^{5}\right) + x^{8}\left(15a^{3}c^{3}d^{7} + \frac{315a^{2}bc^{4}d^{6}}{4} + \frac{189ab^{2}c^{5}d^{5}}{2} + \frac{105b^{3}c^{6}d^{4}}{4}\right) + x^{7}\left(30a^{3}c^{4}d^{6} + 108a^{2}bc^{5}d^{5} + 90ab^{2}c^{6}d^{4} + \frac{120b^{3}c^{7}d^{3}}{7}\right) + x^{6}\left(42a^{3}c^{5}d^{5} + 105a^{2}bc^{6}d^{4} + 60ab^{2}c^{7}d^{3} + \frac{15b^{3}c^{8}d^{2}}{2}\right) + x^{5}\cdot\left(42a^{3}c^{6}d^{4} + 72a^{2}bc^{7}d^{3} + 27ab^{2}c^{8}d^{2} + 2b^{3}c^{9}d\right) + x^{4}\left(30a^{3}c^{7}d^{3} + \frac{135a^{2}bc^{8}d^{2}}{4} + \frac{15ab^{2}c^{9}d}{2} + \frac{b^{3}c^{10}}{4}\right) + x^{3}\cdot\left(15a^{3}c^{8}d^{2} + 10a^{2}bc^{9}d + ab^{2}c^{10}\right) + x^{2}\left(5a^{3}c^{9}d + \frac{3a^{2}bc^{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**10,x)
```

```
[Out] a**3*c**10*x + b**3*d**10*x**14/14 + x**13*(3*a*b**2*d**10/13 + 10*b**3*c*d
**9/13) + x**12*(a**2*b*d**10/4 + 5*a*b**2*c*d**9/2 + 15*b**3*c**2*d**8/4)
+ x**11*(a**3*d**10/11 + 30*a**2*b*c*d**9/11 + 135*a*b**2*c**2*d**8/11 + 12
0*b**3*c**3*d**7/11) + x**10*(a**3*c*d**9 + 27*a**2*b*c**2*d**8/2 + 36*a*b*
*2*c**3*d**7 + 21*b**3*c**4*d**6) + x**9*(5*a**3*c**2*d**8 + 40*a**2*b*c**3
*d**7 + 70*a*b**2*c**4*d**6 + 28*b**3*c**5*d**5) + x**8*(15*a**3*c**3*d**7
+ 315*a**2*b*c**4*d**6/4 + 189*a*b**2*c**5*d**5/2 + 105*b**3*c**6*d**4/4) +
 x**7*(30*a**3*c**4*d**6 + 108*a**2*b*c**5*d**5 + 90*a*b**2*c**6*d**4 + 120
*b**3*c**7*d**3/7) + x**6*(42*a**3*c**5*d**5 + 105*a**2*b*c**6*d**4 + 60*a*
b**2*c**7*d**3 + 15*b**3*c**8*d**2/2) + x**5*(42*a**3*c**6*d**4 + 72*a**2*b
*c**7*d**3 + 27*a*b**2*c**8*d**2 + 2*b**3*c**9*d) + x**4*(30*a**3*c**7*d**3
 + 135*a**2*b*c**8*d**2/4 + 15*a*b**2*c**9*d/2 + b**3*c**10/4) + x**3*(15*a
**3*c**8*d**2 + 10*a**2*b*c**9*d + a*b**2*c**10) + x**2*(5*a**3*c**9*d + 3*
a**2*b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(84) = 168$.

time = 0.00, size = 634, normalized size = 6.89

$$\frac{1}{14}x^{14}b^{3}d^{10} + \frac{10}{13}x^{13}b^{3}d^{9}c + \frac{3}{13}x^{13}b^{2}ad^{10} + \frac{15}{4}x^{12}b^{3}d^{8}c^{2} + \frac{5}{2}x^{12}b^{2}ad^{9}c + \frac{1}{4}x^{12}ba^{2}d^{10} + \frac{120}{11}x^{11}b^{3}d^{7}c^{3} + \frac{135}{11}x^{11}b^{2}ad^{8}c^{2} + \frac{30}{11}x^{11}ba^{2}d^{9}c + \frac{1}{11}x^{11}a^{3}d^{10} + 21x^{10}b^{3}d^{6}c^{4} + 36x^{10}b^{2}ad^{7}c^{3} + \frac{27}{2}x^{10}ba^{2}d^{8}c^{2} + x^{10}a^{3}d^{9}c + 28x^{9}b^{3}d^{5}c^{5} + 70x^{9}b^{2}ad^{6}c^{4} + 40x^{9}ba^{2}d^{7}c^{3} + 5x^{9}a^{3}d^{8}c^{2} + \frac{105}{4}x^{8}b^{3}d^{4}c^{6} + \frac{189}{2}x^{8}b^{2}ad^{5}c^{5} + \frac{315}{4}x^{8}ba^{2}d^{6}c^{4} + 15x^{8}a^{3}d^{7}c^{3} + \frac{120}{7}x^{7}b^{3}d^{3}c^{7} + 90x^{7}b^{2}ad^{4}c^{6} + 108x^{7}ba^{2}d^{5}c^{5} + 30x^{7}a^{3}d^{6}c^{4} + \frac{15}{2}x^{6}b^{3}d^{2}c^{8} + 60x^{6}b^{2}ad^{3}c^{7} + 105x^{6}ba^{2}d^{4}c^{6} + 42x^{6}a^{3}d^{5}c^{5} + 2x^{5}b^{3}dc^{9} + 27x^{5}b^{2}ad^{2}c^{8} + 72x^{5}ba^{2}d^{3}c^{7} + 42x^{5}a^{3}d^{4}c^{6} + \frac{1}{4}x^{4}b^{3}c^{10} + \frac{15}{2}x^{4}b^{2}adc^{9} + \frac{135}{4}x^{4}ba^{2}d^{2}c^{8} + 30x^{4}a^{3}d^{3}c^{7} + x^{3}b^{2}ac^{10} + 10x^{3}ba^{2}dc^{9} + 15x^{3}a^{3}d^{2}c^{8} + \frac{3}{2}x^{2}ba^{2}c^{10} + 5x^{2}a^{3}dc^{9} + xa^{3}c^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x)
```

```
[Out] 1/14*b^3*d^10*x^14 + 10/13*b^3*c*d^9*x^13 + 3/13*a*b^2*d^10*x^13 + 15/4*b^3
*c^2*d^8*x^12 + 5/2*a*b^2*c*d^9*x^12 + 1/4*a^2*b*d^10*x^12 + 120/11*b^3*c^3
*d^7*x^11 + 135/11*a*b^2*c^2*d^8*x^11 + 30/11*a^2*b*c*d^9*x^11 + 1/11*a^3*d
^10*x^11 + 21*b^3*c^4*d^6*x^10 + 36*a*b^2*c^3*d^7*x^10 + 27/2*a^2*b*c^2*d^8
*x^10 + a^3*c*d^9*x^10 + 28*b^3*c^5*d^5*x^9 + 70*a*b^2*c^4*d^6*x^9 + 40*a^2
*b*c^3*d^7*x^9 + 5*a^3*c^2*d^8*x^9 + 105/4*b^3*c^6*d^4*x^8 + 189/2*a*b^2*c^
5*d^5*x^8 + 315/4*a^2*b*c^4*d^6*x^8 + 15*a^3*c^3*d^7*x^8 + 120/7*b^3*c^7*d^
3*x^7 + 90*a*b^2*c^6*d^4*x^7 + 108*a^2*b*c^5*d^5*x^7 + 30*a^3*c^4*d^6*x^7 +
 15/2*b^3*c^8*d^2*x^6 + 60*a*b^2*c^7*d^3*x^6 + 105*a^2*b*c^6*d^4*x^6 + 42*a
^3*c^5*d^5*x^6 + 2*b^3*c^9*d*x^5 + 27*a*b^2*c^8*d^2*x^5 + 72*a^2*b*c^7*d^3*
x^5 + 42*a^3*c^6*d^4*x^5 + 1/4*b^3*c^10*x^4 + 15/2*a*b^2*c^9*d*x^4 + 135/4*
a^2*b*c^8*d^2*x^4 + 30*a^3*c^7*d^3*x^4 + a*b^2*c^10*x^3 + 10*a^2*b*c^9*d*x^
3 + 15*a^3*c^8*d^2*x^3 + 3/2*a^2*b*c^10*x^2 + 5*a^3*c^9*d*x^2 + a^3*c^10*x
```

Mupad [B]
time = 0.23, size = 495, normalized size = 5.38

$$x^4\left(30a^3c^7d^3+\frac{135a^2bc^8d^2}{4}+\frac{15ab^2c^9d}{2}+\frac{b^3c^{10}}{4}\right)+x^{11}\left(\frac{a^3d^{10}}{11}+\frac{30a^2bcd^9}{11}+\frac{135ab^2c^2d^8}{11}+\frac{120b^3c^3d^7}{11}\right)+a^3c^{10}x+\frac{b^3d^{10}x^{14}}{14}+\frac{3c^5d^2x^6\left(28a^3d^3+70a^2bcd^2+40ab^2c^2d+5b^3c^3\right)}{2}+c^2d^5x^9\left(5a^3d^3+40a^2bcd^2+70ab^2c^2d+28b^3c^3\right)+\frac{6c^4d^3x^7\left(35a^3d^3+126a^2bcd^2+105ab^2c^2d+20b^3c^3\right)}{7}+\frac{3c^3d^4x^8\left(20a^3d^3+105a^2bcd^2+126ab^2c^2d+35b^3c^3\right)}{4}+\frac{a^2c^9x^2\left(10ad+3bc\right)}{2}+\frac{b^2d^9x^{13}\left(3ad+10bc\right)}{13}+ac^8x^3\left(15a^2d^2+10abcd+b^2c^2\right)+\frac{bd^8x^{12}\left(a^2d^2+10abcd+15b^2c^2\right)}{4}+c^6dx^5\left(42a^3d^3+72a^2bcd^2+27ab^2c^2d+2b^3c^3\right)+\frac{cd^6x^{10}\left(2a^3d^3+27a^2bcd^2+72ab^2c^2d+42b^3c^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^10,x)
```

```
[Out] x^4*((b^3*c^10)/4 + 30*a^3*c^7*d^3 + (135*a^2*b*c^8*d^2)/4 + (15*a*b^2*c^9*
d)/2) + x^11*((a^3*d^10)/11 + (120*b^3*c^3*d^7)/11 + (135*a*b^2*c^2*d^8)/11
 + (30*a^2*b*c*d^9)/11) + a^3*c^10*x + (b^3*d^10*x^14)/14 + (3*c^5*d^2*x^6*
(28*a^3*d^3 + 5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2))/2 + c^2*d^5*x^9
*(5*a^3*d^3 + 28*b^3*c^3 + 70*a*b^2*c^2*d + 40*a^2*b*c*d^2) + (6*c^4*d^3*x^
7*(35*a^3*d^3 + 20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d^2))/7 + (3*c^3
*d^4*x^8*(20*a^3*d^3 + 35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^2*b*c*d^2))/4 +
 (a^2*c^9*x^2*(10*a*d + 3*b*c))/2 + (b^2*d^9*x^13*(3*a*d + 10*b*c))/13 + a*
c^8*x^3*(15*a^2*d^2 + b^2*c^2 + 10*a*b*c*d) + (b*d^8*x^12*(a^2*d^2 + 15*b^2
*c^2 + 10*a*b*c*d))/4 + c^6*d*x^5*(42*a^3*d^3 + 2*b^3*c^3 + 27*a*b^2*c^2*d
+ 72*a^2*b*c*d^2) + (c*d^6*x^10*(2*a^3*d^3 + 42*b^3*c^3 + 72*a*b^2*c^2*d +
27*a^2*b*c*d^2))/2
```

3.1309 $\int (a+bx)^2(c+dx)^{10}\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

```
[Out] 1/11*(-a*d+b*c)^2*(d*x+c)^11/d^3-1/6*b*(-a*d+b*c)*(d*x+c)^12/d^3+1/13*b^2*(
d*x+c)^13/d^3
```

Rubi [A]
time = 0.18, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^11)/(11*d^3) - (b*(b*c - a*d)*(c + d*x)^12)/(6*d^3
) + (b^2*(c + d*x)^13)/(13*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{10}}{d^2}-\frac{2b(bc-ad)(c+dx)^{11}}{d^2}+\frac{b^2(c+dx)^{12}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 358 vs. 2(65) = 130.
time = 0.03, size = 358, normalized size = 5.51

$$a^2c^{10}x+ac^9(bc+5ad)x^2+\frac{1}{3}c^8\left(b^2c^2+20abcd+45a^2d^2\right)x^3+\frac{5}{2}c^7d\left(b^2c^2+9abcd+12a^2d^2\right)x^4+3c^6d^2\left(3b^2c^2+16abcd+14a^2d^2\right)x^5+2c^5d^3\left(10b^2c^2+35abcd+21a^2d^2\right)x^6+6c^4d^4\left(5b^2c^2+12abcd+5a^2d^2\right)x^7+\frac{3}{2}c^3d^5\left(21b^2c^2+35abcd+10a^2d^2\right)x^8+\frac{5}{3}c^2d^6\left(14b^2c^2+16abcd+3a^2d^2\right)x^9+cd^7\left(12b^2c^2+9abcd+a^2d^2\right)x^{10}+\frac{1}{11}d^8\left(45b^2c^2+20abcd+a^2d^2\right)x^{11}+\frac{1}{6}bd^9(5bc+ad)x^{12}+\frac{1}{13}b^2d^{10}x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] a^2*c^10*x + a*c^9*(b*c + 5*a*d)*x^2 + (c^8*(b^2*c^2 + 20*a*b*c*d + 45*a^2*
d^2)*x^3)/3 + (5*c^7*d*(b^2*c^2 + 9*a*b*c*d + 12*a^2*d^2)*x^4)/2 + 3*c^6*d^
2*(3*b^2*c^2 + 16*a*b*c*d + 14*a^2*d^2)*x^5 + 2*c^5*d^3*(10*b^2*c^2 + 35*a*
b*c*d + 21*a^2*d^2)*x^6 + 6*c^4*d^4*(5*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*x^
7 + (3*c^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^2)*x^8)/2 + (5*c^2*d^6*(
14*b^2*c^2 + 16*a*b*c*d + 3*a^2*d^2)*x^9)/3 + c*d^7*(12*b^2*c^2 + 9*a*b*c*d
 + a^2*d^2)*x^10 + (d^8*(45*b^2*c^2 + 20*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^9*(5*b*c + a*d)*x^12)/6 + (b^2*d^10*x^13)/13
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 344 vs. $2(65) = 130$.
time = 4.41, size = 342, normalized size = 5.26

$$\frac{x\left(858a^2c^{10}+858ac^9x\left(5ad+bc\right)+143bd^9x^{11}\left(ad+5bc\right)+c^8x^2\left(12870a^2d^2+5720abcd+286b^2c^2\right)+2145c^7dx^3\left(12a^2d^2+9abcd+b^2c^2\right)+c^6d^2x^4\left(36036a^2d^2+41184abcd+7722b^2c^2\right)+c^5d^3x^5\left(36036a^2d^2+60060abcd+17160b^2c^2\right)+c^4d^4x^6\left(25740a^2d^2+61776abcd+25740b^2c^2\right)+c^3d^5x^7\left(12870a^2d^2+45045abcd+27027b^2c^2\right)+858cd^7x^9\left(a^2d^2+9abcd+12b^2c^2\right)+d^8x^{10}\left(78a^2d^2+1560abcd+3510b^2c^2\right)+66b^2d^{10}x^{12}+c^2d^6x^8\left(4290a^2d^2+22880abcd+20020b^2c^2\right)\right)}{858}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^10,x]')
```

```
[Out] x (858 a ^ 2 c ^ 10 + 858 a c ^ 9 x (5 a d + b c) + 143 b d ^ 9 x ^ 11 (a d
 + 5 b c) + c ^ 8 x ^ 2 (12870 a ^ 2 d ^ 2 + 5720 a b c d + 286 b ^ 2 c ^ 2
) + 2145 c ^ 7 d x ^ 3 (12 a ^ 2 d ^ 2 + 9 a b c d + b ^ 2 c ^ 2) + c ^ 6 d
 ^ 2 x ^ 4 (36036 a ^ 2 d ^ 2 + 41184 a b c d + 7722 b ^ 2 c ^ 2) + c ^ 5 d
 ^ 3 x ^ 5 (36036 a ^ 2 d ^ 2 + 60060 a b c d + 17160 b ^ 2 c ^ 2) + c ^ 4
d ^ 4 x ^ 6 (25740 a ^ 2 d ^ 2 + 61776 a b c d + 25740 b ^ 2 c ^ 2) + c ^ 3
 d ^ 5 x ^ 7 (12870 a ^ 2 d ^ 2 + 45045 a b c d + 27027 b ^ 2 c ^ 2) + 858
c d ^ 7 x ^ 9 (a ^ 2 d ^ 2 + 9 a b c d + 12 b ^ 2 c ^ 2) + d ^ 8 x ^ 10 (78
 a ^ 2 d ^ 2 + 1560 a b c d + 3510 b ^ 2 c ^ 2) + 66 b ^ 2 d ^ 10 x ^ 12 +
c ^ 2 d ^ 6 x ^ 8 (4290 a ^ 2 d ^ 2 + 22880 a b c d + 20020 b ^ 2 c ^ 2)) /
 858
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(59) = 118$.
time = 0.14, size = 391, normalized size = 6.02

method	result
norman	$\frac{b^2d^{10}x^{13}}{13}+\left(\frac{1}{6}ab\,d^{10}+\frac{5}{6}b^2c\,d^9\right)x^{12}+\left(\frac{1}{11}a^2d^{10}+\frac{20}{11}abc\,d^9+\frac{45}{11}b^2c^2d^8\right)x^{11}+\left(a^2c\,d^9+9ab\,c^2d^8+12\right.$
default	$\frac{b^2d^{10}x^{13}}{13}+\frac{(2ab\,d^{10}+10b^2c\,d^9)x^{12}}{12}+\frac{(a^2d^{10}+20abc\,d^9+45b^2c^2d^8)x^{11}}{11}+\frac{(10a^2c\,d^9+90ab\,c^2d^8+120b^2c^3d^7)x^{10}}{10}+\frac{(45a^2c^2d^8}{}$
gosper	$\frac{1}{6}x^{12}ab\,d^{10}+\frac{5}{6}x^{12}b^2c\,d^9+\frac{45}{11}x^{11}b^2c^2d^8+5x^9a^2c^2d^8+\frac{70}{3}x^9b^2c^4d^6+15x^8a^2c^3d^7+\frac{63}{2}x^8b^2c^5d^5+30$
risch	$\frac{1}{6}x^{12}ab\,d^{10}+\frac{5}{6}x^{12}b^2c\,d^9+\frac{45}{11}x^{11}b^2c^2d^8+5x^9a^2c^2d^8+\frac{70}{3}x^9b^2c^4d^6+15x^8a^2c^3d^7+\frac{63}{2}x^8b^2c^5d^5+30$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*d^10*x^13+1/12*(2*a*b*d^10+10*b^2*c*d^9)*x^12+1/11*(a^2*d^10+20*a*
b*c*d^9+45*b^2*c^2*d^8)*x^11+1/10*(10*a^2*c*d^9+90*a*b*c^2*d^8+120*b^2*c^3*
d^7)*x^10+1/9*(45*a^2*c^2*d^8+240*a*b*c^3*d^7+210*b^2*c^4*d^6)*x^9+1/8*(120
*a^2*c^3*d^7+420*a*b*c^4*d^6+252*b^2*c^5*d^5)*x^8+1/7*(210*a^2*c^4*d^6+504*
a*b*c^5*d^5+210*b^2*c^6*d^4)*x^7+1/6*(252*a^2*c^5*d^5+420*a*b*c^6*d^4+120*b
^2*c^7*d^3)*x^6+1/5*(210*a^2*c^6*d^4+240*a*b*c^7*d^3+45*b^2*c^8*d^2)*x^5+1/
4*(120*a^2*c^7*d^3+90*a*b*c^8*d^2+10*b^2*c^9*d)*x^4+1/3*(45*a^2*c^8*d^2+20*
a*b*c^9*d+b^2*c^10)*x^3+1/2*(10*a^2*c^9*d+2*a*b*c^10)*x^2+a^2*c^10*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(59) = 118$.
time = 0.27, size = 384, normalized size = 5.91

$$\frac{1}{13}\,b^2d^{10}x^{13}+a^2c^{10}x+\frac{1}{6}\,(5\,b^2cd^9+abd^{10})x^{12}+\frac{1}{11}\,(45\,b^2c^2d^8+20\,abcd^9+a^2d^{10})x^{11}+(12\,b^2c^3d^7+9\,abc^2d^8+a^2cd^9)x^{10}+\frac{5}{3}\,(14\,b^2c^4d^6+16\,abc^3d^7+3\,a^2c^2d^8)x^9+\frac{3}{2}\,(21\,b^2c^5d^5+35\,abc^4d^6+10\,a^2c^3d^7)x^8+6\,(5\,b^2c^6d^4+12\,abc^5d^5+5\,a^2c^4d^6)x^7+2\,(10\,b^2c^7d^3+35\,abc^6d^4+21\,a^2c^5d^5)x^6+3\,(3\,b^2c^8d^2+16\,abc^7d^3+14\,a^2c^6d^4)x^5+\frac{5}{2}\,(b^2c^9d+9\,abc^8d^2+12\,a^2c^7d^3)x^4+\frac{1}{3}\,(b^2c^{10}+20\,abc^9d+45\,a^2c^8d^2)x^3+(abc^{10}+5\,a^2c^9d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
+ 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(59) = 118$.
time = 0.29, size = 384, normalized size = 5.91

$$\frac{1}{13}\,b^2d^{10}x^{13}+a^2c^{10}x+\frac{1}{6}\,(5\,b^2cd^9+abd^{10})x^{12}+\frac{1}{11}\,(45\,b^2c^2d^8+20\,abcd^9+a^2d^{10})x^{11}+(12\,b^2c^3d^7+9\,abc^2d^8+a^2cd^9)x^{10}+\frac{5}{3}\,(14\,b^2c^4d^6+16\,abc^3d^7+3\,a^2c^2d^8)x^9+\frac{3}{2}\,(21\,b^2c^5d^5+35\,abc^4d^6+10\,a^2c^3d^7)x^8+6\,(5\,b^2c^6d^4+12\,abc^5d^5+5\,a^2c^4d^6)x^7+2\,(10\,b^2c^7d^3+35\,abc^6d^4+21\,a^2c^5d^5)x^6+3\,(3\,b^2c^8d^2+16\,abc^7d^3+14\,a^2c^6d^4)x^5+\frac{5}{2}\,(b^2c^9d+9\,abc^8d^2+12\,a^2c^7d^3)x^4+\frac{1}{3}\,(b^2c^{10}+20\,abc^9d+45\,a^2c^8d^2)x^3+(abc^{10}+5\,a^2c^9d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
```

```
+ 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(54) = 108$.
time = 0.07, size = 415, normalized size = 6.38

$$a^{2}c^{10}x+\frac{b^{2}d^{10}x^{13}}{13}+x^{12}\left(\frac{abd^{10}}{6}+\frac{5b^{2}cd^{9}}{6}\right)+x^{11}\left(\frac{a^{2}d^{10}}{11}+\frac{20abcd^{9}}{11}+\frac{45b^{2}c^{2}d^{8}}{11}\right)+x^{10}\left(a^{2}cd^{9}+9abc^{2}d^{8}+12b^{2}c^{3}d^{7}\right)+x^{9}\cdot\left(5a^{2}c^{2}d^{8}+\frac{80abc^{3}d^{7}}{3}+\frac{70b^{2}c^{4}d^{6}}{3}\right)+x^{8}\cdot\left(15a^{2}c^{3}d^{7}+\frac{105abc^{4}d^{6}}{2}+\frac{63b^{2}c^{5}d^{5}}{2}\right)+x^{7}\cdot\left(30a^{2}c^{4}d^{6}+72abc^{5}d^{5}+30b^{2}c^{6}d^{4}\right)+x^{6}\cdot\left(42a^{2}c^{5}d^{5}+70abc^{6}d^{4}+20b^{2}c^{7}d^{3}\right)+x^{5}\cdot\left(42a^{2}c^{6}d^{4}+48abc^{7}d^{3}+9b^{2}c^{8}d^{2}\right)+x^{4}\cdot\left(30a^{2}c^{7}d^{3}+\frac{45abc^{8}d^{2}}{2}+\frac{5b^{2}c^{9}d}{2}\right)+x^{3}\cdot\left(15a^{2}c^{8}d^{2}+\frac{20abc^{9}d}{3}+\frac{b^{2}c^{10}}{3}\right)+x^{2}\cdot\left(5a^{2}c^{9}d+abc^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**10,x)
```

```
[Out] a**2*c**10*x + b**2*d**10*x**13/13 + x**12*(a*b*d**10/6 + 5*b**2*c*d**9/6)
+ x**11*(a**2*d**10/11 + 20*a*b*c*d**9/11 + 45*b**2*c**2*d**8/11) + x**10*(
a**2*c*d**9 + 9*a*b*c**2*d**8 + 12*b**2*c**3*d**7) + x**9*(5*a**2*c**2*d**8
 + 80*a*b*c**3*d**7/3 + 70*b**2*c**4*d**6/3) + x**8*(15*a**2*c**3*d**7 + 10
5*a*b*c**4*d**6/2 + 63*b**2*c**5*d**5/2) + x**7*(30*a**2*c**4*d**6 + 72*a*b
*c**5*d**5 + 30*b**2*c**6*d**4) + x**6*(42*a**2*c**5*d**5 + 70*a*b*c**6*d**
4 + 20*b**2*c**7*d**3) + x**5*(42*a**2*c**6*d**4 + 48*a*b*c**7*d**3 + 9*b**
2*c**8*d**2) + x**4*(30*a**2*c**7*d**3 + 45*a*b*c**8*d**2/2 + 5*b**2*c**9*d
/2) + x**3*(15*a**2*c**8*d**2 + 20*a*b*c**9*d/3 + b**2*c**10/3) + x**2*(5*a
**2*c**9*d + a*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(59) = 118$.
time = 0.00, size = 445, normalized size = 6.85

$$\frac{1}{13}x^{13}b^{2}d^{10}+\frac{5}{6}x^{12}b^{2}d^{9}c+\frac{1}{6}x^{12}bad^{10}+\frac{45}{11}x^{11}b^{2}d^{8}c^{2}+\frac{20}{11}x^{11}bad^{9}c+\frac{1}{11}x^{11}a^{2}d^{10}+12x^{10}b^{2}d^{7}c^{3}+9x^{10}bad^{8}c^{2}+x^{10}a^{2}d^{9}c+\frac{70}{3}x^{9}b^{2}d^{6}c^{4}+\frac{80}{3}x^{9}bad^{7}c^{3}+5x^{9}a^{2}d^{8}c^{2}+\frac{63}{2}x^{8}b^{2}d^{5}c^{5}+\frac{105}{2}x^{8}bad^{6}c^{4}+15x^{8}a^{2}d^{7}c^{3}+30x^{7}b^{2}d^{4}c^{6}+72x^{7}bad^{5}c^{5}+30x^{7}a^{2}d^{6}c^{4}+20x^{6}b^{2}d^{3}c^{7}+70x^{6}bad^{4}c^{6}+42x^{6}a^{2}d^{5}c^{5}+9x^{5}b^{2}d^{2}c^{8}+48x^{5}bad^{3}c^{7}+42x^{5}a^{2}d^{4}c^{6}+\frac{5}{2}x^{4}b^{2}dc^{9}+\frac{45}{2}x^{4}bad^{2}c^{8}+30x^{4}a^{2}d^{3}c^{7}+\frac{1}{3}x^{3}b^{2}c^{10}+\frac{20}{3}x^{3}badc^{9}+15x^{3}a^{2}d^{2}c^{8}+x^{2}bac^{10}+5x^{2}a^{2}dc^{9}+xa^{2}c^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x)
```

```
[Out] 1/13*b^2*d^10*x^13 + 5/6*b^2*c*d^9*x^12 + 1/6*a*b*d^10*x^12 + 45/11*b^2*c^2
*d^8*x^11 + 20/11*a*b*c*d^9*x^11 + 1/11*a^2*d^10*x^11 + 12*b^2*c^3*d^7*x^10
 + 9*a*b*c^2*d^8*x^10 + a^2*c*d^9*x^10 + 70/3*b^2*c^4*d^6*x^9 + 80/3*a*b*c^
3*d^7*x^9 + 5*a^2*c^2*d^8*x^9 + 63/2*b^2*c^5*d^5*x^8 + 105/2*a*b*c^4*d^6*x^
8 + 15*a^2*c^3*d^7*x^8 + 30*b^2*c^6*d^4*x^7 + 72*a*b*c^5*d^5*x^7 + 30*a^2*c
^4*d^6*x^7 + 20*b^2*c^7*d^3*x^6 + 70*a*b*c^6*d^4*x^6 + 42*a^2*c^5*d^5*x^6 +
 9*b^2*c^8*d^2*x^5 + 48*a*b*c^7*d^3*x^5 + 42*a^2*c^6*d^4*x^5 + 5/2*b^2*c^9*
d*x^4 + 45/2*a*b*c^8*d^2*x^4 + 30*a^2*c^7*d^3*x^4 + 1/3*b^2*c^10*x^3 + 20/3
*a*b*c^9*d*x^3 + 15*a^2*c^8*d^2*x^3 + a*b*c^10*x^2 + 5*a^2*c^9*d*x^2 + a^2*
c^10*x
```

Mupad [B]
time = 0.32, size = 348, normalized size = 5.35

$$x^{3}\left(15a^{2}c^{8}d^{2}+\frac{20abc^{9}d}{3}+\frac{b^{2}c^{10}}{3}\right)+x^{11}\left(\frac{a^{2}d^{10}}{11}+\frac{20abcd^{9}}{11}+\frac{45b^{2}c^{2}d^{8}}{11}\right)+a^{2}c^{10}x+\frac{b^{2}d^{10}x^{13}}{13}+ac^{9}x^{2}\left(5ad+bc\right)+\frac{bd^{9}x^{12}\left(ad+5bc\right)}{6}+\frac{5c^{7}dx^{4}\left(12a^{2}d^{2}+9abcd+b^{2}c^{2}\right)}{2}+cd^{7}x^{10}\left(a^{2}d^{2}+9abcd+12b^{2}c^{2}\right)+6c^{4}d^{4}x^{7}\left(5a^{2}d^{2}+12abcd+5b^{2}c^{2}\right)+3c^{6}d^{2}x^{5}\left(14a^{2}d^{2}+16abcd+3b^{2}c^{2}\right)+\frac{5c^{2}d^{6}x^{9}\left(3a^{2}d^{2}+16abcd+14b^{2}c^{2}\right)}{3}+2c^{5}d^{3}x^{6}\left(21a^{2}d^{2}+35abcd+10b^{2}c^{2}\right)+\frac{3c^{3}d^{5}x^{8}\left(10a^{2}d^{2}+35abcd+21b^{2}c^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^2*(c + d*x)^10,x)

[Out] x^3*((b^2*c^10)/3 + 15*a^2*c^8*d^2 + (20*a*b*c^9*d)/3) + x^11*((a^2*d^10)/1
1 + (45*b^2*c^2*d^8)/11 + (20*a*b*c*d^9)/11) + a^2*c^10*x + (b^2*d^10*x^13)
/13 + a*c^9*x^2*(5*a*d + b*c) + (b*d^9*x^12*(a*d + 5*b*c))/6 + (5*c^7*d*x^4
*(12*a^2*d^2 + b^2*c^2 + 9*a*b*c*d))/2 + c*d^7*x^10*(a^2*d^2 + 12*b^2*c^2 +
 9*a*b*c*d) + 6*c^4*d^4*x^7*(5*a^2*d^2 + 5*b^2*c^2 + 12*a*b*c*d) + 3*c^6*d^
2*x^5*(14*a^2*d^2 + 3*b^2*c^2 + 16*a*b*c*d) + (5*c^2*d^6*x^9*(3*a^2*d^2 + 1
4*b^2*c^2 + 16*a*b*c*d))/3 + 2*c^5*d^3*x^6*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*
b*c*d) + (3*c^3*d^5*x^8*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/2

3.1310 $\int (a+bx)(c+dx)^{10}\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}$$

```
[Out] -1/11*(-a*d+b*c)*(d*x+c)^11/d^2+1/12*b*(d*x+c)^12/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)*(c + d*x)^11)/d^2 + (b*(c + d*x)^12)/(12*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{10}}{d}+\frac{b(c+dx)^{11}}{d}\right)\,dx\\ &= -\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(38) = 76.
time = 0.02, size = 220, normalized size = 5.79

$$ac^{10}x+\frac{1}{2}c^9(bc+10ad)x^2+\frac{5}{3}c^8d(2bc+9ad)x^3+\frac{15}{4}c^7d^2(3bc+8ad)x^4+6c^6d^3(4bc+7ad)x^5+7c^5d^4(5bc+6ad)x^6+6c^4d^5(6bc+5ad)x^7+\frac{15}{4}c^3d^6(7bc+4ad)x^8+\frac{5}{3}c^2d^7(8bc+3ad)x^9+\frac{1}{2}cd^8(9bc+2ad)x^{10}+\frac{1}{11}d^9(10bc+ad)x^{11}+\frac{1}{12}bd^{10}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] a*c^10*x + (c^9*(b*c + 10*a*d)*x^2)/2 + (5*c^8*d*(2*b*c + 9*a*d)*x^3)/3 + (
15*c^7*d^2*(3*b*c + 8*a*d)*x^4)/4 + 6*c^6*d^3*(4*b*c + 7*a*d)*x^5 + 7*c^5*d
^4*(5*b*c + 6*a*d)*x^6 + 6*c^4*d^5*(6*b*c + 5*a*d)*x^7 + (15*c^3*d^6*(7*b*c
 + 4*a*d)*x^8)/4 + (5*c^2*d^7*(8*b*c + 3*a*d)*x^9)/3 + (c*d^8*(9*b*c + 2*a*
d)*x^10)/2 + (d^9*(10*b*c + a*d)*x^11)/11 + (b*d^10*x^12)/12
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 201 vs. $2(38) = 76$.
time = 3.23, size = 199, normalized size = 5.24

$$\frac{x\left(132ac^{10}+d^9x^{10}\left(12ad+120bc\right)+11bd^{10}x^{11}+66c^9x\left(10ad+bc\right)+220c^8dx^2\left(9ad+2bc\right)+c^7d^2x^3\left(3960ad+1485bc\right)+c^6d^3x^4\left(5544ad+3168bc\right)+c^5d^4x^5\left(5544ad+4620bc\right)+c^4d^5x^6\left(3960ad+4752bc\right)+c^3d^6x^7\left(1980ad+3465bc\right)+c^2d^7x^8\left(660ad+1760bc\right)+66cd^8x^9\left(2ad+9bc\right)\right)}{132}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^10,x]')
```

```
[Out] x (132 a c ^ 10 + d ^ 9 x ^ 10 (12 a d + 120 b c) + 11 b d ^ 10 x ^ 11 + 66
 c ^ 9 x (10 a d + b c) + 220 c ^ 8 d x ^ 2 (9 a d + 2 b c) + c ^ 7 d ^ 2 x
 ^ 3 (3960 a d + 1485 b c) + c ^ 6 d ^ 3 x ^ 4 (5544 a d + 3168 b c) + c ^
5 d ^ 4 x ^ 5 (5544 a d + 4620 b c) + c ^ 4 d ^ 5 x ^ 6 (3960 a d + 4752 b
c) + c ^ 3 d ^ 6 x ^ 7 (1980 a d + 3465 b c) + c ^ 2 d ^ 7 x ^ 8 (660 a d +
 1760 b c) + 66 c d ^ 8 x ^ 9 (2 a d + 9 b c)) / 132
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.
time = 0.12, size = 241, normalized size = 6.34

method	result
norman	$\frac{b\,d^{10}x^{12}}{12}+\left(\frac{1}{11}a\,d^{10}+\frac{10}{11}bc\,d^{9}\right)x^{11}+\left(ac\,d^{9}+\frac{9}{2}b\,c^{2}d^{8}\right)x^{10}+\left(5a\,c^{2}d^{8}+\frac{40}{3}b\,c^{3}d^{7}\right)x^{9}+\left(15a\,c^{3}d^{7}+\right.$
default	$\frac{b\,d^{10}x^{12}}{12}+\frac{\left(a\,d^{10}+10bc\,d^{9}\right)x^{11}}{11}+\frac{\left(10ac\,d^{9}+45b\,c^{2}d^{8}\right)x^{10}}{10}+\frac{\left(45a\,c^{2}d^{8}+120b\,c^{3}d^{7}\right)x^{9}}{9}+\frac{\left(120a\,c^{3}d^{7}+210b\,c^{4}d^{6}\right)x^{8}}{8}+$
gosper	$\frac{1}{12}b\,d^{10}x^{12}+\frac{1}{11}x^{11}a\,d^{10}+\frac{10}{11}x^{11}bc\,d^{9}+x^{10}ac\,d^{9}+\frac{9}{2}x^{10}b\,c^{2}d^{8}+5x^{9}a\,c^{2}d^{8}+\frac{40}{3}x^{9}b\,c^{3}d^{7}+15x^{8}a\,c^{3}d^{7}$
risch	$\frac{1}{12}b\,d^{10}x^{12}+\frac{1}{11}x^{11}a\,d^{10}+\frac{10}{11}x^{11}bc\,d^{9}+x^{10}ac\,d^{9}+\frac{9}{2}x^{10}b\,c^{2}d^{8}+5x^{9}a\,c^{2}d^{8}+\frac{40}{3}x^{9}b\,c^{3}d^{7}+15x^{8}a\,c^{3}d^{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b*d^10*x^12+1/11*(a*d^10+10*b*c*d^9)*x^11+1/10*(10*a*c*d^9+45*b*c^2*d^
8)*x^10+1/9*(45*a*c^2*d^8+120*b*c^3*d^7)*x^9+1/8*(120*a*c^3*d^7+210*b*c^4*d
^6)*x^8+1/7*(210*a*c^4*d^6+252*b*c^5*d^5)*x^7+1/6*(252*a*c^5*d^5+210*b*c^6*
d^4)*x^6+1/5*(210*a*c^6*d^4+120*b*c^7*d^3)*x^5+1/4*(120*a*c^7*d^3+45*b*c^8*
d^2)*x^4+1/3*(45*a*c^8*d^2+10*b*c^9*d)*x^3+1/2*(10*a*c^9*d+b*c^10)*x^2+a*c^
10*x
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.

time = 0.28, size = 240, normalized size = 6.32

$$\frac{1}{12}\,bd^{10}x^{12}+ac^{10}x+\frac{1}{11}\,(10\,bcd^{9}+ad^{10})x^{11}+\frac{1}{2}\,(9\,bc^{2}d^{8}+2\,acd^{9})x^{10}+\frac{5}{3}\,(8\,bc^{3}d^{7}+3\,ac^{2}d^{8})x^{9}+\frac{15}{4}\,(7\,bc^{4}d^{6}+4\,ac^{3}d^{7})x^{8}+6\,(6\,bc^{5}d^{5}+5\,ac^{4}d^{6})x^{7}+7\,(5\,bc^{6}d^{4}+6\,ac^{5}d^{5})x^{6}+6\,(4\,bc^{7}d^{3}+7\,ac^{6}d^{4})x^{5}+\frac{15}{4}\,(3\,bc^{8}d^{2}+8\,ac^{7}d^{3})x^{4}+\frac{5}{3}\,(2\,bc^{9}d+9\,ac^{8}d^{2})x^{3}+\frac{1}{2}\,(bc^{10}+10\,ac^{9}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.
time = 0.29, size = 240, normalized size = 6.32

$$\frac{1}{12}\,bd^{10}x^{12}+ac^{10}x+\frac{1}{11}\,(10\,bcd^{9}+ad^{10})x^{11}+\frac{1}{2}\,(9\,bc^{2}d^{8}+2\,acd^{9})x^{10}+\frac{5}{3}\,(8\,bc^{3}d^{7}+3\,ac^{2}d^{8})x^{9}+\frac{15}{4}\,(7\,bc^{4}d^{6}+4\,ac^{3}d^{7})x^{8}+6\,(6\,bc^{5}d^{5}+5\,ac^{4}d^{6})x^{7}+7\,(5\,bc^{6}d^{4}+6\,ac^{5}d^{5})x^{6}+6\,(4\,bc^{7}d^{3}+7\,ac^{6}d^{4})x^{5}+\frac{15}{4}\,(3\,bc^{8}d^{2}+8\,ac^{7}d^{3})x^{4}+\frac{5}{3}\,(2\,bc^{9}d+9\,ac^{8}d^{2})x^{3}+\frac{1}{2}\,(bc^{10}+10\,ac^{9}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(32) = 64$.
time = 0.06, size = 248, normalized size = 6.53

$$ac^{10}x+\frac{bd^{10}x^{12}}{12}+x^{11}\left(\frac{ad^{10}}{11}+\frac{10bcd^{9}}{11}\right)+x^{10}\left(acd^{9}+\frac{9bc^{2}d^{8}}{2}\right)+x^{9}\cdot\left(5ac^{2}d^{8}+\frac{40bc^{3}d^{7}}{3}\right)+x^{8}\cdot\left(15ac^{3}d^{7}+\frac{105bc^{4}d^{6}}{4}\right)+x^{7}\cdot(30ac^{4}d^{6}+36bc^{5}d^{5})+x^{6}\cdot(42ac^{5}d^{5}+35bc^{6}d^{4})+x^{5}\cdot(42ac^{6}d^{4}+24bc^{7}d^{3})+x^{4}\cdot\left(30ac^{7}d^{3}+\frac{45bc^{8}d^{2}}{4}\right)+x^{3}\cdot\left(15ac^{8}d^{2}+\frac{10bc^{9}d}{3}\right)+x^{2}\cdot\left(5ac^{9}d+\frac{bc^{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**10,x)
```

```
[Out] a*c**10*x + b*d**10*x**12/12 + x**11*(a*d**10/11 + 10*b*c*d**9/11) + x**10*
(a*c*d**9 + 9*b*c**2*d**8/2) + x**9*(5*a*c**2*d**8 + 40*b*c**3*d**7/3) + x*
*8*(15*a*c**3*d**7 + 105*b*c**4*d**6/4) + x**7*(30*a*c**4*d**6 + 36*b*c**5*
d**5) + x**6*(42*a*c**5*d**5 + 35*b*c**6*d**4) + x**5*(42*a*c**6*d**4 + 24*
b*c**7*d**3) + x**4*(30*a*c**7*d**3 + 45*b*c**8*d**2/4) + x**3*(15*a*c**8*d
**2 + 10*b*c**9*d/3) + x**2*(5*a*c**9*d + b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(34) = 68.

time = 0.00, size = 259, normalized size = 6.82

$$\frac{1}{12}x^{12}bd^{10} + \frac{10}{11}x^{11}bd^9c + \frac{1}{11}x^{11}ad^{10} + \frac{9}{2}x^{10}bd^8c^2 + x^{10}ad^9c + \frac{40}{3}x^9bd^7c^3 + 5x^9ad^8c^2 + \frac{105}{4}x^8bd^6c^4 + 15x^8ad^7c^3 + 36x^7bd^5c^5 + 30x^7ad^6c^4 + 35x^6bd^4c^6 + 42x^6ad^5c^5 + 24x^5bd^3c^7 + 42x^5ad^4c^6 + \frac{45}{4}x^4bd^2c^8 + 30x^4ad^3c^7 + \frac{10}{3}x^3bdc^9 + 15x^3ad^2c^8 + \frac{1}{2}x^2bc^{10} + 5x^2adc^9 + xac^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x)
```

```
[Out] 1/12*b*d^10*x^12 + 10/11*b*c*d^9*x^11 + 1/11*a*d^10*x^11 + 9/2*b*c^2*d^8*x^
10 + a*c*d^9*x^10 + 40/3*b*c^3*d^7*x^9 + 5*a*c^2*d^8*x^9 + 105/4*b*c^4*d^6*
x^8 + 15*a*c^3*d^7*x^8 + 36*b*c^5*d^5*x^7 + 30*a*c^4*d^6*x^7 + 35*b*c^6*d^4
*x^6 + 42*a*c^5*d^5*x^6 + 24*b*c^7*d^3*x^5 + 42*a*c^6*d^4*x^5 + 45/4*b*c^8*
d^2*x^4 + 30*a*c^7*d^3*x^4 + 10/3*b*c^9*d*x^3 + 15*a*c^8*d^2*x^3 + 1/2*b*c^
10*x^2 + 5*a*c^9*d*x^2 + a*c^10*x
```

Mupad [B]

time = 0.13, size = 208, normalized size = 5.47

$$x^2\left(\frac{b\,c^{10}}{2}+5\,a\,d\,c^9\right)+x^{11}\left(\frac{a\,d^{10}}{11}+\frac{10\,b\,c\,d^9}{11}\right)+\frac{b\,d^{10}\,x^{12}}{12}+a\,c^{10}\,x+\frac{5\,c^8\,d\,x^3\,\left(9\,a\,d+2\,b\,c\right)}{3}+\frac{c\,d^8\,x^{10}\,\left(2\,a\,d+9\,b\,c\right)}{2}+\frac{15\,c^7\,d^2\,x^4\,\left(8\,a\,d+3\,b\,c\right)}{4}+6\,c^6\,d^3\,x^5\,\left(7\,a\,d+4\,b\,c\right)+7\,c^5\,d^4\,x^6\,\left(6\,a\,d+5\,b\,c\right)+6\,c^4\,d^5\,x^7\,\left(5\,a\,d+6\,b\,c\right)+\frac{15\,c^3\,d^6\,x^8\,\left(4\,a\,d+7\,b\,c\right)}{4}+\frac{5\,c^2\,d^7\,x^9\,\left(3\,a\,d+8\,b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^10,x)
```

```
[Out] x^2*((b*c^10)/2 + 5*a*c^9*d) + x^11*((a*d^10)/11 + (10*b*c*d^9)/11) + (b*d^
10*x^12)/12 + a*c^10*x + (5*c^8*d*x^3*(9*a*d + 2*b*c))/3 + (c*d^8*x^10*(2*a
*d + 9*b*c))/2 + (15*c^7*d^2*x^4*(8*a*d + 3*b*c))/4 + 6*c^6*d^3*x^5*(7*a*d
+ 4*b*c) + 7*c^5*d^4*x^6*(6*a*d + 5*b*c) + 6*c^4*d^5*x^7*(5*a*d + 6*b*c) +
(15*c^3*d^6*x^8*(4*a*d + 7*b*c))/4 + (5*c^2*d^7*x^9*(3*a*d + 8*b*c))/3
```

3.1311 $\int (c+dx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^{11}}{11d}$$

```
[Out] 1/11*(d*x+c)^11/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{10}\,dx = \frac{(c+dx)^{11}}{11d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(14) = 28.
time = 2.01, size = 106, normalized size = 7.57

$$x\left(c^{10}+5c^9dx+15c^8d^2x^2+30c^7d^3x^3+42c^6d^4x^4+42c^5d^5x^5+30c^4d^6x^6+15c^3d^7x^7+5c^2d^8x^8+cd^9x^9+\frac{d^{10}x^{10}}{11}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^10,x]')
```

```
[Out] x (c ^ 10 + 5 c ^ 9 d x + 15 c ^ 8 d ^ 2 x ^ 2 + 30 c ^ 7 d ^ 3 x ^ 3 + 42
c ^ 6 d ^ 4 x ^ 4 + 42 c ^ 5 d ^ 5 x ^ 5 + 30 c ^ 4 d ^ 6 x ^ 6 + 15 c ^ 3
d ^ 7 x ^ 7 + 5 c ^ 2 d ^ 8 x ^ 8 + c d ^ 9 x ^ 9 + d ^ 10 x ^ 10 / 11)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.93

method	result
default	$\frac{(dx+c)^{11}}{11d}$
gosper	$\frac{1}{11}d^{10}x^{11}+c\,d^{9}x^{10}+5c^{2}d^{8}x^{9}+15c^{3}d^{7}x^{8}+30c^{4}d^{6}x^{7}+42c^{5}d^{5}x^{6}+42c^{6}d^{4}x^{5}+30c^{7}d^{3}x^{4}+15c^{8}d^{2}x$
norman	$\frac{1}{11}d^{10}x^{11}+c\,d^{9}x^{10}+5c^{2}d^{8}x^{9}+15c^{3}d^{7}x^{8}+30c^{4}d^{6}x^{7}+42c^{5}d^{5}x^{6}+42c^{6}d^{4}x^{5}+30c^{7}d^{3}x^{4}+15c^{8}d^{2}x$
risch	$\frac{d^{10}x^{11}}{11}+c\,d^{9}x^{10}+5c^{2}d^{8}x^{9}+15c^{3}d^{7}x^{8}+30c^{4}d^{6}x^{7}+42c^{5}d^{5}x^{6}+42c^{6}d^{4}x^{5}+30c^{7}d^{3}x^{4}+15c^{8}d^{2}x^{3}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*(d*x+c)^11/d
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(d*x + c)^11/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.
time = 0.29, size = 108, normalized size = 7.71

$$\frac{1}{11}\,d^{10}x^{11}+cd^{9}x^{10}+5\,c^{2}d^{8}x^{9}+15\,c^{3}d^{7}x^{8}+30\,c^{4}d^{6}x^{7}+42\,c^{5}d^{5}x^{6}+42\,c^{6}d^{4}x^{5}+30\,c^{7}d^{3}x^{4}+15\,c^{8}d^{2}x^{3}+5\,c^{9}dx^{2}+c^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/11*d^10*x^11 + c*d^9*x^10 + 5*c^2*d^8*x^9 + 15*c^3*d^7*x^8 + 30*c^4*d^6*x
^7 + 42*c^5*d^5*x^6 + 42*c^6*d^4*x^5 + 30*c^7*d^3*x^4 + 15*c^8*d^2*x^3 + 5*
c^9*d*x^2 + c^10*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(8) = 16$.
time = 0.04, size = 114, normalized size = 8.14

$$c^{10}x + 5c^9dx^2 + 15c^8d^2x^3 + 30c^7d^3x^4 + 42c^6d^4x^5 + 42c^5d^5x^6 + 30c^4d^6x^7 + 15c^3d^7x^8 + 5c^2d^8x^9 + cd^9x^{10} + \frac{d^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10,x)
```

```
[Out] c**10*x + 5*c**9*d*x**2 + 15*c**8*d**2*x**3 + 30*c**7*d**3*x**4 + 42*c**6*d
**4*x**5 + 42*c**5*d**5*x**6 + 30*c**4*d**6*x**7 + 15*c**3*d**7*x**8 + 5*c*
*2*d**8*x**9 + c*d**9*x**10 + d**10*x**11/11
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{(dx + c)^{11}}{11d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x)
```

```
[Out] 1/11*(d*x + c)^11/d
```

Mupad [B]
time = 0.08, size = 108, normalized size = 7.71

$$c^{10}\,x + 5\,c^9\,d\,x^2 + 15\,c^8\,d^2\,x^3 + 30\,c^7\,d^3\,x^4 + 42\,c^6\,d^4\,x^5 + 42\,c^5\,d^5\,x^6 + 30\,c^4\,d^6\,x^7 + 15\,c^3\,d^7\,x^8 + 5\,c^2\,d^8\,x^9 + c\,d^9\,x^{10} + \frac{d^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10,x)
```

```
[Out] c^10*x + (d^10*x^11)/11 + 5*c^9*d*x^2 + c*d^9*x^10 + 15*c^8*d^2*x^3 + 30*c^
7*d^3*x^4 + 42*c^6*d^4*x^5 + 42*c^5*d^5*x^6 + 30*c^4*d^6*x^7 + 15*c^3*d^7*x
^8 + 5*c^2*d^8*x^9
```

3.1312 $\int \frac{(c+dx)^{10}}{a+bx}\,dx$

Optimal. Leaf size=241

$$\frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc-ad)^5(c+dx)^5}{5b^6}+\frac{(bc-\dots}{}$$

```
[Out] d*(-a*d+b*c)^9*x/b^10+1/2*(-a*d+b*c)^8*(d*x+c)^2/b^9+1/3*(-a*d+b*c)^7*(d*x+
c)^3/b^8+1/4*(-a*d+b*c)^6*(d*x+c)^4/b^7+1/5*(-a*d+b*c)^5*(d*x+c)^5/b^6+1/6*
(-a*d+b*c)^4*(d*x+c)^6/b^5+1/7*(-a*d+b*c)^3*(d*x+c)^7/b^4+1/8*(-a*d+b*c)^2*
(d*x+c)^8/b^3+1/9*(-a*d+b*c)*(d*x+c)^9/b^2+1/10*(d*x+c)^10/b+(-a*d+b*c)^10*
ln(b*x+a)/b^11
```

Rubi [A]

time = 0.07, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}+\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5(bc-ad)^5}{5b^6}+\frac{(c+dx)^6(bc-ad)^4}{6b^5}+\frac{(c+dx)^7(bc-ad)^3}{7b^4}+\frac{(c+dx)^8(bc-ad)^2}{8b^3}+\frac{(c+dx)^9(bc-ad)}{9b^2}+\frac{(c+dx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^9*x)/b^10 + ((b*c - a*d)^8*(c + d*x)^2)/(2*b^9) + ((b*c - a*
d)^7*(c + d*x)^3)/(3*b^8) + ((b*c - a*d)^6*(c + d*x)^4)/(4*b^7) + ((b*c - a
*d)^5*(c + d*x)^5)/(5*b^6) + ((b*c - a*d)^4*(c + d*x)^6)/(6*b^5) + ((b*c -
a*d)^3*(c + d*x)^7)/(7*b^4) + ((b*c - a*d)^2*(c + d*x)^8)/(8*b^3) + ((b*c -
 a*d)*(c + d*x)^9)/(9*b^2) + (c + d*x)^10/(10*b) + ((b*c - a*d)^10*Log[a +
b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{a+bx}\,dx = \int \left(\frac{d(bc-ad)^9}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)}+\frac{d(bc-ad)^8(c+dx)}{b^9}+\frac{d(bc-ad)^7(c+dx)^2}{b^8}+\frac{d(bc-\dots}{}\right.$$

$$=\frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc-\dots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 591 vs. 2(241) = 482.
time = 0.18, size = 591, normalized size = 2.45

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*x*(-2520*a^9*d^9 + 1260*a^8*b*d^8*(20*c + d*x) - 840*a^7*b^2*d^7*(135*c^
2 + 15*c*d*x + d^2*x^2) + 210*a^6*b^3*d^6*(1440*c^3 + 270*c^2*d*x + 40*c*d^
2*x^2 + 3*d^3*x^3) - 252*a^5*b^4*d^5*(2100*c^4 + 600*c^3*d*x + 150*c^2*d^2*
x^2 + 25*c*d^3*x^3 + 2*d^4*x^4) + 210*a^4*b^5*d^4*(3024*c^5 + 1260*c^4*d*x
+ 480*c^3*d^2*x^2 + 135*c^2*d^3*x^3 + 24*c*d^4*x^4 + 2*d^5*x^5) - 120*a^3*b
^6*d^3*(4410*c^6 + 2646*c^5*d*x + 1470*c^4*d^2*x^2 + 630*c^3*d^3*x^3 + 189*
c^2*d^4*x^4 + 35*c*d^5*x^5 + 3*d^6*x^6) + 45*a^2*b^7*d^2*(6720*c^7 + 5880*c
^6*d*x + 4704*c^5*d^2*x^2 + 2940*c^4*d^3*x^3 + 1344*c^3*d^4*x^4 + 420*c^2*d
^5*x^5 + 80*c*d^6*x^6 + 7*d^7*x^7) - 10*a*b^8*d*(11340*c^8 + 15120*c^7*d*x
+ 17640*c^6*d^2*x^2 + 15876*c^5*d^3*x^3 + 10584*c^4*d^4*x^4 + 5040*c^3*d^5*
x^5 + 1620*c^2*d^6*x^6 + 315*c*d^7*x^7 + 28*d^8*x^8) + b^9*(25200*c^9 + 567
00*c^8*d*x + 100800*c^7*d^2*x^2 + 132300*c^6*d^3*x^3 + 127008*c^5*d^4*x^4 +
 88200*c^4*d^5*x^5 + 43200*c^3*d^6*x^6 + 14175*c^2*d^7*x^7 + 2800*c*d^8*x^8
 + 252*d^9*x^9)))/(2520*b^10) + ((b*c - a*d)^10*Log[a + b*x])/b^11
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 710 vs. 2(241) = 482.
time = 8.44, size = 708, normalized size = 2.94

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^1,x]')
```

```
[Out] (-2520 b d x (a ^ 9 d ^ 9 - 10 a ^ 8 b c d ^ 8 + 45 a ^ 7 b ^ 2 c ^ 2 d ^ 7
 - 120 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 210 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 252 a ^ 4 b
^ 5 c ^ 5 d ^ 4 + 210 a ^ 3 b ^ 6 c ^ 6 d ^ 3 - 120 a ^ 2 b ^ 7 c ^ 7 d ^ 2
 + 45 a b ^ 8 c ^ 8 d - 10 b ^ 9 c ^ 9) + 1260 b ^ 2 d ^ 2 x ^ 2 (a ^ 8 d ^
 8 - 10 a ^ 7 b c d ^ 7 + 45 a ^ 6 b ^ 2 c ^ 2 d ^ 6 - 120 a ^ 5 b ^ 3 c ^
3 d ^ 5 + 210 a ^ 4 b ^ 4 c ^ 4 d ^ 4 - 252 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 210 a
 ^ 2 b ^ 6 c ^ 6 d ^ 2 - 120 a b ^ 7 c ^ 7 d + 45 b ^ 8 c ^ 8) + 840 b ^ 3
d ^ 3 x ^ 3 (-a ^ 7 d ^ 7 + 10 a ^ 6 b c d ^ 6 - 45 a ^ 5 b ^ 2 c ^ 2 d ^ 5
 + 120 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 210 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 252 a ^ 2 b
^ 5 c ^ 5 d ^ 2 - 210 a b ^ 6 c ^ 6 d + 120 b ^ 7 c ^ 7) + 630 b ^ 4 d ^ 4
x ^ 4 (a ^ 6 d ^ 6 - 10 a ^ 5 b c d ^ 5 + 45 a ^ 4 b ^ 2 c ^ 2 d ^ 4 - 120
a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 210 a ^ 2 b ^ 4 c ^ 4 d ^ 2 - 252 a b ^ 5 c ^ 5 d
 + 210 b ^ 6 c ^ 6) + 504 b ^ 5 d ^ 5 x ^ 5 (-a ^ 5 d ^ 5 + 10 a ^ 4 b c d
```

```
^ 4 - 45 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 120 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 210 a b ^
4 c ^ 4 d + 252 b ^ 5 c ^ 5) + 420 b ^ 6 d ^ 6 x ^ 6 (a ^ 4 d ^ 4 - 10 a ^
3 b c d ^ 3 + 45 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 120 a b ^ 3 c ^ 3 d + 210 b ^ 4
c ^ 4) + 360 b ^ 7 d ^ 7 x ^ 7 (-a ^ 3 d ^ 3 + 10 a ^ 2 b c d ^ 2 - 45 a b
^ 2 c ^ 2 d + 120 b ^ 3 c ^ 3) + 315 b ^ 8 d ^ 8 x ^ 8 (a ^ 2 d ^ 2 - 10 a
b c d + 45 b ^ 2 c ^ 2) + 280 b ^ 9 d ^ 9 x ^ 9 (-a d + 10 b c) + 252 b ^ 1
0 d ^ 10 x ^ 10 + 2520 Log[a + b x] (a d - b c) ^ 10) / (2520 b ^ 11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2928 vs. $2(223) = 446$.

time = 0.52, size = 2929, normalized size = 12.15

method	result
norman	$\frac{d^{10}x^{10}}{10b}+\frac{d^{2}\left(a^{8}d^{8}-10a^{7}bc\,d^{7}+45a^{6}b^{2}c^{2}d^{6}-120a^{5}b^{3}c^{3}d^{5}+210a^{4}b^{4}c^{4}d^{4}-252a^{3}b^{5}c^{5}d^{3}+210a^{2}b^{6}c^{6}d^{2}-120a\,b^{7}c^{7}d+45b^{8}c^{8}\right)x^{2}}{2b^{9}}-d^{3}$
risch	$\frac{d^{10}x^{10}}{10b}-\frac{d^{10}x^{9}a}{9b^{2}}+\frac{10d^{9}x^{9}c}{9b}+\frac{d^{10}x^{8}a^{2}}{8b^{3}}+\frac{45d^{8}x^{8}c^{2}}{8b}-\frac{d^{10}x^{7}a^{3}}{7b^{4}}+\frac{120d^{7}x^{7}c^{3}}{7b}+\frac{d^{10}x^{6}a^{4}}{6b^{5}}+\frac{35d^{6}x^{6}c^{4}}{b}-\frac{d^{10}x^{5}a^{5}}{5b^{6}}+25$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^10*(-1/10*d^9*x^10*b^9+1/9*(((a*d-2*b*c)*d^4*b^4-b*d*(-a*b^3*d^4+5*b^4
*c*d^3))*d^4*b^4-b^5*d^5*(a*b^3*d^4+3*b^4*c*d^3))*x^9+1/8*(((a*d-2*b*c)*(-a
*b^3*d^4+5*b^4*c*d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*d^4*b
^4+((a*d-2*b*c)*d^4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a*b^3*d^4+3*b^4*c*d^
3)-b^5*d^5*(a^2*b^2*d^4+a*b^3*c*d^3+4*b^4*c^2*d^2))*x^8+1/7*(((a*d-2*b*c)*(
a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*(-a^3*b*d^4+5*a^2*b^2*c*d^3-1
0*a*b^3*c^2*d^2+10*b^4*c^3*d))*d^4*b^4+((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*d^3
)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*(a*b^3*d^4+3*b^4*c*d^3)+(
(a*d-2*b*c)*d^4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a^2*b^2*d^4+a*b^3*c*d^3+
4*b^4*c^2*d^2)-b^5*d^5*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d
))*x^7+1/6*(((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^
4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4
*c^4))*d^4*b^4+((a*d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*
(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*(a*b^3*d^4+3*b^
4*c*d^3)+((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d
^3+10*b^4*c^2*d^2))*(a^2*b^2*d^4+a*b^3*c*d^3+4*b^4*c^2*d^2)+((a*d-2*b*c)*d^
4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^
2+2*b^4*c^3*d)-b^5*d^5*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3
*d+b^4*c^4))*x^6+1/5*((a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2
-10*a*b^3*c^3*d+5*b^4*c^4)*d^4*b^4+((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3
-10*a*b^3*c^2*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d
^2-10*a*b^3*c^3*d+5*b^4*c^4))*(a*b^3*d^4+3*b^4*c*d^3)+((a*d-2*b*c)*(a^2*b^2
*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3
*c^2*d^2+10*b^4*c^3*d))*(a^2*b^2*d^4+a*b^3*c*d^3+4*b^4*c^2*d^2)+((a*d-2*b*c
```

```
)*(-a*b^3*d^4+5*b^4*c*d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*
(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d)+((a*d-2*b*c)*d^4*b^4-
b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*
b^3*c^3*d+b^4*c^4))*x^5+1/4*((a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*
c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4)*(a*b^3*d^4+3*b^4*c*d^3)+((a*d-2*b*c)*(-a^
3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b
*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4))*(a^2*b^2*d^4+a*b^3*c*d
^3+4*b^4*c^2*d^2)+((a*d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b
*d*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*(a^3*b*d^4-a
^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d)+((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*
d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*(a^4*d^4-3*a^3*b*c*d^3
+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d+b^4*c^4))*x^4+1/3*((a*d-2*b*c)*(a^4*d^4-5*
a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4)*(a^2*b^2*d^4+a*b^3
*c*d^3+4*b^4*c^2*d^2)+((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2
*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c
^3*d+5*b^4*c^4))*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d)+((a*
d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*(-a^3*b*d^4+5*a^2*b
^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c
^2*d^2-2*a*b^3*c^3*d+b^4*c^4))*x^3+1/2*((a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+
10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4)*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b
^3*c^2*d^2+2*b^4*c^3*d)+((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c
^2*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3
*c^3*d+5*b^4*c^4))*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d+b
^4*c^4))*x^2+(a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3
*c^3*d+5*b^4*c^4)*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d+b^
4*c^4)*x)+(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+
210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7
*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(223) = 446$.
time = 0.31, size = 866, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 280*(10*b^9*c*d^9 - a*b^8*d^10)*x^9 + 315*(45*b
^9*c^2*d^8 - 10*a*b^8*c*d^9 + a^2*b^7*d^10)*x^8 + 360*(120*b^9*c^3*d^7 - 45
*a*b^8*c^2*d^8 + 10*a^2*b^7*c*d^9 - a^3*b^6*d^10)*x^7 + 420*(210*b^9*c^4*d^
6 - 120*a*b^8*c^3*d^7 + 45*a^2*b^7*c^2*d^8 - 10*a^3*b^6*c*d^9 + a^4*b^5*d^1
0)*x^6 + 504*(252*b^9*c^5*d^5 - 210*a*b^8*c^4*d^6 + 120*a^2*b^7*c^3*d^7 - 4
5*a^3*b^6*c^2*d^8 + 10*a^4*b^5*c*d^9 - a^5*b^4*d^10)*x^5 + 630*(210*b^9*c^6
*d^4 - 252*a*b^8*c^5*d^5 + 210*a^2*b^7*c^4*d^6 - 120*a^3*b^6*c^3*d^7 + 45*a
```

```
^4*b^5*c^2*d^8 - 10*a^5*b^4*c*d^9 + a^6*b^3*d^10)*x^4 + 840*(120*b^9*c^7*d^
3 - 210*a*b^8*c^6*d^4 + 252*a^2*b^7*c^5*d^5 - 210*a^3*b^6*c^4*d^6 + 120*a^4
*b^5*c^3*d^7 - 45*a^5*b^4*c^2*d^8 + 10*a^6*b^3*c*d^9 - a^7*b^2*d^10)*x^3 +
1260*(45*b^9*c^8*d^2 - 120*a*b^8*c^7*d^3 + 210*a^2*b^7*c^6*d^4 - 252*a^3*b^
6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 - 120*a^5*b^4*c^3*d^7 + 45*a^6*b^3*c^2*d^8
- 10*a^7*b^2*c*d^9 + a^8*b*d^10)*x^2 + 2520*(10*b^9*c^9*d - 45*a*b^8*c^8*d^
2 + 120*a^2*b^7*c^7*d^3 - 210*a^3*b^6*c^6*d^4 + 252*a^4*b^5*c^5*d^5 - 210*a
^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - 45*a^7*b^2*c^2*d^8 + 10*a^8*b*c*d^9
- a^9*d^10)*x)/b^10 + (b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 12
0*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4
*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10
*d^10)*log(b*x + a)/b^11
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(223) = 446$.
time = 0.30, size = 868, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^10*d^10*x^10 + 280*(10*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(45
*b^10*c^2*d^8 - 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(120*b^10*c^3*d^7
- 45*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(210*b^10*c
^4*d^6 - 120*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 - 10*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 504*(252*b^10*c^5*d^5 - 210*a*b^9*c^4*d^6 + 120*a^2*b^8*c^3*d
^7 - 45*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(210*b
^10*c^6*d^4 - 252*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 - 120*a^3*b^7*c^3*d^7
 + 45*a^4*b^6*c^2*d^8 - 10*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(120*b^1
0*c^7*d^3 - 210*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 - 210*a^3*b^7*c^4*d^6 +
 120*a^4*b^6*c^3*d^7 - 45*a^5*b^5*c^2*d^8 + 10*a^6*b^4*c*d^9 - a^7*b^3*d^10
)*x^3 + 1260*(45*b^10*c^8*d^2 - 120*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 - 2
52*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 120*a^5*b^5*c^3*d^7 + 45*a^6*b^4
*c^2*d^8 - 10*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2520*(10*b^10*c^9*d - 45*
a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d^3 - 210*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5
*d^5 - 210*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 - 45*a^7*b^3*c^2*d^8 + 10*
a^8*b^2*c*d^9 - a^9*b*d^10)*x + 2520*(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b
^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^
5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9
*b*c*d^9 + a^10*d^10)*log(b*x + a))/b^11
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(206) = 412$.
time = 0.85, size = 799, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**10/(b*x+a),x)

[Out] x**9*(-a*d**10/(9*b**2) + 10*c*d**9/(9*b)) + x**8*(a**2*d**10/(8*b**3) - 5*a*c*d**9/(4*b**2) + 45*c**2*d**8/(8*b)) + x**7*(-a**3*d**10/(7*b**4) + 10*a**2*c*d**9/(7*b**3) - 45*a*c**2*d**8/(7*b**2) + 120*c**3*d**7/(7*b)) + x**6*(a**4*d**10/(6*b**5) - 5*a**3*c*d**9/(3*b**4) + 15*a**2*c**2*d**8/(2*b**3) - 20*a*c**3*d**7/b**2 + 35*c**4*d**6/b) + x**5*(-a**5*d**10/(5*b**6) + 2*a**4*c*d**9/b**5 - 9*a**3*c**2*d**8/b**4 + 24*a**2*c**3*d**7/b**3 - 42*a*c**4*d**6/b**2 + 252*c**5*d**5/(5*b)) + x**4*(a**6*d**10/(4*b**7) - 5*a**5*c*d**9/(2*b**6) + 45*a**4*c**2*d**8/(4*b**5) - 30*a**3*c**3*d**7/b**4 + 105*a**2*c**4*d**6/(2*b**3) - 63*a*c**5*d**5/b**2 + 105*c**6*d**4/(2*b)) + x**3*(-a**7*d**10/(3*b**8) + 10*a**6*c*d**9/(3*b**7) - 15*a**5*c**2*d**8/b**6 + 40*a**4*c**3*d**7/b**5 - 70*a**3*c**4*d**6/b**4 + 84*a**2*c**5*d**5/b**3 - 70*a*c**6*d**4/b**2 + 40*c**7*d**3/b) + x**2*(a**8*d**10/(2*b**9) - 5*a**7*c*d**9/b**8 + 45*a**6*c**2*d**8/(2*b**7) - 60*a**5*c**3*d**7/b**6 + 105*a**4*c**4*d**6/b**5 - 126*a**3*c**5*d**5/b**4 + 105*a**2*c**6*d**4/b**3 - 60*a*c**7*d**3/b**2 + 45*c**8*d**2/(2*b)) + x*(-a**9*d**10/b**10 + 10*a**8*c*d**9/b**9 - 45*a**7*c**2*d**8/b**8 + 120*a**6*c**3*d**7/b**7 - 210*a**5*c**4*d**6/b**6 + 252*a**4*c**5*d**5/b**5 - 210*a**3*c**6*d**4/b**4 + 120*a**2*c**7*d**3/b**3 - 45*a*c**8*d**2/b**2 + 10*c**9*d/b) + d**10*x**10/(10*b) + (a*d - b*c)**10*log(a + b*x)/b**11

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(223) = 446.
time = 0.00, size = 1041, normalized size = 4.32

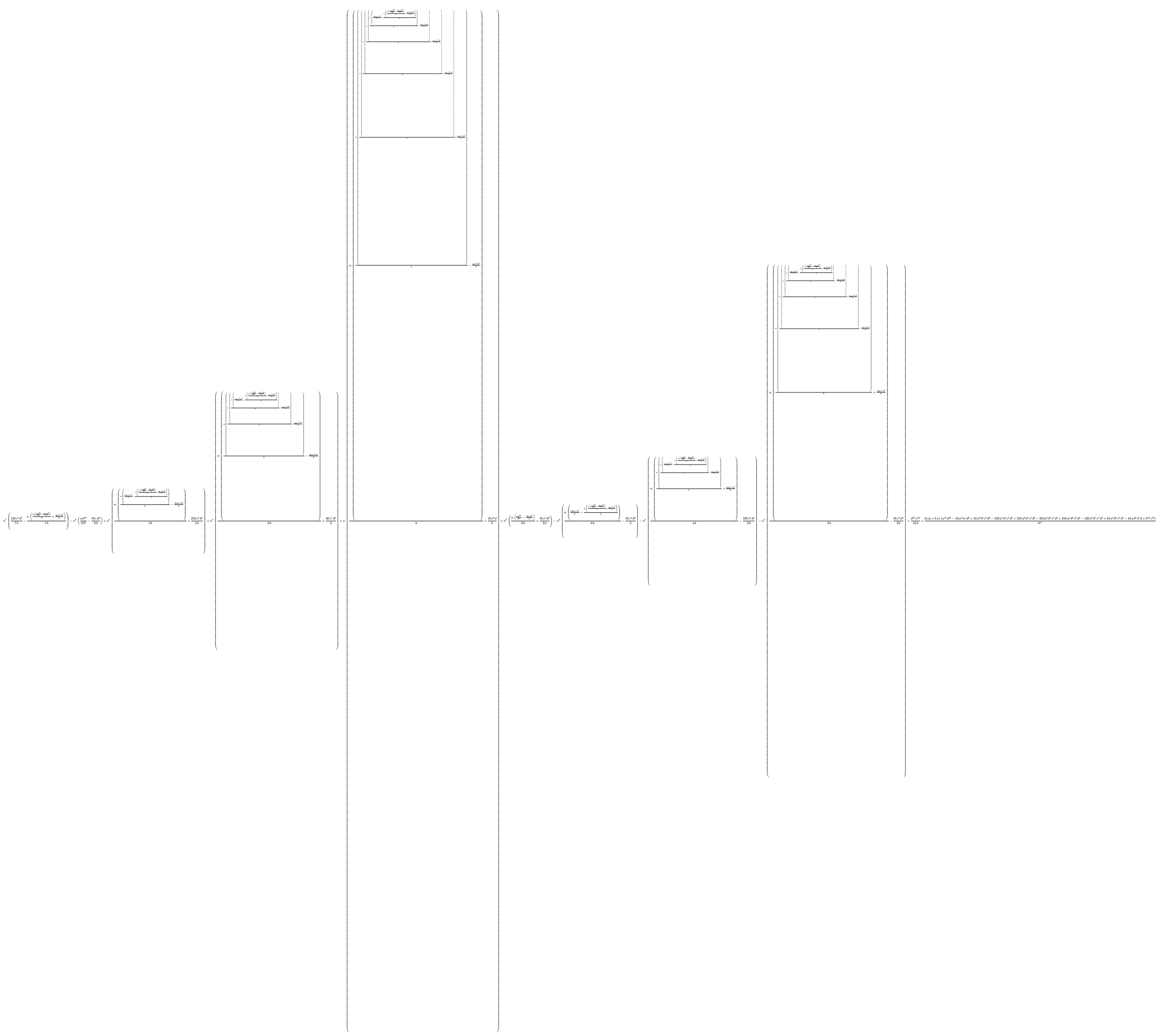
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a),x)

[Out] 1/2520*(252*b^9*d^10*x^10 + 2800*b^9*c*d^9*x^9 - 280*a*b^8*d^10*x^9 + 14175*b^9*c^2*d^8*x^8 - 3150*a*b^8*c*d^9*x^8 + 315*a^2*b^7*d^10*x^8 + 43200*b^9*c^3*d^7*x^7 - 16200*a*b^8*c^2*d^8*x^7 + 3600*a^2*b^7*c*d^9*x^7 - 360*a^3*b^6*d^10*x^7 + 88200*b^9*c^4*d^6*x^6 - 50400*a*b^8*c^3*d^7*x^6 + 18900*a^2*b^7*c^2*d^8*x^6 - 4200*a^3*b^6*c*d^9*x^6 + 420*a^4*b^5*d^10*x^6 + 127008*b^9*c^5*d^5*x^5 - 105840*a*b^8*c^4*d^6*x^5 + 60480*a^2*b^7*c^3*d^7*x^5 - 22680*a^3*b^6*c^2*d^8*x^5 + 5040*a^4*b^5*c*d^9*x^5 - 504*a^5*b^4*d^10*x^5 + 132300*b^9*c^6*d^4*x^4 - 158760*a*b^8*c^5*d^5*x^4 + 132300*a^2*b^7*c^4*d^6*x^4 - 75600*a^3*b^6*c^3*d^7*x^4 + 28350*a^4*b^5*c^2*d^8*x^4 - 6300*a^5*b^4*c*d^9*x^4 + 630*a^6*b^3*d^10*x^4 + 100800*b^9*c^7*d^3*x^3 - 176400*a*b^8*c^6*d^4*x^3 + 211680*a^2*b^7*c^5*d^5*x^3 - 176400*a^3*b^6*c^4*d^6*x^3 + 100800*a^4*b^5*c^3*d^7*x^3 - 37800*a^5*b^4*c^2*d^8*x^3 + 8400*a^6*b^3*c*d^9*x^3 - 840*a^7*b^2*d^10*x^3 + 56700*b^9*c^8*d^2*x^2 - 151200*a*b^8*c^7*d^3*x^2 + 2646

```
00*a^2*b^7*c^6*d^4*x^2 - 317520*a^3*b^6*c^5*d^5*x^2 + 264600*a^4*b^5*c^4*d^
6*x^2 - 151200*a^5*b^4*c^3*d^7*x^2 + 56700*a^6*b^3*c^2*d^8*x^2 - 12600*a^7*
b^2*c*d^9*x^2 + 1260*a^8*b*d^10*x^2 + 25200*b^9*c^9*d*x - 113400*a*b^8*c^8*
d^2*x + 302400*a^2*b^7*c^7*d^3*x - 529200*a^3*b^6*c^6*d^4*x + 635040*a^4*b^
5*c^5*d^5*x - 529200*a^5*b^4*c^4*d^6*x + 302400*a^6*b^3*c^3*d^7*x - 113400*
a^7*b^2*c^2*d^8*x + 25200*a^8*b*c*d^9*x - 2520*a^9*d^10*x)/b^10 + (b^10*c^1
0 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6
*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7
+ 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*log(abs(b*x + a))/b^11
```

Mupad [B]
time = 0.13, size = 979, normalized size = 4.06



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x),x)
```

```
[Out] x^7*((120*c^3*d^7)/(7*b) - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^
2*d^8)/b))/(7*b)) - x^9*((a*d^10)/(9*b^2) - (10*c*d^9)/(9*b)) + x^5*((a*((a
*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/
b))/b))/b - (210*c^4*d^6)/b))/(5*b) + (252*c^5*d^5)/(5*b)) + x^3*((a*((a*((
a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*
d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/
b))/(3*b) + (40*c^7*d^3)/b) + x*((a*((a*((a*((a*((a*((a*((120*c^3*d^7)/b -
(a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4
*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))
/b - (45*c^8*d^2)/b))/b + (10*c^9*d)/b) + x^8*((a*((a*d^10)/b^2 - (10*c*d^9
)/b))/(8*b) + (45*c^2*d^8)/(8*b)) - x^6*((a*((120*c^3*d^7)/b - (a*((a*((a*d
^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/(6*b) - (35*c^4*d^6)/b)
- x^4*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/
b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/(4*b)
- (105*c^6*d^4)/(2*b)) - x^2*((a*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*
((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b
))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))/(2*b)
- (45*c^8*d^2)/(2*b)) + (d^10*x^10)/(10*b) + (log(a + b*x)*(a^10*d^10 + b^1
0*c^10 + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 2
52*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2
*c^2*d^8 - 10*a*b^9*c^9*d - 10*a^9*b*c*d^9))/b^11
```

3.1313 $\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx$

Optimal. Leaf size=258

$$\frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+\frac{63d^5(bc-ad)^5(a+bx)^4}{b^{11}}$$

```
[Out] 45*d^2*(-a*d+b*c)^8*x/b^10-(-a*d+b*c)^10/b^11/(b*x+a)+60*d^3*(-a*d+b*c)^7*(
b*x+a)^2/b^11+70*d^4*(-a*d+b*c)^6*(b*x+a)^3/b^11+63*d^5*(-a*d+b*c)^5*(b*x+a
)^4/b^11+42*d^6*(-a*d+b*c)^4*(b*x+a)^5/b^11+20*d^7*(-a*d+b*c)^3*(b*x+a)^6/b
^11+45/7*d^8*(-a*d+b*c)^2*(b*x+a)^7/b^11+5/4*d^9*(-a*d+b*c)*(b*x+a)^8/b^11+
1/9*d^10*(b*x+a)^9/b^11+10*d*(-a*d+b*c)^9*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.32, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx)^4(bc-ad)^5}{b^{11}}+\frac{70d^4(a+bx)^3(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^2(bc-ad)^7}{b^{11}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{10d(bc-ad)^9\log(a+bx)}{b^{11}}+\frac{d^{10}(a+bx)^9}{9b^{11}}+\frac{45d^2x(bc-ad)^8}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (45*d^2*(b*c - a*d)^8*x)/b^10 - (b*c - a*d)^10/(b^11*(a + b*x)) + (60*d^3*(
b*c - a*d)^7*(a + b*x)^2)/b^11 + (70*d^4*(b*c - a*d)^6*(a + b*x)^3)/b^11 +
(63*d^5*(b*c - a*d)^5*(a + b*x)^4)/b^11 + (42*d^6*(b*c - a*d)^4*(a + b*x)^5
)/b^11 + (20*d^7*(b*c - a*d)^3*(a + b*x)^6)/b^11 + (45*d^8*(b*c - a*d)^2*(a
 + b*x)^7)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^8)/(4*b^11) + (d^10*(a +
 b*x)^9)/(9*b^11) + (10*d*(b*c - a*d)^9*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx = \int \left(\frac{45d^2(bc-ad)^8}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^2}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)}+\frac{120d^3(bc-ad)^7(a+bx)}{b^{10}}+\frac{210d^4}{}\right.$$

$$= \frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 708 vs. 2(258) = 516.
time = 0.14, size = 708, normalized size = 2.74

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (-252*a^10*d^10 + 252*a^9*b*d^9*(10*c + 9*d*x) + 1260*a^8*b^2*d^8*(-9*c^2 -
 16*c*d*x + d^2*x^2) - 420*a^7*b^3*d^7*(-72*c^3 - 189*c^2*d*x + 27*c*d^2*x^
2 + d^3*x^3) + 210*a^6*b^4*d^6*(-252*c^4 - 864*c^3*d*x + 216*c^2*d^2*x^2 +
18*c*d^3*x^3 + d^4*x^4) - 126*a^5*b^5*d^5*(-504*c^5 - 2100*c^4*d*x + 840*c^
3*d^2*x^2 + 120*c^2*d^3*x^3 + 15*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(-12
60*c^6 - 6048*c^5*d*x + 3780*c^4*d^2*x^2 + 840*c^3*d^3*x^3 + 180*c^2*d^4*x^
4 + 27*c*d^5*x^5 + 2*d^6*x^6) - 12*a^3*b^7*d^3*(-2520*c^7 - 13230*c^6*d*x +
 13230*c^5*d^2*x^2 + 4410*c^4*d^3*x^3 + 1470*c^3*d^4*x^4 + 378*c^2*d^5*x^5
+ 63*c*d^6*x^6 + 5*d^7*x^7) + 9*a^2*b^8*d^2*(-1260*c^8 - 6720*c^7*d*x + 117
60*c^6*d^2*x^2 + 5880*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 1176*c^3*d^5*x^5 + 3
36*c^2*d^6*x^6 + 60*c*d^7*x^7 + 5*d^8*x^8) - a*b^9*d*(-2520*c^9 - 11340*c^8
*d*x + 45360*c^7*d^2*x^2 + 35280*c^6*d^3*x^3 + 26460*c^5*d^4*x^4 + 15876*c^
4*d^5*x^5 + 7056*c^3*d^6*x^6 + 2160*c^2*d^7*x^7 + 405*c*d^8*x^8 + 35*d^9*x^
9) + b^10*(-252*c^10 + 11340*c^8*d^2*x^2 + 15120*c^7*d^3*x^3 + 17640*c^6*d^
4*x^4 + 15876*c^5*d^5*x^5 + 10584*c^4*d^6*x^6 + 5040*c^3*d^7*x^7 + 1620*c^2
*d^8*x^8 + 315*c*d^9*x^9 + 28*d^10*x^10) - 2520*d*(-(b*c) + a*d)^9*(a + b*x
)*Log[a + b*x])/(252*b^11*(a + b*x))
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 782 vs. 2(258) = 516.
time = 9.26, size = 770, normalized size = 2.98

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^2,x]')
```

```
[Out] (-10 d Log[a + b x] (a + b x) (a d - b c) ^ 9 - a ^ 10 d ^ 10 + 10 a ^ 9 b
c d ^ 9 - 45 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 120 a ^ 7 b ^ 3 c ^ 3 d ^ 7 - 210 a
^ 6 b ^ 4 c ^ 4 d ^ 6 + 252 a ^ 5 b ^ 5 c ^ 5 d ^ 5 - 210 a ^ 4 b ^ 6 c ^ 6
 d ^ 4 + 120 a ^ 3 b ^ 7 c ^ 7 d ^ 3 - 45 a ^ 2 b ^ 8 c ^ 8 d ^ 2 + 10 a b
^ 9 c ^ 9 d - b ^ 10 c ^ 10 + b d ^ 2 x (a + b x) (9 a ^ 8 d ^ 8 - 80 a ^ 7
 b c d ^ 7 + 315 a ^ 6 b ^ 2 c ^ 2 d ^ 6 - 720 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 10
50 a ^ 4 b ^ 4 c ^ 4 d ^ 4 - 1008 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 630 a ^ 2 b ^ 6
 c ^ 6 d ^ 2 - 240 a b ^ 7 c ^ 7 d + 45 b ^ 8 c ^ 8) - b ^ 2 d ^ 3 x ^ 2 (a
 + b x) (4 a ^ 7 d ^ 7 - 35 a ^ 6 b c d ^ 6 + 135 a ^ 5 b ^ 2 c ^ 2 d ^ 5 -
 300 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 420 a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 378 a ^ 2 b ^
```

```
5 c ^ 5 d ^ 2 + 210 a b ^ 6 c ^ 6 d - 60 b ^ 7 c ^ 7) + b ^ 3 d ^ 4 x ^ 3 (
a + b x) (7 a ^ 6 d ^ 6 - 60 a ^ 5 b c d ^ 5 + 225 a ^ 4 b ^ 2 c ^ 2 d ^ 4
- 480 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 630 a ^ 2 b ^ 4 c ^ 4 d ^ 2 - 504 a b ^ 5 c
 ^ 5 d + 210 b ^ 6 c ^ 6) / 3 - b ^ 4 d ^ 5 x ^ 4 (a + b x) (3 a ^ 5 d ^ 5
- 25 a ^ 4 b c d ^ 4 + 90 a ^ 3 b ^ 2 c ^ 2 d ^ 3 - 180 a ^ 2 b ^ 3 c ^ 3 d
 ^ 2 + 210 a b ^ 4 c ^ 4 d - 126 b ^ 5 c ^ 5) / 2 + b ^ 5 d ^ 6 x ^ 5 (a +
b x) (a ^ 4 d ^ 4 - 8 a ^ 3 b c d ^ 3 + 27 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 48 a b
 ^ 3 c ^ 3 d + 42 b ^ 4 c ^ 4) - b ^ 6 d ^ 7 x ^ 6 (a + b x) (2 a ^ 3 d ^ 3
 - 15 a ^ 2 b c d ^ 2 + 45 a b ^ 2 c ^ 2 d - 60 b ^ 3 c ^ 3) / 3 + b ^ 7 d
^ 8 x ^ 7 (a + b x) (3 a ^ 2 d ^ 2 - 20 a b c d + 45 b ^ 2 c ^ 2) / 7 - b ^
 8 d ^ 9 x ^ 8 (a + b x) (a d - 5 b c) / 4 + b ^ 9 d ^ 10 x ^ 9 (a + b x) /
 9) / (b ^ 11 (a + b x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(252) = 504$.

time = 0.15, size = 933, normalized size = 3.62

method	result
norman	$\frac{\frac{\left(10a^{10}d^{10}-90a^{9}bc\,d^{9}+360a^{8}b^{2}c^{2}d^{8}-840a^{7}b^{3}c^{3}d^{7}+1260a^{6}b^{4}c^{4}d^{6}-1260a^{5}b^{5}c^{5}d^{5}+840a^{4}b^{6}c^{6}d^{4}-360a^{3}b^{7}c^{7}d^{3}+90a^{2}b^{8}c^{8}d^{2}-10a\,b^{9}c^{9}d+b^{10}c^{10}\right)}{b^{10}a}}{}$
default	$\frac{d^{2}\left(45b^{8}c^{8}x+9a^{8}d^{8}x+\frac{1}{9}d^{8}x^{9}b^{8}-\frac{20}{7}a\,b^{7}c\,d^{7}x^{7}+5a^{2}b^{6}c\,d^{7}x^{6}-15a\,b^{7}c^{2}d^{6}x^{6}-8a^{3}b^{5}c\,d^{7}x^{5}+27a^{2}b^{6}c^{2}d^{6}x^{5}-48a\,b^{7}c^{3}d^{5}x^{5}+\frac{25}{2}a^{4}b^{4}c\,d^{7}x^{4}\right.}{}$
risch	$\frac{10a^{9}c\,d^{9}}{b^{10}(bx+a)}-\frac{45a^{8}c^{2}d^{8}}{b^{9}(bx+a)}+\frac{120a^{7}c^{3}d^{7}}{b^{8}(bx+a)}-\frac{210a^{6}c^{4}d^{6}}{b^{7}(bx+a)}+\frac{252a^{5}c^{5}d^{5}}{b^{6}(bx+a)}-\frac{210a^{4}c^{6}d^{4}}{b^{5}(bx+a)}+\frac{120a^{3}c^{7}d^{3}}{b^{4}(bx+a)}-\frac{45a^{2}c^{8}d^{2}}{b^{3}(bx+a)}+\frac{10a\,c^{9}d}{b^{2}(bx+a)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/b^10*(45*b^8*c^8*x+9*a^8*d^8*x+1/9*d^8*x^9*b^8-20/7*a*b^7*c*d^7*x^7+5*a
^2*b^6*c*d^7*x^6-15*a*b^7*c^2*d^6*x^6-8*a^3*b^5*c*d^7*x^5+27*a^2*b^6*c^2*d^
6*x^5-48*a*b^7*c^3*d^5*x^5+25/2*a^4*b^4*c*d^7*x^4-45*a^3*b^5*c^2*d^6*x^4+90
*a^2*b^6*c^3*d^5*x^4-105*a*b^7*c^4*d^4*x^4-20*a^5*b^3*c*d^7*x^3+75*a^4*b^4*
c^2*d^6*x^3-160*a^3*b^5*c^3*d^5*x^3+210*a^2*b^6*c^4*d^4*x^3-168*a*b^7*c^5*d
^3*x^3+35*a^6*b^2*c*d^7*x^2-135*a^5*b^3*c^2*d^6*x^2+300*a^4*b^4*c^3*d^5*x^2
-420*a^3*b^5*c^4*d^4*x^2+378*a^2*b^6*c^5*d^3*x^2-210*a*b^7*c^6*d^2*x^2-80*a
^7*b*c*d^7*x+315*a^6*b^2*c^2*d^6*x-720*a^5*b^3*c^3*d^5*x+1050*a^4*b^4*c^4*d
^4*x-1008*a^3*b^5*c^5*d^3*x+630*a^2*b^6*c^6*d^2*x-240*a*b^7*c^7*d*x+a^4*b^4
*d^8*x^5-1/4*a*b^7*d^8*x^8+5/4*b^8*c*d^7*x^8+3/7*a^2*b^6*d^8*x^7+45/7*b^8*c
^2*d^6*x^7-2/3*a^3*b^5*d^8*x^6+20*b^8*c^3*d^5*x^6+42*b^8*c^4*d^4*x^5-3/2*a^
5*b^3*d^8*x^4+63*b^8*c^5*d^3*x^4+7/3*a^6*b^2*d^8*x^3+70*b^8*c^6*d^2*x^3-4*a
^7*b*d^8*x^2+60*b^8*c^7*d*x^2)-(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8
-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^
6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11
/(b*x+a)-10/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3
*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*
c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(252) = 504$.
time = 0.28, size = 874, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 2
10*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^
3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)/(b^12*x + a*b^
11) + 1/252*(28*b^8*d^10*x^9 + 63*(5*b^8*c*d^9 - a*b^7*d^10)*x^8 + 36*(45*b
^8*c^2*d^8 - 20*a*b^7*c*d^9 + 3*a^2*b^6*d^10)*x^7 + 84*(60*b^8*c^3*d^7 - 45
*a*b^7*c^2*d^8 + 15*a^2*b^6*c*d^9 - 2*a^3*b^5*d^10)*x^6 + 252*(42*b^8*c^4*d
^6 - 48*a*b^7*c^3*d^7 + 27*a^2*b^6*c^2*d^8 - 8*a^3*b^5*c*d^9 + a^4*b^4*d^10
)*x^5 + 126*(126*b^8*c^5*d^5 - 210*a*b^7*c^4*d^6 + 180*a^2*b^6*c^3*d^7 - 90
*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 - 3*a^5*b^3*d^10)*x^4 + 84*(210*b^8*c^6
*d^4 - 504*a*b^7*c^5*d^5 + 630*a^2*b^6*c^4*d^6 - 480*a^3*b^5*c^3*d^7 + 225*
a^4*b^4*c^2*d^8 - 60*a^5*b^3*c*d^9 + 7*a^6*b^2*d^10)*x^3 + 252*(60*b^8*c^7*
d^3 - 210*a*b^7*c^6*d^4 + 378*a^2*b^6*c^5*d^5 - 420*a^3*b^5*c^4*d^6 + 300*a
^4*b^4*c^3*d^7 - 135*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9 - 4*a^7*b*d^10)*x^2
 + 252*(45*b^8*c^8*d^2 - 240*a*b^7*c^7*d^3 + 630*a^2*b^6*c^6*d^4 - 1008*a^3
*b^5*c^5*d^5 + 1050*a^4*b^4*c^4*d^6 - 720*a^5*b^3*c^3*d^7 + 315*a^6*b^2*c^2
*d^8 - 80*a^7*b*c*d^9 + 9*a^8*d^10)*x)/b^10 + 10*(b^9*c^9*d - 9*a*b^8*c^8*d
^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^
5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a
^9*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(252) = 504$.
time = 0.31, size = 1124, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*d^10*x^10 - 252*b^10*c^10 + 2520*a*b^9*c^9*d - 11340*a^2*b^8
*c^8*d^2 + 30240*a^3*b^7*c^7*d^3 - 52920*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^
5*d^5 - 52920*a^6*b^4*c^4*d^6 + 30240*a^7*b^3*c^3*d^7 - 11340*a^8*b^2*c^2*d
^8 + 2520*a^9*b*c*d^9 - 252*a^10*d^10 + 35*(9*b^10*c*d^9 - a*b^9*d^10)*x^9
+ 45*(36*b^10*c^2*d^8 - 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(84*b^10*c^3
*d^7 - 36*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(126*b^1
0*c^4*d^6 - 84*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 - 9*a^3*b^7*c*d^9 + a^4*b
```

```
^6*d^10)*x^6 + 126*(126*b^10*c^5*d^5 - 126*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 - 36*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(84*b^1
0*c^6*d^4 - 126*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 - 84*a^3*b^7*c^3*d^7 +
36*a^4*b^6*c^2*d^8 - 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 420*(36*b^10*c^7
*d^3 - 84*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 - 126*a^3*b^7*c^4*d^6 + 84*a^
4*b^6*c^3*d^7 - 36*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 +
1260*(9*b^10*c^8*d^2 - 36*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 - 126*a^3*b^7*
c^5*d^5 + 126*a^4*b^6*c^4*d^6 - 84*a^5*b^5*c^3*d^7 + 36*a^6*b^4*c^2*d^8 - 9
*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 252*(45*a*b^9*c^8*d^2 - 240*a^2*b^8*c^
7*d^3 + 630*a^3*b^7*c^6*d^4 - 1008*a^4*b^6*c^5*d^5 + 1050*a^5*b^5*c^4*d^6 -
 720*a^6*b^4*c^3*d^7 + 315*a^7*b^3*c^2*d^8 - 80*a^8*b^2*c*d^9 + 9*a^9*b*d^1
0)*x + 2520*(a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 - 84*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 - 126*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^
7 - 36*a^8*b^2*c^2*d^8 + 9*a^9*b*c*d^9 - a^10*d^10 + (b^10*c^9*d - 9*a*b^9*
c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 1
26*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*
d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^12*x + a*b^11)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(240) = 480$.

time = 1.51, size = 816, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**2,x)
```

```
[Out] x**8*(-a*d**10/(4*b**3) + 5*c*d**9/(4*b**2)) + x**7*(3*a**2*d**10/(7*b**4)
- 20*a*c*d**9/(7*b**3) + 45*c**2*d**8/(7*b**2)) + x**6*(-2*a**3*d**10/(3*b*
*5) + 5*a**2*c*d**9/b**4 - 15*a*c**2*d**8/b**3 + 20*c**3*d**7/b**2) + x**5*
(a**4*d**10/b**6 - 8*a**3*c*d**9/b**5 + 27*a**2*c**2*d**8/b**4 - 48*a*c**3*
d**7/b**3 + 42*c**4*d**6/b**2) + x**4*(-3*a**5*d**10/(2*b**7) + 25*a**4*c*d
**9/(2*b**6) - 45*a**3*c**2*d**8/b**5 + 90*a**2*c**3*d**7/b**4 - 105*a*c**4
*d**6/b**3 + 63*c**5*d**5/b**2) + x**3*(7*a**6*d**10/(3*b**8) - 20*a**5*c*d
**9/b**7 + 75*a**4*c**2*d**8/b**6 - 160*a**3*c**3*d**7/b**5 + 210*a**2*c**4
*d**6/b**4 - 168*a*c**5*d**5/b**3 + 70*c**6*d**4/b**2) + x**2*(-4*a**7*d**1
0/b**9 + 35*a**6*c*d**9/b**8 - 135*a**5*c**2*d**8/b**7 + 300*a**4*c**3*d**7
/b**6 - 420*a**3*c**4*d**6/b**5 + 378*a**2*c**5*d**5/b**4 - 210*a*c**6*d**4
/b**3 + 60*c**7*d**3/b**2) + x*(9*a**8*d**10/b**10 - 80*a**7*c*d**9/b**9 +
315*a**6*c**2*d**8/b**8 - 720*a**5*c**3*d**7/b**7 + 1050*a**4*c**4*d**6/b**
6 - 1008*a**3*c**5*d**5/b**5 + 630*a**2*c**6*d**4/b**4 - 240*a*c**7*d**3/b*
*3 + 45*c**8*d**2/b**2) + (-a**10*d**10 + 10*a**9*b*c*d**9 - 45*a**8*b**2*c
**2*d**8 + 120*a**7*b**3*c**3*d**7 - 210*a**6*b**4*c**4*d**6 + 252*a**5*b**
5*c**5*d**5 - 210*a**4*b**6*c**6*d**4 + 120*a**3*b**7*c**7*d**3 - 45*a**2*b
**8*c**8*d**2 + 10*a*b**9*c**9*d - b**10*c**10)/(a*b**11 + b**12*x) + d**10
*x**9/(9*b**2) - 10*d*(a*d - b*c)**9*log(a + b*x)/b**11
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(252) = 504.
time = 0.00, size = 998, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x)
```

```
[Out] 10*(b^9*c^9*d - 9*a*b^8*c^8*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 +
 126*a^4*b^5*c^5*d^5 - 126*a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^
2*c^2*d^8 + 9*a^8*b*c*d^9 - a^9*d^10)*log(abs(b*x + a))/b^11 - (b^10*c^10 -
 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^
6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 4
5*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)*b^11) + 1/252*(2
8*b^16*d^10*x^9 + 315*b^16*c*d^9*x^8 - 63*a*b^15*d^10*x^8 + 1620*b^16*c^2*d
^8*x^7 - 720*a*b^15*c*d^9*x^7 + 108*a^2*b^14*d^10*x^7 + 5040*b^16*c^3*d^7*x
^6 - 3780*a*b^15*c^2*d^8*x^6 + 1260*a^2*b^14*c*d^9*x^6 - 168*a^3*b^13*d^10*
x^6 + 10584*b^16*c^4*d^6*x^5 - 12096*a*b^15*c^3*d^7*x^5 + 6804*a^2*b^14*c^2
*d^8*x^5 - 2016*a^3*b^13*c*d^9*x^5 + 252*a^4*b^12*d^10*x^5 + 15876*b^16*c^5
*d^5*x^4 - 26460*a*b^15*c^4*d^6*x^4 + 22680*a^2*b^14*c^3*d^7*x^4 - 11340*a^
3*b^13*c^2*d^8*x^4 + 3150*a^4*b^12*c*d^9*x^4 - 378*a^5*b^11*d^10*x^4 + 1764
0*b^16*c^6*d^4*x^3 - 42336*a*b^15*c^5*d^5*x^3 + 52920*a^2*b^14*c^4*d^6*x^3
- 40320*a^3*b^13*c^3*d^7*x^3 + 18900*a^4*b^12*c^2*d^8*x^3 - 5040*a^5*b^11*c
*d^9*x^3 + 588*a^6*b^10*d^10*x^3 + 15120*b^16*c^7*d^3*x^2 - 52920*a*b^15*c^
6*d^4*x^2 + 95256*a^2*b^14*c^5*d^5*x^2 - 105840*a^3*b^13*c^4*d^6*x^2 + 7560
0*a^4*b^12*c^3*d^7*x^2 - 34020*a^5*b^11*c^2*d^8*x^2 + 8820*a^6*b^10*c*d^9*x
^2 - 1008*a^7*b^9*d^10*x^2 + 11340*b^16*c^8*d^2*x - 60480*a*b^15*c^7*d^3*x
+ 158760*a^2*b^14*c^6*d^4*x - 254016*a^3*b^13*c^5*d^5*x + 264600*a^4*b^12*c
^4*d^6*x - 181440*a^5*b^11*c^3*d^7*x + 79380*a^6*b^10*c^2*d^8*x - 20160*a^7
*b^9*c*d^9*x + 2268*a^8*b^8*d^10*x)/b^18
```

Mupad [B]
time = 0.35, size = 3475, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^2,x)
```

```
[Out] x^7*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(7*b) - (a^2*d^10)/(7*b^4) + (
45*c^2*d^8)/(7*b^2)) - x^5*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10
)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*(
(2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/(5*b) - (42*c^4*d^6)/b^2 + (a^2*((2
*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2
))/(5*b^2)) - x^8*((a*d^10)/(4*b^3) - (5*c*d^9)/(4*b^2)) + x^3*((70*c^6*d^4
```

)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(3*b) + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/(3*b^2)) - x^2*((a*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b - (60*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(2*b^2)) + x^6*((20*c^3*d^7)/b^2 - (a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/(3*b) + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(6*b^2)) + x*((45*c^8*d^2)/b^2 - (a^2*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b^2 + (2*a*((2*a*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c *d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b - (120*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b^2))/b) + x^4*((a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/(2*b) - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/(4*b^2) + (63*c^5*d^5)/b^2) - (log(a + b*x)*(10*a^9*d^10 - 10*b^9*c^9*d + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d^4 - 1260*a^4*b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 360*a^7*b^2*c^2*d^8 - 90*a^8*b*c*d^9))/b^11 - (a^10*d^10 + b^10*c^10 + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a*b^9*c^9*d - 10*a^9*b*c*d^9)/(b*(a*b^10 + b^11*x)) + (d^10*x^9)/(9*b^2)

3.1314 $\int \frac{(c+dx)^{10}}{(a+bx)^3} dx$

Optimal. Leaf size=262

$$\frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+\frac{84d^5(bc-ad)^5(a+bx)^3}{b^{11}}+105$$

```
[Out] 120*d^3*(-a*d+b*c)^7*x/b^10-1/2*(-a*d+b*c)^10/b^11/(b*x+a)^2-10*d*(-a*d+b*c
)^9/b^11/(b*x+a)+105*d^4*(-a*d+b*c)^6*(b*x+a)^2/b^11+84*d^5*(-a*d+b*c)^5*(b
*x+a)^3/b^11+105/2*d^6*(-a*d+b*c)^4*(b*x+a)^4/b^11+24*d^7*(-a*d+b*c)^3*(b*x
+a)^5/b^11+15/2*d^8*(-a*d+b*c)^2*(b*x+a)^6/b^11+10/7*d^9*(-a*d+b*c)*(b*x+a)
^7/b^11+1/8*d^10*(b*x+a)^8/b^11+45*d^2*(-a*d+b*c)^8*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.30, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+bx)^3(bc-ad)^5}{b^{11}}+\frac{105d^4(a+bx)^2(bc-ad)^6}{b^{11}}+\frac{45d^2(bc-ad)^8\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}+\frac{d^{10}(a+bx)^8}{8b^{11}}+\frac{120d^3x(bc-ad)^7}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (120*d^3*(b*c - a*d)^7*x)/b^10 - (b*c - a*d)^10/(2*b^11*(a + b*x)^2) - (10*
d*(b*c - a*d)^9)/(b^11*(a + b*x)) + (105*d^4*(b*c - a*d)^6*(a + b*x)^2)/b^1
1 + (84*d^5*(b*c - a*d)^5*(a + b*x)^3)/b^11 + (105*d^6*(b*c - a*d)^4*(a + b
*x)^4)/(2*b^11) + (24*d^7*(b*c - a*d)^3*(a + b*x)^5)/b^11 + (15*d^8*(b*c -
a*d)^2*(a + b*x)^6)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^7)/(7*b^11) +
(d^10*(a + b*x)^8)/(8*b^11) + (45*d^2*(b*c - a*d)^8*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx=\int \left(\frac{120d^3(bc-ad)^7}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^3}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^2}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)}+\frac{210d^4(bc-ad}{b^{10}}\right.$$

$$=\frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+84d^5($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 708 vs. 2(262) = 524.
time = 0.15, size = 708, normalized size = 2.70

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (532*a^10*d^10 - 56*a^9*b*d^9*(85*c + 26*d*x) + 28*a^8*b^2*d^8*(675*c^2 + 3
80*c*d*x - 116*d^2*x^2) - 280*a^7*b^3*d^7*(156*c^3 + 117*c^2*d*x - 91*c*d^2
*x^2 + 3*d^3*x^3) + 210*a^6*b^4*d^6*(308*c^4 + 256*c^3*d*x - 414*c^2*d^2*x^
2 + 32*c*d^3*x^3 + d^4*x^4) - 84*a^5*b^5*d^5*(756*c^5 + 560*c^4*d*x - 2000*
c^3*d^2*x^2 + 280*c^2*d^3*x^3 + 20*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(9
80*c^6 + 336*c^5*d*x - 4760*c^4*d^2*x^2 + 1120*c^3*d^3*x^3 + 140*c^2*d^4*x^
4 + 16*c*d^5*x^5 + d^6*x^6) - 24*a^3*b^7*d^3*(700*c^7 - 490*c^6*d*x - 6174*
c^5*d^2*x^2 + 2450*c^4*d^3*x^3 + 490*c^3*d^4*x^4 + 98*c^2*d^5*x^5 + 14*c*d^
6*x^6 + d^7*x^7) + 3*a^2*b^8*d^2*(1260*c^8 - 4480*c^7*d*x - 21560*c^6*d^2*x
^2 + 15680*c^5*d^3*x^3 + 4900*c^4*d^4*x^4 + 1568*c^3*d^5*x^5 + 392*c^2*d^6*
x^6 + 64*c*d^7*x^7 + 5*d^8*x^8) - 2*a*b^9*d*(140*c^9 - 2520*c^8*d*x - 6720*
c^7*d^2*x^2 + 11760*c^6*d^3*x^3 + 5880*c^5*d^4*x^4 + 2940*c^4*d^5*x^5 + 117
6*c^3*d^6*x^6 + 336*c^2*d^7*x^7 + 60*c*d^8*x^8 + 5*d^9*x^9) + b^10*(-28*c^1
0 - 560*c^9*d*x + 6720*c^7*d^3*x^3 + 5880*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 +
2940*c^4*d^6*x^6 + 1344*c^3*d^7*x^7 + 420*c^2*d^8*x^8 + 80*c*d^9*x^9 + 7*d^
10*x^10) + 2520*d^2*(b*c - a*d)^8*(a + b*x)^2*Log[a + b*x])/(56*b^11*(a + b
*x)^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 891 vs. 2(262) = 524.
time = 11.53, size = 889, normalized size = 3.39

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^3,x]')
```

```
[Out] (2520 d ^ 2 Log[a + b x] (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (a d - b c) ^ 8 +
532 a ^ 10 d ^ 10 - 4760 a ^ 9 b c d ^ 9 + 18900 a ^ 8 b ^ 2 c ^ 2 d ^ 8 -
43680 a ^ 7 b ^ 3 c ^ 3 d ^ 7 + 64680 a ^ 6 b ^ 4 c ^ 4 d ^ 6 - 63504 a ^ 5
 b ^ 5 c ^ 5 d ^ 5 + 41160 a ^ 4 b ^ 6 c ^ 6 d ^ 4 - 16800 a ^ 3 b ^ 7 c ^
7 d ^ 3 + 3780 a ^ 2 b ^ 8 c ^ 8 d ^ 2 - 280 a b ^ 9 c ^ 9 d - 28 b ^ 10 c
^ 10 + 560 b d x (a ^ 9 d ^ 9 - 9 a ^ 8 b c d ^ 8 + 36 a ^ 7 b ^ 2 c ^ 2 d
^ 7 - 84 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 126 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 126 a ^ 4
b ^ 5 c ^ 5 d ^ 4 + 84 a ^ 3 b ^ 6 c ^ 6 d ^ 3 - 36 a ^ 2 b ^ 7 c ^ 7 d ^ 2
 + 9 a b ^ 8 c ^ 8 d - b ^ 9 c ^ 9) - 56 b d ^ 3 x (a ^ 2 + 2 a b x + b ^ 2
 x ^ 2) (36 a ^ 7 d ^ 7 - 280 a ^ 6 b c d ^ 6 + 945 a ^ 5 b ^ 2 c ^ 2 d ^ 5
```

```
- 1800 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 2100 a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 1512 a ^ 2
 b ^ 5 c ^ 5 d ^ 2 + 630 a b ^ 6 c ^ 6 d - 120 b ^ 7 c ^ 7) + 28 b ^ 2 d ^
4 x ^ 2 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (28 a ^ 6 d ^ 6 - 210 a ^ 5 b c d ^
 5 + 675 a ^ 4 b ^ 2 c ^ 2 d ^ 4 - 1200 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 1260 a ^
2 b ^ 4 c ^ 4 d ^ 2 - 756 a b ^ 5 c ^ 5 d + 210 b ^ 6 c ^ 6) - 56 b ^ 3 d ^
 5 x ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (7 a ^ 5 d ^ 5 - 50 a ^ 4 b c d ^
4 + 150 a ^ 3 b ^ 2 c ^ 2 d ^ 3 - 240 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 210 a b ^ 4
 c ^ 4 d - 84 b ^ 5 c ^ 5) + 70 b ^ 4 d ^ 6 x ^ 4 (a ^ 2 + 2 a b x + b ^ 2
x ^ 2) (3 a ^ 4 d ^ 4 - 20 a ^ 3 b c d ^ 3 + 54 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 7
2 a b ^ 3 c ^ 3 d + 42 b ^ 4 c ^ 4) - 56 b ^ 5 d ^ 7 x ^ 5 (a ^ 2 + 2 a b x
 + b ^ 2 x ^ 2) (2 a ^ 3 d ^ 3 - 12 a ^ 2 b c d ^ 2 + 27 a b ^ 2 c ^ 2 d -
24 b ^ 3 c ^ 3) + 28 b ^ 6 d ^ 8 x ^ 6 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2) (2 a
 ^ 2 d ^ 2 - 10 a b c d + 15 b ^ 2 c ^ 2) - 8 b ^ 7 d ^ 9 x ^ 7 (a ^ 2 + 2
a b x + b ^ 2 x ^ 2) (3 a d - 10 b c) + 7 b ^ 8 d ^ 10 x ^ 8 (a ^ 2 + 2 a b
 x + b ^ 2 x ^ 2)) / (56 b ^ 11 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(252) = 504$.

time = 0.16, size = 914, normalized size = 3.49

method	result
norman	$\frac{135a^{10}d^{10}-1080a^9bc\,d^9+3780a^8b^2c^2d^8-7560a^7b^3c^3d^7+9450a^6b^4c^4d^6-7560a^5b^5c^5d^5+3780a^4b^6c^6d^4-1080a^3b^7c^7d^3+135a^2b^8c^8d^2-10a\,b^9c^9d-b}{2b^{11}}$
default	$-\frac{d^3\left(36a^7d^7x-120b^7c^7x-\frac{15}{4}a^4b^3d^7x^4-\frac{105}{2}b^7c^4d^3x^4+7a^5b^2d^7x^3-84b^7c^5d^2x^3-14a^6b\,d^7x^2-\frac{1}{8}d^7x^8b^7-105b^7c^6d\,x^2-\frac{675}{2}a^4b^3c^2d^5x^2\right.}{}$
risch	$\frac{675d^8a^4c^2x^2}{2b^7}-\frac{600d^7a^3c^3x^2}{b^6}-\frac{5d^9ac\,x^6}{b^4}+\frac{12d^9a^2c\,x^5}{b^5}-\frac{360d^9\ln(bx+a)a^7c}{b^{10}}+\frac{1260d^8\ln(bx+a)a^6c^2}{b^9}-\frac{2520d^7\ln(bx+a)a^5c}{b^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3/b^10*(36*a^7*d^7*x-120*b^7*c^7*x-15/4*a^4*b^3*d^7*x^4-105/2*b^7*c^4*d^
3*x^4+7*a^5*b^2*d^7*x^3-84*b^7*c^5*d^2*x^3-14*a^6*b*d^7*x^2-1/8*d^7*x^8*b^7
-105*b^7*c^6*d*x^2-675/2*a^4*b^3*c^2*d^5*x^2+600*a^3*b^4*c^3*d^4*x^2-630*a^
2*b^5*c^4*d^3*x^2+378*a*b^6*c^5*d^2*x^2-280*a^6*b*c*d^6*x+945*a^5*b^2*c^2*d
^5*x-1800*a^4*b^3*c^3*d^4*x+2100*a^3*b^4*c^4*d^3*x-1512*a^2*b^5*c^5*d^2*x+6
30*a*b^6*c^6*d*x+25*a^3*b^4*c*d^6*x^4-135/2*a^2*b^5*c^2*d^5*x^4+90*a*b^6*c^
3*d^4*x^4-50*a^4*b^3*c*d^6*x^3+150*a^3*b^4*c^2*d^5*x^3-240*a^2*b^5*c^3*d^4*
x^3+210*a*b^6*c^4*d^3*x^3+105*a^5*b^2*c*d^6*x^2+3/7*a*b^6*d^7*x^7-10/7*b^7*
c*d^6*x^7-a^2*b^5*d^7*x^6-15/2*b^7*c^2*d^5*x^6+2*a^3*b^4*d^7*x^5-24*b^7*c^3
*d^4*x^5+27*a*b^6*c^2*d^5*x^5+5*a*b^6*c*d^6*x^6-12*a^2*b^5*c*d^6*x^5)+10/b^
11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b
^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^
8*c^8*d-b^9*c^9)/(b*x+a)-1/2*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-1
20*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*
```

```
d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(
b*x+a)^2+45/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c
^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7
*d+b^8*c^8)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(252) = 504$.
time = 0.28, size = 881, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^10*c^10 + 10*a*b^9*c^9*d - 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^
3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 15
60*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 +
 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4
 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*
b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^9*b*d^10)*x)/(b^13*x^2 + 2*a*b^12*x + a^2
*b^11) + 1/56*(7*b^7*d^10*x^8 + 8*(10*b^7*c*d^9 - 3*a*b^6*d^10)*x^7 + 28*(1
5*b^7*c^2*d^8 - 10*a*b^6*c*d^9 + 2*a^2*b^5*d^10)*x^6 + 56*(24*b^7*c^3*d^7 -
 27*a*b^6*c^2*d^8 + 12*a^2*b^5*c*d^9 - 2*a^3*b^4*d^10)*x^5 + 70*(42*b^7*c^4
*d^6 - 72*a*b^6*c^3*d^7 + 54*a^2*b^5*c^2*d^8 - 20*a^3*b^4*c*d^9 + 3*a^4*b^3
*d^10)*x^4 + 56*(84*b^7*c^5*d^5 - 210*a*b^6*c^4*d^6 + 240*a^2*b^5*c^3*d^7 -
 150*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 - 7*a^5*b^2*d^10)*x^3 + 28*(210*b^7
*c^6*d^4 - 756*a*b^6*c^5*d^5 + 1260*a^2*b^5*c^4*d^6 - 1200*a^3*b^4*c^3*d^7
+ 675*a^4*b^3*c^2*d^8 - 210*a^5*b^2*c*d^9 + 28*a^6*b*d^10)*x^2 + 56*(120*b^
7*c^7*d^3 - 630*a*b^6*c^6*d^4 + 1512*a^2*b^5*c^5*d^5 - 2100*a^3*b^4*c^4*d^6
 + 1800*a^4*b^3*c^3*d^7 - 945*a^5*b^2*c^2*d^8 + 280*a^6*b*c*d^9 - 36*a^7*d^
10)*x)/b^10 + 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a
^3*b^5*c^5*d^5 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d
^8 - 8*a^7*b*c*d^9 + a^8*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(252) = 504$.
time = 0.30, size = 1233, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*d^10*x^10 - 28*b^10*c^10 - 280*a*b^9*c^9*d + 3780*a^2*b^8*c^8*
d^2 - 16800*a^3*b^7*c^7*d^3 + 41160*a^4*b^6*c^6*d^4 - 63504*a^5*b^5*c^5*d^5
 + 64680*a^6*b^4*c^4*d^6 - 43680*a^7*b^3*c^3*d^7 + 18900*a^8*b^2*c^2*d^8 -
4760*a^9*b*c*d^9 + 532*a^10*d^10 + 10*(8*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*
```

```
(28*b^10*c^2*d^8 - 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(56*b^10*c^3*d^7
- 28*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(70*b^10*c^4*
d^6 - 56*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 - 8*a^3*b^7*c*d^9 + a^4*b^6*d^1
0)*x^6 + 84*(56*b^10*c^5*d^5 - 70*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 - 28*a
^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(28*b^10*c^6*d^4
 - 56*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 - 56*a^3*b^7*c^3*d^7 + 28*a^4*b^6*
c^2*d^8 - 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(8*b^10*c^7*d^3 - 28*a*
b^9*c^6*d^4 + 56*a^2*b^8*c^5*d^5 - 70*a^3*b^7*c^4*d^6 + 56*a^4*b^6*c^3*d^7
- 28*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 28*(480*a*b^9*
c^7*d^3 - 2310*a^2*b^8*c^6*d^4 + 5292*a^3*b^7*c^5*d^5 - 7140*a^4*b^6*c^4*d^
6 + 6000*a^5*b^5*c^3*d^7 - 3105*a^6*b^4*c^2*d^8 + 910*a^7*b^3*c*d^9 - 116*a
^8*b^2*d^10)*x^2 - 56*(10*b^10*c^9*d - 90*a*b^9*c^8*d^2 + 240*a^2*b^8*c^7*d
^3 - 210*a^3*b^7*c^6*d^4 - 252*a^4*b^6*c^5*d^5 + 840*a^5*b^5*c^4*d^6 - 960*
a^6*b^4*c^3*d^7 + 585*a^7*b^3*c^2*d^8 - 190*a^8*b^2*c*d^9 + 26*a^9*b*d^10)*
x + 2520*(a^2*b^8*c^8*d^2 - 8*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 - 56*a^5
*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 - 56*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8
 - 8*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8
*c^6*d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 2
8*a^6*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2*(a*b^9*c^8*d^2
- 8*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 - 56*a^4*b^6*c^5*d^5 + 70*a^5*b^5*
c^4*d^6 - 56*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 - 8*a^8*b^2*c*d^9 + a^9*b
*d^10)*x)*log(b*x + a))/(b^13*x^2 + 2*a*b^12*x + a^2*b^11)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(243) = 486$.

time = 6.30, size = 843, normalized size = 3.22

$$x^7\left(-\frac{3ad^{10}}{7b^4}+\frac{10cd^9}{7b^3}\right)+x^6\left(\frac{a^2d^{10}}{b^5}-\frac{5acd^9}{b^4}+\frac{15c^2d^8}{2b^3}\right)+x^5\left(-\frac{2a^3d^{10}}{b^6}+\frac{12a^2cd^9}{b^5}-\frac{27ac^2d^8}{b^4}+\frac{24c^3d^7}{b^3}\right)+x^4\cdot\left(\frac{15a^4d^{10}}{4b^7}-\frac{25a^3cd^9}{b^6}+\frac{135a^2c^2d^8}{2b^5}-\frac{90ac^3d^7}{b^4}+\frac{105c^4d^6}{2b^3}\right)+x^3\left(-\frac{7a^5d^{10}}{b^8}+\frac{50a^4cd^9}{b^7}-\frac{150a^3c^2d^8}{b^6}+\frac{240a^2c^3d^7}{b^5}-\frac{210ac^4d^6}{b^4}+\frac{84c^5d^5}{b^3}\right)+x^2\cdot\left(\frac{14a^6d^{10}}{b^9}-\frac{105a^5cd^9}{b^8}+\frac{675a^4c^2d^8}{2b^7}-\frac{600a^3c^3d^7}{b^6}+\frac{630a^2c^4d^6}{b^5}-\frac{378ac^5d^5}{b^4}\ldots$$
[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**3,x)
```

```
[Out] x**7*(-3*a*d**10/(7*b**4) + 10*c*d**9/(7*b**3)) + x**6*(a**2*d**10/b**5 - 5
*a*c*d**9/b**4 + 15*c**2*d**8/(2*b**3)) + x**5*(-2*a**3*d**10/b**6 + 12*a**
2*c*d**9/b**5 - 27*a*c**2*d**8/b**4 + 24*c**3*d**7/b**3) + x**4*(15*a**4*d*
*10/(4*b**7) - 25*a**3*c*d**9/b**6 + 135*a**2*c**2*d**8/(2*b**5) - 90*a*c**
3*d**7/b**4 + 105*c**4*d**6/(2*b**3)) + x**3*(-7*a**5*d**10/b**8 + 50*a**4*
c*d**9/b**7 - 150*a**3*c**2*d**8/b**6 + 240*a**2*c**3*d**7/b**5 - 210*a*c**
4*d**6/b**4 + 84*c**5*d**5/b**3) + x**2*(14*a**6*d**10/b**9 - 105*a**5*c*d*
*9/b**8 + 675*a**4*c**2*d**8/(2*b**7) - 600*a**3*c**3*d**7/b**6 + 630*a**2*
c**4*d**6/b**5 - 378*a*c**5*d**5/b**4 + 105*c**6*d**4/b**3) + x*(-36*a**7*d
**10/b**10 + 280*a**6*c*d**9/b**9 - 945*a**5*c**2*d**8/b**8 + 1800*a**4*c**
3*d**7/b**7 - 2100*a**3*c**4*d**6/b**6 + 1512*a**2*c**5*d**5/b**5 - 630*a*c
**6*d**4/b**4 + 120*c**7*d**3/b**3) + (19*a**10*d**10 - 170*a**9*b*c*d**9 +
 675*a**8*b**2*c**2*d**8 - 1560*a**7*b**3*c**3*d**7 + 2310*a**6*b**4*c**4*d
```

```
**6 - 2268*a**5*b**5*c**5*d**5 + 1470*a**4*b**6*c**6*d**4 - 600*a**3*b**7*c
**7*d**3 + 135*a**2*b**8*c**8*d**2 - 10*a*b**9*c**9*d - b**10*c**10 + x*(20
*a**9*b*d**10 - 180*a**8*b**2*c*d**9 + 720*a**7*b**3*c**2*d**8 - 1680*a**6*
b**4*c**3*d**7 + 2520*a**5*b**5*c**4*d**6 - 2520*a**4*b**6*c**5*d**5 + 1680
*a**3*b**7*c**6*d**4 - 720*a**2*b**8*c**7*d**3 + 180*a*b**9*c**8*d**2 - 20*
b**10*c**9*d))/(2*a**2*b**11 + 4*a*b**12*x + 2*b**13*x**2) + d**10*x**8/(8*
b**3) + 45*d**2*(a*d - b*c)**8*log(a + b*x)/b**11
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(252) = 504.
time = 0.00, size = 976, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x)
```

```
[Out] 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a^3*b^5*c^5*d^5
 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d^8 - 8*a^7*b*c
*d^9 + a^8*d^10)*log(abs(b*x + a))/b^11 - 1/2*(b^10*c^10 + 10*a*b^9*c^9*d -
 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^
5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c
^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 + 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2
+ 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b
^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^
9*b*d^10)*x)/((b*x + a)^2*b^11) + 1/56*(7*b^21*d^10*x^8 + 80*b^21*c*d^9*x^7
 - 24*a*b^20*d^10*x^7 + 420*b^21*c^2*d^8*x^6 - 280*a*b^20*c*d^9*x^6 + 56*a^
2*b^19*d^10*x^6 + 1344*b^21*c^3*d^7*x^5 - 1512*a*b^20*c^2*d^8*x^5 + 672*a^2
*b^19*c*d^9*x^5 - 112*a^3*b^18*d^10*x^5 + 2940*b^21*c^4*d^6*x^4 - 5040*a*b^
20*c^3*d^7*x^4 + 3780*a^2*b^19*c^2*d^8*x^4 - 1400*a^3*b^18*c*d^9*x^4 + 210*
a^4*b^17*d^10*x^4 + 4704*b^21*c^5*d^5*x^3 - 11760*a*b^20*c^4*d^6*x^3 + 1344
0*a^2*b^19*c^3*d^7*x^3 - 8400*a^3*b^18*c^2*d^8*x^3 + 2800*a^4*b^17*c*d^9*x^
3 - 392*a^5*b^16*d^10*x^3 + 5880*b^21*c^6*d^4*x^2 - 21168*a*b^20*c^5*d^5*x^
2 + 35280*a^2*b^19*c^4*d^6*x^2 - 33600*a^3*b^18*c^3*d^7*x^2 + 18900*a^4*b^1
7*c^2*d^8*x^2 - 5880*a^5*b^16*c*d^9*x^2 + 784*a^6*b^15*d^10*x^2 + 6720*b^21
*c^7*d^3*x - 35280*a*b^20*c^6*d^4*x + 84672*a^2*b^19*c^5*d^5*x - 117600*a^3
*b^18*c^4*d^6*x + 100800*a^4*b^17*c^3*d^7*x - 52920*a^5*b^16*c^2*d^8*x + 15
680*a^6*b^15*c*d^9*x - 2016*a^7*b^14*d^10*x)/b^24
```

Mupad [B]
time = 0.38, size = 3299, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^3,x)
```

[Out] x^3*((84*c^5*d^5)/b^3 - (a*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b ^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^ 3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d ^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a* d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/ b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(3*b^3)) - x^7*((3*a*d^10)/(7*b^4) - (10*c*d^9)/(7*b^3)) - ((b^10*c^10 - 19*a^10*d^10 - 135* a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5 *c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^ 8 + 10*a*b^9*c^9*d + 170*a^9*b*c*d^9)/(2*b) - x*(10*a^9*d^10 - 10*b^9*c^9*d + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d^4 - 1260*a^4* b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 360*a^7*b^2*c^2* d^8 - 90*a^8*b*c*d^9))/(a^2*b^10 + b^12*x^2 + 2*a*b^11*x) - x^5*((3*a*((3*a *((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(5*b) + (a^3*d^10)/(5*b^6) - (24*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/(5*b^2)) + x^6*((a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/(2* b) - (a^2*d^10)/(2*b^5) + (15*c^2*d^8)/(2*b^3)) + x^4*((3*a*((3*a*((3*a*((3 *a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9) /b^3))/b^2))/(4*b) + (105*c^4*d^6)/(2*b^3) + (a^3*((3*a*d^10)/b^4 - (10*c*d ^9)/b^3))/(4*b^3) - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3* a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(4*b^2)) - x^2*((3*a^2*((3*a*((3*a*((3*a *((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c* d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/ b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/(2*b^2) + (3*a*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^ 10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b ^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3)) /b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3 *a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9) /b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2 *d^10)/b^5 + (45*c^2*d^8)/b^3))/b^3))/(2*b) - (105*c^6*d^4)/b^3 - (a^3*((3* a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d ^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/(2*b^3)) + x*((3*a*((3*a^2*((3*a*((3*a*((3*a*((3*a* d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^ 3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b

^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
(45*c^2*d^8)/b^3))/b^2))/b^2 + (3*a*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*
((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8
)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^
4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^1
0)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2)
)/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5
+ (45*c^2*d^8)/b^3))/b^3))/b - (210*c^6*d^4)/b^3 - (a^3*((3*a*((3*a*((3*a*d
^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (
a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3
))/b^2))/b^3))/b - (a^3*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3)
)/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d
^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)
/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^1
0)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b
^3 + (120*c^7*d^3)/b^3 - (3*a^2*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/
b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10
)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^
6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2
- (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45
*c^2*d^8)/b^3))/b^3))/b^2) + (log(a + b*x)*(45*a^8*d^10 + 45*b^8*c^8*d^2 -
360*a*b^7*c^7*d^3 + 1260*a^2*b^6*c^6*d^4 - 2520*a^3*b^5*c^5*d^5 + 3150*a^4*
b^4*c^4*d^6 - 2520*a^5*b^3*c^3*d^7 + 1260*a^6*b^2*c^2*d^8 - 360*a^7*b*c*d^9
))/b^11 + (d^10*x^8)/(8*b^3)

3.1315 $\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx$

Optimal. Leaf size=258

$$\frac{210d^4(bc-ad)^6x}{b^{10}}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{126d^5(bc-ad)^5(a+bx)^2}{b^{11}}+\frac{70d^6(bc-ad)}{b^{11}}$$

```
[Out] 210*d^4*(-a*d+b*c)^6*x/b^10-1/3*(-a*d+b*c)^10/b^11/(b*x+a)^3-5*d*(-a*d+b*c)
^9/b^11/(b*x+a)^2-45*d^2*(-a*d+b*c)^8/b^11/(b*x+a)+126*d^5*(-a*d+b*c)^5*(b*
x+a)^2/b^11+70*d^6*(-a*d+b*c)^4*(b*x+a)^3/b^11+30*d^7*(-a*d+b*c)^3*(b*x+a)^
4/b^11+9*d^8*(-a*d+b*c)^2*(b*x+a)^5/b^11+5/3*d^9*(-a*d+b*c)*(b*x+a)^6/b^11+
1/7*d^10*(b*x+a)^7/b^11+120*d^3*(-a*d+b*c)^7*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.30, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+bx)^2(bc-ad)^5}{b^{11}}+\frac{120d^3(bc-ad)^7\log(a+bx)}{b^{11}}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}+\frac{d^{10}(a+bx)^7}{7b^{11}}+\frac{210d^4x(bc-ad)^6}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (210*d^4*(b*c - a*d)^6*x)/b^10 - (b*c - a*d)^10/(3*b^11*(a + b*x)^3) - (5*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^2) - (45*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
) + (126*d^5*(b*c - a*d)^5*(a + b*x)^2)/b^11 + (70*d^6*(b*c - a*d)^4*(a + b
*x)^3)/b^11 + (30*d^7*(b*c - a*d)^3*(a + b*x)^4)/b^11 + (9*d^8*(b*c - a*d)^
2*(a + b*x)^5)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^6)/(3*b^11) + (d^10*(a +
 b*x)^7)/(7*b^11) + (120*d^3*(b*c - a*d)^7*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx &= \int \left(\frac{210d^4(bc-ad)^6}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^4}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^3}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^2}+\frac{120d^3(bc-ad}{b^{10}(a+bx)}\right.\\ &= \frac{210d^4(bc-ad)^6x}{b^{10}}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{126d^5(bc-ad)^5(a}{b^{11}}\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 427, normalized size = 1.66

$$\frac{21bd^4(210b^6c^6-1008ab^5c^5d+2100a^2b^4c^4d^2-2400a^3b^3c^3d^3+1575a^4b^2c^2d^4-560a^5bcd^5+84a^6d^6)x+21b^2d^5(126b^5c^5-420ab^4c^4d+600a^2b^3c^3d^2-450a^3b^2c^2d^3+175a^4bcd^4-28a^5d^5)x^2+35b^3d^6(42b^4c^4-96ab^3c^3d+90a^2b^2c^2d^2-40a^3bcd^3+7a^4d^4)x^3+105b^4d^7(6b^3c^3-9ab^2c^2d+5a^2bcd^2-a^3d^3)x^4+21b^5d^8(9b^2c^2-8abcd+2a^2d^2)x^5+7b^6d^9(5bc-2ad)x^6+3b^7d^{10}x^7-\frac{7(bc-ad)^{10}}{(a+bx)^3}+\frac{105d(-bc+ad)^9}{(a+bx)^2}-\frac{945d^2(bc-ad)^8}{a+bx}+2520d^3(bc-ad)^7\log(a+bx)}{21b^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^10/(a + b*x)^4,x]

[Out] (21*b*d^4*(210*b^6*c^6 - 1008*a*b^5*c^5*d + 2100*a^2*b^4*c^4*d^2 - 2400*a^3*b^3*c^3*d^3 + 1575*a^4*b^2*c^2*d^4 - 560*a^5*b*c*d^5 + 84*a^6*d^6)*x + 21*b^2*d^5*(126*b^5*c^5 - 420*a*b^4*c^4*d + 600*a^2*b^3*c^3*d^2 - 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 - 28*a^5*d^5)*x^2 + 35*b^3*d^6*(42*b^4*c^4 - 96*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 + 7*a^4*d^4)*x^3 + 105*b^4*d^7*(6*b^3*c^3 - 9*a*b^2*c^2*d + 5*a^2*b*c*d^2 - a^3*d^3)*x^4 + 21*b^5*d^8*(9*b^2*c^2 - 8*a*b*c*d + 2*a^2*d^2)*x^5 + 7*b^6*d^9*(5*b*c - 2*a*d)*x^6 + 3*b^7*d^10*x^7 - (7*(b*c - a*d)^10)/(a + b*x)^3 + (105*d*(-(b*c) + a*d)^9)/(a + b*x)^2 - (945*d^2*(b*c - a*d)^8)/(a + b*x) + 2520*d^3*(b*c - a*d)^7*Log[a + b*x])/(21*b^11)

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 987 vs. 2(258) = 516.
time = 28.97, size = 985, normalized size = 3.82

[illegible]

Antiderivative was successfully verified.

[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^4,x]')

[Out] (-2520 d ^ 3 Log[a + b x] (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (a d - b c) ^ 7 - 847 a ^ 10 d ^ 10 + 6685 a ^ 9 b c d ^ 9 - 22995 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 44940 a ^ 7 b ^ 3 c ^ 3 d ^ 7 - 54390 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 41454 a ^ 5 b ^ 5 c ^ 5 d ^ 5 - 19110 a ^ 4 b ^ 6 c ^ 6 d ^ 4 + 4620 a ^ 3 b ^ 7 c ^ 7 d ^ 3 - 315 a ^ 2 b ^ 8 c ^ 8 d ^ 2 - 35 a b ^ 9 c ^ 9 d - 7 b ^ 10 c ^ 10 - 105 b d x (17 a ^ 9 d ^ 9 - 135 a ^ 8 b c d ^ 8 + 468 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 924 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 1134 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 882 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 420 a ^ 3 b ^ 6 c ^ 6 d ^ 3 - 108 a ^ 2 b ^ 7 c ^ 7 d ^ 2 + 9 a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + 945 b ^ 2 d ^ 2 x ^ 2 (-a ^ 8 d ^ 8 + 8 a ^ 7 b c d ^ 7 - 28 a ^ 6 b ^ 2 c ^ 2 d ^ 6 + 56 a ^ 5 b ^ 3 c ^ 3 d ^ 5 - 70 a ^ 4 b ^ 4 c ^ 4 d ^ 4 + 56 a ^ 3 b ^ 5 c ^ 5 d ^ 3 - 28 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 8 a b ^ 7 c ^ 7 d - b ^ 8 c ^ 8) + 21 b d ^ 4 x (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (84 a ^ 6 d ^ 6 - 560 a ^ 5 b c d ^ 5 + 1575 a ^ 4 b ^ 2 c ^ 2 d ^ 4 - 2400 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 2100 a ^ 2 b ^ 4 c ^ 4 d ^ 2 - 1008 a b ^ 5 c ^ 5 d + 210 b ^ 6 c ^ 6) - 21 b ^ 2 d ^ 5 x ^ 2 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (28 a ^ 5 d ^ 5 - 175 a ^ 4 b c d ^ 4 + 450 a ^ 3 b ^ 2 c ^ 2 d ^ 3 - 600 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 420 a b ^ 4 c ^ 4 d - 126 b

```
^ 5 c ^ 5) + 35 b ^ 3 d ^ 6 x ^ 3 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 +
b ^ 3 x ^ 3) (7 a ^ 4 d ^ 4 - 40 a ^ 3 b c d ^ 3 + 90 a ^ 2 b ^ 2 c ^ 2 d ^
 2 - 96 a b ^ 3 c ^ 3 d + 42 b ^ 4 c ^ 4) - 105 b ^ 4 d ^ 7 x ^ 4 (a ^ 3 +
3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (a ^ 3 d ^ 3 - 5 a ^ 2 b c d ^
 2 + 9 a b ^ 2 c ^ 2 d - 6 b ^ 3 c ^ 3) + 21 b ^ 5 d ^ 8 x ^ 5 (a ^ 3 + 3 a
 ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) (2 a ^ 2 d ^ 2 - 8 a b c d + 9 b
^ 2 c ^ 2) - 7 b ^ 6 d ^ 9 x ^ 6 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b
 ^ 3 x ^ 3) (2 a d - 5 b c) + 3 b ^ 7 d ^ 10 x ^ 7 (a ^ 3 + 3 a ^ 2 b x + 3
 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) / (21 b ^ 11 (a ^ 3 + 3 a ^ 2 b x + 3 a b ^
2 x ^ 2 + b ^ 3 x ^ 3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(252) = 504$.

time = 0.17, size = 896, normalized size = 3.47

method	result
norman	$-\frac{660a^{10}d^{10}-4620a^9bc\,d^9+13860a^8b^2c^2d^8-23100a^7b^3c^3d^7+23100a^6b^4c^4d^6-13860a^5b^5c^5d^5+4620a^4b^6c^6d^4-660a^3b^7c^7d^3+45a^2b^8c^8d^2+5a\,b^9c^9}{3b^{11}}$
default	$d^4(-\frac{2}{3}a\,b^5d^6x^6+\frac{5}{3}b^6c\,d^5x^6+2a^2b^4d^6x^5+9b^6c^2d^4x^5-5a^3b^3d^6x^4+30b^6c^3d^3x^4+\frac{35}{3}a^4b^2d^6x^3+70b^6c^4d^2x^3-28a^5b\,d^6x^2+\frac{1}{7}d^6x^7b^6+84a$
risch	$-\frac{420d^6a\,c^4x^2}{b^5}-\frac{560d^9a^5cx}{b^9}+\frac{1575d^8a^4c^2x}{b^8}-\frac{2400d^7a^3c^3x}{b^7}+\frac{2100d^6a^2c^4x}{b^6}-\frac{1008d^5a\,c^5x}{b^5}-\frac{8d^9ac\,x^5}{b^5}+\frac{25d^9a^2c\,x^4}{b^6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^4/b^10*(-2/3*a*b^5*d^6*x^6+5/3*b^6*c*d^5*x^6+2*a^2*b^4*d^6*x^5+9*b^6*c^2*
d^4*x^5-5*a^3*b^3*d^6*x^4+30*b^6*c^3*d^3*x^4+35/3*a^4*b^2*d^6*x^3+70*b^6*c^
4*d^2*x^3-28*a^5*b*d^6*x^2+1/7*d^6*x^7*b^6+84*a^6*d^6*x+210*b^6*c^6*x-45*a*
b^5*c^2*d^4*x^4-200/3*a^3*b^3*c*d^5*x^3+150*a^2*b^4*c^2*d^4*x^3-160*a*b^5*c
^3*d^3*x^3+175*a^4*b^2*c*d^5*x^2-450*a^3*b^3*c^2*d^4*x^2+600*a^2*b^4*c^3*d^
3*x^2-420*a*b^5*c^4*d^2*x^2-560*a^5*b*c*d^5*x+1575*a^4*b^2*c^2*d^4*x-2400*a
^3*b^3*c^3*d^3*x+2100*a^2*b^4*c^4*d^2*x-1008*a*b^5*c^5*d*x+126*b^6*c^5*d*x^
2-8*a*b^5*c*d^5*x^5+25*a^2*b^4*c*d^5*x^4)-45/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^
7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d
^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)+5/b^11*d*(a^9*d^9-9*a^
8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4
*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(
b*x+a)^2-120/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*
c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)*ln(b*x
+a)-1/3*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+21
0*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d
^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(252) = 504$.

time = 0.30, size = 891, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3
+ 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420
*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*d^9 + 121*a^10*d^10 +
 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 56*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c^2*d^8 - 8*a^7*
b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b^9*c^8*d^2 - 108*a^2*
b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^5 + 1134*a^5*b^5*c^4*
d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*a^8*b^2*c*d^9 + 17*a^
9*b*d^10)*x)/(b^14*x^3 + 3*a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11) + 1/21*(3*
b^6*d^10*x^7 + 7*(5*b^6*c*d^9 - 2*a*b^5*d^10)*x^6 + 21*(9*b^6*c^2*d^8 - 8*a
*b^5*c*d^9 + 2*a^2*b^4*d^10)*x^5 + 105*(6*b^6*c^3*d^7 - 9*a*b^5*c^2*d^8 + 5
*a^2*b^4*c*d^9 - a^3*b^3*d^10)*x^4 + 35*(42*b^6*c^4*d^6 - 96*a*b^5*c^3*d^7
+ 90*a^2*b^4*c^2*d^8 - 40*a^3*b^3*c*d^9 + 7*a^4*b^2*d^10)*x^3 + 21*(126*b^6
*c^5*d^5 - 420*a*b^5*c^4*d^6 + 600*a^2*b^4*c^3*d^7 - 450*a^3*b^3*c^2*d^8 +
175*a^4*b^2*c*d^9 - 28*a^5*b*d^10)*x^2 + 21*(210*b^6*c^6*d^4 - 1008*a*b^5*c
^5*d^5 + 2100*a^2*b^4*c^4*d^6 - 2400*a^3*b^3*c^3*d^7 + 1575*a^4*b^2*c^2*d^8
 - 560*a^5*b*c*d^9 + 84*a^6*d^10)*x)/b^10 + 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*
d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^6 + 35*a^4*b^3*c^3*d^7 - 21*a^5
*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. 2(252) = 504.
time = 0.31, size = 1316, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*d^10*x^10 - 7*b^10*c^10 - 35*a*b^9*c^9*d - 315*a^2*b^8*c^8*d^2
 + 4620*a^3*b^7*c^7*d^3 - 19110*a^4*b^6*c^6*d^4 + 41454*a^5*b^5*c^5*d^5 - 5
4390*a^6*b^4*c^4*d^6 + 44940*a^7*b^3*c^3*d^7 - 22995*a^8*b^2*c^2*d^8 + 6685
*a^9*b*c*d^9 - 847*a^10*d^10 + 5*(7*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(21*b^
10*c^2*d^8 - 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 18*(35*b^10*c^3*d^7 - 21*a
*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(35*b^10*c^4*d^6 -
35*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 - 7*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 126*(21*b^10*c^5*d^5 - 35*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 - 21*a^3*b^
7*c^2*d^8 + 7*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(7*b^10*c^6*d^4 - 21*
a*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 - 35*a^3*b^7*c^3*d^7 + 21*a^4*b^6*c^2*d^
```

```
8 - 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 7*(1890*a*b^9*c^6*d^4 - 7938*a^2*
b^8*c^5*d^5 + 15330*a^3*b^7*c^4*d^6 - 16680*a^4*b^6*c^3*d^7 + 10575*a^5*b^5
*c^2*d^8 - 3665*a^6*b^4*c*d^9 + 539*a^7*b^3*d^10)*x^3 - 21*(45*b^10*c^8*d^2
 - 360*a*b^9*c^7*d^3 + 630*a^2*b^8*c^6*d^4 + 378*a^3*b^7*c^5*d^5 - 2730*a^4
*b^6*c^4*d^6 + 4080*a^5*b^5*c^3*d^7 - 3015*a^6*b^4*c^2*d^8 + 1145*a^7*b^3*c
*d^9 - 179*a^8*b^2*d^10)*x^2 - 21*(5*b^10*c^9*d + 45*a*b^9*c^8*d^2 - 540*a^
2*b^8*c^7*d^3 + 1890*a^3*b^7*c^6*d^4 - 3402*a^4*b^6*c^5*d^5 + 3570*a^5*b^5*
c^4*d^6 - 2220*a^6*b^4*c^3*d^7 + 765*a^7*b^3*c^2*d^8 - 115*a^8*b^2*c*d^9 +
a^9*b*d^10)*x + 2520*(a^3*b^7*c^7*d^3 - 7*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*
d^5 - 35*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 - 21*a^8*b^2*c^2*d^8 + 7*a^9*
b*c*d^9 - a^10*d^10 + (b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5
- 35*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*
c*d^9 - a^7*b^3*d^10)*x^3 + 3*(a*b^9*c^7*d^3 - 7*a^2*b^8*c^6*d^4 + 21*a^3*b
^7*c^5*d^5 - 35*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 - 21*a^6*b^4*c^2*d^8 +
 7*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 3*(a^2*b^8*c^7*d^3 - 7*a^3*b^7*c^6*d
^4 + 21*a^4*b^6*c^5*d^5 - 35*a^5*b^5*c^4*d^6 + 35*a^6*b^4*c^3*d^7 - 21*a^7*
b^3*c^2*d^8 + 7*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^14*x^3 + 3*
a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 907 vs. 2(252) = 504.
time = 0.00, size = 957, normalized size = 3.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x)
```

```
[Out] 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^
6 + 35*a^4*b^3*c^3*d^7 - 21*a^5*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log
(abs(b*x + a))/b^11 - 1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a
^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*
d^9 + 121*a^10*d^10 + 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*
d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6
*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b
```

```
^9*c^8*d^2 - 108*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^
5 + 1134*a^5*b^5*c^4*d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*
a^8*b^2*c*d^9 + 17*a^9*b*d^10)*x)/((b*x + a)^3*b^11) + 1/21*(3*b^24*d^10*x^
7 + 35*b^24*c*d^9*x^6 - 14*a*b^23*d^10*x^6 + 189*b^24*c^2*d^8*x^5 - 168*a*b
^23*c*d^9*x^5 + 42*a^2*b^22*d^10*x^5 + 630*b^24*c^3*d^7*x^4 - 945*a*b^23*c^
2*d^8*x^4 + 525*a^2*b^22*c*d^9*x^4 - 105*a^3*b^21*d^10*x^4 + 1470*b^24*c^4*
d^6*x^3 - 3360*a*b^23*c^3*d^7*x^3 + 3150*a^2*b^22*c^2*d^8*x^3 - 1400*a^3*b^
21*c*d^9*x^3 + 245*a^4*b^20*d^10*x^3 + 2646*b^24*c^5*d^5*x^2 - 8820*a*b^23*
c^4*d^6*x^2 + 12600*a^2*b^22*c^3*d^7*x^2 - 9450*a^3*b^21*c^2*d^8*x^2 + 3675
*a^4*b^20*c*d^9*x^2 - 588*a^5*b^19*d^10*x^2 + 4410*b^24*c^6*d^4*x - 21168*a
*b^23*c^5*d^5*x + 44100*a^2*b^22*c^4*d^6*x - 50400*a^3*b^21*c^3*d^7*x + 330
75*a^4*b^20*c^2*d^8*x - 11760*a^5*b^19*c*d^9*x + 1764*a^6*b^18*d^10*x)/b^28
```

Mupad [B]

time = 0.39, size = 2219, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^4,x)
```

```
[Out] x^3*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b
^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*(
(4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/(3*b) - (a^4*d^10)/(3*b^8) + (70*c^
4*d^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(3*b^3) - (2*a^2*((4
*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b
^4))/b^2) - x^6*((2*a*d^10)/(3*b^5) - (5*c*d^9)/(3*b^4)) - x^4*((a*((4*a*((
4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/
b + (a^3*d^10)/b^7 - (30*c^3*d^7)/b^4 - (3*a^2*((4*a*d^10)/b^5 - (10*c*d^9)
/b^4))/(2*b^2)) + x^5*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(5*b) - (6*a
^2*d^10)/(5*b^6) + (9*c^2*d^8)/b^4) - x*((4*a*((252*c^5*d^5)/b^4 - (4*a*((4
*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (4
5*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^
10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (
4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5
 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b + (a^
4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^4 + (6*a^2*((4*a*((4*a*((4*a*d^10)/b
^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*
d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
^2))/b^2 - (4*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)
/b^6 + (45*c^2*d^8)/b^4))/b^3))/b - (210*c^6*d^4)/b^4 + (6*a^2*((4*a*((4*a*
((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8
)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 -
 (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (4*a^3*((4
*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5 - (10*c*
d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b^2 - (4*a^3*((4
```

```
*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*
d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^
5 - (10*c*d^9)/b^4))/b^2))/b^3 + (a^4*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^
4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^4) + x^2*((126*c^5*d^5)/b^4
 - (2*a*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^1
0)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a
^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d
^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*
a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^
2))/b + (a^4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(2*b^4) + (3*a^2*((4*a*((4*
a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^
4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10
*c*d^9)/b^4))/b^2))/b^2 - (2*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
 - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^3) - ((121*a^10*d^10 + b^10*c^10
 + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a
^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2
*c^2*d^8 + 5*a*b^9*c^9*d - 955*a^9*b*c*d^9)/(3*b) + x*(85*a^9*d^10 + 5*b^9*
c^9*d + 45*a*b^8*c^8*d^2 - 540*a^2*b^7*c^7*d^3 + 2100*a^3*b^6*c^6*d^4 - 441
0*a^4*b^5*c^5*d^5 + 5670*a^5*b^4*c^4*d^6 - 4620*a^6*b^3*c^3*d^7 + 2340*a^7*
b^2*c^2*d^8 - 675*a^8*b*c*d^9) + x^2*(45*a^8*b*d^10 + 45*b^9*c^8*d^2 - 360*
a*b^8*c^7*d^3 - 360*a^7*b^2*c*d^9 + 1260*a^2*b^7*c^6*d^4 - 2520*a^3*b^6*c^5
*d^5 + 3150*a^4*b^5*c^4*d^6 - 2520*a^5*b^4*c^3*d^7 + 1260*a^6*b^3*c^2*d^8))
/(a^3*b^10 + b^13*x^3 + 3*a^2*b^11*x + 3*a*b^12*x^2) + (d^10*x^7)/(7*b^4) -
 (log(a + b*x)*(120*a^7*d^10 - 120*b^7*c^7*d^3 + 840*a*b^6*c^6*d^4 - 2520*a
^2*b^5*c^5*d^5 + 4200*a^3*b^4*c^4*d^6 - 4200*a^4*b^3*c^3*d^7 + 2520*a^5*b^2
*c^2*d^8 - 840*a^6*b*c*d^9))/b^11
```

3.1316 $\int \frac{(c+dx)^{10}}{(a+bx)^5} \, dx$

Optimal. Leaf size=262

$$\frac{252d^5(bc-ad)^5x}{b^{10}} - \frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4} - \frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3} - \frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2} - \frac{120d^3(bc-ad)^7}{b^{11}(a+bx)} + \frac{105d^6(bc-ad)^4(a+}{b^{11}}$$

```
[Out] 252*d^5*(-a*d+b*c)^5*x/b^10-1/4*(-a*d+b*c)^10/b^11/(b*x+a)^4-10/3*d*(-a*d+b
*c)^9/b^11/(b*x+a)^3-45/2*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^2-120*d^3*(-a*d+b*c
)^7/b^11/(b*x+a)+105*d^6*(-a*d+b*c)^4*(b*x+a)^2/b^11+40*d^7*(-a*d+b*c)^3*(b
*x+a)^3/b^11+45/4*d^8*(-a*d+b*c)^2*(b*x+a)^4/b^11+2*d^9*(-a*d+b*c)*(b*x+a)^
5/b^11+1/6*d^10*(b*x+a)^6/b^11+210*d^4*(-a*d+b*c)^6*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.29, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}} + \frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}} + \frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}} + \frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}} + \frac{210d^4(bc-ad)^6\log(a+bx)}{b^{11}} - \frac{120d^3(bc-ad)^7}{b^{11}(a+bx)} - \frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2} - \frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3} - \frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4} + \frac{d^{10}(a+bx)^6}{6b^{11}} + \frac{252d^5x(bc-ad)^5}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (252*d^5*(b*c - a*d)^5*x)/b^10 - (b*c - a*d)^10/(4*b^11*(a + b*x)^4) - (10*
d*(b*c - a*d)^9)/(3*b^11*(a + b*x)^3) - (45*d^2*(b*c - a*d)^8)/(2*b^11*(a +
 b*x)^2) - (120*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)) + (105*d^6*(b*c - a*d)^
4*(a + b*x)^2)/b^11 + (40*d^7*(b*c - a*d)^3*(a + b*x)^3)/b^11 + (45*d^8*(b*
c - a*d)^2*(a + b*x)^4)/(4*b^11) + (2*d^9*(b*c - a*d)*(a + b*x)^5)/b^11 + (
d^10*(a + b*x)^6)/(6*b^11) + (210*d^4*(b*c - a*d)^6*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^{10}}{(a+bx)^5}\,dx &= \int \left(\frac{252d^5(bc-ad)^5}{b^{10}} + \frac{(bc-ad)^{10}}{b^{10}(a+bx)^5} + \frac{10d(bc-ad)^9}{b^{10}(a+bx)^4} + \frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^3} + \frac{120d^3(bc-a}{b^{10}(a+bx)}\right. \\ &= \frac{252d^5(bc-ad)^5x}{b^{10}} - \frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4} - \frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3} - \frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2} - \frac{120d^3(bc-ad)}{b^{11}(a+bx)} \end{aligned}$$

Mathematica [A]
time = 0.12, size = 359, normalized size = 1.37

$$\frac{12bd^5\left(252b^5c^5-1050ab^4c^4d+1800a^2b^3c^3d^2-1575a^3b^2c^2d^3+700a^4bcd^4-126a^5d^5\right)x+30b^2d^6\left(42b^4c^4-120ab^3c^3d+135a^2b^2c^2d^2-70a^3bcd^3+14a^4d^4\right)x^2+20b^3d^7\left(24b^3c^3-45ab^2c^2d+30a^2bcd^2-7a^3d^3\right)x^3+15b^4d^8\left(9b^2c^2-10abcd+3a^2d^2\right)x^4+12b^5d^9(2bc-ad)x^5+2b^6d^{10}x^6-\frac{3(bc-ad)^{10}}{(a+bx)^4}+\frac{40d(-bc+ad)^9}{(a+bx)^3}-\frac{270d^2(bc-ad)^8}{(a+bx)^2}+\frac{1440d^3(-bc+ad)^7}{a+bx}+2520d^4(bc-ad)^6\log(a+bx)}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(252*b^5*c^5 - 1050*a*b^4*c^4*d + 1800*a^2*b^3*c^3*d^2 - 1575*a^3
*b^2*c^2*d^3 + 700*a^4*b*c*d^4 - 126*a^5*d^5)*x + 30*b^2*d^6*(42*b^4*c^4 -
120*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 - 70*a^3*b*c*d^3 + 14*a^4*d^4)*x^2 +
20*b^3*d^7*(24*b^3*c^3 - 45*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 7*a^3*d^3)*x^3 +
 15*b^4*d^8*(9*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 12*b^5*d^9*(2*b*c -
a*d)*x^5 + 2*b^6*d^10*x^6 - (3*(b*c - a*d)^10)/(a + b*x)^4 + (40*d*(-(b*c)
+ a*d)^9)/(a + b*x)^3 - (270*d^2*(b*c - a*d)^8)/(a + b*x)^2 + (1440*d^3*(-(
b*c) + a*d)^7)/(a + b*x) + 2520*d^4*(b*c - a*d)^6*Log[a + b*x])/(12*b^11)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1062 vs. 2(262) = 524.
time = 196.77, size = 1060, normalized size = 4.05

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^5,x]')
```

```
[Out] (1207 a ^ 10 d ^ 10 - 8250 a ^ 9 b c d ^ 9 + 23985 a ^ 8 b ^ 2 c ^ 2 d ^ 8
- 38280 a ^ 7 b ^ 3 c ^ 3 d ^ 7 + 35910 a ^ 6 b ^ 4 c ^ 4 d ^ 6 - 19404 a ^
 5 b ^ 5 c ^ 5 d ^ 5 + 5250 a ^ 4 b ^ 6 c ^ 6 d ^ 4 - 360 a ^ 3 b ^ 7 c ^ 7
 d ^ 3 - 45 a ^ 2 b ^ 8 c ^ 8 d ^ 2 - 10 a b ^ 9 c ^ 9 d - 3 b ^ 10 c ^ 10
- 20 b d x (-191 a ^ 9 d ^ 9 + 1314 a ^ 8 b c d ^ 8 - 3852 a ^ 7 b ^ 2 c ^
2 d ^ 7 + 6216 a ^ 6 b ^ 3 c ^ 3 d ^ 6 - 5922 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 327
6 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 924 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 72 a ^ 2 b ^ 7 c
 ^ 7 d ^ 2 + 9 a b ^ 8 c ^ 8 d + 2 b ^ 9 c ^ 9) + 270 b ^ 2 d ^ 2 x ^ 2 (15
a ^ 8 d ^ 8 - 104 a ^ 7 b c d ^ 7 + 308 a ^ 6 b ^ 2 c ^ 2 d ^ 6 - 504 a ^ 5
 b ^ 3 c ^ 3 d ^ 5 + 490 a ^ 4 b ^ 4 c ^ 4 d ^ 4 - 280 a ^ 3 b ^ 5 c ^ 5 d
 ^ 3 + 84 a ^ 2 b ^ 6 c ^ 6 d ^ 2 - 8 a b ^ 7 c ^ 7 d - b ^ 8 c ^ 8) + 1440
b ^ 3 d ^ 3 x ^ 3 (a ^ 7 d ^ 7 - 7 a ^ 6 b c d ^ 6 + 21 a ^ 5 b ^ 2 c ^ 2 d
 ^ 5 - 35 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 35 a ^ 3 b ^ 4 c ^ 4 d ^ 3 - 21 a ^ 2 b
 ^ 5 c ^ 5 d ^ 2 + 7 a b ^ 6 c ^ 6 d - b ^ 7 c ^ 7) + 2520 d ^ 4 Log[a + b
x] (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^
 4) (a d - b c) ^ 6 - 12 b d ^ 5 x (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^
 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4) (126 a ^ 5 d ^ 5 - 700 a ^ 4 b c d ^ 4
+ 1575 a ^ 3 b ^ 2 c ^ 2 d ^ 3 - 1800 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 1050 a b ^
4 c ^ 4 d - 252 b ^ 5 c ^ 5) + 30 b ^ 2 d ^ 6 x ^ 2 (a ^ 4 + 4 a ^ 3 b x +
6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4) (14 a ^ 4 d ^ 4 - 70 a
```

^ 3 b c d ^ 3 + 135 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 120 a b ^ 3 c ^ 3 d + 42 b ^
4 c ^ 4) - 20 b ^ 3 d ^ 7 x ^ 3 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2
+ 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4) (7 a ^ 3 d ^ 3 - 30 a ^ 2 b c d ^ 2 + 45
a b ^ 2 c ^ 2 d - 24 b ^ 3 c ^ 3) + 15 b ^ 4 d ^ 8 x ^ 4 (a ^ 4 + 4 a ^ 3 b
x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4) (3 a ^ 2 d ^ 2 -
10 a b c d + 9 b ^ 2 c ^ 2) - 12 b ^ 5 d ^ 9 x ^ 5 (a ^ 4 + 4 a ^ 3 b x + 6
a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3 + b ^ 4 x ^ 4) (a d - 2 b c) + 2 b ^ 6
d ^ 10 x ^ 6 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4 a b ^ 3 x ^ 3
+ b ^ 4 x ^ 4)) / (12 b ^ 11 (a ^ 4 + 4 a ^ 3 b x + 6 a ^ 2 b ^ 2 x ^ 2 + 4
a b ^ 3 x ^ 3 + b ^ 4 x ^ 4))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(252) = 504$.

time = 0.15, size = 881, normalized size = 3.36

method	result
norman	$\frac{5250a^{10}d^{10}-31500a^9bc\,d^9+78750a^8b^2c^2d^8-105000a^7b^3c^3d^7+78750a^6b^4c^4d^6-31500a^5b^5c^5d^5+5250a^4b^6c^6d^4-360a^3b^7c^7d^3-45a^2b^8c^8d^2-10a}{12b^{11}}$
default	$-\frac{d^5\left(-\frac{1}{6}d^5x^6b^5+a\,b^4d^5x^5-2b^5c\,d^4x^5-\frac{15}{4}a^2b^3d^5x^4+\frac{25}{2}a\,b^4c\,d^4x^4-\frac{45}{4}b^5c^2d^3x^4+\frac{35}{3}a^3b^2d^5x^3-50a^2b^3c\,d^4x^3+75a\,b^4c^2d^3x^3-40b^5c\right.}{}$
risch	$\frac{d^{10}x^6}{6b^5}-\frac{25d^9ac\,x^4}{2b^6}+\frac{\left(120a^7b^2d^{10}-840a^6b^3c\,d^9+2520a^5b^4c^2d^8-4200a^4b^5c^3d^7+4200a^3b^6c^4d^6-2520a^2b^7c^5d^5+840a\,b^8c^6d^4-12\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] -d^5/b^10*(-1/6*d^5*x^6*b^5+a*b^4*d^5*x^5-2*b^5*c*d^4*x^5-15/4*a^2*b^3*d^5*
x^4+25/2*a*b^4*c*d^4*x^4-45/4*b^5*c^2*d^3*x^4+35/3*a^3*b^2*d^5*x^3-50*a^2*b
^3*c*d^4*x^3+75*a*b^4*c^2*d^3*x^3-40*b^5*c^3*d^2*x^3-35*a^4*b*d^5*x^2+175*a
^3*b^2*c*d^4*x^2-675/2*a^2*b^3*c^2*d^3*x^2+300*a*b^4*c^3*d^2*x^2-105*b^5*c^
4*d*x^2+126*a^5*d^5*x-700*a^4*b*c*d^4*x+1575*a^3*b^2*c^2*d^3*x-1800*a^2*b^3
*c^3*d^2*x+1050*a*b^4*c^4*d*x-252*b^5*c^5*x)+120/b^11*d^3*(a^7*d^7-7*a^6*b*
c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c
^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)-1/4*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*
b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210
*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10
*c^10)/b^11/(b*x+a)^4-45/2/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d
^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*
d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^2+210/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15
*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^
6)*ln(b*x+a)+10/3/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b
^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^
2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(252) = 504$.

time = 0.29, size = 903, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^10*c^10 + 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*
d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6
+ 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 8250*a^9*b*c*d^9 - 1207*a
^10*d^10 + 1440*(b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a
^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9
- a^7*b^3*d^10)*x^3 + 270*(b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 - 84*a^2*b^8*c^6*
d^4 + 280*a^3*b^7*c^5*d^5 - 490*a^4*b^6*c^4*d^6 + 504*a^5*b^5*c^3*d^7 - 308
*a^6*b^4*c^2*d^8 + 104*a^7*b^3*c*d^9 - 15*a^8*b^2*d^10)*x^2 + 20*(2*b^10*c^
9*d + 9*a*b^9*c^8*d^2 + 72*a^2*b^8*c^7*d^3 - 924*a^3*b^7*c^6*d^4 + 3276*a^4
*b^6*c^5*d^5 - 5922*a^5*b^5*c^4*d^6 + 6216*a^6*b^4*c^3*d^7 - 3852*a^7*b^3*c
^2*d^8 + 1314*a^8*b^2*c*d^9 - 191*a^9*b*d^10)*x)/(b^15*x^4 + 4*a*b^14*x^3 +
 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11) + 1/12*(2*b^5*d^10*x^6 + 12*(2*b
^5*c*d^9 - a*b^4*d^10)*x^5 + 15*(9*b^5*c^2*d^8 - 10*a*b^4*c*d^9 + 3*a^2*b^3
*d^10)*x^4 + 20*(24*b^5*c^3*d^7 - 45*a*b^4*c^2*d^8 + 30*a^2*b^3*c*d^9 - 7*a
^3*b^2*d^10)*x^3 + 30*(42*b^5*c^4*d^6 - 120*a*b^4*c^3*d^7 + 135*a^2*b^3*c^2
*d^8 - 70*a^3*b^2*c*d^9 + 14*a^4*b*d^10)*x^2 + 12*(252*b^5*c^5*d^5 - 1050*a
*b^4*c^4*d^6 + 1800*a^2*b^3*c^3*d^7 - 1575*a^3*b^2*c^2*d^8 + 700*a^4*b*c*d^
9 - 126*a^5*d^10)*x)/b^10 + 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4
*c^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^
10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(252) = 504$.

time = 0.30, size = 1365, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*d^10*x^10 - 3*b^10*c^10 - 10*a*b^9*c^9*d - 45*a^2*b^8*c^8*d^2
- 360*a^3*b^7*c^7*d^3 + 5250*a^4*b^6*c^6*d^4 - 19404*a^5*b^5*c^5*d^5 + 3591
0*a^6*b^4*c^4*d^6 - 38280*a^7*b^3*c^3*d^7 + 23985*a^8*b^2*c^2*d^8 - 8250*a^
9*b*c*d^9 + 1207*a^10*d^10 + 4*(6*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(15*b^10
*c^2*d^8 - 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(20*b^10*c^3*d^7 - 15*a*b
^9*c^2*d^8 + 6*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(15*b^10*c^4*d^6 - 20
*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 - 6*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 504*(6*b^10*c^5*d^5 - 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 15*a^3*b^7*c
^2*d^8 + 6*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + (12096*a*b^9*c^5*d^5 - 42840
```

```
*a^2*b^8*c^4*d^6 + 66720*a^3*b^7*c^3*d^7 - 54765*a^4*b^6*c^2*d^8 + 23250*a^
5*b^5*c*d^9 - 4043*a^6*b^4*d^10)*x^4 - 4*(360*b^10*c^7*d^3 - 2520*a*b^9*c^6
*d^4 + 3024*a^2*b^8*c^5*d^5 + 5040*a^3*b^7*c^4*d^6 - 16320*a^4*b^6*c^3*d^7
+ 16965*a^5*b^5*c^2*d^8 - 8130*a^6*b^4*c*d^9 + 1523*a^7*b^3*d^10)*x^3 - 6*(
45*b^10*c^8*d^2 + 360*a*b^9*c^7*d^3 - 3780*a^2*b^8*c^6*d^4 + 10584*a^3*b^7*
c^5*d^5 - 13860*a^4*b^6*c^4*d^6 + 8880*a^5*b^5*c^3*d^7 - 1935*a^6*b^4*c^2*d
^8 - 570*a^7*b^3*c*d^9 + 263*a^8*b^2*d^10)*x^2 - 4*(10*b^10*c^9*d + 45*a*b^
9*c^8*d^2 + 360*a^2*b^8*c^7*d^3 - 4620*a^3*b^7*c^6*d^4 + 15624*a^4*b^6*c^5*
d^5 - 26460*a^5*b^5*c^4*d^6 + 25680*a^6*b^4*c^3*d^7 - 14535*a^7*b^3*c^2*d^8
 + 4470*a^8*b^2*c*d^9 - 577*a^9*b*d^10)*x + 2520*(a^4*b^6*c^6*d^4 - 6*a^5*b
^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 - 20*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 -
 6*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c
^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^
4*d^10)*x^4 + 4*(a*b^9*c^6*d^4 - 6*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 - 2
0*a^4*b^6*c^3*d^7 + 15*a^5*b^5*c^2*d^8 - 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^
3 + 6*(a^2*b^8*c^6*d^4 - 6*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 - 20*a^5*b^
5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 - 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 4*(a
^3*b^7*c^6*d^4 - 6*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 - 20*a^6*b^4*c^3*d^
7 + 15*a^7*b^3*c^2*d^8 - 6*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^
15*x^4 + 4*a*b^14*x^3 + 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 895 vs. 2(252) = 504.
time = 0.00, size = 945, normalized size = 3.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x)
```

```
[Out] 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c^4*d^6 - 20*a^3*b^3*c^3*d^
7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10)*log(abs(b*x + a))/b^11 -
 1/12*(3*b^10*c^10 + 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*
d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6
+ 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 8250*a^9*b*c*d^9 - 1207*a
^10*d^10 + 1440*(b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a
```

```
^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9
- a^7*b^3*d^10)*x^3 + 270*(b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 - 84*a^2*b^8*c^6*
d^4 + 280*a^3*b^7*c^5*d^5 - 490*a^4*b^6*c^4*d^6 + 504*a^5*b^5*c^3*d^7 - 308
*a^6*b^4*c^2*d^8 + 104*a^7*b^3*c*d^9 - 15*a^8*b^2*d^10)*x^2 + 20*(2*b^10*c^
9*d + 9*a*b^9*c^8*d^2 + 72*a^2*b^8*c^7*d^3 - 924*a^3*b^7*c^6*d^4 + 3276*a^4
*b^6*c^5*d^5 - 5922*a^5*b^5*c^4*d^6 + 6216*a^6*b^4*c^3*d^7 - 3852*a^7*b^3*c
^2*d^8 + 1314*a^8*b^2*c*d^9 - 191*a^9*b*d^10)*x)/((b*x + a)^4*b^11) + 1/12*
(2*b^25*d^10*x^6 + 24*b^25*c*d^9*x^5 - 12*a*b^24*d^10*x^5 + 135*b^25*c^2*d^
8*x^4 - 150*a*b^24*c*d^9*x^4 + 45*a^2*b^23*d^10*x^4 + 480*b^25*c^3*d^7*x^3
- 900*a*b^24*c^2*d^8*x^3 + 600*a^2*b^23*c*d^9*x^3 - 140*a^3*b^22*d^10*x^3 +
 1260*b^25*c^4*d^6*x^2 - 3600*a*b^24*c^3*d^7*x^2 + 4050*a^2*b^23*c^2*d^8*x^
2 - 2100*a^3*b^22*c*d^9*x^2 + 420*a^4*b^21*d^10*x^2 + 3024*b^25*c^5*d^5*x -
 12600*a*b^24*c^4*d^6*x + 21600*a^2*b^23*c^3*d^7*x - 18900*a^3*b^22*c^2*d^8
*x + 8400*a^4*b^21*c*d^9*x - 1512*a^5*b^20*d^10*x)/b^30
```

Mupad [B]

time = 0.38, size = 1494, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^5,x)
```

```
[Out] x^2*((5*a*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/
b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^
2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/(2*b) - (5*a^4*d^10)/(2*b^9) + (
105*c^4*d^6)/b^5 + (5*a^3*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (5*a^2*(
(5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8
)/b^5))/b^2) - x^5*((a*d^10)/b^6 - (2*c*d^9)/b^5) - x^3*((5*a*((5*a*((5*a*d
^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/(3*b
) + (10*a^3*d^10)/(3*b^8) - (40*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 - (1
0*c*d^9)/b^5))/(3*b^2)) + x^4*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/(4*b
) - (5*a^2*d^10)/(2*b^7) + (45*c^2*d^8)/(4*b^5)) - x*((5*a*((5*a*((5*a*((5*
a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b
^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 -
(10*c*d^9)/b^5))/b^2))/b - (5*a^4*d^10)/b^9 + (210*c^4*d^6)/b^5 + (10*a^3*(
(5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (10*a^2*((5*a*((5*a*d^10)/b^6 - (10
*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^2))/b + (a^5*d^1
0)/b^10 - (252*c^5*d^5)/b^5 - (5*a^4*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^4
 - (10*a^2*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)
/b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a
^2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/b^2 + (10*a^3*((5*a*((5*a*d^10)
/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^3) - (
(3*b^10*c^10 - 1207*a^10*d^10 + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 -
5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6 + 3828
0*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 10*a*b^9*c^9*d + 8250*a^9*b*c*d
```

^9)/(12*b) + x*((10*b^9*c^9*d)/3 - (955*a^9*d^10)/3 + 15*a*b^8*c^8*d^2 + 12
0*a^2*b^7*c^7*d^3 - 1540*a^3*b^6*c^6*d^4 + 5460*a^4*b^5*c^5*d^5 - 9870*a^5*
b^4*c^4*d^6 + 10360*a^6*b^3*c^3*d^7 - 6420*a^7*b^2*c^2*d^8 + 2190*a^8*b*c*d
^9) - x^3*(120*a^7*b^2*d^10 - 120*b^9*c^7*d^3 + 840*a*b^8*c^6*d^4 - 840*a^6
*b^3*c*d^9 - 2520*a^2*b^7*c^5*d^5 + 4200*a^3*b^6*c^4*d^6 - 4200*a^4*b^5*c^3
*d^7 + 2520*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)/2 - (675*a^8*b*d^10)/2
 + 180*a*b^8*c^7*d^3 + 2340*a^7*b^2*c*d^9 - 1890*a^2*b^7*c^6*d^4 + 6300*a^3
*b^6*c^5*d^5 - 11025*a^4*b^5*c^4*d^6 + 11340*a^5*b^4*c^3*d^7 - 6930*a^6*b^3
*c^2*d^8))/(a^4*b^10 + b^14*x^4 + 4*a^3*b^11*x + 4*a*b^13*x^3 + 6*a^2*b^12*
x^2) + (log(a + b*x)*(210*a^6*d^10 + 210*b^6*c^6*d^4 - 1260*a*b^5*c^5*d^5 +
 3150*a^2*b^4*c^4*d^6 - 4200*a^3*b^3*c^3*d^7 + 3150*a^4*b^2*c^2*d^8 - 1260*
a^5*b*c*d^9))/b^11 + (d^10*x^6)/(6*b^5)

3.1317 $\int \frac{(c+dx)^{10}}{(a+bx)^6}\,dx$

Optimal. Leaf size=260

$$\frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}+\frac{60d^7(}{}$$

```
[Out] 210*d^6*(-a*d+b*c)^4*x/b^10-1/5*(-a*d+b*c)^10/b^11/(b*x+a)^5-5/2*d*(-a*d+b*
c)^9/b^11/(b*x+a)^4-15*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^3-60*d^3*(-a*d+b*c)^7/
b^11/(b*x+a)^2-210*d^4*(-a*d+b*c)^6/b^11/(b*x+a)+60*d^7*(-a*d+b*c)^3*(b*x+a
)^2/b^11+15*d^8*(-a*d+b*c)^2*(b*x+a)^3/b^11+5/2*d^9*(-a*d+b*c)*(b*x+a)^4/b^
11+1/5*d^10*(b*x+a)^5/b^11+252*d^5*(-a*d+b*c)^5*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.29, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{252d^5(bc-ad)^5\log(a+bx)}{b^{11}}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}+\frac{d^{10}(a+bx)^5}{5b^{11}}+\frac{210d^6x(bc-ad)^4}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (210*d^6*(b*c - a*d)^4*x)/b^10 - (b*c - a*d)^10/(5*b^11*(a + b*x)^5) - (5*d
*(b*c - a*d)^9)/(2*b^11*(a + b*x)^4) - (15*d^2*(b*c - a*d)^8)/(b^11*(a + b*
x)^3) - (60*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^2) - (210*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)) + (60*d^7*(b*c - a*d)^3*(a + b*x)^2)/b^11 + (15*d^8*(b*c
- a*d)^2*(a + b*x)^3)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^4)/(2*b^11) + (d^
10*(a + b*x)^5)/(5*b^11) + (252*d^5*(b*c - a*d)^5*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^6}\,dx &= \int \left(\frac{210d^6(bc-ad)^4}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^6}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^5}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^4}+\frac{120d^3(bc-ad}{b^{10}(a+bx)^3}\right.\\ &= \frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\end{aligned}$$

Mathematica [A]
time = 0.13, size = 305, normalized size = 1.17

$$\frac{10bd^6\left(210b^4c^4-720ab^3c^3d+945a^2b^2c^2d^2-560a^3bcd^3+126a^4d^4\right)x+10b^2d^7\left(60b^3c^3-135ab^2c^2d+105a^2bcd^2-28a^3d^3\right)x^2+10b^3d^8\left(15b^2c^2-20abcd+7a^2d^2\right)x^3+5b^4d^9(5bc-3ad)x^4+2b^5d^{10}x^5-\frac{2(bc-ad)^{10}}{(a+bx)^5}+\frac{25d(-bc+ad)^9}{(a+bx)^4}-\frac{150d^2(bc-ad)^8}{(a+bx)^3}+\frac{600d^3(-bc+ad)^7}{(a+bx)^2}-\frac{2100d^4(bc-ad)^6}{a+bx}+2520d^5(bc-ad)^5\log(a+bx)}{10b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (10*b*d^6*(210*b^4*c^4 - 720*a*b^3*c^3*d + 945*a^2*b^2*c^2*d^2 - 560*a^3*b*
c*d^3 + 126*a^4*d^4)*x + 10*b^2*d^7*(60*b^3*c^3 - 135*a*b^2*c^2*d + 105*a^2
*b*c*d^2 - 28*a^3*d^3)*x^2 + 10*b^3*d^8*(15*b^2*c^2 - 20*a*b*c*d + 7*a^2*d^
2)*x^3 + 5*b^4*d^9*(5*b*c - 3*a*d)*x^4 + 2*b^5*d^10*x^5 - (2*(b*c - a*d)^10
)/(a + b*x)^5 + (25*d*(-(b*c) + a*d)^9)/(a + b*x)^4 - (150*d^2*(b*c - a*d)^
8)/(a + b*x)^3 + (600*d^3*(-(b*c) + a*d)^7)/(a + b*x)^2 - (2100*d^4*(b*c -
a*d)^6)/(a + b*x) + 2520*d^5*(b*c - a*d)^5*Log[a + b*x])/(10*b^11)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^6,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(252) = 504$.
time = 0.15, size = 870, normalized size = 3.35

method	result
norman	$-\frac{\frac{5754a^{10}d^{10}-28770a^9bc\,d^9+57540a^8b^2c^2d^8-57540a^7b^3c^3d^7+28770a^6b^4c^4d^6-5754a^5b^5c^5d^5+420a^4b^6c^6d^4+60a^3b^7c^7d^3+15a^2b^8c^8d^2+5a\,b^9c^9}{10b^{11}}}{}$
default	$\frac{d^6\left(\frac{1}{5}d^4x^5b^4-\frac{3}{2}a\,b^3d^4x^4+\frac{5}{2}b^4c\,d^3x^4+7a^2b^2d^4x^3-20a\,b^3c\,d^3x^3+15b^4c^2d^2x^3-28a^3b\,d^4x^2+105a^2b^2c\,d^3x^2-135a\,b^3c^2d^2x^2+60b^4c^3d\right.}{b^{10}}$
risch	$\frac{d^{10}x^5}{5b^6}-\frac{3d^{10}a\,x^4}{2b^7}+\frac{5d^9c\,x^4}{2b^6}+\frac{7d^{10}a^2x^3}{b^8}-\frac{20d^9ac\,x^3}{b^7}+\frac{15d^8c^2x^3}{b^6}-\frac{28d^{10}a^3x^2}{b^9}+\frac{105d^9a^2c\,x^2}{b^8}-\frac{135d^8a\,c^2x^2}{b^7}+60d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] d^6/b^10*(1/5*d^4*x^5*b^4-3/2*a*b^3*d^4*x^4+5/2*b^4*c*d^3*x^4+7*a^2*b^2*d^4
*x^3-20*a*b^3*c*d^3*x^3+15*b^4*c^2*d^2*x^3-28*a^3*b*d^4*x^2+105*a^2*b^2*c*d
^3*x^2-135*a*b^3*c^2*d^2*x^2+60*b^4*c^3*d*x^2+126*a^4*d^4*x-560*a^3*b*c*d^3
*x+945*a^2*b^2*c^2*d^2*x-720*a*b^3*c^3*d*x+210*b^4*c^4*x)-210/b^11*d^4*(a^6
```

```
*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2
-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)+5/2/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^
2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3
*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^4-1/5*(a^10*
d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*
d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*
c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^5+60/b^11*d^3*(a^7*d^7-7*a^6
*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^
5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^2-252/b^11*d^5*(a^5*d^5-5*a^4*b*c*
d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)*ln(b*x+a)-
15/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70
*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^
8)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(252) = 504$.

time = 0.33, size = 912, normalized size = 3.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/10*(2*b^10*c^10 + 5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^
3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27
540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*
d^10 + 2100*(b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b
^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600
*(b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6
 - 245*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^
3*d^10)*x^3 + 150*(b^10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 30
8*a^3*b^7*c^5*d^5 + 910*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b
^4*c^2*d^8 - 428*a^7*b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*
a*b^9*c^8*d^2 + 12*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*
d^5 + 3234*a^5*b^5*c^4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 -
1599*a^8*b^2*c*d^9 + 275*a^9*b*d^10)*x)/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b
^14*x^3 + 10*a^3*b^13*x^2 + 5*a^4*b^12*x + a^5*b^11) + 1/10*(2*b^4*d^10*x^5
 + 5*(5*b^4*c*d^9 - 3*a*b^3*d^10)*x^4 + 10*(15*b^4*c^2*d^8 - 20*a*b^3*c*d^9
 + 7*a^2*b^2*d^10)*x^3 + 10*(60*b^4*c^3*d^7 - 135*a*b^3*c^2*d^8 + 105*a^2*b
^2*c*d^9 - 28*a^3*b*d^10)*x^2 + 10*(210*b^4*c^4*d^6 - 720*a*b^3*c^3*d^7 + 9
45*a^2*b^2*c^2*d^8 - 560*a^3*b*c*d^9 + 126*a^4*d^10)*x)/b^10 + 252*(b^5*c^5
*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^8 + 5*a^4*b*
c*d^9 - a^5*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(252) = 504$.

time = 0.30, size = 1395, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="fricas")

[Out] 1/10*(2*b^10*d^10*x^10 - 2*b^10*c^10 - 5*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2 - 60*a^3*b^7*c^7*d^3 - 420*a^4*b^6*c^6*d^4 + 5754*a^5*b^5*c^5*d^5 - 18270*a^6*b^4*c^4*d^6 + 27540*a^7*b^3*c^3*d^7 - 22290*a^8*b^2*c^2*d^8 + 9395*a^9*b*c*d^9 - 1627*a^10*d^10 + 5*(5*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*(10*b^10*c^2*d^8 - 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(10*b^10*c^3*d^7 - 10*a*b^9*c^2*d^8 + 5*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(5*b^10*c^4*d^6 - 10*a*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 - 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + (10500*a*b^9*c^4*d^6 - 30000*a^2*b^8*c^3*d^7 + 35250*a^3*b^7*c^2*d^8 - 19375*a^4*b^6*c*d^9 + 4127*a^5*b^5*d^10)*x^5 - 5*(420*b^10*c^6*d^4 - 2520*a*b^9*c^5*d^5 + 2100*a^2*b^8*c^4*d^6 + 4800*a^3*b^7*c^3*d^7 - 10050*a^4*b^6*c^2*d^8 + 6775*a^5*b^5*c*d^9 - 1607*a^6*b^4*d^10)*x^4 - 10*(60*b^10*c^7*d^3 + 420*a*b^9*c^6*d^4 - 3780*a^2*b^8*c^5*d^5 + 8400*a^3*b^7*c^4*d^6 - 7800*a^4*b^6*c^3*d^7 + 2550*a^5*b^5*c^2*d^8 + 475*a^6*b^4*c*d^9 - 347*a^7*b^3*d^10)*x^3 - 10*(15*b^10*c^8*d^2 + 60*a*b^9*c^7*d^3 + 420*a^2*b^8*c^6*d^4 - 4620*a^3*b^7*c^5*d^5 + 12600*a^4*b^6*c^4*d^6 - 16200*a^5*b^5*c^3*d^7 + 10950*a^6*b^4*c^2*d^8 - 3725*a^7*b^3*c*d^9 + 493*a^8*b^2*d^10)*x^2 - 5*(5*b^10*c^9*d + 15*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 5250*a^4*b^6*c^5*d^5 + 15750*a^5*b^5*c^4*d^6 - 22500*a^6*b^4*c^3*d^7 + 17250*a^7*b^3*c^2*d^8 - 6875*a^8*b^2*c*d^9 + 1123*a^9*b*d^10)*x + 2520*(a^5*b^5*c^5*d^5 - 5*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 - 10*a^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 - a^10*d^10 + (b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 5*(a*b^9*c^5*d^5 - 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 - 10*a^4*b^6*c^2*d^8 + 5*a^5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 10*(a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 - 10*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 10*(a^3*b^7*c^5*d^5 - 5*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 - 10*a^6*b^4*c^2*d^8 + 5*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 5*(a^4*b^6*c^5*d^5 - 5*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 - 10*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b^14*x^3 + 10*a^3*b^13*x^2 + 5*a^4*b^12*x + a^5*b^11)

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**10/(b*x+a)**6,x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(252) = 504.
time = 0.00, size = 934, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x)
```

```
[Out] 252*(b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^
8 + 5*a^4*b*c*d^9 - a^5*d^10)*log(abs(b*x + a))/b^11 - 1/10*(2*b^10*c^10 +
5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d
^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 +
 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*d^10 + 2100*(b^10*c^6
*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b
^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600*(b^10*c^7*d^3 + 7*a*
b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6 - 245*a^4*b^6*c^3*d^
7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^3*d^10)*x^3 + 150*(b^
10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 308*a^3*b^7*c^5*d^5 + 9
10*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b^4*c^2*d^8 - 428*a^7*
b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*a*b^9*c^8*d^2 + 12*a^
2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*d^5 + 3234*a^5*b^5*c^
4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 - 1599*a^8*b^2*c*d^9 +
275*a^9*b*d^10)*x)/((b*x + a)^5*b^11) + 1/10*(2*b^24*d^10*x^5 + 25*b^24*c*d
^9*x^4 - 15*a*b^23*d^10*x^4 + 150*b^24*c^2*d^8*x^3 - 200*a*b^23*c*d^9*x^3 +
 70*a^2*b^22*d^10*x^3 + 600*b^24*c^3*d^7*x^2 - 1350*a*b^23*c^2*d^8*x^2 + 10
50*a^2*b^22*c*d^9*x^2 - 280*a^3*b^21*d^10*x^2 + 2100*b^24*c^4*d^6*x - 7200*
a*b^23*c^3*d^7*x + 9450*a^2*b^22*c^2*d^8*x - 5600*a^3*b^21*c*d^9*x + 1260*a
^4*b^20*d^10*x)/b^30
```

Mupad [**B**]
time = 0.40, size = 1141, normalized size = 4.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^6,x)
```

```
[Out] x^3*((2*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (5*a^2*d^10)/b^8 + (15*c^2
*d^8)/b^6) - x^2*((3*a*((6*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2
*d^10)/b^8 + (45*c^2*d^8)/b^6))/b + (10*a^3*d^10)/b^9 - (60*c^3*d^7)/b^6 -
(15*a^2*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/(2*b^2)) - x^4*((3*a*d^10)/(2*b^
7) - (5*c*d^9)/(2*b^6)) - (x^4*(210*a^6*b^3*d^10 + 210*b^9*c^6*d^4 - 1260*a
*b^8*c^5*d^5 - 1260*a^5*b^4*c*d^9 + 3150*a^2*b^7*c^4*d^6 - 4200*a^3*b^6*c^3
*d^7 + 3150*a^4*b^5*c^2*d^8) + (1627*a^10*d^10 + 2*b^10*c^10 + 15*a^2*b^8*c
```

^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 + 5*a *b^9*c^9*d - 9395*a^9*b*c*d^9)/(10*b) + x*((1375*a^9*d^10)/2 + (5*b^9*c^9*d)/2 + (15*a*b^8*c^8*d^2)/2 + 30*a^2*b^7*c^7*d^3 + 210*a^3*b^6*c^6*d^4 - 262 5*a^4*b^5*c^5*d^5 + 8085*a^5*b^4*c^4*d^6 - 11970*a^6*b^3*c^3*d^7 + 9570*a^7 *b^2*c^2*d^8 - (7995*a^8*b*c*d^9)/2) + x^3*(780*a^7*b^2*d^10 + 60*b^9*c^7*d ^3 + 420*a*b^8*c^6*d^4 - 4620*a^6*b^3*c*d^9 - 3780*a^2*b^7*c^5*d^5 + 10500* a^3*b^6*c^4*d^6 - 14700*a^4*b^5*c^3*d^7 + 11340*a^5*b^4*c^2*d^8) + x^2*(109 5*a^8*b*d^10 + 15*b^9*c^8*d^2 + 60*a*b^8*c^7*d^3 - 6420*a^7*b^2*c*d^9 + 420 *a^2*b^7*c^6*d^4 - 4620*a^3*b^6*c^5*d^5 + 13650*a^4*b^5*c^4*d^6 - 19740*a^5 *b^4*c^3*d^7 + 15540*a^6*b^3*c^2*d^8))/(a^5*b^10 + b^15*x^5 + 5*a^4*b^11*x + 5*a*b^14*x^4 + 10*a^3*b^12*x^2 + 10*a^2*b^13*x^3) + x*((6*a*((6*a*((6*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6))/b + (20*a^3*d^10)/b^9 - (120*c^3*d^7)/b^6 - (15*a^2*((6*a*d^10)/b^7 - (10 *c*d^9)/b^6))/b^2))/b - (15*a^4*d^10)/b^10 + (210*c^4*d^6)/b^6 + (20*a^3*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b^3 - (15*a^2*((6*a*((6*a*d^10)/b^7 - (10* c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6))/b^2) + (d^10*x^5)/(5*b^6) - (log(a + b*x)*(252*a^5*d^10 - 252*b^5*c^5*d^5 + 1260*a*b^4*c^4*d^6 - 2520*a^2*b^3*c^3*d^7 + 2520*a^3*b^2*c^2*d^8 - 1260*a^4*b*c*d^9))/b^11

3.1318 $\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx$

Optimal. Leaf size=262

$$\frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{252d^5}{b^{11}(}$$

```
[Out] 120*d^7*(-a*d+b*c)^3*x/b^10-1/6*(-a*d+b*c)^10/b^11/(b*x+a)^6-2*d*(-a*d+b*c)
^9/b^11/(b*x+a)^5-45/4*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^4-40*d^3*(-a*d+b*c)^7/
b^11/(b*x+a)^3-105*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^2-252*d^5*(-a*d+b*c)^5/b^1
1/(b*x+a)+45/2*d^8*(-a*d+b*c)^2*(b*x+a)^2/b^11+10/3*d^9*(-a*d+b*c)*(b*x+a)^
3/b^11+1/4*d^10*(b*x+a)^4/b^11+210*d^6*(-a*d+b*c)^4*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.27, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{210d^6(bc-ad)^4\log(a+bx)}{b^{11}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}+\frac{d^{10}(a+bx)^4}{4b^{11}}+\frac{120d^7x(bc-ad)^3}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (120*d^7*(b*c - a*d)^3*x)/b^10 - (b*c - a*d)^10/(6*b^11*(a + b*x)^6) - (2*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^5) - (45*d^2*(b*c - a*d)^8)/(4*b^11*(a + b*
x)^4) - (40*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^3) - (105*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)^2) - (252*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)) + (45*d^8*(b
*c - a*d)^2*(a + b*x)^2)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^3)/(3*b^1
1) + (d^10*(a + b*x)^4)/(4*b^11) + (210*d^6*(b*c - a*d)^4*Log[a + b*x])/b^1
1
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx &= \int \left(\frac{120d^7(bc-ad)^3}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^7}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^6}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^5}+\frac{120d^3(bc-ad}{b^{10}(a+bx)^4}\right.\\ &= \frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\end{aligned}$$

Mathematica [A]
time = 0.13, size = 265, normalized size = 1.01

$$\frac{12bd^7\left(120b^3c^3-315ab^2c^2d+280a^2bcd^2-84a^3d^3\right)x+6b^2d^8\left(45b^2c^2-70abcd+28a^2d^2\right)x^2+4b^3d^9(10bc-7ad)x^3+3b^4d^{10}x^4-\frac{2(bc-ad)^{10}}{(a+bx)^6}+\frac{24d(-bc+ad)^9}{(a+bx)^5}-\frac{135d^2(bc-ad)^8}{(a+bx)^4}+\frac{480d^3(-bc+ad)^7}{(a+bx)^3}-\frac{1260d^4(bc-ad)^6}{(a+bx)^2}+\frac{3024d^5(-bc+ad)^5}{a+bx}+2520d^6(bc-ad)^4\log(a+bx)}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (12*b*d^7*(120*b^3*c^3 - 315*a*b^2*c^2*d + 280*a^2*b*c*d^2 - 84*a^3*d^3)*x
+ 6*b^2*d^8*(45*b^2*c^2 - 70*a*b*c*d + 28*a^2*d^2)*x^2 + 4*b^3*d^9*(10*b*c
- 7*a*d)*x^3 + 3*b^4*d^10*x^4 - (2*(b*c - a*d)^10)/(a + b*x)^6 + (24*d*(-(b
*c) + a*d)^9)/(a + b*x)^5 - (135*d^2*(b*c - a*d)^8)/(a + b*x)^4 + (480*d^3*
(-(b*c) + a*d)^7)/(a + b*x)^3 - (1260*d^4*(b*c - a*d)^6)/(a + b*x)^2 + (302
4*d^5*(-(b*c) + a*d)^5)/(a + b*x) + 2520*d^6*(b*c - a*d)^4*Log[a + b*x])/(1
2*b^11)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^7,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(252) = 504$.
time = 0.15, size = 862, normalized size = 3.29

method	result
norman	$\frac{6174a^{10}d^{10}-24696a^9bc\,d^9+37044a^8b^2c^2d^8-24696a^7b^3c^3d^7+6174a^6b^4c^4d^6-504a^5b^5c^5d^5-84a^4b^6c^6d^4-24a^3b^7c^7d^3-9a^2b^8c^8d^2-4a\,b^9c^9d-2b^{1}}{12b^{11}}$
default	$-\frac{d^7\left(-\frac{1}{4}d^3x^4b^3+\frac{7}{3}a\,b^2d^3x^3-\frac{10}{3}b^3c\,d^2x^3-14a^2b\,d^3x^2+35a\,b^2c\,d^2x^2-\frac{45}{2}b^3c^2d\,x^2+84a^3d^3x-280a^2bc\,d^2x+315a\,b^2c^2dx-120b^3c^3x\right)}{b^{10}}$
risch	$\frac{d^{10}x^4}{4b^7}-\frac{7d^{10}a\,x^3}{3b^8}+\frac{10d^9c\,x^3}{3b^7}+\frac{14d^{10}a^2x^2}{b^9}-\frac{35d^9ac\,x^2}{b^8}+\frac{45d^8c^2x^2}{2b^7}-\frac{84d^{10}a^3x}{b^{10}}+\frac{280d^9a^2cx}{b^9}-\frac{315d^8a\,c^2x}{b^8}+\frac{120d^7}{b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -d^7/b^10*(-1/4*d^3*x^4*b^3+7/3*a*b^2*d^3*x^3-10/3*b^3*c*d^2*x^3-14*a^2*b*d
^3*x^2+35*a*b^2*c*d^2*x^2-45/2*b^3*c^2*d*x^2+84*a^3*d^3*x-280*a^2*b*c*d^2*x
+315*a*b^2*c^2*d*x-120*b^3*c^3*x)+252/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^
3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)-45/4/b^11*d
```

```
^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*
c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a
)^4+2/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+1
26*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^
2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^5-105/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a
^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
/(b*x+a)^2-1/6*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3
*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^
7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^6+210/b
^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(
b*x+a)+40/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3
*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(252) = 504$.
time = 0.37, size = 925, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3
 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6174*a^6*b^4*c^4*d^6 + 16056*
a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 10036*a^9*b*c*d^9 - 2131*a^10*d^1
0 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*
c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 1260*(b^10*c^6*d^4 + 6*a*b^
9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^7*c^3*d^7 - 105*a^4*b^6*c^2*d^8
+ 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 + 240*(2*b^10*c^7*d^3 + 7*a*b^9*c
^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c^4*d^6 + 910*a^4*b^6*c^3*d^7 - 9
87*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 107*a^7*b^3*d^10)*x^3 + 45*(3*b^10
*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c^5*d^5 - 175
0*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 - 4788*a^6*b^4*c^2*d^8 + 2552*a^7*
b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 24*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c
^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9
 - 1879*a^9*b*d^10)*x)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*
b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 1/12*(3*b^3*d^10*x^
4 + 4*(10*b^3*c*d^9 - 7*a*b^2*d^10)*x^3 + 6*(45*b^3*c^2*d^8 - 70*a*b^2*c*d^
9 + 28*a^2*b*d^10)*x^2 + 12*(120*b^3*c^3*d^7 - 315*a*b^2*c^2*d^8 + 280*a^2*
b*c*d^9 - 84*a^3*d^10)*x)/b^10 + 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2
*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(252) = 504$.
time = 0.31, size = 1386, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*d^10*x^10 - 2*b^10*c^10 - 4*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
24*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 504*a^5*b^5*c^5*d^5 + 6174*a^6*b^
4*c^4*d^6 - 16056*a^7*b^3*c^3*d^7 + 18414*a^8*b^2*c^2*d^8 - 10036*a^9*b*c*d
^9 + 2131*a^10*d^10 + 10*(4*b^10*c*d^9 - a*b^9*d^10)*x^9 + 45*(6*b^10*c^2*d
^8 - 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(4*b^10*c^3*d^7 - 6*a*b^9*c^2*
d^8 + 4*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + (8640*a*b^9*c^3*d^7 - 18630*a^2
*b^8*c^2*d^8 + 14660*a^3*b^7*c*d^9 - 4043*a^4*b^6*d^10)*x^6 - 6*(504*b^10*c
^5*d^5 - 2520*a*b^9*c^4*d^6 + 1440*a^2*b^8*c^3*d^7 + 3510*a^3*b^7*c^2*d^8 -
 4580*a^4*b^6*c*d^9 + 1523*a^5*b^5*d^10)*x^5 - 15*(84*b^10*c^6*d^4 + 504*a*
b^9*c^5*d^5 - 3780*a^2*b^8*c^4*d^6 + 6480*a^3*b^7*c^3*d^7 - 4050*a^4*b^6*c^
2*d^8 + 460*a^5*b^5*c*d^9 + 263*a^6*b^4*d^10)*x^4 - 20*(24*b^10*c^7*d^3 + 8
4*a*b^9*c^6*d^4 + 504*a^2*b^8*c^5*d^5 - 4620*a^3*b^7*c^4*d^6 + 9840*a^4*b^6
*c^3*d^7 - 9090*a^5*b^5*c^2*d^8 + 3820*a^6*b^4*c*d^9 - 577*a^7*b^3*d^10)*x^
3 - 15*(9*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 504*a^3*b^
7*c^5*d^5 - 5250*a^4*b^6*c^4*d^6 + 12360*a^5*b^5*c^3*d^7 - 12870*a^6*b^4*c^
2*d^8 + 6340*a^7*b^3*c*d^9 - 1207*a^8*b^2*d^10)*x^2 - 6*(4*b^10*c^9*d + 9*a
*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^
5 - 5754*a^5*b^5*c^4*d^6 + 14376*a^6*b^4*c^3*d^7 - 15894*a^7*b^3*c^2*d^8 +
8356*a^8*b^2*c*d^9 - 1711*a^9*b*d^10)*x + 2520*(a^6*b^4*c^4*d^6 - 4*a^7*b^3
*c^3*d^7 + 6*a^8*b^2*c^2*d^8 - 4*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^4*d^6 -
4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 6*(a*b^9*c^4*d^6 - 4*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 - 4*a^4*b^6*c*d^9
 + a^5*b^5*d^10)*x^5 + 15*(a^2*b^8*c^4*d^6 - 4*a^3*b^7*c^3*d^7 + 6*a^4*b^6*
c^2*d^8 - 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 20*(a^3*b^7*c^4*d^6 - 4*a^4
*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 - 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 15
*(a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 - 4*a^7*b^3*c*d^9
 + a^8*b^2*d^10)*x^2 + 6*(a^5*b^5*c^4*d^6 - 4*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c
^2*d^8 - 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^17*x^6 + 6*a*b^1
6*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x
+ a^6*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 878 vs. 2(252) = 504.
time = 0.00, size = 928, normalized size = 3.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x)
```

```
[Out] 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^
4*d^10)*log(abs(b*x + a))/b^11 - 1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*
b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5
 - 6174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 1
0036*a^9*b*c*d^9 - 2131*a^10*d^10 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 +
10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x
^5 + 1260*(b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^
7*c^3*d^7 - 105*a^4*b^6*c^2*d^8 + 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 +
 240*(2*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c
^4*d^6 + 910*a^4*b^6*c^3*d^7 - 987*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 10
7*a^7*b^3*d^10)*x^3 + 45*(3*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6
*d^4 + 168*a^3*b^7*c^5*d^5 - 1750*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 -
4788*a^6*b^4*c^2*d^8 + 2552*a^7*b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^
10*c^9*d + 9*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*
a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*
b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9 - 1879*a^9*b*d^10)*x)/((b*x + a)^6*b^11) +
 1/12*(3*b^21*d^10*x^4 + 40*b^21*c*d^9*x^3 - 28*a*b^20*d^10*x^3 + 270*b^21*
c^2*d^8*x^2 - 420*a*b^20*c*d^9*x^2 + 168*a^2*b^19*d^10*x^2 + 1440*b^21*c^3*
d^7*x - 3780*a*b^20*c^2*d^8*x + 3360*a^2*b^19*c*d^9*x - 1008*a^3*b^18*d^10*
x)/b^28
```

Mupad [B]
time = 0.42, size = 997, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^7,x)
```

```
[Out] x^2*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/(2*b) - (21*a^2*d^10)/(2*b^9)
+ (45*c^2*d^8)/(2*b^7)) - (x^4*(105*b^9*c^6*d^4 - 1155*a^6*b^3*d^10 + 630*a
*b^8*c^5*d^5 + 5670*a^5*b^4*c*d^9 - 4725*a^2*b^7*c^4*d^6 + 10500*a^3*b^6*c^
3*d^7 - 11025*a^4*b^5*c^2*d^8) + (2*b^10*c^10 - 2131*a^10*d^10 + 9*a^2*b^8*
c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6
174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 4*a*b
^9*c^9*d + 10036*a^9*b*c*d^9)/(12*b) + x*(2*b^9*c^9*d - (1879*a^9*d^10)/2 +
```

(9*a*b^8*c^8*d^2)/2 + 12*a^2*b^7*c^7*d^3 + 42*a^3*b^6*c^6*d^4 + 252*a^4*b^5*c^5*d^5 - 2877*a^5*b^4*c^4*d^6 + 7308*a^6*b^3*c^3*d^7 - 8262*a^7*b^2*c^2*d^8 + 4458*a^8*b*c*d^9) + x^3*(40*b^9*c^7*d^3 - 2140*a^7*b^2*d^10 + 140*a*b^8*c^6*d^4 + 10360*a^6*b^3*c*d^9 + 840*a^2*b^7*c^5*d^5 - 7700*a^3*b^6*c^4*d^6 + 18200*a^4*b^5*c^3*d^7 - 19740*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)/4 - (7995*a^8*b*d^10)/4 + 30*a*b^8*c^7*d^3 + 9570*a^7*b^2*c*d^9 + 105*a^2*b^7*c^6*d^4 + 630*a^3*b^6*c^5*d^5 - (13125*a^4*b^5*c^4*d^6)/2 + 16170*a^5*b^4*c^3*d^7 - 17955*a^6*b^3*c^2*d^8) - x^5*(252*a^5*b^4*d^10 - 252*b^9*c^5*d^5 + 1260*a*b^8*c^4*d^6 - 1260*a^4*b^5*c*d^9 - 2520*a^2*b^7*c^3*d^7 + 2520*a^3*b^6*c^2*d^8))/(a^6*b^10 + b^16*x^6 + 6*a^5*b^11*x + 6*a*b^15*x^5 + 15*a^4*b^12*x^2 + 20*a^3*b^13*x^3 + 15*a^2*b^14*x^4) - x^3*((7*a*d^10)/(3*b^8) - (10*c*d^9)/(3*b^7)) - x*((7*a*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b - (21*a^2*d^10)/b^9 + (45*c^2*d^8)/b^7))/b + (35*a^3*d^10)/b^10 - (120*c^3*d^7)/b^7 - (21*a^2*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b^2) + (log(a + b*x)*(210*a^4*d^10 + 210*b^4*c^4*d^6 - 840*a*b^3*c^3*d^7 + 1260*a^2*b^2*c^2*d^8 - 840*a^3*b*c*d^9))/b^11 + (d^10*x^4)/(4*b^7)

3.1319 $\int \frac{(c+dx)^{10}}{(a+bx)^8} dx$

Optimal. Leaf size=258

$$\frac{45d^8(bc-ad)^2x}{b^{10}} - \frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7} - \frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6} - \frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5} - \frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4} - \frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3} - \frac{126d^5(bc}{b^{11}(a+}$$

```
[Out] 45*d^8*(-a*d+b*c)^2*x/b^10-1/7*(-a*d+b*c)^10/b^11/(b*x+a)^7-5/3*d*(-a*d+b*c
)^9/b^11/(b*x+a)^6-9*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^5-30*d^3*(-a*d+b*c)^7/b^
11/(b*x+a)^4-70*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^3-126*d^5*(-a*d+b*c)^5/b^11/(
b*x+a)^2-210*d^6*(-a*d+b*c)^4/b^11/(b*x+a)+5*d^9*(-a*d+b*c)*(b*x+a)^2/b^11+
1/3*d^10*(b*x+a)^3/b^11+120*d^7*(-a*d+b*c)^3*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.25, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}} + \frac{120d^7(bc-ad)^3\log(a+bx)}{b^{11}} - \frac{210d^6(bc-ad)^4}{b^{11}(a+bx)} - \frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2} - \frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3} - \frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4} - \frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5} - \frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6} - \frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7} + \frac{d^{10}(a+bx)^3}{3b^{11}} + \frac{45d^8x(bc-ad)^2}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (45*d^8*(b*c - a*d)^2*x)/b^10 - (b*c - a*d)^10/(7*b^11*(a + b*x)^7) - (5*d*
(b*c - a*d)^9)/(3*b^11*(a + b*x)^6) - (9*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
^5) - (30*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^4) - (70*d^4*(b*c - a*d)^6)/(b
^11*(a + b*x)^3) - (126*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^2) - (210*d^6*(b
*c - a*d)^4)/(b^11*(a + b*x)) + (5*d^9*(b*c - a*d)*(a + b*x)^2)/b^11 + (d^1
0*(a + b*x)^3)/(3*b^11) + (120*d^7*(b*c - a*d)^3*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^{10}}{(a+bx)^8} dx &= \int \left(\frac{45d^8(bc-ad)^2}{b^{10}} + \frac{(bc-ad)^{10}}{b^{10}(a+bx)^8} + \frac{10d(bc-ad)^9}{b^{10}(a+bx)^7} + \frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^6} + \frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^5} \right. \\ &= \frac{45d^8(bc-ad)^2x}{b^{10}} - \frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7} - \frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6} - \frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5} - \frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4} - \end{aligned}$$

Mathematica [A]
time = 0.16, size = 239, normalized size = 0.93

$$\frac{21bd^8\left(45b^2c^2-80abcd+36a^2d^2\right)x+21b^2d^9(5bc-4ad)x^2+7b^3d^{10}x^3-\frac{3(bc-ad)^{10}}{(a+bx)^7}+\frac{35d(-bc+ad)^9}{(a+bx)^6}-\frac{189d^2(bc-ad)^8}{(a+bx)^5}+\frac{630d^3(-bc+ad)^7}{(a+bx)^4}-\frac{1470d^4(bc-ad)^6}{(a+bx)^3}+\frac{2646d^5(-bc+ad)^5}{(a+bx)^2}-\frac{4410d^6(bc-ad)^4}{a+bx}+2520d^7(bc-ad)^3\log(a+bx)}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (21*b*d^8*(45*b^2*c^2 - 80*a*b*c*d + 36*a^2*d^2)*x + 21*b^2*d^9*(5*b*c - 4*
a*d)*x^2 + 7*b^3*d^10*x^3 - (3*(b*c - a*d)^10)/(a + b*x)^7 + (35*d*(-(b*c)
+ a*d)^9)/(a + b*x)^6 - (189*d^2*(b*c - a*d)^8)/(a + b*x)^5 + (630*d^3*(-(b
*c) + a*d)^7)/(a + b*x)^4 - (1470*d^4*(b*c - a*d)^6)/(a + b*x)^3 + (2646*d^
5*(-(b*c) + a*d)^5)/(a + b*x)^2 - (4410*d^6*(b*c - a*d)^4)/(a + b*x) + 2520
*d^7*(b*c - a*d)^3*Log[a + b*x])/(21*b^11)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^8,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(252) = 504$.
time = 0.14, size = 856, normalized size = 3.32

method	result
norman	$-\frac{6534a^{10}d^{10}-19602a^9bc\,d^9+19602a^8b^2c^2d^8-6534a^7b^3c^3d^7+630a^6b^4c^4d^6+126a^5b^5c^5d^5+42a^4b^6c^6d^4+18a^3b^7c^7d^3+9a^2b^8c^8d^2+5a\,b^9c^9d+3b^{1}}{21b^{11}}$
default	$\frac{d^8\left(\frac{1}{3}d^2x^3b^2-4ab\,d^2x^2+5b^2cd\,x^2+36a^2d^2x-80abcdx+45b^2c^2x\right)}{b^{10}}-\frac{210d^6\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{b^{11}(bx+a)}+\frac{30d^3(a}{}$
risch	$\frac{d^{10}x^3}{3b^8}-\frac{4d^{10}a\,x^2}{b^9}+\frac{5d^9c\,x^2}{b^8}+\frac{36d^{10}a^2x}{b^{10}}-\frac{80d^9acx}{b^9}+\frac{45d^8c^2x}{b^8}+\frac{(-210a^4b^5d^{10}+840a^3b^6c\,d^9-1260a^2b^7c^2d^8+840a\,b^8c^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] d^8/b^10*(1/3*d^2*x^3*b^2-4*a*b*d^2*x^2+5*b^2*c*d*x^2+36*a^2*d^2*x-80*a*b*c
*d*x+45*b^2*c^2*x)-210/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*
a*b^3*c^3*d+b^4*c^4)/(b*x+a)+30/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*
c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^
6*d-b^7*c^7)/(b*x+a)^4-1/7*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120
```

```
*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^
4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*
x+a)^7-9/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*
d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+
b^8*c^8)/(b*x+a)^5+126/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-1
0*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^2+5/3/b^11*d*(a^9*d^9-9*a^
8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4
*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(
b*x+a)^6-120/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*x+
a)-70/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3
+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(252) = 504$.
time = 0.35, size = 934, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3
 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^
7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 2761*a^10*d^10
+ 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^
9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 - 30*a^2*b^8*c
^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10)*x^5 + 1470
*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3*b^7*c^3*d^7
 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x^4 + 210*(3*
b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 -
 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^9 + 319*a^7*
b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 +
 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^7 + 3654*a^6
*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(5*b^10*c^9*d
 + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*
c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^7*b^3*c^2*d^
8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/(b^18*x^7 + 7*a*b^17*x^6 + 21*
a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*x^2 + 7*a^6*
b^12*x + a^7*b^11) + 1/3*(b^2*d^10*x^3 + 3*(5*b^2*c*d^9 - 4*a*b*d^10)*x^2 +
 3*(45*b^2*c^2*d^8 - 80*a*b*c*d^9 + 36*a^2*d^10)*x)/b^10 + 120*(b^3*c^3*d^7
 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(b*x + a)/b^11
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(252) = 504$.
time = 0.31, size = 1362, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*d^10*x^10 - 3*b^10*c^10 - 5*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
18*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 126*a^5*b^5*c^5*d^5 - 630*a^6*b^4
*c^4*d^6 + 6534*a^7*b^3*c^3*d^7 - 12987*a^8*b^2*c^2*d^8 + 10047*a^9*b*c*d^9
 - 2761*a^10*d^10 + 35*(3*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(3*b^10*c^2*d^
8 - 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 49*(135*a*b^9*c^2*d^8 - 195*a^2*b^8
*c*d^9 + 77*a^3*b^7*d^10)*x^7 - 49*(90*b^10*c^4*d^6 - 360*a*b^9*c^3*d^7 + 1
35*a^2*b^8*c^2*d^8 + 285*a^3*b^7*c*d^9 - 179*a^4*b^6*d^10)*x^6 - 147*(18*b^
10*c^5*d^5 + 90*a*b^9*c^4*d^6 - 540*a^2*b^8*c^3*d^7 + 675*a^3*b^7*c^2*d^8 -
 255*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 - 245*(6*b^10*c^6*d^4 + 18*a*b^9*c^5
*d^5 + 90*a^2*b^8*c^4*d^6 - 660*a^3*b^7*c^3*d^7 + 1035*a^4*b^6*c^2*d^8 - 61
5*a^5*b^5*c*d^9 + 121*a^6*b^4*d^10)*x^4 - 35*(18*b^10*c^7*d^3 + 42*a*b^9*c^
6*d^4 + 126*a^2*b^8*c^5*d^5 + 630*a^3*b^7*c^4*d^6 - 5250*a^4*b^6*c^3*d^7 +
9135*a^5*b^5*c^2*d^8 - 6195*a^6*b^4*c*d^9 + 1477*a^7*b^3*d^10)*x^3 - 21*(9*
b^10*c^8*d^2 + 18*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
+ 630*a^4*b^6*c^4*d^6 - 5754*a^5*b^5*c^3*d^7 + 10647*a^6*b^4*c^2*d^8 - 7707
*a^7*b^3*c*d^9 + 1981*a^8*b^2*d^10)*x^2 - 7*(5*b^10*c^9*d + 9*a*b^9*c^8*d^2
 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 630*a^5*
b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 11907*a^7*b^3*c^2*d^8 - 8967*a^8*b^2*c
*d^9 + 2401*a^9*b*d^10)*x + 2520*(a^7*b^3*c^3*d^7 - 3*a^8*b^2*c^2*d^8 + 3*a
^9*b*c*d^9 - a^10*d^10 + (b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9
- a^3*b^7*d^10)*x^7 + 7*(a*b^9*c^3*d^7 - 3*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^
9 - a^4*b^6*d^10)*x^6 + 21*(a^2*b^8*c^3*d^7 - 3*a^3*b^7*c^2*d^8 + 3*a^4*b^6
*c*d^9 - a^5*b^5*d^10)*x^5 + 35*(a^3*b^7*c^3*d^7 - 3*a^4*b^6*c^2*d^8 + 3*a^
5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 35*(a^4*b^6*c^3*d^7 - 3*a^5*b^5*c^2*d^8 +
 3*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 21*(a^5*b^5*c^3*d^7 - 3*a^6*b^4*c^2*
d^8 + 3*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 7*(a^6*b^4*c^3*d^7 - 3*a^7*b^3*
c^2*d^8 + 3*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^18*x^7 + 7*a*b^
17*x^6 + 21*a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*
x^2 + 7*a^6*b^12*x + a^7*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(252) = 504.

time = 0.00, size = 924, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x)
```

```
[Out] 120*(b^3*c^3*d^7 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(abs(b*x
+ a))/b^11 - 1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3
*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d
^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 276
1*a^10*d^10 + 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 -
30*a^2*b^8*c^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10
)*x^5 + 1470*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3
*b^7*c^3*d^7 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x
^4 + 210*(3*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b
^7*c^4*d^6 - 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^
9 + 319*a^7*b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^
7 + 3654*a^6*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(
5*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^
7*b^3*c^2*d^8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/((b*x + a)^7*b^11)
 + 1/3*(b^16*d^10*x^3 + 15*b^16*c*d^9*x^2 - 12*a*b^15*d^10*x^2 + 135*b^16*c
^2*d^8*x - 240*a*b^15*c*d^9*x + 108*a^2*b^14*d^10*x)/b^24
```

Mupad [B]
time = 0.43, size = 950, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^8,x)
```

```
[Out] x*((8*a*((8*a*d^10)/b^9 - (10*c*d^9)/b^8))/b - (28*a^2*d^10)/b^10 + (45*c^2
*d^8)/b^8) - (x^4*(2590*a^6*b^3*d^10 + 70*b^9*c^6*d^4 + 210*a*b^8*c^5*d^5 -
 9870*a^5*b^4*c*d^9 + 1050*a^2*b^7*c^4*d^6 - 7700*a^3*b^6*c^3*d^7 + 13650*a
^4*b^5*c^2*d^8) + x^6*(210*a^4*b^5*d^10 + 210*b^9*c^4*d^6 - 840*a*b^8*c^3*d
^7 - 840*a^3*b^6*c*d^9 + 1260*a^2*b^7*c^2*d^8) + (2761*a^10*d^10 + 3*b^10*c
^10 + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5
*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c
^2*d^8 + 5*a*b^9*c^9*d - 10047*a^9*b*c*d^9)/(21*b) + x*((2509*a^9*d^10)/3 +
 (5*b^9*c^9*d)/3 + 3*a*b^8*c^8*d^2 + 6*a^2*b^7*c^7*d^3 + 14*a^3*b^6*c^6*d^4
 + 42*a^4*b^5*c^5*d^5 + 210*a^5*b^4*c^4*d^6 - 2058*a^6*b^3*c^3*d^7 + 4014*a
```

```
^7*b^2*c^2*d^8 - 3069*a^8*b*c*d^9) + x^3*(3190*a^7*b^2*d^10 + 30*b^9*c^7*d^
3 + 70*a*b^8*c^6*d^4 - 11970*a^6*b^3*c*d^9 + 210*a^2*b^7*c^5*d^5 + 1050*a^3
*b^6*c^4*d^6 - 8750*a^4*b^5*c^3*d^7 + 16170*a^5*b^4*c^2*d^8) + x^2*(2229*a^
8*b*d^10 + 9*b^9*c^8*d^2 + 18*a*b^8*c^7*d^3 - 8262*a^7*b^2*c*d^9 + 42*a^2*b
^7*c^6*d^4 + 126*a^3*b^6*c^5*d^5 + 630*a^4*b^5*c^4*d^6 - 5754*a^5*b^4*c^3*d
^7 + 10962*a^6*b^3*c^2*d^8) + x^5*(1134*a^5*b^4*d^10 + 126*b^9*c^5*d^5 + 63
0*a*b^8*c^4*d^6 - 4410*a^4*b^5*c*d^9 - 3780*a^2*b^7*c^3*d^7 + 6300*a^3*b^6*
c^2*d^8))/(a^7*b^10 + b^17*x^7 + 7*a^6*b^11*x + 7*a*b^16*x^6 + 21*a^5*b^12*
x^2 + 35*a^4*b^13*x^3 + 35*a^3*b^14*x^4 + 21*a^2*b^15*x^5) - x^2*((4*a*d^10
)/b^9 - (5*c*d^9)/b^8) - (log(a + b*x)*(120*a^3*d^10 - 120*b^3*c^3*d^7 + 36
0*a*b^2*c^2*d^8 - 360*a^2*b*c*d^9))/b^11 + (d^10*x^3)/(3*b^8)
```

3.1320 $\int \frac{(c+dx)^{10}}{(a+bx)^9} \, dx$

Optimal. Leaf size=258

$$\frac{d^9(10bc-9ad)x}{b^{10}}+\frac{d^{10}x^2}{2b^9}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}$$

```
[Out] d^9*(-9*a*d+10*b*c)*x/b^10+1/2*d^10*x^2/b^9-1/8*(-a*d+b*c)^10/b^11/(b*x+a)^
8-10/7*d*(-a*d+b*c)^9/b^11/(b*x+a)^7-15/2*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^6-2
4*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^5-105/2*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^4-84*
d^5*(-a*d+b*c)^5/b^11/(b*x+a)^3-105*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^2-120*d^7
*(-a*d+b*c)^3/b^11/(b*x+a)+45*d^8*(-a*d+b*c)^2*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{45d^8(bc-ad)^2\log(a+bx)}{b^{11}}-\frac{120d^7(bc-ad)^3}{b^{11}(a+bx)}-\frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2}-\frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}+\frac{d^9x(10bc-9ad)}{b^{10}}+\frac{d^{10}x^2}{2b^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (d^9*(10*b*c - 9*a*d)*x)/b^10 + (d^10*x^2)/(2*b^9) - (b*c - a*d)^10/(8*b^11
*(a + b*x)^8) - (10*d*(b*c - a*d)^9)/(7*b^11*(a + b*x)^7) - (15*d^2*(b*c -
a*d)^8)/(2*b^11*(a + b*x)^6) - (24*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^5) -
(105*d^4*(b*c - a*d)^6)/(2*b^11*(a + b*x)^4) - (84*d^5*(b*c - a*d)^5)/(b^11
*(a + b*x)^3) - (105*d^6*(b*c - a*d)^4)/(b^11*(a + b*x)^2) - (120*d^7*(b*c
- a*d)^3)/(b^11*(a + b*x)) + (45*d^8*(b*c - a*d)^2*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx &= \int \left(\frac{d^9(10bc-9ad)}{b^{10}}+\frac{d^{10}x}{b^9}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^9}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^8}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^7}+\frac{120d^3(b}{b^{10}(a}\right.\\ &= \frac{d^9(10bc-9ad)x}{b^{10}}+\frac{d^{10}x^2}{2b^9}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{24d^3(bc-}{b^{11}(a+}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 712 vs. 2(258) = 516.
time = 0.20, size = 712, normalized size = 2.76

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (3601*a^10*d^10 + 2*a^9*b*d^9*(-4609*c + 13144*d*x) + a^8*b^2*d^8*(6849*c^2
 - 68704*c*d*x + 81928*d^2*x^2) + 8*a^7*b^3*d^7*(-105*c^3 + 6534*c^2*d*x -
27538*c*d^2*x^2 + 17542*d^3*x^3) + 14*a^6*b^4*d^6*(-15*c^4 - 480*c^3*d*x +
12348*c^2*d^2*x^2 - 28112*c*d^3*x^3 + 10010*d^4*x^4) - 28*a^5*b^5*d^5*(3*c^
5 + 60*c^4*d*x + 840*c^3*d^2*x^2 - 11508*c^2*d^3*x^3 + 15050*c*d^4*x^4 - 27
44*d^5*x^5) - 14*a^4*b^6*d^4*(3*c^6 + 48*c^5*d*x + 420*c^4*d^2*x^2 + 3360*c
^3*d^3*x^3 - 26250*c^2*d^4*x^4 + 19040*c*d^5*x^5 - 1064*d^6*x^6) - 8*a^3*b^
7*d^3*(3*c^7 + 42*c^6*d*x + 294*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 7350*c^3*d
^4*x^4 - 32340*c^2*d^5*x^5 + 10780*c*d^6*x^6 + 728*d^7*x^7) - a^2*b^8*d^2*(
15*c^8 + 192*c^7*d*x + 1176*c^6*d^2*x^2 + 4704*c^5*d^3*x^3 + 14700*c^4*d^4*
x^4 + 47040*c^3*d^5*x^5 - 105840*c^2*d^6*x^6 + 4480*c*d^7*x^7 + 3248*d^8*x^
8) - 2*a*b^9*d*(5*c^9 + 60*c^8*d*x + 336*c^7*d^2*x^2 + 1176*c^6*d^3*x^3 + 2
940*c^5*d^4*x^4 + 5880*c^4*d^5*x^5 + 11760*c^3*d^6*x^6 - 10080*c^2*d^7*x^7
- 2240*c*d^8*x^8 + 140*d^9*x^9) - b^10*(7*c^10 + 80*c^9*d*x + 420*c^8*d^2*x
^2 + 1344*c^7*d^3*x^3 + 2940*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 + 5880*c^4*d^6*
x^6 + 6720*c^3*d^7*x^7 - 560*c*d^9*x^9 - 28*d^10*x^10) + 2520*d^8*(b*c - a*
d)^2*(a + b*x)^8*Log[a + b*x])/(56*b^11*(a + b*x)^8)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^9,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(248) = 496.
time = 0.17, size = 854, normalized size = 3.31

method	result
default	$-\frac{d^{9}\left(-\frac{1}{2}bd\,x^{2}+9adx-10bcx\right)}{b^{10}}+\frac{120d^{7}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}{b^{11}(bx+a)}-\frac{105d^{4}\left(a^{6}d^{6}-6a^{5}bc\,d^{5}+15a^{4}b^{2}c^{2}d^{4}-20a^{3}b^{3}c^{3}d^{3}+15\right.}{2b^{11}(bx+a)^{4}}$

norman	$\frac{6849a^{10}d^{10}-13698a^{9}bc\,d^{9}+6849a^{8}b^{2}c^{2}d^{8}-840a^{7}b^{3}c^{3}d^{7}-210a^{6}b^{4}c^{4}d^{6}-84a^{5}b^{5}c^{5}d^{5}-42a^{4}b^{6}c^{6}d^{4}-24a^{3}b^{7}c^{7}d^{3}-15a^{2}b^{8}c^{8}d^{2}-10a\,b^{9}c^{9}d-7b^{10}c^{10}}{56b^{11}}$
risch	$\frac{d^{10}x^{2}}{2b^{9}}-\frac{9d^{10}ax}{b^{10}}+\frac{10d^{9}cx}{b^{9}}+\frac{(120a^{3}b^{6}d^{10}-360a^{2}b^{7}c\,d^{9}+360a\,b^{8}c^{2}d^{8}-120b^{9}c^{3}d^{7})x^{7}+105b^{5}d^{6}(7a^{4}d^{4}-20a^{3}bc\,d^{3}+18a^{2}b^{2}c^{2}d^{2}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^9,x,method=_RETURNVERBOSE)

[Out] -d^9/b^10*(-1/2*b*d*x^2+9*a*d*x-10*b*c*x)+120/b^11*d^7*(a^3*d^3-3*a^2*b*c*d ^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)-105/2/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15* a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^4+10/7/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^ 3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2 *b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^7-1/8*(a^10*d^10-10*a^9*b*c*d^9 +45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5 *d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^ 9*d+b^10*c^10)/b^11/(b*x+a)^8+24/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2 *c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c ^6*d-b^7*c^7)/(b*x+a)^5-105/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d ^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^2-15/2/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28 *a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+2 8*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^6+45/b^11*d^8*(a^2*d^2-2*a *b*c*d+b^2*c^2)*ln(b*x+a)+84/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2 *d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(248) = 496$.
time = 0.39, size = 945, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="maxima")

[Out] -1/56*(7*b^10*c^10 + 10*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d ^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^ 7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 + 6720*(b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 5880*(b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c* d^9 - 7*a^4*b^6*d^10)*x^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2 *b^8*c^3*d^7 - 110*a^3*b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x ^5 + 2940*(b^10*c^6*d^4 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7* c^3*d^7 - 125*a^4*b^6*c^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 + 336*(4*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4

```
*d^6 + 140*a^4*b^6*c^3*d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459
*a^7*b^3*d^10)*x^3 + 84*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*
d^4 + 28*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*
a^6*b^4*c^2*d^8 + 2676*a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*
c^9*d + 15*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4
*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2
*d^8 + 8658*a^8*b^2*c*d^9 - 3349*a^9*b*d^10)*x)/(b^19*x^8 + 8*a*b^18*x^7 +
28*a^2*b^17*x^6 + 56*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*
a^6*b^13*x^2 + 8*a^7*b^12*x + a^8*b^11) + 1/2*(b*d^10*x^2 + 2*(10*b*c*d^9 -
 9*a*d^10)*x)/b^10 + 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(b*x + a)
/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1296 vs. $2(248) = 496$.
time = 0.31, size = 1296, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*d^10*x^10 - 7*b^10*c^10 - 10*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2
 - 24*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 84*a^5*b^5*c^5*d^5 - 210*a^6*b
^4*c^4*d^6 - 840*a^7*b^3*c^3*d^7 + 6849*a^8*b^2*c^2*d^8 - 9218*a^9*b*c*d^9
+ 3601*a^10*d^10 + 280*(2*b^10*c*d^9 - a*b^9*d^10)*x^9 + 112*(40*a*b^9*c*d^
9 - 29*a^2*b^8*d^10)*x^8 - 448*(15*b^10*c^3*d^7 - 45*a*b^9*c^2*d^8 + 10*a^2
*b^8*c*d^9 + 13*a^3*b^7*d^10)*x^7 - 392*(15*b^10*c^4*d^6 + 60*a*b^9*c^3*d^7
 - 270*a^2*b^8*c^2*d^8 + 220*a^3*b^7*c*d^9 - 38*a^4*b^6*d^10)*x^6 - 784*(6*
b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 60*a^2*b^8*c^3*d^7 - 330*a^3*b^7*c^2*d^8
+ 340*a^4*b^6*c*d^9 - 98*a^5*b^5*d^10)*x^5 - 980*(3*b^10*c^6*d^4 + 6*a*b^9*
c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 60*a^3*b^7*c^3*d^7 - 375*a^4*b^6*c^2*d^8 + 4
30*a^5*b^5*c*d^9 - 143*a^6*b^4*d^10)*x^4 - 112*(12*b^10*c^7*d^3 + 21*a*b^9*
c^6*d^4 + 42*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 -
2877*a^5*b^5*c^2*d^8 + 3514*a^6*b^4*c*d^9 - 1253*a^7*b^3*d^10)*x^3 - 28*(15
*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 84*a^3*b^7*c^5*d^5
+ 210*a^4*b^6*c^4*d^6 + 840*a^5*b^5*c^3*d^7 - 6174*a^6*b^4*c^2*d^8 + 7868*a
^7*b^3*c*d^9 - 2926*a^8*b^2*d^10)*x^2 - 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^2
 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*b
^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8588*a^8*b^2*c*d^
9 - 3286*a^9*b*d^10)*x + 2520*(a^8*b^2*c^2*d^8 - 2*a^9*b*c*d^9 + a^10*d^10
+ (b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 8*(a*b^9*c^2*d^8 - 2*
a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 28*(a^2*b^8*c^2*d^8 - 2*a^3*b^7*c*d^9 +
 a^4*b^6*d^10)*x^6 + 56*(a^3*b^7*c^2*d^8 - 2*a^4*b^6*c*d^9 + a^5*b^5*d^10)*
x^5 + 70*(a^4*b^6*c^2*d^8 - 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 56*(a^5*b
^5*c^2*d^8 - 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 28*(a^6*b^4*c^2*d^8 - 2*
a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 8*(a^7*b^3*c^2*d^8 - 2*a^8*b^2*c*d^9 +
```

```
a^9*b*d^10)*x)*log(b*x + a))/(b^19*x^8 + 8*a*b^18*x^7 + 28*a^2*b^17*x^6 + 5
6*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*a^6*b^13*x^2 + 8*a^
7*b^12*x + a^8*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**9,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(248) = 496.
time = 0.00, size = 922, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x)
```

```
[Out] 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(abs(b*x + a))/b^11 + 1/2*(b^9
*d^10*x^2 + 20*b^9*c*d^9*x - 18*a*b^8*d^10*x)/b^18 - 1/56*(7*b^10*c^10 + 10
*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4
 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^
8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 + 6720*(b^10*c^3*d^7 - 3*
a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 5880*(b^10*c^4*d^6 +
4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*d^9 - 7*a^4*b^6*d^10)*x
^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 110*a^3*
b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x^5 + 2940*(b^10*c^6*d^4
 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 - 125*a^4*b^6*c
^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 + 336*(4*b^10*c^7*d^3 + 7
*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*
d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459*a^7*b^3*d^10)*x^3 + 84
*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 + 28*a^3*b^7*c^5*d^
5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*a^6*b^4*c^2*d^8 + 2676*
a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^
2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*
b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8658*a^8*b^2*c*d
^9 - 3349*a^9*b*d^10)*x)/((b*x + a)^8*b^11)
```

Mupad [**B**]
time = 0.26, size = 946, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^10/(a + b*x)^9,x)

[Out] (log(a + b*x)*(45*a^2*d^10 + 45*b^2*c^2*d^8 - 90*a*b*c*d^9))/b^11 - (x^4*((105*b^9*c^6*d^4)/2 - (5985*a^6*b^3*d^10)/2 + 105*a*b^8*c^5*d^5 + 8085*a^5*b^4*c*d^9 + (525*a^2*b^7*c^4*d^6)/2 + 1050*a^3*b^6*c^3*d^7 - (13125*a^4*b^5*c^2*d^8)/2) + x^6*(105*b^9*c^4*d^6 - 735*a^4*b^5*d^10 + 420*a*b^8*c^3*d^7 + 2100*a^3*b^6*c*d^9 - 1890*a^2*b^7*c^2*d^8) + (7*b^10*c^10 - 3601*a^10*d^10 + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^8 + 10*a*b^9*c^9*d + 9218*a^9*b*c*d^9)/(56*b) + x*((10*b^9*c^9*d)/7 - (3349*a^9*d^10)/7 + (15*a*b^8*c^8*d^2)/7 + (24*a^2*b^7*c^7*d^3)/7 + 6*a^3*b^6*c^6*d^4 + 12*a^4*b^5*c^5*d^5 + 30*a^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - (6534*a^7*b^2*c^2*d^8)/7 + (8658*a^8*b*c*d^9)/7) + x^3*(24*b^9*c^7*d^3 - 2754*a^7*b^2*d^10 + 42*a*b^8*c^6*d^4 + 7308*a^6*b^3*c*d^9 + 84*a^2*b^7*c^5*d^5 + 210*a^3*b^6*c^4*d^6 + 840*a^4*b^5*c^3*d^7 - 5754*a^5*b^4*c^2*d^8) + x^2*((15*b^9*c^8*d^2)/2 - (3069*a^8*b*d^10)/2 + 12*a*b^8*c^7*d^3 + 4014*a^7*b^2*c*d^9 + 21*a^2*b^7*c^6*d^4 + 42*a^3*b^6*c^5*d^5 + 105*a^4*b^5*c^4*d^6 + 420*a^5*b^4*c^3*d^7 - 3087*a^6*b^3*c^2*d^8) + x^5*(84*b^9*c^5*d^5 - 1974*a^5*b^4*d^10 + 210*a*b^8*c^4*d^6 + 5460*a^4*b^5*c*d^9 + 840*a^2*b^7*c^3*d^7 - 4620*a^3*b^6*c^2*d^8) - x^7*(120*a^3*b^6*d^10 - 120*b^9*c^3*d^7 + 360*a*b^8*c^2*d^8 - 360*a^2*b^7*c*d^9))/(a^8*b^10 + b^18*x^8 + 8*a^7*b^11*x + 8*a*b^17*x^7 + 28*a^6*b^12*x^2 + 56*a^5*b^13*x^3 + 70*a^4*b^14*x^4 + 56*a^3*b^15*x^5 + 28*a^2*b^16*x^6) - x*((9*a*d^10)/b^10 - (10*c*d^9)/b^9) + (d^10*x^2)/(2*b^9)

3.1321 $\int \frac{(c+dx)^{10}}{(a+bx)^{10}} dx$

Optimal. Leaf size=257

$$\frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{70 \dots}{b \dots}$$

```
[Out] d^10*x/b^10-1/9*(-a*d+b*c)^10/b^11/(b*x+a)^9-5/4*d*(-a*d+b*c)^9/b^11/(b*x+a
)^8-45/7*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^7-20*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^6
-42*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^5-63*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^4-70*d
^6*(-a*d+b*c)^4/b^11/(b*x+a)^3-60*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^2-45*d^8*(-
a*d+b*c)^2/b^11/(b*x+a)+10*d^9*(-a*d+b*c)*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{10d^9(bc-ad)\log(a+bx)}{b^{11}}-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}+\frac{d^{10}x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] (d^10*x)/b^10 - (b*c - a*d)^10/(9*b^11*(a + b*x)^9) - (5*d*(b*c - a*d)^9)/(
4*b^11*(a + b*x)^8) - (45*d^2*(b*c - a*d)^8)/(7*b^11*(a + b*x)^7) - (20*d^3
*(b*c - a*d)^7)/(b^11*(a + b*x)^6) - (42*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)
^5) - (63*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^4) - (70*d^6*(b*c - a*d)^4)/(b
^11*(a + b*x)^3) - (60*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^2) - (45*d^8*(b*c
 - a*d)^2)/(b^11*(a + b*x)) + (10*d^9*(b*c - a*d)*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx = \int \left(\frac{d^{10}}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{10}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^9}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^8}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^7}+\frac{210d^4(b \dots}{b^{10}(a \dots}\right.$$
$$= \frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-a \dots}{b^{11}(a+bx \dots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 708 vs. $2(257) = 514$.
time = 0.25, size = 708, normalized size = 2.75

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10*d^10 + a^9*b*d^9*(-7129*c + 41229*d*x) + 9*a^8*b^2*d^8*(1
40*c^2 - 6849*c*d*x + 17064*d^2*x^2) + 12*a^7*b^3*d^7*(35*c^3 + 945*c^2*d*x
 - 19602*c*d^2*x^2 + 27342*d^3*x^3) + 42*a^6*b^4*d^6*(5*c^4 + 90*c^3*d*x +
1080*c^2*d^2*x^2 - 12348*c*d^3*x^3 + 10458*d^4*x^4) + 126*a^5*b^5*d^5*(c^5
+ 15*c^4*d*x + 120*c^3*d^2*x^2 + 840*c^2*d^3*x^3 - 5754*c*d^4*x^4 + 2982*d^
5*x^5) + 42*a^4*b^6*d^4*(2*c^6 + 27*c^5*d*x + 180*c^4*d^2*x^2 + 840*c^3*d^3
*x^3 + 3780*c^2*d^4*x^4 - 15750*c*d^5*x^5 + 4704*d^6*x^6) + 12*a^3*b^7*d^3*
(5*c^7 + 63*c^6*d*x + 378*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 4410*c^3*d^4*x^4
 + 13230*c^2*d^5*x^5 - 32340*c*d^6*x^6 + 4536*d^7*x^7) + 9*a^2*b^8*d^2*(5*c
^8 + 60*c^7*d*x + 336*c^6*d^2*x^2 + 1176*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 5
880*c^3*d^5*x^5 + 11760*c^2*d^6*x^6 - 15120*c*d^7*x^7 + 252*d^8*x^8) + a*b^
9*d*(35*c^9 + 405*c^8*d*x + 2160*c^7*d^2*x^2 + 7056*c^6*d^3*x^3 + 15876*c^5
*d^4*x^4 + 26460*c^4*d^5*x^5 + 35280*c^3*d^6*x^6 + 45360*c^2*d^7*x^7 - 2268
0*c*d^8*x^8 - 2268*d^9*x^9) + b^10*(28*c^10 + 315*c^9*d*x + 1620*c^8*d^2*x^
2 + 5040*c^7*d^3*x^3 + 10584*c^6*d^4*x^4 + 15876*c^5*d^5*x^5 + 17640*c^4*d^
6*x^6 + 15120*c^3*d^7*x^7 + 11340*c^2*d^8*x^8 - 252*d^10*x^10) + 2520*d^9*(
-(b*c) + a*d)*(a + b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^10,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(251) = 502$.
time = 0.17, size = 859, normalized size = 3.34

method	result
risch	$\frac{d^{10}x}{b^{10}} + \frac{\left(-45a^2b^7d^{10}+90a\,b^8c\,d^9-45b^9c^2d^8\right)x^8-60b^6d^7\left(5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)x^7-70b^5d^6\left(13a^4d^4-22a^3bc\,d^3+6a^2b^2c\right.}{}$

default	$\frac{d^{10}x}{b^{10}} - \frac{45d^8(a^2d^2-2abcd+b^2c^2)}{b^{11}(bx+a)} + \frac{63d^5(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5)}{b^{11}(bx+a)^4} - \frac{a^{10}d^{10}-10a^9bc\,d^9+45a^8}{}$
norman	$\frac{\frac{d^{10}x^{10}}{b} - \frac{7129a^{10}d^{10}-7129a^9bc\,d^9+1260a^8b^2c^2d^8+420a^7b^3c^3d^7+210a^6b^4c^4d^6+126a^5b^5c^5d^5+84a^4b^6c^6d^4+60a^3b^7c^7d^3+45a^2b^8c^8d^2+35a\,b^9c^9d}{252b^{11}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] d^10*x/b^10-45/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+63/b^11*d^5*(a^
5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5
*c^5)/(b*x+a)^4-1/9*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^
3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a
^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^9-
45/7/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+
70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*
c^8)/(b*x+a)^7+5/4/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*
b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a
^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^8-42/b^11*d^4*(a^6*d^6-6*a^5*
b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^
5*d+b^6*c^6)/(b*x+a)^5+60/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)/(b*x+a)^2+20/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^
4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/
(b*x+a)^6-10/b^11*d^9*(a*d-b*c)*ln(b*x+a)-70/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^
3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(251) = 502$.
time = 0.36, size = 957, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] d^10*x/b^10 - 1/252*(28*b^10*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 6
0*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*
c^4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4
861*a^10*d^10 + 11340*(b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1
5120*(b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^
7 + 17640*(b^10*c^4*d^6 + 2*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*
c*d^9 + 13*a^4*b^6*d^10)*x^6 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*
a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)
*x^5 + 5292*(2*b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*
b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4
 + 504*(10*b^10*c^7*d^3 + 14*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^
```

```
7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 +
 669*a^7*b^3*d^10)*x^3 + 108*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7
+ 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35
*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*
b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4609*a^9*b*d^10)*x)/(b^20*x^9 + 9*a*b^19
*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*
x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + 10*(b*
c*d^9 - a*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(251) = 502$.

time = 0.30, size = 1216, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*d^10*x^10 + 2268*a*b^9*d^10*x^9 - 28*b^10*c^10 - 35*a*b^9*c
^9*d - 45*a^2*b^8*c^8*d^2 - 60*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 126*a
^5*b^5*c^5*d^5 - 210*a^6*b^4*c^4*d^6 - 420*a^7*b^3*c^3*d^7 - 1260*a^8*b^2*c
^2*d^8 + 7129*a^9*b*c*d^9 - 4861*a^10*d^10 - 2268*(5*b^10*c^2*d^8 - 10*a*b^
9*c*d^9 + a^2*b^8*d^10)*x^8 - 3024*(5*b^10*c^3*d^7 + 15*a*b^9*c^2*d^8 - 45*
a^2*b^8*c*d^9 + 18*a^3*b^7*d^10)*x^7 - 3528*(5*b^10*c^4*d^6 + 10*a*b^9*c^3*
d^7 + 30*a^2*b^8*c^2*d^8 - 110*a^3*b^7*c*d^9 + 56*a^4*b^6*d^10)*x^6 - 5292*
(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8
 - 125*a^4*b^6*c*d^9 + 71*a^5*b^5*d^10)*x^5 - 5292*(2*b^10*c^6*d^4 + 3*a*b^
9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 1
37*a^5*b^5*c*d^9 + 83*a^6*b^4*d^10)*x^4 - 504*(10*b^10*c^7*d^3 + 14*a*b^9*c
^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210
*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 651*a^7*b^3*d^10)*x^3 - 108*(15*b^1
0*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70
*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3
*c*d^9 + 1422*a^8*b^2*d^10)*x^2 - 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^
4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4
581*a^9*b*d^10)*x + 2520*(a^9*b*c*d^9 - a^10*d^10 + (b^10*c*d^9 - a*b^9*d^1
0)*x^9 + 9*(a*b^9*c*d^9 - a^2*b^8*d^10)*x^8 + 36*(a^2*b^8*c*d^9 - a^3*b^7*d
^10)*x^7 + 84*(a^3*b^7*c*d^9 - a^4*b^6*d^10)*x^6 + 126*(a^4*b^6*c*d^9 - a^5
*b^5*d^10)*x^5 + 126*(a^5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 84*(a^6*b^4*c*d^9
 - a^7*b^3*d^10)*x^3 + 36*(a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 9*(a^8*b^2*c
*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*
x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x
^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**10,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(251) = 502.
time = 0.00, size = 924, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x)
```

```
[Out] d^10*x/b^10 + 10*(b*c*d^9 - a*d^10)*log(abs(b*x + a))/b^11 - 1/252*(28*b^10
*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 84*a^4*b
^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 420*a^7*b^3*c^3*d^
7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4861*a^10*d^10 + 11340*(b^10*
c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 15120*(b^10*c^3*d^7 + 3*a*b^9
*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^7 + 17640*(b^10*c^4*d^6 + 2*
a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*c*d^9 + 13*a^4*b^6*d^10)*x^6
 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7
*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)*x^5 + 5292*(2*b^10*c^6*d^4
+ 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2
*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4 + 504*(10*b^10*c^7*d^3 + 14
*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d
^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 669*a^7*b^3*d^10)*x^3 + 108
*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178
*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d
^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^
5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c
*d^9 + 4609*a^9*b*d^10)*x)/((b*x + a)^9*b^11)
```

Mupad [**B**]
time = 0.50, size = 955, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^10,x)

[Out] (d^10*x)/b^10 - (log(a + b*x)*(10*a*d^10 - 10*b*c*d^9))/b^11 - (x^4*(1827*a
^6*b^3*d^10 + 42*b^9*c^6*d^4 + 63*a*b^8*c^5*d^5 - 2877*a^5*b^4*c*d^9 + 105*
a^2*b^7*c^4*d^6 + 210*a^3*b^6*c^3*d^7 + 630*a^4*b^5*c^2*d^8) + x^6*(910*a^4
*b^5*d^10 + 70*b^9*c^4*d^6 + 140*a*b^8*c^3*d^7 - 1540*a^3*b^6*c*d^9 + 420*a
^2*b^7*c^2*d^8) + (4861*a^10*d^10 + 28*b^10*c^10 + 45*a^2*b^8*c^8*d^2 + 60*
a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^
4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 35*a*b^9*c^9*d - 7129*
a^9*b*c*d^9)/(252*b) + x*((4609*a^9*d^10)/28 + (5*b^9*c^9*d)/4 + (45*a*b^8*
c^8*d^2)/28 + (15*a^2*b^7*c^7*d^3)/7 + 3*a^3*b^6*c^6*d^4 + (9*a^4*b^5*c^5*d
^5)/2 + (15*a^5*b^4*c^4*d^6)/2 + 15*a^6*b^3*c^3*d^7 + 45*a^7*b^2*c^2*d^8 -
(6849*a^8*b*c*d^9)/28) + x^8*(45*a^2*b^7*d^10 + 45*b^9*c^2*d^8 - 90*a*b^8*c
*d^9) + x^3*(1338*a^7*b^2*d^10 + 20*b^9*c^7*d^3 + 28*a*b^8*c^6*d^4 - 2058*a
^6*b^3*c*d^9 + 42*a^2*b^7*c^5*d^5 + 70*a^3*b^6*c^4*d^6 + 140*a^4*b^5*c^3*d^
7 + 420*a^5*b^4*c^2*d^8) + x^2*((4329*a^8*b*d^10)/7 + (45*b^9*c^8*d^2)/7 +
(60*a*b^8*c^7*d^3)/7 - (6534*a^7*b^2*c*d^9)/7 + 12*a^2*b^7*c^6*d^4 + 18*a^3
*b^6*c^5*d^5 + 30*a^4*b^5*c^4*d^6 + 60*a^5*b^4*c^3*d^7 + 180*a^6*b^3*c^2*d^
8) + x^5*(1617*a^5*b^4*d^10 + 63*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 - 2625*a^4
*b^5*c*d^9 + 210*a^2*b^7*c^3*d^7 + 630*a^3*b^6*c^2*d^8) + x^7*(300*a^3*b^6*
d^10 + 60*b^9*c^3*d^7 + 180*a*b^8*c^2*d^8 - 540*a^2*b^7*c*d^9))/(a^9*b^10 +
 b^19*x^9 + 9*a^8*b^11*x + 9*a*b^18*x^8 + 36*a^7*b^12*x^2 + 84*a^6*b^13*x^3
 + 126*a^5*b^14*x^4 + 126*a^4*b^15*x^5 + 84*a^3*b^16*x^6 + 36*a^2*b^17*x^7)
```

3.1322 $\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx$

Optimal. Leaf size=271

$$-\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{10\ldots}{2\ldots}$$

```
[Out] -1/10*(-a*d+b*c)^10/b^11/(b*x+a)^10-10/9*d*(-a*d+b*c)^9/b^11/(b*x+a)^9-45/8
*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^8-120/7*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^7-35*d
^4*(-a*d+b*c)^6/b^11/(b*x+a)^6-252/5*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^5-105/2*
d^6*(-a*d+b*c)^4/b^11/(b*x+a)^4-40*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^3-45/2*d^8
*(-a*d+b*c)^2/b^11/(b*x+a)^2-10*d^9*(-a*d+b*c)/b^11/(b*x+a)+d^10*ln(b*x+a)/
b^11
```

Rubi [A]

time = 0.20, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}+\frac{d^{10}\log(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -1/10*(b*c - a*d)^10/(b^11*(a + b*x)^10) - (10*d*(b*c - a*d)^9)/(9*b^11*(a
+ b*x)^9) - (45*d^2*(b*c - a*d)^8)/(8*b^11*(a + b*x)^8) - (120*d^3*(b*c - a
*d)^7)/(7*b^11*(a + b*x)^7) - (35*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^6) - (
252*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^5) - (105*d^6*(b*c - a*d)^4)/(2*b^
11*(a + b*x)^4) - (40*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^3) - (45*d^8*(b*c
- a*d)^2)/(2*b^11*(a + b*x)^2) - (10*d^9*(b*c - a*d))/(b^11*(a + b*x)) + (d
^10*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{11}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{10}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^9}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^8}+\frac{210d^4(bc-a\ldots}{b^{10}(a+bx)^7\ldots}\right.$$

$$= -\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad\ldots}{b^{11}(a+bx)^6}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 591 vs. 2(271) = 542.
time = 0.22, size = 591, normalized size = 2.18

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -1/2520*((b*c - a*d)*(7381*a^9*d^9 + a^8*b*d^8*(4861*c + 71290*d*x) + a^7*b
^2*d^7*(3601*c^2 + 46090*c*d*x + 308205*d^2*x^2) + a^6*b^3*d^6*(2761*c^3 +
33490*c^2*d*x + 194805*c*d^2*x^2 + 784080*d^3*x^3) + a^5*b^4*d^5*(2131*c^4
+ 25090*c^3*d*x + 138105*c^2*d^2*x^2 + 481680*c*d^3*x^3 + 1296540*d^4*x^4)
+ a^4*b^5*d^4*(1627*c^5 + 18790*c^4*d*x + 100305*c^3*d^2*x^2 + 330480*c^2*d
^3*x^3 + 767340*c*d^4*x^4 + 1450008*d^5*x^5) + a^3*b^6*d^3*(1207*c^6 + 1375
0*c^5*d*x + 71955*c^4*d^2*x^2 + 229680*c^3*d^3*x^3 + 502740*c^2*d^4*x^4 + 8
14968*c*d^5*x^5 + 1102500*d^6*x^6) + a^2*b^7*d^2*(847*c^7 + 9550*c^6*d*x +
49275*c^5*d^2*x^2 + 154080*c^4*d^3*x^3 + 326340*c^3*d^4*x^4 + 497448*c^2*d^
5*x^5 + 573300*c*d^6*x^6 + 554400*d^7*x^7) + a*b^8*d*(532*c^8 + 5950*c^7*d*
x + 30375*c^6*d^2*x^2 + 93600*c^5*d^3*x^3 + 194040*c^4*d^4*x^4 + 285768*c^3
*d^5*x^5 + 308700*c^2*d^6*x^6 + 252000*c*d^7*x^7 + 170100*d^8*x^8) + b^9*(2
52*c^9 + 2800*c^8*d*x + 14175*c^7*d^2*x^2 + 43200*c^6*d^3*x^3 + 88200*c^5*d
^4*x^4 + 127008*c^4*d^5*x^5 + 132300*c^3*d^6*x^6 + 100800*c^2*d^7*x^7 + 567
00*c*d^8*x^8 + 25200*d^9*x^9)))/(b^11*(a + b*x)^10) + (d^10*Log[a + b*x])/b
^11
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^11,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(257) = 514.
time = 0.24, size = 865, normalized size = 3.19

method	result
risch	$\frac{\frac{10d^9(ad-bc)x^9}{b^2}+\frac{45d^8\left(3a^2d^2-2abcd-b^2c^2\right)x^8}{2b^3}+\frac{20d^7\left(11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3\right)x^7}{b^4}+\frac{35d^6\left(25a^4d^4-12a^3bc\,d^3-6a^2b^2c^2d^2-4a\,b^3c^3d\right.}{2b^5}}{}$
norman	$\frac{7381a^{10}d^{10}-2520a^9bc\,d^9-1260a^8b^2c^2d^8-840a^7b^3c^3d^7-630a^6b^4c^4d^6-504a^5b^5c^5d^5-420a^4b^6c^6d^4-360a^3b^7c^7d^3-315a^2b^8c^8d^2-280a\,b^9c^9d-25}{2520b^{11}}$

default	$\frac{10d^9(ad-bc)}{b^{11}(bx+a)} - \frac{105d^6(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}{2b^{11}(bx+a)^4} + \frac{10d(a^9d^9-9a^8bc\,d^8+36a^7b^2c^2d^7-84a^6b^3c^3d^6+126a^5b^4c^4d^5}{9b^{11}(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^11,x,method=_RETURNVERBOSE)
```

```
[Out] 10/b^11*d^9*(a*d-b*c)/(b*x+a)-105/2/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^4+10/9/b^11*d*(a^9*d^9-9*a^8*b*c*
d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c
^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)
^9+120/7/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*
d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^7-
45/8/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+
70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*
c^8)/(b*x+a)^8-1/10*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^
3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a
^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^10
+252/5/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^
2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^5-45/2/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/(b*x+a)^2-35/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^
3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^6+d^10*ln(b*x+a
)/b^11+40/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(257) = 514$.
time = 0.32, size = 975, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
```

```
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x)/(b^21*x^10 + 10*a*b^20*x^9 + 45*a^2*b^19
*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b^16*x^5 + 210*a^6*b^1
5*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^12*x + a^10*b^11) + d
^10*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(257) = 514$.
time = 0.30, size = 1107, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x - 2520*(b^10*d^10*x^10 + 10*a*b^9*d^10*x^
9 + 45*a^2*b^8*d^10*x^8 + 120*a^3*b^7*d^10*x^7 + 210*a^4*b^6*d^10*x^6 + 252
*a^5*b^5*d^10*x^5 + 210*a^6*b^4*d^10*x^4 + 120*a^7*b^3*d^10*x^3 + 45*a^8*b^
2*d^10*x^2 + 10*a^9*b*d^10*x + a^10*d^10)*log(b*x + a))/(b^21*x^10 + 10*a*b
^20*x^9 + 45*a^2*b^19*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b
^16*x^5 + 210*a^6*b^15*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^
12*x + a^10*b^11)
```

Sympy [F(-1)]

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**11,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(257) = 514.
time = 0.00, size = 926, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x)
```

```
[Out] d^10*log(abs(b*x + a))/b^11 - 1/2520*(25200*(b^9*c*d^9 - a*b^8*d^10)*x^9 +
56700*(b^9*c^2*d^8 + 2*a*b^8*c*d^9 - 3*a^2*b^7*d^10)*x^8 + 50400*(2*b^9*c^3
*d^7 + 3*a*b^8*c^2*d^8 + 6*a^2*b^7*c*d^9 - 11*a^3*b^6*d^10)*x^7 + 44100*(3*
b^9*c^4*d^6 + 4*a*b^8*c^3*d^7 + 6*a^2*b^7*c^2*d^8 + 12*a^3*b^6*c*d^9 - 25*a
^4*b^5*d^10)*x^6 + 10584*(12*b^9*c^5*d^5 + 15*a*b^8*c^4*d^6 + 20*a^2*b^7*c^
3*d^7 + 30*a^3*b^6*c^2*d^8 + 60*a^4*b^5*c*d^9 - 137*a^5*b^4*d^10)*x^5 + 882
0*(10*b^9*c^6*d^4 + 12*a*b^8*c^5*d^5 + 15*a^2*b^7*c^4*d^6 + 20*a^3*b^6*c^3*
d^7 + 30*a^4*b^5*c^2*d^8 + 60*a^5*b^4*c*d^9 - 147*a^6*b^3*d^10)*x^4 + 720*(
60*b^9*c^7*d^3 + 70*a*b^8*c^6*d^4 + 84*a^2*b^7*c^5*d^5 + 105*a^3*b^6*c^4*d^
6 + 140*a^4*b^5*c^3*d^7 + 210*a^5*b^4*c^2*d^8 + 420*a^6*b^3*c*d^9 - 1089*a^
7*b^2*d^10)*x^3 + 135*(105*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 140*a^2*b^7*c^
6*d^4 + 168*a^3*b^6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 + 280*a^5*b^4*c^3*d^7 + 4
20*a^6*b^3*c^2*d^8 + 840*a^7*b^2*c*d^9 - 2283*a^8*b*d^10)*x^2 + 10*(280*b^9
*c^9*d + 315*a*b^8*c^8*d^2 + 360*a^2*b^7*c^7*d^3 + 420*a^3*b^6*c^6*d^4 + 50
4*a^4*b^5*c^5*d^5 + 630*a^5*b^4*c^4*d^6 + 840*a^6*b^3*c^3*d^7 + 1260*a^7*b^
2*c^2*d^8 + 2520*a^8*b*c*d^9 - 7129*a^9*d^10)*x + (252*b^10*c^10 + 280*a*b^
9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 +
 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 + 1260*a^8
*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^10*d^10)/b)/((b*x + a)^10*b^10)
```

Mupad [B]
time = 0.56, size = 866, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^11,x)
```

[Out] (d^10*log(a + b*x))/b^11 - (x^4*(35*b^10*c^6*d^4 - (1029*a^6*b^4*d^10)/2 + 42*a*b^9*c^5*d^5 + 210*a^5*b^5*c*d^9 + (105*a^2*b^8*c^4*d^6)/2 + 70*a^3*b^7*c^3*d^7 + 105*a^4*b^6*c^2*d^8) - x^9*(10*a*b^9*d^10 - 10*b^10*c*d^9) + x*((10*b^10*c^9*d)/9 - (7129*a^9*b*d^10)/252 + (5*a*b^9*c^8*d^2)/4 + 10*a^8*b^2*c*d^9 + (10*a^2*b^8*c^7*d^3)/7 + (5*a^3*b^7*c^6*d^4)/3 + 2*a^4*b^6*c^5*d^5 + (5*a^5*b^5*c^4*d^6)/2 + (10*a^6*b^4*c^3*d^7)/3 + 5*a^7*b^3*c^2*d^8) + x^6*((105*b^10*c^4*d^6)/2 - (875*a^4*b^6*d^10)/2 + 70*a*b^9*c^3*d^7 + 210*a^3*b^7*c*d^9 + 105*a^2*b^8*c^2*d^8) + x^8*((45*b^10*c^2*d^8)/2 - (135*a^2*b^8*d^10)/2 + 45*a*b^9*c*d^9) + x^3*((120*b^10*c^7*d^3)/7 - (2178*a^7*b^3*d^10)/7 + 20*a*b^9*c^6*d^4 + 120*a^6*b^4*c*d^9 + 24*a^2*b^8*c^5*d^5 + 30*a^3*b^7*c^4*d^6 + 40*a^4*b^6*c^3*d^7 + 60*a^5*b^5*c^2*d^8) + x^5*((252*b^10*c^5*d^5)/5 - (2877*a^5*b^5*d^10)/5 + 63*a*b^9*c^4*d^6 + 252*a^4*b^6*c*d^9 + 84*a^2*b^8*c^3*d^7 + 126*a^3*b^7*c^2*d^8) - (7381*a^10*d^10)/2520 + (b^10*c^10)/10 + x^7*(40*b^10*c^3*d^7 - 220*a^3*b^7*d^10 + 60*a*b^9*c^2*d^8 + 120*a^2*b^8*c*d^9) + x^2*((45*b^10*c^8*d^2)/8 - (6849*a^8*b^2*d^10)/56 + (45*a*b^9*c^7*d^3)/7 + 45*a^7*b^3*c*d^9 + (15*a^2*b^8*c^6*d^4)/2 + 9*a^3*b^7*c^5*d^5 + (45*a^4*b^6*c^4*d^6)/4 + 15*a^5*b^5*c^3*d^7 + (45*a^6*b^4*c^2*d^8)/2) + (a^2*b^8*c^8*d^2)/8 + (a^3*b^7*c^7*d^3)/7 + (a^4*b^6*c^6*d^4)/6 + (a^5*b^5*c^5*d^5)/5 + (a^6*b^4*c^4*d^6)/4 + (a^7*b^3*c^3*d^7)/3 + (a^8*b^2*c^2*d^8)/2 + (a*b^9*c^9*d)/9 + a^9*b*c*d^9)/(b^11*(a + b*x)^10)

3.1323 $\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

```
[Out] -1/11*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^11
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -1/11*(c + d*x)^11/((b*c - a*d)*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx = -\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 665 vs. 2(28) = 56.
time = 0.17, size = 665, normalized size = 23.75

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -1/11*(a^10*d^10 + a^9*b*d^9*(c + 11*d*x) + a^8*b^2*d^8*(c^2 + 11*c*d*x + 5
5*d^2*x^2) + a^7*b^3*d^7*(c^3 + 11*c^2*d*x + 55*c*d^2*x^2 + 165*d^3*x^3) +
a^6*b^4*d^6*(c^4 + 11*c^3*d*x + 55*c^2*d^2*x^2 + 165*c*d^3*x^3 + 330*d^4*x^
4) + a^5*b^5*d^5*(c^5 + 11*c^4*d*x + 55*c^3*d^2*x^2 + 165*c^2*d^3*x^3 + 330
*c*d^4*x^4 + 462*d^5*x^5) + a^4*b^6*d^4*(c^6 + 11*c^5*d*x + 55*c^4*d^2*x^2
+ 165*c^3*d^3*x^3 + 330*c^2*d^4*x^4 + 462*c*d^5*x^5 + 462*d^6*x^6) + a^3*b^
7*d^3*(c^7 + 11*c^6*d*x + 55*c^5*d^2*x^2 + 165*c^4*d^3*x^3 + 330*c^3*d^4*x^
4 + 462*c^2*d^5*x^5 + 462*c*d^6*x^6 + 330*d^7*x^7) + a^2*b^8*d^2*(c^8 + 11*
c^7*d*x + 55*c^6*d^2*x^2 + 165*c^5*d^3*x^3 + 330*c^4*d^4*x^4 + 462*c^3*d^5*
x^5 + 462*c^2*d^6*x^6 + 330*c*d^7*x^7 + 165*d^8*x^8) + a*b^9*d*(c^9 + 11*c^
8*d*x + 55*c^7*d^2*x^2 + 165*c^6*d^3*x^3 + 330*c^5*d^4*x^4 + 462*c^4*d^5*x^
5 + 462*c^3*d^6*x^6 + 330*c^2*d^7*x^7 + 165*c*d^8*x^8 + 55*d^9*x^9) + b^10*
(c^10 + 11*c^9*d*x + 55*c^8*d^2*x^2 + 165*c^7*d^3*x^3 + 330*c^6*d^4*x^4 + 4
62*c^5*d^5*x^5 + 462*c^4*d^6*x^6 + 330*c^3*d^7*x^7 + 165*c^2*d^8*x^8 + 55*c
*d^9*x^9 + 11*d^10*x^10))/(b^11*(a + b*x)^11)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^12,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(26) = 52$.
time = 0.14, size = 866, normalized size = 30.93

method	result
risch	$\frac{-\frac{d^{10} x^{10}}{b}-\frac{5 d^{9} \left(a d +b c \right) x^{9}}{b^{2}}-\frac{15 d^{8} \left(a^{2} d^{2}+a b c d +b^{2} c^{2}\right) x^{8}}{b^{3}}-\frac{30 d^{7} \left(a^{3} d^{3}+a^{2} b c \,d^{2}+a \,b^{2} c^{2} d +b^{3} c^{3}\right) x^{7}}{b^{4}}-\frac{42 d^{6} \left(a^{4} d^{4}+a^{3} b c \,d^{3}+a^{2} b^{2} c^{2} d^{2}+a \,b^{3} c^{3} d +b^{4}\right.}{b^{5}}}{}$
norman	$\frac{-\frac{d^{10} x^{10}}{b}+\frac{5 \left(-a \,d^{10}-b c \,d^{9}\right) x^{9}}{b^{2}}+\frac{15 \left(-a^{2} d^{10}-a b c \,d^{9}-b^{2} c^{2} d^{8}\right) x^{8}}{b^{3}}+\frac{30 \left(-a^{3} d^{10}-a^{2} b c \,d^{9}-a \,b^{2} c^{2} d^{8}-b^{3} c^{3} d^{7}\right) x^{7}}{b^{4}}+\frac{42 \left(-a^{4} d^{10}-a^{3} b c \,d^{9}-b^{2} a^{2} c^{2}\right.}{b^{5}}}{}$
default	$-\frac{d^{10}}{b^{11} \left(b x +a \right)}+\frac{30 d^{7} \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right)}{b^{11} \left(b x +a \right)^{4}}-\frac{5 d^{2} \left(a^{8} d^{8}-8 a^{7} b c \,d^{7}+28 a^{6} b^{2} c^{2} d^{6}-56 a^{5} b^{3} c^{3} d^{5}+70 a^{4} b^{4} c^{4} d^{4}-56 a^{3} b^{5} c^{5}\right.}{b^{11} \left(b x +a \right)^{9}}$
gosper	$-\frac{11 d^{10} x^{10} b^{10}+55 a \,b^{9} d^{10} x^{9}+55 b^{10} c \,d^{9} x^{9}+165 a^{2} b^{8} d^{10} x^{8}+165 a \,b^{9} c \,d^{9} x^{8}+165 b^{10} c^{2} d^{8} x^{8}+330 a^{3} b^{7} d^{10} x^{7}+330 a^{2} b^{8} c \,d^{9} x^{7}+330 a \,b^{9} c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] -d^10/b^11/(b*x+a)+30/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)/(b*x+a)^4-5/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3
```

```
*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c
^7*d+b^8*c^8)/(b*x+a)^9-30/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d
^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^7+1
5/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*
a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^8+1/b^11*
d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*
c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c
^8*d-b^9*c^9)/(b*x+a)^10-42/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^5-1/11*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b
^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*
a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*
c^10)/b^11/(b*x+a)^11+5/b^11*d^9*(a*d-b*c)/(b*x+a)^2+42/b^11*d^5*(a^5*d^5-5
*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(
b*x+a)^6-15/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(26) = 52$.
time = 0.34, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(26) = 52$.

time = 0.31, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**12,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(26) = 52.
time = 0.00, size = 1019, normalized size = 36.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x)
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + 55*b^10*c*d^9*x^9 + 55*a*b^9*d^10*x^9 + 165*b^10
*c^2*d^8*x^8 + 165*a*b^9*c*d^9*x^8 + 165*a^2*b^8*d^10*x^8 + 330*b^10*c^3*d^
7*x^7 + 330*a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7 + 330*a^3*b^7*d^10*x^
7 + 462*b^10*c^4*d^6*x^6 + 462*a*b^9*c^3*d^7*x^6 + 462*a^2*b^8*c^2*d^8*x^6
+ 462*a^3*b^7*c*d^9*x^6 + 462*a^4*b^6*d^10*x^6 + 462*b^10*c^5*d^5*x^5 + 462
*a*b^9*c^4*d^6*x^5 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 46
2*a^4*b^6*c*d^9*x^5 + 462*a^5*b^5*d^10*x^5 + 330*b^10*c^6*d^4*x^4 + 330*a*b
^9*c^5*d^5*x^4 + 330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^
4*b^6*c^2*d^8*x^4 + 330*a^5*b^5*c*d^9*x^4 + 330*a^6*b^4*d^10*x^4 + 165*b^10
*c^7*d^3*x^3 + 165*a*b^9*c^6*d^4*x^3 + 165*a^2*b^8*c^5*d^5*x^3 + 165*a^3*b^
7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 + 165*a^6
*b^4*c*d^9*x^3 + 165*a^7*b^3*d^10*x^3 + 55*b^10*c^8*d^2*x^2 + 55*a*b^9*c^7*
d^3*x^2 + 55*a^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*
d^6*x^2 + 55*a^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 55*a^7*b^3*c*d^
9*x^2 + 55*a^8*b^2*d^10*x^2 + 11*b^10*c^9*d*x + 11*a*b^9*c^8*d^2*x + 11*a^2
*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*c
^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 11*a^8*b^2*c*d^9*x
 + 11*a^9*b*d^10*x + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*b^7*c^
7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d
^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^11*b^11)
```

Mupad [B]
time = 0.46, size = 1066, normalized size = 38.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^12,x)
```

```
[Out] -(a^10*d^10 + b^10*c^10 + 11*b^10*d^10*x^10 + 55*a*b^9*d^10*x^9 + 55*b^10*c
*d^9*x^9 + a^2*b^8*c^8*d^2 + a^3*b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^
5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d^7 + a^8*b^2*c^2*d^8 + 55*a^8*b^2*d^
10*x^2 + 165*a^7*b^3*d^10*x^3 + 330*a^6*b^4*d^10*x^4 + 462*a^5*b^5*d^10*x^5
 + 462*a^4*b^6*d^10*x^6 + 330*a^3*b^7*d^10*x^7 + 165*a^2*b^8*d^10*x^8 + 55*
b^10*c^8*d^2*x^2 + 165*b^10*c^7*d^3*x^3 + 330*b^10*c^6*d^4*x^4 + 462*b^10*c
^5*d^5*x^5 + 462*b^10*c^4*d^6*x^6 + 330*b^10*c^3*d^7*x^7 + 165*b^10*c^2*d^8
*x^8 + a*b^9*c^9*d + a^9*b*c*d^9 + 11*a^9*b*d^10*x + 11*b^10*c^9*d*x + 55*a
^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*d^6*x^2 + 55*a
^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 165*a^2*b^8*c^5*d^5*x^3 + 165
*a^3*b^7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 +
330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^4*b^6*c^2*d^8*x^4
 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 462*a^2*b^8*c^2*d^8*
x^6 + 11*a*b^9*c^8*d^2*x + 11*a^8*b^2*c*d^9*x + 165*a*b^9*c*d^9*x^8 + 11*a^
2*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*
c^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 55*a*b^9*c^7*d^3*
```

x^2 + 55*a^7*b^3*c*d^9*x^2 + 165*a*b^9*c^6*d^4*x^3 + 165*a^6*b^4*c*d^9*x^3 + 330*a*b^9*c^5*d^5*x^4 + 330*a^5*b^5*c*d^9*x^4 + 462*a*b^9*c^4*d^6*x^5 + 462*a^4*b^6*c*d^9*x^5 + 462*a*b^9*c^3*d^7*x^6 + 462*a^3*b^7*c*d^9*x^6 + 330*a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7)/(11*a^11*b^11 + 11*b^22*x^11 + 121*a^10*b^12*x + 121*a*b^21*x^10 + 605*a^9*b^13*x^2 + 1815*a^8*b^14*x^3 + 3630*a^7*b^15*x^4 + 5082*a^6*b^16*x^5 + 5082*a^5*b^17*x^6 + 3630*a^4*b^18*x^7 + 1815*a^3*b^19*x^8 + 605*a^2*b^20*x^9)

3.1324 $\int \frac{(c+dx)^{10}}{(a+bx)^{13}} dx$

Optimal. Leaf size=58

$$-\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}}$$

```
[Out] -1/12*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^12+1/132*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^11
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -1/12*(c + d*x)^11/((b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^11)/(132*(b*c
- a*d)^2*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx = -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{12(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(58) = 116$.
time = 0.17, size = 684, normalized size = 11.79

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -1/132*(a^10*d^10 + 2*a^9*b*d^9*(c + 6*d*x) + 3*a^8*b^2*d^8*(c^2 + 8*c*d*x
+ 22*d^2*x^2) + 4*a^7*b^3*d^7*(c^3 + 9*c^2*d*x + 33*c*d^2*x^2 + 55*d^3*x^3)
 + a^6*b^4*d^6*(5*c^4 + 48*c^3*d*x + 198*c^2*d^2*x^2 + 440*c*d^3*x^3 + 495*
d^4*x^4) + 6*a^5*b^5*d^5*(c^5 + 10*c^4*d*x + 44*c^3*d^2*x^2 + 110*c^2*d^3*x
^3 + 165*c*d^4*x^4 + 132*d^5*x^5) + a^4*b^6*d^4*(7*c^6 + 72*c^5*d*x + 330*c
^4*d^2*x^2 + 880*c^3*d^3*x^3 + 1485*c^2*d^4*x^4 + 1584*c*d^5*x^5 + 924*d^6*
x^6) + 4*a^3*b^7*d^3*(2*c^7 + 21*c^6*d*x + 99*c^5*d^2*x^2 + 275*c^4*d^3*x^3
 + 495*c^3*d^4*x^4 + 594*c^2*d^5*x^5 + 462*c*d^6*x^6 + 198*d^7*x^7) + 3*a^2
*b^8*d^2*(3*c^8 + 32*c^7*d*x + 154*c^6*d^2*x^2 + 440*c^5*d^3*x^3 + 825*c^4*
d^4*x^4 + 1056*c^3*d^5*x^5 + 924*c^2*d^6*x^6 + 528*c*d^7*x^7 + 165*d^8*x^8)
 + 2*a*b^9*d*(5*c^9 + 54*c^8*d*x + 264*c^7*d^2*x^2 + 770*c^6*d^3*x^3 + 1485
*c^5*d^4*x^4 + 1980*c^4*d^5*x^5 + 1848*c^3*d^6*x^6 + 1188*c^2*d^7*x^7 + 495
*c*d^8*x^8 + 110*d^9*x^9) + b^10*(11*c^10 + 120*c^9*d*x + 594*c^8*d^2*x^2 +
 1760*c^7*d^3*x^3 + 3465*c^6*d^4*x^4 + 4752*c^5*d^5*x^5 + 4620*c^4*d^6*x^6
+ 3168*c^3*d^7*x^7 + 1485*c^2*d^8*x^8 + 440*c*d^9*x^9 + 66*d^10*x^10))/(b^1
1*(a + b*x)^12)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^13,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(54) = 108$.

time = 0.14, size = 867, normalized size = 14.95

method	result
risch	$\frac{-\frac{d^{10}x^{10}}{2b}-\frac{5d^9(ad+2bc)x^9}{3b^2}-\frac{15d^8\left(a^2d^2+2abcd+3b^2c^2\right)x^8}{4b^3}-\frac{6d^7\left(a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3\right)x^7}{b^4}-\frac{7d^6\left(a^4d^4+2a^3bc\,d^3+3a^2b^2c^2d^2+4a\,b^3c^3\right.}{b^5}}{}$
default	$-\frac{a^{10}d^{10}-10a^9bc\,d^9+45a^8b^2c^2d^8-120a^7b^3c^3d^7+210a^6b^4c^4d^6-252a^5b^5c^5d^5+210a^4b^6c^6d^4-120a^3b^7c^7d^3+45a^2b^8c^8d^2-10a\,b^9c^9d+b}{12b^{11}(bx+a)^{12}}$
norman	$\frac{-\frac{d^{10}x^{10}}{2b}+\frac{5\left(-ab\,d^{10}-2b^2c\,d^9\right)x^9}{3b^3}+\frac{15\left(-a^2b\,d^{10}-2a\,b^2c\,d^9-3b^3c^2d^8\right)x^8}{4b^4}+\frac{6\left(-a^3b\,d^{10}-2b^2a^2c\,d^9-3a\,b^3c^2d^8-4b^4c^3d^7\right)x^7}{b^5}+\frac{7\left(-a^4b\,d^{10}-2a^3b\right.}{}}{}$
gosper	$-\frac{66d^{10}x^{10}b^{10}+220a\,b^9d^{10}x^9+440b^{10}c\,d^9x^9+495a^2b^8d^{10}x^8+990a\,b^9c\,d^9x^8+1485b^{10}c^2d^8x^8+792a^3b^7d^{10}x^7+1584a^2b^8c\,d^9x^7+2376a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*
a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3
+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^12-45/4/b^11*d^8
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4+36/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+1
0*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^7+40/3/
b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^
3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^9-105/4/b^1
1*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b
^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^8-9/2/b^11*d^2*(a^8*d^8-8*a^7*b*c
*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^
5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^10+24/b^11*d^7*(a^3
*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^5+10/11/b^11*d*(a^9*d^9-9
*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*
a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9
)/(b*x+a)^11-1/2*d^10/b^11/(b*x+a)^2-35/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a
^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^6+10/3/b^11*d^9*(a*d-b*c)/(b*
x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(54) = 108$.

time = 0.32, size = 986, normalized size = 17.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(54) = 108$.
time = 0.30, size = 986, normalized size = 17.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="fricas")
```


```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
```

```
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**13,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(54) = 108.
time = 0.00, size = 1029, normalized size = 17.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x)
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 440*b^10*c*d^9*x^9 + 220*a*b^9*d^10*x^9 + 1485*
b^10*c^2*d^8*x^8 + 990*a*b^9*c*d^9*x^8 + 495*a^2*b^8*d^10*x^8 + 3168*b^10*c
^3*d^7*x^7 + 2376*a*b^9*c^2*d^8*x^7 + 1584*a^2*b^8*c*d^9*x^7 + 792*a^3*b^7*
d^10*x^7 + 4620*b^10*c^4*d^6*x^6 + 3696*a*b^9*c^3*d^7*x^6 + 2772*a^2*b^8*c^
2*d^8*x^6 + 1848*a^3*b^7*c*d^9*x^6 + 924*a^4*b^6*d^10*x^6 + 4752*b^10*c^5*d
^5*x^5 + 3960*a*b^9*c^4*d^6*x^5 + 3168*a^2*b^8*c^3*d^7*x^5 + 2376*a^3*b^7*c
^2*d^8*x^5 + 1584*a^4*b^6*c*d^9*x^5 + 792*a^5*b^5*d^10*x^5 + 3465*b^10*c^6*
d^4*x^4 + 2970*a*b^9*c^5*d^5*x^4 + 2475*a^2*b^8*c^4*d^6*x^4 + 1980*a^3*b^7*
c^3*d^7*x^4 + 1485*a^4*b^6*c^2*d^8*x^4 + 990*a^5*b^5*c*d^9*x^4 + 495*a^6*b^
4*d^10*x^4 + 1760*b^10*c^7*d^3*x^3 + 1540*a*b^9*c^6*d^4*x^3 + 1320*a^2*b^8*
c^5*d^5*x^3 + 1100*a^3*b^7*c^4*d^6*x^3 + 880*a^4*b^6*c^3*d^7*x^3 + 660*a^5*
b^5*c^2*d^8*x^3 + 440*a^6*b^4*c*d^9*x^3 + 220*a^7*b^3*d^10*x^3 + 594*b^10*c
^8*d^2*x^2 + 528*a*b^9*c^7*d^3*x^2 + 462*a^2*b^8*c^6*d^4*x^2 + 396*a^3*b^7*
c^5*d^5*x^2 + 330*a^4*b^6*c^4*d^6*x^2 + 264*a^5*b^5*c^3*d^7*x^2 + 198*a^6*b
^4*c^2*d^8*x^2 + 132*a^7*b^3*c*d^9*x^2 + 66*a^8*b^2*d^10*x^2 + 120*b^10*c^9
*d*x + 108*a*b^9*c^8*d^2*x + 96*a^2*b^8*c^7*d^3*x + 84*a^3*b^7*c^6*d^4*x +
72*a^4*b^6*c^5*d^5*x + 60*a^5*b^5*c^4*d^6*x + 48*a^6*b^4*c^3*d^7*x + 36*a^7
*b^3*c^2*d^8*x + 24*a^8*b^2*c*d^9*x + 12*a^9*b*d^10*x + 11*b^10*c^10 + 10*a
*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*
```

```
a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8
 + 2*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^12*b^11)
```

Mupad [B]
time = 0.39, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^{11}\,(12\,a\,d-11\,b\,c+b\,d\,x)}{132\,(a\,d-b\,c)^2\,(a+b\,x)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^13,x)
[Out] ((c + d*x)^11*(12*a*d - 11*b*c + b*d*x))/(132*(a*d - b*c)^2*(a + b*x)^12)
```

3.1325 $\int \frac{(c+dx)^{10}}{(a+bx)^{14}} dx$

Optimal. Leaf size=89

$$-\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}+\frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}}-\frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}$$

```
[Out] -1/13*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^13+1/78*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x+
a)^12-1/858*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^11
```

Rubi [A]

time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -1/13*(c + d*x)^11/((b*c - a*d)*(a + b*x)^13) + (d*(c + d*x)^11)/(78*(b*c -
 a*d)^2*(a + b*x)^12) - (d^2*(c + d*x)^11)/(858*(b*c - a*d)^3*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx = -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} - \frac{(2d)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{13(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} + \frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{78(bc-ad)^2}$$
$$= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} + \frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}} - \frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(89) = 178.
time = 0.18, size = 690, normalized size = 7.75

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -1/858*(a^10*d^10 + a^9*b*d^9*(3*c + 13*d*x) + 3*a^8*b^2*d^8*(2*c^2 + 13*c*
d*x + 26*d^2*x^2) + 2*a^7*b^3*d^7*(5*c^3 + 39*c^2*d*x + 117*c*d^2*x^2 + 143
*d^3*x^3) + a^6*b^4*d^6*(15*c^4 + 130*c^3*d*x + 468*c^2*d^2*x^2 + 858*c*d^3
*x^3 + 715*d^4*x^4) + 3*a^5*b^5*d^5*(7*c^5 + 65*c^4*d*x + 260*c^3*d^2*x^2 +
 572*c^2*d^3*x^3 + 715*c*d^4*x^4 + 429*d^5*x^5) + a^4*b^6*d^4*(28*c^6 + 273
*c^5*d*x + 1170*c^4*d^2*x^2 + 2860*c^3*d^3*x^3 + 4290*c^2*d^4*x^4 + 3861*c*
d^5*x^5 + 1716*d^6*x^6) + 2*a^3*b^7*d^3*(18*c^7 + 182*c^6*d*x + 819*c^5*d^2
*x^2 + 2145*c^4*d^3*x^3 + 3575*c^3*d^4*x^4 + 3861*c^2*d^5*x^5 + 2574*c*d^6*
x^6 + 858*d^7*x^7) + 3*a^2*b^8*d^2*(15*c^8 + 156*c^7*d*x + 728*c^6*d^2*x^2
+ 2002*c^5*d^3*x^3 + 3575*c^4*d^4*x^4 + 4290*c^3*d^5*x^5 + 3432*c^2*d^6*x^6
 + 1716*c*d^7*x^7 + 429*d^8*x^8) + a*b^9*d*(55*c^9 + 585*c^8*d*x + 2808*c^7
*d^2*x^2 + 8008*c^6*d^3*x^3 + 15015*c^5*d^4*x^4 + 19305*c^4*d^5*x^5 + 17160
*c^3*d^6*x^6 + 10296*c^2*d^7*x^7 + 3861*c*d^8*x^8 + 715*d^9*x^9) + b^10*(66
*c^10 + 715*c^9*d*x + 3510*c^8*d^2*x^2 + 10296*c^7*d^3*x^3 + 20020*c^6*d^4*
x^4 + 27027*c^5*d^5*x^5 + 25740*c^4*d^6*x^6 + 17160*c^3*d^7*x^7 + 7722*c^2*
d^8*x^8 + 2145*c*d^9*x^9 + 286*d^10*x^10))/(b^11*(a + b*x)^13)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^14,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(83) = 166$.

time = 0.15, size = 867, normalized size = 9.74

method	result
risch	$-\frac{d^{10}x^{10}}{3b} - \frac{5d^9(ad+3bc)x^9}{6b^2} - \frac{3d^8\left(a^2d^2+3abcd+6b^2c^2\right)x^8}{2b^3} - \frac{2d^7\left(a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3\right)x^7}{b^4} - \frac{2d^6\left(a^4d^4+3a^3bc\,d^3+6a^2b^2c^2d^2+10a\,b^3c\right.}{b^5}$
default	$-\frac{a^{10}d^{10}-10a^9bc\,d^9+45a^8b^2c^2d^8-120a^7b^3c^3d^7+210a^6b^4c^4d^6-252a^5b^5c^5d^5+210a^4b^6c^6d^4-120a^3b^7c^7d^3+45a^2b^8c^8d^2-10a\,b^9c^9d+b}{13b^{11}(bx+a)^{13}}$
norman	$-\frac{d^{10}x^{10}}{3b} + \frac{5\left(-a\,b^2d^{10}-3b^3c\,d^9\right)x^9}{6b^4} + \frac{3\left(-b^2a^2d^{10}-3a\,b^3c\,d^9-6b^4c^2d^8\right)x^8}{2b^5} + \frac{2\left(-a^3b^2d^{10}-3a^2b^3c\,d^9-6a\,b^4c^2d^8-10b^5c^3d^7\right)x^7}{b^6} + \frac{2\left(-a^4b^2d^{10}-3\right.}{}$
gosper	$-\frac{286d^{10}x^{10}b^{10}+715a\,b^9d^{10}x^9+2145b^{10}c\,d^9x^9+1287a^2b^8d^{10}x^8+3861a\,b^9c\,d^9x^8+7722b^{10}c^2d^8x^8+1716a^3b^7d^{10}x^7+5148a^2b^8c\,d^9x^7+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*
a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3
+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^13-30/b^11*d^6*(
a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^7+5/
6/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a
^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*
a*b^8*c^8*d-b^9*c^9)/(b*x+a)^12+5/2/b^11*d^9*(a*d-b*c)/(b*x+a)^4-70/3/b^11*
d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4
*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^9+63/2/b^11*d^5*(a^5*d^5-5*a^4*b*c*
d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^8+
12/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35
*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^10-9/b^1
1*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^5-45/11/b^11*d^2*(a^8*d^8-8*a^7*b
*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*
c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^11+20/b^11*d^7*(a
^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^6-1/3*d^10/b^11/(b*x+a)
^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(83) = 166$.

time = 0.37, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(83) = 166$.
time = 0.30, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="fricas")
```


```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
```

```
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**14,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(83) = 166.
time = 0.00, size = 1029, normalized size = 11.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x)
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 2145*b^10*c*d^9*x^9 + 715*a*b^9*d^10*x^9 + 772
2*b^10*c^2*d^8*x^8 + 3861*a*b^9*c*d^9*x^8 + 1287*a^2*b^8*d^10*x^8 + 17160*b
^10*c^3*d^7*x^7 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7 + 1716*a
^3*b^7*d^10*x^7 + 25740*b^10*c^4*d^6*x^6 + 17160*a*b^9*c^3*d^7*x^6 + 10296*
a^2*b^8*c^2*d^8*x^6 + 5148*a^3*b^7*c*d^9*x^6 + 1716*a^4*b^6*d^10*x^6 + 2702
7*b^10*c^5*d^5*x^5 + 19305*a*b^9*c^4*d^6*x^5 + 12870*a^2*b^8*c^3*d^7*x^5 +
7722*a^3*b^7*c^2*d^8*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 1287*a^5*b^5*d^10*x^5 +
 20020*b^10*c^6*d^4*x^4 + 15015*a*b^9*c^5*d^5*x^4 + 10725*a^2*b^8*c^4*d^6*x
^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 2145*a^5*b^5*c*d
^9*x^4 + 715*a^6*b^4*d^10*x^4 + 10296*b^10*c^7*d^3*x^3 + 8008*a*b^9*c^6*d^4
*x^3 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^6*x^3 + 2860*a^4*b^6*c
^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 858*a^6*b^4*c*d^9*x^3 + 286*a^7*b^3
*d^10*x^3 + 3510*b^10*c^8*d^2*x^2 + 2808*a*b^9*c^7*d^3*x^2 + 2184*a^2*b^8*c
^6*d^4*x^2 + 1638*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*
b^5*c^3*d^7*x^2 + 468*a^6*b^4*c^2*d^8*x^2 + 234*a^7*b^3*c*d^9*x^2 + 78*a^8*
b^2*d^10*x^2 + 715*b^10*c^9*d*x + 585*a*b^9*c^8*d^2*x + 468*a^2*b^8*c^7*d^3
*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^5*c^4*d^6*x
+ 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 39*a^8*b^2*c*d^9*x + 13*a^
9*b*d^10*x + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^
```

```
7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 +
10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10)/((b*x +
 a)^13*b^11)
```

Mupad [B]
time = 0.48, size = 1098, normalized size = 12.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^14,x)
```

```
[Out] -(a^10*d^10 + 66*b^10*c^10 + 286*b^10*d^10*x^10 + 715*a*b^9*d^10*x^9 + 2145
*b^10*c*d^9*x^9 + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*
d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*
b^2*c^2*d^8 + 78*a^8*b^2*d^10*x^2 + 286*a^7*b^3*d^10*x^3 + 715*a^6*b^4*d^10
*x^4 + 1287*a^5*b^5*d^10*x^5 + 1716*a^4*b^6*d^10*x^6 + 1716*a^3*b^7*d^10*x^
7 + 1287*a^2*b^8*d^10*x^8 + 3510*b^10*c^8*d^2*x^2 + 10296*b^10*c^7*d^3*x^3
+ 20020*b^10*c^6*d^4*x^4 + 27027*b^10*c^5*d^5*x^5 + 25740*b^10*c^4*d^6*x^6
+ 17160*b^10*c^3*d^7*x^7 + 7722*b^10*c^2*d^8*x^8 + 55*a*b^9*c^9*d + 3*a^9*b
*c*d^9 + 13*a^9*b*d^10*x + 715*b^10*c^9*d*x + 2184*a^2*b^8*c^6*d^4*x^2 + 16
38*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*b^5*c^3*d^7*x^2
 + 468*a^6*b^4*c^2*d^8*x^2 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^
6*x^3 + 2860*a^4*b^6*c^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 10725*a^2*b^8
*c^4*d^6*x^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 12870*
a^2*b^8*c^3*d^7*x^5 + 7722*a^3*b^7*c^2*d^8*x^5 + 10296*a^2*b^8*c^2*d^8*x^6
+ 585*a*b^9*c^8*d^2*x + 39*a^8*b^2*c*d^9*x + 3861*a*b^9*c*d^9*x^8 + 468*a^2
*b^8*c^7*d^3*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^
5*c^4*d^6*x + 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 2808*a*b^9*c^7
*d^3*x^2 + 234*a^7*b^3*c*d^9*x^2 + 8008*a*b^9*c^6*d^4*x^3 + 858*a^6*b^4*c*d
^9*x^3 + 15015*a*b^9*c^5*d^5*x^4 + 2145*a^5*b^5*c*d^9*x^4 + 19305*a*b^9*c^4
*d^6*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 17160*a*b^9*c^3*d^7*x^6 + 5148*a^3*b^7*
c*d^9*x^6 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7)/(858*a^13*b^1
1 + 858*b^24*x^13 + 11154*a^12*b^12*x + 11154*a*b^23*x^12 + 66924*a^11*b^13
*x^2 + 245388*a^10*b^14*x^3 + 613470*a^9*b^15*x^4 + 1104246*a^8*b^16*x^5 +
1472328*a^7*b^17*x^6 + 1472328*a^6*b^18*x^7 + 1104246*a^5*b^19*x^8 + 613470
*a^4*b^20*x^9 + 245388*a^3*b^21*x^10 + 66924*a^2*b^22*x^11)
```

3.1326 $\int \frac{(c+dx)^{10}}{(a+bx)^{15}} dx$

Optimal. Leaf size=120

$$-\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}-\frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}}+\frac{d^3(c+dx)^{11}}{4004(bc-ad)^4(a+bx)^{11}}$$

```
[Out] -1/14*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^14+3/182*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^13-1/364*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^12+1/4004*d^3*(d*x+c)^11/(
-a*d+b*c)^4/(b*x+a)^11
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -1/14*(c + d*x)^11/((b*c - a*d)*(a + b*x)^14) + (3*d*(c + d*x)^11)/(182*(b*
c - a*d)^2*(a + b*x)^13) - (d^2*(c + d*x)^11)/(364*(b*c - a*d)^3*(a + b*x)^
12) + (d^3*(c + d*x)^11)/(4004*(b*c - a*d)^4*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx = -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} - \frac{(3d)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{14(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} + \frac{(3d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{91(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} - \frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}} - \frac{d^3\int \frac{(c}{(a}}{364(b}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} - \frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}} + \frac{}{4004(b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 692 vs. $2(120) = 240$.
time = 0.17, size = 692, normalized size = 5.77

$$-\frac{a^{10}d^{10}+2a^9bd^9(2c+7dx)+a^8b^2d^8(10c^2+56cdx+91d^2x^2)+4a^7b^3d^7(5c^3+35c^2dx+91cd^2x^2+91d^3x^3)+7a^6b^4d^6(5c^4+40c^3dx+130c^2d^2x^2+208cd^3x^3+143d^4x^4)+14a^5b^5d^5(4c^5+35c^4dx+130c^3d^2x^2+260c^2d^3x^3+286cd^4x^4+143d^5x^5)+7a^4b^6d^4(12c^6+112c^5dx+455c^4d^2x^2+1040c^3d^3x^3+1430c^2d^4x^4+1144cd^5x^5+429d^6x^6)+4a^3b^7d^3(30c^7+294c^6dx+1274c^5d^2x^2+3185c^4d^3x^3+5005c^3d^4x^4+5005c^2d^5x^5+3003cd^6x^6+858d^7x^7)+a^2b^8d^2(165c^8+1680c^7dx+7644c^6d^2x^2+20384c^5d^3x^3+35035c^4d^4x^4+40040c^3d^5x^5+30030c^2d^6x^6+13728cd^7x^7+3003d^8x^8)+2ab^9d(110c^9+1155c^8dx+5460c^7d^2x^2+15288c^6d^3x^3+28028c^5d^4x^4+35035c^4d^5x^5+30030c^3d^6x^6+17160c^2d^7x^7+6006cd^8x^8+1001d^9x^9)+b^{10}(286c^{10}+3080c^9dx+15015c^8d^2x^2+43680c^7d^3x^3+84084c^6d^4x^4+112112c^5d^5x^5+105105c^4d^6x^6+68640c^3d^7x^7+30030c^2d^8x^8+8008cd^9x^9+1001d^{10}x^{10})}{4004b^{11}(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -1/4004*(a^10*d^10 + 2*a^9*b*d^9*(2*c + 7*d*x) + a^8*b^2*d^8*(10*c^2 + 56*c
*d*x + 91*d^2*x^2) + 4*a^7*b^3*d^7*(5*c^3 + 35*c^2*d*x + 91*c*d^2*x^2 + 91*
d^3*x^3) + 7*a^6*b^4*d^6*(5*c^4 + 40*c^3*d*x + 130*c^2*d^2*x^2 + 208*c*d^3*
x^3 + 143*d^4*x^4) + 14*a^5*b^5*d^5*(4*c^5 + 35*c^4*d*x + 130*c^3*d^2*x^2 +
 260*c^2*d^3*x^3 + 286*c*d^4*x^4 + 143*d^5*x^5) + 7*a^4*b^6*d^4*(12*c^6 + 1
12*c^5*d*x + 455*c^4*d^2*x^2 + 1040*c^3*d^3*x^3 + 1430*c^2*d^4*x^4 + 1144*c
*d^5*x^5 + 429*d^6*x^6) + 4*a^3*b^7*d^3*(30*c^7 + 294*c^6*d*x + 1274*c^5*d^
2*x^2 + 3185*c^4*d^3*x^3 + 5005*c^3*d^4*x^4 + 5005*c^2*d^5*x^5 + 3003*c*d^6
*x^6 + 858*d^7*x^7) + a^2*b^8*d^2*(165*c^8 + 1680*c^7*d*x + 7644*c^6*d^2*x^
2 + 20384*c^5*d^3*x^3 + 35035*c^4*d^4*x^4 + 40040*c^3*d^5*x^5 + 30030*c^2*d
^6*x^6 + 13728*c*d^7*x^7 + 3003*d^8*x^8) + 2*a*b^9*d*(110*c^9 + 1155*c^8*d*
x + 5460*c^7*d^2*x^2 + 15288*c^6*d^3*x^3 + 28028*c^5*d^4*x^4 + 35035*c^4*d^
5*x^5 + 30030*c^3*d^6*x^6 + 17160*c^2*d^7*x^7 + 6006*c*d^8*x^8 + 1001*d^9*x
^9) + b^10*(286*c^10 + 3080*c^9*d*x + 15015*c^8*d^2*x^2 + 43680*c^7*d^3*x^3
 + 84084*c^6*d^4*x^4 + 112112*c^5*d^5*x^5 + 105105*c^4*d^6*x^6 + 68640*c^3*
d^7*x^7 + 30030*c^2*d^8*x^8 + 8008*c*d^9*x^9 + 1001*d^10*x^10))/(b^11*(a +
b*x)^14)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^15,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(112) = 224$.
time = 0.15, size = 867, normalized size = 7.22

method	result
risch	$\frac{-\frac{d^{10}x^{10}}{4b}-\frac{d^{9}(ad+4bc)x^{9}}{2b^{2}}-\frac{3d^{8}\left(a^{2}d^{2}+4abcd+10b^{2}c^{2}\right)x^{8}}{4b^{3}}-\frac{6d^{7}\left(a^{3}d^{3}+4a^{2}bc\,d^{2}+10a\,b^{2}c^{2}d+20b^{3}c^{3}\right)x^{7}}{7b^{4}}-\frac{3d^{6}\left(a^{4}d^{4}+4a^{3}bc\,d^{3}+10a^{2}b^{2}c^{2}d^{2}+20a\,b\cdots\right)}{4b^{5}}\cdots}{\cdots}$
default	$\frac{10d\left(a^{9}d^{9}-9a^{8}bc\,d^{8}+36a^{7}b^{2}c^{2}d^{7}-84a^{6}b^{3}c^{3}d^{6}+126a^{5}b^{4}c^{4}d^{5}-126a^{4}b^{5}c^{5}d^{4}+84a^{3}b^{6}c^{6}d^{3}-36a^{2}b^{7}c^{7}d^{2}+9a\,b^{8}c^{8}d-b^{9}c^{9}\right)}{13b^{11}(bx+a)^{13}}-\frac{15d^{2}\left(a^{8}d^{8}\cdots\right)}{\cdots}$
norman	$\frac{\frac{-a^{10}b^{3}d^{10}-4a^{9}b^{4}c\,d^{9}-10a^{8}b^{5}c^{2}d^{8}-20a^{7}b^{6}c^{3}d^{7}-35a^{6}b^{7}c^{4}d^{6}-56a^{5}b^{8}c^{5}d^{5}-84a^{4}b^{9}c^{6}d^{4}-120a^{3}b^{10}c^{7}d^{3}-165a^{2}b^{11}c^{8}d^{2}-220a\,b^{12}c^{9}d-286b^{13}c^{10}\cdots}{4004b^{14}}\cdots}{\cdots}$
gosper	$-\frac{1001d^{10}x^{10}b^{10}+2002a\,b^{9}d^{10}x^{9}+8008b^{10}c\,d^{9}x^{9}+3003a^{2}b^{8}d^{10}x^{8}+12012a\,b^{9}c\,d^{9}x^{8}+30030b^{10}c^{2}d^{8}x^{8}+3432a^{3}b^{7}d^{10}x^{7}+13728a^{2}b^{8}c\,d\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^15,x,method=_RETURNVERBOSE)
```

```
[Out] 10/13/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+1
26*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^
2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^13-15/4/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28
*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+2
8*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^12-1/14*(a^10*d^10-10*a^9*
b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*
b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a
*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^14-1/4*d^10/b^11/(b*x+a)^4+28/b^11*d^5*(
a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b
^5*c^5)/(b*x+a)^9-105/4/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^8-21/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b
^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*
x+a)^10+2/b^11*d^9*(a*d-b*c)/(b*x+a)^5+120/11/b^11*d^3*(a^7*d^7-7*a^6*b*c*d
^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*
d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^11-15/2/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*
c^2)/(b*x+a)^6+120/7/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/(b*x+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(112) = 224$.
time = 0.33, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(112) = 224$.
time = 0.30, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
```

```
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**15,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(112) = 224.
time = 0.00, size = 1029, normalized size = 8.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x)
[Out] -1/4004*(1001*b^10*d^10*x^10 + 8008*b^10*c*d^9*x^9 + 2002*a*b^9*d^10*x^9 +
30030*b^10*c^2*d^8*x^8 + 12012*a*b^9*c*d^9*x^8 + 3003*a^2*b^8*d^10*x^8 + 68
640*b^10*c^3*d^7*x^7 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2*b^8*c*d^9*x^7 +
3432*a^3*b^7*d^10*x^7 + 105105*b^10*c^4*d^6*x^6 + 60060*a*b^9*c^3*d^7*x^6 +
 30030*a^2*b^8*c^2*d^8*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 3003*a^4*b^6*d^10*x^
6 + 112112*b^10*c^5*d^5*x^5 + 70070*a*b^9*c^4*d^6*x^5 + 40040*a^2*b^8*c^3*d
^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 2002*a^5*b^5*
d^10*x^5 + 84084*b^10*c^6*d^4*x^4 + 56056*a*b^9*c^5*d^5*x^4 + 35035*a^2*b^8
*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^2*d^8*x^4 + 4004
*a^5*b^5*c*d^9*x^4 + 1001*a^6*b^4*d^10*x^4 + 43680*b^10*c^7*d^3*x^3 + 30576
*a*b^9*c^6*d^4*x^3 + 20384*a^2*b^8*c^5*d^5*x^3 + 12740*a^3*b^7*c^4*d^6*x^3
+ 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3 + 1456*a^6*b^4*c*d^9*
x^3 + 364*a^7*b^3*d^10*x^3 + 15015*b^10*c^8*d^2*x^2 + 10920*a*b^9*c^7*d^3*x
^2 + 7644*a^2*b^8*c^6*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4
```

```
*d^6*x^2 + 1820*a^5*b^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 364*a^7*b^3
*c*d^9*x^2 + 91*a^8*b^2*d^10*x^2 + 3080*b^10*c^9*d*x + 2310*a*b^9*c^8*d^2*x
 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6*c^5*d^5*x
+ 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^2*d^8*x + 5
6*a^8*b^2*c*d^9*x + 14*a^9*b*d^10*x + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165
*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^
5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^
9*b*c*d^9 + a^10*d^10)/((b*x + a)^14*b^11)
```

Mupad [B]
time = 1.30, size = 1109, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^15,x)
```

```
[Out] -(a^10*d^10 + 286*b^10*c^10 + 1001*b^10*d^10*x^10 + 2002*a*b^9*d^10*x^9 + 8
008*b^10*c*d^9*x^9 + 165*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6
*c^6*d^4 + 56*a^5*b^5*c^5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 1
0*a^8*b^2*c^2*d^8 + 91*a^8*b^2*d^10*x^2 + 364*a^7*b^3*d^10*x^3 + 1001*a^6*b
^4*d^10*x^4 + 2002*a^5*b^5*d^10*x^5 + 3003*a^4*b^6*d^10*x^6 + 3432*a^3*b^7*
d^10*x^7 + 3003*a^2*b^8*d^10*x^8 + 15015*b^10*c^8*d^2*x^2 + 43680*b^10*c^7*
d^3*x^3 + 84084*b^10*c^6*d^4*x^4 + 112112*b^10*c^5*d^5*x^5 + 105105*b^10*c^
4*d^6*x^6 + 68640*b^10*c^3*d^7*x^7 + 30030*b^10*c^2*d^8*x^8 + 220*a*b^9*c^9
*d + 4*a^9*b*c*d^9 + 14*a^9*b*d^10*x + 3080*b^10*c^9*d*x + 7644*a^2*b^8*c^6
*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4*d^6*x^2 + 1820*a^5*b
^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 20384*a^2*b^8*c^5*d^5*x^3 + 1274
0*a^3*b^7*c^4*d^6*x^3 + 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3
 + 35035*a^2*b^8*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^
2*d^8*x^4 + 40040*a^2*b^8*c^3*d^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 30030*a
^2*b^8*c^2*d^8*x^6 + 2310*a*b^9*c^8*d^2*x + 56*a^8*b^2*c*d^9*x + 12012*a*b^
9*c*d^9*x^8 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6
*c^5*d^5*x + 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^
2*d^8*x + 10920*a*b^9*c^7*d^3*x^2 + 364*a^7*b^3*c*d^9*x^2 + 30576*a*b^9*c^6
*d^4*x^3 + 1456*a^6*b^4*c*d^9*x^3 + 56056*a*b^9*c^5*d^5*x^4 + 4004*a^5*b^5*
c*d^9*x^4 + 70070*a*b^9*c^4*d^6*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 60060*a*b^9*
c^3*d^7*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2
*b^8*c*d^9*x^7)/(4004*a^14*b^11 + 4004*b^25*x^14 + 56056*a^13*b^12*x + 5605
6*a*b^24*x^13 + 364364*a^12*b^13*x^2 + 1457456*a^11*b^14*x^3 + 4008004*a^10
*b^15*x^4 + 8016008*a^9*b^16*x^5 + 12024012*a^8*b^17*x^6 + 13741728*a^7*b^1
8*x^7 + 12024012*a^6*b^19*x^8 + 8016008*a^5*b^20*x^9 + 4008004*a^4*b^21*x^1
0 + 1457456*a^3*b^22*x^11 + 364364*a^2*b^23*x^12)
```

3.1327 $\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$

Optimal. Leaf size=151

$$-\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}+\frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}}-\frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}}+\frac{d^3(c+dx)^{11}}{1365(bc-ad)^4(a+bx)^{12}}-150$$

```
[Out] -1/15*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^15+2/105*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^14-2/455*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^13+1/1365*d^3*(d*x+c)^11/(
-a*d+b*c)^4/(b*x+a)^12-1/15015*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^11
```

Rubi [A]
time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-\frac{(c+dx)^{11}}{15(a+bx)^{15}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -1/15*(c + d*x)^11/((b*c - a*d)*(a + b*x)^15) + (2*d*(c + d*x)^11)/(105*(b*
c - a*d)^2*(a + b*x)^14) - (2*d^2*(c + d*x)^11)/(455*(b*c - a*d)^3*(a + b*x
)^13) + (d^3*(c + d*x)^11)/(1365*(b*c - a*d)^4*(a + b*x)^12) - (d^4*(c + d*
x)^11)/(15015*(b*c - a*d)^5*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx = -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} - \frac{(4d)\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{15(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} + \frac{(2d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} - \frac{(4d^3)}{455(}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{}{1365(}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{}{1365(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 690 vs. $2(151) = 302$.
time = 0.18, size = 690, normalized size = 4.57

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -1/15015*(a^10*d^10 + 5*a^9*b*d^9*(c + 3*d*x) + 15*a^8*b^2*d^8*(c^2 + 5*c*d
*x + 7*d^2*x^2) + 5*a^7*b^3*d^7*(7*c^3 + 45*c^2*d*x + 105*c*d^2*x^2 + 91*d^
3*x^3) + 35*a^6*b^4*d^6*(2*c^4 + 15*c^3*d*x + 45*c^2*d^2*x^2 + 65*c*d^3*x^3
 + 39*d^4*x^4) + 21*a^5*b^5*d^5*(6*c^5 + 50*c^4*d*x + 175*c^3*d^2*x^2 + 325
*c^2*d^3*x^3 + 325*c*d^4*x^4 + 143*d^5*x^5) + 35*a^4*b^6*d^4*(6*c^6 + 54*c^
5*d*x + 210*c^4*d^2*x^2 + 455*c^3*d^3*x^3 + 585*c^2*d^4*x^4 + 429*c*d^5*x^5
 + 143*d^6*x^6) + 5*a^3*b^7*d^3*(66*c^7 + 630*c^6*d*x + 2646*c^5*d^2*x^2 +
6370*c^4*d^3*x^3 + 9555*c^3*d^4*x^4 + 9009*c^2*d^5*x^5 + 5005*c*d^6*x^6 + 1
287*d^7*x^7) + 15*a^2*b^8*d^2*(33*c^8 + 330*c^7*d*x + 1470*c^6*d^2*x^2 + 38
22*c^5*d^3*x^3 + 6370*c^4*d^4*x^4 + 7007*c^3*d^5*x^5 + 5005*c^2*d^6*x^6 + 2
145*c*d^7*x^7 + 429*d^8*x^8) + 5*a*b^9*d*(143*c^9 + 1485*c^8*d*x + 6930*c^7
*d^2*x^2 + 19110*c^6*d^3*x^3 + 34398*c^5*d^4*x^4 + 42042*c^4*d^5*x^5 + 3503
5*c^3*d^6*x^6 + 19305*c^2*d^7*x^7 + 6435*c*d^8*x^8 + 1001*d^9*x^9) + b^10*(
1001*c^10 + 10725*c^9*d*x + 51975*c^8*d^2*x^2 + 150150*c^7*d^3*x^3 + 286650
*c^6*d^4*x^4 + 378378*c^5*d^5*x^5 + 350350*c^4*d^6*x^6 + 225225*c^3*d^7*x^7
 + 96525*c^2*d^8*x^8 + 25025*c*d^9*x^9 + 3003*d^10*x^10))/(b^11*(a + b*x)^1
5)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^16,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(141) = 282$.
time = 0.14, size = 867, normalized size = 5.74

method	result
risch	$\frac{-\frac{a^{10}d^{10}+5a^9bc\,d^9+15a^8b^2c^2d^8+35a^7b^3c^3d^7+70a^6b^4c^4d^6+126a^5b^5c^5d^5+210a^4b^6c^6d^4+330a^3b^7c^7d^3+495a^2b^8c^8d^2+715a\,b^9c^9d+1001b^{10}c^{10}}{15015b^{11}}-}{}$
default	$-\frac{45d^2\left(a^8d^8-8a^7bc\,d^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6d^2-8a\,b^7c^7d+b^8c^8\right)}{13b^{11}(bx+a)^{13}}+\frac{10d^3\left(a^7d^7-7a^6bc\,d^6+\right.}{}$
norman	$\frac{\frac{-a^{10}b^4d^{10}-5a^9b^5c\,d^9-15a^8b^6c^2d^8-35a^7b^7c^3d^7-70a^6b^8c^4d^6-126a^5b^9c^5d^5-210a^4b^{10}c^6d^4-330a^3b^{11}c^7d^3-495a^2b^{12}c^8d^2-715a\,b^{13}c^9d-1001b^1}{15015b^{15}}}{}$
gosper	$-\frac{3003d^{10}x^{10}b^{10}+5005a\,b^9d^{10}x^9+25025b^{10}c\,d^9x^9+6435a^2b^8d^{10}x^8+32175a\,b^9c\,d^9x^8+96525b^{10}c^2d^8x^8+6435a^3b^7d^{10}x^7+32175a^2b^8c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^16,x,method=_RETURNVERBOSE)
```

```
[Out] -45/13/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^
5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^
8*c^8)/(b*x+a)^13+10/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*
a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7
)/(b*x+a)^12+5/7/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^
3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2
*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^14-70/3/b^11*d^6*(a^4*d^4-4*a^3
*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^9+15/b^11*d^7*(a^
3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^8+126/5/b^11*d^5*(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)/(b*x+a)^10-1/5*d^10/b^11/(b*x+a)^5-210/11/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5
+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6
*c^6)/(b*x+a)^11-45/7/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^7+5/3/b^
11*d^9*(a*d-b*c)/(b*x+a)^6-1/15*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^
8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c
^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^1
1/(b*x+a)^15
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(141) = 282$.
time = 0.33, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(141) = 282$.
time = 0.30, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
```

```
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**16,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(141) = 282.
time = 0.00, size = 1029, normalized size = 6.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x)
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 25025*b^10*c*d^9*x^9 + 5005*a*b^9*d^10*x^9
+ 96525*b^10*c^2*d^8*x^8 + 32175*a*b^9*c*d^9*x^8 + 6435*a^2*b^8*d^10*x^8 +
225225*b^10*c^3*d^7*x^7 + 96525*a*b^9*c^2*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7
 + 6435*a^3*b^7*d^10*x^7 + 350350*b^10*c^4*d^6*x^6 + 175175*a*b^9*c^3*d^7*x
^6 + 75075*a^2*b^8*c^2*d^8*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 5005*a^4*b^6*d^1
0*x^6 + 378378*b^10*c^5*d^5*x^5 + 210210*a*b^9*c^4*d^6*x^5 + 105105*a^2*b^8
*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8*x^5 + 15015*a^4*b^6*c*d^9*x^5 + 3003*a
^5*b^5*d^10*x^5 + 286650*b^10*c^6*d^4*x^4 + 171990*a*b^9*c^5*d^5*x^4 + 9555
0*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 20475*a^4*b^6*c^2*d^8*x
^4 + 6825*a^5*b^5*c*d^9*x^4 + 1365*a^6*b^4*d^10*x^4 + 150150*b^10*c^7*d^3*x
^3 + 95550*a*b^9*c^6*d^4*x^3 + 57330*a^2*b^8*c^5*d^5*x^3 + 31850*a^3*b^7*c^
4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^5*c^2*d^8*x^3 + 2275*a^6
```

```
*b^4*c*d^9*x^3 + 455*a^7*b^3*d^10*x^3 + 51975*b^10*c^8*d^2*x^2 + 34650*a*b^
9*c^7*d^3*x^2 + 22050*a^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 735
0*a^4*b^6*c^4*d^6*x^2 + 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2
 + 525*a^7*b^3*c*d^9*x^2 + 105*a^8*b^2*d^10*x^2 + 10725*b^10*c^9*d*x + 7425
*a*b^9*c^8*d^2*x + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x + 1890*a
^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x + 225*a^7
*b^3*c^2*d^8*x + 75*a^8*b^2*c*d^9*x + 15*a^9*b*d^10*x + 1001*b^10*c^10 + 71
5*a*b^9*c^9*d + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6
*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a
^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^15*b^11)
```

Mupad [B]
time = 2.28, size = 1120, normalized size = 7.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^16,x)
```

```
[Out] -(a^10*d^10 + 1001*b^10*c^10 + 3003*b^10*d^10*x^10 + 5005*a*b^9*d^10*x^9 +
25025*b^10*c*d^9*x^9 + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7
 + 15*a^8*b^2*c^2*d^8 + 105*a^8*b^2*d^10*x^2 + 455*a^7*b^3*d^10*x^3 + 1365*
a^6*b^4*d^10*x^4 + 3003*a^5*b^5*d^10*x^5 + 5005*a^4*b^6*d^10*x^6 + 6435*a^3
*b^7*d^10*x^7 + 6435*a^2*b^8*d^10*x^8 + 51975*b^10*c^8*d^2*x^2 + 150150*b^1
0*c^7*d^3*x^3 + 286650*b^10*c^6*d^4*x^4 + 378378*b^10*c^5*d^5*x^5 + 350350*
b^10*c^4*d^6*x^6 + 225225*b^10*c^3*d^7*x^7 + 96525*b^10*c^2*d^8*x^8 + 715*a
*b^9*c^9*d + 5*a^9*b*c*d^9 + 15*a^9*b*d^10*x + 10725*b^10*c^9*d*x + 22050*a
^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 7350*a^4*b^6*c^4*d^6*x^2 +
 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2 + 57330*a^2*b^8*c^5*d^
5*x^3 + 31850*a^3*b^7*c^4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^
5*c^2*d^8*x^3 + 95550*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 204
75*a^4*b^6*c^2*d^8*x^4 + 105105*a^2*b^8*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8
*x^5 + 75075*a^2*b^8*c^2*d^8*x^6 + 7425*a*b^9*c^8*d^2*x + 75*a^8*b^2*c*d^9*
x + 32175*a*b^9*c*d^9*x^8 + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x
 + 1890*a^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x
+ 225*a^7*b^3*c^2*d^8*x + 34650*a*b^9*c^7*d^3*x^2 + 525*a^7*b^3*c*d^9*x^2 +
 95550*a*b^9*c^6*d^4*x^3 + 2275*a^6*b^4*c*d^9*x^3 + 171990*a*b^9*c^5*d^5*x^
4 + 6825*a^5*b^5*c*d^9*x^4 + 210210*a*b^9*c^4*d^6*x^5 + 15015*a^4*b^6*c*d^9
*x^5 + 175175*a*b^9*c^3*d^7*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 96525*a*b^9*c^2
*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7)/(15015*a^15*b^11 + 15015*b^26*x^15 + 22
5225*a^14*b^12*x + 225225*a*b^25*x^14 + 1576575*a^13*b^13*x^2 + 6831825*a^1
2*b^14*x^3 + 20495475*a^11*b^15*x^4 + 45090045*a^10*b^16*x^5 + 75150075*a^9
*b^17*x^6 + 96621525*a^8*b^18*x^7 + 96621525*a^7*b^19*x^8 + 75150075*a^6*b^
20*x^9 + 45090045*a^5*b^21*x^10 + 20495475*a^4*b^22*x^11 + 6831825*a^3*b^23
*x^12 + 1576575*a^2*b^24*x^13)
```

3.1328 $\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx$

Optimal. Leaf size=182

$$-\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}-\frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}}+\frac{d^3(c+dx)^{11}}{728(bc-ad)^4(a+bx)^{13}}-\frac{}{4368(}$$

```
[Out] -1/16*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^16+1/48*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x+
a)^15-1/168*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^14+1/728*d^3*(d*x+c)^11/(-a
*d+b*c)^4/(b*x+a)^13-1/4368*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^12+1/48048*
d^5*(d*x+c)^11/(-a*d+b*c)^6/(b*x+a)^11
```

Rubi [A]
time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{d(c+dx)^{11}}{48(a+bx)^{15}(bc-ad)^2}-\frac{(c+dx)^{11}}{16(a+bx)^{16}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -1/16*(c + d*x)^11/((b*c - a*d)*(a + b*x)^16) + (d*(c + d*x)^11)/(48*(b*c -
 a*d)^2*(a + b*x)^15) - (d^2*(c + d*x)^11)/(168*(b*c - a*d)^3*(a + b*x)^14)
 + (d^3*(c + d*x)^11)/(728*(b*c - a*d)^4*(a + b*x)^13) - (d^4*(c + d*x)^11)
/(4368*(b*c - a*d)^5*(a + b*x)^12) + (d^5*(c + d*x)^11)/(48048*(b*c - a*d)^
6*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx = -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} - \frac{(5d)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{16(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{12(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} - \frac{d^3\int \frac{(c+}{(a+}}{56(bc}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d}{728(bc}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d}{728(bc}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d}{728(bc}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 694 vs. $2(182) = 364$.
time = 0.17, size = 694, normalized size = 3.81

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -1/48048*(a^10*d^10 + 2*a^9*b*d^9*(3*c + 8*d*x) + 3*a^8*b^2*d^8*(7*c^2 + 32
*c*d*x + 40*d^2*x^2) + 8*a^7*b^3*d^7*(7*c^3 + 42*c^2*d*x + 90*c*d^2*x^2 + 7
0*d^3*x^3) + 14*a^6*b^4*d^6*(9*c^4 + 64*c^3*d*x + 180*c^2*d^2*x^2 + 240*c*d
^3*x^3 + 130*d^4*x^4) + 84*a^5*b^5*d^5*(3*c^5 + 24*c^4*d*x + 80*c^3*d^2*x^2
 + 140*c^2*d^3*x^3 + 130*c*d^4*x^4 + 52*d^5*x^5) + 14*a^4*b^6*d^4*(33*c^6 +
 288*c^5*d*x + 1080*c^4*d^2*x^2 + 2240*c^3*d^3*x^3 + 2730*c^2*d^4*x^4 + 187
2*c*d^5*x^5 + 572*d^6*x^6) + 8*a^3*b^7*d^3*(99*c^7 + 924*c^6*d*x + 3780*c^5
*d^2*x^2 + 8820*c^4*d^3*x^3 + 12740*c^3*d^4*x^4 + 11466*c^2*d^5*x^5 + 6006*
c*d^6*x^6 + 1430*d^7*x^7) + 3*a^2*b^8*d^2*(429*c^8 + 4224*c^7*d*x + 18480*c
^6*d^2*x^2 + 47040*c^5*d^3*x^3 + 76440*c^4*d^4*x^4 + 81536*c^3*d^5*x^5 + 56
056*c^2*d^6*x^6 + 22880*c*d^7*x^7 + 4290*d^8*x^8) + 2*a*b^9*d*(1001*c^9 + 1
0296*c^8*d*x + 47520*c^7*d^2*x^2 + 129360*c^6*d^3*x^3 + 229320*c^5*d^4*x^4
+ 275184*c^4*d^5*x^5 + 224224*c^3*d^6*x^6 + 120120*c^2*d^7*x^7 + 38610*c*d^
8*x^8 + 5720*d^9*x^9) + b^10*(3003*c^10 + 32032*c^9*d*x + 154440*c^8*d^2*x^
2 + 443520*c^7*d^3*x^3 + 840840*c^6*d^4*x^4 + 1100736*c^5*d^5*x^5 + 1009008
```

```
*c^4*d^6*x^6 + 640640*c^3*d^7*x^7 + 270270*c^2*d^8*x^8 + 68640*c*d^9*x^9 +
8008*d^10*x^10))/(b^11*(a + b*x)^16)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^17,x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(170) = 340.
time = 0.21, size = 867, normalized size = 4.76

method	result
risch	$-\frac{\frac{a^{10}d^{10}+6a^9bc\,d^9+21a^8b^2c^2d^8+56a^7b^3c^3d^7+126a^6b^4c^4d^6+252a^5b^5c^5d^5+462a^4b^6c^6d^4+792a^3b^7c^7d^3+1287a^2b^8c^8d^2+2002a\,b^9c^9d+3003b^{10}c^{10}}{48048b^{11}}}{}$
default	$\frac{120d^3\left(a^7d^7-7a^6bc\,d^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7a\,b^6c^6d-b^7c^7\right)}{13b^{11}(bx+a)^{13}}-\frac{35d^4\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-\right.}{2b^{11}}$
norman	$\frac{\frac{-a^{10}b^5d^{10}-6a^9b^6c\,d^9-21a^8b^7c^2d^8-56a^7b^8c^3d^7-126a^6b^9c^4d^6-252a^5b^{10}c^5d^5-462a^4b^{11}c^6d^4-792a^3b^{12}c^7d^3-1287a^2b^{13}c^8d^2-2002a\,b^{14}c^9d-30}{48048b^{16}}}{}$
gosper	$-\frac{8008d^{10}x^{10}b^{10}+11440a\,b^9d^{10}x^9+68640b^{10}c\,d^9x^9+12870a^2b^8d^{10}x^8+77220a\,b^9c\,d^9x^8+270270b^{10}c^2d^8x^8+11440a^3b^7d^{10}x^7+68640a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^17,x,method=_RETURNVERBOSE)
```

```
[Out] 120/13/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^
4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^13-3
5/2/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+1
5*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^12-45/14/b^11*d^2*(a^8*d^8
-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*
a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^14+40/3/b
^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^9-45/8/b^11*d
^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^8-21/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3
+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^10+252/11/b^11*d^5*(a^5*d
^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^
5)/(b*x+a)^11-1/16*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3
*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^
3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^16-
1/6*d^10/b^11/(b*x+a)^6+2/3/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^
7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6
```

```
*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^15+10/7/b^11*d^9*(a*
d-b*c)/(b*x+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(170) = 340$.
time = 0.34, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(170) = 340$.
time = 0.31, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**17,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(170) = 340.
time = 0.00, size = 1029, normalized size = 5.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x)
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 68640*b^10*c*d^9*x^9 + 11440*a*b^9*d^10*x^9
 + 270270*b^10*c^2*d^8*x^8 + 77220*a*b^9*c*d^9*x^8 + 12870*a^2*b^8*d^10*x^8
 + 640640*b^10*c^3*d^7*x^7 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9
```

```
*x^7 + 11440*a^3*b^7*d^10*x^7 + 1009008*b^10*c^4*d^6*x^6 + 448448*a*b^9*c^3
*d^7*x^6 + 168168*a^2*b^8*c^2*d^8*x^6 + 48048*a^3*b^7*c*d^9*x^6 + 8008*a^4*
b^6*d^10*x^6 + 1100736*b^10*c^5*d^5*x^5 + 550368*a*b^9*c^4*d^6*x^5 + 244608
*a^2*b^8*c^3*d^7*x^5 + 91728*a^3*b^7*c^2*d^8*x^5 + 26208*a^4*b^6*c*d^9*x^5
+ 4368*a^5*b^5*d^10*x^5 + 840840*b^10*c^6*d^4*x^4 + 458640*a*b^9*c^5*d^5*x^
4 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^3*d^7*x^4 + 38220*a^4*b^6
*c^2*d^8*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 1820*a^6*b^4*d^10*x^4 + 443520*b^1
0*c^7*d^3*x^3 + 258720*a*b^9*c^6*d^4*x^3 + 141120*a^2*b^8*c^5*d^5*x^3 + 705
60*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 + 11760*a^5*b^5*c^2*d^8*
x^3 + 3360*a^6*b^4*c*d^9*x^3 + 560*a^7*b^3*d^10*x^3 + 154440*b^10*c^8*d^2*x
^2 + 95040*a*b^9*c^7*d^3*x^2 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^
5*d^5*x^2 + 15120*a^4*b^6*c^4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6
*b^4*c^2*d^8*x^2 + 720*a^7*b^3*c*d^9*x^2 + 120*a^8*b^2*d^10*x^2 + 32032*b^1
0*c^9*d*x + 20592*a*b^9*c^8*d^2*x + 12672*a^2*b^8*c^7*d^3*x + 7392*a^3*b^7*
c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*a^6*b^4*c
^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 96*a^8*b^2*c*d^9*x + 16*a^9*b*d^10*x + 3
003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d
^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a
^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)
^16*b^11)
```

Mupad [B]
time = 0.58, size = 1131, normalized size = 6.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^17,x)
```

```
[Out] -(a^10*d^10 + 3003*b^10*c^10 + 8008*b^10*d^10*x^10 + 11440*a*b^9*d^10*x^9 +
 68640*b^10*c*d^9*x^9 + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^
4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*
d^7 + 21*a^8*b^2*c^2*d^8 + 120*a^8*b^2*d^10*x^2 + 560*a^7*b^3*d^10*x^3 + 18
20*a^6*b^4*d^10*x^4 + 4368*a^5*b^5*d^10*x^5 + 8008*a^4*b^6*d^10*x^6 + 11440
*a^3*b^7*d^10*x^7 + 12870*a^2*b^8*d^10*x^8 + 154440*b^10*c^8*d^2*x^2 + 4435
20*b^10*c^7*d^3*x^3 + 840840*b^10*c^6*d^4*x^4 + 1100736*b^10*c^5*d^5*x^5 +
1009008*b^10*c^4*d^6*x^6 + 640640*b^10*c^3*d^7*x^7 + 270270*b^10*c^2*d^8*x^
8 + 2002*a*b^9*c^9*d + 6*a^9*b*c*d^9 + 16*a^9*b*d^10*x + 32032*b^10*c^9*d*x
 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^5*d^5*x^2 + 15120*a^4*b^6*c^
4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6*b^4*c^2*d^8*x^2 + 141120*a^
2*b^8*c^5*d^5*x^3 + 70560*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 +
 11760*a^5*b^5*c^2*d^8*x^3 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^
3*d^7*x^4 + 38220*a^4*b^6*c^2*d^8*x^4 + 244608*a^2*b^8*c^3*d^7*x^5 + 91728*
a^3*b^7*c^2*d^8*x^5 + 168168*a^2*b^8*c^2*d^8*x^6 + 20592*a*b^9*c^8*d^2*x +
96*a^8*b^2*c*d^9*x + 77220*a*b^9*c*d^9*x^8 + 12672*a^2*b^8*c^7*d^3*x + 7392
```

*a^3*b^7*c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*a^6*b^4*c^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 95040*a*b^9*c^7*d^3*x^2 + 720*a^7*b^3*c*d^9*x^2 + 258720*a*b^9*c^6*d^4*x^3 + 3360*a^6*b^4*c*d^9*x^3 + 458640*a*b^9*c^5*d^5*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 550368*a*b^9*c^4*d^6*x^5 + 26208*a^4*b^6*c*d^9*x^5 + 448448*a*b^9*c^3*d^7*x^6 + 48048*a^3*b^7*c*d^9*x^6 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9*x^7)/(48048*a^16*b^11 + 48048*b^27*x^16 + 768768*a^15*b^12*x + 768768*a*b^26*x^15 + 5765760*a^14*b^13*x^2 + 26906880*a^13*b^14*x^3 + 87447360*a^12*b^15*x^4 + 209873664*a^11*b^16*x^5 + 384768384*a^10*b^17*x^6 + 549669120*a^9*b^18*x^7 + 618377760*a^8*b^19*x^8 + 549669120*a^7*b^20*x^9 + 384768384*a^6*b^21*x^10 + 209873664*a^5*b^22*x^11 + 87447360*a^4*b^23*x^12 + 26906880*a^3*b^24*x^13 + 5765760*a^2*b^25*x^14)

3.1329 $\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx$

Optimal. Leaf size=213

$$-\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}}+\frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}}-\frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}}+\frac{d^3(c+dx)^{11}}{476(bc-ad)^4(a+bx)^{14}}-61$$

```
[Out] -1/17*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^17+3/136*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^16-1/136*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^15+1/476*d^3*(d*x+c)^11/(-
a*d+b*c)^4/(b*x+a)^14-3/6188*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^13+1/12376
*d^5*(d*x+c)^11/(-a*d+b*c)^6/(b*x+a)^12-1/136136*d^6*(d*x+c)^11/(-a*d+b*c)^
7/(b*x+a)^11
```

Rubi [A]
time = 0.06, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{136(a+bx)^{15}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{136(a+bx)^{16}(bc-ad)^2}-\frac{(c+dx)^{11}}{17(a+bx)^{17}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -1/17*(c + d*x)^11/((b*c - a*d)*(a + b*x)^17) + (3*d*(c + d*x)^11)/(136*(b*
c - a*d)^2*(a + b*x)^16) - (d^2*(c + d*x)^11)/(136*(b*c - a*d)^3*(a + b*x)^
15) + (d^3*(c + d*x)^11)/(476*(b*c - a*d)^4*(a + b*x)^14) - (3*d^4*(c + d*x
)^11)/(6188*(b*c - a*d)^5*(a + b*x)^13) + (d^5*(c + d*x)^11)/(12376*(b*c -
a*d)^6*(a + b*x)^12) - (d^6*(c + d*x)^11)/(136136*(b*c - a*d)^7*(a + b*x)^1
1)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx &= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} - \frac{(6d)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{17(bc-ad)} \\
&= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} + \frac{(15d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{136(bc-ad)^2} \\
&= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} - \frac{d^3\int \frac{(c+\cdots}{(a+\cdots}}{34(bc\cdots} \\
&= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^{3}\cdots}{476(bc\cdots} \\
&= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^{3}\cdots}{476(bc\cdots} \\
&= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^{3}\cdots}{476(bc\cdots} \\
&= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^{3}\cdots}{476(bc\cdots}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(213) = 426.
time = 0.18, size = 690, normalized size = 3.24

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -1/136136*(a^10*d^10 + a^9*b*d^9*(7*c + 17*d*x) + a^8*b^2*d^8*(28*c^2 + 119
*c*d*x + 136*d^2*x^2) + 4*a^7*b^3*d^7*(21*c^3 + 119*c^2*d*x + 238*c*d^2*x^2
 + 170*d^3*x^3) + 14*a^6*b^4*d^6*(15*c^4 + 102*c^3*d*x + 272*c^2*d^2*x^2 +
340*c*d^3*x^3 + 170*d^4*x^4) + 14*a^5*b^5*d^5*(33*c^5 + 255*c^4*d*x + 816*c
^3*d^2*x^2 + 1360*c^2*d^3*x^3 + 1190*c*d^4*x^4 + 442*d^5*x^5) + 14*a^4*b^6*
d^4*(66*c^6 + 561*c^5*d*x + 2040*c^4*d^2*x^2 + 4080*c^3*d^3*x^3 + 4760*c^2*
d^4*x^4 + 3094*c*d^5*x^5 + 884*d^6*x^6) + 4*a^3*b^7*d^3*(429*c^7 + 3927*c^6
*d*x + 15708*c^5*d^2*x^2 + 35700*c^4*d^3*x^3 + 49980*c^3*d^4*x^4 + 43316*c^
2*d^5*x^5 + 21658*c*d^6*x^6 + 4862*d^7*x^7) + a^2*b^8*d^2*(3003*c^8 + 29172
*c^7*d*x + 125664*c^6*d^2*x^2 + 314160*c^5*d^3*x^3 + 499800*c^4*d^4*x^4 + 5
```

```
19792*c^3*d^5*x^5 + 346528*c^2*d^6*x^6 + 136136*c*d^7*x^7 + 24310*d^8*x^8)
+ a*b^9*d*(5005*c^9 + 51051*c^8*d*x + 233376*c^7*d^2*x^2 + 628320*c^6*d^3*x
^3 + 1099560*c^5*d^4*x^4 + 1299480*c^4*d^5*x^5 + 1039584*c^3*d^6*x^6 + 5445
44*c^2*d^7*x^7 + 170170*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(8008*c^10 + 8508
5*c^9*d*x + 408408*c^8*d^2*x^2 + 1166880*c^7*d^3*x^3 + 2199120*c^6*d^4*x^4
+ 2858856*c^5*d^5*x^5 + 2598960*c^4*d^6*x^6 + 1633632*c^3*d^7*x^7 + 680680*
c^2*d^8*x^8 + 170170*c*d^9*x^9 + 19448*d^10*x^10))/(b^11*(a + b*x)^17)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^18,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(199) = 398$.
time = 0.14, size = 867, normalized size = 4.07

method	result
risch	$-\frac{a^{10}d^{10}+7a^9bc\,d^9+28a^8b^2c^2d^8+84a^7b^3c^3d^7+210a^6b^4c^4d^6+462a^5b^5c^5d^5+924a^4b^6c^6d^4+1716a^3b^7c^7d^3+3003a^2b^8c^8d^2+5005a\,b^9c^9d+8008b^{10}}{136136b^{11}}$
default	$-\frac{210d^4\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)}{13b^{11}(bx+a)^{13}}+\frac{21d^5\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3\right.}{b^{11}(bx+a)^{12}}$
norman	$\frac{-a^{10}b^6d^{10}-7a^9b^7c\,d^9-28a^8b^8c^2d^8-84a^7b^9c^3d^7-210a^6b^{10}c^4d^6-462a^5b^{11}c^5d^5-924a^4b^{12}c^6d^4-1716a^3b^{13}c^7d^3-3003a^2b^{14}c^8d^2-5005a\,b^{15}c^9}{136136b^{17}}$
gosper	$-\frac{19448d^{10}x^{10}b^{10}+24310a\,b^9d^{10}x^9+170170b^{10}c\,d^9x^9+24310a^2b^8d^{10}x^8+170170a\,b^9c\,d^9x^8+680680b^{10}c^2d^8x^8+19448a^3b^7d^{10}x^7+1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^18,x,method=_RETURNVERBOSE)
```

```
[Out] -210/13/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d
^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^13+21/b^11*d^5*(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)/(b*x+a)^12+60/7/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4
*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(
b*x+a)^14-5/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^9+5/4/b^11*d^9*(a*
d-b*c)/(b*x+a)^8+12/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/
(b*x+a)^10-210/11/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3
*c^3*d+b^4*c^4)/(b*x+a)^11+5/8/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2
*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*
```

```
c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^16-3/b^11*d^2*(a^
8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^
4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^15-1
/7*d^10/b^11/(b*x+a)^7-1/17*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-12
0*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d
^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b
*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(199) = 398$.

time = 0.33, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(199) = 398$.

time = 0.30, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**18,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(199) = 398.
time = 0.00, size = 1029, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x)
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 170170*b^10*c*d^9*x^9 + 24310*a*b^9*d^10*
x^9 + 680680*b^10*c^2*d^8*x^8 + 170170*a*b^9*c*d^9*x^8 + 24310*a^2*b^8*d^10
*x^8 + 1633632*b^10*c^3*d^7*x^7 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8
*c*d^9*x^7 + 19448*a^3*b^7*d^10*x^7 + 2598960*b^10*c^4*d^6*x^6 + 1039584*a*
b^9*c^3*d^7*x^6 + 346528*a^2*b^8*c^2*d^8*x^6 + 86632*a^3*b^7*c*d^9*x^6 + 12
376*a^4*b^6*d^10*x^6 + 2858856*b^10*c^5*d^5*x^5 + 1299480*a*b^9*c^4*d^6*x^5
 + 519792*a^2*b^8*c^3*d^7*x^5 + 173264*a^3*b^7*c^2*d^8*x^5 + 43316*a^4*b^6*
c*d^9*x^5 + 6188*a^5*b^5*d^10*x^5 + 2199120*b^10*c^6*d^4*x^4 + 1099560*a*b^
9*c^5*d^5*x^4 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*a^3*b^7*c^3*d^7*x^4 + 6
6640*a^4*b^6*c^2*d^8*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 2380*a^6*b^4*d^10*x^4
+ 1166880*b^10*c^7*d^3*x^3 + 628320*a*b^9*c^6*d^4*x^3 + 314160*a^2*b^8*c^5*
d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3*d^7*x^3 + 19040*a^
5*b^5*c^2*d^8*x^3 + 4760*a^6*b^4*c*d^9*x^3 + 680*a^7*b^3*d^10*x^3 + 408408*
b^10*c^8*d^2*x^2 + 233376*a*b^9*c^7*d^3*x^2 + 125664*a^2*b^8*c^6*d^4*x^2 +
62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d
^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 + 952*a^7*b^3*c*d^9*x^2 + 136*a^8*b^2*d^1
0*x^2 + 85085*b^10*c^9*d*x + 51051*a*b^9*c^8*d^2*x + 29172*a^2*b^8*c^7*d^3*
x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^4*d^6
*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 119*a^8*b^2*c*d^9*x +
 17*a^9*b*d^10*x + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*a^2*b^8*c^8*d^2
 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a
^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7*a^9*b*c*d^9 +
a^10*d^10)/((b*x + a)^17*b^11)
```

Mupad [B]
time = 0.66, size = 1142, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^18,x)
```

```
[Out] -(a^10*d^10 + 8008*b^10*c^10 + 19448*b^10*d^10*x^10 + 24310*a*b^9*d^10*x^9
+ 170170*b^10*c*d^9*x^9 + 3003*a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924
*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c
^3*d^7 + 28*a^8*b^2*c^2*d^8 + 136*a^8*b^2*d^10*x^2 + 680*a^7*b^3*d^10*x^3 +
 2380*a^6*b^4*d^10*x^4 + 6188*a^5*b^5*d^10*x^5 + 12376*a^4*b^6*d^10*x^6 + 1
9448*a^3*b^7*d^10*x^7 + 24310*a^2*b^8*d^10*x^8 + 408408*b^10*c^8*d^2*x^2 +
1166880*b^10*c^7*d^3*x^3 + 2199120*b^10*c^6*d^4*x^4 + 2858856*b^10*c^5*d^5*
x^5 + 2598960*b^10*c^4*d^6*x^6 + 1633632*b^10*c^3*d^7*x^7 + 680680*b^10*c^2
*d^8*x^8 + 5005*a*b^9*c^9*d + 7*a^9*b*c*d^9 + 17*a^9*b*d^10*x + 85085*b^10*
c^9*d*x + 125664*a^2*b^8*c^6*d^4*x^2 + 62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^
4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 +
314160*a^2*b^8*c^5*d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3
```

*d^7*x^3 + 19040*a^5*b^5*c^2*d^8*x^3 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*a^3*b^7*c^3*d^7*x^4 + 66640*a^4*b^6*c^2*d^8*x^4 + 519792*a^2*b^8*c^3*d^7*x^5 + 173264*a^3*b^7*c^2*d^8*x^5 + 346528*a^2*b^8*c^2*d^8*x^6 + 51051*a*b^9*c^8*d^2*x + 119*a^8*b^2*c*d^9*x + 170170*a*b^9*c*d^9*x^8 + 29172*a^2*b^8*c^7*d^3*x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^4*d^6*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 233376*a*b^9*c^7*d^3*x^2 + 952*a^7*b^3*c*d^9*x^2 + 628320*a*b^9*c^6*d^4*x^3 + 4760*a^6*b^4*c*d^9*x^3 + 1099560*a*b^9*c^5*d^5*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 1299480*a*b^9*c^4*d^6*x^5 + 43316*a^4*b^6*c*d^9*x^5 + 1039584*a*b^9*c^3*d^7*x^6 + 86632*a^3*b^7*c*d^9*x^6 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8*c*d^9*x^7)/(136136*a^17*b^11 + 136136*b^28*x^17 + 2314312*a^16*b^12*x + 2314312*a*b^27*x^16 + 18514496*a^15*b^13*x^2 + 92572480*a^14*b^14*x^3 + 324003680*a^13*b^15*x^4 + 842409568*a^12*b^16*x^5 + 1684819136*a^11*b^17*x^6 + 2647572928*a^10*b^18*x^7 + 3309466160*a^9*b^19*x^8 + 3309466160*a^8*b^20*x^9 + 2647572928*a^7*b^21*x^10 + 1684819136*a^6*b^22*x^11 + 842409568*a^5*b^23*x^12 + 324003680*a^4*b^24*x^13 + 92572480*a^3*b^25*x^14 + 18514496*a^2*b^26*x^15)

3.1330 $\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$

Optimal. Leaf size=244

$$-\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}}+\frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}}-\frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}}+\frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx)^{15}}-122$$

```
[Out] -1/18*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^18+7/306*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^17-7/816*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^16+7/2448*d^3*(d*x+c)^11/(
-a*d+b*c)^4/(b*x+a)^15-1/1224*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^14+1/5304
*d^5*(d*x+c)^11/(-a*d+b*c)^6/(b*x+a)^13-1/31824*d^6*(d*x+c)^11/(-a*d+b*c)^7
/(b*x+a)^12+1/350064*d^7*(d*x+c)^11/(-a*d+b*c)^8/(b*x+a)^11
```

Rubi [A]
time = 0.07, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+\frac{7d^3(c+dx)^{11}}{2448(a+bx)^{15}(bc-ad)^4}-\frac{7d^2(c+dx)^{11}}{816(a+bx)^{16}(bc-ad)^3}+\frac{7d(c+dx)^{11}}{306(a+bx)^{17}(bc-ad)^2}-\frac{(c+dx)^{11}}{18(a+bx)^{18}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -1/18*(c + d*x)^11/((b*c - a*d)*(a + b*x)^18) + (7*d*(c + d*x)^11)/(306*(b*
c - a*d)^2*(a + b*x)^17) - (7*d^2*(c + d*x)^11)/(816*(b*c - a*d)^3*(a + b*x
)^16) + (7*d^3*(c + d*x)^11)/(2448*(b*c - a*d)^4*(a + b*x)^15) - (d^4*(c +
d*x)^11)/(1224*(b*c - a*d)^5*(a + b*x)^14) + (d^5*(c + d*x)^11)/(5304*(b*c
- a*d)^6*(a + b*x)^13) - (d^6*(c + d*x)^11)/(31824*(b*c - a*d)^7*(a + b*x)^
12) + (d^7*(c + d*x)^11)/(350064*(b*c - a*d)^8*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx = -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} - \frac{(7d)\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx}{18(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} + \frac{(7d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{51(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} - \frac{(35d^3)}{816}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{}{2448(b}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{}{2448(b}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{}{2448(b}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{}{2448(b}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{}{2448(b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 694 vs. 2(244) = 488.
time = 0.17, size = 694, normalized size = 2.84

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -1/350064*(a^10*d^10 + 2*a^9*b*d^9*(4*c + 9*d*x) + 9*a^8*b^2*d^8*(4*c^2 + 1
6*c*d*x + 17*d^2*x^2) + 24*a^7*b^3*d^7*(5*c^3 + 27*c^2*d*x + 51*c*d^2*x^2 +
 34*d^3*x^3) + 6*a^6*b^4*d^6*(55*c^4 + 360*c^3*d*x + 918*c^2*d^2*x^2 + 1088
*c*d^3*x^3 + 510*d^4*x^4) + 36*a^5*b^5*d^5*(22*c^5 + 165*c^4*d*x + 510*c^3*
d^2*x^2 + 816*c^2*d^3*x^3 + 680*c*d^4*x^4 + 238*d^5*x^5) + 6*a^4*b^6*d^4*(2
86*c^6 + 2376*c^5*d*x + 8415*c^4*d^2*x^2 + 16320*c^3*d^3*x^3 + 18360*c^2*d^
4*x^4 + 11424*c*d^5*x^5 + 3094*d^6*x^6) + 24*a^3*b^7*d^3*(143*c^7 + 1287*c^
```

```
6*d*x + 5049*c^5*d^2*x^2 + 11220*c^4*d^3*x^3 + 15300*c^3*d^4*x^4 + 12852*c^
2*d^5*x^5 + 6188*c*d^6*x^6 + 1326*d^7*x^7) + 9*a^2*b^8*d^2*(715*c^8 + 6864*
c^7*d*x + 29172*c^6*d^2*x^2 + 71808*c^5*d^3*x^3 + 112200*c^4*d^4*x^4 + 1142
40*c^3*d^5*x^5 + 74256*c^2*d^6*x^6 + 28288*c*d^7*x^7 + 4862*d^8*x^8) + 2*a*
b^9*d*(5720*c^9 + 57915*c^8*d*x + 262548*c^7*d^2*x^2 + 700128*c^6*d^3*x^3 +
 1211760*c^5*d^4*x^4 + 1413720*c^4*d^5*x^5 + 1113840*c^3*d^6*x^6 + 572832*c
^2*d^7*x^7 + 175032*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(19448*c^10 + 205920*
c^9*d*x + 984555*c^8*d^2*x^2 + 2800512*c^7*d^3*x^3 + 5250960*c^6*d^4*x^4 +
6785856*c^5*d^5*x^5 + 6126120*c^4*d^6*x^6 + 3818880*c^3*d^7*x^7 + 1575288*c
^2*d^8*x^8 + 388960*c*d^9*x^9 + 43758*d^10*x^10))/(b^11*(a + b*x)^18)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^19,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(228) = 456$.
time = 0.16, size = 867, normalized size = 3.55

method	result
risch	$-\frac{a^{10}d^{10}+8a^9bc\,d^9+36a^8b^2c^2d^8+120a^7b^3c^3d^7+330a^6b^4c^4d^6+792a^5b^5c^5d^5+1716a^4b^6c^6d^4+3432a^3b^7c^7d^3+6435a^2b^8c^8d^2+11440a\,b^9c^9d+19448b}{350064b^{11}}$
default	$\frac{252d^5\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{13b^{11}\left(bx+a\right)^{13}}-\frac{35d^6\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{2b^{11}\left(bx+a\right)^{12}}-\frac{15d^4\left(a^6\right.}{}$
norman	$\frac{-a^{10}b^7d^{10}-8a^9b^8c\,d^9-36a^8b^9c^2d^8-120a^7b^{10}c^3d^7-330a^6b^{11}c^4d^6-792a^5b^{12}c^5d^5-1716a^4b^{13}c^6d^4-3432a^3b^{14}c^7d^3-6435a^2b^{15}c^8d^2-11440a\,b^{16}}{350064b^{18}}$
gosper	$-\frac{43758d^{10}x^{10}b^{10}+48620a\,b^9d^{10}x^9+388960b^{10}c\,d^9x^9+43758a^2b^8d^{10}x^8+350064a\,b^9c\,d^9x^8+1575288b^{10}c^2d^8x^8+31824a^3b^7d^{10}x^7+25}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^19,x,method=_RETURNVERBOSE)
```

```
[Out] 252/13/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^
2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^13-35/2/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^12-15/b^11*d^4*(a^6*d^6-6*a^
5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*
c^5*d+b^6*c^6)/(b*x+a)^14+10/9/b^11*d^9*(a*d-b*c)/(b*x+a)^9-1/8*d^10/b^11/(
b*x+a)^8-9/2/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^10+120/11/b^11*d^
7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^11-45/16/b^11*d^2*(
```

```
a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*
d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^16
+8/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35
*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^15-1/18*
(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^
4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^
2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^18+10/17/b^11*d*(a^9*d
^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-
126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9
*c^9)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(228) = 456$.
time = 0.33, size = 1052, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(228) = 456$.

time = 0.31, size = 1052, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**19,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(228) = 456.
time = 0.00, size = 1029, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x)
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 388960*b^10*c*d^9*x^9 + 48620*a*b^9*d^10*
x^9 + 1575288*b^10*c^2*d^8*x^8 + 350064*a*b^9*c*d^9*x^8 + 43758*a^2*b^8*d^1
0*x^8 + 3818880*b^10*c^3*d^7*x^7 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a^2*b
^8*c*d^9*x^7 + 31824*a^3*b^7*d^10*x^7 + 6126120*b^10*c^4*d^6*x^6 + 2227680*
a*b^9*c^3*d^7*x^6 + 668304*a^2*b^8*c^2*d^8*x^6 + 148512*a^3*b^7*c*d^9*x^6 +
 18564*a^4*b^6*d^10*x^6 + 6785856*b^10*c^5*d^5*x^5 + 2827440*a*b^9*c^4*d^6*
x^5 + 1028160*a^2*b^8*c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 68544*a^4*
b^6*c*d^9*x^5 + 8568*a^5*b^5*d^10*x^5 + 5250960*b^10*c^6*d^4*x^4 + 2423520*
a*b^9*c^5*d^5*x^4 + 1009800*a^2*b^8*c^4*d^6*x^4 + 367200*a^3*b^7*c^3*d^7*x^
4 + 110160*a^4*b^6*c^2*d^8*x^4 + 24480*a^5*b^5*c*d^9*x^4 + 3060*a^6*b^4*d^1
0*x^4 + 2800512*b^10*c^7*d^3*x^3 + 1400256*a*b^9*c^6*d^4*x^3 + 646272*a^2*b
^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*b^6*c^3*d^7*x^3 + 2
9376*a^5*b^5*c^2*d^8*x^3 + 6528*a^6*b^4*c*d^9*x^3 + 816*a^7*b^3*d^10*x^3 +
984555*b^10*c^8*d^2*x^2 + 525096*a*b^9*c^7*d^3*x^2 + 262548*a^2*b^8*c^6*d^4
*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 50490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b
^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 153*a^
8*b^2*d^10*x^2 + 205920*b^10*c^9*d*x + 115830*a*b^9*c^8*d^2*x + 61776*a^2*b
^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 5940*a^5
*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 144*a^8*b
^2*c*d^9*x + 18*a^9*b*d^10*x + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 6435*a
^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b^5*
c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 +
8*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^18*b^11)
```

Mupad [B]
time = 12.02, size = 1153, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^19,x)
```

```
[Out] -(a^10*d^10 + 19448*b^10*c^10 + 43758*b^10*d^10*x^10 + 48620*a*b^9*d^10*x^9
 + 388960*b^10*c*d^9*x^9 + 6435*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 17
16*a^4*b^6*c^6*d^4 + 792*a^5*b^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^
3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 + 153*a^8*b^2*d^10*x^2 + 816*a^7*b^3*d^10*x^
3 + 3060*a^6*b^4*d^10*x^4 + 8568*a^5*b^5*d^10*x^5 + 18564*a^4*b^6*d^10*x^6
+ 31824*a^3*b^7*d^10*x^7 + 43758*a^2*b^8*d^10*x^8 + 984555*b^10*c^8*d^2*x^2
 + 2800512*b^10*c^7*d^3*x^3 + 5250960*b^10*c^6*d^4*x^4 + 6785856*b^10*c^5*d
^5*x^5 + 6126120*b^10*c^4*d^6*x^6 + 3818880*b^10*c^3*d^7*x^7 + 1575288*b^10
*c^2*d^8*x^8 + 11440*a*b^9*c^9*d + 8*a^9*b*c*d^9 + 18*a^9*b*d^10*x + 205920
*b^10*c^9*d*x + 262548*a^2*b^8*c^6*d^4*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 5
```

0490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8*x^2 + 646272*a^2*b^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*b^6*c^3*d^7*x^3 + 29376*a^5*b^5*c^2*d^8*x^3 + 1009800*a^2*b^8*c^4*d^6*x^4 + 367200*a^3*b^7*c^3*d^7*x^4 + 110160*a^4*b^6*c^2*d^8*x^4 + 1028160*a^2*b^8*c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 668304*a^2*b^8*c^2*d^8*x^6 + 115830*a*b^9*c^8*d^2*x + 144*a^8*b^2*c*d^9*x + 350064*a*b^9*c*d^9*x^8 + 61776*a^2*b^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 5940*a^5*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 525096*a*b^9*c^7*d^3*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 1400256*a*b^9*c^6*d^4*x^3 + 6528*a^6*b^4*c*d^9*x^3 + 2423520*a*b^9*c^5*d^5*x^4 + 24480*a^5*b^5*c*d^9*x^4 + 2827440*a*b^9*c^4*d^6*x^5 + 68544*a^4*b^6*c*d^9*x^5 + 2227680*a*b^9*c^3*d^7*x^6 + 148512*a^3*b^7*c*d^9*x^6 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a^2*b^8*c*d^9*x^7)/(350064*a^18*b^11 + 350064*b^29*x^18 + 6301152*a^17*b^12*x + 6301152*a*b^28*x^17 + 53559792*a^16*b^13*x^2 + 285652224*a^15*b^14*x^3 + 1071195840*a^14*b^15*x^4 + 2999348352*a^13*b^16*x^5 + 6498588096*a^12*b^17*x^6 + 11140436736*a^11*b^18*x^7 + 15318100512*a^10*b^19*x^8 + 17020111680*a^9*b^20*x^9 + 15318100512*a^8*b^21*x^10 + 11140436736*a^7*b^22*x^11 + 6498588096*a^6*b^23*x^12 + 2999348352*a^5*b^24*x^13 + 1071195840*a^4*b^25*x^14 + 285652224*a^3*b^26*x^15 + 53559792*a^2*b^27*x^16)

3.1331 $\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$

Optimal. Leaf size=273

$$-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{21}{13}$$

```
[Out] -1/19*(-a*d+b*c)^10/b^11/(b*x+a)^19-5/9*d*(-a*d+b*c)^9/b^11/(b*x+a)^18-45/1
7*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^17-15/2*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^16-14
*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^15-18*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^14-210/1
3*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^13-10*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^12-45/1
1*d^8*(-a*d+b*c)^2/b^11/(b*x+a)^11-d^9*(-a*d+b*c)/b^11/(b*x+a)^10-1/9*d^10/
b^11/(b*x+a)^9
```

Rubi [A]

time = 0.20, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{d^{10}}{9b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -1/19*(b*c - a*d)^10/(b^11*(a + b*x)^19) - (5*d*(b*c - a*d)^9)/(9*b^11*(a +
 b*x)^18) - (45*d^2*(b*c - a*d)^8)/(17*b^11*(a + b*x)^17) - (15*d^3*(b*c -
a*d)^7)/(2*b^11*(a + b*x)^16) - (14*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^15)
- (18*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^14) - (210*d^6*(b*c - a*d)^4)/(13*
b^11*(a + b*x)^13) - (10*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^12) - (45*d^8*(
b*c - a*d)^2)/(11*b^11*(a + b*x)^11) - (d^9*(b*c - a*d))/(b^11*(a + b*x)^10
) - d^10/(9*b^11*(a + b*x)^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{20}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{19}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{18}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{17}}+\frac{210d^4(bc-}{b^{10}(a+bx}\right.$$

$$= -\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-ad}{b^{11}(a+bx)^1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. 2(273) = 546.
time = 0.17, size = 692, normalized size = 2.53

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -1/831402*(a^10*d^10 + a^9*b*d^9*(9*c + 19*d*x) + 9*a^8*b^2*d^8*(5*c^2 + 19
*c*d*x + 19*d^2*x^2) + 3*a^7*b^3*d^7*(55*c^3 + 285*c^2*d*x + 513*c*d^2*x^2
+ 323*d^3*x^3) + 3*a^6*b^4*d^6*(165*c^4 + 1045*c^3*d*x + 2565*c^2*d^2*x^2 +
 2907*c*d^3*x^3 + 1292*d^4*x^4) + 9*a^5*b^5*d^5*(143*c^5 + 1045*c^4*d*x + 3
135*c^3*d^2*x^2 + 4845*c^2*d^3*x^3 + 3876*c*d^4*x^4 + 1292*d^5*x^5) + 3*a^4
*b^6*d^4*(1001*c^6 + 8151*c^5*d*x + 28215*c^4*d^2*x^2 + 53295*c^3*d^3*x^3 +
 58140*c^2*d^4*x^4 + 34884*c*d^5*x^5 + 9044*d^6*x^6) + 3*a^3*b^7*d^3*(2145*
c^7 + 19019*c^6*d*x + 73359*c^5*d^2*x^2 + 159885*c^4*d^3*x^3 + 213180*c^3*d
^4*x^4 + 174420*c^2*d^5*x^5 + 81396*c*d^6*x^6 + 16796*d^7*x^7) + 9*a^2*b^8*
d^2*(1430*c^8 + 13585*c^7*d*x + 57057*c^6*d^2*x^2 + 138567*c^5*d^3*x^3 + 21
3180*c^4*d^4*x^4 + 213180*c^3*d^5*x^5 + 135660*c^2*d^6*x^6 + 50388*c*d^7*x^
7 + 8398*d^8*x^8) + a*b^9*d*(24310*c^9 + 244530*c^8*d*x + 1100385*c^7*d^2*x
^2 + 2909907*c^6*d^3*x^3 + 4988412*c^5*d^4*x^4 + 5755860*c^4*d^5*x^5 + 4476
780*c^3*d^6*x^6 + 2267460*c^2*d^7*x^7 + 680238*c*d^8*x^8 + 92378*d^9*x^9) +
 b^10*(43758*c^10 + 461890*c^9*d*x + 2200770*c^8*d^2*x^2 + 6235515*c^7*d^3*
x^3 + 11639628*c^6*d^4*x^4 + 14965236*c^5*d^5*x^5 + 13430340*c^4*d^6*x^6 +
8314020*c^3*d^7*x^7 + 3401190*c^2*d^8*x^8 + 831402*c*d^9*x^9 + 92378*d^10*x
^10))/(b^11*(a + b*x)^19)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^20,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(259) = 518.
time = 0.15, size = 866, normalized size = 3.17

method	result
risch	$-\frac{a^{10} d^{10}+9 a^{9} b c \,d^{9}+45 a^{8} b^{2} c^{2} d^{8}+165 a^{7} b^{3} c^{3} d^{7}+495 a^{6} b^{4} c^{4} d^{6}+1287 a^{5} b^{5} c^{5} d^{5}+3003 a^{4} b^{6} c^{6} d^{4}+6435 a^{3} b^{7} c^{7} d^{3}+12870 a^{2} b^{8} c^{8} d^{2}+24310 a \,b^{9} c^{9} d+4375}{831402 b^{11}}$

default	$-\frac{210d^6(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}{13b^{11}(bx+a)^{13}}+\frac{10d^7(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{b^{11}(bx+a)^{12}}+\frac{18d^5(a^5d^5-5a^4bc\,d^4+10a^3b^2c}{b^{11}(}$
norman	$\frac{\frac{-a^{10}b^8d^{10}-9a^9b^9c\,d^9-45a^8b^{10}c^2d^8-165a^7b^{11}c^3d^7-495a^6b^{12}c^4d^6-1287a^5b^{13}c^5d^5-3003a^4b^{14}c^6d^4-6435a^3b^{15}c^7d^3-12870a^2b^{16}c^8d^2-24310}{831402b^{19}}}{}$
gosper	$-\frac{92378d^{10}x^{10}b^{10}+92378a\,b^9d^{10}x^9+831402b^{10}c\,d^9x^9+75582a^2b^8d^{10}x^8+680238a\,b^9c\,d^9x^8+3401190b^{10}c^2d^8x^8+50388a^3b^7d^{10}x^7+}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^20,x,method=_RETURNVERBOSE)

[Out] -210/13/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^13+10/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^12+18/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^14-1/9*d^10/b^11/(b*x+a)^9-1/19*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^19+1/b^11*d^9*(a*d-b*c)/(b*x+a)^10-45/11/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^11-14/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^15+5/9/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^18+15/2/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^16-45/17/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^17

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(259) = 518$.
time = 0.33, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="maxima")

[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(495*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^

```
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(259) = 518$.
time = 0.30, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
```

```
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**20,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(259) = 518.
time = 0.00, size = 1029, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x)
[Out] -1/831402*(92378*b^10*d^10*x^10 + 831402*b^10*c*d^9*x^9 + 92378*a*b^9*d^10*
x^9 + 3401190*b^10*c^2*d^8*x^8 + 680238*a*b^9*c*d^9*x^8 + 75582*a^2*b^8*d^1
0*x^8 + 8314020*b^10*c^3*d^7*x^7 + 2267460*a*b^9*c^2*d^8*x^7 + 453492*a^2*b
^8*c*d^9*x^7 + 50388*a^3*b^7*d^10*x^7 + 13430340*b^10*c^4*d^6*x^6 + 4476780
*a*b^9*c^3*d^7*x^6 + 1220940*a^2*b^8*c^2*d^8*x^6 + 244188*a^3*b^7*c*d^9*x^6
 + 27132*a^4*b^6*d^10*x^6 + 14965236*b^10*c^5*d^5*x^5 + 5755860*a*b^9*c^4*d
^6*x^5 + 1918620*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 104652*
a^4*b^6*c*d^9*x^5 + 11628*a^5*b^5*d^10*x^5 + 11639628*b^10*c^6*d^4*x^4 + 49
88412*a*b^9*c^5*d^5*x^4 + 1918620*a^2*b^8*c^4*d^6*x^4 + 639540*a^3*b^7*c^3*
d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 34884*a^5*b^5*c*d^9*x^4 + 3876*a^6*b
^4*d^10*x^4 + 6235515*b^10*c^7*d^3*x^3 + 2909907*a*b^9*c^6*d^4*x^3 + 124710
3*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 159885*a^4*b^6*c^3*d^7
*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 969*a^7*b^3*d^1
0*x^3 + 2200770*b^10*c^8*d^2*x^2 + 1100385*a*b^9*c^7*d^3*x^2 + 513513*a^2*b
^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 2
8215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*c^2*d^8*x^2 + 1539*a^7*b^3*c*d^9*x^
2 + 171*a^8*b^2*d^10*x^2 + 461890*b^10*c^9*d*x + 244530*a*b^9*c^8*d^2*x + 1
22265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*d^5*x
 + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d^8*x
```

```
+ 171*a^8*b^2*c*d^9*x + 19*a^9*b*d^10*x + 43758*b^10*c^10 + 24310*a*b^9*c^9
*d + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 +
1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b
^2*c^2*d^8 + 9*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^19*b^11)
```

Mupad [B]
time = 25.72, size = 1164, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^20,x)
```

```
[Out] -(a^10*d^10 + 43758*b^10*c^10 + 92378*b^10*d^10*x^10 + 92378*a*b^9*d^10*x^9
 + 831402*b^10*c*d^9*x^9 + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3
003*a^4*b^6*c^6*d^4 + 1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*
b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 + 171*a^8*b^2*d^10*x^2 + 969*a^7*b^3*d^10*
x^3 + 3876*a^6*b^4*d^10*x^4 + 11628*a^5*b^5*d^10*x^5 + 27132*a^4*b^6*d^10*x
^6 + 50388*a^3*b^7*d^10*x^7 + 75582*a^2*b^8*d^10*x^8 + 2200770*b^10*c^8*d^2
*x^2 + 6235515*b^10*c^7*d^3*x^3 + 11639628*b^10*c^6*d^4*x^4 + 14965236*b^10
*c^5*d^5*x^5 + 13430340*b^10*c^4*d^6*x^6 + 8314020*b^10*c^3*d^7*x^7 + 34011
90*b^10*c^2*d^8*x^8 + 24310*a*b^9*c^9*d + 9*a^9*b*c*d^9 + 19*a^9*b*d^10*x +
 461890*b^10*c^9*d*x + 513513*a^2*b^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*
x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 28215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*
c^2*d^8*x^2 + 1247103*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 15
9885*a^4*b^6*c^3*d^7*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 1918620*a^2*b^8*c^4*
d^6*x^4 + 639540*a^3*b^7*c^3*d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 1918620
*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 1220940*a^2*b^8*c^2*d^8
*x^6 + 244530*a*b^9*c^8*d^2*x + 171*a^8*b^2*c*d^9*x + 680238*a*b^9*c*d^9*x^
8 + 122265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*
d^5*x + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d
^8*x + 1100385*a*b^9*c^7*d^3*x^2 + 1539*a^7*b^3*c*d^9*x^2 + 2909907*a*b^9*c
^6*d^4*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 4988412*a*b^9*c^5*d^5*x^4 + 34884*a^5
*b^5*c*d^9*x^4 + 5755860*a*b^9*c^4*d^6*x^5 + 104652*a^4*b^6*c*d^9*x^5 + 447
6780*a*b^9*c^3*d^7*x^6 + 244188*a^3*b^7*c*d^9*x^6 + 2267460*a*b^9*c^2*d^8*x
^7 + 453492*a^2*b^8*c*d^9*x^7)/(831402*a^19*b^11 + 831402*b^30*x^19 + 15796
638*a^18*b^12*x + 15796638*a*b^29*x^18 + 142169742*a^17*b^13*x^2 + 80562853
8*a^16*b^14*x^3 + 3222514152*a^15*b^15*x^4 + 9667542456*a^14*b^16*x^5 + 225
57599064*a^13*b^17*x^6 + 41892683976*a^12*b^18*x^7 + 62839025964*a^11*b^19*
x^8 + 76803253956*a^10*b^20*x^9 + 76803253956*a^9*b^21*x^10 + 62839025964*a
^8*b^22*x^11 + 41892683976*a^7*b^23*x^12 + 22557599064*a^6*b^24*x^13 + 9667
542456*a^5*b^25*x^14 + 3222514152*a^4*b^26*x^15 + 805628538*a^3*b^27*x^16 +
 142169742*a^2*b^28*x^17)
```

3.1332 $\int \frac{(c+dx)^{10}}{(a+bx)^{21}} \, dx$

Optimal. Leaf size=279

$$-\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-$$

```
[Out] -1/20*(-a*d+b*c)^10/b^11/(b*x+a)^20-10/19*d*(-a*d+b*c)^9/b^11/(b*x+a)^19-5/
2*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^18-120/17*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^17-
105/8*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^16-84/5*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^1
5-15*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^14-120/13*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^
13-15/4*d^8*(-a*d+b*c)^2/b^11/(b*x+a)^12-10/11*d^9*(-a*d+b*c)/b^11/(b*x+a)^
11-1/10*d^10/b^11/(b*x+a)^10
```

Rubi [A]

time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{d^{10}}{10b^{11}(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -1/20*(b*c - a*d)^10/(b^11*(a + b*x)^20) - (10*d*(b*c - a*d)^9)/(19*b^11*(a
 + b*x)^19) - (5*d^2*(b*c - a*d)^8)/(2*b^11*(a + b*x)^18) - (120*d^3*(b*c -
 a*d)^7)/(17*b^11*(a + b*x)^17) - (105*d^4*(b*c - a*d)^6)/(8*b^11*(a + b*x)
^16) - (84*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^15) - (15*d^6*(b*c - a*d)^4
)/(b^11*(a + b*x)^14) - (120*d^7*(b*c - a*d)^3)/(13*b^11*(a + b*x)^13) - (1
5*d^8*(b*c - a*d)^2)/(4*b^11*(a + b*x)^12) - (10*d^9*(b*c - a*d))/(11*b^11*
(a + b*x)^11) - d^10/(10*b^11*(a + b*x)^10)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{21}} \, dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{21}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{20}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{19}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{18}}+\frac{210d^4(bc-}{b^{10}(a+bx}\right.\\ &= -\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-}{8b^{11}(a+b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. $2(279) = 558$.
time = 0.18, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -1/1847560*(a^10*d^10 + 10*a^9*b*d^9*(c + 2*d*x) + 5*a^8*b^2*d^8*(11*c^2 +
40*c*d*x + 38*d^2*x^2) + 20*a^7*b^3*d^7*(11*c^3 + 55*c^2*d*x + 95*c*d^2*x^2
 + 57*d^3*x^3) + 5*a^6*b^4*d^6*(143*c^4 + 880*c^3*d*x + 2090*c^2*d^2*x^2 +
2280*c*d^3*x^3 + 969*d^4*x^4) + 2*a^5*b^5*d^5*(1001*c^5 + 7150*c^4*d*x + 20
900*c^3*d^2*x^2 + 31350*c^2*d^3*x^3 + 24225*c*d^4*x^4 + 7752*d^5*x^5) + 5*a
^4*b^6*d^4*(1001*c^6 + 8008*c^5*d*x + 27170*c^4*d^2*x^2 + 50160*c^3*d^3*x^3
 + 53295*c^2*d^4*x^4 + 31008*c*d^5*x^5 + 7752*d^6*x^6) + 20*a^3*b^7*d^3*(57
2*c^7 + 5005*c^6*d*x + 19019*c^5*d^2*x^2 + 40755*c^4*d^3*x^3 + 53295*c^3*d^
4*x^4 + 42636*c^2*d^5*x^5 + 19380*c*d^6*x^6 + 3876*d^7*x^7) + 5*a^2*b^8*d^2
*(4862*c^8 + 45760*c^7*d*x + 190190*c^6*d^2*x^2 + 456456*c^5*d^3*x^3 + 6928
35*c^4*d^4*x^4 + 682176*c^3*d^5*x^5 + 426360*c^2*d^6*x^6 + 155040*c*d^7*x^7
 + 25194*d^8*x^8) + 10*a*b^9*d*(4862*c^9 + 48620*c^8*d*x + 217360*c^7*d^2*x
^2 + 570570*c^6*d^3*x^3 + 969969*c^5*d^4*x^4 + 1108536*c^4*d^5*x^5 + 852720
*c^3*d^6*x^6 + 426360*c^2*d^7*x^7 + 125970*c*d^8*x^8 + 16796*d^9*x^9) + b^1
0*(92378*c^10 + 972400*c^9*d*x + 4618900*c^8*d^2*x^2 + 13041600*c^7*d^3*x^3
 + 24249225*c^6*d^4*x^4 + 31039008*c^5*d^5*x^5 + 27713400*c^4*d^6*x^6 + 170
54400*c^3*d^7*x^7 + 6928350*c^2*d^8*x^8 + 1679600*c*d^9*x^9 + 184756*d^10*x
^10))/(b^11*(a + b*x)^20)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^21,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(259) = 518$.
time = 0.16, size = 867, normalized size = 3.11

method	result
risch	$-\frac{\frac{a^{10}d^{10}+10a^{9}bc\,d^{9}+55a^{8}b^{2}c^{2}d^{8}+220a^{7}b^{3}c^{3}d^{7}+715a^{6}b^{4}c^{4}d^{6}+2002a^{5}b^{5}c^{5}d^{5}+5005a^{4}b^{6}c^{6}d^{4}+11440a^{3}b^{7}c^{7}d^{3}+24310a^{2}b^{8}c^{8}d^{2}+48620a\,b^{9}c^{9}d+92}{1847560b^{11}}}{}$

default	$\frac{120d^7(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{13b^{11}(bx+a)^{13}}-\frac{15d^8(a^2d^2-2abcd+b^2c^2)}{4b^{11}(bx+a)^{12}}-\frac{15d^6(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}{b^{11}(bx+a)^{14}}+10$
norman	$\frac{\frac{-a^{10}b^9d^{10}-10a^9b^{10}c\,d^9-55a^8b^{11}c^2d^8-220a^7b^{12}c^3d^7-715a^6b^{13}c^4d^6-2002a^5b^{14}c^5d^5-5005a^4b^{15}c^6d^4-11440a^3b^{16}c^7d^3-24310a^2b^{17}c^8d^2-48}{1847560b^{20}}}{}$
gosper	$-\frac{184756d^{10}x^{10}b^{10}+167960a\,b^9d^{10}x^9+1679600b^{10}c\,d^9x^9+125970a^2b^8d^{10}x^8+1259700a\,b^9c\,d^9x^8+6928350b^{10}c^2d^8x^8+77520a^3b^7d^1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^21,x,method=_RETURNVERBOSE)
```

```
[Out] 120/13/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^13-15
/4/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^12-15/b^11*d^6*(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^14+10/19/b^11*d
*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c
^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^
8*d-b^9*c^9)/(b*x+a)^19-1/10*d^10/b^11/(b*x+a)^10+10/11/b^11*d^9*(a*d-b*c)/
(b*x+a)^11-1/20*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^
3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b
^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^20-105
/8/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15
*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^16+84/5/b^11*d^5*(a^5*d^5-5
*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(
b*x+a)^15-5/2/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3
*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c
^7*d+b^8*c^8)/(b*x+a)^18+120/17/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*
c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^
6*d-b^7*c^7)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(259) = 518$.

time = 0.34, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="maxima")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
```

```
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(259) = 518$.

time = 0.30, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="fricas")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
```

```
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**21,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(259) = 518.
time = 0.00, size = 1029, normalized size = 3.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x)
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 1679600*b^10*c*d^9*x^9 + 167960*a*b^9*d
^10*x^9 + 6928350*b^10*c^2*d^8*x^8 + 1259700*a*b^9*c*d^9*x^8 + 125970*a^2*b
^8*d^10*x^8 + 17054400*b^10*c^3*d^7*x^7 + 4263600*a*b^9*c^2*d^8*x^7 + 77520
0*a^2*b^8*c*d^9*x^7 + 77520*a^3*b^7*d^10*x^7 + 27713400*b^10*c^4*d^6*x^6 +
8527200*a*b^9*c^3*d^7*x^6 + 2131800*a^2*b^8*c^2*d^8*x^6 + 387600*a^3*b^7*c*
d^9*x^6 + 38760*a^4*b^6*d^10*x^6 + 31039008*b^10*c^5*d^5*x^5 + 11085360*a*b
^9*c^4*d^6*x^5 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 +
 155040*a^4*b^6*c*d^9*x^5 + 15504*a^5*b^5*d^10*x^5 + 24249225*b^10*c^6*d^4*
x^4 + 9699690*a*b^9*c^5*d^5*x^4 + 3464175*a^2*b^8*c^4*d^6*x^4 + 1065900*a^3
*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 4
845*a^6*b^4*d^10*x^4 + 13041600*b^10*c^7*d^3*x^3 + 5705700*a*b^9*c^6*d^4*x^
3 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d^6*x^3 + 250800*a^4*b
^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 1140
*a^7*b^3*d^10*x^3 + 4618900*b^10*c^8*d^2*x^2 + 2173600*a*b^9*c^7*d^3*x^2 +
950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7*c^5*d^5*x^2 + 135850*a^4*b^6*c^
4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 10450*a^6*b^4*c^2*d^8*x^2 + 1900*a^
7*b^3*c*d^9*x^2 + 190*a^8*b^2*d^10*x^2 + 972400*b^10*c^9*d*x + 486200*a*b^9
*c^8*d^2*x + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 40040*a^
4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 1100*a
```

^7*b^3*c^2*d^8*x + 200*a^8*b^2*c*d^9*x + 20*a^9*b*d^10*x + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^20*b^11)

Mupad [B]
time = 0.80, size = 1175, normalized size = 4.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^21,x)
```

```
[Out] -(a^10*d^10 + 92378*b^10*c^10 + 184756*b^10*d^10*x^10 + 167960*a*b^9*d^10*x
^9 + 1679600*b^10*c*d^9*x^9 + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3
 + 5005*a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*
a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 190*a^8*b^2*d^10*x^2 + 1140*a^7*b^3*
d^10*x^3 + 4845*a^6*b^4*d^10*x^4 + 15504*a^5*b^5*d^10*x^5 + 38760*a^4*b^6*d
^10*x^6 + 77520*a^3*b^7*d^10*x^7 + 125970*a^2*b^8*d^10*x^8 + 4618900*b^10*c
^8*d^2*x^2 + 13041600*b^10*c^7*d^3*x^3 + 24249225*b^10*c^6*d^4*x^4 + 310390
08*b^10*c^5*d^5*x^5 + 27713400*b^10*c^4*d^6*x^6 + 17054400*b^10*c^3*d^7*x^7
 + 6928350*b^10*c^2*d^8*x^8 + 48620*a*b^9*c^9*d + 10*a^9*b*c*d^9 + 20*a^9*b
*d^10*x + 972400*b^10*c^9*d*x + 950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7
*c^5*d^5*x^2 + 135850*a^4*b^6*c^4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 104
50*a^6*b^4*c^2*d^8*x^2 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d
^6*x^3 + 250800*a^4*b^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 3464175*a
^2*b^8*c^4*d^6*x^4 + 1065900*a^3*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x
^4 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 + 2131800*a^2
*b^8*c^2*d^8*x^6 + 486200*a*b^9*c^8*d^2*x + 200*a^8*b^2*c*d^9*x + 1259700*a
*b^9*c*d^9*x^8 + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 4004
0*a^4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 11
00*a^7*b^3*c^2*d^8*x + 2173600*a*b^9*c^7*d^3*x^2 + 1900*a^7*b^3*c*d^9*x^2 +
 5705700*a*b^9*c^6*d^4*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 9699690*a*b^9*c^5*d^
5*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 11085360*a*b^9*c^4*d^6*x^5 + 155040*a^4*b
^6*c*d^9*x^5 + 8527200*a*b^9*c^3*d^7*x^6 + 387600*a^3*b^7*c*d^9*x^6 + 42636
00*a*b^9*c^2*d^8*x^7 + 775200*a^2*b^8*c*d^9*x^7)/(1847560*a^20*b^11 + 18475
60*b^31*x^20 + 36951200*a^19*b^12*x + 36951200*a*b^30*x^19 + 351036400*a^18
*b^13*x^2 + 2106218400*a^17*b^14*x^3 + 8951428200*a^16*b^15*x^4 + 286445702
40*a^15*b^16*x^5 + 71611425600*a^14*b^17*x^6 + 143222851200*a^13*b^18*x^7 +
 232737133200*a^12*b^19*x^8 + 310316177600*a^11*b^20*x^9 + 341347795360*a^1
0*b^21*x^10 + 310316177600*a^9*b^22*x^11 + 232737133200*a^8*b^23*x^12 + 143
222851200*a^7*b^24*x^13 + 71611425600*a^6*b^25*x^14 + 28644570240*a^5*b^26*
x^15 + 8951428200*a^4*b^27*x^16 + 2106218400*a^3*b^28*x^17 + 351036400*a^2*
b^29*x^18)
```

3.1333 $\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx$

Optimal. Leaf size=279

$$-\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-1$$

```
[Out] -1/21*(-a*d+b*c)^10/b^11/(b*x+a)^21-1/2*d*(-a*d+b*c)^9/b^11/(b*x+a)^20-45/1
9*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^19-20/3*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^18-21
0/17*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^17-63/4*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^16
-14*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^15-60/7*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^14-
45/13*d^8*(-a*d+b*c)^2/b^11/(b*x+a)^13-5/6*d^9*(-a*d+b*c)/b^11/(b*x+a)^12-1
/11*d^10/b^11/(b*x+a)^11
```

Rubi [A]
time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d^{10}}{11b^{11}(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -1/21*(b*c - a*d)^10/(b^11*(a + b*x)^21) - (d*(b*c - a*d)^9)/(2*b^11*(a + b
*x)^20) - (45*d^2*(b*c - a*d)^8)/(19*b^11*(a + b*x)^19) - (20*d^3*(b*c - a*
d)^7)/(3*b^11*(a + b*x)^18) - (210*d^4*(b*c - a*d)^6)/(17*b^11*(a + b*x)^17
) - (63*d^5*(b*c - a*d)^5)/(4*b^11*(a + b*x)^16) - (14*d^6*(b*c - a*d)^4)/(
b^11*(a + b*x)^15) - (60*d^7*(b*c - a*d)^3)/(7*b^11*(a + b*x)^14) - (45*d^8
*(b*c - a*d)^2)/(13*b^11*(a + b*x)^13) - (5*d^9*(b*c - a*d))/(6*b^11*(a + b
*x)^12) - d^10/(11*b^11*(a + b*x)^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{22}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{21}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{20}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{19}}+\frac{210d^4(bc-\ldots}{b^{10}(a+bx\ldots}\right.\\ &= -\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-\ldots}{17b^{11}(a+bx\ldots}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. $2(279) = 558$.
time = 0.18, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -1/3879876*(a^10*d^10 + a^9*b*d^9*(11*c + 21*d*x) + 3*a^8*b^2*d^8*(22*c^2 +
 77*c*d*x + 70*d^2*x^2) + 2*a^7*b^3*d^7*(143*c^3 + 693*c^2*d*x + 1155*c*d^2
*x^2 + 665*d^3*x^3) + 7*a^6*b^4*d^6*(143*c^4 + 858*c^3*d*x + 1980*c^2*d^2*x
^2 + 2090*c*d^3*x^3 + 855*d^4*x^4) + 21*a^5*b^5*d^5*(143*c^5 + 1001*c^4*d*x
 + 2860*c^3*d^2*x^2 + 4180*c^2*d^3*x^3 + 3135*c*d^4*x^4 + 969*d^5*x^5) + 7*
a^4*b^6*d^4*(1144*c^6 + 9009*c^5*d*x + 30030*c^4*d^2*x^2 + 54340*c^3*d^3*x^
3 + 56430*c^2*d^4*x^4 + 31977*c*d^5*x^5 + 7752*d^6*x^6) + 2*a^3*b^7*d^3*(97
24*c^7 + 84084*c^6*d*x + 315315*c^5*d^2*x^2 + 665665*c^4*d^3*x^3 + 855855*c
^3*d^4*x^4 + 671517*c^2*d^5*x^5 + 298452*c*d^6*x^6 + 58140*d^7*x^7) + 3*a^2
*b^8*d^2*(14586*c^8 + 136136*c^7*d*x + 560560*c^6*d^2*x^2 + 1331330*c^5*d^3
*x^3 + 1996995*c^4*d^4*x^4 + 1939938*c^3*d^5*x^5 + 1193808*c^2*d^6*x^6 + 42
6360*c*d^7*x^7 + 67830*d^8*x^8) + a*b^9*d*(92378*c^9 + 918918*c^8*d*x + 408
4080*c^7*d^2*x^2 + 10650640*c^6*d^3*x^3 + 17972955*c^5*d^4*x^4 + 20369349*c
^4*d^5*x^5 + 15519504*c^3*d^6*x^6 + 7674480*c^2*d^7*x^7 + 2238390*c*d^8*x^8
 + 293930*d^9*x^9) + b^10*(184756*c^10 + 1939938*c^9*d*x + 9189180*c^8*d^2*
x^2 + 25865840*c^7*d^3*x^3 + 47927880*c^6*d^4*x^4 + 61108047*c^5*d^5*x^5 +
54318264*c^4*d^6*x^6 + 33256080*c^3*d^7*x^7 + 13430340*c^2*d^8*x^8 + 323323
0*c*d^9*x^9 + 352716*d^10*x^10))/(b^11*(a + b*x)^21)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^10/(a + b*x)^22,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(259) = 518$.
time = 0.18, size = 867, normalized size = 3.11

method	result
risch	$-\frac{a^{10}d^{10}+11a^{9}bc\,d^{9}+66a^{8}b^{2}c^{2}d^{8}+286a^{7}b^{3}c^{3}d^{7}+1001a^{6}b^{4}c^{4}d^{6}+3003a^{5}b^{5}c^{5}d^{5}+8008a^{4}b^{6}c^{6}d^{4}+19448a^{3}b^{7}c^{7}d^{3}+43758a^{2}b^{8}c^{8}d^{2}+92378a\,b^{9}c^{9}d+1}{3879876b^{11}}$

default	$-\frac{45d^8(a^2d^2-2abcd+b^2c^2)}{13b^{11}(bx+a)^{13}}+\frac{5d^9(ad-bc)}{6b^{11}(bx+a)^{12}}+\frac{60d^7(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{7b^{11}(bx+a)^{14}}+\frac{63d^5(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10}{4b^{11}(bx+a)^{1}}$
norman	$\frac{-a^{10}b^{10}d^{10}-11a^9b^{11}c\,d^9-66a^8b^{12}c^2d^8-286a^7b^{13}c^3d^7-1001a^6b^{14}c^4d^6-3003a^5b^{15}c^5d^5-8008a^4b^{16}c^6d^4-19448a^3b^{17}c^7d^3-43758a^2b^{18}c^8d^2-}{3879876b^{21}}$
gosper	$-\frac{352716d^{10}x^{10}b^{10}+293930a\,b^9d^{10}x^9+3233230b^{10}c\,d^9x^9+203490a^2b^8d^{10}x^8+2238390a\,b^9c\,d^9x^8+13430340b^{10}c^2d^8x^8+116280a^3b^7}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^22,x,method=_RETURNVERBOSE)
```

```
[Out] -45/13/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^13+5/6/b^11*d^9*(a*d-b*
c)/(b*x+a)^12+60/7/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(
b*x+a)^14+63/4/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^
3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^16-45/19/b^11*d^2*(a^8*d^8-8*a^7*b
*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*
c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^19-1/11*d^10/b^11
/(b*x+a)^11+1/2/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3
*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*
b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^20-1/21*(a^10*d^10-10*a^9*b*c*d^
9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^
5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c
^9*d+b^10*c^10)/b^11/(b*x+a)^21-14/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^
2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^15+20/3/b^11*d^3*(a^7*d^7-7*a^6*b*
c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c
^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^18-210/17/b^11*d^4*(a^6*d^6-6*a^5*b*c
*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d
+b^6*c^6)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(259) = 518$.
time = 0.33, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="maxima")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
```

```
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(259) = 518$.
time = 0.31, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="fricas")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
```

```
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**22,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(259) = 518.
time = 0.00, size = 1029, normalized size = 3.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x)
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 3233230*b^10*c*d^9*x^9 + 293930*a*b^9*d
^10*x^9 + 13430340*b^10*c^2*d^8*x^8 + 2238390*a*b^9*c*d^9*x^8 + 203490*a^2*
b^8*d^10*x^8 + 33256080*b^10*c^3*d^7*x^7 + 7674480*a*b^9*c^2*d^8*x^7 + 1279
080*a^2*b^8*c*d^9*x^7 + 116280*a^3*b^7*d^10*x^7 + 54318264*b^10*c^4*d^6*x^6
 + 15519504*a*b^9*c^3*d^7*x^6 + 3581424*a^2*b^8*c^2*d^8*x^6 + 596904*a^3*b^
7*c*d^9*x^6 + 54264*a^4*b^6*d^10*x^6 + 61108047*b^10*c^5*d^5*x^5 + 20369349
*a*b^9*c^4*d^6*x^5 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*
x^5 + 223839*a^4*b^6*c*d^9*x^5 + 20349*a^5*b^5*d^10*x^5 + 47927880*b^10*c^6
*d^4*x^4 + 17972955*a*b^9*c^5*d^5*x^4 + 5990985*a^2*b^8*c^4*d^6*x^4 + 17117
10*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^2*d^8*x^4 + 65835*a^5*b^5*c*d^9*x
^4 + 5985*a^6*b^4*d^10*x^4 + 25865840*b^10*c^7*d^3*x^3 + 10650640*a*b^9*c^6
*d^4*x^3 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^7*c^4*d^6*x^3 + 3803
80*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 14630*a^6*b^4*c*d^9*x^
3 + 1330*a^7*b^3*d^10*x^3 + 9189180*b^10*c^8*d^2*x^2 + 4084080*a*b^9*c^7*d^
3*x^2 + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a^3*b^7*c^5*d^5*x^2 + 210210*a
^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2 + 13860*a^6*b^4*c^2*d^8*x^2
+ 2310*a^7*b^3*c*d^9*x^2 + 210*a^8*b^2*d^10*x^2 + 1939938*b^10*c^9*d*x + 91
```

8918*a*b^9*c^8*d^2*x + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*x + 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^7*x + 1386*a^7*b^3*c^2*d^8*x + 231*a^8*b^2*c*d^9*x + 21*a^9*b*d^10*x + 184756*b^10*c^10 + 92378*a*b^9*c^9*d + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^21*b^11)

Mupad [B]
time = 1.04, size = 1186, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^22,x)
```

```
[Out] -(a^10*d^10 + 184756*b^10*c^10 + 352716*b^10*d^10*x^10 + 293930*a*b^9*d^10*
x^9 + 3233230*b^10*c*d^9*x^9 + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^
3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 28
6*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 210*a^8*b^2*d^10*x^2 + 1330*a^7*b^
3*d^10*x^3 + 5985*a^6*b^4*d^10*x^4 + 20349*a^5*b^5*d^10*x^5 + 54264*a^4*b^6
*d^10*x^6 + 116280*a^3*b^7*d^10*x^7 + 203490*a^2*b^8*d^10*x^8 + 9189180*b^1
0*c^8*d^2*x^2 + 25865840*b^10*c^7*d^3*x^3 + 47927880*b^10*c^6*d^4*x^4 + 611
08047*b^10*c^5*d^5*x^5 + 54318264*b^10*c^4*d^6*x^6 + 33256080*b^10*c^3*d^7*
x^7 + 13430340*b^10*c^2*d^8*x^8 + 92378*a*b^9*c^9*d + 11*a^9*b*c*d^9 + 21*a
^9*b*d^10*x + 1939938*b^10*c^9*d*x + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a
^3*b^7*c^5*d^5*x^2 + 210210*a^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2
 + 13860*a^6*b^4*c^2*d^8*x^2 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^
7*c^4*d^6*x^3 + 380380*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 59
90985*a^2*b^8*c^4*d^6*x^4 + 1711710*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^
2*d^8*x^4 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*x^5 + 358
1424*a^2*b^8*c^2*d^8*x^6 + 918918*a*b^9*c^8*d^2*x + 231*a^8*b^2*c*d^9*x + 2
238390*a*b^9*c*d^9*x^8 + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*
x + 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^
7*x + 1386*a^7*b^3*c^2*d^8*x + 4084080*a*b^9*c^7*d^3*x^2 + 2310*a^7*b^3*c*d
^9*x^2 + 10650640*a*b^9*c^6*d^4*x^3 + 14630*a^6*b^4*c*d^9*x^3 + 17972955*a*
b^9*c^5*d^5*x^4 + 65835*a^5*b^5*c*d^9*x^4 + 20369349*a*b^9*c^4*d^6*x^5 + 22
3839*a^4*b^6*c*d^9*x^5 + 15519504*a*b^9*c^3*d^7*x^6 + 596904*a^3*b^7*c*d^9*
x^6 + 7674480*a*b^9*c^2*d^8*x^7 + 1279080*a^2*b^8*c*d^9*x^7)/(3879876*a^21*
b^11 + 3879876*b^32*x^21 + 81477396*a^20*b^12*x + 81477396*a*b^31*x^20 + 81
4773960*a^19*b^13*x^2 + 5160235080*a^18*b^14*x^3 + 23221057860*a^17*b^15*x^
4 + 78951596724*a^16*b^16*x^5 + 210537591264*a^15*b^17*x^6 + 451151981280*a
^14*b^18*x^7 + 789515967240*a^13*b^19*x^8 + 1140411952680*a^12*b^20*x^9 + 1
368494343216*a^11*b^21*x^10 + 1368494343216*a^10*b^22*x^11 + 1140411952680*
a^9*b^23*x^12 + 789515967240*a^8*b^24*x^13 + 451151981280*a^7*b^25*x^14 + 2
```

```
10537591264*a^6*b^26*x^15 + 78951596724*a^5*b^27*x^16 + 23221057860*a^4*b^2
8*x^17 + 5160235080*a^3*b^29*x^18 + 814773960*a^2*b^30*x^19)
```

3.1334 $\int \frac{(a+bx)^5}{c+dx} \, dx$

Optimal. Leaf size=122

$$\frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+bx)^5}{5d}-\frac{(bc-ad)^5\log(c+}{d^6}$$

```
[Out] b*(-a*d+b*c)^4*x/d^5-1/2*(-a*d+b*c)^3*(b*x+a)^2/d^4+1/3*(-a*d+b*c)^2*(b*x+a
)^3/d^3-1/4*(-a*d+b*c)*(b*x+a)^4/d^2+1/5*(b*x+a)^5/d-(-a*d+b*c)^5*ln(d*x+c)
/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{c+dx} \, dx &= \int \left(\frac{b(bc-ad)^4}{d^5}-\frac{b(bc-ad)^3(a+bx)}{d^4}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)(a+bx)^3}{d^2}+ b\right. \\ &= \frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+b}{5d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 167, normalized size = 1.37

$$\frac{bdx\left(300a^4d^4+300a^3bd^3(-2c+dx)+100a^2b^2d^2\left(6c^2-3cdx+2d^2x^2\right)+25ab^3d\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)+b^4\left(60c^4-30c^3dx+20c^2d^2x^2-15cd^3x^3+12d^4x^4\right)\right)-60(bc-ad)^5\log(c+dx)}{60d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4
*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

Mathics [A]
time = 3.70, size = 191, normalized size = 1.57

$$\frac{60bdx\left(5a^4d^4-10a^3bcd^3+10a^2b^2c^2d^2-5ab^3c^3d+b^4c^4\right)+30b^2d^2x^2\left(10a^3d^3-10a^2bcd^2+5ab^2c^2d-b^3c^3\right)+20b^3d^3x^3\left(10a^2d^2-5abcd+b^2c^2\right)+15b^4d^4x^4\left(5ad-bc\right)+12b^5d^5x^5+60\text{Log}\left[c+dx\right]\left(ad-bc\right)^5}{60d^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x),x]')
```

```
[Out] (60 b d x (5 a ^ 4 d ^ 4 - 10 a ^ 3 b c d ^ 3 + 10 a ^ 2 b ^ 2 c ^ 2 d ^ 2
- 5 a b ^ 3 c ^ 3 d + b ^ 4 c ^ 4) + 30 b ^ 2 d ^ 2 x ^ 2 (10 a ^ 3 d ^ 3 -
 10 a ^ 2 b c d ^ 2 + 5 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3) + 20 b ^ 3 d ^ 3 x ^
 3 (10 a ^ 2 d ^ 2 - 5 a b c d + b ^ 2 c ^ 2) + 15 b ^ 4 d ^ 4 x ^ 4 (5 a d
 - b c) + 12 b ^ 5 d ^ 5 x ^ 5 + 60 Log[c + d x] (a d - b c) ^ 5) / (60 d ^
 6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(114) = 228$.
time = 0.14, size = 266, normalized size = 2.18

method	result
norman	$\frac{b\left(5a^4d^4-10a^3bc\,d^3+10a^2b^2c^2d^2-5a\,b^3c^3d+b^4c^4\right)x}{d^5}+\frac{b^5x^5}{5d}+\frac{b^2\left(10a^3d^3-10a^2bc\,d^2+5a\,b^2c^2d-b^3c^3\right)x^2}{2d^4}+\frac{b^3\left(10a^2d^2-5abcd+\right.}{3d^3}$
default	$\frac{b\left(\frac{1}{5}d^4x^5b^4+\frac{5}{4}a\,b^3d^4x^4-\frac{1}{4}b^4c\,d^3x^4+\frac{10}{3}a^2b^2d^4x^3-\frac{5}{3}a\,b^3c\,d^3x^3+\frac{1}{3}b^4c^2d^2x^3+5a^3b\,d^4x^2-5a^2b^2c\,d^3x^2+\frac{5}{2}a\,b^3c^2d^2x^2-\frac{1}{2}b^4c^3d\,x^2+5a^4\right.}{d^5}$
risch	$\frac{b^5x^5}{5d}+\frac{5b^4a\,x^4}{4d}-\frac{b^5c\,x^4}{4d^2}+\frac{10b^3a^2x^3}{3d}-\frac{5b^4ac\,x^3}{3d^2}+\frac{b^5c^2x^3}{3d^3}+\frac{5b^2a^3x^2}{d}-\frac{5b^3a^2c\,x^2}{d^2}+\frac{5b^4a\,c^2x^2}{2d^3}-\frac{b^5c^3x^2}{2d^4}+\frac{5b\,a^4x}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^5*(1/5*d^4*x^5*b^4+5/4*a*b^3*d^4*x^4-1/4*b^4*c*d^3*x^4+10/3*a^2*b^2*d^4
*x^3-5/3*a*b^3*c*d^3*x^3+1/3*b^4*c^2*d^2*x^3+5*a^3*b*d^4*x^2-5*a^2*b^2*c*d^
3*x^2+5/2*a*b^3*c^2*d^2*x^2-1/2*b^4*c^3*d*x^2+5*a^4*d^4*x-10*a^3*b*c*d^3*x+
10*a^2*b^2*c^2*d^2*x-5*a*b^3*c^3*d*x+b^4*c^4*x)+(a^5*d^5-5*a^4*b*c*d^4+10*a
^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6*ln(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(114) = 228$.
time = 0.27, size = 258, normalized size = 2.11

$$\frac{12\,b^5d^4x^5-15\,(b^5cd^3-5\,ab^4d^4)x^4+20\,(b^5c^2d^2-5\,ab^4cd^3+10\,a^2b^3d^4)x^3-30\,(b^5c^3d-5\,ab^4c^2d^2+10\,a^2b^3cd^3-10\,a^3b^2d^4)x^2+60\,(b^5c^4-5\,ab^4c^3d+10\,a^2b^3c^2d^2-10\,a^3b^2cd^3+5\,a^4bd^4)x}{60\,d^5}-\frac{(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\log\left(dx+c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2 -
 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 + 10
*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*a^2
*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*b^4*
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
log(d*x + c)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(114) = 228$.
time = 0.29, size = 259, normalized size = 2.12

$$\frac{12\,b^5d^5x^5 - 15\,(b^5cd^4 - 5\,ab^4d^5)x^4 + 20\,(b^5c^2d^3 - 5\,ab^4cd^4 + 10\,a^2b^3d^5)x^3 - 30\,(b^5c^3d^2 - 5\,ab^4c^2d^3 + 10\,a^2b^3cd^4 - 10\,a^3b^2d^5)x^2 + 60\,(b^5c^4d - 5\,ab^4c^3d^2 + 10\,a^2b^3c^2d^3 - 10\,a^3b^2cd^4 + 5\,a^4bd^5)x - 60\,(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)\log(dx + c)}{60\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 -
 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 +
10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2 +
10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*log(d*x + c))/d^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(104) = 208$.
time = 0.30, size = 209, normalized size = 1.71

$$\frac{b^5x^5}{5d} + x^4\cdot\left(\frac{5ab^4}{4d} - \frac{b^5c}{4d^2}\right) + x^3\cdot\left(\frac{10a^2b^3}{3d} - \frac{5ab^4c}{3d^2} + \frac{b^5c^2}{3d^3}\right) + x^2\cdot\left(\frac{5a^3b^2}{d} - \frac{5a^2b^3c}{d^2} + \frac{5ab^4c^2}{2d^3} - \frac{b^5c^3}{2d^4}\right) + x\left(\frac{5a^4b}{d} - \frac{10a^3b^2c}{d^2} + \frac{10a^2b^3c^2}{d^3} - \frac{5ab^4c^3}{d^4} + \frac{b^5c^4}{d^5}\right) + \frac{(ad - bc)^5\log(c + dx)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c),x)
```

```
[Out] b**5*x**5/(5*d) + x**4*(5*a*b**4/(4*d) - b**5*c/(4*d**2)) + x**3*(10*a**2*b
**3/(3*d) - 5*a*b**4*c/(3*d**2) + b**5*c**2/(3*d**3)) + x**2*(5*a**3*b**2/d
 - 5*a**2*b**3*c/d**2 + 5*a*b**4*c**2/(2*d**3) - b**5*c**3/(2*d**4)) + x*(5
*a**4*b/d - 10*a**3*b**2*c/d**2 + 10*a**2*b**3*c**2/d**3 - 5*a*b**4*c**3/d*
*4 + b**5*c**4/d**5) + (a*d - b*c)**5*log(c + d*x)/d**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(114) = 228$.
time = 0.00, size = 296, normalized size = 2.43

$$\frac{\frac{1}{5}x^5b^5d^4 - \frac{1}{4}x^4b^5d^3c + \frac{5}{4}x^4b^4ad^4 + \frac{1}{3}x^3b^5d^2c^2 - \frac{5}{3}x^3b^4ad^3c + \frac{10}{3}x^3b^3a^2d^4 - \frac{1}{2}x^2b^5dc^3 + \frac{5}{2}x^2b^4ad^2c^2 - 5x^2b^3a^2d^3c + 5x^2b^2a^3d^4 + xb^5c^4 - 5xb^4adc^3 + 10xb^3a^2d^2c^2 - 10xb^2a^3d^3c + 5xba^4d^4}{d^5} + \frac{(-b^5c^5 + 5b^4adc^4 - 10b^3a^2d^2c^3 + 10b^2a^3d^3c^2 - 5ba^4d^4c + a^5d^5)\ln|xd + c|}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x)
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^2
*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150*a
*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*c^4
*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x + 300*
a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

Mupad [B]
time = 0.07, size = 280, normalized size = 2.30

$$x\left(\frac{5\,a^4\,b}{d}-\frac{c\,\left(\frac{10\,a^3\,b^2}{d}+\frac{c\,\left(\frac{c\,\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{d}-\frac{10\,a^2\,b^3}{d}\right)}{d}\right)}{d}\right)+x^4\,\left(\frac{5\,a\,b^4}{4\,d}-\frac{b^5\,c}{4\,d^2}\right)+x^2\,\left(\frac{5\,a^3\,b^2}{d}+\frac{c\,\left(\frac{c\,\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{d}-\frac{10\,a^2\,b^3}{d}\right)}{2\,d}\right)-x^3\,\left(\frac{c\,\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{3\,d}-\frac{10\,a^2\,b^3}{3\,d}\right)+\frac{b^5\,x^5}{5\,d}+\frac{\ln\left(c+d\,x\right)\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x),x)
```

```
[Out] x*((5*a^4*b)/d - (c*((10*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d
 - (10*a^2*b^3)/d))/d))/d) + x^4*((5*a*b^4)/(4*d) - (b^5*c)/(4*d^2)) + x^2*
((5*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d - (10*a^2*b^3)/d))/(
2*d)) - x^3*((c*((5*a*b^4)/d - (b^5*c)/d^2))/(3*d) - (10*a^2*b^3)/(3*d)) +
(b^5*x^5)/(5*d) + (log(c + d*x)*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 1
0*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/d^6
```

3.1335 $\int \frac{(a+bx)^4}{c+dx}\,dx$

Optimal. Leaf size=98

$$-\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c+dx)}{d^5}$$

```
[Out] -b*(-a*d+b*c)^3*x/d^4+1/2*(-a*d+b*c)^2*(b*x+a)^2/d^3-1/3*(-a*d+b*c)*(b*x+a)
^3/d^2+1/4*(b*x+a)^4/d+(-a*d+b*c)^4*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int\frac{(a+bx)^4}{c+dx}\,dx&=\int\left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)dx\\&=-\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 115, normalized size = 1.17

$$\frac{bdx\left(48a^3d^3+36a^2bd^2(-2c+dx)+8ab^2d\left(6c^2-3cdx+2d^2x^2\right)+b^3\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)\right)+12(bc-ad)^4\log(c+dx)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x),x]
```

```
[Out] (b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*x
 + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(
b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

Mathics [A]
time = 3.02, size = 127, normalized size = 1.30

$$\frac{bdx\left(4a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3\right)+\frac{b^2d^2x^2\left(6a^2d^2-4abcd+b^2c^2\right)}{2}+\frac{b^3d^3x^3(4ad-bc)}{3}+\frac{b^4d^4x^4}{4}+\mathrm{Log}\left[c+dx\right](ad-bc)^4}{d^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x),x]')
```

```
[Out] (b d x (4 a ^ 3 d ^ 3 - 6 a ^ 2 b c d ^ 2 + 4 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3
) + b ^ 2 d ^ 2 x ^ 2 (6 a ^ 2 d ^ 2 - 4 a b c d + b ^ 2 c ^ 2) / 2 + b ^ 3
 d ^ 3 x ^ 3 (4 a d - b c) / 3 + b ^ 4 d ^ 4 x ^ 4 / 4 + Log[c + d x] (a d
- b c) ^ 4) / d ^ 5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(92) = 184$.
time = 0.15, size = 189, normalized size = 1.93

method	result
norman	$\frac{b\left(4a^3d^3-6a^2bcd^2+4ab^2c^2d-b^3c^3\right)x}{d^4}+\frac{b^4x^4}{4d}+\frac{b^2\left(6a^2d^2-4abcd+b^2c^2\right)x^2}{2d^3}+\frac{b^3(4ad-bc)x^3}{3d^2}+\frac{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4}{d^5}$
default	$\frac{b\left(\frac{d^3x^4b^3}{4}+\frac{\left((2ad-bc)b^2d^2+2ab^2d^3\right)x^3}{3}+\frac{\left(2(2ad-bc)abd^2+bd\left(2a^2d^2-2abcd+b^2c^2\right)\right)x^2}{2}+(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)x\right)}{d^4}+\frac{(a^4d^4-}{}$
risch	$\frac{b^4x^4}{4d}+\frac{4b^3ax^3}{3d}-\frac{b^4cx^3}{3d^2}+\frac{3b^2a^2x^2}{d}-\frac{2b^3acx^2}{d^2}+\frac{b^4c^2x^2}{2d^3}+\frac{4ba^3x}{d}-\frac{6b^2a^2cx}{d^2}+\frac{4b^3ac^2x}{d^3}-\frac{b^4c^3x}{d^4}+\frac{\ln(dx+c)a^4}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^4*(1/4*d^3*x^4*b^3+1/3*((2*a*d-b*c)*b^2*d^2+2*a*b^2*d^3)*x^3+1/2*(2*(2*
a*d-b*c)*a*b*d^2+b*d*(2*a^2*d^2-2*a*b*c*d+b^2*c^2))*x^2+(2*a*d-b*c)*(2*a^2*
d^2-2*a*b*c*d+b^2*c^2)*x)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)/d^5*ln(d*x+c)
```

Maxima [A]
time = 0.26, size = 177, normalized size = 1.81

$$\frac{3\,b^4d^3x^4-4\left(b^4cd^2-4\,ab^3d^3\right)x^3+6\left(b^4c^2d-4\,ab^3cd^2+6\,a^2b^2d^3\right)x^2-12\left(b^4c^3-4\,ab^3c^2d+6\,a^2b^2cd^2-4\,a^3bd^3\right)x}{12\,d^4}+\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a*
b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

Fricas [A]
time = 0.30, size = 179, normalized size = 1.83

$$\frac{3\,b^4d^4x^4-4\,(b^4cd^3-4\,ab^3d^4)x^3+6\,(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2d^4)x^2-12\,(b^4c^3d-4\,ab^3c^2d^2+6\,a^2b^2cd^3-4\,a^3bd^4)x+12\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log\left(dx+c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4*
a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*
b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

Sympy [A]
time = 0.23, size = 136, normalized size = 1.39

$$\frac{b^4x^4}{4d}+x^3\cdot\left(\frac{4ab^3}{3d}-\frac{b^4c}{3d^2}\right)+x^2\cdot\left(\frac{3a^2b^2}{d}-\frac{2ab^3c}{d^2}+\frac{b^4c^2}{2d^3}\right)+x\left(\frac{4a^3b}{d}-\frac{6a^2b^2c}{d^2}+\frac{4ab^3c^2}{d^3}-\frac{b^4c^3}{d^4}\right)+\frac{(ad-bc)^4\log\left(c+dx\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c),x)
```

```
[Out] b**4*x**4/(4*d) + x**3*(4*a*b**3/(3*d) - b**4*c/(3*d**2)) + x**2*(3*a**2*b*
*2/d - 2*a*b**3*c/d**2 + b**4*c**2/(2*d**3)) + x*(4*a**3*b/d - 6*a**2*b**2*
c/d**2 + 4*a*b**3*c**2/d**3 - b**4*c**3/d**4) + (a*d - b*c)**4*log(c + d*x)
/d**5
```

Giac [A]
time = 0.00, size = 198, normalized size = 2.02

$$\frac{\frac{1}{4}x^4b^4d^3-\frac{1}{3}x^3b^4d^2c+\frac{4}{3}x^3b^3ad^3+\frac{1}{2}x^2b^4dc^2-2x^2b^3ad^2c+3x^2b^2a^2d^3-xb^4c^3+4xb^3adc^2-6xb^2a^2d^2c+4xba^3d^3}{d^4}+\frac{(b^4c^4-4b^3adc^3+6b^2a^2d^2c^2-4ba^3d^3c+a^4d^4)\ln|xd+c|}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x)
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
- 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d*x
 - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

Mupad [B]
time = 0.22, size = 189, normalized size = 1.93

$$x^3\left(\frac{4\,a\,b^3}{3\,d}-\frac{b^4\,c}{3\,d^2}\right)+x\left(\frac{4\,a^3\,b}{d}+\frac{c\left(\frac{c\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{d}-\frac{6\,a^2\,b^2}{d}\right)}{d}\right)-x^2\left(\frac{c\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{2\,d}-\frac{3\,a^2\,b^2}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{b^4\,x^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x),x)
```

```
[Out] x^3*((4*a*b^3)/(3*d) - (b^4*c)/(3*d^2)) + x*((4*a^3*b)/d + (c*((c*((4*a*b^3
)/d - (b^4*c)/d^2))/d - (6*a^2*b^2)/d))/d) - x^2*((c*((4*a*b^3)/d - (b^4*c)
/d^2))/(2*d) - (3*a^2*b^2)/d) + (log(c + d*x)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d^5 + (b^4*x^4)/(4*d)
```

3.1336 $\int \frac{(a+bx)^3}{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}$$

```
[Out] b*(-a*d+b*c)^2*x/d^3-1/2*(-a*d+b*c)*(b*x+a)^2/d^2+1/3*(b*x+a)^3/d-(-a*d+b*c
)^3*ln(d*x+c)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{c+dx}\,dx &= \int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx\\ &= \frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.00

$$\frac{bdx\left(18a^2d^2+9abd(-2c+dx)+b^2\left(6c^2-3cdx+2d^2x^2\right)\right)-6(bc-ad)^3\log(c+dx)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^
2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

Mathics [A]
time = 2.49, size = 79, normalized size = 1.07

$$\frac{bdx\left(3a^2d^2-3abcd+b^2c^2\right)+\frac{b^2d^2x^2(3ad-bc)}{2}+\frac{b^3d^3x^3}{3}+\text{Log}\left[c+dx\right]\left(ad-bc\right)^3}{d^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/(c + d*x),x]')
```

```
[Out] (b d x (3 a ^ 2 d ^ 2 - 3 a b c d + b ^ 2 c ^ 2) + b ^ 2 d ^ 2 x ^ 2 (3 a d
 - b c) / 2 + b ^ 3 d ^ 3 x ^ 3 / 3 + Log[c + d x] (a d - b c) ^ 3) / d ^ 4
```

Maple [A]
time = 0.14, size = 109, normalized size = 1.47

method	result
norman	$\frac{b(3a^2d^2-3abcd+b^2c^2)x}{d^3}+\frac{b^3x^3}{3d}+\frac{b^2(3ad-bc)x^2}{2d^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(dx+c)}{d^4}$
default	$\frac{b(\frac{1}{3}d^2x^3b^2+\frac{3}{2}ab\,d^2x^2-\frac{1}{2}b^2cd\,x^2+3a^2d^2x-3abcdx+b^2c^2x)}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(dx+c)}{d^4}$
risch	$\frac{b^3x^3}{3d}+\frac{3b^2a\,x^2}{2d}-\frac{b^3c\,x^2}{2d^2}+\frac{3b\,a^2x}{d}-\frac{3b^2acx}{d^2}+\frac{b^3c^2x}{d^3}+\frac{\ln(dx+c)a^3}{d}-\frac{3\ln(dx+c)a^2bc}{d^2}+\frac{3\ln(dx+c)a\,b^2c^2}{d^3}-\frac{\ln(dx+c)}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(1/3*d^2*x^3*b^2+3/2*a*b*d^2*x^2-1/2*b^2*c*d*x^2+3*a^2*d^2*x-3*a*b*c*
d*x+b^2*c^2*x)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(d*x+c)
```

Maxima [A]
time = 0.27, size = 114, normalized size = 1.54

$$\frac{2\,b^3d^2x^3-3\,(b^3cd-3\,ab^2d^2)x^2+6\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)x}{6\,d^3}-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*c
*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(d*x + c)/d^4
```

Fricas [A]

time = 0.29, size = 115, normalized size = 1.55

$$\frac{2\,b^3d^3x^3-3\,(b^3cd^2-3\,ab^2d^3)x^2+6\,(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3)x-6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log{(dx+c)}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*b
^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(d*x + c))/d^4
```

Sympy [A]

time = 0.18, size = 83, normalized size = 1.12

$$\frac{b^3x^3}{3d}+x^2\cdot\left(\frac{3ab^2}{2d}-\frac{b^3c}{2d^2}\right)+x\left(\frac{3a^2b}{d}-\frac{3ab^2c}{d^2}+\frac{b^3c^2}{d^3}\right)+\frac{(ad-bc)^3\log{(c+dx)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c),x)
```

```
[Out] b**3*x**3/(3*d) + x**2*(3*a*b**2/(2*d) - b**3*c/(2*d**2)) + x*(3*a**2*b/d -
 3*a*b**2*c/d**2 + b**3*c**2/d**3) + (a*d - b*c)**3*log(c + d*x)/d**4
```

Giac [A]

time = 0.00, size = 124, normalized size = 1.68

$$\frac{\frac{1}{3}x^3b^3d^2-\frac{1}{2}x^2b^3dc+\frac{3}{2}x^2b^2ad^2+xb^3c^2-3xb^2adc+3xba^2d^2}{d^3}+\frac{(-b^3c^3+3b^2adc^2-3ba^2d^2c+a^3d^3)\ln{|xd+c|}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*b
^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(d*x + c))/d^4
```

Mupad [B]

time = 0.07, size = 118, normalized size = 1.59

$$x^2\,\left(\frac{3\,a\,b^2}{2\,d}-\frac{b^3\,c}{2\,d^2}\right)+x\,\left(\frac{3\,a^2\,b}{d}-\frac{c\,\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right)+\frac{\ln{(c+d\,x)}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{d^4}+\frac{b^3\,x^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x),x)
```

```
[Out] x^2*((3*a*b^2)/(2*d) - (b^3*c)/(2*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d
- (b^3*c)/d^2))/d) + (log(c + d*x)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/d^4 + (b^3*x^3)/(3*d)
```

3.1337 $\int \frac{(a+bx)^2}{c+dx} \, dx$

Optimal. Leaf size=50

$$-\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}$$

```
[Out] -b*(-a*d+b*c)*x/d^2+1/2*(b*x+a)^2/d+(-a*d+b*c)^2*ln(d*x+c)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.86

$$\frac{bdx(-2bc+4ad+bdx)+2(bc-ad)^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x),x]
```

```
[Out] (b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

Mathics [A]
time = 2.09, size = 46, normalized size = 0.92

$$\frac{bdx\,(2ad-bc)+\frac{b^2d^2x^2}{2}+\mathrm{Log}\,[c+dx]\,(ad-bc)^2}{d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x),x]')
```

```
[Out] (b d x (2 a d - b c) + b ^ 2 d ^ 2 x ^ 2 / 2 + Log[c + d x] (a d - b c) ^ 2
) / d ^ 3
```

Maple [A]
time = 0.18, size = 56, normalized size = 1.12

method	result	size
default	$\frac{b\left(\frac{1}{2}bd\,x^2+2adx-bcx\right)}{d^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^3}$	56
norman	$\frac{b(2ad-bc)x}{d^2}+\frac{b^2x^2}{2d}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^3}$	59
risch	$\frac{b^2x^2}{2d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\ln(dx+c)a^2}{d}-\frac{2\ln(dx+c)abc}{d^2}+\frac{\ln(dx+c)b^2c^2}{d^3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(1/2*b*d*x^2+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*ln(d*x+c)
```

Maxima [A]
time = 0.27, size = 60, normalized size = 1.20

$$\frac{b^2dx^2-2\,(b^2c-2\,abd)x}{2\,d^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log\,(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(d*x + c)/d^3
```

Fricas [A]
time = 0.29, size = 62, normalized size = 1.24

$$\frac{b^2d^2x^2-2\,(b^2cd-2\,abd^2)x+2\,(b^2c^2-2\,abcd+a^2d^2)\log\,(dx+c)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(d*x + c))/d^3
```

Sympy [A]
time = 0.13, size = 44, normalized size = 0.88

$$\frac{b^2x^2}{2d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) + \frac{(ad-bc)^2\log(c+dx)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c),x)
```

```
[Out] b**2*x**2/(2*d) + x*(2*a*b/d - b**2*c/d**2) + (a*d - b*c)**2*log(c + d*x)/d
**3
```

Giac [A]
time = 0.00, size = 65, normalized size = 1.30

$$\frac{\frac{1}{2}x^2b^2d - xb^2c + 2xbad}{d^2} + \frac{(b^2c^2 - 2badc + a^2d^2)\ln|xd+c|}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(d*x + c))/d^3
```

Mupad [B]
time = 0.22, size = 62, normalized size = 1.24

$$\frac{\ln(c+d\,x)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{d^3} - x\,\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - x*((b^2*c)/d^2 - (2*a*
b)/d) + (b^2*x^2)/(2*d)
```

3.1338 $\int \frac{a+bx}{c+dx} \, dx$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

[Out] b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c + d*x),x]

[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{c+dx} \, dx &= \int \left(\frac{b}{d} + \frac{-bc+ad}{d(c+dx)}\right) dx \\ &= \frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.96

$$\frac{bx}{d} + \frac{(-bc+ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(c + d*x),x]

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Mathics [A]
time = 1.86, size = 24, normalized size = 0.92

$$\frac{bdx + \mathrm{Log}\left[c + dx\right](ad - bc)}{d^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x),x]')
[Out] (b d x + Log[c + d x] (a d - b c)) / d ^ 2
```

Maple [A]
time = 0.12, size = 26, normalized size = 1.00

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
risch	$\frac{bx}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
[Out] b*x/d+(a*d-b*c)/d^2*ln(d*x+c)
```

Maxima [A]
time = 0.26, size = 26, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A]
time = 0.30, size = 25, normalized size = 0.96

$$\frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A]
time = 0.08, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log{(c + dx)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{xb}{d} + \frac{(-bc + ad)\ln|xd + c|}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x)
```

```
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

Mupad [B]
time = 0.20, size = 25, normalized size = 0.96

$$\frac{\ln(c + d\,x)\,(a\,d - b\,c)}{d^2} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

3.1339 $\int \frac{1}{c+dx}\,dx$

Optimal. Leaf size=10

$$\frac{\log(c+dx)}{d}$$

[Out] ln(d*x+c)/d

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-1),x]

[Out] Log[c + d*x]/d

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{c+dx}\,dx = \frac{\log(c+dx)}{d}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-1),x]

[Out] Log[c + d*x]/d

Mathics [A]
time = 1.68, size = 10, normalized size = 1.00

$$\frac{\text{Log}\left[c+dx\right]}{d}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x),x]')
```

```
[Out] Log[c + d x] / d
```

Maple [A]
time = 0.12, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(dx+c)}{d}$	11
norman	$\frac{\ln(dx+c)}{d}$	11
risch	$\frac{\ln(dx+c)}{d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] ln(d*x+c)/d
```

Maxima [A]
time = 0.27, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(d*x + c)/d
```

Fricas [A]
time = 0.31, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="fricas")
```

```
[Out] log(d*x + c)/d
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.70

$$\frac{\log(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x)
```

```
[Out] log(c + d*x)/d
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\ln|xd+c|}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x)
```

```
[Out] log(abs(d*x + c))/d
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x),x)
```

```
[Out] log(c + d*x)/d
```

3.1340 $\int \frac{1}{(a+bx)(c+dx)} dx$

Optimal. Leaf size=36

$$\frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}$$

```
[Out] ln(b*x+a)/(-a*d+b*c)-ln(d*x+c)/(-a*d+b*c)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {36, 31}

$$\frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)}\,dx &= \frac{b\int \frac{1}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx}\,dx}{bc-ad}\\ &= \frac{\log(a+bx)}{bc-ad} - \frac{\log(c+dx)}{bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.72

$$\frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

Mathics [A]
time = 3.59, size = 30, normalized size = 0.83

$$\frac{\text{Log}\left[\frac{c}{d}+x\right]-\text{Log}\left[\frac{a}{b}+x\right]}{ad-bc}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-1)/(c + d*x),x]')
```

```
[Out] (Log[c / d + x] - Log[a / b + x]) / (a d - b c)
```

Maple [A]
time = 0.16, size = 37, normalized size = 1.03

method	result	size
default	$\frac{\ln(dx+c)}{ad-bc}-\frac{\ln(bx+a)}{ad-bc}$	37
norman	$\frac{\ln(dx+c)}{ad-bc}-\frac{\ln(bx+a)}{ad-bc}$	37
risch	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(-dx-c)}{ad-bc}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)*ln(d*x+c)-1/(a*d-b*c)*ln(b*x+a)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc-ad}-\frac{\log\left(dx+c\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

Fricas [A]
time = 0.30, size = 26, normalized size = 0.72

$$\frac{\log\left(bx+a\right)-\log\left(dx+c\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(26) = 52$.

time = 0.19, size = 128, normalized size = 3.56

$$\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x)
```

```
[Out] log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(
a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) - 2
*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d - b
*c)
```

Giac [A]

time = 0.00, size = 43, normalized size = 1.19

$$\frac{d\ln|xd+c|}{-bdc+ad^2}+\frac{b\ln|xb+a|}{b^2c-bad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x)
```

```
[Out] b*log(abs(b*x + a))/(b^2*c - a*b*d) - d*log(abs(d*x + c))/(b*c*d - a*d^2)
```

Mupad [B]

time = 0.26, size = 25, normalized size = 0.69

$$\frac{\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{a\,d-b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)),x)
```

```
[Out] log((c + d*x)/(a + b*x))/(a*d - b*c)
```

3.1341 $\int \frac{1}{(a+bx)^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -1/(-a*d+b*c)/(b*x+a)-d*ln(b*x+a)/(-a*d+b*c)^2+d*ln(d*x+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{46\}$

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.93

$$\frac{-bc+ad-d(a+bx)\log(a+bx)+d(a+bx)\log(c+dx)}{(bc-ad)^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

Mathics [A]
time = 4.81, size = 93, normalized size = 1.63

$$\frac{d\left(a^2d-abc+bx\left(ad-bc\right)\right)\left(\mathrm{Log}\left[\frac{c+dx}{d}\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)+\left(ad-bc\right)^2}{\left(ad-bc\right)^2\left(a^2d-abc+bx\left(ad-bc\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-2)/(c + d*x),x]')
```

```
[Out] (d (a ^ 2 d - a b c + b x (a d - b c)) (Log[(c + d x) / d] - Log[(a + b x)
/ b]) + (a d - b c) ^ 2) / ((a d - b c) ^ 2 (a ^ 2 d - a b c + b x (a d - b
 c)))
```

Maple [A]
time = 0.16, size = 57, normalized size = 1.00

method	result	size
default	$\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}$	57
risch	$\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}+\frac{d\ln(-dx-c)}{a^2d^2-2abcd+b^2c^2}$	86
norman	$-\frac{bx}{a(ad-bc)(bx+a)}+\frac{d\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A]
time = 0.31, size = 92, normalized size = 1.61

$$-\frac{d\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A]
time = 0.30, size = 93, normalized size = 1.63

$$-\frac{bc-ad+(bdx+ad)\log\left(bx+a\right)-(bdx+ad)\log\left(dx+c\right)}{ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.
time = 0.40, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}+\frac{1}{a^2d-abc+x\,(abd-b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A]
time = 0.00, size = 101, normalized size = 1.77

$$\frac{d^2\ln|xd+c|}{b^2dc^2-2bad^2c+a^2d^3}-\frac{bd\ln|xb+a|}{b^3c^2-2b^2adc+ba^2d^2}+\frac{-bc+da}{(bc-da)^2\,(xb+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] -b*d*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + d^2*log(abs(d*
x + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/((b*c - a*d)*(b*x + a))
```

Mupad [B]
time = 0.14, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)}-\frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

3.1342 $\int \frac{1}{(a+bx)^3(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}$$

```
[Out] -1/2/(-a*d+b*c)/(b*x+a)^2+d/(-a*d+b*c)^2/(b*x+a)+d^2*ln(b*x+a)/(-a*d+b*c)^3
-d^2*ln(d*x+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] -1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a
 + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3}\right.\\ &= -\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(-bc+3ad+2bdx)}{(a+bx)^2}+2d^2\log(a+bx)-2d^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] (((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
- 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(82) = 164.
time = 6.84, size = 227, normalized size = 2.77

$$\frac{d^2\left(a^4d^2-2a^3bcd+a^2b^2c^2+2abx\left(a^2d^2-2abcd+b^2c^2\right)+b^2x^2\left(a^2d^2-2abcd+b^2c^2\right)\right)\left(\mathrm{Log}\left[\frac{c+dx}{d}\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)+\frac{(3ad-bc+2bdx)(ad-bc)^3}{2}}{(ad-bc)^3\left(a^4d^2-2a^3bcd+a^2b^2c^2+2abx\left(a^2d^2-2abcd+b^2c^2\right)+b^2x^2\left(a^2d^2-2abcd+b^2c^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-3)/(c + d*x),x]')
```

```
[Out] (d ^ 2 (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + a ^ 2 b ^ 2 c ^ 2 + 2 a b x (a ^ 2 d
^ 2 - 2 a b c d + b ^ 2 c ^ 2) + b ^ 2 x ^ 2 (a ^ 2 d ^ 2 - 2 a b c d + b ^
 2 c ^ 2)) (Log[(c + d x) / d] - Log[(a + b x) / b]) + (3 a d - b c + 2 b d
 x) (a d - b c) ^ 3 / 2) / ((a d - b c) ^ 3 (a ^ 4 d ^ 2 - 2 a ^ 3 b c d +
a ^ 2 b ^ 2 c ^ 2 + 2 a b x (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) + b ^ 2
 x ^ 2 (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2)))
```

Maple [A]
time = 0.17, size = 81, normalized size = 0.99

method	result	size
default	$\frac{d^2\ln(dx+c)}{(ad-bc)^3}+\frac{1}{2(ad-bc)(bx+a)^2}+\frac{d}{(ad-bc)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-bc)^3}$	81
risch	$\frac{\frac{bdx}{a^2d^2-2abcd+b^2c^2}+\frac{3ad-bc}{2a^2d^2-4abcd+2b^2c^2}}{(bx+a)^2}-\frac{d^2\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{d^2\ln(-dx-c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	172
norman	$\frac{\frac{bdx}{a^2d^2-2abcd+b^2c^2}+\frac{3a\,b^2d-b^3c}{2b^2\left(a^2d^2-2abcd+b^2c^2\right)}}{(bx+a)^2}+\frac{d^2\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{d^2\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2
/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(80) = 160.
time = 0.29, size = 202, normalized size = 2.46

$$\frac{d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx-bc+3\,ad}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.
time = 0.30, size = 242, normalized size = 2.95

$$-\frac{b^2c^2 - 4\,abcd + 3\,a^2d^2 - 2\,(b^2cd - abd^2)x - 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log\left(bx + a\right) + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log\left(dx + c\right)}{2\,(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3 + (b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3)x^2 + 2\,(ab^4c^3 - 3\,a^2b^3c^2d + 3\,a^3b^2cd^2 - a^4bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2 + 2*
(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(68) = 136$.
time = 0.62, size = 381, normalized size = 4.65

$$\frac{d^2\log\left(x + \frac{-\frac{a^4d^6}{(ad-bc)^3} + \frac{4a^3bcd^5}{(ad-bc)^3} - \frac{6a^2b^2c^2d^4}{(ad-bc)^3} + \frac{4ab^3c^3d^3}{(ad-bc)^3} + ad^3 - \frac{b^4c^4d^2}{(ad-bc)^3} + bcd^2}{2bd^3}\right)}{(ad-bc)^3} - \frac{d^2\log\left(x + \frac{\frac{a^4d^6}{(ad-bc)^3} - \frac{4a^3bcd^5}{(ad-bc)^3} + \frac{6a^2b^2c^2d^4}{(ad-bc)^3} - \frac{4ab^3c^3d^3}{(ad-bc)^3} + ad^3 + \frac{b^4c^4d^2}{(ad-bc)^3} + bcd^2}{2bd^3}\right)}{(ad-bc)^3} + \frac{3ad - bc + 2bdx}{2a^4d^2 - 4a^3bcd + 2a^2b^2c^2 + x^2\cdot(2a^2b^2d^2 - 4ab^3cd + 2b^4c^2) + x\,(4a^3bd^2 - 8a^2b^2cd + 4ab^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c),x)
```

```
[Out] d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3 +
a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*c)*
*3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*c)**
3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b*c)**
3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b
*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**2*b**2*
c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*a**3*b*d
**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.
time = 0.00, size = 172, normalized size = 2.10

$$\frac{d^3\ln|xd + c|}{-b^3dc^3 + 3b^2ad^2c^2 - 3ba^2d^3c + a^3d^4} - \frac{bd^2\ln|xb + a|}{-b^4c^3 + 3b^3adc^2 - 3b^2a^2d^2c + ba^3d^3} + \frac{\frac{1}{2}\left(-b^2c^2 + 4bdca - 3d^2a^2 + (2b^2dc - 2bd^2a)\,x\right)}{(bc - da)^3\,(xb + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x)
```

```
[Out] b*d^2*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3) - d^3*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) - 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(b*x + a)^2)
```

Mupad [B]
time = 0.16, size = 182, normalized size = 2.22

$$\frac{\frac{3\,a\,d-b\,c}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{a^2+2\,a\,b\,x+b^2\,x^2}-\frac{2\,d^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)),x)
```

```
[Out] ((3*a*d - b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x)/(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a^2 + b^2*x^2 + 2*a*b*x) - (2*d^2*atanh((a^3*d^3 + b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

3.1343 $\int \frac{(a+bx)^5}{(c+dx)^2} \, dx$

Optimal. Leaf size=130

$$-\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c+dx)^4}{4d^6}+\frac{5b(bc-ad)^4 \text{l}}{d^6}$$

```
[Out] -10*b^2*(-a*d+b*c)^3*x/d^5+(-a*d+b*c)^5/d^6/(d*x+c)+5*b^3*(-a*d+b*c)^2*(d*x
+c)^2/d^6-5/3*b^4*(-a*d+b*c)*(d*x+c)^3/d^6+1/4*b^5*(d*x+c)^4/d^6+5*b*(-a*d+
b*c)^4*ln(d*x+c)/d^6
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+dx)^4}{4d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (-10*b^2*(b*c - a*d)^3*x)/d^5 + (b*c - a*d)^5/(d^6*(c + d*x)) + (5*b^3*(b*c
 - a*d)^2*(c + d*x)^2)/d^6 - (5*b^4*(b*c - a*d)*(c + d*x)^3)/(3*d^6) + (b^5
*(c + d*x)^4)/(4*d^6) + (5*b*(b*c - a*d)^4*Log[c + d*x])/d^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^2}\,dx &= \int \left(-\frac{10b^2(bc-ad)^3}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^2}+\frac{5b(bc-ad)^4}{d^5(c+dx)}+\frac{10b^3(bc-ad)^2(c+dx)}{d^5}-\frac{5b^4(bc-}{}\right.\\ &= -\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 228, normalized size = 1.75

$$\frac{60a^4bcd^4-12a^5d^5+120a^3b^2d^3\left(-c^2+cdx+d^2x^2\right)+60a^2b^3d^2\left(2c^3-4c^2dx-3cd^2x^2+d^3x^3\right)+20ab^4d\left(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4\right)+b^5\left(12c^5-48c^4dx-30c^3d^2x^2+10c^2d^3x^3-5cd^4x^4+3d^5x^5\right)+60b(bc-ad)^4(c+dx)\log(c+dx)}{12d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (60*a^4*b*c*d^4 - 12*a^5*d^5 + 120*a^3*b^2*d^3*(-c^2 + c*d*x + d^2*x^2) + 6
0*a^2*b^3*d^2*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + 20*a*b^4*d*(-3*
c^4 + 9*c^3*d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) + b^5*(12*c^5 - 48
*c^4*d*x - 30*c^3*d^2*x^2 + 10*c^2*d^3*x^3 - 5*c*d^4*x^4 + 3*d^5*x^5) + 60*
b*(b*c - a*d)^4*(c + d*x)*Log[c + d*x])/(12*d^6*(c + d*x))
```

Mathics [A]
time = 4.09, size = 227, normalized size = 1.75

$$\frac{-a^5d^5+5b\mathrm{Log}\left[c+dx\right](c+dx)(ad-bc)^4+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5+b^2dx\left(10a^3d^3-20a^2bcd^2+15ab^2c^2d-4b^3c^3\right)(c+dx)+\frac{b^3d^2x^2\left(10a^2d^2-10abcd+3b^2c^2\right)(c+dx)}{2}+\frac{b^4d^3x^3(5ad-2bc)(c+dx)}{3}+\frac{b^5d^4x^4(c+dx)}{4}}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x)^2,x]')
```

```
[Out] (-a ^ 5 d ^ 5 + 5 b Log[c + d x] (c + d x) (a d - b c) ^ 4 + 5 a ^ 4 b c d
^ 4 - 10 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 10 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 5 a b ^ 4 c
 ^ 4 d + b ^ 5 c ^ 5 + b ^ 2 d x (10 a ^ 3 d ^ 3 - 20 a ^ 2 b c d ^ 2 + 15
a b ^ 2 c ^ 2 d - 4 b ^ 3 c ^ 3) (c + d x) + b ^ 3 d ^ 2 x ^ 2 (10 a ^ 2 d
^ 2 - 10 a b c d + 3 b ^ 2 c ^ 2) (c + d x) / 2 + b ^ 4 d ^ 3 x ^ 3 (5 a d
- 2 b c) (c + d x) / 3 + b ^ 5 d ^ 4 x ^ 4 (c + d x) / 4) / (d ^ 6 (c + d x
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(126) = 252$.
time = 0.18, size = 259, normalized size = 1.99

method	result
norman	$\frac{\frac{\left(a^5d^5-5a^4bc\,d^4+20a^3b^2c^2d^3-30a^2b^3c^3d^2+20a\,b^4c^4d-5b^5c^5\right)x}{d^5c}+\frac{b^5x^5}{4d}+\frac{5b^2\left(4a^3d^3-6a^2bc\,d^2+4a\,b^2c^2d-b^3c^3\right)x^2}{2d^4}+\frac{5b^3\left(6a^2d^2-4abcd+b^2c^2\right)}{6d^3}}{dx+c}$
default	$\frac{b^2\left(\frac{1}{4}d^3x^4b^3+\frac{5}{3}a\,b^2d^3x^3-\frac{2}{3}b^3c\,d^2x^3+5a^2b\,d^3x^2-5a\,b^2c\,d^2x^2+\frac{3}{2}b^3c^2d\,x^2+10a^3d^3x-20a^2bc\,d^2x+15a\,b^2c^2dx-4b^3c^3x\right)}{d^5}-\frac{a^5d^5-5a}{}$
risch	$\frac{b^5x^4}{4d^2}+\frac{5b^4a\,x^3}{3d^2}-\frac{2b^5c\,x^3}{3d^3}+\frac{5b^3a^2x^2}{d^2}-\frac{5b^4ac\,x^2}{d^3}+\frac{3b^5c^2x^2}{2d^4}+\frac{10b^2a^3x}{d^2}-\frac{20b^3a^2cx}{d^3}+\frac{15b^4a\,c^2x}{d^4}-\frac{4b^5c^3x}{d^5}-\frac{a^5}{d(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^5*(1/4*d^3*x^4*b^3+5/3*a*b^2*d^3*x^3-2/3*b^3*c*d^2*x^3+5*a^2*b*d^3*x^
2-5*a*b^2*c*d^2*x^2+3/2*b^3*c^2*d*x^2+10*a^3*d^3*x-20*a^2*b*c*d^2*x+15*a*b^
2*c^2*d*x-4*b^3*c^3*x)-(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3
*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)+5*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(126) = 252$.
time = 0.36, size = 264, normalized size = 2.03

$$\frac{b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5}{d^7x + cd^6} + \frac{3\,b^5d^3x^4 - 4\,(2\,b^5cd^2 - 5\,ab^4d^3)x^3 + 6\,(3\,b^5c^2d - 10\,ab^4cd^2 + 10\,a^2b^3d^3)x^2 - 12\,(4\,b^5c^3 - 15\,ab^4c^2d + 20\,a^2b^3cd^2 - 10\,a^3b^2d^3)x}{12\,d^5} + \frac{5\,(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4)\log(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*
b*c*d^4 - a^5*d^5)/(d^7*x + c*d^6) + 1/12*(3*b^5*d^3*x^4 - 4*(2*b^5*c*d^2 -
 5*a*b^4*d^3)*x^3 + 6*(3*b^5*c^2*d - 10*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^2 -
 12*(4*b^5*c^3 - 15*a*b^4*c^2*d + 20*a^2*b^3*c*d^2 - 10*a^3*b^2*d^3)*x)/d^5
 + 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4)*log(d*x + c)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(126) = 252$.
time = 0.30, size = 373, normalized size = 2.87

$$\frac{3\,b^5d^5x^5 + 12\,b^5c^5 - 60\,ab^4c^4d + 120\,a^2b^3c^3d^2 - 120\,a^3b^2c^2d^3 + 60\,a^4bcd^4 - 12\,a^5d^5 - 5\,(b^5cd^4 - 4\,ab^4d^5)x^4 + 10\,(b^5c^2d^3 - 4\,ab^4cd^4 + 6\,a^2b^3d^5)x^3 - 30\,(b^5c^3d^2 - 4\,ab^4c^2d^3 + 6\,a^2b^3cd^4 - 4\,a^3b^2d^5)x^2 - 12\,(4\,b^5c^4d - 15\,ab^4c^3d^2 + 20\,a^2b^3c^2d^3 - 10\,a^3b^2cd^4)x + 60\,(b^5c^5 - 4\,ab^4c^4d + 6\,a^2b^3c^3d^2 - 4\,a^3b^2c^2d^3 + a^4bcd^4 + (b^5c^4d - 4\,ab^4c^3d^2 + 6\,a^2b^3c^2d^3 - 4\,a^3b^2cd^4 + a^4bd^5)x)\log(dx + c)}{12\,(d^7x + cd^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*d^5*x^5 + 12*b^5*c^5 - 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 - 1
20*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 - 12*a^5*d^5 - 5*(b^5*c*d^4 - 4*a*b^4*d
^5)*x^4 + 10*(b^5*c^2*d^3 - 4*a*b^4*c*d^4 + 6*a^2*b^3*d^5)*x^3 - 30*(b^5*c^
3*d^2 - 4*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 - 4*a^3*b^2*d^5)*x^2 - 12*(4*b^5*
c^4*d - 15*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4)*x + 60*(b
^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*c*d^
4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^
4*b*d^5)*x)*log(d*x + c))/(d^7*x + c*d^6)
```

Sympy [A]
time = 0.54, size = 231, normalized size = 1.78

$$\frac{b^5x^4}{4d^2} + \frac{5b\,(ad - bc)^4\log(c + dx)}{d^6} + x^3 \cdot \left(\frac{5ab^4}{3d^2} - \frac{2b^5c}{3d^3}\right) + x^2 \cdot \left(\frac{5a^2b^3}{d^2} - \frac{5ab^4c}{d^3} + \frac{3b^5c^2}{2d^4}\right) + x\left(\frac{10a^3b^2}{d^2} - \frac{20a^2b^3c}{d^3} + \frac{15ab^4c^2}{d^4} - \frac{4b^5c^3}{d^5}\right) + \frac{-a^5d^5 + 5a^4bcd^4 - 10a^3b^2c^2d^3 + 10a^2b^3c^3d^2 - 5ab^4c^4d + b^5c^5}{cd^6 + d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**2,x)
```

```
[Out] b**5*x**4/(4*d**2) + 5*b*(a*d - b*c)**4*log(c + d*x)/d**6 + x**3*(5*a*b**4/
(3*d**2) - 2*b**5*c/(3*d**3)) + x**2*(5*a**2*b**3/d**2 - 5*a*b**4*c/d**3 +
3*b**5*c**2/(2*d**4)) + x*(10*a**3*b**2/d**2 - 20*a**2*b**3*c/d**3 + 15*a*b
```

```
**4*c**2/d**4 - 4*b**5*c**3/d**5) + (-a**5*d**5 + 5*a**4*b*c*d**4 - 10*a**3
*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d + b**5*c**5)/(c*
d**6 + d**7*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(126) = 252.
time = 0.00, size = 290, normalized size = 2.23

$$\frac{\frac{1}{4}x^4b^5d^6 - \frac{2}{3}x^3b^5d^5c + \frac{5}{3}x^3b^4ad^6 + \frac{3}{2}x^2b^5d^4c^2 - 5x^2b^4ad^5c + 5x^2b^3a^2d^6 - 4xb^5d^3c^3 + 15xb^4ad^4c^2 - 20xb^3a^2d^5c + 10xb^2a^3d^6}{d^8} + \frac{b^5c^5 - 5b^4dc^4a + 10b^3d^2c^3a^2 - 10b^2d^3c^2a^3 + 5bd^4ca^4 - d^5a^5}{d^6(xd+c)} + \frac{(5b^5c^4 - 20b^4adc^3 + 30b^3a^2d^2c^2 - 20b^2a^3d^3c + 5ba^4d^4)\ln|xd+c|}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x)
```

```
[Out] 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^
4)*log(abs(d*x + c))/d^6 + (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 -
10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)/((d*x + c)*d^6) + 1/12*(3*b^5
*d^6*x^4 - 8*b^5*c*d^5*x^3 + 20*a*b^4*d^6*x^3 + 18*b^5*c^2*d^4*x^2 - 60*a*b
^4*c*d^5*x^2 + 60*a^2*b^3*d^6*x^2 - 48*b^5*c^3*d^3*x + 180*a*b^4*c^2*d^4*x
- 240*a^2*b^3*c*d^5*x + 120*a^3*b^2*d^6*x)/d^8
```

Mupad [B]
time = 0.25, size = 327, normalized size = 2.52

$$x^3\left(\frac{5\,a\,b^4}{3\,d^2}-\frac{2\,b^5\,c}{3\,d^3}\right)+x\left(\frac{2\,c\,\left(\frac{2\,c\,\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d}-\frac{10\,a^2\,b^3}{d^2}+\frac{b^5\,c^2}{d^4}\right)}{d}+\frac{10\,a^3\,b^2}{d^2}-\frac{c^2\,\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d^2}\right)-x^2\left(\frac{c\,\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d}-\frac{5\,a^2\,b^3}{d^2}+\frac{b^5\,c^2}{2\,d^4}\right)+\frac{\ln\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)}{d^6}-\frac{a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5}{d\,\left(x\,d^6+c\,d^5\right)}+\frac{b^5\,x^4}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^2,x)
```

```
[Out] x^3*((5*a*b^4)/(3*d^2) - (2*b^5*c)/(3*d^3)) + x*((2*c*((2*c*((5*a*b^4)/d^2
- (2*b^5*c)/d^3))/d - (10*a^2*b^3)/d^2 + (b^5*c^2)/d^4))/d + (10*a^3*b^2)/d
^2 - (c^2*((5*a*b^4)/d^2 - (2*b^5*c)/d^3))/d^2) - x^2*((c*((5*a*b^4)/d^2 -
(2*b^5*c)/d^3))/d - (5*a^2*b^3)/d^2 + (b^5*c^2)/(2*d^4)) + (log(c + d*x)*(5
*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c
^3*d))/d^6 - (a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)/(d*(c*d^5 + d^6*x)) + (b^5*x^4)/(4*d^2)
```

3.1344 $\int \frac{(a+bx)^4}{(c+dx)^2} \, dx$

Optimal. Leaf size=104

$$\frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}$$

```
[Out] 6*b^2*(-a*d+b*c)^2*x/d^4-(-a*d+b*c)^4/d^5/(d*x+c)-2*b^3*(-a*d+b*c)*(d*x+c)^
2/d^5+1/3*b^4*(d*x+c)^3/d^5-4*b*(-a*d+b*c)^3*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^2}\,dx &= \int \left(\frac{6b^2(bc-ad)^2}{d^4}+\frac{(-bc+ad)^4}{d^4(c+dx)^2}-\frac{4b(bc-ad)^3}{d^4(c+dx)}-\frac{4b^3(bc-ad)(c+dx)}{d^4}+\frac{b^4(c+dx)^2}{d^4}\right)\\ &= \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c}{d^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 165, normalized size = 1.59

$$\frac{12a^3bcd^3-3a^4d^4+18a^2b^2d^2\left(-c^2+cdx+d^2x^2\right)+6ab^3d\left(2c^3-4c^2dx-3cd^2x^2+d^3x^3\right)+b^4\left(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4\right)-12b(bc-ad)^3(c+dx)\log(c+dx)}{3d^5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*a
*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*d*
x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x)*L
og[c + d*x])/(3*d^5*(c + d*x))
```

Mathics [A]
time = 3.32, size = 160, normalized size = 1.54

$$\frac{-a^4d^4+4b\text{Log}\left[c+dx\right](c+dx)\,(ad-bc)^3+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d-b^4c^4+b^2dx\,(6a^2d^2-8abcd+3b^2c^2)\,(c+dx)+b^3d^2x^2\,(2ad-bc)\,(c+dx)+\frac{b^4d^3x^3(c+dx)}{3}}{d^5\,(c+dx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x)^2,x]')
```

```
[Out] (-a ^ 4 d ^ 4 + 4 b Log[c + d x] (c + d x) (a d - b c) ^ 3 + 4 a ^ 3 b c d
^ 3 - 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 4 a b ^ 3 c ^ 3 d - b ^ 4 c ^ 4 + b ^ 2 d
 x (6 a ^ 2 d ^ 2 - 8 a b c d + 3 b ^ 2 c ^ 2) (c + d x) + b ^ 3 d ^ 2 x ^
2 (2 a d - b c) (c + d x) + b ^ 4 d ^ 3 x ^ 3 (c + d x) / 3) / (d ^ 5 (c +
d x))
```

Maple [A]
time = 0.14, size = 175, normalized size = 1.68

method	result
default	$\frac{b^2\left(\frac{1}{3}d^2x^3b^2+2ab\,d^2x^2-b^2cd\,x^2+6a^2d^2x-8abcdx+3b^2c^2x\right)}{d^4}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}{d^5(dx+c)}+\frac{4b\left(a^3d^3-3a^2bc\,d^2+\right.}{}$
norman	$\frac{\frac{\left(a^4d^4-4a^3bc\,d^3+12a^2b^2c^2d^2-12a\,b^3c^3d+4b^4c^4\right)x}{d^4c}+\frac{b^4x^4}{3d}+\frac{2b^2\left(3a^2d^2-3abcd+b^2c^2\right)x^2}{d^3}+\frac{2b^3(3ad-bc)x^3}{3d^2}}{dx+c}+\frac{4b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d\right.}{d^5}$
risch	$\frac{b^4x^3}{3d^2}+\frac{2b^3a\,x^2}{d^2}-\frac{b^4c\,x^2}{d^3}+\frac{6b^2a^2x}{d^2}-\frac{8b^3acx}{d^3}+\frac{3b^4c^2x}{d^4}-\frac{a^4}{d(dx+c)}+\frac{4a^3bc}{d^2(dx+c)}-\frac{6a^2b^2c^2}{d^3(dx+c)}+\frac{4a\,b^3c^3}{d^4(dx+c)}-\frac{b^4c^4}{d^5(dx+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^4*(1/3*d^2*x^3*b^2+2*a*b*d^2*x^2-b^2*c*d*x^2+6*a^2*d^2*x-8*a*b*c*d*x+
3*b^2*c^2*x)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)/d^5/(d*x+c)+4*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*x+
c)
```

Maxima [A]
time = 0.26, size = 183, normalized size = 1.76

$$-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{d^6x+cd^5}+\frac{b^4d^2x^3-3\,(b^4cd-2\,ab^3d^2)x^2+3\,(3\,b^4c^2-8\,ab^3cd+6\,a^2b^2d^2)x}{3\,d^4}-\frac{4\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(d
^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b^4
*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(102) = 204$.
time = 0.29, size = 267, normalized size = 2.57

$$\frac{b^4d^4x^4 - 3\,b^4c^4 + 12\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 12\,a^3bcd^3 - 3\,a^4d^4 - 2\,(b^4cd^3 - 3\,ab^3d^4)x^3 + 6\,(b^4c^2d^2 - 3\,ab^3cd^3 + 3\,a^2b^2d^4)x^2 + 3\,(3\,b^4c^3d - 8\,ab^3c^2d^2 + 6\,a^2b^2cd^3)x - 12\,(b^4c^4 - 3\,ab^3c^3d + 3\,a^2b^2c^2d^2 - a^3bcd^3 + (b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)x)\log(dx + c)}{3\,(d^6x + cd^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 3
*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*b*c*
d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*log(d*
x + c))/(d^6*x + c*d^5)
```

Sympy [A]
time = 0.41, size = 155, normalized size = 1.49

$$\frac{b^4x^3}{3d^2} + \frac{4b\,(ad - bc)^3\log{(c + dx)}}{d^5} + x^2\cdot\left(\frac{2ab^3}{d^2} - \frac{b^4c}{d^3}\right) + x\left(\frac{6a^2b^2}{d^2} - \frac{8ab^3c}{d^3} + \frac{3b^4c^2}{d^4}\right) + \frac{-a^4d^4 + 4a^3bcd^3 - 6a^2b^2c^2d^2 + 4ab^3c^3d - b^4c^4}{cd^5 + d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**2,x)
```

```
[Out] b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 + x**2*(2*a*b**3/
d**2 - b**4*c/d**3) + x*(6*a**2*b**2/d**2 - 8*a*b**3*c/d**3 + 3*b**4*c**2/d
**4) + (-a**4*d**4 + 4*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 + 4*a*b**3*c**
3*d - b**4*c**4)/(c*d**5 + d**6*x)
```

Giac [A]
time = 0.00, size = 197, normalized size = 1.89

$$\frac{\frac{1}{3}x^3b^4d^4 - x^2b^4d^3c + 2x^2b^3ad^4 + 3xb^4d^2c^2 - 8xb^3ad^3c + 6xb^2a^2d^4}{d^6} + \frac{-b^4c^4 + 4b^3dc^3a - 6b^2d^2c^2a^2 + 4bd^3ca^3 - d^4a^4}{d^5\,(xd + c)} + \frac{(-4b^4c^3 + 12b^3adc^2 - 12b^2a^2d^2c + 4ba^3d^3)\ln|xd + c|}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x)
```

```
[Out] -4*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c)
)/d^5 + 1/3*(b^4*d^4*x^3 - 3*b^4*c*d^3*x^2 + 6*a*b^3*d^4*x^2 + 9*b^4*c^2*d^
2*x - 24*a*b^3*c*d^3*x + 18*a^2*b^2*d^4*x)/d^6 - (b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/((d*x + c)*d^5)
```

Mupad [B]
time = 0.07, size = 203, normalized size = 1.95

$$x^2\left(\frac{2\,a\,b^3}{d^2}-\frac{b^4\,c}{d^3}\right)-x\left(\frac{2\,c\,\left(\frac{4\,a\,b^3}{d^2}-\frac{2\,b^4\,c}{d^3}\right)}{d}-\frac{6\,a^2\,b^2}{d^2}+\frac{b^4\,c^2}{d^4}\right)+\frac{b^4\,x^3}{3\,d^2}-\frac{\ln\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{d^5}-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{d\,\left(x\,d^5+c\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^2,x)
```

```
[Out] x^2*((2*a*b^3)/d^2 - (b^4*c)/d^3) - x*((2*c*((4*a*b^3)/d^2 - (2*b^4*c)/d^3)
)/d - (6*a^2*b^2)/d^2 + (b^4*c^2)/d^4) + (b^4*x^3)/(3*d^2) - (log(c + d*x)*
(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/d^5 - (a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(d*(c*d^4
 + d^5*x))
```

3.1345 $\int \frac{(a+bx)^3}{(c+dx)^2}\,dx$

Optimal. Leaf size=75

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}$$

```
[Out] -b^2*(-3*a*d+2*b*c)*x/d^3+1/2*b^3*x^2/d^2+(-a*d+b*c)^3/d^4/(d*x+c)+3*b*(-a*
d+b*c)^2*ln(d*x+c)/d^4
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x}{d^2}+\frac{(-bc+ad)^3}{d^3(c+dx)^2}+\frac{3b(bc-ad)^2}{d^3(c+dx)}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 114, normalized size = 1.52

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{d^4(c+dx)}+\frac{3\left(b^3c^2-2ab^2cd+a^2bd^2\right)\log(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*Log[c + d*x])/d^4
```

Mathics [A]
time = 2.65, size = 107, normalized size = 1.43

$$\frac{-a^3d^3+3b\text{Log}\left[c+dx\right]\left(c+dx\right)\left(ad-bc\right)^2+3a^2bcd^2-3ab^2c^2d+b^3c^3+b^2dx\left(3ad-2bc\right)\left(c+dx\right)+\frac{b^3d^2x^2(c+dx)}{2}}{d^4\left(c+dx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/(c + d*x)^2,x]')
```

```
[Out] (-a ^ 3 d ^ 3 + 3 b Log[c + d x] (c + d x) (a d - b c) ^ 2 + 3 a ^ 2 b c d
^ 2 - 3 a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3 + b ^ 2 d x (3 a d - 2 b c) (c + d x)
 + b ^ 3 d ^ 2 x ^ 2 (c + d x) / 2) / (d ^ 4 (c + d x))
```

Maple [A]
time = 0.14, size = 108, normalized size = 1.44

method	result
default	$\frac{b^2\left(\frac{1}{2}bd\,x^2+3adx-2bcx\right)}{d^3}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{d^4(dx+c)}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}$
norman	$\frac{-\frac{a^3d^3-3a^2bc\,d^2+6a\,b^2c^2d-3b^3c^3}{d^4}+\frac{b^3x^3}{2d}+\frac{3b^2(2ad-bc)x^2}{2d^2}}{dx+c}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}$
risch	$\frac{b^3x^2}{2d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}-\frac{a^3}{d(dx+c)}+\frac{3a^2bc}{d^2(dx+c)}-\frac{3a\,b^2c^2}{d^3(dx+c)}+\frac{b^3c^3}{d^4(dx+c)}+\frac{3b\ln(dx+c)a^2}{d^2}-\frac{6b^2\ln(dx+c)ac}{d^3}+$ 3b^3 ln(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^3*(1/2*b*d*x^2+3*a*d*x-2*b*c*x)-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-
b^3*c^3)/d^4/(d*x+c)+3*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*x+c)
```

Maxima [A]
time = 0.28, size = 117, normalized size = 1.56

$$\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{d^5x+cd^4}+\frac{b^3dx^2-2\,(2\,b^3c-3\,ab^2d)x}{2\,d^3}+\frac{3\,(b^3c^2-2\,ab^2cd+a^2bd^2)\log\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*(
b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*log(d*x + c)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.
time = 0.29, size = 172, normalized size = 2.29

$$\frac{b^3d^3x^3 + 2\,b^3c^3 - 6\,ab^2c^2d + 6\,a^2bcd^2 - 2\,a^3d^3 - 3\,(b^3cd^2 - 2\,ab^2d^3)x^2 - 2\,(2\,b^3c^2d - 3\,ab^2cd^2)x + 6\,(b^3c^3 - 2\,ab^2c^2d + a^2bcd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x)\log\left(dx + c\right)}{2\,(d^5x + cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(b^
3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

Sympy [A]
time = 0.30, size = 102, normalized size = 1.36

$$\frac{b^3x^2}{2d^2} + \frac{3b\,(ad - bc)^2\log\left(c + dx\right)}{d^4} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 + x*(3*a*b**2/d**
2 - 2*b**3*c/d**3) + (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3
*c**3)/(c*d**4 + d**5*x)
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.68

$$\frac{\frac{1}{2}x^2b^3d^2 - 2xb^3dc + 3xb^2ad^2}{d^4} + \frac{b^3c^3 - 3b^2dc^2a + 3bd^2ca^2 - d^3a^3}{d^4\,(xd + c)} + \frac{(3b^3c^2 - 6b^2adc + 3ba^2d^2)\ln|xd + c|}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c))/d^4 + 1/2*(b^3*d^2*
x^2 - 4*b^3*c*d*x + 6*a*b^2*d^2*x)/d^4 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)/((d*x + c)*d^4)
```

Mupad [B]
time = 0.08, size = 123, normalized size = 1.64

$$x\left(\frac{3\,a\,b^2}{d^2} - \frac{2\,b^3\,c}{d^3}\right) + \frac{\ln\left(c + d\,x\right)\,\left(3\,a^2\,b\,d^2 - 6\,a\,b^2\,c\,d + 3\,b^3\,c^2\right)}{d^4} - \frac{a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3}{d\,\left(x\,d^4 + c\,d^3\right)} + \frac{b^3\,x^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (log(c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d))/d^4 - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
d*(c*d^3 + d^4*x)) + (b^3*x^2)/(2*d^2)
```

3.1346 $\int \frac{(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=51

$$\frac{b^2x}{d^2} - \frac{(bc-ad)^2}{d^3(c+dx)} - \frac{2b(bc-ad)\log(c+dx)}{d^3}$$

```
[Out] b^2*x/d^2-(-a*d+b*c)^2/d^3/(d*x+c)-2*b*(-a*d+b*c)*ln(d*x+c)/d^3
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{(bc-ad)^2}{d^3(c+dx)} - \frac{2b(bc-ad)\log(c+dx)}{d^3} + \frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{(c+dx)^2}\,dx &= \int \left(\frac{b^2}{d^2} + \frac{(-bc+ad)^2}{d^2(c+dx)^2} - \frac{2b(bc-ad)}{d^2(c+dx)}\right) dx\\
&= \frac{b^2x}{d^2} - \frac{(bc-ad)^2}{d^3(c+dx)} - \frac{2b(bc-ad)\log(c+dx)}{d^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.92

$$\frac{b^2dx - \frac{(bc-ad)^2}{c+dx} + 2b(-bc+ad)\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

Mathics [A]
time = 2.28, size = 67, normalized size = 1.31

$$\frac{-a^2d^2 + 2b\text{Log}\left[c + dx\right]\left(ad - bc\right)\left(c + dx\right) + 2abcd - b^2c^2 + b^2dx\left(c + dx\right)}{d^3\left(c + dx\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x)^2,x]')
```

```
[Out] (-a ^ 2 d ^ 2 + 2 b Log[c + d x] (a d - b c) (c + d x) + 2 a b c d - b ^ 2
c ^ 2 + b ^ 2 d x (c + d x)) / (d ^ 3 (c + d x))
```

Maple [A]
time = 0.14, size = 63, normalized size = 1.24

method	result	size
default	$\frac{b^2x}{d^2} - \frac{a^2d^2-2abcd+b^2c^2}{d^3(dx+c)} + \frac{2b(ad-bc)\ln(dx+c)}{d^3}$	63
norman	$\frac{\frac{b^2x^2}{d} - \frac{a^2d^2-2abcd+2b^2c^2}{d^3}}{dx+c} + \frac{2b(ad-bc)\ln(dx+c)}{d^3}$	68
risch	$\frac{b^2x}{d^2} - \frac{a^2}{d(dx+c)} + \frac{2abc}{d^2(dx+c)} - \frac{b^2c^2}{d^3(dx+c)} + \frac{2b\ln(dx+c)a}{d^2} - \frac{2b^2\ln(dx+c)c}{d^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2-(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)+2*b/d^3*(a*d-b*c)*ln(d*x+
c)
```

Maxima [A]
time = 0.27, size = 67, normalized size = 1.31

$$\frac{b^2x}{d^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{d^4x + cd^3} - \frac{2\left(b^2c - abd\right)\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a*
b*d)*log(d*x + c)/d^3
```

Fricas [A]
time = 0.30, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2 + b^2cdx - b^2c^2 + 2\,abcd - a^2d^2 - 2\,(b^2c^2 - abcd + (b^2cd - abd^2)x)\log{(dx + c)}}{d^4x + cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

Sympy [A]
time = 0.20, size = 60, normalized size = 1.18

$$\frac{b^2x}{d^2} + \frac{2b\,(ad - bc)\log{(c + dx)}}{d^3} + \frac{-a^2d^2 + 2abcd - b^2c^2}{cd^3 + d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 + (-a**2*d**2 + 2*a*b*c*d -
 b**2*c**2)/(c*d**3 + d**4*x)
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.35

$$\frac{xb^2}{d^2} + \frac{-b^2c^2 + 2bdca - d^2a^2}{d^3\,(xd + c)} + \frac{(-2b^2c + 2bad)\ln{|xd + c|}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] b^2*x/d^2 - 2*(b^2*c - a*b*d)*log(abs(d*x + c))/d^3 - (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/((d*x + c)*d^3)
```

Mupad [B]
time = 0.24, size = 71, normalized size = 1.39

$$\frac{b^2\,x}{d^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{d\,\left(x\,d^3 + c\,d^2\right)} - \frac{\ln{(c + d\,x)}\,\left(2\,b^2\,c - 2\,a\,b\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] (b^2*x)/d^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(d*(c*d^2 + d^3*x)) - (log(c
+ d*x)*(2*b^2*c - 2*a*b*d))/d^3
```

3.1347 $\int \frac{a+bx}{(c+dx)^2}\,dx$

Optimal. Leaf size=31

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

```
[Out] (-a*d+b*c)/d^2/(d*x+c)+b*ln(d*x+c)/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^2}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^2}+\frac{b}{d(c+dx)}\right)dx\\ &= \frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Mathics [A]
time = 1.94, size = 32, normalized size = 1.03

$$\frac{-ad + b\text{Log}\,[c + dx]\,(c + dx) + bc}{d^2\,(c + dx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x)^2,x]')
[Out] (-a d + b Log[c + d x] (c + d x) + b c) / (d ^ 2 (c + d x))
```

Maple [A]
time = 0.12, size = 33, normalized size = 1.06

method	result	size
default	$-\frac{ad-bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	33
norman	$-\frac{ad-bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	33
risch	$-\frac{a}{d(dx+c)}+\frac{bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -(a*d-b*c)/d^2/(d*x+c)+b*ln(d*x+c)/d^2
```

Maxima [A]
time = 0.28, size = 34, normalized size = 1.10

$$\frac{bc - ad}{d^3x + cd^2} + \frac{b\log\left(dx + c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] (b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

Fricas [A]
time = 0.30, size = 37, normalized size = 1.19

$$\frac{bc - ad + (bdx + bc)\log\left(dx + c\right)}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

Sympy [**A**]
time = 0.11, size = 27, normalized size = 0.87

$$\frac{b\log(c+dx)}{d^2}+\frac{-ad+bc}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**2,x)
```

```
[Out] b*log(c + d*x)/d**2 + (-a*d + b*c)/(c*d**2 + d**3*x)
```

Giac [**A**]
time = 0.00, size = 33, normalized size = 1.06

$$\frac{bc-ad}{dd(xd+c)}+\frac{b\ln|xd+c|}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x)
```

```
[Out] b*log(abs(d*x + c))/d^2 + (b*c - a*d)/((d*x + c)*d^2)
```

Mupad [**B**]
time = 0.04, size = 32, normalized size = 1.03

$$\frac{b\,\ln(c+d\,x)}{d^2}-\frac{a\,d-b\,c}{d^2\,(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^2,x)
```

```
[Out] (b*log(c + d*x))/d^2 - (a*d - b*c)/(d^2*(c + d*x))
```

3.1348 $\int \frac{1}{(c+dx)^2} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{d(c+dx)}$$

```
[Out] -1/d/(d*x+c)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-2),x]
[Out] -(1/(d*(c + d*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2} \, dx = -\frac{1}{d(c+dx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-2),x]
[Out] -(1/(d*(c + d*x)))
```

Mathics [A]
time = 1.69, size = 12, normalized size = 1.00

$$-\frac{1}{d\,(c+dx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x)^2,x]')
```

```
[Out] -1 / (d (c + d x))
```

Maple [A]
time = 0.13, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{d(dx+c)}$	13
default	$-\frac{1}{d(dx+c)}$	13
norman	$\frac{x}{c(dx+c)}$	13
risch	$-\frac{1}{d(dx+c)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/(d*x+c)
```

Maxima [A]
time = 0.35, size = 12, normalized size = 1.00

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/((d*x + c)*d)
```

Fricas [A]
time = 0.29, size = 13, normalized size = 1.08

$$-\frac{1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/(d^2*x + c*d)
```

Sympy [A]
time = 0.07, size = 10, normalized size = 0.83

$$-\frac{1}{cd+d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2,x)
```

```
[Out] -1/(c*d + d**2*x)
```

Giac [A]
time = 0.00, size = 9, normalized size = 0.75

$$-\frac{1}{d\,(xd+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x)
```

```
[Out] -1/((d*x + c)*d)
```

Mupad [B]
time = 0.19, size = 12, normalized size = 1.00

$$-\frac{1}{d\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^2,x)
```

```
[Out] -1/(d*(c + d*x))
```

3.1349 $\int \frac{1}{(a+bx)(c+dx)^2}\,dx$

Optimal. Leaf size=56

$$\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

```
[Out] 1/(-a*d+b*c)/(d*x+c)+b*ln(b*x+a)/(-a*d+b*c)^2-b*ln(d*x+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right)dx\\ &= \frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.95

$$\frac{bc-ad+b(c+dx)\log(a+bx)-b(c+dx)\log(c+dx)}{(bc-ad)^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c - a
*d)^2*(c + d*x))
```

Mathics [A]
time = 5.09, size = 95, normalized size = 1.70

$$\frac{b\left(acd-bc^2+dx\left(ad-bc\right)\right)\left(\operatorname{Log}\left[\frac{a+bx}{b}\right]-\operatorname{Log}\left[\frac{c+dx}{d}\right]\right)-\left(ad-bc\right)^2}{\left(ad-bc\right)^2\left(acd-bc^2+dx\left(ad-bc\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-1)/(c + d*x)^2,x]')
```

```
[Out] (b (a c d - b c ^ 2 + d x (a d - b c)) (Log[(a + b x) / b] - Log[(c + d x)
/ d]) - (a d - b c) ^ 2) / ((a d - b c) ^ 2 (a c d - b c ^ 2 + d x (a d - b
 c)))
```

Maple [A]
time = 0.16, size = 58, normalized size = 1.04

method	result	size
default	$-\frac{1}{(ad-bc)(dx+c)}-\frac{b\ln(dx+c)}{(ad-bc)^2}+\frac{b\ln(bx+a)}{(ad-bc)^2}$	58
risch	$-\frac{1}{(ad-bc)(dx+c)}+\frac{b\ln(-bx-a)}{a^2d^2-2abcd+b^2c^2}-\frac{b\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}$	87
norman	$\frac{dx}{c(ad-bc)(dx+c)}+\frac{b\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}-\frac{b\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A]
time = 0.30, size = 90, normalized size = 1.61

$$\frac{b\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{b\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Fricas [A]
time = 0.29, size = 92, normalized size = 1.64

$$\frac{bc-ad+(bdx+bc)\log\left(bx+a\right)-(bdx+bc)\log\left(dx+c\right)}{b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2*
c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.
time = 0.40, size = 233, normalized size = 4.16

$$-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}-\frac{1}{acd-bc^2+x(ad^2-bcd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**2,x)
```

```
[Out] -b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2
 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2
*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*
a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d -
b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c*d -
b*c**2 + x*(a*d**2 - b*c*d))
```

Giac [A]
time = 0.00, size = 101, normalized size = 1.80

$$\frac{b^2\ln|xb+a|}{b^3c^2-2b^2adc+ba^2d^2}-\frac{bd\ln|xd+c|}{b^2dc^2-2bad^2c+a^2d^3}+\frac{bc-da}{(bc-da)^2(xd+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x)
```

```
[Out] b^2*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - b*d*log(abs(d*x
 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) + 1/((b*c - a*d)*(d*x + c))
```

Mupad [B]
time = 0.29, size = 47, normalized size = 0.84

$$-\frac{1}{(a\,d-b\,c)\,(c+d\,x)}-\frac{b\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^2),x)
```

```
[Out] - 1/((a*d - b*c)*(c + d*x)) - (b*log((c + d*x)/(a + b*x)))/(a*d - b*c)^2
```

3.1350 $\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

```
[Out] -b/(-a*d+b*c)^2/(b*x+a)-d/(-a*d+b*c)^2/(d*x+c)-2*b*d*ln(b*x+a)/(-a*d+b*c)^3
+2*b*d*ln(d*x+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] -(b/((b*c - a*d)^2*(a + b*x))) - d/((b*c - a*d)^2*(c + d*x)) - (2*b*d*Log[a
 + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\frac{2\ldots}{(bc-ad\ldots}\right.\\ &= -\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.81

$$\frac{\frac{b(-bc+ad)}{a+bx}+\frac{d(-bc+ad)}{c+dx}-2bd\log(a+bx)+2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
+ b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 248 vs. $2(81) = 162$.
time = 6.85, size = 248, normalized size = 3.06

$$\frac{2bd\left(a^3cd^2-2a^2bc^2d+ab^2c^3+x\left(a^3d^3-a^2bcd^2-ab^2c^2d+b^3c^3\right)+bdx^2\left(a^2d^2-2abcd+b^2c^2\right)\right)\left(\operatorname{Log}\left[\frac{a+bx}{b}\right]-\operatorname{Log}\left[\frac{c+dx}{d}\right]\right)+(-ad-bc-2bdx)(ad-bc)^3}{(ad-bc)^3\left(a^3cd^2-2a^2bc^2d+ab^2c^3+x\left(a^3d^3-a^2bcd^2-ab^2c^2d+b^3c^3\right)+bdx^2\left(a^2d^2-2abcd+b^2c^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-2)/(c + d*x)^2,x]')
```

```
[Out] (2 b d (a ^ 3 c d ^ 2 - 2 a ^ 2 b c ^ 2 d + a b ^ 2 c ^ 3 + x (a ^ 3 d ^ 3
- a ^ 2 b c d ^ 2 - a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) + b d x ^ 2 (a ^ 2 d ^ 2
 - 2 a b c d + b ^ 2 c ^ 2)) (Log[(a + b x) / b] - Log[(c + d x) / d]) + (-
a d - b c - 2 b d x) (a d - b c) ^ 3) / ((a d - b c) ^ 3 (a ^ 3 c d ^ 2 - 2
 a ^ 2 b c ^ 2 d + a b ^ 2 c ^ 3 + x (a ^ 3 d ^ 3 - a ^ 2 b c d ^ 2 - a b ^
 2 c ^ 2 d + b ^ 3 c ^ 3) + b d x ^ 2 (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^
2)))
```

Maple [A]
time = 0.16, size = 82, normalized size = 1.01

method	result	size
default	$-\frac{d}{(ad-bc)^2(dx+c)}-\frac{2db\ln(dx+c)}{(ad-bc)^3}-\frac{b}{(ad-bc)^2(bx+a)}+\frac{2db\ln(bx+a)}{(ad-bc)^3}$	82
risch	$\frac{-\frac{2bdx}{a^2d^2-2abcd+b^2c^2}-\frac{ad+bc}{a^2d^2-2abcd+b^2c^2}}{(bx+a)(dx+c)}-\frac{2bd\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{2bd\ln(-bx-a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	177
norman	$\frac{\frac{-ab\,d^2-b^2cd}{db\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{2bdx}{a^2d^2-2abcd+b^2c^2}}{(bx+a)(dx+c)}+\frac{2bd\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{2bd\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*
d/(a*d-b*c)^3*b*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.
time = 0.29, size = 208, normalized size = 2.57

$$-\frac{2\,bd\log\left(bx+a\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bd\log\left(dx+c\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,bdx+bc+ad}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 2
*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (2*
b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(81) = 162.
time = 0.30, size = 241, normalized size = 2.98

$$-\frac{b^2c^2 - a^2d^2 + 2\,(b^2cd - abd^2)x + 2\,(b^2d^2x^2 + abcd + (b^2cd + abd^2)x)\log\left(bx + a\right) - 2\,(b^2d^2x^2 + abcd + (b^2cd + abd^2)x)\log\left(dx + c\right)}{ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + (b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)x^2 + (b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
 a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(70) = 140.
time = 0.65, size = 406, normalized size = 5.01

$$-\frac{2bd\log\left(x + \frac{-\frac{2a^4bd^5}{(ad-bc)^3} + \frac{8a^3b^2cd^4}{(ad-bc)^3} - \frac{12a^2b^3c^2d^3}{(ad-bc)^3} + \frac{8ab^4c^3d^2}{(ad-bc)^3} + 2abd^2 - \frac{2b^5c^4d}{(ad-bc)^3} + 2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3} + \frac{2bd\log\left(x + \frac{\frac{2a^4bd^5}{(ad-bc)^3} - \frac{8a^3b^2cd^4}{(ad-bc)^3} + \frac{12a^2b^3c^2d^3}{(ad-bc)^3} - \frac{8ab^4c^3d^2}{(ad-bc)^3} + 2abd^2 + \frac{2b^5c^4d}{(ad-bc)^3} + 2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3} + \frac{-ad - bc - 2bdx}{a^3cd^2 - 2a^2bc^2d + ab^2c^3 + x^2\left(a^2bd^3 - 2ab^2cd^2 + b^3c^2d\right) + x\left(a^3d^3 - a^2bcd^2 - ab^2c^2d + b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] -2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d -
b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**2*d
**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*a**3
*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*a*b
**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)**3 +
2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x)/(a**3*c*
d**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2*c*d**2
+ b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b**3*c**3))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(81) = 162.
time = 0.00, size = 165, normalized size = 2.04

$$\frac{2bd^2\ln|xd + c|}{b^3dc^3 - 3b^2ad^2c^2 + 3ba^2d^3c - a^3d^4} + \frac{2b^2d\ln|xb + a|}{-b^4c^3 + 3b^3adc^2 - 3b^2a^2d^2c + ba^3d^3} + \frac{2xbd + bc + ad}{(-b^2c^2 + 2badc - a^2d^2)\,(x^2bd + xbc + xad + ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] -2*b^2*d*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3) + 2*b*d^2*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*
c*d^3 - a^3*d^4) - (2*b*d*x + b*c + a*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(
b*d*x^2 + b*c*x + a*d*x + a*c))
```

Mupad [B]
time = 0.33, size = 74, normalized size = 0.91

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)\,(c+d\,x)}-\frac{2\,d}{{(a\,d-b\,c)}^2\,(c+d\,x)}-\frac{2\,b\,d\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{{(a\,d-b\,c)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)*(c + d*x)) - (2*d)/((a*d - b*c)^2*(c + d*x)) - (2*
b*d*log((c + d*x)/(a + b*x)))/(a*d - b*c)^3
```

3.1351 $\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}$$

```
[Out] -1/2*b/(-a*d+b*c)^2/(b*x+a)^2+2*b*d/(-a*d+b*c)^3/(b*x+a)+d^2/(-a*d+b*c)^3/(
d*x+c)+3*b*d^2*ln(b*x+a)/(-a*d+b*c)^4-3*b*d^2*ln(d*x+c)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] -1/2*b/((b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^
2/((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d
^2*Log[c + d*x])/(b*c - a*d)^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^3}-\frac{2b^2d}{(bc-ad)^3(a+bx)^2}+\frac{3b^2d^2}{(bc-ad)^4(a+bx)}-\frac{}{(bc-ad}\right.\\ &= -\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(a}{(bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.90

$$\frac{-\frac{b(bc-ad)^2}{(a+bx)^2}+\frac{4bd(bc-ad)}{a+bx}+\frac{2d^2(bc-ad)}{c+dx}+6bd^2\log(a+bx)-6bd^2\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] (-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2*
(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*(b
*c - a*d)^4)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 497 vs. $2(109) = 218$.
time = 9.78, size = 495, normalized size = 4.54

$$\frac{6bd^2\left(a^5cd^3-3a^4bc^2d^2+3a^3b^2c^3d-a^2b^3c^4-ax\left(-a^4d^4+a^3bcd^3+3a^2b^2c^2d^2-5ab^3c^3d+2b^4c^4\right)+bx^2\left(2a^4d^4-5a^3bcd^3+3a^2b^2c^2d^2+ab^3c^3d-b^4c^4\right)+b^2dx^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\right)\left(\mathrm{Log}\left[\frac{a+bx}{b}\right]-\mathrm{Log}\left[\frac{c+dx}{d}\right]\right)+\left(-2a^2d^2-5abcd+b^2c^2-3bdx\left(3ad+bc\right)-6b^2d^2x^2\right)\left(ad-bc\right)^4}{2\left(ad-bc\right)^4\left(a^5cd^3-3a^4bc^2d^2+3a^3b^2c^3d-a^2b^3c^4-ax\left(-a^4d^4+a^3bcd^3+3a^2b^2c^2d^2-5ab^3c^3d+2b^4c^4\right)+bx^2\left(2a^4d^4-5a^3bcd^3+3a^2b^2c^2d^2+ab^3c^3d-b^4c^4\right)+b^2dx^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-3)/(c + d*x)^2,x]')
```

```
[Out] (6 b d ^ 2 (a ^ 5 c d ^ 3 - 3 a ^ 4 b c ^ 2 d ^ 2 + 3 a ^ 3 b ^ 2 c ^ 3 d -
 a ^ 2 b ^ 3 c ^ 4 - a x (-a ^ 4 d ^ 4 + a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c
^ 2 d ^ 2 - 5 a b ^ 3 c ^ 3 d + 2 b ^ 4 c ^ 4) + b x ^ 2 (2 a ^ 4 d ^ 4 - 5
 a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + a b ^ 3 c ^ 3 d - b ^ 4 c ^
4) + b ^ 2 d x ^ 3 (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b ^ 2 c ^ 2 d - b
 ^ 3 c ^ 3)) (Log[(a + b x) / b] - Log[(c + d x) / d]) + (-2 a ^ 2 d ^ 2 -
5 a b c d + b ^ 2 c ^ 2 - 3 b d x (3 a d + b c) - 6 b ^ 2 d ^ 2 x ^ 2) (a d
 - b c) ^ 4) / (2 (a d - b c) ^ 4 (a ^ 5 c d ^ 3 - 3 a ^ 4 b c ^ 2 d ^ 2 +
3 a ^ 3 b ^ 2 c ^ 3 d - a ^ 2 b ^ 3 c ^ 4 - a x (-a ^ 4 d ^ 4 + a ^ 3 b c d
 ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 5 a b ^ 3 c ^ 3 d + 2 b ^ 4 c ^ 4) + b x
 ^ 2 (2 a ^ 4 d ^ 4 - 5 a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + a b ^
 3 c ^ 3 d - b ^ 4 c ^ 4) + b ^ 2 d x ^ 3 (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2
+ 3 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3)))
```

Maple [**A**]
time = 0.16, size = 109, normalized size = 1.00

method	result
default	$-\frac{d^2}{(ad-bc)^3(dx+c)}-\frac{3d^2b\ln(dx+c)}{(ad-bc)^4}-\frac{b}{2(ad-bc)^2(bx+a)^2}+\frac{3d^2b\ln(bx+a)}{(ad-bc)^4}-\frac{2bd}{(ad-bc)^3(bx+a)}$
risch	$\frac{-\frac{3b^2d^2x^2}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{3(3ad+bc)dbx}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{2a^2d^2+5abcd-b^2c^2}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx+a)^2(dx+c)}+\frac{3d^2b\ln(-bx-}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2}$
norman	$\frac{-\frac{3b^2d^2x^2}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{-2a^2b^2d^3-5a\,b^3c\,d^2+b^4c^2d}{2d\,b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(-9a\,b^3d^3-3b^4c\,d^2\right)x}{2d\,b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx+a)^2(dx+c)}+\frac{3d^2b}{a^4d^4-4a^3bc\,d^3+6a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*d-b*c)^2/(b
*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(107) = 214$.
time = 0.31, size = 386, normalized size = 3.54

$$\frac{3\,bd^2\log\left(bx+a\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{3\,bd^2\log\left(dx+c\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{6\,b^2d^2x^2-b^2c^2+5\,abcd+2\,a^2d^2+3\,(b^2cd+3\,abd^2)x}{2\,(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^3+(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4)x^2+(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b
^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5*a*
b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3
*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*
a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b
^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(107) = 214$.
time = 0.30, size = 494, normalized size = 4.53

$$-\frac{b^3c^3-6\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3-6\,(b^3cd^2-ab^2d^3)x^2-3\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x-6\,(b^3d^3x^3+a^2bcd^2+(b^3cd^2+2\,ab^2d^3)x^2+(2\,ab^2cd^2+a^2bd^3)x)\log\left(bx+a\right)+6\,(b^3d^3x^3+a^2bcd^2+(b^3cd^2+2\,ab^2d^3)x^2+(2\,ab^2cd^2+a^2bd^3)x)\log\left(dx+c\right)}{2\,(a^2b^4c^5-4\,a^3b^3c^4d+6\,a^4b^2c^3d^2-4\,a^5bc^2d^3+a^6cd^4+(b^6c^4d-4\,ab^5c^3d^2+6\,a^2b^4c^2d^3-4\,a^3b^3cd^4+a^4b^2d^5)x^3+(b^6c^5-2\,ab^5c^4d-2\,a^2b^4c^3d^2+8\,a^3b^3c^2d^3-7\,a^4b^2cd^4+2\,a^5bd^5)x^2+(2\,ab^5c^5-7\,a^2b^4c^4d+8\,a^3b^3c^3d^2-2\,a^4b^2c^2d^3-2\,a^5bcd^4+a^6d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5 - 4*a
^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6*c^4*d
 - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^3
 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4
*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*
c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(97) = 194$.
time = 1.04, size = 634, normalized size = 5.82

$$-\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2d^5}{(ad-bc)^4}-\frac{30a^2b^4c^3d^4}{(ad-bc)^4}+\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3-\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{-2a^2d^2-5abcd+b^2c^2-6b^2d^2x^2+x\left(-9abd^2-3b^2cd\right)}{2a^5cd^3-6a^4bc^2d^2+6a^3b^2c^3d-2a^2b^3c^4+x^3\cdot\left(2a^3b^2d^4-6a^2b^3cd^3+6ab^4c^2d^2-2b^5c^3d\right)+x^2\cdot\left(4a^4bd^4-10a^3b^2cd^3+6a^2b^3c^2d^2+2ab^4c^3d-2b^5c^4\right)+x\left(2a^5d^4-2a^4bcd^3-6a^3b^2c^2d^2+10a^2b^3c^3d-4ab^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] -3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*d
 - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4
/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b**6*
c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + 3
*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a*d -
b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4/(a
*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b**6*c**
5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + (-2*
a**2*d**2 - 5*a*b*c*d + b**2*c**2 - 6*b**2*d**2*x**2 + x*(-9*a*b*d**2 - 3*b
**2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*d - 2*a**2
*b**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b**4*c**2*d*
*2 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3 + 6*a**2*b*
*3*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4 - 2*a**4*b*c
*d**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**4*c**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(107) = 214$.

time = 0.00, size = 264, normalized size = 2.42

$$-\frac{3bd^3\ln|xd+c|}{b^4dc^4-4b^3ad^2c^3+6b^2a^2d^3c^2-4ba^3d^4c+a^4d^5}+\frac{3b^2d^2\ln|xb+a|}{b^5c^4-4b^4adc^3+6b^3a^2d^2c^2-4b^2a^3d^3c+ba^4d^4}+\frac{\frac{1}{2}\left((6b^3d^2c-6b^2d^3a)x^2+(3b^3dc^2+6b^2d^2ca-9bd^3a^2)x-b^3c^3+6b^2dc^2a-3bd^2ca^2-2d^3a^3\right)}{(bc-da)^4(xd+c)(xb+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 3*b^2*d^2*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
4*a^3*b^2*c*d^3 + a^4*b*d^4) - 3*b*d^3*log(abs(d*x + c))/(b^4*c^4*d - 4*a*b
^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/2*(b^3*c^3 -
6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 -
 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x + a)^2*
(d*x + c))
```

Mupad [B]

time = 0.40, size = 330, normalized size = 3.03

$$\frac{6\,b\,d^2\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{\frac{2\,a^2\,d^2+5\,a\,b\,c\,d-b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,d\,x\,\left(c\,b^2+3\,a\,d\,b\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(d\,a^2+2\,b\,c\,a\right)+a^2\,c+x^2\,\left(c\,b^2+2\,a\,d\,b\right)+b^2\,d\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^2),x)
```

```
[Out] (6*b*d^2*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c*d^3)/(a*d - b
*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d
- b*c)^4))/(a*d - b*c)^4 - ((2*a^2*d^2 - b^2*c^2 + 5*a*b*c*d)/(2*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*d*x*(b^2*c + 3*a*b*d))/(2*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(x*(a^2*d + 2*a*b*c) + a^2*
c + x^2*(b^2*c + 2*a*b*d) + b^2*d*x^3)
```

3.1352 $\int \frac{(a+bx)^6}{(c+dx)^3}\,dx$

Optimal. Leaf size=158

$$-\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-ad)(c+dx)^3}{d^7}+\frac{b^6(c+dx)^4}{4d^7}$$

```
[Out] -20*b^3*(-a*d+b*c)^3*x/d^6-1/2*(-a*d+b*c)^6/d^7/(d*x+c)^2+6*b*(-a*d+b*c)^5/
d^7/(d*x+c)+15/2*b^4*(-a*d+b*c)^2*(d*x+c)^2/d^7-2*b^5*(-a*d+b*c)*(d*x+c)^3/
d^7+1/4*b^6*(d*x+c)^4/d^7+15*b^2*(-a*d+b*c)^4*ln(d*x+c)/d^7
```

Rubi [A]
time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{b^6(c+dx)^4}{4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-20*b^3*(b*c - a*d)^3*x)/d^6 - (b*c - a*d)^6/(2*d^7*(c + d*x)^2) + (6*b*(b
*c - a*d)^5)/(d^7*(c + d*x)) + (15*b^4*(b*c - a*d)^2*(c + d*x)^2)/(2*d^7) -
 (2*b^5*(b*c - a*d)*(c + d*x)^3)/d^7 + (b^6*(c + d*x)^4)/(4*d^7) + (15*b^2*
(b*c - a*d)^4*Log[c + d*x])/d^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{(c+dx)^3}\,dx &= \int \left(-\frac{20b^3(bc-ad)^3}{d^6}+\frac{(-bc+ad)^6}{d^6(c+dx)^3}-\frac{6b(bc-ad)^5}{d^6(c+dx)^2}+\frac{15b^2(bc-ad)^4}{d^6(c+dx)}+\frac{15b^4(bc-ad)^2(c\right.\\ &= -\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-a\end{aligned}$$

Mathematica [A]
time = 0.07, size = 303, normalized size = 1.92

$$\frac{-2a^6d^6-12a^5bd^5(c+2dx)+30a^4b^2cd^4(3c+4dx)+40a^3b^3d^3\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+30a^2b^4d^2\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)+4ab^5d\left(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5\right)+b^6\left(22c^6-16c^5dx-68c^4d^2x^2-20c^3d^3x^3+5c^2d^4x^4-2cd^5x^5+d^6x^6\right)+60b^2(bc-ad)^4(c+dx)^2\log(c+dx)}{4d^7(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-2*a^6*d^6 - 12*a^5*b*d^5*(c + 2*d*x) + 30*a^4*b^2*c*d^4*(3*c + 4*d*x) + 4
0*a^3*b^3*d^3*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 30*a^2*b^4*d
^2*(7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^5*d
*(-27*c^5 + 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d
^5*x^5) + b^6*(22*c^6 - 16*c^5*d*x - 68*c^4*d^2*x^2 - 20*c^3*d^3*x^3 + 5*c^
2*d^4*x^4 - 2*c*d^5*x^5 + d^6*x^6) + 60*b^2*(b*c - a*d)^4*(c + d*x)^2*Log[c
 + d*x])/(4*d^7*(c + d*x)^2)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 382 vs. $2(158) = 316$.
time = 7.45, size = 380, normalized size = 2.41

$$\frac{-2a^6d^6-12a^5bcd^5+60b^2\text{Log}\left[c+dx\right]\left(c^2+2cdx+d^2x^2\right)\left(ad-bc\right)^4+90a^4b^2c^2d^4-200a^3b^3c^3d^3+210a^2b^4c^4d^2-108ab^5c^5d+22b^6c^6-24bdx\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)+4b^3dx\left(20a^3d^3-45a^2bcd^2+36ab^2c^2d-10b^3c^3\right)\left(c^2+2cdx+d^2x^2\right)+6b^4d^2x^2\left(5a^2d^2-6abcd+2b^2c^2\right)\left(c^2+2cdx+d^2x^2\right)+4b^5d^3x^3\left(2ad-bc\right)\left(c^2+2cdx+d^2x^2\right)+b^6d^4x^4\left(c^2+2cdx+d^2x^2\right)}{4d^7\left(c^2+2cdx+d^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^6/(c + d*x)^3,x]')
```

```
[Out] (-2 a ^ 6 d ^ 6 - 12 a ^ 5 b c d ^ 5 + 60 b ^ 2 Log[c + d x] (c ^ 2 + 2 c d
 x + d ^ 2 x ^ 2) (a d - b c) ^ 4 + 90 a ^ 4 b ^ 2 c ^ 2 d ^ 4 - 200 a ^ 3
b ^ 3 c ^ 3 d ^ 3 + 210 a ^ 2 b ^ 4 c ^ 4 d ^ 2 - 108 a b ^ 5 c ^ 5 d + 22
b ^ 6 c ^ 6 - 24 b d x (a ^ 5 d ^ 5 - 5 a ^ 4 b c d ^ 4 + 10 a ^ 3 b ^ 2 c
^ 2 d ^ 3 - 10 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 5 a b ^ 4 c ^ 4 d - b ^ 5 c ^ 5) +
 4 b ^ 3 d x (20 a ^ 3 d ^ 3 - 45 a ^ 2 b c d ^ 2 + 36 a b ^ 2 c ^ 2 d - 10
 b ^ 3 c ^ 3) (c ^ 2 + 2 c d x + d ^ 2 x ^ 2) + 6 b ^ 4 d ^ 2 x ^ 2 (5 a ^
2 d ^ 2 - 6 a b c d + 2 b ^ 2 c ^ 2) (c ^ 2 + 2 c d x + d ^ 2 x ^ 2) + 4 b
^ 5 d ^ 3 x ^ 3 (2 a d - b c) (c ^ 2 + 2 c d x + d ^ 2 x ^ 2) + b ^ 6 d ^ 4
 x ^ 4 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2)) / (4 d ^ 7 (c ^ 2 + 2 c d x + d ^ 2
 x ^ 2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(152) = 304$.
time = 0.14, size = 351, normalized size = 2.22

method	result
norman	$\frac{-\frac{a^6d^6+6a^5bc\,d^5-45a^4b^2c^2d^4+180a^3b^3c^3d^3-270a^2b^4c^4d^2+180a\,b^5c^5d-45b^6c^6}{2d^7}+\frac{b^6x^6}{4d}-\frac{2\left(3a^5b\,d^5-15a^4b^2c\,d^4+60a^3b^3c^2d^3-90a^2b^4c^3d^2+60 \ldots\right.}{d^6}}{(dx+c)^2}$
default	$\frac{b^3\left(\frac{1}{4}d^3x^4b^3+2a\,b^2d^3x^3-b^3c\,d^2x^3+\frac{15}{2}a^2b\,d^3x^2-9a\,b^2c\,d^2x^2+3b^3c^2d\,x^2+20a^3d^3x-45a^2bc\,d^2x+36a\,b^2c^2dx-10b^3c^3x\right)}{d^6}-\frac{6b\left(a^5d^5- \ldots\right.}{}$
risch	$\frac{b^6x^4}{4d^3}+\frac{2b^5a\,x^3}{d^3}-\frac{b^6c\,x^3}{d^4}+\frac{15b^4a^2x^2}{2d^3}-\frac{9b^5ac\,x^2}{d^4}+\frac{3b^6c^2x^2}{d^5}+\frac{20b^3a^3x}{d^3}-\frac{45b^4a^2cx}{d^4}+\frac{36b^5a\,c^2x}{d^5}-\frac{10b^6c^3x}{d^6}+\frac{(-6a \ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d^6*(1/4*d^3*x^4*b^3+2*a*b^2*d^3*x^3-b^3*c*d^2*x^3+15/2*a^2*b*d^3*x^2-9
*a*b^2*c*d^2*x^2+3*b^3*c^2*d*x^2+20*a^3*d^3*x-45*a^2*b*c*d^2*x+36*a*b^2*c^2
*d*x-10*b^3*c^3*x)-6*b/d^7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2
*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)+15*b^2/d^7*(a^4*d^4-4*a^3*b*c*d
^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(d*x+c)-1/2/d^7*(a^6*d^6-6*a^
5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*
c^5*d+b^6*c^6)/(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(152) = 304$.
time = 0.33, size = 364, normalized size = 2.30

$$\frac{11\,b^6c^6 - 54\,ab^5c^5d + 105\,a^2b^4c^4d^2 - 100\,a^3b^3c^3d^3 + 45\,a^4b^2c^2d^4 - 6\,a^5bcd^5 - a^6d^6 + 12\,(b^6c^5d - 5\,ab^5c^4d^2 + 10\,a^2b^4c^3d^3 - 10\,a^3b^3c^2d^4 + 5\,a^4b^2cd^5 - a^5bd^6)x}{2\,(d^9x^2 + 2\,cd^8x + c^2d^7)} + \frac{b^6d^3x^4 - 4\,(b^6cd^2 - 2\,ab^5d^3)x^3 + 6\,(2\,b^6c^2d - 6\,ab^5cd^2 + 5\,a^2b^4d^3)x^2 - 4\,(10\,b^6c^3 - 36\,ab^5c^2d + 45\,a^2b^4cd^2 - 20\,a^3b^3d^3)x}{4\,d^6} + \frac{15\,(b^6c^4 - 4\,ab^5c^3d + 6\,a^2b^4c^2d^2 - 4\,a^3b^3cd^3 + a^4b^2d^4)\log(dx + c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 - 100*a^3*b^3*c^3*d^
3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*d^6 + 12*(b^6*c^5*d - 5*a*b^5*
c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2*d^4 + 5*a^4*b^2*c*d^5 - a^5*b
*d^6)*x)/(d^9*x^2 + 2*c*d^8*x + c^2*d^7) + 1/4*(b^6*d^3*x^4 - 4*(b^6*c*d^2
- 2*a*b^5*d^3)*x^3 + 6*(2*b^6*c^2*d - 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^2 -
4*(10*b^6*c^3 - 36*a*b^5*c^2*d + 45*a^2*b^4*c*d^2 - 20*a^3*b^3*d^3)*x)/d^6
+ 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b
^2*d^4)*log(d*x + c)/d^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(152) = 304$.
time = 0.30, size = 548, normalized size = 3.47

$$\frac{b^6d^6x^6 + 22\,b^6c^6 - 108\,ab^5c^5d + 210\,a^2b^4c^4d^2 - 200\,a^3b^3c^3d^3 + 90\,a^4b^2c^2d^4 - 12\,a^5bcd^5 - 2\,a^6d^6 - 2\,(b^6cd^5 - 4\,ab^5d^6)x^5 + 5\,(b^6c^2d^4 - 4\,ab^5cd^5 + 6\,a^2b^4d^6)x^4 - 20\,(b^6c^3d^3 - 4\,ab^5c^2d^4 + 6\,a^2b^4cd^5 - 4\,a^3b^3d^6)x^3 - 2\,(34\,b^6c^4d^2 - 126\,ab^5c^3d^3 + 165\,a^2b^4c^2d^4 - 80\,a^3b^3cd^5)x^2 - 4\,(4\,b^6c^5d - 6\,ab^5c^4d^2 - 15\,a^2b^4c^3d^3 + 40\,a^3b^3c^2d^4 - 30\,a^4b^2cd^5 + 6\,a^5bd^6)x + 60\,(b^6c^6 - 4\,ab^5c^5d + 6\,a^2b^4c^4d^2 - 4\,a^3b^3c^3d^3 + a^4b^2c^2d^4 + (b^6c^4d^2 - 4\,ab^5c^3d^3 + 6\,a^2b^4c^2d^4 - 4\,a^3b^3cd^5 + a^4b^2d^6)x^2 + 2\,(b^6c^5d - 4\,ab^5c^4d^2 + 6\,a^2b^4c^3d^3 - 4\,a^3b^3c^2d^4 + a^4b^2cd^5)x)\log(dx + c)}{4\,(d^9x^2 + 2\,cd^8x + c^2d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*d^6*x^6 + 22*b^6*c^6 - 108*a*b^5*c^5*d + 210*a^2*b^4*c^4*d^2 - 200
*a^3*b^3*c^3*d^3 + 90*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 2*a^6*d^6 - 2*(b^6
*c*d^5 - 4*a*b^5*d^6)*x^5 + 5*(b^6*c^2*d^4 - 4*a*b^5*c*d^5 + 6*a^2*b^4*d^6)
*x^4 - 20*(b^6*c^3*d^3 - 4*a*b^5*c^2*d^4 + 6*a^2*b^4*c*d^5 - 4*a^3*b^3*d^6)
*x^3 - 2*(34*b^6*c^4*d^2 - 126*a*b^5*c^3*d^3 + 165*a^2*b^4*c^2*d^4 - 80*a^3
*b^3*c*d^5)*x^2 - 4*(4*b^6*c^5*d - 6*a*b^5*c^4*d^2 - 15*a^2*b^4*c^3*d^3 + 4
0*a^3*b^3*c^2*d^4 - 30*a^4*b^2*c*d^5 + 6*a^5*b*d^6)*x + 60*(b^6*c^6 - 4*a*b
^5*c^5*d + 6*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + (b^6*c
^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^
6)*x^2 + 2*(b^6*c^5*d - 4*a*b^5*c^4*d^2 + 6*a^2*b^4*c^3*d^3 - 4*a^3*b^3*c^2
*d^4 + a^4*b^2*c*d^5)*x)*log(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(144) = 288$.

time = 1.31, size = 340, normalized size = 2.15

$$\frac{b^6x^4}{4d^3} + \frac{15b^2(ad-bc)^4\log{(c+dx)}}{d^7} + x^3\cdot\left(\frac{2ab^5}{d^3} - \frac{b^6c}{d^4}\right) + x^2\cdot\left(\frac{15a^2b^4}{2d^3} - \frac{9ab^5c}{d^4} + \frac{3b^6c^2}{d^5}\right) + x\left(\frac{20a^3b^3}{d^3} - \frac{45a^2b^4c}{d^4} + \frac{36ab^5c^2}{d^5} - \frac{10b^6c^3}{d^6}\right) + \frac{-a^6d^6 - 6a^5bcd^5 + 45a^4b^2c^2d^4 - 100a^3b^3c^3d^3 + 105a^2b^4c^4d^2 - 54ab^5c^5d + 11b^6c^6 + x(-12a^5bd^6 + 60a^4b^2cd^5 - 120a^3b^3c^2d^4 + 120a^2b^4c^3d^3 - 60ab^5c^4d^2 + 12b^6c^5d)}{2c^2d^7 + 4cd^8x + 2d^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**3,x)
```

```
[Out] b**6*x**4/(4*d**3) + 15*b**2*(a*d - b*c)**4*log(c + d*x)/d**7 + x**3*(2*a*b
**5/d**3 - b**6*c/d**4) + x**2*(15*a**2*b**4/(2*d**3) - 9*a*b**5*c/d**4 + 3
*b**6*c**2/d**5) + x*(20*a**3*b**3/d**3 - 45*a**2*b**4*c/d**4 + 36*a*b**5*c
**2/d**5 - 10*b**6*c**3/d**6) + (-a**6*d**6 - 6*a**5*b*c*d**5 + 45*a**4*b**
2*c**2*d**4 - 100*a**3*b**3*c**3*d**3 + 105*a**2*b**4*c**4*d**2 - 54*a*b**5
*c**5*d + 11*b**6*c**6 + x*(-12*a**5*b*d**6 + 60*a**4*b**2*c*d**5 - 120*a**
3*b**3*c**2*d**4 + 120*a**2*b**4*c**3*d**3 - 60*a*b**5*c**4*d**2 + 12*b**6*
c**5*d))/(2*c**2*d**7 + 4*c*d**8*x + 2*d**9*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(152) = 304$.

time = 0.00, size = 384, normalized size = 2.43

$$\frac{\frac{1}{4}x^4b^6d^9 - x^3b^6d^8c + 2x^3b^5ad^9 + 3x^2b^6d^7c^2 - 9x^2b^5ad^8c + \frac{15}{2}x^2b^4a^2d^9 - 10xb^6d^6c^3 + 36xb^5ad^7c^2 - 45xb^4a^2d^8c + 20xb^3a^3d^9}{d^{12}} + \frac{\frac{1}{2}\left(11b^6c^6 - 54b^5dc^5a + 105b^4d^2c^4a^2 - 100b^3d^3c^3a^3 + 45b^2d^4c^2a^4 - 6bd^5ca^5 - d^6a^6 + (12b^6dc^5 - 60b^5d^2c^4a + 120b^4d^3c^3a^2 - 120b^3d^4c^2a^3 + 60b^2d^5ca^4 - 12bd^6a^5)x\right)}{d^7(xd+c)^2} + \frac{(15b^6c^4 - 60b^5adc^3 + 90b^4a^2d^2c^2 - 60b^3a^3d^3c + 15b^2a^4d^4)\ln|xd+c|}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x)
```

```
[Out] 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2
*d^4)*log(abs(d*x + c))/d^7 + 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^
4*c^4*d^2 - 100*a^3*b^3*c^3*d^3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*
d^6 + 12*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2
*d^4 + 5*a^4*b^2*c*d^5 - a^5*b*d^6)*x)/((d*x + c)^2*d^7) + 1/4*(b^6*d^9*x^4
 - 4*b^6*c*d^8*x^3 + 8*a*b^5*d^9*x^3 + 12*b^6*c^2*d^7*x^2 - 36*a*b^5*c*d^8*
x^2 + 30*a^2*b^4*d^9*x^2 - 40*b^6*c^3*d^6*x + 144*a*b^5*c^2*d^7*x - 180*a^2
*b^4*c*d^8*x + 80*a^3*b^3*d^9*x)/d^12
```

Mupad [B]

time = 0.27, size = 441, normalized size = 2.79

$$x^3\left(\frac{2\,a\,b^5}{d^3}-\frac{b^6\,c}{d^4}\right)-\frac{\frac{a^6\,d^6+6\,a^5\,b\,c\,d^5-45\,a^4\,b^2\,c^2\,d^4+100\,a^3\,b^3\,c^3\,d^3-105\,a^2\,b^4\,c^4\,d^2+54\,a\,b^5\,c^5\,d-11\,b^6\,c^6}{2\,d}-x\,\left(-6\,a^5\,b\,d^5+30\,a^4\,b^2\,c\,d^4-60\,a^3\,b^3\,c^2\,d^3+60\,a^2\,b^4\,c^3\,d^2-30\,a\,b^5\,c^4\,d+6\,b^6\,c^5\right)}{c^2\,d^6+2\,c\,d^7\,x+d^8\,x^2}-x^2\,\left(\frac{3\,c\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{2\,d}-\frac{15\,a^2\,b^4}{2\,d^3}+\frac{3\,b^6\,c^2}{2\,d^5}\right)+x\,\left(\frac{3\,c\,\left(\frac{3\,c\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d}-\frac{15\,a^2\,b^4}{d^3}+\frac{3\,b^6\,c^2}{d^5}\right)}{d}+\frac{20\,a^3\,b^3}{d^3}-\frac{b^6\,c^3}{d^6}-\frac{3\,c^2\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d^2}\right)+\frac{\ln\left(c+d\,x\right)\,\left(15\,a^4\,b^2\,d^4-60\,a^3\,b^3\,c\,d^3+90\,a^2\,b^4\,c^2\,d^2-60\,a\,b^5\,c^3\,d+15\,b^6\,c^4\right)}{d^7}+\frac{b^6\,x^4}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^3,x)
```

```
[Out] x^3*((2*a*b^5)/d^3 - (b^6*c)/d^4) - ((a^6*d^6 - 11*b^6*c^6 - 105*a^2*b^4*c^
4*d^2 + 100*a^3*b^3*c^3*d^3 - 45*a^4*b^2*c^2*d^4 + 54*a*b^5*c^5*d + 6*a^5*b
```

```
*c*d^5)/(2*d) - x*(6*b^6*c^5 - 6*a^5*b*d^5 + 30*a^4*b^2*c*d^4 + 60*a^2*b^4*
c^3*d^2 - 60*a^3*b^3*c^2*d^3 - 30*a*b^5*c^4*d))/(c^2*d^6 + d^8*x^2 + 2*c*d^
7*x) - x^2*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/(2*d) - (15*a^2*b^4)/(2*d
^3) + (3*b^6*c^2)/(2*d^5)) + x*((3*c*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))
/d - (15*a^2*b^4)/d^3 + (3*b^6*c^2)/d^5))/d + (20*a^3*b^3)/d^3 - (b^6*c^3)/
d^6 - (3*c^2*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/d^2) + (log(c + d*x)*(15*b^6*
c^4 + 15*a^4*b^2*d^4 - 60*a^3*b^3*c*d^3 + 90*a^2*b^4*c^2*d^2 - 60*a*b^5*c^3
*d))/d^7 + (b^6*x^4)/(4*d^3)
```

3.1353 $\int \frac{(a+bx)^5}{(c+dx)^3}\,dx$

Optimal. Leaf size=133

$$\frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}-\frac{10b^2(bc-ad)^3\log(c+$$

```
[Out] 10*b^3*(-a*d+b*c)^2*x/d^5+1/2*(-a*d+b*c)^5/d^6/(d*x+c)^2-5*b*(-a*d+b*c)^4/d
^6/(d*x+c)-5/2*b^4*(-a*d+b*c)*(d*x+c)^2/d^6+1/3*b^5*(d*x+c)^3/d^6-10*b^2*(-
a*d+b*c)^3*ln(d*x+c)/d^6
```

Rubi [A]
time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^3}\,dx &= \int \left(\frac{10b^3(bc-ad)^2}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^3}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}-\frac{5b^4(bc-ad)(c+\right.\\ &= \frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 230, normalized size = 1.73

$$\frac{-3a^5d^5-15a^4bd^4(c+2dx)+30a^3b^2cd^3(3c+4dx)+30a^2b^3d^2\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+15ab^4d\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)+b^5\left(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5\right)-60b^2(bc-ad)^3(c+dx)^2\log(c+dx)}{6d^6(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) + 3
0*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d*(
7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c^5
+ 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5) -
60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(133) = 266.
time = 4.77, size = 288, normalized size = 2.17

$$\frac{-3a^5d^5-15a^4bcd^4+60b^2\text{Log}\left[c+dx\right]\left(c^2+2cdx+d^2x^2\right)\left(ad-bc\right)^3+90a^3b^2c^2d^3-150a^2b^3c^3d^2+105ab^4c^4d-27b^5c^5-30bdx\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)+6b^3dx\left(10a^2d^2-15abcd+6b^2c^2\right)\left(c^2+2cdx+d^2x^2\right)+3b^4d^2x^2\left(5ad-3bc\right)\left(c^2+2cdx+d^2x^2\right)+2b^5d^3x^3\left(c^2+2cdx+d^2x^2\right)}{6d^6\left(c^2+2cdx+d^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x)^3,x]')
```

```
[Out] (-3 a ^ 5 d ^ 5 - 15 a ^ 4 b c d ^ 4 + 60 b ^ 2 Log[c + d x] (c ^ 2 + 2 c d
 x + d ^ 2 x ^ 2) (a d - b c) ^ 3 + 90 a ^ 3 b ^ 2 c ^ 2 d ^ 3 - 150 a ^ 2
b ^ 3 c ^ 3 d ^ 2 + 105 a b ^ 4 c ^ 4 d - 27 b ^ 5 c ^ 5 - 30 b d x (a ^ 4
d ^ 4 - 4 a ^ 3 b c d ^ 3 + 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 4 a b ^ 3 c ^ 3 d +
 b ^ 4 c ^ 4) + 6 b ^ 3 d x (10 a ^ 2 d ^ 2 - 15 a b c d + 6 b ^ 2 c ^ 2) (
c ^ 2 + 2 c d x + d ^ 2 x ^ 2) + 3 b ^ 4 d ^ 2 x ^ 2 (5 a d - 3 b c) (c ^ 2
 + 2 c d x + d ^ 2 x ^ 2) + 2 b ^ 5 d ^ 3 x ^ 3 (c ^ 2 + 2 c d x + d ^ 2 x
^ 2)) / (6 d ^ 6 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2))
```

Maple [**A**]
time = 0.14, size = 254, normalized size = 1.91

method	result
default	$\frac{b^3\left(\frac{1}{3}d^2x^3b^2+\frac{5}{2}ab\,d^2x^2-\frac{3}{2}b^2cd\,x^2+10a^2d^2x-15abcdx+6b^2c^2x\right)}{d^5}-\frac{5b\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{d^6(dx+c)}+\frac{10b^2\left(a^3d^3-3\right.}{}$
norman	$\frac{-\frac{a^5d^5+5a^4bc\,d^4-30a^3b^2c^2d^3+90a^2b^3c^3d^2-90a\,b^4c^4d+30b^5c^5}{2d^6}+\frac{b^5x^5}{3d}-\frac{\left(5a^4b\,d^4-20a^3b^2c\,d^3+60a^2b^3c^2d^2-60a\,b^4c^3d+20b^5c^4\right)x}{d^5}+\frac{10b^3\left(3a^2d^2-\right.}{}}{(dx+c)^2}$
risch	$\frac{b^5x^3}{3d^3}+\frac{5b^4a\,x^2}{2d^3}-\frac{3b^5c\,x^2}{2d^4}+\frac{10b^3a^2x}{d^3}-\frac{15b^4acx}{d^4}+\frac{6b^5c^2x}{d^5}+\frac{\left(-5a^4b\,d^4+20a^3b^2c\,d^3-30a^2b^3c^2d^2+20a\,b^4c^3d-5b^5c^4\right)x-\frac{a^5d}{}}{d^5(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d^5*(1/3*d^2*x^3*b^2+5/2*a*b*d^2*x^2-3/2*b^2*c*d*x^2+10*a^2*d^2*x-15*a*
b*c*d*x+6*b^2*c^2*x)-5*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b
^3*c^3*d+b^4*c^4)/(d*x+c)+10*b^2/d^6*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)*ln(d*x+c)-1/2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*
c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(127) = 254$.
time = 0.27, size = 271, normalized size = 2.04

$$-\frac{9\,b^5c^5-35\,ab^4c^4d+50\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3+5\,a^4bcd^4+a^5d^5+10\,(b^5c^4d-4\,ab^4c^3d^2+6\,a^2b^3c^2d^3-4\,a^3b^2cd^4+a^4bd^5)x}{2\,(d^8x^2+2\,cd^7x+c^2d^6)}+\frac{2\,b^5d^2x^3-3\,(3\,b^5cd-5\,ab^4d^2)x^2+6\,(6\,b^5c^2-15\,ab^4cd+10\,a^2b^3d^2)x}{6\,d^5}-\frac{10\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)\log\left(dx+c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
+ 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2
*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/
6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*a*b^
4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(127) = 254$.
time = 0.29, size = 416, normalized size = 3.13

$$\frac{2\,b^5d^5x^5-27\,b^5c^5+105\,ab^4c^4d-150\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-15\,a^4bcd^4-3\,a^5d^5-5\,(b^5cd^4-3\,ab^4d^5)x^4+20\,(b^5c^2d^3-3\,ab^4cd^4+3\,a^2b^3d^5)x^3+3\,(21\,b^5c^3d^2-55\,ab^4c^2d^3+40\,a^2b^3cd^4)x^2+6\,(b^5c^4d+5\,ab^4c^3d^2-20\,a^2b^3c^2d^3+20\,a^3b^2cd^4-5\,a^4bd^5)x-60\,(b^5c^5-3\,ab^4c^4d+3\,a^2b^3c^3d^2-a^3b^2c^2d^3+(b^5c^3d^2-3\,ab^4c^2d^3+3\,a^2b^3cd^4-a^3b^2d^5)x^2+2\,(b^5c^4d-3\,ab^4c^3d^2+3\,a^2b^3c^2d^3-a^3b^2cd^4)x)\log\left(dx+c\right)}{6\,(d^8x^2+2\,cd^7x+c^2d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 + 9
0*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d^5
)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5*c^
3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b^4*c
^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(b^5*c
^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d^2 - 3
*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d - 3*a*b^
4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^8*x^2 +
2*c*d^7*x + c^2*d^6)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(121) = 242$.
time = 1.00, size = 258, normalized size = 1.94

$$\frac{b^5x^3}{3d^3}+\frac{10b^2\left(ad-bc\right)^3\log\left(c+dx\right)}{d^6}+x^2\cdot\left(\frac{5ab^4}{2d^3}-\frac{3b^5c}{2d^4}\right)+x\left(\frac{10a^2b^3}{d^3}-\frac{15ab^4c}{d^4}+\frac{6b^5c^2}{d^5}\right)+\frac{-a^5d^5-5a^4bcd^4+30a^3b^2c^2d^3-50a^2b^3c^3d^2+35ab^4c^4d-9b^5c^5+x\left(-10a^4bd^5+40a^3b^2cd^4-60a^2b^3c^2d^3+40ab^4c^3d^2-10b^5c^4d\right)}{2c^2d^6+4cd^7x+2d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**3,x)
```

```
[Out] b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 + x**2*(5*a*b
**4/(2*d**3) - 3*b**5*c/(2*d**4)) + x*(10*a**2*b**3/d**3 - 15*a*b**4*c/d**4
```

```
 + 6*b**5*c**2/d**5) + (-a**5*d**5 - 5*a**4*b*c*d**4 + 30*a**3*b**2*c**2*d*
*3 - 50*a**2*b**3*c**3*d**2 + 35*a*b**4*c**4*d - 9*b**5*c**5 + x*(-10*a**4*
b*d**5 + 40*a**3*b**2*c*d**4 - 60*a**2*b**3*c**2*d**3 + 40*a*b**4*c**3*d**2
 - 10*b**5*c**4*d))/(2*c**2*d**6 + 4*c*d**7*x + 2*d**8*x**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(127) = 254$.
time = 0.00, size = 287, normalized size = 2.16

$$\frac{\frac{1}{3}x^3b^5d^6 - \frac{3}{2}x^2b^5d^5c + \frac{5}{2}x^2b^4ad^6 + 6xb^5d^4c^2 - 15xb^4ad^5c + 10xb^3a^2d^6}{d^9} + \frac{\frac{1}{2}\left(-9b^5c^5 + 35b^4dc^4a - 50b^3d^2c^3a^2 + 30b^2d^3c^2a^3 - 5bd^4ca^4 - d^5a^5 + \left(-10b^5dc^4 + 40b^4d^2c^3a - 60b^3d^3c^2a^2 + 40b^2d^4ca^3 - 10bd^5a^4\right)x\right)}{d^6\left(xd + c\right)^2} + \frac{\left(-10b^5c^3 + 30b^4adc^2 - 30b^3a^2d^2c + 10b^2a^3d^3\right)\ln|xd + c|}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x)
```

```
[Out] -10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a
^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2
*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x - 90*a
*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

Mupad [**B**]
time = 0.10, size = 291, normalized size = 2.19

$$x^2\left(\frac{5\,a\,b^4}{2\,d^3} - \frac{3\,b^5\,c}{2\,d^4}\right) - \frac{\frac{a^5\,d^5+5\,a^4\,b\,c\,d^4-30\,a^3\,b^2\,c^2\,d^3+50\,a^2\,b^3\,c^3\,d^2-35\,a\,b^4\,c^4\,d+9\,b^5\,c^5}{2\,d} + x\,\left(5\,a^4\,b\,d^4 - 20\,a^3\,b^2\,c\,d^3 + 30\,a^2\,b^3\,c^2\,d^2 - 20\,a\,b^4\,c^3\,d + 5\,b^5\,c^4\right)}{c^2\,d^5 + 2\,c\,d^6\,x + d^7\,x^2} - x\,\left(\frac{3\,c\,\left(\frac{5\,a\,b^4}{d^3} - \frac{3\,b^5\,c}{d^4}\right)}{d} - \frac{10\,a^2\,b^3}{d^3} + \frac{3\,b^5\,c^2}{d^5}\right) - \frac{\ln\left(c + d\,x\right)\,\left(-10\,a^3\,b^2\,d^3 + 30\,a^2\,b^3\,c\,d^2 - 30\,a\,b^4\,c^2\,d + 10\,b^5\,c^3\right)}{d^6} + \frac{b^5\,x^3}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^3,x)
```

```
[Out] x^2*((5*a*b^4)/(2*d^3) - (3*b^5*c)/(2*d^4)) - ((a^5*d^5 + 9*b^5*c^5 + 50*a^
2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 35*a*b^4*c^4*d + 5*a^4*b*c*d^4)/(2*d)
+ x*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a
*b^4*c^3*d))/(c^2*d^5 + d^7*x^2 + 2*c*d^6*x) - x*((3*c*((5*a*b^4)/d^3 - (3*
b^5*c)/d^4))/d - (10*a^2*b^3)/d^3 + (3*b^5*c^2)/d^5) - (log(c + d*x)*(10*b^
5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d))/d^6 + (b^5*x^3
)/(3*d^3)
```

3.1354 $\int \frac{(a+bx)^4}{(c+dx)^3} \, dx$

Optimal. Leaf size=103

$$-\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}$$

```
[Out] -b^3*(-4*a*d+3*b*c)*x/d^4+1/2*b^4*x^2/d^3-1/2*(-a*d+b*c)^4/d^5/(d*x+c)^2+4*
b*(-a*d+b*c)^3/d^5/(d*x+c)+6*b^2*(-a*d+b*c)^2*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^3}\,dx &= \int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{b^4x}{d^3}+\frac{(-bc+ad)^4}{d^4(c+dx)^3}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}\right)dx\\ &= -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 167, normalized size = 1.62

$$\frac{-a^4d^4-4a^3bd^3(c+2dx)+6a^2b^2cd^2(3c+4dx)+4ab^3d\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+b^4\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)+12b^2(bc-ad)^2(c+dx)^2\log(c+dx)}{2d^5(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*a
*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^3*
d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c + d
*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 212 vs. $2(103) = 206$.
time = 3.92, size = 210, normalized size = 2.04

$$\frac{-a^4d^4 - 4a^3bcd^3 + 12b^2\text{Log}\left[c + dx\right]\left(c^2 + 2cdx + d^2x^2\right)(ad - bc)^2 + 18a^2b^2c^2d^2 - 20ab^3c^3d + 7b^4c^4 - 8bdx\left(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3\right) + 2b^3dx\left(4ad - 3bc\right)\left(c^2 + 2cdx + d^2x^2\right) + b^4d^2x^2\left(c^2 + 2cdx + d^2x^2\right)}{2d^5\left(c^2 + 2cdx + d^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x)^3,x]')
```

```
[Out] (-a ^ 4 d ^ 4 - 4 a ^ 3 b c d ^ 3 + 12 b ^ 2 Log[c + d x] (c ^ 2 + 2 c d x
+ d ^ 2 x ^ 2) (a d - b c) ^ 2 + 18 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 20 a b ^ 3 c
^ 3 d + 7 b ^ 4 c ^ 4 - 8 b d x (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b ^
2 c ^ 2 d - b ^ 3 c ^ 3) + 2 b ^ 3 d x (4 a d - 3 b c) (c ^ 2 + 2 c d x + d
 ^ 2 x ^ 2) + b ^ 4 d ^ 2 x ^ 2 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2)) / (2 d ^ 5
 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2))
```

Maple [A]
time = 0.14, size = 172, normalized size = 1.67

method	result
default	$\frac{b^3\left(\frac{1}{2}bd\,x^2+4adx-3bcx\right)}{d^4}-\frac{4b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^5(dx+c)}+\frac{6b^2\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^5}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2}{2d^5(dx+c)}$
norman	$\frac{-\frac{a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+36a\,b^3c^3d-18b^4c^4}{2d^5}+\frac{b^4x^4}{2d}-\frac{2\left(2a^3b\,d^3-6b^2a^2c\,d^2+12a\,b^3c^2d-6b^4c^3\right)x}{d^4}+\frac{2b^3(2ad-bc)x^3}{d^2}}{(dx+c)^2}+\frac{6b^2\left(a^2d^2-2abcd+\right.}{d^5}$
risch	$\frac{b^4x^2}{2d^3}+\frac{4a\,b^3x}{d^3}-\frac{3b^4cx}{d^4}+\frac{\left(-4a^3b\,d^3+12b^2a^2c\,d^2-12a\,b^3c^2d+4b^4c^3\right)x-\frac{a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+20a\,b^3c^3d-7b^4c^4}{2d}}{d^4(dx+c)^2}+\frac{6b^2\ln(dx}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d^4*(1/2*b*d*x^2+4*a*d*x-3*b*c*x)-4*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/(d*x+c)+6*b^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*x+c)-1/
2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^5/(d*x+
c)^2
```

Maxima [A]
time = 0.27, size = 191, normalized size = 1.85

$$\frac{7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4 + 8\,(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4)x}{2\,(d^7x^2 + 2\,cd^6x + c^2d^5)} + \frac{b^4dx^2 - 2\,(3\,b^4c - 4\,ab^3d)x}{2\,d^4} + \frac{6\,(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)\log\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*
d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d^7
*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x)/d
^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(99) = 198$.
time = 0.30, size = 291, normalized size = 2.83

$$\frac{b^4d^4x^4 + 7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4 - 4\,(b^4cd^3 - 2\,ab^3d^4)x^3 - (11\,b^4c^2d^2 - 16\,ab^3cd^3)x^2 + 2\,(b^4c^3d - 8\,ab^3c^2d^2 + 12\,a^2b^2cd^3 - 4\,a^3bd^4)x + 12\,(b^4c^4 - 2\,ab^3c^3d + a^2b^2c^2d^2 + (b^4c^2d^2 - 2\,ab^3cd^3 + a^2b^2d^4)x^2 + 2\,(b^4c^3d - 2\,ab^3c^2d^2 + a^2b^2cd^3)x)\log(dx + c)}{2\,(d^7x^2 + 2\,cd^6x + c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 16*
a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4*a^
3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d^2 -
 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a^2*b^
2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A]
time = 0.72, size = 185, normalized size = 1.80

$$\frac{b^4x^2}{2d^3} + \frac{6b^2\,(ad - bc)^2\log{(c + dx)}}{d^5} + x\left(\frac{4ab^3}{d^3} - \frac{3b^4c}{d^4}\right) + \frac{-a^4d^4 - 4a^3bcd^3 + 18a^2b^2c^2d^2 - 20ab^3c^3d + 7b^4c^4 + x\,(-8a^3bd^4 + 24a^2b^2cd^3 - 24ab^3c^2d^2 + 8b^4c^3d)}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**3,x)
```

```
[Out] b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 + x*(4*a*b**3/
d**3 - 3*b**4*c/d**4) + (-a**4*d**4 - 4*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d
**2 - 20*a*b**3*c**3*d + 7*b**4*c**4 + x*(-8*a**3*b*d**4 + 24*a**2*b**2*c*d
**3 - 24*a*b**3*c**2*d**2 + 8*b**4*c**3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d
**7*x**2)
```

Giac [A]
time = 0.00, size = 198, normalized size = 1.92

$$\frac{\frac{1}{2}x^2b^4d^3 - 3xb^4d^2c + 4xb^3ad^3}{d^6} + \frac{\frac{1}{2}\,(7b^4c^4 - 20b^3dc^3a + 18b^2d^2c^2a^2 - 4bd^3ca^3 - d^4a^4 + (8b^4dc^3 - 24b^3d^2c^2a + 24b^2d^3ca^2 - 8bd^4a^3)\,x)}{d^5\,(xd + c)^2} + \frac{(6b^4c^2 - 12b^3adc + 6b^2a^2d^2)\ln|xd + c|}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x)
```

[Out] 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)

Mupad [B]

time = 0.10, size = 196, normalized size = 1.90

$$x\left(\frac{4\,a\,b^3}{d^3}-\frac{3\,b^4\,c}{d^4}\right)-\frac{\frac{a^4\,d^4+4\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d-7\,b^4\,c^4}{2\,d}-x\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{c^2\,d^4+2\,c\,d^5\,x+d^6\,x^2}+\frac{b^4\,x^2}{2\,d^3}+\frac{\ln\left(c+d\,x\right)\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^4/(c + d*x)^3,x)

[Out] x*((4*a*b^3)/d^3 - (3*b^4*c)/d^4) - ((a^4*d^4 - 7*b^4*c^4 - 18*a^2*b^2*c^2*d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3)/(2*d) - x*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(c^2*d^4 + d^6*x^2 + 2*c*d^5*x) + (b^4*x^2)/(2*d^3) + (log(c + d*x)*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d))/d^5

3.1355 $\int \frac{(a+bx)^3}{(c+dx)^3}\,dx$

Optimal. Leaf size=78

$$\frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}$$

```
[Out] b^3*x/d^3+1/2*(-a*d+b*c)^3/d^4/(d*x+c)^2-3*b*(-a*d+b*c)^2/d^4/(d*x+c)-3*b^2
*(-a*d+b*c)*ln(d*x+c)/d^4
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int\frac{(a+bx)^3}{(c+dx)^3}\,dx&=\int\left(\frac{b^3}{d^3}+\frac{(-bc+ad)^3}{d^3(c+dx)^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}-\frac{3b^2(bc-ad)}{d^3(c+dx)}\right)dx\\&=\frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.46

$$\frac{-a^3d^3-3a^2bd^2(c+2dx)+3ab^2cd(3c+4dx)+b^3\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)-6b^2(bc-ad)(c+dx)^2\log(c+dx)}{2d^4(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/(c + d*x)^3,x]

[Out] (-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)^2*Log[c + d*x])/(2*d^4*(c + d*x)^2)

Mathics [A]
time = 3.20, size = 144, normalized size = 1.85

$$\frac{-a^3d^3-3a^2bcd^2+6b^2\text{Log}\,[c+dx]\,(ad-bc)\,(c^2+2cdx+d^2x^2)+9ab^2c^2d-5b^3c^3-6bdx\,(a^2d^2-2abcd+b^2c^2)+2b^3dx\,(c^2+2cdx+d^2x^2)}{2d^4\,(c^2+2cdx+d^2x^2)}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^3/(c + d*x)^3,x]')

[Out] (-a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 6 b ^ 2 Log[c + d x] (a d - b c) (c ^ 2 + 2 c d x + d ^ 2 x ^ 2) + 9 a b ^ 2 c ^ 2 d - 5 b ^ 3 c ^ 3 - 6 b d x (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) + 2 b ^ 3 d x (c ^ 2 + 2 c d x + d ^ 2 x ^ 2)) / (2 d ^ 4 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2))

Maple [A]
time = 0.15, size = 114, normalized size = 1.46

method	result	size
default	$\frac{b^3x}{d^3}-\frac{3b(a^2d^2-2abcd+b^2c^2)}{d^4(dx+c)}+\frac{3b^2(ad-bc)\ln(dx+c)}{d^4}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2d^4(dx+c)^2}$	114
norman	$\frac{\frac{b^3x^3}{d}-\frac{a^3d^3+3a^2bc\,d^2-9a\,b^2c^2d+9b^3c^3}{2d^4}-\frac{\left(3a^2b\,d^2-6a\,b^2cd+6b^3c^2\right)x}{d^3}}{(dx+c)^2}+\frac{3b^2(ad-bc)\ln(dx+c)}{d^4}$	116
risch	$\frac{b^3x}{d^3}+\frac{(-3a^2b\,d^2+6a\,b^2cd-3b^3c^2)x-\frac{a^3d^3+3a^2bc\,d^2-9a\,b^2c^2d+5b^3c^3}{2d}}{d^3(dx+c)^2}+\frac{3b^2\ln(dx+c)a}{d^3}-\frac{3b^3\ln(dx+c)c}{d^4}$	121

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] b^3*x/d^3-3*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+3*b^2/d^4*(a*d-b*c)*ln(d*x+c)-1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^2

Maxima [A]
time = 0.28, size = 125, normalized size = 1.60

$$\frac{b^3x}{d^3}-\frac{5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(d^6x^2+2\,cd^5x+c^2d^4)}-\frac{3\,(b^3c-ab^2d)\log\,(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")

```
[Out] b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) -
3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(76) = 152$.
time = 0.29, size = 188, normalized size = 2.41

$$\frac{2\,b^3d^3x^3 + 4\,b^3cd^2x^2 - 5\,b^3c^3 + 9\,ab^2c^2d - 3\,a^2bcd^2 - a^3d^3 - 2\,(2\,b^3c^2d - 6\,ab^2cd^2 + 3\,a^2bd^3)x - 6\,(b^3c^3 - ab^2c^2d + (b^3cd^2 - ab^2d^3)x^2 + 2\,(b^3c^2d - ab^2cd^2)x)\log{(dx + c)}}{2\,(d^6x^2 + 2\,cd^5x + c^2d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b*
c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*
c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c*d^
2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [A]
time = 0.49, size = 128, normalized size = 1.64

$$\frac{b^3x}{d^3} + \frac{3b^2\left(ad - bc\right)\log{\left(c + dx \right)}}{d^4} + \frac{-a^3d^3 - 3a^2bcd^2 + 9ab^2c^2d - 5b^3c^3 + x\left(-6a^2bd^3 + 12ab^2cd^2 - 6b^3c^2d\right)}{2c^2d^4 + 4cd^5x + 2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 + (-a**3*d**3 - 3*a**2*b
*c*d**2 + 9*a*b**2*c**2*d - 5*b**3*c**3 + x*(-6*a**2*b*d**3 + 12*a*b**2*c*d
**2 - 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)
```

Giac [A]
time = 0.00, size = 125, normalized size = 1.60

$$\frac{xb^3}{d^3} + \frac{\frac{1}{2}\left(-5b^3c^3 + 9b^2dc^2a - 3bd^2ca^2 - d^3a^3 + \left(-6b^3dc^2 + 12b^2d^2ca - 6bd^3a^2\right)x\right)}{d^4\left(xd + c\right)^2} + \frac{\left(-3b^3c + 3b^2ad\right)\ln\left|xd + c\right|}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9*
a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((d*x + c)^2*d^4)
```

Mupad [B]
time = 0.11, size = 130, normalized size = 1.67

$$\frac{b^3\,x}{d^3} - \frac{\ln\left(c + d\,x\right)\,\left(3\,b^3\,c - 3\,a\,b^2\,d\right)}{d^4} - \frac{\frac{a^3\,d^3 + 3\,a^2\,b\,c\,d^2 - 9\,a\,b^2\,c^2\,d + 5\,b^3\,c^3}{2\,d} + x\,\left(3\,a^2\,b\,d^2 - 6\,a\,b^2\,c\,d + 3\,b^3\,c^2\right)}{c^2\,d^3 + 2\,c\,d^4\,x + d^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^3/(c + d*x)^3,x)

[Out] (b^3*x)/d^3 - (log(c + d*x)*(3*b^3*c - 3*a*b^2*d))/d^4 - ((a^3*d^3 + 5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(2*d) + x*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d))/(c^2*d^3 + d^5*x^2 + 2*c*d^4*x)

3.1356 $\int \frac{(a+bx)^2}{(c+dx)^3} \, dx$

Optimal. Leaf size=59

$$-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}$$

```
[Out] -1/2*(-a*d+b*c)^2/d^3/(d*x+c)^2+2*b*(-a*d+b*c)/d^3/(d*x+c)+b^2*ln(d*x+c)/d^
3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] -1/2*(b*c - a*d)^2/(d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) +
(b^2*Log[c + d*x])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^3}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3}-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{b^2}{d^2(c+dx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.81

$$\frac{\frac{(bc-ad)(3bc+ad+4bdx)}{(c+dx)^2}+2b^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(2
*d^3)
```

Mathics [A]
time = 2.58, size = 84, normalized size = 1.42

$$\frac{-\frac{a^2d^2}{2} - abcd + b^2\text{Log}\left[c + dx\right]\left(c^2 + 2cdx + d^2x^2\right) + \frac{3b^2c^2}{2} - 2bdx\left(ad - bc\right)}{d^3\left(c^2 + 2cdx + d^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x)^3,x]')
```

```
[Out] (-a ^ 2 d ^ 2 / 2 - a b c d + b ^ 2 Log[c + d x] (c ^ 2 + 2 c d x + d ^ 2 x
 ^ 2) + 3 b ^ 2 c ^ 2 / 2 - 2 b d x (a d - b c)) / (d ^ 3 (c ^ 2 + 2 c d x
+ d ^ 2 x ^ 2))
```

Maple [A]
time = 0.13, size = 69, normalized size = 1.17

method	result	size
risch	$\frac{-\frac{2b(ad-bc)x}{d^2}-\frac{a^2d^2+2abcd-3b^2c^2}{2d^3}}{(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}$	66
norman	$\frac{-\frac{a^2d^2+2abcd-3b^2c^2}{2d^3}-\frac{2\left(abd-b^2c\right)x}{d^2}}{(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}$	68
default	$-\frac{2b(ad-bc)}{d^3(dx+c)}+\frac{b^2\ln(dx+c)}{d^3}-\frac{a^2d^2-2abcd+b^2c^2}{2d^3(dx+c)^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/d^3*(a*d-b*c)/(d*x+c)+b^2*ln(d*x+c)/d^3-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/d^3/(d*x+c)^2
```

Maxima [A]
time = 0.27, size = 80, normalized size = 1.36

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\,(b^2cd - abd^2)x}{2\,(d^5x^2 + 2\,cd^4x + c^2d^3)} + \frac{b^2\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

Fricas [A]
time = 0.29, size = 100, normalized size = 1.69

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log{(dx+c)}}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A]
time = 0.27, size = 80, normalized size = 1.36

$$\frac{b^2\log{(c+dx)}}{d^3}+\frac{-a^2d^2-2abcd+3b^2c^2+x\,(-4abd^2+4b^2cd)}{2c^2d^3+4cd^4x+2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c + d*x)/d**3 + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x*(-4*a*b*
d**2 + 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

Giac [A]
time = 0.00, size = 75, normalized size = 1.27

$$\frac{\frac{1}{2}\left((4b^2c-4bda)\,x+\frac{3b^2c^2-2bdca-d^2a^2}{d}\right)}{d^2\,(xd+c)^2}+\frac{b^2\ln{|xd+c|}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

Mupad [B]
time = 0.23, size = 77, normalized size = 1.31

$$\frac{b^2\,\ln{(c+d\,x)}}{d^3}-\frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{2\,d^3}+\frac{2\,b\,x\,(a\,d-b\,c)}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3 - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(2*d^3) + (2*b*
x*(a*d - b*c))/d^2)/(c^2 + d^2*x^2 + 2*c*d*x)
```

3.1357 $\int \frac{a+bx}{(c+dx)^3} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

```
[Out] 1/2*(b*x+a)^2/(-a*d+b*c)/(d*x+c)^2
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{37\}$

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{(c+dx)^3} dx = \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] -1/2*(a*d + b*(c + 2*d*x))/(d^2*(c + d*x)^2)
```

Mathics [A]
time = 2.01, size = 37, normalized size = 1.32

$$\frac{-ad - bc - 2bdx}{2d^2\left(c^2 + 2cdx + d^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x)^3,x]')
[Out] (-a d - b c - 2 b d x) / (2 d ^ 2 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2))
```

Maple [A]
time = 0.12, size = 35, normalized size = 1.25

method	result	size
gosper	$-\frac{2bdx+ad+bc}{2d^2(dx+c)^2}$	25
norman	$\frac{-\frac{bx}{d}-\frac{ad+bc}{2d^2}}{(dx+c)^2}$	29
risch	$\frac{-\frac{bx}{d}-\frac{ad+bc}{2d^2}}{(dx+c)^2}$	29
default	$-\frac{b}{d^2(dx+c)}-\frac{ad-bc}{2d^2(dx+c)^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -b/d^2/(d*x+c)-1/2*(a*d-b*c)/d^2/(d*x+c)^2
```

Maxima [A]
time = 0.28, size = 38, normalized size = 1.36

$$-\frac{2\,bdx + bc + ad}{2\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="maxima")
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Fricas [A]
time = 0.28, size = 38, normalized size = 1.36

$$-\frac{2\,bdx + bc + ad}{2\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="fricas")
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A]
time = 0.15, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2c^2d^2+4cd^3x+2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**3,x)
[Out] (-a*d - b*c - 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

Giac [A]
time = 0.00, size = 29, normalized size = 1.04

$$\frac{-2xbd-bc-ad}{2d^2\left(xd+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x)
[Out] -1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

Mupad [B]
time = 0.03, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,d^2}+\frac{b\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^3,x)
[Out] -((a*d + b*c)/(2*d^2) + (b*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x)
```

3.1358 $\int \frac{1}{(c+dx)^3} dx$

Optimal. Leaf size=14

$$-\frac{1}{2d(c+dx)^2}$$

[Out] -1/2/d/(d*x+c)^2

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-3),x]

[Out] -1/2*1/(d*(c + d*x)^2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3} dx = -\frac{1}{2d(c+dx)^2}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-3),x]

[Out] -1/2*1/(d*(c + d*x)^2)

Mathics [A]
time = 1.82, size = 23, normalized size = 1.64

$$-\frac{1}{2d\left(c^2+2cdx+d^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x)^3,x]')
[Out] -1 / (2 d (c ^ 2 + 2 c d x + d ^ 2 x ^ 2))
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{2d(dx+c)^2}$	13
default	$-\frac{1}{2d(dx+c)^2}$	13
norman	$-\frac{1}{2d(dx+c)^2}$	13
risch	$-\frac{1}{2d(dx+c)^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/d/(d*x+c)^2
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{2(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="maxima")
[Out] -1/2/((d*x + c)^2*d)
```

Fricas [A]
time = 0.30, size = 24, normalized size = 1.71

$$-\frac{1}{2(d^3x^2+2cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="fricas")
[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.09, size = 26, normalized size = 1.86

$$-\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3,x)
```

```
[Out] -1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{2d\left(xd+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x)
```

```
[Out] -1/2/((d*x + c)^2*d)
```

Mupad [B]
time = 0.02, size = 26, normalized size = 1.86

$$-\frac{1}{2\,c^2\,d+4\,c\,d^2\,x+2\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^3,x)
```

```
[Out] -1/(2*c^2*d + 2*d^3*x^2 + 4*c*d^2*x)
```

3.1359 $\int \frac{1}{(a+bx)(c+dx)^3} \, dx$

Optimal. Leaf size=82

$$\frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}$$

```
[Out] 1/2/(-a*d+b*c)/(d*x+c)^2+b/(-a*d+b*c)^2/(d*x+c)+b^2*ln(b*x+a)/(-a*d+b*c)^3-
b^2*ln(d*x+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3}\right.\\ &= \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(3bc-ad+2bdx)}{(c+dx)^2}+2b^2\log(a+bx)-2b^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] (((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] - 2
*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 229 vs. $2(82) = 164$.
time = 6.63, size = 227, normalized size = 2.77

$$\frac{b^2\left(a^2c^2d^2-2abc^3d+b^2c^4+2cdx\left(a^2d^2-2abcd+b^2c^2\right)+d^2x^2\left(a^2d^2-2abcd+b^2c^2\right)\right)\left(\text{Log}\left[\frac{c+dx}{d}\right]-\text{Log}\left[\frac{a+bx}{b}\right]\right)+\frac{(-ad+3bc+2bdx)(ad-bc)^3}{2}}{(ad-bc)^3\left(a^2c^2d^2-2abc^3d+b^2c^4+2cdx\left(a^2d^2-2abcd+b^2c^2\right)+d^2x^2\left(a^2d^2-2abcd+b^2c^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-1)/(c + d*x)^3,x]')
```

```
[Out] (b ^ 2 (a ^ 2 c ^ 2 d ^ 2 - 2 a b c ^ 3 d + b ^ 2 c ^ 4 + 2 c d x (a ^ 2 d
^ 2 - 2 a b c d + b ^ 2 c ^ 2) + d ^ 2 x ^ 2 (a ^ 2 d ^ 2 - 2 a b c d + b ^
 2 c ^ 2)) (Log[(c + d x) / d] - Log[(a + b x) / b]) + (-a d + 3 b c + 2 b
d x) (a d - b c) ^ 3 / 2) / ((a d - b c) ^ 3 (a ^ 2 c ^ 2 d ^ 2 - 2 a b c ^
 3 d + b ^ 2 c ^ 4 + 2 c d x (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) + d ^
2 x ^ 2 (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2)))
```

Maple [A]
time = 0.17, size = 81, normalized size = 0.99

method	result	size
default	$-\frac{1}{2(ad-bc)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(ad-bc)^3}+\frac{b}{(ad-bc)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(ad-bc)^3}$	81
risch	$\frac{\frac{bdx}{a^2d^2-2abcd+b^2c^2}-\frac{ad-3bc}{2\left(a^2d^2-2abcd+b^2c^2\right)}}{(dx+c)^2}+\frac{b^2\ln(-dx-c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{b^2\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	171
norman	$\frac{\frac{bdx}{a^2d^2-2abcd+b^2c^2}+\frac{-a\,d^3+3bc\,d^2}{2d^2\left(a^2d^2-2abcd+b^2c^2\right)}}{(dx+c)^2}+\frac{b^2\ln(dx+c)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{b^2\ln(bx+a)}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(d*x+c)-b^
2/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(80) = 160$.
time = 0.28, size = 202, normalized size = 2.46

$$\frac{b^2\log\left(bx+a\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{b^2\log\left(dx+c\right)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx+3\,bc-ad}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.
time = 0.30, size = 242, normalized size = 2.95

$$\frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\,(b^2cd - abd^2)x + 2\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\log\left(bx + a\right) - 2\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\log\left(dx + c\right)}{2\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3 + (b^3c^3d^2 - 3\,ab^2c^2d^3 + 3\,a^2bcd^4 - a^3d^5)x^2 + 2\,(b^3c^4d - 3\,ab^2c^3d^2 + 3\,a^2bc^2d^3 - a^3cd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^
2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 + 2*(
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(68) = 136$.
time = 0.61, size = 381, normalized size = 4.65

$$\frac{b^2\log\left(x + \frac{-\frac{a^4b^2d^4}{(ad-bc)^3} + \frac{4a^3b^3cd^3}{(ad-bc)^3} - \frac{6a^2b^4c^2d^2}{(ad-bc)^3} + \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d - \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3} - \frac{b^2\log\left(x + \frac{\frac{a^4b^2d^4}{(ad-bc)^3} - \frac{4a^3b^3cd^3}{(ad-bc)^3} + \frac{6a^2b^4c^2d^2}{(ad-bc)^3} - \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d + \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3} + \frac{-ad + 3bc + 2bdx}{2a^2c^2d^2 - 4abc^3d + 2b^2c^4 + x^2\cdot\left(2a^2d^4 - 4abcd^3 + 2b^2c^2d^2\right) + x\left(4a^2cd^3 - 8abc^2d^2 + 4b^2c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**3,x)
```

```
[Out] b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b*
c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c)*
*3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*c)*
*3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b
*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b
*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d + 2*b**2
*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4*a**2*c*
d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.
time = 0.00, size = 171, normalized size = 2.09

$$\frac{b^3\ln|xb + a|}{b^4c^3 - 3b^3adc^2 + 3b^2a^2d^2c - ba^3d^3} - \frac{b^2d\ln|xd + c|}{b^3dc^3 - 3b^2ad^2c^2 + 3ba^2d^3c - a^3d^4} + \frac{\frac{1}{2}\left(3b^2c^2 - 4bdca + d^2a^2 + \left(2b^2dc - 2bd^2a\right)x\right)}{\left(bc - da\right)^3\left(xd + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x)
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(d*x + c)^2)
```

Mupad [B]
time = 0.30, size = 183, normalized size = 2.23

$$-\frac{\frac{a\,d-3\,b\,c}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{c^2+2\,c\,d\,x+d^2\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^3),x)
```

```
[Out] - ((a*d - 3*b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(c^2 + d^2*x^2 + 2*c*d*x) - (2*b^2*atanh((a^3*d^3 + b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

3.1360 $\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx$

Optimal. Leaf size=110

$$-\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}$$

```
[Out] -b^2/(-a*d+b*c)^3/(b*x+a)-1/2*d/(-a*d+b*c)^2/(d*x+c)^2-2*b*d/(-a*d+b*c)^3/(
d*x+c)-3*b^2*d*ln(b*x+a)/(-a*d+b*c)^4+3*b^2*d*ln(d*x+c)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{2}{(bc-ad}\right.\\ &= -\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a}{(bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bc-ad)}{a+bx}+\frac{d(bc-ad)^2}{(c+dx)^2}+\frac{4bd(bc-ad)}{c+dx}+6b^2d\log(a+bx)-6b^2d\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/2*((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b*
d*(b*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(b*
c - a*d)^4
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 498 vs. $2(110) = 220$.
time $= 9.91$, size $= 496$, normalized size $= 4.51$

$$\frac{6b^2d\left(a^4c^2d^3-3a^3bc^3d^2+3a^2b^2c^4d-ab^3c^5+bd^2x^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)+cx\left(2a^4d^4-5a^3bcd^3+3a^2b^2c^2d^2+ab^3c^3d-b^4c^4\right)-dx^2\left(-a^4d^4+a^3bcd^3+3a^2b^2c^2d^2-5ab^3c^3d+2b^4c^4\right)\right)\left(\mathrm{Log}\left[\frac{c+dx}{d}\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)+\left(-a^2d^2+5abcd+2b^2c^2+3bdx\left(ad+3bc\right)+6b^2d^2x^2\right)\left(ad-bc\right)^4}{2\left(ad-bc\right)^4\left(a^4c^2d^3-3a^3bc^3d^2+3a^2b^2c^4d-ab^3c^5+bd^2x^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)+cx\left(2a^4d^4-5a^3bcd^3+3a^2b^2c^2d^2+ab^3c^3d-b^4c^4\right)-dx^2\left(-a^4d^4+a^3bcd^3+3a^2b^2c^2d^2-5ab^3c^3d+2b^4c^4\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-2)/(c + d*x)^3,x]')
```

```
[Out] (6 b ^ 2 d (a ^ 4 c ^ 2 d ^ 3 - 3 a ^ 3 b c ^ 3 d ^ 2 + 3 a ^ 2 b ^ 2 c ^ 4
 d - a b ^ 3 c ^ 5 + b d ^ 2 x ^ 3 (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b
 ^ 2 c ^ 2 d - b ^ 3 c ^ 3) + c x (2 a ^ 4 d ^ 4 - 5 a ^ 3 b c d ^ 3 + 3 a
^ 2 b ^ 2 c ^ 2 d ^ 2 + a b ^ 3 c ^ 3 d - b ^ 4 c ^ 4) - d x ^ 2 (-a ^ 4 d
^ 4 + a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 5 a b ^ 3 c ^ 3 d + 2 b
 ^ 4 c ^ 4)) (Log[(c + d x) / d] - Log[(a + b x) / b]) + (-a ^ 2 d ^ 2 + 5
a b c d + 2 b ^ 2 c ^ 2 + 3 b d x (a d + 3 b c) + 6 b ^ 2 d ^ 2 x ^ 2) (a d
 - b c) ^ 4) / (2 (a d - b c) ^ 4 (a ^ 4 c ^ 2 d ^ 3 - 3 a ^ 3 b c ^ 3 d ^
2 + 3 a ^ 2 b ^ 2 c ^ 4 d - a b ^ 3 c ^ 5 + b d ^ 2 x ^ 3 (a ^ 3 d ^ 3 - 3
a ^ 2 b c d ^ 2 + 3 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3) + c x (2 a ^ 4 d ^ 4 - 5
 a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + a b ^ 3 c ^ 3 d - b ^ 4 c ^
4) - d x ^ 2 (-a ^ 4 d ^ 4 + a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 -
5 a b ^ 3 c ^ 3 d + 2 b ^ 4 c ^ 4)))
```

Maple [**A**]
time $= 0.17$, size $= 108$, normalized size $= 0.98$

method	result
default	$-\frac{d}{2(ad-bc)^2(dx+c)^2}+\frac{3db^2\ln(dx+c)}{(ad-bc)^4}+\frac{2db}{(ad-bc)^3(dx+c)}+\frac{b^2}{(ad-bc)^3(bx+a)}-\frac{3db^2\ln(bx+a)}{(ad-bc)^4}$
risch	$\frac{\frac{3b^2d^2x^2}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+\frac{3(ad+3bc)bdx}{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}-\frac{a^2d^2-5abcd-2b^2c^2}{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}}{(bx+a)(dx+c)^2}+\frac{3b^2d\ln(-dx-\cdots}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-\cdots}$
norman	$\frac{\frac{3b^2d^2x^2}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+\frac{-a^2bd^4+5ab^2cd^3+2b^3c^2d^2}{2bd^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(3ab^2d^4+9b^3cd^3\right)x}{2bd^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}}{(bx+a)(dx+c)^2}-\frac{3b^2d\ln\cdots}{a^4d^4-4a^3bcd^3+6a^2b\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*
b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(108) = 216$.
time = 0.27, size = 386, normalized size = 3.51

$$-\frac{3\,b^{2}d\log\left(bx+a\right)}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}+\frac{3\,b^{2}d\log\left(dx+c\right)}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}-\frac{6\,b^{2}d^{2}x^{2}+2\,b^{2}c^{2}+5\,abcd-a^{2}d^{2}+3\,(3\,b^{2}cd+abd^{2})x}{2\,(ab^{3}c^{5}-3\,a^{2}b^{2}c^{4}d+3\,a^{3}bc^{3}d^{2}-a^{4}c^{2}d^{3}+(b^{4}c^{3}d^{2}-3\,ab^{3}c^{2}d^{3}+3\,a^{2}b^{2}cd^{4}-a^{3}bd^{5})x^{3}+(2\,b^{4}c^{4}d-5\,ab^{3}c^{3}d^{2}+3\,a^{2}b^{2}c^{2}d^{3}+a^{3}bcd^{4}-a^{4}d^{5})x^{2}+(b^{4}c^{5}-ab^{3}c^{4}d-3\,a^{2}b^{2}c^{3}d^{2}+5\,a^{3}bc^{2}d^{3}-2\,a^{4}cd^{4})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2 + 5
*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(108) = 216$.
time = 0.30, size = 495, normalized size = 4.50

$$-\frac{2\,b^{3}c^{3}+3\,ab^{2}c^{2}d-6\,a^{2}bcd^{2}+a^{3}d^{3}+6\,(b^{3}cd^{2}-ab^{2}d^{3})x^{2}+3\,(3\,b^{3}c^{2}d-2\,ab^{2}cd^{2}-a^{2}bd^{3})x+6\,(b^{3}d^{3}x^{3}+ab^{2}c^{2}d+(2\,b^{3}cd^{2}+ab^{2}d^{3})x^{2}+(b^{3}c^{2}d+2\,ab^{2}cd^{2})x)\log\left(bx+a\right)-6\,(b^{3}d^{3}x^{3}+ab^{2}c^{2}d+(2\,b^{3}cd^{2}+ab^{2}d^{3})x^{2}+(b^{3}c^{2}d+2\,ab^{2}cd^{2})x)\log\left(dx+c\right)}{2\,(ab^{4}c^{6}-4\,a^{2}b^{3}c^{5}d+6\,a^{3}b^{2}c^{4}d^{2}-4\,a^{4}bc^{3}d^{3}+a^{5}c^{2}d^{4}+(b^{5}c^{4}d^{2}-4\,ab^{4}c^{3}d^{3}+6\,a^{2}b^{3}c^{2}d^{4}-4\,a^{3}b^{2}cd^{5}+a^{4}bd^{6})x^{3}+(2\,b^{5}c^{5}d-7\,ab^{4}c^{4}d^{2}+8\,a^{2}b^{3}c^{3}d^{3}-2\,a^{3}b^{2}c^{2}d^{4}-2\,a^{4}bcd^{5}+a^{5}d^{6})x^{2}+(b^{5}c^{6}-2\,ab^{4}c^{5}d-2\,a^{2}b^{3}c^{4}d^{2}+8\,a^{3}b^{2}c^{3}d^{3}-7\,a^{4}bc^{2}d^{4}+2\,a^{5}cd^{5})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 - 4*a^2
*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d
^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^3
 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 -
 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^
2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(97) = 194$.
time = 0.99, size = 632, normalized size = 5.75

$$\frac{3b^{2}d\log\left(x+\frac{-\frac{3a^{5}b^{2}d^{6}}{(ad-bc)^{4}}+\frac{15a^{4}b^{3}cd^{5}}{(ad-bc)^{4}}-\frac{30a^{3}b^{4}c^{2}d^{4}}{(ad-bc)^{4}}+\frac{30a^{2}b^{5}c^{3}d^{3}}{(ad-bc)^{4}}-\frac{15ab^{6}c^{4}d^{2}}{(ad-bc)^{4}}+3ab^{2}d^{2}+\frac{3b^{7}c^{5}d}{(ad-bc)^{4}}+3b^{3}cd}{6b^{3}d^{2}}\right)}{(ad-bc)^{4}}-\frac{3b^{2}d\log\left(x+\frac{\frac{3a^{5}b^{2}d^{6}}{(ad-bc)^{4}}-\frac{15a^{4}b^{3}cd^{5}}{(ad-bc)^{4}}+\frac{30a^{3}b^{4}c^{2}d^{4}}{(ad-bc)^{4}}-\frac{30a^{2}b^{5}c^{3}d^{3}}{(ad-bc)^{4}}+\frac{15ab^{6}c^{4}d^{2}}{(ad-bc)^{4}}+3ab^{2}d^{2}-\frac{3b^{7}c^{5}d}{(ad-bc)^{4}}+3b^{3}cd}{6b^{3}d^{2}}\right)}{(ad-bc)^{4}}+\frac{-a^{2}d^{2}+5abcd+2b^{2}c^{2}+6b^{2}d^{2}x^{2}+x\left(3abd^{2}+9b^{2}cd\right)}{2a^{4}c^{2}d^{3}-6a^{3}bc^{3}d^{2}+6a^{2}b^{2}c^{4}d-2ab^{3}c^{5}+x^{3}\cdot\left(2a^{3}bd^{5}-6a^{2}b^{2}cd^{4}+6ab^{3}c^{2}d^{3}-2b^{4}c^{3}d^{2}\right)+x^{2}\cdot\left(2a^{4}d^{5}-2a^{3}bcd^{4}-6a^{2}b^{2}c^{2}d^{3}+10ab^{3}c^{3}d^{2}-4b^{4}c^{4}d\right)+x\left(4a^{4}cd^{4}-10a^{3}bc^{2}d^{3}+6a^{2}b^{2}c^{3}d^{2}+2ab^{3}c^{4}d-2b^{4}c^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] 3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(a
*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d*
*3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 + 3*
b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 - 3*
b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d
- b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 - 3*b**
7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 + (-a**
2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**2 + 9*b**
2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4*d - 2*a*b
**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c**2*d**3 -
2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**2*b**2*c**2*
d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4 - 10*a**3*b*
c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*c**5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(108) = 216$.
time = 0.00, size = 264, normalized size = 2.40

$$-\frac{3b^3d\ln|xb+a|}{b^5c^4-4b^4adc^3+6b^3a^2d^2c^2-4b^2a^3d^3c+ba^4d^4}+\frac{3b^2d^2\ln|xd+c|}{b^4dc^4-4b^3ad^2c^3+6b^2a^2d^3c^2-4ba^3d^4c+a^4d^5}+\frac{\frac{1}{2}\left(\left(-6b^3d^2c+6b^2d^3a\right)x^2+\left(-9b^3dc^2+6b^2d^2ca+3bd^3a^2\right)x-2b^3c^3-3b^2dc^2a+6bd^2ca^2-d^3a^3\right)}{(bc-da)^4(xd+c)^2(xb+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -3*b^3*d*log(abs(b*x + a))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4) + 3*b^2*d^2*log(abs(d*x + c))/(b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/2*(2*b^3*c^3
 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 - a*b^2*d^3)*x^2
+ 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x)/((b*c - a*d)^4*(b*x + a)*(
d*x + c)^2)
```

Mupad [B]
time = 0.40, size = 329, normalized size = 2.99

$$\frac{\frac{-a^2d^2+5abcd+2b^2c^2}{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{3bx\left(ad^2+3bcd\right)}{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{3b^2d^2x^2}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}}{x\left(bc^2+2adc\right)+ac^2+x^2\left(ad^2+2bcd\right)+bd^2x^3}-\frac{6b^2d\,\mathrm{atanh}\left(\frac{a^4d^4-2a^3bcd^3+2ab^3c^3d-b^4c^4}{(ad-bc)^4}+\frac{2bdx\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{(ad-bc)^4}\right)}{(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] ((2*b^2*c^2 - a^2*d^2 + 5*a*b*c*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (3*b*x*(a*d^2 + 3*b*c*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(x*(b*c^2 + 2*a*c*d) + a*c^2 + x^2*(a*d^2 + 2*b*c*d) +
 b*d^2*x^3) - (6*b^2*d*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c
*d^3)/(a*d - b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))/(a*d - b*c)^4))/(a*d - b*c)^4
```

3.1361 $\int \frac{1}{(a+bx)^3(c+dx)^3} dx$

Optimal. Leaf size=143

$$-\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+dx)}+\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-$$

```
[Out] -1/2*b^2/(-a*d+b*c)^3/(b*x+a)^2+3*b^2*d/(-a*d+b*c)^4/(b*x+a)+1/2*d^2/(-a*d+
b*c)^3/(d*x+c)^2+3*b*d^2/(-a*d+b*c)^4/(d*x+c)+6*b^2*d^2*ln(b*x+a)/(-a*d+b*c
)^5-6*b^2*d^2*ln(d*x+c)/(-a*d+b*c)^5
```

Rubi [A]
time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/2*b^2/((b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d)/((b*c - a*d)^4*(a + b*x))
+ d^2/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2)/((b*c - a*d)^4*(c + d*x)) +
 (6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b*c - a
*d)^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^3}-\frac{3b^3d}{(bc-ad)^4(a+bx)^2}+\frac{6b^3d^2}{(bc-ad)^5(a+bx)}-\frac{}{(bc-ad}\right.\\ &= -\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3b}{(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 128, normalized size = 0.90

$$\frac{-\frac{b^2(bc-ad)^2}{(a+bx)^2}+\frac{6b^2d(bc-ad)}{a+bx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}+\frac{6bd^2(bc-ad)}{c+dx}+12b^2d^2\log(a+bx)-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] (-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d^
2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d^2
*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 785 vs. $2(143) = 286$.
time = 13.60, size = 783, normalized size = 5.48

$$\frac{12b^2d^2\left(a^6c^2d^4-4a^5bc^3d^3+6a^4b^2c^4d^2-4a^3b^3c^5d+a^2b^4c^6+2acx\left(a^5d^5-3a^4bcd^4+2a^3b^2c^2d^3+2a^2b^3c^3d^2-3ab^4c^4d+b^5c^5\right)+x^2\left(a^6d^6-9a^4b^2c^2d^4+16a^3b^3c^3d^3-9a^2b^4c^4d^2+b^6c^6\right)+2bdx^3\left(a^5d^5-3a^4bcd^4+2a^3b^2c^2d^3+2a^2b^3c^3d^2-3ab^4c^4d+b^5c^5\right)+b^2d^2x^4\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)\right)\left(\mathrm{Log}\left[\frac{c+dx}{d}\right]-\mathrm{Log}\left[\frac{a+bx}{b}\right]\right)+\left(-a^3d^3+7a^2bcd^2+7ab^2c^2d-b^3c^3+4bdx\left(a^2d^2+7abcd+b^2c^2\right)+18b^2d^2x^2\left(ad+bc\right)+12b^3d^3x^3\right)\left(ad-bc\right)^5}{2\left(ad-bc\right)^5\left(a^6c^2d^4-4a^5bc^3d^3+6a^4b^2c^4d^2-4a^3b^3c^5d+a^2b^4c^6+2acx\left(a^5d^5-3a^4bcd^4+2a^3b^2c^2d^3+2a^2b^3c^3d^2-3ab^4c^4d+b^5c^5\right)+x^2\left(a^6d^6-9a^4b^2c^2d^4+16a^3b^3c^3d^3-9a^2b^4c^4d^2+b^6c^6\right)+2bdx^3\left(a^5d^5-3a^4bcd^4+2a^3b^2c^2d^3+2a^2b^3c^3d^2-3ab^4c^4d+b^5c^5\right)+b^2d^2x^4\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-3)/(c + d*x)^3,x]')
```

```
[Out] (12 b ^ 2 d ^ 2 (a ^ 6 c ^ 2 d ^ 4 - 4 a ^ 5 b c ^ 3 d ^ 3 + 6 a ^ 4 b ^ 2
c ^ 4 d ^ 2 - 4 a ^ 3 b ^ 3 c ^ 5 d + a ^ 2 b ^ 4 c ^ 6 + 2 a c x (a ^ 5 d
^ 5 - 3 a ^ 4 b c d ^ 4 + 2 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 2 a ^ 2 b ^ 3 c ^ 3 d
 ^ 2 - 3 a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5) + x ^ 2 (a ^ 6 d ^ 6 - 9 a ^ 4 b ^
2 c ^ 2 d ^ 4 + 16 a ^ 3 b ^ 3 c ^ 3 d ^ 3 - 9 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + b
^ 6 c ^ 6) + 2 b d x ^ 3 (a ^ 5 d ^ 5 - 3 a ^ 4 b c d ^ 4 + 2 a ^ 3 b ^ 2 c
 ^ 2 d ^ 3 + 2 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 3 a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5) +
 b ^ 2 d ^ 2 x ^ 4 (a ^ 4 d ^ 4 - 4 a ^ 3 b c d ^ 3 + 6 a ^ 2 b ^ 2 c ^ 2 d
 ^ 2 - 4 a b ^ 3 c ^ 3 d + b ^ 4 c ^ 4)) (Log[(c + d x) / d] - Log[(a + b x
) / b]) + (-a ^ 3 d ^ 3 + 7 a ^ 2 b c d ^ 2 + 7 a b ^ 2 c ^ 2 d - b ^ 3 c ^
 3 + 4 b d x (a ^ 2 d ^ 2 + 7 a b c d + b ^ 2 c ^ 2) + 18 b ^ 2 d ^ 2 x ^ 2
 (a d + b c) + 12 b ^ 3 d ^ 3 x ^ 3) (a d - b c) ^ 5) / (2 (a d - b c) ^ 5
(a ^ 6 c ^ 2 d ^ 4 - 4 a ^ 5 b c ^ 3 d ^ 3 + 6 a ^ 4 b ^ 2 c ^ 4 d ^ 2 - 4
a ^ 3 b ^ 3 c ^ 5 d + a ^ 2 b ^ 4 c ^ 6 + 2 a c x (a ^ 5 d ^ 5 - 3 a ^ 4 b
c d ^ 4 + 2 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 2 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 3 a b ^ 4
 c ^ 4 d + b ^ 5 c ^ 5) + x ^ 2 (a ^ 6 d ^ 6 - 9 a ^ 4 b ^ 2 c ^ 2 d ^ 4 +
16 a ^ 3 b ^ 3 c ^ 3 d ^ 3 - 9 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + b ^ 6 c ^ 6) + 2 b
 d x ^ 3 (a ^ 5 d ^ 5 - 3 a ^ 4 b c d ^ 4 + 2 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 2 a
 ^ 2 b ^ 3 c ^ 3 d ^ 2 - 3 a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5) + b ^ 2 d ^ 2 x ^
 4 (a ^ 4 d ^ 4 - 4 a ^ 3 b c d ^ 3 + 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 4 a b ^ 3
 c ^ 3 d + b ^ 4 c ^ 4)))
```

Maple [A]
time = 0.18, size = 140, normalized size = 0.98

method	result
default	$-\frac{d^2}{2(ad-bc)^3(dx+c)^2}+\frac{6d^2b^2\ln(dx+c)}{(ad-bc)^5}+\frac{3d^2b}{(ad-bc)^4(dx+c)}+\frac{b^2}{2(ad-bc)^3(bx+a)^2}-\frac{6d^2b^2\ln(bx+a)}{(ad-bc)^5}+\frac{3b^2d}{(ad-bc)^4(bx+a)}$
risch	$\frac{\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}+\frac{9b^2d^2(ad+bc)x^2}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}+\frac{2\left(a^2d^2+7abcd+b^2c^2\right)bdx}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}-\frac{}{2(a^{}}}{(bx+a)^2(dx+c)^2}$

norman	$\frac{\frac{(9a\,b^4d^5+9b^5c\,d^4)x^2}{d^2b^2(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}+\frac{-a^3b^2d^5+7a^2b^3c\,d^4+7a\,b^4c^2d^3-b^5c^3d^2}{2d^2b^2(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}+\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d}}{(bx+a)^2(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3*d^2/(a*d-b
*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*b^2*ln(b*x+
a)+3*b^2/(a*d-b*c)^4*d/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(139) = 278$.
time = 0.27, size = 594, normalized size = 4.15

$$\frac{6\,b^2d^2\log\left(bx+a\right)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}-\frac{6\,b^2d^2\log\left(dx+c\right)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}+\frac{12\,b^3d^3x^3-b^3c^3+7\,ab^2c^2d+7\,a^2bcd^2-a^3d^3+18\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+7\,ab^2cd^2+a^2bd^3)x}{2\,(a^2b^4c^6-4\,a^3b^3c^5d+6\,a^4b^2c^4d^2-4\,a^5bc^3d^3+a^6c^2d^4+(b^6c^4d^2-4\,ab^5c^3d^3+6\,a^2b^4c^2d^4-4\,a^3b^3cd^5+a^4b^2d^6)x^4+2\,(b^6c^5d-3\,ab^5c^4d^2+2\,a^2b^4c^3d^3+2\,a^3b^3c^2d^4-3\,a^4b^2cd^5+a^5bd^6)x^3+(b^6c^6-9\,a^2b^4c^4d^2+16\,a^3b^3c^3d^3-9\,a^4b^2c^2d^4+a^6d^6)x^2+2\,(ab^5c^6-3\,a^2b^4c^5d+2\,a^3b^3c^4d^2+2\,a^4b^2c^3d^3-3\,a^5bc^2d^4+a^6cd^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
- a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
- a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 +
 a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4
 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*
a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b
^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d
^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^
3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(139) = 278$.
time = 0.31, size = 760, normalized size = 5.31

$$-\frac{b^4c^4-8\,ab^3c^3d+8\,a^3bcd^3-a^4d^4-12\,(b^4cd^3-ab^3d^4)x^3-18\,(b^4c^2d^2-a^2b^2d^4)x^2-4\,(b^4c^3d+6\,ab^3c^2d^2-6\,a^2b^2cd^3-a^3bd^4)x-12\,(b^4d^4x^4+a^2b^2c^2d^2+2\,(b^4cd^3+ab^3d^4)x^3+(b^4c^2d^2+4\,ab^3cd^3+a^2b^2d^4)x^2+2\,(ab^3c^2d^2+a^2b^2cd^3)x)\log\left(bx+a\right)+12\,(b^4d^4x^4+a^2b^2c^2d^2+2\,(b^4cd^3+ab^3d^4)x^3+(b^4c^2d^2+4\,ab^3cd^3+a^2b^2d^4)x^2+2\,(ab^3c^2d^2+a^2b^2cd^3)x)\log\left(dx+c\right)}{2\,(a^2b^5c^7-5\,a^3b^4c^6d+10\,a^4b^3c^5d^2-10\,a^5b^2c^4d^3+5\,a^6bc^3d^4-a^7c^2d^5+(b^7c^5d^2-5\,ab^6c^4d^3+10\,a^2b^5c^3d^4-10\,a^3b^4c^2d^5+5\,a^4b^3cd^6-a^5b^2d^7)x^4+2\,(b^7c^6d-4\,ab^6c^5d^2+5\,a^2b^5c^4d^3-5\,a^4b^3c^2d^5+4\,a^5b^2cd^6-a^6bd^7)x^3+(b^7c^7-ab^6c^6d-9\,a^2b^5c^5d^2+25\,a^3b^4c^4d^3-25\,a^4b^3c^3d^4+9\,a^5b^2c^2d^5+a^6bcd^6-a^7d^7)x^2+2\,(ab^6c^7-4\,a^2b^5c^6d+5\,a^3b^4c^5d^2-5\,a^5b^2c^3d^4+4\,a^6bc^2d^5-a^7cd^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 - a
*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b^3
*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d
```

```
^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2
*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^4*d^4
*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a
*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*
x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^
4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10
*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^4
+ 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 +
4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d
^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*
d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5
*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(128) = 256$.
time = 1.46, size = 881, normalized size = 6.16

$$\frac{6b^2d^2\log\left(x+\frac{-\frac{6a^6b^2d^8}{(ad-bc)^5}+\frac{36a^5b^3cd^7}{(ad-bc)^5}-\frac{90a^4b^4c^2d^6}{(ad-bc)^5}+\frac{120a^3b^5c^3d^5}{(ad-bc)^5}-\frac{90a^2b^6c^4d^4}{(ad-bc)^5}+\frac{36ab^7c^5d^3}{(ad-bc)^5}+6ab^2d^3-\frac{6b^8c^6d^2}{(ad-bc)^5}+6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5}-\frac{6b^2d^2\log\left(x+\frac{\frac{6a^6b^2d^8}{(ad-bc)^5}-\frac{36a^5b^3cd^7}{(ad-bc)^5}+\frac{90a^4b^4c^2d^6}{(ad-bc)^5}-\frac{120a^3b^5c^3d^5}{(ad-bc)^5}+\frac{90a^2b^6c^4d^4}{(ad-bc)^5}-\frac{36ab^7c^5d^3}{(ad-bc)^5}+6ab^2d^3+\frac{6b^8c^6d^2}{(ad-bc)^5}+6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5}+\frac{-a^3d^3+7a^2bcd^2+7ab^2c^2d-b^3c^3+12b^3d^3x^3+x^2\cdot(18ab^2d^3+18b^3cd^2)+x(4a^2bd^3+28ab^2cd^2+4b^3c^2d)}{2a^6c^2d^4-8a^5bc^3d^3+12a^4b^2c^4d^2-8a^3b^3c^5d+2a^2b^4c^6+x^4\cdot(2a^4b^2d^6-8a^3b^3cd^5+12a^2b^4c^2d^4-8ab^5c^3d^3+2b^6c^4d^2)+x^3\cdot(4a^5bd^6-12a^4b^2cd^5+8a^3b^3c^2d^4+8a^2b^4c^3d^3-12ab^5c^4d^2+4b^6c^5d)+x^2\cdot(2a^6d^6-18a^4b^2c^2d^4+32a^3b^3c^3d^3-18a^2b^4c^4d^2+2b^6c^6)+x(4a^6cd^5-12a^5bc^2d^4+8a^4b^2c^3d^3+8a^3b^3c^4d^2-12a^2b^4c^5d+4ab^5c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] 6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**7
/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c**
3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**7*c
**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)**5 +
 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (6*a**
6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*a**4*b
**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*
a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6
*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(12*b**3*d*
*3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**
3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d**2) + x*(4*
a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d**4 - 8*a**5
*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2*a**2*b**4*c*
*6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b**4*c**2*d**4 -
 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**6 - 12*a**4*b**
2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 - 12*a*b**5*c**4*d
**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c**2*d**4 + 32*a**3
*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) + x*(4*a**6*c*d**5
- 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b**3*c**4*d**2 - 12*
a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(139) = 278$.

time = 0.00, size = 357, normalized size = 2.50

$$\frac{6b^2d^3\ln|xd+c|}{-b^5dc^5+5b^4ad^2c^4-10b^3a^2d^3c^3+10b^2a^3d^4c^2-5ba^4d^5c+a^5d^6}+\frac{6b^3d^2\ln|xb+a|}{b^6c^5-5b^5adc^4+10b^4a^2d^2c^3-10b^3a^3d^3c^2+5b^2a^4d^4c-ba^5d^5}+\frac{-12x^3b^3d^3-18x^2b^3d^2c-18x^2b^2ad^3-4xb^3dc^2-28xb^2ad^2c-4xba^2d^3+b^3c^3-7b^2adc^2-7ba^2d^2c+a^3d^3}{(-2b^4c^4+8b^3adc^3-12b^2a^2d^2c^2+8ba^3d^3c-2a^4d^4)(x^2bd+xbc+xad+ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 6*b^3*d^2*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 -
 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - 6*b^2*d^3*log(abs(d*x
+ c))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^
4 + 5*a^4*b*c*d^5 - a^5*d^6) + 1/2*(12*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*
a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 28*a*b^2*c*d^2*x + 4*a^2*b*d^3*x - b^3*c^3
+ 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*d*x^2 + b*c*x + a*d*x + a*c)^2)
```

Mupad [B]
time = 0.53, size = 542, normalized size = 3.79

$$\frac{\frac{6\,b^3\,d^3\,x^3}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}-\frac{a^3\,d^3-7\,a^2\,b\,c\,d^2-7\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}+\frac{9\,b\,d\,x^2\,(c\,b^2\,d+a\,b\,d^2)}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}+\frac{2\,b\,d\,x\,(a^2\,d^2+7\,a\,b\,c\,d+b^2\,c^2)}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}}{x\,(2\,d\,a^2\,c+2\,b\,a\,c^2)+x^2\,(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)+x^3\,(2\,c\,b^2\,d+2\,a\,b\,d^2)+a^2\,c^2+b^2\,d^2\,x^4}-\frac{12\,b^2\,d^2\,\mathrm{atanh}\left(\frac{a^5\,d^5-3\,a^4\,b\,c\,d^4+2\,a^3\,b^2\,c^2\,d^3+2\,a^2\,b^3\,c^3\,d^2-3\,a\,b^4\,c^4\,d+b^5\,c^5}{(a\,d-b\,c)^5}+\frac{2\,b\,d\,x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{(a\,d-b\,c)^5}\right)}{(a\,d-b\,c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^3),x)
```

```
[Out] ((6*b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3) - (a^3*d^3 + b^3*c^3 - 7*a*b^2*c^2*d - 7*a^2*b*c*d^2)/(2*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (9*b
*d*x^2*(a*b*d^2 + b^2*c*d))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 + 7*a*b*c*d))/(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(x*(2*a*
b*c^2 + 2*a^2*c*d) + x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^3*(2*a*b*d^2 +
 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^4) - (12*b^2*d^2*atanh((a^5*d^5 + b^5*c^5
 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3*a*b^4*c^4*d - 3*a^4*b*c*d^4)/(
a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d - b*c)^5
```

3.1362 $\int \frac{(a+bx)^9}{(c+dx)^8} \, dx$

Optimal. Leaf size=232

$$-\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-$$

```
[Out] -b^8*(-9*a*d+8*b*c)*x/d^9+1/2*b^9*x^2/d^8+1/7*(-a*d+b*c)^9/d^10/(d*x+c)^7-3
/2*b*(-a*d+b*c)^8/d^10/(d*x+c)^6+36/5*b^2*(-a*d+b*c)^7/d^10/(d*x+c)^5-21*b^
3*(-a*d+b*c)^6/d^10/(d*x+c)^4+42*b^4*(-a*d+b*c)^5/d^10/(d*x+c)^3-63*b^5*(-a
*d+b*c)^4/d^10/(d*x+c)^2+84*b^6*(-a*d+b*c)^3/d^10/(d*x+c)+36*b^7*(-a*d+b*c)
^2*ln(d*x+c)/d^10
```

Rubi [A]
time = 0.25, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{36b^7(bc-ad)^2\log(c+dx)}{d^{10}}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}+\frac{b^9x^2}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -((b^8*(8*b*c - 9*a*d)*x)/d^9) + (b^9*x^2)/(2*d^8) + (b*c - a*d)^9/(7*d^10*
(c + d*x)^7) - (3*b*(b*c - a*d)^8)/(2*d^10*(c + d*x)^6) + (36*b^2*(b*c - a*
d)^7)/(5*d^10*(c + d*x)^5) - (21*b^3*(b*c - a*d)^6)/(d^10*(c + d*x)^4) + (4
2*b^4*(b*c - a*d)^5)/(d^10*(c + d*x)^3) - (63*b^5*(b*c - a*d)^4)/(d^10*(c +
 d*x)^2) + (84*b^6*(b*c - a*d)^3)/(d^10*(c + d*x)) + (36*b^7*(b*c - a*d)^2*
Log[c + d*x])/d^10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^9}{(c+dx)^8}\,dx = \int \left(-\frac{b^8(8bc-9ad)}{d^9}+\frac{b^9x}{d^8}+\frac{(-bc+ad)^9}{d^9(c+dx)^8}+\frac{9b(bc-ad)^8}{d^9(c+dx)^7}-\frac{36b^2(bc-ad)^7}{d^9(c+dx)^6}+\frac{84b^3(bc-}{d^9(c+}\right.$$

$$= -\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-}{d^{10}(c+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 584 vs. 2(232) = 464.
time = 0.18, size = 584, normalized size = 2.52

$$-\frac{10a^9d^9+15a^8bd^8(c+7dx)+24a^7b^2d^7(c^2+7cdx+21d^2x^2)+42a^6b^3d^6(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+84a^5b^4d^5(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+210a^4b^5d^4(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+840a^3b^6d^3(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)-6a^2b^7cd^2(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)+6ab^8d(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8)-b^9(3349c^9+20923c^8dx+53949c^7d^2x^2+72275c^6d^3x^3+50225c^5d^4x^4+12495c^4d^5x^5-4655c^3d^6x^6-3185c^2d^7x^7-315cd^8x^8+35d^9x^9)-2520b^7(bc-ad)^2(c+dx)^7\log(c+dx)}{70d^{10}(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -1/70*(10*a^9*d^9 + 15*a^8*b*d^8*(c + 7*d*x) + 24*a^7*b^2*d^7*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 42*a^6*b^3*d^6*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x
^3) + 84*a^5*b^4*d^5*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*
d^4*x^4) + 210*a^4*b^5*d^4*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x
^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 840*a^3*b^6*d^3*(c^6 + 7*c^5*d*x + 21*c^4
*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 6*
a^2*b^7*c*d^2*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*
x^3 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + 6*a*b^8*d*(1443
*c^8 + 9261*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4
*x^4 + 11025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) -
 b^9*(3349*c^9 + 20923*c^8*d*x + 53949*c^7*d^2*x^2 + 72275*c^6*d^3*x^3 + 50
225*c^5*d^4*x^4 + 12495*c^4*d^5*x^5 - 4655*c^3*d^6*x^6 - 3185*c^2*d^7*x^7 -
 315*c*d^8*x^8 + 35*d^9*x^9) - 2520*b^7*(b*c - a*d)^2*(c + d*x)^7*Log[c + d
*x])/(d^10*(c + d*x)^7)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^9/(c + d*x)^8,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(224) = 448.
time = 0.14, size = 705, normalized size = 3.04

method	result
default	$\frac{b^8\left(\frac{1}{2}bd\,x^2+9adx-8bcx\right)}{d^9}-\frac{42b^4\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{d^{10}(dx+c)^3}-\frac{3b\left(a^8d^8-8a^7bc\,d^7+28a^6b^2c^2d^6-56\cdots\right.}{\cdots}$
norman	$\frac{-\frac{10a^9d^9+15a^8bc\,d^8+24a^7b^2c^2d^7+42a^6b^3c^3d^6+84a^5b^4c^4d^5+210a^4b^5c^5d^4+840a^3b^6d^3c^6-6534a^2b^7d^2c^7+13068a\,b^8c^8d-6534b^9c^9}{70d^{10}}+\frac{b^9x^9}{2d}-\frac{7(12\cdots}{\cdots}}{\cdots}$

risch	$\frac{b^9x^2}{2d^8}+\frac{9b^8ax}{d^8}-\frac{8b^9cx}{d^9}+\frac{(-84a^3b^6d^8+252a^2b^7c\,d^7-252a\,b^8c^2d^6+84b^9c^3d^5)x^6-63b^5d^4\left(a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+20a\,b^3c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^9/(d*x+c)^8,x,method=_RETURNVERBOSE)

[Out] b^8/d^9*(1/2*b*d*x^2+9*a*d*x-8*b*c*x)-42*b^4/d^10*(a^5*d^5-5*a^4*b*c*d^4+10
*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)^3-3/2*b/
d^10*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^
4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(d*x
+c)^6-36/5*b^2/d^10*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^
3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(d*x+c)^
5-21*b^3/d^10*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+
15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(d*x+c)^4-84*b^6/d^10*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+36*b^7/d^10*(a^2*d^2-2*a*b*c*d+b
^2*c^2)*ln(d*x+c)-63*b^5/d^10*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*
b^3*c^3*d+b^4*c^4)/(d*x+c)^2-1/7/d^10*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2
*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*
c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(d*x+c)^7

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(224) = 448$.
time = 0.34, size = 786, normalized size = 3.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="maxima")

[Out] 1/70*(3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*
c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 2
4*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b
^8*c^2*d^7 + 3*a^2*b^7*c*d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*
a*b^8*c^3*d^6 + 18*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1
470*(47*b^9*c^5*d^4 - 130*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*
c^2*d^7 - 5*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154
*a*b^8*c^5*d^4 + 125*a^2*b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d
^7 - 2*a^5*b^4*c*d^8 - a^6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8
*c^6*d^3 + 959*a^2*b^7*c^5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 -
 14*a^5*b^4*c^2*d^7 - 7*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c
^8*d - 2676*a*b^8*c^7*d^2 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70
*a^4*b^5*c^4*d^5 - 28*a^5*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^
8 - 5*a^8*b*d^9)*x)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^1
4*x^4 + 35*c^4*d^13*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10) + 1/2*

```
(b^9*d*x^2 - 2*(8*b^9*c - 9*a*b^8*d)*x)/d^9 + 36*(b^9*c^2 - 2*a*b^8*c*d + a
^2*b^7*d^2)*log(d*x + c)/d^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(224) = 448$.
time = 0.30, size = 1093, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/70*(35*b^9*d^9*x^9 + 3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d
^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^
6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 - 315*(b^9
*c*d^8 - 2*a*b^8*d^9)*x^8 - 245*(13*b^9*c^2*d^7 - 18*a*b^8*c*d^8)*x^7 - 245
*(19*b^9*c^3*d^6 + 18*a*b^8*c^2*d^7 - 72*a^2*b^7*c*d^8 + 24*a^3*b^6*d^9)*x^
6 + 735*(17*b^9*c^4*d^5 - 90*a*b^8*c^3*d^6 + 108*a^2*b^7*c^2*d^7 - 24*a^3*b
^6*c*d^8 - 6*a^4*b^5*d^9)*x^5 + 245*(205*b^9*c^5*d^4 - 690*a*b^8*c^4*d^5 +
660*a^2*b^7*c^3*d^6 - 120*a^3*b^6*c^2*d^7 - 30*a^4*b^5*c*d^8 - 12*a^5*b^4*d
^9)*x^4 + 245*(295*b^9*c^6*d^3 - 870*a*b^8*c^5*d^4 + 750*a^2*b^7*c^4*d^5 -
120*a^3*b^6*c^3*d^6 - 30*a^4*b^5*c^2*d^7 - 12*a^5*b^4*c*d^8 - 6*a^6*b^3*d^9
)*x^3 + 21*(2569*b^9*c^7*d^2 - 7098*a*b^8*c^6*d^3 + 5754*a^2*b^7*c^5*d^4 -
840*a^3*b^6*c^4*d^5 - 210*a^4*b^5*c^3*d^6 - 84*a^5*b^4*c^2*d^7 - 42*a^6*b^3
*c*d^8 - 24*a^7*b^2*d^9)*x^2 + 7*(2989*b^9*c^8*d - 7938*a*b^8*c^7*d^2 + 617
4*a^2*b^7*c^6*d^3 - 840*a^3*b^6*c^5*d^4 - 210*a^4*b^5*c^4*d^5 - 84*a^5*b^4*
c^3*d^6 - 42*a^6*b^3*c^2*d^7 - 24*a^7*b^2*c*d^8 - 15*a^8*b*d^9)*x + 2520*(b
^9*c^9 - 2*a*b^8*c^8*d + a^2*b^7*c^7*d^2 + (b^9*c^2*d^7 - 2*a*b^8*c*d^8 + a
^2*b^7*d^9)*x^7 + 7*(b^9*c^3*d^6 - 2*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 2
1*(b^9*c^4*d^5 - 2*a*b^8*c^3*d^6 + a^2*b^7*c^2*d^7)*x^5 + 35*(b^9*c^5*d^4 -
 2*a*b^8*c^4*d^5 + a^2*b^7*c^3*d^6)*x^4 + 35*(b^9*c^6*d^3 - 2*a*b^8*c^5*d^4
 + a^2*b^7*c^4*d^5)*x^3 + 21*(b^9*c^7*d^2 - 2*a*b^8*c^6*d^3 + a^2*b^7*c^5*d
^4)*x^2 + 7*(b^9*c^8*d - 2*a*b^8*c^7*d^2 + a^2*b^7*c^6*d^3)*x)*log(d*x + c)
)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^14*x^4 + 35*c^4*d^1
3*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(224) = 448.
time = 0.00, size = 764, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x)
```

```
[Out] 36*(b^9*c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*log(abs(d*x + c))/d^10 + 1/2*(b^9*
d^8*x^2 - 16*b^9*c*d^7*x + 18*a*b^8*d^8*x)/d^16 + 1/70*(3349*b^9*c^9 - 8658
*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5
*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^
8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b^8*c^2*d^7 + 3*a^2*b^7*c*
d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*a*b^8*c^3*d^6 + 18*a^2*b^
7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1470*(47*b^9*c^5*d^4 - 130
*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*c^2*d^7 - 5*a^4*b^5*c*d^8
 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154*a*b^8*c^5*d^4 + 125*a^2*
b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d^7 - 2*a^5*b^4*c*d^8 - a^
6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8*c^6*d^3 + 959*a^2*b^7*c^
5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 - 14*a^5*b^4*c^2*d^7 - 7*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c^8*d - 2676*a*b^8*c^7*d^2
 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70*a^4*b^5*c^4*d^5 - 28*a^5
*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^8 - 5*a^8*b*d^9)*x)/((d*x
 + c)^7*d^10)
```

Mupad [B]
time = 0.26, size = 784, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/(c + d*x)^8,x)
```

```
[Out] x*((9*a*b^8)/d^8 - (8*b^9*c)/d^9) - ((10*a^9*d^9 - 3349*b^9*c^9 - 6534*a^2*
b^7*c^7*d^2 + 840*a^3*b^6*c^6*d^3 + 210*a^4*b^5*c^5*d^4 + 84*a^5*b^4*c^4*d^
5 + 42*a^6*b^3*c^3*d^6 + 24*a^7*b^2*c^2*d^7 + 8658*a*b^8*c^8*d + 15*a^8*b*c
*d^8)/(70*d) + x*((3*a^8*b*d^8)/2 - (3069*b^9*c^8)/10 + (12*a^7*b^2*c*d^7)/
5 - (3087*a^2*b^7*c^6*d^2)/5 + 84*a^3*b^6*c^5*d^3 + 21*a^4*b^5*c^4*d^4 + (4
2*a^5*b^4*c^3*d^5)/5 + (21*a^6*b^3*c^2*d^6)/5 + (4014*a*b^8*c^7*d)/5) + x^3
*(21*a^6*b^3*d^8 - 1197*b^9*c^6*d^2 + 3234*a*b^8*c^5*d^3 + 42*a^5*b^4*c*d^7
 - 2625*a^2*b^7*c^4*d^4 + 420*a^3*b^6*c^3*d^5 + 105*a^4*b^5*c^2*d^6) + x^2*
((36*a^7*b^2*d^8)/5 - (4131*b^9*c^7*d)/5 + (10962*a*b^8*c^6*d^2)/5 + (63*a^
6*b^3*c*d^7)/5 - (8631*a^2*b^7*c^5*d^3)/5 + 252*a^3*b^6*c^4*d^4 + 63*a^4*b^
5*c^3*d^5 + (126*a^5*b^4*c^2*d^6)/5) + x^5*(63*a^4*b^5*d^8 - 441*b^9*c^4*d^
```

```
4 + 1260*a*b^8*c^3*d^5 + 252*a^3*b^6*c*d^7 - 1134*a^2*b^7*c^2*d^6) + x^4*(4
2*a^5*b^4*d^8 - 987*b^9*c^5*d^3 + 2730*a*b^8*c^4*d^4 + 105*a^4*b^5*c*d^7 -
2310*a^2*b^7*c^3*d^5 + 420*a^3*b^6*c^2*d^6) + x^6*(84*a^3*b^6*d^8 - 84*b^9*
c^3*d^5 + 252*a*b^8*c^2*d^6 - 252*a^2*b^7*c*d^7))/(c^7*d^9 + d^16*x^7 + 7*c
^6*d^10*x + 7*c*d^15*x^6 + 21*c^5*d^11*x^2 + 35*c^4*d^12*x^3 + 35*c^3*d^13*
x^4 + 21*c^2*d^14*x^5) + (b^9*x^2)/(2*d^8) + (log(c + d*x)*(36*b^9*c^2 + 36
*a^2*b^7*d^2 - 72*a*b^8*c*d))/d^10
```

3.1363 $\int \frac{(a+bx)^8}{(c+dx)^8} \, dx$

Optimal. Leaf size=209

$$\frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{28b^6}{d^9}$$

```
[Out] b^8*x/d^8-1/7*(-a*d+b*c)^8/d^9/(d*x+c)^7+4/3*b*(-a*d+b*c)^7/d^9/(d*x+c)^6-2
8/5*b^2*(-a*d+b*c)^6/d^9/(d*x+c)^5+14*b^3*(-a*d+b*c)^5/d^9/(d*x+c)^4-70/3*b
^4*(-a*d+b*c)^4/d^9/(d*x+c)^3+28*b^5*(-a*d+b*c)^3/d^9/(d*x+c)^2-28*b^6*(-a*
d+b*c)^2/d^9/(d*x+c)-8*b^7*(-a*d+b*c)*ln(d*x+c)/d^9
```

Rubi [A]
time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{b^8x}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] (b^8*x)/d^8 - (b*c - a*d)^8/(7*d^9*(c + d*x)^7) + (4*b*(b*c - a*d)^7)/(3*d^
9*(c + d*x)^6) - (28*b^2*(b*c - a*d)^6)/(5*d^9*(c + d*x)^5) + (14*b^3*(b*c
- a*d)^5)/(d^9*(c + d*x)^4) - (70*b^4*(b*c - a*d)^4)/(3*d^9*(c + d*x)^3) +
(28*b^5*(b*c - a*d)^3)/(d^9*(c + d*x)^2) - (28*b^6*(b*c - a*d)^2)/(d^9*(c +
 d*x)) - (8*b^7*(b*c - a*d)*Log[c + d*x])/d^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^8}{(c+dx)^8}\,dx = \int \left(\frac{b^8}{d^8}+\frac{(-bc+ad)^8}{d^8(c+dx)^8}-\frac{8b(bc-ad)^7}{d^8(c+dx)^7}+\frac{28b^2(bc-ad)^6}{d^8(c+dx)^6}-\frac{56b^3(bc-ad)^5}{d^8(c+dx)^5}+\frac{70b^4(bc-a}{d^8(c+dx}\right.$$

$$=\frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^{}}{3d^9(c+dx)^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 474 vs. $2(209) = 418$.
time = 0.13, size = 474, normalized size = 2.27

$$-\frac{15a^8d^8+20a^7bd^7(c+7dx)+28a^6b^2d^6(c^2+7cdx+21d^2x^2)+42a^5b^3d^5(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+70a^4b^4d^4(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+140a^3b^5d^3(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+420a^2b^6d^2(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)-2ab^7cd(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)+b^8(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8)+840b^7(bc-ad)(c+dx)^7\log(c+dx)}{105d^9(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^8*d^8 + 20*a^7*b*d^7*(c + 7*d*x) + 28*a^6*b^2*d^6*(c^2 + 7*c*d
*x + 21*d^2*x^2) + 42*a^5*b^3*d^5*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*
x^3) + 70*a^4*b^4*d^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35
*d^4*x^4) + 140*a^3*b^5*d^3*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*
x^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 420*a^2*b^6*d^2*(c^6 + 7*c^5*d*x + 21*c^
4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 2
*a*b^7*c*d*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3
 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + b^8*(1443*c^8 + 92
61*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4*x^4 + 11
025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) + 840*b^7*
(b*c - a*d)*(c + d*x)^7*Log[c + d*x])/(d^9*(c + d*x)^7)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^8/(c + d*x)^8,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(201) = 402$.
time = 0.14, size = 576, normalized size = 2.76

method	result
risch	$\frac{b^8x}{d^8}+\frac{\left(-28a^2b^6d^7+56ab^7cd^6-28b^8c^2d^5\right)x^6-28b^5d^4\left(a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3\right)x^5-\frac{70b^4d^3\left(a^4d^4+2a^3bcd^3+6a^2b^2c^2d^2-22ab^3\right.}{3}$
default	$\frac{b^8x}{d^8}-\frac{4b\left(a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7\right)}{3d^9(dx+c)^6}-\frac{28b^6\left(a^2d^2-2abcd+b^2c^2\right)}{d^9(dx+c)}-28$
norman	$\frac{\frac{b^8x^8}{d}-\frac{15a^8d^8+20a^7bcd^7+28a^6b^2c^2d^6+42a^5b^3c^3d^5+70a^4b^4c^4d^4+140a^3b^5c^5d^3+420a^2b^6c^6d^2-2178ab^7c^7d+2178b^8c^8}{105d^9}-\frac{7\left(4a^2b^6d^2-8ab^7cd+8b^8\right.}{d^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] b^8*x/d^8-4/3*b/d^9*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^
3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(d*x+c)^
6-28*b^6/d^9*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-28/5*b^2/d^9*(a^6*d^6-6*a^
5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*
c^5*d+b^6*c^6)/(d*x+c)^5+8*b^7/d^9*(a*d-b*c)*ln(d*x+c)-70/3*b^4/d^9*(a^4*d^
4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^3-14*b^3/d
^9*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4
*d-b^5*c^5)/(d*x+c)^4-28*b^5/d^9*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/(d*x+c)^2-1/7/d^9*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*
c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^
7*d+b^8*c^8)/(d*x+c)^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(201) = 402$.
time = 0.30, size = 649, normalized size = 3.11

$$\frac{b^8 x}{d^8} - \frac{1443\,b^8c^8 - 2178\,ab^7c^7d + 420\,a^2b^6c^6d^2 + 140\,a^3b^5c^5d^3 + 70\,a^4b^4c^4d^4 + 42\,a^5b^3c^3d^5 + 28\,a^6b^2c^2d^6 + 20\,a^7bcd^7 + 15\,a^8d^8 + 2940\,(b^8c^2d^6 - 2\,ab^7cd^7 + a^2b^6d^8)x^6 + 2940\,(5\,b^8c^3d^5 - 9\,ab^7c^2d^6 + 3\,a^2b^6cd^7 + a^3b^5d^8)x^5 + 2450\,(13\,b^8c^4d^4 - 22\,ab^7c^3d^5 + 6\,a^2b^6c^2d^6 + 2\,a^3b^5cd^7 + a^4b^4d^8)x^4 + 490\,(77\,b^8c^5d^3 - 125\,ab^7c^4d^4 + 30\,a^2b^6c^3d^5 + 10\,a^3b^5c^2d^6 + 5\,a^4b^4cd^7 + 3\,a^5b^3d^8)x^3 + 294\,(87\,b^8c^6d^2 - 137\,ab^7c^5d^3 + 30\,a^2b^6c^4d^4 + 10\,a^3b^5c^3d^5 + 5\,a^4b^4c^2d^6 + 3\,a^5b^3cd^7 + 2\,a^6b^2d^8)x^2 + 14\,(669\,b^8c^7d - 1029\,ab^7c^6d^2 + 210\,a^2b^6c^5d^3 + 70\,a^3b^5c^4d^4 + 35\,a^4b^4c^3d^5 + 21\,a^5b^3c^2d^6 + 14\,a^6b^2cd^7 + 10\,a^7bd^8)x}{105\,(d^{16}x^7 + 7\,cd^{15}x^6 + 21\,c^2d^{14}x^5 + 35\,c^3d^{13}x^4 + 35\,c^4d^{12}x^3 + 21\,c^5d^{11}x^2 + 7\,c^6d^{10}x + c^7d^9)} - \frac{8\,(b^8c - ab^7d)\log(dx + c)}{d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^8*x/d^8 - 1/105*(1443*b^8*c^8 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 +
140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 + 20*a^7*b*c*d^7 + 15*a^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 +
 a^2*b^6*d^8)*x^6 + 2940*(5*b^8*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7
 + a^3*b^5*d^8)*x^5 + 2450*(13*b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c
^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b
^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*
a^5*b^3*d^8)*x^3 + 294*(87*b^8*c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4
*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2
*d^8)*x^2 + 14*(669*b^8*c^7*d - 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 +
70*a^3*b^5*c^4*d^4 + 35*a^4*b^4*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c
*d^7 + 10*a^7*b*d^8)*x)/(d^16*x^7 + 7*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3
*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*d^11*x^2 + 7*c^6*d^10*x + c^7*d^9) - 8
*(b^8*c - a*b^7*d)*log(d*x + c)/d^9
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(201) = 402$.
time = 0.30, size = 852, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="fricas")
```

[Out] 1/105*(105*b^8*d^8*x^8 + 735*b^8*c*d^7*x^7 - 1443*b^8*c^8 + 2178*a*b^7*c^7*d - 420*a^2*b^6*c^6*d^2 - 140*a^3*b^5*c^5*d^3 - 70*a^4*b^4*c^4*d^4 - 42*a^5*b^3*c^3*d^5 - 28*a^6*b^2*c^2*d^6 - 20*a^7*b*c*d^7 - 15*a^8*d^8 - 735*(b^8*c^2*d^6 - 8*a*b^7*c*d^7 + 4*a^2*b^6*d^8)*x^6 - 735*(15*b^8*c^3*d^5 - 36*a*b^7*c^2*d^6 + 12*a^2*b^6*c*d^7 + 4*a^3*b^5*d^8)*x^5 - 1225*(23*b^8*c^4*d^4 - 44*a*b^7*c^3*d^5 + 12*a^2*b^6*c^2*d^6 + 4*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x^4 - 245*(145*b^8*c^5*d^3 - 250*a*b^7*c^4*d^4 + 60*a^2*b^6*c^3*d^5 + 20*a^3*b^5*c^2*d^6 + 10*a^4*b^4*c*d^7 + 6*a^5*b^3*d^8)*x^3 - 147*(169*b^8*c^6*d^2 - 274*a*b^7*c^5*d^3 + 60*a^2*b^6*c^4*d^4 + 20*a^3*b^5*c^3*d^5 + 10*a^4*b^4*c^2*d^6 + 6*a^5*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 - 7*(1323*b^8*c^7*d - 2058*a*b^7*c^6*d^2 + 420*a^2*b^6*c^5*d^3 + 140*a^3*b^5*c^4*d^4 + 70*a^4*b^4*c^3*d^5 + 42*a^5*b^3*c^2*d^6 + 28*a^6*b^2*c*d^7 + 20*a^7*b*d^8)*x - 840*(b^8*c^8 - a*b^7*c^7*d + (b^8*c*d^7 - a*b^7*d^8)*x^7 + 7*(b^8*c^2*d^6 - a*b^7*c*d^7)*x^6 + 21*(b^8*c^3*d^5 - a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 - a*b^7*c^3*d^5)*x^4 + 35*(b^8*c^5*d^3 - a*b^7*c^4*d^4)*x^3 + 21*(b^8*c^6*d^2 - a*b^7*c^5*d^3)*x^2 + 7*(b^8*c^7*d - a*b^7*c^6*d^2)*x)*log(d*x + c))/(d^16*x^7 + 7*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*d^11*x^2 + 7*c^6*d^10*x + c^7*d^9)

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**8/(d*x+c)**8,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(201) = 402.
time = 0.00, size = 621, normalized size = 2.97

$$\frac{xb^8}{d^8} + \frac{\frac{1}{105}\left(\left(-2940b^8d^6c^2 + 5880b^7d^7ca - 2940b^6d^8a^2\right)x^6 + \left(-14700b^8d^5c^3 + 26460b^7d^6c^2a - 8820b^6d^7ca^2 - 2940b^5d^8a^3\right)x^5 + \left(-31850b^8d^4c^4 + 53900b^7d^5c^3a - 14700b^6d^6c^2a^2 - 4900b^5d^7ca^3 - 2450b^4d^8a^4\right)x^4 + \left(-37730b^8d^3c^5 + 61250b^7d^4c^4a - 14700b^6d^5c^3a^2 - 4900b^5d^6c^2a^3 - 2450b^4d^7ca^4 - 1470b^3d^8a^5\right)x^3 + \left(-25578b^8d^2c^6 + 40278b^7d^3c^5a - 8820b^6d^4c^4a^2 - 2940b^5d^5c^3a^3 - 1470b^4d^6c^2a^4 - 882b^3d^7ca^5 - 588b^2d^8a^6\right)x^2 + \left(-9366b^8dc^7 + 14406b^7d^2c^6a - 2940b^6d^3c^5a^2 - 980b^5d^4c^4a^3 - 490b^4d^5c^3a^4 - 294b^3d^6c^2a^5 - 196b^2d^7ca^6 - 140bd^8a^7\right)x - 1443b^8c^8 + 2178b^7dc^7a - 420b^6d^2c^6a^2 - 140b^5d^3c^5a^3 - 70b^4d^4c^4a^4 - 42b^3d^5c^3a^5 - 28b^2d^6c^2a^6 - 20bd^7ca^7 - 15d^8a^8\right)}{d^9\left(xd+c\right)^7} + \frac{\left(-8b^8c + 8b^7ad\right)\ln\left|xd+c\right|}{d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^8/(d*x+c)^8,x)

[Out] b^8*x/d^8 - 8*(b^8*c - a*b^7*d)*log(abs(d*x + c))/d^9 - 1/105*(1443*b^8*c^8 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 + 20*a^7*b*c*d^7 + 15*a^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 + a^2*b^6*d^8)*x^6 + 2940*(5*b^8*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 2450*(13*b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 294*(87*b^8*

```
c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a
^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2*d^8)*x^2 + 14*(669*b^8*c^7*d -
 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 + 70*a^3*b^5*c^4*d^4 + 35*a^4*b^4
*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x)/((d*x +
 c)^7*d^9)
```

Mupad [B]
time = 0.43, size = 649, normalized size = 3.11

$$\frac{b^8\,x}{d^8}-\frac{\ln\left(c+d\,x\right)\,\left(8\,b^8\,c-8\,a\,b^7\,d\right)}{d^9}-\frac{x^4\,\left(\frac{70\,a^4\,b^4\,d^7}{3}+\frac{140\,a^3\,b^5\,c\,d^6}{3}+140\,a^2\,b^6\,c^2\,d^5-\frac{1540\,a\,b^7\,c^3\,d^4}{3}+\frac{910\,b^8\,c^4\,d^3}{3}\right)+x^6\,\left(28\,a^2\,b^6\,d^7-56\,a\,b^7\,c\,d^6+28\,b^8\,c^2\,d^5\right)+\frac{15\,a^8\,d^8+20\,a^7\,b\,c\,d^7+28\,a^6\,b^2\,c^2\,d^6+42\,a^5\,b^3\,c^3\,d^5+70\,a^4\,b^4\,c^4\,d^4+140\,a^3\,b^5\,c^5\,d^3+420\,a^2\,b^6\,c^6\,d^2-2178\,a\,b^7\,c^7\,d+1443\,b^8\,c^8}{105\,d}+x\,\left(\frac{4\,a^7\,b\,d^7}{3}+\frac{28\,a^6\,b^2\,c\,d^6}{15}+\frac{14\,a^5\,b^3\,c^2\,d^5}{5}+\frac{14\,a^4\,b^4\,c^3\,d^4}{3}+\frac{28\,a^3\,b^5\,c^4\,d^3}{3}+28\,a^2\,b^6\,c^5\,d^2-\frac{686\,a\,b^7\,c^6\,d}{5}+\frac{446\,b^8\,c^7}{5}\right)+x^3\,\left(14\,a^5\,b^3\,d^7+\frac{70\,a^4\,b^4\,c\,d^6}{3}+\frac{140\,a^3\,b^5\,c^2\,d^5}{3}+140\,a^2\,b^6\,c^3\,d^4-\frac{1750\,a\,b^7\,c^4\,d^3}{3}+\frac{1078\,b^8\,c^5\,d^2}{3}\right)+x^2\,\left(\frac{28\,a^6\,b^2\,d^7}{5}+\frac{42\,a^5\,b^3\,c\,d^6}{5}+14\,a^4\,b^4\,c^2\,d^5+28\,a^3\,b^5\,c^3\,d^4+84\,a^2\,b^6\,c^4\,d^3-\frac{1918\,a\,b^7\,c^5\,d^2}{5}+\frac{1218\,b^8\,c^6\,d}{5}\right)+x^5\,\left(28\,a^3\,b^5\,d^7+84\,a^2\,b^6\,c\,d^6-252\,a\,b^7\,c^2\,d^5+140\,b^8\,c^3\,d^4\right)}{c^7\,d^8+7\,c^6\,d^9\,x+21\,c^5\,d^{10}\,x^2+35\,c^4\,d^{11}\,x^3+35\,c^3\,d^{12}\,x^4+21\,c^2\,d^{13}\,x^5+7\,c\,d^{14}\,x^6+d^{15}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/(c + d*x)^8,x)
```

```
[Out] (b^8*x)/d^8 - (log(c + d*x)*(8*b^8*c - 8*a*b^7*d))/d^9 - (x^4*((70*a^4*b^4*
d^7)/3 + (910*b^8*c^4*d^3)/3 - (1540*a*b^7*c^3*d^4)/3 + (140*a^3*b^5*c*d^6)
/3 + 140*a^2*b^6*c^2*d^5) + x^6*(28*a^2*b^6*d^7 + 28*b^8*c^2*d^5 - 56*a*b^7
*c*d^6) + (15*a^8*d^8 + 1443*b^8*c^8 + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^
5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 2178
*a*b^7*c^7*d + 20*a^7*b*c*d^7)/(105*d) + x*((446*b^8*c^7)/5 + (4*a^7*b*d^7)
/3 + (28*a^6*b^2*c*d^6)/15 + 28*a^2*b^6*c^5*d^2 + (28*a^3*b^5*c^4*d^3)/3 +
(14*a^4*b^4*c^3*d^4)/3 + (14*a^5*b^3*c^2*d^5)/5 - (686*a*b^7*c^6*d)/5) + x^
3*(14*a^5*b^3*d^7 + (1078*b^8*c^5*d^2)/3 - (1750*a*b^7*c^4*d^3)/3 + (70*a^4
*b^4*c*d^6)/3 + 140*a^2*b^6*c^3*d^4 + (140*a^3*b^5*c^2*d^5)/3) + x^2*((1218
*b^8*c^6*d)/5 + (28*a^6*b^2*d^7)/5 - (1918*a*b^7*c^5*d^2)/5 + (42*a^5*b^3*c
*d^6)/5 + 84*a^2*b^6*c^4*d^3 + 28*a^3*b^5*c^3*d^4 + 14*a^4*b^4*c^2*d^5) + x
^5*(28*a^3*b^5*d^7 + 140*b^8*c^3*d^4 - 252*a*b^7*c^2*d^5 + 84*a^2*b^6*c*d^6
))/(c^7*d^8 + d^15*x^7 + 7*c^6*d^9*x + 7*c*d^14*x^6 + 21*c^5*d^10*x^2 + 35*
c^4*d^11*x^3 + 35*c^3*d^12*x^4 + 21*c^2*d^13*x^5)
```

3.1364 $\int \frac{(a+bx)^7}{(c+dx)^8}\,dx$

Optimal. Leaf size=194

$$\frac{(bc-ad)^7}{7d^8(c+dx)^7}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{7b^6(bc-ad)}{d^8(c+dx)}$$

```
[Out] 1/7*(-a*d+b*c)^7/d^8/(d*x+c)^7-7/6*b*(-a*d+b*c)^6/d^8/(d*x+c)^6+21/5*b^2*(-
a*d+b*c)^5/d^8/(d*x+c)^5-35/4*b^3*(-a*d+b*c)^4/d^8/(d*x+c)^4+35/3*b^4*(-a*d
+b*c)^3/d^8/(d*x+c)^3-21/2*b^5*(-a*d+b*c)^2/d^8/(d*x+c)^2+7*b^6*(-a*d+b*c)/
d^8/(d*x+c)+b^7*ln(d*x+c)/d^8
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7b^6(bc-ad)}{d^8(c+dx)}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{(bc-ad)^7}{7d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^7/(7*d^8*(c + d*x)^7) - (7*b*(b*c - a*d)^6)/(6*d^8*(c + d*x)^6)
 + (21*b^2*(b*c - a*d)^5)/(5*d^8*(c + d*x)^5) - (35*b^3*(b*c - a*d)^4)/(4*d
^8*(c + d*x)^4) + (35*b^4*(b*c - a*d)^3)/(3*d^8*(c + d*x)^3) - (21*b^5*(b*c
 - a*d)^2)/(2*d^8*(c + d*x)^2) + (7*b^6*(b*c - a*d))/(d^8*(c + d*x)) + (b^7
*Log[c + d*x])/d^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^7}{d^7(c+dx)^8}+\frac{7b(bc-ad)^6}{d^7(c+dx)^7}-\frac{21b^2(bc-ad)^5}{d^7(c+dx)^6}+\frac{35b^3(bc-ad)^4}{d^7(c+dx)^5}-\frac{35b^4(bc-ad)^3}{d^7(c+dx)^4}+\right.\\ &= \frac{(bc-ad)^7}{7d^8(c+dx)^7}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\end{aligned}$$

Mathematica [A]
time = 0.10, size = 308, normalized size = 1.59

$$\frac{(bc-ad)\left(60a^6d^6+10a^5bd^5(13c+49dx)+2a^4b^2d^4\left(107c^2+539cdx+882d^2x^2\right)+a^3b^3d^3\left(319c^3+1813c^2dx+3969cd^2x^2+3675d^3x^3\right)+a^2b^4d^2\left(459c^4+2793c^3dx+6909c^2d^2x^2+8575cd^3x^3+4900d^4x^4\right)+ab^5d\left(669c^5+4263c^4dx+11319c^3d^2x^2+15925c^2d^3x^3+12250cd^4x^4+4410d^5x^5\right)+b^6\left(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6\right)\right)}{420d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] ((b*c - a*d)*(60*a^6*d^6 + 10*a^5*b*d^5*(13*c + 49*d*x) + 2*a^4*b^2*d^4*(10
7*c^2 + 539*c*d*x + 882*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 1813*c^2*d*x + 39
69*c*d^2*x^2 + 3675*d^3*x^3) + a^2*b^4*d^2*(459*c^4 + 2793*c^3*d*x + 6909*c
^2*d^2*x^2 + 8575*c*d^3*x^3 + 4900*d^4*x^4) + a*b^5*d*(669*c^5 + 4263*c^4*d
*x + 11319*c^3*d^2*x^2 + 15925*c^2*d^3*x^3 + 12250*c*d^4*x^4 + 4410*d^5*x^5
) + b^6*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 +
26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6)))/(420*d^8*(c + d*x)^7)
 + (b^7*Log[c + d*x])/d^8
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^7/(c + d*x)^8,x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(182) = 364$.
time = 0.14, size = 462, normalized size = 2.38

method	result
risch	$-\frac{7b^6(ad-bc)x^6}{d^2}-\frac{21b^5\left(a^2d^2+2abcd-3b^2c^2\right)x^5}{2d^3}-\frac{35b^4\left(2a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d-11b^3c^3\right)x^4}{6d^4}-\frac{35b^3\left(3a^4d^4+4a^3bc\,d^3+6a^2b^2c^2d^2+12a\,b^3c^3d-\right.}{12d^5}$
norman	$-\frac{60a^7d^7+70a^6bc\,d^6+84a^5b^2c^2d^5+105a^4b^3c^3d^4+140a^3b^4c^4d^3+210a^2b^5c^5d^2+420a\,b^6c^6d-1089b^7c^7}{420d^8}-\frac{7\left(a\,b^6d-b^7c\right)x^6}{d^2}-\frac{21\left(a^2b^5d^2+2a\,b^6cd-\right.}{2d^3}$
default	$-\frac{7b\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)}{6d^8(dx+c)^6}-\frac{7b^6(ad-bc)}{d^8(dx+c)}-\frac{35b^4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-\right.}{3d^8(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -7/6*b*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*
b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/d^8/(d*x+c)^6-7*b^6/d^8*(a*d-b*c)/(d*x+c
)-35/3*b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^8/(d*x+c)^3+b^7*
```

```
ln(d*x+c)/d^8-35/4*b^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3
*d+b^4*c^4)/d^8/(d*x+c)^4-21/5*b^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^
3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^8/(d*x+c)^5-21/2*b^5*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/d^8/(d*x+c)^2-1/7*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2
*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d
-b^7*c^7)/d^8/(d*x+c)^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(182) = 364$.
time = 0.28, size = 535, normalized size = 2.76

$$\frac{1089\,b^7c^7-420\,ab^6c^6d-210\,a^2b^5c^5d^2-140\,a^3b^4c^4d^3-105\,a^4b^3c^3d^4-84\,a^5b^2c^2d^5-70\,a^6bcd^6-60\,a^7d^7+2940\,(b^7cd^6-ab^6d^7)x^6+4410\,(3\,b^7c^2d^5-2\,ab^6cd^6-a^2b^5d^7)x^5+2450\,(11\,b^7c^3d^4-6\,ab^6c^2d^5-3\,a^2b^5cd^6-2\,a^3b^4d^7)x^4+1225\,(25\,b^7c^4d^3-12\,ab^6c^3d^4-6\,a^2b^5c^2d^5-4\,a^3b^4cd^6-3\,a^4b^3d^7)x^3+147\,(137\,b^7c^5d^2-60\,ab^6c^4d^3-30\,a^2b^5c^3d^4-20\,a^3b^4c^2d^5-15\,a^4b^3cd^6-12\,a^5b^2d^7)x^2+49\,(147\,b^7c^6d-60\,ab^6c^5d^2-30\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4-15\,a^4b^3c^2d^5-12\,a^5b^2cd^6-10\,a^6bd^7)x}{420\,(d^{15}x^7+7\,cd^{14}x^6+21\,c^2d^{13}x^5+35\,c^3d^{12}x^4+35\,c^4d^{11}x^3+21\,c^5d^{10}x^2+7\,c^6d^9x+c^7d^8)}+\frac{b^7\log(dx+c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x)/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 35*c^3*d^12*
x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8) + b^7*log(
d*x + c)/d^8
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(182) = 364$.
time = 0.31, size = 625, normalized size = 3.22

$$\frac{1089\,b^7c^7-420\,ab^6c^6d-210\,a^2b^5c^5d^2-140\,a^3b^4c^4d^3-105\,a^4b^3c^3d^4-84\,a^5b^2c^2d^5-70\,a^6bcd^6-60\,a^7d^7+2940\,(b^7cd^6-ab^6d^7)x^6+4410\,(3\,b^7c^2d^5-2\,ab^6cd^6-a^2b^5d^7)x^5+2450\,(11\,b^7c^3d^4-6\,ab^6c^2d^5-3\,a^2b^5cd^6-2\,a^3b^4d^7)x^4+1225\,(25\,b^7c^4d^3-12\,ab^6c^3d^4-6\,a^2b^5c^2d^5-4\,a^3b^4cd^6-3\,a^4b^3d^7)x^3+147\,(137\,b^7c^5d^2-60\,ab^6c^4d^3-30\,a^2b^5c^3d^4-20\,a^3b^4c^2d^5-15\,a^4b^3cd^6-12\,a^5b^2d^7)x^2+49\,(147\,b^7c^6d-60\,ab^6c^5d^2-30\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4-15\,a^4b^3c^2d^5-12\,a^5b^2cd^6-10\,a^6bd^7)x+420\,(b^7d^7x^7+7\,b^7cd^6x^6+21\,b^7c^2d^5x^5+35\,b^7c^3d^4x^4+35\,b^7c^4d^3x^3+21\,b^7c^5d^2x^2+7\,b^7c^6dx+b^7c^7)\log(dx+c)}{420\,(d^{15}x^7+7\,cd^{14}x^6+21\,c^2d^{13}x^5+35\,c^3d^{12}x^4+35\,c^4d^{11}x^3+21\,c^5d^{10}x^2+7\,c^6d^9x+c^7d^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
```

```
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 21*b^7*c^2*d^5*x^5
+ 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^2*x^2 + 7*b^7*c^6*
d*x + b^7*c^7)*log(d*x + c))/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 3
5*c^3*d^12*x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(182) = 364.
time = 0.00, size = 494, normalized size = 2.55

$$\frac{\frac{1}{420}\left((2940b^7d^5c-2940b^6d^6a)x^6+(13230b^7d^4c^2-8820b^6d^5ca-4410b^5d^6a^2)x^5+(26950b^7d^3c^3-14700b^6d^4c^2a-7350b^5d^5ca^2-4900b^4d^6a^3)x^4+(30625b^7d^2c^4-14700b^6d^3c^3a-7350b^5d^4c^2a^2-4900b^4d^5ca^3-3675b^3d^6a^4)x^3+(20139b^7dc^5-8820b^6d^2c^4a-4410b^5d^3c^3a^2-2940b^4d^4c^2a^3-2205b^3d^5ca^4-1764b^2d^6a^5)x^2+(7203b^7c^6-2940b^6dc^5a-1470b^5d^2c^4a^2-980b^4d^3c^3a^3-735b^3d^4c^2a^4-588b^2d^5ca^5-490bd^6a^6)x+\frac{1089b^7c^7-420b^6dc^6a-210b^5d^2c^5a^2-140b^4d^3c^4a^3-105b^3d^4c^3a^4-84b^2d^5c^2a^5-70bd^6ca^6-60d^7a^7}{d}\right)}{d^7(xd+c)^7}+\frac{b^7\ln|xd+c|}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x)
```

```
[Out] b^7*log(abs(d*x + c))/d^8 + 1/420*(2940*(b^7*c*d^5 - a*b^6*d^6)*x^6 + 4410*
(3*b^7*c^2*d^4 - 2*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 2450*(11*b^7*c^3*d^3 -
6*a*b^6*c^2*d^4 - 3*a^2*b^5*c*d^5 - 2*a^3*b^4*d^6)*x^4 + 1225*(25*b^7*c^4*d
^2 - 12*a*b^6*c^3*d^3 - 6*a^2*b^5*c^2*d^4 - 4*a^3*b^4*c*d^5 - 3*a^4*b^3*d^6
)*x^3 + 147*(137*b^7*c^5*d - 60*a*b^6*c^4*d^2 - 30*a^2*b^5*c^3*d^3 - 20*a^3
*b^4*c^2*d^4 - 15*a^4*b^3*c*d^5 - 12*a^5*b^2*d^6)*x^2 + 49*(147*b^7*c^6 - 6
0*a*b^6*c^5*d - 30*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 - 15*a^4*b^3*c^2*d^
4 - 12*a^5*b^2*c*d^5 - 10*a^6*b*d^6)*x + (1089*b^7*c^7 - 420*a*b^6*c^6*d -
210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^
2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7*d^7)/d)/((d*x + c)^7*d^7)
```

Mupad [**B**]
time = 0.38, size = 460, normalized size = 2.37

$$\frac{b^7\ln(c+dx)}{d^8}-\frac{x\left(\frac{7a^6bd^7}{6}+\frac{7a^5b^2cd^6}{5}+\frac{7a^4b^3c^2d^5}{4}+\frac{7a^3b^4c^3d^4}{3}+\frac{7a^2b^5c^4d^3}{2}+7ab^6c^5d^2-\frac{343b^7c^6d}{20}\right)+x^6(7ab^6d^7-7b^7cd^6)+x^3\left(\frac{35a^4b^3d^7}{4}+\frac{35a^3b^4cd^6}{3}+\frac{35a^2b^5c^2d^5}{2}+35ab^6c^3d^4-\frac{875b^7c^4d^3}{12}\right)+x^5\left(\frac{21a^2b^5d^7}{2}+21ab^6cd^6-\frac{63b^7c^2d^5}{2}\right)+x^2\left(\frac{21a^5b^2d^7}{5}+\frac{21a^4b^3cd^6}{4}+7a^3b^4c^2d^5+\frac{21a^2b^5c^3d^4}{2}+21ab^6c^4d^3-\frac{959b^7c^5d^2}{20}\right)+\frac{a^7d^7}{7}-\frac{363b^7c^7}{140}+x^4\left(\frac{35a^3b^4d^7}{3}+\frac{35a^2b^5cd^6}{2}+35ab^6c^2d^5-\frac{385b^7c^3d^4}{6}\right)+\frac{a^2b^5c^5d^2}{2}+\frac{a^3b^4c^4d^3}{3}+\frac{a^4b^3c^3d^4}{4}+\frac{a^5b^2c^2d^5}{5}+ab^6c^6d+\frac{a^6bcd^6}{6}}{d^8(c+dx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/(c + d*x)^8,x)
```

```
[Out] (b^7*log(c + d*x))/d^8 - (x*((7*a^6*b*d^7)/6 - (343*b^7*c^6*d)/20 + 7*a*b^6
*c^5*d^2 + (7*a^5*b^2*c*d^6)/5 + (7*a^2*b^5*c^4*d^3)/2 + (7*a^3*b^4*c^3*d^4
```

)/3 + (7*a^4*b^3*c^2*d^5)/4) + x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*a
^4*b^3*d^7)/4 - (875*b^7*c^4*d^3)/12 + 35*a*b^6*c^3*d^4 + (35*a^3*b^4*c*d^6
)/3 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*a^2*b^5*d^7)/2 - (63*b^7*c^2*d^5)/
2 + 21*a*b^6*c*d^6) + x^2*((21*a^5*b^2*d^7)/5 - (959*b^7*c^5*d^2)/20 + 21*a
*b^6*c^4*d^3 + (21*a^4*b^3*c*d^6)/4 + (21*a^2*b^5*c^3*d^4)/2 + 7*a^3*b^4*c^
2*d^5) + (a^7*d^7)/7 - (363*b^7*c^7)/140 + x^4*((35*a^3*b^4*d^7)/3 - (385*b
^7*c^3*d^4)/6 + 35*a*b^6*c^2*d^5 + (35*a^2*b^5*c*d^6)/2) + (a^2*b^5*c^5*d^2
)/2 + (a^3*b^4*c^4*d^3)/3 + (a^4*b^3*c^3*d^4)/4 + (a^5*b^2*c^2*d^5)/5 + a*b
^6*c^6*d + (a^6*b*c*d^6)/6)/(d^8*(c + d*x)^7)

3.1365 $\int \frac{(a+bx)^6}{(c+dx)^8} \, dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

```
[Out] 1/7*(b*x+a)^7/(-a*d+b*c)/(d*x+c)^7
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^7/(7*(b*c - a*d)*(c + d*x)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^8} \, dx = \frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 271 vs. 2(28) = 56.
time = 0.06, size = 271, normalized size = 9.68

$$-\frac{a^6d^6+a^5bd^5(c+7dx)+a^4b^2d^4\left(c^2+7cdx+21d^2x^2\right)+a^3b^3d^3\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+a^2b^4d^2\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)+ab^5d\left(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5\right)+b^6\left(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6\right)}{7d^7(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] -1/7*(a^6*d^6 + a^5*b*d^5*(c + 7*d*x) + a^4*b^2*d^4*(c^2 + 7*c*d*x + 21*d^2
*x^2) + a^3*b^3*d^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + a^2*b^4
*d^2*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4) + a*b^5
*d*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 21*d
^5*x^5) + b^6*(c^6 + 7*c^5*d*x + 21*c^4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d
^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6))/(d^7*(c + d*x)^7)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 384 vs. $2(28) = 56$.
time = 193.32, size = 375, normalized size = 13.39

$$-\frac{a^6d^6+a^5bcd^5+a^4b^2c^2d^4+a^3b^3c^3d^3+a^2b^4c^4d^2+ab^5c^5d+b^6c^6+7bdx\left(a^5d^5+a^4bcd^4+a^3b^2c^2d^3+a^2b^3c^3d^2+ab^4c^4d+b^5c^5\right)+21b^2d^2x^2\left(a^4d^4+a^3bcd^3+a^2b^2c^2d^2+ab^3c^3d+b^4c^4\right)+35b^3d^3x^3\left(a^3d^3+a^2bcd^2+ab^2c^2d+b^3c^3\right)+35b^4d^4x^4\left(a^2d^2+abcd+b^2c^2\right)+21b^5d^5x^5\left(ad+bc\right)+7b^6d^6x^6}{7d^7\left(c^7+7c^6dx+21c^5d^2x^2+35c^4d^3x^3+35c^3d^4x^4+21c^2d^5x^5+7cd^6x^6+d^7x^7\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^6/(c + d*x)^8,x]')
```

```
[Out] -(a ^ 6 d ^ 6 + a ^ 5 b c d ^ 5 + a ^ 4 b ^ 2 c ^ 2 d ^ 4 + a ^ 3 b ^ 3 c ^
 3 d ^ 3 + a ^ 2 b ^ 4 c ^ 4 d ^ 2 + a b ^ 5 c ^ 5 d + b ^ 6 c ^ 6 + 7 b d
x (a ^ 5 d ^ 5 + a ^ 4 b c d ^ 4 + a ^ 3 b ^ 2 c ^ 2 d ^ 3 + a ^ 2 b ^ 3 c
^ 3 d ^ 2 + a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5) + 21 b ^ 2 d ^ 2 x ^ 2 (a ^ 4 d
^ 4 + a ^ 3 b c d ^ 3 + a ^ 2 b ^ 2 c ^ 2 d ^ 2 + a b ^ 3 c ^ 3 d + b ^ 4 c
 ^ 4) + 35 b ^ 3 d ^ 3 x ^ 3 (a ^ 3 d ^ 3 + a ^ 2 b c d ^ 2 + a b ^ 2 c ^ 2
 d + b ^ 3 c ^ 3) + 35 b ^ 4 d ^ 4 x ^ 4 (a ^ 2 d ^ 2 + a b c d + b ^ 2 c ^
 2) + 21 b ^ 5 d ^ 5 x ^ 5 (a d + b c) + 7 b ^ 6 d ^ 6 x ^ 6) / (7 d ^ 7 (c
 ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2 x ^ 2 + 35 c ^ 4 d ^ 3 x ^ 3 + 35 c ^ 3
 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5 + 7 c d ^ 6 x ^ 6 + d ^ 7 x ^ 7))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(26) = 52$.
time = 0.14, size = 357, normalized size = 12.75

method	result
risch	$\frac{-\frac{b^6x^6}{d}-\frac{3b^5(ad+bc)x^5}{d^2}-\frac{5b^4\left(a^2d^2+abcd+b^2c^2\right)x^4}{d^3}-\frac{5b^3\left(a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{d^4}-\frac{3b^2\left(a^4d^4+a^3bc\,d^3+a^2b^2c^2d^2+a\,b^3c^3d+b^4c^4\right)x^2}{d^5}-}{(dx+c)^7}$
norman	$\frac{-\frac{b^6x^6}{d}-\frac{3\left(a\,b^5d+b^6c\right)x^5}{d^2}-\frac{5\left(a^2b^4d^2+a\,b^5cd+c^2b^6\right)x^4}{d^3}-\frac{5\left(a^3b^3d^3+a^2c\,b^4d^2+a\,c^2b^5d+c^3b^6\right)x^3}{d^4}-\frac{3\left(a^4b^2d^4+a^3b^3c\,d^3+a^2b^4c^2d^2+a\,b^5c^3d+b^6c^4\right.}{d^5}}{(dx+c)^7}$
default	$-\frac{b\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{d^7(dx+c)^6}-\frac{b^6}{d^7(dx+c)}-\frac{5b^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^7(dx+c)^4}-\frac{3b^2\left(a^4d^4\right.}{}$
gosper	$-\frac{7x^6b^6d^6+21a\,b^5d^6x^5+21b^6c\,d^5x^5+35a^2b^4d^6x^4+35a\,b^5c\,d^5x^4+35b^6c^2d^4x^4+35a^3b^3d^6x^3+35a^2b^4c\,d^5x^3+35a\,b^5c^2d^4x^3+35b^6c^3d^3x^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -b*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4
*d-b^5*c^5)/d^7/(d*x+c)^6-b^6/d^7/(d*x+c)-5*b^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*
```

```
b^2*c^2*d-b^3*c^3)/d^7/(d*x+c)^4-3*b^2/d^7*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^5-5*b^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/d^7/(d*x+c)^3-3*b^5*(a*d-b*c)/d^7/(d*x+c)^2-1/7*(a^6*d^6-6*a^5*b*c*d^5+15*
a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6
)/d^7/(d*x+c)^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(26) = 52$.
time = 0.27, size = 398, normalized size = 14.21

$$-\frac{7\,b^{6}d^{6}x^{6}+b^{6}c^{6}+ab^{5}c^{5}d+a^{2}b^{4}c^{4}d^{2}+a^{3}b^{3}c^{3}d^{3}+a^{4}b^{2}c^{2}d^{4}+a^{5}bcd^{5}+a^{6}d^{6}+21\,(b^{6}cd^{5}+ab^{5}d^{6})x^{5}+35\,(b^{6}c^{2}d^{4}+ab^{5}cd^{5}+a^{2}b^{4}d^{6})x^{4}+35\,(b^{6}c^{3}d^{3}+ab^{5}c^{2}d^{4}+a^{2}b^{4}cd^{5}+a^{3}b^{3}d^{6})x^{3}+21\,(b^{6}c^{4}d^{2}+ab^{5}c^{3}d^{3}+a^{2}b^{4}c^{2}d^{4}+a^{3}b^{3}cd^{5}+a^{4}b^{2}d^{6})x^{2}+7\,(b^{6}c^{5}d+ab^{5}c^{4}d^{2}+a^{2}b^{4}c^{3}d^{3}+a^{3}b^{3}c^{2}d^{4}+a^{4}b^{2}cd^{5}+a^{5}bd^{6})x}{7\,(d^{14}x^{7}+7\,cd^{13}x^{6}+21\,c^{2}d^{12}x^{5}+35\,c^{3}d^{11}x^{4}+35\,c^{4}d^{10}x^{3}+21\,c^{5}d^{9}x^{2}+7\,c^{6}d^{8}x+c^{7}d^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(26) = 52$.
time = 0.29, size = 398, normalized size = 14.21

$$-\frac{7\,b^{6}d^{6}x^{6}+b^{6}c^{6}+ab^{5}c^{5}d+a^{2}b^{4}c^{4}d^{2}+a^{3}b^{3}c^{3}d^{3}+a^{4}b^{2}c^{2}d^{4}+a^{5}bcd^{5}+a^{6}d^{6}+21\,(b^{6}cd^{5}+ab^{5}d^{6})x^{5}+35\,(b^{6}c^{2}d^{4}+ab^{5}cd^{5}+a^{2}b^{4}d^{6})x^{4}+35\,(b^{6}c^{3}d^{3}+ab^{5}c^{2}d^{4}+a^{2}b^{4}cd^{5}+a^{3}b^{3}d^{6})x^{3}+21\,(b^{6}c^{4}d^{2}+ab^{5}c^{3}d^{3}+a^{2}b^{4}c^{2}d^{4}+a^{3}b^{3}cd^{5}+a^{4}b^{2}d^{6})x^{2}+7\,(b^{6}c^{5}d+ab^{5}c^{4}d^{2}+a^{2}b^{4}c^{3}d^{3}+a^{3}b^{3}c^{2}d^{4}+a^{4}b^{2}cd^{5}+a^{5}bd^{6})x}{7\,(d^{14}x^{7}+7\,cd^{13}x^{6}+21\,c^{2}d^{12}x^{5}+35\,c^{3}d^{11}x^{4}+35\,c^{4}d^{10}x^{3}+21\,c^{5}d^{9}x^{2}+7\,c^{6}d^{8}x+c^{7}d^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(26) = 52.
time = 0.00, size = 399, normalized size = 14.25

$$\frac{-7x^6b^6d^6 - 21x^5b^6d^5c - 21x^5b^5ad^6 - 35x^4b^6d^4c^2 - 35x^4b^5ad^5c - 35x^4b^4a^2d^6 - 35x^3b^6d^3c^3 - 35x^3b^5ad^4c^2 - 35x^3b^4a^2d^5c - 35x^3b^3a^3d^6 - 21x^2b^6d^2c^4 - 21x^2b^5ad^3c^3 - 21x^2b^4a^2d^4c^2 - 21x^2b^3a^3d^5c - 21x^2b^2a^4d^6 - 7xb^6dc^5 - 7xb^5ad^2c^4 - 7xb^4a^2d^3c^3 - 7xb^3a^3d^4c^2 - 7xb^2a^4d^5c - 7xba^5d^6 - b^6c^6 - b^5adc^5 - b^4a^2d^2c^4 - b^3a^3d^3c^3 - b^2a^4d^4c^2 - ba^5d^5c - a^6d^6}{7d^7\,(xd + c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x)
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + 21*b^6*c*d^5*x^5 + 21*a*b^5*d^6*x^5 + 35*b^6*c^2*d^4*
x^4 + 35*a*b^5*c*d^5*x^4 + 35*a^2*b^4*d^6*x^4 + 35*b^6*c^3*d^3*x^3 + 35*a*b
^5*c^2*d^4*x^3 + 35*a^2*b^4*c*d^5*x^3 + 35*a^3*b^3*d^6*x^3 + 21*b^6*c^4*d^2
*x^2 + 21*a*b^5*c^3*d^3*x^2 + 21*a^2*b^4*c^2*d^4*x^2 + 21*a^3*b^3*c*d^5*x^2
 + 21*a^4*b^2*d^6*x^2 + 7*b^6*c^5*d*x + 7*a*b^5*c^4*d^2*x + 7*a^2*b^4*c^3*d
^3*x + 7*a^3*b^3*c^2*d^4*x + 7*a^4*b^2*c*d^5*x + 7*a^5*b*d^6*x + b^6*c^6 +
a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c
*d^5 + a^6*d^6)/((d*x + c)^7*d^7)
```

Mupad [B]
time = 0.15, size = 378, normalized size = 13.50

$$-\frac{\frac{a^6\,d^6+a^5\,b\,c\,d^5+a^4\,b^2\,c^2\,d^4+a^3\,b^3\,c^3\,d^3+a^2\,b^4\,c^4\,d^2+a\,b^5\,c^5\,d+b^6\,c^6}{7\,d^7}+\frac{b^6\,x^6}{d}+\frac{5\,b^3\,x^3\,\left(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}{d^4}+\frac{b\,x\,\left(a^5\,d^5+a^4\,b\,c\,d^4+a^3\,b^2\,c^2\,d^3+a^2\,b^3\,c^3\,d^2+a\,b^4\,c^4\,d+b^5\,c^5\right)}{d^6}+\frac{3\,b^5\,x^5\,\left(a\,d+b\,c\right)}{d^2}+\frac{3\,b^2\,x^2\,\left(a^4\,d^4+a^3\,b\,c\,d^3+a^2\,b^2\,c^2\,d^2+a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{5\,b^4\,x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^8,x)
```

```
[Out] -((a^6*d^6 + b^6*c^6 + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4
+ a*b^5*c^5*d + a^5*b*c*d^5)/(7*d^7) + (b^6*x^6)/d + (5*b^3*x^3*(a^3*d^3 +
b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2))/d^4 + (b*x*(a^5*d^5 + b^5*c^5 + a^2*b
^3*c^3*d^2 + a^3*b^2*c^2*d^3 + a*b^4*c^4*d + a^4*b*c*d^4))/d^6 + (3*b^5*x^5
*(a*d + b*c))/d^2 + (3*b^2*x^2*(a^4*d^4 + b^4*c^4 + a^2*b^2*c^2*d^2 + a*b^3
*c^3*d + a^3*b*c*d^3))/d^5 + (5*b^4*x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/d^3)
/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^
4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1366 $\int \frac{(a+bx)^5}{(c+dx)^8} \, dx$

Optimal. Leaf size=58

$$\frac{(a+bx)^6}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}$$

```
[Out] 1/7*(b*x+a)^6/(-a*d+b*c)/(d*x+c)^7+1/42*b*(b*x+a)^6/(-a*d+b*c)^2/(d*x+c)^6
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^6/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^6)/(42*(b*c - a*d)^2
*(c + d*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^8}\,dx = \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7} + \frac{b\int \frac{(a+bx)^5}{(c+dx)^7}\,dx}{7(bc-ad)}$$
$$= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 205 vs. $2(58) = 116$.
time = 0.04, size = 205, normalized size = 3.53

$$-\frac{6a^5d^5+5a^4bd^4(c+7dx)+4a^3b^2d^3(c^2+7cdx+21d^2x^2)+3a^2b^3d^2(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+2ab^4d(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+b^5(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)}{42d^6(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a^5*d^5 + 5*a^4*b*d^4*(c + 7*d*x) + 4*a^3*b^2*d^3*(c^2 + 7*c*d*x +
 21*d^2*x^2) + 3*a^2*b^3*d^2*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3)
+ 2*a*b^4*d*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4)
+ b^5*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 2
1*d^5*x^5))/(d^6*(c + d*x)^7)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 310 vs. $2(58) = 116$.
time = 39.22, size = 307, normalized size = 5.29

$$-\frac{6a^5d^5+5a^4bcd^4+4a^3b^2c^2d^3+3a^2b^3c^3d^2+2ab^4c^4d+b^5c^5+7bdx(5a^4d^4+4a^3bcd^3+3a^2b^2c^2d^2+2ab^3c^3d+b^4c^4)+21b^2d^2x^2(4a^3d^3+3a^2bcd^2+2ab^2c^2d+b^3c^3)+35b^3d^3x^3(3a^2d^2+2abcd+b^2c^2)+35b^4d^4x^4(2ad+bc)+21b^5d^5x^5}{42d^6(c^7+7c^6dx+21c^5d^2x^2+35c^4d^3x^3+35c^3d^4x^4+21c^2d^5x^5+7cd^6x^6+d^7x^7)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x)^8,x]')
```

```
[Out] -(6 a ^ 5 d ^ 5 + 5 a ^ 4 b c d ^ 4 + 4 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 3 a ^ 2 b
 ^ 3 c ^ 3 d ^ 2 + 2 a b ^ 4 c ^ 4 d + b ^ 5 c ^ 5 + 7 b d x (5 a ^ 4 d ^ 4
 + 4 a ^ 3 b c d ^ 3 + 3 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 2 a b ^ 3 c ^ 3 d + b ^
4 c ^ 4) + 21 b ^ 2 d ^ 2 x ^ 2 (4 a ^ 3 d ^ 3 + 3 a ^ 2 b c d ^ 2 + 2 a b
^ 2 c ^ 2 d + b ^ 3 c ^ 3) + 35 b ^ 3 d ^ 3 x ^ 3 (3 a ^ 2 d ^ 2 + 2 a b c
d + b ^ 2 c ^ 2) + 35 b ^ 4 d ^ 4 x ^ 4 (2 a d + b c) + 21 b ^ 5 d ^ 5 x ^
5) / (42 d ^ 6 (c ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2 x ^ 2 + 35 c ^ 4 d ^ 3
 x ^ 3 + 35 c ^ 3 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5 + 7 c d ^ 6 x ^ 6 + d
^ 7 x ^ 7))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(54) = 108$.
time = 0.16, size = 265, normalized size = 4.57

method	result
risch	$\frac{-\frac{b^5x^5}{2d}-\frac{5b^4(2ad+bc)x^4}{6d^2}-\frac{5b^3\left(3a^2d^2+2abcd+b^2c^2\right)x^3}{6d^3}-\frac{b^2\left(4a^3d^3+3a^2bc\,d^2+2a\,b^2c^2d+b^3c^3\right)x^2}{2d^4}-\frac{b\left(5a^4d^4+4a^3bc\,d^3+3a^2b^2c^2d^2+2a\,b^3c^3d+b\cdots\right)}{6d^5}\cdots}{(dx+c)^7}$
default	$-\frac{5b^4(ad-bc)}{3d^6(dx+c)^3}-\frac{5b\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{6d^6(dx+c)^6}-\frac{2b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^6(dx+c)^5}-\frac{5b^3\left(a^2d^2-2abcd+\cdots\right)}{2d^6(dx+c)^4}\cdots$
norman	$\frac{-\frac{b^5x^5}{2d}-\frac{5\left(2a\,b^4d^2+b^5cd\right)x^4}{6d^3}-\frac{5\left(3a^2b^3d^3+2a\,b^4c\,d^2+b^5c^2d\right)x^3}{6d^4}-\frac{\left(4a^3b^2d^4+3a^2b^3c\,d^3+2a\,b^4c^2d^2+b^5c^3d\right)x^2}{2d^5}-\frac{\left(5a^4b\,d^5+4a^3b^2c\,d^4+3a^2b^3c^2\cdots\right)}{6d^6}\cdots}{(dx+c)^7}$
gosper	$-\frac{21b^5x^5d^5+70a\,b^4d^5x^4+35b^5c\,d^4x^4+105a^2b^3d^5x^3+70a\,b^4c\,d^4x^3+35b^5c^2d^3x^3+84a^3b^2d^5x^2+63a^2b^3c\,d^4x^2+42a\,b^4c^2d^3x^2+21b^5c^3\cdots}{42d^6(dx\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3*b^4*(a*d-b*c)/d^6/(d*x+c)^3-5/6*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^6-2*b^2/d^6*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^5-5/2*b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^6/(d
*x+c)^4-1/2*b^5/d^6/(d*x+c)^2-1/7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3
-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)^7
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(54) = 108$.
time = 0.29, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,(b^5cd^4+2\,ab^4d^5)x^4+35\,(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5)x^3+21\,(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5)x^2+7\,(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5)x}{42\,(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(54) = 108$.
time = 0.29, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,(b^5cd^4+2\,ab^4d^5)x^4+35\,(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5)x^3+21\,(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5)x^2+7\,(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5)x}{42\,(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(54) = 108.
time = 0.00, size = 294, normalized size = 5.07

$$\frac{-21x^5b^5d^5 - 35x^4b^5d^4c - 70x^4b^4ad^5 - 35x^3b^5d^3c^2 - 70x^3b^4ad^4c - 105x^3b^3a^2d^5 - 21x^2b^5d^2c^3 - 42x^2b^4ad^3c^2 - 63x^2b^3a^2d^4c - 84x^2b^2a^3d^5 - 7xb^5dc^4 - 14xb^4ad^2c^3 - 21xb^3a^2d^3c^2 - 28xb^2a^3d^4c - 35xba^4d^5 - b^5c^5 - 2b^4adc^4 - 3b^3a^2d^2c^3 - 4b^2a^3d^3c^2 - 5ba^4d^4c - 6a^5d^5}{42d^6(xd+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x)
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + 35*b^5*c*d^4*x^4 + 70*a*b^4*d^5*x^4 + 35*b^5*c^2*d^
3*x^3 + 70*a*b^4*c*d^4*x^3 + 105*a^2*b^3*d^5*x^3 + 21*b^5*c^3*d^2*x^2 + 42*
a*b^4*c^2*d^3*x^2 + 63*a^2*b^3*c*d^4*x^2 + 84*a^3*b^2*d^5*x^2 + 7*b^5*c^4*d
*x + 14*a*b^4*c^3*d^2*x + 21*a^2*b^3*c^2*d^3*x + 28*a^3*b^2*c*d^4*x + 35*a^
4*b*d^5*x + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3*b^2*c^2*d^3
 + 5*a^4*b*c*d^4 + 6*a^5*d^5)/((d*x + c)^7*d^6)
```

Mupad [B]
time = 0.28, size = 39, normalized size = 0.67

$$\frac{(a+b\,x)^6\,\left(7\,b\,c-6\,a\,d+b\,d\,x\right)}{42\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^8,x)
```

```
[Out] ((a + b*x)^6*(7*b*c - 6*a*d + b*d*x))/(42*(a*d - b*c)^2*(c + d*x)^7)
```

3.1367 $\int \frac{(a+bx)^4}{(c+dx)^8} dx$

Optimal. Leaf size=89

$$\frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}$$

```
[Out] 1/7*(b*x+a)^5/(-a*d+b*c)/(d*x+c)^7+1/21*b*(b*x+a)^5/(-a*d+b*c)^2/(d*x+c)^6+
1/105*b^2*(b*x+a)^5/(-a*d+b*c)^3/(d*x+c)^5
```

Rubi [A]
time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^5/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^5)/(21*(b*c - a*d)^2
*(c + d*x)^6) + (b^2*(a + b*x)^5)/(105*(b*c - a*d)^3*(c + d*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^8}\,dx &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7} + \frac{(2b)\int \frac{(a+bx)^4}{(c+dx)^7}\,dx}{7(bc-ad)} \\ &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6} + \frac{b^2\int \frac{(a+bx)^4}{(c+dx)^6}\,dx}{21(bc-ad)^2} \\ &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6} + \frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 144, normalized size = 1.62

$$-\frac{15a^4d^4+10a^3bd^3(c+7dx)+6a^2b^2d^2\left(c^2+7cdx+21d^2x^2\right)+3ab^3d\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+b^4\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)}{105d^5(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 6*a^2*b^2*d^2*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 3*a*b^3*d*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) +
 b^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4))/(d^5*(
c + d*x)^7)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(89) = 178.
time = 9.40, size = 232, normalized size = 2.61

$$-\frac{15a^4d^4+10a^3bcd^3+6a^2b^2c^2d^2+3ab^3c^3d+b^4c^4+7bdx\left(10a^3d^3+6a^2bcd^2+3ab^2c^2d+b^3c^3\right)+21b^2d^2x^2\left(6a^2d^2+3abcd+b^2c^2\right)+35b^3d^3x^3\left(3ad+bc\right)+35b^4d^4x^4}{105d^5\left(c^7+7c^6dx+21c^5d^2x^2+35c^4d^3x^3+35c^3d^4x^4+21c^2d^5x^5+7cd^6x^6+d^7x^7\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x)^8,x]')
```

```
[Out] -(15 a ^ 4 d ^ 4 + 10 a ^ 3 b c d ^ 3 + 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 + 3 a b ^
 3 c ^ 3 d + b ^ 4 c ^ 4 + 7 b d x (10 a ^ 3 d ^ 3 + 6 a ^ 2 b c d ^ 2 + 3
a b ^ 2 c ^ 2 d + b ^ 3 c ^ 3) + 21 b ^ 2 d ^ 2 x ^ 2 (6 a ^ 2 d ^ 2 + 3 a
b c d + b ^ 2 c ^ 2) + 35 b ^ 3 d ^ 3 x ^ 3 (3 a d + b c) + 35 b ^ 4 d ^ 4
x ^ 4) / (105 d ^ 5 (c ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2 x ^ 2 + 35 c ^ 4
d ^ 3 x ^ 3 + 35 c ^ 3 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5 + 7 c d ^ 6 x ^ 6
 + d ^ 7 x ^ 7))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(83) = 166.
time = 0.14, size = 186, normalized size = 2.09

method	result
risch	$\frac{-\frac{b^4x^4}{3d}-\frac{b^3(3ad+bc)x^3}{3d^2}-\frac{b^2\left(6a^2d^2+3abcd+b^2c^2\right)x^2}{5d^3}-\frac{b\left(10a^3d^3+6a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)x}{15d^4}-\frac{15a^4d^4+10a^3bc\,d^3+6a^2b^2c^2d^2+3a\,b^3c^3d+b^4c^4}{105d^5}}{(dx+c)^7}$
gosper	$-\frac{35d^4x^4b^4+105a\,b^3d^4x^3+35b^4c\,d^3x^3+126a^2b^2d^4x^2+63a\,b^3c\,d^3x^2+21b^4c^2d^2x^2+70a^3b\,d^4x+42a^2b^2c\,d^3x+21a\,b^3c^2d^2x+7b^4c^3dx+}{105d^5(dx+c)^7}$
default	$-\frac{b^4}{3d^5(dx+c)^3}-\frac{2b(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{3d^5(dx+c)^6}-\frac{b^3(ad-bc)}{d^5(dx+c)^4}-\frac{6b^2\left(a^2d^2-2abcd+b^2c^2\right)}{5d^5(dx+c)^5}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-}{7d^5(dx+c)^7}$
norman	$\frac{-\frac{b^4x^4}{3d}-\frac{\left(3a\,b^3d^3+b^4c\,d^2\right)x^3}{3d^4}-\frac{\left(6b^2a^2d^4+3a\,b^3c\,d^3+b^4c^2d^2\right)x^2}{5d^5}-\frac{\left(10a^3b\,d^5+6a^2b^2c\,d^4+3a\,b^3c^2d^3+b^4c^3d^2\right)x}{15d^6}-\frac{15a^4d^6+10a^3bc\,d^5+6a^2b^2c^2d}{105d^7}}{(dx+c)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^4/d^5/(d*x+c)^3-2/3*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/(d*x+c)^6-b^3*(a*d-b*c)/d^5/(d*x+c)^4-6/5*b^2/d^5*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/(d*x+c)^5-1/7*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d
+b^4*c^4)/d^5/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.
time = 0.27, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\,(b^4cd^3+3\,ab^3d^4)x^3+21\,(b^4c^2d^2+3\,ab^3cd^3+6\,a^2b^2d^4)x^2+7\,(b^4c^3d+3\,ab^3c^2d^2+6\,a^2b^2cd^3+10\,a^3bd^4)x}{105\,(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+7\,c^6d^6x+c^7d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.
time = 0.30, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\,(b^4cd^3+3\,ab^3d^4)x^3+21\,(b^4c^2d^2+3\,ab^3cd^3+6\,a^2b^2d^4)x^2+7\,(b^4c^3d+3\,ab^3c^2d^2+6\,a^2b^2cd^3+10\,a^3bd^4)x}{105\,(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+7\,c^6d^6x+c^7d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
```

```
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(73) = 146$.
time = 108.56, size = 267, normalized size = 3.00

$$\frac{-15a^4d^4 - 10a^3bcd^3 - 6a^2b^2c^2d^2 - 3ab^3c^3d - b^4c^4 - 35b^4d^4x^4 + x^3\left(-105ab^3d^4 - 35b^4cd^3\right) + x^2\left(-126a^2b^2d^4 - 63ab^3cd^3 - 21b^4c^2d^2\right) + x\left(-70a^3bd^4 - 42a^2b^2cd^3 - 21ab^3c^2d^2 - 7b^4c^3d\right)}{105c^7d^5 + 735c^6d^6x + 2205c^5d^7x^2 + 3675c^4d^8x^3 + 3675c^3d^9x^4 + 2205c^2d^{10}x^5 + 735cd^{11}x^6 + 105d^{12}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**8,x)
```

```
[Out] (-15*a**4*d**4 - 10*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 - 3*a*b**3*c**3*d
 - b**4*c**4 - 35*b**4*d**4*x**4 + x**3*(-105*a*b**3*d**4 - 35*b**4*c*d**3)
 + x**2*(-126*a**2*b**2*d**4 - 63*a*b**3*c*d**3 - 21*b**4*c**2*d**2) + x*(-
70*a**3*b*d**4 - 42*a**2*b**2*c*d**3 - 21*a*b**3*c**2*d**2 - 7*b**4*c**3*d)
)/(105*c**7*d**5 + 735*c**6*d**6*x + 2205*c**5*d**7*x**2 + 3675*c**4*d**8*x
**3 + 3675*c**3*d**9*x**4 + 2205*c**2*d**10*x**5 + 735*c*d**11*x**6 + 105*d
**12*x**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(83) = 166$.
time = 0.00, size = 201, normalized size = 2.26

$$\frac{-35x^4b^4d^4 - 35x^3b^4d^3c - 105x^3b^3ad^4 - 21x^2b^4d^2c^2 - 63x^2b^3ad^3c - 126x^2b^2a^2d^4 - 7xb^4dc^3 - 21xb^3ad^2c^2 - 42xb^2a^2d^3c - 70xba^3d^4 - b^4c^4 - 3b^3adc^3 - 6b^2a^2d^2c^2 - 10ba^3d^3c - 15a^4d^4}{105d^5\left(xd + c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x)
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + 35*b^4*c*d^3*x^3 + 105*a*b^3*d^4*x^3 + 21*b^4*c^2*
d^2*x^2 + 63*a*b^3*c*d^3*x^2 + 126*a^2*b^2*d^4*x^2 + 7*b^4*c^3*d*x + 21*a*b
^3*c^2*d^2*x + 42*a^2*b^2*c*d^3*x + 70*a^3*b*d^4*x + b^4*c^4 + 3*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + 15*a^4*d^4)/((d*x + c)^7*d^5)
```

Mupad [B]
time = 0.11, size = 237, normalized size = 2.66

$$-\frac{\frac{15\,a^4\,d^4+10\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+3\,a\,b^3\,c^3\,d+b^4\,c^4}{105\,d^5}+\frac{b^4\,x^4}{3\,d}+\frac{b^3\,x^3\,\left(3\,a\,d+b\,c\right)}{3\,d^2}+\frac{b\,x\,\left(10\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{15\,d^4}+\frac{b^2\,x^2\,\left(6\,a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5\,d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^8,x)
```

```
[Out] -((15*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 3*a*b^3*c^3*d + 10*a^3*b*c*d^
3)/(105*d^5) + (b^4*x^4)/(3*d) + (b^3*x^3*(3*a*d + b*c))/(3*d^2) + (b*x*(10
```

```
*a^3*d^3 + b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(15*d^4) + (b^2*x^2*(6
*a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/(5*d^3))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21
*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x
)
```

3.1368 $\int \frac{(a+bx)^3}{(c+dx)^8}\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b^3}{4d^4(c+dx)^4}$$

```
[Out] 1/7*(-a*d+b*c)^3/d^4/(d*x+c)^7-1/2*b*(-a*d+b*c)^2/d^4/(d*x+c)^6+3/5*b^2*(-a
*d+b*c)/d^4/(d*x+c)^5-1/4*b^3/d^4/(d*x+c)^4
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {45}

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b^3}{4d^4(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^3/(7*d^4*(c + d*x)^7) - (b*(b*c - a*d)^2)/(2*d^4*(c + d*x)^6) +
 (3*b^2*(b*c - a*d))/(5*d^4*(c + d*x)^5) - b^3/(4*d^4*(c + d*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^8}+\frac{3b(bc-ad)^2}{d^3(c+dx)^7}-\frac{3b^2(bc-ad)}{d^3(c+dx)^6}+\frac{b^3}{d^3(c+dx)^5}\right)dx\\ &= \frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b^3}{4d^4(c+dx)^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 94, normalized size = 1.02

$$-\frac{20a^3d^3+10a^2bd^2(c+7dx)+4ab^2d\left(c^2+7cdx+21d^2x^2\right)+b^3\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)}{140d^4(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] -1/140*(20*a^3*d^3 + 10*a^2*b*d^2*(c + 7*d*x) + 4*a*b^2*d*(c^2 + 7*c*d*x + 21*d^2*x^2) + b^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3))/(d^4*(c + d*x)^7)
```

Mathics [A]
time = 4.83, size = 173, normalized size = 1.88

$$\frac{-20a^3d^3-10a^2bcd^2-4ab^2c^2d-b^3c^3-7bdx\left(10a^2d^2+4abcd+b^2c^2\right)+21b^2d^2x^2\left(-4ad-bc\right)-35b^3d^3x^3}{140d^4\left(c^7+7c^6dx+21c^5d^2x^2+35c^4d^3x^3+35c^3d^4x^4+21c^2d^5x^5+7cd^6x^6+d^7x^7\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3/(c + d*x)^8,x]')
```

```
[Out] (-20 a ^ 3 d ^ 3 - 10 a ^ 2 b c d ^ 2 - 4 a b ^ 2 c ^ 2 d - b ^ 3 c ^ 3 - 7 b d x (10 a ^ 2 d ^ 2 + 4 a b c d + b ^ 2 c ^ 2) + 21 b ^ 2 d ^ 2 x ^ 2 (-4 a d - b c) - 35 b ^ 3 d ^ 3 x ^ 3) / (140 d ^ 4 (c ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2 x ^ 2 + 35 c ^ 4 d ^ 3 x ^ 3 + 35 c ^ 3 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5 + 7 c d ^ 6 x ^ 6 + d ^ 7 x ^ 7))
```

Maple [A]
time = 0.14, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{b^3x^3}{4d}-\frac{3b^2(4ad+bc)x^2}{20d^2}-\frac{b\left(10a^2d^2+4abcd+b^2c^2\right)x}{20d^3}-\frac{20a^3d^3+10a^2bc\,d^2+4a\,b^2c^2d+b^3c^3}{140d^4}}{(dx+c)^7}$	110
gosper	$-\frac{35b^3x^3d^3+84a\,b^2d^3x^2+21b^3c\,d^2x^2+70a^2b\,d^3x+28a\,b^2c\,d^2x+7b^3c^2dx+20a^3d^3+10a^2bc\,d^2+4a\,b^2c^2d+b^3c^3}{140d^4(dx+c)^7}$	115
default	$-\frac{b(a^2d^2-2abcd+b^2c^2)}{2d^4(dx+c)^6}-\frac{3b^2(ad-bc)}{5d^4(dx+c)^5}-\frac{b^3}{4d^4(dx+c)^4}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{7d^4(dx+c)^7}$	122
norman	$\frac{-\frac{b^3x^3}{4d}-\frac{3\left(4a\,b^2d^4+b^3c\,d^3\right)x^2}{20d^5}-\frac{\left(10a^2b\,d^5+4a\,b^2c\,d^4+b^3c^2d^3\right)x}{20d^6}-\frac{20a^3d^6+10a^2bc\,d^5+4a\,b^2c^2d^4+b^3c^3d^3}{140d^7}}{(dx+c)^7}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^6-3/5*b^2/d^4*(a*d-b*c)/(d*x+c)^5-1/4*b^3/d^4/(d*x+c)^4-1/7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(84) = 168.
time = 0.27, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3+21\,(b^3cd^2+4\,ab^2d^3)x^2+7\,(b^3c^2d+4\,ab^2cd^2+10\,a^2bd^3)x}{140\,(d^{11}x^7+7\,cd^{10}x^6+21\,c^2d^9x^5+35\,c^3d^8x^4+35\,c^4d^7x^3+21\,c^5d^6x^2+7\,c^6d^5x+c^7d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.
time = 0.30, size = 182, normalized size = 1.98

$$-\frac{35\,b^{3}d^{3}x^{3}+b^{3}c^{3}+4\,ab^{2}c^{2}d+10\,a^{2}bcd^{2}+20\,a^{3}d^{3}+21\,(b^{3}cd^{2}+4\,ab^{2}d^{3})x^{2}+7\,(b^{3}c^{2}d+4\,ab^{2}cd^{2}+10\,a^{2}bd^{3})x}{140\,(d^{11}x^{7}+7\,cd^{10}x^{6}+21\,c^{2}d^{9}x^{5}+35\,c^{3}d^{8}x^{4}+35\,c^{4}d^{7}x^{3}+21\,c^{5}d^{6}x^{2}+7\,c^{6}d^{5}x+c^{7}d^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(80) = 160$.
time = 2.01, size = 196, normalized size = 2.13

$$\frac{-20a^{3}d^{3}-10a^{2}bcd^{2}-4ab^{2}c^{2}d-b^{3}c^{3}-35b^{3}d^{3}x^{3}+x^{2}\left(-84ab^{2}d^{3}-21b^{3}cd^{2}\right)+x\left(-70a^{2}bd^{3}-28ab^{2}cd^{2}-7b^{3}c^{2}d\right)}{140c^{7}d^{4}+980c^{6}d^{5}x+2940c^{5}d^{6}x^{2}+4900c^{4}d^{7}x^{3}+4900c^{3}d^{8}x^{4}+2940c^{2}d^{9}x^{5}+980cd^{10}x^{6}+140d^{11}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] (-20*a**3*d**3 - 10*a**2*b*c*d**2 - 4*a*b**2*c**2*d - b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-84*a*b**2*d**3 - 21*b**3*c*d**2) + x*(-70*a**2*b*d**3 - 2
8*a*b**2*c*d**2 - 7*b**3*c**2*d))/(140*c**7*d**4 + 980*c**6*d**5*x + 2940*c
**5*d**6*x**2 + 4900*c**4*d**7*x**3 + 4900*c**3*d**8*x**4 + 2940*c**2*d**9*
x**5 + 980*c*d**10*x**6 + 140*d**11*x**7)
```

Giac [A]
time = 0.00, size = 126, normalized size = 1.37

$$\frac{-35x^{3}b^{3}d^{3}-21x^{2}b^{3}d^{2}c-84x^{2}b^{2}ad^{3}-7xb^{3}dc^{2}-28xb^{2}ad^{2}c-70xba^{2}d^{3}-b^{3}c^{3}-4b^{2}adc^{2}-10ba^{2}d^{2}c-20a^{3}d^{3}}{140d^{4}\left(xd+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 21*b^3*c*d^2*x^2 + 84*a*b^2*d^3*x^2 + 7*b^3*c^2*d*
x + 28*a*b^2*c*d^2*x + 70*a^2*b*d^3*x + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*
c*d^2 + 20*a^3*d^3)/((d*x + c)^7*d^4)
```

Mupad [B]
time = 0.10, size = 176, normalized size = 1.91

$$-\frac{\frac{20\,a^3\,d^3+10\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d+b^3\,c^3}{140\,d^4}+\frac{b^3\,x^3}{4\,d}+\frac{b\,x\,\left(10\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{20\,d^3}+\frac{3\,b^2\,x^2\,\left(4\,a\,d+b\,c\right)}{20\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^8,x)
```

```
[Out] -((20*a^3*d^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2)/(140*d^4) + (b^3*
x^3)/(4*d) + (b*x*(10*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/(20*d^3) + (3*b^2*x^2
*(4*a*d + b*c))/(20*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 3
5*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1369 $\int \frac{(a+bx)^2}{(c+dx)^8} \, dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5}$$

```
[Out] -1/7*(-a*d+b*c)^2/d^3/(d*x+c)^7+1/3*b*(-a*d+b*c)/d^3/(d*x+c)^6-1/5*b^2/d^3/
(d*x+c)^5
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -1/7*(b*c - a*d)^2/(d^3*(c + d*x)^7) + (b*(b*c - a*d))/(3*d^3*(c + d*x)^6)
- b^2/(5*d^3*(c + d*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{(c+dx)^8} \, dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^8}-\frac{2b(bc-ad)}{d^2(c+dx)^7}+\frac{b^2}{d^2(c+dx)^6}\right) dx \\ &= -\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.85

$$-\frac{15a^2d^2+5abd(c+7dx)+b^2\left(c^2+7cdx+21d^2x^2\right)}{105d^3(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(c^2 + 7*c*d*x + 21*d^2*x^2)
)/(d^3*(c + d*x)^7)
```

Mathics [A]
time = 3.57, size = 125, normalized size = 1.92

$$\frac{-15a^2d^2-5abcd-b^2c^2-7bdx\,(5ad+bc)-21b^2d^2x^2}{105d^3\,(c^7+7c^6dx+21c^5d^2x^2+35c^4d^3x^3+35c^3d^4x^4+21c^2d^5x^5+7cd^6x^6+d^7x^7)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x)^8,x]')
```

```
[Out] (-15 a ^ 2 d ^ 2 - 5 a b c d - b ^ 2 c ^ 2 - 7 b d x (5 a d + b c) - 21 b ^
 2 d ^ 2 x ^ 2) / (105 d ^ 3 (c ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2 x ^ 2 +
35 c ^ 4 d ^ 3 x ^ 3 + 35 c ^ 3 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5 + 7 c d
^ 6 x ^ 6 + d ^ 7 x ^ 7))
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{21b^2x^2d^2+35ab\,d^2x+7b^2cdx+15a^2d^2+5abcd+b^2c^2}{105d^3(dx+c)^7}$	62
risch	$\frac{-\frac{b^2x^2}{5d}-\frac{b(5ad+bc)x}{15d^2}-\frac{15a^2d^2+5abcd+b^2c^2}{105d^3}}{(dx+c)^7}$	63
default	$-\frac{b(ad-bc)}{3d^3(dx+c)^6}-\frac{b^2}{5d^3(dx+c)^5}-\frac{a^2d^2-2abcd+b^2c^2}{7d^3(dx+c)^7}$	71
norman	$\frac{-\frac{b^2x^2}{5d}-\frac{\left(5ab\,d^5+b^2c\,d^4\right)x}{15d^6}-\frac{15a^2d^6+5abc\,d^5+b^2c^2d^4}{105d^7}}{(dx+c)^7}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/d^3*(a*d-b*c)/(d*x+c)^6-1/5*b^2/d^3/(d*x+c)^5-1/7*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/d^3/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(59) = 118.
time = 0.28, size = 131, normalized size = 2.02

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\,(b^2cd+5\,abd^2)x}{105\,(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.
time = 0.29, size = 131, normalized size = 2.02

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\,(b^2cd+5\,abd^2)x}{105\,(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(54) = 108$.
time = 0.82, size = 139, normalized size = 2.14

$$\frac{-15a^2d^2-5abcd-b^2c^2-21b^2d^2x^2+x\,(-35abd^2-7b^2cd)}{105c^7d^3+735c^6d^4x+2205c^5d^5x^2+3675c^4d^6x^3+3675c^3d^7x^4+2205c^2d^8x^5+735cd^9x^6+105d^{10}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] (-15*a**2*d**2 - 5*a*b*c*d - b**2*c**2 - 21*b**2*d**2*x**2 + x*(-35*a*b*d**
2 - 7*b**2*c*d))/(105*c**7*d**3 + 735*c**6*d**4*x + 2205*c**5*d**5*x**2 + 3
675*c**4*d**6*x**3 + 3675*c**3*d**7*x**4 + 2205*c**2*d**8*x**5 + 735*c*d**9
*x**6 + 105*d**10*x**7)
```

Giac [A]
time = 0.00, size = 69, normalized size = 1.06

$$\frac{-21x^2b^2d^2-7xb^2dc-35xbad^2-b^2c^2-5badc-15a^2d^2}{105d^3\,(xd+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 7*b^2*c*d*x + 35*a*b*d^2*x + b^2*c^2 + 5*a*b*c*d +
 15*a^2*d^2)/((d*x + c)^7*d^3)
```

Mupad [B]
time = 0.09, size = 129, normalized size = 1.98

$$-\frac{\frac{15\,a^2\,d^2+5\,a\,b\,c\,d+b^2\,c^2}{105\,d^3}+\frac{b^2\,x^2}{5\,d}+\frac{b\,x\,(5\,a\,d+b\,c)}{15\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^8,x)
```

```
[Out] -((15*a^2*d^2 + b^2*c^2 + 5*a*b*c*d)/(105*d^3) + (b^2*x^2)/(5*d) + (b*x*(5*
a*d + b*c))/(15*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^
4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1370 $\int \frac{a+bx}{(c+dx)^8}\,dx$

Optimal. Leaf size=38

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

```
[Out] 1/7*(-a*d+b*c)/d^2/(d*x+c)^7-1/6*b/d^2/(d*x+c)^6
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)/(7*d^2*(c + d*x)^7) - b/(6*d^2*(c + d*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^8}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^8}+\frac{b}{d(c+dx)^7}\right)dx\\ &= \frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{6ad+b(c+7dx)}{42d^2(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a*d + b*(c + 7*d*x))/(d^2*(c + d*x)^7)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 94 vs. $2(38) = 76$.
time = 2.84, size = 92, normalized size = 2.42

$$\frac{-6ad - bc - 7bdx}{42d^2 (c^7 + 7c^6dx + 21c^5d^2x^2 + 35c^4d^3x^3 + 35c^3d^4x^4 + 21c^2d^5x^5 + 7cd^6x^6 + d^7x^7)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x)^8,x]')
```

```
[Out] (-6 a d - b c - 7 b d x) / (42 d ^ 2 (c ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2
x ^ 2 + 35 c ^ 4 d ^ 3 x ^ 3 + 35 c ^ 3 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5
+ 7 c d ^ 6 x ^ 6 + d ^ 7 x ^ 7))
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{7bdx+6ad+bc}{42d^2(dx+c)^7}$	26
risch	$\frac{-\frac{bx}{6d}-\frac{6ad+bc}{42d^2}}{(dx+c)^7}$	30
default	$-\frac{b}{6d^2(dx+c)^6}-\frac{ad-bc}{7d^2(dx+c)^7}$	35
norman	$\frac{-\frac{bx}{6d}-\frac{6a\,d^6+bc\,d^5}{42d^7}}{(dx+c)^7}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b/d^2/(d*x+c)^6-1/7*(a*d-b*c)/d^2/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(34) = 68$.
time = 0.27, size = 94, normalized size = 2.47

$$-\frac{7\,bdx + bc + 6\,ad}{42\,(d^9x^7 + 7\,cd^8x^6 + 21\,c^2d^7x^5 + 35\,c^3d^6x^4 + 35\,c^4d^5x^3 + 21\,c^5d^4x^2 + 7\,c^6d^3x + c^7d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(34) = 68$.
time = 0.29, size = 94, normalized size = 2.47

$$-\frac{7\,bdx + bc + 6\,ad}{42\,(d^9x^7 + 7\,cd^8x^6 + 21\,c^2d^7x^5 + 35\,c^3d^6x^4 + 35\,c^4d^5x^3 + 21\,c^5d^4x^2 + 7\,c^6d^3x + c^7d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.
time = 0.42, size = 100, normalized size = 2.63

$$\frac{-6ad - bc - 7bdx}{42c^7d^2 + 294c^6d^3x + 882c^5d^4x^2 + 1470c^4d^5x^3 + 1470c^3d^6x^4 + 882c^2d^7x^5 + 294cd^8x^6 + 42d^9x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**8,x)
```

```
[Out] (-6*a*d - b*c - 7*b*d*x)/(42*c**7*d**2 + 294*c**6*d**3*x + 882*c**5*d**4*x*
*2 + 1470*c**4*d**5*x**3 + 1470*c**3*d**6*x**4 + 882*c**2*d**7*x**5 + 294*c
*d**8*x**6 + 42*d**9*x**7)
```

Giac [A]
time = 0.00, size = 30, normalized size = 0.79

$$\frac{-7xbd - bc - 6ad}{42d^2\,(xd + c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/((d*x + c)^7*d^2)
```

Mupad [B]
time = 0.23, size = 96, normalized size = 2.53

$$-\frac{\frac{6\,a\,d+b\,c}{42\,d^2} + \frac{b\,x}{6\,d}}{c^7 + 7\,c^6\,d\,x + 21\,c^5\,d^2\,x^2 + 35\,c^4\,d^3\,x^3 + 35\,c^3\,d^4\,x^4 + 21\,c^2\,d^5\,x^5 + 7\,c\,d^6\,x^6 + d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^8,x)
```

```
[Out] -((6*a*d + b*c)/(42*d^2) + (b*x)/(6*d))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c
^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1371 $\int \frac{1}{(c+dx)^8} dx$

Optimal. Leaf size=14

$$-\frac{1}{7d(c+dx)^7}$$

```
[Out] -1/7/d/(d*x+c)^7
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-8),x]
[Out] -1/7*1/(d*(c + d*x)^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^8} dx = -\frac{1}{7d(c+dx)^7}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-8),x]
[Out] -1/7*1/(d*(c + d*x)^7)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(14) = 28.
time = 2.46, size = 78, normalized size = 5.57

$$-\frac{1}{7d\left(c^7+7c^6dx+21c^5d^2x^2+35c^4d^3x^3+35c^3d^4x^4+21c^2d^5x^5+7cd^6x^6+d^7x^7\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x)^8,x]')
```

```
[Out] -1 / (7 d (c ^ 7 + 7 c ^ 6 d x + 21 c ^ 5 d ^ 2 x ^ 2 + 35 c ^ 4 d ^ 3 x ^
3 + 35 c ^ 3 d ^ 4 x ^ 4 + 21 c ^ 2 d ^ 5 x ^ 5 + 7 c d ^ 6 x ^ 6 + d ^ 7 x
 ^ 7))
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{7d(dx+c)^{7}}$	13
default	$-\frac{1}{7d(dx+c)^{7}}$	13
norman	$-\frac{1}{7d(dx+c)^{7}}$	13
risch	$-\frac{1}{7d(dx+c)^{7}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/d/(d*x+c)^7
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$-\frac{1}{7\,(dx+c)^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(12) = 24$.
time = 0.30, size = 79, normalized size = 5.64

$$-\frac{1}{7\,(d^{8}x^{7}+7\,cd^{7}x^{6}+21\,c^{2}d^{6}x^{5}+35\,c^{3}d^{5}x^{4}+35\,c^{4}d^{4}x^{3}+21\,c^{5}d^{3}x^{2}+7\,c^{6}d^{2}x+c^{7}d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(d^8*x^7 + 7*c*d^7*x^6 + 21*c^2*d^6*x^5 + 35*c^3*d^5*x^4 + 35*c^4*d^4*
x^3 + 21*c^5*d^3*x^2 + 7*c^6*d^2*x + c^7*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(12) = 24$.
time = 0.24, size = 85, normalized size = 6.07

$$-\frac{1}{7c^7d + 49c^6d^2x + 147c^5d^3x^2 + 245c^4d^4x^3 + 245c^3d^5x^4 + 147c^2d^6x^5 + 49cd^7x^6 + 7d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**8,x)
```

```
[Out] -1/(7*c**7*d + 49*c**6*d**2*x + 147*c**5*d**3*x**2 + 245*c**4*d**4*x**3 + 2
45*c**3*d**5*x**4 + 147*c**2*d**6*x**5 + 49*c*d**7*x**6 + 7*d**8*x**7)
```

Giac [A]
time = 0.00, size = 12, normalized size = 0.86

$$-\frac{1}{7d\left(xd+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x)
```

```
[Out] -1/7/((d*x + c)^7*d)
```

Mupad [B]
time = 0.22, size = 81, normalized size = 5.79

$$-\frac{1}{7\,c^7\,d+49\,c^6\,d^2\,x+147\,c^5\,d^3\,x^2+245\,c^4\,d^4\,x^3+245\,c^3\,d^5\,x^4+147\,c^2\,d^6\,x^5+49\,c\,d^7\,x^6+7\,d^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^8,x)
```

```
[Out] -1/(7*c^7*d + 7*d^8*x^7 + 49*c^6*d^2*x + 49*c*d^7*x^6 + 147*c^5*d^3*x^2 + 2
45*c^4*d^4*x^3 + 245*c^3*d^5*x^4 + 147*c^2*d^6*x^5)
```

3.1372 $\int \frac{1}{(a+bx)(c+dx)^8} \, dx$

Optimal. Leaf size=202

$$\frac{1}{7(bc-ad)(c+dx)^7}+\frac{b}{6(bc-ad)^2(c+dx)^6}+\frac{b^2}{5(bc-ad)^3(c+dx)^5}+\frac{b^3}{4(bc-ad)^4(c+dx)^4}+\frac{b^4}{3(bc-ad)^5(c+d}$$

```
[Out] 1/7/(-a*d+b*c)/(d*x+c)^7+1/6*b/(-a*d+b*c)^2/(d*x+c)^6+1/5*b^2/(-a*d+b*c)^3/
(d*x+c)^5+1/4*b^3/(-a*d+b*c)^4/(d*x+c)^4+1/3*b^4/(-a*d+b*c)^5/(d*x+c)^3+1/2
*b^5/(-a*d+b*c)^6/(d*x+c)^2+b^6/(-a*d+b*c)^7/(d*x+c)+b^7*ln(b*x+a)/(-a*d+b*
c)^8-b^7*ln(d*x+c)/(-a*d+b*c)^8
```

Rubi [A]
time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\frac{b^7 \log(a+bx)}{(bc-ad)^8}-\frac{b^7 \log(c+dx)}{(bc-ad)^8}+\frac{b^6}{(c+dx)(bc-ad)^7}+\frac{b^5}{2(c+dx)^2(bc-ad)^6}+\frac{b^4}{3(c+dx)^3(bc-ad)^5}+\frac{b^3}{4(c+dx)^4(bc-ad)^4}+\frac{b^2}{5(c+dx)^5(bc-ad)^3}+\frac{b}{6(c+dx)^6(bc-ad)^2}+\frac{1}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] 1/(7*(b*c - a*d)*(c + d*x)^7) + b/(6*(b*c - a*d)^2*(c + d*x)^6) + b^2/(5*(b
*c - a*d)^3*(c + d*x)^5) + b^3/(4*(b*c - a*d)^4*(c + d*x)^4) + b^4/(3*(b*c
- a*d)^5*(c + d*x)^3) + b^5/(2*(b*c - a*d)^6*(c + d*x)^2) + b^6/((b*c - a*d
)^7*(c + d*x)) + (b^7*Log[a + b*x])/(b*c - a*d)^8 - (b^7*Log[c + d*x])/(b*c
 - a*d)^8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^8} \, dx &= \int \left(\frac{b^8}{(bc-ad)^8(a+bx)}-\frac{d}{(bc-ad)(c+dx)^8}-\frac{bd}{(bc-ad)^2(c+dx)^7}-\frac{b^2}{(bc-ad)^3}\right. \\ &= \frac{1}{7(bc-ad)(c+dx)^7}+\frac{b}{6(bc-ad)^2(c+dx)^6}+\frac{b^2}{5(bc-ad)^3(c+dx)^5}+\frac{b}{4(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 196, normalized size = 0.97

$$\frac{60(bc-ad)^7+70b(bc-ad)^6(c+dx)+84b^2(bc-ad)^5(c+dx)^2+105b^3(bc-ad)^4(c+dx)^3+140b^4(bc-ad)^3(c+dx)^4+210b^5(bc-ad)^2(c+dx)^5+420b^6(bc-ad)(c+dx)^6+420b^7(c+dx)^7\log(a+bx)-420b^7(c+dx)^7\log(c+dx)}{420(bc-ad)^8(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] (60*(b*c - a*d)^7 + 70*b*(b*c - a*d)^6*(c + d*x) + 84*b^2*(b*c - a*d)^5*(c
+ d*x)^2 + 105*b^3*(b*c - a*d)^4*(c + d*x)^3 + 140*b^4*(b*c - a*d)^3*(c + d
*x)^4 + 210*b^5*(b*c - a*d)^2*(c + d*x)^5 + 420*b^6*(b*c - a*d)*(c + d*x)^6
 + 420*b^7*(c + d*x)^7*Log[a + b*x] - 420*b^7*(c + d*x)^7*Log[c + d*x])/(42
0*(b*c - a*d)^8*(c + d*x)^7)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 1884 vs. $2(202) = 404$.
time = 27.84, size = 1882, normalized size = 9.32

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-1)/(c + d*x)^8,x]')
```

```
[Out] (-60 a ^ 7 d ^ 7 + 490 a ^ 6 b c d ^ 6 + 70 a ^ 6 b d ^ 7 x - 1764 a ^ 5 b
^ 2 c ^ 2 d ^ 5 - 588 a ^ 5 b ^ 2 c d ^ 6 x - 84 a ^ 5 b ^ 2 d ^ 7 x ^ 2 +
3675 a ^ 4 b ^ 3 c ^ 3 d ^ 4 + 2205 a ^ 4 b ^ 3 c ^ 2 d ^ 5 x + 735 a ^ 4 b
 ^ 3 c d ^ 6 x ^ 2 + 105 a ^ 4 b ^ 3 d ^ 7 x ^ 3 - 4900 a ^ 3 b ^ 4 c ^ 4 d
 ^ 3 - 4900 a ^ 3 b ^ 4 c ^ 3 d ^ 4 x - 2940 a ^ 3 b ^ 4 c ^ 2 d ^ 5 x ^ 2
- 980 a ^ 3 b ^ 4 c d ^ 6 x ^ 3 - 140 a ^ 3 b ^ 4 d ^ 7 x ^ 4 + 4410 a ^ 2
b ^ 5 c ^ 5 d ^ 2 + 7350 a ^ 2 b ^ 5 c ^ 4 d ^ 3 x + 7350 a ^ 2 b ^ 5 c ^ 3
 d ^ 4 x ^ 2 + 4410 a ^ 2 b ^ 5 c ^ 2 d ^ 5 x ^ 3 + 1470 a ^ 2 b ^ 5 c d ^
6 x ^ 4 + 210 a ^ 2 b ^ 5 d ^ 7 x ^ 5 - 2940 a b ^ 6 c ^ 6 d - 8820 a b ^ 6
 c ^ 5 d ^ 2 x - 14700 a b ^ 6 c ^ 4 d ^ 3 x ^ 2 - 14700 a b ^ 6 c ^ 3 d ^
4 x ^ 3 - 8820 a b ^ 6 c ^ 2 d ^ 5 x ^ 4 - 2940 a b ^ 6 c d ^ 6 x ^ 5 - 420
 a b ^ 6 d ^ 7 x ^ 6 - 420 b ^ 7 c ^ 7 Log[c / d + x] + 420 b ^ 7 c ^ 7 Log
[a / b + x] + 1089 b ^ 7 c ^ 7 - 2940 b ^ 7 c ^ 6 d x Log[c / d + x] + 2940
 b ^ 7 c ^ 6 d x Log[a / b + x] + 4683 b ^ 7 c ^ 6 d x - 8820 b ^ 7 c ^ 5 d
 ^ 2 x ^ 2 Log[c / d + x] + 8820 b ^ 7 c ^ 5 d ^ 2 x ^ 2 Log[a / b + x] + 9
639 b ^ 7 c ^ 5 d ^ 2 x ^ 2 - 14700 b ^ 7 c ^ 4 d ^ 3 x ^ 3 Log[c / d + x]
+ 11165 b ^ 7 c ^ 4 d ^ 3 x ^ 3 + 14700 b ^ 7 c ^ 4 d ^ 3 x ^ 3 Log[a / b +
 x] - 14700 b ^ 7 c ^ 3 d ^ 4 x ^ 4 Log[c / d + x] + 7490 b ^ 7 c ^ 3 d ^ 4
 x ^ 4 + 14700 b ^ 7 c ^ 3 d ^ 4 x ^ 4 Log[a / b + x] - 8820 b ^ 7 c ^ 2 d
^ 5 x ^ 5 Log[c / d + x] + 2730 b ^ 7 c ^ 2 d ^ 5 x ^ 5 + 8820 b ^ 7 c ^ 2
d ^ 5 x ^ 5 Log[a / b + x] - 2940 b ^ 7 c d ^ 6 x ^ 6 Log[c / d + x] + 420
b ^ 7 c d ^ 6 x ^ 6 + 2940 b ^ 7 c d ^ 6 x ^ 6 Log[a / b + x] - 420 b ^ 7 d
 ^ 7 x ^ 7 Log[c / d + x] + 420 b ^ 7 d ^ 7 x ^ 7 Log[a / b + x]) / (420 (a
 ^ 8 c ^ 7 d ^ 8 + 7 a ^ 8 c ^ 6 d ^ 9 x + 21 a ^ 8 c ^ 5 d ^ 10 x ^ 2 + 35
 a ^ 8 c ^ 4 d ^ 11 x ^ 3 + 35 a ^ 8 c ^ 3 d ^ 12 x ^ 4 + 21 a ^ 8 c ^ 2 d
^ 13 x ^ 5 + 7 a ^ 8 c d ^ 14 x ^ 6 + a ^ 8 d ^ 15 x ^ 7 - 8 a ^ 7 b c ^ 8
d ^ 7 - 56 a ^ 7 b c ^ 7 d ^ 8 x - 168 a ^ 7 b c ^ 6 d ^ 9 x ^ 2 - 280 a ^
7 b c ^ 5 d ^ 10 x ^ 3 - 280 a ^ 7 b c ^ 4 d ^ 11 x ^ 4 - 168 a ^ 7 b c ^ 3
 d ^ 12 x ^ 5 - 56 a ^ 7 b c ^ 2 d ^ 13 x ^ 6 - 8 a ^ 7 b c d ^ 14 x ^ 7 +
```

```
28 a ^ 6 b ^ 2 c ^ 9 d ^ 6 + 196 a ^ 6 b ^ 2 c ^ 8 d ^ 7 x + 588 a ^ 6 b ^
2 c ^ 7 d ^ 8 x ^ 2 + 980 a ^ 6 b ^ 2 c ^ 6 d ^ 9 x ^ 3 + 980 a ^ 6 b ^ 2 c
 ^ 5 d ^ 10 x ^ 4 + 588 a ^ 6 b ^ 2 c ^ 4 d ^ 11 x ^ 5 + 196 a ^ 6 b ^ 2 c
^ 3 d ^ 12 x ^ 6 + 28 a ^ 6 b ^ 2 c ^ 2 d ^ 13 x ^ 7 - 56 a ^ 5 b ^ 3 c ^ 1
0 d ^ 5 - 392 a ^ 5 b ^ 3 c ^ 9 d ^ 6 x - 1176 a ^ 5 b ^ 3 c ^ 8 d ^ 7 x ^
2 - 1960 a ^ 5 b ^ 3 c ^ 7 d ^ 8 x ^ 3 - 1960 a ^ 5 b ^ 3 c ^ 6 d ^ 9 x ^ 4
 - 1176 a ^ 5 b ^ 3 c ^ 5 d ^ 10 x ^ 5 - 392 a ^ 5 b ^ 3 c ^ 4 d ^ 11 x ^ 6
 - 56 a ^ 5 b ^ 3 c ^ 3 d ^ 12 x ^ 7 + 70 a ^ 4 b ^ 4 c ^ 11 d ^ 4 + 490 a
^ 4 b ^ 4 c ^ 10 d ^ 5 x + 1470 a ^ 4 b ^ 4 c ^ 9 d ^ 6 x ^ 2 + 2450 a ^ 4
b ^ 4 c ^ 8 d ^ 7 x ^ 3 + 2450 a ^ 4 b ^ 4 c ^ 7 d ^ 8 x ^ 4 + 1470 a ^ 4 b
 ^ 4 c ^ 6 d ^ 9 x ^ 5 + 490 a ^ 4 b ^ 4 c ^ 5 d ^ 10 x ^ 6 + 70 a ^ 4 b ^
4 c ^ 4 d ^ 11 x ^ 7 - 56 a ^ 3 b ^ 5 c ^ 12 d ^ 3 - 392 a ^ 3 b ^ 5 c ^ 11
 d ^ 4 x - 1176 a ^ 3 b ^ 5 c ^ 10 d ^ 5 x ^ 2 - 1960 a ^ 3 b ^ 5 c ^ 9 d ^
 6 x ^ 3 - 1960 a ^ 3 b ^ 5 c ^ 8 d ^ 7 x ^ 4 - 1176 a ^ 3 b ^ 5 c ^ 7 d ^
8 x ^ 5 - 392 a ^ 3 b ^ 5 c ^ 6 d ^ 9 x ^ 6 - 56 a ^ 3 b ^ 5 c ^ 5 d ^ 10 x
 ^ 7 + 28 a ^ 2 b ^ 6 c ^ 13 d ^ 2 + 196 a ^ 2 b ^ 6 c ^ 12 d ^ 3 x + 588 a
 ^ 2 b ^ 6 c ^ 11 d ^ 4 x ^ 2 + 980 a ^ 2 b ^ 6 c ^ 10 d ^ 5 x ^ 3 + 980 a
^ 2 b ^ 6 c ^ 9 d ^ 6 x ^ 4 + 588 a ^ 2 b ^ 6 c ^ 8 d ^ 7 x ^ 5 + 196 a ^ 2
 b ^ 6 c ^ 7 d ^ 8 x ^ 6 + 28 a ^ 2 b ^ 6 c ^ 6 d ^ 9 x ^ 7 - 8 a b ^ 7 c ^
 14 d - 56 a b ^ 7 c ^ 13 d ^ 2 x - 168 a b ^ 7 c ^ 12 d ^ 3 x ^ 2 - 280 a
b ^ 7 c ^ 11 d ^ 4 x ^ 3 - 280 a b ^ 7 c ^ 10 d ^ 5 x ^ 4 - 168 a b ^ 7 c ^
 9 d ^ 6 x ^ 5 - 56 a b ^ 7 c ^ 8 d ^ 7 x ^ 6 - 8 a b ^ 7 c ^ 7 d ^ 8 x ^ 7
 + b ^ 8 c ^ 15 + 7 b ^ 8 c ^ 14 d x + 21 b ^ 8 c ^ 13 d ^ 2 x ^ 2 + 35 b ^
 8 c ^ 12 d ^ 3 x ^ 3 + 35 b ^ 8 c ^ 11 d ^ 4 x ^ 4 + 21 b ^ 8 c ^ 10 d ^ 5
 x ^ 5 + 7 b ^ 8 c ^ 9 d ^ 6 x ^ 6 + b ^ 8 c ^ 8 d ^ 7 x ^ 7))
```

Maple [A]

time = 0.20, size = 192, normalized size = 0.95

method	result
default	$-\frac{1}{7(ad-bc)(dx+c)^{7}}-\frac{b^{2}}{5(ad-bc)^{3}(dx+c)^{5}}-\frac{b^{4}}{3(ad-bc)^{5}(dx+c)^{3}}-\frac{b^{6}}{(ad-bc)^{7}(dx+c)}+\frac{b^{3}}{4(ad-bc)^{4}(dx+c)^{4}}+\frac{b^{5}}{2(ad-bc)^{6}(dx+c}$
risch	$-\frac{b^{6}d^{6}x^{6}}{a^{7}d^{7}-7a^{6}bc\,d^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35a^{3}b^{4}c^{4}d^{3}-21a^{2}b^{5}c^{5}d^{2}+7a\,b^{6}c^{6}d-b^{7}c^{7}}+\frac{(ad-13bc)b^{5}d^{5}x^{5}}{2a^{7}d^{7}-14a^{6}bc\,d^{6}+42a^{5}b^{2}c^{2}d^{5}-70a^{4}b^{3}c^{3}d^{4}+70a^{3}b^{4}c^{4}d}$
norman	$-\frac{b^{6}d^{6}x^{6}}{a^{7}d^{7}-7a^{6}bc\,d^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35a^{3}b^{4}c^{4}d^{3}-21a^{2}b^{5}c^{5}d^{2}+7a\,b^{6}c^{6}d-b^{7}c^{7}}+\frac{-60a^{6}d^{13}+430a^{5}bc\,d^{12}-1334a^{4}b^{2}c^{2}d^{11}+2341a^{3}b^{3}c^{3}d^{10}}{420d^{7}\left(a^{7}d^{7}-7a^{6}bc\,d^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/(a*d-b*c)/(d*x+c)^7-1/5*b^2/(a*d-b*c)^3/(d*x+c)^5-1/3*b^4/(a*d-b*c)^5/
(d*x+c)^3-b^6/(a*d-b*c)^7/(d*x+c)+1/4*b^3/(a*d-b*c)^4/(d*x+c)^4+1/2*b^5/(a*
d-b*c)^6/(d*x+c)^2-b^7/(a*d-b*c)^8*ln(d*x+c)+1/6*b/(a*d-b*c)^2/(d*x+c)^6+b^
7/(a*d-b*c)^8*ln(b*x+a)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(190) = 380$.
time = 0.36, size = 1418, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^7*log(b*x + a)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8) - b^7*log(d*x + c)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8) + 1/420*(420*b^6*d^6*x^6
+ 1089*b^6*c^6 - 1851*a*b^5*c^5*d + 2559*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3
*d^3 + 1334*a^4*b^2*c^2*d^4 - 430*a^5*b*c*d^5 + 60*a^6*d^6 + 210*(13*b^6*c*
d^5 - a*b^5*d^6)*x^5 + 70*(107*b^6*c^2*d^4 - 19*a*b^5*c*d^5 + 2*a^2*b^4*d^6
)*x^4 + 35*(319*b^6*c^3*d^3 - 101*a*b^5*c^2*d^4 + 25*a^2*b^4*c*d^5 - 3*a^3*
b^3*d^6)*x^3 + 21*(459*b^6*c^4*d^2 - 241*a*b^5*c^3*d^3 + 109*a^2*b^4*c^2*d^
4 - 31*a^3*b^3*c*d^5 + 4*a^4*b^2*d^6)*x^2 + 7*(669*b^6*c^5*d - 591*a*b^5*c^
4*d^2 + 459*a^2*b^4*c^3*d^3 - 241*a^3*b^3*c^2*d^4 + 74*a^4*b^2*c*d^5 - 10*a
^5*b*d^6)*x)/(b^7*c^14 - 7*a*b^6*c^13*d + 21*a^2*b^5*c^12*d^2 - 35*a^3*b^4*
c^11*d^3 + 35*a^4*b^3*c^10*d^4 - 21*a^5*b^2*c^9*d^5 + 7*a^6*b*c^8*d^6 - a^7
*c^7*d^7 + (b^7*c^7*d^7 - 7*a*b^6*c^6*d^8 + 21*a^2*b^5*c^5*d^9 - 35*a^3*b^4
*c^4*d^10 + 35*a^4*b^3*c^3*d^11 - 21*a^5*b^2*c^2*d^12 + 7*a^6*b*c*d^13 - a^
7*d^14)*x^7 + 7*(b^7*c^8*d^6 - 7*a*b^6*c^7*d^7 + 21*a^2*b^5*c^6*d^8 - 35*a^
3*b^4*c^5*d^9 + 35*a^4*b^3*c^4*d^10 - 21*a^5*b^2*c^3*d^11 + 7*a^6*b*c^2*d^1
2 - a^7*c*d^13)*x^6 + 21*(b^7*c^9*d^5 - 7*a*b^6*c^8*d^6 + 21*a^2*b^5*c^7*d^
7 - 35*a^3*b^4*c^6*d^8 + 35*a^4*b^3*c^5*d^9 - 21*a^5*b^2*c^4*d^10 + 7*a^6*b
*c^3*d^11 - a^7*c^2*d^12)*x^5 + 35*(b^7*c^10*d^4 - 7*a*b^6*c^9*d^5 + 21*a^2
*b^5*c^8*d^6 - 35*a^3*b^4*c^7*d^7 + 35*a^4*b^3*c^6*d^8 - 21*a^5*b^2*c^5*d^9
 + 7*a^6*b*c^4*d^10 - a^7*c^3*d^11)*x^4 + 35*(b^7*c^11*d^3 - 7*a*b^6*c^10*d
^4 + 21*a^2*b^5*c^9*d^5 - 35*a^3*b^4*c^8*d^6 + 35*a^4*b^3*c^7*d^7 - 21*a^5*
b^2*c^6*d^8 + 7*a^6*b*c^5*d^9 - a^7*c^4*d^10)*x^3 + 21*(b^7*c^12*d^2 - 7*a*
b^6*c^11*d^3 + 21*a^2*b^5*c^10*d^4 - 35*a^3*b^4*c^9*d^5 + 35*a^4*b^3*c^8*d^
6 - 21*a^5*b^2*c^7*d^7 + 7*a^6*b*c^6*d^8 - a^7*c^5*d^9)*x^2 + 7*(b^7*c^13*d
 - 7*a*b^6*c^12*d^2 + 21*a^2*b^5*c^11*d^3 - 35*a^3*b^4*c^10*d^4 + 35*a^4*b^
3*c^9*d^5 - 21*a^5*b^2*c^8*d^6 + 7*a^6*b*c^7*d^7 - a^7*c^6*d^8)*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1589 vs. $2(190) = 380$.
time = 0.33, size = 1589, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^
4*c^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 -
 60*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*
b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21
*a^2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d
^4 + 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*
b^7*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5
 + 35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^
5*d^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 -
84*a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 2
1*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^
2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(b*x + a) - 420*(b^7*d^7*x^7 + 7*b^7*c*
d^6*x^6 + 21*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21
*b^7*c^5*d^2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(d*x + c))/(b^8*c^15 - 8*a*b
^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4
 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8
 + (b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^1
0 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7
*b*c*d^14 + a^8*d^15)*x^7 + 7*(b^8*c^9*d^6 - 8*a*b^7*c^8*d^7 + 28*a^2*b^6*c
^7*d^8 - 56*a^3*b^5*c^6*d^9 + 70*a^4*b^4*c^5*d^10 - 56*a^5*b^3*c^4*d^11 + 2
8*a^6*b^2*c^3*d^12 - 8*a^7*b*c^2*d^13 + a^8*c*d^14)*x^6 + 21*(b^8*c^10*d^5
- 8*a*b^7*c^9*d^6 + 28*a^2*b^6*c^8*d^7 - 56*a^3*b^5*c^7*d^8 + 70*a^4*b^4*c^
6*d^9 - 56*a^5*b^3*c^5*d^10 + 28*a^6*b^2*c^4*d^11 - 8*a^7*b*c^3*d^12 + a^8*
c^2*d^13)*x^5 + 35*(b^8*c^11*d^4 - 8*a*b^7*c^10*d^5 + 28*a^2*b^6*c^9*d^6 -
56*a^3*b^5*c^8*d^7 + 70*a^4*b^4*c^7*d^8 - 56*a^5*b^3*c^6*d^9 + 28*a^6*b^2*c
^5*d^10 - 8*a^7*b*c^4*d^11 + a^8*c^3*d^12)*x^4 + 35*(b^8*c^12*d^3 - 8*a*b^7
*c^11*d^4 + 28*a^2*b^6*c^10*d^5 - 56*a^3*b^5*c^9*d^6 + 70*a^4*b^4*c^8*d^7 -
 56*a^5*b^3*c^7*d^8 + 28*a^6*b^2*c^6*d^9 - 8*a^7*b*c^5*d^10 + a^8*c^4*d^11)
*x^3 + 21*(b^8*c^13*d^2 - 8*a*b^7*c^12*d^3 + 28*a^2*b^6*c^11*d^4 - 56*a^3*b
^5*c^10*d^5 + 70*a^4*b^4*c^9*d^6 - 56*a^5*b^3*c^8*d^7 + 28*a^6*b^2*c^7*d^8
- 8*a^7*b*c^6*d^9 + a^8*c^5*d^10)*x^2 + 7*(b^8*c^14*d - 8*a*b^7*c^13*d^2 +
28*a^2*b^6*c^12*d^3 - 56*a^3*b^5*c^11*d^4 + 70*a^4*b^4*c^10*d^5 - 56*a^5*b^
3*c^9*d^6 + 28*a^6*b^2*c^8*d^7 - 8*a^7*b*c^7*d^8 + a^8*c^6*d^9)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1776 vs. $2(170) = 340$.
time = 7.99, size = 1776, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**8,x)
```

```
[Out] -b**7*log(x + (-a**9*b**7*d**9/(a*d - b*c)**8 + 9*a**8*b**8*c*d**8/(a*d - b
*c)**8 - 36*a**7*b**9*c**2*d**7/(a*d - b*c)**8 + 84*a**6*b**10*c**3*d**6/(a
*d - b*c)**8 - 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 + 126*a**4*b**12*c**
```

```
5*d**4/(a*d - b*c)**8 - 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 + 36*a**2*b*
*14*c**7*d**2/(a*d - b*c)**8 - 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d +
 b**16*c**9/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + b**7*log(
x + (a**9*b**7*d**9/(a*d - b*c)**8 - 9*a**8*b**8*c*d**8/(a*d - b*c)**8 + 36
*a**7*b**9*c**2*d**7/(a*d - b*c)**8 - 84*a**6*b**10*c**3*d**6/(a*d - b*c)**
8 + 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 - 126*a**4*b**12*c**5*d**4/(a*d
 - b*c)**8 + 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 - 36*a**2*b**14*c**7*d*
*2/(a*d - b*c)**8 + 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d - b**16*c**9
/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + (-60*a**6*d**6 + 430
*a**5*b*c*d**5 - 1334*a**4*b**2*c**2*d**4 + 2341*a**3*b**3*c**3*d**3 - 2559
*a**2*b**4*c**4*d**2 + 1851*a*b**5*c**5*d - 1089*b**6*c**6 - 420*b**6*d**6*
x**6 + x**5*(210*a*b**5*d**6 - 2730*b**6*c*d**5) + x**4*(-140*a**2*b**4*d**
6 + 1330*a*b**5*c*d**5 - 7490*b**6*c**2*d**4) + x**3*(105*a**3*b**3*d**6 -
875*a**2*b**4*c*d**5 + 3535*a*b**5*c**2*d**4 - 11165*b**6*c**3*d**3) + x**2
*(-84*a**4*b**2*d**6 + 651*a**3*b**3*c*d**5 - 2289*a**2*b**4*c**2*d**4 + 50
61*a*b**5*c**3*d**3 - 9639*b**6*c**4*d**2) + x*(70*a**5*b*d**6 - 518*a**4*b
**2*c*d**5 + 1687*a**3*b**3*c**2*d**4 - 3213*a**2*b**4*c**3*d**3 + 4137*a*b
**5*c**4*d**2 - 4683*b**6*c**5*d))/(420*a**7*c**7*d**7 - 2940*a**6*b*c**8*d
**6 + 8820*a**5*b**2*c**9*d**5 - 14700*a**4*b**3*c**10*d**4 + 14700*a**3*b*
*4*c**11*d**3 - 8820*a**2*b**5*c**12*d**2 + 2940*a*b**6*c**13*d - 420*b**7*
c**14 + x**7*(420*a**7*d**14 - 2940*a**6*b*c*d**13 + 8820*a**5*b**2*c**2*d*
*12 - 14700*a**4*b**3*c**3*d**11 + 14700*a**3*b**4*c**4*d**10 - 8820*a**2*b
**5*c**5*d**9 + 2940*a*b**6*c**6*d**8 - 420*b**7*c**7*d**7) + x**6*(2940*a*
*7*c*d**13 - 20580*a**6*b*c**2*d**12 + 61740*a**5*b**2*c**3*d**11 - 102900*
a**4*b**3*c**4*d**10 + 102900*a**3*b**4*c**5*d**9 - 61740*a**2*b**5*c**6*d*
*8 + 20580*a*b**6*c**7*d**7 - 2940*b**7*c**8*d**6) + x**5*(8820*a**7*c**2*d
**12 - 61740*a**6*b*c**3*d**11 + 185220*a**5*b**2*c**4*d**10 - 308700*a**4*
b**3*c**5*d**9 + 308700*a**3*b**4*c**6*d**8 - 185220*a**2*b**5*c**7*d**7 +
61740*a*b**6*c**8*d**6 - 8820*b**7*c**9*d**5) + x**4*(14700*a**7*c**3*d**11
 - 102900*a**6*b*c**4*d**10 + 308700*a**5*b**2*c**5*d**9 - 514500*a**4*b**3
*c**6*d**8 + 514500*a**3*b**4*c**7*d**7 - 308700*a**2*b**5*c**8*d**6 + 1029
00*a*b**6*c**9*d**5 - 14700*b**7*c**10*d**4) + x**3*(14700*a**7*c**4*d**10
- 102900*a**6*b*c**5*d**9 + 308700*a**5*b**2*c**6*d**8 - 514500*a**4*b**3*c
**7*d**7 + 514500*a**3*b**4*c**8*d**6 - 308700*a**2*b**5*c**9*d**5 + 102900
*a*b**6*c**10*d**4 - 14700*b**7*c**11*d**3) + x**2*(8820*a**7*c**5*d**9 - 6
1740*a**6*b*c**6*d**8 + 185220*a**5*b**2*c**7*d**7 - 308700*a**4*b**3*c**8*
d**6 + 308700*a**3*b**4*c**9*d**5 - 185220*a**2*b**5*c**10*d**4 + 61740*a*b
**6*c**11*d**3 - 8820*b**7*c**12*d**2) + x*(2940*a**7*c**6*d**8 - 20580*a**
6*b*c**7*d**7 + 61740*a**5*b**2*c**8*d**6 - 102900*a**4*b**3*c**9*d**5 + 10
2900*a**3*b**4*c**10*d**4 - 61740*a**2*b**5*c**11*d**3 + 20580*a*b**6*c**12
*d**2 - 2940*b**7*c**13*d))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(190) = 380$.

time = 0.01, size = 725, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x)
```

```
[Out] b^8*log(abs(b*x + a))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^
3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^
6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8) - b^7*d*log(abs(d*x + c))/(b^8*c^8*d - 8*a
*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5
 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8 + a^8*d^9) + 1/4
20*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^4*c
^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 - 60
*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*b^6
*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21*a^
2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4
+ 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*b^7
*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5 +
35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^5*d
^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 - 84*
a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x)/((b*c - a*d)^8*(d*x + c)^7)
```

Mupad [B]
time = 0.87, size = 1299, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^8),x)
```

```
[Out] (2*b^7*atanh((a^8*d^8 - b^8*c^8 - 14*a^2*b^6*c^6*d^2 + 14*a^3*b^5*c^5*d^3 -
 14*a^5*b^3*c^3*d^5 + 14*a^6*b^2*c^2*d^6 + 6*a*b^7*c^7*d - 6*a^7*b*c*d^7)/(
a*d - b*c)^8 + (2*b*d*x*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^
4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6
*b*c*d^6))/(a*d - b*c)^8))/(a*d - b*c)^8 - ((60*a^6*d^6 + 1089*b^6*c^6 + 25
59*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3*d^3 + 1334*a^4*b^2*c^2*d^4 - 1851*a*b
^5*c^5*d - 430*a^5*b*c*d^5)/(420*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 +
35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*
d - 7*a^6*b*c*d^6)) - (b^3*x^3*(3*a^3*d^6 - 319*b^3*c^3*d^3 + 101*a*b^2*c^2
*d^4 - 25*a^2*b*c*d^5))/(12*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6)) + (b^6*d^6*x^6)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35
*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d
- 7*a^6*b*c*d^6) - (b^5*x^5*(a*d^6 - 13*b*c*d^5))/(2*(a^7*d^7 - b^7*c^7 - 2
```

```
1*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^
2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^2*x^2*(4*a^4*d^6 + 459*b^4*c^4
*d^2 - 241*a*b^3*c^3*d^3 + 109*a^2*b^2*c^2*d^4 - 31*a^3*b*c*d^5))/(20*(a^7*
d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^
4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^4*x^4*(2*a^2*
d^6 + 107*b^2*c^2*d^4 - 19*a*b*c*d^5))/(6*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c
^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a
*b^6*c^6*d - 7*a^6*b*c*d^6)) - (b*x*(10*a^5*d^6 - 669*b^5*c^5*d + 591*a*b^4
*c^4*d^2 - 459*a^2*b^3*c^3*d^3 + 241*a^3*b^2*c^2*d^4 - 74*a^4*b*c*d^5))/(60
*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*
c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)))/(c^7 + d^7*
x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c
^2*d^5*x^5 + 7*c^6*d*x)
```

3.1373 $\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$

Optimal. Leaf size=231

$$-\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{3b^2d}{5(bc-ad)^4(c+dx)^5}-\frac{b^3d}{(bc-ad)^5(c+d}$$

```
[Out] -b^7/(-a*d+b*c)^8/(b*x+a)-1/7*d/(-a*d+b*c)^2/(d*x+c)^7-1/3*b*d/(-a*d+b*c)^3
/(d*x+c)^6-3/5*b^2*d/(-a*d+b*c)^4/(d*x+c)^5-b^3*d/(-a*d+b*c)^5/(d*x+c)^4-5/
3*b^4*d/(-a*d+b*c)^6/(d*x+c)^3-3*b^5*d/(-a*d+b*c)^7/(d*x+c)^2-7*b^6*d/(-a*d
+b*c)^8/(d*x+c)-8*b^7*d*ln(b*x+a)/(-a*d+b*c)^9+8*b^7*d*ln(d*x+c)/(-a*d+b*c)
^9
```

Rubi [A]
time = 0.19, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{8b^7d\log(a+bx)}{(bc-ad)^9}+\frac{8b^7d\log(c+dx)}{(bc-ad)^9}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc-ad)^6}-\frac{b^3d}{(c+dx)^4(bc-ad)^5}-\frac{3b^2d}{5(c+dx)^5(bc-ad)^4}-\frac{bd}{3(c+dx)^6(bc-ad)^3}-\frac{d}{7(c+dx)^7(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -(b^7/((b*c - a*d)^8*(a + b*x))) - d/(7*(b*c - a*d)^2*(c + d*x)^7) - (b*d)/
(3*(b*c - a*d)^3*(c + d*x)^6) - (3*b^2*d)/(5*(b*c - a*d)^4*(c + d*x)^5) - (
b^3*d)/((b*c - a*d)^5*(c + d*x)^4) - (5*b^4*d)/(3*(b*c - a*d)^6*(c + d*x)^3
) - (3*b^5*d)/((b*c - a*d)^7*(c + d*x)^2) - (7*b^6*d)/((b*c - a*d)^8*(c + d
*x)) - (8*b^7*d*Log[a + b*x])/(b*c - a*d)^9 + (8*b^7*d*Log[c + d*x])/(b*c -
 a*d)^9
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx=\int \left(\frac{b^8}{(bc-ad)^8(a+bx)^2}-\frac{8b^8d}{(bc-ad)^9(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^8}+\frac{2}{(bc-ad}\right.$$
$$=-\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{3}{5(bc-a}$$

Mathematica [A]
time = 0.15, size = 213, normalized size = 0.92

$$-\frac{\frac{105b^7(bc-ad)}{a+bx}-\frac{15d(-bc+ad)^7}{(c+dx)^7}+\frac{35bd(bc-ad)^6}{(c+dx)^6}+\frac{63b^2d(bc-ad)^5}{(c+dx)^5}+\frac{105b^3d(bc-ad)^4}{(c+dx)^4}+\frac{175b^4d(bc-ad)^3}{(c+dx)^3}+\frac{315b^5d(bc-ad)^2}{(c+dx)^2}+\frac{735b^6d(bc-ad)}{c+dx}+840b^7d\log(a+bx)-840b^7d\log(c+dx)}{105(bc-ad)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -1/105*((105*b^7*(b*c - a*d))/(a + b*x) - (15*d*(-(b*c) + a*d)^7)/(c + d*x)
^7 + (35*b*d*(b*c - a*d)^6)/(c + d*x)^6 + (63*b^2*d*(b*c - a*d)^5)/(c + d*x
)^5 + (105*b^3*d*(b*c - a*d)^4)/(c + d*x)^4 + (175*b^4*d*(b*c - a*d)^3)/(c
+ d*x)^3 + (315*b^5*d*(b*c - a*d)^2)/(c + d*x)^2 + (735*b^6*d*(b*c - a*d))/
(c + d*x) + 840*b^7*d*Log[a + b*x] - 840*b^7*d*Log[c + d*x])/(b*c - a*d)^9
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 2739 vs. 2(231) = 462.
time = 42.52, size = 2737, normalized size = 11.85

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-2)/(c + d*x)^8,x]')
```

```
[Out] (8 b ^ 7 d (a ^ 9 c ^ 7 d ^ 8 - 8 a ^ 8 b c ^ 8 d ^ 7 + 28 a ^ 7 b ^ 2 c ^
9 d ^ 6 - 56 a ^ 6 b ^ 3 c ^ 10 d ^ 5 + 70 a ^ 5 b ^ 4 c ^ 11 d ^ 4 - 56 a
^ 4 b ^ 5 c ^ 12 d ^ 3 + 28 a ^ 3 b ^ 6 c ^ 13 d ^ 2 - 8 a ^ 2 b ^ 7 c ^ 14
 d + a b ^ 8 c ^ 15 + b d ^ 7 x ^ 8 (a ^ 8 d ^ 8 - 8 a ^ 7 b c d ^ 7 + 28 a
 ^ 6 b ^ 2 c ^ 2 d ^ 6 - 56 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 70 a ^ 4 b ^ 4 c ^ 4
d ^ 4 - 56 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 28 a ^ 2 b ^ 6 c ^ 6 d ^ 2 - 8 a b ^ 7
 c ^ 7 d + b ^ 8 c ^ 8) + c ^ 6 x (7 a ^ 9 d ^ 9 - 55 a ^ 8 b c d ^ 8 + 188
 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 364 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 434 a ^ 5 b ^ 4 c
^ 4 d ^ 5 - 322 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 140 a ^ 3 b ^ 6 c ^ 6 d ^ 3 - 28
a ^ 2 b ^ 7 c ^ 7 d ^ 2 - a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + 7 c ^ 5 d x ^ 2
(3 a ^ 9 d ^ 9 - 23 a ^ 8 b c d ^ 8 + 76 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 140 a ^
6 b ^ 3 c ^ 3 d ^ 6 + 154 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 98 a ^ 4 b ^ 5 c ^ 5 d
^ 4 + 28 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 4 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 5 a b ^ 8 c
^ 8 d + b ^ 9 c ^ 9) + c ^ 4 d ^ 2 x ^ 3 (35 a ^ 9 d ^ 9 - 259 a ^ 8 b c d
^ 8 + 812 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 1372 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 1274 a ^
 5 b ^ 4 c ^ 4 d ^ 5 - 490 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 196 a ^ 3 b ^ 6 c ^ 6
d ^ 3 + 308 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 133 a b ^ 8 c ^ 8 d + 21 b ^ 9 c ^ 9)
 + c ^ 3 d ^ 3 x ^ 4 (35 a ^ 9 d ^ 9 - 245 a ^ 8 b c d ^ 8 + 700 a ^ 7 b ^
2 c ^ 2 d ^ 7 - 980 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 490 a ^ 5 b ^ 4 c ^ 4 d ^ 5 +
 490 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 980 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 700 a ^ 2 b ^
7 c ^ 7 d ^ 2 - 245 a b ^ 8 c ^ 8 d + 35 b ^ 9 c ^ 9) + c ^ 2 d ^ 4 x ^ 5 (
21 a ^ 9 d ^ 9 - 133 a ^ 8 b c d ^ 8 + 308 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 196 a
^ 6 b ^ 3 c ^ 3 d ^ 6 - 490 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 1274 a ^ 4 b ^ 5 c ^
5 d ^ 4 - 1372 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 812 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 259
```

a b ^ 8 c ^ 8 d + 35 b ^ 9 c ^ 9) + 7 c d ^ 5 x ^ 6 (a ^ 9 d ^ 9 - 5 a ^ 8 b c d ^ 8 + 4 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 28 a ^ 6 b ^ 3 c ^ 3 d ^ 6 - 98 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 154 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 140 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 76 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 23 a b ^ 8 c ^ 8 d + 3 b ^ 9 c ^ 9) + d ^ 6 x ^ 7 (a ^ 9 d ^ 9 - a ^ 8 b c d ^ 8 - 28 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 1 40 a ^ 6 b ^ 3 c ^ 3 d ^ 6 - 322 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 434 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 364 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 188 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 5 5 a b ^ 8 c ^ 8 d + 7 b ^ 9 c ^ 9)) (Log[(a + b x) / b] - Log[(c + d x) / d]) + (-15 a ^ 7 d ^ 7 + 125 a ^ 6 b c d ^ 6 - 463 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 1007 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 1443 a ^ 3 b ^ 4 c ^ 4 d ^ 3 + 1497 a ^ 2 b ^ 5 c ^ 5 d ^ 2 - 1443 a b ^ 6 c ^ 6 d - 105 b ^ 7 c ^ 7 - 2 b d x (-10 a ^ 6 d ^ 6 + 88 a ^ 5 b c d ^ 5 - 353 a ^ 4 b ^ 2 c ^ 2 d ^ 4 + 872 a ^ 3 b ^ 3 c ^ 3 d ^ 3 - 1578 a ^ 2 b ^ 4 c ^ 4 d ^ 2 + 2832 a b ^ 5 c ^ 5 d + 108 9 b ^ 6 c ^ 6) + b ^ 2 d ^ 2 x ^ 2 (-28 a ^ 5 d ^ 5 + 266 a ^ 4 b c d ^ 4 - 1204 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 3696 a ^ 2 b ^ 3 c ^ 3 d ^ 2 - 11004 a b ^ 4 c ^ 4 d - 9366 b ^ 5 c ^ 5) + b ^ 3 d ^ 3 x ^ 3 (42 a ^ 4 d ^ 4 - 448 a ^ 3 b c d ^ 3 + 2492 a ^ 2 b ^ 2 c ^ 2 d ^ 2 - 12208 a b ^ 3 c ^ 3 d - 19278 b ^ 4 c ^ 4) + b ^ 4 d ^ 4 x ^ 4 (-70 a ^ 3 d ^ 3 + 910 a ^ 2 b c d ^ 2 - 7910 a b ^ 2 c ^ 2 d - 22330 b ^ 3 c ^ 3) + b ^ 5 d ^ 5 x ^ 5 (140 a ^ 2 d ^ 2 - 2800 a b c d - 14980 b ^ 2 c ^ 2) + b ^ 6 d ^ 6 x ^ 6 (-420 a d - 546 0 b c) - 840 b ^ 7 d ^ 7 x ^ 7) (a d - b c) ^ 9 / 105) / ((a d - b c) ^ 9 (a ^ 9 c ^ 7 d ^ 8 - 8 a ^ 8 b c ^ 8 d ^ 7 + 28 a ^ 7 b ^ 2 c ^ 9 d ^ 6 - 56 a ^ 6 b ^ 3 c ^ 10 d ^ 5 + 70 a ^ 5 b ^ 4 c ^ 11 d ^ 4 - 56 a ^ 4 b ^ 5 c ^ 12 d ^ 3 + 28 a ^ 3 b ^ 6 c ^ 13 d ^ 2 - 8 a ^ 2 b ^ 7 c ^ 14 d + a b ^ 8 c ^ 15 + b d ^ 7 x ^ 8 (a ^ 8 d ^ 8 - 8 a ^ 7 b c d ^ 7 + 28 a ^ 6 b ^ 2 c ^ 2 d ^ 6 - 56 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 70 a ^ 4 b ^ 4 c ^ 4 d ^ 4 - 56 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 28 a ^ 2 b ^ 6 c ^ 6 d ^ 2 - 8 a b ^ 7 c ^ 7 d + b ^ 8 c ^ 8) + c ^ 6 x (7 a ^ 9 d ^ 9 - 55 a ^ 8 b c d ^ 8 + 188 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 364 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 434 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 322 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 140 a ^ 3 b ^ 6 c ^ 6 d ^ 3 - 28 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + 7 c ^ 5 d x ^ 2 (3 a ^ 9 d ^ 9 - 23 a ^ 8 b c d ^ 8 + 76 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 140 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 154 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 98 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 28 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 4 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 5 a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + 7 c ^ 4 d ^ 2 x ^ 3 (5 a ^ 9 d ^ 9 - 37 a ^ 8 b c d ^ 8 + 116 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 196 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 182 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 70 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 28 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 44 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 19 a b ^ 8 c ^ 8 d + 3 b ^ 9 c ^ 9) + 35 c ^ 3 d ^ 3 x ^ 4 (a ^ 9 d ^ 9 - 7 a ^ 8 b c d ^ 8 + 20 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 28 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 14 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 14 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 28 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 20 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 7 a b ^ 8 c ^ 8 d + b ^ 9 c ^ 9) + 7 c ^ 2 d ^ 4 x ^ 5 (3 a ^ 9 d ^ 9 - 19 a ^ 8 b c d ^ 8 + 44 a ^ 7 b ^ 2 c ^ 2 d ^ 7 - 28 a ^ 6 b ^ 3 c ^ 3 d ^ 6 - 70 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 182 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 196 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 116 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 37 a b ^ 8 c ^ 8 d + 5 b ^ 9 c ^ 9) + 7 c

```
d ^ 5 x ^ 6 (a ^ 9 d ^ 9 - 5 a ^ 8 b c d ^ 8 + 4 a ^ 7 b ^ 2 c ^ 2 d ^ 7 +
28 a ^ 6 b ^ 3 c ^ 3 d ^ 6 - 98 a ^ 5 b ^ 4 c ^ 4 d ^ 5 + 154 a ^ 4 b ^ 5
c ^ 5 d ^ 4 - 140 a ^ 3 b ^ 6 c ^ 6 d ^ 3 + 76 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 23
a b ^ 8 c ^ 8 d + 3 b ^ 9 c ^ 9) + d ^ 6 x ^ 7 (a ^ 9 d ^ 9 - a ^ 8 b c d
^ 8 - 28 a ^ 7 b ^ 2 c ^ 2 d ^ 7 + 140 a ^ 6 b ^ 3 c ^ 3 d ^ 6 - 322 a ^ 5
b ^ 4 c ^ 4 d ^ 5 + 434 a ^ 4 b ^ 5 c ^ 5 d ^ 4 - 364 a ^ 3 b ^ 6 c ^ 6 d ^
3 + 188 a ^ 2 b ^ 7 c ^ 7 d ^ 2 - 55 a b ^ 8 c ^ 8 d + 7 b ^ 9 c ^ 9)))
```

Maple [A]
time = 0.20, size = 223, normalized size = 0.97

method	result
default	$-\frac{d}{7(ad-bc)^{2}(dx+c)^{7}}-\frac{8d\,b^{7}\ln(dx+c)}{(ad-bc)^{9}}-\frac{7d\,b^{6}}{(ad-bc)^{8}(dx+c)}+\frac{3d\,b^{5}}{(ad-bc)^{7}(dx+c)^{2}}-\frac{5d\,b^{4}}{3(ad-bc)^{6}(dx+c)^{3}}+\frac{d\,b^{3}}{(ad-bc)^{5}(dx+c)^{4}}$
risch	$-\frac{8b^{7}d^{7}x^{7}}{a^{8}d^{8}-8a^{7}bc\,d^{7}+28a^{6}b^{2}c^{2}d^{6}-56a^{5}b^{3}c^{3}d^{5}+70a^{4}b^{4}c^{4}d^{4}-56a^{3}b^{5}c^{5}d^{3}+28a^{2}b^{6}c^{6}d^{2}-8a\,b^{7}c^{7}d+b^{8}c^{8}}-\frac{}{a^{8}d^{8}-8a^{7}bc\,d^{7}+28a^{6}b^{2}c^{2}d^{6}-56a^{5}b^{3}c^{3}d}$
norman	$-\frac{8b^{7}d^{7}x^{7}}{a^{8}d^{8}-8a^{7}bc\,d^{7}+28a^{6}b^{2}c^{2}d^{6}-56a^{5}b^{3}c^{3}d^{5}+70a^{4}b^{4}c^{4}d^{4}-56a^{3}b^{5}c^{5}d^{3}+28a^{2}b^{6}c^{6}d^{2}-8a\,b^{7}c^{7}d+b^{8}c^{8}}+\frac{}{d^{2}b\left(a^{8}d^{8}-8a^{7}bc\,d^{7}+28a^{6}b^{2}c^{2}d^{6}-56a^{5}b\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*d/(a*d-b*c)^2/(d*x+c)^7-8*d/(a*d-b*c)^9*b^7*ln(d*x+c)-7*d/(a*d-b*c)^8*
b^6/(d*x+c)+3*d/(a*d-b*c)^7*b^5/(d*x+c)^2-5/3*d/(a*d-b*c)^6*b^4/(d*x+c)^3+d
/(a*d-b*c)^5*b^3/(d*x+c)^4-3/5*d/(a*d-b*c)^4*b^2/(d*x+c)^5+1/3*d/(a*d-b*c)^
3*b/(d*x+c)^6-b^7/(a*d-b*c)^8/(b*x+a)+8*d/(a*d-b*c)^9*b^7*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1881 vs. $2(223) = 446$.
time = 0.46, size = 1881, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -8*b^7*d*log(b*x + a)/(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9) + 8*b^7*d*log(d*x + c)/
(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^
4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d
^7 + 9*a^8*b*c*d^8 - a^9*d^9) - 1/105*(840*b^7*d^7*x^7 + 105*b^7*c^7 + 1443
*a*b^6*c^6*d - 1497*a^2*b^5*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c
^3*d^4 + 463*a^5*b^2*c^2*d^5 - 125*a^6*b*c*d^6 + 15*a^7*d^7 + 420*(13*b^7*c
*d^6 + a*b^6*d^7)*x^6 + 140*(107*b^7*c^2*d^5 + 20*a*b^6*c*d^6 - a^2*b^5*d^7
```

```
)*x^5 + 70*(319*b^7*c^3*d^4 + 113*a*b^6*c^2*d^5 - 13*a^2*b^5*c*d^6 + a^3*b^
4*d^7)*x^4 + 14*(1377*b^7*c^4*d^3 + 872*a*b^6*c^3*d^4 - 178*a^2*b^5*c^2*d^5
 + 32*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 14*(669*b^7*c^5*d^2 + 786*a*b^6*
c^4*d^3 - 264*a^2*b^5*c^3*d^4 + 86*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 2*a
^5*b^2*d^7)*x^2 + 2*(1089*b^7*c^6*d + 2832*a*b^6*c^5*d^2 - 1578*a^2*b^5*c^4
*d^3 + 872*a^3*b^4*c^3*d^4 - 353*a^4*b^3*c^2*d^5 + 88*a^5*b^2*c*d^6 - 10*a^
6*b*d^7)*x)/(a*b^8*c^15 - 8*a^2*b^7*c^14*d + 28*a^3*b^6*c^13*d^2 - 56*a^4*b
^5*c^12*d^3 + 70*a^5*b^4*c^11*d^4 - 56*a^6*b^3*c^10*d^5 + 28*a^7*b^2*c^9*d^
6 - 8*a^8*b*c^8*d^7 + a^9*c^7*d^8 + (b^9*c^8*d^7 - 8*a*b^8*c^7*d^8 + 28*a^2
*b^7*c^6*d^9 - 56*a^3*b^6*c^5*d^10 + 70*a^4*b^5*c^4*d^11 - 56*a^5*b^4*c^3*d
^12 + 28*a^6*b^3*c^2*d^13 - 8*a^7*b^2*c*d^14 + a^8*b*d^15)*x^8 + (7*b^9*c^9
*d^6 - 55*a*b^8*c^8*d^7 + 188*a^2*b^7*c^7*d^8 - 364*a^3*b^6*c^6*d^9 + 434*a
^4*b^5*c^5*d^10 - 322*a^5*b^4*c^4*d^11 + 140*a^6*b^3*c^3*d^12 - 28*a^7*b^2*
c^2*d^13 - a^8*b*c*d^14 + a^9*d^15)*x^7 + 7*(3*b^9*c^10*d^5 - 23*a*b^8*c^9*
d^6 + 76*a^2*b^7*c^8*d^7 - 140*a^3*b^6*c^7*d^8 + 154*a^4*b^5*c^6*d^9 - 98*a
^5*b^4*c^5*d^10 + 28*a^6*b^3*c^4*d^11 + 4*a^7*b^2*c^3*d^12 - 5*a^8*b*c^2*d^
13 + a^9*c*d^14)*x^6 + 7*(5*b^9*c^11*d^4 - 37*a*b^8*c^10*d^5 + 116*a^2*b^7*
c^9*d^6 - 196*a^3*b^6*c^8*d^7 + 182*a^4*b^5*c^7*d^8 - 70*a^5*b^4*c^6*d^9 -
28*a^6*b^3*c^5*d^10 + 44*a^7*b^2*c^4*d^11 - 19*a^8*b*c^3*d^12 + 3*a^9*c^2*d
^13)*x^5 + 35*(b^9*c^12*d^3 - 7*a*b^8*c^11*d^4 + 20*a^2*b^7*c^10*d^5 - 28*a
^3*b^6*c^9*d^6 + 14*a^4*b^5*c^8*d^7 + 14*a^5*b^4*c^7*d^8 - 28*a^6*b^3*c^6*d
^9 + 20*a^7*b^2*c^5*d^10 - 7*a^8*b*c^4*d^11 + a^9*c^3*d^12)*x^4 + 7*(3*b^9*
c^13*d^2 - 19*a*b^8*c^12*d^3 + 44*a^2*b^7*c^11*d^4 - 28*a^3*b^6*c^10*d^5 -
70*a^4*b^5*c^9*d^6 + 182*a^5*b^4*c^8*d^7 - 196*a^6*b^3*c^7*d^8 + 116*a^7*b^
2*c^6*d^9 - 37*a^8*b*c^5*d^10 + 5*a^9*c^4*d^11)*x^3 + 7*(b^9*c^14*d - 5*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 + 28*a^3*b^6*c^11*d^4 - 98*a^4*b^5*c^10*d^
5 + 154*a^5*b^4*c^9*d^6 - 140*a^6*b^3*c^8*d^7 + 76*a^7*b^2*c^7*d^8 - 23*a^8
*b*c^6*d^9 + 3*a^9*c^5*d^10)*x^2 + (b^9*c^15 - a*b^8*c^14*d - 28*a^2*b^7*c^
13*d^2 + 140*a^3*b^6*c^12*d^3 - 322*a^4*b^5*c^11*d^4 + 434*a^5*b^4*c^10*d^5
 - 364*a^6*b^3*c^9*d^6 + 188*a^7*b^2*c^8*d^7 - 55*a^8*b*c^7*d^8 + 7*a^9*c^6
*d^9)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. $2(223) = 446$.

time = 0.35, size = 2264, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*b^8*c^8 + 1338*a*b^7*c^7*d - 2940*a^2*b^6*c^6*d^2 + 2940*a^3*b^
5*c^5*d^3 - 2450*a^4*b^4*c^4*d^4 + 1470*a^5*b^3*c^3*d^5 - 588*a^6*b^2*c^2*d
^6 + 140*a^7*b*c*d^7 - 15*a^8*d^8 + 840*(b^8*c*d^7 - a*b^7*d^8)*x^7 + 420*(
13*b^8*c^2*d^6 - 12*a*b^7*c*d^7 - a^2*b^6*d^8)*x^6 + 140*(107*b^8*c^3*d^5 -
 87*a*b^7*c^2*d^6 - 21*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 70*(319*b^8*c^4*d
```

```
^4 - 206*a*b^7*c^3*d^5 - 126*a^2*b^6*c^2*d^6 + 14*a^3*b^5*c*d^7 - a^4*b^4*d
^8)*x^4 + 14*(1377*b^8*c^5*d^3 - 505*a*b^7*c^4*d^4 - 1050*a^2*b^6*c^3*d^5 +
 210*a^3*b^5*c^2*d^6 - 35*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 14*(669*b^8*
c^6*d^2 + 117*a*b^7*c^5*d^3 - 1050*a^2*b^6*c^4*d^4 + 350*a^3*b^5*c^3*d^5 -
105*a^4*b^4*c^2*d^6 + 21*a^5*b^3*c*d^7 - 2*a^6*b^2*d^8)*x^2 + 2*(1089*b^8*c
^7*d + 1743*a*b^7*c^6*d^2 - 4410*a^2*b^6*c^5*d^3 + 2450*a^3*b^5*c^4*d^4 - 1
225*a^4*b^4*c^3*d^5 + 441*a^5*b^3*c^2*d^6 - 98*a^6*b^2*c*d^7 + 10*a^7*b*d^8
)*x + 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d^8)*x^7 + 7*(3
*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*c^2*d^6)*x^5 +
 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*a*b^7*c^4*d^4)
*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7*a*b^7*c^6*d^2
)*x)*log(b*x + a) - 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d
^8)*x^7 + 7*(3*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*
c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*
a*b^7*c^4*d^4)*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7
*a*b^7*c^6*d^2)*x)*log(d*x + c))/(a*b^9*c^16 - 9*a^2*b^8*c^15*d + 36*a^3*b^
7*c^14*d^2 - 84*a^4*b^6*c^13*d^3 + 126*a^5*b^5*c^12*d^4 - 126*a^6*b^4*c^11*
d^5 + 84*a^7*b^3*c^10*d^6 - 36*a^8*b^2*c^9*d^7 + 9*a^9*b*c^8*d^8 - a^10*c^7
*d^9 + (b^10*c^9*d^7 - 9*a*b^9*c^8*d^8 + 36*a^2*b^8*c^7*d^9 - 84*a^3*b^7*c^
6*d^10 + 126*a^4*b^6*c^5*d^11 - 126*a^5*b^5*c^4*d^12 + 84*a^6*b^4*c^3*d^13
- 36*a^7*b^3*c^2*d^14 + 9*a^8*b^2*c*d^15 - a^9*b*d^16)*x^8 + (7*b^10*c^10*d
^6 - 62*a*b^9*c^9*d^7 + 243*a^2*b^8*c^8*d^8 - 552*a^3*b^7*c^7*d^9 + 798*a^4
*b^6*c^6*d^10 - 756*a^5*b^5*c^5*d^11 + 462*a^6*b^4*c^4*d^12 - 168*a^7*b^3*c
^3*d^13 + 27*a^8*b^2*c^2*d^14 + 2*a^9*b*c*d^15 - a^10*d^16)*x^7 + 7*(3*b^10
*c^11*d^5 - 26*a*b^9*c^10*d^6 + 99*a^2*b^8*c^9*d^7 - 216*a^3*b^7*c^8*d^8 +
294*a^4*b^6*c^7*d^9 - 252*a^5*b^5*c^6*d^10 + 126*a^6*b^4*c^5*d^11 - 24*a^7*
b^3*c^4*d^12 - 9*a^8*b^2*c^3*d^13 + 6*a^9*b*c^2*d^14 - a^10*c*d^15)*x^6 + 7
*(5*b^10*c^12*d^4 - 42*a*b^9*c^11*d^5 + 153*a^2*b^8*c^10*d^6 - 312*a^3*b^7*
c^9*d^7 + 378*a^4*b^6*c^8*d^8 - 252*a^5*b^5*c^7*d^9 + 42*a^6*b^4*c^6*d^10 +
 72*a^7*b^3*c^5*d^11 - 63*a^8*b^2*c^4*d^12 + 22*a^9*b*c^3*d^13 - 3*a^10*c^2
*d^14)*x^5 + 35*(b^10*c^13*d^3 - 8*a*b^9*c^12*d^4 + 27*a^2*b^8*c^11*d^5 - 4
8*a^3*b^7*c^10*d^6 + 42*a^4*b^6*c^9*d^7 - 42*a^6*b^4*c^7*d^9 + 48*a^7*b^3*c
^6*d^10 - 27*a^8*b^2*c^5*d^11 + 8*a^9*b*c^4*d^12 - a^10*c^3*d^13)*x^4 + 7*(
3*b^10*c^14*d^2 - 22*a*b^9*c^13*d^3 + 63*a^2*b^8*c^12*d^4 - 72*a^3*b^7*c^11
*d^5 - 42*a^4*b^6*c^10*d^6 + 252*a^5*b^5*c^9*d^7 - 378*a^6*b^4*c^8*d^8 + 31
2*a^7*b^3*c^7*d^9 - 153*a^8*b^2*c^6*d^10 + 42*a^9*b*c^5*d^11 - 5*a^10*c^4*d
^12)*x^3 + 7*(b^10*c^15*d - 6*a*b^9*c^14*d^2 + 9*a^2*b^8*c^13*d^3 + 24*a^3*
b^7*c^12*d^4 - 126*a^4*b^6*c^11*d^5 + 252*a^5*b^5*c^10*d^6 - 294*a^6*b^4*c^
9*d^7 + 216*a^7*b^3*c^8*d^8 - 99*a^8*b^2*c^7*d^9 + 26*a^9*b*c^6*d^10 - 3*a^
10*c^5*d^11)*x^2 + (b^10*c^16 - 2*a*b^9*c^15*d - 27*a^2*b^8*c^14*d^2 + 168*
a^3*b^7*c^13*d^3 - 462*a^4*b^6*c^12*d^4 + 756*a^5*b^5*c^11*d^5 - 798*a^6*b^
4*c^10*d^6 + 552*a^7*b^3*c^9*d^7 - 243*a^8*b^2*c^8*d^8 + 62*a^9*b*c^7*d^9 -
 7*a^10*c^6*d^10)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2336 vs.

2(209) = 418.
time = 25.48, size = 2336, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] -8*b**7*d*log(x + (-8*a**10*b**7*d**11/(a*d - b*c)**9 + 80*a**9*b**8*c*d**1
0/(a*d - b*c)**9 - 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 + 960*a**7*b**10*
c**3*d**8/(a*d - b*c)**9 - 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 + 2016*
a**5*b**12*c**5*d**6/(a*d - b*c)**9 - 1680*a**4*b**13*c**6*d**5/(a*d - b*c)
**9 + 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 - 360*a**2*b**15*c**8*d**3/(a
*d - b*c)**9 + 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 - 8*b**1
7*c**10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + 8*b
**7*d*log(x + (8*a**10*b**7*d**11/(a*d - b*c)**9 - 80*a**9*b**8*c*d**10/(a*
d - b*c)**9 + 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 - 960*a**7*b**10*c**3*
d**8/(a*d - b*c)**9 + 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 - 2016*a**5*
b**12*c**5*d**6/(a*d - b*c)**9 + 1680*a**4*b**13*c**6*d**5/(a*d - b*c)**9 -
 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 + 360*a**2*b**15*c**8*d**3/(a*d -
b*c)**9 - 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 + 8*b**17*c**
10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + (-15*a**
7*d**7 + 125*a**6*b*c*d**6 - 463*a**5*b**2*c**2*d**5 + 1007*a**4*b**3*c**3*
d**4 - 1443*a**3*b**4*c**4*d**3 + 1497*a**2*b**5*c**5*d**2 - 1443*a*b**6*c*
*6*d - 105*b**7*c**7 - 840*b**7*d**7*x**7 + x**6*(-420*a*b**6*d**7 - 5460*b
**7*c*d**6) + x**5*(140*a**2*b**5*d**7 - 2800*a*b**6*c*d**6 - 14980*b**7*c*
*2*d**5) + x**4*(-70*a**3*b**4*d**7 + 910*a**2*b**5*c*d**6 - 7910*a*b**6*c*
*2*d**5 - 22330*b**7*c**3*d**4) + x**3*(42*a**4*b**3*d**7 - 448*a**3*b**4*c
*d**6 + 2492*a**2*b**5*c**2*d**5 - 12208*a*b**6*c**3*d**4 - 19278*b**7*c**4
*d**3) + x**2*(-28*a**5*b**2*d**7 + 266*a**4*b**3*c*d**6 - 1204*a**3*b**4*c
**2*d**5 + 3696*a**2*b**5*c**3*d**4 - 11004*a*b**6*c**4*d**3 - 9366*b**7*c*
*5*d**2) + x*(20*a**6*b*d**7 - 176*a**5*b**2*c*d**6 + 706*a**4*b**3*c**2*d*
*5 - 1744*a**3*b**4*c**3*d**4 + 3156*a**2*b**5*c**4*d**3 - 5664*a*b**6*c**5
*d**2 - 2178*b**7*c**6*d))/(105*a**9*c**7*d**8 - 840*a**8*b*c**8*d**7 + 294
0*a**7*b**2*c**9*d**6 - 5880*a**6*b**3*c**10*d**5 + 7350*a**5*b**4*c**11*d*
*4 - 5880*a**4*b**5*c**12*d**3 + 2940*a**3*b**6*c**13*d**2 - 840*a**2*b**7*
c**14*d + 105*a*b**8*c**15 + x**8*(105*a**8*b*d**15 - 840*a**7*b**2*c*d**14
 + 2940*a**6*b**3*c**2*d**13 - 5880*a**5*b**4*c**3*d**12 + 7350*a**4*b**5*c
**4*d**11 - 5880*a**3*b**6*c**5*d**10 + 2940*a**2*b**7*c**6*d**9 - 840*a*b*
*8*c**7*d**8 + 105*b**9*c**8*d**7) + x**7*(105*a**9*d**15 - 105*a**8*b*c*d*
*14 - 2940*a**7*b**2*c**2*d**13 + 14700*a**6*b**3*c**3*d**12 - 33810*a**5*b
**4*c**4*d**11 + 45570*a**4*b**5*c**5*d**10 - 38220*a**3*b**6*c**6*d**9 + 1
9740*a**2*b**7*c**7*d**8 - 5775*a*b**8*c**8*d**7 + 735*b**9*c**9*d**6) + x*
*6*(735*a**9*c*d**14 - 3675*a**8*b*c**2*d**13 + 2940*a**7*b**2*c**3*d**12 +
 20580*a**6*b**3*c**4*d**11 - 72030*a**5*b**4*c**5*d**10 + 113190*a**4*b**5
*c**6*d**9 - 102900*a**3*b**6*c**7*d**8 + 55860*a**2*b**7*c**8*d**7 - 16905
```

```
*a*b**8*c**9*d**6 + 2205*b**9*c**10*d**5) + x**5*(2205*a**9*c**2*d**13 - 13
965*a**8*b*c**3*d**12 + 32340*a**7*b**2*c**4*d**11 - 20580*a**6*b**3*c**5*d
**10 - 51450*a**5*b**4*c**6*d**9 + 133770*a**4*b**5*c**7*d**8 - 144060*a**3
*b**6*c**8*d**7 + 85260*a**2*b**7*c**9*d**6 - 27195*a*b**8*c**10*d**5 + 367
5*b**9*c**11*d**4) + x**4*(3675*a**9*c**3*d**12 - 25725*a**8*b*c**4*d**11 +
 73500*a**7*b**2*c**5*d**10 - 102900*a**6*b**3*c**6*d**9 + 51450*a**5*b**4*
c**7*d**8 + 51450*a**4*b**5*c**8*d**7 - 102900*a**3*b**6*c**9*d**6 + 73500*
a**2*b**7*c**10*d**5 - 25725*a*b**8*c**11*d**4 + 3675*b**9*c**12*d**3) + x*
*3*(3675*a**9*c**4*d**11 - 27195*a**8*b*c**5*d**10 + 85260*a**7*b**2*c**6*d
**9 - 144060*a**6*b**3*c**7*d**8 + 133770*a**5*b**4*c**8*d**7 - 51450*a**4*
b**5*c**9*d**6 - 20580*a**3*b**6*c**10*d**5 + 32340*a**2*b**7*c**11*d**4 -
13965*a*b**8*c**12*d**3 + 2205*b**9*c**13*d**2) + x**2*(2205*a**9*c**5*d**1
0 - 16905*a**8*b*c**6*d**9 + 55860*a**7*b**2*c**7*d**8 - 102900*a**6*b**3*c
**8*d**7 + 113190*a**5*b**4*c**9*d**6 - 72030*a**4*b**5*c**10*d**5 + 20580*
a**3*b**6*c**11*d**4 + 2940*a**2*b**7*c**12*d**3 - 3675*a*b**8*c**13*d**2 +
 735*b**9*c**14*d) + x*(735*a**9*c**6*d**9 - 5775*a**8*b*c**7*d**8 + 19740*
a**7*b**2*c**8*d**7 - 38220*a**6*b**3*c**9*d**6 + 45570*a**5*b**4*c**10*d**
5 - 33810*a**4*b**5*c**11*d**4 + 14700*a**3*b**6*c**12*d**3 - 2940*a**2*b**
7*c**13*d**2 - 105*a*b**8*c**14*d + 105*b**9*c**15))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(223) = 446$.
time = 0.01, size = 894, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -8*b^8*d*log(abs(b*x + a))/(b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 -
 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^
4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9) + 8*b^7*d^2*l
og(abs(d*x + c))/(b^9*c^9*d - 9*a*b^8*c^8*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3
*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d
^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a^9*d^10) - 1/105*(105*b^8*c^8 +
1338*a*b^7*c^7*d - 2940*a^2*b^6*c^6*d^2 + 2940*a^3*b^5*c^5*d^3 - 2450*a^4*b
^4*c^4*d^4 + 1470*a^5*b^3*c^3*d^5 - 588*a^6*b^2*c^2*d^6 + 140*a^7*b*c*d^7 -
 15*a^8*d^8 + 840*(b^8*c*d^7 - a*b^7*d^8)*x^7 + 420*(13*b^8*c^2*d^6 - 12*a*
b^7*c*d^7 - a^2*b^6*d^8)*x^6 + 140*(107*b^8*c^3*d^5 - 87*a*b^7*c^2*d^6 - 21
*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 70*(319*b^8*c^4*d^4 - 206*a*b^7*c^3*d^5
 - 126*a^2*b^6*c^2*d^6 + 14*a^3*b^5*c*d^7 - a^4*b^4*d^8)*x^4 + 14*(1377*b^8
*c^5*d^3 - 505*a*b^7*c^4*d^4 - 1050*a^2*b^6*c^3*d^5 + 210*a^3*b^5*c^2*d^6 -
 35*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 14*(669*b^8*c^6*d^2 + 117*a*b^7*c^
5*d^3 - 1050*a^2*b^6*c^4*d^4 + 350*a^3*b^5*c^3*d^5 - 105*a^4*b^4*c^2*d^6 +
21*a^5*b^3*c*d^7 - 2*a^6*b^2*d^8)*x^2 + 2*(1089*b^8*c^7*d + 1743*a*b^7*c^6*
```

```
d^2 - 4410*a^2*b^6*c^5*d^3 + 2450*a^3*b^5*c^4*d^4 - 1225*a^4*b^4*c^3*d^5 +
441*a^5*b^3*c^2*d^6 - 98*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x)/((b*c - a*d)^9*(b
*x + a)*(d*x + c)^7)
```

Mupad [B]
time = 1.39, size = 1738, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^8),x)
```

```
[Out] (16*b^7*d*atanh((a^9*d^9 + b^9*c^9 + 20*a^2*b^7*c^7*d^2 - 28*a^3*b^6*c^6*d^
3 + 14*a^4*b^5*c^5*d^4 + 14*a^5*b^4*c^4*d^5 - 28*a^6*b^3*c^3*d^6 + 20*a^7*b
^2*c^2*d^7 - 7*a*b^8*c^8*d - 7*a^8*b*c*d^8)/(a*d - b*c)^9 + (2*b*d*x*(a^8*d
^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)
)/(a*d - b*c)^9))/(a*d - b*c)^9 - ((15*a^7*d^7 + 105*b^7*c^7 - 1497*a^2*b^5
*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c^3*d^4 + 463*a^5*b^2*c^2*d^
5 + 1443*a*b^6*c^6*d - 125*a^6*b*c*d^6)/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*b^
6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (4*b^5*x^5*(107*b^2*
c^2*d^5 - a^2*d^7 + 20*a*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^2*x^2*(2*a^5*d^7 + 669
*b^5*c^5*d^2 + 786*a*b^4*c^4*d^3 - 264*a^2*b^3*c^3*d^4 + 86*a^3*b^2*c^2*d^5
 - 19*a^4*b*c*d^6))/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^4*x^4*(a^3*d^7 + 319*b^3*c^3*d^4 + 1
13*a*b^2*c^2*d^5 - 13*a^2*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b*x*(1089*b^6*c^6*d - 1
0*a^6*d^7 + 2832*a*b^5*c^5*d^2 - 1578*a^2*b^4*c^4*d^3 + 872*a^3*b^3*c^3*d^4
 - 353*a^4*b^2*c^2*d^5 + 88*a^5*b*c*d^6))/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*
b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (8*b^7*d^7*x^7)/(a
^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*
d^7) + (4*b^6*x^6*(a*d^7 + 13*b*c*d^6))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^
6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (2*b^3*x^3*(1377*b^4*c^4*d
^3 - 3*a^4*d^7 + 872*a*b^3*c^3*d^4 - 178*a^2*b^2*c^2*d^5 + 32*a^3*b*c*d^6))
/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a
^7*b*c*d^7)))/(x^7*(a*d^7 + 7*b*c*d^6) + x^3*(35*a*c^4*d^3 + 21*b*c^5*d^2)
+ x^5*(21*a*c^2*d^5 + 35*b*c^3*d^4) + x^4*(35*a*c^3*d^4 + 35*b*c^4*d^3) + a
```

```
*c^7 + x*(b*c^7 + 7*a*c^6*d) + x^2*(21*a*c^5*d^2 + 7*b*c^6*d) + x^6*(21*b*c
^2*d^5 + 7*a*c*d^6) + b*d^7*x^8)
```

3.1374 $\int \frac{1}{(a+bx)^3(c+dx)^8}\,dx$

Optimal. Leaf size=276

$$-\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+\frac{bd^2}{2(bc-ad)^4(c+dx)^6}+\frac{6b^2d^2}{5(bc-ad)^5(c+dx)^5}$$

```
[Out] -1/2*b^7/(-a*d+b*c)^8/(b*x+a)^2+8*b^7*d/(-a*d+b*c)^9/(b*x+a)+1/7*d^2/(-a*d+
b*c)^3/(d*x+c)^7+1/2*b*d^2/(-a*d+b*c)^4/(d*x+c)^6+6/5*b^2*d^2/(-a*d+b*c)^5/
(d*x+c)^5+5/2*b^3*d^2/(-a*d+b*c)^6/(d*x+c)^4+5*b^4*d^2/(-a*d+b*c)^7/(d*x+c)
^3+21/2*b^5*d^2/(-a*d+b*c)^8/(d*x+c)^2+28*b^6*d^2/(-a*d+b*c)^9/(d*x+c)+36*b
^7*d^2*ln(b*x+a)/(-a*d+b*c)^10-36*b^7*d^2*ln(d*x+c)/(-a*d+b*c)^10
```

Rubi [A]

time = 0.25, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$\frac{36b^7d^2\log(a+bx)}{(bc-ad)^{10}}-\frac{36b^7d^2\log(c+dx)}{(bc-ad)^{10}}+\frac{8b^7d}{(a+bx)(bc-ad)^9}-\frac{b^7}{2(a+bx)^2(bc-ad)^8}+\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21b^5d^2}{2(c+dx)^2(bc-ad)^8}+\frac{5b^4d^2}{(c+dx)^3(bc-ad)^7}+\frac{5b^3d^2}{2(c+dx)^4(bc-ad)^6}+\frac{6b^2d^2}{5(c+dx)^5(bc-ad)^5}+\frac{bd^2}{2(c+dx)^6(bc-ad)^4}+\frac{d^2}{7(c+dx)^7(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] -1/2*b^7/((b*c - a*d)^8*(a + b*x)^2) + (8*b^7*d)/((b*c - a*d)^9*(a + b*x))
+ d^2/(7*(b*c - a*d)^3*(c + d*x)^7) + (b*d^2)/(2*(b*c - a*d)^4*(c + d*x)^6)
 + (6*b^2*d^2)/(5*(b*c - a*d)^5*(c + d*x)^5) + (5*b^3*d^2)/(2*(b*c - a*d)^6
*(c + d*x)^4) + (5*b^4*d^2)/((b*c - a*d)^7*(c + d*x)^3) + (21*b^5*d^2)/(2*(
b*c - a*d)^8*(c + d*x)^2) + (28*b^6*d^2)/((b*c - a*d)^9*(c + d*x)) + (36*b^
7*d^2*Log[a + b*x])/(b*c - a*d)^10 - (36*b^7*d^2*Log[c + d*x])/(b*c - a*d)^
10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^8}\,dx = \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^3}-\frac{8b^8d}{(bc-ad)^9(a+bx)^2}+\frac{36b^8d^2}{(bc-ad)^{10}(a+bx)}-\right.$$
$$= -\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+$$

Mathematica [**A**]
time = 0.12, size = 254, normalized size = 0.92

$$\frac{-\frac{35b^7(bc-ad)^2}{(a+bx)^2}+\frac{560b^7d(bc-ad)}{a+bx}+\frac{10d^2(bc-ad)^7}{(c+dx)^7}+\frac{35bd^2(bc-ad)^6}{(c+dx)^6}+\frac{84b^2d^2(bc-ad)^5}{(c+dx)^5}+\frac{175b^3d^2(bc-ad)^4}{(c+dx)^4}+\frac{350b^4d^2(bc-ad)^3}{(c+dx)^3}+\frac{735b^5d^2(bc-ad)^2}{(c+dx)^2}+\frac{1960b^6d^2(bc-ad)}{c+dx}+2520b^7d^2\log(a+bx)-2520b^7d^2\log(c+dx)}{70(bc-ad)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] ((-35*b^7*(b*c - a*d)^2)/(a + b*x)^2 + (560*b^7*d*(b*c - a*d))/(a + b*x) +
(10*d^2*(b*c - a*d)^7)/(c + d*x)^7 + (35*b*d^2*(b*c - a*d)^6)/(c + d*x)^6 +
 (84*b^2*d^2*(b*c - a*d)^5)/(c + d*x)^5 + (175*b^3*d^2*(b*c - a*d)^4)/(c +
d*x)^4 + (350*b^4*d^2*(b*c - a*d)^3)/(c + d*x)^3 + (735*b^5*d^2*(b*c - a*d)
^2)/(c + d*x)^2 + (1960*b^6*d^2*(b*c - a*d))/(c + d*x) + 2520*b^7*d^2*Log[a
 + b*x] - 2520*b^7*d^2*Log[c + d*x])/(70*(b*c - a*d)^10)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 2097 vs. 2(276) = 552.
time = 65.95, size = 2095, normalized size = 7.59

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^(-3)/(c + d*x)^8,x]')
```

```
[Out] -36 b ^ 7 d ^ 2 Log[c / d + x] / (a d - b c) ^ 10 + 36 b ^ 7 d ^ 2 Log[a /
b + x] / (a d - b c) ^ 10 - (10 a ^ 8 d ^ 8 - 95 a ^ 7 b c d ^ 7 + 409 a ^
6 b ^ 2 c ^ 2 d ^ 6 - 1061 a ^ 5 b ^ 3 c ^ 3 d ^ 5 + 1879 a ^ 4 b ^ 4 c ^ 4
 d ^ 4 - 2531 a ^ 3 b ^ 5 c ^ 5 d ^ 3 + 3349 a ^ 2 b ^ 6 c ^ 6 d ^ 2 + 595
a b ^ 7 c ^ 7 d - 35 b ^ 8 c ^ 8 + 3 b d x (-5 a ^ 7 d ^ 7 + 51 a ^ 6 b c d
 ^ 6 - 243 a ^ 5 b ^ 2 c ^ 2 d ^ 5 + 737 a ^ 4 b ^ 3 c ^ 3 d ^ 4 - 1713 a ^
 3 b ^ 4 c ^ 4 d ^ 3 + 4167 a ^ 2 b ^ 5 c ^ 5 d ^ 2 + 3621 a b ^ 6 c ^ 6 d
+ 105 b ^ 7 c ^ 7) + 6 b ^ 2 d ^ 2 x ^ 2 (4 a ^ 6 d ^ 6 - 45 a ^ 5 b c d ^
5 + 249 a ^ 4 b ^ 2 c ^ 2 d ^ 4 - 976 a ^ 3 b ^ 3 c ^ 3 d ^ 3 + 3924 a ^ 2
b ^ 4 c ^ 4 d ^ 2 + 7515 a b ^ 5 c ^ 5 d + 1089 b ^ 6 c ^ 6) + 42 b ^ 3 d ^
 3 x ^ 3 (-a ^ 5 d ^ 5 + 13 a ^ 4 b c d ^ 4 - 92 a ^ 3 b ^ 2 c ^ 2 d ^ 3 +
608 a ^ 2 b ^ 3 c ^ 3 d ^ 2 + 2163 a b ^ 4 c ^ 4 d + 669 b ^ 5 c ^ 5) + 42
b ^ 4 d ^ 4 x ^ 4 (2 a ^ 4 d ^ 4 - 33 a ^ 3 b c d ^ 3 + 387 a ^ 2 b ^ 2 c ^
 2 d ^ 2 + 2467 a b ^ 3 c ^ 3 d + 1377 b ^ 4 c ^ 4) + 210 b ^ 5 d ^ 5 x ^ 5
 (-a ^ 3 d ^ 3 + 27 a ^ 2 b c d ^ 2 + 327 a b ^ 2 c ^ 2 d + 319 b ^ 3 c ^ 3
) + 420 b ^ 6 d ^ 6 x ^ 6 (2 a ^ 2 d ^ 2 + 59 a b c d + 107 b ^ 2 c ^ 2) +
1260 b ^ 7 d ^ 7 x ^ 7 (3 a d + 13 b c) + 2520 b ^ 8 d ^ 8 x ^ 8) / (70 (a
^ 11 c ^ 7 d ^ 9 - 9 a ^ 10 b c ^ 8 d ^ 8 + 36 a ^ 9 b ^ 2 c ^ 9 d ^ 7 - 84
 a ^ 8 b ^ 3 c ^ 10 d ^ 6 + 126 a ^ 7 b ^ 4 c ^ 11 d ^ 5 - 126 a ^ 6 b ^ 5
c ^ 12 d ^ 4 + 84 a ^ 5 b ^ 6 c ^ 13 d ^ 3 - 36 a ^ 4 b ^ 7 c ^ 14 d ^ 2 +
9 a ^ 3 b ^ 8 c ^ 15 d - a ^ 2 b ^ 9 c ^ 16 - a c ^ 6 x (-7 a ^ 10 d ^ 10 +
 61 a ^ 9 b c d ^ 9 - 234 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 516 a ^ 7 b ^ 3 c ^ 3 d
 ^ 7 - 714 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 630 a ^ 5 b ^ 5 c ^ 5 d ^ 5 - 336 a ^
```

4 b ^ 6 c ^ 6 d ^ 4 + 84 a ^ 3 b ^ 7 c ^ 7 d ^ 3 + 9 a ^ 2 b ^ 8 c ^ 8 d ^
2 - 11 a b ^ 9 c ^ 9 d + 2 b ^ 10 c ^ 10) + b d ^ 6 x ^ 8 (2 a ^ 10 d ^ 10
- 11 a ^ 9 b c d ^ 9 + 9 a ^ 8 b ^ 2 c ^ 2 d ^ 8 + 84 a ^ 7 b ^ 3 c ^ 3 d ^
7 - 336 a ^ 6 b ^ 4 c ^ 4 d ^ 6 + 630 a ^ 5 b ^ 5 c ^ 5 d ^ 5 - 714 a ^ 4
b ^ 6 c ^ 6 d ^ 4 + 516 a ^ 3 b ^ 7 c ^ 7 d ^ 3 - 234 a ^ 2 b ^ 8 c ^ 8 d ^
2 + 61 a b ^ 9 c ^ 9 d - 7 b ^ 10 c ^ 10) + c ^ 5 x ^ 2 (21 a ^ 11 d ^ 11
- 175 a ^ 10 b c d ^ 10 + 631 a ^ 9 b ^ 2 c ^ 2 d ^ 9 - 1269 a ^ 8 b ^ 3 c
^ 3 d ^ 8 + 1506 a ^ 7 b ^ 4 c ^ 4 d ^ 7 - 966 a ^ 6 b ^ 5 c ^ 5 d ^ 6 + 12
6 a ^ 5 b ^ 6 c ^ 6 d ^ 5 + 294 a ^ 4 b ^ 7 c ^ 7 d ^ 4 - 231 a ^ 3 b ^ 8 c
^ 8 d ^ 3 + 69 a ^ 2 b ^ 9 c ^ 9 d ^ 2 - 5 a b ^ 10 c ^ 10 d - b ^ 11 c ^
11) + 7 c ^ 4 d x ^ 3 (5 a ^ 11 d ^ 11 - 39 a ^ 10 b c d ^ 10 + 127 a ^ 9 b
^ 2 c ^ 2 d ^ 9 - 213 a ^ 8 b ^ 3 c ^ 3 d ^ 8 + 162 a ^ 7 b ^ 4 c ^ 4 d ^
7 + 42 a ^ 6 b ^ 5 c ^ 5 d ^ 6 - 210 a ^ 5 b ^ 6 c ^ 6 d ^ 5 + 198 a ^ 4 b
^ 7 c ^ 7 d ^ 4 - 87 a ^ 3 b ^ 8 c ^ 8 d ^ 3 + 13 a ^ 2 b ^ 9 c ^ 9 d ^ 2 +
3 a b ^ 10 c ^ 10 d - b ^ 11 c ^ 11) - 7 c ^ 3 d ^ 2 x ^ 4 (-5 a ^ 11 d ^
11 + 35 a ^ 10 b c d ^ 10 - 93 a ^ 9 b ^ 2 c ^ 2 d ^ 9 + 87 a ^ 8 b ^ 3 c ^
3 d ^ 8 + 102 a ^ 7 b ^ 4 c ^ 4 d ^ 7 - 378 a ^ 6 b ^ 5 c ^ 5 d ^ 6 + 462
a ^ 5 b ^ 6 c ^ 6 d ^ 5 - 282 a ^ 4 b ^ 7 c ^ 7 d ^ 4 + 63 a ^ 3 b ^ 8 c ^
8 d ^ 3 + 23 a ^ 2 b ^ 9 c ^ 9 d ^ 2 - 17 a b ^ 10 c ^ 10 d + 3 b ^ 11 c ^
11) + 7 c ^ 2 d ^ 3 x ^ 5 (3 a ^ 11 d ^ 11 - 17 a ^ 10 b c d ^ 10 + 23 a ^
9 b ^ 2 c ^ 2 d ^ 9 + 63 a ^ 8 b ^ 3 c ^ 3 d ^ 8 - 282 a ^ 7 b ^ 4 c ^ 4 d
^ 7 + 462 a ^ 6 b ^ 5 c ^ 5 d ^ 6 - 378 a ^ 5 b ^ 6 c ^ 6 d ^ 5 + 102 a ^ 4
b ^ 7 c ^ 7 d ^ 4 + 87 a ^ 3 b ^ 8 c ^ 8 d ^ 3 - 93 a ^ 2 b ^ 9 c ^ 9 d ^
2 + 35 a b ^ 10 c ^ 10 d - 5 b ^ 11 c ^ 11) + d ^ 5 x ^ 7 (a ^ 11 d ^ 11 +
5 a ^ 10 b c d ^ 10 - 69 a ^ 9 b ^ 2 c ^ 2 d ^ 9 + 231 a ^ 8 b ^ 3 c ^ 3 d
^ 8 - 294 a ^ 7 b ^ 4 c ^ 4 d ^ 7 - 126 a ^ 6 b ^ 5 c ^ 5 d ^ 6 + 966 a ^ 5
b ^ 6 c ^ 6 d ^ 5 - 1506 a ^ 4 b ^ 7 c ^ 7 d ^ 4 + 1269 a ^ 3 b ^ 8 c ^ 8
d ^ 3 - 631 a ^ 2 b ^ 9 c ^ 9 d ^ 2 + 175 a b ^ 10 c ^ 10 d - 21 b ^ 11 c ^
11) + b ^ 2 d ^ 7 x ^ 9 (a ^ 9 d ^ 9 - 9 a ^ 8 b c d ^ 8 + 36 a ^ 7 b ^ 2
c ^ 2 d ^ 7 - 84 a ^ 6 b ^ 3 c ^ 3 d ^ 6 + 126 a ^ 5 b ^ 4 c ^ 4 d ^ 5 - 12
6 a ^ 4 b ^ 5 c ^ 5 d ^ 4 + 84 a ^ 3 b ^ 6 c ^ 6 d ^ 3 - 36 a ^ 2 b ^ 7 c ^
7 d ^ 2 + 9 a b ^ 8 c ^ 8 d - b ^ 9 c ^ 9) - 7 c d ^ 4 x ^ 6 (-a ^ 11 d ^
11 + 3 a ^ 10 b c d ^ 10 + 13 a ^ 9 b ^ 2 c ^ 2 d ^ 9 - 87 a ^ 8 b ^ 3 c ^
3 d ^ 8 + 198 a ^ 7 b ^ 4 c ^ 4 d ^ 7 - 210 a ^ 6 b ^ 5 c ^ 5 d ^ 6 + 42 a
^ 5 b ^ 6 c ^ 6 d ^ 5 + 162 a ^ 4 b ^ 7 c ^ 7 d ^ 4 - 213 a ^ 3 b ^ 8 c ^ 8
d ^ 3 + 127 a ^ 2 b ^ 9 c ^ 9 d ^ 2 - 39 a b ^ 10 c ^ 10 d + 5 b ^ 11 c ^
11)))

Maple [A]

time = 0.24, size = 265, normalized size = 0.96

method	result
default	$-\frac{d^2}{7(ad-bc)^3(dx+c)^7}-\frac{36d^2b^7\ln(dx+c)}{(ad-bc)^{10}}-\frac{28d^2b^6}{(ad-bc)^9(dx+c)}+\frac{21d^2b^5}{2(ad-bc)^8(dx+c)^2}-\frac{5d^2b^4}{(ad-bc)^7(dx+c)^3}+\frac{5d^2b^3}{2(ad-bc)^6(dx+c)^4}$
risch	Expression too large to display

norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*d^2/(a*d-b*c)^3/(d*x+c)^7-36*d^2/(a*d-b*c)^10*b^7*ln(d*x+c)-28*d^2/(a*
d-b*c)^9*b^6/(d*x+c)+21/2*d^2/(a*d-b*c)^8*b^5/(d*x+c)^2-5*d^2/(a*d-b*c)^7*b
^4/(d*x+c)^3+5/2*d^2/(a*d-b*c)^6*b^3/(d*x+c)^4-6/5*d^2/(a*d-b*c)^5*b^2/(d*x
+c)^5+1/2*d^2/(a*d-b*c)^4*b/(d*x+c)^6-1/2*b^7/(a*d-b*c)^8/(b*x+a)^2+36*d^2/
(a*d-b*c)^10*b^7*ln(b*x+a)-8*b^7/(a*d-b*c)^9*d/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2399 vs. $2(264) = 528$.
time = 0.51, size = 2399, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 36*b^7*d^2*log(b*x + a)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b
^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^
10*d^10) - 36*b^7*d^2*log(d*x + c)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8
*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5
+ 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b
*c*d^9 + a^10*d^10) + 1/70*(2520*b^8*d^8*x^8 - 35*b^8*c^8 + 595*a*b^7*c^7*d
 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b^4*c^4*d^4 - 106
1*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 - 95*a^7*b*c*d^7 + 10*a^8*d^8 + 126
0*(13*b^8*c*d^7 + 3*a*b^7*d^8)*x^7 + 420*(107*b^8*c^2*d^6 + 59*a*b^7*c*d^7
+ 2*a^2*b^6*d^8)*x^6 + 210*(319*b^8*c^3*d^5 + 327*a*b^7*c^2*d^6 + 27*a^2*b^
6*c*d^7 - a^3*b^5*d^8)*x^5 + 42*(1377*b^8*c^4*d^4 + 2467*a*b^7*c^3*d^5 + 38
7*a^2*b^6*c^2*d^6 - 33*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x^4 + 42*(669*b^8*c^5
*d^3 + 2163*a*b^7*c^4*d^4 + 608*a^2*b^6*c^3*d^5 - 92*a^3*b^5*c^2*d^6 + 13*a
^4*b^4*c*d^7 - a^5*b^3*d^8)*x^3 + 6*(1089*b^8*c^6*d^2 + 7515*a*b^7*c^5*d^3
+ 3924*a^2*b^6*c^4*d^4 - 976*a^3*b^5*c^3*d^5 + 249*a^4*b^4*c^2*d^6 - 45*a^5
*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 + 3*(105*b^8*c^7*d + 3621*a*b^7*c^6*d^2 + 4
167*a^2*b^6*c^5*d^3 - 1713*a^3*b^5*c^4*d^4 + 737*a^4*b^4*c^3*d^5 - 243*a^5*
b^3*c^2*d^6 + 51*a^6*b^2*c*d^7 - 5*a^7*b*d^8)*x)/(a^2*b^9*c^16 - 9*a^3*b^8*
c^15*d + 36*a^4*b^7*c^14*d^2 - 84*a^5*b^6*c^13*d^3 + 126*a^6*b^5*c^12*d^4 -
 126*a^7*b^4*c^11*d^5 + 84*a^8*b^3*c^10*d^6 - 36*a^9*b^2*c^9*d^7 + 9*a^10*b
*c^8*d^8 - a^11*c^7*d^9 + (b^11*c^9*d^7 - 9*a*b^10*c^8*d^8 + 36*a^2*b^9*c^7
*d^9 - 84*a^3*b^8*c^6*d^10 + 126*a^4*b^7*c^5*d^11 - 126*a^5*b^6*c^4*d^12 +
84*a^6*b^5*c^3*d^13 - 36*a^7*b^4*c^2*d^14 + 9*a^8*b^3*c*d^15 - a^9*b^2*d^16
)*x^9 + (7*b^11*c^10*d^6 - 61*a*b^10*c^9*d^7 + 234*a^2*b^9*c^8*d^8 - 516*a^
3*b^8*c^7*d^9 + 714*a^4*b^7*c^6*d^10 - 630*a^5*b^6*c^5*d^11 + 336*a^6*b^5*c
```

```
^4*d^12 - 84*a^7*b^4*c^3*d^13 - 9*a^8*b^3*c^2*d^14 + 11*a^9*b^2*c*d^15 - 2*
a^10*b*d^16)*x^8 + (21*b^11*c^11*d^5 - 175*a*b^10*c^10*d^6 + 631*a^2*b^9*c^
9*d^7 - 1269*a^3*b^8*c^8*d^8 + 1506*a^4*b^7*c^7*d^9 - 966*a^5*b^6*c^6*d^10
+ 126*a^6*b^5*c^5*d^11 + 294*a^7*b^4*c^4*d^12 - 231*a^8*b^3*c^3*d^13 + 69*a
^9*b^2*c^2*d^14 - 5*a^10*b*c*d^15 - a^11*d^16)*x^7 + 7*(5*b^11*c^12*d^4 - 3
9*a*b^10*c^11*d^5 + 127*a^2*b^9*c^10*d^6 - 213*a^3*b^8*c^9*d^7 + 162*a^4*b^
7*c^8*d^8 + 42*a^5*b^6*c^7*d^9 - 210*a^6*b^5*c^6*d^10 + 198*a^7*b^4*c^5*d^1
1 - 87*a^8*b^3*c^4*d^12 + 13*a^9*b^2*c^3*d^13 + 3*a^10*b*c^2*d^14 - a^11*c*
d^15)*x^6 + 7*(5*b^11*c^13*d^3 - 35*a*b^10*c^12*d^4 + 93*a^2*b^9*c^11*d^5 -
 87*a^3*b^8*c^10*d^6 - 102*a^4*b^7*c^9*d^7 + 378*a^5*b^6*c^8*d^8 - 462*a^6*
b^5*c^7*d^9 + 282*a^7*b^4*c^6*d^10 - 63*a^8*b^3*c^5*d^11 - 23*a^9*b^2*c^4*d
^12 + 17*a^10*b*c^3*d^13 - 3*a^11*c^2*d^14)*x^5 + 7*(3*b^11*c^14*d^2 - 17*a
*b^10*c^13*d^3 + 23*a^2*b^9*c^12*d^4 + 63*a^3*b^8*c^11*d^5 - 282*a^4*b^7*c^
10*d^6 + 462*a^5*b^6*c^9*d^7 - 378*a^6*b^5*c^8*d^8 + 102*a^7*b^4*c^7*d^9 +
87*a^8*b^3*c^6*d^10 - 93*a^9*b^2*c^5*d^11 + 35*a^10*b*c^4*d^12 - 5*a^11*c^3
*d^13)*x^4 + 7*(b^11*c^15*d - 3*a*b^10*c^14*d^2 - 13*a^2*b^9*c^13*d^3 + 87*
a^3*b^8*c^12*d^4 - 198*a^4*b^7*c^11*d^5 + 210*a^5*b^6*c^10*d^6 - 42*a^6*b^5
*c^9*d^7 - 162*a^7*b^4*c^8*d^8 + 213*a^8*b^3*c^7*d^9 - 127*a^9*b^2*c^6*d^10
 + 39*a^10*b*c^5*d^11 - 5*a^11*c^4*d^12)*x^3 + (b^11*c^16 + 5*a*b^10*c^15*d
 - 69*a^2*b^9*c^14*d^2 + 231*a^3*b^8*c^13*d^3 - 294*a^4*b^7*c^12*d^4 - 126*
a^5*b^6*c^11*d^5 + 966*a^6*b^5*c^10*d^6 - 1506*a^7*b^4*c^9*d^7 + 1269*a^8*b
^3*c^8*d^8 - 631*a^9*b^2*c^7*d^9 + 175*a^10*b*c^6*d^10 - 21*a^11*c^5*d^11)*
x^2 + (2*a*b^10*c^16 - 11*a^2*b^9*c^15*d + 9*a^3*b^8*c^14*d^2 + 84*a^4*b^7*
c^13*d^3 - 336*a^5*b^6*c^12*d^4 + 630*a^6*b^5*c^11*d^5 - 714*a^7*b^4*c^10*d
^6 + 516*a^8*b^3*c^9*d^7 - 234*a^9*b^2*c^8*d^8 + 61*a^10*b*c^7*d^9 - 7*a^11
*c^6*d^10)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3016 vs. $2(264) = 528$.

time = 0.37, size = 3016, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/70*(35*b^9*c^9 - 630*a*b^8*c^8*d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c
^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 2940*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6
 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a
*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7
- 420*(107*b^9*c^3*d^6 - 48*a*b^8*c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^
9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*
a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 - 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^
5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b
^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3 + 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d
^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3
```

*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 + 6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 - 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 + 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x - 2520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(b*x + a) + 2520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(d*x + c))/(a^2*b^10*c^17 - 10*a^3*b^9*c^16*d + 45*a^4*b^8*c^15*d^2 - 120*a^5*b^7*c^14*d^3 + 210*a^6*b^6*c^13*d^4 - 252*a^7*b^5*c^12*d^5 + 210*a^8*b^4*c^11*d^6 - 120*a^9*b^3*c^10*d^7 + 45*a^10*b^2*c^9*d^8 - 10*a^11*b*c^8*d^9 + a^12*c^7*d^10 + (b^12*c^10*d^7 - 10*a*b^11*c^9*d^8 + 45*a^2*b^10*c^8*d^9 - 120*a^3*b^9*c^7*d^10 + 210*a^4*b^8*c^6*d^11 - 252*a^5*b^7*c^5*d^12 + 210*a^6*b^6*c^4*d^13 - 120*a^7*b^5*c^3*d^14 + 45*a^8*b^4*c^2*d^15 - 10*a^9*b^3*c*d^16 + a^10*b^2*d^17)*x^9 + (7*b^12*c^11*d^6 - 68*a*b^11*c^10*d^7 + 295*a^2*b^10*c^9*d^8 - 750*a^3*b^9*c^8*d^9 + 1230*a^4*b^8*c^7*d^10 - 1344*a^5*b^7*c^6*d^11 + 966*a^6*b^6*c^5*d^12 - 420*a^7*b^5*c^4*d^13 + 75*a^8*b^4*c^3*d^14 + 20*a^9*b^3*c^2*d^15 - 13*a^10*b^2*c*d^16 + 2*a^11*b*d^17)*x^8 + (21*b^12*c^12*d^5 - 196*a*b^11*c^11*d^6 + 806*a^2*b^10*c^10*d^7 - 1900*a^3*b^9*c^9*d^8 + 2775*a^4*b^8*c^8*d^9 - 2472*a^5*b^7*c^7*d^10 + 1092*a^6*b^6*c^6*d^11 + 168*a^7*b^5*c^5*d^12 - 525*a^8*b^4*c^4*d^13 + 300*a^9*b^3*c^3*d^14 - 74*a^10*b^2*c^2*d^15 + 4*a^11*b*c*d^16 + a^12*d^17)*x^7 + 7*(5*b^12*c^13*d^4 - 44*a*b^11*c^12*d^5 + 166*a^2*b^10*c^11*d^6 - 340*a^3*b^9*c^10*d^7 + 375*a^4*b^8*c^9*d^8 - 120*a^5*b^7*c^8*d^9 - 252*a^6*b^6*c^7*d^10 + 408*a^7*b^5*c^6*d^11 - 285*a^8*b^4*c^5*d^12 + 100*a^9*b^3*c^4*d^13 - 10*a^10*b^2*c^3*d^14 - 4*a^11*b*c^2*d^15 + a^12*c*d^16)*x^6 + 7*(5*b^12*c^14*d^3 - 40*a*b^11*c^13*d^4 + 128*a^2*b^10*c^12*d^5 - 180*a^3*b^9*c^11*d^6 - 15*a^4*b^8*c^10*d^7 + 480*a^5*b^7*c^9*d^8 - 840*a^6*b^6*c^8*d^9 + 744*a^7*b^5*c^7*d^10 - 345*a^8*b^4*c^6*d^11 + 40*a^9*b^3*c^5*d^12 + 40*a^10*b^2*c^4*d^13 - 20*a^11*b*c^3*d^14 + 3*a^12*c^2*d^15)*x^5 + 7*(3*b^12*c^15*d^2 - 20*a*b^11*c^14*d^3 + 40*a^2*b^10*c^13*d^4 + 40*a^3*b^9*c^12*d^5 - 345*a^4*b^8*c^11*d^6 + 744*a^5*b^7*c^10*d^7 - 840*a^6*b^6*c^9*d^8 + 480*a^7*b^5*c^8*d^9 - 15*a^8*b^4*c^7*d^10 - 180*a^9*b^3*c^6*d^11 + 128*a^10*b^2*c^5*d^12 - 40*a^11*b*c^4*d^13 + 5*a^12*c^3*d^14)*x^4 + 7*(b^12*c^16*d - 4*a*b^11*c^15*d^2 - 10*a^2*b^10*c^14*d^3 + 100*a^3*b^9*c^13*d^4 - 285*a^4*b^8*c^12*d^5 + 408*a^5*b^7*c^11*d^6 - 252*a^6*b^6*c

```
^10*d^7 - 120*a^7*b^5*c^9*d^8 + 375*a^8*b^4*c^8*d^9 - 340*a^9*b^3*c^7*d^10
+ 166*a^10*b^2*c^6*d^11 - 44*a^11*b*c^5*d^12 + 5*a^12*c^4*d^13)*x^3 + (b^12
*c^17 + 4*a*b^11*c^16*d - 74*a^2*b^10*c^15*d^2 + 300*a^3*b^9*c^14*d^3 - 525
*a^4*b^8*c^13*d^4 + 168*a^5*b^7*c^12*d^5 + 1092*a^6*b^6*c^11*d^6 - 2472*a^7
*b^5*c^10*d^7 + 2775*a^8*b^4*c^9*d^8 - 1900*a^9*b^3*c^8*d^9 + 806*a^10*b^2*
c^7*d^10 - 196*a^11*b*c^6*d^11 + 21*a^12*c^5*d^12)*x^2 + (2*a*b^11*c^17 - 1
3*a^2*b^10*c^16*d + 20*a^3*b^9*c^15*d^2 + 75*a^4*b^8*c^14*d^3 - 420*a^5*b^7
*c^13*d^4 + 966*a^6*b^6*c^12*d^5 - 1344*a^7*b^5*c^11*d^6 + 1230*a^8*b^4*c^1
0*d^7 - 750*a^9*b^3*c^9*d^8 + 295*a^10*b^2*c^8*d^9 - 68*a^11*b*c^7*d^10 + 7
*a^12*c^6*d^11)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. 2(264) = 528.
time = 0.01, size = 1064, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] 36*b^8*d^2*log(abs(b*x + a))/(b^11*c^10 - 10*a*b^10*c^9*d + 45*a^2*b^9*c^8*
d^2 - 120*a^3*b^8*c^7*d^3 + 210*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 + 210
*a^6*b^5*c^4*d^6 - 120*a^7*b^4*c^3*d^7 + 45*a^8*b^3*c^2*d^8 - 10*a^9*b^2*c*
d^9 + a^10*b*d^10) - 36*b^7*d^3*log(abs(d*x + c))/(b^10*c^10*d - 10*a*b^9*c
^9*d^2 + 45*a^2*b^8*c^8*d^3 - 120*a^3*b^7*c^7*d^4 + 210*a^4*b^6*c^6*d^5 - 2
52*a^5*b^5*c^5*d^6 + 210*a^6*b^4*c^4*d^7 - 120*a^7*b^3*c^3*d^8 + 45*a^8*b^2
*c^2*d^9 - 10*a^9*b*c*d^10 + a^10*d^11) - 1/70*(35*b^9*c^9 - 630*a*b^8*c^8*
d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 29
40*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b
*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d
^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7 - 420*(107*b^9*c^3*d^6 - 48*a*b^8*
c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*
a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 -
 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3
*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3
+ 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4
```

```
*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 +
6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4
*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2
- 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 + 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^
6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^
7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x)/((b*c - a*d)^10*(b*x + a)^2*(d*x + c
)^7)
```

Mupad [B]
time = 1.91, size = 2224, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^8),x)
```

```
[Out] (72*b^7*d^2*atanh((a^10*d^10 - b^10*c^10 - 27*a^2*b^8*c^8*d^2 + 48*a^3*b^7*
c^7*d^3 - 42*a^4*b^6*c^6*d^4 + 42*a^6*b^4*c^4*d^6 - 48*a^7*b^3*c^3*d^7 + 27
*a^8*b^2*c^2*d^8 + 8*a*b^9*c^9*d - 8*a^9*b*c*d^9)/(a*d - b*c)^10 + (2*b*d*x
*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5
*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
9*a*b^8*c^8*d - 9*a^8*b*c*d^8))/(a*d - b*c)^10))/(a*d - b*c)^10 - ((10*a^8*
d^8 - 35*b^8*c^8 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b
^4*c^4*d^4 - 1061*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 + 595*a*b^7*c^7*d -
 95*a^7*b*c*d^7)/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c
^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 3
6*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b^2*x^2*(4*a^6*d^8
 + 1089*b^6*c^6*d^2 + 7515*a*b^5*c^5*d^3 + 3924*a^2*b^4*c^4*d^4 - 976*a^3*b
^3*c^3*d^5 + 249*a^4*b^2*c^2*d^6 - 45*a^5*b*c*d^7))/(35*(a^9*d^9 - b^9*c^9
- 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b
^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^
8*b*c*d^8)) + (3*b^4*x^4*(2*a^4*d^8 + 1377*b^4*c^4*d^4 + 2467*a*b^3*c^3*d^5
 + 387*a^2*b^2*c^2*d^6 - 33*a^3*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^
7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5
- 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8))
 + (3*b*x*(105*b^7*c^7*d - 5*a^7*d^8 + 3621*a*b^6*c^6*d^2 + 4167*a^2*b^5*c^
5*d^3 - 1713*a^3*b^4*c^4*d^4 + 737*a^4*b^3*c^3*d^5 - 243*a^5*b^2*c^2*d^6 +
51*a^6*b*c*d^7))/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c
^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 3
6*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (6*b^6*x^6*(2*a^2*d^8
 + 107*b^2*c^2*d^6 + 59*a*b*c*d^7))/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2
 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*
b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*b^3*
x^3*(669*b^5*c^5*d^3 - a^5*d^8 + 2163*a*b^4*c^4*d^4 + 608*a^2*b^3*c^3*d^5 -
 92*a^3*b^2*c^2*d^6 + 13*a^4*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c
^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 8
```

```
4*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) +
(3*b^5*x^5*(319*b^3*c^3*d^5 - a^3*d^8 + 327*a*b^2*c^2*d^6 + 27*a^2*b*c*d^7)
)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^
5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (36*b^8*d^8*x^8)/(a^9*d^9 - b^9*c^9 - 36*
a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^
4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c
*d^8) + (18*b^6*d*x^7*(13*b^2*c*d^6 + 3*a*b*d^7))/(a^9*d^9 - b^9*c^9 - 36*a
^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4
*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*
d^8))/(x^3*(7*b^2*c^6*d + 35*a^2*c^4*d^3 + 42*a*b*c^5*d^2) + x^6*(7*a^2*c*d
^6 + 35*b^2*c^3*d^4 + 42*a*b*c^2*d^5) + x*(7*a^2*c^6*d + 2*a*b*c^7) + x^2*(
b^2*c^7 + 21*a^2*c^5*d^2 + 14*a*b*c^6*d) + x^7*(a^2*d^7 + 21*b^2*c^2*d^5 +
14*a*b*c*d^6) + x^4*(35*a^2*c^3*d^4 + 21*b^2*c^5*d^2 + 70*a*b*c^4*d^3) + x^
5*(21*a^2*c^2*d^5 + 35*b^2*c^4*d^3 + 70*a*b*c^3*d^4) + x^8*(7*b^2*c*d^6 + 2
*a*b*d^7) + a^2*c^7 + b^2*d^7*x^9)
```

3.1375 $\int (a+bx)^5\sqrt{c+dx}\,dx$

Optimal. Leaf size=156

$$-\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{9/2}}{9d^6}-$$

```
[Out] -2/3*(-a*d+b*c)^5*(d*x+c)^(3/2)/d^6+2*b*(-a*d+b*c)^4*(d*x+c)^(5/2)/d^6-20/7
*b^2*(-a*d+b*c)^3*(d*x+c)^(7/2)/d^6+20/9*b^3*(-a*d+b*c)^2*(d*x+c)^(9/2)/d^6
-10/11*b^4*(-a*d+b*c)*(d*x+c)^(11/2)/d^6+2/13*b^5*(d*x+c)^(13/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = $\{45\}$

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+dx)^{3/2}(bc-ad)^5}{3d^6}+\frac{2b^5(c+dx)^{13/2}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b*c - a*d)^4*(c + d*x)^(
5/2))/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(b*c -
 a*d)^2*(c + d*x)^(9/2))/(9*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(11/2))/(1
1*d^6) + (2*b^5*(c + d*x)^(13/2))/(13*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5\sqrt{c+dx}\,dx=\int\left(\frac{(-bc+ad)^5\sqrt{c+dx}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{3/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^5}{d^5}\right.$$

$$=-\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}$$

Mathematica [A]
time = 0.11, size = 217, normalized size = 1.39

$$\frac{2(c+dx)^{3/2}\left(3003a^5d^5+3003a^4bd^4(-2c+3dx)+858a^3b^2d^3\left(8c^2-12cdx+15d^2x^2\right)+286a^2b^3d^2\left(-16c^3+24c^2dx-30cd^2x^2+35d^3x^3\right)+13ab^4d\left(128c^4-192c^3dx+240c^2d^2x^2-280cd^3x^3+315d^4x^4\right)+b^5\left(-256c^5+384c^4dx-480c^3d^2x^2+560c^2d^3x^3-630cd^4x^4+693d^5x^5\right)\right)}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(3003*a^5*d^5 + 3003*a^4*b*d^4*(-2*c + 3*d*x) + 858*a^3*
b^2*d^3*(8*c^2 - 12*c*d*x + 15*d^2*x^2) + 286*a^2*b^3*d^2*(-16*c^3 + 24*c^2
*d*x - 30*c*d^2*x^2 + 35*d^3*x^3) + 13*a*b^4*d*(128*c^4 - 192*c^3*d*x + 240
*c^2*d^2*x^2 - 280*c*d^3*x^3 + 315*d^4*x^4) + b^5*(-256*c^5 + 384*c^4*d*x -
 480*c^3*d^2*x^2 + 560*c^2*d^3*x^3 - 630*c*d^4*x^4 + 693*d^5*x^5)))/(9009*d
^6)
```

Mathics [A]
time = 7.07, size = 247, normalized size = 1.58

$$\frac{2\left(3003a^5d^5+9009b\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)(c+dx)-15015a^4bcd^4+12870b^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)(c+dx)^2+30030a^3b^2c^2d^3+10010b^3\left(a^2d^2-2abcd+b^2c^2\right)(c+dx)^3-30030a^2b^3c^3d^2+4095b^4(ad-bc)(c+dx)^4+15015ab^4c^4d+693b^5(c+dx)^5-3003b^5c^5\right)(c+dx)^{\frac{3}{2}}}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(c + d*x)^(1/2),x]')
```

```
[Out] 2 (3003 a ^ 5 d ^ 5 + 9009 b (a ^ 4 d ^ 4 - 4 a ^ 3 b c d ^ 3 + 6 a ^ 2 b ^
 2 c ^ 2 d ^ 2 - 4 a b ^ 3 c ^ 3 d + b ^ 4 c ^ 4) (c + d x) - 15015 a ^ 4 b
 c d ^ 4 + 12870 b ^ 2 (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b ^ 2 c ^ 2 d
 - b ^ 3 c ^ 3) (c + d x) ^ 2 + 30030 a ^ 3 b ^ 2 c ^ 2 d ^ 3 + 10010 b ^ 3
 (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (c + d x) ^ 3 - 30030 a ^ 2 b ^ 3
c ^ 3 d ^ 2 + 4095 b ^ 4 (a d - b c) (c + d x) ^ 4 + 15015 a b ^ 4 c ^ 4 d
+ 693 b ^ 5 (c + d x) ^ 5 - 3003 b ^ 5 c ^ 5) (c + d x) ^ (3 / 2) / (9009 d
 ^ 6)
```

Maple [A]
time = 0.14, size = 121, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{13}{2}}}{13}+\frac{10(ad-bc)b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{7}{2}}}{7}+2(ad-bc)^4b(dx+c)^{\frac{5}{2}}+\frac{2(ad-bc)^5(dx+c}{3}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{13}{2}}}{13}+\frac{10(ad-bc)b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{7}{2}}}{7}+2(ad-bc)^4b(dx+c)^{\frac{5}{2}}+\frac{2(ad-bc)^5(dx+c}{3}}{d^6}$
gosper	$2(dx+c)^{\frac{3}{2}}\left(693b^5x^5d^5+4095a\,b^4d^5x^4-630b^5c\,d^4x^4+10010a^2b^3d^5x^3-3640a\,b^4c\,d^4x^3+560b^5c^2d^3x^3+12870a^3b^2d^5x^2-858\right.$
trager	$2\left(693b^5d^6x^6+4095a\,b^4d^6x^5+63b^5c\,d^5x^5+10010a^2b^3d^6x^4+455a\,b^4c\,d^5x^4-70b^5c^2d^4x^4+12870a^3b^2d^6x^3+1430a^2b^3c\,d^5x^3\right.$
risch	$2\left(693b^5d^6x^6+4095a\,b^4d^6x^5+63b^5c\,d^5x^5+10010a^2b^3d^6x^4+455a\,b^4c\,d^5x^4-70b^5c^2d^4x^4+12870a^3b^2d^6x^3+1430a^2b^3c\,d^5x^3\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/13*b^5*(d*x+c)^(13/2)+5/11*(a*d-b*c)*b^4*(d*x+c)^(11/2)+10/9*(a*d-
b*c)^2*b^3*(d*x+c)^(9/2)+10/7*(a*d-b*c)^3*b^2*(d*x+c)^(7/2)+(a*d-b*c)^4*b*(
d*x+c)^(5/2)+1/3*(a*d-b*c)^5*(d*x+c)^(3/2))
```

Maxima [A]
time = 0.27, size = 259, normalized size = 1.66

$$\frac{2\left(693\,(dx+c)^{\frac{13}{2}}b^5-4095\,(b^5c-ab^4d)(dx+c)^{\frac{11}{2}}+10010\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)(dx+c)^{\frac{9}{2}}-12870\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{7}{2}}+9009\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)(dx+c)^{\frac{5}{2}}-3003\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)(dx+c)^{\frac{3}{2}}\right)}{9009\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*(d*x + c)^(13/2)*b^5 - 4095*(b^5*c - a*b^4*d)*(d*x + c)^(11/2)
+ 10010*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(9/2) - 12870*(b^5*
c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(7/2) + 9009
*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
)*(d*x + c)^(5/2) - 3003*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(3/2))/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(134) = 268$.
time = 0.30, size = 338, normalized size = 2.17

$$\frac{2\,(693\,b^5d^6x^6-256\,b^5c^6+1664\,ab^4c^5d-4576\,a^2b^3c^4d^2+6864\,a^3b^2c^3d^3-6006\,a^4bc^2d^4+3003\,a^5cd^5+63\,(b^5cd^5+65\,ab^4d^6)x^5-35\,(2\,b^5c^2d^4-13\,ab^4cd^5-286\,a^2b^3d^6)x^4+10\,(8\,b^5c^3d^3-52\,ab^4c^2d^4+143\,a^2b^3cd^5+1287\,a^3b^2d^6)x^3-3\,(32\,b^5c^4d^2-208\,ab^4c^3d^3+572\,a^2b^3c^2d^4-858\,a^3b^2cd^5-3003\,a^4bd^6)x^2+(128\,b^5c^5d-832\,ab^4c^4d^2+2288\,a^2b^3c^3d^3-3432\,a^3b^2c^2d^4+3003\,a^4bcd^5+3003\,a^5d^6)x)\sqrt{dx+c}}{9009\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9009*(693*b^5*d^6*x^6 - 256*b^5*c^6 + 1664*a*b^4*c^5*d - 4576*a^2*b^3*c^4
*d^2 + 6864*a^3*b^2*c^3*d^3 - 6006*a^4*b*c^2*d^4 + 3003*a^5*c*d^5 + 63*(b^5
*c*d^5 + 65*a*b^4*d^6)*x^5 - 35*(2*b^5*c^2*d^4 - 13*a*b^4*c*d^5 - 286*a^2*b
^3*d^6)*x^4 + 10*(8*b^5*c^3*d^3 - 52*a*b^4*c^2*d^4 + 143*a^2*b^3*c*d^5 + 12
87*a^3*b^2*d^6)*x^3 - 3*(32*b^5*c^4*d^2 - 208*a*b^4*c^3*d^3 + 572*a^2*b^3*c
^2*d^4 - 858*a^3*b^2*c*d^5 - 3003*a^4*b*d^6)*x^2 + (128*b^5*c^5*d - 832*a*b
^4*c^4*d^2 + 2288*a^2*b^3*c^3*d^3 - 3432*a^3*b^2*c^2*d^4 + 3003*a^4*b*c*d^5
 + 3003*a^5*d^6)*x)*sqrt(d*x + c)/d^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(144) = 288$.
time = 2.41, size = 314, normalized size = 2.01

$$\frac{2\left(\frac{b^5(c+dx)^{\frac{13}{2}}}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\cdot(5ab^4d-5b^5c)}{11d^5}+\frac{(c+dx)^{\frac{9}{2}}\cdot(10a^2b^3d^2-20ab^4cd+10b^5c^2)}{9d^5}+\frac{(c+dx)^{\frac{7}{2}}\cdot(10a^3b^2d^3-30a^2b^3cd^2+30ab^4c^2d-10b^5c^3)}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\cdot(5a^4bd^4-20a^3b^2cd^3+30a^2b^3c^2d^2-20ab^4c^3d+5b^5c^4)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\,(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5)}{3d^5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**5*(c + d*x)**(13/2)/(13*d**5) + (c + d*x)**(11/2)*(5*a*b**4*d - 5*b**
5*c)/(11*d**5) + (c + d*x)**(9/2)*(10*a**2*b**3*d**2 - 20*a*b**4*c*d + 10*b
**5*c**2)/(9*d**5) + (c + d*x)**(7/2)*(10*a**3*b**2*d**3 - 30*a**2*b**3*c*d
**2 + 30*a*b**4*c**2*d - 10*b**5*c**3)/(7*d**5) + (c + d*x)**(5/2)*(5*a**4*
b*d**4 - 20*a**3*b**2*c*d**3 + 30*a**2*b**3*c**2*d**2 - 20*a*b**4*c**3*d +
5*b**5*c**4)/(5*d**5) + (c + d*x)**(3/2)*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*
a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)
/(3*d**5))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(134) = 268$.
time = 0.01, size = 1039, normalized size = 6.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x)
```

```
[Out] 2/9009*(9009*sqrt(d*x + c)*a^5*c + 3003*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^5 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c/d + 6006*(3*(d
*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*b^2*c/d^2
+ 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
4*b/d + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)
*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(
d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2/d
^2 + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*
c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c/d^4 + 286*(3
5*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(
d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^3/d^3 + 13*(63*(d*x + c)^
(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(
5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c/d^5 + 65
*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1
386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)
*a*b^4/d^4 + 3*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5/d^5)/d
```

Mupad [B]
time = 0.08, size = 137, normalized size = 0.88

$$\frac{2\,b^5\,(c+d\,x)^{13/2}}{13\,d^6}-\frac{(10\,b^5\,c-10\,a\,b^4\,d)\,(c+d\,x)^{11/2}}{11\,d^6}+\frac{2\,(a\,d-b\,c)^5\,(c+d\,x)^{3/2}}{3\,d^6}+\frac{20\,b^2\,(a\,d-b\,c)^3\,(c+d\,x)^{7/2}}{7\,d^6}+\frac{20\,b^3\,(a\,d-b\,c)^2\,(c+d\,x)^{9/2}}{9\,d^6}+\frac{2\,b\,(a\,d-b\,c)^4\,(c+d\,x)^{5/2}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(13/2))/(13*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(11/
2))/(11*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(3/2))/(3*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(9/2))/(9*
d^6) + (2*b*(a*d - b*c)^4*(c + d*x)^(5/2))/d^6
```

3.1376 $\int (a+bx)^4 \sqrt{c+dx}\, dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-\frac{8b^3(bc-ad)(c+dx)^{9/2}}{9d^5}+\frac{2b^4(c$$

```
[Out] 2/3*(-a*d+b*c)^4*(d*x+c)^(3/2)/d^5-8/5*b*(-a*d+b*c)^3*(d*x+c)^(5/2)/d^5+12/
7*b^2*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^5-8/9*b^3*(-a*d+b*c)*(d*x+c)^(9/2)/d^5+2
/11*b^4*(d*x+c)^(11/2)/d^5
```

Rubi [A]

time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(3/2))/(3*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(5
/2))/(5*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^5) - (8*b^3*(b*c
 - a*d)*(c + d*x)^(9/2))/(9*d^5) + (2*b^4*(c + d*x)^(11/2))/(11*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4\sqrt{c+dx}\, dx &= \int \left(\frac{(-bc+ad)^4\sqrt{c+dx}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{3/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{5/2}}{d^4}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-\end{aligned}$$

Mathematica [A]

time = 0.08, size = 154, normalized size = 1.19

$$\frac{2(c+dx)^{3/2}\left(1155a^4d^4+924a^3bd^3(-2c+3dx)+198a^2b^2d^2\left(8c^2-12cdx+15d^2x^2\right)+44ab^3d\left(-16c^3+24c^2dx-30cd^2x^2+35d^3x^3\right)+b^4\left(128c^4-192c^3dx+240c^2d^2x^2-280cd^3x^3+315d^4x^4\right)\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*a^4*d^4 + 924*a^3*b*d^3*(-2*c + 3*d*x) + 198*a^2*b
^2*d^2*(8*c^2 - 12*c*d*x + 15*d^2*x^2) + 44*a*b^3*d*(-16*c^3 + 24*c^2*d*x -
 30*c*d^2*x^2 + 35*d^3*x^3) + b^4*(128*c^4 - 192*c^3*d*x + 240*c^2*d^2*x^2
- 280*c*d^3*x^3 + 315*d^4*x^4)))/(3465*d^5)
```

Mathics [A]
time = 5.57, size = 172, normalized size = 1.33

$$\frac{2\left(1155a^4d^4+2772b\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)(c+dx)-4620a^3bcd^3+2970b^2\left(a^2d^2-2abcd+b^2c^2\right)(c+dx)^2+6930a^2b^2c^2d^2+1540b^3(ad-bc)(c+dx)^3-4620ab^3c^3d+315b^4(c+dx)^4+1155b^4c^4\right)(c+dx)^{\frac{3}{2}}}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^(1/2),x]')
```

```
[Out] 2 (1155 a ^ 4 d ^ 4 + 2772 b (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b ^ 2 c
 ^ 2 d - b ^ 3 c ^ 3) (c + d x) - 4620 a ^ 3 b c d ^ 3 + 2970 b ^ 2 (a ^ 2
d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (c + d x) ^ 2 + 6930 a ^ 2 b ^ 2 c ^ 2 d ^
 2 + 1540 b ^ 3 (a d - b c) (c + d x) ^ 3 - 4620 a b ^ 3 c ^ 3 d + 315 b ^
4 (c + d x) ^ 4 + 1155 b ^ 4 c ^ 4) (c + d x) ^ (3 / 2) / (3465 d ^ 5)
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{8(ad-bc)^3b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^4(dx+c)^{\frac{3}{2}}}{3}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{8(ad-bc)^3b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^4(dx+c)^{\frac{3}{2}}}{3}}{d^5}$
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(315d^4x^4b^4+1540a\,b^3d^4x^3-280b^4c\,d^3x^3+2970a^2b^2d^4x^2-1320a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+2772a^3b\,d^4x-2376}{3465d^5}$
trager	$2(315b^4d^5x^5+1540a\,b^3d^5x^4+35b^4c\,d^4x^4+2970a^2b^2d^5x^3+220a\,b^3c\,d^4x^3-40b^4c^2d^3x^3+2772a^3b\,d^5x^2+594a^2b^2c\,d^4x^2-$
risch	$2(315b^4d^5x^5+1540a\,b^3d^5x^4+35b^4c\,d^4x^4+2970a^2b^2d^5x^3+220a\,b^3c\,d^4x^3-40b^4c^2d^3x^3+2772a^3b\,d^5x^2+594a^2b^2c\,d^4x^2-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/11*b^4*(d*x+c)^(11/2)+4/9*(a*d-b*c)*b^3*(d*x+c)^(9/2)+6/7*(a*d-b*c
)^2*b^2*(d*x+c)^(7/2)+4/5*(a*d-b*c)^3*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)^4*(d*x+
c)^(3/2))
```

Maxima [A]
time = 0.28, size = 181, normalized size = 1.40

$$\frac{2\left(315\,(dx+c)^{\frac{11}{2}}b^4-1540\,(b^4c-ab^3d)(dx+c)^{\frac{9}{2}}+2970\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(dx+c)^{\frac{7}{2}}-2772\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)(dx+c)^{\frac{5}{2}}+1155\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{3}{2}}\right)}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(d*x + c)^(11/2)*b^4 - 1540*(b^4*c - a*b^3*d)*(d*x + c)^(9/2) +
 2970*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(7/2) - 2772*(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(5/2) + 1155*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x +
c)^(3/2))/d^5
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(109) = 218$.
time = 0.30, size = 245, normalized size = 1.90

$$\frac{2\left(315\,b^{4}d^{5}x^{5}+128\,b^{4}c^{5}-704\,ab^{3}c^{4}d+1584\,a^{2}b^{2}c^{3}d^{2}-1848\,a^{3}bc^{2}d^{3}+1155\,a^{4}cd^{4}+35\left(b^{4}cd^{4}+44\,ab^{3}d^{5}\right)x^{4}-10\left(4\,b^{4}c^{2}d^{3}-22\,ab^{3}cd^{4}-297\,a^{2}b^{2}d^{5}\right)x^{3}+6\left(8\,b^{4}c^{3}d^{2}-44\,ab^{3}c^{2}d^{3}+99\,a^{2}b^{2}cd^{4}+462\,a^{3}bd^{5}\right)x^{2}-\left(64\,b^{4}c^{4}d-352\,ab^{3}c^{3}d^{2}+792\,a^{2}b^{2}c^{2}d^{3}-924\,a^{3}bcd^{4}-1155\,a^{4}d^{5}\right)x\right)\sqrt{dx+c}}{3465\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^4*d^5*x^5 + 128*b^4*c^5 - 704*a*b^3*c^4*d + 1584*a^2*b^2*c^3*
d^2 - 1848*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 + 35*(b^4*c*d^4 + 44*a*b^3*d^5)*x
^4 - 10*(4*b^4*c^2*d^3 - 22*a*b^3*c*d^4 - 297*a^2*b^2*d^5)*x^3 + 6*(8*b^4*c
^3*d^2 - 44*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 + 462*a^3*b*d^5)*x^2 - (64*b^4
*c^4*d - 352*a*b^3*c^3*d^2 + 792*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 - 1155*a
^4*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [**A**]
time = 2.00, size = 223, normalized size = 1.73

$$\frac{2\left(\frac{b^{4}(c+dx)^{\frac{11}{2}}}{11d^{4}}+\frac{(c+dx)^{\frac{9}{2}}\cdot(4ab^{3}d-4b^{4}c)}{9d^{4}}+\frac{(c+dx)^{\frac{7}{2}}\cdot(6a^{2}b^{2}d^{2}-12ab^{3}cd+6b^{4}c^{2})}{7d^{4}}+\frac{(c+dx)^{\frac{5}{2}}\cdot(4a^{3}bd^{3}-12a^{2}b^{2}cd^{2}+12ab^{3}c^{2}d-4b^{4}c^{3})}{5d^{4}}+\frac{(c+dx)^{\frac{3}{2}}\left(a^{4}d^{4}-4a^{3}bcd^{3}+6a^{2}b^{2}c^{2}d^{2}-4ab^{3}c^{3}d+b^{4}c^{4}\right)}{3d^{4}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**4*(c + d*x)**(11/2)/(11*d**4) + (c + d*x)**(9/2)*(4*a*b**3*d - 4*b**4
*c)/(9*d**4) + (c + d*x)**(7/2)*(6*a**2*b**2*d**2 - 12*a*b**3*c*d + 6*b**4*
c**2)/(7*d**4) + (c + d*x)**(5/2)*(4*a**3*b*d**3 - 12*a**2*b**2*c*d**2 + 12
*a*b**3*c**2*d - 4*b**4*c**3)/(5*d**4) + (c + d*x)**(3/2)*(a**4*d**4 - 4*a*
*3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*d**4)
)/d
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(109) = 218$.
time = 0.00, size = 748, normalized size = 5.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x)
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^4*c + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^4 + 4620*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c/d + 1386*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b^2*c/d^2 +
 924*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*
b/d + 396*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c/d^3 + 594*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2/d^2 + 1
1*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 4
20*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c/d^4 + 44*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3/d^3 + 5*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4/d^4)/d
```

Mupad [B]
time = 0.22, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(11/2))/(11*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(9/2))
/(9*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^5) + (12*b^2*(a*d - b*c)^
2*(c + d*x)^(7/2))/(7*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^5)
```

3.1377 $\int (a+bx)^3\sqrt{c+dx}\,dx$

Optimal. Leaf size=100

$$-\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

```
[Out] -2/3*(-a*d+b*c)^3*(d*x+c)^(3/2)/d^4+6/5*b*(-a*d+b*c)^2*(d*x+c)^(5/2)/d^4-6/
7*b^2*(-a*d+b*c)*(d*x+c)^(7/2)/d^4+2/9*b^3*(d*x+c)^(9/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
5/2))/(5*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^4) + (2*b^3*(c + d
*x)^(9/2))/(9*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3\sqrt{c+dx}\,dx=\int \left(\frac{(-bc+ad)^3\sqrt{c+dx}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{3/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{5/2}}{d^3}\right.$$
$$=-\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+$$

Mathematica [A]
time = 0.06, size = 102, normalized size = 1.02

$$\frac{2(c+dx)^{3/2}\left(105a^3d^3+63a^2bd^2(-2c+3dx)+9ab^2d\left(8c^2-12cdx+15d^2x^2\right)+b^3\left(-16c^3+24c^2dx-30cd^2x^2+35d^3x^3\right)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(105*a^3*d^3 + 63*a^2*b*d^2*(-2*c + 3*d*x) + 9*a*b^2*d*(
8*c^2 - 12*c*d*x + 15*d^2*x^2) + b^3*(-16*c^3 + 24*c^2*d*x - 30*c*d^2*x^2 +
 35*d^3*x^3)))/(315*d^4)
```

Mathics [A]
time = 4.38, size = 110, normalized size = 1.10

$$\frac{2\left(105a^3d^3+189b\left(a^2d^2-2abcd+b^2c^2\right)(c+dx)-315a^2bcd^2+135b^2\left(ad-bc\right)(c+dx)^2+315ab^2c^2d+35b^3\left(c+dx\right)^3-105b^3c^3\right)(c+dx)^{\frac{3}{2}}}{315d^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^(1/2),x]')
```

```
[Out] 2 (105 a ^ 3 d ^ 3 + 189 b (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (c + d x
) - 315 a ^ 2 b c d ^ 2 + 135 b ^ 2 (a d - b c) (c + d x) ^ 2 + 315 a b ^ 2
 c ^ 2 d + 35 b ^ 3 (c + d x) ^ 3 - 105 b ^ 3 c ^ 3) (c + d x) ^ (3 / 2) /
(315 d ^ 4)
```

Maple [A]
time = 0.15, size = 78, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6(ad-bc)b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)^2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^3(dx+c)^{\frac{3}{2}}}{3}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6(ad-bc)b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)^2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^3(dx+c)^{\frac{3}{2}}}{3}}{d^4}$
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(35b^3x^3d^3+135a\,b^2d^3x^2-30b^3c\,d^2x^2+189a^2b\,d^3x-108a\,b^2c\,d^2x+24b^3c^2dx+105a^3d^3-126a^2bc\,d^2+72a\,b^2c^2\cdots\right.}{315d^4}$
trager	$\frac{2\left(35b^3d^4x^4+135a\,b^2d^4x^3+5b^3c\,d^3x^3+189a^2b\,d^4x^2+27a\,b^2c\,d^3x^2-6b^3c^2d^2x^2+105a^3d^4x+63a^2bc\,d^3x-36a\,b^2c^2d^2x+8\cdots\right.}{315d^4}$
risch	$\frac{2\left(35b^3d^4x^4+135a\,b^2d^4x^3+5b^3c\,d^3x^3+189a^2b\,d^4x^2+27a\,b^2c\,d^3x^2-6b^3c^2d^2x^2+105a^3d^4x+63a^2bc\,d^3x-36a\,b^2c^2d^2x+8\cdots\right.}{315d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/9*b^3*(d*x+c)^(9/2)+3/7*(a*d-b*c)*b^2*(d*x+c)^(7/2)+3/5*(a*d-b*c)^
2*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)^3*(d*x+c)^(3/2))
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.18

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^3-135\,(b^3c-ab^2d)(dx+c)^{\frac{7}{2}}+189\,(b^3c^2-2\,ab^2cd+a^2bd^2)(dx+c)^{\frac{5}{2}}-105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx+c)^{\frac{3}{2}}\right)}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^3 - 135*(b^3*c - a*b^2*d)*(d*x + c)^(7/2) + 189
*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(5/2) - 105*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))/d^4
```

Fricas [A]
time = 0.30, size = 164, normalized size = 1.64

$$\frac{2\,{\left(35\,b^{3} d^{4} x^{4} - 16\,b^{3} c^{4} + 72\,a b^{2} c^{3} d - 126\,a^{2} b c^{2} d^{2} + 105\,a^{3} c d^{3} + 5\,{\left(b^{3} c d^{3} + 27\,a b^{2} d^{4}\right)} x^{3} - 3\,{\left(2\,b^{3} c^{2} d^{2} - 9\,a b^{2} c d^{3} - 63\,a^{2} b d^{4}\right)} x^{2} + {\left(8\,b^{3} c^{3} d - 36\,a b^{2} c^{2} d^{2} + 63\,a^{2} b c d^{3} + 105\,a^{3} d^{4}\right)} x\right)} \sqrt{d x + c}}{315\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*d^4*x^4 - 16*b^3*c^4 + 72*a*b^2*c^3*d - 126*a^2*b*c^2*d^2 + 1
05*a^3*c*d^3 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*x^3 - 3*(2*b^3*c^2*d^2 - 9*a*b^
2*c*d^3 - 63*a^2*b*d^4)*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*
d^3 + 105*a^3*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A]
time = 1.55, size = 146, normalized size = 1.46

$$\frac{2 \left(\frac{b^{3} \left(c + d x\right)^{\frac{9}{2}}}{9 d^{3}} + \frac{\left(c + d x\right)^{\frac{7}{2}} \cdot \left(3 a b^{2} d - 3 b^{3} c\right)}{7 d^{3}} + \frac{\left(c + d x\right)^{\frac{5}{2}} \cdot \left(3 a^{2} b d^{2} - 6 a b^{2} c d + 3 b^{3} c^{2}\right)}{5 d^{3}} + \frac{\left(c + d x\right)^{\frac{3}{2}} \left(a^{3} d^{3} - 3 a^{2} b c d^{2} + 3 a b^{2} c^{2} d - b^{3} c^{3}\right)}{3 d^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**3*(c + d*x)**(9/2)/(9*d**3) + (c + d*x)**(7/2)*(3*a*b**2*d - 3*b**3*c
)/(7*d**3) + (c + d*x)**(5/2)*(3*a**2*b*d**2 - 6*a*b**2*c*d + 3*b**3*c**2)/
(5*d**3) + (c + d*x)**(3/2)*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3)/(3*d**3))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(84) = 168.
time = 0.00, size = 500, normalized size = 5.00

$$\frac{\frac{2 b^{3} d \left(\frac{1}{9} \sqrt{c + d x} (c+dx)^{4} - \frac{4}{7} \sqrt{c + d x} (c+dx)^{3} c + \frac{6}{5} \sqrt{c + d x} (c+dx)^{2} c^{2} - \frac{4}{3} \sqrt{c + d x} (c+dx) c^{3} + \sqrt{c + d x} c^{4}\right)}{d^{4}} + \frac{2 b^{3} c \left(\frac{1}{7} \sqrt{c + d x} (c+dx)^{3} - \frac{3}{5} \sqrt{c + d x} (c+dx)^{2} c + \sqrt{c + d x} (c+dx) c^{2} - \sqrt{c + d x} c^{3}\right)}{d^{3}} + \frac{6 a b^{2} d \left(\frac{1}{7} \sqrt{c + d x} (c+dx)^{3} - \frac{3}{5} \sqrt{c + d x} (c+dx)^{2} c + \sqrt{c + d x} (c+dx) c^{2} - \sqrt{c + d x} c^{3}\right)}{d^{3}} + \frac{6 a b^{2} c \left(\frac{1}{5} \sqrt{c + d x} (c+dx)^{2} - \frac{2}{3} \sqrt{c + d x} (c+dx) c + \sqrt{c + d x} c^{2}\right)}{d^{2}} + \frac{6 a^{2} b d \left(\frac{1}{5} \sqrt{c + d x} (c+dx)^{2} - \frac{2}{3} \sqrt{c + d x} (c+dx) c + \sqrt{c + d x} c^{2}\right)}{d^{2}} + 2 a^{3} \left(\frac{1}{3} \sqrt{c + d x} \left(c + d x\right) - c \sqrt{c + d x}\right) + \frac{6 a^{2} b c \left(\frac{1}{3} \sqrt{c + d x} (c+dx) - c \sqrt{c + d x}\right)}{d} + 2 a^{3} c \sqrt{c + d x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^3*c + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c/d + 63*(3*(d*x + c)
```

```
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c/d^2 + 63*(3*(
d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b/d + 9*(
5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt
(d*x + c)*c^3)*b^3*c/d^3 + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2/d^2 + (35*(d*x + c)^(9/
2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*
c^3 + 315*sqrt(d*x + c)*c^4)*b^3/d^3)/d
```

Mupad [B]
time = 0.07, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(9/2))/(9*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(7/2))/(
7*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(a*d - b*c)^2*(c
+ d*x)^(5/2))/(5*d^4)
```

3.1378 $\int (a+bx)^2 \sqrt{c+dx}\, dx$

Optimal. Leaf size=71

$$\frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

```
[Out] 2/3*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^3-4/5*b*(-a*d+b*c)*(d*x+c)^(5/2)/d^3+2/7*b
^2*(d*x+c)^(7/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(5/2
))/(5*d^3) + (2*b^2*(c + d*x)^(7/2))/(7*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\sqrt{c+dx}\, dx &= \int \left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right) dx\\ &= \frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(-2c+3dx)+b^2\left(8c^2-12cdx+15d^2x^2\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x) + b^2*(8*c^2 - 12*
c*d*x + 15*d^2*x^2)))/(105*d^3)
```

Mathics [A]
time = 3.44, size = 63, normalized size = 0.89

$$\frac{2\left(35a^2d^2+42b\left(ad-bc\right)\left(c+dx\right)-70abcd+15b^2\left(c+dx\right)^2+35b^2c^2\right)\left(c+dx\right)^{\frac{3}{2}}}{105d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^(1/2),x]')
```

```
[Out] 2 (35 a ^ 2 d ^ 2 + 42 b (a d - b c) (c + d x) - 70 a b c d + 15 b ^ 2 (c +
 d x) ^ 2 + 35 b ^ 2 c ^ 2) (c + d x) ^ (3 / 2) / (105 d ^ 3)
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{4(ad-bc)b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^2(dx+c)^{\frac{3}{2}}}{3}}{d^3}$	56
default	$\frac{\frac{2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{4(ad-bc)b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^2(dx+c)^{\frac{3}{2}}}{3}}{d^3}$	56
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(15b^2x^2d^2+42ab\,d^2x-12b^2cdx+35a^2d^2-28abcd+8b^2c^2\right)}{105d^3}$	63
trager	$\frac{2\left(15b^2d^3x^3+42ab\,d^3x^2+3b^2c\,d^2x^2+35a^2d^3x+14abc\,d^2x-4b^2c^2dx+35a^2c\,d^2-28ab\,c^2d+8b^2c^3\right)\sqrt{dx+c}}{105d^3}$	100
risch	$\frac{2\left(15b^2d^3x^3+42ab\,d^3x^2+3b^2c\,d^2x^2+35a^2d^3x+14abc\,d^2x-4b^2c^2dx+35a^2c\,d^2-28ab\,c^2d+8b^2c^3\right)\sqrt{dx+c}}{105d^3}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/7*b^2*(d*x+c)^(7/2)+2/5*(a*d-b*c)*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)^2*
(d*x+c)^(3/2))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.96

$$\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^2-42\,(b^2c-abd)(dx+c)^{\frac{5}{2}}+35\,(b^2c^2-2\,abcd+a^2d^2)(dx+c)^{\frac{3}{2}}\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(d*x + c)^(7/2)*b^2 - 42*(b^2*c - a*b*d)*(d*x + c)^(5/2) + 35*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))/d^3
```

Fricas [A]
time = 0.30, size = 99, normalized size = 1.39

$$\frac{2\,(15\,b^2d^3x^3 + 8\,b^2c^3 - 28\,abc^2d + 35\,a^2cd^2 + 3\,(b^2cd^2 + 14\,abd^3)x^2 - (4\,b^2c^2d - 14\,abcd^2 - 35\,a^2d^3)x)\sqrt{dx + c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d^3*x^3 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^2 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x)*sqrt(d
*x + c)/d^3
```

Sympy [A]
time = 1.19, size = 85, normalized size = 1.20

$$\frac{2\left(\frac{b^2(c+dx)^{\frac{7}{2}}}{7d^2} + \frac{(c+dx)^{\frac{5}{2}}\cdot(2abd-2b^2c)}{5d^2} + \frac{(c+dx)^{\frac{3}{2}}\,(a^2d^2-2abcd+b^2c^2)}{3d^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**2*(c + d*x)**(7/2)/(7*d**2) + (c + d*x)**(5/2)*(2*a*b*d - 2*b**2*c)/(
5*d**2) + (c + d*x)**(3/2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*d**2))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(59) = 118.
time = 0.00, size = 300, normalized size = 4.23

$$\frac{\frac{2b^2d\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3-\frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c+\sqrt{c+dx}\,(c+dx)c^2-\sqrt{c+dx}\,c^3\right)}{d^3}+\frac{2b^2c\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+\frac{4abd\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+2a^2\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)+\frac{4abc\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)}{d}+2a^2c\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a^2*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c/d^2 + 14*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b/d + 3*(5*(d*x +
c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)
*c^3)*b^2/d^2)/d
```

Mupad [B]
time = 0.24, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}\,\left(15\,b^2\,{\left(c+d\,x\right)}^2+35\,a^2\,d^2+35\,b^2\,c^2-42\,b^2\,c\,\left(c+d\,x\right)+42\,a\,b\,d\,\left(c+d\,x\right)-70\,a\,b\,c\,d\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(15*b^2*(c + d*x)^2 + 35*a^2*d^2 + 35*b^2*c^2 - 42*b^2*c
*(c + d*x) + 42*a*b*d*(c + d*x) - 70*a*b*c*d))/(105*d^3)
```

3.1379 $\int (a+bx)\sqrt{c+dx}\,dx$

Optimal. Leaf size=42

$$-\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}$$

```
[Out] -2/3*(-a*d+b*c)*(d*x+c)^(3/2)/d^2+2/5*b*(d*x+c)^(5/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(3/2))/(3*d^2) + (2*b*(c + d*x)^(5/2))/(5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{3/2}(-2bc+5ad+3bdx)}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x],x]
[Out] (2*(c + d*x)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x))/(15*d^2)
```

Mathics [A]
time = 2.74, size = 29, normalized size = 0.69

$$\frac{2\left(5ad+3b\left(c+dx\right)-5bc\right)\left(c+dx\right)^{\frac{3}{2}}}{15d^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^(1/2),x]')
[Out] 2 (5 a d + 3 b (c + d x) - 5 b c) (c + d x) ^ (3 / 2) / (15 d ^ 2)
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(3bdx+5ad-2bc)}{15d^2}$	27
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)(dx+c)^{\frac{3}{2}}}{3}}{d^2}$	34
default	$\frac{\frac{2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)(dx+c)^{\frac{3}{2}}}{3}}{d^2}$	34
trager	$\frac{2(3b\,d^2x^2+5a\,d^2x+bcdx+5acd-2b\,c^2)\sqrt{dx+c}}{15d^2}$	46
risch	$\frac{2(3b\,d^2x^2+5a\,d^2x+bcdx+5acd-2b\,c^2)\sqrt{dx+c}}{15d^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/5*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)*(d*x+c)^(3/2))
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.79

$$\frac{2\left(3\left(dx+c\right)^{\frac{5}{2}}b-5\left(bc-ad\right)\left(dx+c\right)^{\frac{3}{2}}\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*(d*x + c)^(5/2)*b - 5*(b*c - a*d)*(d*x + c)^(3/2))/d^2
```

Fricas [A]
time = 0.31, size = 46, normalized size = 1.10

$$\frac{2\,(3\,bd^2x^2 - 2\,bc^2 + 5\,acd + (bcd + 5\,ad^2)x)\sqrt{dx+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*d^2*x^2 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x)*sqrt(d*x + c)/
d^2
```

Sympy [A]
time = 0.92, size = 36, normalized size = 0.86

$$\frac{2\left(\frac{b(c+dx)^{\frac{5}{2}}}{5d} + \frac{(c+dx)^{\frac{3}{2}}(ad-bc)}{3d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b*(c + d*x)**(5/2)/(5*d) + (c + d*x)**(3/2)*(a*d - b*c)/(3*d))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(34) = 68.
time = 0.00, size = 147, normalized size = 3.50

$$\frac{\frac{2bd\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2 - \frac{2}{3}\sqrt{c+dx}\,(c+dx)c + \sqrt{c+dx}\,c^2\right)}{d^2} + 2a\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx) - c\sqrt{c+dx}\right) + \frac{2bc\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx) - c\sqrt{c+dx}\right)}{d} + 2ac\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a*c + 5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a + 5*
((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c/d + (3*(d*x + c)^(5/2) - 10*(d*x
+ c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b/d)/d
```

Mupad [B]
time = 0.04, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}\,\left(5\,a\,d-5\,b\,c+3\,b\,\left(c+d\,x\right)\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(5*a*d - 5*b*c + 3*b*(c + d*x)))/(15*d^2)
```

3.1380 $\int \sqrt{c+dx}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{3/2}}{3d}$$

[Out] 2/3*(d*x+c)^(3/2)/d

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*x],x]

[Out] (2*(c + d*x)^(3/2))/(3*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+dx}\,dx = \frac{2(c+dx)^{3/2}}{3d}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d*x],x]

[Out] (2*(c + d*x)^(3/2))/(3*d)

Mathics [A]
time = 1.62, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{3}{2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^(1/2),x]')
[Out] 2 (c + d x) ^ (3 / 2) / (3 d)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
derivativedivides	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
default	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
trager	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
risch	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(d*x+c)^(3/2)/d
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(d*x + c)^(3/2)/d
```

Fricas [A]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(d*x + c)^(3/2)/d
```

Sympy [**A**]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*d)
```

Giac [**A**]
time = 0.00, size = 21, normalized size = 1.31

$$\frac{\sqrt{c+dx}\,\left(c+dx\right)}{\frac{3}{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x)
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\left(c+d\,x\right)^{3/2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

3.1381 $\int \frac{\sqrt{c+dx}}{a+bx}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(3/2)
+2*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 65, 214}

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{a+bx}\,dx &= \frac{2\sqrt{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b}\\ &= \frac{2\sqrt{c+dx}}{b}+\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{bd}\\ &= \frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[-(b*c) + a*d]])/b^(3/2)
```

Mathics [A]
time = 4.18, size = 48, normalized size = 0.77

$$\frac{2\left(\sqrt{c+dx}-\text{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{\frac{ad}{b}-c}}\right]\sqrt{\frac{ad}{b}-c}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^1,x]')
```

```
[Out] 2 (Sqrt[c + d x] - ArcTan[Sqrt[c + d x] / Sqrt[a d / b - c]] Sqrt[a d / b -
 c]) / b
```

Maple [A]
time = 0.16, size = 61, normalized size = 0.98

method	result	size
derivativedivides	$\frac{2\sqrt{dx+c}}{b}+\frac{2(-ad+bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{b\sqrt{(ad-bc)\,b}}$	61
default	$\frac{2\sqrt{dx+c}}{b}+\frac{2(-ad+bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{b\sqrt{(ad-bc)\,b}}$	61
risch	$\frac{2\sqrt{dx+c}}{b}-\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)ad}{b\sqrt{(ad-bc)\,b}}+\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)c}{\sqrt{(ad-bc)\,b}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)^(1/2)/b+2*(-a*d+b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)
/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 143, normalized size = 2.31

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,\sqrt{dx+c}}{b},\ -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx+c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*
c - a*d)/b))/(b*x + a)) + 2*sqrt(d*x + c))/b, -2*(sqrt(-(b*c - a*d)/b)*arct
an(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x + c))/b]
```

Sympy [A]
time = 1.95, size = 61, normalized size = 0.98

$$\frac{2\left(\frac{d\sqrt{c+dx}}{b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x)/b - d*(a*d - b*c)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b)
)/(b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 0.00, size = 73, normalized size = 1.18

$$\frac{2\sqrt{c+dx}}{b}+\frac{(4cb-4ad)\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{b\cdot 2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2*(b*c - a*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + 2*sqrt(d*x + c)/b
```

Mupad [B]
time = 0.07, size = 50, normalized size = 0.81

$$\frac{2\sqrt{c+d\,x}}{b}-\frac{2\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(1/2))/b - (2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)
)*(a*d - b*c)^(1/2))/b^(3/2)
```

3.1382 $\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{c+dx}}{b(a+bx)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
-(d*x+c)^(1/2)/b/(b*x+a)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {43, 65, 214}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx &= -\frac{\sqrt{c+dx}}{b(a+bx)}+\frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b}\\ &= -\frac{\sqrt{c+dx}}{b(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b}\\ &= -\frac{\sqrt{c+dx}}{b(a+bx)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [**A**]
time = 0.18, size = 69, normalized size = 0.99

$$-\frac{\sqrt{c+dx}}{b(a+bx)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b
*c) + a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 655 vs. $2(70) = 140$.
time = 22.41, size = 597, normalized size = 8.53

$$\frac{-abd\sqrt{\frac{ad-bc}{b}}\sqrt{c+dx}+\frac{abd^2(a^2d-abc+abdx-b^2cx)\left(\text{Log}\left[-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right]-\text{Log}\left[a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right]\right)\sqrt{-\frac{1}{b(ad-bc)^3}}\sqrt{\frac{ad-bc}{b}}}{2}+b^2c\sqrt{\frac{ad-bc}{b}}\sqrt{c+dx}+2d\text{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right](a^2d-abc+abdx-b^2cx)+\frac{b^2cd(a^2d-abc+abdx-b^2cx)\left(\text{Log}\left[a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right]-\text{Log}\left[-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right]\right)\sqrt{-\frac{1}{b(ad-bc)^3}}\sqrt{\frac{ad-bc}{b}}}{2}}{b^2\sqrt{\frac{ad-bc}{b}}(a^2d-abc+abdx-b^2cx)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^2,x]')
```

```
[Out] (-a b d Sqrt[(a d - b c) / b] Sqrt[c + d x] + a b d ^ 2 (a ^ 2 d - a b c +
a b d x - b ^ 2 c x) (Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a
 b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c
) ^ 3)] + Sqrt[c + d x]] - Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] -
 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d -
 b c) ^ 3)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d - b c) ^ 3)] Sqrt[(a d - b
c) / b] / 2 + b ^ 2 c Sqrt[(a d - b c) / b] Sqrt[c + d x] + 2 d ArcTan[Sqrt
[c + d x] / Sqrt[(a d - b c) / b]] (a ^ 2 d - a b c + a b d x - b ^ 2 c x)
+ b ^ 2 c d (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (Log[a ^ 2 d ^ 2 Sqrt[-
```

```
1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2
 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] - Log[-a ^ 2 d ^ 2 S
qrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] -
b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]]) Sqrt[-1 / (b (
a d - b c) ^ 3)] Sqrt[(a d - b c) / b] / 2) / (b ^ 2 Sqrt[(a d - b c) / b]
(a ^ 2 d - a b c + a b d x - b ^ 2 c x))
```

Maple [A]
time = 0.16, size = 73, normalized size = 1.04

method	result	size
derivativedivides	$2d\left(-\frac{\sqrt{dx+c}}{2b((dx+c)b+ad-bc)}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}\right)$	73
default	$2d\left(-\frac{\sqrt{dx+c}}{2b((dx+c)b+ad-bc)}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}\right)$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/2/b*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+1/2/b/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.32, size = 232, normalized size = 3.31

$$\left[\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(b^2c-abd)\sqrt{dx+c}}{2\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x)},\ \frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(b^2c-abd)\sqrt{dx+c}}{ab^3c-a^2b^2d+(b^4c-ab^3d)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b
^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(b^2*c - a*b*d)*sqrt(d*x + c))/
(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x), (sqrt(-b^2*c + a*b*d)*(b*d*x +
 a*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (b^2*c - a
*b*d)*sqrt(d*x + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(58) = 116$.
time = 21.97, size = 573, normalized size = 8.19

$$-\frac{2ad^2\sqrt{c+dx}}{2a^2bd^2-2ab^2cd+2ab^2d^2x-2b^3cdx}+\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{2cd\sqrt{c+dx}}{2a^2d^2-2abcd+2abd^2x-2b^2cdx}+\frac{2d\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad}{b}-c}}\right)}{b^2\sqrt{\frac{ad}{b}-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2
*b**3*c*d*x) + a*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b
*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(
-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*
b) - c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)*
*3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d -
b*c)**3)) + sqrt(c + d*x))/2 + c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**
2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**
2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*c*d*sqrt(c + d*x)
/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2*d*atan(sqrt(c
+ d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c))
```

Giac [A]
time = 0.00, size = 82, normalized size = 1.17

$$-\frac{\sqrt{c+dx}\,d}{b\left((c+dx)\,b-cb+da\right)}+\frac{2d\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{b\cdot 2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x)
```

```
[Out] d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - s
qrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*b)
```

Mupad [B]
time = 0.24, size = 61, normalized size = 0.87

$$\frac{d \operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}}-\frac{d\,\sqrt{c+d\,x}}{d\,x\,b^2+a\,d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^2,x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(3/2)*(a*d - b*c)^
(1/2)) - (d*(c + d*x)^(1/2))/(a*b*d + b^2*d*x)
```

3.1383 $\int \frac{\sqrt{c+dx}}{(a+bx)^3} dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(3/2)-1/2*(d*x+c)^(1/2)/b/(b*x+a)^2-1/4*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b*x+a
)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)}-\frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[c + d*x]/(b*(a + b*x)^2) - (d*Sqrt[c + d*x])/(4*b*(b*c - a*d)*(a
+ b*x)) + (d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(3/2)
*(b*c - a*d)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}+\frac{d\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4b}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 99, normalized size = 0.90

$$\frac{\frac{\sqrt{b}\sqrt{c+dx}\,(2bc-ad+bdx)}{(-bc+ad)(a+bx)^2}+\frac{d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x]*(2*b*c - a*d + b*d*x))/((-(b*c) + a*d)*(a + b*x)^2)
 + (d^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^
(3/2))/(4*b^(3/2))
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1383 vs. 2(110) = 220.
time = 57.12, size = 1307, normalized size = 11.88

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^3,x]')
```

```
[Out] (8 d (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 -
 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b
^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) Sqrt[c + d x] + d ^ 2 (-3 a d Log[a ^ 3
d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d
- b c) ^ 5)] + 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] - b ^ 3 c ^
 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c)
 ^ 5)] + 3 a d Log[-a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a ^ 2 b
c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a
d - b c) ^ 5)] + b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]
] Sqrt[-1 / (b (a d - b c) ^ 5)] - 4 Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b
 c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1
/ (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + 4
Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a
d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]
] Sqrt[-1 / (b (a d - b c) ^ 3)]) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (
a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^
 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c
d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) - 10 a ^ 2 d ^ 2 (a ^ 2 d - a b c + a b d x -
b ^ 2 c x) Sqrt[c + d x] + 2 b (-3 a d (c + d x) + 3 b c (c + d x) - 5 b c
^ 2) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[c + d x] + 20 a b c d (a
^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[c + d x] + 3 b c d ^ 2 (a ^ 2 d -
 a b c + a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^
2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2
 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) (Log[a ^ 3 d ^
3 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b
c) ^ 5)] + 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] - b ^ 3 c ^ 3 S
qrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] - Log[-a ^ 3 d ^ 3 Sqrt[-1 /
 (b (a d - b c) ^ 5)] + 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] -
3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] + b ^ 3 c ^ 3 Sqrt[-1 / (b
 (a d - b c) ^ 5)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d - b c) ^ 5)]) / (8 b
 (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a
 ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^
2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2))
```

Maple [A]
time = 0.14, size = 106, normalized size = 0.96

method	result	size
derivativedivides	$2d^{2}\left(\frac{\frac{(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\sqrt{dx+c}}{8b}}{((dx+c)b+ad-bc)^{2}}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8(ad-bc)b\sqrt{(ad-bc)\,b}}\right)$	106

default	$2d^{2}\left(\frac{\frac{(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\sqrt{dx+c}}{8b}}{((dx+c)b+ad-bc)^{2}}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8(ad-bc)b\sqrt{(ad-bc)\,b}}\right)$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*((1/8/(a*d-b*c)*(d*x+c)^(3/2)-1/8*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c
)^2+1/8/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b
)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(90) = 180$.
time = 0.31, size = 456, normalized size = 4.15

$$\left[-\frac{(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(2\,b^3c^2-3\,ab^2cd+a^2bd^2+(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{8\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^2+2\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)x)},\ -\frac{(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(2\,b^3c^2-3\,ab^2cd+a^2bd^2+(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{4\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^2+2\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x
 + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(2*b^3
*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x), -1/4*((b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*
d)*sqrt(d*x + c)/(b*d*x + b*c)) + (2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b
^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^
4*c*d + a^3*b^3*d^2)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(88) = 176$.

time = 95.19, size = 1658, normalized size = 15.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**3,x)
```

```
[Out] -10*a**2*d**4*sqrt(c + d*x)/(8*a**4*b*d**4 - 16*a**3*b**2*c*d**3 + 16*a**3*
b**2*d**4*x - 48*a**2*b**3*c*d**3*x + 8*a**2*b**3*d**2*(c + d*x)**2 + 16*a*
b**4*c**3*d + 48*a*b**4*c**2*d**2*x - 16*a*b**4*c*d*(c + d*x)**2 - 8*b**5*c
**4 - 16*b**5*c**3*d*x + 8*b**5*c**2*(c + d*x)**2) + 20*a*c*d**3*sqrt(c + d
*x)/(8*a**4*d**4 - 16*a**3*b*c*d**3 + 16*a**3*b*d**4*x - 48*a**2*b**2*c*d**
3*x + 8*a**2*b**2*d**2*(c + d*x)**2 + 16*a*b**3*c**3*d + 48*a*b**3*c**2*d**
2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*
c**2*(c + d*x)**2) - 6*a*d**3*(c + d*x)**(3/2)/(8*a**4*d**4 - 16*a**3*b*c*d
**3 + 16*a**3*b*d**4*x - 48*a**2*b**2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)
**2 + 16*a*b**3*c**3*d + 48*a*b**3*c**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2
 - 8*b**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*c**2*(c + d*x)**2) + 3*a*d**3*sq
rt(-1/(b*(a*d - b*c)**5))*log(-a**3*d**3*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a*
*2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**5)) - 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d -
 b*c)**5)) + b**3*c**3*sqrt(-1/(b*(a*d - b*c)**5)) + sqrt(c + d*x))/(8*b) -
 3*a*d**3*sqrt(-1/(b*(a*d - b*c)**5))*log(a**3*d**3*sqrt(-1/(b*(a*d - b*c)*
*5)) - 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a*b**2*c**2*d*sqrt(-
1/(b*(a*d - b*c)**5)) - b**3*c**3*sqrt(-1/(b*(a*d - b*c)**5)) + sqrt(c + d*
x))/(8*b) - 10*b*c**2*d**2*sqrt(c + d*x)/(8*a**4*d**4 - 16*a**3*b*c*d**3 +
16*a**3*b*d**4*x - 48*a**2*b**2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)**2 +
16*a*b**3*c**3*d + 48*a*b**3*c**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b
**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*c**2*(c + d*x)**2) + 6*b*c*d**2*(c + d
*x)**(3/2)/(8*a**4*d**4 - 16*a**3*b*c*d**3 + 16*a**3*b*d**4*x - 48*a**2*b**
2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)**2 + 16*a*b**3*c**3*d + 48*a*b**3*c
**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b**4*c**4 - 16*b**4*c**3*d*x +
8*b**4*c**2*(c + d*x)**2) - 3*c*d**2*sqrt(-1/(b*(a*d - b*c)**5))*log(-a**3*
d**3*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**
5)) - 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d - b*c)**5)) + b**3*c**3*sqrt(-1/(b*(a
*d - b*c)**5)) + sqrt(c + d*x))/8 + 3*c*d**2*sqrt(-1/(b*(a*d - b*c)**5))*lo
g(a**3*d**3*sqrt(-1/(b*(a*d - b*c)**5)) - 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d -
 b*c)**5)) + 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d - b*c)**5)) - b**3*c**3*sqrt(-
1/(b*(a*d - b*c)**5)) + sqrt(c + d*x))/8 + 2*d**2*sqrt(c + d*x)/(2*a**2*b*d
**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) - d**2*sqrt(-1/(b*(a*d
 - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) + d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d -
 b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(
a*d - b*c)**3)) + sqrt(c + d*x))/(2*b)
```

Giac [A]
time = 0.00, size = 149, normalized size = 1.35

$$\frac{\sqrt{c+dx}\,(c+dx)\,bd^2+\sqrt{c+dx}\,cbd^2-\sqrt{c+dx}\,d^3a}{(-4cb^2+4bda)\,((c+dx)\,b-cb+da)^2}-\frac{d^2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(2cb^2-2bda)\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x)
```

```
[Out] -1/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt
(-b^2*c + a*b*d)) - 1/4*((d*x + c)^(3/2)*b*d^2 + sqrt(d*x + c)*b*c*d^2 - sq
rt(d*x + c)*a*d^3)/((b^2*c - a*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [B]
time = 0.30, size = 135, normalized size = 1.23

$$\frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{d^2\,\sqrt{c+d\,x}}{4\,b}-\frac{d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,\left(a\,d-b\,c\right)}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^3,x)
```

```
[Out] (d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(3/2)*(a*d - b
*c)^(3/2)) - ((d^2*(c + d*x)^(1/2))/(4*b) - (d^2*(c + d*x)^(3/2))/(4*(a*d -
 b*c)))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)
```

3.1384 $\int \frac{\sqrt{c+dx}}{(a+bx)^4} \, dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}$$

```
[Out] -1/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(5/2)-1/3*(d*x+c)^(1/2)/b/(b*x+a)^3-1/12*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b*x
+a)^2+1/8*d^2*(d*x+c)^(1/2)/b/(-a*d+b*c)^2/(b*x+a)
```

Rubi [A]

time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] -1/3*Sqrt[c + d*x]/(b*(a + b*x)^3) - (d*Sqrt[c + d*x])/(12*b*(b*c - a*d)*(a
 + b*x)^2) + (d^2*Sqrt[c + d*x])/(8*b*(b*c - a*d)^2*(a + b*x)) - (d^3*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}+\frac{d\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6b} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}-\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b(bc-ad)} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}+\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b(bc-ad)^2} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}+\frac{d^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx \right.}{8b(bc-ad} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.62, size = 131, normalized size = 0.90

$$\frac{\sqrt{c+dx}\,\left(-3a^2d^2+2abd(7c+4dx)+b^2\left(-8c^2-2cdx+3d^2x^2\right)\right)}{24b(bc-ad)^2(a+bx)^3}+\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8b^{3/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (Sqrt[c + d*x]*(-3*a^2*d^2 + 2*a*b*d*(7*c + 4*d*x) + b^2*(-8*c^2 - 2*c*d*x
+ 3*d^2*x^2)))/(24*b*(b*c - a*d)^2*(a + b*x)^3) + (d^3*ArcTan[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*b^(3/2)*(-(b*c) + a*d)^(5/2))
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 2681 vs. 2(146) = 292.
time = 132.51, size = 2591, normalized size = 17.75

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^4,x]')
```

```
[Out] (60 a d ^ 2 (a ^ 6 d ^ 3 - 3 a ^ 5 b c d ^ 2 + 3 a ^ 5 b d ^ 3 x + 3 a ^ 4
b ^ 2 c ^ 2 d - 9 a ^ 4 b ^ 2 c d ^ 2 x + 3 a ^ 4 b ^ 2 d ^ 3 x ^ 2 - a ^ 3
 b ^ 3 c ^ 3 + 9 a ^ 3 b ^ 3 c ^ 2 d x - 9 a ^ 3 b ^ 3 c d ^ 2 x ^ 2 + a ^
3 b ^ 3 d ^ 3 x ^ 3 - 3 a ^ 2 b ^ 4 c ^ 3 x + 9 a ^ 2 b ^ 4 c ^ 2 d x ^ 2 -
 3 a ^ 2 b ^ 4 c d ^ 2 x ^ 3 - 3 a b ^ 5 c ^ 3 x ^ 2 + 3 a b ^ 5 c ^ 2 d x
^ 3 - b ^ 6 c ^ 3 x ^ 3) Sqrt[c + d x] + 6 b (a ^ 4 d ^ 2 - 2 a ^ 3 b c d +
 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2
d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2)
(33 a ^ 2 c d ^ 2 - 5 a b d (c + d x) ^ 2 - 33 a b c ^ 2 d + 5 b ^ 2 c (c +
 d x) ^ 2 + 11 b ^ 2 c ^ 3) Sqrt[c + d x] + 80 b (-a ^ 2 d ^ 2 + 2 a b c d
- b ^ 2 c ^ 2) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^
 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2
x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) (c + d x) ^ (3 / 2) + 3 d ^ 3
(-5 a d Log[a ^ 4 d ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] - 4 a ^ 3 b c d ^ 3
Sqrt[-1 / (b (a d - b c) ^ 7)] + 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 Sqrt[-1 / (b (a
d - b c) ^ 7)] - 4 a b ^ 3 c ^ 3 d Sqrt[-1 / (b (a d - b c) ^ 7)] + b ^ 4 c
 ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b
c) ^ 7)] + 5 a d Log[-a ^ 4 d ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] + 4 a ^ 3
b c d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 7)] - 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 Sqrt[-
1 / (b (a d - b c) ^ 7)] + 4 a b ^ 3 c ^ 3 d Sqrt[-1 / (b (a d - b c) ^ 7)]
 - b ^ 4 c ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] + Sqrt[c + d x]] Sqrt[-1 / (b
 (a d - b c) ^ 7)] - 6 Log[-a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3
a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a b ^ 2 c ^ 2 d Sqrt[-1
/ (b (a d - b c) ^ 5)] + b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[
c + d x]] Sqrt[-1 / (b (a d - b c) ^ 5)] + 6 Log[a ^ 3 d ^ 3 Sqrt[-1 / (b (
a d - b c) ^ 5)] - 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a b
 ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] - b ^ 3 c ^ 3 Sqrt[-1 / (b (a d
 - b c) ^ 5)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 5)]) (a ^ 4 d ^ 2
 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c
d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b
 ^ 4 c ^ 2 x ^ 2) (a ^ 6 d ^ 3 - 3 a ^ 5 b c d ^ 2 + 3 a ^ 5 b d ^ 3 x + 3
a ^ 4 b ^ 2 c ^ 2 d - 9 a ^ 4 b ^ 2 c d ^ 2 x + 3 a ^ 4 b ^ 2 d ^ 3 x ^ 2 -
 a ^ 3 b ^ 3 c ^ 3 + 9 a ^ 3 b ^ 3 c ^ 2 d x - 9 a ^ 3 b ^ 3 c d ^ 2 x ^ 2
+ a ^ 3 b ^ 3 d ^ 3 x ^ 3 - 3 a ^ 2 b ^ 4 c ^ 3 x + 9 a ^ 2 b ^ 4 c ^ 2 d x
 ^ 2 - 3 a ^ 2 b ^ 4 c d ^ 2 x ^ 3 - 3 a b ^ 5 c ^ 3 x ^ 2 + 3 a b ^ 5 c ^
2 d x ^ 3 - b ^ 6 c ^ 3 x ^ 3) - 66 a ^ 3 d ^ 3 (a ^ 4 d ^ 2 - 2 a ^ 3 b c
d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^
 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^
```

2) Sqrt[c + d x] + 12 b d (a ^ 6 d ^ 3 - 3 a ^ 5 b c d ^ 2 + 3 a ^ 5 b d ^ 3 x + 3 a ^ 4 b ^ 2 c ^ 2 d - 9 a ^ 4 b ^ 2 c d ^ 2 x + 3 a ^ 4 b ^ 2 d ^ 3 x ^ 2 - a ^ 3 b ^ 3 c ^ 3 + 9 a ^ 3 b ^ 3 c ^ 2 d x - 9 a ^ 3 b ^ 3 c d ^ 2 x ^ 2 + a ^ 3 b ^ 3 d ^ 3 x ^ 3 - 3 a ^ 2 b ^ 4 c ^ 3 x + 9 a ^ 2 b ^ 4 c ^ 2 d x ^ 2 - 3 a ^ 2 b ^ 4 c d ^ 2 x ^ 3 - 3 a b ^ 5 c ^ 3 x ^ 2 + 3 a b ^ 5 c ^ 2 d x ^ 3 - b ^ 6 c ^ 3 x ^ 3) (-2 c + 3 d x) Sqrt[c + d x] + 15 b c d ^ 3 (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) (a ^ 6 d ^ 3 - 3 a ^ 5 b c d ^ 2 + 3 a ^ 5 b d ^ 3 x + 3 a ^ 4 b ^ 2 c ^ 2 d - 9 a ^ 4 b ^ 2 c d ^ 2 x + 3 a ^ 4 b ^ 2 d ^ 3 x ^ 2 - a ^ 3 b ^ 3 c ^ 3 + 9 a ^ 3 b ^ 3 c ^ 2 d x - 9 a ^ 3 b ^ 3 c d ^ 2 x ^ 2 + a ^ 3 b ^ 3 d ^ 3 x ^ 3 - 3 a ^ 2 b ^ 4 c ^ 3 x + 9 a ^ 2 b ^ 4 c ^ 2 d x ^ 2 - 3 a ^ 2 b ^ 4 c d ^ 2 x ^ 3 - 3 a b ^ 5 c ^ 3 x ^ 2 + 3 a b ^ 5 c ^ 2 d x ^ 3 - b ^ 6 c ^ 3 x ^ 3) (Log[a ^ 4 d ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] - 4 a ^ 3 b c d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 7)] + 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 7)] - 4 a b ^ 3 c ^ 3 d Sqrt[-1 / (b (a d - b c) ^ 7)] + b ^ 4 c ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] + Sqrt[c + d x]] - Log[-a ^ 4 d ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] + 4 a ^ 3 b c d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 7)] - 6 a ^ 2 b ^ 2 c ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 7)] + 4 a b ^ 3 c ^ 3 d Sqrt[-1 / (b (a d - b c) ^ 7)] - b ^ 4 c ^ 4 Sqrt[-1 / (b (a d - b c) ^ 7)] + Sqrt[c + d x]]) Sqrt [-1 / (b (a d - b c) ^ 7)]) / (48 b (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) (a ^ 6 d ^ 3 - 3 a ^ 5 b c d ^ 2 + 3 a ^ 5 b d ^ 3 x + 3 a ^ 4 b ^ 2 c ^ 2 d - 9 a ^ 4 b ^ 2 c d ^ 2 x + 3 a ^ 4 b ^ 2 d ^ 3 x ^ 2 - a ^ 3 b ^ 3 c ^ 3 + 9 a ^ 3 b ^ 3 c ^ 2 d x - 9 a ^ 3 b ^ 3 c d ^ 2 x ^ 2 + a ^ 3 b ^ 3 d ^ 3 x ^ 3 - 3 a ^ 2 b ^ 4 c ^ 3 x + 9 a ^ 2 b ^ 4 c ^ 2 d x ^ 2 - 3 a ^ 2 b ^ 4 c d ^ 2 x ^ 3 - 3 a b ^ 5 c ^ 3 x ^ 2 + 3 a b ^ 5 c ^ 2 d x ^ 3 - b ^ 6 c ^ 3 x ^ 3))

Maple [A]
time = 0.15, size = 152, normalized size = 1.04

method	result	size
derivativedivides	$2d^3\left(\frac{\frac{b(dx+c)^{\frac{5}{2}}}{16a^2d^2-32abcd+16b^2c^2}+\frac{(dx+c)^{\frac{3}{2}}}{6ad-6bc}-\frac{\sqrt{dx+c}}{16b}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16b(a^2d^2-2abcd+b^2c^2)\sqrt{(ad-bc)b}}\right)$	152
default	$2d^3\left(\frac{\frac{b(dx+c)^{\frac{5}{2}}}{16a^2d^2-32abcd+16b^2c^2}+\frac{(dx+c)^{\frac{3}{2}}}{6ad-6bc}-\frac{\sqrt{dx+c}}{16b}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16b(a^2d^2-2abcd+b^2c^2)\sqrt{(ad-bc)b}}\right)$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*((1/16*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+1/6/(a*d-b*c)*(d*x
+c)^(3/2)-1/16*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c)^3+1/16/b/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(122) = 244.
time = 0.32, size = 785, normalized size = 5.38

$$\left[\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(8\,b^4c^3-22\,ab^3c^2d+17\,a^2b^2cd^2-3\,a^3bd^3-3\,(b^4cd^2-ab^3d^3)x^2+2\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^3+3\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^2+3\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x)},\ \frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(8\,b^4c^3-22\,ab^3c^2d+17\,a^2b^2cd^2-3\,a^3bd^3-3\,(b^4cd^2-ab^3d^3)x^2+2\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^3+3\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^2+3\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) - 2*(8*b^4*c^3 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^
3 - 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^
2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^
3 + 3*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3
*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x), 1/24*(
3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a
*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3
 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^3 - 3*(b^4*c*d^2 - a*b^3*d
^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^3 + 3*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3*(a^2*b^6*c^3 - 3*a^3*b^5*
c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 254, normalized size = 1.74

$$\frac{3\sqrt{c+dx}\,(c+dx)^2b^2d^3-8\sqrt{c+dx}\,(c+dx)cb^2d^3+8\sqrt{c+dx}\,(c+dx)bd^4a-3\sqrt{c+dx}\,c^2b^2d^3+6\sqrt{c+dx}\,cbd^4a-3\sqrt{c+dx}\,d^5a^2}{(24c^2b^3-48cb^2da+24bd^2a^2)((c+dx)b-cb+da)^3}-\frac{d^3\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2(-4c^2b^3+8cb^2da-4bd^2a^2)\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 - 8*
(d*x + c)^(3/2)*b^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 + 8*(d*x + c)^(3/2)
*a*b*d^4 + 6*sqrt(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*((d*x + c)*b - b*c + a*d)^3)
```

Mupad [B]
time = 0.37, size = 207, normalized size = 1.42

$$\frac{\frac{d^3\,(c+d\,x)^{3/2}}{3\,(a\,d-b\,c)}-\frac{d^3\,\sqrt{c+d\,x}}{8\,b}+\frac{b\,d^3\,(c+d\,x)^{5/2}}{8\,(a\,d-b\,c)^2}}{(c+d\,x)\,(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2)+b^3\,(c+d\,x)^3-(3\,b^3\,c-3\,a\,b^2\,d)\,(c+d\,x)^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}+\frac{d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{3/2}\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^4,x)
```

```
[Out] ((d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)) - (d^3*(c + d*x)^(1/2))/(8*b) + (b*d
^3*(c + d*x)^(5/2))/(8*(a*d - b*c)^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (d^3*atan((b^(1/2)*(c + d*
x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(3/2)*(a*d - b*c)^(5/2))
```

3.1385 $\int \frac{\sqrt{c+dx}}{(a+bx)^5} \, dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}}$$

```
[Out] 5/64*d^4*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(7/2)-1/4*(d*x+c)^(1/2)/b/(b*x+a)^4-1/24*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b*x
+a)^3+5/96*d^2*(d*x+c)^(1/2)/b/(-a*d+b*c)^2/(b*x+a)^2-5/64*d^3*(d*x+c)^(1/2
)/b/(-a*d+b*c)^3/(b*x+a)
```

Rubi [A]

time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}}-\frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3}+\frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2}-\frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)}-\frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -1/4*Sqrt[c + d*x]/(b*(a + b*x)^4) - (d*Sqrt[c + d*x])/(24*b*(b*c - a*d)*(a
 + b*x)^3) + (5*d^2*Sqrt[c + d*x])/(96*b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^
3*Sqrt[c + d*x])/(64*b*(b*c - a*d)^3*(a + b*x)) + (5*d^4*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(64*b^(3/2)*(b*c - a*d)^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx = -\frac{\sqrt{c+dx}}{4b(a+bx)^4} + \frac{d\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8b}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} - \frac{(5d^2)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} + \frac{(5d^3)\int \frac{1}{(a+bx)^2\sqrt{c+d}}}{64b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4} - \frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3} + \frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx}$$

Mathematica [A]
time = 0.92, size = 170, normalized size = 0.93

$$\frac{\sqrt{c+dx}\,\left(-15a^3d^3+a^2bd^2(118c+73dx)+ab^2d\left(-136c^2-36cdx+55d^2x^2\right)+b^3\left(48c^3+8c^2dx-10cd^2x^2+15d^3x^3\right)\right)}{192b(-bc+ad)^3(a+bx)^4}+\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{64b^{3/2}(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] (Sqrt[c + d*x]*(-15*a^3*d^3 + a^2*b*d^2*(118*c + 73*d*x) + a*b^2*d*(-136*c^
2 - 36*c*d*x + 55*d^2*x^2) + b^3*(48*c^3 + 8*c^2*d*x - 10*c*d^2*x^2 + 15*d^
```

```
3*x^3)))/(192*b*(-(b*c) + a*d)^3*(a + b*x)^4) + (5*d^4*ArcTan[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[-(b*c) + a*d]])/(64*b^(3/2)*(-(b*c) + a*d)^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^5,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.14, size = 217, normalized size = 1.19

method	result
derivativedivides	$2d^4\left(\frac{\frac{5b^2(dx+c)^{\frac{7}{2}}}{128\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{55b(dx+c)^{\frac{5}{2}}}{384\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{73(dx+c)^{\frac{3}{2}}}{384(ad-bc)}-\frac{5\sqrt{dx+c}}{128b}}{((dx+c)b+ad-bc)^4}+\frac{5\,\mathrm{arct}}{128b(a^3d^3-3a^2bc}\right.$
default	$2d^4\left(\frac{\frac{5b^2(dx+c)^{\frac{7}{2}}}{128\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{55b(dx+c)^{\frac{5}{2}}}{384\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{73(dx+c)^{\frac{3}{2}}}{384(ad-bc)}-\frac{5\sqrt{dx+c}}{128b}}{((dx+c)b+ad-bc)^4}+\frac{5\,\mathrm{arct}}{128b(a^3d^3-3a^2bc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^4*((5/128*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d*x+c)^(7/
2)+55/384*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+73/384/(a*d-b*c)*(d*x
+c)^(3/2)-5/128*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c)^4+5/128/b/(a^3*d^3-3*a
^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2
)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(154) = 308$.
time = 0.32, size = 1176, normalized size = 6.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 184*a*b^4*c^3*d + 254*
a^2*b^3*c^2*d^2 - 133*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*
d^4)*x^3 - 5*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5
*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3
 + a^8*b^2*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*
c*d^3 + a^4*b^6*d^4)*x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d
^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d
+ 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 -
 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x), -
1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)
/(b*d*x + b*c)) + (48*b^5*c^4 - 184*a*b^4*c^3*d + 254*a^2*b^3*c^2*d^2 - 133
*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*x^3 - 5*(2*b^5*c
^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5*c^3*d - 44*a*b^4*c^2
*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4 + (b^1
0*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3
+ a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 -
 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*
a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(154) = 308$.

time = 0.01, size = 388, normalized size = 2.13

$$\frac{-15\sqrt{c+dx}\,(c+dx)^3b^3d^4+55\sqrt{c+dx}\,(c+dx)^2cb^3d^4-55\sqrt{c+dx}\,(c+dx)^2b^2d^5a-73\sqrt{c+dx}\,(c+dx)\,c^2b^3d^4+146\sqrt{c+dx}\,(c+dx)\,cb^2d^5a-73\sqrt{c+dx}\,(c+dx)\,bd^6a^2-15\sqrt{c+dx}\,c^3b^3d^4+45\sqrt{c+dx}\,c^2b^2d^5a-45\sqrt{c+dx}\,cbd^6a^2+15\sqrt{c+dx}\,d^7a^3}{(192c^3b^4-576c^2b^3da+576cb^2d^2a^2-192bd^3a^3)\,((c+dx)\,b-cb+da)^4}-\frac{5d^4\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(32c^3b^4-96c^2b^3da+96cb^2d^2a^2-32bd^3a^3)\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x)
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x
 + c)^(7/2)*b^3*d^4 - 55*(d*x + c)^(5/2)*b^3*c*d^4 + 73*(d*x + c)^(3/2)*b^3
*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^4 + 55*(d*x + c)^(5/2)*a*b^2*d^5 - 14
6*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqrt(d*x + c)*a*b^2*c^2*d^5 + 73*(d*x +
c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^
7)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*((d*x + c)*b -
b*c + a*d)^4)
```

Mupad [**B**]

time = 0.22, size = 297, normalized size = 1.63

$$\frac{\frac{73\,d^4\,(c+d\,x)^{3/2}}{192\,(a\,d-b\,c)}-\frac{5\,d^4\,\sqrt{c+d\,x}}{64\,b}+\frac{5\,b^2\,d^4\,(c+d\,x)^{7/2}}{64\,(a\,d-b\,c)^3}+\frac{55\,b\,d^4\,(c+d\,x)^{5/2}}{192\,(a\,d-b\,c)^2}}{b^4\,(c+d\,x)^4-(4\,b^4\,c-4\,a\,b^3\,d)\,(c+d\,x)^3-(c+d\,x)\,(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3)+a^4\,d^4+b^4\,c^4+(c+d\,x)^2\,(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{3/2}\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^5,x)
```

```
[Out] ((73*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (5*d^4*(c + d*x)^(1/2))/(64*b
) + (5*b^2*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^3) + (55*b*d^4*(c + d*x)^(5
/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x
)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*
d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*
c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + (5*d^4*atan((b^
(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(3/2)*(a*d - b*c)^(7/2))
```

3.1386 $\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx$

Optimal. Leaf size=218

$$-\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}-\frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2}+\frac{7d^4\sqrt{c+d}}{128b(bc-ad)^4(}$$

```
[Out] -7/128*d^5*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*
c)^(9/2)-1/5*(d*x+c)^(1/2)/b/(b*x+a)^5-1/40*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b
*x+a)^4+7/240*d^2*(d*x+c)^(1/2)/b/(-a*d+b*c)^2/(b*x+a)^3-7/192*d^3*(d*x+c)^
(1/2)/b/(-a*d+b*c)^3/(b*x+a)^2+7/128*d^4*(d*x+c)^(1/2)/b/(-a*d+b*c)^4/(b*x+
a)
```

Rubi [A]
time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{7d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c+dx}}{40b(a+bx)^4(bc-ad)}-\frac{\sqrt{c+dx}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] -1/5*Sqrt[c + d*x]/(b*(a + b*x)^5) - (d*Sqrt[c + d*x])/(40*b*(b*c - a*d)*(a
 + b*x)^4) + (7*d^2*Sqrt[c + d*x])/(240*b*(b*c - a*d)^2*(a + b*x)^3) - (7*d
^3*Sqrt[c + d*x])/(192*b*(b*c - a*d)^3*(a + b*x)^2) + (7*d^4*Sqrt[c + d*x])
/(128*b*(b*c - a*d)^4*(a + b*x)) - (7*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[b*c - a*d]])/(128*b^(3/2)*(b*c - a*d)^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx &= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}+\frac{d\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx}{10b}\\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}-\frac{(7d^2)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}+\frac{(7d^3)\int \frac{1}{(a+bx)^3\sqrt{c+\cdots}}}{96b(bc-ad)\cdots}\\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}-\frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}\\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}-\frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}\\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}-\frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}\\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}-\frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}
\end{aligned}$$

Mathematica [A]
time = 1.29, size = 224, normalized size = 1.03

$$\frac{\sqrt{c+dx}\,\left(-105a^4d^4+10a^3bd^3(121c+79dx)+2a^2b^2d^2\left(-1052c^2-289cdx+448d^2x^2\right)+2ab^3d\left(744c^3+128c^2dx-161cd^2x^2+245d^3x^3\right)+b^4\left(-384c^4-48c^3dx+56c^2d^2x^2-70cd^3x^3+105d^4x^4\right)\right)}{1920b(bc-ad)^4(a+bx)^5}+\frac{7d^5\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{128b^{3/2}(-bc+ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (Sqrt[c + d*x]*(-105*a^4*d^4 + 10*a^3*b*d^3*(121*c + 79*d*x) + 2*a^2*b^2*d^
2*(-1052*c^2 - 289*c*d*x + 448*d^2*x^2) + 2*a*b^3*d*(744*c^3 + 128*c^2*d*x
- 161*c*d^2*x^2 + 245*d^3*x^3) + b^4*(-384*c^4 - 48*c^3*d*x + 56*c^2*d^2*x^
2 - 70*c*d^3*x^3 + 105*d^4*x^4)))/(1920*b*(b*c - a*d)^4*(a + b*x)^5) + (7*d
^5*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(128*b^(3/2)*(-(b*c)
 + a*d)^(9/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^6,x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.16, size = 293, normalized size = 1.34

method	result
derivativedivides	$2d^5\left(\frac{\frac{7b^3(dx+c)^{\frac{9}{2}}}{256\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{49b^2(dx+c)^{\frac{7}{2}}}{384\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7b(dx+c)^{\frac{5}{2}}}{30\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{79(d\cdots}{384(\cdots}}{((dx+c)b+ad-bc)^5}\right.$
default	$2d^5\left(\frac{\frac{7b^3(dx+c)^{\frac{9}{2}}}{256\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{49b^2(dx+c)^{\frac{7}{2}}}{384\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7b(dx+c)^{\frac{5}{2}}}{30\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{79(d\cdots}{384(\cdots}}{((dx+c)b+ad-bc)^5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^5*((7/256*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^
4*c^4)*(d*x+c)^(9/2)+49/384*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)*(d*x+c)^(7/2)+7/30*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+79/384/(a
*d-b*c)*(d*x+c)^(3/2)-7/256*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c)^5+7/256/b/
(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/((a*d-b*c)*
b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(186) = 372$.
time = 0.34, size = 1673, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(384*b^6*c^5
- 1872*a*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4
*b^2*c*d^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*
d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2
*d^3 + 87*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5
*c^3*d^2 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sq
rt(d*x + c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b
^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d +
10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x
^5 + 5*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2
*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^
4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d
^5)*x^3 + 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b
^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^
7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b
^3*d^5)*x), 1/1920*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3
 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)*arcta
n(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (384*b^6*c^5 - 1872*a
*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4*b^2*c*d
^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 8
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5*c^3*d^2
 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sqrt(d*x +
 c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d
^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b
^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x^5 + 5*(
a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5
```

```
*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10
*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*x^3
+ 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d
^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d
+ 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*
x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(186) = 372.
time = 0.01, size = 551, normalized size = 2.53

$$\frac{-105\sqrt{c+dx}\,(c+dx)^4b^4d^5+490\sqrt{c+dx}\,(c+dx)^3cb^4d^5-490\sqrt{c+dx}\,(c+dx)^3b^3d^6a-896\sqrt{c+dx}\,(c+dx)^2c^2b^4d^5+1792\sqrt{c+dx}\,(c+dx)^2cb^3d^6a-896\sqrt{c+dx}\,(c+dx)^2b^2d^7a^2+790\sqrt{c+dx}\,(c+dx)c^3b^4d^5-2370\sqrt{c+dx}\,(c+dx)c^2b^3d^6a+2370\sqrt{c+dx}\,(c+dx)cb^2d^7a^2-790\sqrt{c+dx}\,(c+dx)bd^8a^3+105\sqrt{c+dx}\,c^4b^4d^5-420\sqrt{c+dx}\,c^3b^3d^6a+630\sqrt{c+dx}\,c^2b^2d^7a^2-420\sqrt{c+dx}\,cbd^8a^3+105\sqrt{c+dx}\,d^9a^4}{(-1920c^4b^5+7680c^3b^4da-11520c^2b^3d^2a^2+7680cb^2d^3a^3-1920bd^4a^4)((c+dx)b-cb+da)^5}-\frac{7d^5\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2(-64c^4b^5+256c^3b^4da-384c^2b^3d^2a^2+256cb^2d^3a^3-64bd^4a^4)\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x)
```

```
[Out] 7/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*c + a*b*
d)) + 1/1920*(105*(d*x + c)^(9/2)*b^4*d^5 - 490*(d*x + c)^(7/2)*b^4*c*d^5 +
 896*(d*x + c)^(5/2)*b^4*c^2*d^5 - 790*(d*x + c)^(3/2)*b^4*c^3*d^5 - 105*sq
rt(d*x + c)*b^4*c^4*d^5 + 490*(d*x + c)^(7/2)*a*b^3*d^6 - 1792*(d*x + c)^(5
/2)*a*b^3*c*d^6 + 2370*(d*x + c)^(3/2)*a*b^3*c^2*d^6 + 420*sqrt(d*x + c)*a*
b^3*c^3*d^6 + 896*(d*x + c)^(5/2)*a^2*b^2*d^7 - 2370*(d*x + c)^(3/2)*a^2*b^
2*c*d^7 - 630*sqrt(d*x + c)*a^2*b^2*c^2*d^7 + 790*(d*x + c)^(3/2)*a^3*b*d^8
 + 420*sqrt(d*x + c)*a^3*b*c*d^8 - 105*sqrt(d*x + c)*a^4*d^9)/((b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*((d*x + c)*
b - b*c + a*d)^5)
```

Mupad [**B**]
time = 0.49, size = 401, normalized size = 1.84

$$\frac{\frac{79\,d^5\,(c+d\,x)^{3/2}}{192\,(a\,d-b\,c)}-\frac{7\,d^5\,\sqrt{c+d\,x}}{128\,b}+\frac{49\,b^2\,d^5\,(c+d\,x)^{7/2}}{192\,(a\,d-b\,c)^3}+\frac{7\,b^3\,d^5\,(c+d\,x)^{9/2}}{128\,(a\,d-b\,c)^4}+\frac{7\,b\,d^5\,(c+d\,x)^{5/2}}{15\,(a\,d-b\,c)^2}}{b^5\,(c+d\,x)^5-(c+d\,x)^2\,(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3)-(5\,b^5\,c-5\,a\,b^4\,d)\,(c+d\,x)^4+a^5\,d^5-b^5\,c^5+(c+d\,x)^3\,(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2)+(c+d\,x)\,(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{7\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{3/2}\,(a\,d-b\,c)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^6,x)
```

[Out] ((79*d^5*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (7*d^5*(c + d*x)^(1/2))/(128*b) + (49*b^2*d^5*(c + d*x)^(7/2))/(192*(a*d - b*c)^3) + (7*b^3*d^5*(c + d*x)^(9/2))/(128*(a*d - b*c)^4) + (7*b*d^5*(c + d*x)^(5/2))/(15*(a*d - b*c)^2))/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4) + (7*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(3/2)*(a*d - b*c)^(9/2))

3.1387 $\int (a+bx)^5(c+dx)^{3/2}\,dx$

Optimal. Leaf size=158

$$-\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/2}}{9d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{11/2}}{11d^6}-$$

```
[Out] -2/5*(-a*d+b*c)^5*(d*x+c)^(5/2)/d^6+10/7*b*(-a*d+b*c)^4*(d*x+c)^(7/2)/d^6-2
0/9*b^2*(-a*d+b*c)^3*(d*x+c)^(9/2)/d^6+20/11*b^3*(-a*d+b*c)^2*(d*x+c)^(11/2
)/d^6-10/13*b^4*(-a*d+b*c)*(d*x+c)^(13/2)/d^6+2/15*b^5*(d*x+c)^(15/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c+dx)^{5/2}(bc-ad)^5}{5d^6}+\frac{2b^5(c+dx)^{15/2}}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(5/2))/(5*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(7/2))/(7*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(
b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(13
/2))/(13*d^6) + (2*b^5*(c + d*x)^(15/2))/(15*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{3/2}\,dx = \int \left(\frac{(-bc+ad)^5(c+dx)^{3/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{5/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx}{d^5}\right.$$

$$= -\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/}}{9d^6}$$

Mathematica [A]
time = 0.12, size = 217, normalized size = 1.37

$$\frac{2(c+dx)^{5/2}\left(9009a^5d^5+6435a^4bd^4(-2c+5dx)+1430a^3b^2d^3\left(8c^2-20cdx+35d^2x^2\right)+390a^2b^3d^2\left(-16c^3+40c^2dx-70cd^2x^2+105d^3x^3\right)+15ab^4d\left(128c^4-320c^3dx+560c^2d^2x^2-840cd^3x^3+1155d^4x^4\right)+b^5\left(-256c^5+640c^4dx-1120c^3d^2x^2+1680c^2d^3x^3-2310cd^4x^4+3003d^5x^5\right)\right)}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(9009*a^5*d^5 + 6435*a^4*b*d^4*(-2*c + 5*d*x) + 1430*a^3
*b^2*d^3*(8*c^2 - 20*c*d*x + 35*d^2*x^2) + 390*a^2*b^3*d^2*(-16*c^3 + 40*c^
2*d*x - 70*c*d^2*x^2 + 105*d^3*x^3) + 15*a*b^4*d*(128*c^4 - 320*c^3*d*x + 5
60*c^2*d^2*x^2 - 840*c*d^3*x^3 + 1155*d^4*x^4) + b^5*(-256*c^5 + 640*c^4*d*
x - 1120*c^3*d^2*x^2 + 1680*c^2*d^3*x^3 - 2310*c*d^4*x^4 + 3003*d^5*x^5)))/
(45045*d^6)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 456 vs. $2(158) = 316$.
time $= 22.90$, size $= 452$, normalized size $= 2.86$

$$\frac{2\left(9009a^5c^2d^5+18018a^5cd^6x+9009a^5d^7x^2-12870a^4bc^3d^4+6435a^4bc^2d^5x+51480a^4bcd^6x^2+32175a^4bd^7x^3+11440a^3b^2c^4d^3-5720a^3b^2c^3d^4x+4290a^3b^2c^2d^5x^2+71500a^3b^2cd^6x^3+50050a^3b^2d^7x^4-6240a^2b^3c^5d^2+3120a^2b^3c^4d^3x-2340a^2b^3c^3d^4x^2+1950a^2b^3c^2d^5x^3+54600a^2b^3cd^6x^4+40950a^2b^3d^7x^5+1920ab^4c^6d-960ab^4c^5d^2x+720ab^4c^4d^3x^2-600ab^4c^3d^4x^3+525ab^4c^2d^5x^4+22050ab^4cd^6x^5+17325ab^4d^7x^6-256b^5c^7+128b^5c^6dx-96b^5c^5d^2x^2+80b^5c^4d^3x^3-70b^5c^3d^4x^4+63b^5c^2d^5x^5+3696b^5cd^6x^6+3003b^5d^7x^7\right)\sqrt{c+dx}}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(c + d*x)^(3/2),x]')
```

```
[Out] 2 (9009 a ^ 5 c ^ 2 d ^ 5 + 18018 a ^ 5 c d ^ 6 x + 9009 a ^ 5 d ^ 7 x ^ 2
- 12870 a ^ 4 b c ^ 3 d ^ 4 + 6435 a ^ 4 b c ^ 2 d ^ 5 x + 51480 a ^ 4 b c
d ^ 6 x ^ 2 + 32175 a ^ 4 b d ^ 7 x ^ 3 + 11440 a ^ 3 b ^ 2 c ^ 4 d ^ 3 - 5
720 a ^ 3 b ^ 2 c ^ 3 d ^ 4 x + 4290 a ^ 3 b ^ 2 c ^ 2 d ^ 5 x ^ 2 + 71500
a ^ 3 b ^ 2 c d ^ 6 x ^ 3 + 50050 a ^ 3 b ^ 2 d ^ 7 x ^ 4 - 6240 a ^ 2 b ^
3 c ^ 5 d ^ 2 + 3120 a ^ 2 b ^ 3 c ^ 4 d ^ 3 x - 2340 a ^ 2 b ^ 3 c ^ 3 d ^
 4 x ^ 2 + 1950 a ^ 2 b ^ 3 c ^ 2 d ^ 5 x ^ 3 + 54600 a ^ 2 b ^ 3 c d ^ 6 x
 ^ 4 + 40950 a ^ 2 b ^ 3 d ^ 7 x ^ 5 + 1920 a b ^ 4 c ^ 6 d - 960 a b ^ 4 c
 ^ 5 d ^ 2 x + 720 a b ^ 4 c ^ 4 d ^ 3 x ^ 2 - 600 a b ^ 4 c ^ 3 d ^ 4 x ^
3 + 525 a b ^ 4 c ^ 2 d ^ 5 x ^ 4 + 22050 a b ^ 4 c d ^ 6 x ^ 5 + 17325 a b
 ^ 4 d ^ 7 x ^ 6 - 256 b ^ 5 c ^ 7 + 128 b ^ 5 c ^ 6 d x - 96 b ^ 5 c ^ 5 d
 ^ 2 x ^ 2 + 80 b ^ 5 c ^ 4 d ^ 3 x ^ 3 - 70 b ^ 5 c ^ 3 d ^ 4 x ^ 4 + 63 b
 ^ 5 c ^ 2 d ^ 5 x ^ 5 + 3696 b ^ 5 c d ^ 6 x ^ 6 + 3003 b ^ 5 d ^ 7 x ^ 7)
 Sqrt[c + d x] / (45045 d ^ 6)
```

Maple [**A**]
time $= 0.16$, size $= 122$, normalized size $= 0.77$

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{15}{2}}}{15}+\frac{10(ad-bc)b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{10(ad-bc)^4b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^5(d\ldots}{5}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{15}{2}}}{15}+\frac{10(ad-bc)b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{10(ad-bc)^4b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^5(d\ldots}{5}}{d^6}$
gosper	$2(dx+c)^{\frac{5}{2}}(3003b^5x^5d^5+17325ab^4d^5x^4-2310b^5cd^4x^4+40950a^2b^3d^5x^3-12600ab^4cd^4x^3+1680b^5c^2d^3x^3+50050a^3b^2d^{\ldots}$
trager	$2(3003b^5d^7x^7+17325ab^4d^7x^6+3696b^5cd^6x^6+40950a^2b^3d^7x^5+22050ab^4cd^6x^5+63b^5c^2d^5x^5+50050a^3b^2d^7x^4+54600a\ldots$

risch	$\frac{2(3003b^5d^7x^7+17325a\,b^4d^7x^6+3696b^5c\,d^6x^6+40950a^2b^3d^7x^5+22050a\,b^4c\,d^6x^5+63b^5c^2d^5x^5+50050a^3b^2d^7x^4+54600a^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/15*b^5*(d*x+c)^(15/2)+5/13*(a*d-b*c)*b^4*(d*x+c)^(13/2)+10/11*(a*d
-b*c)^2*b^3*(d*x+c)^(11/2)+10/9*(a*d-b*c)^3*b^2*(d*x+c)^(9/2)+5/7*(a*d-b*c)
^4*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^5*(d*x+c)^(5/2))
```

Maxima [A]
time = 0.26, size = 259, normalized size = 1.64

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^5-17325\,(b^5c-ab^4d)(dx+c)^{\frac{13}{2}}+40950\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)(dx+c)^{\frac{11}{2}}-50050\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{9}{2}}+32175\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)(dx+c)^{\frac{7}{2}}-9009\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)(dx+c)^{\frac{5}{2}}\right)}{45045\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^5 - 17325*(b^5*c - a*b^4*d)*(d*x + c)^(13/
2) + 40950*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(11/2) - 50050*(
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(9/2) +
32175*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*(d*x + c)^(7/2) - 9009*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2
 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(5/2))/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(134) = 268$.
time = 0.30, size = 418, normalized size = 2.65

$$\frac{2\left(3003\,b^5d^7x^7-256\,b^5c^7+1920\,ab^4c^6d-6240\,a^2b^3c^5d^2+11440\,a^3b^2c^4d^3-12870\,a^4bc^3d^4+9009\,a^5c^2d^5+231\,(16\,b^5cd^6+75\,ab^4d^7)x^6+63\,(b^5c^2d^5+350\,ab^4cd^6+650\,a^2b^3d^7)x^5-35\,(2\,b^5c^3d^4-15\,ab^4c^2d^5-1560\,a^2b^3cd^6-1430\,a^3b^2d^7)x^4+5\,(16\,b^5c^4d^3-120\,ab^4c^3d^4+390\,a^2b^3c^2d^5+14300\,a^3b^2cd^6+6435\,a^4bd^7)x^3-3\,(32\,b^5c^5d^2-240\,ab^4c^4d^3+780\,a^2b^3c^3d^4-1430\,a^3b^2c^2d^5-17160\,a^4bcd^6-3003\,a^5d^7)x^2+(128\,b^5c^6d-960\,ab^4c^5d^2+3120\,a^2b^3c^4d^3-5720\,a^3b^2c^3d^4+6435\,a^4bc^2d^5+18018\,a^5cd^6)x\right)\sqrt{dx+c}}{45045\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^5*d^7*x^7 - 256*b^5*c^7 + 1920*a*b^4*c^6*d - 6240*a^2*b^3*c
^5*d^2 + 11440*a^3*b^2*c^4*d^3 - 12870*a^4*b*c^3*d^4 + 9009*a^5*c^2*d^5 + 2
31*(16*b^5*c*d^6 + 75*a*b^4*d^7)*x^6 + 63*(b^5*c^2*d^5 + 350*a*b^4*c*d^6 +
650*a^2*b^3*d^7)*x^5 - 35*(2*b^5*c^3*d^4 - 15*a*b^4*c^2*d^5 - 1560*a^2*b^3*
c*d^6 - 1430*a^3*b^2*d^7)*x^4 + 5*(16*b^5*c^4*d^3 - 120*a*b^4*c^3*d^4 + 390
*a^2*b^3*c^2*d^5 + 14300*a^3*b^2*c*d^6 + 6435*a^4*b*d^7)*x^3 - 3*(32*b^5*c^
5*d^2 - 240*a*b^4*c^4*d^3 + 780*a^2*b^3*c^3*d^4 - 1430*a^3*b^2*c^2*d^5 - 17
160*a^4*b*c*d^6 - 3003*a^5*d^7)*x^2 + (128*b^5*c^6*d - 960*a*b^4*c^5*d^2 +
3120*a^2*b^3*c^4*d^3 - 5720*a^3*b^2*c^3*d^4 + 6435*a^4*b*c^2*d^5 + 18018*a^
5*c*d^6)*x)*sqrt(d*x + c)/d^6
```

Sympy [A]
time = 13.41, size = 763, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(3/2),x)
```

```
[Out] a**5*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**5*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 10*a**4*b*c*(-c*(c
 + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 10*a**4*b*(c**2*(c + d*x)**(3
/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 20*a**3*b**2*c*
(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**
3 + 20*a**3*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*
c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**2*b**3*c*(-c**3*(c
+ d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c +
 d*x)**(9/2)/9)/d**4 + 20*a**2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c +
d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d
*x)**(11/2)/11)/d**4 + 10*a*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d
*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*
x)**(11/2)/11)/d**5 + 10*a*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**5*c*(-c**5*(c + d*x)**
(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c +
 d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**6 + 2*
b**5*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*
x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6
*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(134) = 268$.
time = 0.01, size = 1798, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x)
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^5*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^5*c + 75075*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^5 +
 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
3*b^2*c^2/d^2 + 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^4*b*c/d + 12870*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c^2/d^3 + 25740*(5*(
d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*a^3*b^2*c/d^2 + 6435*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c
+ 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4*b/d + 715*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c^2/d^4 + 2860*(35*(d*x + c)^(9/2)
```

```
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a^2*b^3*c/d^3 + 1430*(35*(d*x + c)^(9/2) - 180*(d
*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*s
qrt(d*x + c)*c^4)*a^3*b^2/d^2 + 65*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/
2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^
(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c^2/d^5 + 650*(63*(d*x + c)^(11/2) -
 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3
 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c/d^4 + 650*(63*
(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(
d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*
b^3/d^3 + 30*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x +
c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(
d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c/d^5 + 75*(231*(d*x + c)^
(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c
)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sq
rt(d*x + c)*c^6)*a*b^4/d^4 + 7*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2
)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x +
 c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 643
5*sqrt(d*x + c)*c^7)*b^5/d^5)/d
```

Mupad [B]

time = 0.24, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{15/2}}{15\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{5/2}}{5\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(15/2))/(15*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(13/
2))/(13*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(5/2))/(5*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(11/2))/(1
1*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^6)
```

3.1388 $\int (a+bx)^4(c+dx)^{3/2}\,dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}-\frac{8b^3(bc-ad)(c+dx)^{11/2}}{11d^5}+\frac{2b^4(}{}$$

```
[Out] 2/5*(-a*d+b*c)^4*(d*x+c)^(5/2)/d^5-8/7*b*(-a*d+b*c)^3*(d*x+c)^(7/2)/d^5+4/3
*b^2*(-a*d+b*c)^2*(d*x+c)^(9/2)/d^5-8/11*b^3*(-a*d+b*c)*(d*x+c)^(11/2)/d^5+
2/13*b^4*(d*x+c)^(13/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(5/2))/(5*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(7
/2))/(7*d^5) + (4*b^2*(b*c - a*d)^2*(c + d*x)^(9/2))/(3*d^5) - (8*b^3*(b*c
- a*d)*(c + d*x)^(11/2))/(11*d^5) + (2*b^4*(c + d*x)^(13/2))/(13*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{3/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{5/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx}{d^4}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 154, normalized size = 1.19

$$\frac{2(c+dx)^{5/2}\left(3003a^4d^4+1716a^3bd^3(-2c+5dx)+286a^2b^2d^2\left(8c^2-20cdx+35d^2x^2\right)+52ab^3d\left(-16c^3+40c^2dx-70cd^2x^2+105d^3x^3\right)+b^4\left(128c^4-320c^3dx+560c^2d^2x^2-840cd^3x^3+1155d^4x^4\right)\right)}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*a^4*d^4 + 1716*a^3*b*d^3*(-2*c + 5*d*x) + 286*a^2*
b^2*d^2*(8*c^2 - 20*c*d*x + 35*d^2*x^2) + 52*a*b^3*d*(-16*c^3 + 40*c^2*d*x
- 70*c*d^2*x^2 + 105*d^3*x^3) + b^4*(128*c^4 - 320*c^3*d*x + 560*c^2*d^2*x^
2 - 840*c*d^3*x^3 + 1155*d^4*x^4)))/(15015*d^5)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(129) = 258$.
time = 17.46, size = 331, normalized size = 2.57

$$\frac{2\left(3003a^4c^2d^4+6006a^4cd^5x+3003a^4d^6x^2-3432a^3bc^3d^3+1716a^3bc^2d^4x+13728a^3bcd^5x^2+8580a^3bd^6x^3+2288a^2b^2c^4d^2-1144a^2b^2c^3d^3x+858a^2b^2c^2d^4x^2+14300a^2b^2cd^5x^3+10010a^2b^2d^6x^4-832ab^3c^5d+416ab^3c^4d^2x-312ab^3c^3d^3x^2+260ab^3c^2d^4x^3+7280ab^3cd^5x^4+5460ab^3d^6x^5+128b^4c^6-64b^4c^5dx+48b^4c^4d^2x^2-40b^4c^3d^3x^3+35b^4c^2d^4x^4+1470b^4cd^5x^5+1155b^4d^6x^6\right)\sqrt{c+dx}}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^(3/2),x]')
```

```
[Out] 2 (3003 a ^ 4 c ^ 2 d ^ 4 + 6006 a ^ 4 c d ^ 5 x + 3003 a ^ 4 d ^ 6 x ^ 2 -
 3432 a ^ 3 b c ^ 3 d ^ 3 + 1716 a ^ 3 b c ^ 2 d ^ 4 x + 13728 a ^ 3 b c d
^ 5 x ^ 2 + 8580 a ^ 3 b d ^ 6 x ^ 3 + 2288 a ^ 2 b ^ 2 c ^ 4 d ^ 2 - 1144
a ^ 2 b ^ 2 c ^ 3 d ^ 3 x + 858 a ^ 2 b ^ 2 c ^ 2 d ^ 4 x ^ 2 + 14300 a ^ 2
 b ^ 2 c d ^ 5 x ^ 3 + 10010 a ^ 2 b ^ 2 d ^ 6 x ^ 4 - 832 a b ^ 3 c ^ 5 d
+ 416 a b ^ 3 c ^ 4 d ^ 2 x - 312 a b ^ 3 c ^ 3 d ^ 3 x ^ 2 + 260 a b ^ 3 c
 ^ 2 d ^ 4 x ^ 3 + 7280 a b ^ 3 c d ^ 5 x ^ 4 + 5460 a b ^ 3 d ^ 6 x ^ 5 +
128 b ^ 4 c ^ 6 - 64 b ^ 4 c ^ 5 d x + 48 b ^ 4 c ^ 4 d ^ 2 x ^ 2 - 40 b ^
4 c ^ 3 d ^ 3 x ^ 3 + 35 b ^ 4 c ^ 2 d ^ 4 x ^ 4 + 1470 b ^ 4 c d ^ 5 x ^ 5
 + 1155 b ^ 4 d ^ 6 x ^ 6) Sqrt[c + d x] / (15015 d ^ 5)
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{8(ad-bc)b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)^2b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{8(ad-bc)^3b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^4(dx+c)^{\frac{5}{2}}}{5}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{8(ad-bc)b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)^2b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{8(ad-bc)^3b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^4(dx+c)^{\frac{5}{2}}}{5}}{d^5}$
gosper	$\frac{2(dx+c)^{\frac{5}{2}}\left(1155d^4x^4b^4+5460a\,b^3d^4x^3-840b^4c\,d^3x^3+10010a^2b^2d^4x^2-3640a\,b^3c\,d^3x^2+560b^4c^2d^2x^2+8580a^3b\,d^4x-5720\right.}{15015d^5}$
trager	$2\left(1155b^4d^6x^6+5460a\,b^3d^6x^5+1470b^4c\,d^5x^5+10010a^2b^2d^6x^4+7280a\,b^3c\,d^5x^4+35b^4c^2d^4x^4+8580a^3b\,d^6x^3+14300a^2b^2c\right.$
risch	$2\left(1155b^4d^6x^6+5460a\,b^3d^6x^5+1470b^4c\,d^5x^5+10010a^2b^2d^6x^4+7280a\,b^3c\,d^5x^4+35b^4c^2d^4x^4+8580a^3b\,d^6x^3+14300a^2b^2c\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/13*b^4*(d*x+c)^(13/2)+4/11*(a*d-b*c)*b^3*(d*x+c)^(11/2)+2/3*(a*d-b
*c)^2*b^2*(d*x+c)^(9/2)+4/7*(a*d-b*c)^3*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^4*(d*
x+c)^(5/2))
```

Maxima [A]
time = 0.30, size = 181, normalized size = 1.40

$$\frac{2\left(1155\,(dx+c)^{\frac{13}{2}}b^4-5460\,(b^4c-ab^3d)(dx+c)^{\frac{11}{2}}+10010\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(dx+c)^{\frac{9}{2}}-8580\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)(dx+c)^{\frac{7}{2}}+3003\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{5}{2}}\right)}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(d*x + c)^(13/2)*b^4 - 5460*(b^4*c - a*b^3*d)*(d*x + c)^(11/2
) + 10010*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(9/2) - 8580*(b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(7/2) + 3003*
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*
x + c)^(5/2))/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(109) = 218$.
time = 0.30, size = 311, normalized size = 2.41

$$\frac{2\,(1155\,b^4d^6x^6+128\,b^4c^6-832\,ab^3c^5d+2288\,a^2b^2c^4d^2-3432\,a^3bc^3d^3+3003\,a^4c^2d^4+210\,(7\,b^4cd^5+26\,ab^3d^6)x^5+35\,(b^4c^2d^4+208\,ab^3cd^5+286\,a^2b^2d^6)x^4-20\,(2\,b^4c^3d^3-13\,ab^3c^2d^4-715\,a^2b^2cd^5-429\,a^3bd^6)x^3+3\,(16\,b^4c^4d^2-104\,ab^3c^3d^3+286\,a^2b^2c^2d^4+4576\,a^3bcd^5+1001\,a^4d^6)x^2-2\,(32\,b^4c^5d-208\,ab^3c^4d^2+572\,a^2b^2c^3d^3-858\,a^3bc^2d^4-3003\,a^4cd^5)x)\sqrt{dx+c}}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^4*d^6*x^6 + 128*b^4*c^6 - 832*a*b^3*c^5*d + 2288*a^2*b^2*c^
4*d^2 - 3432*a^3*b*c^3*d^3 + 3003*a^4*c^2*d^4 + 210*(7*b^4*c*d^5 + 26*a*b^3
*d^6)*x^5 + 35*(b^4*c^2*d^4 + 208*a*b^3*c*d^5 + 286*a^2*b^2*d^6)*x^4 - 20*(
2*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4 - 715*a^2*b^2*c*d^5 - 429*a^3*b*d^6)*x^3 +
 3*(16*b^4*c^4*d^2 - 104*a*b^3*c^3*d^3 + 286*a^2*b^2*c^2*d^4 + 4576*a^3*b*c
*d^5 + 1001*a^4*d^6)*x^2 - 2*(32*b^4*c^5*d - 208*a*b^3*c^4*d^2 + 572*a^2*b^
2*c^3*d^3 - 858*a^3*b*c^2*d^4 - 3003*a^4*c*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A]
time = 10.07, size = 559, normalized size = 4.33

$$a^4c\left(\begin{cases}\sqrt{c}\,x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^4\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{8a^3bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{8a^3b\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{12a^2b^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}+\frac{12a^2b^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{8ab^3c\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}+\frac{8ab^3\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^4}+\frac{2b^4c\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^5}+\frac{2b^4\left(-\frac{c^5(c+dx)^{\frac{3}{2}}}{3}+c^4\,(c+dx)^{\frac{5}{2}}-\frac{10c^3(c+dx)^{\frac{7}{2}}}{7}+\frac{10c^2(c+dx)^{\frac{9}{2}}}{9}-\frac{5c(c+dx)^{\frac{11}{2}}}{11}+\frac{(c+dx)^{\frac{13}{2}}}{13}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(3/2),x)
```

```
[Out] a**4*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**4*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 8*a**3*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 8*a**3*b*(c**2*(c + d*x)**(3/2
```

```
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 12*a**2*b**2*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3
+ 12*a**2*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*
(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 8*a*b**3*c*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 8*a*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2
)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2
)/11)/d**4 + 2*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
+ 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11
)/d**5 + 2*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3
*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11
 + (c + d*x)**(13/2)/13)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(109) = 218$.
time = 0.01, size = 1320, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x)
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4 +
 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
2*b^2*c^2/d^2 + 24024*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^3*b*c/d + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c^2/d^3 + 15444*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x +
 c)*c^3)*a^2*b^2*c/d^2 + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b/d + 143*(35*(d*x + c)^(
9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2
)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^2/d^4 + 1144*(35*(d*x + c)^(9/2) - 180
*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 31
5*sqrt(d*x + c)*c^4)*a*b^3*c/d^3 + 858*(35*(d*x + c)^(9/2) - 180*(d*x + c)^
(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
+ c)*c^4)*a^2*b^2/d^2 + 130*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*b^4*c/d^4 + 260*(63*(d*x + c)^(11/2) - 385*(d*x
 + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(
d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^3/d^3 + 15*(231*(d*x + c)^(
13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)
^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqr
t(d*x + c)*c^6)*b^4/d^4)/d
```

Mupad [B]

time = 0.24, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,(c+d\,x)^{13/2}}{13\,d^5}-\frac{(8\,b^4\,c-8\,a\,b^3\,d)\,\,(c+d\,x)^{11/2}}{11\,d^5}+\frac{2\,(a\,d-b\,c)^4\,(c+d\,x)^{5/2}}{5\,d^5}+\frac{4\,b^2\,(a\,d-b\,c)^2\,(c+d\,x)^{9/2}}{3\,d^5}+\frac{8\,b\,(a\,d-b\,c)^3\,(c+d\,x)^{7/2}}{7\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(13/2))/(13*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(11/2)
)/(11*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(5/2))/(5*d^5) + (4*b^2*(a*d - b*c)
^2*(c + d*x)^(9/2))/(3*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^5)
```

3.1389 $\int (a+bx)^3(c+dx)^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

```
[Out] -2/5*(-a*d+b*c)^3*(d*x+c)^(5/2)/d^4+6/7*b*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^4-2/
3*b^2*(-a*d+b*c)*(d*x+c)^(9/2)/d^4+2/11*b^3*(d*x+c)^(11/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
7/2))/(7*d^4) - (2*b^2*(b*c - a*d)*(c + d*x)^(9/2))/(3*d^4) + (2*b^3*(c + d
*x)^(11/2))/(11*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^{3/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{5/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{7/}}{d^3}\right.\\ &= -\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}+\end{aligned}$$

Mathematica [A]
time = 0.06, size = 102, normalized size = 1.02

$$\frac{2(c+dx)^{5/2}\left(231a^3d^3+99a^2bd^2(-2c+5dx)+11ab^2d\left(8c^2-20cdx+35d^2x^2\right)+b^3\left(-16c^3+40c^2dx-70cd^2x^2+105d^3x^3\right)\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(231*a^3*d^3 + 99*a^2*b*d^2*(-2*c + 5*d*x) + 11*a*b^2*d*
(8*c^2 - 20*c*d*x + 35*d^2*x^2) + b^3*(-16*c^3 + 40*c^2*d*x - 70*c*d^2*x^2
+ 105*d^3*x^3)))/(1155*d^4)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.
time = 12.95, size = 227, normalized size = 2.27

$$\frac{2\left(231a^3c^2d^3 + 462a^3cd^4x + 231a^3d^5x^2 - 198a^2bc^3d^2 + 99a^2bc^2d^3x + 792a^2bcd^4x^2 + 495a^2bd^5x^3 + 88ab^2c^4d - 44ab^2c^3d^2x + 33ab^2c^2d^3x^2 + 550ab^2cd^4x^3 + 385ab^2d^5x^4 - 16b^3c^5 + 8b^3c^4dx - 6b^3c^3d^2x^2 + 5b^3c^2d^3x^3 + 140b^3cd^4x^4 + 105b^3d^5x^5\right)\sqrt{c+dx}}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^(3/2),x]')
```

```
[Out] 2 (231 a ^ 3 c ^ 2 d ^ 3 + 462 a ^ 3 c d ^ 4 x + 231 a ^ 3 d ^ 5 x ^ 2 - 19
8 a ^ 2 b c ^ 3 d ^ 2 + 99 a ^ 2 b c ^ 2 d ^ 3 x + 792 a ^ 2 b c d ^ 4 x ^
2 + 495 a ^ 2 b d ^ 5 x ^ 3 + 88 a b ^ 2 c ^ 4 d - 44 a b ^ 2 c ^ 3 d ^ 2 x
 + 33 a b ^ 2 c ^ 2 d ^ 3 x ^ 2 + 550 a b ^ 2 c d ^ 4 x ^ 3 + 385 a b ^ 2 d
 ^ 5 x ^ 4 - 16 b ^ 3 c ^ 5 + 8 b ^ 3 c ^ 4 d x - 6 b ^ 3 c ^ 3 d ^ 2 x ^ 2
 + 5 b ^ 3 c ^ 2 d ^ 3 x ^ 3 + 140 b ^ 3 c d ^ 4 x ^ 4 + 105 b ^ 3 d ^ 5 x
^ 5) Sqrt[c + d x] / (1155 d ^ 4)
```

Maple [**A**]
time = 0.17, size = 78, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{6(ad-bc)^2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^3(dx+c)^{\frac{5}{2}}}{5}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{6(ad-bc)^2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^3(dx+c)^{\frac{5}{2}}}{5}}{d^4}$
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(105b^3x^3d^3+385a\,b^2d^3x^2-70b^3c\,d^2x^2+495a^2b\,d^3x-220a\,b^2c\,d^2x+40b^3c^2dx+231a^3d^3-198a^2bc\,d^2+88a\,b^2c^2}{1155d^4}$
trager	$2(105b^3d^5x^5+385a\,b^2d^5x^4+140b^3c\,d^4x^4+495a^2b\,d^5x^3+550a\,b^2c\,d^4x^3+5b^3c^2d^3x^3+231a^3d^5x^2+792a^2bc\,d^4x^2+33a\,b^2c$
risch	$2(105b^3d^5x^5+385a\,b^2d^5x^4+140b^3c\,d^4x^4+495a^2b\,d^5x^3+550a\,b^2c\,d^4x^3+5b^3c^2d^3x^3+231a^3d^5x^2+792a^2bc\,d^4x^2+33a\,b^2c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/11*b^3*(d*x+c)^(11/2)+1/3*(a*d-b*c)*b^2*(d*x+c)^(9/2)+3/7*(a*d-b*c
)^2*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^3*(d*x+c)^(5/2))
```

Maxima [**A**]
time = 0.27, size = 118, normalized size = 1.18

$$\frac{2\left(105\,(dx+c)^{\frac{11}{2}}b^3 - 385\,(b^3c - ab^2d)(dx+c)^{\frac{9}{2}} + 495\,(b^3c^2 - 2\,ab^2cd + a^2bd^2)(dx+c)^{\frac{7}{2}} - 231\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)(dx+c)^{\frac{5}{2}}\right)}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*(d*x + c)^(11/2)*b^3 - 385*(b^3*c - a*b^2*d)*(d*x + c)^(9/2) +
495*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(7/2) - 231*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(5/2))/d^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(84) = 168$.
time = 0.30, size = 216, normalized size = 2.16

$$\frac{2\left(105\,b^{3}d^{5}x^{5}-16\,b^{3}c^{5}+88\,ab^{2}c^{4}d-198\,a^{2}bc^{3}d^{2}+231\,a^{3}c^{2}d^{3}+35\left(4\,b^{3}cd^{4}+11\,ab^{2}d^{5}\right)x^{4}+5\left(b^{3}c^{2}d^{3}+110\,ab^{2}cd^{4}+99\,a^{2}bd^{5}\right)x^{3}-3\left(2\,b^{3}c^{3}d^{2}-11\,ab^{2}c^{2}d^{3}-264\,a^{2}bcd^{4}-77\,a^{3}d^{5}\right)x^{2}+\left(8\,b^{3}c^{4}d-44\,ab^{2}c^{3}d^{2}+99\,a^{2}bc^{2}d^{3}+462\,a^{3}cd^{4}\right)x\right)\sqrt{dx+c}}{1155\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*d^5*x^5 - 16*b^3*c^5 + 88*a*b^2*c^4*d - 198*a^2*b*c^3*d^2 +
 231*a^3*c^2*d^3 + 35*(4*b^3*c*d^4 + 11*a*b^2*d^5)*x^4 + 5*(b^3*c^2*d^3 + 1
10*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - 3*(2*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3 -
264*a^2*b*c*d^4 - 77*a^3*d^5)*x^2 + (8*b^3*c^4*d - 44*a*b^2*c^3*d^2 + 99*a^
2*b*c^2*d^3 + 462*a^3*c*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [**A**]
time = 7.30, size = 386, normalized size = 3.86

$$a^{3}c\left(\begin{cases}\sqrt{c}\,x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^{3}\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{6a^{2}bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^{2}}+\frac{6a^{2}b\left(\frac{c^{2}(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^{2}}+\frac{6ab^{2}c\left(\frac{c^{2}(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^{3}}+\frac{6ab^{2}\left(-\frac{c^{3}(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^{2}(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^{3}}+\frac{2b^{3}c\left(-\frac{c^{3}(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^{2}(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^{4}}+\frac{2b^{3}\left(\frac{c^{4}(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^{3}(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^{2}(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(3/2),x)
```

```
[Out] a**3*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**3*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 6*a**2*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 6*a**2*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 6*a*b**2*c*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 6*
a*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x
)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**3*c*(-c**3*(c + d*x)**(3/2)/3
+ 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/
d**4 + 2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2
*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**4
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(84) = 168$.
time = 0.01, size = 911, normalized size = 9.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x)
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^3*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^3*c + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^2/d + 231*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3 + 693*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c^2
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b*c/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^3*c^2/d^3 + 594*(5*(d*x + c)^(7/2) - 21
*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c
/d^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c
^2 - 35*sqrt(d*x + c)*c^3)*a^2*b/d + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)
^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
 + c)*c^4)*b^3*c/d^3 + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378
*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b
^2/d^2 + 5*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/
2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x
 + c)*c^5)*b^3/d^3)/d
```

Mupad [B]
time = 0.25, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(11/2))/(11*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(9/2))
/(9*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(a*d - b*c)^2*(
c + d*x)^(7/2))/(7*d^4)
```

3.1390 $\int (a+bx)^2(c+dx)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

```
[Out] 2/5*(-a*d+b*c)^2*(d*x+c)^(5/2)/d^3-4/7*b*(-a*d+b*c)*(d*x+c)^(7/2)/d^3+2/9*b
^2*(d*x+c)^(9/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(7/2
))/(7*d^3) + (2*b^2*(c + d*x)^(9/2))/(9*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(-2c+5dx)+b^2\left(8c^2-20cdx+35d^2x^2\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x) + b^2*(8*c^2 - 20*
c*d*x + 35*d^2*x^2)))/(315*d^3)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 146 vs. $2(71) = 142$.
time = 8.95, size = 142, normalized size = 2.00

$$\frac{2\left(21a^2d^2\left(-2c+3dx\right)+105a^2cd^2+6abd\left(-42c\left(c+dx\right)-7c\left(2c-3dx\right)+35c^2+15\left(c+dx\right)^2\right)+b^2\left(-135c\left(c+dx\right)^2+3c\left(-42c\left(c+dx\right)+35c^2+15\left(c+dx\right)^2\right)+189c^2\left(c+dx\right)-105c^3+35\left(c+dx\right)^3\right)\right)\left(c+dx\right)^{\frac{3}{2}}}{315d^3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^(3/2),x]')
```

```
[Out] 2 (21 a ^ 2 d ^ 2 (-2 c + 3 d x) + 105 a ^ 2 c d ^ 2 + 6 a b d (-42 c (c +
d x) - 7 c (2 c - 3 d x) + 35 c ^ 2 + 15 (c + d x) ^ 2) + b ^ 2 (-135 c (c
+ d x) ^ 2 + 3 c (-42 c (c + d x) + 35 c ^ 2 + 15 (c + d x) ^ 2) + 189 c ^
2 (c + d x) - 105 c ^ 3 + 35 (c + d x) ^ 3)) (c + d x) ^ (3 / 2) / (315 d ^
 3)
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{4(ad-bc)b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^2(dx+c)^{\frac{5}{2}}}{5}}{d^3}$
default	$\frac{\frac{2b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{4(ad-bc)b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^2(dx+c)^{\frac{5}{2}}}{5}}{d^3}$
gosper	$\frac{2(dx+c)^{\frac{5}{2}}\left(35b^2x^2d^2+90ab\,d^2x-20b^2cdx+63a^2d^2-36abcd+8b^2c^2\right)}{315d^3}$
trager	$\frac{2\left(35b^2d^4x^4+90ab\,d^4x^3+50b^2c\,d^3x^3+63a^2d^4x^2+144abc\,d^3x^2+3b^2c^2d^2x^2+126a^2c\,d^3x+18ab\,c^2d^2x-4b^2c^3dx+63a^2c^2d\right.}{315d^3}$
risch	$\frac{2\left(35b^2d^4x^4+90ab\,d^4x^3+50b^2c\,d^3x^3+63a^2d^4x^2+144abc\,d^3x^2+3b^2c^2d^2x^2+126a^2c\,d^3x+18ab\,c^2d^2x-4b^2c^3dx+63a^2c^2d\right.}{315d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/9*b^2*(d*x+c)^(9/2)+2/7*(a*d-b*c)*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^2*
(d*x+c)^(5/2))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.96

$$\frac{2\left(35\left(dx+c\right)^{\frac{9}{2}}b^2-90\left(b^2c-abd\right)\left(dx+c\right)^{\frac{7}{2}}+63\left(b^2c^2-2\,abcd+a^2d^2\right)\left(dx+c\right)^{\frac{5}{2}}\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^2 - 90*(b^2*c - a*b*d)*(d*x + c)^(7/2) + 63*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(5/2))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(59) = 118$.
time = 0.31, size = 137, normalized size = 1.93

$$\frac{2\,(35\,b^2d^4x^4 + 8\,b^2c^4 - 36\,abc^3d + 63\,a^2c^2d^2 + 10\,(5\,b^2cd^3 + 9\,abd^4)x^3 + 3\,(b^2c^2d^2 + 48\,abcd^3 + 21\,a^2d^4)x^2 - 2\,(2\,b^2c^3d - 9\,abc^2d^2 - 63\,a^2cd^3)x)\sqrt{dx+c}}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*d^4*x^4 + 8*b^2*c^4 - 36*a*b*c^3*d + 63*a^2*c^2*d^2 + 10*(5*b
^2*c*d^3 + 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^2
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x)*sqrt(d*x + c)/d^3
```

Sympy [A]
time = 4.86, size = 240, normalized size = 3.38

$$a^2c\left(\begin{cases}\sqrt{c}\,x & \text{for } d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right) + \frac{2a^2\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3} + \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d} + \frac{4abc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3} + \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} + \frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3} - \frac{2c(c+dx)^{\frac{5}{2}}}{5} + \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2} + \frac{2b^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3} - \frac{2c(c+dx)^{\frac{5}{2}}}{5} + \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3} + \frac{2b^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3} + \frac{3c^2(c+dx)^{\frac{5}{2}}}{5} - \frac{3c(c+dx)^{\frac{7}{2}}}{7} + \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 4*a*b*c*(-c*(c + d
*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 -
2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*c*(c**2*(c + d*x
)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 2*b**2*(-c
**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7
 + (c + d*x)**(9/2)/9)/d**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(59) = 118$.
time = 0.00, size = 571, normalized size = 8.04

$$\frac{\frac{2b^2d^2\left(\frac{1}{9}\sqrt{c+dx}\,(c+dx)^4 - \frac{4}{7}\sqrt{c+dx}\,(c+dx)^3c + \frac{6}{5}\sqrt{c+dx}\,(c+dx)^2c^2 - \frac{4}{3}\sqrt{c+dx}\,(c+dx)c^3 + \sqrt{c+dx}\,c^4\right)}{d^4} + \frac{4abd^2\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3 - \frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c + \sqrt{c+dx}\,(c+dx)c^2 - \sqrt{c+dx}\,c^3\right)}{d^3} + \frac{4b^2cd\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3 - \frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c + \sqrt{c+dx}\,(c+dx)c^2 - \sqrt{c+dx}\,c^3\right)}{d^3} + \frac{2a^2d^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2 - \frac{2}{3}\sqrt{c+dx}\,(c+dx)c + \sqrt{c+dx}\,c^2\right)}{d^2} + \frac{2b^2c^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2 - \frac{2}{3}\sqrt{c+dx}\,(c+dx)c + \sqrt{c+dx}\,c^2\right)}{d^2} + \frac{8abcd\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2 - \frac{2}{3}\sqrt{c+dx}\,(c+dx)c + \sqrt{c+dx}\,c^2\right)}{d^2} + 4a^2c\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx) - c\sqrt{c+dx}\right) + \frac{4abc^2\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx) - c\sqrt{c+dx}\right)}{d} + 2a^2c^2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x)
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^2*c^2 + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c
)*a^2*c + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^2/d + 21*(3*(d*x
```

```
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2 + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^2/d^2 + 84*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d +
 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35
*sqrt(d*x + c)*c^3)*b^2*c/d^2 + 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*
c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b/d + (35*(d*x + c)^(9
/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)
*c^3 + 315*sqrt(d*x + c)*c^4)*b^2/d^2)/d
```

Mupad [B]
time = 0.06, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}\,\left(35\,b^2\,{\left(c+d\,x\right)}^2+63\,a^2\,d^2+63\,b^2\,c^2-90\,b^2\,c\,\left(c+d\,x\right)+90\,a\,b\,d\,\left(c+d\,x\right)-126\,a\,b\,c\,d\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(35*b^2*(c + d*x)^2 + 63*a^2*d^2 + 63*b^2*c^2 - 90*b^2*c
*(c + d*x) + 90*a*b*d*(c + d*x) - 126*a*b*c*d))/(315*d^3)
```

3.1391 $\int (a+bx)(c+dx)^{3/2}\,dx$

Optimal. Leaf size=42

$$-\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}$$

```
[Out] -2/5*(-a*d+b*c)*(d*x+c)^(5/2)/d^2+2/7*b*(d*x+c)^(7/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^2) + (2*b*(c + d*x)^(7/2))/(7*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{5/2}(-2bc+7ad+5bdx)}{35d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(c + d*x)^(3/2),x]

[Out] (2*(c + d*x)^(5/2)*(-2*b*c + 7*a*d + 5*b*d*x))/(35*d^2)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.33, size = 78, normalized size = 1.86

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2bc^3+c^2d\left(7a+bx\right)+d^2x\left(14ac+7adx+8bcx+5bdx^2\right)\right)\sqrt{c+dx}}{35d^2},d!{=}0\right\}\right\},c^{\frac{3}{2}}\left(ax+\frac{bx^2}{2}\right)\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^(3/2),x]')

[Out] Piecewise[{{2 (-2 b c ^ 3 + c ^ 2 d (7 a + b x) + d ^ 2 x (14 a c + 7 a d x + 8 b c x + 5 b d x ^ 2)) Sqrt[c + d x] / (35 d ^ 2), d != 0}}, c ^ (3 / 2) (a x + b x ^ 2 / 2)]

Maple [A]
time = 0.13, size = 34, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(5bdx+7ad-2bc)}{35d^2}$	27
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)(dx+c)^{\frac{5}{2}}}{5}}{d^2}$	34
default	$\frac{\frac{2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)(dx+c)^{\frac{5}{2}}}{5}}{d^2}$	34
trager	$\frac{2\left(5b\,d^3x^3+7a\,d^3x^2+8bc\,d^2x^2+14ac\,d^2x+b\,c^2dx+7a\,c^2d-2b\,c^3\right)\sqrt{dx+c}}{35d^2}$	70
risch	$\frac{2\left(5b\,d^3x^3+7a\,d^3x^2+8bc\,d^2x^2+14ac\,d^2x+b\,c^2dx+7a\,c^2d-2b\,c^3\right)\sqrt{dx+c}}{35d^2}$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d^2*(1/7*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)*(d*x+c)^(5/2))

Maxima [A]
time = 0.26, size = 33, normalized size = 0.79

$$\frac{2\left(5\left(dx+c\right)^{\frac{7}{2}}b-7\left(bc-ad\right)\left(dx+c\right)^{\frac{5}{2}}\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="maxima")

```
[Out] 2/35*(5*(d*x + c)^(7/2)*b - 7*(b*c - a*d)*(d*x + c)^(5/2))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.30, size = 69, normalized size = 1.64

$$\frac{2\,(5\,bd^3x^3 - 2\,bc^3 + 7\,ac^2d + (8\,bcd^2 + 7\,ad^3)x^2 + (bc^2d + 14\,acd^2)x)\sqrt{dx + c}}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*d^3*x^3 - 2*b*c^3 + 7*a*c^2*d + (8*b*c*d^2 + 7*a*d^3)*x^2 + (b*c^
2*d + 14*a*c*d^2)*x)*sqrt(d*x + c)/d^2
```

Sympy [A]
time = 0.17, size = 146, normalized size = 3.48

$$\begin{cases} \frac{2ac^2\sqrt{c + dx}}{5d} + \frac{4acx\sqrt{c + dx}}{5} + \frac{2adx^2\sqrt{c + dx}}{5} - \frac{4bc^3\sqrt{c + dx}}{35d^2} + \frac{2bc^2x\sqrt{c + dx}}{35d} + \frac{16bcx^2\sqrt{c + dx}}{35} + \frac{2bdx^3\sqrt{c + dx}}{7} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(ax + \frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((2*a*c**2*sqrt(c + d*x)/(5*d) + 4*a*c*x*sqrt(c + d*x)/5 + 2*a*d*x
**2*sqrt(c + d*x)/5 - 4*b*c**3*sqrt(c + d*x)/(35*d**2) + 2*b*c**2*x*sqrt(c
+ d*x)/(35*d) + 16*b*c*x**2*sqrt(c + d*x)/35 + 2*b*d*x**3*sqrt(c + d*x)/7,
Ne(d, 0)), (c**(3/2)*(a*x + b*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(34) = 68$.
time = 0.00, size = 300, normalized size = 7.14

$$\frac{\frac{2bd^2\left(\frac{1}{7}\sqrt{c + dx}\,(c+dx)^3 - \frac{3}{5}\sqrt{c + dx}\,(c+dx)^2c + \sqrt{c + dx}\,(c+dx)c^2 - \sqrt{c + dx}\,c^3\right)}{d^3} + \frac{2ad^2\left(\frac{1}{5}\sqrt{c + dx}\,(c+dx)^2 - \frac{2}{3}\sqrt{c + dx}\,(c+dx)c + \sqrt{c + dx}\,c^2\right)}{d^2} + \frac{4bcd\left(\frac{1}{5}\sqrt{c + dx}\,(c+dx)^2 - \frac{2}{3}\sqrt{c + dx}\,(c+dx)c + \sqrt{c + dx}\,c^2\right)}{d^2} + 4ac\left(\frac{1}{3}\sqrt{c + dx}\,(c + dx) - c\sqrt{c + dx}\right) + \frac{2bc^2\left(\frac{1}{3}\sqrt{c + dx}\,(c+dx) - c\sqrt{c + dx}\right)}{d} + 2ac^2\sqrt{c + dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x)
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a*c^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^2/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a + 14*(3*(d*x + c)^(5/2)
- 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c/d + 3*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b/
d)/d
```

Mupad [B]
time = 0.21, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}\,\left(7\,a\,d-7\,b\,c+5\,b\,\left(c+d\,x\right)\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(3/2),x)
[Out] (2*(c + d*x)^(5/2)*(7*a*d - 7*b*c + 5*b*(c + d*x)))/(35*d^2)
```

3.1392 $\int (c+dx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{5/2}}{5d}$$

```
[Out] 2/5*(d*x+c)^(5/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{3/2}\,dx = \frac{2(c+dx)^{5/2}}{5d}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Mathics [A]
time = 1.63, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{5}{2}}}{5d}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^(3/2),x]')
```

```
[Out] 2 (c + d x) ^ (5 / 2) / (5 d)
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{5}{2}}}{5d}$	13
derivativedivides	$\frac{2(dx+c)^{\frac{5}{2}}}{5d}$	13
default	$\frac{2(dx+c)^{\frac{5}{2}}}{5d}$	13
trager	$\frac{2(d^2x^2+2cdx+c^2)\sqrt{dx+c}}{5d}$	29
risch	$\frac{2(d^2x^2+2cdx+c^2)\sqrt{dx+c}}{5d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(d*x+c)^(5/2)/d
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(d*x + c)^(5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.30, size = 28, normalized size = 1.75

$$\frac{2\,(d^2x^2+2\,cdx+c^2)\sqrt{dx+c}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(d*x + c)/d
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2(c+dx)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2),x)

[Out] 2*(c + d*x)**(5/2)/(5*d)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.
time = 0.00, size = 113, normalized size = 7.06

$$\frac{\frac{2d^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+4c\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)+2c^2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2),x)

[Out] 2/15*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 30*sqrt(d*x + c)*c^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c)/d

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(3/2),x)

[Out] (2*(c + d*x)^(5/2))/(5*d)

3.1393 $\int \frac{(c+dx)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=86

$$\frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

[Out] 2/3*(d*x+c)^(3/2)/b-2*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)+2*(-a*d+b*c)*(d*x+c)^(1/2)/b^2

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 65, 214}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)}{b^2}+\frac{2(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(3/2)/(a + b*x),x]

[Out] (2*(b*c - a*d)*Sqrt[c + d*x])/b^2 + (2*(c + d*x)^(3/2))/(3*b) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{a+bx}\,dx &= \frac{2(c+dx)^{3/2}}{3b}+\frac{(bc-ad)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b}\\ &= \frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}+\frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^2}\\ &= \frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}+\frac{(2(bc-ad)^2)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b^2d}\\ &= \frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.90

$$\frac{2\sqrt{c+dx}\,(4bc-3ad+bdx)}{3b^2}+\frac{2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(4*b*c - 3*a*d + b*d*x))/(3*b^2) + (2*(-(b*c) + a*d)^(3/2)
*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

Mathics [A]
time = 8.94, size = 86, normalized size = 1.00

$$\frac{-2ad\sqrt{c+dx}}{b^2}+\frac{2\,(c+dx)^{\frac{3}{2}}}{3b}+\frac{2b\text{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{\frac{ad}{b}-c}}\right]\left(\frac{ad}{b}-c\right)^{\frac{7}{2}}}{(ad-bc)^2}+\frac{2c\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^1,x]')
```

```
[Out] -2 a d Sqrt[c + d x] / b ^ 2 + 2 (c + d x) ^ (3 / 2) / (3 b) + 2 b ArcTan[S
qrt[c + d x] / Sqrt[a d / b - c]] (a d / b - c) ^ (7 / 2) / (a d - b c) ^ 2
 + 2 c Sqrt[c + d x] / b
```

Maple [A]
time = 0.17, size = 99, normalized size = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{b(dx+c)^{\frac{3}{2}}}{3}+ad\sqrt{dx+c}-bc\sqrt{dx+c}\right)}{b^{2}}+\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{b^{2}\sqrt{(ad-bc)\,b}}$
default	$-\frac{2\left(-\frac{b(dx+c)^{\frac{3}{2}}}{3}+ad\sqrt{dx+c}-bc\sqrt{dx+c}\right)}{b^{2}}+\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{b^{2}\sqrt{(ad-bc)\,b}}$
risch	$-\frac{2(-bdx+3ad-4bc)\sqrt{dx+c}}{3b^{2}}+\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)a^{2}d^{2}}{b^{2}\sqrt{(ad-bc)\,b}}-\frac{4\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)acd}{b\sqrt{(ad-bc)\,b}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2*(-1/3*b*(d*x+c)^(3/2)+a*d*(d*x+c)^(1/2)-b*c*(d*x+c)^(1/2))+2*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b
*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 188, normalized size = 2.19

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\,(bdx+4\,bc-3\,ad)\sqrt{dx+c}}{3\,b^{2}},\ -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(bdx+4\,bc-3\,ad)\sqrt{dx+c}\right)}{3\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(
d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(b*d*x + 4*b*c - 3*a*d)*sqrt
(d*x + c))/b^2, -2/3*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x + 4*b*c - 3*a*d)*sqrt(d*x
+ c))/b^2]
```

Sympy [A]
time = 6.89, size = 82, normalized size = 0.95

$$\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx}\,\left(-2ad+2bc\right)}{b^{2}}+\frac{2\left(ad-bc\right)^{2}\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^{3}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*b) + sqrt(c + d*x)*(-2*a*d + 2*b*c)/b**2 + 2*(a*d - b
*c)**2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 0.00, size = 133, normalized size = 1.55

$$\frac{\frac{2}{3}\sqrt{c+dx}\,\left(c+dx\right)b^{2}-2\sqrt{c+dx}\,dba+2\sqrt{c+dx}\,cb^{2}}{b^{3}}+\frac{\left(4d^{2}a^{2}-8dcba+4c^{2}b^{2}\right)\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^{2}c+abd}}\right)}{2b^{2}\sqrt{-b^{2}c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x)
```

```
[Out] 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/3*((d*x + c)^(3/2)*b^2 + 3*sqrt(d*x + c)
*b^2*c - 3*sqrt(d*x + c)*a*b*d)/b^3
```

Mupad [B]
time = 0.07, size = 93, normalized size = 1.08

$$\frac{2\left(c+d\,x\right)^{3/2}}{3\,b}-\frac{2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{b^{2}}+\frac{2\operatorname{atan}\left(\frac{\sqrt{b}\,\left(a\,d-b\,c\right)^{3/2}\,\sqrt{c+d\,x}}{a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}}\right)\,\left(a\,d-b\,c\right)^{3/2}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b) - (2*(a*d - b*c)*(c + d*x)^(1/2))/b^2 + (2*atan((
b^(1/2)*(a*d - b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
*(a*d - b*c)^(3/2))/b^(5/2)
```

3.1394 $\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$\frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] -(d*x+c)^(3/2)/b/(b*x+a)-3*d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)
)*(-a*d+b*c)^(1/2)/b^(5/2)+3*d*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {43, 52, 65, 214}

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/b^2 - (c + d*x)^(3/2)/(b*(a + b*x)) - (3*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3(bc-ad))\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b^2}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.23, size = 83, normalized size = 0.98

$$
\frac{\sqrt{c+dx}\,(-bc+3ad+2bdx)}{b^2(a+bx)}-\frac{3d\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(b^2*(a + b*x)) - (3*d*Sqrt[-(b*
c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 1013 vs. 2(85) = 170.
time = 77.12, size = 935, normalized size = 11.00

[illegible]

Antiderivative was successfully verified.

[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^2,x]')

[Out] (d (a d - b c) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (-a ^ 2 d ^ 2 Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + a ^ 2 d ^ 2 Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + 4 Sqrt[c + d x]) - 8 a d ^ 2 ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] (a ^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[(a d - b c) / b] + 2 a ^ 2 d ^ 2 (a d - b c) Sqrt[c + d x] - 4 a b c d (a d - b c) Sqrt[c + d x] + 2 b ^ 2 c ^ 2 (a d - b c) Sqrt[c + d x] + 8 b c d ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] (a ^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[(a d - b c) / b] + 2 a b c d ^ 2 (a d - b c) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] - Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 d (a d - b c) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] - Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d - b c) ^ 3)]) / (2 b ^ 2 (a d - b c) (a ^ 2 d - a b c + a b d x - b ^ 2 c x))

Maple [A]
time = 0.22, size = 100, normalized size = 1.18

method	result
derivativedivides	$2d\left(\frac{\sqrt{dx+c}}{b^{2}}-\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{3(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2\sqrt{(ad-bc)\,b}}}{b^{2}}\right)$
default	$2d\left(\frac{\sqrt{dx+c}}{b^{2}}-\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{3(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2\sqrt{(ad-bc)\,b}}}{b^{2}}\right)$

risch	$\frac{2d\sqrt{dx+c}}{b^2}+\frac{d^2\sqrt{dx+c}\,a}{b^2(bdx+ad)}-\frac{d\sqrt{dx+c}\,c}{b(bdx+ad)}-\frac{3d^2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)a}{b^2\sqrt{(ad-bc)\,b}}+\frac{3d\arctan\left(\frac{b\sqrt{\ldots}}{\sqrt{(a\ldots}}\right.}{b\sqrt{(ad-\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(1/b^2*(d*x+c)^(1/2)-1/b^2*((-1/2*a*d+1/2*b*c)*(d*x+c)^(1/2)/((d*x+c)*b
+a*d-b*c)+3/2*(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*
c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 210, normalized size = 2.47

$$\left[\frac{3\,(bdx+ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{2\,(b^3x+ab^2)},\ -\frac{3\,(bdx+ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d*x + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(2*b*d*x - b*c + 3*a*d)*sqr
t(d*x + c))/(b^3*x + a*b^2), -(3*(b*d*x + a*d)*sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b*d*x - b*c + 3*a*d
)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(73) = 146$.
time = 67.90, size = 923, normalized size = 10.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**2,x)
```

```
[Out] 2*a**2*d**3*sqrt(c + d*x)/(2*a**2*b**2*d**2 - 2*a*b**3*c*d + 2*a*b**3*d**2*
x - 2*b**4*c*d*x) - a**2*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sq
rt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c*
*2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + a**2*d**3*sqrt(-
1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d
*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt
(c + d*x))/(2*b**2) - 4*a*c*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*
d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) + a*c*d**2*sqrt(-1/(b*(a*d - b*c)**3))*
log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*
c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/b - a*c*d*
*2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) -
2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3
)) + sqrt(c + d*x))/b - 4*a*d**2*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**3*
sqrt(a*d/b - c)) - c**2*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c**2*d*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 +
 2*c**2*d*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*
d*x) + 4*c*d*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + 2
*d*sqrt(c + d*x)/b**2
```

Giac [A]
time = 0.01, size = 131, normalized size = 1.54

$$\frac{2\sqrt{c+dx}\,d}{b^2}+\frac{\sqrt{c+dx}\,d^2a-\sqrt{c+dx}\,dcb}{b^2\left((c+dx)\,b+da-cb\right)}+\frac{\left(-6d^2a+6dcb\right)\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2b^2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*sqrt(d*x + c)*d/b^2 + 3*(b*c*d - a*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - (sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*a*d^2)/(((d*x + c)*b - b*c + a*d)*b^2)
```

Mupad [B]
time = 0.11, size = 109, normalized size = 1.28

$$\frac{\left(a\,d^2-b\,c\,d\right)\,\sqrt{c+d\,x}}{b^3\,\left(c+d\,x\right)-b^3\,c+a\,b^2\,d}+\frac{2\,d\,\sqrt{c+d\,x}}{b^2}-\frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^2,x)
```

```
[Out] ((a*d^2 - b*c*d)*(c + d*x)^(1/2))/(b^3*(c + d*x) - b^3*c + a*b^2*d) + (2*d*
(c + d*x)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(a*d - b*c)^(1/2)*(c + d*x)^(1/
2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/b^(5/2)
```

3.1395 $\int \frac{(c+dx)^{3/2}}{(a+bx)^3} dx$

Optimal. Leaf size=100

$$-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

```
[Out] -1/2*(d*x+c)^(3/2)/b/(b*x+a)^2-3/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-3/4*d*(d*x+c)^(1/2)/b^2/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {43, 65, 214}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(4*b^2*(a + b*x)) - (c + d*x)^(3/2)/(2*b*(a + b*x)^2)
- (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx = -\frac{(c+dx)^{3/2}}{2b(a+bx)^2} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{4b}$$

$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)} - \frac{(c+dx)^{3/2}}{2b(a+bx)^2} + \frac{(3d^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)} - \frac{(c+dx)^{3/2}}{2b(a+bx)^2} + \frac{(3d)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)} - \frac{(c+dx)^{3/2}}{2b(a+bx)^2} - \frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

Mathematica [**A**]
time = 0.34, size = 90, normalized size = 0.90

$$-\frac{\sqrt{c+dx}\,(2bc+3ad+5bdx)}{4b^2(a+bx)^2} + \frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{4b^{5/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*(2*b*c + 3*a*d + 5*b*d*x))/(b^2*(a + b*x)^2) + (3*d^2*A
rcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) +
 a*d])
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 2490 vs. $2(100) = 200$.
time = 246.50, size = 2342, normalized size = 23.42

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^3,x]')
```

```
[Out] (16 d ^ 2 ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] (a ^ 2 d - a b c +
a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^
 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3
c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) - 16 a b d ^ 2 (a ^ 4 d
^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2
 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2
+ b ^ 4 c ^ 2 x ^ 2) Sqrt[(a d - b c) / b] Sqrt[c + d x] + 6 a ^ 2 b ^ 2 d
```

^ 2 (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (-4 c + d x) Sqrt[(a d - b c) / b] Sqrt[c + d x] + a b d ^ 3 (-3 a d Log[-a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] + b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a d Log[a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] - b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 5)] - 8 Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + 8 Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)]) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) Sqrt[(a d - b c) / b] + 10 a ^ 3 b d ^ 3 (a ^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[(a d - b c) / b] Sqrt[c + d x] + 2 b ^ 3 c ^ 2 (15 a d + 3 b (c + d x) - 5 b c) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[(a d - b c) / b] Sqrt[c + d x] + 16 b ^ 2 c d (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) Sqrt[(a d - b c) / b] Sqrt[c + d x] - 12 a b ^ 3 c d (a ^ 2 d - a b c + a b d x - b ^ 2 c x) Sqrt[(a d - b c) / b] (c + d x) ^ (3 / 2) + 2 b ^ 2 c d ^ 2 (-3 a d Log[a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] - b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a d Log[-a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] + b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 5)] - 4 Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + 4 Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)]) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) Sqrt[(a d - b c) / b] + 3 b ^ 3 c ^ 2 d ^ 2 (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d ^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 + 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2) (Log[a ^ 3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] - 3 a ^ 2 b c d ^ 2 Sqrt [-1 / (b (a d - b c) ^ 5)] + 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] - b ^ 3 c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]] - Log[-a ^

```
3 d ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + 3 a ^ 2 b c d ^ 2 Sqrt[-1 / (b (a
 d - b c) ^ 5)] - 3 a b ^ 2 c ^ 2 d Sqrt[-1 / (b (a d - b c) ^ 5)] + b ^ 3
c ^ 3 Sqrt[-1 / (b (a d - b c) ^ 5)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d -
b c) ^ 5)] Sqrt[(a d - b c) / b]) / (8 b ^ 3 Sqrt[(a d - b c) / b] (a ^ 2 d
 - a b c + a b d x - b ^ 2 c x) (a ^ 4 d ^ 2 - 2 a ^ 3 b c d + 2 a ^ 3 b d
^ 2 x + a ^ 2 b ^ 2 c ^ 2 - 4 a ^ 2 b ^ 2 c d x + a ^ 2 b ^ 2 d ^ 2 x ^ 2 +
 2 a b ^ 3 c ^ 2 x - 2 a b ^ 3 c d x ^ 2 + b ^ 4 c ^ 2 x ^ 2))
```

Maple [A]
time = 0.16, size = 97, normalized size = 0.97

method	result	size
derivativedivides	$2d^{2}\left(\frac{-\frac{5(dx+c)^{\frac{3}{2}}}{8b}-\frac{3(ad-bc)\sqrt{dx+c}}{8b^{2}}}{\left((dx+c)b+ad-bc\right)^{2}}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8b^{2}\sqrt{(ad-bc)\,b}}\right)$	97
default	$2d^{2}\left(\frac{-\frac{5(dx+c)^{\frac{3}{2}}}{8b}-\frac{3(ad-bc)\sqrt{dx+c}}{8b^{2}}}{\left((dx+c)b+ad-bc\right)^{2}}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8b^{2}\sqrt{(ad-bc)\,b}}\right)$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*((-5/8*(d*x+c)^(3/2)/b-3/8*(a*d-b*c)/b^2*(d*x+c)^(1/2))/((d*x+c)*b+a*
d-b*c)^2+3/8/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(
1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(80) = 160.
time = 0.32, size = 383, normalized size = 3.83

$$\left[\frac{3\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^{2}c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^{3}c^{2}+ab^{2}cd-3\,a^{2}bd^{2}+5\left(b^{3}cd-ab^{2}d^{2}\right)x\right)\sqrt{dx+c}}{8\left(a^{2}b^{4}c-a^{3}b^{3}d+\left(b^{6}c-ab^{5}d\right)x^{2}+2\left(ab^{5}c-a^{2}b^{4}d\right)x\right)},\ \frac{3\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{-b^{2}c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(2\,b^{3}c^{2}+ab^{2}cd-3\,a^{2}bd^{2}+5\left(b^{3}cd-ab^{2}d^{2}\right)x\right)\sqrt{dx+c}}{4\left(a^{2}b^{4}c-a^{3}b^{3}d+\left(b^{6}c-ab^{5}d\right)x^{2}+2\left(ab^{5}c-a^{2}b^{4}d\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))
/(a^2*b^4*c - a^3*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x
), 1/4*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (2*b^3*c^2 + a*b^2*c*d
 - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c - a^3
*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.01, size = 137, normalized size = 1.37

$$\frac{-5\sqrt{c+dx}\,(c+dx)\,d^2b-3\sqrt{c+dx}\,d^3a+3\sqrt{c+dx}\,d^2cb}{4b^2\left((c+dx)\,b+da-cb\right)^2}+\frac{3d^2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2b^2\cdot 2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x)
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^2) - 1/4*(5*(d*x + c)^(3/2)*b*d^2 - 3*sqrt(d*x + c)*b*c*d^2 + 3*sqrt(d*x
+ c)*a*d^3)/(((d*x + c)*b - b*c + a*d)^2*b^2)
```

Mupad [**B**]
time = 0.28, size = 135, normalized size = 1.35

$$\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{5\,d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,b}+\frac{3\,d^2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{4\,b^2}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(5/2)*(a*d -
 b*c)^(1/2)) - ((5*d^2*(c + d*x)^(3/2))/(4*b) + (3*d^2*(a*d - b*c)*(c + d*x
)^(1/2))/(4*b^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)
```

3.1396 $\int \frac{(c+dx)^{3/2}}{(a+bx)^4} \, dx$

Optimal. Leaf size=136

$$-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/3*(d*x+c)^(3/2)/b/(b*x+a)^3+1/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(3/2)-1/4*d*(d*x+c)^(1/2)/b^2/(b*x+a)^2-1/8*
d^2*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {43, 44, 65, 214}

$$\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)}-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -1/4*(d*Sqrt[c + d*x])/(b^2*(a + b*x)^2) - (d^2*Sqrt[c + d*x])/(8*b^2*(b*c
- a*d)*(a + b*x)) - (c + d*x)^(3/2)/(3*b*(a + b*x)^3) + (d^3*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{2b}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b^2}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^2(bc-ad)}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c}\right.}{8b^2(bc-ad)}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.54, size = 129, normalized size = 0.95

$$\frac{\sqrt{c+dx}\,\left(-3a^2d^2-2abd(c+4dx)+b^2\left(8c^2+14cdx+3d^2x^2\right)\right)}{24b^2(-bc+ad)(a+bx)^3}+\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8b^{5/2}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^4,x]
[Out] (Sqrt[c + d*x]*(-3*a^2*d^2 - 2*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 14*c*d*x +
3*d^2*x^2)))/(24*b^2*(-(b*c) + a*d)*(a + b*x)^3) + (d^3*ArcTan[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*b^(5/2)*(-(b*c) + a*d)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^4,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 126, normalized size = 0.93

method	result	size
derivativedivides	$2d^3\left(\frac{\frac{(dx+c)^{\frac{5}{2}}}{16ad-16bc}-\frac{(dx+c)^{\frac{3}{2}}}{6b}-\frac{(ad-bc)\sqrt{dx+c}}{16b^2}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16(ad-bc)b^2\sqrt{(ad-bc)\,b}}\right)$	126
default	$2d^3\left(\frac{\frac{(dx+c)^{\frac{5}{2}}}{16ad-16bc}-\frac{(dx+c)^{\frac{3}{2}}}{6b}-\frac{(ad-bc)\sqrt{dx+c}}{16b^2}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16(ad-bc)b^2\sqrt{(ad-bc)\,b}}\right)$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*((1/16/(a*d-b*c)*(d*x+c)^(5/2)-1/6*(d*x+c)^(3/2)/b-1/16*(a*d-b*c)/b^2
*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^3+1/16/(a*d-b*c)/b^2/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(112) = 224.
time = 0.32, size = 666, normalized size = 4.90

$$\left[-\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(8\,b^4c^3-10\,ab^3c^2d-a^2b^2cd^2+3\,a^3bd^3+3\,(b^4cd^2-ab^3d^3)x^2+2\,(7\,b^4c^2d-11\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c^2-2\,a^4b^4cd+a^5b^3d^2+(b^8c^2-2\,ab^7cd+a^2b^6d^2)x^3+3\,(ab^7c^2-2\,a^2b^6cd+a^3b^5d^2)x^2+3\,(a^2b^6c^2-2\,a^3b^5cd+a^4b^4d^2)x)},\ -\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(8\,b^4c^3-10\,ab^3c^2d-a^2b^2cd^2+3\,a^3bd^3+3\,(b^4cd^2-ab^3d^3)x^2+2\,(7\,b^4c^2d-11\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c^2-2\,a^4b^4cd+a^5b^3d^2+(b^8c^2-2\,ab^7cd+a^2b^6d^2)x^3+3\,(ab^7c^2-2\,a^2b^6cd+a^3b^5d^2)x^2+3\,(a^2b^6c^2-2\,a^3b^5cd+a^4b^4d^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*(8*b^4*c^3 - 10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3
+ 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b
^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*
b^5*d^2)*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*x), -1/24*(3*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*
d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (8*b^4*c^3 -
10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3 + 3*(b^4*c*d^2 - a*b^3*d^3)*x^
2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3
*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d
^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*x^2 + 3*(a^2*b^6*c^2
- 2*a^3*b^5*c*d + a^4*b^4*d^2)*x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.01, size = 228, normalized size = 1.68

$$\frac{3\sqrt{c+dx}\,(c+dx)^2d^3b^2-8\sqrt{c+dx}\,(c+dx)\,d^4ba+8\sqrt{c+dx}\,(c+dx)\,d^3cb^2-3\sqrt{c+dx}\,d^5a^2+6\sqrt{c+dx}\,d^4cba-3\sqrt{c+dx}\,d^3c^2b^2}{(24db^2a-24cb^3)\left((c+dx)\,b+da-cb\right)^3}+\frac{d^3\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(4db^2a-4cb^3)\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x)
```

```
[Out] -1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c - a*b^2*d)*sq
rt(-b^2*c + a*b*d)) - 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 + 8*(d*x + c)^(3/2)*b
^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 - 8*(d*x + c)^(3/2)*a*b*d^4 + 6*sqrt
(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c - a*b^2*d)*((d*x + c
)*b - b*c + a*d)^3)
```

Mupad [**B**]
time = 0.34, size = 209, normalized size = 1.54

$$\frac{d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{d^3\,{\left(c+d\,x\right)}^{3/2}}{3\,b}-\frac{d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,\left(a\,d-b\,c\right)}+\frac{d^3\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{8\,b^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^4,x)
```

```
[Out] (d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(5/2)*(a*d - b
*c)^(3/2)) - ((d^3*(c + d*x)^(3/2))/(3*b) - (d^3*(c + d*x)^(5/2))/(8*(a*d -
 b*c)) + (d^3*(a*d - b*c)*(c + d*x)^(1/2))/(8*b^2))/((c + d*x)*(3*b^3*c^2 +
 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c +
d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
```

3.1397 $\int \frac{(c+dx)^{3/2}}{(a+bx)^5} dx$

Optimal. Leaf size=172

$$-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2}+\frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)/b/(b*x+a)^4-3/64*d^4*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d
+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(5/2)-1/8*d*(d*x+c)^(1/2)/b^2/(b*x+a)^3-1/3
2*d^2*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)^2+3/64*d^3*(d*x+c)^(1/2)/b^2/(-a
*d+b*c)^2/(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}+\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -1/8*(d*Sqrt[c + d*x])/(b^2*(a + b*x)^3) - (d^2*Sqrt[c + d*x])/(32*b^2*(b*c
 - a*d)*(a + b*x)^2) + (3*d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x
)) - (c + d*x)^(3/2)/(4*b*(a + b*x)^4) - (3*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[b*c - a*d]])/(64*b^(5/2)*(b*c - a*d)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{d^2\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{16b^2}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{(3d^3)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^2(bc-ad)}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{(3d}{}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{(3d}{}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{3d^4}{}$$

Mathematica [A]
time = 0.86, size = 171, normalized size = 0.99

$$-\frac{\sqrt{c+dx}\left(3a^3d^3+a^2bd^2(2c+11dx)-ab^2d\left(24c^2+44cdx+11d^2x^2\right)+b^3\left(16c^3+24c^2dx+2cd^2x^2-3d^3x^3\right)\right)}{64b^2(bc-ad)^2(a+bx)^4}+\frac{3d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{64b^{5/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -1/64*(Sqrt[c + d*x]*(3*a^3*d^3 + a^2*b*d^2*(2*c + 11*d*x) - a*b^2*d*(24*c^
2 + 44*c*d*x + 11*d^2*x^2) + b^3*(16*c^3 + 24*c^2*d*x + 2*c*d^2*x^2 - 3*d^3
```

```
*x^3)))/(b^2*(b*c - a*d)^2*(a + b*x)^4) + (3*d^4*ArcTan[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[-(b*c) + a*d]])/(64*b^(5/2)*(-(b*c) + a*d)^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^5,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 172, normalized size = 1.00

method	result
derivativedivides	$2d^4\left(\frac{\frac{3b(dx+c)^{\frac{7}{2}}}{128\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{11(dx+c)^{\frac{5}{2}}}{128(ad-bc)}-\frac{11(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{128b^2}}{((dx+c)b+ad-bc)^4}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\ldots}}\right.}{128(a^2d^2-2abcd+b^2c^2)b^2\sqrt{(ad\ldots}}\right.$
default	$2d^4\left(\frac{\frac{3b(dx+c)^{\frac{7}{2}}}{128\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{11(dx+c)^{\frac{5}{2}}}{128(ad-bc)}-\frac{11(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{128b^2}}{((dx+c)b+ad-bc)^4}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\ldots}}\right.}{128(a^2d^2-2abcd+b^2c^2)b^2\sqrt{(ad\ldots}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^4*((3/128/(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(d*x+c)^(7/2)+11/128/(a*d-b*c)*
(d*x+c)^(5/2)-11/128*(d*x+c)^(3/2)/b-3/128*(a*d-b*c)/b^2*(d*x+c)^(1/2))/((d
*x+c)*b+a*d-b*c)^4+3/128/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(144) = 288$.
time = 0.32, size = 1043, normalized size = 6.06

$$\left[\frac{3\left(b^{4}d^{4}x^{4}+4ab^{3}d^{4}x^{3}+6a^{2}b^{2}d^{4}x^{2}+4a^{3}bd^{4}x+a^{4}d^{4}\right)\sqrt{b^{2}c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^{2}c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(16b^{5}c^{4}-40ab^{4}c^{3}d+26a^{2}b^{3}c^{2}d^{2}+a^{3}b^{2}cd^{3}-3a^{4}bd^{4}-3\left(b^{5}cd^{3}-ab^{4}d^{4}\right)x^{3}+\left(2b^{5}c^{2}d^{2}-13ab^{4}cd^{3}+11a^{2}b^{3}d^{4}\right)x^{2}+\left(24b^{5}c^{3}d-68ab^{4}c^{2}d^{2}+55a^{2}b^{3}cd^{3}-11a^{3}b^{2}d^{4}\right)x\right)\sqrt{dx+c}}{128\left(a^{4}b^{6}c^{3}-3a^{5}b^{5}c^{2}d+3a^{6}b^{4}cd^{2}-a^{7}b^{3}d^{3}+\left(b^{10}c^{3}-3ab^{9}c^{2}d+3a^{2}b^{8}cd^{2}-a^{3}b^{7}d^{3}\right)x^{4}+4\left(ab^{9}c^{3}-3a^{2}b^{8}c^{2}d+3a^{3}b^{7}cd^{2}-a^{4}b^{6}d^{3}\right)x^{3}+6\left(a^{2}b^{8}c^{3}-3a^{3}b^{7}c^{2}d+3a^{4}b^{6}cd^{2}-a^{5}b^{5}d^{3}\right)x^{2}+4\left(a^{3}b^{7}c^{3}-3a^{4}b^{6}c^{2}d+3a^{5}b^{5}cd^{2}-a^{6}b^{4}d^{3}\right)x\right)},\ \frac{3\left(b^{4}d^{4}x^{4}+4ab^{3}d^{4}x^{3}+6a^{2}b^{2}d^{4}x^{2}+4a^{3}bd^{4}x+a^{4}d^{4}\right)\sqrt{-b^{2}c+abd}\arctan\left(\frac{\sqrt{-b^{2}c+abd}\sqrt{dx+c}}{bdx+bc}\right)-\left(16b^{5}c^{4}-40ab^{4}c^{3}d+26a^{2}b^{3}c^{2}d^{2}+a^{3}b^{2}cd^{3}-3a^{4}bd^{4}-3\left(b^{5}cd^{3}-ab^{4}d^{4}\right)x^{3}+\left(2b^{5}c^{2}d^{2}-13ab^{4}cd^{3}+11a^{2}b^{3}d^{4}\right)x^{2}+\left(24b^{5}c^{3}d-68ab^{4}c^{2}d^{2}+55a^{2}b^{3}cd^{3}-11a^{3}b^{2}d^{4}\right)x\right)\sqrt{dx+c}}{64\left(a^{4}b^{6}c^{3}-3a^{5}b^{5}c^{2}d+3a^{6}b^{4}cd^{2}-a^{7}b^{3}d^{3}+\left(b^{10}c^{3}-3ab^{9}c^{2}d+3a^{2}b^{8}cd^{2}-a^{3}b^{7}d^{3}\right)x^{4}+4\left(ab^{9}c^{3}-3a^{2}b^{8}c^{2}d+3a^{3}b^{7}cd^{2}-a^{4}b^{6}d^{3}\right)x^{3}+6\left(a^{2}b^{8}c^{3}-3a^{3}b^{7}c^{2}d+3a^{4}b^{6}cd^{2}-a^{5}b^{5}d^{3}\right)x^{2}+4\left(a^{3}b^{7}c^{3}-3a^{4}b^{6}c^{2}d+3a^{5}b^{5}cd^{2}-a^{6}b^{4}d^{3}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/128*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c -
a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(16*b^5*c^4 - 40*a*b^4*c^3*d + 26*a^2*
b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 - a*b^4*d^4)*x^3 +
 (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (24*b^5*c^3*d - 68
*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*
b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 + (b^10*c^3 - 3*a
*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*
c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2
*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d
+ 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x), 1/64*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3
 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (16*b^5*c^4 - 40*a*b^4
*c^3*d + 26*a^2*b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 -
a*b^4*d^4)*x^3 + (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (2
4*b^5*c^3*d - 68*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt
(d*x + c))/(a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 +
 (b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*
c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3
 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 -
3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 360, normalized size = 2.09

$$\frac{3\sqrt{c+dx}\,(c+dx)^{3}d^{4}b^{3}+11\sqrt{c+dx}\,(c+dx)^{2}d^{5}b^{2}a-11\sqrt{c+dx}\,(c+dx)^{2}d^{4}cb^{3}-11\sqrt{c+dx}\,(c+dx)d^{6}ba^{2}+22\sqrt{c+dx}\,(c+dx)d^{5}cb^{2}a-11\sqrt{c+dx}\,(c+dx)d^{4}c^{2}b^{3}-3\sqrt{c+dx}\,d^{7}a^{3}+9\sqrt{c+dx}\,d^{6}cba^{2}-9\sqrt{c+dx}\,d^{5}c^{2}b^{2}a+3\sqrt{c+dx}\,d^{4}c^{3}b^{3}}{(64d^{2}b^{2}a^{2}-128dcb^{3}a+64c^{2}b^{4})((c+dx)b+da-cb)^{4}}+\frac{3d^{4}\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^{2}c+abd}}\right)}{2(32d^{2}b^{2}a^{2}-64dcb^{3}a+32c^{2}b^{4})\sqrt{-b^{2}c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x)
```

```
[Out] 3/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/64*(3*(d*x + c)^(7/2)*b^3*d^4 -
 11*(d*x + c)^(5/2)*b^3*c*d^4 - 11*(d*x + c)^(3/2)*b^3*c^2*d^4 + 3*sqrt(d*x
 + c)*b^3*c^3*d^4 + 11*(d*x + c)^(5/2)*a*b^2*d^5 + 22*(d*x + c)^(3/2)*a*b^2
*c*d^5 - 9*sqrt(d*x + c)*a*b^2*c^2*d^5 - 11*(d*x + c)^(3/2)*a^2*b*d^6 + 9*s
qrt(d*x + c)*a^2*b*c*d^6 - 3*sqrt(d*x + c)*a^3*d^7)/((b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*((d*x + c)*b - b*c + a*d)^4)
```

Mupad [B]
time = 0.37, size = 296, normalized size = 1.72

$$\frac{3\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{11\,d^4\,{\left(c+d\,x\right)}^{3/2}}{64\,b}-\frac{11\,d^4\,{\left(c+d\,x\right)}^{5/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^4\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{64\,b^2}-\frac{3\,b\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^5,x)
```

```
[Out] (3*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(5/2)*(a*d
- b*c)^(5/2)) - ((11*d^4*(c + d*x)^(3/2))/(64*b) - (11*d^4*(c + d*x)^(5/2))
/(64*(a*d - b*c)) + (3*d^4*(a*d - b*c)*(c + d*x)^(1/2))/(64*b^2) - (3*b*d^4
*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3
*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 1
2*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2
 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
```

3.1398 $\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx$

Optimal. Leaf size=208

$$-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+bx)}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+$$

```
[Out] -1/5*(d*x+c)^(3/2)/b/(b*x+a)^5+3/128*d^5*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*
d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(7/2)-3/40*d*(d*x+c)^(1/2)/b^2/(b*x+a)^4-1
/80*d^2*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)^3+1/64*d^3*(d*x+c)^(1/2)/b^2/(
-a*d+b*c)^2/(b*x+a)^2-3/128*d^4*(d*x+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x+a)
```

Rubi [A]
time = 0.07, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}}-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)}-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(40*b^2*(a + b*x)^4) - (d^2*Sqrt[c + d*x])/(80*b^2*(b*
c - a*d)*(a + b*x)^3) + (d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x)
^2) - (3*d^4*Sqrt[c + d*x])/(128*b^2*(b*c - a*d)^3*(a + b*x)) - (c + d*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(128*b^(5/2)*(b*c - a*d)^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+\frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+\frac{(3d^2)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}-\frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+}}{128b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+}}{128b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+}}{128b^2(bc-ad)}$$

Mathematica [A]
time = 1.57, size = 223, normalized size = 1.07

$$\frac{\frac{\sqrt{b}\sqrt{c+dx}\left(-15a^4d^4-10a^3bd^3(c+7dx)+2a^2b^2d^2\left(124c^2+233cdx+64d^2x^2\right)-2ab^3d\left(168c^3+256c^2dx+23cd^2x^2-35d^3x^3\right)+b^4\left(128c^4+176c^3dx+8c^2d^2x^2-10cd^3x^3+15d^4x^4\right)\right)}{(-bc+ad)^3(a+bx)^5}+\frac{15d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x]*(-15*a^4*d^4 - 10*a^3*b*d^3*(c + 7*d*x) + 2*a^2*b^2
*d^2*(124*c^2 + 233*c*d*x + 64*d^2*x^2) - 2*a*b^3*d*(168*c^3 + 256*c^2*d*x
+ 23*c*d^2*x^2 - 35*d^3*x^3) + b^4*(128*c^4 + 176*c^3*d*x + 8*c^2*d^2*x^2 -
 10*c*d^3*x^3 + 15*d^4*x^4)))/((-(b*c) + a*d)^3*(a + b*x)^5) + (15*d^5*ArcT
an[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2))/(640*
b^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^6,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 237, normalized size = 1.14

method	result
derivativedivides	$2d^5\left(\frac{\frac{3b^2(dx+c)^{\frac{9}{2}}}{256\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7b(dx+c)^{\frac{7}{2}}}{128\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{(dx+c)^{\frac{5}{2}}}{10ad-10bc}-\frac{7(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{256b^2}}{((dx+c)b+ad-bc)^5}+\right.$
default	$2d^5\left(\frac{\frac{3b^2(dx+c)^{\frac{9}{2}}}{256\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7b(dx+c)^{\frac{7}{2}}}{128\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{(dx+c)^{\frac{5}{2}}}{10ad-10bc}-\frac{7(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{256b^2}}{((dx+c)b+ad-bc)^5}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^5*((3/256/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^2*(d*x+c)^(9/
2)+7/128/(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(d*x+c)^(7/2)+1/10/(a*d-b*c)*(d*x+c)
^(5/2)-7/128*(d*x+c)^(3/2)/b-3/256*(a*d-b*c)/b^2*(d*x+c)^(1/2))/((d*x+c)*b+
a*d-b*c)^5+3/256/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^2/((a*d-b*
c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(176) = 352$.
time = 0.32, size = 1492, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(128*b^6*c^5
- 464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c
*d^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87
*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2
+ 489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c
))/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a
^9*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*
d^3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d
^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*
d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^
4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^
2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3
+ a^8*b^4*d^4)*x), -1/640*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d
^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)
*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (128*b^6*c^5 -
464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c*d
^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*a*
b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87*a
^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2 +
489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c))
/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9
*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^
3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2
 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d
+ 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^4
- 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^2
```

```
+ 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 +
a^8*b^4*d^4)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(176) = 352$.
time = 0.02, size = 523, normalized size = 2.51

$$\frac{15\sqrt{c+dx}\,(c+dx)^4 d^5b^4+70\sqrt{c+dx}\,(c+dx)^3 d^6b^3a-70\sqrt{c+dx}\,(c+dx)^3 d^5cb^4+128\sqrt{c+dx}\,(c+dx)^2 d^7b^2a^2-256\sqrt{c+dx}\,(c+dx)^2 d^6cb^3a+128\sqrt{c+dx}\,(c+dx)^2 d^5c^2b^4-70\sqrt{c+dx}\,(c+dx)\,d^8ba^3+210\sqrt{c+dx}\,(c+dx)\,d^7cb^2a^2-210\sqrt{c+dx}\,(c+dx)\,d^6c^2b^3a+70\sqrt{c+dx}\,(c+dx)\,d^5c^3b^4-15\sqrt{c+dx}\,d^9a^4+60\sqrt{c+dx}\,d^8cba^3-90\sqrt{c+dx}\,d^7c^2b^2a^2+60\sqrt{c+dx}\,d^6c^3b^3a-15\sqrt{c+dx}\,d^5c^4b^4}{(640d^3b^2a^3-1920d^2cb^3a^2+1920dc^2b^4a-640c^3b^5)\,((c+dx)\,b+da-cb)^5}+\frac{3d^5\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(64d^3b^2a^3-192d^2cb^3a^2+192dc^2b^4a-64c^3b^5)\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x)
```

```
[Out] -3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^3 - 3*a*b^4
*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*c + a*b*d)) - 1/640*(15*(
d*x + c)^(9/2)*b^4*d^5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 + 128*(d*x + c)^(5/2)
*b^4*c^2*d^5 + 70*(d*x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^
5 + 70*(d*x + c)^(7/2)*a*b^3*d^6 - 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d
*x + c)^(3/2)*a*b^3*c^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 + 128*(d*x + c
)^(5/2)*a^2*b^2*d^7 + 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*
a^2*b^2*c^2*d^7 - 70*(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d
^8 - 15*sqrt(d*x + c)*a^4*d^9)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
- a^3*b^2*d^3)*((d*x + c)*b - b*c + a*d)^5)
```

Mupad [B]
time = 0.47, size = 398, normalized size = 1.91

$$\frac{\frac{d^5\,(c+d\,x)^{5/2}}{5\,(a\,d-b\,c)}-\frac{7\,d^5\,(c+d\,x)^{3/2}}{64\,b}+\frac{3\,b^2\,d^5\,(c+d\,x)^{9/2}}{128\,(a\,d-b\,c)^3}-\frac{3\,d^5\,(a\,d-b\,c)\,\sqrt{c+d\,x}}{128\,b^2}+\frac{7\,b\,d^5\,(c+d\,x)^{7/2}}{64\,(a\,d-b\,c)^2}}{b^5\,(c+d\,x)^5-(c+d\,x)^2\,(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3)-(5\,b^5\,c-5\,a\,b^4\,d)\,(c+d\,x)^4+a^5\,d^5-b^5\,c^5+(c+d\,x)^3\,(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2)+(c+d\,x)\,(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{5/2}\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^6,x)
```

```
[Out] ((d^5*(c + d*x)^(5/2))/(5*(a*d - b*c)) - (7*d^5*(c + d*x)^(3/2))/(64*b) + (
3*b^2*d^5*(c + d*x)^(9/2))/(128*(a*d - b*c)^3) - (3*d^5*(a*d - b*c)*(c + d*
x)^(1/2))/(128*b^2) + (7*b*d^5*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^5*(c
 + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 3
0*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c
```

```
 + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c
^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d)
- 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
+ (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(5/2)*(a
*d - b*c)^(7/2))
```

3.1399 $\int (a+bx)^5(c+dx)^{5/2}\,dx$

Optimal. Leaf size=158

$$-\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11/2}}{11d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{13/2}}{13d^6}$$

```
[Out] -2/7*(-a*d+b*c)^5*(d*x+c)^(7/2)/d^6+10/9*b*(-a*d+b*c)^4*(d*x+c)^(9/2)/d^6-2
0/11*b^2*(-a*d+b*c)^3*(d*x+c)^(11/2)/d^6+20/13*b^3*(-a*d+b*c)^2*(d*x+c)^(13
/2)/d^6-2/3*b^4*(-a*d+b*c)*(d*x+c)^(15/2)/d^6+2/17*b^5*(d*x+c)^(17/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c+dx)^{7/2}(bc-ad)^5}{7d^6}+\frac{2b^5(c+dx)^{17/2}}{17d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(7/2))/(7*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(9/2))/(9*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3
*(b*c - a*d)^2*(c + d*x)^(13/2))/(13*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(1
5/2))/(3*d^6) + (2*b^5*(c + d*x)^(17/2))/(17*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^5(c+dx)^{5/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{7/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx}{d^5}\right.\\ &= -\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11}}{11d^6}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 217, normalized size = 1.37

$$\frac{2(c+dx)^{7/2}\left(21879a^5d^5+12155a^4bd^4(-2c+7dx)+2210a^3b^2d^3\left(8c^2-28cdx+63d^2x^2\right)+510a^2b^3d^2\left(-16c^3+56c^2dx-126cd^2x^2+231d^3x^3\right)+17ab^4d\left(128c^4-448c^3dx+1008c^2d^2x^2-1848cd^3x^3+3003d^4x^4\right)+b^5\left(-256c^5+896c^4dx-2016c^3d^2x^2+3696c^2d^3x^3-6006cd^4x^4+9009d^5x^5\right)\right)}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(21879*a^5*d^5 + 12155*a^4*b*d^4*(-2*c + 7*d*x) + 2210*a
^3*b^2*d^3*(8*c^2 - 28*c*d*x + 63*d^2*x^2) + 510*a^2*b^3*d^2*(-16*c^3 + 56*
c^2*d*x - 126*c*d^2*x^2 + 231*d^3*x^3) + 17*a*b^4*d*(128*c^4 - 448*c^3*d*x
+ 1008*c^2*d^2*x^2 - 1848*c*d^3*x^3 + 3003*d^4*x^4) + b^5*(-256*c^5 + 896*c
^4*d*x - 2016*c^3*d^2*x^2 + 3696*c^2*d^3*x^3 - 6006*c*d^4*x^4 + 9009*d^5*x^
5)))/(153153*d^6)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 548 vs. $2(158) = 316$.
time = 38.50, size = 544, normalized size = 3.44

$$\frac{2\left(21879a^5c^3d^5+65637a^5c^2d^6x+65637a^5cd^7x^2+21879a^5d^8x^3-24310a^4bc^4d^4+12155a^4bc^3d^5x+182325a^4bc^2d^6x^2+230945a^4bcd^7x^3+85085a^4bd^8x^4+17680a^3b^2c^5d^3-8840a^3b^2c^4d^4x+6630a^3b^2c^3d^5x^2+249730a^3b^2c^2d^6x^3+355810a^3b^2cd^7x^4+139230a^3b^2d^8x^5-8160a^2b^3c^6d^2+4080a^2b^3c^5d^3x-3060a^2b^3c^4d^4x^2+2550a^2b^3c^3d^5x^3+189210a^2b^3c^2d^6x^4+289170a^2b^3cd^7x^5+117810a^2b^3d^8x^6+2176ab^4c^7d-1088ab^4c^6d^2x+816ab^4c^5d^3x^2-680ab^4c^4d^4x^3+595ab^4c^3d^5x^4+76041ab^4c^2d^6x^5+121737ab^4cd^7x^6+51051ab^4d^8x^7-256b^5c^8+128b^5c^7dx-96b^5c^6d^2x^2+80b^5c^5d^3x^3-70b^5c^4d^4x^4+63b^5c^3d^5x^5+12705b^5c^2d^6x^6+21021b^5cd^7x^7+9009b^5d^8x^8\right)\sqrt{c+dx}}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5*(c + d*x)^(5/2),x]')
```

```
[Out] 2 (21879 a ^ 5 c ^ 3 d ^ 5 + 65637 a ^ 5 c ^ 2 d ^ 6 x + 65637 a ^ 5 c d ^
7 x ^ 2 + 21879 a ^ 5 d ^ 8 x ^ 3 - 24310 a ^ 4 b c ^ 4 d ^ 4 + 12155 a ^ 4
 b c ^ 3 d ^ 5 x + 182325 a ^ 4 b c ^ 2 d ^ 6 x ^ 2 + 230945 a ^ 4 b c d ^
7 x ^ 3 + 85085 a ^ 4 b d ^ 8 x ^ 4 + 17680 a ^ 3 b ^ 2 c ^ 5 d ^ 3 - 8840
a ^ 3 b ^ 2 c ^ 4 d ^ 4 x + 6630 a ^ 3 b ^ 2 c ^ 3 d ^ 5 x ^ 2 + 249730 a ^
 3 b ^ 2 c ^ 2 d ^ 6 x ^ 3 + 355810 a ^ 3 b ^ 2 c d ^ 7 x ^ 4 + 139230 a ^
3 b ^ 2 d ^ 8 x ^ 5 - 8160 a ^ 2 b ^ 3 c ^ 6 d ^ 2 + 4080 a ^ 2 b ^ 3 c ^ 5
 d ^ 3 x - 3060 a ^ 2 b ^ 3 c ^ 4 d ^ 4 x ^ 2 + 2550 a ^ 2 b ^ 3 c ^ 3 d ^
5 x ^ 3 + 189210 a ^ 2 b ^ 3 c ^ 2 d ^ 6 x ^ 4 + 289170 a ^ 2 b ^ 3 c d ^ 7
 x ^ 5 + 117810 a ^ 2 b ^ 3 d ^ 8 x ^ 6 + 2176 a b ^ 4 c ^ 7 d - 1088 a b ^
 4 c ^ 6 d ^ 2 x + 816 a b ^ 4 c ^ 5 d ^ 3 x ^ 2 - 680 a b ^ 4 c ^ 4 d ^ 4
x ^ 3 + 595 a b ^ 4 c ^ 3 d ^ 5 x ^ 4 + 76041 a b ^ 4 c ^ 2 d ^ 6 x ^ 5 + 1
21737 a b ^ 4 c d ^ 7 x ^ 6 + 51051 a b ^ 4 d ^ 8 x ^ 7 - 256 b ^ 5 c ^ 8 +
 128 b ^ 5 c ^ 7 d x - 96 b ^ 5 c ^ 6 d ^ 2 x ^ 2 + 80 b ^ 5 c ^ 5 d ^ 3 x
^ 3 - 70 b ^ 5 c ^ 4 d ^ 4 x ^ 4 + 63 b ^ 5 c ^ 3 d ^ 5 x ^ 5 + 12705 b ^ 5
 c ^ 2 d ^ 6 x ^ 6 + 21021 b ^ 5 c d ^ 7 x ^ 7 + 9009 b ^ 5 d ^ 8 x ^ 8) Sq
rt[c + d x] / (153153 d ^ 6)
```

Maple [**A**]
time = 0.15, size = 122, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{17}{2}}}{17}+\frac{2(ad-bc)b^4(dx+c)^{\frac{15}{2}}}{3}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)^4b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^5(d}{7}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{17}{2}}}{17}+\frac{2(ad-bc)b^4(dx+c)^{\frac{15}{2}}}{3}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)^4b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^5(d}{7}}{d^6}$

gosper	$\frac{2(dx+c)^{\frac{7}{2}}(9009b^5x^5d^5+51051a\,b^4d^5x^4-6006b^5c\,d^4x^4+117810a^2b^3d^5x^3-31416a\,b^4c\,d^4x^3+3696b^5c^2d^3x^3+139230a^3b^2d}{}$
trager	$\frac{2(9009b^5d^8x^8+51051a\,b^4d^8x^7+21021b^5c\,d^7x^7+117810a^2b^3d^8x^6+121737a\,b^4c\,d^7x^6+12705b^5c^2d^6x^6+139230a^3b^2d^8x^5+}{}$
risch	$\frac{2(9009b^5d^8x^8+51051a\,b^4d^8x^7+21021b^5c\,d^7x^7+117810a^2b^3d^8x^6+121737a\,b^4c\,d^7x^6+12705b^5c^2d^6x^6+139230a^3b^2d^8x^5+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/17*b^5*(d*x+c)^(17/2)+1/3*(a*d-b*c)*b^4*(d*x+c)^(15/2)+10/13*(a*d-
b*c)^2*b^3*(d*x+c)^(13/2)+10/11*(a*d-b*c)^3*b^2*(d*x+c)^(11/2)+5/9*(a*d-b*c
)^4*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^5*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.29, size = 259, normalized size = 1.64

$$\frac{2\left(9009\,(dx+c)^{\frac{17}{2}}b^5-51051\,(b^5c-ab^4d)(dx+c)^{\frac{15}{2}}+117810\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)(dx+c)^{\frac{13}{2}}-139230\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{11}{2}}+85085\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)(dx+c)^{\frac{9}{2}}-21879\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)(dx+c)^{\frac{7}{2}}\right)}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/153153*(9009*(d*x + c)^(17/2)*b^5 - 51051*(b^5*c - a*b^4*d)*(d*x + c)^(15
/2) + 117810*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(13/2) - 13923
0*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(11/2
) + 85085*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*(d*x + c)^(9/2) - 21879*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(7/2))/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(134) = 268$.
time = 0.30, size = 497, normalized size = 3.15

$$\frac{2\,(9009\,b^5d^8x^8-256\,b^5c^8+2176\,ab^4c^7d-8160\,a^2b^3c^6d^2+17680\,a^3b^2c^5d^3-24310\,a^4bc^4d^4+21879\,a^5c^3d^5+3003\,(7\,b^5cd^7+17\,ab^4d^8)x^7+231\,(55\,b^5c^2d^6+527\,ab^4cd^7+510\,a^2b^3d^8)x^6+63\,(b^5c^3d^5+1207\,ab^4c^2d^6+4590\,a^2b^3cd^7+2210\,a^3b^2d^8)x^5-35\,(2\,b^5c^4d^4-17\,ab^4c^3d^5-5406\,a^2b^3c^2d^6-10166\,a^3b^2cd^7-2431\,a^4bd^8)x^4+(80\,b^5c^5d^3-680\,ab^4c^4d^4+2550\,a^2b^3c^3d^5+249730\,a^3b^2c^2d^6+230945\,a^4bcd^7+21879\,a^5d^8)x^3-3\,(32\,b^5c^6d^2-272\,ab^4c^5d^3+1020\,a^2b^3c^4d^4-2210\,a^3b^2c^3d^5-60775\,a^4bc^2d^6-21879\,a^5cd^7)x^2+(128\,b^5c^7d-1088\,ab^4c^6d^2+4080\,a^2b^3c^5d^3-8840\,a^3b^2c^4d^4+12155\,a^4bc^3d^5+65637\,a^5c^2d^6)x)\sqrt{dx+c}}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/153153*(9009*b^5*d^8*x^8 - 256*b^5*c^8 + 2176*a*b^4*c^7*d - 8160*a^2*b^3*
c^6*d^2 + 17680*a^3*b^2*c^5*d^3 - 24310*a^4*b*c^4*d^4 + 21879*a^5*c^3*d^5 +
 3003*(7*b^5*c*d^7 + 17*a*b^4*d^8)*x^7 + 231*(55*b^5*c^2*d^6 + 527*a*b^4*c*
d^7 + 510*a^2*b^3*d^8)*x^6 + 63*(b^5*c^3*d^5 + 1207*a*b^4*c^2*d^6 + 4590*a^
2*b^3*c*d^7 + 2210*a^3*b^2*d^8)*x^5 - 35*(2*b^5*c^4*d^4 - 17*a*b^4*c^3*d^5
- 5406*a^2*b^3*c^2*d^6 - 10166*a^3*b^2*c*d^7 - 2431*a^4*b*d^8)*x^4 + (80*b^
5*c^5*d^3 - 680*a*b^4*c^4*d^4 + 2550*a^2*b^3*c^3*d^5 + 249730*a^3*b^2*c^2*d
```

```
^6 + 230945*a^4*b*c*d^7 + 21879*a^5*d^8)*x^3 - 3*(32*b^5*c^6*d^2 - 272*a*b^
4*c^5*d^3 + 1020*a^2*b^3*c^4*d^4 - 2210*a^3*b^2*c^3*d^5 - 60775*a^4*b*c^2*d
^6 - 21879*a^5*c*d^7)*x^2 + (128*b^5*c^7*d - 1088*a*b^4*c^6*d^2 + 4080*a^2*
b^3*c^5*d^3 - 8840*a^3*b^2*c^4*d^4 + 12155*a^4*b*c^3*d^5 + 65637*a^5*c^2*d^
6)*x)*sqrt(d*x + c)/d^6
```

Sympy [A]
time = 21.93, size = 1292, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(5/2),x)
```

```
[Out] a**5*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**5*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**5*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 10*a*
*4*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 20*a**4*b*c*(
c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
 + 10*a**4*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c
 + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 20*a**3*b**2*c**2*(c**2*(c +
d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 40*a**3
*b**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*
x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**3*b**2*(c**4*(c + d*x)**(3/2
)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)
**(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 20*a**2*b**3*c**2*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 40*a**2*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)
**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)*
*(11/2)/11)/d**4 + 20*a**2*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**4 + 10*a*b**4*c**2*(c**4*(c + d
*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*
(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**5 + 20*a*b**4*c*(-c**5*(c + d
*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2
*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5
 + 10*a*b**4*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4
*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2
)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**5 + 2*b**5*c**2*
(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2
)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(1
3/2)/13)/d**6 + 4*b**5*c*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)
/5 + 15*c**4*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c +
 d*x)**(11/2)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6 +
2*b**5*(-c**7*(c + d*x)**(3/2)/3 + 7*c**6*(c + d*x)**(5/2)/5 - 3*c**5*(c +
d*x)**(7/2) + 35*c**4*(c + d*x)**(9/2)/9 - 35*c**3*(c + d*x)**(11/2)/11 + 2
```

```
1*c**2*(c + d*x)**(13/2)/13 - 7*c*(c + d*x)**(15/2)/15 + (c + d*x)**(17/2)/
17)/d**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1599 vs. $2(134) = 268$.
time = 0.02, size = 2696, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x)
```

```
[Out] 2/765765*(765765*sqrt(d*x + c)*a^5*c^3 + 765765*((d*x + c)^(3/2) - 3*sqrt(d
*x + c)*c)*a^5*c^2 + 1276275*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^
3/d + 153153*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^5*c + 510510*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x
+ c)*c^2)*a^3*b^2*c^3/d^2 + 765765*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*
c + 15*sqrt(d*x + c)*c^2)*a^4*b*c^2/d + 21879*(5*(d*x + c)^(7/2) - 21*(d*x
+ c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^5 + 218790*
(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqr
t(d*x + c)*c^3)*a^2*b^3*c^3/d^3 + 656370*(5*(d*x + c)^(7/2) - 21*(d*x + c)^
(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2*c^2/d^2 +
328185*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*a^4*b*c/d + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c
)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*
x + c)*c^4)*a*b^4*c^3/d^4 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)
*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*
c^4)*a^2*b^3*c^2/d^3 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c +
378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*
a^3*b^2*c/d^2 + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*
x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^4*b/d
 + 1105*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*
c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x +
c)*c^5)*b^5*c^3/d^5 + 16575*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c^2/d^4 + 33150*(63*(d*x + c)^(11/2) - 38
5*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 +
1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*b^3*c/d^3 + 11050*(63
*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*
(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^3
*b^2/d^2 + 765*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c^2/d^5 + 3825*(231*(d*x
 + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d
*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3
003*sqrt(d*x + c)*c^6)*a*b^4*c/d^4 + 2550*(231*(d*x + c)^(13/2) - 1638*(d*x
```

```
 + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009
*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a
^2*b^3/d^3 + 357*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12285*(d
*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2)*c^4
- 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d*x + c
)*c^7)*b^5*c/d^5 + 595*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12
285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2
)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d
*x + c)*c^7)*a*b^4/d^4 + 7*(6435*(d*x + c)^(17/2) - 58344*(d*x + c)^(15/2)*
c + 235620*(d*x + c)^(13/2)*c^2 - 556920*(d*x + c)^(11/2)*c^3 + 850850*(d*x
 + c)^(9/2)*c^4 - 875160*(d*x + c)^(7/2)*c^5 + 612612*(d*x + c)^(5/2)*c^6 -
 291720*(d*x + c)^(3/2)*c^7 + 109395*sqrt(d*x + c)*c^8)*b^5/d^5)/d
```

Mupad [B]
time = 0.27, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{17/2}}{17\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{15/2}}{15\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(17/2))/(17*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(15/
2))/(15*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(7/2))/(7*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(13/2))/
(13*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(9/2))/(9*d^6)
```

3.1400 $\int (a+bx)^4(c+dx)^{5/2}\,dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}-\frac{8b^3(bc-ad)(c+dx)^{13/2}}{13d^5}+\frac{2b^4}{}$$

[Out] 2/7*(-a*d+b*c)^4*(d*x+c)^(7/2)/d^5-8/9*b*(-a*d+b*c)^3*(d*x+c)^(9/2)/d^5+12/11*b^2*(-a*d+b*c)^2*(d*x+c)^(11/2)/d^5-8/13*b^3*(-a*d+b*c)*(d*x+c)^(13/2)/d^5+2/15*b^4*(d*x+c)^(15/2)/d^5

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^4*(c + d*x)^(5/2),x]

[Out] (2*(b*c - a*d)^4*(c + d*x)^(7/2))/(7*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^5) - (8*b^3*(b*c - a*d)*(c + d*x)^(13/2))/(13*d^5) + (2*b^4*(c + d*x)^(15/2))/(15*d^5)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{5/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{7/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{9/2}}{d^4}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 154, normalized size = 1.19

$$\frac{2(c+dx)^{7/2}\left(6435a^4d^4+2860a^3bd^3(-2c+7dx)+390a^2b^2d^2\left(8c^2-28cdx+63d^2x^2\right)+60ab^3d\left(-16c^3+56c^2dx-126cd^2x^2+231d^3x^3\right)+b^4\left(128c^4-448c^3dx+1008c^2d^2x^2-1848cd^3x^3+3003d^4x^4\right)\right)}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*a^4*d^4 + 2860*a^3*b*d^3*(-2*c + 7*d*x) + 390*a^2*
b^2*d^2*(8*c^2 - 28*c*d*x + 63*d^2*x^2) + 60*a*b^3*d*(-16*c^3 + 56*c^2*d*x
- 126*c*d^2*x^2 + 231*d^3*x^3) + b^4*(128*c^4 - 448*c^3*d*x + 1008*c^2*d^2*
x^2 - 1848*c*d^3*x^3 + 3003*d^4*x^4)))/(45045*d^5)
```

Mathics [**B**] Leaf count is larger than twice the leaf count of optimal. 410 vs. $2(129) = 258$.
time = 27.92, size = 406, normalized size = 3.15

$$\frac{2\left(6435a^4c^3d^4+19305a^4c^2d^5x+19305a^4cd^6x^2+6435a^4d^7x^3-5720a^3bc^4d^3+2860a^3bc^3d^4x+42900a^3bc^2d^5x^2+54340a^3bcd^6x^3+20020a^3bd^7x^4+3120a^2b^2c^5d^2-1560a^2b^2c^4d^3x+1170a^2b^2c^3d^4x^2+44070a^2b^2c^2d^5x^3+62790a^2b^2cd^6x^4+24570a^2b^2d^7x^5-960ab^3c^6d+480ab^3c^5d^2x-360ab^3c^4d^3x^2+300ab^3c^3d^4x^3+22260ab^3c^2d^5x^4+34020ab^3cd^6x^5+13860ab^3d^7x^6+128b^4c^7-64b^4c^6dx+48b^4c^5d^2x^2-40b^4c^4d^3x^3+35b^4c^3d^4x^4+4473b^4c^2d^5x^5+7161b^4cd^6x^6+3003b^4d^7x^7\right)\sqrt{c+dx}}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4*(c + d*x)^(5/2),x]')
```

```
[Out] 2 (6435 a ^ 4 c ^ 3 d ^ 4 + 19305 a ^ 4 c ^ 2 d ^ 5 x + 19305 a ^ 4 c d ^ 6
 x ^ 2 + 6435 a ^ 4 d ^ 7 x ^ 3 - 5720 a ^ 3 b c ^ 4 d ^ 3 + 2860 a ^ 3 b c
 ^ 3 d ^ 4 x + 42900 a ^ 3 b c ^ 2 d ^ 5 x ^ 2 + 54340 a ^ 3 b c d ^ 6 x ^
3 + 20020 a ^ 3 b d ^ 7 x ^ 4 + 3120 a ^ 2 b ^ 2 c ^ 5 d ^ 2 - 1560 a ^ 2 b
 ^ 2 c ^ 4 d ^ 3 x + 1170 a ^ 2 b ^ 2 c ^ 3 d ^ 4 x ^ 2 + 44070 a ^ 2 b ^ 2
 c ^ 2 d ^ 5 x ^ 3 + 62790 a ^ 2 b ^ 2 c d ^ 6 x ^ 4 + 24570 a ^ 2 b ^ 2 d
^ 7 x ^ 5 - 960 a b ^ 3 c ^ 6 d + 480 a b ^ 3 c ^ 5 d ^ 2 x - 360 a b ^ 3 c
 ^ 4 d ^ 3 x ^ 2 + 300 a b ^ 3 c ^ 3 d ^ 4 x ^ 3 + 22260 a b ^ 3 c ^ 2 d ^
5 x ^ 4 + 34020 a b ^ 3 c d ^ 6 x ^ 5 + 13860 a b ^ 3 d ^ 7 x ^ 6 + 128 b ^
 4 c ^ 7 - 64 b ^ 4 c ^ 6 d x + 48 b ^ 4 c ^ 5 d ^ 2 x ^ 2 - 40 b ^ 4 c ^ 4
 d ^ 3 x ^ 3 + 35 b ^ 4 c ^ 3 d ^ 4 x ^ 4 + 4473 b ^ 4 c ^ 2 d ^ 5 x ^ 5 +
7161 b ^ 4 c d ^ 6 x ^ 6 + 3003 b ^ 4 d ^ 7 x ^ 7) Sqrt[c + d x] / (45045 d
 ^ 5)
```

Maple [**A**]
time = 0.15, size = 100, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{15}{2}}}{15}+\frac{8(ad-bc)b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)^3b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^4(dx+c)^{\frac{7}{2}}}{7}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{15}{2}}}{15}+\frac{8(ad-bc)b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)^3b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^4(dx+c)^{\frac{7}{2}}}{7}}{d^5}$
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(3003d^4x^4b^4+13860a\,b^3d^4x^3-1848b^4c\,d^3x^3+24570a^2b^2d^4x^2-7560a\,b^3c\,d^3x^2+1008b^4c^2d^2x^2+20020a^3b\,d^4x}{45045d^5}$
trager	$2(3003b^4d^7x^7+13860a\,b^3d^7x^6+7161b^4c\,d^6x^6+24570b^2a^2d^7x^5+34020a\,b^3c\,d^6x^5+4473b^4c^2d^5x^5+20020a^3b\,d^7x^4+6279$
risch	$2(3003b^4d^7x^7+13860a\,b^3d^7x^6+7161b^4c\,d^6x^6+24570b^2a^2d^7x^5+34020a\,b^3c\,d^6x^5+4473b^4c^2d^5x^5+20020a^3b\,d^7x^4+6279$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/15*b^4*(d*x+c)^(15/2)+4/13*(a*d-b*c)*b^3*(d*x+c)^(13/2)+6/11*(a*d-
b*c)^2*b^2*(d*x+c)^(11/2)+4/9*(a*d-b*c)^3*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^4*(
d*x+c)^(7/2))
```

Maxima [A]
time = 0.26, size = 181, normalized size = 1.40

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^4-13860\,(b^4c-ab^3d)(dx+c)^{\frac{13}{2}}+24570\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(dx+c)^{\frac{11}{2}}-20020\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)(dx+c)^{\frac{9}{2}}+6435\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{7}{2}}\right)}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^4 - 13860*(b^4*c - a*b^3*d)*(d*x + c)^(13/
2) + 24570*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(11/2) - 20020*(
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(9/2) + 64
35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
(d*x + c)^(7/2))/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(109) = 218$.
time = 0.30, size = 377, normalized size = 2.92

$$\frac{2\,(3003\,b^4d^7x^7+128\,b^4c^7-960\,ab^3c^6d+3120\,a^2b^2c^5d^2-5720\,a^3bc^4d^3+6435\,a^4c^3d^4+231\,(31\,b^4cd^6+60\,ab^3d^7)x^6+63\,(71\,b^4c^2d^5+540\,ab^3cd^6+390\,a^2b^2d^7)x^5+35\,(b^4c^3d^4+636\,ab^3c^2d^5+1794\,a^2b^2cd^6+572\,a^3bd^7)x^4-5\,(8\,b^4c^4d^3-60\,ab^3c^3d^4-8814\,a^2b^2c^2d^5-10868\,a^3bcd^6-1287\,a^4d^7)x^3+3\,(16\,b^4c^5d^2-120\,ab^3c^4d^3+390\,a^2b^2c^3d^4+14300\,a^3bc^2d^5+6435\,a^4cd^6)x^2-(64\,b^4c^6d-480\,ab^3c^5d^2+1560\,a^2b^2c^4d^3-2860\,a^3bc^3d^4-19305\,a^4c^2d^5)x)\sqrt{dx+c}}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^4*d^7*x^7 + 128*b^4*c^7 - 960*a*b^3*c^6*d + 3120*a^2*b^2*c^
5*d^2 - 5720*a^3*b*c^4*d^3 + 6435*a^4*c^3*d^4 + 231*(31*b^4*c*d^6 + 60*a*b^
3*d^7)*x^6 + 63*(71*b^4*c^2*d^5 + 540*a*b^3*c*d^6 + 390*a^2*b^2*d^7)*x^5 +
35*(b^4*c^3*d^4 + 636*a*b^3*c^2*d^5 + 1794*a^2*b^2*c*d^6 + 572*a^3*b*d^7)*x
^4 - 5*(8*b^4*c^4*d^3 - 60*a*b^3*c^3*d^4 - 8814*a^2*b^2*c^2*d^5 - 10868*a^3
*b*c*d^6 - 1287*a^4*d^7)*x^3 + 3*(16*b^4*c^5*d^2 - 120*a*b^3*c^4*d^3 + 390*
a^2*b^2*c^3*d^4 + 14300*a^3*b*c^2*d^5 + 6435*a^4*c*d^6)*x^2 - (64*b^4*c^6*d
 - 480*a*b^3*c^5*d^2 + 1560*a^2*b^2*c^4*d^3 - 2860*a^3*b*c^3*d^4 - 19305*a^
4*c^2*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A]
time = 16.79, size = 960, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**4*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**4*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 8*a**
3*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 16*a**3*b*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
+ 8*a**3*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c +
 d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 12*a**2*b**2*c**2*(c**2*(c + d*
x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 24*a**2*b
**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)
**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 12*a**2*b**2*(c**4*(c + d*x)**(3/2)/
3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**
(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 8*a*b**3*c**2*(-c**3*(c + d*x)**(3/2
)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)
/9)/d**4 + 16*a*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/1
1)/d**4 + 8*a*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c
**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)
/11 + (c + d*x)**(13/2)/13)/d**4 + 2*b**4*c**2*(c**4*(c + d*x)**(3/2)/3 - 4
*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)
/9 + (c + d*x)**(11/2)/11)/d**5 + 4*b**4*c*(-c**5*(c + d*x)**(3/2)/3 + c**4
*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9
 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**4*(c**6*(c
+ d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*x)**(7/2)/7 -
20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6*c*(c + d*x)**
(13/2)/13 + (c + d*x)**(15/2)/15)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(109) = 218$.
time = 0.01, size = 2008, normalized size = 15.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x)
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^3 + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c^2 + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^3/d +
 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4
*c + 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2
)*a^2*b^2*c^3/d^2 + 36036*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sq
rt(d*x + c)*c^2)*a^3*b*c^2/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)
*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4 + 5148*(5*(d*x + c)
^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*a*b^3*c^3/d^3 + 23166*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d
*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2*c^2/d^2 + 15444*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
```

)*c^3)*a^3*b*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^3/d^4 + 1716*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3*c^2/d^3 + 2574*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^2*c/d^2 + 572*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^3*b/d + 195*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4*c^2/d^4 + 780*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^3*c/d^3 + 390*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*b^2/d^2 + 45*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^4*c/d^4 + 60*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a*b^3/d^3 + 7*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d*x + c)*c^7)*b^4/d^4)/d

Mupad [B]
time = 0.23, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{15/2}}{15\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^4*(c + d*x)^(5/2),x)

[Out] (2*b^4*(c + d*x)^(15/2))/(15*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(13/2))/(13*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^5) + (12*b^2*(a*d - b*c)^2*(c + d*x)^(11/2))/(11*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(9/2))/(9*d^5)

3.1401 $\int (a+bx)^3(c+dx)^{5/2}\,dx$

Optimal. Leaf size=100

$$-\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

```
[Out] -2/7*(-a*d+b*c)^3*(d*x+c)^(7/2)/d^4+2/3*b*(-a*d+b*c)^2*(d*x+c)^(9/2)/d^4-6/
11*b^2*(-a*d+b*c)*(d*x+c)^(11/2)/d^4+2/13*b^3*(d*x+c)^(13/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(b*c - a*d)^2*(c + d*x)^(
9/2))/(3*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(11/2))/(11*d^4) + (2*b^3*(c +
 d*x)^(13/2))/(13*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^{5/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{7/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)}{d^3}\right.\\ &= -\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 1.02

$$\frac{2(c+dx)^{7/2}\left(429a^3d^3+143a^2bd^2(-2c+7dx)+13ab^2d\left(8c^2-28cdx+63d^2x^2\right)+b^3\left(-16c^3+56c^2dx-126cd^2x^2+231d^3x^3\right)\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(429*a^3*d^3 + 143*a^2*b*d^2*(-2*c + 7*d*x) + 13*a*b^2*d
*(8*c^2 - 28*c*d*x + 63*d^2*x^2) + b^3*(-16*c^3 + 56*c^2*d*x - 126*c*d^2*x^
2 + 231*d^3*x^3)))/(3003*d^4)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.60, size = 276, normalized size = 2.76

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-16b^3c^6+8b^2c^5d\left(13a+bx\right)-2bc^4d^2\left(143a^2+26abx+3b^2x^2\right)+c^3d^3\left(429a^3+143a^2bx+39ab^2x^2+5b^3x^3\right)+d^4x\left(1287a^3c^2+1287a^3cdx+429a^3d^2x^2+2145a^2bc^2x+2717a^2bcdx^2+1001a^2bd^2x^3+1469ab^2c^2x^2+2093ab^2cdx^3+819ab^2d^2x^4+371b^3c^2x^3+567b^3cdx^4+231b^3d^2x^5\right)\right)\sqrt{c+dx}}{3003d^4},d!=0\right\}\right\},c^{\frac{5}{2}}\left(a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}\right)\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (-16 b ^ 3 c ^ 6 + 8 b ^ 2 c ^ 5 d (13 a + b x) - 2 b c ^ 4 d
 ^ 2 (143 a ^ 2 + 26 a b x + 3 b ^ 2 x ^ 2) + c ^ 3 d ^ 3 (429 a ^ 3 + 143
a ^ 2 b x + 39 a b ^ 2 x ^ 2 + 5 b ^ 3 x ^ 3) + d ^ 4 x (1287 a ^ 3 c ^ 2 +
 1287 a ^ 3 c d x + 429 a ^ 3 d ^ 2 x ^ 2 + 2145 a ^ 2 b c ^ 2 x + 2717 a ^
 2 b c d x ^ 2 + 1001 a ^ 2 b d ^ 2 x ^ 3 + 1469 a b ^ 2 c ^ 2 x ^ 2 + 2093
 a b ^ 2 c d x ^ 3 + 819 a b ^ 2 d ^ 2 x ^ 4 + 371 b ^ 3 c ^ 2 x ^ 3 + 567
b ^ 3 c d x ^ 4 + 231 b ^ 3 d ^ 2 x ^ 5)) Sqrt[c + d x] / (3003 d ^ 4), d !
= 0}}, c ^ (5 / 2) (a ^ 3 x + 3 a ^ 2 b x ^ 2 / 2 + a b ^ 2 x ^ 3 + b ^ 3 x
 ^ 4 / 4)]
```

Maple [A]
time = 0.14, size = 78, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{6(ad-bc)b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)^2b(dx+c)^{\frac{9}{2}}}{3}+\frac{2(ad-bc)^3(dx+c)^{\frac{7}{2}}}{7}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{6(ad-bc)b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)^2b(dx+c)^{\frac{9}{2}}}{3}+\frac{2(ad-bc)^3(dx+c)^{\frac{7}{2}}}{7}}{d^4}$
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(231b^3x^3d^3+819ab^2d^3x^2-126b^3cd^2x^2+1001a^2bd^3x-364ab^2cd^2x+56b^3c^2dx+429a^3d^3-286a^2bcd^2+104ab^2 \ldots}{3003d^4}$
trager	$2(231b^3d^6x^6+819ab^2d^6x^5+567b^3cd^5x^5+1001a^2bd^6x^4+2093ab^2cd^5x^4+371b^3c^2d^4x^4+429a^3d^6x^3+2717a^2bcd^5x^3+146 \ldots$
risch	$2(231b^3d^6x^6+819ab^2d^6x^5+567b^3cd^5x^5+1001a^2bd^6x^4+2093ab^2cd^5x^4+371b^3c^2d^4x^4+429a^3d^6x^3+2717a^2bcd^5x^3+146 \ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/13*b^3*(d*x+c)^(13/2)+3/11*(a*d-b*c)*b^2*(d*x+c)^(11/2)+1/3*(a*d-b
*c)^2*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^3*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.18

$$\frac{2\left(231\,(dx+c)^{\frac{13}{2}}b^3-819\,(b^3c-ab^2d)(dx+c)^{\frac{11}{2}}+1001\,(b^3c^2-2\,ab^2cd+a^2bd^2)(dx+c)^{\frac{9}{2}}-429\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx+c)^{\frac{7}{2}}\right)}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*(d*x + c)^(13/2)*b^3 - 819*(b^3*c - a*b^2*d)*(d*x + c)^(11/2) +
 1001*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(9/2) - 429*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(7/2))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(84) = 168.
time = 0.29, size = 268, normalized size = 2.68

$$\frac{2\,(231\,b^3d^6x^6-16\,b^3c^6+104\,ab^2c^5d-286\,a^2bc^4d^2+429\,a^3c^3d^3+63\,(9\,b^3cd^5+13\,ab^2d^6)x^5+7\,(53\,b^3c^2d^4+299\,ab^2cd^5+143\,a^2bd^6)x^4+(5\,b^3c^3d^3+1469\,ab^2c^2d^4+2717\,a^2bcd^5+429\,a^3d^6)x^3-3\,(2\,b^3c^4d^2-13\,ab^2c^3d^3-715\,a^2bc^2d^4-429\,a^3cd^5)x^2+(8\,b^3c^5d-52\,ab^2c^4d^2+143\,a^2bc^3d^3+1287\,a^3c^2d^4)x)\sqrt{dx+c}}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*d^6*x^6 - 16*b^3*c^6 + 104*a*b^2*c^5*d - 286*a^2*b*c^4*d^2
+ 429*a^3*c^3*d^3 + 63*(9*b^3*c*d^5 + 13*a*b^2*d^6)*x^5 + 7*(53*b^3*c^2*d^4
 + 299*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 + (5*b^3*c^3*d^3 + 1469*a*b^2*c^2*d
^4 + 2717*a^2*b*c*d^5 + 429*a^3*d^6)*x^3 - 3*(2*b^3*c^4*d^2 - 13*a*b^2*c^3*
d^3 - 715*a^2*b*c^2*d^4 - 429*a^3*c*d^5)*x^2 + (8*b^3*c^5*d - 52*a*b^2*c^4*
d^2 + 143*a^2*b*c^3*d^3 + 1287*a^3*c^2*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A]
time = 0.45, size = 549, normalized size = 5.49

$$\begin{cases} \frac{2a^3c^3\sqrt{c+dx}}{7d}+\frac{6a^3c^2x\sqrt{c+dx}}{7}+\frac{6a^3cdx^2\sqrt{c+dx}}{7}+\frac{2a^3d^2x^3\sqrt{c+dx}}{7}-\frac{4a^2bc^4\sqrt{c+dx}}{21d^2}+\frac{2a^2bc^3x\sqrt{c+dx}}{21d}+\frac{10a^2bc^2x^2\sqrt{c+dx}}{7}+\frac{38a^2bcdx^3\sqrt{c+dx}}{21}+\frac{2a^2bd^2x^4\sqrt{c+dx}}{3}+\frac{16ab^2c^5\sqrt{c+dx}}{231d^3}-\frac{8ab^2c^4x\sqrt{c+dx}}{231d^2}+\frac{2ab^2c^3x^2\sqrt{c+dx}}{77d}+\frac{226ab^2c^2x^3\sqrt{c+dx}}{231}+\frac{46ab^2cdx^4\sqrt{c+dx}}{33}+\frac{6ab^2d^2x^5\sqrt{c+dx}}{11}-\frac{32b^3c^6\sqrt{c+dx}}{3003d^4}+\frac{16b^3c^5x\sqrt{c+dx}}{3003d^3}-\frac{4b^3c^4x^2\sqrt{c+dx}}{1001d^2}+\frac{10b^3c^3x^3\sqrt{c+dx}}{3003d}+\frac{106b^3c^2x^4\sqrt{c+dx}}{429}+\frac{54b^3cdx^5\sqrt{c+dx}}{143}+\frac{2b^3d^2x^6\sqrt{c+dx}}{13} & \text{for}\; d\neq 0 \\ c^{\frac{5}{2}}\left(a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**3*c**3*sqrt(c + d*x)/(7*d) + 6*a**3*c**2*x*sqrt(c + d*x)/7
+ 6*a**3*c*d*x**2*sqrt(c + d*x)/7 + 2*a**3*d**2*x**3*sqrt(c + d*x)/7 - 4*a*
*2*b*c**4*sqrt(c + d*x)/(21*d**2) + 2*a**2*b*c**3*x*sqrt(c + d*x)/(21*d) +
10*a**2*b*c**2*x**2*sqrt(c + d*x)/7 + 38*a**2*b*c*d*x**3*sqrt(c + d*x)/21 +
 2*a**2*b*d**2*x**4*sqrt(c + d*x)/3 + 16*a*b**2*c**5*sqrt(c + d*x)/(231*d**
3) - 8*a*b**2*c**4*x*sqrt(c + d*x)/(231*d**2) + 2*a*b**2*c**3*x**2*sqrt(c +
 d*x)/(77*d) + 226*a*b**2*c**2*x**3*sqrt(c + d*x)/231 + 46*a*b**2*c*d*x**4*
sqrt(c + d*x)/33 + 6*a*b**2*d**2*x**5*sqrt(c + d*x)/11 - 32*b**3*c**6*sqrt(
c + d*x)/(3003*d**4) + 16*b**3*c**5*x*sqrt(c + d*x)/(3003*d**3) - 4*b**3*c*
```

```
*4*x**2*sqrt(c + d*x)/(1001*d**2) + 10*b**3*c**3*x**3*sqrt(c + d*x)/(3003*d
) + 106*b**3*c**2*x**4*sqrt(c + d*x)/429 + 54*b**3*c*d*x**5*sqrt(c + d*x)/1
43 + 2*b**3*d**2*x**6*sqrt(c + d*x)/13, Ne(d, 0)), (c**(5/2)*(a**3*x + 3*a*
*2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(84) = 168.
time = 0.01, size = 1412, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x)
```

```
[Out] 2/15015*(15015*sqrt(d*x + c)*a^3*c^3 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^3*c^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^3/d +
 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3
*c + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b^2*c^3/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^2*b*c^2/d + 429*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3 + 429*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^
3*c^3/d^3 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(
3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c^2/d^2 + 3861*(5*(d*x + c)^(7/2) -
21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b
*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2
)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^3*c^2/d^3 + 429*
(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420
*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^2*c/d^2 + 143*(35*(d*x +
c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^
(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b/d + 65*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^3*c/d^3 + 65*(63*(d*x + c
)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)
^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^2/d^2 +
5*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^
2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3
/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^3/d^3)/d
```

Mupad [B]
time = 0.08, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{13/2}}{13\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(13/2))/(13*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(11/2)
)/(11*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(a*d - b*c)^2
*(c + d*x)^(9/2))/(3*d^4)
```

3.1402 $\int (a+bx)^2(c+dx)^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

```
[Out] 2/7*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^3-4/9*b*(-a*d+b*c)*(d*x+c)^(9/2)/d^3+2/11*
b^2*(d*x+c)^(11/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(9/2
))/(9*d^3) + (2*b^2*(c + d*x)^(11/2))/(11*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(-2c+7dx)+b^2\left(8c^2-28cdx+63d^2x^2\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x) + b^2*(8*c^2 - 28*
c*d*x + 63*d^2*x^2)))/(693*d^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.64, size = 183, normalized size = 2.58

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(8b^2c^5-4bc^4d\left(11a+bx\right)+c^3d^2\left(99a^2+22abx+3b^2x^2\right)+d^3x\left(297a^2c^2+297a^2cdx+99a^2d^2x^2+330abc^2x+418abcdx^2+154abd^2x^3+113b^2c^2x^2+161b^2cdx^3+63b^2d^2x^4\right)\right)\sqrt{c+dx}}{693d^3},d!=0\right\}\right\},c^{\frac{5}{2}}\left(a^2x+abx^2+\frac{b^2x^3}{3}\right)\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (8 b ^ 2 c ^ 5 - 4 b c ^ 4 d (11 a + b x) + c ^ 3 d ^ 2 (99 a
 ^ 2 + 22 a b x + 3 b ^ 2 x ^ 2) + d ^ 3 x (297 a ^ 2 c ^ 2 + 297 a ^ 2 c d
 x + 99 a ^ 2 d ^ 2 x ^ 2 + 330 a b c ^ 2 x + 418 a b c d x ^ 2 + 154 a b d
 ^ 2 x ^ 3 + 113 b ^ 2 c ^ 2 x ^ 2 + 161 b ^ 2 c d x ^ 3 + 63 b ^ 2 d ^ 2 x
 ^ 4)) Sqrt[c + d x] / (693 d ^ 3), d != 0}}, c ^ (5 / 2) (a ^ 2 x + a b x
^ 2 + b ^ 2 x ^ 3 / 3)]
```

Maple [A]
time = 0.15, size = 56, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^2(dx+c)^{\frac{7}{2}}}{7}}{d^3}$
default	$\frac{\frac{2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^2(dx+c)^{\frac{7}{2}}}{7}}{d^3}$
gosper	$\frac{2(dx+c)^{\frac{7}{2}}\left(63b^2x^2d^2+154ab\,d^2x-28b^2cdx+99a^2d^2-44abcd+8b^2c^2\right)}{693d^3}$
trager	$\frac{2(63b^2d^5x^5+154ab\,d^5x^4+161b^2c\,d^4x^4+99a^2d^5x^3+418abc\,d^4x^3+113b^2c^2d^3x^3+297a^2c\,d^4x^2+330ab\,c^2d^3x^2+3b^2c^3d^2x^2}{693d^3}$
risch	$\frac{2(63b^2d^5x^5+154ab\,d^5x^4+161b^2c\,d^4x^4+99a^2d^5x^3+418abc\,d^4x^3+113b^2c^2d^3x^3+297a^2c\,d^4x^2+330ab\,c^2d^3x^2+3b^2c^3d^2x^2}{693d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/11*b^2*(d*x+c)^(11/2)+2/9*(a*d-b*c)*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^
2*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.26, size = 68, normalized size = 0.96

$$\frac{2\left(63\,(dx+c)^{\frac{11}{2}}b^2-154\,(b^2c-abd)(dx+c)^{\frac{9}{2}}+99\,(b^2c^2-2\,abcd+a^2d^2)(dx+c)^{\frac{7}{2}}\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(d*x + c)^(11/2)*b^2 - 154*(b^2*c - a*b*d)*(d*x + c)^(9/2) + 99*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(7/2))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(59) = 118$.
time = 0.29, size = 174, normalized size = 2.45

$$\frac{2\,(63\,b^2d^5x^5 + 8\,b^2c^5 - 44\,abc^4d + 99\,a^2c^3d^2 + 7\,(23\,b^2cd^4 + 22\,abd^5)x^4 + (113\,b^2c^2d^3 + 418\,abcd^4 + 99\,a^2d^5)x^3 + 3\,(b^2c^3d^2 + 110\,abc^2d^3 + 99\,a^2cd^4)x^2 - (4\,b^2c^4d - 22\,abc^3d^2 - 297\,a^2c^2d^3)x)\sqrt{dx + c}}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*d^5*x^5 + 8*b^2*c^5 - 44*a*b*c^4*d + 99*a^2*c^3*d^2 + 7*(23*b
^2*c*d^4 + 22*a*b*d^5)*x^4 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5)
*x^3 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^2 - (4*b^2*c^4*d
- 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x)*sqrt(d*x + c)/d^3
```

Sympy [A]
time = 0.38, size = 355, normalized size = 5.00

$$\begin{cases} \frac{2a^2c^3\sqrt{c+dx}}{7d} + \frac{6a^2c^2x\sqrt{c+dx}}{7} + \frac{6a^2cdx^2\sqrt{c+dx}}{7} + \frac{2a^2d^2x^3\sqrt{c+dx}}{7} - \frac{8abc^4\sqrt{c+dx}}{63d^2} + \frac{4abc^3x\sqrt{c+dx}}{63d} + \frac{20abc^2x^2\sqrt{c+dx}}{21} + \frac{76abcdx^3\sqrt{c+dx}}{63} + \frac{4abd^2x^4\sqrt{c+dx}}{9} + \frac{16b^2c^5\sqrt{c+dx}}{693d^3} - \frac{8b^2c^4x\sqrt{c+dx}}{693d^2} + \frac{2b^2c^3x^2\sqrt{c+dx}}{231d} + \frac{226b^2c^2x^3\sqrt{c+dx}}{693} + \frac{46b^2cdx^4\sqrt{c+dx}}{99} + \frac{2b^2d^2x^5\sqrt{c+dx}}{11} & \text{for}\ d \neq 0 \\ c^{\frac{5}{2}}\left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**2*c**3*sqrt(c + d*x)/(7*d) + 6*a**2*c**2*x*sqrt(c + d*x)/7
+ 6*a**2*c*d*x**2*sqrt(c + d*x)/7 + 2*a**2*d**2*x**3*sqrt(c + d*x)/7 - 8*a*
b*c**4*sqrt(c + d*x)/(63*d**2) + 4*a*b*c**3*x*sqrt(c + d*x)/(63*d) + 20*a*b
*c**2*x**2*sqrt(c + d*x)/21 + 76*a*b*c*d*x**3*sqrt(c + d*x)/63 + 4*a*b*d**2
*x**4*sqrt(c + d*x)/9 + 16*b**2*c**5*sqrt(c + d*x)/(693*d**3) - 8*b**2*c**4
*x*sqrt(c + d*x)/(693*d**2) + 2*b**2*c**3*x**2*sqrt(c + d*x)/(231*d) + 226*
b**2*c**2*x**3*sqrt(c + d*x)/693 + 46*b**2*c*d*x**4*sqrt(c + d*x)/99 + 2*b*
*2*d**2*x**5*sqrt(c + d*x)/11, Ne(d, 0)), (c**(5/2)*(a**2*x + a*b*x**2 + b*
*2*x**3/3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(59) = 118$.
time = 0.01, size = 911, normalized size = 12.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x)
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^2*c^3 + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^2*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^3/d + 693*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*c + 23
1*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^3
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a*b*c^2/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^2*c^2/d^2 + 5
94*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*
sqrt(d*x + c)*c^3)*a*b*c/d + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c
 + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^
4)*b^2*c/d^2 + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x +
c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d + 5*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b
^2/d^2)/d
```

Mupad [B]
time = 0.07, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(63\,b^2\,{\left(c+d\,x\right)}^2+99\,a^2\,d^2+99\,b^2\,c^2-154\,b^2\,c\,\left(c+d\,x\right)+154\,a\,b\,d\,\left(c+d\,x\right)-198\,a\,b\,c\,d\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(63*b^2*(c + d*x)^2 + 99*a^2*d^2 + 99*b^2*c^2 - 154*b^2*
c*(c + d*x) + 154*a*b*d*(c + d*x) - 198*a*b*c*d))/(693*d^3)
```

3.1403 $\int (a+bx)(c+dx)^{5/2}\,dx$

Optimal. Leaf size=42

$$-\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}$$

```
[Out] -2/7*(-a*d+b*c)*(d*x+c)^(7/2)/d^2+2/9*b*(d*x+c)^(9/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^2) + (2*b*(c + d*x)^(9/2))/(9*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{5/2}}{d}+\frac{b(c+dx)^{7/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{7/2}(-2bc+9ad+7bdx)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2)*(-2*b*c + 9*a*d + 7*b*d*x))/(63*d^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.91, size = 102, normalized size = 2.43

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2bc^4+c^3d\left(9a+bx\right)+d^2x\left(27ac^2+27acdx+9ad^2x^2+15bc^2x+19bcdx^2+7bd^2x^3\right)\right)\sqrt{c+dx}}{63d^2},d!=0\right\}\right\},c^{\frac{5}{2}}\left(ax+\frac{bx^2}{2}\right)\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^(5/2),x]')
[Out] Piecewise[{{2 (-2 b c ^ 4 + c ^ 3 d (9 a + b x) + d ^ 2 x (27 a c ^ 2 + 27
a c d x + 9 a d ^ 2 x ^ 2 + 15 b c ^ 2 x + 19 b c d x ^ 2 + 7 b d ^ 2 x ^ 3
)) Sqrt[c + d x] / (63 d ^ 2), d != 0}}, c ^ (5 / 2) (a x + b x ^ 2 / 2)]
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(7bdx+9ad-2bc)}{63d^2}$	27
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)(dx+c)^{\frac{7}{2}}}{7}}{d^2}$	34
default	$\frac{\frac{2b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)(dx+c)^{\frac{7}{2}}}{7}}{d^2}$	34
trager	$\frac{2\left(7b\,d^4x^4+9a\,d^4x^3+19bc\,d^3x^3+27ac\,d^3x^2+15b\,c^2d^2x^2+27a\,c^2d^2x+b\,c^3dx+9a\,c^3d-2b\,c^4\right)\sqrt{dx+c}}{63d^2}$	94
risch	$\frac{2\left(7b\,d^4x^4+9a\,d^4x^3+19bc\,d^3x^3+27ac\,d^3x^2+15b\,c^2d^2x^2+27a\,c^2d^2x+b\,c^3dx+9a\,c^3d-2b\,c^4\right)\sqrt{dx+c}}{63d^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/9*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.79

$$\frac{2\left(7\left(dx+c\right)^{\frac{9}{2}}b-9\left(bc-ad\right)\left(dx+c\right)^{\frac{7}{2}}\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="maxima")
```

[Out] 2/63*(7*(d*x + c)^(9/2)*b - 9*(b*c - a*d)*(d*x + c)^(7/2))/d^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.
time = 0.30, size = 93, normalized size = 2.21

$$\frac{2\,{\left(7\,b d^{4} x^{4} - 2\,b c^{4} + 9\,a c^{3} d + {\left(19\,b c d^{3} + 9\,a d^{4}\right)} x^{3} + 3\,{\left(5\,b c^{2} d^{2} + 9\,a c d^{3}\right)} x^{2} + {\left(b c^{3} d + 27\,a c^{2} d^{2}\right)} x\right)} \sqrt{d x + c}}{63\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*d^4*x^4 - 2*b*c^4 + 9*a*c^3*d + (19*b*c*d^3 + 9*a*d^4)*x^3 + 3*(5
*b*c^2*d^2 + 9*a*c*d^3)*x^2 + (b*c^3*d + 27*a*c^2*d^2)*x)*sqrt(d*x + c)/d^2
```

Sympy [A]
time = 0.30, size = 194, normalized size = 4.62

$$\begin{cases} \frac{2 a c^{3} \sqrt{c + d x}}{7 d} + \frac{6 a c^{2} x \sqrt{c + d x}}{7} + \frac{6 a c d x^{2} \sqrt{c + d x}}{7} + \frac{2 a d^{2} x^{3} \sqrt{c + d x}}{7} - \frac{4 b c^{4} \sqrt{c + d x}}{63 d^{2}} + \frac{2 b c^{3} x \sqrt{c + d x}}{63 d} + \frac{10 b c^{2} x^{2} \sqrt{c + d x}}{21} + \frac{38 b c d x^{3} \sqrt{c + d x}}{63} + \frac{2 b d^{2} x^{4} \sqrt{c + d x}}{9} & \text{for}\: d \neq 0 \\ c^{\frac{5}{2}} \left(a x + \frac{b x^{2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a*c**3*sqrt(c + d*x)/(7*d) + 6*a*c**2*x*sqrt(c + d*x)/7 + 6*a*
c*d*x**2*sqrt(c + d*x)/7 + 2*a*d**2*x**3*sqrt(c + d*x)/7 - 4*b*c**4*sqrt(c
+ d*x)/(63*d**2) + 2*b*c**3*x*sqrt(c + d*x)/(63*d) + 10*b*c**2*x**2*sqrt(c
+ d*x)/21 + 38*b*c*d*x**3*sqrt(c + d*x)/63 + 2*b*d**2*x**4*sqrt(c + d*x)/9,
 Ne(d, 0)), (c**(5/2)*(a*x + b*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(34) = 68$.
time = 0.00, size = 500, normalized size = 11.90

$$\frac{\frac{2 b d^{3} \left(\frac{1}{9} \sqrt{c + d x} (c+dx)^{4} - \frac{4}{7} \sqrt{c + d x} (c+dx)^{3} c + \frac{6}{5} \sqrt{c + d x} (c+dx)^{2} c^{2} - \frac{4}{3} \sqrt{c + d x} (c+dx) c^{3} + \sqrt{c + d x}\, c^{4}\right)}{d^{4}} + \frac{2 a d^{3} \left(\frac{1}{7} \sqrt{c + d x} (c+dx)^{3} - \frac{3}{5} \sqrt{c + d x} (c+dx)^{2} c + \sqrt{c + d x} (c+dx) c^{2} - \sqrt{c + d x}\, c^{3}\right)}{d^{3}} + \frac{6 b c d^{2} \left(\frac{1}{7} \sqrt{c + d x} (c+dx)^{3} - \frac{3}{5} \sqrt{c + d x} (c+dx)^{2} c + \sqrt{c + d x} (c+dx) c^{2} - \sqrt{c + d x}\, c^{3}\right)}{d^{3}} + \frac{6 a c d^{2} \left(\frac{1}{5} \sqrt{c + d x} (c+dx)^{2} - \frac{2}{3} \sqrt{c + d x} (c+dx) c + \sqrt{c + d x}\, c^{2}\right)}{d^{2}} + \frac{6 b c^{2} d \left(\frac{1}{5} \sqrt{c + d x} (c+dx)^{2} - \frac{2}{3} \sqrt{c + d x} (c+dx) c + \sqrt{c + d x}\, c^{2}\right)}{d^{2}} + 6 a c^{2} \left(\frac{1}{3} \sqrt{c + d x} \,(c + d x) - c \sqrt{c + d x}\right) + \frac{2 b c^{3} \left(\frac{1}{3} \sqrt{c + d x} (c+dx) - c \sqrt{c + d x}\right)}{d} + 2 a c^{3} \sqrt{c + d x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x)
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a*c^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a*c^2 + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^3/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*c + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c^2/d + 9*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2
)*c^2 - 35*sqrt(d*x + c)*c^3)*b*c/d + (35*(d*x + c)^(9/2) - 180*(d*x + c)^(
```

```
7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x +
 c)*c^4)*b/d)/d
```

Mupad [B]
time = 0.05, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(9\,a\,d-9\,b\,c+7\,b\,\left(c+d\,x\right)\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(5/2),x)
[Out] (2*(c + d*x)^(7/2)*(9*a*d - 9*b*c + 7*b*(c + d*x)))/(63*d^2)
```

3.1404 $\int (c+dx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{7/2}}{7d}$$

```
[Out] 2/7*(d*x+c)^(7/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{5/2}\,dx = \frac{2(c+dx)^{7/2}}{7d}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

Mathics [A]
time = 1.75, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{7}{2}}}{7d}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^(5/2),x]')
```

```
[Out] 2 (c + d x) ^ (7 / 2) / (7 d)
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{7}{2}}}{7d}$	13
derivativedivides	$\frac{2(dx+c)^{\frac{7}{2}}}{7d}$	13
default	$\frac{2(dx+c)^{\frac{7}{2}}}{7d}$	13
trager	$\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)\sqrt{dx+c}}{7d}$	40
risch	$\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)\sqrt{dx+c}}{7d}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(d*x+c)^(7/2)/d
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(d*x + c)^(7/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.
time = 0.29, size = 39, normalized size = 2.44

$$\frac{2\,(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3)\sqrt{dx+c}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(d*x + c)/d
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2(c+dx)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2),x)
```

```
[Out] 2*(c + d*x)**(7/2)/(7*d)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(12) = 24$.
time = 0.00, size = 199, normalized size = 12.44

$$\frac{\frac{2d^3\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3-\frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c+\sqrt{c+dx}\,(c+dx)c^2-\sqrt{c+dx}\,c^3\right)}{d^3}+\frac{6cd^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+6c^2\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)+2c^3\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x)
```

```
[Out] 2/35*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 + 3
5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c^2 + 7*(3*(d*x + c)^(5/2) - 10*(d*
x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*c)/d
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

3.1405 $\int \frac{(c+dx)^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=112

$$\frac{2(bc-ad)^2\sqrt{c+dx}}{b^3}+\frac{2(bc-ad)(c+dx)^{3/2}}{3b^2}+\frac{2(c+dx)^{5/2}}{5b}-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

```
[Out] 2/3*(-a*d+b*c)*(d*x+c)^(3/2)/b^2+2/5*(d*x+c)^(5/2)/b-2*(-a*d+b*c)^(5/2)*arc
tanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+2*(-a*d+b*c)^2*(d*x+c)
^(1/2)/b^3
```

Rubi [A]

time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 65, 214}

$$-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}+\frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/b^3 + (2*(b*c - a*d)*(c + d*x)^(3/2))/(3*b^
2) + (2*(c + d*x)^(5/2))/(5*b) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{a+bx}\,dx = \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{b}$$

$$= \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b^2}$$

$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx)\sqrt{c+}}}{b^3}$$

$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} - \frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\right.}{b^{7/2}}$$

Mathematica [A]
time = 0.11, size = 108, normalized size = 0.96

$$\frac{2\sqrt{c+dx}\,\left(15a^2d^2-5abd(7c+dx)+b^2\left(23c^2+11cdx+3d^2x^2\right)\right)}{15b^3} - \frac{2(-bc+ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 - 5*a*b*d*(7*c + d*x) + b^2*(23*c^2 + 11*c*d*x
 + 3*d^2*x^2)))/(15*b^3) - (2*(-(b*c) + a*d)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[-(b*c) + a*d]])/b^(7/2)
```

Mathics [A]
time = 15.91, size = 140, normalized size = 1.25

$$\frac{2\left(\frac{15\left(a^2d^2-2abcd+b^2c^2\right)(ad-bc)^5\sqrt{c+dx}}{b} + 3b\,(ad-bc)^5\,(c+dx)^{\frac{5}{2}} - 15b^6\text{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right]\left(\frac{ad-bc}{b}\right)^{\frac{15}{2}} - 5\,(ad-bc)^6\,(c+dx)^{\frac{3}{2}}\right)}{15b^2\,(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^1,x]')
```

```
[Out] 2 (15 (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (a d - b c) ^ 5 Sqrt[c + d x]
 / b + 3 b (a d - b c) ^ 5 (c + d x) ^ (5 / 2) - 15 b ^ 6 ArcTan[Sqrt[c + d
 x] / Sqrt[(a d - b c) / b]] ((a d - b c) / b) ^ (15 / 2) - 5 (a d - b c) ^
 6 (c + d x) ^ (3 / 2)) / (15 b ^ 2 (a d - b c) ^ 5)
```

Maple [A]
time = 0.17, size = 161, normalized size = 1.44

method	result
derivativedivides	$\frac{\frac{2(dx+c)^{\frac{5}{2}}b^2}{5}-\frac{2abd(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c(dx+c)^{\frac{3}{2}}}{3}+2a^2d^2\sqrt{dx+c}-4abcd\sqrt{dx+c}+2b^2c^2\sqrt{dx+c}}{b^3}+\frac{2(-a^3d^3+3}{}$
default	$\frac{\frac{2(dx+c)^{\frac{5}{2}}b^2}{5}-\frac{2abd(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c(dx+c)^{\frac{3}{2}}}{3}+2a^2d^2\sqrt{dx+c}-4abcd\sqrt{dx+c}+2b^2c^2\sqrt{dx+c}}{b^3}+\frac{2(-a^3d^3+3}{}$
risch	$\frac{2(3b^2x^2d^2-5ab\,d^2x+11b^2cdx+15a^2d^2-35abcd+23b^2c^2)\sqrt{dx+c}}{15b^3}-\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)a^3d^3}{b^3\sqrt{(ad-bc)\,b}}+\frac{6\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*(d*x+c)^(5/2)*b^2-1/3*a*b*d*(d*x+c)^(3/2)+1/3*b^2*c*(d*x+c)^(3/2
)+a^2*d^2*(d*x+c)^(1/2)-2*a*b*c*d*(d*x+c)^(1/2)+b^2*c^2*(d*x+c)^(1/2))+2*(-
a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/((a*d-b*c)*b)^(1/2)*arctan
(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 290, normalized size = 2.59

$$\left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(3\,b^2d^2x^2+23\,b^2c^2-35\,abcd+15\,a^2d^2+(11\,b^2cd-5\,abd^2)x)\sqrt{dx+c}}{15\,b^3},\ -\frac{2\left(15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^2d^2x^2+23\,b^2c^2-35\,abcd+15\,a^2d^2+(11\,b^2cd-5\,abd^2)x)\sqrt{dx+c}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="fricas")

[Out] [1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(3*b^2*d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/b^3, -2/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/b^3]

Sympy [A]
time = 12.45, size = 121, normalized size = 1.08

$$\frac{2\left(c+dx\right)^{\frac{5}{2}}}{5b}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(-2ad+2bc\right)}{3b^{2}}+\frac{\sqrt{c+dx}\,\left(2a^{2}d^{2}-4abcd+2b^{2}c^{2}\right)}{b^{3}}-\frac{2\left(ad-bc\right)^{3}\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^{4}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)/(b*x+a),x)

[Out] 2*(c + d*x)**(5/2)/(5*b) + (c + d*x)**(3/2)*(-2*a*d + 2*b*c)/(3*b**2) + sqrt(c + d*x)*(2*a**2*d**2 - 4*a*b*c*d + 2*b**2*c**2)/b**3 - 2*(a*d - b*c)**3*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))

Giac [A]
time = 0.00, size = 221, normalized size = 1.97

$$\frac{\frac{2}{5}\sqrt{c+dx}\,\left(c+dx\right)^{2}b^{4}-\frac{2}{3}\sqrt{c+dx}\,\left(c+dx\right)db^{3}a+\frac{2}{3}\sqrt{c+dx}\,\left(c+dx\right)cb^{4}+2\sqrt{c+dx}\,d^{2}b^{2}a^{2}-4\sqrt{c+dx}\,dcb^{3}a+2\sqrt{c+dx}\,c^{2}b^{4}}{b^{5}}+\frac{\left(-4d^{3}a^{3}+12d^{2}cba^{2}-12dc^{2}b^{2}a+4c^{3}b^{3}\right)\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^{2}c+abd}}\right)}{2b^{3}\sqrt{-b^{2}c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a),x)

[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/15*(3*(d*x + c)^(5/2)*b^4 + 5*(d*x + c)^(3/2)*b^4*c + 15*sqrt(d*x + c)*b^4*c^2 - 5*(d*x + c)^(3/2)*a*b^3*d - 30*sqrt(d*x + c)*a*b^3*c*d + 15*sqrt(d*x + c)*a^2*b^2*d^2)/b^5

Mupad [B]
time = 0.08, size = 130, normalized size = 1.16

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{2\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}+\frac{2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{b^3}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}\,\sqrt{c+d\,x}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) - (2*(a*d - b*c)*(c + d*x)^(3/2))/(3*b^2) + (2*(a
*d - b*c)^2*(c + d*x)^(1/2))/b^3 - (2*atan((b^(1/2)*(a*d - b*c)^(5/2)*(c +
d*x)^(1/2))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c
)^(5/2))/b^(7/2)
```

3.1406 $\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=110

$$\frac{5d(bc-ad)\sqrt{c+dx}}{b^3}+\frac{5d(c+dx)^{3/2}}{3b^2}-\frac{(c+dx)^{5/2}}{b(a+bx)}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

```
[Out] 5/3*d*(d*x+c)^(3/2)/b^2-(d*x+c)^(5/2)/b/(b*x+a)-5*d*(-a*d+b*c)^(3/2)*arctan
h(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+5*d*(-a*d+b*c)*(d*x+c)^(1
/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {43, 52, 65, 214}

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{5d\sqrt{c+dx}\,(bc-ad)}{b^3}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x])/b^3 + (5*d*(c + d*x)^(3/2))/(3*b^2) - (c +
d*x)^(5/2)/(b*(a + b*x)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx = -\frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d(bc-ad))\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b^2}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d(bc-ad)^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}}{2b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx}{d}}\right.}{b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{bc-}}\right.}{b^{7/2}}$$

Mathematica [A]
time = 0.29, size = 116, normalized size = 1.05

$$-\frac{\sqrt{c+dx}\,\left(15a^2d^2+10abd(-2c+dx)+b^2\left(3c^2-14cdx-2d^2x^2\right)\right)}{3b^3(a+bx)} + \frac{5d(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] -1/3*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(3*c^2 - 14*c
*d*x - 2*d^2*x^2)))/(b^3*(a + b*x)) + (5*d*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(7/2)
```

Mathics [B] Leaf count is larger than twice the leaf count of optimal. 1486 vs. 2(110) = 220.
time = 134.05, size = 1358, normalized size = 12.35

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^2,x]')
```

```
[Out] (6 a ^ 2 d ^ 3 ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] (a d - b c) (a
 ^ 2 d - a b c + a b d x - b ^ 2 c x) + a b ^ 2 d ^ 2 (a ^ 2 d - a b c + a
b d x - b ^ 2 c x) (-a ^ 2 d ^ 2 Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^
 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b
(a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + a ^ 2 d
 ^ 2 Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sqrt[-1 /
(b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c
+ d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] - 8 Sqrt[c + d x]) ((a d - b c) / b)
 ^ (3 / 2) / 2 - a ^ 3 b ^ 2 d ^ 3 ((a d - b c) / b) ^ (3 / 2) Sqrt[c + d x
] - 12 a b c d ^ 2 ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] (a d - b c
) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) + 3 a ^ 2 b ^ 3 c d ^ 2 ((a d - b
 c) / b) ^ (3 / 2) Sqrt[c + d x] + 3 a b ^ 4 c ^ 2 d ^ 2 (a ^ 2 d - a b c +
 a b d x - b ^ 2 c x) (Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2
a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b
c) ^ 3)] + Sqrt[c + d x]] - Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)]
- 2 a b c d Sqrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d
- b c) ^ 3)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d - b c) ^ 3)] ((a d - b c)
/ b) ^ (3 / 2) / 2 + b ^ 5 c ^ 3 ((a d - b c) / b) ^ (3 / 2) Sqrt[c + d x]
+ b ^ 3 d (a ^ 2 d - a b c + a b d x - b ^ 2 c x) (24 c Sqrt[c + d x] - 9 a
 ^ 2 c d ^ 2 Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b c d Sq
rt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] +
 Sqrt[c + d x]] Sqrt[-1 / (b (a d - b c) ^ 3)] + 9 a ^ 2 c d ^ 2 Log[a ^ 2
d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d Sqrt[-1 / (b (a d - b c) ^
 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + Sqrt[c + d x]] Sqrt[-1
/ (b (a d - b c) ^ 3)] + 4 (c + d x) ^ (3 / 2)) ((a d - b c) / b) ^ (3 / 2)
 / 6 + 6 b ^ 2 c ^ 2 d ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] (a d -
 b c) (a ^ 2 d - a b c + a b d x - b ^ 2 c x) - 3 a b ^ 4 c ^ 2 d ((a d - b
 c) / b) ^ (3 / 2) Sqrt[c + d x] + b ^ 5 c ^ 3 d (a ^ 2 d - a b c + a b d x
 - b ^ 2 c x) (Log[a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] - 2 a b c d S
qrt[-1 / (b (a d - b c) ^ 3)] + b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)]
+ Sqrt[c + d x]] - Log[-a ^ 2 d ^ 2 Sqrt[-1 / (b (a d - b c) ^ 3)] + 2 a b
c d Sqrt[-1 / (b (a d - b c) ^ 3)] - b ^ 2 c ^ 2 Sqrt[-1 / (b (a d - b c) ^
 3)] + Sqrt[c + d x]]) Sqrt[-1 / (b (a d - b c) ^ 3)] ((a d - b c) / b) ^ (
3 / 2) / 2) / (b ^ 5 ((a d - b c) / b) ^ (3 / 2) (a ^ 2 d - a b c + a b d x
 - b ^ 2 c x))
```

Maple [A]
time = 0.19, size = 152, normalized size = 1.38

method	result
derivativedivides	$2d\left(-\frac{-\frac{b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{b^3}+\frac{\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{5\left(a^2d^2-2abcd+b^2c^{2}\ldots\right.}{2\sqrt{\ldots}}}{b^3}\right.$
default	$2d\left(-\frac{-\frac{b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{b^3}+\frac{\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{5\left(a^2d^2-2abcd+b^2c^{2}\ldots\right.}{2\sqrt{\ldots}}}{b^3}\right.$
risch	$-\frac{2d(-bdx+6ad-7bc)\sqrt{dx+c}}{3b^3}-\frac{d^3\sqrt{dx+c}\,a^2}{b^3(bdx+ad)}+\frac{2d^2\sqrt{dx+c}\,ac}{b^2(bdx+ad)}-\frac{d\sqrt{dx+c}\,c^2}{b(bdx+ad)}+\frac{5d^3\arctan\ldots}{b^3\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/b^3*(-1/3*b*(d*x+c)^(3/2)+2*a*d*(d*x+c)^(1/2)-2*b*c*(d*x+c)^(1/2))+
1/b^3*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)
+5/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)
/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 0.31, size = 330, normalized size = 3.00

$$\left[-\frac{15\,(abcd-a^2d^2+(b^2cd-abd^2)x)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\,(2\,b^2d^2x^2-3\,b^2c^2+20\,abcd-15\,a^2d^2+2\,(7\,b^2cd-5\,abd^2)x)\sqrt{dx+c}}{6\,(b^4x+ab^3)},-\frac{15\,(abcd-a^2d^2+(b^2cd-abd^2)x)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,b^2d^2x^2-3\,b^2c^2+20\,abcd-15\,a^2d^2+2\,(7\,b^2cd-5\,abd^2)x)\sqrt{dx+c}}{3\,(b^4x+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="fricas")

[Out] [-1/6*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3), -1/3*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. $2(97) = 194$.
time = 120.02, size = 1312, normalized size = 11.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)/(b*x+a)**2,x)

[Out] -2*a**3*d**4*sqrt(c + d*x)/(2*a**2*b**3*d**2 - 2*a*b**4*c*d + 2*a*b**4*d**2*x - 2*b**5*c*d*x) + a**3*d**4*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**3) - a**3*d**4*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**3) + 6*a**2*c*d**3*sqrt(c + d*x)/(2*a**2*b**2*d**2 - 2*a*b**3*c*d + 2*a*b**3*d**2*x - 2*b**4*c*d*x) - 3*a**2*c*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + 3*a**2*c*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + 6*a**2*d**3*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**4*sqrt(a*d/b - c)) - 6*a*c**2*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) + 3*a*c**2*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - 3*a*c**2*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - 12*a*c*d**2*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**3*sqrt(a*d/b - c)) - 4*a*d**2*sqrt(c + d*x)/b**3 - c**3*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c**3*d*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c +

```
d*x))/2 + 2*c**3*d*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x -
 2*b**2*c*d*x) + 6*c**2*d*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*
d/b - c)) + 4*c*d*sqrt(c + d*x)/b**2 + 2*d*(c + d*x)**(3/2)/(3*b**2)
```

Giac [A]
time = 0.01, size = 216, normalized size = 1.96

$$\frac{\frac{2}{3}\sqrt{c+dx}\,(c+dx)\,db^4-4\sqrt{c+dx}\,d^2b^3a+4\sqrt{c+dx}\,dcb^4}{b^6}+\frac{-\sqrt{c+dx}\,d^3a^2+2\sqrt{c+dx}\,d^2cba-\sqrt{c+dx}\,dc^2b^2}{b^3\left((c+dx)\,b+da-cb\right)}+\frac{\left(10d^3a^2-20d^2cba+10dc^2b^2\right)\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2b^3\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x)
```

```
[Out] 5*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - (sqrt(d*x + c)*b^2*c^2*d - 2*sqrt(d*x
+ c)*a*b*c*d^2 + sqrt(d*x + c)*a^2*d^3)/(((d*x + c)*b - b*c + a*d)*b^3) + 2
/3*((d*x + c)^(3/2)*b^4*d + 6*sqrt(d*x + c)*b^4*c*d - 6*sqrt(d*x + c)*a*b^3
*d^2)/b^6
```

Mupad [B]
time = 0.12, size = 161, normalized size = 1.46

$$\frac{2\,d\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}-\frac{\sqrt{c+d\,x}\,\left(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{b^4\,\left(c+d\,x\right)-b^4\,c+a\,b^3\,d}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{7/2}}+\frac{2\,d\,\left(2\,b^2\,c-2\,a\,b\,d\right)\,\sqrt{c+d\,x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*d*(c + d*x)^(3/2))/(3*b^2) - ((c + d*x)^(1/2)*(a^2*d^3 + b^2*c^2*d - 2*a
*b*c*d^2))/(b^4*(c + d*x) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/2)*d*(a*d -
b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))*(a*d - b*c
)^(3/2))/b^(7/2) + (2*d*(2*b^2*c - 2*a*b*d)*(c + d*x)^(1/2))/b^4
```

3.1407 $\int \frac{(c+dx)^{5/2}}{(a+bx)^3} dx$

Optimal. Leaf size=119

$$\frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} - \frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}$$

```
[Out] -5/4*d*(d*x+c)^(3/2)/b^2/(b*x+a)-1/2*(d*x+c)^(5/2)/b/(b*x+a)^2-15/4*d^2*arc
tanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+15/4*
d^2*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 52, 65, 214}

$$-\frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{15d^2\sqrt{c+dx}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x])/(4*b^3) - (5*d*(c + d*x)^(3/2))/(4*b^2*(a + b*x)) -
(c + d*x)^(5/2)/(2*b*(a + b*x)^2) - (15*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^3}\,dx = -\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx}{4b}$$

$$= -\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d^2)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{8b^2}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d^2(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^3}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d(bc-ad))\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{4b^3}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}-\frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right.}{4b^{7/2}}$$

Mathematica [A]
time = 0.42, size = 119, normalized size = 1.00

$$\frac{\sqrt{c+dx}\,\left(15a^2d^2-5abd(c-5dx)+b^2\left(-2c^2-9cdx+8d^2x^2\right)\right)}{4b^3(a+bx)^2}-\frac{15d^2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (Sqrt[c + d*x]*(15*a^2*d^2 - 5*a*b*d*(c - 5*d*x) + b^2*(-2*c^2 - 9*c*d*x +
8*d^2*x^2)))/(4*b^3*(a + b*x)^2) - (15*d^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^3,x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.20, size = 138, normalized size = 1.16

method	result
derivativedivides	$2d^2\left(\frac{\sqrt{dx+c}}{b^3}-\frac{\frac{\left(-\frac{9}{8}abd+\frac{9}{8}b^2c\right)(dx+c)^{\frac{3}{2}}+\left(-\frac{7}{8}a^2d^2+\frac{7}{4}abcd-\frac{7}{8}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15(ad-bc)\arctan\left(\frac{b\sqrt{dx+}}{\sqrt{(ad-b}}\right.}{8\sqrt{(ad-bc)b}}}{b^3}\right.$
default	$2d^2\left(\frac{\sqrt{dx+c}}{b^3}-\frac{\frac{\left(-\frac{9}{8}abd+\frac{9}{8}b^2c\right)(dx+c)^{\frac{3}{2}}+\left(-\frac{7}{8}a^2d^2+\frac{7}{4}abcd-\frac{7}{8}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15(ad-bc)\arctan\left(\frac{b\sqrt{dx+}}{\sqrt{(ad-b}}\right.}{8\sqrt{(ad-bc)b}}}{b^3}\right.$
risch	$\frac{2d^2\sqrt{dx+c}}{b^3}+\frac{9d^3(dx+c)^{\frac{3}{2}}a}{4b^2(bdx+ad)^2}-\frac{9d^2(dx+c)^{\frac{3}{2}}c}{4b(bdx+ad)^2}+\frac{7d^4\sqrt{dx+c}\,a^2}{4b^3(bdx+ad)^2}-\frac{7d^3\sqrt{dx+c}\,ac}{2b^2(bdx+ad)^2}+\frac{7d^2\sqrt{dx+c}}{4b(bdx+ad)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(1/b^3*(d*x+c)^(1/2)-1/b^3*(((-9/8*a*b*d+9/8*b^2*c)*(d*x+c)^(3/2)+(-7
/8*a^2*d^2+7/4*a*b*c*d-7/8*b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^2+15
/8*(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail

Fricas [A]
time = 0.31, size = 344, normalized size = 2.89

$$\left[\frac{15\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-(9\,b^2cd-25\,abd^2)x)\sqrt{dx+c}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)},\ -\frac{15\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-(9\,b^2cd-25\,abd^2)x)\sqrt{dx+c}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="fricas")

[Out] [1/8*(15*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d
*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(8
*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2
)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(
-(b*c - a*d)/b)/(b*c - a*d)) - (8*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*
a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3)]

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)/(b*x+a)**3,x)

[Out] Timed out

Giac [A]
time = 0.01, size = 213, normalized size = 1.79

$$\frac{2\sqrt{c+dx}\,d^2}{b^3}+\frac{9\sqrt{c+dx}\,(c+dx)\,d^3ba-9\sqrt{c+dx}\,(c+dx)\,d^2cb^2+7\sqrt{c+dx}\,d^4a^2-14\sqrt{c+dx}\,d^3cba+7\sqrt{c+dx}\,d^2c^2b^2}{4b^3\,((c+dx)\,b+da-cb)^2}+\frac{(-15d^3a+15d^2cb)\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2b^3\cdot 2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x)

[Out] 2*sqrt(d*x + c)*d^2/b^3 + 15/4*(b*c*d^2 - a*d^3)*arctan(sqrt(d*x + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/4*(9*(d*x + c)^(3/2)*b^2*
c*d^2 - 7*sqrt(d*x + c)*b^2*c^2*d^2 - 9*(d*x + c)^(3/2)*a*b*d^3 + 14*sqrt(d

```
*x + c)*a*b*c*d^3 - 7*sqrt(d*x + c)*a^2*d^4)/(((d*x + c)*b - b*c + a*d)^2*b
^3)
```

Mupad [B]
time = 0.16, size = 199, normalized size = 1.67

$$\frac{2\,d^2\,\sqrt{c+d\,x}}{b^3}-\frac{\left(\frac{9\,b^2\,c\,d^2}{4}-\frac{9\,a\,b\,d^3}{4}\right)\,{\left(c+d\,x\right)}^{3/2}-\sqrt{c+d\,x}\,\left(\frac{7\,a^2\,d^4}{4}-\frac{7\,a\,b\,c\,d^3}{2}+\frac{7\,b^2\,c^2\,d^2}{4}\right)}{b^5\,{\left(c+d\,x\right)}^2-\left(2\,b^5\,c-2\,a\,b^4\,d\right)\,\left(c+d\,x\right)+b^5\,c^2+a^2\,b^3\,d^2-2\,a\,b^4\,c\,d}-\frac{15\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,d^2\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^3-b\,c\,d^2}\right)\,\sqrt{a\,d-b\,c}}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^3,x)
```

```
[Out] (2*d^2*(c + d*x)^(1/2))/b^3 - (((9*b^2*c*d^2)/4 - (9*a*b*d^3)/4)*(c + d*x)^
(3/2) - (c + d*x)^(1/2)*((7*a^2*d^4)/4 + (7*b^2*c^2*d^2)/4 - (7*a*b*c*d^3)/
2))/(b^5*(c + d*x)^2 - (2*b^5*c - 2*a*b^4*d)*(c + d*x) + b^5*c^2 + a^2*b^3*
d^2 - 2*a*b^4*c*d) - (15*d^2*atan((b^(1/2)*d^2*(a*d - b*c)^(1/2)*(c + d*x)^
(1/2))/(a*d^3 - b*c*d^2))*(a*d - b*c)^(1/2))/(4*b^(7/2))
```

3.1408 $\int \frac{(c+dx)^{5/2}}{(a+bx)^4} \, dx$

Optimal. Leaf size=126

$$-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}$$

```
[Out] -5/12*d*(d*x+c)^(3/2)/b^2/(b*x+a)^2-1/3*(d*x+c)^(5/2)/b/(b*x+a)^3-5/8*d^3*a
rctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(1/2)-5/8
*d^2*(d*x+c)^(1/2)/b^3/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {43, 65, 214}

$$-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(8*b^3*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (c + d*x)^(5/2)/(3*b*(a + b*x)^3) - (5*d^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx}{6b}\\
&= -\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{8b^2}\\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^3\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^3}\\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b^3}\\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A]
time = 0.53, size = 119, normalized size = 0.94

$$
-\frac{\sqrt{c+dx}\,\left(15a^2d^2+10abd(c+4dx)+b^2\left(8c^2+26cdx+33d^2x^2\right)\right)}{24b^3(a+bx)^3}+\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8b^{7/2}\sqrt{-bc+ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 26*c
*d*x + 33*d^2*x^2)))/(b^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*
x])/Sqrt[-(b*c) + a*d]])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^4,x]')
```

[Out] Timed out

Maple [A]
time = 0.16, size = 130, normalized size = 1.03

method	result
derivativedivides	$2d^{3}\left(\frac{-\frac{11(dx+c)^{\frac{5}{2}}}{16b}-\frac{5(ad-bc)(dx+c)^{\frac{3}{2}}}{6b^{2}}-\frac{5\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\sqrt{dx+c}}{16b^{3}}}{\left((dx+c)b+ad-bc\right)^{3}}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16b^{3}\sqrt{(ad-bc)\,b}}\right)$
default	$2d^{3}\left(\frac{-\frac{11(dx+c)^{\frac{5}{2}}}{16b}-\frac{5(ad-bc)(dx+c)^{\frac{3}{2}}}{6b^{2}}-\frac{5\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\sqrt{dx+c}}{16b^{3}}}{\left((dx+c)b+ad-bc\right)^{3}}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16b^{3}\sqrt{(ad-bc)\,b}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*((-11/16*(d*x+c)^(5/2)/b-5/6*(a*d-b*c)/b^2*(d*x+c)^(3/2)-5/16/b^3*(a^
2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^3+5/16/b^3/((a*
d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(102) = 204.
time = 0.31, size = 563, normalized size = 4.47

$$\left[\frac{15\,(b^{3}d^{3}x^{3}+3\,ab^{2}d^{3}x^{2}+3\,a^{2}bd^{3}x+a^{3}d^{3})\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^{2}c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(8\,b^{4}c^{3}+2\,ab^{3}c^{2}d+5\,a^{2}b^{2}cd^{2}-15\,a^{3}bd^{3}+33\,(b^{4}cd^{2}-ab^{3}d^{3})x^{2}+2\,(13\,b^{4}c^{2}d+7\,ab^{3}cd^{2}-20\,a^{2}b^{2}d^{3})x)\sqrt{dx+c}}{48\,(a^{3}b^{5}c-a^{4}b^{4}d+(b^{8}c-ab^{7}d)x^{3}+3\,(ab^{7}c-a^{2}b^{6}d)x^{2}+3\,(a^{2}b^{6}c-a^{3}b^{5}d)x)},\ \frac{15\,(b^{3}d^{3}x^{3}+3\,ab^{2}d^{3}x^{2}+3\,a^{2}bd^{3}x+a^{3}d^{3})\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{-b^{2}c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(8\,b^{4}c^{3}+2\,ab^{3}c^{2}d+5\,a^{2}b^{2}cd^{2}-15\,a^{3}bd^{3}+33\,(b^{4}cd^{2}-ab^{3}d^{3})x^{2}+2\,(13\,b^{4}c^{2}d+7\,ab^{3}cd^{2}-20\,a^{2}b^{2}d^{3})x)\sqrt{dx+c}}{24\,(a^{3}b^{5}c-a^{4}b^{4}d+(b^{8}c-ab^{7}d)x^{3}+3\,(ab^{7}c-a^{2}b^{6}d)x^{2}+3\,(a^{2}b^{6}c-a^{3}b^{5}d)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) - 2*(8*b^4*c^3 + 2*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^
3 + 33*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^
3 + 3*(a*b^7*c - a^2*b^6*d)*x^2 + 3*(a^2*b^6*c - a^3*b^5*d)*x), 1/24*(15*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*d
)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3 + 2
*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^3 + 33*(b^4*c*d^2 - a*b^3*d^3)*
x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^3 + 3*(a*b^7*c - a^2*b^6*d)*x^2
 + 3*(a^2*b^6*c - a^3*b^5*d)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 210, normalized size = 1.67

$$\frac{-33\sqrt{c+dx}\,(c+dx)^2d^3b^2-40\sqrt{c+dx}\,(c+dx)\,d^4ba+40\sqrt{c+dx}\,(c+dx)\,d^3cb^2-15\sqrt{c+dx}\,d^5a^2+30\sqrt{c+dx}\,d^4cba-15\sqrt{c+dx}\,d^3c^2b^2}{24b^3\left((c+dx)\,b+da-cb\right)^3}+\frac{5d^3\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{4b^3\cdot 2\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x)
```

```
[Out] 5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^3) - 1/24*(33*(d*x + c)^(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 15
*sqrt(d*x + c)*b^2*c^2*d^3 + 40*(d*x + c)^(3/2)*a*b*d^4 - 30*sqrt(d*x + c)*
a*b*c*d^4 + 15*sqrt(d*x + c)*a^2*d^5)/(((d*x + c)*b - b*c + a*d)^3*b^3)
```

Mupad [B]
time = 0.36, size = 222, normalized size = 1.76

$$\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{7/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{11\,d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,b}+\frac{5\,d^3\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,b^3}+\frac{5\,d^3\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^4,x)
```

```
[Out] (5*d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(7/2)*(a*d -
 b*c)^(1/2)) - ((11*d^3*(c + d*x)^(5/2))/(8*b) + (5*d^3*(c + d*x)^(1/2)*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d))/(8*b^3) + (5*d^3*(a*d - b*c)*(c + d*x)^(3/2))
/(3*b^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x
)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)
```

3.1409 $\int \frac{(c+dx)^{5/2}}{(a+bx)^5} dx$

Optimal. Leaf size=162

$$-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}$$

```
[Out] -5/24*d*(d*x+c)^(3/2)/b^2/(b*x+a)^3-1/4*(d*x+c)^(5/2)/b/(b*x+a)^4+5/64*d^4*
arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(3/2)-5/
32*d^2*(d*x+c)^(1/2)/b^3/(b*x+a)^2-5/64*d^3*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b
*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)}-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(32*b^3*(a + b*x)^2) - (5*d^3*Sqrt[c + d*x])/(64*b^3
*(b*c - a*d)*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(24*b^2*(a + b*x)^3) - (c +
 d*x)^(5/2)/(4*b*(a + b*x)^4) + (5*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[b*c - a*d]])/(64*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx &= -\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{(5d)\int\frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx}{8b}\\
&= -\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{(5d^2)\int\frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{16b^2}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{(5d^3)\int\frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^3}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}-\frac{(5d^4)\int\frac{\cdots}{(a+\cdots}}{128b^3\cdots}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}-\frac{(5d^3)\,\mathrm{Subs}\cdots}{\cdots}\\
&= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{5d^4\tanh^{-1}\cdots}{64b^{7/2}\cdots}\end{aligned}$$

Mathematica [A]
time = 0.90, size = 172, normalized size = 1.06

$$\frac{\sqrt{c+dx}\,\left(15a^3d^3+5a^2bd^2(2c+11dx)+ab^2d\left(8c^2+36cdx+73d^2x^2\right)-b^3\left(48c^3+136c^2dx+118cd^2x^2+15d^3x^3\right)\right)}{192b^3(bc-ad)(a+bx)^4}+\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{64b^{7/2}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (Sqrt[c + d*x]*(15*a^3*d^3 + 5*a^2*b*d^2*(2*c + 11*d*x) + a*b^2*d*(8*c^2 +
36*c*d*x + 73*d^2*x^2) - b^3*(48*c^3 + 136*c^2*d*x + 118*c*d^2*x^2 + 15*d^3
```

```
*x^3)))/(192*b^3*(b*c - a*d)*(a + b*x)^4) + (5*d^4*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[-(b*c) + a*d]])/(64*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^5,x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 159, normalized size = 0.98

method	result
derivativedivides	$2d^4\left(\frac{\frac{5(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{73(dx+c)^{\frac{5}{2}}}{384b}-\frac{55(ad-bc)(dx+c)^{\frac{3}{2}}}{384b^2}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{128b^3}}{((dx+c)b+ad-bc)^4}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{128(ad-bc)b^3\sqrt{(ad-bc)b}}\right)$
default	$2d^4\left(\frac{\frac{5(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{73(dx+c)^{\frac{5}{2}}}{384b}-\frac{55(ad-bc)(dx+c)^{\frac{3}{2}}}{384b^2}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{128b^3}}{((dx+c)b+ad-bc)^4}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{128(ad-bc)b^3\sqrt{(ad-bc)b}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 2*d^4*((5/128/(a*d-b*c)*(d*x+c)^(7/2)-73/384*(d*x+c)^(5/2)/b-55/384*(a*d-b*
c)/b^2*(d*x+c)^(3/2)-5/128/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(1/2))/(
(d*x+c)*b+a*d-b*c)^4+5/128/(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+
c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(134) = 268$.
time = 0.31, size = 894, normalized size = 5.52

$$\left[-\frac{15\,(b^4d^4x^4+4\,ab^3d^4x^3+6\,a^2b^2d^4x^2+4\,a^3bd^4x+a^4d^4)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(48\,b^5c^4-56\,ab^4c^3d-2\,a^2b^3c^2d^2-5\,a^3b^2cd^3+15\,a^4bd^4+15\,(b^5cd^3-ab^4d^4)x^3+(118\,b^5c^2d^2-191\,ab^4cd^3+73\,a^2b^3d^4)x^2+(136\,b^5c^3d-172\,ab^4c^2d^2-19\,a^2b^3cd^3+55\,a^3b^2d^4)x)\sqrt{dx+c}}{384\,(a^4b^6c^2-2\,a^5b^5cd+a^6b^4d^2+(b^{10}c^2-2\,ab^9cd+a^2b^8d^2)x^4+4\,(ab^9c^2-2\,a^2b^8cd+a^3b^7d^2)x^3+6\,(a^2b^8c^2-2\,a^3b^7cd+a^4b^6d^2)x^2+4\,(a^3b^7c^2-2\,a^4b^6cd+a^5b^5d^2)x)},\ -\frac{15\,(b^4d^4x^4+4\,ab^3d^4x^3+6\,a^2b^2d^4x^2+4\,a^3bd^4x+a^4d^4)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(48\,b^5c^4-56\,ab^4c^3d-2\,a^2b^3c^2d^2-5\,a^3b^2cd^3+15\,a^4bd^4+15\,(b^5cd^3-ab^4d^4)x^3+(118\,b^5c^2d^2-191\,ab^4cd^3+73\,a^2b^3d^4)x^2+(136\,b^5c^3d-172\,ab^4c^2d^2-19\,a^2b^3cd^3+55\,a^3b^2d^4)x)\sqrt{dx+c}}{192\,(a^4b^6c^2-2\,a^5b^5cd+a^6b^4d^2+(b^{10}c^2-2\,ab^9cd+a^2b^8d^2)x^4+4\,(ab^9c^2-2\,a^2b^8cd+a^3b^7d^2)x^3+6\,(a^2b^8c^2-2\,a^3b^7cd+a^4b^6d^2)x^2+4\,(a^3b^7c^2-2\,a^4b^6cd+a^5b^5d^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a^2
*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*
x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5*c
^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x +
 c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d +
 a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(a^
2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6*c
*d + a^5*b^5*d^2)*x), -1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
 + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a
^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4
)*x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5
*c^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d
 + a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(
a^2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6
*c*d + a^5*b^5*d^2)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.01, size = 332, normalized size = 2.05

$$\frac{15\sqrt{c+dx}\,(c+dx)^3d^4b^3-73\sqrt{c+dx}\,(c+dx)^2d^5b^2a+73\sqrt{c+dx}\,(c+dx)^2d^4cb^3-55\sqrt{c+dx}\,(c+dx)\,d^6ba^2+110\sqrt{c+dx}\,(c+dx)\,d^5cb^2a-55\sqrt{c+dx}\,(c+dx)\,d^4c^2b^3-15\sqrt{c+dx}\,d^7a^3+45\sqrt{c+dx}\,d^6cba^2-45\sqrt{c+dx}\,d^5c^2b^2a+15\sqrt{c+dx}\,d^4c^3b^3}{(192db^3a-192cb^4)\,((c+dx)\,b+da-cb)^4}+\frac{5d^4\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(32db^3a-32cb^4)\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x)
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c - a*b^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x + c)^(7/2)*b^3*d^4 + 73*(d*x + c)^(5/
2)*b^3*c*d^4 - 55*(d*x + c)^(3/2)*b^3*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^
4 - 73*(d*x + c)^(5/2)*a*b^2*d^5 + 110*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqr
t(d*x + c)*a*b^2*c^2*d^5 - 55*(d*x + c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*
a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^7)/((b^4*c - a*b^3*d)*((d*x + c)*b - b
*c + a*d)^4)
```

Mupad [B]

time = 0.41, size = 309, normalized size = 1.91

$$\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{73\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,b}-\frac{5\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{5\,d^4\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{64\,b^3}+\frac{55\,d^4\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{192\,b^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^5,x)
```

```
[Out] (5*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(7/2)*(a*d
- b*c)^(3/2)) - ((73*d^4*(c + d*x)^(5/2))/(192*b) - (5*d^4*(c + d*x)^(7/2))
/(64*(a*d - b*c)) + (5*d^4*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
/(64*b^3) + (55*d^4*(a*d - b*c)*(c + d*x)^(3/2))/(192*b^2))/(b^4*(c + d*x)^
4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3
+ 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b
^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)
```

3.1410 $\int \frac{(c+dx)^{5/2}}{(a+bx)^6} dx$

Optimal. Leaf size=198

$$-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}-\frac{3d^5\tanh^{-1}}{128b}$$

```
[Out] -1/8*d*(d*x+c)^(3/2)/b^2/(b*x+a)^4-1/5*(d*x+c)^(5/2)/b/(b*x+a)^5-3/128*d^5*
arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(5/2)-1/
16*d^2*(d*x+c)^(1/2)/b^3/(b*x+a)^3-1/64*d^3*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b
*x+a)^2+3/128*d^4*(d*x+c)^(1/2)/b^3/(-a*d+b*c)^2/(b*x+a)
```

Rubi [A]
time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}+\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -1/16*(d^2*Sqrt[c + d*x])/(b^3*(a + b*x)^3) - (d^3*Sqrt[c + d*x])/(64*b^3*(
b*c - a*d)*(a + b*x)^2) + (3*d^4*Sqrt[c + d*x])/(128*b^3*(b*c - a*d)^2*(a +
 b*x)) - (d*(c + d*x)^(3/2))/(8*b^2*(a + b*x)^4) - (c + d*x)^(5/2)/(5*b*(a
+ b*x)^5) - (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(128*b
^(7/2)*(b*c - a*d)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx}{2b}$$

$$= -\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{(3d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{16b^2}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^3}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} - \frac{(3d^4)\int \frac{}{(a+bx}}{128b^3(}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} -$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} -$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} -$$

Mathematica [A]
time = 1.21, size = 222, normalized size = 1.12

$$-\frac{\sqrt{c+dx}\,\left(15a^4d^4+10a^3bd^3(c+7dx)+2a^2b^2d^2\left(4c^2+23cdx+64d^2x^2\right)-2ab^3d\left(88c^3+256c^2dx+233cd^2x^2+35d^3x^3\right)+b^4\left(128c^4+336c^3dx+248c^2d^2x^2+10cd^3x^3-15d^4x^4\right)\right)}{640b^3(bc-ad)^2(a+bx)^5}+\frac{3d^5\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{128b^{7/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -1/640*(Sqrt[c + d*x]*(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 2*a^2*b^2*d^
2*(4*c^2 + 23*c*d*x + 64*d^2*x^2) - 2*a*b^3*d*(88*c^3 + 256*c^2*d*x + 233*c
*d^2*x^2 + 35*d^3*x^3) + b^4*(128*c^4 + 336*c^3*d*x + 248*c^2*d^2*x^2 + 10*
c*d^3*x^3 - 15*d^4*x^4)))/(b^3*(b*c - a*d)^2*(a + b*x)^5) + (3*d^5*ArcTan[(
Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(128*b^(7/2)*(-(b*c) + a*d)^(5/
2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^6,x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.16, size = 205, normalized size = 1.04

method	result
derivativedivides	$2d^5\left(\frac{\frac{3b(dx+c)^{\frac{9}{2}}}{256\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{7(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{(dx+c)^{\frac{5}{2}}}{10b}-\frac{7(ad-bc)(dx+c)^{\frac{3}{2}}}{128b^2}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{256b^3}}{((dx+c)b+ad-bc)^5}+\frac{}{256b^3}\right.$
default	$2d^5\left(\frac{\frac{3b(dx+c)^{\frac{9}{2}}}{256\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{7(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{(dx+c)^{\frac{5}{2}}}{10b}-\frac{7(ad-bc)(dx+c)^{\frac{3}{2}}}{128b^2}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{256b^3}}{((dx+c)b+ad-bc)^5}+\frac{}{256b^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^5*((3/256*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(9/2)+7/128/(a*d-b*c)*(
d*x+c)^(7/2)-1/10*(d*x+c)^(5/2)/b-7/128*(a*d-b*c)/b^2*(d*x+c)^(3/2)-3/256/b
^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^5+3/256/b
^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/(
(a*d-b*c)*b)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(166) = 332$.
time = 0.32, size = 1337, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^
2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c
 - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(128*b^6*c^5 -
 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^
4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 10*(b^6*c^2*d^3 - 8*a*b
^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3*d^2 - 357*a*b^5*c^2*d^3 + 29
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168*b^6*c^4*d - 424*a*b^5*c^3*d^
2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 35*a^4*b^2*d^5)*x)*sqrt(d*x +
c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5*c*d^2 - a^8*b^4*d^3 + (b^12*
c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*b^9*d^3)*x^5 + 5*(a*b^11*c^3
- 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8*d^3)*x^4 + 10*(a^2*b^10*c^3
- 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*d^3)*x^3 + 10*(a^3*b^9*c^3 -
3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^3)*x^2 + 5*(a^4*b^8*c^3 - 3*a
^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*x), 1/640*(15*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
 a^5*d^5)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b
*d*x + b*c)) - (128*b^6*c^5 - 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*
x^4 + 10*(b^6*c^2*d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3
*d^2 - 357*a*b^5*c^2*d^3 + 297*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168
*b^6*c^4*d - 424*a*b^5*c^3*d^2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 3
5*a^4*b^2*d^5)*x)*sqrt(d*x + c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5
*c*d^2 - a^8*b^4*d^3 + (b^12*c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*
b^9*d^3)*x^5 + 5*(a*b^11*c^3 - 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8
*d^3)*x^4 + 10*(a^2*b^10*c^3 - 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*
d^3)*x^3 + 10*(a^3*b^9*c^3 - 3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^
3)*x^2 + 5*(a^4*b^8*c^3 - 3*a^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*
x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(166) = 332.
time = 0.02, size = 494, normalized size = 2.49

$$\frac{15\sqrt{c+dx}\,(c+dx)^4d^5b^4+70\sqrt{c+dx}\,(c+dx)^3d^6b^3a-70\sqrt{c+dx}\,(c+dx)^3d^5cb^4-128\sqrt{c+dx}\,(c+dx)^2d^7b^2a^2+256\sqrt{c+dx}\,(c+dx)^2d^6cb^3a-128\sqrt{c+dx}\,(c+dx)^2d^5c^2b^4-70\sqrt{c+dx}\,(c+dx)d^8ba^3+210\sqrt{c+dx}\,(c+dx)d^7cb^2a^2-210\sqrt{c+dx}\,(c+dx)d^6c^2b^3a+70\sqrt{c+dx}\,(c+dx)d^5c^3b^4-15\sqrt{c+dx}\,d^9a^4+60\sqrt{c+dx}\,d^8cba^3-90\sqrt{c+dx}\,d^7c^2b^2a^2+60\sqrt{c+dx}\,d^6c^3b^3a-15\sqrt{c+dx}\,d^5c^4b^4}{(640d^2b^3a^2-1280dcb^4a+640c^2b^5)\left((c+dx)b+da-cb\right)^5}+\frac{3d^5\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(64d^2b^3a^2-128dcb^4a+64c^2b^5)\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^2 - 2*a*b^4*
c*d + a^2*b^3*d^2)*sqrt(-b^2*c + a*b*d)) + 1/640*(15*(d*x + c)^(9/2)*b^4*d^
5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 - 128*(d*x + c)^(5/2)*b^4*c^2*d^5 + 70*(d*
x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^5 + 70*(d*x + c)^(7/2
)*a*b^3*d^6 + 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d*x + c)^(3/2)*a*b^3*c
^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 - 128*(d*x + c)^(5/2)*a^2*b^2*d^7 +
 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*a^2*b^2*c^2*d^7 - 70*
(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d^8 - 15*sqrt(d*x + c)
*a^4*d^9)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*((d*x + c)*b - b*c + a*d)^
5)
```

Mupad [**B**]
time = 0.50, size = 411, normalized size = 2.08

$$\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{d^5\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{7\,d^5\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^5\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{128\,b^3}+\frac{7\,d^5\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{64\,b^2}-\frac{3\,b\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^6,x)
```

```
[Out] (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(7/2)*(a*d
 - b*c)^(5/2)) - ((d^5*(c + d*x)^(5/2))/(5*b) - (7*d^5*(c + d*x)^(7/2))/(64
*(a*d - b*c)) + (3*d^5*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12
8*b^3) + (7*d^5*(a*d - b*c)*(c + d*x)^(3/2))/(64*b^2) - (3*b*d^5*(c + d*x)^
(9/2))/(128*(a*d - b*c)^2))/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10
*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(
c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 -
 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30
*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^
3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
```

3.1411 $\int \frac{\sqrt{-1+x}}{(1+x)^2} dx$

Optimal. Leaf size=35

$$-\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-(-1+x)^(1/2)/(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 209}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx &= -\frac{\sqrt{-1+x}}{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx\\ &= -\frac{\sqrt{-1+x}}{1+x}+\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\ &= -\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.00

$$-\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.83, size = 87, normalized size = 2.49

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-\sqrt{\frac{1-x}{1+x}}+\frac{\text{ArcCosh}\left[\frac{\sqrt{2}}{\sqrt{1+x}}\right]\sqrt{2+2x}}{2}\right)}{\sqrt{1+x}},\frac{1}{\text{Abs}\left[1+x\right]}>\frac{1}{2}\right\}\right\},-\frac{\sqrt{1-\frac{2}{1+x}}}{\sqrt{1+x}}-\frac{\sqrt{2}\,\text{ArcSin}\left[\frac{\sqrt{2}}{\sqrt{1+x}}\right]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[-1 + x]/(1 + x)^2,x]')
```

```
[Out] Piecewise[{{I (-Sqrt[(1 - x) / (1 + x)] + ArcCosh[Sqrt[2] / Sqrt[1 + x]] Sq
rt[2 + 2 x] / 2) / Sqrt[1 + x], 1 / Abs[1 + x] > 1 / 2}}, -Sqrt[1 - 2 / (1
+ x)] / Sqrt[1 + x] - Sqrt[2] ArcSin[Sqrt[2] / Sqrt[1 + x]] / 2]
```

Maple [A]
time = 0.17, size = 30, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\sqrt{-1+x}}{1+x}$	30
default	$\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\sqrt{-1+x}}{1+x}$	30
risch	$\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\sqrt{-1+x}}{1+x}$	30
trager	$-\frac{\sqrt{-1+x}}{1+x}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+2\right)x+4\sqrt{-1+x}+3\,\mathrm{RootOf}\left(\_Z^2+2\right)}{1+x}\right)}{4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-(-1+x)^(1/2)/(1+x)
```

Maxima [A]

time = 0.35, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

Fricas [A]

time = 0.30, size = 33, normalized size = 0.94

$$\frac{\sqrt{2}\,(x+1)\arctan\left(\frac{1}{2}\sqrt{2}\,\sqrt{x-1}\right)-2\sqrt{x-1}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - 2*sqrt(x - 1))/(x +
1)
```

Sympy [A]
time = 0.75, size = 105, normalized size = 3.00

$$\begin{cases} \frac{\sqrt{2}\, i \operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} + \frac{i}{\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}} - \frac{2i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{\sqrt{1-\frac{2}{x+1}}}{\sqrt{x+1}} - \frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**2,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/2 + I/(sqrt(-1 + 2/(x + 1))
*sqrt(x + 1)) - 2*I/(sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)), 1/Abs(x + 1) > 1
/2), (-sqrt(1 - 2/(x + 1))/sqrt(x + 1) - sqrt(2)*asin(sqrt(2)/sqrt(x + 1))/
2, True))
```

Giac [A]
time = 0.00, size = 36, normalized size = 1.03

$$-\frac{\sqrt{x-1}}{x+1} + \frac{\arctan\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x)
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

Mupad [B]
time = 0.06, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}\ \operatorname{atan}\left(\frac{\sqrt{2}\ \sqrt{x-1}}{2}\right)}{2} - \frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^2,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/2 - (x - 1)^(1/2)/(x + 1)
```

3.1412 $\int \frac{\sqrt{-1+x}}{(1+x)^3}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/16*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-1/2*(-1+x)^(1/2)/(1+x)^2+1/8*
(-1+x)^(1/2)/(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 209}

$$\frac{\sqrt{x-1}}{8(x+1)}-\frac{\sqrt{x-1}}{2(x+1)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] -1/2*Sqrt[-1 + x]/(1 + x)^2 + Sqrt[-1 + x]/(8*(1 + x)) + ArcTan[Sqrt[-1 + x
]/Sqrt[2]]/(8*Sqrt[2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}}{(1+x)^3}\,dx &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,(1+x)^2}\,dx\\
&= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{16}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx\\
&= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.77

$$\frac{(-3+x)\sqrt{-1+x}}{8(1+x)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] ((-3 + x)*Sqrt[-1 + x])/(8*(1 + x)^2) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/(8*Sqr
t[2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.94, size = 165, normalized size = 2.95

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(-16\,(1+x)\,(1-x)+2\,(-1+x)\,(1+x)^3+12\,(1-x)\,(1+x)^2+\sqrt{2}\,\text{ArcCosh}\left[\frac{\sqrt{2}}{\sqrt{1+x}}\right](1+x)^{\frac{9}{2}}\left(\frac{1-x}{1+x}\right)^{\frac{3}{2}}\right)}{16\,(1+x)^{\frac{9}{2}}\left(\frac{1-x}{1+x}\right)^{\frac{3}{2}}},\frac{1}{\text{Abs}\,[1+x]}>\frac{1}{2}\right\}\right\},\frac{-3}{4\,(1+x)^{\frac{3}{2}}\sqrt{1-\frac{2}{1+x}}}-\frac{\sqrt{2}\,\text{ArcSin}\left[\frac{\sqrt{2}}{\sqrt{1+x}}\right]}{16}+\frac{1}{8\sqrt{1+x}\sqrt{1-\frac{2}{1+x}}}+\frac{1}{(1+x)^{\frac{5}{2}}\sqrt{1-\frac{2}{1+x}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[Sqrt[-1 + x]/(1 + x)^3,x]')
```

```
[Out] Piecewise[{{I / 16 (-16 (1 + x) (1 - x) + 2 (-1 + x) (1 + x) ^ 3 + 12 (1 -
x) (1 + x) ^ 2 + Sqrt[2] ArcCosh[Sqrt[2] / Sqrt[1 + x]] (1 + x) ^ (9 / 2) (
(1 - x) / (1 + x)) ^ (3 / 2)) / ((1 + x) ^ (9 / 2) ((1 - x) / (1 + x)) ^ (3
 / 2)), 1 / Abs[1 + x] > 1 / 2}}, -3 / (4 (1 + x) ^ (3 / 2) Sqrt[1 - 2 / (1
 + x)]) - Sqrt[2] ArcSin[Sqrt[2] / Sqrt[1 + x]] / 16 + 1 / (8 Sqrt[1 + x] S
qrt[1 - 2 / (1 + x)]) + 1 / ((1 + x) ^ (5 / 2) Sqrt[1 - 2 / (1 + x)])]
```

Maple [A]
time = 0.20, size = 40, normalized size = 0.71

method	result	size
risch	$\frac{x^2-4x+3}{8(1+x)^2\sqrt{-1+x}}+\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
derivativedivides	$\frac{\frac{(-1+x)^{\frac{3}{2}}}{8}-\frac{\sqrt{-1+x}}{4}}{(1+x)^2}+\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{16}$	40
default	$\frac{\frac{(-1+x)^{\frac{3}{2}}}{8}-\frac{\sqrt{-1+x}}{4}}{(1+x)^2}+\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{16}$	40
trager	$\frac{(-3+x)\sqrt{-1+x}}{8(1+x)^2}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+2\right)x+4\sqrt{-1+x}+3\,\mathrm{RootOf}\left(\_Z^2+2\right)}{1+x}\right)}{32}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/16*(-1+x)^(3/2)-1/8*(-1+x)^(1/2))/(1+x)^2+1/16*arctan(1/2*(-1+x)^(1/2)
*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.37, size = 43, normalized size = 0.77

$$\frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\,\sqrt{x-1}}{8\,\left((x-1)^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/((x - 1)^2 + 4*x)
```

Fricas [A]
time = 0.31, size = 46, normalized size = 0.82

$$\frac{\sqrt{2}\,(x^2+2\,x+1)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+2\,\sqrt{x-1}\,(x-3)}{16\,(x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 + 2*x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x -
1)*(x - 3))/(x^2 + 2*x + 1)
```

Sympy [A]
time = 1.62, size = 168, normalized size = 3.00

$$\begin{cases} \frac{\sqrt{2}\,i\,\operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} - \frac{i}{8\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}} + \frac{3i}{4\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} - \frac{i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{5}{2}}} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{\sqrt{2}\,\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} + \frac{1}{8\sqrt{1-\frac{2}{x+1}}\,\sqrt{x+1}} - \frac{3}{4\sqrt{1-\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} + \frac{1}{\sqrt{1-\frac{2}{x+1}}\,(x+1)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**3,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/16 - I/(8*sqrt(-1 + 2/(x +
1))*sqrt(x + 1)) + 3*I/(4*sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)) - I/(sqrt(-1
 + 2/(x + 1))*(x + 1)**(5/2)), 1/Abs(x + 1) > 1/2), (-sqrt(2)*asin(sqrt(2)/
sqrt(x + 1))/16 + 1/(8*sqrt(1 - 2/(x + 1))*sqrt(x + 1)) - 3/(4*sqrt(1 - 2/(
x + 1))*(x + 1)**(3/2)) + 1/(sqrt(1 - 2/(x + 1))*(x + 1)**(5/2)), True))
```

Giac [A]
time = 0.00, size = 56, normalized size = 1.00

$$\frac{\sqrt{x-1}\,(x-1)-2\sqrt{x-1}}{8\,(x+1)^2}+\frac{\arctan\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x)
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/(x + 1)^2
```

Mupad [B]
time = 0.04, size = 45, normalized size = 0.80

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \sqrt{x-1}}{2}\right)}{16}-\frac{\frac{\sqrt{x-1}}{4}-\frac{(x-1)^{3/2}}{8}}{4\,x+(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^3,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/16 - ((x - 1)^(1/2)/4 - (x - 1)^(
3/2)/8)/(4*x + (x - 1)^2)
```

3.1413 $\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=154

$$-\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{7/2}}{7d^6}-1$$

```
[Out] 10/3*b*(-a*d+b*c)^4*(d*x+c)^(3/2)/d^6-4*b^2*(-a*d+b*c)^3*(d*x+c)^(5/2)/d^6+
20/7*b^3*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^6-10/9*b^4*(-a*d+b*c)*(d*x+c)^(9/2)/d
^6+2/11*b^5*(d*x+c)^(11/2)/d^6-2*(-a*d+b*c)^5*(d*x+c)^(1/2)/d^6
```

Rubi [A]
time = 0.03, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+dx}\,(bc-ad)^5}{d^6}+\frac{2b^5(c+dx)^{11/2}}{11d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*Sqrt[c + d*x])/d^6 + (10*b*(b*c - a*d)^4*(c + d*x)^(3/2))
/(3*d^6) - (4*b^2*(b*c - a*d)^3*(c + d*x)^(5/2))/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(7/2))/(7*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(9/2))/(9*d^6) +
 (2*b^5*(c + d*x)^(11/2))/(11*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5\sqrt{c+dx}}+\frac{5b(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{3/2}}{d^5}+\frac{10b^3(bc-ad)^2}{d^5}\right.\\ &= -\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(b}{}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 216, normalized size = 1.40

$$\frac{2\sqrt{c+dx}\,(693a^5d^5+1155a^4bd^4(-2c+dx)+462a^3b^2d^3(8c^2-4cdx+3d^2x^2)+198a^2b^3d^2(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3)+11ab^4d(128c^4-64c^3dx+48c^2d^2x^2-40cd^3x^3+35d^4x^4)+b^5(-256c^5+128c^4dx-96c^3d^2x^2+80c^2d^3x^3-70cd^4x^4+63d^5x^5))}{693d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(693*a^5*d^5 + 1155*a^4*b*d^4*(-2*c + d*x) + 462*a^3*b^2*d
^3*(8*c^2 - 4*c*d*x + 3*d^2*x^2) + 198*a^2*b^3*d^2*(-16*c^3 + 8*c^2*d*x - 6
*c*d^2*x^2 + 5*d^3*x^3) + 11*a*b^4*d*(128*c^4 - 64*c^3*d*x + 48*c^2*d^2*x^2
 - 40*c*d^3*x^3 + 35*d^4*x^4) + b^5*(-256*c^5 + 128*c^4*d*x - 96*c^3*d^2*x^
2 + 80*c^2*d^3*x^3 - 70*c*d^4*x^4 + 63*d^5*x^5)))/(693*d^6)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 45.97, size = 395, normalized size = 2.56

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(693a^5cd^5+693a^5d^6x-2310a^4bc^2d^4-1155a^4bcd^5x+1155a^4bd^6x^2+3696a^3b^2c^3d^3+1848a^3b^2c^2d^4x-462a^3b^2cd^5x^2+1386a^3b^2d^6x^3-3168a^2b^3c^4d^2-1584a^2b^3c^3d^3x+396a^2b^3c^2d^4x^2-198a^2b^3cd^5x^3+990a^2b^3d^6x^4+1408ab^4c^5d+704ab^4c^4d^2x-176ab^4c^3d^3x^2+88ab^4c^2d^4x^3-55ab^4cd^5x^4+385ab^4d^6x^5-256b^5c^6-128b^5c^5dx+32b^5c^4d^2x^2-16b^5c^3d^3x^3+10b^5c^2d^4x^4-7b^5cd^5x^5+63b^5d^6x^6\right)}{693d^6\sqrt{c+dx}},d!=0\right\}\right\},\frac{\text{Piecewise}\left[\left\{\left\{a^5x,b==0\right\},\left\{\frac{(a+bx)^6}{6b},\text{True}\right\}\right\}\right]}{\sqrt{c}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x)^(1/2),x]')
```

```
[Out] Piecewise[{{2 (693 a ^ 5 c d ^ 5 + 693 a ^ 5 d ^ 6 x - 2310 a ^ 4 b c ^ 2 d
 ^ 4 - 1155 a ^ 4 b c d ^ 5 x + 1155 a ^ 4 b d ^ 6 x ^ 2 + 3696 a ^ 3 b ^ 2
 c ^ 3 d ^ 3 + 1848 a ^ 3 b ^ 2 c ^ 2 d ^ 4 x - 462 a ^ 3 b ^ 2 c d ^ 5 x ^
 2 + 1386 a ^ 3 b ^ 2 d ^ 6 x ^ 3 - 3168 a ^ 2 b ^ 3 c ^ 4 d ^ 2 - 1584 a ^
 2 b ^ 3 c ^ 3 d ^ 3 x + 396 a ^ 2 b ^ 3 c ^ 2 d ^ 4 x ^ 2 - 198 a ^ 2 b ^
3 c d ^ 5 x ^ 3 + 990 a ^ 2 b ^ 3 d ^ 6 x ^ 4 + 1408 a b ^ 4 c ^ 5 d + 704
a b ^ 4 c ^ 4 d ^ 2 x - 176 a b ^ 4 c ^ 3 d ^ 3 x ^ 2 + 88 a b ^ 4 c ^ 2 d
^ 4 x ^ 3 - 55 a b ^ 4 c d ^ 5 x ^ 4 + 385 a b ^ 4 d ^ 6 x ^ 5 - 256 b ^ 5
c ^ 6 - 128 b ^ 5 c ^ 5 d x + 32 b ^ 5 c ^ 4 d ^ 2 x ^ 2 - 16 b ^ 5 c ^ 3 d
 ^ 3 x ^ 3 + 10 b ^ 5 c ^ 2 d ^ 4 x ^ 4 - 7 b ^ 5 c d ^ 5 x ^ 5 + 63 b ^ 5
d ^ 6 x ^ 6) / (693 d ^ 6 Sqrt[c + d x]), d != 0}}, Piecewise[{{a ^ 5 x, b
== 0}, {(a + b x) ^ 6 / (6 b), True}}] / Sqrt[c]]
```

Maple [A]
time = 0.15, size = 121, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{7}{2}}}{7}+4(ad-bc)^3b^2(dx+c)^{\frac{5}{2}}+\frac{10(ad-bc)^4b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^5\sqrt{d}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{7}{2}}}{7}+4(ad-bc)^3b^2(dx+c)^{\frac{5}{2}}+\frac{10(ad-bc)^4b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^5\sqrt{d}}{d^6}$
gosper	$2\sqrt{dx+c}\ (63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a^2b$
trager	$2\sqrt{dx+c}\ (63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a^2b$
risch	$2\sqrt{dx+c}\ (63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a^2b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/11*b^5*(d*x+c)^(11/2)+5/9*(a*d-b*c)*b^4*(d*x+c)^(9/2)+10/7*(a*d-b*
c)^2*b^3*(d*x+c)^(7/2)+2*(a*d-b*c)^3*b^2*(d*x+c)^(5/2)+5/3*(a*d-b*c)^4*b*(d
*x+c)^(3/2)+(a*d-b*c)^5*(d*x+c)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.
time = 0.27, size = 283, normalized size = 1.84

$$\frac{2\left(693\sqrt{dx+c}\,a^5+\frac{1155\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^4b}{d}+\frac{462\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^3b^2}{d^2}+\frac{198\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a^2b^3}{d^3}+\frac{11\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)ab^4}{d^4}+\frac{\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^2-1386(dx+c)^{\frac{5}{2}}c^3+1155(dx+c)^{\frac{3}{2}}c^4-693\sqrt{dx+c}\,c^5\right)b^5}{d^5}\right)}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

Fricas [A]
time = 0.29, size = 261, normalized size = 1.69

$$\frac{2\left(63b^5d^5x^5-256b^5c^5+1408ab^4c^4d-3168a^2b^3c^3d^2+3696a^3b^2c^2d^3-2310a^4bcd^4+693a^5d^5-35\left(2b^5cd^4-11ab^4d^5\right)x^4+10\left(8b^5c^2d^3-44ab^4cd^4+99a^2b^3d^5\right)x^3-6\left(16b^5c^3d^2-88ab^4c^2d^3+198a^2b^3cd^4-231a^3b^2d^5\right)x^2+\left(128b^5c^4d-704ab^4c^3d^2+1584a^2b^3c^2d^3-1848a^3b^2cd^4+1155a^4bd^5\right)x\right)\sqrt{dx+c}}{693\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*d^5*x^5 - 256*b^5*c^5 + 1408*a*b^4*c^4*d - 3168*a^2*b^3*c^3*d
^2 + 3696*a^3*b^2*c^2*d^3 - 2310*a^4*b*c*d^4 + 693*a^5*d^5 - 35*(2*b^5*c*d^
4 - 11*a*b^4*d^5)*x^4 + 10*(8*b^5*c^2*d^3 - 44*a*b^4*c*d^4 + 99*a^2*b^3*d^5
)*x^3 - 6*(16*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 + 198*a^2*b^3*c*d^4 - 231*a^3*
b^2*d^5)*x^2 + (128*b^5*c^4*d - 704*a*b^4*c^3*d^2 + 1584*a^2*b^3*c^2*d^3 -
1848*a^3*b^2*c*d^4 + 1155*a^4*b*d^5)*x)*sqrt(d*x + c)/d^6
```

Sympy [A]
time = 36.74, size = 728, normalized size = 4.73

$$\begin{cases}\frac{-\frac{2a^5c}{\sqrt{c+dx}}-2a^5\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{10a^4bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{10a^4b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{20a^3b^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{20a^3b^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{20a^2b^3c\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3}-\frac{20a^2b^3\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{10ab^4c\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4}-\frac{10ab^4\left(-\frac{c^5}{\sqrt{c+dx}}-5c^4\sqrt{c+dx}+\frac{10c^3(c+dx)^{\frac{3}{2}}}{3}-2c^2(c+dx)^{\frac{5}{2}}+\frac{5c(c+dx)^{\frac{7}{2}}}{7}-\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{2b^5c\left(-\frac{c^5}{\sqrt{c+dx}}-5c^4\sqrt{c+dx}+\frac{10c^3(c+dx)^{\frac{3}{2}}}{3}-2c^2(c+dx)^{\frac{5}{2}}+\frac{5c(c+dx)^{\frac{7}{2}}}{7}-\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^5}-\frac{2b^5\left(\frac{c^6}{\sqrt{c+dx}}+6c^5\sqrt{c+dx}-5c^4(c+dx)^{\frac{3}{2}}+4c^3(c+dx)^{\frac{5}{2}}-\frac{15c^2(c+dx)^{\frac{7}{2}}}{7}+\frac{2c(c+dx)^{\frac{9}{2}}}{3}-\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^5}}{d} & \text{for}\ d\neq 0\\ \frac{\begin{cases}a^5x & \text{for}\ b=0\\ \frac{(a+bx)^6}{6b} & \text{otherwise}\end{cases}}{\sqrt{c}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**5*c/sqrt(c + d*x) - 2*a**5*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 10*a**4*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 10*a**4*b*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 20*a**3*b**2*c*(
c**2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 20*a**3
*b**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c
 + d*x)**(5/2)/5)/d**2 - 20*a**2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(
c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 20*a**2*b**3*(c*
*4/sqrt(c + d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c
+ d*x)**(5/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 10*a*b**4*c*(c**4/sqrt(c + d*x
) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5
 - (c + d*x)**(7/2)/7)/d**4 - 10*a*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(
c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c +
d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**4 - 2*b**5*c*(-c**5/sqrt(c + d*x) -
5*c**4*sqrt(c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2)
 + 5*c*(c + d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**5 - 2*b**5*(c**6/sqrt(c
+ d*x) + 6*c**5*sqrt(c + d*x) - 5*c**4*(c + d*x)**(3/2) + 4*c**3*(c + d*x)*
*(5/2) - 15*c**2*(c + d*x)**(7/2)/7 + 2*c*(c + d*x)**(9/2)/3 - (c + d*x)**(
11/2)/11)/d**5)/d, Ne(d, 0)), (Piecewise((a**5*x, Eq(b, 0)), ((a + b*x)**6/
(6*b), True))/sqrt(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(134) = 268.
time = 0.00, size = 448, normalized size = 2.91

$$\frac{\frac{2b^5\left(\frac{1}{11}\sqrt{c+dx}\,(c+dx)^5-\frac{5}{9}\sqrt{c+dx}\,(c+dx)^4c+\frac{10}{7}\sqrt{c+dx}\,(c+dx)^3c^2-2\sqrt{c+dx}\,(c+dx)^2c^3+\frac{5}{3}\sqrt{c+dx}\,(c+dx)c^4-\sqrt{c+dx}\,c^5\right)}{d^5}+\frac{10ab^4\left(\frac{1}{9}\sqrt{c+dx}\,(c+dx)^4-\frac{4}{7}\sqrt{c+dx}\,(c+dx)^3c+\frac{6}{5}\sqrt{c+dx}\,(c+dx)^2c^2-\frac{4}{3}\sqrt{c+dx}\,(c+dx)c^3+\sqrt{c+dx}\,c^4\right)}{d^4}+\frac{20a^2b^3\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3-\frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c+\sqrt{c+dx}\,(c+dx)c^2-\sqrt{c+dx}\,c^3\right)}{d^3}+\frac{20a^3b^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+\frac{10a^4b\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)}{d}+2a^5\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

Mupad [B]
time = 0.07, size = 137, normalized size = 0.89

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,\sqrt{c+d\,x}}{d^6}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(11/2))/(11*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(9/2
))/(9*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(1/2))/d^6 + (4*b^2*(a*d - b*c)^3*(
c + d*x)^(5/2))/d^6 + (20*b^3*(a*d - b*c)^2*(c + d*x)^(7/2))/(7*d^6) + (10*
b*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^6)
```

3.1414 $\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=127

$$\frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-ad)(c+dx)^{7/2}}{7d^5}+\frac{2b^4(c+\ldots}{9\ldots}$$

```
[Out] -8/3*b*(-a*d+b*c)^3*(d*x+c)^(3/2)/d^5+12/5*b^2*(-a*d+b*c)^2*(d*x+c)^(5/2)/d
^5-8/7*b^3*(-a*d+b*c)*(d*x+c)^(7/2)/d^5+2/9*b^4*(d*x+c)^(9/2)/d^5+2*(-a*d+b
*c)^4*(d*x+c)^(1/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}\,(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*Sqrt[c + d*x])/d^5 - (8*b*(b*c - a*d)^3*(c + d*x)^(3/2))/(
3*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^5) - (8*b^3*(b*c - a*d
)*(c + d*x)^(7/2))/(7*d^5) + (2*b^4*(c + d*x)^(9/2))/(9*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4\sqrt{c+dx}}-\frac{4b(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{4b^3(bc-ad)(c+\ldots}{d^4}\right.\\ &= \frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-\ldots}{}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 153, normalized size = 1.20

$$\frac{2\sqrt{c+dx}\,\left(315a^4d^4+420a^3bd^3(-2c+dx)+126a^2b^2d^2\left(8c^2-4cdx+3d^2x^2\right)+36ab^3d\left(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3\right)+b^4\left(128c^4-64c^3dx+48c^2d^2x^2-40cd^3x^3+35d^4x^4\right)\right)}{315d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*a^4*d^4 + 420*a^3*b*d^3*(-2*c + d*x) + 126*a^2*b^2*d^
2*(8*c^2 - 4*c*d*x + 3*d^2*x^2) + 36*a*b^3*d*(-16*c^3 + 8*c^2*d*x - 6*c*d^2
*x^2 + 5*d^3*x^3) + b^4*(128*c^4 - 64*c^3*d*x + 48*c^2*d^2*x^2 - 40*c*d^3*x
^3 + 35*d^4*x^4)))/(315*d^5)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 32.61, size = 291, normalized size = 2.29

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(315a^4cd^4+315a^4d^5x-840a^3bc^2d^3-420a^3bcd^4x+420a^3bd^5x^2+1008a^2b^2c^3d^2+504a^2b^2c^2d^3x-126a^2b^2cd^4x^2+378a^2b^2d^5x^3-576ab^3c^4d-288ab^3c^3d^2x+72ab^3c^2d^3x^2-36ab^3cd^4x^3+180ab^3d^5x^4+128b^4c^5+64b^4c^4dx-16b^4c^3d^2x^2+8b^4c^2d^3x^3-5b^4cd^4x^4+35b^4d^5x^5\right)}{315d^5\sqrt{c+dx}},d!=0\right\}\right\},\frac{\text{Piecewise}\left[\left\{\left\{a^4x,b==0\right\},\left\{\frac{(a+bx)^5}{5b},\text{True}\right\}\right\}\right]}{\sqrt{c}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x)^(1/2),x]')
```

```
[Out] Piecewise[{{2 (315 a ^ 4 c d ^ 4 + 315 a ^ 4 d ^ 5 x - 840 a ^ 3 b c ^ 2 d
^ 3 - 420 a ^ 3 b c d ^ 4 x + 420 a ^ 3 b d ^ 5 x ^ 2 + 1008 a ^ 2 b ^ 2 c
^ 3 d ^ 2 + 504 a ^ 2 b ^ 2 c ^ 2 d ^ 3 x - 126 a ^ 2 b ^ 2 c d ^ 4 x ^ 2 +
 378 a ^ 2 b ^ 2 d ^ 5 x ^ 3 - 576 a b ^ 3 c ^ 4 d - 288 a b ^ 3 c ^ 3 d ^
2 x + 72 a b ^ 3 c ^ 2 d ^ 3 x ^ 2 - 36 a b ^ 3 c d ^ 4 x ^ 3 + 180 a b ^ 3
 d ^ 5 x ^ 4 + 128 b ^ 4 c ^ 5 + 64 b ^ 4 c ^ 4 d x - 16 b ^ 4 c ^ 3 d ^ 2
x ^ 2 + 8 b ^ 4 c ^ 2 d ^ 3 x ^ 3 - 5 b ^ 4 c d ^ 4 x ^ 4 + 35 b ^ 4 d ^ 5
x ^ 5) / (315 d ^ 5 Sqrt[c + d x]), d != 0}}, Piecewise[{{a ^ 4 x, b == 0},
 {(a + b x) ^ 5 / (5 b), True}}] / Sqrt[c]]
```

Maple [A]
time = 0.14, size = 99, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{8(ad-bc)b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{5}{2}}}{5}+\frac{8(ad-bc)^3b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^4\sqrt{dx+c}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{8(ad-bc)b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{5}{2}}}{5}+\frac{8(ad-bc)^3b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^4\sqrt{dx+c}}{d^5}$
gosper	$\frac{2\sqrt{dx+c}\,(35d^4x^4b^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c}{315d^5}$
trager	$\frac{2\sqrt{dx+c}\,(35d^4x^4b^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c}{315d^5}$
risch	$\frac{2\sqrt{dx+c}\,(35d^4x^4b^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c}{315d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/9*b^4*(d*x+c)^(9/2)+4/7*(a*d-b*c)*b^3*(d*x+c)^(7/2)+6/5*(a*d-b*c)^
2*b^2*(d*x+c)^(5/2)+4/3*(a*d-b*c)^3*b*(d*x+c)^(3/2)+(a*d-b*c)^4*(d*x+c)^(1/
2))
```

Maxima [A]
time = 0.27, size = 204, normalized size = 1.61

$$\frac{2 \left(315 \sqrt{dx+c}\, a^4 + \frac{420 \left((dx+c)^{\frac{3}{2}} - 3 \sqrt{dx+c}\, c\right) a^3 b}{d} + \frac{126 \left(3 (dx+c)^{\frac{5}{2}} - 10 (dx+c)^{\frac{3}{2}} c + 15 \sqrt{dx+c}\, c^2\right) a^2 b^2}{d^2} + \frac{36 \left(5 (dx+c)^{\frac{7}{2}} - 21 (dx+c)^{\frac{5}{2}} c + 35 (dx+c)^{\frac{3}{2}} c^2 - 35 \sqrt{dx+c}\, c^3\right) a b^3}{d^3} + \frac{\left(35 (dx+c)^{\frac{9}{2}} - 180 (dx+c)^{\frac{7}{2}} c + 378 (dx+c)^{\frac{5}{2}} c^2 - 420 (dx+c)^{\frac{3}{2}} c^3 + 315 \sqrt{dx+c}\, c^4\right) b^4}{d^4}\right)}{315 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

Fricas [A]
time = 0.29, size = 182, normalized size = 1.43

$$\frac{2 \left(35 b^4 d^4 x^4 + 128 b^4 c^4 - 576 a b^3 c^3 d + 1008 a^2 b^2 c^2 d^2 - 840 a^3 b c d^3 + 315 a^4 d^4 - 20 \left(2 b^4 c d^3 - 9 a b^3 d^4\right) x^3 + 6 \left(8 b^4 c^2 d^2 - 36 a b^3 c d^3 + 63 a^2 b^2 d^4\right) x^2 - 4 \left(16 b^4 c^3 d - 72 a b^3 c^2 d^2 + 126 a^2 b^2 c d^3 - 105 a^3 b d^4\right) x\right) \sqrt{dx+c}}{315 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*d^4*x^4 + 128*b^4*c^4 - 576*a*b^3*c^3*d + 1008*a^2*b^2*c^2*d^
2 - 840*a^3*b*c*d^3 + 315*a^4*d^4 - 20*(2*b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 6*
(8*b^4*c^2*d^2 - 36*a*b^3*c*d^3 + 63*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 7
2*a*b^3*c^2*d^2 + 126*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(d*x + c)/d^5
```

Sympy [A]
time = 25.95, size = 532, normalized size = 4.19

$$\begin{cases} \frac{- \frac{2 a^4 c}{\sqrt{c+dx}} - 2 a^4 \left(- \frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{8 a^3 b c \left(- \frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{8 a^3 b \left(\frac{c^2}{\sqrt{c+dx}} + 2 c \sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{12 a^2 b^2 c \left(\frac{c^2}{\sqrt{c+dx}} + 2 c \sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{12 a^2 b^2 \left(- \frac{c^3}{\sqrt{c+dx}} - 3 c^2 \sqrt{c+dx} + c (c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{8 a b^3 c \left(- \frac{c^3}{\sqrt{c+dx}} - 3 c^2 \sqrt{c+dx} + c (c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3} - \frac{8 a b^3 \left(\frac{c^4}{\sqrt{c+dx}} + 4 c^3 \sqrt{c+dx} - 2 c^2 (c+dx)^{\frac{3}{2}} + \frac{4 c (c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3} - \frac{2 b^4 c \left(\frac{c^4}{\sqrt{c+dx}} + 4 c^3 \sqrt{c+dx} - 2 c^2 (c+dx)^{\frac{3}{2}} + \frac{4 c (c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4} - \frac{2 b^4 \left(- \frac{c^5}{\sqrt{c+dx}} - 5 c^4 \sqrt{c+dx} + \frac{10 c^3 (c+dx)^{\frac{3}{2}}}{3} - 2 c^2 (c+dx)^{\frac{5}{2}} + \frac{5 c (c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}}{d} & \text{for}\ d \neq 0 \\ \frac{\begin{cases} a^4 x & \text{for}\ b = 0 \\ \frac{(a+bx)^5}{5b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**4*c/sqrt(c + d*x) - 2*a**4*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 8*a**3*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 8*a**3*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 12*a**2*b**2*c*(c*
*2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 12*a**2*b
**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c +
```

```
d*x)**(5/2)/5)/d**2 - 8*a*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*
x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 8*a*b**3*(c**4/sqrt(c
+ d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5
/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 2*b**4*c*(c**4/sqrt(c + d*x) + 4*c**3*sq
rt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c + d*x)*
*(7/2)/7)/d**4 - 2*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(c + d*x) + 10*c*
*3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c + d*x)**(7/2)/7 -
(c + d*x)**(9/2)/9)/d**4)/d, Ne(d, 0)), (Piecewise((a**4*x, Eq(b, 0)), ((a
+ b*x)**5/(5*b), True))/sqrt(c), True))
```

Giac [A]
time = 0.00, size = 314, normalized size = 2.47

$$\frac{\frac{2b^4\left(\frac{1}{9}\sqrt{c+dx}\,(c+dx)^4-\frac{4}{7}\sqrt{c+dx}\,(c+dx)^3c+\frac{6}{5}\sqrt{c+dx}\,(c+dx)^2c^2-\frac{4}{3}\sqrt{c+dx}\,(c+dx)c^3+\sqrt{c+dx}\,c^4\right)}{d^4}+\frac{8ab^3\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3-\frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c+\sqrt{c+dx}\,(c+dx)c^2-\sqrt{c+dx}\,c^3\right)}{d^3}+\frac{12a^2b^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+\frac{8a^3b\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)}{d}+2a^4\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

Mupad [B]
time = 0.24, size = 112, normalized size = 0.88

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(9/2))/(9*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(7/2))/(
7*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(1/2))/d^5 + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(5/2))/(5*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^5)
```

3.1415 $\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=96

$$-\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

```
[Out] 2*b*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^4-6/5*b^2*(-a*d+b*c)*(d*x+c)^(5/2)/d^4+2/7
*b^3*(d*x+c)^(7/2)/d^4-2*(-a*d+b*c)^3*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}\,(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*Sqrt[c + d*x])/d^4 + (2*b*(b*c - a*d)^2*(c + d*x)^(3/2))/
d^4 - (6*b^2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^4) + (2*b^3*(c + d*x)^(7/2))
/(7*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3\sqrt{c+dx}}+\frac{3b(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{b^3(c+dx)^{5/2}}{d^3}\right) dx\\ &= -\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 1.05

$$\frac{2\sqrt{c+dx}\,\left(35a^3d^3+35a^2bd^2(-2c+dx)+7ab^2d\left(8c^2-4cdx+3d^2x^2\right)+b^3\left(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3\right)\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^3*d^3 + 35*a^2*b*d^2*(-2*c + d*x) + 7*a*b^2*d*(8*c^2
 - 4*c*d*x + 3*d^2*x^2) + b^3*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^2 + 5*d^3*x^
3)))/(35*d^4)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 21.99, size = 204, normalized size = 2.12

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(35a^3cd^3+35a^3d^4x-70a^2bc^2d^2-35a^2bcd^3x+35a^2bd^4x^2+56ab^2c^3d+28ab^2c^2d^2x-7ab^2cd^3x^2+21ab^2d^4x^3-16b^3c^4-8b^3c^3dx+2b^3c^2d^2x^2-b^3cd^3x^3+5b^3d^4x^4\right)}{35d^4\sqrt{c+dx}},d!=0\right\}\right\},\frac{\text{Piecewise}\left[\left\{\left\{a^3x,b==0\right\},\left\{\frac{(a+bx)^4}{4b},\text{True}\right\}\right\}\right]}{\sqrt{c}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^3/(c + d*x)^(1/2),x]')
```

```
[Out] Piecewise[{{2 (35 a ^ 3 c d ^ 3 + 35 a ^ 3 d ^ 4 x - 70 a ^ 2 b c ^ 2 d ^ 2
 - 35 a ^ 2 b c d ^ 3 x + 35 a ^ 2 b d ^ 4 x ^ 2 + 56 a b ^ 2 c ^ 3 d + 28
a b ^ 2 c ^ 2 d ^ 2 x - 7 a b ^ 2 c d ^ 3 x ^ 2 + 21 a b ^ 2 d ^ 4 x ^ 3 -
16 b ^ 3 c ^ 4 - 8 b ^ 3 c ^ 3 d x + 2 b ^ 3 c ^ 2 d ^ 2 x ^ 2 - b ^ 3 c d
^ 3 x ^ 3 + 5 b ^ 3 d ^ 4 x ^ 4) / (35 d ^ 4 Sqrt[c + d x]), d != 0}}, Piec
ewise[{{a ^ 3 x, b == 0}, {(a + b x) ^ 4 / (4 b), True}}] / Sqrt[c]]
```

Maple [A]
time = 0.15, size = 76, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)b^2(dx+c)^{\frac{5}{2}}}{5}+2(ad-bc)^2b(dx+c)^{\frac{3}{2}}+2(ad-bc)^3\sqrt{dx+c}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)b^2(dx+c)^{\frac{5}{2}}}{5}+2(ad-bc)^2b(dx+c)^{\frac{3}{2}}+2(ad-bc)^3\sqrt{dx+c}}{d^4}$
gosper	$\frac{2\sqrt{dx+c}\,(5b^3x^3d^3+21d^3ax^2b^2-6b^3cd^2x^2+35a^2bd^3x-28ab^2cd^2x+8b^3c^2dx+35a^3d^3-70a^2bcd^2+56ab^2c^2d-16}{35d^4}$
trager	$\frac{2\sqrt{dx+c}\,(5b^3x^3d^3+21d^3ax^2b^2-6b^3cd^2x^2+35a^2bd^3x-28ab^2cd^2x+8b^3c^2dx+35a^3d^3-70a^2bcd^2+56ab^2c^2d-16}{35d^4}$
risch	$\frac{2\sqrt{dx+c}\,(5b^3x^3d^3+21d^3ax^2b^2-6b^3cd^2x^2+35a^2bd^3x-28ab^2cd^2x+8b^3c^2dx+35a^3d^3-70a^2bcd^2+56ab^2c^2d-16}{35d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/7*b^3*(d*x+c)^(7/2)+3/5*(a*d-b*c)*b^2*(d*x+c)^(5/2)+(a*d-b*c)^2*b*
(d*x+c)^(3/2)+(a*d-b*c)^3*(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 137, normalized size = 1.43

$$\frac{2\left(35\,\sqrt{dx+c}\,a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2b}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab^2}{d^2}+\frac{\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

Fricas [A]
time = 0.30, size = 115, normalized size = 1.20

$$\frac{2\,(5\,b^3d^3x^3-16\,b^3c^3+56\,ab^2c^2d-70\,a^2bcd^2+35\,a^3d^3-3\,(2\,b^3cd^2-7\,ab^2d^3)x^2+(8\,b^3c^2d-28\,ab^2cd^2+35\,a^2bd^3)x)\sqrt{dx+c}}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*d^3*x^3 - 16*b^3*c^3 + 56*a*b^2*c^2*d - 70*a^2*b*c*d^2 + 35*a^3
*d^3 - 3*(2*b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 28*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(d*x + c)/d^4
```

Sympy [A]
time = 16.89, size = 366, normalized size = 3.81

$$\begin{cases} \frac{-\frac{2a^3c}{\sqrt{c+dx}}-2a^3\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{6a^2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{6a^2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{6ab^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{6ab^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{2b^3c\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3}-\frac{2b^3\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for}\ d\neq 0 \\ \frac{\begin{cases} a^3x & \text{for}\ b=0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**3*c/sqrt(c + d*x) - 2*a**3*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 6*a**2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 6*a**2*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 6*a*b**2*c*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 6*a*b**2*(-c*
*3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(
5/2)/5)/d**2 - 2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c
+ d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 2*b**3*(c**4/sqrt(c + d*x) + 4*c
**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c +
 d*x)**(7/2)/7)/d**3)/d, Ne(d, 0)), (Piecewise((a**3*x, Eq(b, 0)), ((a + b*
x)**4/(4*b), True))/sqrt(c), True))
```

Giac [A]
time = 0.00, size = 202, normalized size = 2.10

$$\frac{\frac{2b^3\left(\frac{1}{7}\sqrt{c+dx}\,(c+dx)^3-\frac{3}{5}\sqrt{c+dx}\,(c+dx)^2c+\sqrt{c+dx}\,(c+dx)c^2-\sqrt{c+dx}\,c^3\right)}{d^3}+\frac{6ab^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+\frac{6a^2b\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)}{d}+2a^3\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

Mupad [B]
time = 0.26, size = 87, normalized size = 0.91

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(7/2))/(7*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(5/2))/(
5*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^4 + (2*b*(a*d - b*c)^2*(c + d*
x)^(3/2))/d^4
```

3.1416 $\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=69

$$\frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

```
[Out] -4/3*b*(-a*d+b*c)*(d*x+c)^(3/2)/d^3+2/5*b^2*(d*x+c)^(5/2)/d^3+2*(-a*d+b*c)^
2*(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/d^3 - (4*b*(b*c - a*d)*(c + d*x)^(3/2))/(3*
d^3) + (2*b^2*(c + d*x)^(5/2))/(5*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.87

$$\frac{2\sqrt{c+dx}\,\left(15a^2d^2+10abd(-2c+dx)+b^2\left(8c^2-4cdx+3d^2x^2\right)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(8*c^2 - 4*c*d*x
 + 3*d^2*x^2)))/(15*d^3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 13.23, size = 134, normalized size = 1.94

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(15a^2cd^2+15a^2d^3x-20abc^2d-10abcd^2x+10abd^3x^2+8b^2c^3+4b^2c^2dx-b^2cd^2x^2+3b^2d^3x^3\right)}{15d^3\sqrt{c+dx}},d!=0\right\}\right\},\frac{\text{Piecewise}\left[\left\{\left\{a^2x,b==0\right\},\left\{\frac{(a+bx)^3}{3b},\text{True}\right\}\right\}\right]}{\sqrt{c}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x)^(1/2),x]')
```

```
[Out] Piecewise[{{2 (15 a ^ 2 c d ^ 2 + 15 a ^ 2 d ^ 3 x - 20 a b c ^ 2 d - 10 a
b c d ^ 2 x + 10 a b d ^ 3 x ^ 2 + 8 b ^ 2 c ^ 3 + 4 b ^ 2 c ^ 2 d x - b ^
2 c d ^ 2 x ^ 2 + 3 b ^ 2 d ^ 3 x ^ 3) / (15 d ^ 3 Sqrt[c + d x]), d != 0}}
, Piecewise[{{a ^ 2 x, b == 0}, {(a + b x) ^ 3 / (3 b), True}}] / Sqrt[c]]
```

Maple [A]
time = 0.14, size = 55, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{2(dx+c)^{\frac{5}{2}}b^2}{5}+\frac{4(ad-bc)b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^2\sqrt{dx+c}}{d^3}$	55
default	$\frac{\frac{2(dx+c)^{\frac{5}{2}}b^2}{5}+\frac{4(ad-bc)b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^2\sqrt{dx+c}}{d^3}$	55
gosper	$\frac{2\sqrt{dx+c}\,(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$	63
trager	$\frac{2\sqrt{dx+c}\,(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$	63
risch	$\frac{2\sqrt{dx+c}\,(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/5*(d*x+c)^(5/2)*b^2+2/3*(a*d-b*c)*b*(d*x+c)^(3/2)+(a*d-b*c)^2*(d*x
+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.19

$$\frac{2\left(15\sqrt{dx+c}\,a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)ab}{d}+\frac{\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

Fricas [A]
time = 0.29, size = 64, normalized size = 0.93

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\left(2\,b^2cd-5\,abd^2\right)x\right)\sqrt{dx+c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c)/d^3
```

Sympy [A]
time = 9.65, size = 231, normalized size = 3.35

$$\begin{cases} \frac{-\frac{2a^2c}{\sqrt{c+dx}}-2a^2\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{4abc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{4ab\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{2b^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{2b^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for } d\neq 0 \\ \frac{\begin{cases} a^2x & \text{for } b=0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**2*c/sqrt(c + d*x) - 2*a**2*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 4*a*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 4*a*b*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 2*b**2*c*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 2*b**2*(-c**3/sqrt(c
+ d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d*
*2)/d, Ne(d, 0)), (Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), True)
)/sqrt(c), True))
```

Giac [A]
time = 0.00, size = 116, normalized size = 1.68

$$\frac{\frac{2b^2\left(\frac{1}{5}\sqrt{c+dx}\,(c+dx)^2-\frac{2}{3}\sqrt{c+dx}\,(c+dx)c+\sqrt{c+dx}\,c^2\right)}{d^2}+\frac{4ab\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)}{d}+2a^2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

Mupad [B]
time = 0.07, size = 68, normalized size = 0.99

$$\frac{2\sqrt{c+d\,x}\,\left(3\,b^2\,{\left(c+d\,x\right)}^2+15\,a^2\,d^2+15\,b^2\,c^2-10\,b^2\,c\,\left(c+d\,x\right)+10\,a\,b\,d\,\left(c+d\,x\right)-30\,a\,b\,c\,d\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2)*(3*b^2*(c + d*x)^2 + 15*a^2*d^2 + 15*b^2*c^2 - 10*b^2*c*
(c + d*x) + 10*a*b*d*(c + d*x) - 30*a*b*c*d))/(15*d^3)
```

3.1417 $\int \frac{a+bx}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=40

$$-\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}$$

```
[Out] 2/3*b*(d*x+c)^(3/2)/d^2-2*(-a*d+b*c)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[c + d*x])/d^2 + (2*b*(c + d*x)^(3/2))/(3*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{c+dx}}\,dx &= \int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx\\ &= -\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.72

$$\frac{2\sqrt{c+dx}\,(-2bc+3ad+bdx)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/(3*d^2)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 4.62, size = 75, normalized size = 1.88

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(3ad\left(c+dx\right)-b\left(-3c\left(2c+dx\right)+3c^2+\left(c+dx\right)\left(5c-dx\right)\right)\right)}{3d^2\sqrt{c+dx}},d!{=}0\right\}\right\},\frac{ax+\frac{bx^2}{2}}{\sqrt{c}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x)^(1/2),x]')
```

```
[Out] Piecewise[{{2 (3 a d (c + d x) - b (-3 c (2 c + d x) + 3 c ^ 2 + (c + d x)
(5 c - d x))) / (3 d ^ 2 Sqrt[c + d x]), d != 0}}, (a x + b x ^ 2 / 2) / Sq
rt[c]]
```

Maple [A]
time = 0.13, size = 38, normalized size = 0.95

method	result	size
gosper	$\frac{2\sqrt{dx+c}\,(bdx+3ad-2bc)}{3d^2}$	26
trager	$\frac{2\sqrt{dx+c}\,(bdx+3ad-2bc)}{3d^2}$	26
risch	$\frac{2\sqrt{dx+c}\,(bdx+3ad-2bc)}{3d^2}$	26
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{d^2}$	38
default	$\frac{\frac{2b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{d^2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(1/3*b*(d*x+c)^(3/2)+a*d*(d*x+c)^(1/2)-b*c*(d*x+c)^(1/2))
```

Maxima [A]
time = 0.26, size = 39, normalized size = 0.98

$$\frac{2\left(3\sqrt{dx+c}\,a+\frac{\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

Fricas [A]
time = 0.29, size = 25, normalized size = 0.62

$$\frac{2\,(bdx-2\,bc+3\,ad)\sqrt{dx+c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x - 2*b*c + 3*a*d)*sqrt(d*x + c)/d^2
```

Sympy [A]
time = 2.11, size = 121, normalized size = 3.02

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{c+dx}}-2a\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2),x)
[Out] Piecewise(((-2*a*c/sqrt(c + d*x) - 2*a*(-c/sqrt(c + d*x) - sqrt(c + d*x)) -
 2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 2*b*(c**2/sqrt(c + d*x) + 2*c
*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d)/d, Ne(d, 0)), ((a*x + b*x**2/2)/sqr
t(c), True))
```

Giac [A]
time = 0.00, size = 51, normalized size = 1.28

$$\frac{\frac{2b\left(\frac{1}{3}\sqrt{c+dx}\,(c+dx)-c\sqrt{c+dx}\right)}{d}+2a\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x)
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{c+d\,x}\,\left(3\,a\,d-3\,b\,c+b\,\left(c+d\,x\right)\right)}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(1/2),x)
[Out] (2*(c + d*x)^(1/2)*(3*a*d - 3*b*c + b*(c + d*x)))/(3*d^2)
```

3.1418 $\int \frac{1}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{c+dx}}{d}$$

```
[Out] 2*(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}}{d}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

Mathics [A]
time = 1.64, size = 12, normalized size = 0.86

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x)^(1/2),x]')
```

```
[Out] 2 Sqrt[c + d x] / d
```

Maple [A]
time = 0.13, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{2\sqrt{dx+c}}{d}$	13
derivativedivides	$\frac{2\sqrt{dx+c}}{d}$	13
default	$\frac{2\sqrt{dx+c}}{d}$	13
trager	$\frac{2\sqrt{dx+c}}{d}$	13
risch	$\frac{2\sqrt{dx+c}}{d}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)^(1/2)/d
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Fricas [A]
time = 0.29, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(1/2),x)
```

```
[Out] 2*sqrt(c + d*x)/d
```

Giac [A]
time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x)
```

```
[Out] 2*sqrt(d*x + c)/d
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\,\sqrt{c+d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2))/d
```

3.1419 $\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}$$

```
[Out] -2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{65, 214\}$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Mathics [A]
time = 4.09, size = 49, normalized size = 1.04

$$\frac{-2\mathrm{ArcTan}\left[\frac{1}{\sqrt{\frac{b}{ad-bc}}\sqrt{c+dx}}\right]}{\sqrt{\frac{b}{ad-bc}}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^1*(c + d*x)^(1/2)),x]')
[Out] -2 ArcTan[1 / (Sqrt[b / (a d - b c)] Sqrt[c + d x])] / (Sqrt[b / (a d - b c
)] (a d - b c))
```

Maple [A]
time = 0.16, size = 37, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}$	37
default	$\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 119, normalized size = 2.53

$$\left[\frac{\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)}{\sqrt{b^2c-abd}},\ \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)}{b^2c-abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a))
/sqrt(b^2*c - a*b*d), 2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sq
rt(d*x + c)/(b*d*x + b*c))/(b^2*c - a*b*d)]
```

Sympy [A]
time = 2.22, size = 44, normalized size = 0.94

$$-\frac{2\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{b}{ad-bc}}\,\sqrt{c+dx}}\right)}{\sqrt{\frac{b}{ad-bc}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] -2*atan(1/(sqrt(b/(a*d - b*c))*sqrt(c + d*x)))/(sqrt(b/(a*d - b*c))*(a*d -
b*c))
```

Giac [A]
time = 0.00, size = 46, normalized size = 0.98

$$\frac{2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x)
[Out] 2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 0.27, size = 38, normalized size = 0.81

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\sqrt{c+d\,x}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/2)),x)
[Out] (2*atan((b*(c + d*x)^(1/2))/(a*b*d - b^2*c)^(1/2)))/(a*b*d - b^2*c)^(1/2)
```

3.1420 $\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}$$

```
[Out] d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^(1/2)-
(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{bc-ad}$$
$$= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}$$

Mathematica [**A**]
time = 0.17, size = 75, normalized size = 0.99

$$\frac{\sqrt{c+dx}}{(-bc+ad)(a+bx)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] Sqrt[c + d*x]/((-(b*c) + a*d)*(a + b*x)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^2*(c + d*x)^(1/2)),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.16, size = 87, normalized size = 1.14

method	result	size

derivativedivides	$2d\left(\frac{\sqrt{dx+c}}{2(ad-bc)((dx+c)b+ad-bc)}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2(ad-bc)\sqrt{(ad-bc)\,b}}\right)$	87
default	$2d\left(\frac{\sqrt{dx+c}}{2(ad-bc)((dx+c)b+ad-bc)}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2(ad-bc)\sqrt{(ad-bc)\,b}}\right)$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(1/2*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*
c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(64) = 128.
time = 0.31, size = 280, normalized size = 3.68

$$\left[-\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(b^2c-abd)\sqrt{dx+c}}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x)},-\frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(b^2c-abd)\sqrt{dx+c}}{ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(
b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(b^2*c - a*b*d)*sqrt(d*x + c))
/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*x), -(sqrt(-b^2*c + a*b*d)*(b*d*x + a*d)*arctan(sqrt(-b^2*c + a*b*d)*s
qrt(d*x + c)/(b*d*x + b*c)) + (b^2*c - a*b*d)*sqrt(d*x + c))/(a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*sqrt(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 98, normalized size = 1.29

$$2\left(\frac{\sqrt{c+dx}\,d}{(-2bc+2da)\,((c+dx)\,b-bc+da)}+\frac{d\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(-bc+da)\,\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] -d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
- a*d)) - sqrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*(b*c - a*d))
```

Mupad [B]
time = 0.09, size = 74, normalized size = 0.97

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\,\left(a\,d-b\,c\right)^{3/2}}+\frac{d\,\sqrt{c+d\,x}}{\left(a\,d-b\,c\right)\,\left(a\,d-b\,c+b\,\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^
(3/2)) + (d*(c + d*x)^(1/2))/((a*d - b*c)*(a*d - b*c + b*(c + d*x)))
```

3.1421 $\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}$$

```
[Out] -3/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(5/2)/b
^(1/2)-1/2*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^2+3/4*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] -1/2*Sqrt[c + d*x]/((b*c - a*d)*(a + b*x)^2) + (3*d*Sqrt[c + d*x])/(4*(b*c
- a*d)^2*(a + b*x)) - (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(4*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx &= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}-\frac{(3d)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}+\frac{(3d^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8(bc-ad)^2}\\
&= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}+\frac{(3d)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c}\right.}{4(bc-ad)^2}\\
&= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 96, normalized size = 0.84

$$\frac{1}{4}\left(\frac{\sqrt{c+dx}\,(-2bc+5ad+3bdx)}{(bc-ad)^2(a+bx)^2}+\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*Sqrt[c + d*x]),x]
[Out] ((Sqrt[c + d*x]*(-2*b*c + 5*a*d + 3*b*d*x))/((b*c - a*d)^2*(a + b*x)^2) + (
3*d^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c)
+ a*d)^(5/2)))/4
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^3*(c + d*x)^(1/2)),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 138, normalized size = 1.21

method	result	size
derivativedivides	$2d^{2}\left(\frac{\sqrt{dx+c}}{4(ad-bc)\left((dx+c)b+ad-bc\right)^{2}}+\frac{\frac{3\sqrt{dx+c}}{8(ad-bc)\left((dx+c)b+ad-bc\right)}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8(ad-bc)\sqrt{(ad-bc)\,b}}}{ad-bc}\right)$	138
default	$2d^{2}\left(\frac{\sqrt{dx+c}}{4(ad-bc)\left((dx+c)b+ad-bc\right)^{2}}+\frac{\frac{3\sqrt{dx+c}}{8(ad-bc)\left((dx+c)b+ad-bc\right)}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8(ad-bc)\sqrt{(ad-bc)\,b}}}{ad-bc}\right)$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(1/4*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^2+3/4/(a*d-b*c)*(1/2
*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(94) = 188.
time = 0.31, size = 549, normalized size = 4.82

$$\left[\frac{3\,(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2})\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^{2}c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(2\,b^{3}c^{2}-7\,ab^{2}cd+5\,a^{2}bd^{2}-3\,(b^{3}cd-ab^{2}d^{2})x)\sqrt{dx+c}}{8\,(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}+(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3})x^{2}+2\,(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3})x)},\ \frac{3\,(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2})\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{-b^{2}c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(2\,b^{3}c^{2}-7\,ab^{2}cd+5\,a^{2}bd^{2}-3\,(b^{3}cd-ab^{2}d^{2})x)\sqrt{dx+c}}{4\,(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}+(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3})x^{2}+2\,(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c
))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3 + (b^6*c^3
- 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^2 + 2*(a*b^5*c^3 - 3*a^2
*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x), 1/4*(3*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x
 + c)/(b*d*x + b*c)) - (2*b^3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d
- a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c
*d^2 - a^5*b*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*x^2 + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**3*sqrt(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 178, normalized size = 1.56

$$2\left(\frac{-3\sqrt{c+dx}\,(c+dx)\,bd^2+5\sqrt{c+dx}\,bd^2c-5\sqrt{c+dx}\,d^3a}{(-8b^2c^2+16bdca-8d^2a^2)\left((c+dx)\,b-bc+da\right)^2}+\frac{3d^2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(4b^2c^2-8bdca+4d^2a^2)\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/4*(3*(d*x + c)^(3/2)*b*d^2 - 5*sqrt(d*
x + c)*b*c*d^2 + 5*sqrt(d*x + c)*a*d^3)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((
d*x + c)*b - b*c + a*d)^2)
```

Mupad [B]
time = 0.33, size = 142, normalized size = 1.25

$$\frac{\frac{5\,d^2\,\sqrt{c+d\,x}}{4\,(a\,d-b\,c)}+\frac{3\,b\,d^2\,(c+d\,x)^{3/2}}{4\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^2-(2\,b^2\,c-2\,a\,b\,d)\,(c+d\,x)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}+\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(1/2)),x)
```

```
[Out] ((5*d^2*(c + d*x)^(1/2))/(4*(a*d - b*c)) + (3*b*d^2*(c + d*x)^(3/2))/(4*(a*
d - b*c)^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) + (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/
2)))/(4*b^(1/2)*(a*d - b*c)^(5/2))
```

3.1422 $\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx$

Optimal. Leaf size=147

$$-\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3}+\frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2}-\frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}$$

```
[Out] 5/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(7/2)/b^
(1/2)-1/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^3+5/12*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)^2-5/8*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)
```

Rubi [A]

time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] -1/3*Sqrt[c + d*x]/((b*c - a*d)*(a + b*x)^3) + (5*d*Sqrt[c + d*x])/(12*(b*c
 - a*d)^2*(a + b*x)^2) - (5*d^2*Sqrt[c + d*x])/(8*(b*c - a*d)^3*(a + b*x))
+ (5*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c
- a*d)^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} - \frac{(5d)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} + \frac{(5d^2)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{(5d^3)}{}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{(5d^2)}{}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} + \frac{5d^3\ \mathrm{ta}}{}$$

Mathematica [A]
time = 0.32, size = 128, normalized size = 0.87

$$\frac{\sqrt{c+dx}\,\left(33a^2d^2+2abd(-13c+20dx)+b^2\left(8c^2-10cdx+15d^2x^2\right)\right)}{24(-bc+ad)^3(a+bx)^3} + \frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8\sqrt{b}\,(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(33*a^2*d^2 + 2*a*b*d*(-13*c + 20*d*x) + b^2*(8*c^2 - 10*c*d
*x + 15*d^2*x^2)))/(24*(-(b*c) + a*d)^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*Sqrt[b]*(-(b*c) + a*d)^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^4*(c + d*x)^(1/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 187, normalized size = 1.27

method	result
derivativedivides	$2d^3\left(\frac{\sqrt{dx+c}}{6(ad-bc)((dx+c)b+ad-bc)^3}+\frac{\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2}+\frac{5\left(\frac{3\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)}+\frac{3\arctan\left(\frac{b\sqrt{dx-}}{\sqrt{(ad-}}\right)}{8(ad-bc)\sqrt{(ad-}}\right)}{6(ad-bc)}}{ad-bc}\right.$
default	$2d^3\left(\frac{\sqrt{dx+c}}{6(ad-bc)((dx+c)b+ad-bc)^3}+\frac{\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2}+\frac{5\left(\frac{3\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)}+\frac{3\arctan\left(\frac{b\sqrt{dx-}}{\sqrt{(ad-}}\right)}{8(ad-bc)\sqrt{(ad-}}\right)}{6(ad-bc)}}{ad-bc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(1/6*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^3+5/6/(a*d-b*c)*(1/4
*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^2+3/4/(a*d-b*c)*(1/2*(d*x+c)^(
1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan
(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(123) = 246$.
time = 0.31, size = 884, normalized size = 6.01

$$\left[-\frac{15\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(8\,b^4c^3-34\,ab^3c^2d+59\,a^2b^2cd^2-33\,a^3bd^3+15\,(b^4cd^2-ab^3d^3)x^2-10\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^3+3\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^2+3\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x)},\ -\frac{15\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(8\,b^4c^3-34\,ab^3c^2d+59\,a^2b^2cd^2-33\,a^3bd^3+15\,(b^4cd^2-ab^3d^3)x^2-10\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^3+3\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^2+3\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b
^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c
))/(b*x + a)) + 2*(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b
*d^3 + 15*(b^4*c*d^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*
c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3
*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^
4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)
, -1/24*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) +
(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b*d^3 + 15*(b^4*c*d
^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqr
t(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*
c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^
5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2
*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*
d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 280, normalized size = 1.90

$$2\left(\frac{-15\sqrt{c+dx}\,(c+dx)^2b^2d^3+40\sqrt{c+dx}\,(c+dx)\,b^2d^3c-40\sqrt{c+dx}\,(c+dx)\,bd^4a-33\sqrt{c+dx}\,b^2d^3c^2+66\sqrt{c+dx}\,bd^4ca-33\sqrt{c+dx}\,d^5a^2}{(48b^3c^3-144b^2dc^2a+144bd^2ca^2-48d^3a^3)\left((c+dx)\,b-bc+da\right)^3}+\frac{5d^3\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\left(-8b^3c^3+24b^2dc^2a-24bd^2ca^2+8d^3a^3\right)\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] -5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/24*(15*(d*x + c)^
(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 33*sqrt(d*x + c)*b^2*c^2*d^3
 + 40*(d*x + c)^(3/2)*a*b*d^4 - 66*sqrt(d*x + c)*a*b*c*d^4 + 33*sqrt(d*x +
c)*a^2*d^5)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)
*b - b*c + a*d)^3)
```

Mupad [B]
time = 0.39, size = 218, normalized size = 1.48

$$\frac{\frac{11\,d^3\,\sqrt{c+d\,x}}{8\,\left(a\,d-b\,c\right)}+\frac{5\,b^2\,d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,{\left(a\,d-b\,c\right)}^3}+\frac{5\,b\,d^3\,{\left(c+d\,x\right)}^{3/2}}{3\,{\left(a\,d-b\,c\right)}^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}+\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(1/2)),x)
```

```
[Out] ((11*d^3*(c + d*x)^(1/2))/(8*(a*d - b*c)) + (5*b^2*d^3*(c + d*x)^(5/2))/(8*
(a*d - b*c)^3) + (5*b*d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)^2))/((c + d*x)*(3
*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^
2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (5*
d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(7/2))
```

3.1423 $\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx$

Optimal. Leaf size=180

$$-\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4}+\frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3}-\frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2}+\frac{35d^3\sqrt{c+dx}}{64(bc-ad)^4(a+bx)}-\frac{35d^4\tanh^{-1}}{64\sqrt{b}}$$

```
[Out] -35/64*d^4*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(9/2)
/b^(1/2)-1/4*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^4+7/24*d*(d*x+c)^(1/2)/(-a*d+
b*c)^2/(b*x+a)^3-35/96*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^2+35/64*d^3*(
d*x+c)^(1/2)/(-a*d+b*c)^4/(b*x+a)
```

Rubi [A]

time = 0.05, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$-\frac{35d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}\,(bc-ad)^{9/2}}+\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] -1/4*Sqrt[c + d*x]/((b*c - a*d)*(a + b*x)^4) + (7*d*Sqrt[c + d*x])/(24*(b*c
 - a*d)^2*(a + b*x)^3) - (35*d^2*Sqrt[c + d*x])/(96*(b*c - a*d)^3*(a + b*x)
^2) + (35*d^3*Sqrt[c + d*x])/(64*(b*c - a*d)^4*(a + b*x)) - (35*d^4*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(64*Sqrt[b]*(b*c - a*d)^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} - \frac{(7d)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} + \frac{(35d^2)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} - \frac{(35d^3}{}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35}{64(bc}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35}{64(bc}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35}{64(bc}$$

Mathematica [A]
time = 0.52, size = 166, normalized size = 0.92

$$\frac{1}{192}\left(\frac{\sqrt{c+dx}\,\left(279a^3d^3+a^2bd^2(-326c+511dx)+ab^2d\left(200c^2-252cdx+385d^2x^2\right)+b^3\left(-48c^3+56c^2dx-70cd^2x^2+105d^3x^3\right)\right)}{(bc-ad)^4(a+bx)^4}+\frac{105d^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{9/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] ((Sqrt[c + d*x]*(279*a^3*d^3 + a^2*b*d^2*(-326*c + 511*d*x) + a*b^2*d*(200*
c^2 - 252*c*d*x + 385*d^2*x^2) + b^3*(-48*c^3 + 56*c^2*d*x - 70*c*d^2*x^2 +
 105*d^3*x^3)))/((b*c - a*d)^4*(a + b*x)^4) + (105*d^4*ArcTan[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(9/2)))/192
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^5*(c + d*x)^(1/2)),x]')

[Out] Timed out

Maple [A]

time = 0.16, size = 236, normalized size = 1.31

method	result
derivativedivides	$2d^4 \left(\frac{\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)^4} + \frac{\frac{7\sqrt{dx+c}}{48(ad-bc)((dx+c)b+ad-bc)^3} + \frac{7\left(\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2} + \frac{5\left(\frac{3\sqrt{dx}}{8(ad-bc)((dx+}\right.}{}\right.}{8(ad-}}{ad-bc}\right.$
default	$2d^4 \left(\frac{\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)^4} + \frac{\frac{7\sqrt{dx+c}}{48(ad-bc)((dx+c)b+ad-bc)^3} + \frac{7\left(\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2} + \frac{5\left(\frac{3\sqrt{dx}}{8(ad-bc)((dx+}\right.}{}\right.}{8(ad-}}{ad-bc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^5/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] 2*d^4*(1/8*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^4+7/8/(a*d-b*c)*(1/6
*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^3+5/6/(a*d-b*c)*(1/4*(d*x+c)^(
1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^2+3/4/(a*d-b*c)*(1/2*(d*x+c)^(1/2)/(a*d-
b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(152) = 304.
time = 0.32, size = 1325, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] [1/384*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(48*b^5*c^4 - 248*a*b^4*c^3*d + 526*
a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*(b^5*c*d^3 - a*b^
4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 - 7*(
8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqr
t(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^
3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^
2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^4 + 4
*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5
*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a
^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^2 +
4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3
```

```
+ 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x), 1/192*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*
x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arc
tan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (48*b^5*c^4 - 248*a
*b^4*c^3*d + 526*a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*
(b^5*c*d^3 - a*b^4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b
^3*d^4)*x^2 - 7*(8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^
3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^
3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*
b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5
*b^5*d^5)*x^4 + 4*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^
4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3
*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^
7*b^3*d^5)*x^2 + 4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10
*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**5/(d*x+c)**(1/2),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(152) = 304.
time = 0.00, size = 409, normalized size = 2.27

$$2\left(\frac{105\sqrt{c+dx}\,(c+dx)^3b^3d^4-385\sqrt{c+dx}\,(c+dx)^2b^3d^4c+385\sqrt{c+dx}\,(c+dx)^2b^2d^5a+511\sqrt{c+dx}\,(c+dx)b^3d^4c^2-1022\sqrt{c+dx}\,(c+dx)b^2d^5ca+511\sqrt{c+dx}\,(c+dx)bd^6a^2-279\sqrt{c+dx}\,b^3d^4c^3+837\sqrt{c+dx}\,b^2d^5c^2a-837\sqrt{c+dx}\,bd^6ca^2+279\sqrt{c+dx}\,d^7a^3}{(384b^4c^4-1536b^3dc^3a+2304b^2d^2c^2a^2-1536bd^3ca^3+384d^4a^4)((c+dx)b-bc+da)^4}+\frac{35d^4\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2(64b^4c^4-256b^3dc^3a+384b^2d^2c^2a^2-256bd^3ca^3+64d^4a^4)\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x)
[Out] 35/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d))
+ 1/192*(105*(d*x + c)^(7/2)*b^3*d^4 - 385*(d*x + c)^(5/2)*b^3*c*d^4 + 511*
(d*x + c)^(3/2)*b^3*c^2*d^4 - 279*sqrt(d*x + c)*b^3*c^3*d^4 + 385*(d*x + c)
^(5/2)*a*b^2*d^5 - 1022*(d*x + c)^(3/2)*a*b^2*c*d^5 + 837*sqrt(d*x + c)*a*b
^2*c^2*d^5 + 511*(d*x + c)^(3/2)*a^2*b*d^6 - 837*sqrt(d*x + c)*a^2*b*c*d^6
+ 279*sqrt(d*x + c)*a^3*d^7)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b - b*c + a*d)^4)
```

Mupad [**B**]
time = 0.46, size = 307, normalized size = 1.71

$$\frac{\frac{93\,d^4\,\sqrt{c+d\,x}}{64\,(a\,d-b\,c)}+\frac{385\,b^2\,d^4\,(c+d\,x)^{5/2}}{192\,(a\,d-b\,c)^3}+\frac{35\,b^3\,d^4\,(c+d\,x)^{7/2}}{64\,(a\,d-b\,c)^4}+\frac{511\,b\,d^4\,(c+d\,x)^{3/2}}{192\,(a\,d-b\,c)^2}}{b^4\,(c+d\,x)^4-(4\,b^4\,c-4\,a\,b^3\,d)\,(c+d\,x)^3-(c+d\,x)\,(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3)+a^4\,d^4+b^4\,c^4+(c+d\,x)^2\,(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{35\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,\sqrt{b}\,(a\,d-b\,c)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^5*(c + d*x)^(1/2)),x)
```

```
[Out] ((93*d^4*(c + d*x)^(1/2))/(64*(a*d - b*c)) + (385*b^2*d^4*(c + d*x)^(5/2))/
(192*(a*d - b*c)^3) + (35*b^3*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^4) + (51
1*b*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c -
 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c
*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2
*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3) + (35*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(1/2)
*(a*d - b*c)^(9/2))
```

3.1424 $\int \frac{(a+bx)^5}{(c+dx)^{3/2}} \, dx$

Optimal. Leaf size=152

$$\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+dx)^{5/2}}{d^6}-\frac{10b^4(bc-ad)}{7d}$$

```
[Out] -20/3*b^2*(-a*d+b*c)^3*(d*x+c)^(3/2)/d^6+4*b^3*(-a*d+b*c)^2*(d*x+c)^(5/2)/d
^6-10/7*b^4*(-a*d+b*c)*(d*x+c)^(7/2)/d^6+2/9*b^5*(d*x+c)^(9/2)/d^6+2*(-a*d+
b*c)^5/d^6/(d*x+c)^(1/2)+10*b*(-a*d+b*c)^4*(d*x+c)^(1/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}\,(bc-ad)^4}{d^6}+\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{2b^5(c+dx)^{9/2}}{9d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(d^6*Sqrt[c + d*x]) + (10*b*(b*c - a*d)^4*Sqrt[c + d*x])/
d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(b*c - a*d)^2
*(c + d*x)^(5/2))/d^6 - (10*b^4*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^6) + (2*b
^5*(c + d*x)^(9/2))/(9*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{3/2}} \, dx = \int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{3/2}}+\frac{5b(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{10b^2(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)}{d^5}\right.$$

$$=\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+}{d^6}$$

Mathematica [A]
time = 0.11, size = 214, normalized size = 1.41

$$\frac{2\left(-63a^5d^5+315a^4bd^4(2c+dx)+210a^3b^2d^3\left(-8c^2-4cdx+d^2x^2\right)+126a^2b^3d^2\left(16c^3+8c^2dx-2cd^2x^2+d^3x^3\right)+9ab^4d\left(-128c^4-64c^3dx+16c^2d^2x^2-8cd^3x^3+5d^4x^4\right)+b^5\left(256c^5+128c^4dx-32c^3d^2x^2+16c^2d^3x^3-10cd^4x^4+7d^5x^5\right)\right)}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-63*a^5*d^5 + 315*a^4*b*d^4*(2*c + d*x) + 210*a^3*b^2*d^3*(-8*c^2 - 4*c
*d*x + d^2*x^2) + 126*a^2*b^3*d^2*(16*c^3 + 8*c^2*d*x - 2*c*d^2*x^2 + d^3*x
^3) + 9*a*b^4*d*(-128*c^4 - 64*c^3*d*x + 16*c^2*d^2*x^2 - 8*c*d^3*x^3 + 5*d
^4*x^4) + b^5*(256*c^5 + 128*c^4*d*x - 32*c^3*d^2*x^2 + 16*c^2*d^3*x^3 - 10
*c*d^4*x^4 + 7*d^5*x^5)))/(63*d^6*Sqrt[c + d*x])
```

Mathics [A]
time = 21.12, size = 192, normalized size = 1.26

$$\frac{2\left(b\left(315a^4d^4+210b\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)(c+dx)-1260a^3bcd^3+126b^2\left(a^2d^2-2abcd+b^2c^2\right)(c+dx)^2+1890a^2b^2c^2d^2+45b^3(ad-bc)(c+dx)^3-1260ab^3c^3d+7b^4(c+dx)^4+315b^4c^4\right)(c+dx)-63(ad-bc)^5\right)}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x)^(3/2),x]')
```

```
[Out] 2 (b (315 a ^ 4 d ^ 4 + 210 b (a ^ 3 d ^ 3 - 3 a ^ 2 b c d ^ 2 + 3 a b ^ 2
c ^ 2 d - b ^ 3 c ^ 3) (c + d x) - 1260 a ^ 3 b c d ^ 3 + 126 b ^ 2 (a ^ 2
d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (c + d x) ^ 2 + 1890 a ^ 2 b ^ 2 c ^ 2 d ^
 2 + 45 b ^ 3 (a d - b c) (c + d x) ^ 3 - 1260 a b ^ 3 c ^ 3 d + 7 b ^ 4 (c
 + d x) ^ 4 + 315 b ^ 4 c ^ 4) (c + d x) - 63 (a d - b c) ^ 5) / (63 d ^ 6
Sqrt[c + d x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(134) = 268$.
time = 0.15, size = 324, normalized size = 2.13

method	result
risch	$\frac{2b(7d^4x^4b^4+45ab^3d^4x^3-17b^4cd^3x^3+126a^2b^2d^4x^2-117ab^3cd^3x^2+33b^4c^2d^2x^2+210a^3bd^4x-378a^2b^2cd^3x+261ab^3c^2d}{63d^6}$
gosper	$-\frac{2(-7b^5x^5d^5-45ab^4d^5x^4+10b^5cd^4x^4-126a^2b^3d^5x^3+72ab^4cd^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+252a^2b^3cd^4x^2-144a}{}$
trager	$-\frac{2(-7b^5x^5d^5-45ab^4d^5x^4+10b^5cd^4x^4-126a^2b^3d^5x^3+72ab^4cd^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+252a^2b^3cd^4x^2-144a}{}$
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{9}{2}}}{9}+\frac{10ab^4d(dx+c)^{\frac{7}{2}}}{7}-\frac{10b^5c(dx+c)^{\frac{7}{2}}}{7}+4a^2b^3d^2(dx+c)^{\frac{5}{2}}-8ab^4cd(dx+c)^{\frac{5}{2}}+4b^5c^2(dx+c)^{\frac{5}{2}}+\frac{20a^3b^2d^3(dx+c)^{\frac{3}{2}}}{3}-20}{}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{9}{2}}}{9}+\frac{10ab^4d(dx+c)^{\frac{7}{2}}}{7}-\frac{10b^5c(dx+c)^{\frac{7}{2}}}{7}+4a^2b^3d^2(dx+c)^{\frac{5}{2}}-8ab^4cd(dx+c)^{\frac{5}{2}}+4b^5c^2(dx+c)^{\frac{5}{2}}+\frac{20a^3b^2d^3(dx+c)^{\frac{3}{2}}}{3}-20}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/9*b^5*(d*x+c)^(9/2)+5/7*a*b^4*d*(d*x+c)^(7/2)-5/7*b^5*c*(d*x+c)^(7
/2)+2*a^2*b^3*d^2*(d*x+c)^(5/2)-4*a*b^4*c*d*(d*x+c)^(5/2)+2*b^5*c^2*(d*x+c)
^(5/2)+10/3*a^3*b^2*d^3*(d*x+c)^(3/2)-10*a^2*b^3*c*d^2*(d*x+c)^(3/2)+10*a*b
^4*c^2*d*(d*x+c)^(3/2)-10/3*b^5*c^3*(d*x+c)^(3/2)+5*a^4*b*d^4*(d*x+c)^(1/2)
-20*a^3*b^2*c*d^3*(d*x+c)^(1/2)+30*a^2*b^3*c^2*d^2*(d*x+c)^(1/2)-20*a*b^4*c
^3*d*(d*x+c)^(1/2)+5*b^5*c^4*(d*x+c)^(1/2)-(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^
2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.29, size = 267, normalized size = 1.76

$$\frac{2\left(\frac{7\,(dx+c)^{\frac{9}{2}}b^5-45\,(b^5c-ab^4d)\,(dx+c)^{\frac{7}{2}}+126\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)\,(dx+c)^{\frac{5}{2}}-210\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)\,(dx+c)^{\frac{3}{2}}+315\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)\,\sqrt{dx+c}}{d^5}+\frac{63\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)}{\sqrt{dx+c}\,d^5}\right)}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*((7*(d*x + c)^(9/2)*b^5 - 45*(b^5*c - a*b^4*d)*(d*x + c)^(7/2) + 126*(
b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(5/2) - 210*(b^5*c^3 - 3*a*b
^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(3/2) + 315*(b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(d*x +
 c))/d^5 + 63*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(134) = 268$.
time = 0.30, size = 271, normalized size = 1.78

$$\frac{2\,(7\,b^5d^5x^5+256\,b^5c^5-1152\,ab^4c^4d+2016\,a^2b^3c^3d^2-1680\,a^3b^2c^2d^3+630\,a^4bcd^4-63\,a^5d^5-5\,(2\,b^5cd^4-9\,ab^4d^5)x^4+2\,(8\,b^5c^2d^3-36\,ab^4cd^4+63\,a^2b^3d^5)x^3-2\,(16\,b^5c^3d^2-72\,ab^4c^2d^3+126\,a^2b^3cd^4-105\,a^3b^2d^5)x^2+(128\,b^5c^4d-576\,ab^4c^3d^2+1008\,a^2b^3c^2d^3-840\,a^3b^2cd^4+315\,a^4bd^5)x)\sqrt{dx+c}}{63\,(d^7x+cd^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^5*d^5*x^5 + 256*b^5*c^5 - 1152*a*b^4*c^4*d + 2016*a^2*b^3*c^3*d^2
 - 1680*a^3*b^2*c^2*d^3 + 630*a^4*b*c*d^4 - 63*a^5*d^5 - 5*(2*b^5*c*d^4 - 9
*a*b^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 36*a*b^4*c*d^4 + 63*a^2*b^3*d^5)*x^3 -
 2*(16*b^5*c^3*d^2 - 72*a*b^4*c^2*d^3 + 126*a^2*b^3*c*d^4 - 105*a^3*b^2*d^5
)*x^2 + (128*b^5*c^4*d - 576*a*b^4*c^3*d^2 + 1008*a^2*b^3*c^2*d^3 - 840*a^3
*b^2*c*d^4 + 315*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^7*x + c*d^6)
```

Sympy [A]
time = 19.45, size = 243, normalized size = 1.60

$$\frac{2b^5\,(c+dx)^{\frac{9}{2}}}{9d^6}+\frac{(c+dx)^{\frac{7}{2}}\cdot(10ab^4d-10b^5c)}{7d^6}+\frac{(c+dx)^{\frac{5}{2}}\cdot(20a^2b^3d^2-40ab^4cd+20b^5c^2)}{5d^6}+\frac{(c+dx)^{\frac{3}{2}}\cdot(20a^3b^2d^3-60a^2b^3cd^2+60ab^4c^2d-20b^5c^3)}{3d^6}+\frac{\sqrt{c+dx}\,(10a^4bd^4-40a^3b^2cd^3+60a^2b^3c^2d^2-40ab^4c^3d+10b^5c^4)}{d^6}-\frac{2\,(ad-bc)^5}{d^6\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(9/2)/(9*d**6) + (c + d*x)**(7/2)*(10*a*b**4*d - 10*b**5*
c)/(7*d**6) + (c + d*x)**(5/2)*(20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5
*c**2)/(5*d**6) + (c + d*x)**(3/2)*(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2
 + 60*a*b**4*c**2*d - 20*b**5*c**3)/(3*d**6) + sqrt(c + d*x)*(10*a**4*b*d**
4 - 40*a**3*b**2*c*d**3 + 60*a**2*b**3*c**2*d**2 - 40*a*b**4*c**3*d + 10*b*
*5*c**4)/d**6 - 2*(a*d - b*c)**5/(d**6*sqrt(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(134) = 268$.
time = 0.01, size = 465, normalized size = 3.06

$$\frac{\frac{2}{9}\sqrt{c+dx}\,(c+dx)^4 b^5 d^{48} - \frac{10}{7}\sqrt{c+dx}\,(c+dx)^3 b^5 c d^{48} + \frac{10}{7}\sqrt{c+dx}\,(c+dx)^3 b^4 d^{49} a + 4\sqrt{c+dx}\,(c+dx)^2 b^5 c^2 d^{48} - 8\sqrt{c+dx}\,(c+dx)^2 b^4 c d^{49} a + 4\sqrt{c+dx}\,(c+dx)^2 b^3 d^{50} a^2 - \frac{20}{3}\sqrt{c+dx}\,(c+dx) b^5 c^3 d^{48} + 20\sqrt{c+dx}\,(c+dx) b^4 c^2 d^{49} a - 20\sqrt{c+dx}\,(c+dx) b^3 c d^{50} a^2 + \frac{20}{3}\sqrt{c+dx}\,(c+dx) b^2 d^{51} a^3 + 10\sqrt{c+dx}\, b^5 c^4 d^{48} - 40\sqrt{c+dx}\, b^4 c^3 d^{49} a + 60\sqrt{c+dx}\, b^3 c^2 d^{50} a^2 - 40\sqrt{c+dx}\, b^2 c d^{51} a^3 + 10\sqrt{c+dx}\, b d^{52} a^4}{d^{54}} + \frac{2b^5c^5 - 10b^4c^4da + 20b^3c^3d^2a^2 - 20b^2c^2d^3a^3 + 10bcd^4a^4 - 2d^5a^5}{d^6\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x)
```

```
[Out] 2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^
4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^6) + 2/63*(7*(d*x + c)^(9/2)*b^5*d^48
 - 45*(d*x + c)^(7/2)*b^5*c*d^48 + 126*(d*x + c)^(5/2)*b^5*c^2*d^48 - 210*(
d*x + c)^(3/2)*b^5*c^3*d^48 + 315*sqrt(d*x + c)*b^5*c^4*d^48 + 45*(d*x + c)
^(7/2)*a*b^4*d^49 - 252*(d*x + c)^(5/2)*a*b^4*c*d^49 + 630*(d*x + c)^(3/2)*
a*b^4*c^2*d^49 - 1260*sqrt(d*x + c)*a*b^4*c^3*d^49 + 126*(d*x + c)^(5/2)*a^
2*b^3*d^50 - 630*(d*x + c)^(3/2)*a^2*b^3*c*d^50 + 1890*sqrt(d*x + c)*a^2*b^
3*c^2*d^50 + 210*(d*x + c)^(3/2)*a^3*b^2*d^51 - 1260*sqrt(d*x + c)*a^3*b^2*
c*d^51 + 315*sqrt(d*x + c)*a^4*b*d^52)/d^54
```

Mupad [B]
time = 0.08, size = 192, normalized size = 1.26

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{2\,a^5\,d^5-10\,a^4\,b\,c\,d^4+20\,a^3\,b^2\,c^2\,d^3-20\,a^2\,b^3\,c^3\,d^2+10\,a\,b^4\,c^4\,d-2\,b^5\,c^5}{d^6\,\sqrt{c+d\,x}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{4\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(9/2))/(9*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(7/2))
/(7*d^6) - (2*a^5*d^5 - 2*b^5*c^5 - 20*a^2*b^3*c^3*d^2 + 20*a^3*b^2*c^2*d^3
 + 10*a*b^4*c^4*d - 10*a^4*b*c*d^4)/(d^6*(c + d*x)^(1/2)) + (20*b^2*(a*d -
b*c)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(a*d - b*c)^2*(c + d*x)^(5/2))/d^6
 + (10*b*(a*d - b*c)^4*(c + d*x)^(1/2))/d^6
```

3.1425 $\int \frac{(a+bx)^4}{(c+dx)^{3/2}} \, dx$

Optimal. Leaf size=123

$$-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{5/2}}{5d^5}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

```
[Out] 4*b^2*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^5-8/5*b^3*(-a*d+b*c)*(d*x+c)^(5/2)/d^5+2
/7*b^4*(d*x+c)^(7/2)/d^5-2*(-a*d+b*c)^4/d^5/(d*x+c)^(1/2)-8*b*(-a*d+b*c)^3*
(d*x+c)^(1/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}\,(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(d^5*Sqrt[c + d*x]) - (8*b*(b*c - a*d)^3*Sqrt[c + d*x])/
d^5 + (4*b^2*(b*c - a*d)^2*(c + d*x)^(3/2))/d^5 - (8*b^3*(b*c - a*d)*(c + d
*x)^(5/2))/(5*d^5) + (2*b^4*(c + d*x)^(7/2))/(7*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{3/2}}-\frac{4b(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b^2(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{3/2}}{d^4}\right.\\ &= -\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+}{5d^5}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 151, normalized size = 1.23

$$\frac{2\left(-35a^4d^4+140a^3bd^3(2c+dx)+70a^2b^2d^2\left(-8c^2-4cdx+d^2x^2\right)+28ab^3d\left(16c^3+8c^2dx-2cd^2x^2+d^3x^3\right)+b^4\left(-128c^4-64c^3dx+16c^2d^2x^2-8cd^3x^3+5d^4x^4\right)\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*a^4*d^4 + 140*a^3*b*d^3*(2*c + d*x) + 70*a^2*b^2*d^2*(-8*c^2 - 4*c*
d*x + d^2*x^2) + 28*a*b^3*d*(16*c^3 + 8*c^2*d*x - 2*c*d^2*x^2 + d^3*x^3) +
b^4*(-128*c^4 - 64*c^3*d*x + 16*c^2*d^2*x^2 - 8*c*d^3*x^3 + 5*d^4*x^4)))/(3
5*d^5*Sqrt[c + d*x])
```

Mathics [A]
time = 14.84, size = 130, normalized size = 1.06

$$\frac{2\left(b\left(140a^3d^3+70b\left(a^2d^2-2abcd+b^2c^2\right)(c+dx)-420a^2bcd^2+28b^2\left(ad-bc\right)(c+dx)^2+420ab^2c^2d+5b^3\left(c+dx\right)^3-140b^3c^3\right)(c+dx)-35\left(ad-bc\right)^4\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x)^(3/2),x]')
```

```
[Out] 2 (b (140 a ^ 3 d ^ 3 + 70 b (a ^ 2 d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (c + d
 x) - 420 a ^ 2 b c d ^ 2 + 28 b ^ 2 (a d - b c) (c + d x) ^ 2 + 420 a b ^
2 c ^ 2 d + 5 b ^ 3 (c + d x) ^ 3 - 140 b ^ 3 c ^ 3) (c + d x) - 35 (a d -
b c) ^ 4) / (35 d ^ 5 Sqrt[c + d x])
```

Maple [A]
time = 0.16, size = 219, normalized size = 1.78

method	result
risch	$\frac{2b\left(5b^3x^3d^3+28d^3a\,x^2b^2-13b^3c\,d^2x^2+70a^2b\,d^3x-84a\,b^2c\,d^2x+29b^3c^2dx+140a^3d^3-350a^2bc\,d^2+308a\,b^2c^2d-93b^3c^3\right)\sqrt{c}}{35d^5}$
gosper	$-\frac{2\left(-5d^4x^4b^4-28a\,b^3d^4x^3+8b^4c\,d^3x^3-70a^2b^2d^4x^2+56a\,b^3c\,d^3x^2-16b^4c^2d^2x^2-140a^3b\,d^4x+280a^2b^2c\,d^3x-224a\,b^3c^2d\right.}{35\sqrt{dx+c}\;d^5}$
trager	$-\frac{2\left(-5d^4x^4b^4-28a\,b^3d^4x^3+8b^4c\,d^3x^3-70a^2b^2d^4x^2+56a\,b^3c\,d^3x^2-16b^4c^2d^2x^2-140a^3b\,d^4x+280a^2b^2c\,d^3x-224a\,b^3c^2d\right.}{35\sqrt{dx+c}\;d^5}$
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{7}{2}}}{7}+\frac{8a\,b^3d(dx+c)^{\frac{5}{2}}}{5}-\frac{8b^4c(dx+c)^{\frac{5}{2}}}{5}+4a^2b^2d^2(dx+c)^{\frac{3}{2}}-8a\,b^3cd(dx+c)^{\frac{3}{2}}+4b^4c^2(dx+c)^{\frac{3}{2}}+8a^3b\,d^3\sqrt{dx+c}-2}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{7}{2}}}{7}+\frac{8a\,b^3d(dx+c)^{\frac{5}{2}}}{5}-\frac{8b^4c(dx+c)^{\frac{5}{2}}}{5}+4a^2b^2d^2(dx+c)^{\frac{3}{2}}-8a\,b^3cd(dx+c)^{\frac{3}{2}}+4b^4c^2(dx+c)^{\frac{3}{2}}+8a^3b\,d^3\sqrt{dx+c}-2}{d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/7*b^4*(d*x+c)^(7/2)+4/5*a*b^3*d*(d*x+c)^(5/2)-4/5*b^4*c*(d*x+c)^(5
/2)+2*a^2*b^2*d^2*(d*x+c)^(3/2)-4*a*b^3*c*d*(d*x+c)^(3/2)+2*b^4*c^2*(d*x+c)
^(3/2)+4*a^3*b*d^3*(d*x+c)^(1/2)-12*a^2*b^2*c*d^2*(d*x+c)^(1/2)+12*a*b^3*c^
2*d*(d*x+c)^(1/2)-4*b^4*c^3*(d*x+c)^(1/2)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 189, normalized size = 1.54

$$\frac{2\left(\frac{5(dx+c)^{\frac{7}{2}}b^4-28(b^4c-ab^3d)(dx+c)^{\frac{5}{2}}+70(b^4c^2-2ab^3cd+a^2b^2d^2)(dx+c)^{\frac{3}{2}}-140(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)\sqrt{dx+c}}{d^4}-\frac{35(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)}{\sqrt{dx+c}\,d^4}\right)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(d*x + c)^(7/2)*b^4 - 28*(b^4*c - a*b^3*d)*(d*x + c)^(5/2) + 70*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(3/2) - 140*(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(d*x + c))/d^4 - 35*(b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(sqrt(d*x + c)*
d^4))/d
```

Fricas [A]
time = 0.29, size = 192, normalized size = 1.56

$$\frac{2\,(5\,b^4d^4x^4-128\,b^4c^4+448\,ab^3c^3d-560\,a^2b^2c^2d^2+280\,a^3bcd^3-35\,a^4d^4-4\,(2\,b^4cd^3-7\,ab^3d^4)x^3+2\,(8\,b^4c^2d^2-28\,ab^3cd^3+35\,a^2b^2d^4)x^2-4\,(16\,b^4c^3d-56\,ab^3c^2d^2+70\,a^2b^2cd^3-35\,a^3bd^4)x)\sqrt{dx+c}}{35\,(d^6x+cd^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*d^4*x^4 - 128*b^4*c^4 + 448*a*b^3*c^3*d - 560*a^2*b^2*c^2*d^2 +
 280*a^3*b*c*d^3 - 35*a^4*d^4 - 4*(2*b^4*c*d^3 - 7*a*b^3*d^4)*x^3 + 2*(8*b^
4*c^2*d^2 - 28*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 56*a*b
^3*c^2*d^2 + 70*a^2*b^2*c*d^3 - 35*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^6*x + c*d
^5)
```

Sympy [A]
time = 14.34, size = 168, normalized size = 1.37

$$\frac{2b^4(c+dx)^{\frac{7}{2}}}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\cdot(8ab^3d-8b^4c)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\cdot(12a^2b^2d^2-24ab^3cd+12b^4c^2)}{3d^5}+\frac{\sqrt{c+dx}\,(8a^3bd^3-24a^2b^2cd^2+24ab^3c^2d-8b^4c^3)}{d^5}-\frac{2\,(ad-bc)^4}{d^5\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(7/2)/(7*d**5) + (c + d*x)**(5/2)*(8*a*b**3*d - 8*b**4*c)
/(5*d**5) + (c + d*x)**(3/2)*(12*a**2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c
**2)/(3*d**5) + sqrt(c + d*x)*(8*a**3*b*d**3 - 24*a**2*b**2*c*d**2 + 24*a*b
**3*c**2*d - 8*b**4*c**3)/d**5 - 2*(a*d - b*c)**4/(d**5*sqrt(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(109) = 218.
time = 0.00, size = 314, normalized size = 2.55

$$\frac{\frac{2}{7}\sqrt{c+dx}\,(c+dx)^3b^4d^{30}-\frac{8}{5}\sqrt{c+dx}\,(c+dx)^2b^4cd^{30}+\frac{8}{5}\sqrt{c+dx}\,(c+dx)^2b^3d^{31}a+4\sqrt{c+dx}\,(c+dx)b^4c^2d^{30}-8\sqrt{c+dx}\,(c+dx)b^3cd^{31}a+4\sqrt{c+dx}\,(c+dx)b^2d^{32}a^2-8\sqrt{c+dx}\,b^4c^3d^{30}+24\sqrt{c+dx}\,b^3c^2d^{31}a-24\sqrt{c+dx}\,b^2cd^{32}a^2+8\sqrt{c+dx}\,bd^{33}a^3}{d^{35}}+\frac{-2b^4c^4+8b^3c^3da-12b^2c^2d^2a^2+8bcd^3a^3-2d^4a^4}{d^5\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
(sqrt(d*x + c)*d^5) + 2/35*(5*(d*x + c)^(7/2)*b^4*d^30 - 28*(d*x + c)^(5/2)
*b^4*c*d^30 + 70*(d*x + c)^(3/2)*b^4*c^2*d^30 - 140*sqrt(d*x + c)*b^4*c^3*d
^30 + 28*(d*x + c)^(5/2)*a*b^3*d^31 - 140*(d*x + c)^(3/2)*a*b^3*c*d^31 + 42
0*sqrt(d*x + c)*a*b^3*c^2*d^31 + 70*(d*x + c)^(3/2)*a^2*b^2*d^32 - 420*sqrt
(d*x + c)*a^2*b^2*c*d^32 + 140*sqrt(d*x + c)*a^3*b*d^33)/d^35
```

Mupad [B]
time = 0.06, size = 153, normalized size = 1.24

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{2\,a^4\,d^4-8\,a^3\,b\,c\,d^3+12\,a^2\,b^2\,c^2\,d^2-8\,a\,b^3\,c^3\,d+2\,b^4\,c^4}{d^5\,\sqrt{c+d\,x}}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(7/2))/(7*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(5/2))/(
5*d^5) - (2*a^4*d^4 + 2*b^4*c^4 + 12*a^2*b^2*c^2*d^2 - 8*a*b^3*c^3*d - 8*a^
3*b*c*d^3)/(d^5*(c + d*x)^(1/2)) + (4*b^2*(a*d - b*c)^2*(c + d*x)^(3/2))/d^
5 + (8*b*(a*d - b*c)^3*(c + d*x)^(1/2))/d^5
```

3.1426 $\int \frac{(a+bx)^3}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=94

$$\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

```
[Out] -2*b^2*(-a*d+b*c)*(d*x+c)^(3/2)/d^4+2/5*b^3*(d*x+c)^(5/2)/d^4+2*(-a*d+b*c)^
3/d^4/(d*x+c)^(1/2)+6*b*(-a*d+b*c)^2*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}\,(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(d^4*Sqrt[c + d*x]) + (6*b*(b*c - a*d)^2*Sqrt[c + d*x])/d
^4 - (2*b^2*(b*c - a*d)*(c + d*x)^(3/2))/d^4 + (2*b^3*(c + d*x)^(5/2))/(5*d
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{3/2}}+\frac{3b(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{3b^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{b^3(c+dx)^{3/2}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 1.05

$$\frac{2\left(-5a^3d^3+15a^2bd^2(2c+dx)+5ab^2d\left(-8c^2-4cdx+d^2x^2\right)+b^3\left(16c^3+8c^2dx-2cd^2x^2+d^3x^3\right)\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/(c + d*x)^(3/2),x]

[Out] (2*(-5*a^3*d^3 + 15*a^2*b*d^2*(2*c + d*x) + 5*a*b^2*d*(-8*c^2 - 4*c*d*x + d^2*x^2) + b^3*(16*c^3 + 8*c^2*d*x - 2*c*d^2*x^2 + d^3*x^3)))/(5*d^4*Sqrt[c + d*x])

Mathics [A]
time = 10.56, size = 82, normalized size = 0.87

$$\frac{2\left(b\left(15a^2d^2+5b\left(ad-bc\right)\left(c+dx\right)-30abcd+b^2\left(c+dx\right)^2+15b^2c^2\right)\left(c+dx\right)-5\left(ad-bc\right)^3\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^3/(c + d*x)^(3/2),x]')

[Out] 2 (b (15 a ^ 2 d ^ 2 + 5 b (a d - b c) (c + d x) - 30 a b c d + b ^ 2 (c + d x) ^ 2 + 15 b ^ 2 c ^ 2) (c + d x) - 5 (a d - b c) ^ 3) / (5 d ^ 4 Sqrt[c + d x])

Maple [A]
time = 0.19, size = 136, normalized size = 1.45

method	result
risch	$\frac{2b\left(b^2x^2d^2+5abd^2x-3b^2cdx+15a^2d^2-25abcd+11b^2c^2\right)\sqrt{dx+c}}{5d^4}-\frac{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{d^4\sqrt{dx+c}}$
gosper	$-\frac{2\left(-b^3x^3d^3-5d^3ax^2b^2+2b^3cd^2x^2-15a^2bd^3x+20ab^2cd^2x-8b^3c^2dx+5a^3d^3-30a^2bcd^2+40ab^2c^2d-16b^3c^3\right)}{5\sqrt{dx+c}\,d^4}$
trager	$-\frac{2\left(-b^3x^3d^3-5d^3ax^2b^2+2b^3cd^2x^2-15a^2bd^3x+20ab^2cd^2x-8b^3c^2dx+5a^3d^3-30a^2bcd^2+40ab^2c^2d-16b^3c^3\right)}{5\sqrt{dx+c}\,d^4}$
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{5}{2}}}{5}+2ab^2d(dx+c)^{\frac{3}{2}}-2b^3c(dx+c)^{\frac{3}{2}}+6a^2bd^2\sqrt{dx+c}-12ab^2cd\sqrt{dx+c}+6b^3c^2\sqrt{dx+c}-\frac{2\left(a^3d^3-3\right.}{}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{5}{2}}}{5}+2ab^2d(dx+c)^{\frac{3}{2}}-2b^3c(dx+c)^{\frac{3}{2}}+6a^2bd^2\sqrt{dx+c}-12ab^2cd\sqrt{dx+c}+6b^3c^2\sqrt{dx+c}-\frac{2\left(a^3d^3-3\right.}{}}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d^4*(1/5*b^3*(d*x+c)^(5/2)+a*b^2*d*(d*x+c)^(3/2)-b^3*c*(d*x+c)^(3/2)+3*a^2*b*d^2*(d*x+c)^(1/2)-6*a*b^2*c*d*(d*x+c)^(1/2)+3*b^3*c^2*(d*x+c)^(1/2)-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^(1/2))

Maxima [A]
time = 0.28, size = 125, normalized size = 1.33

$$\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^3-5\,(b^3c-ab^2d)(dx+c)^{\frac{3}{2}}+15\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{dx+c}}{d^3}+\frac{5\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}{\sqrt{dx+c}\,d^3}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(((d*x + c)^(5/2)*b^3 - 5*(b^3*c - a*b^2*d)*(d*x + c)^(3/2) + 15*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(d*x + c))/d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^3))/d
```

Fricas [A]
time = 0.29, size = 124, normalized size = 1.32

$$\frac{2\,(b^3d^3x^3+16\,b^3c^3-40\,ab^2c^2d+30\,a^2bcd^2-5\,a^3d^3-(2\,b^3cd^2-5\,ab^2d^3)x^2+(8\,b^3c^2d-20\,ab^2cd^2+15\,a^2bd^3)x)\sqrt{dx+c}}{5\,(d^5x+cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*d^3*x^3 + 16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3
 - (2*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 20*a*b^2*c*d^2 + 15*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^5*x + c*d^4)
```

Sympy [A]
time = 10.18, size = 109, normalized size = 1.16

$$\frac{2b^3\,(c+dx)^{\frac{5}{2}}}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\cdot(6ab^2d-6b^3c)}{3d^4}+\frac{\sqrt{c+dx}\,\left(6a^2bd^2-12ab^2cd+6b^3c^2\right)}{d^4}-\frac{2\,(ad-bc)^3}{d^4\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**3*(c + d*x)**(5/2)/(5*d**4) + (c + d*x)**(3/2)*(6*a*b**2*d - 6*b**3*c)
/(3*d**4) + sqrt(c + d*x)*(6*a**2*b*d**2 - 12*a*b**2*c*d + 6*b**3*c**2)/d**
4 - 2*(a*d - b*c)**3/(d**4*sqrt(c + d*x))
```

Giac [A]
time = 0.00, size = 192, normalized size = 2.04

$$\frac{\frac{2}{5}\sqrt{c+dx}\,(c+dx)^2\,b^3d^{16}-2\sqrt{c+dx}\,(c+dx)\,b^3cd^{16}+2\sqrt{c+dx}\,(c+dx)\,b^2d^{17}a+6\sqrt{c+dx}\,b^3c^2d^{16}-12\sqrt{c+dx}\,b^2cd^{17}a+6\sqrt{c+dx}\,bd^{18}a^2}{d^{20}}+\frac{2b^3c^3-6b^2c^2da+6bcd^2a^2-2d^3a^3}{d^4\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^4) +
 2/5*((d*x + c)^(5/2)*b^3*d^16 - 5*(d*x + c)^(3/2)*b^3*c*d^16 + 15*sqrt(d*x
 + c)*b^3*c^2*d^16 + 5*(d*x + c)^(3/2)*a*b^2*d^17 - 30*sqrt(d*x + c)*a*b^2*
c*d^17 + 15*sqrt(d*x + c)*a^2*b*d^18)/d^20
```

Mupad [B]
time = 0.08, size = 114, normalized size = 1.21

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}-\frac{2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-2\,b^3\,c^3}{d^4\,\sqrt{c+d\,x}}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(5/2))/(5*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(3/2))/(
3*d^4) - (2*a^3*d^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(d^4*(c +
d*x)^(1/2)) + (6*b*(a*d - b*c)^2*(c + d*x)^(1/2))/d^4
```

3.1427 $\int \frac{(a+bx)^2}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=67

$$-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

```
[Out] 2/3*b^2*(d*x+c)^(3/2)/d^3-2*(-a*d+b*c)^2/d^3/(d*x+c)^(1/2)-4*b*(-a*d+b*c)*(
d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = $\{45\}$

$$-\frac{4b\sqrt{c+dx}\,(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x]) - (4*b*(b*c - a*d)*Sqrt[c + d*x])/d^
3 + (2*b^2*(c + d*x)^(3/2))/(3*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx\\ &= -\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.88

$$\frac{2\left(-3a^2d^2+6abd(2c+dx)+b^2\left(-8c^2-4cdx+d^2x^2\right)\right)}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2*d^2 + 6*a*b*d*(2*c + d*x) + b^2*(-8*c^2 - 4*c*d*x + d^2*x^2)))/(
3*d^3*Sqrt[c + d*x])
```

Mathics [A]
time = 7.24, size = 48, normalized size = 0.72

$$\frac{2\left(b\left(6ad+b\left(c+dx\right)-6bc\right)\left(c+dx\right)-3\left(ad-bc\right)^{2}\right)}{3d^{3}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x)^(3/2),x]')
```

```
[Out] 2 (b (6 a d + b (c + d x) - 6 b c) (c + d x) - 3 (a d - b c) ^ 2) / (3 d ^
3 Sqrt[c + d x])
```

Maple [A]
time = 0.17, size = 74, normalized size = 1.10

method	result	size
risch	$\frac{2b(bdx+6ad-5bc)\sqrt{dx+c}}{3d^{3}}-\frac{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}{d^{3}\sqrt{dx+c}}$	61
gosper	$-\frac{2(-b^{2}x^{2}d^{2}-6ab\,d^{2}x+4b^{2}cdx+3a^{2}d^{2}-12abcd+8b^{2}c^{2})}{3\sqrt{dx+c}\,d^{3}}$	63
trager	$-\frac{2(-b^{2}x^{2}d^{2}-6ab\,d^{2}x+4b^{2}cdx+3a^{2}d^{2}-12abcd+8b^{2}c^{2})}{3\sqrt{dx+c}\,d^{3}}$	63
derivativedivides	$\frac{\frac{2b^{2}(dx+c)^{\frac{3}{2}}}{3}+4adb\sqrt{dx+c}-4b^{2}c\sqrt{dx+c}-\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{\sqrt{dx+c}}}{d^{3}}$	74
default	$\frac{\frac{2b^{2}(dx+c)^{\frac{3}{2}}}{3}+4adb\sqrt{dx+c}-4b^{2}c\sqrt{dx+c}-\frac{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{\sqrt{dx+c}}}{d^{3}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/3*b^2*(d*x+c)^(3/2)+2*a*d*b*(d*x+c)^(1/2)-2*b^2*c*(d*x+c)^(1/2)-(a
^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 1.12

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^{2}-6\left(b^{2}c-abd\right)\sqrt{dx+c}}{d^{2}}-\frac{3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}{\sqrt{dx+c}\,d^{2}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^2 - 6*(b^2*c - a*b*d)*sqrt(d*x + c))/d^2 - 3*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^2))/d
```

Fricas [A]
time = 0.29, size = 73, normalized size = 1.09

$$\frac{2\left(b^2d^2x^2-8\,b^2c^2+12\,abcd-3\,a^2d^2-2\left(2\,b^2cd-3\,abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^4x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*d^2*x^2 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x)*sqrt(d*x + c)/(d^4*x + c*d^3)
```

Sympy [A]
time = 6.29, size = 65, normalized size = 0.97

$$\frac{2b^2\left(c+dx\right)^{\frac{3}{2}}}{3d^3}+\frac{\sqrt{c+dx}\,\left(4abd-4b^2c\right)}{d^3}-\frac{2\left(ad-bc\right)^2}{d^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**2*(c + d*x)**(3/2)/(3*d**3) + sqrt(c + d*x)*(4*a*b*d - 4*b**2*c)/d**3
- 2*(a*d - b*c)**2/(d**3*sqrt(c + d*x))
```

Giac [A]
time = 0.00, size = 106, normalized size = 1.58

$$\frac{\frac{2}{3}\sqrt{c+dx}\,\left(c+dx\right)b^2d^6-4\sqrt{c+dx}\,b^2cd^6+4\sqrt{c+dx}\,bd^7a}{d^9}+\frac{-2b^2c^2+4bcda-2d^2a^2}{d^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^3) + 2/3*((d*x + c)^(3/
2)*b^2*d^6 - 6*sqrt(d*x + c)*b^2*c*d^6 + 6*sqrt(d*x + c)*a*b*d^7)/d^9
```

Mupad [B]
time = 0.26, size = 67, normalized size = 1.00

$$\frac{\frac{2\,b^2\,{\left(c+d\,x\right)}^2}{3}-2\,a^2\,d^2-2\,b^2\,c^2-4\,b^2\,c\,\left(c+d\,x\right)+4\,a\,b\,d\,\left(c+d\,x\right)+4\,a\,b\,c\,d}{d^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(3/2),x)
```

```
[Out] ((2*b^2*(c + d*x)^2)/3 - 2*a^2*d^2 - 2*b^2*c^2 - 4*b^2*c*(c + d*x) + 4*a*b*
d*(c + d*x) + 4*a*b*c*d)/(d^3*(c + d*x)^(1/2))
```

3.1428 $\int \frac{a+bx}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

```
[Out] 2*(-a*d+b*c)/d^2/(d*x+c)^(1/2)+2*b*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d))/(d^2*Sqrt[c + d*x]) + (2*b*Sqrt[c + d*x])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right)dx\\ &= \frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.71

$$\frac{2(2bc-ad+bdx)}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(2*b*c - a*d + b*d*x))/(d^2*Sqrt[c + d*x])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 1.99, size = 46, normalized size = 1.21

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(2bc+d\left(-a+bx\right)\right)}{d^2\sqrt{c+dx}},d!{=}0\right\}\right\},\frac{ax+\frac{bx^2}{2}}{c^{\frac{3}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x)^(3/2),x]')
```

```
[Out] Piecewise[{{2 (2 b c + d (-a + b x)) / (d ^ 2 Sqrt[c + d x]), d != 0}}, (a
x + b x ^ 2 / 2) / c ^ (3 / 2)]
```

Maple [A]
time = 0.16, size = 33, normalized size = 0.87

method	result	size
gosper	$-\frac{2(-bdx+ad-2bc)}{\sqrt{dx+c}\, d^2}$	26
trager	$-\frac{2(-bdx+ad-2bc)}{\sqrt{dx+c}\, d^2}$	26
derivativedivides	$\frac{2b\sqrt{dx+c}-\frac{2(ad-bc)}{\sqrt{dx+c}}}{d^2}$	33
default	$\frac{2b\sqrt{dx+c}-\frac{2(ad-bc)}{\sqrt{dx+c}}}{d^2}$	33
risch	$\frac{2b\sqrt{dx+c}}{d^2}-\frac{2(ad-bc)}{d^2\sqrt{dx+c}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(b*(d*x+c)^(1/2)-(a*d-b*c)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{dx+c}\,b}{d}+\frac{bc-ad}{\sqrt{dx+c}\,d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 2*(sqrt(d*x + c)*b/d + (b*c - a*d)/(sqrt(d*x + c)*d))/d
```

Fricas [A]
time = 0.30, size = 35, normalized size = 0.92

$$\frac{2\,(bdx+2\,bc-ad)\sqrt{dx+c}}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 2*(b*d*x + 2*b*c - a*d)*sqrt(d*x + c)/(d^3*x + c*d^2)
```

Sympy [A]
time = 0.33, size = 60, normalized size = 1.58

$$\begin{cases} -\frac{2a}{d\sqrt{c+dx}}+\frac{4bc}{d^2\sqrt{c+dx}}+\frac{2bx}{d\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(3/2),x)
[Out] Piecewise((-2*a/(d*sqrt(c + d*x)) + 4*b*c/(d**2*sqrt(c + d*x)) + 2*b*x/(d*s
qrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(3/2), True))
```

Giac [A]
time = 0.00, size = 42, normalized size = 1.11

$$\frac{2\sqrt{c+dx}\,b}{d^2}+\frac{2bc-2da}{d^2\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x)
[Out] 2*sqrt(d*x + c)*b/d^2 + 2*(b*c - a*d)/(sqrt(d*x + c)*d^2)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.66

$$\frac{4\,b\,c-2\,a\,d+2\,b\,d\,x}{d^2\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(3/2),x)
[Out] (4*b*c - 2*a*d + 2*b*d*x)/(d^2*(c + d*x)^(1/2))
```

3.1429 $\int \frac{1}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{d\sqrt{c+dx}}$$

[Out] -2/d/(d*x+c)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-3/2),x]

[Out] -2/(d*Sqrt[c + d*x])

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{3/2}}\,dx = -\frac{2}{d\sqrt{c+dx}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-3/2),x]

[Out] -2/(d*Sqrt[c + d*x])

Mathics [A]
time = 1.59, size = 12, normalized size = 0.86

$$\frac{-2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x)^(3/2),x]')
```

```
[Out] -2 / (d Sqrt[c + d x])
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{2}{d\sqrt{dx+c}}$	13
derivativedivides	$-\frac{2}{d\sqrt{dx+c}}$	13
default	$-\frac{2}{d\sqrt{dx+c}}$	13
trager	$-\frac{2}{d\sqrt{dx+c}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.30, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{dx+c}}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*x + c)/(d^2*x + c*d)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.86

$$-\frac{2}{d\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(3/2),x)
```

```
[Out] -2/(d*sqrt(c + d*x))
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.07

$$-\frac{2}{d\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x)
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{d\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(3/2),x)
```

```
[Out] -2/(d*(c + d*x)^(1/2))
```

3.1430 $\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(3/2)
+2/(-a*d+b*c)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 65, 214}

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx &= \frac{2}{(bc-ad)\sqrt{c+dx}}+\frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{bc-ad}\\ &= \frac{2}{(bc-ad)\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d(bc-ad)}\\ &= \frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 1.00

$$\frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2)
```

Mathics [A]
time = 6.27, size = 82, normalized size = 1.19

$$\frac{2\left(-\text{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right]\sqrt{c+dx}-\sqrt{\frac{ad-bc}{b}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^1*(c + d*x)^(3/2)),x]')
```

```
[Out] 2 (-ArcTan[Sqrt[c + d x] / Sqrt[(a d - b c) / b]] Sqrt[c + d x] - Sqrt[(a d
 - b c) / b]) / (Sqrt[(a d - b c) / b] (a d - b c) Sqrt[c + d x])
```

Maple [A]
time = 0.19, size = 68, normalized size = 0.99

method	result	size
derivativedivides	$-\frac{2}{(ad-bc)\sqrt{dx+c}}-\frac{2b\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)\sqrt{(ad-bc)\,b}}$	68
default	$-\frac{2}{(ad-bc)\sqrt{dx+c}}-\frac{2b\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)\sqrt{(ad-bc)\,b}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a*d-b*c)/(d*x+c)^(1/2)-2*b/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+
c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.30, size = 214, normalized size = 3.10

$$\left[-\frac{(dx+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\sqrt{dx+c}}{bc^2-acd+(bcd-ad^2)x}, -\frac{2\left((dx+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\sqrt{dx+c}\right)}{bc^2-acd+(bcd-ad^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((d*x + c)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*s
qrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*sqrt(d*x + c))/(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x), -2*((d*x + c)*sqrt(-b/(b*c - a*d))*arctan(-(b*c -
 a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - sqrt(d*x + c))/(b
*c^2 - a*c*d + (b*c*d - a*d^2)*x)]
```

Sympy [A]
time = 5.30, size = 60, normalized size = 0.87

$$-\frac{2}{\sqrt{c+dx}\,(ad-bc)}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] -2/(sqrt(c + d*x)*(a*d - b*c)) - 2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/
(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [A]
time = 0.00, size = 83, normalized size = 1.20

$$2\left(-\frac{1}{(-bc+da)\,\sqrt{c+dx}}-\frac{2b\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(-bc+da)\,\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] 2*b*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 2/((b*c - a*d)*sqrt(d*x + c))
```

Mupad [B]
time = 0.27, size = 57, normalized size = 0.83

$$-\frac{2}{(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{{(a\,d-b\,c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(3/2)),x)
```

```
[Out] - 2/((a*d - b*c)*(c + d*x)^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*(c + d*x)^(1/2
))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

3.1431 $\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3d}{(bc-ad)^2\sqrt{c+dx}}-\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] 3*d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2
)-3*d/(-a*d+b*c)^2/(d*x+c)^(1/2)-1/(-a*d+b*c)/(b*x+a)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$-\frac{3d}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d)/((b*c - a*d)^2*Sqrt[c + d*x]) - 1/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c -
 a*d)^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3bd)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d$$

$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,d\right.}{(bc-ad}$$

$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} + \frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{bc-}}\right.}{(bc-ad)^{5/2}}$$

Mathematica [A]
time = 0.29, size = 90, normalized size = 0.91

$$-\frac{2ad+b(c+3dx)}{(bc-ad)^2(a+bx)\sqrt{c+dx}} - \frac{3\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] -((2*a*d + b*(c + 3*d*x))/((b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x])) - (3*Sqr
t[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(
5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^2*(c + d*x)^(3/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.19, size = 100, normalized size = 1.01

method	result	size
derivativedivides	$2d\left(-\frac{b\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}\right)}{(ad-bc)^2}-\frac{1}{(ad-bc)^2\sqrt{dx+c}}\right)$	100
default	$2d\left(-\frac{b\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}\right)}{(ad-bc)^2}-\frac{1}{(ad-bc)^2\sqrt{dx+c}}\right)$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/(a*d-b*c)^2*b*(1/2*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+3/2/((a*d-b*c)
*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))-1/(a*d-b*c)^2/(d*x+c
)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(85) = 170$.

time = 0.31, size = 423, normalized size = 4.27

$$\left[\frac{3\,(bd^2x^2 + acd + (bcd + ad^2)x)\sqrt{\frac{b}{bc - ad}}\log\left(\frac{bdx + 2\,bc - ad + 2\,(bc - ad)\sqrt{dx + c}\sqrt{\frac{b}{bc - ad}}}{bx + a}\right) - 2\,(3\,bdx + bc + 2\,ad)\sqrt{dx + c}}{2\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x)}, \frac{3\,(bd^2x^2 + acd + (bcd + ad^2)x)\sqrt{-\frac{b}{bc - ad}}\arctan\left(-\frac{(bc - ad)\sqrt{dx + c}\sqrt{-\frac{b}{bc - ad}}}{bdx + bc}\right) - (3\,bdx + bc + 2\,ad)\sqrt{dx + c}}{ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*
d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x +
 a)) - 2*(3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x), (3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2
)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c -
a*d))/(b*d*x + b*c)) - (3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 -
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^2\,(c + dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(3/2)), x)
```

Giac [A]

time = 0.00, size = 169, normalized size = 1.71

$$2\left(\frac{3\,(c + dx)\,bd - 2bdc + 2d^2a}{(2b^2c^2 - 4bdca + 2d^2a^2)\left(-\sqrt{c + dx}\,(c + dx)\,b + \sqrt{c + dx}\,bc - \sqrt{c + dx}\,da\right)} + \frac{3bd\arctan\left(\frac{b\sqrt{c + dx}}{\sqrt{-b^2c + abd}}\right)}{2\,(-b^2c^2 + 2bdca - d^2a^2)\sqrt{-b^2c + abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -3*b*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - (3*(d*x + c)*b*d - 2*b*c*d + 2*a*d^2)/((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + sqrt
(d*x + c)*a*d))
```

Mupad [B]
time = 0.19, size = 123, normalized size = 1.24

$$-\frac{\frac{2\,d}{a\,d-b\,c}+\frac{3\,b\,d\,(c+d\,x)}{{(a\,d-b\,c)}^2}}{b\,{(c+d\,x)}^{3/2}+(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{(a\,d-b\,c)}^{5/2}}\right)}{{(a\,d-b\,c)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d)/(a*d - b*c) + (3*b*d*(c + d*x))/(a*d - b*c)^2)/(b*(c + d*x)^(3/2)
+ (a*d - b*c)*(c + d*x)^(1/2)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

3.1432 $\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}}-\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}}+\frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}$$

```
[Out] -15/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c
)^(7/2)+15/4*d^2/(-a*d+b*c)^3/(d*x+c)^(1/2)-1/2/(-a*d+b*c)/(b*x+a)^2/(d*x+c
)^(1/2)+5/4*d/(-a*d+b*c)^2/(b*x+a)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$\frac{15d^2}{4\sqrt{c+dx}\,(bc-ad)^3}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (15*d^2)/(4*(b*c - a*d)^3*Sqrt[c + d*x]) - 1/(2*(b*c - a*d)*(a + b*x)^2*Sqr
t[c + d*x]) + (5*d)/(4*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) - (15*Sqrt[b]
*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(7/2)
)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{(15d^2)\int \cdots}{8(bc \cdots}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\cdots}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\cdots}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\cdots}$$

Mathematica [A]
time = 0.50, size = 126, normalized size = 0.90

$$\frac{1}{4}\left(\frac{8a^2d^2+abd(9c+25dx)+b^2\left(-2c^2+5cdx+15d^2x^2\right)}{(bc-ad)^3(a+bx)^2\sqrt{c+dx}} - \frac{15\sqrt{b}\,d^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

[Out] ((8*a^2*d^2 + a*b*d*(9*c + 25*d*x) + b^2*(-2*c^2 + 5*c*d*x + 15*d^2*x^2))/((b*c - a*d)^3*(a + b*x)^2*Sqrt[c + d*x]) - (15*Sqrt[b]*d^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2))/4

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^3*(c + d*x)^(3/2)),x]')

[Out] Timed out

Maple [A]
time = 0.20, size = 122, normalized size = 0.87

method	result
derivativedivides	$2d^2\left(-\frac{1}{(ad-bc)^3\sqrt{dx+c}}-\frac{b\left(\frac{\frac{7b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{9ad}{8}-\frac{9bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\right)$
default	$2d^2\left(-\frac{1}{(ad-bc)^3\sqrt{dx+c}}-\frac{b\left(\frac{\frac{7b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{9ad}{8}-\frac{9bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*d^2*(-1/(a*d-b*c)^3/(d*x+c)^(1/2)-b/(a*d-b*c)^3*((7/8*b*(d*x+c)^(3/2)+(9/8*a*d-9/8*b*c)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^2+15/8/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(116) = 232$.
time = 0.32, size = 782, normalized size = 5.59

$$\left[-\frac{15\,(b^2d^3x^3 + a^2cd^2 + (b^2cd^2 + 2\,abd^3)x^2 + (2\,abcd^2 + a^2d^3)x)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right) - 2\,(15\,b^2d^2x^2 - 2\,b^2c^2 + 9\,abcd + 8\,a^2d^2 + 5\,(b^2cd + 5\,abd^2)x)\sqrt{dx+c}}{8\,(a^2b^3c^4 - 3\,a^3b^2c^3d + 3\,a^4bc^2d^2 - a^5cd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^3 + (b^5c^4 - ab^4c^3d - 3\,a^2b^3c^2d^2 + 5\,a^3b^2cd^3 - 2\,a^4bd^4)x^2 + (2\,ab^4c^4 - 5\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 + a^4bcd^3 - a^5d^4)x)}, -\frac{15\,(b^2d^3x^3 + a^2cd^2 + (b^2cd^2 + 2\,abd^3)x^2 + (2\,abcd^2 + a^2d^3)x)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right) - (15\,b^2d^2x^2 - 2\,b^2c^2 + 9\,abcd + 8\,a^2d^2 + 5\,(b^2cd + 5\,abd^2)x)\sqrt{dx+c}}{4\,(a^2b^3c^4 - 3\,a^3b^2c^3d + 3\,a^4bc^2d^2 - a^5cd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^3 + (b^5c^4 - ab^4c^3d - 3\,a^2b^3c^2d^2 + 5\,a^3b^2cd^3 - 2\,a^4bd^4)x^2 + (2\,ab^4c^4 - 5\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 + a^4bcd^3 - a^5d^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c
*d^2 + a^2*d^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c -
a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*(15*b^2*d^2*x^2 - 2*
b^2*c^2 + 9*a*b*c*d + 8*a^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))
/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c
^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^
4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x), -1/4*(
15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c*d^2 +
a^2*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/
(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 - 2*b^2*c^2 + 9*a*b*c*d + 8*a
^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))/(a^2*b^3*c^4 - 3*a^3*b^2
*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2
*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2
+ 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a
^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(116) = 232$.

time = 0.00, size = 267, normalized size = 1.91

$$2\left(-\frac{d^2}{\left(-b^3c^3+3b^2dc^2a-3bd^2ca^2+d^3a^3\right)\sqrt{c+dx}}+\frac{-7\sqrt{c+dx}\,(c+dx)\,b^2d^2+9\sqrt{c+dx}\,b^2d^2c-9\sqrt{c+dx}\,bd^3a}{\left(-8b^3c^3+24b^2dc^2a-24bd^2ca^2+8d^3a^3\right)\left(-(c+dx)\,b+bc-da\right)^2}+\frac{15bd^2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\left(4b^3c^3-12b^2dc^2a+12bd^2ca^2-4d^3a^3\right)\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] 15/4*b*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*d^2/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(d*x + c)) + 1/4*(7*(d*x + c)
^(3/2)*b^2*d^2 - 9*sqrt(d*x + c)*b^2*c*d^2 + 9*sqrt(d*x + c)*a*b*d^3)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c + a*d)^2
)
```

Mupad [**B**]

time = 0.44, size = 205, normalized size = 1.46

$$-\frac{\frac{2\,d^2}{a\,d-b\,c}+\frac{15\,b^2\,d^2\,(c+d\,x)^2}{4\,(a\,d-b\,c)^3}+\frac{25\,b\,d^2\,(c+d\,x)}{4\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^{5/2}-\left(2\,b^2\,c-2\,a\,b\,d\right)\,(c+d\,x)^{3/2}+\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{15\,\sqrt{b}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^{7/2}}\right)}{4\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^2)/(a*d - b*c) + (15*b^2*d^2*(c + d*x)^2)/(4*(a*d - b*c)^3) + (25*b
*d^2*(c + d*x))/(4*(a*d - b*c)^2))/(b^2*(c + d*x)^(5/2) - (2*b^2*c - 2*a*b*
d)*(c + d*x)^(3/2) + (c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (15
*b^(1/2)*d^2*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(4*(a*d - b*c)^(7/2))
```

3.1433 $\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}}-\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}}+\frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}}-\frac{35d^2}{24(bc-ad)^3(a+}$$

```
[Out] 35/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)
^(9/2)-35/8*d^3/(-a*d+b*c)^4/(d*x+c)^(1/2)-1/3/(-a*d+b*c)/(b*x+a)^3/(d*x+c)
^(1/2)+7/12*d/(-a*d+b*c)^2/(b*x+a)^2/(d*x+c)^(1/2)-35/24*d^2/(-a*d+b*c)^3/(
b*x+a)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$-\frac{35d^3}{8\sqrt{c+dx}\,(bc-ad)^4}+\frac{35\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}\,(bc-ad)^3}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{3(a+bx)^3\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*Sqrt[c + d*x]) - 1/(3*(b*c - a*d)*(a + b*x)^3*Sq
rt[c + d*x]) + (7*d)/(12*(b*c - a*d)^2*(a + b*x)^2*Sqrt[c + d*x]) - (35*d^2
)/(24*(b*c - a*d)^3*(a + b*x)*Sqrt[c + d*x]) + (35*Sqrt[b]*d^3*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*(b*c - a*d)^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx}{6(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} + \frac{(35d^2)\int \cdots}{24(\cdots}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{\cdots}{24(bc-a\cdots}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx\cdots}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx\cdots}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx\cdots}$$

Mathematica [A]
time = 0.66, size = 170, normalized size = 0.98

$$\frac{-48a^3d^3-3a^2bd^2(29c+77dx)-2ab^2d\left(-19c^2+49cdx+140d^2x^2\right)-b^3\left(8c^3-14c^2dx+35cd^2x^2+105d^3x^3\right)}{24(bc-ad)^4(a+bx)^3\sqrt{c+dx}} - \frac{35\sqrt{b}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8(-bc+ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

[Out] (-48*a^3*d^3 - 3*a^2*b*d^2*(29*c + 77*d*x) - 2*a*b^2*d*(-19*c^2 + 49*c*d*x + 140*d^2*x^2) - b^3*(8*c^3 - 14*c^2*d*x + 35*c*d^2*x^2 + 105*d^3*x^3))/(24*(b*c - a*d)^4*(a + b*x)^3*Sqrt[c + d*x]) - (35*Sqrt[b]*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*(-(b*c) + a*d)^(9/2))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^4*(c + d*x)^(3/2)),x]')

[Out] Timed out

Maple [A]
time = 0.19, size = 156, normalized size = 0.90

method	result
derivativedivides	$2d^3\left(-\frac{b\left(\frac{\frac{19(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{17(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{29}{16}a^2d^2-\frac{29}{8}abcd+\frac{29}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{35\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^4}\right.$
default	$2d^3\left(-\frac{b\left(\frac{\frac{19(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{17(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{29}{16}a^2d^2-\frac{29}{8}abcd+\frac{29}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{35\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^4/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*d^3*(-1/(a*d-b*c)^4*b*((19/16*(d*x+c)^(5/2)*b^2+17/6*(a*d-b*c)*b*(d*x+c)^(3/2)+(29/16*a^2*d^2-29/8*a*b*c*d+29/16*b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^3+35/16/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))-1/(a*d-b*c)^4/(d*x+c)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(145) = 290.
time = 0.32, size = 1204, normalized size = 6.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*x^3 + 3*(a*
b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b/(b*c - a*d
))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d
)))/(b*x + a)) - 2*(105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b
*c*d^2 + 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 1
4*a*b^2*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x), 1/24*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*
x^3 + 3*(a*b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(-
b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*
x + b*c)) - (105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b*c*d^2
+ 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 14*a*b^2
*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6
*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*
b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a
^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^
4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*
```

```
b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)
*x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(145) = 290.
time = 0.01, size = 381, normalized size = 2.20

$$2\left(-\frac{d^3}{(b^4c^4-4b^3dc^3a+6b^2d^2c^2a^2-4bd^3ca^3+d^4a^4)\sqrt{c+dx}}+\frac{57\sqrt{c+dx}\,(c+dx)^2b^3d^3-136\sqrt{c+dx}\,(c+dx)b^3d^3c+136\sqrt{c+dx}\,(c+dx)b^2d^4a+87\sqrt{c+dx}\,b^3d^3c^2-174\sqrt{c+dx}\,b^2d^4ca+87\sqrt{c+dx}\,bd^5a^2}{(48b^4c^4-192b^3dc^3a+288b^2d^2c^2a^2-192bd^3ca^3+48d^4a^4)(-(c+dx)b+bc-da)^3}-\frac{35bd^3\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2(8b^4c^4-32b^3dc^3a+48b^2d^2c^2a^2-32bd^3ca^3+8d^4a^4)\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -35/8*b*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)
) - 2*d^3/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(d*x + c)) - 1/24*(57*(d*x + c)^(5/2)*b^3*d^3 - 136*(d*x + c)^(
3/2)*b^3*c*d^3 + 87*sqrt(d*x + c)*b^3*c^2*d^3 + 136*(d*x + c)^(3/2)*a*b^2*d
^4 - 174*sqrt(d*x + c)*a*b^2*c*d^4 + 87*sqrt(d*x + c)*a^2*b*d^5)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b
 - b*c + a*d)^3)
```

Mupad [**B**]
time = 0.54, size = 294, normalized size = 1.70

$$-\frac{\frac{2\,d^3}{a\,d-b\,c}+\frac{35\,b^2\,d^3\,{(c+d\,x)}^2}{3\,{(a\,d-b\,c)}^3}+\frac{35\,b^3\,d^3\,{(c+d\,x)}^3}{8\,{(a\,d-b\,c)}^4}+\frac{77\,b\,d^3\,(c+d\,x)}{8\,{(a\,d-b\,c)}^2}}{\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{(c+d\,x)}^{7/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{(c+d\,x)}^{5/2}+{(c+d\,x)}^{3/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}-\frac{35\,\sqrt{b}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{{(a\,d-b\,c)}^{9/2}}\right)}{8\,{(a\,d-b\,c)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^3)/(a*d - b*c) + (35*b^2*d^3*(c + d*x)^2)/(3*(a*d - b*c)^3) + (35*b
^3*d^3*(c + d*x)^3)/(8*(a*d - b*c)^4) + (77*b*d^3*(c + d*x))/(8*(a*d - b*c)
^2))/((c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) +
 b^3*(c + d*x)^(7/2) - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^(5/2) + (c + d*x)^(3
/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d)) - (35*b^(1/2)*d^3*atan((b^(1/2
)*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
4*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(8*(a*d - b*c)^(9/2))
```

3.1434 $\int \frac{(a+bx)^5}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=152

$$\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/2}}{3d^6}-\frac{2b^4(bc-ad)(c+dx)^{5/2}}{d^6}$$

```
[Out] 2/3*(-a*d+b*c)^5/d^6/(d*x+c)^(3/2)+20/3*b^3*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^6-
2*b^4*(-a*d+b*c)*(d*x+c)^(5/2)/d^6+2/7*b^5*(d*x+c)^(7/2)/d^6-10*b*(-a*d+b*c
)^4/d^6/(d*x+c)^(1/2)-20*b^2*(-a*d+b*c)^3*(d*x+c)^(1/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}\,(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b^5(c+dx)^{7/2}}{7d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(3*d^6*(c + d*x)^(3/2)) - (10*b*(b*c - a*d)^4)/(d^6*Sqrt[
c + d*x]) - (20*b^2*(b*c - a*d)^3*Sqrt[c + d*x])/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(3/2))/(3*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(5/2))/d^6 + (2*b
^5*(c + d*x)^(7/2))/(7*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{5/2}}+\frac{5b(bc-ad)^4}{d^5(c+dx)^{3/2}}-\frac{10b^2(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{10b^3(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{5b^4(b\\ &= \frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/2}}{3d^6}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 217, normalized size = 1.43

$$-\frac{2\left(7a^5d^5+35a^4bd^4(2c+3dx)-70a^3b^2d^3\left(8c^2+12cdx+3d^2x^2\right)+70a^2b^3d^2\left(16c^3+24c^2dx+6cd^2x^2-d^3x^3\right)-7ab^4d\left(128c^4+192c^3dx+48c^2d^2x^2-8cd^3x^3+3d^4x^4\right)+b^5\left(256c^5+384c^4dx+96c^3d^2x^2-16c^2d^3x^3+6cd^4x^4-3d^5x^5\right)\right)}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(7*a^5*d^5 + 35*a^4*b*d^4*(2*c + 3*d*x) - 70*a^3*b^2*d^3*(8*c^2 + 12*c*
d*x + 3*d^2*x^2) + 70*a^2*b^3*d^2*(16*c^3 + 24*c^2*d*x + 6*c*d^2*x^2 - d^3*
x^3) - 7*a*b^4*d*(128*c^4 + 192*c^3*d*x + 48*c^2*d^2*x^2 - 8*c*d^3*x^3 + 3*
d^4*x^4) + b^5*(256*c^5 + 384*c^4*d*x + 96*c^3*d^2*x^2 - 16*c^2*d^3*x^3 + 6
*c*d^4*x^4 - 3*d^5*x^5)))/(21*d^6*(c + d*x)^(3/2))
```

Mathics [A]
time = 25.16, size = 152, normalized size = 1.00

$$\frac{2\left(-105b\left(c+dx\right)\left(ad-bc\right)^{4}+b^{2}\left(210a^{3}d^{3}+70b\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(c+dx\right)-630a^{2}bcd^{2}+21b^{2}\left(ad-bc\right)\left(c+dx\right)^{2}+630ab^{2}c^{2}d+3b^{3}\left(c+dx\right)^{3}-210b^{3}c^{3}\right)\left(c+dx\right)^{2}-7\left(ad-bc\right)^{5}\right)}{21d^{6}\left(c+dx\right)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^5/(c + d*x)^(5/2),x]')
```

```
[Out] 2 (-105 b (c + d x) (a d - b c) ^ 4 + b ^ 2 (210 a ^ 3 d ^ 3 + 70 b (a ^ 2
d ^ 2 - 2 a b c d + b ^ 2 c ^ 2) (c + d x) - 630 a ^ 2 b c d ^ 2 + 21 b ^ 2
 (a d - b c) (c + d x) ^ 2 + 630 a b ^ 2 c ^ 2 d + 3 b ^ 3 (c + d x) ^ 3 -
210 b ^ 3 c ^ 3) (c + d x) ^ 2 - 7 (a d - b c) ^ 5) / (21 d ^ 6 (c + d x) ^
 (3 / 2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(134) = 268$.
time = 0.17, size = 294, normalized size = 1.93

method	result
risch	$\frac{2b^{2}\left(3b^{3}x^{3}d^{3}+21d^{3}a\,x^{2}b^{2}-12b^{3}c\,d^{2}x^{2}+70a^{2}b\,d^{3}x-98a\,b^{2}c\,d^{2}x+37b^{3}c^{2}dx+210a^{3}d^{3}-560a^{2}bc\,d^{2}+511a\,b^{2}c^{2}d-158b^{3}c^{3}}{21d^{6}}$
gosper	$-\frac{2\left(-3b^{5}x^{5}d^{5}-21a\,b^{4}d^{5}x^{4}+6b^{5}c\,d^{4}x^{4}-70a^{2}b^{3}d^{5}x^{3}+56a\,b^{4}c\,d^{4}x^{3}-16b^{5}c^{2}d^{3}x^{3}-210a^{3}b^{2}d^{5}x^{2}+420a^{2}b^{3}c\,d^{4}x^{2}-336a\,b}{}$
trager	$-\frac{2\left(-3b^{5}x^{5}d^{5}-21a\,b^{4}d^{5}x^{4}+6b^{5}c\,d^{4}x^{4}-70a^{2}b^{3}d^{5}x^{3}+56a\,b^{4}c\,d^{4}x^{3}-16b^{5}c^{2}d^{3}x^{3}-210a^{3}b^{2}d^{5}x^{2}+420a^{2}b^{3}c\,d^{4}x^{2}-336a\,b}{}$
derivativedivides	$\frac{\frac{2b^{5}(dx+c)^{\frac{7}{2}}}{7}+2a\,b^{4}d(dx+c)^{\frac{5}{2}}-2b^{5}c(dx+c)^{\frac{5}{2}}+\frac{20a^{2}b^{3}d^{2}(dx+c)^{\frac{3}{2}}}{3}-\frac{40a\,b^{4}cd(dx+c)^{\frac{3}{2}}}{3}+\frac{20b^{5}c^{2}(dx+c)^{\frac{3}{2}}}{3}+20a^{3}b^{2}d^{3}\sqrt{dx+}}{}$
default	$\frac{\frac{2b^{5}(dx+c)^{\frac{7}{2}}}{7}+2a\,b^{4}d(dx+c)^{\frac{5}{2}}-2b^{5}c(dx+c)^{\frac{5}{2}}+\frac{20a^{2}b^{3}d^{2}(dx+c)^{\frac{3}{2}}}{3}-\frac{40a\,b^{4}cd(dx+c)^{\frac{3}{2}}}{3}+\frac{20b^{5}c^{2}(dx+c)^{\frac{3}{2}}}{3}+20a^{3}b^{2}d^{3}\sqrt{dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/7*b^5*(d*x+c)^(7/2)+a*b^4*d*(d*x+c)^(5/2)-b^5*c*(d*x+c)^(5/2)+10/3
*a^2*b^3*d^2*(d*x+c)^(3/2)-20/3*a*b^4*c*d*(d*x+c)^(3/2)+10/3*b^5*c^2*(d*x+c
```

```
)^(3/2)+10*a^3*b^2*d^3*(d*x+c)^(1/2)-30*a^2*b^3*c*d^2*(d*x+c)^(1/2)+30*a*b^
4*c^2*d*(d*x+c)^(1/2)-10*b^5*c^3*(d*x+c)^(1/2)-5*b*(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^(1/2)-1/3*(a^5*d^5-5*a^4*b*
c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)^
(3/2))
```

Maxima [A]
time = 0.27, size = 265, normalized size = 1.74

$$\frac{2\left(\frac{3\,(dx+c)^{\frac{7}{2}}b^5-21\,(b^5c-ab^4d)(dx+c)^{\frac{5}{2}}+70\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)(dx+c)^{\frac{3}{2}}-210\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)\sqrt{dx+c}}{d^5}+\frac{7\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5-15\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)(dx+c))}{(dx+c)^{\frac{3}{2}}d^5}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*((3*(d*x + c)^(7/2)*b^5 - 21*(b^5*c - a*b^4*d)*(d*x + c)^(5/2) + 70*(b
^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(3/2) - 210*(b^5*c^3 - 3*a*b^
4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(d*x + c))/d^5 + 7*(b^5*c^5 -
5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a
^5*d^5 - 15*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4)*(d*x + c))/((d*x + c)^(3/2)*d^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.
time = 0.30, size = 283, normalized size = 1.86

$$\frac{2\,(3\,b^5d^5x^5-256\,b^5c^5+896\,ab^4c^4d-1120\,a^2b^3c^3d^2+560\,a^3b^2c^2d^3-70\,a^4bcd^4-7\,a^5d^5-3\,(2\,b^5cd^4-7\,ab^4d^5)x^4+2\,(8\,b^5c^2d^3-28\,ab^4cd^4+35\,a^2b^3d^5)x^3-6\,(16\,b^5c^3d^2-56\,ab^4c^2d^3+70\,a^2b^3cd^4-35\,a^3b^2d^5)x^2-3\,(128\,b^5c^4d-448\,ab^4c^3d^2+560\,a^2b^3c^2d^3-280\,a^3b^2cd^4+35\,a^4bd^5)x)\sqrt{dx+c}}{21\,(d^8x^2+2\,cd^7x+c^2d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^5*d^5*x^5 - 256*b^5*c^5 + 896*a*b^4*c^4*d - 1120*a^2*b^3*c^3*d^2
+ 560*a^3*b^2*c^2*d^3 - 70*a^4*b*c*d^4 - 7*a^5*d^5 - 3*(2*b^5*c*d^4 - 7*a*b
^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 28*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 6*(
16*b^5*c^3*d^2 - 56*a*b^4*c^2*d^3 + 70*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
- 3*(128*b^5*c^4*d - 448*a*b^4*c^3*d^2 + 560*a^2*b^3*c^2*d^3 - 280*a^3*b^2*
c*d^4 + 35*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6)
```

Sympy [A]
time = 29.40, size = 196, normalized size = 1.29

$$\frac{2b^5\,(c+dx)^{\frac{7}{2}}}{7d^6}-\frac{10b\,(ad-bc)^4}{d^6\sqrt{c+dx}}+\frac{(c+dx)^{\frac{5}{2}}\cdot(10ab^4d-10b^5c)}{5d^6}+\frac{(c+dx)^{\frac{3}{2}}\cdot(20a^2b^3d^2-40ab^4cd+20b^5c^2)}{3d^6}+\frac{\sqrt{c+dx}\,\left(20a^3b^2d^3-60a^2b^3cd^2+60ab^4c^2d-20b^5c^3\right)}{d^6}-\frac{2\,(ad-bc)^5}{3d^6\,(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(7/2)/(7*d**6) - 10*b*(a*d - b*c)**4/(d**6*sqrt(c + d*x))
 + (c + d*x)**(5/2)*(10*a*b**4*d - 10*b**5*c)/(5*d**6) + (c + d*x)**(3/2)*(
20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5*c**2)/(3*d**6) + sqrt(c + d*x)*
(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2 + 60*a*b**4*c**2*d - 20*b**5*c**3)
/d**6 - 2*(a*d - b*c)**5/(3*d**6*(c + d*x)**(3/2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(134) = 268.
time = 0.01, size = 421, normalized size = 2.77

$$\frac{\frac{2}{7}\sqrt{c+dx}\,(c+dx)^3b^5d^{36}-2\sqrt{c+dx}\,(c+dx)^2b^5cd^{36}+2\sqrt{c+dx}\,(c+dx)^2b^4d^{37}a+\frac{20}{3}\sqrt{c+dx}\,(c+dx)b^5c^2d^{36}-\frac{40}{3}\sqrt{c+dx}\,(c+dx)b^4cd^{37}a+\frac{20}{3}\sqrt{c+dx}\,(c+dx)b^3d^{38}a^2-20\sqrt{c+dx}\,b^5c^3d^{36}+60\sqrt{c+dx}\,b^4c^2d^{37}a-60\sqrt{c+dx}\,b^3cd^{38}a^2+20\sqrt{c+dx}\,b^2d^{39}a^3}{d^{42}}+\frac{-30(c+dx)b^5c^4+120(c+dx)b^4c^3da-180(c+dx)b^3c^2d^2a^2+120(c+dx)b^2cd^3a^3-30(c+dx)bd^4a^4+2b^5c^5-10b^4c^4da+20b^3c^3d^2a^2-20b^2c^2d^3a^3+10bcd^4a^4-2d^5a^5}{3d^6\sqrt{c+dx}\,(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(15*(d*x + c)*b^5*c^4 - b^5*c^5 - 60*(d*x + c)*a*b^4*c^3*d + 5*a*b^4*c
^4*d + 90*(d*x + c)*a^2*b^3*c^2*d^2 - 10*a^2*b^3*c^3*d^2 - 60*(d*x + c)*a^3
*b^2*c*d^3 + 10*a^3*b^2*c^2*d^3 + 15*(d*x + c)*a^4*b*d^4 - 5*a^4*b*c*d^4 +
a^5*d^5)/((d*x + c)^(3/2)*d^6) + 2/21*(3*(d*x + c)^(7/2)*b^5*d^36 - 21*(d*x
 + c)^(5/2)*b^5*c*d^36 + 70*(d*x + c)^(3/2)*b^5*c^2*d^36 - 210*sqrt(d*x + c
)*b^5*c^3*d^36 + 21*(d*x + c)^(5/2)*a*b^4*d^37 - 140*(d*x + c)^(3/2)*a*b^4*
c*d^37 + 630*sqrt(d*x + c)*a*b^4*c^2*d^37 + 70*(d*x + c)^(3/2)*a^2*b^3*d^38
 - 630*sqrt(d*x + c)*a^2*b^3*c*d^38 + 210*sqrt(d*x + c)*a^3*b^2*d^39)/d^42
```

Mupad [**B**]
time = 0.08, size = 229, normalized size = 1.51

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^6}-\frac{\frac{2\,a^5\,d^5}{3}-\frac{2\,b^5\,c^5}{3}+\left(c+d\,x\right)\,\left(10\,a^4\,b\,d^4-40\,a^3\,b^2\,c\,d^3+60\,a^2\,b^3\,c^2\,d^2-40\,a\,b^4\,c^3\,d+10\,b^5\,c^4\right)-\frac{20\,a^2\,b^3\,c^3\,d^2}{3}+\frac{20\,a^3\,b^2\,c^2\,d^3}{3}+\frac{10\,a\,b^4\,c^4\,d}{3}-\frac{10\,a^4\,b\,c\,d^4}{3}}{d^6\,{\left(c+d\,x\right)}^{3/2}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(7/2))/(7*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(5/2))
/(5*d^6) - ((2*a^5*d^5)/3 - (2*b^5*c^5)/3 + (c + d*x)*(10*b^5*c^4 + 10*a^4*
b*d^4 - 40*a^3*b^2*c*d^3 + 60*a^2*b^3*c^2*d^2 - 40*a*b^4*c^3*d) - (20*a^2*b
^3*c^3*d^2)/3 + (20*a^3*b^2*c^2*d^3)/3 + (10*a*b^4*c^4*d)/3 - (10*a^4*b*c*d
^4)/3)/(d^6*(c + d*x)^(3/2)) + (20*b^2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^6 +
 (20*b^3*(a*d - b*c)^2*(c + d*x)^(3/2))/(3*d^6)
```

3.1435 $\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

```
[Out] -2/3*(-a*d+b*c)^4/d^5/(d*x+c)^(3/2)-8/3*b^3*(-a*d+b*c)*(d*x+c)^(3/2)/d^5+2/
5*b^4*(d*x+c)^(5/2)/d^5+8*b*(-a*d+b*c)^3/d^5/(d*x+c)^(1/2)+12*b^2*(-a*d+b*c
)^2*(d*x+c)^(1/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}\,(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(3*d^5*(c + d*x)^(3/2)) + (8*b*(b*c - a*d)^3)/(d^5*Sqrt[
c + d*x]) + (12*b^2*(b*c - a*d)^2*Sqrt[c + d*x])/d^5 - (8*b^3*(b*c - a*d)*(
c + d*x)^(3/2))/(3*d^5) + (2*b^4*(c + d*x)^(5/2))/(5*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{5/2}}-\frac{4b(bc-ad)^3}{d^4(c+dx)^{3/2}}+\frac{6b^2(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{4b^3(bc-ad)\sqrt{c+dx}}{d^4}+\frac{b^4(c+d\right.\\ &= -\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}+\end{aligned}$$

Mathematica [A]
time = 0.09, size = 153, normalized size = 1.22

$$\frac{2\left(-5a^4d^4-20a^3bd^3(2c+3dx)+30a^2b^2d^2\left(8c^2+12cdx+3d^2x^2\right)+20ab^3d\left(-16c^3-24c^2dx-6cd^2x^2+d^3x^3\right)+b^4\left(128c^4+192c^3dx+48c^2d^2x^2-8cd^3x^3+3d^4x^4\right)\right)}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*a^4*d^4 - 20*a^3*b*d^3*(2*c + 3*d*x) + 30*a^2*b^2*d^2*(8*c^2 + 12*c*
d*x + 3*d^2*x^2) + 20*a*b^3*d*(-16*c^3 - 24*c^2*d*x - 6*c*d^2*x^2 + d^3*x^3
) + b^4*(128*c^4 + 192*c^3*d*x + 48*c^2*d^2*x^2 - 8*c*d^3*x^3 + 3*d^4*x^4))
)/(15*d^5*(c + d*x)^(3/2))
```

Mathics [A]
time = 19.43, size = 105, normalized size = 0.84

$$\frac{2\left(-60b\left(c+dx\right)\left(ad-bc\right)^{3}+b^{2}\left(90a^{2}d^{2}+20b\left(ad-bc\right)\left(c+dx\right)-180abcd+3b^{2}\left(c+dx\right)^{2}+90b^{2}c^{2}\right)\left(c+dx\right)^{2}-5\left(ad-bc\right)^{4}\right)}{15d^{5}\left(c+dx\right)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^4/(c + d*x)^(5/2),x]')
```

```
[Out] 2 (-60 b (c + d x) (a d - b c) ^ 3 + b ^ 2 (90 a ^ 2 d ^ 2 + 20 b (a d - b
c) (c + d x) - 180 a b c d + 3 b ^ 2 (c + d x) ^ 2 + 90 b ^ 2 c ^ 2) (c + d
 x) ^ 2 - 5 (a d - b c) ^ 4) / (15 d ^ 5 (c + d x) ^ (3 / 2))
```

Maple [A]
time = 0.16, size = 198, normalized size = 1.58

method	result
risch	$\frac{2b^{2}\left(3b^{2}x^{2}d^{2}+20ab\,d^{2}x-14b^{2}cdx+90a^{2}d^{2}-160abcd+73b^{2}c^{2}\right)\sqrt{dx+c}}{15d^{5}}-\frac{2(12bdx+ad+11bc)\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\cdots\right.}{3d^{5}(dx+c)^{\frac{3}{2}}}$
gosper	$-\frac{2\left(-3d^{4}x^{4}b^{4}-20a\,b^{3}d^{4}x^{3}+8b^{4}c\,d^{3}x^{3}-90a^{2}b^{2}d^{4}x^{2}+120a\,b^{3}c\,d^{3}x^{2}-48b^{4}c^{2}d^{2}x^{2}+60a^{3}b\,d^{4}x-360a^{2}b^{2}c\,d^{3}x+480a\,b^{3}c\cdots\right.}{15(dx+c)^{\frac{3}{2}}d^{5}}$
trager	$-\frac{2\left(-3d^{4}x^{4}b^{4}-20a\,b^{3}d^{4}x^{3}+8b^{4}c\,d^{3}x^{3}-90a^{2}b^{2}d^{4}x^{2}+120a\,b^{3}c\,d^{3}x^{2}-48b^{4}c^{2}d^{2}x^{2}+60a^{3}b\,d^{4}x-360a^{2}b^{2}c\,d^{3}x+480a\,b^{3}c\cdots\right.}{15(dx+c)^{\frac{3}{2}}d^{5}}$
derivativedivides	$\frac{\frac{2b^{4}(dx+c)^{\frac{5}{2}}}{5}+\frac{8a\,b^{3}d(dx+c)^{\frac{3}{2}}}{3}-\frac{8b^{4}c(dx+c)^{\frac{3}{2}}}{3}+12a^{2}b^{2}d^{2}\sqrt{dx+c}-24a\,b^{3}cd\sqrt{dx+c}+12b^{4}c^{2}\sqrt{dx+c}-\frac{2\left(a^{4}d\cdots\right.}{\cdots}}{d^{5}}$
default	$\frac{\frac{2b^{4}(dx+c)^{\frac{5}{2}}}{5}+\frac{8a\,b^{3}d(dx+c)^{\frac{3}{2}}}{3}-\frac{8b^{4}c(dx+c)^{\frac{3}{2}}}{3}+12a^{2}b^{2}d^{2}\sqrt{dx+c}-24a\,b^{3}cd\sqrt{dx+c}+12b^{4}c^{2}\sqrt{dx+c}-\frac{2\left(a^{4}d\cdots\right.}{\cdots}}{d^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/5*b^4*(d*x+c)^(5/2)+4/3*a*b^3*d*(d*x+c)^(3/2)-4/3*b^4*c*(d*x+c)^(3
/2)+6*a^2*b^2*d^2*(d*x+c)^(1/2)-12*a*b^3*c*d*(d*x+c)^(1/2)+6*b^4*c^2*(d*x+c
)^(1/2)-1/3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
/(d*x+c)^(3/2)-4*b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^(1
/2))
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.50

$$\frac{2\left(\frac{3\,(dx+c)^{\frac{5}{2}}b^4-20\,(b^4c-ab^3d)\,(dx+c)^{\frac{3}{2}}+90\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\,\sqrt{dx+c}}{d^4}-\frac{5\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4-12\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\,(dx+c))}{(dx+c)^{\frac{3}{2}}d^4}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(d*x + c)^(5/2)*b^4 - 20*(b^4*c - a*b^3*d)*(d*x + c)^(3/2) + 90*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(d*x + c))/d^4 - 5*(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c))/((d*x + c)^(3/2)*d^4))/
d
```

Fricas [A]
time = 0.31, size = 203, normalized size = 1.62

$$\frac{2\,(3\,b^4d^4x^4+128\,b^4c^4-320\,ab^3c^3d+240\,a^2b^2c^2d^2-40\,a^3bcd^3-5\,a^4d^4-4\,(2\,b^4cd^3-5\,ab^3d^4)x^3+6\,(8\,b^4c^2d^2-20\,ab^3cd^3+15\,a^2b^2d^4)x^2+12\,(16\,b^4c^3d-40\,ab^3c^2d^2+30\,a^2b^2cd^3-5\,a^3bd^4)x)\sqrt{dx+c}}{15\,(d^7x^2+2\,cd^6x+c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*d^4*x^4 + 128*b^4*c^4 - 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 -
 40*a^3*b*c*d^3 - 5*a^4*d^4 - 4*(2*b^4*c*d^3 - 5*a*b^3*d^4)*x^3 + 6*(8*b^4*
c^2*d^2 - 20*a*b^3*c*d^3 + 15*a^2*b^2*d^4)*x^2 + 12*(16*b^4*c^3*d - 40*a*b^
3*c^2*d^2 + 30*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^7*x^2 + 2*c
*d^6*x + c^2*d^5)
```

Sympy [A]
time = 22.29, size = 136, normalized size = 1.09

$$\frac{2b^4\,(c+dx)^{\frac{5}{2}}}{5d^5}-\frac{8b\,(ad-bc)^3}{d^5\sqrt{c+dx}}+\frac{(c+dx)^{\frac{3}{2}}\cdot(8ab^3d-8b^4c)}{3d^5}+\frac{\sqrt{c+dx}\,\,(12a^2b^2d^2-24ab^3cd+12b^4c^2)}{d^5}-\frac{2\,(ad-bc)^4}{3d^5\,(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(5/2)/(5*d**5) - 8*b*(a*d - b*c)**3/(d**5*sqrt(c + d*x))
 + (c + d*x)**(3/2)*(8*a*b**3*d - 8*b**4*c)/(3*d**5) + sqrt(c + d*x)*(12*a**
2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c**2)/d**5 - 2*(a*d - b*c)**4/(3*d**5
*(c + d*x)**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(109) = 218.
time = 0.01, size = 282, normalized size = 2.26

$$\frac{\frac{2}{5}\sqrt{c+dx}\,\,(c+dx)^2\,b^4d^{20}-\frac{8}{3}\sqrt{c+dx}\,\,(c+dx)\,b^4cd^{20}+\frac{8}{3}\sqrt{c+dx}\,\,(c+dx)\,b^3d^{21}a+12\sqrt{c+dx}\,b^4c^2d^{20}-24\sqrt{c+dx}\,b^3cd^{21}a+12\sqrt{c+dx}\,b^2d^{22}a^2}{d^{25}}+\frac{24\,(c+dx)\,b^4c^3-72\,(c+dx)\,b^3c^2da+72\,(c+dx)\,b^2cd^2a^2-24\,(c+dx)\,bd^3a^3-2b^4c^4+8b^3c^3da-12b^2c^2d^2a^2+8bcd^3a^3-2d^4a^4}{3d^5\sqrt{c+dx}\,\,(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(12*(d*x + c)*b^4*c^3 - b^4*c^4 - 36*(d*x + c)*a*b^3*c^2*d + 4*a*b^3*c^
3*d + 36*(d*x + c)*a^2*b^2*c*d^2 - 6*a^2*b^2*c^2*d^2 - 12*(d*x + c)*a^3*b*d
^3 + 4*a^3*b*c*d^3 - a^4*d^4)/((d*x + c)^(3/2)*d^5) + 2/15*(3*(d*x + c)^(5/
2)*b^4*d^20 - 20*(d*x + c)^(3/2)*b^4*c*d^20 + 90*sqrt(d*x + c)*b^4*c^2*d^20
 + 20*(d*x + c)^(3/2)*a*b^3*d^21 - 180*sqrt(d*x + c)*a*b^3*c*d^21 + 90*sqrt
(d*x + c)*a^2*b^2*d^22)/d^25
```

Mupad [B]
time = 0.30, size = 175, normalized size = 1.40

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{\left(c+d\,x\right)\,\left(-8\,a^3\,b\,d^3+24\,a^2\,b^2\,c\,d^2-24\,a\,b^3\,c^2\,d+8\,b^4\,c^3\right)-\frac{2\,a^4\,d^4}{3}-\frac{2\,b^4\,c^4}{3}-4\,a^2\,b^2\,c^2\,d^2+\frac{8\,a\,b^3\,c^3\,d}{3}+\frac{8\,a^3\,b\,c\,d^3}{3}}{d^5\,{\left(c+d\,x\right)}^{3/2}}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(5/2))/(5*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(3/2))/(
3*d^5) + ((c + d*x)*(8*b^4*c^3 - 8*a^3*b*d^3 + 24*a^2*b^2*c*d^2 - 24*a*b^3*
c^2*d) - (2*a^4*d^4)/3 - (2*b^4*c^4)/3 - 4*a^2*b^2*c^2*d^2 + (8*a*b^3*c^3*d
)/3 + (8*a^3*b*c*d^3)/3)/(d^5*(c + d*x)^(3/2)) + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(1/2))/d^5
```

3.1436 $\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

```
[Out] 2/3*(-a*d+b*c)^3/d^4/(d*x+c)^(3/2)+2/3*b^3*(d*x+c)^(3/2)/d^4-6*b*(-a*d+b*c)
^2/d^4/(d*x+c)^(1/2)-6*b^2*(-a*d+b*c)*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$-\frac{6b^2\sqrt{c+dx}\,(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(3*d^4*(c + d*x)^(3/2)) - (6*b*(b*c - a*d)^2)/(d^4*Sqrt[c
 + d*x]) - (6*b^2*(b*c - a*d)*Sqrt[c + d*x])/d^4 + (2*b^3*(c + d*x)^(3/2))/
(3*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{5/2}}+\frac{3b(bc-ad)^2}{d^3(c+dx)^{3/2}}-\frac{3b^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{b^3\sqrt{c+dx}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 1.05

$$-\frac{2\left(a^3d^3+3a^2bd^2(2c+3dx)-3ab^2d\left(8c^2+12cdx+3d^2x^2\right)+b^3\left(16c^3+24c^2dx+6cd^2x^2-d^3x^3\right)\right)}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a^3*d^3 + 3*a^2*b*d^2*(2*c + 3*d*x) - 3*a*b^2*d*(8*c^2 + 12*c*d*x + 3*
d^2*x^2) + b^3*(16*c^3 + 24*c^2*d*x + 6*c*d^2*x^2 - d^3*x^3)))/(3*d^4*(c +
d*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 5.14, size = 136, normalized size = 1.42

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-16b^3c^3+24b^2c^2d\left(a-bx\right)+6bcd^2\left(-a^2+6abx-b^2x^2\right)+d^3\left(-a^3-9a^2bx+9ab^2x^2+b^3x^3\right)\right)}{3d^4\left(c+dx\right)^{\frac{3}{2}}},d!=0\right\}\right\},\frac{a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}}{c^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^3/(c + d*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (-16 b ^ 3 c ^ 3 + 24 b ^ 2 c ^ 2 d (a - b x) + 6 b c d ^ 2 (
-a ^ 2 + 6 a b x - b ^ 2 x ^ 2) + d ^ 3 (-a ^ 3 - 9 a ^ 2 b x + 9 a b ^ 2 x
 ^ 2 + b ^ 3 x ^ 3)) / (3 d ^ 4 (c + d x) ^ (3 / 2)), d != 0}}, (a ^ 3 x +
3 a ^ 2 b x ^ 2 / 2 + a b ^ 2 x ^ 3 + b ^ 3 x ^ 4 / 4) / c ^ (5 / 2)]
```

Maple [A]
time = 0.17, size = 122, normalized size = 1.27

method	result	size
risch	$\frac{2b^2(bdx+9ad-8bc)\sqrt{dx+c}}{3d^4}-\frac{2(9bdx+ad+8bc)(a^2d^2-2abcd+b^2c^2)}{3d^4(dx+c)^{\frac{3}{2}}}$	76
gosper	$-\frac{2\left(-b^3x^3d^3-9d^3a\,x^2b^2+6b^3c\,d^2x^2+9a^2b\,d^3x-36a\,b^2c\,d^2x+24b^3c^2dx+a^3d^3+6a^2bc\,d^2-24a\,b^2c^2d+16b^3c^3\right)}{3(dx+c)^{\frac{3}{2}}d^4}$	115
trager	$-\frac{2\left(-b^3x^3d^3-9d^3a\,x^2b^2+6b^3c\,d^2x^2+9a^2b\,d^3x-36a\,b^2c\,d^2x+24b^3c^2dx+a^3d^3+6a^2bc\,d^2-24a\,b^2c^2d+16b^3c^3\right)}{3(dx+c)^{\frac{3}{2}}d^4}$	115
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{3}{2}}}{3}+6ad\,b^2\sqrt{dx+c}-6b^3c\sqrt{dx+c}-\frac{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{6b\left(a^2d^2-2abcd+b^2c^2\right)}{\sqrt{dx+c}}}{d^4}$	122
default	$\frac{\frac{2b^3(dx+c)^{\frac{3}{2}}}{3}+6ad\,b^2\sqrt{dx+c}-6b^3c\sqrt{dx+c}-\frac{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{6b\left(a^2d^2-2abcd+b^2c^2\right)}{\sqrt{dx+c}}}{d^4}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/3*b^3*(d*x+c)^(3/2)+3*a*d*b^2*(d*x+c)^(1/2)-3*b^3*c*(d*x+c)^(1/2)-
1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^(3/2)-3*b*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.27, size = 122, normalized size = 1.27

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^3-9\left(b^3c-ab^2d\right)\sqrt{dx+c}}{d^3}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3-9\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)}{(dx+c)^{\frac{3}{2}}d^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^3 - 9*(b^3*c - a*b^2*d)*sqrt(d*x + c))/d^3 + (b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*(d*x + c))/((d*x + c)^(3/2)*d^3))/d
```

Fricas [A]
time = 0.29, size = 136, normalized size = 1.42

$$\frac{2\left(b^3d^3x^3-16\,b^3c^3+24\,ab^2c^2d-6\,a^2bcd^2-a^3d^3-3\left(2\,b^3cd^2-3\,ab^2d^3\right)x^2-3\left(8\,b^3c^2d-12\,ab^2cd^2+3\,a^2bd^3\right)x\right)\sqrt{dx+c}}{3\left(d^6x^2+2\,cd^5x+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*d^3*x^3 - 16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a^3*d^3 -
3*(2*b^3*c*d^2 - 3*a*b^2*d^3)*x^2 - 3*(8*b^3*c^2*d - 12*a*b^2*c*d^2 + 3*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [A]
time = 0.60, size = 461, normalized size = 4.80

$$\begin{cases} -\frac{2a^3d^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{12a^2bcd^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{18a^2bd^3x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{48ab^2c^2d}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{72ab^2cd^2x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{18ab^2d^3x^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{32b^3c^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{48b^3c^2dx}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}-\frac{12b^3cd^2x^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}}+\frac{2b^3d^3x^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**3*d**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) -
 12*a**2*b*c*d**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 18*a*
*2*b*d**3*x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 48*a*b**2*c
**2*d/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 72*a*b**2*c*d**2*
x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 18*a*b**2*d**3*x**2/(
3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 32*b**3*c**3/(3*c*d**4*s
qrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 48*b**3*c**2*d*x/(3*c*d**4*sqrt(c
+ d*x) + 3*d**5*x*sqrt(c + d*x)) - 12*b**3*c*d**2*x**2/(3*c*d**4*sqrt(c + d
*x) + 3*d**5*x*sqrt(c + d*x)) + 2*b**3*d**3*x**3/(3*c*d**4*sqrt(c + d*x) +
3*d**5*x*sqrt(c + d*x)), Ne(d, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**
3 + b**3*x**4/4)/c**(5/2), True))
```

Giac [A]
time = 0.00, size = 172, normalized size = 1.79

$$\frac{\frac{2}{3}\sqrt{c+dx}\,(c+dx)\,b^3d^8-6\sqrt{c+dx}\,b^3cd^8+6\sqrt{c+dx}\,b^2d^9a}{d^{12}}+\frac{-18\,(c+dx)\,b^3c^2+36\,(c+dx)\,b^2cda-18\,(c+dx)\,bd^2a^2+2b^3c^3-6b^2c^2da+6bcd^2a^2-2d^3a^3}{3d^4\sqrt{c+dx}\,(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(9*(d*x + c)*b^3*c^2 - b^3*c^3 - 18*(d*x + c)*a*b^2*c*d + 3*a*b^2*c^2*
d + 9*(d*x + c)*a^2*b*d^2 - 3*a^2*b*c*d^2 + a^3*d^3)/((d*x + c)^(3/2)*d^4)
+ 2/3*((d*x + c)^(3/2)*b^3*d^8 - 9*sqrt(d*x + c)*b^3*c*d^8 + 9*sqrt(d*x + c
)*a*b^2*d^9)/d^12
```

Mupad [B]
time = 0.09, size = 128, normalized size = 1.33

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^3-2\,a^3\,d^3+2\,b^3\,c^3-18\,b^3\,c\,{\left(c+d\,x\right)}^2-18\,b^3\,c^2\,\left(c+d\,x\right)+18\,a\,b^2\,d\,{\left(c+d\,x\right)}^2-18\,a^2\,b\,d^2\,\left(c+d\,x\right)-6\,a\,b^2\,c^2\,d+6\,a^2\,b\,c\,d^2+36\,a\,b^2\,c\,d\,\left(c+d\,x\right)}{3\,d^4\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^3 - 2*a^3*d^3 + 2*b^3*c^3 - 18*b^3*c*(c + d*x)^2 - 18*b^3*
c^2*(c + d*x) + 18*a*b^2*d*(c + d*x)^2 - 18*a^2*b*d^2*(c + d*x) - 6*a*b^2*c
^2*d + 6*a^2*b*c*d^2 + 36*a*b^2*c*d*(c + d*x))/(3*d^4*(c + d*x)^(3/2))
```

3.1437 $\int \frac{(a+bx)^2}{(c+dx)^{5/2}} \, dx$

Optimal. Leaf size=67

$$-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

```
[Out] -2/3*(-a*d+b*c)^2/d^3/(d*x+c)^(3/2)+4*b*(-a*d+b*c)/d^3/(d*x+c)^(1/2)+2*b^2*
(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(3*d^3*(c + d*x)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x]) + (2*b^2*Sqrt[c + d*x])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{5/2}} \, dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}}-\frac{2b(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{b^2}{d^2\sqrt{c+dx}}\right) dx\\ &= -\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}+\frac{2b^2\sqrt{c+dx}}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.93

$$\frac{-2a^2d^2-4abd(2c+3dx)+2b^2\left(8c^2+12cdx+3d^2x^2\right)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*a^2*d^2 - 4*a*b*d*(2*c + 3*d*x) + 2*b^2*(8*c^2 + 12*c*d*x + 3*d^2*x^2))
/(3*d^3*(c + d*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.78, size = 87, normalized size = 1.30

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(8b^2c^2+4bcd\left(-a+3bx\right)+d^2\left(-a^2-6abx+3b^2x^2\right)\right)}{3d^3\left(c+dx\right)^{\frac{3}{2}}},d!=0\right\}\right\},\frac{a^2x+abx^2+\frac{b^2x^3}{3}}{c^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^2/(c + d*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (8 b ^ 2 c ^ 2 + 4 b c d (-a + 3 b x) + d ^ 2 (-a ^ 2 - 6 a b
 x + 3 b ^ 2 x ^ 2)) / (3 d ^ 3 (c + d x) ^ (3 / 2)), d != 0}}, (a ^ 2 x +
a b x ^ 2 + b ^ 2 x ^ 3 / 3) / c ^ (5 / 2)]
```

Maple [A]
time = 0.16, size = 66, normalized size = 0.99

method	result	size
risch	$\frac{2b^2\sqrt{dx+c}}{d^3}-\frac{2(6bdx+ad+5bc)(ad-bc)}{3d^3(dx+c)^{\frac{3}{2}}}$	50
gosper	$-\frac{2\left(-3b^2x^2d^2+6ab\,d^2x-12b^2cdx+a^2d^2+4abcd-8b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}d^3}$	62
trager	$-\frac{2\left(-3b^2x^2d^2+6ab\,d^2x-12b^2cdx+a^2d^2+4abcd-8b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}d^3}$	62
derivativedivides	$\frac{2b^2\sqrt{dx+c}-\frac{4b(ad-bc)}{\sqrt{dx+c}}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}}}{d^3}$	66
default	$\frac{2b^2\sqrt{dx+c}-\frac{4b(ad-bc)}{\sqrt{dx+c}}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}}}{d^3}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(b^2*(d*x+c)^(1/2)-2*b*(a*d-b*c)/(d*x+c)^(1/2)-1/3*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/(d*x+c)^(3/2))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.07

$$\frac{2\left(\frac{3\sqrt{dx+c}\,b^2}{d^2}-\frac{b^2c^2-2\,abcd+a^2d^2-6\,(b^2c-abd)\,(dx+c)}{(dx+c)^{\frac{3}{2}}\,d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*b^2/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 6*(b^2*c -
a*b*d)*(d*x + c))/((d*x + c)^(3/2)*d^2))/d
```

Fricas [A]
time = 0.30, size = 85, normalized size = 1.27

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-4\,abcd-a^2d^2+6\left(2\,b^2cd-abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x)*sqrt(d*x + c)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A]
time = 0.60, size = 265, normalized size = 3.96

$$\begin{cases} -\frac{2a^2d^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}-\frac{8abcd}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}-\frac{12abd^2x}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}+\frac{16b^2c^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}+\frac{24b^2cdx}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}}+\frac{6b^2d^2x^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*d**2/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) -
 8*a*b*c*d/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) - 12*a*b*d**2*
x/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 16*b**2*c**2/(3*c*d**
3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 24*b**2*c*d*x/(3*c*d**3*sqrt(c
+ d*x) + 3*d**4*x*sqrt(c + d*x)) + 6*b**2*d**2*x**2/(3*c*d**3*sqrt(c + d*x)
 + 3*d**4*x*sqrt(c + d*x)), Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/c
**(5/2), True))
```

Giac [A]
time = 0.00, size = 89, normalized size = 1.33

$$\frac{2\sqrt{c+dx}\,b^2}{d^3}+\frac{12\left(c+dx\right)b^2c-12\left(c+dx\right)bda-2b^2c^2+4bcda-2d^2a^2}{3d^3\sqrt{c+dx}\,\left(c+dx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] 2*sqrt(d*x + c)*b^2/d^3 + 2/3*(6*(d*x + c)*b^2*c - b^2*c^2 - 6*(d*x + c)*a*
b*d + 2*a*b*c*d - a^2*d^2)/((d*x + c)^(3/2)*d^3)
```

Mupad [B]
time = 0.07, size = 68, normalized size = 1.01

$$\frac{6\,b^2\,{\left(c+d\,x\right)}^2-2\,a^2\,d^2-2\,b^2\,c^2+12\,b^2\,c\,\left(c+d\,x\right)-12\,a\,b\,d\,\left(c+d\,x\right)+4\,a\,b\,c\,d}{3\,d^3\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(5/2),x)

[Out] (6*b^2*(c + d*x)^2 - 2*a^2*d^2 - 2*b^2*c^2 + 12*b^2*c*(c + d*x) - 12*a*b*d*
(c + d*x) + 4*a*b*c*d)/(3*d^3*(c + d*x)^(3/2))
```

3.1438 $\int \frac{a+bx}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=40

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

```
[Out] 2/3*(-a*d+b*c)/d^2/(d*x+c)^(3/2)-2*b/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d))/(3*d^2*(c + d*x)^(3/2)) - (2*b)/(d^2*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{5/2}}+\frac{b}{d(c+dx)^{3/2}}\right)dx\\ &= \frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.72

$$-\frac{2(2bc+ad+3bdx)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(2*b*c + a*d + 3*b*d*x))/(3*d^2*(c + d*x)^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.76, size = 47, normalized size = 1.18

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(-2bc+d\left(-a-3bx\right)\right)}{3d^2\left(c+dx\right)^{\frac{3}{2}}},d!{=}0\right\}\right\},\frac{ax+\frac{bx^2}{2}}{c^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^1/(c + d*x)^(5/2),x]')
```

```
[Out] Piecewise[{{2 (-2 b c + d (-a - 3 b x)) / (3 d ^ 2 (c + d x) ^ (3 / 2)), d
!= 0}}, (a x + b x ^ 2 / 2) / c ^ (5 / 2)]
```

Maple [A]
time = 0.14, size = 34, normalized size = 0.85

method	result	size
gosper	$-\frac{2(3bdx+ad+2bc)}{3(dx+c)^{\frac{3}{2}}d^2}$	26
trager	$-\frac{2(3bdx+ad+2bc)}{3(dx+c)^{\frac{3}{2}}d^2}$	26
derivativedivides	$\frac{-\frac{2(ad-bc)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b}{\sqrt{dx+c}}}{d^2}$	34
default	$\frac{-\frac{2(ad-bc)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b}{\sqrt{dx+c}}}{d^2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(-1/3*(a*d-b*c)/(d*x+c)^(3/2)-b/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.70

$$-\frac{2\left(3\left(dx+c\right)b-bc+ad\right)}{3\left(dx+c\right)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

Fricas [A]
time = 0.29, size = 46, normalized size = 1.15

$$-\frac{2\,(3\,bdx+2\,bc+ad)\sqrt{dx+c}}{3\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*d*x + 2*b*c + a*d)*sqrt(d*x + c)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A]
time = 0.50, size = 124, normalized size = 3.10

$$\begin{cases} -\frac{2ad}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}}-\frac{4bc}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}}-\frac{6bdx}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} & \text{for}\ d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(5/2),x)
[Out] Piecewise((-2*a*d/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 4*b*c
/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 6*b*d*x/(3*c*d**2*sqrt
(c + d*x) + 3*d**3*x*sqrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(5/2),
 True))
```

Giac [A]
time = 0.00, size = 42, normalized size = 1.05

$$\frac{-6\,(c+dx)\,b+2bc-2da}{3d^2\sqrt{c+dx}\,\,(c+dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x)
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

Mupad [B]
time = 0.25, size = 29, normalized size = 0.72

$$-\frac{2\,a\,d-2\,b\,c+6\,b\,\left(c+d\,x\right)}{3\,d^2\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(5/2),x)
[Out] -(2*a*d - 2*b*c + 6*b*(c + d*x))/(3*d^2*(c + d*x)^(3/2))
```

3.1439 $\int \frac{1}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=16

$$-\frac{2}{3d(c+dx)^{3/2}}$$

[Out] -2/3/d/(d*x+c)^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-5/2),x]

[Out] -2/(3*d*(c + d*x)^(3/2))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{5/2}} dx = -\frac{2}{3d(c+dx)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-5/2),x]

[Out] -2/(3*d*(c + d*x)^(3/2))

Mathics [A]
time = 1.57, size = 12, normalized size = 0.75

$$\frac{-2}{3d\,(c+dx)^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^0/(c + d*x)^(5/2),x]')
```

```
[Out] -2 / (3 d (c + d x) ^ (3 / 2))
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13
default	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13
trager	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d/(d*x+c)^(3/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.29, size = 31, normalized size = 1.94

$$-\frac{2\,\sqrt{dx+c}}{3\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d*x + c)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.88

$$-\frac{2}{3d\left(c+dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(5/2),x)
```

```
[Out] -2/(3*d*(c + d*x)**(3/2))
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.44

$$-\frac{2}{3d\sqrt{c+dx}\,\left(c+dx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.75

$$-\frac{2}{3\,d\,\left(c+d\,x\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(5/2),x)
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

3.1440 $\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] 2/3/(-a*d+b*c)/(d*x+c)^(3/2)-2*b^(3/2)*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/(-a*d+b*c)^(5/2)+2*b/(-a*d+b*c)^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {53, 65, 214}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}\,(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b)/((b*c - a*d)^2*Sqrt[c + d*x]) - (
2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{b\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{bc-ad}\\ &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{(bc-ad)^2}\\ &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}+\frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\right.}{d(bc-ad)^2}\\ &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 85, normalized size = 0.91

$$\frac{2(4bc-ad+3bdx)}{3(bc-ad)^2(c+dx)^{3/2}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)),x]
[Out] (2*(4*b*c - a*d + 3*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) + (2*b^(3/2)*
ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2)
```

Mathics [A]
time = 7.51, size = 107, normalized size = 1.15

$$\frac{2b\left(-\frac{(ad-bc)^2\sqrt{c+dx}}{b}+3b\mathrm{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right]\sqrt{\frac{ad-bc}{b}}\,(c+dx)^2+3\,(ad-bc)\,(c+dx)^{\frac{3}{2}}\right)}{3\,(ad-bc)^3\,(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((a + b*x)^1*(c + d*x)^(5/2)),x]')
```

```
[Out] 2 b (-(a d - b c) ^ 2 Sqrt[c + d x] / b + 3 b ArcTan[Sqrt[c + d x] / Sqrt[(
a d - b c) / b]] Sqrt[(a d - b c) / b] (c + d x) ^ 2 + 3 (a d - b c) (c + d
 x) ^ (3 / 2)) / (3 (a d - b c) ^ 3 (c + d x) ^ 2)
```

Maple [A]
time = 0.17, size = 90, normalized size = 0.97

method	result	size
derivativedivides	$-\frac{2}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^{2}\sqrt{dx+c}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)^{2}\sqrt{(ad-bc)\,b}}$	90
default	$-\frac{2}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^{2}\sqrt{dx+c}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)^{2}\sqrt{(ad-bc)\,b}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)+2/(a*d-b*c)^2*b/(d*x+c)^(1/2)+2*b^2/(a*d-b*c)^
2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(77) = 154$.
time = 0.31, size = 398, normalized size = 4.28

$$\left[\frac{3\,(bd^{2}x^{2}+2\,bcdx+bc^{2})\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\,(3\,bdx+4\,bc-ad)\sqrt{dx+c}}{3\,(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4})x^{2}+2\,(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3})x)},\ -\frac{2\left(3\,(bd^{2}x^{2}+2\,bcdx+bc^{2})\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+4\,bc-ad)\sqrt{dx+c}\right)}{3\,(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4})x^{2}+2\,(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*
b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2
*(3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^
2 + a^2*c*d^3)*x), -2/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/(b*c - a
*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c))
- (3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d
^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x)]
```

Sympy [A]
time = 7.04, size = 83, normalized size = 0.89

$$\frac{2b}{\sqrt{c+dx}\,(ad-bc)^2}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{2}{3\,(c+dx)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] 2*b/(sqrt(c + d*x)*(a*d - b*c)**2) + 2*b*atan(sqrt(c + d*x)/sqrt((a*d - b*c
)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 2/(3*(c + d*x)**(3/2)*(a*d - b
*c))
```

Giac [A]
time = 0.00, size = 135, normalized size = 1.45

$$2\left(\frac{3\,(c+dx)\,b+bc-da}{(3b^2c^2-6bdca+3d^2a^2)\,\sqrt{c+dx}\,(c+dx)}+\frac{2b^2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(b^2c^2-2bdca+d^2a^2)\,\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] 2*b^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 2/3*(3*(d*x + c)*b + b*c - a*d)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))
```

Mupad [B]
time = 0.33, size = 100, normalized size = 1.08

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{2}{3\,\left(a\,d-b\,c\right)}-\frac{2\,b\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(5/2)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(
a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (2/(3*(a*d - b*c)) - (2*b*(c + d*x))
/(a*d - b*c)^2)/(c + d*x)^(3/2)
```

3.1441 $\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}}-\frac{5bd}{(bc-ad)^3\sqrt{c+dx}}+\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}$$

```
[Out] -5/3*d/(-a*d+b*c)^2/(d*x+c)^(3/2)-1/(-a*d+b*c)/(b*x+a)/(d*x+c)^(3/2)+5*b^(3
/2)*d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(7/2)-5*b*
d/(-a*d+b*c)^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {44, 53, 65, 214}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}\,(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-5*d)/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - 1/((b*c - a*d)*(a + b*x)*(c + d*
x)^(3/2)) - (5*b*d)/((b*c - a*d)^3*Sqrt[c + d*x]) + (5*b^(3/2)*d*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5bd)\int \frac{1}{(a+bx)(c+dx)^3}}{2(bc-ad)^2}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}}$$

Mathematica [A]
time = 0.30, size = 125, normalized size = 1.01

$$\frac{2a^2d^2 - 2abd(7c+5dx) - b^2\left(3c^2+20cdx+15d^2x^2\right)}{3(bc-ad)^3(a+bx)(c+dx)^{3/2}} + \frac{5b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 2*a*b*d*(7*c + 5*d*x) - b^2*(3*c^2 + 20*c*d*x + 15*d^2*x^2))/(
3*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) + (5*b^(3/2)*d*ArcTan[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^2*(c + d*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.17, size = 121, normalized size = 0.98

method	result
derivativedivides	$2d\left(-\frac{1}{3(ad-bc)^{2}(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^{3}\sqrt{dx+c}}+\frac{b^{2}\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}\right)}{(ad-bc)^{3}}\right)$
default	$2d\left(-\frac{1}{3(ad-bc)^{2}(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^{3}\sqrt{dx+c}}+\frac{b^{2}\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}\right)}{(ad-bc)^{3}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/3/(a*d-b*c)^2/(d*x+c)^(3/2)+2/(a*d-b*c)^3*b/(d*x+c)^(1/2)+1/(a*d-b*
c)^3*b^2*(1/2*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+5/2/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(106) = 212$.
time = 0.33, size = 782, normalized size = 6.31

$$\left[-\frac{15\,(b^2d^3x^3 + abc^2d + (2\,b^2cd^2 + abd^3)x^2 + (b^2c^2d + 2\,abcd^2)x)\sqrt{\frac{b}{bc - ad}}\log\left(\frac{bdx + 2\,bc - ad - 2\,(bc - ad)\sqrt{dx + c}\sqrt{\frac{b}{bc - ad}}}{bx + a}\right) + 2\,(15\,b^2d^2x^2 + 3\,b^2c^2 + 14\,abcd - 2\,a^2d^2 + 10\,(2\,b^2cd + abd^2)x)\sqrt{dx + c}}{6\,(ab^3c^5 - 3\,a^2b^2c^4d + 3\,a^3bc^3d^2 - a^4c^2d^3 + (b^4c^3d^2 - 3\,ab^3c^2d^3 + 3\,a^2b^2cd^4 - a^3bd^5)x^3 + (2\,b^4c^4d - 5\,ab^3c^3d^2 + 3\,a^2b^2c^2d^3 + a^3bcd^4 - a^4d^5)x^2 + (b^4c^5 - ab^3c^4d - 3\,a^2b^2c^3d^2 + 5\,a^3bc^2d^3 - 2\,a^4cd^4)x)}, \frac{15\,(b^2d^3x^3 + abc^2d + (2\,b^2cd^2 + abd^3)x^2 + (b^2c^2d + 2\,abcd^2)x)\sqrt{-\frac{b}{bc - ad}}\arctan\left(-\frac{(bc - ad)\sqrt{dx + c}\sqrt{-\frac{b}{bc - ad}}}{bdx + bc}\right) - (15\,b^2d^2x^2 + 3\,b^2c^2 + 14\,abcd - 2\,a^2d^2 + 10\,(2\,b^2cd + abd^2)x)\sqrt{dx + c}}{3\,(ab^3c^5 - 3\,a^2b^2c^4d + 3\,a^3bc^3d^2 - a^4c^2d^3 + (b^4c^3d^2 - 3\,ab^3c^2d^3 + 3\,a^2b^2cd^4 - a^3bd^5)x^3 + (2\,b^4c^4d - 5\,ab^3c^3d^2 + 3\,a^2b^2c^2d^3 + a^3bcd^4 - a^4d^5)x^2 + (b^4c^5 - ab^3c^4d - 3\,a^2b^2c^3d^2 + 5\,a^3bc^2d^3 - 2\,a^4cd^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2
*d + 2*a*b*c*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c
- a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(15*b^2*d^2*x^2 +
3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x +
 c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d -
 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^
5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x), 1/
3*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2*d +
 2*a*b*c*d^2)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqr
t(-b/(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 + 3*b^2*c^2 + 14*a*b*c*d
 - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x + c))/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^2\,(c + dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(106) =$ 212.

time = 0.00, size = 241, normalized size = 1.94

$$2\left(\frac{\sqrt{c+dx}\,b^2d}{(2b^3c^3-6b^2dc^2a+6bd^2ca^2-2d^3a^3)\,(-(c+dx)\,b+bc-da)}+\frac{6\,(c+dx)\,bd+bdc-d^2a}{(-3b^3c^3+9b^2dc^2a-9bd^2ca^2+3d^3a^3)\,\sqrt{c+dx}\,\,(c+dx)}+\frac{5b^2d\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(-b^3c^3+3b^2dc^2a-3bd^2ca^2+d^3a^3)\,\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -5*b^2*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x + c)*b^2*d
/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c +
a*d)) - 2/3*(6*(d*x + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))
```

Mupad [B]
time = 0.38, size = 161, normalized size = 1.30

$$\frac{\frac{10\,b\,d\,(c+d\,x)}{3\,(a\,d-b\,c)^2}-\frac{2\,d}{3\,(a\,d-b\,c)}+\frac{5\,b^2\,d\,(c+d\,x)^2}{(a\,d-b\,c)^3}}{b\,(c+d\,x)^{5/2}+(a\,d-b\,c)\,\,(c+d\,x)^{3/2}}+\frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\,\sqrt{c+d\,x}\,\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^{7/2}}\right)}{(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(5/2)),x)
```

```
[Out] ((10*b*d*(c + d*x))/(3*(a*d - b*c)^2) - (2*d)/(3*(a*d - b*c)) + (5*b^2*d*(c
 + d*x)^2)/(a*d - b*c)^3)/(b*(c + d*x)^(5/2) + (a*d - b*c)*(c + d*x)^(3/2))
 + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(a*d - b*c)^(7/2)
```

3.1442 $\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=167

$$\frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}}-\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}}+\frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}}+\frac{35bd^2}{4(bc-ad)^4\sqrt{c+}$$

```
[Out] 35/12*d^2/(-a*d+b*c)^3/(d*x+c)^(3/2)-1/2/(-a*d+b*c)/(b*x+a)^2/(d*x+c)^(3/2)
+7/4*d/(-a*d+b*c)^2/(b*x+a)/(d*x+c)^(3/2)-35/4*b^(3/2)*d^2*arctanh(b^(1/2)*
(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(9/2)+35/4*b*d^2/(-a*d+b*c)^4/(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}\,(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{2(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (35*d^2)/(12*(b*c - a*d)^3*(c + d*x)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x)^2*
(c + d*x)^(3/2)) + (7*d)/(4*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(3/2)) + (35*
b*d^2)/(4*(b*c - a*d)^4*Sqrt[c + d*x]) - (35*b^(3/2)*d^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}} + \frac{(35d^2)\,\ldots}{8\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+\ldots}$$

Mathematica [A]
time = 0.60, size = 168, normalized size = 1.01

$$\frac{-8a^3d^3+8a^2bd^2(10c+7dx)+ab^2d\left(39c^2+238cdx+175d^2x^2\right)+b^3\left(-6c^3+21c^2dx+140cd^2x^2+105d^3x^3\right)}{12(bc-ad)^4(a+bx)^2(c+dx)^{3/2}}+\frac{35b^{3/2}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{4(-bc+ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (-8*a^3*d^3 + 8*a^2*b*d^2*(10*c + 7*d*x) + a*b^2*d*(39*c^2 + 238*c*d*x + 17
5*d^2*x^2) + b^3*(-6*c^3 + 21*c^2*d*x + 140*c*d^2*x^2 + 105*d^3*x^3))/(12*(
b*c - a*d)^4*(a + b*x)^2*(c + d*x)^(3/2)) + (35*b^(3/2)*d^2*ArcTan[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*(-(b*c) + a*d)^(9/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^3*(c + d*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.17, size = 143, normalized size = 0.86

method	result
derivativedivides	$2d^2\left(-\frac{1}{3(ad-bc)^3(dx+c)^{\frac{3}{2}}}+\frac{3b}{(ad-bc)^4\sqrt{dx+c}}+\frac{b^2\left(\frac{\frac{11b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{13ad}{8}-\frac{13bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{35\arctan\left(\frac{}{\sqrt{}}\right)}{8\sqrt{(a}}\right)}{(ad-bc)^4}\right.$
default	$2d^2\left(-\frac{1}{3(ad-bc)^3(dx+c)^{\frac{3}{2}}}+\frac{3b}{(ad-bc)^4\sqrt{dx+c}}+\frac{b^2\left(\frac{\frac{11b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{13ad}{8}-\frac{13bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{35\arctan\left(\frac{}{\sqrt{}}\right)}{8\sqrt{(a}}\right)}{(ad-bc)^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-1/3/(a*d-b*c)^3/(d*x+c)^(3/2)+3/(a*d-b*c)^4*b/(d*x+c)^(1/2)+b^2/(a*
d-b*c)^4*((11/8*b*(d*x+c)^(3/2)+(13/8*a*d-13/8*b*c)*(d*x+c)^(1/2))/((d*x+c)
*b+a*d-b*c)^2+35/8/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(139) = 278.
time = 0.32, size = 1226, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d^3 + a*b^2*d^4)*x^3 + (
b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*b^2*c^2*d^2 + a^2*b*c*d
^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*
x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(105*b^3*d^3*x^3 - 6*b^3*c^3 + 3
9*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2*d^3)
*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^
2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2
*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 +
2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4
*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5
*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^
2*d^4 + a^6*c*d^5)*x), -1/12*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d
^3 + a*b^2*d^4)*x^3 + (b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*
b^2*c^2*d^2 + a^2*b*c*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (105*b^3*d^3*x^3 - 6*b^3*c^
3 + 39*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2
*d^3)*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c)
)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^
6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*
c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d
^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^
2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(
a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5
*b*c^2*d^4 + a^6*c*d^5)*x)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(139) = 278.
time = 0.01, size = 344, normalized size = 2.06

$$2\left(\frac{-11\sqrt{c+dx}\,(c+dx)b^3d^2+13\sqrt{c+dx}\,b^3d^2c-13\sqrt{c+dx}\,b^2d^3a}{(-8b^4c^4+32b^3dc^3a-48b^2d^2c^2a^2+32bd^3ca^3-8d^4a^4)\left(-(c+dx)b+bc-da\right)^2}+\frac{9(c+dx)bd^2+bd^2c-d^3a}{(3b^4c^4-12b^3dc^3a+18b^2d^2c^2a^2-12bd^3ca^3+3d^4a^4)\sqrt{c+dx}\,(c+dx)}-\frac{35b^2d^2\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2(-4b^4c^4+16b^3dc^3a-24b^2d^2c^2a^2+16bd^3ca^3-4d^4a^4)\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] 35/4*b^2*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d
)) + 2/3*(9*(d*x + c)*b*d^2 + b*c*d^2 - a*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x + c)^(3/2)) + 1/4*(11*(d*
x + c)^(3/2)*b^3*d^2 - 13*sqrt(d*x + c)*b^3*c*d^2 + 13*sqrt(d*x + c)*a*b^2*
d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [B]
time = 0.28, size = 243, normalized size = 1.46

$$\frac{\frac{175\,b^2\,d^2\,(c+d\,x)^2}{12\,(a\,d-b\,c)^3}-\frac{2\,d^2}{3\,(a\,d-b\,c)}+\frac{35\,b^3\,d^2\,(c+d\,x)^3}{4\,(a\,d-b\,c)^4}+\frac{14\,b\,d^2\,(c+d\,x)}{3\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^{7/2}-(2\,b^2\,c-2\,a\,b\,d)\,(c+d\,x)^{5/2}+(c+d\,x)^{3/2}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{35\,b^{3/2}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{(a\,d-b\,c)^{9/2}}\right)}{4\,(a\,d-b\,c)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(5/2)),x)
```

```
[Out] ((175*b^2*d^2*(c + d*x)^2)/(12*(a*d - b*c)^3) - (2*d^2)/(3*(a*d - b*c)) + (
35*b^3*d^2*(c + d*x)^3)/(4*(a*d - b*c)^4) + (14*b*d^2*(c + d*x))/(3*(a*d -
b*c)^2))/(b^2*(c + d*x)^(7/2) - (2*b^2*c - 2*a*b*d)*(c + d*x)^(5/2) + (c +
d*x)^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (35*b^(3/2)*d^2*atan((b^(1/2)
*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(4*(a*d - b*c)^(9/2))
```

3.1443 $\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}}-\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}}+\frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}}-\frac{2}{8(bc-ad)^3(a}$$

```
[Out] -35/8*d^3/(-a*d+b*c)^4/(d*x+c)^(3/2)-1/3/(-a*d+b*c)/(b*x+a)^3/(d*x+c)^(3/2)
+3/4*d/(-a*d+b*c)^2/(b*x+a)^2/(d*x+c)^(3/2)-21/8*d^2/(-a*d+b*c)^3/(b*x+a)/(
d*x+c)^(3/2)+105/8*b^(3/2)*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/
2))/(-a*d+b*c)^(11/2)-105/8*b*d^3/(-a*d+b*c)^5/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}\,(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{3d}{4(a+bx)^2(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{3(a+bx)^3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*(c + d*x)^(3/2)) - 1/(3*(b*c - a*d)*(a + b*x)^3*
(c + d*x)^(3/2)) + (3*d)/(4*(b*c - a*d)^2*(a + b*x)^2*(c + d*x)^(3/2)) - (2
1*d^2)/(8*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - (105*b*d^3)/(8*(b*c -
a*d)^5*Sqrt[c + d*x]) + (105*b^(3/2)*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/(8*(b*c - a*d)^(11/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} - \frac{(3d)\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} + \frac{(21d^2)\int}{8}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} - \frac{}{8(bc-a}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+b}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+b}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+b}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+b}$$

Mathematica [A]
time = 0.91, size = 220, normalized size = 1.10

$$\frac{1}{24}\left(\frac{-16a^4d^4+16a^3bd^3(13c+9dx)+3a^2b^2d^2\left(55c^2+318cdx+231d^2x^2\right)+2ab^3d\left(-25c^3+90c^2dx+567cd^2x^2+420d^3x^3\right)+b^4\left(8c^4-18c^3dx+63c^2d^2x^2+420cd^3x^3+315d^4x^4\right)}{(-bc+ad)^5(a+bx)^3(c+dx)^{3/2}}+\frac{315b^{3/2}d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{11/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^4*(c + d*x)^(5/2)),x]

[Out] ((-16*a^4*d^4 + 16*a^3*b*d^3*(13*c + 9*d*x) + 3*a^2*b^2*d^2*(55*c^2 + 318*c *d*x + 231*d^2*x^2) + 2*a*b^3*d*(-25*c^3 + 90*c^2*d*x + 567*c*d^2*x^2 + 420 *d^3*x^3) + b^4*(8*c^4 - 18*c^3*d*x + 63*c^2*d^2*x^2 + 420*c*d^3*x^3 + 315* d^4*x^4))/((-(b*c) + a*d)^5*(a + b*x)^3*(c + d*x)^(3/2)) + (315*b^(3/2)*d^3 *ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(11/2)) /24

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^4*(c + d*x)^(5/2)),x]')

[Out] Timed out

Maple [**A**]
time = 0.17, size = 177, normalized size = 0.88

method	result
derivativedivides	$2d^3\left(\frac{b^2\left(\frac{\frac{41(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{35(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{55}{16}a^2d^2-\frac{55}{8}abcd+\frac{55}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{105\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^5}\right.$
default	$2d^3\left(\frac{b^2\left(\frac{\frac{41(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{35(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{55}{16}a^2d^2-\frac{55}{8}abcd+\frac{55}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{105\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^4/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2*d^3*(1/(a*d-b*c)^5*b^2*((41/16*(d*x+c)^(5/2)*b^2+35/6*(a*d-b*c)*b*(d*x+c) ^(3/2)+(55/16*a^2*d^2-55/8*a*b*c*d+55/16*b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b

```
+a*d-b*c)^3+105/16/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2)))-1/3/(a*d-b*c)^4/(d*x+c)^(3/2)+4/(a*d-b*c)^5*b/(d*x+c)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(168) = 336$.
time = 0.32, size = 1840, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 + (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4
+ (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*
a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqr
t(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqr
t(b/(b*c - a*d)))/(b*x + a)) + 2*(315*b^4*d^4*x^4 + 8*b^4*c^4 - 50*a*b^3*c^
3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 420*(b^4*c*d^3 +
 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2
- 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*x)*sqr
t(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^
2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3
+ 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*
x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d
^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7
+ a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^
4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9
*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4
+ 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3
*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 +
7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x), 1/24*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 +
 (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4 + (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2
*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^
2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
```

```
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (315*b^4*d^4*x^4 + 8*b^4*c^
4 - 50*a*b^3*c^3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 4
20*(b^4*c*d^3 + 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^
2*b^2*d^4)*x^2 - 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^
3*b*d^4)*x)*sqrt(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*
d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5
*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6
- a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 1
0*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*
x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25
*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (
3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a
^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b
^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6
*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(168) = 336.
time = 0.01, size = 469, normalized size = 2.34

$$2\left(\frac{315\,(c+dx)^4\,b^4d^3-840\,(c+dx)^3\,b^4d^3c+840\,(c+dx)^3\,b^3d^4a+693\,(c+dx)^2\,b^4d^3c^2-1386\,(c+dx)^2\,b^3d^4ca+693\,(c+dx)^2\,b^2d^5a^2-144\,(c+dx)\,b^4d^3c^3+432\,(c+dx)\,b^3d^4c^2a-432\,(c+dx)\,b^2d^5ca^2+144\,(c+dx)\,bd^6a^3-16b^4d^3c^4+64b^3d^4c^3a-96b^2d^5c^2a^2+64bd^6ca^3-16d^7a^4}{(48b^5c^5-240b^4dc^4a+480b^3d^2c^3a^2-480b^2d^3c^2a^3+240bd^4ca^4-48d^5a^5)\left(-\sqrt{c+dx}\,(c+dx)\,b+\sqrt{c+dx}\,bc-\sqrt{c+dx}\,da\right)^3}+\frac{105b^2d^3\arctan\left(\frac{b\sqrt{c+dx}}{\sqrt{-b^2c+abd}}\right)}{2\,(-8b^5c^5+40b^4dc^4a-80b^3d^2c^3a^2+80b^2d^3c^2a^3-40bd^4ca^4+8d^5a^5)\,\sqrt{-b^2c+abd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -105/8*b^2*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^5 - 5*a
*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*
d^5)*sqrt(-b^2*c + a*b*d)) - 1/24*(315*(d*x + c)^4*b^4*d^3 - 840*(d*x + c)^
3*b^4*c*d^3 + 693*(d*x + c)^2*b^4*c^2*d^3 - 144*(d*x + c)*b^4*c^3*d^3 - 16*
b^4*c^4*d^3 + 840*(d*x + c)^3*a*b^3*d^4 - 1386*(d*x + c)^2*a*b^3*c*d^4 + 43
2*(d*x + c)*a*b^3*c^2*d^4 + 64*a*b^3*c^3*d^4 + 693*(d*x + c)^2*a^2*b^2*d^5
- 432*(d*x + c)*a^2*b^2*c*d^5 - 96*a^2*b^2*c^2*d^5 + 144*(d*x + c)*a^3*b*d^
6 + 64*a^3*b*c*d^6 - 16*a^4*d^7)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*((d*x + c)^(3/2)*b - s
qrt(d*x + c)*b*c + sqrt(d*x + c)*a*d)^3)
```

Mupad [B]
time = 0.64, size = 334, normalized size = 1.67

$$\frac{\frac{231\,b^2\,d^3\,{\left(c+d\,x\right)}^2}{8\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^3}{3\,\left(a\,d-b\,c\right)}+\frac{35\,b^3\,d^3\,{\left(c+d\,x\right)}^3}{{\left(a\,d-b\,c\right)}^4}+\frac{105\,b^4\,d^3\,{\left(c+d\,x\right)}^4}{8\,{\left(a\,d-b\,c\right)}^5}+\frac{6\,b\,d^3\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{\left(c+d\,x\right)}^{9/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}+{\left(c+d\,x\right)}^{5/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}+\frac{105\,b^{3/2}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{{\left(a\,d-b\,c\right)}^{11/2}}\right)}{8\,{\left(a\,d-b\,c\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(5/2)),x)
```

```
[Out] ((231*b^2*d^3*(c + d*x)^2)/(8*(a*d - b*c)^3) - (2*d^3)/(3*(a*d - b*c)) + (3
5*b^3*d^3*(c + d*x)^3)/(a*d - b*c)^4 + (105*b^4*d^3*(c + d*x)^4)/(8*(a*d -
b*c)^5) + (6*b*d^3*(c + d*x))/(a*d - b*c)^2)/((c + d*x)^(3/2)*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + b^3*(c + d*x)^(9/2) - (3*b^3*c - 3
*a*b^2*d)*(c + d*x)^(7/2) + (c + d*x)^(5/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*
b^2*c*d)) + (105*b^(3/2)*d^3*atan((b^(1/2)*(c + d*x)^(1/2)*(a^5*d^5 - b^5*c
^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^
4))/(a*d - b*c)^(11/2)))/(8*(a*d - b*c)^(11/2))
```

3.1444 $\int (a+bx)^5(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

```
[Out] 2/15*(b*c*x+a*c)^(15/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^{3/2}\,dx &= \frac{\int (ac+bcx)^{13/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{15/2}}{15bc^6}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6(c(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]

[Out] (2*(a + b*x)^6*(c*(a + b*x))^(3/2))/(15*b)

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.28, size = 81, normalized size = 3.68

$$\text{Piecewise}\left[\left\{\left\{\frac{2b^{\frac{13}{2}}c^{\frac{3}{2}}\left(\frac{a}{b}+x\right)^{\frac{15}{2}}}{15},\text{Abs}\left[\frac{a}{b}+x\right]<1\right\}\right\},b^{\frac{13}{2}}c^{\frac{3}{2}}\text{meijerg}\left[\left\{\{1\},\left\{\frac{17}{2}\right\}\right\},\left\{\left\{\frac{15}{2}\right\},\{0\}\right\},\frac{a}{b}+x\right]+b^{\frac{13}{2}}c^{\frac{3}{2}}\text{meijerg}\left[\left\{\left\{\frac{17}{2},1\right\},\{\}\right\},\left\{\{\},\left\{\frac{15}{2},0\right\}\right\},\frac{a}{b}+x\right]\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]')

[Out] Piecewise[{{2 b ^ (13 / 2) c ^ (3 / 2) (a / b + x) ^ (15 / 2) / 15, Abs[a / b + x] < 1}}, b ^ (13 / 2) c ^ (3 / 2) meijerg[{{1}, {17 / 2}}, {{15 / 2}, {0}}, a / b + x] + b ^ (13 / 2) c ^ (3 / 2) meijerg[{{17 / 2, 1}, {}}, {{} , {15 / 2, 0}}, a / b + x]]

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{15}{2}}}{15b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{15}{2}}}{15b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}(bcx+ac)^{\frac{3}{2}}}{15b}$	23
trager	$\frac{2c(b^{7}x^{7}+7a\,b^{6}x^{6}+21a^{2}b^{5}x^{5}+35a^{3}b^{4}x^{4}+35a^{4}b^{3}x^{3}+21a^{5}b^{2}x^{2}+7a^{6}bx+a^{7})\sqrt{bcx+ac}}{15b}$	88
risch	$\frac{2c^{2}(b^{7}x^{7}+7a\,b^{6}x^{6}+21a^{2}b^{5}x^{5}+35a^{3}b^{4}x^{4}+35a^{4}b^{3}x^{3}+21a^{5}b^{2}x^{2}+7a^{6}bx+a^{7})(bx+a)}{15b\sqrt{c(bx+a)}}$	94

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^5*(b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/15*(b*c*x+a*c)^(15/2)/b/c^6

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{15}{2}}}{15\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*c*x + a*c)^(15/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(18) = 36$.
time = 0.30, size = 95, normalized size = 4.32

$$\frac{2\left(b^7cx^7+7\,ab^6cx^6+21\,a^2b^5cx^5+35\,a^3b^4cx^4+35\,a^4b^3cx^3+21\,a^5b^2cx^2+7\,a^6bcx+a^7c\right)\sqrt{bcx+ac}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(b^7*c*x^7 + 7*a*b^6*c*x^6 + 21*a^2*b^5*c*x^5 + 35*a^3*b^4*c*x^4 + 35*
a^4*b^3*c*x^3 + 21*a^5*b^2*c*x^2 + 7*a^6*b*c*x + a^7*c)*sqrt(b*c*x + a*c)/b
```

Sympy [A]
time = 0.72, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{13}{2}}c^{\frac{3}{2}}\left(\frac{a}{b}+x\right)^{\frac{15}{2}}}{15} & \text{for } \left|\frac{a}{b}+x\right|<1 \\ b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{17}{2} \\ \frac{15}{2} & 0 \end{matrix}\middle|\frac{a}{b}+x\right)+b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{17}{2},1 & \\ & \frac{15}{2},0 \end{matrix}\middle|\frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(13/2)*c**(3/2)*(a/b + x)**(15/2)/15, Abs(a/b + x) < 1), (b
**(13/2)*c**(3/2)*meijerg(((1,), (17/2,)), ((15/2,), (0,)), a/b + x) + b**(
13/2)*c**(3/2)*meijerg(((17/2, 1), ()), ((), (15/2, 0)), a/b + x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(18) = 36$.
time = 0.01, size = 1080, normalized size = 49.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x)
```

```
[Out] 2/6435*(6435*sqrt(b*c*x + a*c)*a^7*c - 15015*(3*sqrt(b*c*x + a*c)*a*c - (b*
c*x + a*c)^(3/2))*a^6 + 9009*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*
c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^5/c - 6435*(35*sqrt(b*c*x + a*c)*a^
3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*
c*x + a*c)^(7/2))*a^4/c^2 + 715*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x
```

```
 + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)
^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^3/c^3 - 195*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a^2/c^4 + 15*(3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*
x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a
*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(
11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a/c^5 - (6435*sqrt(b*c*x + a*c)*a^7*c
^7 - 15015*(b*c*x + a*c)^(3/2)*a^6*c^6 + 27027*(b*c*x + a*c)^(5/2)*a^5*c^5
- 32175*(b*c*x + a*c)^(7/2)*a^4*c^4 + 25025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 1
2285*(b*c*x + a*c)^(11/2)*a^2*c^2 + 3465*(b*c*x + a*c)^(13/2)*a*c - 429*(b*
c*x + a*c)^(15/2))/c^6)/b
```

Mupad [B]
time = 0.05, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{15/2}}{15\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(3/2)*(a + b*x)^5,x)
[Out] (2*(c*(a + b*x))^(15/2))/(15*b*c^6)
```

3.1445 $\int (a+bx)^5 \sqrt{ac+bcx}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

[Out] 2/13*(b*c*x+a*c)^(13/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5*Sqrt[a*c + b*c*x],x]

[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a+bx)^5 \sqrt{ac+bcx}\,dx &= \frac{\int (ac+bcx)^{11/2}\,dx}{c^5} \\ &= \frac{2(ac+bcx)^{13/2}}{13bc^6} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6\sqrt{c(a+bx)}}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6*Sqrt[c*(a + b*x)])/(13*b)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.13, size = 81, normalized size = 3.68

$$\text{Piecewise}\left[\left\{\left\{\frac{2b^{\frac{11}{2}}\sqrt{c}\left(\frac{a}{b}+x\right)^{\frac{13}{2}}}{13},\text{Abs}\left[\frac{a}{b}+x\right]<1\right\}\right\},b^{\frac{11}{2}}\sqrt{c}\,\text{meijerg}\left[\left\{\{1\},\left\{\frac{15}{2}\right\}\right\},\left\{\left\{\frac{13}{2}\right\},\{0\}\right\},\frac{a}{b}+x\right]+b^{\frac{11}{2}}\sqrt{c}\,\text{meijerg}\left[\left\{\left\{\frac{15}{2},1\right\},\{\}\right\},\left\{\{\},\left\{\frac{13}{2},0\right\}\right\},\frac{a}{b}+x\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5*(a*c + b*c*x)^(1/2),x]')
```

```
[Out] Piecewise[{{2 b ^ (11 / 2) Sqrt[c] (a / b + x) ^ (13 / 2) / 13, Abs[a / b +
 x] < 1}}, b ^ (11 / 2) Sqrt[c] meijerg[{{1}, {15 / 2}}, {{13 / 2}, {0}}, a
 / b + x] + b ^ (11 / 2) Sqrt[c] meijerg[{{15 / 2, 1}, {}}, {{}, {13 / 2, 0
}}, a / b + x]]
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{13}{2}}}{13b\,c^6}$	19
default	$\frac{2(bcx+ac)^{\frac{13}{2}}}{13b\,c^6}$	19
gosper	$\frac{2(bx+a)^6\sqrt{bcx+ac}}{13b}$	23
trager	$\frac{2\left(x^6b^6+6a\,x^5b^5+15a^2x^4b^4+20a^3b^3x^3+15a^4x^2b^2+6a^5xb+a^6\right)\sqrt{bcx+ac}}{13b}$	76
risch	$\frac{2c\left(x^6b^6+6a\,x^5b^5+15a^2x^4b^4+20a^3b^3x^3+15a^4x^2b^2+6a^5xb+a^6\right)(bx+a)}{13b\sqrt{c\,(bx+a)}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*(b*c*x+a*c)^(13/2)/b/c^6
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{13}{2}}}{13\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*c*x + a*c)^(13/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(18) = 36$.
time = 0.29, size = 75, normalized size = 3.41

$$\frac{2\left(b^{6}x^{6}+6\,ab^{5}x^{5}+15\,a^{2}b^{4}x^{4}+20\,a^{3}b^{3}x^{3}+15\,a^{4}b^{2}x^{2}+6\,a^{5}bx+a^{6}\right)\sqrt{bcx+ac}}{13\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*
x^2 + 6*a^5*b*x + a^6)*sqrt(b*c*x + a*c)/b
```

Sympy [A]
time = 0.66, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{11}{2}}\sqrt{c}\left(\frac{a}{b}+x\right)^{\frac{13}{2}}}{13} & \text{for}\ \left|\frac{a}{b}+x\right|<1 \\ b^{\frac{11}{2}}\sqrt{c}\,G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{15}{2} \\ \frac{13}{2} & 0 \end{matrix}\middle|\,\frac{a}{b}+x\right)+b^{\frac{11}{2}}\sqrt{c}\,G_{2,2}^{0,2}\left(\begin{matrix} \frac{15}{2},1 & \\ & \frac{13}{2},0 \end{matrix}\middle|\,\frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(11/2)*sqrt(c)*(a/b + x)**(13/2)/13, Abs(a/b + x) < 1), (b*
*(11/2)*sqrt(c)*meijerg(((1,), (15/2,)), ((13/2,), (0,)), a/b + x) + b**(11
/2)*sqrt(c)*meijerg(((15/2, 1), ()), ((), (13/2, 0)), a/b + x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(18) = 36$.
time = 0.00, size = 820, normalized size = 37.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x)
```

```
[Out] 2/3003*(3003*sqrt(b*c*x + a*c)*a^6 - 6006*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x
 + a*c)^(3/2))*a^5/c + 3003*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c
)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^4/c^2 - 1716*(35*sqrt(b*c*x + a*c)*a
^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b
*c*x + a*c)^(7/2))*a^3/c^3 + 143*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*
```

```
x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c
)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^2/c^4 - 26*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a/c^5 + (3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*x + a
*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a*c)^(
7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(11/2)
*a*c + 231*(b*c*x + a*c)^(13/2))/c^6)/b
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\left(c\,\left(a+b\,x\right)\right)^{13/2}}{13\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(1/2)*(a + b*x)^5,x)
[Out] (2*(c*(a + b*x))^(13/2))/(13*b*c^6)
```

3.1446 $\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

```
[Out] 2/11*(b*c*x+a*c)^(11/2)/b/c^6
```

Rubi [A]

time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx &= \frac{\int (ac+bcx)^{9/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{11/2}}{11bc^6}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{11b\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6)/(11*b*Sqrt[c*(a + b*x)])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.64, size = 119, normalized size = 5.41

$$\text{Piecewise}\left[\left\{\left\{0,\text{Abs}\left[\frac{a}{b}+x\right]<1\&\&\text{Abs}\left[\frac{b}{a+bx}\right]<1\right\},\left\{\frac{2b^{\frac{9}{2}}\left(\frac{a}{b}+x\right)^{\frac{11}{2}}}{11\sqrt{c}},\text{Abs}\left[\frac{a}{b}+x\right]<1||\text{Abs}\left[\frac{b}{a+bx}\right]<1\right\}\right\},\frac{b^{\frac{9}{2}}\text{meijerg}\left[\left\{\{1\},\left\{\frac{13}{2}\right\}\right\},\left\{\left\{\frac{11}{2}\right\},\{0\}\right\},\frac{a}{b}+x\right]}{\sqrt{c}}+\frac{b^{\frac{9}{2}}\text{meijerg}\left[\left\{\left\{\frac{13}{2},1\right\},\{\}\right\},\left\{\{\},\left\{\frac{11}{2},0\right\}\right\},\frac{a}{b}+x\right]}{\sqrt{c}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(1/2),x]')
```

```
[Out] Piecewise[{{0, Abs[a / b + x] < 1 && Abs[b / (a + b x)] < 1}, {2 b ^ (9 / 2
) (a / b + x) ^ (11 / 2) / (11 Sqrt[c]), Abs[a / b + x] < 1 || Abs[b / (a +
 b x)] < 1}}, b ^ (9 / 2) meijerg[{{1}, {13 / 2}}, {{11 / 2}, {0}}, a / b +
 x] / Sqrt[c] + b ^ (9 / 2) meijerg[{{13 / 2, 1}, {}}, {{}, {11 / 2, 0}}, a
 / b + x] / Sqrt[c]]
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{11}{2}}}{11b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{11}{2}}}{11b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{11b\sqrt{bcx+ac}}$	23
trager	$\frac{2\left(b^{5}x^{5}+5a\,b^{4}x^{4}+10a^{2}b^{3}x^{3}+10a^{3}b^{2}x^{2}+5a^{4}bx+a^{5}\right)\sqrt{bcx+ac}}{11cb}$	68
risch	$\frac{2\left(b^{5}x^{5}+5a\,b^{4}x^{4}+10a^{2}b^{3}x^{3}+10a^{3}b^{2}x^{2}+5a^{4}bx+a^{5}\right)(bx+a)}{11b\sqrt{c\,(bx+a)}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*(b*c*x+a*c)^(11/2)/b/c^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(18) = 36.
time = 0.27, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^{5}-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^{4}}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^{2}c^{2}-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^{3}}{c^{2}}-\frac{198\left(35\sqrt{bcx+ac}\,a^{3}c^{3}-35\,(bcx+ac)^{\frac{3}{2}}a^{2}c^{2}+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^{2}}{c^{3}}+\frac{11\left(315\sqrt{bcx+ac}\,a^{4}c^{4}-420\,(bcx+ac)^{\frac{3}{2}}a^{3}c^{3}+378\,(bcx+ac)^{\frac{5}{2}}a^{2}c^{2}-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a}{c^{4}}-\frac{693\sqrt{bcx+ac}\,a^{5}c^{5}-1155\,(bcx+ac)^{\frac{3}{2}}a^{4}c^{4}+1386\,(bcx+ac)^{\frac{5}{2}}a^{3}c^{3}-990\,(bcx+ac)^{\frac{7}{2}}a^{2}c^{2}+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}}{c^{5}}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(18) = 36$.
time = 0.30, size = 67, normalized size = 3.05

$$\frac{2\left(b^5x^5 + 5\,ab^4x^4 + 10\,a^2b^3x^3 + 10\,a^3b^2x^2 + 5\,a^4bx + a^5\right)\sqrt{bcx + ac}}{11\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*c*x + a*c)/(b*c)
```

Sympy [A]
time = 0.68, size = 88, normalized size = 4.00

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \wedge \left|\frac{a}{b}+x\right| < 1 \\ \frac{2b^{\frac{9}{2}}\left(\frac{a}{b}+x\right)^{\frac{11}{2}}}{11\sqrt{c}} & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \vee \left|\frac{a}{b}+x\right| < 1 \\ \frac{b^{\frac{9}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{13}{2} \\ \frac{11}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{\sqrt{c}} + \frac{b^{\frac{9}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{13}{2}, 1 & \\ & \frac{11}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((0, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (2*b**(9/2)*(a/b
+ x)**(11/2)/(11*sqrt(c)), (Abs(a/b + x) < 1) | (1/Abs(a/b + x) < 1)), (b**
(9/2)*meijerg(((1,), (13/2,)), ((11/2,), (0,)), a/b + x)/sqrt(c) + b**(9/2)
*meijerg(((13/2, 1), ()), ((), (11/2, 0)), a/b + x)/sqrt(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(18) = 36.
time = 0.00, size = 608, normalized size = 27.64

$$\frac{\frac{2b^5\left(\frac{1}{11}\sqrt{ac+bcx}\,(ac+bcx)^5-\frac{5}{9}\sqrt{ac+bcx}\,(ac+bcx)^4ac+\frac{10}{7}\sqrt{ac+bcx}\,(ac+bcx)^3a^2c^2-2\sqrt{ac+bcx}\,(ac+bcx)^2a^3c^3+\frac{5}{3}\sqrt{ac+bcx}\,(ac+bcx)a^4c^4-\sqrt{ac+bcx}\,a^5c^5\right)}{c^5b^5}+\frac{10ab^4\left(\frac{1}{9}\sqrt{ac+bcx}\,(ac+bcx)^4-\frac{4}{7}\sqrt{ac+bcx}\,(ac+bcx)^3ac+\frac{6}{5}\sqrt{ac+bcx}\,(ac+bcx)^2a^2c^2-\frac{4}{3}\sqrt{ac+bcx}\,(ac+bcx)a^3c^3+\sqrt{ac+bcx}\,a^4c^4\right)}{c^4b^4}+\frac{20a^2b^3\left(\frac{1}{7}\sqrt{ac+bcx}\,(ac+bcx)^3-\frac{3}{5}\sqrt{ac+bcx}\,(ac+bcx)^2ac+\sqrt{ac+bcx}\,(ac+bcx)a^2c^2-\sqrt{ac+bcx}\,a^3c^3\right)}{c^3b^3}+\frac{20a^3b^2\left(\frac{1}{5}\sqrt{ac+bcx}\,(ac+bcx)^2-\frac{2}{3}\sqrt{ac+bcx}\,(ac+bcx)ac+\sqrt{ac+bcx}\,a^2c^2\right)}{c^2b^2}+2a^5\sqrt{ac+bcx}+\frac{10a^4\left(\frac{1}{3}\sqrt{ac+bcx}\,(ac+bcx)-ac\sqrt{ac+bcx}\right)}{c}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x)
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\left(c\,\left(a+b\,x\right)\right)^{11/2}}{11\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(1/2),x)
```

```
[Out] (2*(c*(a + b*x))^(11/2))/(11*b*c^6)
```

3.1447 $\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

[Out] 2/9*(b*c*x+a*c)^(9/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]

[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx &= \frac{\int (ac+bcx)^{7/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{9/2}}{9bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{9b(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6)/(9*b*(c*(a + b*x))^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.67, size = 119, normalized size = 5.41

$$\text{Piecewise}\left[\left\{\left\{0,\text{Abs}\left[\frac{a}{b}+x\right]<1\&\&\text{Abs}\left[\frac{b}{a+bx}\right]<1\right\},\left\{\frac{2b^{\frac{7}{2}}\left(\frac{a}{b}+x\right)^{\frac{9}{2}}}{9c^{\frac{3}{2}}},\text{Abs}\left[\frac{a}{b}+x\right]<1||\text{Abs}\left[\frac{b}{a+bx}\right]<1\right\}\right\},\frac{b^{\frac{7}{2}}\text{meijerg}\left[\left\{\{1\},\left\{\frac{11}{2}\right\}\right\},\left\{\left\{\frac{9}{2}\right\},\{0\}\right\},\frac{a}{b}+x\right]}{c^{\frac{3}{2}}}+\frac{b^{\frac{7}{2}}\text{meijerg}\left[\left\{\left\{\frac{11}{2},1\right\},\{\}\right\},\left\{\{\},\left\{\frac{9}{2},0\right\}\right\},\frac{a}{b}+x\right]}{c^{\frac{3}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]')
```

```
[Out] Piecewise[{{0, Abs[a / b + x] < 1 && Abs[b / (a + b x)] < 1}, {2 b ^ (7 / 2
) (a / b + x) ^ (9 / 2) / (9 c ^ (3 / 2)), Abs[a / b + x] < 1 || Abs[b / (a
 + b x)] < 1}}, b ^ (7 / 2) meijerg[{{1}, {11 / 2}}, {{9 / 2}, {0}}, a / b
+ x] / c ^ (3 / 2) + b ^ (7 / 2) meijerg[{{11 / 2, 1}, {}}, {{}, {9 / 2, 0}
}, a / b + x] / c ^ (3 / 2)]
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{9}{2}}}{9b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{9}{2}}}{9b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{9b(bcx+ac)^{\frac{3}{2}}}$	23
trager	$\frac{2\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4}\right)\sqrt{bcx+ac}}{9c^{2}b}$	57
risch	$\frac{2\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4}\right)(bx+a)}{9cb\sqrt{c\,(bx+a)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(b*c*x+a*c)^(9/2)/b/c^6
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{9}{2}}}{9\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*c*x + a*c)^(9/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.
time = 0.30, size = 56, normalized size = 2.55

$$\frac{2\left(b^{4}x^{4}+4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+4\,a^{3}bx+a^{4}\right)\sqrt{bcx+ac}}{9\,bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(b*c*x +
a*c)/(b*c^2)
```

Sympy [A]
time = 0.60, size = 88, normalized size = 4.00

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|}<1\wedge\left|\frac{a}{b}+x\right|<1 \\ \frac{2b^{\frac{7}{2}}\left(\frac{a}{b}+x\right)^{\frac{9}{2}}}{9c^{\frac{3}{2}}} & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|}<1\vee\left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{7}{2}}G_{2,2}^{1,1}\left(\begin{matrix}1 & \frac{11}{2} \\ \frac{9}{2} & 0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{3}{2}}}+\frac{b^{\frac{7}{2}}G_{2,2}^{0,2}\left(\begin{matrix}\frac{11}{2},1 & \\ & \frac{9}{2},0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((0, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (2*b**(7/2)*(a/b
+ x)**(9/2)/(9*c**(3/2)), (Abs(a/b + x) < 1) | (1/Abs(a/b + x) < 1)), (b**(
7/2)*meijerg(((1,), (11/2,)), ((9/2,), (0,)), a/b + x)/c**(3/2) + b**(7/2)*
meijerg(((11/2, 1), ()), ((), (9/2, 0)), a/b + x)/c**(3/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(18) = 36$.
time = 0.00, size = 426, normalized size = 19.36

$$\frac{\frac{2b^{4}\left(\frac{1}{9}\sqrt{ac+bcx}\,(ac+bcx)^{4}-\frac{4}{7}\sqrt{ac+bcx}\,(ac+bcx)^{3}ac+\frac{6}{5}\sqrt{ac+bcx}\,(ac+bcx)^{2}a^{2}c^{2}-\frac{4}{3}\sqrt{ac+bcx}\,(ac+bcx)a^{3}c^{3}+\sqrt{ac+bcx}\,a^{4}c^{4}\right)}{c^{4}b^{4}}+\frac{8ab^{3}\left(\frac{1}{7}\sqrt{ac+bcx}\,(ac+bcx)^{3}-\frac{3}{5}\sqrt{ac+bcx}\,(ac+bcx)^{2}ac+\sqrt{ac+bcx}\,(ac+bcx)a^{2}c^{2}-\sqrt{ac+bcx}\,a^{3}c^{3}\right)}{c^{3}b^{3}}+\frac{12a^{2}b^{2}\left(\frac{1}{5}\sqrt{ac+bcx}\,(ac+bcx)^{2}-\frac{2}{3}\sqrt{ac+bcx}\,(ac+bcx)ac+\sqrt{ac+bcx}\,a^{2}c^{2}\right)}{c^{2}b^{2}}+2a^{4}\sqrt{ac+bcx}+\frac{8a^{3}\left(\frac{1}{3}\sqrt{ac+bcx}\,(ac+bcx)-ac\sqrt{ac+bcx}\right)}{c}}{c^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x)
```

```
[Out] 2/315*(315*sqrt(b*c*x + a*c)*a^4 - 420*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
a*c)^(3/2))*a^3/c + 126*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3
/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^2/c^2 - 36*(35*sqrt(b*c*x + a*c)*a^3*c^3
 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x +
 a*c)^(7/2))*a/c^3 + (315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/
2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c
+ 35*(b*c*x + a*c)^(9/2))/c^4)/(b*c^2)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{9/2}}{9\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(3/2),x)
```

```
[Out] (2*(c*(a + b*x))^(9/2))/(9*b*c^6)
```

3.1448 $\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

```
[Out] 2/7*(b*c*x+a*c)^(7/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx &= \frac{\int (ac+bcx)^{5/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{7/2}}{7bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{7b(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^6)/(7*b*(c*(a + b*x))^(5/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.65, size = 119, normalized size = 5.41

$$\text{Piecewise}\left[\left\{\left\{0,\text{Abs}\left[\frac{a}{b}+x\right]<1\&\&\text{Abs}\left[\frac{b}{a+bx}\right]<1\right\},\left\{\frac{2b^{\frac{5}{2}}\left(\frac{a}{b}+x\right)^{\frac{7}{2}}}{7c^{\frac{5}{2}}},\text{Abs}\left[\frac{a}{b}+x\right]<1||\text{Abs}\left[\frac{b}{a+bx}\right]<1\right\}\right\},\frac{b^{\frac{5}{2}}\text{meijerg}\left[\left\{\{1\},\left\{\frac{9}{2}\right\}\right\},\left\{\left\{\frac{7}{2}\right\},\{0\}\right\},\frac{a}{b}+x\right]}{c^{\frac{5}{2}}}+\frac{b^{\frac{5}{2}}\text{meijerg}\left[\left\{\left\{\frac{9}{2},1\right\},\{\}\right\},\left\{\{\},\left\{\frac{7}{2},0\right\}\right\},\frac{a}{b}+x\right]}{c^{\frac{5}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]')
```

```
[Out] Piecewise[{{0, Abs[a / b + x] < 1 && Abs[b / (a + b x)] < 1}, {2 b ^ (5 / 2
) (a / b + x) ^ (7 / 2) / (7 c ^ (5 / 2)), Abs[a / b + x] < 1 || Abs[b / (a
 + b x)] < 1}}, b ^ (5 / 2) meijerg[{{1}, {9 / 2}}, {{7 / 2}, {0}}, a / b +
 x] / c ^ (5 / 2) + b ^ (5 / 2) meijerg[{{9 / 2, 1}, {}}, {{}, {7 / 2, 0}},
 a / b + x] / c ^ (5 / 2)]
```

Maple [A]
time = 0.15, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{7}{2}}}{7b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{7}{2}}}{7b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{7b(bcx+ac)^{\frac{5}{2}}}$	23
trager	$\frac{2(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})\sqrt{bcx+ac}}{7c^{3}b}$	46
risch	$\frac{2(b^{3}x^{3}+3a\,b^{2}x^{2}+3a^{2}bx+a^{3})(bx+a)}{7c^{2}b\sqrt{c\,(bx+a)}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(b*c*x+a*c)^(7/2)/b/c^6
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{7}{2}}}{7\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*c*x + a*c)^(7/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.
time = 0.29, size = 45, normalized size = 2.05

$$\frac{2\,(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\sqrt{bcx + ac}}{7\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*c*x + a*c)/(b*c^3)
```

Sympy [A]
time = 0.69, size = 88, normalized size = 4.00

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \wedge \left|\frac{a}{b} + x\right| < 1 \\ \frac{2b^{\frac{5}{2}}\left(\frac{a}{b}+x\right)^{\frac{7}{2}}}{7c^{\frac{5}{2}}} & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \vee \left|\frac{a}{b} + x\right| < 1 \\ \frac{b^{\frac{5}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{9}{2} \\ \frac{7}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x \right)}{c^{\frac{5}{2}}} + \frac{b^{\frac{5}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{9}{2}, 1 & \\ & \frac{7}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x \right)}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(5/2),x)
```

```
[Out] Piecewise((0, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (2*b**(5/2)*(a/b
+ x)**(7/2)/(7*c**(5/2)), (Abs(a/b + x) < 1) | (1/Abs(a/b + x) < 1)), (b**(
5/2)*meijerg(((1,), (9/2,)), ((7/2,), (0,)), a/b + x)/c**(5/2) + b**(5/2)*m
eijerg(((9/2, 1), ()), ((), (7/2, 0)), a/b + x)/c**(5/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(18) = 36$.
time = 0.00, size = 273, normalized size = 12.41

$$\frac{\frac{2b^3\left(\frac{1}{7}\sqrt{ac+bcx}\,(ac+bcx)^3 - \frac{3}{5}\sqrt{ac+bcx}\,(ac+bcx)^2ac + \sqrt{ac+bcx}\,(ac+bcx)a^2c^2 - \sqrt{ac+bcx}\,a^3c^3\right)}{c^3b^3} + \frac{6ab^2\left(\frac{1}{5}\sqrt{ac+bcx}\,(ac+bcx)^2 - \frac{2}{3}\sqrt{ac+bcx}\,(ac+bcx)ac + \sqrt{ac+bcx}\,a^2c^2\right)}{c^2b^2} + 2a^3\sqrt{ac+bcx} + \frac{6a^2\left(\frac{1}{3}\sqrt{ac+bcx}\,(ac+bcx) - ac\sqrt{ac+bcx}\right)}{c}}{c^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x)
```

```
[Out] 2/35*(35*sqrt(b*c*x + a*c)*a^3 - 35*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a^2/c + 7*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a
*c + 3*(b*c*x + a*c)^(5/2))*a/c^2 - (35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c
*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2
))/c^3)/(b*c^3)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{7/2}}{7\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(5/2),x)
```

```
[Out] (2*(c*(a + b*x))^(7/2))/(7*b*c^6)
```

3.1449 $\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

```
[Out] 2/5*(b*c*x+a*c)^(5/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx &= \frac{\int (ac+bcx)^{3/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{5/2}}{5bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{5b(c(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a + b*x)^6)/(5*b*(c*(a + b*x))^(7/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.49, size = 47, normalized size = 2.14

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(a^2+bx\left(2a+bx\right)\right)\sqrt{c\left(a+bx\right)}}{5bc^4},b!{=}0\right\}\right\},\frac{a^5x}{\left(ac\right)^{\frac{7}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]')
```

```
[Out] Piecewise[{{2 (a ^ 2 + b x (2 a + b x)) Sqrt[c (a + b x)] / (5 b c ^ 4), b
!= 0}}, a ^ 5 x / (a c) ^ (7 / 2)]
```

Maple [A]
time = 0.18, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{5}{2}}}{5b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{5}{2}}}{5b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{5b(bcx+ac)^{\frac{7}{2}}}$	23
trager	$\frac{2(x^{2}b^{2}+2abx+a^{2})\sqrt{bcx+ac}}{5c^{4}b}$	35
risch	$\frac{2(x^{2}b^{2}+2abx+a^{2})(bx+a)}{5c^{3}b\sqrt{c(bx+a)}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*c*x+a*c)^(5/2)/b/c^6
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.82

$$\frac{2\left(bcx+ac\right)^{\frac{5}{2}}}{5\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*c*x + a*c)^(5/2)/(b*c^6)
```

Fricas [A]
time = 0.29, size = 34, normalized size = 1.55

$$\frac{2\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{bcx+ac}}{5\,bc^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*c*x + a*c)/(b*c^4)
```

Sympy [A]
time = 0.89, size = 80, normalized size = 3.64

$$\begin{cases} \frac{2a^{2}\sqrt{ac+bcx}}{5bc^{4}}+\frac{4ax\sqrt{ac+bcx}}{5c^{4}}+\frac{2bx^{2}\sqrt{ac+bcx}}{5c^{4}} & \text{for}\ b\neq 0 \\ \frac{a^{5}x}{(ac)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(7/2),x)
```

```
[Out] Piecewise((2*a**2*sqrt(a*c + b*c*x)/(5*b*c**4) + 4*a*x*sqrt(a*c + b*c*x)/(5
*c**4) + 2*b*x**2*sqrt(a*c + b*c*x)/(5*c**4), Ne(b, 0)), (a**5*x/(a*c)**(7/
2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(18) = 36.
time = 0.00, size = 155, normalized size = 7.05

$$\frac{\frac{2b^{2}\left(\frac{1}{5}\sqrt{ac+bcx}\,(ac+bcx)^{2}-\frac{2}{3}\sqrt{ac+bcx}\,(ac+bcx)ac+\sqrt{ac+bcx}\,a^{2}c^{2}\right)}{c^{2}b^{2}}+2a^{2}\sqrt{ac+bcx}+\frac{4a\left(\frac{1}{3}\sqrt{ac+bcx}\,(ac+bcx)-ac\sqrt{ac+bcx}\right)}{c}}{c^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x)
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c +
 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c^4)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{5/2}}{5\,b\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(7/2),x)
```

```
[Out] (2*(c*(a + b*x))^(5/2))/(5*b*c^6)
```

3.1450 $\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}} dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

```
[Out] 2/3*(b*c*x+a*c)^(3/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}}\,dx &= \frac{\int \sqrt{ac+bcx}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{3/2}}{3bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.18

$$\frac{2(a+bx)\sqrt{c(a+bx)}}{3bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)])/(3*b*c^5)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.60, size = 38, normalized size = 1.73

$$\text{Piecewise}\left[\left\{\left\{\frac{2\left(a+bx\right)\sqrt{c\left(a+bx\right)}}{3bc^{5}},b!{=}0\right\}\right\},\frac{a^{5}x}{\left(ac\right)^{\frac{9}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]')
```

```
[Out] Piecewise[{{2 (a + b x) Sqrt[c (a + b x)] / (3 b c ^ 5), b != 0}}, a ^ 5 x
/ (a c) ^ (9 / 2)]
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{3}{2}}}{3b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{3}{2}}}{3b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{3b(bcx+ac)^{\frac{9}{2}}}$	23
trager	$\frac{2(bx+a)\sqrt{bcx+ac}}{3c^{5}b}$	24
risch	$\frac{2(bx+a)^{2}}{3c^{4}b\sqrt{c\left(bx+a\right)}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*c*x+a*c)^(3/2)/b/c^6
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.82

$$\frac{2\left(bcx+ac\right)^{\frac{3}{2}}}{3\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="maxima")
[Out] 2/3*(b*c*x + a*c)^(3/2)/(b*c^6)
```

Fricas [A]
time = 0.30, size = 23, normalized size = 1.05

$$\frac{2\sqrt{bcx+ac}\,(bx+a)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*c*x + a*c)*(b*x + a)/(b*c^5)
```

Sympy [A]
time = 1.20, size = 53, normalized size = 2.41

$$\begin{cases} \frac{2a\sqrt{ac+bcx}}{3bc^5} + \frac{2x\sqrt{ac+bcx}}{3c^5} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(9/2),x)
[Out] Piecewise((2*a*sqrt(a*c + b*c*x)/(3*b*c**5) + 2*x*sqrt(a*c + b*c*x)/(3*c**5
), Ne(b, 0)), (a**5*x/(a*c)**(9/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(18) = 36.
time = 0.00, size = 67, normalized size = 3.05

$$\frac{\frac{2\left(\frac{1}{3}\sqrt{ac+bcx}\,(ac+bcx)-ac\sqrt{ac+bcx}\right)}{c}+2a\sqrt{ac+bcx}}{c^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x)
[Out] 2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2)
)/c)/(b*c^5)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{3/2}}{3\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(9/2),x)
[Out] (2*(c*(a + b*x))^(3/2))/(3*b*c^6)
```

3.1451 $\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

[Out] 2*(b*c*x+a*c)^(1/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]

[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} dx &= \frac{\int \frac{1}{\sqrt{ac+bcx}} dx}{c^5}\\ &= \frac{2\sqrt{ac+bcx}}{bc^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.20

$$\frac{2(a+bx)}{bc^5\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*(a + b*x))/(b*c^5*Sqrt[c*(a + b*x)])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.91, size = 33, normalized size = 1.65

$$\text{Piecewise}\left[\left\{\left\{\frac{2\sqrt{c\,(a+bx)}}{bc^6},b!{=}0\right\}\right\},\frac{a^5x}{(ac)^{\frac{11}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]')
```

```
[Out] Piecewise[{{2 Sqrt[c (a + b x)] / (b c ^ 6), b != 0}}, a ^ 5 x / (a c) ^ (1
1 / 2)]
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.95

method	result	size
derivativedivides	$\frac{2\sqrt{bcx+ac}}{b\,c^{6}}$	19
default	$\frac{2\sqrt{bcx+ac}}{b\,c^{6}}$	19
trager	$\frac{2\sqrt{bcx+ac}}{b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{b(bcx+ac)^{\frac{11}{2}}}$	23
risch	$\frac{2bx+2a}{c^{5}b\sqrt{c\,(bx+a)}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*c*x+a*c)^(1/2)/b/c^6
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.90

$$\frac{2\,\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="maxima")
```

[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)

Fricas [A]
time = 0.30, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="fricas")

[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)

Sympy [A]
time = 1.80, size = 29, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{ac+bcx}}{bc^6} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{11}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**5/(b*c*x+a*c)**(11/2),x)

[Out] Piecewise((2*sqrt(a*c + b*c*x)/(b*c**6), Ne(b, 0)), (a**5*x/(a*c)**(11/2), True))

Giac [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{2\sqrt{ac+bcx}}{c^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x)

[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)

Mupad [B]
time = 0.03, size = 17, normalized size = 0.85

$$\frac{2\sqrt{c\,(a+b\,x)}}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^5/(a*c + b*c*x)^(11/2),x)

[Out] (2*(c*(a + b*x))^(1/2))/(b*c^6)

3.1452 $\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

```
[Out] -2/b/c^6/(b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] -2/(b*c^6*Sqrt[a*c + b*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx &= \frac{\int \frac{1}{(ac+bcx)^{3/2}}\,dx}{c^5}\\ &= -\frac{2}{bc^6\sqrt{ac+bcx}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.20

$$-\frac{2(a+bx)}{bc^5(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x))/(b*c^5*(c*(a + b*x))^(3/2))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 3.64, size = 40, normalized size = 2.00

$$\text{Piecewise}\left[\left\{\left\{\frac{-2\sqrt{c\,(a+bx)}}{bc^7\,(a+bx)},b!{=}0\right\}\right\},\frac{a^5x}{(ac)^{\frac{13}{2}}}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]')
```

```
[Out] Piecewise[{{-2 Sqrt[c (a + b x)] / (b c ^ 7 (a + b x)), b != 0}}, a ^ 5 x /
 (a c) ^ (13 / 2)]
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2}{b\,c^{6}\sqrt{bcx+ac}}$	19
default	$-\frac{2}{b\,c^{6}\sqrt{bcx+ac}}$	19
gosper	$-\frac{2(bx+a)^{6}}{b(bcx+ac)^{\frac{13}{2}}}$	23
trager	$-\frac{2\sqrt{bcx+ac}}{c^{7}b(bx+a)}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/c^6/(b*c*x+a*c)^(1/2)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx+ac}\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

Fricas [A]
time = 0.29, size = 29, normalized size = 1.45

$$-\frac{2\sqrt{bcx+ac}}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*c*x + a*c)/(b^2*c^7*x + a*b*c^7)
```

Sympy [A]
time = 3.00, size = 41, normalized size = 2.05

$$\begin{cases} -\frac{2\sqrt{ac+bcx}}{abc^7+b^2c^7x} & \text{for } b\neq 0 \\ \frac{a^5x}{(ac)^{\frac{13}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(13/2),x)
```

```
[Out] Piecewise((-2*sqrt(a*c + b*c*x)/(a*b*c**7 + b**2*c**7*x), Ne(b, 0)), (a**5*
x/(a*c)**(13/2), True))
```

Giac [A]
time = 0.00, size = 22, normalized size = 1.10

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x)
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.85

$$-\frac{2}{b\,c^6\,\sqrt{c\,\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(13/2),x)
```

```
[Out] -2/(b*c^6*(c*(a + b*x))^(1/2))
```

3.1453 $\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx$

Optimal. Leaf size=14

$$-\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)$$

```
[Out] -arctanh(1/2*(2+x)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 213}

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx &= 2\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= -\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 1.91, size = 29, normalized size = 2.07

$$\text{Piecewise}\left[\left\{\left\{-\text{ArcCoth}\left[\frac{\sqrt{2+x}}{2}\right], \text{Abs}\left[2+x\right]>4\right\}\right\}, -\text{ArcTanh}\left[\frac{\sqrt{2+x}}{2}\right]\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/((-2 + x)*Sqrt[2 + x]),x]')
```

```
[Out] Piecewise[{{-ArcCoth[Sqrt[2 + x] / 2], Abs[2 + x] > 4}}, -ArcTanh[Sqrt[2 +
x] / 2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.16, size = 22, normalized size = 1.57

method	result	size
trager	$-\frac{\ln\left(-\frac{6+x+4\sqrt{2+x}}{-2+x}\right)}{2}$	21
derivativedivides	$\frac{\ln\left(\sqrt{2+x}-2\right)}{2}-\frac{\ln\left(\sqrt{2+x}+2\right)}{2}$	22
default	$\frac{\ln\left(\sqrt{2+x}-2\right)}{2}-\frac{\ln\left(\sqrt{2+x}+2\right)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((2+x)^(1/2)-2)-1/2*ln((2+x)^(1/2)+2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.28, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.29, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

Sympy [A]
time = 0.35, size = 26, normalized size = 1.86

$$\begin{cases} -\operatorname{acoth}\left(\frac{\sqrt{x+2}}{2}\right) & \text{for}\ |x+2|>4 \\ -\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)**(1/2),x)
```

```
[Out] Piecewise((-acoth(sqrt(x + 2)/2), Abs(x + 2) > 4), (-atanh(sqrt(x + 2)/2),
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.00, size = 31, normalized size = 2.21

$$2\left(\frac{\ln\left|\sqrt{x+2}-2\right|}{4}-\frac{\ln\left(\sqrt{x+2}+2\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x)
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(abs(sqrt(x + 2) - 2))
```

Mupad [B]
time = 0.05, size = 10, normalized size = 0.71

$$-\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(x + 2)^(1/2)),x)
[Out] -atanh((x + 2)^(1/2)/2)
```

3.1454 $\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}$$

```
[Out] 2/21*arctan(1/7*21^(1/2)*(1+5*x)^(1/2))*21^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 209}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx = \frac{2}{5}\text{Subst}\left(\int \frac{1}{\frac{7}{5}+\frac{3x^2}{5}}\,dx, x, \sqrt{1+5x}\right)$$
$$= \frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.24, size = 49, normalized size = 1.96

$$\text{Piecewise}\left[\left\{\left\{\frac{2I\sqrt{21}\,\text{ArcCosh}\left[\frac{\sqrt{35}}{5\sqrt{2+3x}}\right]}{21}, \frac{3}{\text{Abs}\left[2+3x\right]} > \frac{15}{7}\right\}\right\}, \frac{-2\sqrt{21}\,\text{ArcSin}\left[\frac{\sqrt{105}}{15\sqrt{\frac{2}{3}+x}}\right]}{21}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]')
```

```
[Out] Piecewise[{{2 I / 21 Sqrt[21] ArcCosh[Sqrt[35] / (5 Sqrt[2 + 3 x])], 3 / Ab
s[2 + 3 x] > 15 / 7}}, -2 Sqrt[21] ArcSin[Sqrt[105] / (15 Sqrt[2 / 3 + x])]
 / 21]
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{21}\,\sqrt{5x+1}}{7}\right)\sqrt{21}}{21}$	19
default	$\frac{2\arctan\left(\frac{\sqrt{21}\,\sqrt{5x+1}}{7}\right)\sqrt{21}}{21}$	19
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+21\right)\ln\left(-\frac{15\operatorname{RootOf}\left(\_Z^2+21\right)x-4\operatorname{RootOf}\left(\_Z^2+21\right)-42\sqrt{5x+1}}{2+3x}\right)}{21}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)/(5*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*arctan(1/7*21^(1/2)*(5*x+1)^(1/2))*21^(1/2)
```

Maxima [A]
time = 0.38, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Fricas [A]
time = 0.30, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Sympy [A]

time = 0.57, size = 61, normalized size = 2.44

$$\begin{cases} \frac{2\sqrt{21}\, i \operatorname{acosh}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{for } \frac{1}{|x+\frac{2}{3}|} > \frac{15}{7} \\ -\frac{2\sqrt{21}\operatorname{asin}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(21)*I*acosh(sqrt(105)/(15*sqrt(x + 2/3)))/21, 1/Abs(x + 2
/3) > 15/7), (-2*sqrt(21)*asin(sqrt(105)/(15*sqrt(x + 2/3)))/21, True))
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.08

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{3\sqrt{5x+1}}{\sqrt{21}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x)
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Mupad [B]
time = 0.06, size = 15, normalized size = 0.60

$$\frac{2\sqrt{21}\operatorname{atan}\left(\frac{\sqrt{105\,x+21}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x + 2)*(5*x + 1)^(1/2)),x)
```

```
[Out] (2*21^(1/2)*atan((105*x + 21)^(1/2)/7))/21
```

3.1455 $\int \frac{\sqrt[3]{1-x}}{1+x} \, dx$

Optimal. Leaf size=84

$$3\sqrt[3]{1-x}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}$$

```
[Out] 3*(1-x)^(1/3)+3/2*ln(2^(1/3)-(1-x)^(1/3))*2^(1/3)-1/2*ln(1+x)*2^(1/3)-2^(1/
3)*arctan(1/3*(1+2^(2/3)*(1-x)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 59, 631, 210, 31}

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + (3*Log[2^(1/3) - (1 - x)^(1/3)])/2^(2/3) - Log[1 + x]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x}}{1+x}\,dx = 3\sqrt[3]{1-x}+2\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx$$

$$= 3\sqrt[3]{1-x}-\frac{\log(1+x)}{2^{2/3}}-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{3\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\right.}{\sqrt[3]{2}}$$

$$= 3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}+\left(3\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.$$

$$= 3\sqrt[3]{1-x}-\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}$$

Mathematica [A]
time = 0.10, size = 112, normalized size = 1.33

$$3\sqrt[3]{1-x}-\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\sqrt[3]{2}\,\log\left(-2+2^{2/3}\sqrt[3]{1-x}\right)-\frac{\log\left(2+2^{2/3}\sqrt[3]{1-x}+\sqrt[3]{2}\,(1-x)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + 2^(1/3)*Log[-2 + 2^(2/3)*(1 - x)^(1/3)] - Log[2 + 2^(2/3)*(1 - x)^(1/3
) + 2^(1/3)*(1 - x)^(2/3)]/2^(2/3)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.55, size = 78, normalized size = 0.93

$$3+\mathrm{Log}\left[1-\frac{2^{\frac{2}{3}}(-1+x)^{\frac{1}{3}}\,\mathrm{exp\_polar}[I\mathrm{Pi}]}{2}\right]-2^{\frac{1}{3}}\mathrm{Log}\left[1-\frac{2^{\frac{2}{3}}(-1+x)^{\frac{1}{3}}\,\mathrm{exp\_polar}\left[\frac{5I}{3}\mathrm{Pi}\right]}{2}\right]-(-1+x)^{\frac{1}{3}}+2^{\frac{1}{3}}\mathrm{Log}\left[1-\frac{2^{\frac{2}{3}}(-1+x)^{\frac{1}{3}}\,\mathrm{exp\_polar}\left[\frac{I}{3}\mathrm{Pi}\right]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^(1/3)/(1 + x),x]')
```

```
[Out] -2 ^ (1 / 3) Log[1 - 2 ^ (2 / 3) (-1 + x) ^ (1 / 3) exp_polar[I Pi] / 2] +
3 -1 ^ (1 / 3) (-1 + x) ^ (1 / 3) + -1 ^ (2 / 3) 2 ^ (1 / 3) Log[1 - 2 ^ (2
 / 3) (-1 + x) ^ (1 / 3) exp_polar[5 I / 3 Pi] / 2] + 2 ^ (1 / 3) Log[1 - 2
 ^ (2 / 3) (-1 + x) ^ (1 / 3) exp_polar[I / 3 Pi] / 2]
```

Maple [A]
time = 3.09, size = 84, normalized size = 1.00

method	result
derivativedivides	$3(1-x)^{\frac{1}{3}}+2^{\frac{1}{3}}\ln\left((1-x)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\frac{2^{\frac{1}{3}}\ln\left((1-x)^{\frac{2}{3}}+2^{\frac{1}{3}}(1-x)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{2}-2^{\frac{1}{3}}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(1-x)\right.}{3}\right.$
default	$3(1-x)^{\frac{1}{3}}+2^{\frac{1}{3}}\ln\left((1-x)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\frac{2^{\frac{1}{3}}\ln\left((1-x)^{\frac{2}{3}}+2^{\frac{1}{3}}(1-x)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{2}-2^{\frac{1}{3}}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(1-x)\right.}{3}\right.$
trager	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/3)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 3*(1-x)^(1/3)+2^(1/3)*ln((1-x)^(1/3)-2^(1/3))-1/2*2^(1/3)*ln((1-x)^(2/3)+2^
(1/3)*(1-x)^(1/3)+2^(2/3))-2^(1/3)*arctan(1/3*(1+2^(2/3)*(1-x)^(1/3))*3^(1/
2))*3^(1/2)
```

Maxima [A]
time = 0.35, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x+1)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

Fricas [A]
time = 0.30, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3\,(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(-x + 1)^(1/3) + 1/3*sqrt(3)) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.44, size = 170, normalized size = 2.02

$$\frac{4\sqrt[3]{-1}\,\sqrt[3]{x-1}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{-\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{-2}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{i\pi}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{5i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)/(1+x),x)
```

```
[Out] 4*(-1)**(1/3)*(x - 1)**(1/3)*gamma(4/3)/gamma(7/3) + 4*(-2)**(1/3)*exp(-I*p
i/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi/3)/2 + 1)*gamma(4/3)/(3*ga
mma(7/3)) - 4*(-2)**(1/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi)/2 +
1)*gamma(4/3)/(3*gamma(7/3)) + 4*(-2)**(1/3)*exp(I*pi/3)*log(-2**(2/3)*(x -
 1)**(1/3)*exp_polar(5*I*pi/3)/2 + 1)*gamma(4/3)/(3*gamma(7/3))
```

Giac [A]
time = 0.00, size = 127, normalized size = 1.51

$$-\frac{1}{2}\cdot 2^{\frac{1}{3}}\ln\left(\left((-x+1)^{\frac{1}{3}}\right)^{2}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+2^{\frac{1}{3}}\cdot 2^{\frac{1}{3}}\right)-2^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2\left((-x+1)^{\frac{1}{3}}+\frac{2^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\cdot 2^{\frac{1}{3}}}\right)+\frac{6}{6}\cdot 2^{\frac{1}{3}}\ln\left|(-x+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right|+3\,(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x)
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(abs(-2^(1/3) + (-x + 1)^(1/3))) + 3*(-x + 1)^(1/3)
```

Mupad [B]
time = 0.07, size = 104, normalized size = 1.24

$$2^{1/3}\ln\left(18\,(1-x)^{1/3}-18\,2^{1/3}\right)+3\,(1-x)^{1/3}+\frac{2^{1/3}\ln\left(18\,(1-x)^{1/3}-9\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{2^{1/3}\ln\left(18\,(1-x)^{1/3}+9\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)/(x + 1),x)
```

```
[Out] 2^(1/3)*log(18*(1 - x)^(1/3) - 18*2^(1/3)) + 3*(1 - x)^(1/3) + (2^(1/3)*log
(18*(1 - x)^(1/3) - 9*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/2 - (2^(1
/3)*log(18*(1 - x)^(1/3) + 9*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/2
```

3.1456 $\int \sqrt[3]{3-2x}\,(7+x)\,dx$

Optimal. Leaf size=27

$$-\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}$$

```
[Out] -51/16*(3-2*x)^(4/3)+3/28*(3-2*x)^(7/3)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(1/3)*(7 + x),x]
```

```
[Out] (-51*(3 - 2*x)^(4/3))/16 + (3*(3 - 2*x)^(7/3))/28
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{3-2x}\,(7+x)\,dx &= \int \left(\frac{17}{2}\sqrt[3]{3-2x}-\frac{1}{2}(3-2x)^{4/3}\right)dx\\ &= -\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.67

$$-\frac{3}{112}(3-2x)^{4/3}(107+8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(1/3)*(7 + x),x]
```

```
[Out] (-3*(3 - 2*x)^(4/3)*(107 + 8*x))/112
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.37, size = 67, normalized size = 2.48

$$\text{Piecewise}\left[\left\{\left\{3-\frac{1^{\frac{1}{3}}\left(-321+190x+16x^2\right)(-3+2x)^{\frac{1}{3}}}{112},\text{Abs}[7+x]>\frac{17}{2}\right\}\right\},\frac{-2601\,(3-2x)^{\frac{1}{3}}}{112}-\frac{51\,(7+x)\,(3-2x)^{\frac{1}{3}}}{56}+\frac{3\,(3-2x)^{\frac{1}{3}}\,(7+x)^2}{7}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(3 - 2*x)^(1/3)*(7 + x),x]')
```

```
[Out] Piecewise[{{3 -1 ^ (1 / 3) (-321 + 190 x + 16 x ^ 2) (-3 + 2 x) ^ (1 / 3) /
 112, Abs[7 + x] > 17 / 2}}, -2601 (3 - 2 x) ^ (1 / 3) / 112 - 51 (7 + x) (
3 - 2 x) ^ (1 / 3) / 56 + 3 (3 - 2 x) ^ (1 / 3) (7 + x) ^ 2 / 7]
```

Maple [A]
time = 0.15, size = 20, normalized size = 0.74

method	result	size
gosper	$-\frac{3(8x+107)(3-2x)^{\frac{4}{3}}}{112}$	15
trager	$\left(\frac{3}{7}x^2+\frac{285}{56}x-\frac{963}{112}\right)(3-2x)^{\frac{1}{3}}$	19
derivativedivides	$-\frac{51(3-2x)^{\frac{4}{3}}}{16}+\frac{3(3-2x)^{\frac{7}{3}}}{28}$	20
default	$-\frac{51(3-2x)^{\frac{4}{3}}}{16}+\frac{3(3-2x)^{\frac{7}{3}}}{28}$	20
risch	$-\frac{3(16x^2+190x-321)(2x-3)}{112(3-2x)^{\frac{2}{3}}}$	25
meijerg	$7\,3^{\frac{1}{3}}x\,\text{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],\frac{2x}{3}\right)+\frac{3^{\frac{1}{3}}x^2\,\text{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],\frac{2x}{3}\right)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(1/3)*(7+x),x,method=_RETURNVERBOSE)
```

```
[Out] -51/16*(3-2*x)^(4/3)+3/28*(3-2*x)^(7/3)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.70

$$\frac{3}{28}\,(-2\,x+3)^{\frac{7}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="maxima")
```

```
[Out] 3/28*(-2*x + 3)^(7/3) - 51/16*(-2*x + 3)^(4/3)
```

Fricas [A]
time = 0.29, size = 19, normalized size = 0.70

$$\frac{3}{112}\left(16\,x^{2}+190\,x-321\right)\left(-2\,x+3\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="fricas")
[Out] 3/112*(16*x^2 + 190*x - 321)*(-2*x + 3)^(1/3)
```

Sympy [A]
time = 0.71, size = 112, normalized size = 4.15

$$\begin{cases} \frac{3(x+7)^2\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{7} - \frac{51(x+7)\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{56} - \frac{2601\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{112} & \text{for}\ |x+7| > \frac{17}{2} \\ \frac{3\sqrt[3]{3-2x}\,(x+7)^2}{7} - \frac{51\sqrt[3]{3-2x}\,(x+7)}{56} - \frac{2601\sqrt[3]{3-2x}}{112} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(1/3)*(7+x),x)
[Out] Piecewise((3*(x + 7)**2*(2*x - 3)**(1/3)*exp(I*pi/3)/7 - 51*(x + 7)*(2*x -
3)**(1/3)*exp(I*pi/3)/56 - 2601*(2*x - 3)**(1/3)*exp(I*pi/3)/112, Abs(x + 7
) > 17/2), (3*(3 - 2*x)**(1/3)*(x + 7)**2/7 - 51*(3 - 2*x)**(1/3)*(x + 7)/5
6 - 2601*(3 - 2*x)**(1/3)/112, True))
```

Giac [A]
time = 0.00, size = 114, normalized size = 4.22

$$\frac{-\frac{33}{2}\left(\frac{1}{4}\left(-2x+3\right)^{\frac{1}{3}}\left(-2x+3\right)-3\left(-2x+3\right)^{\frac{1}{3}}\right)+\frac{6}{4}\left(\frac{1}{7}\left(-2x+3\right)^{\frac{1}{3}}\left(-2x+3\right)^{2}-\frac{3}{2}\left(-2x+3\right)^{\frac{1}{3}}\left(-2x+3\right)+9\left(-2x+3\right)^{\frac{1}{3}}\right)-63\left(-2x+3\right)^{\frac{1}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x)
[Out] 3/28*(2*x - 3)^2*(-2*x + 3)^(1/3) - 51/16*(-2*x + 3)^(4/3)
```

Mupad [B]
time = 0.26, size = 14, normalized size = 0.52

$$-\frac{3\,{\left(3-2\,x\right)}^{4/3}\,\left(8\,x+107\right)}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 2*x)^(1/3)*(x + 7),x)
[Out] -(3*(3 - 2*x)^(4/3)*(8*x + 107))/112
```

3.1457 $\int \sqrt[3]{1-x}\,(1+x)^2\,dx$

Optimal. Leaf size=38

$$-3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}$$

```
[Out] -3*(1-x)^(4/3)+12/7*(1-x)^(7/3)-3/10*(1-x)^(10/3)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)*(1 + x)^2,x]
```

```
[Out] -3*(1 - x)^(4/3) + (12*(1 - x)^(7/3))/7 - (3*(1 - x)^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{1-x}\,(1+x)^2\,dx &= \int \left(4\sqrt[3]{1-x}-4(1-x)^{4/3}+(1-x)^{7/3}\right)dx\\ &= -3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.61

$$-\frac{3}{70}(1-x)^{4/3}\left(37+26x+7x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)*(1 + x)^2,x]
```

[Out] (-3*(1 - x)^(4/3)*(37 + 26*x + 7*x^2))/70

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.
time = 2.58, size = 84, normalized size = 2.21

$$\text{Piecewise}\left[\left\{\left\{3-\frac{1^{\frac{1}{3}}\left(-37+11x+19x^2+7x^3\right)\left(-1+x\right)^{\frac{1}{3}}}{70},\text{Abs}\left[1+x\right]>2\right\}\right\},\frac{-54\left(1-x\right)^{\frac{1}{3}}}{35}-\frac{9\left(1+x\right)\left(1-x\right)^{\frac{1}{3}}}{35}-\frac{3\left(1+x\right)^2\left(1-x\right)^{\frac{1}{3}}}{35}+\frac{3\left(1+x\right)^3\left(1-x\right)^{\frac{1}{3}}}{10}\right]$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(1 - x)^(1/3)*(1 + x)^2,x]')

[Out] Piecewise[{{3 -1 ^ (1 / 3) (-37 + 11 x + 19 x ^ 2 + 7 x ^ 3) (-1 + x) ^ (1 / 3) / 70, Abs[1 + x] > 2}}, -54 (1 - x) ^ (1 / 3) / 35 - 9 (1 + x) (1 - x) ^ (1 / 3) / 35 - 3 (1 + x) ^ 2 (1 - x) ^ (1 / 3) / 35 + 3 (1 + x) ^ 3 (1 - x) ^ (1 / 3) / 10]

Maple [A]
time = 0.17, size = 29, normalized size = 0.76

method	result
gosper	$-\frac{3(7x^2+26x+37)(1-x)^{\frac{4}{3}}}{70}$
trager	$\left(\frac{3}{10}x^3+\frac{57}{70}x^2+\frac{33}{70}x-\frac{111}{70}\right)(1-x)^{\frac{1}{3}}$
risch	$-\frac{3(7x^3+19x^2+11x-37)(-1+x)}{70(1-x)^{\frac{2}{3}}}$
derivativedivides	$-3(1-x)^{\frac{4}{3}}+\frac{12(1-x)^{\frac{7}{3}}}{7}-\frac{3(1-x)^{\frac{10}{3}}}{10}$
default	$-3(1-x)^{\frac{4}{3}}+\frac{12(1-x)^{\frac{7}{3}}}{7}-\frac{3(1-x)^{\frac{10}{3}}}{10}$
meijerg	$x\,\text{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],x\right)+x^2\,\text{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],x\right)+\frac{x^3\,\text{hypergeom}\left(\left[-\frac{1}{3},3\right],[4],x\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)^(1/3)*(1+x)^2,x,method=_RETURNVERBOSE)

[Out] -3*(1-x)^(4/3)+12/7*(1-x)^(7/3)-3/10*(1-x)^(10/3)

Maxima [A]
time = 0.26, size = 28, normalized size = 0.74

$$-\frac{3}{10}\left(-x+1\right)^{\frac{10}{3}}+\frac{12}{7}\left(-x+1\right)^{\frac{7}{3}}-3\left(-x+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="maxima")

[Out] -3/10*(-x + 1)^(10/3) + 12/7*(-x + 1)^(7/3) - 3*(-x + 1)^(4/3)

Fricas [A]
time = 0.29, size = 24, normalized size = 0.63

$$\frac{3}{70}\left(7\,x^{3}+19\,x^{2}+11\,x-37\right)\left(-x+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="fricas")
[Out] 3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(-x + 1)^(1/3)
```

Sympy [A]
time = 0.97, size = 144, normalized size = 3.79

$$\begin{cases} -\frac{3\sqrt[3]{x-1}\,(x+1)^{3}e^{-\frac{2i\pi}{3}}}{10}+\frac{3\sqrt[3]{x-1}\,(x+1)^{2}e^{-\frac{2i\pi}{3}}}{35}+\frac{9\sqrt[3]{x-1}\,(x+1)e^{-\frac{2i\pi}{3}}}{35}+\frac{54\sqrt[3]{x-1}\,e^{-\frac{2i\pi}{3}}}{35} & \text{for } |x+1|>2 \\ \frac{3\sqrt[3]{1-x}\,(x+1)^{3}}{10}-\frac{3\sqrt[3]{1-x}\,(x+1)^{2}}{35}-\frac{9\sqrt[3]{1-x}\,(x+1)}{35}-\frac{54\sqrt[3]{1-x}}{35} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)*(1+x)**2,x)
[Out] Piecewise((-3*(x - 1)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(x - 1)**(1/3
)*(x + 1)**2*exp(-2*I*pi/3)/35 + 9*(x - 1)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35
 + 54*(x - 1)**(1/3)*exp(-2*I*pi/3)/35, Abs(x + 1) > 2), (3*(1 - x)**(1/3)*
(x + 1)**3/10 - 3*(1 - x)**(1/3)*(x + 1)**2/35 - 9*(1 - x)**(1/3)*(x + 1)/3
5 - 54*(1 - x)**(1/3)/35, True))
```

Giac [A]
time = 0.00, size = 148, normalized size = 3.89

$$3\left(-\frac{1}{10}(-x+1)^{\frac{1}{3}}(-x+1)^{3}+\frac{3}{7}(-x+1)^{\frac{1}{3}}(-x+1)^{2}-\frac{3}{4}(-x+1)^{\frac{1}{3}}(-x+1)+(-x+1)^{\frac{1}{3}}\right)-3\left(-\frac{1}{4}(-x+1)^{\frac{1}{3}}(-x+1)+(-x+1)^{\frac{1}{3}}\right)+3\left(\frac{1}{7}(-x+1)^{\frac{1}{3}}(-x+1)^{2}-\frac{1}{2}(-x+1)^{\frac{1}{3}}(-x+1)+(-x+1)^{\frac{1}{3}}\right)-3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x)
[Out] 3/10*(x - 1)^3*(-x + 1)^(1/3) + 12/7*(x - 1)^2*(-x + 1)^(1/3) - 3*(-x + 1)^
(4/3)
```

Mupad [B]
time = 0.05, size = 21, normalized size = 0.55

$$-\frac{3\,{\left(1-x\right)}^{4/3}\,\left(40\,x+7\,{\left(x-1\right)}^2+30\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)*(x + 1)^2,x)
[Out] -(3*(1 - x)^(4/3)*(40*x + 7*(x - 1)^2 + 30))/70
```

3.1458 $\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*ln(b*x+a)/b^(2/3)/(-a*d+b*c)^(1/3)+3/2*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))/b^(2/3)/(-a*d+b*c)^(1/3)+arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/
(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/(-a*d+b*c)^(1/3)
```

Rubi [A]

time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {57, 631, 210, 31}

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(b^(2/3)*(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3))
 + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a
*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx = -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2b} - \text{3Su}$$

$$= -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{b^{2/3}\sqrt[3]{bc}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}} - \frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A]
time = 0.19, size = 154, normalized size = 1.11

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{-bc+ad}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{-bc+ad}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)+\log\left((-bc+ad)^{2/3}-\sqrt[3]{b}\sqrt[3]{-bc+ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)}{2b^{2/3}\sqrt[3]{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/S
qrt[3]] - 2*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)] + Log[(-(b*
c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
 + d*x)^(2/3)])/(2*b^(2/3)*(-(b*c) + a*d)^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)*(c + d*x)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.17, size = 161, normalized size = 1.16

method	result
derivativedivides	$-\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c}{\left(\frac{ad-bc}{b}\right.}\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}$
default	$-\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c}{\left(\frac{ad-bc}{b}\right.}\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/((a*d-b*c)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/2/b/((a*d-
b*c)/b)^(1/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)
/b)^(2/3))+3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b
)^(1/3)*(d*x+c)^(1/3)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(108) = 216$.
time = 0.32, size = 570, normalized size = 4.10

$$\left[\frac{\sqrt{3}\,(b^2c-abd)\sqrt{-\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,b^2dx+3\,b^2c-abd-\sqrt{3}\left((b^3c-ab^2d)^{\frac{1}{3}}(bc-ad)+(b^2c-abd)(dx+c)^{\frac{1}{3}}-2\,(b^3c-ab^2d)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\right)\sqrt{-\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}-3\,(b^3c-ab^2d)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)-(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^2+(b^3c-ab^2d)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{2}{3}}\right)+2\,(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-(b^3c-ab^2d)^{\frac{1}{3}}\right)}{2\,(b^3c-ab^2d)}, \frac{2\,\sqrt{3}\,(b^2c-abd)\sqrt{\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{1}{3}}\right)\sqrt{\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}}{3\,b}\right)-(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^2+(b^3c-ab^2d)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{2}{3}}\right)+2\,(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-(b^3c-ab^2d)^{\frac{1}{3}}\right)}{2\,(b^3c-ab^2d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))*lo
g((2*b^2*d*x + 3*b^2*c - a*b*d - sqrt(3)*((b^3*c - a*b^2*d)^(1/3)*(b*c - a*
d) + (b^2*c - a*b*d)*(d*x + c)^(1/3) - 2*(b^3*c - a*b^2*d)^(2/3)*(d*x + c)^
(2/3))*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d)) - 3*(b^3*c - a*b^2*d)^(2/
3)*(d*x + c)^(1/3))/(b*x + a)) - (b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(2/3
)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)*b + (b^3*c - a*b^2*d)^(2/3)
) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/3)*b - (b^3*c - a*b^2*d)^(1/
3)))/(b^3*c - a*b^2*d), 1/2*(2*sqrt(3)*(b^2*c - a*b*d)*sqrt((b^3*c - a*b^2*
d)^(1/3)/(b*c - a*d))*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3)*b + (b^3*c - a*
b^2*d)^(1/3))*sqrt((b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))/b) - (b^3*c - a*b^2
*d)^(2/3)*log((d*x + c)^(2/3)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)
*b + (b^3*c - a*b^2*d)^(2/3)) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/
3)*b - (b^3*c - a*b^2*d)^(1/3)))/(b^3*c - a*b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(1/3)), x)
```

Giac [A]
time = 0.01, size = 280, normalized size = 2.01

$$3\left(\frac{\left((-ab^2d+b^3c)^{\frac{1}{3}}\right)^2\ln\left(\left((c+dx)^{\frac{1}{3}}\right)^2+\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}(c+dx)^{\frac{1}{3}}+\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\right)}{6ab^2d-6b^3c}-\frac{4\left((-ab^2d+b^3c)^{\frac{1}{3}}\right)^2\arctan\left(\frac{2\left((c+dx)^{\frac{1}{3}}+\frac{\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}}\right)}{4\sqrt{3}\,ab^2d-4\sqrt{3}\,b^3c}-\frac{\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\ln\left|(c+dx)^{\frac{1}{3}}-\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\right|}{3(-bc+ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x)
```

```
[Out] 3*(b^3*c - a*b^2*d)^(2/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a
*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b^2*d) - 1/
2*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d
)/b)^(2/3))/(b^3*c - a*b^2*d)^(1/3) + ((b*c - a*d)/b)^(2/3)*log(abs((d*x +
c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

Mupad [B]
time = 0.21, size = 204, normalized size = 1.47

$$\frac{\ln\left(9\,b\,(c+d\,x)^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{4/3}\,(b\,c-a\,d)^{2/3}}\right)}{b^{2/3}\,(b\,c-a\,d)^{1/3}}+\frac{\ln\left(9\,b\,(c+d\,x)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,b^3\,c-9\,a\,b^2\,d)}{4\,b^{4/3}\,(b\,c-a\,d)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{2/3}\,(b\,c-a\,d)^{1/3}}-\frac{\ln\left(9\,b\,(c+d\,x)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,b^3\,c-9\,a\,b^2\,d)}{4\,b^{4/3}\,(b\,c-a\,d)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{2/3}\,(b\,c-a\,d)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/3)),x)
```

```
[Out] log(9*b*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(4/3)*(b*c - a*d)^(2/3))
)/(b^(2/3)*(b*c - a*d)^(1/3)) + (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i - 1
)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i - 1))
/(2*b^(2/3)*(b*c - a*d)^(1/3)) - (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i +
1)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i + 1)
)/(2*b^(2/3)*(b*c - a*d)^(1/3))
```

3.1459 $\int \frac{1}{(a+bx)(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

```
[Out] -1/2*ln(b*x+a)/b^(1/3)/(-a*d+b*c)^(2/3)+3/2*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))/b^(1/3)/(-a*d+b*c)^(2/3)-arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/
(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)/(-a*d+b*c)^(2/3)
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {59, 631, 210, 31}

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx = -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\text{Subst}\left(\right.}{}$$

$$= -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,c\right.}{\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{}\right.}{2\sqrt[3]{b}\,(bc-a}$$

Mathematica [A]
time = 0.15, size = 154, normalized size = 1.10

$$-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{-bc+ad}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{-bc+ad}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\right) + \log\left((-bc+ad)^{2/3}-\sqrt[3]{b}\,\sqrt[3]{-bc+ad}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)}{2\sqrt[3]{b}\,(-bc+ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3
))/Sqrt[3]] - 2*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)] + Log[(
-(b*c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3
)*(c + d*x)^(2/3)])/(b^(1/3)*(-(b*c) + a*d)^(2/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)*(c + d*x)^(2/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.17, size = 160, normalized size = 1.14

method	result
derivativedivides	$\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}$
default	$\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/((a*d-b*c)/b)^(2/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))-1/2/b/((a*d-b
*c)/b)^(2/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/
b)^(2/3))+1/b/((a*d-b*c)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/
b)^(1/3)*(d*x+c)^(1/3)-1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(109) = 218$.
time = 0.32, size = 900, normalized size = 6.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1
/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + sq
rt(3)*(2*(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)
^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + (b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3)
 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2), -1/2*(2*sqrt(3)*
(b^2*c - a*b*d)*sqrt((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*arctan(1/
3*sqrt(3)*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) + 2*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)^(1/3)/b)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(
2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(2/3)), x)
```

Giac [A]
time = 0.01, size = 256, normalized size = 1.83

$$3\left(\frac{\left(-ab^{2}d+b^{3}c\right)^{\frac{1}{3}}\ln\left(\left((c+dx)^{\frac{1}{3}}\right)^{2}+\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}(c+dx)^{\frac{1}{3}}+\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\right)}{6abd-6b^{2}c}+\frac{2\left(-ab^{2}d+b^{3}c\right)^{\frac{1}{3}}\arctan\left(\frac{2\left((c+dx)^{\frac{1}{3}}+\frac{\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}}{2}\right)}{\sqrt{3}\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}}\right)}{2\sqrt{3}\,abd-2\sqrt{3}\,b^{2}c}-\frac{\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\ln\left|(c+dx)^{\frac{1}{3}}-\left(\frac{-ad+bc}{b}\right)^{\frac{1}{3}}\right|}{3(-bc+ad)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x)
```

```
[Out] -3*(b^3*c - a*b^2*d)^(1/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c -
a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b*d) - 1/2
*(b^3*c - a*b^2*d)^(1/3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)
/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/(b^2*c - a*b*d) + ((b*c - a*d)/b)^(1/3)*
log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

Mupad [B]
time = 0.37, size = 206, normalized size = 1.47

$$\frac{\ln\left(9\,b^2\,(c+d\,x)^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{1/3}\,(a\,d-b\,c)^{2/3}}\right)}{b^{1/3}\,(a\,d-b\,c)^{2/3}}+\frac{\ln\left(9\,b^2\,(c+d\,x)^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,(9\,b^3\,c-9\,a\,b^2\,d)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}-\frac{\ln\left(9\,b^2\,(c+d\,x)^{1/3}+\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,(9\,b^3\,c-9\,a\,b^2\,d)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(2/3)),x)
```

```
[Out] log(9*b^2*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(1/3)*(a*d - b*c)^(2/3
)))/(b^(1/3)*(a*d - b*c)^(2/3)) + (log(9*b^2*(c + d*x)^(1/3) - ((3^(1/2)*1i
 - 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i - 1
))/(2*b^(1/3)*(a*d - b*c)^(2/3)) - (log(9*b^2*(c + d*x)^(1/3) + ((3^(1/2)*1
i + 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i +
1))/(2*b^(1/3)*(a*d - b*c)^(2/3))
```

3.1460 $\int (a+bx)^{7/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=230

$$-\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4}+\frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3}-\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2}+(bc-$$

```
[Out] 7/128*(-a*d+b*c)^5*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(
3/2)/d^(9/2)+7/192*(-a*d+b*c)^3*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d^3-7/240*(-a
*d+b*c)^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b/d^2+1/40*(-a*d+b*c)*(b*x+a)^(7/2)*(
d*x+c)^(1/2)/b/d+1/5*(b*x+a)^(9/2)*(d*x+c)^(1/2)/b-7/128*(-a*d+b*c)^4*(b*x+
a)^(1/2)*(d*x+c)^(1/2)/b/d^4
```

Rubi [A]

time = 0.11, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{240bd^2}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40bd}+\frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] (-7*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b*d^4) + (7*(b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(192*b*d^3) - (7*(b*c - a*d)^2*(a + b*x)^
(5/2)*Sqrt[c + d*x])/(240*b*d^2) + ((b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*
x])/(40*b*d) + ((a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b) + (7*(b*c - a*d)^5*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(3/2)*d^(9/2)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{7/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} + \frac{(bc-ad)\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx}{10b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} - \frac{(7(bc-ad)^2)\int \frac{(a+bx)^5}{\sqrt{c+}}}{80bd}$$

$$= -\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^9}{}$$

$$= \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-a}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-a}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-a}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-a}{}$$

Mathematica [A]

time = 0.41, size = 166, normalized size = 0.72

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(105d^4(a+bx)^4+790bd^3(a+bx)^3(c+dx)-896b^2d^2(a+bx)^2(c+dx)^2+490b^3d(a+bx)(c+dx)^3-105b^4(c+dx)^4\right)}{1920bd^4}+\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{128b^{3/2}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(105*d^4*(a + b*x)^4 + 790*b*d^3*(a + b*x)^3*(
c + d*x) - 896*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 490*b^3*d*(a + b*x)*(c + d
*x)^3 - 105*b^4*(c + d*x)^4))/(1920*b*d^4) + (7*(b*c - a*d)^5*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(3/2)*d^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/2)*(c + d*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 239, normalized size = 1.04

method	result

default	$\frac{(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}}{5d} - \frac{7(-ad+bc)\left(\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}{4d} - \frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d} - \frac{(-ad+bc)\left(\frac{\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(b*x+a)^(7/2)*(d*x+c)^(3/2)-7/10*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(5/2)*(d
*x+c)^(3/2)-5/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)-1/2*(-a*d+b
*c)/d*(1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d
*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)
^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/
2))/(b*d)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.34, size = 702, normalized size = 3.05

$$\left[-\frac{105\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(384\,b^5d^5x^4-105\,b^5c^4d+490\,ab^4c^3d^2-896\,a^2b^3c^2d^3+790\,a^3b^2cd^4+105\,a^4bd^5+48\,(b^5cd^4+31\,ab^4d^5)x^3-8\,(7\,b^5c^2d^3-32\,ab^4cd^4-263\,a^2b^3d^5)x^2+2\,(35\,b^5c^3d^2-161\,ab^4c^2d^3+289\,a^2b^3cd^4+605\,a^3b^2d^5)x)\sqrt{bx+a}\sqrt{dx+c}}{7680\,b^2d^5},\;-\frac{105\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(384\,b^5d^5x^4-105\,b^5c^4d+490\,ab^4c^3d^2-896\,a^2b^3c^2d^3+790\,a^3b^2cd^4+105\,a^4bd^5+48\,(b^5cd^4+31\,ab^4d^5)x^3-8\,(7\,b^5c^2d^3-32\,ab^4cd^4-263\,a^2b^3d^5)x^2+2\,(35\,b^5c^3d^2-161\,ab^4c^2d^3+289\,a^2b^3cd^4+605\,a^3b^2d^5)x)\sqrt{bx+a}\sqrt{dx+c}}{3840\,b^2d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*
x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 49
0*a*b^4*c^3*d^2 - 896*a^2*b^3*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 +
 48*(b^5*c*d^4 + 31*a*b^4*d^5)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 26
3*a^2*b^3*d^5)*x^2 + 2*(35*b^5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*
d^4 + 605*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^5), -1/3840*(
105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*
d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x)) - 2*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 490*a*b^4*c^3*d^2 - 896*a^2*b^3
*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 + 48*(b^5*c*d^4 + 31*a*b^4*d^5
)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 263*a^2*b^3*d^5)*x^2 + 2*(35*b^
5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*d^4 + 605*a^3*b^2*d^5)*x)*sqr
t(b*x + a)*sqrt(d*x + c))/(b^2*d^5)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(186) = 372.
time = 0.10, size = 1471, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/1920*(480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^4*abs(b)/
b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*
(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4
+ 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a
*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x +
 a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 3
5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d -
a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*b*abs(b) + (sqrt(b^2*c + (b*x + a)*b*d - a*
b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^1
9*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/
(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 4
47*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3
*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^
8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*
a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x
+ a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*b^2*abs(b
) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a^3*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{7/2}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)*(c + d*x)^(1/2), x)
```

3.1461 $\int (a+bx)^{5/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=192

$$\frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3}-\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2}+\frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd}+\frac{(a+bx)}{}$$

```
[Out] -5/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(
3/2)/d^(7/2)-5/96*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d^2+1/24*(-a*d
+b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b/d+1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b+5/6
4*(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^3
```

Rubi [A]
time = 0.07, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24bd}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b*d^3) - (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*b*d^2) + ((b*c - a*d)*(a + b*x)^(5/2)*S
qrt[c + d*x])/(24*b*d) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - (5*(b*c -
a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(3/2
)*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} - \frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3}}{\sqrt{c+d}}}{48bd}$$

$$= -\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7}}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(}{}$$

Mathematica [A]
time = 0.34, size = 144, normalized size = 0.75

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(15d^3(a+bx)^3+73bd^2(a+bx)^2(c+dx)-55b^2d(a+bx)(c+dx)^2+15b^3(c+dx)^3\right)}{192bd^3} - \frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^3*(a + b*x)^3 + 73*b*d^2*(a + b*x)^2*(c
+ d*x) - 55*b^2*d*(a + b*x)*(c + d*x)^2 + 15*b^3*(c + d*x)^3))/(192*b*d^3)
- (5*(b*c - a*d)^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])]
)/(64*b^(3/2)*d^(7/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)*(c + d*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.18, size = 206, normalized size = 1.07

method	result
default	$\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}{4d} - \frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d} - \frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b} - \frac{(ad}{}\right.}{}\right.}{}\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)-5/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(3/2)*(d*
x+c)^(3/2)-1/2*(-a*d+b*c)/d*(1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*
c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)
/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^
2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.32, size = 540, normalized size = 2.81

$$\left[\frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8(b^2cd+abd^2)x\right)+4\left(48b^4d^4x^3+15b^4c^3d-55ab^3c^2d^2+73a^2b^2cd^3+15a^3bd^4+8(b^4cd^3+17ab^3d^4)x^2-2(5b^4c^2d^2-18ab^3cd^3-59a^2b^2d^4)x\right)\sqrt{bx+a}\sqrt{dx+c}}{768b^2d^4}, \frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\left(48b^4d^4x^3+15b^4c^3d-55ab^3c^2d^2+73a^2b^2cd^3+15a^3bd^4+8(b^4cd^3+17ab^3d^4)x^2-2(5b^4c^2d^2-18ab^3cd^3-59a^2b^2d^4)x\right)\sqrt{bx+a}\sqrt{dx+c}}{384b^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2
 - 18*a*b^3*c*d^3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^
4), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2 - 18*a*b^3*c*d^
3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^4)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(154) = 308.
time = 0.07, size = 972, normalized size = 5.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/192*(24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*abs(b) - 192*((b^2*c - a*b*d)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt
(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*abs(b)/b^2 +
 (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a
)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b
^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^
2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*
(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^
4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*b^2*d^3))*b*abs(b) + 144*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(
2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
 - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)))/(sqrt(b*d)*d))*a^2*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{5/2}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(1/2), x)
```

3.1462 $\int (a+bx)^{3/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=154

$$-\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2}+\frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}+\frac{(bc-ad)^3\tanh^{-1}\Big(}{8b^{3/2}d^5}$$

```
[Out] 1/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/
2)/d^(5/2)+1/12*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d+1/3*(b*x+a)^(5/2
)*(d*x+c)^(1/2)/b-1/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -1/8*((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(b*d^2) + ((b*c - a*d)*(a
+ b*x)^(3/2)*Sqrt[c + d*x])/(12*b*d) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b
) + ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(8*b^(3/2)*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} - \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{8bd}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3}$$

Mathematica [A]
time = 0.24, size = 128, normalized size = 0.83

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(3a^2d^2+2abd(4c+7dx)+b^2\left(-3c^2+2cdx+8d^2x^2\right)\right)}{24bd^2} + \frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{8b^{3/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(3*a^2*d^2 + 2*a*b*d*(4*c + 7*d*x) + b^2*(-3*c
^2 + 2*c*d*x + 8*d^2*x^2)))/(24*b*d^2) + ((b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*b^(3/2)*d^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.15, size = 173, normalized size = 1.12

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}}{2b\sqrt{d}}\right.}{4d}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)-1/2*(-a*d+b*c)/d*(1/2/d*(b*x+a)^(1/2)*(d*
x+c)^(3/2)-1/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*
((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d
*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.32, size = 410, normalized size = 2.66

$$\left[-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2-3\,b^3c^2d+8\,ab^2cd^2+3\,a^2bd^3+2\,(b^3cd^2+7\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,b^2d^3},\,-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(8\,b^3d^3x^2-3\,b^3c^2d+8\,ab^2cd^2+3\,a^2bd^3+2\,(b^3cd^2+7\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b*d^3 + 2*(b^3*c*d^2 + 7*a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3), -1/48*(3*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a
*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b
*d^3 + 2*(b^3*c*d^2 + 7*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3
)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(122) = 244.
time = 0.04, size = 586, normalized size = 3.81

$$\frac{\frac{2b^2|b|\left(2\left(\left(\frac{\frac{1}{2304}\cdot 192b^5d^4\sqrt{a+bx}\sqrt{a+bx}}{b^7d^4}-\frac{\frac{1}{2304}\left(-48b^6d^3c+624b^5d^4a\right)}{b^7d^4}\right)\sqrt{a+bx}\sqrt{a+bx}-\frac{\frac{1}{2304}\left(72b^7d^2c^2+144b^6d^3ac-792b^5d^4a^2\right)}{b^7d^4}\right)\sqrt{a+bx}\sqrt{-abd+b^2c+bd(a+bx)}+\frac{2\left(5a^3d^3-3a^2bcd^2-ab^2c^2d-b^3c^3\right)\ln\left|\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right|}{32bd^2\sqrt{bd}}\right)}{b^2}+\frac{4ab|b|\left(2\left(\frac{\frac{1}{24}\cdot 8d^2\sqrt{a+bx}\sqrt{a+bx}}{d^2}-\frac{\frac{1}{24}\left(-4bdc+20d^2a\right)}{d^2}\right)\sqrt{a+bx}\sqrt{-abd+b^2c+bd(a+bx)}+\frac{2\left(-3a^2bd^2+2ab^2cd+b^3c^2\right)\ln\left|\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right|}{16d\sqrt{bd}}\right)}{b^2b}+\frac{2a^2|b|\left(\frac{1}{2}\sqrt{a+bx}\sqrt{-abd+b^2c+bd(a+bx)}+\frac{2\left(abd-b^2c\right)\ln\left|\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right|}{4\sqrt{bd}}\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/24*((sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*abs(b) - 24*((b^2*c - a*b*d)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*abs(b)/b^2 + 12*(sq
rt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqr
t(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a*abs(b)/b^
2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/2),x)
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/2), x)
```

3.1463 $\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$\frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

```
[Out] -1/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3
/2)/d^(3/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b+1/4*(-a*d+b*c)*(b*x+a)^(1/2)*
(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c\cdots}}}{8bd}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{\cdots}{\sqrt{c\cdots}}\cdots\right.}{\cdots}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\cdots\right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\cdots}{\sqrt{b}\cdots}\right.}{4b^{3/2}d^{3/2}}$$

Mathematica [A]
time = 0.22, size = 95, normalized size = 0.82

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(ad+b(c+2dx))}{4bd} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]

[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(a*d + b*(c + 2*d*x)))/(4*b*d) - ((b*c - a*d)^
2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^(3
/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.16, size = 140, normalized size = 1.21

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x}\right.}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(
1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*
ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*
d)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.32, size = 300, normalized size = 2.59

$$\left[\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x+b^2cd+abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,b^2d^2},\ \frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(2\,b^2d^2x+b^2cd+abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx}\,\sqrt{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(90) = 180.
time = 0.02, size = 299, normalized size = 2.58

$$\frac{\frac{2b|b|\left(2\left(\frac{\frac{1}{64}\cdot 8d^2\sqrt{a+bx}\,\sqrt{a+bx}}{d^2}-\frac{\frac{1}{64}\left(-4bdc+20d^2a\right)}{d^2}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}+\frac{2\left(-3a^2bd^2+2ab^2cd+b^3c^2\right)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{16d\sqrt{bd}}\right)}{b^2b}+\frac{2a|b|\left(\frac{1}{2}\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}+\frac{2\left(abd-b^2c\right)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{4\sqrt{bd}}\right)}{b^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

Mupad [B]
time = 0.14, size = 88, normalized size = 0.76

$$\left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

```
[Out] (x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*
c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c
)^2)/(8*b^(3/2)*d^(3/2))
```

3.1464 $\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}$$

```
[Out] (-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)/d^(
1/2)+(b*x+a)^(1/2)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {52, 65, 223, 212}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx &= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^2}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 74, normalized size = 1.03

$$
\frac{b\sqrt{a+bx}\,\sqrt{c+dx}+\sqrt{\frac{b}{d}}\,(-bc+ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/Sqrt[a + b*x],x]
[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x] + Sqrt[b/d]*(-(b*c) + a*d)*Log[Sqrt[a + b*x]
 - Sqrt[b/d]*Sqrt[c + d*x]])/b^2
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [A]
time = 0.16, size = 107, normalized size = 1.49

method	result	s
default	$\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)\,(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}+(ad+bc)\,x+ac}\right)}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 236, normalized size = 3.28

$$\left[\frac{4\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{2}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^{2}cd+abd^{2})x\right)}{4\,b^{2}d},\ \frac{2\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^{2}d^{2}x^{2}+abcd+(b^{2}cd+abd^{2})x)}\right)}{2\,b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b^2*d), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
```

```
b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)))/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/sqrt(a + b*x), x)
```

Giac [A]
time = 0.01, size = 117, normalized size = 1.62

$$\frac{d^2\left(\frac{\frac{1}{2}\cdot 2\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{bd}+\frac{2(ad-bc)\ln\left|\sqrt{ad^2-bcd+bd\,(c+dx)}-\sqrt{bd}\,\sqrt{c+dx}\right|}{2b\sqrt{bd}}\right)}{|d|\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] -d*((b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c
*d + a*d^2)))/(sqrt(b*d)*b) - sqrt((d*x + c)*b*d - b*c*d + a*d^2)*sqrt(d*x
+ c)/(b*d))/abs(d)
```

Mupad [B]
time = 4.01, size = 260, normalized size = 3.61

$$\frac{\frac{(2\,a\,d+2\,b\,c)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)}+\frac{(2\,a\,d+2\,b\,c)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{b\,d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}-\frac{8\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)(a\,d-b\,c)}{b^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2)))/(d^2*((c + d*x)^(1/2) - c^(1
/2))) + ((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*d*((c + d*x)^(1/
2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c
 + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d*x)^(1/2
) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c + d*x
)^(1/2) - c^(1/2))^2)) - (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^
(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(3/2)*d^(1/2))
```

3.1465 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}$$

```
[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*d^(1/2)/b^(3/2)-2*(d
*x+c)^(1/2)/b/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {49, 65, 223, 212}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b}\\ &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^2}\\ &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}\\ &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 1.00

$$-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(50) = 100.
time = 0.34, size = 241, normalized size = 3.65

$$\left[\frac{(bx+a)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2x+ab)}, -\frac{(bx+a)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)+2\,\sqrt{bx+a}\,\sqrt{dx+c}}{b^2x+ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x + a)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*x + a*b), -((b
*x + a)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(b^2*x + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(3/2), x)
```

Giac [A]
time = 0.02, size = 127, normalized size = 1.92

$$-\frac{2d^2\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{b\,|d|\,(ad^2-bcd+bd\,(c+dx))}-\frac{2d^2\ln\left|\sqrt{ad^2-bcd+bd\,(c+dx)}-\sqrt{bd}\,\sqrt{c+dx}\right|}{b\sqrt{bd}\,\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*d^2*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^
2)))/(sqrt(b*d)*b*abs(d)) - 2*sqrt(d*x + c)*d^2/(sqrt((d*x + c)*b*d - b*c*d
 + a*d^2)*b*abs(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c+d\,x}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/2)/(a + b*x)^(3/2), x)
```

3.1466 $\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

```
[Out] -2/3*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(5/2),x]')

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.
time = 0.16, size = 88, normalized size = 2.75

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}}{3(bx+a)^{\frac{3}{2}}(ad-bc)}$	27
default	$-\frac{\sqrt{dx+c}}{b(bx+a)^{\frac{3}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{2b}$	88

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(d*x+c)^(1/2)/(b*x+a)^(3/2)+1/2*(a*d-b*c)/b*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more detail

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.34, size = 65, normalized size = 2.03

$$-\frac{2\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{3\,(a^2bc-a^3d+(b^3c-ab^2d)x^2+2\,(ab^2c-a^2bd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)*(d*x + c)^(3/2)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2
 + 2*(a*b^2*c - a^2*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(5/2), x)
```

Giac [A]
time = 0.04, size = 96, normalized size = 3.00

$$\frac{6bd^4\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{(-9b^2c\,|d|+9bda\,|d|)\,(ad^2-bcd+bd\,(c+dx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(d*x + c)^(3/2)*b*d^4/((b^2*c*abs(d) - a*b*d*abs(d))*((d*x + c)*b*d -
b*c*d + a*d^2)^(3/2))
```

Mupad [B]
time = 0.72, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{\left(3\,a\,d-3\,b\,c\right)\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/((3*a*d - 3*b*c)*(a + b*x)^(3/2))
```

3.1467 $\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}}+\frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}$$

```
[Out] -2/5*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(5/2)+4/15*d*(d*x+c)^(3/2)/(-a*d+b*c)
^2/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2)
)/(15*(b*c - a*d)^2*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{5(bc-ad)}$$
$$= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} + \frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{3/2}(-3bc+5ad+2bdx)}{15(bc-ad)^2(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
[Out] (2*(c + d*x)^(3/2)*(-3*b*c + 5*a*d + 2*b*d*x))/(15*(b*c - a*d)^2*(a + b*x)^
(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(7/2),x]')
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(54) = 108.
time = 0.16, size = 128, normalized size = 1.94

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(2bdx+5ad-3bc)}{15(bx+a)^{\frac{5}{2}}(a^2d^2-2abcd+b^2c^2)}$	54
default	$-\frac{\sqrt{dx+c}}{2b(bx+a)^{\frac{5}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{4b}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*x+c)^(1/2)/(b*x+a)^(5/2)+1/4*(a*d-b*c)/b*(-2/5*(d*x+c)^(1/2)/(-a*
d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a
)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(54) = 108.
time = 0.45, size = 175, normalized size = 2.65

$$\frac{2\left(2\,bd^2x^2-3\,bc^2+5\,acd-(bcd-5\,ad^2)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{15\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^3+3\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)x^2+3\,(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b*d^2*x^2 - 3*b*c^2 + 5*a*c*d - (b*c*d - 5*a*d^2)*x)*sqrt(b*x + a)*
sqrt(d*x + c)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c+dx}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(54) = 108.
time = 0.06, size = 195, normalized size = 2.95

$$\frac{2\left(\frac{30b^3d^6\sqrt{c+dx}\sqrt{c+dx}}{225b^4c^2|d|-450b^3dac|d|+225b^2d^2a^2|d|}-\frac{75b^3d^6c-75b^2d^7a}{225b^4c^2|d|-450b^3dac|d|+225b^2d^2a^2|d|}\right)\sqrt{c+dx}\sqrt{c+dx}\sqrt{c+dx}\sqrt{ad^2-bcd+bd(c+dx)}}{(ad^2-bcd+bd(c+dx))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/15*(2*(d*x + c)*b^3*d^6/(b^4*c^2*abs(d) - 2*a*b^3*c*d*abs(d) + a^2*b^2*d^
2*abs(d)) - 5*(b^3*c*d^6 - a*b^2*d^7)/(b^4*c^2*abs(d) - 2*a*b^3*c*d*abs(d)
+ a^2*b^2*d^2*abs(d)))*(d*x + c)^(3/2)/((d*x + c)*b*d - b*c*d + a*d^2)^(5/2
)
```

Mupad [B]
time = 0.82, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(10\,a\,d^2-2\,b\,c\,d\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{6\,b\,c^2-10\,a\,c\,d}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,d^2\,x^2}{15\,b\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(7/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(10*a*d^2 - 2*b*c*d))/(15*b^2*(a*d - b*c)^2) - (6*b*c^
2 - 10*a*c*d)/(15*b^2*(a*d - b*c)^2) + (4*d^2*x^2)/(15*b*(a*d - b*c)^2)))/(
x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*x)^(1/2))/b
)
```

3.1468 $\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}$$

```
[Out] -2/7*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(7/2)+8/35*d*(d*x+c)^(3/2)/(-a*d+b*c)
^2/(b*x+a)^(5/2)-16/105*d^2*(d*x+c)^(3/2)/(-a*d+b*c)^3/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (8*d*(c + d*x)^(3/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(4d)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}$$
$$= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{(8d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{35(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}$$

Mathematica [**A**]
time = 0.09, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(-3c+2dx)+b^2\left(15c^2-12cdx+8d^2x^2\right)\right)}{105(bc-ad)^3(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-3*c + 2*d*x) + b^2*(15*c^2 - 1
2*c*d*x + 8*d^2*x^2)))/(105*(b*c - a*d)^3*(a + b*x)^(7/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(9/2),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.
time = 0.16, size = 168, normalized size = 1.66

method	result
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(8b^2x^2d^2+28ab\,d^2x-12b^2cdx+35a^2d^2-42abcd+15b^2c^2\right)}{105(bx+a)^{\frac{7}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$

default	$-\frac{\sqrt{dx+c}}{3b(bx+a)^{\frac{7}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{7(-ad+bc)}\right)}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(d*x+c)^(1/2)/(b*x+a)^(7/2)+1/6*(a*d-b*c)/b*(-2/7*(d*x+c)^(1/2)/(-a*
d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a
)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d
*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(83) = 166$.

time = 0.78, size = 337, normalized size = 3.34

$$-\frac{2\left(8b^{2}d^{3}x^{3}+15b^{2}c^{3}-42abc^{2}d+35a^{2}cd^{2}-4(b^{2}cd^{2}-7abd^{3})x^{2}+(3b^{2}c^{2}d-14abcd^{2}+35a^{2}d^{3})x\right)\sqrt{bx+a}\sqrt{dx+c}}{105\left(a^{4}b^{3}c^{3}-3a^{5}b^{2}c^{2}d+3a^{6}bcd^{2}-a^{7}d^{3}+(b^{7}c^{3}-3ab^{6}c^{2}d+3a^{2}b^{5}cd^{2}-a^{3}b^{4}d^{3})x^{4}+4(ab^{6}c^{3}-3a^{2}b^{5}c^{2}d+3a^{3}b^{4}cd^{2}-a^{4}b^{3}d^{3})x^{3}+6(a^{2}b^{5}c^{3}-3a^{3}b^{4}c^{2}d+3a^{4}b^{3}cd^{2}-a^{5}b^{2}d^{3})x^{2}+4(a^{3}b^{4}c^{3}-3a^{4}b^{3}c^{2}d+3a^{5}b^{2}cd^{2}-a^{6}bd^{3})x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^2*d^3*x^3 + 15*b^2*c^3 - 42*a*b*c^2*d + 35*a^2*c*d^2 - 4*(b^2*c
*d^2 - 7*a*b*d^3)*x^2 + (3*b^2*c^2*d - 14*a*b*c*d^2 + 35*a^2*d^3)*x)*sqrt(b
*x + a)*sqrt(d*x + c)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5
```

```
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^
3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(83) = 166.
time = 0.11, size = 339, normalized size = 3.36

$$\frac{2\left(\left(\left(\frac{280b^5d^8\sqrt{c+dx}\,\sqrt{c+dx}}{-3675b^6c^3|d|+11025b^5dac^2|d|-11025b^4d^2a^2c|d|+3675b^3d^3a^3|d|}-\frac{980b^5d^8c-980b^4d^9a}{-3675b^6c^3|d|+11025b^5dac^2|d|-11025b^4d^2a^2c|d|+3675b^3d^3a^3|d|}\right)\sqrt{c+dx}\,\sqrt{c+dx}-\frac{-1225b^5d^8c^2+2450b^4d^9ac-1225b^3d^{10}a^2}{-3675b^6c^3|d|+11025b^5dac^2|d|-11025b^4d^2a^2c|d|+3675b^3d^3a^3|d|}\right)\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{\left(ad^2-bcd+bd\,(c+dx)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x)
```

```
[Out] -2/105*(4*(2*(d*x + c)*b^5*d^8/(b^6*c^3*abs(d) - 3*a*b^5*c^2*d*abs(d) + 3*a
^2*b^4*c*d^2*abs(d) - a^3*b^3*d^3*abs(d)) - 7*(b^5*c*d^8 - a*b^4*d^9)/(b^6*
c^3*abs(d) - 3*a*b^5*c^2*d*abs(d) + 3*a^2*b^4*c*d^2*abs(d) - a^3*b^3*d^3*ab
s(d)))*(d*x + c) + 35*(b^5*c^2*d^8 - 2*a*b^4*c*d^9 + a^2*b^3*d^10)/(b^6*c^3
*abs(d) - 3*a*b^5*c^2*d*abs(d) + 3*a^2*b^4*c*d^2*abs(d) - a^3*b^3*d^3*abs(d
)))*(d*x + c)^(3/2)/((d*x + c)*b*d - b*c*d + a*d^2)^(7/2)
```

Mupad [B]
time = 0.97, size = 203, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{70\,a^2\,c\,d^2-84\,a\,b\,c^2\,d+30\,b^2\,c^3}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x\,\left(70\,a^2\,d^3-28\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^3\,x^3}{105\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^3}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((30*b^2*c^3 + 70*a^2*c*d^2 - 84*a*b*c^2*d)/(105*b^3*(a*d
- b*c)^3) + (x*(70*a^2*d^3 + 6*b^2*c^2*d - 28*a*b*c*d^2))/(105*b^3*(a*d - b
*c)^3) + (16*d^3*x^3)/(105*b*(a*d - b*c)^3) + (8*d^2*x^2*(7*a*d - b*c))/(10
5*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (
3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

3.1469 $\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{32d^3(c+dx)^{3/2}}{315(bc-ad)^4(a+bx)^{3/2}}$$

```
[Out] -2/9*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(9/2)+4/21*d*(d*x+c)^(3/2)/(-a*d+b*c)
^2/(b*x+a)^(7/2)-16/105*d^2*(d*x+c)^(3/2)/(-a*d+b*c)^3/(b*x+a)^(5/2)+32/315
*d^3*(d*x+c)^(3/2)/(-a*d+b*c)^4/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(3/2)
)/(21*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(5/2)) + (32*d^3*(c + d*x)^(3/2))/(315*(b*c - a*d)^4*(a +
 b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} + \frac{(8d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{21(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} - \frac{(16c}{}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{}{315(}$$

Mathematica [A]
time = 0.13, size = 93, normalized size = 0.68

$$-\frac{2(c+dx)^{3/2}\left(-105d^3(a+bx)^3+189bd^2(a+bx)^2(c+dx)-135b^2d(a+bx)(c+dx)^2+35b^3(c+dx)^3\right)}{315(bc-ad)^4(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
[Out] (-2*(c + d*x)^(3/2)*(-105*d^3*(a + b*x)^3 + 189*b*d^2*(a + b*x)^2*(c + d*x)
 - 135*b^2*d*(a + b*x)*(c + d*x)^2 + 35*b^3*(c + d*x)^3))/(315*(b*c - a*d)^
4*(a + b*x)^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(11/2),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 208, normalized size = 1.53

method	result
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(16b^3x^3d^3+72d^3a\,x^2b^2-24b^3c\,d^2x^2+126a^2b\,d^3x-108a\,b^2c\,d^2x+30b^3c^2dx+105a^3d^3-189a^2bc\,d^2+135a\,b^2c^2d-35b^3c^3\right)}{315(bx+a)^{\frac{9}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

default	$-\frac{\sqrt{dx+c}}{4b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{\cdots}{3(-\cdots}\right.}{5(-ad+b\cdots}\right.}{7(-ad+bc)}\right.}{9(-ad+bc)}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(d*x+c)^(1/2)/(b*x+a)^(9/2)+1/8*(a*d-b*c)/b*(-2/9*(d*x+c)^(1/2)/(-a*
d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a
)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d
/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2
)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(112) = 224$.
time = 2.43, size = 532, normalized size = 3.91

$$\frac{2\,(16\,b^3d^4x^4-35\,b^3c^4+135\,ab^2c^3d-189\,a^2bc^2d^2+105\,a^3cd^3-8\,(b^3cd^3-9\,ab^2d^4)x^3+6\,(b^3c^2d^2-6\,ab^2cd^3+21\,a^2bd^4)x^2-(5\,b^3c^3d-27\,ab^2c^2d^2+63\,a^2bcd^3-105\,a^3d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{315\,(a^5b^4c^4-4\,a^6b^3c^3d+6\,a^7b^2c^2d^2-4\,a^8bcd^3+a^9d^4+(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4)x^5+5\,(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2d^2-4\,a^4b^5cd^3+a^5b^4d^4)x^4+10\,(a^2b^7c^4-4\,a^3b^6c^3d+6\,a^4b^5c^2d^2-4\,a^5b^4cd^3+a^6b^3d^4)x^3+10\,(a^3b^6c^4-4\,a^4b^5c^3d+6\,a^5b^4c^2d^2-4\,a^6b^3cd^3+a^7b^2d^4)x^2+5\,(a^4b^5c^4-4\,a^5b^4c^3d+6\,a^6b^3c^2d^2-4\,a^7b^2cd^3+a^8bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^3*d^4*x^4 - 35*b^3*c^4 + 135*a*b^2*c^3*d - 189*a^2*b*c^2*d^2 +
105*a^3*c*d^3 - 8*(b^3*c*d^3 - 9*a*b^2*d^4)*x^3 + 6*(b^3*c^2*d^2 - 6*a*b^2*
c*d^3 + 21*a^2*b*d^4)*x^2 - (5*b^3*c^3*d - 27*a*b^2*c^2*d^2 + 63*a^2*b*c*d^
3 - 105*a^3*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 - 4*a*b^8*c^3*
d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5*(a*b^8*c^4 -
 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^4 +
 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a
^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(112) = 224.
time = 0.15, size = 527, normalized size = 3.88

$$\frac{2\left(\left(\left(\left(\frac{45360b^7d^{10}\sqrt{c+dx}\sqrt{c+dx}}{893025b^8c^4|d|-3572100b^7dac^3|d|+5358150b^6d^2a^2c^2|d|-3572100b^5d^3a^3c|d|+893025b^4d^4a^4|d|}-\frac{204120b^7d^{10}c-204120b^6d^{11}a}{893025b^8c^4|d|-3572100b^7dac^3|d|+5358150b^6d^2a^2c^2|d|-3572100b^5d^3a^3c|d|+893025b^4d^4a^4|d|}\right)\sqrt{c+dx}\sqrt{c+dx}-\frac{-357210b^7d^{10}c^2+714420b^6d^{11}ac-357210b^5d^{12}a^2}{893025b^8c^4|d|-3572100b^7dac^3|d|+5358150b^6d^2a^2c^2|d|-3572100b^5d^3a^3c|d|+893025b^4d^4a^4|d|}\right)\sqrt{c+dx}\sqrt{c+dx}-\frac{297675b^7d^{10}c^3-893025b^6d^{11}ac^2+893025b^5d^{12}a^2c-297675b^4d^{13}a^3}{893025b^8c^4|d|-3572100b^7dac^3|d|+5358150b^6d^2a^2c^2|d|-3572100b^5d^3a^3c|d|+893025b^4d^4a^4|d|}\right)\sqrt{c+dx}\sqrt{c+dx}\sqrt{c+dx}\sqrt{ad^2-bcd+bd\left(c+dx\right)}}{\left(ad^2-bcd+bd\left(c+dx\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(2*(4*(2*(d*x + c)*b^7*d^10/(b^8*c^4*abs(d) - 4*a*b^7*c^3*d*abs(d) +
6*a^2*b^6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d) + a^4*b^4*d^4*abs(d)) - 9
*(b^7*c*d^10 - a*b^6*d^11)/(b^8*c^4*abs(d) - 4*a*b^7*c^3*d*abs(d) + 6*a^2*b
^6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d) + a^4*b^4*d^4*abs(d)))*(d*x + c)
 + 63*(b^7*c^2*d^10 - 2*a*b^6*c*d^11 + a^2*b^5*d^12)/(b^8*c^4*abs(d) - 4*a*
b^7*c^3*d*abs(d) + 6*a^2*b^6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d) + a^4*
b^4*d^4*abs(d)))*(d*x + c) - 105*(b^7*c^3*d^10 - 3*a*b^6*c^2*d^11 + 3*a^2*b
^5*c*d^12 - a^3*b^4*d^13)/(b^8*c^4*abs(d) - 4*a*b^7*c^3*d*abs(d) + 6*a^2*b^
6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d) + a^4*b^4*d^4*abs(d)))*(d*x + c)^
(3/2)/((d*x + c)*b*d - b*c*d + a*d^2)^(9/2)
```

Mupad [**B**]
time = 1.18, size = 292, normalized size = 2.15

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^4\,x^4}{315\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{-210\,a^3\,c\,d^3+378\,a^2\,b\,c^2\,d^2-270\,a\,b^2\,c^3\,d+70\,b^3\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(210\,a^3\,d^4-126\,a^2\,b\,c\,d^3+54\,a\,b^2\,c^2\,d^2-10\,b^3\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^4}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(1/2)/(a + b*x)^(11/2),x)

[Out] ((c + d*x)^(1/2)*((32*d^4*x^4)/(315*b*(a*d - b*c)^4) - (70*b^3*c^4 - 210*a^3*c*d^3 + 378*a^2*b*c^2*d^2 - 270*a*b^2*c^3*d)/(315*b^4*(a*d - b*c)^4) + (x*(210*a^3*d^4 - 10*b^3*c^3*d + 54*a*b^2*c^2*d^2 - 126*a^2*b*c*d^3))/(315*b^4*(a*d - b*c)^4) + (16*d^3*x^3*(9*a*d - b*c))/(315*b^2*(a*d - b*c)^4) + (4*d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^3*(a*d - b*c)^4)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)

3.1470 $\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{128d^3(c+dx)^{3/2}}{1155(bc-ad)^4(a+bx)^{5/2}}$$

```
[Out] -2/11*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(11/2)+16/99*d*(d*x+c)^(3/2)/(-a*d+b
*c)^2/(b*x+a)^(9/2)-32/231*d^2*(d*x+c)^(3/2)/(-a*d+b*c)^3/(b*x+a)^(7/2)+128
/1155*d^3*(d*x+c)^(3/2)/(-a*d+b*c)^4/(b*x+a)^(5/2)-256/3465*d^4*(d*x+c)^(3/
2)/(-a*d+b*c)^5/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (16*d*(c + d*x)^(3
/2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (32*d^2*(c + d*x)^(3/2))/(231*(b*
c - a*d)^3*(a + b*x)^(7/2)) + (128*d^3*(c + d*x)^(3/2))/(1155*(b*c - a*d)^4
*(a + b*x)^(5/2)) - (256*d^4*(c + d*x)^(3/2))/(3465*(b*c - a*d)^5*(a + b*x)
^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(8d)\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{(16d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{(64}{}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{}{115}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{}{115}$$

Mathematica [A]
time = 0.14, size = 117, normalized size = 0.68

$$-\frac{2(c+dx)^{3/2}\left(1155d^4 - \frac{2772bd^3(c+dx)}{a+bx} + \frac{2970b^2d^2(c+dx)^2}{(a+bx)^2} - \frac{1540b^3d(c+dx)^3}{(a+bx)^3} + \frac{315b^4(c+dx)^4}{(a+bx)^4}\right)}{3465(bc-ad)^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*d^4 - (2772*b*d^3*(c + d*x))/(a + b*x) + (2970*b^
2*d^2*(c + d*x)^2)/(a + b*x)^2 - (1540*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (31
5*b^4*(c + d*x)^4)/(a + b*x)^4))/(3465*(b*c - a*d)^5*(a + b*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/2)/(a + b*x)^(13/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 248, normalized size = 1.45

method	result
default	$-\frac{\sqrt{dx+c}}{5b(bx+a)^{\frac{11}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}-\frac{10d\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}\right.}{}\right.}{9}\right.}{11(-ad+bc)}\right.}{10b}$
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(128d^{4}x^{4}b^{4}+704a\,b^{3}d^{4}x^{3}-192b^{4}c\,d^{3}x^{3}+1584a^{2}b^{2}d^{4}x^{2}-1056a\,b^{3}c\,d^{3}x^{2}+240b^{4}c^{2}d^{2}x^{2}+1848a^{3}b\,d^{4}x-2376a^{2}b^{2}c\,d^{3}x+132\right.}{3465(bx+a)^{\frac{11}{2}}\left(a^{5}d^{5}-5a^{4}bc\,d^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*(d*x+c)^(1/2)/(b*x+a)^(11/2)+1/10*(a*d-b*c)/b*(-2/11/(-a*d+b*c)/(b*x
+a)^(11/2)*(d*x+c)^(1/2)-10/11*d/(-a*d+b*c)*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/
(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)
-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+
b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d
+b*c)^2/(b*x+a)^(1/2))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(141) = 282$.
time = 5.20, size = 781, normalized size = 4.57

$$-\frac{2\,\left(128\,b^{4}d^{5}x^{5}+315\,b^{4}c^{5}-1540\,ab^{3}c^{4}d+2970\,a^{2}b^{2}c^{3}d^{2}-2772\,a^{3}bc^{2}d^{3}+1155\,a^{4}cd^{4}-64\,(b^{4}cd^{4}-11\,ab^{3}d^{5})x^{4}+16\,(3\,b^{4}c^{2}d^{3}-22\,ab^{3}cd^{4}+99\,a^{2}b^{2}d^{5})x^{3}-8\,(5\,b^{4}c^{3}d^{2}-33\,ab^{3}c^{2}d^{3}+99\,a^{2}b^{2}cd^{4}-231\,a^{3}bd^{5})x^{2}+(35\,b^{4}c^{4}d-220\,ab^{3}c^{3}d^{2}+594\,a^{2}b^{2}c^{2}d^{3}-924\,a^{3}bcd^{4}+1155\,a^{4}d^{5})x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3465\,\left(a^{6}b^{5}c^{5}-5\,a^{7}b^{4}c^{4}d+10\,a^{8}b^{3}c^{3}d^{2}-10\,a^{9}b^{2}c^{2}d^{3}+5\,a^{10}bcd^{4}-a^{11}d^{5}+(b^{11}c^{5}-5\,ab^{10}c^{4}d+10\,a^{2}b^{9}c^{3}d^{2}-10\,a^{3}b^{8}c^{2}d^{3}+5\,a^{4}b^{7}cd^{4}-a^{5}b^{6}d^{5})x^{6}+6\,(ab^{10}c^{5}-5\,a^{2}b^{9}c^{4}d+10\,a^{3}b^{8}c^{3}d^{2}-10\,a^{4}b^{7}c^{2}d^{3}+5\,a^{5}b^{6}cd^{4}-a^{6}b^{5}d^{5})x^{5}+15\,(a^{2}b^{9}c^{5}-5\,a^{3}b^{8}c^{4}d+10\,a^{4}b^{7}c^{3}d^{2}-10\,a^{5}b^{6}c^{2}d^{3}+5\,a^{6}b^{5}cd^{4}-a^{7}b^{4}d^{5})x^{4}+20\,(a^{3}b^{8}c^{5}-5\,a^{4}b^{7}c^{4}d+10\,a^{5}b^{6}c^{3}d^{2}-10\,a^{6}b^{5}c^{2}d^{3}+5\,a^{7}b^{4}cd^{4}-a^{8}b^{3}d^{5})x^{3}+15\,(a^{4}b^{7}c^{5}-5\,a^{5}b^{6}c^{4}d+10\,a^{6}b^{5}c^{3}d^{2}-10\,a^{7}b^{4}c^{2}d^{3}+5\,a^{8}b^{3}cd^{4}-a^{9}b^{2}d^{5})x^{2}+6\,(a^{5}b^{6}c^{5}-5\,a^{6}b^{5}c^{4}d+10\,a^{7}b^{4}c^{3}d^{2}-10\,a^{8}b^{3}c^{2}d^{3}+5\,a^{9}b^{2}cd^{4}-a^{10}bd^{5})x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/3465*(128*b^4*d^5*x^5 + 315*b^4*c^5 - 1540*a*b^3*c^4*d + 2970*a^2*b^2*c^
3*d^2 - 2772*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 - 64*(b^4*c*d^4 - 11*a*b^3*d^5)
*x^4 + 16*(3*b^4*c^2*d^3 - 22*a*b^3*c*d^4 + 99*a^2*b^2*d^5)*x^3 - 8*(5*b^4*
c^3*d^2 - 33*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 - 231*a^3*b*d^5)*x^2 + (35*b^
4*c^4*d - 220*a*b^3*c^3*d^2 + 594*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 + 1155*
a^4*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^5*c^5 - 5*a^7*b^4*c^4*d + 10
*a^8*b^3*c^3*d^2 - 10*a^9*b^2*c^2*d^3 + 5*a^10*b*c*d^4 - a^11*d^5 + (b^11*c
^5 - 5*a*b^10*c^4*d + 10*a^2*b^9*c^3*d^2 - 10*a^3*b^8*c^2*d^3 + 5*a^4*b^7*c
*d^4 - a^5*b^6*d^5)*x^6 + 6*(a*b^10*c^5 - 5*a^2*b^9*c^4*d + 10*a^3*b^8*c^3*
d^2 - 10*a^4*b^7*c^2*d^3 + 5*a^5*b^6*c*d^4 - a^6*b^5*d^5)*x^5 + 15*(a^2*b^9
*c^5 - 5*a^3*b^8*c^4*d + 10*a^4*b^7*c^3*d^2 - 10*a^5*b^6*c^2*d^3 + 5*a^6*b^
5*c*d^4 - a^7*b^4*d^5)*x^4 + 20*(a^3*b^8*c^5 - 5*a^4*b^7*c^4*d + 10*a^5*b^6
*c^3*d^2 - 10*a^6*b^5*c^2*d^3 + 5*a^7*b^4*c*d^4 - a^8*b^3*d^5)*x^3 + 15*(a^
4*b^7*c^5 - 5*a^5*b^6*c^4*d + 10*a^6*b^5*c^3*d^2 - 10*a^7*b^4*c^2*d^3 + 5*a
^8*b^3*c*d^4 - a^9*b^2*d^5)*x^2 + 6*(a^5*b^6*c^5 - 5*a^6*b^5*c^4*d + 10*a^7
*b^4*c^3*d^2 - 10*a^8*b^3*c^2*d^3 + 5*a^9*b^2*c*d^4 - a^10*b*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(141) = 282.

time = 0.20, size = 767, normalized size = 4.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x)
```

```
[Out] -2/3465*(8*(2*(4*(2*(d*x + c)*b^9*d^12/(b^10*c^5*abs(d) - 5*a*b^9*c^4*d*abs
(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d
^4*abs(d) - a^5*b^5*d^5*abs(d)) - 11*(b^9*c*d^12 - a*b^8*d^13)/(b^10*c^5*ab
s(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^
3*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)))*(d*x + c) + 99*(b^
9*c^2*d^12 - 2*a*b^8*c*d^13 + a^2*b^7*d^14)/(b^10*c^5*abs(d) - 5*a*b^9*c^4*
d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4*b^
6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)))*(d*x + c) - 231*(b^9*c^3*d^12 - 3*a*b
^8*c^2*d^13 + 3*a^2*b^7*c*d^14 - a^3*b^6*d^15)/(b^10*c^5*abs(d) - 5*a*b^9*c
^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4
*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)))*(d*x + c) + 1155*(b^9*c^4*d^12 - 4
*a*b^8*c^3*d^13 + 6*a^2*b^7*c^2*d^14 - 4*a^3*b^6*c*d^15 + a^4*b^5*d^16)/(b^
10*c^5*abs(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b
^7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)))*(d*x + c)
^(3/2)/((d*x + c)*b*d - b*c*d + a*d^2)^(11/2)
```

Mupad [B]

time = 1.43, size = 397, normalized size = 2.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{2310\,a^4\,c\,d^4-5544\,a^3\,b\,c^2\,d^3+5940\,a^2\,b^2\,c^3\,d^2-3080\,a\,b^3\,c^4\,d+630\,b^4\,c^5}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(2310\,a^4\,d^5-1848\,a^3\,b\,c\,d^4+1188\,a^2\,b^2\,c^2\,d^3-440\,a\,b^3\,c^3\,d^2+70\,b^4\,c^4\,d\right)}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,d^5\,x^5}{3465\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{16\,d^2\,x^2\,\left(231\,a^3\,d^3-99\,a^2\,b\,c\,d^2+33\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}{3465\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{3465\,b^2\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3465\,b^3\,{\left(a\,d-b\,c\right)}^5}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((630*b^4*c^5 + 2310*a^4*c*d^4 - 5544*a^3*b*c^2*d^3 + 5940
*a^2*b^2*c^3*d^2 - 3080*a*b^3*c^4*d)/(3465*b^5*(a*d - b*c)^5) + (x*(2310*a^
4*d^5 + 70*b^4*c^4*d - 440*a*b^3*c^3*d^2 + 1188*a^2*b^2*c^2*d^3 - 1848*a^3*
b*c*d^4))/(3465*b^5*(a*d - b*c)^5) + (256*d^5*x^5)/(3465*b*(a*d - b*c)^5) +
 (16*d^2*x^2*(231*a^3*d^3 - 5*b^3*c^3 + 33*a*b^2*c^2*d - 99*a^2*b*c*d^2))/(
3465*b^4*(a*d - b*c)^5) + (128*d^4*x^4*(11*a*d - b*c))/(3465*b^2*(a*d - b*c
)^5) + (32*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2 - 22*a*b*c*d))/(3465*b^3*(a*d -
b*c)^5)))/(x^5*(a + b*x)^(1/2) + (a^5*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a
 + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^
(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

3.1471 $\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=227

$$\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3}-\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d}+\frac{3(bc-ad}{}$$

```
[Out] 1/5*(b*x+a)^(7/2)*(d*x+c)^(3/2)/b-3/128*(-a*d+b*c)^5*arctanh(d^(1/2)*(b*x+a
)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(7/2)-1/64*(-a*d+b*c)^3*(b*x+a)^(3
/2)*(d*x+c)^(1/2)/b^2/d^2+1/80*(-a*d+b*c)^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^2
/d+3/40*(-a*d+b*c)*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b^2+3/128*(-a*d+b*c)^4*(b*x+
a)^(1/2)*(d*x+c)^(1/2)/b^2/d^3
```

Rubi [A]
time = 0.08, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 223, 212}

$$-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{80b^2d}+\frac{3(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40b^2}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] (3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^2*d^3) - ((b*c - a*d)^
3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^2*(a + b*x)^(5
/2)*Sqrt[c + d*x])/(80*b^2*d) + (3*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x
])/(40*b^2) + ((a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b) - (3*(b*c - a*d)^5*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2
))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad))\int (a+bx)^{5/2}\sqrt{c+dx}\,dx}{10b}$$

$$= \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad)^2)\int}{80b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)}{}$$

$$= -\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad}{}$$

Mathematica [A]

time = 0.50, size = 166, normalized size = 0.73

$$-\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(15d^4(a+bx)^4-70bd^3(a+bx)^3(c+dx)-128b^2d^2(a+bx)^2(c+dx)^2+70b^3d(a+bx)(c+dx)^3-15b^4(c+dx)^4\right)}{640b^2d^3}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] -1/640*(Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^4*(a + b*x)^4 - 70*b*d^3*(a + b*x
)^3*(c + d*x) - 128*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 70*b^3*d*(a + b*x)*(c
 + d*x)^3 - 15*b^4*(c + d*x)^4))/(b^2*d^3) - (3*(b*c - a*d)^5*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 239, normalized size = 1.05

method	result

default	$\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{2}}}{5d} - \frac{(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{4d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad-bc)}{}\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(b*x+a)^(5/2)*(d*x+c)^(5/2)-1/2*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(3/2)*(d*
x+c)^(5/2)-3/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)-1/6*(-a*d+b*
c)/d*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x
+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.33, size = 702, normalized size = 3.09

$$\left[-\frac{15\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(128\,b^5d^5x^4+15\,b^5c^4d-70\,ab^4c^3d^2+128\,a^2b^3c^2d^3+70\,a^3b^2cd^4-15\,a^4bd^5+16\,(11\,b^5cd^4+21\,ab^4d^5)x^3+8\,(b^5c^2d^3+64\,ab^4cd^4+31\,a^2b^3d^5)x^2-2\,(5\,b^5c^3d^2-23\,ab^4c^2d^3-233\,a^2b^3cd^4-5\,a^3b^2d^5)x)\sqrt{bx+a}\,\sqrt{dx+c}}{2560\,b^3d^4},\ \frac{15\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(128\,b^5d^5x^4+15\,b^5c^4d-70\,ab^4c^3d^2+128\,a^2b^3c^2d^3+70\,a^3b^2cd^4-15\,a^4bd^5+16\,(11\,b^5cd^4+21\,ab^4d^5)x^3+8\,(b^5c^2d^3+64\,ab^4cd^4+31\,a^2b^3d^5)x^2-2\,(5\,b^5c^3d^2-23\,ab^4c^2d^3-233\,a^2b^3cd^4-5\,a^3b^2d^5)x)\sqrt{bx+a}\,\sqrt{dx+c}}{1280\,b^3d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(
11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2
*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^4), 1/1280*(15*(b^5*c^
5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(1
28*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70
*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b
^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a
*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x
 + c))/(b^3*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(183) = 366$.
time = 0.16, size = 2330, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/1920*(240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c*abs(b
)/b^2 + 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^
4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9
*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x
 + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 -
 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c*abs(b) + 30*(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b
^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6
)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 -
93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b
*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d*ab
s(b) + 240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*d*abs(b)/b + (sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*
c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 51
3*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^
2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4
 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^
19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b
^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3
*d^4))*b*d*abs(b) + 1440*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
+ (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d
^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*d))*a^2*c*abs(b)/b^2 + 480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*d*abs(b)/b^3)/b
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(3/2), x)
```

3.1472 $\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=189

$$-\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2}+\frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d}+\frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}+(a+bx$$

```
[Out] 1/4*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b+3/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)
^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(5/2)+1/32*(-a*d+b*c)^2*(b*x+a)^(3/
2)*(d*x+c)^(1/2)/b^2/d+1/8*(-a*d+b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^2-3/64*
(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-3*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^2*d) + ((b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(5
/2)*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(3(bc-ad))\int (a+bx)^{3/2}\sqrt{c+dx}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(bc-ad)^2\int \frac{(a+bx)^3}{\sqrt{c+d}}}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}}{4}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad}{}$$

Mathematica [A]
time = 0.38, size = 144, normalized size = 0.76

$$-\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(3d^3(a+bx)^3-11bd^2(a+bx)^2(c+dx)-11b^2d(a+bx)(c+dx)^2+3b^3(c+dx)^3\right)}{64b^2d^2}+\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64b^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] -1/64*(Sqrt[a + b*x]*Sqrt[c + d*x]*(3*d^3*(a + b*x)^3 - 11*b*d^2*(a + b*x)^
2*(c + d*x) - 11*b^2*d*(a + b*x)*(c + d*x)^2 + 3*b^3*(c + d*x)^3))/(b^2*d^2
) + (3*(b*c - a*d)^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x]
)])/(64*b^(5/2)*d^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 206, normalized size = 1.09

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{4d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b} - \dots\right)}{\dots}\right)}{\dots}\right)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)-3/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(1/2)*(d*
x+c)^(5/2)-1/6*(-a*d+b*c)/d*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c
)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/
(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2
+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.33, size = 534, normalized size = 2.83

$$\left[\frac{3\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\,\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bdx+bc+ad\right)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(16b^4d^4x^3-3b^4c^3d+11ab^3c^2d^2+11a^2b^2cd^3-3a^3bd^4+24\left(b^4cd^3+ab^3d^4\right)x^2+2\left(b^4c^2d^2+22ab^3cd^3+a^2b^2d^4\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{256\,b^3d^3}, -\frac{3\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(16b^4d^4x^3-3b^4c^3d+11ab^3c^2d^2+11a^2b^2cd^3-3a^3bd^4+24\left(b^4cd^3+ab^3d^4\right)x^2+2\left(b^4c^2d^2+22ab^3cd^3+a^2b^2d^4\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{128\,b^3d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(16*b^4*d^4*x^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c
*d^3 - 3*a^3*b*d^4 + 24*(b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a
*b^3*c*d^3 + a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3), -1/128
*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(
d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(16*b^4*d^4*x
^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 3*a^3*b*d^4 + 24*(
b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a*b^3*c*d^3 + a^2*b^2*d^4)
*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(151) = 302$.
time = 0.11, size = 1443, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/192*(8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4
*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2
*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*abs(b) - 192*((b^2*c - a*b*d)*log
(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(
b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c*abs(b)/b^2
+ (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x +
a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*
b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c
^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3
*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d
^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*b^2*d^3))*d*abs(b) + 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d*abs
(b)/b + 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a
*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-s
qrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d
))*a*c*abs(b)/b^2 + 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
(b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*d))*a^2*d*abs(b)/b^3)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(3/2), x)
```

3.1473 $\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx$

Optimal. Leaf size=151

$$\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d}+\frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}$$

```
[Out] 1/3*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b-1/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^
(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(3/2)+1/4*(-a*d+b*c)*(b*x+a)^(3/2)*(
d*x+c)^(1/2)/b^2+1/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 223, 212}

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^2*d) + ((b*c - a*d)*(a + b
*x)^(3/2)*Sqrt[c + d*x])/(4*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b)
- ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(5/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{\sqrt{a+}}{\sqrt{c+}}}{8b^2}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(}{3b}$$

Mathematica [A]
time = 0.24, size = 127, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(-3a^2d^2+2abd(4c+dx)+b^2\left(3c^2+14cdx+8d^2x^2\right)\right)}{24b^2d} - \frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(-3*a^2*d^2 + 2*a*b*d*(4*c + d*x) + b^2*(3*c^2
 + 14*c*d*x + 8*d^2*x^2)))/(24*b^2*d) - ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(5/2)*d^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 173, normalized size = 1.15

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d}-\frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)\,(dx+c}}{2b}\right.}{4b}\right.}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)-1/6*(-a*d+b*c)/d*(1/2*(d*x+c)^(3/2)*(b*x+
a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*(
(b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*
x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.33, size = 410, normalized size = 2.72

$$\left[-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2+3\,b^3c^2d+8\,ab^2cd^2-3\,a^2bd^3+2\,(7\,b^3cd^2+ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,b^3d^2},\ \frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(8\,b^3d^3x^2+3\,b^3c^2d+8\,ab^2cd^2-3\,a^2bd^3+2\,(7\,b^3cd^2+ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,b^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*d^3 + 2*(7*b^3*c*d^2 + a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2), 1/48*(3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*
d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*
d^3 + 2*(7*b^3*c*d^2 + a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2)
]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(119) = 238.
time = 0.06, size = 768, normalized size = 5.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x)
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*a*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d*abs(b)/b - 6
*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)
*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*s
qrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*abs(b
)/b^2 - 6*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*
d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sq
```

```
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d)
)*a*d*abs(b)/b^3)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+bx}\, (c+dx)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(3/2), x)
```

3.1474 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=113

$$\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

```
[Out] 3/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/
2)/d^(1/2)+1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b+3/4*(-a*d+b*c)*(b*x+a)^(1/2)*(
d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^2) + (Sqrt[a + b*x]*(c + d
*x)^(3/2))/(2*b) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt[d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+\ldots}}}{8b^2}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{\ldots}{\sqrt{c-\ldots}}\ldots\right.}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx\ldots\right.}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\ldots}}{\sqrt{b}\,\sqrt{c+\ldots}}\right.}{4b^{5/2}\sqrt{d}}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 0.83

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(5bc-3ad+2bdx)}{4b^2} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(5*b*c - 3*a*d + 2*b*d*x))/(4*b^2) + (3*(b*c -
 a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2
)*Sqrt[d])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 140, normalized size = 1.24

method	result
default	$\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}}\right.}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1
/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*l
n((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d
)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 306, normalized size = 2.71

$$\left[\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x+5\,b^2cd-3\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,b^3d},-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(2\,b^2d^2x+5\,b^2cd-3\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + 5*b^2*c*d - 3*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x + 5*b^2*c*d - 3*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.02, size = 186, normalized size = 1.65

$$\frac{d^2\left(2\left(\frac{\frac{1}{8}\cdot 2b^2\sqrt{c+dx}\,\sqrt{c+dx}}{b^3d}-\frac{\frac{1}{8}\left(-3b^2c+3bda\right)}{b^3d}\right)\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\left(c+dx\right)}+\frac{2\left(-3a^2d^2+6abcd-3b^2c^2\right)\ln\left|\sqrt{ad^2-bcd+bd\left(c+dx\right)}-\sqrt{bd}\,\sqrt{c+dx}\right|}{8b^2\sqrt{bd}}\right)}{|d|\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/4*(sqrt((d*x + c)*b*d - b*c*d + a*d^2)*sqrt(d*x + c)*(2*(d*x + c)/(b*d) +
 3*(b^2*c - a*b*d)/(b^3*d)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-sq
rt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^2)))/(sqrt(b*d)*b^
2))*d/abs(d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(1/2), x)
```

3.1475 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}$$

```
[Out] 3*(-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*d^(1/2)/b
^(5/2)-2*(d*x+c)^(3/2)/b/(b*x+a)^(1/2)+3*d*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}+\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (3*d*Sqrt[a + b*x]*Sqrt[c + d*x])/b^2 - (2*(c + d*x)^(3/2))/(b*Sqrt[a + b*x
]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\right.}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.41, size = 89, normalized size = 0.91

$$\frac{\frac{b\sqrt{c+dx}\,(-2bc+3ad+bdx)}{\sqrt{a+bx}}+3\sqrt{\frac{b}{d}}\,d(-bc+ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] ((b*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/Sqrt[a + b*x] + 3*Sqrt[b/d]*d*(
-(b*c) + a*d)*Log[Sqrt[a + b*x] - Sqrt[b/d]*Sqrt[c + d*x]])/b^3
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.35, size = 311, normalized size = 3.17

$$\left[-\frac{3\,(abc - a^2d + (b^2c - abd)x)\sqrt{\frac{d}{b}}\,\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 - 4\,(2\,b^2dx + b^2c + abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}} + 8\,(b^2cd + abd^2)x\right) - 4\,(bdx - 2\,bc + 3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\,(b^3x + ab^2)}, -\frac{3\,(abc - a^2d + (b^2c - abd)x)\sqrt{-\frac{d}{b}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right) - 2\,(bdx - 2\,bc + 3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^3x + ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b*c - a^2*d + (b^2*c - a*b*d)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x - 2*b*c + 3*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2), -1/2*(3*(a*b*c - a^2*d +
 (b^2*c - a*b*d)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(b
*d*x - 2*b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(3/2), x)
```

Giac [A]
time = 0.03, size = 198, normalized size = 2.02

$$\frac{2\left(\frac{\frac{1}{2}b^2d^2\sqrt{c+dx}\,\sqrt{c+dx}}{b^3|d|} + \frac{\frac{1}{2}(-3b^2d^2c+3bd^3a)}{b^3|d|}\right)\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{ad^2-bcd+bd\,(c+dx)} + \frac{2\,(3ad^3-3bcd^2)\ln\left|\sqrt{ad^2-bcd+bd\,(c+dx)} - \sqrt{bd}\,\sqrt{c+dx}\right|}{2b^2\sqrt{bd}\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] sqrt(d*x + c)*((d*x + c)*d^2/(b*abs(d)) - 3*(b^2*c*d^2 - a*b*d^3)/(b^3*abs(
d)))/sqrt((d*x + c)*b*d - b*c*d + a*d^2) - 3*(b*c*d^2 - a*d^3)*log(abs(-sqr
t(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^2)))/(sqrt(b*d)*b^2
*abs(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(3/2)/(a + b*x)^(3/2), x)
```

3.1476 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}$$

```
[Out] -2/3*(d*x+c)^(3/2)/b/(b*x+a)^(3/2)+2*d^(3/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/
b^(1/2)/(d*x+c)^(1/2))/b^(5/2)-2*d*(d*x+c)^(1/2)/b^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 223, 212}

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*d*Sqrt[c + d*x])/(b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(3/2))/(3*b*(a + b*
x)^(3/2)) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b^2}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 0.88

$$-\frac{2\sqrt{c+dx}\,(3ad+b(c+4dx))}{3b^2(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(3*a*d + b*(c + 4*d*x)))/(3*b^2*(a + b*x)^(3/2)) + (2*d^(
3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]')
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(70) = 140.
time = 0.39, size = 325, normalized size = 3.53

$$\left[\frac{3\,(b^2dx^2+2\,abdx+a^2d)\sqrt{\frac{d}{b}}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,(4\,bdx+bc+3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}, -\frac{3\,(b^2dx^2+2\,abdx+a^2d)\sqrt{-\frac{d}{b}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)+2\,(4\,bdx+bc+3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*(b^2*d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt
```

```
(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + b*c + 3*a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2), -1/3*(3*(b^2*
d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x))
 + 2*(4*b*d*x + b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^
3*x + a^2*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(70) = 140.
time = 0.05, size = 237, normalized size = 2.58

$$\frac{2\left(-\frac{(-12b^3d^4c+12b^2d^5a)\sqrt{c+dx}\sqrt{c+dx}}{-9b^4|d|c+9b^3d|d|a}-\frac{9b^3d^4c^2-18b^2d^5ac+9bd^6a^2}{-9b^4|d|c+9b^3d|d|a}\right)\sqrt{c+dx}\sqrt{ad^2-bcd+bd(c+dx)}}{(ad^2-bcd+bd(c+dx))^2}-\frac{2d^3\ln\left|\sqrt{ad^2-bcd+bd(c+dx)}-\sqrt{bd}\sqrt{c+dx}\right|}{b^2\sqrt{bd}\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
[Out] -2*d^3*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^
2)))/(sqrt(b*d)*b^2*abs(d)) - 2/3*sqrt(d*x + c)*(4*(b^3*c*d^4 - a*b^2*d^5)*
(d*x + c)/(b^4*c*abs(d) - a*b^3*d*abs(d)) - 3*(b^3*c^2*d^4 - 2*a*b^2*c*d^5
+ a^2*b*d^6)/(b^4*c*abs(d) - a*b^3*d*abs(d)))/((d*x + c)*b*d - b*c*d + a*d^
2)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(3/2)/(a + b*x)^(5/2), x)
```

3.1477 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

```
[Out] -2/5*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]')

[Out] Timed out

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(26) = 52$.
time = 0.18, size = 161, normalized size = 5.03

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}}{5(bx+a)^{\frac{5}{2}}(ad-bc)}$
default	$-\frac{(dx+c)^{\frac{3}{2}}}{b(bx+a)^{\frac{5}{2}}}+\frac{3(ad-bc)\left(-\frac{\sqrt{dx+c}}{2b(bx+a)^{\frac{5}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{4b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)/(b*x+a)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(d*x+c)^(3/2)/(b*x+a)^(5/2)+3/2*(a*d-b*c)/b*(-1/2/b*(d*x+c)^(1/2)/(b*x+a)^(5/2)+1/4*(a*d-b*c)/b*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(26) = 52$.
time = 0.47, size = 104, normalized size = 3.25

$$-\frac{2\,(d^2x^2 + 2\,cdx + c^2)\sqrt{bx+a}\,\sqrt{dx+c}}{5\,(a^3bc - a^4d + (b^4c - ab^3d)x^3 + 3\,(ab^3c - a^2b^2d)x^2 + 3\,(a^2b^2c - a^3bd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b*c - a^4*d
 + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c - a^3
*b*d)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(26) = 52$.
time = 0.08, size = 149, normalized size = 4.66

$$-\frac{2\,(45b^3d^6c - 45b^2d^7a)\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{ad^2 - bcd + bd\,(c+dx)}}{(225b^4c^2\,|d| - 450b^3dac\,|d| + 225b^2d^2a^2\,|d|)\,(ad^2 - bcd + bd\,(c+dx))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x)
```

```
[Out] -2/5*(b^3*c*d^6 - a*b^2*d^7)*(d*x + c)^(5/2)/((b^4*c^2*abs(d) - 2*a*b^3*c*d
*abs(d) + a^2*b^2*d^2*abs(d))*((d*x + c)*b*d - b*c*d + a*d^2)^(5/2))
```

Mupad [B]
time = 0.80, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(a+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(7/2),x)
```

```
[Out] (2*(c + d*x)^(5/2))/((5*a*d - 5*b*c)*(a + b*x)^(5/2))
```

3.1478 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}$$

```
[Out] -2/7*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(7/2)+4/35*d*(d*x+c)^(5/2)/(-a*d+b*c)
^2/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (4*d*(c + d*x)^(5/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(2d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}$$

Mathematica [**A**]
time = 0.09, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{5/2}(-5bc+7ad+2bdx)}{35(bc-ad)^2(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-5*b*c + 7*a*d + 2*b*d*x))/(35*(b*c - a*d)^2*(a + b*x)^
(7/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(54) = 108.
time = 0.16, size = 201, normalized size = 3.05

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(2bdx+7ad-5bc)}{35(bx+a)^{\frac{7}{2}}(a^2d^2-2abcd+b^2c^2)}$

default	$-\frac{(dx+c)^{\frac{3}{2}}}{2b(bx+a)^{\frac{7}{2}}}+\frac{3(ad-bc)\left(-\frac{\sqrt{dx+c}}{3b(bx+a)^{\frac{7}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\cdots\right)}{5(-ad+\cdots}\right)}{7(-ad+bc)}\right)}{6b}\right)}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*x+c)^(3/2)/(b*x+a)^(7/2)+3/4*(a*d-b*c)/b*(-1/3/b*(d*x+c)^(1/2)/(b
*x+a)^(7/2)+1/6*(a*d-b*c)/b*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/
7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c
)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*
c)^2/(b*x+a)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 0.82, size = 235, normalized size = 3.56

$$\frac{2\left(2bd^3x^3-5bc^3+7ac^2d-(bcd^2-7ad^3)x^2-2(4bc^2d-7acd^2)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{35\left(a^4b^2c^2-2a^5bcd+a^6d^2+(b^6c^2-2ab^5cd+a^2b^4d^2)x^4+4(ab^5c^2-2a^2b^4cd+a^3b^3d^2)x^3+6(a^2b^4c^2-2a^3b^3cd+a^4b^2d^2)x^2+4(a^3b^3c^2-2a^4b^2cd+a^5bd^2)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b*d^3*x^3 - 5*b*c^3 + 7*a*c^2*d - (b*c*d^2 - 7*a*d^3)*x^2 - 2*(4*b*
c^2*d - 7*a*c*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^2*c^2 - 2*a^5*b*c*
d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*c^2 - 2*
a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(54) = 108.
time = 0.12, size = 276, normalized size = 4.18

$$\frac{2\left(-\frac{(210b^5d^8c-210b^4d^9a)\sqrt{c+dx}\,\sqrt{c+dx}}{-3675b^6c^3|d|+11025b^5dac^2|d|-11025b^4d^2a^2c|d|+3675b^3d^3a^3|d|}-\frac{-735b^5d^8c^2+1470b^4d^9ac-735b^3d^{10}a^2}{-3675b^6c^3|d|+11025b^5dac^2|d|-11025b^4d^2a^2c|d|+3675b^3d^3a^3|d|}\right)\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{(ad^2-bcd+bd\,(c+dx))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/35*(d*x + c)^(5/2)*(2*(b^5*c*d^8 - a*b^4*d^9)*(d*x + c)/(b^6*c^3*abs(d) -
 3*a*b^5*c^2*d*abs(d) + 3*a^2*b^4*c*d^2*abs(d) - a^3*b^3*d^3*abs(d)) - 7*(b
^5*c^2*d^8 - 2*a*b^4*c*d^9 + a^2*b^3*d^10)/(b^6*c^3*abs(d) - 3*a*b^5*c^2*d*
abs(d) + 3*a^2*b^4*c*d^2*abs(d) - a^3*b^3*d^3*abs(d)))/((d*x + c)*b*d - b*c
*d + a*d^2)^(7/2)
```

Mupad [B]
time = 0.93, size = 178, normalized size = 2.70

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^3\,x^3}{35\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{10\,b\,c^3-14\,a\,c^2\,d}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{x^2\,\left(14\,a\,d^3-2\,b\,c\,d^2\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,c\,d\,x\,\left(7\,a\,d-4\,b\,c\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^3*x^3)/(35*b^2*(a*d - b*c)^2) - (10*b*c^3 - 14*a*c^2
*d)/(35*b^3*(a*d - b*c)^2) + (x^2*(14*a*d^3 - 2*b*c*d^2))/(35*b^3*(a*d - b*
c)^2) + (4*c*d*x*(7*a*d - 4*b*c))/(35*b^3*(a*d - b*c)^2)))/(x^3*(a + b*x)^(
1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(
a + b*x)^(1/2))/b^2)
```

3.1479 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}$$

```
[Out] -2/9*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(9/2)+8/63*d*(d*x+c)^(5/2)/(-a*d+b*c)
^2/(b*x+a)^(7/2)-16/315*d^2*(d*x+c)^(5/2)/(-a*d+b*c)^3/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (8*d*(c + d*x)^(5/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(5/2))/(315*(b*c -
 a*d)^3*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(4d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{(8d^2)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{63(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}$$

Mathematica [**A**]
time = 0.12, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{9/2}\left(35b^2 + \frac{63d^2(a+bx)^2}{(c+dx)^2} - \frac{90bd(a+bx)}{c+dx}\right)}{315(bc-ad)^3(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
[Out] (-2*(c + d*x)^(9/2)*(35*b^2 + (63*d^2*(a + b*x)^2)/(c + d*x)^2 - (90*b*d*(a
 + b*x))/(c + d*x)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]')
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(83) = 166.
time = 0.17, size = 241, normalized size = 2.39

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}\left(8b^2x^2d^2+36ab\,d^2x-20b^2cdx+63a^2d^2-90abcd+35b^2c^2\right)}{315(bx+a)^{\frac{9}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$

default	$-\frac{(dx+c)^{\frac{3}{2}}}{3b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{\sqrt{dx+c}}{4b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(\cdots\right.}{\cdots}\right)}{\cdots}\right)}{9(-ad\cdots}\right)}{8b}\cdots\right)}{2b}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(d*x+c)^(3/2)/(b*x+a)^(9/2)+1/2*(a*d-b*c)/b*(-1/4/b*(d*x+c)^(1/2)/(b
*x+a)^(9/2)+1/8*(a*d-b*c)/b*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/
9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c
)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+
c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^
(1/2))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(83) = 166$.
time = 2.59, size = 426, normalized size = 4.22

$$-\frac{2 \, {\left(8 \, b^{2} d^{4} x^{4} + 35 \, b^{2} c^{4} - 90 \, a b c^{3} d + 63 \, a^{2} c^{2} d^{2} - 4 \, {\left(b^{2} c d^{3} - 9 \, a b d^{4}\right)} x^{3} + 3 \, {\left(b^{2} c^{2} d^{2} - 6 \, a b c d^{3} + 21 \, a^{2} d^{4}\right)} x^{2} + 2 \, {\left(25 \, b^{2} c^{3} d - 72 \, a b c^{2} d^{2} + 63 \, a^{2} c d^{3}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{315 \, {\left(a^{5} b^{3} c^{3} - 3 \, a^{6} b^{2} c^{2} d + 3 \, a^{7} b c d^{2} - a^{8} d^{3} + {\left(b^{8} c^{3} - 3 \, a b^{7} c^{2} d + 3 \, a^{2} b^{6} c d^{2} - a^{3} b^{5} d^{3}\right)} x^{5} + 5 \, {\left(a b^{7} c^{3} - 3 \, a^{2} b^{6} c^{2} d + 3 \, a^{3} b^{5} c d^{2} - a^{4} b^{4} d^{3}\right)} x^{4} + 10 \, {\left(a^{2} b^{6} c^{3} - 3 \, a^{3} b^{5} c^{2} d + 3 \, a^{4} b^{4} c d^{2} - a^{5} b^{3} d^{3}\right)} x^{3} + 10 \, {\left(a^{3} b^{5} c^{3} - 3 \, a^{4} b^{4} c^{2} d + 3 \, a^{5} b^{3} c d^{2} - a^{6} b^{2} d^{3}\right)} x^{2} + 5 \, {\left(a^{4} b^{4} c^{3} - 3 \, a^{5} b^{3} c^{2} d + 3 \, a^{6} b^{2} c d^{2} - a^{7} b d^{3}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^2*d^4*x^4 + 35*b^2*c^4 - 90*a*b*c^3*d + 63*a^2*c^2*d^2 - 4*(b^2
*c*d^3 - 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 - 6*a*b*c*d^3 + 21*a^2*d^4)*x^2 +
2*(25*b^2*c^3*d - 72*a*b*c^2*d^2 + 63*a^2*c*d^3)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c^3 - 3*a^3*b^5
*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3 - 3*a^4*b^4*c
^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*
d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(83) = 166$.
time = 0.18, size = 448, normalized size = 4.44

$$\frac{2 \left(\left(\left(-\frac{\left(22680 b^{7} d^{10} c - 22680 b^{6} d^{11} a\right) \sqrt{c+dx} \sqrt{c+dx}}{893025 b^{8} c^{4} |d| - 3572100 b^{7} d a c^{3} |d| + 5358150 b^{6} d^{2} a^{2} c^{2} |d| - 3572100 b^{5} d^{3} a^{3} c |d| + 893025 b^{4} d^{4} a^{4} |d|} - \frac{-102060 b^{7} d^{10} c^{2} + 204120 b^{6} d^{11} a c - 102060 b^{5} d^{12} a^{2}}{893025 b^{8} c^{4} |d| - 3572100 b^{7} d a c^{3} |d| + 5358150 b^{6} d^{2} a^{2} c^{2} |d| - 3572100 b^{5} d^{3} a^{3} c |d| + 893025 b^{4} d^{4} a^{4} |d|}\right) \sqrt{c+dx} \sqrt{c+dx} - \frac{178605 b^{7} d^{10} c^{3} - 535815 b^{6} d^{11} a c^{2} + 535815 b^{5} d^{12} a^{2} c - 178605 b^{4} d^{13} a^{3}}{893025 b^{8} c^{4} |d| - 3572100 b^{7} d a c^{3} |d| + 5358150 b^{6} d^{2} a^{2} c^{2} |d| - 3572100 b^{5} d^{3} a^{3} c |d| + 893025 b^{4} d^{4} a^{4} |d|}\right) \sqrt{c+dx} \sqrt{c+dx} \sqrt{c+dx} \sqrt{c+dx} \sqrt{c+dx} \sqrt{a d^{2} - b c d + b d (c+dx)}}{\left(a d^{2} - b c d + b d (c+dx)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x)
```

```
[Out] -2/315*(4*(d*x + c)*(2*(b^7*c*d^10 - a*b^6*d^11)*(d*x + c)/(b^8*c^4*abs(d)
- 4*a*b^7*c^3*d*abs(d) + 6*a^2*b^6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d)
+ a^4*b^4*d^4*abs(d)) - 9*(b^7*c^2*d^10 - 2*a*b^6*c*d^11 + a^2*b^5*d^12)/(b
^8*c^4*abs(d) - 4*a*b^7*c^3*d*abs(d) + 6*a^2*b^6*c^2*d^2*abs(d) - 4*a^3*b^5
*c*d^3*abs(d) + a^4*b^4*d^4*abs(d))) + 63*(b^7*c^3*d^10 - 3*a*b^6*c^2*d^11
+ 3*a^2*b^5*c*d^12 - a^3*b^4*d^13)/(b^8*c^4*abs(d) - 4*a*b^7*c^3*d*abs(d) +
 6*a^2*b^6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d) + a^4*b^4*d^4*abs(d)))*(
d*x + c)^(5/2)/((d*x + c)*b*d - b*c*d + a*d^2)^(9/2)
```

Mupad [B]
time = 1.11, size = 268, normalized size = 2.65

$$\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^2\,c^2\,d^2-180\,a\,b\,c^3\,d+70\,b^2\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,\left(126\,a^2\,d^4-36\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^4\,x^4}{315\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,c\,d\,x\,\left(63\,a^2\,d^2-72\,a\,b\,c\,d+25\,b^2\,c^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((70*b^2*c^4 + 126*a^2*c^2*d^2 - 180*a*b*c^3*d)/(315*b^4*(
a*d - b*c)^3) + (x^2*(126*a^2*d^4 + 6*b^2*c^2*d^2 - 36*a*b*c*d^3))/(315*b^4
*(a*d - b*c)^3) + (16*d^4*x^4)/(315*b^2*(a*d - b*c)^3) + (8*d^3*x^3*(9*a*d
- b*c))/(315*b^3*(a*d - b*c)^3) + (4*c*d*x*(63*a^2*d^2 + 25*b^2*c^2 - 72*a*
b*c*d))/(315*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/
2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (
4*a^3*x*(a + b*x)^(1/2))/b^3)
```

3.1480 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}} dx$

Optimal. Leaf size=136

$$-\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{32d^3(c+dx)^{5/2}}{1155(bc-ad)^4(a+bx)^{5/2}}$$

```
[Out] -2/11*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(11/2)+4/33*d*(d*x+c)^(5/2)/(-a*d+b*
c)^2/(b*x+a)^(9/2)-16/231*d^2*(d*x+c)^(5/2)/(-a*d+b*c)^3/(b*x+a)^(7/2)+32/1
155*d^3*(d*x+c)^(5/2)/(-a*d+b*c)^4/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (4*d*(c + d*x)^(5/
2))/(33*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(5/2))/(231*(b*c
 - a*d)^3*(a + b*x)^(7/2)) + (32*d^3*(c + d*x)^(5/2))/(1155*(b*c - a*d)^4*(
a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(6d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} + \frac{(8d^2)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{(1}{}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{}{11}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.70

$$-\frac{2(c+dx)^{11/2}\left(105b^3 - \frac{231d^3(a+bx)^3}{(c+dx)^3} + \frac{495bd^2(a+bx)^2}{(c+dx)^2} - \frac{385b^2d(a+bx)}{c+dx}\right)}{1155(bc-ad)^4(a+bx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
[Out] (-2*(c + d*x)^(11/2)*(105*b^3 - (231*d^3*(a + b*x)^3)/(c + d*x)^3 + (495*b*
d^2*(a + b*x)^2)/(c + d*x)^2 - (385*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
 - a*d)^4*(a + b*x)^(11/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]')
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(112) = 224.
time = 0.17, size = 281, normalized size = 2.07

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}\left(16b^3x^3d^3+88d^3a\,x^2b^2-40b^3c\,d^2x^2+198a^2b\,d^3x-220a\,b^2c\,d^2x+70b^3c^2dx+231a^3d^3-495a^2bc\,d^2+385a\,b^2c^2d-105b^3c^3\right)}{1155(bx+a)^{\frac{11}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

$$8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\right.$$

$$10d\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}-\right.$$

$$3(ad-bc)\left(-\frac{\sqrt{dx+c}}{5b(bx+a)^{\frac{11}{2}}}+\frac{\cdots}{1\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(d*x+c)^(3/2)/(b*x+a)^(11/2)+3/8*(a*d-b*c)/b*(-1/5/b*(d*x+c)^(1/2)/(
b*x+a)^(11/2)+1/10*(a*d-b*c)/b*(-2/11/(-a*d+b*c)/(b*x+a)^(11/2)*(d*x+c)^(1/
2)-10/11*d/(-a*d+b*c)*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-
a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/
5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/
2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))
)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(112) = 224$.
time = 5.50, size = 649, normalized size = 4.77

$$\frac{2\left(16\,b^{3}d^{5}x^{5}-105\,b^{3}c^{5}+385\,ab^{2}c^{4}d-495\,a^{2}bc^{3}d^{2}+231\,a^{3}c^{2}d^{3}-8\left(b^{3}cd^{4}-11\,ab^{2}d^{5}\right)x^{4}+2\left(3\,b^{3}c^{2}d^{3}-22\,ab^{2}cd^{4}+99\,a^{2}bd^{5}\right)x^{3}-\left(5\,b^{3}c^{3}d^{2}-33\,ab^{2}c^{2}d^{3}+99\,a^{2}bcd^{4}-231\,a^{3}d^{5}\right)x^{2}-2\left(70\,b^{3}c^{4}d-275\,ab^{2}c^{3}d^{2}+396\,a^{2}bc^{2}d^{3}-231\,a^{3}cd^{4}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{1155\left(a^{6}b^{4}c^{4}-4\,a^{7}b^{3}c^{3}d+6\,a^{8}b^{2}c^{2}d^{2}-4\,a^{9}bcd^{3}+a^{10}d^{4}+\left(b^{10}c^{4}-4\,ab^{9}c^{3}d+6\,a^{2}b^{8}c^{2}d^{2}-4\,a^{3}b^{7}cd^{3}+a^{4}b^{6}d^{4}\right)x^{6}+6\left(ab^{9}c^{4}-4\,a^{2}b^{8}c^{3}d+6\,a^{3}b^{7}c^{2}d^{2}-4\,a^{4}b^{6}cd^{3}+a^{5}b^{5}d^{4}\right)x^{5}+15\left(a^{2}b^{8}c^{4}-4\,a^{3}b^{7}c^{3}d+6\,a^{4}b^{6}c^{2}d^{2}-4\,a^{5}b^{5}cd^{3}+a^{6}b^{4}d^{4}\right)x^{4}+20\left(a^{3}b^{7}c^{4}-4\,a^{4}b^{6}c^{3}d+6\,a^{5}b^{5}c^{2}d^{2}-4\,a^{6}b^{4}cd^{3}+a^{7}b^{3}d^{4}\right)x^{3}+15\left(a^{4}b^{6}c^{4}-4\,a^{5}b^{5}c^{3}d+6\,a^{6}b^{4}c^{2}d^{2}-4\,a^{7}b^{3}cd^{3}+a^{8}b^{2}d^{4}\right)x^{2}+6\left(a^{5}b^{5}c^{4}-4\,a^{6}b^{4}c^{3}d+6\,a^{7}b^{3}c^{2}d^{2}-4\,a^{8}b^{2}cd^{3}+a^{9}bd^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^3*d^5*x^5 - 105*b^3*c^5 + 385*a*b^2*c^4*d - 495*a^2*b*c^3*d^2
+ 231*a^3*c^2*d^3 - 8*(b^3*c*d^4 - 11*a*b^2*d^5)*x^4 + 2*(3*b^3*c^2*d^3 - 2
2*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - (5*b^3*c^3*d^2 - 33*a*b^2*c^2*d^3 + 99*
a^2*b*c*d^4 - 231*a^3*d^5)*x^2 - 2*(70*b^3*c^4*d - 275*a*b^2*c^3*d^2 + 396*
a^2*b*c^2*d^3 - 231*a^3*c*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^4*c^4
- 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 + a^10*d^4 + (b^10*c^
4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^6
+ 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^
5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*
a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^
```

```
5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*(a^4*b^6*c^4 - 4*a^
5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*x^2 + 6*(a
^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*
d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(112) = 224.
time = 0.25, size = 671, normalized size = 4.93

$$\frac{2\left(\left(\left(-\frac{(1496880b^9d^{12}c-1496880b^8d^{13}a)\sqrt{c+dx}\sqrt{c+dx}}{-108056025b^{10}c^5|d|+540280125b^9dac^4|d|-1080560250b^8d^2a^2c^3|d|+1080560250b^7d^3a^3c^2|d|-540280125b^6d^4a^4c|d|+108056025b^5d^5a^5|d|}-\frac{-8232840b^9d^{12}c^2+16465680b^8d^{13}ac-8232840b^7d^{14}a^2}{-108056025b^{10}c^5|d|+540280125b^9dac^4|d|-1080560250b^8d^2a^2c^3|d|+1080560250b^7d^3a^3c^2|d|-540280125b^6d^4a^4c|d|+108056025b^5d^5a^5|d|}\right)\sqrt{c+dx}\sqrt{c+dx}-\frac{18523890b^9d^{12}c^3-55571670b^8d^{13}ac^2+55571670b^7d^{14}a^2c-18523890b^6d^{15}a^3}{-108056025b^{10}c^5|d|+540280125b^9dac^4|d|-1080560250b^8d^2a^2c^3|d|+1080560250b^7d^3a^3c^2|d|-540280125b^6d^4a^4c|d|+108056025b^5d^5a^5|d|}\right)\sqrt{c+dx}\sqrt{c+dx}-\frac{-21611205b^9d^{12}c^4+86444820b^8d^{13}ac^3-129667230b^7d^{14}a^2c^2+86444820b^6d^{15}a^3c-21611205b^5d^{16}a^4}{-108056025b^{10}c^5|d|+540280125b^9dac^4|d|-1080560250b^8d^2a^2c^3|d|+1080560250b^7d^3a^3c^2|d|-540280125b^6d^4a^4c|d|+108056025b^5d^5a^5|d|}\right)\sqrt{c+dx}\sqrt{c+dx}\sqrt{c+dx}\sqrt{c+dx}\sqrt{c+dx}\sqrt{ad^2-bcd+bd(c+dx)}}{(ad^2-bcd+bd(c+dx))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/1155*(2*(4*(d*x + c)*(2*(b^9*c*d^12 - a*b^8*d^13)*(d*x + c)/(b^10*c^5*abs
(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3
*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)) - 11*(b^9*c^2*d^12 -
 2*a*b^8*c*d^13 + a^2*b^7*d^14)/(b^10*c^5*abs(d) - 5*a*b^9*c^4*d*abs(d) + 1
0*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d^4*abs(
d) - a^5*b^5*d^5*abs(d))) + 99*(b^9*c^3*d^12 - 3*a*b^8*c^2*d^13 + 3*a^2*b^7
*c*d^14 - a^3*b^6*d^15)/(b^10*c^5*abs(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^
8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5
*b^5*d^5*abs(d)))*(d*x + c) - 231*(b^9*c^4*d^12 - 4*a*b^8*c^3*d^13 + 6*a^2*
b^7*c^2*d^14 - 4*a^3*b^6*c*d^15 + a^4*b^5*d^16)/(b^10*c^5*abs(d) - 5*a*b^9*
c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^
4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)))*(d*x + c)^(5/2)/((d*x + c)*b*d -
b*c*d + a*d^2)^(11/2)
```

Mupad [B]
time = 1.33, size = 376, normalized size = 2.76

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(462\,a^3\,d^5-198\,a^2\,b\,c\,d^4+66\,a\,b^2\,c^2\,d^3-10\,b^3\,c^3\,d^2\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}-\frac{-462\,a^3\,c^2\,d^3+990\,a^2\,b\,c^3\,d^2-770\,a\,b^2\,c^4\,d+210\,b^3\,c^5}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(924\,a^3\,c\,d^4-1584\,a^2\,b\,c^2\,d^3+1100\,a\,b^2\,c^3\,d^2-280\,b^3\,c^4\,d\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^5\,x^5}{1155\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{1155\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{1155\,b^4\,{\left(a\,d-b\,c\right)}^4}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(462*a^3*d^5 - 10*b^3*c^3*d^2 + 66*a*b^2*c^2*d^3 - 1
98*a^2*b*c*d^4))/(1155*b^5*(a*d - b*c)^4) - (210*b^3*c^5 - 462*a^3*c^2*d^3
+ 990*a^2*b*c^3*d^2 - 770*a*b^2*c^4*d)/(1155*b^5*(a*d - b*c)^4) + (x*(924*a
^3*c*d^4 - 280*b^3*c^4*d + 1100*a*b^2*c^3*d^2 - 1584*a^2*b*c^2*d^3))/(1155*
b^5*(a*d - b*c)^4) + (32*d^5*x^5)/(1155*b^2*(a*d - b*c)^4) + (16*d^4*x^4*(1
1*a*d - b*c))/(1155*b^3*(a*d - b*c)^4) + (4*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2
 - 22*a*b*c*d))/(1155*b^4*(a*d - b*c)^4)))/(x^5*(a + b*x)^(1/2) + (a^5*(a +
 b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)
^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

3.1481 $\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3}-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d}+\frac{(bc-ad)}{}$$

```
[Out] 1/12*(-a*d+b*c)*(b*x+a)^(7/2)*(d*x+c)^(3/2)/b^2+1/6*(b*x+a)^(7/2)*(d*x+c)^(
5/2)/b-5/512*(-a*d+b*c)^6*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/
2))/b^(7/2)/d^(7/2)-5/768*(-a*d+b*c)^4*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d^2+
1/192*(-a*d+b*c)^3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3/d+1/32*(-a*d+b*c)^2*(b*x
+a)^(7/2)*(d*x+c)^(1/2)/b^3+5/512*(-a*d+b*c)^5*(b*x+a)^(1/2)*(d*x+c)^(1/2)/
b^3/d^3
```

Rubi [A]

time = 0.11, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}{768b^3d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}{192b^3d}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x])/(512*b^3*d^3) - (5*(b*c - a*d
)^4*(a + b*x)^(3/2)*Sqrt[c + d*x])/(768*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)
^(5/2)*Sqrt[c + d*x])/(192*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(32*b^3) + ((b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(12*b^2) +
((a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b) - (5*(b*c - a*d)^6*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(512*b^(7/2)*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(5(bc-ad))\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(bc-ad)^2\int(a+}{8}$$

$$= \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)}{}$$

$$= \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad}{}$$

$$= -\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ac}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ac}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ac}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ac}{}$$

Mathematica [A]
time = 0.61, size = 191, normalized size = 0.73

$$\frac{\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx}\,\sqrt{c+dx}\,\left(15d^5(a+bx)^5-85bd^4(a+bx)^4(c+dx)+198b^2d^3(a+bx)^3(c+dx)^2+198b^3d^2(a+bx)^2(c+dx)^3-85b^4d(a+bx)(c+dx)^4+15b^5(c+dx)^5\right)-15(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{1536b^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^5*(a + b*x)^5 - 85*b*d^4
*(a + b*x)^4*(c + d*x) + 198*b^2*d^3*(a + b*x)^3*(c + d*x)^2 + 198*b^3*d^2*
(a + b*x)^2*(c + d*x)^3 - 85*b^4*d*(a + b*x)*(c + d*x)^4 + 15*b^5*(c + d*x)
^5) - 15*(b*c - a*d)^6*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*
x])])/(1536*b^(7/2)*d^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 272, normalized size = 1.04

method	result

$$5(ad-bc$$

$$(-ad+bc)\left(\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\right.$$

$$3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d}-\right.$$

$$5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{2}}}{5d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(b*x+a)^(5/2)*(d*x+c)^(7/2)-5/12*(-a*d+b*c)/d*(1/5/d*(b*x+a)^(3/2)*(d
*x+c)^(7/2)-3/10*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)-1/8*(-a*d+
b*c)/d*(1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*c)/b*(1/2*(d*x+c)^(3/2
)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b
*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b
*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(212) = 424$.
time = 0.35, size = 882, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6144*(15*(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sq
rt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(256*b^6*d^6*x^5 +
 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^3*d^3 + 198*a^3*b^3*c^2*d^
4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*d^5 + a*b^5*d^6)*x^4 + 16*
(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^6)*x^3 + 8*(b^6*c^3*d^3 +
159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^2 - 2*(5*b^6*c^4*d^2
 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^3*b^3*c*d^5 + 5*a^4*b^2*d^
6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4), 1/3072*(15*(b^6*c^6 - 6*a*b^5
*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
```

```
^5*b*c*d^5 + a^6*d^6)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d
)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)) + 2*(256*b^6*d^6*x^5 + 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^
3*d^3 + 198*a^3*b^3*c^2*d^4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*
d^5 + a*b^5*d^6)*x^4 + 16*(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^
6)*x^3 + 8*(b^6*c^3*d^3 + 159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d
^6)*x^2 - 2*(5*b^6*c^4*d^2 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^
3*b^3*c*d^5 + 5*a^4*b^2*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}} (c+dx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3120 vs. $2(212) = 424$.
time = 0.30, size = 4138, normalized size = 15.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/7680*(960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c^2*abs(b) - 7680*((b^2*c - a*
b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c^2*a
bs(b)/b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x +
a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^
2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt
(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d
^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*
b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c^2*abs(b) + 240*(sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 -
 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b
^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c
*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*
```

c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)
*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))
*a*c*d*abs(b) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*
(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7
*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*
c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*c*d*abs(b)/b + 8*(s
qrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a
)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b
^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c
^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(
7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^
7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c
^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*b^3*d^4))*b*c*d*abs(b) + 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
(2(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^
8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^2
3*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a
^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5
 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*
sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*
b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*a*d^2*abs(b)
+ 320*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^3*d^2*abs(b)/b^2 + 120*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*
c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 16
3*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2
*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^
2*d^3))*a^2*d^2*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(2*(b
x + a)(8*(b*x + a)*(10*(b*x + a)/b^5 + (b^30*c*d^9 - 61*a*b^29*d^10)/(b^3
4*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*c*d^9 - 417*a^2*b^29*d^10)/(b^34*d
^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2*d^8 + 183*a^2*b^30*c*d^9 - 3481*a^
3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5*(21*b^33*c^4*d^6 + 56*a*b^32*c^3*d^
7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^30*c*d^9 - 2279*a^4*b^29*d^10)/(b^34*d
^10))*(b*x + a) + 15*(21*b^34*c^5*d^5 + 35*a*b^33*c^4*d^6 + 50*a^2*b^32*c^3
*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4*b^30*c*d^9 - 793*a^5*b^29*d^10)/(b^34*
d^10))*sqrt(b*x + a) + 15*(21*b^6*c^6 + 14*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 - 231*a^6*d^6)

```
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b^4*d^5))*b*d^2*abs(b) + 5760*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^2*c^2*abs(b)/b^2 + 3840*(sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (
b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqr
t(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*c*d*abs(b)/b^3)/b
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(5/2), x)
```

3.1482 $\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=224

$$-\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2}+\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d}+\frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3}+(bc-a$$

```
[Out] 1/8*(-a*d+b*c)*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b^2+1/5*(b*x+a)^(5/2)*(d*x+c)^(5
/2)/b+3/128*(-a*d+b*c)^5*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2
))/b^(7/2)/d^(5/2)+1/64*(-a*d+b*c)^3*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d+1/16
*(-a*d+b*c)^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3-3/128*(-a*d+b*c)^4*(b*x+a)^(1
/2)*(d*x+c)^(1/2)/b^3/d^2
```

Rubi [A]

time = 0.09, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}{128b^3d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{16b^3}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] (-3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^3*d^2) + ((b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(5/
2)*Sqrt[c + d*x])/(16*b^3) + ((b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/
(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b) + (3*(b*c - a*d)^5*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(7/2)*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(3(bc-ad)^2)\int (c$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)}{}$$

$$= \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

Mathematica [A]

time = 0.48, size = 169, normalized size = 0.75

$$\frac{\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx}\,\sqrt{c+dx}\,\left(15d^4(a+bx)^4-70bd^3(a+bx)^3(c+dx)+128b^2d^2(a+bx)^2(c+dx)^2+70b^3d(a+bx)(c+dx)^3-15b^4(c+dx)^4\right)+15(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{640b^{7/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^4*(a + b*x)^4 - 70*b*d^3
*(a + b*x)^3*(c + d*x) + 128*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 70*b^3*d*(a
+ b*x)*(c + d*x)^3 - 15*b^4*(c + d*x)^4) + 15*(b*c - a*d)^5*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(640*b^(7/2)*d^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.17, size = 239, normalized size = 1.07

method	result

default	$\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{2}}}{5d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b} - \frac{5(ad-bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)-3/10*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(1/2)*(d
*x+c)^(7/2)-1/8*(-a*d+b*c)/d*(1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*
c)/b*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x
+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.33, size = 702, normalized size = 3.13

$$\left[-\frac{15\left(b^5c^5 - 5ab^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4bcd^4 - a^5d^5\right)\sqrt{bd}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 - 4\left(2bdx + bc + ad\right)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c} + 8\left(b^2cd + abd^2\right)x\right) - 4\left(128b^5d^5x^4 - 15b^5c^4d + 70ab^4c^3d^2 + 128a^2b^3c^2d^3 - 70a^3b^2cd^4 + 15a^4bd^5 + 16\left(21b^5cd^4 + 11ab^4d^5\right)x^3 + 8\left(31b^5c^2d^3 + 64ab^4cd^4 + a^2b^3d^5\right)x^2 + 2\left(5b^5c^3d^2 + 233ab^4c^2d^3 + 23a^2b^3cd^4 - 5a^3b^2d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{2560\,b^4d^3}, -\frac{15\left(b^5c^5 - 5ab^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4bcd^4 - a^5d^5\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx + bc + ad\right)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2 + abcd + \left(b^2cd + abd^2\right)x\right)}\right) - 2\left(128b^5d^5x^4 - 15b^5c^4d + 70ab^4c^3d^2 + 128a^2b^3c^2d^3 - 70a^3b^2cd^4 + 15a^4bd^5 + 16\left(21b^5cd^4 + 11ab^4d^5\right)x^3 + 8\left(31b^5c^2d^3 + 64ab^4cd^4 + a^2b^3d^5\right)x^2 + 2\left(5b^5c^3d^2 + 233ab^4c^2d^3 + 23a^2b^3cd^4 - 5a^3b^2d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{1280\,b^4d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 70*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(
21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2
*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^3), -1/1280*(15*(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^
4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*
x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(
128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 7
0*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(
31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233
*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*
x + c))/(b^4*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1962 vs. 2(180) = 360.
time = 0.19, size = 2623, normalized size = 11.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/1920*(80*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c^2*abs(b) - 1920*((b^2*c - a*b*d
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c^2*abs(
b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*
(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d
^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 +
9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*
x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3
- 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)))/(sqrt(b*d)*b^2*d^3))*c*d*abs(b) + 320*(sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a
*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^
7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*
d^2))*a*c*d*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)
*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) -
 (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b
^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^
23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c
^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) -
15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35
*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*d^2*abs(b) + 80*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*
c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*
b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b*d^2))*a^2*d^2*abs(b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^
6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^1
4*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3
*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d^2*abs(b)
/b + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
```

```
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a*c^2*abs(b)/b^2 + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*a^2*c*d*abs(b)/b^3)/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/2), x)
```

3.1483 $\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx$

Optimal. Leaf size=186

$$\frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d}+\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3}+\frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2}+\frac{(a+b$$

```
[Out] 5/24*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^2+1/4*(b*x+a)^(3/2)*(d*x+c)^(
5/2)/b-5/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2
))/b^(7/2)/d^(3/2)+5/32*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3+5/64*(
-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}{24b^2}+\frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^3*d) + (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^3) + (5*(b*c - a*d)*(a + b*x)^(3/2)*(
c + d*x)^(3/2))/(24*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b) - (5*(b*
c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^
(7/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad))\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad)^2)\int \sqrt{}}{1}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)}{}$$

Mathematica [A]
time = 0.33, size = 144, normalized size = 0.77

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(15d^3(a+bx)^3-55bd^2(a+bx)^2(c+dx)+73b^2d(a+bx)(c+dx)^2+15b^3(c+dx)^3\right)}{192b^3d} - \frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^3*(a + b*x)^3 - 55*b*d^2*(a + b*x)^2*(c + d*x) + 73*b^2*d*(a + b*x)*(c + d*x)^2 + 15*b^3*(c + d*x)^3))/(192*b^3*d) - (5*(b*c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(7/2)*d^(3/2))

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(5/2),x]')

[Out] Timed out

Maple [**A**]
time = 0.16, size = 206, normalized size = 1.11

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d}-\frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5(ad-bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}\right.}{}\right.}{}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/2)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)-1/8*(-a*d+b*c)/d*(1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*c)/b*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.32, size = 540, normalized size = 2.90

$$\left[\frac{15\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(48\,b^4d^4x^3+15\,b^4c^3d+73\,ab^3c^2d^2-55\,a^2b^2cd^3+15\,a^3bd^4+8\,(17\,b^4cd^3+ab^3d^4)x^2+2\,(59\,b^4c^2d^2+18\,ab^3cd^3-5\,a^2b^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{768\,b^4d^2}, \frac{15\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(48\,b^4d^4x^3+15\,b^4c^3d+73\,ab^3c^2d^2-55\,a^2b^2cd^3+15\,a^3bd^4+8\,(17\,b^4cd^3+ab^3d^4)x^2+2\,(59\,b^4c^2d^2+18\,ab^3cd^3-5\,a^2b^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{384\,b^4d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^
2 + 18*a*b^3*c*d^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^
2), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^2 + 18*a*b^3*c*d
^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^2)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(148) = 296.
time = 0.11, size = 1443, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x)
```

```
[Out] -1/192*(192*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*
sqrt(b*x + a))*a*c^2*abs(b)/b^2 - 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*d*abs
(b)/b - 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d^2*abs(b)/b^2 - (sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*
d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*
a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b
^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt
(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*
d^3))*d^2*abs(b)/b - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*c^2*abs(b)/b^2 - 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2
*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
- 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d))*a*c*d*abs(b)/b^3)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/2), x)
```

3.1484 $\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=148

$$\frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3}+\frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}+\frac{5(bc-ad)^3\tanh^{-1}\left(\right.}{8b^{7/2}\sqrt{\ }}$$

```
[Out] 5/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/
2)/d^(1/2)+5/12*(-a*d+b*c)*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b^2+1/3*(d*x+c)^(5/2
)*(b*x+a)^(1/2)/b+5/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^3) + (5*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2))/(12*b^2) + (Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b)
 + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*b^(7/2)*Sqrt[d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad))\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad)^2)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5}}{3b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5}}{3b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5}}{3b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5}}{3b}$$

Mathematica [A]
time = 0.22, size = 124, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(15a^2d^2-10abd(4c+dx)+b^2\left(33c^2+26cdx+8d^2x^2\right)\right)}{24b^3} + \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(4*c + d*x) + b^2*(33*c ^2 + 26*c*d*x + 8*d^2*x^2)))/(24*b^3) + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*S qrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(7/2)*Sqrt[d])

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(1/2),x]')

[Out] Timed out

Maple [**A**]
time = 0.16, size = 173, normalized size = 1.17

method	result
default	$\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5(ad-bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\ \cdots}{2b\sqrt{\cdots}}\right)}{4b}\right)}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*c)/b*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [A]
time = 0.34, size = 412, normalized size = 2.78

$$\left[-\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2+33\,b^3c^2d-40\,ab^2cd^2+15\,a^2bd^3+2\,(13\,b^3cd^2-5\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,b^4d},-\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(8\,b^3d^3x^2+33\,b^3c^2d-40\,ab^2cd^2+15\,a^2bd^3+2\,(13\,b^3cd^2-5\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 + 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 -
5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d), -1/48*(15*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b
*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (
b^2*c*d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 +
 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 - 5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x +
c))/(b^4*d)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.02, size = 263, normalized size = 1.78

$$\frac{d^2\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 24b^4\sqrt{c+dx}\,\sqrt{c+dx}}{b^5d}-\frac{\frac{1}{144}(-30b^4c+30b^3da)}{b^5d}\right)\sqrt{c+dx}\,\sqrt{c+dx}-\frac{\frac{1}{144}(-45b^4c^2+90b^3dac-45b^2d^2a^2)}{b^5d}\right)\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}+\frac{2(5a^3d^3-15a^2bcd^2+15ab^2c^2d-5b^3c^3)\ln\left|\sqrt{ad^2-bcd+bd\,(c+dx)}-\sqrt{bd}\,\sqrt{c+dx}\right|}{16b^3\sqrt{bd}}\right)\right)}{|d|\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/24*(sqrt((d*x + c)*b*d - b*c*d + a*d^2)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)/(b*d) + 5*(b^4*c - a*b^3*d)/(b^5*d)) + 15*(b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)/(b^5*d)) - 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^2)))
/(sqrt(b*d)*b^3))*d/abs(d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(1/2), x)
```

3.1485 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}$$

```
[Out] 15/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*d^(1
/2)/b^(7/2)-2*(d*x+c)^(5/2)/b/(b*x+a)^(1/2)+5/2*d*(d*x+c)^(3/2)*(b*x+a)^(1/
2)/b^2+15/4*d*(-a*d+b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}+\frac{15d\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (15*d*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^3) + (5*d*Sqrt[a + b*x]
*(c + d*x)^(3/2))/(2*b^2) - (2*(c + d*x)^(5/2))/(b*Sqrt[a + b*x]) + (15*Sqr
t[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(4*b^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{b}$$

$$=\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(15d(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b^2}$$

$$=\frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(15d(bc-a}{}$$

$$=\frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(15d(bc-a}{}$$

$$=\frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(15d(bc-a}{}$$

$$=\frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{15\sqrt{d}\,(bc-}{}$$

Mathematica [A]
time = 0.25, size = 124, normalized size = 0.90

$$\frac{\sqrt{c+dx}\,\left(-15a^2d^2-5abd(-5c+dx)+b^2\left(-8c^2+9cdx+2d^2x^2\right)\right)}{4b^3\sqrt{a+bx}}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x]*(-15*a^2*d^2 - 5*a*b*d*(-5*c + d*x) + b^2*(-8*c^2 + 9*c*d*x
+ 2*d^2*x^2)))/(4*b^3*Sqrt[a + b*x]) + (15*Sqrt[d]*(b*c - a*d)^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.38, size = 439, normalized size = 3.18

$$\left[\frac{15\,(ab^2c^2 - 2\,a^2bcd + a^3d^2 + (b^3c^2 - 2\,ab^2cd + a^2bd^2)x)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,b^2dx + b^2c + abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}} + 8\,(b^2cd + abd^2)x\right) + 4\,(2\,b^2d^2x^2 - 8\,b^2c^2 + 25\,abcd - 15\,a^2d^2 + (9\,b^2cd - 5\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,(b^4x + ab^3)}, -\frac{15\,(ab^2c^2 - 2\,a^2bcd + a^3d^2 + (b^3c^2 - 2\,ab^2cd + a^2bd^2)x)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right) - 2\,(2\,b^2d^2x^2 - 8\,b^2c^2 + 25\,abcd - 15\,a^2d^2 + (9\,b^2cd - 5\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,(b^4x + ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2
+ (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3),
-1/8*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(2*b^2*d^2*x
^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2 + (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(3/2), x)
```

Giac [A]
time = 0.04, size = 282, normalized size = 2.04

$$\frac{2\left(\left(\frac{\frac{1}{8}\cdot 2b^4d^2\sqrt{c+dx}\,\sqrt{c+dx}}{b^5|d|} + \frac{\frac{1}{8}(5b^4d^2c-5b^3d^3a)}{b^5|d|}\right)\sqrt{c+dx}\,\sqrt{c+dx} + \frac{\frac{1}{8}(-15b^4d^2c^2+30b^3d^3ac-15b^2d^4a^2)}{b^5|d|}\right)\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{ad^2-bcd+bd\,(c+dx)} + \frac{2\,(-15a^2d^4+30abcd^3-15b^2c^2d^2)\ln\left|\sqrt{ad^2-bcd+bd\,(c+dx)} - \sqrt{bd}\,\sqrt{c+dx}\right|}{8b^3\sqrt{bd}\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] 1/4*sqrt(d*x + c)*((d*x + c)*(2*(d*x + c)*d^2/(b*abs(d)) + 5*(b^4*c*d^2 - a
*b^3*d^3)/(b^5*abs(d))) - 15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(b
^5*abs(d)))/sqrt((d*x + c)*b*d - b*c*d + a*d^2) - 15/4*(b^2*c^2*d^2 - 2*a*b
```

```
*c*d^3 + a^2*d^4)*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b
*c*d + a*d^2)))/(sqrt(b*d)*b^3*abs(d))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(3/2), x)
```

3.1486 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=128

$$\frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}$$

```
[Out] -2/3*(d*x+c)^(5/2)/b/(b*x+a)^(3/2)+5*d^(3/2)*(-a*d+b*c)*arctanh(d^(1/2)*(b*
x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)-10/3*d*(d*x+c)^(3/2)/b^2/(b*x+a)^
(1/2)+5*d^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}+\frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (5*d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/b^3 - (10*d*(c + d*x)^(3/2))/(3*b^2*Sqr
t[a + b*x]) - (2*(c + d*x)^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*d^(3/2)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}$$

$$= -\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{(5d^2)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b^2}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{(5d^2(bc-ad))\int \frac{\ldots}{\sqrt{a+bx}\ldots}}{2b^3}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{(5d^2(bc-ad))\,\mathrm{Subst}\left(\int \ldots\right)}{\ldots}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{(5d^2(bc-ad))\,\mathrm{Subst}\left(\int \frac{\ldots}{1-\ldots}\right)}{b^4}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right)}{b^{7/2}}$$

Mathematica [A]

time = 0.69, size = 121, normalized size = 0.95

$$\frac{\frac{\sqrt{c+dx}\,(15a^2d^2-10abd(c-2dx)+b^2(-2c^2-14cdx+3d^2x^2))}{(a+bx)^{3/2}}+\frac{15d(-bc+ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{\sqrt{\frac{b}{d}}}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] ((Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(-2*c^2 - 14*c*d*x
 + 3*d^2*x^2)))/(a + b*x)^(3/2) + (15*d*(-(b*c) + a*d)*Log[Sqrt[a + b*x] -
Sqrt[b/d]*Sqrt[c + d*x]])/Sqrt[b/d])/(3*b^3)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(100) = 200$.
time = 0.43, size = 475, normalized size = 3.71

$$\left[-\frac{15\left(a^{2}bcd-a^{3}d^{2}+\left(b^{3}cd-ab^{2}d^{2}\right)x^{2}+2\left(ab^{2}cd-a^{2}bd^{2}\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^{2}d^{2}x^{2}+b^{2}c^{2}+6abcd+a^{2}d^{2}-4\left(2b^{2}dx+b^{2}c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^{2}cd+abd^{2}\right)x\right)-4\left(3b^{2}d^{2}x^{2}-2b^{2}c^{2}-10abcd+15a^{2}d^{2}-2\left(7b^{2}cd-10abd^{2}\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{12\left(b^{5}x^{2}+2ab^{4}x+a^{2}b^{3}\right)}, -\frac{15\left(a^{2}bcd-a^{3}d^{2}+\left(b^{3}cd-ab^{2}d^{2}\right)x^{2}+2\left(ab^{2}cd-a^{2}bd^{2}\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^{2}x^{2}+acd+\left(bcd+ad^{2}\right)x\right)}\right)-2\left(3b^{2}d^{2}x^{2}-2b^{2}c^{2}-10abcd+15a^{2}d^{2}-2\left(7b^{2}cd-10abd^{2}\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(b^{5}x^{2}+2ab^{4}x+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^2)*x^2 + 2*(a*b^2*c*d
- a^2*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2
*d^2 - 2*(7*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/6*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^
2)*x^2 + 2*(a*b^2*c*d - a^2*b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c
+ a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d +
 a*d^2)*x)) - 2*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 2*(7
*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx\right)^{\frac{5}{2}}}{\left(a+bx\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(100) = 200$.
time = 0.07, size = 352, normalized size = 2.75

$$\frac{2\left(\left(-\frac{\left(-9b^{5}d^{5}c+9b^{4}d^{6}a\right)\sqrt{c+dx}\sqrt{c+dx}}{18b^{6}d|d|c-18b^{5}d^{2}|d|a}-\frac{60b^{5}d^{5}c^{2}-120b^{4}d^{6}ac+60b^{3}d^{7}a^{2}}{18b^{6}d|d|c-18b^{5}d^{2}|d|a}\right)\sqrt{c+dx}\sqrt{c+dx}-\frac{-45b^{5}d^{5}c^{3}+135b^{4}d^{6}ac^{2}-135b^{3}d^{7}a^{2}c+45b^{2}d^{8}a^{3}}{18b^{6}d|d|c-18b^{5}d^{2}|d|a}\right)\sqrt{c+dx}\sqrt{ad^{2}-bcd+bd\left(c+dx\right)}}{\left(ad^{2}-bcd+bd\left(c+dx\right)\right)^{2}}+\frac{2\left(5ad^{4}-5bcd^{3}\right)\ln\left|\sqrt{ad^{2}-bcd+bd\left(c+dx\right)}-\sqrt{bd}\sqrt{c+dx}\right|}{2b^{3}\sqrt{bd}\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] 1/3*((d*x + c)*(3*(b^5*c*d^5 - a*b^4*d^6)*(d*x + c)/(b^6*c*d*abs(d) - a*b^5
*d^2*abs(d)) - 20*(b^5*c^2*d^5 - 2*a*b^4*c*d^6 + a^2*b^3*d^7)/(b^6*c*d*abs(
```

```
d) - a*b^5*d^2*abs(d))) + 15*(b^5*c^3*d^5 - 3*a*b^4*c^2*d^6 + 3*a^2*b^3*c*d
^7 - a^3*b^2*d^8)/(b^6*c*d*abs(d) - a*b^5*d^2*abs(d)))*sqrt(d*x + c)/((d*x
+ c)*b*d - b*c*d + a*d^2)^(3/2) - 5*(b*c*d^3 - a*d^4)*log(abs(-sqrt(b*d)*sq
rt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^2)))/(sqrt(b*d)*b^3*abs(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(5/2), x)
```

3.1487 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}$$

```
[Out] -2/3*d*(d*x+c)^(3/2)/b^2/(b*x+a)^(3/2)-2/5*(d*x+c)^(5/2)/b/(b*x+a)^(5/2)+2*
d^(5/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)-2*d^2*
(d*x+c)^(1/2)/b^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 223, 212}

$$\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*d^2*Sqrt[c + d*x])/(b^3*Sqrt[a + b*x]) - (2*d*(c + d*x)^(3/2))/(3*b^2*(
a + b*x)^(3/2)) - (2*(c + d*x)^(5/2))/(5*b*(a + b*x)^(5/2)) + (2*d^(5/2)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx}{b}$$

$$= -\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^2\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b^2}$$

$$= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^3\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b^3}$$

$$= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx\right.}{b^4}$$

$$= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right.}{b^4}$$

$$= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}$$

Mathematica [A]
time = 0.13, size = 111, normalized size = 0.92

$$-\frac{2\sqrt{c+dx}\,\left(15a^2d^2+5abd(c+7dx)+b^2\left(3c^2+11cdx+23d^2x^2\right)\right)}{15b^3(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(3*c^2 + 11*c*d*x
 + 23*d^2*x^2)))/(15*b^3*(a + b*x)^(5/2)) + (2*d^(5/2)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(7/2)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(92) = 184.
time = 0.59, size = 463, normalized size = 3.86

$$\left[\frac{15\,(b^3d^2x^3+3\,ab^2d^2x^2+3\,a^2bd^2x+a^3d^2)\sqrt{\frac{d}{b}}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,(23\,b^2d^2x^2+3\,b^2c^2+5\,abcd+15\,a^2d^2+(11\,b^2cd+35\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{30\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)},\ -\frac{15\,(b^3d^2x^3+3\,ab^2d^2x^2+3\,a^2bd^2x+a^3d^2)\sqrt{-\frac{d}{b}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)+2\,(23\,b^2d^2x^2+3\,b^2c^2+5\,abcd+15\,a^2d^2+(11\,b^2cd+35\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{15\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x + a^3*d^2)*sqrt(d/
b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c
 + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x)
- 4*(23*b^2*d^2*x^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^
4*x + a^3*b^3), -1/15*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x +
a^3*d^2)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
 + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*(23*b^2*d^2*x
^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35*a*b*d^2)*x)*sqrt
(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(92) = 184.
time = 0.10, size = 421, normalized size = 3.51

$$\frac{2\left(\left(\left(-\frac{\left(345b^6d^6c^2-690b^5d^7ac+345b^4d^8a^2\right)\sqrt{c+dx}\,\sqrt{c+dx}}{225b^7|d|c^2-450b^6d|d|ac+225b^5d^2|d|a^2}-\frac{-525b^6d^6c^3+1575b^5d^7ac^2-1575b^4d^8a^2c+525b^3d^9a^3}{225b^7|d|c^2-450b^6d|d|ac+225b^5d^2|d|a^2}\right)\sqrt{c+dx}\,\sqrt{c+dx}-\frac{225b^6d^6c^4-900b^5d^7ac^3+1350b^4d^8a^2c^2-900b^3d^9a^3c+225b^2d^{10}a^4}{225b^7|d|c^2-450b^6d|d|ac+225b^5d^2|d|a^2}\right)\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\,(c+dx)}}{\left(ad^2-bcd+bd\,(c+dx)\right)^3}-\frac{2d^4\ln\left|\sqrt{ad^2-bcd+bd\,(c+dx)}-\sqrt{bd}\,\sqrt{c+dx}\right|}{b^3\sqrt{bd}\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

```
[Out] -2*d^4*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*c*d + a*d^
2)))/(sqrt(b*d)*b^3*abs(d)) - 2/15*((d*x + c)*(23*(b^6*c^2*d^6 - 2*a*b^5*c*
d^7 + a^2*b^4*d^8)*(d*x + c)/(b^7*c^2*abs(d) - 2*a*b^6*c*d*abs(d) + a^2*b^5
*d^2*abs(d)) - 35*(b^6*c^3*d^6 - 3*a*b^5*c^2*d^7 + 3*a^2*b^4*c*d^8 - a^3*b^
3*d^9)/(b^7*c^2*abs(d) - 2*a*b^6*c*d*abs(d) + a^2*b^5*d^2*abs(d))) + 15*(b^
6*c^4*d^6 - 4*a*b^5*c^3*d^7 + 6*a^2*b^4*c^2*d^8 - 4*a^3*b^3*c*d^9 + a^4*b^2
*d^10)/(b^7*c^2*abs(d) - 2*a*b^6*c*d*abs(d) + a^2*b^5*d^2*abs(d)))*sqrt(d*x
 + c)/((d*x + c)*b*d - b*c*d + a*d^2)^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(7/2), x)
```

3.1488 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}} dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

```
[Out] -2/7*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}} dx = -\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]')
```

```
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(26) = 52$.
time = 0.16, size = 234, normalized size = 7.31

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}}{7(bx+a)^{\frac{7}{2}}(ad-bc)}$

default	$-\frac{(dx+c)^{\frac{5}{2}}}{b(bx+a)^{\frac{7}{2}}}+\frac{5(ad-bc)\left(-\frac{(dx+c)^{\frac{3}{2}}}{2b(bx+a)^{\frac{7}{2}}}+\frac{3(ad-bc)\left(-\frac{\sqrt{dx+c}}{3b(bx+a)^{\frac{7}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{6b}\right.}{4b}\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(d*x+c)^(5/2)/(b*x+a)^(7/2)+5/2*(a*d-b*c)/b*(-1/2/b*(d*x+c)^(3/2)/(b*x
+a)^(7/2)+3/4*(a*d-b*c)/b*(-1/3/b*(d*x+c)^(1/2)/(b*x+a)^(7/2)+1/6*(a*d-b*c)
/b*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x
+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a
*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(26) = 52$.
time = 0.80, size = 138, normalized size = 4.31

$$-\frac{2\,(d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3)\sqrt{bx+a}\,\sqrt{dx+c}}{7\,(a^4bc - a^5d + (b^5c - ab^4d)x^4 + 4\,(ab^4c - a^2b^3d)x^3 + 6\,(a^2b^3c - a^3b^2d)x^2 + 4\,(a^3b^2c - a^4bd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(b*x + a)*sqrt(d*x + c)/
(a^4*b*c - a^5*d + (b^5*c - a*b^4*d)*x^4 + 4*(a*b^4*c - a^2*b^3*d)*x^3 + 6*
(a^2*b^3*c - a^3*b^2*d)*x^2 + 4*(a^3*b^2*c - a^4*b*d)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.
time = 0.16, size = 199, normalized size = 6.22

$$-\frac{2\,(-525b^5d^8c^2 + 1050b^4d^9ac - 525b^3d^{10}a^2)\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{ad^2 - bcd + bd\,(c+dx)}}{(-3675b^6c^3\,|d| + 11025b^5dac^2\,|d| - 11025b^4d^2a^2c\,|d| + 3675b^3d^3a^3\,|d|)\,(ad^2 - bcd + bd\,(c+dx))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x)
```

```
[Out] -2/7*(b^5*c^2*d^8 - 2*a*b^4*c*d^9 + a^2*b^3*d^10)*(d*x + c)^(7/2)/((b^6*c^3
*abs(d) - 3*a*b^5*c^2*d*abs(d) + 3*a^2*b^4*c*d^2*abs(d) - a^3*b^3*d^3*abs(d
))*((d*x + c)*b*d - b*c*d + a*d^2)^(7/2))
```

Mupad [B]
time = 0.97, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{\left(7\,a\,d-7\,b\,c\right)\,{\left(a+b\,x\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(9/2),x)
[Out] (2*(c + d*x)^(7/2))/((7*a*d - 7*b*c)*(a + b*x)^(7/2))
```

3.1489 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}$$

```
[Out] -2/9*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(9/2)+4/63*d*(d*x+c)^(7/2)/(-a*d+b*c)
^2/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(7/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}$$

Mathematica [A]
time = 0.10, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{7/2}(-7bc+9ad+2bdx)}{63(bc-ad)^2(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-7*b*c + 9*a*d + 2*b*d*x))/(63*(b*c - a*d)^2*(a + b*x)^
(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]')
```

```
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(54) = 108.
time = 0.16, size = 274, normalized size = 4.15

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(2bdx+9ad-7bc)}{63(bx+a)^{\frac{9}{2}}(a^2d^2-2abcd+b^2c^2)}$

$$5(ad-bc)\left(-\frac{(dx+c)^{\frac{3}{2}}}{3b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{\sqrt{dx+c}}{4b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*x+c)^(5/2)/(b*x+a)^(9/2)+5/4*(a*d-b*c)/b*(-1/3/b*(d*x+c)^(3/2)/(b
*x+a)^(9/2)+1/2*(a*d-b*c)/b*(-1/4/b*(d*x+c)^(1/2)/(b*x+a)^(9/2)+1/8*(a*d-b*
c)/b*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d
*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(
-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*
x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(54) = 108$.
time = 2.70, size = 295, normalized size = 4.47

$$\frac{2\,{\left(2\,bd^{4}x^{4} - 7\,bc^{4} + 9\,ac^{3}d - {\left(bcd^{3} - 9\,ad^{4}\right)}x^{3} - 3\,{\left(5\,bc^{2}d^{2} - 9\,acd^{3}\right)}x^{2} - {\left(19\,bc^{3}d - 27\,ac^{2}d^{2}\right)}x\right)}\sqrt{bx + a}\,\sqrt{dx + c}}{63\,{\left(a^{5}b^{2}c^{2} - 2\,a^{6}bcd + a^{7}d^{2} + {\left(b^{7}c^{2} - 2\,ab^{6}cd + a^{2}b^{5}d^{2}\right)}x^{5} + 5\,{\left(ab^{6}c^{2} - 2\,a^{2}b^{5}cd + a^{3}b^{4}d^{2}\right)}x^{4} + 10\,{\left(a^{2}b^{5}c^{2} - 2\,a^{3}b^{4}cd + a^{4}b^{3}d^{2}\right)}x^{3} + 10\,{\left(a^{3}b^{4}c^{2} - 2\,a^{4}b^{3}cd + a^{5}b^{2}d^{2}\right)}x^{2} + 5\,{\left(a^{4}b^{3}c^{2} - 2\,a^{5}b^{2}cd + a^{6}bd^{2}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b*d^4*x^4 - 7*b*c^4 + 9*a*c^3*d - (b*c*d^3 - 9*a*d^4)*x^3 - 3*(5*b*
c^2*d^2 - 9*a*c*d^3)*x^2 - (19*b*c^3*d - 27*a*c^2*d^2)*x)*sqrt(b*x + a)*sqr
t(d*x + c)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (b^7*c^2 - 2*a*b^6*c*d +
a^2*b^5*d^2)*x^5 + 5*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*x^4 + 10*(a^
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*x^3 + 10*(a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*x^2 + 5*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(54) = 108$.
time = 0.24, size = 355, normalized size = 5.38

$$\frac{2\left(-\frac{\left(-28350b^7d^{10}c^2+56700b^6d^{11}ac-28350b^5d^{12}a^2\right)\sqrt{c+dx}\,\sqrt{c+dx}}{893025b^8c^4|d|-3572100b^7dac^3|d|+5358150b^6d^2a^2c^2|d|-3572100b^5d^3a^3c|d|+893025b^4d^4a^4|d|}-\frac{127575b^7d^{10}c^3-382725b^6d^{11}ac^2+382725b^5d^{12}a^2c-127575b^4d^{13}a^3}{893025b^8c^4|d|-3572100b^7dac^3|d|+5358150b^6d^2a^2c^2|d|-3572100b^5d^3a^3c|d|+893025b^4d^4a^4|d|}\right)\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{c+dx}\,\sqrt{ad^2-bcd+bd\left(c+dx\right)}}{\left(ad^2-bcd+bd\left(c+dx\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/63*(d*x + c)^(7/2)*(2*(b^7*c^2*d^10 - 2*a*b^6*c*d^11 + a^2*b^5*d^12)*(d*x
 + c)/(b^8*c^4*abs(d) - 4*a*b^7*c^3*d*abs(d) + 6*a^2*b^6*c^2*d^2*abs(d) - 4
*a^3*b^5*c*d^3*abs(d) + a^4*b^4*d^4*abs(d)) - 9*(b^7*c^3*d^10 - 3*a*b^6*c^2
*d^11 + 3*a^2*b^5*c*d^12 - a^3*b^4*d^13)/(b^8*c^4*abs(d) - 4*a*b^7*c^3*d*ab
s(d) + 6*a^2*b^6*c^2*d^2*abs(d) - 4*a^3*b^5*c*d^3*abs(d) + a^4*b^4*d^4*abs(
d)))/((d*x + c)*b*d - b*c*d + a*d^2)^(9/2)
```

Mupad [B]
time = 1.14, size = 229, normalized size = 3.47

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^4\,x^4}{63\,b^3\,{\left(a\,d-b\,c\right)}^2}-\frac{14\,b\,c^4-18\,a\,c^3\,d}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{x^3\,\left(18\,a\,d^4-2\,b\,c\,d^3\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c^2\,d\,x\,\left(27\,a\,d-19\,b\,c\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c\,d^2\,x^2\,\left(9\,a\,d-5\,b\,c\right)}{21\,b^4\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^4*x^4)/(63*b^3*(a*d - b*c)^2) - (14*b*c^4 - 18*a*c^3
*d)/(63*b^4*(a*d - b*c)^2) + (x^3*(18*a*d^4 - 2*b*c*d^3))/(63*b^4*(a*d - b*
c)^2) + (2*c^2*d*x*(27*a*d - 19*b*c))/(63*b^4*(a*d - b*c)^2) + (2*c*d^2*x^2
*(9*a*d - 5*b*c))/(21*b^4*(a*d - b*c)^2)))/(x^4*(a + b*x)^(1/2) + (a^4*(a +
 b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/
2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

3.1490 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}} dx$

Optimal. Leaf size=101

$$-\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}$$

```
[Out] -2/11*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(11/2)+8/99*d*(d*x+c)^(7/2)/(-a*d+b*
c)^2/(b*x+a)^(9/2)-16/693*d^2*(d*x+c)^(7/2)/(-a*d+b*c)^3/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (8*d*(c + d*x)^(7/
2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(7/2))/(693*(b*c
 - a*d)^3*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(4d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{(8d^2)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{99(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}$$

Mathematica [**A**]
time = 0.12, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{7/2}\left(99d^2-\frac{154bd(c+dx)}{a+bx}+\frac{63b^2(c+dx)^2}{(a+bx)^2}\right)}{693(bc-ad)^3(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
[Out] (-2*(c + d*x)^(7/2)*(99*d^2 - (154*b*d*(c + d*x))/(a + b*x) + (63*b^2*(c + d*x)^2)/(a + b*x)^2))/(693*(b*c - a*d)^3*(a + b*x)^(7/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]')
[Out] Timed out
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(83) = 166.
time = 0.16, size = 314, normalized size = 3.11

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}\left(8b^2x^2d^2+44ab\,d^2x-28b^2cdx+99a^2d^2-154abcd+63b^2c^2\right)}{693(bx+a)^{\frac{11}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$

$$10d\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}-\right.$$

$$3(ad-bc)\left(-\frac{\sqrt{dx+c}}{5b(bx+a)^{\frac{11}{2}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(d*x+c)^(5/2)/(b*x+a)^(11/2)+5/6*(a*d-b*c)/b*(-1/4/b*(d*x+c)^(3/2)/(
b*x+a)^(11/2)+3/8*(a*d-b*c)/b*(-1/5/b*(d*x+c)^(1/2)/(b*x+a)^(11/2)+1/10*(a*
d-b*c)/b*(-2/11/(-a*d+b*c)/(b*x+a)^(11/2)*(d*x+c)^(1/2)-10/11*d/(-a*d+b*c)*
(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)
^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+
b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^
(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(83) = 166$.
time = 6.03, size = 513, normalized size = 5.08

$$-\frac{2\,{\left(8\,b^{2}d^{5}x^{5}+63\,b^{2}c^{5}-154\,abc^{4}d+99\,a^{2}c^{3}d^{2}-4\,{\left(b^{2}cd^{4}-11\,abd^{5}\right)}x^{4}+{\left(3\,b^{2}c^{2}d^{3}-22\,abcd^{4}+99\,a^{2}d^{5}\right)}x^{3}+{\left(113\,b^{2}c^{3}d^{2}-330\,abc^{2}d^{3}+297\,a^{2}cd^{4}\right)}x^{2}+{\left(161\,b^{2}c^{4}d-418\,abc^{3}d^{2}+297\,a^{2}c^{2}d^{3}\right)}x\right)}\sqrt{bx+a}\,\sqrt{dx+c}}{693\,{\left(a^{6}b^{3}c^{3}-3\,a^{7}b^{2}c^{2}d+3\,a^{8}bcd^{2}-a^{9}d^{3}+{\left(b^{9}c^{3}-3\,ab^{8}c^{2}d+3\,a^{2}b^{7}cd^{2}-a^{3}b^{6}d^{3}\right)}x^{6}+6\,{\left(ab^{8}c^{3}-3\,a^{2}b^{7}c^{2}d+3\,a^{3}b^{6}cd^{2}-a^{4}b^{5}d^{3}\right)}x^{5}+15\,{\left(a^{2}b^{7}c^{3}-3\,a^{3}b^{6}c^{2}d+3\,a^{4}b^{5}cd^{2}-a^{5}b^{4}d^{3}\right)}x^{4}+20\,{\left(a^{3}b^{6}c^{3}-3\,a^{4}b^{5}c^{2}d+3\,a^{5}b^{4}cd^{2}-a^{6}b^{3}d^{3}\right)}x^{3}+15\,{\left(a^{4}b^{5}c^{3}-3\,a^{5}b^{4}c^{2}d+3\,a^{6}b^{3}cd^{2}-a^{7}b^{2}d^{3}\right)}x^{2}+6\,{\left(a^{5}b^{4}c^{3}-3\,a^{6}b^{3}c^{2}d+3\,a^{7}b^{2}cd^{2}-a^{8}bd^{3}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^2*d^5*x^5 + 63*b^2*c^5 - 154*a*b*c^4*d + 99*a^2*c^3*d^2 - 4*(b^
2*c*d^4 - 11*a*b*d^5)*x^4 + (3*b^2*c^2*d^3 - 22*a*b*c*d^4 + 99*a^2*d^5)*x^3
 + (113*b^2*c^3*d^2 - 330*a*b*c^2*d^3 + 297*a^2*c*d^4)*x^2 + (161*b^2*c^4*d
 - 418*a*b*c^3*d^2 + 297*a^2*c^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b
^3*c^3 - 3*a^7*b^2*c^2*d + 3*a^8*b*c*d^2 - a^9*d^3 + (b^9*c^3 - 3*a*b^8*c^2
*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*x^6 + 6*(a*b^8*c^3 - 3*a^2*b^7*c^2*d +
3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*x^5 + 15*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*x^4 + 20*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2 - a^6*b^3*d^3)*x^3 + 15*(a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*
b^3*c*d^2 - a^7*b^2*d^3)*x^2 + 6*(a^5*b^4*c^3 - 3*a^6*b^3*c^2*d + 3*a^7*b^2
*c*d^2 - a^8*b*d^3)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(83) = 166.
time = 0.31, size = 561, normalized size = 5.55

$$\frac{2\left(\left(\left(-\frac{\left(-1247400 b^{9} d^{12} c^{2}+2494800 b^{8} d^{13} a c-1247400 b^{7} d^{14} a^{2}\right) \sqrt{c+d x} \sqrt{c+d x}}{-108056025 b^{10} c^{5}|d|+540280125 b^{9} d a c^{4}|d|-1080560250 b^{8} d^{2} a^{2} c^{3}|d|+1080560250 b^{7} d^{3} a^{3} c^{2}|d|-540280125 b^{6} d^{4} a^{4} c|d|+108056025 b^{5} d^{5} a^{5}|d|}-\frac{6860700 b^{9} d^{12} c^{3}-20582100 b^{8} d^{13} a c^{2}+20582100 b^{7} d^{14} a^{2} c-6860700 b^{6} d^{15} a^{3}}{-108056025 b^{10} c^{5}|d|+540280125 b^{9} d a c^{4}|d|-1080560250 b^{8} d^{2} a^{2} c^{3}|d|+1080560250 b^{7} d^{3} a^{3} c^{2}|d|-540280125 b^{6} d^{4} a^{4} c|d|+108056025 b^{5} d^{5} a^{5}|d|}\right) \sqrt{c+d x} \sqrt{c+d x}-\frac{-15436575 b^{9} d^{12} c^{4}+61746300 b^{8} d^{13} a c^{3}-92619450 b^{7} d^{14} a^{2} c^{2}+61746300 b^{6} d^{15} a^{3} c-15436575 b^{5} d^{16} a^{4}}{-108056025 b^{10} c^{5}|d|+540280125 b^{9} d a c^{4}|d|-1080560250 b^{8} d^{2} a^{2} c^{3}|d|+1080560250 b^{7} d^{3} a^{3} c^{2}|d|-540280125 b^{6} d^{4} a^{4} c|d|+108056025 b^{5} d^{5} a^{5}|d|}\right) \sqrt{c+d x} \sqrt{c+d x} \sqrt{c+d x} \sqrt{c+d x} \sqrt{c+d x} \sqrt{c+d x} \sqrt{c+d x} \sqrt{a d^{2}-b c d+b d(c+d x)}\right)}{\left(a d^{2}-b c d+b d(c+d x)\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x)
```

```
[Out] -2/693*(4*(d*x + c)*(2*(b^9*c^2*d^12 - 2*a*b^8*c*d^13 + a^2*b^7*d^14)*(d*x
+ c)/(b^10*c^5*abs(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) -
10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)) -
11*(b^9*c^3*d^12 - 3*a*b^8*c^2*d^13 + 3*a^2*b^7*c*d^14 - a^3*b^6*d^15)/(b^1
0*c^5*abs(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d) - 10*a^3*b^
7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d))) + 99*(b^9*
c^4*d^12 - 4*a*b^8*c^3*d^13 + 6*a^2*b^7*c^2*d^14 - 4*a^3*b^6*c*d^15 + a^4*b
^5*d^16)/(b^10*c^5*abs(d) - 5*a*b^9*c^4*d*abs(d) + 10*a^2*b^8*c^3*d^2*abs(d
) - 10*a^3*b^7*c^2*d^3*abs(d) + 5*a^4*b^6*c*d^4*abs(d) - a^5*b^5*d^5*abs(d)
))*(d*x + c)^(7/2)/((d*x + c)*b*d - b*c*d + a*d^2)^(11/2)
```

Mupad [B]
time = 1.35, size = 333, normalized size = 3.30

$$\frac{\sqrt{c+d\,x}\,\left(\frac{198\,a^2\,c^3\,d^2-308\,a\,b\,c^4\,d+126\,b^2\,c^5}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{x^3\,\left(198\,a^2\,d^5-44\,a\,b\,c\,d^4+6\,b^2\,c^2\,d^3\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^5\,x^5}{693\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{693\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c\,d^2\,x^2\,\left(297\,a^2\,d^2-330\,a\,b\,c\,d+113\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c^2\,d\,x\,\left(297\,a^2\,d^2-418\,a\,b\,c\,d+161\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((126*b^2*c^5 + 198*a^2*c^3*d^2 - 308*a*b*c^4*d)/(693*b^5*
(a*d - b*c)^3) + (x^3*(198*a^2*d^5 + 6*b^2*c^2*d^3 - 44*a*b*c*d^4))/(693*b^
5*(a*d - b*c)^3) + (16*d^5*x^5)/(693*b^3*(a*d - b*c)^3) + (8*d^4*x^4*(11*a*
d - b*c))/(693*b^4*(a*d - b*c)^3) + (2*c*d^2*x^2*(297*a^2*d^2 + 113*b^2*c^2
 - 330*a*b*c*d))/(693*b^5*(a*d - b*c)^3) + (2*c^2*d*x*(297*a^2*d^2 + 161*b^
2*c^2 - 418*a*b*c*d))/(693*b^5*(a*d - b*c)^3)))/(x^5*(a + b*x)^(1/2) + (a^5
*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a +
 b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/
b^4)
```

3.1491 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}} dx$

Optimal. Leaf size=136

$$-\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}}+\frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}}-\frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}}+\frac{32d^3(c+dx)^{7/2}}{3003(bc-ad)^4(a+bx)^{7/2}}$$

```
[Out] -2/13*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(13/2)+12/143*d*(d*x+c)^(7/2)/(-a*d+
b*c)^2/(b*x+a)^(11/2)-16/429*d^2*(d*x+c)^(7/2)/(-a*d+b*c)^3/(b*x+a)^(9/2)+3
2/3003*d^3*(d*x+c)^(7/2)/(-a*d+b*c)^4/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(13*(b*c - a*d)*(a + b*x)^(13/2)) + (12*d*(c + d*x)^(7
/2))/(143*(b*c - a*d)^2*(a + b*x)^(11/2)) - (16*d^2*(c + d*x)^(7/2))/(429*(
b*c - a*d)^3*(a + b*x)^(9/2)) + (32*d^3*(c + d*x)^(7/2))/(3003*(b*c - a*d)^
4*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx = -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} - \frac{(6d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx}{13(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} + \frac{(24d^2)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{143(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} - \frac{(1}{}$$
$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} + \frac{}{30}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.70

$$-\frac{2(c+dx)^{7/2}\left(-429d^3+\frac{1001bd^2(c+dx)}{a+bx}-\frac{819b^2d(c+dx)^2}{(a+bx)^2}+\frac{231b^3(c+dx)^3}{(a+bx)^3}\right)}{3003(bc-ad)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(-429*d^3 + (1001*b*d^2*(c + d*x))/(a + b*x) - (819*b^2
*d*(c + d*x)^2)/(a + b*x)^2 + (231*b^3*(c + d*x)^3)/(a + b*x)^3))/(3003*(b*
c - a*d)^4*(a + b*x)^(7/2))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(112) = 224.
time = 0.17, size = 354, normalized size = 2.60

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}\left(16b^3x^3d^3+104d^3a\,x^2b^2-56b^3c\,d^2x^2+286a^2b\,d^3x-364a\,b^2c\,d^2x+126b^3c^2dx+429a^3d^3-1001a^2bc\,d^2+819a\,b^2c^2d-231b^3c^3\right)}{3003(bx+a)^{\frac{13}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

$$12d\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{13(-ad+bc)(bx+a)^{\frac{13}{2}}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(d*x+c)^(5/2)/(b*x+a)^(13/2)+5/8*(a*d-b*c)/b*(-1/5/b*(d*x+c)^(3/2)/(
b*x+a)^(13/2)+3/10*(a*d-b*c)/b*(-1/6/b*(d*x+c)^(1/2)/(b*x+a)^(13/2)+1/12*(a
*d-b*c)/b*(-2/13/(-a*d+b*c)/(b*x+a)^(13/2)*(d*x+c)^(1/2)-12/13*d/(-a*d+b*c)
*(-2/11/(-a*d+b*c)/(b*x+a)^(11/2)*(d*x+c)^(1/2)-10/11*d/(-a*d+b*c)*(-2/9*(d
*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(
-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*
x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/
3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(112) = 224$.
time = 12.24, size = 765, normalized size = 5.62

$$\frac{2\,(16\,b^3d^6x^6 - 231\,b^3c^6 + 819\,ab^2c^5d - 1001\,a^2bc^4d^2 + 429\,a^3c^3d^3 - 8\,(b^3cd^5 - 13\,ab^2d^6)x^5 + 2\,(3\,b^3c^2d^4 - 26\,ab^2cd^5 + 143\,a^2bd^6)x^4 - (5\,b^3c^3d^3 - 39\,ab^2c^2d^4 + 143\,a^2bcd^5 - 429\,a^3d^6)x^3 - (371\,b^3c^4d^2 - 1469\,ab^2c^3d^3 + 2145\,a^2bc^2d^4 - 1287\,a^3cd^5)x^2 - (567\,b^3c^5d - 2093\,ab^2c^4d^2 + 2717\,a^2bc^3d^3 - 1287\,a^3c^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{3003\,(a^7b^4c^4 - 4\,a^8b^3c^3d + 6\,a^9b^2c^2d^2 - 4\,a^{10}bcd^3 + a^{11}d^4 + (b^{11}c^4 - 4\,ab^{10}c^3d + 6\,a^2b^9c^2d^2 - 4\,a^3b^8cd^3 + a^4b^7d^4)x^7 + 7\,(ab^{10}c^4 - 4\,a^2b^9c^3d + 6\,a^3b^8c^2d^2 - 4\,a^4b^7cd^3 + a^5b^6d^4)x^6 + 21\,(a^2b^9c^4 - 4\,a^3b^8c^3d + 6\,a^4b^7c^2d^2 - 4\,a^5b^6cd^3 + a^6b^5d^4)x^5 + 35\,(a^3b^8c^4 - 4\,a^4b^7c^3d + 6\,a^5b^6c^2d^2 - 4\,a^6b^5cd^3 + a^7b^4d^4)x^4 + 35\,(a^4b^7c^4 - 4\,a^5b^6c^3d + 6\,a^6b^5c^2d^2 - 4\,a^7b^4cd^3 + a^8b^3d^4)x^3 + 21\,(a^5b^6c^4 - 4\,a^6b^5c^3d + 6\,a^7b^4c^2d^2 - 4\,a^8b^3cd^3 + a^9b^2d^4)x^2 + 7\,(a^6b^5c^4 - 4\,a^7b^4c^3d + 6\,a^8b^3c^2d^2 - 4\,a^9b^2cd^3 + a^{10}bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^3*d^6*x^6 - 231*b^3*c^6 + 819*a*b^2*c^5*d - 1001*a^2*b*c^4*d^2
 + 429*a^3*c^3*d^3 - 8*(b^3*c*d^5 - 13*a*b^2*d^6)*x^5 + 2*(3*b^3*c^2*d^4 -
26*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 - (5*b^3*c^3*d^3 - 39*a*b^2*c^2*d^4 + 1
43*a^2*b*c*d^5 - 429*a^3*d^6)*x^3 - (371*b^3*c^4*d^2 - 1469*a*b^2*c^3*d^3 +
 2145*a^2*b*c^2*d^4 - 1287*a^3*c*d^5)*x^2 - (567*b^3*c^5*d - 2093*a*b^2*c^4
*d^2 + 2717*a^2*b*c^3*d^3 - 1287*a^3*c^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
)/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^1
1*d^4 + (b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 +
a^4*b^7*d^4)*x^7 + 7*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*
```

```
a^4*b^7*c*d^3 + a^5*b^6*d^4)*x^6 + 21*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^
4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*x^5 + 35*(a^3*b^8*c^4 - 4*a^
4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*x^4 + 35*(
a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b
^3*d^4)*x^3 + 21*(a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8
*b^3*c*d^3 + a^9*b^2*d^4)*x^2 + 7*(a^6*b^5*c^4 - 4*a^7*b^4*c^3*d + 6*a^8*b^
3*c^2*d^2 - 4*a^9*b^2*c*d^3 + a^10*b*d^4)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(15/2),x)
[Out] Exception raised: SystemError
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(112) = 224.
time = 0.48, size = 811, normalized size = 5.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x)
[Out] 2/3003*(2*(4*(d*x + c)*(2*(b^11*c^2*d^14 - 2*a*b^10*c*d^15 + a^2*b^9*d^16)*
(d*x + c)/(b^12*c^6*abs(d) - 6*a*b^11*c^5*d*abs(d) + 15*a^2*b^10*c^4*d^2*ab
s(d) - 20*a^3*b^9*c^3*d^3*abs(d) + 15*a^4*b^8*c^2*d^4*abs(d) - 6*a^5*b^7*c*
d^5*abs(d) + a^6*b^6*d^6*abs(d)) - 13*(b^11*c^3*d^14 - 3*a*b^10*c^2*d^15 +
3*a^2*b^9*c*d^16 - a^3*b^8*d^17)/(b^12*c^6*abs(d) - 6*a*b^11*c^5*d*abs(d) +
 15*a^2*b^10*c^4*d^2*abs(d) - 20*a^3*b^9*c^3*d^3*abs(d) + 15*a^4*b^8*c^2*d^
4*abs(d) - 6*a^5*b^7*c*d^5*abs(d) + a^6*b^6*d^6*abs(d))) + 143*(b^11*c^4*d^
14 - 4*a*b^10*c^3*d^15 + 6*a^2*b^9*c^2*d^16 - 4*a^3*b^8*c*d^17 + a^4*b^7*d^
18)/(b^12*c^6*abs(d) - 6*a*b^11*c^5*d*abs(d) + 15*a^2*b^10*c^4*d^2*abs(d) -
 20*a^3*b^9*c^3*d^3*abs(d) + 15*a^4*b^8*c^2*d^4*abs(d) - 6*a^5*b^7*c*d^5*ab
s(d) + a^6*b^6*d^6*abs(d)))*(d*x + c) - 429*(b^11*c^5*d^14 - 5*a*b^10*c^4*d
^15 + 10*a^2*b^9*c^3*d^16 - 10*a^3*b^8*c^2*d^17 + 5*a^4*b^7*c*d^18 - a^5*b^
6*d^19)/(b^12*c^6*abs(d) - 6*a*b^11*c^5*d*abs(d) + 15*a^2*b^10*c^4*d^2*abs(
d) - 20*a^3*b^9*c^3*d^3*abs(d) + 15*a^4*b^8*c^2*d^4*abs(d) - 6*a^5*b^7*c*d^
5*abs(d) + a^6*b^6*d^6*abs(d)))*(d*x + c)^(7/2)/((d*x + c)*b*d - b*c*d + a*
d^2)^(13/2)
```

Mupad [**B**]
time = 1.62, size = 459, normalized size = 3.38

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(2574\,a^3\,c\,d^5-4290\,a^2\,b\,c^2\,d^4+2938\,a\,b^2\,c^3\,d^3-742\,b^3\,c^4\,d^2\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}-\frac{-858\,a^3\,c^3\,d^3+2002\,a^2\,b\,c^4\,d^2-1638\,a\,b^2\,c^5\,d+462\,b^3\,c^6}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{x^3\,\left(858\,a^3\,d^6-286\,a^2\,b\,c\,d^5+78\,a\,b^2\,c^2\,d^4-10\,b^3\,c^3\,d^3\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^6\,x^6}{3003\,b^3\,{\left(a\,d-b\,c\right)}^4}-\frac{x\,\left(-2574\,a^3\,c^2\,d^4+5434\,a^2\,b\,c^3\,d^3-4186\,a\,b^2\,c^4\,d^2+1134\,b^3\,c^5\,d\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^5\,x^5\,\left(13\,a\,d-b\,c\right)}{3003\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^4\,x^4\,\left(143\,a^2\,d^2-26\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3003\,b^5\,{\left(a\,d-b\,c\right)}^4}\right)}{x^6\,\sqrt{a+b\,x}+\frac{a^6\,\sqrt{a+b\,x}}{b^6}+\frac{15\,a^2\,x^4\,\sqrt{a+b\,x}}{b^2}+\frac{20\,a^3\,x^3\,\sqrt{a+b\,x}}{b^3}+\frac{15\,a^4\,x^2\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a\,x^5\,\sqrt{a+b\,x}}{b}+\frac{6\,a^5\,x\,\sqrt{a+b\,x}}{b^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(15/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(2574*a^3*c*d^5 - 742*b^3*c^4*d^2 + 2938*a*b^2*c^3*d
^3 - 4290*a^2*b*c^2*d^4))/(3003*b^6*(a*d - b*c)^4) - (462*b^3*c^6 - 858*a^3
*c^3*d^3 + 2002*a^2*b*c^4*d^2 - 1638*a*b^2*c^5*d)/(3003*b^6*(a*d - b*c)^4)
+ (x^3*(858*a^3*d^6 - 10*b^3*c^3*d^3 + 78*a*b^2*c^2*d^4 - 286*a^2*b*c*d^5))
/(3003*b^6*(a*d - b*c)^4) + (32*d^6*x^6)/(3003*b^3*(a*d - b*c)^4) - (x*(113
4*b^3*c^5*d - 2574*a^3*c^2*d^4 - 4186*a*b^2*c^4*d^2 + 5434*a^2*b*c^3*d^3))/
(3003*b^6*(a*d - b*c)^4) + (16*d^5*x^5*(13*a*d - b*c))/(3003*b^4*(a*d - b*c
)^4) + (4*d^4*x^4*(143*a^2*d^2 + 3*b^2*c^2 - 26*a*b*c*d))/(3003*b^5*(a*d -
b*c)^4)))/(x^6*(a + b*x)^(1/2) + (a^6*(a + b*x)^(1/2))/b^6 + (15*a^2*x^4*(a
 + b*x)^(1/2))/b^2 + (20*a^3*x^3*(a + b*x)^(1/2))/b^3 + (15*a^4*x^2*(a + b*
x)^(1/2))/b^4 + (6*a*x^5*(a + b*x)^(1/2))/b + (6*a^5*x*(a + b*x)^(1/2))/b^5
)
```

3.1492 $\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=183

$$-\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4}+\frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3}-\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2}+\frac{(a$$

```
[Out] 35/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(
9/2)/b^(1/2)+35/96*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^3-7/24*(-a*d+
b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d^2+1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d-35/6
4*(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A]

time = 0.07, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64\sqrt{b}\,d^{9/2}}-\frac{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24d^2}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (-35*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*d^4) + (35*(b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*d^3) - (7*(b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(24*d^2) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (35*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*Sqrt
[b]*d^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} + \frac{(35(bc-ad)^2)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,d}{48d^2}$$

$$= \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+}}{4d}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+}{24}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+}{24}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+}{24}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+}{24}$$

Mathematica [A]
time = 0.19, size = 164, normalized size = 0.90

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(279a^3d^3+a^2bd^2(-511c+326dx)+ab^2d\left(385c^2-252cdx+200d^2x^2\right)+b^3\left(-105c^3+70c^2dx-56cd^2x^2+48d^3x^3\right)\right)}{192d^4}+\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64\sqrt{b}\,d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(279*a^3*d^3 + a^2*b*d^2*(-511*c + 326*d*x) +
a*b^2*d*(385*c^2 - 252*c*d*x + 200*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*x -
56*c*d^2*x^2 + 48*d^3*x^3)))/(192*d^4) + (35*(b*c - a*d)^4*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*Sqrt[b]*d^(9/2))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/2)/(c + d*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.16, size = 206, normalized size = 1.13

method	result
default	$\frac{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}{4d} - \frac{7(-ad+bc)\left(\frac{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}{3d} - \frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\sqrt{dx+\ldots}}{d}\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d-7/8*(-a*d+b*c)/d*(1/3*(b*x+a)^(5/2)*(d*x+
c)^(1/2)/d-5/6*(-a*d+b*c)/d*(1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d-3/4*(-a*d+b*
c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2
)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x
^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.33, size = 542, normalized size = 2.96

$$\left[\frac{105\,{\left(b^{4} c^{4} - 4\,a b^{3} c^{3} d + 6\,a^{2} b^{2} c^{2} d^{2} - 4\,a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{b d} \log\left(8\,b^{2} d^{2} x^{2} + b^{2} c^{2} + 6\,a b c d + a^{2} d^{2} + 4\,{\left(2\,b d x + b c + a d\right)} \sqrt{b d} \sqrt{b x + a} \sqrt{d x + c} + 8\,{\left(b^{2} c d + a b d^{2}\right)} x\right) + 4\,{\left(48\,b^{4} d^{4} x^{3} - 105\,b^{4} c^{3} d + 385\,a b^{3} c^{2} d^{2} - 511\,a^{2} b^{2} c d^{3} + 279\,a^{3} b d^{4} - 8\,{\left(7\,b^{4} c d^{3} - 25\,a b^{3} d^{4}\right)} x^{2} + 2\,{\left(35\,b^{4} c^{2} d^{2} - 126\,a b^{3} c d^{3} + 163\,a^{2} b^{2} d^{4}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{768\,b d^{5}}, -\frac{105\,{\left(b^{4} c^{4} - 4\,a b^{3} c^{3} d + 6\,a^{2} b^{2} c^{2} d^{2} - 4\,a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{-b d} \arctan\left(\frac{{\left(2\,b d x + b c + a d\right)} \sqrt{-b d} \sqrt{b x + a} \sqrt{d x + c}}{2\,{\left(b^{2} d^{2} x^{2} + a b c d + {\left(b^{2} c d + a b d^{2}\right)} x\right)}}\right) - 2\,{\left(48\,b^{4} d^{4} x^{3} - 105\,b^{4} c^{3} d + 385\,a b^{3} c^{2} d^{2} - 511\,a^{2} b^{2} c d^{3} + 279\,a^{3} b d^{4} - 8\,{\left(7\,b^{4} c d^{3} - 25\,a b^{3} d^{4}\right)} x^{2} + 2\,{\left(35\,b^{4} c^{2} d^{2} - 126\,a b^{3} c d^{3} + 163\,a^{2} b^{2} d^{4}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{384\,b d^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2
*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*
b*d^2)*x) + 4*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2
*b^2*c*d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4
*c^2*d^2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^5), -1/384*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)
*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)) - 2*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2*b^2*c*
d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4*c^2*d^
2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d
^5)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.02, size = 353, normalized size = 1.93

$$\frac{b^2\left(2\left(\left(\left(\left(\frac{\frac{1}{5760}\cdot 720d^6\sqrt{a+bx}\,\sqrt{a+bx}}{bd^7}-\frac{\frac{1}{5760}\left(840bd^5c-840d^6a\right)}{bd^7}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{\frac{1}{5760}\left(-1050b^2d^4c^2+2100bd^5ac-1050d^6a^2\right)}{bd^7}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{\frac{1}{5760}\left(1575b^3d^3c^3-4725b^2d^4ac^2+4725bd^5a^2c-1575d^6a^3\right)}{bd^7}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}+\frac{2\left(-35a^4d^4+140a^3bcd^3-210a^2b^2c^2d^2+140ab^3c^3d-35b^4c^4\right)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{128d^4\sqrt{bd}}\right)\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/192*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)/(b*d) - 7*(b*c*d^5 - a*d^6)/(b*d^7)) + 35*(b^2*c^2*d^4 - 2*a*b*c*d^5
 + a^2*d^6)/(b*d^7)) - 105*(b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 -
 a^3*d^6)/(b*d^7))*sqrt(b*x + a) - 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4))*b/abs(b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(1/2), x)
```

3.1493 $\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=148

$$\frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3}-\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\frac{5(bc-ad)^3\tanh^{-1}(}{8\sqrt{b}\,d}$$

```
[Out] -5/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(7
/2)/b^(1/2)-5/12*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^2+1/3*(b*x+a)^(5/
2)*(d*x+c)^(1/2)/d+5/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^3) - (5*(b*c - a*d)*(a +
 b*x)^(3/2)*Sqrt[c + d*x])/(12*d^2) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d)
 - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*Sqrt[b]*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} + \frac{(5(bc-ad)^2)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,d}{8d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+d}}{3d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+d}}{3d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+d}}{3d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+d}}{3d}$$

Mathematica [A]
time = 0.19, size = 125, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,\left(33a^2d^2+2abd(-20c+13dx)+b^2\left(15c^2-10cdx+8d^2x^2\right)\right)}{24d^3} - \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{8\sqrt{b}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x) + b^2*(
15*c^2 - 10*c*d*x + 8*d^2*x^2)))/(24*d^3) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*Sqrt[b]*d^(7/2))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/2),x]')

[Out] Timed out

Maple [A]
time = 0.16, size = 173, normalized size = 1.17

method	result
default	$\frac{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}{3d}-\frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d}-\frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}-\frac{(-ad+bc)\sqrt{(bx+a)\,(dx}}{}\right.}{4d}\right.}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(5/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d-5/6*(-a*d+b*c)/d*(1/2*(b*x+a)^(3/2)*(d*x+
c)^(1/2)/d-3/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d
*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*
d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.33, size = 412, normalized size = 2.78

$$\left[-\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2+15\,b^3c^2d-40\,ab^2cd^2+33\,a^2bd^3-2\,(5\,b^3cd^2-13\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,bd^4},\ \frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(8\,b^3d^3x^2+15\,b^3c^2d-40\,ab^2cd^2+33\,a^2bd^3-2\,(5\,b^3cd^2-13\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 1
3*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^4), 1/48*(15*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*
c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 +
33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^4)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.02, size = 263, normalized size = 1.78

$$\frac{b^2\left(2\left(\left(\left(\frac{\frac{1}{144}\cdot 24d^4\sqrt{a+bx}\,\sqrt{a+bx}}{bd^5}-\frac{\frac{1}{144}(30bd^3c-30d^4a)}{bd^5}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{\frac{1}{144}(-45b^2d^2c^2+90bd^3ac-45d^4a^2)}{bd^5}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}+\frac{2(-5a^3d^3+15a^2bcd^2-15ab^2c^2d+5b^3c^3)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{16d^3\sqrt{bd}}\right)\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d^3))*b/abs(b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/2),x)
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/2), x)
```

3.1494 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=113

$$-\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}$$

```
[Out] 3/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(5/
2)/b^(1/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d-3/4*(-a*d+b*c)*(b*x+a)^(1/2)*(
d*x+c)^(1/2)/d^2
```

Rubi [A]

time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^2) + ((a + b*x)^(3/2)*Sqr
t[c + d*x])/(2*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[
b]*Sqrt[c + d*x])])/(4*Sqrt[b]*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{(3(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c-}}}{8d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{(3(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{4b}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{(3(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-}}{\sqrt{b}\,\sqrt{c-}}\right.}{4\sqrt{b}\,d^{5/2}}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 0.83

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(-3bc+5ad+2bdx)}{4d^2} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4\sqrt{b}\,d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(-3*b*c + 5*a*d + 2*b*d*x))/(4*d^2) + (3*(b*c
- a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*Sqrt[
b]*d^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.15, size = 140, normalized size = 1.24

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d}-\frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}-\frac{(-ad+bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd}\right.}{2d\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{b}}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d-3/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(
1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)
*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b
*d)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 306, normalized size = 2.71

$$\left[\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x-3\,b^2cd+5\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,bd^3},\ -\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(2\,b^2d^2x-3\,b^2cd+5\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x - 3*b^2*c*d + 5*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/sqrt(c + d*x), x)
```

Giac [A]
time = 0.01, size = 186, normalized size = 1.65

$$\frac{b^2\left(2\left(\frac{\frac{1}{8}\cdot 2d^2\sqrt{a+bx}\,\sqrt{a+bx}}{bd^3}-\frac{\frac{1}{8}(3bdc-3d^2a)}{bd^3}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}+\frac{2(-3a^2d^2+6abcd-3b^2c^2)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{8d^2\sqrt{bd}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b*d) -
 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^
2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/2), x)
```

3.1495 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

```
[Out] -(-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(3/2)/b^
(1/2)+(b*x+a)^(1/2)*(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx &= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d} \\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{bd} \\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{bd} \\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 74, normalized size = 1.01

$$\frac{d\sqrt{a+bx}\,\sqrt{c+dx} + \frac{(bc-ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{\sqrt{\frac{b}{d}}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (d*Sqrt[a + b*x]*Sqrt[c + d*x] + ((b*c - a*d)*Log[Sqrt[a + b*x] - Sqrt[b/d]
*Sqrt[c + d*x]])/Sqrt[b/d])/d^2
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [A]
time = 0.16, size = 107, normalized size = 1.47

method	result
default	$\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}-\frac{(-ad+bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}+(ad+bc)\,x+ac}\right)}{2d\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x
+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*
d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.31, size = 235, normalized size = 3.22

$$\left[\frac{4\,\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{2}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^{2}cd+abd^{2})x\right)}{4\,bd^{2}},\ \frac{2\,\sqrt{bx+a}\,\sqrt{dx+c}\,bd+(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^{2}d^{2}x^{2}+abcd+(b^{2}cd+abd^{2})x)}\right)}{2\,bd^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b*d^2), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/sqrt(c + d*x), x)
```

Giac [A]
time = 0.01, size = 117, normalized size = 1.60

$$\frac{b^2\left(\frac{\frac{1}{2}\cdot 2\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{bd}+\frac{2(-ad+bc)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{2d\sqrt{bd}}\right)}{|b|\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] b*((b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
 a)/(b*d))/abs(b)
```

Mupad [B]
time = 3.80, size = 261, normalized size = 3.58

$$\frac{\frac{(2ad+2bc)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^3}+\frac{(2cb^2+2adb)\left(\sqrt{a+bx}-\sqrt{a}\right)}{d^3\left(\sqrt{c+dx}-\sqrt{c}\right)}-\frac{8\sqrt{a}\,b\sqrt{c}\,\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2b\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{c+dx}-\sqrt{c}\right)}\right)(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(d^2*((c + d*x)^(1/2) - c^
(1/2))^3) + ((2*b^2*c + 2*a*b*d)*((a + b*x)^(1/2) - a^(1/2)))/(d^3*((c + d*
x)^(1/2) - c^(1/2))) - (8*a^(1/2)*b*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/
```

```
(d^2*((c + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d
*x)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*(
(c + d*x)^(1/2) - c^(1/2))^2)) + (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/
2)))/(b^(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(1/2)*d^(3/2))
```

3.1496 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [**A**]
time = 0.06, size = 42, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[
d])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(1/2)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.
time = 0.16, size = 76, normalized size = 1.81

method	result	size

default	$\frac{\sqrt{(bx+a)\,(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^2+(ad+bc)\,x+ac}\right)}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d
*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(30) = 60$.
time = 0.31, size = 178, normalized size = 4.24

$$\left[\frac{\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)}{2\,bd}, -\frac{\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d
*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^
2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b
*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x))/(b*d
)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)), x)
```

Giac [A]
time = 0.01, size = 63, normalized size = 1.50

$$-\frac{2b^2\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{|b|\,b\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -2*b*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*abs(b))
```

Mupad [B]
time = 0.29, size = 45, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x)^(1/2) - a
^(1/2)))))/(-b*d)^(1/2)
```

3.1497 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/2)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**A**]
time = 0.16, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{2\sqrt{dx+c}}{\sqrt{bx+a}\,(ad-bc)}$	27
default	$-\frac{2\sqrt{dx+c}}{(-ad+bc)\sqrt{bx+a}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.30, size = 42, normalized size = 1.40

$$-\frac{2\,\sqrt{bx+a}\,\sqrt{dx+c}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)*sqrt(d*x + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.
time = 0.01, size = 71, normalized size = 2.37

$$\frac{8b\sqrt{bd}}{2\,|b|\left(\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2+abd-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] -4*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)*abs(b))
```

Mupad [B]
time = 0.73, size = 26, normalized size = 0.87

$$\frac{2\,\sqrt{c+d\,x}}{\left(a\,d-b\,c\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(1/2))/((a*d - b*c)*(a + b*x)^(1/2))
```

3.1498 $\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^
2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (4*d*Sqrt[c + d*x])/(3
*(b*c - a*d)^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{3(bc-ad)}$$
$$= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.70

$$\frac{2\sqrt{c+dx}\,(-bc+3ad+2bdx)}{3(bc-ad)^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
[Out] (2*Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(a + b*x)^(3/
2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/2)),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 55, normalized size = 0.83

method	result	size
gosper	$\frac{2\sqrt{dx+c}\,(2bdx+3ad-bc)}{3(bx+a)^{\frac{3}{2}}(a^2d^2-2abcd+b^2c^2)}$	54
default	$-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^
2/(b*x+a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.34, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx - bc + 3\,ad)\sqrt{bx + a}\,\sqrt{dx + c}}{3\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x - b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^2*c^2 - 2*a^
3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^
2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{5}{2}}\,\sqrt{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*sqrt(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(54) = 108$.
time = 0.01, size = 136, normalized size = 2.06

$$-\frac{32b\sqrt{bd}\,bd\left(-3\left(\sqrt{-abd + b^2c + bd\,(a + bx)} - \sqrt{bd}\,\sqrt{a + bx}\right)^2 - abd + b^2c\right)}{2\cdot 6\,|b|\left(\left(\sqrt{-abd + b^2c + bd\,(a + bx)} - \sqrt{bd}\,\sqrt{a + bx}\right)^2 + abd - b^2c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 8/3*(b^2*c - a*b*d - 3*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*abs(b))
```

Mupad [B]
time = 0.89, size = 71, normalized size = 1.08

$$\frac{\left(\frac{4\,d\,x}{3\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d-2\,b\,c}{3\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{a\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (((4*d*x)/(3*(a*d - b*c)^2) + (6*a*d - 2*b*c)/(3*b*(a*d - b*c)^2))*(c + d*x
)^(1/2))/(x*(a + b*x)^(1/2) + (a*(a + b*x)^(1/2))/b)
```

3.1499 $\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}+\frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}}-\frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}$$

```
[Out] -2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)+8/15*d*(d*x+c)^(1/2)/(-a*d+b*c)
^2/(b*x+a)^(3/2)-16/15*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}\,(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (8*d*Sqrt[c + d*x])/(1
5*(b*c - a*d)^2*(a + b*x)^(3/2)) - (16*d^2*Sqrt[c + d*x])/(15*(b*c - a*d)^3
*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{5(bc-ad)}$$
$$= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} + \frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}} + \frac{(8d^2)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{15(bc-ad)^2}$$
$$= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} + \frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}} - \frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}$$

Mathematica [A]
time = 0.08, size = 75, normalized size = 0.74

$$-\frac{2\sqrt{c+dx}\,\left(15a^2d^2-10abd(c-2dx)+b^2\left(3c^2-4cdx+8d^2x^2\right)\right)}{15(bc-ad)^3(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(3*c^2 - 4*c*d*x
 + 8*d^2*x^2)))/(15*(b*c - a*d)^3*(a + b*x)^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(1/2)),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 95, normalized size = 0.94

method	result	size
default	$-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}$	95
gosper	$\frac{2\sqrt{dx+c}\,(8b^2x^2d^2+20ab\,d^2x-4b^2cdx+15a^2d^2-10abcd+3b^2c^2)}{15(bx+a)^{\frac{5}{2}}(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^
(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/
2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(83) = 166.
time = 0.46, size = 251, normalized size = 2.49

$$-\frac{2\left(8\,b^{2}d^{2}x^{2}+3\,b^{2}c^{2}-10\,abcd+15\,a^{2}d^{2}-4\left(b^{2}cd-5\,abd^{2}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{15\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{3}+3\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3}\right)x^{2}+3\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*d^2*x^2 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3
*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^
3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{\frac{7}{2}}\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(83) = 166.
time = 0.03, size = 254, normalized size = 2.51

$$-\frac{128b\sqrt{bd}\,(bd)^2\left(-10\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^4-5\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2abd+5\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2b^2c-a^2b^2d^2+2ab^3dc-b^4c^2\right)}{60\,|b|\left(\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2+abd-b^2c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] -32/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((b^2*c - a*b*
d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*ab
s(b))
```

Mupad [B]
time = 1.01, size = 133, normalized size = 1.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d^2\,x^2}{15\,{\left(a\,d-b\,c\right)}^3}+\frac{30\,a^2\,d^2-20\,a\,b\,c\,d+6\,b^2\,c^2}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d\,x\,\left(5\,a\,d-b\,c\right)}{15\,b\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*d^2*x^2)/(15*(a*d - b*c)^3) + (30*a^2*d^2 + 6*b^2*c^2
 - 20*a*b*c*d)/(15*b^2*(a*d - b*c)^3) + (8*d*x*(5*a*d - b*c))/(15*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*
x)^(1/2))/b)
```

3.1500 $\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}}+\frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}+\frac{32d^3\sqrt{c+dx}}{35(bc-ad)^4\sqrt{a+bx}}$$

```
[Out] -2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)+12/35*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)^(5/2)-16/35*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(3/2)+32/35*
d^3*(d*x+c)^(1/2)/(-a*d+b*c)^4/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}\,(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (12*d*Sqrt[c + d*x])/(
35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*Sqrt[c + d*x])/(35*(b*c - a*d)^
3*(a + b*x)^(3/2)) + (32*d^3*Sqrt[c + d*x])/(35*(b*c - a*d)^4*Sqrt[a + b*x]
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx}{7(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{(24d^2)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}}{35(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}$$

<u>**Mathematica**</u> [A]
time = 0.12, size = 93, normalized size = 0.68

$$-\frac{2\sqrt{c+dx}\,(-35d^3(a+bx)^3+35bd^2(a+bx)^2(c+dx)-21b^2d(a+bx)(c+dx)^2+5b^3(c+dx)^3)}{35(bc-ad)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(-35*d^3*(a + b*x)^3 + 35*b*d^2*(a + b*x)^2*(c + d*x) - 2
1*b^2*d*(a + b*x)*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35*(b*c - a*d)^4*(a +
b*x)^(7/2))
```

<u>**Mathics**</u> [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(1/2)),x]')
```

```
[Out] Timed out
```

<u>**Maple**</u> [A]
time = 0.20, size = 135, normalized size = 0.99

method	result	size

default	$-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{7(-ad+bc)}$	135
gosper	$\frac{2\sqrt{dx+c}\,(16b^{3}x^{3}d^{3}+56d^{3}a\,x^{2}b^{2}-8b^{3}c\,d^{2}x^{2}+70a^{2}b\,d^{3}x-28a\,b^{2}c\,d^{2}x+6b^{3}c^{2}dx+35a^{3}d^{3}-35a^{2}bc\,d^{2}+21a\,b^{2}c^{2}d-5b^{3}c^{3})}{35(bx+a)^{\frac{7}{2}}(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4})}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^
(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b
*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(112) = 224$.
time = 0.73, size = 419, normalized size = 3.08

$$\frac{2\,(16\,b^{3}d^{3}x^{3}-5\,b^{3}c^{3}+21\,ab^{2}c^{2}d-35\,a^{2}bcd^{2}+35\,a^{3}d^{3}-8\,(b^{3}cd^{2}-7\,ab^{2}d^{3})x^{2}+2\,(3\,b^{3}c^{2}d-14\,ab^{2}cd^{2}+35\,a^{2}bd^{3})x)\sqrt{bx+a}\,\sqrt{dx+c}}{35\,(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4})x^{4}+4\,(ab^{7}c^{4}-4\,a^{2}b^{6}c^{3}d+6\,a^{3}b^{5}c^{2}d^{2}-4\,a^{4}b^{4}cd^{3}+a^{5}b^{3}d^{4})x^{3}+6\,(a^{2}b^{6}c^{4}-4\,a^{3}b^{5}c^{3}d+6\,a^{4}b^{4}c^{2}d^{2}-4\,a^{5}b^{3}cd^{3}+a^{6}b^{2}d^{4})x^{2}+4\,(a^{3}b^{5}c^{4}-4\,a^{4}b^{4}c^{3}d+6\,a^{5}b^{3}c^{2}d^{2}-4\,a^{6}b^{2}cd^{3}+a^{7}bd^{4})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*d^3*x^3 - 5*b^3*c^3 + 21*a*b^2*c^2*d - 35*a^2*b*c*d^2 + 35*a^3
*d^3 - 8*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 2*(3*b^3*c^2*d - 14*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d
 + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d +
 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*
a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*
(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*
```

```
b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6
*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(112) = 224.
time = 0.05, size = 429, normalized size = 3.15

$$-\frac{512b\sqrt{bd}\,(bd)^3\left(-35\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^6-21\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^4abd+21\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^4b^2c-7\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2a^2b^2d^2+14\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2ab^3dc-7\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2b^4c^2-a^3b^3d^3+3a^2b^4d^2c-3ab^5dc^2+b^6c^3\right)}{280\,|b|\left(\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2+abd-b^2c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x)
```

```
[Out] 64/35*(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3 - 7*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2 + 14*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^3*c*d - 7
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*
d^2 + 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
b^2*c - 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
4*a*b*d - 35*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6)*sqrt(b*d)*b^4*d^3/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2)^7*abs(b))
```

Mupad [B]
time = 1.19, size = 209, normalized size = 1.54

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^3\,x^3}{35\,{\left(a\,d-b\,c\right)}^4}+\frac{70\,a^3\,d^3-70\,a^2\,b\,c\,d^2+42\,a\,b^2\,c^2\,d-10\,b^3\,c^3}{35\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d\,x\,\left(35\,a^2\,d^2-14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{35\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{35\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^3*x^3)/(35*(a*d - b*c)^4) + (70*a^3*d^3 - 10*b^3*c^
3 + 42*a*b^2*c^2*d - 70*a^2*b*c*d^2)/(35*b^3*(a*d - b*c)^4) + (4*d*x*(35*a^
2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(35*b^2*(a*d - b*c)^4) + (16*d^2*x^2*(7*a*
d - b*c))/(35*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2
))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

3.1501 $\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=171

$$-\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}+\frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{128d^3\sqrt{c+dx}}{315(bc-ad)^4(a+bx)^{3/2}}-\frac{}{3}$$

```
[Out] -2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)+16/63*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)^(7/2)-32/105*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(5/2)+128/3
15*d^3*(d*x+c)^(1/2)/(-a*d+b*c)^4/(b*x+a)^(3/2)-256/315*d^4*(d*x+c)^(1/2)/(
-a*d+b*c)^5/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}\,(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (16*d*Sqrt[c + d*x])/(
63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (32*d^2*Sqrt[c + d*x])/(105*(b*c - a*d)
^3*(a + b*x)^(5/2)) + (128*d^3*Sqrt[c + d*x])/(315*(b*c - a*d)^4*(a + b*x)^
(3/2)) - (256*d^4*Sqrt[c + d*x])/(315*(b*c - a*d)^5*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{(16d^2)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}}{21(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}$$

Mathematica [A]
time = 0.13, size = 115, normalized size = 0.67

$$-\frac{2\sqrt{c+dx}\,\left(315d^4(a+bx)^4-420bd^3(a+bx)^3(c+dx)+378b^2d^2(a+bx)^2(c+dx)^2-180b^3d(a+bx)(c+dx)^3+35b^4(c+dx)^4\right)}{315(bc-ad)^5(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(315*d^4*(a + b*x)^4 - 420*b*d^3*(a + b*x)^3*(c + d*x) +
378*b^2*d^2*(a + b*x)^2*(c + d*x)^2 - 180*b^3*d*(a + b*x)*(c + d*x)^3 + 35*
b^4*(c + d*x)^4))/(315*(b*c - a*d)^5*(a + b*x)^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(1/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 175, normalized size = 1.02

method	result

default	$-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}\right.}{7(-ad+bc)}\right.}{9(-ad+bc)}$
gosper	$\frac{2\sqrt{dx+c}\,(128d^4x^4b^4+576a\,b^3d^4x^3-64b^4c\,d^3x^3+1008a^2b^2d^4x^2-288a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+840a^3b\,d^4x-504a^2b^2c\,d^3x+216a}{315(bx+a)^{\frac{9}{2}}(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^
(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b
*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(
3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(141) = 282$.
time = 2.27, size = 638, normalized size = 3.73

$$-\frac{2\,(128\,b^4d^4x^4+35\,b^4c^4-180\,ab^3c^3d+378\,a^2b^2c^2d^2-420\,a^3bcd^3+315\,a^4d^4-64\,(b^4cd^3-9\,ab^3d^4)x^3+48\,(b^4c^2d^2-6\,ab^3cd^3+21\,a^2b^2d^4)x^2-8\,(5\,b^4c^3d-27\,ab^3c^2d^2+63\,a^2b^2cd^3-105\,a^3bd^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{315\,(a^5b^5c^5-5\,a^6b^4c^4d+10\,a^7b^3c^3d^2-10\,a^8b^2c^2d^3+5\,a^9bcd^4-a^{10}d^5+(b^{10}c^5-5\,ab^9c^4d+10\,a^2b^8c^3d^2-10\,a^3b^7c^2d^3+5\,a^4b^6cd^4-a^5b^5d^5)x^5+5\,(ab^9c^5-5\,a^2b^8c^4d+10\,a^3b^7c^3d^2-10\,a^4b^6c^2d^3+5\,a^5b^5cd^4-a^6b^4d^5)x^4+10\,(a^2b^8c^5-5\,a^3b^7c^4d+10\,a^4b^6c^3d^2-10\,a^5b^5c^2d^3+5\,a^6b^4cd^4-a^7b^3d^5)x^3+10\,(a^3b^7c^5-5\,a^4b^6c^4d+10\,a^5b^5c^3d^2-10\,a^6b^4c^2d^3+5\,a^7b^3cd^4-a^8b^2d^5)x^2+5\,(a^4b^6c^5-5\,a^5b^5c^4d+10\,a^6b^4c^3d^2-10\,a^7b^3c^2d^3+5\,a^8b^2cd^4-a^9bd^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*d^4*x^4 + 35*b^4*c^4 - 180*a*b^3*c^3*d + 378*a^2*b^2*c^2*d^
2 - 420*a^3*b*c*d^3 + 315*a^4*d^4 - 64*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 48*(
```

```
b^4*c^2*d^2 - 6*a*b^3*c*d^3 + 21*a^2*b^2*d^4)*x^2 - 8*(5*b^4*c^3*d - 27*a*b
^3*c^2*d^2 + 63*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x +
c)/(a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3
 + 5*a^9*b*c*d^4 - a^10*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^
2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^5 + 5*(a*b^9*c^5
- 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d
^4 - a^6*b^4*d^5)*x^4 + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*
d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^3 + 10*(a^3*b^7
*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^
3*c*d^4 - a^8*b^2*d^5)*x^2 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*
c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(141) = 282.
time = 0.08, size = 663, normalized size = 3.88

$$-\frac{2048 b\sqrt{bd}\,(bd)^4\left(-126\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^8-84\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^6abd+84\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^6b^2c-36\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^4a^2b^2d^2+72\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^4ab^3dc-36\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^4b^4c^2-9\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2a^3b^3d^3+27\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2a^2b^4d^2c-27\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2ab^5dc^2+9\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2b^6c^3-a^4b^4d^4+4a^3b^5d^3c-6a^2b^6d^2c^2+4ab^7dc^3-b^8c^4\right)}{1260\,|b|\left(\left(\sqrt{-abd+b^2c+bd(a+bx)}-\sqrt{bd}\sqrt{a+bx}\right)^2+abd-b^2c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x)
```

```
[Out] -512/315*(b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a
^4*b^4*d^4 - 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^6*c^3 + 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^5*c^2*d - 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^3*d^3 + 36*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^4*c^2 - 72*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d + 36*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 84*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^2*c + 84*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b*d + 126*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8)*sqrt(b*d)*b^5*d^
4/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2)^9*abs(b))
```

Mupad [B]
time = 1.37, size = 303, normalized size = 1.77

$$\frac{\sqrt{c+d\,x}\,\left(\frac{256\,d^4\,x^4}{315\,{\left(a\,d-b\,c\right)}^5}+\frac{630\,a^4\,d^4-840\,a^3\,b\,c\,d^3+756\,a^2\,b^2\,c^2\,d^2-360\,a\,b^3\,c^3\,d+70\,b^4\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(1680\,a^3\,b\,d^4-1008\,a^2\,b^2\,c\,d^3+432\,a\,b^3\,c^2\,d^2-80\,b^4\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((256*d^4*x^4)/(315*(a*d - b*c)^5) + (630*a^4*d^4 + 70*b^4
*c^4 + 756*a^2*b^2*c^2*d^2 - 360*a*b^3*c^3*d - 840*a^3*b*c*d^3)/(315*b^4*(a
*d - b*c)^5) + (x*(1680*a^3*b*d^4 - 80*b^4*c^3*d + 432*a*b^3*c^2*d^2 - 1008
*a^2*b^2*c*d^3))/(315*b^4*(a*d - b*c)^5) + (128*d^3*x^3*(9*a*d - b*c))/(315
*b*(a*d - b*c)^5) + (32*d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^
2*(a*d - b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^
2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*
x)^(1/2))/b^3)
```

3.1502 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}}+\frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4}-\frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}+\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}$$

```
[Out] -35/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(
1/2)/d^(9/2)-2*(b*x+a)^(7/2)/d/(d*x+c)^(1/2)-35/12*b*(-a*d+b*c)*(b*x+a)^(3/
2)*(d*x+c)^(1/2)/d^3+7/3*b*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d^2+35/8*b*(-a*d+b*c
)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}+\frac{35b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^4}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^3}+\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}-\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(d*Sqrt[c + d*x]) + (35*b*(b*c - a*d)^2*Sqrt[a + b*x]*
Sqrt[c + d*x])/(8*d^4) - (35*b*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(
12*d^3) + (7*b*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d^2) - (35*Sqrt[b]*(b*c -
a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2)
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} - \frac{(35b(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} + \frac{(35b(bc-}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

Mathematica [A]
time = 0.29, size = 166, normalized size = 0.95

$$\frac{\sqrt{a+bx}\,(-48a^3d^3+3a^2bd^2(77c+29dx)+2ab^2d(-140c^2-49cdx+19d^2x^2)+b^3(105c^3+35c^2dx-14cd^2x^2+8d^3x^3))}{24d^4\sqrt{c+dx}} - \frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-48*a^3*d^3 + 3*a^2*b*d^2*(77*c + 29*d*x) + 2*a*b^2*d*(-140
*c^2 - 49*c*d*x + 19*d^2*x^2) + b^3*(105*c^3 + 35*c^2*d*x - 14*c*d^2*x^2 +
8*d^3*x^3)))/(24*d^4*Sqrt[c + d*x]) - (35*Sqrt[b]*(b*c - a*d)^3*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(138) = 276.
time = 0.45, size = 603, normalized size = 3.47

$$\left[-\frac{105\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3+\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^3+105b^3c^3-280ab^2c^2d+231a^2bcd^2-48a^3d^3-2\left(7b^3cd^2-19ab^2d^3\right)x^2+\left(35b^3c^2d-98ab^2cd^2+87a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96\left(d^5x+cd^4\right)}, \frac{105\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3+\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\left(8b^3d^3x^3+105b^3c^3-280ab^2c^2d+231a^2bcd^2-48a^3d^3-2\left(7b^3cd^2-19ab^2d^3\right)x^2+\left(35b^3c^2d-98ab^2cd^2+87a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48\left(d^5x+cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(105*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(
b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d^3*
x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48*a^3*d^3 - 2*(7*b
^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*d^2 + 87*a^2*b*d^
3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4), 1/48*(105*(b^3*c^4 - 3*
```

```
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqr
t(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)
) + 2*(8*b^3*d^3*x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48
*a^3*d^3 - 2*(7*b^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*
d^2 + 87*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(138) = 276.
time = 0.05, size = 379, normalized size = 2.18

$$\frac{2\left(\left(\left(\left(\frac{\frac{1}{48}\cdot 8b^2d^6\sqrt{a+bx}\,\sqrt{a+bx}}{d^7|b|}-\frac{\frac{1}{48}(14b^3d^5c-14b^2d^6a)}{d^7|b|}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{\frac{1}{48}(-35b^4d^4c^2+70b^3d^5ac-35b^2d^6a^2)}{d^7|b|}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{\frac{1}{48}(-105b^5d^3c^3+315b^4d^4ac^2-315b^3d^5a^2c+105b^2d^6a^3)}{d^7|b|}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{-abd+b^2c+bd\,(a+bx)}+\frac{2\,(-35a^3b^2d^3+105a^2b^3cd^2-105ab^4c^2d+35b^5c^3)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{16d^4\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/24*((b*x + a)*(2*(b*x + a)*(4*(b*x + a)*b^2/(d*abs(b)) - 7*(b^3*c*d^5 - a
*b^2*d^6)/(d^7*abs(b))) + 35*(b^4*c^2*d^4 - 2*a*b^3*c*d^5 + a^2*b^2*d^6)/(d
^7*abs(b))) + 105*(b^5*c^3*d^3 - 3*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^
2*d^6)/(d^7*abs(b)))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 35
/8*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(-sqrt(
b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4*a
bs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(3/2), x)
```

3.1503 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}-\frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}+\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}$$

```
[Out] 15/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(1
/2)/d^(7/2)-2*(b*x+a)^(5/2)/d/(d*x+c)^(1/2)+5/2*b*(b*x+a)^(3/2)*(d*x+c)^(1/
2)/d^2-15/4*b*(-a*d+b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{15b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(d*Sqrt[c + d*x]) - (15*b*(b*c - a*d)*Sqrt[a + b*x]*Sq
rt[c + d*x])/(4*d^3) + (5*b*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d^2) + (15*Sq
rt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(4*d^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} - \frac{(15b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{(15b(bc-}{}$$

$$(15(bc-a$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + $$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{(15(bc-a}{}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{15\sqrt{b}\,(bc}{}$$

Mathematica [A]
time = 0.23, size = 124, normalized size = 0.90

$$\frac{\sqrt{a+bx}\,\left(-8a^2d^2+abd(25c+9dx)+b^2\left(-15c^2-5cdx+2d^2x^2\right)\right)}{4d^3\sqrt{c+dx}}+\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-8*a^2*d^2 + a*b*d*(25*c + 9*d*x) + b^2*(-15*c^2 - 5*c*d*x
+ 2*d^2*x^2)))/(4*d^3*Sqrt[c + d*x]) + (15*Sqrt[b]*(b*c - a*d)^2*ArcTanh[(S
qrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.38, size = 441, normalized size = 3.20

$$\left[\frac{15\,(b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bd^2x + bcd + ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}} + 8\,(b^2cd + abd^2)x\right) + 4\,(2\,b^2d^2x^2 - 15\,b^2c^2 + 25\,abcd - 8\,a^2d^2 - (5\,b^2cd - 9\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,(d^4x + cd^3)}, -\frac{15\,(b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right) - 2\,(2\,b^2d^2x^2 - 15\,b^2c^2 + 25\,abcd - 8\,a^2d^2 - (5\,b^2cd - 9\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,(d^4x + cd^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2
- (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3),
-1/8*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(2*b^2*d^2*x
^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2 - (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(3/2), x)
```

Giac [A]
time = 0.04, size = 282, normalized size = 2.04

$$\frac{2\left(\left(\left(\frac{\frac{1}{8}\cdot 2b^2d^4\sqrt{a+bx}\,\sqrt{a+bx}}{d^5|b|} + \frac{\frac{1}{8}(-5b^3d^3c+5b^2d^4a)}{d^5|b|}\right)\sqrt{a+bx}\,\sqrt{a+bx} + \frac{\frac{1}{8}(-15b^4d^2c^2+30b^3d^3ac-15b^2d^4a^2)}{d^5|b|}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{-abd+b^2c+bd\,(a+bx)} + \frac{2\,(-15a^2b^2d^2+30ab^3cd-15b^4c^2)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)} - \sqrt{bd}\,\sqrt{a+bx}\right|}{8d^3\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/4*sqrt(b*x + a)*((b*x + a)*(2*(b*x + a)*b^2/(d*abs(b)) - 5*(b^3*c*d^3 - a
*b^2*d^4)/(d^5*abs(b))) - 15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(d
^5*abs(b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) - 15/4*(b^4*c^2 - 2*a*b^3*c
```

```
*d + a^2*b^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(3/2),x)
[Out] int((a + b*x)^(5/2)/(c + d*x)^(3/2), x)
```

3.1504 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}$$

```
[Out] -3*(-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(1/2)/
d^(5/2)-2*(b*x+a)^(3/2)/d/(d*x+c)^(1/2)+3*b*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x]) + (3*b*Sqrt[a + b*x]*Sqrt[c + d*x])/
d^2 - (3*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[
c + d*x])])/d^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{(3b)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.38, size = 87, normalized size = 0.89

$$\frac{\frac{\sqrt{a+bx}\,(3bc-2ad+bdx)}{\sqrt{c+dx}}+3\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] ((Sqrt[a + b*x]*(3*b*c - 2*a*d + b*d*x))/Sqrt[c + d*x] + 3*Sqrt[b/d]*(b*c -
 a*d)*Log[Sqrt[a + b*x] - Sqrt[b/d]*Sqrt[c + d*x]])/d^2
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.35, size = 311, normalized size = 3.17

$$\left[-\frac{3\,(bc^2-acd+(bcd-ad^2)x)\sqrt{\frac{b}{d}}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\,(d^3x+cd^2)},\ \frac{3\,(bc^2-acd+(bcd-ad^2)x)\sqrt{-\frac{b}{d}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(d^3x+cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x + 3*b*c - 2*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2), 1/2*(3*(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a
)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(b*
d*x + 3*b*c - 2*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/2), x)
```

Giac [A]
time = 0.03, size = 199, normalized size = 2.03

$$\frac{2\left(\frac{\frac{1}{2}b^2d^2\sqrt{a+bx}\,\sqrt{a+bx}}{d^3|b|}-\frac{\frac{1}{2}(-3b^3dc+3b^2d^2a)}{d^3|b|}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{-abd+b^2c+bd\,(a+bx)}+\frac{2\,(-3ab^2d+3b^3c)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{2d^2\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] sqrt(b*x + a)*((b*x + a)*b^2/(d*abs(b)) + 3*(b^3*c*d - a*b^2*d^2)/(d^3*abs(
b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 3*(b^3*c - a*b^2*d)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2
*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(3/2), x)
```

3.1505 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}$$

```
[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(1/2)/d^(3/2)-2*(b
*x+a)^(1/2)/d/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {49, 65, 223, 212}

$$\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{b\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d}\\ &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d}\\ &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(50) = 100.
time = 0.33, size = 241, normalized size = 3.65

$$\left[\frac{(dx+c)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\sqrt{bx+a}\sqrt{dx+c}}{2\left(d^2x+cd\right)},\ -\frac{(dx+c)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\sqrt{bx+a}\sqrt{dx+c}}{d^2x+cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((d*x + c)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(d^2*x + c*d), -((d
*x + c)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(d^2*x + c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx}}{\left(c + dx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [A]
time = 0.02, size = 127, normalized size = 1.92

$$-\frac{2b^2\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{d\,|b|\,(-abd+b^2c+bd\,(a+bx))}-\frac{2b^2\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{d\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*b^2*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d*abs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*d*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(3/2), x)
```

3.1506 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

```
[Out] 2*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(3/2)),x]')

[Out] Timed out

Maple [A]
time = 0.16, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{2\sqrt{bx+a}}{\sqrt{dx+c}\,(ad-bc)}$	27
default	$-\frac{2\sqrt{bx+a}}{\sqrt{dx+c}\,(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail

Fricas [A]
time = 0.31, size = 42, normalized size = 1.40

$$\frac{2\,\sqrt{bx+a}\,\sqrt{dx+c}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(d*x + c)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/2)), x)
```

Giac [**A**]
time = 0.01, size = 72, normalized size = 2.40

$$\frac{4b^2\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(2bc\,|b|-2da\,|b|)\,(-abd+b^2c+bd\,(a+bx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(b*c*abs(b) - a*d*
abs(b)))
```

Mupad [**B**]
time = 0.74, size = 26, normalized size = 0.87

$$-\frac{2\sqrt{a+b\,x}}{(a\,d-b\,c)\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(2*(a + b*x)^(1/2))/((a*d - b*c)*(c + d*x)^(1/2))
```

3.1507 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}-\frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}$$

```
[Out] -2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)-4*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x]) - (4*d*Sqrt[a + b*x])/((b*c -
a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{bc-ad}$$
$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}} - \frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.68

$$-\frac{2(ad+b(c+2dx))}{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-2*(a*d + b*(c + 2*d*x)))/((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 65, normalized size = 1.05

method	result	size
gosper	$-\frac{2(2bdx+ad+bc)}{\sqrt{bx+a}\,\sqrt{dx+c}\,(a^2d^2-2abcd+b^2c^2)}$	52
default	$-\frac{2}{(-ad+bc)\sqrt{bx+a}\,\sqrt{dx+c}}+\frac{4d\sqrt{bx+a}}{(-ad+bc)\sqrt{dx+c}\,(ad-bc)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x
+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(54) = 108.
time = 0.34, size = 125, normalized size = 2.02

$$-\frac{2\,(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(54) = 108.
time = 0.02, size = 177, normalized size = 2.85

$$2\left(-\frac{2b^2d\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(2b^2c^2\,|b|-4bdac\,|b|+2d^2a^2\,|b|)\,(-abd+b^2c+bd\,(a+bx))}-\frac{4b^2\sqrt{bd}}{2\,(ad\,|b|-|b|\,bc)\left(\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2+adb-b^2c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*sqrt(b*x + a)*b^2*d/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b)
)*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - 4*sqrt(b*d)*b^2/((b^2*c - a*b*d -
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*(b*c*abs
(b) - a*d*abs(b)))
```

Mupad [B]
time = 0.86, size = 71, normalized size = 1.15

$$-\frac{\left(\frac{4\,b\,x}{{\left(a\,d-b\,c\right)}^2}+\frac{2\,a\,d+2\,b\,c}{d\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{c\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(((4*b*x)/(a*d - b*c)^2 + (2*a*d + 2*b*c)/(d*(a*d - b*c)^2))*(c + d*x)^(1/
2))/(x*(a + b*x)^(1/2) + (c*(a + b*x)^(1/2))/d)
```

3.1508 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}+\frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}+\frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/2)+8/3*d/(-a*d+b*c)^2/(b*x+a)^(1/2
)/(d*x+c)^(1/2)+16/3*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (8*d)/(3*(b*c - a*d)^2*S
qrt[a + b*x]*Sqrt[c + d*x]) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} + \frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{(8d^2)\ldots}{\ldots}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} + \frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{16\ldots}{3(bc-\ldots}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.70

$$-\frac{2(c+dx)^{3/2}\left(b^2-\frac{3d^2(a+bx)^2}{(c+dx)^2}-\frac{6bd(a+bx)}{c+dx}\right)}{3(bc-ad)^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
[Out] (-2*(c + d*x)^(3/2)*(b^2 - (3*d^2*(a + b*x)^2)/(c + d*x)^2 - (6*b*d*(a + b*
x))/(c + d*x)))/(3*(b*c - a*d)^3*(a + b*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{2\left(8b^2x^2d^2+12ab\,d^2x+4b^2cdx+3a^2d^2+6abcd-b^2c^2\right)}{3(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105
default	$-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}-\frac{4d\left(-\frac{2}{(-ad+bc)\sqrt{bx+a}\,\sqrt{dx+c}}+\frac{4d\sqrt{bx+a}}{(-ad+bc)\sqrt{dx+c}\,(ad-bc)}\right)}{3(-ad+bc)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)
/(b*x+a)^(1/2)/(d*x+c)^(1/2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*
d-b*c))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(83) = 166$.
time = 0.41, size = 273, normalized size = 2.70

$$\frac{2 \left(8 b^{2} d^{2} x^{2} - b^{2} c^{2} + 6 abcd + 3 a^{2} d^{2} + 4 \left(b^{2} cd + 3 abd^{2}\right) x\right) \sqrt{bx + a} \sqrt{dx + c}}{3 \left(a^{2} b^{3} c^{4} - 3 a^{3} b^{2} c^{3} d + 3 a^{4} b c^{2} d^{2} - a^{5} c d^{3} + \left(b^{5} c^{3} d - 3 a b^{4} c^{2} d^{2} + 3 a^{2} b^{3} c d^{3} - a^{3} b^{2} d^{4}\right) x^{3} + \left(b^{5} c^{4} - a b^{4} c^{3} d - 3 a^{2} b^{3} c^{2} d^{2} + 5 a^{3} b^{2} c d^{3} - 2 a^{4} b d^{4}\right) x^{2} + \left(2 a b^{4} c^{4} - 5 a^{2} b^{3} c^{3} d + 3 a^{3} b^{2} c^{2} d^{2} + a^{4} b c d^{3} - a^{5} d^{4}\right) x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*d^2*x^2 - b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2 + 4*(b^2*c*d + 3*a*b*d
^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b
*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3
*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 +
 a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{\frac{5}{2}} \left(c + dx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(83) = 166.
time = 0.07, size = 433, normalized size = 4.29

$$2\left(\frac{2b^2d^2\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(2b^3c^3\,|b|-6b^2dac^2\,|b|+6bd^2a^2c\,|b|-2d^3a^3\,|b|)\,(-abd+b^2c+bd\,(a+bx))}+\frac{2\left(-3b^2d\sqrt{bd}\,\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^4-12b^3d^2\sqrt{bd}\,\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2a+12b^4d\sqrt{bd}\,\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2c-5b^4d^3\sqrt{bd}\,a^2+10b^5d^2\sqrt{bd}\,ac-5b^6d\sqrt{bd}\,c^2\right)}{(3d^2a^2\,|b|-6bdac\,|b|+3b^2c^2\,|b|)\left(\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2+bda-b^2c\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^2/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c
*d^2*abs(b) - a^3*d^3*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/3*(5
*sqrt(b*d)*b^6*c^2*d - 10*sqrt(b*d)*a*b^5*c*d^2 + 5*sqrt(b*d)*a^2*b^4*d^3 -
 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^4*c*d + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^2*a*b^3*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^2*d)/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b
) + a^2*d^2*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)^3)
```

Mupad [B]
time = 1.06, size = 141, normalized size = 1.40

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{8\,x\,\left(3\,a\,d+b\,c\right)}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,b\,d\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{6\,a^2\,d^2+12\,a\,b\,c\,d-2\,b^2\,c^2}{3\,b\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a\,c\,\sqrt{a+b\,x}}{b\,d}+\frac{x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((8*x*(3*a*d + b*c))/(3*(a*d - b*c)^3) + (16*b*d*x^2)/(3*
(a*d - b*c)^3) + (6*a^2*d^2 - 2*b^2*c^2 + 12*a*b*c*d)/(3*b*d*(a*d - b*c)^3)
))/(x^2*(a + b*x)^(1/2) + (a*c*(a + b*x)^(1/2))/(b*d) + (x*(a*d + b*c)*(a +
 b*x)^(1/2))/(b*d))
```

3.1509 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}+\frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}-\frac{16d^2}{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}-\frac{32d}{5(bc-}$$

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(1/2)+4/5*d/(-a*d+b*c)^2/(b*x+a)^(3/2
)/(d*x+c)^(1/2)-16/5*d^2/(-a*d+b*c)^3/(b*x+a)^(1/2)/(d*x+c)^(1/2)-32/5*d^3*
(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}\,(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*Sqrt[c + d*x]) + (4*d)/(5*(b*c - a*d)^2*(
a + b*x)^(3/2)*Sqrt[c + d*x]) - (16*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqr
t[c + d*x]) - (32*d^3*Sqrt[a + b*x])/(5*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx}{5(bc-ad)}$$
$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} + \frac{(8d^2}{}$$
$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{}{5(bc}$$
$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{}{5(bc}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 0.68

$$-\frac{2(c+dx)^{5/2}\left(b^3+\frac{5d^3(a+bx)^3}{(c+dx)^3}+\frac{15bd^2(a+bx)^2}{(c+dx)^2}-\frac{5b^2d(a+bx)}{c+dx}\right)}{5(bc-ad)^4(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*(b^3 + (5*d^3*(a + b*x)^3)/(c + d*x)^3 + (15*b*d^2*(a +
 b*x)^2)/(c + d*x)^2 - (5*b^2*d*(a + b*x))/(c + d*x)))/(5*(b*c - a*d)^4*(a
+ b*x)^(5/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 145, normalized size = 1.07

method	result

default	$-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}-\frac{6d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}-\frac{4d\left(-\frac{2}{(-ad+bc)\sqrt{bx+a}\,\sqrt{dx+c}}+\frac{4d\sqrt{}}{(-ad+bc)\sqrt{}}\right.}{3(-ad+bc)}\right.}{5(-ad+bc)}$
gosper	$-\frac{2(16b^3x^3d^3+40d^3a\,x^2b^2+8b^3c\,d^2x^2+30a^2b\,d^3x+20a\,b^2c\,d^2x-2b^3c^2dx+5a^3d^3+15a^2bc\,d^2-5a\,b^2c^2d+b^3c^3)}{5(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(1/2)-6/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*
c)/(b*x+a)^(3/2)/(d*x+c)^(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(b*x+a)^(1/2
)/(d*x+c)^(1/2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(112) = 224$.
time = 0.60, size = 455, normalized size = 3.35

$$-\frac{2\left(16\,b^{3}d^{3}x^{3}+b^{3}c^{3}-5\,ab^{2}c^{2}d+15\,a^{2}bcd^{2}+5\,a^{3}d^{3}+8\left(b^{3}cd^{2}+5\,ab^{2}d^{3}\right)x^{2}-2\left(b^{3}c^{2}d-10\,ab^{2}cd^{2}-15\,a^{2}bd^{3}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{5\left(a^{3}b^{4}c^{5}-4\,a^{4}b^{3}c^{4}d+6\,a^{5}b^{2}c^{3}d^{2}-4\,a^{6}bc^{2}d^{3}+a^{7}cd^{4}+\left(b^{7}c^{4}d-4\,ab^{6}c^{3}d^{2}+6\,a^{2}b^{5}c^{2}d^{3}-4\,a^{3}b^{4}cd^{4}+a^{4}b^{3}d^{5}\right)x^{4}+\left(b^{7}c^{5}-ab^{6}c^{4}d-6\,a^{2}b^{5}c^{3}d^{2}+14\,a^{3}b^{4}c^{2}d^{3}-11\,a^{4}b^{3}cd^{4}+3\,a^{5}b^{2}d^{5}\right)x^{3}+3\left(ab^{6}c^{5}-3\,a^{2}b^{5}c^{4}d+2\,a^{3}b^{4}c^{3}d^{2}+2\,a^{4}b^{3}c^{2}d^{3}-3\,a^{5}b^{2}cd^{4}+a^{6}bd^{5}\right)x^{2}+\left(3\,a^{2}b^{5}c^{5}-11\,a^{3}b^{4}c^{4}d+14\,a^{4}b^{3}c^{3}d^{2}-6\,a^{5}b^{2}c^{2}d^{3}-a^{6}bcd^{4}+a^{7}d^{5}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
 + 8*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 10*a*b^2*c*d^2 - 15*a^2
*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a
^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d^2
 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*b^
6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5
*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*
```

```
b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(112) = 224.
time = 0.18, size = 955, normalized size = 7.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^3/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^
2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)) - 4/5*(11*sqrt(b*d)*b^10*c^4*d^2 - 44*sqrt(b*d)*a*b^9*c
^3*d^3 + 66*sqrt(b*d)*a^2*b^8*c^2*d^4 - 44*sqrt(b*d)*a^3*b^7*c*d^5 + 11*sqr
t(b*d)*a^4*b^6*d^6 - 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*b^8*c^3*d^2 + 150*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^7*c^2*d^3 - 150*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*
d^4 + 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a^3*b^5*d^5 + 80*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*b^6*c^2*d^2 - 160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^5*c*d^3 + 80*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^
4 - 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^6*b^4*c*d^2 + 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^3*d^3 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^2*d^2)/((b^3*c^3*abs(b) - 3*a*b^2*
c^2*d*abs(b) + 3*a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

Mupad [B]
time = 1.31, size = 227, normalized size = 1.67

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d\,x^2\,\left(5\,a\,d+b\,c\right)}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,a^3\,d^3+6\,a^2\,b\,c\,d^2-2\,a\,b^2\,c^2\,d+\frac{2\,b^3\,c^3}{5}}{b^2\,d\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b\,d^2\,x^3}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(15\,a^2\,d^2+10\,a\,b\,c\,d-b^2\,c^2\right)}{5\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^2\,c\,\sqrt{a+b\,x}}{b^2\,d}+\frac{x^2\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a\,x\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((16*d*x^2*(5*a*d + b*c))/(5*(a*d - b*c)^4) + (2*a^3*d^3
+ (2*b^3*c^3)/5 - 2*a*b^2*c^2*d + 6*a^2*b*c*d^2)/(b^2*d*(a*d - b*c)^4) + (3
2*b*d^2*x^3)/(5*(a*d - b*c)^4) + (4*x*(15*a^2*d^2 - b^2*c^2 + 10*a*b*c*d))/
(5*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^2*c*(a + b*x)^(1/2))/(b^2*d
) + (x^2*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a*x*(a*d + 2*b*c)*(a + b*x
)^(1/2))/(b^2*d))
```

3.1510 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}+\frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}-\frac{32d^2}{35(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}+$$

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(1/2)+16/35*d/(-a*d+b*c)^2/(b*x+a)^(5
/2)/(d*x+c)^(1/2)-32/35*d^2/(-a*d+b*c)^3/(b*x+a)^(3/2)/(d*x+c)^(1/2)+128/35
*d^3/(-a*d+b*c)^4/(b*x+a)^(1/2)/(d*x+c)^(1/2)+256/35*d^4*(b*x+a)^(1/2)/(-a*
d+b*c)^5/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}\,(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x]) + (16*d)/(35*(b*c - a*d)^2
*(a + b*x)^(5/2)*Sqrt[c + d*x]) - (32*d^2)/(35*(b*c - a*d)^3*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + (128*d^3)/(35*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (256*d^4*Sqrt[a + b*x])/(35*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} + \frac{(48d}{}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{}{35(b}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{}{35(b}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{}{35(b}$$

Mathematica [A]
time = 0.14, size = 117, normalized size = 0.68

$$-\frac{2(c+dx)^{7/2}\left(5b^4 - \frac{35d^4(a+bx)^4}{(c+dx)^4} - \frac{140bd^3(a+bx)^3}{(c+dx)^3} + \frac{70b^2d^2(a+bx)^2}{(c+dx)^2} - \frac{28b^3d(a+bx)}{c+dx}\right)}{35(bc-ad)^5(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(5*b^4 - (35*d^4*(a + b*x)^4)/(c + d*x)^4 - (140*b*d^3*
(a + b*x)^3)/(c + d*x)^3 + (70*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 - (28*b^3*d
*(a + b*x))/(c + d*x)))/(35*(b*c - a*d)^5*(a + b*x)^(7/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 185, normalized size = 1.08

method	result

default	$-\frac{2}{7(-ad+bc)(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}-\frac{8d\left(-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}-\frac{6d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}-\frac{4d\left(-\frac{\cdots}{(-ad+b\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{7(-ad+bc)\cdots}$
gosper	$-\frac{2\left(128d^4x^4b^4+448a\,b^3d^4x^3+64b^4c\,d^3x^3+560a^2b^2d^4x^2+224a\,b^3c\,d^3x^2-16b^4c^2d^2x^2+280a^3b\,d^4x+280a^2b^2c\,d^3x-56a\,b^3c^2d^2x+8b^4\cdots\right.}{35(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\,\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(1/2)-8/7*d/(-a*d+b*c)*(-2/5/(-a*d+b*
c)/(b*x+a)^(5/2)/(d*x+c)^(1/2)-6/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*c)/(b*x+a)^(3
/2)/(d*x+c)^(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/
2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(141) = 282$.
time = 1.50, size = 689, normalized size = 4.03

$$\frac{2\,(128\,b^4d^4x^4-5\,b^4c^4+28\,ab^3c^3d-70\,a^2b^2c^2d^2+140\,a^3bcd^3+35\,a^4d^4+64\,(b^4cd^3+7\,ab^3d^4)x^3-16\,(b^4c^2d^2-14\,ab^3cd^3-35\,a^2b^2d^4)x^2+8\,(b^4c^3d-7\,ab^3c^2d^2+35\,a^2b^2cd^3+35\,a^3bd^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{35\,(a^4b^5c^6-5\,a^5b^4c^5d+10\,a^6b^3c^4d^2-10\,a^7b^2c^3d^3+5\,a^8bc^2d^4-a^9cd^5+(b^9c^5d-5\,ab^8c^4d^2+10\,a^2b^7c^3d^3-10\,a^3b^6c^2d^4+5\,a^4b^5cd^5-a^5b^4d^6)x^5+(b^9c^6-ab^8c^5d-10\,a^2b^7c^4d^2+30\,a^3b^6c^3d^3-35\,a^4b^5c^2d^4+19\,a^5b^4cd^5-4\,a^6b^3d^6)x^4+2\,(2\,ab^8c^6-7\,a^2b^7c^5d+5\,a^3b^6c^4d^2+10\,a^4b^5c^3d^3-20\,a^5b^4c^2d^4+13\,a^6b^3cd^5-3\,a^7b^2d^6)x^3+2\,(3\,a^2b^7c^6-13\,a^3b^6c^5d+20\,a^4b^5c^4d^2-10\,a^5b^4c^3d^3-5\,a^6b^3c^2d^4+7\,a^7b^2cd^5-2\,a^8bd^6)x^2+(4\,a^3b^6c^6-19\,a^4b^5c^5d+35\,a^5b^4c^4d^2-30\,a^6b^3c^3d^3+10\,a^7b^2c^2d^4+a^8bcd^5-a^9d^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(128*b^4*d^4*x^4 - 5*b^4*c^4 + 28*a*b^3*c^3*d - 70*a^2*b^2*c^2*d^2 + 1
40*a^3*b*c*d^3 + 35*a^4*d^4 + 64*(b^4*c*d^3 + 7*a*b^3*d^4)*x^3 - 16*(b^4*c^
2*d^2 - 14*a*b^3*c*d^3 - 35*a^2*b^2*d^4)*x^2 + 8*(b^4*c^3*d - 7*a*b^3*c^2*d
```

```
^2 + 35*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b
^5*c^6 - 5*a^5*b^4*c^5*d + 10*a^6*b^3*c^4*d^2 - 10*a^7*b^2*c^3*d^3 + 5*a^8*
b*c^2*d^4 - a^9*c*d^5 + (b^9*c^5*d - 5*a*b^8*c^4*d^2 + 10*a^2*b^7*c^3*d^3 -
 10*a^3*b^6*c^2*d^4 + 5*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^5 + (b^9*c^6 - a*b^8
*c^5*d - 10*a^2*b^7*c^4*d^2 + 30*a^3*b^6*c^3*d^3 - 35*a^4*b^5*c^2*d^4 + 19*
a^5*b^4*c*d^5 - 4*a^6*b^3*d^6)*x^4 + 2*(2*a*b^8*c^6 - 7*a^2*b^7*c^5*d + 5*a
^3*b^6*c^4*d^2 + 10*a^4*b^5*c^3*d^3 - 20*a^5*b^4*c^2*d^4 + 13*a^6*b^3*c*d^5
 - 3*a^7*b^2*d^6)*x^3 + 2*(3*a^2*b^7*c^6 - 13*a^3*b^6*c^5*d + 20*a^4*b^5*c^
4*d^2 - 10*a^5*b^4*c^3*d^3 - 5*a^6*b^3*c^2*d^4 + 7*a^7*b^2*c*d^5 - 2*a^8*b*
d^6)*x^2 + (4*a^3*b^6*c^6 - 19*a^4*b^5*c^5*d + 35*a^5*b^4*c^4*d^2 - 30*a^6*
b^3*c^3*d^3 + 10*a^7*b^2*c^2*d^4 + a^8*b*c*d^5 - a^9*d^6)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. 2(141) = 282.
time = 0.39, size = 1731, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x)
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^4/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^
3*c^3*d^2*abs(b) - 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d
^5*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/35*(93*sqrt(b*d)*b^14*c
^6*d^3 - 558*sqrt(b*d)*a*b^13*c^5*d^4 + 1395*sqrt(b*d)*a^2*b^12*c^4*d^5 - 1
860*sqrt(b*d)*a^3*b^11*c^3*d^6 + 1395*sqrt(b*d)*a^4*b^10*c^2*d^7 - 558*sqrt
(b*d)*a^5*b^9*c*d^8 + 93*sqrt(b*d)*a^6*b^8*d^9 - 616*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^12*c^5*d^3 + 3080*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a*b^11*c^4*d^4 - 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^2*a^2*b^10*c^3*d^5 + 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^9*c^2*d^6 - 3080*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4
*b^8*c*d^7 + 616*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^5*b^7*d^8 + 1673*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^4*d^3 - 6692*sqrt(b*d)*(sqrt(b
```

```
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^9*c^3*d^4 +
10038*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*a^2*b^8*c^2*d^5 - 6692*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c*d^6 + 1673*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^6*d^7 - 2240*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^3*d^3 + 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^6*a*b^7*c^2*d^4 - 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c*d^5 + 2240*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^5*d^6 +
 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^8*b^6*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^8*a*b^5*c*d^4 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*d^5 - 280*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^4*c*d^
3 + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^10*a*b^3*d^4 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^12*b^2*d^3)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b)
 + 6*a^2*b^2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c
 - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2)^7)
```

Mupad [B]
time = 1.50, size = 337, normalized size = 1.97

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{256\,b\,d^3\,x^4}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^2\,x^3\,\left(7\,a\,d+b\,c\right)}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{70\,a^4\,d^4+280\,a^3\,b\,c\,d^3-140\,a^2\,b^2\,c^2\,d^2+56\,a\,b^3\,c^3\,d-10\,b^4\,c^4}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(560\,a^3\,b\,d^4+560\,a^2\,b^2\,c\,d^3-112\,a\,b^3\,c^2\,d^2+16\,b^4\,c^3\,d\right)}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d\,x^2\,\left(35\,a^2\,d^2+14\,a\,b\,c\,d-b^2\,c^2\right)}{35\,b\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^3\,c\,\sqrt{a+b\,x}}{b^3\,d}+\frac{x^3\,\left(3\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{3\,a\,x^2\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^2\,x\,\left(a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((256*b*d^3*x^4)/(35*(a*d - b*c)^5) + (128*d^2*x^3*(7*a*d
 + b*c))/(35*(a*d - b*c)^5) + (70*a^4*d^4 - 10*b^4*c^4 - 140*a^2*b^2*c^2*d^
2 + 56*a*b^3*c^3*d + 280*a^3*b*c*d^3)/(35*b^3*d*(a*d - b*c)^5) + (x*(560*a^
3*b*d^4 + 16*b^4*c^3*d - 112*a*b^3*c^2*d^2 + 560*a^2*b^2*c*d^3))/(35*b^3*d*
(a*d - b*c)^5) + (32*d*x^2*(35*a^2*d^2 - b^2*c^2 + 14*a*b*c*d))/(35*b*(a*d
- b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^3*c*(a + b*x)^(1/2))/(b^3*d) + (x^3*(
3*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (3*a*x^2*(a*d + b*c)*(a + b*x)^(1/2))
/(b^2*d) + (a^2*x*(a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

3.1511 $\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}}+\frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}-\frac{32d^2}{63(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}+\frac{\cdots}{63(\cdots}$$

```
[Out] -2/9/(-a*d+b*c)/(b*x+a)^(9/2)/(d*x+c)^(1/2)+20/63*d/(-a*d+b*c)^2/(b*x+a)^(7
/2)/(d*x+c)^(1/2)-32/63*d^2/(-a*d+b*c)^3/(b*x+a)^(5/2)/(d*x+c)^(1/2)+64/63*
d^3/(-a*d+b*c)^4/(b*x+a)^(3/2)/(d*x+c)^(1/2)-256/63*d^4/(-a*d+b*c)^5/(b*x+a
)^(1/2)/(d*x+c)^(1/2)-512/63*d^5*(b*x+a)^(1/2)/(-a*d+b*c)^6/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{20d}{63(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{9(a+bx)^{9/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) - (256*d^4)/(63*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x]) - (512*d^5*Sqr
t[a + b*x])/(63*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx = -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} + \frac{(8\cdots}{\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

Mathematica [A]
time = 0.13, size = 139, normalized size = 0.67

$$-\frac{2(c+dx)^{9/2}\left(7b^5+\frac{63d^5(a+bx)^5}{(c+dx)^5}+\frac{315bd^4(a+bx)^4}{(c+dx)^4}-\frac{210b^2d^3(a+bx)^3}{(c+dx)^3}+\frac{126b^3d^2(a+bx)^2}{(c+dx)^2}-\frac{45b^4d(a+bx)}{c+dx}\right)}{63(bc-ad)^6(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(7*b^5 + (63*d^5*(a + b*x)^5)/(c + d*x)^5 + (315*b*d^4*
(a + b*x)^4)/(c + d*x)^4 - (210*b^2*d^3*(a + b*x)^3)/(c + d*x)^3 + (126*b^3
*d^2*(a + b*x)^2)/(c + d*x)^2 - (45*b^4*d*(a + b*x))/(c + d*x)))/(63*(b*c -
 a*d)^6*(a + b*x)^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 225, normalized size = 1.09

method	result
default	$-\frac{2}{9(-ad+bc)(bx+a)^{\frac{9}{2}}\sqrt{dx+c}}-\frac{10d\left(-\frac{2}{7(-ad+bc)(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}-\frac{8d\left(-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}-\frac{6d\left(-\frac{}{3(-ad+b}\right.}{}\right)}{}\right)}{}$
gosper	$-\frac{2(256b^5d^5x^5+1152a\,b^4d^5x^4+128b^5c\,d^4x^4+2016a^2b^3d^5x^3+576a\,b^4c\,d^4x^3-32b^5c^2d^3x^3+1680a^3b^2d^5x^2+1008a^2b^3c\,d^4x^2-144a\,b^4c^2}{63(bx+a)^{\frac{9}{2}}\sqrt{dx+c}\,(a^6d^6-6a^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/(-a*d+b*c)/(b*x+a)^(9/2)/(d*x+c)^(1/2)-10/9*d/(-a*d+b*c)*(-2/7/(-a*d+b
*c)/(b*x+a)^(7/2)/(d*x+c)^(1/2)-8/7*d/(-a*d+b*c)*(-2/5/(-a*d+b*c)/(b*x+a)^(
5/2)/(d*x+c)^(1/2)-6/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^
(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+4*d/(-a*d
+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(170) = 340$.
time = 3.59, size = 955, normalized size = 4.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(256*b^5*d^5*x^5 + 7*b^5*c^5 - 45*a*b^4*c^4*d + 126*a^2*b^3*c^3*d^2 -
 210*a^3*b^2*c^2*d^3 + 315*a^4*b*c*d^4 + 63*a^5*d^5 + 128*(b^5*c*d^4 + 9*a*
b^4*d^5)*x^4 - 32*(b^5*c^2*d^3 - 18*a*b^4*c*d^4 - 63*a^2*b^3*d^5)*x^3 + 16*
(b^5*c^3*d^2 - 9*a*b^4*c^2*d^3 + 63*a^2*b^3*c*d^4 + 105*a^3*b^2*d^5)*x^2 -
2*(5*b^5*c^4*d - 36*a*b^4*c^3*d^2 + 126*a^2*b^3*c^2*d^3 - 420*a^3*b^2*c*d^4
 - 315*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^6*c^7 - 6*a^6*b^5*c
^6*d + 15*a^7*b^4*c^5*d^2 - 20*a^8*b^3*c^4*d^3 + 15*a^9*b^2*c^3*d^4 - 6*a^1
0*b*c^2*d^5 + a^11*c*d^6 + (b^11*c^6*d - 6*a*b^10*c^5*d^2 + 15*a^2*b^9*c^4*
d^3 - 20*a^3*b^8*c^3*d^4 + 15*a^4*b^7*c^2*d^5 - 6*a^5*b^6*c*d^6 + a^6*b^5*d
^7)*x^6 + (b^11*c^7 - a*b^10*c^6*d - 15*a^2*b^9*c^5*d^2 + 55*a^3*b^8*c^4*d^
3 - 85*a^4*b^7*c^3*d^4 + 69*a^5*b^6*c^2*d^5 - 29*a^6*b^5*c*d^6 + 5*a^7*b^4*
d^7)*x^5 + 5*(a*b^10*c^7 - 4*a^2*b^9*c^6*d + 3*a^3*b^8*c^5*d^2 + 10*a^4*b^7
*c^4*d^3 - 25*a^5*b^6*c^3*d^4 + 24*a^6*b^5*c^2*d^5 - 11*a^7*b^4*c*d^6 + 2*a
^8*b^3*d^7)*x^4 + 10*(a^2*b^9*c^7 - 5*a^3*b^8*c^6*d + 9*a^4*b^7*c^5*d^2 - 5
*a^5*b^6*c^4*d^3 - 5*a^6*b^5*c^3*d^4 + 9*a^7*b^4*c^2*d^5 - 5*a^8*b^3*c*d^6
+ a^9*b^2*d^7)*x^3 + 5*(2*a^3*b^8*c^7 - 11*a^4*b^7*c^6*d + 24*a^5*b^6*c^5*d
^2 - 25*a^6*b^5*c^4*d^3 + 10*a^7*b^4*c^3*d^4 + 3*a^8*b^3*c^2*d^5 - 4*a^9*b^
2*c*d^6 + a^10*b*d^7)*x^2 + (5*a^4*b^7*c^7 - 29*a^5*b^6*c^6*d + 69*a^6*b^5*
c^5*d^2 - 85*a^7*b^4*c^4*d^3 + 55*a^8*b^3*c^3*d^4 - 15*a^9*b^2*c^2*d^5 - a^
10*b*c*d^6 + a^11*d^7)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2438 vs. $2(170) = 340$.
time = 0.77, size = 2771, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^5/((b^6*c^6*abs(b) - 6*a*b^5*c^5*d*abs(b) + 15*a^2*b
^4*c^4*d^2*abs(b) - 20*a^3*b^3*c^3*d^3*abs(b) + 15*a^4*b^2*c^2*d^4*abs(b) -
 6*a^5*b*c*d^5*abs(b) + a^6*d^6*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
```

) - 4/63*(193*sqrt(b*d)*b^18*c^8*d^4 - 1544*sqrt(b*d)*a*b^17*c^7*d^5 + 5404
*sqrt(b*d)*a^2*b^16*c^6*d^6 - 10808*sqrt(b*d)*a^3*b^15*c^5*d^7 + 13510*sqrt
(b*d)*a^4*b^14*c^4*d^8 - 10808*sqrt(b*d)*a^5*b^13*c^3*d^9 + 5404*sqrt(b*d)*
a^6*b^12*c^2*d^10 - 1544*sqrt(b*d)*a^7*b^11*c*d^11 + 193*sqrt(b*d)*a^8*b^10
*d^12 - 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^16*c^7*d^4 + 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^6*d^5 - 35154*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^14*c^5*d^
6 + 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^13*c^4*d^7 - 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^3*d^8 + 35154*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^11*c^2
*d^9 - 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*a^6*b^10*c*d^10 + 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^9*d^11 + 6318*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^6*d^4 - 3
7908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^13*c^5*d^5 + 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^4*d^6 - 126360*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^3*d^7 +
 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^4*a^4*b^10*c^2*d^8 - 37908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c*d^9 + 6318*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*d^10 - 1331
4*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^6*b^12*c^5*d^4 + 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^6*a*b^11*c^4*d^5 - 133140*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^3*d^6 + 133140
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a^3*b^9*c^2*d^7 - 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c*d^8 + 13314*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*d^9 + 16128*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10
*c^4*d^4 - 64512*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*a*b^9*c^3*d^5 + 96768*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^2*d^6 - 64512*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c
*d^7 + 16128*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^8*a^4*b^6*d^8 - 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^3*d^4 + 24570*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^2*d^5 - 24
570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^10*a^2*b^6*c*d^6 + 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*d^7 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^2*d^4 - 5796*sqrt(b

```
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^
5*c*d^5 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^12*a^2*b^4*d^6 - 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c*d^4 + 630*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*d^5 + 63*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2
*d^4)/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) -
 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c
- a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
)^9)
```

Mupad [B]
time = 1.96, size = 454, normalized size = 2.20

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^5\,d^5+630\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+252\,a^2\,b^3\,c^3\,d^2-90\,a\,b^4\,c^4\,d+14\,b^5\,c^5}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b\,d^4\,x^5}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,d^3\,x^4\,\left(9\,a\,d+b\,c\right)}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(1260\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4-504\,a^2\,b^3\,c^2\,d^3+144\,a\,b^4\,c^3\,d^2-20\,b^5\,c^4\,d\right)}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d^2\,x^3\,\left(63\,a^2\,d^2+18\,a\,b\,c\,d-b^2\,c^2\right)}{63\,b\,{\left(a\,d-b\,c\right)}^6}+\frac{32\,d\,x^2\,\left(105\,a^3\,d^3+63\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{63\,b^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^4\,c\,\sqrt{a+b\,x}}{b^4\,d}+\frac{x^4\,\left(4\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{2\,a\,x^3\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^3\,x\,\left(a\,d+4\,b\,c\right)\,\sqrt{a+b\,x}}{b^4\,d}+\frac{2\,a^2\,x^2\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((126*a^5*d^5 + 14*b^5*c^5 + 252*a^2*b^3*c^3*d^2 - 420*a^
3*b^2*c^2*d^3 - 90*a*b^4*c^4*d + 630*a^4*b*c*d^4)/(63*b^4*d*(a*d - b*c)^6)
+ (512*b*d^4*x^5)/(63*(a*d - b*c)^6) + (256*d^3*x^4*(9*a*d + b*c))/(63*(a*d
 - b*c)^6) + (x*(1260*a^4*b*d^5 - 20*b^5*c^4*d + 144*a*b^4*c^3*d^2 + 1680*a
^3*b^2*c*d^4 - 504*a^2*b^3*c^2*d^3))/(63*b^4*d*(a*d - b*c)^6) + (64*d^2*x^3
*(63*a^2*d^2 - b^2*c^2 + 18*a*b*c*d))/(63*b*(a*d - b*c)^6) + (32*d*x^2*(105
*a^3*d^3 + b^3*c^3 - 9*a*b^2*c^2*d + 63*a^2*b*c*d^2))/(63*b^2*(a*d - b*c)^6
)))/(x^5*(a + b*x)^(1/2) + (a^4*c*(a + b*x)^(1/2))/(b^4*d) + (x^4*(4*a*d +
b*c)*(a + b*x)^(1/2))/(b*d) + (2*a*x^3*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b^
2*d) + (a^3*x*(a*d + 4*b*c)*(a + b*x)^(1/2))/(b^4*d) + (2*a^2*x^2*(2*a*d +
3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

3.1512 $\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}}-\frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}}+\frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5}-\frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4}+\frac{7b}{}$$

```
[Out] -2/3*(b*x+a)^(9/2)/d/(d*x+c)^(3/2)-105/8*b^(3/2)*(-a*d+b*c)^3*arctanh(d^(1/
2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(11/2)-6*b*(b*x+a)^(7/2)/d^2/(d*x
+c)^(1/2)-35/4*b^2*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^4+7*b^2*(b*x+a)
^(5/2)*(d*x+c)^(1/2)/d^3+105/8*b^2*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)
/d^5
```

Rubi [A]

time = 0.07, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{11/2}}+\frac{105b^2\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^5}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}-\frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(9/2))/(3*d*(c + d*x)^(3/2)) - (6*b*(a + b*x)^(7/2))/(d^2*Sqr
t[c + d*x]) + (105*b^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^5) -
 (35*b^2*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(4*d^4) + (7*b^2*(a + b
*x)^(5/2)*Sqrt[c + d*x])/d^3 - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(11/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} + \frac{(3b)\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{(21b^2)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3} - \frac{(35b^2(bc-ad))\int \frac{(a+bx)^{3/}}{\sqrt{c+d}}}{2d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} + \frac{7b^2(a+bx)^{5/2}\sqrt{}}{d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

Mathematica [A]
time = 0.37, size = 165, normalized size = 0.81

$$-\frac{(a+bx)^{9/2}\left(16d^4+\frac{144bd^3(c+dx)}{a+bx}-\frac{693b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{840b^3d(c+dx)^3}{(a+bx)^3}-\frac{315b^4(c+dx)^4}{(a+bx)^4}\right)}{24d^5(c+dx)^{3/2}}-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{8d^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/24*((a + b*x)^(9/2)*(16*d^4 + (144*b*d^3*(c + d*x))/(a + b*x) - (693*b^2
*d^2*(c + d*x)^2)/(a + b*x)^2 + (840*b^3*d*(c + d*x)^3)/(a + b*x)^3 - (315*
b^4*(c + d*x)^4)/(a + b*x)^4))/(d^5*(c + d*x)^(3/2)) - (105*b^(3/2)*(b*c -
a*d)^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*d^(11/2
))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{9}{2}}}{(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(164) = 328.
time = 0.82, size = 879, normalized size = 4.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 +
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*
```

```
c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 - a^3*b*c*d^4)*x)*sqrt(b/d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d
^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5), 1/48*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a
^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2
*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^
3 - a^3*b*c*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)]
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(164) = 328.
time = 0.10, size = 613, normalized size = 3.00

$$\frac{2\left(\left(\left(\left(\left(-\frac{\left(-216b^6d^8c+216b^5d^9a\right)\sqrt{a+bx}\,\sqrt{a+bx}}{1296b^2d^9|b|c-1296bd^{10}|b|a}-\frac{486b^7d^7c^2-972b^6d^8ac+486b^5d^9a^2}{1296b^2d^9|b|c-1296bd^{10}|b|a}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{-1701b^8d^6c^3+5103b^7d^7ac^2-5103b^6d^8a^2c+1701b^5d^9a^3}{1296b^2d^9|b|c-1296bd^{10}|b|a}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{-11340b^9d^5c^4+45360b^8d^6ac^3-68040b^7d^7a^2c^2+45360b^6d^8a^3c-11340b^5d^9a^4}{1296b^2d^9|b|c-1296bd^{10}|b|a}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{-8505b^{10}d^4c^5+42525b^9d^5ac^4-85050b^8d^6a^2c^3+85050b^7d^7a^3c^2-42525b^6d^8a^4c+8505b^5d^9a^5}{1296b^2d^9|b|c-1296bd^{10}|b|a}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\left(a+bx\right)}\right)}{\left(-abd+b^2c+bd\left(a+bx\right)\right)^2}+\frac{2\left(-105a^3b^3d^3+315a^2b^4cd^2-315ab^5c^2d+105b^6c^3\right)\ln\left|\sqrt{-abd+b^2c+bd\left(a+bx\right)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{16d^5\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] 1/24*(((2*(b*x + a)*(4*(b^6*c*d^8 - a*b^5*d^9)*(b*x + a)/(b^2*c*d^9*abs(b)
- a*b*d^10*abs(b)) - 9*(b^7*c^2*d^7 - 2*a*b^6*c*d^8 + a^2*b^5*d^9)/(b^2*c*d
^9*abs(b) - a*b*d^10*abs(b))) + 63*(b^8*c^3*d^6 - 3*a*b^7*c^2*d^7 + 3*a^2*b
^6*c*d^8 - a^3*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 4
20*(b^9*c^4*d^5 - 4*a*b^8*c^3*d^6 + 6*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 + a
^4*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 315*(b^10*c^5
*d^4 - 5*a*b^9*c^4*d^5 + 10*a^2*b^8*c^3*d^6 - 10*a^3*b^7*c^2*d^7 + 5*a^4*b^
6*c*d^8 - a^5*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*sqrt(b*x + a)/
(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 105/8*(b^6*c^3 - 3*a*b^5*c^2*d + 3*
```

```
a^2*b^4*c*d^2 - a^3*b^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^5*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{9/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/(c + d*x)^(5/2),x)
[Out] int((a + b*x)^(9/2)/(c + d*x)^(5/2), x)
```

3.1513 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=170

$$-\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}+\frac{35b^{3/2}(bc-a}{}$$

[Out] -2/3*(b*x+a)^(7/2)/d/(d*x+c)^(3/2)+35/4*b^(3/2)*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(9/2)-14/3*b*(b*x+a)^(5/2)/d^2/(d*x+c)^(1/2)+35/6*b^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^3-35/4*b^2*(-a*d+b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^4

Rubi [A]
time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]

[Out] (-2*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/2)) - (14*b*(a + b*x)^(5/2))/(3*d^2*Sqrt[c + d*x]) - (35*b^2*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^4) + (35*b^2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(6*d^3) + (35*b^(3/2)*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(9/2))

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{(35b^2)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} - \frac{(35b^2(bc-ad))\int \frac{\sqrt{a+}}{\sqrt{c+}}}{4d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}}{6d^3}$$

Mathematica [A]
time = 0.25, size = 166, normalized size = 0.98

$$-\frac{\sqrt{a+bx}\,\left(8a^3d^3+8a^2bd^2(7c+10dx)-ab^2d\left(175c^2+238cdx+39d^2x^2\right)+b^3\left(105c^3+140c^2dx+21cd^2x^2-6d^3x^3\right)\right)}{12d^4(c+dx)^{3/2}} + \frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[a + b*x]*(8*a^3*d^3 + 8*a^2*b*d^2*(7*c + 10*d*x) - a*b^2*d*(175
*c^2 + 238*c*d*x + 39*d^2*x^2) + b^3*(105*c^3 + 140*c^2*d*x + 21*c*d^2*x^2
- 6*d^3*x^3)))/(d^4*(c + d*x)^(3/2)) + (35*b^(3/2)*(b*c - a*d)^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*d^(9/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(132) = 264.
time = 0.57, size = 657, normalized size = 3.86

$$\left[\frac{105\,(b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 + (b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4)x^2 + 2\,(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bd^2x + bcd + ad^2)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}} + 8\,(b^2cd + abd^2)x\right) + 4\,(6\,b^3d^3x^3 - 105\,b^3c^3 + 175\,ab^2c^2d - 56\,a^2bcd^2 - 8\,a^3d^3 - 3\,(7\,b^3cd^2 - 13\,ab^2d^3)x^2 - 2\,(70\,b^3c^2d - 119\,ab^2cd^2 + 40\,a^2bd^3)x)\sqrt{bx+a}\sqrt{dx+c}}{48\,(d^6x^2 + 2\,cd^5x + c^2d^4)}, -\frac{105\,(b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 + (b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4)x^2 + 2\,(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx + bc + ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2 + abc + (b^2c + abd)x)}\right) - 2\,(6\,b^3d^3x^3 - 105\,b^3c^3 + 175\,ab^2c^2d - 56\,a^2bcd^2 - 8\,a^3d^3 - 3\,(7\,b^3cd^2 - 13\,ab^2d^3)x^2 - 2\,(70\,b^3c^2d - 119\,ab^2cd^2 + 40\,a^2bd^3)x)\sqrt{bx+a}\sqrt{dx+c}}{24\,(d^6x^2 + 2\,cd^5x + c^2d^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3*c^2*d^2 - 2*a*b^
2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3)*x)
*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x
 + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(6*b^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2
 - 8*a^3*d^3 - 3*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a
*b^2*c*d^2 + 40*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d
```

```
^5*x + c^2*d^4), -1/24*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3
*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2
+ a^2*b*c*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(6*b
^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2 - 8*a^3*d^3 - 3
*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a*b^2*c*d^2 + 40*
a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)]
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(132) = 264.
time = 0.08, size = 475, normalized size = 2.79

$$\frac{2\left(\left(\left(\left(-\frac{\left(-18b^6d^6c+18b^5d^7a\right)\sqrt{a+bx}\,\sqrt{a+bx}}{72b^2d^7|b|c-72bd^8|b|a}-\frac{63b^7d^5c^2-126b^6d^6ac+63b^5d^7a^2}{72b^2d^7|b|c-72bd^8|b|a}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{420b^8d^4c^3-1260b^7d^5ac^2+1260b^6d^6a^2c-420b^5d^7a^3}{72b^2d^7|b|c-72bd^8|b|a}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{315b^9d^3c^4-1260b^8d^4ac^3+1890b^7d^5a^2c^2-1260b^6d^6a^3c+315b^5d^7a^4}{72b^2d^7|b|c-72bd^8|b|a}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{\left(-abd+b^2c+bd\,(a+bx)\right)^2}+\frac{2\left(-35a^2b^3d^2+70ab^4cd-35b^5c^2\right)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{8d^4\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] 1/12*((3*(b*x + a)*(2*(b^6*c*d^6 - a*b^5*d^7)*(b*x + a)/(b^2*c*d^7*abs(b) -
 a*b*d^8*abs(b)) - 7*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)/(b^2*c*d^7
*abs(b) - a*b*d^8*abs(b))) - 140*(b^8*c^3*d^4 - 3*a*b^7*c^2*d^5 + 3*a^2*b^6
*c*d^6 - a^3*b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*(b*x + a) - 105*
(b^9*c^4*d^3 - 4*a*b^8*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 4*a^3*b^6*c*d^6 + a^4*
b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x +
 a)*b*d - a*b*d)^(3/2) - 35/4*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*log(abs
(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d
)*d^4*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(5/2), x)
```

3.1514 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{7/2}}$$

```
[Out] -2/3*(b*x+a)^(5/2)/d/(d*x+c)^(3/2)-5*b^(3/2)*(-a*d+b*c)*arctanh(d^(1/2)*(b*
x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(7/2)-10/3*b*(b*x+a)^(3/2)/d^2/(d*x+c)^
(1/2)+5*b^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^3
```

Rubi [A]

time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{7/2}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*(a + b*x)^(3/2))/(3*d^2*
Sqrt[c + d*x]) + (5*b^2*Sqrt[a + b*x]*Sqrt[c + d*x])/d^3 - (5*b^(3/2)*(b*c
- a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} + \frac{(5b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{(5b^2)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{(5b^2(bc-ad))\int \frac{1}{\sqrt{a+bx}\ldots}}{2d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{(5b(bc-ad))\text{Subst}\left(\int \frac{\ldots}{\sqrt{\ldots}}\ldots\right)}{\ldots}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{(5b(bc-ad))\text{Subst}\left(\int \frac{1}{1-\ldots}\ldots\right)}{d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\ldots\right)}{d^{7/2}}$$

Mathematica [A]

time = 0.65, size = 123, normalized size = 0.96

$$\frac{\frac{\sqrt{a+bx}\left(-2a^2d^2-2abd(5c+7dx)+b^2(15c^2+20cdx+3d^2x^2)\right)}{(c+dx)^{3/2}}+15b\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] ((Sqrt[a + b*x]*(-2*a^2*d^2 - 2*a*b*d*(5*c + 7*d*x) + b^2*(15*c^2 + 20*c*d*
x + 3*d^2*x^2)))/(c + d*x)^(3/2) + 15*b*Sqrt[b/d]*(b*c - a*d)*Log[Sqrt[a +
b*x] - Sqrt[b/d]*Sqrt[c + d*x]])/(3*d^3)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(100) = 200$.
time = 0.45, size = 475, normalized size = 3.71

$$\left[-\frac{15\,(b^2c^3-abc^2d+(b^2cd^2-abd^3)x^2+2\,(b^2c^2d-abcd^2)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,(3\,b^2d^2x^2+15\,b^2c^2-10\,abcd-2\,a^2d^2+2\,(10\,b^2cd-7\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{12\,(d^5x^2+2\,cd^4x+c^2d^3)},\ \frac{15\,(b^2c^3-abc^2d+(b^2cd^2-abd^3)x^2+2\,(b^2c^2d-abcd^2)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,(3\,b^2d^2x^2+15\,b^2c^2-10\,abcd-2\,a^2d^2+2\,(10\,b^2cd-7\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{6\,(d^5x^2+2\,cd^4x+c^2d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3)*x^2 + 2*(b^2*c^2*d
- a*b*c*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2
*d^2 + 2*(10*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2
+ 2*c*d^4*x + c^2*d^3), 1/6*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3
)*x^2 + 2*(b^2*c^2*d - a*b*c*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c +
 a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c +
a*b*d)*x)) + 2*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2*d^2 + 2*(10
*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(100) = 200$.
time = 0.07, size = 353, normalized size = 2.76

$$\frac{2\left(\left(-\frac{(-9b^6d^4c+9b^5d^5a)\sqrt{a+bx}\,\sqrt{a+bx}}{18b^2d^5|b|c-18bd^6|b|a}-\frac{-60b^7d^3c^2+120b^6d^4ac-60b^5d^5a^2}{18b^2d^5|b|c-18bd^6|b|a}\right)\sqrt{a+bx}\,\sqrt{a+bx}-\frac{-45b^8d^2c^3+135b^7d^3ac^2-135b^6d^4a^2c+45b^5d^5a^3}{18b^2d^5|b|c-18bd^6|b|a}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(-abd+b^2c+bd\,(a+bx))^2}+\frac{2\,(-5ab^3d+5b^4c)\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{2d^3\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] 1/3*((b*x + a)*(3*(b^6*c*d^4 - a*b^5*d^5)*(b*x + a)/(b^2*c*d^5*abs(b) - a*b
*d^6*abs(b)) + 20*(b^7*c^2*d^3 - 2*a*b^6*c*d^4 + a^2*b^5*d^5)/(b^2*c*d^5*ab
s(b) - a*b*d^6*abs(b))) + 15*(b^8*c^3*d^2 - 3*a*b^7*c^2*d^3 + 3*a^2*b^6*c*d
```

```
^4 - a^3*b^5*d^5)/(b^2*c*d^5*abs(b) - a*b*d^6*abs(b)))*sqrt(b*x + a)/(b^2*c
 + (b*x + a)*b*d - a*b*d)^(3/2) + 5*(b^4*c - a*b^3*d)*log(abs(-sqrt(b*d)*sq
rt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/2),x)
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/2), x)
```

3.1515 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}$$

```
[Out] -2/3*(b*x+a)^(3/2)/d/(d*x+c)^(3/2)+2*b^(3/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/
b^(1/2)/(d*x+c)^(1/2))/d^(5/2)-2*b*(b*x+a)^(1/2)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 223, 212}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*Sqrt[a + b*x])/(d^2*Sqrt[
c + d*x]) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/d^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}+\frac{b\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx}{d} \\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{d^2} \\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2} \\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2} \\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 0.88

$$
-\frac{2\sqrt{a+bx}\,(3bc+ad+4bdx)}{3d^2(c+dx)^{3/2}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*b*c + a*d + 4*b*d*x))/(3*d^2*(c + d*x)^(3/2)) + (2*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/d^(5/2)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(70) = 140.
time = 0.40, size = 325, normalized size = 3.53

$$\left[\frac{3\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,(4\,bdx+3\,bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{6\,(d^4x^2+2\,cd^3x+c^2d^2)},\ -\frac{3\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,(4\,bdx+3\,bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(d^4x^2+2\,cd^3x+c^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt
```

```
(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + 3*b*c + a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2), -1/3*(3*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x))
 + 2*(4*b*d*x + 3*b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^
3*x + c^2*d^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(70) = 140.
time = 0.05, size = 238, normalized size = 2.59

$$\frac{2\left(-\frac{(-12b^5d^2c+12b^4d^3a)\sqrt{a+bx}\,\sqrt{a+bx}}{-9bd^3|b|c+9d^4|b|a}-\frac{-9b^6dc^2+18b^5d^2ac-9b^4d^3a^2}{-9bd^3|b|c+9d^4|b|a}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(-abd+b^2c+bd\,(a+bx))^2}-\frac{2b^3\ln\left|\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right|}{d^2\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2*b^3*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d^2*abs(b)) - 2/3*sqrt(b*x + a)*(4*(b^5*c*d^2 - a*b^4*d^3)*
(b*x + a)/(b*c*d^3*abs(b) - a*d^4*abs(b)) + 3*(b^6*c^2*d - 2*a*b^5*c*d^2 +
a^2*b^4*d^3)/(b*c*d^3*abs(b) - a*d^4*abs(b)))/(b^2*c + (b*x + a)*b*d - a*b*
d)^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/2), x)
```

3.1516 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

[Out] 2/3*(b*x+a)^(3/2)/(-a*d+b*c)/(d*x+c)^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/2),x]

[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx = \frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/2),x]

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(5/2),x]')
[Out] Timed out
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.
time = 0.16, size = 88, normalized size = 2.75

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}}{3(dx+c)^{\frac{3}{2}}(ad-bc)}$	27
default	$-\frac{\sqrt{bx+a}}{d(dx+c)^{\frac{3}{2}}}+\frac{(-ad+bc)\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^{2}\sqrt{dx+c}}\right)}{2d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/d*(b*x+a)^(1/2)/(d*x+c)^(3/2)+1/2*(-a*d+b*c)/d*(-2/3/(a*d-b*c)/(d*x+c)^(
3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.34, size = 65, normalized size = 2.03

$$\frac{2\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{3\,(bc^{3}-ac^{2}d+(bcd^{2}-ad^{3})x^{2}+2\,(bc^{2}d-acd^{2})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)*sqrt(d*x + c)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2
+ 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [A]
time = 0.04, size = 97, normalized size = 3.03

$$-\frac{6b^4d\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(-9bdc\,|b|+9d^2a\,|b|)\,(-abd+b^2c+bd\,(a+bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(b*x + a)^(3/2)*b^4*d/((b*c*d*abs(b) - a*d^2*abs(b))*(b^2*c + (b*x + a)
*b*d - a*b*d)^(3/2))
```

Mupad [B]
time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{2\,a\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{2\,b\,x\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}\right)\,\sqrt{c+d\,x}}{x^2-\frac{3\,b\,c^3-3\,a\,c^2\,d}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{6\,c\,d\,x\,(a\,d-b\,c)}{3\,a\,d^3-3\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/2),x)
```

```
[Out] -((((2*a*(a + b*x)^(1/2))/(3*a*d^3 - 3*b*c*d^2) + (2*b*x*(a + b*x)^(1/2))/(3
*a*d^3 - 3*b*c*d^2))*(c + d*x)^(1/2))/(x^2 - (3*b*c^3 - 3*a*c^2*d)/(3*a*d^3
 - 3*b*c*d^2) + (6*c*d*x*(a*d - b*c))/(3*a*d^3 - 3*b*c*d^2))
```

3.1517 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}}+\frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}$$

```
[Out] 2/3*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(3/2)+4/3*b*(b*x+a)^(1/2)/(-a*d+b*c)^2
/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b*x])/(3*
(b*c - a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx = \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{3(bc-ad)}$$
$$= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}} + \frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.70

$$\frac{2\sqrt{a+bx}\,(3bc-ad+2bdx)}{3(bc-ad)^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
[Out] (2*Sqrt[a + b*x]*(3*b*c - a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(5/2)),x]')
[Out] Timed out
```

Maple [A]
time = 0.16, size = 55, normalized size = 0.83

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-2bdx+ad-3bc)}{3(dx+c)^{\frac{3}{2}}(a^2d^2-2abcd+b^2c^2)}$	53
default	$-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^2\sqrt{dx+c}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/
(d*x+c)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(54) = 108.
time = 0.37, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx + 3\,bc - ad)\sqrt{bx + a}\,\sqrt{dx + c}}{3\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x + 3*b*c - a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*c^4 - 2*a*b*c^
3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*
d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx}\,(c + dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(54) = 108.
time = 0.02, size = 174, normalized size = 2.64

$$\frac{4\left(-\frac{6b^4d^2\sqrt{a+bx}\,\sqrt{a+bx}}{-18b^2dc^2|b|+36bd^2ac|b|-18d^3a^2|b|} - \frac{9b^5dc-9b^4d^2a}{-18b^2dc^2|b|+36bd^2ac|b|-18d^3a^2|b|}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{(-abd+b^2c+bd\,(a+bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(2*(b*x + a)*b^4*d^2/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a^2*d^3*a
bs(b)) + 3*(b^5*c*d - a*b^4*d^2)/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a
^2*d^3*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

Mupad [B]
time = 0.90, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(6\,c\,b^2+2\,a\,d\,b\right)}{3\,d^2\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,a^2\,d-6\,a\,b\,c}{3\,d^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,b^2\,x^2}{3\,d\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(6*b^2*c + 2*a*b*d))/(3*d^2*(a*d - b*c)^2) - (2*a^2*d
- 6*a*b*c)/(3*d^2*(a*d - b*c)^2) + (4*b^2*x^2)/(3*d*(a*d - b*c)^2)))/(x^2*(
a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(1/2))/d)
```

3.1518 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}-\frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{16bd\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(3/2)/(b*x+a)^(1/2)-8/3*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/
(d*x+c)^(3/2)-16/3*b*d*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) - (8*d*Sqrt[a + b*x])/(3*(b*
c - a*d)^2*(c + d*x)^(3/2)) - (16*b*d*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{(4d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx}{bc-ad}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{(8bd)\int \frac{1}{\sqrt{a+bx}\cdots}}{3(bc-a\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{16bd\sqrt{a+bx}\cdots}{3(bc-ad)^3\sqrt{c+\cdots}}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.74

$$-\frac{2(a+bx)^{3/2}\left(-d^2+\frac{6bd(c+dx)}{a+bx}+\frac{3b^2(c+dx)^2}{(a+bx)^2}\right)}{3(bc-ad)^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-d^2 + (6*b*d*(c + d*x))/(a + b*x) + (3*b^2*(c + d*x)^
2)/(a + b*x)^2))/(3*(b*c - a*d)^3*(c + d*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 95, normalized size = 0.97

method	result	size
default	$-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4d\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^{2}\sqrt{dx+c}}\right)}{-ad+bc}$	95
gosper	$-\frac{2\left(-8b^{2}x^{2}d^{2}-4ab\,d^{2}x-12b^{2}cdx+a^{2}d^{2}-6abcd-3b^{2}c^{2}\right)}{3\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(3/2)/(b*x+a)^(1/2)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d
*x+c)^(3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(82) = 164.
time = 0.51, size = 273, normalized size = 2.79

$$-\frac{2\,{\left(8\,b^{2} d^{2} x^{2} + 3\,b^{2} c^{2} + 6\,a b c d - a^{2} d^{2} + 4\,{\left(3\,b^{2} c d + a b d^{2}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{3\,{\left(a b^{3} c^{5} - 3\,a^{2} b^{2} c^{4} d + 3\,a^{3} b c^{3} d^{2} - a^{4} c^{2} d^{3} + {\left(b^{4} c^{3} d^{2} - 3\,a b^{3} c^{2} d^{3} + 3\,a^{2} b^{2} c d^{4} - a^{3} b d^{5}\right)} x^{3} + {\left(2\,b^{4} c^{4} d - 5\,a b^{3} c^{3} d^{2} + 3\,a^{2} b^{2} c^{2} d^{3} + a^{3} b c d^{4} - a^{4} d^{5}\right)} x^{2} + {\left(b^{4} c^{5} - a b^{3} c^{4} d - 3\,a^{2} b^{2} c^{3} d^{2} + 5\,a^{3} b c^{2} d^{3} - 2\,a^{4} c d^{4}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*d^2*x^2 + 3*b^2*c^2 + 6*a*b*c*d - a^2*d^2 + 4*(3*b^2*c*d + a*b*
d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b x\right)^{\frac{3}{2}} \left(c + d x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(82) = 164.
time = 0.04, size = 459, normalized size = 4.68

$$2\left(\frac{2\left(-\frac{\left(-15b^6d^3c^2|b|+30b^5d^4ac|b|-15b^4d^5a^2|b|\right)\sqrt{a+bx}\,\sqrt{a+bx}}{-18b^5dc^5b^2+90b^4d^2ac^4b^2-180b^3d^3a^2c^3b^2+180b^2d^4a^3c^2b^2-90bd^5a^4cb^2+18d^6a^5b^2}-\frac{-18b^7d^2c^3|b|+54b^6d^3ac^2|b|-54b^5d^4a^2c|b|+18b^4d^5a^3|b|}{-18b^5dc^5b^2+90b^4d^2ac^4b^2-180b^3d^3a^2c^3b^2+180b^2d^4a^3c^2b^2-90bd^5a^4cb^2+18d^6a^5b^2}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{\left(-abd+b^2c+bd\,(a+bx)\right)^2}+\frac{4b^3\sqrt{bd}}{2\left(a^2d^2\,|b|-2ad\,|b|\,bc+|b|\,b^2c^2\right)\left(\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2+adb-b^2c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] -4*sqrt(b*d)*b^3/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b))*(b^2
*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2)) - 2/3*sqrt(b*x + a)*(5*(b^6*c^2*d^3*abs(b) - 2*a*b^5*c*d^4*abs(b) + a
^2*b^4*d^5*abs(b))*(b*x + a)/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*
d^3 - 10*a^3*b^4*c^2*d^4 + 5*a^4*b^3*c*d^5 - a^5*b^2*d^6) + 6*(b^7*c^3*d^2*
abs(b) - 3*a*b^6*c^2*d^3*abs(b) + 3*a^2*b^5*c*d^4*abs(b) - a^3*b^4*d^5*abs(
b))/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*d^3 - 10*a^3*b^4*c^2*d^4
+ 5*a^4*b^3*c*d^5 - a^5*b^2*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

Mupad [B]
time = 1.03, size = 132, normalized size = 1.35

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b^2\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{-2\,a^2\,d^2+12\,a\,b\,c\,d+6\,b^2\,c^2}{3\,d^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,b\,x\,\left(a\,d+3\,b\,c\right)}{3\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b^2*x^2)/(3*(a*d - b*c)^3) + (6*b^2*c^2 - 2*a^2*d^2 +
 12*a*b*c*d)/(3*d^2*(a*d - b*c)^3) + (8*b*x*(a*d + 3*b*c))/(3*d*(a*d - b*c)
^3)))/(x^2*(a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(
1/2))/d)
```

3.1519 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}+\frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}}+\frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3(c+dx)^{3/2}}+\frac{32bd^2\sqrt{a+bx}}{3(bc-ad)^4\sqrt{c+dx}}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(3/2)+4*d/(-a*d+b*c)^2/(d*x+c)^(3/2)/
(b*x+a)^(1/2)+16/3*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(3/2)+32/3*b*d^2*
(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/2)) + (4*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/2)) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*(c
+ d*x)^(3/2)) + (32*b*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx}{bc-ad}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{(8d^2)}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{1}{3(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{1}{3(bc-}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(d^3 - \frac{9bd^2(c+dx)}{a+bx} - \frac{9b^2d(c+dx)^2}{(a+bx)^2} + \frac{b^3(c+dx)^3}{(a+bx)^3}\right)}{3(bc-ad)^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(d^3 - (9*b*d^2*(c + d*x))/(a + b*x) - (9*b^2*d*(c + d*
x)^2)/(a + b*x)^2 + (b^3*(c + d*x)^3)/(a + b*x)^3))/(3*(b*c - a*d)^4*(c + d
*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 135, normalized size = 1.00

method	result	siz

default	$-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4d\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^{2}\sqrt{dx+c}}\right)}{-ad+bc}\right)}{-ad+bc}$	1
gosper	$-\frac{2\left(-16b^{3}x^{3}d^{3}-24d^{3}a\,x^{2}b^{2}-24b^{3}c\,d^{2}x^{2}-6a^{2}b\,d^{3}x-36a\,b^{2}c\,d^{2}x-6b^{3}c^{2}dx+a^{3}d^{3}-9a^{2}bc\,d^{2}-9a\,b^{2}c^{2}d+b^{3}c^{3}\right)}{3(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(3/2)-2*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(
d*x+c)^(3/2)/(b*x+a)^(1/2)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*
x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(113) = 226$.
time = 0.66, size = 447, normalized size = 3.31

$$\frac{2\left(16\,b^{3}d^{3}x^{3}-b^{3}c^{3}+9\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}-a^{3}d^{3}+24\left(b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+6\left(b^{3}c^{2}d+6\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(a^{2}b^{4}c^{6}-4\,a^{3}b^{3}c^{5}d+6\,a^{4}b^{2}c^{4}d^{2}-4\,a^{5}bc^{3}d^{3}+a^{6}c^{2}d^{4}+\left(b^{6}c^{4}d^{2}-4\,ab^{5}c^{3}d^{3}+6\,a^{2}b^{4}c^{2}d^{4}-4\,a^{3}b^{3}cd^{5}+a^{4}b^{2}d^{6}\right)x^{4}+2\left(b^{6}c^{5}d-3\,ab^{5}c^{4}d^{2}+2\,a^{2}b^{4}c^{3}d^{3}+2\,a^{3}b^{3}c^{2}d^{4}-3\,a^{4}b^{2}cd^{5}+a^{5}bd^{6}\right)x^{3}+\left(b^{6}c^{6}-9\,a^{2}b^{4}c^{4}d^{2}+16\,a^{3}b^{3}c^{3}d^{3}-9\,a^{4}b^{2}c^{2}d^{4}+a^{6}d^{6}\right)x^{2}+2\left(ab^{5}c^{6}-3\,a^{2}b^{4}c^{5}d+2\,a^{3}b^{3}c^{4}d^{2}+2\,a^{4}b^{2}c^{3}d^{3}-3\,a^{5}bc^{2}d^{4}+a^{6}cd^{5}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*d^3*x^3 - b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - a^3*d^3 + 2
4*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(b^3*c^2*d + 6*a*b^2*c*d^2 + a^2*b*d^3)*x
)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^
4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*
a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b
^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*
b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
```

```
c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(113) = 226.
time = 0.13, size = 799, normalized size = 5.92

$$2\left(\frac{2\left(-\frac{\left(24b^7d^4c^3|b|-72b^6d^5ac^2|b|+72b^5d^6a^2c|b|-24b^4d^7a^3|b|\right)\sqrt{a+bx}\,\sqrt{a+bx}}{-18b^7dc^7b^2+126b^6d^2ac^6b^2-378b^5d^3a^2c^5b^2+630b^4d^4a^3c^4b^2-630b^3d^5a^4c^3b^2+378b^2d^6a^5c^2b^2-126bd^7a^6cb^2+18d^8a^7b^2}-\frac{27b^8d^3c^4|b|-108b^7d^4ac^3|b|+162b^6d^5a^2c^2|b|-108b^5d^6a^3c|b|+27b^4d^7a^4|b|}{-18b^7dc^7b^2+126b^6d^2ac^6b^2-378b^5d^3a^2c^5b^2+630b^4d^4a^3c^4b^2-630b^3d^5a^4c^3b^2+378b^2d^6a^5c^2b^2-126bd^7a^6cb^2+18d^8a^7b^2}\right)\sqrt{a+bx}\,\sqrt{-abd+b^2c+bd\,(a+bx)}}{\left(-abd+b^2c+bd\,(a+bx)\right)^2}+\frac{2\left(6b^3d\sqrt{bd}\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^4+18b^4d^2\sqrt{bd}\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2a-18b^5d\sqrt{bd}\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2c+8b^5d^3\sqrt{bd}\,a^2-16b^6d^2\sqrt{bd}\,ac+8b^7d\sqrt{bd}\,c^2\right)}{\left(3d^3a^3\,|b|-9bd^2a^2c\,|b|+9b^2dac^2\,|b|-3b^3c^3\,|b|\right)\left(\left(\sqrt{-abd+b^2c+bd\,(a+bx)}-\sqrt{bd}\,\sqrt{a+bx}\right)^2+bda-b^2c\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b^7*c^3*d^4*abs(b) - 3*a*b^6*c^2*d^5*abs(b) + 3*a^2*b
^5*c*d^6*abs(b) - a^3*b^4*d^7*abs(b))*(b*x + a)/(b^9*c^7*d - 7*a*b^8*c^6*d^
2 + 21*a^2*b^7*c^5*d^3 - 35*a^3*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b
^4*c^2*d^6 + 7*a^6*b^3*c*d^7 - a^7*b^2*d^8) + 9*(b^8*c^4*d^3*abs(b) - 4*a*b
^7*c^3*d^4*abs(b) + 6*a^2*b^6*c^2*d^5*abs(b) - 4*a^3*b^5*c*d^6*abs(b) + a^4
*b^4*d^7*abs(b))/(b^9*c^7*d - 7*a*b^8*c^6*d^2 + 21*a^2*b^7*c^5*d^3 - 35*a^3
*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b^4*c^2*d^6 + 7*a^6*b^3*c*d^7 -
a^7*b^2*d^8))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 8/3*(4*sqrt(b*d)*b^7*
c^2*d - 8*sqrt(b*d)*a*b^6*c*d^2 + 4*sqrt(b*d)*a^2*b^5*d^3 - 9*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c*d + 9*
sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
*a*b^4*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^3*d)/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c*
d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3)
```

Mupad [B]
time = 1.29, size = 224, normalized size = 1.66

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b\,x^2\,\left(a\,d+b\,c\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{2\,a^3\,d^3-18\,a^2\,b\,c\,d^2-18\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{3\,b\,d^2\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b^2\,d\,x^3}{3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{d\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a\,c^2\,\sqrt{a+b\,x}}{b\,d^2}+\frac{x^2\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{c\,x\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b*x^2*(a*d + b*c))/(a*d - b*c)^4 - (2*a^3*d^3 + 2*b^3
*c^3 - 18*a*b^2*c^2*d - 18*a^2*b*c*d^2)/(3*b*d^2*(a*d - b*c)^4) + (32*b^2*d
*x^3)/(3*(a*d - b*c)^4) + (4*x*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(d*(a*d - b
*c)^4)))/(x^3*(a + b*x)^(1/2) + (a*c^2*(a + b*x)^(1/2))/(b*d^2) + (x^2*(a*d
 + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (c*x*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d
^2))
```

3.1520 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}+\frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}-\frac{32d^2}{5(bc-ad)^3\sqrt{a+bx}\,(c+dx)^{3/2}}-1$$

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(3/2)+16/15*d/(-a*d+b*c)^2/(b*x+a)^(3
/2)/(d*x+c)^(3/2)-32/5*d^2/(-a*d+b*c)^3/(d*x+c)^(3/2)/(b*x+a)^(1/2)-128/15*
d^3*(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(3/2)-256/15*b*d^3*(b*x+a)^(1/2)/(-a
*d+b*c)^5/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}\,(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2)) + (16*d)/(15*(b*c - a*d)
^2*(a + b*x)^(3/2)*(c + d*x)^(3/2)) - (32*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*
x]*(c + d*x)^(3/2)) - (128*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^4*(c + d*x)^(
3/2)) - (256*b*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} + $$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - $$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - $$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - $$

Mathematica [A]
time = 0.14, size = 117, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(-5d^4+\frac{60bd^3(c+dx)}{a+bx}+\frac{90b^2d^2(c+dx)^2}{(a+bx)^2}-\frac{20b^3d(c+dx)^3}{(a+bx)^3}+\frac{3b^4(c+dx)^4}{(a+bx)^4}\right)}{15(bc-ad)^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-5*d^4 + (60*b*d^3*(c + d*x))/(a + b*x) + (90*b^2*d^2*
(c + d*x)^2)/(a + b*x)^2 - (20*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (3*b^4*(c +
 d*x)^4)/(a + b*x)^4))/(15*(b*c - a*d)^5*(c + d*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 175, normalized size = 1.02

method	result

default	$-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}-\frac{8d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4d\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\cdots\right)}{-ad\cdots}\right)}{-ad+bc}\right)}{5(-ad+bc)}$
gosper	$-\frac{2(-128d^4x^4b^4-320a\,b^3d^4x^3-192b^4c\,d^3x^3-240a^2b^2d^4x^2-480a\,b^3c\,d^3x^2-48b^4c^2d^2x^2-40a^3b\,d^4x-360a^2b^2c\,d^3x-120a\,b^3c^2d^2x+8b\cdots}{15(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(3/2)-8/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*
c)/(b*x+a)^(3/2)/(d*x+c)^(3/2)-2*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(d*x+c)^(3/2)/
(b*x+a)^(1/2)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*x+a)^(1/2)+4/
3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(142) = 284.
time = 1.95, size = 715, normalized size = 4.16

$$-\frac{2\,(128\,b^4d^4x^4+3\,b^4c^4-20\,ab^3c^3d+90\,a^2b^2c^2d^2+60\,a^3bcd^3-5\,a^4d^4+64\,(3\,b^4cd^3+5\,ab^3d^4)x^3+48\,(b^4c^2d^2+10\,ab^3cd^3+5\,a^2b^2d^4)x^2-8\,(b^4c^3d-15\,ab^3c^2d^2-45\,a^2b^2cd^3-5\,a^3bd^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{15\,(a^3b^5c^7-5\,a^4b^4c^6d+10\,a^5b^3c^5d^2-10\,a^6b^2c^4d^3+5\,a^7bc^3d^4-a^8c^2d^5+(b^8c^5d^2-5\,ab^7c^4d^3+10\,a^2b^6c^3d^4-10\,a^3b^5c^2d^5+5\,a^4b^4cd^6-a^5b^3d^7)x^5+(2\,b^8c^6d-7\,ab^7c^5d^2+5\,a^2b^6c^4d^3+10\,a^3b^5c^3d^4-20\,a^4b^4c^2d^5+13\,a^5b^3cd^6-3\,a^6b^2d^7)x^4+(b^8c^7+ab^7c^6d-17\,a^2b^6c^5d^2+35\,a^3b^5c^4d^3-25\,a^4b^4c^3d^4-a^5b^3c^2d^5+9\,a^6b^2cd^6-3\,a^7bd^7)x^3+(3\,ab^7c^7-9\,a^2b^6c^6d+a^3b^5c^5d^2+25\,a^4b^4c^4d^3-35\,a^5b^3c^3d^4+17\,a^6b^2c^2d^5-a^7bcd^6-a^8d^7)x^2+(3\,a^2b^6c^7-13\,a^3b^5c^6d+20\,a^4b^4c^5d^2-10\,a^5b^3c^4d^3-5\,a^6b^2c^3d^4+7\,a^7bc^2d^5-2\,a^8cd^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*d^4*x^4 + 3*b^4*c^4 - 20*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 +
60*a^3*b*c*d^3 - 5*a^4*d^4 + 64*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 48*(b^4*c
^2*d^2 + 10*a*b^3*c*d^3 + 5*a^2*b^2*d^4)*x^2 - 8*(b^4*c^3*d - 15*a*b^3*c^2*
```

```
d^2 - 45*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b
^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*
b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d
^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d
 - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^
2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17
*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^
5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a
^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d
^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^
4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 -
2*a^8*c*d^6)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. 2(142) = 284.
time = 0.34, size = 1406, normalized size = 8.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*sqrt(b*x + a)*(11*(b^8*c^4*d^5*abs(b) - 4*a*b^7*c^3*d^6*abs(b) + 6*a^2
*b^6*c^2*d^7*abs(b) - 4*a^3*b^5*c*d^8*abs(b) + a^4*b^4*d^9*abs(b))*(b*x + a
)/(b^11*c^9*d - 9*a*b^10*c^8*d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4
+ 126*a^4*b^7*c^5*d^5 - 126*a^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b
^4*c^2*d^8 + 9*a^8*b^3*c*d^9 - a^9*b^2*d^10) + 12*(b^9*c^5*d^4*abs(b) - 5*a
*b^8*c^4*d^5*abs(b) + 10*a^2*b^7*c^3*d^6*abs(b) - 10*a^3*b^6*c^2*d^7*abs(b)
 + 5*a^4*b^5*c*d^8*abs(b) - a^5*b^4*d^9*abs(b))/(b^11*c^9*d - 9*a*b^10*c^8*
d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4 + 126*a^4*b^7*c^5*d^5 - 126*a
^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b^4*c^2*d^8 + 9*a^8*b^3*c*d^9
- a^9*b^2*d^10))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 4/15*(73*sqrt(b*d)
*b^11*c^4*d^2 - 292*sqrt(b*d)*a*b^10*c^3*d^3 + 438*sqrt(b*d)*a^2*b^9*c^2*d^
4 - 292*sqrt(b*d)*a^3*b^8*c*d^5 + 73*sqrt(b*d)*a^4*b^7*d^6 - 320*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^9*c^3*d
```

```
^2 + 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^8*c^2*d^3 - 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^7*c*d^4 + 320*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^6*d^5 + 490*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^7*
c^2*d^2 - 980*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^6*c*d^3 + 490*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^5*d^4 - 240*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^5*c*d^2 + 240*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^4
*d^3 + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^8*b^3*d^2)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^2*c^2
*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

Mupad [B]

time = 1.53, size = 346, normalized size = 2.01

$$\frac{\sqrt{c+dx}\,\left(\frac{32\,x^2\,\left(5\,a^2\,d^2+10\,a\,b\,c\,d+b^2\,c^2\right)}{5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,b^2\,d^2\,x^4}{15\,{\left(a\,d-b\,c\right)}^5}+\frac{-10\,a^4\,d^4+120\,a^3\,b\,c\,d^3+180\,a^2\,b^2\,c^2\,d^2-40\,a\,b^3\,c^3\,d+6\,b^4\,c^4}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(80\,a^3\,b\,d^4+720\,a^2\,b^2\,c\,d^3+240\,a\,b^3\,c^2\,d^2-16\,b^4\,c^3\,d\right)}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,b\,d\,x^3\,\left(5\,a\,d+3\,b\,c\right)}{15\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{2\,x^3\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^2\,c^2\,\sqrt{a+b\,x}}{b^2\,d^2}+\frac{2\,a\,c\,x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(5*a^2*d^2 + b^2*c^2 + 10*a*b*c*d))/(5*(a*d - b*c
)^5) + (256*b^2*d^2*x^4)/(15*(a*d - b*c)^5) + (6*b^4*c^4 - 10*a^4*d^4 + 180
*a^2*b^2*c^2*d^2 - 40*a*b^3*c^3*d + 120*a^3*b*c*d^3)/(15*b^2*d^2*(a*d - b*c
)^5) + (x*(80*a^3*b*d^4 - 16*b^4*c^3*d + 240*a*b^3*c^2*d^2 + 720*a^2*b^2*c*
d^3))/(15*b^2*d^2*(a*d - b*c)^5) + (128*b*d*x^3*(5*a*d + 3*b*c))/(15*(a*d -
 b*c)^5)))/(x^4*(a + b*x)^(1/2) + (x^2*(a + b*x)^(1/2)*(a^2*d^2 + b^2*c^2 +
 4*a*b*c*d))/(b^2*d^2) + (2*x^3*(a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a^2*c
^2*(a + b*x)^(1/2))/(b^2*d^2) + (2*a*c*x*(a*d + b*c)*(a + b*x)^(1/2))/(b^2*
d^2))
```

3.1521 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}+\frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}}-\frac{32d^2}{21(bc-ad)^3(a+bx)^{3/2}(c+dx)^{3/2}}$$

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(3/2)+4/7*d/(-a*d+b*c)^2/(b*x+a)^(5/2
)/(d*x+c)^(3/2)-32/21*d^2/(-a*d+b*c)^3/(b*x+a)^(3/2)/(d*x+c)^(3/2)+64/7*d^3
/(-a*d+b*c)^4/(d*x+c)^(3/2)/(b*x+a)^(1/2)+256/21*d^4*(b*x+a)^(1/2)/(-a*d+b*
c)^5/(d*x+c)^(3/2)+512/21*b*d^4*(b*x+a)^(1/2)/(-a*d+b*c)^6/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}\,(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{7(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2)) + (4*d)/(7*(b*c - a*d)^2
*(a + b*x)^(5/2)*(c + d*x)^(3/2)) - (32*d^2)/(21*(b*c - a*d)^3*(a + b*x)^(3
/2)*(c + d*x)^(3/2)) + (64*d^3)/(7*(b*c - a*d)^4*Sqrt[a + b*x]*(c + d*x)^(3
/2)) + (256*d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^5*(c + d*x)^(3/2)) + (512*b*
d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} - \frac{(10d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{(16}{}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(}$$

Mathematica [A]
time = 0.13, size = 139, normalized size = 0.67

$$-\frac{2(a+bx)^{3/2}\left(7d^5-\frac{105bd^4(c+dx)}{a+bx}-\frac{210b^2d^3(c+dx)^2}{(a+bx)^2}+\frac{70b^3d^2(c+dx)^3}{(a+bx)^3}-\frac{21b^4d(c+dx)^4}{(a+bx)^4}+\frac{3b^5(c+dx)^5}{(a+bx)^5}\right)}{21(bc-ad)^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(7*d^5 - (105*b*d^4*(c + d*x))/(a + b*x) - (210*b^2*d^3
*(c + d*x)^2)/(a + b*x)^2 + (70*b^3*d^2*(c + d*x)^3)/(a + b*x)^3 - (21*b^4*
d*(c + d*x)^4)/(a + b*x)^4 + (3*b^5*(c + d*x)^5)/(a + b*x)^5))/(21*(b*c - a
*d)^6*(c + d*x)^(3/2))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.18, size = 215, normalized size = 1.04

method	result
default	$-\frac{2}{7(-ad+bc)(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}}-\frac{10d\left(-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}-\frac{8d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{\cdots}}\cdots\right)}{\cdots}\right)}{5(\cdots}\right)}{7(-ad+bc)\cdots}$
gosper	$-\frac{2(-256b^5d^5x^5-896a\,b^4d^5x^4-384b^5c\,d^4x^4-1120a^2b^3d^5x^3-1344a\,b^4c\,d^4x^3-96b^5c^2d^3x^3-560a^3b^2d^5x^2-1680a^2b^3c\,d^4x^2-336a\,b^4\cdots}{21(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}(a^6d^6-6a^5bc\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(3/2)-10/7*d/(-a*d+b*c)*(-2/5/(-a*d+b
*c)/(b*x+a)^(5/2)/(d*x+c)^(3/2)-8/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*c)/(b*x+a)^(
3/2)/(d*x+c)^(3/2)-2*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(d*x+c)^(3/2)/(b*x+a)^(1/2
)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c
)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. $2(171) = 342$.
time = 4.34, size = 999, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(256*b^5*d^5*x^5 - 3*b^5*c^5 + 21*a*b^4*c^4*d - 70*a^2*b^3*c^3*d^2 + 2
10*a^3*b^2*c^2*d^3 + 105*a^4*b*c*d^4 - 7*a^5*d^5 + 128*(3*b^5*c*d^4 + 7*a*b
^4*d^5)*x^4 + 32*(3*b^5*c^2*d^3 + 42*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 16
*(b^5*c^3*d^2 - 21*a*b^4*c^2*d^3 - 105*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
+ 2*(3*b^5*c^4*d - 28*a*b^4*c^3*d^2 + 210*a^2*b^3*c^2*d^3 + 420*a^3*b^2*c*d
^4 + 35*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^6*c^8 - 6*a^5*b^5*
c^7*d + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4 - 6*a^
9*b*c^3*d^5 + a^10*c^2*d^6 + (b^10*c^6*d^2 - 6*a*b^9*c^5*d^3 + 15*a^2*b^8*c
^4*d^4 - 20*a^3*b^7*c^3*d^5 + 15*a^4*b^6*c^2*d^6 - 6*a^5*b^5*c*d^7 + a^6*b^
4*d^8)*x^6 + 2*(b^10*c^7*d - 4*a*b^9*c^6*d^2 + 3*a^2*b^8*c^5*d^3 + 10*a^3*b
^7*c^4*d^4 - 25*a^4*b^6*c^3*d^5 + 24*a^5*b^5*c^2*d^6 - 11*a^6*b^4*c*d^7 + 2
*a^7*b^3*d^8)*x^5 + (b^10*c^8 + 2*a*b^9*c^7*d - 27*a^2*b^8*c^6*d^2 + 64*a^3
*b^7*c^5*d^3 - 55*a^4*b^6*c^4*d^4 - 6*a^5*b^5*c^3*d^5 + 43*a^6*b^4*c^2*d^6
- 28*a^7*b^3*c*d^7 + 6*a^8*b^2*d^8)*x^4 + 4*(a*b^9*c^8 - 3*a^2*b^8*c^7*d -
2*a^3*b^7*c^6*d^2 + 19*a^4*b^6*c^5*d^3 - 30*a^5*b^5*c^4*d^4 + 19*a^6*b^4*c^
3*d^5 - 2*a^7*b^3*c^2*d^6 - 3*a^8*b^2*c*d^7 + a^9*b*d^8)*x^3 + (6*a^2*b^8*c
^8 - 28*a^3*b^7*c^7*d + 43*a^4*b^6*c^6*d^2 - 6*a^5*b^5*c^5*d^3 - 55*a^6*b^4
*c^4*d^4 + 64*a^7*b^3*c^3*d^5 - 27*a^8*b^2*c^2*d^6 + 2*a^9*b*c*d^7 + a^10*d
^8)*x^2 + 2*(2*a^3*b^7*c^8 - 11*a^4*b^6*c^7*d + 24*a^5*b^5*c^6*d^2 - 25*a^6
*b^4*c^5*d^3 + 10*a^7*b^3*c^4*d^4 + 3*a^8*b^2*c^3*d^5 - 4*a^9*b*c^2*d^6 + a
^10*c*d^7)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(171) = 342$.
time = 0.74, size = 2268, normalized size = 10.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] 2/3*sqrt(b*x + a)*(14*(b^9*c^5*d^6*abs(b) - 5*a*b^8*c^4*d^7*abs(b) + 10*a^2
*b^7*c^3*d^8*abs(b) - 10*a^3*b^6*c^2*d^9*abs(b) + 5*a^4*b^5*c*d^10*abs(b) -
```

a^5*b^4*d^11*abs(b))*(b*x + a)/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*
b^11*c^9*d^3 - 165*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6
*d^6 + 462*a^6*b^7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55
*a^9*b^4*c^2*d^10 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12) + 15*(b^10*c^6*d^5*
abs(b) - 6*a*b^9*c^5*d^6*abs(b) + 15*a^2*b^8*c^4*d^7*abs(b) - 20*a^3*b^7*c^
3*d^8*abs(b) + 15*a^4*b^6*c^2*d^9*abs(b) - 6*a^5*b^5*c*d^10*abs(b) + a^6*b^
4*d^11*abs(b))/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*b^11*c^9*d^3 - 16
5*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6*d^6 + 462*a^6*b^
7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55*a^9*b^4*c^2*d^10
 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/
2) + 8/21*(79*sqrt(b*d)*b^15*c^6*d^3 - 474*sqrt(b*d)*a*b^14*c^5*d^4 + 1185*
sqrt(b*d)*a^2*b^13*c^4*d^5 - 1580*sqrt(b*d)*a^3*b^12*c^3*d^6 + 1185*sqrt(b*
d)*a^4*b^11*c^2*d^7 - 474*sqrt(b*d)*a^5*b^10*c*d^8 + 79*sqrt(b*d)*a^6*b^9*d
^9 - 511*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*b^13*c^5*d^3 + 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^12*c^4*d^4 - 5110*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^11*c^3*d^5 + 51
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^3*b^10*c^2*d^6 - 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a^4*b^9*c*d^7 + 511*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^8*d^8 + 1344*sqrt(b
d)(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^11*
c^4*d^3 - 5376*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a*b^10*c^3*d^4 + 8064*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^9*c^2*d^5 - 5376*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^8*c*d^
6 + 1344*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^7*d^7 - 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*b^9*c^3*d^3 + 5250*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^8*c^2*d^4 - 5250*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2
*b^7*c*d^5 + 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^6*a^3*b^6*d^6 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^7*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^6*c*d^4 + 10
15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^8*a^2*b^5*d^5 - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^10*b^5*c*d^3 + 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^4*d^4 + 42*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^3*d^3)/((b^5*c
^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) - 10*a^3*b^2*c
^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c - a*b*d - (sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7)

Mupad [B]

time = 1.91, size = 478, normalized size = 2.31

$$\frac{\sqrt{c+dx}\,\left(\frac{32\,x^2\,\left(35\,a^3\,d^3+105\,a^2\,b\,c\,d^2+21\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{21\,b\,{\left(a\,d-b\,c\right)}^6}-\frac{14\,a^5\,d^5-210\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+140\,a^2\,b^3\,c^3\,d^2-42\,a\,b^4\,c^4\,d+6\,b^5\,c^5}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d\,x^3\,\left(35\,a^2\,d^2+42\,a\,b\,c\,d+3\,b^2\,c^2\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b^2\,d^3\,x^5}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,b\,d^2\,x^4\,\left(7\,a\,d+3\,b\,c\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(140\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4+840\,a^2\,b^3\,c^2\,d^3-112\,a\,b^4\,c^3\,d^2+12\,b^5\,c^4\,d\right)}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{x^3\,\sqrt{a+b\,x}\,\left(3\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{x^4\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^3\,c^2\,\sqrt{a+b\,x}}{b^3\,d^2}+\frac{a\,x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+6\,a\,b\,c\,d+3\,b^2\,c^2\right)}{b^3\,d^2}+\frac{a^2\,c\,x\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(35*a^3*d^3 - b^3*c^3 + 21*a*b^2*c^2*d + 105*a^2*
b*c*d^2))/(21*b*(a*d - b*c)^6) - (14*a^5*d^5 + 6*b^5*c^5 + 140*a^2*b^3*c^3*
d^2 - 420*a^3*b^2*c^2*d^3 - 42*a*b^4*c^4*d - 210*a^4*b*c*d^4)/(21*b^3*d^2*(
a*d - b*c)^6) + (64*d*x^3*(35*a^2*d^2 + 3*b^2*c^2 + 42*a*b*c*d))/(21*(a*d -
 b*c)^6) + (512*b^2*d^3*x^5)/(21*(a*d - b*c)^6) + (256*b*d^2*x^4*(7*a*d + 3
*b*c))/(21*(a*d - b*c)^6) + (x*(140*a^4*b*d^5 + 12*b^5*c^4*d - 112*a*b^4*c^
3*d^2 + 1680*a^3*b^2*c*d^4 + 840*a^2*b^3*c^2*d^3))/(21*b^3*d^2*(a*d - b*c)^
6)))/(x^5*(a + b*x)^(1/2) + (x^3*(a + b*x)^(1/2)*(3*a^2*d^2 + b^2*c^2 + 6*a
*b*c*d))/(b^2*d^2) + (x^4*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (a^3*c^2
*(a + b*x)^(1/2))/(b^3*d^2) + (a*x^2*(a + b*x)^(1/2)*(a^2*d^2 + 3*b^2*c^2 +
 6*a*b*c*d))/(b^3*d^2) + (a^2*c*x*(2*a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d^2
))
```

3.1522 $\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x+a)^(1/2))/b
```

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 1.37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{4+a+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
[Out] (2*ArcTanh[Sqrt[a + b*x]/Sqrt[4 + a + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 + a + b*x]*Sqrt[a + b*x]),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.
time = 0.17, size = 86, normalized size = 4.53

method	result	size
default	$\frac{\sqrt{(bx+a)(bx+a+4)}\,\ln\left(\frac{\frac{ab}{2}+\frac{b(a+4)}{2}+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+(ab+b(a+4))x+a(a+4)}\right)}{\sqrt{bx+a}\,\sqrt{bx+a+4}\,\sqrt{b^2}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((b*x+a)*(b*x+a+4))^(1/2)/(b*x+a)^(1/2)/(b*x+a+4)^(1/2)*ln((1/2*a*b+1/2*b*(
a+4)+b^2*x)/(b^2)^(1/2)+(x^2*b^2+(a*b+b*(a+4))*x+a*(a+4))^(1/2))/(b^2)^(1/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.27, size = 48, normalized size = 2.53

$$\frac{\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+a^2+2\,(ab+2\,b)x+4\,a}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + a^2 + 2*(a*b + 2*b)*x + 4*a)*b + 4*b
)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.29, size = 31, normalized size = 1.63

$$-\frac{\log\left(-bx + \sqrt{bx + a + 4}\,\sqrt{bx + a} - a - 2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + a + 4)*sqrt(b*x + a) - a - 2)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx}\,\sqrt{a + bx + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*x+a+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(a + b*x + 4)), x)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.42

$$-\frac{2\ln\left(\sqrt{a + bx + 4} - \sqrt{a + bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x)
```

```
[Out] -2*log(sqrt(b*x + a + 4) - sqrt(b*x + a))/b
```

Mupad [B]
time = 0.31, size = 50, normalized size = 2.63

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{a+4}-\sqrt{a+b\,x+4}\right)}{\sqrt{-b^2}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(a + b*x + 4)^(1/2)),x)
```

```
[Out] (4*atan((b*((a + 4)^(1/2) - (a + b*x + 4)^(1/2)))/((-b^2)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(-b^2)^(1/2)
```

3.1523 $\int \frac{1}{\sqrt{2+bx}\,\sqrt{6+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x+2)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{65, 221\}$

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\,\sqrt{6+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{6+bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[6 + b*x]/Sqrt[2 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 + 2 + b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(15) = 30$.
time = 0.17, size = 66, normalized size = 3.47

method	result	size
default	$\frac{\sqrt{(bx+2)(bx+6)}\,\ln\left(\frac{b^2x+4b}{\sqrt{b^2}}+\sqrt{x^2b^2+8bx+12}\right)}{\sqrt{bx+2}\,\sqrt{bx+6}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+2)*(b*x+6))^(1/2)/(b*x+2)^(1/2)/(b*x+6)^(1/2)*ln((b^2*x+4*b)/(b^2)^(1
/2)+(b^2*x^2+8*b*x+12)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.26, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+8\,bx+12}\,b+8\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 8*b*x + 12)*b + 8*b)/b
```

Fricas [A]
time = 0.29, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+6}\,\sqrt{bx+2}\,-4\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 6)*sqrt(b*x + 2) - 4)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+2}\,\sqrt{bx+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+6)**(1/2),x)
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 6)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.37

$$-\frac{2\ln\left(\sqrt{bx+6}\,-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x)
[Out] -2*log(sqrt(b*x + 6) - sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.34, size = 47, normalized size = 2.47

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{6}-\sqrt{b\,x+6}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 6)^(1/2)),x)
[Out] -(4*atan((b*(6^(1/2) - (b*x + 6)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

3.1524 $\int \frac{1}{\sqrt{1+bx}\,\sqrt{5+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x+1)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+bx}\,\sqrt{5+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\
&= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{5+bx}}{\sqrt{1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[5 + b*x]/Sqrt[1 + b*x]])/b
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 + 1 + b*x]*Sqrt[1 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(15) = 30$.
time = 0.16, size = 66, normalized size = 3.47

method	result	size
default	$\frac{\sqrt{(bx+1)(bx+5)}\,\ln\left(\frac{b^2x+3b}{\sqrt{b^2}}+\sqrt{x^2b^2+6bx+5}\right)}{\sqrt{bx+1}\,\sqrt{bx+5}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+1)*(b*x+5))^(1/2)/(b*x+1)^(1/2)/(b*x+5)^(1/2)*ln((b^2*x+3*b)/(b^2)^(1
/2)+(b^2*x^2+6*b*x+5)^(1/2))/(b^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.26, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+6\,bx+5}\,b+6\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 6*b*x + 5)*b + 6*b)/b
```

Fricas [A]
time = 0.30, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+5}\,\sqrt{bx+1}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 5)*sqrt(b*x + 1) - 3)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+1}\,\sqrt{bx+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+5)**(1/2),x)
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 5)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.37

$$-\frac{2\ln\left(\sqrt{bx+5}-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x)
[Out] -2*log(sqrt(b*x + 5) - sqrt(b*x + 1))/b
```

Mupad [B]
time = 0.33, size = 43, normalized size = 2.26

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{5}-\sqrt{b\,x+5}\right)}{\left(\sqrt{b\,x+1}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 5)^(1/2)),x)
[Out] (4*atan((b*(5^(1/2) - (b*x + 5)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

3.1525 $\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=17

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x)^(1/2))/b
```

Rubi [**A**]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.
time = 0.03, size = 42, normalized size = 2.47

$$-\frac{2\sqrt{x}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{4+bx}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (-2*Sqrt[x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[4 + b*x]])/(Sqrt[b]*Sqrt[b*x])
```

Mathics [A]
time = 2.38, size = 14, normalized size = 0.82

$$\frac{2\text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{2}\right]}{b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 + 0 + b*x]*Sqrt[0 + b*x]),x]')
```

```
[Out] 2 ArcSinh[Sqrt[b] Sqrt[x] / 2] / b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(13) = 26.
time = 0.14, size = 60, normalized size = 3.53

method	result	size
meijerg	$\frac{2\sqrt{x}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}\,\sqrt{bx}}$	23
default	$\frac{\sqrt{bx\left(bx+4\right)}\,\ln\left(\frac{b^{2}x+2b}{\sqrt{b^{2}}}+\sqrt{x^{2}b^{2}+4bx}\right)}{\sqrt{bx}\,\sqrt{bx+4}\,\sqrt{b^{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x*(b*x+4))^(1/2)/(b*x)^(1/2)/(b*x+4)^(1/2)*ln((b^2*x+2*b)/(b^2)^(1/2)+(b
^2*x^2+4*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

time = 0.27, size = 32, normalized size = 1.88

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+4\,bx}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 4*b*x)*b + 4*b)/b
```

Fricas [**A**]
time = 0.29, size = 25, normalized size = 1.47

$$-\frac{\log\left(-bx+\sqrt{bx+4}\,\sqrt{bx}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x) - 2)/b
```

Sympy [**A**]
time = 0.68, size = 15, normalized size = 0.88

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+4)**(1/2),x)
[Out] 2*asinh(sqrt(b)*sqrt(x)/2)/b
```

Giac [**A**]
time = 0.00, size = 24, normalized size = 1.41

$$-\frac{2\ln\left(\sqrt{bx+4}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x)
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x))/b
```

Mupad [B]
time = 0.31, size = 33, normalized size = 1.94

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x+4}-2\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 4)^(1/2)),x)
[Out] -(4*atan((b*((b*x + 4)^(1/2) - 2))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)^(1/2
)
```

3.1526 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{3+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}$$

[Out] 2*arcsinh(1/2*(b*x-1)^(1/2))/b

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]

[Out] (2*ArcSinh[Sqrt[-1 + b*x]/2])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\,\sqrt{3+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+bx}}{\sqrt{-1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[-1 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 - 1 + b*x]*Sqrt[-1 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(15) = 30.
time = 0.16, size = 64, normalized size = 3.37

method	result	size
default	$\frac{\sqrt{(bx-1)(bx+3)}\,\ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{x^2b^2+2bx-3}\right)}{\sqrt{bx-1}\,\sqrt{bx+3}\,\sqrt{b^2}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-1)*(b*x+3))^(1/2)/(b*x-1)^(1/2)/(b*x+3)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2
)+(b^2*x^2+2*b*x-3)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.27, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+2\,bx-3}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x - 3)*b + 2*b)/b
```

Fricas [A]
time = 0.29, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+3}\,\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 3)*sqrt(b*x - 1) - 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-1}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+3)**(1/2),x)
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 3)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.37

$$-\frac{2\ln\left(\sqrt{bx+3}-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x)
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x - 1))/b
```

Mupad [B]
time = 0.32, size = 44, normalized size = 2.32

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{3}-\sqrt{b\,x+3}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 3)^(1/2)),x)
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((3^(1/2) - (b*x + 3)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

3.1527 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] arccosh(1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{54\}$

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.04, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.21, size = 69, normalized size = 6.27

$$\frac{I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{4}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{4\text{exp\_polar}[2I\text{Pi}]}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/(Sqrt[4 - 2 + b*x]*Sqrt[-2 + b*x]),x]')

[Out] (I meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 / 2, 1}, {}}, {{-1 / 4, 1 / 4}, {-1 / 2, 0, 0, 0}}, 4 / (b ^ 2 x ^ 2)] + meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1 / 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 / 4, 1, 0}, {}}, 4 exp_polar[2 I Pi] / (b ^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(9) = 18$.
time = 0.16, size = 57, normalized size = 5.18

method	result	size
default	$\frac{\sqrt{(bx-2)(bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b^2*x^2-4)^(1/2))/(b^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.26, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")

[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.30, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 18.01, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.
time = 0.00, size = 26, normalized size = 2.36

$$-\frac{2\ln\left(\sqrt{bx+2}-\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

Mupad [B]
time = 0.30, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{b\,x-2}+\sqrt{2}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1528 $\int \frac{1}{\sqrt{-3+bx}\,\sqrt{1+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}$$

[Out] 2*arcsinh(1/2*(b*x-3)^(1/2))/b

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]

[Out] (2*ArcSinh[Sqrt[-3 + b*x]/2])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\,\sqrt{1+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{1+bx}}{\sqrt{-3+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[1 + b*x]/Sqrt[-3 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 - 3 + b*x]*Sqrt[-3 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(15) = 30$.
time = 0.16, size = 66, normalized size = 3.47

method	result	size
default	$\frac{\sqrt{(bx-3)(bx+1)}\,\ln\left(\frac{b^2x-b}{\sqrt{b^2}}+\sqrt{x^2b^2-2bx-3}\right)}{\sqrt{bx-3}\,\sqrt{bx+1}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-3)*(b*x+1))^(1/2)/(b*x-3)^(1/2)/(b*x+1)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2
)+(b^2*x^2-2*b*x-3)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.26, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-2\,bx-3}\,b-2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x - 3)*b - 2*b)/b
```

Fricas [A]
time = 0.29, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+1}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 1)*sqrt(b*x - 3) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+1)**(1/2),x)
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 1)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.37

$$-\frac{2\ln\left(\sqrt{bx+1}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x)
[Out] -2*log(sqrt(b*x + 1) - sqrt(b*x - 3))/b
```

Mupad [B]
time = 0.29, size = 46, normalized size = 2.42

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x - 3)^(1/2)),x)
[Out] (4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1
/2))))/(-b^2)^(1/2)
```

3.1529 $\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}} \, dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b}$$

```
[Out] 2*arcsinh((b*x+2)^(1/2))/b
```

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}} \, dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[2 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[3 + b*x]*Sqrt[2 + b*x]),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(13) = 26$.
time = 0.16, size = 66, normalized size = 4.40

method	result	size
default	$\frac{\sqrt{(bx+2)(bx+3)}\,\ln\left(\frac{\frac{5}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+5bx+6}\right)}{\sqrt{bx+2}\,\sqrt{bx+3}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((b*x+2)*(b*x+3))^(1/2)/(b*x+2)^(1/2)/(b*x+3)^(1/2)*ln((5/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+5*b*x+6)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.
time = 0.27, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+5\,bx+6}\,b+5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 5*b*x + 6)*b + 5*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.30, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+3}\,\sqrt{bx+2}\,-5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 3)*sqrt(b*x + 2) - 5)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+2}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 3)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.73

$$-\frac{2\ln\left(\sqrt{bx+3}\,-\sqrt{bx+2}\,\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x)
```

```
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.29, size = 47, normalized size = 3.13

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{3}-\sqrt{b\,x+3}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (b*x + 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

3.1530 $\int \frac{1}{2+bx} \, dx$

Optimal. Leaf size=10

$$\frac{\log(2+bx)}{b}$$

[Out] ln(b*x+2)/b

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

[In] Int[(2 + b*x)^(-1),x]

[Out] Log[2 + b*x]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+bx} \, dx = \frac{\log(2+bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(2+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + b*x)^(-1),x]

[Out] Log[2 + b*x]/b

Mathics [A]
time = 1.64, size = 10, normalized size = 1.00

$$\frac{\operatorname{Log}[2+bx]}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Sqrt[2 + b*x]*Sqrt[2 + b*x]),x]')
[Out] Log[2 + b x] / b
```

Maple [A]
time = 0.14, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+2)}{b}$	11
norman	$\frac{\ln(bx+2)}{b}$	11
risch	$\frac{\ln(bx+2)}{b}$	11
meijerg	$\frac{\ln\left(\frac{bx}{2}+1\right)}{b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2),x,method=_RETURNVERBOSE)
[Out] ln(b*x+2)/b
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="maxima")
[Out] log(b*x + 2)/b
```

Fricas [A]
time = 0.29, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="fricas")
[Out] log(b*x + 2)/b
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.70

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x)
```

```
[Out] log(b*x + 2)/b
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\ln |xb+2|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x)
```

```
[Out] log(abs(b*x + 2))/b
```

Mupad [B]
time = 0.26, size = 10, normalized size = 1.00

$$\frac{\ln (b\,x+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + 2),x)
```

```
[Out] log(b*x + 2)/b
```

3.1531 $\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}$$

```
[Out] 2*arcsinh((b*x+1)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[1 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(13) = 26$.
time = 0.16, size = 66, normalized size = 4.40

method	result	size
default	$\frac{\sqrt{(bx+1)(bx+2)}\,\ln\left(\frac{\frac{3}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+3bx+2}\right)}{\sqrt{bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((b*x+1)*(b*x+2))^(1/2)/(b*x+1)^(1/2)/(b*x+2)^(1/2)*ln((3/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+3*b*x+2)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.
time = 0.28, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+3\,bx+2}\,b+3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 3*b*x + 2)*b + 3*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.30, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx+1}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x + 1) - 3)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+2)**(1/2),x)
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.73

$$-\frac{2\ln\left(\sqrt{bx+2}-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x)
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x + 1))/b
```

Mupad [B]
time = 0.29, size = 43, normalized size = 2.87

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 2)^(1/2)),x)
[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

3.1532 $\int \frac{1}{\sqrt{bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x)^(1/2)*2^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/Sqrt[2]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{bx}\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\
&= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.
time = 0.00, size = 42, normalized size = 2.21

$$-\frac{2\sqrt{x}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*Sqrt[x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(Sqrt[b]*Sqrt[b*x])
```

Mathics [A]
time = 2.34, size = 17, normalized size = 0.89

$$\frac{2\text{ArcSinh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[0 + b*x]*Sqrt[2 + b*x]),x]')
[Out] 2 ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(16) = 32.
time = 0.14, size = 58, normalized size = 3.05

method	result	size
meijerg	$\frac{2\sqrt{x}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{\sqrt{b}\sqrt{bx}}$	26
default	$\frac{\sqrt{bx(bx+2)}\,\ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{x^2b^2+2bx}\right)}{\sqrt{bx}\sqrt{bx+2}\sqrt{b^2}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (b*x*(b*x+2))^(1/2)/(b*x)^(1/2)/(b*x+2)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2)+(b^2
*x^2+2*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 32, normalized size = 1.68

$$\frac{\log\left(2\,b^2x+2\sqrt{b^2x^2+2\,bx}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x)*b + 2*b)/b
```

Fricas [A]
time = 0.29, size = 25, normalized size = 1.32

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x) - 1)/b
```

Sympy [A]
time = 0.69, size = 20, normalized size = 1.05

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

Giac [A]
time = 0.00, size = 24, normalized size = 1.26

$$-\frac{2\ln\left(\sqrt{bx+2}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x))/b
```

Mupad [B]
time = 0.28, size = 37, normalized size = 1.95

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)
^(1/2)
```

3.1533 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}$$

```
[Out] 2*arcsinh(1/3*(b*x-1)^(1/2)*3^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/Sqrt[3]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-1 + b*x]])/b
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(18) = 36$.
time = 0.15, size = 65, normalized size = 3.10

method	result	size
default	$\frac{\sqrt{(bx-1)(bx+2)}\,\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+bx-2}\right)}{\sqrt{bx-1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((b*x-1)*(b*x+2))^(1/2)/(b*x-1)^(1/2)/(b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+b*x-2)^(1/2))/(b^2)^(1/2)
```

Maxima [**A**]
time = 0.27, size = 30, normalized size = 1.43

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-2}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 2)*b + b)/b
```

Fricas [A]
time = 0.29, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 1) - 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+2)**(1/2),x)
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.24

$$-\frac{2\ln\left(\sqrt{bx+2}-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x)
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 1))/b
```

Mupad [B]
time = 0.29, size = 44, normalized size = 2.10

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

3.1534 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] arccosh(1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{54\}$

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.02, size = 69, normalized size = 6.27

$$\frac{I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{4}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{4\text{exp\_polar}[2I\text{Pi}]}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] (I meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 / 2, 1}, {}}, {{-1 / 4, 1 / 4}, {-
1 / 2, 0, 0, 0}}, 4 / (b ^ 2 x ^ 2)] + meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1 /
 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 / 4, 1, 0}, {}}, 4 exp_polar[2 I Pi] / (b
^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(9) = 18$.
time = 0.15, size = 57, normalized size = 5.18

method	result	size
default	$\frac{\sqrt{(bx-2)(bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.28, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.30, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx + \sqrt{bx+2}\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 16.78, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.
time = 0.00, size = 26, normalized size = 2.36

$$-\frac{2\ln\left(\sqrt{bx+2} - \sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

Mupad [B]
time = 0.00, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1535 $\int \frac{1}{\sqrt{-3+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}$$

```
[Out] 2*arcsinh(1/5*(b*x-3)^(1/2)*5^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/Sqrt[5]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-3+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-3 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(18) = 36.
time = 0.16, size = 66, normalized size = 3.14

method	result	size
default	$\frac{\sqrt{(bx-3)(bx+2)}\,\ln\left(\frac{-\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-bx-6}\right)}{\sqrt{bx-3}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-3)*(b*x+2))^(1/2)/(b*x-3)^(1/2)/(b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(b^2)
^(1/2)+(b^2*x^2-b*x-6)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 1.57

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-bx-6}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 6)*b - b)/b
```

Fricas [A]
time = 0.30, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx + 2\,\sqrt{bx+2}\,\sqrt{bx-3} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 3) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+2)**(1/2),x)
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 26, normalized size = 1.24

$$-\frac{2\ln\left(\sqrt{bx+2} - \sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x)
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 3))/b
```

Mupad [B]
time = 0.28, size = 50, normalized size = 2.38

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x - 3)^(1/2)),x)
[Out] -(4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1536 $\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

```
[Out] arcsin(2/5*b*x-1/5)/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 633, 222}

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(1 - 2*b*x)/5]/b)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{6+bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25b^2}}}\,dx, x, b-2b^2x\right)}{5b^2}$$
$$= -\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[3 - b*x]/Sqrt[2 + b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(11) = 22$.
time = 0.16, size = 65, normalized size = 4.06

method	result	size
default	$\frac{\sqrt{(-bx+3)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x-\frac{1}{2b}\right)}{\sqrt{-x^2b^2+bx+6}}\right)}{\sqrt{-bx+3}\,\sqrt{bx+2}\,\sqrt{b^2}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+3)*(b*x+2))^(1/2)/(-b*x+3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x+6)^(1/2))
```

Maxima [A]
time = 0.35, size = 21, normalized size = 1.31

$$-\frac{\arcsin\left(-\frac{2\,b^2x-b}{5\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/5*(2*b^2*x - b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.
time = 0.30, size = 44, normalized size = 2.75

$$-\frac{\arctan\left(\frac{(2\,bx-1)\sqrt{bx+2}\,\sqrt{-bx+3}}{2\,(b^2x^2-bx-6)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x - 1)*sqrt(b*x + 2)*sqrt(-b*x + 3)/(b^2*x^2 - b*x - 6))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx+3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 3)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.44

$$-\frac{2\arcsin\left(\frac{\sqrt{-bx+3}}{\sqrt{5}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*arcsin(1/5*sqrt(5)*sqrt(-b*x + 3))/b
```

Mupad [B]
time = 0.08, size = 44, normalized size = 2.75

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

3.1537 $\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] arcsin(1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{41, 222\}$

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
[Out] ArcSin[(b*x)/2]/b
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{4-b^2x^2}}\,dx\\ &= \frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(11) = 22.
time = 0.03, size = 39, normalized size = 3.55

$$-\frac{\log\left(-\sqrt{-b^2}\,x+\sqrt{4-b^2x^2}\right)}{\sqrt{-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(Log[-(Sqrt[-b^2]*x) + Sqrt[4 - b^2*x^2]]/Sqrt[-b^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 15.09, size = 69, normalized size = 6.27

$$\frac{-I\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{4}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{4\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] (-I meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1 / 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 /
4, 1, 0}, {}}, 4 / (b ^ 2 x ^ 2)] + meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 /
 2, 1}, {}}, {{-1 / 4, 1 / 4}, {-1 / 2, 0, 0, 0}}, 4 exp_polar[-2 I Pi] / (
b ^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(9) = 18$.

time = 0.16, size = 56, normalized size = 5.09

method	result	size
default	$\frac{\sqrt{(-bx+2)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-x^2b^2+4}}\right)}{\sqrt{-bx+2}\,\sqrt{bx+2}\,\sqrt{b^2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+2)*(b*x+2))^(1/2)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*x/(-b^2*x^2+4)^(1/2))
```

Maxima [A]

time = 0.35, size = 9, normalized size = 0.82

$$\frac{\arcsin\left(\frac{1}{2}\,bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*b*x)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.
time = 0.29, size = 31, normalized size = 2.82

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx+2}-2}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(b*x + 2)*sqrt(-b*x + 2) - 2)/(b*x))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 17.17, size = 76, normalized size = 6.91

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
4/(b**2*x**2))/(4*pi**(3/2)*b) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.73

$$-\frac{2\arcsin\left(\frac{\sqrt{-bx+2}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*arcsin(1/2*sqrt(-b*x + 2))/b
```

Mupad [B]
time = 0.08, size = 44, normalized size = 4.00

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

3.1538 $\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}} dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}$$

```
[Out] arcsin(2/3*b*x+1/3)/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 633, 222}

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(-1 - 2*b*x)/3]/b)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{2-bx-b^2x^2}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9b^2}}}\,dx, x, -b-2b^2x\right)}{3b^2}$$

$$= -\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}$$

Mathematica [**A**]
time = 0.04, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[1 - b*x]/Sqrt[2 + b*x]])/b
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(11) = 22.
time = 0.17, size = 66, normalized size = 4.12

method	result	size
default	$\frac{\sqrt{(-bx+1)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{1}{2b}\right)}{\sqrt{-x^2b^2-bx+2}}\right)}{\sqrt{-bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+1)*(b*x+2))^(1/2)/(-b*x+1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+1/2/b)/(-b^2*x^2-b*x+2)^(1/2))
```

Maxima [A]
time = 0.34, size = 19, normalized size = 1.19

$$-\frac{\arcsin\left(-\frac{2\,b^2x+b}{3\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*(2*b^2*x + b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(11) = 22$.
time = 0.31, size = 43, normalized size = 2.69

$$-\frac{\arctan\left(\frac{(2\,bx+1)\sqrt{bx+2}\,\sqrt{-bx+1}}{2\,(b^2x^2+bx-2)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(-b*x + 1)/(b^2*x^2 + b*x - 2))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.44

$$-\frac{2\arcsin\left(\frac{\sqrt{-bx+1}}{\sqrt{3}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*arcsin(1/3*sqrt(3)*sqrt(-b*x + 1))/b
```

Mupad [B]
time = 0.32, size = 40, normalized size = 2.50

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{1-b\,x}-1\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(b^2)^(1/2)
)))/(b^2)^(1/2)
```

3.1539 $\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=10

$$\frac{\sin^{-1}(1+bx)}{b}$$

[Out] arcsin(b*x+1)/b

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {55, 633, 222}

$$\frac{\sin^{-1}(bx+1)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]

[Out] ArcSin[1 + b*x]/b

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-bx}\,\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}$$
$$= \frac{\sin^{-1}(1+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(10) = 20.
time = 0.01, size = 57, normalized size = 5.70

$$-\frac{2\sqrt{x}\,\sqrt{2+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}\,\sqrt{-bx(2+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[2 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(Sqrt[b]
*Sqrt[-(b*x*(2 + b*x))])
```

Mathics [C] Result contains complex when optimal does not.
time = 2.32, size = 17, normalized size = 1.70

$$\frac{-2I\,\text{ArcSinh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right]}{b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[0 - b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] -2 I ArcSinh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(10) = 20.
time = 0.15, size = 58, normalized size = 5.80

method	result	size

meijerg	$\frac{2\sqrt{x}\ \operatorname{arcsinh}\left(\frac{\sqrt{b}\ \sqrt{x}\ \sqrt{2}}{2}\right)}{\sqrt{b}\ \sqrt{-bx}}$	27
default	$\frac{\sqrt{-bx\,(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\ \left(x+\frac{1}{b}\right)}{\sqrt{-x^2b^2-2bx}}\right)}{\sqrt{-bx}\ \sqrt{bx+2}\ \sqrt{b^2}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x*(b*x+2))^(1/2)/(-b*x)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

Maxima [A]
time = 0.36, size = 18, normalized size = 1.80

$$-\frac{\arcsin\left(-\frac{b^2x+b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(b^2*x + b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.30, size = 26, normalized size = 2.60

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\ \sqrt{-bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(b*x + 2)*sqrt(-b*x)/(b*x))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.83, size = 24, normalized size = 2.40

$$-\frac{2i\operatorname{asinh}\left(\frac{\sqrt{2}\ \sqrt{b}\ \sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*I*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

Giac [A]
time = 0.00, size = 21, normalized size = 2.10

$$-\frac{2\arcsin\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*arcsin(1/2*sqrt(2)*sqrt(-b*x))/b
```

Mupad [B]
time = 0.29, size = 34, normalized size = 3.40

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{-b\,x}\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((-b*x)^(1/2)*(b^2)^(1/2))))/(b^2)
^(1/2)
```

3.1540 $\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(3+2bx)}{b}$$

[Out] arcsin(2*b*x+3)/b

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 633, 222}

$$\frac{\sin^{-1}(2bx+3)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]

[Out] ArcSin[3 + 2*b*x]/b

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{-2-3bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, -3b-2b^2x\right)}{b^2}$$
$$= \frac{\sin^{-1}(3+2bx)}{b}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(11) = 22.
time = 0.01, size = 59, normalized size = 5.36

$$\frac{2\sqrt{1+bx}\sqrt{2+bx}\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{1+bx}}\right)}{b\sqrt{-((1+bx)(2+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*Sqrt[2 + b*x]*ArcTanh[Sqrt[2 + b*x]/Sqrt[1 + b*x]])/(b*Sqr
t[-((1 + b*x)*(2 + b*x))])
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(11) = 22.
time = 0.17, size = 66, normalized size = 6.00

method	result	size

default	$\frac{\sqrt{(-bx-1)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{3}{2b}\right)}{\sqrt{-x^2b^2-3bx-2}}\right)}{\sqrt{-bx-1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-1)*(b*x+2))^(1/2)/(-b*x-1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+3/2/b)/(-b^2*x^2-3*b*x-2)^(1/2))
```

Maxima [A]
time = 0.35, size = 21, normalized size = 1.91

$$-\frac{\arcsin\left(-\frac{2\,b^2x+3\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 3*b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.
time = 0.30, size = 44, normalized size = 4.00

$$-\frac{\arctan\left(\frac{(2\,bx+3)\sqrt{bx+2}\,\sqrt{-bx-1}}{2\,(b^2x^2+3\,bx+2)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 3)*sqrt(b*x + 2)*sqrt(-b*x - 1)/(b^2*x^2 + 3*b*x + 2))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 16, normalized size = 1.45

$$-\frac{2\arcsin\left(\sqrt{-bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*arcsin(sqrt(-b*x - 1))/b
```

Mupad [B]
time = 0.30, size = 41, normalized size = 3.73

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)^(1/
2))))/(b^2)^(1/2)
```

3.1541 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{2+bx}\,\log(2+bx)}{b\sqrt{-2-bx}}$$

```
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {23, 31}

$$\frac{\sqrt{bx+2}\,\log(bx+2)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*Log[2 + b*x])/(b*Sqrt[-2 - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx &= \frac{\sqrt{2+bx}\,\int \frac{1}{2+bx}\,dx}{\sqrt{-2-bx}}\\ &= \frac{\sqrt{2+bx}\,\log(2+bx)}{b\sqrt{-2-bx}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.97

$$\frac{(2+bx)\log(2+bx)}{b\sqrt{-(2+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ((2 + b*x)*Log[2 + b*x])/(b*Sqrt[-(2 + b*x)^2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.37, size = 153, normalized size = 5.28

$$\text{Piecewise}\left[\left\{\left\{\frac{I\left(\text{Log}\left[\frac{b}{2+bx}\right]-\text{Log}\left[\frac{2+bx}{b}\right]\right)}{b},\text{Abs}\left[\frac{2}{b}+x\right]<1\&\&\text{Abs}\left[\frac{b}{2+bx}\right]<1\right\},\left\{-\frac{I\text{Log}\left[\frac{2}{b}+x\right]}{b},\text{Abs}\left[\frac{2}{b}+x\right]<1\right\},\left\{\frac{I\text{Log}\left[\frac{b}{2+bx}\right]}{b},\text{Abs}\left[\frac{b}{2+bx}\right]<1\right\}\right\},-\frac{I\text{meijerg}\left[\{\{1,1\},\{\}\},\{\{\},\{0,0\}\},\frac{2}{b}+x\right]}{b}+\frac{I\text{meijerg}\left[\{\{\},\{1,1\}\},\{\{0,0\},\{\}\},\frac{2}{b}+x\right]}{b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]')
```

```
[Out] Piecewise[{{I (Log[b / (2 + b x)] - Log[(2 + b x) / b]) / b, Abs[2 / b + x]
 < 1 && Abs[b / (2 + b x)] < 1}, {-I Log[2 / b + x] / b, Abs[2 / b + x] < 1
}, {I Log[b / (2 + b x)] / b, Abs[b / (2 + b x)] < 1}}, -I meijerg[{{1, 1},
 {}}, {{}, {0, 0}}, 2 / b + x] / b + I meijerg[{{}, {1, 1}}, {{0, 0}, {}},
2 / b + x] / b]
```

Maple [A]
time = 0.24, size = 26, normalized size = 0.90

method	result	size
default	$\frac{\ln(bx+2)\sqrt{bx+2}}{b\sqrt{-bx-2}}$	26
meijerg	$\frac{\sqrt{\text{signum}\left(\frac{bx}{2}+1\right)}\,\ln\left(\frac{bx}{2}+1\right)}{\sqrt{-\text{signum}\left(\frac{bx}{2}+1\right)}\,b}$	32
risch	$-\frac{i\sqrt{\frac{-bx-2}{bx+2}}\,\sqrt{bx+2}\,\ln(bx+2)}{\sqrt{-bx-2}\,b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.55

$$\sqrt{-\frac{1}{b^2}}\log\left(x+\frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-1/b^2)*log(x + 2/b)
```

Fricas [A]
time = 0.29, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 1.14, size = 87, normalized size = 3.00

$$\begin{cases} \frac{i \log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} - \frac{i \log\left(x+\frac{2}{b}\right)}{b} & \text{for}\ \frac{1}{\left|x+\frac{2}{b}\right|} < 1 \wedge \left|x+\frac{2}{b}\right| < 1 \\ -\frac{i \log\left(x+\frac{2}{b}\right)}{b} & \text{for}\ \left|x+\frac{2}{b}\right| < 1 \\ \frac{i \log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} & \text{for}\ \frac{1}{\left|x+\frac{2}{b}\right|} < 1 \\ \frac{i G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x+\frac{2}{b}\right)}{b} - \frac{i G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x+\frac{2}{b}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*log(1/(x + 2/b))/b - I*log(x + 2/b)/b, (Abs(x + 2/b) < 1) & (1
/Abs(x + 2/b) < 1)), (-I*log(x + 2/b)/b, Abs(x + 2/b) < 1), (I*log(1/(x + 2
/b))/b, 1/Abs(x + 2/b) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/b
)/b - I*meijerg(((1, 1), ()), ((), (0, 0)), x + 2/b)/b, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.00, size = 17, normalized size = 0.59

$$-\frac{\operatorname{sign}(b)\operatorname{sign}(x)\,\mathrm{i}\ln|-bx-2|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -I*log(abs(-b*x - 2))*sgn(b)*sgn(x)/b
```

Mupad [B]
time = 0.07, size = 47, normalized size = 1.62

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 2)^(1/2)),x)
[Out] -(4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

3.1542 $\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

```
[Out] -2*arctan((-b*x-3)^(1/2)/(b*x+2)^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {65, 223, 209}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

Mathematica [**A**]
time = 0.03, size = 26, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.
time = 0.16, size = 66, normalized size = 2.54

method	result	size
default	$\frac{\sqrt{(-bx-3)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{5}{2b}\right)}{\sqrt{-x^2b^2-5bx-6}}\right)}{\sqrt{-bx-3}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-3)*(b*x+2))^(1/2)/(-b*x-3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+5/2/b)/(-b^2*x^2-5*b*x-6)^(1/2))
```

Maxima [A]
time = 0.38, size = 21, normalized size = 0.81

$$-\frac{\arcsin\left(-\frac{2\,b^2x+5\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 5*b)/b)/b
```

Fricas [A]
time = 0.32, size = 44, normalized size = 1.69

$$-\frac{\arctan\left(\frac{(2\,bx+5)\sqrt{bx+2}\,\sqrt{-bx-3}}{2\,(b^2x^2+5\,bx+6)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 5)*sqrt(b*x + 2)*sqrt(-b*x - 3)/(b^2*x^2 + 5*b*x + 6))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(b*x + 2)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.00, size = 28, normalized size = 1.08

$$-\frac{2i\ln\left(\sqrt{-bx-2}-\sqrt{-bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] -2*I*log(sqrt(-b*x - 2) - sqrt(-b*x - 3))/b
```

Mupad [B]
time = 0.30, size = 47, normalized size = 1.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

3.1543 $\int \frac{1}{\sqrt{2-bx}\,\sqrt{3-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

```
[Out] -2*arcsinh((-b*x+2)^(1/2))/b
```

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2-bx}\,\sqrt{3-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(16) = 32.
time = 0.04, size = 59, normalized size = 3.69

$$\frac{\log\left(-1+\frac{\sqrt{3-bx}}{\sqrt{2-bx}}\right)}{b}-\frac{\log\left(b+\frac{b\sqrt{3-bx}}{\sqrt{2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[3 - b*x]/Sqrt[2 - b*x]]/b - Log[b + (b*Sqrt[3 - b*x])/Sqrt[2
 - b*x]]/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 - b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(14) = 28.
time = 0.16, size = 70, normalized size = 4.38

method	result	size
default	$\frac{\sqrt{(-bx+2)(-bx+3)}\,\ln\left(\frac{-\frac{5}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-5bx+6}\right)}{\sqrt{-bx+2}\,\sqrt{-bx+3}\,\sqrt{b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+2)*(-b*x+3))^(1/2)/(-b*x+2)^(1/2)/(-b*x+3)^(1/2)*ln((-5/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-5*b*x+6)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.
time = 0.27, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-5\,bx+6}\,b-5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 5*b*x + 6)*b - 5*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

time = 0.30, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+3}\,\sqrt{-bx+2}+5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 3)*sqrt(-b*x + 2) + 5)/b
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx+2}\,\sqrt{-bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(-b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 2)*sqrt(-b*x + 3)), x)
```

Giac [A]

time = 0.00, size = 27, normalized size = 1.69

$$\frac{2\ln\left(\sqrt{-bx+3}-\sqrt{-bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x)
```

```
[Out] 2*log(sqrt(-b*x + 3) - sqrt(-b*x + 2))/b
```

Mupad [B]

time = 0.31, size = 49, normalized size = 3.06

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)
^(1/2))))/(-b^2)^(1/2)
```

3.1544 $\int \frac{1}{2-bx} dx$

Optimal. Leaf size=12

$$-\frac{\log(2-bx)}{b}$$

[Out] -ln(-b*x+2)/b

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {31}

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(2 - b*x)^(-1),x]

[Out] -(Log[2 - b*x]/b)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2-bx} dx = -\frac{\log(2-bx)}{b}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - b*x)^(-1),x]

[Out] -(Log[2 - b*x]/b)

Mathics [A]
time = 1.69, size = 11, normalized size = 0.92

$$-\frac{\text{Log}[-2+bx]}{b}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 - b*x]),x]')
[Out] -Log[-2 + b x] / b
```

Maple [A]
time = 0.15, size = 13, normalized size = 1.08

method	result	size
norman	$-\frac{\ln(bx-2)}{b}$	12
risch	$-\frac{\ln(bx-2)}{b}$	12
default	$-\frac{\ln(-bx+2)}{b}$	13
meijerg	$-\frac{\ln\left(-\frac{bx}{2}+1\right)}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2),x,method=_RETURNVERBOSE)
[Out] -ln(-b*x+2)/b
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="maxima")
[Out] -log(b*x - 2)/b
```

Fricas [A]
time = 0.28, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="fricas")
[Out] -log(b*x - 2)/b
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x)
```

```
[Out] -log(b*x - 2)/b
```

Giac [A]
time = 0.00, size = 11, normalized size = 0.92

$$-\frac{\ln|xb-2|}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x)
```

```
[Out] -log(abs(b*x - 2))/b
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.92

$$-\frac{\ln\left(b\,x-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x - 2),x)
```

```
[Out] -log(b*x - 2)/b
```

3.1545 $\int \frac{1}{\sqrt{1-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

```
[Out] -2*arcsinh((-b*x+1)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-bx}\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(16) = 32.
time = 0.04, size = 59, normalized size = 3.69

$$\frac{\log\left(-1+\frac{\sqrt{2-bx}}{\sqrt{1-bx}}\right)}{b}-\frac{\log\left(b+\frac{b\sqrt{2-bx}}{\sqrt{1-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[2 - b*x]/Sqrt[1 - b*x]]/b - Log[b + (b*Sqrt[2 - b*x])/Sqrt[1
- b*x]]/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(14) = 28.
time = 0.16, size = 70, normalized size = 4.38

method	result	size
default	$\frac{\sqrt{(-bx+1)(-bx+2)}\,\ln\left(\frac{-\frac{3}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-3bx+2}\right)}{\sqrt{-bx+1}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+1)*(-b*x+2))^(1/2)/(-b*x+1)^(1/2)/(-b*x+2)^(1/2)*ln((-3/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-3*b*x+2)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.
time = 0.28, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-3\,bx+2}\,b-3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 3*b*x + 2)*b - 3*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.29, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx+1}\,+3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x + 1) + 3)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx+1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(-b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.69

$$\frac{2\ln\left(\sqrt{-bx+2}\,-\sqrt{-bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x + 1))/b
```

Mupad [B]
time = 0.31, size = 45, normalized size = 2.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{1-b\,x}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

3.1546 $\int \frac{1}{\sqrt{-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

[Out] -2*arcsinh(1/2*(-b*x)^(1/2)*2^(1/2))/b

Rubi [A]

time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]

[Out] (-2*ArcSinh[Sqrt[-(b*x)]/Sqrt[2]])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{-bx}\right)}{b}\\
&= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.
time = 0.01, size = 48, normalized size = 2.40

$$-\frac{2\sqrt{x}\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}\sqrt{-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*Sqrt[x]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(Sqrt[-b]*Sqrt[-(b*x)
])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.61, size = 43, normalized size = 2.15

$$\text{Piecewise}\left[\left\{\left\{\frac{-2\text{ArcCosh}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b},\text{Abs}\left[bx\right]>2\right\}\right\},\frac{-2I\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right]}{b}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[0 - b*x]*Sqrt[2 - b*x]),x]')
```

```
[Out] Piecewise[{{-2 ArcCosh[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b, Abs[b x] > 2}}, -2
 I ArcSin[Sqrt[2] Sqrt[b] Sqrt[x] / 2] / b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.
time = 0.14, size = 64, normalized size = 3.20

method	result	size
meijerg	$\frac{2\sqrt{x}\,\arcsin\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{\sqrt{b}\sqrt{-bx}}$	27
default	$\frac{\sqrt{-bx\left(-bx+2\right)}\,\ln\left(\frac{b^{2}x-b}{\sqrt{b^{2}}}+\sqrt{x^{2}b^{2}-2bx}\right)}{\sqrt{-bx}\sqrt{-bx+2}\sqrt{b^{2}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x*(-b*x+2))^(1/2)/(-b*x)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2)+
(b^2*x^2-2*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 32, normalized size = 1.60

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 - 2\,bx}\,b - 2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x)*b - 2*b)/b
```

Fricas [A]
time = 0.29, size = 27, normalized size = 1.35

$$-\frac{\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-bx} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x) + 1)/b
```

Sympy [A]
time = 0.76, size = 51, normalized size = 2.55

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b} & \text{for}\ |bx| > 2 \\ -\frac{2i\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, Abs(b*x) > 2), (-2*I*asin
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, True))
```

Giac [A]
time = 0.00, size = 25, normalized size = 1.25

$$\frac{2\ln\left(\sqrt{-bx+2} - \sqrt{-bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x))/b
```

Mupad [B]
time = 0.28, size = 39, normalized size = 1.95

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\sqrt{-b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((-b*x)^(1/2)*(-b^2)^(1/2))))/(-b^
2)^(1/2)
```

3.1547 $\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}$$

```
[Out] -2*arcsinh(1/3*(-b*x-1)^(1/2)*3^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-1 - b*x]/Sqrt[3]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1-bx}\right)}{b}\\
&= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-1-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-1 - b*x]])/b
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.
time = 0.16, size = 70, normalized size = 3.18

method	result	size
default	$\frac{\sqrt{(-bx-1)(-bx+2)}\,\ln\left(\frac{-\frac{1}{2}b+b^{2}x}{\sqrt{b^{2}}}+\sqrt{x^{2}b^{2}-bx-2}\right)}{\sqrt{-bx-1}\,\sqrt{-bx+2}\,\sqrt{b^{2}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-1)*(-b*x+2))^(1/2)/(-b*x-1)^(1/2)/(-b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-b*x-2)^(1/2))/(b^2)^(1/2)
```

Maxima [**A**]
time = 0.28, size = 33, normalized size = 1.50

$$\frac{\log\left(2\,b^{2}x+2\,\sqrt{b^{2}x^{2}-bx-2}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 2)*b - b)/b
```

Fricas [A]
time = 0.29, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-1}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 1) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx-1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(-b*x+2)**(1/2),x)
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(-b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.23

$$\frac{2\ln\left(\sqrt{-bx+2}-\sqrt{-bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x)
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 1))/b
```

Mupad [B]
time = 0.28, size = 46, normalized size = 2.09

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(2 - b*x)^(1/2)),x)
[Out] -(4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)^(
1/2))))/(-b^2)^(1/2)
```

3.1548 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

```
[Out] -arccosh(-1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{54\}$

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] -(ArcCosh[-1/2*(b*x)]/b)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx = -\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.
time = 0.04, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-2 - b*x]])/b

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.85, size = 71, normalized size = 5.92

$$\frac{-\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{4}{b^2x^2}\right]-I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{4\text{exp\_polar}[-2I\text{Pi}]}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]')
```

```
[Out] (-meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1 / 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 / 4,
 1, 0}, {}}, 4 / (b ^ 2 x ^ 2)] - I meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 /
 2, 1}, {}}, {{-1 / 4, 1 / 4}, {-1 / 2, 0, 0, 0}}, 4 exp_polar[-2 I Pi] / (
b ^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(10) = 20$.
time = 0.16, size = 61, normalized size = 5.08

method	result	size
default	$\frac{\sqrt{(-bx-2)(-bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-4}\right)}{\sqrt{-bx-2}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-2)*(-b*x+2))^(1/2)/(-b*x-2)^(1/2)/(-b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2
)+(b^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.30, size = 26, normalized size = 2.17

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(10) = 20$.
time = 0.29, size = 28, normalized size = 2.33

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x - 2))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 16.51, size = 78, normalized size = 6.50

$$-\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] -meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4/
(b**2*x**2))/(4*pi**(3/2)*b) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 0.00, size = 27, normalized size = 2.25

$$\frac{2\ln\left(\sqrt{-bx+2}-\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 2))/b
```

Mupad [B]
time = 0.29, size = 52, normalized size = 4.33

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1549 $\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}$$

```
[Out] -2*arcsinh(1/5*(-b*x-3)^(1/2)*5^(1/2))/b
```

Rubi [**A**]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-3 - b*x]/Sqrt[5]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}\\
&= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-3-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-3 - b*x]])/b
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.
time = 0.17, size = 69, normalized size = 3.14

method	result	size
default	$\frac{\sqrt{(-bx-3)(-bx+2)}\,\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+bx-6}\right)}{\sqrt{-bx-3}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-3)*(-b*x+2))^(1/2)/(-b*x-3)^(1/2)/(-b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b
^2)^(1/2)+(b^2*x^2+b*x-6)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.25, size = 30, normalized size = 1.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-6}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 6)*b + b)/b
```

Fricas [A]
time = 0.29, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-3}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 3) - 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx-3}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(-b*x+2)**(1/2),x)
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(-b*x + 2)), x)
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.23

$$\frac{2\ln\left(\sqrt{-bx+2}-\sqrt{-bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x)
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 3))/b
```

Mupad [B]
time = 0.29, size = 52, normalized size = 2.36

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 3)^(1/2)),x)
[Out] (4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1550 $\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

```
[Out] arccosh(1/4*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {54}

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/4]/b
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.04, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{4+bx}}{\sqrt{-4+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[4 + b*x]/Sqrt[-4 + b*x]])/b
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 16.03, size = 69, normalized size = 6.27

$$\frac{I\text{meijerg}\left[\left\{\left\{-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1\right\},\{\}\right\},\left\{\left\{-\frac{1}{4},\frac{1}{4}\right\},\left\{-\frac{1}{2},0,0,0\right\}\right\},\frac{16}{b^2x^2}\right]+\text{meijerg}\left[\left\{\left\{\frac{1}{4},\frac{3}{4}\right\},\left\{\frac{1}{2},\frac{1}{2},1,1\right\}\right\},\left\{\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0\right\},\{\}\right\},\frac{16\text{exp\_polar}[2I\text{Pi}]}{b^2x^2}\right]}{4\text{Pi}^{\frac{3}{2}}b}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 + b*x]*Sqrt[-4 + b*x]),x]')
```

```
[Out] (I meijerg[{{-1 / 2, -1 / 4, 0, 1 / 4, 1 / 2, 1}, {}}, {{-1 / 4, 1 / 4}, {-
1 / 2, 0, 0, 0}}, 16 / (b ^ 2 x ^ 2)] + meijerg[{{1 / 4, 3 / 4}, {1 / 2, 1
/ 2, 1, 1}}, {{0, 1 / 4, 1 / 2, 3 / 4, 1, 0}, {}}, 16 exp_polar[2 I Pi] / (
b ^ 2 x ^ 2)]) / (4 Pi ^ (3 / 2) b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(9) = 18$.
time = 0.17, size = 57, normalized size = 5.18

method	result	size
default	$\frac{\sqrt{(bx-4)(bx+4)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-16}\right)}{\sqrt{bx-4}\,\sqrt{bx+4}\,\sqrt{b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-4)*(b*x+4))^(1/2)/(b*x-4)^(1/2)/(b*x+4)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-16)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.25, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-16}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 16)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.29, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+4}\,\sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x - 4))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 17.29, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{16e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{16}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 16*
exp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 16/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.
time = 0.00, size = 26, normalized size = 2.36

$$-\frac{2\ln\left(\sqrt{bx+4} - \sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x)
```

```
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x - 4))/b
```

Mupad [B]
time = 0.32, size = 40, normalized size = 3.64

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-4}-2{}\mathrm{i}\right)}{\left(\sqrt{b\,x+4}-2\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 4)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x - 4)^(1/2) - 2i))/(((b*x + 4)^(1/2) - 2)*(-b^2)^(1/2))))/
(-b^2)^(1/2)
```

3.1551 $\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[Out] 2*arcsinh(d^(1/2)*(-b*(1-c)/d+b*x)^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[d]*Sqrt[-((b*(1 - c))/d) + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{-b+bc}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{-b+bc}{d}+bx}\right)}{b}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 1.19

$$\frac{2\sqrt{-1+c+dx}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-1+c+dx}}\right)}{d\sqrt{\frac{b(-1+c+dx)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-1 + c + d*x]*ArcTanh[Sqrt[c + d*x]/Sqrt[-1 + c + d*x]])/(d*Sqrt[(b
*(-1 + c + d*x))/d])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(33) = 66$.
time = 0.19, size = 100, normalized size = 2.33

method	result	size
default	$\frac{\sqrt{\left(bx+\frac{b(c-1)}{d}\right)(dx+c)}\,\ln\left(\frac{\frac{b(c-1)}{2}+\frac{bc}{2}+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}+\left(b\left(c-1\right)+bc\right)x+\frac{b(c-1)c}{d}}\right)}{\sqrt{bx+\frac{b(c-1)}{d}}\,\sqrt{dx+c}\,\sqrt{bd}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+b*(c-1)/d)*(d*x+c))^(1/2)/(b*x+b*(c-1)/d)^(1/2)/(d*x+c)^(1/2)*ln((1/2
*b*(c-1)+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(b*(c-1)+b*c)*x+b*(c-1)/d*c)^(
1/2))/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(30) = 60$.
time = 0.31, size = 175, normalized size = 4.07

$$\left[\frac{\sqrt{bd}\,\log\left(8\,bd^{2}x^{2}+8\,bc^{2}+8\,(2\,bc-b)dx+4\,\sqrt{bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}}{2\,(bd^{2}x^{2}+bc^{2}+(2\,bc-b)dx-bc)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b*d^2*x^2 + 8*b*c^2 + 8*(2*b*c - b)*d*x + 4*sqrt(b*d)*
(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((b*d*x + b*c - b)/d) - 8*b*c + b)/(b*d
), -sqrt(-b*d)*arctan(1/2*sqrt(-b*d)*(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((
b*d*x + b*c - b)/d)/(b*d^2*x^2 + b*c^2 + (2*b*c - b)*d*x - b*c))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\frac{c}{d}+x-\frac{1}{d}\right)}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(c/d + x - 1/d))*sqrt(c + d*x)), x)
```

Giac [A]
time = 0.02, size = 57, normalized size = 1.33

$$-\frac{2d^2\ln\left|\sqrt{bd\left(c+dx\right)-bd}-\sqrt{bd}\,\sqrt{c+dx}\right|}{|d|\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -2*d*log(abs(-sqrt(b*d)*sqrt(d*x + c) + sqrt((d*x + c)*b*d - b*d)))/(sqrt(b
*d)*abs(d))
```

Mupad [B]
time = 0.50, size = 66, normalized size = 1.53

$$\frac{4\,\mathrm{atan}\left(-\frac{d\left(\sqrt{b\,x-\frac{b-b\,c}{d}}-\sqrt{-\frac{b-b\,c}{d}}\right)}{\sqrt{-b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - (b - b*c)/d)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (4*atan(-(d*((b*x - (b - b*c)/d)^(1/2) - (-(b - b*c)/d)^(1/2)))/((-b*d)^(1/
2)*((c + d*x)^(1/2) - c^(1/2)))))/(-b*d)^(1/2)
```

3.1552 $\int \frac{1}{\sqrt{x}\,\sqrt{-3+2x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{2}\,\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)$$

```
[Out] arcsinh(1/3*(-3+2*x)^(1/2)*3^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {56, 221}

$$\sqrt{2}\,\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,\sqrt{-3+2x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{2}\,\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.36

$$-\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt{x}+\sqrt{-3+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] -(Sqrt[2]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[-3 + 2*x]])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.16, size = 34, normalized size = 1.55

$$\text{Piecewise}\left[\left\{\left\{\sqrt{2}\,\text{ArcCosh}\left[\frac{\sqrt{6}\,\sqrt{x}}{3}\right],\text{Abs}\left[x\right]>\frac{3}{2}\right\}\right\},-I\sqrt{2}\,\text{ArcSin}\left[\frac{\sqrt{6}\,\sqrt{x}}{3}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]')
```

```
[Out] Piecewise[{{Sqrt[2] ArcCosh[Sqrt[6] Sqrt[x] / 3], Abs[x] > 3 / 2}}, -I Sqrt
[2] ArcSin[Sqrt[6] Sqrt[x] / 3]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.
time = 0.16, size = 48, normalized size = 2.18

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(x-\frac{3}{2}\right)}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)}{\sqrt{\operatorname{signum}\left(x-\frac{3}{2}\right)}}$	31
default	$\frac{\sqrt{x\left(2x-3\right)}\,\ln\left(\frac{\left(-\frac{3}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^{2}-3x}\right)\sqrt{2}}{2\sqrt{x}\,\sqrt{2x-3}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x*(2*x-3))^(1/2)/x^(1/2)/(2*x-3)^(1/2)*ln(1/2*(-3/2+2*x)*2^(1/2)+(2*x^
2-3*x)^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.38, size = 41, normalized size = 1.86

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2\,x-3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2\,x-3}}{\sqrt{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x - 3)/sqrt(x))/(sqrt(2) + sqrt(2*x - 3
)/sqrt(x)))
```

Fricas [A]
time = 0.29, size = 26, normalized size = 1.18

$$\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{2x-3}\sqrt{x}-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(2*x - 3)*sqrt(x) - 4*x + 3)
```

Sympy [A]
time = 0.55, size = 42, normalized size = 1.91

$$\begin{cases}\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for } |x|>\frac{3}{2}\\ -\sqrt{2}\,i\operatorname{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-3+2*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x)/3), Abs(x) > 3/2), (-sqrt(2)*I*asi
n(sqrt(6)*sqrt(x)/3), True))
```

Giac [A]
time = 0.00, size = 33, normalized size = 1.50

$$-\frac{2\ln\left|\sqrt{2x-3}-\sqrt{2}\sqrt{x}\right|}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x)
```

```
[Out] -sqrt(2)*log(abs(-sqrt(2)*sqrt(x) + sqrt(2*x - 3)))
```

Mupad [B]
time = 0.44, size = 30, normalized size = 1.36

$$-2\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\left(-\sqrt{2x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2*x - 3)^(1/2)),x)
[Out] -2*2^(1/2)*atanh((2^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*x^(1/2)))
```

3.1553 $\int \frac{1}{\sqrt{-3+2x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)$$

```
[Out] 1/3*arcsinh(1/13*39^(1/2)*(-3+2*x)^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {56, 221}

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+2x}\,\sqrt{2+3x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13+3x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 26, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-9+6x}{4+6x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcTanh[1/Sqrt[(-9 + 6*x)/(4 + 6*x)]]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.37, size = 43, normalized size = 1.65

$$\text{Piecewise}\left[\left\{\left\{\frac{\sqrt{6}\,\text{ArcCosh}\left[\frac{\sqrt{26}\sqrt{2+3x}}{13}\right]}{3},\text{Abs}\left[\frac{2}{3}+x\right]>\frac{13}{6}\right\}\right\},-\frac{I\sqrt{6}\,\text{ArcSin}\left[\frac{\sqrt{78}\sqrt{\frac{2}{3}+x}}{13}\right]}{3}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[2+3*x]*Sqrt[-3+2*x]),x]')
```

```
[Out] Piecewise[{{Sqrt[6] ArcCosh[Sqrt[26] Sqrt[2 + 3 x] / 13] / 3, Abs[2 / 3 + x
] > 13 / 6}}, -I Sqrt[6] ArcSin[Sqrt[78] Sqrt[2 / 3 + x] / 13] / 3]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.
time = 0.17, size = 57, normalized size = 2.19

method	result	size
default	$\frac{\sqrt{(2x-3)(2+3x)}\,\ln\left(\frac{\left(-\frac{5}{2}+6x\right)\sqrt{6}}{6}+\sqrt{6x^{2}-5x-6}\right)\sqrt{6}}{6\sqrt{2x-3}\,\sqrt{2+3x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-3)^(1/2)/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*x-3)*(2+3*x))^(1/2)/(2*x-3)^(1/2)/(2+3*x)^(1/2)*ln(1/6*(-5/2+6*x)*6
^(1/2)+(6*x^2-5*x-6)^(1/2))*6^(1/2)
```

Maxima [A]
time = 0.36, size = 28, normalized size = 1.08

$$\frac{1}{6}\sqrt{6}\log\left(2\sqrt{6}\sqrt{6x^2-5x-6}+12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*log(2*sqrt(6)*sqrt(6*x^2 - 5*x - 6) + 12*x - 5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.
time = 0.29, size = 46, normalized size = 1.77

$$\frac{1}{12}\sqrt{3}\sqrt{2}\log\left(4\sqrt{3}\sqrt{2}(12x-5)\sqrt{3x+2}\sqrt{2x-3}+288x^2-240x-119\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*log(4*sqrt(3)*sqrt(2)*(12*x - 5)*sqrt(3*x + 2)*sqrt(2*
x - 3) + 288*x^2 - 240*x - 119)
```

Sympy [A]
time = 0.66, size = 56, normalized size = 2.15

$$\begin{cases} \frac{\sqrt{6}\,\operatorname{acosh}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{for}\ \left|x+\frac{2}{3}\right| > \frac{13}{6} \\ -\frac{\sqrt{6}\,i\operatorname{asin}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(6)*acosh(sqrt(78)*sqrt(x + 2/3)/13)/3, Abs(x + 2/3) > 13/6)
, (-sqrt(6)*I*asin(sqrt(78)*sqrt(x + 2/3)/13)/3, True))
```

Giac [A]
time = 0.00, size = 46, normalized size = 1.77

$$-\frac{2\ln\left(\sqrt{3(2x-3)+13}-\sqrt{3}\sqrt{2x-3}\right)}{\sqrt{2}\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x)
```

```
[Out] -1/3*sqrt(3)*sqrt(2)*log(-sqrt(3)*sqrt(2*x - 3) + sqrt(6*x + 4))
```

Mupad [B]
time = 0.12, size = 43, normalized size = 1.65

$$\frac{2\sqrt{6}\,\operatorname{atanh}\left(\frac{\sqrt{6}\,\left(-\sqrt{2\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\left(\sqrt{2}-\sqrt{3\,x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 3)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] (2*6^(1/2)*atanh((6^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*(2^(1/2) - (3*
x + 2)^(1/2)))))/3
```

3.1554 $\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

[Out] 2*arcsin(d^(1/2)*(b*(1-c)/d+b*x)^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {65, 222}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]

[Out] (2*ArcSin[(Sqrt[d]*Sqrt[(b*(1 - c))/d + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\,\sqrt{c-dx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{b-bc}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{b-bc}{d}+bx}\right)}{b}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 1.38

$$-\frac{2\sqrt{1-c+dx}\,\tan^{-1}\left(\frac{\sqrt{c-dx}}{\sqrt{1-c+dx}}\right)}{d\sqrt{\frac{b(1-c+dx)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (-2*Sqrt[1 - c + d*x]*ArcTan[Sqrt[c - d*x]/Sqrt[1 - c + d*x]])/(d*Sqrt[(b*(
1 - c + d*x))/d])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(32) = 64$.
time = 0.18, size = 109, normalized size = 2.60

method	result	size
default	$\frac{\sqrt{\left(bx-\frac{b(c-1)}{d}\right)(-dx+c)}\,\arctan\left(\frac{\sqrt{bd}\,\left(x-\frac{b(c-1)+bc}{2bd}\right)}{\sqrt{-bd\,x^{2}+(b\,(c-1)+bc)\,x-\frac{b(c-1)c}{d}}}\right)}{\sqrt{bx-\frac{b(c-1)}{d}}\,\sqrt{-dx+c}\,\sqrt{bd}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-b*(c-1)/d)*(-d*x+c))^(1/2)/(b*x-b*(c-1)/d)^(1/2)/(-d*x+c)^(1/2)/(b*d)
^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(b*(c-1)+b*c)/b/d)/(-b*d*x^2+(b*(c-1)+b*c)
*x-b*(c-1)/d*c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.
time = 0.31, size = 176, normalized size = 4.19

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,bd^{2}x^{2}+8\,bc^{2}-8\,(2\,bc-b)dx-4\,\sqrt{-bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{bd}\,\arctan\left(\frac{\sqrt{bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}}{2\,(bd^{2}x^{2}+bc^{2}-(2\,bc-b)dx-bc)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b*d^2*x^2 + 8*b*c^2 - 8*(2*b*c - b)*d*x - 4*sqrt(-b*
d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqrt((b*d*x - b*c + b)/d) - 8*b*c + b)/
(b*d), -sqrt(b*d)*arctan(1/2*sqrt(b*d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqr
t((b*d*x - b*c + b)/d)/(b*d^2*x^2 + b*c^2 - (2*b*c - b)*d*x - b*c))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(-\frac{c}{d}+x+\frac{1}{d}\right)}\,\sqrt{c-dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)**(1/2)/(-d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(-c/d + x + 1/d))*sqrt(c - d*x)), x)
```

Giac [A]
time = 0.02, size = 60, normalized size = 1.43

$$\frac{2d^2\ln\left|\sqrt{-bd\,(c-dx)+bd}-\sqrt{-bd}\,\sqrt{c-dx}\right|}{|d|\,\sqrt{-bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x)
```

```
[Out] 2*d*log(abs(-sqrt(-b*d)*sqrt(-d*x + c) + sqrt((d*x - c)*b*d + b*d)))/(sqrt(
-b*d)*abs(d))
```

Mupad [B]
time = 0.51, size = 63, normalized size = 1.50

$$-\frac{4\,\mathrm{atan}\left(-\frac{d\,\left(\sqrt{\frac{b-b\,c}{d}+b\,x}-\sqrt{\frac{b-b\,c}{d}}\right)}{\sqrt{b\,d}\,\left(\sqrt{c-d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((b - b*c)/d + b*x)^(1/2)*(c - d*x)^(1/2)),x)
```

```
[Out] -(4*atan(-(d*(((b - b*c)/d + b*x)^(1/2) - ((b - b*c)/d)^(1/2)))/((b*d)^(1/2
)*((c - d*x)^(1/2) - c^(1/2)))))/(b*d)^(1/2)
```

3.1555 $\int \frac{1}{\sqrt{4-x}\,\sqrt{x}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

[Out] arcsin(-1+1/2*x)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 633, 222}

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[4 - x]*Sqrt[x]),x]

[Out] -ArcSin[1 - x/2]

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = \int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)\right)$$
$$= -\sin^{-1}\left(1-\frac{x}{2}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(10) = 20$.
time = 0.03, size = 38, normalized size = 3.80

$$\frac{2\sqrt{-4+x}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-4+x}}\right)}{\sqrt{-((-4+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[-4 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-4 + x]])/Sqrt[-((-4 + x)*x)]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.08, size = 23, normalized size = 2.30

$$\text{Piecewise}\left[\left\{\left\{-2I\text{ArcCosh}\left[\frac{\sqrt{x}}{2}\right], \text{Abs}[x] > 4\right\}\right\}, 2\text{ArcSin}\left[\frac{\sqrt{x}}{2}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]')
```

```
[Out] Piecewise[{{-2 I ArcCosh[Sqrt[x] / 2], Abs[x] > 4}}, 2 ArcSin[Sqrt[x] / 2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.
time = 0.15, size = 27, normalized size = 2.70

method	result	size
meijerg	$2\arcsin\left(\frac{\sqrt{x}}{2}\right)$	9

default	$\frac{\sqrt{(4-x)\,x}\,\arcsin\left(-1+\frac{x}{2}\right)}{\sqrt{4-x}\,\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] ((4-x)*x)^(1/2)/(4-x)^(1/2)/x^(1/2)*arcsin(-1+1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.35, size = 14, normalized size = 1.40

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.30, size = 14, normalized size = 1.40

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

Sympy [A]
time = 0.50, size = 24, normalized size = 2.40

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{x}}{2}\right) & \text{for}\ |x| > 4 \\ 2\operatorname{asin}\left(\frac{\sqrt{x}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)**(1/2)/x**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(x)/2), Abs(x) > 4), (2*asin(sqrt(x)/2), True))
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.50

$$-2\arcsin\left(\frac{\sqrt{-x+4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x)
[Out] -2*arcsin(1/2*sqrt(-x + 4))
```

Mupad [B]
time = 0.29, size = 16, normalized size = 1.60

$$-4\,\mathrm{atan}\left(\frac{\sqrt{4-x}-2}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(4 - x)^(1/2)),x)
[Out] -4*atan(((4 - x)^(1/2) - 2)/x^(1/2))
```

3.1556 $\int \frac{1}{\sqrt{3-2x}\,\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{2}\,\sin^{-1}\left(\sqrt{\frac{2}{3}}\,\sqrt{x}\right)$$

```
[Out] arcsin(1/3*6^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {56, 222}

$$\sqrt{2}\,\sin^{-1}\left(\sqrt{\frac{2}{3}}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

```
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\,\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{2}\,\sin^{-1}\left(\sqrt{\frac{2}{3}}\,\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 1.90

$$-2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{3}-\sqrt{3-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

```
[Out] -2*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[x])/(Sqrt[3] - Sqrt[3 - 2*x])]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.20, size = 34, normalized size = 1.70

$$\text{Piecewise}\left[\left\{\left\{-I\sqrt{2}\,\text{ArcCosh}\left[\frac{\sqrt{6}\,\sqrt{x}}{3}\right],\text{Abs}\,[x]>\frac{3}{2}\right\}\right\},\sqrt{2}\,\text{ArcSin}\left[\frac{\sqrt{6}\,\sqrt{x}}{3}\right]\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[x]*Sqrt[3 - 2*x]),x]')
```

```
[Out] Piecewise[{{-I Sqrt[2] ArcCosh[Sqrt[6] Sqrt[x] / 3], Abs[x] > 3 / 2}}, Sqrt
[2] ArcSin[Sqrt[6] Sqrt[x] / 3]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.
time = 0.15, size = 31, normalized size = 1.55

method	result	size
meijerg	$\sqrt{2}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)$	17
default	$\frac{\sqrt{(3-2x)\,x}\,\sqrt{2}\,\arcsin\left(\frac{4x}{3}-1\right)}{2\sqrt{3-2x}\,\sqrt{x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((3-2*x)*x)^(1/2)/(3-2*x)^(1/2)/x^(1/2)*2^(1/2)*arcsin(4/3*x-1)
```

Maxima [A]
time = 0.34, size = 21, normalized size = 1.05

$$-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-2\,x+3}}{2\,\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Fricas [A]
time = 0.29, size = 21, normalized size = 1.05

$$-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-2\,x+3}}{2\,\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Sympy [A]
time = 0.55, size = 42, normalized size = 2.10

$$\begin{cases} -\sqrt{2}\,i\,\operatorname{acosh}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{for } |x| > \frac{3}{2} \\ \sqrt{2}\,\operatorname{asin}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/x**(1/2),x)
[Out] Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x)/3), Abs(x) > 3/2), (sqrt(2)*asi
n(sqrt(6)*sqrt(x)/3), True))
```

Giac [A]
time = 0.00, size = 27, normalized size = 1.35

$$-\frac{2\arcsin\left(\frac{\sqrt{-2x+3}}{\sqrt{3}}\right)}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x)
[Out] -sqrt(2)*arcsin(1/3*sqrt(3)*sqrt(-2*x + 3))
```

Mupad [B]
time = 0.30, size = 27, normalized size = 1.35

$$2\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\sqrt{3}-\sqrt{3-2\,x}\right)}{2\,\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(3 - 2*x)^(1/2)),x)
[Out] 2*2^(1/2)*atan((2^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*x^(1/2)))
```

3.1557 $\int \frac{1}{\sqrt{3-2x}\,\sqrt{3+5x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{5}}\,\sin^{-1}\left(\sqrt{\frac{2}{21}}\,\sqrt{3+5x}\right)$$

```
[Out] 1/5*arcsin(1/21*42^(1/2)*(3+5*x)^(1/2))*10^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {56, 222}

$$\sqrt{\frac{2}{5}}\,\sin^{-1}\left(\sqrt{\frac{2}{21}}\,\sqrt{5x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] Sqrt[2/5]*ArcSin[Sqrt[2/21]*Sqrt[3 + 5*x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\,\sqrt{3+5x}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{21-2x^2}}\,dx, x, \sqrt{3+5x}\right)}{\sqrt{5}}\\ &= \sqrt{\frac{2}{5}}\,\sin^{-1}\left(\sqrt{\frac{2}{21}}\,\sqrt{3+5x}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.19

$$-\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{15}{2}-5x}}{\sqrt{3+5x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -(Sqrt[2/5]*ArcTan[Sqrt[15/2 - 5*x]/Sqrt[3 + 5*x]])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 2.36, size = 43, normalized size = 1.65

$$\text{Piecewise}\left[\left\{\left\{\left(-\frac{I}{5}\right)\sqrt{10}\,\text{ArcCosh}\left[\frac{\sqrt{42}\sqrt{3+5x}}{21}\right],\text{Abs}\left[\frac{3}{5}+x\right]>\frac{21}{10}\right\}\right\},\frac{\sqrt{10}\,\text{ArcSin}\left[\frac{\sqrt{210}\sqrt{\frac{3}{5}+x}}{21}\right]}{5}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[3+5*x]*Sqrt[3-2*x]),x]')
```

```
[Out] Piecewise[{{(-I / 5) Sqrt[10] ArcCosh[Sqrt[42] Sqrt[3 + 5 x] / 21], Abs[3 /
 5 + x] > 21 / 10}}, Sqrt[10] ArcSin[Sqrt[210] Sqrt[3 / 5 + x] / 21] / 5]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.17, size = 39, normalized size = 1.50

method	result	size
default	$\frac{\sqrt{(3-2x)(3+5x)}\,\sqrt{10}\,\arcsin\left(\frac{20x}{21}-\frac{3}{7}\right)}{10\sqrt{3-2x}\,\sqrt{3+5x}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*((3-2*x)*(3+5*x))^(1/2)/(3-2*x)^(1/2)/(3+5*x)^(1/2)*10^(1/2)*arcsin(20
/21*x-3/7)
```

Maxima [A]
time = 0.36, size = 11, normalized size = 0.42

$$-\frac{1}{10}\sqrt{10}\arcsin\left(-\frac{20}{21}x+\frac{3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(10)*arcsin(-20/21*x + 3/7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.
time = 0.30, size = 44, normalized size = 1.69

$$-\frac{1}{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{5x+3}\sqrt{-2x+3}-3\sqrt{5}\sqrt{2}}{10x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2)*arctan(1/10*(sqrt(5)*sqrt(2)*sqrt(5*x + 3)*sqrt(-2*x +
 3) - 3*sqrt(5)*sqrt(2))/x)
```

Sympy [A]
time = 0.60, size = 56, normalized size = 2.15

$$\begin{cases} -\frac{\sqrt{10}\, i\, \operatorname{acosh}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{for } \left|x+\frac{3}{5}\right| > \frac{21}{10} \\ \frac{\sqrt{10}\operatorname{asin}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/(3+5*x)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(10)*I*acosh(sqrt(210)*sqrt(x + 3/5)/21)/5, Abs(x + 3/5) >
21/10), (sqrt(10)*asin(sqrt(210)*sqrt(x + 3/5)/21)/5, True))
```

Giac [A]
time = 0.00, size = 35, normalized size = 1.35

$$-\frac{2\sqrt{5}\arcsin\left(\frac{5\sqrt{-2x+3}}{\sqrt{105}}\right)}{\sqrt{2}\cdot 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x)
```

```
[Out] -1/5*sqrt(5)*sqrt(2)*arcsin(1/21*sqrt(105)*sqrt(-2*x + 3))
```

Mupad [B]
time = 0.08, size = 40, normalized size = 1.54

$$-\frac{2\,\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,\left(\sqrt{3}-\sqrt{3-2\,x}\right)}{2\left(\sqrt{3}-\sqrt{5\,x+3}\right)}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(1/2)*(5*x + 3)^(1/2)),x)
```

```
[Out] -(2*10^(1/2)*atan((10^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*(3^(1/2) - (5*x
 + 3)^(1/2)))))/5
```

3.1558 $\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[Out] -2*arctan(d^(1/2)*(-b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {65, 223, 209}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx}\right)}{b}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx}}{\sqrt{c+dx}}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a-bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a - b*x])])/(Sqrt[b]*Sqrt[d
])
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/(Sqrt[a-b*x]*Sqrt[c+d*x]),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(31) = 62$.
time = 0.17, size = 84, normalized size = 1.95

method	result	size

default	$\frac{\sqrt{(-bx+a)(dx+c)}\,\arctan\left(\frac{\sqrt{bd}\,\left(x-\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\,x^2+(ad-bc)\,x+ac}}\right)}{\sqrt{-bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+a)*(d*x+c))^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2)/(b*d)^(1/2)*arctan((b
*d)^(1/2)*(x-1/2*(a*d-b*c)/b/d)/(-b*d*x^2+(a*d-b*c)*x+a*c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.
time = 0.31, size = 185, normalized size = 4.30

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,(2\,bdx+bc-ad)\sqrt{-bd}\,\sqrt{-bx+a}\,\sqrt{dx+c}+8\,(b^2cd-abd^2)x\right)}{2\,bd},-\frac{\sqrt{bd}\,\arctan\left(\frac{(2\,bdx+bc-ad)\sqrt{bd}\,\sqrt{-bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2-abcd+(b^2cd-abd^2)x)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*(2*b
*d*x + b*c - a*d)*sqrt(-b*d)*sqrt(-b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d - a*
b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*(2*b*d*x + b*c - a*d)*sqrt(b*d)*sqrt
(-b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x))/(
b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x)*sqrt(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 66, normalized size = 1.53

$$\frac{2b^2 \ln \left| \sqrt{abd + b^2c - bd\,(a - bx)} - \sqrt{-bd}\,\sqrt{a - bx} \right|}{|b|\, b\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2*b*log(abs(-sqrt(-b*d)*sqrt(-b*x + a) + sqrt(b^2*c + (b*x - a)*b*d + a*b*d
)))/(sqrt(-b*d)*abs(b))
```

Mupad [B]
time = 0.34, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{d\left(\sqrt{a-b\,x}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((d*((a - b*x)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x)^(1/2) - c^
(1/2)))))/(b*d)^(1/2)
```

3.1559 $\int (a+bx)^{3/2}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=457

$$-\frac{108(bc-ad)^2\sqrt{a+bx}\,\sqrt[3]{c+dx}}{935bd^2}+\frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}-\frac{108\ 3^{3/4}\sqrt{2-}}{}$$

```
[Out] 12/187*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/3)/b/d+6/17*(b*x+a)^(5/2)*(d*x+c
)^(1/3)/b-108/935*(-a*d+b*c)^2*(d*x+c)^(1/3)*(b*x+a)^(1/2)/b/d^2-108/935*3^
(3/4)*(-a*d+b*c)^3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(
1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-
a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d
+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-
a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(4/3)/d^3/
(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(
-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$-\frac{108\ 3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^3\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{935b^{4/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{108\sqrt{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)^2}{935bd^2}+\frac{12(a+bx)^{3/2}\sqrt[3]{c+dx}\,(bc-ad)}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]
```

```
[Out] (-108*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/3))/(935*b*d^2) + (12*(b*c -
 a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/3))/(187*b*d) + (6*(a + b*x)^(5/2)*(c +
d*x)^(1/3))/(17*b) - (108*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^3*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^(4/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[3]{c+dx}\,dx = \frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b} + \frac{(2(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx}{17b}$$

$$= \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + \frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b} - \frac{(18(bc-ad)^2)\int \sqrt{}}{187bd}$$

$$= -\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2} + \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + 6(a+$$

$$= -\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2} + \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + 6(a+$$

$$= -\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2} + \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + 6(a+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.16

$$\frac{2(a+bx)^{5/2}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} \sqrt[3]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/3), x)
```

3.1560 $\int \sqrt{a+bx}\,\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=419

$$\frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}+\frac{12\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{\ldots}\right.}{\ldots}$$

```
[Out] 6/11*(b*x+a)^(3/2)*(d*x+c)^(1/3)/b+12/55*(-a*d+b*c)*(d*x+c)^(1/3)*(b*x+a)^(
1/2)/b/d+12/55*3^(3/4)*(-a*d+b*c)^2*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
)*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2
/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1
/2))/b^(4/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

Rubi [A]
time = 0.27, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$\frac{12\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{55b^{4/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{12\sqrt{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/3),x]
```

```
[Out] (12*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*b*d) + (6*(a + b*x)^(3/2
)*(c + d*x)^(1/3))/(11*b) + (12*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(4/3)*d^2*Sq
rt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt[3]{c+dx}\,dx = \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} + \frac{(2(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx}{11b}$$

$$= \frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} - \frac{(6(bc-ad)^2)\int \frac{}{\sqrt{a}}}{55bd}$$

$$= \frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} - \frac{(18(bc-ad)^2)\,\text{Subst}}{}$$

$$= \frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b} + \frac{12\ 3^{3/4}\sqrt{2-\sqrt{3}}\ (b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.17

$$\frac{2(a+bx)^{3/2}\sqrt[3]{c+dx}\ {}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\, (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\, \sqrt[3]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/3),x)
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,\,{\left(c+d\,x\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(1/3), x)
```

3.1561 $\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=381

$$\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-}\right.}}}{5b^{4/3}d\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{bc-}}{\left(\left(1-\right.\right.}}}$$

```
[Out] 6/5*(d*x+c)^(1/3)*(b*x+a)^(1/2)/b-4/5*3^(3/4)*(-a*d+b*c)*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(
1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*
x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))/b^(4/3)/d/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)}{5b^{4/3}d\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/Sqrt[a + b*x],x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*b) - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(4
/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b} + \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{5b}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b} + \frac{(6(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{5bd}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b} - \frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-}{}}}{5b^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$\frac{2\sqrt{a+bx}\,\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{\sqrt{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(1/3)/sqrt(a + b*x), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{1/3}}{\sqrt{a+b\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(1/2), x)
```

3.1562 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}}-\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right.}}}{\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-a\right.}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-a\right.}}}}$$

[Out] -2*(d*x+c)^(1/3)/b/(b*x+a)^(1/2)-4/3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(4/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.19, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 65, 225}

$$-\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]

[Out] (-2*(c + d*x)^(1/3))/(b*Sqrt[a + b*x]) - (4*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{3b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} - \frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc\cdots}\right.}}}{\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\,\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$-\frac{2\sqrt[3]{c+dx}\ {}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\ \sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{(a + bx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(3/2),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(3/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x)^{1/3}}{(a + b\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(3/2), x)
```

3.1563 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=417

$$-\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}-\frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}+\frac{4\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}\cdots}{9\sqrt[4]{3}\,b^{4/3}(bc-ad)\sqrt{a\cdots}}$$

```
[Out] -2/3*(d*x+c)^(1/3)/b/(b*x+a)^(3/2)-4/9*d*(d*x+c)^(1/3)/b/(-a*d+b*c)/(b*x+a)
^(1/2)+4/27*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*
(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b
*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)
^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b
*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(4/3)/(
-a*d+b*c)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {49, 53, 65, 225}

$$\frac{4\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\,(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{4d\sqrt[3]{c+dx}}{9b\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/(3*b*(a + b*x)^(3/2)) - (4*d*(c + d*x)^(1/3))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) + (4*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c
+ d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(4/3)*(b*c - a*d)*Sqrt[a + b
*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}+\frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{9b}$$
$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}-\frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}-\frac{(2d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{27b(bc-ad)}$$
$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}-\frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}-\frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx,x,\sqrt[3]{c+d\right.}{9b(bc-ad)}$$
$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}-\frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}+\frac{4\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.18

$$-\frac{2\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(5/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(5/2), x)
```

3.1564 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=457

$$-\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}-\frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}}+\frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}}-\frac{28\sqrt{2-\sqrt{3}}\,d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{\ldots}\right.}{\ldots}$$

[Out] -2/5*(d*x+c)^(1/3)/b/(b*x+a)^(5/2)-4/45*d*(d*x+c)^(1/3)/b/(-a*d+b*c)/(b*x+a)^(3/2)+28/135*d^2*(d*x+c)^(1/3)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)-28/405*d^2*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(4/3)/(-a*d+b*c)^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.32, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 53, 65, 225}

$$-\frac{28\sqrt{2-\sqrt{3}}\,d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\,(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{28d^2\sqrt[3]{c+dx}}{135b\sqrt{a+bx}\,(bc-ad)^2}-\frac{4d\sqrt[3]{c+dx}}{45b(a+bx)^{3/2}(bc-ad)}-\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]

[Out] (-2*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/2)) - (4*d*(c + d*x)^(1/3))/(45*b*(b*c - a*d)*(a + b*x)^(3/2)) + (28*d^2*(c + d*x)^(1/3))/(135*b*(b*c - a*d)^2*Sqrt[a + b*x]) - (28*Sqrt[2 - Sqrt[3]]*d^2*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(135*3^(1/4)*b^(4/3)*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} + \frac{(2d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx}{15b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} - \frac{(14d^2)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{135b(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} + \frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}} + \frac{(14d^3)\int \frac{}{\sqrt{a-}}}{405b(b}$$

$$(14d^2)\,\text{Subst}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} + \frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}} + \frac{}{}$$

$$28\sqrt{2-\sqrt{3}}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} + \frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}} - \frac{}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.16

$$-\frac{2\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{3};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-5/2, -1/3, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(7/2), x)
```

3.1565 $\int \frac{(a+bx)^{3/2}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=839

$$-\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2}+\frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d}-\frac{162(bc-ad)^2\sqrt{a+bx}}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}$$

```
[Out] 6/13*(b*x+a)^(3/2)*(d*x+c)^(2/3)/d-54/91*(-a*d+b*c)*(d*x+c)^(2/3)*(b*x+a)^(
1/2)/d^2-162/91*(-a*d+b*c)^2*(b*x+a)^(1/2)/b^(2/3)/d^2/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))-54/91*3^(3/4)*(-a*d+b*c)^(7/3)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2
*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(
1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(
1/2)))^2)^(1/2)/b^(2/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1
/2)))^2)^(1/2)+81/91*3^(1/4)*(-a*d+b*c)^(7/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a
*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))/b^(2/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 310, 225, 1893}

$$\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)(bc-ad)^{7/3}}{91b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{54\sqrt{2}\,3^{3/4}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)(bc-ad)^{7/3}}{91b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{162\sqrt{a+bx}\,(bc-ad)^2}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{54\sqrt{a+bx}\,(c+dx)^{2/3}(bc-ad)}{91d^2}+\frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]
```

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(2/3))/(91*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(2/3))/(13*d) - (162*(b*c - a*d)^2*Sqrt[a + b*x])/(91*b^(2/3)*
d^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) + (81*3^(1
```

```
/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))], -7 + 4*Sqrt[3]])/(91*b^(2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^
(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (54*Sqrt[2]*3^(3/4)*(b*c - a*d
)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^(
2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[3]{c+dx}}\,dx = \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx}{13d}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} + \frac{(27(bc-ad)^2)\int \frac{}{\sqrt{a+b}}}{91d^2}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} + \frac{(81(bc-ad)^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{(81(bc-ad)^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{162(bc-a}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.37, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/3), x)
```

3.1566 $\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=804

$$\frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d}+\frac{18(bc-ad)\sqrt{a+bx}}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(bc-ad)^{4/3}\left(\sqrt[3]{b}\right.}{}$$

```
[Out] 6/7*(d*x+c)^(2/3)*(b*x+a)^(1/2)/d+18/7*(-a*d+b*c)*(b*x+a)^(1/2)/b^(2/3)/d/(
-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))+6/7*3^(3/4)*(-a*d+b*c)
^(4/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)
^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/
3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c
)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+
b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^(2/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/
3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b
*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)-9/7*3^(1/4)*(-a*d+b*c)^(4/3)*((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/
3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I
*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)
*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 804, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 310, 225, 1893}

$$-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)(bc-ad)^{4/3}}{7b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{6\sqrt{2}\,3^{3/4}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)(bc-ad)^{4/3}}{7b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{18\sqrt{a+bx}\,(bc-ad)}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(2/3))/(7*d) + (18*(b*c - a*d)*Sqrt[a + b*x])/(7
*b^(2/3)*d*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (
9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*
```

```
(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d
*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c -
a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (6*Sqrt[2]*3^(3/4)*(b*c -
 a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*
b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[3]{c+dx}}\,dx}{7d}$$

$$= \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{7d^2}$$

$$= \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} + \frac{(9(bc-ad))\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right.}{7\sqrt[3]{b}\,d^2}$$

$$= \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} + \frac{18(bc-ad)\sqrt{a+bx}}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{9\sqrt[4]{3}\,\sqrt{2+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Fricas [**F**]
time = 0.37, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/3),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/3), x)
```

3.1567 $\int \frac{1}{\sqrt{a+bx}\,\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=762

$$-\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\cdots}$$

```
[Out] -6*(b*x+a)^(1/2)/b^(2/3)/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2
)))-2*3^(3/4)*(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*Ell
ipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x
+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)
^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1
/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^(2/3)/d/(b*x+a)^
(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)+3*3^(1/4)*(-a*d+b*c)^
(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^
(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3
)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(
d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3
)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/d/(b*x+a)^(1/2)/(
-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c
)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {65, 310, 225, 1893}

$$-\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{2\sqrt{2}\,3^{3/4}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/3)),x]
```

```
[Out] (-6*Sqrt[a + b*x])/(b^(2/3)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(1/3)*((b*c - a*d)
^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(
```

```
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sq
rt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1
- Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*3
^(3/4)*(b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*
Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[3]{c+dx}}\,dx = \frac{3\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{\sqrt[3]{b}\,d} + \frac{\left(3\sqrt{2\left(2+\sqrt{3}\right)}\right.}{}$$

$$= -\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} + \frac{3\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{bc-ad}\,\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\,\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.37, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/3)), x)
```

3.1568 $\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=796

$$-\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}}-\frac{2d\sqrt{a+bx}}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}\right.}{}$$

```
[Out] -2*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(1/2)-2*d*(b*x+a)^(1/2)/b^(2/3)/(-a*d+b
*c)/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))-2/3*((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1
/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-
I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3
)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2
)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1
/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+
b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^
(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*
c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^
(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))/b^(2/3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+
b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*
(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 310, 225, 1893}

$$-\frac{2\sqrt{a+bx}\,d}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt{2}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*(c + d*x)^(2/3))/((b*c - a*d)*Sqrt[a + b*x]) - (2*d*Sqrt[a + b*x])/(b^(
2/3)*(b*c - a*d)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^
```

```
(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))],
-7 + 4*Sqrt[3]])/(b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*
d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2
/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx = -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} + \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{\operatorname{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{\sqrt[3]{b}\,(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt{a+bx}}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 71, normalized size = 0.09

$$-\frac{2\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\ \sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]

[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/3))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)

[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/3)), x)

Fricas [**F**]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/3)), x)
```

3.1569 $\int \frac{1}{(a+bx)^{5/2}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=842

$$-\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}}+\frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}}+\frac{10d^2\sqrt{a+bx}}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-$$

```
[Out] -2/3*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(3/2)+10/9*d*(d*x+c)^(2/3)/(-a*d+b*c)
^2/(b*x+a)^(1/2)+10/9*d^2*(b*x+a)^(1/2)/b^(2/3)/(-a*d+b*c)^2/(-b^(1/3)*(d*x
+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))+10/27*d*((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)
))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(
1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x
+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*3
^(3/4)/b^(2/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-
3^(1/2)))^2)^(1/2)-5/9*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE
((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)
*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4
)/b^(2/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 842, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 310, 225, 1893}

$$\frac{10\sqrt{a+bx}\,d^2}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)d}{3\,3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{10\sqrt{2}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)d}{9\sqrt[4]{3}\,b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{10(c+dx)^{2/3}d}{9(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*(c + d*x)^(2/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (10*d*(c + d*x)^(2/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (10*d^2*Sqrt[a + b*x])/(9*b^(2/3)*(b*c
 - a*d)^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (5
```

```
*Sqrt[2 + Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b
*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c +
d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*
EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d
)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (10*Sqrt[2]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(2/3)*(b*c - a*d
)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[3]{c+dx}}\,dx = -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,d}{27(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(5d)\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{b}{}}}\right.}{9(bc-a}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(5d)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc}}{\sqrt{a-\frac{bc}{d}}}\right.}{9\sqrt[3]{b}\,(}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{10d^2\sqrt{}}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 73, normalized size = 0.09

$$-\frac{2\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}\,(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/3)), x)
```

Fricas [**F**]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/3)), x)
```

3.1570 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=416

$$-\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{}$$

[Out] 6/11*(b*x+a)^(3/2)*(d*x+c)^(1/3)/d-54/55*(-a*d+b*c)*(d*x+c)^(1/3)*(b*x+a)^(1/2)/d^2-54/55*3^(3/4)*(-a*d+b*c)^2*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{55\sqrt[3]{b}\,d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{54\sqrt{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{55d^2}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]

[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*d^2) + (6*(a + b*x)^(3/2)*(c + d*x)^(1/3))/(11*d) - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(1/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx = \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx}{11d}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} + \frac{(27(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}}}{55d^2}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} + \frac{(81(bc-ad)^2)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} - \frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(2/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(2/3), x)
```

3.1571 $\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=381

$$\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d}+\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-\cdots}\right.}}}{5\sqrt[3]{b}\,d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

[Out] 6/5*(d*x+c)^(1/3)*(b*x+a)^(1/2)/d+6/5*3^(3/4)*(-a*d+b*c)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.21, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{5\sqrt[3]{b}\,d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]/(c + d*x)^(2/3),x]

[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*d) + (6*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(1/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx = \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{5d}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{5d^2}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d} + \frac{6\ 3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc}{}}}{5\sqrt[3]{b}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.19

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(2/3),x]
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(2/3),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx}}{(c + dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(2/3),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(2/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\,x}}{(c + d\,x)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(2/3), x)
```

3.1572 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=345

$$-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}{\sqrt[3]{b}\,d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}}}$$

[Out] -2*3^(3/4)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/3)/d/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.16, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 225}

$$-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\,d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(2/3)),x]

[Out] (-2*3^(3/4)*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(b^(1/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx = \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc\cdots}\right.}}}{\sqrt[3]{b}\,d\sqrt{a+bx}\,\sqrt{-\frac{\cdots}{(\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.21

$$\frac{2\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(2/3)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[1/2, 2
/3, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in ___instancecheck___

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(2/3)),x]')
[Out] cought exception: maximum recursion depth exceeded in __instancecheck__
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(2/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(2/3)), x)
```

3.1573 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=383

$$-\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}\ \cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{\cdots}\right.}}\ \cdots}{\sqrt[4]{3}\sqrt[3]{b}(bc-ad)\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

```
[Out] -2*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(1/2)+2/3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a
*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2
)-1/2*2^(1/2))*3^(3/4)/b^(1/3)/(-a*d+b*c)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[3]{c+dx}}{\sqrt{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[2 - Sqrt[3]]*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*(
b*c - a*d)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx = -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{3(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-\cdots}{\cdots}}\cdots}{\sqrt[4]{3}\,\sqrt[3]{\cdots}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(a+b\,x)}^{3/2}\,{(c+d\,x)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(2/3)), x)
```

3.1574 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=421

$$-\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}}-\frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/}}{((1}}}{9\sqrt[4]{3}\,\sqrt[3]{b}\,(bc-}$$

```
[Out] -2/3*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(3/2)+14/9*d*(d*x+c)^(1/3)/(-a*d+b*c)
^2/(b*x+a)^(1/2)-14/27*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF
((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)
*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4
)/b^(1/3)/(-a*d+b*c)^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))
^2)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 225}

$$-\frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{a+bx}\,(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{14d\sqrt[3]{c+dx}}{9\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt[3]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (14*d*(c + d*x)^(1/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) - (14*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(1/3)*(b*c - a*d
)^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx = -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(7d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,d}{27(bc-ad)^2}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(7d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+}}\right.}{9(bc-}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.17

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(-\frac{3}{2},\frac{2}{3};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(2/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/2}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(2/3)), x)
```

3.1575 $\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=219

$$\frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}}+\frac{(bc-$$

```
[Out] 1/6*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(1/3)/b/d+1/2*(b*x+a)^(5/3)*(d*x+c)^(1
/3)/b+1/18*(-a*d+b*c)^2*ln(d*x+c)/b^(4/3)/d^(5/3)+1/6*(-a*d+b*c)^2*ln(-1+d^
(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(4/3)/d^(5/3)+1/9*(-a*d+b*c)^2
*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3)*3^(1/2)
)/b^(4/3)/d^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*b*d) + ((a + b*x)^(5/3)*(c
 + d*x)^(1/3))/(2*b) + ((b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*
x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) +
 ((b*c - a*d)^2*Log[c + d*x])/(18*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[-1
+ (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(4/3)*d^(5/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a+bx}\,(}}{9bd}$$

$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{3\sqrt{3}\,b}$$

Mathematica [A]
time = 0.52, size = 220, normalized size = 1.00

$$\frac{(bc-ad)^2\left(\frac{3\sqrt[3]{b}\,d^{2/3}(a+bx)^{2/3}\sqrt[3]{c+dx}\,(2ad+b(c+3dx))}{(bc-ad)^2}-2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)-\log\left(d^{2/3}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)\right)}{18b^{4/3}d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)^2*((3*b^(1/3)*d^(2/3)*(a + b*x)^(2/3)*(c + d*x)^(1/3)*(2*a*d +
 b*(c + 3*d*x)))/(b*c - a*d)^2 - 2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)
^(1/3))/(d^(1/3)*(a + b*x)^(1/3)))/Sqrt[3]] + 2*Log[d^(1/3) - (b^(1/3)*(c +
 d*x)^(1/3))/(a + b*x)^(1/3)] - Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1
/3))/(a + b*x)^(1/3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x)^(2/3)]))/(18*b^(
4/3)*d^(5/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

Fricas [A]
time = 0.32, size = 717, normalized size = 3.27

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(b^{3}c^{2}d-2ab^{2}cd^{2}+a^{2}bd^{3}\right)\sqrt{-\frac{\left(bd^{2}\right)^{\frac{1}{3}}}{b}}\log\left(-3bd^{2}x-2bcd-ad^{2}+3\left(bd^{2}\right)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(bd^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(bd^{2}\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{\left(bd^{2}\right)^{\frac{1}{3}}}{b}}\right)+2\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(bd^{2}\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(bd^{2}\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)-\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(bd^{2}\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(bd^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+\left(bd^{2}\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)+3\left(3b^{2}d^{3}x+b^{2}cd^{2}+2abd^{3}\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{18\,b^{2}d^{3}},\ -\frac{6\sqrt{\frac{1}{3}}\left(b^{3}c^{2}d-2ab^{2}cd^{2}+a^{2}bd^{3}\right)\sqrt{\frac{\left(bd^{2}\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(bd^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+\left(bd^{2}\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{\left(bd^{2}\right)^{\frac{1}{3}}}{b}}}{bd^{2}x+ad^{2}}\right)-2\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(bd^{2}\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(bd^{2}\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(bd^{2}\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(bd^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+\left(bd^{2}\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)-3\left(3b^{2}d^{3}x+b^{2}cd^{2}+2abd^{3}\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{18\,b^{2}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(b*d^2)^(1
/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1/3)*(b*x + a)^(2/3)*(
d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (b*
d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(1/3)*(b*d*x + a*d))*s
qrt(-(b*d^2)^(1/3)/b)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*lo
g(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)
) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(
1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x + b^2*c*d^2 + 2*a*b*d^3)*(b
*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3), -1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt(
(b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*
d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x +
a))/(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3
) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) - 3*(3*b^2*d^3*x + b^2*c*d^2 +
2*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)*(c + d*x)**(1/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,x\right)}^{2/3}\,{\left(c+d\,x\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(2/3)*(c + d*x)^(1/3), x)
```

3.1576 $\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=172

$$\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\right.}{}$$

```
[Out] (b*x+a)^(2/3)*(d*x+c)^(1/3)/b-1/6*(-a*d+b*c)*ln(d*x+c)/b^(4/3)/d^(2/3)-1/2*
(-a*d+b*c)*ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(4/3)/d^(2/
3)-1/3*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x
+c)^(1/3)*3^(1/2))/b^(4/3)/d^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/b - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^
(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(4/3)
*d^(2/3)) - ((b*c - a*d)*Log[c + d*x])/(6*b^(4/3)*d^(2/3)) - ((b*c - a*d)*L
og[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3)*d^
(2/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx = \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3b}$$

$$= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}} - \frac{(bc-ad)\log(c+}{6b^{4/3}d^{2/3}}$$

Mathematica [A]
time = 5.65, size = 278, normalized size = 1.62

$$\frac{(a+bx)^{2/3}\left(6\sqrt[3]{b}\,(d(a+bx))^{2/3}\sqrt[3]{c+dx}+2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)+(-2bc+2ad)\log\left(\sqrt[3]{d(a+bx)}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)+bc\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)-ad\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)\right)}{6b^{4/3}(d(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(6*b^(1/3)*(d*(a + b*x))^(2/3)*(c + d*x)^(1/3) + 2*Sqrt[3]
*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(2*(d*(a + b*x))^(1/3
) + b^(1/3)*(c + d*x)^(1/3))] + (-2*b*c + 2*a*d)*Log[(d*(a + b*x))^(1/3) -
b^(1/3)*(c + d*x)^(1/3)] + b*c*Log[(d*(a + b*x))^(2/3) + b^(1/3)*(d*(a + b*
x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)] - a*d*Log[(d*(a + b*x)
)^(2/3) + b^(1/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(
2/3)]))/(6*b^(4/3)*(d*(a + b*x))^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(132) = 264$.
time = 0.32, size = 596, normalized size = 3.47

$$\left[\frac{6\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 3\sqrt{\frac{1}{3}}\,(b^2cd-abd^2)\sqrt{-\frac{(bd^2)^{\frac{1}{3}}}{b}}\log\left(-3\,bd^2x - 2\,bcd - ad^2 + 3\,(bd^2)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d + 3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd - (bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - (bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{(bd^2)^{\frac{1}{3}}}{b}}\right) - 2\,(bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + (bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{6\,b^2d^2}, \frac{6\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 + 6\sqrt{\frac{1}{3}}\,(b^2cd-abd^2)\sqrt{\frac{(bd^2)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{(bd^2)^{\frac{1}{3}}}{b}}}{bd^2x+ad^2}\right) - 2\,(bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + (bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{6\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt(-(b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1
/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d - (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(
1/3)*(b*d*x + a*d))*sqrt(-(b*d^2)^(1/3)/b)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*l
og(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a
)) + (b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (
b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))
/(b*x + a)))/(b^2*d^2), 1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 + 6*sq
rt(1/3)*(b^2*c*d - a*b*d^2)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^
2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqr
t((b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + (
b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x
+ a)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(1/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(1/3), x)
```

3.1577 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{b^{4/3}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}-\frac{3\sqrt[3]{d}\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2b^{4/3}}$$

```
[Out] -3*(d*x+c)^(1/3)/b/(b*x+a)^(1/3)-1/2*d^(1/3)*ln(d*x+c)/b^(4/3)-3/2*d^(1/3)*
ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(4/3)-d^(1/3)*arctan(1
/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3)*3^(1/2))*3^(1/2)
/b^(4/3)
```

Rubi [A]
time = 0.02, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 61}

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) - (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/b^(4/3)
 - (d^(1/3)*Log[c + d*x])/(2*b^(4/3)) - (3*d^(1/3)*Log[-1 + (d^(1/3)*(a + b
*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{b}$$
$$= -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} - \frac{\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{b^{4/3}} - \frac{\sqrt[3]{d}\,\log(c+dx)}{2b^{4/3}} - \frac{3\sqrt[3]{d}}{}$$

Mathematica [A]
time = 0.16, size = 191, normalized size = 1.28

$$\frac{-\frac{6\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+2\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}}{\sqrt{3}}\right)-2\sqrt[3]{d}\,\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)+\sqrt[3]{d}\,\log\left(d^{2/3}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] ((-6*b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + 2*Sqrt[3]*d^(1/3)*ArcTan[(1
 + (2*b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3)))/Sqrt[3]] - 2*d^(1
/3)*Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)] + d^(1/3)*Log[
d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c +
 d*x)^(2/3))/(a + b*x)^(2/3)])/(2*b^(4/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.
time = 0.30, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}\,(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}b\left(-\frac{d}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+ad)}{3\,(bdx+ad)}\right)+(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{2}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bx+a}\right)-2\,(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)+6\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{2\,(b^2x+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(b*x + a)*(-d/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)*b*(-d/b)^(2/3) + sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) +
(b*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(2/3) - (b*x + a)^(2/3)*(d*x +
 c)^(1/3)*(-d/b)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(b*x + a)) - 2*(b
*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(1/3) + (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a)) + 6*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*x + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(4/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(4/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(4/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(4/3), x)
```

3.1578 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}} dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

```
[Out] -3/4*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}} dx = -\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.18, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{3(dx+c)^{\frac{4}{3}}}{4(bx+a)^{\frac{4}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/(b*x+a)^(4/3)*(d*x+c)^(4/3)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.
time = 0.29, size = 65, normalized size = 2.03

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}{4\,(a^2bc-a^3d+(b^3c-ab^2d)x^2+2\,(ab^2c-a^2bd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/4*(b*x + a)^(2/3)*(d*x + c)^(4/3)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x
^2 + 2*(a*b^2*c - a^2*b*d)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.71, size = 92, normalized size = 2.88

$$-\frac{\left(\frac{3\,c}{4\,b^2\,c-4\,a\,b\,d}+\frac{3\,d\,x}{4\,b^2\,c-4\,a\,b\,d}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}-\frac{\left(4\,a^2\,d-4\,a\,b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}{4\,b^2\,c-4\,a\,b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(7/3),x)
```

```
[Out] -(((3*c)/(4*b^2*c - 4*a*b*d) + (3*d*x)/(4*b^2*c - 4*a*b*d))*(c + d*x)^(1/3)
)/(x*(a + b*x)^(1/3) - ((4*a^2*d - 4*a*b*c)*(a + b*x)^(1/3))/(4*b^2*c - 4*a
*b*d))
```

3.1579 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}$$

```
[Out] -3/7*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(7/3)+9/28*d*(d*x+c)^(4/3)/(-a*d+b*c)
^2/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(4/3)
)/(28*(b*c - a*d)^2*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx = -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{7(bc-ad)}$$
$$= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{4/3}(-4bc+7ad+3bdx)}{28(bc-ad)^2(a+bx)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
[Out] (3*(c + d*x)^(4/3)*(-4*b*c + 7*a*d + 3*b*d*x))/(28*(b*c - a*d)^2*(a + b*x)^
(7/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]')
[Out] Timed out
```

Maple [A]
time = 0.19, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{3(dx+c)^{\frac{4}{3}}(3bdx+7ad-4bc)}{28(bx+a)^{\frac{7}{3}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(10/3),x,method=_RETURNVERBOSE)
[Out] 3/28*(d*x+c)^(4/3)*(3*b*d*x+7*a*d-4*b*c)/(b*x+a)^(7/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(54) = 108$.
time = 0.30, size = 175, normalized size = 2.65

$$\frac{3\,(3\,bd^2x^2 - 4\,bc^2 + 7\,acd - (bcd - 7\,ad^2)x)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{28\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2 + (b^5c^2 - 2\,ab^4cd + a^2b^3d^2)x^3 + 3\,(ab^4c^2 - 2\,a^2b^3cd + a^3b^2d^2)x^2 + 3\,(a^2b^3c^2 - 2\,a^3b^2cd + a^4bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 3/28*(3*b*d^2*x^2 - 4*b*c^2 + 7*a*c*d - (b*c*d - 7*a*d^2)*x)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4
*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 +
 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(10/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(10/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.03, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{x\,(21\,a\,d^2-3\,b\,c\,d)}{28\,b^2\,(a\,d-b\,c)^2}-\frac{12\,b\,c^2-21\,a\,c\,d}{28\,b^2\,(a\,d-b\,c)^2}+\frac{9\,d^2\,x^2}{28\,b\,(a\,d-b\,c)^2}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(10/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((x*(21*a*d^2 - 3*b*c*d))/(28*b^2*(a*d - b*c)^2) - (12*b*c
^2 - 21*a*c*d)/(28*b^2*(a*d - b*c)^2) + (9*d^2*x^2)/(28*b*(a*d - b*c)^2)))/
(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*x)^(1/3))/
b)
```

3.1580 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}+\frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}}-\frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}$$

```
[Out] -3/10*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(10/3)+9/35*d*(d*x+c)^(4/3)/(-a*d+b*
c)^2/(b*x+a)^(7/3)-27/140*d^2*(d*x+c)^(4/3)/(-a*d+b*c)^3/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (9*d*(c + d*x)^(4/
3))/(35*(b*c - a*d)^2*(a + b*x)^(7/3)) - (27*d^2*(c + d*x)^(4/3))/(140*(b*c
 - a*d)^3*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx = -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{5(bc-ad)}$$

$$= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} + \frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}} + \frac{(9d^2)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} + \frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}} - \frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{4/3}\left(35d^2-\frac{40bd(c+dx)}{a+bx}+\frac{14b^2(c+dx)^2}{(a+bx)^2}\right)}{140(bc-ad)^3(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
[Out] (-3*(c + d*x)^(4/3)*(35*d^2 - (40*b*d*(c + d*x))/(a + b*x) + (14*b^2*(c + d
*x)^2)/(a + b*x)^2))/(140*(b*c - a*d)^3*(a + b*x)^(4/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]')
[Out] Timed out
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{3(dx+c)^{\frac{4}{3}}(9b^2x^2d^2+30ab\,d^2x-12b^2cdx+35a^2d^2-40abcd+14b^2c^2)}{140(bx+a)^{\frac{10}{3}}(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/140*(d*x+c)^(4/3)*(9*b^2*d^2*x^2+30*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-40*
a*b*c*d+14*b^2*c^2)/(b*x+a)^(10/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(83) = 166.
time = 0.30, size = 337, normalized size = 3.34

$$-\frac{3\,(9\,b^2d^3x^3+14\,b^2c^3-40\,abc^2d+35\,a^2cd^2-3\,(b^2cd^2-10\,abd^3)x^2+(2\,b^2c^2d-10\,abcd^2+35\,a^2d^3)x)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\,(a^4b^3c^3-3\,a^5b^2c^2d+3\,a^6bcd^2-a^7d^3+(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)x^4+4\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)x^3+6\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)x^2+4\,(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

```
[Out] -3/140*(9*b^2*d^3*x^3 + 14*b^2*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2 - 3*(b^2*c
*d^2 - 10*a*b*d^3)*x^2 + (2*b^2*c^2*d - 10*a*b*c*d^2 + 35*a^2*d^3)*x)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 -
 a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 +
4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^
2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b
^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{(a + bx)^{\frac{13}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(13/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(13/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.02, size = 203, normalized size = 2.01

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{105\,a^2\,c\,d^2-120\,a\,b\,c^2\,d+42\,b^2\,c^3}{140\,b^3\,(a\,d-b\,c)^3}+\frac{x\,(105\,a^2\,d^3-30\,a\,b\,c\,d^2+6\,b^2\,c^2\,d)}{140\,b^3\,(a\,d-b\,c)^3}+\frac{27\,d^3\,x^3}{140\,b\,(a\,d-b\,c)^3}+\frac{9\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(13/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((42*b^2*c^3 + 105*a^2*c*d^2 - 120*a*b*c^2*d)/(140*b^3*(a*
d - b*c)^3) + (x*(105*a^2*d^3 + 6*b^2*c^2*d - 30*a*b*c*d^2))/(140*b^3*(a*d
- b*c)^3) + (27*d^3*x^3)/(140*b*(a*d - b*c)^3) + (9*d^2*x^2*(10*a*d - b*c))
/(140*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/3) + (a^3*(a + b*x)^(1/3))/b^3
 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/b^2)
```

3.1581 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}}+\frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}}-\frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}}+\frac{243d^3(c+dx)^{4/3}}{1820(bc-ad)^4(a+bx)^{4/3}}$$

```
[Out] -3/13*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(13/3)+27/130*d*(d*x+c)^(4/3)/(-a*d+
b*c)^2/(b*x+a)^(10/3)-81/455*d^2*(d*x+c)^(4/3)/(-a*d+b*c)^3/(b*x+a)^(7/3)+2
43/1820*d^3*(d*x+c)^(4/3)/(-a*d+b*c)^4/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(13*(b*c - a*d)*(a + b*x)^(13/3)) + (27*d*(c + d*x)^(4
/3))/(130*(b*c - a*d)^2*(a + b*x)^(10/3)) - (81*d^2*(c + d*x)^(4/3))/(455*(
b*c - a*d)^3*(a + b*x)^(7/3)) + (243*d^3*(c + d*x)^(4/3))/(1820*(b*c - a*d)
^4*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx = -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} - \frac{(9d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx}{13(bc-ad)}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} + \frac{(27d^2)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{65(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} -$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} +$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{4/3}\left(-455d^3+\frac{780bd^2(c+dx)}{a+bx}-\frac{546b^2d(c+dx)^2}{(a+bx)^2}+\frac{140b^3(c+dx)^3}{(a+bx)^3}\right)}{1820(bc-ad)^4(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(-455*d^3 + (780*b*d^2*(c + d*x))/(a + b*x) - (546*b^2*
d*(c + d*x)^2)/(a + b*x)^2 + (140*b^3*(c + d*x)^3)/(a + b*x)^3))/(1820*(b*c
 - a*d)^4*(a + b*x)^(4/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4496 deep
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$\frac{3(dx+c)^{\frac{4}{3}}\left(81b^3x^3d^3+351d^3a\,x^2b^2-108b^3c\,d^2x^2+585a^2b\,d^3x-468a\,b^2c\,d^2x+126b^3c^2dx+455a^3d^3-780a^2bc\,d^2+546a\,b^2c^2d-140b^3c^3\right.}{1820(bx+a)^{\frac{13}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/1820*(d*x+c)^(4/3)*(81*b^3*d^3*x^3+351*a*b^2*d^3*x^2-108*b^3*c*d^2*x^2+58
5*a^2*b*d^3*x-468*a*b^2*c*d^2*x+126*b^3*c^2*d*x+455*a^3*d^3-780*a^2*b*c*d^2
+546*a*b^2*c^2*d-140*b^3*c^3)/(b*x+a)^(13/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(112) = 224$.
time = 0.30, size = 533, normalized size = 3.92

$$\frac{3\,(81\,b^3d^4x^4 - 140\,b^3c^4 + 546\,ab^2c^3d - 780\,a^2bc^2d^2 + 455\,a^3cd^3 - 27\,(b^3cd^3 - 13\,ab^2d^4)x^3 + 9\,(2\,b^3c^2d^2 - 13\,ab^2cd^3 + 65\,a^2bd^4)x^2 - (14\,b^3c^3d - 78\,ab^2c^2d^2 + 195\,a^2bcd^3 - 455\,a^3d^4)x)(bx + a)^{\frac{2}{3}}(dx + c)^{\frac{1}{3}}}{1820\,(a^5b^4c^4 - 4\,a^6b^3c^3d + 6\,a^7b^2c^2d^2 - 4\,a^8bcd^3 + a^9d^4 + (b^9c^4 - 4\,ab^8c^3d + 6\,a^2b^7c^2d^2 - 4\,a^3b^6cd^3 + a^4b^5d^4)x^5 + 5\,(ab^8c^4 - 4\,a^2b^7c^3d + 6\,a^3b^6c^2d^2 - 4\,a^4b^5cd^3 + a^5b^4d^4)x^4 + 10\,(a^2b^7c^4 - 4\,a^3b^6c^3d + 6\,a^4b^5c^2d^2 - 4\,a^5b^4cd^3 + a^6b^3d^4)x^3 + 10\,(a^3b^6c^4 - 4\,a^4b^5c^3d + 6\,a^5b^4c^2d^2 - 4\,a^6b^3cd^3 + a^7b^2d^4)x^2 + 5\,(a^4b^5c^4 - 4\,a^5b^4c^3d + 6\,a^6b^3c^2d^2 - 4\,a^7b^2cd^3 + a^8bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(81*b^3*d^4*x^4 - 140*b^3*c^4 + 546*a*b^2*c^3*d - 780*a^2*b*c^2*d^2
+ 455*a^3*c*d^3 - 27*(b^3*c*d^3 - 13*a*b^2*d^4)*x^3 + 9*(2*b^3*c^2*d^2 - 13
*a*b^2*c*d^3 + 65*a^2*b*d^4)*x^2 - (14*b^3*c^3*d - 78*a*b^2*c^2*d^2 + 195*a
^2*b*c*d^3 - 455*a^3*d^4)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^5*b^4*c^4 -
 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 -
 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5
*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b
^4*d^4)*x^4 + 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(16/3),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.15, size = 293, normalized size = 2.15

$$\frac{(c+d\,x)^{1/3}\left(\frac{243\,d^4\,x^4}{1820\,b\,(a\,d-b\,c)^4}-\frac{-1365\,a^3\,c\,d^3+2340\,a^2\,b\,c^2\,d^2-1638\,a\,b^2\,c^3\,d+420\,b^3\,c^4}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{x\,(1365\,a^3\,d^4-585\,a^2\,b\,c\,d^3+234\,a\,b^2\,c^2\,d^2-42\,b^3\,c^3\,d)}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{81\,d^3\,x^3\,(13\,a\,d-b\,c)}{1820\,b^2\,(a\,d-b\,c)^4}+\frac{27\,d^2\,x^2\,(65\,a^2\,d^2-13\,a\,b\,c\,d+2\,b^2\,c^2)}{1820\,b^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/3}+\frac{a^4\,(a+b\,x)^{1/3}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/3}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/3}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/3}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(16/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^4*x^4)/(1820*b*(a*d - b*c)^4) - (420*b^3*c^4 - 136
5*a^3*c*d^3 + 2340*a^2*b*c^2*d^2 - 1638*a*b^2*c^3*d)/(1820*b^4*(a*d - b*c)^
4) + (x*(1365*a^3*d^4 - 42*b^3*c^3*d + 234*a*b^2*c^2*d^2 - 585*a^2*b*c*d^3)
)/(1820*b^4*(a*d - b*c)^4) + (81*d^3*x^3*(13*a*d - b*c))/(1820*b^2*(a*d - b
*c)^4) + (27*d^2*x^2*(65*a^2*d^2 + 2*b^2*c^2 - 13*a*b*c*d))/(1820*b^3*(a*d
- b*c)^4)))/(x^4*(a + b*x)^(1/3) + (a^4*(a + b*x)^(1/3))/b^4 + (6*a^2*x^2*(
a + b*x)^(1/3))/b^2 + (4*a*x^3*(a + b*x)^(1/3))/b + (4*a^3*x*(a + b*x)^(1/3
))/b^3)
```

3.1582 $\int (a+bx)^{4/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=655

$$-\frac{3(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20bd^2}+\frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(b}{}$$

[Out] -3/20*(-a*d+b*c)^2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/b/d^2+3/40*(-a*d+b*c)*(b*x+a
)^(4/3)*(d*x+c)^(1/3)/b/d+3/8*(b*x+a)^(7/3)*(d*x+c)^(1/3)/b+1/20*3^(3/4)*(-
a*d+b*c)^3*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3
)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b
c)^2)^(1/2)(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d
^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*(
(b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-
a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(4/3)/d^(7/3)/(b*x+a)^(2/3)/
(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)

Rubi [A]
time = 0.95, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{10\,2^{2/3}b^{4/3}d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)^2}{20bd^2}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}\,(bc-ad)}{40bd}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]

[Out] (-3*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*b*d^2) + (3*(b*c - a
d)(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*b*d) + (3*(a + b*x)^(7/3)*(c + d*x
)^(1/3))/(8*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c + d
*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^

```
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(4/3)*d^(7/3)*(
a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int (a+bx)^{4/3}\sqrt[3]{c+dx}\,dx = \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{(bc-ad)\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}dx}{8b}$$

$$= \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} - \frac{(bc-ad)^2\int \frac{\sqrt[3]{a+b}}{(c+dx)^{2/}}}{10bd}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)}{}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)}{}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)}{}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.11

$$\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 10/3, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{4}{3}} (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{4}{3}} \sqrt[3]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)*(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(4/3)*(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{4/3}\,(c+d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(4/3)*(c + d*x)^(1/3), x)
```

3.1583 $\int \sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=617

$$\frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(b\cdots}}{\cdots}$$

[Out] 3/10*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(1/3)/b/d+3/5*(b*x+a)^(4/3)*(d*x+c)^(1/3)/b-1/10*3^(3/4)*(-a*d+b*c)^2*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(4/3)/d^(4/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.58, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5\,2^{2/3}b^{4/3}d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{10bd}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]

[Out] (3*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(10*b*d) + (3*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(5*b) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip

```
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)
*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,dx = \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} + \frac{(bc-ad)\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}dx}{5b}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{(bc-ad)^2\int \frac{1}{(a+bx)^{2/3}(c}}{10bd}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{((bc-ad)^2((a+bx)(}{10bd(}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{\left(3(bc-ad)^2((a+bx)\right.}{}$$

$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.12

$$\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(4*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{3}} (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+bx}\, \sqrt[3]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)*(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x)^{1/3}\,(c + d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(1/3)*(c + d*x)^(1/3), x)
```

3.1584 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=576

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\left((bc-ad)^{2/3}+2^{2/\cdots}\right.}{2^{2/3}b^{4/3}\cdots}$$

```
[Out] 3/2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/b+1/2*3^(3/4)*(-a*d+b*c)*((b*x+a)*(d*x+c))^
(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x
+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)*2^(1/3)/b^(4/3)/d^(1/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*
c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 576, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2^{2/3}b^{4/3}\sqrt[3]{d}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c
 - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
```

```
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(4/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2b}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{\left((bc-ad)((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{\left(3(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int\right.}{2b(a+bx)^{2/3}(c+dx)}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.12

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
[Out] int((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(2/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(2/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(2/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(2/3), x)
```

3.1585 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/3}}\,dx$

Optimal. Leaf size=568

$$-\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\right.}{2^{2/3}b^{4/3}(a+}$$

```
[Out] -3/2*(d*x+c)^(1/3)/b/(b*x+a)^(2/3)+1/2*3^(3/4)*d^(2/3)*((b*x+a)*(d*x+c))^(2
/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*Elli
pticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-
3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2
^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a
)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^
(1/2)*2^(1/3)/b^(4/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b
*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 64, 637, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(2*b*(a + b*x)^(2/3)) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(
2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
```

```
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3
)*b^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
- a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{d\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2b}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{\left(d((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{\left(3d((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd}}\right.}{2b(a+bx)^{2/3}(c+dx)^{2/3}(bc+}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\left((bc-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.13

$$-\frac{3\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b(a+bx)^{2/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, (d*(a + b*x))/(-(b*c
) + a*d)])/(2*b*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
[Out] int((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/3), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(5/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(5/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(5/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(5/3), x)
```

3.1586 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{8/3}}\,dx$

Optimal. Leaf size=617

$$-\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}}-\frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\Big((b}{}$$

```
[Out] -3/5*(d*x+c)^(1/3)/b/(b*x+a)^(5/3)-3/10*d*(d*x+c)^(1/3)/b/(-a*d+b*c)/(b*x+a
)^(2/3)-1/10*3^(3/4)*d^(5/3)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(
1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)
-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)
*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(4/3)/(-a*
d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^
(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 64, 637, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{5\,2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3d\sqrt[3]{c+dx}}{10b(a+bx)^{2/3}(bc-ad)}-\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/3)) - (3*d*(c + d*x)^(1/3))/(10*b*(b
*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(5/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
```

```
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)*(b*c
 - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
- 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{8/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} + \frac{d\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{5b}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{d^2\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{10b(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{\left(d^2((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx}}{10b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{\left(3d^2((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bc}\right.}{10b(bc-}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.12

$$-\frac{3\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{5}{3},-\frac{1}{3};-\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(8/3), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(8/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(8/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(8/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(8/3), x)
```

3.1587 $\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=216

$$-\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}-$$

```
[Out] -2/3*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(2/3)/d^2+1/2*(b*x+a)^(4/3)*(d*x+c)^(
2/3)/d-1/9*(-a*d+b*c)^2*ln(b*x+a)/b^(2/3)/d^(7/3)-1/3*(-a*d+b*c)^2*ln(-1+b^
(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/b^(2/3)/d^(7/3)-2/9*(-a*d+b*c)^2
*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2)
)/b^(2/3)/d^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {52, 61}

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-2*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(2/3))/(3*d^2) + ((a + b*x)^(4/3)
*(c + d*x)^(2/3))/(2*d) - (2*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c
 + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/
3)) - ((b*c - a*d)^2*Log[a + b*x])/(9*b^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*b^(2/3)*d^(7
/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
```

```
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx &= \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{(2(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d}\\ &= -\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} + \frac{(2(bc-ad)^2)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c}}}{9d^2}\\ &= -\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\right.}{3\sqrt{3}\,b^{2/3}c}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 231, normalized size = 1.07

$$\frac{3b^{2/3}\sqrt[3]{d}\,\sqrt[3]{a+bx}\,(c+dx)^{2/3}(-4bc+7ad+3bdx)+4\sqrt{3}\,(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}}{\sqrt{3}}\right)-4(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)+2(bc-ad)^2\log\left(b^{2/3}+\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{18b^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*b^(2/3)*d^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(2/3)*(-4*b*c + 7*a*d + 3*b*d*
x) + 4*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/
3)*(c + d*x)^(1/3)))/Sqrt[3]] - 4*(b*c - a*d)^2*Log[b^(1/3) - (d^(1/3)*(a +
 b*x)^(1/3))/(c + d*x)^(1/3)] + 2*(b*c - a*d)^2*Log[b^(2/3) + (d^(2/3)*(a +
 b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3)])/(18*b^(2/3)*d^(7/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(166) = 332.
time = 0.32, size = 740, normalized size = 3.43

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(b^{3}c^{2}d-2ab^{2}cd^{2}+a^{2}bd^{3}\right)\sqrt{\frac{\left(-b^{2}d\right)^{\frac{1}{3}}}{d}}\log\left(3b^{2}dx+b^{2}c+2abd+3\left(-b^{2}d\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-b^{2}d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}+\left(-b^{2}d\right)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{\frac{\left(-b^{2}d\right)^{\frac{1}{3}}}{d}}\right)+2\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(-b^{2}d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+\left(-b^{2}d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^{2}d\right)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)-4\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(-b^{2}d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-b^{2}d\right)^{\frac{2}{3}}(dx+c)}{dx+c}\right)+3\left(3b^{3}d^{2}x-4b^{3}cd+7ab^{2}d^{2}\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{18\,b^{2}d^{3}}, \frac{12\sqrt{\frac{1}{3}}\left(b^{3}c^{2}d-2ab^{2}cd^{2}+a^{2}bd^{3}\right)\sqrt{-\frac{\left(-b^{2}d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-b^{2}d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^{2}d\right)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{-\frac{\left(-b^{2}d\right)^{\frac{1}{3}}}{d}}}{b^{2}dx+b^{2}c}\right)+2\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(-b^{2}d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+\left(-b^{2}d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^{2}d\right)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)-4\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\left(-b^{2}d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-b^{2}d\right)^{\frac{2}{3}}(dx+c)}{dx+c}\right)+3\left(3b^{3}d^{2}x-4b^{3}cd+7ab^{2}d^{2}\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{18\,b^{2}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b^2*d)^(1
/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)^(1/3)*(b*d*x + b*c))
*sqrt((-b^2*d)^(1/3)/d)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)
*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d
 - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*b^3*d^2*x - 4*b^3*c*d + 7*a*
b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*d^3), 1/18*(12*sqrt(1/3)*(b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*
x + b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + 2*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))
```

```
/(d*x + c)) - 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*
b^3*d^2*x - 4*b^3*c*d + 7*a*b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*
d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(4/3)/(c + d*x)^(1/3), x)
```

3.1588 $\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\right.}{}$$

```
[Out] (b*x+a)^(1/3)*(d*x+c)^(2/3)/d+1/6*(-a*d+b*c)*ln(b*x+a)/b^(2/3)/d^(4/3)+1/2*
(-a*d+b*c)*ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/b^(2/3)/d^(4/
3)+1/3*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x
+a)^(1/3)*3^(1/2))/b^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(1/3)*(c + d*x)^(2/3))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^
(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*b^(2/3)
*d^(4/3)) + ((b*c - a*d)*Log[a + b*x])/(6*b^(2/3)*d^(4/3)) + ((b*c - a*d)*L
og[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3)*d^
(4/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d}$$
$$= \frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}} + \frac{(bc-ad)\log(a+}{6b^{2/3}d^{4/3}}$$

Mathematica [A]
time = 5.82, size = 278, normalized size = 1.63

$$\frac{(a+bx)^{4/3}\left(6b^{2/3}\sqrt[3]{d(a+bx)}\,(c+dx)^{2/3} + 2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right) + 2(bc-ad)\log\left(\sqrt[3]{d(a+bx)} - \sqrt[3]{b}\,\sqrt[3]{c+dx}\right) - bc\log\left((d(a+bx))^{2/3} + \sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx} + b^{2/3}(c+dx)^{2/3}\right) + ad\log\left((d(a+bx))^{2/3} + \sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx} + b^{2/3}(c+dx)^{2/3}\right)\right)}{6b^{2/3}(d(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(4/3)*(6*b^(2/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(2/3) + 2*Sqrt[3]
*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(2*(d*(a + b*x))^(1/3
) + b^(1/3)*(c + d*x)^(1/3))] + 2*(b*c - a*d)*Log[(d*(a + b*x))^(1/3) - b^(
1/3)*(c + d*x)^(1/3)] - b*c*Log[(d*(a + b*x))^(2/3) + b^(1/3)*(d*(a + b*x))
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)] + a*d*Log[(d*(a + b*x))^(
2/3) + b^(1/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
)]))/(6*b^(2/3)*(d*(a + b*x))^(4/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(132) = 264$.
time = 0.32, size = 618, normalized size = 3.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1
/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x
 + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)
^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) - (-b^2*d)^(2/3)*(b*c - a*d)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)^(2/3
)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*
x + c))/(d*x + c)))/(b^2*d^2), 1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d
 - 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) - (-b^2*d)^(2/3)*(b*c -
a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1
/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)
^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3
)*(d*x + c))/(d*x + c)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx}}{\sqrt[3]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(1/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{1/3}}{{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(1/3)/(c + d*x)^(1/3), x)
```

3.1589 $\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

```
[Out] -1/2*ln(b*x+a)/b^(2/3)/d^(1/3)-3/2*ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x
+a)^(1/3))/b^(2/3)/d^(1/3)-arctan(1/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(
1/3)/(b*x+a)^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/d^(1/3)
```

Rubi [A]

time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {61}

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*
(a + b*x)^(1/3))])/(b^(2/3)*d^(1/3))) - Log[a + b*x]/(2*b^(2/3)*d^(1/3)) -
(3*Log[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3
)*d^(1/3))
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

Mathematica [A]
time = 0.13, size = 155, normalized size = 1.23

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)+\log\left(b^{2/3}+\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3)
))/Sqrt[3]] - 2*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)] +
Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(2*b^(2/3)*d^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(90) = 180$.

time = 0.31, size = 519, normalized size = 4.12

$$\left[\frac{\sqrt{3}\, bd \sqrt{\frac{(-b^2 d)^{\frac{1}{3}}}{d}} \log\left(3\, b^2 dx + b^2 c + 2\, abd + 3\, (-b^2 d)^{\frac{1}{3}} (bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} b + \sqrt{3} \left(2\, (bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}} bd - (-b^2 d)^{\frac{2}{3}} (bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} + (-b^2 d)^{\frac{1}{3}} (bdx+bc)\right) \sqrt{\frac{(-b^2 d)^{\frac{1}{3}}}{d}}\right) + (-b^2 d)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}} bd + (-b^2 d)^{\frac{2}{3}} (bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} - (-b^2 d)^{\frac{1}{3}} (bdx+bc)}{dx+c}\right) - 2\, (-b^2 d)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} bd - (-b^2 d)^{\frac{2}{3}} (dx+c)}{dx+c}\right)}{2\, b^2 d}, \frac{2\, \sqrt{3}\, bd \sqrt{-\frac{(-b^2 d)^{\frac{1}{3}}}{d}} \arctan\left(\frac{\sqrt{3} \left(2\, (-b^2 d)^{\frac{2}{3}} (bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} - (-b^2 d)^{\frac{1}{3}} (bdx+bc)\right) \sqrt{-\frac{(-b^2 d)^{\frac{1}{3}}}{d}}}{3\, (b^2 dx + b^2 c)}\right) + (-b^2 d)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}} bd + (-b^2 d)^{\frac{2}{3}} (bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} - (-b^2 d)^{\frac{1}{3}} (bdx+bc)}{dx+c}\right) - 2\, (-b^2 d)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{1}{3}} (dx+c)^{\frac{2}{3}} bd - (-b^2 d)^{\frac{2}{3}} (dx+c)}{dx+c}\right)}{2\, b^2 d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d +
3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + sqrt(3)*(2*(b*x + a)^(
2/3)*(d*x + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) +
 (-b^2*d)^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) + (-b^2*d)^(2/3)*log
(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x
 + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*l
og(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x +
c)))/(b^2*d), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b^2*d)^(1/3)/d)*arctan(1/3*sqrt(3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + (-b^2*d)^(2/3)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*log((
(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)))
/(b^2*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}} \sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{2/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)), x)
```

3.1590 $\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

```
[Out] -3/2*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{3(dx+c)^{\frac{2}{3}}}{2(bx+a)^{\frac{2}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2/(b*x+a)^(2/3)*(d*x+c)^(2/3)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

Fricas [A]
time = 0.29, size = 42, normalized size = 1.31

$$-\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\,(abc-a^2d+(b^2c-abd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] -3/2*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/3}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)), x)
```

3.1591 $\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=66

$$-\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}}+\frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}$$

```
[Out] -3/5*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(5/3)+9/10*d*(d*x+c)^(2/3)/(-a*d+b*c)
^2/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (9*d*(c + d*x)^(2/3)
)/(10*(b*c - a*d)^2*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{5(bc-ad)}$$
$$= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}} + \frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{2/3}(-2bc+5ad+3bdx)}{10(bc-ad)^2(a+bx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(-2*b*c + 5*a*d + 3*b*d*x))/(10*(b*c - a*d)^2*(a + b*x)^
(5/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{3(dx+c)^{\frac{2}{3}}(3bdx+5ad-2bc)}{10(bx+a)^{\frac{5}{3}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*(d*x+c)^(2/3)*(3*b*d*x+5*a*d-2*b*c)/(b*x+a)^(5/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.30, size = 118, normalized size = 1.79

$$\frac{3\,(3\,bdx - 2\,bc + 5\,ad)(bx + a)^{\frac{1}{3}}(dx + c)^{\frac{2}{3}}}{10\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(3*b*d*x - 2*b*c + 5*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{8}{3}}\,\sqrt[3]{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(a + b\,x)}^{8/3}\,{(c + d\,x)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)), x)
```

3.1592 $\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}+\frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}}-\frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}$$

```
[Out] -3/8*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(8/3)+9/20*d*(d*x+c)^(2/3)/(-a*d+b*c)
^2/(b*x+a)^(5/3)-27/40*d^2*(d*x+c)^(2/3)/(-a*d+b*c)^3/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (9*d*(c + d*x)^(2/3)
)/(20*(b*c - a*d)^2*(a + b*x)^(5/3)) - (27*d^2*(c + d*x)^(2/3))/(40*(b*c -
a*d)^3*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{4(bc-ad)}$$
$$= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}} + \frac{(9d^2)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}}{20(bc-ad)^2}$$
$$= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}} - \frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{8/3}\left(5b^2+\frac{20d^2(a+bx)^2}{(c+dx)^2}-\frac{16bd(a+bx)}{c+dx}\right)}{40(bc-ad)^3(a+bx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
[Out] (-3*(c + d*x)^(8/3)*(5*b^2 + (20*d^2*(a + b*x)^2)/(c + d*x)^2 - (16*b*d*(a
+ b*x))/(c + d*x)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]')
[Out] Timed out
```

Maple [A]
time = 0.18, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{3(dx+c)^{\frac{2}{3}}\left(9b^2x^2d^2+24ab\,d^2x-6b^2cdx+20a^2d^2-16abcd+5b^2c^2\right)}{40(bx+a)^{\frac{8}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/40*(d*x+c)^(2/3)*(9*b^2*d^2*x^2+24*a*b*d^2*x-6*b^2*c*d*x+20*a^2*d^2-16*a*
b*c*d+5*b^2*c^2)/(b*x+a)^(8/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(83) = 166$.
time = 0.33, size = 251, normalized size = 2.49

$$-\frac{3\,(9\,b^2d^2x^2+5\,b^2c^2-16\,abcd+20\,a^2d^2-6\,(b^2cd-4\,abd^2)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/40*(9*b^2*d^2*x^2 + 5*b^2*c^2 - 16*a*b*c*d + 20*a^2*d^2 - 6*(b^2*c*d - 4
*a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a
^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^
2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)), x)
```

3.1593 $\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=136

$$-\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}}+\frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}}-\frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}}+\frac{243d^3(c+dx)^{2/3}}{440(bc-ad)^4(a+bx)^{2/3}}$$

```
[Out] -3/11*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(11/3)+27/88*d*(d*x+c)^(2/3)/(-a*d+b
*c)^2/(b*x+a)^(8/3)-81/220*d^2*(d*x+c)^(2/3)/(-a*d+b*c)^3/(b*x+a)^(5/3)+243
/440*d^3*(d*x+c)^(2/3)/(-a*d+b*c)^4/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(11*(b*c - a*d)*(a + b*x)^(11/3)) + (27*d*(c + d*x)^(2
/3))/(88*(b*c - a*d)^2*(a + b*x)^(8/3)) - (81*d^2*(c + d*x)^(2/3))/(220*(b*
c - a*d)^3*(a + b*x)^(5/3)) + (243*d^3*(c + d*x)^(2/3))/(440*(b*c - a*d)^4*
(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+}}}{44(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{11/3}\left(40b^3-\frac{220d^3(a+bx)^3}{(c+dx)^3}+\frac{264bd^2(a+bx)^2}{(c+dx)^2}-\frac{165b^2d(a+bx)}{c+dx}\right)}{440(bc-ad)^4(a+bx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(11/3)*(40*b^3 - (220*d^3*(a + b*x)^3)/(c + d*x)^3 + (264*b*d
^2*(a + b*x)^2)/(c + d*x)^2 - (165*b^2*d*(a + b*x))/(c + d*x)))/(440*(b*c -
 a*d)^4*(a + b*x)^(11/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$\frac{3(dx+c)^{\frac{2}{3}}\left(81b^3x^3d^3+297d^3a\,x^2b^2-54b^3c\,d^2x^2+396a^2b\,d^3x-198a\,b^2c\,d^2x+45b^3c^2dx+220a^3d^3-264a^2bc\,d^2+165a\,b^2c^2d-40b^3c^3\right)}{440(bx+a)^{\frac{11}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/440*(d*x+c)^(2/3)*(81*b^3*d^3*x^3+297*a*b^2*d^3*x^2-54*b^3*c*d^2*x^2+396*
a^2*b*d^3*x-198*a*b^2*c*d^2*x+45*b^3*c^2*d*x+220*a^3*d^3-264*a^2*b*c*d^2+16
5*a*b^2*c^2*d-40*b^3*c^3)/(b*x+a)^(11/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(112) = 224.
time = 0.37, size = 420, normalized size = 3.09

$$\frac{3\,(81\,b^3d^3x^3-40\,b^3c^3+165\,ab^2c^2d-264\,a^2bcd^2+220\,a^3d^3-27\,(2\,b^3cd^2-11\,ab^2d^3)x^2+9\,(5\,b^3c^2d-22\,ab^2cd^2+44\,a^2bd^3)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{440\,(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^4+4\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^3+6\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x^2+4\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(81*b^3*d^3*x^3 - 40*b^3*c^3 + 165*a*b^2*c^2*d - 264*a^2*b*c*d^2 + 22
0*a^3*d^3 - 27*(2*b^3*c*d^2 - 11*a*b^2*d^3)*x^2 + 9*(5*b^3*c^2*d - 22*a*b^2
*c*d^2 + 44*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(14/3)/(d*x+c)**(1/3),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{14/3}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)), x)
```

3.1594 $\int \frac{(a+bx)^{8/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1365

$$\frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3}-\frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2}+\frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d}-\frac{\quad}{7b^{2/3}d^{11/3}}$$

```
[Out] 3/7*(-a*d+b*c)^2*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d^3-12/35*(-a*d+b*c)*(b*x+a)^(
5/3)*(d*x+c)^(2/3)/d^2+3/10*(b*x+a)^(8/3)*(d*x+c)^(2/3)/d-3/7*2^(2/3)*(-a*d
+b*c)^3*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+
c))^2)^(1/2)/b^(2/3)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)
/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/
2)))-2/7*2^(1/6)*3^(3/4)*(-a*d+b*c)^(11/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(
1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(
2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^(11/3)/(b*x+a)^(1/3)/(
d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3
)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)+3/14*3^(1/4)*(-a*d+b*c)^(11/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(
2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^
(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b
*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+
2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(
2/3)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+
c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a
*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.92, antiderivative size = 1365, normalized size of antiderivative = 1.00, number

of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{11/3}}{7\sqrt[3]{2}b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}} - \frac{2\sqrt[6]{2}3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{11/3}}{7b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}} - \frac{3\,2^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)^3}{7b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)} + \frac{3(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^2}{7d^3} - \frac{12(a+bx)^{5/3}(c+dx)^{2/3}(bc-ad)}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(7*d^3) - (12*(b*c - a*d)
*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(35*d^2) + (3*(a + b*x)^(8/3)*(c + d*x)^(
2/3))/(10*d) - (3*2^(2/3)*(b*c - a*d)^3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*b^(2/3)*d^(11/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (3*3^(
1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(11/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(
b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((
a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(1/3)*b^(2/3)*d^(11/3)*(a + b*x)^(1/3)*(
c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2
]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*2^(1/6)*3^(3/4)*(b*c - a*d)^(11/3)*((
a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4
/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
 + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b
*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*b^(2/3)*d^(
11/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*
d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))
^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{8/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \frac{(4(bc-ad))\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx}{5d}$$

$$= -\frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} + \frac{(4(bc-ad)^2)\int \frac{(a+bx)^2}{\sqrt[3]{c+a}}}{7d^2}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c}{10d}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c}{10d}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c}{10d}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c}{10d}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c}{10d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{11/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{d(a+bx)}{-bc+ad}\right)}{11b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(11/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3
, 11/3, 14/3, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]')
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{8}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(8/3)/(c + d*x)**(1/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{8/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(8/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(8/3)/(c + d*x)^(1/3), x)
```

3.1595 $\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1330

$$-\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2}+\frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d}+\frac{15(bc-ad)^2\sqrt[3]{(a+bx)(c}}{14\sqrt[3]{2}\,b^{2/3}d^{8/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+}$$

```
[Out] -15/28*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d^2+3/7*(b*x+a)^(5/3)*(d*x+c)
^(2/3)/d+15/28*(-a*d+b*c)^2*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(
1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x
+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+5/14*3^(3/4)*(-a*d+b*c)^(8/3)*((b*x+a)*(d*x
+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2
/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c
)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c
)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*
2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/b^(2/
3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))
^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x
+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+
b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-15/56*3^(1/4)*(-a*d+b*c)^(8/3)*((b*x+a)*(d
*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^
(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b
*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1
/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3
)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3)
)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1
/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x
+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.40, antiderivative size = 1330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {52, 64, 637, 309, 224, 1891}

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{28\sqrt[3]{2}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{5\,3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{7\,2^{5/6}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{15\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)^2}{14\sqrt[3]{2}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{15(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)}{28d^2}+\frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-15*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(28*d^2) + (3*(a + b*x)^(
5/3)*(c + d*x)^(2/3))/(7*d) + (15*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b
^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(8/3)*((a + b*x)*(c + d
*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*(b*c - a*d)^(
8/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5
/6)*b^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{(5(bc-ad)^2)\int \frac{}{\sqrt[3]{a+b}}}{14d^2}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(5(bc-ad)^2\sqrt[3]{(a+bx}\right.}{1}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(15(bc-ad)^2\sqrt[3]{(a+b}\right.}{}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(15(bc-ad)^2\sqrt[3]{(a+b}\right.}{}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{15(bc}{14\sqrt[3]{2}\,b^{2/3}d^{8/3}\sqrt[3]{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{8/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{-bc+ad}\right)}{8b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{\sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(1/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(5/3)/(c + d*x)^(1/3), x)
```

3.1596 $\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1293

$$\frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c}}{2\sqrt[3]{2}\,b^{2/3}d^{5/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{}\right.}$$

```
[Out] 3/4*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d-3/4*(-a*d+b*c)*((b*x+a)*(d*x+c))^(1/3)*((
2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(5/
3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))-1/2*3^(3/4)*(-a*d+b*c)
^(5/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^
2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+
a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)*2^(1/6)/b^(2/3)/d^(5/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c
)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b
^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)+3/8*3^(1/4)*(-a*d+b*c)
^(5/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^
2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/
3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x
+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+
b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(5/3)/(b*x+a)^(1/3)/(d*x
+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*(
(-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A]

time = 1.12, antiderivative size = 1293, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {52, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{4\sqrt[3]{2}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}} - \frac{3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{2^{5/6}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}} - \frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)}{2\sqrt[3]{2}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)} + \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(4*d) - (3*(b*c - a*d)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2
*2^(1/3)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d
*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(5/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*d
^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a
*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*(b*c -
a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((
b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
+ d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(
2^(5/6)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{2d}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left((bc-ad)\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}}{2d\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left(3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{2d\sqrt[3]{a+bx}\sqrt[3]{c+}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left(3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{2\,2^{2/3}\sqrt[3]{b}\,d^{4/3}\sqrt[3]{a+bx}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+}}{2\sqrt[3]{2}\,b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{5/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(1/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(2/3)/(c + d*x)^(1/3), x)
```

3.1597 $\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1257

$$\frac{3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}\,b^{2/3}d^{2/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}\right.}$$

```
[Out] 3/2*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^
2)^(1/2)*2^(2/3)/b^(2/3)/d^(2/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b
*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^
(1/2)))+2^(1/6)*3^(3/4)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c
)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(
2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/
3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/
3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^(2/3)/(b*x+a)^(1/3)/(d*x
+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*(
(-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1
/2)-3/4*3^(1/4)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+
2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(
1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(
4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(
1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*
x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/
d^(2/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)
^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c
)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.85, antiderivative size = 1257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$,

Rules used = {64, 637, 309, 224, 1891}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[3]{2}b^{2/3}d^{2/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{\sqrt[6]{2}\,3^{3/4}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}d^{2/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}b^{2/3}d^{2/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*
(c + 2*d*x))^2])/(2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(
b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*
d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*
c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*
2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) +
 (2^(1/6)*3^(3/4)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c +
 a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c -
a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*
x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))], -7 - 4*Sqrt[3]])/(b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b
*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(
c + 2*d*x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{(a+bx)(c+dx)}\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}$$

$$= \frac{\left(3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+ad)^2 -}}\right.}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)}$$

$$= \frac{\left(3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{}}{\sqrt{-4abcd+(bc+ad)^2 -}}\right.}{2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bd}$$

$$= \frac{3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2}}{\sqrt[3]{2}\,b^{2/3}d^{2/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{2/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 2/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}} (dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(1/3),x)
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(1/3)), x)
```

3.1598 $\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1297

$$-\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{3\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+\cdots)}}{\sqrt[3]{2}\,b^{2/3}(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\cdots\right)}$$

```
[Out] -3*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(1/3)+3/2*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/
(-a*d+b*c)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d
^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+2^(1/6)*3^(3/4
)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+
c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*
c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b
*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^
(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))
^2)^(1/2)/b^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d
+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+
a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-3/4*3^(1/4)*d^(1/3
)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a
)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^
(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-
a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(
d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^
(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/
(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 1.10, antiderivative size = 1297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[3]{2}b^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{\sqrt[6]{2}3^{3/4}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{3\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}b^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/((b*c - a*d)*(a + b*x)^(1/3)) + (3*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2
])/(2^(1/3)*b^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d
+ 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*b^(2/3)*(b*c
 - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((
b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2^(1/6)*
3^(3/4)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sq
rt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x
)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3
]])/(b^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{bc-ad}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{\left(d\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,d}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{\left(3d\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\Big(}{(bc-ad)\sqrt[3]{a+bx}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{\left(3d^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subs}}{2^{2/3}\sqrt[3]{b}(bc-ad)\sqrt[3]{a}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{3\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc-}}{\sqrt[3]{2}b^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\Big(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/3)*(c + d*x)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}} (dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}} \sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(1/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{4/3}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(4/3)*(c + d*x)^(1/3)), x)
```

3.1599 $\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1335

$$-\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}}+\frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}}-\frac{3d^{4/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+a}}{2\sqrt[3]{2}\,b^{2/3}(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(\right.\right.}$$

```
[Out] -3/4*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(4/3)+3/2*d*(d*x+c)^(2/3)/(-a*d+b*c)^
2/(b*x+a)^(1/3)-3/4*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(
1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^2/(b*x+a)^(1/3)
/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))-1/2*3^(3/4)*d^(4/3)*((b*x+a)*(d*x+c))^
(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/
3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/
3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+
a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/b^(2/3)/(-
a*d+b*c)^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x
+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-
a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)+3/8*3^(1/4)*d^(4/3)*((b*x+a)*(d*x+c))^
(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2
*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x
+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d
*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)
+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.37, antiderivative size = 1335, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{4\sqrt[3]{2}b^{2/3}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{2^{5/6}b^{2/3}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{4/3}}{2\sqrt[3]{2}b^{2/3}(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{3(c+dx)^{2/3}d}{2(bc-ad)^2\sqrt[3]{a+bx}}-\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (3*d*(c + d*x)^(2/3)
)/(2*(b*c - a*d)^2*(a + b*x)^(1/3)) - (3*d^(4/3)*((a + b*x)*(c + d*x))^(1/3
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*b
^(2/3)*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*(b*c - a*d)^(4/
3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^
(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*d^(4/3)*((a
 + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/
3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
+ 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*b^(2/3
)*(b*c - a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} - \frac{d\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{d^2\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(d^2\sqrt[3]{(a+bx)(c+dx)}\right)\int}{2(bc-ad)^2\sqrt[3]{a}}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(3d^2\sqrt[3]{(a+bx)(c+dx)}\sqrt{(}\right.}{}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(3d^{5/3}\sqrt[3]{(a+bx)(c+dx)}\right.}{}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{3d^{4/}}{2\sqrt[3]{2}\,b^{2/3}(bc-ad)^2\sqrt[3]{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};-\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b(a+bx)^{4/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(4*b*(a + b*x)^(4/3)*(c + d*x)^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}} (dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{3}} \sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/3}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(7/3)*(c + d*x)^(1/3)), x)
```

3.1600 $\int \frac{1}{(a+bx)^{10/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1372

$$-\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}}+\frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}}-\frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}+\frac{15d^{7/3}\sqrt[3]{}}{14\sqrt[3]{2}\,b^{2/3}(bc-ad)^3\sqrt[3]{a+bx}\,\sqrt[3]{}}$$

```
[Out] -3/7*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(7/3)+15/28*d*(d*x+c)^(2/3)/(-a*d+b*c
)^2/(b*x+a)^(4/3)-15/14*d^2*(d*x+c)^(2/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)+15/28*
d^(7/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+
c))^2)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*
d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2
/3)*(1+3^(1/2)))+5/14*3^(3/4)*d^(7/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(
2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^
(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*
d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*
((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(
-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/b^(2/3)/(-a*d+b*c)^(7/3)/(b*x
+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*
d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3
^(1/2)))^2)^(1/2)-15/56*3^(1/4)*d^(7/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)
^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d
+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3
)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/
3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b
^(2/3)/(-a*d+b*c)^(7/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d
+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*
d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.67, antiderivative size = 1372, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{7/3}}{28\sqrt[3]{2}b^{2/3}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{5\,3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{7/3}}{7\,2^{5/6}b^{2/3}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{15\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{7/3}}{14\sqrt[3]{2}b^{2/3}(bc-ad)^3\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{15(c+dx)^{2/3}d^2}{14(bc-ad)^3\sqrt[3]{a+bx}}+\frac{15(c+dx)^{2/3}d}{28(bc-ad)^2(a+bx)^{4/3}}-\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (15*d*(c + d*x)^(2/3
))/(28*(b*c - a*d)^2*(a + b*x)^(4/3)) - (15*d^2*(c + d*x)^(2/3))/(14*(b*c -
 a*d)^3*(a + b*x)^(1/3)) + (15*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b^(2/3)*(b
*c - a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sq
rt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3
)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)
*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*(b*c - a*d)^(7/3)*(a +
 b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/(
(1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*d^(7/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5/6)*b^(2/3)*(
b*c - a*d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt
[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(5d)\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx}{7(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} + \frac{(5d^2)\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}}{14(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} +$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} +$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} +$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} +$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{7}{3},\frac{1}{3};-\frac{4}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-7/3, 1/3, -4/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(7*b*(a + b*x)^(7/3)*(c + d*x)^(1/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{10}{3}} (dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
[Out] int(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(1/3)), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^4*d*x^5 + a^4*c + (b^4*c + 4*a*
b^3*d)*x^4 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 2*a^3*b*d)*
x^2 + (4*a^3*b*c + a^4*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{10}{3}} \sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(10/3)*(c + d*x)**(1/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{10/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(10/3)*(c + d*x)^(1/3)), x)
```

3.1601 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=216

$$-\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}\sqrt[3]{b}\,d^{8/3}}-$$

```
[Out] -5/6*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(1/3)/d^2+1/2*(b*x+a)^(5/3)*(d*x+c)^(
1/3)/d-5/18*(-a*d+b*c)^2*ln(d*x+c)/b^(1/3)/d^(8/3)-5/6*(-a*d+b*c)^2*ln(-1+d
^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(1/3)/d^(8/3)-5/9*(-a*d+b*c)^
2*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3)*3^(1/2
))/b^(1/3)/d^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}\,d^{8/3}}-\frac{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}{6d^2}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*d^2) + ((a + b*x)^(5/3)
*(c + d*x)^(1/3))/(2*d) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a
 + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^(8/
3)) - (5*(b*c - a*d)^2*Log[c + d*x])/(18*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^
2*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(1/3)
*d^(8/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
```

```
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6d}$$
$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} + \frac{(5(bc-ad)^2)\int \frac{1}{\sqrt[3]{a+bx}\,(c+\cdots}}{9d^2}$$
$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\cdots\right.}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8\cdots}}$$

Mathematica [A]
time = 0.33, size = 231, normalized size = 1.07

$$\frac{3\sqrt[3]{b}\,d^{2/3}(a+bx)^{2/3}\sqrt[3]{c+dx}\,(-5bc+8ad+3bdx)+10\sqrt{3}\,(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}}{\sqrt{3}}\right)-10(bc-ad)^2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)+5(bc-ad)^2\log\left(d^{2/3}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)}{18\sqrt[3]{b}\,d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*b^(1/3)*d^(2/3)*(a + b*x)^(2/3)*(c + d*x)^(1/3)*(-5*b*c + 8*a*d + 3*b*d*
x) + 10*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(d^(1
/3)*(a + b*x)^(1/3)))/Sqrt[3]] - 10*(b*c - a*d)^2*Log[d^(1/3) - (b^(1/3)*(c
 + d*x)^(1/3))/(a + b*x)^(1/3)] + 5*(b*c - a*d)^2*Log[d^(2/3) + (b^(1/3)*d^
(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x
)^(2/3)])/(18*b^(1/3)*d^(8/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(166) = 332.
time = 0.32, size = 741, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b*d^2)^(
1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)
*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2)^(1/3)*(b*d*x + a*d
))*sqrt((-b*d^2)^(1/3)/b)) - 10*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2
/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b
*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
- (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x - 5*b^2*c*d^2 +
 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*d^4), 1/18*(30*sqrt(1/3)*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d
*x + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 10*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b
*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(
3*b^2*d^3*x - 5*b^2*c*d^2 + 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*
d^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(2/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(5/3)/(c + d*x)^(2/3), x)
```

3.1602 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=169

$$\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}+\frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log}{}$$

```
[Out] (b*x+a)^(2/3)*(d*x+c)^(1/3)/d+1/3*(-a*d+b*c)*ln(d*x+c)/b^(1/3)/d^(5/3)+(-a*
d+b*c)*ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(1/3)/d^(5/3)+2
/3*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^
(1/3)*3^(1/2))/b^(1/3)/d^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}\,d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/d + (2*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*
d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(1/
3)*d^(5/3)) + ((b*c - a*d)*Log[c + d*x])/(3*b^(1/3)*d^(5/3)) + ((b*c - a*d)
*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(b^(1/3)*d^
(5/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3d}\\ &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} + \frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d^{5/3}} + \frac{(bc-ad)\log(c}{3\sqrt[3]{b}\,d^{5/3}}\end{aligned}$$

Mathematica [**A**]
time = 6.55, size = 278, normalized size = 1.64

$$\frac{(a+bx)^{5/3}\left(3\sqrt[3]{b}\,(d(a+bx))^{2/3}\sqrt[3]{c+dx}-2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)+2(bc-ad)\log\left(\sqrt[3]{d(a+bx)}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)-bc\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)+ad\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)\right)}{3\sqrt[3]{b}\,(d(a+bx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((a + b*x)^(5/3)*(3*b^(1/3)*(d*(a + b*x))^(2/3)*(c + d*x)^(1/3) - 2*Sqrt[3]
*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(2*(d*(a + b*x))^(1/3
) + b^(1/3)*(c + d*x)^(1/3))] + 2*(b*c - a*d)*Log[(d*(a + b*x))^(1/3) - b^(
1/3)*(c + d*x)^(1/3)] - b*c*Log[(d*(a + b*x))^(2/3) + b^(1/3)*(d*(a + b*x))
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)] + a*d*Log[(d*(a + b*x))^(
2/3) + b^(1/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
)]))/(3*b^(1/3)*(d*(a + b*x))^(5/3))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(131) = 262$.
time = 0.32, size = 619, normalized size = 3.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(
1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*
x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2
)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) + 2*(-b*d^2)^(2/3)*(b*c - a*
d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*
x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b
*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))/(b*x + a)))/(b*d^3), 1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2
- 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x +
 a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) + 2*(-b*d^2)^(2/3)*(b*c -
 a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/
(b*x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3
)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*
d*x + a*d))/(b*x + a)))/(b*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(2/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(2/3)/(c + d*x)^(2/3), x)
```

3.1603 $\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2\sqrt[3]{b}\,d^{2/3}}$$

```
[Out] -1/2*ln(d*x+c)/b^(1/3)/d^(2/3)-3/2*ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x
+c)^(1/3))/b^(1/3)/d^(2/3)-arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(
1/3)/(d*x+c)^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/d^(2/3)
```

Rubi [A]

time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {61}

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*
(c + d*x)^(1/3))])/(b^(1/3)*d^(2/3))) - Log[c + d*x]/(2*b^(1/3)*d^(2/3)) -
(3*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(1/3
)*d^(2/3))
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2\sqrt[3]{b}\,d^{2/3}}$$

Mathematica [A]
time = 0.12, size = 155, normalized size = 1.23

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)+\log\left(d^{2/3}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)}{2\sqrt[3]{b}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3)
))/Sqrt[3]] - 2*Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)] +
Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*
(c + d*x)^(2/3))/(a + b*x)^(2/3)])/(2*b^(1/3)*d^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in ___instancecheck___

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in __instancecheck__
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(90) = 180$.
time = 0.31, size = 521, normalized size = 4.13

$$\left[\frac{\sqrt{3}\, bd\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\log\left(-3\, bd^2x - 2\, bcd - ad^2 - 3\left(-bd^2\right)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d - \sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd - \left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} + \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\right) - 2\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd - \left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + \left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd + \left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\, bd^2}, \frac{2\sqrt{3}\, bd\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{3}\left(2\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}}{3\left(bd^2x+ad^2\right)}\right) - 2\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd - \left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + \left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd + \left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\, bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 -
 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - sqrt(3)*(2*(b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (-b*d^2)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) - 2*(-b*d^2)^(2/3)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x +
 a)) + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(
2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x +
 a)))/(b*d^2), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b*d^2)^(1/3)/b)*arctan(1/3*sqrt(3
)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(-b*d^2)^(2/3)*log(
((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x + a))
 + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)
*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a))
)/(b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{1/3}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)), x)
```

3.1604 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=30

$$-\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

```
[Out] -3*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(1/3)
```

Rubi [A]

time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

Mathematica [A]

time = 0.04, size = 30, normalized size = 1.00

$$-\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{3(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/(b*x+a)^(1/3)*(d*x+c)^(1/3)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

Fricas [A]
time = 0.29, size = 42, normalized size = 1.40

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(2/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.83, size = 26, normalized size = 0.87

$$\frac{3\,{\left(c+d\,x\right)}^{1/3}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x)
```

```
[Out] (3*(c + d*x)^(1/3))/((a*d - b*c)*(a + b*x)^(1/3))
```

3.1605 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}}+\frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}$$

```
[Out] -3/4*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(4/3)+9/4*d*(d*x+c)^(1/3)/(-a*d+b*c)^
2/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}\,(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (9*d*(c + d*x)^(1/3)
)/(4*(b*c - a*d)^2*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{4(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}} + \frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}$$

Mathematica [**A**]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{3\sqrt[3]{c+dx}\,(-bc+4ad+3bdx)}{4(bc-ad)^2(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(-(b*c) + 4*a*d + 3*b*d*x))/(4*(b*c - a*d)^2*(a + b*x)^(
4/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]')
```

```
[Out] Timed out
```

Maple [**A**]
time = 0.24, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{3(dx+c)^{\frac{1}{3}}(3bdx+4ad-bc)}{4(bx+a)^{\frac{4}{3}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(d*x+c)^(1/3)*(3*b*d*x+4*a*d-b*c)/(b*x+a)^(4/3)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.29, size = 118, normalized size = 1.79

$$\frac{3\,(3\,bdx - bc + 4\,ad)(bx + a)^{\frac{2}{3}}(dx + c)^{\frac{1}{3}}}{4\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(3*b*d*x - b*c + 4*a*d)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{7}{3}}\,(c + dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.98, size = 71, normalized size = 1.08

$$\frac{\left(\frac{9\,d\,x}{4\,{\left(a\,d-b\,c\right)}^2}+\frac{12\,a\,d-3\,b\,c}{4\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}+\frac{a\,{\left(a+b\,x\right)}^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x)

[Out] (((9*d*x)/(4*(a*d - b*c)^2) + (12*a*d - 3*b*c)/(4*b*(a*d - b*c)^2))*(c + d*x)^(1/3))/(x*(a + b*x)^(1/3) + (a*(a + b*x)^(1/3))/b)

3.1606 $\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}}-\frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

```
[Out] -3/7*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(7/3)+9/14*d*(d*x+c)^(1/3)/(-a*d+b*c)
^2/(b*x+a)^(4/3)-27/14*d^2*(d*x+c)^(1/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}\,(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(1/3)
)/(14*(b*c - a*d)^2*(a + b*x)^(4/3)) - (27*d^2*(c + d*x)^(1/3))/(14*(b*c -
a*d)^3*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{7(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}} + \frac{(9d^2)\int \frac{1}{(a+bx)^{4/3}(c+dx)^2}}{14(bc-ad)^2}$$
$$= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}} - \frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

Mathematica [A]
time = 0.09, size = 75, normalized size = 0.74

$$-\frac{3\sqrt[3]{c+dx}\,(14a^2d^2-7abd(c-3dx)+b^2(2c^2-3cdx+9d^2x^2))}{14(bc-ad)^3(a+bx)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
[Out] (-3*(c + d*x)^(1/3)*(14*a^2*d^2 - 7*a*b*d*(c - 3*d*x) + b^2*(2*c^2 - 3*c*d*
x + 9*d^2*x^2)))/(14*(b*c - a*d)^3*(a + b*x)^(7/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]')
[Out] Timed out
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{3(dx+c)^{\frac{1}{3}}(9b^2x^2d^2+21ab\,d^2x-3b^2cdx+14a^2d^2-7abcd+2b^2c^2)}{14(bx+a)^{\frac{7}{3}}(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/14*(d*x+c)^(1/3)*(9*b^2*d^2*x^2+21*a*b*d^2*x-3*b^2*c*d*x+14*a^2*d^2-7*a*b
*c*d+2*b^2*c^2)/(b*x+a)^(7/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(83) = 166.
time = 0.30, size = 251, normalized size = 2.49

$$-\frac{3\,(9\,b^2d^2x^2+2\,b^2c^2-7\,abcd+14\,a^2d^2-3\,(b^2cd-7\,abd^2)x)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{14\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/14*(9*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 14*a^2*d^2 - 3*(b^2*c*d - 7*
a*b*d^2)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^
3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2
*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{10}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(10/3)*(c + d*x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.51, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{27\,d^2\,x^2}{14\,(a\,d-b\,c)^3}+\frac{42\,a^2\,d^2-21\,a\,b\,c\,d+6\,b^2\,c^2}{14\,b^2\,(a\,d-b\,c)^3}+\frac{9\,d\,x\,(7\,a\,d-b\,c)}{14\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((27*d^2*x^2)/(14*(a*d - b*c)^3) + (42*a^2*d^2 + 6*b^2*c^2
 - 21*a*b*c*d)/(14*b^2*(a*d - b*c)^3) + (9*d*x*(7*a*d - b*c))/(14*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*
x)^(1/3))/b)
```

3.1607 $\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}}+\frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}}-\frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}}+\frac{243d^3\sqrt[3]{c+dx}}{140(bc-ad)^4\sqrt[3]{a+bx}}$$

```
[Out] -3/10*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(10/3)+27/70*d*(d*x+c)^(1/3)/(-a*d+b
*c)^2/(b*x+a)^(7/3)-81/140*d^2*(d*x+c)^(1/3)/(-a*d+b*c)^3/(b*x+a)^(4/3)+243
/140*d^3*(d*x+c)^(1/3)/(-a*d+b*c)^4/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}\,(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (27*d*(c + d*x)^(1
/3))/(70*(b*c - a*d)^2*(a + b*x)^(7/3)) - (81*d^2*(c + d*x)^(1/3))/(140*(b*
c - a*d)^3*(a + b*x)^(4/3)) + (243*d^3*(c + d*x)^(1/3))/(140*(b*c - a*d)^4*
(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx}{10(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{7/3}(c+}}{35(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+}$$

$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.70

$$-\frac{3\sqrt[3]{c+dx}\left(-140d^3+\frac{105bd^2(c+dx)}{a+bx}-\frac{60b^2d(c+dx)^2}{(a+bx)^2}+\frac{14b^3(c+dx)^3}{(a+bx)^3}\right)}{140(bc-ad)^4\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(-140*d^3 + (105*b*d^2*(c + d*x))/(a + b*x) - (60*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (14*b^3*(c + d*x)^3)/(a + b*x)^3))/(140*(b*c -
a*d)^4*(a + b*x)^(1/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.19, size = 171, normalized size = 1.26

method	result	
gosper	$\frac{3(dx+c)^{\frac{1}{3}}\left(81b^3x^3d^3+270d^3a\,x^2b^2-27b^3c\,d^2x^2+315a^2b\,d^3x-90a\,b^2c\,d^2x+18b^3c^2dx+140a^3d^3-105a^2bc\,d^2+60a\,b^2c^2d-14b^3c^3\right)}{140(bx+a)^{\frac{10}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/140*(d*x+c)^(1/3)*(81*b^3*d^3*x^3+270*a*b^2*d^3*x^2-27*b^3*c*d^2*x^2+315*
a^2*b*d^3*x-90*a*b^2*c*d^2*x+18*b^3*c^2*d*x+140*a^3*d^3-105*a^2*b*c*d^2+60*
a*b^2*c^2*d-14*b^3*c^3)/(b*x+a)^(10/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(112) = 224.
time = 0.30, size = 419, normalized size = 3.08

$$\frac{3\left(81\,b^3d^3x^3-14\,b^3c^3+60\,ab^2c^2d-105\,a^2bcd^2+140\,a^3d^3-27\,(b^3cd^2-10\,ab^2d^3)x^2+9\,(2\,b^3c^2d-10\,ab^2cd^2+35\,a^2bd^3)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^4+4\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^3+6\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x^2+4\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(81*b^3*d^3*x^3 - 14*b^3*c^3 + 60*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 140
*a^3*d^3 - 27*(b^3*c*d^2 - 10*a*b^2*d^3)*x^2 + 9*(2*b^3*c^2*d - 10*a*b^2*c*
d^2 + 35*a^2*b*d^3)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^4*c^4 - 4*a^5
*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b
^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^
7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4
)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*
d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d
^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/3)/(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.27, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{243\,d^3\,x^3}{140\,(a\,d-b\,c)^4}+\frac{420\,a^3\,d^3-315\,a^2\,b\,c\,d^2+180\,a\,b^2\,c^2\,d-42\,b^3\,c^3}{140\,b^3\,(a\,d-b\,c)^4}+\frac{27\,d\,x\,(35\,a^2\,d^2-10\,a\,b\,c\,d+2\,b^2\,c^2)}{140\,b^2\,(a\,d-b\,c)^4}+\frac{81\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^3*x^3)/(140*(a*d - b*c)^4) + (420*a^3*d^3 - 42*b^3
*c^3 + 180*a*b^2*c^2*d - 315*a^2*b*c*d^2)/(140*b^3*(a*d - b*c)^4) + (27*d*x
*(35*a^2*d^2 + 2*b^2*c^2 - 10*a*b*c*d))/(140*b^2*(a*d - b*c)^4) + (81*d^2*x
^2*(10*a*d - b*c))/(140*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/3) + (a^3*(a +
 b*x)^(1/3))/b^3 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/
b^2)
```

3.1608 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=649

$$\frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3}-\frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d}-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}}{}$$

```
[Out] 21/20*(-a*d+b*c)^2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/d^3-21/40*(-a*d+b*c)*(b*x+a)
^(4/3)*(d*x+c)^(1/3)/d^2+3/8*(b*x+a)^(7/3)*(d*x+c)^(1/3)/d-7/20*3^(3/4)*(-a
*d+b*c)^3*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+
c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*
c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^
(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((
b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a
*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/d^(10/3)/(b*x+a)^(2/3)/
(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$,
Rules used = {52, 64, 637, 224}

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{10\,2^{2/3}\sqrt[3]{b}\,d^{10/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{21\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)^2}{20d^3}-\frac{21(a+bx)^{4/3}\sqrt[3]{c+dx}\,(bc-ad)}{40d^2}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]
```

```
[Out] (21*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*d^3) - (21*(b*c - a*
d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*d^2) + (3*(a + b*x)^(7/3)*(c + d*x)
^(1/3))/(8*d) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c +
d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*
b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b
```

```
^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*d^(10/3)
*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}}\,dx = \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx}{8d}$$

$$= -\frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} + \frac{(7(bc-ad)^2)\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}}{10d^2}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.11

$$\frac{3(a+bx)^{10/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{10}{3};\frac{13}{3};\frac{d(a+bx)}{-bc+ad}\right)}{10b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(10/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3
, 10/3, 13/3, (d*(a + b*x))/(-(b*c) + a*d)])/(10*b*(c + d*x)^(2/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(2/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(7/3)/(c + d*x)^(2/3), x)
```

3.1609 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=614

$$-\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d}+\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}}{}$$

[Out] -6/5*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(1/3)/d^2+3/5*(b*x+a)^(4/3)*(d*x+c)^(1/3)/d+2/5*2^(1/3)*3^(3/4)*(-a*d+b*c)^2*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/d^(7/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.58, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\,d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{6\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{5d^2}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]

[Out] (-6*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(5*d^2) + (3*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(5*d) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)

```
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b^(1/
3)*d^(7/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx = \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} - \frac{(4(bc-ad))\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx}{5d}$$
$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{(2(bc-ad)^2)\int \frac{1}{(a+bx)^{2/3}(c+d}}{5d^2}$$
$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{(2(bc-ad)^2((a+bx)(c+}{5d^2(a+bx}$$
$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{\Big(6(bc-ad)^2((a+bx)(c+}{}$$
$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.12

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(2/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(2/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(4/3)/(c + d*x)^(2/3), x)
```

3.1610 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=577

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\left((bc-ad)^{2/3}+2^{2/}\right.}{2^{2/3}\sqrt[3]{b}}$$

[Out] 3/2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/d-1/2*3^(3/4)*(-a*d+b*c)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/d^(4/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.39, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{2^{2/3}\sqrt[3]{b}\,d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]

[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*d) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3

```
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(1/3)*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2d}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{\left((bc-ad)((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2d(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{\left(3(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\Big(}{2d(a+bx)^{2/3}(c+dx)}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.13

$$\frac{3(a+bx)^{4/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(2/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(1/3)/(c + d*x)^(2/3), x)
```

3.1611 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=542

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+a}\right.}{\sqrt[3]{b}\,\sqrt[3]{d}\,(a+bx)^{2/3}(c+dx)^{2/}}$$

[Out] 2^(1/3)*3^(3/4)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/d^(1/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.31, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {64, 637, 224}

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\,\sqrt[3]{d}\,(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]

[Out] (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqr

```
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))], -7 - 4*Sqrt[3]])/(b^(1/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*
c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx = \frac{((a+bx)(c+dx))^{2/3}\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{\left(3((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+}\right.}{(a+bx)^{2/3}(c+dx)^{2/3}(bc+ad+2bd}$$

$$= \frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\left((bc-ad)^{2/3}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.13

$$\frac{3\sqrt[3]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[1/3,
 2/3, 4/3, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{2/3}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(2/3)), x)
```

3.1612 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=586

$$-\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\left((bc-ad)^{2/3}+2^{2/3}\right.}{2^{2/3}\sqrt[3]{b}\,(b}$$

```
[Out] -3/2*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(2/3)-1/2*3^(3/4)*d^(2/3)*((b*x+a)*(d
*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^
(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b
*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1
/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3
)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3)
)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1
/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/(-a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*
d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3
)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{c+dx}}{2(a+bx)^{2/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(2*(b*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sq
rt[3]]*d^(2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
```

```
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(2^(2/3)*b^(1/3)*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d
+ 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x
))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{d\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{\left(d((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{\left(3d((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Su}}{2(bc-ad)(a+}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+a}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.12

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(2*b*(a + b*x)^(2/3)*(c + d*x)^(2/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(2/3)), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(2/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/3}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(2/3)), x)
```

3.1613 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=621

$$-\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}}+\frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}}+\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad}}{}$$

```
[Out] -3/5*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(5/3)+6/5*d*(d*x+c)^(1/3)/(-a*d+b*c)^
2/(b*x+a)^(2/3)+2/5*2^(1/3)*3^(3/4)*d^(5/3)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+
b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(
2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((
-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^
(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d
^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/
3)/(-a*d+b*c)^2/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*
x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(
-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {53, 64, 637, 224}

$$\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^2(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{6d\sqrt[3]{c+dx}}{5(a+bx)^{2/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{5(a+bx)^{5/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (6*d*(c + d*x)^(1/3)
)/(5*(b*c - a*d)^2*(a + b*x)^(2/3)) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*
d^(5/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
```

```
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b
^(1/3)*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{5(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{(2d^2)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}}{5(bc-ad)^2}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{(2d^2((a+bx)(c+dx))^2}{5(bc-ad)^2(a}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{\left(6d^2((a+bx)(c+dx))\right.}{}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.12

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(-\frac{5}{3},\frac{2}{3};-\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}} (dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
[Out] int(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}} (c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(2/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{8/3}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(2/3)), x)
```

3.1614 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=656

$$-\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}}+\frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}}-\frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/3}}-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{8/3}((a+bx)(\cdots}{\cdots}$$

```
[Out] -3/8*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(8/3)+21/40*d*(d*x+c)^(1/3)/(-a*d+b*c
)^2/(b*x+a)^(5/3)-21/20*d^2*(d*x+c)^(1/3)/(-a*d+b*c)^3/(b*x+a)^(2/3)-7/20*3
^(3/4)*d^(8/3)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x
+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a
*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/
3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/
3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/(-a*d+b*c)^3/(b*x+
a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d
+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 656, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 224}

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{8/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{10\,2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^3(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{21d^2\sqrt[3]{c+dx}}{20(a+bx)^{2/3}(bc-ad)^3}+\frac{21d\sqrt[3]{c+dx}}{40(a+bx)^{5/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{8(a+bx)^{8/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (21*d*(c + d*x)^(1/3
))/(40*(b*c - a*d)^2*(a + b*x)^(5/3)) - (21*d^2*(c + d*x)^(1/3))/(20*(b*c -
 a*d)^3*(a + b*x)^(2/3)) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(8/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
```

```
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*(b*
c - a*d)^3*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} - \frac{(7d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx}{8(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} + \frac{(7d^2)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}}{10(bc-ad)^2}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.11

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};\frac{d(a+bx)}{-bc+ad}\right)}{8b(a+bx)^{8/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(8*b*(a + b*x)^(8/3)*(c + d*x)^(2/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+a\right)^{\frac{11}{3}} \left(dx+c\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^4*d*x^5 + a^4*c + (b^4*c + 4*a*
b^3*d)*x^4 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 2*a^3*b*d)*
x^2 + (4*a^3*b*c + a^4*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{11}{3}} \left(c+dx\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(2/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{11/3}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(2/3)), x)
```

3.1615 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=241

$$-\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}}-\frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3}+\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{3\sqrt{3}\,d^{10}}$$

```
[Out] -3*(b*x+a)^(7/3)/d/(d*x+c)^(1/3)-14/3*b*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(2
/3)/d^3+7/2*b*(b*x+a)^(4/3)*(d*x+c)^(2/3)/d^2-7/9*b^(1/3)*(-a*d+b*c)^2*ln(b
*x+a)/d^(10/3)-7/3*b^(1/3)*(-a*d+b*c)^2*ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)
/(b*x+a)^(1/3))/d^(10/3)-14/9*b^(1/3)*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*b
^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2))/d^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 52, 61}

$$-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log(a+bx)}{9d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3d^{10/3}}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}-\frac{14b\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^3}+\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2}-\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(7/3))/(d*(c + d*x)^(1/3)) - (14*b*(b*c - a*d)*(a + b*x)^(1/3
)*(c + d*x)^(2/3))/(3*d^3) + (7*b*(a + b*x)^(4/3)*(c + d*x)^(2/3))/(2*d^2)
- (14*b^(1/3)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/
(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*d^(10/3)) - (7*b^(1/3)*(b*c
- a*d)^2*Log[a + b*x])/(9*d^(10/3)) - (7*b^(1/3)*(b*c - a*d)^2*Log[-1 + (b^
(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*d^(10/3))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx &= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx}{d} \\
&= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{(14b(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d^2} \\
&= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} + \frac{(14b(bc \cdots}{\cdots} \\
&= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14\sqrt[3]{b}\,(\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.48, size = 267, normalized size = 1.11

$$\frac{-\frac{3\sqrt[3]{d}\,\sqrt[3]{a+bx}\,\left(18a^2d^2-abd(49c+13dx)+b^2\left(28c^2+7cdx-3d^2x^2\right)\right)}{\sqrt[3]{c+dx}}+28\sqrt{3}\,\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}}{\sqrt{3}}\right)-28\sqrt[3]{b}\,(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)+14\sqrt[3]{b}\,(bc-ad)^2\log\left(b^{2/3}+\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{18d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] ((-3*d^(1/3)*(a + b*x)^(1/3)*(18*a^2*d^2 - a*b*d*(49*c + 13*d*x) + b^2*(28*
c^2 + 7*c*d*x - 3*d^2*x^2)))/(c + d*x)^(1/3) + 28*Sqrt[3]*b^(1/3)*(b*c - a*
d)^2*ArcTan[(1 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3)))/Sqr
t[3]] - 28*b^(1/3)*(b*c - a*d)^2*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c
 + d*x)^(1/3)] + 14*b^(1/3)*(b*c - a*d)^2*Log[b^(2/3) + (d^(2/3)*(a + b*x)^
```

```
(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)]
)/(18*d^(10/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
[Out] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(187) = 374$.
time = 0.31, size = 423, normalized size = 1.76

$$-\frac{28\sqrt{3}\,(b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right) + 14\,(b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}} - (bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}} + (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right) - 28\,(b^2c^3 - 2\,abc^2d + a^2cd^2 + (b^2c^2d - 2\,abcd^2 + a^2d^3)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}} + (bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right) - 3\,(3\,b^2d^2x^2 - 28\,b^2c^2 + 49\,abcd - 18\,a^2d^2 - (7\,b^2cd - 13\,abd^2)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{18\,(d^4x + cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] -1/18*(28*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) + 14*(b^2
```

```
*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b
/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b
/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 28*(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*
log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) -
 3*(3*b^2*d^2*x^2 - 28*b^2*c^2 + 49*a*b*c*d - 18*a^2*d^2 - (7*b^2*c*d - 13*
a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^4*x + c*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(4/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(7/3)/(c + d*x)^(4/3), x)
```

3.1616 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=195

$$-\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}+\frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2}+\frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{2\sqrt[3]{b}\,(bc-ad}{3d^{7}}$$

```
[Out] -3*(b*x+a)^(4/3)/d/(d*x+c)^(1/3)+4*b*(b*x+a)^(1/3)*(d*x+c)^(2/3)/d^2+2/3*b^
(1/3)*(-a*d+b*c)*ln(b*x+a)/d^(7/3)+2*b^(1/3)*(-a*d+b*c)*ln(-1+b^(1/3)*(d*x+
c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/d^(7/3)+4/3*b^(1/3)*(-a*d+b*c)*arctan(1/3*3
^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2))/d^(7/3)*3^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 52, 61}

$$\frac{2\sqrt[3]{b}\,(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}\,(bc-ad)\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2}-\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(4/3))/(d*(c + d*x)^(1/3)) + (4*b*(a + b*x)^(1/3)*(c + d*x)^(
2/3))/d^2 + (4*b^(1/3)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^
(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*d^(7/3)) + (2*b^(1/3)*(
b*c - a*d)*Log[a + b*x])/(3*d^(7/3)) + (2*b^(1/3)*(b*c - a*d)*Log[-1 + (b^(
1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/d^(7/3)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx &= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}+\frac{(4b)\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{d}\\ &= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}+\frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2}-\frac{(4b(bc-ad))\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d^2}\\ &= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}+\frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2}+\frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+}}\right.}{\sqrt{3}\,d^{7/3}}\end{aligned}$$

Mathematica [A]
time = 10.09, size = 245, normalized size = 1.26

$$\frac{(a+bx)^{7/3}\left(\frac{3\sqrt[3]{d(a+bx)}\,(4bc-3ad+bdx)}{\sqrt[3]{c+dx}}+4\sqrt{3}\,\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)+4\sqrt[3]{b}\,(bc-ad)\log\left(\sqrt[3]{d(a+bx)}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)+2\sqrt[3]{b}\,(-bc+ad)\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)\right)}{3(d(a+bx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] ((a + b*x)^(7/3)*((3*(d*(a + b*x))^(1/3)*(4*b*c - 3*a*d + b*d*x))/(c + d*x)
^(1/3) + 4*Sqrt[3]*b^(1/3)*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1
/3))/(2*(d*(a + b*x))^(1/3) + b^(1/3)*(c + d*x)^(1/3))] + 4*b^(1/3)*(b*c -
a*d)*Log[(d*(a + b*x))^(1/3) - b^(1/3)*(c + d*x)^(1/3)] + 2*b^(1/3)*(-(b*c)
 + a*d)*Log[(d*(a + b*x))^(2/3) + b^(1/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(1/
3) + b^(2/3)*(c + d*x)^(2/3)]))/(3*(d*(a + b*x))^(7/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.31, size = 306, normalized size = 1.57

$$\frac{4\sqrt{3}\,(bc^2-acd+(bcd-ad^2)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right)+2\,(bc^2-acd+(bcd-ad^2)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-4\,(bc^2-acd+(bcd-ad^2)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)+3\,(bdx+4\,bc-3\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{3\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)*arctan(1/3*
(2*sqrt(3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x
+ b*c))/(b*d*x + b*c)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)
*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b/d)^(1/3)
 + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 4*(b*c^2 - a*c*d + (b*c*d
- a*d^2)*x)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x
 + c)^(2/3))/(d*x + c)) + 3*(b*d*x + 4*b*c - 3*a*d)*(b*x + a)^(1/3)*(d*x +
c)^(2/3))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(4/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(4/3)/(c + d*x)^(4/3), x)
```

3.1617 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{d^{4/3}}-\frac{\sqrt[3]{b}\,\log(a+bx)}{2d^{4/3}}-\frac{3\sqrt[3]{b}\,\log\left(-1+\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{2d^{4/3}}$$

```
[Out] -3*(b*x+a)^(1/3)/d/(d*x+c)^(1/3)-1/2*b^(1/3)*ln(b*x+a)/d^(4/3)-3/2*b^(1/3)*
ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/d^(4/3)-b^(1/3)*arctan(1
/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2))*3^(1/2)
/d^(4/3)
```

Rubi [A]
time = 0.02, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 61}

$$-\frac{3\sqrt[3]{b}\,\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\,\log(a+bx)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) - (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/d^(4/3)
 - (b^(1/3)*Log[a + b*x])/(2*d^(4/3)) - (3*b^(1/3)*Log[-1 + (b^(1/3)*(c + d
*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*d^(4/3))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx = -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} - \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{d^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2d^{4/3}} - \frac{3\sqrt[3]{b}\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.18, size = 191, normalized size = 1.28

$$\frac{-\frac{6\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + 2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}}{\sqrt{3}}\right) - 2\sqrt[3]{b}\,\log\left(\sqrt[3]{b} - \frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right) + \sqrt[3]{b}\,\log\left(b^{2/3} + \frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}} + \frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] ((-6*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + 2*Sqrt[3]*b^(1/3)*ArcTan[(1
 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3)))/Sqrt[3]] - 2*b^(1
/3)*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)] + b^(1/3)*Log[
b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a +
 b*x)^(1/3))/(c + d*x)^(1/3)])/(2*d^(4/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.
time = 0.30, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}\,(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right)+(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-2\,(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)+6\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\,(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(d*x + c)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3
)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) +
(d*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x +
 c)^(2/3)*(-b/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 2*(d
*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c)) + 6*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(4/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(1/3)/(c + d*x)^(4/3), x)
```

3.1618 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=30

$$\frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

```
[Out] 3*(b*x+a)^(1/3)/(-a*d+b*c)/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.20, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
[Out] -3*(b*x+a)^(1/3)/(d*x+c)^(1/3)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

Fricas [A]
time = 0.30, size = 42, normalized size = 1.40

$$\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] 3*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{2}{3}} (c + dx)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(4/3),x)
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(4/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{2/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)), x)
```

3.1619 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}$$

```
[Out] -3/2/(-a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)-9/2*d*(b*x+a)^(1/3)/(-a*d+b*c)^
2/(d*x+c)^(1/3)
```

Rubi [A]

time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(2*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3)) - (9*d*(a + b*x)^(1/3))/
(2*(b*c - a*d)^2*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx = -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx}{2(bc-ad)}$$
$$= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}} - \frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.68

$$-\frac{3(2ad+b(c+3dx))}{2(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(2*a*d + b*(c + 3*d*x)))/(2*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(1/
3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.16, size = 53, normalized size = 0.80

method	result	size
gosper	$-\frac{3(3bdx+2ad+bc)}{2(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*(3*b*d*x+2*a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.
time = 0.30, size = 126, normalized size = 1.91

$$-\frac{3\,(3\,bdx + bc + 2\,ad)(bx + a)^{\frac{1}{3}}(dx + c)^{\frac{2}{3}}}{2\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(3*b*d*x + b*c + 2*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{5}{3}}\,(c + dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(4/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/3}\,{\left(c+d\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)), x)
```

3.1620 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}+\frac{27d^2\sqrt[3]{a+bx}}{5(bc-ad)^3\sqrt[3]{c+dx}}$$

```
[Out] -3/5/(-a*d+b*c)/(b*x+a)^(5/3)/(d*x+c)^(1/3)+9/5*d/(-a*d+b*c)^2/(b*x+a)^(2/3
)/(d*x+c)^(1/3)+27/5*d^2*(b*x+a)^(1/3)/(-a*d+b*c)^3/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(5*(b*c - a*d)*(a + b*x)^(5/3)*(c + d*x)^(1/3)) + (9*d)/(5*(b*c - a*d)^2
*(a + b*x)^(2/3)*(c + d*x)^(1/3)) + (27*d^2*(a + b*x)^(1/3))/(5*(b*c - a*d)
^3*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx = -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx}{5(bc-ad)}$$
$$= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}} + \frac{(9d^2)}{}$$
$$= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}} + \frac{27}{5(bc-}$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.70

$$-\frac{3(c+dx)^{5/3}\left(b^2-\frac{5d^2(a+bx)^2}{(c+dx)^2}-\frac{5bd(a+bx)}{c+dx}\right)}{5(bc-ad)^3(a+bx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(5/3)*(b^2 - (5*d^2*(a + b*x)^2)/(c + d*x)^2 - (5*b*d*(a + b*
x))/(c + d*x)))/(5*(b*c - a*d)^3*(a + b*x)^(5/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{3\left(9b^2x^2d^2+15ab\,d^2x+3b^2cdx+5a^2d^2+5abcd-b^2c^2\right)}{5(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/5*(9*b^2*d^2*x^2+15*a*b*d^2*x+3*b^2*c*d*x+5*a^2*d^2+5*a*b*c*d-b^2*c^2)/(
b*x+a)^(5/3)/(d*x+c)^(1/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(83) = 166.
time = 0.30, size = 273, normalized size = 2.70

$$\frac{3\,(9\,b^2d^2x^2-b^2c^2+5\,abcd+5\,a^2d^2+3\,(b^2cd+5\,abd^2)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{5\,(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^3+(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4)x^2+(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(9*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 5*a^2*d^2 + 3*(b^2*c*d + 5*a*b*d
^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a
^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 -
 a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*
c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(4/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{8/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)), x)
```

3.1621 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}}+\frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}}-\frac{81d^2}{40(bc-ad)^3(a+bx)^{2/3}\sqrt[3]{c+dx}}-$$

```
[Out] -3/8/(-a*d+b*c)/(b*x+a)^(8/3)/(d*x+c)^(1/3)+27/40*d/(-a*d+b*c)^2/(b*x+a)^(5
/3)/(d*x+c)^(1/3)-81/40*d^2/(-a*d+b*c)^3/(b*x+a)^(2/3)/(d*x+c)^(1/3)-243/40
*d^3*(b*x+a)^(1/3)/(-a*d+b*c)^4/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}\,(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(8*(b*c - a*d)*(a + b*x)^(8/3)*(c + d*x)^(1/3)) + (27*d)/(40*(b*c - a*d)
^2*(a + b*x)^(5/3)*(c + d*x)^(1/3)) - (81*d^2)/(40*(b*c - a*d)^3*(a + b*x)^
(2/3)*(c + d*x)^(1/3)) - (243*d^3*(a + b*x)^(1/3))/(40*(b*c - a*d)^4*(c + d
*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx = -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} - \frac{(9d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx}{8(bc-ad)}$$
$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{(27c}{}$$
$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{}{40(b}$$
$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{}{40(b}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{8/3}\left(5b^3+\frac{40d^3(a+bx)^3}{(c+dx)^3}+\frac{60bd^2(a+bx)^2}{(c+dx)^2}-\frac{24b^2d(a+bx)}{c+dx}\right)}{40(bc-ad)^4(a+bx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^3 + (40*d^3*(a + b*x)^3)/(c + d*x)^3 + (60*b*d^2*(
a + b*x)^2)/(c + d*x)^2 - (24*b^2*d*(a + b*x))/(c + d*x)))/(40*(b*c - a*d)^
4*(a + b*x)^(8/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.26

method	result	size
gosper	$-\frac{3\left(81b^3x^3d^3+216d^3a\,x^2b^2+27b^3c\,d^2x^2+180a^2b\,d^3x+72a\,b^2c\,d^2x-9b^3c^2dx+40a^3d^3+60a^2bc\,d^2-24a\,b^2c^2d+5b^3c^3\right)}{40(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/40*(81*b^3*d^3*x^3+216*a*b^2*d^3*x^2+27*b^3*c*d^2*x^2+180*a^2*b*d^3*x+72
*a*b^2*c*d^2*x-9*b^3*c^2*d*x+40*a^3*d^3+60*a^2*b*c*d^2-24*a*b^2*c^2*d+5*b^3
*c^3)/(b*x+a)^(8/3)/(d*x+c)^(1/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(112) = 224$.
time = 0.33, size = 456, normalized size = 3.35

$$-\frac{3\,(81\,b^{3}d^{3}x^{3}+5\,b^{3}c^{3}-24\,ab^{2}c^{2}d+60\,a^{2}bcd^{2}+40\,a^{3}d^{3}+27\,(b^{3}cd^{2}+8\,ab^{2}d^{3})x^{2}-9\,(b^{3}c^{2}d-8\,ab^{2}cd^{2}-20\,a^{2}bd^{3})x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\,(a^{3}b^{4}c^{5}-4\,a^{4}b^{3}c^{4}d+6\,a^{5}b^{2}c^{3}d^{2}-4\,a^{6}bc^{2}d^{3}+a^{7}cd^{4}+(b^{7}c^{4}d-4\,ab^{6}c^{3}d^{2}+6\,a^{2}b^{5}c^{2}d^{3}-4\,a^{3}b^{4}cd^{4}+a^{4}b^{3}d^{5})x^{4}+(b^{7}c^{5}-ab^{6}c^{4}d-6\,a^{2}b^{5}c^{3}d^{2}+14\,a^{3}b^{4}c^{2}d^{3}-11\,a^{4}b^{3}cd^{4}+3\,a^{5}b^{2}d^{5})x^{3}+3\,(ab^{6}c^{5}-3\,a^{2}b^{5}c^{4}d+2\,a^{3}b^{4}c^{3}d^{2}+2\,a^{4}b^{3}c^{2}d^{3}-3\,a^{5}b^{2}cd^{4}+a^{6}bd^{5})x^{2}+(3\,a^{2}b^{5}c^{5}-11\,a^{3}b^{4}c^{4}d+14\,a^{4}b^{3}c^{3}d^{2}-6\,a^{5}b^{2}c^{2}d^{3}-a^{6}bcd^{4}+a^{7}d^{5})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/40*(81*b^3*d^3*x^3 + 5*b^3*c^3 - 24*a*b^2*c^2*d + 60*a^2*b*c*d^2 + 40*a^
3*d^3 + 27*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 9*(b^3*c^2*d - 8*a*b^2*c*d^2 - 2
0*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(4/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)), x)
```

3.1622 $\int \frac{(a+bx)^{8/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1355

$$-\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}}-\frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3}+\frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2}+\frac{30\ 2^{2/3}\sqrt[3]{b}\,(bc-}{7d^{11/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(b}$$

```
[Out] -3*(b*x+a)^(8/3)/d/(d*x+c)^(1/3)-30/7*b*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(2
/3)/d^3+24/7*b*(b*x+a)^(5/3)*(d*x+c)^(2/3)/d^2+30/7*2^(2/3)*b^(1/3)*(-a*d+b
*c)^2*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c)
)^2)^(1/2)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+20/7*
2^(1/6)*3^(3/4)*b^(1/3)*(-a*d+b*c)^(8/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c
)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(
2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/
3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/
3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/
3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-15/
7*2^(2/3)*3^(1/4)*b^(1/3)*(-a*d+b*c)^(8/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-
a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(
1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/d^(11/
3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2
)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.73, antiderivative size = 1355, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 64, 637, 309, 224, 1891}

$$-\frac{15\, 2^{2/3}\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{20\sqrt[6]{2}\,3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{30\, 2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)^2}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{30b(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)}{7d^3}+\frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2}-\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(8/3))/(d*(c + d*x)^(1/3)) - (30*b*(b*c - a*d)*(a + b*x)^(2/3
)*(c + d*x)^(2/3))/(7*d^3) + (24*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(7*d^2)
 + (30*2^(2/3)*b^(1/3)*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c
+ a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*d^(11/3)*(a + b*x)^(1
/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) - (15*2^(2/3)*3^(1/
4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(8/3)*((a + b*x)*(c + d*x))^(1/3)*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1
/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/
3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a + b*x)^(1/3)*(c + d*x)^(1
/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d
)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d
 + b*(c + 2*d*x))^2]) + (20*2^(1/6)*3^(3/4)*b^(1/3)*(b*c - a*d)^(8/3)*((a +
 b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
 - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a +
b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{8/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} + \frac{(8b)\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} - \frac{(40b(bc-ad))\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d^2}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{(20b\ldots}{\ldots}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(20\ldots\right.}{\ldots}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(60\ldots\right.}{\ldots}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\left(30\ldots\right.}{\ldots}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{\ldots}{7d^{11\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{11/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\ {}_2F_1\left(\frac{4}{3},\frac{11}{3};\frac{14}{3};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(11/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3
, 11/3, 14/3, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(4/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(2/3)*(d*x + c)^(2/3)/(d^2*x^2
 + 2*c*d*x + c^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{8}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(8/3)/(c + d*x)**(4/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{8/3}}{{\left(c+d\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(8/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(8/3)/(c + d*x)^(4/3), x)
```

3.1623 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1317

$$-\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{15\sqrt[3]{b}\,(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+}}{2\sqrt[3]{2}\,d^{8/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-\right.}$$

```
[Out] -3*(b*x+a)^(5/3)/d/(d*x+c)^(1/3)+15/4*b*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d^2-15/
4*b^(1/3)*(-a*d+b*c)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((
a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*
d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2
/3)*(1+3^(1/2)))-5/2*3^(3/4)*b^(1/3)*(-a*d+b*c)^(5/3)*((b*x+a)*(d*x+c))^(1/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*Ellip
ticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3
^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-
2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*
b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/d^(8/3)/(b*x+
a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d
+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^
(1/2)))^2)^(1/2)+15/8*3^(1/4)*b^(1/3)*(-a*d+b*c)^(5/3)*((b*x+a)*(d*x+c))^(1
/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*Elli
pticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-
3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2
^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a
)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^
(1/2)*2^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b
*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.37, antiderivative size = 1317, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$,

Rules used = {49, 52, 64, 637, 309, 224, 1891}

$$\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{4\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{5\,3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{2^{5/6}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{15\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)}{2\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(5/3))/(d*(c + d*x)^(1/3)) + (15*b*(a + b*x)^(2/3)*(c + d*x)^
(2/3))/(4*d^2) - (15*b^(1/3)*(b*c - a*d)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (15*3^
(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/
3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d
^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^
(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*d^(8/3)*(a + b*x)^(1/3)*(c
 + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]
*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*(b*c - a*d)^(5/3)*((a
+ b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3
) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*d^(8/3)
*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
```

```
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{(5b)\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{(5b(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{2d^2}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{\left(5b(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{}{\sqrt[3]{ac}}}{2d^2\sqrt[3]{a+bx}\sqrt[3]{c+d}}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{\left(15b(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+}\right.}{}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{\left(15b^{2/3}(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc}\right.}{}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{15\sqrt[3]{b}(bc-ad)\sqrt[3]{(a+bx)(c}}{2\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bd}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{8/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{4}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{-bc+ad}\right)}{8b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(4/3))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
[Out] int((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(4/3), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/3)*(d*x + c)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(4/3), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{5/3}}{{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(5/3)/(c + d*x)^(4/3), x)
```

3.1624 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1279

$$-\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}}+\frac{3\ 2^{2/3}\sqrt[3]{b}\ \sqrt[3]{(a+bx)(c+dx)}\ \sqrt{(bc+ad+2bdx)^2}\ \sqrt{(ad+b(c+2dx))^2}}{d^{5/3}\sqrt[3]{a+bx}\ \sqrt[3]{c+dx}\ (bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\ \sqrt[3]{d}\ \sqrt[3]{(a+bx)}\right.}$$

```
[Out] -3*(b*x+a)^(2/3)/d/(d*x+c)^(1/3)+3*2^(2/3)*b^(1/3)*((b*x+a)*(d*x+c))^(1/3)*
((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(5/3)/(b*x+a)^(1/
3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c
))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+2*2^(1/6)*3^(3/4)*b^(1/3)*(-a*d+b*c)
^(2/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^
2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+
a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)/d^(5/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x
+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-
a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*b^(1/3)*(-a*d+b*c)^(2/3)*(
(b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(
-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)
*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d
+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x
+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/d^(5/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d
*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)
+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.06, antiderivative size = 1279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316,

Rules used = {49, 64, 637, 309, 224, 1891}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{2}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt[6]{2}3^{3/4}\sqrt[3]{b}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(2/3))/(d*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
/(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
 d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)
*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*
b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/
3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*d^(5/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*(b*c -
 a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[(
(b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
 + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/
(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{(2b)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(2b\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{d\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(6b\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd+(}}\right.}{d\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(3\sqrt[3]{2}\,b^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{\right.}}{\sqrt{-4a}}\right.}{d^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad}}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{4}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(4/3))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(4/3), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(2/3)/(c + d*x)^(4/3), x)
```

3.1625 $\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1298

$$\frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}}-\frac{3\sqrt[3]{b}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2c}}{\sqrt[3]{2}\,d^{2/3}(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt{}\right.}$$

[Out] 3*(b*x+a)^(2/3)/(-a*d+b*c)/(d*x+c)^(1/3)-3/2*b^(1/3)*((b*x+a)*(d*x+c))^(1/3
)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/d^(2/3)/(
-a*d+b*c)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))-2^(1/6)*3^(3/4)
b^(1/3)((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c
))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d
*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c
)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*
x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(
2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^
2)^(1/2)/d^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+
b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a
)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)+3/4*3^(1/4)*b^(1/3)
*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)
+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/
2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a
*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d
*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(
2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/d^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/(
d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3
)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)

Rubi [A]

time = 1.07, antiderivative size = 1298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[3]{2}d^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{\sqrt[6]{2}3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}d^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3))/((b*c - a*d)*(c + d*x)^(1/3)) - (3*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]
)/(2^(1/3)*d^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2
/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/
3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*d^(2/3)*(b*c
- a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b
*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2^(1/6)*3
^(3/4)*b^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))
^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{4/3}}\,dx = \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{b\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(b\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(3b\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \right.}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(3b^{2/3}\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{2^{2/3}\sqrt[3]{d}\,(bc-ad)\sqrt[3]{a+}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{3\sqrt[3]{b}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+}}{\sqrt[3]{2}\,d^{2/3}(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{2/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[2/3,
 4/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(4/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}} (dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\, (c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(4/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(4/3)), x)
```

3.1626 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1327

$$-\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}-\frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}}+\frac{3\;2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(\cdots}}{(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\cdots\right.\right.}$$

```
[Out] -3/(-a*d+b*c)/(b*x+a)^(1/3)/(d*x+c)^(1/3)-6*d*(b*x+a)^(2/3)/(-a*d+b*c)^2/(d
*x+c)^(1/3)+3*2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d
+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/3)/(d*x+
c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)
+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+2*2^(1/6)*3^(3/4)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+
b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+
b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)
+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3
)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/(-a*d+b*c)
^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^
(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^
(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2
*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x
+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)*2^(2/3)/(-a*d+b*c)^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b
*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.40, antiderivative size = 1327, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$-\frac{6(a+bx)^{2/3}d}{(bc-ad)^2\sqrt[3]{c+dx}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{2}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt[6]{2}3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)\sqrt[3]{d}}{(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}\sqrt[3]{d}}{(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/((b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)) - (6*d*(a + b*x)^(2/3))/((
b*c - a*d)^2*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((
b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*(b*c - a*d)^(4/3)*(a + b*
x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/((b*c -
 a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*
c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx = -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{4/3}}\,dx}{bc-ad}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{(2bd)\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c}}}{(bc-ad)^2}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(2bd\sqrt[3]{(a+bx)(c+}\right.}{(bc-}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(6bd\sqrt[3]{(a+bx)(c+}\right.}{}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(3\sqrt[3]{2}\,b^{2/3}d^{2/3}\sqrt[3]{(a+}\right.}{}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{3\,2^{2/3}}{(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.05

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{a+bx}\,(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/3)*(c + d*x)^(4/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}} (dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}} (c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(4/3)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{4/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(4/3)*(c + d*x)^(4/3)), x)
```

3.1627 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1370

$$-\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}+\frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}+\frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}}-\frac{}{2\sqrt[3]{2}\,(bc-ad)^3}$$

```
[Out] -3/4/(-a*d+b*c)/(b*x+a)^(4/3)/(d*x+c)^(1/3)+15/4*d/(-a*d+b*c)^2/(b*x+a)^(1/
3)/(d*x+c)^(1/3)+15/2*d^2*(b*x+a)^(2/3)/(-a*d+b*c)^3/(d*x+c)^(1/3)-15/4*b^(
1/3)*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2
*d*x+c))^2)^(1/2)*2^(2/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x
+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)
*(1+3^(1/2)))-5/2*3^(3/4)*b^(1/3)*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*
c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^
(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1
/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2
/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/(-a*d+b*c)^(7/3)/(b*x+a)^
(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*
c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))
/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/
2)))^2)^(1/2)+15/8*3^(1/4)*b^(1/3)*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-
a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(
1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3
)/(-a*d+b*c)^(7/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2
*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/
3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.64, antiderivative size = 1370, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$\frac{15(a+bx)^{2/3}d^2}{2(bc-ad)^3\sqrt[3]{c+dx}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{4\sqrt[3]{2}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{5\,3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{2^{5/6}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{15\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{4/3}}{2\sqrt[3]{2}(bc-ad)^3\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}-\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(4*(b*c - a*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3)) + (15*d)/(4*(b*c - a*d)^
2*(a + b*x)^(1/3)*(c + d*x)^(1/3)) + (15*d^2*(a + b*x)^(2/3))/(2*(b*c - a*d
)^3*(c + d*x)^(1/3)) - (15*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*(b*c -
a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))
+ (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(
1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2
/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*(b*c - a*d)^(7/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*d^(4/3)*((a +
b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*(b*c - a*
d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}(c+dx)^{4/3}}\,dx = -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx}{4(bc-ad)}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{(5d^2)\int$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d^2}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d^2}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d^2}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d^2}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d^2}{2(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.05

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},\frac{4}{3};-\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b(a+bx)^{4/3}(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[-4/3, 4/3, -1/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(4*b*(a + b*x)^(4/3)*(c + d*x)^(4/3))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}} (dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*
d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{3}} (c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(4/3)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(7/3)*(c + d*x)^(4/3)), x)
```

3.1628 $\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=77

$$\sqrt[3]{-1+x}\,(1+x)^{2/3}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{\sqrt{3}}+\frac{1}{3}\log(-1+x)+\log\left(-1+\frac{\sqrt[3]{1+x}}{\sqrt[3]{-1+x}}\right)$$

```
[Out] (-1+x)^(1/3)*(1+x)^(2/3)+1/3*ln(-1+x)+ln(-1+(1+x)^(1/3)/(-1+x)^(1/3))+2/3*a
rctan(1/3*3^(1/2)+2/3*(1+x)^(1/3)/(-1+x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {52, 61}

$$\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\,\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqr
t[3]*(-1 + x)^(1/3))])/Sqrt[3] + Log[-1 + x]/3 + Log[-1 + (1 + x)^(1/3)/(-1
 + x)^(1/3)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx = \sqrt[3]{-1+x}\,(1+x)^{2/3} - \frac{2}{3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{1+x}}\,dx$$

$$= \sqrt[3]{-1+x}\,(1+x)^{2/3} + \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1+x}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{\sqrt{3}} + \frac{1}{3}\log(-1+x) + \log\left(-1+\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(77) = 154.
time = 0.18, size = 158, normalized size = 2.05

$$\frac{\sqrt[3]{\frac{-1+x}{1+x}}\left(3\sqrt[3]{-1+x}+3\sqrt[3]{-1+x}\,x+2\sqrt{3}\,\sqrt[3]{1+x}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{1+x}}{2\sqrt[3]{-1+x}+\sqrt[3]{1+x}}\right)+2\sqrt[3]{1+x}\,\log\left(\sqrt[3]{-1+x}-\sqrt[3]{1+x}\right)-\sqrt[3]{1+x}\,\log\left((-1+x)^{2/3}+\sqrt[3]{-1+x}\,\sqrt[3]{1+x}+(1+x)^{2/3}\right)\right)}{3\sqrt[3]{-1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (((-1 + x)/(1 + x))^(1/3)*(3*(-1 + x)^(1/3) + 3*(-1 + x)^(1/3)*x + 2*Sqrt[3
]*(1 + x)^(1/3)*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2*(-1 + x)^(1/3) + (1 + x)^
(1/3))] + 2*(1 + x)^(1/3)*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)] - (1 + x)^(1/
3)*Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(3*
(-1 + x)^(1/3))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.36, size = 25, normalized size = 0.32

$$\frac{32^{\frac{2}{3}}(-1+x)^{\frac{4}{3}}\,\mathrm{hyper}\left[\left\{\frac{1}{3},\frac{4}{3}\right\},\left\{\frac{7}{3}\right\},\frac{(-1+x)\mathrm{exp\_polar}[I\mathrm{Pi}]}{2}\right]}{8}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(-1 + x)^(1/3)/(1 + x)^(1/3),x]')
```

```
[Out] 3 2 ^ (2 / 3) (-1 + x) ^ (4 / 3) hyper[{1 / 3, 4 / 3}, {7 / 3}, (-1 + x) ex
p_polar[I Pi] / 2] / 8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 577, normalized size = 7.49

method	result

risch	$(-1+x)^{\frac{1}{3}}(1+x)^{\frac{2}{3}}+\frac{\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^2-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x+3\,\mathrm{RootOf}(}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/3)/(1+x)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x)^(1/3)*(1+x)^(2/3)+(2/3*RootOf(_Z^2+_Z+1)*ln(-(2*RootOf(_Z^2+_Z+1)^2*
x^2-2*RootOf(_Z^2+_Z+1)^2*x+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+3*RootO
f(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x+5*RootOf(_Z^2+_Z+1)*x^2-3*RootOf(_Z^2+_Z
+1)*(x^3-x^2-x+1)^(1/3)-4*RootOf(_Z^2+_Z+1)*x+2*x^2-RootOf(_Z^2+_Z+1)-2)/(-
1+x))-2/3*ln(-(2*RootOf(_Z^2+_Z+1)^2*x^2-2*RootOf(_Z^2+_Z+1)^2*x-3*RootOf(_
Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x-Roo
tOf(_Z^2+_Z+1)*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)-3*(x^3-x^2-x+1)^
(2/3)-3*(x^3-x^2-x+1)^(1/3)*x-x^2+RootOf(_Z^2+_Z+1)+3*(x^3-x^2-x+1)^(1/3)+2
*x-1)/(-1+x))*RootOf(_Z^2+_Z+1)-2/3*ln(-(2*RootOf(_Z^2+_Z+1)^2*x^2-2*RootOf
(_Z^2+_Z+1)^2*x-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)
*(x^3-x^2-x+1)^(1/3)*x-RootOf(_Z^2+_Z+1)*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x
+1)^(1/3)-3*(x^3-x^2-x+1)^(2/3)-3*(x^3-x^2-x+1)^(1/3)*x-x^2+RootOf(_Z^2+_Z+
1)+3*(x^3-x^2-x+1)^(1/3)+2*x-1)/(-1+x)))/(-1+x)^(2/3)*((-1+x)^2*(1+x))^(1/3
)/(1+x)^(1/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

Fricas [A]
time = 0.30, size = 107, normalized size = 1.39

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1)+2\sqrt{3}(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}}{3(x+1)}\right)+(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-\frac{1}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}+(x+1)^{\frac{1}{3}}(x-1)^{\frac{2}{3}}+x+1}{x+1}\right)+\frac{2}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-x-1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x + 1)^(2/3)*(x - 1)^
(1/3))/(x + 1)) + (x + 1)^(2/3)*(x - 1)^(1/3) - 1/3*log(((x + 1)^(2/3)*(x -
 1)^(1/3) + (x + 1)^(1/3)*(x - 1)^(2/3) + x + 1)/(x + 1)) + 2/3*log(((x + 1
)^(2/3)*(x - 1)^(1/3) - x - 1)/(x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.72, size = 39, normalized size = 0.51

$$\frac{2^{\frac{2}{3}}\left(x-1\right)^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{(x-1)e^{i\pi}}{2}\right)}{2\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/3)/(1+x)**(1/3),x)
```

```
[Out] 2**(2/3)*(x - 1)**(4/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), (x - 1)*exp_po
lar(I*pi)/2)/(2*gamma(7/3))
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)^{1/3}}{(x+1)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/3)/(x + 1)^(1/3),x)
```

```
[Out] int((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

3.1629 $\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx$

Optimal. Leaf size=185

$$-\frac{8(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{77bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b}+\frac{16(bc-ad)^{13/4}}{}$$

```
[Out] 4/77*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b/d+4/11*(b*x+a)^(5/2)*(d*x+c)^
(1/4)/b-8/77*(-a*d+b*c)^2*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b/d^2+16/77*(-a*d+b*c
)^(13/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-
a*d+b*c))^(1/2)/b^(5/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{16(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{77b^{5/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}\,(bc-ad)^2}{77bd^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}{77bd}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]
```

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(77*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(1/4))/(11*b) + (16*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(77*b^(5
/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx = \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{11b}$$

$$= \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b} - \frac{(6(bc-ad)^2)\int \frac{\sqrt{a-}}{(c+d}}{77bd}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)}{1}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)}{1}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)}{1}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd} + \frac{4(a+bx)}{1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.39

$$\frac{2(a+bx)^{5/2}\sqrt[4]{c+dx}\ {}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\sqrt[4]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/4), x)
```

3.1630 $\int \sqrt{a+bx}\sqrt[4]{c+dx}\,dx$

Optimal. Leaf size=147

$$\frac{4(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{21bd}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b}-\frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+d}}{\sqrt[4]{bc-ad}}\right.\right.}{21b^{5/4}d^2\sqrt{a+bx}}$$

```
[Out] 4/7*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b+4/21*(-a*d+b*c)*(d*x+c)^(1/4)*(b*x+a)^(1/
2)/b/d-8/21*(-a*d+b*c)^(9/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/
4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/d^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$-\frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{21b^{5/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{21bd}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/4),x]
```

```
[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b*d) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*b) - (8*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(21*b^(5/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt[4]{c+dx}\,dx = \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{7b}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{(2(bc-ad)^2)\int \frac{}{\sqrt{a+b}}}{21bd}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{(8(bc-ad)^2)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{\left(8(bc-ad)^2\sqrt{\frac{d(a+b}{-bc+}}\right.}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+}{bc-}}}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.50

$$\frac{2(a+bx)^{3/2}\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(1/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,\left(dx+c\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt[4]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/4),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(1/4), x)
```

3.1631 $\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=111

$$\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b}+\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}$$

```
[Out] 4/3*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b+4/3*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/d/(b*
x+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/Sqrt[a + b*x],x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b) + (4*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(5/4)*d*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3b}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{(4(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3bd}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{\left(4(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c\right.}{3bd\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\,\sqrt[4]{c+dx}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/4)/(a + b*x)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/sqrt(a + b*x), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{1/4}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(1/2), x)
```

3.1632 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(1/4)/b/(b*x+a)^(1/2)+2*(-a*d+b*c)^(1/4)*EllipticF(b^(1/4)*(d*x+
c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/(b*x+a)^
(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 230, 227}

$$\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}-\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(b*Sqrt[a + b*x]) + (2*(b*c - a*d)^(1/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(5/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{2b}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b\sqrt{a+bx}}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.68

$$-\frac{2\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(3/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(3/2), x)
```

3.1633 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}-\frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(1/4)/b/(b*x+a)^(3/2)-1/3*d*(d*x+c)^(1/4)/b/(-a*d+b*c)/(b*x+a)
^(1/2)-1/3*d*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a
)/(-a*d+b*c))^(1/2)/b^(5/4)/(-a*d+b*c)^(3/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}\sqrt{a+bx}\,(bc-ad)^{3/4}}-\frac{d\sqrt[4]{c+dx}}{3b\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(1/4))/(3*b*(b*c
- a*d)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(5/4)*(b*c - a*
d)^(3/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

ntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{6b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d^2\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{12b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right.}{3b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right.}}}\right.}{3b(bc-ad)\sqrt{a+bx}}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right.}{3b^{5/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.50

$$-\frac{2\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(5/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(5/2), x)
```

3.1634 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}+\frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}}+\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{6b^{5/4}(bc-ad)^{7/4}\sqrt{a+bx}}$$

```
[Out] -2/5*(d*x+c)^(1/4)/b/(b*x+a)^(5/2)-1/15*d*(d*x+c)^(1/4)/b/(-a*d+b*c)/(b*x+a
)^(3/2)+1/6*d^2*(d*x+c)^(1/4)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)+1/6*d^2*Elliptic
F(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b
^(5/4)/(-a*d+b*c)^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{6b^{5/4}\sqrt{a+bx}\,(bc-ad)^{7/4}}+\frac{d^2\sqrt[4]{c+dx}}{6b\sqrt{a+bx}\,(bc-ad)^2}-\frac{d\sqrt[4]{c+dx}}{15b(a+bx)^{3/2}(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(1/4))/(15*b*(b*c
 - a*d)*(a + b*x)^(3/2)) + (d^2*(c + d*x)^(1/4))/(6*b*(b*c - a*d)^2*Sqrt[a
+ b*x]) + (d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)
*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(5/4)*(b*c - a*d)^(7/4)*Sqr
t[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx}{10b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} - \frac{d^2\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{12b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^3\int \frac{1}{\sqrt{a+bx}\,(c+\cdots}}{24b(bc-ad\cdots}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^2\text{Subst}\left(\int \frac{\cdots}{\sqrt{a\cdots}}\right.}{\cdots}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\cdots}{6b^{5/4}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.39

$$-\frac{2\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(7/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(7/2), x)
```

3.1635 $\int (a+bx)^{3/2}(c+dx)^{3/4}\,dx$

Optimal. Leaf size=270

$$-\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}+\frac{16(bc-ad}{}$$

```
[Out] 4/39*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/4)/b/d+4/13*(b*x+a)^(5/2)*(d*x+c)^
(3/4)/b-8/65*(-a*d+b*c)^2*(d*x+c)^(3/4)*(b*x+a)^(1/2)/b/d^2+16/65*(-a*d+b*c
)^(15/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-
a*d+b*c))^(1/2)/b^(7/4)/d^3/(b*x+a)^(1/2)-16/65*(-a*d+b*c)^(15/4)*EllipticF
(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^
(7/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)^2}{65bd^2}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}{39bd}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]
```

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(65*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(39*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*b) + (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(65*b^(7
/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(65*b^(7/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/4}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} + \frac{(3(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{13b}$$

$$= \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} - \frac{(2(bc-ad)^2)\,\int}{13}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.27

$$\frac{2(a+bx)^{5/2}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(3/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} (c+dx)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{3/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(3/4), x)
```

3.1636 $\int \sqrt{a+bx}\,(c+dx)^{3/4}\,dx$

Optimal. Leaf size=232

$$\frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b}-\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+}}{\sqrt[4]{bc-}}\right.\right.}{15b^{7/4}d^2\sqrt{a+bx}}$$

[Out] 4/9*(b*x+a)^(3/2)*(d*x+c)^(3/4)/b+4/15*(-a*d+b*c)*(d*x+c)^(3/4)*(b*x+a)^(1/2)/b/d-8/15*(-a*d+b*c)^(11/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d^2/(b*x+a)^(1/2)+8/15*(-a*d+b*c)^(11/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d^2/(b*x+a)^(1/2)

Rubi [A]

time = 0.18, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{15bd}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/4),x]

[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*b*d) + (4*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*b) - (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*b^(7/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*b^(7/4)*d^2*Sqrt[a + b*x])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{3/4}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3b}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{(2(bc-ad)^2)\int \frac{}{\sqrt{}}}{15}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{(8(bc-ad)^2)\,\text{Subs}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{\left(8(bc-ad)^{5/2}\right)\text{Su}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{\left(8(bc-ad)^{5/2}\sqrt{\cdots}\right.}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{8(bc-ad)^{11/4}\sqrt{-\cdots}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{8(bc-ad)^{11/4}\sqrt{-\cdots}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.31

$$\frac{2(a+bx)^{3/2}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(3/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\ (dx+c)^{\frac{3}{4}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,(c+dx)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/4),x)
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(3/4), x)
```

3.1637 $\int \frac{(c+dx)^{3/4}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=196

$$\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b}+\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}-\frac{12(bc-ad)^{7/4}\sqrt{-\cdots}}{\cdots}$$

```
[Out] 4/5*(d*x+c)^(3/4)*(b*x+a)^(1/2)/b+12/5*(-a*d+b*c)^(7/4)*EllipticE(b^(1/4)*(
d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d/(b
*x+a)^(1/2)-12/5*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c
)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/Sqrt[a + b*x],x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*b) + (12*(b*c - a*d)^(7/4)*Sqrt[-((d*(
a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a
*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x]) - (12*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} + \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5b}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} + \frac{(12(bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5bd}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} - \frac{\left(12(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5b^{3/2}d}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} - \frac{\left(12(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,d\right.}{5b^{3/2}d\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} - \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle| -1\right.}{5b^{7/4}d\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} + \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle| -1\right.}{5b^{7/4}d\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.36

$$\frac{2\sqrt{a+bx}\,(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/Sqrt[a + b*x],x]
```

[Out] (2*Sqrt[a + b*x]*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(3/4))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(c + d*x)^(3/4)/(a + b*x)^(1/2),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/4)/(b*x+a)^(1/2),x)

[Out] int((d*x+c)^(3/4)/(b*x+a)^(1/2),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x + c)^(3/4)/sqrt(b*x + a), x)

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] integral((d*x + c)^(3/4)/sqrt(b*x + a), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(3/4)/sqrt(a + b*x), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/4}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(3/4)/(a + b*x)^(1/2), x)
```

3.1638 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{b^{7/4}}$$

```
[Out] -2*(d*x+c)^(3/4)/b/(b*x+a)^(1/2)+6*(-a*d+b*c)^(3/4)*EllipticE(b^(1/4)*(d*x+
c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(b*x+a)^
(1/2)-6*(-a*d+b*c)^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I
)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {49, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(b*Sqrt[a + b*x]) + (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x]) - (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{(3d)\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{2b}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{6\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} - \frac{\left(6\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b^{3/2}} + \frac{\left(6\sqrt{bc\right.}{}}{}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} - \frac{\left(6\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+\right.}{b^{3/2}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} - \frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}} + \frac{\left(6\sqrt{\right.}}{}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}} - \frac{6(bc}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.39

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]
```

[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/4))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python o bject

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/4)/(b*x+a)^(3/2),x)

[Out] int((d*x+c)^(3/4)/(b*x+a)^(3/2),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(3/2), x)

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*x^2 + 2*a*b*x + a^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(3/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/4}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(3/4)/(a + b*x)^(3/2), x)
```

3.1639 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt[4]{bc-ad}\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\cdots}{b^{7}\cdots}$$

```
[Out] -2/3*(d*x+c)^(3/4)/b/(b*x+a)^(3/2)-d*(d*x+c)^(3/4)/b/(-a*d+b*c)/(b*x+a)^(1/
2)+d*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+
b*c))^(1/2)/b^(7/4)/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)-d*EllipticF(b^(1/4)*(d*x
+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(-a*d+b
*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{d(c+dx)^{3/4}}{b\sqrt{a+bx}(bc-ad)}-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(3/4))/(b*(b*c -
a*d)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx}{2b}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{4b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b^{3/2}\sqrt{bc-ad}}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} - \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)}}}\right.}{b^{3/2}\sqrt{bc-ad}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|}{b^{7/4}\sqrt[4]{bc-ad}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|}{b^{7/4}\sqrt[4]{bc-ad}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.33

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]
[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(3/4)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(5/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{3/4}}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(3/4)/(a + b*x)^(5/2), x)
```

3.1640 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}}-\frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}}+\frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}}-\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+d}}{\sqrt[4]{bc-ad}}\right.\right.}{10b^{7/4}(bc-ad)^{5/4}\sqrt{a+bx}}$$

```
[Out] -2/5*(d*x+c)^(3/4)/b/(b*x+a)^(5/2)-1/5*d*(d*x+c)^(3/4)/b/(-a*d+b*c)/(b*x+a)
^(3/2)+3/10*d^2*(d*x+c)^(3/4)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)-3/10*d^2*Ellipti
cE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/
b^(7/4)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)+3/10*d^2*EllipticF(b^(1/4)*(d*x+c)^(
1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(-a*d+b*c)^(
5/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {49, 53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{3d^2(c+dx)^{3/4}}{10b\sqrt{a+bx}\,(bc-ad)^2}-\frac{d(c+dx)^{3/4}}{5b(a+bx)^{3/2}(bc-ad)}-\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(3/4))/(5*b*(b*c
- a*d)*(a + b*x)^(3/2)) + (3*d^2*(c + d*x)^(3/4))/(10*b*(b*c - a*d)^2*Sqrt[
a + b*x]) - (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(
1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*d)^(5/4
)*Sqrt[a + b*x]) + (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*
d)^(5/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} + \frac{(3d)\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx}{10b}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} - \frac{(3d^2)\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx}{20b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{(3d^3)\int \frac{}{\sqrt{a+bx}\ldots}}{40b(bc-\ldots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{(3d^2)\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{(3d^2)\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(3d^2\sqrt{\frac{d(a+bx\ldots}{-bc+a\ldots}}\ldots\right.}{\ldots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{3d^2\sqrt{-\frac{d(a+bx\ldots}{bc-ad\ldots}}\ldots}{10b\ldots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{3d^2\sqrt{-\frac{d(a+bx\ldots}{bc-ad\ldots}}\ldots}{10b\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.27

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(7/2), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(7/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{3/4}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(3/4)/(a + b*x)^(7/2), x)
```

3.1641 $\int (a+bx)^{3/2}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=220

$$-\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2}+\frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2}+\frac{4(a+b$$

```
[Out] 4/231*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b^2/d+4/33*(-a*d+b*c)*(b*x+a
)^(5/2)*(d*x+c)^(1/4)/b^2+4/15*(b*x+a)^(5/2)*(d*x+c)^(5/4)/b-8/231*(-a*d+b*
c)^3*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2/d^2+16/231*(-a*d+b*c)^(17/4)*EllipticF
(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^
(9/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 230, 227}

$$\frac{16(bc-ad)^{17/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{231b^{9/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^3}{231b^2d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)^2}{231b^2d}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}\,(bc-ad)}{33b^2}+\frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]
```

```
[Out] (-8*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d^2) + (4*(b*c -
a*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(231*b^2*d) + (4*(b*c - a*d)*(a + b
*x)^(5/2)*(c + d*x)^(1/4))/(33*b^2) + (4*(a + b*x)^(5/2)*(c + d*x)^(5/4))/(
15*b) + (16*(b*c - a*d)^(17/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF
[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b^(9/4)*d^3
*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/4}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b} + \frac{(bc-ad)\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx}{3b}$$

$$= \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2} + \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b} + \frac{(bc-ad)^2\int \frac{(a+}{(c+}}{33b^2}$$

$$= \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2} + \frac{4(a+b}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d} + \frac{4(bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]

[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/4))

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]')

[Out] Timed out

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(3/2)*(d*x+c)^(5/4),x)

[Out] int((b*x+a)^(3/2)*(d*x+c)^(5/4),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="maxima")

[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/4), x)

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/4), x)
```

3.1642 $\int \sqrt{a+bx}\,(c+dx)^{5/4}\,dx$

Optimal. Leaf size=182

$$\frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d}+\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}-\frac{40(bc-ad)^{13/4}}{}$$

```
[Out] 20/77*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b^2+4/11*(b*x+a)^(3/2)*(d*x+c)
^(5/4)/b+20/231*(-a*d+b*c)^2*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2/d-40/231*(-a*d
+b*c)^(13/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a
)/(-a*d+b*c))^(1/2)/b^(9/4)/d^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$-\frac{40(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{231b^{9/4}d^2\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{231b^2d}+\frac{20(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/4),x]
```

```
[Out] (20*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d) + (20*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b^2) + (4*(a + b*x)^(3/2)*(c + d*x
)^(5/4))/(11*b) - (40*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b
^(9/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/4}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b} + \frac{(5(bc-ad))\int \sqrt{a+bx}\,\sqrt[4]{c+dx}\,dx}{11b}$$

$$= \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b} + \frac{(5(bc-ad)^2)\int}{77b^2}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+b}{}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+b}{}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+b}{}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.40

$$\frac{2(a+bx)^{3/2}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(5/4),x]')
```

```
[Out] Exception raised: AttributeError >> 'SympyExpression' object has no attribu
te 'expr'
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx + a}\ (dx + c)^{\frac{5}{4}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Sympy [A]
time = 10.37, size = 218, normalized size = 1.20

$$-\frac{2ad\,(a+bx)^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{3b^2}+\frac{2c\,(a+bx)^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{3b}+\frac{2d\,(a+bx)^{\frac{5}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\operatorname{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/4),x)
```

```
[Out] -2*a*d*(a + b*x)**(3/2)*hyper((-1/4, 3/2), (5/2,), a*d*exp_polar(I*pi)/(b*p
olar_lift(-a*d/b + c)) + d*x*exp_polar(I*pi)/polar_lift(-a*d/b + c))*polar_
lift(-a*d/b + c)**(1/4)/(3*b**2) + 2*c*(a + b*x)**(3/2)*hyper((-1/4, 3/2),
(5/2,), a*d*exp_polar(I*pi)/(b*polar_lift(-a*d/b + c)) + d*x*exp_polar(I*pi
)/polar_lift(-a*d/b + c))*polar_lift(-a*d/b + c)**(1/4)/(3*b) + 2*d*(a + b*
x)**(5/2)*hyper((-1/4, 5/2), (7/2,), a*d*exp_polar(I*pi)/(b*polar_lift(-a*d
/b + c)) + d*x*exp_polar(I*pi)/polar_lift(-a*d/b + c))*polar_lift(-a*d/b +
c)**(1/4)/(5*b**2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,\,{\left(c+d\,x\right)}^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/4), x)
```

3.1643 $\int \frac{(c+dx)^{5/4}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=144

$$\frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2}+\frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b}+\frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|\left.-1\right)}{21b^{9/4}d\sqrt{a+bx}}$$

[Out] 20/21*(-a*d+b*c)*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2+4/7*(d*x+c)^(5/4)*(b*x+a)^(1/2)/b+20/21*(-a*d+b*c)^(9/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/d/(b*x+a)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|\left.-1\right)}{21b^{9/4}d\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{21b^2}+\frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/Sqrt[a + b*x],x]
```

```
[Out] (20*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b^2) + (4*Sqrt[a + b*x]*(c + d*x)^(5/4))/(7*b) + (20*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(21*b^(9/4)*d*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx}{7b}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,(c+\cdots}}{21b^2}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{(20(bc-ad)^2)\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{\left(20(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\cdots\right.}{\cdots}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\cdots}{21b^{9/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.49

$$\frac{2\sqrt{a+bx}\,(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(5/4)/sqrt(a + b*x), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(1/2), x)
```

3.1644 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2}-\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}}+\frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(5/4)/b/(b*x+a)^(1/2)+10/3*d*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2+10/
3*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 230, 227}

$$\frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}+\frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2}-\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]
```

```
[Out] (10*d*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b^2) - (2*(c + d*x)^(5/4))/(b*Sqrt[
a + b*x]) + (10*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellipt
icF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/4)*Sq
rt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx}{2b}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{6b^2}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(10(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \right.}{3b^2}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{\left(10(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{3b^2\sqrt{a+bx}}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{}}{\sqrt[4]{bc}}\right.\right.}{3b^{9/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.54

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(3/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(3/2), x)
```

3.1645 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(5/4)/b/(b*x+a)^(3/2)-5/3*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(1/2)+5/
3*d*(-a*d+b*c)^(1/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-
d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 230, 227}

$$\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}-\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-5*d*(c + d*x)^(1/4))/(3*b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(3*b*(a
+ b*x)^(3/2)) + (5*d*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/
4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx}{6b}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{12b^2}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{(5d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3b^2}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{\left(5d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{5d\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{3b^{9/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.54

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/2), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/4}}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(5/2), x)
```

3.1646 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{9/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] -1/3*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(3/2)-2/5*(d*x+c)^(5/4)/b/(b*x+a)^(5/2)-1/
6*d^2*(d*x+c)^(1/4)/b^2/(-a*d+b*c)/(b*x+a)^(1/2)-1/6*d^2*EllipticF(b^(1/4)*
(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(-a
*d+b*c)^(3/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{9/4}\sqrt{a+bx}\,(bc-ad)^{3/4}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2\sqrt{a+bx}\,(bc-ad)}-\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]
```

```
[Out] -1/3*(d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(3/2)) - (d^2*(c + d*x)^(1/4))/(6*b
^2*(b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(5*b*(a + b*x)^(5/2)) -
 (d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x
)^(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(9/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x
])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx}{2b}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} + \frac{d^2\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{12b^2}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^3\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{24b^2(bc-ad)}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\cdots}}\cdots\right)}{6b^2(bc-\cdots}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{\left(d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\cdots}{6b^2\cdots}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-}\cdots\right.}{6b^{9/4}(bc-ad\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.42

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{4};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(7/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(7/2), x)
```

3.1647 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}}-\frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}}+\frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}+\frac{5d^3\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\,(}{84b^{9/4}(bc}$$

```
[Out] -1/7*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(5/2)-1/42*d^2*(d*x+c)^(1/4)/b^2/(-a*d+b*c
)/(b*x+a)^(3/2)-2/7*(d*x+c)^(5/4)/b/(b*x+a)^(7/2)+5/84*d^3*(d*x+c)^(1/4)/b^
2/(-a*d+b*c)^2/(b*x+a)^(1/2)+5/84*d^3*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d
+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(-a*d+b*c)^(7/4)/(b*x+
a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$\frac{5d^3\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{84b^{9/4}\sqrt{a+bx}\,(bc-ad)^{7/4}}+\frac{5d^3\sqrt[4]{c+dx}}{84b^2\sqrt{a+bx}\,(bc-ad)^2}-\frac{d^2\sqrt[4]{c+dx}}{42b^2(a+bx)^{3/2}(bc-ad)}-\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}}-\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]
```

```
[Out] -1/7*(d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(5/2)) - (d^2*(c + d*x)^(1/4))/(42*
b^2*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d^3*(c + d*x)^(1/4))/(84*b^2*(b*c - a
*d)^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(7*b*(a + b*x)^(7/2)) + (5*d^3*S
qrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4)
)/(b*c - a*d)^(1/4)], -1])/(84*b^(9/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx}{14b}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{d^2\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx}{28b^2}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} - \frac{(5d^3)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,d}{168b^2(bc-ad)}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.34

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{7}{2},-\frac{5}{4};-\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-7/2, -5/4, -5/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(7*b*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/2), x)
```

Fricas [**F**]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*
x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(9/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{5/4}}{{\left(a+b\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(9/2),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(9/2), x)
```

3.1648 $\int \frac{(a+bx)^{5/2}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=264

$$\frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3}-\frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d}-\frac{32(bc-ad}{}$$

```
[Out] -40/117*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d^2+4/13*(b*x+a)^(5/2)*(d*x+
c)^(3/4)/d+16/39*(-a*d+b*c)^2*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^3-32/39*(-a*d+b
*c)^(15/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/
(-a*d+b*c))^(1/2)/b^(3/4)/d^4/(b*x+a)^(1/2)+32/39*(-a*d+b*c)^(15/4)*Ellipti
cF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/
b^(3/4)/d^4/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}-\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}+\frac{16\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)^2}{39d^3}-\frac{40(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}{117d^2}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]
```

```
[Out] (16*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(39*d^3) - (40*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(117*d^2) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*d) - (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(39*b^(3
/4)*d^4*Sqrt[a + b*x]) + (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(39*b^(3/4)*d^4*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt[4]{c+dx}}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} - \frac{(10(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{13d}$$

$$= -\frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} + \frac{(20(bc-ad)^2)\int \frac{\sqrt{a}}{\sqrt[4]{c}}}{39d^2}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(}{13d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.28

$$\frac{2(a+bx)^{7/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/4), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/4), x)
```

3.1649 $\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=229

$$-\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d}+\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{}}{\sqrt[4]{bc}}\right.\right.}{15b^{3/4}d^3\sqrt{a+bx}}$$

```
[Out] 4/9*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d-8/15*(-a*d+b*c)*(d*x+c)^(3/4)*(b*x+a)^(1/
2)/d^2+16/15*(-a*d+b*c)^(11/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(
1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d^3/(b*x+a)^(1/2)-16/15*(-a*d
+b*c)^(11/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a
)/(-a*d+b*c))^(1/2)/b^(3/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{15d^2}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]
```

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*d^2) + (4*(a + b*x)^(3/2
)*(c + d*x)^(3/4))/(9*d) + (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c
 - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1
])/(15*b^(3/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b
*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(
1/4)], -1])/(15*b^(3/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{(2(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3d}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{(4(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\ldots}}{15d^2}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{(16(bc-ad)^2)\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{\left(16(bc-ad)^{5/2}\right)\mathrm{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{\left(16(bc-ad)^{5/2}\sqrt{\frac{d(a+\ldots}{-bc+\ldots}}\right.}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+\ldots}{bc-\ldots}}}{15\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+\ldots}{bc-\ldots}}}{15\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 73, normalized size = 0.32

$$\frac{2(a+bx)^{5/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/4),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/4), x)
```

3.1650 $\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=196

$$\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d}-\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}+\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d}{}}}{}$$

```
[Out] 4/5*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d-8/5*(-a*d+b*c)^(7/4)*EllipticE(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d^2/(
b*x+a)^(1/2)+8/5*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c
)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d^2/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/4),x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d) - (8*(b*c - a*d)^(7/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} - \frac{(8(bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^2}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} + \frac{\left(8(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5\sqrt{b}\,d^2} -$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} + \frac{\left(8(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,\right.}{5\sqrt{b}\,d^2\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} + \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} - \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.37

$$\frac{2(a+bx)^{3/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(1/4),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/4), x)
```

3.1651 $\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}-\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}$$

```
[Out] 4*(-a*d+b*c)^(3/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d/(b*x+a)^(1/2)-4*(-a*d+b*c)^(3/4)*Ellipt
icF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)
/b^(3/4)/d/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {65, 313, 230, 227, 1214, 1213, 435}

$$\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}-\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x])
- (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[
(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x]
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx = \frac{4\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d}$$

$$= -\frac{\left(4\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\,d} + \frac{\left(4\sqrt{bc-ad}\right)}{}$$

$$= -\frac{\left(4\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\,d\sqrt{a+bx}} +$$

$$= -\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}} + \frac{\left(4\sqrt{bc-ad}\right.}{}$$

$$= \frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}} - \frac{4(bc-ad)^{3/4}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.43

$$\frac{2\sqrt{a+bx}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(1/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/4)), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/4)), x)
```

3.1652 $\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=191

$$-\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}}+\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(1/2)+2*EllipticE(b^(1/4)*(d*x+c)^(1/4)
/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/(-a*d+b*c)^(1/4)
/(b*x+a)^(1/2)-2*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b
*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,\sqrt[4]{bc-ad}}+\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,\sqrt[4]{bc-ad}}-\frac{2(c+dx)^{3/4}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx = -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\sqrt{bc-ad}} + \text{—} \quad 2\mathrm{Su}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x,\right.}{\sqrt{b}\sqrt{bc-ad}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}} +$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}} -$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.37

$$-\frac{2\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/4)), x)
```

3.1653 $\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=224

$$-\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}}+\frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}}+\frac{d\sqrt{-\frac{d(\cdots}{b\cdots}}}{\cdots}$$

```
[Out] -2/3*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(3/2)+d*(d*x+c)^(3/4)/(-a*d+b*c)^2/(b
*x+a)^(1/2)-d*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+
a)/(-a*d+b*c))^(1/2)/b^(3/4)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)+d*EllipticF(b^(
1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4
)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{d(c+dx)^{3/4}}{\sqrt{a+bx}\,(bc-ad)^2}-\frac{2(c+dx)^{3/4}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (d*(c + d*x)^(3/4))/
((b*c - a*d)^2*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx = -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} - \frac{d\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{4(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{\sqrt{b}\,(bc-ad)^{3}}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int\right.}{\sqrt{b}\,(bc-}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\right.\right.}{b^{3/4}(bc-ad)^{5/4}}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\right.\right.}{b^{3/4}(bc-ad)^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.33

$$-\frac{2\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/4)), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}} \sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/4)), x)
```

3.1654 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=144

$$-\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d}+\frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c-}}{\sqrt[4]{bc-}}\right.\right.}{7\sqrt[4]{b}\,d^3\sqrt{a+bx}}$$

```
[Out] 4/7*(b*x+a)^(3/2)*(d*x+c)^(1/4)/d-8/7*(-a*d+b*c)*(d*x+c)^(1/4)*(b*x+a)^(1/2
)/d^2+16/7*(-a*d+b*c)^(9/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4
),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{7\sqrt[4]{b}\,d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{7d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]
```

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(7*d^2) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*d) + (16*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(7*b^(1/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx = \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} - \frac{(6(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{7d}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{(4(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,(c+d}}{7d^2}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{(16(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{\left(16(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right.}{}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{7\sqrt[4]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.51

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(3/4), x)
```

3.1655 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=111

$$\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d}-\frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,d^2\sqrt{a+bx}}$$

```
[Out] 4/3*(d*x+c)^(1/4)*(b*x+a)^(1/2)/d-8/3*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/d^2/(
b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 230, 227}

$$\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d}-\frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,d^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/4),x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*d) - (8*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(1/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx &= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3d}\\
&= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{(8(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2}\\
&= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{\left(8(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{\cdots}\right.}{3d^2\sqrt{a+bx}}\\
&= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,d^2\sqrt{a+bx}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(3/4),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

<u>**Sympy**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/4), x)
```

<u>**Giac**</u> [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(3/4), x)
```

3.1656 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{4\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,d\sqrt{a+bx}}$$

```
[Out] 4*(-a*d+b*c)^(1/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/d/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {65, 230, 227}

$$\frac{4\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,d\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(1/4)*d*Sqrt[a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx = \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d}$$

$$= \frac{\left(4\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{a+bx}}$$

$$= \frac{4\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,d\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.86

$$\frac{2\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/4)),x]
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[1/2, 3
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(3/4)),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\left(c+dx\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/4),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/4)), x)
```

3.1657 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=111

$$-\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}}-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(1/2)-2*EllipticF(b^(1/4)*(d*x+c)^(1/4)
/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/(-a*d+b*c)^(3/4)
/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$-\frac{2\sqrt[4]{c+dx}}{\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{a+bx}\,(bc-ad)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/((b*c - a*d)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx = -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x,\right.}{(bc-ad)\sqrt{a+bx}}$$

$$= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.64

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/4)), x)
```

3.1658 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}+\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}(bc-ad)^{7/4}\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(3/2)+5/3*d*(d*x+c)^(1/4)/(-a*d+b*c)^
2/(b*x+a)^(1/2)+5/3*d*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(
-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/(-a*d+b*c)^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {53, 65, 230, 227}

$$\frac{5d\sqrt[4]{c+dx}}{3\sqrt{a+bx}(bc-ad)^2}-\frac{2\sqrt[4]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}+\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\sqrt{a+bx}(bc-ad)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d*(c + d*x)^(1/4)
)/(3*(b*c - a*d)^2*Sqrt[a + b*x]) + (5*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(
1/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx = -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,d}{12(bc-ad)^2}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{(5d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+}}\right.}{3(bc-}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{\left(5d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}}{3(b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-}\right.}{3\sqrt[4]{b}\,(bc-ad}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.49

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(3/4)), x)
```

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/4)), x)
```

3.1659 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=254

$$-\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}}-\frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3}+\frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2}+\frac{32\sqrt[4]{b}\,(bc-ad)^{11/4}\sqrt{-\frac{d(a}{bc}}}{}$$

```
[Out] -4*(b*x+a)^(5/2)/d/(d*x+c)^(1/4)+40/9*b*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d^2-16/
3*b*(-a*d+b*c)*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^3+32/3*b^(1/4)*(-a*d+b*c)^(11/
4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*
c))^(1/2)/d^4/(b*x+a)^(1/2)-32/3*b^(1/4)*(-a*d+b*c)^(11/4)*EllipticF(b^(1/4
)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^4/(b*x+
a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{32\sqrt[4]{b}\,(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^4\sqrt{a+bx}}+\frac{32\sqrt[4]{b}\,(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^4\sqrt{a+bx}}-\frac{16b\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{3d^3}+\frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2}-\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/2))/(d*(c + d*x)^(1/4)) - (16*b*(b*c - a*d)*Sqrt[a + b*x]*
(c + d*x)^(3/4))/(3*d^3) + (40*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^2) +
 (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Elliptic
E[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*Sqrt[a +
 b*x]) - (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*
EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*
Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 227

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b, 4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rule 230

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

Rule 313

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]}, Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]

Rule 435

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 1213

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 1214

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} + \frac{(10b)\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{(20b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{(8b(\cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{(32b\cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{\left(32\cdots\right.}{\cdots}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{\left(32\cdots\right.}{\cdots}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{32\sqrt[4]{\cdots}}{\cdots}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{32\sqrt[4]{\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 10.05, size = 73, normalized size = 0.29

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/4), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(3/4)/(d^2*x^2 +
 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/4), x)
```

3.1660 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=220

$$-\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}}+\frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2}-\frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}$$

```
[Out] -4*(b*x+a)^(3/2)/d/(d*x+c)^(1/4)+24/5*b*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^2-48/
5*b^(1/4)*(-a*d+b*c)^(7/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4)
,I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)+48/5*b^(1/4)*(-a*d+b*c)
^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*
d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}+\frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2}-\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(d*(c + d*x)^(1/4)) + (24*b*Sqrt[a + b*x]*(c + d*x)^(3
/4))/(5*d^2) - (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*
d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5
*d^3*Sqrt[a + b*x]) + (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b
*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)],
-1])/(5*d^3*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ [m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{(6b)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} - \frac{(12b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} - \frac{(48b(bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,d\right.}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} + \frac{\left(48\sqrt{b}\,(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\right.}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} + \frac{\left(48\sqrt{b}\,(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\right.}{5d^3\sqrt{a+}}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} + \frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} - \frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\right.}{5d^3\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/4), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/4), x)
```

3.1661 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=190

$$-\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}}+\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}-\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(}{b}}$$

```
[Out] -4*(b*x+a)^(1/2)/d/(d*x+c)^(1/4)+8*b^(1/4)*(-a*d+b*c)^(3/4)*EllipticE(b^(1/
4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^2/(b*x
+a)^(1/2)-8*b^(1/4)*(-a*d+b*c)^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+
b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^2/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {49, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}+\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(d*(c + d*x)^(1/4)) + (8*b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[
-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b
*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x]) - (8*b^(1/4)*(b*c - a*d)^(3/4)*S
qrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4)
)/(b*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx = -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{(8b)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{\left(8\sqrt{b}\,\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2} + \ldots$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{\left(8\sqrt{b}\,\sqrt{bc-ad}\,\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \ldots\right)}{d^2\sqrt{a+bx}}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}} + \ldots$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}} - \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.38

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(5/4),x]')
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(5/4), x)
```

3.1662 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=197

$$\frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}-\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\sqrt{a+bx}}+\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\right.}{d\sqrt[4]{bc-ad}\sqrt{c}}$$

```
[Out] 4*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(1/4)-4*b^(1/4)*EllipticE(b^(1/4)*(d*x+c
)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^(1/4
)/(b*x+a)^(1/2)+4*b^(1/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),
I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$,
Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{4\sqrt{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}+\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/4)) - (4*b^(1/4)*Sqrt[-((d*(a +
 b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)
^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) + (4*b^(1/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx = \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{b\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{(4b)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{bc-ad}} -$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(4\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x\right.}{d\sqrt{bc-ad}\,\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\,\sqrt{a+bx}} -$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\,\sqrt{a+bx}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.36

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[1/2, 5
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/4)), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/4),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/4)), x)
```

3.1663 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=222

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}-\frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}}+\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{(bc-ad)^{5/4}\sqrt{a+bx}}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(1/4)/(b*x+a)^(1/2)-6*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/(d
*x+c)^(1/4)+6*b^(1/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(
-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)-6*b^(1/4)*Ellip
ticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2
)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{6d\sqrt{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}-\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4)) - (6*d*Sqrt[a + b*x])/((b*c
- a*d)^2*(c + d*x)^(1/4)) + (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x]) - (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{(3d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(3bd)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{}}}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(6b)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{\left(6\sqrt{b}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{\left(6\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{(bc}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{(bc}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.32

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(a+b\,x)}^{3/2}\,{(c+d\,x)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/4)), x)
```

3.1664 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=261

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}+\frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}+\frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}}-\frac{7\sqrt[4]{b}\,d\sqrt{-\frac{d(a+bx)}{bc-ad}}}{(b\cdots}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/4)+7/3*d/(-a*d+b*c)^2/(d*x+c)^(1/4
)/(b*x+a)^(1/2)+7*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/4)-7*b^(1/4)*d*
EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))
^(1/2)/(-a*d+b*c)^(9/4)/(b*x+a)^(1/2)+7*b^(1/4)*d*EllipticF(b^(1/4)*(d*x+c)
^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(9/4)/(
b*x+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{7d^2\sqrt{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{7d}{3\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}+\frac{7\sqrt[4]{b}\,d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{9/4}}-\frac{7\sqrt[4]{b}\,d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4)) + (7*d)/(3*(b*c - a*d)^2
*Sqrt[a + b*x]*(c + d*x)^(1/4)) + (7*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c +
 d*x)^(1/4)) - (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Ar
cSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4)*
Sqrt[a + b*x]) + (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4
)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 227

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b, 4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

Rule 230

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

Rule 313

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]}, Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]

Rule 435

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 1213

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sqrt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 1214

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{(7d^2)\int$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 73, normalized size = 0.28

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}\,(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(a+b\,x)}^{5/2}\,{(c+d\,x)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/4)), x)
```

3.1665 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{7/4}} \, dx$

Optimal. Leaf size=207

$$-\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}}+\frac{160b(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{33d^4}-\frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3}+\frac{56b(a+bx)^{5/2}\sqrt[4]{c}}{33d^2}$$

```
[Out] -4/3*(b*x+a)^(7/2)/d/(d*x+c)^(3/4)-80/33*b*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)
^(1/4)/d^3+56/33*b*(b*x+a)^(5/2)*(d*x+c)^(1/4)/d^2+160/33*b*(-a*d+b*c)^2*(d
*x+c)^(1/4)*(b*x+a)^(1/2)/d^4-320/33*b^(3/4)*(-a*d+b*c)^(13/4)*EllipticF(b^
(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^5/(
b*x+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 230, 227}

$$-\frac{320b^{3/4}(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{33d^5\sqrt{a+bx}}+\frac{160b\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{33d^4}-\frac{80b(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}{33d^3}+\frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2}-\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]
```

```
[Out] (-4*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/4)) + (160*b*(b*c - a*d)^2*Sqrt[a +
b*x]*(c + d*x)^(1/4))/(33*d^4) - (80*b*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x
)^(1/4))/(33*d^3) + (56*b*(a + b*x)^(5/2)*(c + d*x)^(1/4))/(33*d^2) - (320*
b^(3/4)*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(33*d^5*Sqrt[a + b*x
])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b, 4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{7/4}}\,dx = -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{(14b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/4}}\,dx}{3d}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2} - \frac{(140b(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{33d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2} + \frac{(40b}{}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.35

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[7/4,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(7/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(7/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*(d*x + c)^
(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(7/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(7/4),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(7/4), x)
```

3.1666 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=137

$$-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2}-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^3\sqrt{a+bx}}$$

```
[Out] -4/3*(b*x+a)^(3/2)/d/(d*x+c)^(3/4)+8/3*b*(d*x+c)^(1/4)*(b*x+a)^(1/2)/d^2-16
/3*b^(3/4)*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4
),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 230, 227}

$$-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^3\sqrt{a+bx}}+\frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2}-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/4)) + (8*b*Sqrt[a + b*x]*(c + d*x)^(
1/4))/(3*d^2) - (16*b^(3/4)*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
3*d^3*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}}\,dx = -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{(2b)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{(4b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{(16b(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx\right.}{3d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{\left(16b(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{3d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\right.\right.}{3d^3\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.53

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}{}_2F_1\left(\frac{7}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[7/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(7/4),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(7/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(7/4),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(7/4), x)
```

3.1667 $\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=111

$$-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}+\frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}$$

```
[Out] -4/3*(b*x+a)^(1/2)/d/(d*x+c)^(3/4)+8/3*b^(3/4)*(-a*d+b*c)^(1/4)*EllipticF(b
^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^2/
(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 230, 227}

$$\frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(7/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(3*d*(c + d*x)^(3/4)) + (8*b^(3/4)*(b*c - a*d)^(1/4)*Sqr
t[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/
(b*c - a*d)^(1/4)], -1])/(3*d^2*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}}\,dx = -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3d}$$

$$= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{(8b)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2}$$

$$= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{\left(8b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2\sqrt{a+bx}}$$

$$= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{8b^{3/4}\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}{}_2F_1\left(\frac{3}{2},\frac{7}{4};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(7/4),x]
```

[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[3/2, 7/4, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(7/4))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(7/4),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python object

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/2)/(d*x+c)^(7/4),x)

[Out] int((b*x+a)^(1/2)/(d*x+c)^(7/4),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="maxima")

[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/4), x)

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(7/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(7/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(7/4), x)
```

3.1668 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=118

$$\frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] 4/3*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(3/4)+4/3*b^(3/4)*EllipticF(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^
(3/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {53, 65, 230, 227}

$$\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d\sqrt{a+bx}\,(bc-ad)^{3/4}}+\frac{4\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(7/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/4)) + (4*b^(3/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*d*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/4}}\,dx = \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}} + \frac{b\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}} + \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}} + \frac{\left(4b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x\right.}{3d(bc-ad)\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}} + \frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d(bc-ad)^{3/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.60

$$\frac{2\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(7/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[1/2, 7
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(7/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(7/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/4)), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(7/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/4)), x)
```

3.1669 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=146

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}-\frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}}-\frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{3(bc-ad)^{7/4}\sqrt{a+bx}}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(3/4)/(b*x+a)^(1/2)-10/3*d*(b*x+a)^(1/2)/(-a*d+b*c)^2
/(d*x+c)^(3/4)-10/3*b^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4
),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$-\frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{3\sqrt{a+bx}\,(bc-ad)^{7/4}}-\frac{10d\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4)) - (10*d*Sqrt[a + b*x])/(3*(b
*c - a*d)^2*(c + d*x)^(3/4)) - (10*b^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d)
)]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*(
b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{(5d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{(5bd)\int \frac{1}{\sqrt{a+bx}\cdots}}{6(bc-a\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{(10b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{\left(10b\sqrt{\frac{d(a+bx\cdots}{-bc+a\cdots}}\right.}{\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{10b^{3/4}\sqrt{-\frac{d(a+\cdots}{bc-\cdots}}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.49

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4} {}_2F_1\left(-\frac{1}{2},\frac{7}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[-1/2, 7/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(7/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/4)), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(7/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/4)), x)
```

3.1670 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=178

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}+\frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}+\frac{5d^2\sqrt{a+bx}}{(bc-ad)^3(c+dx)^{3/4}}+\frac{5b^{3/4}d\sqrt{-\frac{d(a}{bc}}}{}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(3/4)+3*d/(-a*d+b*c)^2/(d*x+c)^(3/4)/
(b*x+a)^(1/2)+5*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(3/4)+5*b^(3/4)*d*El
lipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(
1/2)/(-a*d+b*c)^(11/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$\frac{5b^{3/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{11/4}}+\frac{5d^2\sqrt{a+bx}}{(c+dx)^{3/4}(bc-ad)^3}+\frac{3d}{\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/4)) + (3*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/4)) + (5*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c + d
*x)^(3/4)) + (5*b^(3/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(11/4)*S
qrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} - \frac{(3d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{(15d^2}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.41

$$-\frac{2 \left(\frac{b(c+dx)}{bc-ad}\right)^{7/4} {}_2F_1\left(-\frac{3}{2},\frac{7}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[-3/2, 7/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(7/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/4)), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(7/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/4)), x)
```

3.1671 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=286

$$-\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}}-\frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}}-\frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4}+\frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3}+\frac{448b^5}{}$$

```
[Out] -4/5*(b*x+a)^(7/2)/d/(d*x+c)^(5/4)-56/5*b*(b*x+a)^(5/2)/d^2/(d*x+c)^(1/4)+1
12/9*b^2*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d^3-224/15*b^2*(-a*d+b*c)*(d*x+c)^(3/4
)*(b*x+a)^(1/2)/d^4+448/15*b^(5/4)*(-a*d+b*c)^(11/4)*EllipticE(b^(1/4)*(d*x
+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^5/(b*x+a)^(1/
2)-448/15*b^(5/4)*(-a*d+b*c)^(11/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b
*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^5/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{448b^{5/4}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15d^5\sqrt{a+bx}}+\frac{448b^{5/4}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15d^5\sqrt{a+bx}}-\frac{224b^2\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{15d^4}+\frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3}-\frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}}-\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(7/2))/(5*d*(c + d*x)^(5/4)) - (56*b*(a + b*x)^(5/2))/(5*d^2*
(c + d*x)^(1/4)) - (224*b^2*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*
d^4) + (112*b^2*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^3) + (448*b^(5/4)*(b*
c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4
)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]) - (448*b
^(5/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} + \frac{(14b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} + \frac{(28b^2)\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} - \frac{(56b^2(bc-ad))\int \frac{\sqrt{a}}{\sqrt[4]{c}}}{3d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 73, normalized size = 0.26

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(9/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(9/4), x)
```

Fricas [**F**]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*(d*x + c)^
(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(9/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(9/4),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(9/4), x)
```

3.1672 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=248

$$-\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}}-\frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}}+\frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3}-\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\sqrt[4]{\right.\right.}{5d^4\sqrt{a+bx}}$$

```
[Out] -4/5*(b*x+a)^(5/2)/d/(d*x+c)^(5/4)-8*b*(b*x+a)^(3/2)/d^2/(d*x+c)^(1/4)+48/5
*b^2*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^3-96/5*b^(5/4)*(-a*d+b*c)^(7/4)*Elliptic
E(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d
^4/(b*x+a)^(1/2)+96/5*b^(5/4)*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1
/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^4/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^4\sqrt{a+bx}}-\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^4\sqrt{a+bx}}+\frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3}-\frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}}-\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(5/2))/(5*d*(c + d*x)^(5/4)) - (8*b*(a + b*x)^(3/2))/(d^2*(c
+ d*x)^(1/4)) + (48*b^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d^3) - (96*b^(5/4
)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]) + (96
*b^(5/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} + \frac{(2b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{(12b^2)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d^2}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} - \frac{(24b^2(bc-ad))\int \frac{}{\sqrt{a+}}}{5d^3}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} - \frac{(96b^2(bc-ad))\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} + \frac{\left(96b^{3/2}(bc-ad)^{3/2}\right)\mathrm{Su}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} + \frac{\Big(96b^{3/2}(bc-ad)^{3/2}\sqrt{\frac{c}{}}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} + \frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{c}{}}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} - \frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{c}{}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 10.06, size = 73, normalized size = 0.29

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(\frac{9}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(9/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(9/4), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(3/4)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(9/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(9/4),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(9/4), x)
```

3.1673 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=222

$$-\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}}-\frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}}+\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{48b^{5/4}(\cdots}{\cdots}$$

```
[Out] -4/5*(b*x+a)^(3/2)/d/(d*x+c)^(5/4)-24/5*b*(b*x+a)^(1/2)/d^2/(d*x+c)^(1/4)+4
8/5*b^(5/4)*(-a*d+b*c)^(3/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/
4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)-48/5*b^(5/4)*(-a*d+b*
c)^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-
a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {49, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}+\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}}-\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(5*d*(c + d*x)^(5/4)) - (24*b*Sqrt[a + b*x])/(5*d^2*(c
 + d*x)^(1/4)) + (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(5*d^3*Sqrt[a + b*x]) - (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(5*d^3*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} + \frac{(6b)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{(12b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{(48b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{\left(48b^{3/2}\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \right.}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{\left(48b^{3/2}\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+\frac{}{(}}}\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c-}}{\sqrt[4]{bc-}}\right.\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c-}}{\sqrt[4]{bc-}}\right.\right.}{5d^3\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(9/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(9/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(9/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(9/4),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(9/4), x)
```

3.1674 $\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=232

$$-\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}}+\frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}}-\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt[4]{bc-ad}\,\sqrt{a+bx}}+\frac{8b^{5/4}\sqrt{-\ldots}}{\ldots}$$

```
[Out] -4/5*(b*x+a)^(1/2)/d/(d*x+c)^(5/4)+8/5*b*(b*x+a)^(1/2)/d/(-a*d+b*c)/(d*x+c)
^(1/4)-8/5*b^(5/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/d^2/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)+8/5*b^(5/4)*El
lipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(
1/2)/d^2/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\,\sqrt[4]{bc-ad}}-\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\,\sqrt[4]{bc-ad}}+\frac{8b\sqrt{a+bx}}{5d\sqrt[4]{c+dx}\,(bc-ad)}-\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(9/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(5*d*(c + d*x)^(5/4)) + (8*b*Sqrt[a + b*x])/(5*d*(b*c -
a*d)*(c + d*x)^(1/4)) - (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2*(b*c
- a*d)^(1/4)*Sqrt[a + b*x]) + (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2
*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}}\,dx = -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{(2b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5d(bc-ad)}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c-\right.}{5d^2(bc-ad)}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{(8b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{\right.}{5d^2\sqrt{bc-ad}}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(8b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{b}{(a-}}}\right.}{5d^2\sqrt{bc-ad}\,\sqrt{a+b}}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{5d^2\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{5d^2\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.31

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(\frac{3}{2},\frac{9}{4};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[3/2,
 9/4, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(9/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(9/4),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(9/4), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(9/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(9/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(9/4), x)
```

3.1675 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=236

$$\frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}}+\frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}}-\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d(bc-ad)^{5/4}\sqrt{a+bx}}+\frac{12b^{5}}{}$$

```
[Out] 4/5*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(5/4)+12/5*b*(b*x+a)^(1/2)/(-a*d+b*c)^
2/(d*x+c)^(1/4)-12/5*b^(5/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/
4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)+12/5*b
^(5/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*
d+b*c))^(1/2)/d/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{12b\sqrt{a+bx}}{5\sqrt[4]{c+dx}\,(bc-ad)^2}+\frac{4\sqrt{a+bx}}{5(c+dx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(9/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/(5*(b*c - a*d)*(c + d*x)^(5/4)) + (12*b*Sqrt[a + b*x])/(5
*(b*c - a*d)^2*(c + d*x)^(1/4)) - (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x]) + (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*
c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -
1])/(5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{9/4}}\,dx = \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{(3b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx}{5(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{(3b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5(bc-ad)^2}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{(12b^2)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+}}\right.}{5d(bc-a}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(12b^{3/2})\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}}}\right.}{5d(bc-a}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(12b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}}{5d(bc-}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-}\right.}{5d(bc-ad)^{5/4}}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-}\right.}{5d(bc-ad)^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.30

$$\frac{2\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(9/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[1/2, 9
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(9/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(9/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(9/4)), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 + a*
d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(9/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(9/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(9/4)), x)
```

3.1676 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=262

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}}-\frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}}-\frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}}+\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\Big(}{5(bc-ad}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(5/4)/(b*x+a)^(1/2)-14/5*d*(b*x+a)^(1/2)/(-a*d+b*c)^2
/(d*x+c)^(5/4)-42/5*b*d*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/4)+42/5*b^(5/
4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*
c))^(1/2)/(-a*d+b*c)^(9/4)/(b*x+a)^(1/2)-42/5*b^(5/4)*EllipticF(b^(1/4)*(d*
x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(9/
4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{9/4}}+\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{9/4}}-\frac{42bd\sqrt{a+bx}}{5\sqrt[4]{c+dx}\,(bc-ad)^3}-\frac{14d\sqrt{a+bx}}{5(c+dx)^{5/4}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/4)) - (14*d*Sqrt[a + b*x])/(5*(b
*c - a*d)^2*(c + d*x)^(5/4)) - (42*b*d*Sqrt[a + b*x])/(5*(b*c - a*d)^3*(c +
 d*x)^(1/4)) + (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9/4)
*Sqrt[a + b*x]) - (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9
/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{(7d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{9/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{(21bd)\int \frac{}{\sqrt{a+}}}{10(bc-}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 71, normalized size = 0.27

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(-\frac{1}{2},\frac{9}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[-1/2, 9/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(9/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(9/4)), x)
```

Fricas [**F**]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d^3*x^5 + a^2*c^3 + (3*b^2*c*d^
2 + 2*a*b*d^3)*x^4 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 +
 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(9/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(9/4)), x)
```

3.1677 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=303

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}}+\frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}}+\frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/4}}+\frac{77bd^2\sqrt{a}}{5(bc-ad)^4}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(5/4)+11/3*d/(-a*d+b*c)^2/(d*x+c)^(5/
4)/(b*x+a)^(1/2)+77/15*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(5/4)+77/5*b*
d^2*(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(1/4)-77/5*b^(5/4)*d*EllipticE(b^(1/
4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*
c)^(13/4)/(b*x+a)^(1/2)+77/5*b^(5/4)*d*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*
d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(13/4)/(b*x+a)^(1/
2)
```

Rubi [A]

time = 0.21, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{13/4}}-\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{13/4}}+\frac{77bd^2\sqrt{a+bx}}{5\sqrt[4]{c+dx}\,(bc-ad)^4}+\frac{77d^2\sqrt{a+bx}}{15(c+dx)^{5/4}(bc-ad)^3}+\frac{11d}{3\sqrt{a+bx}\,(c+dx)^{5/4}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(5/4)) + (11*d)/(3*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(5/4)) + (77*d^2*Sqrt[a + b*x])/(15*(b*c - a*d)^3
*(c + d*x)^(5/4)) + (77*b*d^2*Sqrt[a + b*x])/(5*(b*c - a*d)^4*(c + d*x)^(1/
4)) - (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)*Sqrt[
a + b*x]) + (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)
*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{9/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} - \frac{(11d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{(77d}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.24

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(-\frac{3}{2},\frac{9}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[-3/2, 9/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(9/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(9/4)), x)
```

Fricas [**F**]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d^3*x^6 + a^3*c^3 + 3*(b^3*c*d^
2 + a*b^2*d^3)*x^5 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c
^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^3 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*x^2 + 3*(a^2*b*c^3 + a^3*c^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(9/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(a+b\,x)}^{5/2}\,{(c+d\,x)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(9/4)), x)
```

3.1678 $\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=205

$$\frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}+\frac{5(bc-ad)^3\tan}{64b}$$

```
[Out] 5/96*(-a*d+b*c)^2*(b*x+a)^(3/4)*(d*x+c)^(1/4)/b^2/d+5/24*(-a*d+b*c)*(b*x+a)
^(7/4)*(d*x+c)^(1/4)/b^2+1/3*(b*x+a)^(7/4)*(d*x+c)^(5/4)/b+5/64*(-a*d+b*c)^
3*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)/d^(7/4)-5/64*
(-a*d+b*c)^3*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)/d
^(7/4)
```

Rubi [A]
time = 0.10, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (5*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(96*b^2*d) + (5*(b*c - a*
d)*(a + b*x)^(7/4)*(c + d*x)^(1/4))/(24*b^2) + ((a + b*x)^(7/4)*(c + d*x)^(
5/4))/(3*b) + (5*(b*c - a*d)^3*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c
 + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c - a*d)^3*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx = \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int (a+bx)^{3/4}\sqrt[4]{c+dx}\,dx}{12b}$$

$$= \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad)^2)\int \frac{(a}{(c}}{96b^2}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

Mathematica [A]
time = 0.51, size = 174, normalized size = 0.85

$$\frac{(bc-ad)^3\left(\frac{2\sqrt[4]{b}\,d^{3/4}(a+bx)^{3/4}\sqrt[4]{c+dx}\,(-15a^2d^2+6abd(7c+2dx)+b^2(5c^2+52cdx+32d^2x^2))}{(bc-ad)^3}-15\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)-15\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)\right)}{192b^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^3*((2*b^(1/4)*d^(3/4)*(a + b*x)^(3/4)*(c + d*x)^(1/4)*(-15*a^2
*d^2 + 6*a*b*d*(7*c + 2*d*x) + b^2*(5*c^2 + 52*c*d*x + 32*d^2*x^2)))/(b*c -
 a*d)^3 - 15*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))] -
15*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]))/(192*b^(9
/4)*d^(7/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2151 vs. 2(159) = 318.
time = 0.42, size = 2151, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/384*(60*b^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*arc
tan(((b^10*c^3*d^5 - 3*a*b^9*c^2*d^6 + 3*a^2*b^8*c*d^7 - a^3*b^7*d^8)*(b*x
+ a)^(3/4)*(d*x + c)^(1/4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^
3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))
^(3/4) + (b^8*d^5*x + a*b^7*d^5)*sqrt(((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^
4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d
^6)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^4*x + a*b^4*d^4)*sqrt((b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
```

```
 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^
11*b*c*d^11 + a^12*d^12)/(b^9*d^7)))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^9*d^7))^(3/4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^1
0*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^
5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a
^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^
13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b
^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^
10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 15*b^2*d*((b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d^2*x + a*b^2*d^2
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4))/(b*x + a)) - 15*b
^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4
) - (b^3*d^2*x + a*b^2*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7)
)^(1/4))/(b*x + a)) - 4*(32*b^2*d^2*x^2 + 5*b^2*c^2 + 42*a*b*c*d - 15*a^2*d
^2 + 4*(13*b^2*c*d + 3*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)*(c + d*x)**(5/4), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/4}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/4)*(c + d*x)^(5/4), x)
```

3.1679 $\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal. Leaf size=167

$$\frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2}{}$$

```
[Out] 5/8*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^(1/4)/b^2+1/2*(b*x+a)^(3/4)*(d*x+c)^(5
/4)/b-5/16*(-a*d+b*c)^2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))
/b^(9/4)/d^(3/4)+5/16*(-a*d+b*c)^2*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d
*x+c)^(1/4))/b^(9/4)/d^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*b^2) + ((a + b*x)^(3/4)*
(c + d*x)^(5/4))/(2*b) - (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/
(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh
[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx = \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\int \frac{1}{\sqrt[4]{a+bx}\,(c+\ldots}}{32b^2}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\left(c-\frac{ad}{b}\ldots\right.}\right.}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\ldots\right.}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{b}-\ldots}\right.}{16b^2\sqrt{\ldots}}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a-\ldots}}{\sqrt[4]{b}\,\sqrt[4]{c+\ldots}}\right.}{16b^{9/4}d^{3/4}}$$

Mathematica [A]
time = 0.39, size = 143, normalized size = 0.86

$$\frac{(bc-ad)^2\left(\frac{2\sqrt[4]{b}\,(a+bx)^{3/4}\sqrt[4]{c+dx}\,(9bc-5ad+4bdx)}{(bc-ad)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{d^{3/4}}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{d^{3/4}}\right)}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] ((b*c - a*d)^2*((2*b^(1/4)*(a + b*x)^(3/4)*(c + d*x)^(1/4)*(9*b*c - 5*a*d +
 4*b*d*x))/(b*c - a*d)^2 + (5*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a
+ b*x)^(1/4))])/d^(3/4) + (5*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a
+ b*x)^(1/4))])/d^(3/4)))/(16*b^(9/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. $2(127) = 254$.

time = 0.38, size = 1468, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*arctan(-((b^9*c^2*d^2 - 2*a*b^8*c*d^
3 + a^2*b^7*d^4)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4) -
 (b^8*d^2*x + a*b^7*d^2)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^2*x + a*b^
4*d^2)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^9*d^3)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4))/(a*b^8*
c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^
4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d
^8 + (b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 7
0*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d
^7 + a^8*b*d^8)*x)) - 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d*x + a*b^2*d)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^9*d^3))^(1/4))/(b*x + a)) + 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b^3*d*x +
a*b^2*d)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4))/(b*x + a)) - 4*(4*b*d*x + 9*b*c - 5*a*d)
*(b*x + a)^(3/4)*(d*x + c)^(1/4))/b^2
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt[4]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(1/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(1/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(1/4), x)
```

3.1680 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}-\frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}$$

```
[Out] 5*d*(b*x+a)^(3/4)*(d*x+c)^(1/4)/b^2-4*(d*x+c)^(5/4)/b/(b*x+a)^(1/4)-5/2*d^(
1/4)*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)
+5/2*d^(1/4)*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4)
)/b^(9/4)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 65, 338, 304, 211, 214}

$$-\frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (5*d*(a + b*x)^(3/4)*(c + d*x)^(1/4))/b^2 - (4*(c + d*x)^(5/4))/(b*(a + b*x
)^(1/4)) - (5*d^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)
*(c + d*x)^(1/4))])/(2*b^(9/4)) + (5*d^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(9/4))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx = -\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{b}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4b^2}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a}\right.}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{\left(5\sqrt{d}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \right.}{2b^2}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} - \frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}} + \frac{5\sqrt[4]{d}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.04, size = 71, normalized size = 0.47

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]')
```

[Out] Timed out

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)

[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")

[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(116) = 232.
time = 0.35, size = 857, normalized size = 5.64

$$\frac{20\,(b^3x+ab^2)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{(b^8c-ab^7d)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{3}{4}}+(b^8x+ab^7)\sqrt{\frac{(b^2c^2-2abcd+a^2d^2)\sqrt{bx+a}\sqrt{dx+c}+(b^5x+ab^4)\sqrt{\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}}}{bx+a}}\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{3}{4}}}{ab^4c^4d-4a^2b^3c^3d^2+6a^3b^2c^2d^3-4a^4bcd^4+a^5d^5+(b^5c^4d-4ab^4c^3d^2+6a^2b^3c^2d^3-4a^3b^2cd^4+a^4bd^5)x}\right)+5\,(b^3x+ab^2)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}+(b^3x+ab^2)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-5\,(b^3x+ab^2)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}-(b^3x+ab^2)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)+4\,(bdx-4\,bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{4\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")

[Out] 1/4*(20*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*arctan(((b^8*c - a*b^7*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(3/4) + (b^8*x + a*b^7)*sqrt(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*x + a*b^4)*sqrt((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9))/(b*x + a))*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(3/4))/(a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3 - 4*a^4*b*c*d^4 + a^5*d^5 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-5*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/

```
4))/(b*x + a)) - 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^
2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
a)^(3/4)*(d*x + c)^(1/4) - (b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4))/(b*x + a)) + 4*(b*
d*x - 4*b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^3*x + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(5/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(5/4), x)
```

3.1681 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx$

Optimal. Leaf size=134

$$-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}$$

```
[Out] -4*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(1/4)-4/5*(d*x+c)^(5/4)/b/(b*x+a)^(5/4)-2*d^
(5/4)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)+2*d^(5/4)
*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)
```

Rubi [A]

time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 65, 338, 304, 211, 214}

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(1/4)) - (4*(c + d*x)^(5/4))/(5*b*(a
+ b*x)^(5/4)) - (2*d^(5/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d
*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/b^(9/4)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx = -\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/4}}\,dx}{b}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{b^2}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(4d^2)\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b^3}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(4d^2)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(2d^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^2} - \frac{(2$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} - \frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}} + \frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{}}{\sqrt[4]{b}\,\sqrt[4]{}}\right.}{b^{9/4}}$$

Mathematica [A]
time = 0.27, size = 122, normalized size = 0.91

$$\frac{2\left(-\frac{2\sqrt[4]{b}\sqrt[4]{c+dx}(bc+5ad+6bdx)}{(a+bx)^{5/4}} + 5d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right) + 5d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)\right)}{5b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (2*((-2*b^(1/4)*(c + d*x)^(1/4)*(b*c + 5*a*d + 6*b*d*x))/(a + b*x)^(5/4) +
5*d^(5/4)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))] + 5*d
^(5/4)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]))/(5*b^
(9/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]')
```

[Out] Timed out

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(100) = 200.
time = 0.33, size = 368, normalized size = 2.75

$$-\frac{20\left(b^{4} x^{2}+2\,a b^{3} x+a^{2} b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}} \arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}} b^{7} d\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}-(b^{8}x+ab^{7})\sqrt{\frac{\sqrt{bx+a}\,\sqrt{dx+c}\,d^{2}+(b^{5}x+ab^{4})\sqrt{\frac{d^{5}}{b^{9}}}}{bx+a}}\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}}{bd^{5}x+ad^{5}}\right)-5\left(b^{4} x^{2}+2\,a b^{3} x+a^{2} b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}} \log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}} d+(b^{3}x+ab^{2})\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+5\left(b^{4} x^{2}+2\,a b^{3} x+a^{2} b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}} \log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}} d-(b^{3}x+ab^{2})\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+4\,(6\,bdx+bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(b^{4} x^{2}+2\,a b^{3} x+a^{2} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="fricas")
[Out] -1/5*(20*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*arctan(-((b*x + a)
^(3/4)*(d*x + c)^(1/4)*b^7*d*(d^5/b^9)^(3/4) - (b^8*x + a*b^7)*sqrt((sqrt(b
*x + a)*sqrt(d*x + c)*d^2 + (b^5*x + a*b^4)*sqrt(d^5/b^9))/(b*x + a))*(d^5/
b^9)^(3/4))/(b*d^5*x + a*d^5)) - 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9
)^(1/4)*log(((b*x + a)^(3/4)*(d*x + c)^(1/4)*d + (b^3*x + a*b^2)*(d^5/b^9)^
(1/4))/(b*x + a)) + 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*log((
(b*x + a)^(3/4)*(d*x + c)^(1/4)*d - (b^3*x + a*b^2)*(d^5/b^9)^(1/4))/(b*x +
 a)) + 4*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(9/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(9/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(9/4), x)
```

3.1682 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}} dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

```
[Out] -4/9*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}} dx = -\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]')

[Out] Timed out

Maple [A]
time = 0.17, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{4(dx+c)^{\frac{9}{4}}}{9(bx+a)^{\frac{9}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/4)/(b*x+a)^(13/4),x,method=_RETURNVERBOSE)

[Out] 4/9/(b*x+a)^(9/4)*(d*x+c)^(9/4)/(a*d-b*c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="maxima")

[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(26) = 52.
time = 0.33, size = 104, normalized size = 3.25

$$-\frac{4\left(d^{2}x^{2}+2\,cdx+c^{2}\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{9\left(a^{3}bc-a^{4}d+(b^{4}c-ab^{3}d)x^{3}+3\left(ab^{3}c-a^{2}b^{2}d\right)x^{2}+3\left(a^{2}b^{2}c-a^{3}bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="fricas")

[Out] -4/9*(d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b*c - a^4*d + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c - a^3*b*d)*x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(13/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(13/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.81, size = 99, normalized size = 3.09

$$\frac{4\,c^2\,{\left(c+d\,x\right)}^{1/4}+4\,d^2\,x^2\,{\left(c+d\,x\right)}^{1/4}+8\,c\,d\,x\,{\left(c+d\,x\right)}^{1/4}}{{\left(a+b\,x\right)}^{1/4}\,\left(9\,d\,a^3+18\,d\,a^2\,b\,x-9\,c\,a^2\,b+9\,d\,a\,b^2\,x^2-18\,c\,a\,b^2\,x-9\,c\,b^3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(13/4),x)
```

```
[Out] (4*c^2*(c + d*x)^(1/4) + 4*d^2*x^2*(c + d*x)^(1/4) + 8*c*d*x*(c + d*x)^(1/4
))/((a + b*x)^(1/4)*(9*a^3*d - 9*b^3*c*x^2 - 9*a^2*b*c - 18*a*b^2*c*x + 18*
a^2*b*d*x + 9*a*b^2*d*x^2))
```

3.1683 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}$$

```
[Out] -4/13*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(13/4)+16/117*d*(d*x+c)^(9/4)/(-a*d+
b*c)^2/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(9
/4))/(117*(b*c - a*d)^2*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx = -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{13(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}$$

Mathematica [A]
time = 0.13, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{9/4}(-9bc+13ad+4bdx)}{117(bc-ad)^2(a+bx)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
[Out] (4*(c + d*x)^(9/4)*(-9*b*c + 13*a*d + 4*b*d*x))/(117*(b*c - a*d)^2*(a + b*x
)^(13/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 4961 deep
```

Maple [A]
time = 0.22, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{4(dx+c)^{\frac{9}{4}}(4bdx+13ad-9bc)}{117(bx+a)^{\frac{13}{4}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(17/4),x,method=_RETURNVERBOSE)
[Out] 4/117*(d*x+c)^(9/4)*(4*b*d*x+13*a*d-9*b*c)/(b*x+a)^(13/4)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 0.37, size = 235, normalized size = 3.56

$$\frac{4\,(4\,bd^3x^3 - 9\,bc^3 + 13\,ac^2d - (bcd^2 - 13\,ad^3)x^2 - 2\,(7\,bc^2d - 13\,acd^2)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{117\,(a^4b^2c^2 - 2\,a^5bcd + a^6d^2 + (b^6c^2 - 2\,ab^5cd + a^2b^4d^2)x^4 + 4\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)x^3 + 6\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)x^2 + 4\,(a^3b^3c^2 - 2\,a^4b^2cd + a^5bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="fricas")
```

```
[Out] 4/117*(4*b*d^3*x^3 - 9*b*c^3 + 13*a*c^2*d - (b*c*d^2 - 13*a*d^3)*x^2 - 2*(7
*b*c^2*d - 13*a*c*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^2*c^2 - 2*
a^5*b*c*d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(17/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.95, size = 178, normalized size = 2.70

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{16\,d^3\,x^3}{117\,b^2\,(a\,d-b\,c)^2}-\frac{36\,b\,c^3-52\,a\,c^2\,d}{117\,b^3\,(a\,d-b\,c)^2}+\frac{x^2\,(52\,a\,d^3-4\,b\,c\,d^2)}{117\,b^3\,(a\,d-b\,c)^2}+\frac{8\,c\,d\,x\,(13\,a\,d-7\,b\,c)}{117\,b^3\,(a\,d-b\,c)^2}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(17/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((16*d^3*x^3)/(117*b^2*(a*d - b*c)^2) - (36*b*c^3 - 52*a*c
^2*d)/(117*b^3*(a*d - b*c)^2) + (x^2*(52*a*d^3 - 4*b*c*d^2))/(117*b^3*(a*d
- b*c)^2) + (8*c*d*x*(13*a*d - 7*b*c))/(117*b^3*(a*d - b*c)^2)))/(x^3*(a +
b*x)^(1/4) + (a^3*(a + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a
^2*x*(a + b*x)^(1/4))/b^2)
```

3.1684 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}} dx$

Optimal. Leaf size=101

$$-\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}+\frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}}-\frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}$$

```
[Out] -4/17*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(17/4)+32/221*d*(d*x+c)^(9/4)/(-a*d+
b*c)^2/(b*x+a)^(13/4)-128/1989*d^2*(d*x+c)^(9/4)/(-a*d+b*c)^3/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(17*(b*c - a*d)*(a + b*x)^(17/4)) + (32*d*(c + d*x)^(9
/4))/(221*(b*c - a*d)^2*(a + b*x)^(13/4)) - (128*d^2*(c + d*x)^(9/4))/(1989
*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx = -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} - \frac{(8d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{17(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} + \frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}} + \frac{(32d^2)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{221(bc-ad)^2}$$
$$= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} + \frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}} - \frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}$$

Mathematica [A]
time = 0.16, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{9/4}\left(221d^2-\frac{306bd(c+dx)}{a+bx}+\frac{117b^2(c+dx)^2}{(a+bx)^2}\right)}{1989(bc-ad)^3(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*d^2 - (306*b*d*(c + d*x))/(a + b*x) + (117*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(1989*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 9881 deep
```

Maple [A]
time = 0.22, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{4(dx+c)^{\frac{9}{4}}\left(32b^2x^2d^2+136ab\,d^2x-72b^2cdx+221a^2d^2-306abcd+117b^2c^2\right)}{1989(bx+a)^{\frac{17}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(21/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/1989*(d*x+c)^(9/4)*(32*b^2*d^2*x^2+136*a*b*d^2*x-72*b^2*c*d*x+221*a^2*d^2
-306*a*b*c*d+117*b^2*c^2)/(b*x+a)^(17/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(83) = 166$.
time = 0.38, size = 426, normalized size = 4.22

$$-\frac{4\,(32\,b^2d^4x^4+117\,b^2c^4-306\,abc^3d+221\,a^2c^2d^2-8\,(b^2cd^3-17\,abd^4)x^3+(5\,b^2c^2d^2-34\,abcd^3+221\,a^2d^4)x^2+2\,(81\,b^2c^3d-238\,abc^2d^2+221\,a^2cd^3)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{1989\,(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^5+5\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^4+10\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x^3+10\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3)x^2+5\,(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="fricas")
```

```
[Out] -4/1989*(32*b^2*d^4*x^4 + 117*b^2*c^4 - 306*a*b*c^3*d + 221*a^2*c^2*d^2 - 8
*(b^2*c*d^3 - 17*a*b*d^4)*x^3 + (5*b^2*c^2*d^2 - 34*a*b*c*d^3 + 221*a^2*d^4
)*x^2 + 2*(81*b^2*c^3*d - 238*a*b*c^2*d^2 + 221*a^2*c*d^3)*x)*(b*x + a)^(3/
4)*(d*x + c)^(1/4)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3
 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7
*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c
^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(21/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.13, size = 268, normalized size = 2.65

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{884\,a^2\,c^2\,d^2-1224\,a\,b\,c^3\,d+468\,b^2\,c^4}{1989\,b^4\,(a\,d-b\,c)^3}+\frac{x^2\,(884\,a^2\,d^4-136\,a\,b\,c\,d^3+20\,b^2\,c^2\,d^2)}{1989\,b^4\,(a\,d-b\,c)^3}+\frac{128\,d^4\,x^4}{1989\,b^2\,(a\,d-b\,c)^3}+\frac{32\,d^3\,x^3\,(17\,a\,d-b\,c)}{1989\,b^3\,(a\,d-b\,c)^3}+\frac{8\,c\,d\,x\,(221\,a^2\,d^2-238\,a\,b\,c\,d+81\,b^2\,c^2)}{1989\,b^4\,(a\,d-b\,c)^3}\right)}{x^4\,(a+b\,x)^{1/4}+\frac{a^4\,(a+b\,x)^{1/4}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/4}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/4}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/4}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(21/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((468*b^2*c^4 + 884*a^2*c^2*d^2 - 1224*a*b*c^3*d)/(1989*b^
4*(a*d - b*c)^3) + (x^2*(884*a^2*d^4 + 20*b^2*c^2*d^2 - 136*a*b*c*d^3))/(19
89*b^4*(a*d - b*c)^3) + (128*d^4*x^4)/(1989*b^2*(a*d - b*c)^3) + (32*d^3*x^
3*(17*a*d - b*c))/(1989*b^3*(a*d - b*c)^3) + (8*c*d*x*(221*a^2*d^2 + 81*b^2
*c^2 - 238*a*b*c*d))/(1989*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/4) + (a^4
*(a + b*x)^(1/4))/b^4 + (6*a^2*x^2*(a + b*x)^(1/4))/b^2 + (4*a*x^3*(a + b*x
)^(1/4))/b + (4*a^3*x*(a + b*x)^(1/4))/b^3)
```

3.1685 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}} dx$

Optimal. Leaf size=136

$$-\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}}+\frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}}-\frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}}+\frac{512d^3(c+dx)^{9/4}}{13923(bc-ad)^4(a+bx)^{9/4}}$$

```
[Out] -4/21*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(21/4)+16/119*d*(d*x+c)^(9/4)/(-a*d+
b*c)^2/(b*x+a)^(17/4)-128/1547*d^2*(d*x+c)^(9/4)/(-a*d+b*c)^3/(b*x+a)^(13/4
)+512/13923*d^3*(d*x+c)^(9/4)/(-a*d+b*c)^4/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(21*(b*c - a*d)*(a + b*x)^(21/4)) + (16*d*(c + d*x)^(9
/4))/(119*(b*c - a*d)^2*(a + b*x)^(17/4)) - (128*d^2*(c + d*x)^(9/4))/(1547
*(b*c - a*d)^3*(a + b*x)^(13/4)) + (512*d^3*(c + d*x)^(9/4))/(13923*(b*c -
a*d)^4*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx = -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx}{7(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} + \frac{(32d^2)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{119(bc-ad)^2}$$
$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}} -$$
$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}} +$$

Mathematica [A]
time = 0.18, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{9/4}\left(-1547d^3+\frac{3213bd^2(c+dx)}{a+bx}-\frac{2457b^2d(c+dx)^2}{(a+bx)^2}+\frac{663b^3(c+dx)^3}{(a+bx)^3}\right)}{13923(bc-ad)^4(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(-1547*d^3 + (3213*b*d^2*(c + d*x))/(a + b*x) - (2457*b
^2*d*(c + d*x)^2)/(a + b*x)^2 + (663*b^3*(c + d*x)^3)/(a + b*x)^3))/(13923*
(b*c - a*d)^4*(a + b*x)^(9/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.26

method	result
gosper	$\frac{4(dx+c)^{\frac{9}{4}}\left(128b^3x^3d^3+672d^3a\,x^2b^2-288b^3c\,d^2x^2+1428a^2b\,d^3x-1512a\,b^2c\,d^2x+468b^3c^2dx+1547a^3d^3-3213a^2bc\,d^2+2457a\,b^2c^2d-66\right.}{13923(bx+a)^{\frac{21}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/4)/(b*x+a)^(25/4),x,method=_RETURNVERBOSE)

[Out] 4/13923*(d*x+c)^(9/4)*(128*b^3*d^3*x^3+672*a*b^2*d^3*x^2-288*b^3*c*d^2*x^2+1428*a^2*b*d^3*x-1512*a*b^2*c*d^2*x+468*b^3*c^2*d*x+1547*a^3*d^3-3213*a^2*b*c*d^2+2457*a*b^2*c^2*d-663*b^3*c^3)/(b*x+a)^(21/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="maxima")

[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(112) = 224$.
time = 0.46, size = 649, normalized size = 4.77

$$\frac{4\,(128\,b^3d^5x^5 - 663\,b^3c^5 + 2457\,ab^2c^4d - 3213\,a^2bc^3d^2 + 1547\,a^3c^2d^3 - 32\,(b^3cd^4 - 21\,ab^2d^5)x^4 + 4\,(5\,b^3c^2d^3 - 42\,ab^2cd^4 + 357\,a^2bd^5)x^3 - (15\,b^3c^3d^2 - 105\,ab^2c^2d^3 + 357\,a^2bcd^4 - 1547\,a^3d^5)x^2 - 2\,(429\,b^3c^4d - 1701\,ab^2c^3d^2 + 2499\,a^2bc^2d^3 - 1547\,a^3cd^4)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{13923\,(a^6b^4c^4 - 4\,a^7b^3c^3d + 6\,a^8b^2c^2d^2 - 4\,a^9bcd^3 + a^{10}d^4 + (b^{10}c^4 - 4\,ab^9c^3d + 6\,a^2b^8c^2d^2 - 4\,a^3b^7cd^3 + a^4b^6d^4)x^6 + 6\,(ab^9c^4 - 4\,a^2b^8c^3d + 6\,a^3b^7c^2d^2 - 4\,a^4b^6cd^3 + a^5b^5d^4)x^5 + 15\,(a^2b^8c^4 - 4\,a^3b^7c^3d + 6\,a^4b^6c^2d^2 - 4\,a^5b^5cd^3 + a^6b^4d^4)x^4 + 20\,(a^3b^7c^4 - 4\,a^4b^6c^3d + 6\,a^5b^5c^2d^2 - 4\,a^6b^4cd^3 + a^7b^3d^4)x^3 + 15\,(a^4b^6c^4 - 4\,a^5b^5c^3d + 6\,a^6b^4c^2d^2 - 4\,a^7b^3cd^3 + a^8b^2d^4)x^2 + 6\,(a^5b^5c^4 - 4\,a^6b^4c^3d + 6\,a^7b^3c^2d^2 - 4\,a^8b^2cd^3 + a^9bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="fricas")

[Out] 4/13923*(128*b^3*d^5*x^5 - 663*b^3*c^5 + 2457*a*b^2*c^4*d - 3213*a^2*b*c^3*d^2 + 1547*a^3*c^2*d^3 - 32*(b^3*c*d^4 - 21*a*b^2*d^5)*x^4 + 4*(5*b^3*c^2*d^3 - 42*a*b^2*c*d^4 + 357*a^2*b*d^5)*x^3 - (15*b^3*c^3*d^2 - 105*a*b^2*c^2*d^3 + 357*a^2*b*c*d^4 - 1547*a^3*d^5)*x^2 - 2*(429*b^3*c^4*d - 1701*a*b^2*c^3*d^2 + 2499*a^2*b*c^2*d^3 - 1547*a^3*c*d^4)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^6*b^4*c^4 - 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 + a^10*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^6 + 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*x^2 + 6*(a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d^4)*x)

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(25/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.36, size = 376, normalized size = 2.76

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{x^2\,\left(6188\,a^3\,d^5-1428\,a^2\,b\,c\,d^4+420\,a\,b^2\,c^2\,d^3-60\,b^3\,c^3\,d^2\right)}{13923\,b^5\,(a\,d-b\,c)^4}-\frac{-6188\,a^3\,c^2\,d^3+12852\,a^2\,b\,c^3\,d^2-9828\,a\,b^2\,c^4\,d+2652\,b^3\,c^5}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{x\,\left(12376\,a^3\,c\,d^4-19992\,a^2\,b\,c^2\,d^3+13608\,a\,b^2\,c^3\,d^2-3432\,b^3\,c^4\,d\right)}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{512\,d^5\,x^5}{13923\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^4\,x^4\,(21\,a\,d-b\,c)}{13923\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d^3\,x^3\,\left(357\,a^2\,d^2-42\,a\,b\,c\,d+5\,b^2\,c^2\right)}{13923\,b^4\,(a\,d-b\,c)^4}\right)}{x^5\,(a+b\,x)^{1/4}+\frac{a^5\,(a+b\,x)^{1/4}}{b^5}+\frac{10\,a^2\,x^3\,(a+b\,x)^{1/4}}{b^2}+\frac{10\,a^3\,x^2\,(a+b\,x)^{1/4}}{b^3}+\frac{5\,a\,x^4\,(a+b\,x)^{1/4}}{b}+\frac{5\,a^4\,x\,(a+b\,x)^{1/4}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(25/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((x^2*(6188*a^3*d^5 - 60*b^3*c^3*d^2 + 420*a*b^2*c^2*d^3 -
 1428*a^2*b*c*d^4))/(13923*b^5*(a*d - b*c)^4) - (2652*b^3*c^5 - 6188*a^3*c^
2*d^3 + 12852*a^2*b*c^3*d^2 - 9828*a*b^2*c^4*d)/(13923*b^5*(a*d - b*c)^4) +
 (x*(12376*a^3*c*d^4 - 3432*b^3*c^4*d + 13608*a*b^2*c^3*d^2 - 19992*a^2*b*c
^2*d^3))/(13923*b^5*(a*d - b*c)^4) + (512*d^5*x^5)/(13923*b^2*(a*d - b*c)^4
) + (128*d^4*x^4*(21*a*d - b*c))/(13923*b^3*(a*d - b*c)^4) + (16*d^3*x^3*(3
57*a^2*d^2 + 5*b^2*c^2 - 42*a*b*c*d))/(13923*b^4*(a*d - b*c)^4)))/(x^5*(a +
 b*x)^(1/4) + (a^5*(a + b*x)^(1/4))/b^5 + (10*a^2*x^3*(a + b*x)^(1/4))/b^2
+ (10*a^3*x^2*(a + b*x)^(1/4))/b^3 + (5*a*x^4*(a + b*x)^(1/4))/b + (5*a^4*x
*(a + b*x)^(1/4))/b^4)
```

3.1686 $\int (a+bx)^{5/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=408

$$-\frac{5(bc-ad)^3\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{84b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2}+\frac{2(a+b$$

```
[Out] -5/84*(-a*d+b*c)^3*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2/d^2+1/42*(-a*d+b*c)^2*(b
*x+a)^(5/4)*(d*x+c)^(1/4)/b^2/d+1/7*(-a*d+b*c)*(b*x+a)^(9/4)*(d*x+c)^(1/4)/
b^2+2/7*(b*x+a)^(9/4)*(d*x+c)^(5/4)/b+5/336*(-a*d+b*c)^(9/2)*((b*x+a)*(d*x+
c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a
*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/
4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+
a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1
/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b
*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a
*d+b*c))^2)^(1/2)/b^(9/4)/d^(9/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+
b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{5(bc-ad)^{9/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{168\sqrt{2}b^{9/4}d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{5\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^3}{84b^2d^2}+\frac{(a+bx)^{5/4}\sqrt[4]{c+dx}(bc-ad)^2}{42b^2d}+\frac{(a+bx)^{9/4}\sqrt[4]{c+dx}(bc-ad)}{7b^2}+\frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]
```

```
[Out] (-5*(b*c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(84*b^2*d^2) + ((b*c - a
*d)^2*(a + b*x)^(5/4)*(c + d*x)^(1/4))/(42*b^2*d) + ((b*c - a*d)*(a + b*x)^
(9/4)*(c + d*x)^(1/4))/(7*b^2) + (2*(a + b*x)^(9/4)*(c + d*x)^(5/4))/(7*b)
+ (5*(b*c - a*d)^(9/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*
x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[
(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*
x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(168*Sqrt[2]*b^(9/4)*
d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b
*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int (a+bx)^{5/4}(c+dx)^{5/4}\,dx = \frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad))\int (a+bx)^{5/4}\sqrt[4]{c+dx}\,dx}{14b}$$

$$= \frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2} + \frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b} + \frac{(bc-ad)^2\int \frac{(a+bx}{(c+dx}}{28b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2} + \frac{2(a+bx)}{}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-a}{}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-a}{}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-a}{}$$

$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.18

$$\frac{4(a+bx)^{9/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{-bc+ad}\right)}{9b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, (d*(a
 + b*x))/(-(b*c) + a*d)])/(9*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)*(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(1/4)*(d*x + c)^(1/4), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{4}} (c+dx)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)*(c + d*x)**(5/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/4}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(5/4)*(c + d*x)^(5/4), x)
```

3.1687 $\int \sqrt[4]{a+bx}\,(c+dx)^{5/4}\,dx$

Optimal. Leaf size=370

$$\frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d}+\frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2}+\frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b}-\frac{(bc-ad)^{7/2}((a+bx\cdots}{\cdots}$$

```
[Out] 1/6*(-a*d+b*c)^2*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2/d+1/3*(-a*d+b*c)*(b*x+a)^(
5/4)*(d*x+c)^(1/4)/b^2+2/5*(b*x+a)^(5/4)*(d*x+c)^(5/4)/b-1/24*(-a*d+b*c)^(7
/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))
^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b
*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))
*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c
)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*
(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/d^(5/4)/(b*x+a)^(3/4)/(d*x+c)^(
3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$-\frac{(bc-ad)^{7/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{12\sqrt{2}\,b^{9/4}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{6b^2d}+\frac{(a+bx)^{5/4}\sqrt[4]{c+dx}\,(bc-ad)}{3b^2}+\frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(6*b^2*d) + ((b*c - a*d)*(a
 + b*x)^(5/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(5/4)*(c + d*x)^(5/4)
)/(5*b) - ((b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(12*Sqrt[2]*b^(
9/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*
d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \sqrt[4]{a+bx}\,(c+dx)^{5/4}\,dx = \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} + \frac{(bc-ad)\int \sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt[4]{a+}}{(c+dx}}{12b^2}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}}{5b}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}}{5b}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}}{5b}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}}{5b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.20

$$\frac{4(a+bx)^{5/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 5/4, 9/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{4}}\,(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{a + bx} \left(c + dx\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)*(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(1/4)*(c + d*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,x\right)}^{1/4}\,{\left(c+d\,x\right)}^{5/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)*(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(1/4)*(c + d*x)^(5/4), x)
```

3.1688 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}}\,dx$

Optimal. Leaf size=332

$$\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2}+\frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b}+\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)}}{}$$

```
[Out] 5/3*(-a*d+b*c)*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2+2/3*(b*x+a)^(1/4)*(d*x+c)^(5
/4)/b+5/12*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d
^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*Ell
ipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b
*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d
+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b
^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/d^(1/4)
/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2
)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{6\sqrt{2}\,b^{9/4}\sqrt[4]{d}\,(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{5\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{3b^2}+\frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(1/4
)*(c + d*x)^(5/4))/(3*b) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqr
t[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan
[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/
2])/(6*Sqrt[2]*b^(9/4)*d^(1/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}}\,dx = \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/4}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad)^2)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3}}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad)^2((a+bx)(c+dx}{12b^2(a+bx)}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{\Big(5(bc-ad)^2((a+bx)(c+dx}{}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{5(bc-ad)^{5/2}((a+bx)(c+dx}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.21

$$\frac{4\sqrt[4]{a+bx}\,(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(1/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 1/4, 5/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="fricas")
[Out] integral((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(3/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(3/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(3/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(3/4), x)
```

3.1689 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/4}}\,dx$

Optimal. Leaf size=325

$$\frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\left(1+\frac{2\sqrt{}}{}\right.}{3\sqrt{}}$$

[Out] 10/3*d*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2-4/3*(d*x+c)^(5/4)/b/(b*x+a)^(3/4)+5/6*d^(3/4)*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 64, 637, 226}

$$\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]

[Out] (10*d*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) - (4*(c + d*x)^(5/4))/(3*b*(a + b*x)^(3/4)) + (5*d^(3/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(3*Sqrt[2]*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/4}}\,dx = -\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/4}}\,dx}{3b}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{(5d(bc-ad))\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{6b^2}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{\left(5d(bc-ad)((a+bx)(c+dx))^{3/4}\right)\int \frac{\ }{(ac+(bc+a}}{6b^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{\left(10d(bc-ad)((a+bx)(c+dx))^{3/4}\sqrt{(bc+ac}\right.}{3b}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+}}{\ }$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.22

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -3/4, 1/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(3*b*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/4), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(7/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(7/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(7/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(7/4), x)
```

3.1690 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{11/4}} \, dx$

Optimal. Leaf size=325

$$-\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{5\sqrt{2}\, d^{7/4}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\Big(1+\frac{2\sqrt{b}\cdots}{\cdots}}{21b\cdots}$$

```
[Out] -20/21*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(3/4)-4/7*(d*x+c)^(5/4)/b/(b*x+a)^(7/4)+
5/21*d^(7/4)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)
*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^
(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*ar
ctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2
*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(
1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c
)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/
4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2
)
```

Rubi [A]

time = 0.22, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 64, 637, 226}

$$\frac{5\sqrt{2}\, d^{7/4}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{21b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]
```

```
[Out] (-20*d*(c + d*x)^(1/4))/(21*b^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(7*b
*(a + b*x)^(7/4)) + (5*Sqrt[2]*d^(7/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x)
)^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2
*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*
d]], 1/2])/(21*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{11/4}}\,dx = -\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/4}}\,dx}{7b}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{(5d^2)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{21b^2}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{\left(5d^2((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{21b^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{\left(20d^2((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\cdots}{21b^2(a+bx\cdots}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}} + \frac{5\sqrt{2}\,d^{7/4}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+a\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.22

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-7/4, -5/4, -3/4, (d*(a + b*x))/(-(b*
c) + a*d)])/(7*b*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(11/4), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{11}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(11/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(11/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{11/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(11/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(11/4), x)
```

3.1691 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{15/4}}\,dx$

Optimal. Leaf size=363

$$-\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}}-\frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}}-\frac{10\sqrt{2}\,d^{11/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+a}}{}$$

```
[Out] -20/77*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(7/4)-20/231*d^2*(d*x+c)^(1/4)/b^2/(-a*d
+b*c)/(b*x+a)^(3/4)-4/11*(d*x+c)^(5/4)/b/(b*x+a)^(11/4)-10/231*d^(11/4)*((b
*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*
2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(
d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(
1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2
)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*
c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)
*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2
*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 64, 637, 226}

$$-\frac{10\sqrt{2}\,d^{11/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{20d^2\sqrt[4]{c+dx}}{231b^2(a+bx)^{3/4}(bc-ad)}-\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}}-\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]
```

```
[Out] (-20*d*(c + d*x)^(1/4))/(77*b^2*(a + b*x)^(7/4)) - (20*d^2*(c + d*x)^(1/4))
/(231*b^2*(b*c - a*d)*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(11*b*(a + b*x
)^(11/4)) - (10*Sqrt[2]*d^(11/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*
d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a
*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sq
rt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/
4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4
)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqr
t[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{15/4}}\,dx = -\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{11/4}}\,dx}{11b}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} + \frac{(5d^2)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{(10d^3)\int \frac{1}{(a+bx)^{3/4}(}}{231b^2(bc-}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{(10d^3((a+bx)(c}{231b^2(bc-}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{\left(40d^3((a+bx)(c\right.}{}$$

$$10\sqrt{2}\,d^{11/4}((a+$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.20

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{11}{4},-\frac{5}{4};-\frac{7}{4};\frac{d(a+bx)}{-bc+ad}\right)}{11b(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-11/4, -5/4, -7/4, (d*(a + b*x))/(-(b
*c) + a*d)])/(11*b*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3277 deep
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(15/4), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2
*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(15/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{15/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(15/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(15/4), x)
```

3.1692 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{19/4}}\, dx$

Optimal. Leaf size=401

$$-\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+\frac{4\sqrt{2}\,d^{15/4}((a}{}$$

```
[Out] -4/33*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(11/4)-4/231*d^2*(d*x+c)^(1/4)/b^2/(-a*d+
b*c)/(b*x+a)^(7/4)+8/231*d^3*(d*x+c)^(1/4)/b^2/(-a*d+b*c)^2/(b*x+a)^(3/4)-4
/15*(d*x+c)^(5/4)/b/(b*x+a)^(15/4)+4/231*d^(15/4)*((b*x+a)*(d*x+c))^(3/4)*(
cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/
2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/
(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))
^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)
*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2
*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+
b*c))^2)^(1/2)/b^(9/4)/(-a*d+b*c)^(3/2)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*
x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 64, 637, 226}

$$\frac{4\sqrt{2}\,d^{15/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(a+bx)^{3/4}(bc-ad)^2}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(a+bx)^{7/4}(bc-ad)}-\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(33*b^2*(a + b*x)^(11/4)) - (4*d^2*(c + d*x)^(1/4))/
(231*b^2*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d^3*(c + d*x)^(1/4))/(231*b^2*(b
*c - a*d)^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(15*b*(a + b*x)^(15/4))
+ (4*Sqrt[2]*d^(15/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x
)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(
a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*
((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4)*(b*c - a*
d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d +
b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{19/4}}\,dx = -\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx}{3b}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}} + \frac{d^2\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx}{33b^2}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} - \frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}} - \frac{(2d^3)\int \frac{1}{(a+bx)^{7/4}(c+}}{77b^2(bc-a}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c+}{15b(a-}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c+}{15b(a-}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c+}{15b(a-}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c+}{15b(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.18

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{15}{4},-\frac{5}{4};-\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{15b(a+bx)^{15/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-15/4, -5/4, -11/4, (d*(a + b*x))/(-(
b*c) + a*d)])/(15*b*(a + b*x)^(15/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 7141 deep
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(19/4), x)
```

Fricas [F]
time = 0.39, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^
3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(19/4),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{19/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(19/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(19/4), x)
```

3.1693 $\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$-\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)}{}$$

```
[Out] -5/8*(-a*d+b*c)*(b*x+a)^(1/4)*(d*x+c)^(3/4)/d^2+1/2*(b*x+a)^(5/4)*(d*x+c)^(
3/4)/d+5/16*(-a*d+b*c)^2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4)
)/b^(3/4)/d^(9/4)+5/16*(-a*d+b*c)^2*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(
d*x+c)^(1/4))/b^(3/4)/d^(9/4)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 246, 218, 214, 211}

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}\,(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(3/4))/(8*d^2) + ((a + b*x)^(5/4)
*(c + d*x)^(3/4))/(2*d) + (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))
/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTan
h[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx = \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{8d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{\cdots}}}{32d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{1-\cdots}\right.}{8bd^{\cdots}}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \cdots\right.}{16\cdots}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\cdots}}{\sqrt[4]{b}\,\sqrt[4]{\cdots}}\right.}{16b^{3/4}d^{9/4}}$$

Mathematica [A]
time = 0.34, size = 143, normalized size = 0.86

$$\frac{2\sqrt[4]{d}\,\sqrt[4]{a+bx}\,(c+dx)^{3/4}(-5bc+9ad+4bdx) + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}} + \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}}}{16d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (2*d^(1/4)*(a + b*x)^(1/4)*(c + d*x)^(3/4)*(-5*b*c + 9*a*d + 4*b*d*x) + (5*
(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
b^(3/4) + (5*(b*c - a*d)^2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c +
d*x)^(1/4))])/b^(3/4))/(16*d^(9/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]')

[Out] Timed out

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)

[Out] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")

[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. 2(127) = 254.
time = 0.38, size = 1468, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")

[Out] -1/32*(20*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*arctan(-((b^4*c^2*d^7 - 2*a*b^3*c*d^8 + a^2*b^2*d^9)*(b*x + a)^(1/4)*(d*x + c)^(3/4)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4) - (b^2*d^8*x + b^2*c*d^7)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^5*x + b^2*c*d^4)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9)))/(d*x + c))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4))/(b^8*c^

```
9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^
5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d
^8 + (b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4
 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c
*d^8 + a^8*d^9)*x)) - 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^3*x + b*c*d^2)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^3*d^9))^(1/4))/(d*x + c)) + 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) - (b*d^3*x +
b*c*d^2)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4))/(d*x + c)) - 4*(4*b*d*x - 5*b*c + 9*a*d)
*(b*x + a)^(1/4)*(d*x + c)^(3/4))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/4}}{{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(5/4)/(c + d*x)^(1/4), x)
```

3.1694 $\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}$$

```
[Out] (b*x+a)^(1/4)*(d*x+c)^(3/4)/d-1/2*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b
^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(5/4)-1/2*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a
)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(5/4)
```

Rubi [A]

time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 246, 218, 214, 211}

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((a + b*x)^(1/4)*(c + d*x)^(3/4))/d - ((b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x
)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/4)) - ((b*c - a*d)*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/
4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2\sqrt{b}\,d} - \frac{(bc-ad)\text{Subs}}{}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+}}{\sqrt[4]{b}\,\sqrt[4]{c+}}\right.}{2b^{3/4}d^{5/4}}$$

Mathematica [A]
time = 6.51, size = 129, normalized size = 1.02

$$\frac{(a+bx)^{5/4}\left(2b^{3/4}\sqrt[4]{d(a+bx)}\,(c+dx)^{3/4}+(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d(a+bx)}}\right)+(-bc+ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d(a+bx)}}\right)\right)}{2b^{3/4}(d(a+bx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((a + b*x)^(5/4)*(2*b^(3/4)*(d*(a + b*x))^(1/4)*(c + d*x)^(3/4) + (b*c - a*
d)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d*(a + b*x))^(1/4)] + (-(b*c) + a*d)*A
rcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d*(a + b*x))^(1/4)]))/(2*b^(3/4)*(d*(a +
b*x))^(5/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(95) = 190$.
time = 0.34, size = 814, normalized size = 6.41

$$-\frac{4\,d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{(b^3cd^4-ab^2d^5)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{3}{4}}+(b^2d^5x+b^2cd^4)\sqrt{\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bx+a}\,\sqrt{dx+c}+(b^2d^3x+b^2cd^2)\sqrt{\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}}}{dx+c}}\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{3}{4}}}{b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4+(b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5)x}\right)+d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}\log\left(-\frac{(bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}+(bd^2x+bcd)\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}}{dx+c}\right)-d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}\log\left(-\frac{(bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}-(bd^2x+bcd)\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}}{dx+c}\right)-4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b^3*d^5))^(1/4)*arctan(((b^3*c*d^4 - a*b^2*d^5)*(b*x + a)^(1/4)*(d
*x + c)^(3/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(b^3*d^5))^(3/4) + (b^2*d^5*x + b^2*c*d^4)*sqrt(((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^3*x + b^2*c*d^2)*sqr
t((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
b^3*d^5)))/(d*x + c))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(3/4))/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*
c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4 + (b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*x)) + d*((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c
- a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/
(d*x + c)) - d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(
3/4) - (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/(d*x + c)) - 4*(b*x + a)^(1/4)*(d*
x + c)^(3/4))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a + bx}}{\sqrt[4]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(1/4), x)
```

3.1695 $\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

```
[Out] 2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(1/4)+2*arc
tanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(1/4)
```

Rubi [A]

time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {65, 246, 218, 214, 211}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx = \frac{4\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$

$$= \frac{4\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a}}{\sqrt[4]{c}}\right.}{\sqrt{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Mathematica [A]
time = 0.08, size = 73, normalized size = 0.86

$$\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
[Out] (2*(ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))] + ArcTanh[(
d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))]))/(b^(3/4)*d^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}} (dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(61) = 122.
time = 0.32, size = 234, normalized size = 2.75

$$-4\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}b^2d\left(\frac{1}{b^3d}\right)^{\frac{3}{4}}-(b^2d^2x+b^2cd)\sqrt{\frac{(b^2dx+b^2c)\sqrt{\frac{1}{b^3d}}+\sqrt{bx+a}\,\sqrt{dx+c}}{dx+c}}\left(\frac{1}{b^3d}\right)^{\frac{3}{4}}}{dx+c}\right)+\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)-\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] -4*(1/(b^3*d))^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*b^2*d*(1/(b^3
*d))^(3/4) - (b^2*d^2*x + b^2*c*d)*sqrt(((b^2*d*x + b^2*c)*sqrt(1/(b^3*d))
+ sqrt(b*x + a)*sqrt(d*x + c))/(d*x + c))*(1/(b^3*d))^(3/4))/(d*x + c)) + (
1/(b^3*d))^(1/4)*log(((b*d*x + b*c)*(1/(b^3*d))^(1/4) + (b*x + a)^(1/4)*(d*
```

```
x + c)^(3/4))/(d*x + c)) - (1/(b^3*d))^(1/4)*log(-((b*d*x + b*c)*(1/(b^3*d)
)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{3/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)), x)
```

3.1696 $\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

```
[Out] -4/3*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.19, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}}{3(bx+a)^{\frac{3}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/3/(b*x+a)^(3/4)*(d*x+c)^(3/4)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

Fricas [A]
time = 0.33, size = 42, normalized size = 1.31

$$-\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\,(abc-a^2d+(b^2c-abd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] -4/3*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{(a+b\,x)}^{7/4}\,{(c+d\,x)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)), x)
```

3.1697 $\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=66

$$-\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}}+\frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}$$

```
[Out] -4/7*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(7/4)+16/21*d*(d*x+c)^(3/4)/(-a*d+b*c
)^2/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (16*d*(c + d*x)^(3/4
))/(21*(b*c - a*d)^2*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{7(bc-ad)}$$
$$= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}} + \frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{3/4}(-3bc+7ad+4bdx)}{21(bc-ad)^2(a+bx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
[Out] (4*(c + d*x)^(3/4)*(-3*b*c + 7*a*d + 4*b*d*x))/(21*(b*c - a*d)^2*(a + b*x)^
(7/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.22, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}(4bdx+7ad-3bc)}{21(bx+a)^{\frac{7}{4}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/21*(d*x+c)^(3/4)*(4*b*d*x+7*a*d-3*b*c)/(b*x+a)^(7/4)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.42, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx-3\,bc+7\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{21\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(4*b*d*x - 3*b*c + 7*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(a+b\,x)}^{11/4}\,{(c+d\,x)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)), x)
```

3.1698 $\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}+\frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}}-\frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+bx)^{3/4}}$$

```
[Out] -4/11*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(11/4)+32/77*d*(d*x+c)^(3/4)/(-a*d+b
*c)^2/(b*x+a)^(7/4)-128/231*d^2*(d*x+c)^(3/4)/(-a*d+b*c)^3/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(11*(b*c - a*d)*(a + b*x)^(11/4)) + (32*d*(c + d*x)^(3
/4))/(77*(b*c - a*d)^2*(a + b*x)^(7/4)) - (128*d^2*(c + d*x)^(3/4))/(231*(b
*c - a*d)^3*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} - \frac{(8d)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} + \frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c-}}}{77(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} + \frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}} - \frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+ba}$$

Mathematica [**A**]
time = 0.13, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{11/4}\left(21b^2+\frac{77d^2(a+bx)^2}{(c+dx)^2}-\frac{66bd(a+bx)}{c+dx}\right)}{231(bc-ad)^3(a+bx)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(21*b^2 + (77*d^2*(a + b*x)^2)/(c + d*x)^2 - (66*b*d*(
a + b*x))/(c + d*x)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Maple [**A**]
time = 0.21, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}\left(32b^2x^2d^2+88ab\,d^2x-24b^2cdx+77a^2d^2-66abcd+21b^2c^2\right)}{231(bx+a)^{\frac{11}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/231*(d*x+c)^(3/4)*(32*b^2*d^2*x^2+88*a*b*d^2*x-24*b^2*c*d*x+77*a^2*d^2-66
*a*b*c*d+21*b^2*c^2)/(b*x+a)^(11/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(83) = 166$.
time = 0.65, size = 252, normalized size = 2.50

$$-\frac{4\,(32\,b^2d^2x^2 + 21\,b^2c^2 - 66\,abcd + 77\,a^2d^2 - 8\,(3\,b^2cd - 11\,abd^2)x)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{231\,(a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3 + (b^6c^3 - 3\,ab^5c^2d + 3\,a^2b^4cd^2 - a^3b^3d^3)x^3 + 3\,(ab^5c^3 - 3\,a^2b^4c^2d + 3\,a^3b^3cd^2 - a^4b^2d^3)x^2 + 3\,(a^2b^4c^3 - 3\,a^3b^3c^2d + 3\,a^4b^2cd^2 - a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/231*(32*b^2*d^2*x^2 + 21*b^2*c^2 - 66*a*b*c*d + 77*a^2*d^2 - 8*(3*b^2*c*
d - 11*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^3*c^3 - 3*a^4*b^2
*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d
^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5
*b*d^3)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(1/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{15/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)), x)
```

3.1699 $\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=136

$$-\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}}+\frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}}-\frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}}+\frac{512d^3(c+dx)^{3/4}}{1155(bc-ad)^4(a+bx)^{3/4}}$$

```
[Out] -4/15*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(15/4)+16/55*d*(d*x+c)^(3/4)/(-a*d+b
*c)^2/(b*x+a)^(11/4)-128/385*d^2*(d*x+c)^(3/4)/(-a*d+b*c)^3/(b*x+a)^(7/4)+5
12/1155*d^3*(d*x+c)^(3/4)/(-a*d+b*c)^4/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(15*(b*c - a*d)*(a + b*x)^(15/4)) + (16*d*(c + d*x)^(3
/4))/(55*(b*c - a*d)^2*(a + b*x)^(11/4)) - (128*d^2*(c + d*x)^(3/4))/(385*(
b*c - a*d)^3*(a + b*x)^(7/4)) + (512*d^3*(c + d*x)^(3/4))/(1155*(b*c - a*d)
^4*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c}}}{55(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/}}{385(bc-ad)^3(a+b}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/}}{385(bc-ad)^3(a+b}$$

Mathematica [A]
time = 0.16, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{15/4}\left(77b^3-\frac{385d^3(a+bx)^3}{(c+dx)^3}+\frac{495bd^2(a+bx)^2}{(c+dx)^2}-\frac{315b^2d(a+bx)}{c+dx}\right)}{1155(bc-ad)^4(a+bx)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
[Out] (-4*(c + d*x)^(15/4)*(77*b^3 - (385*d^3*(a + b*x)^3)/(c + d*x)^3 + (495*b*d
^2*(a + b*x)^2)/(c + d*x)^2 - (315*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
- a*d)^4*(a + b*x)^(15/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 7772 deep
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$\frac{4(dx+c)^{\frac{3}{4}}\left(128b^3x^3d^3+480d^3a\,x^2b^2-96b^3c\,d^2x^2+660a^2b\,d^3x-360a\,b^2c\,d^2x+84b^3c^2dx+385a^3d^3-495a^2bc\,d^2+315a\,b^2c^2d-77b^3c^3\right)}{1155(bx+a)^{\frac{15}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/1155*(d*x+c)^(3/4)*(128*b^3*d^3*x^3+480*a*b^2*d^3*x^2-96*b^3*c*d^2*x^2+66
0*a^2*b*d^3*x-360*a*b^2*c*d^2*x+84*b^3*c^2*d*x+385*a^3*d^3-495*a^2*b*c*d^2+
315*a*b^2*c^2*d-77*b^3*c^3)/(b*x+a)^(15/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(112) = 224$.
time = 1.26, size = 419, normalized size = 3.08

$$\frac{4\,(128\,b^3d^3x^3-77\,b^3c^3+315\,ab^2c^2d-495\,a^2bcd^2+385\,a^3d^3-96\,(b^3cd^2-5\,ab^2d^3)x^2+12\,(7\,b^3c^2d-30\,ab^2cd^2+55\,a^2bd^3)x)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{1155\,(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^4+4\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^3+6\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x^2+4\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(128*b^3*d^3*x^3 - 77*b^3*c^3 + 315*a*b^2*c^2*d - 495*a^2*b*c*d^2 +
385*a^3*d^3 - 96*(b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 12*(7*b^3*c^2*d - 30*a*b^2
*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(19/4)/(d*x+c)**(1/4),x)
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{19/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)), x)
```

3.1700 $\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}} \, dx$

Optimal. Leaf size=751

$$-\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2}+\frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d}+\frac{7(bc-ad)\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad}}{10\sqrt{b}\,d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)}$$

```
[Out] -7/15*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^(3/4)/d^2+2/5*(b*x+a)^(7/4)*(d*x+c)^
(3/4)/d+7/10*(-a*d+b*c)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)
*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(5/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a
*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))-7/
20*(-a*d+b*c)^(7/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*(
(b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(s
in(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2
))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*(
(2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d
^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(11/4)/(b*x+a
)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
+7/40*(-a*d+b*c)^(7/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4
)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*Elliptic
F(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(
1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c)
)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2
)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(11/4)/(b*
x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1
/2)
```

Rubi [A]

time = 0.56, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 64, 637, 311, 226, 1210}

$$\frac{7(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{20\sqrt{2}\,b^{3/4}d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{7(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{10\sqrt{2}\,b^{3/4}d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{7(bc-ad)\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{10\sqrt{b}\,d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{7(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)}{15d^2}+\frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(15*d^2) + (2*(a + b*x)^(7
/4)*(c + d*x)^(3/4))/(5*d) + (7*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[
```

```
(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*Sqrt[b]*d^(5/2)
*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (7*(b*c - a*d)^(7/2)*((a + b
*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c
- a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)
]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/
Sqrt[b*c - a*d]], 1/2])/(10*Sqrt[2]*b^(3/4)*d^(11/4)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (7*(b*c -
a*d)^(7/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (
2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c
+ 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*
(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*b^(3/4)*d^(11/4)*(a +
 b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx = \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{10d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{(7(bc-ad)^2)\int \frac{}{\sqrt[4]{a+bx}}}{20d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\sqrt[4]{(a+bx)}\right.}{20}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\sqrt[4]{(a+bx)}\right.}{}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^3\sqrt[4]{(a+bx)}\right.}{}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{7(bc-ad)\sqrt{(a+b}}{10\sqrt{b}\,d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{11/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{-bc+ad}\right)}{11b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(1/4),x)
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(7/4)/(c + d*x)^(1/4), x)
```

3.1701 $\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=705

$$\frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d}-\frac{\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}-$$

```
[Out] 2/3*(b*x+a)^(3/4)*(d*x+c)^(3/4)/d-((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c
)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(3/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/
(2*b*d*x+a*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*
d+b*c))+1/2*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*El
lipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+
b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*
d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*
b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(7/4
)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^
2)^(1/2)-1/4*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)
*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*E
llipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d
+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a
*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2
*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(7/
4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))
^2)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 705, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 64, 637, 311, 226, 1210}

$$-\frac{(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\,b^{3/4}d^{7/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,b^{3/4}d^{7/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(3*d) - (Sqrt[(a + b*x)*(c + d*x)]*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(Sqrt[b]*d^(3/2)*(
```

```
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d
]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + ((b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4
)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - ((b*c - a*d)^(5/2)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(2*Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx = \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{2d}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left((bc-ad)\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}}{2d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left(2(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{d\sqrt[4]{a+bx}\sqrt[4]{c+}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left((bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}}{}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{7/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Fricas [**F**]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(1/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(3/4)/(c + d*x)^(1/4), x)
```

3.1702 $\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=688

$$\frac{2\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\sqrt{d}(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}-\frac{\sqrt{2}(bc-ad)^{3/2}}{}$$

```
[Out] 2*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)
^(1/2)/(-a*d+b*c)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/b^(1/2)/d^(
1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))+1/2*(-a*d+b*c
)^(3/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x
+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)
*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(
b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1
/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d
+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x
+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(3/4)/(b*x+a)^(1/4)/(d*x+
c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)-(-a*d+b*c)^(
3/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c)
)^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2)
)*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*
x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*
((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(3/4)/(b*x+a)^(1/4)/
(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 688, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {64, 637, 311, 226, 1210}

$$\frac{(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}b^{3/4}d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{\sqrt{2}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{b^{3/4}d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\sqrt{d}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])/(Sqrt[b]*Sqrt[d]*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)
*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
```

```
b*c - a*d))) - (Sqrt[2]*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*S
qrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt
[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b
^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(
a*d + b*(c + 2*d*x))^2]) + ((b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*S
qrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[
b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(
Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2]
)/(Sqrt[2]*b^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b
*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{(a+bx)(c+dx)}\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

$$= \frac{\left(4\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2 -}}\right.}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)}$$

$$= \frac{\left(2(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd+(b}}\right.}{\sqrt{b}\,\sqrt{d}\,\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2}$$

$$= \frac{2\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,\sqrt{d}\,(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(}}{bc-ad}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.11

$$\frac{4(a+bx)^{3/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(1/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(1/4)), x)
```

3.1703 $\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=718

$$-\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}}+\frac{4\sqrt{d}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right.}$$

```
[Out] -4*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(1/4)+4*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)
*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^2/(b*x+
a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x
+a)*(d*x+c))^(1/2)/(-a*d+b*c))-2*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arc
tan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c
)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^
(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)
*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((
a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2
)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c
)/((a*d+b*(2*d*x+c))^2)^(1/2)+d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan
(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(
1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/
2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^
(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d
+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(
-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(
(a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$\frac{\sqrt{2}\sqrt[4]{d}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{2\sqrt{2}\sqrt[4]{d}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4(c+dx)^{3/4}}{\sqrt[4]{a+bx}(bc-ad)}+\frac{4\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^2(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/((b*c - a*d)*(a + b*x)^(1/4)) + (4*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

```
/(Sqrt[b]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*
x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (2*Sq
rt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d)
)*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[
(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*
d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(3/4)*(a +
b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))
^2]) + (Sqrt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2
]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(
3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(2d\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(8d\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{(bc-ad)\sqrt[4]{a+b}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(4\sqrt{d}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Sub}}{\sqrt{b}\sqrt[4]{a+bx}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{4\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx}}{\sqrt{b}(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.10

$$-\frac{4\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/4)*(c + d*x)^(1/4)), x)
```

3.1704 $\int \frac{1}{(a+bx)^{9/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=760

$$-\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}}+\frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}}-\frac{8d^{3/2}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(}}{5\sqrt{b}(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{}}{}\right.}$$

```
[Out] -4/5*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(5/4)+8/5*d*(d*x+c)^(3/4)/(-a*d+b*c)^
2/(b*x+a)^(1/4)-8/5*d^(3/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(
1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(
2*b*d*x+a*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d
+b*c))+4/5*d^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(si
n(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)
)),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+
b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^
(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)^
(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2
)^(1/2)-2/5*d^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*(
(b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(s
in(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2
))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d
+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b
^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)
^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^
2)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$-\frac{2\sqrt{2}d^{5/4}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{4\sqrt{2}d^{5/4}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{8d^{3/2}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{5\sqrt{b}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^3(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{8d(c+dx)^{3/4}}{5\sqrt[4]{a+bx}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (8*d*(c + d*x)^(3/4)
)/(5*(b*c - a*d)^2*(a + b*x)^(1/4)) - (8*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*
```

```
Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*Sqrt[b]*(b*
c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (4*Sqrt[2]*d^(5/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*S
qrt[2]*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b
*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b
*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a
 + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} - \frac{(2d)\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{(4d^2)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,d}{5(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(4d^2\sqrt[4]{(a+bx)(c+dx)}\right)\int}{5(bc-ad)^2\sqrt[4]{}}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(16d^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{}\right.}{}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(8d^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{}\right.}{}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{8d^{3/2}\sqrt{(a+bx)(c}}{5\sqrt{b}\,(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 73, normalized size = 0.10

$$-\frac{4\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/4}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(1/4)), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(1/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{9/4}\,{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(9/4)*(c + d*x)^(1/4)), x)
```

3.1705 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=167

$$-\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad}{}$$

[Out] -7/8*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^(1/4)/d^2+1/2*(b*x+a)^(7/4)*(d*x+c)^(1/4)/d-21/16*(-a*d+b*c)^2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(11/4)+21/16*(-a*d+b*c)^2*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(11/4)

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8d^2}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]

[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*d^2) + ((a + b*x)^(7/4)*(c + d*x)^(1/4))/(2*d) - (21*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4)) + (21*(b*c - a*d)^2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4))

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\int \frac{1}{\sqrt[4]{a+bx}\,(\cdots}}{32d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{(c\cdots}\right.}{8\cdots}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{1-\cdots}\right.}{8bd^2\cdots}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{\cdots}}\right.}{16\cdots}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{\cdots}}{\sqrt[4]{b}\,\sqrt[4]{\cdots}}\right.}{16\sqrt[4]{b}\,d^{11/4}}$$

Mathematica [A]
time = 0.24, size = 143, normalized size = 0.86

$$\frac{2d^{3/4}(a+bx)^{3/4}\sqrt[4]{c+dx}\,(-7bc+11ad+4bdx) + \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}} + \frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}}}{16d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (2*d^(3/4)*(a + b*x)^(3/4)*(c + d*x)^(1/4)*(-7*b*c + 11*a*d + 4*b*d*x) + (2
1*(b*c - a*d)^2*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]
)/b^(1/4) + (21*(b*c - a*d)^2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a
 + b*x)^(1/4))])/b^(1/4))/(16*d^(11/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]')
```

[Out] Timed out

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(127) = 254$.
time = 0.37, size = 1457, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/32*(84*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*arctan(-((b^3*c^2*d^8 - 2*a*b^2*c*d^9
 + a^2*b*d^10)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4) - (b
^2*d^8*x + a*b*d^8)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^6*x + a*d^6)*sqrt
((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(b*d^11)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4))/(a*b^8*c^8 - 8*a^2*b
^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 5
6*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8 + (b^9*c^8
```

```
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8)*x)) - 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b*d^3*x + a*d^3)*((b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^
(1/4))/(b*x + a)) + 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^3*x + a*d^3)*((b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(b*d^11))^(1/4))/(b*x + a)) - 4*(4*b*d*x - 7*b*c + 11*a*d)*(b*x + a)^(3/4)
*(d*x + c)^(1/4))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(7/4)/(c + d*x)^(3/4), x)
```

3.1706 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=127

$$\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}+\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}$$

```
[Out] (b*x+a)^(3/4)*(d*x+c)^(1/4)/d+3/2*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b
^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(7/4)-3/2*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a
)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(7/4)
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {52, 65, 338, 304, 211, 214}

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((a + b*x)^(3/4)*(c + d*x)^(1/4))/d + (3*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b
*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)
*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d
^(7/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx &= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4d} \\
&= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd} \\
&= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd} \\
&= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^{3/2}} + \frac{(3(bc-
\\
&= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} + \frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}} - \frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{}}{\sqrt[4]{b}\,\sqrt[4]{}}\right.}{2\sqrt[4]{b}\,d^{7/4}}
\end{aligned}$$

Mathematica [A]
time = 7.41, size = 131, normalized size = 1.03

$$\frac{(a+bx)^{7/4}\left(2\sqrt[4]{b}\,(d(a+bx))^{3/4}\sqrt[4]{c+dx}+(-3bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d(a+bx)}}\right)+(-3bc+3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d(a+bx)}}\right)\right)}{2\sqrt[4]{b}\,(d(a+bx))^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((a + b*x)^(7/4)*(2*b^(1/4)*(d*(a + b*x))^(3/4)*(c + d*x)^(1/4) + (-3*b*c +
 3*a*d)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d*(a + b*x))^(1/4)] + (-3*b*c + 3
*a*d)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d*(a + b*x))^(1/4)]))/(2*b^(1/4)*(
d*(a + b*x))^(7/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(95) = 190$.
time = 0.34, size = 808, normalized size = 6.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/4*(12*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(1/4)*arctan(((b^2*c*d^5 - a*b*d^6)*(b*x + a)^(3/4)*(d*x
+ c)^(1/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(3/4) + (b^2*d^5*x + a*b*d^5)*sqrt(((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^4*x + a*d^4)*sqrt((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7)))/(b*x
 + a))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)/(b*d^7))^(3/4))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a
^4*b*c*d^3 + a^5*d^4 + (b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4)*x)) + 3*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a
)^(3/4)*(d*x + c)^(1/4) + (b*d^2*x + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 3*d*
((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b
*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^2*x
 + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 4*(b*x + a)^(3/4)*(d*x + c)^(1/4))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(3/4)/(c + d*x)^(3/4), x)
```

3.1707 $\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

```
[Out] -2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(3/4)+2*ar
ctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(3/4)
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {65, 338, 304, 211, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(1/4)*d
^(3/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
(b^(1/4)*d^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx &= \frac{4\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}\\ &= \frac{4\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+}}{\sqrt[4]{c+}}\right.}{\sqrt{d}}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 73, normalized size = 0.86

$$\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (2*(ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))] + ArcTanh[(
b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]))/(b^(1/4)*d^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}\,(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(61) = 122$.
time = 0.31, size = 234, normalized size = 2.75

$$-4\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}bd^2\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}-(b^2d^2x+abd^2)\sqrt{\frac{(bd^2x+ad^2)\sqrt{\frac{1}{bd^3}}+\sqrt{bx+a}\,\sqrt{dx+c}}{bx+a}}\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}}{bx+a}\right)+\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(\frac{(bdx+ad)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}+(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{bx+a}\right)-\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(-\frac{(bdx+ad)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}-(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] -4*(1/(b*d^3))^(1/4)*arctan(-((b*x + a)^(3/4)*(d*x + c)^(1/4)*b*d^2*(1/(b*d
^3))^(3/4) - (b^2*d^2*x + a*b*d^2)*sqrt(((b*d^2*x + a*d^2)*sqrt(1/(b*d^3))
+ sqrt(b*x + a)*sqrt(d*x + c))/(b*x + a))*(1/(b*d^3))^(3/4))/(b*x + a)) + (
1/(b*d^3))^(1/4)*log(((b*d*x + a*d)*(1/(b*d^3))^(1/4) + (b*x + a)^(3/4)*(d*
```

```
x + c)^(1/4))/(b*x + a)) - (1/(b*d^3))^(1/4)*log(-((b*d*x + a*d)*(1/(b*d^3)
)^(1/4) - (b*x + a)^(3/4)*(d*x + c)^(1/4))/(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(3/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)), x)
```

3.1708 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

```
[Out] -4*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
[Out] 4/(b*x+a)^(1/4)*(d*x+c)^(1/4)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

Fricas [A]
time = 0.29, size = 42, normalized size = 1.40

$$-\frac{4\,(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] -4*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(3/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.71, size = 26, normalized size = 0.87

$$\frac{4\,{\left(c+d\,x\right)}^{1/4}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x)
```

```
[Out] (4*(c + d*x)^(1/4))/((a*d - b*c)*(a + b*x)^(1/4))
```

3.1709 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}}+\frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}$$

```
[Out] -4/5*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(5/4)+16/5*d*(d*x+c)^(1/4)/(-a*d+b*c)
^2/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}\,(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (16*d*(c + d*x)^(1/4
))/(5*(b*c - a*d)^2*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{5(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}} + \frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{4\sqrt[4]{c+dx}\,(-bc+5ad+4bdx)}{5(bc-ad)^2(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
[Out] (4*(c + d*x)^(1/4)*(-(b*c) + 5*a*d + 4*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(
5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]')
[Out] Timed out
```

Maple [A]
time = 0.19, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}(4bdx+5ad-bc)}{5(bx+a)^{\frac{5}{4}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
[Out] 4/5*(d*x+c)^(1/4)*(4*b*d*x+5*a*d-b*c)/(b*x+a)^(5/4)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.29, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx - bc + 5\,ad)(bx + a)^{\frac{3}{4}}(dx + c)^{\frac{1}{4}}}{5\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(4*b*d*x - b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{9}{4}}\,(c + dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.87, size = 71, normalized size = 1.08

$$\frac{\left(\frac{16\,d\,x}{5\,(a\,d-b\,c)^2} + \frac{20\,a\,d-4\,b\,c}{5\,b\,(a\,d-b\,c)^2}\right)\,(c + d\,x)^{1/4}}{x\,(a + b\,x)^{1/4} + \frac{a\,(a+b\,x)^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x)
```

```
[Out] (((16*d*x)/(5*(a*d - b*c)^2) + (20*a*d - 4*b*c)/(5*b*(a*d - b*c)^2))*(c + d
*x)^(1/4))/(x*(a + b*x)^(1/4) + (a*(a + b*x)^(1/4))/b)
```

3.1710 $\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}+\frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}}-\frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}$$

```
[Out] -4/9*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(9/4)+32/45*d*(d*x+c)^(1/4)/(-a*d+b*c
)^2/(b*x+a)^(5/4)-128/45*d^2*(d*x+c)^(1/4)/(-a*d+b*c)^3/(b*x+a)^(1/4)
```

Rubi [A]

time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}\,(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(9*(b*c - a*d)*(a + b*x)^(9/4)) + (32*d*(c + d*x)^(1/4
))/(45*(b*c - a*d)^2*(a + b*x)^(5/4)) - (128*d^2*(c + d*x)^(1/4))/(45*(b*c
- a*d)^3*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{9(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} + \frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{5/4}(c+dx}$$
$$= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} + \frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}} - \frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.74

$$-\frac{4\sqrt[4]{c+dx}\,\left(45a^2d^2-18abd(c-4dx)+b^2\left(5c^2-8cdx+32d^2x^2\right)\right)}{45(bc-ad)^3(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(45*a^2*d^2 - 18*a*b*d*(c - 4*d*x) + b^2*(5*c^2 - 8*c*d
*x + 32*d^2*x^2)))/(45*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}\left(32b^2x^2d^2+72ab\,d^2x-8b^2cdx+45a^2d^2-18abcd+5b^2c^2\right)}{45(bx+a)^{\frac{9}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/45*(d*x+c)^(1/4)*(32*b^2*d^2*x^2+72*a*b*d^2*x-8*b^2*c*d*x+45*a^2*d^2-18*a
*b*c*d+5*b^2*c^2)/(b*x+a)^(9/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(83) = 166$.
time = 0.30, size = 251, normalized size = 2.49

$$-\frac{4\,(32\,b^2d^2x^2+5\,b^2c^2-18\,abcd+45\,a^2d^2-8\,(b^2cd-9\,abd^2)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{45\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/45*(32*b^2*d^2*x^2 + 5*b^2*c^2 - 18*a*b*c*d + 45*a^2*d^2 - 8*(b^2*c*d -
9*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*
d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b
^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^
3)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.02, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{128\,d^2\,x^2}{45\,(a\,d-b\,c)^3}+\frac{180\,a^2\,d^2-72\,a\,b\,c\,d+20\,b^2\,c^2}{45\,b^2\,(a\,d-b\,c)^3}+\frac{32\,d\,x\,(9\,a\,d-b\,c)}{45\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/4}+\frac{a^2\,(a+b\,x)^{1/4}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((128*d^2*x^2)/(45*(a*d - b*c)^3) + (180*a^2*d^2 + 20*b^2*
c^2 - 72*a*b*c*d)/(45*b^2*(a*d - b*c)^3) + (32*d*x*(9*a*d - b*c))/(45*b*(a*
d - b*c)^3)))/(x^2*(a + b*x)^(1/4) + (a^2*(a + b*x)^(1/4))/b^2 + (2*a*x*(a
+ b*x)^(1/4))/b)
```

3.1711 $\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}}-\frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}}+\frac{512d^3\sqrt[4]{c+dx}}{195(bc-ad)^4\sqrt[4]{a+bx}}$$

```
[Out] -4/13*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(13/4)+16/39*d*(d*x+c)^(1/4)/(-a*d+b
*c)^2/(b*x+a)^(9/4)-128/195*d^2*(d*x+c)^(1/4)/(-a*d+b*c)^3/(b*x+a)^(5/4)+51
2/195*d^3*(d*x+c)^(1/4)/(-a*d+b*c)^4/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}\,(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(1
/4))/(39*(b*c - a*d)^2*(a + b*x)^(9/4)) - (128*d^2*(c + d*x)^(1/4))/(195*(b
*c - a*d)^3*(a + b*x)^(5/4)) + (512*d^3*(c + d*x)^(1/4))/(195*(b*c - a*d)^4
*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(12d)\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx}{13(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{9/4}(c+}}{39(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+}$$
$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+}$$

Mathematica [A]
time = 0.13, size = 93, normalized size = 0.68

$$-\frac{4\sqrt[4]{c+dx}\,\left(-195d^3(a+bx)^3+117bd^2(a+bx)^2(c+dx)-65b^2d(a+bx)(c+dx)^2+15b^3(c+dx)^3\right)}{195(bc-ad)^4(a+bx)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(-195*d^3*(a + b*x)^3 + 117*b*d^2*(a + b*x)^2*(c + d*x)
 - 65*b^2*d*(a + b*x)*(c + d*x)^2 + 15*b^3*(c + d*x)^3))/(195*(b*c - a*d)^4
*(a + b*x)^(13/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Maple [A]
time = 0.18, size = 171, normalized size = 1.26

method	result
gosper	$\frac{4(dx+c)^{\frac{1}{4}}\left(128b^3x^3d^3+416d^3a\,x^2b^2-32b^3c\,d^2x^2+468a^2b\,d^3x-104a\,b^2c\,d^2x+20b^3c^2dx+195a^3d^3-117a^2bc\,d^2+65a\,b^2c^2d-15b^3c^3\right)}{195(bx+a)^{\frac{13}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/195*(d*x+c)^(1/4)*(128*b^3*d^3*x^3+416*a*b^2*d^3*x^2-32*b^3*c*d^2*x^2+468
*a^2*b*d^3*x-104*a*b^2*c*d^2*x+20*b^3*c^2*d*x+195*a^3*d^3-117*a^2*b*c*d^2+6
5*a*b^2*c^2*d-15*b^3*c^3)/(b*x+a)^(13/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(112) = 224.
time = 0.31, size = 419, normalized size = 3.08

$$\frac{4\left(128\,b^3d^3x^3-15\,b^3c^3+65\,ab^2c^2d-117\,a^2bcd^2+195\,a^3d^3-32\,(b^3cd^2-13\,ab^2d^3)x^2+4\,(5\,b^3c^2d-26\,ab^2cd^2+117\,a^2bd^3)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{195\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^4+4\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^3+6\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x^2+4\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/195*(128*b^3*d^3*x^3 - 15*b^3*c^3 + 65*a*b^2*c^2*d - 117*a^2*b*c*d^2 + 19
5*a^3*d^3 - 32*(b^3*c*d^2 - 13*a*b^2*d^3)*x^2 + 4*(5*b^3*c^2*d - 26*a*b^2*c
*d^2 + 117*a^2*b*d^3)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^4*c^4 - 4*a
^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a
*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*
b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d
^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*
c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2
*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(17/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.26, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{512\,d^3\,x^3}{195\,(a\,d-b\,c)^4}+\frac{780\,a^3\,d^3-468\,a^2\,b\,c\,d^2+260\,a\,b^2\,c^2\,d-60\,b^3\,c^3}{195\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d\,x\,(117\,a^2\,d^2-26\,a\,b\,c\,d+5\,b^2\,c^2)}{195\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^2\,x^2\,(13\,a\,d-b\,c)}{195\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((512*d^3*x^3)/(195*(a*d - b*c)^4) + (780*a^3*d^3 - 60*b^3
*c^3 + 260*a*b^2*c^2*d - 468*a^2*b*c*d^2)/(195*b^3*(a*d - b*c)^4) + (16*d*x
*(117*a^2*d^2 + 5*b^2*c^2 - 26*a*b*c*d))/(195*b^2*(a*d - b*c)^4) + (128*d^2
*x^2*(13*a*d - b*c))/(195*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/4) + (a^3*(a
 + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a^2*x*(a + b*x)^(1/4)
)/b^2)
```

3.1712 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=332

$$-\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2}+\frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d}+\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bd}}{}$$

```
[Out] -5/3*(-a*d+b*c)*(b*x+a)^(1/4)*(d*x+c)^(1/4)/d^2+2/3*(b*x+a)^(5/4)*(d*x+c)^(
1/4)/d+5/12*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*El
lipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+
b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*
d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*
b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)/d^(9/4
)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^
2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{6\sqrt{2}\sqrt[4]{b}\,d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{5\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)}{3d^2}+\frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*d^2) + (2*(a + b*x)^(5/
4)*(c + d*x)^(1/4))/(3*d) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(6*Sqrt[2]*b^(1/4)*d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d
+ 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx = \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{(5(bc-ad)^2)\int \frac{1}{(a+bx)^{3/4}(c+d}}{12d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{(5(bc-ad)^2((a+bx)(c+}{12d^2(a+b}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{\Big(5(bc-ad)^2((a+bx)(c+}{}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{5(bc-ad)^{5/2}((a+bx)(c+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.22

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(3/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(5/4)/(c + d*x)^(3/4), x)
```

3.1713 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=295

$$\frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d}-\frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+a}}{bc-ad}\right.}{\sqrt{2}\,\sqrt[4]{b}\,d^{5/4}(a+bx)^{3/4}(c}$$

[Out] 2*(b*x+a)^(1/4)*(d*x+c)^(1/4)/d-1/2*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)/d^(5/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d}-\frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt[4]{b}\,d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]

[Out] (2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/d - ((b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[2]*b^(1/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx = \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{2d}$$

$$= \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{\left((bc-ad)((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{2d(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{\left(2(bc-ad)((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\Big(}{d(a+bx)^{3/4}(c+dx}$$

$$= \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\Big(1+\frac{2\sqrt{}}{}}{\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 73, normalized size = 0.25

$$\frac{4(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
[Out] int((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(3/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(3/4), x)
```

3.1714 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2}\,\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)\sqrt{\frac{\quad}{(bc-ad)^2}}}{\sqrt[4]{b}\,\sqrt[4]{d}\,(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2b}$$

```
[Out] ((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/
4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a
)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(
1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*
c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1
/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)/d^(1/4)/(b
*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {64, 637, 226}

$$\frac{\sqrt{2}\,\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{d}\,(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*
d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqr
t[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/
4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(1/4)*d^(1/4)*(a
 + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx = \frac{((a+bx)(c+dx))^{3/4}\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{\left(4((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+}\right.}{(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bc}$$

$$= \frac{\sqrt{2}\,\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(}}{}\right.}{\sqrt[4]{b}\,\sqrt[4]{d}\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.26

$$\frac{4\sqrt[4]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[1/4,
 3/4, 5/4, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}} (dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}} (c+dx)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/4}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(3/4)), x)
```

3.1715 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=306

$$-\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}}-\frac{2\sqrt{2}\,d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+\cdots}}{bc-ad}\cdots\right.}{3\sqrt[4]{b}\sqrt{bc-ad}\,(a+bx)^{3\cdots}}$$

```
[Out] -4/3*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(3/4)-2/3*d^(3/4)*((b*x+a)*(d*x+c))^(
3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*
c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^
(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d
*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d
^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*
d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/
(-a*d+b*c))^2)^(1/2)/b^(1/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/
(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 226}

$$-\frac{2\sqrt{2}\,d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/4)) - (2*Sqrt[2]*d^(3/4)*(
(a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/
((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*
d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(
1/4))/Sqrt[b*c - a*d]], 1/2])/(3*b^(1/4)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c
 + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{3(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{\left(2d((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{3(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{\left(8d((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right) \mathrm{S}}{3(bc-ad)(a+}$$

$$= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{2\sqrt{2}\,d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.24

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/4}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/4)*(c + d*x)^(3/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}} (dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(3/4)), x)
```

Fricas [**F**]
time = 0.30, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(3/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{7/4}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(3/4)), x)
```

3.1716 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=339

$$-\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}}+\frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}}+\frac{4\sqrt{2}\,d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\Big(1+\cdots}{7\cdots}$$

```
[Out] -4/7*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(7/4)+8/7*d*(d*x+c)^(1/4)/(-a*d+b*c)^
2/(b*x+a)^(3/4)+4/7*d^(7/4)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d
^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*Ell
ipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b
*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/
2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^
2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)
/(-a*d+b*c)^(3/2)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*
d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 226}

$$\frac{4\sqrt{2}\,d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{8d\sqrt[4]{c+dx}}{7(a+bx)^{3/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d*(c + d*x)^(1/4)
)/(7*(b*c - a*d)^2*(a + b*x)^(3/4)) + (4*Sqrt[2]*d^(7/4)*((a + b*x)*(c + d*
x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF
[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c -
a*d]], 1/2])/(7*b^(1/4)*(b*c - a*d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(
b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{(4d^2)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\ldots}{7(bc-ad)^2}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{(4d^2((a+bx)(c+dx))\ldots}{7(bc-ad)^2(a\ldots}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{\left(16d^2((a+bx)(c+dx\ldots\right.}{\ldots}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{4\sqrt{2}\,d^{7/4}((a+bx)(c+\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.22

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/4}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(7*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{4}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/4}\,{\left(c+d\,x\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(3/4)), x)
```

3.1717 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}$$

```
[Out] -4*(b*x+a)^(5/4)/d/(d*x+c)^(1/4)+5*b*(b*x+a)^(1/4)*(d*x+c)^(3/4)/d^2-5/2*b^
(1/4)*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/d^(9/4
)-5/2*b^(1/4)*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4
))/d^(9/4)
```

Rubi [A]

time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 65, 246, 218, 214, 211}

$$-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/4))/(d*(c + d*x)^(1/4)) + (5*b*(a + b*x)^(1/4)*(c + d*x)^(
3/4))/d^2 - (5*b^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/(2*d^(9/4)) - (5*b^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*d^(9/4))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{(5b)\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5b(bc-ad))\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x\right.}{d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{\left(5\sqrt{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x\right.}{2d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}} - \frac{5\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.05, size = 73, normalized size = 0.48

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]')

[Out] Timed out

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)

[Out] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")

[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(116) = 232.
time = 0.38, size = 857, normalized size = 5.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")

[Out] -1/4*(20*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*arctan(((b*c*d^7 - a*d^8)*(b*x + a)^(1/4)*(d*x + c)^(3/4)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(3/4) + (d^8*x + c*d^7)*sqrt(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (d^5*x + c*d^4)*sqrt((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9))/(d*x + c))*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(3/4))/(b^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(-5*((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (d^3*x + c*d^2)*((b^5*c^4

```
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1
/4))/(d*x + c)) - 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
 a)^(1/4)*(d*x + c)^(3/4) - (d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4))/(d*x + c)) - 4*(b
*d*x + 5*b*c - 4*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(5/4)/(c + d*x)^(5/4), x)
```

3.1718 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=108

$$-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}$$

```
[Out] -4*(b*x+a)^(1/4)/d/(d*x+c)^(1/4)+2*b^(1/4)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(
1/4)/(d*x+c)^(1/4))/d^(5/4)+2*b^(1/4)*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)
/(d*x+c)^(1/4))/d^(5/4)
```

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 65, 246, 218, 214, 211}

$$\frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx &= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{b\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx,x,\sqrt[4]{a+bx}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}+\frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 105, normalized size = 0.97

$$\frac{2\left(-\frac{2\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}+\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)+\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)\right)}{d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
[Out] (2*((-2*d^(1/4)*(a + b*x)^(1/4))/(c + d*x)^(1/4) + b^(1/4)*ArcTan[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))] + b^(1/4)*ArcTanh[(d^(1/4)*(a +
 b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))]))/d^(5/4)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]')
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(80) = 160$.
time = 0.31, size = 273, normalized size = 2.53

$$-\frac{4\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}d^{4}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}-\left(d^{5}x+cd^{4}\right)\sqrt{\frac{\left(d^{3}x+cd^{2}\right)\sqrt{\frac{b}{d^{5}}}+\sqrt{bx+a}\sqrt{dx+c}}{dx+c}}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}}{bdx+bc}\right)-\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(-\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -(4*(d^2*x + c*d)*(b/d^5)^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*d^
4*(b/d^5)^(3/4) - (d^5*x + c*d^4)*sqrt(((d^3*x + c*d^2)*sqrt(b/d^5) + sqrt(
b*x + a)*sqrt(d*x + c))/(d*x + c))*(b/d^5)^(3/4))/(b*d*x + b*c)) - (d^2*x +
 c*d)*(b/d^5)^(1/4)*log(((d^2*x + c*d)*(b/d^5)^(1/4) + (b*x + a)^(1/4)*(d*x
 + c)^(3/4))/(d*x + c)) + (d^2*x + c*d)*(b/d^5)^(1/4)*log(-((d^2*x + c*d)*(
b/d^5)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c)) + 4*(b*x + a)^(1
/4)*(d*x + c)^(3/4))/(d^2*x + c*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(5/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(5/4), x)
```

3.1719 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=30

$$\frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

```
[Out] 4*(b*x+a)^(1/4)/(-a*d+b*c)/(d*x+c)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx = \frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**A**]
time = 0.21, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
[Out] -4*(b*x+a)^(1/4)/(d*x+c)^(1/4)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

Fricas [**A**]
time = 0.30, size = 42, normalized size = 1.40

$$\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] 4*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(5/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)), x)
```

3.1720 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}} dx$

Optimal. Leaf size=66

$$-\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}$$

```
[Out] -4/3/(-a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)-16/3*d*(b*x+a)^(1/4)/(-a*d+b*c)
^2/(d*x+c)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(3*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4)) - (16*d*(a + b*x)^(1/4))
/(3*(b*c - a*d)^2*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx = -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx}{3(bc-ad)}$$
$$= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}} - \frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.68

$$-\frac{4(3ad+b(c+4dx))}{3(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(3*a*d + b*(c + 4*d*x)))/(3*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/
4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.18, size = 53, normalized size = 0.80

method	result	size
gosper	$-\frac{4(4bdx+3ad+bc)}{3(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(4*b*d*x+3*a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.
time = 0.30, size = 126, normalized size = 1.91

$$-\frac{4\,(4\,bdx+bc+3\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] -4/3*(4*b*d*x + b*c + 3*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(5/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)), x)
```

3.1721 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}+\frac{128d^2\sqrt[4]{a+bx}}{21(bc-ad)^3\sqrt[4]{c+dx}}$$

```
[Out] -4/7/(-a*d+b*c)/(b*x+a)^(7/4)/(d*x+c)^(1/4)+32/21*d/(-a*d+b*c)^2/(b*x+a)^(3
/4)/(d*x+c)^(1/4)+128/21*d^2*(b*x+a)^(1/4)/(-a*d+b*c)^3/(d*x+c)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(7*(b*c - a*d)*(a + b*x)^(7/4)*(c + d*x)^(1/4)) + (32*d)/(21*(b*c - a*d)
^2*(a + b*x)^(3/4)*(c + d*x)^(1/4)) + (128*d^2*(a + b*x)^(1/4))/(21*(b*c -
a*d)^3*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx = -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx}{7(bc-ad)}$$
$$= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}} + \frac{(32c}{}$$
$$= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}} + \frac{1}{21(b}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{7/4}\left(3b^2 - \frac{21d^2(a+bx)^2}{(c+dx)^2} - \frac{14bd(a+bx)}{c+dx}\right)}{21(bc-ad)^3(a+bx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(7/4)*(3*b^2 - (21*d^2*(a + b*x)^2)/(c + d*x)^2 - (14*b*d*(a
+ b*x))/(c + d*x)))/(21*(b*c - a*d)^3*(a + b*x)^(7/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.19, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{4\left(32b^2x^2d^2+56ab\,d^2x+8b^2cdx+21a^2d^2+14abcd-3b^2c^2\right)}{21(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4/21*(32*b^2*d^2*x^2+56*a*b*d^2*x+8*b^2*c*d*x+21*a^2*d^2+14*a*b*c*d-3*b^2*
c^2)/(b*x+a)^(7/4)/(d*x+c)^(1/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(83) = 166.
time = 0.42, size = 273, normalized size = 2.70

$$\frac{4\,(32\,b^2d^2x^2 - 3\,b^2c^2 + 14\,abcd + 21\,a^2d^2 + 8\,(b^2cd + 7\,abd^2)x)(bx + a)^{\frac{1}{4}}(dx + c)^{\frac{3}{4}}}{21\,(a^2b^3c^4 - 3\,a^3b^2c^3d + 3\,a^4bc^2d^2 - a^5cd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^3 + (b^5c^4 - ab^4c^3d - 3\,a^2b^3c^2d^2 + 5\,a^3b^2cd^3 - 2\,a^4bd^4)x^2 + (2\,ab^4c^4 - 5\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 + a^4bcd^3 - a^5d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4/21*(32*b^2*d^2*x^2 - 3*b^2*c^2 + 14*a*b*c*d + 21*a^2*d^2 + 8*(b^2*c*d + 7
*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c
*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^
3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{11}{4}}\,(c + dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(5/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)), x)
```

3.1722 $\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}}+\frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}}-\frac{128d^2}{77(bc-ad)^3(a+bx)^{3/4}\sqrt[4]{c+dx}}-$$

```
[Out] -4/11/(-a*d+b*c)/(b*x+a)^(11/4)/(d*x+c)^(1/4)+48/77*d/(-a*d+b*c)^2/(b*x+a)^
(7/4)/(d*x+c)^(1/4)-128/77*d^2/(-a*d+b*c)^3/(b*x+a)^(3/4)/(d*x+c)^(1/4)-512
/77*d^3*(b*x+a)^(1/4)/(-a*d+b*c)^4/(d*x+c)^(1/4)
```

Rubi [A]

time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}\,(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(11*(b*c - a*d)*(a + b*x)^(11/4)*(c + d*x)^(1/4)) + (48*d)/(77*(b*c - a*
d)^2*(a + b*x)^(7/4)*(c + d*x)^(1/4)) - (128*d^2)/(77*(b*c - a*d)^3*(a + b*
x)^(3/4)*(c + d*x)^(1/4)) - (512*d^3*(a + b*x)^(1/4))/(77*(b*c - a*d)^4*(c
+ d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx = -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} - \frac{(12d)\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx}{11(bc-ad)}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{(9}{}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{}{77}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{}{77}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{11/4}\left(7b^3+\frac{77d^3(a+bx)^3}{(c+dx)^3}+\frac{77bd^2(a+bx)^2}{(c+dx)^2}-\frac{33b^2d(a+bx)}{c+dx}\right)}{77(bc-ad)^4(a+bx)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(7*b^3 + (77*d^3*(a + b*x)^3)/(c + d*x)^3 + (77*b*d^2*
(a + b*x)^2)/(c + d*x)^2 - (33*b^2*d*(a + b*x))/(c + d*x)))/(77*(b*c - a*d)
^4*(a + b*x)^(11/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5458 deep
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result	size
gosper	$-\frac{4\left(128 b^{3} x^{3} d^{3}+352 d^{3} a \,x^{2} b^{2}+32 b^{3} c \,d^{2} x^{2}+308 a^{2} b \,d^{3} x +88 a \,b^{2} c \,d^{2} x -12 b^{3} c^{2} d x +77 a^{3} d^{3}+77 a^{2} b c \,d^{2}-33 a \,b^{2} c^{2} d +7 b^{3} c^{3}\right)}{77 \left(b x +a \right)^{\frac{11}{4}} \left(d x +c \right)^{\frac{1}{4}} \left(a^{4} d^{4}-4 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a \,b^{3} c^{3} d +b^{4} c^{4}\right)}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4/77*(128*b^3*d^3*x^3+352*a*b^2*d^3*x^2+32*b^3*c*d^2*x^2+308*a^2*b*d^3*x+8
8*a*b^2*c*d^2*x-12*b^3*c^2*d*x+77*a^3*d^3+77*a^2*b*c*d^2-33*a*b^2*c^2*d+7*b
^3*c^3)/(b*x+a)^(11/4)/(d*x+c)^(1/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(112) = 224$.
time = 0.62, size = 457, normalized size = 3.36

$$-\frac{4\left(128\,b^{3}d^{3}x^{3}+7\,b^{3}c^{3}-33\,ab^{2}c^{2}d+77\,a^{2}bcd^{2}+77\,a^{3}d^{3}+32\left(b^{3}cd^{2}+11\,ab^{2}d^{3}\right)x^{2}-4\left(3\,b^{3}c^{2}d-22\,ab^{2}cd^{2}-77\,a^{2}bd^{3}\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{77\left(a^{3}b^{4}c^{5}-4\,a^{4}b^{3}c^{4}d+6\,a^{5}b^{2}c^{3}d^{2}-4\,a^{6}bc^{2}d^{3}+a^{7}cd^{4}+\left(b^{7}c^{4}d-4\,ab^{6}c^{3}d^{2}+6\,a^{2}b^{5}c^{2}d^{3}-4\,a^{3}b^{4}cd^{4}+a^{4}b^{3}d^{5}\right)x^{4}+\left(b^{7}c^{5}-ab^{6}c^{4}d-6\,a^{2}b^{5}c^{3}d^{2}+14\,a^{3}b^{4}c^{2}d^{3}-11\,a^{4}b^{3}cd^{4}+3\,a^{5}b^{2}d^{5}\right)x^{3}+3\left(ab^{6}c^{5}-3\,a^{2}b^{5}c^{4}d+2\,a^{3}b^{4}c^{3}d^{2}+2\,a^{4}b^{3}c^{2}d^{3}-3\,a^{5}b^{2}cd^{4}+a^{6}bd^{5}\right)x^{2}+\left(3\,a^{2}b^{5}c^{5}-11\,a^{3}b^{4}c^{4}d+14\,a^{4}b^{3}c^{3}d^{2}-6\,a^{5}b^{2}c^{2}d^{3}-a^{6}bcd^{4}+a^{7}d^{5}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/77*(128*b^3*d^3*x^3 + 7*b^3*c^3 - 33*a*b^2*c^2*d + 77*a^2*b*c*d^2 + 77*a
^3*d^3 + 32*(b^3*c*d^2 + 11*a*b^2*d^3)*x^2 - 4*(3*b^3*c^2*d - 22*a*b^2*c*d^
2 - 77*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^4*c^5 - 4*a^4*b
^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4
*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (
b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^
3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c
^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^
4 + a^7*d^5)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(5/4),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{15/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)), x)
```

3.1723 $\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=776

$$-\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}}-\frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3}+\frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2}+\frac{77\sqrt{b}\,(bc-ad)\sqrt{(a+bx}}{10d^{7/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

```
[Out] -4*(b*x+a)^(11/4)/d/(d*x+c)^(1/4)-77/15*b*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^
(3/4)/d^3+22/5*b*(b*x+a)^(7/4)*(d*x+c)^(3/4)/d^2+77/10*(-a*d+b*c)*b^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1
/2)/d^(7/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1
/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))-77/20*b^(1/4)*(-a*d+b*c)^(7/2)*((b*
x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2
^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d
*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1
/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^
(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/
2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))
^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(15/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a
*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)+77/40*b^(1/4)*(-a*d+b*c)^(7/2)*
((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/
4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a
)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+
2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)
^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x
+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(15/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d
*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 64, 637, 311, 226, 1210}

$$\frac{77\sqrt{b}\,(bc-ad)\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{10d^{7/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{77\sqrt[4]{b}\,(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{20\sqrt{2}\,d^{15/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{77\sqrt[4]{b}\,(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{10\sqrt{2}\,d^{15/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{77b(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)}{15d^3}+\frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2}-\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(11/4))/(d*(c + d*x)^(1/4)) - (77*b*(b*c - a*d)*(a + b*x)^(3/
4)*(c + d*x)^(3/4))/(15*d^3) + (22*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))/(5*d^
```

```
2) + (77*Sqrt[b]*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*
b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*d^(7/2)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))) - (77*b^(1/4)*(b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))
^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)
*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
(2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*
ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d
]], 1/2])/(10*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (77*b^(1/4)*(b*c - a*d)^(7/2)*((
a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/(
(b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d
))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1
/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)
^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} + \frac{(11b)\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} - \frac{(77b(bc-ad))\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{10d^2}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{(7\cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{(7\cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{(7\cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{(7\cdots}{\cdots}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \frac{77\cdots}{10\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.09

$$\frac{4(a+bx)^{15/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{19}{4};\frac{d(a+bx)}{-bc+ad}\right)}{15b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(15/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4
, 15/4, 19/4, (d*(a + b*x))/(-(b*c) + a*d)])/(15*b*(c + d*x)^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{11}{4}}}{(dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(11/4)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(3/4)*(d*x + c)^(3/4)/(d^2*x^2
 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(11/4)/(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{11/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(11/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(11/4)/(c + d*x)^(5/4), x)
```

3.1724 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=730

$$-\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}}+\frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2}-\frac{7\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c}{d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+b}{bc-ad}\right.}$$

```
[Out] -4*(b*x+a)^(7/4)/d/(d*x+c)^(1/4)+14/3*b*(b*x+a)^(3/4)*(d*x+c)^(3/4)/d^2-7*b
^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c
))^2)^(1/2)/d^(5/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1
/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))+7/2*b^(1/4)*(-a*d+b*c)^(5/2
)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(
1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x
+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4
)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(
1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^
2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d
*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(11/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b
*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)-7/4*b^(1/4)*(-a*d+b*c)^(5
/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))
^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b
*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))
*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c
)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*
(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(11/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2
*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 64, 637, 311, 226, 1210}

$$-\frac{7\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{7\sqrt[4]{b}\,(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\,d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{7\sqrt[4]{b}\,(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2}-\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(7/4))/(d*(c + d*x)^(1/4)) + (14*b*(a + b*x)^(3/4)*(c + d*x)^
(3/4))/(3*d^2) - (7*Sqrt[b]*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b
```

```
*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(5/2)*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x
)])/(b*c - a*d))) + (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(1/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(2*Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{(7b(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{2d^2}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(7b(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{}{\sqrt[4]{ac}}}{2d^2\sqrt[4]{a+bx}\sqrt[4]{c+d}}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(14b(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+}\right.}{}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(7\sqrt{b}(bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc}\right.}{}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{7\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+}}{d^{5/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/4))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.37, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/4)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(5/4), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(7/4)/(c + d*x)^(5/4), x)
```

3.1725 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=712

$$-\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}}+\frac{6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{3/2}(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}-\ldots 3\ldots$$

```
[Out] -4*(b*x+a)^(3/4)/d/(d*x+c)^(1/4)+6*b^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*
x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(3/2)/(-a*d+b*c)/(b*x+a)^
(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c)
)^(1/2)/(-a*d+b*c))+3/2*b^(1/4)*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(1/4)*(c
os(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(
-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^
(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)
*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^
2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^
(1/2)/d^(7/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b
*(2*d*x+c))^2)^(1/2)-3*b^(1/4)*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(1/4)*(co
s(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-
a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(
1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d
*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*
c))^2)^(1/2)/d^(7/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*
(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 64, 637, 311, 226, 1210}

$$\frac{6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{3\sqrt[4]{b}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt{2}\sqrt[4]{b}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(3/4))/(d*(c + d*x)^(1/4)) + (6*Sqrt[b]*Sqrt[(a + b*x)*(c + d
*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(3/2)*
```

```
(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*S
qrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (3*Sqrt[2]*b^(1/4
)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^
2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*
d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*
(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((
a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(7/4)*(a + b*x)^(1/4)
*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (3*
b^(1/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b
*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sq
rt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1
/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[2]*d^(7/4)*(
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d
*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
- 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{(3b)\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(3b\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(12b\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd\,-}}\right.}{d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(6\sqrt{b}\,(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int\right.}{d^{3/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{6\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+}}{d^{3/2}(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a}}{bc}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(5/4), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/4)/(c + d*x)^(5/4), x)
```

3.1726 $\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=719

$$\frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}}-\frac{4\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{d}\,(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}$$

```
[Out] 4*(b*x+a)^(3/4)/(-a*d+b*c)/(d*x+c)^(1/4)-4*b^(1/2)*((b*x+a)*(d*x+c))^(1/2)*
((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^2/(b*x+a
)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/d^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+
a)*(d*x+c))^(1/2)/(-a*d+b*c))+2*b^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arct
an(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(
1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*
d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a
*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)
/(-a*d+b*c))^2)^(1/2)/d^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)
/((a*d+b*(2*d*x+c))^2)^(1/2)-b^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(
b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1
/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2
)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^(
1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+
b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-
a*d+b*c))^2)^(1/2)/d^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((
a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt{2}\sqrt[4]{b}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{d}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^2(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{4(a+bx)^{3/4}}{\sqrt[4]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4))/((b*c - a*d)*(c + d*x)^(1/4)) - (4*Sqrt[b]*Sqrt[(a + b*
x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/
```

```
(Sqrt[d]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (2*Sqr
t[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3/4)*(a + b
*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^
2]) - (Sqrt[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b
*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]
*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3
/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
+ 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{5/4}}\,dx = \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{(2b)\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(2b\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(8b\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\operatorname{Subst}\left(\int \right.}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(4\sqrt{b}\,\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\operatorname{Subst}}{\sqrt{d}\,\sqrt[4]{a+bx}\,\sqrt[4]{}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{4\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}}{\sqrt{d}\,(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[3/4,
 5/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}} (dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(5/4)), x)
```

Fricas [**F**]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/4}\,{\left(c+d\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(5/4)), x)
```

3.1727 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=750

$$-\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}-\frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}}+\frac{8\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx}}{(bc-ad)^3\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\right.}$$

```
[Out] -4/(-a*d+b*c)/(b*x+a)^(1/4)/(d*x+c)^(1/4)-8*d*(b*x+a)^(3/4)/(-a*d+b*c)^2/(d
*x+c)^(1/4)+8*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)
^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(1/4)/(d*x+c)^(1/4)
/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))
-4*b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(si
n(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)
)),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+
b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^
(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(b*x+a)^(1/4)/(d
*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)+
2*b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b
*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin
(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))
),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b
*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(
1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(b*x+a)^(1/4)/(d*
x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$-\frac{8d(a+bx)^{3/4}}{\sqrt[4]{c+dx}\,(bc-ad)^2}+\frac{8\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^3(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{4}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}+\frac{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/((b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)) - (8*d*(a + b*x)^(3/4))/((
b*c - a*d)^2*(c + d*x)^(1/4)) + (8*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((b*c - a*d)
```

```
^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (4*Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*Sqrt[2]*b^(
1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4
)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx = -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{5/4}}\,dx}{bc-ad}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(4bd)\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{\cdots}}}{(bc-ad)^{\cdots}}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(4bd\sqrt[4]{(a+bx)(c+\cdots}\right.}{(bc-\cdots}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(16bd\sqrt[4]{(a+bx)(c\cdots}\right.}{\cdots}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(8\sqrt{b}\,\sqrt{d}\,\sqrt[4]{(a+b\cdots}\right.}{\cdots}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{8\sqrt{b}\,\sqrt{d}\,\sqrt{(a\cdots}}{(bc-ad)^3\sqrt[4]{a+bx}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{a+bx}\,(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/4)*(c + d*x)^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}} (dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(5/4)), x)
```

Fricas [**F**]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(5/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/4}\,{\left(c+d\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(5/4)*(c + d*x)^(5/4)), x)
```

3.1728 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=795

$$-\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}+\frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}+\frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}}-\frac{48\sqrt{b}\,d^{3/2}\sqrt{}}{5(bc-ad)^4\sqrt[4]{a+}}$$

```
[Out] -4/5/(-a*d+b*c)/(b*x+a)^(5/4)/(d*x+c)^(1/4)+24/5*d/(-a*d+b*c)^2/(b*x+a)^(1/
4)/(d*x+c)^(1/4)+48/5*d^2*(b*x+a)^(3/4)/(-a*d+b*c)^3/(d*x+c)^(1/4)-48/5*d^(
3/2)*b^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2
*d*x+c))^2)^(1/2)/(-a*d+b*c)^4/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c
)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))+24/5*b^(1/4)*d^(
5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c)
)^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2)
)*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*
x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*
((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(-a*d+b*c)^(3/2)/(b*x+a)^(1/4)
/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)-12/5*b^(1/4)*d
^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+
c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*
((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b
^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/
2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*
d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2
)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(-a*d+b*c)^(3/2)/(b*x+a)^(1/
4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$\frac{48(a+bx)^{3/4}d^2}{5(bc-ad)^3\sqrt[4]{c+dx}}-\frac{48\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}\,d^{3/2}}{5(bc-ad)^4\sqrt[4]{a+bx}\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{24\sqrt{2}\sqrt[4]{b}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)d^{5/4}}{5(bc-ad)^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\,(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{12\sqrt{2}\sqrt[4]{b}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)d^{5/4}}{5(bc-ad)^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\,(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}-\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(5*(b*c - a*d)*(a + b*x)^(5/4)*(c + d*x)^(1/4)) + (24*d)/(5*(b*c - a*d)^
2*(a + b*x)^(1/4)*(c + d*x)^(1/4)) + (48*d^2*(a + b*x)^(3/4))/(5*(b*c - a*d
```

```
)^3*(c + d*x)^(1/4)) - (48*Sqrt[b]*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*(b*c - a*d)^4*(a
+ b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (24*Sqrt[2]*b^(1/4)*d^(5/4)*((a
+ b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[
d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b
*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4
))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^(1/4)*(c + d*x)^(
1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (12*Sqrt[2]*b^(
1/4)*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^
(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}(c+dx)^{5/4}}\,dx = -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx}{5(bc-ad)}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{(12d^2)\int \cdots}{\cdots}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d^2 \cdots}{5(bc-\cdots}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d^2 \cdots}{5(bc-\cdots}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d^2 \cdots}{5(bc-\cdots}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d^2 \cdots}{5(bc-\cdots}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d^2 \cdots}{5(bc-\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.09

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/4}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/4)*(c + d*x)^(5/4))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}} (dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(5/4)), x)
```

Fricas [**F**]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*
d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(5/4)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(a+b\,x)}^{9/4}\,{(c+d\,x)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(9/4)*(c + d*x)^(5/4)), x)
```

3.1729 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx$

Optimal. Leaf size=279

$$\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\right.}{\sqrt{2}\sqrt[4]{a}\,b}$$

```
[Out] -1/2*ln(-a^(1/4)*b^(1/4)*(-a*x+1)^(1/4)*2^(1/2)/(b*x+1)^(1/4)+a^(1/2)+b^(1/
2)*(-a*x+1)^(1/2)/(b*x+1)^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)+1/2*ln(a^(1/4)*b^(
1/4)*(-a*x+1)^(1/4)*2^(1/2)/(b*x+1)^(1/4)+a^(1/2)+b^(1/2)*(-a*x+1)^(1/2)/(b
*x+1)^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)+arctan(1-b^(1/4)*(-a*x+1)^(1/4)*2^(1/2
)/a^(1/4)/(b*x+1)^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)-arctan(1+b^(1/4)*(-a*x+1)^
(1/4)*2^(1/2)/a^(1/4)/(b*x+1)^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)
```

Rubi [A]
time = 0.22, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}+1\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*(1 - a*x)^(1/4))/(a^(1/4)*(1 + b*x)^(1
/4))])/(a^(1/4)*b^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*(1 - a*x)^(
1/4))/(a^(1/4)*(1 + b*x)^(1/4))])/(a^(1/4)*b^(3/4)) - Log[Sqrt[a] + (Sqrt[b
]*Sqrt[1 - a*x])/Sqrt[1 + b*x] - (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/
(1 + b*x)^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqrt[a] + (Sqrt[b]*Sqrt[1
- a*x])/Sqrt[1 + b*x] + (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/(1 + b*x)
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx = -\frac{4\text{Subst}\left(\int \frac{x^2}{\left(1+\frac{b}{a}-\frac{bx^4}{a}\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a}$$
$$= \frac{2\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}} - \frac{2\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{a\sqrt{b}}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}\right.}{b}$$
$$= -\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{}}{\sqrt{1}}\right.}{}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Mathematica [A]
time = 0.20, size = 156, normalized size = 0.56

$$\frac{\sqrt{2}\left(\tan^{-1}\left(\frac{-\sqrt{b}\sqrt{1-ax}+\sqrt{a}\sqrt{1+bx}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}\sqrt[4]{1+bx}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}\sqrt[4]{1+bx}}{\sqrt{b}\sqrt{1-ax}+\sqrt{a}\sqrt{1+bx}}\right)\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*(ArcTan[(-(Sqrt[b]*Sqrt[1 - a*x]) + Sqrt[a]*Sqrt[1 + b*x])/(Sqrt[2
]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4)*(1 + b*x)^(1/4))] + ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*(1 - a*x)^(1/4)*(1 + b*x)^(1/4))/(Sqrt[b]*Sqrt[1 - a*x] + Sqrt[
a]*Sqrt[1 + b*x])]))/(a^(1/4)*b^(3/4))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-ax+1\right)^{\frac{1}{4}} \left(bx+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

Fricas [A]
time = 0.34, size = 247, normalized size = 0.89

$$-4 \left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} \arctan\left(-\frac{(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}} - (a^2b^2x-ab^2)\sqrt{\frac{(ab^2x-b^2)\sqrt{-\frac{1}{ab^3}} - \sqrt{-ax+1}\sqrt{bx+1}}{ax-1}}\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}}{ax-1}\right) - \left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} \log\left(\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} + (-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right) + \left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} \log\left(-\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} - (-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(-1/(a*b^3))^(1/4)*arctan(-((-a*x + 1)^(3/4)*(b*x + 1)^(1/4)*a*b^2*(-1/(
a*b^3))^(3/4) - (a^2*b^2*x - a*b^2)*sqrt(((a*b^2*x - b^2)*sqrt(-1/(a*b^3))
- sqrt(-a*x + 1)*sqrt(b*x + 1))/(a*x - 1))*(-1/(a*b^3))^(3/4))/(a*x - 1)) -
 (-1/(a*b^3))^(1/4)*log(((a*b*x - b)*(-1/(a*b^3))^(1/4) + (-a*x + 1)^(3/4)*
(b*x + 1)^(1/4))/(a*x - 1)) + (-1/(a*b^3))^(1/4)*log(-((a*b*x - b)*(-1/(a*b
^3))^(1/4) - (-a*x + 1)^(3/4)*(b*x + 1)^(1/4))/(a*x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,\left(bx+1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(b*x+1)**(3/4),x)
```

```
[Out] Integral(1/((-a*x + 1)**(1/4)*(b*x + 1)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-a\,x)^{1/4}\,(b\,x+1)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)),x)
```

```
[Out] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)), x)
```

3.1730 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}-\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a}-$$

```
[Out] -1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)/a*2^(1/2)+1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a
*x+1)^(1/2))/a*2^(1/2)-arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1
/2)/a-arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)/a
```

Rubi [A]

time = 0.10, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx = -\frac{4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{a}$$
$$= -\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}\,a}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\log\left(1\right.}{}$$

Mathematica [A]
time = 0.10, size = 96, normalized size = 0.50

$$\frac{\sqrt{2}\left(\tan^{-1}\left(\frac{-\sqrt{1-ax}+\sqrt{1+ax}}{\sqrt{2}\sqrt[4]{1-a^2x^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-a^2x^2}}{\sqrt{1-ax}+\sqrt{1+ax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*(ArcTan[(-Sqrt[1 - a*x] + Sqrt[1 + a*x])/(Sqrt[2]*(1 - a^2*x^2)^(1
/4))] + ArcTanh[(Sqrt[2]*(1 - a^2*x^2)^(1/4))/(Sqrt[1 - a*x] + Sqrt[1 + a*x
])]))/a
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]')
```

[Out] cought exception: maximum recursion depth exceeded

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(154) = 308$.
time = 0.33, size = 448, normalized size = 2.32

$$2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}(a^4x-a^3)\sqrt{\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+(a^3x-a^2)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+ax-1}{ax-1}\right)+2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}(a^4x-a^3)\sqrt{-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-(a^3x-a^2)\sqrt{\frac{1}{a^4}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-ax+1}{ax-1}\right)-\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+(a^3x-a^2)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-(a^3x-a^2)\sqrt{\frac{1}{a^4}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a^3*(a^(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt((sqrt(2)*(a*x + 1)^(1/4)*(-
a*x + 1)^(3/4)*a*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1
)*sqrt(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) + a*x - 1)/(a*x - 1)) + 2*sqrt(
2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a^3*(a^
(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x +
1)^(3/4)*a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt
(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) - a*x + 1)/(a*x - 1)) - 1/2*sqrt(2)*(
a^(-4))^(1/4)*log((sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a*(a^(-4))^(1/4
) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x - 1)) +
 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt(-a*x + 1
))/(a*x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(a*x+1)**(3/4),x)
```

```
[Out] Integral(1/((-a*x + 1)**(1/4)*(a*x + 1)**(3/4)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-a\,x\right)}^{1/4}\,{\left(a\,x+1\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)),x)
```

```
[Out] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)), x)
```

3.1731 $\int \frac{(a+bx)^{3/2}}{\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

```
[Out] 2/5*(b*x+a)^(5/2)*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([1/5, 5/2],[7/2],-
d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/5)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{(a+bx)^{3/2}}{\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= \frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

Mathematica [**A**]
time = 10.02, size = 73, normalized size = 0.99

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/5))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Fricas [F]
time = 1.72, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/5),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/5), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/5),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/5), x)
```

3.1732 $\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

[Out] 2/3*(b*x+a)^(3/2)*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([1/5, 3/2],[5/2],-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/5)

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/5),x]

[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 3/2, 5/2, -((d*(a + b*x))/(b*c - a*d))])/(3*b*(c + d*x)^(1/5))

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{\sqrt{a+bx}}{\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$

$$= \frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/5))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(1/5),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Fricas [F]
time = 1.74, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx}}{\sqrt[5]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/5),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/5), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/5}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/5),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/5), x)
```

3.1733 $\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

```
[Out] 2*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([1/5, 1/2],[3/2],-d*(b*x+a)/(-a*d+
b*c))*(b*x+a)^(1/2)/b/(d*x+c)^(1/5)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/5)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt{a+bx}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= \frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 71, normalized size = 0.99

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/5)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/5))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(1/5)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/5)), x)
```

Fricas [F]
time = 1.68, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx}\,\sqrt[5]{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/5),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/5)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/5)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/5)), x)
```

3.1734 $\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

```
[Out] -2*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([-1/2, 1/5],[1/2],-d*(b*x+a)/(-a*
d+b*c))/b/(d*x+c)^(1/5)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, -((
d*(a + b*x))/(b*c - a*d))])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= -\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 71, normalized size = 0.99

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]')
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/5)), x)
```

Fricas [F]
time = 1.73, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/5),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/5)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/5)), x)
```

3.1735 $\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

```
[Out] -2/3*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([-3/2, 1/5],[-1/2],-d*(b*x+a)/(
-a*d+b*c))/b/(b*x+a)^(3/2)/(d*x+c)^(1/5)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, -(
(d*(a + b*x))/(b*c - a*d))])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= -\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/5)), x)
```

Fricas [**F**]
time = 1.59, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/5),x)
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/5)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x)
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/5)), x)
```

3.1736 $\int (a+bx)^{5/2}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=487

$$\frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+b}{}$$

```
[Out] -9/352*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/6)/b/d^2+3/176*(-a*d+b*c)*(b*x
+a)^(5/2)*(d*x+c)^(1/6)/b/d+3/11*(b*x+a)^(7/2)*(d*x+c)^(1/6)/b+81/1408*(-a*
d+b*c)^3*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b/d^3-81/2816*3^(3/4)*(-a*d+b*c)^(11/3
)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)
))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(
2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b
*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d^4/(b*x+a)^(1/2)/(
-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {52, 65, 231}

$$-\frac{81\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{11/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816bd^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{81\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^3}{1408bd^3}-\frac{9(a+bx)^{3/2}\sqrt[6]{c+dx}\,(bc-ad)^2}{352bd^2}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}\,(bc-ad)}{176bd}+\frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]
```

```
[Out] (81*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(1408*b*d^3) - (9*(b*c - a
*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(352*b*d^2) + (3*(b*c - a*d)*(a + b*
x)^(5/2)*(c + d*x)^(1/6))/(176*b*d) + (3*(a + b*x)^(7/2)*(c + d*x)^(1/6))/(
11*b) - (81*3^(3/4)*(b*c - a*d)^(11/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
 b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/
3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
```

```
- Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2816*b*d^4*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}+\frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx}{22b}$$

$$= \frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}-\frac{(15(bc-ad)^2)\int \frac{(a+}{(c+}}{352bd}$$

$$= -\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+bx}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.15

$$\frac{2(a+bx)^{7/2}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/2, 9/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx + a)^{\frac{5}{2}} (dx + c)^{\frac{1}{6}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.38, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{5}{2}} \sqrt[6]{c + dx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(1/6), x)
```

3.1737 $\int (a+bx)^{3/2}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=449

$$-\frac{27(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{320bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}+\frac{27\ 3^{3/4}(bc-a}{}$$

[Out] 3/80*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/6)/b/d+3/8*(b*x+a)^(5/2)*(d*x+c)^(1/6)/b-27/320*(-a*d+b*c)^2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b/d^2+27/640*3^(3/4)*(-a*d+b*c)^(8/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.25, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{27\ 3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640bd^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{27\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{320bd^2}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}\,(bc-ad)}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]

[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(320*b*d^2) + (3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(80*b*d) + (3*(a + b*x)^(5/2)*(c + d*x)^(1/6))/(8*b) + (27*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(640*b*d^3*Sqrt[a + b*x]*S

```
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}dx}{16b}$$

$$= \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b} - \frac{(9(bc-ad)^2)\int \frac{\sqrt{a}}{(c+}}{160bd}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{320bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+b}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{320bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+b}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{320bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+b}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.16

$$\frac{2(a+bx)^{5/2}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{1}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} \sqrt[6]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x)^{3/2}\,(c + d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/6), x)
```

3.1738 $\int \sqrt{a+bx}\,\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=411

$$\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20bd}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b}-\frac{3\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+}\right.}{}$$

```
[Out] 3/5*(b*x+a)^(3/2)*(d*x+c)^(1/6)/b+3/20*(-a*d+b*c)*(d*x+c)^(1/6)*(b*x+a)^(1/
2)/b/d-3/40*3^(3/4)*(-a*d+b*c)^(5/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1
/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {52, 65, 231}

$$-\frac{3\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40bd^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{20bd}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/6),x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*b*d) + (3*(a + b*x)^(3/2)
*(c + d*x)^(1/6))/(5*b) - (3*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*((b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)
*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c -
a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*b*d^2*Sqrt[a +
```

```
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}dx}{10b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{(3(bc-ad)^2)\int \frac{}{\sqrt{a+b}}}{40bd}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{(9(bc-ad)^2)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{3\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{3/2}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(1/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,\left(dx+c\right)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/6),x)
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,\left(c+d\,x\right)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(1/6), x)
```

3.1739 $\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=375

$$\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b}+\frac{3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\sqrt[3]{bc-ad}-\left(1+\cdots\right.\right.}}\cdots}{4bd\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c\cdots}}{\left(\sqrt[3]{bc-\cdots}\right.}}}$$

[Out] 3/2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b+1/4*3^(3/4)*(-a*d+b*c)^(2/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4bd\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(1/6)/Sqrt[a + b*x],x]

[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*b) + (3^(3/4)*(b*c - a*d)^(2/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4*b

```
*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{4b}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{2bd}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b} + \frac{3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-}{}}}{4bd\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.19

$$\frac{2\sqrt{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(1/6)/sqrt(a + b*x), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(1/6)/(a + b*x)^(1/2), x)
```

3.1740 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}+\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+d}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}{\sqrt[4]{3}\,b\sqrt[3]{bc-ad}\,\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc}\right.}{\left(\sqrt[3]{bc-ad}-\left(1\right.\right.}}}$$

```
[Out] -2*(d*x+c)^(1/6)/b/(b*x+a)^(1/2)+1/3*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1
/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/b/(-a*d+b*c)^(1/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1
/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 65, 231}

$$\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,b\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(b*Sqrt[a + b*x]) + ((c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b*(b*c - a*d)^(1/3)*Sq
```

```
rt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}dx}{3b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}dx, x, \sqrt[6]{c+dx}\right)}{b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}} + \frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\cdots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{\cdots}\right.\right.\cdots}}\cdots}{\sqrt[4]{3}\,b\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.19

$$-\frac{2\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(3/2),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(3/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/6)/(a + b*x)^(3/2), x)
```

3.1741 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=409

$$-\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}}-\frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}-\frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{\cdots}}{\left(\sqrt[3]{bc-ad}-\cdots\right.}}}{9\sqrt[4]{3}\,b(bc-ad)^{4/3}\sqrt{a\cdots}}$$

```
[Out] -2/3*(d*x+c)^(1/6)/b/(b*x+a)^(3/2)-2/9*d*(d*x+c)^(1/6)/b/(-a*d+b*c)/(b*x+a)
^(1/2)-2/27*d*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipti
cF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((
(-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/
3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b
/(-a*d+b*c)^(4/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 53, 65, 231}

$$-\frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,b\sqrt{a+bx}\,(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2d\sqrt[6]{c+dx}}{9b\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(3*b*(a + b*x)^(3/2)) - (2*d*(c + d*x)^(1/6))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) - (2*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
```

```
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b*(b*c - a*d)^(4/3)*Sqrt[a + b*
x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx}{9b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(2d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(4d)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+\right.}{9b(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.18

$$-\frac{2\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(5/2), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(1/6)/(a + b*x)^(5/2), x)
```

3.1742 $\int (a+bx)^{3/2}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=896

$$-\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b}-\frac{81\left(1+\right.}{448b^{5/3}d^2\left(\right.}$$

[Out] 3/28*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(5/6)/b/d+3/10*(b*x+a)^(5/2)*(d*x+c)^(5/6)/b-27/224*(-a*d+b*c)^2*(d*x+c)^(5/6)*(b*x+a)^(1/2)/b/d^2-81/448*(-a*d+b*c)^3*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)/d^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-81/448*3^(1/4)*(-a*d+b*c)^(10/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/896*3^(3/4)*(-a*d+b*c)^(10/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.78, antiderivative size = 896, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$-\frac{81\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{448b^{5/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{896b^{5/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{81\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}(bc-ad)^3}{448b^{5/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{27\sqrt{a+bx}(c+dx)^{5/6}(bc-ad)^2}{224bd^2}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}(bc-ad)}{28bd}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]
```

```
[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*b*d) + (3*(a + b*x)^(5/2)*(c + d*x
)^(5/6))/(10*b) - (81*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(
1/6))/(448*b^(5/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(5/3)*d^3*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (27*
3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(5/3)*d^3*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 231</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
```

```
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/6}\,dx = \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b} - \frac{(9(bc-ad)^2)}{56}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + 3($$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + 3($$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + 3($$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + 3($$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{5/2}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Fricas [**F**]
time = 0.42, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/6), x)
```

3.1743 $\int \sqrt{a+bx}\,(c+dx)^{5/6}\,dx$

Optimal. Leaf size=858

$$\frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b}+\frac{45\left(1+\sqrt{3}\right)(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{112b^{5/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right.}$$

```
[Out] 3/7*(b*x+a)^(3/2)*(d*x+c)^(5/6)/b+15/56*(-a*d+b*c)*(d*x+c)^(5/6)*(b*x+a)^(1
/2)/b/d+45/112*(-a*d+b*c)^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)
/d/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+45/112*3^(1/4)*(-a*
d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Elliptic
E((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((
-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3
))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d^
2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+15/224*
3^(3/4)*(-a*d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1
/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2
)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1
/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(5/3)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$\frac{45\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{112b^{5/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{15\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{224b^{5/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{45\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{112b^{5/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{15\sqrt{a+bx}\,(c+dx)^{5/6}(bc-ad)}{56bd}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/6),x]
```

```
[Out] (15*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*b*d) + (3*(a + b*x)^(3/2
)*(c + d*x)^(5/6))/(7*b) + (45*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(c
 + d*x)^(1/6))/(112*b^(5/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))) + (45*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
 a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1
/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(5/3)*d^2*Sqrt[a +
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
+ (15*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(5/3)*d^2*Sqrt[a + b
*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^
(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/6}\,dx = \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{(5(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14b}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} - \frac{(15(bc-ad)^2)\int}{}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} - \frac{(45(bc-ad)^2)\,S}{}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{(45(bc-ad)^2)\,S}{}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{45\left(1+\sqrt{3}\right.}{112b^{5/3}d\left(\sqrt[3]{bc-}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)*(c + d*x)^(5/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\ (dx+c)^{\frac{5}{6}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.43, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\ (c+dx)^{\frac{5}{6}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/6),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(5/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,\left(c+d\,x\right)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/6), x)
```

3.1744 $\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}}\, dx$

Optimal. Leaf size=817

$$\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b}-\frac{15\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{15\sqrt[4]{3}\,(bc-ad)^{4/3}\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-\right.}{}$$

```
[Out] 3/4*(d*x+c)^(5/6)*(b*x+a)^(1/2)/b-15/8*(-a*d+b*c)*(d*x+c)^(1/6)*(1+3^(1/2))
*(b*x+a)^(1/2)/b^(5/3)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))
-15/8*3^(1/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b
*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))
^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^
(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2
)^(1/2)/b^(5/3)/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^
2)^(1/2)-5/16*3^(3/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1
/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)
*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 817, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$-\frac{15\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{8b^{5/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{16b^{5/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{15\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/Sqrt[a + b*x],x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*b) - (15*(1 + Sqrt[3])*(b*c - a*d)*Sqr
t[a + b*x]*(c + d*x)^(1/6))/(8*b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))) - (15*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(5/3)*d*Sqr
t[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^
2)]) - (5*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(5/3)*d*Sqrt[a +
b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}}\,dx = \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} + \frac{(5(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{8b}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} + \frac{(15(bc-ad))\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4bd}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} - \frac{(15(bc-ad))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+d\right.}{8b^{5/3}d}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} - \frac{15\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{15\sqrt[4]{3}\,(bc-a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\,(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(1/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.42, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^{\frac{5}{6}}}{\sqrt{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(5/6)/sqrt(a + b*x), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x)^{5/6}}{\sqrt{a + b\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(1/2), x)
```

3.1745 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=798

$$-\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}}-\frac{5\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{5\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{}$$

```
[Out] -2*(d*x+c)^(5/6)/b/(b*x+a)^(1/2)-5*d*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2
)/b^(5/3)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-5*3^(1/4)*(-
a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipt
icE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2
/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/
(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/6*(-a*d
+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+
b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*
(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF
((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3
^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(
d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 798, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 65, 314, 231, 1895}

$$-\frac{5\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{5\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(b*Sqrt[a + b*x]) - (5*(1 + Sqrt[3])*d*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (5*3^(1/4)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(5/3)*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(1 - Sqrt
[3])*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/
3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1
/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(
c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{3b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} + \frac{10\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} - \frac{5\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{5/3}} - \frac{\left(5\left(1-\sqrt{\right.\right.}{}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} - \frac{5\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{5\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.09

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(3/2), x)
```

Fricas [**F**]
time = 0.43, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(3/2),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(3/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(3/2), x)
```

3.1746 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=854

$$-\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}}-\frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}}-\frac{10\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{10d\sqrt[6]{c+\cdots}}{\cdots}$$

```
[Out] -2/3*(d*x+c)^(5/6)/b/(b*x+a)^(3/2)-10/9*d*(d*x+c)^(5/6)/b/(-a*d+b*c)/(b*x+a
)^(1/2)-10/9*d^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)/(-a*d+b*c)
/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-10/9*d*(d*x+c)^(1/6)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^
2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a
*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(-a*d+b*c)^(2/3)/(b*x
+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/27*d*(d*x+c
)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*
c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(
-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 854, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 53, 65, 314, 231, 1895}

$$-\frac{10\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^2}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{10\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{3\,3^{3/4}b^{5/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{5\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{9\sqrt[4]{3}\,b^{5/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{10(c+dx)^{5/6}d}{9b(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(3*b*(a + b*x)^(3/2)) - (10*d*(c + d*x)^(5/6))/(9*b*(b
*c - a*d)*Sqrt[a + b*x]) - (10*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*b^(5/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))) - (10*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3
) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
+ d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(
1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(
2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c +
d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (
2 + Sqrt[3])/4])/(9*3^(1/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{9b}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} + \frac{(10d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} + \frac{(20d)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+}\right.}{9b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(10d)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx\right.}{9b^{5/3}(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \frac{10\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.09

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-3/2, -5/6, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.42, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(5/2), x)
```

3.1747 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=896

$$-\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}}-\frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}}+\frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}}+\frac{8\left(1+\sqrt{3}\right)d^3\sqrt{a+bx}\,\sqrt[6]{c-}}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right.\right.}$$

```
[Out] -2/5*(d*x+c)^(5/6)/b/(b*x+a)^(5/2)-2/9*d*(d*x+c)^(5/6)/b/(-a*d+b*c)/(b*x+a)
^(3/2)+8/27*d^2*(d*x+c)^(5/6)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)+8/27*d^3*(d*x+c)
^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)/(-a*d+b*c)^2/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+8/27*d^2*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^
2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c
)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*
(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/81*d^2*(d*x+c)^(1/6)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-
3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/(
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))
^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(
-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(-a*d+b*c)^(5/3)/(b
*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 896, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 53, 65, 314, 231, 1895}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^3}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d^2}{9\,3^{3/4}b^{5/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{4\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d^2}{27\sqrt[4]{3}\,b^{5/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{8(c+dx)^{5/6}d^2}{27b(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/6}d}{9b(bc-ad)(a+bx)^{3/2}}-\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(5*b*(a + b*x)^(5/2)) - (2*d*(c + d*x)^(5/6))/(9*b*(b*
c - a*d)*(a + b*x)^(3/2)) + (8*d^2*(c + d*x)^(5/6))/(27*b*(b*c - a*d)^2*Sqr
t[a + b*x]) + (8*(1 + Sqrt[3])*d^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(27*b^(5/
3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)) + (8*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*S
qrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3
)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x
)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 +
 Sqrt[3])/4])/(9*3^(3/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (4*(1 - Sqrt[3
])*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[(
(b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
+ d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2
]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt
[3])/4])/(27*3^(1/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3
)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx}{3b}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} - \frac{(4d^2)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} - \frac{(8d^3)\int \frac{}{\sqrt{a+bx}}}{81b(bc-}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} - \frac{(16d^2)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \frac{(8d^2)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \frac{8\left(1\right.}{27b^{5/3}(bc-ad)^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.08

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{6};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-5/2, -5/6, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.44, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(7/2), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(5/6)/(a + b*x)^(7/2), x)
```

3.1748 $\int \frac{(a+bx)^{5/2}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=890

$$\frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3}-\frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d}+\frac{243\left(1+\right.}{448b^{2/3}d^3\left(\sqrt[3]{\right.}}$$

```
[Out] -9/28*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(5/6)/d^2+3/10*(b*x+a)^(5/2)*(d*x+c)
^(5/6)/d+81/224*(-a*d+b*c)^2*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d^3+243/448*(-a*d+
b*c)^3*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)/d^3/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+243/448*3^(1/4)*(-a*d+b*c)^(10/3)*(d*x
+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b
^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^4/(b*x+a)^(1/2)/(
-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+81/896*3^(3/4)*(-a*d+b*c
)^(10/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-
3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1
-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1
/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x
+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^
(2/3)/d^4/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 890, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$\frac{243\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{448b^{2/3}d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{81\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{896b^{2/3}d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{243\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}(bc-ad)^3}{448b^{2/3}d^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{81\sqrt{a+bx}(c+dx)^{5/6}(bc-ad)^2}{224d^3}-\frac{9(a+bx)^{3/2}(c+dx)^{5/6}(bc-ad)}{28d^2}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]
```

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*d^3) - (9*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(
5/6))/(10*d) + (243*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/
6))/(448*b^(2/3)*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))) + (243*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(2/3)*d^4*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (81*3
^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(2/3)*d^4*Sqrt[a + b*x]*Sq
rt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
```

```
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt[6]{c+dx}}\,dx = \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} - \frac{(3(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{4d}$$

$$= -\frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} + \frac{(27(bc-ad)^2)\int \frac{\sqrt{a+}}{\sqrt[6]{c+}}}{56d^2}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{7/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.47, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{\sqrt[6]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/6), x)
```

3.1749 $\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=855

$$-\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d}-\frac{81\left(1+\sqrt{3}\right)(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+d}\right.}$$

```
[Out] 3/7*(b*x+a)^(3/2)*(d*x+c)^(5/6)/d-27/56*(-a*d+b*c)*(d*x+c)^(5/6)*(b*x+a)^(1
/2)/d^2-81/112*(-a*d+b*c)^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)
/d^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-81/112*3^(1/4)*(-
a*d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipt
icE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2
/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/
d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/22
4*3^(3/4)*(-a*d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/(
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^
(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)/b^(2/3)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$-\frac{81\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{112b^{2/3}d^3\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{224b^{2/3}d^3\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{81\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{27\sqrt{a+bx}\,(c+dx)^{5/6}(bc-ad)}{56d^2}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]
```

```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(5/6))/(7*d) - (81*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(112*b^(2/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)
*(c + d*x)^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(2/3)*d^3*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
]) - (27*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*
d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
- a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
- (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(
1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(2/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx = \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14d}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} + \frac{(27(bc-ad)^2)\int \frac{}{\sqrt{a+}}}{112d^2}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} + \frac{(81(bc-ad)^2)\,\text{Subst}\Big(}{}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{(81(bc-ad)^2)\,\text{Subst}\Big(}{}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{81\left(1+\sqrt{3}\right)(bc-}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.42, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/6), x)
```

3.1750 $\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=820

$$\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d}+\frac{9\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{9\sqrt[4]{3}\,(bc-ad)^{4/3}\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-\right.}{}$$

```
[Out] 3/4*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d+9/8*(-a*d+b*c)*(d*x+c)^(1/6)*(1+3^(1/2))*
(b*x+a)^(1/2)/b^(2/3)/d/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)+9/8*3^(1/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b
*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))
^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^
(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2
)^(1/2)/b^(2/3)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)+3/16*3^(3/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^
2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^
(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(
1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 820, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$\frac{9\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{8b^{2/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{16b^{2/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{9\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/6),x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*d) + (9*(1 + Sqrt[3])*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(1/6))/(8*b^(2/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))) + (9*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(2/3)*d^2*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)]) + (3*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b
*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/
3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d
)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(2/3)*d^2*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx = \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{8d}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4d^2}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} + \frac{(9(bc-ad))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{8b^{2/3}d^2}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} + \frac{9\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} + \frac{9\sqrt[4]{3}\,(bc-ad}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(1/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.43, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/6),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/6), x)
```

3.1751 $\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=780

$$-\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{3\sqrt[4]{3}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{}}{}$$

```
[Out] -3*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))-3*3^(1/4)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1
/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*
c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^
(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(3/4)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6
)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+
c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/
3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d/(b*x+a)^(1/2)
/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {65, 314, 231, 1895}

$$-\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt[6]{c+dx}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/6)),x]
```

```
[Out] (-3*(1 + Sqrt[3])*Sqrt[a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (3*3^(1/4)*(b*c - a*d)^(1/3)*
(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c -
a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^
(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*Ellip
ticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4]
)/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2)]) - (3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(1/3)*(c + d*x)^(
1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3)
+ b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos
[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*b^(2/3
)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx = \frac{6\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{2/3}d} - \frac{\left(3\left(1-\sqrt{3}\right)(bc \cdots}{\cdots}$$

$$= -\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{3\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\,\left(\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\,\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/6)),x]
```

[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/6))

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(1/6)),x]')

[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)

[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/6)), x)

Fricas [**F**]
time = 0.41, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/6),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/6)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/6)), x)
```

3.1752 $\displaystyle \int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=813

$$-\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}-\frac{2\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{2\sqrt[4]{3}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\right.}{}$$

```
[Out] -2*(d*x+c)^(5/6)/(-a*d+b*c)/(b*x+a)^(1/2)-2*d*(d*x+c)^(1/6)*(1+3^(1/2))*(b*
x+a)^(1/2)/b^(2/3)/(-a*d+b*c)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))-2*3^(1/4)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*((
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ell
ipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)
^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)-1/3*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipt
icF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3
)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/b^(2/3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 813, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$-\frac{2\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{2\sqrt[4]{3}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(1 + Sqrt[3])*d*Sqrt[
a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))) - (2*3^(1/4)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(2/3)*(b*c - a*d)^(2/3)*
Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3
))^2)]) - ((1 - Sqrt[3])*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(
1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*
(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]
*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx &= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}+\frac{(2d)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{3(bc-ad)}\\
&= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}+\frac{4\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx,x,\sqrt[6]{c+dx}\right)}{bc-ad}\\
&= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}-\frac{2\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx,x,\sqrt[6]{c+dx}\right)}{b^{2/3}(bc-ad)}\\
&= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}-\frac{2\left(1+\sqrt{3}\right)d\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$-\frac{2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]')
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.43, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/6)), x)
```

3.1753 $\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=858

$$-\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}}+\frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}}+\frac{8\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+$$

```
[Out] -2/3*(d*x+c)^(5/6)/(-a*d+b*c)/(b*x+a)^(3/2)+8/9*d*(d*x+c)^(5/6)/(-a*d+b*c)^
2/(b*x+a)^(1/2)+8/9*d^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)/(-a
*d+b*c)^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+8/9*d*(d*x+c
)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(
1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(-a*d+b*c)^(5
/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/27*
d*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)
))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((
(-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/
3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b
^(2/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^2}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{3\,3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{4\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{9\sqrt[4]{3}\,b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{8(c+dx)^{5/6}d}{9(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (8*d*(c + d*x)^(5/6)
)/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (8*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c +
 d*x)^(1/6))/(9*b^(2/3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))) + (8*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
]) + (4*(1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx = -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(8d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,d}{27(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(16d)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+}}\right.}{9(bc-a}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(8d)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-}{\sqrt{a-\frac{bc}{d}}}\right.}{9b^{2/3}(}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{8\left(1+\sqrt{3}\right)d^2\sqrt{}}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.09

$$-\frac{2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.41, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/6)), x)
```

3.1754 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=440

$$\frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3}-\frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d}-\frac{81\ 3^{3/4}(bc-ad)}{}$$

```
[Out] -9/16*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/6)/d^2+3/8*(b*x+a)^(5/2)*(d*x+c)^
(1/6)/d+81/64*(-a*d+b*c)^2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/d^3-81/128*3^(3/4)*(
-a*d+b*c)^(8/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1
/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellip
ticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(
2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^4/(b*
x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))
/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$-\frac{81\ 3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{128d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{81\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{64d^3}-\frac{9(a+bx)^{3/2}\sqrt[6]{c+dx}\,(bc-ad)}{16d^2}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]
```

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(64*d^3) - (9*(b*c - a*d)*
(a + b*x)^(3/2)*(c + d*x)^(1/6))/(16*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(1
/6))/(8*d) - (81*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
```

```
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(128*d^4*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx = \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} - \frac{(15(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx}{16d}$$

$$= -\frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} + \frac{(27(bc-ad)^2)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}}{32d^2}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+}}{8d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+}}{8d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+}}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.17

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/6), x)
```

3.1755 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=405

$$-\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d}+\frac{27\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c\cdots}\right.}{\cdots}$$

```
[Out] 3/5*(b*x+a)^(3/2)*(d*x+c)^(1/6)/d-27/20*(-a*d+b*c)*(d*x+c)^(1/6)*(b*x+a)^(1
/2)/d^2+27/40*3^(3/4)*(-a*d+b*c)^(5/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^
(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{27\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{27\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{20d^2}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]
```

```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(1/6))/(5*d) + (27*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*(
(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1
/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c
 - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*d^3*Sqrt[a
```

```
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx = \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx}{10d}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{(27(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,(\cdots}}{40d^2}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{(81(bc-ad)^2)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{27\ 3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/6), x)
```

3.1756 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=372

$$\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d}-\frac{3\,3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-}}{\left(\sqrt[3]{bc-ad}-(1\right.}}}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c}}{\left(\sqrt[3]{bc-\right.}}}}$$

[Out] 3/2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/d-3/4*3^(3/4)*(-a*d+b*c)^(2/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d}-\frac{3\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/6),x]

[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*d) - (3*3^(3/4)*(b*c - a*d)^(2/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4*d^2*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(

```
1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx = \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{4d}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{2d^2}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d} - \frac{3\,3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc}{}}}{4d^2}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.20

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(5/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx}}{(c + dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{5/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(5/6), x)
```

3.1757 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=343

$$\frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\Bigg(}{d\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}}}$$

```
[Out] 3^(3/4)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+
c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+
b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d/(-a*d+b*c)^(1
/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 231}

$$\frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/6)),x]
```

```
[Out] (3^(3/4)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sq
rt[3])/4])/(d*(b*c - a*d)^(1/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/
3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx = \frac{6\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d}$$

$$= \frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{\cdots}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\cdots\right.\right.}}\cdots}{d\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{\cdots}}{(\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.21

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/2, 5
/6, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(5/6)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{5}{6}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/6),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/6)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/6)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/6)), x)
```

3.1758 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}}-\frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+\cdots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c-\cdots}\right.}}}{\sqrt[4]{3}\,(bc-ad)^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+\cdots}}{\left(\sqrt[3]{bc-\cdots}\right.}}}$$

```
[Out] -2*(d*x+c)^(1/6)/(-a*d+b*c)/(b*x+a)^(1/2)-2/3*(d*x+c)^(1/6)*((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/
2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(
d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(-a*d+b*c)^(4/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*
x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 231}

$$-\frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{a+bx}\,(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[6]{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(1/6)*((b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*
(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a
*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
+ Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d
)^(4/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) -
```

```
b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx = -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{3(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)}{(}}}{\sqrt[4]{3}\,(bc}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(5/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/6)), x)
```

3.1759 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=410

$$-\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}}+\frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}\cdots}{\left(\sqrt[3]{bc\cdots\right.}}}\cdots}{9\sqrt[4]{3}\,(bc-a\cdots}$$

```
[Out] -2/3*(d*x+c)^(1/6)/(-a*d+b*c)/(b*x+a)^(3/2)+16/9*d*(d*x+c)^(1/6)/(-a*d+b*c)
^2/(b*x+a)^(1/2)+16/27*d*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1
/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2
)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*
(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/(-a*d+b*c)^(7/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 231}

$$\frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\sqrt{a+bx}\,(bc-ad)^{7/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{16d\sqrt[6]{c+dx}}{9\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt[6]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (16*d*(c + d*x)^(1/6
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (16*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(7/3)*S
```

```
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/6}}\,dx = -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(16d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/}}}{27(bc-ad)^2}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(32d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}}}\right.}{9(bc}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}\right.}{}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.18

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(-\frac{3}{2},\frac{5}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}} (c+dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/2}\,(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/6)), x)
```

3.1760 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=880

$$-\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}}-\frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3}+\frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2}-\frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-a\ldots}{112d^3\left(\sqrt[3]{bc-ad}-\left(1+\ldots\right.\right.}$$

```
[Out] -6*(b*x+a)^(5/2)/d/(d*x+c)^(1/6)+45/7*b*(b*x+a)^(3/2)*(d*x+c)^(5/6)/d^2-405
/56*b*(-a*d+b*c)*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d^3-1215/112*b^(1/3)*(-a*d+b*c
)^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/d^3/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1+3^(1/2)))-1215/112*3^(1/4)*b^(1/3)*(-a*d+b*c)^(7/3)*(d*x+c)
^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1
/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^4/(b*x+a)^(1/2)/(-b^(1/3)*(d
*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-405/224*3^(3/4)*b^(1/3)*(-a*d+b*c)^
(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2)
)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)
^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^4/(
b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 880, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 52, 65, 314, 231, 1895}

$$-\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}}+\frac{45b(c+dx)^{5/6}(a+bx)^{3/2}}{7d^2}-\frac{405b(bc-ad)(c+dx)^{5/6}\sqrt{a+bx}}{56d^3}-\frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)^2\sqrt[6]{c+dx}\,\sqrt{a+bx}}{112d^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{1215\sqrt[4]{3}\,\sqrt[3]{b}\,(bc-ad)^{7/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112d^4\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,\sqrt{a+bx}}-\frac{405\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)^{7/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224d^4\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(5/2))/(d*(c + d*x)^(1/6)) - (405*b*(b*c - a*d)*Sqrt[a + b*x]
*(c + d*x)^(5/6))/(56*d^3) + (45*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(7*d^2)
 - (1215*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))
/(112*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (1
215*3^(1/4)*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (405*3^(3/4)*(1
- Sqrt[3])*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)
*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} + \frac{(15b)\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{(135b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14d^2}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} + \frac{(40}{}$$

$$(12$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} + \frac{}{}$$

$$\Big(1$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{}{}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{12}{1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.46, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(5/6)/(d^2*x^2 +
 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(7/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(7/6),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(7/6), x)
```

3.1761 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=844

$$-\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}}+\frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2}+\frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{81\sqrt[4]{3}\,\sqrt[3]{b}\,(bc-a}{}$$

```
[Out] -6*(b*x+a)^(3/2)/d/(d*x+c)^(1/6)+27/4*b*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d^2+81/
8*b^(1/3)*(-a*d+b*c)*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/d^2/((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+81/8*3^(1/4)*b^(1/3)*(-a*d+b*c)^
(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*
c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^3/(b*x+a)^(1/2)
/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+27/16*3^(3/4)*b^(1/3)*
(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Elli
pticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2
/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^
(1/2)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.56, antiderivative size = 844, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 52, 65, 314, 231, 1895}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(3/2))/(d*(c + d*x)^(1/6)) + (27*b*Sqrt[a + b*x]*(c + d*x)^(5
/6))/(4*d^2) + (81*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x
)^(1/6))/(8*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
) + (81*3^(1/4)*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*d^3*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (27*3^(3/4)*(1
 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*d^3*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{(9b)\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} - \frac{(27b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{8d^2}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} - \frac{(81b(bc-ad))\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\right.}{4d^3}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} + \frac{\left(81\sqrt[3]{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc\text{-}}{\sqrt{a-\frac{bc}{d}}}\right.}{8d^3}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} + \frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+d}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.44, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(7/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(7/6), x)
```

3.1762 $\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=806

$$-\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}}-\frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{9\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\cdots\right.}{\cdots}$$

```
[Out] -6*(b*x+a)^(1/2)/d/(d*x+c)^(1/6)-9*b^(1/3)*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a
)^(1/2)/d/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-9*3^(1/4)*b^
(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^
(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d
*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3
^(3/4)*b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+
b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2))
)^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*
x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(
1+3^(1/2)))^2)^(1/2)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 806, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 65, 314, 231, 1895}

$$-\frac{9\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}}-\frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(7/6),x]
```

```
[Out] (-6*Sqrt[a + b*x])/(d*(c + d*x)^(1/6)) - (9*(1 + Sqrt[3])*b^(1/3)*Sqrt[a +
b*x]*(c + d*x)^(1/6))/(d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (9*3^(1/4)*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d^2*Sqrt[a + b*x]*Sqrt
[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (3*3^(3/
4)*(1 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*d^2*Sqrt[a + b*x]*Sqrt[-((b
^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
 - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} + \frac{(3b)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} + \frac{(18b)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d^2}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\left(9\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d^2} - \frac{\left(9\left(1\right.\right.}{}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} - \frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{9\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(7/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/2)/(c + d*x)^(7/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.46, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(7/6),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(7/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(7/6), x)
```

3.1763 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=817

$$\frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}+\frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{6\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\right.}{}$$

[Out] 6*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(1/6)+6*b^(1/3)*(d*x+c)^(1/6)*(1+3^(1/2)
)*(b*x+a)^(1/2)/(-a*d+b*c)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))+6*3^(1/4)*b^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^
(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d
*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))
^2)^(1/2)+3^(3/4)*b^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4
2^(1/2))(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(
1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))^2)^(1/2)/d/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.48, antiderivative size = 817, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$\frac{6\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}+\frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(7/6)),x]
```

```
[Out] (6*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/6)) + (6*(1 + Sqrt[3])*b^(1/3)*
Sqrt[a + b*x]*(c + d*x)^(1/6))/((b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))) + (6*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)]) + (3^(3/4)*(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/6}}\,dx = \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} - \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} - \frac{(12b)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d(bc-ad)}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} + \frac{\left(6\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c-\right.}{d(bc-ad)}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} + \frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} + 6$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(7/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[1/2, 7
/6, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/2)*(c + d*x)^(7/6)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.45, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(7/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/6)), x)
```

3.1764 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=844

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}-\frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}}-\frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}\,d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(1/6)/(b*x+a)^(1/2)-8*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/(d
*x+c)^(1/6)-8*b^(1/3)*d*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/(-a*d+b*c)^
2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-8*3^(1/4)*b^(1/3)*(d
*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(
1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*(
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)
+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/(-a*d+b*c)^(5/3)/(b*x+a)^
(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/3*b^(1/3)*(d*x+
c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b
*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(-a*d+b*
c)^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 844, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$-\frac{8\sqrt{a+bx}\,d}{(bc-ad)^2\sqrt[6]{c+dx}}-\frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{8\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{(bc-ad)^{5/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{4\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,(bc-ad)^{5/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6)) - (8*d*Sqrt[a + b*x])/((b*c
- a*d)^2*(c + d*x)^(1/6)) - (8*(1 + Sqrt[3])*b^(1/3)*d*Sqrt[a + b*x]*(c + d
*x)^(1/6))/((b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))) - (8*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c
+ d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))], (2 + Sqrt[3])/4])/((b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (4*(1 - Sqr
t[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2
/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2
 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c
 + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{(4d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/6}}\,dx}{3(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} + \frac{(8bd)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{\ldots}}}{3(bc-ad)\ldots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} + \frac{(16b)\text{Subst}\left(\int \frac{}{\sqrt{a\ldots}}\right.}{\ldots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} - \frac{\left(8\sqrt[3]{b}\right)\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} - \frac{8\left(1+\sqrt{3}\right)\ldots}{(bc-ad)^2\left(\sqrt[3]{bc-\ldots}\right.}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.08

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(-\frac{1}{2},\frac{7}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.44, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}} (c+dx)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(7/6)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/6)), x)
```

3.1765 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=893

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}+\frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}+\frac{80d^2\sqrt{a+bx}}{9(bc-ad)^3\sqrt[6]{c+dx}}+\frac{80\left(1+\sqrt{3}\right.}{9(bc-ad)^3\left(\sqrt[3]{bc\right.}}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/6)+20/9*d/(-a*d+b*c)^2/(d*x+c)^(1/
6)/(b*x+a)^(1/2)+80/9*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/6)+80/9*b^(
1/3)*d^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/(-a*d+b*c)^3/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+80/9*b^(1/3)*d*(d*x+c)^(1/6)*((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/
2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c
)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/(-a*d+b*c)^(8/3)/(b*x+a)^(1/2)/(-b^
(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+40/27*b^(1/3)*d*(d*x+c)^(1/
6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)
))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2
/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(-a*d+b*c)^(8/
3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 893, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$\frac{80\sqrt{a+bx}\,d^2}{9(bc-ad)^3\sqrt[6]{c+dx}}+\frac{80\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^2}{9(bc-ad)^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{80\sqrt[3]{b}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{3\,3^{3/4}(bc-ad)^{8/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{40\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{9\sqrt[4]{3}\,(bc-ad)^{8/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6)) + (20*d)/(9*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(1/6)) + (80*d^2*Sqrt[a + b*x])/(9*(b*c - a*d)^3*
(c + d*x)^(1/6)) + (80*(1 + Sqrt[3])*b^(1/3)*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*(b*c - a*d)^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))) + (80*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-
((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((
b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (40*(1 - S
qrt[3])*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) +
b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))]
, (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
```

```
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/6}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{(40d^2)}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80}{9(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80}{9(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80}{9(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80}{9(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.08

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(-\frac{3}{2},\frac{7}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+a\right)^{\frac{5}{2}} \left(dx+c\right)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.43, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{5}{2}} \left(c+dx\right)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(7/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/6)), x)
```

3.1766 $\int \sqrt[6]{a+bx}\,(c+dx)^{13/6}\,dx$

Optimal. Leaf size=84

$$\frac{6(bc-ad)^2(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/7*(-a*d+b*c)^2*(b*x+a)^(7/6)*(d*x+c)^(1/6)*hypergeom([-13/6, 7/6],[13/6],
-d*(b*x+a)/(-a*d+b*c))/b^3/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,(bc-ad)^2\;{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 7
/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^3*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{13/6}\,dx = \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int \sqrt[6]{a+bx}\,\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}\,dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)^2(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.06, size = 73, normalized size = 0.87

$$\frac{6(a+bx)^{7/6}(c+dx)^{13/6}\;{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 7/6, 13/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 5984 deep
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
[Out] int((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(13/6), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(13/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{13/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(13/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(13/6), x)
```

3.1767 $\int \sqrt[6]{a+bx}\,(c+dx)^{7/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/7*(-a*d+b*c)*(b*x+a)^(7/6)*(d*x+c)^(1/6)*hypergeom([-7/6, 7/6],[13/6],-d*
(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,(bc-ad)\;{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 7/6,
 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^2*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{7/6}\,dx = \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int \sqrt[6]{a+bx}\,\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{7/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,(c+dx)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(7/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{7/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(7/6), x)
```

3.1768 $\int \sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/7*(b*x+a)^(7/6)*(d*x+c)^(1/6)*hypergeom([-1/6, 7/6],[13/6],-d*(b*x+a)/(-a
*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, -((d*
(a + b*x))/(b*c - a*d))])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,dx = \frac{\sqrt[6]{c+dx}\int \sqrt[6]{a+bx}\,\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [**A**]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(1/6), x)
```

3.1769 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

```
[Out] 6/7*(b*x+a)^(7/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([5/6, 7/6],[13/6],
-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{5/6}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}$$

$$= \frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

Mathematica [**A**]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.38, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{5/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(5/6), x)
```

3.1770 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6/7*(b*x+a)^(7/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([7/6, 11/6],[13/6]
,-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/6,
 11/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{11/6}}\,dx = \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}$$
$$= \frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.06, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6} {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[7/6
, 11/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(11/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(11/6), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a + bx}}{(c + dx)^{\frac{11}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(11/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(11/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x)^{1/6}}{(c + d\,x)^{11/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(11/6), x)
```

3.1771 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6/7*b*(b*x+a)^(7/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([7/6, 17/6],[13/
6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$
$$= \frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [**A**]
time = 10.03, size = 81, normalized size = 0.99

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(17/6), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(17/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{17/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(17/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(17/6), x)
```

3.1772 $\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx$

Optimal. Leaf size=427

$$\frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(\cdots}{\cdots}$$

```
[Out] 5/12*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(5/6)/b/d+1/2*(b*x+a)^(7/6)*(d*x+c)^(
5/6)/b-5/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6
))/b^(11/6)/d^(7/6)+5/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*
x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(7/6)-5/
144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(
1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(7/6)-5/72*(-a*d+b*c)^2*arctan
(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(1
1/6)/d^(7/6)*3^(1/2)-5/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+
a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(11/6)/d^(7/6)*3^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}}+\frac{5\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12bd}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b*d) + ((a + b*x)^(7/6)
*(c + d*x)^(5/6))/(2*b) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(
7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(11/6)*d^(7/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(11/6)*d^(7/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12b}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\int \frac{\quad}{(a+b}}{72bc}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\,\text{Subst}}{\quad}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\,\text{Subst}}{\quad}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\,\text{Subst}}{\quad}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}}{36b^{11/6}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}}{36b^{11/6}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{5(bc-ad)^2\tan^{-1}\Bigg(}{24\sqrt{3}\,b}$$

Mathematica [A]
time = 0.80, size = 267, normalized size = 0.63

$$\frac{(bc-ad)^2\left(\frac{6b^{5/6}\sqrt[6]{d}\sqrt[6]{a+bx}\,(c+dx)^{5/6}(5bc+ad+6bdx)}{(bc-ad)^2}+5\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)-5\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)-10\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)-5\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right)}{72b^{11/6}d^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(5/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(5/6)*(5*b*c +
 a*d + 6*b*d*x))/(b*c - a*d)^2 + 5*Sqrt[3]*ArcTan[(1 - (2*d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] - 5*Sqrt[3]*ArcTan[(1 + (2*d^(1
/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] - 10*ArcTanh[(d^(1
/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))] - 5*ArcTanh[(b^(1/6)*d^(1/6
)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c +
d*x)^(1/3))]))/(72*b^(11/6)*d^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(5/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. $2(321) = 642$.

time = 0.43, size = 5633, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d^7 + a^2*b^9*d^8)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x
 + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x
 + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(5/6) + sqrt(3)*(b
^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 +
 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*
b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d
^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2
+ 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^
5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4
*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^1
2*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*
b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*
d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*
a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*
```

b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^
8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 4
95*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*
b*c*d^12 + a^12*d^13)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^11*d^7))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d
^7 + a^2*b^9*d^8)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^11*d^7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(-((
b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
 c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^11*d^7))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 -
220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*
b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d
^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d
 - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^
4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5
*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^
10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*
d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220
*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12
+ (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^
9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 7
92*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b
^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c

*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(25*((b^4*c^2*d - 2*a*b^3*c*d^2 + a
^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x + b^4*
c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x + c))
+ 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(-25*((b
^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^11*d^7))^(1/3))/(d*x + c)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6) + (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^11*d^7))^(1/6))/(d*x + c)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6) - (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^11*d^7))^(1/6))/(d*x + c)) + 12*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(1
/6)*(d*x + c)^(5/6))/(b*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(5/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(1/6)*(c + d*x)^(5/6), x)
```

3.1773 $\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=378

$$\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+b}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+}}\right.}{2\sqrt{3}\,b^{5/6}d^{7/6}}$$

```
[Out] (b*x+a)^(1/6)*(d*x+c)^(5/6)/d-1/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(7/6)+1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(
b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(
5/6)/d^(7/6)-1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)
+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(7/6)-1/6*(-a*d+b*c
)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/
2))/b^(5/6)/d^(7/6)*3^(1/2)-1/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(
b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(5/6)/d^(7/6)*3^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/
6)*d^(7/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/6)*d^(7/6)) - ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(5/6)
*d^(7/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(
7/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(7/6)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx &= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d} \\
&= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd} \\
&= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd} \\
&= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{5/6}d} - \cdots \\
&= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \cdots \\
&= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \cdots \\
&= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}} - \cdots
\end{aligned}
$$

Mathematica [A]

time = 10.21, size = 291, normalized size = 0.77

$$
\frac{(a+bx)^{7/6}\left(6b^{5/6}\sqrt[6]{d(a+bx)}\,(c+dx)^{5/6}+\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{2\sqrt[6]{d(a+bx)}-\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{2\sqrt[6]{d(a+bx)}+\sqrt[6]{b}\sqrt[6]{c+dx}}\right)-2(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d(a+bx)}}\right)-(bc-ad)\tanh^{-1}\left(\frac{\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[6]{b}\sqrt[6]{d(a+bx)}\sqrt[6]{c+dx}}\right)\right)}{6b^{5/6}(d(a+bx))^{7/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(7/6)*(6*b^(5/6)*(d*(a + b*x))^(1/6)*(c + d*x)^(5/6) + Sqrt[3]*(
b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(2*(d*(a + b*x))^(1/6)
- b^(1/6)*(c + d*x)^(1/6))] + Sqrt[3]*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(
c + d*x)^(1/6))/(2*(d*(a + b*x))^(1/6) + b^(1/6)*(c + d*x)^(1/6))] - 2*(b*c
 - a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d*(a + b*x))^(1/6)] - (b*c - a*d
)*ArcTanh[((d*(a + b*x))^(1/3) + b^(1/3)*(c + d*x)^(1/3))/(b^(1/6)*(d*(a +
b*x))^(1/6)*(c + d*x)^(1/6))]))/(6*b^(5/6)*(d*(a + b*x))^(7/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3025 vs. 2(280) = 560.
time = 0.45, size = 3025, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + 2*sqrt(3)*
(b^4*d^7*x + b^4*c*d^6)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)
^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^3*x + b^2*
c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)
)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + sqrt(3)*(b^6*
c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*
c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a
^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4
*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*
b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5
- 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^5*d^7))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^
5*d^7))^(5/6) + 2*sqrt(3)*(b^4*d^7*x + b^4*c*d^6)*sqrt(-((b^2*c*d - a*b*d^2
)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*
d^7))^(5/6) - sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^
3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6
*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2
*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b
^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^
6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^
3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + d*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
```

```
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x
+ a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3)
 + (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/3))/(d*x + c)) - d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7
))^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)) + 2*d*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6) + (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^5*d^7))^(1/6))/(d*x + c)) - 2*d*((b^6*c^6 - 6*a*b^5*c^5*d +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^
5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(
5/6) - (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7)
)^(1/6))/(d*x + c)) - 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+bx)^{1/6}}{(c+dx)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(1/6), x)
```

3.1774 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\cdots$$

```
[Out] -6*(b*x+a)^(1/6)/d/(d*x+c)^(1/6)+2*b^(1/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^
(1/6)/(d*x+c)^(1/6))/d^(7/6)-1/2*b^(1/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(
d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(7/6)+1/2*b^(1/
6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(
1/6)/(d*x+c)^(1/6))/d^(7/6)+b^(1/6)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)
^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(7/6)+b^(1/6)*arctan(1/3*3^
(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(7
/6)
```

Rubi [A]
time = 0.34, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\,\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\,\sqrt[6]{b}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{7/6}}+\frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) - (b^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*d^(7/6)) + (b^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*d^(7/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{\left(2\sqrt[6]{b}\right)\mathrm{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d} + \frac{\left(2\sqrt[6]{b}\right)\mathrm{S}}{}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\,\mathrm{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,d\right.}{2d^{7/6}}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}}{}\right.}{2d^{7/6}}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+b}}{\sqrt[6]{b}\sqrt[6]{c+d}}}{\sqrt{3}}\right.}{d^{7/6}}$$

Mathematica [A]

time = 0.25, size = 239, normalized size = 0.72

$$\frac{-\frac{6\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}-\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{d^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] ((-6*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6) - Sqrt[3]*b^(1/6)*ArcTan[(1 -
 (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] + Sqrt[3]*
b^(1/6)*ArcTan[(1 + (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/
Sqrt[3]] + 2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(
1/6))] + b^(1/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/
(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/d^(7/6)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(239) = 478$.

time = 0.36, size = 663, normalized size = 2.00

$$-\frac{4\sqrt{3}\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}d^{6}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(d^{7}x+cd^{6}\right)\sqrt{\frac{\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}+\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}+\left(bx+a\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}}{dx+c}}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}+\sqrt{3}\left(bdx+bc\right)}{3\left(bdx+bc\right)}\right)+4\sqrt{3}\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}d^{6}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(d^{7}x+cd^{6}\right)\sqrt{-\frac{\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}-\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}-\left(bx+a\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}}{dx+c}}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}-\sqrt{3}\left(bdx+bc\right)}{3\left(bdx+bc\right)}\right)-\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(\frac{4\left(\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}+\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}+\left(bx+a\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}\right)}{dx+c}\right)+\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{4\left(\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}-\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}-\left(bx+a\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}\right)}{dx+c}\right)-2\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}+\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}}{dx+c}\right)+2\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}-\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}}{dx+c}\right)+12\left(bx+a\right)^{\frac{1}{6}}\left(dx+c\right)^{\frac{5}{6}}}{2\left(d^{2}x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sqrt(3)*(d^7*x + c*d^6)*sqrt(
((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3*x + c*d^2)*(b/d^7)^
(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(b/d^7)^(5/6) + sqrt(3)
*(b*d*x + b*c))/(b*d*x + b*c)) + 4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sq
rt(3)*(d^7*x + c*d^6)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6
) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x +
 c))*(b/d^7)^(5/6) - sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) - (d^2*x + c*d)*
(b/d^7)^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3
*x + c*d^2)*(b/d^7)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) + (
d^2*x + c*d)*(b/d^7)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7
)^(1/6) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/
(d*x + c)) - 2*(d^2*x + c*d)*(b/d^7)^(1/6)*log(((d^2*x + c*d)*(b/d^7)^(1/6)
 + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) + 2*(d^2*x + c*d)*(b/d^7)^(1
/6)*log(-((d^2*x + c*d)*(b/d^7)^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d
*x + c)) + 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^2*x + c*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(7/6), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+bx)^{1/6}}{(c+dx)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(7/6),x)
```

```
[Out] int((a + b*x)^(1/6)/(c + d*x)^(7/6), x)
```

3.1775 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

```
[Out] 6/7*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx = \frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]')
[Out] Timed out
```

Maple [**A**]
time = 0.17, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}}{7(dx+c)^{\frac{7}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(13/6),x,method=_RETURNVERBOSE)
[Out] -6/7*(b*x+a)^(7/6)/(d*x+c)^(7/6)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.
time = 0.30, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}{7\,(bc^3-ac^2d+(bcd^2-ad^3)x^2+2\,(bc^2d-acd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] 6/7*(b*x + a)^(7/6)*(d*x + c)^(5/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^
2 + 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(13/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(13/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,{\left(a+b\,x\right)}^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{6\,b\,x\,{\left(a+b\,x\right)}^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}\right)\,{\left(c+d\,x\right)}^{5/6}}{x^2-\frac{7\,b\,c^3-7\,a\,c^2\,d}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{14\,c\,d\,x\,\left(a\,d-b\,c\right)}{7\,a\,d^3-7\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(13/6),x)
```

```
[Out] -(((6*a*(a + b*x)^(1/6))/(7*a*d^3 - 7*b*c*d^2) + (6*b*x*(a + b*x)^(1/6))/(7
*a*d^3 - 7*b*c*d^2))*(c + d*x)^(5/6))/(x^2 - (7*b*c^3 - 7*a*c^2*d)/(7*a*d^3
 - 7*b*c*d^2) + (14*c*d*x*(a*d - b*c))/(7*a*d^3 - 7*b*c*d^2))
```

3.1776 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}}+\frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}$$

```
[Out] 6/13*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(13/6)+36/91*b*(b*x+a)^(7/6)/(-a*d+b*
c)^2/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (36*b*(a + b*x)^(7/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}} + \frac{(6b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{13(bc-ad)}$$
$$= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}} + \frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}$$

Mathematica [A]
time = 0.12, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}(13bc-7ad+6bdx)}{91(bc-ad)^2(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(13*b*c - 7*a*d + 6*b*d*x))/(91*(b*c - a*d)^2*(c + d*x)^
(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Maple [A]
time = 0.20, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}(-6bdx+7ad-13bc)}{91(dx+c)^{\frac{13}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(19/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/91*(b*x+a)^(7/6)*(-6*b*d*x+7*a*d-13*b*c)/(d*x+c)^(13/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(54) = 108$.
time = 0.33, size = 175, normalized size = 2.65

$$\frac{6\,(6\,b^2dx^2 + 13\,abc - 7\,a^2d + (13\,b^2c - abd)x)(bx + a)^{\frac{1}{6}}(dx + c)^{\frac{5}{6}}}{91\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2 + (b^2c^2d^3 - 2\,abcd^4 + a^2d^5)x^3 + 3\,(b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^2 + 3\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(6*b^2*d*x^2 + 13*a*b*c - 7*a^2*d + (13*b^2*c - a*b*d)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 - 2
*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2
 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.75, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{5/6}\,\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{1/6}}{91\,d^2\,(a\,d-b\,c)^2}-\frac{(42\,a^2\,d-78\,a\,b\,c)\,(a+b\,x)^{1/6}}{91\,d^3\,(a\,d-b\,c)^2}+\frac{x\,(78\,b^2\,c-6\,a\,b\,d)\,(a+b\,x)^{1/6}}{91\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(19/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((36*b^2*x^2*(a + b*x)^(1/6))/(91*d^2*(a*d - b*c)^2) - ((4
2*a^2*d - 78*a*b*c)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2) + (x*(78*b^2*c
- 6*a*b*d)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 + (3*c*
x^2)/d + (3*c^2*x)/d^2)
```

3.1777 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}$$

```
[Out] 6/19*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(19/6)+72/247*b*(b*x+a)^(7/6)/(-a*d+b
*c)^2/(d*x+c)^(13/6)+432/1729*b^2*(b*x+a)^(7/6)/(-a*d+b*c)^3/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (72*b*(a + b*x)^(7/
6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (432*b^2*(a + b*x)^(7/6))/(1729*
(b*c - a*d)^3*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{(12b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)} \\
&= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{(72b^2)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{247(bc-ad)^2} \\
&= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 73, normalized size = 0.72

$$
\frac{6(a+bx)^{7/6}\left(247b^2+\frac{91d^2(a+bx)^2}{(c+dx)^2}-\frac{266bd(a+bx)}{c+dx}\right)}{1729(bc-ad)^3(c+dx)^{7/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
[Out] (6*(a + b*x)^(7/6)*(247*b^2 + (91*d^2*(a + b*x)^2)/(c + d*x)^2 - (266*b*d*(
a + b*x))/(c + d*x)))/(1729*(b*c - a*d)^3*(c + d*x)^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]')
[Out] Timed out
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}\left(72b^2x^2d^2-84ab\,d^2x+228b^2cdx+91a^2d^2-266abcd+247b^2c^2\right)}{1729(dx+c)^{\frac{19}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(25/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/1729*(b*x+a)^(7/6)*(72*b^2*d^2*x^2-84*a*b*d^2*x+228*b^2*c*d*x+91*a^2*d^2
-266*a*b*c*d+247*b^2*c^2)/(d*x+c)^(19/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(83) = 166.
time = 0.36, size = 338, normalized size = 3.35

$$\frac{6\,(72\,b^3d^2x^3+247\,ab^2c^2-266\,a^2bcd+91\,a^3d^2+12\,(19\,b^3cd-ab^2d^2)x^2+(247\,b^3c^2-38\,ab^2cd+7\,a^2bd^2)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\,(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7)x^4+4\,(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d^5-a^3cd^6)x^3+6\,(b^3c^5d^2-3\,ab^2c^4d^3+3\,a^2bc^3d^4-a^3c^2d^5)x^2+4\,(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(72*b^3*d^2*x^3 + 247*a*b^2*c^2 - 266*a^2*b*c*d + 91*a^3*d^2 + 12*(1
9*b^3*c*d - a*b^2*d^2)*x^2 + (247*b^3*c^2 - 38*a*b^2*c*d + 7*a^2*b*d^2)*x)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
- a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*x
^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x^3 +
6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2 + 4*(
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.95, size = 213, normalized size = 2.11

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{(a+b\,x)^{1/6}\,(546\,a^3\,d^2-1596\,a^2\,b\,c\,d+1482\,a\,b^2\,c^2)}{1729\,d^4\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3\,(a+b\,x)^{1/6}}{1729\,d^2\,(a\,d-b\,c)^3}+\frac{x\,(a+b\,x)^{1/6}\,(42\,a^2\,b\,d^2-228\,a\,b^2\,c\,d+1482\,b^3\,c^2)}{1729\,d^4\,(a\,d-b\,c)^3}-\frac{72\,b^2\,x^2\,(a\,d-19\,b\,c)\,(a+b\,x)^{1/6}}{1729\,d^3\,(a\,d-b\,c)^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(546*a^3*d^2 + 1482*a*b^2*c^2 - 1596*a^
2*b*c*d))/(1729*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(1/6))/(1729*d^
2*(a*d - b*c)^3) + (x*(a + b*x)^(1/6)*(1482*b^3*c^2 + 42*a^2*b*d^2 - 228*a*
b^2*c*d))/(1729*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(a*d - 19*b*c)*(a + b*x)^(
1/6))/(1729*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d
^3 + (6*c^2*x^2)/d^2)
```

3.1778 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}}+\frac{7776b^3(a+bx)^{7/6}}{43225(bc-ad)^4(c+dx)^{7/6}}$$

```
[Out] 6/25*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(25/6)+108/475*b*(b*x+a)^(7/6)/(-a*d+
b*c)^2/(d*x+c)^(19/6)+1296/6175*b^2*(b*x+a)^(7/6)/(-a*d+b*c)^3/(d*x+c)^(13/
6)+7776/43225*b^3*(b*x+a)^(7/6)/(-a*d+b*c)^4/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (108*b*(a + b*x)^(7
/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (1296*b^2*(a + b*x)^(7/6))/(617
5*(b*c - a*d)^3*(c + d*x)^(13/6)) + (7776*b^3*(a + b*x)^(7/6))/(43225*(b*c
- a*d)^4*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(18b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$
$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{(216b^2)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$
$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} +$$
$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} +$$

Mathematica [A]
time = 0.16, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}\left(6175b^3 - \frac{1729d^3(a+bx)^3}{(c+dx)^3} + \frac{6825bd^2(a+bx)^2}{(c+dx)^2} - \frac{9975b^2d(a+bx)}{c+dx}\right)}{43225(bc-ad)^4(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(6175*b^3 - (1729*d^3*(a + b*x)^3)/(c + d*x)^3 + (6825*b
*d^2*(a + b*x)^2)/(c + d*x)^2 - (9975*b^2*d*(a + b*x))/(c + d*x)))/(43225*(
b*c - a*d)^4*(c + d*x)^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}\left(-1296b^3x^3d^3+1512d^3a\,x^2b^2-5400b^3c\,d^2x^2-1638a^2b\,d^3x+6300a\,b^2c\,d^2x-8550b^3c^2dx+1729a^3d^3-6825a^2bc\,d^2+9975\right.}{43225(dx+c)^{\frac{25}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(31/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/43225*(b*x+a)^(7/6)*(-1296*b^3*d^3*x^3+1512*a*b^2*d^3*x^2-5400*b^3*c*d^2
*x^2-1638*a^2*b*d^3*x+6300*a*b^2*c*d^2*x-8550*b^3*c^2*d*x+1729*a^3*d^3-6825
*a^2*b*c*d^2+9975*a*b^2*c^2*d-6175*b^3*c^3)/(d*x+c)^(25/6)/(a^4*d^4-4*a^3*b
*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(112) = 224$.
time = 0.34, size = 533, normalized size = 3.92

$$\frac{6\,{\left(1296\,b^{4}d^{3}x^{4} + 6175\,ab^{3}c^{3} - 9975\,a^{2}b^{2}c^{2}d + 6825\,a^{3}bcd^{2} - 1729\,a^{4}d^{3} + 216\,{\left(25\,b^{4}cd^{2} - ab^{3}d^{3}\right)}x^{3} + 18\,{\left(475\,b^{4}c^{2}d - 50\,ab^{3}cd^{2} + 7\,a^{2}b^{2}d^{3}\right)}x^{2} + {\left(6175\,b^{4}c^{3} - 1425\,ab^{3}c^{2}d + 525\,a^{2}b^{2}cd^{2} - 91\,a^{3}bd^{3}\right)}x\right)}{\left(bx + a\right)}^{\frac{1}{6}}{\left(dx + c\right)}^{\frac{5}{6}}}{43225\,{\left(b^{4}c^{9} - 4\,ab^{3}c^{8}d + 6\,a^{2}b^{2}c^{7}d^{2} - 4\,a^{3}bc^{6}d^{3} + a^{4}c^{5}d^{4} + {\left(b^{4}c^{4}d^{5} - 4\,ab^{3}c^{3}d^{6} + 6\,a^{2}b^{2}c^{2}d^{7} - 4\,a^{3}bcd^{8} + a^{4}d^{9}\right)}x^{5} + 5\,{\left(b^{4}c^{5}d^{4} - 4\,ab^{3}c^{4}d^{5} + 6\,a^{2}b^{2}c^{3}d^{6} - 4\,a^{3}bc^{2}d^{7} + a^{4}cd^{8}\right)}x^{4} + 10\,{\left(b^{4}c^{6}d^{3} - 4\,ab^{3}c^{5}d^{4} + 6\,a^{2}b^{2}c^{4}d^{5} - 4\,a^{3}bc^{3}d^{6} + a^{4}c^{2}d^{7}\right)}x^{3} + 10\,{\left(b^{4}c^{7}d^{2} - 4\,ab^{3}c^{6}d^{3} + 6\,a^{2}b^{2}c^{5}d^{4} - 4\,a^{3}bc^{4}d^{5} + a^{4}c^{3}d^{6}\right)}x^{2} + 5\,{\left(b^{4}c^{8}d - 4\,ab^{3}c^{7}d^{2} + 6\,a^{2}b^{2}c^{6}d^{3} - 4\,a^{3}bc^{5}d^{4} + a^{4}c^{4}d^{5}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/43225*(1296*b^4*d^3*x^4 + 6175*a*b^3*c^3 - 9975*a^2*b^2*c^2*d + 6825*a^3*
b*c*d^2 - 1729*a^4*d^3 + 216*(25*b^4*c*d^2 - a*b^3*d^3)*x^3 + 18*(475*b^4*c
^2*d - 50*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + (6175*b^4*c^3 - 1425*a*b^3*c^2
*d + 525*a^2*b^2*c*d^2 - 91*a^3*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4
*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*
c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^
4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 + 5*(b^4*c^8*d
 - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.15, size = 302, normalized size = 2.22

$$\frac{(c+d\,x)^{5/6}\,\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{1/6}}{43225\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\,(10374\,a^4\,d^3-40950\,a^3\,b\,c\,d^2+59850\,a^2\,b^2\,c^2\,d-37050\,a\,b^3\,c^3)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\,(-546\,a^3\,b\,d^3+3150\,a^2\,b^2\,c\,d^2-8550\,a\,b^3\,c^2\,d+37050\,b^4\,c^3)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{1/6}\,(7\,a^2\,d^2-50\,a\,b\,c\,d+475\,b^2\,c^2)}{43225\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{43225\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(31/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^4*x^4*(a + b*x)^(1/6))/(43225*d^2*(a*d - b*c)^4)
- ((a + b*x)^(1/6)*(10374*a^4*d^3 - 37050*a*b^3*c^3 + 59850*a^2*b^2*c^2*d -
 40950*a^3*b*c*d^2))/(43225*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(37050*
b^4*c^3 - 546*a^3*b*d^3 + 3150*a^2*b^2*c*d^2 - 8550*a*b^3*c^2*d))/(43225*d^
5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(1/6)*(7*a^2*d^2 + 475*b^2*c^2 -
50*a*b*c*d))/(43225*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(a*d - 25*b*c)*(a +
b*x)^(1/6))/(43225*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*x^4)/d + (5*c
^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

3.1779 $\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=427

$$\frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}+\frac{5(bc\cdots}{\cdots}$$

```
[Out] 1/12*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b/d+1/2*(b*x+a)^(11/6)*(d*x+c)^
(1/6)/b-5/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/
6))/b^(7/6)/d^(11/6)+5/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d
*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(11/6)-5
/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^
(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(11/6)+5/72*(-a*d+b*c)^2*arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(
7/6)/d^(11/6)*3^(1/2)+5/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x
+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(7/6)/d^(11/6)*3^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b*d) + ((a + b*x)^(11/6)*
(c + d*x)^(1/6))/(2*b) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
+ b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11
/6)) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqr
t[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a
*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(
7/6)*d^(11/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(
c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b
^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(7/6)*d^(11/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx = \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}+\frac{(bc-ad)\int\frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{\left(5(bc-ad)^2\right)\int\frac{}{\sqrt[6]{a+}}}{72bd}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{\left(5(bc-ad)^2\right)\mathrm{Subst}\Big(}{}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{\left(5(bc-ad)^2\right)\mathrm{Subst}\Big(}{1}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{\left(5(bc-ad)^2\right)\mathrm{Subst}\Big(}{}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{5(bc-ad)^2\tanh^{-1}\Big(}{36b^{7/6}d^{11}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{5(bc-ad)^2\tanh^{-1}\Big(}{36b^{7/6}d^{11}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{5(bc-ad)^2\tan^{-1}\Bigg(\frac{1-}{}}{24\sqrt{3}\,b^{7/}}$$

Mathematica [A]
time = 0.77, size = 267, normalized size = 0.63

$$\frac{(bc-ad)^2\left(\frac{6\sqrt[6]{b}\,d^{5/6}(a+bx)^{5/6}\sqrt[6]{c+dx}\,(5ad+b(c+6dx))}{(bc-ad)^2}+5\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)-5\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)-10\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)-5\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right)}{72b^{7/6}d^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(1/6)*d^(5/6)*(a + b*x)^(5/6)*(c + d*x)^(1/6)*(5*a*d +
 b*(c + 6*d*x)))/(b*c - a*d)^2 + 5*Sqrt[3]*ArcTan[(1 - (2*b^(1/6)*(c + d*x)
^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] - 5*Sqrt[3]*ArcTan[(1 + (2*b^(1
/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] - 10*ArcTanh[(b^(1
/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))] - 5*ArcTanh[(b^(1/6)*d^(1/6
)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c +
d*x)^(1/3))]))/(72*b^(7/6)*d^(11/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}} (dx+c)^{\frac{1}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="maxima")
```

[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. $2(321) = 642$.
time = 0.44, size = 5633, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="fricas")

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(((b^3*c^2*d^2 - 2*a*b^2*
c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x
 + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x
 + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) + sqrt(3)*(a
*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a
^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c
^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66
*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b
^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^
8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*
b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*
a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c
^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 +
 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^
```

12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d
^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*
a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*
c*d^11 + a^12*b*d^12)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^7*d^11))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^1
0 + a^2*b^6*d^11)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^7*d^11))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(-((
b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
 c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^7*d^11))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^
2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a
^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c
^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 -
 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b
^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^
7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a
^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^
3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c
^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 -
220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12
+ (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d
^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*
a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c
^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c

*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(25*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x + a)) + 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(-25*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x + a)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6))/(b*x + a)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6))/(b*x + a)) + 12*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(5/6)*(c + d*x)^(1/6), x)
```

3.1780 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=378

$$\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c}}\right.}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}$$

```
[Out] (b*x+a)^(5/6)*(d*x+c)^(1/6)/d-5/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(11/6)+5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*
(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^
(1/6)/d^(11/6)-5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/
3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(11/6)+5/6*(-a*d+
b*c)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^
(1/2))/b^(1/6)/d^(11/6)*3^(1/2)+5/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/
6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(1/6)/d^(11/6)*3^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/d - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
1/6)*d^(11/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(1/6)*d^(11/6)) - (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(1/6)*d^(11/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(1/6)*d^(11/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6d}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3d^{5/3}} - \frac{(5(bc-a\ldots}{}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{(5(bc-ad))\text{Subst}\left(\int \ldots\right.}{}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\ldots\right.}{1\ldots}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\ldots}{}\right.}{2\sqrt{3}\sqrt[6]{b}\,d\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.03, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2997 vs. 2(280) = 560.
time = 0.40, size = 2997, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")

[Out] -1/12*(20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b^2*d^9*x + a*b*d^9)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b^2*d^9*x + a*b*d^9)*sqrt(-((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 5*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/

```
(b*d^11))^(1/6)*log(25*((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a)) - 5*d*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-25*((b*c*d^2 - a*d^3)
*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^
(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11)
)^(1/3))/(b*x + a)) + 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11
))^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b*d^2*x + a
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a))
- 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-5*((b*
c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b*d^2*x + a*d^2)*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a)) - 12*(b*x + a)^(5/6)
*(d*x + c)^(1/6))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(5/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(5/6), x)
```

3.1781 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=334

$$-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}}-$$

```
[Out] -6/5*(b*x+a)^(5/6)/d/(d*x+c)^(5/6)+2*b^(5/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/d^(11/6)-1/2*b^(5/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3
)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(11/6)+1/2*b
^(5/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+
a)^(1/6)/(d*x+c)^(1/6))/d^(11/6)-b^(5/6)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b
*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(11/6)-b^(5/6)*arctan(
1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2
)/d^(11/6)
```

Rubi [A]
time = 0.39, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {49, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{11/6}}+\frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11
/6) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt
[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(d^(1/6)*(a +
b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) - (b^(5/6)*Log[b^(1/3) + (
d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6)
)/(c + d*x)^(1/6)])/(2*d^(11/6)) + (b^(5/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(2*d^(11/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx = -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{b\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{5/3}} + \frac{\left(2b^{5/6}\right)\cdots}{\cdots}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,\cdots}\cdots\right)}{2d^{11/6}}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\cdots\right)}{2d^{11/6}}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} - \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+\cdots}}{\sqrt[6]{b}\sqrt[6]{c+\cdots}}}{\sqrt{3}}\right)}{d^{11/6}}\cdots$$

Mathematica [A]
time = 0.30, size = 244, normalized size = 0.73

$$\frac{-\frac{6d^{5/6}(a+bx)^{5/6}}{(c+dx)^{5/6}} - 5\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 5\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 10b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right) + 5b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{5d^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] ((-6*d^(5/6)*(a + b*x)^(5/6))/(c + d*x)^(5/6) - 5*Sqrt[3]*b^(5/6)*ArcTan[(1
 - (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] + 5*Sqrt
[3]*b^(5/6)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6
)))/Sqrt[3]] + 10*b^(5/6)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b
*x)^(1/6))] + 5*b^(5/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^
(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(5*d^(11/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(241) = 482.
time = 0.41, size = 755, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -1/10*(20*sqrt(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x +
a*d^9)*sqrt(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) + (b*x
+ a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x +
 a))*(b^5/d^11)^(5/6) + sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) + 20*sqrt
(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x + a*d^9)*sqrt(-(
(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d
*x + c)^(1/3)*b^2 - (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a))*(b^5/d^1
1)^(5/6) - sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) - 5*(d^2*x + c*d)*(b^5
/d^11)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6)
+ (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))
/(b*x + a)) + 5*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x
 + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 -
(b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a)) - 10*(d^2*x + c*d)*(b^5/d^11
)^(1/6)*log(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b + (b*d^2*x + a*d^2)*(b^5/d^1
1)^(1/6))/(b*x + a)) + 10*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(((b*x + a)^(5/
6)*(d*x + c)^(1/6)*b - (b*d^2*x + a*d^2)*(b^5/d^11)^(1/6))/(b*x + a)) + 12*
(b*x + a)^(5/6)*(d*x + c)^(1/6))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(11/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(11/6),x)
```

```
[Out] int((a + b*x)^(5/6)/(c + d*x)^(11/6), x)
```

3.1782 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

```
[Out] 6/11*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(11/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx = \frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Maple [**A**]
time = 0.18, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}}{11(dx+c)^{\frac{11}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(17/6),x,method=_RETURNVERBOSE)
[Out] -6/11*(b*x+a)^(11/6)/(d*x+c)^(11/6)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.34, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{11}{6}}(dx+c)^{\frac{1}{6}}}{11\,(bc^3-ac^2d+(bcd^2-ad^3)x^2+2\,(bc^2d-acd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
[Out] 6/11*(b*x + a)^(11/6)*(d*x + c)^(1/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*
x^2 + 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(17/6),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x)
[Out] Could not integrate
```

Mupad [**B**]
time = 0.59, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,{\left(a+b\,x\right)}^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{6\,b\,x\,{\left(a+b\,x\right)}^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}\right)\,{\left(c+d\,x\right)}^{1/6}}{x^2-\frac{11\,b\,c^3-11\,a\,c^2\,d}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{22\,c\,d\,x\,\left(a\,d-b\,c\right)}{11\,a\,d^3-11\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(17/6),x)
[Out] -(((6*a*(a + b*x)^(5/6))/(11*a*d^3 - 11*b*c*d^2) + (6*b*x*(a + b*x)^(5/6))/
(11*a*d^3 - 11*b*c*d^2))*(c + d*x)^(1/6))/(x^2 - (11*b*c^3 - 11*a*c^2*d)/(1
1*a*d^3 - 11*b*c*d^2) + (22*c*d*x*(a*d - b*c))/(11*a*d^3 - 11*b*c*d^2))
```

3.1783 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}$$

```
[Out] 6/17*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(17/6)+36/187*b*(b*x+a)^(11/6)/(-a*d
+b*c)^2/(d*x+c)^(11/6)
```

Rubi [A]

time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (36*b*(a + b*x)^(1
1/6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx = \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{(6b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{17(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}$$

Mathematica [A]
time = 0.13, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{11/6}(17bc-11ad+6bdx)}{187(bc-ad)^2(c+dx)^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
[Out] (6*(a + b*x)^(11/6)*(17*b*c - 11*a*d + 6*b*d*x))/(187*(b*c - a*d)^2*(c + d*
x)^(17/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 11481 deep
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}(-6bdx+11ad-17bc)}{187(dx+c)^{\frac{17}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(23/6),x,method=_RETURNVERBOSE)
[Out] -6/187*(b*x+a)^(11/6)*(-6*b*d*x+11*a*d-17*b*c)/(d*x+c)^(17/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(54) = 108$.
time = 0.33, size = 175, normalized size = 2.65

$$\frac{6\,(6\,b^2dx^2+17\,abc-11\,a^2d+(17\,b^2c-5\,abd)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{187\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^2d^3-2\,abcd^4+a^2d^5)x^3+3\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2+3\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/187*(6*b^2*d*x^2 + 17*a*b*c - 11*a^2*d + (17*b^2*c - 5*a*b*d)*x)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3
 - 2*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)
*x^2 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.74, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{5/6}}{187\,d^2\,(a\,d-b\,c)^2}-\frac{(66\,a^2\,d-102\,a\,b\,c)\,(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}+\frac{x\,(102\,b^2\,c-30\,a\,b\,d)\,(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(5/6)/(c + d*x)^(23/6),x)

[Out] ((c + d*x)^(1/6)*((36*b^2*x^2*(a + b*x)^(5/6))/(187*d^2*(a*d - b*c)^2) - ((66*a^2*d - 102*a*b*c)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2) + (x*(102*b^2*c - 30*a*b*d)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 + (3*c*x^2)/d + (3*c^2*x)/d^2)

3.1784 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}$$

```
[Out] 6/23*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(23/6)+72/391*b*(b*x+a)^(11/6)/(-a*d
+b*c)^2/(d*x+c)^(17/6)+432/4301*b^2*(b*x+a)^(11/6)/(-a*d+b*c)^3/(d*x+c)^(11
/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{47, 37\}$

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (72*b*(a + b*x)^(1
1/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (432*b^2*(a + b*x)^(11/6))/(43
01*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(12b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{(72b^2)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}$$
$$= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}$$

Mathematica [A]
time = 0.15, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{23/6}\left(187d^2-\frac{506bd(c+dx)}{a+bx}+\frac{391b^2(c+dx)^2}{(a+bx)^2}\right)}{4301(bc-ad)^3(c+dx)^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
[Out] (6*(a + b*x)^(23/6)*(187*d^2 - (506*b*d*(c + d*x))/(a + b*x) + (391*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]')
[Out] Timed out
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}\left(72b^2x^2d^2-132ab\,d^2x+276b^2cdx+187a^2d^2-506abcd+391b^2c^2\right)}{4301(dx+c)^{\frac{23}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(29/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/4301*(b*x+a)^(11/6)*(72*b^2*d^2*x^2-132*a*b*d^2*x+276*b^2*c*d*x+187*a^2*
d^2-506*a*b*c*d+391*b^2*c^2)/(d*x+c)^(23/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(83) = 166.
time = 0.34, size = 338, normalized size = 3.35

$$\frac{6\,(72\,b^3d^2x^3+391\,ab^2c^2-506\,a^2bcd+187\,a^3d^2+12\,(23\,b^3cd-5\,ab^2d^2)x^2+(391\,b^3c^2-230\,ab^2cd+55\,a^2bd^2)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{4301\,(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7)x^4+4\,(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d^5-a^3cd^6)x^3+6\,(b^3c^5d^2-3\,ab^2c^4d^3+3\,a^2bc^3d^4-a^3c^2d^5)x^2+4\,(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/4301*(72*b^3*d^2*x^3 + 391*a*b^2*c^2 - 506*a^2*b*c*d + 187*a^3*d^2 + 12*(
23*b^3*c*d - 5*a*b^2*d^2)*x^2 + (391*b^3*c^2 - 230*a*b^2*c*d + 55*a^2*b*d^2
)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5
*d^2 - a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d
^7)*x^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x
^3 + 6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2
+ 4*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.94, size = 214, normalized size = 2.12

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{(a+b\,x)^{5/6}\,\left(1122\,a^3\,d^2-3036\,a^2\,b\,c\,d+2346\,a\,b^2\,c^2\right)}{4301\,d^4\,{\left(a\,d-b\,c\right)}^3}+\frac{432\,b^3\,x^3\,(a+b\,x)^{5/6}}{4301\,d^2\,{\left(a\,d-b\,c\right)}^3}+\frac{x\,(a+b\,x)^{5/6}\,\left(330\,a^2\,b\,d^2-1380\,a\,b^2\,c\,d+2346\,b^3\,c^2\right)}{4301\,d^4\,{\left(a\,d-b\,c\right)}^3}-\frac{72\,b^2\,x^2\,\left(5\,a\,d-23\,b\,c\right)\,(a+b\,x)^{5/6}}{4301\,d^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(29/6),x)
```

```
[Out] -((c + d*x)^(1/6)*(((a + b*x)^(5/6)*(1122*a^3*d^2 + 2346*a*b^2*c^2 - 3036*a
^2*b*c*d))/(4301*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(5/6))/(4301*d
^2*(a*d - b*c)^3) + (x*(a + b*x)^(5/6)*(2346*b^3*c^2 + 330*a^2*b*d^2 - 1380
*a*b^2*c*d))/(4301*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(5*a*d - 23*b*c)*(a + b
*x)^(5/6))/(4301*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3
*x)/d^3 + (6*c^2*x^2)/d^2)
```

3.1785 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}} dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}}+\frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}}+\frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}}+\frac{7776b^3(a+bx)^{11/6}}{124729(bc-ad)^4(c+dx}$$

```
[Out] 6/29*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(29/6)+108/667*b*(b*x+a)^(11/6)/(-a*
d+b*c)^2/(d*x+c)^(23/6)+1296/11339*b^2*(b*x+a)^(11/6)/(-a*d+b*c)^3/(d*x+c)^
(17/6)+7776/124729*b^3*(b*x+a)^(11/6)/(-a*d+b*c)^4/(d*x+c)^(11/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(29*(b*c - a*d)*(c + d*x)^(29/6)) + (108*b*(a + b*x)^(
11/6))/(667*(b*c - a*d)^2*(c + d*x)^(23/6)) + (1296*b^2*(a + b*x)^(11/6))/(
11339*(b*c - a*d)^3*(c + d*x)^(17/6)) + (7776*b^3*(a + b*x)^(11/6))/(124729
*(b*c - a*d)^4*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx = \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{(18b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx}{29(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{(216b^2)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{667(bc-ad)^2}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} +$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} +$$

Mathematica [A]
time = 0.17, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{29/6}\left(-4301d^3+\frac{16269bd^2(c+dx)}{a+bx}-\frac{22011b^2d(c+dx)^2}{(a+bx)^2}+\frac{11339b^3(c+dx)^3}{(a+bx)^3}\right)}{124729(bc-ad)^4(c+dx)^{29/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
[Out] (6*(a + b*x)^(29/6)*(-4301*d^3 + (16269*b*d^2*(c + d*x))/(a + b*x) - (22011
*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (11339*b^3*(c + d*x)^3)/(a + b*x)^3))/(12
4729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]')
[Out] Timed out
```

Maple [A]
time = 0.20, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}\left(-1296b^3x^3d^3+2376d^3a\,x^2b^2-6264b^3c\,d^2x^2-3366a^2b\,d^3x+11484a\,b^2c\,d^2x-12006b^3c^2dx+4301a^3d^3-16269a^2bc\,d^2+\right.}{124729(dx+c)^{\frac{29}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(35/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/124729*(b*x+a)^(11/6)*(-1296*b^3*d^3*x^3+2376*a*b^2*d^3*x^2-6264*b^3*c*d
^2*x^2-3366*a^2*b*d^3*x+11484*a*b^2*c*d^2*x-12006*b^3*c^2*d*x+4301*a^3*d^3-
16269*a^2*b*c*d^2+22011*a*b^2*c^2*d-11339*b^3*c^3)/(d*x+c)^(29/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(112) = 224$.
time = 0.35, size = 533, normalized size = 3.92

$$\frac{6\,(1296\,b^4d^3x^4+11339\,ab^3c^3-22011\,a^2b^2c^2d+16269\,a^3bcd^2-4301\,a^4d^3+216\,(29\,b^4cd^2-5\,ab^3d^3)x^3+18\,(667\,b^4c^2d-290\,ab^3cd^2+55\,a^2b^2d^3)x^2+(11339\,b^4c^3-10005\,ab^3c^2d+4785\,a^2b^2cd^2-935\,a^3bd^3)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{124729\,(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9)x^5+5\,(b^4c^5d^4-4\,ab^3c^4d^5+6\,a^2b^2c^3d^6-4\,a^3bc^2d^7+a^4cd^8)x^4+10\,(b^4c^6d^3-4\,ab^3c^5d^4+6\,a^2b^2c^4d^5-4\,a^3bc^3d^6+a^4c^2d^7)x^3+10\,(b^4c^7d^2-4\,ab^3c^6d^3+6\,a^2b^2c^5d^4-4\,a^3bc^4d^5+a^4c^3d^6)x^2+5\,(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="fricas")
```

```
[Out] 6/124729*(1296*b^4*d^3*x^4 + 11339*a*b^3*c^3 - 22011*a^2*b^2*c^2*d + 16269*
a^3*b*c*d^2 - 4301*a^4*d^3 + 216*(29*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 18*(667
*b^4*c^2*d - 290*a*b^3*c*d^2 + 55*a^2*b^2*d^3)*x^2 + (11339*b^4*c^3 - 10005
*a*b^3*c^2*d + 4785*a^2*b^2*c*d^2 - 935*a^3*b*d^3)*x)*(b*x + a)^(5/6)*(d*x
+ c)^(1/6)/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^
6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 +
6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 -
4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 +
5*(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*
c^4*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(35/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.16, size = 303, normalized size = 2.23

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{5/6}}{124729\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{5/6}\,(25806\,a^4\,d^3-97614\,a^3\,b\,c\,d^2+132066\,a^2\,b^2\,c^2\,d-68034\,a\,b^3\,c^3)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{5/6}\,(-5610\,a^3\,b\,d^3+28710\,a^2\,b^2\,c\,d^2-60030\,a\,b^3\,c^2\,d+68034\,b^4\,c^3)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{5/6}\,(55\,a^2\,d^2-290\,a\,b\,c\,d+667\,b^2\,c^2)}{124729\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(5\,a\,d-29\,b\,c)\,(a+b\,x)^{5/6}}{124729\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(35/6),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4*(a + b*x)^(5/6))/(124729*d^2*(a*d - b*c)^4)
 - ((a + b*x)^(5/6)*(25806*a^4*d^3 - 68034*a*b^3*c^3 + 132066*a^2*b^2*c^2*d
 - 97614*a^3*b*c*d^2))/(124729*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(5/6)*(680
34*b^4*c^3 - 5610*a^3*b*d^3 + 28710*a^2*b^2*c*d^2 - 60030*a*b^3*c^2*d))/(12
4729*d^5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(5/6)*(55*a^2*d^2 + 667*b^
2*c^2 - 290*a*b*c*d))/(124729*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(5*a*d - 2
9*b*c)*(a + b*x)^(5/6))/(124729*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*
x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

3.1786 $\int (a+bx)^{5/6}(c+dx)^{11/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/11*(-a*d+b*c)*(b*x+a)^(11/6)*(d*x+c)^(5/6)*hypergeom([-11/6, 11/6],[17/6]
,-d*(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]

time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 11
/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b^2*((b*(c + d*x))/(b*c - a*d)
)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{5/6}(c+dx)^{11/6}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int (a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}\,dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

$$= \frac{6(bc-ad)(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{11/6}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 11/6, 17/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5984 deep
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(11/6), x)
```

Fricas [F]
time = 0.58, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(11/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(11/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{5/6}\,(c+d\,x)^{11/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(5/6)*(c + d*x)^(11/6), x)
```

3.1787 $\int (a+bx)^{5/6}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/11*(b*x+a)^(11/6)*(d*x+c)^(5/6)*hypergeom([-5/6, 11/6],[17/6],-d*(b*x+a)/
(-a*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, -((
d*(a + b*x))/(b*c - a*d))])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{5/6}(c+dx)^{5/6}\,dx = \frac{(c+dx)^{5/6}\int (a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= \frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.61, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{5}{6}} (c + dx)^{\frac{5}{6}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x)^{5/6}\,(c + d\,x)^{5/6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(5/6)*(c + d*x)^(5/6), x)
```

3.1788 $\int \frac{(a+bx)^{5/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

```
[Out] 6/11*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([1/6, 11/6],[17/
6],-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{(a+bx)^{5/6}}{\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}$$

$$= \frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.59, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/6}}{{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(1/6), x)
```

3.1789 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)\sqrt[6]{c+dx}}$$

```
[Out] 6/11*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([7/6, 11/6],[17/
6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)*(c + d*x)^(1/6
))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{7/6}}\,dx = \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.04, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\ {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.57, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(7/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(7/6), x)
```

3.1790 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^2\sqrt[6]{c+dx}}$$

```
[Out] 6/11*b*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([11/6, 13/6],[
17/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1
1/6, 13/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^2*(c + d*x)
^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{13/6}}\,dx = \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}$$
$$= \frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^2\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\ {}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[11
/6, 13/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(13/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(13/6), x)
```

Fricas [F]
time = 0.63, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(13/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(13/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/6}}{{\left(c+d\,x\right)}^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(13/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(13/6), x)
```

3.1791 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=84

$$\frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^3\sqrt[6]{c+dx}}$$

```
[Out] 6/11*b^2*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([11/6, 19/6]
,[17/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {72, 71}

$$\frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*b^2*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1
[11/6, 19/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^3*(c + d*
x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{19/6}}\,dx = \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}$$
$$= \frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^3\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.04, size = 81, normalized size = 0.96

$$\frac{6b(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\ {}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11(bc-ad)^2(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[1
1/6, 19/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*(b*c - a*d)^2*(c + d*x)
^(7/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(19/6), x)
```

Fricas [**F**]
time = 0.56, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2
*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(19/6),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(19/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(19/6), x)
```

3.1792 $\int (a+bx)^{7/6}(c+dx)^{13/6}\,dx$

Optimal. Leaf size=84

$$\frac{6(bc-ad)^2(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/13*(-a*d+b*c)^2*(b*x+a)^(13/6)*(d*x+c)^(1/6)*hypergeom([-13/6, 13/6],[19/
6],-d*(b*x+a)/(-a*d+b*c))/b^3/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,(bc-ad)^2\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6,
13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^3*((b*(c + d*x))/(b*c - a*
d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{7/6}(c+dx)^{13/6}\,dx = \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int (a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}\,dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)^2(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.08, size = 73, normalized size = 0.87

$$\frac{6(a+bx)^{13/6}(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 13/6, 19/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 9880 deep
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
[Out] int((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(13/6), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^2)*x^2 + (b*c^2 + 2*a*c*d)*x)*
(b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(13/6),x)
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{7/6}\,(c+d\,x)^{13/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)*(c + d*x)^(13/6),x)
[Out] int((a + b*x)^(7/6)*(c + d*x)^(13/6), x)
```

3.1793 $\int (a+bx)^{7/6}(c+dx)^{7/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/13*(-a*d+b*c)*(b*x+a)^(13/6)*(d*x+c)^(1/6)*hypergeom([-7/6, 13/6],[19/6],
-d*(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]

time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,(bc-ad)\;{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 13/
6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^2*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{7/6}(c+dx)^{7/6}\,dx = \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int (a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [**A**]
time = 10.05, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{13/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 13/6, 19/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3276 deep
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(1/6), x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(7/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{7/6}\,(c+d\,x)^{7/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)*(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(7/6)*(c + d*x)^(7/6), x)
```

3.1794 $\int (a+bx)^{7/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/13*(b*x+a)^(13/6)*(d*x+c)^(1/6)*hypergeom([-1/6, 13/6],[19/6],-d*(b*x+a)/
(-a*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, -((
d*(a + b*x))/(b*c - a*d))])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{7/6}\sqrt[6]{c+dx}\,dx = \frac{\sqrt[6]{c+dx}\int (a+bx)^{7/6}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

$$= \frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.02, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{7}{6}} \sqrt[6]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(7/6)*(c + d*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{7/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(7/6)*(c + d*x)^(1/6), x)
```

3.1795 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

```
[Out] 6/13*(b*x+a)^(13/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([5/6, 13/6],[19/
6],-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{5/6}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}$$

$$= \frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\, {}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\, {}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.39, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/6)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(5/6), x)
```

3.1796 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6/13*(b*x+a)^(13/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([11/6, 13/6],[19
/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[11/
6, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)*(c + d*x)^(5/
6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{11/6}}\,dx = \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}$$
$$= \frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}\,{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[11
/6, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(11/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(11/6), x)
```

Fricas [**F**]
time = 0.39, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(11/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/6}}{{\left(c+d\,x\right)}^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(11/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(11/6), x)
```

3.1797 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6/13*b*(b*x+a)^(13/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([13/6, 17/6],[
19/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {72, 71}

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$
$$= \frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [**A**]
time = 10.04, size = 81, normalized size = 0.99

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4496 deep
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(17/6), x)
```

Fricas [**F**]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(17/6),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{17/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(17/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(17/6), x)
```

3.1798 $\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=424

$$-\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+$$

```
[Out] -7/12*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(5/6)/d^2+1/2*(b*x+a)^(7/6)*(d*x+c)^
(5/6)/d+7/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/
6))/b^(5/6)/d^(13/6)-7/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d
*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(13/6)+7
/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^
(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(13/6)+7/72*(-a*d+b*c)^2*arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(
5/6)/d^(13/6)*3^(1/2)+7/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x
+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(5/6)/d^(13/6)*3^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}}-\frac{7\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12d^2}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*d^2) + ((a + b*x)^(7/6
)*(c + d*x)^(5/6))/(2*d) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(
13/6)) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(S
qrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c -
 a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b
^(5/6)*d^(13/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(1
44*b^(5/6)*d^(13/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ [m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 65

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ [b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

Rule 214

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 216

Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a /b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k* Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2 *k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 246

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int [1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/ n]

Rule 632

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{(7(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12d}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c}}}{72d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{\sqrt[6]{}}\right.}{}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{1-}\right.}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{\sqrt[3]{b}}\right.}{}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{}}{\sqrt[6]{b}\,\sqrt[6]{}}\right.}{36b^{5/6}d^{13/6}}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{}}{\sqrt[6]{b}\,\sqrt[6]{}}\right.}{36b^{5/6}d^{13/6}}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}}{\sqrt[6]{b}}}{}\right.}{24\sqrt{3}\,b^{5/6}d^{13/6}}$$

Mathematica [A]
time = 0.68, size = 288, normalized size = 0.68

$$\frac{6b^{5/6}\sqrt[6]{d}\sqrt[6]{a+bx}(c+dx)^{5/6}(-7bc+13ad+6bdx)-7\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+7\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+14(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{72b^{5/6}d^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*b^(5/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(5/6)*(-7*b*c + 13*a*d + 6*b*d
*x) - 7*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1
/6)*(c + d*x)^(1/6)))/Sqrt[3]] + 7*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 + (2*d^(
1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] + 14*(b*c - a*d)^
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))] + 7*(b*c - a
*d)^2*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a
 + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(72*b^(5/6)*d^(13/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. $2(318) = 636$.

time = 0.46, size = 5633, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")

```
[Out] -1/144*(28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d
^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(((b^3*c^2*d^2 - 2*a*
b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d
^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/
(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) + sqrt(3
)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d
^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*
a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c
^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*
d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 79
2*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4
*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 +
 a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*
a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*
c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 +
 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 1
```

2*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) + 28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(-((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a

^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c)) + 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(-49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c)) - 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6))/(d*x + c)) + 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6))/(d*x + c)) - 12*(6*b*d*x - 7*b*c + 13*a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6))/d^2

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(1/6), x)
```

3.1799 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$-\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}+\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\,\mathrm{t}}{}$$

```
[Out] -6*(b*x+a)^(7/6)/d/(d*x+c)^(1/6)+7*b*(b*x+a)^(1/6)*(d*x+c)^(5/6)/d^2-7/3*b^
(1/6)*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/d^(13
/6)+7/12*b^(1/6)*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-
b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(13/6)-7/12*b^(1/6)*(-a*d+b*
c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(
1/6)/(d*x+c)^(1/6))/d^(13/6)-7/6*b^(1/6)*(-a*d+b*c)*arctan(-1/3*3^(1/2)+2/3
*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/d^(13/6)*3^(1/2)-7/6*
b^(1/6)*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*
x+c)^(1/6)*3^(1/2))/d^(13/6)*3^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {49, 52, 65, 246, 216, 648, 632, 210, 642, 214}

$$\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}+\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(d*(c + d*x)^(1/6)) + (7*b*(a + b*x)^(1/6)*(c + d*x)^(
5/6))/d^2 + (7*b^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6
)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/
6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6)*(b*c - a*d)*ArcTanh
[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*d^(13/6)) + (7*b^
(1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
 (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6)) - (7*b^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) +
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{(7b)\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{(7b(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{(7(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx \right.}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{(7(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}} \right.}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{\left(7\sqrt[6]{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+ \cdots} \right.}{3d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3d^{13/6}}+\cdots$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3d^{13/6}}+\cdots$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}+\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.04, size = 73, normalized size = 0.18

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3084 vs. $2(301) = 602$.
time = 0.40, size = 3084, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/12*(28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^
4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d
^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*d^11 - a*d^12)*(b*x + a)^(1/6)*(
d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
```

c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + 2
sqrt(3)(d^12*x + c*d^11)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c
)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*(
(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4
*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c))*((b^7*c
^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c
^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + sqrt(3)*(b^7*c^7 - 6*a*
b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 -
6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5
*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*
b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d
^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*
a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^
5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6
 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2
*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*
d^11 - a*d^12)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d +
15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*
d^5 + a^6*b*d^6)/d^13)^(5/6) + 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*c*d^2
 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2
*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
a^6*b*d^6)/d^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d
*x + c)^(2/3) - (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/3))/(d*x + c))*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(5/6) - sqrt(3)*(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^
4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6
*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3
*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a
^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^
5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^
3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(
d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log
(49*((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*
c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^
5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
 + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 1
5*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d
^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c)) - 7*(d^3*x + c*d^2)*((b^7*c^6 - 6*a
*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 -
 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log(-49*((b*c*d^2 - a*d^3)*(b*x +

```
a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) -
 (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^
4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/
(d*x + c)) + 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (d^3*x
+ c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^
3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c
)) - 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^
(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (d^3*x + c*d^2)
*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a
^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c)) - 12*
(b*d*x + 7*b*c - 6*a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^3*x + c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(7/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(7/6), x)
```

3.1800 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=358

$$-\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}}{\sqrt{3}\,\sqrt[6]{}}\right.}{d^{13/6}}$$

```
[Out] -6/7*(b*x+a)^(7/6)/d/(d*x+c)^(7/6)-6*b*(b*x+a)^(1/6)/d^2/(d*x+c)^(1/6)+2*b^
(7/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/d^(13/6)-1/2*b^(
7/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)
^(1/6)/(d*x+c)^(1/6))/d^(13/6)+1/2*b^(7/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)
/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(13/6)+b^(7/6
)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/
2))*3^(1/2)/d^(13/6)+b^(7/6)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b
^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(13/6)
```

Rubi [A]
time = 0.37, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{13/6}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(7*d*(c + d*x)^(7/6)) - (6*b*(a + b*x)^(1/6))/(d^2*(c
+ d*x)^(1/6)) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1
/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcTan
[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]
)/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x
)^(1/6))])/d^(13/6) - (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c +
 d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/
6)) + (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx = -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}+\frac{b\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{b^2\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{(6b)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx,x,\sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{(6b)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{\left(2b^{7/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx,x,\frac{\sqrt[6]{a+}}{\sqrt[6]{c+}}\right.}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{b^{7/6}\text{Subst}\left(\int \frac{-\sqrt{}}{\sqrt[3]{b}-}\right.}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{b^{7/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{}}{}\right.}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-}}{}$$

Mathematica [A]

time = 0.41, size = 257, normalized size = 0.72

$$\frac{-\frac{6\sqrt[6]{d}\,\sqrt[6]{a+bx}\,(7bc+ad+8bdx)}{(c+dx)^{7/6}}-7\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+7\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+14b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)+7b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{7d^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] ((-6*d^(1/6)*(a + b*x)^(1/6)*(7*b*c + a*d + 8*b*d*x))/(c + d*x)^(7/6) - 7*S
qrt[3]*b^(7/6)*ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(
1/6)))/Sqrt[3]] + 7*Sqrt[3]*b^(7/6)*ArcTan[(1 + (2*d^(1/6)*(a + b*x)^(1/6))
/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] + 14*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x
)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))] + 7*b^(7/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a
 + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)
^(1/3))])/(7*d^(13/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(259) = 518$.
time = 0.34, size = 855, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] -1/14*(28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*arctan(-
1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/6) - 2*
sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/
d^13)^(1/6) + (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^
13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) + sqrt(3)*(b^7*d*x + b^7*c))/(b^7*d*
x + b^7*c)) + 28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*a
rctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/
6) - 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d
^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 - (d^5*x + c*d^4)
*(b^7/d^13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) - sqrt(3)*(b^7*d*x + b^7*c))
/(b^7*d*x + b^7*c)) - 7*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*lo
g(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) + (b*x + a)^(1/
3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) + 7*(
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*
x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 -
 (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) - 14*(d^4*x^2 + 2*c*d^3*x + c
^2*d^2)*(b^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b + (d^3*x +
c*d^2)*(b^7/d^13)^(1/6))/(d*x + c)) + 14*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b
^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b - (d^3*x + c*d^2)*(b^
7/d^13)^(1/6))/(d*x + c)) + 12*(8*b*d*x + 7*b*c + a*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(13/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(13/6), x)
```

3.1801 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

```
[Out] 6/13*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(13/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Maple [A]
time = 0.17, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}}{13(dx+c)^{\frac{13}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(19/6),x,method=_RETURNVERBOSE)
[Out] -6/13*(b*x+a)^(13/6)/(d*x+c)^(13/6)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(26) = 52.
time = 0.34, size = 104, normalized size = 3.25

$$\frac{6\,(b^2x^2 + 2\,abx + a^2)(bx + a)^{\frac{1}{6}}(dx + c)^{\frac{5}{6}}}{13\,(bc^4 - ac^3d + (bcd^3 - ad^4)x^3 + 3\,(bc^2d^2 - acd^3)x^2 + 3\,(bc^3d - ac^2d^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
[Out] 6/13*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^4 - a*c
^3*d + (b*c*d^3 - a*d^4)*x^3 + 3*(b*c^2*d^2 - a*c*d^3)*x^2 + 3*(b*c^3*d - a
*c^2*d^2)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.76, size = 199, normalized size = 6.22

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{6\,a^2\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{6\,b^2\,x^2\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{12\,a\,b\,x\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}\right)}{x^3-\frac{13\,b\,c^4-13\,a\,c^3\,d}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c\,d^2\,x^2\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c^2\,d\,x\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(19/6),x)
```

```
[Out] -((c + d*x)^(5/6)*((6*a^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (6*b^2
*x^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (12*a*b*x*(a + b*x)^(1/6))/
(13*a*d^4 - 13*b*c*d^3)))/(x^3 - (13*b*c^4 - 13*a*c^3*d)/(13*a*d^4 - 13*b*c
*d^3) + (39*c*d^2*x^2*(a*d - b*c))/(13*a*d^4 - 13*b*c*d^3) + (39*c^2*d*x*(a
*d - b*c))/(13*a*d^4 - 13*b*c*d^3))
```

3.1802 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}$$

```
[Out] 6/19*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(19/6)+36/247*b*(b*x+a)^(13/6)/(-a*d
+b*c)^2/(d*x+c)^(13/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (36*b*(a + b*x)^(1
3/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx = \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(6b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}(19bc-13ad+6bdx)}{247(bc-ad)^2(c+dx)^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
[Out] (6*(a + b*x)^(13/6)*(19*b*c - 13*a*d + 6*b*d*x))/(247*(b*c - a*d)^2*(c + d*
x)^(19/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]')
[Out] Timed out
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}(-6bdx+13ad-19bc)}{247(dx+c)^{\frac{19}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(25/6),x,method=_RETURNVERBOSE)
[Out] -6/247*(b*x+a)^(13/6)*(-6*b*d*x+13*a*d-19*b*c)/(d*x+c)^(19/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 0.34, size = 235, normalized size = 3.56

$$\frac{6\left(6\,b^3dx^3+19\,a^2bc-13\,a^3d+(19\,b^3c-ab^2d)x^2+2\,(19\,ab^2c-10\,a^2bd)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{247\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^2d^4-2\,abcd^5+a^2d^6)x^4+4\,(b^2c^3d^3-2\,abc^2d^4+a^2cd^5)x^3+6\,(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^2+4\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/247*(6*b^3*d*x^3 + 19*a^2*b*c - 13*a^3*d + (19*b^3*c - a*b^2*d)*x^2 + 2*(
19*a*b^2*c - 10*a^2*b*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^6 - 2*a*
b*c^5*d + a^2*c^4*d^2 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^4 + 4*(b^2*
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x^3 + 6*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 +
 a^2*c^2*d^4)*x^2 + 4*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.91, size = 189, normalized size = 2.86

$$-\frac{{\left(c+d\,x\right)}^{5/6}\,\left(\frac{\left(78\,a^3\,d-114\,a^2\,b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}{247\,d^4\,{\left(a\,d-b\,c\right)}^2}-\frac{36\,b^3\,x^3\,{\left(a+b\,x\right)}^{1/6}}{247\,d^3\,{\left(a\,d-b\,c\right)}^2}-\frac{x^2\,\left(114\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(a+b\,x\right)}^{1/6}}{247\,d^4\,{\left(a\,d-b\,c\right)}^2}+\frac{12\,a\,b\,x\,\left(10\,a\,d-19\,b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}{247\,d^4\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((78*a^3*d - 114*a^2*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d
 - b*c)^2) - (36*b^3*x^3*(a + b*x)^(1/6))/(247*d^3*(a*d - b*c)^2) - (x^2*(1
14*b^3*c - 6*a*b^2*d)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2) + (12*a*b*x*
(10*a*d - 19*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2)))/(x^4 + c^4/d^4
 + (4*c*x^3)/d + (4*c^3*x)/d^3 + (6*c^2*x^2)/d^2)
```

3.1803 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}$$

```
[Out] 6/25*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(25/6)+72/475*b*(b*x+a)^(13/6)/(-a*d
+b*c)^2/(d*x+c)^(19/6)+432/6175*b^2*(b*x+a)^(13/6)/(-a*d+b*c)^3/(d*x+c)^(13
/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{47, 37\}$

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (72*b*(a + b*x)^(1
3/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (432*b^2*(a + b*x)^(13/6))/(61
75*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(12b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{(72b^2)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$
$$= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}$$

Mathematica [A]
time = 0.16, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{13/6}\left(475b^2+\frac{247d^2(a+bx)^2}{(c+dx)^2}-\frac{650bd(a+bx)}{c+dx}\right)}{6175(bc-ad)^3(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
[Out] (6*(a + b*x)^(13/6)*(475*b^2 + (247*d^2*(a + b*x)^2)/(c + d*x)^2 - (650*b*d
*(a + b*x))/(c + d*x)))/(6175*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]')
[Out] Timed out
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}\left(72b^2x^2d^2-156ab\,d^2x+300b^2cdx+247a^2d^2-650abcd+475b^2c^2\right)}{6175(dx+c)^{\frac{25}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(31/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/6175*(b*x+a)^(13/6)*(72*b^2*d^2*x^2-156*a*b*d^2*x+300*b^2*c*d*x+247*a^2*
d^2-650*a*b*c*d+475*b^2*c^2)/(d*x+c)^(25/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(83) = 166$.
time = 0.36, size = 427, normalized size = 4.23

$$\frac{6\,(72\,b^4d^2x^4+475\,a^2b^2c^2-650\,a^3bcd+247\,a^4d^2+12\,(25\,b^4cd-ab^3d^2)x^3+(475\,b^4c^2-50\,ab^3cd+7\,a^2b^2d^2)x^2+2\,(475\,ab^3c^2-500\,a^2b^2cd+169\,a^3bd^2)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{6175\,(b^3c^8-3\,ab^2c^7d+3\,a^2bc^6d^2-a^3c^5d^3+(b^3c^3d^5-3\,ab^2c^2d^6+3\,a^2bcd^7-a^3d^8)x^5+5\,(b^3c^4d^4-3\,ab^2c^3d^5+3\,a^2bc^2d^6-a^3cd^7)x^4+10\,(b^3c^5d^3-3\,ab^2c^4d^4+3\,a^2bc^3d^5-a^3c^2d^6)x^3+10\,(b^3c^6d^2-3\,ab^2c^5d^3+3\,a^2bc^4d^4-a^3c^3d^5)x^2+5\,(b^3c^7d-3\,ab^2c^6d^2+3\,a^2bc^5d^3-a^3c^4d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/6175*(72*b^4*d^2*x^4 + 475*a^2*b^2*c^2 - 650*a^3*b*c*d + 247*a^4*d^2 + 12
*(25*b^4*c*d - a*b^3*d^2)*x^3 + (475*b^4*c^2 - 50*a*b^3*c*d + 7*a^2*b^2*d^2
)*x^2 + 2*(475*a*b^3*c^2 - 500*a^2*b^2*c*d + 169*a^3*b*d^2)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3 + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*x^5 + 5*(b^3*
c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 10*(b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^3 + 10*(b^3*c^6*d^
2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2 + 5*(b^3*c^7*d - 3
*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.14, size = 278, normalized size = 2.75

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{(a+b\,x)^{1/6}\,(1482\,a^4\,d^2-3900\,a^3\,b\,c\,d+2850\,a^2\,b^2\,c^2)}{6175\,d^5\,(a\,d-b\,c)^3}+\frac{432\,b^4\,x^4\,(a+b\,x)^{1/6}}{6175\,d^3\,(a\,d-b\,c)^3}+\frac{x^2\,(a+b\,x)^{1/6}\,(42\,a^2\,b^2\,d^2-300\,a\,b^3\,c\,d+2850\,b^4\,c^2)}{6175\,d^5\,(a\,d-b\,c)^3}-\frac{72\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{6175\,d^4\,(a\,d-b\,c)^3}+\frac{12\,a\,b\,x\,(a+b\,x)^{1/6}\,(169\,a^2\,d^2-500\,a\,b\,c\,d+475\,b^2\,c^2)}{6175\,d^5\,(a\,d-b\,c)^3}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(31/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(1482*a^4*d^2 + 2850*a^2*b^2*c^2 - 3900
*a^3*b*c*d))/(6175*d^5*(a*d - b*c)^3) + (432*b^4*x^4*(a + b*x)^(1/6))/(6175
*d^3*(a*d - b*c)^3) + (x^2*(a + b*x)^(1/6)*(2850*b^4*c^2 + 42*a^2*b^2*d^2 -
 300*a*b^3*c*d))/(6175*d^5*(a*d - b*c)^3) - (72*b^3*x^3*(a*d - 25*b*c)*(a +
 b*x)^(1/6))/(6175*d^4*(a*d - b*c)^3) + (12*a*b*x*(a + b*x)^(1/6)*(169*a^2*
d^2 + 475*b^2*c^2 - 500*a*b*c*d))/(6175*d^5*(a*d - b*c)^3)))/(x^5 + c^5/d^5
 + (5*c*x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

3.1804 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}}+\frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}}+\frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}}+\frac{7776b^3(a+bx)^{13/6}}{191425(bc-ad)^4(c+dx}$$

```
[Out] 6/31*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(31/6)+108/775*b*(b*x+a)^(13/6)/(-a*
d+b*c)^2/(d*x+c)^(25/6)+1296/14725*b^2*(b*x+a)^(13/6)/(-a*d+b*c)^3/(d*x+c)^
(19/6)+7776/191425*b^3*(b*x+a)^(13/6)/(-a*d+b*c)^4/(d*x+c)^(13/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(31*(b*c - a*d)*(c + d*x)^(31/6)) + (108*b*(a + b*x)^(
13/6))/(775*(b*c - a*d)^2*(c + d*x)^(25/6)) + (1296*b^2*(a + b*x)^(13/6))/(
14725*(b*c - a*d)^3*(c + d*x)^(19/6)) + (7776*b^3*(a + b*x)^(13/6))/(191425
*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx = \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{(18b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx}{31(bc-ad)}$$

$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{(216b^2)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{775(bc-ad)^2}$$

$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} +$$

$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} +$$

Mathematica [A]
time = 0.20, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}\left(14725b^3 - \frac{6175d^3(a+bx)^3}{(c+dx)^3} + \frac{22971bd^2(a+bx)^2}{(c+dx)^2} - \frac{30225b^2d(a+bx)}{c+dx}\right)}{191425(bc-ad)^4(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(14725*b^3 - (6175*d^3*(a + b*x)^3)/(c + d*x)^3 + (2297
1*b*d^2*(a + b*x)^2)/(c + d*x)^2 - (30225*b^2*d*(a + b*x))/(c + d*x)))/(191
425*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.18, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}\left(-1296b^3x^3d^3+2808d^3a\,x^2b^2-6696b^3c\,d^2x^2-4446a^2b\,d^3x+14508a\,b^2c\,d^2x-13950b^3c^2dx+6175a^3d^3-22971a^2bc\,d^2+\right.}{191425(dx+c)^{\frac{31}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(37/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/191425*(b*x+a)^(13/6)*(-1296*b^3*d^3*x^3+2808*a*b^2*d^3*x^2-6696*b^3*c*d
^2*x^2-4446*a^2*b*d^3*x+14508*a*b^2*c*d^2*x-13950*b^3*c^2*d*x+6175*a^3*d^3-
22971*a^2*b*c*d^2+30225*a*b^2*c^2*d-14725*b^3*c^3)/(d*x+c)^(31/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(112) = 224$.
time = 0.32, size = 649, normalized size = 4.77

$$\frac{6\,(1296\,b^5d^3x^5 + 14725\,a^2b^3c^3 - 30225\,a^3b^2c^2d + 22971\,a^4bcd^2 - 6175\,a^5d^3 + 216\,(31\,b^5cd^2 - ab^4d^3)x^4 + 18\,(775\,b^5c^2d - 62\,ab^4cd^2 + 7\,a^2b^3d^3)x^3 + (14725\,b^5c^3 - 2325\,ab^4c^2d + 651\,a^2b^3cd^2 - 91\,a^3b^2d^3)x^2 + 2\,(14725\,ab^4c^3 - 23250\,a^2b^3c^2d + 15717\,a^3b^2cd^2 - 3952\,a^4bd^3)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{191425\,(b^4c^{10} - 4\,ab^3c^9d + 6\,a^2b^2c^8d^2 - 4\,a^3bc^7d^3 + a^4c^6d^4 + (b^4c^4d^6 - 4\,ab^3c^3d^7 + 6\,a^2b^2c^2d^8 - 4\,a^3bcd^9 + a^4d^{10})x^6 + 6\,(b^4c^5d^5 - 4\,ab^3c^4d^6 + 6\,a^2b^2c^3d^7 - 4\,a^3bc^2d^8 + a^4cd^9)x^5 + 15\,(b^4c^6d^4 - 4\,ab^3c^5d^5 + 6\,a^2b^2c^4d^6 - 4\,a^3bc^3d^7 + a^4c^2d^8)x^4 + 20\,(b^4c^7d^3 - 4\,ab^3c^6d^4 + 6\,a^2b^2c^5d^5 - 4\,a^3bc^4d^6 + a^4c^3d^7)x^3 + 15\,(b^4c^8d^2 - 4\,ab^3c^7d^3 + 6\,a^2b^2c^6d^4 - 4\,a^3bc^5d^5 + a^4c^4d^6)x^2 + 6\,(b^4c^9d - 4\,ab^3c^8d^2 + 6\,a^2b^2c^7d^3 - 4\,a^3bc^6d^4 + a^4c^5d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="fricas")
```

```
[Out] 6/191425*(1296*b^5*d^3*x^5 + 14725*a^2*b^3*c^3 - 30225*a^3*b^2*c^2*d + 2297
1*a^4*b*c*d^2 - 6175*a^5*d^3 + 216*(31*b^5*c*d^2 - a*b^4*d^3)*x^4 + 18*(775
*b^5*c^2*d - 62*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*x^3 + (14725*b^5*c^3 - 2325*a*
b^4*c^2*d + 651*a^2*b^3*c*d^2 - 91*a^3*b^2*d^3)*x^2 + 2*(14725*a*b^4*c^3 -
23250*a^2*b^3*c^2*d + 15717*a^3*b^2*c*d^2 - 3952*a^4*b*d^3)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b
*c^7*d^3 + a^4*c^6*d^4 + (b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8
 - 4*a^3*b*c*d^9 + a^4*d^10)*x^6 + 6*(b^4*c^5*d^5 - 4*a*b^3*c^4*d^6 + 6*a^2
*b^2*c^3*d^7 - 4*a^3*b*c^2*d^8 + a^4*c*d^9)*x^5 + 15*(b^4*c^6*d^4 - 4*a*b^3
*c^5*d^5 + 6*a^2*b^2*c^4*d^6 - 4*a^3*b*c^3*d^7 + a^4*c^2*d^8)*x^4 + 20*(b^4
*c^7*d^3 - 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^5*d^5 - 4*a^3*b*c^4*d^6 + a^4*c^3*
d^7)*x^3 + 15*(b^4*c^8*d^2 - 4*a*b^3*c^7*d^3 + 6*a^2*b^2*c^6*d^4 - 4*a^3*b*
c^5*d^5 + a^4*c^4*d^6)*x^2 + 6*(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7
*d^3 - 4*a^3*b*c^6*d^4 + a^4*c^5*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(37/6),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.43, size = 385, normalized size = 2.83

$$\frac{(c+d\,x)^{5/6}\,\left(\frac{7776\,b^5\,x^5\,(a+b\,x)^{1/6}}{191425\,d^3\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\,(37050\,a^5\,d^3-137826\,a^4\,b\,c\,d^2+181350\,a^3\,b^2\,c^2\,d-88350\,a^2\,b^3\,c^3)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x^2\,(a+b\,x)^{1/6}\,(-546\,a^3\,b^2\,d^3+3906\,a^2\,b^3\,c\,d^2-13950\,a\,b^4\,c^2\,d+88350\,b^5\,c^3)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\,(-47424\,a^4\,b\,d^3+188604\,a^3\,b^2\,c\,d^2-279000\,a^2\,b^3\,c^2\,d+176700\,a\,b^4\,c^3)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{108\,b^3\,x^3\,(a+b\,x)^{1/6}\,(7\,a^2\,d^2-62\,a\,b\,c\,d+775\,b^2\,c^2)}{191425\,d^5\,(a\,d-b\,c)^4}-\frac{1296\,b^4\,x^4\,(a\,d-31\,b\,c)\,(a+b\,x)^{1/6}}{191425\,d^4\,(a\,d-b\,c)^4}\right)}{x^6+\frac{c^6}{d^6}+\frac{6\,c\,x^5}{d}+\frac{6\,c^5\,x}{d^5}+\frac{15\,c^2\,x^4}{d^2}+\frac{20\,c^3\,x^3}{d^3}+\frac{15\,c^4\,x^2}{d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(37/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^5*x^5*(a + b*x)^(1/6))/(191425*d^3*(a*d - b*c)^4)
 - ((a + b*x)^(1/6)*(37050*a^5*d^3 - 88350*a^2*b^3*c^3 + 181350*a^3*b^2*c^2
*d - 137826*a^4*b*c*d^2))/(191425*d^6*(a*d - b*c)^4) + (x^2*(a + b*x)^(1/6)
*(88350*b^5*c^3 - 546*a^3*b^2*d^3 + 3906*a^2*b^3*c*d^2 - 13950*a*b^4*c^2*d)
)/(191425*d^6*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(176700*a*b^4*c^3 - 47424
*a^4*b*d^3 - 279000*a^2*b^3*c^2*d + 188604*a^3*b^2*c*d^2))/(191425*d^6*(a*d
 - b*c)^4) + (108*b^3*x^3*(a + b*x)^(1/6)*(7*a^2*d^2 + 775*b^2*c^2 - 62*a*b
*c*d))/(191425*d^5*(a*d - b*c)^4) - (1296*b^4*x^4*(a*d - 31*b*c)*(a + b*x)^
(1/6))/(191425*d^4*(a*d - b*c)^4)))/(x^6 + c^6/d^6 + (6*c*x^5)/d + (6*c^5*x
)/d^5 + (15*c^2*x^4)/d^2 + (20*c^3*x^3)/d^3 + (15*c^4*x^2)/d^4)
```

3.1805 $\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=424

$$\frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2}+\frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}-\frac{7(}{}$$

```
[Out] 7/12*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b^2+1/2*(b*x+a)^(5/6)*(d*x+c)^(
7/6)/b+7/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6
))/b^(13/6)/d^(5/6)-7/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*
x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(13/6)/d^(5/6)+7/
144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(
1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(13/6)/d^(5/6)-7/72*(-a*d+b*c)^2*arctan
(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(1
3/6)/d^(5/6)*3^(1/2)-7/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+
a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(13/6)/d^(5/6)*3^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}}+\frac{7(a+bx)^{5/6}\sqrt[6]{c+dx}\,(bc-ad)}{12b^2}+\frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (7*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^2) + ((a + b*x)^(5/6)
*(c + d*x)^(7/6))/(2*b) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(
5/6)) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(13/6)*d^(5/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(13/6)*d^(5/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+\cdots}}{72b^2}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{\cdots}{\left(c-\frac{ac}{b}\cdots\right.}\cdots\right.}{12b\cdots}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}\cdots}\cdots\right.}{12b^3}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt[3]{b}-\cdots}\cdots\right.}{36\cdots}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a\cdots}}{\sqrt[6]{b}\,\sqrt[6]{c\cdots}}\right.}{36b^{13/6}d^{5/6}}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a\cdots}}{\sqrt[6]{b}\,\sqrt[6]{c\cdots}}\right.}{36b^{13/6}d^{5/6}}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{\cdots}}{\sqrt[6]{b}\,\sqrt[6]{\cdots}}}{\sqrt{\cdots}}\right.}{24\sqrt{3}\,b^{13/6}d^{5/6}}$$

Mathematica [A]
time = 0.70, size = 278, normalized size = 0.66

$$\frac{(bc-ad)^2\left(\frac{6\sqrt[6]{b}\,(a+bx)^{5/6}\sqrt[6]{c+dx}\,(13bc-7ad+6bdx)}{(bc-ad)^2}-\frac{7\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right)}{d^{5/6}}+\frac{7\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right)}{d^{5/6}}+\frac{14\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{d^{5/6}}+\frac{7\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{d^{5/6}}\right)}{72b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(1/6)*(a + b*x)^(5/6)*(c + d*x)^(1/6)*(13*b*c - 7*a*d
+ 6*b*d*x))/(b*c - a*d)^2 - (7*Sqrt[3]*ArcTan[(1 - (2*b^(1/6)*(c + d*x)^(1/
6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]])/d^(5/6) + (7*Sqrt[3]*ArcTan[(1 + (
2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]])/d^(5/6) + (
14*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/d^(5/6) +
(7*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a +
b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/d^(5/6)))/(72*b^(13/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="maxima")

[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. $2(318) = 636$.
time = 0.41, size = 5633, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="fricas")

[Out] -1/144*(28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*sqrt(((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) + sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d

^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*sqrt(-((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 7*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^

7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(49*((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d^2*x
+ a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x
+ a)) + 7*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(
-49*((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*
(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(b^13*d^5))^(1/3))/(b*x + a)) - 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
5/6)*(d*x + c)^(1/6) + (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/6))/(b*x + a)) + 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(5/6)*(d*x + c)^(1/6) - (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6))/(b*x + a)) - 12*(6*b*d*x + 13*b*c - 7*a*d)*(b*x +

```
a)^(5/6)*(d*x + c)^(1/6))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(1/6),x)
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(1/6), x)
```

3.1806 $\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=378

$$\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+b}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+}}\right.}{2\sqrt{3}\,b^{7/6}d^{5/6}}$$

```
[Out] (b*x+a)^(5/6)*(d*x+c)^(1/6)/b+1/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(5/6)-1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(
b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(
7/6)/d^(5/6)+1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)
+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(5/6)-1/6*(-a*d+b*c
)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/
2))/b^(7/6)/d^(5/6)*3^(1/2)-1/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(
b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(7/6)/d^(5/6)*3^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/b + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/
6)*d^(5/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/6)*d^(5/6)) + ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(7/6)
*d^(5/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(
5/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(5/6)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{2/3}} + \frac{(bc \cdots}{\cdots}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-\sqrt[6]{b}\cdots}{\sqrt[3]{b}-\sqrt[6]{b}\cdots}\right.}{12b\cdots}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt[3]{c\cdots}}\right.}{12b^{7\cdots}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}}{\sqrt{\cdots}}\right.}{2\sqrt{3}\,b^{7/6}d^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.02, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{5/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3025 vs. 2(280) = 560.
time = 0.40, size = 3025, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="fricas")

[Out] 1/12*(4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4

```
- 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x +
 a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5
))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5
))^(1/3))/(b*x + a)) - b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5)
)^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) + 2*b*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6) + (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^7*d^5))^(1/6))/(b*x + a)) - 2*b*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1
/6) - (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))
^(1/6))/(b*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(1/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(1/6), x)
```

3.1807 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right.}{\sqrt[6]{b}\,d^{5/6}}$$

```
[Out] 2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(5/6)-1/2*
ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6
)/(d*x+c)^(1/6))/b^(1/6)/d^(5/6)+1/2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+
c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(5/6)-arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(
1/2)/b^(1/6)/d^(5/6)-arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(
d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(1/6)/d^(5/6)
```

Rubi [A]
time = 0.36, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c
 + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) + (
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^
(5/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)
*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6)) + Log[b^(1/3
) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^
(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx = \frac{6\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{2/3}} + \frac{2\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\right.}{\sqrt[6]{b}\,d^{2/3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right.}{2\sqrt[6]{b}\,d^{5/6}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} + \frac{2\tanh}{}$$

Mathematica [A]

time = 0.17, size = 194, normalized size = 0.63

$$\frac{\sqrt{3}\left(-\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)+\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*(-ArcTan[(1 - (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)
))/Sqrt[3]] + ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1
/6)))/Sqrt[3]]) + 2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1
/6))] + ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*
(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(b^(1/6)*d^(5/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(222) = 444.
time = 0.33, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}bd^4\left(\frac{1}{bd^5}\right)^{\frac{5}{6}} - 2\sqrt{3}\left(b^2d^4x+abd^4\right)\sqrt{\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}} + \left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}} + (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{1}{bd^5}\right)^{\frac{5}{6}} + \sqrt{3}(bx+a)}{3(bx+a)}\right) - 2\sqrt{3}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}bd^4\left(\frac{1}{bd^5}\right)^{\frac{5}{6}} - 2\sqrt{3}\left(b^2d^4x+abd^4\right)\sqrt{-\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}} - \left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}} - (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{1}{bd^5}\right)^{\frac{5}{6}} - \sqrt{3}(bx+a)}{3(bx+a)}\right) + \frac{1}{2}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}} + \left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}} + (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right) - \frac{1}{2}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}} - \left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}} - (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right) + \left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+ad)\left(\frac{1}{bd^5}\right)^{\frac{1}{6}} + (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right) - \left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+ad)\left(\frac{1}{bd^5}\right)^{\frac{1}{6}} - (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x +
c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqrt(3)*(b^2*d^4*x + a*b*d^4)*sqrt(((b
*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/6) + (b*d^2*x + a*d^2)*(1/(b
*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6
) + sqrt(3)*(b*x + a))/(b*x + a)) - 2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqr
```

```
t(3)*(b^2*d^4*x + a*b*d^4)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d
^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6) - sqrt(3)*(b*x + a))/(b*x + a)) + 1/2
*(1/(b*d^5))^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/
6) + (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))
/(b*x + a)) - 1/2*(1/(b*d^5))^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)
*d*(1/(b*d^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3
)*(d*x + c)^(1/3))/(b*x + a)) + (1/(b*d^5))^(1/6)*log(((b*d*x + a*d)*(1/(b*
d^5))^(1/6) + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) - (1/(b*d^5))^(1/
6)*log(-((b*d*x + a*d)*(1/(b*d^5))^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))
/(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(5/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/6}\,(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)), x)
```

3.1808 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6/5*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx = \frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]')
[Out] Timed out
```

Maple [**A**]
time = 0.19, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}}{5(dx+c)^{\frac{5}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x,method=_RETURNVERBOSE)
[Out] -6/5*(b*x+a)^(5/6)/(d*x+c)^(5/6)/(a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

Fricas [**A**]
time = 0.33, size = 42, normalized size = 1.31

$$\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\,(bc^2-acd+(bcd-ad^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] 6/5*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(11/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.76, size = 27, normalized size = 0.84

$$-\frac{6\,{\left(a+b\,x\right)}^{5/6}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(c+d\,x\right)}^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x)
```

```
[Out] -(6*(a + b*x)^(5/6))/((5*a*d - 5*b*c)*(c + d*x)^(5/6))
```

3.1809 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}}+\frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6/11*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(11/6)+36/55*b*(b*x+a)^(5/6)/(-a*d+b*
c)^2/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{47, 37\}$

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(11*(b*c - a*d)*(c + d*x)^(11/6)) + (36*b*(a + b*x)^(5/
6))/(55*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx = \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}} + \frac{(6b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{11(bc-ad)}$$
$$= \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}} + \frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [**A**]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{5/6}(11bc-5ad+6bdx)}{55(bc-ad)^2(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(11*b*c - 5*a*d + 6*b*d*x))/(55*(b*c - a*d)^2*(c + d*x)^
(11/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4497 deep
```

Maple [**A**]
time = 0.20, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}(-6bdx+5ad-11bc)}{55(dx+c)^{\frac{11}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/55*(b*x+a)^(5/6)*(-6*b*d*x+5*a*d-11*b*c)/(d*x+c)^(11/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.33, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx + 11\,bc - 5\,ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")

[Out] 6/55*(6*b*d*x + 11*b*c - 5*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(17/6),x)

[Out] Exception raised: SystemError

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x)

[Out] Could not integrate

Mupad [B]
time = 0.86, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{x\,(66\,c\,b^2+6\,a\,d\,b)}{55\,d^2\,(a\,d-b\,c)^2}-\frac{30\,a^2\,d-66\,a\,b\,c}{55\,d^2\,(a\,d-b\,c)^2}+\frac{36\,b^2\,x^2}{55\,d\,(a\,d-b\,c)^2}\right)}{x^2\,(a+b\,x)^{1/6}+\frac{c^2\,(a+b\,x)^{1/6}}{d^2}+\frac{2\,c\,x\,(a+b\,x)^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((x*(66*b^2*c + 6*a*b*d))/(55*d^2*(a*d - b*c)^2) - (30*a^2
*d - 66*a*b*c)/(55*d^2*(a*d - b*c)^2) + (36*b^2*x^2)/(55*d*(a*d - b*c)^2)))
/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(a + b*x)^(1/6))
/d)
```

3.1810 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}}+\frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)^{5/6}}$$

```
[Out] 6/17*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(17/6)+72/187*b*(b*x+a)^(5/6)/(-a*d+b
*c)^2/(d*x+c)^(11/6)+432/935*b^2*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (72*b*(a + b*x)^(5/
6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6)) + (432*b^2*(a + b*x)^(5/6))/(935*(
b*c - a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx = \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{(12b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{17(bc-ad)}$$

$$= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}} + \frac{(72b^2)\int \frac{1}{\sqrt[6]{a+bx}\,(c+d}}{187(bc-ad)^2}$$

$$= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}} + \frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{17/6}\left(55d^2 - \frac{170bd(c+dx)}{a+bx} + \frac{187b^2(c+dx)^2}{(a+bx)^2}\right)}{935(bc-ad)^3(c+dx)^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
[Out] (6*(a + b*x)^(17/6)*(55*d^2 - (170*b*d*(c + d*x))/(a + b*x) + (187*b^2*(c +
 d*x)^2)/(a + b*x)^2))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]')
[Out] Timed out
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}\left(72b^2x^2d^2-60ab\,d^2x+204b^2cdx+55a^2d^2-170abcd+187b^2c^2\right)}{935(dx+c)^{\frac{17}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/935*(b*x+a)^(5/6)*(72*b^2*d^2*x^2-60*a*b*d^2*x+204*b^2*c*d*x+55*a^2*d^2-
170*a*b*c*d+187*b^2*c^2)/(d*x+c)^(17/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(83) = 166.
time = 0.33, size = 252, normalized size = 2.50

$$\frac{6\,(72\,b^2d^2x^2+187\,b^2c^2-170\,abcd+55\,a^2d^2+12\,(17\,b^2cd-5\,abd^2)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)x^3+3\,(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5)x^2+3\,(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/935*(72*b^2*d^2*x^2 + 187*b^2*c^2 - 170*a*b*c*d + 55*a^2*d^2 + 12*(17*b^2
*c*d - 5*a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^6 - 3*a*b^2*c^5
*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2
*b*c*d^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^
4 - a^3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3
*c^2*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x)

[Out] Could not integrate

Mupad [B]
time = 1.03, size = 203, normalized size = 2.01

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{330\,a^3\,d^2-1020\,a^2\,b\,c\,d+1122\,a\,b^2\,c^2}{935\,d^3\,(a\,d-b\,c)^3}+\frac{x\,(-30\,a^2\,b\,d^2+204\,a\,b^2\,c\,d+1122\,b^3\,c^2)}{935\,d^3\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3}{935\,d\,(a\,d-b\,c)^3}+\frac{72\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x)

[Out] -((c + d*x)^(1/6)*((330*a^3*d^2 + 1122*a*b^2*c^2 - 1020*a^2*b*c*d)/(935*d^3*(a*d - b*c)^3) + (x*(1122*b^3*c^2 - 30*a^2*b*d^2 + 204*a*b^2*c*d))/(935*d^3*(a*d - b*c)^3) + (432*b^3*x^3)/(935*d*(a*d - b*c)^3) + (72*b^2*x^2*(a*d + 17*b*c))/(935*d^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/6) + (c^3*(a + b*x)^(1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*x)^(1/6))/d^2)

3.1811 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}}+\frac{7776b^3(a+bx)^{5/6}}{21505(bc-ad)^4(c+dx)}$$

```
[Out] 6/23*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(23/6)+108/391*b*(b*x+a)^(5/6)/(-a*d+
b*c)^2/(d*x+c)^(17/6)+1296/4301*b^2*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(11/
6)+7776/21505*b^3*(b*x+a)^(5/6)/(-a*d+b*c)^4/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (108*b*(a + b*x)^(5
/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (1296*b^2*(a + b*x)^(5/6))/(430
1*(b*c - a*d)^3*(c + d*x)^(11/6)) + (7776*b^3*(a + b*x)^(5/6))/(21505*(b*c
- a*d)^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(18b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$
$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{(216b^2)\int \frac{1}{\sqrt[6]{a+bx}\,(c+}}{391(bc-ad)^2}$$
$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx}$$
$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{23/6}\left(-935d^3+\frac{3795bd^2(c+dx)}{a+bx}-\frac{5865b^2d(c+dx)^2}{(a+bx)^2}+\frac{4301b^3(c+dx)^3}{(a+bx)^3}\right)}{21505(bc-ad)^4(c+dx)^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
[Out] (6*(a + b*x)^(23/6)*(-935*d^3 + (3795*b*d^2*(c + d*x))/(a + b*x) - (5865*b^
2*d*(c + d*x)^2)/(a + b*x)^2 + (4301*b^3*(c + d*x)^3)/(a + b*x)^3))/(21505*
(b*c - a*d)^4*(c + d*x)^(23/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]')
[Out] Timed out
```

Maple [A]
time = 0.23, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}\left(-1296b^3x^3d^3+1080d^3a\,x^2b^2-4968b^3c\,d^2x^2-990a^2b\,d^3x+4140a\,b^2c\,d^2x-7038b^3c^2dx+935a^3d^3-3795a^2bc\,d^2+5865a\,b^2\right.}{21505(dx+c)^{\frac{23}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/21505*(b*x+a)^(5/6)*(-1296*b^3*d^3*x^3+1080*a*b^2*d^3*x^2-4968*b^3*c*d^2
*x^2-990*a^2*b*d^3*x+4140*a*b^2*c*d^2*x-7038*b^3*c^2*d*x+935*a^3*d^3-3795*a
^2*b*c*d^2+5865*a*b^2*c^2*d-4301*b^3*c^3)/(d*x+c)^(23/6)/(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(112) = 224.
time = 0.32, size = 420, normalized size = 3.09

$$\frac{6\left(1296\,b^3d^3x^3+4301\,b^3c^3-5865\,ab^2c^2d+3795\,a^2bcd^2-935\,a^3d^3+216\,(23\,b^3cd^2-5\,ab^2d^3)x^2+18\,(391\,b^3c^2d-230\,ab^2cd^2+55\,a^2bd^3)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{21505\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8)x^4+4\,(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7)x^3+6\,(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6)x^2+4\,(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/21505*(1296*b^3*d^3*x^3 + 4301*b^3*c^3 - 5865*a*b^2*c^2*d + 3795*a^2*b*c*
d^2 - 935*a^3*d^3 + 216*(23*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 18*(391*b^3*c^2*
d - 230*a*b^2*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^4
*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 +
(b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^
8)*x^4 + 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2
*d^6 + a^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^
4 - 4*a^3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.20, size = 292, normalized size = 2.15

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{7776\,b^4\,x^4}{21505\,d\,(a\,d-b\,c)^4}-\frac{5610\,a^4\,d^3-22770\,a^3\,b\,c\,d^2+35190\,a^2\,b^2\,c^2\,d-25806\,a\,b^3\,c^3}{21505\,d^4\,(a\,d-b\,c)^4}+\frac{x\,(330\,a^3\,b\,d^3-2070\,a^2\,b^2\,c\,d^2+7038\,a\,b^3\,c^2\,d+25806\,b^4\,c^3)}{21505\,d^4\,(a\,d-b\,c)^4}+\frac{1296\,b^3\,x^3\,(a\,d+23\,b\,c)}{21505\,d^2\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(-5\,a^2\,d^2+46\,a\,b\,c\,d+391\,b^2\,c^2)}{21505\,d^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/6}+\frac{c^4\,(a+b\,x)^{1/6}}{d^4}+\frac{6\,c^2\,x^2\,(a+b\,x)^{1/6}}{d^2}+\frac{4\,c\,x^3\,(a+b\,x)^{1/6}}{d}+\frac{4\,c^3\,x\,(a+b\,x)^{1/6}}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4)/(21505*d*(a*d - b*c)^4) - (5610*a^4*d^3 -
25806*a*b^3*c^3 + 35190*a^2*b^2*c^2*d - 22770*a^3*b*c*d^2)/(21505*d^4*(a*d
- b*c)^4) + (x*(25806*b^4*c^3 + 330*a^3*b*d^3 - 2070*a^2*b^2*c*d^2 + 7038*a
*b^3*c^2*d))/(21505*d^4*(a*d - b*c)^4) + (1296*b^3*x^3*(a*d + 23*b*c))/(215
05*d^2*(a*d - b*c)^4) + (108*b^2*x^2*(391*b^2*c^2 - 5*a^2*d^2 + 46*a*b*c*d)
)/(21505*d^3*(a*d - b*c)^4)))/(x^4*(a + b*x)^(1/6) + (c^4*(a + b*x)^(1/6))/
d^4 + (6*c^2*x^2*(a + b*x)^(1/6))/d^2 + (4*c*x^3*(a + b*x)^(1/6))/d + (4*c^
3*x*(a + b*x)^(1/6))/d^3)
```

3.1812 $\int \frac{(c+dx)^{11/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/5*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(5/6)*hypergeom([-11/6, 5/6],[11/6],-d
*(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 5/6
, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b^2*((b*(c + d*x))/(b*c - a*d))^(
5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{\sqrt[6]{a+bx}}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}{\sqrt[6]{a+bx}}\,dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= \frac{6(bc-ad)(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [**A**]
time = 10.04, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{5/6}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 5/6, 11/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
[Out] int((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(1/6), x)
```

Fricas [F]
time = 0.60, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(11/6)/(b*x + a)^(1/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(1/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(11/6)/(a + b*x)^(1/6), x)
```

3.1813 $\int \frac{(c+dx)^{5/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/5*(b*x+a)^(5/6)*(d*x+c)^(5/6)*hypergeom([-5/6, 5/6],[11/6],-d*(b*x+a)/(-a
*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{72, 71\}$

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, -((d*
(a + b*x))/(b*c - a*d))])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{\sqrt[6]{a+bx}}\,dx = \frac{(c+dx)^{5/6}\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}{\sqrt[6]{a+bx}}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}} = \frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Fricas [F]
time = 0.57, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^{\frac{5}{6}}}{\sqrt[6]{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(1/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(1/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/6}}{{\left(a+b\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(1/6), x)
```

3.1814 $\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

```
[Out] 6/5*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([1/6, 5/6],[11/6],
-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.02, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in ___instancecheck___

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]')
[Out] cought exception: maximum recursion depth exceeded in __instancecheck__
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.60, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(1/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(1/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(1/6)), x)
```

3.1815 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)\sqrt[6]{c+dx}}$$

```
[Out] 6/5*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([5/6, 7/6],[11/6],
-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(1/6)
```

Rubi [A]

time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{7/6}}\,dx = \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.03, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\;{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.56, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a + bx}\,(c + dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(7/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(7/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(7/6)), x)
```

3.1816 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^2\sqrt[6]{c+dx}}$$

```
[Out] 6/5*b*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([5/6, 13/6],[11/
6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/
6, 13/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^2*(c + d*x)^(1
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{13/6}}\,dx = \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}$$
$$= \frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^2\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.04, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\;{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[5/6
, 13/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(13/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(13/6)), x)
```

Fricas [F]
time = 0.55, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 +
a*d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(13/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(13/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(13/6)), x)
```

3.1817 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=84

$$\frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^3\sqrt[6]{c+dx}}$$

```
[Out] 6/5*b^2*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([5/6, 19/6],[1
1/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(d*x+c)^(1/6)
```

Rubi [A]

time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*b^2*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[
5/6, 19/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^3*(c + d*x)^
(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{19/6}}\,dx = \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}$$
$$= \frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^3\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.05, size = 81, normalized size = 0.96

$$\frac{6b(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\;{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5(bc-ad)^2(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[5/
6, 19/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*(b*c - a*d)^2*(c + d*x)^(7
/6))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(19/6)), x)
```

Fricas [F]
time = 0.56, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^4*x^5 + a*c^4 + (4*b*c*d^3 +
a*d^4)*x^4 + 2*(3*b*c^2*d^2 + 2*a*c*d^3)*x^3 + 2*(2*b*c^3*d + 3*a*c^2*d^2)*
x^2 + (b*c^4 + 4*a*c^3*d)*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(19/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{1/6}\,(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(19/6)), x)
```

3.1818 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)^2\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6*(-a*d+b*c)^2*(b*x+a)^(1/6)*(d*x+c)^(1/6)*hypergeom([-13/6, 1/6],[7/6],-d*
(b*x+a)/(-a*d+b*c))/b^3/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2\;{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 1
/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^3*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{5/6}}\,dx = \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}{(a+bx)^{5/6}}\,dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)^2\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.04, size = 71, normalized size = 0.87

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 1/6, 7/6, (d*(
a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(5/6), x)
```

Fricas [F]
time = 0.37, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(5/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{13/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(13/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(13/6)/(a + b*x)^(5/6), x)
```

3.1819 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=80

$$\frac{6(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(1/6)*hypergeom([-7/6, 1/6],[7/6],-d*(b*
x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)\;{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 1/6,
 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{5/6}}\,dx = \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}{(a+bx)^{5/6}}\,dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.03, size = 71, normalized size = 0.89

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]')
[Out] Timed out
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
[Out] int((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(5/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(5/6), x)
```

3.1820 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6*(b*x+a)^(1/6)*(d*x+c)^(1/6)*hypergeom([-1/6, 1/6],[7/6],-d*(b*x+a)/(-a*d+
b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]

time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, -((d*(
a + b*x))/(b*c - a*d))])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{c+dx}\int \frac{\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}{(a+bx)^{5/6}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.03, size = 71, normalized size = 0.99

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(5/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(5/6), x)
```

3.1821 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

```
[Out] 6*(b*x+a)^(1/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([1/6, 5/6],[7/6],-d*
(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{5/6}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}$$
$$= \frac{6\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.02, size = 71, normalized size = 0.99

$$\frac{6\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{5}{6}}\left(c+dx\right)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(5/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{5/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(5/6)), x)
```

3.1822 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=79

$$\frac{6\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6*(b*x+a)^(1/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([1/6, 11/6],[7/6],-d
*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]

time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 11/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{11/6}}\,dx = \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}$$
$$= \frac{6\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 71, normalized size = 0.90

$$\frac{6\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[1/6
, 11/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(11/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(11/6)), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(11/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(11/6)), x)
```

3.1823 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=80

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6*b*(b*x+a)^(1/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([1/6, 17/6],[7/6],
-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(5/6)
```

Rubi [A]

time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/
6, 17/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(c + d*x)^(5/6)
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$
$$= \frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.03, size = 79, normalized size = 0.99

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/
6, 17/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(c + d*x)^(5/6)
)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(17/6)), x)
```

Fricas [F]
time = 0.35, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 +
a*d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(17/6),x)
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{17/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(17/6)), x)
```

3.1824 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=424

$$\frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b}-\frac{55(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+\cdots$$

```
[Out] 11/12*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(5/6)/b^2+1/2*(b*x+a)^(1/6)*(d*x+c)^
(11/6)/b+55/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(
1/6))/b^(17/6)/d^(1/6)-55/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)
/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(17/6)/d^(1/6
)+55/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6
)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(17/6)/d^(1/6)+55/72*(-a*d+b*c)^2*
arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2)
)/b^(17/6)/d^(1/6)*3^(1/2)+55/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6
)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(17/6)/d^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{55(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}-\frac{55(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}}+\frac{11\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12b^2}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (11*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b^2) + ((a + b*x)^(1/6
)*(c + d*x)^(11/6))/(2*b) - (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*
d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))
/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b
*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(
36*b^(17/6)*d^(1/6)) - (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(
1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])
/(144*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(144*b^(17/6)*d^(1/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(11(bc-ad))\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx}{12b}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c \cdots}}}{72b^2}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\,\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{1-\cdots}\right)}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{\sqrt[3]{\cdots}}\right)}{\cdots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}\right)}{36b^{17/6}\sqrt[6]{d}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}\right)}{36b^{17/6}\sqrt[6]{d}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} - \frac{55(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{b}\cdots}}{\cdots}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}$$

Mathematica [A]
time = 0.44, size = 278, normalized size = 0.66

$$\frac{(bc-ad)^2\left(\frac{6b^{5/6}\sqrt[6]{a+bx}\,(c+dx)^{5/6}(17bc-11ad+6bdx)}{(bc-ad)^2}-\frac{55\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{d}}+\frac{55\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{d}}+\frac{110\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{d}}+\frac{55\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[6]{d}}\right)}{72b^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(5/6)*(a + b*x)^(1/6)*(c + d*x)^(5/6)*(17*b*c - 11*a*d
 + 6*b*d*x))/(b*c - a*d)^2 - (55*Sqrt[3]*ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(
1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]])/d^(1/6) + (55*Sqrt[3]*ArcTan[(1
+ (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]])/d^(1/6)
+ (110*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(1/6
) + (55*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*
(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/d^(1/6)))/(72*b^(17/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="maxima")

[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(5/6), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5591 vs. $2(318) = 636$.
time = 0.42, size = 5591, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="fricas")

[Out] -1/144*(220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 -

12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*
a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7
*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
- 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^1
3)*x)) + 220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*
d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924
*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*
c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d)
)^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d +
a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12)/(b^17*d))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)
*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b
^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*
d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495
*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c
^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^1
0*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 9
24*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^
3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12
*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 49
5*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5
*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10
- 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^
9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d
^13)*x)) - 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9

+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*log
(3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3) + (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17
*d))^(1/3))/(d*x + c)) + 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17
*d))^(1/6)*log(-3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) - (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x
+ a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^17*d))^(1/3))/(d*x + c)) - 110*b^2*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a
)^(1/6)*(d*x + c)^(5/6) + (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6))/(d*x + c)) + 110*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) - (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^17*d))^(1/6))/(d*x + c)) - 12*(6*b*d*x + 17*b*c - 11*a*d)*(b*x + a)^
(1/6)*(d*x + c)^(5/6))/b^2

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(5/6),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(11/6)/(a + b*x)^(5/6), x)
```

3.1825 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}} dx$

Optimal. Leaf size=378

$$\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+\cdots}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c\cdots}}\right.}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}$$

```
[Out] (b*x+a)^(1/6)*(d*x+c)^(5/6)/b+5/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(1/6)-5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*
(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^
(11/6)/d^(1/6)+5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/
3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(1/6)+5/6*(-a*d+
b*c)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^
(1/2))/b^(11/6)/d^(1/6)*3^(1/2)+5/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/
6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(11/6)/d^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/b - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
11/6)*d^(1/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(11/6)*d^(1/6)) + (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(11/6)*d^(1/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(11/6)*d^(1/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(11/6)*d^(1/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6b}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{11/6}} + \cdots$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} + \frac{(5(bc-ad))\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} - \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\cdots\right.}{1\cdots}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\cdots}{\cdots}\right.}{2\sqrt{3}\,b^{11/6}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.02, size = 71, normalized size = 0.19

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2997 vs. 2(280) = 560.
time = 0.42, size = 2997, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/12*(20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/
6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*sqrt(3)*(b^
9*d^2*x + b^9*c*d)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + sqrt(3)*(b^6*c^7 - 6*a*b^5*c^
6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*
b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^
6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^
2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15
*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6
+ a^6*d^7)*x)) + 20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^1
1*d))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*
sqrt(3)*(b^9*d^2*x + b^9*c*d)*sqrt(-((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) - (b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^4*d*x + b^
4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) - sqrt(3)*(b^6*c^7
- 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*
d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b
^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*
d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3
 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*
c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*
a^5*b*c*d^6 + a^6*d^7)*x)) + 5*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
```

```
b^11*d))^(1/6)*log(25*((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c)) - 5*b*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-25*((b^3*c - a*b^2*d)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b
^11*d))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(
2/3) - (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))
^(1/3))/(d*x + c)) + 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d)
)^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (b^2*d*x + b^
2*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) -
 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-5*((b*c
 - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (b^2*d*x + b^2*c)*((b^6*c^6 - 6*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) + 12*(b*x + a)^(1/6)*
(d*x + c)^(5/6))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/6),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(5/6), x)
```

3.1826 $\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+b}}{\sqrt[6]{b}\,\sqrt[6]{c+d}}\right.}{b^{5/6}\sqrt[6]{d}}$$

```
[Out] 2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(1/6)-1/2*
ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6
)/(d*x+c)^(1/6))/b^(5/6)/d^(1/6)+1/2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+
c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(1/6)+arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(
1/2)/b^(5/6)/d^(1/6)+arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(
d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(5/6)/d^(1/6)
```

Rubi [A]
time = 0.32, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1
/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6)) + Log[b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*
x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx = \frac{6\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{5/6}} + \frac{2\text{Subst}\left(\int \frac{\sqrt[6]{b}+\cdots}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{\cdots}}\right.}{\cdots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\cdots\right.}{2b^{2/3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{5/6}\sqrt[6]{d}}\cdots$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{2\tan\cdots}{\cdots}$$

Mathematica [A]
time = 0.16, size = 194, normalized size = 0.63

$$\frac{\sqrt{3}\left(-\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (Sqrt[3]*(-ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)
))/Sqrt[3]] + ArcTan[(1 + (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1
/6)))/Sqrt[3]]) + 2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1
/6))] + ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*
(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(b^(5/6)*d^(1/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}} (dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(220) = 440.
time = 0.36, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4d\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-2\sqrt{3}(b^4d^2x+b^4cd)\sqrt{\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+(b^2dx+b^2c)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}+\sqrt{3}(dx+c)}{3(dx+c)}\right)-2\sqrt{3}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4d\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-2\sqrt{3}(b^4d^2x+b^4cd)\sqrt{-\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-(b^2dx+b^2c)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-\sqrt{3}(dx+c)}{3(dx+c)}\right)+\frac{1}{2}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+(b^2dx+b^2c)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)-\frac{1}{2}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-(b^2dx+b^2c)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)+\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)-\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x +
c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqrt(3)*(b^4*d^2*x + b^4*c*d)*sqrt(((b
*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/6) + (b^2*d*x + b^2*c)*(1/(b
^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6
) + sqrt(3)*(d*x + c))/(d*x + c)) - 2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3
```

```
*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqr
t(3)*(b^4*d^2*x + b^4*c*d)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5
*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6) - sqrt(3)*(d*x + c))/(d*x + c)) + 1/2
*(1/(b^5*d))^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/
6) + (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))
/(d*x + c)) - 1/2*(1/(b^5*d))^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)
*b*(1/(b^5*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3
)*(d*x + c)^(2/3))/(d*x + c)) + (1/(b^5*d))^(1/6)*log(((b*d*x + b*c)*(1/(b^
5*d))^(1/6) + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) - (1/(b^5*d))^(1/
6)*log(-((b*d*x + b*c)*(1/(b^5*d))^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))
/(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(1/6),x)
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(1/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)), x)
```

3.1827 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=30

$$\frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

```
[Out] 6*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
```

[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]')

[Out] Timed out

Maple [A]
time = 0.17, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x,method=_RETURNVERBOSE)

[Out] -6*(b*x+a)^(1/6)/(d*x+c)^(1/6)/(a*d-b*c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)

Fricas [A]
time = 0.31, size = 42, normalized size = 1.40

$$\frac{6(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")

[Out] 6*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(7/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)), x)
```

3.1828 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}}+\frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}$$

```
[Out] 6/7*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(7/6)+36/7*b*(b*x+a)^(1/6)/(-a*d+b*c)^
2/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}\,(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(7*(b*c - a*d)*(c + d*x)^(7/6)) + (36*b*(a + b*x)^(1/6)
)/(7*(b*c - a*d)^2*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}} + \frac{(6b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx}{7(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}} + \frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{6\sqrt[6]{a+bx}\,(7bc-ad+6bdx)}{7(bc-ad)^2(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
[Out] (6*(a + b*x)^(1/6)*(7*b*c - a*d + 6*b*d*x))/(7*(b*c - a*d)^2*(c + d*x)^(7/6
))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]')
[Out] Timed out
```

Maple [A]
time = 0.17, size = 53, normalized size = 0.80

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}(-6bdx+ad-7bc)}{7(dx+c)^{\frac{7}{6}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x,method=_RETURNVERBOSE)
[Out] -6/7*(b*x+a)^(1/6)*(-6*b*d*x+a*d-7*b*c)/(d*x+c)^(7/6)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.34, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx+7\,bc-ad)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{7\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] 6/7*(6*b*d*x + 7*b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*
c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(a+b\,x)}^{5/6}\,{(c+d\,x)}^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)), x)
```

3.1829 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}}+\frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}$$

[Out] 6/13*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(13/6)+72/91*b*(b*x+a)^(1/6)/(-a*d+b*c)^2/(d*x+c)^(7/6)+432/91*b^2*(b*x+a)^(1/6)/(-a*d+b*c)^3/(d*x+c)^(1/6)

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}\,(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]

[Out] (6*(a + b*x)^(1/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (72*b*(a + b*x)^(1/6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6)) + (432*b^2*(a + b*x)^(1/6))/(91*(b*c - a*d)^3*(c + d*x)^(1/6))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{(12b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx}{13(bc-ad)}$$

$$= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}} + \frac{(72b^2)\int \frac{1}{(a+bx)^{5/6}(c+dx}}{91(bc-ad)^2}$$

$$= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}} + \frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 0.10, size = 77, normalized size = 0.76

$$\frac{6\sqrt[6]{a+bx}\,\left(7a^2d^2-2abd(13c+6dx)+b^2\left(91c^2+156cdx+72d^2x^2\right)\right)}{91(bc-ad)^3(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
[Out] (6*(a + b*x)^(1/6)*(7*a^2*d^2 - 2*a*b*d*(13*c + 6*d*x) + b^2*(91*c^2 + 156*
c*d*x + 72*d^2*x^2)))/(91*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 9141 deep
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}\left(72b^2x^2d^2-12ab\,d^2x+156b^2cdx+7a^2d^2-26abcd+91b^2c^2\right)}{91(dx+c)^{\frac{13}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x,method=_RETURNVERBOSE)
[Out] -6/91*(b*x+a)^(1/6)*(72*b^2*d^2*x^2-12*a*b*d^2*x+156*b^2*c*d*x+7*a^2*d^2-26
*a*b*c*d+91*b^2*c^2)/(d*x+c)^(13/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(83) = 166$.
time = 0.33, size = 252, normalized size = 2.50

$$\frac{6\,(72\,b^2d^2x^2+91\,b^2c^2-26\,abcd+7\,a^2d^2+12\,(13\,b^2cd-abd^2)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)x^3+3\,(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5)x^2+3\,(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(72*b^2*d^2*x^2 + 91*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2 + 12*(13*b^2*c*d
 - a*b*d^2)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^6 - 3*a*b^2*c^5*d + 3
*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d
^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^
3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d
^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)), x)
```

3.1830 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}}+\frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}}+\frac{7776b^3\sqrt[6]{a+bx}}{1729(bc-ad)^4\sqrt[6]{c+dx}}$$

```
[Out] 6/19*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(19/6)+108/247*b*(b*x+a)^(1/6)/(-a*d+
b*c)^2/(d*x+c)^(13/6)+1296/1729*b^2*(b*x+a)^(1/6)/(-a*d+b*c)^3/(d*x+c)^(7/6
)+7776/1729*b^3*(b*x+a)^(1/6)/(-a*d+b*c)^4/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}\,(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (108*b*(a + b*x)^(1
/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (1296*b^2*(a + b*x)^(1/6))/(172
9*(b*c - a*d)^3*(c + d*x)^(7/6)) + (7776*b^3*(a + b*x)^(1/6))/(1729*(b*c -
a*d)^4*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(18b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$

$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{(216b^2)\int \frac{1}{(a+bx)^{5/6}(c}}{247(bc-ac}$$

$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+b}}{1729(bc-ad)^3(c+}$$

$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+b}}{1729(bc-ad)^3(c+}$$

Mathematica [A]
time = 0.13, size = 93, normalized size = 0.68

$$\frac{6\sqrt[6]{a+bx}\,\left(-91d^3(a+bx)^3+399bd^2(a+bx)^2(c+dx)-741b^2d(a+bx)(c+dx)^2+1729b^3(c+dx)^3\right)}{1729(bc-ad)^4(c+dx)^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(-91*d^3*(a + b*x)^3 + 399*b*d^2*(a + b*x)^2*(c + d*x) -
 741*b^2*d*(a + b*x)*(c + d*x)^2 + 1729*b^3*(c + d*x)^3))/(1729*(b*c - a*d)
^4*(c + d*x)^(19/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}\left(-1296b^3x^3d^3+216d^3a\,x^2b^2-4104b^3c\,d^2x^2-126a^2b\,d^3x+684a\,b^2c\,d^2x-4446b^3c^2dx+91a^3d^3-399a^2bc\,d^2+741a\,b^2c^2d-\right.}{1729(dx+c)^{\frac{19}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/1729*(b*x+a)^(1/6)*(-1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2-4104*b^3*c*d^2*x
^2-126*a^2*b*d^3*x+684*a*b^2*c*d^2*x-4446*b^3*c^2*d*x+91*a^3*d^3-399*a^2*b*
c*d^2+741*a*b^2*c^2*d-1729*b^3*c^3)/(d*x+c)^(19/6)/(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(112) = 224.
time = 0.32, size = 420, normalized size = 3.09

$$\frac{6\,(1296\,b^3d^3x^3+1729\,b^3c^3-741\,ab^2c^2d+399\,a^2bcd^2-91\,a^3d^3+216\,(19\,b^3cd^2-ab^2d^3)x^2+18\,(247\,b^3c^2d-38\,ab^2cd^2+7\,a^2bd^3)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8)x^4+4\,(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7)x^3+6\,(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6)x^2+4\,(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(1296*b^3*d^3*x^3 + 1729*b^3*c^3 - 741*a*b^2*c^2*d + 399*a^2*b*c*d^2
 - 91*a^3*d^3 + 216*(19*b^3*c*d^2 - a*b^2*d^3)*x^2 + 18*(247*b^3*c^2*d - 38
*a*b^2*c*d^2 + 7*a^2*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^4*c^8 - 4
*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (b^4*c^4
*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*x^4 +
 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2*d^6 + a
^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^4 - 4*a^
3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2
*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{25/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)), x)
```

3.1831 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=449

$$\frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3}+\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}-\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{24\sqrt{3}\,b^{19}}$$

```
[Out] 91/12*d*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b^3+13/2*d*(b*x+a)^(5/6)*(d*
x+c)^(7/6)/b^2-6*(d*x+c)^(13/6)/b/(b*x+a)^(1/6)+91/36*d^(1/6)*(-a*d+b*c)^2*
arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(19/6)-91/144*d^(1/6
)*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/
6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(19/6)+91/144*d^(1/6)*(-a*d+b*c)^2*ln(b^(
1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x
+c)^(1/6))/b^(19/6)-91/72*d^(1/6)*(-a*d+b*c)^2*arctan(-1/3*3^(1/2)+2/3*d^(1
/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(19/6)*3^(1/2)-91/72*d^(
1/6)*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x
+c)^(1/6)*3^(1/2))/b^(19/6)*3^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {49, 52, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{19/6}}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{19/6}}+\frac{91d(a+bx)^{5/6}\sqrt[6]{c+dx}\,(bc-ad)}{12b^3}+\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}-\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (91*d*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^3) + (13*d*(a + b*
x)^(5/6)*(c + d*x)^(7/6))/(2*b^2) - (6*(c + d*x)^(13/6))/(b*(a + b*x)^(1/6)
) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6)
)/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) - (91*d^(1/6)*(
b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcT
anh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(19/6)) - (
91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6))
+ (91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*
x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6
))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(13d)\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b^2}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d \cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d \cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d \cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91 \cdots\right.}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{\cdots}}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{\cdots}}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{\cdots}}{\cdots}$$

Mathematica [A]
time = 0.76, size = 308, normalized size = 0.69

$$\frac{(bc-ad)^2\left(\frac{6\sqrt[6]{b}\sqrt[6]{c+dx}\left(-91a^2d^2-13abd(-13c+dx)+b^2\left(-72c^2+25cdx+6d^2x^2\right)\right)}{(bc-ad)^2\sqrt[6]{a+bx}} - 91\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 91\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 182\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right) + 91\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right)}{72b^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(1/6)*(c + d*x)^(1/6)*(-91*a^2*d^2 - 13*a*b*d*(-13*c +
 d*x) + b^2*(-72*c^2 + 25*c*d*x + 6*d^2*x^2)))/((b*c - a*d)^2*(a + b*x)^(1/
6)) - 91*Sqrt[3]*d^(1/6)*ArcTan[(1 - (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(
a + b*x)^(1/6)))/Sqrt[3]] + 91*Sqrt[3]*d^(1/6)*ArcTan[(1 + (2*b^(1/6)*(c +
d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] + 182*d^(1/6)*ArcTanh[(b^(1
/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))] + 91*d^(1/6)*ArcTanh[(b^(1/
6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1
/3)*(c + d*x)^(1/3))]))/(72*b^(19/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5690 vs. 2(339) = 678.
time = 0.42, size = 5690, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="fricas")

[Out] -1/144*(364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) + sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) + 364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2

- 2*a*b^17*c*d + a^2*b^16*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) - 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^

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3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a
^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^
3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/
(b*x + a)) + 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2
*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^
7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 2
20*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/
b^19)^(1/6)*log(-8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6
)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3
 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^
6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3
*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*
x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^
11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 -
 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*
b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^1
2 + a^12*d^13)/b^19)^(1/3))/(b*x + a)) - 182*(b^4*x + a*b^3)*((b^12*c^12*d
- 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4
*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*
d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 1
2*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^4*x + a*b^3)*((b^12*c^12*d - 12*
a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*
c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^1
1*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) + 182*(b^4*x + a*b^3)*((b^1
2*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4
+ 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7
*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2
*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^4*x + a*b^3)*((b^12*c^1
2*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*
c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) - 12*(6*b^2*d^2*x^
2 - 72*b^2*c^2 + 169*a*b*c*d - 91*a^2*d^2 + (25*b^2*c*d - 13*a*b*d^2)*x)*(b
*x + a)^(5/6)*(d*x + c)^(1/6))/(b^4*x + a*b^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(7/6),x)
```

```
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{13/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(13/6)/(a + b*x)^(7/6),x)
```

```
[Out] int((c + d*x)^(13/6)/(a + b*x)^(7/6), x)
```

3.1832 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)}{}$$

```
[Out] 7*d*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b^2-6*(d*x+c)^(7/6)/b/(b*x+a)^(1/6)+7/3*d^(
1/6)*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(13/
6)-7/12*d^(1/6)*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b
^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(13/6)+7/12*d^(1/6)*(-a*d+b*c
)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1
/6)/(d*x+c)^(1/6))/b^(13/6)-7/6*d^(1/6)*(-a*d+b*c)*arctan(-1/3*3^(1/2)+2/3*
d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(13/6)*3^(1/2)-7/6*d
^(1/6)*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x
+c)^(1/6)*3^(1/2))/b^(13/6)*3^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {49, 52, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}}+\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (7*d*(a + b*x)^(5/6)*(c + d*x)^(1/6))/b^2 - (6*(c + d*x)^(7/6))/(b*(a + b*x
)^(1/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(
1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(13/6)) - (7*d^(1/6)
*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(13/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTanh[
(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(13/6)) - (7*d^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(13/6)) + (7*d^(1
/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (
b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(13/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx &= -\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d)\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{b} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b^2} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a \cdots}\right.}{b^3} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right) \cdots}{b^3} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{\left(7\sqrt[3]{d}\,(bc-ad)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d} \cdots}\right.}{3b^{13/6}} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{(7 \cdots}{\cdots} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{7\sqrt[6]{\cdots}}{\cdots} \\
&= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}} - \cdots
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.05, size = 71, normalized size = 0.18

$$
-\frac{6(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3084 vs. $2(301) = 602$.
time = 0.38, size = 3084, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="fricas")
[Out] 1/12*(28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4
*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^
7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*c - a*b^11*d)*(b*x + a)^(5/6)*(d
*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
```

^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + 2*
sqrt(3)*(b^12*x + a*b^11)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)
^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((
b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*
a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a))*((b^6*c^
6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^
2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + sqrt(3)*(a*b^6*c^6*d - 6
*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2
*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*
c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b
*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^
3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a
*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5
 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*
d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*
c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*
c - a*b^11*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c
*d^6 + a^6*d^7)/b^13)^(5/6) + 2*sqrt(3)*(b^12*x + a*b^11)*sqrt(-((b^3*c - a
*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*
a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
 a^6*d^7)/b^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*
x + c)^(1/3) - (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/3))/(b*x + a))*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(5/6) - sqrt(3)*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20
*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*
d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*
c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2
 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c
*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3
*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(b
^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(
49*((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5
*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
+ 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*
d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a)) - 7*(b^3*x + a*b^2)*((b^6*c^6*d - 6*
a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^
5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(-49*((b^3*c - a*b^2*d)*(b*x +

```
a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) -
(b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3
*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(
b*x + a)) + 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^3*x +
 a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3
*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)
) - 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(
1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^3*x + a*b^2)*
((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 1
5*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)) + 12*(
b*d*x - 6*b*c + 7*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^3*x + a*b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(7/6),x)
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(7/6), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(7/6), x)
```

3.1833 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{7/6}}+ 2\cdots$$

```
[Out] -6*(d*x+c)^(1/6)/b/(b*x+a)^(1/6)+2*d^(1/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^
(1/6)/(d*x+c)^(1/6))/b^(7/6)-1/2*d^(1/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(
d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)+1/2*d^(1/
6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(
1/6)/(d*x+c)^(1/6))/b^(7/6)-d^(1/6)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)
^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(7/6)-d^(1/6)*arctan(1/3*3^
(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(7
/6)
```

Rubi [A]
time = 0.40, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{\sqrt[6]{d}\,\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\,\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{7/6}}+\frac{2\sqrt[6]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6)
 - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) + (2*d^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) - (d^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*b^(7/6)) + (d^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*b^(7/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

& IntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{d\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{b}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{(6d)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx,x,\sqrt[6]{a+bx}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{(6d)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{7/6}}+\frac{\left(2\sqrt[3]{d}\right)\text{Su}}{}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt[6]{d}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx\right.}{2b^{7/6}}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt[6]{d}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\sqrt[6]{b}\right.}{2b^{7/6}}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}}-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right.}{b^{7/6}}$$

Mathematica [A]

time = 0.23, size = 239, normalized size = 0.72

$$\frac{-\frac{6\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}-\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right)+\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right)+2\sqrt[6]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)+\sqrt[6]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] ((-6*b^(1/6)*(c + d*x)^(1/6))/(a + b*x)^(1/6) - Sqrt[3]*d^(1/6)*ArcTan[(1 -
 (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] + Sqrt[3]*
d^(1/6)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/
Sqrt[3]] + 2*d^(1/6)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(
1/6))] + d^(1/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/
(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/b^(7/6)
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(241) = 482$.

time = 0.35, size = 663, normalized size = 2.00

$$-\frac{4\sqrt{3}\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b^{6}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^{7}x+ab^{6}\right)\sqrt{\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}+\sqrt{3}\,(bdx+ad)}{3\,(bdx+ad)}\right)+4\sqrt{3}\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b^{6}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^{7}x+ab^{6}\right)\sqrt{-\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-\sqrt{3}\,(bdx+ad)}{3\,(bdx+ad)}\right)-\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)+\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)-2\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(\frac{\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)+2\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)+12\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{2\left(b^{2}x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sqrt(3)*(b^7*x + a*b^6)*sqrt(
((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3*x + a*b^2)*(d/b^7)^
(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(d/b^7)^(5/6) + sqrt(3)
*(b*d*x + a*d))/(b*d*x + a*d)) + 4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sq
rt(3)*(b^7*x + a*b^6)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6
) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x +
 a))*(d/b^7)^(5/6) - sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) - (b^2*x + a*b)*
(d/b^7)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3
*x + a*b^2)*(d/b^7)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a)) + (
b^2*x + a*b)*(d/b^7)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7
)^(1/6) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/
(b*x + a)) - 2*(b^2*x + a*b)*(d/b^7)^(1/6)*log(((b^2*x + a*b)*(d/b^7)^(1/6)
 + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) + 2*(b^2*x + a*b)*(d/b^7)^(1
/6)*log(-((b^2*x + a*b)*(d/b^7)^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b
*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^2*x + a*b)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c + dx}}{(a + bx)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(7/6),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(7/6), x)
```

Giac [F] N/A

time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(7/6),x)
```

```
[Out] int((c + d*x)^(1/6)/(a + b*x)^(7/6), x)
```

3.1834 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}} dx$

Optimal. Leaf size=30

$$-\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

[Out] -6*(d*x+c)^(1/6)/(-a*d+b*c)/(b*x+a)^(1/6)

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]

[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]

[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]')

[Out] Timed out

Maple [A]
time = 0.17, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{6(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x,method=_RETURNVERBOSE)

[Out] 6/(b*x+a)^(1/6)*(d*x+c)^(1/6)/(a*d-b*c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)

Fricas [A]
time = 0.31, size = 42, normalized size = 1.40

$$-\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")

[Out] -6*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(5/6)), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x)
[Out] Could not integrate
```

Mupad [**B**]
time = 0.68, size = 26, normalized size = 0.87

$$\frac{6\,{\left(c+d\,x\right)}^{1/6}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x)
[Out] (6*(c + d*x)^(1/6))/((a*d - b*c)*(a + b*x)^(1/6))
```

3.1835 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=64

$$-\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}-\frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] -6/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)-36/5*d*(b*x+a)^(5/6)/(-a*d+b*c)^2
/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6)) - (36*d*(a + b*x)^(5/6))/(
5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}} - \frac{(6d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{bc-ad}$$
$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}} - \frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [**A**]
time = 0.09, size = 45, normalized size = 0.70

$$-\frac{6(5bc+ad+6bdx)}{5(bc-ad)^2\sqrt[6]{a+bx}\,(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
[Out] (-6*(5*b*c + a*d + 6*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(5/
6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]')
[Out] Timed out
```

Maple [**A**]
time = 0.18, size = 53, normalized size = 0.83

method	result	size
gosper	$-\frac{6(6bdx+ad+5bc)}{5(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x,method=_RETURNVERBOSE)
[Out] -6/5*(6*b*d*x+a*d+5*b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.
time = 0.31, size = 126, normalized size = 1.97

$$-\frac{6\,(6\,bdx+5\,bc+ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -6/5*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(11/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.83, size = 72, normalized size = 1.12

$$-\frac{\left(\frac{36\,b\,x}{5\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d+30\,b\,c}{5\,d\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/6}}{x\,{\left(a+b\,x\right)}^{1/6}+\frac{c\,{\left(a+b\,x\right)}^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x)
```

```
[Out] -(((36*b*x)/(5*(a*d - b*c)^2) + (6*a*d + 30*b*c)/(5*d*(a*d - b*c)^2))*(c +
d*x)^(1/6))/(x*(a + b*x)^(1/6) + (c*(a + b*x)^(1/6))/d)
```

3.1836 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}} dx$

Optimal. Leaf size=98

$$-\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}}-\frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}}-\frac{432bd(a+bx)^{5/6}}{55(bc-ad)^3(c+dx)^{5/6}}$$

```
[Out] -6/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(11/6)-72/11*d*(b*x+a)^(5/6)/(-a*d+b*c)
^2/(d*x+c)^(11/6)-432/55*b*d*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(11/6)) - (72*d*(a + b*x)^(5/6))/
(11*(b*c - a*d)^2*(c + d*x)^(11/6)) - (432*b*d*(a + b*x)^(5/6))/(55*(b*c -
a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{(12d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{bc-ad}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}} - \frac{(72bd)\int \frac{}{\sqrt[6]{}}}{11}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}} - \frac{432bd(a}{55(bc-ad)}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.74

$$-\frac{6(a+bx)^{11/6}\left(-5d^2+\frac{22bd(c+dx)}{a+bx}+\frac{55b^2(c+dx)^2}{(a+bx)^2}\right)}{55(bc-ad)^3(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(a + b*x)^(11/6)*(-5*d^2 + (22*b*d*(c + d*x))/(a + b*x) + (55*b^2*(c +
d*x)^2)/(a + b*x)^2))/(55*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7772 deep
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.07

method	result	size
gosper	$-\frac{6\left(-72b^2x^2d^2-12ab\,d^2x-132b^2cdx+5a^2d^2-22abcd-55b^2c^2\right)}{55(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/55*(-72*b^2*d^2*x^2-12*a*b*d^2*x-132*b^2*c*d*x+5*a^2*d^2-22*a*b*c*d-55*b
^2*c^2)/(b*x+a)^(1/6)/(d*x+c)^(11/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(82) = 164$.
time = 0.32, size = 273, normalized size = 2.79

$$-\frac{6\,(72\,b^2d^2x^2 + 55\,b^2c^2 + 22\,abcd - 5\,a^2d^2 + 12\,(11\,b^2cd + abd^2)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\,(ab^3c^5 - 3\,a^2b^2c^4d + 3\,a^3bc^3d^2 - a^4c^2d^3 + (b^4c^3d^2 - 3\,ab^3c^2d^3 + 3\,a^2b^2cd^4 - a^3bd^5)x^3 + (2\,b^4c^4d - 5\,ab^3c^3d^2 + 3\,a^2b^2c^2d^3 + a^3bcd^4 - a^4d^5)x^2 + (b^4c^5 - ab^3c^4d - 3\,a^2b^2c^3d^2 + 5\,a^3bc^2d^3 - 2\,a^4cd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] -6/55*(72*b^2*d^2*x^2 + 55*b^2*c^2 + 22*a*b*c*d - 5*a^2*d^2 + 12*(11*b^2*c*
d + a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.96, size = 132, normalized size = 1.35

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{432\,b^2\,x^2}{55\,(a\,d-b\,c)^3}+\frac{-30\,a^2\,d^2+132\,a\,b\,c\,d+330\,b^2\,c^2}{55\,d^2\,(a\,d-b\,c)^3}+\frac{72\,b\,x\,(a\,d+11\,b\,c)}{55\,d\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/6}+\frac{c^2\,(a+b\,x)^{1/6}}{d^2}+\frac{2\,c\,x\,(a+b\,x)^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((432*b^2*x^2)/(55*(a*d - b*c)^3) + (330*b^2*c^2 - 30*a^2*
d^2 + 132*a*b*c*d)/(55*d^2*(a*d - b*c)^3) + (72*b*x*(a*d + 11*b*c))/(55*d*(
a*d - b*c)^3)))/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(
a + b*x)^(1/6))/d)
```

3.1837 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=134

$$-\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}}-\frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}}-\frac{1296bd(a+bx)^{5/6}}{187(bc-ad)^3(c+dx)^{11/6}}-\frac{7776b^2d(a+bx)}{935(bc-ad)^4(c+}$$

```
[Out] -6/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(17/6)-108/17*d*(b*x+a)^(5/6)/(-a*d+b*c
)^2/(d*x+c)^(17/6)-1296/187*b*d*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(11/6)-7
776/935*b^2*d*(b*x+a)^(5/6)/(-a*d+b*c)^4/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(17/6)) - (108*d*(a + b*x)^(5/6))
/(17*(b*c - a*d)^2*(c + d*x)^(17/6)) - (1296*b*d*(a + b*x)^(5/6))/(187*(b*c
 - a*d)^3*(c + d*x)^(11/6)) - (7776*b^2*d*(a + b*x)^(5/6))/(935*(b*c - a*d)
^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{(18d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{bc-ad}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{(216bd)\int \cdots}{17\cdots}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd\cdots}{187(bc-a\cdots}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd\cdots}{187(bc-a\cdots}$$

Mathematica [A]
time = 0.12, size = 95, normalized size = 0.71

$$-\frac{6(a+bx)^{17/6}\left(55d^3-\frac{255bd^2(c+dx)}{a+bx}+\frac{561b^2d(c+dx)^2}{(a+bx)^2}+\frac{935b^3(c+dx)^3}{(a+bx)^3}\right)}{935(bc-ad)^4(c+dx)^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(a + b*x)^(17/6)*(55*d^3 - (255*b*d^2*(c + d*x))/(a + b*x) + (561*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (935*b^3*(c + d*x)^3)/(a + b*x)^3))/(935*(b*c -
 a*d)^4*(c + d*x)^(17/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.28

method	result
gosper	$-\frac{6\left(1296b^3x^3d^3+216d^3a\,x^2b^2+3672b^3c\,d^2x^2-90a^2b\,d^3x+612a\,b^2c\,d^2x+3366b^3c^2dx+55a^3d^3-255a^2bc\,d^2+561a\,b^2c^2d+935b^3c^3\right)}{935(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/935*(1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2+3672*b^3*c*d^2*x^2-90*a^2*b*d^3*
x+612*a*b^2*c*d^2*x+3366*b^3*c^2*d*x+55*a^3*d^3-255*a^2*b*c*d^2+561*a*b^2*c
^2*d+935*b^3*c^3)/(b*x+a)^(1/6)/(d*x+c)^(17/6)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(112) = 224.
time = 0.33, size = 457, normalized size = 3.41

$$-\frac{6\,(1296\,b^3d^3x^3+935\,b^3c^3+561\,ab^2c^2d-255\,a^2bcd^2+55\,a^3d^3+216\,(17\,b^3cd^2+ab^2d^3)x^2+18\,(187\,b^3c^2d+34\,ab^2cd^2-5\,a^2bd^3)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\,(ab^4c^7-4\,a^2b^3c^6d+6\,a^3b^2c^5d^2-4\,a^4bc^4d^3+a^5c^3d^4+(b^5c^4d^3-4\,ab^4c^3d^4+6\,a^2b^3c^2d^5-4\,a^3b^2cd^6+a^4bd^7)x^4+(3\,b^5c^5d^2-11\,ab^4c^4d^3+14\,a^2b^3c^3d^4-6\,a^3b^2c^2d^5-a^4bcd^6+a^5d^7)x^3+3\,(b^5c^6d-3\,ab^4c^5d^2+2\,a^2b^3c^4d^3+2\,a^3b^2c^3d^4-3\,a^4bc^2d^5+a^5cd^6)x^2+(b^5c^7-ab^4c^6d-6\,a^2b^3c^5d^2+14\,a^3b^2c^4d^3-11\,a^4bc^3d^4+3\,a^5c^2d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] -6/935*(1296*b^3*d^3*x^3 + 935*b^3*c^3 + 561*a*b^2*c^2*d - 255*a^2*b*c*d^2
+ 55*a^3*d^3 + 216*(17*b^3*c*d^2 + a*b^2*d^3)*x^2 + 18*(187*b^3*c^2*d + 34*
a*b^2*c*d^2 - 5*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^4*c^7 -
4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*
c^4*d^3 - 4*a*b^4*c^3*d^4 + 6*a^2*b^3*c^2*d^5 - 4*a^3*b^2*c*d^6 + a^4*b*d^7
)*x^4 + (3*b^5*c^5*d^2 - 11*a*b^4*c^4*d^3 + 14*a^2*b^3*c^3*d^4 - 6*a^3*b^2*
c^2*d^5 - a^4*b*c*d^6 + a^5*d^7)*x^3 + 3*(b^5*c^6*d - 3*a*b^4*c^5*d^2 + 2*a
^2*b^3*c^4*d^3 + 2*a^3*b^2*c^3*d^4 - 3*a^4*b*c^2*d^5 + a^5*c*d^6)*x^2 + (b^
5*c^7 - a*b^4*c^6*d - 6*a^2*b^3*c^5*d^2 + 14*a^3*b^2*c^4*d^3 - 11*a^4*b*c^3
*d^4 + 3*a^5*c^2*d^5)*x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.15, size = 209, normalized size = 1.56

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{7776\,b^3\,x^3}{935\,(a\,d-b\,c)^4}+\frac{330\,a^3\,d^3-1530\,a^2\,b\,c\,d^2+3366\,a\,b^2\,c^2\,d+5610\,b^3\,c^3}{935\,d^3\,(a\,d-b\,c)^4}+\frac{108\,b\,x\,(-5\,a^2\,d^2+34\,a\,b\,c\,d+187\,b^2\,c^2)}{935\,d^2\,(a\,d-b\,c)^4}+\frac{1296\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((7776*b^3*x^3)/(935*(a*d - b*c)^4) + (330*a^3*d^3 + 5610
*b^3*c^3 + 3366*a*b^2*c^2*d - 1530*a^2*b*c*d^2)/(935*d^3*(a*d - b*c)^4) + (
108*b*x*(187*b^2*c^2 - 5*a^2*d^2 + 34*a*b*c*d))/(935*d^2*(a*d - b*c)^4) + (
1296*b^2*x^2*(a*d + 17*b*c))/(935*d*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/6) +
 (c^3*(a + b*x)^(1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*
x)^(1/6))/d^2)
```

3.1838 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{7/6}} dx$

Optimal. Leaf size=80

$$-\frac{6(bc-ad)(c+dx)^{5/6} {}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] -6*(-a*d+b*c)*(d*x+c)^(5/6)*hypergeom([-11/6, -1/6],[5/6],-d*(b*x+a)/(-a*d+
b*c))/b^2/(b*x+a)^(1/6)/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(b*c - a*d)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, -((d*(a
 + b*x))/(b*c - a*d))])/(b^2*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{7/6}}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}{(a+bx)^{7/6}}\,dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

$$= -\frac{6(bc-ad)(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 71, normalized size = 0.89

$$-\frac{6(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]
[Out] (-6*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, (d*(a + b*x))/(-(b
*c) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(7/6), x)
```

Fricas [F]
time = 0.55, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(11/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(7/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(11/6)/(a + b*x)^(7/6), x)
```

3.1839 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=72

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] -6*(d*x+c)^(5/6)*hypergeom([-5/6, -1/6],[5/6],-d*(b*x+a)/(-a*d+b*c))/b/(b*x
+a)^(1/6)/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, -((d*(a + b*x))/(b*c
 - a*d))])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/6}}\,dx = \frac{(c+dx)^{5/6}\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}{(a+bx)^{7/6}}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= -\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [**A**]
time = 10.03, size = 71, normalized size = 0.99

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]')
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/6), x)
```

Fricas [F]
time = 0.58, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(7/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(7/6), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/6}}{{\left(a+b\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(7/6), x)
```

3.1840 $\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=72

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 1/6],[5/6],-d*(b*x+a)/(-a*
d+b*c))/b/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]

time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}$$
$$= -\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.02, size = 71, normalized size = 0.99

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]')
[Out] Timed out
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.58, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(1/6),x)
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(1/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{7/6}\,(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(1/6)), x)
```

3.1841 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=79

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 7/6],[5/6],-d*(b*x+a)/(-a*
d+b*c))/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{7/6}}\,dx = \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}$$
$$= -\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.04, size = 71, normalized size = 0.90

$$-\frac{6\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\,(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(7/6))
```

Mathics [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]')
```

```
[Out] Timed out
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.54, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(7/6),x)
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(7/6)), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/6}\,{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x)
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(7/6)), x)
```

3.1842 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=80

$$-\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*b*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 13/6],[5/6],-d*(b*x+a)/(
-a*d+b*c))/(-a*d+b*c)^2/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 13/6, 5/6,
-((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/
6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{13/6}}\,dx = \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}$$
$$= -\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\;\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 71, normalized size = 0.89

$$-\frac{6\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\,(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[-1/6, 13/6, 5/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(13/6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]')
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}\,(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(13/6)), x)
```

Fricas [F]
time = 0.54, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^3*x^5 + a^2*c^3 + (3*b^2*c*
d^2 + 2*a*b*d^3)*x^4 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(13/6),x)
[Out] Exception raised: SystemError
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{7/6}\,(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x)
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(13/6)), x)
```

3.1843 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=82

$$-\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3\sqrt[6]{a+bx}\ \sqrt[6]{c+dx}}$$

```
[Out] -6*b^2*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 19/6],[5/6],-d*(b*x+a)
/(-a*d+b*c))/(-a*d+b*c)^3/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\ \sqrt[6]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (-6*b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 19/6, 5/6
, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(
1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{19/6}}\,dx = \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}$$
$$= -\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3\sqrt[6]{a+bx}\;\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 79, normalized size = 0.96

$$-\frac{6b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\;{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\,(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/6, 19/6, 5/6,
(d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(7/
6))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 10662 deep
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}\,(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(19/6)), x)
```

Fricas [F]
time = 0.56, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^4*x^6 + a^2*c^4 + 2*(2*b^2*
c*d^3 + a*b*d^4)*x^5 + (6*b^2*c^2*d^2 + 8*a*b*c*d^3 + a^2*d^4)*x^4 + 4*(b^2
*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^3)*x^3 + (b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^
2*d^2)*x^2 + 2*(a*b*c^4 + 2*a^2*c^3*d)*x), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/6}\,{\left(c+d\,x\right)}^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x)
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(19/6)), x)
```

3.1844 $\int (a+bx)^m(a+b(2+m)x)\,dx$

Optimal. Leaf size=11

$$x(a+bx)^{1+m}$$

[Out] x*(b*x+a)^(1+m)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {34}

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^m*(a + b*(2 + m)*x),x]

[Out] x*(a + b*x)^(1 + m)

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int (a+bx)^m(a+b(2+m)x)\,dx = x(a+bx)^{1+m}$$

Mathematica [A]
time = 0.04, size = 11, normalized size = 1.00

$$x(a+bx)^{1+m}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^m*(a + b*(2 + m)*x),x]

[Out] x*(a + b*x)^(1 + m)

Mathics [A]
time = 1.85, size = 11, normalized size = 1.00

$$x\,(a+bx)^{1+m}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^m*(a + b*(m + 2)*x),x]')
```

```
[Out] x (a + b x) ^ (1 + m)
```

Maple [A]
time = 0.15, size = 12, normalized size = 1.09

method	result	size
gosper	$x\left(bx+a\right)^{1+m}$	12
risch	$\left(bx+a\right)^{m} x\left(bx+a\right)$	15
norman	$ax\,{\mathrm e}^{m\ln\left(bx+a\right)}+x^{2} b\,{\mathrm e}^{m\ln\left(bx+a\right)}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a+b*(2+m)*x),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*x+a)^(1+m)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(11) = 22$.
time = 0.26, size = 106, normalized size = 9.64

$$\frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m m}{(m^2+3\,m+2)b}+\frac{2\,(b^2(m+1)x^2+abmx-a^2)(bx+a)^m}{(m^2+3\,m+2)b}+\frac{(bx+a)^{m+1}a}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*m/((m^2 + 3*m + 2)*b) + 2*(b^
2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m/((m^2 + 3*m + 2)*b) + (b*x + a)^
(m + 1)*a/(b*(m + 1))
```

Fricas [A]
time = 0.31, size = 17, normalized size = 1.55

$$\left(bx^{2}+ax\right)\left(bx+a\right)^{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^m
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.
time = 0.13, size = 20, normalized size = 1.82

$$ax\left(a+bx\right)^{m}+bx^{2}\left(a+bx\right)^{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a+b*(2+m)*x),x)
```

```
[Out] a*x*(a + b*x)**m + b*x**2*(a + b*x)**m
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.00, size = 27, normalized size = 2.45

$$axe^{m\ln(a+bx)} + bx^2e^{m\ln(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x)
```

```
[Out] (b*x + a)^m*b*x^2 + (b*x + a)^m*a*x
```

Mupad [B]
time = 0.46, size = 11, normalized size = 1.00

$$x\,(a+b\,x)^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x*(m + 2))*(a + b*x)^m,x)
```

```
[Out] x*(a + b*x)^(m + 1)
```

3.1845 $\int (a+bx)^m (c+dx)^n \, dx$

Optimal. Leaf size=61

$$-\frac{(a+bx)^{1+m}(c+dx)^{1+n}\,{}_2F_1\left(1,2+m+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+n],b*(d*x+c)/(-a*d+b*c
))/(-a*d+b*c)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {72, 71}

$$\frac{(a+bx)^{m+1}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a
+ b*x))/(b*c - a*d))])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+m)}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.20

$$\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n,x]
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m*(c + d*x)^n,x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (bx+a)^m (dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n,x)
[Out] int((b*x+a)^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a+b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^m*(c + d*x)^n, x)
```

3.1846 $\int (a+bx)^m (c+dx)^3 \, dx$

Optimal. Leaf size=110

$$\frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}+\frac{d^3(a+bx)^{4+m}}{b^4(4+m)}$$

```
[Out] (-a*d+b*c)^3*(b*x+a)^(1+m)/b^4/(1+m)+3*d*(-a*d+b*c)^2*(b*x+a)^(2+m)/b^4/(2+
m)+3*d^2*(-a*d+b*c)*(b*x+a)^(3+m)/b^4/(3+m)+d^3*(b*x+a)^(4+m)/b^4/(4+m)
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^(1 + m))/(b^4*(1 + m)) + (3*d*(b*c - a*d)^2*(a + b
*x)^(2 + m))/(b^4*(2 + m)) + (3*d^2*(b*c - a*d)*(a + b*x)^(3 + m))/(b^4*(3
+ m)) + (d^3*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^m}{b^3}+\frac{3d(bc-ad)^2(a+bx)^{1+m}}{b^3}+\frac{3d^2(bc-ad)(a+bx)^{2+m}}{b^3}\right.\\ &= \frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}+\end{aligned}$$

Mathematica [A]
time = 0.09, size = 94, normalized size = 0.85

$$\frac{(a+bx)^{1+m}\left(\frac{(bc-ad)^3}{1+m}+\frac{3d(bc-ad)^2(a+bx)}{2+m}+\frac{3d^2(bc-ad)(a+bx)^2}{3+m}+\frac{d^3(a+bx)^3}{4+m}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^3/(1 + m) + (3*d*(b*c - a*d)^2*(a + b*x))/(
2 + m) + (3*d^2*(b*c - a*d)*(a + b*x)^2)/(3 + m) + (d^3*(a + b*x)^3)/(4 + m
)))/b^4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 15.83, size = 2986, normalized size = 27.15

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^m*(c + d*x)^3,x]')
```

```
[Out] Piecewise[{{x (4 c ^ 3 + 6 c ^ 2 d x + 4 c d ^ 2 x ^ 2 + d ^ 3 x ^ 3) a ^ m
 / 4, b == 0}, {(a ^ 3 d ^ 3 (11 + 6 Log[(a + b x) / b]) + 3 a ^ 2 b d ^ 2
(-2 c + 6 d x Log[(a + b x) / b] + 9 d x) + 3 a b ^ 2 d (-c ^ 2 - 6 c d x +
 6 d ^ 2 x ^ 2 + 6 d ^ 2 x ^ 2 Log[(a + b x) / b]) + b ^ 3 (-2 c ^ 3 - 9 c
^ 2 d x - 18 c d ^ 2 x ^ 2 + 6 d ^ 3 x ^ 3 Log[(a + b x) / b])) / (6 b ^ 4
(a ^ 3 + 3 a ^ 2 b x + 3 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)), m == -4}, {(-3 a ^
3 d ^ 3 (3 + 2 Log[(a + b x) / b]) + 3 a ^ 2 b d ^ 2 (2 c Log[(a + b x) / b
] + 3 c - 4 d x - 4 d x Log[(a + b x) / b]) + 3 a b ^ 2 d (-c ^ 2 + 4 c d x
 + 4 c d x Log[(a + b x) / b] - 2 d ^ 2 x ^ 2 Log[(a + b x) / b]) + b ^ 3 (
-c ^ 3 - 6 c ^ 2 d x + 6 c d ^ 2 x ^ 2 Log[(a + b x) / b] + 2 d ^ 3 x ^ 3))
 / (2 b ^ 4 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)), m == -3}, {(6 a ^ 3 d ^ 3 (1
+ Log[(a + b x) / b]) + 6 a ^ 2 b d ^ 2 (-2 c - 2 c Log[(a + b x) / b] + d
x Log[(a + b x) / b]) + 3 a b ^ 2 d (2 c ^ 2 + 2 c ^ 2 Log[(a + b x) / b] -
 4 c d x Log[(a + b x) / b] - d ^ 2 x ^ 2) + b ^ 3 (-2 c ^ 3 + 6 c ^ 2 d x
Log[(a + b x) / b] + 6 c d ^ 2 x ^ 2 + d ^ 3 x ^ 3)) / (2 b ^ 4 (a + b x)),
 m == -2}, {(-a ^ 3 d ^ 3 Log[(a + b x) / b] + a ^ 2 b d ^ 2 (3 c Log[(a +
b x) / b] + d x) - a b ^ 2 d (6 c ^ 2 Log[(a + b x) / b] + 6 c d x + d ^ 2
x ^ 2) / 2 + b ^ 3 (6 c ^ 3 Log[(a + b x) / b] + 18 c ^ 2 d x + 9 c d ^ 2 x
 ^ 2 + 2 d ^ 3 x ^ 3) / 6) / b ^ 4, m == -1}}, -6 a ^ 4 d ^ 3 (a + b x) ^ m
 / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4)
+ 24 a ^ 3 b c d ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^
2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 6 a ^ 3 b c d ^ 2 m (a + b x) ^ m / (24
 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 6 a
^ 3 b d ^ 3 m x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 1
0 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) - 36 a ^ 2 b ^ 2 c ^ 2 d (a + b x) ^ m / (24 b
 ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) - 21 a ^
 2 b ^ 2 c ^ 2 d m (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2
+ 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) - 3 a ^ 2 b ^ 2 c ^ 2 d m ^ 2 (a + b x) ^ m
 / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4)
- 24 a ^ 2 b ^ 2 c d ^ 2 m x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b
```

^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) - 6 a ^ 2 b ^ 2 c d ^ 2 m ^ 2 x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) - 3 a ^ 2 b ^ 2 d ^ 3 m x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) - 3 a ^ 2 b ^ 2 d ^ 3 m ^ 2 x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 24 a b ^ 3 c ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 26 a b ^ 3 c ^ 3 m (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 9 a b ^ 3 c ^ 3 m ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + a b ^ 3 c ^ 3 m ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 36 a b ^ 3 c ^ 2 d m x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 21 a b ^ 3 c ^ 2 d m ^ 2 x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 3 a b ^ 3 c ^ 2 d m ^ 3 x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 12 a b ^ 3 c d ^ 2 m x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 15 a b ^ 3 c d ^ 2 m ^ 2 x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 3 a b ^ 3 c d ^ 2 m ^ 3 x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 2 a b ^ 3 d ^ 3 m x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 3 a b ^ 3 d ^ 3 m ^ 2 x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + a b ^ 3 d ^ 3 m ^ 3 x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 24 b ^ 4 c ^ 3 x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 26 b ^ 4 c ^ 3 m x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 9 b ^ 4 c ^ 3 m ^ 2 x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + b ^ 4 c ^ 3 m ^ 3 x (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 36 b ^ 4 c ^ 2 d x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 57 b ^ 4 c ^ 2 d m x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 24 b ^ 4 c ^ 2 d m ^ 2 x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 3 b ^ 4 c ^ 2 d m ^ 3 x ^ 2 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 24 b ^ 4 c d ^ 2 x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 42 b ^ 4 c d ^ 2 m x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 21 b ^ 4 c d ^ 2 m ^ 2 x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 3 b ^ 4 c d ^ 2 m ^ 3 x ^ 3 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 6 b ^ 4 d ^ 3 x ^ 4 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 11

```
b ^ 4 d ^ 3 m x ^ 4 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2
 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) + 6 b ^ 4 d ^ 3 m ^ 2 x ^ 4 (a + b x) ^ m
/ (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4 m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4) +
 b ^ 4 d ^ 3 m ^ 3 x ^ 4 (a + b x) ^ m / (24 b ^ 4 + 50 b ^ 4 m + 35 b ^ 4
m ^ 2 + 10 b ^ 4 m ^ 3 + b ^ 4 m ^ 4)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(110) = 220$.
time = 0.18, size = 389, normalized size = 3.54

method	result
gosper	$-\frac{(bx+a)^{1+m}\left(-b^3d^3m^3x^3-3b^3c\,d^2m^3x^2-6b^3d^3m^2x^3+3a\,b^2d^3m^2x^2-3b^3c^2d\,m^3x-21b^3c\,d^2m^2x^2-11b^3d^3m\,x^3+6a\,b^2c\,d^2m^2x+9\right.}{}$
norman	$\frac{d^3x^4\mathrm{e}^{m\ln(bx+a)}}{4+m}+\frac{(3a\,b^2c^2d\,m^3+b^3c^3m^3-6a^2bc\,d^2m^2+21a\,b^2c^2d\,m^2+9b^3c^3m^2+6a^3d^3m-24a^2bc\,d^2m+36a\,b^2c^2dm+26b^3c^3m+}{b^3(m^4+10m^3+35m^2+50m+24)}$
risch	$-\frac{\left(-b^4d^3m^3x^4-a\,b^3d^3m^3x^3-3b^4c\,d^2m^3x^3-6b^4d^3m^2x^4-3a\,b^3c\,d^2m^3x^2-3a\,b^3d^3m^2x^3-3b^4c^2d\,m^3x^2-21b^4c\,d^2m^2x^3-11b^4d^3m\,x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(-b^3*d^3*m^3*x^3-3*b^3*c*d^2*m^3*x^2-6*b^3*d^3*m^2*x^3+3*a*
b^2*d^3*m^2*x^2-3*b^3*c^2*d*m^3*x-21*b^3*c*d^2*m^2*x^2-11*b^3*d^3*m*x^3+6*a
*b^2*c*d^2*m^2*x+9*a*b^2*d^3*m*x^2-b^3*c^3*m^3-24*b^3*c^2*d*m^2*x-42*b^3*c*
d^2*m*x^2-6*b^3*d^3*x^3-6*a^2*b*d^3*m*x+3*a*b^2*c^2*d*m^2+30*a*b^2*c*d^2*m*
x+6*a*b^2*d^3*x^2-9*b^3*c^3*m^2-57*b^3*c^2*d*m*x-24*b^3*c*d^2*x^2-6*a^2*b*c
*d^2*m-6*a^2*b*d^3*x+21*a*b^2*c^2*d*m+24*a*b^2*c*d^2*x-26*b^3*c^3*m-36*b^3*
c^2*d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/b^4/(m^4+10*m^3
+35*m^2+50*m+24)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(110) = 220$.
time = 0.29, size = 246, normalized size = 2.24

$$\frac{3\,(b^2(m+1)x^2+abmx-a^2)(bx+a)^m c^2 d}{(m^2+3\,m+2)b^2}+\frac{(bx+a)^{m+1}c^3}{b(m+1)}+\frac{3\,((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m cd^2}{(m^3+6\,m^2+11\,m+6)b^3}+\frac{((m^3+6\,m^2+11\,m+6)b^4x^4+(m^3+3\,m^2+2\,m)ab^3x^3-3\,(m^2+m)a^2b^2x^2+6\,a^3bmx-6\,a^4)(bx+a)^m d^3}{(m^4+10\,m^3+35\,m^2+50\,m+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c^2*d/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c^3/(b*(m + 1)) + 3*((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*d^2/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m^2 + 2*m)*a*b^3*x
^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x + a)^m*d^3/((m^4 +
 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(110) = 220$.

time = 0.33, size = 497, normalized size = 4.52

$$\frac{(ab^3c^3m^3 + 24\,ab^3c^3 - 36\,a^2b^2c^2d + 24\,a^3bcd^2 - 6\,a^4d^3 + (b^4d^3m^3 + 6\,b^4d^3m^2 + 11\,b^4d^3m + 6\,b^4d^3)x^4 + (24\,b^4cd^2 + (3\,b^4cd^2 + ab^3d^3)m^3 + 3\,(7\,b^4cd^2 + ab^3d^3)m^2 + 2\,(21\,b^4cd^2 + ab^3d^3)m)x^3 + 3\,(3\,ab^3c^3 - a^2b^2c^2d)m^2 + 3\,(12\,b^4c^2d + (b^4c^2d + ab^3cd^2)m^3 + (8\,b^4c^2d + 5\,ab^3cd^2 - a^2b^2d^3)m^2 + (19\,b^4c^2d + 4\,ab^3cd^2 - a^2b^2d^3)m)x^2 + (26\,ab^3c^3 - 21\,a^2b^2c^2d + 6\,a^3bcd^2)m + (24\,b^4c^3 + (b^4c^3 + 3\,ab^3c^2d)m^3 + 3\,(3\,b^4c^3 + 7\,ab^3c^2d - 2\,a^2b^2cd^2)m^2 + 2\,(13\,b^4c^3 + 18\,ab^3c^2d - 12\,a^2b^2cd^2 + 3\,a^3bd^3)m)x)(bx + a)^m}{b^4m^4 + 10\,b^4m^3 + 35\,b^4m^2 + 50\,b^4m + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^3*c^3*m^3 + 24*a*b^3*c^3 - 36*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 6*a^4*d
^3 + (b^4*d^3*m^3 + 6*b^4*d^3*m^2 + 11*b^4*d^3*m + 6*b^4*d^3)*x^4 + (24*b^4
*c*d^2 + (3*b^4*c*d^2 + a*b^3*d^3)*m^3 + 3*(7*b^4*c*d^2 + a*b^3*d^3)*m^2 +
2*(21*b^4*c*d^2 + a*b^3*d^3)*m)*x^3 + 3*(3*a*b^3*c^3 - a^2*b^2*c^2*d)*m^2 +
 3*(12*b^4*c^2*d + (b^4*c^2*d + a*b^3*c*d^2)*m^3 + (8*b^4*c^2*d + 5*a*b^3*c
*d^2 - a^2*b^2*d^3)*m^2 + (19*b^4*c^2*d + 4*a*b^3*c*d^2 - a^2*b^2*d^3)*m)*x
^2 + (26*a*b^3*c^3 - 21*a^2*b^2*c^2*d + 6*a^3*b*c*d^2)*m + (24*b^4*c^3 + (b
^4*c^3 + 3*a*b^3*c^2*d)*m^3 + 3*(3*b^4*c^3 + 7*a*b^3*c^2*d - 2*a^2*b^2*c*d^
2)*m^2 + 2*(13*b^4*c^3 + 18*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*m
)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Sympy [A]

time = 2.65, size = 4058, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**3,x)
```

```
[Out] Piecewise((a**m*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(
b, 0)), (6*a**3*d**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 11*a**3*d**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 6*a**2*b*c*d**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d**3*x*log(a/b + x)/(6*a**3*b**4 +
18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d**3*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*a*b**2*c**2*d/(6*a
**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 18*a*b**2*c*d**
2*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b*
*2*d**3*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3) + 18*a*b**2*d**3*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) - 2*b**3*c**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 9*b**3*c**2*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) - 18*b**3*c*d**2*x**2/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d**3*x**3*log(a/b + x)/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(m, -4)), (-6*a*
*3*d**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d**3
/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 6*a**2*b*c*d**2*log(a/b + x)/(2
```

```
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 9*a**2*b*c*d**2/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b
**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*
x**2) - 3*a*b**2*c**2*d/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d**3*x**2*lo
g(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c**3/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 6*b**3*c**2*d*x/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 6*b**3*c*d**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 2*b**3*d**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2),
Eq(m, -3)), (6*a**3*d**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d**3/(
2*a*b**4 + 2*b**5*x) - 12*a**2*b*c*d**2*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
- 12*a**2*b*c*d**2/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d**3*x*log(a/b + x)/(2*
a*b**4 + 2*b**5*x) + 6*a*b**2*c**2*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6
*a*b**2*c**2*d/(2*a*b**4 + 2*b**5*x) - 12*a*b**2*c*d**2*x*log(a/b + x)/(2*a
*b**4 + 2*b**5*x) - 3*a*b**2*d**3*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c**3/
(2*a*b**4 + 2*b**5*x) + 6*b**3*c**2*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 6*b**3*c*d**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d**3*x**3/(2*a*b**4 + 2*b
**5*x), Eq(m, -2)), (-a**3*d**3*log(a/b + x)/b**4 + 3*a**2*c*d**2*log(a/b +
 x)/b**3 + a**2*d**3*x/b**3 - 3*a*c**2*d*log(a/b + x)/b**2 - 3*a*c*d**2*x/b
**2 - a*d**3*x**2/(2*b**2) + c**3*log(a/b + x)/b + 3*c**2*d*x/b + 3*c*d**2*
x**2/(2*b) + d**3*x**3/(3*b), Eq(m, -1)), (-6*a**4*d**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*a**3*b*c*d**2
*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 24*a**3*b*c*d**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m*
*2 + 50*b**4*m + 24*b**4) + 6*a**3*b*d**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*c**2*d*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 21*a**2*b**2*c**2*d*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) - 36*a**2*b**2*c**2*d*(a + b*x)**m/(b**4*m**4 +
10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 6*a**2*b**2*c*d**2*m**
2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) - 24*a**2*b**2*c*d**2*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35
*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d**3*m**2*x**2*(a + b*x)**m
/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b
**2*d**3*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + a*b**3*c**3*m**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c**3*m**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*a*b**3*c
**3*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 24*a*b**3*c**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c**2*d*m**3*x*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*a*b**3*c**2*d*m*
*2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 36*a*b**3*c**2*d*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
```

```
**4*m**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c*d**2*m**3*x**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 15*a*b**3*c
*d**2*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*d**2*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d**3*m**3*x**3*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3
*a*b**3*d**3*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**
2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d**3*m*x**3*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c**3*m**3*x*(a + b
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*
b**4*c**3*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50
*b**4*m + 24*b**4) + 26*b**4*c**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**
3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c**3*x*(a + b*x)**m/(b**4
*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3*b**4*c**2*d*
m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) + 24*b**4*c**2*d*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m*
*3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 57*b**4*c**2*d*m*x**2*(a + b*x)*
*m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 36*b**
4*c**2*d*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b*
*4*m + 24*b**4) + 3*b**4*c*d**2*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*b**4*c*d**2*m**2*x**3*(a +
 b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
42*b**4*c*d**2*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
 + 50*b**4*m + 24*b**4) + 24*b**4*c*d**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d**3*m**3*x**4*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6
*b**4*d**3*m**2*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 11*b**4*d**3*m*x**4*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d**3*x**4*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(110) = 220.

time = 0.00, size = 920, normalized size = 8.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x)
```

```
[Out] ((b*x + a)^m*b^4*d^3*m^3*x^4 + 3*(b*x + a)^m*b^4*c*d^2*m^3*x^3 + (b*x + a)^
m*a*b^3*d^3*m^3*x^3 + 6*(b*x + a)^m*b^4*d^3*m^2*x^4 + 3*(b*x + a)^m*b^4*c^2
*d*m^3*x^2 + 3*(b*x + a)^m*a*b^3*c*d^2*m^3*x^2 + 21*(b*x + a)^m*b^4*c*d^2*m
^2*x^3 + 3*(b*x + a)^m*a*b^3*d^3*m^2*x^3 + 11*(b*x + a)^m*b^4*d^3*m*x^4 + (
b*x + a)^m*b^4*c^3*m^3*x + 3*(b*x + a)^m*a*b^3*c^2*d*m^3*x + 24*(b*x + a)^m
```

```
*b^4*c^2*d*m^2*x^2 + 15*(b*x + a)^m*a*b^3*c*d^2*m^2*x^2 - 3*(b*x + a)^m*a^2
*b^2*d^3*m^2*x^2 + 42*(b*x + a)^m*b^4*c*d^2*m*x^3 + 2*(b*x + a)^m*a*b^3*d^3
*m*x^3 + 6*(b*x + a)^m*b^4*d^3*x^4 + (b*x + a)^m*a*b^3*c^3*m^3 + 9*(b*x + a
)^m*b^4*c^3*m^2*x + 21*(b*x + a)^m*a*b^3*c^2*d*m^2*x - 6*(b*x + a)^m*a^2*b^
2*c*d^2*m^2*x + 57*(b*x + a)^m*b^4*c^2*d*m*x^2 + 12*(b*x + a)^m*a*b^3*c*d^2
*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d^3*m*x^2 + 24*(b*x + a)^m*b^4*c*d^2*x^3 + 9
*(b*x + a)^m*a*b^3*c^3*m^2 - 3*(b*x + a)^m*a^2*b^2*c^2*d*m^2 + 26*(b*x + a)
^m*b^4*c^3*m*x + 36*(b*x + a)^m*a*b^3*c^2*d*m*x - 24*(b*x + a)^m*a^2*b^2*c*
d^2*m*x + 6*(b*x + a)^m*a^3*b*d^3*m*x + 36*(b*x + a)^m*b^4*c^2*d*x^2 + 26*(
b*x + a)^m*a*b^3*c^3*m - 21*(b*x + a)^m*a^2*b^2*c^2*d*m + 6*(b*x + a)^m*a^3
*b*c*d^2*m + 24*(b*x + a)^m*b^4*c^3*x + 24*(b*x + a)^m*a*b^3*c^3 - 36*(b*x
+ a)^m*a^2*b^2*c^2*d + 24*(b*x + a)^m*a^3*b*c*d^2 - 6*(b*x + a)^m*a^4*d^3)/
(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Mupad [B]
time = 0.94, size = 478, normalized size = 4.35

$$\frac{d^3\,x^4\,{\left(a+b\,x\right)}^m\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{a\,{\left(a+b\,x\right)}^m\,\left(-6\,a^3\,d^3+6\,a^2\,b\,c\,d^2\,m+24\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d\,m^2-21\,a\,b^2\,c^2\,d\,m-36\,a\,b^2\,c^2\,d+b^3\,c^3\,m^3+9\,b^3\,c^3\,m^2+26\,b^3\,c^3\,m+24\,b^3\,c^3\right)}{b^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}+\frac{x\,{\left(a+b\,x\right)}^m\,\left(6\,a^3\,b\,d^3\,m-6\,a^2\,b^2\,c\,d^2\,m^2-24\,a^2\,b^2\,c\,d^2\,m+3\,a\,b^3\,c^2\,d\,m^3+21\,a\,b^3\,c^2\,d\,m^2+36\,a\,b^3\,c^2\,d\,m+b^4\,c^3\,m^3+9\,b^4\,c^3\,m^2+26\,b^4\,c^3\,m+24\,b^4\,c^3\right)}{b^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}+\frac{3\,d\,x^2\,\left(m+1\right)\,{\left(a+b\,x\right)}^m\,\left(-a^2\,d^2\,m+a\,b\,c\,d\,m^2+4\,a\,b\,c\,d\,m+b^2\,c^2\,m^2+7\,b^2\,c^2\,m+12\,b^2\,c^2\right)}{b^2\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}+\frac{d^2\,x^3\,{\left(a+b\,x\right)}^m\,\left(12\,b\,c+a\,d\,m+3\,b\,c\,m\right)\,\left(m^2+3\,m+2\right)}{b\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^3,x)
```

```
[Out] (d^3*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (a*(a + b*x)^m*(24*b^3*c^3 - 6*a^3*d^3 + 26*b^3*c^3*m + 9*b^3*c^3
*m^2 + b^3*c^3*m^3 - 36*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 21*a*b^2*c^2*d*m + 6
*a^2*b*c*d^2*m - 3*a*b^2*c^2*d*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4)) + (x*(a + b*x)^m*(24*b^4*c^3 + 26*b^4*c^3*m + 9*b^4*c^3*m^2 + b^4*c^3*m
^3 + 6*a^3*b*d^3*m + 36*a*b^3*c^2*d*m - 24*a^2*b^2*c*d^2*m + 21*a*b^3*c^2*d
*m^2 + 3*a*b^3*c^2*d*m^3 - 6*a^2*b^2*c*d^2*m^2))/(b^4*(50*m + 35*m^2 + 10*m
^3 + m^4 + 24)) + (3*d*x^2*(m + 1)*(a + b*x)^m*(12*b^2*c^2 - a^2*d^2*m + 7*
b^2*c^2*m + b^2*c^2*m^2 + 4*a*b*c*d*m + a*b*c*d*m^2))/(b^2*(50*m + 35*m^2 +
 10*m^3 + m^4 + 24)) + (d^2*x^3*(a + b*x)^m*(12*b*c + a*d*m + 3*b*c*m)*(3*m
 + m^2 + 2))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

3.1847 $\int (a+bx)^m (c+dx)^2\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}$$

```
[Out] (-a*d+b*c)^2*(b*x+a)^(1+m)/b^3/(1+m)+2*d*(-a*d+b*c)*(b*x+a)^(2+m)/b^3/(2+m)
+d^2*(b*x+a)^(3+m)/b^3/(3+m)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1 + m))/(b^3*(1 + m)) + (2*d*(b*c - a*d)*(a + b*x
)^(2 + m))/(b^3*(2 + m)) + (d^2*(a + b*x)^(3 + m))/(b^3*(3 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^m}{b^2}+\frac{2d(bc-ad)(a+bx)^{1+m}}{b^2}+\frac{d^2(a+bx)^{2+m}}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 0.86

$$\frac{(a+bx)^{1+m}\left(\frac{(bc-ad)^2}{1+m}+\frac{2d(bc-ad)(a+bx)}{2+m}+\frac{d^2(a+bx)^2}{3+m}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^2/(1 + m) + (2*d*(b*c - a*d)*(a + b*x))/(2
+ m) + (d^2*(a + b*x)^2)/(3 + m)))/b^3
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.81, size = 1256, normalized size = 16.10

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^m*(c + d*x)^2,x]')
```

```
[Out] Piecewise[{{x (3 c ^ 2 + 3 c d x + d ^ 2 x ^ 2) a ^ m / 3, b == 0}, {(a ^ 2
 d ^ 2 (3 + 2 Log[(a + b x) / b]) + 2 a b d (-c + 2 d x + 2 d x Log[(a + b
x) / b]) + b ^ 2 (-c ^ 2 - 4 c d x + 2 d ^ 2 x ^ 2 Log[(a + b x) / b])) / (
2 b ^ 3 (a ^ 2 + 2 a b x + b ^ 2 x ^ 2)), m == -3}, {(-2 a ^ 2 d ^ 2 (1 + L
og[(a + b x) / b]) + 2 a b d (c + c Log[(a + b x) / b] - d x Log[(a + b x)
/ b]) + b ^ 2 (-c ^ 2 + 2 c d x Log[(a + b x) / b] + d ^ 2 x ^ 2)) / (b ^ 3
 (a + b x)), m == -2}, {(a ^ 2 d ^ 2 Log[(a + b x) / b] - a b d (2 c Log[(a
 + b x) / b] + d x) + b ^ 2 (2 c ^ 2 Log[(a + b x) / b] + 4 c d x + d ^ 2 x
 ^ 2) / 2) / b ^ 3, m == -1}}, 2 a ^ 3 d ^ 2 (a + b x) ^ m / (6 b ^ 3 + 11
b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) - 6 a ^ 2 b c d (a + b x) ^ m / (6 b
 ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) - 2 a ^ 2 b c d m (a + b x
) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) - 2 a ^ 2 b d
^ 2 m x (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3
) + 6 a b ^ 2 c ^ 2 (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 +
 b ^ 3 m ^ 3) + 5 a b ^ 2 c ^ 2 m (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6
 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + a b ^ 2 c ^ 2 m ^ 2 (a + b x) ^ m / (6 b ^ 3
+ 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + 6 a b ^ 2 c d m x (a + b x) ^
 m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + 2 a b ^ 2 c d m
 ^ 2 x (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3)
 + a b ^ 2 d ^ 2 m x ^ 2 (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m
^ 2 + b ^ 3 m ^ 3) + a b ^ 2 d ^ 2 m ^ 2 x ^ 2 (a + b x) ^ m / (6 b ^ 3 + 1
1 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + 6 b ^ 3 c ^ 2 x (a + b x) ^ m /
(6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + 5 b ^ 3 c ^ 2 m x (a
 + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + b ^ 3
c ^ 2 m ^ 2 x (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3
 m ^ 3) + 6 b ^ 3 c d x ^ 2 (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3
 m ^ 2 + b ^ 3 m ^ 3) + 8 b ^ 3 c d m x ^ 2 (a + b x) ^ m / (6 b ^ 3 + 11 b
 ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + 2 b ^ 3 c d m ^ 2 x ^ 2 (a + b x) ^
 m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) + 2 b ^ 3 d ^ 2 x
 ^ 3 (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3) +
```

```
3 b ^ 3 d ^ 2 m x ^ 3 (a + b x) ^ m / (6 b ^ 3 + 11 b ^ 3 m + 6 b ^ 3 m ^
2 + b ^ 3 m ^ 3) + b ^ 3 d ^ 2 m ^ 2 x ^ 3 (a + b x) ^ m / (6 b ^ 3 + 11 b
^ 3 m + 6 b ^ 3 m ^ 2 + b ^ 3 m ^ 3)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.
time = 0.19, size = 159, normalized size = 2.04

method	result
gosper	$\frac{(bx+a)^{1+m}(b^2d^2m^2x^2+2b^2cd\,m^2x+3b^2d^2m\,x^2-2ab\,d^2mx+b^2c^2m^2+8b^2cdmx+2b^2x^2d^2-2abcdm-2ab\,d^2x+5b^2c^2m+6b^2cdx+2a^2d^2 \ldots}{b^3(m^3+6m^2+11m+6)}$
norman	$\frac{d^2x^3\mathrm{e}^{m\ln(bx+a)}}{3+m}+\frac{a(b^2c^2m^2-2abcdm+5b^2c^2m+2a^2d^2-6abcd+6b^2c^2)\mathrm{e}^{m\ln(bx+a)}}{b^3(m^3+6m^2+11m+6)}+\frac{(adm+2bcm+6bc)d\,x^2\mathrm{e}^{m\ln(bx+a)}}{b(m^2+5m+6)}-\frac{(-2a \ldots}{\ldots}$
risch	$\frac{(b^3d^2m^2x^3+a\,b^2d^2m^2x^2+2b^3cd\,m^2x^2+3b^3d^2m\,x^3+2a\,b^2cd\,m^2x+a\,b^2d^2m\,x^2+b^3c^2m^2x+8b^3cdm\,x^2+2d^2x^3b^3-2a^2b\,d^2mx+a\,b^2c^2m \ldots}{(2+m)(3+m)(1+m)b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+m)*(b^2*d^2*m^2*x^2+2*b^2*c*d*m^2*x+3*b^2*d^2*m*x^2-2*a*b*d^2*m*
x+b^2*c^2*m^2+8*b^2*c*d*m*x+2*b^2*d^2*x^2-2*a*b*c*d*m-2*a*b*d^2*x+5*b^2*c^2
*m+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/b^3/(m^3+6*m^2+11*m+6)
```

Maxima [A]
time = 0.29, size = 138, normalized size = 1.77

$$\frac{2\,(b^2(m+1)x^2+abmx-a^2)(bx+a)^m cd}{(m^2+3\,m+2)b^2}+\frac{(bx+a)^{m+1}c^2}{b(m+1)}+\frac{((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m d^2}{(m^3+6\,m^2+11\,m+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*d/((m^2 + 3*m + 2)*b^2) +
 (b*x + a)^(m + 1)*c^2/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a
*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d^2/((m^3 + 6*m^2 + 11*m + 6)*b
^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(78) = 156$.
time = 0.32, size = 235, normalized size = 3.01

$$\frac{(ab^2c^2m^2+6\,ab^2c^2-6\,a^2bcd+2\,a^3d^2+(b^3d^2m^2+3\,b^3d^2m+2\,b^3d^2)x^3+(6\,b^3cd+(2\,b^3cd+ab^2d^2)m^2+(8\,b^3cd+ab^2d^2)m)x^2+(5\,ab^2c^2-2\,a^2bcd)m+(6\,b^3c^2+(b^3c^2+2\,ab^2cd)m^2+(5\,b^3c^2+6\,ab^2cd-2\,a^2bd^2)m)x)(bx+a)^m}{b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2*m^2 + 6*a*b^2*c^2 - 6*a^2*b*c*d + 2*a^3*d^2 + (b^3*d^2*m^2 + 3*b
^3*d^2*m + 2*b^3*d^2)*x^3 + (6*b^3*c*d + (2*b^3*c*d + a*b^2*d^2)*m^2 + (8*b
```

```
^3*c*d + a*b^2*d^2)*m)*x^2 + (5*a*b^2*c^2 - 2*a^2*b*c*d)*m + (6*b^3*c^2 + (
b^3*c^2 + 2*a*b^2*c*d)*m^2 + (5*b^3*c^2 + 6*a*b^2*c*d - 2*a^2*b*d^2)*m)*x)*
(b*x + a)^m/(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)
```

Sympy [A]
time = 0.73, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**2,x)
```

```
[Out] Piecewise((a**m*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(b, 0)), (2*a**2*d**2*
log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d**2/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*a*b*c*d/(2*a**2*b**3 + 4*a*b**4*x + 2
*b**5*x**2) + 4*a*b*d**2*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 4*a*b*d**2*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c**2/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 4*b**2*c*d*x/(2*a**2*b**3 + 4*a*b
**4*x + 2*b**5*x**2) + 2*b**2*d**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**
4*x + 2*b**5*x**2), Eq(m, -3)), (-2*a**2*d**2*log(a/b + x)/(a*b**3 + b**4*x
) - 2*a**2*d**2/(a*b**3 + b**4*x) + 2*a*b*c*d*log(a/b + x)/(a*b**3 + b**4*x
) + 2*a*b*c*d/(a*b**3 + b**4*x) - 2*a*b*d**2*x*log(a/b + x)/(a*b**3 + b**4*
x) - b**2*c**2/(a*b**3 + b**4*x) + 2*b**2*c*d*x*log(a/b + x)/(a*b**3 + b**4
*x) + b**2*d**2*x**2/(a*b**3 + b**4*x), Eq(m, -2)), (a**2*d**2*log(a/b + x)
/b**3 - 2*a*c*d*log(a/b + x)/b**2 - a*d**2*x/b**2 + c**2*log(a/b + x)/b + 2
*c*d*x/b + d**2*x**2/(2*b), Eq(m, -1)), (2*a**3*d**2*(a + b*x)**m/(b**3*m**
3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*c*d*m*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 6*a**2*b*c*d*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*d**2*m*x*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*c**2*m**2*(a + b*x)**
m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 5*a*b**2*c**2*m*(a + b*x
)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c**2*(a + b*
x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*a*b**2*c*d*m**2*x*
(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c*d*
m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*d*
*2*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) +
a*b**2*d**2*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b*
*3) + b**3*c**2*m**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m +
6*b**3) + 5*b**3*c**2*m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m
 + 6*b**3) + 6*b**3*c**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*
m + 6*b**3) + 2*b**3*c*d*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 +
11*b**3*m + 6*b**3) + 8*b**3*c*d*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m*
*2 + 11*b**3*m + 6*b**3) + 6*b**3*c*d*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3
*m**2 + 11*b**3*m + 6*b**3) + b**3*d**2*m**2*x**3*(a + b*x)**m/(b**3*m**3 +
 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 3*b**3*d**2*m*x**3*(a + b*x)**m/(b**3*
m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*b**3*d**2*x**3*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(78) = 156.
time = 0.00, size = 427, normalized size = 5.47

$$\frac{2a^3d^2e^{m\ln(a+bx)} - 2a^2bcdme^{m\ln(a+bx)} - 6a^2bcde^{m\ln(a+bx)} - 2a^2bd^2mxe^{m\ln(a+bx)} + ab^2c^2m^2e^{m\ln(a+bx)} + 5ab^2c^2me^{m\ln(a+bx)} + 6ab^2c^2e^{m\ln(a+bx)} + 2ab^2cdm^2xe^{m\ln(a+bx)} + 6ab^2cdmxe^{m\ln(a+bx)} + ab^2d^2m^2x^2e^{m\ln(a+bx)} + ab^2d^2mx^2e^{m\ln(a+bx)} + b^3c^2m^2xe^{m\ln(a+bx)} + 5b^3c^2mxe^{m\ln(a+bx)} + 6b^3c^2xe^{m\ln(a+bx)} + 2b^3cdm^2x^2e^{m\ln(a+bx)} + 8b^3cdmx^2e^{m\ln(a+bx)} + 6b^3cdx^2e^{m\ln(a+bx)} + b^3d^2m^2x^3e^{m\ln(a+bx)} + 3b^3d^2mx^3e^{m\ln(a+bx)} + 2b^3d^2x^3e^{m\ln(a+bx)}}{b^3m^3 + 6b^3m^2 + 11b^3m + 6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x)
```

```
[Out] ((b*x + a)^m*b^3*d^2*m^2*x^3 + 2*(b*x + a)^m*b^3*c*d*m^2*x^2 + (b*x + a)^m*
a*b^2*d^2*m^2*x^2 + 3*(b*x + a)^m*b^3*d^2*m*x^3 + (b*x + a)^m*b^3*c^2*m^2*x
 + 2*(b*x + a)^m*a*b^2*c*d*m^2*x + 8*(b*x + a)^m*b^3*c*d*m*x^2 + (b*x + a)^
m*a*b^2*d^2*m*x^2 + 2*(b*x + a)^m*b^3*d^2*x^3 + (b*x + a)^m*a*b^2*c^2*m^2 +
 5*(b*x + a)^m*b^3*c^2*m*x + 6*(b*x + a)^m*a*b^2*c*d*m*x - 2*(b*x + a)^m*a^
2*b*d^2*m*x + 6*(b*x + a)^m*b^3*c*d*x^2 + 5*(b*x + a)^m*a*b^2*c^2*m - 2*(b*
x + a)^m*a^2*b*c*d*m + 6*(b*x + a)^m*b^3*c^2*x + 6*(b*x + a)^m*a*b^2*c^2 -
6*(b*x + a)^m*a^2*b*c*d + 2*(b*x + a)^m*a^3*d^2)/(b^3*m^3 + 6*b^3*m^2 + 11*
b^3*m + 6*b^3)
```

Mupad [**B**]
time = 0.66, size = 226, normalized size = 2.90

$$(a+b\,x)^m\,\left(\frac{a\,\left(2\,a^2\,d^2-2\,a\,b\,c\,d\,m-6\,a\,b\,c\,d+b^2\,c^2\,m^2+5\,b^2\,c^2\,m+6\,b^2\,c^2\right)}{b^3\,\left(m^3+6\,m^2+11\,m+6\right)}+\frac{d^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{x\,\left(-2\,a^2\,b\,d^2\,m+2\,a\,b^2\,c\,d\,m^2+6\,a\,b^2\,c\,d\,m+b^3\,c^2\,m^2+5\,b^3\,c^2\,m+6\,b^3\,c^2\right)}{b^3\,\left(m^3+6\,m^2+11\,m+6\right)}+\frac{d\,x^2\,\left(m+1\right)\,\left(6\,b\,c+a\,d\,m+2\,b\,c\,m\right)}{b\,\left(m^3+6\,m^2+11\,m+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^2,x)
```

```
[Out] (a + b*x)^m*((a*(2*a^2*d^2 + 6*b^2*c^2 + 5*b^2*c^2*m + b^2*c^2*m^2 - 6*a*b*
c*d - 2*a*b*c*d*m))/(b^3*(11*m + 6*m^2 + m^3 + 6)) + (d^2*x^3*(3*m + m^2 +
2))/(11*m + 6*m^2 + m^3 + 6) + (x*(6*b^3*c^2 + 5*b^3*c^2*m + b^3*c^2*m^2 -
2*a^2*b*d^2*m + 2*a*b^2*c*d*m^2 + 6*a*b^2*c*d*m))/(b^3*(11*m + 6*m^2 + m^3
+ 6)) + (d*x^2*(m + 1)*(6*b*c + a*d*m + 2*b*c*m))/(b*(11*m + 6*m^2 + m^3 +
6)))
```

3.1848 $\int (a+bx)^m(c+dx)\,dx$

Optimal. Leaf size=46

$$\frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}$$

[Out] (-a*d+b*c)*(b*x+a)^(1+m)/b^2/(1+m)+d*(b*x+a)^(2+m)/b^2/(2+m)

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^m*(c + d*x),x]

[Out] ((b*c - a*d)*(a + b*x)^(1 + m))/(b^2*(1 + m)) + (d*(a + b*x)^(2 + m))/(b^2*(2 + m))

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^m}{b}+\frac{d(a+bx)^{1+m}}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.89

$$\frac{(a+bx)^{1+m}(-ad+bc(2+m)+bd(1+m)x)}{b^2(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(a*d) + b*c*(2 + m) + b*d*(1 + m)*x))/(b^2*(1 + m)*(2
+ m))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.04, size = 383, normalized size = 8.33

$$\text{Piecewise}\left[\left\{\left\{\frac{x(2c+dx)a^m}{2},b==0\right\},\left\{\frac{ad\left(1+\text{Log}\left[\frac{a+bx}{b}\right]\right)+b\left(-c+dx\text{Log}\left[\frac{a+bx}{b}\right]\right)}{b^2(a+bx)},m==-2\right\},\left\{\frac{-ad\text{Log}\left[\frac{a}{b}+x\right]+bc\text{Log}\left[\frac{a}{b}+x\right]+bdx}{b^2},m==-1\right\}\right\},-\frac{a^2d(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{2abc(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{abcm(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{abdmx(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{2b^2cx(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{b^2cmx(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{b^2dx^2(a+bx)^m}{2b^2+3b^2m+b^2m^2}+\frac{b^2dmx^2(a+bx)^m}{2b^2+3b^2m+b^2m^2}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^m*(c + d*x)^1,x]')
```

```
[Out] Piecewise[{{x (2 c + d x) a ^ m / 2, b == 0}, {(a d (1 + Log[(a + b x) / b]
) + b (-c + d x Log[(a + b x) / b])) / (b ^ 2 (a + b x)), m == -2}, {(-a d
Log[a / b + x] + b c Log[a / b + x] + b d x) / b ^ 2, m == -1}}, -a ^ 2 d (
a + b x) ^ m / (2 b ^ 2 + 3 b ^ 2 m + b ^ 2 m ^ 2) + 2 a b c (a + b x) ^ m
/ (2 b ^ 2 + 3 b ^ 2 m + b ^ 2 m ^ 2) + a b c m (a + b x) ^ m / (2 b ^ 2 +
3 b ^ 2 m + b ^ 2 m ^ 2) + a b d m x (a + b x) ^ m / (2 b ^ 2 + 3 b ^ 2 m +
 b ^ 2 m ^ 2) + 2 b ^ 2 c x (a + b x) ^ m / (2 b ^ 2 + 3 b ^ 2 m + b ^ 2 m
^ 2) + b ^ 2 c m x (a + b x) ^ m / (2 b ^ 2 + 3 b ^ 2 m + b ^ 2 m ^ 2) + b
^ 2 d x ^ 2 (a + b x) ^ m / (2 b ^ 2 + 3 b ^ 2 m + b ^ 2 m ^ 2) + b ^ 2 d m
 x ^ 2 (a + b x) ^ m / (2 b ^ 2 + 3 b ^ 2 m + b ^ 2 m ^ 2)]
```

Maple [A]
time = 0.15, size = 49, normalized size = 1.07

method	result	size
gosper	$-\frac{(bx+a)^{1+m}(-bdmx-bcm-bdx+ad-2bc)}{b^2(m^2+3m+2)}$	49
risch	$-\frac{(-dx^2b^2m-abdmx-b^2cmx-dx^2b^2-abcm-2b^2cx+a^2d-2abc)(bx+a)^m}{b^2(2+m)(1+m)}$	81
norman	$\frac{dx^2e^{m\ln(bx+a)}}{2+m}+\frac{(adm+bcm+2bc)xe^{m\ln(bx+a)}}{b(m^2+3m+2)}-\frac{a(-bcm+ad-2bc)e^{m\ln(bx+a)}}{b^2(m^2+3m+2)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(-b*d*m*x-b*c*m-b*d*x+a*d-2*b*c)/b^2/(m^2+3*m+2)
```

Maxima [A]
time = 0.27, size = 63, normalized size = 1.37

$$\frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m d}{(m^2+3m+2)b^2}+\frac{(bx+a)^{m+1}c}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d/((m^2 + 3*m + 2)*b^2) + (b*
x + a)^(m + 1)*c/(b*(m + 1))
```

Fricas [A]
time = 0.33, size = 83, normalized size = 1.80

$$\frac{(abcm + 2\,abc - a^2d + (b^2dm + b^2d)x^2 + (2\,b^2c + (b^2c + abd)m)x)(bx + a)^m}{b^2m^2 + 3\,b^2m + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*b*c*m + 2*a*b*c - a^2*d + (b^2*d*m + b^2*d)*x^2 + (2*b^2*c + (b^2*c + a*
b*d)*m)*x)*(b*x + a)^m/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

Sympy [A]
time = 0.44, size = 377, normalized size = 8.20

$$\begin{cases} a^m \left(cx + \frac{dx^2}{2}\right) & \text{for}\ b = 0 \\ \frac{ad\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{ad}{ab^2+b^3x} - \frac{bc}{ab^2+b^3x} + \frac{bdx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ m = -2 \\ -\frac{ad\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{c\log\left(\frac{a}{b}+x\right)}{b} + \frac{dx}{b} & \text{for}\ m = -1 \\ -\frac{a^2d(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{abcm(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{2abc(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{abdmx(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{b^2cmx(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{2b^2cx(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{b^2dmx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2} + \frac{b^2dx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c),x)
```

```
[Out] Piecewise((a**m*(c*x + d*x**2/2), Eq(b, 0)), (a*d*log(a/b + x)/(a*b**2 + b*
*3*x) + a*d/(a*b**2 + b**3*x) - b*c/(a*b**2 + b**3*x) + b*d*x*log(a/b + x)/
(a*b**2 + b**3*x), Eq(m, -2)), (-a*d*log(a/b + x)/b**2 + c*log(a/b + x)/b +
 d*x/b, Eq(m, -1)), (-a**2*d*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 a*b*c*m*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + 2*a*b*c*(a + b*x)**
m/(b**2*m**2 + 3*b**2*m + 2*b**2) + a*b*d*m*x*(a + b*x)**m/(b**2*m**2 + 3*b
**2*m + 2*b**2) + b**2*c*m*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 2*b**2*c*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*m*x**2*(a
 + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*x**2*(a + b*x)**m/(b**2
*m**2 + 3*b**2*m + 2*b**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(46) = 92.
time = 0.00, size = 147, normalized size = 3.20

$$\frac{-a^2de^{m\ln(a+bx)} + abcme^{m\ln(a+bx)} + 2abce^{m\ln(a+bx)} + abdmxe^{m\ln(a+bx)} + b^2cmxe^{m\ln(a+bx)} + 2b^2cxe^{m\ln(a+bx)} + b^2dmx^2e^{m\ln(a+bx)} + b^2dx^2e^{m\ln(a+bx)}}{b^2m^2 + 3b^2m + 2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x)
```

```
[Out] ((b*x + a)^m*b^2*d*m*x^2 + (b*x + a)^m*b^2*c*m*x + (b*x + a)^m*a*b*d*m*x +
(b*x + a)^m*b^2*d*x^2 + (b*x + a)^m*a*b*c*m + 2*(b*x + a)^m*b^2*c*x + 2*(b*
x + a)^m*a*b*c - (b*x + a)^m*a^2*d)/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

Mupad [B]
time = 0.48, size = 88, normalized size = 1.91

$$(a+b\,x)^m\,\left(\frac{a\,\left(2\,b\,c-a\,d+b\,c\,m\right)}{b^2\,\left(m^2+3\,m+2\right)}+\frac{x\,\left(2\,b^2\,c+b^2\,c\,m+a\,b\,d\,m\right)}{b^2\,\left(m^2+3\,m+2\right)}+\frac{d\,x^2\,\left(m+1\right)}{m^2+3\,m+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x),x)
```

```
[Out] (a + b*x)^m*((a*(2*b*c - a*d + b*c*m))/(b^2*(3*m + m^2 + 2)) + (x*(2*b^2*c
+ b^2*c*m + a*b*d*m))/(b^2*(3*m + m^2 + 2)) + (d*x^2*(m + 1))/(3*m + m^2 +
2))
```

3.1849 $\int \frac{(a+bx)^m}{c+dx} \, dx$

Optimal. Leaf size=51

$$\frac{(a+bx)^{1+m} {}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

```
[Out] (b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(1
+m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$\frac{(a+bx)^{m+1} {}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
- a*d))])/((b*c - a*d)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{c+dx} \, dx = \frac{(a+bx)^{1+m} {}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.00

$$-\frac{(a+bx)^{1+m} {}_2F_1\left(1,1+m;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d)])/((-(b*c) + a*d)*(1 + m)))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m/(c + d*x)^1,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c),x)
```

```
[Out] int((b*x+a)^m/(d*x+c),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c),x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x),x)
```

```
[Out] int((a + b*x)^m/(c + d*x), x)
```

3.1850 $\int \frac{(a+bx)^m}{(c+dx)^2} dx$

Optimal. Leaf size=52

$$\frac{b(a+bx)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

```
[Out] b*(b*x+a)^(1+m)*hypergeom([2, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^
2/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$\frac{b(a+bx)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x)^2,x]
```

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)^2}\,dx = \frac{b(a+bx)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 1.00

$$\frac{b(a+bx)^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x)^2,x]
```

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m/(c + d*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)^2,x)
```

```
[Out] int((b*x+a)^m/(d*x+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)**2,x)
[Out] Integral((a + b*x)**m/(c + d*x)**2, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x)^2,x)
[Out] int((a + b*x)^m/(c + d*x)^2, x)
```

3.1851 $\int \frac{(a+bx)^m}{(c+dx)^3} \, dx$

Optimal. Leaf size=54

$$\frac{b^2(a+bx)^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

```
[Out] b^2*(b*x+a)^(1+m)*hypergeom([3, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c
)^3/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$\frac{b^2(a+bx)^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)^3} \, dx = \frac{b^2(a+bx)^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{b^2(a+bx)^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m/(c + d*x)^3,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)^3,x)
```

```
[Out] int((b*x+a)^m/(d*x+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x)**3, x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,x\right)}^m}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x)^3,x)
```

```
[Out] int((a + b*x)^m/(c + d*x)^3, x)
```

3.1852 $\int (a+bx)^3(c+dx)^n\,dx$

Optimal. Leaf size=111

$$-\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}+\frac{b^3(c+dx)^{4+n}}{d^4(4+n)}$$

```
[Out] -(-a*d+b*c)^3*(d*x+c)^(1+n)/d^4/(1+n)+3*b*(-a*d+b*c)^2*(d*x+c)^(2+n)/d^4/(2
+n)-3*b^2*(-a*d+b*c)*(d*x+c)^(3+n)/d^4/(3+n)+b^3*(d*x+c)^(4+n)/d^4/(4+n)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)^3*(c + d*x)^(1 + n))/(d^4*(1 + n))) + (3*b*(b*c - a*d)^2*(c
+ d*x)^(2 + n))/(d^4*(2 + n)) - (3*b^2*(b*c - a*d)*(c + d*x)^(3 + n))/(d^4*
(3 + n)) + (b^3*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^n}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{1+n}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{2+n}}{d^3}\right.\\ &= -\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}+\end{aligned}$$

Mathematica [A]
time = 0.09, size = 95, normalized size = 0.86

$$\frac{(c+dx)^{1+n}\left(-\frac{(bc-ad)^3}{1+n}+\frac{3b(bc-ad)^2(c+dx)}{2+n}-\frac{3b^2(bc-ad)(c+dx)^2}{3+n}+\frac{b^3(c+dx)^3}{4+n}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-((b*c - a*d)^3/(1 + n)) + (3*b*(b*c - a*d)^2*(c + d*x)
)/(2 + n) - (3*b^2*(b*c - a*d)*(c + d*x)^2)/(3 + n) + (b^3*(c + d*x)^3)/(4
+ n)))/d^4
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 16.79, size = 2986, normalized size = 26.90

result too large to display

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^3*(c + d*x)^n,x]')
```

```
[Out] Piecewise[{{x (4 a ^ 3 + 6 a ^ 2 b x + 4 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3) c ^ n
 / 4, d == 0}, {(b ^ 3 c ^ 3 (11 + 6 Log[(c + d x) / d]) + 3 b ^ 2 c ^ 2 d
(-2 a + 6 b x Log[(c + d x) / d] + 9 b x) + 3 b c d ^ 2 (-a ^ 2 - 6 a b x +
 6 b ^ 2 x ^ 2 + 6 b ^ 2 x ^ 2 Log[(c + d x) / d]) + d ^ 3 (-2 a ^ 3 - 9 a
^ 2 b x - 18 a b ^ 2 x ^ 2 + 6 b ^ 3 x ^ 3 Log[(c + d x) / d])) / (6 d ^ 4
(c ^ 3 + 3 c ^ 2 d x + 3 c d ^ 2 x ^ 2 + d ^ 3 x ^ 3)), n == -4}, {(-3 b ^
3 c ^ 3 (3 + 2 Log[(c + d x) / d]) + 3 b ^ 2 c ^ 2 d (2 a Log[(c + d x) / d
] + 3 a - 4 b x - 4 b x Log[(c + d x) / d]) + 3 b c d ^ 2 (-a ^ 2 + 4 a b x
 + 4 a b x Log[(c + d x) / d] - 2 b ^ 2 x ^ 2 Log[(c + d x) / d]) + d ^ 3 (
-a ^ 3 - 6 a ^ 2 b x + 6 a b ^ 2 x ^ 2 Log[(c + d x) / d] + 2 b ^ 3 x ^ 3))
 / (2 d ^ 4 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2)), n == -3}, {(6 b ^ 3 c ^ 3 (1
+ Log[(c + d x) / d]) + 6 b ^ 2 c ^ 2 d (-2 a - 2 a Log[(c + d x) / d] + b
x Log[(c + d x) / d]) + 3 b c d ^ 2 (2 a ^ 2 + 2 a ^ 2 Log[(c + d x) / d] -
 4 a b x Log[(c + d x) / d] - b ^ 2 x ^ 2) + d ^ 3 (-2 a ^ 3 + 6 a ^ 2 b x
Log[(c + d x) / d] + 6 a b ^ 2 x ^ 2 + b ^ 3 x ^ 3)) / (2 d ^ 4 (c + d x)),
 n == -2}, {(-b ^ 3 c ^ 3 Log[(c + d x) / d] + b ^ 2 c ^ 2 d (3 a Log[(c +
d x) / d] + b x) - b c d ^ 2 (6 a ^ 2 Log[(c + d x) / d] + 6 a b x + b ^ 2
x ^ 2) / 2 + d ^ 3 (6 a ^ 3 Log[(c + d x) / d] + 18 a ^ 2 b x + 9 a b ^ 2 x
 ^ 2 + 2 b ^ 3 x ^ 3) / 6) / d ^ 4, n == -1}}, 24 a ^ 3 c d ^ 3 (c + d x) ^
 n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4
) + 26 a ^ 3 c d ^ 3 n (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n
^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 9 a ^ 3 c d ^ 3 n ^ 2 (c + d x) ^ n /
 (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) +
a ^ 3 c d ^ 3 n ^ 3 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2
 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 24 a ^ 3 d ^ 4 x (c + d x) ^ n / (24 d ^
 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 26 a ^ 3
 d ^ 4 n x (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^
 4 n ^ 3 + d ^ 4 n ^ 4) + 9 a ^ 3 d ^ 4 n ^ 2 x (c + d x) ^ n / (24 d ^ 4 +
 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + a ^ 3 d ^ 4
n ^ 3 x (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4
```

n ^ 3 + d ^ 4 n ^ 4) - 36 a ^ 2 b c ^ 2 d ^ 2 (c + d x) ^ n / (24 d ^ 4 + 5
0 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) - 21 a ^ 2 b c ^
2 d ^ 2 n (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^
4 n ^ 3 + d ^ 4 n ^ 4) - 3 a ^ 2 b c ^ 2 d ^ 2 n ^ 2 (c + d x) ^ n / (24 d
^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 36 a ^
2 b c d ^ 3 n x (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 +
10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 21 a ^ 2 b c d ^ 3 n ^ 2 x (c + d x) ^ n /
(24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 3
a ^ 2 b c d ^ 3 n ^ 3 x (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4
n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 36 a ^ 2 b d ^ 4 x ^ 2 (c + d x) ^
n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4)
+ 57 a ^ 2 b d ^ 4 n x ^ 2 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^
4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 24 a ^ 2 b d ^ 4 n ^ 2 x ^ 2 (c
+ d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^
4 n ^ 4) + 3 a ^ 2 b d ^ 4 n ^ 3 x ^ 2 (c + d x) ^ n / (24 d ^ 4 + 50 d ^
4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 24 a b ^ 2 c ^ 3 d (
c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d
^ 4 n ^ 4) + 6 a b ^ 2 c ^ 3 d n (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n +
35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) - 24 a b ^ 2 c ^ 2 d ^ 2 n x
(c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 +
d ^ 4 n ^ 4) - 6 a b ^ 2 c ^ 2 d ^ 2 n ^ 2 x (c + d x) ^ n / (24 d ^ 4 + 5
0 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 12 a b ^ 2 c d
^ 3 n x ^ 2 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d
^ 4 n ^ 3 + d ^ 4 n ^ 4) + 15 a b ^ 2 c d ^ 3 n ^ 2 x ^ 2 (c + d x) ^ n /
(24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 3
a b ^ 2 c d ^ 3 n ^ 3 x ^ 2 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d
^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 24 a b ^ 2 d ^ 4 x ^ 3 (c + d x
) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n
^ 4) + 42 a b ^ 2 d ^ 4 n x ^ 3 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35
d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 21 a b ^ 2 d ^ 4 n ^ 2 x ^ 3
(c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 +
d ^ 4 n ^ 4) + 3 a b ^ 2 d ^ 4 n ^ 3 x ^ 3 (c + d x) ^ n / (24 d ^ 4 + 50
d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) - 6 b ^ 3 c ^ 4 (c
+ d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d
^ 4 n ^ 4) + 6 b ^ 3 c ^ 3 d n x (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 3
5 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) - 3 b ^ 3 c ^ 2 d ^ 2 n x ^ 2
(c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 +
d ^ 4 n ^ 4) - 3 b ^ 3 c ^ 2 d ^ 2 n ^ 2 x ^ 2 (c + d x) ^ n / (24 d ^ 4 +
50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 2 b ^ 3 c d
^ 3 n x ^ 3 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d
^ 4 n ^ 3 + d ^ 4 n ^ 4) + 3 b ^ 3 c d ^ 3 n ^ 2 x ^ 3 (c + d x) ^ n / (24
d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + b ^ 3
c d ^ 3 n ^ 3 x ^ 3 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^
2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 6 b ^ 3 d ^ 4 x ^ 4 (c + d x) ^ n / (24
d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 11 b

```
^ 3 d ^ 4 n x ^ 4 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2
+ 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) + 6 b ^ 3 d ^ 4 n ^ 2 x ^ 4 (c + d x) ^ n /
 (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4) +
b ^ 3 d ^ 4 n ^ 3 x ^ 4 (c + d x) ^ n / (24 d ^ 4 + 50 d ^ 4 n + 35 d ^ 4 n
 ^ 2 + 10 d ^ 4 n ^ 3 + d ^ 4 n ^ 4)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(111) = 222$.
time = 0.18, size = 386, normalized size = 3.48

method	result
gosper	$\frac{(dx+c)^{1+n}\left(b^3d^3n^3x^3+3a\,b^2d^3n^3x^2+6b^3d^3n^2x^3+3a^2b\,d^3n^3x+21a\,b^2d^3n^2x^2-3b^3c\,d^2n^2x^2+11b^3d^3n\,x^3+a^3d^3n^3+24a^2b\,d^3n^2x-6a\right.}{}$
norman	$\frac{b^3x^4\mathrm{e}^{n\ln(dx+c)}}{4+n}+\frac{c\left(a^3d^3n^3+9a^3d^3n^2-3a^2bc\,d^2n^2+26a^3d^3n-21a^2bc\,d^2n+6a\,b^2c^2dn+24a^3d^3-36a^2bc\,d^2+24a\,b^2c^2d-6b^3c^3\right)\mathrm{e}^{n\ln}}{d^4(n^4+10n^3+35n^2+50n+24)}$
risch	$\frac{\left(b^3d^4n^3x^4+3a\,b^2d^4n^3x^3+b^3c\,d^3n^3x^3+6b^3d^4n^2x^4+3a^2b\,d^4n^3x^2+3a\,b^2c\,d^3n^3x^2+21a\,b^2d^4n^2x^3+3b^3c\,d^3n^2x^3+11b^3d^4n\,x^4+a^3d^4n^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)*(b^3*d^3*n^3*x^3+3*a*b^2*d^3*n^3*x^2+6*b^3*d^3*n^2*x^3+3*a^2*
b*d^3*n^3*x+21*a*b^2*d^3*n^2*x^2-3*b^3*c*d^2*n^2*x^2+11*b^3*d^3*n*x^3+a^3*d
^3*n^3+24*a^2*b*d^3*n^2*x-6*a*b^2*c*d^2*n^2*x+42*a*b^2*d^3*n*x^2-9*b^3*c*d^
2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-3*a^2*b*c*d^2*n^2+57*a^2*b*d^3*n*x-30*a
*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2+6*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*
n-21*a^2*b*c*d^2*n+36*a^2*b*d^3*x+6*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x+6*b^3*c^
2*d*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/d^4/(n^4+10*n^3+3
5*n^2+50*n+24)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(111) = 222$.
time = 0.27, size = 246, normalized size = 2.22

$$\frac{3\,(d^2(n+1)x^2+cdnx-c^2)(dx+c)^n a^2 b}{(n^2+3\,n+2)d^2}+\frac{(dx+c)^{n+1}a^3}{d(n+1)}+\frac{3\,((n^2+3\,n+2)d^3x^3+(n^2+n)cd^2x^2-2\,c^2dnx+2\,c^3)(dx+c)^n ab^2}{(n^3+6\,n^2+11\,n+6)d^3}+\frac{((n^3+6\,n^2+11\,n+6)d^4x^4+(n^3+3\,n^2+2\,n)cd^3x^3-3\,(n^2+n)c^2d^2x^2+6\,c^3dnx-6\,c^4)(dx+c)^n b^3}{(n^4+10\,n^3+35\,n^2+50\,n+24)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 3*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a^2*b/((n^2 + 3*n + 2)*d^2)
 + (d*x + c)^(n + 1)*a^3/(d*(n + 1)) + 3*((n^2 + 3*n + 2)*d^3*x^3 + (n^2 +
n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*a*b^2/((n^3 + 6*n^2 + 11*n
+ 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x
^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*b^3/((n^4 +
 10*n^3 + 35*n^2 + 50*n + 24)*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(111) = 222$.
time = 0.35, size = 496, normalized size = 4.47

$$\frac{\left(a^{3} c d^{3} n^{3} - 6 b^{3} c^{4} + 24 a b^{2} c^{3} d - 36 a^{2} b c^{2} d^{2} + 24 a^{3} c d^{3} + \left(b^{3} d^{4} n^{3} + 6 b^{3} d^{4} n^{2} + 11 b^{3} d^{4} n + 6 b^{3} d^{4}\right) x^{4} + \left(24 a b^{2} d^{4} + \left(b^{3} c d^{3} + 3 a b^{2} d^{4}\right) n^{3} + 3 \left(b^{3} c d^{3} + 7 a b^{2} d^{4}\right) n^{2} + 2 \left(b^{3} c d^{3} + 21 a b^{2} d^{4}\right) n\right) x^{3} - 3 \left(a^{2} b c^{2} d^{2} - 3 a^{3} c d^{3}\right) n^{2} + 3 \left(12 a^{2} b d^{4} + \left(a b^{2} c d^{3} + a^{2} b d^{4}\right) n^{3} - \left(b^{3} c^{2} d^{2} - 5 a b^{2} c d^{3} - 8 a^{2} b d^{4}\right) n^{2} - \left(b^{3} c^{2} d^{2} - 4 a b^{2} c d^{3} - 19 a^{2} b d^{4}\right) n\right) x^{2} + \left(6 a b^{2} c^{3} d - 21 a^{2} b c^{2} d^{2} + 26 a^{3} c d^{3}\right) n + \left(24 a^{3} d^{4} + \left(3 a^{2} b c d^{3} + a^{3} d^{4}\right) n^{3} - 3 \left(2 a b^{2} c^{2} d^{2} - 7 a^{2} b c d^{3} - 3 a^{3} d^{4}\right) n^{2} + 2 \left(3 b^{3} c^{3} d - 12 a b^{2} c^{2} d^{2} + 18 a^{2} b c d^{3} + 13 a^{3} d^{4}\right) n\right) x\right) (dx + c)^{n}}{d^{4} n^{4} + 10 d^{4} n^{3} + 35 d^{4} n^{2} + 50 d^{4} n + 24 d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^3*c*d^3*n^3 - 6*b^3*c^4 + 24*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 24*a^3*c*d
^3 + (b^3*d^4*n^3 + 6*b^3*d^4*n^2 + 11*b^3*d^4*n + 6*b^3*d^4)*x^4 + (24*a*b
^2*d^4 + (b^3*c*d^3 + 3*a*b^2*d^4)*n^3 + 3*(b^3*c*d^3 + 7*a*b^2*d^4)*n^2 +
2*(b^3*c*d^3 + 21*a*b^2*d^4)*n)*x^3 - 3*(a^2*b*c^2*d^2 - 3*a^3*c*d^3)*n^2 +
 3*(12*a^2*b*d^4 + (a*b^2*c*d^3 + a^2*b*d^4)*n^3 - (b^3*c^2*d^2 - 5*a*b^2*c
*d^3 - 8*a^2*b*d^4)*n^2 - (b^3*c^2*d^2 - 4*a*b^2*c*d^3 - 19*a^2*b*d^4)*n)*x
^2 + (6*a*b^2*c^3*d - 21*a^2*b*c^2*d^2 + 26*a^3*c*d^3)*n + (24*a^3*d^4 + (3
*a^2*b*c*d^3 + a^3*d^4)*n^3 - 3*(2*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 - 3*a^3*d^
4)*n^2 + 2*(3*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 + 13*a^3*d^4)*n
)*x)*(d*x + c)^n/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

Sympy [A]
time = 1.31, size = 4058, normalized size = 36.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), Eq(
d, 0)), (-2*a**3*d**3/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d*
*7*x**3) - 3*a**2*b*c*d**2/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 +
 6*d**7*x**3) - 9*a**2*b*d**3*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x
**2 + 6*d**7*x**3) - 6*a*b**2*c**2*d/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d
**6*x**2 + 6*d**7*x**3) - 18*a*b**2*c*d**2*x/(6*c**3*d**4 + 18*c**2*d**5*x
+ 18*c*d**6*x**2 + 6*d**7*x**3) - 18*a*b**2*d**3*x**2/(6*c**3*d**4 + 18*c**
2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*c**3*log(c/d + x)/(6*c**3
*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 11*b**3*c**3/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c**2*d*x
*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3)
 + 27*b**3*c**2*d*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7
*x**3) + 18*b**3*c*d**2*x**2*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 1
8*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c*d**2*x**2/(6*c**3*d**4 + 18*c**2*d
**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*d**3*x**3*log(c/d + x)/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3), Eq(n, -4)), (-a**
3*d**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 3*a**2*b*c*d**2/(2*c**2*d
**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*a**2*b*d**3*x/(2*c**2*d**4 + 4*c*d**5*x
```

```
 + 2*d**6*x**2) + 6*a*b**2*c**2*d*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x +
2*d**6*x**2) + 9*a*b**2*c**2*d/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 6*a*b**2*d**3*
x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*b**3*c**3*lo
g(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 9*b**3*c**3/(2*c**2*d
**4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x*log(c/d + x)/(2*c**2*d**
4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) - 6*b**3*c*d**2*x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) + 2*b**3*d**3*x**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2),
 Eq(n, -3)), (-2*a**3*d**3/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2*log(c/d
+ x)/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2/(2*c*d**4 + 2*d**5*x) + 6*a**2
*b*d**3*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d*log(c/d + x
)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d/(2*c*d**4 + 2*d**5*x) - 12*a*b**
2*c*d**2*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*a*b**2*d**3*x**2/(2*c*d**
4 + 2*d**5*x) + 6*b**3*c**3*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**
3/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**2*d*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x
) - 3*b**3*c*d**2*x**2/(2*c*d**4 + 2*d**5*x) + b**3*d**3*x**3/(2*c*d**4 + 2
*d**5*x), Eq(n, -2)), (a**3*log(c/d + x)/d - 3*a**2*b*c*log(c/d + x)/d**2 +
 3*a**2*b*x/d + 3*a*b**2*c**2*log(c/d + x)/d**3 - 3*a*b**2*c*x/d**2 + 3*a*b
**2*x**2/(2*d) - b**3*c**3*log(c/d + x)/d**4 + b**3*c**2*x/d**3 - b**3*c*x*
*2/(2*d**2) + b**3*x**3/(3*d), Eq(n, -1)), (a**3*c*d**3*n**3*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9*a**3*c*d
**3*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n
+ 24*d**4) + 26*a**3*c*d**3*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d
**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*c*d**3*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + a**3*d**4*n**3*x*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9
*a**3*d**4*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 5
0*d**4*n + 24*d**4) + 26*a**3*d**4*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n*
*3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*d**4*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*a**2*b*c**2
*d**2*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*
n + 24*d**4) - 21*a**2*b*c**2*d**2*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 36*a**2*b*c**2*d**2*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**2*b*c
*d**3*n**3*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**
4*n + 24*d**4) + 21*a**2*b*c*d**3*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*
n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 36*a**2*b*c*d**3*n*x*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**
2*b*d**4*n**3*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
50*d**4*n + 24*d**4) + 24*a**2*b*d**4*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 1
0*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 57*a**2*b*d**4*n*x**2*(
c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4)
 + 36*a**2*b*d**4*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**
```

```
2 + 50*d**4*n + 24*d**4) + 6*a*b**2*c**3*d*n*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a*b**2*c**3*d*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 6*a*b
**2*c**2*d**2*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) - 24*a*b**2*c**2*d**2*n*x*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2*c*d**3*n**3*x
**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*
d**4) + 15*a*b**2*c*d**3*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 +
 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 12*a*b**2*c*d**3*n*x**2*(c + d*x)**n
/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2
*d**4*n**3*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*
d**4*n + 24*d**4) + 21*a*b**2*d**4*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 42*a*b**2*d**4*n*x**3*(c +
 d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) +
24*a*b**2*d**4*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
 50*d**4*n + 24*d**4) - 6*b**3*c**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
+ 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*c**3*d*n*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*b**3*c**2*d
**2*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d*
*4*n + 24*d**4) - 3*b**3*c**2*d**2*n*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4
*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + b**3*c*d**3*n**3*x**3*(c + d*
x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*b
**3*c*d**3*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) + 2*b**3*c*d**3*n*x**3*(c + d*x)**n/(d**4*n**4 + 10*
d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + b**3*d**4*n**3*x**4*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6
*b**3*d**4*n**2*x**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) + 11*b**3*d**4*n*x**4*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*d**4*x**4*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(111) = 222.
time = 0.00, size = 920, normalized size = 8.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x)
```

```
[Out] ((d*x + c)^n*b^3*d^4*n^3*x^4 + (d*x + c)^n*b^3*c*d^3*n^3*x^3 + 3*(d*x + c)^
n*a*b^2*d^4*n^3*x^3 + 6*(d*x + c)^n*b^3*d^4*n^2*x^4 + 3*(d*x + c)^n*a*b^2*c
*d^3*n^3*x^2 + 3*(d*x + c)^n*a^2*b*d^4*n^3*x^2 + 3*(d*x + c)^n*b^3*c*d^3*n^
2*x^3 + 21*(d*x + c)^n*a*b^2*d^4*n^2*x^3 + 11*(d*x + c)^n*b^3*d^4*n*x^4 + 3
*(d*x + c)^n*a^2*b*c*d^3*n^3*x + (d*x + c)^n*a^3*d^4*n^3*x - 3*(d*x + c)^n*
```

```
b^3*c^2*d^2*n^2*x^2 + 15*(d*x + c)^n*a*b^2*c*d^3*n^2*x^2 + 24*(d*x + c)^n*a
^2*b*d^4*n^2*x^2 + 2*(d*x + c)^n*b^3*c*d^3*n*x^3 + 42*(d*x + c)^n*a*b^2*d^4
*n*x^3 + 6*(d*x + c)^n*b^3*d^4*x^4 + (d*x + c)^n*a^3*c*d^3*n^3 - 6*(d*x + c
)^n*a*b^2*c^2*d^2*n^2*x + 21*(d*x + c)^n*a^2*b*c*d^3*n^2*x + 9*(d*x + c)^n*
a^3*d^4*n^2*x - 3*(d*x + c)^n*b^3*c^2*d^2*n*x^2 + 12*(d*x + c)^n*a*b^2*c*d^
3*n*x^2 + 57*(d*x + c)^n*a^2*b*d^4*n*x^2 + 24*(d*x + c)^n*a*b^2*d^4*x^3 - 3
*(d*x + c)^n*a^2*b*c^2*d^2*n^2 + 9*(d*x + c)^n*a^3*c*d^3*n^2 + 6*(d*x + c)^
n*b^3*c^3*d*n*x - 24*(d*x + c)^n*a*b^2*c^2*d^2*n*x + 36*(d*x + c)^n*a^2*b*c
*d^3*n*x + 26*(d*x + c)^n*a^3*d^4*n*x + 36*(d*x + c)^n*a^2*b*d^4*x^2 + 6*(d
*x + c)^n*a*b^2*c^3*d*n - 21*(d*x + c)^n*a^2*b*c^2*d^2*n + 26*(d*x + c)^n*a
^3*c*d^3*n + 24*(d*x + c)^n*a^3*d^4*x - 6*(d*x + c)^n*b^3*c^4 + 24*(d*x + c
)^n*a*b^2*c^3*d - 36*(d*x + c)^n*a^2*b*c^2*d^2 + 24*(d*x + c)^n*a^3*c*d^3)/
(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

Mupad [B]
time = 0.91, size = 478, normalized size = 4.31

$$\frac{x\,(c+d\,x)^n\,\left(a^3\,d^4\,n^3+9\,a^3\,d^4\,n^2+26\,a^3\,d^4\,n+24\,a^3\,d^4+3\,a^2\,b\,c\,d^3\,n^3+21\,a^2\,b\,c\,d^3\,n^2+36\,a^2\,b\,c\,d^3\,n-6\,a\,b^2\,c^2\,d^2\,n^2-24\,a\,b^2\,c^2\,d^2\,n+6\,b^3\,c^3\,d\,n\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^3\,x^4\,(c+d\,x)^n\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{c\,(c+d\,x)^n\,\left(a^3\,d^3\,n^3+9\,a^3\,d^3\,n^2+26\,a^3\,d^3\,n+24\,a^3\,d^3-3\,a^2\,b\,c\,d^2\,n^2-21\,a^2\,b\,c\,d^2\,n-36\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d\,n+24\,a\,b^2\,c^2\,d-6\,b^3\,c^3\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,b\,x^2\,(n+1)\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2+a\,b\,c\,d\,n^2+4\,a\,b\,c\,d\,n-b^2\,c^2\,n\right)}{d^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^2\,x^3\,(c+d\,x)^n\,(12\,a\,d+3\,a\,d\,n+b\,c\,n)\,\left(n^2+3\,n+2\right)}{d\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^n,x)
```

```
[Out] (x*(c + d*x)^n*(24*a^3*d^4 + 26*a^3*d^4*n + 9*a^3*d^4*n^2 + a^3*d^4*n^3 + 6
*b^3*c^3*d*n + 36*a^2*b*c*d^3*n - 24*a*b^2*c^2*d^2*n + 21*a^2*b*c*d^3*n^2 +
 3*a^2*b*c*d^3*n^3 - 6*a*b^2*c^2*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24)) + (b^3*x^4*(c + d*x)^n*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) + (c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n
 + 9*a^3*d^3*n^2 + a^3*d^3*n^3 + 24*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 6*a*b^2*
c^2*d*n - 21*a^2*b*c*d^2*n - 3*a^2*b*c*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n
^3 + n^4 + 24)) + (3*b*x^2*(n + 1)*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n -
b^2*c^2*n + a^2*d^2*n^2 + 4*a*b*c*d*n + a*b*c*d*n^2))/(d^2*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24)) + (b^2*x^3*(c + d*x)^n*(12*a*d + 3*a*d*n + b*c*n)*(3*n
 + n^2 + 2))/(d*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

3.1853 $\int (a+bx)^2(c+dx)^n\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}$$

```
[Out] (-a*d+b*c)^2*(d*x+c)^(1+n)/d^3/(1+n)-2*b*(-a*d+b*c)*(d*x+c)^(2+n)/d^3/(2+n)
+b^2*(d*x+c)^(3+n)/d^3/(3+n)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^(1 + n))/(d^3*(1 + n)) - (2*b*(b*c - a*d)*(c + d*x
)^(2 + n))/(d^3*(2 + n)) + (b^2*(c + d*x)^(3 + n))/(d^3*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^n}{d^2}-\frac{2b(bc-ad)(c+dx)^{1+n}}{d^2}+\frac{b^2(c+dx)^{2+n}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 0.86

$$\frac{(c+dx)^{1+n}\left(\frac{(bc-ad)^2}{1+n}-\frac{2b(bc-ad)(c+dx)}{2+n}+\frac{b^2(c+dx)^2}{3+n}\right)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2*(c + d*x)^n,x]

[Out] ((c + d*x)^(1 + n)*((b*c - a*d)^2/(1 + n) - (2*b*(b*c - a*d)*(c + d*x))/(2 + n) + (b^2*(c + d*x)^2)/(3 + n)))/d^3

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 6.90, size = 1256, normalized size = 16.10

Antiderivative was successfully verified.

[In] mathics('Integrate[(a + b*x)^2*(c + d*x)^n,x]')

[Out] Piecewise[{{x (3 a ^ 2 + 3 a b x + b ^ 2 x ^ 2) c ^ n / 3, d == 0}, {(b ^ 2 c ^ 2 (3 + 2 Log[(c + d x) / d]) + 2 b c d (-a + 2 b x + 2 b x Log[(c + d x) / d]) + d ^ 2 (-a ^ 2 - 4 a b x + 2 b ^ 2 x ^ 2 Log[(c + d x) / d])) / (2 d ^ 3 (c ^ 2 + 2 c d x + d ^ 2 x ^ 2)), n == -3}, {(-2 b ^ 2 c ^ 2 (1 + Log[(c + d x) / d]) + 2 b c d (a + a Log[(c + d x) / d] - b x Log[(c + d x) / d]) + d ^ 2 (-a ^ 2 + 2 a b x Log[(c + d x) / d] + b ^ 2 x ^ 2)) / (d ^ 3 (c + d x)), n == -2}, {(b ^ 2 c ^ 2 Log[(c + d x) / d] - b c d (2 a Log[(c + d x) / d] + b x) + d ^ 2 (2 a ^ 2 Log[(c + d x) / d] + 4 a b x + b ^ 2 x ^ 2) / 2) / d ^ 3, n == -1}}, 6 a ^ 2 c d ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 5 a ^ 2 c d ^ 2 n (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + a ^ 2 c d ^ 2 n ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 6 a ^ 2 d ^ 3 x (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 5 a ^ 2 d ^ 3 n x (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + a ^ 2 d ^ 3 n ^ 2 x (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) - 6 a b c ^ 2 d (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) - 2 a b c ^ 2 d n (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 6 a b c d ^ 2 n x (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 2 a b c d ^ 2 n ^ 2 x (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 6 a b d ^ 3 x ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 8 a b d ^ 3 n x ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 2 a b d ^ 3 n ^ 2 x ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 2 b ^ 2 c ^ 3 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) - 2 b ^ 2 c ^ 2 d n x (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + b ^ 2 c d ^ 2 n x ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + b ^ 2 c d ^ 2 n ^ 2 x ^ 2 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) + 2 b ^ 2 d ^ 3 x ^ 3 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3) +

```
3 b ^ 2 d ^ 3 n x ^ 3 (c + d x) ^ n / (6 d ^ 3 + 11 d ^ 3 n + 6 d ^ 3 n ^
2 + d ^ 3 n ^ 3) + b ^ 2 d ^ 3 n ^ 2 x ^ 3 (c + d x) ^ n / (6 d ^ 3 + 11 d
^ 3 n + 6 d ^ 3 n ^ 2 + d ^ 3 n ^ 3)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.
time = 0.19, size = 159, normalized size = 2.04

method	result
gosper	$\frac{(dx+c)^{1+n}(b^2d^2n^2x^2+2ab\,d^2n^2x+3b^2d^2n\,x^2+a^2d^2n^2+8ab\,d^2nx-2b^2cdnx+2b^2x^2d^2+5a^2d^2n-2abcdn+6ab\,d^2x-2b^2cdx+6a^2d^2-6a}{d^3(n^3+6n^2+11n+6)}$
norman	$\frac{b^2x^3e^{n\ln(dx+c)}}{3+n}+\frac{c(a^2d^2n^2+5a^2d^2n-2abcdn+6a^2d^2-6abcd+2b^2c^2)e^{n\ln(dx+c)}}{d^3(n^3+6n^2+11n+6)}+\frac{(a^2d^2n^2+2abcd\,n^2+5a^2d^2n+6abcdn-2b^2c^2n+}{d^2(n^3+6n^2+11n+6)}$
risch	$\frac{(b^2d^3n^2x^3+2ab\,d^3n^2x^2+b^2c\,d^2n^2x^2+3b^2d^3n\,x^3+a^2d^3n^2x+2abc\,d^2n^2x+8ab\,d^3n\,x^2+b^2c\,d^2n\,x^2+2b^2x^3d^3+a^2c\,d^2n^2+5a^2d^3nx+6abc}{(2+n)(3+n)(1+n)d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)*(b^2*d^2*n^2*x^2+2*a*b*d^2*n^2*x+3*b^2*d^2*n*x^2+a^2*d^2*n^2+
8*a*b*d^2*n*x-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-2*a*b*c*d*n+6*a*b*d^2
*x-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/d^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.29, size = 138, normalized size = 1.77

$$\frac{2\,(d^2(n+1)x^2+cdnx-c^2)(dx+c)^n ab}{(n^2+3\,n+2)d^2}+\frac{(dx+c)^{n+1}a^2}{d(n+1)}+\frac{((n^2+3\,n+2)d^3x^3+(n^2+n)cd^2x^2-2\,c^2dnx+2\,c^3)(dx+c)^n b^2}{(n^3+6\,n^2+11\,n+6)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 2*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a*b/((n^2 + 3*n + 2)*d^2) +
 (d*x + c)^(n + 1)*a^2/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c
*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b^2/((n^3 + 6*n^2 + 11*n + 6)*d
^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(78) = 156$.
time = 0.33, size = 237, normalized size = 3.04

$$\frac{(a^2cd^2n^2+2\,b^2c^3-6\,abc^2d+6\,a^2cd^2+(b^2d^3n^2+3\,b^2d^3n+2\,b^2d^3)x^3+(6\,abd^3+(b^2cd^2+2\,abd^3)n^2+(b^2cd^2+8\,abd^3)n)x^2-(2\,abc^2d-5\,a^2cd^2)n+(6\,a^2d^3+(2\,abcd^2+a^2d^3)n^2-(2\,b^2c^2d-6\,abcd^2-5\,a^2d^3)n)x)(dx+c)^n}{d^3n^3+6\,d^3n^2+11\,d^3n+6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^2*c*d^2*n^2 + 2*b^2*c^3 - 6*a*b*c^2*d + 6*a^2*c*d^2 + (b^2*d^3*n^2 + 3*b
^2*d^3*n + 2*b^2*d^3)*x^3 + (6*a*b*d^3 + (b^2*c*d^2 + 2*a*b*d^3)*n^2 + (b^2
```

```
*c*d^2 + 8*a*b*d^3)*n)*x^2 - (2*a*b*c^2*d - 5*a^2*c*d^2)*n + (6*a^2*d^3 + (
2*a*b*c*d^2 + a^2*d^3)*n^2 - (2*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*n)*x)*
(d*x + c)^n/(d^3*n^3 + 6*d^3*n^2 + 11*d^3*n + 6*d^3)
```

Sympy [**A**]

time = 0.81, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(d, 0)), (-a**2*d**2/(
2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) - 2*a*b*c*d/(2*c**2*d**3 + 4*c*d**4
*x + 2*d**5*x**2) - 4*a*b*d**2*x/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) +
 2*b**2*c**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 3*b**2
*c**2/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x*log(c/d + x)/
(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x/(2*c**2*d**3 + 4*c*
d**4*x + 2*d**5*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d*
*4*x + 2*d**5*x**2), Eq(n, -3)), (-a**2*d**2/(c*d**3 + d**4*x) + 2*a*b*c*d*
log(c/d + x)/(c*d**3 + d**4*x) + 2*a*b*c*d/(c*d**3 + d**4*x) + 2*a*b*d**2*x
*log(c/d + x)/(c*d**3 + d**4*x) - 2*b**2*c**2*log(c/d + x)/(c*d**3 + d**4*x
) - 2*b**2*c**2/(c*d**3 + d**4*x) - 2*b**2*c*d*x*log(c/d + x)/(c*d**3 + d**
4*x) + b**2*d**2*x**2/(c*d**3 + d**4*x), Eq(n, -2)), (a**2*log(c/d + x)/d -
 2*a*b*c*log(c/d + x)/d**2 + 2*a*b*x/d + b**2*c**2*log(c/d + x)/d**3 - b**2
*c*x/d**2 + b**2*x**2/(2*d), Eq(n, -1)), (a**2*c*d**2*n**2*(c + d*x)**n/(d*
*3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*c*d**2*n*(c + d*x)**n/
(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*c*d**2*(c + d*x)**n
/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + a**2*d**3*n**2*x*(c + d*x
)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*d**3*n*x*(c +
d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*d**3*x*(c +
 d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 2*a*b*c**2*d*n*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 6*a*b*c**2*d*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*b*c*d**2*n*
*2*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a*b*c*
d**2*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*
b*d**3*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3
) + 8*a*b*d**3*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6
*d**3) + 6*a*b*d**3*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n
+ 6*d**3) + 2*b**2*c**3*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n +
 6*d**3) - 2*b**2*c**2*d*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**
3*n + 6*d**3) + b**2*c*d**2*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2
 + 11*d**3*n + 6*d**3) + b**2*c*d**2*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**
3*n**2 + 11*d**3*n + 6*d**3) + b**2*d**3*n**2*x**3*(c + d*x)**n/(d**3*n**3
+ 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 3*b**2*d**3*n*x**3*(c + d*x)**n/(d**3
*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*b**2*d**3*x**3*(c + d*x)**n/(
d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(78) = 156.

time = 0.00, size = 427, normalized size = 5.47

$$\frac{a^{2} c d^{2} n^{2} e^{n \ln\left(c+d x\right)} + 5 a^{2} c d^{2} n e^{n \ln\left(c+d x\right)} + 6 a^{2} c d^{2} e^{n \ln\left(c+d x\right)} + a^{2} d^{3} n^{2} x e^{n \ln\left(c+d x\right)} + 5 a^{2} d^{3} n x e^{n \ln\left(c+d x\right)} + 6 a^{2} d^{3} x e^{n \ln\left(c+d x\right)} - 2 a b c^{2} d n e^{n \ln\left(c+d x\right)} - 6 a b c^{2} d e^{n \ln\left(c+d x\right)} + 2 a b c d^{2} n^{2} x e^{n \ln\left(c+d x\right)} + 6 a b c d^{2} n x e^{n \ln\left(c+d x\right)} + 2 a b d^{3} n^{2} x^{2} e^{n \ln\left(c+d x\right)} + 8 a b d^{3} n x^{2} e^{n \ln\left(c+d x\right)} + 6 a b d^{3} x^{2} e^{n \ln\left(c+d x\right)} + 2 b^{2} c^{3} e^{n \ln\left(c+d x\right)} - 2 b^{2} c^{2} d n x e^{n \ln\left(c+d x\right)} + b^{2} c d^{2} n^{2} x^{2} e^{n \ln\left(c+d x\right)} + b^{2} c d^{2} n x^{2} e^{n \ln\left(c+d x\right)} + b^{2} d^{3} n^{2} x^{3} e^{n \ln\left(c+d x\right)} + 3 b^{2} d^{3} n x^{3} e^{n \ln\left(c+d x\right)} + 2 b^{2} d^{3} x^{3} e^{n \ln\left(c+d x\right)}}{d^{3} n^{3} + 6 d^{3} n^{2} + 11 d^{3} n + 6 d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x)
```

```
[Out] ((d*x + c)^n*b^2*d^3*n^2*x^3 + (d*x + c)^n*b^2*c*d^2*n^2*x^2 + 2*(d*x + c)^
n*a*b*d^3*n^2*x^2 + 3*(d*x + c)^n*b^2*d^3*n*x^3 + 2*(d*x + c)^n*a*b*c*d^2*n
^2*x + (d*x + c)^n*a^2*d^3*n^2*x + (d*x + c)^n*b^2*c*d^2*n*x^2 + 8*(d*x + c
)^n*a*b*d^3*n*x^2 + 2*(d*x + c)^n*b^2*d^3*x^3 + (d*x + c)^n*a^2*c*d^2*n^2 -
 2*(d*x + c)^n*b^2*c^2*d*n*x + 6*(d*x + c)^n*a*b*c*d^2*n*x + 5*(d*x + c)^n*
a^2*d^3*n*x + 6*(d*x + c)^n*a*b*d^3*x^2 - 2*(d*x + c)^n*a*b*c^2*d*n + 5*(d*
x + c)^n*a^2*c*d^2*n + 6*(d*x + c)^n*a^2*d^3*x + 2*(d*x + c)^n*b^2*c^3 - 6*
(d*x + c)^n*a*b*c^2*d + 6*(d*x + c)^n*a^2*c*d^2)/(d^3*n^3 + 6*d^3*n^2 + 11*
d^3*n + 6*d^3)
```

Mupad [B]

time = 0.62, size = 226, normalized size = 2.90

$$(c+d\,x)^n\,\left(\frac{c\,\left(a^2\,d^2\,n^2+5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+2\,b^2\,c^2\right)}{d^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b^2\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{x\,\left(a^2\,d^3\,n^2+5\,a^2\,d^3\,n+6\,a^2\,d^3+2\,a\,b\,c\,d^2\,n^2+6\,a\,b\,c\,d^2\,n-2\,b^2\,c^2\,d\,n\right)}{d^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,x^2\,\left(n+1\right)\,\left(6\,a\,d+2\,a\,d\,n+b\,c\,n\right)}{d\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(6*a^2*d^2 + 2*b^2*c^2 + 5*a^2*d^2*n + a^2*d^2*n^2 - 6*a*b*
c*d - 2*a*b*c*d*n))/(d^3*(11*n + 6*n^2 + n^3 + 6)) + (b^2*x^3*(3*n + n^2 +
2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*a^2*d^3 + 5*a^2*d^3*n + a^2*d^3*n^2 -
2*b^2*c^2*d*n + 2*a*b*c*d^2*n^2 + 6*a*b*c*d^2*n))/(d^3*(11*n + 6*n^2 + n^3
+ 6)) + (b*x^2*(n + 1)*(6*a*d + 2*a*d*n + b*c*n))/(d*(11*n + 6*n^2 + n^3 +
6)))
```

3.1854 $\int (a+bx)(c+dx)^n\,dx$

Optimal. Leaf size=47

$$-\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}$$

```
[Out] -(-a*d+b*c)*(d*x+c)^(1+n)/d^2/(1+n)+b*(d*x+c)^(2+n)/d^2/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)*(c + d*x)^(1 + n))/(d^2*(1 + n))) + (b*(c + d*x)^(2 + n))/(d
^2*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^n}{d}+\frac{b(c+dx)^{1+n}}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.87

$$\frac{(c+dx)^{1+n}(-bc+ad(2+n)+bd(1+n)x)}{d^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(b*c) + a*d*(2 + n) + b*d*(1 + n)*x))/(d^2*(1 + n)*(2
+ n))
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 3.07, size = 383, normalized size = 8.15

$$\text{Piecewise}\left[\left\{\left\{\frac{x\left(2a+bx\right)c^{n}}{2},d==0\right\},\left\{\frac{bc\left(1+\text{Log}\left[\frac{c+dx}{d}\right]\right)+d\left(-a+bx\text{Log}\left[\frac{c+dx}{d}\right]\right)}{d^{2}\left(c+dx\right)},n==-2\right\},\left\{\frac{ad\text{Log}\left[\frac{c}{d}+x\right]-bc\text{Log}\left[\frac{c}{d}+x\right]+bdx}{d^{2}},n==-1\right\}\right\},\frac{2acd\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}+\frac{acdn\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}+\frac{2ad^{2}x\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}+\frac{ad^{2}nx\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}-\frac{bc^{2}\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}+\frac{bcdnx\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}+\frac{bd^{2}x^{2}\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}+\frac{bd^{2}nx^{2}\left(c+dx\right)^{n}}{2d^{2}+3d^{2}n+d^{2}n^{2}}\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[(a + b*x)^1*(c + d*x)^n,x]')
```

```
[Out] Piecewise[{{x (2 a + b x) c ^ n / 2, d == 0}, {(b c (1 + Log[(c + d x) / d]
) + d (-a + b x Log[(c + d x) / d])) / (d ^ 2 (c + d x)), n == -2}, {(a d L
og[c / d + x] - b c Log[c / d + x] + b d x) / d ^ 2, n == -1}}, 2 a c d (c
+ d x) ^ n / (2 d ^ 2 + 3 d ^ 2 n + d ^ 2 n ^ 2) + a c d n (c + d x) ^ n /
(2 d ^ 2 + 3 d ^ 2 n + d ^ 2 n ^ 2) + 2 a d ^ 2 x (c + d x) ^ n / (2 d ^ 2
+ 3 d ^ 2 n + d ^ 2 n ^ 2) + a d ^ 2 n x (c + d x) ^ n / (2 d ^ 2 + 3 d ^ 2
 n + d ^ 2 n ^ 2) - b c ^ 2 (c + d x) ^ n / (2 d ^ 2 + 3 d ^ 2 n + d ^ 2 n
^ 2) + b c d n x (c + d x) ^ n / (2 d ^ 2 + 3 d ^ 2 n + d ^ 2 n ^ 2) + b d
^ 2 x ^ 2 (c + d x) ^ n / (2 d ^ 2 + 3 d ^ 2 n + d ^ 2 n ^ 2) + b d ^ 2 n x
 ^ 2 (c + d x) ^ n / (2 d ^ 2 + 3 d ^ 2 n + d ^ 2 n ^ 2)]
```

Maple [A]
time = 0.16, size = 46, normalized size = 0.98

method	result	size
gosper	$\frac{\left(dx+c\right)^{1+n} \left(bdnx+adn+bdx+2ad-bc\right)}{d^{2} \left(n^{2}+3n+2\right)}$	46
risch	$\frac{\left(x^{2} b\,d^{2} n+a\,d^{2} n x+bcdnx+x^{2} b\,d^{2}+acdn+2a\,d^{2} x+2acd-b\,c^{2}\right) \left(dx+c\right)^{n}}{d^{2} \left(2+n\right) \left(1+n\right)}$	76
norman	$\frac{b\,x^{2} {\mathrm e}^{n \ln \left(dx+c\right)}}{2+n}+\frac{c \left(adn+2ad-bc\right) {\mathrm e}^{n \ln \left(dx+c\right)}}{d^{2} \left(n^{2}+3n+2\right)}+\frac{\left(adn+bcn+2ad\right) x\,{\mathrm e}^{n \ln \left(dx+c\right)}}{d \left(n^{2}+3n+2\right)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)*(b*d*n*x+a*d*n+b*d*x+2*a*d-b*c)/d^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.26, size = 63, normalized size = 1.34

$$\frac{{\left(d^{2} {\left(n + 1\right)} x^{2} + c d n x - c^{2}\right)} {\left(d x + c\right)}^{n} b}{{\left(n^{2} + 3 \, n + 2\right)} d^{2}} + \frac{{\left(d x + c\right)}^{n + 1} a}{d {\left(n + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] (d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*b/((n^2 + 3*n + 2)*d^2) + (d*
x + c)^(n + 1)*a/(d*(n + 1))
```

Fricas [A]
time = 0.32, size = 83, normalized size = 1.77

$$\frac{\left(acdn - bc^2 + 2\,acd + (bd^2n + bd^2)x^2 + (2\,ad^2 + (bcd + ad^2)n)x\right)(dx + c)^n}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a*c*d*n - b*c^2 + 2*a*c*d + (b*d^2*n + b*d^2)*x^2 + (2*a*d^2 + (b*c*d + a*
d^2)*n)*x)*(d*x + c)^n/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

Sympy [A]
time = 0.33, size = 377, normalized size = 8.02

$$\begin{cases} c^n \left(ax + \frac{bx^2}{2}\right) & \text{for } d = 0 \\ -\frac{ad}{cd^2+d^3x} + \frac{bc\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} + \frac{bc}{cd^2+d^3x} + \frac{bdx\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} & \text{for } n = -2 \\ \frac{a\log\left(\frac{c}{d}+x\right)}{d} - \frac{bc\log\left(\frac{c}{d}+x\right)}{d^2} + \frac{bx}{d} & \text{for } n = -1 \\ \frac{acdn(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{2acd(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{ad^2nx(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{2ad^2x(c+dx)^n}{d^2n^2+3d^2n+2d^2} - \frac{bc^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{bcdnx(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{bd^2nx^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{bd^2x^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a*x + b*x**2/2), Eq(d, 0)), (-a*d/(c*d**2 + d**3*x) + b*c*
log(c/d + x)/(c*d**2 + d**3*x) + b*c/(c*d**2 + d**3*x) + b*d*x*log(c/d + x)
/(c*d**2 + d**3*x), Eq(n, -2)), (a*log(c/d + x)/d - b*c*log(c/d + x)/d**2 +
 b*x/d, Eq(n, -1)), (a*c*d*n*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 2*a*c*d*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + a*d**2*n*x*(c + d*x
)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + 2*a*d**2*x*(c + d*x)**n/(d**2*n**2 +
 3*d**2*n + 2*d**2) - b*c**2*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 b*c*d*n*x*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*n*x**2*(c
+ d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*x**2*(c + d*x)**n/(d**2*
n**2 + 3*d**2*n + 2*d**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(47) = 94.
time = 0.00, size = 147, normalized size = 3.13

$$\frac{acdne^{n\ln(c+dx)} + 2acde^{n\ln(c+dx)} + ad^2nxe^{n\ln(c+dx)} + 2ad^2xe^{n\ln(c+dx)} - bc^2e^{n\ln(c+dx)} + bcdnxe^{n\ln(c+dx)} + bd^2nx^2e^{n\ln(c+dx)} + bd^2x^2e^{n\ln(c+dx)}}{d^2n^2 + 3d^2n + 2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x)
```

```
[Out] ((d*x + c)^n*b*d^2*n*x^2 + (d*x + c)^n*b*c*d*n*x + (d*x + c)^n*a*d^2*n*x +
(d*x + c)^n*b*d^2*x^2 + (d*x + c)^n*a*c*d*n + 2*(d*x + c)^n*a*d^2*x - (d*x
+ c)^n*b*c^2 + 2*(d*x + c)^n*a*c*d)/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

Mupad [B]
time = 0.49, size = 88, normalized size = 1.87

$$(c+d\,x)^n\,\left(\frac{c\,\left(2\,a\,d-b\,c+a\,d\,n\right)}{d^2\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,\left(n+1\right)}{n^2+3\,n+2}+\frac{x\,\left(2\,a\,d^2+a\,d^2\,n+b\,c\,d\,n\right)}{d^2\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(2*a*d - b*c + a*d*n))/(d^2*(3*n + n^2 + 2)) + (b*x^2*(n +
1))/(3*n + n^2 + 2) + (x*(2*a*d^2 + a*d^2*n + b*c*d*n))/(d^2*(3*n + n^2 + 2
)))
```

3.1855 $\int (c+dx)^n \, dx$

Optimal. Leaf size=18

$$\frac{(c+dx)^{1+n}}{d(1+n)}$$

```
[Out] (d*x+c)^(1+n)/d/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n,x]
[Out] (c + d*x)^(1 + n)/(d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^n \, dx = \frac{(c+dx)^{1+n}}{d(1+n)}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.94

$$\frac{(c+dx)^{1+n}}{d+dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n,x]
[Out] (c + d*x)^(1 + n)/(d + d*n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception:

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^0*(c + d*x)^n,x]')
[Out] cought exception:
```

Maple [A]
time = 0.15, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{(dx+c)^{1+n}}{d(1+n)}$	19
default	$\frac{(dx+c)^{1+n}}{d(1+n)}$	19
risch	$\frac{(dx+c)(dx+c)^{n}}{d(1+n)}$	22
norman	$\frac{x\,\mathrm{e}^{n\ln(dx+c)}}{1+n}+\frac{c\,\mathrm{e}^{n\ln(dx+c)}}{d(1+n)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n,x,method=_RETURNVERBOSE)
[Out] (d*x+c)^(1+n)/d/(1+n)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 1.00

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="maxima")
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

Fricas [A]
time = 0.30, size = 20, normalized size = 1.11

$$\frac{(dx+c)(dx+c)^{n}}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="fricas")
[Out] (d*x + c)*(d*x + c)^n/(d*n + d)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(c+dx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log{(c+dx)} & \text{otherwise} \end{cases}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n,x)
[Out] Piecewise(((c + d*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(c + d*x), True))/d
```

Giac [A]
time = 0.00, size = 16, normalized size = 0.89

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x)
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

Mupad [B]
time = 0.38, size = 18, normalized size = 1.00

$$\frac{(c+d\,x)^{n+1}}{d\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n,x)
[Out] (c + d*x)^(n + 1)/(d*(n + 1))
```

3.1856 $\int \frac{(c+dx)^n}{a+bx}\,dx$

Optimal. Leaf size=51

$$-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

```
[Out] -(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c)/(1
+n)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x),x]
```

```
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{a+bx}\,dx = -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.00

$$-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x),x]
```

```
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^1,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^n}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a),x)
```

```
[Out] int((d*x+c)^n/(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^n}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a),x)
```

```
[Out] Integral((c + d*x)**n/(a + b*x), x)
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x)^n}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x),x)
```

```
[Out] int((c + d*x)^n/(a + b*x), x)
```

3.1857 $\int \frac{(c+dx)^n}{(a+bx)^2} dx$

Optimal. Leaf size=51

$$\frac{d(c+dx)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^2(1+n)}$$

```
[Out] d*(d*x+c)^(1+n)*hypergeom([2, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c)^2
/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {70}

$$\frac{d(c+dx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^2,x]
```

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{(a+bx)^2} dx = \frac{d(c+dx)^{1+n} {}_2F_1\left(2,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^2(1+n)}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 1.02

$$\frac{d(c+dx)^{1+n} {}_2F_1\left(2,1+n;2+n;-\frac{b(c+dx)}{-bc+ad}\right)}{(-bc+ad)^2(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^2,x]
```

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((b*(c + d*x))/(-(
b*c) + a*d))])/((-(b*c) + a*d)^2*(1 + n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^2,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{n}}{(bx+a)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^n/(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^2, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^n}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a)**2,x)
[Out] Integral((c + d*x)**n/(a + b*x)**2, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x)^n}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^2,x)
[Out] int((c + d*x)^n/(a + b*x)^2, x)
```

3.1858 $\int \frac{(c+dx)^n}{(a+bx)^3}\,dx$

Optimal. Leaf size=54

$$-\frac{d^2(c+dx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^3(1+n)}$$

```
[Out] -d^2*(d*x+c)^(1+n)*hypergeom([3, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c
)^3/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$-\frac{d^2(c+dx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^3,x]
```

```
[Out] -((d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, (b*(c + d*x))/(
b*c - a*d)])/((b*c - a*d)^3*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{(a+bx)^3}\,dx = -\frac{d^2(c+dx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^3(1+n)}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{d^2(c+dx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;-\frac{b(c+dx)}{-bc+ad}\right)}{(-bc+ad)^3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^3,x]
```

```
[Out] (d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, -((b*(c + d*x))/(
-(b*c) + a*d))])/((-(b*c) + a*d)^3*(1 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded in comparison

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^3,x]')
```

```
[Out] cought exception: maximum recursion depth exceeded in comparison
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^n}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^n/(b*x+a)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^3, x)
```

Fricas [**F**]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^n}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a)**3,x)
[Out] Integral((c + d*x)**n/(a + b*x)**3, x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x)^n}{(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^3,x)
[Out] int((c + d*x)^n/(a + b*x)^3, x)
```

3.1859 $\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=143

$$-\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}+\frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)}-\frac{2d^2(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^3(1-n)(2-n)(3-n)}$$

```
[Out] -(b*x+a)^(-3+n)*(d*x+c)^(1-n)/(-a*d+b*c)/(3-n)+2*d*(b*x+a)^(-2+n)*(d*x+c)^(
1-n)/(-a*d+b*c)^2/(2-n)/(3-n)-2*d^2*(b*x+a)^(-1+n)*(d*x+c)^(1-n)/(-a*d+b*c)
^3/(1-n)/(2-n)/(3-n)
```

Rubi [A]
time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(3 - n))) + (2*d*(a +
 b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(2 - n)*(3 - n)) - (2*d^2*
(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^3*(1 - n)*(2 - n)*(3 - n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} - \frac{(2d)\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx}{(bc-ad)(3-n)}$$
$$= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} + \frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)} + \frac{(2d^2)\int (a+bx)^{-}}{(bc-ad)^2(2}$$
$$= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} + \frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)} - \frac{2d^2(a+bx)^{-1}}{(bc-ad)^3(1-n}$$

Mathematica [**A**]
time = 0.08, size = 112, normalized size = 0.78

$$\frac{(a+bx)^{-3+n}(c+dx)^{1-n}\left(a^2d^2\left(6-5n+n^2\right)-2abd(-3+n)(c(-1+n)+dx)+b^2\left(c^2\left(2-3n+n^2\right)+2cd(-1+n)x+2d^2x^2\right)\right)}{(bc-ad)^3(-3+n)(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-3 + n)*(c + d*x)^(1 - n)*(a^2*d^2*(6 - 5*n + n^2) - 2*a*b*d*(-
3 + n)*(c*(-1 + n) + d*x) + b^2*(c^2*(2 - 3*n + n^2) + 2*c*d*(-1 + n)*x + 2
*d^2*x^2)))/((b*c - a*d)^3*(-3 + n)*(-2 + n)*(-1 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(-4 + n)/(c + d*x)^n,x]')
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(143) = 286.
time = 0.22, size = 322, normalized size = 2.25

method	result
gosper	$-\frac{(bx+a)^{-3+n}(dx+c)\left(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2-5a^2d^2n+8abcdn+6ab\,d^2x-3b^2c^2n-2b^2cd\right.}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3-6a^3d^3n^2+18a^2bc\,d^2n^2-18a\,b^2c^2d\,n^2+6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4+n)/((d*x+c)^n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(-3+n)*(d*x+c)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*c^2*n^
2+2*b^2*c*d*n*x+2*b^2*d^2*x^2-5*a^2*d^2*n+8*a*b*c*d*n+6*a*b*d^2*x-3*b^2*c^2
```

```
*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d^2*n^
3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3-6*a^3*d^3*n^2+18*a^2*b*c*d^2*n^2-18*a*b^2*c
^2*d*n^2+6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-11*b^
3*c^3*n-6*a^3*d^3+18*a^2*b*c*d^2-18*a*b^2*c^2*d+6*b^3*c^3)/((d*x+c)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(130) = 260.
time = 0.35, size = 512, normalized size = 3.58

$$-\frac{\left(2\,b^{3}d^{3}x^{4}+2\,ab^{2}c^{3}-6\,a^{2}bc^{2}d+6\,a^{3}cd^{2}+2\,\left(4\,ab^{2}d^{3}+\left(b^{3}cd^{2}-ab^{2}d^{3}\right)n\right)x^{3}+\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}\right)n^{2}+\left(12\,a^{2}bd^{3}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)n^{2}-\left(b^{3}c^{2}d-8\,ab^{2}cd^{2}+7\,a^{2}bd^{3}\right)n\right)x^{2}-\left(3\,ab^{2}c^{3}-8\,a^{2}bc^{2}d+5\,a^{3}cd^{2}\right)n+\left(2\,b^{3}c^{3}-6\,ab^{2}c^{2}d+6\,a^{2}bcd^{2}+6\,a^{3}d^{3}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)n^{2}-\left(3\,b^{3}c^{3}-7\,ab^{2}c^{2}d-a^{2}bcd^{2}+5\,a^{3}d^{3}\right)n\right)x\right)(bx+a)^{n-4}}{\left(6\,b^{3}c^{3}-18\,ab^{2}c^{2}d+18\,a^{2}bcd^{2}-6\,a^{3}d^{3}-\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)n^{3}+6\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)n^{2}-11\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)n\right)(dx+c)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="fricas")
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 - (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 - (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 - (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(n - 4)/((6*b^3*c^3 - 18*
a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*n^2 - 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n)*(d*x +
 c)^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4+n)/((d*x+c)**n),x)
[Out] Exception raised: SystemError
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.10, size = 528, normalized size = 3.69

$$-\frac{x\,{\left(a+b\,x\right)}^{n-4}\,\left(a^3\,d^3\,n^2-5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2+a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2+7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2-3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{a\,c\,{\left(a+b\,x\right)}^{n-4}\,\left(a^2\,d^2\,n^2-5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2-3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^3\,d^3\,x^4\,{\left(a+b\,x\right)}^{n-4}}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{b\,d\,x^2\,{\left(a+b\,x\right)}^{n-4}\,\left(a^2\,d^2\,n^2-7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2-b^2\,c^2\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^2\,d^2\,x^3\,{\left(a+b\,x\right)}^{n-4}\,\left(4\,a\,d-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 4)/(c + d*x)^n,x)
```

```
[Out] - (x*(a + b*x)^(n - 4)*(6*a^3*d^3 + 2*b^3*c^3 - 5*a^3*d^3*n - 3*b^3*c^3*n +
 a^3*d^3*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 7*a*b^2*c^2*d*
n + a^2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c +
 d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (a*c*(a + b*x)^(n - 4)*(6*a^2*d^2 + 2*b
^2*c^2 - 5*a^2*d^2*n - 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d
+ 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 +
n^3 - 6)) - (2*b^3*d^3*x^4*(a + b*x)^(n - 4))/((a*d - b*c)^3*(c + d*x)^n*(1
1*n - 6*n^2 + n^3 - 6)) - (b*d*x^2*(a + b*x)^(n - 4)*(12*a^2*d^2 - 7*a^2*d^
2*n - b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 + 8*a*b*c*d*n - 2*a*b*c*d*n^2))
/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^(n - 4)*(4*a*d - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6
*n^2 + n^3 - 6))
```

3.1860 $\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=86

$$-\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}+\frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}$$

```
[Out] -(b*x+a)^(-2+n)*(d*x+c)^(1-n)/(-a*d+b*c)/(2-n)+d*(b*x+a)^(-1+n)*(d*x+c)^(1-n)/(-a*d+b*c)^2/(1-n)/(2-n)
```

Rubi [A]
time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(2 - n))) + (d*(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(Simplify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)} - \frac{d\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx}{(bc-ad)(2-n)}$$
$$= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)} + \frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.69

$$\frac{(a+bx)^{-2+n}(c+dx)^{1-n}(-ad(-2+n)+bc(-1+n)+bdx)}{(bc-ad)^2(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
[Out] ((a + b*x)^(-2 + n)*(c + d*x)^(1 - n)*(-(a*d*(-2 + n)) + b*c*(-1 + n) + b*d
*x))/((b*c - a*d)^2*(-2 + n)*(-1 + n))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(-3 + n)/(c + d*x)^n,x]')
[Out] Timed out
```

Maple [A]
time = 0.19, size = 127, normalized size = 1.48

method	result	size
gosper	$-\frac{(bx+a)^{-2+n}(dx+c)(adn-bcn-bdx-2ad+bc)(dx+c)^{-n}}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2-3a^2d^2n+6abcdn-3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3+n)/((d*x+c)^n),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(-2+n)*(d*x+c)*(a*d*n-b*c*n-b*d*x-2*a*d+b*c)/(a^2*d^2*n^2-2*a*b*c*
d*n^2+b^2*c^2*n^2-3*a^2*d^2*n+6*a*b*c*d*n-3*b^2*c^2*n+2*a^2*d^2-4*a*b*c*d+2
*b^2*c^2)/((d*x+c)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.
time = 0.33, size = 206, normalized size = 2.40

$$\frac{(b^2d^2x^3 - abc^2 + 2\,a^2cd + (3\,abd^2 + (b^2cd - abd^2)n)x^2 + (abc^2 - a^2cd)n - (b^2c^2 - 2\,abcd - 2\,a^2d^2 - (b^2c^2 - a^2d^2)n)x)(bx + a)^{n-3}}{(2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + (b^2c^2 - 2\,abcd + a^2d^2)n^2 - 3\,(b^2c^2 - 2\,abcd + a^2d^2)n)(dx + c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 - (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(n - 3)/((2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*n^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n)*(d*x
+ c)^n)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.77, size = 220, normalized size = 2.56

$$(a+b\,x)^{n-3}\,\left(\frac{x\,\left(2\,a^2\,d^2-b^2\,c^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}+\frac{b^2\,d^2\,x^3}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}+\frac{a\,c\,\left(2\,a\,d-b\,c-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}+\frac{b\,d\,x^2\,\left(3\,a\,d-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 3)/(c + d*x)^n,x)
```

```
[Out] (a + b*x)^(n - 3)*((x*(2*a^2*d^2 - b^2*c^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*
c*d))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b^2*d^2*x^3)/((a*d - b
*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (a*c*(2*a*d - b*c - a*d*n + b*c*n))/((
a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b*d*x^2*(3*a*d - a*d*n + b*c*n
))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)))
```

3.1861 $\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

```
[Out] -(b*x+a)^(-1+n)*(d*x+c)^(1-n)/(-a*d+b*c)/(1-n)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.92

$$\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(-1 + n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(-2 + n)/(c + d*x)^n,x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [A]
time = 0.19, size = 45, normalized size = 1.15

method	result	size
gosper	$-\frac{(bx+a)^{-1+n}(dx+c)(dx+c)^{-n}}{adn-bcn-ad+bc}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2+n)/((d*x+c)^n),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(-1+n)*(d*x+c)/(a*d*n-b*c*n-a*d+b*c)/((d*x+c)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

Fricas [A]
time = 0.32, size = 60, normalized size = 1.54

$$-\frac{(bdx^2+ac+(bc+ad)x)(bx+a)^{n-2}}{(bc-ad-(bc-ad)n)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="fricas")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(n - 2)/((b*c - a*d - (b*c - a*d
)*n)*(d*x + c)^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.56, size = 102, normalized size = 2.62

$$-(a+b\,x)^{n-2}\,\left(\frac{a\,c}{(a\,d-b\,c)\,(n-1)\,{(c+d\,x)}^n}+\frac{x\,(a\,d+b\,c)}{(a\,d-b\,c)\,(n-1)\,{(c+d\,x)}^n}+\frac{b\,d\,x^2}{(a\,d-b\,c)\,(n-1)\,{(c+d\,x)}^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 2)/(c + d*x)^n,x)
```

```
[Out] -(a + b*x)^(n - 2)*((a*c)/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (x*(a*d + b*c
))/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (b*d*x^2)/((a*d - b*c)*(n - 1)*(c +
d*x)^n))
```

3.1862 $\int (a+bx)^{-1+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=66

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;1+n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

```
[Out] (b*x+a)^n*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, n],[1+n],-d*(b*x+a)/(-a*d+
b*c))/b/n/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;n+1;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, -
((d*(a + b*x))/(b*c - a*d))])/(b*n*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-1+n}(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{-1+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;1+n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 0.98

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;1+n;\frac{d(a+bx)}{-bc+ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, (
d*(a + b*x))/(-(b*c) + a*d)])/(b*n*(c + d*x)^n)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(-1 + n)/(c + d*x)^n,x]')
```

```
[Out] Timed out
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{-1+n}(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1+n)/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^(-1+n)/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{n-1}}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 1)/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^(n - 1)/(c + d*x)^n, x)
```

3.1863 $\int (a+bx)^n(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

```
[Out] (b*x+a)^(1+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 1+n],[2+n],-d*(b*x+a)/
(-a*d+b*c))/b/(1+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^n\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

Mathematica [**A**]
time = 0.04, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c + d*x)^n,x]
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + n)*(c + d*x)^n)
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(0 + n)/(c + d*x)^n,x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^n\,(dx+c)^{-n}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/((d*x+c)^n),x)
[Out] int((b*x+a)^n/((d*x+c)^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/(d*x + c)^n, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^n}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^n/(c + d*x)^n, x)
```

3.1864 $\int (a+bx)^{1+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{2+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,2+n;3+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(2+n)}$$

```
[Out] (b*x+a)^(2+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 2+n],[3+n],-d*(b*x+a)/
(-a*d+b*c))/b/(2+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+2}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(2 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 2 + n
, 3 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(2 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{1+n}(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{1+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}\,dx$$
$$= \frac{(a+bx)^{2+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,2+n;3+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(2+n)}$$

Mathematica [A]
time = 0.06, size = 89, normalized size = 1.24

$$\frac{(bc-ad)(a+bx)^{n}\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{1-n}\,{}_2F_1\left(-1-n,1-n;2-n;\frac{b(c+dx)}{bc-ad}\right)}{d^{2}(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1 + n)/(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^n*(c + d*x)^(1 - n)*Hypergeometric2F1[-1 - n, 1 - n,
 2 - n, (b*(c + d*x))/(b*c - a*d)])/(d^2*(-1 + n)*((d*(a + b*x))/(-(b*c) +
a*d))^n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(1 + n)/(c + d*x)^n,x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{1+n}(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1+n)/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^(1+n)/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{n+1}}{{\left(c+d\,x\right)}^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n + 1)/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^(n + 1)/(c + d*x)^n, x)
```

3.1865 $\int (a+bx)^{2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{3+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,3+n;4+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(3+n)}$$

```
[Out] (b*x+a)^(3+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 3+n],[4+n],-d*(b*x+a)/
(-a*d+b*c))/b/(3+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+3}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+3;n+4;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(3 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 3 + n
, 4 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(3 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{2+n}(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{2+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{3+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,3+n;4+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(3+n)}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 1.28

$$-\frac{(bc-ad)^2(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{1-n}\,{}_2F_1\left(-2-n,1-n;2-n;\frac{b(c+dx)}{bc-ad}\right)}{d^3(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2 + n)/(c + d*x)^n,x]
[Out] -(((b*c - a*d)^2*(a + b*x)^n*(c + d*x)^(1 - n)*Hypergeometric2F1[-2 - n, 1
- n, 2 - n, (b*(c + d*x))/(b*c - a*d)])/(d^3*(-1 + n)*((d*(a + b*x))/(-(b*c
) + a*d))^n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(2 + n)/(c + d*x)^n,x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{2+n}(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2+n)/((d*x+c)^n),x)
[Out] int((b*x+a)^(2+n)/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{n+2}}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n + 2)/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^(n + 2)/(c + d*x)^n, x)
```

3.1866 $\int (a+bx)^{-n}(c+dx)^n\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^n(c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

[Out] (-d*(b*x+a)/(-a*d+b*c))^n*(d*x+c)^(1+n)*hypergeom([n, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/d/(1+n)/((b*x+a)^n)

Rubi [A]

time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {72, 71}

$$\frac{(a+bx)^{-n}(c+dx)^{n+1}\left(-\frac{d(a+bx)}{bc-ad}\right)^n\,{}_2F_1\left(n,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^n/(a + b*x)^n,x]

[Out] ((-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-n}(c+dx)^n\,dx = \left((a+bx)^{-n}\left(\frac{d(a+bx)}{-bc+ad}\right)^n\right)\int (c+dx)^n\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^n (c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{-n}\left(\frac{d(a+bx)}{-bc+ad}\right)^n (c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^n,x]
[Out] (((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1
+ n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^(n + 0),x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (dx+c)^n\,(bx+a)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/((b*x+a)^n),x)
[Out] int((d*x+c)^n/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^n, x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b*x + a)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/((b*x+a)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^n}{(a+b\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^n,x)
```

```
[Out] int((c + d*x)^n/(a + b*x)^n, x)
```

3.1867 $\int (a+bx)^{-1-n}(c+dx)^n\,dx$

Optimal. Leaf size=75

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

```
[Out] -(d*x+c)^n*hypergeom([-n, -n],[1-n],-d*(b*x+a)/(-a*d+b*c))/b/n/((b*x+a)^n)/
((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - n)*(c + d*x)^n,x]
```

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, -((d*(a + b*x))/(b*c - a*d)
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-1-n}(c+dx)^n\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{-1-n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx$$
$$= -\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.99

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{d(a+bx)}{-bc+ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - n)*(c + d*x)^n,x]
```

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (d*(a + b*x))/(-(b*c) + a*d
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^(n + 1),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (bx+a)^{-1-n}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

```
[Out] int((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^n}{(a+b\,x)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 1),x)
```

```
[Out] int((c + d*x)^n/(a + b*x)^(n + 1), x)
```

3.1868 $\int (a+bx)^{-2-n}(c+dx)^n\,dx$

Optimal. Leaf size=37

$$-\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

```
[Out] -(b*x+a)^(-1-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(1 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.03

$$\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(-1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(-1 - n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^(n + 2),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [**A**]
time = 0.19, size = 41, normalized size = 1.11

method	result	size
gosper	$\frac{(bx+a)^{-1-n}(dx+c)^{1+n}}{adn-bcn+ad-bc}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-1-n)*(d*x+c)^(1+n)/(a*d*n-b*c*n+a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

Fricas [**A**]
time = 0.35, size = 59, normalized size = 1.59

$$-\frac{(bdx^2+ac+(bc+ad)x)(bx+a)^{-n-2}(dx+c)^n}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(-n - 2)*(d*x + c)^n/(b*c - a*d
+ (b*c - a*d)*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2-n)*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.53, size = 97, normalized size = 2.62

$$\frac{\frac{a\,c\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{x\,\left(a\,d+b\,c\right)\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{b\,d\,x^2\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}}{{\left(a+b\,x\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 2),x)
```

```
[Out] ((a*c*(c + d*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(c + d*x)^n)/((a*
d - b*c)*(n + 1)) + (b*d*x^2*(c + d*x)^n)/((a*d - b*c)*(n + 1)))/(a + b*x)^
(n + 2)
```

3.1869 $\int (a+bx)^{-3-n}(c+dx)^n\,dx$

Optimal. Leaf size=80

$$-\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}+\frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}$$

```
[Out] -(b*x+a)^(-2-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(2+n)+d*(b*x+a)^(-1-n)*(d*x+c)^(1+
n)/(-a*d+b*c)^2/(1+n)/(2+n)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(2 + n))) + (d*(a + b
*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-3-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)} - \frac{d\int (a+bx)^{-2-n}(c+dx)^n\,dx}{(bc-ad)(2+n)}$$
$$= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)} + \frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.75

$$\frac{(a+bx)^{-2-n}(c+dx)^{1+n}(ad(2+n)-b(c+cn-dx))}{(bc-ad)^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
[Out] ((a + b*x)^(-2 - n)*(c + d*x)^(1 + n)*(a*d*(2 + n) - b*(c + c*n - d*x)))/((
b*c - a*d)^2*(1 + n)*(2 + n))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^(n + 3),x]')
[Out] Timed out
```

Maple [A]
time = 0.19, size = 123, normalized size = 1.54

method	result	size
gosper	$\frac{(bx+a)^{-2-n}(dx+c)^{1+n}(adn-bcn+bdx+2ad-bc)}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
[Out] (b*x+a)^(-2-n)*(d*x+c)^(1+n)*(a*d*n-b*c*n+b*d*x+2*a*d-b*c)/(a^2*d^2*n^2-2*a
*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b*
c*d+2*b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(80) = 160$.
time = 0.32, size = 207, normalized size = 2.59

$$\frac{(b^2d^2x^3 - abc^2 + 2\,a^2cd + (3\,abd^2 - (b^2cd - abd^2)n)x^2 - (abc^2 - a^2cd)n - (b^2c^2 - 2\,abcd - 2\,a^2d^2 + (b^2c^2 - a^2d^2)n)x)(bx + a)^{-n-3}(dx + c)^n}{2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + (b^2c^2 - 2\,abcd + a^2d^2)n^2 + 3\,(b^2c^2 - 2\,abcd + a^2d^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 - (b^2*c*d - a*b*d^2)*n)*x^
2 - (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(-n - 3)*(d*x + c)^n/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.74, size = 214, normalized size = 2.68

$$\frac{\frac{x\,{\left(c+d\,x\right)}^n\,\left(2\,a^2\,d^2-b^2\,c^2+a^2\,d^2\,n-b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}+\frac{a\,c\,{\left(c+d\,x\right)}^n\,\left(2\,a\,d-b\,c+a\,d\,n-b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}+\frac{b^2\,d^2\,x^3\,{\left(c+d\,x\right)}^n}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}+\frac{b\,d\,x^2\,{\left(c+d\,x\right)}^n\,\left(3\,a\,d+a\,d\,n-b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}}{{\left(a+b\,x\right)}^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 3),x)
```

```
[Out] ((x*(c + d*x)^n*(2*a^2*d^2 - b^2*c^2 + a^2*d^2*n - b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) + (a*c*(c + d*x)^n*(2*a*d - b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(c + d*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(c + d*x)^n*(3*a*d + a*d*n - b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(a + b*x)^(n + 3)
```

3.1870 $\int (a+bx)^{-4-n}(c+dx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}+\frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)}-\frac{2d^2(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

```
[Out] -(b*x+a)^(-3-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(3+n)+2*d*(b*x+a)^(-2-n)*(d*x+c)^(
1+n)/(-a*d+b*c)^2/(2+n)/(3+n)-2*d^2*(b*x+a)^(-1-n)*(d*x+c)^(1+n)/(-a*d+b*c)
^3/(1+n)/(2+n)/(3+n)
```

Rubi [A]

time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(3 + n))) + (2*d*(a +
 b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(2 + n)*(3 + n)) - (2*d^2*
(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-4-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} - \frac{(2d)\int (a+bx)^{-3-n}(c+dx)^n\,dx}{(bc-ad)(3+n)}$$

$$= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} + \frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)} + \frac{(2d^2)\int (a+bx)^{-2}}{(bc-ad)^2(2}$$

$$= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} + \frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)} - \frac{2d^2(a+bx)^{-1-}}{(bc-ad)^3(1+n)}$$

Mathematica [A]
time = 0.07, size = 113, normalized size = 0.86

$$-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}\left(a^2d^2\left(6+5n+n^2\right)-2abd(3+n)(c+cn-dx)+b^2\left(c^2\left(2+3n+n^2\right)-2cd(1+n)x+2d^2x^2\right)\right)}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n)*(a^2*d^2*(6 + 5*n + n^2) - 2*a*b*d*
(3 + n)*(c + c*n - d*x) + b^2*(c^2*(2 + 3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^(n + 4),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(131) = 262$.
time = 0.19, size = 318, normalized size = 2.43

method	result
gosper	$\frac{(bx+a)^{-3-n}(dx+c)^{1+n}\left(a^2d^2n^2-2abcd\,n^2+2ab\,d^2nx+b^2c^2n^2-2b^2cdnx+2b^2x^2d^2+5a^2d^2n-8abcdn+6ab\,d^2x+3b^2c^2n-2\right.}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-3-n)*(d*x+c)^(1+n)*(a^2*d^2*n^2-2*a*b*c*d*n^2+2*a*b*d^2*n*x+b^2*c
^2*n^2-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-8*a*b*c*d*n+6*a*b*d^2*x+3*b^
```

```
2*c^2*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d
^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a*
b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-
11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(131) = 262$.
time = 0.33, size = 509, normalized size = 3.89

$$-\frac{\left(2\,b^{3}d^{3}x^{4}+2\,ab^{2}c^{3}-6\,a^{2}bc^{2}d+6\,a^{3}cd^{2}+2\,(4\,ab^{2}d^{3}-(b^{3}cd^{2}-ab^{2}d^{3})n)x^{3}+(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2})n^{2}+(12\,a^{2}bd^{3}+(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3})n^{2}+(b^{3}c^{2}d-8\,ab^{2}cd^{2}+7\,a^{2}bd^{3})n)x^{2}+(3\,ab^{2}c^{3}-8\,a^{2}bc^{2}d+5\,a^{3}cd^{2})n+(2\,b^{3}c^{3}-6\,ab^{2}c^{2}d+6\,a^{2}bcd^{2}+6\,a^{3}d^{3}+(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3})n^{2}+(3\,b^{3}c^{3}-7\,ab^{2}c^{2}d-a^{2}bcd^{2}+5\,a^{3}d^{3})n)x\right)(bx+a)^{-n-4}(dx+c)^{n}}{6\,b^{3}c^{3}-18\,ab^{2}c^{2}d+18\,a^{2}bcd^{2}-6\,a^{3}d^{3}+(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})n^{3}+6\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})n^{2}+11\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 - (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 + (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(-n - 4)*(d*x + c)^n/(6*b
^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*n)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.99, size = 525, normalized size = 4.01

$$\frac{x\,(c+d\,x)^n\,\left(a^3\,d^3\,n^2+5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^3\,d^3\,x^4\,(c+d\,x)^n}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+b^2\,c^2\,n\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^2\,d^2\,x^3\,(c+d\,x)^n\,\left(4\,a\,d+a\,d\,n-b\,c\,n\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 4),x)
```

```
[Out] (x*(c + d*x)^n*(6*a^3*d^3 + 2*b^3*c^3 + 5*a^3*d^3*n + 3*b^3*c^3*n + a^3*d^3
*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 7*a*b^2*c^2*d*n - a^2*
b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(a + b*x)^(n
 + 4)*(11*n + 6*n^2 + n^3 + 6)) + (a*c*(c + d*x)^n*(6*a^2*d^2 + 2*b^2*c^2 +
 5*a^2*d^2*n + 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*b*
c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^
3 + 6)) + (2*b^3*d^3*x^4*(c + d*x)^n)/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*
n + 6*n^2 + n^3 + 6)) + (b*d*x^2*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n + b^
2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d -
 b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (2*b^2*d^2*x^3*(c + d
*x)^n*(4*a*d + a*d*n - b*c*n))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n
^2 + n^3 + 6))
```

3.1871 $\int (a+bx)^{-5-n}(c+dx)^n\,dx$

Optimal. Leaf size=186

$$-\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)}+\frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)}-\frac{6d^2(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^3(2+n)(3+n)(4+n)}+\frac{6d^3(a+\cdots}{(bc-ad)^4(1+\cdots}$$

```
[Out] -(b*x+a)^(-4-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(4+n)+3*d*(b*x+a)^(-3-n)*(d*x+c)^(
1+n)/(-a*d+b*c)^2/(3+n)/(4+n)-6*d^2*(b*x+a)^(-2-n)*(d*x+c)^(1+n)/(-a*d+b*c)
^3/(2+n)/(3+n)/(4+n)+6*d^3*(b*x+a)^(-1-n)*(d*x+c)^(1+n)/(-a*d+b*c)^4/(1+n)/
(2+n)/(3+n)/(4+n)
```

Rubi [A]
time = 0.07, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-4 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(4 + n))) + (3*d*(a +
 b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(3 + n)*(4 + n)) - (6*d^2*
(a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n
)) + (6*d^3*(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int (a+bx)^{-5-n}(c+dx)^n\,dx &= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} - \frac{(3d)\int (a+bx)^{-4-n}(c+dx)^n\,dx}{(bc-ad)(4+n)} \\
&= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} + \frac{(6d^2)\int (a+bx)^{-3}}{(bc-ad)^2(3}\\
&= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2-}}{(bc-ad)^3(2+n)}\\
&= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2-}}{(bc-ad)^3(2+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 195, normalized size = 1.05

$$
\frac{(a+bx)^{-4-n}(c+dx)^{1+n}\left(a^3d^3\left(24+26n+9n^2+n^3\right)-3a^2bd^2\left(12+7n+n^2\right)(c+cn-dx)+3ab^2d(4+n)\left(c^2\left(2+3n+n^2\right)-2cd(1+n)x+2d^2x^2\right)-b^3\left(c^3\left(6+11n+6n^2+n^3\right)-3c^2d\left(2+3n+n^2\right)x+6cd^2(1+n)x^2-6d^3x^3\right)\right)}{(bc-ad)^4(1+n)(2+n)(3+n)(4+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-4 - n)*(c + d*x)^(1 + n)*(a^3*d^3*(24 + 26*n + 9*n^2 + n^3) -
3*a^2*b*d^2*(12 + 7*n + n^2)*(c + c*n - d*x) + 3*a*b^2*d*(4 + n)*(c^2*(2 +
3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^2*x^2) - b^3*(c^3*(6 + 11*n + 6*n^2 + n^
3) - 3*c^2*d*(2 + 3*n + n^2)*x + 6*c*d^2*(1 + n)*x^2 - 6*d^3*x^3)))/((b*c -
 a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(c + d*x)^n/(a + b*x)^(n + 5),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(186) = 372.
time = 0.21, size = 661, normalized size = 3.55

method	result
gosper	$\frac{(bx+a)^{-4-n}(dx+c)^{1+n}\left(a^3d^3n^3-3a^2bc\,d^2n^3+3a^2b\,d^3n^2x+3a\,b^2c^2d\,n^3-6a\,b^2c\,d^2n^2x+6a\,b^2d^3n\,x^2-b^3c^3n^3+3b^3c^2d\,n^2x-6b^3c\,d^2n\,x^2\right.}{a^4d^4n^4-4a^3bc\,d^3n^4+6a^2b^2c^2d^2n^4-4a\,b^3c^3d\,n^4+b^4c^4n^4+10a^4d^4n^3-40a^3bc\,d^3n^3+60a^2b^2c^2d^2n^3-40}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-5-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-4-n)*(d*x+c)^(1+n)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3+3*a^2*b*d^3*n^2
*x+3*a*b^2*c^2*d*n^3-6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3+3*b^
3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-24*a^2*b*c*d^2*
n^2+21*a^2*b*d^3*n*x+21*a*b^2*c^2*d*n^2-30*a*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2
-6*b^3*c^3*n^2+9*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*n-57*a^2*b*c*d^2*
n+36*a^2*b*d^3*x+42*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x-11*b^3*c^3*n+6*b^3*c^2*d
*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/(a^4*d^4*n^4-4*a^3*b
*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*n
^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4*
n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c^
3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*n
-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d
^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(186) = 372$.
time = 0.33, size = 959, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 - 6*a*b^3*c^4 + 24*a^2*b^2*c^3*d - 36*a^3*b*c^2*d^2 + 24*a^4
*c*d^3 + 6*(5*a*b^3*d^4 - (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 - (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*a^2*b^2*d^4 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (b^4*c^2*d^2 - 10*a*b^3*c*d^3
+ 9*a^2*b^2*d^4)*n)*x^3 - 3*(2*a*b^3*c^4 - 7*a^2*b^2*c^3*d + 8*a^3*b*c^2*d^
2 - 3*a^4*c*d^3)*n^2 + (60*a^3*b*d^4 - (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 - 3*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 9*a^2*b^2*c*
```

```
d^3 - 4*a^3*b*d^4)*n^2 - (2*b^4*c^3*d - 15*a*b^3*c^2*d^2 + 60*a^2*b^2*c*d^3
 - 47*a^3*b*d^4)*n)*x^2 - (11*a*b^3*c^4 - 42*a^2*b^2*c^3*d + 57*a^3*b*c^2*d
^2 - 26*a^4*c*d^3)*n - (6*b^4*c^4 - 24*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 2
4*a^3*b*c*d^3 - 24*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(2*b^4*c^4 - 6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 - 3*a^4*d^4)*n^2 + (11*b^4*c^4 - 40*a*b^3*c^3*d + 45*a^2*b^2*c^2*d^2 + 10
*a^3*b*c*d^3 - 26*a^4*d^4)*n)*x)*(b*x + a)^(-n - 5)*(d*x + c)^n/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-5-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.64, size = 944, normalized size = 5.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 5),x)
```

```
[Out] (a*c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n - 11*b^3*c^3*n + 9*
a^3*d^3*n^2 - 6*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 24*a*b^2*c^2*d -
36*a^2*b*c*d^2 + 42*a*b^2*c^2*d*n - 57*a^2*b*c*d^2*n + 21*a*b^2*c^2*d*n^2 -
 24*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^
```

```
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (x*(c + d*x)^n*(
6*b^4*c^4 - 24*a^4*d^4 - 26*a^4*d^4*n + 11*b^4*c^4*n - 9*a^4*d^4*n^2 + 6*b^
4*c^4*n^2 - a^4*d^4*n^3 + b^4*c^4*n^3 + 36*a^2*b^2*c^2*d^2 - 24*a*b^3*c^3*d
 - 24*a^3*b*c*d^3 - 40*a*b^3*c^3*d*n + 10*a^3*b*c*d^3*n + 9*a^2*b^2*c^2*d^2
*n^2 - 18*a*b^3*c^3*d*n^2 + 12*a^3*b*c*d^3*n^2 - 2*a*b^3*c^3*d*n^3 + 2*a^3*
b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n
 + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^4*d^4*x^5*(c + d*x)^n)/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
c + d*x)^n*(20*a^2*d^2 + 9*a^2*d^2*n + b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(c + d*x)^n*(5*a*d + a*d*n -
 b*c*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(c + d*x)^n*(60*a^3*d^3 + 47*a^3*d^3*n - 2*b^3*c^3*n + 12*a^
3*d^3*n^2 - 3*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 15*a*b^2*c^2*d*n -
60*a^2*b*c*d^2*n + 18*a*b^2*c^2*d*n^2 - 27*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*
n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

3.1872 $\int (a+bx)^n(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

```
[Out] (b*x+a)^(1+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 1+n],[2+n],-d*(b*x+a)/
(-a*d+b*c))/b/(1+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^n\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\ {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

Mathematica [A]
time = 0.00, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\ {}_2F_1\left(n,1+n;2+n;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c + d*x)^n,x]
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + n)*(c + d*x)^n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c + d*x)^(n + 0),x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n\,(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/((d*x+c)^n),x)
[Out] int((b*x+a)^n/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^n}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^n/(c + d*x)^n, x)
```

3.1873 $\int (a+bx)^n (c+dx)^{-1-n}\,dx$

Optimal. Leaf size=75

$$-\frac{(a+bx)^n \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} (c+dx)^{-n} {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

```
[Out] -(b*x+a)^n*hypergeom([-n, -n],[1-n],b*(d*x+c)/(-a*d+b*c))/d/n/((-d*(b*x+a)/
(-a*d+b*c))^n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{(a+bx)^n (c+dx)^{-n} \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-1 - n),x]
```

```
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-1-n}\,dx = \left((a+bx)^n \left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}\right) \int (c+dx)^{-1-n} \left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^n dx$$
$$= -\frac{(a+bx)^n \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} (c+dx)^{-n}\,{}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 0.99

$$-\frac{(a+bx)^n \left(\frac{d(a+bx)}{-bc+ad}\right)^{-n} (c+dx)^{-n}\,{}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-1 - n),x]
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c + d*x)^(n + 1),x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-1-n),x)
[Out] int((b*x+a)^n*(d*x+c)^(-1-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Fricas [**F**]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^n}{(c+d\,x)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 1),x)
```

```
[Out] int((a + b*x)^n/(c + d*x)^(n + 1), x)
```

3.1874 $\int (a+bx)^n(c+dx)^{-2-n}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-1-n)/(-a*d+b*c)/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-2 - n),x]
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-2-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 1.00

$$\frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-2 - n),x]
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c + d*x)^(n + 2),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [**A**]
time = 0.19, size = 42, normalized size = 1.17

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-1-n}}{adn-bcn+ad-bc}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-2-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(a*d*n-b*c*n+a*d-b*c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

Fricas [**A**]
time = 0.33, size = 58, normalized size = 1.61

$$\frac{(bdx^2+ac+(bc+ad)x)(bx+a)^n(dx+c)^{-n-2}}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="fricas")
```

```
[Out] (b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^n*(d*x + c)^(-n - 2)/(b*c - a*d +
 (b*c - a*d)*n)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-2-n),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

$$\text{Could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.56, size = 98, normalized size = 2.72

$$-\frac{\frac{a\,c\,{\left(a+b\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{x\,\left(a\,d+b\,c\right)\,{\left(a+b\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{b\,d\,x^2\,{\left(a+b\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}}{{\left(c+d\,x\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 2),x)
```

```
[Out] -((a*c*(a + b*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(a + b*x)^n)/((a
*d - b*c)*(n + 1)) + (b*d*x^2*(a + b*x)^n)/((a*d - b*c)*(n + 1)))/(c + d*x)
^(n + 2)
```

3.1875 $\int (a+bx)^n(c+dx)^{-3-n}\,dx$

Optimal. Leaf size=79

$$\frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-2-n)/(-a*d+b*c)/(2+n)+b*(b*x+a)^(1+n)*(d*x+c)^(-1-n
)/(-a*d+b*c)^2/(1+n)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{47, 37\}$

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)*(2 + n)) + (b*(a + b*x)
^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-3-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)} + \frac{b\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)(2+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)} + \frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.75

$$\frac{(a+bx)^{1+n}(c+dx)^{-2-n}(-ad(1+n)+bc(2+n)+bdx)}{(bc-ad)^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n)*(-(a*d*(1 + n)) + b*c*(2 + n) + b*d*x
))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c + d*x)^(n + 3),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.23, size = 124, normalized size = 1.57

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-2-n}(adn-bcn-bdx+ad-2bc)}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-3-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-2-n)*(a*d*n-b*c*n-b*d*x+a*d-2*b*c)/(a^2*d^2*n^2-2*
a*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b
*c*d+2*b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(79) = 158$.
time = 0.32, size = 205, normalized size = 2.59

$$\frac{(b^2d^2x^3+2\,abc^2-a^2cd+(3\,b^2cd+(b^2cd-abd^2)n)x^2+(abc^2-a^2cd)n+(2\,b^2c^2+2\,abcd-a^2d^2+(b^2c^2-a^2d^2)n)x)(bx+a)^n(dx+c)^{-n-3}}{2\,b^2c^2-4\,abcd+2\,a^2d^2+(b^2c^2-2\,abcd+a^2d^2)n^2+3\,(b^2c^2-2\,abcd+a^2d^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 + 2*a*b*c^2 - a^2*c*d + (3*b^2*c*d + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n + (2*b^2*c^2 + 2*a*b*c*d - a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 3)/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-3-n),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 0.74, size = 214, normalized size = 2.71

$$\frac{\frac{x\,(a+b\,x)^n\,(2\,b^2\,c^2-a^2\,d^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d)}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}-\frac{a\,c\,(a+b\,x)^n\,(a\,d-2\,b\,c+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}+\frac{b^2\,d^2\,x^3\,(a+b\,x)^n}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}+\frac{b\,d\,x^2\,(a+b\,x)^n\,(3\,b\,c-a\,d\,n+b\,c\,n)}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}}{(c+d\,x)^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 3),x)
```

```
[Out] ((x*(a + b*x)^n*(2*b^2*c^2 - a^2*d^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) - (a*c*(a + b*x)^n*(a*d - 2*b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(a + b*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(a + b*x)^n*(3*b*c - a*d*n + b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(c + d*x)^(n + 3)
```

3.1876 $\int (a+bx)^n (c+dx)^{-4-n}\,dx$

Optimal. Leaf size=130

$$\frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)}+\frac{2b^2(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-3-n)/(-a*d+b*c)/(3+n)+2*b*(b*x+a)^(1+n)*(d*x+c)^(-2
-n)/(-a*d+b*c)^2/(2+n)/(3+n)+2*b^2*(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(-a*d+b*c)^
3/(1+n)/(2+n)/(3+n)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)*(3 + n)) + (2*b*(a + b*
x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^2*(2 + n)*(3 + n)) + (2*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-4-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{(2b)\int (a+bx)^n (c+dx)^{-3-n}\,dx}{(bc-ad)(3+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)} + \frac{(2b^2)\int (a+bx)^n (c\cdot}{(bc-ad)^2(2+r}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)} + \frac{2b^2(a+bx)^{1+n}(c}{(bc-ad)^3(1+n)(2}$$

Mathematica [**A**]
time = 0.07, size = 112, normalized size = 0.86

$$\frac{(a+bx)^{1+n}(c+dx)^{-3-n}\left(a^2d^2\left(2+3n+n^2\right)-2abd(1+n)(c(3+n)+dx)+b^2\left(c^2\left(6+5n+n^2\right)+2cd(3+n)x+2d^2x^2\right)\right)}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n)*(a^2*d^2*(2 + 3*n + n^2) - 2*a*b*d*(1
 + n)*(c*(3 + n) + d*x) + b^2*(c^2*(6 + 5*n + n^2) + 2*c*d*(3 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Mathics [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c + d*x)^(n + 4),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(130) = 260.
time = 0.24, size = 319, normalized size = 2.45

method	result
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-3-n}\left(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2+3a^2d^2n-8abcdn-2ab\,d^2x+5b^2c^2n-\right.}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-4-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-3-n)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*
c^2*n^2+2*b^2*c*d*n*x+2*b^2*d^2*x^2+3*a^2*d^2*n-8*a*b*c*d*n-2*a*b*d^2*x+5*b
```

```
^2*c^2*n+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*
d^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a
*b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n
-11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(130) = 260$.
time = 0.33, size = 507, normalized size = 3.90

$$\frac{\left(2\,b^3d^3x^4+6\,ab^2c^3-6\,a^2bc^2d+2\,a^3cd^2+2\left(4\,b^3cd^2+(b^3cd^2-ab^2d^3)n\right)x^3+(ab^2c^3-2\,a^2bc^2d+a^3cd^2)n^2+\left(12\,b^3c^2d+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)n^2+(7\,b^3c^2d-8\,ab^2cd^2+a^2bd^3)n\right)x^2+(5\,ab^2c^3-8\,a^2bc^2d+3\,a^3cd^2)n+\left(6\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)n^2+(5\,b^3c^3-ab^2c^2d-7\,a^2bcd^2+3\,a^3d^3)n\right)x\right)(bx+a)^n(dx+c)^{-n-4}}{6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3+(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)n^3+6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)n^2+11\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="fricas")
[Out] (2*b^3*d^3*x^4 + 6*a*b^2*c^3 - 6*a^2*b*c^2*d + 2*a^3*c*d^2 + 2*(4*b^3*c*d^2
 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)
*n^2 + (12*b^3*c^2*d + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (7*b^3
*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*n)*x^2 + (5*a*b^2*c^3 - 8*a^2*b*c^2*d +
 3*a^3*c*d^2)*n + (6*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (5*b^3*c^3 - a*b^2*c^
2*d - 7*a^2*b*c*d^2 + 3*a^3*d^3)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 4)/(6*b^
3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*n)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-4-n),x)
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 1.02, size = 528, normalized size = 4.06

$$-\frac{x\,(a+b\,x)^n\,\left(a^3\,d^3\,n^2+3\,a^3\,d^3\,n+2\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-7\,a^2\,b\,c\,d^2\,n-6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-a\,b^2\,c^2\,d\,n+6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+5\,b^3\,c^3\,n+6\,b^3\,c^3\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{a\,c\,(a+b\,x)^n\,\left(a^2\,d^2\,n^2+3\,a^2\,d^2\,n+2\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+5\,b^2\,c^2\,n+6\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^3\,d^3\,x^4\,(a+b\,x)^n}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{b\,d\,x^2\,(a+b\,x)^n\,\left(a^2\,d^2\,n^2+a^2\,d^2\,n-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+7\,b^2\,c^2\,n+12\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^2\,d^2\,x^3\,(a+b\,x)^n\,\left(4\,b\,c-a\,d\,n+b\,c\,n\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 4),x)
```

```
[Out] - (x*(a + b*x)^n*(2*a^3*d^3 + 6*b^3*c^3 + 3*a^3*d^3*n + 5*b^3*c^3*n + a^3*d
^3*n^2 + b^3*c^3*n^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a*b^2*c^2*d*n - 7*a^
2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c + d*x)^
(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (a*c*(a + b*x)^n*(2*a^2*d^2 + 6*b^2*c^2
 + 3*a^2*d^2*n + 5*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*
b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 +
n^3 + 6)) - (2*b^3*d^3*x^4*(a + b*x)^n)/((a*d - b*c)^3*(c + d*x)^(n + 4)*(1
1*n + 6*n^2 + n^3 + 6)) - (b*d*x^2*(a + b*x)^n*(12*b^2*c^2 + a^2*d^2*n + 7*
b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d
 - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^n*(4*b*c - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6
*n^2 + n^3 + 6))
```

3.1877 $\int (a+bx)^n (c+dx)^{-5-n}\,dx$

Optimal. Leaf size=185

$$\frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)}+\frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)}+\frac{6b^2(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^3(2+n)(3+n)(4+n)}+\frac{6b^3(a+bx}{(bc-ad)^4(1+n}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-4-n)/(-a*d+b*c)/(4+n)+3*b*(b*x+a)^(1+n)*(d*x+c)^(-3
-n)/(-a*d+b*c)^2/(3+n)/(4+n)+6*b^2*(b*x+a)^(1+n)*(d*x+c)^(-2-n)/(-a*d+b*c)^
3/(2+n)/(3+n)/(4+n)+6*b^3*(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(-a*d+b*c)^4/(1+n)/(
2+n)/(3+n)/(4+n)
```

Rubi [A]
time = 0.05, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n))/((b*c - a*d)*(4 + n)) + (3*b*(a + b*
x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)^2*(3 + n)*(4 + n)) + (6*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n))
+ (6*b^3*(a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^4*(1 + n)*(2 +
n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-5-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{(3b)\int (a+bx)^n (c+dx)^{-4-n}\,dx}{(bc-ad)(4+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{(6b^2)\int (a+bx)^n (c-}{(bc-ad)^2(3+n}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(c}{(bc-ad)^3(2+n)(3}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(c}{(bc-ad)^3(2+n)(3}$$

Mathematica [A]
time = 0.09, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{1+n}(c+dx)^{-4-n}\left(-a^3d^3\left(6+11n+6n^2+n^3\right)+3a^2bd^2\left(2+3n+n^2\right)(c(4+n)+dx)-3ab^2d(1+n)\left(c^2\left(12+7n+n^2\right)+2cd(4+n)x+2d^2x^2\right)+b^3\left(c^3\left(24+26n+9n^2+n^3\right)+3c^2d\left(12+7n+n^2\right)x+6cd^2(4+n)x^2+6d^3x^3\right)\right)}{(bc-ad)^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n)*(-(a^3*d^3*(6 + 11*n + 6*n^2 + n^3))
+ 3*a^2*b*d^2*(2 + 3*n + n^2)*(c*(4 + n) + d*x) - 3*a*b^2*d*(1 + n)*(c^2*(1
2 + 7*n + n^2) + 2*c*d*(4 + n)*x + 2*d^2*x^2) + b^3*(c^3*(24 + 26*n + 9*n^2
 + n^3) + 3*c^2*d*(12 + 7*n + n^2)*x + 6*c*d^2*(4 + n)*x^2 + 6*d^3*x^3)))/(
(b*c - a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^n/(c + d*x)^(n + 5),x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(185) = 370$.
time = 0.23, size = 662, normalized size = 3.58

method	result
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-4-n}\left(a^3d^3n^3-3a^2bc\,d^2n^3-3a^2b\,d^3n^2x+3a\,b^2c^2d\,n^3+6a\,b^2c\,d^2n^2x+6a\,b^2d^3n\,x^2-b^3c^3n^3-3b^3c^2d\,n^2x-6b^3c\,d^2n\right.}{a^4d^4n^4-4a^3bc\,d^3n^4+6a^2b^2c^2d^2n^4-4a\,b^3c^3d\,n^4+b^4c^4n^4+10a^4d^4n^3-40a^3bc\,d^3n^3+60a^2b^2c^2d^2n^3-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-5-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-4-n)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3-3*a^2*b*d^3*n^
2*x+3*a*b^2*c^2*d*n^3+6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3-3*b
^3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2-6*b^3*d^3*x^3+6*a^3*d^3*n^2-21*a^2*b*c*d^2
*n^2-9*a^2*b*d^3*n*x+24*a*b^2*c^2*d*n^2+30*a*b^2*c*d^2*n*x+6*a*b^2*d^3*x^2-
9*b^3*c^3*n^2-21*b^3*c^2*d*n*x-24*b^3*c*d^2*x^2+11*a^3*d^3*n-42*a^2*b*c*d^2
*n-6*a^2*b*d^3*x+57*a*b^2*c^2*d*n+24*a*b^2*c*d^2*x-26*b^3*c^3*n-36*b^3*c^2*
d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/(a^4*d^4*n^4-4*a^3*
b*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*
n^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4
*n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c
^3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*
n-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*
d^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(185) = 370.
time = 0.34, size = 954, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 + 24*a*b^3*c^4 - 36*a^2*b^2*c^3*d + 24*a^3*b*c^2*d^2 - 6*a^4
*c*d^3 + 6*(5*b^4*c*d^3 + (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 + (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*b^4*c^2*d^2 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (9*b^4*c^2*d^2 - 10*a*b^3*c*d^
3 + a^2*b^2*d^4)*n)*x^3 + 3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^
2 - 2*a^4*c*d^3)*n^2 + (60*b^4*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 + 3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*
```

```
c*d^3 - a^3*b*d^4)*n^2 + (47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2*b^2*c*d^
3 - 2*a^3*b*d^4)*n)*x^2 + (26*a*b^3*c^4 - 57*a^2*b^2*c^3*d + 42*a^3*b*c^2*d
^2 - 11*a^4*c*d^3)*n + (24*b^4*c^4 + 24*a*b^3*c^3*d - 36*a^2*b^2*c^2*d^2 +
24*a^3*b*c*d^3 - 6*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c*d^
3 - 2*a^4*d^4)*n^2 + (26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 - 11*a^4*d^4)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 5)/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-5-n),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x)
```

```
[Out] Could not integrate
```

Mupad [**B**]
time = 1.61, size = 945, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 5),x)
```

```
[Out] (6*b^4*d^4*x^5*(a + b*x)^n)/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2
 + 10*n^3 + n^4 + 24)) - (a*c*(a + b*x)^n*(6*a^3*d^3 - 24*b^3*c^3 + 11*a^3*
d^3*n - 26*b^3*c^3*n + 6*a^3*d^3*n^2 - 9*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^
3*n^3 + 36*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 57*a*b^2*c^2*d*n - 42*a^2*b*c*d^2
```

*n + 24*a*b^2*c^2*d*n^2 - 21*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*
c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24)) - (x*(a + b*x)^n*(6*a^4*d^4 - 24*b^4*c^4 + 11*a^4*d^4*n - 26*b^4*c^4
*n + 6*a^4*d^4*n^2 - 9*b^4*c^4*n^2 + a^4*d^4*n^3 - b^4*c^4*n^3 + 36*a^2*b^2
*c^2*d^2 - 24*a*b^3*c^3*d - 24*a^3*b*c*d^3 + 10*a*b^3*c^3*d*n - 40*a^3*b*c*
d^3*n + 9*a^2*b^2*c^2*d^2*n^2 + 12*a*b^3*c^3*d*n^2 - 18*a^3*b*c*d^3*n^2 + 2
*a*b^3*c^3*d*n^3 - 2*a^3*b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^
4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
a + b*x)^n*(20*b^2*c^2 + a^2*d^2*n + 9*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n +
35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(a + b*x)^n*(5*b*c - a*d*n +
b*c*n))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(a + b*x)^n*(60*b^3*c^3 - 2*a^3*d^3*n + 47*b^3*c^3*n - 3*a^3
*d^3*n^2 + 12*b^3*c^3*n^2 - a^3*d^3*n^3 + b^3*c^3*n^3 - 60*a*b^2*c^2*d*n +
15*a^2*b*c*d^2*n - 27*a*b^2*c^2*d*n^2 + 18*a^2*b*c*d^2*n^2 - 3*a*b^2*c^2*d*
n^3 + 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))

3.1878 $\int (a+bx)^{-2+n}(c+dx)^{1-n}\,dx$

Optimal. Leaf size=83

$$-\frac{(bc-ad)(a+bx)^{-1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(-1+n,-1+n;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}$$

```
[Out] -(-a*d+b*c)*(b*x+a)^(-1+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([-1+n, -1+n],
[n],-d*(b*x+a)/(-a*d+b*c))/b^2/(1-n)/((d*x+c)^n)
```

Rubi [A]

time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {72, 71}

$$-\frac{(bc-ad)(a+bx)^{n-1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n-1,n-1;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)*(c + d*x)^(1 - n),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x)^(-1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeomet
ric2F1[-1 + n, -1 + n, n, -((d*(a + b*x))/(b*c - a*d))])/(b^2*(1 - n)*(c +
d*x)^n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{1-n}\,dx = \frac{\left((bc-ad)(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^{-2+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{1-n}dx}{b}$$
$$= -\frac{(bc-ad)(a+bx)^{-1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(-1+n,-1+n;n;-\frac{d}{}\right.}{b^2(1-n)}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.90

$$\frac{(a+bx)^{-1+n}(c+dx)^{1-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-1+n} {}_2F_1\left(-1+n,-1+n;n;\frac{d(a+bx)}{-bc+ad}\right)}{b(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)*(c + d*x)^(1 - n),x]
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n)*((b*(c + d*x))/(b*c - a*d))^(-1 + n)*
Hypergeometric2F1[-1 + n, -1 + n, n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(-1
+ n))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(n - 2)/(c + d*x)^(n - 1),x]')
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{-2+n}(dx+c)^{1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
[Out] int((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Fricas [F]
time = 0.33, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)*(d*x+c)**(1-n),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{n-2}\,(c+d\,x)^{1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 2)*(c + d*x)^(1 - n),x)
```

```
[Out] int((a + b*x)^(n - 2)*(c + d*x)^(1 - n), x)
```

3.1879 $\int (a+bx)^{1+n}(c+dx)^{-1-n}\,dx$

Optimal. Leaf size=84

$$\frac{(bc-ad)(a+bx)^n\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

```
[Out] (-a*d+b*c)*(b*x+a)^n*hypergeom([-n, -1-n],[1-n],b*(d*x+c)/(-a*d+b*c))/d^2/n
/((-d*(b*x+a)/(-a*d+b*c))^n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {72, 71}

$$\frac{(bc-ad)(a+bx)^n(c+dx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(-n-1,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1 + n)*(c + d*x)^(-1 - n),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{1+n}(c+dx)^{-1-n}\,dx = \frac{\left((-bc+ad)(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}\right)\int (c+dx)^{-1-n}\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^{1+n}\,dx}{d}$$

$$= \frac{(bc-ad)(a+bx)^n\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right.}{d^2n}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.99

$$\frac{(bc-ad)(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1 + n)*(c + d*x)^(-1 - n),x]
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^n)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(n + 1)/(c + d*x)^(n + 1),x]')
[Out] Timed out
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{1+n}(dx+c)^{-1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
[Out] int((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1+n)*(d*x+c)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{n+1}}{(c+d\,x)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n + 1)/(c + d*x)^(n + 1),x)
```

```
[Out] int((a + b*x)^(n + 1)/(c + d*x)^(n + 1), x)
```

3.1880 $\int (a+bx)^m(c+dx)^{1+2n-2(1+n)}\,dx$

Optimal. Leaf size=51

$$\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

```
[Out] (b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(1
+m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7, 70}

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]
```

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
- a*d))])/((b*c - a*d)*(1 + m))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^{1+2n-2(1+n)}\,dx &= \int \frac{(a+bx)^m}{c+dx}\,dx\\ &= \frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 51, normalized size = 1.00

$$-\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]
[Out] -(((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d)])/((-(b*c) + a*d)*(1 + m)))
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]')
[Out] cought exception: maximum recursion depth exceeded
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c),x)
[Out] int((b*x+a)^m/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="maxima")
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c),x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x),x)
```

```
[Out] int((a + b*x)^m/(c + d*x), x)
```

3.1881 $\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx$

Optimal. Leaf size=57

$$-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -1/(-a*d+b*c)/(b*x+a)-d*ln(b*x+a)/(-a*d+b*c)^2+d*ln(d*x+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7, 46}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c + d*x])/(b*c - a*d)^2
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2}\,dx &= \int \frac{1}{(a+bx)^2(c+dx)}\,dx\\ &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.93

$$\frac{-bc+ad-d(a+bx)\log(a+bx)+d(a+bx)\log(c+dx)}{(bc-ad)^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
 - a*d)^2*(a + b*x))
```

Mathics [A]
time = 4.68, size = 93, normalized size = 1.63

$$\frac{d\left(a^2d-abc+bx\left(ad-bc\right)\right)\left(\operatorname{Log}\left[\frac{c+dx}{d}\right]-\operatorname{Log}\left[\frac{a+bx}{b}\right]\right)+\left(ad-bc\right)^2}{\left(ad-bc\right)^2\left(a^2d-abc+bx\left(ad-bc\right)\right)}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(a + b*x)^2*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]')
```

```
[Out] (d (a ^ 2 d - a b c + b x (a d - b c)) (Log[(c + d x) / d] - Log[(a + b x)
/ b]) + (a d - b c) ^ 2) / ((a d - b c) ^ 2 (a ^ 2 d - a b c + b x (a d - b
 c)))
```

Maple [A]
time = 0.19, size = 57, normalized size = 1.00

method	result	size
default	$\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}$	57
risch	$\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}+\frac{d\ln(-dx-c)}{a^2d^2-2abcd+b^2c^2}$	86
norman	$-\frac{bx}{a(ad-bc)(bx+a)}+\frac{d\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A]
time = 0.27, size = 92, normalized size = 1.61

$$-\frac{d\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A]
time = 0.31, size = 93, normalized size = 1.63

$$-\frac{bc - ad + (bdx + ad)\log(bx + a) - (bdx + ad)\log(dx + c)}{ab^2c^2 - 2\,a^2bcd + a^3d^2 + (b^3c^2 - 2\,ab^2cd + a^2bd^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.
time = 0.48, size = 233, normalized size = 4.09

$$\frac{d\log\left(x + \frac{-\frac{a^3d^4}{(ad-bc)^2} + \frac{3a^2bcd^3}{(ad-bc)^2} - \frac{3ab^2c^2d^2}{(ad-bc)^2} + ad^2 + \frac{b^3c^3d}{(ad-bc)^2} + bcd}{2bd^2}\right)}{(ad - bc)^2} - \frac{d\log\left(x + \frac{\frac{a^3d^4}{(ad-bc)^2} - \frac{3a^2bcd^3}{(ad-bc)^2} + \frac{3ab^2c^2d^2}{(ad-bc)^2} + ad^2 - \frac{b^3c^3d}{(ad-bc)^2} + bcd}{2bd^2}\right)}{(ad - bc)^2} + \frac{1}{a^2d - abc + x(abd - b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A]
time = 0.00, size = 101, normalized size = 1.77

$$\frac{d^2\ln|xd + c|}{b^2dc^2 - 2bad^2c + a^2d^3} - \frac{bd\ln|xb + a|}{b^3c^2 - 2b^2adc + ba^2d^2} + \frac{-bc + da}{(bc - da)^2\,(xb + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] -b*d*log(abs(b*x + a))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + d^2*log(abs(d*
x + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/((b*c - a*d)*(b*x + a))
```

Mupad [B]
time = 0.44, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)}-\frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

3.1882 $\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx$

Optimal. Leaf size=95

$$-\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)}+\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-1-m}}{a^2bc^2(1+m)(2+m)}$$

```
[Out] -(b*x+a)^(1+m)*(a*c*(1+m)+b*c*(2+m)*x)^(-2-m)/a/b/c/(2+m)+(b*x+a)^(1+m)*(a*
c*(1+m)+b*c*(2+m)*x)^(-1-m)/a^2/b/c^2/(m^2+3*m+2)
```

Rubi [A]

time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {47, 37}

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-2 - m))/(a*b*c*(2 + m)
)) + ((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-1 - m))/(a^2*b*c^2*
(1 + m)*(2 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx = -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)} - \frac{\int (a+bx)}{}$$
$$= -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)} + \frac{(a+bx)^{1+}}{}$$

Mathematica [A]
time = 0.17, size = 54, normalized size = 0.57

$$\frac{x(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-m}}{a^2c^3(1+m)(a(1+m)+b(2+m)x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] (x*(a + b*x)^(1 + m))/(a^2*c^3*(1 + m)*(a*(1 + m) + b*(2 + m)*x)^2*(a*c*(1
+ m) + b*c*(2 + m)*x)^m)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^m/(a*c*(m + 1) + b*c*(m + 2)*x)^(m + 3),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.28, size = 57, normalized size = 0.60

method	result	size
gosper	$\frac{(bx+a)^{1+m}(bxm+am+2bx+a)x(bcxm+acm+2bcx+ac)^{-3-m}}{a^2(1+m)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+m)*(b*m*x+a*m+2*b*x+a)/a^2/(1+m)*x*(b*c*m*x+a*c*m+2*b*c*x+a*c)^(
-3-m)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="maxima")
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

Fricas [A]
time = 0.32, size = 85, normalized size = 0.89

$$\frac{((b^2m+2\,b^2)x^3+(2\,abm+3\,ab)x^2+(a^2m+a^2)x)(acm+ac+(bcm+2\,bc)x)^{-m-3}(bx+a)^m}{a^2m+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="fricas")
[Out] ((b^2*m + 2*b^2)*x^3 + (2*a*b*m + 3*a*b)*x^2 + (a^2*m + a^2)*x)*(a*c*m + a*
c + (b*c*m + 2*b*c)*x)^(-m - 3)*(b*x + a)^m/(a^2*m + a^2)
```

Sympy [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a*c*(1+m)+b*c*(2+m)*x)**(-3-m),x)
[Out] Timed out
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x)
[Out] Could not integrate
```

Mupad [B]
time = 1.04, size = 81, normalized size = 0.85

$$\frac{x\,{\left(a+b\,x\right)}^m+\frac{b\,x^2\,\left(2\,m+3\right)\,{\left(a+b\,x\right)}^m}{a\,\left(m+1\right)}+\frac{b^2\,x^3\,\left(m+2\right)\,{\left(a+b\,x\right)}^m}{a^2\,\left(m+1\right)}}{{\left(a\,c\,\left(m+1\right)+b\,c\,x\,\left(m+2\right)\right)}^{m+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3),x)
[Out] (x*(a + b*x)^m + (b*x^2*(2*m + 3)*(a + b*x)^m)/(a*(m + 1)) + (b^2*x^3*(m +
2)*(a + b*x)^m)/(a^2*(m + 1)))/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3)
```

3.1883 $\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$

Optimal. Leaf size=97

$$-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

```
[Out] -(d*x+c)^(a*d/(-a*d+b*c))/b/c/((b*x+a)^(b*c/(-a*d+b*c)))+(d*x+c)^(a*d/(-a*d
+b*c))/a/b/c/((b*x+a)^(a*d/(-a*d+b*c)))
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {47, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a*d)),x
]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx = -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} - \frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}}{bc}$$
$$= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} + \frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

Mathematica [A]
time = 0.25, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a
*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

cought exception: maximum recursion depth exceeded while calling a Python object

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)
/(b*c - a*d)),x]')
```

```
[Out] cought exception: maximum recursion depth exceeded while calling a Python o
bject
```

Maple [A]
time = 0.20, size = 66, normalized size = 0.68

method	result	size
gosper	$\frac{(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}x}{ac}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x,method=_RETUR
NVERBOSE)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

Fricas [A]
time = 0.32, size = 84, normalized size = 0.87

$$\frac{bdx^3 + acx + (bc + ad)x^2}{(bx + a)^{\frac{2\,bc - ad}{bc - ad}}(dx + c)^{\frac{bc - 2\,ad}{bc - ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{-\frac{bc}{-ad + bc} - 1}(c + dx)^{\frac{ad}{-ad + bc} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-b*c/(-a*d+b*c))*(d*x+c)**(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] Integral((a + b*x)**(-b*c/(-a*d + b*c) - 1)*(c + d*x)**(a*d/(-a*d + b*c) -
1), x)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 2.14, size = 119, normalized size = 1.23

$$\frac{x\,{\left(a+b\,x\right)}^{\frac{b\,c}{a\,d-b\,c}-1}+\frac{x^2\,\left(a\,d+b\,c\right)\,{\left(a+b\,x\right)}^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}+\frac{b\,d\,x^3\,{\left(a+b\,x\right)}^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}}{{\left(c+d\,x\right)}^{\frac{a\,d}{a\,d-b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^((b*c)/(a*d - b*c) - 1)/(c + d*x)^((a*d)/(a*d - b*c) + 1),x)
```

```
[Out] (x*(a + b*x)^((b*c)/(a*d - b*c) - 1) + (x^2*(a*d + b*c)*(a + b*x)^((b*c)/(a
*d - b*c) - 1))/(a*c) + (b*d*x^3*(a + b*x)^((b*c)/(a*d - b*c) - 1))/(a*c))/
(c + d*x)^((a*d)/(a*d - b*c) + 1)
```

3.1884 $\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$

Optimal. Leaf size=97

$$-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

```
[Out] -(d*x+c)^(a*d/(-a*d+b*c))/b/c/((b*x+a)^(b*c/(-a*d+b*c)))+(d*x+c)^(a*d/(-a*d
+b*c))/a/b/c/((b*x+a)^(a*d/(-a*d+b*c)))
```

Rubi [A]
time = 0.01, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {47, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(-(b*c)
 + a*d)),x]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx = -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} - \frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx}{bc}$$
$$= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} + \frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

Mathematica [A]
time = 0.12, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(
-(b*c) + a*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

Mathics [F(-1)]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c -
 2*a*d)/((-b)*c + a*d)),x]')
```

```
[Out] Timed out
```

Maple [A]
time = 0.21, size = 66, normalized size = 0.68

method	result	size
gosper	$\frac{(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}x}{ac}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c)),x,met
hod=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

Fricas [**A**]
time = 0.36, size = 84, normalized size = 0.87

$$\frac{bdx^3 + acx + (bc + ad)x^2}{(bx + a)^{\frac{2bc - ad}{bc - ad}}(dx + c)^{\frac{bc - 2ad}{bc - ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

Sympy [**F(-1)**]
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)**((-2*a*d+b*c)/(a*d-b*c
)),x)
```

```
[Out] Timed out
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x)
```

```
[Out] Could not integrate
```

Mupad [B]
time = 0.85, size = 142, normalized size = 1.46

$$\frac{\frac{x}{(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{x^2\,(a\,d+b\,c)}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{b\,d\,x^3}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}}{(c+d\,x)^{\frac{2\,a\,d-b\,c}{a\,d-b\,c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^((a*d - 2*b*c)/(a*d - b*c))*(c + d*x)^((2*a*d - b*c)/(a*d
- b*c))),x)
```

```
[Out] (x/(a + b*x)^((a*d - 2*b*c)/(a*d - b*c)) + (x^2*(a*d + b*c))/(a*c*(a + b*x)
^((a*d - 2*b*c)/(a*d - b*c))) + (b*d*x^3)/(a*c*(a + b*x)^((a*d - 2*b*c)/(a*
d - b*c))))/(c + d*x)^((2*a*d - b*c)/(a*d - b*c))
```

3.1885 $\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=30

$$2^{1+n}\sqrt{1+x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

```
[Out] 2^(1+n)*hypergeom([1/2, -n],[3/2],1/2+1/2*x)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{71\}$

$$2^{n+1}\sqrt{x+1}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^n/Sqrt[1 + x],x]
```

```
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx = 2^{1+n}\sqrt{1+x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 1.00

$$2^{1+n}\sqrt{1+x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^n/Sqrt[1 + x],x]
```

```
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.78, size = 28, normalized size = 0.93

$$2^{1+n}\sqrt{1+x}\,\text{hyper}\left[\left\{\frac{1}{2},-n\right\},\left\{\frac{3}{2}\right\},\frac{(1+x)\,\text{exp\_polar}\,[2I\text{Pi}]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^n/Sqrt[1 + x],x]')
```

```
[Out] 2 ^ (1 + n) Sqrt[1 + x] hyper[{1 / 2, -n}, {3 / 2}, (1 + x) exp_polar[2 I P
i] / 2]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(1-x)^{n}}{\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^n/(1+x)^(1/2),x)
```

```
[Out] int((1-x)^n/(1+x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x + 1)^n/sqrt(x + 1), x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x + 1)^n/sqrt(x + 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.30, size = 29, normalized size = 0.97

$$2\cdot 2^n\sqrt{x+1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|\frac{(x+1)\,e^{2i\pi}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**n/(1+x)**(1/2),x)
```

```
[Out] 2*2**n*sqrt(x + 1)*hyper((1/2, -n), (3/2,), (x + 1)*exp_polar(2*I*pi)/2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(1-x)^n}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^n/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^n/(x + 1)^(1/2), x)
```

3.1886 $\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=35

$$-2^{1+n}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

```
[Out] -2^(1+n)*hypergeom([1/2, -n],[3/2],1/2-1/2*x)*(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {71}

$$-2^{n+1}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^n/Sqrt[1 - x],x]
```

```
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx = -2^{1+n}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 1.00

$$-2^{1+n}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^n/Sqrt[1 - x],x]
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 2.78, size = 29, normalized size = 0.83

$$-I2^{1+n}\sqrt{-1+x}\,\text{hyper}\left[\left\{\frac{1}{2},-n\right\},\left\{\frac{3}{2}\right\},\frac{(-1+x)\,\text{exp\_polar}\,[I\text{Pi}]}{2}\right]$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^n/Sqrt[1 - x],x]')
[Out] -I 2 ^ (1 + n) Sqrt[-1 + x] hyper[{1 / 2, -n}, {3 / 2}, (-1 + x) exp_polar[
I Pi] / 2]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^n/(1-x)^(1/2),x)
[Out] int((1+x)^n/(1-x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)^n/sqrt(-x + 1), x)
```

Fricas [F]
time = 0.34, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="fricas")
[Out] integral(-(x + 1)^n*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.28, size = 31, normalized size = 0.89

$$-2 \cdot 2^n i \sqrt{x-1}\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, -n \\ \frac{3}{2} \end{matrix}\middle| \frac{(x-1)\,e^{i\pi}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**n/(1-x)**(1/2),x)
[Out] -2*2**n*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi)/2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(x+1)^n}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^n/(1 - x)^(1/2),x)
[Out] int((x + 1)^n/(1 - x)^(1/2), x)
```

3.1887 $\int (1-x)^n(1+x)^{7/3}\,dx$

Optimal. Leaf size=33

$$\frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

```
[Out] 3/5*2^(-1+n)*(1+x)^(10/3)*hypergeom([10/3, -n],[13/3],1/2+1/2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {71}

$$\frac{3}{5}2^{n-1}(x+1)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^n*(1 + x)^(7/3),x]
```

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int (1-x)^n(1+x)^{7/3}\,dx = \frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

Mathematica [A]
time = 0.07, size = 33, normalized size = 1.00

$$\frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^n*(1 + x)^(7/3),x]
```

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 52.35, size = 27, normalized size = 0.82

$$\frac{32^n (1+x)^{\frac{10}{3}} \text{hyper}\left[\left\{\frac{10}{3},-n\right\},\left\{\frac{13}{3}\right\},\frac{(1+x)\text{exp\_polar}[2I\text{Pi}]}{2}\right]}{10}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 - x)^n*(1 + x)^(7/3),x]')
```

```
[Out] 3 2 ^ n (1 + x) ^ (10 / 3) hyper[{10 / 3, -n}, {13 / 3}, (1 + x) exp_polar[
2 I Pi] / 2] / 10
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (1-x)^n (1+x)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^n*(1+x)^(7/3),x)
```

```
[Out] int((1-x)^n*(1+x)^(7/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(7/3)*(-x + 1)^n, x)
```

Fricas [F]
time = 0.36, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 2*x + 1)*(x + 1)^(1/3)*(-x + 1)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 77.09, size = 37, normalized size = 1.12

$$\frac{2^{n}\left(x+1\right)^{\frac{10}{3}}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{10}{3},-n\\ \frac{13}{3}\end{matrix}\middle|\frac{(x+1)e^{2i\pi}}{2}\right)}{\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**n*(1+x)**(7/3),x)
[Out] 2**n*(x + 1)**(10/3)*gamma(10/3)*hyper((10/3, -n), (13/3,), (x + 1)*exp_pol
ar(2*I*pi)/2)/gamma(13/3)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (1-x)^{n}\,(x+1)^{7/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^n*(x + 1)^(7/3),x)
[Out] int((1 - x)^n*(x + 1)^(7/3), x)
```

3.1888 $\int (1-x)^{7/3}(1+x)^n \, dx$

Optimal. Leaf size=37

$$-\frac{3}{5}2^{-1+n}(1-x)^{10/3} {}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

```
[Out] -3/5*2^(-1+n)*(1-x)^(10/3)*hypergeom([10/3, -n],[13/3],1/2-1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {71}

$$-\frac{3}{5}2^{n-1}(1-x)^{10/3} {}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/3)*(1 + x)^n,x]
```

```
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int (1-x)^{7/3}(1+x)^n \, dx = -\frac{3}{5}2^{-1+n}(1-x)^{10/3} {}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 1.00

$$-\frac{3}{5}2^{-1+n}(1-x)^{10/3} {}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/3)*(1 + x)^n,x]
```

```
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Mathics [**C**] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 52.44, size = 30, normalized size = 0.81

$$3 - \frac{2^n(-1+x)^{\frac{10}{3}} \operatorname{hyper}\left[\left\{\frac{10}{3}, -n\right\}, \left\{\frac{13}{3}\right\}, \frac{(-1+x)\text{exp\_polar}[I\text{Pi}]}{2}\right]}{10}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(1 + x)^n*(1 - x)^(7/3),x]')
```

```
[Out] 3 -1 ^ (1 / 3) 2 ^ n (-1 + x) ^ (10 / 3) hyper[{10 / 3, -n}, {13 / 3}, (-1
+ x) exp_polar[I Pi] / 2] / 10
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int (1-x)^{\frac{7}{3}} (1+x)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/3)*(1+x)^n,x)
```

```
[Out] int((1-x)^(7/3)*(1+x)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^n*(-x + 1)^(7/3), x)
```

Fricas [**F**]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="fricas")
```

```
[Out] integral((x^2 - 2*x + 1)*(x + 1)^n*(-x + 1)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 76.09, size = 42, normalized size = 1.14

$$\frac{\sqrt[3]{-1}\cdot 2^{n}\left(x-1\right)^{\frac{10}{3}}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{10}{3},-n\\\frac{13}{3}\end{matrix}\middle|\frac{(x-1)e^{i\pi}}{2}\right)}{\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/3)*(1+x)**n,x)
[Out] (-1)**(1/3)*2**n*(x - 1)**(10/3)*gamma(10/3)*hyper((10/3, -n), (13/3,), (x
- 1)*exp_polar(I*pi)/2)/gamma(13/3)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x)
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (1-x)^{7/3}\,(x+1)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/3)*(x + 1)^n,x)
[Out] int((1 - x)^(7/3)*(x + 1)^n, x)
```

3.1889 $\int (1+2x)^{-m}(2+3x)^m\,dx$

Optimal. Leaf size=47

$$\frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

```
[Out] 2^(-1-m)*(1+2*x)^(1-m)*hypergeom([-m, 1-m],[2-m],-3-6*x)/(1-m)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {71}

$$\frac{2^{-m-1}(2x+1)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(2x+1))}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^m/(1 + 2*x)^m,x]
```

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int (1+2x)^{-m}(2+3x)^m\,dx = \frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.00

$$\frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^m/(1 + 2*x)^m,x]
```

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Mathics [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 16.34, size = 41, normalized size = 0.87

$$\frac{3^{-1+m}E^{-I\mathrm{Pi}m}\left(2+3x\right)^{1+m}\mathrm{hyper}\left[\left\{m,1+m\right\},\left\{2+m\right\},4+6x\right]}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[(2 + 3*x)^m/(1 + 2*x)^m,x]')
```

```
[Out] 3 ^ (-1 + m) E ^ (-I Pi m) (2 + 3 x) ^ (1 + m) hyper[{m, 1 + m}, {2 + m}, 4
 + 6 x] / (1 + m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(2+3x\right)^{m}\left(1+2x\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^m/((1+2*x)^m),x)
```

```
[Out] int((2+3*x)^m/((1+2*x)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)^m/(2*x + 1)^m, x)
```

Fricas [F]
time = 0.31, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="fricas")
```

```
[Out] integral((3*x + 2)^m/(2*x + 1)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.80, size = 42, normalized size = 0.89

$$\frac{3^{2m} \left(x+\frac{2}{3}\right) \left(x+\frac{2}{3}\right)^m e^{-i\pi m}\Gamma\left(m+1\right) {}_2F_1\left(\begin{matrix} m, m+1 \\ m+2 \end{matrix}\middle| 6x+4\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**m/((1+2*x)**m),x)
```

```
[Out] 3**(2*m)*(x + 2/3)*(x + 2/3)**m*exp(-I*pi*m)*gamma(m + 1)*hyper((m, m + 1),
 (m + 2,), 6*x + 4)/gamma(m + 2)
```

Giac [F] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x)
```

```
[Out] Could not integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(3\,x+2\right)}^m}{{\left(2\,x+1\right)}^m} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^m/(2*x + 1)^m,x)
```

```
[Out] int((3*x + 2)^m/(2*x + 1)^m, x)
```

3.1890 $\int \left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n\,dx$

Optimal. Leaf size=45

$$\frac{(c+dx)^{1+n}\,{}_2F_1\left(-m,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

[Out] (d*x+c)^(1+n)*hypergeom([-m, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/d/(1+n)

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {192, 71}

$$\frac{(c+dx)^{n+1}\,{}_2F_1\left(-m,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Int[((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n,x]

[Out] ((c + d*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n))

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 192

```
Int[(u_)^(m_.)*(v_)^(n_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^n, x] /; FreeQ[{m, n}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u
, v}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n\,dx &= \int (c+dx)^n \left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^m dx\\ &= \frac{(c+dx)^{1+n}\,{}_2F_1\left(-m,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 88, normalized size = 1.96

$$\frac{(a+bx)\left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)*((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n*Hypergeometric2F1[1
 + m, -n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + m)*((b*(c + d*x))/(
b*c - a*d))^n)
```

Mathics [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Warning: Unable to verify antiderivative.

```
[In] mathics('Integrate[((d*(a + b*x))/((-b)*c + a*d))^m*(c + d*x)^n,x]')
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\frac{d\,(bx+a)}{ad-bc}\right)^m (dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
```

```
[Out] int((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*(-(b*x + a)*d/(b*c - a*d))^m, x)
```

Fricas [F]
time = 0.32, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*(-(b*d*x + a*d)/(b*c - a*d))^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))**m*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

Giac [**F**] N/A
time = 0.00, size = 0, normalized size = 0.00

Could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
```

```
[Out] Could not integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c+d\,x)^n\,\left(\frac{d\,\left(a+b\,x\right)}{a\,d-b\,c}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n*((d*(a + b*x))/(a*d - b*c))^m,x)
```

```
[Out] int((c + d*x)^n*((d*(a + b*x))/(a*d - b*c))^m, x)
```

3.1891 $\int (a + bx + cx^2 + dx^3) \, dx$

Optimal. Leaf size=28

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/4*d*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x + c*x^2 + d*x^3,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Rubi steps

$$\int (a + bx + cx^2 + dx^3) \, dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x + c*x^2 + d*x^3,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Mathics [A]
time = 1.68, size = 20, normalized size = 0.71

$$x\left(a + \frac{bx}{2} + \frac{cx^2}{3} + \frac{dx^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[a + b*x + c*x^2 + d*x^3,x]')
[Out] x (a + b x / 2 + c x ^ 2 / 3 + d x ^ 3 / 4)
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.82

method	result	size
gosper	$ax+\frac{1}{2}x^{2}b+\frac{1}{3}c\,x^{3}+\frac{1}{4}d\,x^{4}$	23
default	$ax+\frac{1}{2}x^{2}b+\frac{1}{3}c\,x^{3}+\frac{1}{4}d\,x^{4}$	23
norman	$ax+\frac{1}{2}x^{2}b+\frac{1}{3}c\,x^{3}+\frac{1}{4}d\,x^{4}$	23
risch	$ax+\frac{1}{2}x^{2}b+\frac{1}{3}c\,x^{3}+\frac{1}{4}d\,x^{4}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3+c*x^2+b*x+a,x,method=_RETURNVERBOSE)
[Out] a*x+1/2*x^2*b+1/3*c*x^3+1/4*d*x^4
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^{4}+\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="maxima")
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.30, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^{4}+\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="fricas")
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.79

$$ax+\frac{bx^{2}}{2}+\frac{cx^{3}}{3}+\frac{dx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3+c*x**2+b*x+a,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + d*x**4/4
```

Giac [**A**]
time = 0.00, size = 25, normalized size = 0.89

$$\frac{1}{4}dx^4 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x)
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [**B**]
time = 0.04, size = 22, normalized size = 0.79

$$\frac{d\,x^4}{4} + \frac{c\,x^3}{3} + \frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x + c*x^2 + d*x^3,x)
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

3.1892 $\int(-x^3+x^4)\,dx$

Optimal. Leaf size=15

$$-\frac{x^4}{4}+\frac{x^5}{5}$$

```
[Out] -1/4*x^4+1/5*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[-x^3 + x^4,x]
```

```
[Out] -1/4*x^4 + x^5/5
```

Rubi steps

$$\int(-x^3+x^4)\,dx=-\frac{x^4}{4}+\frac{x^5}{5}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{x^4}{4}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^3 + x^4,x]
```

```
[Out] -1/4*x^4 + x^5/5
```

Mathics [A]
time = 1.68, size = 10, normalized size = 0.67

$$\frac{x^4(-5+4x)}{20}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-x^3 + x^4,x]')
[Out] x ^ 4 (-5 + 4 x) / 20
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{x^4(-5+4x)}{20}$	11
default	$-\frac{1}{4}x^4+\frac{1}{5}x^5$	12
norman	$-\frac{1}{4}x^4+\frac{1}{5}x^5$	12
risch	$-\frac{1}{4}x^4+\frac{1}{5}x^5$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4-x^3,x,method=_RETURNVERBOSE)
[Out] -1/4*x^4+1/5*x^5
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5-\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="maxima")
[Out] 1/5*x^5 - 1/4*x^4
```

Fricas [A]
time = 0.29, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5-\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="fricas")
[Out] 1/5*x^5 - 1/4*x^4
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.53

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4-x**3,x)
```

```
[Out] x**5/5 - x**4/4
```

Giac [A]
time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x)
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^4\,\left(4\,x-5\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4 - x^3,x)
```

```
[Out] (x^4*(4*x - 5))/20
```

3.1893 $\int (-1+x^5)\,dx$

Optimal. Leaf size=11

$$-x+\frac{x^6}{6}$$

```
[Out] -x+1/6*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

```
[In] Int[-1 + x^5,x]
```

```
[Out] -x + x^6/6
```

Rubi steps

$$\int (-1+x^5)\,dx = -x+\frac{x^6}{6}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-x+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + x^5,x]
```

```
[Out] -x + x^6/6
```

Mathics [A]
time = 1.60, size = 9, normalized size = 0.82

$$-x+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-1 + x^5,x]')
```

```
[Out] -x + x ^ 6 / 6
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
gosper	$\frac{x(x^5-6)}{6}$	9
default	$-x+\frac{1}{6}x^6$	10
norman	$-x+\frac{1}{6}x^6$	10
risch	$-x+\frac{1}{6}x^6$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5-1,x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/6*x^6
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - x
```

Fricas [A]
time = 0.29, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - x
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.45

$$\frac{x^6}{6} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5-1,x)
```

```
[Out] x**6/6 - x
```

Giac [**A**]
time = 0.00, size = 9, normalized size = 0.82

$$\frac{x^6}{6} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x)
```

```
[Out] 1/6*x^6 - x
```

Mupad [**B**]
time = 0.02, size = 8, normalized size = 0.73

$$\frac{x\,\left(x^5-6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5 - 1,x)
```

```
[Out] (x*(x^5 - 6))/6
```

3.1894 $\int (7+4x)\,dx$

Optimal. Leaf size=9

$$7x+2x^2$$

[Out] 2*x^2+7*x

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$2x^2+7x$$

Antiderivative was successfully verified.

[In] Int[7 + 4*x,x]

[Out] 7*x + 2*x^2

Rubi steps

$$\int (7+4x)\,dx = 7x+2x^2$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$7x+2x^2$$

Antiderivative was successfully verified.

[In] Integrate[7 + 4*x,x]

[Out] 7*x + 2*x^2

Mathics [A]
time = 1.61, size = 7, normalized size = 0.78

$$x\,(7+2x)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[7 + 4*x,x]')

[Out] x (7 + 2 x)

Maple [A]
time = 0.01, size = 10, normalized size = 1.11

method	result	size
gosper	$2x^2 + 7x$	10
default	$2x^2 + 7x$	10
norman	$2x^2 + 7x$	10
risch	$2x^2 + 7x$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7+4*x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^2+7*x
```

Maxima [A]
time = 0.34, size = 9, normalized size = 1.00

$$2\,x^2 + 7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="maxima")
```

```
[Out] 2*x^2 + 7*x
```

Fricas [A]
time = 0.29, size = 9, normalized size = 1.00

$$2\,x^2 + 7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="fricas")
```

```
[Out] 2*x^2 + 7*x
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.78

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x)
```

```
[Out] 2*x**2 + 7*x
```

Giac [A]
time = 0.00, size = 11, normalized size = 1.22

$$7x + \frac{4}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x)
```

```
[Out] 2*x^2 + 7*x
```

Mupad [B]
time = 0.03, size = 7, normalized size = 0.78

$$x\,(2\,x+7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 7,x)
```

```
[Out] x*(2*x + 7)
```

3.1895 $\int (4x + \pi x^3) \, dx$

Optimal. Leaf size=14

$$2x^2 + \frac{\pi x^4}{4}$$

```
[Out] 2*x^2+1/4*Pi*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{\pi x^4}{4} + 2x^2$$

Antiderivative was successfully verified.

```
[In] Int[4*x + Pi*x^3,x]
```

```
[Out] 2*x^2 + (Pi*x^4)/4
```

Rubi steps

$$\int (4x + \pi x^3) \, dx = 2x^2 + \frac{\pi x^4}{4}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$2x^2 + \frac{\pi x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x + Pi*x^3,x]
```

```
[Out] 2*x^2 + (Pi*x^4)/4
```

Mathics [A]
time = 1.63, size = 12, normalized size = 0.86

$$\frac{x^2 (8 + \text{Pi} x^2)}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[4*x + Pi*x^3,x]')
[Out] x ^ 2 (8 + Pi x ^ 2) / 4
```

Maple [A]
time = 0.04, size = 15, normalized size = 1.07

method	result	size
gosper	$\frac{x^{2}\left(\pi x^{2}+8\right)}{4}$	13
norman	$2x^{2}+\frac{1}{4}\pi x^{4}$	13
risch	$2x^{2}+\frac{1}{4}\pi x^{4}$	13
default	$\frac{\left(\pi x^{2}+4\right)^{2}}{4\pi}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi*x^3+4*x,x,method=_RETURNVERBOSE)
[Out] 1/4*(Pi*x^2+4)^2/Pi
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{1}{4}\pi x^{4}+2x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="maxima")
[Out] 1/4*pi*x^4 + 2*x^2
```

Fricas [A]
time = 0.29, size = 12, normalized size = 0.86

$$\frac{1}{4}\pi x^{4}+2x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="fricas")
[Out] 1/4*pi*x^4 + 2*x^2
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.71

$$\frac{\pi x^{4}}{4}+2x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x**3+4*x,x)
```

```
[Out] pi*x**4/4 + 2*x**2
```

Giac [**A**]
time = 0.00, size = 15, normalized size = 1.07

$$\frac{1}{4}\pi x^4 + \frac{4}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x)
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{\Pi\, x^4}{4} + 2\, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + Pi*x^3,x)
```

```
[Out] (Pi*x^4)/4 + 2*x^2
```

3.1896 $\int (2x+5x^2)\,dx$

Optimal. Leaf size=11

$$x^2+\frac{5x^3}{3}$$

[Out] x^2+5/3*x^3

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5x^3}{3}+x^2$$

Antiderivative was successfully verified.

[In] Int[2*x + 5*x^2,x]

[Out] x^2 + (5*x^3)/3

Rubi steps

$$\int (2x+5x^2)\,dx=x^2+\frac{5x^3}{3}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x^2+\frac{5x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[2*x + 5*x^2,x]

[Out] x^2 + (5*x^3)/3

Mathics [A]
time = 1.65, size = 10, normalized size = 0.91

$$\frac{x^2(3+5x)}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[2*x + 5*x^2,x]')
```

```
[Out] x ^ 2 (3 + 5 x) / 3
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
default	$x^2+\frac{5}{3}x^3$	10
norman	$x^2+\frac{5}{3}x^3$	10
risch	$x^2+\frac{5}{3}x^3$	10
gosper	$\frac{x^2(3+5x)}{3}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x^2+2*x,x,method=_RETURNVERBOSE)
```

```
[Out] x^2+5/3*x^3
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.82

$$\frac{5}{3}\,x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + x^2
```

Fricas [A]
time = 0.29, size = 9, normalized size = 0.82

$$\frac{5}{3}\,x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="fricas")
```

```
[Out] 5/3*x^3 + x^2
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$\frac{5x^3}{3}+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x**2+2*x,x)
```

```
[Out] 5*x**3/3 + x**2
```

Giac [A]
time = 0.00, size = 15, normalized size = 1.36

$$\frac{5}{3}x^3+\frac{2}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x)
```

```
[Out] 5/3*x^3 + x^2
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.91

$$\frac{x^2\,\left(5\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 5*x^2,x)
```

```
[Out] (x^2*(5*x + 3))/3
```

3.1897 $\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$

Optimal. Leaf size=15

$$\frac{x^3}{6}+\frac{x^4}{12}$$

[Out] 1/6*x^3+1/12*x^4

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

[In] Int[x^2/2 + x^3/3,x]

[Out] x^3/6 + x^4/12

Rubi steps

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx=\frac{x^3}{6}+\frac{x^4}{12}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^3}{6}+\frac{x^4}{12}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/2 + x^3/3,x]

[Out] x^3/6 + x^4/12

Mathics [A]
time = 1.61, size = 8, normalized size = 0.53

$$\frac{x^3\,(2+x)}{12}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^2/2 + x^3/3,x]')
```

```
[Out] x ^ 3 (2 + x) / 12
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{x^{3}(2+x)}{12}$	9
default	$\frac{1}{6}x^{3}+\frac{1}{12}x^{4}$	12
norman	$\frac{1}{6}x^{3}+\frac{1}{12}x^{4}$	12
risch	$\frac{1}{6}x^{3}+\frac{1}{12}x^{4}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^2+1/3*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^3+1/12*x^4
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^{4}+\frac{1}{6}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

Fricas [A]
time = 0.29, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^{4}+\frac{1}{6}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.53

$$\frac{x^{4}}{12}+\frac{x^{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**2+1/3*x**3,x)
```

```
[Out] x**4/12 + x**3/6
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.13

$$\frac{x^3}{2 \cdot 3} + \frac{x^4}{3 \cdot 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x)
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.53

$$\frac{x^3\,\left(x+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/2 + x^3/3,x)
```

```
[Out] (x^3*(x + 2))/12
```

3.1898 $\int (3-5x+2x^2)\,dx$

Optimal. Leaf size=18

$$3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

```
[Out] 3*x-5/2*x^2+2/3*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 5*x + 2*x^2,x]
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Rubi steps

$$\int (3-5x+2x^2)\,dx=3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 5*x + 2*x^2,x]
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Mathics [A]
time = 1.64, size = 13, normalized size = 0.72

$$\frac{x\,(18-15x+4x^2)}{6}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[3 - 5*x + 2*x^2,x]')
```

```
[Out] x (18 - 15 x + 4 x ^ 2) / 6
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{x(4x^2-15x+18)}{6}$	14
default	$3x-\frac{5}{2}x^2+\frac{2}{3}x^3$	15
norman	$3x-\frac{5}{2}x^2+\frac{2}{3}x^3$	15
risch	$3x-\frac{5}{2}x^2+\frac{2}{3}x^3$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2-5*x+3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x-5/2*x^2+2/3*x^3
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.78

$$\frac{2}{3}x^3-\frac{5}{2}x^2+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Fricas [A]
time = 0.29, size = 14, normalized size = 0.78

$$\frac{2}{3}x^3-\frac{5}{2}x^2+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.83

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x**2-5*x+3,x)
```

```
[Out] 2*x**3/3 - 5*x**2/2 + 3*x
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.06

$$\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x)
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(4\,x^2-15\,x+18\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2 - 5*x + 3,x)
```

```
[Out] (x*(4*x^2 - 15*x + 18))/6
```

3.1899 $\int (-2x + x^2 + x^3)\, dx$

Optimal. Leaf size=20

$$-x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int (-2x + x^2 + x^3)\, dx = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Mathics [A]
time = 1.65, size = 14, normalized size = 0.70

$$x^2 \left(-1 + \frac{x}{3} + \frac{x^2}{4}\right)$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-2*x + x^2 + x^3,x]')
[Out] x ^ 2 (-1 + x / 3 + x ^ 2 / 4)
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
gosper	$\frac{x^2(3x^2+4x-12)}{12}$	16
default	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
norman	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
risch	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x,method=_RETURNVERBOSE)
[Out] -x^2+1/3*x^3+1/4*x^4
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Fricas [A]
time = 0.30, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.60

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

Giac [A]
time = 0.00, size = 21, normalized size = 1.05

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{2}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.75

$$\frac{x^2\,\left(3\,x^2+4\,x-12\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2 - 2*x + x^3,x)
```

```
[Out] (x^2*(4*x + 3*x^2 - 12))/12
```

3.1900 $\int (1 - x^2 - 3x^5) \, dx$

Optimal. Leaf size=16

$$x - \frac{x^3}{3} - \frac{x^6}{2}$$

```
[Out] x-1/3*x^3-1/2*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^2 - 3*x^5,x]
[Out] x - x^3/3 - x^6/2
```

Rubi steps

$$\int (1 - x^2 - 3x^5) \, dx = x - \frac{x^3}{3} - \frac{x^6}{2}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$x - \frac{x^3}{3} - \frac{x^6}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^2 - 3*x^5,x]
[Out] x - x^3/3 - x^6/2
```

Mathics [A]
time = 1.63, size = 12, normalized size = 0.75

$$x - \frac{x^3}{3} - \frac{x^6}{2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1 - x^2 - 3*x^5,x]')
```

```
[Out] x - x ^ 3 / 3 - x ^ 6 / 2
```

Maple [A]
time = 0.01, size = 13, normalized size = 0.81

method	result	size
default	$x-\frac{1}{3}x^{3}-\frac{1}{2}x^{6}$	13
norman	$x-\frac{1}{3}x^{3}-\frac{1}{2}x^{6}$	13
risch	$x-\frac{1}{3}x^{3}-\frac{1}{2}x^{6}$	13
gosper	$-\frac{x\left(3x^{5}+2x^{2}-6\right)}{6}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3*x^5-x^2+1,x,method=_RETURNVERBOSE)
```

```
[Out] x-1/3*x^3-1/2*x^6
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$-\frac{1}{2}\,x^{6}-\frac{1}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="maxima")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Fricas [A]
time = 0.32, size = 12, normalized size = 0.75

$$-\frac{1}{2}\,x^{6}-\frac{1}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="fricas")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.62

$$-\frac{x^{6}}{2}-\frac{x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x**5-x**2+1,x)
```

```
[Out] -x**6/2 - x**3/3 + x
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.06

$$-\frac{3}{6}x^6-\frac{x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x)
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - 3*x^5 - x^2,x)
```

```
[Out] x - x^3/3 - x^6/2
```

3.1901 $\int (5+2x+3x^2+4x^3)\,dx$

Optimal. Leaf size=13

$$5x+x^2+x^3+x^4$$

[Out] x^4+x^3+x^2+5*x

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x^4+x^3+x^2+5x$$

Antiderivative was successfully verified.

[In] Int[5 + 2*x + 3*x^2 + 4*x^3,x]

[Out] 5*x + x^2 + x^3 + x^4

Rubi steps

$$\int (5+2x+3x^2+4x^3)\,dx = 5x+x^2+x^3+x^4$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$5x+x^2+x^3+x^4$$

Antiderivative was successfully verified.

[In] Integrate[5 + 2*x + 3*x^2 + 4*x^3,x]

[Out] 5*x + x^2 + x^3 + x^4

Mathics [A]
time = 1.62, size = 11, normalized size = 0.85

$$x\left(5+x+x^2+x^3\right)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[5 + 2*x + 3*x^2 + 4*x^3,x]')

[Out] x (5 + x + x ^ 2 + x ^ 3)

Maple [A]
time = 0.01, size = 14, normalized size = 1.08

method	result	size
gosper	$x^4 + x^3 + x^2 + 5x$	14
default	$x^4 + x^3 + x^2 + 5x$	14
norman	$x^4 + x^3 + x^2 + 5x$	14
risch	$x^4 + x^3 + x^2 + 5x$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^3+3*x^2+2*x+5,x,method=_RETURNVERBOSE)
```

```
[Out] x^4+x^3+x^2+5*x
```

Maxima [A]
time = 0.26, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="maxima")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Fricas [A]
time = 0.30, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="fricas")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.92

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**3+3*x**2+2*x+5,x)
```

```
[Out] x**4 + x**3 + x**2 + 5*x
```

Giac [**A**]
time = 0.00, size = 25, normalized size = 1.92

$$\frac{4}{4}x^4 + \frac{3}{3}x^3 + \frac{2}{2}x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x)
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Mupad [**B**]
time = 0.03, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 3*x^2 + 4*x^3 + 5,x)
```

```
[Out] 5*x + x^2 + x^3 + x^4
```

3.1902 $\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$

Optimal. Leaf size=22

$$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

```
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$ax+b\log(x)-\frac{c}{x}-\frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[a + d/x^3 + c/x^2 + b/x,x]
[Out] -1/2*d/x^2 - c/x + a*x + b*Log[x]
```

Rubi steps

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx=-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + d/x^3 + c/x^2 + b/x,x]
[Out] -1/2*d/x^2 - c/x + a*x + b*Log[x]
```

Mathics [A]
time = 1.75, size = 20, normalized size = 0.91

$$ax+b\text{Log}\left[x\right]-\frac{c}{x}-\frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[a + b/x + c/x^2 + d/x^3,x]')
```

```
[Out] a x + b Log[x] - c / x - d / (2 x ^ 2)
```

Maple [A]
time = 0.01, size = 21, normalized size = 0.95

method	result	size
default	$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\ln\left(x\right)$	21
risch	$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\ln\left(x\right)$	21
norman	$\frac{a\,x^3-\frac{1}{2}d-cx}{x^2}+b\ln\left(x\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+d/x^3+c/x^2+b/x,x,method=_RETURNVERBOSE)
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

Maxima [A]
time = 0.25, size = 20, normalized size = 0.91

$$ax+b\log\left(x\right)-\frac{c}{x}-\frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="maxima")
[Out] a*x + b*log(x) - c/x - 1/2*d/x^2
```

Fricas [A]
time = 0.30, size = 27, normalized size = 1.23

$$\frac{2\,ax^3+2\,bx^2\log\left(x\right)-2\,cx-d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="fricas")
[Out] 1/2*(2*a*x^3 + 2*b*x^2*log(x) - 2*c*x - d)/x^2
```

Sympy [A]
time = 0.11, size = 20, normalized size = 0.91

$$ax+b\log\left(x\right)+\frac{-2cx-d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x**3+c/x**2+b/x,x)
```

```
[Out] a*x + b*log(x) + (-2*c*x - d)/(2*x**2)
```

Giac [A]
time = 0.00, size = 23, normalized size = 1.05

$$ax - \frac{d}{2x^2} - \frac{c}{x} + b\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x)
[Out] a*x + b*log(abs(x)) - c/x - 1/2*d/x^2
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.91

$$a\,x - \frac{\frac{d}{2} + c\,x}{x^2} + b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x + c/x^2 + d/x^3,x)
[Out] a*x - (d/2 + c*x)/x^2 + b*log(x)
```

3.1903 $\int \left(\frac{1}{x^5}+x+x^5\right) dx$

Optimal. Leaf size=22

$$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

```
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^6}{6}-\frac{1}{4x^4}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5) + x + x^5,x]
```

```
[Out] -1/4*1/x^4 + x^2/2 + x^6/6
```

Rubi steps

$$\int \left(\frac{1}{x^5}+x+x^5\right) dx = -\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5) + x + x^5,x]
```

```
[Out] -1/4*1/x^4 + x^2/2 + x^6/6
```

Mathics [A]
time = 1.70, size = 17, normalized size = 0.77

$$\frac{-3+2x^6\left(3+x^4\right)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-5) + x + x^5,x]')
```

```
[Out] (-3 + 2 x ^ 6 (3 + x ^ 4)) / (12 x ^ 4)
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
default	$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$	17
norman	$\frac{\frac{1}{6}x^{10}+\frac{1}{2}x^6-\frac{1}{4}}{x^4}$	17
risch	$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$	17
gosper	$\frac{2x^{10}+6x^6-3}{12x^4}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5+x+x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.73

$$\frac{1}{6}\,x^6+\frac{1}{2}\,x^2-\frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

Fricas [A]
time = 0.29, size = 17, normalized size = 0.77

$$\frac{2\,x^{10}+6\,x^6-3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^10 + 6*x^6 - 3)/x^4
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.68

$$\frac{x^6}{6}+\frac{x^2}{2}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5+x+x**5,x)
```

```
[Out] x**6/6 + x**2/2 - 1/(4*x**4)
```

Giac [A]
time = 0.00, size = 20, normalized size = 0.91

$$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x)
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,x^{10}+6\,x^6-3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x^5 + x^5,x)
```

```
[Out] (6*x^6 + 2*x^10 - 3)/(12*x^4)
```

3.1904 $\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx$

Optimal. Leaf size=15

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

```
[Out] -1/2/x^2-1/x+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3) + x^(-2) + x^(-1),x]
[Out] -1/2*1/x^2 - x^(-1) + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx=-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3) + x^(-2) + x^(-1),x]
[Out] -1/2*1/x^2 - x^(-1) + Log[x]
```

Mathics [A]
time = 1.66, size = 13, normalized size = 0.87

$$-\frac{1}{2x^2}-\frac{1}{x}+\mathrm{Log}\,[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-3) + x^(-2) + x^(-1),x]')
```

```
[Out] -1 / (2 x ^ 2) - 1 / x + Log[x]
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.93

method	result	size
norman	$\frac{-\frac{1}{2}-x}{x^{2}}+\ln \left(x \right)$	13
default	$-\frac{1}{2 x^{2}}-\frac{1}{x}+\ln \left(x \right)$	14
risch	$-\frac{1}{2 x^{2}}-\frac{1}{x}+\ln \left(x \right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3+1/x^2+1/x,x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-1/x+ln(x)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$-\frac{1}{x} - \frac{1}{2\,x^2} + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="maxima")
[Out] -1/x - 1/2/x^2 + log(x)
```

Fricas [A]
time = 0.29, size = 17, normalized size = 1.13

$$\frac{2\,x^2\log\left(x\right) - 2\,x - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x) - 2*x - 1)/x^2
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.93

$$\log\left(x\right) + \frac{-2x - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3+1/x**2+1/x,x)
```

```
[Out] log(x) + (-2*x - 1)/(2*x**2)
```

Giac [A]
time = 0.00, size = 14, normalized size = 0.93

$$-\frac{1}{2x^2} - \frac{1}{x} + \ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x)
```

```
[Out] -1/x - 1/2/x^2 + log(abs(x))
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.73

$$\ln(x) - \frac{x + \frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x + 1/x^2 + 1/x^3,x)
```

```
[Out] log(x) - (x + 1/2)/x^2
```

3.1905 $\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$

Optimal. Leaf size=10

$$\frac{2}{x}+3\log(x)$$

```
[Out] 2/x+3*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x^2 + 3/x,x]
[Out] 2/x + 3*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx = \frac{2}{x}+3\log(x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x^2 + 3/x,x]
[Out] 2/x + 3*Log[x]
```

Mathics [A]
time = 1.63, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\text{Log}\left[x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-2/x^2 + 3/x,x]')
```

[Out] 2 / x + 3 Log[x]

Maple [A]
time = 0.01, size = 11, normalized size = 1.10

method	result	size
default	$\frac{2}{x}+3\ln\left(x\right)$	11
norman	$\frac{2}{x}+3\ln\left(x\right)$	11
risch	$\frac{2}{x}+3\ln\left(x\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x^2+3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/x+3*ln(x)
```

Maxima [A]
time = 0.26, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="maxima")
```

```
[Out] 2/x + 3*log(x)
```

Fricas [A]
time = 0.29, size = 11, normalized size = 1.10

$$\frac{3\,x\log\left(x\right)+2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="fricas")
```

```
[Out] (3*x*log(x) + 2)/x
```

Sympy [A]
time = 0.04, size = 7, normalized size = 0.70

$$3\log\left(x\right)+\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x**2+3/x,x)
```

```
[Out] 3*log(x) + 2/x
```

Giac [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x} + 3\ln|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x)
```

```
[Out] 2/x + 3*log(abs(x))
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$3\,\ln\left(x\right) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/x - 2/x^2,x)
```

```
[Out] 3*log(x) + 2/x
```

3.1906 $\int \left(-\frac{1}{7x^6}+x^6\right) \, dx$

Optimal. Leaf size=15

$$\frac{1}{35x^5}+\frac{x^7}{7}$$

```
[Out] 1/35/x^5+1/7*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[-1/7*1/x^6 + x^6,x]
```

```
[Out] 1/(35*x^5) + x^7/7
```

Rubi steps

$$\int \left(-\frac{1}{7x^6}+x^6\right) \, dx = \frac{1}{35x^5}+\frac{x^7}{7}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{35x^5}+\frac{x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/7*1/x^6 + x^6,x]
```

```
[Out] 1/(35*x^5) + x^7/7
```

Mathics [A]
time = 1.64, size = 12, normalized size = 0.80

$$\frac{1+5x^{12}}{35x^5}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-1/(7*x^6) + x^6,x]')
```

```
[Out] (1 + 5 x ^ 12) / (35 x ^ 5)
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.80

method	result	size
default	$\frac{1}{35x^5}+\frac{x^7}{7}$	12
norman	$\frac{\frac{1}{35}+\frac{x^{12}}{7}}{x^5}$	12
risch	$\frac{1}{35x^5}+\frac{x^7}{7}$	12
gosper	$\frac{5x^{12}+1}{35x^5}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/7/x^6+x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35/x^5+1/7*x^7
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^7+\frac{1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 + 1/35/x^5
```

Fricas [A]
time = 0.31, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*x^12 + 1)/x^5
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.67

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x**6+x**6,x)
```

```
[Out] x**7/7 + 1/(35*x**5)
```

Giac [A]
time = 0.00, size = 16, normalized size = 1.07

$$\frac{1}{7\,(5x^5)} + \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x)
```

```
[Out] 1/7*x^7 + 1/35/x^5
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6 - 1/(7*x^6),x)
```

```
[Out] (5*x^12 + 1)/(35*x^5)
```

3.1907 $\int \left(1+\frac{1}{x}+x\right) dx$

Optimal. Leaf size=11

$$x+\frac{x^2}{2}+\log(x)$$

```
[Out] x+1/2*x^2+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

Rubi steps

$$\int \left(1+\frac{1}{x}+x\right) dx = x+\frac{x^2}{2}+\log(x)$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x+\frac{x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

Mathics [A]
time = 1.61, size = 9, normalized size = 0.82

$$x+\frac{x^2}{2}+\mathrm{Log}\,[x]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1 + x^(-1) + x,x]')
```

```
[Out] x + x ^ 2 / 2 + Log[x]
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
default	$x+\frac{x^2}{2}+\ln(x)$	10
norman	$x+\frac{x^2}{2}+\ln(x)$	10
risch	$x+\frac{x^2}{2}+\ln(x)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+1/x+x,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+ln(x)
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Fricas [A]
time = 0.30, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.73

$$\frac{x^2}{2}+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x)
```

```
[Out] x**2/2 + x + log(x)
```

Giac [**A**]
time = 0.00, size = 11, normalized size = 1.00

$$x + \ln|x| + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x)
```

```
[Out] 1/2*x^2 + x + log(abs(x))
```

Mupad [**B**]
time = 0.03, size = 9, normalized size = 0.82

$$x + \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x + 1,x)
```

```
[Out] x + log(x) + x^2/2
```

3.1908 $\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right)\,dx$

Optimal. Leaf size=13

$$\frac{3}{2x^2}-\frac{4}{x}$$

```
[Out] 3/2/x^2-4/x
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-3/x^3 + 4/x^2,x]
```

```
[Out] 3/(2*x^2) - 4/x
```

Rubi steps

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right)\,dx=\frac{3}{2x^2}-\frac{4}{x}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/x^3 + 4/x^2,x]
```

```
[Out] 3/(2*x^2) - 4/x
```

Mathics [A]
time = 1.63, size = 10, normalized size = 0.77

$$\frac{3-8x}{2x^2}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-3/x^3 + 4/x^2,x]')
```

[Out] (3 - 8 x) / (2 x ^ 2)

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
norman	$\frac{-4x+\frac{3}{2}}{x^2}$	10
gosper	$-\frac{8x-3}{2x^2}$	11
default	$\frac{3}{2x^2}-\frac{4}{x}$	12
risch	$\frac{3}{2x^2}-\frac{4}{x}$	12

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-3/x^3+4/x^2,x,method=_RETURNVERBOSE)

[Out] 3/2/x^2-4/x

Maxima [A]
time = 0.27, size = 11, normalized size = 0.85

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/x^3+4/x^2,x, algorithm="maxima")

[Out] -4/x + 3/2/x^2

Fricas [A]
time = 0.29, size = 10, normalized size = 0.77

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/x^3+4/x^2,x, algorithm="fricas")

[Out] -1/2*(8*x - 3)/x^2

Sympy [A]
time = 0.05, size = 8, normalized size = 0.62

$$\frac{3-8x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/x**3+4/x**2,x)

```
[Out] (3 - 8*x)/(2*x**2)
```

Giac [A]
time = 0.00, size = 14, normalized size = 1.08

$$\frac{3}{2x^2} - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x)
```

```
[Out] -4/x + 3/2/x^2
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.77

$$-\frac{8\,x - 3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4/x^2 - 3/x^3,x)
```

```
[Out] -(8*x - 3)/(2*x^2)
```

3.1909 $\int \left(\frac{1}{x}+2x+x^2\right)\,dx$

Optimal. Leaf size=13

$$x^2+\frac{x^3}{3}+\log(x)$$

```
[Out] x^2+1/3*x^3+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^3}{3}+x^2+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x}+2x+x^2\right)\,dx=x^2+\frac{x^3}{3}+\log(x)$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$x^2+\frac{x^3}{3}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Mathics [A]
time = 1.63, size = 11, normalized size = 0.85

$$x^2+\frac{x^3}{3}+\text{Log}\left[x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-1) + 2*x + x^2,x]')
```

```
[Out] x ^ 2 + x ^ 3 / 3 + Log[x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
default	$x^2+\frac{x^3}{3}+\ln\left(x\right)$	12
norman	$x^2+\frac{x^3}{3}+\ln\left(x\right)$	12
risch	$x^2+\frac{x^3}{3}+\ln\left(x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x+2*x+x^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2+1/3*x^3+ln(x)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.85

$$\frac{1}{3}\,x^3+x^2+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

Fricas [A]
time = 0.31, size = 11, normalized size = 0.85

$$\frac{1}{3}\,x^3+x^2+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.77

$$\frac{x^3}{3}+x^2+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x**2,x)
```

```
[Out] x**3/3 + x**2 + log(x)
```

Giac [A]
time = 0.00, size = 17, normalized size = 1.31

$$\ln |x| + \frac{2}{2}x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x)
```

```
[Out] 1/3*x^3 + x^2 + log(abs(x))
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.85

$$\ln (x) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 1/x + x^2,x)
```

```
[Out] log(x) + x^2 + x^3/3
```

3.1910 $\int \left(x^{5/6}-x^3\right) \, dx$

Optimal. Leaf size=17

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

```
[Out] 6/11*x^(11/6)-1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/6) - x^3,x]
[Out] (6*x^(11/6))/11 - x^4/4
```

Rubi steps

$$\int \left(x^{5/6}-x^3\right) \, dx = \frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/6) - x^3,x]
[Out] (6*x^(11/6))/11 - x^4/4
```

Mathics [A]
time = 1.68, size = 11, normalized size = 0.65

$$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(5/6) - x^3,x]')
[Out] 6 x ^ (11 / 6) / 11 - x ^ 4 / 4
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.71

method	result	size
derivativedivides	$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$	12
default	$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$	12
risch	$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$	12
trager	$-\frac{(x^3+x^2+x+1)(-1+x)}{4}+\frac{6x^{\frac{11}{6}}}{11}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6)-x^3,x,method=_RETURNVERBOSE)
[Out] 6/11*x^(11/6)-1/4*x^4
```

Maxima [A]
time = 0.25, size = 11, normalized size = 0.65

$$-\frac{1}{4}\,x^4+\frac{6}{11}\,x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="maxima")
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Fricas [A]
time = 0.31, size = 11, normalized size = 0.65

$$-\frac{1}{4}\,x^4+\frac{6}{11}\,x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="fricas")
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/6)-x**3,x)
```

```
[Out] 6*x**(11/6)/11 - x**4/4
```

Giac [A]
time = 0.00, size = 20, normalized size = 1.18

$$\frac{6}{11}\left(x^{\frac{1}{6}}\right)^{5}x-\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x)
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.65

$$\frac{6\,x^{11/6}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6) - x^3,x)
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

3.1911 $\int \left(33+\sqrt[33]{x}\right) dx$

Optimal. Leaf size=13

$$33x+\frac{33x^{34/33}}{34}$$

[Out] 33*x+33/34*x^(34/33)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

[In] Int[33 + x^(1/33),x]

[Out] 33*x + (33*x^(34/33))/34

Rubi steps

$$\int \left(33+\sqrt[33]{x}\right) dx = 33x+\frac{33x^{34/33}}{34}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$33x+\frac{33x^{34/33}}{34}$$

Antiderivative was successfully verified.

[In] Integrate[33 + x^(1/33),x]

[Out] 33*x + (33*x^(34/33))/34

Mathics [A]
time = 1.60, size = 9, normalized size = 0.69

$$33x+\frac{33x^{\frac{34}{33}}}{34}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[33 + x^(1/33),x]')
[Out] 33 x + 33 x ^ (34 / 33) / 34
```

Maple [A]
time = 0.17, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$33x+\frac{33x^{\frac{34}{33}}}{34}$	10
default	$33x+\frac{33x^{\frac{34}{33}}}{34}$	10
risch	$33x+\frac{33x^{\frac{34}{33}}}{34}$	10
trager	$-33+33x+\frac{33x^{\frac{34}{33}}}{34}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(33+x^(1/33),x,method=_RETURNVERBOSE)
[Out] 33*x+33/34*x^(34/33)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}}+33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="maxima")
[Out] 33/34*x^(34/33) + 33*x
```

Fricas [A]
time = 0.30, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}}+33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="fricas")
[Out] 33/34*x^(34/33) + 33*x
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34}+33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x**(1/33),x)
```

```
[Out] 33*x**(34/33)/34 + 33*x
```

Giac [A]
time = 0.00, size = 20, normalized size = 1.54

$$33x+\frac{x^{\frac{1}{33}+1}}{\frac{1}{33}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x)
```

```
[Out] 33/34*x^(34/33) + 33*x
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.62

$$\frac{33\,x\,\left(x^{1/33}+34\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/33) + 33,x)
```

```
[Out] (33*x*(x^(1/33) + 34))/34
```

3.1912 $\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right)\,dx$

Optimal. Leaf size=15

$$\sqrt{x}+\frac{4x^{3/2}}{3}$$

```
[Out] 4/3*x^(3/2)+x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{4x^{3/2}}{3}+\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] Sqrt[x] + (4*x^(3/2))/3
```

Rubi steps

$$\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right)\,dx=\sqrt{x}+\frac{4x^{3/2}}{3}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.93

$$\frac{1}{3}\sqrt{x}\,(3+4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] (Sqrt[x]*(3 + 4*x))/3
```

Mathics [A]
time = 1.64, size = 10, normalized size = 0.67

$$\frac{\sqrt{x}\,(3+4x)}{3}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[1/(2*Sqrt[x]) + 2*Sqrt[x],x]')
[Out] Sqrt[x] (3 + 4 x) / 3
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.67

method	result	size
derivativedivides	$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$	10
default	$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$	10
risch	$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$	10
gosper	$\frac{\sqrt{x}\,(3+4x)}{3}$	11
trager	$\frac{\left(2+\frac{8x}{3}\right)\sqrt{x}}{2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/x^(1/2)+2*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/3*x^(3/2)+x^(1/2)
```

Maxima [A]
time = 0.27, size = 9, normalized size = 0.60

$$\frac{4}{3}\,x^{\frac{3}{2}}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="maxima")
[Out] 4/3*x^(3/2) + sqrt(x)
```

Fricas [A]
time = 0.31, size = 10, normalized size = 0.67

$$\frac{1}{3}\,(4\,x+3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="fricas")
[Out] 1/3*(4*x + 3)*sqrt(x)
```

Sympy [**A**]
time = 0.04, size = 12, normalized size = 0.80

$$\frac{4x^{\frac{3}{2}}}{3} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x**(1/2)+2*x**(1/2),x)
```

```
[Out] 4*x**(3/2)/3 + sqrt(x)
```

Giac [**A**]
time = 0.00, size = 21, normalized size = 1.40

$$\frac{2}{2}\sqrt{x} + \frac{2}{3} \cdot 2\sqrt{x}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x)
```

```
[Out] 4/3*x^(3/2) + sqrt(x)
```

Mupad [**B**]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{\sqrt{x}\ (4\,x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)) + 2*x^(1/2),x)
```

```
[Out] (x^(1/2)*(4*x + 3))/3
```

3.1913 $\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right)\,dx$

Optimal. Leaf size=15

$$\frac{1}{x}+4x^{3/2}+10\log(x)$$

```
[Out] 1/x+4*x^(3/2)+10*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-x^(-2) + 10/x + 6*Sqrt[x],x]
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

Rubi steps

$$\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right)\,dx=\frac{1}{x}+4x^{3/2}+10\log(x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{x}+4x^{3/2}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^(-2) + 10/x + 6*Sqrt[x],x]
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

Mathics [A]
time = 1.64, size = 13, normalized size = 0.87

$$\frac{1}{x}+4x^{\frac{3}{2}}+10\text{Log}\left[x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-x^(-2) + 10/x + 6*Sqrt[x],x]')
```

```
[Out] 1 / x + 4 x ^ (3 / 2) + 10 Log[x]
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{1}{x}+4x^{\frac{3}{2}}+10\ln\left(x\right)$	14
default	$\frac{1}{x}+4x^{\frac{3}{2}}+10\ln\left(x\right)$	14
risch	$\frac{1}{x}+4x^{\frac{3}{2}}+10\ln\left(x\right)$	14
trager	$-\frac{-1+x}{x}+4x^{\frac{3}{2}}-10\ln\left(\frac{1}{x}\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/x^2+10/x+6*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x+4*x^(3/2)+10*ln(x)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.87

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="maxima")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(x)
```

Fricas [A]
time = 0.29, size = 18, normalized size = 1.20

$$\frac{4\,x^{\frac{5}{2}}+20\,x\log\left(\sqrt{x}\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="fricas")
```

```
[Out] (4*x^(5/2) + 20*x*log(sqrt(x)) + 1)/x
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}}+10\log\left(x\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x**2+10/x+6*x**(1/2),x)
```

```
[Out] 4*x**(3/2) + 10*log(x) + 1/x
```

Giac [A]
time = 0.00, size = 19, normalized size = 1.27

$$\frac{1}{x} + 10 \ln |x| + \frac{2}{3} \cdot 6 \sqrt{x}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x)
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(abs(x))
```

Mupad [B]
time = 0.29, size = 15, normalized size = 1.00

$$20 \ln \left(\sqrt{x}\right) + \frac{1}{x} + 4 x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/x - 1/x^2 + 6*x^(1/2),x)
```

```
[Out] 20*log(x^(1/2)) + 1/x + 4*x^(3/2)
```

3.1914 $\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx$

Optimal. Leaf size=17

$$-\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

[Out] 2/5*x^(5/2)-2/x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[x^(-3/2) + x^(3/2),x]

[Out] -2/Sqrt[x] + (2*x^(5/2))/5

Rubi steps

$$\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx=-\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.82

$$\frac{2\left(-5+x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3/2) + x^(3/2),x]

[Out] (2*(-5 + x^3))/(5*Sqrt[x])

Mathics [A]
time = 1.68, size = 10, normalized size = 0.59

$$\frac{2\left(-5+x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[x^(-3/2) + x^(3/2),x]')
```

```
[Out] 2 (-5 + x ^ 3) / (5 Sqrt[x])
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.71

method	result	size
gosper	$\frac{\frac{2x^3}{5}-2}{\sqrt{x}}$	11
trager	$\frac{\frac{2x^3}{5}-2}{\sqrt{x}}$	11
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$	12
default	$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$	12
risch	$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)+x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)-2/x^(1/2)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

Fricas [A]
time = 0.36, size = 10, normalized size = 0.59

$$\frac{2\,(x^3-5)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(x^3 - 5)/sqrt(x)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)+x**(3/2),x)
[Out] 2*x**(5/2)/5 - 2/sqrt(x)
```

Giac [A]
time = 0.00, size = 22, normalized size = 1.29

$$-\frac{2}{\sqrt{x}} + \frac{2}{5}\sqrt{x}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x)
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{2\,x^3 - 10}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2) + x^(3/2),x)
[Out] (2*x^3 - 10)/(5*x^(1/2))
```

3.1915 $\int \left(-5x^{3/2}+7x^{5/2}\right) \, dx$

Optimal. Leaf size=15

$$-2x^{5/2}+2x^{7/2}$$

[Out] -2*x^(5/2)+2*x^(7/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$2x^{7/2}-2x^{5/2}$$

Antiderivative was successfully verified.

[In] Int[-5*x^(3/2) + 7*x^(5/2),x]

[Out] -2*x^(5/2) + 2*x^(7/2)

Rubi steps

$$\int \left(-5x^{3/2}+7x^{5/2}\right) \, dx = -2x^{5/2}+2x^{7/2}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 0.67

$$2(-1+x)x^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[-5*x^(3/2) + 7*x^(5/2),x]

[Out] 2*(-1 + x)*x^(5/2)

Mathics [A]
time = 1.62, size = 8, normalized size = 0.53

$$2x^{\frac{5}{2}}\left(-1+x\right)$$

Antiderivative was successfully verified.

[In] mathics('Integrate[-5*x^(3/2) + 7*x^(5/2),x]')

[Out] 2 x ^ (5 / 2) (-1 + x)

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
gosper	$2x^{\frac{5}{2}}\left(-1+x\right)$	9
trager	$2x^{\frac{5}{2}}\left(-1+x\right)$	9
derivativedivides	$-2x^{\frac{5}{2}}+2x^{\frac{7}{2}}$	12
default	$-2x^{\frac{5}{2}}+2x^{\frac{7}{2}}$	12
risch	$-2x^{\frac{5}{2}}+2x^{\frac{7}{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-5*x^(3/2)+7*x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2*x^(5/2)+2*x^(7/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}}-2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="maxima")
[Out] 2*x^(7/2) - 2*x^(5/2)
```

Fricas [A]
time = 0.32, size = 14, normalized size = 0.93

$$2\,{\left(x^{3}-x^{2}\right)}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="fricas")
[Out] 2*(x^3 - x^2)*sqrt(x)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.80

$$2x^{\frac{7}{2}}-2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x**(3/2)+7*x**(5/2),x)
[Out] 2*x**(7/2) - 2*x**(5/2)
```

Giac [A]
time = 0.00, size = 28, normalized size = 1.87

$$-\frac{2}{5} \cdot 5\sqrt{x}\,x^2 + \frac{2}{7} \cdot 7\sqrt{x}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x)
[Out] 2*x^(7/2) - 2*x^(5/2)
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.53

$$2\,x^{5/2}\,\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7*x^(5/2) - 5*x^(3/2),x)
[Out] 2*x^(5/2)*(x - 1)
```

3.1916 $\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx$

Optimal. Leaf size=24

$$4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

```
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^{3/2}}{3}-\frac{x^2}{4}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Rubi steps

$$\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx = 4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Mathics [A]
time = 1.64, size = 16, normalized size = 0.67

$$4\sqrt{x}+\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[2/Sqrt[x] + Sqrt[x] - x/2,x]')
[Out] 4 Sqrt[x] + 2 x ^ (3 / 2) / 3 - x ^ 2 / 4
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}+4\sqrt{x}$	17
default	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}+4\sqrt{x}$	17
risch	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}+4\sqrt{x}$	17
trager	$-\frac{(1+x)(-1+x)}{4}+\frac{\left(8+\frac{4x}{3}\right)\sqrt{x}}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*x+2/x^(1/2)+x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.67

$$-\frac{1}{4}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}+4\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

Fricas [A]
time = 0.30, size = 14, normalized size = 0.58

$$-\frac{1}{4}\,x^2+\frac{2}{3}\,(x+6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="fricas")
[Out] -1/4*x^2 + 2/3*(x + 6)*sqrt(x)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.79

$$\frac{2x^{\frac{3}{2}}}{3}+4\sqrt{x}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x**(1/2)+x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 + 4*sqrt(x) - x**2/4
```

Giac [A]
time = 0.00, size = 28, normalized size = 1.17

$$-\frac{x^2}{2\cdot 2}+2\cdot 2\sqrt{x}+\frac{2}{3}\sqrt{x}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x)
```

```
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.62

$$\frac{\sqrt{x}\,\left(8\,x-3\,x^{3/2}+48\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/x^(1/2) - x/2 + x^(1/2),x)
```

```
[Out] (x^(1/2)*(8*x - 3*x^(3/2) + 48))/12
```

3.1917 $\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx$

Optimal. Leaf size=23

$$\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

```
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x + Sqrt[x]/5 + x^(3/2),x]
```

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx=\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x + Sqrt[x]/5 + x^(3/2),x]
```

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Mathics [A]
time = 1.63, size = 15, normalized size = 0.65

$$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\text{Log}\left[x\right]$$

Antiderivative was successfully verified.

```
[In] mathics('Integrate[-2/x + Sqrt[x]/5 + x^(3/2),x]')
[Out] 2 x ^ (3 / 2) / 15 + 2 x ^ (5 / 2) / 5 - 2 Log[x]
```

Maple [A]
time = 0.04, size = 16, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\ln\left(x\right)$	16
default	$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\ln\left(x\right)$	16
trager	$\frac{2x^{\frac{3}{2}}\left(1+3x\right)}{15}-2\ln\left(x\right)$	16
risch	$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\ln\left(x\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x+x^(3/2)+1/5*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

Maxima [A]
time = 0.25, size = 15, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{15}\,x^{\frac{3}{2}}-2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(x)
```

Fricas [A]
time = 0.30, size = 19, normalized size = 0.83

$$\frac{2}{15}\left(3\,x^{2}+x\right)\sqrt{x}-4\,\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + x)*sqrt(x) - 4*log(sqrt(x))
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.87

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{15}-2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x**(3/2)+1/5*x**(1/2),x)
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/15 - 2*log(x)
```

Giac [A]
time = 0.00, size = 31, normalized size = 1.35

$$-2\ln|x| + \frac{2}{5}\sqrt{x}\,x^2 + \frac{2\sqrt{x}\,x}{5\cdot 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x)
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(abs(x))
```

Mupad [B]
time = 0.28, size = 17, normalized size = 0.74

$$\frac{2\,x^{3/2}}{15} - 4\,\ln\left(\sqrt{x}\right) + \frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/5 - 2/x + x^(3/2),x)
[Out] (2*x^(3/2))/15 - 4*log(x^(1/2)) + (2*x^(5/2))/5
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
```

```
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues."
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal
```

```
#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.


if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;


if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A","";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
```

```
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
```

```
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
```

```
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
```

```
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
```

```
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,''+'') or type(expn,''*'')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;
```

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
```

```
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
       print ("m=",m)
       if m:
          print ("func ", func ," is special_function")
       else:
          print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()
       return False

    except AttributeError as error:
       return False


def expnType(expn):
```

```
    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```